Statistics Explained

2nd Edition

Perry R. Hinton

Also available as a printed book
see title verso for ISBN details

Statistics Explained

2nd Edition

Statistics Explained

2nd Edition

▪ Perry R. Hinton

Routledge
Taylor & Francis Group
LONDON AND NEW YORK

ROUTLEDGE

First published 2004
by Routledge
27 Church Road, Hove, East Sussex BN3 2FA

Simultaneously published in the USA and Canada
by Routledge
29 West 35th Street, New York, NY 10001

Routledge is an imprint of T&F Informa plc

This edition published in the Taylor & Francis e-Library, 2005.

"To purchase your own copy of this or any of Taylor & Francis or Routledge's collection of thousands of eBooks please go to www.eBookstore.tandf.co.uk."

Copyright © 2004 Routledge

All rights reserved. No part of this book may be reprinted or reproduced or utilised in any form or by any electronic, mechanical, or other means, now known or hereafter invented, including photocopying and recording, or in any information storage or retrieval system, without permission in writing from the publishers.

This publication has been produced with paper manufactured to strict environmental standards and with pulp derived from sustainable forests.

British Library Cataloguing-in-Publication Data
A catalogue record for this book is available from the British Library

Library of Congress Cataloging-in-Publication Data
Hinton, Perry R. (Perry Roy), 1954–
Statistics explained / by Perry R. Hinton. – 2nd ed.
p. cm.
Includes bibliographical references and index.
ISBN 0-415-33284-2 – ISBN 0-415-33285-0 (pbk.)
1. Psychometrics–Textbooks. I. Title.
BF39.H54 2004
519.5–dc22 2003027026

ISBN 0-203-49678-7 Master e-book ISBN

ISBN 0-203-57455-9 (Adobe eReader Format)
ISBN 0-415-33285-0 (pbk)
ISBN 0-415-33284-2 (hbk)

To Anna, Anthony and Emma

Contents

List of figures xv
Preface xvii
Preface to the Second Edition xix

1 Introduction **1**

2 Descriptive statistics **5**

Measures of 'central tendency' 8
Measures of 'spread' 11
Describing a set of data: in conclusion 17
Comparing two sets of data with descriptive statistics 18
Some important information about numbers 21

3 Standard scores **25**

Comparing scores from different distributions 26
The Normal Distribution 28
The Standard Normal Distribution 30

4 Introduction to hypothesis testing **35**

Testing an hypothesis 36
The logic of hypothesis testing 41
One- and two-tailed predictions 42

5 Sampling **47**

Populations and samples 48
Selecting a sample 49
Sample statistics and population parameters 51
Summary 56

6 Hypothesis testing with one sample **59**

An example 60
When we do not have the known population standard deviation 64
Confidence intervals 69
Hypothesis testing with one sample: in conclusion 72

7 Selecting samples for comparison **73**

Designing experiments to compare samples 74
The interpretation of sample differences 79

8 Hypothesis testing with two samples **81**

The assumptions of the two sample *t* test 85
Related or independent samples 86
The related *t* test 86
The independent *t* test 89
Confidence intervals 93

9 Significance, error and power **95**

Type I and Type II errors 96
Statistical power 98
The power of a test 99
The choice of α level 100
Effect size 101
Sample size 103
Conclusion 108

10 Introduction to the analysis of variance **111**

Factors and conditions 112
The problem of many conditions and the *t* test 112
Why do scores vary in an experiment? 113
The process of analysing variability 118
The *F* distribution 121
Conclusion 123

11 One factor independent measures ANOVA **125**

Analysing variability in the independent measures ANOVA 126
Rejecting the null hypothesis 132
Unequal sample sizes 133
The relationship of *F* to *t* 135

12 Multiple comparisons **137**

The Tukey test (for all pairwise comparisons) 140
The Scheffé test (for complex comparisons) 144

13 One factor repeated measures ANOVA **149**

Deriving the *F* value 150
Multiple comparisons 158

14 The interaction of factors in the analysis of variance **161**

Interactions 164
Dividing up the between conditions sums of squares 167
Simple main effects 169
Conclusion 170

15 Calculating the two factor ANOVA **171**

The two factor independent measures ANOVA 172
The two factor mixed design ANOVA 181
The two factor repeated measures ANOVA 193
A non-significant interaction 205

16 An introduction to nonparametric analysis **207**

Calculating ranks 212

17 Two sample nonparametric analyses **215**

The Mann–Whitney *U* test (for independent samples) 216
The Wilcoxon signed-ranks test (for related samples) 224

18 One factor ANOVA for ranked data **231**

Kruskal–Wallis test (for independent measures) 232
The Friedman test (for related samples) 240

19 Analysing frequency data: chi-square **247**

Nominal data, categories and frequency counts 248
Introduction to χ^2 248
Chi-square (χ^2) as a 'goodness of fit' test 250
Chi-square (χ^2) as a test of independence 254
The chi-square distribution 256
The assumptions of the χ^2 test 257

20 Linear correlation and regression **261**

Introduction 262
Pearson r correlation coefficient 264
Linear regression 270
The interpretation of correlation and regression 275
Problems with correlation and regression 276
The standard error of the estimate 278
The Spearman r_S correlation coefficient 279

21 Multiple correlation and regression **283**

Introduction to multivariate analysis 284
Partial correlation 284
Multiple correlation 289
Multiple regression 291

22 Complex analyses and computers **295**

Undertaking data analysis by computer 296
Complex analyses 299
Reliability 301
Factor analysis 304
Multivariate analysis of variance (MANOVA) 308
Discriminant function analysis 312
Conclusion 314

23 An introduction to the general linear model **315**

Models 316
An example of a linear model 318
Modelling data 320
The model: the regression equation 323
Selecting a good model 327
Comparing samples (the analysis of variance once again) 333
Explaining variations in the data 337
The general linear model 338

Notes 343
Glossary 347
References 357

Appendix: Acknowledgements and statistical tables **359**

A.1 The standard normal distribution tables 362
A.2 Critical values of the *t* distribution 363
A.3 Critical values of the *F* distribution 364
A.4 Critical values of the Studentized range statistic, *q* 366
A.5 Critical values of the Mann–Whitney *U* statistic 367
A.6 Critical values of the Wilcoxon *T* statistic 369

A.7	Critical values of the chi-square (χ^2) distribution	370
A.8	Table of probabilities for χ^2_r when k and n are small	371
A.9	Critical values of the Pearson r correlation coefficient	372
A.10	Critical values of the Spearman r_S ranked correlation coefficient	373

Index 375

Figures

2.1	Frequency distribution of the examination results	7
2.2	Frequency distribution of last year's examination results	19
3.1	The distribution of women's height: histogram	28
3.2	The distribution of women's height	29
3.3	The Standard Normal Distribution	31
4.1	A hothousing effect of 30 IQ points	38
4.2	A one-tailed prediction and the significance level	44
4.3	A two-tailed prediction and the significance level	45
6.1	Hypothesis testing with a sample of Cyadmine-affected babies	63
6.2	Examples of the *t* distribution	67
8.1	The distribution of the difference between sample means	83
9.1	The risk of a Type I and Type II error	96
9.2	The effect of increasing the sample size on the overlap of the distributions	104
10.1	An example of an *F* distribution (degrees of freedom = 4,8)	122
14.1	An interaction of school by gender	164
14.2a	No interaction in the data	165
14.2b	No interaction in the data again	165
14.2c	An example of an interaction	166
14.2d	Another example of an interaction	166

15.1	The interaction of experience and machine on the number of errors	180
15.2	The interaction of time and experience on machine operator errors	191
15.3	The interaction of product and shift on machine operator errors	203
19.1	The chi-square distribution	257
20.1	Scatterplot of study time by examination performance	263
20.2	The distribution of Pearson's r	269
20.3	Finding the regression line by minimising the error values (E)	272
20.4	Regression of Y on X and the regression of X on Y	275
20.5	The coefficient of determination (r^2)	276
20.6	The scatterplot of smiling time by items sold	278
21.1	The variability of the scores on three variables	288
22.1	Scree plot of the eigenvalues	307
23.1	An orrery	317
23.2	A graph of card sorting times	319
23.3	A plot of the children's general knowledge scores by age	321
23.4	A proposed linear relationship between general knowledge and age	322
23.5	Separating each score into predicted score plus residual	323
23.6	Different linear models	324
23.7	The same linear model for two sets of data	330
23.8	Plot of general knowledge scores for each class	334
23.9	A linear model for the class data	335

Preface

This book sets out to explain the major statistical analyses used by undergraduate students in psychology. It should also satisfy many of the statistical requirements of students of social and life sciences, education, health, business and communication; indeed anyone with a need to understand statistical analysis. The examples in the book are varied but there is a focus on analysing data about people. The book examines many important statistical techniques, providing explanations of how and why they are used.

When I was an undergraduate myself I learnt the appropriate statistics to analyse my experimental data but was frustrated that the books I read simply told me to do this, do that, like a cookbook, with no explanation of why I was using such strange formulae or calculating numbers in the manner suggested. As a graduate student I needed a more detailed knowledge of statistics and there were some excellent books for the new researcher. The only problem was that they were very weighty tomes, as thick as an encyclopaedia and comprehensive to match. There were pages and pages of mathematical formulae that tended to make the heart quail.

I discovered that the apparently mysterious formulae and calculations actually made sense: indeed, they made common sense and the logic of statistical tests was no more difficult than understanding a theory in psychology. As a lecturer I noticed that there was a tendency for students to view statistical tests as difficult and esoteric. In part this was because they knew the formulae for calculating the tests but did not know 'how' or 'why' the

tests were as they were. Over the years as a lecturer I have gained much satisfaction from students exclaiming: 'so that's what it means!' when an explanation is given. And often these are students who had the view that statistics was something they would not be able to understand. Yet this is not the case. Statistics are actually remarkably sensible – they make good sense. The key to understanding statistics is to understand how and why they were developed, what they set out to do, and how they do it. Unfortunately this is sometimes missing. Unlike many theories and explanations of human behaviour and experience which by their nature are the subject of fierce debate, statistics are simply techniques to be used where necessary. A statistical test is a tool, like any other, and so can be used wisely or foolishly. If we know what it is for we can use it well. Few of us would choose to use a fork to drink soup yet people choose inappropriate statistical tests to analyse data. But statistical tests are like spoons and forks. If you know what they can do and how to use them, there is no mystery, you just get on and use them. But like any other tool it does take a little while to understand how and why it works as it does, and then to get the hang of using it oneself. Once the tools have been mastered they becomes easier to use.

I hope that, for students facing the purchase of a statistics book, this book will be able to provide an account of statistical analysis where the mysterious formulae are explained, but without weighing down the reader!

I would like to thank Sue Wilkinson for encouraging me to write this book, Margaret Manning for many interesting statistical discussions and David French for helping me find the time to write it. I would especially like to thank Paul Hartmann for many helpful comments on the text. I have taught an undergraduate course on the *Analysis of Experiments* for a number of years and the feedback, questions and criticisms of the many students who have taken the course have helped me to understand the problems and delights of learning statistics. I have learnt a lot. This book is a response to that experience and I thank them too. Finally, I would like to thank Anna, Anthony and Emma, without whose support I could never have written this book.

Preface to the Second Edition

I have been very pleased with the success of the first edition of this book. I found it particularly gratifying when one of my students said that she could hear me speaking when she read the book. I hope other readers have felt like her, that the book reads like someone talking – hopefully a friendly, helpful voice – as I believe that the best way of learning is having someone explain the material to me clearly. Despite the wonders of new technology we should strive to maintain that personal contact between teacher and student, and writer and reader. I was therefore in a quandary when asked to do a second edition. I had received a number of complimentary letters and emails from lecturers and students so I felt the book was doing its job well. As the old adage says 'if it's not broken don't mend it' and the age of a book shouldn't undermine its value – indeed one of my own favourite books on statistics is Siegel (1956) despite the excellent new editions. However, there are changes I want to make based on the experience of the passing years along with the constructive comments of my readers. I have also produced, with colleagues, a companion volume on the computer statistics package SPSS, which informed some of my thoughts on the new edition (Hinton *et al*., 2004). I hope my voice still comes through clearly and comprehensibly.

Since the first edition the debate has developed around the importance of significance testing (Wilkinson and Task Force on Statistical Inference,

1999). Most academic papers are only published if the findings within them are statistically significant. This may be giving a distorted picture of the overall outcome of research, as we do not know how many studies did not find a significant effect. So a greater prominence should be given to both confidence intervals and statistical power in the reporting of findings. The findings of one study showing a significant finding may in the long run be far less important than a broader understanding of the size of an effect that emerges with a number of studies. I have therefore gone into more detail in explaining 'power' in Chapter 9 and also introduced the calculation of confidence intervals into the book. However, I think that students need to understand the basics of significance testing first so that they are able to understand the issues and engage in the debate from a position of knowledge. I have, therefore, maintained the structure and explanation of significance testing from the first edition of the book.

I have also included a final chapter that introduces the reader to the general linear model. Whilst it is quite possible to happily undertake statistical analysis without knowing about the general linear model, I hope this chapter provides, for the interested reader, a bridge between a good understanding of the statistical techniques and a more in-depth understanding of the underlying principles of most of our statistical analyses. I have found that the assumptions of statistical tests often appear strange but that some appreciation of this underlying model reveals what the assumptions are all about.

I would also like to thank the many students (hundreds in fact) on the undergraduate course in *Research and Experimentation II* who, for seven years, were able to interrogate the author of their set textbook. I appreciated their (generally) kind comments and I am glad that the book helped a number of them to realise that statistics were not so alien after all. I have many happy memories of teaching on that course with my colleagues Victoria West and Alfredo Gaitan, who I would also like to thank. Thanks also to Ian Robertson for our many discussions on teaching and learning, especially on how to make topics clear and comprehensible. Finally, I would especially like to thank Charlotte Brownlow, Isabella McMurray and Bob Cozens, the other members of the *SPSS Explained* team, without whose enthusiasm, support and friendship I might not have taken on the task of writing this new edition.