# Notes



1 There is a possible problem when accurately calculating the median when the same score is obtained by the positions that surround the median position. In our case 55 is the score obtained by six positions: 48, 49, 50, 51, 52 and 53. The 47th score is 54 and the 54th score is 56. The median, between positions 50 and 51, is halfway between positions 48 and 53. To be completely accurate, a score of 55 could actually be anything ranging between 54.5 and 55.5 (as we would round the decimal points up or down respectively to the nearest whole number, so the score at position 48 in reality could be as low as 54.5 and the score at position 53 as high as 55.5). As our median value is halfway along the six positions scoring 55, it is also halfway along the range 54.5 to 55.5, and is 55. This might seem a convoluted way of demonstrating what appears obvious, but imagine that the positions scoring 55 were 47, 48, 49, 50, 51 and 52. Here the median is not halfway but 4/6ths of the way along the positions scoring 55. Now, again, a score of 55 could be as low as 54.5 and as high as 55.5 so our median, in this second example, is actually 4/6ths of the way between 54.5 and 55.5, and hence turns out to be 55.167.

2 If we have the same score at positions surrounding a quartile then we have to do the same sort of calculation we did with the median (see Note 1 above) to produce an accurate value. Interestingly, there are different ways to calculate quartiles depending on the method used. The technique suggested here is a simple method. However, statistical programs (such as SPSS) may give you a different result such as 48.25 instead of 48.5 for the first quartile

and 59.75 rather than 59.5 for the third quartile due to the way they weight the values when they divide the set of numbers into four quartiles.

3 The formula for the normal distribution is: $f(X) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(X-\mu)^2}{2\sigma^2}}$ where $\mu$ and $\sigma$ are the mean and standard deviation of the population and $\pi$ and $e$ are constants. You may be more familiar with $\pi$ than $e$ but they are both known fixed values. We can plot the function $f(X)$ by putting values of $X$ into the formula and we get the familiar bell-shaped curve of the normal distribution.

4 Cyadmine is a purely fictitious name that I made up for this example. Any similarity of the name with a real chemical is purely coincidental.

5 Sometimes 'the critical $t$ value' is referred to as $t_{\alpha/2,df}$ as we want the $t$ value from the tables that cuts off $\alpha/2$ of each end of the distribution for a confidence of $(1 - \alpha)\%$ and $df$ is the degrees of freedom.

6 The reference for GPOWER is Faul and Erdfelder (1992). It can be found at the following web page: http://www.psycho.uni-duesseldorf.de/aap/projects/gpower/index.html. Interested readers are referred to the paper by Thomas and Krebs (1997) which reviews a number of different software packages that perform power calculations.

7 When we have the same subjects in each condition we only use part of the within conditions variance as an estimate of the error variance as we are able to produce a more sensitive measure of the systematic differences between conditions. This is explained in Chapter 13.

8 The formula here is a sums of squares as $\sum(X - \bar{X})^2$ can be also be expressed by the alternative formula that does not require us to work out the mean first:

$$\sum X^2 - \frac{(\sum X)^2}{N}.$$

9 $F = t^2$ when $df = 1$. In the calculation we introduce some minor 'rounding errors' when we round to two decimal places. This is why the square of the $t$ of 1.82 (producing $t^2 = 3.31$) is not quite the same as the $F$ of 3.30. If we had performed all calculations to more decimal places we would have found them to be identical.

10 One disadvantage of the repeated measures ANOVA is that it also requires a further assumption called 'sphericity'. Essentially this means that the effects of the factor are consistent across the participants and the conditions. When analysing data by a computer program (e.g. SPSS) we can employ a test of sphericity to check our data. See Hinton *et al.* (2004) for a fuller discussion of this topic.

11 Laying out data for analysis by hand or for a report needs to be done in a manner that is clear and easy to read, so it minimises the possibility of misinterpretation. However, when inputting data into computer statistical analysis programs there are a few simple rules. Each row contains the data from a single subject. Independent measures variables are often referred to as 'grouping variables' and the different conditions are organised by rows. For example, if you had the scores for ten men and ten women on a task, you would not input two columns of ten results with one column labelled 'men' and the second labelled 'women'. You would put the scores in one column of 20 rows and distinguish between the men and women by a second column (the grouping variable) with a category label for men (i.e. 1) and a category label for the women (i.e. 2), so the second column would contain 10 ones

and 10 twos. Thus, when you read along a row you would know whether it was a man or a women by the category label in the appropriate column. A repeated measures variable does have each condition as a separate column. For example, if 20 people performed a task on Monday and then again on Tuesday, the results would be input in two columns the first headed 'Monday' and the second headed 'Tuesday'. Each row would have two values in it – the score for a participant on Monday and the same person's score for Tuesday.

12 I have followed Keppel (1973) in the choice of error term here. The reader is referred to this text for a discussion of the choice of error terms for the simple main effect in this case. The text also provides further details of the simple main effects for all the two factor ANOVAs.

13 Keppel (1973) contains further details of the simple main effects (see Note 12 above).

14 From Winer (1971).

15 If we wish to use a correction for ties we can work out SE using the following formula:

$$\sqrt{\frac{N(N+1)}{12} - \frac{\sum t}{12(N-1)}\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$$

16 From Zar (1996) with permission.

17 See Winer (1971) for further details of $\chi_r^2$.

18 The correlation coefficient $r$ uses $z$ scores in its analysis. The $z$ score requires the population mean and standard deviation. Due to this the scores are viewed as a population and the formula for calculating the standard deviation is the population standard deviation formula given in Chapter 2. Also, as we are working out means and standard deviations the scores must be measured on interval scales.

19 I am working to two decimal places here to aid the clarity of the explanation and the simplicity of the workings out. More usually we would employ more decimal places to improve the accuracy of the calculation. Most statistical analysis computer programs work to an accuracy of many decimal places.

20 With the same number of subjects for $X$ and $Y$, $\frac{s_Y}{s_X}$ will be the same regardless of whether we use the formula for a population standard deviation for both $s_X$ and $s_Y$ or a sample standard deviation for both $s_X$ and $s_Y$ as the degrees of freedom will cancel out in the calculation.

# Glossary

**absolute deviation** When we subtract the mean value from a score the result (the deviation from the mean) is positive (+) if the score is larger than the mean and negative (−) if it is smaller. If we ignore the sign of the deviation and always treat it as positive we produce the absolute deviation.

**ANOVA** An acronym for the **AN**alysis **O**f **VA**riance.

**between subjects** Also known as independent measures. In this design, the samples we select for each condition of the independent variable are *independent*, in that the samples come from different subjects.

**causal relationship** A relationship where variation in one variable causes variation in another. Statistical tests can show a relationship between variables but not that it is causal. Other factors might be involved in the relationship. We might find that it snows more when the leaves have fallen from the trees, but we cannot claim the fallen leaves cause the snow. Factors such as the season and temperature are involved.

**component** The term used in the principal components method of factor analysis for a potential underlying factor.

**condition** A researcher chooses levels or categories of the independent variable to observe its effect on the dependent variable. These are referred to as conditions, levels, treatments or groups. For example, 'morning' and 'afternoon' might be chosen as the conditions for the independent variable of time of day.

**confidence interval** In statistics we use samples to estimate population values, such as the mean or the difference in means. The confidence interval provides a range of values within which we predict lies the population value (to a certain level of confidence). The 95 per cent confidence interval of the mean worked out from a sample indicates that the estimated population mean would fall between the upper and lower limits for 95 per cent of the samples chosen.

**confounding factor** An independent variable (in addition to the one under test) that has a systematic influence on the dependent variable.

**control group** A group of subjects or participants matched with the experimental group on all relevant factors except the experimental manipulation. For example, a placebo group (who do not take a particular drug) could be used as a control group for a drug group (who do) to examine the effect of the drug on performance.

**correlation** The degree to which the scores (from a set of subjects) on two variables co-relate. That is, the extent to which a variation in the scores on one variable results in a corresponding variation in the scores on the second variable. Usually the relationship we are looking for is linear. A multiple correlation examines the relationship between a combination of predictor variables with a dependent variable.

**critical value** We reject the null hypothesis after a statistical test if the probability of the calculated value of the statistic (under the null hypothesis) is lower than the significance level (e.g. 0.05). Textbooks print tables of the critical values of the statistic, which are the values of the statistic at a particular significance level (e.g. 0.05). We then compare our calculated value with the critical value from the table. For example, if the calculated value of a $t$ statistic is 4.20 and the critical value is 2.31 (at the 0.05 level of significance) then clearly the probability of the test statistic is less than 0.05 and the result is significant. Computer programs do not give a critical value but print out the actual probability of the calculated value (e.g. 0.023765) and we can examine this to see if it is higher or lower than the significance level for the significance of the result.

**degrees of freedom** When calculating a statistic we use information from the data (such as the mean or total) in the calculation. The degrees of freedom is the number of scores we need to know before we can work out the rest using the information we already have. It is the number of scores that are free to vary in the analysis.

**dependent variable** The variable measured by the researcher and predicted to be influenced by (that is, depend on) the independent variable.

**descriptive statistics** Usually we wish to describe our data before conducting further analysis or comparisons. Descriptive statistics such as the mean and standard deviation enable us to summarise a set of data.

**deviation** The difference of a score from the mean. When we subtract the mean value from a score the result is the deviation.

**discriminant function** A discriminant function is one derived from a set of independent (or predictor) variables that can be used to discriminate between the conditions of a dependent variable.

**distribution** The range of possible scores on a variable and their frequency of occurrence. In statistical terms we refer to a distribution as a 'probability density function'. We use the mathematical formulae for known distributions to work out the probability of finding a score as high as or as low as a particular score.

**effect size** The size of the difference between the means of two populations, usually expressed in standard deviation units.

**eigenvalue** In a factor analysis an eigenvalue provides a measure of the amount of variance that can be explained by a proposed factor. If a factor has an eigenvalue of 1 then it can explain as much variance as one of the original independent variables.

**equality of variance** see homogeneity of variance.

**factor** Another name for 'variable', used commonly in the analysis of variance to refer to an independent variable. In factor analysis we analyse the variation in the data to see if it can be explained by fewer factors (i.e. 'new' variables) than the original number of independent variables.

**general linear model** The underlying mathematical model employed in parametric statistics. When there are only two variables, $X$ and $Y$, the relationship between them is linear when they satisfy the formula $Y = a + bX$ (where $a$ and $b$ are constants). The general linear model is a general form of this equation allowing as many $X$ and $Y$ variables as we wish in our analysis.

**frequency** The number of times a score, a range of scores, or a category is obtained in a set of data is referred to as its frequency.

**frequency data** The data collected is simply the number of scores that fall into each of certain specified categories. See also 'nominal data'.

**histogram** A plot of data on a graph, where vertical bars are used to represent the frequency of the scores, range of scores or categories under study.

**homogeneity of variance** Underlying parametric tests is the assumption that the populations from which the samples are drawn have the same variance.

We can examine the variances of the samples in our data to see whether this assumption is appropriate with our data or not.

**homoscedasticity** The scores in a scatterplot are evenly distributed along and about a regression line. This is the assumption made in linear correlation and regression. (This is the correlation and regression equivalent of the homogeneity of variance assumption.)

**hypothesis** A predicted relationship between variables. For example: 'As sleep loss increases so the number of errors on a specific monitoring task will increase'.

**independent measures** A term used to indicate that there are different subjects in each condition of an independent variable.

**independent variable** A variable chosen by the researcher for testing, predicted to influence the dependent variable.

**inferential statistics** Statistics that allow us to make inferences about the data – for example whether samples are drawn from different populations or whether two variables correlate.

**interaction** When there are two or more factors in an analysis of variance then we can examine the interactions between the factors. An interaction indicates that the effect of one factor is not the same at each condition of another factor. For example, if we find that more cold drinks are sold in summer and more hot drinks sold in winter then we have an interaction of 'drink temperature' and 'time of year'.

**intercept** A linear regression finds the best fit linear relationship between two variables. This is a straight line based on the formula $Y = a + bX$, where $b$ is the slope of the line and $a$ is the intercept, or point where the line crosses the $Y$-axis.

**interval data** Data produced by the use of an interval scale. Parametric tests require interval data.

**interval scale** A scale of measurement where the interval between consecutive numbers is always the same. Most measuring devices, such as timers, thermometers, tape measures, employ interval scales.

**item** When we employ a test with a number of variables (such as questions in a questionnaire) we refer to these variables as items, particularly in reliability analysis where we are interested in the correlation between items in the test.

**linear correlation** The extent to which two variables correlate in a linear manner. That is, how close their scatterplot is to a straight line.

**main effect** The effect of a factor on the dependent variable in an analysis of variance measured separately from other factors in the analysis.

**MANOVA** A **M**ultivariate **AN**alysis **O**f **VA**riance. An analysis of variance technique where there can be more than one dependent variable in the analysis.

**matching subjects** Subjects are matched on relevant criteria across the conditions of the independent variable to control for possible confounding variables. For example, participants may be matched on intelligence or experience to control for these factors.

**mean** A measure of the 'average' score in a set of data. The mean is found by adding up all the scores and dividing by the number of scores.

**mean square** A term used in the analysis of variance to refer to the variance in the data due to a particular source of variation.

**median** If we order a set of data from lowest to highest the median is the point that divides the scores into two, with half the scores below and half above the median.

**mixed design** A mixed design is one that includes both independent measures factors and repeated measures factors. For example, a group of men and a group of women are tested in the morning and the afternoon. In this test 'gender' is an independent measures variable (also known as 'between subjects') and time of day is a repeated measures factor (also known as 'within subjects'), so we have a mixed design.

**mode** The score which has occurred the highest number of times in a set of data.

**multiple comparisons** The results of a statistical test with more than two conditions will often show a significant result but not where that difference lies. We need to undertake a comparison of conditions to see which ones are causing the effect. If we compare them two at a time this is known as pairwise comparisons. Multiple comparisons are either 'planned' and a specific comparison is planned in advance of the main test or 'unplanned' where comparisons are undertaken after discovering the significant finding.

**multiple correlation** The correlation of one variable with a combination of other variables.

**multivariate** Literally this means 'many variables' but is most commonly used to refer to a test with more than one dependent variable (as in the MANOVA).

**nominal data** When we use numbers as labels for categories we refer to the data collected as nominal (names). We cannot perform mathematical operations on these numbers: for example if we label the category 'men' as 1 and 'women' as 2 we cannot add up two men and claim it equals one woman! The data are usually the frequency of responses in each category.

**nonparametric test** Statistical tests that do not use, or make assumptions about, the characteristics (parameters) of populations.

**normal distribution** A bell-shaped frequency distribution that appears to underlie many human variables. The normal distribution can be worked out mathematically using the population mean and standard deviation.

**null hypothesis** A prediction that there is no relationship between the independent and dependent variables.

**one-tailed test** A prediction that two samples come from different populations, specifying the direction of the difference: that is, which of the two populations will have the larger mean value.

**opportunity sample** An available sample, which is neither randomly chosen nor chosen to be representative of the population.

**ordinal data** When we cannot assume that the intervals between consecutive numbers on a scale of measurement are of equal size we have ordinal data and can only use the data to rank order the subjects. Ratings are assumed to be ordinal data. We perform nonparametric tests on ordinal data.

**outlier** An extreme value in a scatterplot – in that it lies outside the main cluster of scores. When calculating a linear correlation or regression an outlier will have a disproportionate influence on the statistical calculations.

**parameter** A characteristic of a population, such as the population mean.

**parametric tests** Statistical tests that use the characteristics (parameters) of populations or estimates of them (when assumptions are also made about the populations under study).

**partial correlation** The correlation of two variables after having removed the effects of a third variable from both.

**participant** A person taking part as a 'subject' in a study. The term 'participant' is preferred to 'subject' as it acknowledges the person's agency: i.e. that they have consented to take part in the study.

**population** A complete set of objects or events. In statistics this usually refers to the complete set of subjects or scores we are interested in, from which we have drawn a sample.

**post hoc tests** When we have more than two conditions of an independent variable a statistical test (such as an ANOVA) may show a significant result but not the source of the effect. We can perform post hoc tests (literally post hoc means 'after this') to see which conditions are showing significant differences. Post hoc tests should correct for the additional risk of Type I errors when performing multiple tests on the same data.

**power of a test** The probability that, when there is a genuine effect to be found, the test will find it (that is, correctly reject a false null hypothesis). As an illustration, one test might be like a stopwatch that gives the same time for two runners in a race but a more powerful test is like a sensitive electronic timer that more accurately shows the times to differ by a fiftieth of a second.

**probability** The chance of a specific event occurring from a set of possible events, expressed as a proportion. For example, if there were 4 women and 6 men in a room the probability of meeting a woman first on entering the room is 4/10 or 0.4 as there are 4 women out of 10 people in the room. A probability of 0 indicates an event will never occur and a probability of 1 that it will always occur. In a room of only 10 men there is a probability of 0 (0/10) of meeting a woman first and a probability of 1 (10/10) of meeting a man.

**quartile** If we order a set of scores from the lowest to the highest the quartiles are the points that divide the scores into four equal groups, with a quarter of the scores in each group. The second quartile is the median.

**random error** There will always be random factors influencing subjects' scores in an experiment. Random error is the influence of these random factors on the data. Statistical tests take account of random factors.

**random sample** A sample of a population where each member of the population has an equal chance of being chosen for the sample.

**range** The difference between the highest and lowest scores in a set of data.

**rank** When a set of data is ordered from lowest to highest the rank of a score is its position in this order.

**rank order** A method of ordering scores, listing them from lowest to highest.

**ratio data** Data measured on a ratio scale.

**ratio scale** An interval scale with an absolute zero. A stopwatch has an absolute zero as 0 indicates 'no time' and so we can make ratio statements: 20 seconds is twice as long as 10 seconds. The Celsius and Fahrenheit scales of temperature are interval but not ratio scales and indeed have 0 at different temperatures.

**regression** The prediction of subjects' scores on one variable by their scores on a second variable. This prediction is usually based on the relationship between the variables being linear and hence the prediction can be made using the formula $Y = a + bX$. The larger the correlation between the variables the more accurate the prediction. A multiple regression predicts the variation in a variable by a number of predictor variables.

**reliability** A reliable test is one that that will produce the same result when repeated (in the same circumstances). We can investigate the reliability of the items in a test (such as the questions in a questionnaire) by examining the relationship between each item and the overall score on the test.

**repeated measures** A term used to indicate that the same subjects are providing data for all the conditions of an independent variable.

**representative sample** A subset of a population that shares the same key characteristics of the population. For example, the sample has the same ratio of men to women as the population.

**residual** A linear regression provides a prediction of the subjects' scores on one variable by their scores on a second. The residual is the difference between a subject's actual score and their predicted score on the first variable. (A linear regression predicts that the data follow a linear model. The residuals indicate the extent to which the data do not fit the model, so are often referred to as 'errors'.)

**scatterplot** A graph of subjects' scores on one variable plotted against their scores on a second variable. The graph shows how the scores are 'scattered'.

**significance level** The risk (probability) of erroneously claiming a relationship between an independent and a dependent variable when there is not one. Statistical tests are undertaken so that this probability is chosen to be small, usually set at 0.05 indicating that this will occur no more than 5 times in 100. This sets the probability of making a Type I error.

**simple main effects** A significant interaction in a two factor analysis of variance indicates that the effect of one variable is different at the various conditions of the other variable. Calculating simple main effects tell us what these different effects are. A simple main effect is the effect of one variable at a single condition of a second variable.

**standard deviation** A measure of the standard ('average') difference (deviation) of a score from the mean in a set of scores. It is the square root of the variance. (There is a different calculation for standard deviation when the set of scores are a population as opposed to a sample.)

**standard error of the estimate** A measure of the 'average' distance (standard error) of a score from the regression line.

**standard error of the mean** The standard deviation of the distribution of sample means. It is a measure of the standard ('average') difference of a sample mean from the mean of all sample means of samples of the same size from the same population.

**standard normal distribution** A normal distribution with a mean of 0 and a standard deviation of 1.

**standard score** The position of a score within a distribution of scores. It provides a measure of how many standard deviation units a specific score falls above or below the mean. It is also referred to as a $z$ score.

**statistic** Specifically, a characteristic of a sample, such as the sample mean. More generally, statistic and statistics are used to describe techniques for summarising and analysing numerical data.

**subject** The term used for the source of data in a sample. If people are the subjects of the study it is viewed as more respectful to refer to them as participants, which acknowledges their role as helpful contributors to the investigation.

**sums of squares** The sum of the squared deviations of scores from their mean value.

**systematic error** Data that has been systematically influenced by another variable in addition to the independent variable under test is said to contain systematic error. The additional variable is said to confound the experiment.

**two-tailed test** A prediction that two samples come from different populations, but not stating which population has the higher mean value.

**Type I error** The error of rejecting the null hypothesis when it is true. The risk of this occurring is set by the significance level.

**Type II error** The error of accepting the null hypothesis when it is false.

**univariate** A term used to refer to a statistical test where there is only one dependent variable. ANOVA is a univariate analysis as there can be more than one independent variable but only one dependent variable.

**variance** A measure of how much a set of scores vary from their mean value. Variance is the square of the standard deviation.

**within subjects** Also known as repeated measures. We select the same subjects for each condition of an independent variable for a within-subjects design.

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# Appendix

## Acknowledgements

I am grateful to the following sources for allowing me to reprint or adapt the following statistical tables:

### A.1 The standard normal distribution tables

*From:* Table IIi of of R.A. Fisher and F. Yates (1974) *Statistical Tables for Biological, Agricultural, and Medical Research*, 6th edition. London: Pearson Education Limited (previously published by Oliver and Boyd Ltd, Edinburgh).

### A.2 Critical values of the *t* distribution

*From:* Table III of of R.A. Fisher and F. Yates (1974) *Statistical Tables for Biological, Agricultural, and Medical Research*, 6th edition. London: Pearson Education Limited (previously published by Oliver and Boyd Ltd, Edinburgh).

### A.3 Critical values of the *F* distribution

*From:* M. Merrington and C.M. Thompson (1943) Tables of percentage points of the inverted Beta ($F$) distribution, *Biometrika*, 33 (1943–6), 73–88, by permission of the *Biometrika* Trustees and Oxford University Press.

## A.4 Critical values of the Studentized range statistic, *q*

*From:* Tables 2 and 3 in J. Pachares (1959) Table of the upper 10% points of the studentized range, Biometrika, 46, 461–6, by permission of the *Biometrika* Trustees and Oxford University Press.

## A.5 Critical values of the Mann–Whitney *U* statistic

*From:* Table K of S. Siegel (1956) *Nonparametric Statistics for the Behavioral Sciences*, New York: McGraw-Hill. Reproduced with the permission of The McGraw-Hill Companies.

## A.6 Critical values of the Wilcoxon *T* statistic

*From:* Table J of R.P. Runyan and A. Haber (1991) *Fundamentals of Behavioral Statistics*, 7th edition. New York: McGraw-Hill. Reproduced with the permission of The McGraw-Hill Companies.

## A.7 Critical values of the chi-square ($\chi^2$) distribution

*From:* Table IV of R.A. Fisher and F. Yates (1974) *Statistical Tables for Biological, Agricultural, and Medical Research*, 6th edition. London: Pearson Education Limited (previously published by Oliver and Boyd Ltd, Edinburgh).

## A.8 Table of probabilities for $\chi_r^2$ when *k* and *n* are small

*From:* M. Friedman (1937) The use of ranks to avoid the assumption of normality implicit in the analysis of variance, *Journal of the American Statistical Association*, 32, 200, 675–701. Reprinted with permission. Copyright (1937) by the American Statistical Association. All rights reserved.

## A.9 Critical values of the Pearson *r* correlation coefficient

*From:* Table VII of R.A. Fisher and F. Yates (1974) *Statistical Tables for Biological, Agricultural, and Medical Research*, 6th edition. London: Pearson Education Limited (previously published by Oliver and Boyd Ltd, Edinburgh).

## A.10 Critical values of the Spearman $r_S$ ranked correlation coefficient

*From:* E.G. Olds (1949) The 5% significance levels for sums of squares of rank differences and a correction, *Annals of Mathematical Statistics*, volume 9, pages 133–48. With the permission of The Institute of Mathematical Statistics.

Table on page 240 – extract from the larger table of the Q Statistic in Zar, J., BIO-STATISTICAL ANALYSIS, 3/e, copyright © 1996. Adapted by permission of Pearson Education, Inc., Upper Saddle River, New Jersey.

I am grateful to Pearson Education Limited, on behalf of the Literary Executor of the late Sir Ronald Fisher, FRS, and Dr Frank Yates, FRS for permission to reprint Tables IIi, III, IV, V and VII from their book *Statistical Tables for Biological, Agricultural, and Medical Research*, 6th edition, 1974.

## A.1 The standard normal distribution tables

| *z* | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.4960 | 0.4920 | 0.4880 | 0.4840 | 0.4801 | 0.4761 | 0.4721 | 0.4681 | 0.4641 |
| 0.1 | 0.4602 | 0.4562 | 0.4522 | 0.4483 | 0.4443 | 0.4404 | 0.4364 | 0.4325 | 0.4286 | 0.4247 |
| 0.2 | 0.4207 | 0.4168 | 0.4129 | 0.4090 | 0.4052 | 0.4013 | 0.3974 | 0.3936 | 0.3897 | 0.3859 |
| 0.3 | 0.3821 | 0.3783 | 0.3745 | 0.3707 | 0.3669 | 0.3632 | 0.3594 | 0.3557 | 0.3520 | 0.3483 |
| 0.4 | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 |
| 0.5 | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 |
| 0.6 | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 |
| 0.7 | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2296 | 0.2266 | 0.2236 | 0.2206 | 0.2177 | 0.2148 |
| 0.8 | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 |
| 0.9 | 0.1841 | 0.1814 | 0.1788 | 0.1762 | 0.1736 | 0.1711 | 0.1685 | 0.1660 | 0.1635 | 0.1611 |
| 1.0 | 0.1587 | 0.1562 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 |
| 1.1 | 0.1357 | 0.1335 | 0.1314 | 0.1294 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 |
| 1.2 | 0.1151 | 0.1131 | 0.1112 | 0.1093 | 0.1075 | 0.1056 | 0.1038 | 0.1020 | 0.1003 | 0.0985 |
| 1.3 | 0.0968 | 0.0951 | 0.0934 | 0.0918 | 0.0901 | 0.0885 | 0.0869 | 0.0853 | 0.0838 | 0.0823 |
| 1.4 | 0.0808 | 0.0793 | 0.0778 | 0.0764 | 0.0749 | 0.0735 | 0.0721 | 0.0708 | 0.0694 | 0.0681 |
| 1.5 | 0.0668 | 0.0655 | 0.0643 | 0.0630 | 0.0618 | 0.0606 | 0.0594 | 0.0582 | 0.0571 | 0.0559 |
| 1.6 | 0.0548 | 0.0537 | 0.0526 | 0.0516 | 0.0505 | 0.0495 | 0.0485 | 0.0475 | 0.0465 | 0.0455 |
| 1.7 | 0.0446 | 0.0436 | 0.0427 | 0.0418 | 0.0409 | 0.0401 | 0.0392 | 0.0384 | 0.0375 | 0.0367 |
| 1.8 | 0.0359 | 0.0351 | 0.0344 | 0.0336 | 0.0329 | 0.0322 | 0.0314 | 0.0307 | 0.0301 | 0.0294 |
| 1.9 | 0.0287 | 0.0281 | 0.0274 | 0.0268 | 0.0262 | 0.0256 | 0.0250 | 0.0244 | 0.0239 | 0.0233 |
| 2.0 | 0.0228 | 0.0222 | 0.0217 | 0.0212 | 0.0207 | 0.0202 | 0.0197 | 0.0192 | 0.0188 | 0.0183 |
| 2.1 | 0.0179 | 0.0174 | 0.0170 | 0.0166 | 0.0162 | 0.0158 | 0.0154 | 0.0150 | 0.0146 | 0.0143 |
| 2.2 | 0.0139 | 0.0136 | 0.0132 | 0.0129 | 0.0125 | 0.0122 | 0.0119 | 0.0116 | 0.0113 | 0.0110 |
| 2.3 | 0.0107 | 0.0104 | 0.0102 | 0.0099 | 0.0096 | 0.0094 | 0.0091 | 0.0089 | 0.0087 | 0.0084 |
| 2.4 | 0.0082 | 0.0080 | 0.0078 | 0.0075 | 0.0073 | 0.0071 | 0.0069 | 0.0068 | 0.0066 | 0.0064 |
| 2.5 | 0.0062 | 0.0060 | 0.0059 | 0.0057 | 0.0055 | 0.0054 | 0.0052 | 0.0051 | 0.0049 | 0.0048 |
| 2.6 | 0.0047 | 0.0045 | 0.0044 | 0.0043 | 0.0041 | 0.0040 | 0.0039 | 0.0038 | 0.0037 | 0.0036 |
| 2.7 | 0.0035 | 0.0034 | 0.0033 | 0.0032 | 0.0031 | 0.0030 | 0.0029 | 0.0028 | 0.0027 | 0.0026 |
| 2.8 | 0.0026 | 0.0025 | 0.0024 | 0.0023 | 0.0023 | 0.0022 | 0.0021 | 0.0021 | 0.0020 | 0.0019 |
| 2.9 | 0.0019 | 0.0018 | 0.0018 | 0.0017 | 0.0016 | 0.0016 | 0.0015 | 0.0015 | 0.0014 | 0.0014 |
| 3.0 | 0.0013 | 0.0013 | 0.0013 | 0.0012 | 0.0012 | 0.0011 | 0.0011 | 0.0011 | 0.0010 | 0.0010 |
| 3.1 | 0.0010 | 0.0009 | 0.0009 | 0.0009 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0007 | 0.0007 |
| 3.2 | 0.0007 | 0.0007 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0005 | 0.0005 | 0.0005 |
| 3.3 | 0.0005 | 0.0005 | 0.0005 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0003 |
| 3.4 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0002 |
| 3.5 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 |
| 3.6 | 0.0002 | 0.0002 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 |
| 3.7 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 |
| 3.8 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 |
| 3.9 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

To look up the probability of a $z$ score use the first column, headed '$z$', to find the first decimal place of the $z$ score. The other columns represent the second decimal place. For example, if we wish to look up the probability of a $z$ score of 1.8641 we first round it to two decimal places: 1.86. We go down the $z$ column until we find 1.8. We move along the 1.8 row until we are in the column headed '6' (as the second decimal place is 6) and we find the probability of 0.0314. The probability of finding a score as high or higher than 1.86 is 0.0314.

Notice that there are no $z$ scores in the table greater than 3.99 even though we might calculate them in our analyses. Observe also that the probability values of 3.9 or greater are given as 0.0000. The probability of a $z$ score of 3.9 or larger is not actually zero but is so small that we cannot represent it in a table with only four decimal places.

## A.2 Critical values of the *t* distribution

| | 0.05 Level of significance | | 0.01 Level of significance | |
|---|---|---|---|---|
| *df* | One-tailed test | Two-tailed test | One-tailed test | Two-tailed test |
| 1 | 6.314 | 12.706 | 31.821 | 63.657 |
| 2 | 2.920 | 4.303 | 6.965 | 9.925 |
| 3 | 2.353 | 3.182 | 4.541 | 5.841 |
| 4 | 2.132 | 2.776 | 3.747 | 4.604 |
| 5 | 2.015 | 2.571 | 3.365 | 4.032 |
| 6 | 1.943 | 2.447 | 3.143 | 3.707 |
| 7 | 1.895 | 2.365 | 2.998 | 3.499 |
| 8 | 1.860 | 2.306 | 2.896 | 3.355 |
| 9 | 1.833 | 2.262 | 2.821 | 3.250 |
| 10 | 1.812 | 2.228 | 2.764 | 3.169 |
| 11 | 1.796 | 2.201 | 2.718 | 3.106 |
| 12 | 1.782 | 2.179 | 2.681 | 3.055 |
| 13 | 1.771 | 2.160 | 2.650 | 3.012 |
| 14 | 1.761 | 2.145 | 2.624 | 2.977 |
| 15 | 1.753 | 2.131 | 2.602 | 2.947 |
| 16 | 1.746 | 2.120 | 2.583 | 2.921 |
| 17 | 1.740 | 2.110 | 2.567 | 2.898 |
| 18 | 1.734 | 2.101 | 2.552 | 2.878 |
| 19 | 1.729 | 2.093 | 2.539 | 2.861 |
| 20 | 1.725 | 2.086 | 2.528 | 2.845 |
| 21 | 1.721 | 2.080 | 2.518 | 2.831 |
| 22 | 1.717 | 2.074 | 2.508 | 2.819 |
| 23 | 1.714 | 2.069 | 2.500 | 2.807 |
| 24 | 1.711 | 2.064 | 2.492 | 2.797 |
| 25 | 1.708 | 2.060 | 2.485 | 2.787 |
| 26 | 1.706 | 2.056 | 2.479 | 2.779 |
| 27 | 1.703 | 2.052 | 2.473 | 2.771 |
| 28 | 1.701 | 2.048 | 2.467 | 2.763 |
| 29 | 1.699 | 2.045 | 2.462 | 2.756 |
| 30 | 1.697 | 2.042 | 2.457 | 2.750 |
| 40 | 1.684 | 2.021 | 2.423 | 2.704 |
| 60 | 1.671 | 2.000 | 2.390 | 2.660 |
| 120 | 1.656 | 1.980 | 2.358 | 2.617 |
| ∞ | 1.645 | 1.960 | 2.326 | 2.576 |

The values indicate the size of $t$ that cuts off either 0.05 or 0.01 of the $t$ distribution at the different degrees of freedom. For example, for a one-tailed test with $df = 20$, a value of $t = 1.725$ cuts off 0.05 of the distribution. Thus, for a calculated value of $t$ to be significant it must be greater than or equal to the appropriate table value. That is to say, if the calculated value of $t$ is greater than the table value then the probability that such a result occurred by chance is less than 0.05.

When you have calculated a degrees of freedom that is not in the table (i.e. $df = 32$) use the next lowest value in given the table (i.e. $df = 30$ for a calculated $df = 32$). If you really want to you can use linear interpolation if you wish to be a little more accurate. When the degrees of freedom is very large (into the hundreds) use the infinity (∞) value.

## A.3 Critical values of the *F* distribution

0.05 Level of significance

| | *df*1 | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *df*2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 20 | ∞ |
| 1 | 161.45 | 199.50 | 215.71 | 224.58 | 230.16 | 233.99 | 236.77 | 238.88 | 240.54 | 241.88 | 248.01 | 254.32 |
| 2 | 18.51 | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 | 19.38 | 19.40 | 19.45 | 19.50 |
| 3 | 10.13 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 | 8.79 | 8.66 | 8.53 |
| 4 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 | 5.96 | 5.80 | 5.63 |
| 5 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.74 | 4.56 | 4.36 |
| 6 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 | 4.06 | 3.87 | 3.67 |
| 7 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 | 3.64 | 3.44 | 3.23 |
| 8 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 | 3.35 | 3.15 | 2.93 |
| 9 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 | 3.14 | 3.07 | 3.01 |
| 10 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 | 2.98 | 2.77 | 2.54 |
| 11 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 | 2.85 | 2.65 | 2.40 |
| 12 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 | 2.75 | 2.54 | 2.30 |
| 13 | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 | 2.67 | 2.46 | 2.21 |
| 14 | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 | 2.60 | 2.39 | 2.13 |
| 15 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 | 2.54 | 2.33 | 2.07 |
| 16 | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 | 2.49 | 2.28 | 2.01 |
| 17 | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 | 2.45 | 2.23 | 1.96 |
| 18 | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 | 2.41 | 2.19 | 1.92 |
| 19 | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 | 2.38 | 2.16 | 1.88 |
| 20 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 | 2.35 | 2.12 | 1.84 |
| 21 | 4.32 | 3.47 | 3.07 | 2.84 | 2.68 | 2.57 | 2.49 | 2.42 | 2.37 | 2.32 | 2.10 | 1.81 |
| 22 | 4.30 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 | 2.30 | 2.07 | 1.78 |
| 23 | 4.28 | 3.42 | 3.03 | 2.80 | 2.64 | 2.53 | 2.44 | 2.37 | 2.32 | 2.27 | 2.05 | 1.76 |
| 24 | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 | 2.25 | 2.03 | 1.73 |
| 25 | 4.24 | 3.39 | 2.99 | 2.76 | 2.60 | 2.49 | 2.40 | 2.34 | 2.28 | 2.24 | 2.01 | 1.71 |
| 26 | 4.23 | 3.37 | 2.98 | 2.74 | 2.59 | 2.47 | 2.39 | 2.32 | 2.27 | 2.22 | 1.99 | 1.69 |
| 27 | 4.21 | 3.35 | 2.96 | 2.73 | 2.57 | 2.46 | 2.37 | 2.31 | 2.25 | 2.20 | 1.97 | 1.67 |
| 28 | 4.20 | 3.34 | 2.95 | 2.71 | 2.56 | 2.45 | 2.36 | 2.29 | 2.24 | 2.19 | 1.96 | 1.65 |
| 29 | 4.18 | 3.33 | 2.93 | 2.70 | 2.55 | 2.43 | 2.35 | 2.28 | 2.22 | 2.18 | 1.94 | 1.64 |
| 30 | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 | 2.16 | 1.93 | 1.62 |
| 40 | 4.08 | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.25 | 2.18 | 2.12 | 2.08 | 1.84 | 1.51 |
| 60 | 4.00 | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 2.04 | 1.99 | 1.75 | 1.39 |
| 120 | 3.92 | 3.07 | 2.68 | 2.45 | 2.29 | 2.18 | 2.09 | 2.02 | 1.96 | 1.91 | 1.66 | 1.25 |
| ∞ | 3.84 | 3.00 | 2.60 | 2.37 | 2.21 | 2.10 | 2.01 | 1.94 | 1.88 | 1.83 | 1.57 | 1.00 |

The calculated value of *F* must be larger than or equal to the table value for significance.

## A.3 Critical values of the *F* distribution (continued)

0.01 Level of significance

| *df*2 \ *df*1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 20 | ∞ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 4052.2 | 4999.5 | 5403.3 | 5624.6 | 5763.7 | 5859.0 | 5928.3 | 5981.6 | 6022.5 | 6055.8 | 6208.7 | 6366.0 |
| 2 | 98.50 | 99.00 | 99.17 | 99.25 | 99.30 | 99.33 | 99.36 | 99.37 | 99.39 | 99.40 | 99.45 | 99.50 |
| 3 | 34.12 | 30.82 | 29.46 | 28.71 | 28.24 | 27.91 | 27.67 | 27.49 | 27.34 | 27.23 | 26.69 | 26.12 |
| 4 | 21.20 | 18.00 | 16.69 | 15.98 | 15.52 | 15.21 | 14.98 | 14.80 | 14.66 | 14.55 | 14.02 | 13.46 |
| 5 | 16.26 | 13.27 | 12.06 | 11.39 | 10.97 | 10.67 | 10.46 | 10.29 | 10.16 | 10.05 | 9.55 | 9.02 |
| 6 | 13.74 | 10.92 | 9.78 | 9.15 | 8.75 | 8.47 | 8.26 | 8.10 | 7.98 | 7.87 | 7.40 | 6.88 |
| 7 | 12.25 | 9.55 | 8.45 | 7.85 | 7.46 | 7.19 | 6.99 | 6.84 | 6.72 | 6.62 | 6.16 | 5.67 |
| 8 | 11.26 | 8.65 | 7.59 | 7.01 | 6.63 | 6.37 | 6.18 | 6.03 | 5.91 | 5.81 | 5.36 | 4.86 |
| 9 | 10.56 | 8.02 | 6.99 | 6.42 | 6.06 | 5.80 | 5.61 | 5.47 | 5.35 | 5.26 | 4.81 | 4.31 |
| 10 | 10.04 | 7.56 | 6.55 | 5.99 | 5.64 | 5.39 | 5.20 | 5.06 | 4.94 | 4.85 | 4.41 | 3.91 |
| 11 | 9.65 | 7.21 | 6.22 | 5.67 | 5.32 | 5.07 | 4.89 | 4.74 | 4.63 | 4.54 | 4.10 | 3.60 |
| 12 | 9.33 | 6.93 | 5.95 | 5.41 | 5.06 | 4.82 | 4.64 | 4.50 | 4.39 | 4.30 | 3.86 | 3.36 |
| 13 | 9.07 | 6.70 | 5.74 | 5.21 | 4.86 | 4.62 | 4.44 | 4.30 | 4.19 | 4.10 | 3.66 | 3.17 |
| 14 | 8.86 | 6.51 | 5.56 | 5.04 | 4.70 | 4.46 | 4.28 | 4.14 | 4.03 | 3.94 | 3.51 | 3.00 |
| 15 | 8.68 | 6.36 | 5.42 | 4.89 | 4.56 | 4.32 | 4.14 | 4.00 | 3.89 | 3.80 | 3.37 | 2.87 |
| 16 | 8.53 | 6.23 | 5.29 | 4.77 | 4.44 | 4.20 | 4.03 | 3.89 | 3.78 | 3.69 | 3.26 | 2.75 |
| 17 | 8.40 | 6.11 | 5.18 | 4.67 | 4.34 | 4.10 | 3.93 | 3.79 | 3.68 | 3.59 | 3.16 | 2.65 |
| 18 | 8.29 | 6.01 | 5.09 | 4.58 | 4.25 | 4.01 | 3.84 | 3.71 | 3.60 | 3.51 | 3.08 | 2.57 |
| 19 | 8.18 | 5.93 | 5.01 | 4.50 | 4.17 | 3.94 | 3.77 | 3.63 | 3.52 | 3.43 | 3.00 | 2.49 |
| 20 | 8.10 | 5.85 | 4.94 | 4.43 | 4.10 | 3.87 | 3.70 | 3.56 | 3.46 | 3.37 | 2.94 | 2.42 |
| 21 | 8.02 | 5.78 | 4.87 | 4.37 | 4.04 | 3.81 | 3.64 | 3.51 | 3.40 | 3.31 | 2.88 | 2.36 |
| 22 | 7.95 | 5.72 | 4.82 | 4.31 | 3.99 | 3.76 | 3.59 | 3.45 | 3.35 | 3.26 | 2.83 | 2.31 |
| 23 | 7.88 | 5.66 | 4.76 | 4.26 | 3.94 | 3.71 | 3.54 | 3.41 | 3.30 | 3.21 | 2.78 | 2.26 |
| 24 | 7.82 | 5.61 | 4.72 | 4.22 | 3.90 | 3.67 | 3.50 | 3.36 | 3.26 | 3.17 | 2.74 | 2.21 |
| 25 | 7.77 | 5.57 | 4.68 | 4.18 | 3.86 | 3.63 | 3.46 | 3.32 | 3.22 | 3.13 | 2.70 | 2.17 |
| 26 | 7.72 | 5.53 | 4.64 | 4.14 | 3.82 | 3.59 | 3.42 | 3.29 | 3.18 | 3.09 | 2.66 | 2.13 |
| 27 | 7.68 | 5.49 | 4.60 | 4.11 | 3.78 | 3.56 | 3.39 | 3.26 | 3.15 | 3.06 | 2.63 | 2.10 |
| 28 | 7.64 | 5.45 | 4.57 | 4.07 | 3.75 | 3.53 | 3.36 | 3.23 | 3.12 | 3.03 | 2.60 | 2.06 |
| 29 | 7.60 | 5.42 | 4.54 | 4.04 | 3.73 | 3.50 | 3.33 | 3.20 | 3.09 | 3.00 | 2.57 | 2.03 |
| 30 | 7.58 | 5.39 | 4.51 | 4.02 | 3.70 | 3.47 | 3.30 | 3.17 | 3.07 | 2.98 | 2.55 | 2.01 |
| 40 | 7.31 | 5.18 | 4.31 | 3.83 | 3.51 | 3.29 | 3.12 | 2.99 | 2.89 | 2.80 | 2.37 | 1.80 |
| 60 | 7.08 | 4.98 | 4.13 | 3.65 | 3.34 | 3.12 | 2.95 | 2.82 | 2.72 | 2.63 | 2.20 | 1.60 |
| 120 | 6.85 | 4.79 | 3.95 | 3.48 | 3.17 | 2.96 | 2.79 | 2.66 | 2.56 | 2.47 | 2.03 | 1.38 |
| ∞ | 6.63 | 4.61 | 3.78 | 3.32 | 3.02 | 2.80 | 2.64 | 2.51 | 2.41 | 2.32 | 1.88 | 1.00 |

The calculated value of *F* must be larger than or equal to the table value for significance.

## A.4 Critical values of the Studentized range statistic, *q*

0.05 Level of significance

| Error | Number of conditions (*k*) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *df* | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 5 | 3.64 | 4.60 | 5.22 | 5.67 | 6.03 | 6.33 | 6.58 | 6.80 | 6.99 | 7.17 | 7.32 |
| 6 | 3.46 | 4.34 | 4.90 | 5.30 | 5.63 | 5.90 | 6.12 | 6.32 | 6.49 | 6.65 | 6.79 |
| 7 | 3.34 | 4.16 | 4.68 | 5.06 | 5.36 | 5.61 | 5.82 | 6.00 | 6.16 | 6.30 | 6.43 |
| 8 | 3.26 | 4.04 | 4.53 | 4.89 | 5.17 | 5.40 | 5.60 | 5.77 | 5.92 | 6.05 | 6.18 |
| 9 | 3.20 | 3.95 | 4.41 | 4.76 | 5.02 | 5.24 | 5.43 | 5.59 | 5.74 | 5.87 | 5.98 |
| 10 | 3.15 | 3.88 | 4.33 | 4.65 | 4.91 | 5.12 | 5.30 | 5.46 | 5.60 | 5.72 | 5.83 |
| 11 | 3.11 | 3.82 | 4.26 | 4.57 | 4.82 | 5.03 | 5.20 | 5.35 | 5.49 | 5.61 | 5.71 |
| 12 | 3.08 | 3.77 | 4.20 | 4.51 | 4.75 | 4.95 | 5.12 | 5.27 | 5.36 | 5.51 | 5.61 |
| 13 | 3.06 | 3.73 | 4.15 | 4.45 | 4.69 | 4.88 | 5.05 | 5.19 | 5.32 | 5.43 | 5.53 |
| 14 | 3.03 | 3.70 | 4.11 | 4.41 | 4.64 | 4.83 | 4.99 | 5.13 | 5.25 | 5.36 | 5.46 |
| 15 | 3.01 | 3.67 | 4.08 | 4.37 | 4.59 | 4.78 | 4.94 | 5.08 | 5.20 | 5.31 | 5.40 |
| 16 | 3.00 | 3.65 | 4.05 | 4.33 | 4.56 | 4.74 | 4.90 | 5.03 | 5.15 | 5.26 | 5.35 |
| 17 | 2.98 | 3.63 | 4.02 | 4.30 | 4.52 | 4.70 | 4.86 | 4.99 | 5.11 | 5.21 | 5.31 |
| 18 | 2.97 | 3.61 | 4.00 | 4.28 | 4.49 | 4.67 | 4.82 | 4.96 | 5.07 | 5.17 | 5.27 |
| 19 | 2.96 | 3.59 | 3.98 | 4.25 | 4.47 | 4.65 | 4.79 | 4.92 | 5.04 | 5.14 | 5.23 |
| 20 | 2.95 | 3.58 | 3.96 | 4.23 | 4.45 | 4.62 | 4.77 | 4.90 | 5.01 | 5.11 | 5.20 |
| 24 | 2.92 | 3.53 | 3.90 | 4.17 | 4.37 | 4.54 | 4.68 | 4.81 | 4.92 | 5.01 | 5.10 |
| 30 | 2.89 | 3.49 | 3.85 | 4.10 | 4.30 | 4.46 | 4.60 | 4.72 | 4.82 | 4.92 | 5.00 |
| 40 | 2.86 | 3.44 | 3.79 | 4.04 | 4.23 | 4.39 | 4.52 | 4.63 | 4.73 | 4.82 | 4.90 |
| 60 | 2.83 | 3.40 | 3.74 | 3.98 | 4.16 | 4.31 | 4.44 | 4.55 | 4.65 | 4.73 | 4.81 |
| 120 | 2.80 | 3.36 | 3.68 | 3.92 | 4.10 | 4.24 | 4.36 | 4.47 | 4.56 | 4.64 | 4.71 |
| ∞ | 2.77 | 3.31 | 3.63 | 3.86 | 4.03 | 4.17 | 4.29 | 4.39 | 4.47 | 4.55 | 4.62 |

## A.4 Critical values of the Studentized range statistic, *q* (continued)

0.01 Level of significance

| Error | Number of conditions (*k*) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *df* | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 5 | 5.70 | 6.98 | 7.80 | 8.42 | 8.91 | 9.32 | 9.67 | 9.97 | 10.24 | 10.48 | 10.70 |
| 6 | 5.24 | 6.33 | 7.03 | 7.56 | 7.97 | 8.32 | 8.61 | 8.87 | 9.10 | 9.30 | 9.48 |
| 7 | 4.95 | 5.92 | 6.54 | 7.01 | 7.37 | 7.68 | 7.94 | 8.17 | 8.37 | 8.55 | 8.71 |
| 8 | 4.75 | 5.64 | 6.20 | 6.62 | 6.96 | 7.24 | 7.47 | 7.68 | 7.86 | 8.03 | 8.18 |
| 9 | 4.60 | 5.43 | 5.96 | 6.35 | 6.66 | 6.91 | 7.13 | 7.33 | 7.49 | 7.65 | 7.78 |
| 10 | 4.48 | 5.27 | 5.77 | 6.14 | 6.43 | 6.67 | 6.87 | 7.05 | 7.21 | 7.36 | 7.49 |
| 11 | 4.39 | 5.15 | 5.62 | 5.97 | 6.25 | 6.48 | 6.67 | 6.84 | 6.99 | 7.13 | 7.25 |
| 12 | 4.32 | 5.05 | 5.50 | 5.84 | 6.10 | 6.32 | 6.51 | 6.67 | 6.81 | 6.94 | 7.06 |
| 13 | 4.26 | 4.96 | 5.40 | 5.73 | 5.98 | 6.19 | 6.37 | 6.53 | 6.67 | 6.79 | 6.90 |
| 14 | 4.21 | 4.89 | 5.32 | 5.63 | 5.88 | 6.08 | 6.26 | 6.41 | 6.54 | 6.66 | 6.77 |
| 15 | 4.17 | 4.84 | 5.25 | 5.56 | 5.80 | 5.99 | 6.16 | 6.31 | 6.44 | 6.55 | 6.66 |
| 16 | 4.13 | 4.79 | 5.19 | 5.49 | 5.72 | 5.92 | 6.08 | 6.22 | 6.35 | 6.46 | 6.56 |
| 17 | 4.10 | 4.74 | 5.14 | 5.43 | 5.66 | 5.85 | 6.01 | 6.15 | 6.27 | 6.38 | 6.48 |
| 18 | 4.07 | 4.70 | 5.09 | 5.38 | 5.60 | 5.79 | 5.94 | 6.08 | 6.20 | 6.31 | 6.41 |
| 19 | 4.05 | 4.67 | 5.05 | 5.33 | 5.55 | 5.73 | 5.89 | 6.02 | 6.14 | 6.25 | 6.34 |
| 20 | 4.02 | 4.64 | 5.02 | 5.29 | 5.51 | 5.69 | 5.84 | 5.97 | 6.09 | 6.19 | 6.28 |
| 24 | 3.96 | 4.55 | 4.91 | 5.17 | 5.37 | 5.54 | 5.69 | 5.81 | 5.92 | 6.02 | 6.11 |
| 30 | 3.89 | 4.45 | 4.80 | 5.05 | 5.24 | 5.40 | 5.54 | 5.65 | 5.76 | 5.85 | 5.93 |
| 40 | 3.82 | 4.37 | 4.70 | 4.93 | 5.11 | 5.26 | 5.39 | 5.50 | 5.60 | 5.69 | 5.76 |
| 60 | 3.76 | 4.28 | 4.59 | 4.82 | 4.99 | 5.13 | 5.25 | 5.36 | 5.45 | 5.53 | 5.60 |
| 120 | 3.70 | 4.20 | 4.50 | 4.71 | 4.87 | 5.01 | 5.12 | 5.21 | 5.30 | 5.37 | 5.44 |
| ∞ | 3.64 | 4.12 | 4.40 | 4.60 | 4.76 | 4.88 | 4.99 | 5.08 | 5.16 | 5.23 | 5.29 |

## A.5 Critical values of the Mann–Whitney *U* statistic (continued)

The calculated value of *U* must be smaller than or equal to the table value for significance. Dashes in the table indicate that no value is possible for significance.

0.01 Level of significance: One-tailed test

| $n_2$ \ $n_1$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – |
| 2 | – | – | – | – | – | – | – | – | – | – | – | – | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 3 | – | – | – | – | – | – | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 5 |
| 4 | – | – | – | – | 0 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 | 7 | 7 | 8 | 9 | 9 | 10 |
| 5 | – | – | – | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 6 | – | – | – | 1 | 2 | 3 | 4 | 6 | 7 | 8 | 9 | 11 | 12 | 13 | 15 | 16 | 18 | 19 | 20 | 22 |
| 7 | – | – | 0 | 1 | 3 | 4 | 6 | 7 | 9 | 11 | 12 | 14 | 16 | 17 | 19 | 21 | 23 | 24 | 26 | 28 |
| 8 | – | – | 0 | 2 | 4 | 6 | 7 | 9 | 11 | 13 | 15 | 17 | 20 | 22 | 24 | 26 | 28 | 30 | 32 | 34 |
| 9 | – | – | 1 | 3 | 5 | 7 | 9 | 11 | 14 | 16 | 18 | 21 | 23 | 26 | 28 | 31 | 33 | 36 | 38 | 40 |
| 10 | – | – | 1 | 3 | 6 | 8 | 11 | 13 | 16 | 19 | 22 | 24 | 27 | 30 | 33 | 36 | 38 | 41 | 44 | 47 |
| 11 | – | – | 1 | 4 | 7 | 9 | 12 | 15 | 18 | 22 | 25 | 28 | 31 | 34 | 37 | 41 | 44 | 47 | 50 | 53 |
| 12 | – | – | 2 | 5 | 8 | 11 | 14 | 17 | 21 | 24 | 28 | 31 | 35 | 38 | 42 | 46 | 49 | 53 | 56 | 60 |
| 13 | – | 0 | 2 | 5 | 9 | 12 | 16 | 20 | 23 | 27 | 31 | 35 | 39 | 43 | 47 | 51 | 55 | 59 | 63 | 67 |
| 14 | – | 0 | 2 | 6 | 10 | 13 | 17 | 22 | 26 | 30 | 34 | 38 | 43 | 47 | 51 | 56 | 60 | 65 | 69 | 73 |
| 15 | – | 0 | 3 | 7 | 11 | 15 | 19 | 24 | 28 | 33 | 37 | 42 | 47 | 51 | 56 | 61 | 66 | 70 | 75 | 80 |
| 16 | – | 0 | 3 | 7 | 12 | 16 | 21 | 26 | 31 | 36 | 41 | 46 | 51 | 56 | 61 | 66 | 71 | 76 | 82 | 87 |
| 17 | – | 0 | 4 | 8 | 13 | 18 | 23 | 28 | 33 | 38 | 44 | 49 | 55 | 60 | 66 | 71 | 77 | 82 | 88 | 93 |
| 18 | – | 0 | 4 | 9 | 14 | 19 | 24 | 30 | 36 | 41 | 47 | 53 | 59 | 65 | 70 | 76 | 82 | 88 | 94 | 100 |
| 19 | – | 1 | 4 | 9 | 15 | 20 | 26 | 32 | 38 | 44 | 50 | 56 | 63 | 69 | 75 | 82 | 88 | 94 | 101 | 107 |
| 20 | – | 1 | 5 | 10 | 16 | 22 | 28 | 34 | 40 | 47 | 53 | 60 | 67 | 73 | 80 | 87 | 93 | 100 | 107 | 114 |

0.01 Level of significance: Two-tailed test

| $n_2$ \ $n_1$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – |
| 2 | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | 0 | 0 |
| 3 | – | – | – | – | – | – | – | – | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 | 3 |
| 4 | – | – | – | – | – | 0 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 5 | 5 | 6 | 6 | 7 | 8 |
| 5 | – | – | – | – | 0 | 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 6 | – | – | – | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 9 | 10 | 11 | 12 | 13 | 15 | 16 | 17 | 18 |
| 7 | – | – | – | 0 | 1 | 3 | 4 | 6 | 7 | 9 | 10 | 12 | 13 | 15 | 16 | 18 | 19 | 21 | 22 | 24 |
| 8 | – | – | – | 1 | 2 | 4 | 6 | 7 | 9 | 11 | 13 | 15 | 17 | 18 | 20 | 22 | 24 | 26 | 28 | 30 |
| 9 | – | – | 0 | 1 | 3 | 5 | 7 | 9 | 11 | 13 | 16 | 18 | 20 | 22 | 24 | 27 | 29 | 31 | 33 | 36 |
| 10 | – | – | 0 | 2 | 4 | 6 | 9 | 11 | 13 | 16 | 18 | 21 | 24 | 26 | 29 | 31 | 34 | 37 | 39 | 42 |
| 11 | – | – | 0 | 2 | 5 | 7 | 10 | 13 | 16 | 18 | 21 | 24 | 27 | 30 | 33 | 36 | 39 | 42 | 45 | 48 |
| 12 | – | – | 1 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 31 | 34 | 37 | 41 | 44 | 47 | 51 | 54 |
| 13 | – | – | 1 | 3 | 7 | 10 | 13 | 17 | 20 | 24 | 27 | 31 | 34 | 38 | 42 | 45 | 49 | 53 | 56 | 60 |
| 14 | – | – | 1 | 4 | 7 | 11 | 15 | 18 | 22 | 26 | 30 | 34 | 38 | 42 | 46 | 50 | 54 | 58 | 63 | 67 |
| 15 | – | – | 2 | 5 | 8 | 12 | 16 | 20 | 24 | 29 | 33 | 37 | 42 | 46 | 51 | 55 | 60 | 64 | 69 | 73 |
| 16 | – | – | 2 | 5 | 9 | 13 | 18 | 22 | 27 | 31 | 36 | 41 | 45 | 50 | 55 | 60 | 65 | 70 | 74 | 79 |
| 17 | – | – | 2 | 6 | 10 | 15 | 19 | 24 | 29 | 34 | 39 | 44 | 49 | 54 | 60 | 65 | 70 | 75 | 81 | 86 |
| 18 | – | – | 2 | 6 | 11 | 16 | 21 | 26 | 31 | 37 | 42 | 47 | 53 | 58 | 64 | 70 | 75 | 81 | 87 | 92 |
| 19 | – | 0 | 3 | 7 | 12 | 17 | 22 | 28 | 33 | 39 | 45 | 51 | 56 | 63 | 69 | 74 | 81 | 87 | 93 | 99 |
| 20 | – | 0 | 3 | 8 | 13 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 67 | 73 | 79 | 86 | 92 | 99 | 105 |

## A.6 Critical values of the Wilcoxon *T* statistic

The calculated value of *T* must be lower than or equal to the table value for significance. Dashes in the table indicate that no value is possible for significance.

| | 0.05 Level of significance | | 0.01 Level of significance | |
|---|---|---|---|---|
| *n* | One-tailed test | Two-tailed test | One-tailed test | Two-tailed test |
| 5 | 0 | – | – | – |
| 6 | 2 | 0 | – | – |
| 7 | 3 | 2 | 0 | – |
| 8 | 5 | 3 | 1 | 0 |
| 9 | 8 | 5 | 3 | 1 |
| 10 | 10 | 8 | 5 | 3 |
| 11 | 13 | 10 | 7 | 5 |
| 12 | 17 | 13 | 9 | 7 |
| 13 | 21 | 17 | 12 | 9 |
| 14 | 25 | 21 | 15 | 12 |
| 15 | 30 | 25 | 19 | 15 |
| 16 | 35 | 29 | 23 | 19 |
| 17 | 41 | 34 | 27 | 23 |
| 18 | 47 | 40 | 32 | 27 |
| 19 | 53 | 46 | 37 | 32 |
| 20 | 60 | 52 | 43 | 37 |
| 21 | 67 | 58 | 49 | 42 |
| 22 | 75 | 65 | 55 | 48 |
| 23 | 83 | 73 | 62 | 54 |
| 24 | 91 | 81 | 69 | 61 |
| 25 | 100 | 89 | 76 | 68 |
| 26 | 110 | 98 | 84 | 75 |
| 27 | 119 | 107 | 92 | 83 |
| 28 | 130 | 116 | 101 | 91 |
| 29 | 140 | 126 | 110 | 100 |
| 30 | 151 | 137 | 120 | 109 |
| 31 | 163 | 147 | 130 | 118 |
| 32 | 175 | 159 | 140 | 128 |
| 33 | 187 | 170 | 151 | 138 |
| 34 | 200 | 182 | 162 | 148 |
| 35 | 213 | 195 | 173 | 159 |
| 36 | 227 | 208 | 185 | 171 |
| 37 | 241 | 221 | 198 | 182 |
| 38 | 256 | 235 | 211 | 194 |
| 39 | 271 | 249 | 224 | 207 |
| 40 | 286 | 264 | 238 | 220 |
| 41 | 302 | 279 | 252 | 233 |
| 42 | 319 | 294 | 266 | 247 |
| 43 | 336 | 310 | 281 | 261 |
| 44 | 353 | 327 | 296 | 276 |
| 45 | 371 | 343 | 312 | 291 |
| 46 | 389 | 361 | 328 | 307 |
| 47 | 407 | 378 | 345 | 322 |
| 48 | 426 | 396 | 362 | 339 |
| 49 | 446 | 415 | 379 | 355 |
| 50 | 466 | 434 | 397 | 373 |

## A.7 Critical values of the chi-square ($\chi^2$) distribution

| *df* | 0.05 Level of significance | 0.01 Level of significance |
|---|---|---|
| 1 | 3.84 | 6.64 |
| 2 | 5.99 | 9.21 |
| 3 | 7.82 | 11.34 |
| 4 | 9.49 | 13.28 |
| 5 | 11.07 | 15.09 |
| 6 | 12.59 | 16.81 |
| 7 | 14.07 | 18.48 |
| 8 | 15.51 | 20.09 |
| 9 | 16.92 | 21.67 |
| 10 | 18.31 | 23.21 |
| 11 | 19.68 | 24.72 |
| 12 | 21.03 | 26.22 |
| 13 | 22.36 | 27.69 |
| 14 | 23.68 | 29.14 |
| 15 | 25.00 | 30.58 |
| 16 | 26.30 | 32.00 |
| 17 | 27.59 | 33.41 |
| 18 | 28.87 | 34.80 |
| 19 | 30.14 | 36.19 |
| 20 | 31.41 | 37.57 |
| 21 | 32.67 | 38.93 |
| 22 | 33.92 | 40.29 |
| 23 | 35.17 | 41.64 |
| 24 | 36.42 | 42.98 |
| 25 | 37.65 | 44.31 |
| 26 | 38.88 | 45.64 |
| 27 | 40.11 | 46.97 |
| 28 | 41.34 | 48.28 |
| 29 | 42.56 | 49.59 |
| 30 | 43.77 | 50.89 |

The calculated value of $\chi^2$ must be larger than or equal to the table value for significance.

## A.8 Table of probabilities for $\chi_r^2$ when *k* and *n* are small

| | | 0.05 Level of significance | | 0.01 Level of significance | |
|---|---|---|---|---|---|
| *k* | *n* | $\chi_r^2$ | Probability | $\chi_r^2$ | Probability |
| 3 | 2 | – | – | – | – |
| | | – | – | – | – |
| 3 | 3 | 6.00 | 0.028 | – | – |
| | | 4.67 | 0.194 | – | – |
| 3 | 4 | 6.50 | 0.042 | 8.00 | 0.005 |
| | | 6.00 | 0.069 | 6.50 | 0.042 |
| 3 | 5 | 6.40 | 0.039 | 8.40 | 0.009 |
| | | 5.20 | 0.093 | 7.60 | 0.024 |
| 3 | 6 | 7.00 | 0.029 | 9.00 | 0.008 |
| | | 6.33 | 0.052 | 8.33 | 0.012 |
| 3 | 7 | 7.14 | 0.027 | 8.86 | 0.008 |
| | | 6.00 | 0.052 | 8.00 | 0.016 |
| 3 | 8 | 6.25 | 0.047 | 9.00 | 0.010 |
| | | 5.25 | 0.079 | 7.75 | 0.018 |
| 3 | 9 | 6.22 | 0.048 | 8.67 | 0.010 |
| | | 6.00 | 0.057 | 8.22 | 0.016 |
| 4 | 2 | 6.00 | 0.042 | – | – |
| | | 5.40 | 0.167 | – | – |
| 4 | 3 | 7.40 | 0.033 | 9.00 | 0.002 |
| | | 7.00 | 0.054 | 8.20 | 0.017 |

When *k* and *n* are small $\chi_r^2$ can only take a few values. For each combination of *k* and *n* there are two values given for $\chi_r^2$. The table gives the two values closest to the significance level with their actual probabilities. For example, when $k = 3$ and $n = 6$, $\chi_r^2 = 7.00$ or greater with a probability of 0.029. This is less than 0.05 and so is significant. The next value below 7.00 that $\chi_r^2$ can be is 6.33, with a probability of 0.052, which is not quite significant at 0.05. Dashes in the table indicate that no value is possible for significance.

## A.9 Critical values of the Pearson *r* correlation coefficient

| | 0.05 Level of significance | | 0.01 Level of significance | |
|---|---|---|---|---|
| *df* | One-tailed test (directional) | Two-tailed test (non-directional) | One-tailed test (directional) | Two-tailed test (non-directional) |
| 1 | 0.9877 | 0.9969 | 0.9995 | 0.9999 |
| 2 | 0.9000 | 0.9500 | 0.9800 | 0.9900 |
| 3 | 0.8054 | 0.8783 | 0.9343 | 0.9587 |
| 4 | 0.7293 | 0.8114 | 0.8822 | 0.9172 |
| 5 | 0.6694 | 0.7545 | 0.8329 | 0.8745 |
| 6 | 0.6215 | 0.7067 | 0.7887 | 0.8343 |
| 7 | 0.5822 | 0.6664 | 0.7498 | 0.7977 |
| 8 | 0.5494 | 0.6319 | 0.7155 | 0.7646 |
| 9 | 0.5214 | 0.6021 | 0.6851 | 0.7348 |
| 10 | 0.4973 | 0.5760 | 0.6581 | 0.7079 |
| 11 | 0.4762 | 0.5529 | 0.6339 | 0.6835 |
| 12 | 0.4575 | 0.5324 | 0.6120 | 0.6614 |
| 13 | 0.4409 | 0.5139 | 0.5923 | 0.6411 |
| 14 | 0.4259 | 0.4973 | 0.5742 | 0.6226 |
| 15 | 0.4124 | 0.4821 | 0.5577 | 0.6055 |
| 16 | 0.4000 | 0.4683 | 0.5425 | 0.5897 |
| 17 | 0.3887 | 0.4555 | 0.5285 | 0.5751 |
| 18 | 0.3783 | 0.4438 | 0.5155 | 0.5614 |
| 19 | 0.3687 | 0.4329 | 0.5034 | 0.5487 |
| 20 | 0.3598 | 0.4227 | 0.4921 | 0.5368 |
| 25 | 0.3233 | 0.3809 | 0.4451 | 0.4869 |
| 30 | 0.2960 | 0.3494 | 0.4093 | 0.4487 |
| 35 | 0.2746 | 0.3246 | 0.3810 | 0.4182 |
| 40 | 0.2573 | 0.3044 | 0.3578 | 0.3932 |
| 45 | 0.2428 | 0.2875 | 0.3384 | 0.3721 |
| 50 | 0.2306 | 0.2732 | 0.3218 | 0.3541 |
| 60 | 0.2108 | 0.2500 | 0.2948 | 0.3248 |
| 70 | 0.1954 | 0.2319 | 0.2737 | 0.3017 |
| 80 | 0.1829 | 0.2172 | 0.2565 | 0.2830 |
| 90 | 0.1726 | 0.2050 | 0.2422 | 0.2673 |
| 100 | 0.1638 | 0.1946 | 0.2301 | 0.2540 |

The calculated value of *r* must be larger than or equal to the table value for significance.

## A.10 Critical values of the Spearman $r_s$ ranked correlation coefficient

| | 0.05 Level of significance | | 0.01 Level of significance | |
|---|---|---|---|---|
| *N* | One-tailed test (directional) | Two-tailed test (non-directional) | One-tailed test (directional) | Two-tailed test (non-directional) |
| 5 | 0.900 | 1.000 | 1.000 | – |
| 6 | 0.829 | 0.886 | 0.943 | 1.000 |
| 7 | 0.714 | 0.786 | 0.893 | 0.929 |
| 8 | 0.643 | 0.738 | 0.833 | 0.881 |
| 9 | 0.600 | 0.683 | 0.783 | 0.833 |
| 10 | 0.564 | 0.648 | 0.746 | 0.794 |
| 12 | 0.506 | 0.591 | 0.712 | 0.777 |
| 14 | 0.456 | 0.544 | 0.645 | 0.715 |
| 16 | 0.425 | 0.506 | 0.601 | 0.665 |
| 18 | 0.399 | 0.475 | 0.564 | 0.625 |
| 20 | 0.377 | 0.450 | 0.534 | 0.591 |
| 22 | 0.359 | 0.428 | 0.508 | 0.562 |
| 24 | 0.343 | 0.409 | 0.485 | 0.537 |
| 26 | 0.329 | 0.392 | 0.465 | 0.515 |
| 28 | 0.317 | 0.377 | 0.448 | 0.496 |
| 30 | 0.306 | 0.364 | 0.432 | 0.478 |

The calculated value of $r_s$ must be larger than or equal to the table value for significance.

# Index