Chapter 13

One factor repeated measures ANOVA

Chapter 13

THE INDEPENDENT MEASURES ANOVA assumes that the scores in each condition are unrelated and the subjects have contributed a score to only one of them. However, there are many cases when we want to use the same subjects in all conditions. This is particularly useful as it matches subjects with themselves across the conditions. An experiment on memory, comparing retention of different types of words might use the same participants in each condition (as long as the carry-over effects of practice or fatigue are controlled for). The analysis of variance that deals with this form of data is called a repeated measures design and, as we see below, the calculations are a little different to the independent measures design but the general logic of the ANOVA remains the same.

Deriving the *F* value

A research programme was set up to develop user-friendly computer equipment for those people with physical disabilities. Three new designs of computer keyboard for people with difficulties in hand and finger movement were developed and prototypes created. The research task was to decide which of these prototypes is the most successful. Four potential users of the new equipment agreed to take part in a test of the new keyboards. Each participant was asked to use the keyboard to input a piece of text and the number of errors was recorded. Three equally difficult pieces of text were used so that a participant did not improve performance by practice on the same piece of text. The choice of text and the order in which the keyboards were tested by each participant was controlled for, to account for possible confounding variables. The results of the experiment are shown below.

Participant	*Keyboard 1*	*Keyboard 2*	*Keyboard 3*
1	5	6	10
2	1	2	3
3	0	4	5
4	2	4	6

Notice that there is quite a bit of subject variability, with Participant 1 making the most mistakes and Participant 2 the least. Yet the repeated measures design matches the subjects with themselves across the conditions so that, even though they differ markedly from each other, the question is whether they follow a similar pattern across the conditions, i.e. is one condition the worst for all despite their differences in general accuracy?

If we performed an independent measures ANOVA on these data it would not be informative as it assumes that there is subject variability both between and within the conditions. We can see this by considering the way we calculate F for the independent measures design:

$$F = \frac{\text{Between conditions variance}}{\text{Within conditions variance}}$$

$$F = \frac{\text{Systematic differences + Individual differences + Experimental error}}{\text{Individual differences + Experimental error}}$$

Now as there are no individual differences between the conditions in the repeated measures design (as the subjects are the same) the same formula with repeated measures would produce:

$$\begin{aligned} F &= \frac{\text{Between conditions variance}}{\text{Within conditions variance}} \\ &= \frac{\text{Systematic differences + Experimental error}}{\text{Individual differences + Experimental error}} \end{aligned}$$

This is not a very useful measure of the systematic differences between conditions as F is no longer sensitive to only this one factor but to the individual differences which are now only in the bottom of the equation. A large value of F could mean a large treatment effect but it could mean small individual differences. A small value of F might not mean a lack of systematic differences but simply large individual differences swamping the effect. If we can get rid of the individual differences from the within conditions variance (the bottom part of the formula) we will end up with an excellent formula for a repeated measures design as it will be highly sensitive to systematic differences between conditions.

$$F = \frac{\text{Systematic differences + Experimental error}}{\text{Experimental error}}$$

To produce this we need to find a way of removing the individual differences from the within conditions variance so that we can calculate the appropriate F value.

$$F = \frac{\text{Between conditions variance}}{\text{Within conditions variance} - \text{Individual differences}}$$

Removing the individual differences

When we look at the keyboard data we can see that, despite the individual differences in the participants, there is a general pattern across the participants with Keyboard 1 producing the lowest errors, Keyboard 2 more errors and Keyboard 3 the most. So despite the different level of performance the pattern across the conditions is similar for each of the participants. It is the strength of this pattern, the systematic differences between the conditions, we wish to measure.

The key to extracting the subject differences lies in the sums of squares. So far (see Chapter 10) we have only calculated sums of squares for the conditions: between conditions and within conditions. The table below shows the means of the conditions so that we can calculate these sums of squares.

Participant	*Keyboard 1*	*Keyboard 2*	*Keyboard 3*	*Participant mean*
1	5	6	10	7
2	1	2	3	2
3	0	4	5	3
4	2	4	6	4
Condition mean	2	4	6	Overall mean = 4

The sums of squares within each condition is as follows:

Keyboard 1 $(5-2)^2 + (1-2)^2 + (0-2)^2 + (2-2)^2 = 14$
Keyboard 2 $(6-4)^2 + (2-4)^2 + (4-4)^2 + (4-4)^2 = 8$
Keyboard 3 $(10-6)^2 + (3-6)^2 + (5-6)^2 + (6-6)^2 = 26$

The within conditions sums of squares = 14 + 8 + 26 = 48.

The sums of squares between the condition means = $(2 - 4)^2 + (4 - 4)^2 + (6 - 4)^2 = 8$. As there are four participants per condition the between conditions sums of squares = $4 \times 8 = 32$.

In the above calculations of sums of squares we have focused on the conditions, which are the columns in the above table, and we have calculated the within columns variation and the between columns variation in the scores. The same logic can be applied to the rows, where the sums of squares can be calculated within and between the rows. Notice that the rows are the subjects. Within the rows the variability is not due to differences in subjects as within a row it is always the same subject. However, the variation between the rows is the variation between the subjects. This is a measure of the individual differences between the participants, exactly what we are trying to find.

The sums of squares within each subject is as follows:

Subject 1	$(5 - 7)^2 + (6 - 7)^2 + (10 - 7)^2 = 14$
Subject 2	$(1 - 2)^2 + (2 - 2)^2 + (3 - 2)^2 = 2$
Subject 3	$(0 - 3)^2 + (4 - 3)^2 + (5 - 3)^2 = 14$
Subject 4	$(2 - 4)^2 + (4 - 4)^2 + (6 - 4)^2 = 8$

The within subjects sums of squares = $14 + 2 + 14 + 8 = 38$.

The sums of squares between the subject means = $(7 - 4)^2 + (2 - 4)^2 + (3 - 4)^2 + (4 - 4)^2 = 14$. As there are three conditions per subject the between subjects sums of squares = $3 \times 14 = 42$.

Notice that however we work out the sums of squares the total is always 80. We are not interested in the within subjects sums of squares for the ANOVA but we now have a measure of the individual differences (the between subjects sums of squares of 42). We can now remove the individual differences from the within conditions sums of squares. The residual, our error sums of squares, is $48 - 42 = 6$.

As we are able to take out the between subjects variability from the within conditions variability we no longer use the within conditions variance in our calculation of F but employ the new, smaller, error term. Thus, in the repeated measures design we have more chance of finding a significant effect as we have removed the individual differences completely from the calculation.[10]

The ANOVA summary table

The summary table for a repeated measures ANOVA has two extra rows compared to the independent measures ANOVA because we have to separate the within conditions sums of squares into the between subjects sums of squares and the error sums of squares.

THE ANOVA SUMMARY TABLE

Source of variation	Degrees of freedom	Sums of squares	Mean square	Variance ratio (F)	Probability
Between conditions	$df_{bet.conds}$	$SS_{bet.conds}$	$MS_{bet.conds}$	F	p
Within conditions	$df_{with.conds}$	$SS_{with.conds}$			
Between subjects	$df_{bet.subjs}$	$SS_{bet.subjs}$			
Error	df_{error}	SS_{error}	MS_{error}		
Total	df_{total}	SS_{total}			

Below are listed the formulae for the calculations.

Degrees of freedom:

$df_{total} = N - 1$ where N is the total number of scores

$df_{bet.conds} = k - 1$ where k is the number of conditions

$df_{with.conds} = df_{total} - df_{bet.conds}$

$df_{bet.subjs} = n - 1$ where n is the number of subjects per condition

$df_{error} = (n - 1)(k - 1)$

Sums of squares:

$$SS_{total} = \sum X^2 - \frac{(\sum X)^2}{N}$$

where $\sum X^2$ is the sum of the squared scores and $(\sum X)^2$ is the square of the sum of the scores[8]

$$SS_{bet.conds} = \frac{\sum T_c^2}{n} - \frac{(\sum X)^2}{N}$$

where T_c refers to a total of the scores in a condition, e.g. T_{c_1} is the total of the scores in condition 1. $\sum T_c^2$ is the sum of the squared totals of the conditions

(Notice that we use T_c for the condition totals and not just T. This is to distinguish them from the subject totals T_s.)

$$SS_{with.subjs} = SS_{total} - SS_{bet.conds}$$

$$SS_{bet.subjs} = \frac{\sum T_s^2}{n} - \frac{(\sum X)^2}{N}$$

where T_s refers to a total of the scores for a subject, e.g. T_{s_1} is the total of the scores for subject 1. $\sum T_s^2$ is the sum of the squared totals of the subjects

$$SS_{error} = SS_{with.conds} - SS_{bet.subjs}$$

Mean square:

$$MS_{bet.conds} = \frac{SS_{bet.conds}}{df_{bet.conds}}$$

$$MS_{error} = \frac{SS_{error}}{df_{error}}$$

Variance ratio:

$$F = \frac{MS_{bet.conds}}{MS_{error}}$$

The degrees of freedom accompanying F are the between conditions and error degrees of freedom.

$$F(df_{bet.conds}, df_{error}) = \text{calculated value}$$

We compare the calculated value with the critical value in the F distribution tables at our chosen level of significance (Table A.3 in the Appendix).

When we look up the table value we use $df_{bet.conds}$ as our first degrees of freedom (the columns in the table) and df_{error} as our second degrees of freedom (the rows in the table). Our calculated value of F is only significant if it is equal to or larger than the table value.

A worked example

The keyboard example provides us with some illustrative data for calculating the repeated measures ANOVA. First we calculate the totals for the formulae.

Participant	*Keyboard 1*	*Keyboard 2*	*Keyboard 3*	*Participant totals*
1	5	6	10	$T_{s_1} = 21$
2	1	2	3	$T_{s_2} = 6$
3	0	4	5	$T_{s_3} = 9$
4	2	4	6	$T_{s_4} = 12$
Condition totals	$T_{c_1} = 8$	$T_{c_2} = 16$	$T_{c_3} = 24$	Overall total $\sum X = 48$

We also need:

The number of subjects per condition, $n = 4$
The number of conditions, $k = 3$
The total number of scores, $N = 12$
The overall total squared, $(\sum X)^2 = 2304$
The sums of the squared scores, $\sum X^2 = 5^2 + 1^2 + \ldots + 5^2 + 6^2$
$= 272$

We next calculate the degrees of freedom:

$df_{total} = N - 1 = 12 - 1 = 11$
$df_{bet.conds} = k - 1 = 3 - 1 = 2$
$df_{with.conds} = df_{total} - df_{bet.conds} = 11 - 2 = 9$
$df_{bet.subjs} = n - 1 = 4 - 1 = 3$
$df_{error} = (n - 1)(k - 1) = 3 \times 2 = 6$

Sums of squares:

$$SS_{total} = \sum X^2 - \frac{(\sum X)^2}{N} = 272 - \frac{2304}{12} = 272 - 192 = 80$$

$$SS_{bet.conds} = \frac{\sum T_c^2}{n} - \frac{(\sum X)^2}{N} = \frac{8^2 + 16^2 + 24^2}{4} - \frac{2304}{12}$$

$$= 224 - 192 = 32$$

$$SS_{with.conds} = SS_{total} - SS_{bet.conds} = 80 - 32 = 48$$

$$SS_{bet.subjs} = \frac{\sum T_s^2}{n} - \frac{(\sum X)^2}{N}$$

$$= \frac{21^2 + 6^2 + 9^2 + 12^2}{3} - \frac{2304}{12}$$

$$= 234 - 192 = 42$$

$$SS_{error} = SS_{with.conds} - SS_{bet.subjs} = 48 - 42 = 6$$

Note that most of the variability of the scores within the conditions occurs due to individual differences. Our error sums of squares is consequently a lot smaller than the within conditions sums of squares.

We can now work out the appropriate mean squares and variance ratio:

$$MS_{bet.conds} = \frac{SS_{bet.conds}}{df_{bet.conds}} = \frac{32}{2} = 2$$

$$MS_{error} = \frac{SS_{error}}{df_{error}} = \frac{6}{6} = 1$$

$$F = \frac{MS_{bet.conds}}{MS_{error}} = \frac{16}{1} = 16$$

We therefore have the following summary table:

THE ANOVA SUMMARY TABLE

Source of variation	Degrees of freedom	Sums of squares	Mean square	Variance ratio (F)	Probability
Between conditions	2	32	16	16	$p < 0.01$
Within conditions	9	48			
Between subjects	3	42			
Error	6	6	1		
Total	11 (Between + within)	80			

From the F distribution table, Table A.3 in the Appendix, $F(2,6) = 10.92$ at $p = 0.01$. As our calculated value of F is greater than the table value we can reject the null hypothesis at $p = 0.01$. We can conclude that there is a significant difference between the keyboards on the number of errors made.

(This particular example was deliberately chosen so that the calculations are very simple with whole numbers throughout. This is not typical of the numbers we would normally obtain but shows the working of the repeated measures ANOVA very clearly. For interest we can consider what would have happened if these data had come from 12 different people rather than the same four in each condition. We would have had to perform an independent measures ANOVA and used the within conditions mean square as our error variance. We can see from the above table that this value would have been 48 divided by 9, which equals 5.33. This would have resulted in an F value of 3 (16/5.33) which would not have been significant, as the critical value of $F(2,9) = 4.26$ at $p = 0.05$. The effect of different keyboards would have been lost in all the subject variability.)

Multiple comparisons

We can perform post hoc tests on a repeated measures design ANOVA to find the source of the significant differences. The only difference from the independent measures design is choosing the appropriate error term in the comparison. Whilst not universally agreed on, it is reasonable to use the MS_{error} and df_{error}, as calculated in the ANOVA, in the Tukey calculation of HSD and not the within conditions variance.

For the keyboard example, our means are: $\bar{X}_1 = 2$, $\bar{X}_2 = 4$, $\bar{X}_3 = 6$. We have $MS_{error} = 1$, $df_{error} = 6$, $n = 4$, $k = 3$. In the tables of the Studentized range statistic $q = 4.34$ for 3 conditions and 6 error degrees of freedom at $p = 0.05$, so:

$$\text{HSD} = q\sqrt{\frac{MS_{error}}{n}} = 4.34\sqrt{\frac{1}{4}} = 4.34 \times 0.5 = 2.17$$

The difference of 4 between means of Keyboards 1 and 3 is significant at $p = 0.05$ as it is larger than 2.17. The other differences in means are not significant. The size of the difference in means of 2 between Keyboards 1 and 2, and also between Keyboards 2 and 3, might reach significance if more participants were tested so it is worth exploring these non-significant differences further.

We can look at this information in a slightly different way by calculating confidence intervals. Quite simply, the 95% confdence interval of a mean difference is $95\%\text{CI} = \bar{X}_i - \bar{X}_j \pm \text{HSD}$, where $\bar{X}_i$ and $\bar{X}_j$ are any two means (the i and j standing for 1, 2, 3 etc. or which ever means we choose to compare). So,

For $\bar{X}_1 - \bar{X}_2$, 95%CI = −2 ± 2.17, producing 95%CI = (−4.17, +0.17)
For $\bar{X}_1 - \bar{X}_3$, 95%CI = −4 ± 2.17, producing 95%CI = (−6.17, −1.83)
For $\bar{X}_2 - \bar{X}_3$, 95%CI = −2 ± 2.17, producing 95%CI = (−4.17, +0.17)

Notice that the confidence intervals for $\bar{X}_1 - \bar{X}_2$ and $\bar{X}_2 - \bar{X}_3$ contain zero so this shows why we cannot claim a genuine difference in means for these conditions for the population. However, the zero value is close to one end of the confidence interval, plus, with so few participants (as our example is for illustration purposes), we have low power in our test. A more powerful test with larger sample sizes might show a larger effect.

Details on calculating the one factor repeated measures ANOVA using the SPSS computer statistical package can be found in Chapter 10 of Hinton *et al.* (2004).

Chapter 14

The interaction of factors in the analysis of variance

QUITE OFTEN RESEARCHERS wish to study the effects of more than one independent variable in their research rather than just a single factor, such as observing the effects of *age* and *experience* on motorway driving performance. Fortunately, the analysis of variance can be applied to more than a single independent variable. In fact we could consider any number of independent variables in an analysis, the problem being to explain the complexity of the results. However, as we shall see, the two factor analysis of variance offers advantages over studying the two independent variables separately, particularly as the two factor design allows us to examine the effect of the interaction of the two variables on the scores. In this chapter we shall see the importance of an interaction in data analysis. This will be explained via the use of the following example.

It has been suggested to the city Education Committee that one school in the city (Old School) has gained a reputation for discouraging girls from studying the sciences. A researcher is commissioned to investigate the matter. The researcher chooses another school in the city (New School) that matches Old School on the range of subjects pupils can choose to study (and also matches Old School on a number of other appropriate factors, such as size, standards, ages taught, ratio of boys to girls, etc. to control for confounding factors). In this city the maximum choice for pupils occurs at the age of fifteen and this is also when the pupils study the widest range of subjects. The researcher randomly selects 20 fifteen year old boys and 20 fifteen year old girls from each school and finds out how many science subjects they have chosen to study. In this experiment there are two independent variables, *school* and *gender*, and the dependent variable measured is *number of science subjects chosen*.

The researcher is not particularly interested in the separate effects of the independent variables, but a combination of the two: is the difference between the boys and girls, in terms of the number of science subjects chosen, significantly greater for Old School than for New School? A two factor analysis of variance can be performed on the data to answer this question.

The two factor analysis of variance provides us with not one but three variance ratios. The first two of these concern the main effects of the two factors, that is, taking each factor separately and looking at its effect on the dependent variable. The main effect of school will tell us whether there is a significant difference in the number of science subjects chosen at Old School compared to New School (combining the boys' and girls' scores at each school). This might be of interest, as it will tell us which school is more science-oriented but it will not tell us the difference between the boys and girls. The main effect of gender will tell us whether there is a significant difference between the boys and girls on the number of science subjects chosen. This will combine the boys from both schools and the girls from both schools. Again this might tell us something about differences in science subjects chosen based on gender but will not tell us how they differ between the two schools.

What the two factor ANOVA also tells us is whether there is a significant interaction between the factors or not. A significant interaction occurs when the effect of one factor is different at the different conditions of the other factor. Thus, the effect of *school* on the choice of science subjects for the boys is different to the effect of *school* on the choice of science subjects for the girls. If we found that *school* had no effect on the boys then there would be no difference in number of science subjects chosen whichever school they went to. However, if there was an effect of *school* on the girls with the Old School girls taking fewer science subjects than the New School girls then we would find an interaction in support of the experimental hypothesis. Here the effect of *school* is different for the two conditions of *gender*. The best way to understand a significant interaction is to plot the means for the various conditions on a graph, as in Figure 14.1, where the interaction described above is shown.

It is worth noting that if we obtained the significant interaction of the form shown in Figure 14.1 we would almost certainly have a significant main effect of *school*, as overall there are more science subjects taken at New School compared to Old School, and a significant main effect of *gender*, as overall the boys took more science subjects than the girls, but these main effects are only a by-product of the interaction, not important results in their own right. It is clear from this interaction that at Old School the girls are taking fewer science subjects than the boys whereas at New School there is no such difference.

Even if we had found that the boys in New School chose more science subjects than the girls the experimental hypothesis would still be supported if the boy–girl difference was larger at Old School than at New School. The interaction would again show a significant difference between the two schools in the effect of *gender* on the science subjects chosen.

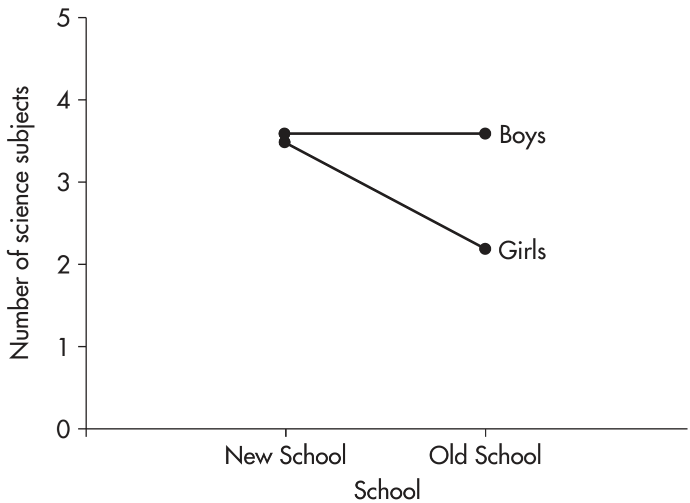

FIGURE 14.1 An interaction of school by gender

Interactions

When the effect of one factor upon another is additive then there is not an interaction in the results. Look at the example data from the schools study in Figure 14.2(a). There is a significant main effect of *gender* here (the girls choose significantly more science subjects than the boys) but no effect of *school* (the same number of science subjects are chosen at the two schools). It does not matter which school we take, the effect of gender is the same: changing from boy to girl adds one science subject to the mean score. In the example data of Figure 14.2(b) there is a main effect of *school*, more science subjects are chosen at New School and a main effect of *gender*, the boys take more science subjects than the girls. But despite having a different pattern of main effects to Figure 14.2(a) there is still no interaction. Going from girls to boys (at either school) simply adds a set amount (0.5) to the mean score. Similarly going from Old School to New School adds a set amount (1) to the mean score, regardless of whether we look at the boys' scores across the two school or the girls' scores. In any graph of means from a two factor experiment we can tell there is not an interaction when the lines on the graph are parallel, as this indicates that the effects of the factors are additive.

The examples in Figures 14.2(c) and 14.2(d) are clearly not additive as the lines on the graphs are not parallel. In these cases we will find an

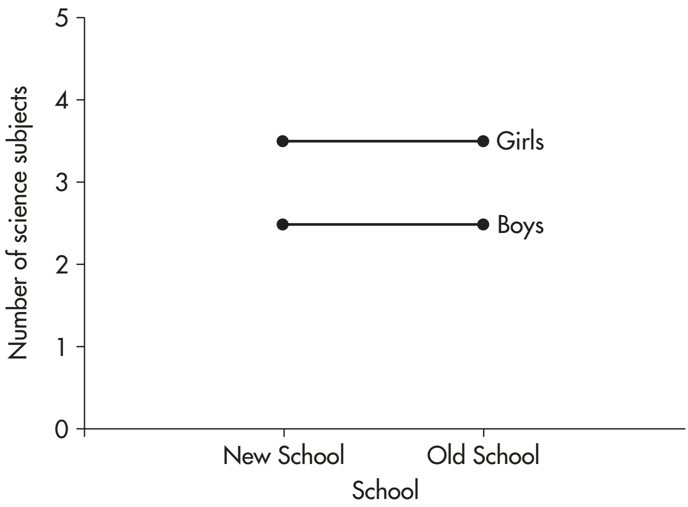

FIGURE 14.2(a) No interaction in the data

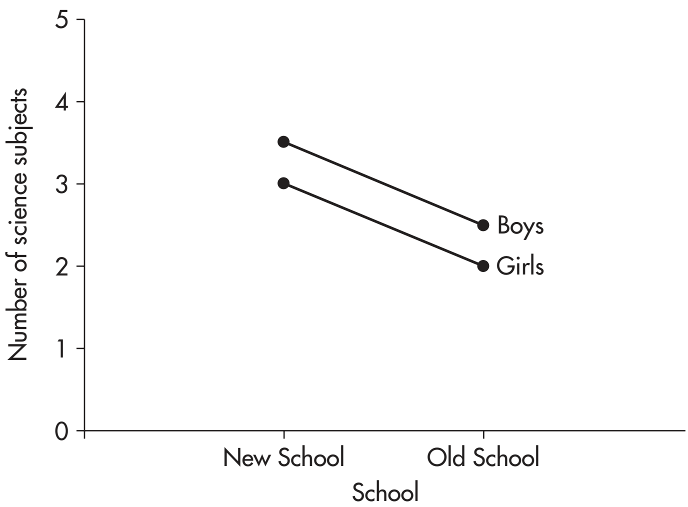

FIGURE 14.2(b) No interaction in the data again

interaction and we can decide on its significance from the two factor ANOVA. In Figure 14.2(c) there are no main effects but the interaction shows that the gender effects reverse as we move from one school to the other. At Old School the boys take one more science subject than the girls but at New School it is the girls who take one more than the boys. In Figure 14.2(d) we

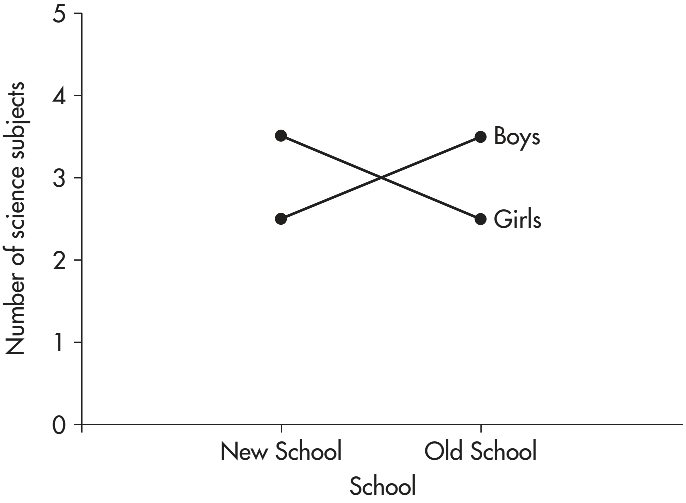

FIGURE 14.2(c) An example of an interaction

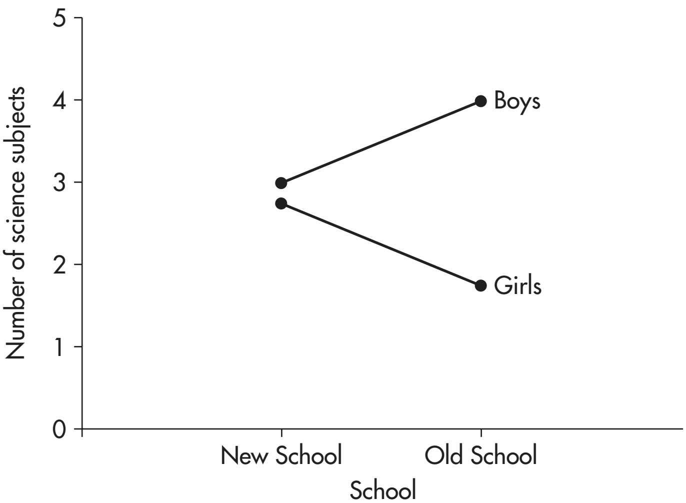

FIGURE 14.2(d) Another example of an interaction

also have an interaction as there is a wider boy–girl gap at Old School compared to New School. There will also be a main effect of *gender* as boys take more science subjects overall but not a main effect of *school* in this example.

The above examples are not exhaustive but the basic rules apply regardless of how many conditions we have for the two factors: parallel lines indicate additivity of factors and hence no interaction. When the lines are not parallel we have an interaction which indicates (if significant) a different effect of one factor at the different conditions of the other factor.

Dividing up the between conditions sums of squares

We have seen in the one factor ANOVA that it is the between conditions variability that contains the systematic differences between conditions. It is only the choice of the error term that differs when we choose repeated measures as opposed to independent measures. The same is true of the two factor ANOVA. However, in the two factor case we have systematic differences that could arise from three possible sources: the effect of the first factor (called Factor A, such as *school*), the effect of the second factor (called Factor B, such as *gender*) and the interaction of the two factors (referred to as Factor $A \times B$).

Just as we are able to partition the total sums of squares into two, the between conditions sums of squares and the within conditions sums of squares, we are also able to divide up the between conditions sums of squares into the sums of squares due to Factor A, Factor B and Factor $A \times B$. Recall that the between conditions sums of squares is:

$$SS_{bet.conds} = \frac{\sum T^2}{n} - \frac{(\sum X)^2}{N}$$

This uses the totals for the conditions in the calculation of variability of the scores between the conditions. If we used this formula for the two factor design then it would indicate a significant difference between conditions but not which factor is producing it. In our example we have four conditions each with 20 subjects ($n = 20$): Old School-Boys, Old School-Girls, New School-Boys and New School-Girls. If we consider for a moment that we are only interested in Factor A (*school*) then we combine the conditions across Factor B to produce conditions of Factor A only: we combine Old School-Boys with Old School-Girls and New School-Boys with New School-Girls to give the conditions of Factor A, Old School (A_1) and New School (A_2). We can then find a sums of squares for Factor A:

$$SS_A = \frac{\sum T_A^2}{bn} - \frac{(\sum X)^2}{N}$$

This formula uses the totals of the conditions of Factor A (in this case T_{A_1} and T_{A_2}) and bn, the number of scores in each of the conditions of Factor A, where b is the number of conditions of Factor B (in this case there are two: Boys and Girls). Combining the 20 Old School-Boys and the 20 Old School-Girls gives 40 (bn) subjects in Old School. We can then work out a mean square using the degrees of freedom for Factor A ($a - 1$, where a is the number of conditions of Factor A which, in this case, is 2).

We can do the same thing for Factor B, by combining the conditions of Factor A within the conditions of Factor A. Old School-Boys are combined with New School-Boys to produce condition B_1, Boys, and Old School-Girls and New School-Girls are combined to produce B_2, Girls. We then work out the formula for the sums of squares for Factor B:

$$SS_B = \frac{\sum T_B^2}{an} - \frac{(\sum X)^2}{N}$$

Dividing by the degrees of freedom ($b - 1$) gives us a mean square for Factor B.

The interaction sums of squares can now be worked out. We do not want to combine any conditions as we are interested in all the different conditions of Factor A and Factor B, referred to as AB conditions. In our example we have Old School-Boys (A_1B_1), Old School-Girls (A_1B_2), New School-Boys (A_2B_1) and New School-Girls (A_2B_2). We can work out the following sums of squares:

$$SS_{bet.conds} = \frac{\sum T_{AB}^2}{n} - \frac{(\sum X)^2}{N}$$

Notice that this is the same formula as the overall between conditions sums of squares. The only difference is one of labelling: the totals of conditions are referred to as T_{AB}, rather than T or T_c, as condition 1 is A_1B_1, condition 2 is A_1B_2, condition 3 is A_2B_1 and condition 4 is A_2B_2. This contains all the variability in the scores due to Factor A, Factor B and the interaction Factor $A \times B$. If we now remove from it the sums of squares from Factor A and Factor B then the remainder will provide us with the sums of squares of the interaction:

$$SS_{A\times B} = \frac{\sum T_{AB}^2}{n} - \frac{(\sum X)^2}{N} - SS_A - SS_B$$

Dividing this by the degrees of freedom of the interaction, $(a - 1)(b - 1)$, gives us the interaction mean square.

All we need to do now is to find the appropriate error variances to compare the mean squares to, in order to calculate F values for the three factors. The choice of error mean square depends on whether the factors are independent or repeated measures and the next chapter describes how this is done.

Simple main effects

If we find a significant interaction in a set of data we know that one factor is having a different effect at the different conditions of the other factor. In our schools example a significant interaction means that the effect of *school* on Boys is different to the effect of *school* on Girls. We can, if we wish, view it the other way round: the effect of *gender* is different on Old School compared to the effect of *gender* on New School. Which way round we choose to look at the interaction depends on our focus of interest. We are concerned here with the effect of *gender* as we want to know what the Boys–Girls difference is at Old School and how it compares to the Boys–Girls difference at New School.

Following the discovery of a significant interaction we may choose to look at the simple main effects of one factor at the conditions of the second factor. Calculating simple main effects is like performing a single factor ANOVA of one factor at each condition of the second factor. We can work out the simple main effects of *gender* on Old School and the simple main effects of *gender* on New School. For the simple main effects of *gender* on Old School we completely ignore the results of New School and work out a sums of squares between the Old School-Boys and the Old School-Girls. We then work out a mean square and an F value for this simple main effect which we compare to an appropriate table value. We can do the same for the simple main effect of gender on New School by ignoring the Old School results. If we had found the interaction as shown in Figure 14.1 we would expect a significant effect of gender at Old School (as the girls take fewer science subjects) but not a significant effect of gender at New School (where boys and girls do not differ in the number of science subjects chosen). These simple main effects would strongly support the experimental hypothesis.

The simple main effects of gender at Old School are only concerned with Old School-Boys (A_1B_1) and Old School-Girls (A_1B_2). Notice that Factor B (*gender*) varies between these two conditions but Factor A does not, it stays at A_1 (Old School), so we term this the simple main effect of B at A_1. The sums of squares of this simple main effect is calculated from the following formula:

$$SS_{B\ at\ A_1} = \frac{\sum T^2_{A_1B}}{n} - \frac{T^2_{A_1}}{bn}$$

where $\sum T^2_{A_1B}$ is sum of the squared totals of the A_1 conditions: the squared total of Old School-Boys ($T^2_{A_1B_1}$) plus the squared totals of Old School-Girls ($T^2_{A_1B_2}$), and $T^2_{A_1}$ is the squared total of all the Old School participants (Boys and Girls combined).

To find the sums of squares for the effects of B at A_2 we work out a similar formula but this time we are only concerned with New School (A_2):

$$SS_{B\ at\ A_2} = \frac{\sum T^2_{A_2B}}{n} - \frac{T^2_{A_2}}{bn}$$

If we had wanted to find the simple main effects for Factor A instead of Factor B all we would have done is use the same formula for the sums of squares but replaced the Bs with As (and the b with a) and vice versa.

Conclusion

A two factor ANOVA allows us to examine the interaction of the two factors. The way we do this is to separate the between conditions sums of squares into the components due to the main effects of each factor and the interaction. We can investigate a significant interaction further by looking at the simple main effects of one factor at the various conditions of the other factor, taken one at a time. In this way we can discover the source of the interaction.

Chapter 15

Calculating the two factor ANOVA

chapter 15

THERE ARE TWO IMPORTANT considerations when calculating the two factor ANOVA: first, it is necessary to lay out the data correctly and second, the correct error terms must be chosen for the variance ratios. In this chapter the three different types of two factor ANOVA are dealt with: the two factor independent measures ANOVA where both the factors, *A* and *B*, are independent measures; the two factor mixed design ANOVA where Factor *A* is independent measures and Factor *B* is repeated measures, and the two factor repeated measures ANOVA where both Factor *A* and Factor *B* are repeated measures.

The two factor independent measures ANOVA

The simplest two factor ANOVA to calculate is where both factors are independent measures. Here the between conditions variance has to be separated into that arising from Factor *A*, Factor *B* and the interaction $A \times B$, as in all two factor ANOVAs. As there are individual differences in all sums of squares calculations we can use the within conditions variance as the error term for all three variance ratios. This makes the calculations relatively easy. We, therefore, complete the following ANOVA summary table.

THE ANOVA SUMMARY TABLE

Source of variation	Degrees of freedom	Sums of squares	Mean square	Variance ratio (F)	Probability
Factor *A*	df_A	SS_A	MS_A	F_A	p_A
Factor *B*	df_B	SS_B	MS_B	F_B	p_B
Interaction $A \times B$	$df_{A\times B}$	$SS_{A\times B}$	$MS_{A\times B}$	$F_{A\times B}$	$p_{A\times B}$
Error (Within conditions)	df_{error}	SS_{error}			
Total	df_{total}	SS_{total}			

The results table

Organising the results table is important for all ANOVAs but which factor we choose as the rows and which as the columns is not as crucial for the two factor independent measures ANOVA as for the other types of two factor ANOVA, but it is important to get the various totals of the different conditions and combination of conditions correct. The following data layout is a good example to use for clarity and organisation.[11]

THE RESULTS TABLE

	Factor B				
Factor A	Condition B_1	Condition B_2	…	Condition B_b	
Condition A_1	X_1	$X_{\dots}$	…	$X_{\dots}$	
	X_2	$X_{\dots}$	…	$X_{\dots}$	
	⋮	⋮	…	⋮	
	X_n	$X_{\dots}$	…	$X_{\dots}$	
	$T_{A_1B_1}$	$T_{A_1B_2}$	…	$T_{A_1B_b}$	T_{A_1}
Condition A_2	$X_{\dots}$	$X_{\dots}$	…	$X_{\dots}$	
	$X_{\dots}$	$X_{\dots}$	…	$X_{\dots}$	
	⋮	⋮		⋮	
	$X_{\dots}$	$X_{\dots}$	…	$X_{\dots}$	
	$T_{A_2B_1}$	$T_{A_2B_2}$	…	$T_{A_2B_b}$	T_{A_2}
⋮	⋮	⋮	⋮	⋮	
Condition A_a	$X_{\dots}$	$X_{\dots}$		$X_{\dots}$	
	$X_{\dots}$	$X_{\dots}$		$X_{\dots}$	
	⋮	⋮		⋮	
	$X_{\dots}$	$X_{\dots}$		X_{abn}	
	$T_{A_aB_1}$	$T_{A_aB_2}$	…	$T_{A_aB_b}$	T_{A_a}
	T_{B_1}	T_{B_2}		T_{B_b}	$\sum X$

The formulae for calculation

Degrees of freedom:

$df_A = a - 1$ where a is the number of condition of Factor A.

$df_B = b - 1$ where b is the number of conditions of Factor B.

$df_{A\times B} = (a - 1)(b - 1)$

$df_{error} = ab(n - 1)$ where n is the number of scores in an AB condition.

$df_{total} = N - 1$ where N is the total number of scores in the data.

Sums of squares:

$$SS_{total} = \sum X^2 - \frac{(\sum X)^2}{N}$$

$$SS_A = \frac{\sum T_A^2}{nb} - \frac{(\sum X)^2}{N}$$ where $\sum T_A^2$ is $T_{A_1}^2 + T_{A_2}^2 + \ldots + T_{A_a}^2$

$$SS_B = \frac{\sum T_B^2}{na} - \frac{(\sum X)^2}{N}$$ where $\sum T_B^2$ is $T_{B_1}^2 + T_{B_2}^2 + \ldots + T_{B_b}^2$

$$SS_{A\times B} = \frac{\sum T_{AB}^2}{n} - \frac{(\sum X)^2}{N} - SS_A - SS_B$$ where $\sum T_{AB}^2$ is $T_{A_1B_1}^2 + T_{A_1B_2}^2 + \ldots + T_{A_aB_b}^2$

$$SS_{error} = SS_{total} - SS_A - SS_B - SS_{A\times B}$$

(There is an alternative formula for SS_{error}:

$$SS_{error} = SS_{with.conds} = \sum X^2 - \frac{\sum T_{AB}^2}{n}$$

Both formulae should give the same answer.)

Mean square:

$$MS_A = \frac{SS_A}{df_A}$$

$$MS_B = \frac{SS_B}{df_B}$$

$$MS_{A\times B} = \frac{SS_{A\times B}}{df_{A\times B}}$$

$$MS_{error} = \frac{SS_{error}}{df_{error}}$$

Variance ratio:

$$F_A(df_A, df_{error}) = \frac{MS_A}{MS_{error}}$$

$$F_B(df_B, df_{error}) = \frac{MS_B}{MS_{error}}$$

$$F_{A\times B}(df_{A\times B}, df_{error}) = \frac{MS_{A\times B}}{MS_{error}}$$

The F values are then compared to the table values (Table A.3 in the Appendix) at the chosen level of significance.

(The above calculations are based on equal numbers of scores, n, in each of the AB conditions. It is possible to perform this analysis with unequal numbers of scores in each condition, as with the single factor independent measures ANOVA, but it will not be dealt with in this book.)

A worked example

An expanding company wanted to know how to introduce a new type of machine into the factory. Should it transfer staff working on the old machine to operate it or employ new staff who had not worked on any machine before? A researcher selected 12 staff who had experience of the old machine and 12 staff who had no such experience. Half the participants from each group were allocated to the new machine and half to the old machine. The number of errors made by the participants over a set time period was measured. These errors are shown below.

Experience on old machine	*Machine*	
	Old	*New*
Novice	4	5
	5	6
	7	5
	6	6
	8	5
	5	6
Experienced	1	8
	2	9
	2	8
	3	8
	2	7
	3	9

What are the effects of the two factors *experience on old machine* and *type of machine* on the dependent variable *number of errors*?

Both factors are independent measures as a participant took part in only one experience/machine condition. I will label *experience on old machine* as Factor A, with two conditions ($a = 2$) 'novice' (A_1) and 'experienced' (A_2), and *type of machine* as Factor B, also with two conditions ($b = 2$), 'old machine' (B_1) and 'new machine' (B_2). There are four AB conditions each with six participants ($n = 6$), giving twenty-four participants in all ($N = 24$).

Factor A	*Factor B*		
	B_1	B_2	
A_1	4	5	
	5	6	
	7	5	
	6	6	
	8	5	
	5	6	
	$T_{A_1B_1} = 35$	$T_{A_1B_2} = 33$	$T_{A_1} = 68$
A_2	1	8	
	2	9	
	2	8	
	3	8	
	2	7	
	3	9	
	$T_{A_2B_1} = 13$	$T_{A_2B_2} = 49$	$T_{A_2} = 62$
	$T_{B_1} = 48$	$T_{B_2} = 82$	$\sum X = 130$

Degrees of freedom:

$$df_A = a - 1 = 2 - 1 = 1$$
$$df_B = b - 1 = 2 - 1 = 1$$
$$df_{A\times B} = (a - 1)(b - 1) = (2 - 1)(2 - 1) = 1$$
$$df_{error} = ab(n - 1) = 2 \times 2 \times (6 - 1) = 20$$
$$df_{total} = N - 1 = 24 - 1 = 23$$

Sums of squares:

$$SS_{total} = \sum X^2 - \frac{(\sum X)^2}{N} = (4^2 + 5^2 + \ldots + 7^2 + 9^2) - \frac{130^2}{24}$$
$$= 127.83$$

$$SS_A = \frac{\sum T_A^2}{nb} - \frac{(\sum X)^2}{N} = \frac{68^2 + 62^2}{6 \times 2} - \frac{130^2}{24} = 1.50$$

$$SS_B = \frac{\sum T_B^2}{na} - \frac{(\sum X)^2}{N} = \frac{48^2 + 82^2}{6 \times 2} - \frac{130^2}{24} = 48.17$$

$$SS_{A\times B} = \frac{\sum T_{AB}^2}{n} - \frac{(\sum X)^2}{N} - SS_A - SS_B$$

$$= \frac{35^2 + 33^2 + 13^2 + 49^2}{6} - \frac{130^2}{24} - 1.50 - 48.17$$

$$= 60.16$$

$$SS_{error} = SS_{total} - SS_A - SS_B - SS_{A\times B}$$

$$= 127.83 - 1.50 - 48.17 - 60.16 = 18.00$$

Mean square:

$$MS_A = \frac{SS_A}{df_A} = \frac{1.50}{1} = 1.50$$

$$MS_B = \frac{SS_B}{df_B} = \frac{48.17}{1} = 48.17$$

$$MS_{A\times B} = \frac{SS_{A\times B}}{df_{A\times B}} = \frac{60.16}{1} = 60.16$$

$$MS_{error} = \frac{SS_{error}}{df_{error}} = \frac{18.00}{20} = 0.90$$

Variance ratio:

$$F_A(1,20) = \frac{MS_A}{MS_{error}} = \frac{1.50}{0.90} = 1.67$$

$$F_B(1,20) = \frac{MS_B}{MS_{error}} = \frac{48.17}{0.90} = 53.52$$

$$F_{A\times B}(1,20) = \frac{MS_{A\times B}}{MS_{error}} = \frac{60.16}{0.90} = 66.84$$

THE ANOVA SUMMARY TABLE

Source of variation	Degrees of freedom	Sums of squares	Mean square	Variance ratio (F)	Probability
Factor A	1	1.50	1.50	1.67	$p > 0.05$
Factor B	1	48.17	48.17	53.52	$p < 0.01$
$A \times B$	1	60.16	60.16	66.84	$p < 0.01$
Error	20	18.00	0.90		
Total	23	127.83			

From the tables of the F distribution (A.3 in the Appendix), $F(1,20) = 4.35$ at $p = 0.05$ and $F(1,20) = 8.10$ at $p = 0.01$. We can conclude that the effect of *experience on an old machine* is not significant at $p = 0.05$ ($F(1,20) = 1.67$), the effect of *type of machine* ($F(1,20) = 53.52$) and the interaction ($F(1,20) = 66.84$) are both highly significant ($p < 0.01$).

We can examine the interaction by calculating the mean values. The table of means is shown below:

Experience on old machine	*Machine*	
	Old machine	*New machine*
Novice	5.83	5.50
Experienced	2.17	8.17

These values are plotted in Figure 15.1. The first point to note is that the lines are not parallel so we have further evidence of the interaction. Notice that the experienced workers, not surprisingly, made fewest errors on the old machine. However, they made most errors on the new machine. This looks like a case of negative transfer, where previously learnt skills can be a hindrance rather than a help. An example of this occurs when a visitor to Britain, experienced in a left-hand drive car, reaches down to change gear with the wrong hand when driving a right-hand drive car. The novice workers appear to perform with equal accuracy on both machines.

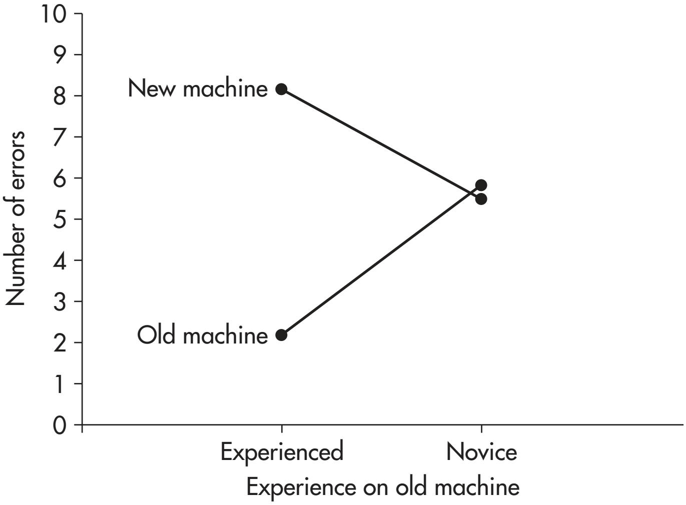

FIGURE 15.1 The interaction of experience and machine on the number of errors

In this case the interaction is quite clear. However, for illustration the simple main effects will be calculated for the effect of *type of machine* on the two levels of experience. In the two factor independent design ANOVA the error term is once again the single error term from the summary table: $MS_{error} = 0.90$, $df_{error} = 20$. This error term is used in all the simple main effects.

The simple main effect of *type of machine* on the novice operators, B at A_1:

$$SS_{B\ at\ A_1} = \frac{\sum T^2_{A_1B}}{n} - \frac{T^2_{A_1}}{bn} = \frac{35^2 + 33^2}{6} - \frac{68^2}{2 \times 6} = 0.33$$

$df_{B\ at\ A_1} = b - 1 = 2 - 1 = 1$ (as it is the effect of B and B has 2 conditions)

$$MS_{B\ at\ A_1} = \frac{SS_{B\ at\ A_1}}{df_{B\ at\ A_1}} = \frac{0.33}{1} = 0.33$$

$$F_{B\ at\ A_1} = \frac{MS_{B\ at\ A_1}}{MS_{error}} = \frac{0.33}{0.90} = 0.37$$

with degrees of freedom $df_{B\ at\ A_1} = 1$ and $df_{error} = 20$.

From the F distribution tables we know that $F(1,20) = 4.35$ at $p = 0.05$, so we can conclude, as the calculated value of F is smaller, that we have not found an effect of *type of machine* on the novice operators.

The simple main effect of *type of machine* on the experienced operators, B at A_2:

$$SS_{B\ at\ A_2} = \frac{\sum T^2_{A_2B}}{n} - \frac{T^2_{A_2}}{bn} = \frac{13^2 + 49^2}{6} - \frac{62^2}{2 \times 6} = 108.00$$

$df_{B\ at\ A_2} = b - 1 = 2 - 1 = 1$ (as it is the effect of B, and B has 2 conditions)

$$MS_{B\ at\ A_2} = \frac{SS_{B\ at\ A_2}}{df_{B\ at\ A_2}} = \frac{108.00}{1} = 108.00$$

$$F_{B\ at\ A_2} = \frac{MS_{B\ at\ A_2}}{MS_{error}} = \frac{108.00}{0.90} = 120.00$$

with degrees of freedom $df_{B\ at\ A_1} = 1$ and $df_{error} = 20$.

From the F distribution tables we know that $F(1,20) = 8.10$ at $p = 0.01$, so we can conclude, as the calculated value of F is considerably larger, that we have a found a highly significant effect of *type of machine* on the experienced operators.

The simple main effects usually explain the cause of an interaction but we can perform post hoc tests such as the Tukey or Scheffé tests if we wish. We need to be careful to select the appropriate comparison and the correct error term although it is particularly easy with the independent measures design as we use just the one error term.

The two factor mixed design ANOVA

The two factor mixed design ANOVA involves one independent measures factor and one repeated measures factor. This design is often used when we want to compare independent groups across a number of 'trials', such as comparing men and women on, say, alertness at different times of the day, or two groups of students on their knowledge at different points throughout the academic year.

For consistency we label the independent measures factor as Factor *A* and the repeated measures factor as Factor *B*. This is important as the error calculations are different for the two types of factor. This leads us to produce two error terms and this makes the calculations a little more complex than for the independent measures design. In the summary table below we see how the subjects' variability, *S*, needs to be considered in the calculations.

THE ANOVA SUMMARY TABLE

Source of variation	Degrees of freedom	Sums of squares	Mean square	Variance ratio (F)	Probability
Factor A	df_A	SS_A	MS_A	F_A	p_A
Error for A (S within A)	df_{errorA}	SS_{errorA}	MS_{errorA}		
Factor B	df_B	SS_B	MS_B	F_B	p_B
Factor $A \times B$	$df_{A\times B}$	$SS_{A\times B}$	$MS_{A\times B}$	$F_{A\times B}$	$p_{A\times B}$
Error for B and $A \times B$ ($B \times AS$)	df_{errorB}	SS_{errorB}	MS_{errorB}		
Total	df_{total}	SS_{total}			

The results table

We designate the independent measures factor (Factor *A*) as the rows and the repeated measures factor (Factor *B*) as the columns in the results table so that the results from a single subject form one row of the table. We must be careful to lay out our results consistently so that we do not analyse the results of the two factors incorrectly. Also if we use a computer program to analyse our data it could analyse the factors the wrong way round if the layout is different.[11]

THE RESULTS TABLE

		Factor B					
Factor A		Condition B_1	Condition B_2	...	Condition B_b		
Condition A_1	S_1	X_1	$X_{...}$	...	$X_{...}$	T_{S_1}	
	S_2	X_2	$X_{...}$	...	$X_{...}$	T_{S_2}	
	⋮	⋮	⋮	⋮	⋮	⋮	
	S_n	X_n	$X_{...}$	...	$X_{...}$	T_{S_n}	
		$T_{A_1B_1}$	$T_{A_1B_2}$	...	$T_{A_1B_b}$		T_{A_1}
Condition A_2	S_{n+1}	$X_{...}$	$X_{...}$	...	$X_{...}$	$T_{S...}$	
	S_{n+2}	$X_{...}$	$X_{...}$	...	$X_{...}$	$T_{S...}$	
	⋮	⋮	⋮	⋮	⋮	⋮	
	S_{2n}	$X_{...}$	$X_{...}$	...	$X_{...}$	$T_{S...}$	
		$T_{A_2B_1}$	$T_{A_2B_2}$	...	$T_{A_2B_b}$		T_{A_2}
⋮		⋮	⋮	⋮	⋮	⋮	
Condition A_a	$S_{...}$	$X_{...}$	$X_{...}$		$X_{...}$	...	
	$S_{...}$	$X_{...}$	$X_{...}$		$X_{...}$	...	
	⋮	⋮	⋮		⋮	⋮	
	S_{an}	$X_{...}$	$X_{...}$		X_{abn}	$T_{S_{an}}$	
		$T_{A_aB_1}$	$T_{A_aB_2}$	...	$T_{A_aB_b}$		T_{A_a}
		T_{B_1}	T_{B_2}		T_{B_b}	...	$\sum X$

The formulae for calculation

Degrees of freedom:

$df_A = a - 1$ where a is the number of conditions of Factor A.

$df_{errorA} = a(n - 1)$ where n is the number of scores in an AB condition.

$df_B = b - 1$ where b is the number of conditions of Factor B.

$df_{A \times B} = (a - 1)(b - 1)$

$df_{errorB} = a(b - 1)(n - 1)$

$df_{total} = N - 1$ where N is the total number of scores in the data.

Sums of squares:

$$SS_{total} = \sum X^2 - \frac{(\sum X)^2}{N}$$

$$SS_A = \frac{\sum T_A^2}{nb} - \frac{(\sum X)^2}{N}$$ where $\sum T_A^2$ is $T_{A_1}^2 + T_{A_2}^2 + \ldots + T_{A_a}^2$

$$SS_{errorA} = \frac{\sum T_S^2}{b} - \frac{(\sum X)^2}{N} - SS_A$$ where $\sum T_S^2$ is $T_{S_1}^2 + T_{S_2}^2 + \ldots + T_{S_{an}}^2$

(The sums of squares between subjects, the first two components of the error A sums of squares, comprises all the Factor A variation. If we take away the variation between the A conditions, SS_A, we are left with the variation within the A conditions as our error term.)

$$SS_B = \frac{\sum T_B^2}{na} - \frac{(\sum X)^2}{N}$$ where $\sum T_B^2$ is $T_{B_1}^2 + T_{B_2}^2 + \ldots + T_{B_b}^2$

$$SS_{A \times B} = \frac{\sum T_{AB}^2}{n} - \frac{(\sum X)^2}{N} - SS_A - SS_B$$ where $\sum T_{AB}^2$ is $T_{A_1B_1}^2 + T_{A_1B_2}^2 + \ldots + T_{A_aB_b}^2$

$$SS_{errorB} = \sum X^2 - \frac{\sum T_S^2}{b} - SS_B - SS_{A \times B}$$

(The variation within subjects, the first two components of the error B sums of squares, contains the B and $A \times B$ variation. Removing the between condition variation for B and $A \times B$ leaves the error sums of squares for B and $A \times B$, unaffected by individual differences.)

Mean square:

$$MS_A = \frac{SS_A}{df_A}$$

$$MS_{errorA} = \frac{SS_{errorA}}{df_{errorA}}$$

$$MS_B = \frac{SS_B}{df_B}$$

$$MS_{A\times B} = \frac{SS_{A\times B}}{df_{A\times B}}$$

$$MS_{errorB} = \frac{SS_{errorB}}{df_{errorB}}$$

Variance ratio:

$$F_A(df_A, df_{errorA}) = \frac{MS_A}{MS_{errorA}}$$

$$F_B(df_B, df_{errorB}) = \frac{MS_B}{MS_{errorB}}$$

$$F_{A\times B}(df_{A\times B}, df_{errorB}) = \frac{MS_{A\times B}}{MS_{errorB}}$$

The F values are then compared to the table values (using Table A.3 in the Appendix) at the chosen level of significance.

A worked example

A company has introduced a new machine on the factory floor and it wants to see how the workers gain skill on the machine. There is particular interest in comparing the performance of workers experienced on the old machine with that of novice operators who have not operated a machine on the factory floor before. A researcher randomly selects 6 experienced operators and 6 novices and monitors the errors they make on the new machine over a three week period to see whether there are differences between the two groups in their performance on the machine. The results are shown below.

Participants	*Time*		
	Week 1	*Week 2*	*Week 3*
Novices			
1	7	6	5
2	4	4	3
3	6	4	4
4	7	6	5
5	6	5	4
6	4	2	2
Experienced			
7	7	3	2
8	8	4	2
9	6	2	1
10	9	6	3
11	7	4	3
12	10	6	3

We have an independent factor *experience* which will be designated Factor A, with ‘novice’ as A_1 and ‘experienced’ as A_2. The repeated measures factor is *time*, so this is Factor B, with ‘Week 1’ as B_1, ‘Week 2’ as B_2 and ‘Week 3’ as B_3. We can draw up the results table as follows.

THE RESULTS TABLE

		Factor B				
Factor A		B_1	B_2	B_3		
A_1	S_1	7	6	5	$T_{S_1} = 18$	
	S_2	4	4	3	$T_{S_2} = 11$	
	S_3	6	4	4	$T_{S_3} = 14$	
	S_4	7	6	5	$T_{S_4} = 18$	
	S_5	6	5	4	$T_{S_5} = 15$	
	S_6	4	2	2	$T_{S_6} = 8$	
		$T_{A_1B_1} = 34$	$T_{A_1B_2} = 27$	$T_{A_1B_3} = 23$		$T_{A_1} = 84$
A_1	S_7	7	3	2	$T_{S_7} = 12$	
	S_8	8	4	2	$T_{S_8} = 14$	
	S_9	6	2	1	$T_{S_9} = 9$	
	S_{10}	9	6	3	$T_{S_{10}} = 18$	
	S_{11}	7	4	3	$T_{S_{11}} = 14$	
	S_{12}	10	6	3	$T_{S_{12}} = 19$	
		$T_{A_2B_1} = 47$	$T_{A_2B_2} = 25$	$T_{A_2B_3} = 14$		$T_{A_2} = 86$
		$T_{B_1} = 81$	$T_{B_2} = 52$	$T_{B_3} = 37$		$\sum X = 170$

Degrees of freedom:

$$df_A = a - 1 = 2 - 1 = 1$$

$$df_{errorA} = a(n - 1) = 2(6 - 1) = 10$$

$$df_B = b - 1 = 3 - 1 = 2$$

$$df_{A\times B} = (a - 1)(b - 1) = (2 - 1)(3 - 1) = 2$$

$$df_{errorB} = a(b - 1)(n - 1) = 2(3 - 1)(6 - 1) = 20$$

$$df_{total} = N - 1 = 36 - 1 = 35$$

Sums of squares:

We can make our calculations easier if we work out the following parts of the formulae first:

$$\frac{(\sum X)^2}{N} = \frac{170^2}{36} = 802.78$$

$$\frac{\sum T_A^2}{nb} = \frac{84^2 + 86^2}{6 \times 3} = 802.89$$

$$\frac{\sum T_B^2}{na} = \frac{81^2 + 52^2 + 37^2}{6 \times 2} = 886.17$$

$$\frac{\sum T_S^2}{b} = \frac{18^2 + 11^2 + \ldots + 14^2 + 19^2}{3} = 852.00$$

$$\frac{\sum T_{AB}^2}{n} = \frac{34^2 + 27^2 + 23^2 + 47^2 + 25^2 + 14^2}{6} = 907.33$$

$$\sum X^2 = 7^2 + 4^2 + \ldots + 3^2 + 3^2 = 962.00$$

Now we can work out the sums of squares:

$$SS_{total} = \sum X^2 - \frac{(\sum X)^2}{N} = 962.00 - 802.78 = 159.22$$

$$SS_A = \frac{\sum T_A^2}{nb} - \frac{(\sum X)^2}{N} = 802.89 - 802.78 = 0.11$$

$$SS_{errorA} = \frac{\sum T_S^2}{b} - \frac{(\sum X)^2}{N} - SS_A = 852.00 - 802.78 - 0.11$$
$$= 49.11$$

$$SS_B = \frac{\sum T_B^2}{na} - \frac{(\sum X)^2}{N} = 886.17 - 802.78 = 83.39$$

$$SS_{A\times B} = \frac{\sum T_{AB}^2}{n} - \frac{(\sum X)^2}{N} - SS_A - SS_B$$
$$= 907.33 - 802.78 - 0.11 - 83.39 = 21.05$$

$$SS_{errorB} = \sum X^2 - \frac{\sum T_S^2}{b} - SS_B - SS_{A\times B}$$
$$= 962.00 - 852.00 - 83.39 - 21.05 = 5.56$$

Mean square:

$$MS_A = \frac{SS_A}{df_A} = \frac{0.11}{1} = 0.11$$

$$MS_{errorA} = \frac{SS_{errorA}}{df_{errorA}} = \frac{49.11}{10} = 4.91$$

$$MS_B = \frac{SS_B}{df_B} = \frac{83.39}{2} = 41.70$$

$$MS_{A\times B} = \frac{SS_{A\times B}}{df_{A\times B}} = \frac{21.05}{2} = 10.53$$

$$MS_{errorB} = \frac{SS_{errorB}}{df_{errorB}} = \frac{5.56}{20} = 0.28$$

Variance ratio:

$$F_A(1{,}10) = \frac{MS_A}{MS_{errorA}} = \frac{0.11}{4.91} = 0.02$$

$$F_B(2{,}20) = \frac{MS_B}{MS_{errorB}} = \frac{41.70}{0.28} = 148.93$$

$$F_{A\times B}(2{,}20) = \frac{MS_{A\times B}}{MS_{errorB}} = \frac{10.53}{0.28} = 37.61$$

THE ANOVA SUMMARY TABLE

Source of variation	Degrees of freedom	Sums of squares	Mean square	Variance ratio (F)	Probability
Factor A	1	0.11	0.11	0.02	$p > 0.05$
Error$A$	10	49.11	4.91		
Factor B	2	83.39	41.70	148.93	$p < 0.01$
Factor $A \times B$	2	21.05	10.53	37.61	$p < 0.01$
Error$B$	20	5.56	0.28		
Total	35	159.22			

In conclusion, the main effect of *experience* ($F(1,10) = 0.02$) is not significant ($F(1,10) = 4.96$ at $p = 0.05$), whereas the main effect of *time* ($F(2,20) = 148.93$) and the interaction ($F(2,20) = 37.61$) are both highly significant ($F(2,20) = 5.85$ at $p = 0.01$).

As we have found a significant interaction we can look at the means to see the source of the interaction. The means are listed in the table below and plotted in Figure 15.2.

Experience	*Time*		
	Week 1	*Week 2*	*Week 3*
Novice	5.67	4.50	3.83
Experienced	7.83	4.17	2.33

We can see that, taken over the three weeks, the total number of errors of the two groups of operators does not differ by very much which is why there was no main effect of *experience*. All the operators made fewer errors over time, which is responsible for the highly significant effect of *time*. The highly significant interaction is interesting, as the experienced operators

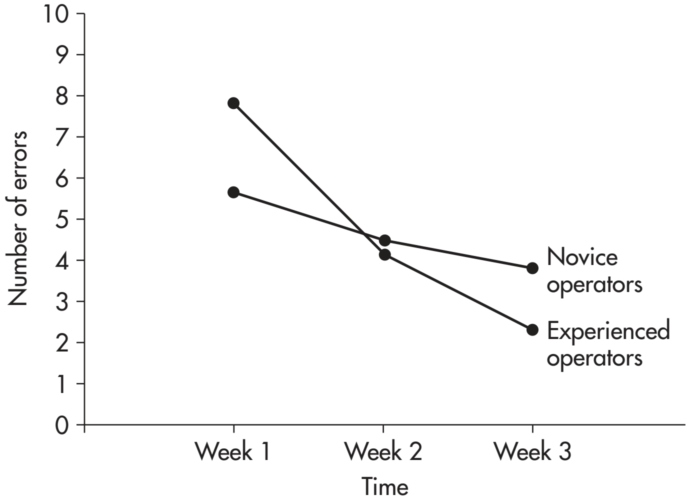

FIGURE 15.2 The interaction of time and experience on machine operator errors

began by making more errors than the novices but by Week 2 had caught them up and at Week 3 were making fewer errors. The initial difficulty for them might have been due to negative transfer (see page 179) from the old machine to the new but after a while their experience began to help them and they leapt ahead. Clearly this is speculation but it is consistent with the outcome of the analysis.

With the mixed design ANOVA, when we have a significant interaction, we are much more likely to look at the simple main effects of the independent measures factor at the various conditions of the repeated measures factor than vice versa. In our example it is more interesting to look at the effect of *experience* at Week 1 and then at Week 2, and Week 3 rather than looking at the effect of *time* on novice operators, and then on experienced operators. I shall therefore only look at the simple main effects of Factor A.[12] The simple main effects allow us to look at the effect of *experience* on the errors at one week only, ignoring the data from the other weeks. In this design we work out a different error term for each simple main effect.

The simple main effect of *experience* at Week 1:

$$SS_{A\ at\ B_1} = \frac{\sum T^2_{AB_1}}{n} - \frac{T^2_{B_1}}{an} = \frac{34^2 + 47^2}{6} - \frac{81^2}{2 \times 6}$$

$$= 560.83 - 546.75 = 14.08$$

$$df_{A\ at\ B_1} = \text{a} - 1 = 2 - 1 = 1$$

$$MS_{A\ at\ B_1} = \frac{SS_{A\ at\ B_1}}{df_{A\ at\ B_1}} = \frac{14.08}{1} = 14.08$$

$$\begin{aligned} SS_{errorA\ at\ B_1} &= \sum T^2_{AB_1S} - \frac{\sum T^2_{AB_1}}{n} \\ &= 7^2 + 4^2 + \ldots + 7^2 + 10^2 - 560.83 \\ &= 581 - 560.83 = 20.17 \end{aligned}$$

where $\sum T^2_{AB_1S}$ is the sum of the squared scores of each subject in each A condition (novice and experienced) at B_1 (Week 1).

$$df_{errorA\ at\ B_1} = a(n - 1) = 2(6 - 1) = 10$$

$$MS_{errorA\ at\ B_1} = \frac{SS_{errorA\ at\ B_1}}{df_{errorA\ at\ B_1}} = \frac{20.17}{10} = 2.02$$

$$\begin{aligned} F_{A\ at\ B_1}(1{,}10) &= \frac{MS_{A\ at\ B_1}}{MS_{errorA\ at\ B_1}} \\ &= \frac{14.08}{2.02} = 6.97 \quad \text{(from Table A.3, } F(1{,}10) = 4.96, p = 0.05) \end{aligned}$$

There is a significant ($p < 0.05$) simple main effect of experience at Week 1. We can conclude that the experienced operators are making significantly more errors than the novice operators in Week 1.

We replace B_1 with B_2 in the above calculations to find the simple main effect of *experience* at Week 2. $F_{A\,at\,B_2}(1{,}10) = 0.14$, so there is not a significant difference between the errors made by the operators in Week 2. We calculate the simple main effect of *experience* at Week 3 in the same way and $F_{A\,at\,B_3}(1{,}10) = 6.64$, which is significant at $p = 0.05$. In Week 3 there is a significant difference in the number of errors made between the two groups of operators, with the experienced operators making significantly fewer errors. Thus, the simple main effects have confirmed the source of the interaction observed by 'eyeballing' the graph in Figure 15.2.

The two factor repeated measures ANOVA

The advantage of having repeated measures on both the factors under study is that we can perform a two factor analysis with relatively few subjects. It also allows us to extract out the subjects' variability and consider whether the subjects are performing at similar levels.

The calculation of the two factor ANOVA is most complex when we have repeated measures on both factors. This is because we have to calculate a different error term for each of the three factors under study (A, B and $A \times B$). In this design we are able to extract the variation between subjects, so subjects (S) can be seen as a random (independent measures) factor in the analysis. To produce an error term for a factor we select the interaction of S with the factor under test. This is shown in the summary table below.

THE ANOVA SUMMARY TABLE

Source of variation	Degrees of freedom	Sums of squares	Mean square	Variance ratio (F)	Probability
Factor A	df_A	SS_A	MS_A	F_A	p_A
Factor B	df_B	SS_B	MS_B	F_B	p_B
Subjects S	df_S	SS_S	(MS_B)	(F_S)	(p_S)
Factor $A \times B$	$df_{A\times B}$	$SS_{A\times B}$	$MS_{A\times B}$	$F_{A\times B}$	$p_{A\times B}$
Error for A ($A \times S$)	df_{errorA}	SS_{errorA}	MS_{errorA}		
Error for B ($B \times S$)	df_{errorB}	SS_{errorB}	MS_{errorB}		
Error for $A \times B$ ($A \times B \times S$)	$df_{errorAB}$	$SS_{errorAB}$	$MS_{errorAB}$		
Total	df_{total}	SS_{total}			

I have include the mean square and F for the subjects in parentheses as we only need to calculate these when we are concerned that there are significant individual differences between the subjects.

The results table

In the mixed design we arranged the data so that columns in the results table refer to the repeated measures factor. We keep the same pattern when both factors are repeated by laying out the table in the format shown below, with the subjects as the rows and the conditions of Factors A and B as the columns. As both factors are repeated measures it does not matter which we choose as Factor A and Factor B as long as we are consistent throughout.[11]

THE RESULTS TABLE

	Condition A_1			…	Condition A_a			
Subjects	Condition B_1	…	Condition B_b	…	Condition B_1	…	Condition B_b	T_S
S_1	X_1		X_b		$X_{..}$		$X_{..}$	T_{S_1}
S_2	$X_{..}$		$X_{..}$		$X_{..}$		$X_{..}$	T_{S_2}
S_3	$X_{..}$		$X_{..}$		$X_{..}$		$X_{..}$	T_{S_3}
$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$	$\vdots$
S_n	$X_{..}$		$X_{..}$		$X_{..}$		X_{abn}	T_{S_n}
	$T_{A_1B_1}$	…	$T_{A_1B_b}$	…	$T_{A_aB_1}$	…	$T_{A_aB_b}$	$\sum X$

We also calculate two additional tables to aid the calculations: the AS matrix and the BS matrix. We work out the former by adding up the scores across B, and the latter by adding up the scores across A. For subject 1, $T_{A_1S_1}$ is the total of the scores in condition A_1 summed across B, so it is the sum of subject 1's scores in conditions A_1B_1 to A_1B_b. Similarly, $T_{B_1S_1}$ is the sum of subject 1's scores in conditions A_1B_1 to A_aB_1.

***AS* Matrix**

Subject	A_1S	...	A_aS
S_1	$T_{A_1S_1}$	...	$T_{A_aS_1}$
S_2	$T_{A_1S_2}$	...	$T_{A_aS_2}$
S_3	$T_{A_1S_3}$	...	$T_{A_aS_3}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_n	$T_{A_1S_n}$	...	$T_{A_aS_n}$
	T_{A_1}	...	T_{A_a}

***BS* Matrix**

Subject	B_1S	...	B_bS
S_1	$T_{B_1S_1}$	...	$T_{B_bS_1}$
S_2	$T_{B_1S_2}$	...	$T_{B_bS_2}$
S_3	$T_{B_1S_3}$	...	$T_{B_bS_3}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_n	$T_{B_1S_n}$	...	$T_{B_bS_n}$
	T_{B_1}	...	T_{B_b}

The formulae for the calculation

Degrees of freedom:

$df_A = a - 1$ where a is the number of conditions of Factor A.

$df_B = b - 1$ where b is the number of conditions of Factor B.

$df_S = n - 1$ where n is the number of subjects.

$df_{A\times B} = (a - 1)(b - 1)$

$df_{errorA} = (a - 1)(n - 1)$

$df_{errorB} = (b - 1)(n - 1)$

$df_{errorAB} = (a - 1)(b - 1)(n - 1)$

$df_{total} = N - 1$ where N is the total number of scores in the data.

Sums of squares:

$$SS_{total} = \sum X^2 - \frac{(\sum X)^2}{N}$$

$$SS_A = \frac{\sum T_A^2}{nb} - \frac{(\sum X)^2}{N}$$ where $\sum T_A^2$ is $T_{A_1}^2 + T_{A_2}^2 + \ldots + T_{A_a}^2$

$$SS_B = \frac{\sum T_B^2}{na} - \frac{(\sum X)^2}{N}$$ where $\sum T_B^2$ is $T_{B_1}^2 + T_{B_2}^2 + \ldots + T_{B_b}^2$

$$SS_S = \frac{\sum T_S^2}{ab} - \frac{(\sum X)^2}{N}$$ where $\sum T_S^2$ is $T_{S_1}^2 + T_{S_2}^2 + \ldots + T_{S_n}^2$

$$SS_{A\times B} = \frac{\sum T_{AB}^2}{n} - \frac{(\sum X)^2}{N} - SS_A - SS_B$$ where $\sum T_{AB}^2$ is $T_{A_1B_1}^2 + T_{A_1B_2}^2 + \ldots + T_{A_aB_b}^2$

$$SS_{errorA} = \frac{\sum T_{AS}^2}{b} - \frac{(\sum X)^2}{N} - SS_A - SS_S$$ where $\sum T_{AS}^2$ is $T_{A_1S_1}^2 + \ldots + T_{A_aS_n}^2 + \ldots + T_{A_aS_1}^2 \ldots + T_{A_aS_n}^2$

$$SS_{errorB} = \frac{\sum T_{BS}^2}{a} - \frac{(\sum X)^2}{N} - SS_B - SS_S$$ where $\sum T_{BS}^2$ is $T_{B_1S_1}^2 + \ldots + T_{B_1S_n}^2 + \ldots + T_{B_bS_1}^2 \ldots + T_{B_bS_n}^2$

$$SS_{errorAB} = SS_{total} - SS_A - SS_B - SS_S - SS_{A\times B} - SS_{errorA} - SS_{errorB}$$

Mean square:

$$MS_A = \frac{SS_A}{df_A}$$

$$MS_B = \frac{SS_B}{df_B}$$

$$MS_S = \frac{SS_S}{df_S}$$

$$MS_{A\times B} = \frac{SS_{A\times B}}{df_{A\times B}}$$

$$MS_{errorA} = \frac{SS_{errorA}}{df_{errorA}}$$

$$MS_{errorB} = \frac{SS_{errorB}}{df_{errorB}}$$

$$MS_{errorAB} = \frac{SS_{errorAB}}{df_{errorAB}}$$

Variance ratio:

$$F_A(df_A, df_{errorA}) = \frac{MS_A}{MS_{errorA}}$$

$$F_B(df_B, df_{errorB}) = \frac{MS_B}{MS_{errorB}}$$

$$F_S(df_S, df_{errorAB}) = \frac{MS_S}{MS_{errorAB}}$$

$$F_{A \times B}(df_{A \times B}, df_{errorAB}) = \frac{MS_{A \times B}}{MS_{errorAB}}$$

The F values are then compared to the table values at the chosen level of significance.

A worked example

In a factory a machine produces two kinds of product, one that requires the operator to follow a complex set of instructions and one that is very simple to make. There are two shifts in the factory, a day shift and a night shift. The factory manager wants the factory to make the products with the minimum of errors. A researcher decides to study the effect of *shift* (day versus night) and *product* (complex versus simple to make) on the errors made by the operators. All operators work both shifts on a rotation system. Six operators are randomly selected and their error performance is measured during a day shift and a night shift. Appropriate balancing is undertaken so that carry-over effects from one shift to another are controlled for by testing three operators on the day shift first and three on the night shift first. The number of errors made during a shift are shown in the table below.

Operator	*Complex product*		*Simple product*	
	Day shift	*Night shift*	*Day shift*	*Night shift*
1	5	9	3	2
2	5	8	2	4
3	7	7	4	5
4	6	10	5	4
5	4	8	3	3
6	6	9	5	6

There are repeated measures on both factors so the repeated measures ANOVA can be used to test the effect of the independent variables on performance. Due to the way I have laid out the conditions above, I shall label *product* as Factor A, with 'complex product' as A_1 and 'simple product' as A_2, and *shift* as Factor B, with 'day shift' as B_1 and 'night shift' as B_2. There are two conditions of Factor A ($a = 2$), two of Factor B ($b = 2$), six participants ($n = 6$) and twenty-four scores in total ($N = 24$).

First we produce the results table:

Participants	*Condition* A_1		*Condition* A_2		
	Condition B_1	*Condition* B_2	*Condition* B_1	*Condition* B_2	T_S
S_1	5	9	3	2	$T_{S_1} = 19$
S_2	5	8	2	4	$T_{S_2} = 19$
S_3	7	7	4	5	$T_{S_3} = 23$
S_4	6	10	5	4	$T_{S_4} = 25$
S_5	4	8	3	3	$T_{S_5} = 18$
S_6	6	9	5	6	$T_{S_6} = 26$
	$T_{A_1B_1} = 33$	$T_{A_1B_2} = 51$	$T_{A_2B_1} = 22$	$T_{A_2B_2} = 24$	$\sum X = 130$

The *AS* and *BS* matrices can be created from the results table.

AS Matrix

Participant	A_1S	A_2S
S_1	14	5
S_2	13	6
S_3	14	9
S_4	16	9
S_5	12	6
S_6	15	11
	$T_{A_1} = 84$	$T_{A_2} = 46$

BS Matrix

Participant	B_1S	B_2S
S_1	8	11
S_2	7	12
S_3	11	12
S_4	11	14
S_5	7	11
S_6	11	15
	$T_{B_1} = 55$	$T_{B_2} = 75$

We can now calculate the F values:

Degrees of freedom:

$$df_A = a - 1 = 2 - 1 = 1$$
$$df_B = b - 1 = 2 - 1 = 1$$
$$df_S = n - 1 = 6 - 1 = 5$$
$$df_{A \times B} = (a - 1)(b - 1) = (2 - 1)(2 - 1) = 1$$
$$df_{errorA} = (a - 1)(n - 1) = (2 - 1)(6 - 1) = 5$$
$$df_{errorB} = (b - 1)(n - 1) = (2 - 1)(6 - 1) = 5$$
$$df_{errorAB} = (a - 1)(b - 1)(n - 1) = (2 - 1)(2 - 1)(6 - 1) = 5$$
$$df_{total} = N - 1 = 24 - 1 = 23$$

Sums of squares:

We can make the calculations easier if we work out the components of the formulae first:

$$\frac{(\sum X)^2}{N} = \frac{130^2}{24} = 704.17$$

$$\frac{\sum T_A^2}{nb} = \frac{84^2 + 46^2}{6 \times 2} = 764.33$$

$$\frac{\sum T_B^2}{na} = \frac{55^2 + 75^2}{6 \times 2} = 720.83$$

$$\frac{\sum T_S^2}{ab} = \frac{19^2 + 19^2 + 23^2 + 25^2 + 18^2 + 26^2}{2 \times 2} = 719.00$$

$$\frac{\sum T_{AB}^2}{n} = \frac{33^2 + 51^2 + 22^2 + 24^2}{6} = 791.67$$

$$\frac{\sum T_{AS}^2}{b} = \frac{14^2 + 13^2 + 14^2 + \ldots + 9^2 + 6^2 + 11^2}{2} = 783.00$$

$$\frac{\sum T_{BS}^2}{a} = \frac{8^2 + 7^2 + 11^2 + \ldots + 14^2 + 11^2 + 15^2}{2} = 738.00$$

$$\sum X^2 = 820.00$$

We can now work out the sums of squares:

$$SS_{total} = \sum X^2 - \frac{(\sum X)^2}{N} = 820.00 - 704.17 = 115.83$$

$$SS_A = \frac{\sum T_A^2}{nb} - \frac{(\sum X)^2}{N} = 764.33 - 704.17 = 60.16$$

$$SS_B = \frac{\sum T_B^2}{na} - \frac{(\sum X)^2}{N} = 720.83 - 704.17 = 16.66$$

$$SS_S = \frac{\sum T_S^2}{ab} - \frac{(\sum X)^2}{N} = 719.00 - 704.17 = 14.83$$

$$SS_{A \times B} = \frac{\sum T_{AB}^2}{n} - \frac{(\sum X)^2}{N} - SS_A - SS_B$$

$$= 791.67 - 704.17 - 60.16 - 16.66 = 10.68$$

$$SS_{errorA} = \frac{\sum T_{AS}^2}{b} - \frac{(\sum X)^2}{N} - SS_A - SS_S$$
$$= 783.00 - 704.17 - 60.16 - 14.83$$
$$= 3.84$$

$$SS_{errorB} = \frac{\sum T_{BS}^2}{a} - \frac{(\sum X)^2}{N} - SS_B - SS_S$$
$$= 738.00 - 704.17 - 16.66 - 14.83$$
$$= 2.34$$

$$SS_{errorAB} = SS_{total} - SS_A - SS_B - SS_S - SS_{A\times B} - SS_{errorA} - SS_{errorB}$$
$$= 820.00 - 60.16 - 16.66 - 14.83 - 10.68 - 3.84 - 2.34$$
$$= 7.32$$

Mean square:

$$MS_A = \frac{SS_A}{df_A} = \frac{60.16}{1} = 60.16$$

$$MS_B = \frac{SS_B}{df_B} = \frac{16.66}{1} = 16.66$$

$$MS_S = \frac{SS_S}{df_S} = \frac{14.83}{5} = 2.97$$

$$MS_{A\times B} = \frac{SS_{A\times B}}{df_{A\times B}} = \frac{10.68}{1} = 10.68$$

$$MS_{errorA} = \frac{SS_{errorA}}{df_{errorA}} = \frac{3.84}{5} = 0.77$$

$$MS_{errorB} = \frac{SS_{errorB}}{df_{errorB}} = \frac{2.34}{5} = 0.47$$

$$MS_{errorAB} = \frac{SS_{errorAB}}{df_{errorAB}} = \frac{7.32}{5} = 1.46$$

Variance ratio:

$$F_A(1{,}5) = \frac{MS_A}{MS_{errorA}} = \frac{60.16}{0.77} = 78.13$$

$$F_B(1{,}5) = \frac{MS_B}{MS_{errorB}} = \frac{16.66}{0.47} = 35.45$$

$$F_S(5{,}5) = \frac{MS_S}{MS_{errorAB}} = \frac{2.97}{1.46} = 2.03$$

$$F_{A\times B}(1{,}5) = \frac{MS_{A\times B}}{MS_{errorAB}} = \frac{10.68}{1.46} = 7.32$$

THE ANOVA SUMMARY TABLE

Source of variation	Degrees of freedom	Sums of squares	Mean square	Variance ratio (F)	Probability
Factor A	1	60.16	60.16	78.13	$p < 0.01$
Factor B	1	16.66	16.66	35.45	$p < 0.01$
Subjects S	5	14.83	2.97	2.03	$p > 0.05$
Factor $A \times B$	1	10.68	10.68	7.32	$p < 0.05$
Error$A$	5	3.84	0.77		
Error$B$	5	2.34	0.47		
ErrorAB	5	7.32	1.46		
Total	23	115.83			

In conclusion there is a highly significant effect of Factor A (*product*) with $F(1{,}5) = 78.13$, and of Factor B (*shift*) with $F(1{,}5) = 35.45$ (compared to a table value of $F(1{,}5) = 16.26$, $p = 0.01$). The interaction of *product* and *shift* ($F(1{,}5) = 7.32$) is significant at the $p = 0.05$ level of significance

($F(1,5) = 6.61$, $p = 0.05$). The effect of *subjects* ($F(5,5) = 2.03$) is not significant ($F(5,5) = 5.05$, $p = 0.05$) which indicates no significant differences between the participants in their level of performance.

The mean number of errors in each condition is shown in the table below.

Complex product		*Simple product*	
Day shift	*Night shift*	*Day shift*	*Night shift*
5.50	8.50	3.67	4.00

These means are plotted in Figure 15.3 to help us interpret the interaction. More errors are made on the complex product than the simple product (producing the effect of *product*) and more errors are made on the night shift (producing the effect of *shift*). However, from Figure 15.3 we can see that the difference in the errors between the day and night shifts is much greater on the complex product. More errors are made at night relative to the day for the complex product in comparison to day–night difference for the simple product.

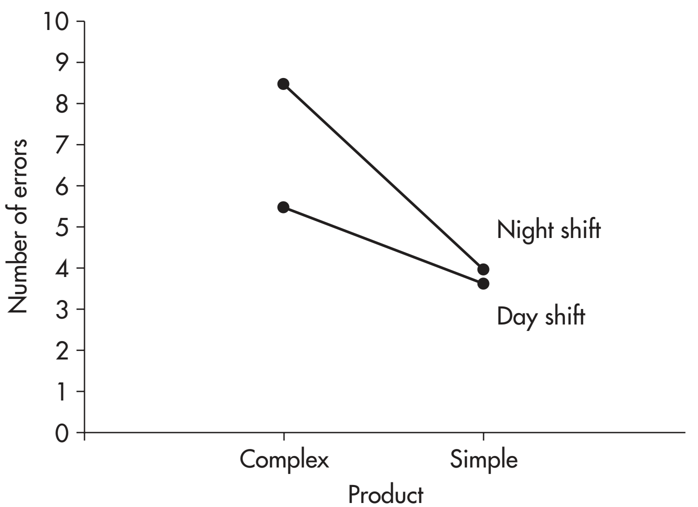

FIGURE 15.3 The interaction of product and shift on machine operator errors

We can perform the simple main effects of shift on the two products separately to confirm the above interpretation of the significant interaction. There is a different error term of each simple main effect but the same formula is used with the As and Bs adjusted accordingly, whichever of the two factors we choose.[13] First, the simple main effect of *shift* (Factor B) on the complex product (A_1).

$$SS_{B\ at\ A_1} = \frac{\sum T^2_{A_1B}}{n} - \frac{T^2_{A_1}}{bn} = \frac{33^2 + 51^2}{6} - \frac{84^2}{2 \times 6} = 615 - 588 = 27$$

$$df_{B\ at\ A_1} = b - 1 = 2 - 1 = 1$$

$$MS_{B\ at\ A_1} = \frac{SS_{B\ at\ A_1}}{df_{B\ at\ A_1}} = \frac{27.00}{1} = 27.00$$

$$SS_{errorB\ at\ A_1} = \sum T^2_{A_1BS} - \frac{\sum T^2_{A_1B}}{n} - \frac{\sum T^2_{A_1S}}{b} + \frac{T^2_{A_1}}{bn}$$

where $\sum T^2_{A_1BS} = 5^2 + 5^2 + 7^2 + \ldots + 10^2 + 8^2 + 9^2 = 626$

$$\frac{\sum T^2_{A_1B}}{n} = 615 \text{ (from above)}$$

$$\frac{\sum T^2_{A_1S}}{b} = \frac{14^2 + 13^2 + 14^2 + 16^2 + 12^2 + 15^2}{2} = 593$$

$$\frac{T^2_{A_1}}{bn} = 588 \text{ (from above)}$$

$$SS_{errorB\ at\ A_1} = 628 - 615 - 593 + 588 = 6$$

$$df_{errorB\ at\ A_1} = (b - 1)(n - 1) = (2 - 1)(6 - 1) = 5$$

$$MS_{errorB\ at\ A_1} = \frac{SS_{errorB\ at\ A_1}}{df_{errorB\ at\ A_1}} = \frac{6}{5} = 1.20$$

$$F_{errorB\ at\ A_1}(1,5) = \frac{MS_{B\ at\ A_1}}{MS_{errorB\ at\ A_1}} = \frac{27.00}{1.20} = 22.50$$

We can conclude that there is a highly significant effect ($p < 0.01$) of *shift* on the errors made on the complex product. Observing the means we see that there is a significant increase in errors during the night shift compared to the day shift.

We can perform the simple main effect of *shift* (Factor B) on the simple product (A_2) in the same way by replacing A_1 in the formulae with A_2. We find that $F_{B\ at\ A_2}(1,5) = 0.47$, so we have not found a significant difference between the number of errors made on the simple product between the two shifts ($p > 0.05$).

A non-significant interaction

In the examples chosen for the three types of two factor ANOVA there has always been a significant interaction. This has been done to illustrate what an interaction entails and also how we can examine the simple main effects to explore the source of the interaction. There will be many cases when the interaction will not be significant, because the effect of the factors is either additive or non-significant. In these cases we can examine the main effects in more detail if we wish by post hoc tests, such as the Tukey or Scheffé as long as we select the appropriate error term for the analysis. In the Tukey for example we would use the mean square error of a significant factor if we wanted to consider the differences in means for the conditions of that factor.

Details on how to calculate the different types of two factor ANOVAs using the SPSS computer statistical package can be found in Chapter 11 of Hinton *et al*. (2004).