Appendix I Additional Topics

PART I

Bayes's Theorem

The Reverend Thomas Bayes (1702–1761) was an English mathematician who discovered an important relation for conditional probabilities. This relation is referred to as *Bayes's rule* or *Bayes's theorem*. It uses conditional probabilities to adjust calculations so that we can accommodate new, relevant information. We will restrict our attention to a special case of Bayes's theorem in which an event B is partitioned into only *two* mutually exclusive events (see Figure AI-1). The general formula is a bit complicated but is a straightforward extension of the basic ideas we will present here. Most advanced texts contain such an extension.

Note: We use the following compact notation in the statement of Bayes's theorem:

Notation	Meaning
A^c	complement of A; *not* A
$P(B\|A)$	probability of event B, *given* event A; $P(B$, *given* $A)$
$P(B\|A^c)$	probability of event B, *given* the complement of A; $P(B$, *given not* $A)$

We will use Figure AI-1 to motivate Bayes's theorem. Let A and B be events in a sample space that have probabilities not equal to 0 or 1. Let A^c be the complement of A.

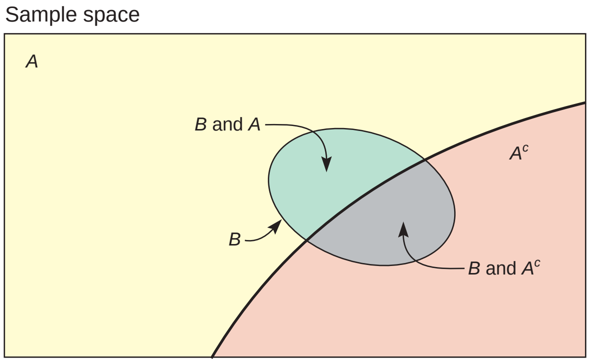
Here is Bayes's theorem: $$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)} \quad (1)$$

Overview of Bayes's Theorem

Suppose we have an event A and we calculate $P(A)$, the unconditional probability of A standing by itself. Now suppose we have a "new" event B and we know the probability of B given that A occurs $P(B|A)$, as well as the probability of B given that A does not occur $P(B|A^c)$. Where does such an event B come from? The event B can be constructed in many possible ways. For example, B can be constructed as

FIGURE A1-1

A Typical Setup for Bayes's Theorem



the result of a consulting service, a testing procedure, or a sorting activity. In the examples and problems, you will find more ways to construct such an event B.

How can we use this "new" information concerning the event B to adjust our calculation of the probability of event A, given B? That is, how can we make our calculation of the probability of A more realistic by including information about the event B? The answer is that we will use Equation (1) of Bayes's theorem.

Let's look at some examples that use Equation (1) of Bayes's theorem. We are grateful to personal friends in the oil and natural gas business in Colorado who provided the basic information in the following example.

EXAMPLE 1 Bayes's theorem

A geologist has examined seismic data and other geologic formations in the vicinity of a proposed site for an oil well. Based on this information, the geologist reports a 65% chance of finding oil. The oil company decides to go ahead and start drilling. As the drilling progresses, sample cores are taken from the well and studied by the geologist. These sample cores have a history of predicting oil *when there is oil* about 85% of the time. However, about 6% of the time the sample cores will predict oil *when there is no oil*. (Note that these probabilities need not add up to 1.) Our geologist is delighted because the sample cores predict oil for this well.

Use the "new" information from the sample cores to revise the geologist's original probability that the well will hit oil. What is the new probability?

SOLUTION: To use Bayes's theorem, we need to identify the events A and B. Then we need to find $P(A)$, $P(A^c)$, $P(B|A)$, and $P(B|A^c)$. From the description of the problem, we have

A is the event that the well strikes oil.
A^c is the event that the well is dry (no oil).
B is the event that the core samples indicate oil.

Again, from the description, we have

$$P(A) = 0.65, \qquad \text{so} \qquad P(A^c) = 1 - 0.65 = 0.35$$

These are our *prior* (before new information) probabilities. New information comes from the sample cores. Probabilities associated with the new information are

$$P(B|A) = 0.85$$

This is the probability that core samples indicate oil when there actually is oil.

$$P(B|A^c) = 0.06$$

This is the probability that core samples indicate oil when there is no oil (dry well).

Now we use Bayes's theorem to revise the probability that the well will hit oil based on the "new" information from core samples. The revised probability is the *posterior* probability we compute that uses the new information from the sample cores:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)} = \frac{(0.85)(0.65)}{(0.85)(0.65) + (0.06)(0.35)} = 0.9634$$

We see that the revised (*posterior*) probability indicates about a 96% chance for the well to hit oil. This is why sample cores that are good can attract money in the form of venture capital (for independent drillers) on a big, expensive well.

GUIDED EXERCISE 1 Bayes's theorem

The Anasazi were prehistoric pueblo people who lived in what is now the southwestern United States. Mesa Verde, Pecos Pueblo, and Chaco Canyon are beautiful national parks and monuments, but long ago they were home to many Anasazi. In prehistoric times, there were several Anasazi migrations, until finally their pueblo homes were completely abandoned. The delightful book *Proceedings of the Anasazi Symposium, 1981*, published by Mesa Verde Museum Association, contains a very interesting discussion about methods anthropologists use to (approximately) date Anasazi objects. There are two popular ways. One is to compare environmental data to other objects of known dates. The other is radioactive carbon dating.

Carbon dating has some variability in its accuracy, depending on how far back in time the age estimate goes and also on the condition of the specimen itself. Suppose experience has shown that the carbon method is correct 75% of the time it is used on an object from a known (given) time period. However, there is a 10% chance that the carbon method will predict that an object is from a certain period even when we already know the object is not from that period.

Using environmental data, an anthropologist reported the probability to be 40% that a fossilized deer bone bracelet was from a certain Anasazi migration period. Then, as a follow-up study, the carbon method also indicated that the bracelet was from this migration period. How can the anthropologist adjust her estimated probability to include the "new" information from the carbon dating?

(a) To use Bayes's theorem, we must identify the events A and B. From the description of the problem, what are A and B?

→ A is the event that the bracelet is from the given migration period. B is the event that carbon dating indicates that the bracelet is from the given migration period.

(b) Find $P(A)$, $P(A^c)$, $P(B|A)$, and $P(B|A^c)$.

→ From the description,

$$P(A) = 0.40$$
$$P(A^c) = 0.60$$
$$P(B|A) = 0.75$$
$$P(B|A^c) = 0.10$$

(c) Compute $P(A|B)$, and explain the meaning of this number.

→ Using Bayes's theorem and the results of part (b), we have

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)}$$
$$= \frac{(0.75)(0.40)}{(0.75)(0.40) + (0.10)(0.60)} = 0.8333$$

The prior (before carbon dating) probability was only 40%. However, the carbon dating enabled us to revise this probability to 83%. Thus, we are about 83% sure that the bracelet came from the given migration period. Perhaps additional research at the site will uncover more information to which Bayes's theorem could be applied again.

The next example is a classic application of Bayes's theorem. Suppose we are faced with two competing hypotheses. Each hypothesis claims to explain the same phenomenon; however, only one hypothesis can be correct. Which hypothesis should we accept? This situation occurs in the natural sciences, the social sciences, medicine, finance, and many other areas of life. Bayes's theorem will help us compute the probabilities that one or the other hypothesis is correct. Then what do we do? Well, the great mathematician and philosopher René Descartes can guide us. Descartes once said, "When it is not in our power to determine what is true, we ought to follow what is most probable." Just knowing probabilities does not allow us with absolute certainty to choose the correct hypothesis, but it does permit us to identify which hypothesis is *most likely* to be correct.

EXAMPLE 2

Competing hypotheses

A large hospital uses two medical labs for blood work, biopsies, throat cultures, and other medical tests. Lab I does 60% of the reports. The other 40% of the reports are done by Lab II. Based on long experience, it is known that about 10% of the reports from Lab I contain errors and that about 7% of the reports from Lab II contain errors. The hospital recently received a lab report that, through additional medical work, was revealed to be incorrect. One hypothesis is that the report with the mistake came from Lab I. The competing hypothesis is that the report with the mistake came from Lab II. Which lab do you suspect is the culprit? Why?

SOLUTION: Let's use the following notation.

A = event report is from Lab I
A^c = event report is from Lab II
B = event report contains a mistake

From the information given,

$P(A) = 0.60 \qquad P(A^c) = 0.40$
$P(B|A) = 0.10 \qquad P(B|A^c) = 0.07$

The probability that the report is from Lab I *given* we have a mistake is $P(A|B)$. Using Bayes's theorem, we get

$$\begin{aligned} P(A|B) &= \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)} \\ &= \frac{(0.10)(0.60)}{(0.10)(0.60) + (0.07)(0.40)} \\ &= \frac{0.06}{0.088} \approx 0.682 \approx 68\% \end{aligned}$$

So, the probability is about 68% that Lab I supplied the report with the error. It follows that the probability is about 100% − 68% = 32% that the erroneous report came from Lab II.

PROBLEM

Bayes's theorem applied to quality control

A company that makes steel bolts knows from long experience that about 12% of its bolts are defective. If the company simply ships all bolts that it produces, then 12% of the shipment the customer receives will be defective. To decrease the percentage of defective bolts shipped to customers, an electronic scanner is installed. The scanner is positioned over the production line and is supposed to pick out the good bolts. However, the scanner itself is not perfect. To test the scanner, a large number of (pretested) "good" bolts were run under the scanner, and it accepted 90% of the bolts as good.

Continued

Then a large number of (pretested) defective bolts were run under the scanner, and it accepted 3% of these as good bolts.

(a) If the company does not use the scanner, what percentage of a shipment is expected to be good? What percentage is expected to be defective?

(b) The scanner itself makes mistakes, and the company is questioning the value of using it. Suppose the company does use the scanner and ships only what the scanner passes as "good" bolts. In this case, what percentage of the shipment is expected to be good? What percentage is expected to be defective?

Partial Answer

To solve this problem, we use Bayes's theorem. The result of using the scanner is a dramatic improvement in the quality of the shipped product. If the scanner is not used, only 88% of the shipped bolts will be good. However, if the scanner is used and only the bolts it passes as good are shipped, then 99.6% of the shipment is expected to be good. Even though the scanner itself makes a considerable number of mistakes, it is definitely worth using. Not only does it increase the quality of a shipment, the bolts it rejects can also be recycled into new bolts.

PART II The Hypergeometric Probability Distribution

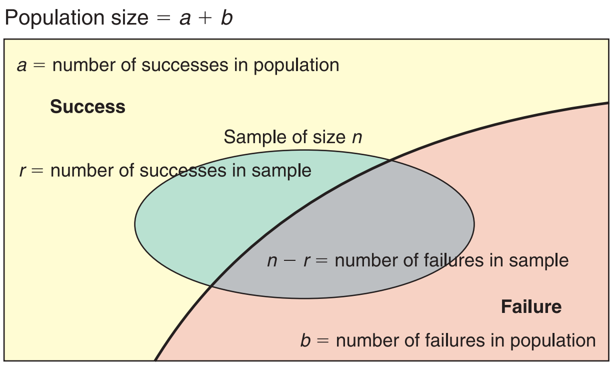
In Chapter 5, we examined the binomial distribution. The binomial probability distribution assumes *independent trials*. If the trials are constructed by drawing samples from a population, then we have two possibilities: We sample either *with replacement* or *without replacement*. If we draw random samples with replacement, the trials can be taken to be independent. If we draw random samples without replacement and the population is very large, then it is reasonable to say that the trials are approximately independent. In this case, we go ahead and use the binomial distribution. However, if the population is relatively small and we draw samples without replacement, the assumption of independent trials is not valid, and we should not use the binomial distribution.

The *hypergeometric distribution* is a probability distribution of a random variable that has two outcomes when sampling is done *without replacement*.

Consider the following notational setup (see Figure AI-2). Suppose we have a population with only *two* distinct types of objects. Such a population might be made up of females and males, students and faculty, residents and nonresidents, defective and nondefective items, and so on. For simplicity of reference, let us call one type of object (your choice) "success" and the other "failure." Let's use the

FIGURE A1-2

Notational Setup for Hypergeometric Distribution



letter a to designate the number of successes in the population and the letter b to designate the number of failures in the population. Thus, the total population size is $a + b$. Next, we draw a random sample (without replacement) of size n from this population. Let r be the number of successes in this sample. Then $n - r$ is the number of failures in the sample. The hypergeometric distribution gives us the probability of r successes in the sample of size n.

Recall from Section 4.3 that the number of combinations of k objects taken j at a time can be computed as

$$C_{k,j} = \frac{k!}{j!(k-j)!}$$

Using the notation of Figure AI-2 and the formula for combinations, the hypergeometric distribution can be calculated.

Hypergeometric distribution

Given that a population has two distinct types of objects, success and failure,

a counts the number of successes in the population.

b counts the number of failures in the population.

For a random sample of size n taken *without replacement* from this population, the probability $P(r)$ of getting r successes in the *sample* is

$$P(r) = \frac{C_{a,r}C_{b,n-r}}{C_{(a+b),n}} \qquad (2)$$

The expected value and standard deviation are

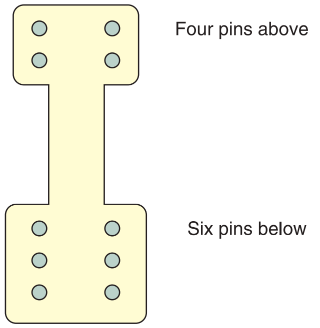
$$\mu = \frac{na}{a+b} \quad \text{and} \quad \sigma = \sqrt{n\left(\frac{a}{a+b}\right)\left(\frac{b}{a+b}\right)\left(\frac{a+b-n}{a+b-1}\right)}$$

EXAMPLE 3 HYPERGEOMETRIC DISTRIBUTION

A section of an Interstate 95 bridge across the Mianus River in Connecticut collapsed suddenly on the morning of June 28, 1983. (See *To Engineer Is Human: The Role of Failure in Successful Designs* by Henry Petroski.) Three people were killed when their vehicles fell off the bridge. It was determined that the collapse was caused by the failure of a metal hanger design that left a section of the bridge with no support when something went wrong with the pins. Subsequent inspection revealed many cracked pins and hangers in bridges across the United States.

FIGURE A1-3

Steel Hanger Design for Bridge Support



(a) Suppose a hanger design uses four pins in the upper part and six pins in the lower part, as shown in Figure AI-3. The hangers come in a kit consisting of the hanger and 10 pins. When a work crew installs a hanger, they start with the top part and randomly select a pin, which is put into place. This is repeated until all four pins are in the top. Then they finish the lower part.

Assume that three pins in the kit are faulty. The other seven are all right. What is the probability that all three faulty pins get put in the top part of the hanger? This means that the support is held up, in effect, by only one good pin.

SOLUTION: The population consists of 10 pins identical in appearance. However, three are faulty and seven are good. The sampling of four pins for the top part of the hanger is done *without replacement*. Since we are interested in the faulty pins, let us label them "success" (only a convenient label). Using the notation of Figure AI-2 and the hypergeometric distribution, we have

a = number of successes in the population (bad pins) = 3
b = number of failures in the population (good pins) = 7
n = sample size (number of pins put in top) = 4
r = number of successes in sample (number of bad pins in top) = 3

The hypergeometric distribution applies because the population is relatively small (10 pins) and sampling is done without replacement. By Equation (2), we compute $P(r)$:

$$P(r) = \frac{C_{a,r}C_{b,n-r}}{C_{(a+b),n}}$$

Using the preceding information about a, b, n, and r, we get

$$P(r = 3) = \frac{C_{3,3}C_{7,1}}{C_{10,4}}$$

Using the formula for $C_{k,j}$, Table 2 of Appendix II, or the combinations key on a calculator, we get

$$P(r = 3) = \frac{1 \cdot 7}{210} = 0.0333$$

We see that there is a better than 3.3% chance of getting three out of four bad pins in the top part of the hanger.

(b) Suppose that all the hanger kits are like the one described in part (a). On a long bridge that uses 200 such hangers, how many do you expect are held up by only one good pin? How might this affect the safety of the bridge?

SOLUTION: We would expect

$$200(0.0333) \approx 6.7$$

That is, between six and seven hangers are expected to be held up by only one good pin. As time goes on, this pin will corrode and show signs of wear as the bridge vibrates. With only one good pin, there is much less margin of safety.

Professor Petroski discusses the bridge on I-95 across the Mianus River in his book mentioned earlier. He points out that this dramatic accidental collapse resulted in better quality control (for hangers and pins) as well as better overall design of bridges. In addition to this, the government has greatly increased programs for maintenance and inspection of bridges.

GUIDED EXERCISE 2 Hypergeometric distribution

The biology club weekend outing has two groups. One group with seven people will camp at Diamond Lake. The other group with 10 people will camp at Arapahoe Pass. Seventeen duffels were prepacked by the outing committee, but six of these had the tents accidentally left out of the duffel. The group going to Diamond Lake picked up their duffels at random from the collection and started off on the trail. The group going to Arapahoe Pass used the remaining duffels. What is the probability that all six duffels without tents were picked up by the group going to Diamond Lake?

Continued

GUIDED EXERCISE 2 *continued*

(a) What is success? Are the duffels selected with or without replacement? Which probability distribution applies?

Success is taking a duffel without a tent. The duffels are selected without replacement. The hypergeometric distribution applies.

(b) Use the hypergeometric distribution to compute the probability of $r = 6$ successes in the sample of seven people going to Diamond Lake.

To use the hypergeometric distribution, we need to know the values of

a = number of successes in population = 6

b = number of failures in population = 11

n = sample size = 7, since seven people are going to Diamond Lake

r = number of successes in sample = 6

$$\text{Then, } P(r = 6) = \frac{C_{6,6}\, C_{11,1}}{C_{17,7}} = \frac{1 \cdot 11}{19448} = 0.0006$$

The probability that all six duffels without tents are taken by the seven hikers to Diamond Lake is 0.0006.

Appendix II Tables

1. Random Numbers
2. Binomial Coefficients $C_{n,r}$
3. Binomial Probability Distribution $C_{n,r}p^r q^{n-r}$
4. Poisson Probability Distribution
5. Areas of a Standard Normal Distribution
6. Critical Values for Student's t Distribution
7. The χ^2 Distribution
8. Critical Values for F Distribution
9. Critical Values for Spearman Rank Correlation, r_s
10. Critical Values for Number of Runs R

TABLE 1 Random Numbers

92630	78240	19267	95457	53497	23894	37708	79862	76471	66418
79445	78735	71549	44843	26104	67318	00701	34986	66751	99723
59654	71966	27386	50004	05358	94031	29281	18544	52429	06080
31524	49587	76612	39789	13537	48086	59483	60680	84675	53014
06348	76938	90379	51392	55887	71015	09209	79157	24440	30244
28703	51709	94456	48396	73780	06436	86641	69239	57662	80181
68108	89266	94730	95761	75023	48464	65544	96583	18911	16391
99938	90704	93621	66330	33393	95261	95349	51769	91616	33238
91543	73196	34449	63513	83834	99411	58826	40456	69268	48562
42103	02781	73920	56297	72678	12249	25270	36678	21313	75767
17138	27584	25296	28387	51350	61664	37893	05363	44143	42677
28297	14280	54524	21618	95320	38174	60579	08089	94999	78460
09331	56712	51333	06289	75345	08811	82711	57392	25252	30333
31295	04204	93712	51287	05754	79396	87399	51773	33075	97061
36146	15560	27592	42089	99281	59640	15221	96079	09961	05371
29553	18432	13630	05529	02791	81017	49027	79031	50912	09399
23501	22642	63081	08191	89420	67800	55137	54707	32945	64522
57888	85846	67967	07835	11314	01545	48535	17142	08552	67457
55336	71264	88472	04334	63919	36394	11196	92470	70543	29776
10087	10072	55980	64688	68239	20461	89381	93809	00796	95945
34101	81277	66090	88872	37818	72142	67140	50785	21380	16703
53362	44940	60430	22834	14130	96593	23298	56203	92671	15925
82975	66158	84731	19436	55790	69229	28661	13675	99318	76873
54827	84673	22898	08094	14326	87038	42892	21127	30712	48489
25464	59098	27436	89421	80754	89924	19097	67737	80368	08795
67609	60214	41475	84950	40133	02546	09570	45682	50165	15609
44921	70924	61295	51137	47596	86735	35561	76649	18217	63446
33170	30972	98130	95828	49786	13301	36081	80761	33985	68621
84687	85445	06208	17654	51333	02878	35010	67578	61574	20749
71886	56450	36567	09395	96951	35507	17555	35212	69106	01679

TABLE 1 *continued*

00475	02224	74722	14721	40215	21351	08596	45625	83981	63748
25993	38881	68361	59560	41274	69742	40703	37993	03435	18873
92882	53178	99195	93803	56985	53089	15305	50522	55900	43026
25138	26810	07093	15677	60688	04410	24505	37890	67186	62829
84631	71882	12991	83028	82484	90339	91950	74579	03539	90122
34003	92326	12793	61453	48121	74271	28363	66561	75220	35908
53775	45749	05734	86169	42762	70175	97310	73894	88606	19994
59316	97885	72807	54966	60859	11932	35265	71601	55577	67715
20479	66557	50705	26999	09854	52591	14063	30214	19890	19292
86180	84931	25455	26044	02227	52015	21820	50599	51671	65411
21451	68001	72710	40261	61281	13172	63819	48970	51732	54113
98062	68375	80089	24135	72355	95428	11808	29740	81644	86610
01788	64429	14430	94575	75153	94576	61393	96192	03227	32258
62465	04841	43272	68702	01274	05437	22953	18946	99053	41690
94324	31089	84159	92933	99989	89500	91586	02802	69471	68274
05797	43984	21575	09908	70221	19791	51578	36432	33494	79888
10395	14289	52185	09721	25789	38562	54794	04897	59012	89251
35177	56986	25549	59730	64718	52630	31100	62384	49483	11409
25633	89619	75882	98256	02126	72099	57183	55887	09320	73463
16464	48280	94254	45777	45150	68865	11382	11782	22695	41988

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TABLE 2 **Binomial Coefficients $C_{n,r}$**

n \ r	0	1	2	3	4	5	6	7	8	9	10
1	1	1									
2	1	2	1								
3	1	3	3	1							
4	1	4	6	4	1						
5	1	5	10	10	5	1					
6	1	6	15	20	15	6	1				
7	1	7	21	35	35	21	7	1			
8	1	8	28	56	70	56	28	8	1		
9	1	9	36	84	126	126	84	36	9	1	
10	1	10	45	120	210	252	210	120	45	10	1
11	1	11	55	165	330	462	462	330	165	55	11
12	1	12	66	220	495	792	924	792	495	220	66
13	1	13	78	286	715	1,287	1,716	1,716	1,287	715	286
14	1	14	91	364	1,001	2,002	3,003	3,432	3,003	2,002	1,001
15	1	15	105	455	1,365	3,003	5,005	6,435	6,435	5,005	3,003
16	1	16	120	560	1,820	4,368	8,008	11,440	12,870	11,440	8,008
17	1	17	136	680	2,380	6,188	12,376	19,448	24,310	24,310	19,448
18	1	18	153	816	3,060	8,568	18,564	31,824	43,758	48,620	43,758
19	1	19	171	969	3,876	11,628	27,132	50,388	75,582	92,378	92,378
20	1	20	190	1,140	4,845	15,504	38,760	77,520	125,970	167,960	184,756

TABLE 3 **Binomial Probability Distribution $C_{n,r}p^r q^{n-r}$**

This table shows the probability of *r* successes in *n* independent trials, each with probability of success *p*.

n	*r*	.01	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70	.75	.80	.85	.90	.95
2	0	.980	.902	.810	.723	.640	.563	.490	.423	.360	.303	.250	.203	.160	.123	.090	.063	.040	.023	.010	.002
	1	.020	.095	.180	.255	.320	.375	.420	.455	.480	.495	.500	.495	.480	.455	.420	.375	.320	.255	.180	.095
	2	.000	.002	.010	.023	.040	.063	.090	.123	.160	.203	.250	.303	.360	.423	.490	.563	.640	.723	.810	.902
3	0	.970	.857	.729	.614	.512	.422	.343	.275	.216	.166	.125	.091	.064	.043	.027	.016	.008	.003	.001	.000
	1	.029	.135	.243	.325	.384	.422	.441	.444	.432	.408	.375	.334	.288	.239	.189	.141	.096	.057	.027	.007
	2	.000	.007	.027	.057	.096	.141	.189	.239	.288	.334	.375	.408	.432	.444	.441	.422	.384	.325	.243	.135
	3	.000	.000	.001	.003	.008	.016	.027	.043	.064	.091	.125	.166	.216	.275	.343	.422	.512	.614	.729	.857
4	0	.961	.815	.656	.522	.410	.316	.240	.179	.130	.092	.062	.041	.026	.015	.008	.004	.002	.001	.000	.000
	1	.039	.171	.292	.368	.410	.422	.412	.384	.346	.300	.250	.200	.154	.112	.076	.047	.026	.011	.004	.000
	2	.001	.014	.049	.098	.154	.211	.265	.311	.346	.368	.375	.368	.346	.311	.265	.211	.154	.098	.049	.014
	3	.000	.000	.004	.011	.026	.047	.076	.112	.154	.200	.250	.300	.346	.384	.412	.422	.410	.368	.292	.171
	4	.000	.000	.000	.001	.002	.004	.008	.015	.026	.041	.062	.092	.130	.179	.240	.316	.410	.522	.656	.815
5	0	.951	.774	.590	.444	.328	.237	.168	.116	.078	.050	.031	.019	.010	.005	.002	.001	.000	.000	.000	.000
	1	.048	.204	.328	.392	.410	.396	.360	.312	.259	.206	.156	.113	.077	.049	.028	.015	.006	.002	.000	.000
	2	.001	.021	.073	.138	.205	.264	.309	.336	.346	.337	.312	.276	.230	.181	.132	.088	.051	.024	.008	.001
	3	.000	.001	.008	.024	.051	.088	.132	.181	.230	.276	.312	.337	.346	.336	.309	.264	.205	.138	.073	.021
	4	.000	.000	.000	.002	.006	.015	.028	.049	.077	.113	.156	.206	.259	.312	.360	.396	.410	.392	.328	.204
	5	.000	.000	.000	.000	.000	.001	.002	.005	.010	.019	.031	.050	.078	.116	.168	.237	.328	.444	.590	.774
6	0	.941	.735	.531	.377	.262	.178	.118	.075	.047	.028	.016	.008	.004	.002	.001	.000	.000	.000	.000	.000
	1	.057	.232	.354	.399	.393	.356	.303	.244	.187	.136	.094	.061	.037	.020	.010	.004	.002	.000	.000	.000
	2	.001	.031	.098	.176	.246	.297	.324	.328	.311	.278	.234	.186	.138	.095	.060	.033	.015	.006	.001	.000
	3	.000	.002	.015	.042	.082	.132	.185	.236	.276	.303	.312	.303	.276	.236	.185	.132	.082	.042	.015	.002
	4	.000	.000	.001	.006	.015	.033	.060	.095	.138	.186	.234	.278	.311	.328	.324	.297	.246	.176	.098	.031
	5	.000	.000	.000	.000	.002	.004	.010	.020	.037	.061	.094	.136	.187	.244	.303	.356	.393	.399	.354	.232
	6	.000	.000	.000	.000	.000	.000	.001	.002	.004	.008	.016	.028	.047	.075	.118	.178	.262	.377	.531	.735
7	0	.932	.698	.478	.321	.210	.133	.082	.049	.028	.015	.008	.004	.002	.001	.000	.000	.000	.000	.000	.000
	1	.066	.257	.372	.396	.367	.311	.247	.185	.131	.087	.055	.032	.017	.008	.004	.001	.000	.000	.000	.000
	2	.002	.041	.124	.210	.275	.311	.318	.299	.261	.214	.164	.117	.077	.047	.025	.012	.004	.001	.000	.000
	3	.000	.004	.023	.062	.115	.173	.227	.268	.290	.292	.273	.239	.194	.144	.097	.058	.029	.011	.003	.000
	4	.000	.000	.003	.011	.029	.058	.097	.144	.194	.239	.273	.292	.290	.268	.227	.173	.115	.062	.023	.004
	5	.000	.000	.000	.001	.004	.012	.025	.047	.077	.117	.164	.214	.261	.299	.318	.311	.275	.210	.124	.041
	6	.000	.000	.000	.000	.000	.001	.004	.008	.017	.032	.055	.087	.131	.185	.247	.311	.367	.396	.372	.257
	7	.000	.000	.000	.000	.000	.000	.000	.001	.002	.004	.008	.015	.028	.049	.082	.133	.210	.321	.478	.698

TABLE 3 *continued*

		p																			
n	*r*	.01	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70	.75	.80	.85	.90	.95
8	0	.923	.663	.430	.272	.168	.100	.058	.032	.017	.008	.004	.002	.001	.000	.000	.000	.000	.000	.000	.000
	1	.075	.279	.383	.385	.336	.267	.198	.137	.090	.055	.031	.016	.008	.003	.001	.000	.000	.000	.000	.000
	2	.003	.051	.149	.238	.294	.311	.296	.259	.209	.157	.109	.070	.041	.022	.010	.004	.001	.000	.000	.000
	3	.000	.005	.033	.084	.147	.208	.254	.279	.279	.257	.219	.172	.124	.081	.047	.023	.009	.003	.000	.000
	4	.000	.000	.005	.018	.046	.087	.136	.188	.232	.263	.273	.263	.232	.188	.136	.087	.046	.018	.005	.000
	5	.000	.000	.000	.003	.009	.023	.047	.081	.124	.172	.219	.257	.279	.279	.254	.208	.147	.084	.033	.005
	6	.000	.000	.000	.000	.001	.004	.010	.022	.041	.070	.109	.157	.209	.259	.296	.311	.294	.238	.149	.051
	7	.000	.000	.000	.000	.000	.000	.001	.003	.008	.016	.031	.055	.090	.137	.198	.267	.336	.385	.383	.279
	8	.000	.000	.000	.000	.000	.000	.000	.000	.001	.002	.004	.008	.017	.032	.058	.100	.168	.272	.430	.663
9	0	.914	.630	.387	.232	.134	.075	.040	.021	.010	.005	.002	.001	.000	.000	.000	.000	.000	.000	.000	.000
	1	.083	.299	.387	.368	.302	.225	.156	.100	.060	.034	.018	.008	.004	.001	.000	.000	.000	.000	.000	.000
	2	.003	.063	.172	.260	.302	.300	.267	.216	.161	.111	.070	.041	.021	.010	.004	.001	.000	.000	.000	.000
	3	.000	.008	.045	.107	.176	.234	.267	.272	.251	.212	.164	.116	.074	.042	.021	.009	.003	.001	.000	.000
	4	.000	.001	.007	.028	.066	.117	.172	.219	.251	.260	.246	.213	.167	.118	.074	.039	.017	.005	.001	.000
	5	.000	.000	.001	.005	.017	.039	.074	.118	.167	.213	.246	.260	.251	.219	.172	.117	.066	.028	.007	.001
	6	.000	.000	.000	.001	.003	.009	.021	.042	.074	.116	.164	.212	.251	.272	.267	.234	.176	.107	.045	.008
	7	.000	.000	.000	.000	.000	.001	.004	.010	.021	.041	.070	.111	.161	.216	.267	.300	.302	.260	.172	.063
	8	.000	.000	.000	.000	.000	.000	.000	.001	.004	.008	.018	.034	.060	.100	.156	.225	.302	.368	.387	.299
	9	.000	.000	.000	.000	.000	.000	.000	.000	.000	.001	.002	.005	.010	.021	.040	.075	.134	.232	.387	.630
10	0	.904	.599	.349	.197	.107	.056	.028	.014	.006	.003	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000
	1	.091	.315	.387	.347	.268	.188	.121	.072	.040	.021	.010	.004	.002	.000	.000	.000	.000	.000	.000	.000
	2	.004	.075	.194	.276	.302	.282	.233	.176	.121	.076	.044	.023	.011	.004	.001	.000	.000	.000	.000	.000
	3	.000	.010	.057	.130	.201	.250	.267	.252	.215	.166	.117	.075	.042	.021	.009	.003	.001	.000	.000	.000
	4	.000	.001	.011	.040	.088	.146	.200	.238	.251	.238	.205	.160	.111	.069	.037	.016	.006	.001	.000	.000
	5	.000	.000	.001	.008	.026	.058	.103	.154	.201	.234	.246	.234	.201	.154	.103	.058	.026	.008	.001	.000
	6	.000	.000	.000	.001	.006	.016	.037	.069	.111	.160	.205	.238	.251	.238	.200	.146	.088	.040	.011	.001
	7	.000	.000	.000	.000	.001	.003	.009	.021	.042	.075	.117	.166	.215	.252	.267	.250	.201	.130	.057	.010
	8	.000	.000	.000	.000	.000	.000	.001	.004	.011	.023	.044	.076	.121	.176	.233	.282	.302	.276	.194	.075
	9	.000	.000	.000	.000	.000	.000	.000	.000	.002	.004	.010	.021	.040	.072	.121	.188	.268	.347	.387	.315
	10	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.001	.003	.006	.014	.028	.056	.107	.197	.349	.599
11	0	.895	.569	.314	.167	.086	.042	.020	.009	.004	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	1	.099	.329	.384	.325	.236	.155	.093	.052	.027	.013	.005	.002	.001	.000	.000	.000	.000	.000	.000	.000
	2	.005	.087	.213	.287	.295	.258	.200	.140	.089	.051	.027	.013	.005	.002	.001	.000	.000	.000	.000	.000

TABLE 3 *continued*

		p																			
n	*r*	.01	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70	.75	.80	.85	.90	.95
11	3	.000	.014	.071	.152	.221	.258	.257	.225	.177	.126	.081	.046	.023	.010	.004	.001	.000	.000	.000	.000
	4	.000	.001	.016	.054	.111	.172	.220	.243	.236	.206	.161	.113	.070	.038	.017	.006	.002	.000	.000	.000
	5	.000	.000	.002	.013	.039	.080	.132	.183	.221	.236	.226	.193	.147	.099	.057	.027	.010	.002	.000	.000
	6	.000	.000	.000	.002	.010	.027	.057	.099	.147	.193	.226	.236	.221	.183	.132	.080	.039	.013	.002	.000
	7	.000	.000	.000	.000	.002	.006	.017	.038	.070	.113	.161	.206	.236	.243	.220	.172	.111	.054	.016	.001
	8	.000	.000	.000	.000	.000	.001	.004	.010	.023	.046	.081	.126	.177	.225	.257	.258	.221	.152	.071	.014
	9	.000	.000	.000	.000	.000	.000	.001	.002	.005	.013	.027	.051	.089	.140	.200	.258	.295	.287	.213	.087
	10	.000	.000	.000	.000	.000	.000	.000	.000	.001	.002	.005	.013	.027	.052	.093	.155	.236	.325	.384	.329
	11	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.001	.004	.009	.020	.042	.086	.167	.314	.569
12	0	.886	.540	.282	.142	.069	.032	.014	.006	.002	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	1	.107	.341	.377	.301	.206	.127	.071	.037	.017	.008	.003	.001	.000	.000	.000	.000	.000	.000	.000	.000
	2	.006	.099	.230	.292	.283	.232	.168	.109	.064	.034	.016	.007	.002	.001	.000	.000	.000	.000	.000	.000
	3	.000	.017	.085	.172	.236	.258	.240	.195	.142	.092	.054	.028	.012	.005	.001	.000	.000	.000	.000	.000
	4	.000	.002	.021	.068	.133	.194	.231	.237	.213	.170	.121	.076	.042	.020	.008	.002	.001	.000	.000	.000
	5	.000	.000	.004	.019	.053	.103	.158	.204	.227	.223	.193	.149	.101	.059	.029	.011	.003	.001	.000	.000
	6	.000	.000	.000	.004	.016	.040	.079	.128	.177	.212	.226	.212	.177	.128	.079	.040	.016	.004	.000	.000
	7	.000	.000	.000	.001	.003	.011	.029	.059	.101	.149	.193	.223	.227	.204	.158	.103	.053	.019	.004	.000
	8	.000	.000	.000	.000	.001	.002	.008	.020	.042	.076	.121	.170	.213	.237	.231	.194	.133	.068	.021	.002
	9	.000	.000	.000	.000	.000	.000	.001	.005	.012	.028	.054	.092	.142	.195	.240	.258	.236	.172	.085	.017
	10	.000	.000	.000	.000	.000	.000	.000	.001	.002	.007	.016	.034	.064	.109	.168	.232	.283	.292	.230	.099
	11	.000	.000	.000	.000	.000	.000	.000	.000	.000	.001	.003	.008	.017	.037	.071	.127	.206	.301	.377	.341
	12	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.001	.002	.006	.014	.032	.069	.142	.282	.540
15	0	.860	.463	.206	.087	.035	.013	.005	.002	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	1	.130	.366	.343	.231	.132	.067	.031	.013	.005	.002	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	2	.009	.135	.267	.286	.231	.156	.092	.048	.022	.009	.003	.001	.000	.000	.000	.000	.000	.000	.000	.000
	3	.000	.031	.129	.218	.250	.225	.170	.111	.063	.032	.014	.005	.002	.000	.000	.000	.000	.000	.000	.000
	4	.000	.005	.043	.116	.188	.225	.219	.179	.127	.078	.042	.019	.007	.002	.001	.000	.000	.000	.000	.000
	5	.000	.001	.010	.045	.103	.165	.206	.212	.186	.140	.092	.051	.024	.010	.003	.001	.000	.000	.000	.000
	6	.000	.000	.002	.013	.043	.092	.147	.191	.207	.191	.153	.105	.061	.030	.012	.003	.001	.000	.000	.000
	7	.000	.000	.000	.003	.014	.039	.081	.132	.177	.201	.196	.165	.118	.071	.035	.013	.003	.001	.000	.000
	8	.000	.000	.000	.001	.003	.013	.035	.071	.118	.165	.196	.201	.177	.132	.081	.039	.014	.003	.000	.000
	9	.000	.000	.000	.000	.001	.003	.012	.030	.061	.105	.153	.191	.207	.191	.147	.092	.043	.013	.002	.000
	10	.000	.000	.000	.000	.000	.001	.003	.010	.024	.051	.092	.140	.186	.212	.206	.165	.103	.045	.010	.001

TABLE 3 *continued*

n	r	p .01	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70	.75	.80	.85	.90	.95
15	11	.000	.000	.000	.000	.000	.000	.001	.002	.007	.019	.042	.078	.127	.179	.219	.225	.188	.116	.043	.005
	12	.000	.000	.000	.000	.000	.000	.000	.000	.002	.005	.014	.032	.063	.111	.170	.225	.250	.218	.129	.031
	13	.000	.000	.000	.000	.000	.000	.000	.000	.000	.001	.003	.009	.022	.048	.092	.156	.231	.286	.267	.135
	14	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.002	.005	.013	.031	.067	.132	.231	.343	.366
	15	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.002	.005	.013	.035	.087	.206	.463
16	0	.851	.440	.185	.074	.028	.010	.003	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	1	.138	.371	.329	.210	.113	.053	.023	.009	.003	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	2	.010	.146	.275	.277	.211	.134	.073	.035	.015	.006	.002	.001	.000	.000	.000	.000	.000	.000	.000	.000
	3	.000	.036	.142	.229	.246	.208	.146	.089	.047	.022	.009	.003	.001	.000	.000	.000	.000	.000	.000	.000
	4	.000	.006	.051	.131	.200	.225	.204	.155	.101	.057	.028	.011	.004	.001	.000	.000	.000	.000	.000	.000
	5	.000	.001	.014	.056	.120	.180	.210	.201	.162	.112	.067	.034	.014	.005	.001	.000	.000	.000	.000	.000
	6	.000	.000	.003	.018	.055	.110	.165	.198	.198	.168	.122	.075	.039	.017	.006	.001	.000	.000	.000	.000
	7	.000	.000	.000	.005	.020	.052	.101	.152	.189	.197	.175	.132	.084	.044	.019	.006	.001	.000	.000	.000
	8	.000	.000	.000	.001	.006	.020	.049	.092	.142	.181	.196	.181	.142	.092	.049	.020	.006	.001	.000	.000
	9	.000	.000	.000	.000	.001	.006	.019	.044	.084	.132	.175	.197	.189	.152	.101	.052	.020	.005	.000	.000
	10	.000	.000	.000	.000	.000	.001	.006	.017	.039	.075	.122	.168	.198	.198	.165	.110	.055	.018	.003	.000
	11	.000	.000	.000	.000	.000	.000	.001	.005	.014	.034	.067	.112	.162	.201	.210	.180	.120	.056	.014	.001
	12	.000	.000	.000	.000	.000	.000	.000	.001	.004	.011	.028	.057	.101	.155	.204	.225	.200	.131	.051	.006
	13	.000	.000	.000	.000	.000	.000	.000	.000	.001	.003	.009	.022	.047	.089	.146	.208	.246	.229	.142	.036
	14	.000	.000	.000	.000	.000	.000	.000	.000	.000	.001	.002	.006	.015	.035	.073	.134	.211	.277	.275	.146
	15	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.001	.003	.009	.023	.053	.113	.210	.329	.371
	16	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.001	.003	.010	.028	.074	.185	.440
20	0	.818	.358	.122	.039	.012	.003	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	1	.165	.377	.270	.137	.058	.021	.007	.002	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	2	.016	.189	.285	.229	.137	.067	.028	.010	.003	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	3	.001	.060	.190	.243	.205	.134	.072	.032	.012	.004	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000
	4	.000	.013	.090	.182	.218	.190	.130	.074	.035	.014	.005	.001	.000	.000	.000	.000	.000	.000	.000	.000
	5	.000	.002	.032	.103	.175	.202	.179	.127	.075	.036	.015	.005	.001	.000	.000	.000	.000	.000	.000	.000
	6	.000	.000	.009	.045	.109	.169	.192	.171	.124	.075	.036	.015	.005	.001	.000	.000	.000	.000	.000	.000
	7	.000	.000	.002	.016	.055	.112	.164	.184	.166	.122	.074	.037	.015	.005	.001	.000	.000	.000	.000	.000
	8	.000	.000	.000	.005	.022	.061	.114	.161	.180	.162	.120	.073	.035	.014	.004	.001	.000	.000	.000	.000
	9	.000	.000	.000	.001	.007	.027	.065	.116	.160	.177	.160	.119	.071	.034	.012	.003	.000	.000	.000	.000

TABLE 3 *continued*

		p																		
n	*r*	.01	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70	.75	.80	.85	.90	.95
20	10	.000	.000	.000	.000	.002	.010	.031	.069	.117	.159	.176	.159	.117	.069	.031	.010	.002	.000	.000	.000
	11	.000	.000	.000	.000	.000	.003	.012	.034	.071	.119	.160	.177	.160	.116	.065	.027	.007	.001	.000	.000
	12	.000	.000	.000	.000	.000	.001	.004	.014	.035	.073	.120	.162	.180	.161	.114	.061	.022	.005	.000	.000
	13	.000	.000	.000	.000	.000	.000	.001	.005	.015	.037	.074	.122	.166	.184	.164	.112	.055	.016	.002	.000
	14	.000	.000	.000	.000	.000	.000	.000	.001	.005	.015	.037	.075	.124	.171	.192	.169	.109	.045	.009	.000
	15	.000	.000	.000	.000	.000	.000	.000	.000	.001	.005	.015	.036	.075	.127	.179	.202	.175	.103	.032	.002
	16	.000	.000	.000	.000	.000	.000	.000	.000	.000	.001	.005	.014	.035	.074	.130	.190	.218	.182	.090	.013
	17	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.001	.004	.012	.032	.072	.134	.205	.243	.190	.060
	18	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.001	.003	.010	.028	.067	.137	.229	.285	.189
	19	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.002	.007	.021	.058	.137	.270	.377
	20	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.001	.003	.012	.039	.122	.358

TABLE 4 Poisson Probability Distribution

For a given value of λ, entry indicates the probability of obtaining a specified value of r.

	λ									
r	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	.9048	.8187	.7408	.6703	.6065	.5488	.4966	.4493	.4066	.3679
1	.0905	.1637	.2222	.2681	.3033	.3293	.3476	.3595	.3659	.3679
2	.0045	.0164	.0333	.0536	.0758	.0988	.1217	.1438	.1647	.1839
3	.0002	.0011	.0033	.0072	.0126	.0198	.0284	.0383	.0494	.0613
4	.0000	.0001	.0003	.0007	.0016	.0030	.0050	.0077	.0111	.0153
5	.0000	.0000	.0000	.0001	.0002	.0004	.0007	.0012	.0020	.0031
6	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0003	.0005
7	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001

	λ									
r	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
0	.3329	.3012	.2725	.2466	.2231	.2019	.1827	.1653	.1496	.1353
1	.3662	.3614	.3543	.3452	.3347	.3230	.3106	.2975	.2842	.2707
2	.2014	.2169	.2303	.2417	.2510	.2584	.2640	.2678	.2700	.2707
3	.0738	.0867	.0998	.1128	.1255	.1378	.1496	.1607	.1710	.1804
4	.0203	.0260	.0324	.0395	.0471	.0551	.0636	.0723	.0812	.0902
5	.0045	.0062	.0084	.0111	.0141	.0176	.0216	.0260	.0309	.0361
6	.0008	.0012	.0018	.0026	.0035	.0047	.0061	.0078	.0098	.0120
7	.0001	.0002	.0003	.0005	.0008	.0011	.0015	.0020	.0027	.0034
8	.0000	.0000	.0001	.0001	.0001	.0002	.0003	.0005	.0006	.0009
9	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0002

	λ									
r	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
0	.1225	.1108	.1003	.0907	.0821	.0743	.0672	.0608	.0550	.0498
1	.2572	.2438	.2306	.2177	.2052	.1931	.1815	.1703	.1596	.1494
2	.2700	.2681	.2652	.2613	.2565	.2510	.2450	.2384	.2314	.2240
3	.1890	.1966	.2033	.2090	.2138	.2176	.2205	.2225	.2237	.2240
4	.0992	.1082	.1169	.1254	.1336	.1414	.1488	.1557	.1622	.1680
5	.0417	.0476	.0538	.0602	.0668	.0735	.0804	.0872	.0940	.1008
6	.0146	.0174	.0206	.0241	.0278	.0319	.0362	.0407	.0455	.0504
7	.0044	.0055	.0068	.0083	.0099	.0118	.0139	.0163	.0188	.0216
8	.0011	.0015	.0019	.0025	.0031	.0038	.0047	.0057	.0068	.0081
9	.0003	.0004	.0005	.0007	.0009	.0011	.0014	.0018	.0022	.0027
10	.0001	.0001	.0001	.0002	.0002	.0003	.0004	.0005	.0006	.0008
11	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0002	.0002
12	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001

TABLE 4 *continued*

r	λ 3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0
0	.0450	.0408	.0369	.0334	.0302	.0273	.0247	.0224	.0202	.0183
1	.1397	.1304	.1217	.1135	.1057	.0984	.0915	.0850	.0789	.0733
2	.2165	.2087	.2008	.1929	.1850	.1771	.1692	.1615	.1539	.1465
3	.2237	.2226	.2209	.2186	.2158	.2125	.2087	.2046	.2001	.1954
4	.1734	.1781	.1823	.1858	.1888	.1912	.1931	.1944	.1951	.1954
5	.1075	.1140	.1203	.1264	.1322	.1377	.1429	.1477	.1522	.1563
6	.0555	.0608	.0662	.0716	.0771	.0826	.0881	.0936	.0989	.1042
7	.0246	.2078	.0312	.0348	.0385	.0425	.0466	.0508	.0551	.0595
8	.0095	.0111	.0129	.0148	.0169	.0191	.0215	.0241	.0269	.0298
9	.0033	.0040	.0047	.0056	.0066	.0076	.0089	.0102	.0116	.0132
10	.0010	.0013	.0016	.0019	.0023	.0028	.0033	.0039	.0045	.0053
11	.0003	.0004	.0005	.0006	.0007	.0009	.0011	.0013	.0016	.0019
12	.0001	.0001	.0001	.0002	.0002	.0003	.0003	.0004	.0005	.0006
13	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001	.0002	.0002
14	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001

r	λ 4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0
0	.0166	.0150	.0136	.0123	.0111	.0101	.0091	.0082	.0074	.0067
1	.0679	.0630	.0583	.0540	.0500	.0462	.0427	.0395	.0365	.0337
2	.1393	.1323	.1254	.1188	.1125	.1063	.1005	.0948	.0894	.0842
3	.1904	.1852	.1798	.1743	.1687	.1631	.1574	.1517	.1460	.1404
4	.1951	.1944	.1933	.1917	.1898	.1875	.1849	.1820	.1789	.1755
5	.1600	.1633	.1662	.1687	.1708	.1725	.1738	.1747	.1753	.1755
6	.1093	.1143	.1191	.1237	.1281	.1323	.1362	.1398	.1432	.1462
7	.0640	.0686	.0732	.0778	.0824	.0869	.0914	.0959	.1002	.1044
8	.0328	.0360	.0393	.0428	.0463	.0500	.0537	.0575	.0614	.0653
9	.0150	.0168	.0188	.0209	.0232	.0255	.0280	.0307	.0334	.0363
10	.0061	.0071	.0081	.0092	.0104	.0118	.0132	.0147	.0164	.0181
11	.0023	.0027	.0032	.0037	.0043	.0049	.0056	.0064	.0073	.0082
12	.0008	.0009	.0011	.0014	.0016	.0019	.0022	.0026	.0030	.0034
13	.0002	.0003	.0004	.0005	.0006	.0007	.0008	.0009	.0011	.0013
14	.0001	.0001	.0001	.0001	.0002	.0002	.0003	.0003	.0004	.0005
15	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001	.0001	.0002

TABLE 4 *continued*

r	λ = 5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0
0	.0061	.0055	.0050	.0045	.0041	.0037	.0033	.0030	.0027	.0025
1	.0311	.0287	.0265	.0244	.0225	.0207	.0191	.0176	.0162	.0149
2	.0793	.0746	.0701	.0659	.0618	.0580	.0544	.0509	.0477	.0446
3	.1348	.1293	.1239	.1185	.1133	.1082	.1033	.0985	.0938	.0892
4	.1719	.1681	.1641	.1600	.1558	.1515	.1472	.1428	.1383	.1339
5	.1753	.1748	.1740	.1728	.1714	.1697	.1678	.1656	.1632	.1606
6	.1490	.1515	.1537	.1555	.1571	.1584	.1594	.1601	.1605	.1606
7	.1086	.1125	.1163	.1200	.1234	.1267	.1298	.1326	.1353	.1377
8	.0692	.0731	.0771	.0810	.0849	.0887	.0925	.0962	.0998	.1033
9	.0392	.0423	.0454	.0486	.0519	.0552	.0586	.0620	.0654	.0688
10	.0200	.0220	.0241	.0262	.0285	.0309	.0334	.0359	.0386	.0413
11	.0093	.0104	.0116	.0129	.0143	.0157	.0173	.0190	.0207	.0225
12	.0039	.0045	.0051	.0058	.0065	.0073	.0082	.0092	.0102	.0113
13	.0015	.0018	.0021	.0024	.0028	.0032	.0036	.0041	.0046	.0052
14	.0006	.0007	.0008	.0009	.0011	.0013	.0015	.0017	.0019	.0022
15	.0002	.0002	.0003	.0003	.0004	.0005	.0006	.0007	.0008	.0009
16	.0001	.0001	.0001	.0001	.0001	.0002	.0002	.0002	.0003	.0003
17	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001

r	λ = 6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	7.0
0	.0022	.0020	.0018	.0017	.0015	.0014	.0012	.0011	.0010	.0009
1	.0137	.0126	.0116	.0106	.0098	.0090	.0082	.0076	.0070	.0064
2	.0417	.0390	.0364	.0340	.0318	.0296	.0276	.0258	.0240	.0223
3	.0848	.0806	.0765	.0726	.0688	.0652	.0617	.0584	.0552	.0521
4	.1294	.1249	.1205	.1162	.1118	.1076	.1034	.0992	.0952	.0912
5	.1579	.1549	.1519	.1487	.1454	.1420	.1385	.1349	.1314	.1277
6	.1605	.1601	.1595	.1586	.1575	.1562	.1546	.1529	.1511	.1490
7	.1399	.1418	.1435	.1450	.1462	.1472	.1480	.1486	.1489	.1490
8	.1066	.1099	.1130	.1160	.1188	.1215	.1240	.1263	.1284	.1304
9	.0723	.0757	.0791	.0825	.0858	.0891	.0923	.0954	.0985	.1014
10	.0441	.0469	.0498	.0528	.0558	.0588	.0618	.0649	.0679	.0710
11	.0245	.0265	.0285	.0307	.0330	.0353	.0377	.0401	.0426	.0452
12	.0124	.0137	.0150	.0164	.0179	.0194	.0210	.0227	.0245	.0264
13	.0058	.0065	.0073	.0081	.0089	.0098	.0108	.0119	.0130	.0142
14	.0025	.0029	.0033	.0037	.0041	.0046	.0052	.0058	.0064	.0071
15	.0010	.0012	.0014	.0016	.0018	.0020	.0023	.0026	.0029	.0033
16	.0004	.0005	.0005	.0006	.0007	.0008	.0010	.0011	.0013	.0014
17	.0001	.0002	.0002	.0002	.0003	.0003	.0004	.0004	.0005	.0006
18	.0000	.0001	.0001	.0001	.0001	.0001	.0001	.0002	.0002	.0002
19	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001

TABLE 4 *continued*

	λ									
r	7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	8.0
0	.0008	.0007	.0007	.0006	.0006	.0005	.0005	.0004	.0004	.0003
1	.0059	.0054	.0049	.0045	.0041	.0038	.0035	.0032	.0029	.0027
2	.0208	.0194	.0180	.0167	.0156	.0145	.0134	.0125	.0116	.0107
3	.0492	.0464	.0438	.0413	.0389	.0366	.0345	.0324	.0305	.0286
4	.0874	.0836	.0799	.0764	.0729	.0696	.0663	.0632	.0602	.0573
5	.1241	.1204	.1167	.1130	.1094	.1057	.1021	.0986	.0951	.0916
6	.1468	.1445	.1420	.1394	.1367	.1339	.1311	.1282	.1252	.1221
7	.1489	.1486	.1481	.1474	.1465	.1454	.1442	.1428	.1413	.1396
8	.1321	.1337	.1351	.1363	.1373	.1382	.1388	.1392	.1395	.1396
9	.1042	.1070	.1096	.1121	.1144	.1167	.1187	.1207	.1224	.1241
10	.0740	.0770	.0800	.0829	.0858	.0887	.0914	.0941	.0967	.0993
11	.0478	.0504	.0531	.0558	.0585	.0613	.0640	.0667	.0695	.0722
12	.0283	.0303	.0323	.0344	.0366	.0388	.0411	.0434	.0457	.0481
13	.0154	.0168	.0181	.0196	.0211	.0227	.0243	.0260	.0278	.0296
14	.0078	.0086	.0095	.0104	.0113	.0123	.0134	.0145	.0157	.0169
15	.0037	.0041	.0046	.0051	.0057	.0062	.0069	.0075	.0083	.0090
16	.0016	.0019	.0021	.0024	.0026	.0030	.0033	.0037	.0041	.0045
17	.0007	.0008	.0009	.0010	.0012	.0013	.0015	.0017	.0019	.0021
18	.0003	.0003	.0004	.0004	.0005	.0006	.0006	.0007	.0008	.0009
19	.0001	.0001	.0001	.0002	.0002	.0002	.0003	.0003	.0003	.0004
20	.0000	.0000	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0002
21	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001

	λ									
r	8.1	8.2	8.3	8.4	8.5	8.6	8.7	8.8	8.9	9.0
0	.0003	.0003	.0002	.0002	.0002	.0002	.0002	.0002	.0001	.0001
1	.0025	.0023	.0021	.0019	.0017	.0016	.0014	.0013	.0012	.0011
2	.0100	.0092	.0086	.0079	.0074	.0068	.0063	.0058	.0054	.0050
3	.0269	.0252	.0237	.0222	.0208	.0195	.0183	.0171	.0160	.0150
4	.0544	.0517	.0491	.0466	.0443	.0420	.0398	.0377	.0357	.0337
5	.0882	.0849	.0816	.0784	.0752	.0722	.0692	.0663	.0635	.0607
6	.1191	.1160	.1128	.1097	.1066	.1034	.1003	.0972	.0941	.0911
7	.1378	.1358	.1338	.1317	.1294	.1271	.1247	.1222	.1197	.1171
8	.1395	.1392	.1388	.1382	.1375	.1366	.1356	.1344	.1332	.1318
9	.1256	.1269	.1280	.1290	.1299	.1306	.1311	.1315	.1317	.1318
10	.1017	.1040	.1063	.1084	.1104	.1123	.1140	.1157	.1172	.1186
11	.0749	.0776	.0802	.0828	.0853	.0878	.0902	.0925	.0948	.0970
12	.0505	.0530	.0555	.0579	.0604	.0629	.0654	.0679	.0703	.0728
13	.0315	.0334	.0354	.0374	.0395	.0416	.0438	.0459	.0481	.0504
14	.0182	.0196	.0210	.0225	.0240	.0256	.0272	.0289	.0306	.0324
15	.0098	.0107	.0116	.0126	.0136	.0147	.0158	.0169	.0182	.0194

TABLE 4 *continued*

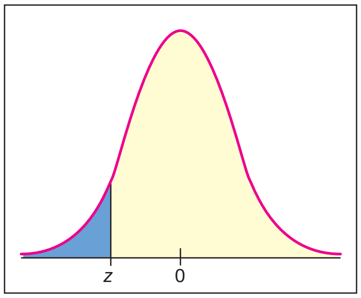
	λ									
r	8.1	8.2	8.3	8.4	8.5	8.6	8.7	8.8	8.9	9.0
16	.0050	.0055	.0060	.0066	.0072	.0079	.0086	.0093	.0101	.0109
17	.0024	.0026	.0029	.0033	.0036	.0040	.0044	.0048	.0053	.0058
18	.0011	.0012	.0014	.0015	.0017	.0019	.0021	.0024	.0026	.0029
19	.0005	.0005	.0006	.0007	.0008	.0009	.0010	.0011	.0012	.0014
20	.0002	.0002	.0002	.0003	.0003	.0004	.0004	.0005	.0005	.0006
21	.0001	.0001	.0001	.0001	.0001	.0002	.0002	.0002	.0002	.0003
22	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001	.0001	.0001

	λ									
r	9.1	9.2	9.3	9.4	9.5	9.6	9.7	9.8	9.9	10
0	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0000
1	.0010	.0009	.0009	.0008	.0007	.0007	.0006	.0005	.0005	.0005
2	.0046	.0043	.0040	.0037	.0034	.0031	.0029	.0027	.0025	.0023
3	.0140	.0131	.0123	.0115	.0107	.0100	.0093	.0087	.0081	.0076
4	.0319	.0302	.0285	.0269	.0254	.0240	.0226	.0213	.0201	.0189
5	.0581	.0555	.0530	.0506	.0483	.0460	.0439	.0418	.0398	.0378
6	.0881	.0851	.0822	.0793	.0764	.0736	.0709	.0682	.0656	.0631
7	.1145	.1118	.1091	.1064	.1037	.1010	.0982	.0955	.0928	.0901
8	.1302	.1286	.1269	.1251	.1232	.1212	.1191	.1170	.1148	.1126
9	.1317	.1315	.1311	.1306	.1300	.1293	.1284	.1274	.1263	.1251
10	.1198	.1210	.1219	.1228	.1235	.1241	.1245	.1249	.1250	.1251
11	.0991	.1012	.1031	.1049	.1067	.1083	.1098	.1112	.1125	.1137
12	.0752	.0776	.0799	.0822	.0844	.0866	.0888	.0908	.0928	.0948
13	.0526	.0549	.0572	.0594	.0617	.0640	.0662	.0685	.0707	.0729
14	.0342	.0361	.0380	.0399	.0419	.0439	.0459	.0479	.0500	.0521
15	.0208	.0221	.0235	.0250	.0265	.0281	.0297	.0313	.0330	.0347
16	.0118	.0127	.0137	.0147	.0157	.0168	.0180	.0192	.0204	.0217
17	.0063	.0069	.0075	.0081	.0088	.0095	.0103	.0111	.0119	.0128
18	.0032	.0035	.0039	.0042	.0046	.0051	.0055	.0060	.0065	.0071
19	.0015	.0017	.0019	.0021	.0023	.0026	.0028	.0031	.0034	.0037
20	.0007	.0008	.0009	.0010	.0011	.0012	.0014	.0015	.0017	.0019
21	.0003	.0003	.0004	.0004	.0005	.0006	.0006	.0007	.0008	.0009
22	.0001	.0001	.0002	.0002	.0002	.0002	.0003	.0003	.0004	.0004
23	.0000	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0002	.0002
24	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001

TABLE 4 *continued*

	λ									
r	11	12	13	14	15	16	17	18	19	20
0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1	.0002	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2	.0010	.0004	.0002	.0001	.0000	.0000	.0000	.0000	.0000	.0000
3	.0037	.0018	.0008	.0004	.0002	.0001	.0000	.0000	.0000	.0000
4	.0102	.0053	.0027	.0013	.0006	.0003	.0001	.0001	.0000	.0000
5	.0224	.0127	.0070	.0037	.0019	.0010	.0005	.0002	.0001	.0001
6	.0411	.0255	.0152	.0087	.0048	.0026	.0014	.0007	.0004	.0002
7	.0646	.0437	.0281	.0174	.0104	.0060	.0034	.0018	.0010	.0005
8	.0888	.0655	.0457	.0304	.0194	.0120	.0072	.0042	.0024	.0013
9	.1085	.0874	.0661	.0473	.0324	.0213	.0135	.0083	.0050	.0029
10	.1194	.1048	.0859	.0663	.0486	.0341	.0230	.0150	.0095	.0058
11	.1194	.1144	.1015	.0844	.0663	.0496	.0355	.0245	.0164	.0106
12	.1094	.1144	.1099	.0984	.0829	.0661	.0504	.0368	.0259	.0176
13	.0926	.1056	.1099	.1060	.0956	.0814	.0658	.0509	.0378	.0271
14	.0728	.0905	.1021	.1060	.1024	.0930	.0800	.0655	.0514	.0387
15	.0534	.0724	.0885	.0989	.1024	.0992	.0906	.0786	.0650	.0516
16	.0367	.0543	.0719	.0866	.0960	.0992	.0963	.0884	.0772	.0646
17	.0237	.0383	.0550	.0713	.0847	.0934	.0963	.0936	.0863	.0760
18	.0145	.0256	.0397	.0554	.0706	.0830	.0909	.0936	.0911	.0844
19	.0084	.0161	.0272	.0409	.0557	.0699	.0814	.0887	.0911	.0888
20	.0046	.0097	.0177	.0286	.0418	.0559	.0692	.0798	.0866	.0888
21	.0024	.0055	.0109	.0191	.0299	.0426	.0560	.0684	.0783	.0846
22	.0012	.0030	.0065	.0121	.0204	.0310	.0433	.0560	.0676	.0769
23	.0006	.0016	.0037	.0074	.0133	.0216	.0320	.0438	.0559	.0669
24	.0003	.0008	.0020	.0043	.0083	.0144	.0226	.0328	.0442	.0557
25	.0001	.0004	.0010	.0024	.0050	.0092	.0154	.0237	.0336	.0446
26	.0000	.0002	.0005	.0013	.0029	.0057	.0101	.0164	.0246	.0343
27	.0000	.0001	.0002	.0007	.0016	.0034	.0063	.0109	.0173	.0254
28	.0000	.0000	.0001	.0003	.0009	.0019	.0038	.0070	.0117	.0181
29	.0000	.0000	.0001	.0002	.0004	.0011	.0023	.0044	.0077	.0125
30	.0000	.0000	.0000	.0001	.0002	.0006	.0013	.0026	.0049	.0083
31	.0000	.0000	.0000	.0000	.0001	.0003	.0007	.0015	.0030	.0054
32	.0000	.0000	.0000	.0000	.0001	.0001	.0004	.0009	.0018	.0034
33	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0005	.0010	.0020
34	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0006	.0012
35	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0003	.0007
36	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0004
37	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002
38	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
39	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001

Source: Biometricka, June 1964, The χ^2 Distribution, H. L. Herter (Table 7). Used by permission of Oxford University Press.



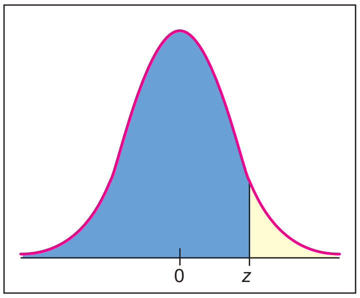
The table entry for *z* is the area to the left of *z*.

TABLE 5 Areas of a Standard Normal Distribution

(a) Table of Areas to the Left of *z*

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

For values of *z* less than −3.49, use 0.000 to approximate the area.



The table entry for z is the area to the left of z.

TABLE 5(a) *continued*

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

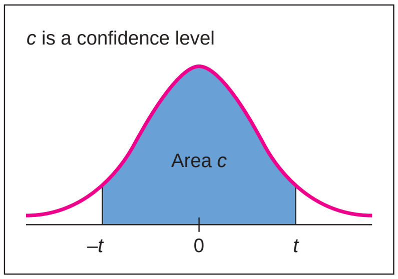
For z values greater than 3.49, use 1.000 to approximate the area.

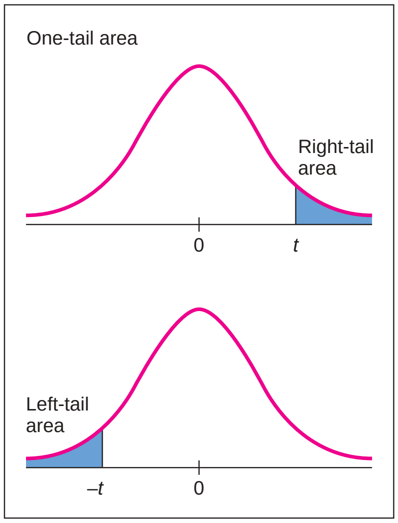
TABLE 5 *continued*

(b) Confidence Interval Critical Values z_c	
Level of Confidence c	Critical Value z_c
0.70, or 70%	1.04
0.75, or 75%	1.15
0.80, or 80%	1.28
0.85, or 85%	1.44
0.90, or 90%	1.645
0.95, or 95%	1.96
0.98, or 98%	2.33
0.99, or 99%	2.58

TABLE 5 *continued*

(c) Hypothesis Testing, Critical Values z_0		
Level of Significance	$\alpha = 0.05$	$\alpha = 0.01$
Critical value z_0 for a left-tailed test	-1.645	-2.33
Critical value z_0 for a right-tailed test	1.645	2.33
Critical values $\pm z_0$ for a two-tailed test	± 1.96	± 2.58





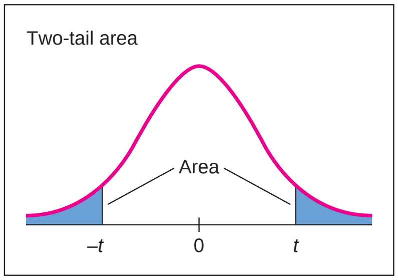
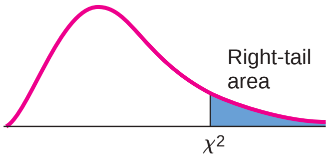


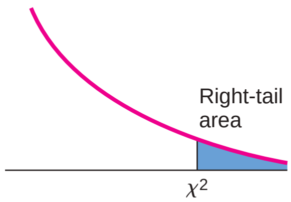
TABLE 6 Critical Values for Student's *t* Distribution

one-tail area	0.250	0.125	0.100	0.075	0.050	0.025	0.010	0.005	0.0005
two-tail area	0.500	0.250	0.200	0.150	0.100	0.050	0.020	0.010	0.0010
d.f. \ *c*	0.500	0.750	0.800	0.850	0.900	0.950	0.980	0.990	0.999
1	1.000	2.414	3.078	4.165	6.314	12.706	31.821	63.657	636.619
2	0.816	1.604	1.886	2.282	2.920	4.303	6.965	9.925	31.599
3	0.765	1.423	1.638	1.924	2.353	3.182	4.541	5.841	12.924
4	0.741	1.344	1.533	1.778	2.132	2.776	3.747	4.604	8.610
5	0.727	1.301	1.476	1.699	2.015	2.571	3.365	4.032	6.869
6	0.718	1.273	1.440	1.650	1.943	2.447	3.143	3.707	5.959
7	0.711	1.254	1.415	1.617	1.895	2.365	2.998	3.499	5.408
8	0.706	1.240	1.397	1.592	1.860	2.306	2.896	3.355	5.041
9	0.703	1.230	1.383	1.574	1.833	2.262	2.821	3.250	4.781
10	0.700	1.221	1.372	1.559	1.812	2.228	2.764	3.169	4.587
11	0.697	1.214	1.363	1.548	1.796	2.201	2.718	3.106	4.437
12	0.695	1.209	1.356	1.538	1.782	2.179	2.681	3.055	4.318
13	0.694	1.204	1.350	1.530	1.771	2.160	2.650	3.012	4.221
14	0.692	1.200	1.345	1.523	1.761	2.145	2.624	2.977	4.140
15	0.691	1.197	1.341	1.517	1.753	2.131	2.602	2.947	4.073
16	0.690	1.194	1.337	1.512	1.746	2.120	2.583	2.921	4.015
17	0.689	1.191	1.333	1.508	1.740	2.110	2.567	2.898	3.965
18	0.688	1.189	1.330	1.504	1.734	2.101	2.552	2.878	3.922
19	0.688	1.187	1.328	1.500	1.729	2.093	2.539	2.861	3.883
20	0.687	1.185	1.325	1.497	1.725	2.086	2.528	2.845	3.850
21	0.686	1.183	1.323	1.494	1.721	2.080	2.518	2.831	3.819
22	0.686	1.182	1.321	1.492	1.717	2.074	2.508	2.819	3.792
23	0.685	1.180	1.319	1.489	1.714	2.069	2.500	2.807	3.768
24	0.685	1.179	1.318	1.487	1.711	2.064	2.492	2.797	3.745
25	0.684	1.198	1.316	1.485	1.708	2.060	2.485	2.787	3.725
26	0.684	1.177	1.315	1.483	1.706	2.056	2.479	2.779	3.707
27	0.684	1.176	1.314	1.482	1.703	2.052	2.473	2.771	3.690
28	0.683	1.175	1.313	1.480	1.701	2.048	2.467	2.763	3.674
29	0.683	1.174	1.311	1.479	1.699	2.045	2.462	2.756	3.659
30	0.683	1.173	1.310	1.477	1.697	2.042	2.457	2.750	3.646
35	0.682	1.170	1.306	1.472	1.690	2.030	2.438	2.724	3.591
40	0.681	1.167	1.303	1.468	1.684	2.021	2.423	2.704	3.551
45	0.680	1.165	1.301	1.465	1.679	2.014	2.412	2.690	3.520
50	0.679	1.164	1.299	1.462	1.676	2.009	2.403	2.678	3.496
60	0.679	1.162	1.296	1.458	1.671	2.000	2.390	2.660	3.460
70	0.678	1.160	1.294	1.456	1.667	1.994	2.381	2.648	3.435
80	0.678	1.159	1.292	1.453	1.664	1.990	2.374	2.639	3.416
100	0.677	1.157	1.290	1.451	1.660	1.984	2.364	2.626	3.390
500	0.675	1.152	1.283	1.442	1.648	1.965	2.334	2.586	3.310
1000	0.675	1.151	1.282	1.441	1.646	1.962	2.330	2.581	3.300
∞	0.674	1.150	1.282	1.440	1.645	1.960	2.326	2.576	3.291

For degrees of freedom *d.f.* not in the table, use the closest *d.f.* that is *smaller.*



For *d.f.* ≥ 3



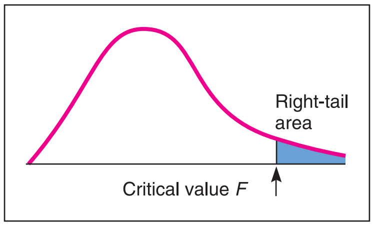
For *d.f.* = 1 or 2

TABLE 7 The χ^2 Distribution

	Right-tail Area									
d.f.	.995	.990	.975	.950	.900	.100	.050	.025	.010	.005
1	$0.0^{4}393$	$0.0^{3}157$	$0.0^{3}982$	$0.0^{2}393$	0.0158	2.71	3.84	5.02	6.63	7.88
2	0.0100	0.0201	0.0506	0.103	0.211	4.61	5.99	7.38	9.21	10.60
3	0.072	0.115	0.216	0.352	0.584	6.25	7.81	9.35	11.34	12.84
4	0.207	0.297	0.484	0.711	1.064	7.78	9.49	11.14	13.28	14.86
5	0.412	0.554	0.831	1.145	1.61	9.24	11.07	12.83	15.09	16.75
6	0.676	0.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	0.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	8.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.21	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	51.80	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4	104.2
80	51.17	53.54	57.15	60.39	64.28	96.58	101.9	106.6	112.3	116.3
90	59.20	61.75	65.65	69.13	73.29	107.6	113.1	118.1	124.1	128.3
100	67.33	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2

Source: Biometricka, June 1964, The χ^2 Distribution, H. L. Herter (Table 7). Used by permission of Oxford University Press.

TABLE 8 Critical Values for *F* Distribution



Degrees of freedom denominator, $d.f._D$	Right-tail area	1	2	3	4	5	6	7	8	9
		Degrees of freedom numerator, $d.f._N$								
	0.100	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86
	0.050	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
1	0.025	647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66	963.28
	0.010	4052.2	4999.5	5403.4	5624.6	5763.6	5859.0	5928.4	5981.1	6022.5
	0.001	405284	500000	540379	562500	576405	585937	592873	598144	602284
	0.100	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
	0.050	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
2	0.025	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39
	0.010	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39
	0.001	998.50	999.00	999.17	999.25	999.30	999.33	999.36	999.37	999.39
	0.100	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
	0.050	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
3	0.025	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47
	0.010	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35
	0.001	167.03	148.50	141.11	137.10	134.58	132.85	131.58	130.62	129.86
	0.100	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
	0.050	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
4	0.025	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90
	0.010	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66
	0.001	74.14	61.25	56.18	53.44	51.71	50.53	49.66	49.00	48.47
	0.100	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32
	0.050	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
5	0.025	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68
	0.010	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16
	0.001	47.18	37.12	33.20	31.09	29.75	28.83	28.16	27.65	27.24
	0.100	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
	0.050	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
6	0.025	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52
	0.010	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98
	0.001	35.51	27.00	23.70	21.92	20.80	20.03	19.46	19.03	18.69
	0.100	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72
	0.050	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
7	0.025	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82
	0.010	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
	0.001	29.25	21.69	18.77	17.20	16.21	15.52	15.02	14.63	14.33
	0.100	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56
	0.050	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
8	0.025	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36
	0.010	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91
	0.001	25.41	18.49	15.83	14.39	13.48	12.86	12.40	12.05	11.77

TABLE 8 *continued*

Degrees of freedom denominator, $d.f._D$	Right-tail area	Degrees of freedom numerator, $d.f._N$: 10	12	15	20	25	30	40	50	60	120	1000
	0.100	60.19	60.71	61.22	61.74	62.05	62.26	62.53	62.69	62.79	63.06	63.30
	0.050	241.88	243.91	245.95	248.01	249.26	250.10	251.14	251.77	252.20	253.25	254.19
1	0.025	968.63	976.71	984.87	993.10	998.08	1001.4	1005.6	1008.1	1009.8	1014.0	1017.7
	0.010	6055.8	6106.3	6157.3	6208.7	6239.8	6260.6	6286.8	6302.5	6313.0	6339.4	6362.7
	0.001	605621	610668	615764	620908	624017	626099	628712	630285	631337	633972	636301
	0.100	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.47	9.47	9.48	9.49
	0.050	19.40	19.41	19.43	19.45	19.46	19.46	19.47	19.48	19.48	19.49	19.49
2	0.025	39.40	39.41	39.43	39.45	39.46	39.46	39.47	39.48	39.48	39.49	39.50
	0.010	99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.48	99.49	99.50
	0.001	999.40	999.42	999.43	999.45	999.46	999.47	999.47	999.48	999.48	999.49	999.50
	0.100	5.23	5.22	5.20	5.18	5.17	5.17	5.16	5.15	5.15	5.14	5.13
	0.050	8.79	8.74	8.70	8.66	8.63	8.62	8.59	8.58	8.57	8.55	8.53
3	0.025	14.42	14.34	14.25	14.17	14.12	14.08	14.04	14.01	13.99	13.95	13.91
	0.010	27.23	27.05	26.87	26.69	26.58	26.50	26.41	26.35	26.32	26.22	26.14
	0.001	129.25	128.32	127.37	126.42	125.84	125.45	124.96	124.66	124.47	123.97	123.53
	0.100	3.92	3.90	3.87	3.84	3.83	3.82	3.80	3.80	3.79	3.78	3.76
	0.050	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.70	5.69	5.66	5.63
4	0.025	8.84	8.75	8.66	8.56	8.50	8.46	8.41	8.38	8.36	8.31	8.26
	0.010	14.55	14.37	14.20	14.02	13.91	13.84	13.75	13.69	13.65	13.56	13.47
	0.001	48.05	47.41	46.76	46.10	45.70	45.43	45.09	44.88	44.75	44.40	44.09
	0.100	3.30	3.27	3.24	3.21	3.19	3.17	3.16	3.15	3.14	3.12	3.11
	0.050	4.74	4.68	4.62	4.56	4.52	4.50	4.46	4.44	4.43	4.40	4.37
5	0.025	6.62	6.52	6.43	6.33	6.27	6.23	6.18	6.14	6.12	6.07	6.02
	0.010	10.05	9.89	9.72	9.55	9.45	9.38	9.29	9.24	9.20	9.11	9.03
	0.001	26.92	26.42	25.91	25.39	25.08	24.87	24.60	24.44	24.33	24.06	23.82
	0.100	2.94	2.90	2.87	2.84	2.81	2.80	2.78	2.77	2.76	2.74	2.72
	0.050	4.06	4.00	3.94	3.87	3.83	3.81	3.77	3.75	3.74	3.70	3.67
6	0.025	5.46	5.37	5.27	5.17	5.11	5.07	5.01	4.98	4.96	4.90	4.86
	0.010	7.87	7.72	7.56	7.40	7.30	7.23	7.14	7.09	7.06	6.97	6.89
	0.001	18.41	17.99	17.56	17.12	16.85	16.67	16.44	16.31	16.21	15.98	15.77
	0.100	2.70	2.67	2.63	2.59	2.57	2.56	2.54	2.52	2.51	2.49	2.47
	0.050	3.64	3.57	3.51	3.44	3.40	3.38	3.34	3.32	3.30	3.27	3.23
7	0.025	4.76	4.67	4.57	4.47	4.40	4.36	4.31	4.28	4.25	4.20	4.15
	0.010	6.62	6.47	6.31	6.16	6.06	5.99	5.91	5.86	5.82	5.74	5.66
	0.001	14.08	13.71	13.32	12.93	12.69	12.53	12.33	12.20	12.12	11.91	11.72
	0.100	2.54	2.50	2.46	2.42	2.40	2.38	2.36	2.35	2.34	2.32	2.30
	0.050	3.35	3.28	3.22	3.15	3.11	3.08	3.04	3.02	3.01	2.97	2.93
8	0.025	4.30	4.20	4.10	4.00	3.94	3.89	3.84	3.81	3.78	3.73	3.68
	0.010	5.81	5.67	5.52	5.36	5.26	5.20	5.12	5.07	5.03	4.95	4.87
	0.001	11.54	11.19	10.84	10.48	10.26	10.11	9.92	9.80	9.73	9.53	9.36

TABLE 8 *continued*

Degrees of freedom denominator, $d.f._D$	Right-tail area	Degrees of freedom numerator, $d.f._N$ 1	2	3	4	5	6	7	8	9
	0.100	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44
	0.050	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
9	0.025	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03
	0.010	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
	0.001	22.86	16.39	13.90	12.56	11.71	11.13	10.70	10.37	10.11
	0.100	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35
	0.050	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
10	0.025	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78
	0.010	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94
	0.001	21.04	14.91	12.55	11.28	10.48	9.93	9.52	9.20	8.96
	0.100	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27
	0.050	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
11	0.025	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59
	0.010	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63
	0.001	19.69	13.81	11.56	10.35	9.58	9.05	8.66	8.35	8.12
	0.100	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21
	0.050	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
12	0.025	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44
	0.010	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39
	0.001	18.64	12.97	10.80	9.63	8.89	8.38	8.00	7.71	7.48
	0.100	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16
	0.050	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
13	0.025	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31
	0.010	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19
	0.001	17.82	12.31	10.21	9.07	8.35	7.86	7.49	7.21	6.98
	0.100	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12
	0.050	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
14	0.025	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21
	0.010	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03
	0.001	17.14	11.78	9.73	8.62	7.92	7.44	7.08	6.80	6.58
	0.100	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09
	0.050	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
15	0.025	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12
	0.010	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89
	0.001	16.59	11.34	9.34	8.25	7.57	7.09	6.74	6.47	6.26
	0.100	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06
	0.050	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
16	0.025	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05
	0.010	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78
	0.001	16.12	10.97	9.01	7.94	7.27	6.80	6.46	6.19	5.98

TABLE 8 *continued*

Degrees of freedom denominator, $d.f._D$	Right-tail area	Degrees of freedom numerator, $d.f._N$ 10	12	15	20	25	30	40	50	60	120	1000
	0.100	2.42	2.38	2.34	2.30	2.27	2.25	2.23	2.22	2.21	2.18	2.16
	0.050	3.14	3.07	3.01	2.94	2.89	2.86	2.83	2.80	2.79	2.75	2.71
9	0.025	3.96	3.87	3.77	3.67	3.60	3.56	3.51	3.47	3.45	3.39	3.34
	0.010	5.26	5.11	4.96	4.81	4.71	4.65	4.57	4.52	4.48	4.40	4.32
	0.001	9.89	9.57	9.24	8.90	8.69	8.55	8.37	8.26	8.19	8.00	7.84
	0.100	2.32	2.28	2.24	2.20	2.17	2.16	2.13	2.12	2.11	2.08	2.06
	0.050	2.98	2.91	2.85	2.77	2.73	2.70	2.66	2.64	2.62	2.58	2.54
10	0.025	3.72	3.62	3.52	3.42	3.35	3.31	3.26	3.22	3.20	3.14	3.09
	0.010	4.85	4.71	4.56	4.41	4.31	4.25	4.17	4.12	4.08	4.00	3.92
	0.001	8.75	8.45	8.13	7.80	7.60	7.47	7.30	7.19	7.12	6.94	6.78
	0.100	2.25	2.21	2.17	2.12	2.10	2.08	2.05	2.04	2.03	2.00	1.98
	0.050	2.85	2.79	2.72	2.65	2.60	2.57	2.53	2.51	2.49	2.45	2.41
11	0.025	3.53	3.43	3.33	3.23	3.16	3.12	3.06	3.03	3.00	2.94	2.89
	0.010	4.54	4.40	4.25	4.10	4.01	3.94	3.86	3.81	3.78	3.69	3.61
	0.001	7.92	7.63	7.32	7.01	6.81	6.68	6.52	6.42	6.35	6.18	6.02
	0.100	2.19	2.15	2.10	2.06	2.03	2.01	1.99	1.97	1.96	1.93	1.91
	0.050	2.75	2.69	2.62	2.54	2.50	2.47	2.43	2.40	2.38	2.34	2.30
12	0.025	3.37	3.28	3.18	3.07	3.01	2.96	2.91	2.87	2.85	2.79	2.73
	0.010	4.30	4.16	4.01	3.86	3.76	3.70	3.62	3.57	3.54	3.45	3.37
	0.001	7.29	7.00	6.71	6.40	6.22	6.09	5.93	5.83	5.76	5.59	5.44
	0.100	2.14	2.10	2.05	2.01	1.98	1.96	1.93	1.92	1.90	1.88	1.85
	0.050	2.67	2.60	2.53	2.46	2.41	2.38	2.34	2.31	2.30	2.25	2.21
13	0.025	3.25	3.15	3.05	2.95	2.88	2.84	2.78	2.74	2.72	2.66	2.60
	0.010	4.10	3.96	3.82	3.66	3.57	3.51	3.43	3.38	3.34	3.25	3.18
	0.001	6.80	6.52	6.23	5.93	5.75	5.63	5.47	5.37	5.30	5.14	4.99
	0.100	2.10	2.05	2.01	1.96	1.93	1.91	1.89	1.87	1.86	1.83	1.80
	0.050	2.60	2.53	2.46	2.39	2.34	2.31	2.27	2.24	2.22	2.18	2.14
14	0.025	3.15	3.05	2.95	2.84	2.78	2.73	2.67	2.64	2.61	2.55	2.50
	0.010	3.94	3.80	3.66	3.51	3.41	3.35	3.27	3.22	3.18	3.09	3.02
	0.001	6.40	6.13	5.85	5.56	5.38	5.25	5.10	5.00	4.94	4.77	4.62
	0.100	2.06	2.02	1.97	1.92	1.89	1.87	1.85	1.83	1.82	1.79	1.76
	0.050	2.54	2.48	2.40	2.33	2.28	2.25	2.20	2.18	2.16	2.11	2.07
15	0.025	3.06	2.96	2.86	2.76	2.69	2.64	2.59	2.55	2.52	2.46	2.40
	0.010	3.80	3.67	3.52	3.37	3.28	3.21	3.13	3.08	3.05	2.96	2.88
	0.001	6.08	5.81	5.54	5.25	5.07	4.95	4.80	4.70	4.64	4.47	4.33
	0.100	2.03	1.99	1.94	1.89	1.86	1.84	1.81	1.79	1.78	1.75	1.72
	0.050	2.49	2.42	2.35	2.28	2.23	2.19	2.15	2.12	2.11	2.06	2.02
16	0.025	2.99	2.89	2.79	2.68	2.61	2.57	2.51	2.47	2.45	2.38	2.32
	0.010	3.69	3.55	3.41	3.26	3.16	3.10	3.02	2.97	2.93	2.84	2.76
	0.001	5.81	5.55	5.27	4.99	4.82	4.70	4.54	4.45	4.39	4.23	4.08

TABLE 8 *continued*

Degrees of freedom denominator, $d.f._D$	Right-tail area	Degrees of freedom numerator, $d.f._N$ 1	2	3	4	5	6	7	8	9
	0.100	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03
	0.050	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
17	0.025	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98
	0.010	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68
	0.001	15.72	10.66	8.73	7.68	7.02	6.56	6.22	5.96	5.75
	0.100	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00
	0.050	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
18	0.025	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93
	0.010	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60
	0.001	15.38	10.39	8.49	7.46	6.81	6.35	6.02	5.76	5.56
	0.100	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98
	0.050	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
19	0.025	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88
	0.010	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52
	0.001	15.08	10.16	8.28	7.27	6.62	6.18	5.85	5.59	5.39
	0.100	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96
	0.050	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
20	0.025	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84
	0.010	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46
	0.001	14.82	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.24
	0.100	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95
	0.050	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
21	0.025	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80
	0.010	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40
	0.001	14.59	9.77	7.94	6.95	6.32	5.88	5.56	5.31	5.11
	0.100	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93
	0.050	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
22	0.025	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76
	0.010	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35
	0.001	14.38	9.61	7.80	6.81	6.19	5.76	5.44	5.19	4.99
	0.100	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92
	0.050	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
23	0.025	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73
	0.010	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30
	0.001	14.20	9.47	7.67	6.70	6.08	5.65	5.33	5.09	4.89
	0.100	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91
	0.050	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
24	0.025	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70
	0.010	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26
	0.001	14.03	9.34	7.55	6.59	5.98	5.55	5.23	4.99	4.80

TABLE 8 *continued*

Degrees of freedom denominator, $d.f._D$	Right-tail area	Degrees of freedom numerator, $d.f._N$										
		10	12	15	20	25	30	40	50	60	120	1000
	0.100	2.00	1.96	1.91	1.86	1.83	1.81	1.78	1.76	1.75	1.72	1.69
	0.050	2.45	2.38	2.31	2.23	2.18	2.15	2.10	2.08	2.06	2.01	1.97
17	0.025	2.92	2.82	2.72	2.62	2.55	2.50	2.44	2.41	2.38	2.32	2.26
	0.010	3.59	3.46	3.31	3.16	3.07	3.00	2.92	2.87	2.83	2.75	2.66
	0.001	5.58	5.32	5.05	4.78	4.60	4.48	4.33	4.24	4.18	4.02	3.87
	0.100	1.98	1.93	1.89	1.84	1.80	1.78	1.75	1.74	1.72	1.69	1.66
	0.050	2.41	2.34	2.27	2.19	2.14	2.11	2.06	2.04	2.02	1.97	1.92
18	0.025	2.87	2.77	2.67	2.56	2.49	2.44	2.38	2.35	2.32	2.26	2.20
	0.010	3.51	3.37	3.23	3.08	2.98	2.92	2.84	2.78	2.75	2.66	2.58
	0.001	5.39	5.13	4.87	4.59	4.42	4.30	4.15	4.06	4.00	3.84	3.69
	0.100	1.96	1.91	1.86	1.81	1.78	1.76	1.73	1.71	1.70	1.67	1.64
	0.050	2.38	2.31	2.23	2.16	2.11	2.07	2.03	2.00	1.98	1.93	1.88
19	0.025	2.82	2.72	2.62	2.51	2.44	2.39	2.33	2.30	2.27	2.20	2.14
	0.010	3.43	3.30	3.15	3.00	2.91	2.84	2.76	2.71	2.67	2.58	2.50
	0.001	5.22	4.97	4.70	4.43	4.26	4.14	3.99	3.90	3.84	3.68	3.53
	0.100	1.94	1.89	1.84	1.79	1.76	1.74	1.71	1.69	1.68	1.64	1.61
	0.050	2.35	2.28	2.20	2.12	2.07	2.04	1.99	1.97	1.95	1.90	1.85
20	0.025	2.77	2.68	2.57	2.46	2.40	2.35	2.29	2.25	2.22	2.16	2.09
	0.010	3.37	3.23	3.09	2.94	2.84	2.78	2.69	2.64	2.61	2.52	2.43
	0.001	5.08	4.82	4.56	4.29	4.12	4.00	3.86	3.77	3.70	3.54	3.40
	0.100	1.92	1.87	1.83	1.78	1.74	1.72	1.69	1.67	1.66	1.62	1.59
	0.050	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.94	1.92	1.87	1.82
21	0.025	2.73	2.64	2.53	2.42	2.36	2.31	2.25	2.21	2.18	2.11	2.05
	0.010	3.31	3.17	3.03	2.88	2.79	2.72	2.64	2.58	2.55	2.46	2.37
	0.001	4.95	4.70	4.44	4.17	4.00	3.88	3.74	3.64	3.58	3.42	3.28
	0.100	1.90	1.86	1.81	1.76	1.73	1.70	1.67	1.65	1.64	1.60	1.57
	0.050	2.30	2.23	2.15	2.07	2.02	1.98	1.94	1.91	1.89	1.84	1.79
22	0.025	2.70	2.60	2.50	2.39	2.32	2.27	2.21	2.17	2.14	2.08	2.01
	0.010	3.26	3.12	2.98	2.83	2.73	2.67	2.58	2.53	2.50	2.40	2.32
	0.001	4.83	4.58	4.33	4.06	3.89	3.78	3.63	3.54	3.48	3.32	3.17
	0.100	1.89	1.84	1.80	1.74	1.71	1.69	1.66	1.64	1.62	1.59	1.55
	0.050	2.27	2.20	2.13	2.05	2.00	1.96	1.91	1.88	1.86	1.81	1.76
23	0.025	2.67	2.57	2.47	2.36	2.29	2.24	2.18	2.14	2.11	2.04	1.98
	0.010	3.21	3.07	2.93	2.78	2.69	2.62	2.54	2.48	2.45	2.35	2.27
	0.001	4.73	4.48	4.23	3.96	3.79	3.68	3.53	3.44	3.38	3.22	3.08
	0.100	1.88	1.83	1.78	1.73	1.70	1.67	1.64	1.62	1.61	1.57	1.54
	0.050	2.25	2.18	2.11	2.03	1.97	1.94	1.89	1.86	1.84	1.79	1.74
24	0.025	2.64	2.54	2.44	2.33	2.26	2.21	2.15	2.11	2.08	2.01	1.94
	0.010	3.17	3.03	2.89	2.74	2.64	2.58	2.49	2.44	2.40	2.31	2.22
	0.001	4.64	4.39	4.14	3.87	3.71	3.59	3.45	3.36	3.29	3.14	2.99

TABLE 8 *continued*

Degrees of freedom denominator, $d.f._D$	Right-tail area	Degrees of freedom numerator, $d.f._N$								
		1	2	3	4	5	6	7	8	9
	0.100	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89
	0.050	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
25	0.025	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68
	0.010	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22
	0.001	13.88	9.22	7.45	6.49	5.89	5.46	5.15	4.91	4.71
	0.100	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88
	0.050	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
26	0.025	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65
	0.010	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18
	0.001	13.74	9.12	7.36	6.41	5.80	5.38	5.07	4.83	4.64
	0.100	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87
	0.050	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
27	0.025	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63
	0.010	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15
	0.001	13.61	9.02	7.27	6.33	5.73	5.31	5.00	4.76	4.57
	0.100	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87
	0.050	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
28	0.025	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61
	0.010	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12
	0.001	13.50	8.93	7.19	6.25	5.66	5.24	4.93	4.69	4.50
	0.100	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86
	0.050	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
29	0.025	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59
	0.010	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09
	0.001	13.39	8.85	7.12	6.19	5.59	5.18	4.87	4.64	4.45
	0.100	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85
	0.050	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
30	0.025	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57
	0.010	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07
	0.001	13.29	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39
	0.100	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79
	0.050	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
40	0.025	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45
	0.010	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89
	0.001	12.61	8.25	6.59	5.70	5.13	4.73	4.44	4.21	4.02
	0.100	2.81	2.41	2.20	2.06	1.97	1.90	1.84	1.80	1.76
	0.050	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07
50	0.025	5.34	3.97	3.39	3.05	2.83	2.67	2.55	2.46	2.38
	0.010	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78
	0.001	12.22	7.96	6.34	5.46	4.90	4.51	4.22	4.00	3.82

TABLE 8 *continued*

Degrees of freedom denominator, $d.f._D$	Right-tail area	Degrees of freedom numerator, $d.f._N$ 10	12	15	20	25	30	40	50	60	120	1000
	0.100	1.87	1.82	1.77	1.72	1.68	1.66	1.63	1.61	1.59	1.56	1.52
	0.050	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.84	1.82	1.77	1.72
25	0.025	2.61	2.51	2.41	2.30	2.23	2.18	2.12	2.08	2.05	1.98	1.91
	0.010	3.13	2.99	2.85	2.70	2.60	2.54	2.45	2.40	2.36	2.27	2.18
	0.001	4.56	4.31	4.06	3.79	3.63	3.52	3.37	3.28	3.22	3.06	2.91
	0.100	1.86	1.81	1.76	1.71	1.67	1.65	1.61	1.59	1.58	1.54	1.51
	0.050	2.22	2.15	2.07	1.99	1.94	1.90	1.85	1.82	1.80	1.75	1.70
26	0.025	2.59	2.49	2.39	2.28	2.21	2.16	2.09	2.05	2.03	1.95	1.89
	0.010	3.09	2.96	2.81	2.66	2.57	2.50	2.42	2.36	2.33	2.23	2.14
	0.001	4.48	4.24	3.99	3.72	3.56	3.44	3.30	3.21	3.15	2.99	2.84
	0.100	1.85	1.80	1.75	1.70	1.66	1.64	1.60	1.58	1.57	1.53	1.50
	0.050	2.20	2.13	2.06	1.97	1.92	1.88	1.84	1.81	1.79	1.73	1.68
27	0.025	2.57	2.47	2.36	2.25	2.18	2.13	2.07	2.03	2.00	1.93	1.86
	0.010	3.06	2.93	2.78	2.63	2.54	2.47	2.38	2.33	2.29	2.20	2.11
	0.001	4.41	4.17	3.92	3.66	3.49	3.38	3.23	3.14	3.08	2.92	2.78
	0.100	1.84	1.79	1.74	1.69	1.65	1.63	1.59	1.57	1.56	1.52	1.48
	0.050	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.79	1.77	1.71	1.66
28	0.025	2.55	2.45	2.34	2.23	2.16	2.11	2.05	2.01	1.98	1.91	1.84
	0.010	3.03	2.90	2.75	2.60	2.51	2.44	2.35	2.30	2.26	2.17	2.08
	0.001	4.35	4.11	3.86	3.60	3.43	3.32	3.18	3.09	3.02	2.86	2.72
	0.100	1.83	1.78	1.73	1.68	1.64	1.62	1.58	1.56	1.55	1.51	1.47
	0.050	2.18	2.10	2.03	1.94	1.89	1.85	1.81	1.77	1.75	1.70	1.65
29	0.025	2.53	2.43	2.32	2.21	2.14	2.09	2.03	1.99	1.96	1.89	1.82
	0.010	3.00	2.87	2.73	2.57	2.48	2.41	2.33	2.27	2.23	2.14	2.05
	0.001	4.29	4.05	3.80	3.54	3.38	3.27	3.12	3.03	2.97	2.81	2.66
	0.100	1.82	1.77	1.72	1.67	1.63	1.61	1.57	1.55	1.54	1.50	1.46
	0.050	2.16	2.09	2.01	1.93	1.88	1.84	1.79	1.76	1.74	1.68	1.63
30	0.025	2.51	2.41	2.31	2.20	2.12	2.07	2.01	1.97	1.94	1.87	1.80
	0.010	2.98	2.84	2.70	2.55	2.45	2.39	2.30	2.25	2.21	2.11	2.02
	0.001	4.24	4.00	3.75	3.49	3.33	3.22	3.07	2.98	2.92	2.76	2.61
	0.100	1.76	1.71	1.66	1.61	1.57	1.54	1.51	1.48	1.47	1.42	1.38
	0.050	2.08	2.00	1.92	1.84	1.78	1.74	1.69	1.66	1.64	1.58	1.52
40	0.025	2.39	2.29	2.18	2.07	1.99	1.94	1.88	1.83	1.80	1.72	1.65
	0.010	2.80	2.66	2.52	2.37	2.27	2.20	2.11	2.06	2.02	1.92	1.82
	0.001	3.87	3.64	3.40	3.14	2.98	2.87	2.73	2.64	2.57	2.41	2.25
	0.100	1.73	1.68	1.63	1.57	1.53	1.50	1.46	1.44	1.42	1.38	1.33
	0.050	2.03	1.95	1.87	1.78	1.73	1.69	1.63	1.60	1.58	1.51	1.45
50	0.025	2.32	2.22	2.11	1.99	1.92	1.87	1.80	1.75	1.72	1.64	1.56
	0.010	2.70	2.56	2.42	2.27	2.17	2.10	2.01	1.95	1.91	1.80	1.70
	0.001	3.67	3.44	3.20	2.95	2.79	2.68	2.53	2.44	2.38	2.21	2.05

TABLE 8 *continued*

Degrees of freedom denominator, $d.f._D$	Right-tail area	1	2	3	4	5	6	7	8	9
		Degrees of freedom numerator, $d.f._N$								
60	0.100	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74
	0.050	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
	0.025	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33
	0.010	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72
	0.001	11.97	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69
100	0.100	2.76	2.36	2.14	2.00	1.91	1.83	1.78	1.73	1.69
	0.050	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97
	0.025	5.18	3.83	3.25	2.92	2.70	2.54	2.42	2.32	2.24
	0.010	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59
	0.001	11.50	7.41	5.86	5.02	4.48	4.11	3.83	3.61	3.44
200	0.100	2.73	2.33	2.11	1.97	1.88	1.80	1.75	1.70	1.66
	0.050	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93
	0.025	5.10	3.76	3.18	2.85	2.63	2.47	2.35	2.26	2.18
	0.010	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50
	0.001	11.15	7.15	5.63	4.81	4.29	3.92	3.65	3.43	3.26
1000	0.100	2.71	2.31	2.09	1.95	1.85	1.78	1.72	1.68	1.64
	0.050	3.85	3.00	2.61	2.38	2.22	2.11	2.02	1.95	1.89
	0.025	5.04	3.70	3.13	2.80	2.58	2.42	2.30	2.20	2.13
	0.010	6.66	4.63	3.80	3.34	3.04	2.82	2.66	2.53	2.43
	0.001	10.89	6.96	5.46	4.65	4.14	3.78	3.51	3.30	3.13

TABLE 8 *continued*

Degrees of freedom denominator, $d.f._D$	Right-tail area	Degrees of freedom numerator, $d.f._N$										
		10	12	15	20	25	30	40	50	60	120	1000
60	0.100	1.71	1.66	1.60	1.54	1.50	1.48	1.44	1.41	1.40	1.35	1.30
	0.050	1.99	1.92	1.84	1.75	1.69	1.65	1.59	1.56	1.53	1.47	1.40
	0.025	2.27	2.17	2.06	1.94	1.87	1.82	1.74	1.70	1.67	1.58	1.49
	0.010	2.63	2.50	2.35	2.20	2.10	2.03	1.94	1.88	1.84	1.73	1.62
	0.001	3.54	3.32	3.08	2.83	2.67	2.55	2.41	2.32	2.25	2.08	1.92
100	0.100	1.66	1.61	1.56	1.49	1.45	1.42	1.38	1.35	1.34	1.28	1.22
	0.050	1.93	1.85	1.77	1.68	1.62	1.57	1.52	1.48	1.45	1.38	1.30
	0.025	2.18	2.08	1.97	1.85	1.77	1.71	1.64	1.59	1.56	1.46	1.36
	0.010	2.50	2.37	2.22	2.07	1.97	1.89	1.80	1.74	1.69	1.57	1.45
	0.001	3.30	3.07	2.84	2.59	2.43	2.32	2.17	2.08	2.01	1.83	1.64
200	0.100	1.63	1.58	1.52	1.46	1.41	1.38	1.34	1.31	1.29	1.23	1.16
	0.050	1.88	1.80	1.72	1.62	1.56	1.52	1.46	1.41	1.39	1.30	1.21
	0.025	2.11	2.01	1.90	1.78	1.70	1.64	1.56	1.51	1.47	1.37	1.25
	0.010	2.41	2.27	2.13	1.97	1.87	1.79	1.69	1.63	1.58	1.45	1.30
	0.001	3.12	2.90	2.67	2.42	2.26	2.15	2.00	1.90	1.83	1.64	1.43
1000	0.100	1.61	1.55	1.49	1.43	1.38	1.35	1.30	1.27	1.25	1.18	1.08
	0.050	1.84	1.76	1.68	1.58	1.52	1.47	1.41	1.36	1.33	1.24	1.11
	0.025	2.06	1.96	1.85	1.72	1.64	1.58	1.50	1.45	1.41	1.29	1.13
	0.010	2.34	2.20	2.06	1.90	1.79	1.72	1.61	1.54	1.50	1.35	1.16
	0.001	2.99	2.77	2.54	2.30	2.14	2.02	1.87	1.77	1.69	1.49	1.22

Source: From Biometrika, Tables of Statisticans, Vol. I; Critical Values for *F* Distribution. (Table 8). Reprinted by permission of Oxford University Press.

TABLE 9 Critical Values for Spearman Rank Correlation, r_s

For a right- (left-) tailed test, use the positive (negative) critical value found in the table under One-tail area. For a two-tailed test, use both the positive and the negative of the critical value found in the table under Two-tail area; n = number of pairs.

	One-tail area			
	0.05	0.025	0.005	0.001
	Two-tail area			
n	0.10	0.05	0.01	0.002
5	0.900	1.000		
6	0.829	0.886	1.000	
7	0.715	0.786	0.929	1.000
8	0.620	0.715	0.881	0.953
9	0.600	0.700	0.834	0.917
10	0.564	0.649	0.794	0.879
11	0.537	0.619	0.764	0.855
12	0.504	0.588	0.735	0.826
13	0.484	0.561	0.704	0.797
14	0.464	0.539	0.680	0.772
15	0.447	0.522	0.658	0.750
16	0.430	0.503	0.636	0.730
17	0.415	0.488	0.618	0.711
18	0.402	0.474	0.600	0.693
19	0.392	0.460	0.585	0.676
20	0.381	0.447	0.570	0.661
21	0.371	0.437	0.556	0.647
22	0.361	0.426	0.544	0.633
23	0.353	0.417	0.532	0.620
24	0.345	0.407	0.521	0.608
25	0.337	0.399	0.511	0.597
26	0.331	0.391	0.501	0.587
27	0.325	0.383	0.493	0.577
28	0.319	0.376	0.484	0.567
29	0.312	0.369	0.475	0.558
30	0.307	0.363	0.467	0.549

Source: From G. J. Glasser and R. F. Winter, "Critical Values of the Coefficient of Rank Correlation for Testing the Hypothesis of Independence," *Biometrika, 48,* 444 (1961). Reprinted by permission of Oxford University Press.

TABLE 10 Critical Values for Number of Runs *R* (Level of significance $\alpha = 0.05$)

Value of n_1	Value of n_2: 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2
	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
3	1	1	1	1	2	2	2	2	2	2	2	2	2	3	3	3	3	3	3
	6	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
4	1	1	1	2	2	2	3	3	3	3	3	3	3	3	4	4	4	4	4
	6	8	9	9	9	10	10	10	10	10	10	10	10	10	10	10	10	10	10
5	1	1	2	2	3	3	3	3	3	4	4	4	4	4	4	4	5	5	5
	6	8	9	10	10	11	11	12	12	12	12	12	12	12	12	12	12	12	12
6	1	2	2	3	3	3	3	4	4	4	4	5	5	5	5	5	5	6	6
	6	8	9	10	11	12	12	13	13	13	13	14	14	14	14	14	14	14	14
7	1	2	2	3	3	3	4	4	5	5	5	5	5	6	6	6	6	6	6
	6	8	10	11	12	13	13	14	14	14	14	15	15	15	16	16	16	16	16
8	1	2	3	3	3	4	4	5	5	5	6	6	6	6	6	7	7	7	7
	6	8	10	11	12	13	14	14	15	15	16	16	16	16	17	17	17	17	17
9	1	2	3	3	4	4	5	5	5	6	6	6	7	7	7	7	8	8	8
	6	8	10	12	13	14	14	15	16	16	16	17	17	18	18	18	18	18	18
10	1	2	3	3	4	5	5	5	6	6	7	7	7	7	8	8	8	8	9
	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20
11	1	2	3	4	4	5	5	6	6	7	7	7	8	8	8	9	9	9	9
	6	8	10	12	13	14	15	16	17	17	18	19	19	19	20	20	20	21	21
12	2	2	3	4	4	5	6	6	7	7	7	8	8	8	9	9	9	10	10
	6	8	10	12	13	14	16	16	17	18	19	19	20	20	21	21	21	22	22
13	2	2	3	4	5	5	6	6	7	7	8	8	9	9	9	10	10	10	10
	6	8	10	12	14	15	16	17	18	19	19	20	20	21	21	22	22	23	23
14	2	2	3	4	5	5	6	7	7	8	8	9	9	9	10	10	10	11	11
	6	8	10	12	14	15	16	17	18	19	20	20	21	22	22	23	23	23	24
15	2	3	3	4	5	6	6	7	7	8	8	9	9	10	10	11	11	11	12
	6	8	10	12	14	15	16	18	18	19	20	21	22	22	23	23	24	24	25
16	2	3	4	4	5	6	6	7	8	8	9	9	10	10	11	11	11	12	12
	6	8	10	12	14	16	17	18	19	20	21	21	22	23	23	24	25	25	25
17	2	3	4	4	5	6	7	7	8	9	9	10	10	11	11	11	12	12	13
	6	8	10	12	14	16	17	18	19	20	21	22	23	23	24	25	25	26	26
18	2	3	4	5	5	6	7	8	8	9	9	10	10	11	11	12	12	13	13
	6	8	10	12	14	16	17	18	19	20	21	22	23	24	25	25	26	26	27
19	2	3	4	5	6	6	7	8	8	9	10	10	11	11	12	12	13	13	13
	6	8	10	12	14	16	17	18	20	21	22	23	23	24	25	26	26	27	27
20	2	3	4	5	6	6	7	8	9	9	10	10	11	12	12	13	13	13	14
	6	8	10	12	14	16	17	18	20	21	22	23	24	25	25	26	27	27	28

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Answers and Key Steps to Odd-Numbered Problems

CHAPTER 1

Section 1.1

1. An individual is a member of the population of interest. A variable is an aspect of an individual subject or object being measured.
3. A parameter is a numerical measurement describing data from a population. A statistic is a numerical measurement describing data from a sample.
5. (a) Nominal level. There is no apparent order relationship among responses.
(b) Ordinal level. There is an increasing relationship from worst to best level of service. The interval between service levels is not meaningful, nor are ratios.
7. (a) Response regarding meal ordered at fast-food restaurants. (b) Qualitative. (c) Responses for *all* adult fast-food customers in the U.S.
9. (a) Nitrogen concentration (mg nitrogen/l water).
(b) Quantitative. (c) Nitrogen concentration (mg nitrogen/l water) in the entire lake.
11. (a) Ratio. (b) Interval. (c) Nominal. (d) Ordinal.
(e) Ratio. (f) Ratio.
13. (a) Nominal. (b) Ratio. (c) Interval. (d) Ordinal.
(e) Ratio. (f) Interval.
15. Answers vary.
(a) For example: Use pounds. Round weights to the nearest pound. Since backpacks might weigh as much as 30 pounds, you might use a high-quality bathroom scale. (b) Some students may not allow you to weigh their backpacks for privacy reasons, etc. (c) Possibly. Some students may want to impress you with the heaviness of their backpacks, or they may be embarrassed about the "junk" they have stowed inside and thus may clean out their backpacks.

Section 1.2

1. In a stratified sample, random samples from each stratum are included. In a cluster sample, the clusters to be included are selected at random and then all members of each selected cluster are included.
3. The advice is wrong. A sampling error accounts only for the difference in results based on the use of a sample rather than of the entire population.
5. Use a random-number table to select four distinct numbers corresponding to people in your class.
(a) Reasons may vary. For instance, the first four students may make a special effort to get to class on time.
(b) Reasons may vary. For instance, four students who come in late might all be nursing students enrolled in an anatomy and physiology class that meets the hour before in a far-away building. They may be more motivated than other students to complete a degree requirement. (c) Reasons may vary. For instance, four students sitting in the back row might be less inclined to participate in class discussions. (d) Reasons may vary. For instance, the tallest students might all be male.
7. Answers vary. Use groups of two digits.
9. Select a starting place in the table and group the digits in groups of four. Scan the table by rows and include the first six groups with numbers between 0001 and 8615.
11. (a) Yes, when a die is rolled several times, the same number may appear more than once. Outcome on the fourth roll is 2. (b) No, for a fair die, the outcomes are random.
13. Since there are five possible outcomes for each question, read single digits from a random-number table. Select a starting place and proceed until you have 10 digits from 1 to 5. Repetition is required. The correct answer for each question will be the letter choice corresponding to the digit chosen for that question.
15. (a) Simple random sample. (b) Cluster sample.
(c) Convenience sample. (d) Systematic sample.
(e) Stratified sample.

Section 1.3

1. Answers vary. People with higher incomes are more likely to have high-speed Internet access and to spend more time online. People with high-speed Internet access might spend less time watching TV news or programming. People with higher incomes might have less time to spend watching TV because of access to other entertainment venues.
3. (a) No, those ages 18–29 in 2006 became ages 20–31 in 2008.
(b) 1977 to 1988 (inclusive).
5. (a) Observational study. (b) Experiment.
(c) Experiment. (d) Observational study.
7. (a) Use random selection to pick 10 calves to inoculate; test all calves; no placebo. (b) Use random selection to pick 9 schools to visit; survey all schools; no placebo.
(c) Use random selection to pick 40 volunteers for skin patch with drug; survey all volunteers; placebo used.
9. Based on the information given, Scheme A is best because it blocks all plots bordering the river together and all plots not bordering the river together.
The blocks of Scheme B do not seem to differ from each other.

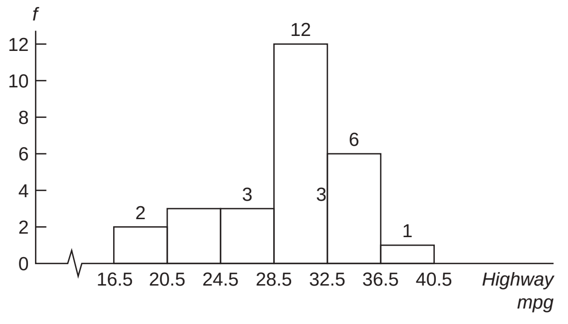
Chapter 1 Review

1. Because of the requirement that each number appear only once in any row, column, or box, it would be very inefficient to use a random-number table to select the numbers. It's better to simply look at existing numbers, list possibilities that meet the requirement, and eliminate numbers that don't work.
3. (a) Stratified. (b) Students on your campus with work-study jobs. (c) Hours scheduled; quantitative; ratio. (d) Rating of applicability of work experience to future employment; qualitative; ordinal. (e) Statistic. (f) 60%; The people choosing not to respond may have some characteristics, such as not working many hours, that would bias the study. (g) No. The sample frame is restricted to one campus.
5. Assign digits so that 3 out of the 10 digits 0 through 9 correspond to the answer "Yes" and 7 of the digits correspond to the answer "No." One assignment is digits 0, 1, and 2 correspond to "Yes," while digits 3, 4, 5, 6, 7, 8, and 9 correspond to "No." Starting with line 1, block 1 of Table 1, this assignment of digits gives the sequence No, Yes, No, No, Yes, No, No.
7. (a) Observational study. (b) Experiment.
9. Possible directions on survey questions: Give height in inches, give age as of last birthday, give GPA to one decimal place, and so forth. Think about the types of responses you wish to have on each question.
11. (a) Experiment, since a treatment is imposed on one colony. (b) The control group receives normal daylight/darkness conditions. The treatment group has light 24 hours per day. (c) The number of fireflies living at the end of 72 hours. (d) Ratio.

CHAPTER 2

Section 2.1

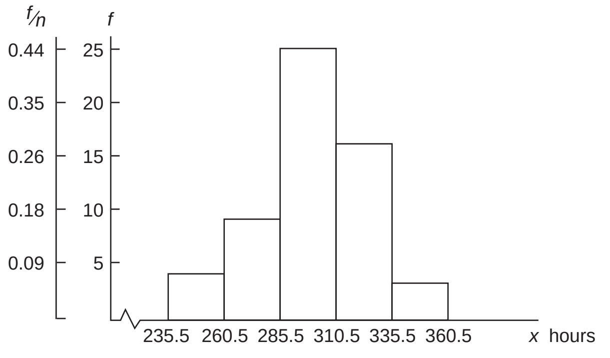
1. Class limits are possible data values. Class limits specify the span of data values that fall within a class. Class boundaries are not possible data values; rather, they are values halfway between the upper class limit of one class and the lower class limit of the next.
3. The classes overlap so that some data values, such as 20, fall within two classes.
5. Class width = 9; class limits: 20–28, 29–37, 38–46, 47–55, 56–64, 65–73, 74–82.
7. (a) Answers vary. Skewed right, if you hope most of the waiting times are low, with only a few times at the higher end of the distribution of waiting times.
(b) A bimodal distribution might reflect the fact that when there are lots of customers, most of the waiting times are longer, especially since the lines are likely to be long. On the other hand, when there are fewer customers, the lines are short or almost nonexistent, and most of the waiting times are briefer.
9. (a) Yes
(b) Histogram of Highway mpg



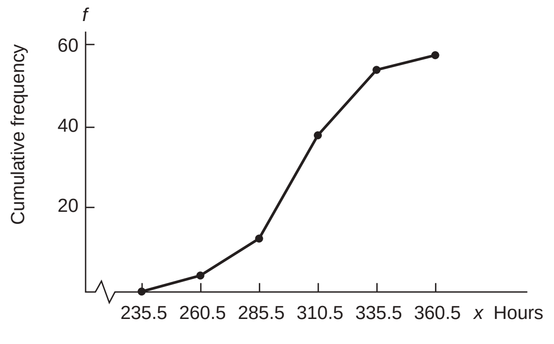
11. (a) Class width = 25.
(b)

Class Limits	Class Boundaries	Midpoint	Frequency	Relative Frequency	Cumulative Frequency
236–260	235.5–260.5	248	4	0.07	4
261–285	260.5–285.5	273	9	0.16	13
286–310	285.5–310.5	298	25	0.44	38
311–335	310.5–335.5	323	16	0.28	54
336–360	335.5–360.5	348	3	0.05	57

(c, d) Hours to Complete the Iditarod—Histogram, Relative-Frequency Histogram



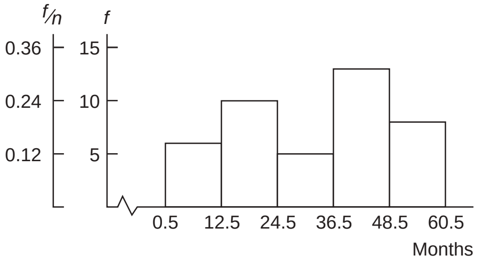
(e) Approximately mound-shaped symmetrical.
(f) Hours to Complete the Iditarod—Ogive



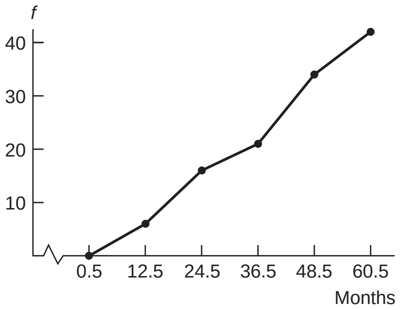
13. (a) Class width = 12.
(b)

Class Limits	Class Boundaries	Midpoint	Frequency	Relative Frequency	Cumulative Frequency
1–12	0.5–12.5	6.5	6	0.14	6
13–24	12.5–24.5	18.5	10	0.24	16
25–36	24.5–36.5	30.5	5	0.12	21
37–48	36.5–48.5	42.5	13	0.31	34
49–60	48.5–60.5	54.5	8	0.19	42

(c, d) Months Before Tumor Recurrence—Histogram, Relative-Frequency Histogram



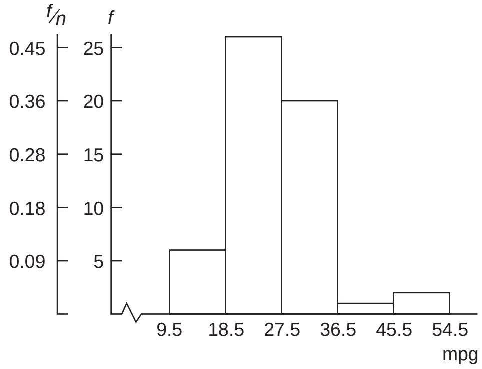
(e) Somewhat bimodal.
(f) Months Before Tumor Recurrence—Ogive



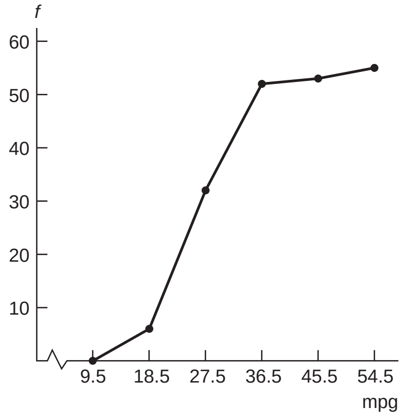
15. (a) Class width = 9.
(b)

Class Limits	Class Boundaries	Midpoint	Frequency	Relative Frequency	Cumulative Frequency
10–18	9.5–18.5	14	6	0.11	6
19–27	18.5–27.5	23	26	0.47	32
28–36	27.5–36.5	32	20	0.36	52
37–45	36.5–45.5	41	1	0.02	53
46–54	45.5–54.5	50	2	0.04	55

(c, d) Fuel Consumption (mpg)—Histogram, Relative-Frequency Histogram



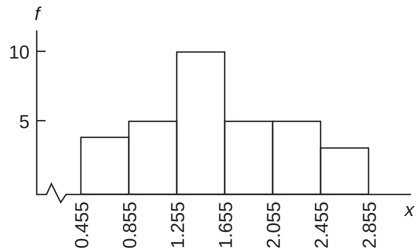
(e) Skewed slightly right.
(f) Fuel Consumption (mpg)—Ogive



17. (a) Clear the decimals.
(b, c) Class width = 0.40.

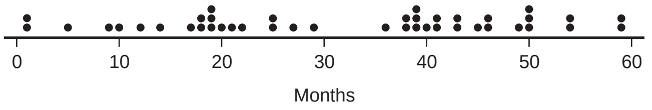
Class Limits	Boundaries	Midpoint	Frequency
0.46–0.85	0.455–0.855	0.655	4
0.86–1.25	0.855–1.255	1.055	5
1.26–1.65	1.255–1.655	1.455	10
1.66–2.05	1.655–2.055	1.855	5
2.06–2.45	2.055–2.455	2.255	5
2.46–2.85	2.455–2.855	2.655	3

(c) Tonnes of Wheat—Histogram



19. (a) One. (b) 5/51 or 9.8%. (c) Interval from 650 to 750.

21. Dotplot for Months Before Tumor Recurrence

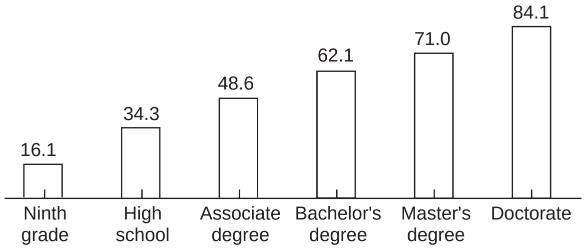


Section 2.2

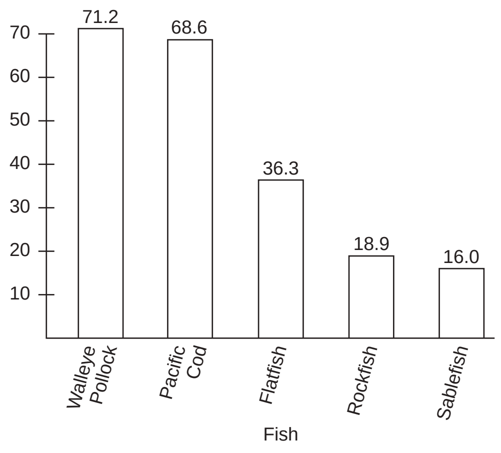
1. (a) Yes, the percentages total more than 100%.
(b) No, in a circle graph the percentages must total 100% (within rounding error).
(c) Yes, the graph is organized in order from most frequently selected reason to least.

3. Pareto chart, because it shows the items in order of importance to the greatest number of employees.

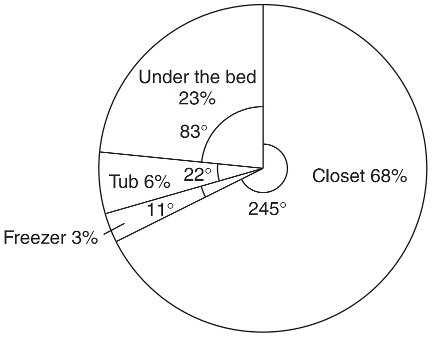
5. Highest Level of Education and Average Annual Household Income (in thousands of dollars).



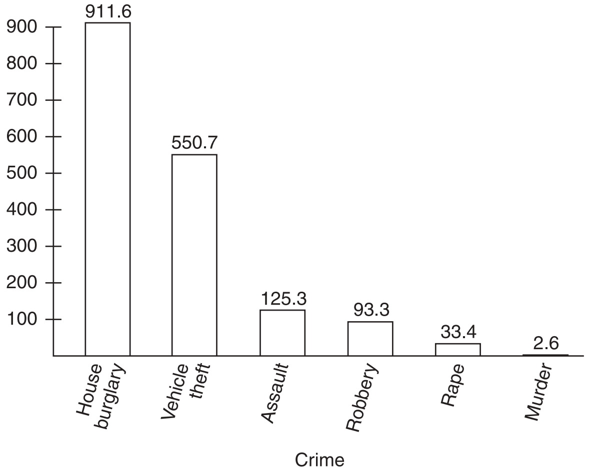
7. Annual Harvest (1000 Metric Tons)—Pareto Chart



9. Where We Hide the Mess

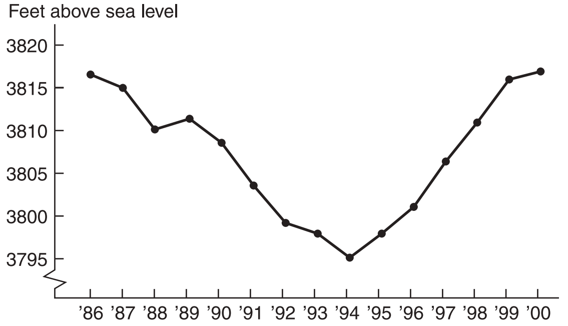


11. (a) Hawaii Crime Rate per 100,000 Population



(b) A circle graph is not appropriate because the data do not reflect all types of crime. Also, the same person may have been the victim of more than one crime.

13. Elevation of Pyramid Lake Surface—Time Plot



Section 2.3

1. (a) Longevity of Cowboys

4	7 = 47 years
4	7
5	2 7 8 8
6	1 6 6 8 8
7	0 2 2 3 3 5 6 7
8	4 4 4 5 6 6 7 9
9	0 1 1 2 3 7

(b) Yes, certainly these cowboys lived long lives.

3. Average Length of Hospital Stay

5	2 = 5.2 years
5	2 3 5 5 6 7
6	0 2 4 6 6 7 7 8 8 8 8 9 9
7	0 0 0 0 0 0 1 1 1 2 2 2 3 3 3 3 4 4 5 5 6 6 8
8	4 5 7
9	4 6 9
10	0 3
11	1

The distribution is skewed right.

5. (a) Minutes Beyond 2 Hours (1961–1980)

0 | 9 = 9 minutes past 2 hours	
0	9 9
1	0 0 2 3 3 4
1	5 5 6 6 7 8 8 9
2	0 2 3 3

(b) Minutes Beyond 2 Hours (1981–2000)

0 | 7 = 7 minutes past 2 hours	
0	7 7 7 8 8 8 8 9 9 9 9 9 9 9
1	0 0 1 1 4

(c) In more recent years, the winning times have been closer to 2 hours, with all the times between 7 and 14 minutes over 2 hours. In the earlier period, more than half the times were more than 2 hours and 14 minutes.

7. Milligrams of Tar per Cigarette

1 | 0 = 1.0 mg tar			
1	0	11	4
2		12	0 4 8
3		13	7
4	1 5	14	1 5 9
5		15	0 1 2 8
6		16	0 6
7	3 8	17	0
8	0 6 8		
9	0		
10		29	8

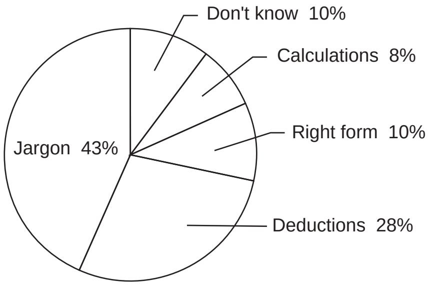
The value 29.8 may be an outlier.

9. Milligrams of Nicotine per Cigarette

0 | 1 = 0.1 milligram	
0	1 4 4
0	5 6 6 6 7 7 7 8 8 9 9 9
1	0 0 0 0 0 0 0 1 2
1	
2	0

Chapter 2 Review

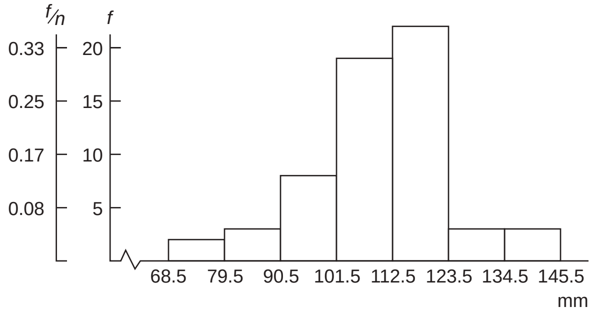
1. (a) Bar graph, Pareto chart, pie chart. (b) All.
3. Any large gaps between bars or stems with leaves at the beginning or end of the data set might indicate that the extreme data values are outliers.
5. (a) Yes, with lines used instead of bars. However, because of the perspective nature of the drawing, the lengths of the bars do not represent the mileages. Thus, the scale for each bar changes. (b) Yes. The scale does not change, and the viewer is not distracted by the graphic of the highway.
7. Problems with Tax Returns



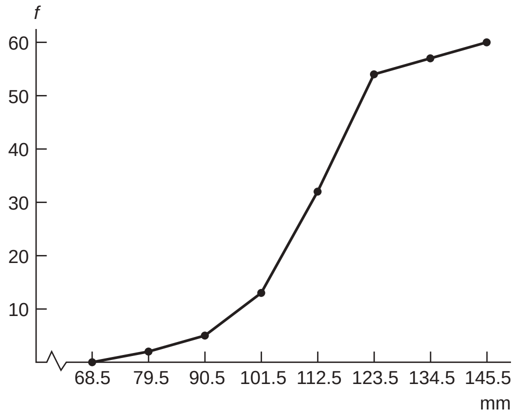
9. (a) Class width = 11.

Class Limits	Class Boundaries	Midpoint	Frequency	Relative Frequency	Cumulative Frequency
69–79	68.5–79.5	74	2	0.03	2
80–90	79.5–90.5	85	3	0.05	5
91–101	90.5–101.5	96	8	0.13	13
102–112	101.5–112.5	107	19	0.32	32
113–123	112.5–123.5	118	22	0.37	54
124–134	123.5–134.5	129	3	0.05	57
135–145	134.5–145.5	140	3	0.05	60

(b, c) Trunk Circumference (mm)—Histogram, Relative-Frequency Histogram



(d) Skewed slightly left.
(e) Trunk Circumference (mm)—Ogive



11. (a) 1240s had 40 data values. (b) 75. (c) From 1203 to 1212. Little if any repairs or new construction.

CHAPTER 3

Section 3.1

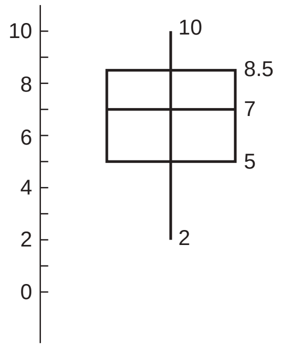
1. Median; mode; mean.
3. $\bar{x} = 5$; median = 6; mode = 2.
5. $\bar{x} = 5$; median = 5.5; mode = 2.
7. Mean, median, and mode are approximately equal.
9. (a) Mode = 5; median = 4; mean = 3.8. (b) Mode. (c) Mean, median, and mode. (d) Mode, median.
11. The supervisor has a legitimate concern because at least half the clients rated the employee below satisfactory. From the information given, it seems that this employee is very inconsistent in her performance.
13. (a) Mode = 2; median = 3; mean = 4.6. (b) Mode = 10; median = 15; mean = 23. (c) Corresponding values are 5 times the original averages. In general, multiplying each data value by a constant c results in the mode, median, and mean changing by a factor of c. (d) Mode = 177.8 cm; median = 172.72 cm; mean = 180.34 cm.
15. $\bar{x} \approx 167.3$°F; median = 171°F; mode = 178°F.
17. (a) $\bar{x} \approx 3.27$; median = 3; mode = 3. (b) $\bar{x} \approx 4.21$; median = 2; mode = 1. (c) Lower Canyon mean is greater; median and mode are less. (d) Trimmed mean = 3.75 and is closer to Upper Canyon mean.
19. (a) $\bar{x}$ = \$136.15; median = \$66.50; mode = \$60. (b) 5% trimmed mean ≈ \$121.28; yes, but still higher than the median. (c) Median. The low and high prices would be useful.
21. 23.
23. $\Sigma wx = 85$; $\Sigma w = 10$; weighted average = 8.5.
25. Approx. 66.67 mph.

Section 3.2

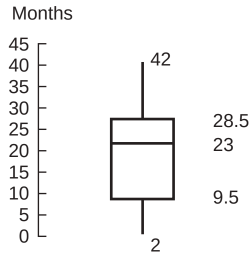
1. Mean.
3. Yes. For the sample standard deviation s, the sum $\Sigma(x - \bar{x})^2$ is divided by $n - 1$, where n is the sample size. For the population standard deviation σ, the sum $\Sigma(x - \mu)^2$ is divided by N, where N is the population size.
5. (a) Range is 4. (b) $s \approx 1.58$. (c) $\sigma \approx 1.41$.
7. For a data set in which not all data values are equal, σ is less than s. The reason is that to compute σ, we divide the sum of the squares by n, and to compute s we divide by the smaller number $n - 1$.
9. (a) (i), (ii), (iii). (b) The data change between data sets (i) and (ii) increased the sum of squared differences $\Sigma(x - \bar{x})^2$ by 10, whereas the data change between data sets (ii) and (iii) increased the sum of squared differences $\Sigma(x - \bar{x})^2$ by only 6.
11. (a) $s \approx 3.6$. (b) $s \approx 18.0$. (c) When each data value is multiplied by 5, the standard deviation is five times greater than that of the original data set. In general, multiplying each data value by the same constant c results in the standard deviation being $|c|$ times as large. (d) No. Multiply 3.1 miles by 1.6 kilometers/mile to obtain $s \approx 4.96$ kilometers.
13. (a) 15. (b) Use a calculator. (c) 37; 6.08. (d) 37; 6.08. (e) $\sigma^2 \approx 29.59$; $\sigma \approx 5.44$.
15. (a) $CV = 10\%$. (b) 14 to 26.
17. (a) 7.87. (b) Use a calculator. (c) $\bar{x} \approx 1.24$; $s^2 \approx 1.78$; $s \approx 1.33$. (d) $CV \approx 107\%$. The standard deviation of the time to failure is just slightly larger than the average time.
19. (a) Use a calculator. (b) $\bar{x} = 49$; $s^2 \approx 687.49$; $s \approx 26.22$. (c) $\bar{y} = 44.8$; $s^2 \approx 508.50$; $s \approx 22.55$. (d) Mallard nests, $CV \approx 53.5\%$; Canada goose nests, $CV \approx 50.3\%$. The CV gives the ratio of the standard deviation to the mean; the CV for mallard nests is slightly higher.
21. Since $CV = s/\bar{x}$, then $s = CV(\bar{x})$; $s = 0.033$.
23. Midpoints: 25.5, 35.5, 45.5; $\bar{x} \approx 35.80$; $s^2 \approx 61.1$; $s \approx 7.82$.
25. Midpoints: 10.55, 14.55, 18.55, 22.55, 26.55; $\bar{x} \approx 15.6$; $s^2 \approx 23.4$; $s \approx 4.8$.

Section 3.3

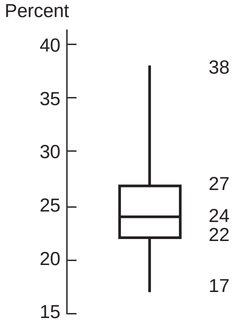
1. 82% or more of the scores were at or below Angela's score; 18% or fewer of the scores were above Angela's score.
3. No, the score 82 might have a percentile rank less than 70.
5. (a) Low = 2; $Q_1 = 5$; median = 7; $Q_3 = 8.5$; high = 10. (b) $IQR = 3.5$. (c) Box-and-Whisker Plot



7. Low = 2; $Q_1 = 9.5$; median = 23; $Q_3 = 28.5$; high = 42; $IQR = 19$.
Nurses' Length of Employment (months)



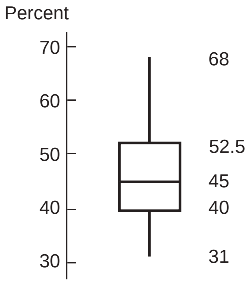
9. (a) Low = 17; $Q_1 = 22$; median = 24; $Q_3 = 27$; high = 38; $IQR = 5$. (b) Third quartile, since it is between the median and Q_3.
Bachelor's Degree Percentage by State



11. (a) California has the lowest premium. Pennsylvania has the highest. (b) Pennsylvania has the highest median premium. (c) California has the smallest range. Texas has the smallest interquartile range. (d) Part (a) is the five-number summary for Texas. It has the smallest IQR. Part (b) is the five-number summary for Pennsylvania. It has the largest minimum. Part (c) is the five-number summary for California. It has the lowest minimum.

Chapter 3 Review

1. (a) Variance and standard deviation. (b) Box-and-whisker plot.
3. (a) For both data sets, mean = 20 and range = 24.
(b) The C1 distribution seems more symmetric because the mean and median are equal, and the median is in the center of the interquartile range. In the C2 distribution, the mean is less than the median.
(c) The C1 distribution has a larger interquartile range that is symmetric around the median. The C2 distribution has a very compressed interquartile range with the median equal to Q_3.
5. (a) Low = 31; $Q_1 = 40$; median = 45; $Q_3 = 52.5$; high = 68; $IQR = 12.5$.
Percentage of Democratic Vote by County



(b) Class width = 8.

Class	Midpoint	f
31–38	34.5	11
39–46	42.5	24
47–54	50.5	15
55–62	58.5	7
63–70	66.5	3

$\bar{x} \approx 46.1$; $s \approx 8.64$; 28.82 to 63.38.

(c) $\bar{x} \approx 46.15$; $s \approx 8.63$.
7. Mean weight = 156.25 pounds.
9. (a) No. (b) \$34,206 to \$68,206. (c) \$10,875.
11. $\Sigma w = 16$, $\Sigma wx = 121$, average = 7.56.

CUMULATIVE REVIEW PROBLEMS

Chapters 1–3

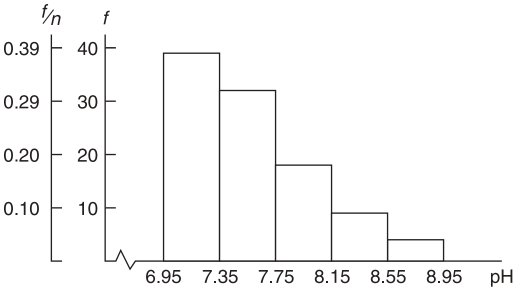
1. (a) Median, percentile. (b) Mean, variance, standard deviation.
2. (a) Gap between first bar and rest of bars or between last bar and rest of bars. (b) Large gap between data on far-left or far-right side and rest of data. (c) Several empty stems after stem including lowest values or before stem including highest values. (d) Data beyond fences placed at $Q_1 - 1.5(IQR)$ and $Q_3 + 1.5(IQR)$.
3. (a) Same. (b) Set B has a higher mean. (c) Set B has a higher standard deviation. (d) Set B has a much longer whisker beyond Q_3.
4. (a) Set A, because 86 is the relatively higher score, since a larger percentage of scores fall below it. (b) Set B, because 86 is more standard deviations above the mean.
5. Assign consecutive numbers to all the wells in the study region. Then use a random-number table, computer, or calculator to select 102 values that are less than or equal to the highest number assigned to a well in the study region. The sample consists of the wells with numbers corresponding to those selected.
6. Ratio.
7. 7 | 0 represents a pH level of 7.0

7	0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1
7	2 2 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 3
7	4 4 4 4 4 4 4 4 4 5 5 5 5 5 5 5 5
7	6 6 6 6 6 6 6 6 6 7 7 7 7 7 7
7	8 8 8 8 8 9 9 9 9 9
8	0 1 1 1 1 1 1 1
8	2 2 2 2 2 2 2
8	4 5
8	6 7
8	8 8

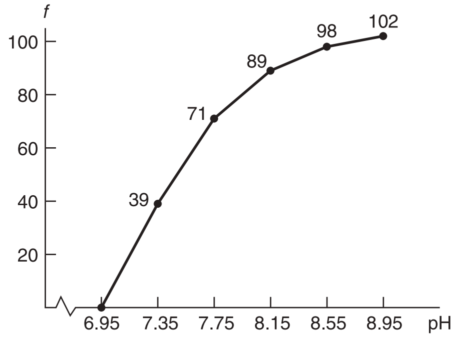
8. Clear the decimals. Then the highest value is 88 and the lowest is 70. The class width for the whole numbers is 4. For the actual data, the class width is 0.4.

Class Limits	Class Boundaries	Midpoint	Frequency	Relative Frequency
7.0–7.3	6.95–7.35	7.15	39	0.38
7.4–7.7	7.35–7.75	7.55	32	0.31
7.8–8.1	7.75–8.15	7.95	18	0.18
8.2–8.5	8.15–8.55	8.35	9	0.09
8.6–8.9	8.55–8.95	8.75	4	0.04

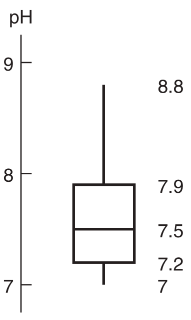
Levels of pH in West Texas Wells—Histogram, Relative-Frequency Histogram



9. Levels of pH in West Texas Wells—Ogive



10. Range = 1.8; $\bar{x} \approx 7.58$; median = 7.5; mode = 7.3.
11. (a) Use a calculator or computer.
(b) $s^2 \approx 0.20$; $s \approx 0.45$; $CV \approx 5.9\%$.
12. 6.68 to 8.48.
13. Levels of pH in West Texas Wells



$IQR = 0.7$.
14. Skewed right. Lower values are more common.
15. 89%; 50%.
16. No, there are no gaps in the plot, but only 6 out of 102, or about 6%, have pH levels at or above 8.4. Eight wells are neutral.
17. Half the wells have pH levels between 7.2 and 7.9. The data are skewed toward the high values, with the upper half of the pH levels spread out more than the lower half. The upper half ranges between 7.5 and 8.8, while the lower half is clustered between 7 and 7.5.
18. The report should emphasize the relatively low mean, median, and mode, and the fact that half the wells have a pH level less than 7.5. The data are clustered at the low end of the range.

Chapter 4

Section 4.1

1. Equally likely outcomes, relative frequency, intuition.
3. (a) 1. (b) 0.
5. 627/1010 ≈ 0.62.
7. Although the probability is high that you will make money, it is not completely certain that you will. In fact, there is a small chance that you could lose your entire investment. If you can afford to lose all of the investment, it might be worthwhile to invest, because there is a high chance of doubling your money.
9. (a) MMM MMF MFM MFF FMM FMF FFM FFF.
(b) P(MMM) = 1/8. P(at least one female) = 1 − P(MMM) = 7/8.
11. No. The probability of heads on the second toss is 0.50 regardless of the outcome on the first toss.
13. Answers vary. Probability as a relative frequency. One concern is whether the students in the class are more or less adept at wiggling their ears than people in the general population.
15. (a) $P(0) = 15/375$; $P(1) = 71/375$; $P(2) = 124/375$; $P(3) = 131/375$; $P(4) = 34/375$. (b) Yes, the listed numbers of similar preferences form the sample space.
17. (a) P(best idea 6 A.M.–12 noon) = 290/966 ≈ 0.30; P(best idea 12 noon–6 P.M.) = 135/966 ≈ 0.14; P(best idea 6 P.M.–12 midnight) = 319/966 ≈ 0.33; P(best idea 12 midnight–6 A.M.) = 222/966 ≈ 0.23. (b) The probabilities add up to 1. They should add up to 1 (within rounding errors), provided the intervals do not overlap and each inventor chose only one interval. The sample space is the set of four time intervals.
19. (b) P(success) = 2/17 ≈ 0.118. (c) P(make shot) = 3/8 or 0.375.
21. (a) P(enter if walks by) = 58/127 ≈ 0.46. (b) P(buy if entered) = 25/58 ≈ 0.43. (c) P(walk in and buy) = 25/127 ≈ 0.20. (d) P(not buy) = 1 − P(buy) ≈ 1 − 0.43 = 0.57.

Section 4.2

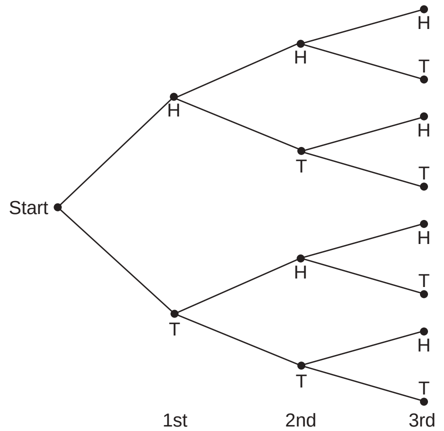
1. No. By definition, mutually exclusive events cannot occur together.
3. (a) 0.7. (b) 0.6.
5. (a) 0.08. (b) 0.04.
7. (a) 0.15. (b) 0.55.
9. (a) Because the events are mutually exclusive, A cannot occur if B occurred. $P(A \mid B) = 0$. (b) Because $P(A \mid B) \neq P(A)$, the events A and B are not independent.
11. (a) $P(A \text{ and } B)$. (b) $P(B \mid A)$. (c) $P(A^c \mid B)$. (d) $P(A \text{ or } B)$. (e) $P(B^c \text{ or } A)$.
13. (a) 0.2; yes. (b) 0.4; yes. (c) 1.0 − 0.2 = 0.8.
15. (a) Yes. (b) P(5 on green *and* 3 on red) = $P(5) \cdot P(3)$ = (1/6)(1/6) = 1/36 ≈ 0.028. (c) P(3 on green *and* 5 on red) = $P(3) \cdot P(5)$ = (1/6)(1/6) = 1/36 ≈ 0.028. (d) P((5 on green *and* 3 on red) *or* (3 on green *and* 5 on red)) = (1/36) + (1/36) = 1/18 ≈ 0.056.
17. (a) P(sum of 6) = P(1 *and* 5) + P(2 *and* 4) + P(3 *and* 3) + P(4 *and* 2) + P(5 *and* 1) = (1/36) + (1/36) + (1/36) + (1/36) + (1/36) = 5/36. (b) P(sum of 4) = P(1 *and* 3) +

P(2 *and* 2) + P(3 *and* 1) = (1/36) + (1/36) + (1/36) = 3/36 or 1/12. (c) P(sum of 6 *or* sum of 4) = P(sum of 6) + P(sum of 4) = (5/36) + (3/36) = 8/36 or 2/9; yes.

19. (a) No, after the first draw the sample space becomes smaller and probabilities for events on the second draw change. (b) P(Ace on 1st *and* King on 2nd) = P(Ace) · P(King | Ace) = (4/52)(4/51) = 4/663. (c) P(King on 1st *and* Ace on 2nd) = P(King) · P(Ace | King) = (4/52)(4/51) = 4/663. (d) P(Ace *and* King in either order) = P(Ace on 1st *and* King on 2nd) + P(King on 1st *and* Ace on 2nd) = (4/663) + (4/663) = 8/663.
21. (a) Yes, replacement of the card restores the sample space and all probabilities for the second draw remain unchanged regardless of the outcome of the first card. (b) P(Ace on 1st *and* King on 2nd) = P(Ace) · P(King) = (4/52)(4/52) = 1/169. (c) P(King on 1st *and* Ace on 2nd) = P(King) · P(Ace) = (4/52)(4/52) = 1/169. (d) P(Ace *and* King in either order) = P(Ace on 1st *and* King on 2nd) + P(King on 1st *and* Ace on 2nd) = (1/169) + (1/169) = 2/169.
23. (a) P(6 years old *or* older) = P(6–9) + P(10–12) + P(13 and over) = 0.27 + 0.14 + 0.22 = 0.63. (b) P(12 years old *or* younger) = P(2 and under) + P(3–5) + P(6–9) + P(10–12) = 0.15 + 0.22 + 0.27 + 0.14 = 0.78. (c) P(between 6 and 12) = P(6–9) + P(10–12) = 0.27 + 0.14 = 0.41. (d) P(between 3 and 9) = P(3–5) + P(6–9) = 0.22 + 0.27 = 0.49. The category 13 and over contains far more ages than the group 10–12. It is not surprising that more toys are purchased for this group, since there are more children in this group.
25. The information from James Burke can be viewed as conditional probabilities. P(reports lie | person is lying) = 0.72 and P(reports lie | person is not lying) = 0.07. (a) P(person is not lying) = 0.90; P(person is not lying *and* polygraph reports lie) = P(person is not lying) × P(reports lie | person not lying) = (0.90)(0.07) = 0.063 or 6.3%. (b) P(person is lying) = 0.10; P(person is lying *and* polygraph reports lie) = P(person is lying) × P(reports lie | person is lying) = (0.10)(0.72) = 0.072 or 7.2%. (c) P(person is not lying) = 0.5; P(person is lying) = 0.5; P(person is not lying *and* polygraph reports lie) = P(person is not lying) × P(reports lie | person not lying) = (0.50)(0.07) = 0.035 or 3.5%. P(person is lying *and* polygraph reports lie) = P(person is lying) × P(reports lie | person is lying) = (0.50)(0.72) = 0.36 or 36%. (d) P(person is not lying) = 0.15; P(person is lying) = 0.85; P(person is not lying *and* polygraph reports lie) = P(person is not lying) × P(reports lie | person is not lying) = (0.15)(0.07) = 0.0105 or 1.05%. P(person is lying *and* polygraph reports lie) = P(person is lying) × P(reports lie | person is lying) = (0.85)(0.72) = 0.612 or 61.2%.
27. (a) 686/1160; 270/580; 416/580. (b) No. (c) 270/1160; 416/1160. (d) 474/1160; 310/580. (e) No. (f) 686/1160 + 580/1160 − 270/1160 = 996/1160.
29. (a) 72/154. (b) 82/154. (c) 79/116. (d) 37/116. (e) 72/270. (f) 82/270.
31. (a) $P(A) = 0.65$. (b) $P(B) = 0.71$. (c) $P(B \mid A) = 0.87$. (d) $P(A \text{ and } B) = P(A) \cdot P(B \mid A) = (0.65)(0.87) \approx 0.57$. (e) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \approx 0.65 + 0.71 - 0.57 = 0.79$. (f) P(not close) = P(profit 1st year *or* profit 2nd year) = $P(A \text{ or } B) \approx 0.79$; P(close) = 1 − P(not close) ≈ 1 − 0.79 = 0.21.
33. (a) P(TB *and* positive) = P(TB)P(positive | TB) = (0.04)(0.82) ≈ 0.033. (b) P(does not have TB) = 1 − P(TB) = 1 − 0.04 = 0.96. (c) P(no TB *and* positive) = P(no TB)P(positive | no TB) = (0.96)(0.09) ≈ 0.086.
35. True. A^c consists of all events not in A.
37. False. If event A^c has occurred, then event A cannot occur.
39. True. $P(A \text{ and } B) = P(B) \cdot P(A \mid B)$. Since $0 < P(B) < 1$, the product $P(B) \cdot P(A \mid B) \le P(A \mid B)$.
41. True. All the outcomes in event A *and* B are also in event A.
43. True. All the outcomes in event A^c *and* B^c are also in event A^c.
45. False. See Problem 9.
47. True. Since $P(A \text{ and } B) = P(A) \cdot P(B) = 0$, either $P(A) = 0$ or $P(B) = 0$.
49. True. All simple events of the sample space under the condition "given B" are included in either the event A or the disjoint event A^c

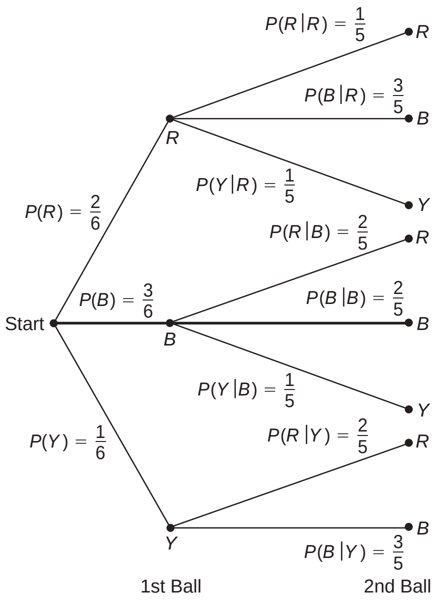
Section 4.3

1. The permutations rule counts the number of different *arrangements* of r items out of n distinct items, whereas the combinations rule counts only the *number* of groups of r items out of n distinct items. The number of permutations is larger than the number of combinations.
3. (a) Use the combinations rule, since only the items in the group and not their arrangement is of concern. (b) Use the permutations rule, since the number of arrangements within each group is of interest.
5. (a) Outcomes for Flipping a Coin Three Times



(b) 3. (c) 3/8.

7. (a) Outcomes for Drawing Two Balls (without replacement)



(b) $P(R \textit{ and } R) = 2/6 \cdot 1/5 = 1/15$.
$P(R \text{ 1st } \textit{and } B \text{ 2nd}) = 2/6 \cdot 3/5 = 1/5$.
$P(R \text{ 1st } \textit{and } Y \text{ 2nd}) = 2/6 \cdot 1/5 = 1/15$.
$P(B \text{ 1st } \textit{and } R \text{ 2nd}) = 3/6 \cdot 2/5 = 1/5$.
$P(B \text{ 1st } \textit{and } B \text{ 2nd}) = 3/6 \cdot 2/5 = 1/5$.
$P(B \text{ 1st } \textit{and } Y \text{ 2nd}) = 3/6 \cdot 1/5 = 1/10$.
$P(Y \text{ 1st } \textit{and } R \text{ 2nd}) = 1/6 \cdot 2/5 = 1/15$.
$P(Y \text{ 1st } \textit{and } B \text{ 2nd}) = 1/6 \cdot 3/5 = 1/10$.

9. $4 \cdot 3 \cdot 2 \cdot 1 = 24$ sequences.
11. $4 \cdot 3 \cdot 3 = 36$.
13. $P_{5,2} = (5!/3!) = 5 \cdot 4 = 20$.
15. $P_{7,7} = (7!/0!) = 7! = 5040$.
17. $C_{5,2} = (5!/(2!3!)) = 10$.
19. $C_{7,7} = (7!/(7!0!)) = 1$.
21. $P_{15,3} = 2730$.
23. $5 \cdot 4 \cdot 3 = 60$.
25. $C_{15,5} = (15!/(5!10!)) = 3003$.
27. (a) $C_{12,6} = (12!/(6!6!)) = 924$.
(b) $C_{7,6} = (7!/(6!1!)) = 7$. (c) $7/924 \approx 0.008$.

Chapter 4 Review

1. (a) The individual does not own a cell phone. (b) The individual owns a cell phone as well as a laptop computer. (c) The individual owns either a cell phone or a laptop computer, and maybe both. (d) The individual owns a cell phone, given he or she owns a laptop computer. (e) The individual owns a laptop computer, given he or she owns a cell phone.
3. For independent events A and B, $P(A) = P(A \mid B)$.
5. (a) P(offer job 1 *and* offer job 2) = 0.56. The probability of getting offers for both jobs is less than the probability of getting each individual job offer.
(b) P(offer job 1 *or* offer job 2) = 0.94. The probability of getting at least one of the job offers is greater than the probability of getting each individual job offer. It seems worthwhile to apply for both jobs since the probability is high of getting at least one offer.
7. (a) No. You need to know that the events are independent or you need to know the value of $P(A \mid B)$ or $P(B \mid A)$. (b) Yes. For independent events, $P(A \textit{ and } B) = P(A) \cdot P(B)$.
9. P(asked) = 24%; P(received | asked) = 45%; P(asked *and* received) = (0.24)(0.45) = 10.8%.
11. (a) Drop a fixed number of tacks and count how many land flat side down. Then form the ratio of the number landing flat side down to the total number dropped.
(b) Up, down. (c) P(up) = 160/500 = 0.32; P(down) = 340/500 = 0.68.
13. (a)

Outcomes x	2	3	4	5	6
$P(x)$	0.028	0.056	0.083	0.111	0.139

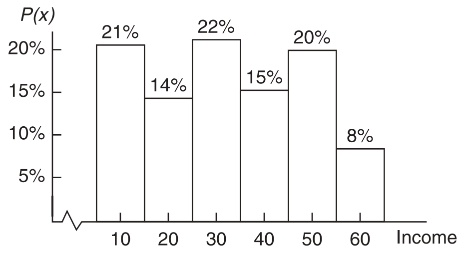
x	7	8	9	10	11	12
$P(x)$	0.167	0.139	0.111	0.083	0.056	0.028

15. $C_{8,2} = (8!/(2!6!)) = (8 \cdot 7/2) = 28$.
17. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 1024$ choices; P(all correct) = $1/1024 \approx 0.00098$.
19. $10 \cdot 10 \cdot 10 = 1000$.

CHAPTER 5

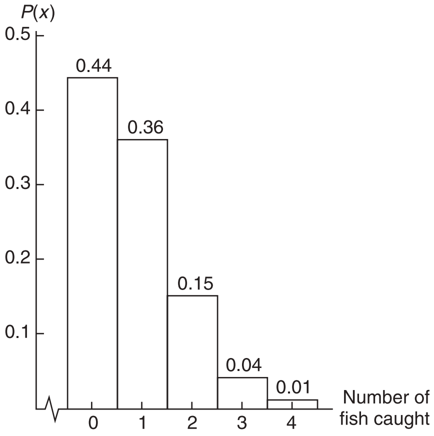
Section 5.1

1. (a) Discrete. (b) Continuous. (c) Continuous. (d) Discrete. (e) Continuous.
3. (a) Yes. (b) No; probabilities total to more than 1.
5. Expected value = 0.9. $\sigma \approx 0.6245$.
7. (a) Yes, 7 of the 10 digits represent "making a basket."
(b) Let S represent "making a basket" and F represent "missing the shot." *F, F, S, S, S, F, F, S, S.*
(c) Yes. Again, 7 of the 10 digits represent "making a basket." *S, S, S, S, S, S, S, S, S, S.*
9. (a) Yes, events are distinct and probabilities total to 1.
(b) Income Distribution ($1000)



(c) 32.3 thousand dollars. (d) 16.12 thousand dollars.

11. (a) Number of Fish Caught in a 6-Hour Period at Pyramid Lake, Nevada



(b) 0.56. (c) 0.20. (d) 0.82. (e) 0.899.

13. (a) 15/719; 704/719. (b) \$0.73; \$14.27.

15. (a) 0.01191; \$595.50. (b) \$646; \$698; \$751.50; \$806.50; \$3497.50 total. (c) \$4197.50. (d) \$1502.50.

17. (a) $\mu_W = 1.5$; $\sigma_W^2 = 208$; $\sigma_W \approx 14.4$.
(b) $\mu_W = 107.5$; $\sigma_W^2 = 52$; $\sigma_W \approx 7.2$.
(c) $\mu_L = 90$; $\sigma_L^2 = 92.16$; $\sigma_L \approx 9.6$.
(d) $\mu_L = 90$; $\sigma_L^2 = 57.76$; $\sigma_L \approx 7.6$.

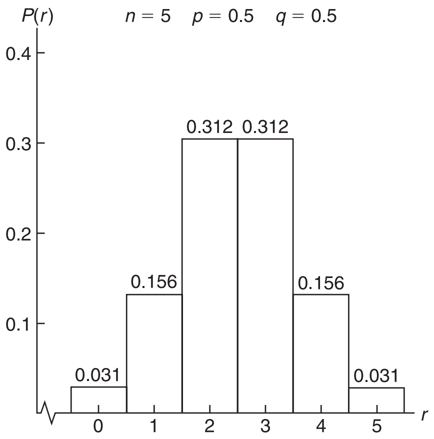
19. (a) $\mu_W = 50.2$; $\sigma_W^2 = 66.125$; $\sigma_W \approx 8.13$. (b) The means are the same. (c) The standard deviation for two policies is smaller. (d) As we include more policies, the coefficients in W decrease, resulting in smaller σ_W^2 and σ_W. For instance, for three policies, $W = (\mu_1 + \mu_2 + \mu_3)/3 \approx 0.33\mu_1 + 0.33\mu_2 + 0.33\mu_3$ and $\sigma_W^2 \approx (0.33)^2\sigma_1^2 + (0.33)^2\sigma_2^2 + (0.33)^2\sigma_3^2$. Yes, the risk appears to decrease by a factor of $1/\sqrt{n}$.

Section 5.2

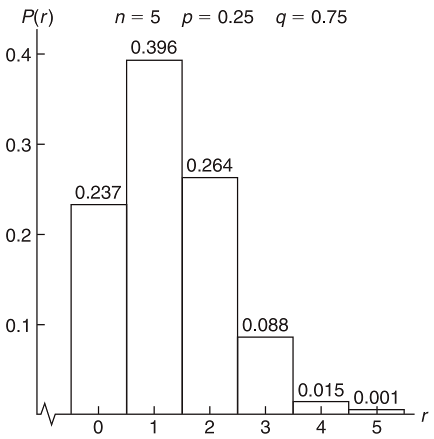
1. The random variable measures the number of successes out of n trials. This text uses the letter r for the random variable.
3. Two outcomes, success or failure.
5. Any monitor failure might endanger patient safely, so you should be concerned about the probability of *at least* one failure, not just exactly one failure.
7. (a) No. A binomial probability model applies to only two outcomes per trial. (b) Yes. Assign outcome A to "success" and outcomes B and C to "failure." $p = 0.40$.
9. (a) A trial consists of looking at the class status of a student enrolled in introductory statistics. Two outcomes are "freshman" and "not freshman." Success is freshman status; failure is any other class status. $P(\text{success}) = 0.40$. (b) Trials are not independent. With a population of only 30 students, in 5 trials without replacement, the probability of success rounded to the nearest hundredth changes for the later trials. Use the hypergeometric distribution for this situation.
11. (a) 0.082. (b) 0.918.
13. (a) 0.000. (b) Yes, the probability of 0 or 1 success is 0.000 to three places after the decimal. It would be a very rare event to get fewer than 2 successes when the probability of success on a single trial is so high.
15. A trial is one flip of a fair quarter. Success = coin shows heads. Failure = coin shows tails. $n = 3$; $p = 0.5$; $q = 0.5$. (a) $P(r = 3 \text{ heads}) = C_{3,3}p^3q^0 = 1(0.5)^3(0.5)^0 = 0.125$. To find this value in Table 3 of Appendix II, use the group in which $n = 3$, the column headed by $p = 0.5$, and the row headed by $r = 3$. (b) $P(r = 2 \text{ heads}) = C_{3,2}p^2q^1 = 3(0.5)^2(0.5)^1 = 0.375$. To find this value in Table 3 of Appendix II, use the group in which $n = 3$, the column headed by $p = 0.5$, and the row headed by $r = 2$. (c) $P(r \text{ is 2 or more}) = P(r = 2 \text{ heads}) + P(r = 3 \text{ heads}) = 0.375 + 0.125 = 0.500$. (d) The probability of getting three tails when you toss a coin three times is the same as getting zero heads. Therefore, $P(3 \text{ tails}) = P(r = 0 \text{ heads}) = C_{3,0}p^0q^3 = 1(0.5)^0(0.5)^3 = 0.125$. To find this value in Table 3 of Appendix II, use the group in which $n = 3$, the column headed by $p = 0.5$, and the row headed by $r = 0$.
17. A trial is recording the gender of one wolf. Success = male. Failure = female. $n = 12$; $p = 0.55$; $q = 0.45$. (a) $P(r \geq 6) = 0.740$. Six or more females means $12 - 6 = 6$ or fewer males; $P(r \leq 6) = 0.473$. Fewer than four females means more than $12 - 4 = 8$ males; $P(r > 8) = 0.135$. (b) A trial is recording the gender of one wolf. Success = male. Failure = female. $n = 12$; $p = 0.70$; $q = 0.30$. $P(r \geq 6) = 0.961$; $P(r \leq 6) = 0.117$; $P(r > 8) = 0.493$.
19. A trial consists of a woman's response regarding her mother-in-law. Success = dislike. Failure = like. $n = 6$; $p = 0.90$; $q = 0.10$. (a) $P(r = 6) = 0.531$. (b) $P(r = 0) = 0.000$ (to three digits). (c) $P(r \geq 4) = P(r = 4) + P(r = 5) + P(r = 6) = 0.098 + 0.354 + 0.531 = 0.983$. (d) $P(r \leq 3) = 1 - P(r \geq 4) \approx 1 - 0.983 = 0.017$ or 0.016 directly from table.
21. A trial is taking a polygraph exam. Success = pass. Failure = fail. $n = 9$; $p = 0.85$; $q = 0.15$. (a) $P(r = 9) = 0.232$. (b) $P(r \geq 5) = P(r = 5) + P(r = 6) + P(r = 7) + P(r = 8) + P(r = 9) = 0.028 + 0.107 + 0.260 + 0.368 + 0.232 = 0.995$. (c) $P(r \leq 4) = 1 - P(r \geq 5) \approx 1 - 0.995 = 0.005$ or 0.006 directly from table. (d) $P(r = 0) = 0.000$ (to three digits).
23. (a) A trial consists of using the Myers–Briggs instrument to determine if a person in marketing is an extrovert. Success = extrovert. Failure = not extrovert. $n = 15$; $p = 0.75$; $q = 0.25$. $P(r \geq 10) = 0.851$; $P(r \geq 5) = 0.999$; $P(r = 15) = 0.013$. (b) A trial consists of using the Myers–Briggs instrument to determine if a computer programmer is an introvert. Success = introvert. Failure = not introvert. $n = 5$; $p = 0.60$; $q = 0.40$. $P(r = 0) = 0.010$; $P(r \geq 3) = 0.683$; $P(r = 5) = 0.078$.
25. $n = 8$; $p = 0.53$; $q = 0.47$. (a) 0.812515; yes, truncated at five digits. (b) 0.187486; 0.18749; yes, rounded to five digits.
27. (a) They are the same. (b) They are the same. (c) $r = 1$. (d) The column headed by $p = 0.80$.
29. (a) $n = 8$; $p = 0.65$; $P(6 \leq r \mid 4 \leq r) = P(6 \leq r)/P(4 \leq r) = 0.428/0.895 \approx 0.478$. (b) $n = 10$; $p = 0.65$; $P(8 \leq r \mid 6 \leq r) = P(8 \leq r)/P(6 \leq r) = 0.262/0.752 \approx 0.348$. (c) Essay. (d) Use event $A = 6 \leq r$ and event $B = 4 \leq r$ in the formula.

Section 5.3

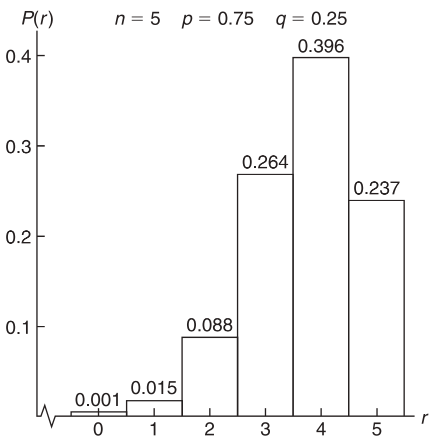
1. The average number of successes.
3. (a) $\mu = 1.6$; $\sigma \approx 1.13$. (b) Yes, 5 successes is more than 2.5σ above the expected value. $P(r \geq 5) = 0.010$.
5. (a) Yes, 120 is more than 2.5 standard deviations above the expected value. (b) Yes, 40 is less than 2.5 standard deviations below the expected value. (c) No, 70 to 90 successes is within 2.5 standard deviations of the expected value.
7. (a) Binomial Distribution
   The distribution is symmetrical.



(b) Binomial Distribution
The distribution is skewed right.

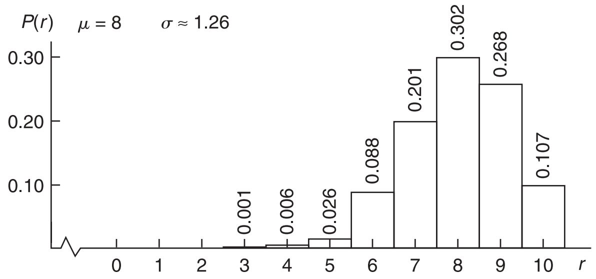


(c) Binomial Distribution
The distribution is skewed left.

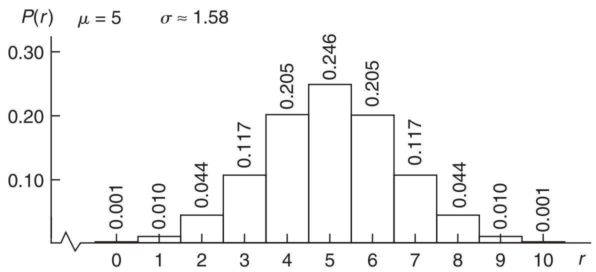


(d) The distributions are mirror images of one another.
(e) The distribution would be skewed left for $p = 0.73$ because the more likely numbers of successes are to the right of the middle.

9. (a) Households with Children Under 2 That Buy Photo Gear

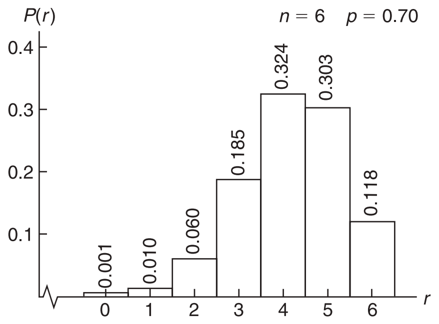


(b) Households with No Children Under 21 That Buy Photo Gear



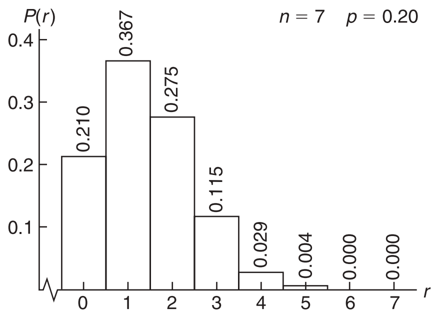
(c) Yes. Adults with children seem to buy more photo gear.

11. (a) Binomial Distribution for Number of Addresses Found



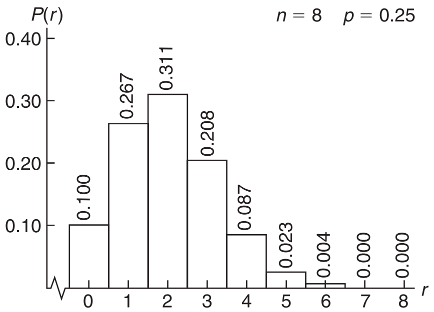
(b) $\mu = 4.2$; $\sigma \approx 1.122$. (c) $n = 5$. Note that $n = 5$ gives $P(r \geq 2) = 0.97$.

13. (a) Binomial Distribution for Number of Illiterate People



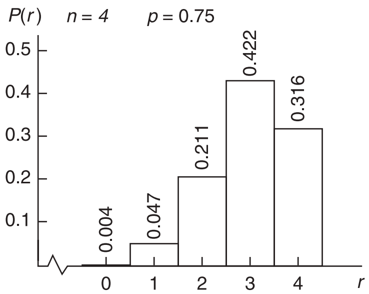
(b) $\mu = 1.4$; $\sigma \approx 1.058$. (c) $n = 12$. Note that $n = 12$ gives $P(r \geq 7) = 0.98$, where success = literate and $p = 0.80$.

15. (a) Binomial Distribution for Number of Gullible Consumers



(b) $\mu = 2$; $\sigma \approx 1.225$. (c) $n = 16$. Note that $n = 16$ gives $P(r \geq 1) = 0.99$.

17. (a) $P(r = 0) = 0.004$; $P(r = 1) = 0.047$; $P(r = 2) = 0.211$; $P(r = 3) = 0.422$; $P(r = 4) = 0.316$.
(b) Binomial Distribution for Number of Parolees Who Do Not Become Repeat Offenders



(c) $\mu = 3$; $\sigma \approx 0.866$. (d) $n = 7$. Note that $n = 7$ gives $P(r \geq 3) = 0.987$.

19. $n = 12$; $p = 0.25$ do not serve; $p = 0.75$ serve.
(a) $P(r = 12 \text{ serve}) = 0.032$. (b) $P(r \geq 6 \text{ do not serve}) = 0.053$. (c) For serving, $\mu = 9$; $\sigma = 1.50$. (d) To be at least 95.9% sure that 12 are available to serve, call 20.

21. $n = 6$; $p = 0.80$ do not solve; $p = 0.20$ solve.
(a) $P(r = 6 \text{ not solved}) = 0.262$. (b) $P(r \geq 1 \text{ solved}) = 0.738$. (c) For solving crime, $\mu = 1.2$; $\sigma \approx 0.98$.
(d) To be 90% sure of solving one or more crimes, investigate $n = 11$ crimes.

23. (a) $P(r = 7 \text{ guilty in U.S.}) = 0.028$; $P(r = 7 \text{ guilty in Japan}) = 0.698$. (b) For guilty in Japan, $\mu = 6.65$; $\sigma \approx 0.58$; for guilty in U.S., $\mu = 4.2$; $\sigma \approx 1.30$. (c) To be 99% sure of at least two guilty convictions in the U.S., look at $n = 8$ trials. To be 99% sure of at least two guilty convictions in Japan, look at $n = 3$ trials.

25. (a) 9. (b) 10.

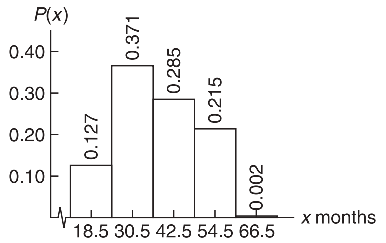
Section 5.4

1. Geometric distribution.
3. No, $n = 50$ is not large enough.
5. 0.144.
7. $\lambda = 8$; 0.1396.
9. (a) $p = 0.77$; $P(n) = (0.77)(0.23)^{n-1}$. (b) $P(1) = 0.77$. (c) $P(2) = 0.1771$. (d) $P(3 \text{ or more tries}) = 1 - P(1) - P(2) = 0.0529$. (e) 1.29, or 1.
11. (a) $P(n) = (0.80)(0.20)^{n-1}$. (b) $P(1) = 0.8$; $P(2) = 0.16$; $P(3) = 0.032$. (c) $P(n \geq 4) = 1 - P(1) - P(2) - P(3) = 1 - 0.8 - 0.16 - 0.032 = 0.008$. (d) $P(n) = (0.04)(0.96)^{n-1}$; $P(1) = 0.04$; $P(2) = 0.0384$; $P(3) = 0.0369$; $P(n \geq 4) = 0.8847$.
13. (a) $P(n) = (0.30)(0.70)^{n-1}$. (b) $P(3) = 0.147$. (c) $P(n > 3) = 1 - P(1) - P(2) - P(3) = 1 - 0.300 - 0.210 - 0.147 = 0.343$. (d) 3.33, or 3.
15. (a) $\lambda = (1.7/10) \times (3/3) = 5.1$ per 30-minute interval; $P(r) = e^{-5.1}(5.1)^r/r!$. (b) Using Table 4 of Appendix II with $\lambda = 5.1$, we find $P(4) = 0.1719$; $P(5) = 0.1753$; $P(6) = 0.1490$. (c) $P(r \geq 4) = 1 - P(0) - P(1) - P(2) - P(3) = 1 - 0.0061 - 0.0311 - 0.0793 - 0.1348 = 0.7487$. (d) $P(r < 4) = 1 - P(r \geq 4) = 1 - 0.7487 = 0.2513$.
17. (a) Births and deaths occur somewhat rarely in a group of 1000 people in a given year. For 1000 people, $\lambda = 16$ births; $\lambda = 8$ deaths. (b) By Table 4 of Appendix II, $P(10 \text{ births}) = 0.0341$; $P(10 \text{ deaths}) = 0.0993$; $P(16 \text{ births}) = 0.0992$; $P(16 \text{ deaths}) = 0.0045$. (c) $\lambda(\text{births}) = (16/1000) \times (1500/1500) = 24$ per 1500 people. $\lambda(\text{deaths}) = (8/1000) \times (1500/1500) = 12$ per 1500 people. By the table, $P(10 \text{ deaths}) = 0.1048$; $P(16 \text{ deaths}) = 0.0543$. Since $\lambda = 24$ is not in the table, use the formula for $P(r)$ to find $P(10 \text{ births}) = 0.00066$; $P(16 \text{ births}) = 0.02186$. (d) $\lambda(\text{births}) = (16/1000) \times (750/750) = 12$ per 750 people. $\lambda(\text{deaths}) = (8/1000) \times (750/750) = 6$ per 750 people. By Table 4 of Appendix II, $P(10 \text{ births}) = 0.1048$; $P(10 \text{ deaths}) = 0.0413$; $P(16 \text{ births}) = 0.0543$; $P(16 \text{ deaths}) = 0.0003$.
19. (a) The Poisson distribution is a good choice for r because gale-force winds occur rather rarely. The occurrences are usually independent. (b) For interval of 108 hours, $\lambda = (1/60) \times (108/108) = 1.8$ per 108 hours. Using Table 4 of Appendix II, we find that $P(2) = 0.2678$; $P(3) = 0.1607$; $P(4) = 0.0723$; $P(r < 2) = P(0) + P(1) = 0.1653 + 0.2975 = 0.4628$. (c) For interval of 180 hours, $\lambda = (1/60) \times (180/180) = 3$ per 180 hours. Table 4 of Appendix II gives $P(3) = 0.2240$; $P(4) = 0.1680$; $P(5) = 0.1008$; $P(r < 3) = P(0) + P(1) + P(2) = 0.0498 + 0.1494 + 0.2240 = 0.4232$.
21. (a) The sales of large buildings are rare events. It is reasonable to assume that they are independent. The variable r = number of sales in a fixed time interval. (b) For a 60-day period, $\lambda = (8/275) \times (60/60) = 1.7$ per 60 days. By Table 4 of Appendix II, $P(0) = 0.1827$; $P(1) = 0.3106$; $P(r \geq 2) = 1 - P(0) - P(1) = 0.5067$. (c) For a 90-day period, $\lambda = (8/275) \times (90/90) = 2.6$ per 90 days. By Table 4 of Appendix II, $P(0) = 0.0743$; $P(2) = 0.2510$; $P(r \geq 3) = 1 - P(0) - P(1) - P(2) = 1 - 0.0743 - 0.1931 - 0.2510 = 0.4816$.
23. (a) The problem satisfies the conditions for a binomial experiment with small $p = 0.0018$ and large $n = 1000$. $np = 1.8$, which is less than 10, so the Poisson approximation to the binomial distribution would be a good choice. $\lambda = np = 1.8$. (b) By Table 4, Appendix II, $P(0) = 0.1653$. (c) $P(r > 1) = 1 - P(0) - P(1) = 1 - 0.1653 - 0.2975 = 0.5372$. (d) $P(r > 2) = 1 - P(0) - P(1) - P(2) = 1 - 0.1653 - 0.2975 - 0.2678 = 0.2694$. (e) $P(r > 3) = 1 - P(0) - P(1) - P(2) - P(3) = 1 - 0.1653 - 0.2975 - 0.2678 - 0.1607 = 0.1087$.

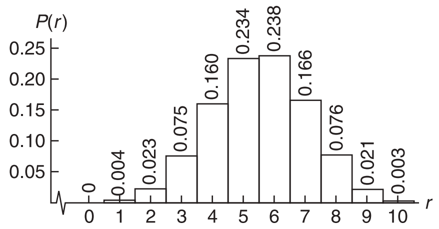
25. (a) The problem satisfies the conditions for a binomial experiment with n large, $n = 175$, and p small. $np = (175)(0.005) = 0.875 < 10$. The Poisson distribution would be a good approximation to the binomial. $n = 175$; $p = 0.005$; $\lambda = np = 0.9$. (b) By Table 4 of Appendix II, $P(0) = 0.4066$. (c) $P(r \geq 1) = 1 - P(0) = 0.5934$. (d) $P(r \geq 2) = 1 - P(0) - P(1) = 0.2275$.
27. (a) $n = 100$; $p = 0.02$; $r = 2$; $P(2) = C_{100,2}(0.02)^2 (0.98)^{98} \approx 0.2734$. (b) $\lambda = np = 2$; $P(2) = [e^{-2}(2)^2]/2! \approx 0.2707$. (c) The approximation is correct to two decimal places. (d) $n = 100$; $p = 0.02$; $r = 3$. By the formula for the binomial distribution, $P(3) \approx 0.1823$. By the Poisson approximation, $P(3) \approx 0.1804$. The approximation is correct to two decimal places.
29. (a) $\lambda \approx 3.4$. (b) $P(r \geq 4 \mid r \geq 2) = P(r \geq 4)/P(r \geq 2) \approx 0.4416/0.8531 \approx 0.5176$. (c) $P(r < 6 \mid r \geq 3) = P(3 \leq r < 6)/P(r \geq 3) \approx 0.5308/0.6602 \approx 0.8040$.
31. (a) $P(n) = C_{n-1,\,11}(0.80^{12})(0.20^{n-12})$.
(b) $P(12) \approx 0.0687$; $P(13) \approx 0.1649$; $P(14) \approx 0.2144$.
(c) 0.4480. (d) 0.5520. (e) $\mu = 15$; $\sigma \approx 1.94$. Susan can expect to get the bonus if she makes 15 contacts, with a standard deviation of about 2 contacts.

Chapter 5 Review

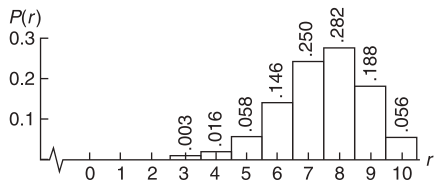
1. A description of all distinct possible values of a random variable x, with a probability assignment $P(x)$ for each value or range of values. $0 \leq P(x) \leq 1$ and $\Sigma P(x) = 1$.
3. (a) Yes. $\mu = 2$ and $\sigma \approx 1.3$. Numbers of successes above 5.25 are unusual. (b) No. It would be unusual to get more than five questions correct.
5. (a) 38; 11.6.
(b) Duration of Leases in Months



7. (a) Number of Claimants Under 25



(b) $P(r \geq 6) = 0.504$. (c) $\mu = 5.5$; $\sigma \approx 1.57$.
9. (a) 0.039. (b) 0.403. (c) 8.
11. (a) Number of Good Grapefruit

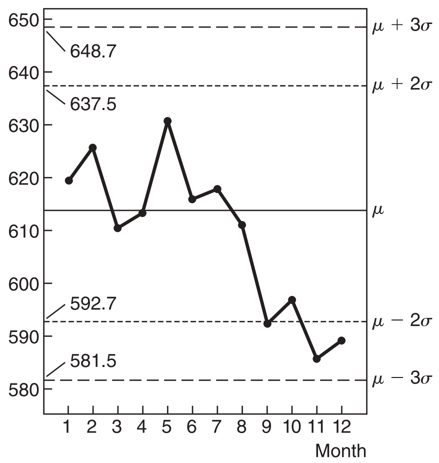


(b) 0.244, 0.999. (c) 7.5. (d) 1.37.
13. $P(r \leq 2) = 0.000$ (to three digits). The data seem to indicate that the percent favoring the increase in fees is less than 85%.
15. (a) Coughs are a relatively rare occurrence. It is reasonable to assume that they are independent events, and the variable is the number of coughs in a fixed time interval. (b) $\lambda = 11$ coughs per minute; $P(r \leq 3) = P(0) + P(1) + P(2) + P(3) = 0.000 + 0.002 + 0.0010 + 0.0037 = 0.0049$. (c) $\lambda = (11/1) \times (0.5/0.5) = 5.5$ coughs per 30-second period. $P(r \geq 3) = 1 - P(0) - P(1) - P(2) = 1 - 0.0041 - 0.0225 - 0.0618 = 0.9116$.
17. The loan-default problem satisfies the conditions for a binomial experiment. Moreover, p is small, n is large, and $np < 10$. Use of the Poisson approximation to the binomial distribution is appropriate. $n = 300$; $p = 1/350 \approx 0.0029$; and $\lambda = np \approx 300(0.0029) = 0.86 \approx 0.9$; $P(r \geq 2) = 1 - P(0) - P(1) = 1 - 0.4066 - 0.3659 = 0.2275$.
19. (a) Use the geometric distribution with $p = 0.5$. $P(n = 2) = (0.5)(0.5) = 0.25$. As long as you toss the coin at least twice, it does not matter how many more times you toss it. To get the first head on the second toss, you must get a tail on the first and a head on the second. (b) $P(n = 4) = (0.5)(0.5)^3 = 0.0625$; $P(n > 4) = 1 - P(1) - P(2) - P(3) - P(4) = 1 - 0.5 - 0.5^2 - 0.5^3 - 0.5^4 = 0.0625$.

CHAPTER 6

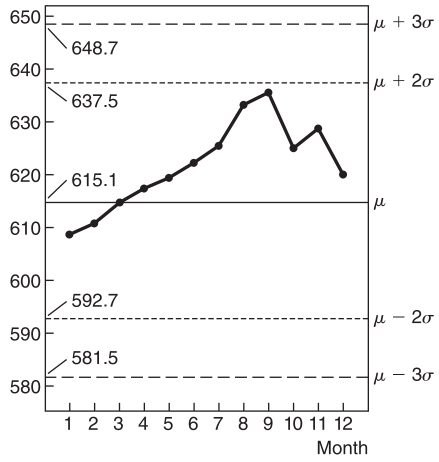
Section 6.1

1. (a) No, it's skewed. (b) No, it crosses the horizontal axis. (c) No, it has three peaks. (d) No, the curve is not smooth.
3. Figure 6-12 has the larger standard deviation. The mean of Figure 6-12 is $\mu = 10$. The mean of Figure 6-13 is $\mu = 4$.
5. (a) 50%. (b) 68%. (c) 99.7%.
7. (a) 50%. (b) 50%. (c) 68%. (d) 95%.
9. (a) From 1207 to 1279. (b) From 1171 to 1315.
(c) From 1135 to 1351.
11. (a) From 1.70 mA to 4.60 mA. (b) From 0.25 mA to 6.05 mA.
13. (a) Tri-County Bank Monthly Loan Request—First Year (thousands of dollars)



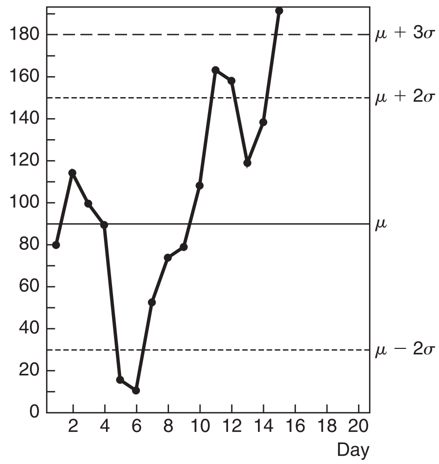
The process is out of control with a type III warning signal, since two of three consecutive points are more than 2 standard deviations below the mean. The trend is down.

(b) Tri-County Bank Monthly Loan Requests—Second Year (thousands of dollars)



The process shows warning signal II, a run of nine consecutive points above the mean. The economy is probably heating up.

15. Visibility Standard Index



There is one point above $\mu + 3\sigma$. Thus control signal I indicates "out of control." Control signal III is present. There are two consecutive points below $\mu - 2\sigma$ and two consecutive points above $\mu + 2\sigma$. The out-of-control signals that cause the most concern are those above the mean. Special pollution regulations may be appropriate for those periods.

17. (a) 0.8000. (b) 0.7000. (c) 0.5000. (d) $\mu = 0$; $\sigma \approx 0.289$. Since $\sigma = 0$, the measurements are unbiased.
19. (a) 0.4493. (b) 0.8454. (c) 0.1857. (d) 120.71.

Section 6.2

1. The number of standard deviations from the mean.
3. 0.
5. (a) −1. (b) 2.4. (c) 20. (d) 36.5.
7. They are the same, since both are 1 standard deviation below the mean.
9. (a) Robert, Juan, and Linda each scored above the mean. (b) Joel scored on the mean. (c) Susan and Jan scored below the mean. (d) Robert, 172; Juan, 184; Susan, 110; Joel, 150; Jan, 134; Linda, 182.
11. (a) $-1.00 < z$. (b) $z < -2.00$. (c) $-2.67 < z < 2.33$. (d) $x < 4.4$. (e) $5.2 < x$. (f) $4.1 < x < 4.5$. (g) A red blood cell count of 5.9 or higher corresponds to a standard z score of 3.67. Practically no data values occur this far above the mean. Such a count would be considered unusually high for a healthy female.
13. 0.5000. 15. 0.0934. 17. 0.6736. 19. 0.0643. 21. 0.8888. 23. 0.4993. 25. 0.8953. 27. 0.3471. 29. 0.0306. 31. 0.5000. 33. 0.4483. 35. 0.8849. 37. 0.0885. 39. 0.8849. 41. 0.8808. 43. 0.3226. 45. 0.4474. 47. 0.2939. 49. 0.6704.

Section 6.3

1. 0.50.
3. Negative.
5. $P(3 \le x \le 6) = P(-0.50 \le z \le 1.00) = 0.5328$.
7. $P(50 \le x \le 70) = P(0.67 \le z \le 2.00) = 0.2286$.
9. $P(8 \le x \le 12) = P(-2.19 \le z \le -0.94) = 0.1593$.
11. $P(x \ge 30) = P(z \ge 2.94) = 0.0016$.
13. $P(x \ge 90) = P(z \ge -0.67) = 0.7486$.
15. −1.555. 17. 0.13. 19. 1.41. 21. −0.92.
23. ±2.33.
25. (a) $P(x > 60) = P(z > -1) = 0.8413$. (b) $P(x < 110) = P(z < 1) = 0.8413$. (c) $P(60 \le x \le 110) = P(-1.00 \le z \le 1.00) = 0.8413 - 0.1587 = 0.6826$. (d) $P(x > 140) = P(z > 2.20) = 0.0139$.
27. (a) $P(x < 3.0 \text{ mm}) = P(z < -2.33) = 0.0099$. (b) $P(x > 7.0 \text{ mm}) = P(z > 2.11) = 0.0174$. (c) $P(3.0 \text{ mm} < x < 7.0 \text{ mm}) = P(-2.33 < z < 2.11) = 0.9727$.
29. (a) $P(x < 36 \text{ months}) = P(z < -1.13) = 0.1292$. The company will replace 13% of its batteries. (b) $P(z < z_0) = 10\%$ for $z_0 = -1.28$; $x = -1.28(8) + 45 = 34.76$. Guarantee the batteries for 35 months.
31. (a) According to the empirical rule, about 95% of the data lies between $\mu - 2\sigma$ and $\mu + 2\sigma$. Since this interval is 4σ wide, we have $4\sigma \approx 6$ years, so $\sigma \approx 1.5$ years. (b) $P(x > 5) = P(z > -2.00) = 0.9772$. (c) $P(x < 10) = P(z < 1.33) = 0.9082$. (d) $P(z < z_0) = 0.10$ for $z_0 = -1.28$; $x = -1.28(1.5) + 8 = 6.08$ years. Guarantee the TVs for about 6.1 years.
33. (a) $\sigma \approx 12$ beats/minute. (b) $P(x < 25) = P(z < -1.75) = 0.0401$. (c) $P(x > 60) = P(z > 1.17) = 0.1210$. (d) $P(25 \le x \le 60) = P(-1.75 \le z \le 1.17) = 0.8389$. (e) $P(z \le z_0) = 0.90$ for $z_0 = 1.28$; $x = 1.28(12) + 46 = 61.36$ beats/minute. A heart rate of 61 beats/minute corresponds to the 90% cutoff point of the distribution.
35. (a) $P(z \ge z_0) = 0.99$ for $z_0 = -2.33$; $x = -2.33(3.7) + 90 \approx 81.38$ months. Guarantee the microchips for 81 months. (b) $P(x \le 84) = P(z \le -1.62) = 0.0526$. (c) Expected loss = (50,000,000)(0.0526) = \$2,630,000. (d) Profit = \$370,000.
37. (a) $z = 1.28$; $x \approx 4.9$ hours. (b) $z = -1.04$; $x \approx 2.9$ hours. (c) Yes; work and/or school schedules may be different on Saturday.
39. (a) In general, $P(A \mid B) = P(A \text{ and } B)/P(B)$; $P(x > 20) = P(z > 0.50) = 0.3085$; $P(x > 15) = P(z > -0.75) = 0.7734$; $P(x > 20 \mid x > 15) = 0.3989$. (b) $P(x > 25) =$

$P(z > 1.75) = 0.0401$; $P(x > 18) = P(z > 0.00) = 0.5000$; $P(x > 25 \mid x > 18) = 0.0802$. (c) Use event $A = x > 20$ and event $B = x > 15$ in the formula.

Section 6.4

1. A set of measurements or counts either existing or conceptual. For example, the population of ages of all people in Colorado; the population of weights of all students in your school; the population count of all antelope in Wyoming.
3. A numerical descriptive measure of a population, such as μ, the population mean; σ, the population standard deviation; or σ^2, the population variance.
5. A statistical inference is a conclusion about the value of a population parameter. We will do both estimation and testing.
7. They help us visualize the sampling distribution through tables and graphs that approximately represent the sampling distribution.
9. We studied the sampling distribution of mean trout lengths based on samples of size 5. Other such sampling distributions abound.

Section 6.5

Note: Answers may differ slightly depending on the number of digits carried in the standard deviation.

1. The standard deviation.
3. $\bar{x}$ is an unbiased estimator for μ; $\hat{p}$ is an unbiased estimator for p.
5. (a) Normal; $\mu_{\bar{x}} = 8$; $\sigma_{\bar{x}} = 2$. (b) 0.50. (c) 0.3085. (d) No, about 30% of all such samples have means exceeding 9.
7. (a) 30 or more. (b) No.
9. The second. The standard error of the first is $\sigma/10$, while that of the second is $\sigma/15$, where σ is the standard deviation of the original x distribution.
11. (a) $\mu_{\bar{x}} = 15$; $\sigma_{\bar{x}} = 2.0$; $P(15 \le \bar{x} \le 17) = P(0 \le z \le 1.00) = 0.3413$. (b) $\mu_{\bar{x}} = 15$; $\sigma_{\bar{x}} = 1.75$; $P(15 \le \bar{x} \le 17) = P(0 \le z \le 1.14) = 0.3729$. (c) The standard deviation is smaller in part (b) because of the larger sample size. Therefore, the distribution about $\mu_{\bar{x}}$ is narrower in part (b).
13. (a) $P(x < 74.5) = P(z < -0.63) = 0.2643$. (b) $P(\bar{x} < 74.5) = P(z < -2.79) = 0.0026$. (c) No. If the weight of coal in only one car were less than 74.5 tons, we could not conclude that the loader is out of adjustment. If the mean weight of coal for a sample of 20 cars were less than 74.5 tons, we would suspect that the loader is malfunctioning. As we see in part (b), the probability of this happening is very low if the loader is correctly adjusted.
15. (a) $P(x < 40) = P(z < -1.80) = 0.0359$. (b) Since the x distribution is approximately normal, the $\bar{x}$ distribution is approximately normal, with mean 85 and standard deviation 17.678. $P(\bar{x} < 40) = P(z < -2.55) = 0.0054$. (c) $P(\bar{x} < 40) = P(z < -3.12) = 0.0009$. (d) $P(\bar{x} < 40) = P(z < -4.02) < 0.0002$. (e) Yes; if the average value based on five tests were less than 40, the patient is almost certain to have excess insulin.
17. (a) $P(x < 54) = P(z < -1.27) = 0.1020$. (b) The expected number undernourished is 2200(0.1020), or about 224. (c) $P(\bar{x} \le 60) = P(z \le -2.99) = 0.0014$. (d) $P(\bar{x} < 64.2) = P(z < 1.20) = 0.8849$. Since the sample average is above the mean, it is quite unlikely that the doe population is undernourished.
19. (a) Since x itself represents a sample mean return based on a large (random) sample of stocks, x has a distribution that is approximately normal (central limit theorem). (b) $P(1\% \le \bar{x} \le 2\%) = P(-1.63 \le z \le 1.09) = 0.8105$. (c) $P(1\% \le \bar{x} \le 2\%) = P(-3.27 \le z \le 2.18) = 0.9849$. (d) Yes. The standard deviation decreases as the sample size increases. (e) $P(\bar{x} < 1\%) = P(z < -3.27) = 0.0005$. This is very unlikely if $\mu = 1.6\%$. One would suspect that μ has slipped below 1.6%.
21. (a) The total checkout time for 30 customers is the sum of the checkout times for each individual customer. Thus, $w = x_1 + x_2 + \cdots + x_{30}$, and the probability that the total checkout time for the next 30 customers is less than 90 is $P(w < 90)$. (b) $w < 90$ is equivalent to $x_1 + x_2 + \cdots + x_{30} < 90$. Divide both sides by 30 to get $\bar{x} < 3$ for samples of size 30. Therefore, $P(w < 90) = P(\bar{x} < 3)$. (c) By the central limit theorem, $\bar{x}$ is approximately normal, with $\mu_{\bar{x}} = 2.7$ minutes and $\sigma_{\bar{x}} = 0.1095$ minute. (d) $P(\bar{x} < 3) = P(z < 2.74) = 0.9969$.
23. (a) $P(w > 90) = P(\bar{x} > 18) = P(z > 0.68) = 0.2483$. (b) $P(w < 80) = P(\bar{x} < 16) = P(z < -0.68) = 0.2483$. (c) $P(80 < w < 90) = P(16 < \bar{x} < 18) = P(-0.68 < z < 0.68) = 0.5034$.

Section 6.6

1. $np > 5$ and $nq > 5$, where $q = 1 - p$.
3. (a) Yes, both $np > 5$ and $nq > 5$. (b) $\mu = 20$; $\sigma \approx 3.162$. (c) $r \ge 23$ corresponds to $x \ge 22.5$. (d) $P(r \ge 23) \approx P(x \ge 22.5) \approx P(z \ge 0.79) \approx 0.2148$. (e) No, the probability that this will occur is about 21%.
5. No, $np = 4.3$ and does not satisfy the criterion that $np > 5$.

Note: Answers may differ slightly depending on how many digits are carried in the computation of the standard deviation and z.

7. $np > 5$; $nq > 5$. (a) $P(r \ge 50) = P(x \ge 49.5) = P(z \ge -27.53) \approx 1$, or almost certain. (b) $P(r \ge 50) = P(x \ge 49.5) = P(z \ge 7.78) \approx 0$, or almost impossible for a random sample.
9. $np > 5$; $nq > 5$. (a) $P(r \ge 15) = P(x \ge 14.5) = P(z \ge -2.35) = 0.9906$. (b) $P(r \ge 30) = P(x \ge 29.5) = P(z \ge 0.62) = 0.2676$. (c) $P(25 \le r \le 35) + P(24.5 \le x \le 35.5) = P(-0.37 \le z \le 1.81) = 0.6092$. (d) $P(r > 40) = P(r \ge 41) = P(x \ge 40.5) = P(z \ge 2.80) = 0.0026$.
11. $np > 5$; $nq > 5$. (a) $P(r \ge 47) = P(x \ge 46.5) = P(z \ge -1.94) = 0.9738$. (b) $P(r \le 58) = P(x \le 58.5) = P(z \le 1.75) = 0.9599$. In parts (c) and (d), let r be the number of products that succeed, and use $p = 1 - 0.80 = 0.20$. (c) $P(r \ge 15) = P(x \ge 14.5) = P(z \ge 0.40) = 0.3446$. (d) $P(r < 10) = P(r \le 9) = P(x \le 9.5) = P(z \le -1.14) = 0.1271$.
13. $np > 5$; $nq > 5$. (a) $P(r > 180) = P(x \ge 180.5) = P(z > -1.11) = 0.8665$. (b) $P(r < 200) = P(x \le 199.5) = P(z \le 1.07) = 0.8577$. (c) P(take sample and

buy product) = P(take sample) · P(buy | take sample) = 0.222. (d) $P(60 \le r \le 80) = P(59.5 \le x \le 80.5) = P(-1.47 \le z \le 1.37) = 0.8439$.

15. $np > 5$; $nq > 5$. (a) 0.94. (b) $P(r \le 255)$. (c) $P(r \le 255) = P(x \le 255.5) = P(z \le 1.16) = 0.8770$.
17. $np > 5$ and $nq > 5$.
19. Yes, since the mean of the approximate sampling distribution is $\mu_{\hat{p}} = p$.
21. (a) Yes, both np and nq exceed 5. $\mu_{\hat{p}} = 0.23$; $\sigma_{\hat{p}} \approx 0.042$. (b) No, $np = 4.6$ and does not exceed 5.

Chapter 6 Review

1. Normal probability distributions are distributions of continuous random variables. They are symmetric about the mean and bell-shaped. Most of the data fall within 3 standard deviations of the mean. The mean and median are the same.
3. No.
5. The points lie close to a straight line.
7. $\sigma_{\bar{x}} = \sigma/\sqrt{n}$.
9. (a) A normal distribution. (b) The mean μ of the x distribution. (c) $\sigma/\sqrt{n}$, where σ is the standard deviation of the x distribution. (d) They will both be approximately normal with the same mean, but the standard deviations will be $\sigma/\sqrt{50}$ and $\sigma/\sqrt{100}$, respectively.
11. (a) 0.9821. (b) 0.3156. (c) 0.2977.
13. 1.645.
15. (a) 0.8665. (b) 0.7330.
17. (a) 0.0166. (b) 0.975.
19. (a) 0.9772. (b) 17.3 hours.
21. (a) $P(x \ge 40) = P(z \ge 0.71) = 0.2389$. (b) $P(\bar{x} \ge 40) = P(z \ge 2.14) = 0.0162$.
23. $P(98 \le \bar{x} \le 102) = P(-1.33 \le z \le 1.33) = 0.8164$.
25. (a) Yes, np and nq both exceed 5. (b) $\mu_{\hat{p}} = 0.4$; $\sigma_{\hat{p}} = 0.1$.

CUMULATIVE REVIEW PROBLEMS

1. The specified ranges of readings are disjoint and cover all possible readings.
2. Essay.
3. Yes; the events constitute the entire sample space.
4. (a) 0.85. (b) 0.70. (c) 0.70. (d) 0.30. (e) 0.15. (f) 0.75. (g) 0.30. (h) 0.05.
5. 0.17
6.

x	$P(x)$
5	0.25
15	0.45
25	0.15
35	0.10
45	0.05

$\mu \approx 17.5$; $\sigma \approx 10.9$.

7. (a) $p = 0.10$. (b) $\mu = 1.2$; $\sigma \approx 1.04$. (c) 0.718. (d) 0.889.
8. (a) 0.05. (b) $P(n) = (0.05)(0.95)^{n-1}$; $n \ge 1$. (c) 0.81.
9. (a) Yes; since $n = 100$ and $np = 5$, the criteria $n \ge 100$ and $np < 10$ are satisfied. $\lambda = 5$. (b) 0.7622. (c) 0.0680.
10. (a) Yes; both np and nq exceed 5. (b) 0.9925. (c) np is too large ($np > 10$) and n is too small ($n < 100$).
11. (a) $\sigma \approx 1.7$. (b) 0.1314. (c) 0.1075.
12. Essay based on material from Chapter 6 and Section 1.2.
13. (a) Because of the large sample size, the central limit theorem describes the $\bar{x}$ distribution (approximately). (b) $P(\bar{x} \le 6820) = P(z \le -2.75) = 0.0030$. (c) The probability that the average white blood cell count for 50 healthy adults is as low as or lower than 6820 is very small, 0.0030. Based on this result, it would be reasonable to gather additional facts.
14. (a) Yes, both np and nq exceed 5. (b) $\mu_{\hat{p}} = p = 0.45$; $\sigma_{\hat{p}} \approx 0.09$.
15. Essay.

CHAPTER 7

Section 7.1

1. True. By definition, critical values z_c are values such that c% of the area under the normal curve falls between $-z_c$ and z_c.
3. True. By definition, the margin of error is the magnitude of the difference between $\bar{x}$ and μ.
5. False. The maximal margin of error is $E = z_c \dfrac{\sigma}{\sqrt{n}}$. As the sample size n increases, the maximal error decreases, resulting in a shorter confidence interval for μ.
7. False. The maximal error of estimate E controls the length of the confidence interval regardless of the value of $\bar{x}$.
9. μ is either in the interval 10.1 to 12.2 or not. Therefore, the probability that μ is in this interval is either 0 or 1, not 0.95.
11. (a) Yes, the x distribution is normal and σ is known so the $\bar{x}$ distribution is also normal. (b) 47.53 to 52.47. (c) You are 90% confident that the confidence interval computed is one that contains μ.
13. (a) 217. (b) Yes, by the central limit theorem.
15. (a) 3.04 gm to 3.26 gm; 0.11 gm. (b) Distribution of weights is normal with known σ. (c) There is an 80% chance that the confidence interval is one of the intervals that contain the population average weight of Allen's hummingbirds in this region. (d) $n = 28$.
17. (a) 34.62 ml/kg to 40.38 ml/kg; 2.88 ml/kg. (b) The sample size is large (30 or more) and σ is known. (c) There is a 99% chance that the confidence interval is one of the intervals that contain the population average blood plasma level for male firefighters. (d) $n = 60$.
19. (a) 125.7 to 151.3 larceny cases; 12.8 larceny cases. (b) 123.3 to 153.7 larceny cases; 15.2 larceny cases. (c) 118.4 to 158.6 larceny cases; 20.1 larceny cases. (d) Yes. (e) Yes.
21. (a) 26.64 to 33.36; 3.36. (b) 27.65 to 32.35; 2.35. (c) 28.43 to 31.57; 1.57. (d) Yes. (e) Yes.
23. (a) The mean rounds to the value given. (b) Using the rounded value of part (a), the 75% interval is from 34.19 thousand to 37.81 thousand. (c) Yes; 30 thousand dollars is below the lower bound of the 75% confidence interval. We can say with 75% confidence that the mean lies between 34.19 thousand and 37.81 thousand. (d) Yes; 40 thousand is above the upper bound of the 75% confidence interval. (e) 33.41 thousand to 38.59 thousand. We can say with 90% confidence that the mean

lies between 33.4 thousand and 38.6 thousand dollars. 30 thousand is below the lower bound and 40 thousand is above the upper bound.

25. (a) 92.5°C to 101.5°C. (b) The balloon will go up.

Section 7.2

1. 2.110.
3. 1.721.
5. $t = 0$.
7. $n = 10$, with $d.f. = 9$.
9. Shorter. For $d.f. = 40$, z_c is less than t_c, and the resulting margin of error E is smaller.
11. (a) Yes, since x has a mound-shaped distribution. (b) 9.12 to 10.88. (c) There is a 90% chance that the confidence interval you computed is one of the confidence intervals that contain μ.
13. (a) The mean and standard deviation round to the values given. (b) Using the rounded values for the mean and standard deviation given in part (a), the interval is from 1249 to 1295. (c) We are 90% confident that the computed interval is one that contains the population mean for the tree-ring date.
15. (a) Use a calculator. (b) 74.7 pounds to 107.3 pounds. (c) We are 75% confident that the computed interval is one that contains the population mean weight of adult mountain lions in the region.
17. (a) The mean and standard deviation round to the given values. (b) 8.41 to 11.49. (c) Since all values in the 99.9% confidence interval are above 6, we can be almost certain that this patient no longer has a calcium deficiency.
19. (a) Boxplots differ in length of interquartile box, location of median, and length of whiskers. The boxplots come from different samples. (b) Yes; no; for 95% confidence intervals, we expect about 95% of the samples to generate intervals that contain the mean of the population.
21. (a) The mean and standard deviation round to the given values. (b) 21.6 to 28.8. (c) 19.4 to 31.0. (d) Using both confidence intervals, we can say that the P/E for Bank One is well below the population average. The P/E for AT&T Wireless is well above the population average. The P/E for Disney is within both confidence intervals. It appears that the P/E for Disney is close to the population average P/E. (e) By the central limit theorem, when n is large, the $\bar{x}$ distribution is approximately normal. In general, $n \geq 30$ is considered large.
23. (a) $d.f. = 30$; 43.59 to 46.82; 43.26 to 47.14; 42.58 to 47.81. (b) 43.63 to 46.77; 43.33 to 47.07; 42.74 to 47.66. (c) Yes; the respective intervals based on the Student's t distribution are slightly longer. (d) For Student's t, $d.f. = 80$; 44.22 to 46.18; 44.03 to 46.37; 43.65 to 46.75. For standard normal, 44.23 to 46.17; 44.05 to 46.35; 43.68 to 46.72. The intervals using the t distribution are still slightly longer than the corresponding intervals using the standard normal distribution. However, with a larger sample size, the differences between the two methods are less pronounced.

Section 7.3

1. $\hat{p} = r/n$.
3. (a) No. (b) The difference between $\hat{p}$ and p. In other words, the margin of error is the difference between results based on a random sample and results based on a population.
5. No, Jerry does not have a random sample of all laptops. In fact, he does not even have a random sample of laptops from the computer science class. Also, because all the laptops he tested for spyware are those of students from the same computer class, it could be that students shared software with classmates and spread the infection among the laptops owned by the students of the class.
7. (a) $n\hat{p} = 30$ and $n\hat{q} = 70$, so both products exceed 5. Also, the trials are binomial trials. (b) 0.225 to 0.375. (c) You are 90% confident that the confidence interval you computed is one of the intervals that contain p.
9. (a) 73. (b) 97.
11. (a) $\hat{p} = 39/62 = 0.6290$. (b) 0.51 to 0.75. If this experiment were repeated many times, about 95% of the intervals would contain p. (c) Both np and nq are greater than 5. If either is less than 5, the normal curve will not necessarily give a good approximation to the binomial.
13. (a) $\hat{p} = 1619/5222 = 0.3100$. (b) 0.29 to 0.33. If we repeat the survey with many different samples of 5222 dwellings, about 99% of the intervals will contain p. (c) Both np and nq are greater than 5. If either is less than 5, the normal curve will not necessarily give a good approximation to the binomial.
15. (a) $\hat{p} = 0.5420$. (b) 0.53 to 0.56. (c) Yes. Both np and nq are greater than 5.
17. (a) $\hat{p} = 0.0304$. (b) 0.02 to 0.05. (c) Yes. Both np and nq are greater than 5.
19. (a) $\hat{p} = 0.8603$. (b) 0.84 to 0.89. (c) A recent study shows that 86% of women shoppers remained loyal to their favorite supermarket last year. The margin of error was 2.5 percentage points.
21. (a) $\hat{p} = 0.25$. (b) 0.22 to 0.28. (c) A survey of 1000 large corporations has shown that 25% will choose a nonsmoking job candidate over an equally qualified smoker. The margin of error was 2.7%.
23. (a) Estimate a proportion; 208. (b) 68.
25. (a) Estimate a proportion; 666. (b) 662.
27. (a) $1/4 - (p - 1/2)^2 = 1/4 - (p^2 - p + 1/4) = -p^2 + p = p(1 - p)$. (b) Since $(p - 1/2)^2 \geq 0$, then $1/4 - (p - 1/2)^2 \leq 1/4$ because we are subtracting $(p - 1/2)^2$ from 1/4.

Section 7.4

1. Two random samples are independent if sample data drawn from one population are completely unrelated to the selection of sample data from the other population.
3. Josh's, because the critical value t_c is smaller based on larger $d.f.$; Kendra's, because her value for t_c is larger.
5. $\mu_1 < \mu_2$.
7. (a) Normal distribution by Theorem 7.1 and the fact that the samples are independent and the population standard deviations are known. (b) $E \approx 1.717$; interval from -3.717 to -0.283. (c) Student's t distribution

with $d.f. = 19$, based on the fact that the original distributions are normal and the samples are independent. (d) $t_{0.90} = 1.729$; $E \approx 1.720$; interval from -3.805 to -0.195. (e) $d.f. \approx 42.85$; interval from -3.755 to -0.245. (f) Since the 90% confidence interval contains all negative values, you can be 90% confident that μ_1 is less than μ_2.

9. (a) Yes, $n_1\hat{p}_1$, $n_1\hat{q}_1$, $n_2\hat{p}_2$, $n_2\hat{q}_2$ all exceed 5. (b) $\hat{\sigma} \approx 0.0943$; $E \approx 0.155$; -0.205 to 0.105. (c) No, the 90% confidence interval contains both negative and positive values.
11. (a) Use a calculator. (b) $d.f. \approx 11$; $E \approx 129.9$; interval from -121.3 to 138.5 ppm. (c) Because the interval contains both positive and negative numbers, we cannot say at the 90% confidence level that one region is more interesting than the other. (d) Student's t because σ_1 and σ_2 are unknown.
13. (a) Use a calculator. (b) $d.f. \approx 15$; $E \approx 5.42$; interval from 12.64% to 23.48% foreign revenue. (c) Because the interval contains only positive values, we can say at the 85% confidence level that technology companies have a higher population mean percentage foreign revenue. (d) Student's t because σ_1 and σ_2 are unknown.
15. (a) Use a calculator. (b) $d.f. \approx 39$; to use Table 6, round down to $d.f. \approx 35$; $E \approx 0.125$; interval from -0.399 to -0.149 feet. (c) Since the interval contains all negative numbers, it seems that at the 90% confidence level the population mean height of pro football players is less than that of pro basketball players. (d) Student's t distribution because σ_1 and σ_2 are unknown. Both samples are large, so no assumptions about the original distributions are needed.
17. (a) Yes, the sample sizes, number of successes, and number of failures are sufficiently large. (b) $\hat{\sigma} \approx 0.0232$; $E = 0.0599$; the interval is from 0.67 to 0.79. (c) The confidence interval contains values that are all positive, so we can be 99% sure that $p_1 > p_2$.
19. (a) Normal distribution since the sample sizes are sufficiently large and both σ_1 and σ_2 are known. (b) $E = 0.3201$; the interval is from -9.12 to -8.48. (c) The interval consists of negative values only. At the 99% confidence level, we can conclude that $\mu_1 < \mu_2$.
21. (a) Yes, the sample sizes, number of successes, and number of failures are sufficiently large. (b) $\hat{p}_1 = 0.3095$; $\hat{p}_2 = 0.1184$; $\hat{\sigma} = 0.0413$; interval from 0.085 to 0.297. (c) The interval contains numbers that are all positive. A greater proportion of hogans exist in Fort Defiance.
23. (a) Use a calculator. (b) Student's t distribution because the population standard deviations are unknown. In addition, since the original distributions are not normal, the sample sizes are too small. (c) $d.f. \approx 9$; $E \approx 5.3$; 3.7 to 14.3 pounds. (d) Interval contains all positive values. At the 85% confidence level, it appears that the population mean weight of gray wolves in Chihuahua is greater than that of gray wolves in Durango.
25. (a) -1.35 to 2.39. (b) 0.06 to 3.86. (c) -0.61 to 3.49. (d) At the 85% confidence level, we can say that the mean index of self-esteem based on competence is greater than the mean index of self-esteem based on physical attractiveness. We cannot conclude that there is a difference between the mean index of self-esteem based on competence and that based on social acceptance. We also cannot conclude that there is a difference in the mean indices based on social acceptance and physical attractiveness.
27. (a) Based on the same data, a 99% confidence interval is longer than a 95% confidence interval. Therefore, if the 95% confidence interval has both positive and negative values, so will the 99% confidence interval. However, for the same data, a 90% confidence interval is shorter than a 95% confidence interval. The 90% confidence interval might contain only positive or only negative values even if the 95% interval contains both. (b) Based on the same data, a 99% confidence interval is longer than a 95% confidence interval. Even if the 95% confidence interval contains values that are all positive, the longer 99% interval could contain both positive and negative values. Since, for the same data, a 90% confidence interval is shorter than a 95% confidence interval, if the 95% confidence interval contains only positive values, so will the 90% confidence interval.
29. (a) $n = 896.1$, or 897 couples in each sample. (b) $n = 768.3$, or 769 couples in each sample.
31. (a) Pooled standard deviation $s \approx 8.6836$; interval from 3.9 to 14.1. (b) The pooled standard deviation method has a shorter interval and a larger $d.f.$

Chapter 7 Review

1. See text.
3. (a) No, the probability that μ is in the interval is either 0 or 1. (b) Yes, 99% confidence intervals are constructed in such a way that 99% of all such confidence intervals based on random samples of the designated size will contain μ.
5. Interval for a mean; 176.91 to 180.49.
7. Interval for a mean. (a) Use a calculator. (b) 64.1 to 84.3.
9. Interval for a proportion; 0.50 to 0.54.
11. Interval for a proportion. (a) $\hat{p} = 0.4072$. (b) 0.333 to 0.482.
13. Difference of means. (a) Use a calculator. (b) $d.f. \approx 71$; to use Table 6, round down to $d.f. \approx 70$; $E \approx 0.83$; interval from -0.06 to 1.6. (c) Because the interval contains both positive and negative values, we cannot conclude at the 95% confidence level that there is any difference in soil water content between the two fields. (d) Student's t distribution because σ_1 and σ_2 are unknown. Both samples are large, so no assumptions about the original distributions are needed.
15. Difference of means. (a) $d.f. \approx 17$; $E \approx 2.5$; interval from 5.5 to 10.5 pounds. (b) Yes, the interval contains values that are all positive. At the 75% level of confidence, it appears that the average weight of adult male wolves from the Northwest Territories is greater.
17. Difference of proportions. (a) $\hat{p}_1 = 0.8495$; $\hat{p}_2 = 0.8916$; -0.1409 to 0.0567. (b) The interval contains both negative and positive numbers. We do not detect a difference in the proportions at the 95% confidence level.

19. (a) $P(A_1 < \mu_1 < B_1 \text{ and } A_2 < \mu_2 < B_2) = (0.80)(0.80) = 0.64$. The complement of the event $A_1 < \mu_1 < B_1$ *and* $A_2 < \mu_2 < B_2$ is that either μ_1 is not in the first interval or μ_2 is not in the second interval, or both. Thus, P(at least one interval fails) $= 1 - P(A_1 < \mu_1 < B_1 \text{ and } A_2 < \mu_2 < B_2) = 1 - 0.64 = 0.36$. (b) Suppose $P(A_1 < \mu_1 < B_1) = c$ and $P(A_2 < \mu_2 < B_2) = c$. If we want the probability that both hold to be 90%, and if x_1 and x_2 are independent, then $P(A_1 < \mu_1 < B_1 \text{ and } A_2 < \mu_2 < B_2) = 0.90$ means $P(A_1 < \mu_1 < B_1) \cdot P(A_2 < \mu_2 < B_2) = 0.90$, so $c^2 = 0.90$, or $c = 0.9487$. (c) In order to have a high probability of success for the whole project, the probability that each component will perform as specified must be significantly higher.

CHAPTER 8

Section 8.1

1. See text.
3. No, if we fail to reject the null hypothesis, we have not proved it beyond all doubt. We have failed only to find sufficient evidence to reject it.
5. Level of significance; α; type I.
7. Fail to reject H_0
9. 0.0184.
11. (a) H_0: $\mu = 40$. (b) H_1: $\mu \neq 40$. (c) H_1: $\mu > 40$. (d) H_1: $\mu < 40$.
13. (a) Yes, because x has a normal distribution. (b) $z \approx 1.12$. (c) 0.2628. (d) Fail to reject H_0 because P-value $> \alpha$.
15. (a) H_0: $\mu = 60$ kg. (b) H_1: $\mu < 60$ kg. (c) H_1: $\mu > 60$ kg. (d) H_1: $\mu \neq 60$ kg. (e) For part (b), the P-value area region is on the left. For part (c), the P-value area is on the right. For part (d), the P-value area is on both sides of the mean.
17. (a) H_0: $\mu = 16.4$ feet. (b) H_1: $\mu > 16.4$ feet. (c) H_1: $\mu < 16.4$ feet. (d) H_1: $\mu \neq 16.4$ feet. (e) For part (b), the P-value area is on the right. For part (c), the P-value area is on the left. For part (d), the P-value area is on both sides of the mean.
19. (a) $\alpha = 0.01$; H_0: $\mu = 4.7\%$; H_1: $\mu > 4.7\%$; right-tailed. (b) Normal; $\bar{x} = 5.38$; $z \approx 0.90$. (c) P-value ≈ 0.1841; on standard normal curve, shade area to the right of 0.90. (d) P-value of $0.1841 > 0.01$ for α; fail to reject H_0. (e) Insufficient evidence at the 0.01 level to reject claim that average yield for bank stocks equals average yield for all stocks.
21. (a) $\alpha = 0.01$; H_0: $\mu = 4.55$ grams; H_1: $\mu < 4.55$ grams; left-tailed. (b) Normal; $\bar{x} = 3.75$ grams; $z \approx -2.80$. (c) P-value ≈ 0.0026; on standard normal curve, shade area to the left of -2.80. (d) P-value of $0.0026 \leq 0.01$ for α; reject H_0. (e) The sample evidence is sufficient at the 0.01 level to justify rejecting H_0. It seems that the hummingbirds in the Grand Canyon region have a lower average weight.
23. (a) $\alpha = 0.01$; H_0: $\mu = 11\%$; H_1: $\mu \neq 11\%$; two-tailed. (b) Normal; $\bar{x} = 12.5\%$; $z = 1.20$. (c) P-value $= 2(0.1151) = 0.2302$; on standard normal curve, shade areas to the right of 1.20 and to the left of -1.20. (d) P-value of $0.2302 > 0.01$ for α; fail to reject H_0. (e) There is insufficient evidence at the 0.01 level to reject H_0. It seems that the average hail damage to wheat crops in Weld County matches the national average.

Section 8.2

1. The P-value for a two-tailed test of μ is twice that for a one-tailed test, based on the same sample data and null hypothesis.
3. $d.f. = n - 1$.
5. Yes. When P-value < 0.01, it is also true that P-value < 0.05.
7. (a) $0.010 < P$-value < 0.020; technology gives P-value ≈ 0.0150. (b) $0.005 < P$-value < 0.010; technology gives P-value ≈ 0.0075.
9. (a) Yes, since the original distribution is mound-shaped and symmetric and σ is unknown; $d.f. = 24$. (b) H_0: $\mu = 9.5$; H_1: $\mu \neq 9.5$. (c) $t \approx 1.250$. (d) $0.200 < P$-value < 0.250; technology gives $t \approx 0.2234$. (e) Fail to reject H_0 because the entire interval containing the P-value > 0.05 for α. (f) The sample evidence is insufficient at the 0.05 level to reject H_0.
11. (a) $\alpha = 0.01$; H_0: $\mu = 16.4$ feet; H_1: $\mu > 16.4$ feet. (b) Normal; $z \approx 1.54$. (c) P-value ≈ 0.618; on standard normal curve, shade area to the right of $z \approx 1.54$. (d) P-value of $0.0618 > 0.01$ for α; fail to reject H_0. (e) At the 1% level, there is insufficient evidence to say that the average storm level is increasing.
13. (a) $\alpha = 0.01$; H_0: $\mu = 1.75$ years; H_1: $\mu > 1.75$ years. (b) Student's t, $d.f. = 45$; $t \approx 2.481$. (c) $0.005 < P$-value < 0.010; on t graph, shade area to the right of 2.481. From TI-84, P-value ≈ 0.0084. (d) Entire P-interval ≤ 0.01 for α; reject H_0. (e) At the 1% level of significance, the sample data indicate that the average age of the Minnesota region coyotes is higher than 1.75 years.
15. (a) $\alpha = 0.05$; H_0: $\mu = 19.4$; H_1: $\mu \neq 19.4$ (b) Student's t, $d.f. = 35$; $t \approx -1.731$. (c) $0.050 < P$-value < 0.100; on t graph, shade area to the right of 1.731 and to the left of -1.731. From TI-84, P-value ≈ 0.0923. (d) P-value interval > 0.05 for α; fail to reject H_0. (e) At the 5% level of significance, the sample evidence does not support rejecting the claim that the average P/E of socially responsible funds is different from that of the S&P stock index.
17. i. Use a calculator. Rounded values are used in part ii. ii. (a) $\alpha = 0.05$; H_0: $\mu = 4.8$; H_1: $\mu < 4.8$. (b) Student's t, $d.f. = 5$; $t \approx -3.499$. (c) $0.005 < P$-value < 0.010; on t graph, shade area to the left of -3.499. From TI-84, P-value ≈ 0.0086. (d) P-value interval ≤ 0.05 for α; reject H_0. (e) At the 5% level of significance, sample evidence supports the claim that the average RBC count for this patient is less than 4.8.
19. i. Use a calculator. Rounded values are used in part ii. ii. (a) $\alpha = 0.01$; H_0: $\mu = 67$; H_1: $\mu \neq 67$. (b) Student's t, $d.f. = 15$; $t \approx -1.962$. (c) $0.050 < P$-value < 0.100; on t graph, shade area to the right of 1.962 and to the left of -1.962. From TI-84, P-value ≈ 0.0686. (d) P-value interval > 0.01; fail to reject H_0. (e) At the 1% level of significance, the sample evidence does not support a claim that the average thickness of slab avalanches in Vail is different from that in Canada.

21. i. Use a calculator. Rounded values are used in part ii. ii. (a) $\alpha = 0.05$; H_0: $\mu = 8.8$; H_1: $\mu \neq 8.8$. (b) Student's t, $d.f. = 13$; $t \approx -1.337$. (c) $0.200 < P\text{-value} < 0.250$; on t graph, shade area to the right of 1.337 and to the left of -1.337. From TI-84, P-value ≈ 0.2042. (d) P-value interval > 0.05; fail to reject H_0. (e) At the 5% level of significance, we cannot conclude that the average catch is different from 8.8 fish per day.
23. (a) The P-value of a one-tailed test is smaller. For a two-tailed test, the P-value is doubled because it includes the area in both tails. (b) Yes; the P-value of a one-tailed test is smaller, so it might be smaller than α, whereas the P-value of a corresponding two-tailed test may be larger than α. (c) Yes; if the two-tailed P-value is less than α, the smaller one-tail area is also less than α. (d) Yes, the conclusions can be different. The conclusion based on the two-tailed test is more conservative in the sense that the sample data must be more extreme (differ more from H_0) in order to reject H_0.
25. (a) For $\alpha = 0.01$, confidence level $c = 0.99$; interval from 20.28 to 23.72; hypothesized $\mu = 20$ is not in the interval; reject H_0. (b) H_0: $\mu = 20$; H_1: $\mu \neq 20$; $z = 3.000$; P-value ≈ 0.0026; P-value of $0.0026 \leq 0.01$ for α; reject H_0; conclusions are the same.
27. Critical value $z_0 = 2.33$; critical region is values to the right of 2.33; since the sample statistic $z = 1.54$ is not in the critical region, fail to reject H_0. At the 1% level, there is insufficient evidence to say that the average storm level is increasing. Conclusion is same as with P-value method.
29. Critical value is $t_0 = 2.412$ for one-tailed test with $d.f. = 45$; critical region is values to the right of 2.412. Since the sample test statistic $t = 2.481$ is in the critical region, reject H_0. At the 1% level, the sample data indicate that the average age of Minnesota region coyotes is higher than 1.75 years. Conclusion is same as with P-value method.

Section 8.3

1. For the conditions $np > 5$ and $nq > 5$, use the value of p from H_0. Note that $q = 1 - p$.
3. Yes. The corresponding P-value for a one-tailed test is half that for a two-tailed test, so the P-value of the one-tailed test is also less than 0.01.
5. (a) Yes, np and nq are both greater than 5. (b) H_0: $p = 0.50$; H_1: $p \neq 0.50$. (c) $\hat{p} = 0.40$; $z \approx -1.10$. (d) 0.2714. (e) Fail to reject H_0 because P-value of $0.2714 > 0.05$ for α. (f) The sample $\hat{p}$ value based on 30 trials is not sufficiently different from 0.50 to justify rejecting H_0 for $\alpha = 0.05$.
7. i. (a) $\alpha = 0.01$; H_0: $p = 0.301$; H_1: $p < 0.301$. (b) Standard normal; yes, $np \approx 64.7 > 5$ and $nq \approx 150.3 > 5$; $\hat{p} \approx 0.214$; $z \approx -2.78$. (c) P-value ≈ 0.0027; on standard normal curve, shade area to the left of -2.78. (d) P-value of $0.0027 \leq 0.01$ for α; reject H_0. (e) At the 1% level of significance, the sample data indicate that the population proportion of numbers with a leading "1" in the revenue file is less than 0.301, predicted by Benford's Law.
ii. Yes; the revenue data file seems to include more numbers with higher first nonzero digits than Benford's Law predicts.
iii. We have not proved H_0 to be false. However, because our sample data led us to reject H_0 and to conclude that there are too few numbers with a leading digit of 1, more investigation is merited.
9. (a) $\alpha = 0.01$; H_0: $p = 0.70$; H_1: $p \neq 0.70$. (b) Standard normal; $\hat{p} = 0.75$; $z \approx 0.62$. (c) P-value $= 2(0.2676) = 0.5352$; on standard normal curve, shade areas to the right of 0.62 and to the left of -0.62. (d) P-value of $0.5352 > 0.01$ for α; fail to reject H_0. (e) At the 1% level of significance, we cannot say that the population proportion of arrests of males aged 15 to 34 in Rock Springs is different from 70%.
11. (a) $\alpha = 0.01$; H_0: $p = 0.77$; H_1: $p < 0.77$. (b) Standard normal; $\hat{p} \approx 0.5556$; $z \approx -2.65$. (c) P-value ≈ 0.004; on standard normal curve, shade area to the left of -2.65. (d) P-value of $0.004 \leq 0.01$ for α; reject H_0. (e) At the 1% level of significance, the data show that the population proportion of driver fatalities related to alcohol is less than 77% in Kit Carson County.
13. (a) $\alpha = 0.01$; H_0: $p = 0.50$; H_1: $p < 0.50$. (b) Standard normal; $\hat{p} \approx 0.2941$; $z \approx -2.40$. (c) P-value $= 0.0082$; on standard normal curve, shade region to the left of -2.40. (d) P-value of $0.0082 \leq 0.01$ for α; reject H_0. (e) At the 1% level of significance, the data indicate that the population proportion of female wolves is now less than 50% in the region.
15. (a) $\alpha = 0.01$; H_0: $p = 0.261$; H_1: $p \neq 0.261$. (b) Standard normal; $\hat{p} \approx 0.1924$; $z \approx -2.78$. (c) P-value $= 2(0.0027) = 0.0054$; on standard normal curve, shade area to the right of 2.78 and to the left of -2.78. (d) P-value of $0.0054 \leq 0.01$ for α; reject H_0. (e) At the 1% level of significance, the sample data indicate that the population proportion of the five-syllable sequence is different from that of Plato's *Republic*.
17. (a) $\alpha = 0.01$; H_0: $p = 0.47$; H_1: $p > 0.47$. (b) Standard normal; $\hat{p} \approx 0.4871$; $z \approx 1.09$. (c) P-value $= 0.1379$; on standard normal curve, shade area to the right of 1.09. (d) P-value of $0.1379 > 0.01$ for α; fail to reject H_0. (e) At the 1% level of significance, there is insufficient evidence to support the claim that the population proportion of customers loyal to Chevrolet is more than 47%.
19. (a) $\alpha = 0.05$; H_0: $p = 0.092$; H_1: $p > 0.092$. (b) Standard normal; $\hat{p} \approx 0.1480$; $z \approx 2.71$. (c) P-value $= 0.0034$; on standard normal curve, shade region to the right of 2.71. (d) P-value of $0.0034 \leq 0.05$ for α; reject H_0. (e) At the 5% level of significance, the data indicate that the population proportion of students with hypertension during final exams week is higher than 9.2%.
21. (a) $\alpha = 0.01$; H_0: $p = 0.82$; H_1: $p \neq 0.82$. (b) Standard normal; $\hat{p} \approx 0.7671$; $z \approx -1.18$. (c) P-value $= 2(0.1190) = 0.2380$; on standard normal curve, shade area to the right of 1.18 and to the left of -1.18. (d) P-value of $0.2380 > 0.01$ for α; fail to reject H_0. (e) At the 1% level of significance, the evidence is insufficient to indicate that the population proportion of

extroverts among college student government leaders is different from 82%.

23. Critical value is $z_0 = -2.33$. The critical region consists of values less than -2.33. The sample test statistic $z = -2.65$ is in the critical region, so we reject H_0. This result is consistent with the *P*-value conclusion.

Section 8.4

1. Paired data are dependent.
3. H_0: $\mu_d = 0$; that is, the mean of the differences is 0, so there is no difference.
5. $d.f. = n - 1$.
7. (a) Yes. The sample size is sufficiently large. Student's t with $d.f. = 35$. (b) H_0: $\mu_d = 0$; H_1: $\mu_d \neq 0$. (c) $t = 2.400$ with $d.f. = 35$. (d) $0.020 < P\text{-value} < 0.050$. TI-84 gives $P\text{-value} \approx 0.0218$. (e) Reject H_0 since the entire interval containing the $P\text{-value} < 0.05$ for α. (f) At the 5% level of significance and for a sample size of 36, the sample mean of the differences is sufficiently different from 0 that we conclude the population mean of the differences is not zero.
9. (a) $\alpha = 0.05$; H_0: $\mu_d = 0$; H_1: $\mu_d \neq 0$. (b) Student's t, $d.f. = 7$; $\bar{d} \approx 2.25$; $t \approx 0.818$. (c) $0.250 < P\text{-value} < 0.500$; on t graph, shade area to the left of -0.818 and to the right of 0.818. From TI-84, $P\text{-value} \approx 0.4402$. (d) P-value interval > 0.05 for α; fail to reject H_0. (e) At the 5% level of significance, the evidence is insufficient to claim a difference in population mean percentage increases for corporate revenue and CEO salary.
11. (a) $\alpha = 0.01$; H_0: $\mu_d = 0$; H_1: $\mu_d > 0$. (b) Student's t, $d.f. = 4$; $\bar{d} \approx 12.6$; $t \approx 1.243$. (c) $0.125 < P\text{-value} < 0.250$; on t graph, shade area to the right of 1.243. From TI-84, $P\text{-value} \approx 0.1408$. (d) P-value interval > 0.01 for α; fail to reject H_0. (e) At the 1% level of significance, the evidence is insufficient to claim that the average peak wind gusts are higher in January.
13. (a) $\alpha = 0.05$; H_0: $\mu_d = 0$; H_1: $\mu_d > 0$. (b) Student's t, $d.f. = 7$; $\bar{d} \approx 6.125$; $t \approx 1.762$. (c) $0.050 < P\text{-value} < 0.075$; on t graph, shade area to the right of 1.762. From TI-84, $P\text{-value} \approx 0.0607$. (d) P-value interval > 0.05 for α; fail to reject H_0. (e) At the 5% level of significance, the evidence is insufficient to indicate that the population average percentage of male wolves in winter is higher.
15. (a) $\alpha = 0.05$; H_0: $\mu_d = 0$; H_1: $\mu_d > 0$. (b) Student's t, $d.f. = 7$; $\bar{d} \approx 6.0$; $t \approx 0.788$. (c) $0.125 < P\text{-value} < 0.250$; on t graph, shade area to the right of 0.788. From TI-84, $P\text{-value} \approx 0.2282$. (d) P-value interval > 0.05 for α; fail to reject H_0. (e) At the 5% level of significance, the evidence is insufficient to show that the population mean number of inhabited houses is greater than that of hogans.
17. i. Use a calculator. Nonrounded results are used in part ii. ii. (a) $\alpha = 0.05$; H_0: $\mu_d = 0$; H_1: $\mu_d > 0$. (b) Student's t, $d.f. = 35$; $\bar{d} \approx 2.472$; $t \approx 1.223$. (c) $0.100 < P\text{-value} < 0.125$; on t graph, shade area to the right of 1.223. From TI-84, $P\text{-value} \approx 0.1147$. (d) P-value interval > 0.05 for α; fail to reject H_0. (e) At the 5% level of significance, the evidence is insufficient to claim that the population mean cost of living index for housing is higher than that for groceries.
19. (a) $\alpha = 0.05$; H_0: $\mu_d = 0$; H_1: $\mu_d > 0$. (b) Student's t, $d.f. = 8$; $\bar{d} = 2.0$; $t \approx 1.333$. (c) $0.100 < P\text{-value} < 0.125$; on t graph, shade area to the right of 1.333. From TI-84, $P\text{-value} \approx 0.1096$. (d) P-value interval > 0.05 for α; fail to reject H_0. (e) At the 5% level of significance, the evidence is insufficient to claim that the population score on the last round is higher than that on the first.
21. (a) $\alpha = 0.05$; H_0: $\mu_d = 0$; H_1: $\mu_d > 0$. (b) Student's t, $d.f. = 7$; $\bar{d} \approx 0.775$; $t \approx 2.080$. (c) $0.025 < P\text{-value} < 0.050$; on t graph, shade area to the right of 2.080. From TI-84, $P\text{-value} \approx 0.0380$. (d) P-value interval ≤ 0.05 for α; reject H_0. (e) At the 5% level of significance, the evidence is sufficient to claim that the population mean time for rats receiving larger rewards to climb the ladder is less.
23. For a two-tailed test with $\alpha = 0.05$ and $d.f. = 7$, the critical values are $\pm t_0 = \pm 2.365$. The sample test statistic $t = 0.818$ is between -2.365 and 2.365, so we do not reject H_0. This conclusion is the same as that reached by the P-value method.

Section 8.5

1. (a) H_0 says that the population means are equal. (b) $z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$. (c) $t = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$ with $d.f.$ = smaller sample size - 1 or $d.f.$ is from Satterthwaite's formula.
3. H_0: $\mu_1 = \mu_2$ or H_0: $\mu_1 - \mu_2 = 0$.
5. $\bar{p} = \dfrac{r_1 + r_2}{n_1 + n_2}$.
7. H_1: $\mu_1 > \mu_2$; H_1: $\mu_1 - \mu_2 > 0$.
9. (a) Student's t with $d.f. = 48$. Samples are independent, population standard deviations are not known, and sample sizes are sufficiently large. (b) H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 \neq \mu_2$. (c) $\bar{x}_1 - \bar{x}_2 = -2$; $t \approx -3.037$. (d) $0.0010 < P\text{-value} < 0.010$ (using $d.f. = 45$ and Table 6). TI-84 gives $P\text{-value} \approx 0.0030$ with $d.f. \approx 110.96$. (e) Because the entire interval containing the $P\text{-value} < 0.01$ for α, reject H_0. (f) At the 1% level of significance, the sample evidence is sufficiently strong to reject H_0 and conclude that the population means are different.
11. (a) Standard normal. Samples are independent, population standard deviations are known, and sample sizes are sufficiently large. (b) H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 \neq \mu_2$. (c) $\bar{x}_1 - \bar{x}_2 = -2$; $z \approx -3.04$. (d) 0.0024. (e) P-value $0.0024 < 0.01$ for α, reject H_0. (f) At the 1% level of significance, the sample evidence is sufficiently strong to reject H_0 and conclude that the population means are different.
13. (a) $\bar{p} \approx 0.657$. (b) Standard normal distribution because $n_1\bar{p}$, $n_1\bar{q}$, $n_2\bar{p}$, $n_2\bar{q}$ are each greater than 5. (c) H_0: $p_1 = p_2$; H_1: $p_1 \neq p_2$ (d) $\hat{p}_1 - \hat{p}_2 = -0.1$; $z \approx -1.38$. (e) $P\text{-value} \approx 0.1676$. (f) Since P-value of $0.1676 \geq 0.05$ for α, fail to reject H_0. (g) At the 5% level of significance, the difference between the sample probabilities of success for the two binomial

experiments is too small to justify rejecting the hypothesis that the probabilities are equal.

15. (a) $\alpha = 0.01$; H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 > \mu_2$. (b) Standard normal; $\bar{x}_1 - \bar{x}_2 = 0.7$; $z \approx 2.57$. (c) P-value $= P(z > 2.57) \approx 0.0051$; on standard normal curve, shade area to the right of 2.57. (d) P-value of $0.0051 \le 0.01$ for α; reject H_0. (e) At the 1% level of significance, the evidence is sufficient to indicate that the population mean REM sleep time for children is more than that for adults.

17. (a) $\alpha = 0.05$; H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 \ne \mu_2$. (b) Standard normal; $\bar{x}_1 - \bar{x}_2 = 0.6$; $z \approx 2.16$. (c) P-value $= 2P(z > 2.16) \approx 2(0.0154) = 0.0308$; on standard normal curve, shade area to the right of 2.16 and to the left of -2.16. (d) P-value of $0.0308 \le 0.05$ for α; reject H_0. (e) At the 5% level of significance, the evidence is sufficient to show that there is a difference between mean responses regarding preference for camping or fishing.

19. i. Use rounded results to compute t.
    ii. (a) $\alpha = 0.01$; H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 < \mu_2$. (b) Student's t, $d.f. = 9$; $\bar{x}_1 - \bar{x}_2 = -0.36$; $t \approx -0.965$. (c) $0.125 < P$-value < 0.250; on t graph, shade area to the left of -0.965. From TI-84, $d.f. \approx 19.96$; P-value ≈ 0.1731. (d) P-value interval > 0.01 for α; do not reject H_0. (e) At the 1% level of significance, the evidence is insufficient to indicate that the violent crime rate in the Rocky Mountain region is higher than that in New England.

21. (a) $\alpha = 0.05$; H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 \ne \mu_2$. (b) Student's t, $d.f. = 29$; $\bar{x}_1 - \bar{x}_2 = -9.7$; $t \approx -0.751$. (c) $0.250 < P$-value < 0.500; on t graph, shade area to the right of 0.751 and to the left of -0.751. From TI-84, $d.f. \approx 57.92$; P-value ≈ 0.4556. (d) P-value interval > 0.05 for α; do not reject H_0. (e) At the 5% level of significance, the evidence is insufficient to indicate that there is a difference between the control and experimental groups in the mean score on the vocabulary portion of the test.

23. i. Use rounded results to compute t.
    ii. (a) $\alpha = 0.05$; H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 \ne \mu_2$. (b) Student's t, $d.f. = 14$; $\bar{x}_1 - \bar{x}_2 = 0.82$; $t \approx 0.869$. (c) $0.250 < P$-value < 0.500; on t graph, shade area to the right of 0.869 and to the left of -0.869. From TI-84, $d.f. \approx 28.81$; P-value ≈ 0.3940. (d) P-value interval > 0.05 for α; do not reject H_0. (e) At the 5% level of significance, the evidence is insufficient to indicate that there is a difference in the mean number of cases of fox rabies between the two regions.

25. i. Use rounded results to compute t.
    ii. (a) $\alpha = 0.05$; H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 \ne \mu_2$. (b) Student's t, $d.f. = 6$; $\bar{x}_1 - \bar{x}_2 = -1.64$; $t \approx -1.041$. (c) $0.250 < P$-value < 0.500; on t graph, shade area to the right of 1.041 and to the left of -1.041. From TI-84, $d.f. \approx 12.28$; P-value ≈ 0.3179. (d) P-value interval > 0.05 for α; do not reject H_0. (e) At the 5% level of significance, the evidence is insufficient to indicate that the mean time lost due to hot tempers is different from that lost due to technical workers' attitudes.

27. (a) $d.f. = 19.96$ (Some software will truncate this to 19.) (b) $d.f. = 9$; the convention of using the smaller of $n_1 - 1$ and $n_2 - 1$ leads to a $d.f.$ that is always less than or equal to that computed by Satterthwaite's formula.

29. (a) $\alpha = 0.05$; H_0: $p_1 = p_2$; H_1: $p_1 \ne p_2$. (b) Standard normal; $\bar{p} \approx 0.2911$; $\hat{p}_1 - \hat{p}_2 \approx -0.052$; $z \approx -1.13$. (c) P-value $\approx 2P(z < -1.13) \approx 2(0.1292) = 0.2584$ on standard normal curve, shade area to the right of 1.13 and to the left of -1.13. (d) P-value of $0.2584 > 0.05$ for α; fail to reject H_0. (e) At the 5% level of significance, there is insufficient evidence to conclude that the population proportion of women favoring more tax dollars for the arts is different from the proportion of men.

31. (a) $\alpha = 0.01$; H_0: $p_1 = p_2$; H_1: $p_1 \ne p_2$. (b) Standard normal; $\bar{p} \approx 0.0676$; $\hat{p}_1 - \hat{p}_2 \approx 0.0237$; $z \approx 0.79$. (c) P-value $\approx 2P(z > 0.79) \approx 2(0.2148) = 0.4296$; on standard normal curve, shade area to the right of 0.79 and to the left of -0.79. (d) P-value of $0.4296 > 0.01$ for α; fail to reject H_0. (e) At the 1% level of significance, there is insufficient evidence to conclude that the population proportion of high school dropouts on Oahu is different from that of Sweetwater County.

33. (a) $\alpha = 0.01$; H_0: $p_1 = p_2$; H_1: $p_1 < p_2$. (b) Standard normal; $\bar{p} = 0.42$; $\hat{p}_1 - \hat{p}_2 = -0.10$; $z \approx -1.43$. (c) P-value $\approx P(z < -1.43) \approx 0.0764$; on standard normal curve, shade area to the left of -1.43. (d) P-value of $0.0764 > 0.01$ for α; fail to reject H_0. (e) At the 1% level of significance, there is insufficient evidence to conclude that the population proportion of adults who believe in extraterrestrials and who attended college is higher than the proportion who believe in extraterrestrials but did not attend college.

35. (a) $\alpha = 0.05$; H_0: $p_1 = p_2$; H_1: $p_1 < p_2$. (b) Standard normal; $\bar{p} \approx 0.2189$; $\hat{p}_1 - \hat{p}_2 \approx -0.074$; $z \approx -2.04$. (c) P-value $\approx P(z < -2.04) \approx 0.0207$; on standard normal curve, shade area to the left of -2.04. (d) P-value of $0.0207 \le 0.05$ for α; reject H_0. (e) At the 5% level of significance, there is sufficient evidence to conclude that the population proportion of trusting people in Chicago is higher for the older group.

37. H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 < \mu_2$; for $d.f. = 9$, $\alpha = 0.01$ in the *one-tail area* row, the critical value is $t_0 = -2.821$; sample test statistic $t = -0.965$ is not in the critical region; fail to reject H_0. This result is consistent with that obtained by the P-value method.

Chapter 8 Review

1. Look at the original x distribution. If it is normal or $n \ge 30$, and σ is known, use the standard normal distribution. If the x distribution is mound-shaped or $n \ge 30$, and σ is unknown, use the Student's t distribution. The $d.f.$ is determined by the application.
3. A larger sample size increases the $|z|$ or $|t|$ value of the sample test statistic.
5. Single mean. (a) $\alpha = 0.05$; H_0: $\mu = 11.1$; H_1: $\mu \ne 11.1$. (b) Standard normal; $z = -3.00$. (c) P-value $= 0.0026$; on standard normal curve, shade area to the right of 3.00 and to the left of -3.00. (d) P-value of $0.0026 \le 0.05$ for α; reject H_0. (e) At the 5% level of significance, the evidence is sufficient to say that the

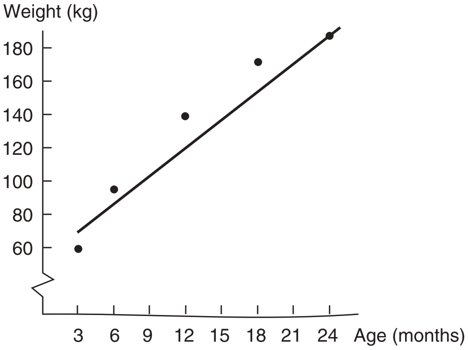
miles driven per vehicle in Chicago is different from the national average.

7. Single mean. (a) $\alpha = 0.01$; H_0: $\mu = 0.8$; H_1: $\mu > 0.8$. (b) Student's t, $d.f. = 8$; $t \approx 4.390$. (c) $0.0005 <$ P-value < 0.005; on t graph, shade area to the right of 4.390. From TI-84, P-value ≈ 0.0012. (d) P-value interval ≤ 0.01 for α; reject H_0. (e) At the 1% level of significance, the evidence is sufficient to say that the Toylot claim of 0.8 A is too low.
9. Single proportion. (a) $\alpha = 0.01$; H_0: $p = 0.60$; H_1: $p < 0.60$. (b) Standard normal; $z = -3.01$. (c) P-value $= 0.0013$; on standard normal curve, shade area to the left of -3.01. (d) P-value of $0.0013 \leq 0.01$ for α; reject H_0. (e) At the 1% level of significance, the evidence is sufficient to show that the mortality rate has dropped.
11. Single mean. (a) $\alpha = 0.01$; H_0: $\mu = 40$; H_1: $\mu > 40$. (b) Standard normal; $z = 3.34$. (c) P-value $= 0.0004$; on standard normal curve, shade area to the right of 3.34. (d) P-value of $0.0004 \leq 0.01$ for α; reject H_0. (e) At the 1% level of significance, the evidence is sufficient to say that the population average number of matches is larger than 40.
13. Difference of means. (a) $\alpha = 0.05$; H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 \neq \mu_2$. (b) Student's t, $d.f. = 50$; $\bar{x}_1 - \bar{x}_2 = 0.3$ cm; $t \approx 1.808$. (c) $0.050 <$ P-value < 0.100; on t graph, shade area to the right of 1.808 and to the left of -1.808. From TI-84, $d.f. \approx 100.27$, P-value ≈ 0.0735. (d) P-value interval > 0.05 for α; do not reject H_0. (e) At the 5% level of significance, the evidence is insufficient to indicate a difference in population mean length between the two types of projectile points.
15. Single mean. (a) $\alpha = 0.05$; H_0: $\mu = 7$ oz; H_1: $\mu \neq 7$ oz. (b) Student's t, $d.f. = 7$; $t \approx 1.697$. (c) $0.100 <$ P-value < 0.150; on t graph, shade area to the right of 1.697 and to the left of -1.697. From TI-84, P-value ≈ 0.1335. (d) P-value interval > 0.05 for α; do not reject H_0. (e) At the 5% level of significance, the evidence is insufficient to show that the population mean amount of coffee per cup is different from 7 oz.
17. Paired difference test. (a) $\alpha = 0.05$; H_0: $\mu_d = 0$; H_1: $\mu_d < 0$. (b) Student's t, $d.f. = 4$; $\bar{d} \approx -4.94$; $t = -2.832$. (c) $0.010 <$ P-value < 0.025; on t graph, shade area to the left of -2.832. From TI-84, P-value ≈ 0.0236. (d) P-value interval ≤ 0.05 for α; reject H_0. (e) At the 5% level of significance, there is sufficient evidence to claim that the population average net sales improved.

CHAPTER 9

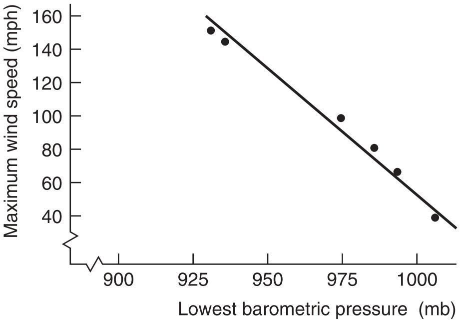
Section 9.1

1. Explanatory variable is placed along horizontal axis, usually x axis. Response variable is placed along vertical axis, usually y axis.
3. Decreases.
5. (a) Moderate. (b) None. (c) High.
7. (a) No. (b) Increasing population might be a lurking variable causing both variables to increase.
9. (a) No. (b) One lurking variable responsible for average annual income increases is inflation. Better training might be a lurking variable responsible for shorter times to run the mile.
11. The correlation coefficient is moderate and negative. It suggests that as gasoline prices increase, consumption decreases, and the relationship is moderately linear. It is risky to apply these results to gasoline prices much higher than \$5.30 per gallon. It could be that many of the discretionary and technical means of reducing consumption have already been applied, so consumers cannot reduce their consumption much more.
13. (a) Ages and Average Weights of Shetland Ponies



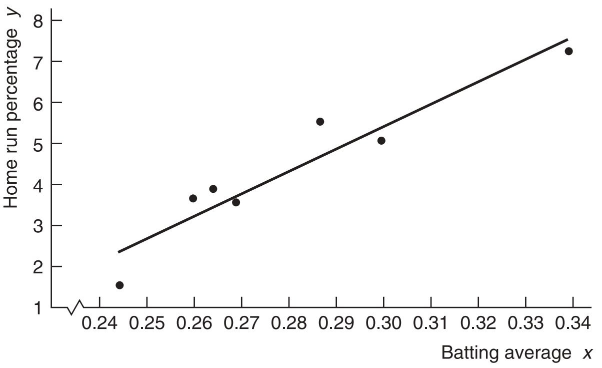
Line slopes upward.
(b) Strong; positive. (c) $r \approx 0.972$; increase.

15. (a) Lowest Barometric Pressure and Maximum Wind Speed for Tropical Cyclones



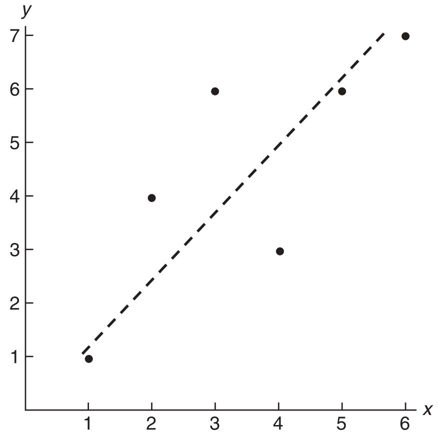
Line slopes downward.
(b) Strong; negative. (c) $r \approx -0.990$; decrease.

17. (a) Batting Average and Home Run Percentage

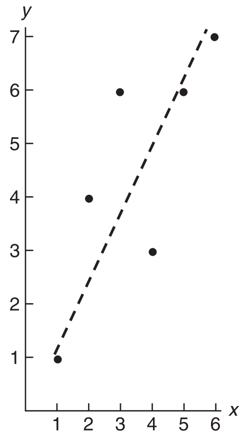


Line slopes upward.
(b) High; positive. (c) $r \approx 0.948$; increase.

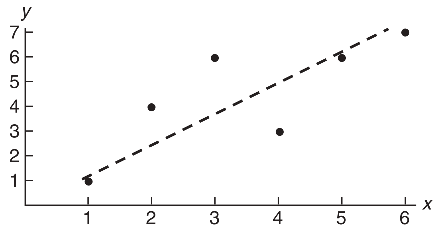
19. (a) Unit Length on y Same as That on x



(b) Unit Length on y Twice That on x



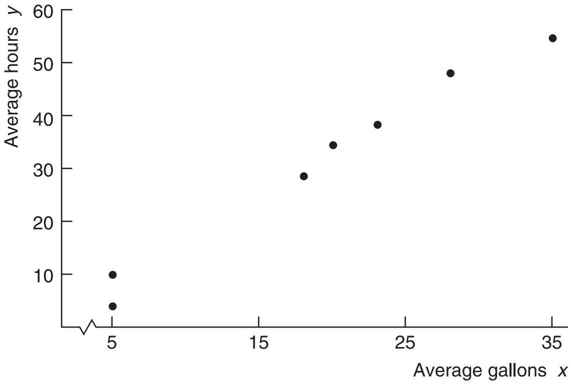
(c) Unit Length on y Half That on x



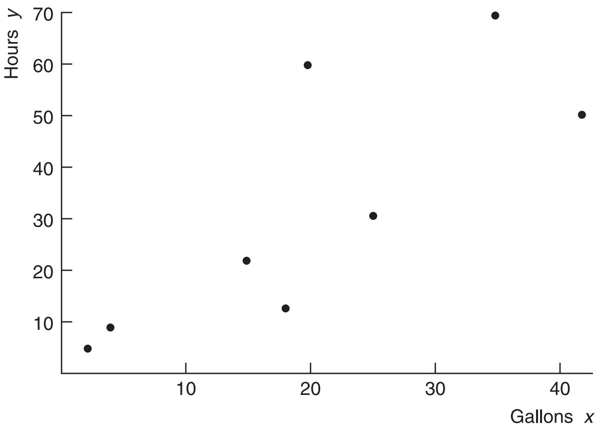
(d) The line in part (b) appears steeper than the line in part (a), whereas the line in part (c) appears flatter than the line in part (a). The slopes actually are all the same, but the lines look different because of the change in unit lengths on the y and x axes.

21. (a) $r \approx 0.972$ with $n = 5$ is significant for $\alpha = 0.05$. For this α, we conclude that age and weight of Shetland ponies are correlated. (b) $r \approx -0.990$ with $n = 6$ is significant for $\alpha = 0.01$. For this α, we conclude that lowest barometric pressure reading and maximum wind speed for cyclones are correlated.

23. (a) Average Hours Lost per Person versus Average Fuel Wasted per Person in Traffic Delays



$r \approx 0.991$.
(b) For variables based on averages, $\bar{x} = 19.25$ hr; $s_x \approx 10.33$ hr; $\bar{y} = 31.13$ gal; $s_y \approx 17.76$ gal. For variables based on single individuals, $\bar{x} = 20.13$ hr; $s_x \approx 13.84$ hr; $\bar{y} = 31.87$ gal; $s_y \approx 25.18$ gal. Dividing by larger numbers results in a smaller value.
(c) Hours Lost per Person versus Fuel Wasted per Person in Traffic Delays

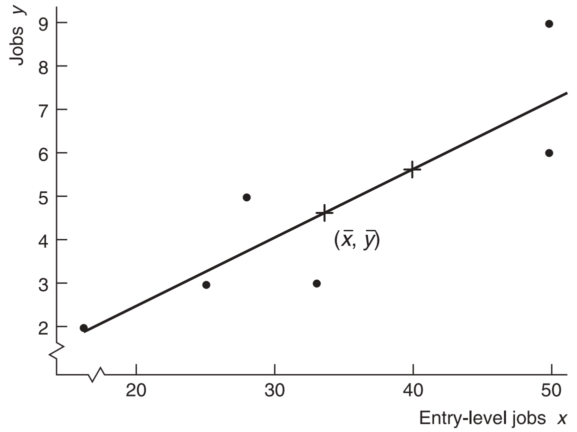


$r \approx 0.794$.
(d) Yes; by the central limit theorem, the $\bar{x}$ distribution has a smaller standard deviation than the corresponding x distribution.

Section 9.2

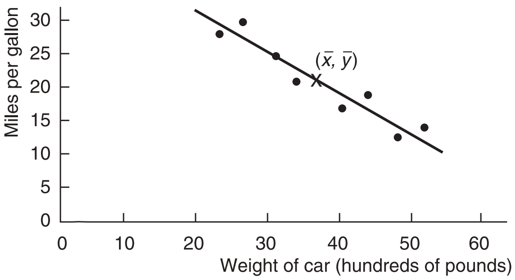
1. $b = -2$. When x changes by 1 unit, y decreases by 2 units.
3. Extrapolating. Extrapolating beyond the range of the data is dangerous because the relationship pattern might change.
5. (a) $\hat{y} \approx 318.16 - 30.878x$. (b) About 31 fewer frost-free days. (c) $r \approx -0.981$. Note that if the slope is negative, r is also negative. (d) 96.3% of variation explained and 3.7% unexplained.

7. (a) Total Number of Jobs and Number of Entry-Level Jobs (Units in 100's)



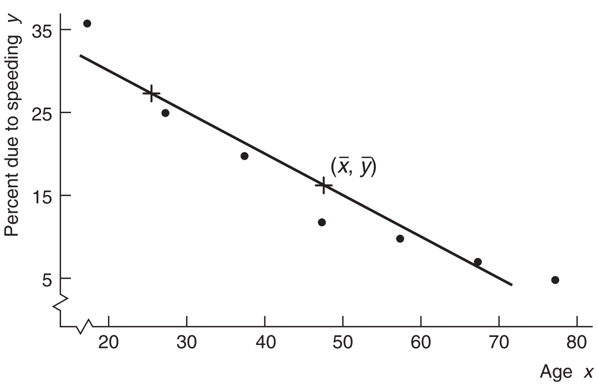
(b) Use a calculator. (c) $\bar{x} \approx 33.67$ jobs; $\bar{y} \approx 4.67$ entry-level jobs; $a \approx -0.748$; $b \approx 0.161$; $\hat{y} \approx -0.748 + 0.161x$ (d) See figure in part (a). (e) $r^2 \approx 0.740$; 74.0% of variation explained and 26.0% unexplained. (f) 5.69 jobs.

9. (a) Weight of Cars and Gasoline Mileage



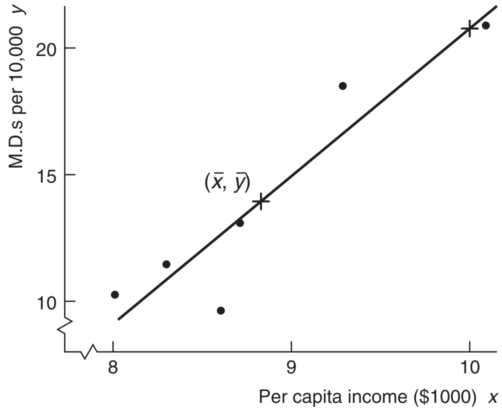
(b) Use a calculator. (c) $\bar{x} \approx 37.375$; $\bar{y} \approx 20.875$ mpg; $a \approx 43.326$; $b \approx -0.6007$; $\hat{y} \approx 43.326 - 0.6007x$. (d) See figure in part (a). (e) $r^2 \approx 0.895$; 89.5% of variation explained and 10.5% unexplained. (f) 20.5 mpg.

11. (a) Age and Percentage of Fatal Accidents Due to Speeding



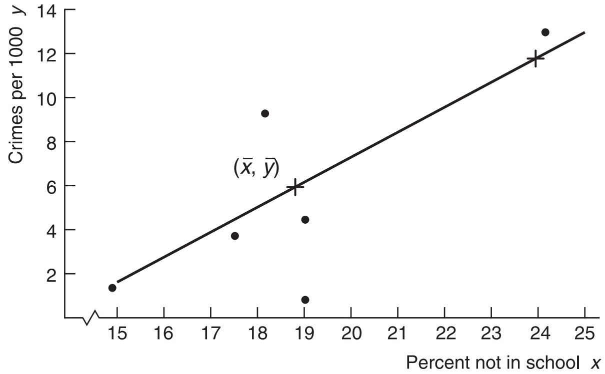
(b) Use a calculator. (c) $\bar{x} \approx 47$ years; $\bar{y} \approx 16.43\%$; $a \approx 39.761$; $b \approx -0.496$; $\hat{y} \approx 39.761 - 0.496x$. (d) See figure in part (a). (e) $r^2 \approx 0.920$; 92.0% of variation explained and 8.0% unexplained. (f) 27.36%.

13. (a) Per Capita Income ($1000) and M.D.s per 10,000 Residents



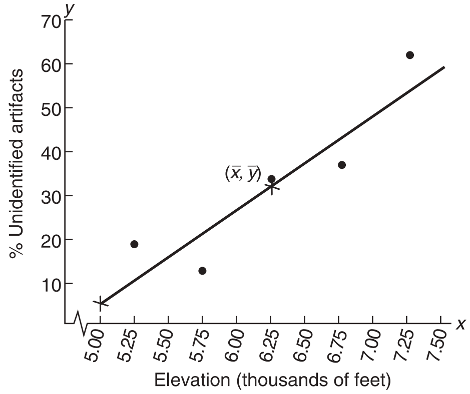
(b) Use a calculator. (c) $\bar{x} = \$8.83$; $\bar{y} \approx 13.95$ M.D.s; $a \approx -36.898$; $b \approx 5.756$; $\hat{y} \approx -36.898 + 5.756x$. (d) See figure in part (a). (e) $r^2 \approx 0.872$; 87.2% of variation explained, 12.8% unexplained. (f) 20.7 M.D.s per 10,000 residents.

15. (a) Percentage of 16- to 19-Year-Olds Not in School and Violent Crime Rate per 1000 Residents



(b) Use a calculator. (c) $\bar{x} = 18.8\%$; $\bar{y} = 5.4$; $a \approx -17.204$; $b \approx 1.202$; $\bar{y} \approx -17.204 + 1.202x$. (d) See figure in part (a). (e) $r^2 \approx 0.584$; 58.4% of variation explained, 41.6% unexplained. (f) 11.6 crimes per 1000 residents.

17. (a) Elevation of Archaeological Sites and Percentage of Unidentified Artifacts

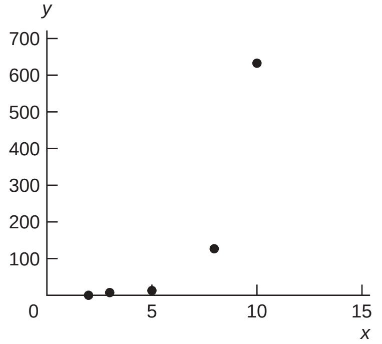


(b) Use a calculator. (c) $\bar{x} = 6.25$; $\bar{y} = 32.8$; $a = -104.7$; $b = 22$; $\hat{y} = -104.7 + 22x$.
(d) See figure in part (a). (e) $r^2 \approx 0.833$; 83.3% of variation explained, 16.7% unexplained. (f) 38.3.

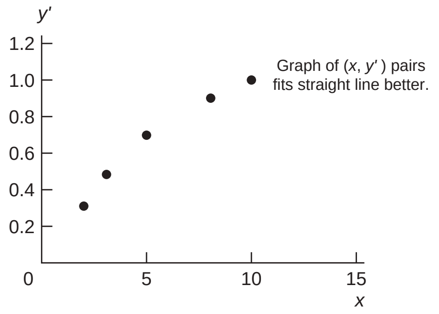
19. (a) Yes. The pattern of residuals appears randomly scattered about the horizontal line at 0. (b) No. There do not appear to be any outliers.

21. (a) Result checks. (b) Result checks. (c) Yes. (d) The equation $x = 0.9337y - 0.1335$ does not match part (b). (e) No. The least-squares equation changes depending on which variable is the explanatory variable and which is the response variable.

23. (a) Model with (x, y) Data Pairs

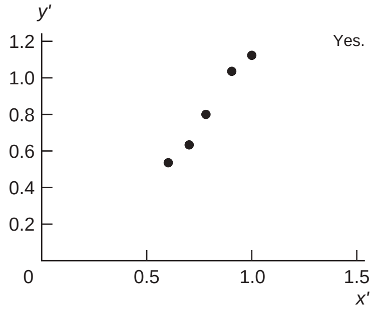


(b) Model with (x, y') Data Pairs



(c) $y' \approx -0.365 + 0.311x$; $r \approx 0.998$.
(d) $\alpha \approx 0.432$; $\beta \approx 2.046$; $y \approx 0.432(2.046)^x$.

25. (a) Model with (x', y') Data Pairs



(b) $y' \approx -0.451 + 1.600x'$; $r \approx 0.991$.
(c) $\alpha \approx 0.354$; $\beta \approx 1.600$; $y \approx 0.354x^{1.600}$.

Section 9.3

1. ρ (Greek letter rho).
3. As x becomes further away from $\bar{x}$, the confidence interval for the predicted y becomes longer.
5. (a) Diameter. (b) $a = -0.223$; $b = 0.7848$; $\hat{y} = -0.223 + 0.7848x$. (c) P-value of b is 0.001. H_0: $\beta = 0$; H_1: $\beta \neq 0$. Since P-value < 0.01, reject H_0 and conclude that the slope is not zero. (d) $r \approx 0.896$. Yes. P-value is 0.001, so we reject H_0 for $\alpha = 0.01$.
7. (a) Use a calculator. (b) $\alpha = 0.05$; H_0: $\rho = 0$; H_1: $\rho > 0$; sample $t \approx 2.522$; $d.f. = 4$; $0.025 < P$-value < 0.050; reject H_0. There seems to be a positive correlation between x and y. From TI-84, P-value ≈ 0.0326. (c) Use a calculator. (d) 45.36%. (e) Interval from 39.05 to 51.67. (f) $\alpha = 0.05$; H_0: $\beta = 0$; H_1: $\beta > 0$; sample $t \approx 2.522$; $d.f. = 4$; $0.025 < P$-value < 0.050; reject H_0. There seems to be a positive slope between x and y. From TI-84, P-value ≈ 0.0326. (g) Interval from 0.064 to 0.760. For every percentage increase in successful free throws, the percentage of successful field goals increases by an amount between 0.06 and 0.76.
9. (a) Use a calculator. (b) $\alpha = 0.01$; H_0: $\rho = 0$; H_1: $\rho < 0$; sample $t \approx -10.06$; $d.f. = 5$; P-value < 0.0005; reject H_0. The sample evidence supports a negative correlation. From TI-84, P-value ≈ 0.00008. (c) Use a calculator. (d) 2.39 hours. (e) Interval from 2.12 to 2.66 hours. (f) $\alpha = 0.01$; H_0: $\beta = 0$; H_1: $\beta < 0$; sample $t \approx -10.06$; $d.f. = 5$; P-value < 0.0005; reject H_0. The sample evidence supports a negative slope. From TI-84, P-value ≈ 0.00008. (g) Interval from -0.065 to -0.044. For every additional meter of depth, the optimal time decreases by between 0.04 and 0.07 hour.
11. (a) Use a calculator. (b) $\alpha = 0.01$; H_0: $\rho = 0$; H_1: $\rho > 0$; sample $t \approx 6.534$; $d.f. = 4$; $0.0005 < P$-value < 0.005; reject H_0. The sample evidence supports a positive correlation. From TI-84, P-value ≈ 0.0014. (c) Use a calculator. (d) \$12.577 thousand. (e) Interval from 12.247 to 12.907 (thousand dollars). (f) $\alpha = 0.01$; H_0: $\beta = 0$; H_1: $\beta > 0$; sample $t \approx 6.534$; $d.f. = 4$; $0.0005 < P$-value < 0.005; reject H_0. The sample evidence supports a positive slope. From TI-84, P-value ≈ 0.0014. (g) Interval from 0.436 to 1.080. For every \$1000 increase in list price, the dealer price increase is between \$436 and \$1080 higher.
13. (a) H_0: $\rho = 0$; H_1: $\rho \neq 0$; $d.f. = 4$; sample $t = 4.129$; $0.01 < P$-value < 0.02; do not reject H_0; r is not significant at the 0.01 level of significance. (b) H_0: $\rho = 0$; H_1: $\rho \neq 0$; $d.f. = 8$; sample $t = 5.840$; P-value < 0.001; reject H_0; r is significant at the 0.01 level of significance. (c) As n increases, the t value corresponding to r also increases, resulting in a smaller P-value.

Section 9.4

1. (a) Response variable is x_1. Explanatory variables are x_2, x_3, x_4. (b) 1.6 is the constant term; 3.5 is the coefficient of x_2; -7.9 is the coefficient of x_3; and 2.0 is

the coefficient of x_4. (c) $x_1 = 10.7$. (d) 3.5 units; 7 units; −14 units. (e) $d.f. = 8$; $t = 1.860$; 2.72 to 4.28. (f) $\alpha = 0.05$; H_0: $\beta_2 = 0$; H_1: $\beta_2 \neq 0$; $d.f. = 8$; $t = 8.35$; P-value < 0.001; reject H_0.

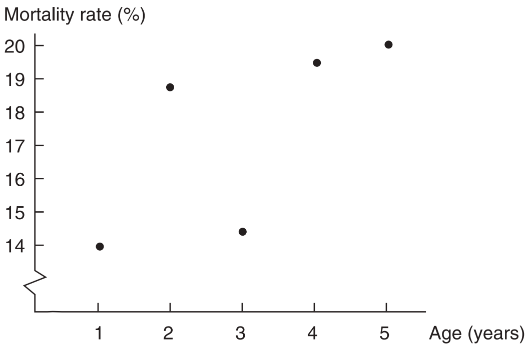
3. (a) $CVx_1 \approx 9.08$; $CVx_2 \approx 14.59$; $CVx_3 \approx 8.88$; x_2 has greatest spread; x_3 has smallest. (b) $r^2x_1x_2 \approx 0.958$; $r^2x_1x_3 \approx 0.942$; $r^2x_2x_3 \approx 0.895$; x_2; yes; 95.8%; 94.2%. (c) 97.7%. (d) $x_1 = 30.99 + 0.861x_2 + 0.335x_3$; 3.35; 8.61. (e) $\alpha = 0.05$; H_0: coefficient = 0; H_1: coefficient ≠ 0; $d.f. = 8$; for β_2, $t = 3.47$ with P-value = 0.008; for β_3, $t = 2.56$ with P-value = 0.034; reject H_0 for each coefficient and conclude that the coefficients of x_2 and x_3 are not zero. (f) $d.f. = 8$; $t = 1.86$; C.I. for β_2 is 0.40 to 1.32; C.I. for β_3 is 0.09 to 0.58. (g) 153.9; 148.3 to 159.4.

5. (a) $CVx_1 \approx 39.64$; $CVx_2 \approx 44.45$; $CVx_3 \approx 50.62$; $CVx_4 \approx 52.15$; x_4; x_1 has a small CV because we divide by a large mean. (b) $r^2x_1x_2 \approx 0.842$; $r^2x_1x_3 \approx 0.865$; $r^2x_1x_4 \approx 0.225$; $r^2x_2x_3 \approx 0.624$; $r^2x_2x_4 \approx 0.184$; $r^2x_3x_4 \approx 0.089$; x_4; 84.2%. (c) 96.7%. (d) $x_1 = 7.68 + 3.66x_2 + 7.62x_3 + 0.83x_4$; 7.62 million dollars. (e) $\alpha = 0.05$; H_0: coefficient = 0; H_1: coefficient ≠ 0; $d.f. = 6$; for β_2, $t = 3.28$ with P-value = 0.017; for β_3, $t = 4.60$ with P-value = 0.004; for β_4, $t = 1.54$ with P-value = 0.175; reject H_0 for β_2 and β_3 and conclude that the coefficients of x_2 and x_3 are not zero. For β_4, fail to reject H_0 and conclude that the coefficient of x_4 could be zero. (f) $d.f. = 6$; $t = 1.943$; C.I. for β_2 is 1.49 to 5.83; C.I. for β_3 is 4.40 to 10.84; C.I. for β_4 is −0.22 to 1.88. (g) 91.95; 77.6 to 106.3. (h) 5.63; 4.21 to 7.04.

7. Depends on data.

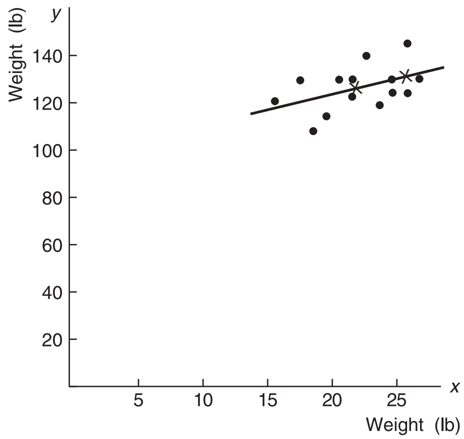
Chapter 9 Review

1. r will be close to 0.
3. Results are more reliable for interpolation.
5. (a) Age and Mortality Rate for Bighorn Sheep



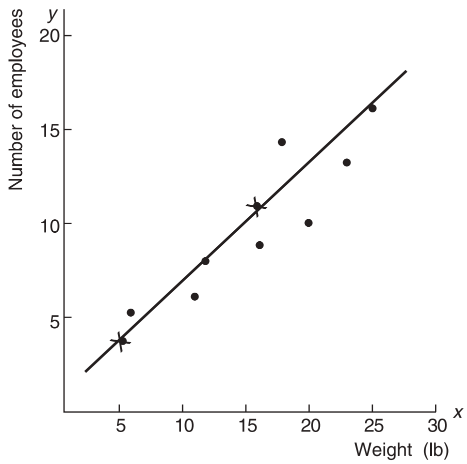
(b) $\bar{x} = 3$; $\bar{y} \approx 17.38$; $b \approx 1.27$; $\hat{y} \approx 13.57 + 1.27x$.
(c) $r \approx 0.685$; $r^2 \approx 0.469$. (d) $\alpha = 0.01$; H_0: $\rho = 0$; H_1: $\rho > 0$; $d.f. = 3$; $t = 1.627$; $0.100 < P$-value < 0.125; do not reject H_0. There does not seem to be a positive correlation between age and mortality rate of bighorn sheep. From TI-84, P-value ≈ 0.1011. (e) No. Based on these limited data, predictions from the least-squares line model might be misleading. There appear to be other lurking variables that affect the mortality rate of sheep in different age groups.

7. (a) Weight of One-Year-Old versus Weight of Adult



(b) $\bar{x} \approx 21.43$; $\bar{y} \approx 126.79$; $b \approx 1.285$; $\hat{y} \approx 99.25 + 1.285x$. (c) $r \approx 0.468$; $r^2 \approx 0.219$; 21.9% explained. (d) $\alpha = 0.01$; H_0: $\rho = 0$; H_1: $\rho > 0$; $d.f. = 12$; $t = 1.835$; $0.025 < P$-value < 0.050; do not reject H_0. At the 1% level of significance, there does not seem to be a positive correlation between weight of baby and weight of adult. From TI-84, P-value ≈ 0.0457. (e) 124.95 pounds. However, since r is not significant, this prediction may not be useful. Other lurking variables seem to have an effect on adult weight. (f) Use a calculator. (g) 105.91 to 143.99 pounds. (h) $\alpha = 0.01$; H_0: $\beta = 0$; H_1: $\beta > 0$; $d.f. = 12$; $t = 1.835$; $0.025 < P$-value < 0.050; do not reject H_0. At the 1% level of significance, there does not seem to be a positive slope between weight of baby x and weight of adult y. From TI-84, P-value ≈ 0.0457. (i) 0.347 to 2.223. At the 80% confidence level, we can say that for each additional pound a female infant weighs at 1 year, the female's adult weight changes by 0.35 to 2.22 pounds.

9. (a) Weight of Mail versus Number of Employees Required

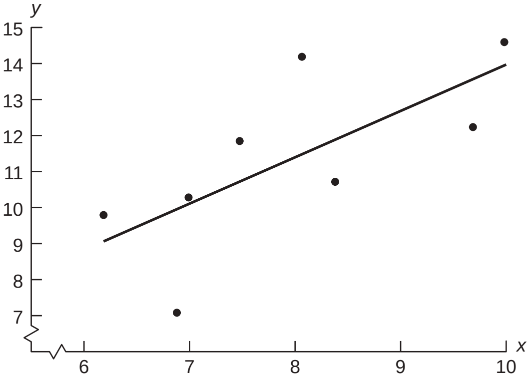


(b) $\bar{x} \approx 16.38$; $\bar{y} \approx 10.13$; $b \approx 0.554$; $\hat{y} \approx 1.051 + 0.554x$.
(c) $r \approx 0.913$; $r^2 \approx 0.833$; 83.3% explained.
(d) $\alpha = 0.01$; H_0: $\rho = 0$; H_1: $\rho > 0$; $d.f. = 6$; $t = 5.467$; $0.0005 < P$-value < 0.005; reject H_0. At the 1% level of

significance, there is sufficient evidence to show a positive correlation between pounds of mail and number of employees required to process the mail. From TI-84, P-value ≈ 0.0008. (e) 9.36. (f) Use a calculator. (g) 4.86 to 13.86. (h) $\alpha = 0.01$; H_0: $\beta = 0$; H_1: $\beta > 0$; $d.f. = 6$; $t = 5.467$; $0.0005 < P\text{-value} < 0.005$; reject H_0. At the 1% level of significance, there is sufficient evidence to show a positive slope between pounds of mail x and number of employees required to process the mail y. From TI-84, P-value ≈ 0.0008. (i) 0.408 to 0.700. At the 80% confidence level, we can say that for each additional pound of mail, between 0.4 and 0.7 additional employees are needed.

CUMULATIVE REVIEW PROBLEMS

1. (a) i. $\alpha = 0.01$; H_0: $\mu = 2.0$ ug/l; H_1: $\mu > 2.0$ ug/l.
   ii. Standard normal; $z = 2.53$.
   iii. P-value ≈ 0.0057; on standard normal curve, shade area to the right of 2.53.
   iv. P-value of $0.0057 \le 0.01$ for α; reject H_0.
   v. At the 1% level of significance, the evidence is sufficient to say that the population mean discharge level of lead is higher.
   (b) 2.13 ug/l to 2.99 ug/l. (c) $n = 48$.
2. (a) Use rounded results to compute t in part (b).
   (b) i. $\alpha = 0.05$; H_0: $\mu = 10\%$; H_1: $\mu > 10\%$.
   ii. Student's t, $d.f. = 11$; $t \approx 1.248$.
   iii. $0.100 < P\text{-value} < 0.125$; on t graph, shade area to the right of 1.248. From TI-84, P-value $\approx$ 0.1190.
   iv. P-value interval > 0.05 for α; fail to reject H_0.
   v. At the 5% level of significance, the evidence does not indicate that the patient is asymptomatic.
   (c) 9.27% to 11.71%.
3. (a) i. $\alpha = 0.05$; H_0: $p = 0.10$; H_1: $p \ne 0.10$; yes, $np > 5$ and $nq > 5$; necessary to use normal approximation to the binominal.
   ii. Standard normal; $\hat{p} \approx 0.147$; $z = 1.29$.
   iii. P-value $= 2P(z > 1.29) \approx 0.1970$; on standard normal curve, shade area to the right of 1.29 and to the left of -1.29.
   iv. P-value of $0.1970 > 0.05$ for α; fail to reject H_0.
   v. At the 5% level of significance, the data do not indicate any difference from the national average for the population proportion of crime victims.
   (b) 0.063 to 0.231. (c) From sample, $p \approx \hat{p} \approx 0.147$; $n = 193$.
4. (a) i. $\alpha = 0.05$; H_0: $\mu_d = 0$; H_1: $\mu_d \ne 0$.
   ii. Student's t, $d.f. = 6$; $\bar{d} \approx -0.0039$, $t \approx -0.771$.
   iii. $0.250 < P\text{-value} < 0.500$; on t graph, shade area to the right of 0.771 and to the left of -0.771. From TI-84, P-value ≈ 0.4699.
   iv. P-value interval > 0.05 for α; fail to reject H_0.
   v. At the 5% level of significance, the evidence does not show a population mean difference in phosphorous reduction between the two methods.
5. (a) i. $\alpha = 0.05$; H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 \ne \mu_2$.
   ii. Student's t, $d.f. = 15$; $t \approx 1.952$.
   iii. $0.050 < P\text{-value} < 0.100$; on t graph, shade area to the right of 1.952 and to the left of -0.1952. From TI-84, P-value ≈ 0.0609.
   iv. P-value interval > 0.05 for α; fail to reject H_0.
   v. At the 5% level of significance, the evidence does not show any difference in the population mean proportion of on-time arrivals in summer versus winter.
   (b) -0.43% to 9.835%. (c) x_1 and x_2 distributions are approximately normal (mound-shaped and symmetric).
6. (a) i. $\alpha = 0.05$; H_0: $p_1 = p_2$; H_1: $p_1 > p_2$.
   ii. Standard normal; $\hat{p}_1 \approx 0.242$; $\hat{p}_2 \approx 0.207$; $\bar{p} \approx 0.2246$; $z \approx 0.58$.
   iii. P-value ≈ 0.2810; on standard normal curve, shade area to the right of 0.58.
   iv. P-value interval > 0.05 for α; fail to reject H_0.
   v. At the 5% level of significance, the evidence does not indicate that the population proportion of single men who go out dancing occasionally differs from the proportion of single women who do so.
   Since $n_1\bar{p}$, $n_1\bar{q}$, $n_2\bar{p}$, and $n_2\bar{q}$ are all greater than 5, the normal approximation to the binomial is justified. (b) -0.065 to 0.139.
7. (a) Essay. (b) Outline of study.
8. Answers vary.
9. (a) Blood Glucose Level



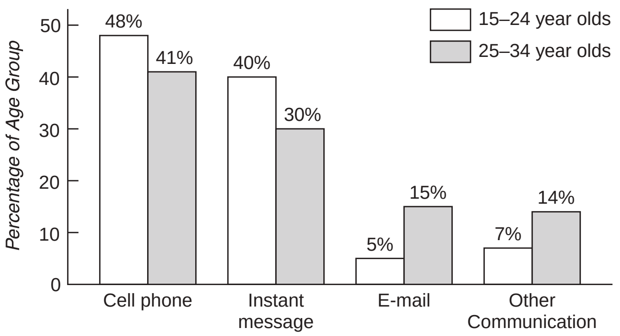
(b) $\hat{y} \approx 1.135 + 1.279x$. (c) $r \approx 0.700$; $r^2 \approx 0.490$; 49% of the variance in y is explained by the model and the variance in x. (d) 12.65; 9.64 to 15.66. (e) $\alpha = 0.01$; H_0: $\rho = 0$; H_1: $\rho \ne 0$; $r \approx 0.700$ with $t \approx 2.40$; $d.f. = 6$; $0.05 < P\text{-value} < 0.10$; do not reject H_0. At the 1% level of significance, the evidence is insufficient to conclude that there is a linear correlation. (f) $S_e \approx 1.901$; $t_c = 1.645$; 0.40 to 2.16.

CHAPTER 10

Section 10.1

1. Skewed right.
3. Right-tailed test.
5. Take random samples from each of the 4 age groups and record the number of people in each age group who recycle each of the 3 product types. Make a contingency table with age groups as labels for rows (or columns) and products as labels for columns (or rows).

7. (a) $d.f. = 6$; $0.005 < P\text{-value} < 0.01$. At the 1% level of significance, we reject H_0 since the P-value is less than 0.01. At the 1% level of significance, we conclude that the age groups differ in the proportions of who recycles each of the specified products.
(b) No. All he can say is that the 4 age groups differ in the proportions of those recycling each specified product. For this study, he cannot determine how the age groups differ regarding the proportions of those recycling the listed products.
9. (a) $\alpha = 0.05$; H_0: Myers–Briggs preference and profession are independent; H_1: Myers–Briggs preference and profession are not independent. (b) $\chi^2 = 8.649$; $d.f. = 2$. (c) $0.010 < P\text{-value} < 0.025$. From TI-84, $P\text{-value} \approx 0.0132$. (d) Reject H_0. (e) At the 5% level of significance, there is sufficient evidence to conclude that Myers–Briggs preference and profession are not independent.
11. (a) $\alpha = 0.01$; H_0: Site type and pottery type are independent; H_1: Site type and pottery type are not independent. (b) $\chi^2 = 0.5552$; $d.f. = 4$. (c) $0.950 < P\text{-value} < 0.975$. From TI-84, $P\text{-value} \approx 0.9679$.
(d) Do not reject H_0. (e) At the 1% level of significance, there is insufficient evidence to conclude that site type and pottery type are not independent.
13. (a) $\alpha = 0.05$; H_0: Age distribution and location are independent; H_1: Age distribution and location are not independent. (b) $\chi^2 = 0.6704$; $d.f. = 4$. (c) $0.950 < P\text{-value} < 0.975$. From TI-84, $P\text{-value} \approx 0.9549$.
(d) Do not reject H_0. (e) At the 5% level of significance, there is insufficient evidence to conclude that age distribution and location are not independent.
15. (a) $\alpha = 0.05$; H_0: Age of young adult and movie preference are independent; H_1: Age of young adult and movie preference are not independent. (b) $\chi^2 = 3.6230$; $d.f. = 4$. (c) $0.100 < P\text{-value} < 0.900$. From TI-84, $P\text{-value} \approx 0.4594$. (d) Do not reject H_0. (e) At the 5% level of significance, there is insufficient evidence to conclude that age of young adult and movie preference are not independent.
17. (a) $\alpha = 0.05$; H_0: Stone tool construction material and site are independent; H_1: Stone tool construction material and site are not independent. (b) $\chi^2 = 11.15$; $d.f. = 3$. (c) $0.010 < P\text{-value} < 0.025$. From TI-84, $P\text{-value} \approx 0.0110$. (d) Reject H_0. (e) At the 5% level of significance, there is sufficient evidence to conclude that stone tool construction material and site are not independent.
19. (i) Communication Preference by Percentage of Age Group



(ii) (a) H_0: The proportions of the different age groups having each communication preference are the same. H_1: The proportions of the different age groups having each communication preference are not the same. (b) $\chi^2 = 9.312$; $d.f. = 3$. (c) $0.025 < P\text{-value} < 0.050$. From TI-84, $P\text{-value} \approx 0.0254$. (d) Reject H_0. (e) At the 5% level of significance, there is sufficient evidence to conclude that the two age groups do not have the same proportions of communications preferences.

Section 10.2

1. $d.f.$ = number of categories − 1.
3. The greater the differences between the observed frequencies and the expected frequencies, the higher the sample χ^2 value. Greater χ^2 values lead to the conclusion that the differences between expected and observed frequencies are too large to be explained by chance alone.
5. (a) $\alpha = 0.05$; H_0: The distributions are the same; H_1: The distributions are different. (b) Sample $\chi^2 = 11.788$; $d.f. = 3$. (c) $0.005 < P\text{-value} < 0.010$.
(d) Reject H_0. (e) At the 5% level of significance, the evidence is sufficient to conclude that the age distribution of the Red Lake Village population does not fit the age distribution of the general Canadian population.
7. (a) $\alpha = 0.01$; H_0: The distributions are the same; H_1: The distributions are different. (b) Sample $\chi^2 = 0.1984$; $d.f. = 4$. (c) $P\text{-value} > 0.995$. (Note that as the χ^2 values decrease, the area in the right tail increases, so $\chi^2 < 0.207$ means that the corresponding $P\text{-value} > 0.995$.) (d) Do not reject H_0. (e) At the 1% level of significance, the evidence is insufficient to conclude that the regional distribution of raw materials does not fit the distribution at the current excavation site.
9. (i) Answers vary. (ii) (a) $\alpha = 0.01$; H_0: The distributions are the same; H_1: The distributions are different. (b) Sample $\chi^2 = 1.5693$; $d.f. = 5$.
(c) $0.900 < P\text{-value} < 0.950$. (d) Do not reject H_0.
(e) At the 1% level of significance, the evidence is insufficient to conclude that the average daily July temperature does not follow a normal distribution.
11. (a) $\alpha = 0.05$; H_0: The distributions are the same; H_1: The distributions are different. (b) Sample $\chi^2 = 9.333$; $d.f. = 3$. (c) $0.025 < P\text{-value} < 0.050$. (d) Reject H_0.
(e) At the 5% level of significance, the evidence is sufficient to conclude that the current fish distribution is different than it was 5 years ago.
13. (a) $\alpha = 0.01$; H_0: The distributions are the same; H_1: The distributions are different. (b) Sample $\chi^2 = 13.70$; $d.f. = 5$. (c) $0.010 < P\text{-value} < 0.025$. (d) Do not reject H_0. (e) At the 1% level of significance, the evidence is insufficient to conclude that the census ethnic origin distribution and the ethnic origin distribution of city residents are different.
15. (a) $\alpha = 0.01$; H_0: The distributions are the same; H_1: The distributions are different. (b) Sample $\chi^2 = 3.559$; $d.f. = 8$. (c) $0.100 < P\text{-value} < 0.900$. (d) Do not reject H_0. (e) At the 1% level of significance, the evidence is insufficient to conclude that the distribution of first nonzero digits in the accounting file does not follow Benford's Law.

17. (a) $P(0) \approx 0.179$; $P(1) \approx 0.308$; $P(2) \approx 0.265$; $P(3) \approx 0.152$; $P(r \geq 4) \approx 0.096$. (b) For $r = 0$, $E \approx 16.11$; for $r = 1$, $E \approx 27.72$; for $r = 2$, $E \approx 23.85$; for $r = 3$, $E \approx 13.68$; for $r \geq 4$, $E \approx 8.64$. (c) $\chi^2 \approx 12.55$ with $d.f. = 4$. (d) $\alpha = 0.01$; H_0: The Poisson distribution fits; H_1: The Poisson distribution does not fit; $0.01 < P$-value < 0.025; do not reject H_0. At the 1% level of significance, we cannot say that the Poisson distribution does not fit the sample data.

Section 10.3

1. Yes. No, the chi-square test of variance requires that the x distribution be a normal distribution.
3. (a) $\alpha = 0.05$; H_0: $\sigma^2 = 42.3$; H_1: $\sigma^2 > 42.3$. (b) $\chi^2 \approx 23.98$; $d.f. = 22$. (c) $0.100 < P$-value < 0.900. (d) Do not reject H_0. (e) At the 5% level of significance, there is insufficient evidence to conclude that the variance is greater in the new section. (f) $\chi^2_U = 36.78$; $\chi^2_L = 10.98$. Interval for σ^2 is from 27.57 to 92.37.
5. (a) $\alpha = 0.01$; H_0: $\sigma^2 = 136.2$; H_1: $\sigma^2 < 136.2$. (b) $\chi^2 \approx 5.92$; $d.f. = 7$. (c) Right-tailed area between 0.900 and 0.100; $0.100 < P$-value < 0.900. (d) Do not reject H_0. (e) At the 1% level of significance, there is insufficient evidence to conclude that the variance for number of mountain climber deaths is less than 136.2. (f) $\chi^2_U = 14.07$; $\chi^2_L = 2.17$. Interval for σ^2 is from 57.26 to 371.29.
7. (a) $\alpha = 0.05$; H_0: $\sigma^2 = 9$; H_1: $\sigma^2 < 9$. (b) $\chi^2 \approx 8.82$; $d.f. = 22$. (c) Right-tail area is between 0.995 and 0.990; $0.005 < P$-value < 0.010. (d) Reject H_0. (e) At the 5% level of significance, there is sufficient evidence to conclude that the variance of protection times for the new typhoid shot is less than 9. (f) $\chi^2_U = 33.92$; $\chi^2_L = 12.34$. Interval for σ is from 1.53 to 2.54.
9. (a) $\alpha = 0.01$; H_0: $\sigma^2 = 0.18$; H_1: $\sigma^2 > 0.18$. (b) $\chi^2 = 90$; $d.f. = 60$. (c) $0.005 < P$-value < 0.010. (d) Reject H_0. (e) At the 1% level of significance, there is sufficient evidence to conclude that the variance of measurements for the fan blades is higher than the specified amount. The inspector is justified in claiming that the blades must be replaced. (f) $\chi^2_U = 79.08$; $\chi^2_L = 43.19$. Interval for σ is from 0.45 mm to 0.61 mm.
11. (i) (a) $\alpha = 0.05$; H_0: $\sigma^2 = 23$; H_1: $\sigma^2 \neq 23$. (b) $\chi^2 \approx 13.06$; $d.f. = 21$. (c) The area to the left of $\chi^2 = 13.06$ is less than 50%, so we double the left-tail area to find the P-value for the two-tailed test. Right-tail area is between 0.950 and 0.900. Subtracting each value from 1, we find that the left-tail area is between 0.050 and 0.100. Doubling the left-tail area for a two-tailed test gives $0.100 < P$-value < 0.200. (d) Do not reject H_0. (e) At the 5% level of significance, there is insufficient evidence to conclude that the variance of battery lifetimes is different from 23. (ii) $\chi^2_U = 32.67$; $\chi^2_L = 11.59$. Interval for σ^2 is from 9.19 to 25.91. (iii) Interval for σ is from 3.03 to 5.09.

Section 10.4

1. Independent.
3. F distributions are not symmetrical. Values of the F distribution are all nonnegative.
5. (a) $\alpha = 0.01$; population 1 is annual production from the first plot; H_0: $\sigma_1^2 = \sigma_2^2$; H_1: $\sigma_1^2 > \sigma_2^2$;. (b) $F \approx 3.73$; $d.f._N = 15$; $d.f._D = 15$. (c) $0.001 < P$-value < 0.010. From TI-84, P-value ≈ 0.0075. (d) Reject H_0. (e) At the 1% level of significance, there is sufficient evidence to show that the variance in annual wheat production of the first plot is greater than that of the second plot.
7. (a) $\alpha = 0.05$; population 1 has data from France; H_0: $\sigma_1^2 = \sigma_2^2$; H_1: $\sigma_1^2 \neq \sigma_2^2$. (b) $F \approx 1.97$; $d.f._N = 20$; $d.f._D = 17$. (c) $0.050 <$ right-tail area < 0.100; $0.100 < P$-value < 0.200. From TI-84, P-value ≈ 0.1631. (d) Do not reject H_0. (e) At the 5% level of significance, there is insufficient evidence to show that the variance in corporate productivity of large companies in France and of those in Germany differ. Volatility of corporate productivity does not appear to differ.
9. (a) $\alpha = 0.05$; population 1 has data from aggressive-growth companies; H_0: $\sigma_1^2 = \sigma_2^2$; H_1: $\sigma_1^2 > \sigma_2^2$. (b) $F \approx 2.54$; $d.f._N = 20$; $d.f._D = 20$. (c) $0.010 < P$-value < 0.025. From TI-84, P-value ≈ 0.0216. (d) Reject H_0. (e) At the 5% level of significance, there is sufficient evidence to show that the variance in percentage annual returns for funds holding aggressive-growth small stocks is larger than that for funds holding value stocks.
11. (a) $\alpha = 0.05$; population 1 has data from the new system; H_0: $\sigma_1^2 = \sigma_2^2$; H_1: $\sigma_1^2 \neq \sigma_2^2$. (b) $F \approx 1.85$; $d.f._N = 30$; $d.f._D = 24$. (c) $0.050 <$ right-tail area < 0.100; $0.100 < P$-value < 0.200. From TI-84, P-value ≈ 0.1266. (d) Do not reject H_0. (e) At the 5% level of significance, there is insufficient evidence to show that the variance in gasoline consumption for the two injection systems is different.

Section 10.5

1. (a) $\alpha = 0.01$; H_0: $\mu_1 = \mu_2 = \mu_3$; H_1: Not all the means are equal. (b–f)

Source of Variation	Sum of Squares	Degrees of Freedom	*MS*	*F* Ratio	*P*-value	Test Decision
Between groups	520.280	2	260.14	0.48	> 0.100	Do not reject H_0
Within groups	7544.190	14	538.87			
Total	8064.470	16				

From TI-84, P-value ≈ 0.6270.

3. (a) $\alpha = 0.05$; H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$; H_1: Not all the means are equal.
(b–f)

Source of Variation	Sum of Squares	Degrees of Freedom	*MS*	*F* Ratio	*P*-value	Test Decision
Between groups	89.637	3	29.879	0.846	> 0.100	Do not reject H_0
Within groups	635.827	18	35.324			
Total	725.464	21				

From TI-84, P-value ≈ 0.4867.

5. (a) $\alpha = 0.05$; H_0: $\mu_1 = \mu_2 = \mu_3$; H_1: Not all the means are equal.
(b–f)

Source of Variation	Sum of Squares	Degrees of Freedom	*MS*	*F* Ratio	*P*-value	Test Decision
Between groups	1303.167	2	651.58	5.005	between 0.025 and 0.050	Reject H_0
Within groups	1171.750	9	130.19			
Total	2474.917	11				

From TI-84, P-value ≈ 0.0346.

7. (a) $\alpha = 0.01$; H_0: $\mu_1 = \mu_2 = \mu_3$; H_1: Not all the means are equal.
(b–f)

Source of Variation	Sum of Squares	Degrees of Freedom	*MS*	*F* Ratio	*P*-value	Test Decision
Between groups	2.042	2	1.021	0.336	> 0.100	Do not reject H_0
Within groups	33.428	11	3.039			
Total	35.470	13				

From TI-84, P-value ≈ 0.7217.

9. (a) $\alpha = 0.05$; H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$; H_1: Not all the means are equal.
(b–f)

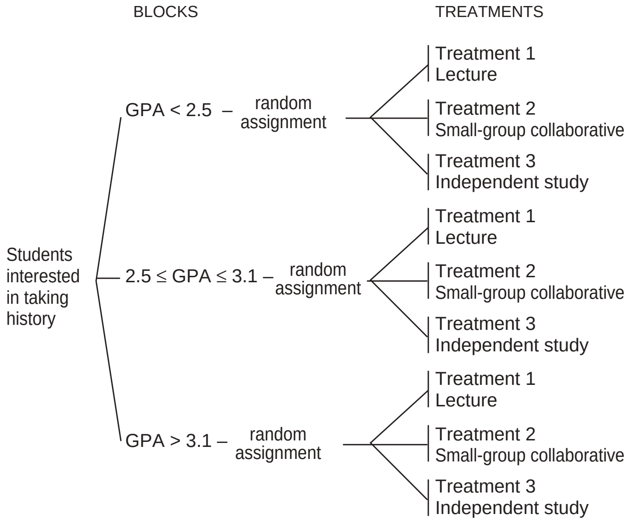
Source of Variation	Sum of Squares	Degrees of Freedom	*MS*	*F* Ratio	*P*-value	Test Decision
Between groups	238.225	3	79.408	4.611	between 0.010 and 0.025	Reject H_0
Within groups	258.340	15	17.223			
Total	496.565	18				

From TI-84, P-value ≈ 0.0177.

Section 10.6

1. Two factors; walking device with 3 levels and task with 2 levels; data table has 6 cells.
3. Since the P-value is less than 0.01, there is a significant difference in mean cadence according to the factor "walking device used."
5. (a) Two factors: income with 4 levels and media type with 5 levels. (b) $\alpha = 0.05$; For income level, H_0: There is no difference in population mean index based on income level; H_1: At least two income levels have different population mean indices; $F_{income} \approx 2.77$ with P-value ≈ 0.088. At the 5% level of significance, do not reject H_0. The data do not indicate any differences in population mean index according to income level. (c) $\alpha = 0.05$; For media, H_0: There is no difference in population mean index according to media type; H_1: At least two media types have different population mean indices; $F_{media} \approx 0.03$ with P-value ≈ 0.998. At the 5% level of significance, do not reject H_0. The data do not indicate any differences in population mean index according to media type.

7. Randomized Block Design



Yes, the design fits the model for randomized block design.

Chapter 10 Review

1. Chi-square, F.
3. Test of homogeneity.
5. One-way ANOVA. $\alpha = 0.05$; H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$; H_1: Not all the means are equal.

Source of Variation	Sum of Squares	Degrees of Freedom	*MS*
Between groups	6149.75	3	2049.917
Within groups	12,454.80	16	778.425
Total	18,604.55	19	

F Ratio	*P*-value	Test Decision
2.633	between 0.050 and 0.100	Do not reject H_0

From TI-84, P-value ≈ 0.0854.

7. (a) Chi-square test of σ^2. (i) $\alpha = 0.01$; H_0: $\sigma^2 = 1{,}040{,}400$; H_1: $\sigma^2 > 1{,}040{,}400$. (ii) $\chi^2 \approx 51.03$; $d.f. = 29$. (iii) $0.005 < P\text{-value} < 0.010$. (iv) Reject H_0. (v) At the 1% level of significance, there is sufficient evidence to conclude that the variance is greater than claimed. (b) $\chi^2_U = 45.72$; $\chi^2_L = 16.05$; $1{,}161{,}147.4 < \sigma^2 < 3{,}307{,}642.4$.
9. Chi-square test of independence. (i) $\alpha = 0.01$; H_0: Student grade and teacher rating are independent; H_1: Student grade and teacher rating are not independent. (ii) $\chi^2 \approx 9.80$; $d.f. = 6$. (iii) $0.100 < P\text{-value} < 0.900$. From TI-84, P-value ≈ 0.1337. (iv) Do not reject H_0. (v) At the 1% level of significance, there is insufficient evidence to claim that student grade and teacher rating are not independent.
11. Chi-square test of goodness of fit. (i) $\alpha = 0.01$; H_0: The distributions are the same; H_1: The distributions are different. (ii) $\chi^2 \approx 11.93$; $d.f. = 4$. (iii) $0.010 < P\text{-value} < 0.025$. (iv) Do not reject H_0. (v) At the 1% level of significance, there is insufficient evidence to claim that the age distribution of the population of Blue Valley has changed.
13. F test for two variances. (i) $\alpha = 0.05$; H_0; $\sigma_1^2 = \sigma_2^2$; H_1: $\sigma_1^2 > \sigma_2^2$. (ii) $F \approx 2.61$; $d.f._N = 15$; $d.f._D = 17$. (iii) $0.025 < P\text{-value} < 0.050$. From TI-84, P-value ≈ 0.0302. (iv) Reject H_0. (v) At the 5% level of significance, there is sufficient evidence to show that the variance for the lifetimes of bulbs manufactured using the new process is larger than that for bulbs made by the old process.

CHAPTER 11

Section 11.1

1. Dependent (matched pairs).
3. (a) $\alpha = 0.05$; H_0: Distributions are the same; H_1: Distributions are different. (b) $x = 7/15 \approx 0.4667$; $z \approx -0.26$. (c) P-value $= 2(0.3974) = 0.7948$. (d) Do not reject H_0. (e) At the 5% level of significance, the data are not significant. The evidence is insufficient to conclude that the economic growth rates are different.
5. (a) $\alpha = 0.05$; H_0: Distributions are the same; H_1: Distributions are different. (b) $x = 10/16 = 0.625$; $z \approx 1.00$. (c) P-value $= 2(0.1587) = 0.3174$. (d) Do not reject H_0. (e) At the 5% level of significance, the data are not significant. The evidence is insufficient to conclude that the lectures had any effect on student awareness of current events.
7. (a) $\alpha = 0.05$; H_0: Distributions are the same; H_1: Distributions are different. (b) $x = 7/12 \approx 0.5833$; $z \approx 0.58$. (c) P-value $= 2(0.2810) = 0.5620$. (d) Do not reject H_0. (e) At the 5% level of significance, the data are not significant. The evidence is insufficient to conclude that the schools are not equally effective.
9. (a) $\alpha = 0.01$; H_0: Distributions are the same; H_1: Distribution after hypnosis is lower. (b) $x = 3/16 = 0.1875$; $z \approx -2.50$. (c) P-value $= 0.0062$. (d) Reject H_0. (e) At the 1% level of significance, the data are significant. The evidence is sufficient to conclude that the number of cigarettes smoked per day was less after hypnosis.
11. (a) $\alpha = 0.01$; H_0: Distributions are the same; H_1: Distributions are different. (b) $x = 10/20 = 0.5000$; $z = 0$. (c) P-value $= 2(0.5000) = 1$. (d) Do not reject H_0. (e) At the 1% level of significance, the data are not significant. The evidence is insufficient to conclude that the distribution of dropout rates is different for males and females.

Section 11.2

1. Independent.
3. (a) $\alpha = 0.05$; H_0: Distributions are the same; H_1: Distributions are different. (b) $R_A = 126$; $\mu_R = 132$;

$\sigma_R \approx 16.25$; $z \approx -0.37$. (c) P-value $\approx 2(0.3557) = 0.7114$. (d) Do not reject H_0. (e) At the 5% level of significance, the evidence is insufficient to conclude that the yield distributions for organic and conventional farming methods are different.

5. (a) $\alpha = 0.05$; H_0: Distributions are the same; H_1: Distributions are different. (b) $R_B = 148$; $\mu_R = 132$; $\sigma_R \approx 16.25$; $z \approx 0.98$. (c) P-value $\approx 2(0.1635) = 0.3270$. (d) Do not reject H_0. (e) At the 5% level of significance, the evidence is insufficient to conclude that the distributions of the training sessions are different.
7. (a) $\alpha = 0.05$; H_0: Distributions are the same; H_1: Distributions are different. (b) $R_A = 92$; $\mu_R = 132$; $\sigma_R \approx 16.25$; $z \approx -2.46$. (c) P-value $\approx 2(0.0069) = 0.0138$. (d) Reject H_0. (e) At the 5% level of significance, the evidence is sufficient to conclude that the completion time distributions for the two settings are different.
9. (a) $\alpha = 0.01$; H_0: Distributions are the same; H_1: Distributions are different. (b) $R_A = 176$; $\mu_R = 132$; $\sigma_R \approx 16.25$; $z \approx 2.71$. (c) P-value $\approx 2(0.0034) = 0.0068$. (d) Reject H_0. (e) At the 1% level of significance, the evidence is sufficient to conclude that the distributions showing percentage of exercisers differ by education level.
11. (a) $\alpha = 0.01$; H_0: Distributions are the same; H_1: Distributions are different. (b) $R_A = 166$; $\mu_R = 150$; $\sigma_R \approx 17.32$; $z \approx 0.92$. (c) P-value $\approx 2(0.1788) = 0.3576$. (d) Do not reject H_0. (e) At the 1% level of significance, the evidence is insufficient to conclude that the distributions of test scores differ according to instruction method.

Section 11.3

1. Monotone increasing.
3. (a) $\alpha = 0.05$; H_0: $\rho_s = 0$; H_1: $\rho_s \neq 0$. (b) $r_s \approx 0.682$. (c) $n = 11$; $0.01 < P\text{-value} < 0.05$. (d) Reject H_0. (e) At the 5% level of significance, we conclude that there is a monotone relationship (either increasing or decreasing) between rank in training class and rank in sales.
5. (a) $\alpha = 0.05$; H_0: $\rho_s = 0$; H_1: $\rho_s > 0$. (b) $r_s \approx 0.571$. (c) $n = 8$; P-value > 0.05. (d) Do not reject H_0. (e) At the 5% level of significance, there is insufficient evidence to indicate a monotone-increasing relationship between crowding and violence.
7. (ii) (a) $\alpha = 0.05$; H_0: $\rho_s = 0$; H_1: $\rho_s < 0$. (b) $r_s \approx -0.214$. (c) $n = 7$; P-value > 0.05. (d) Do not reject H_0. (e) At the 5% level of significance, the evidence is insufficient to conclude that there is a monotone-decreasing relationship between the ranks of humor and aggressiveness.
9. (ii) (a) $\alpha = 0.05$; H_0: $\rho_s = 0$; H_1: $\rho_s \neq 0$. (b) $r_s \approx 0.930$. (c) $n = 13$; P-value < 0.002. (d) Reject H_0. (e) At the 5% level of significance, we conclude that there is a monotone relationship between number of firefighters and number of police.
11. (ii) (a) $\alpha = 0.01$; H_0: $\rho_s = 0$; H_1: $\rho_s \neq 0$. (b) $r_s \approx 0.661$. (c) $n = 8$; $0.05 < P\text{-value} < 0.10$. (d) Do not reject H_0. (e) At the 1% level of significance, we conclude that there is insufficient evidence to reject the null hypothesis of no monotone relationship between rank of insurance sales and rank of per capita income.

Section 11.4

1. Exactly two.
3. (a) $\alpha = 0.05$; H_0: The symbols are randomly mixed in the sequence; H_1: The symbols are not randomly mixed in the sequence. (b) $R = 11$. (c) $n_1 = 12$; $n_2 = 11$; $c_1 = 7$; $c_2 = 18$. (d) Do not reject H_0. (e) At the 5% level of significance, the evidence is insufficient to conclude that the sequence of presidential party affiliations is not random.
5. (a) $\alpha = 0.05$; H_0: The symbols are randomly mixed in the sequence; H_1: The symbols are not randomly mixed in the sequence. (b) $R = 11$. (c) $n_1 = 16$; $n_2 = 7$; $c_1 = 6$; $c_2 = 16$. (d) Do not reject H_0. (e) At the 5% level of significance, the evidence is insufficient to conclude that the sequence of days for seeding and not seeding is not random.
7. (i) Median = 11.7; BBBAAAAABBBA. (ii) (a) $\alpha = 0.05$; H_0: The numbers are randomly mixed about the median; H_1: The numbers are not randomly mixed about the median. (b) $R = 4$. (c) $n_1 = 6$; $n_2 = 6$; $c_1 = 3$; $c_2 = 11$. (d) Do not reject H_0. (e) At the 5% level of significance, the evidence is insufficient to conclude that the sequence of returns is not random about the median.
9. (i) Median = 21.6; BAAAAAABBBBB. (ii) (a) $\alpha = 0.05$; H_0: The numbers are randomly mixed about the median; H_1: The numbers are not randomly mixed about the median. (b) $R = 3$. (c) $n_1 = 6$; $n_2 = 6$; $c_1 = 3$; $c_2 = 11$. (d) Reject H_0. (e) At the 5% level of significance, we can conclude that the sequence of percentages of sand in the soil at successive depths is not random about the median.
11. (a) H_0: The symbols are randomly mixed in the sequence. H_1: The symbols are not randomly mixed in the sequence. (b) $n_1 = 21$; $n_2 = 17$; $R = 18$. (c) $\mu_R \approx 19.80$; $\sigma_R \approx 3.01$; $z \approx -0.60$. (d) Since $-1.96 < z < 1.96$, do not reject H_0; P-value $\approx 2(0.2743) = 0.5486$; at the 5% level of significance, the P-value also tells us not to reject H_0. (e) At the 5% level of significance, the evidence is insufficient to reject the null hypothesis of a random sequence of Democratic and Republican presidential terms.

Chapter 11 Review

1. No assumptions about population distributions are required.
3. (a) Rank-sum test. (b) $\alpha = 0.05$; H_0: Distributions are the same; H_1: Distributions are different. (c) $R_A = 134$; $\mu_R = 132$; $\sigma_R \approx 16.25$; $z \approx 0.12$. (d) P-value $= 2(0.4522) = 0.9044$. (e) Do not reject H_0. At the 5% level of significance, there is insufficient evidence to conclude that the viscosity index distribution has changed with use of the catalyst.
5. (a) Sign test. (b) $\alpha = 0.01$; H_0: Distributions are the same; H_1: Distribution after ads is higher. (c) $x = 0.77$; $z = 1.95$. (d) P-value $= 0.0256$. (e) Do not reject H_0.

At the 1% level of significance, the evidence is insufficient to claim that the distribution is higher after the ads.

7. (a) Spearman rank correlation coefficient test. (b) $\alpha = 0.05$; H_0: $\rho = 0$; H_1: $\rho > 0$. (c) $r_s \approx 0.617$. (d) $n = 9$; $0.025 < P\text{-value} < 0.05$. (e) Reject H_0. At the 5% level of significance, we conclude that there is a monotone-increasing relation between the ranks for the training program and the ranks on the job.
9. (a) Runs test for randomness. (b) $\alpha = 0.05$; H_0: The symbols are randomly mixed in the sequence; H_1: The symbols are not randomly mixed in the sequence. (c) $R = 7$. (d) $n_1 = 16$; $n_2 = 9$; $c_1 = 7$; $c_2 = 18$. (e) Reject H_0. At the 5% level of significance, we can conclude that the sequence of answers is not random.

CUMULATIVE REVIEW PROBLEMS

1. (a) use a calculator. (b) $P(0) \approx 0.543$; $P(1) \approx 0.331$; $P(2) \approx 0.101$; $P(3) \approx 0.025$. (c) 0.3836; $d.f. = 3$. (d) $\alpha = 0.01$; H_0: The distributions are the same; H_1: The distributions are different; $\chi^2 \approx 0.3836$; $0.900 < P\text{-value} < 0.950$; do not reject H_0. At the 1% level of significance, the evidence is insufficient to claim that the distribution does not fit the Poisson distribution.
2. $\alpha = 0.05$; H_0: Yield and fertilizer type are independent; H_1: Yield and fertilizer type are not independent; $\chi^2 \approx 5.005$; $d.f. = 4$; $0.100 < P\text{-value} < 0.900$; do not reject H_0. At the 5% level of significance, the evidence is insufficient to conclude that fertilizer type and yield are not independent.
3. (a) $\alpha = 0.05$; H_0: $\sigma = 0.55$; H_1: $\sigma > 0.55$; $s \approx 0.602$; $d.f. = 9$; $\chi^2 \approx 10.78$; $0.100 < P\text{-value} < 0.900$; do not reject H_0. At the 5% level of significance, there is insufficient evidence to conclude that the standard deviation of petal lengths is greater than 0.55. (b) Interval from 0.44 to 0.99. (c) $\alpha = 0.01$; H_0: $\sigma_1^2 = \sigma_2^2$; H_1: $\sigma_1^2 > \sigma_2^2$; $F \approx 1.95$; $d.f._N = 9$, $d.f._D = 7$; $P\text{-value} > 0.100$; do not reject H_0. At the 1% level of significance, the evidence is insufficient to conclude that the variance of the petal lengths for *Iris virginica* is greater than that for *Iris versicolor*.
4. $\alpha = 0.05$; H_0: $p = 0.5$ (wind direction distributions are the same); H_1: $p \neq 0.5$ (wind direction distributions are different); $x = 11/18$; $z \approx 0.94$; $P\text{-value} = 2(0.1736) = 0.3472$; do not reject H_0. At the 5% level of significance, the evidence is insufficient to conclude that the wind direction distributions are different.
5. $\alpha = 0.01$; H_0: Growth distributions are the same; H_1: Growth distributions are different; $\mu_R = 126.5$; $\sigma_R \approx 15.23$; $R_A = 135$; $z \approx 0.56$; $P\text{-value} = 2(0.2877) = 0.5754$; do not reject H_0. At the 1% level of significance, the evidence is insufficient to conclude that the growth distributions are different for the two root stocks.
6. (b) $\alpha = 0.05$; H_0: $\rho_s = 0$; H_1: $\rho_s \neq 0$; $r_s = 1$; $P\text{-value} < 0.002$; reject H_0. At the 5% level of significance, we can say that there is a monotone relationship between the calcium contents as measured by the labs.
7. Median = 33.45; AABBBBAAAABAABBBBA; $\alpha = 0.05$; H_0: Numbers are random about the median; H_1: Numbers are not random about the median; $R = 7$; $n_1 = n_2 = 9$; $c_1 = 5$; $c_2 = 15$; do not reject H_0. At the 5% level of significance, there is insufficient evidence to conclude that the sunspot activity about the median is not random.

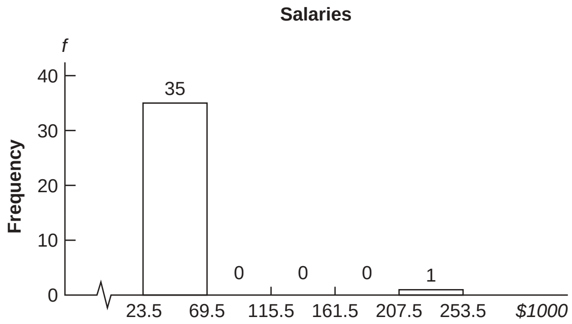
Answers to Selected Even-Numbered Problems

CHAPTER 2

Even-numbered answers not included here appear in the margins of the chapters, next to the problems.

Section 2.1

10. (a) Employee Salaries—Histogram

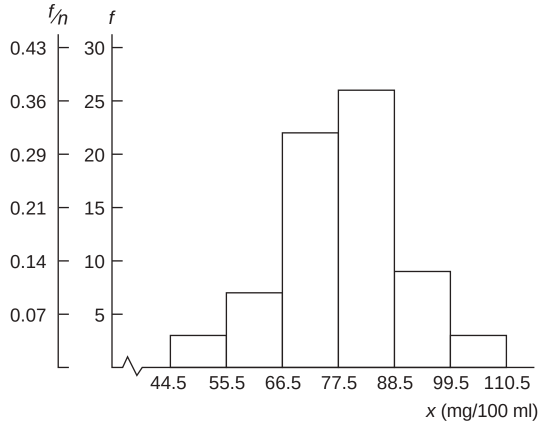


(c) Employee Salaries—Histogram

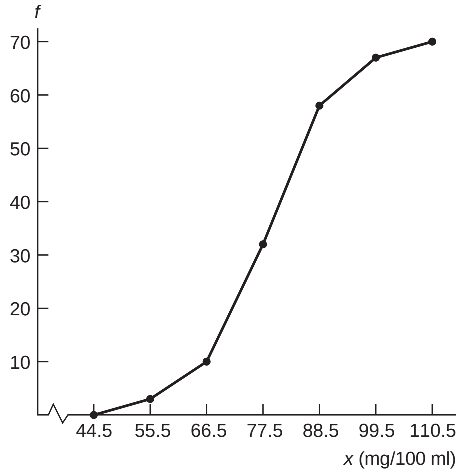
12. (a) Class width = 11.
(b)

Class Limits	Class Boundaries	Midpoint	Frequency	Relative Frequency	Cumulative Frequency
45–55	44.5–55.5	50	3	0.04	3
56–66	55.5–66.5	61	7	0.10	10
67–77	66.5–77.5	72	22	0.31	32
78–88	77.5–88.5	83	26	0.37	58
89–99	88.5–99.5	94	9	0.13	67
100–110	99.5–110.5	103	3	0.04	70

(c, d) Glucose Level (mg/100 ml)—Histogram, Relative-Frequency Histogram



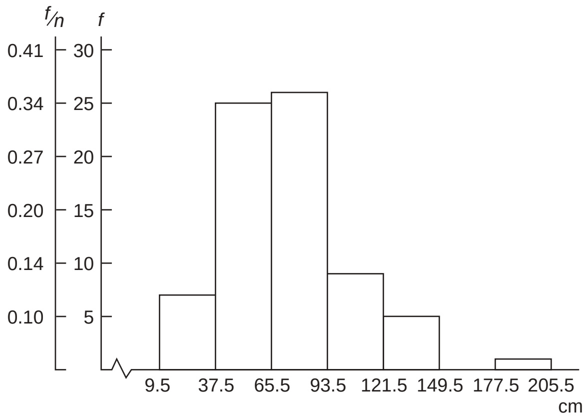
(f) Glucose Level (mg/100 ml)—Ogive



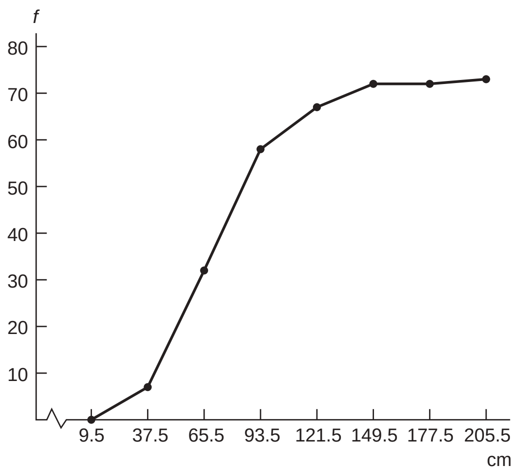
14. (a) Class width = 28.
(b)

Class Limits	Class Boundaries	Midpoint	Frequency	Relative Frequency	Cumulative Frequency
10–37	9.5–37.5	23.5	7	0.10	7
38–65	37.5–65.5	51.5	25	0.34	32
66–93	65.5–93.5	79.5	26	0.36	58
94–121	93.5–121.5	107.5	9	0.12	67
122–149	121.5–149.5	135.5	5	0.07	72
150–177	149.5–177.5	163.5	0	0.00	72
178–205	177.5–205.5	191.5	1	0.01	73

(c, d) Depth of Artifacts (cm)—Histogram, Relative-Frequency Histogram



(f) Depth of Artifacts (cm)—Ogive

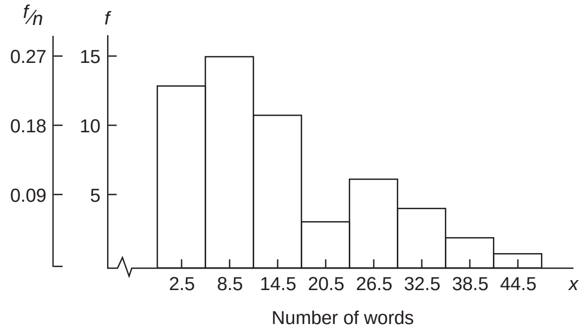


16. (a) Class width = 6.
(b)

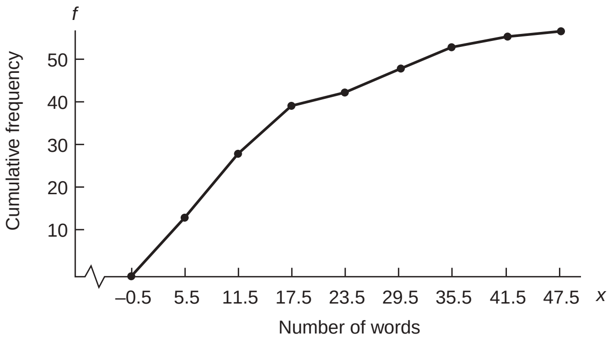
Words of Three Syllables or More

Class Limits	Class Boundaries	Midpoint	Frequency	Relative Frequency	Cumulative Frequency
0–5	−0.5–5.5	2.5	13	0.24	13
6–11	5.5–11.5	8.5	15	0.27	28
12–17	11.5–17.5	14.5	11	0.20	39
18–23	17.5–23.5	20.5	3	0.05	42
24–29	23.5–29.5	26.5	6	0.11	48
30–35	29.5–35.5	32.5	4	0.07	52
36–41	35.5–41.5	38.5	2	0.04	54
42–47	41.5–47.5	44.5	1	0.02	55

(c, d) Words of Three Syllables or More—Histogram, Relative-Frequency Histogram



(f) Ogive for Words of Three Syllables or More

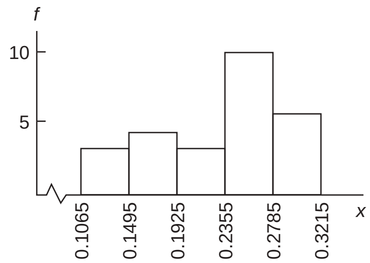


18. (b)

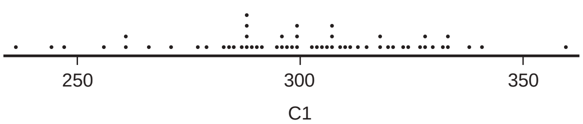
Baseball Batting Averages (class width = 0.043)

Class Limits	Class Boundaries	Midpoint	Frequency
0.107–0.149	0.1065–0.1495	0.128	3
0.150–0.192	0.1495–0.1925	0.171	4
0.193–0.235	0.1925–0.2355	0.214	3
0.236–0.278	0.2355–0.2785	0.257	10
0.279–0.321	0.2785–0.3215	0.3	6

(b, c) Baseball Batting Averages—Histogram

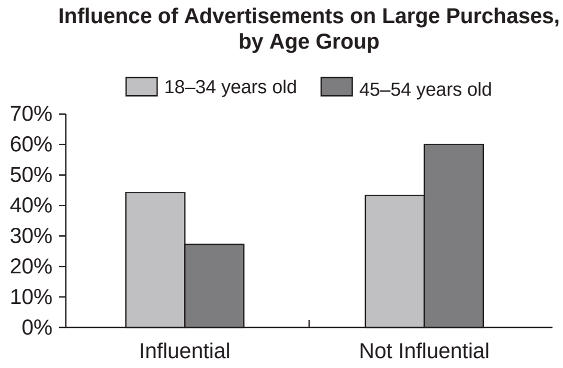


22. Dotplot for Iditarod Finish Time (in hours)

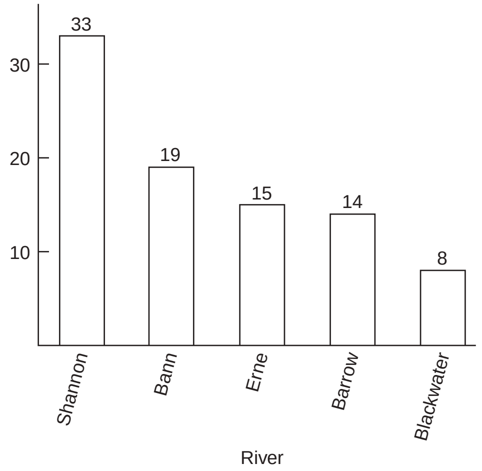


Section 2.2

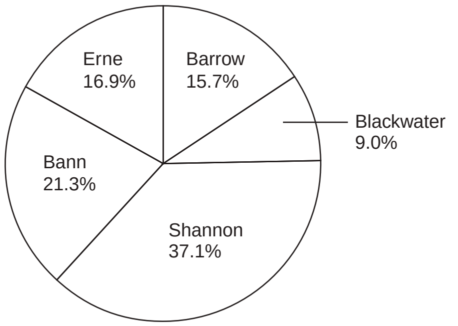
6. (b)



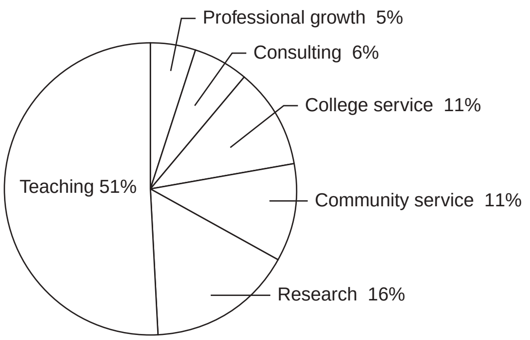
8. (a) Number of Spearheads—Pareto Chart



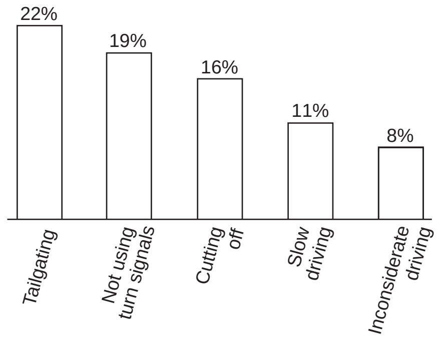
(b) Number of Spearheads—Circle Graph



10. How College Professors Spend Their Time—Circle Graph

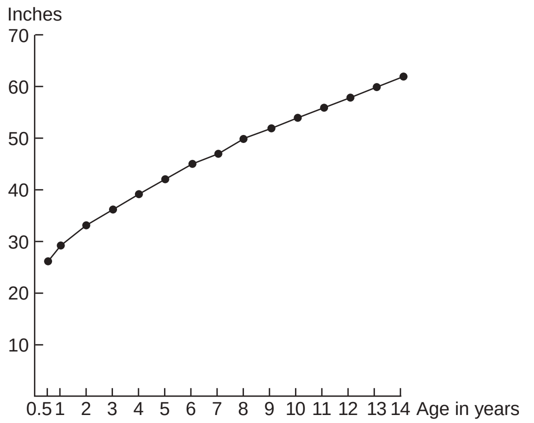


12. Driving Problems—Pareto Chart



No. The total is not 100%, and it is not clear if respondents could mark more than one problem.

14. Changes in Boys' Height with Age—Time-Series Graph



Chapter 2 Review

8. (a)

Age of DUI Arrests

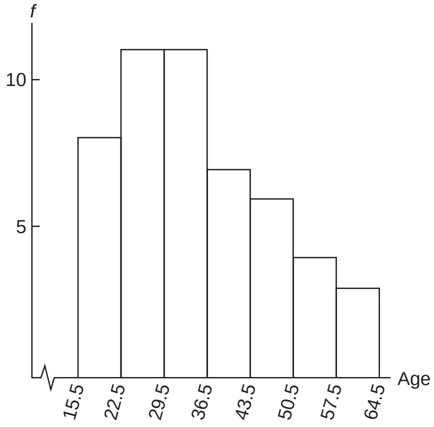
1	6 = 16 years
1	6 8
2	0 1 1 2 2 2 3 4 4 5 6 6 6 7 7 7 9
3	0 0 1 1 2 3 4 4 5 5 6 7 8 9
4	0 0 1 3 5 6 7 7 9 9
5	1 3 5 6 8
6	3 4

(b) Class width = 7.

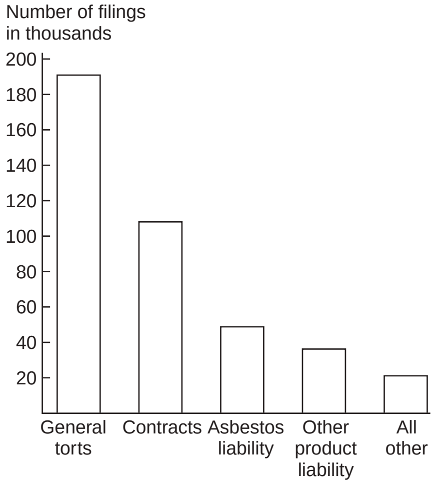
Age Distribution of DUI Arrests

Class Limits	Class Boundaries	Midpoint	Frequency	Relative Frequency	Cumulative Frequency
16–22	15.5–22.5	19	8	0.16	8
23–29	22.5–29.5	26	11	0.22	19
30–36	29.5–36.5	33	11	0.22	30
37–43	36.5–43.5	40	7	0.14	37
44–50	43.5–50.5	47	6	0.12	43
51–57	50.5–57.5	54	4	0.08	47
58–64	57.5–64.5	61	3	0.06	50

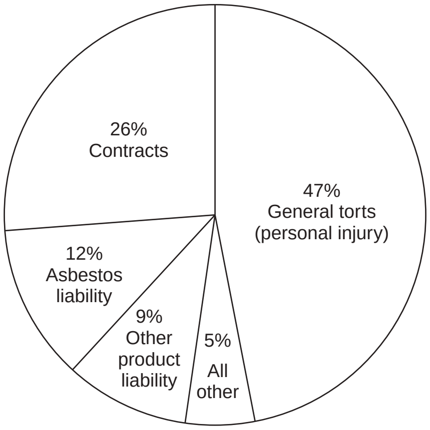
(c) Age Distribution of DUI Arrests—Histogram



10. (a) Distribution of Civil Justice Caseloads Involving Businesses—Pareto Chart



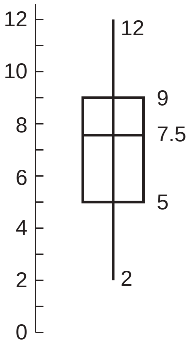
(b) Distribution of Civil Justice Caseloads Involving Businesses—Pie Chart



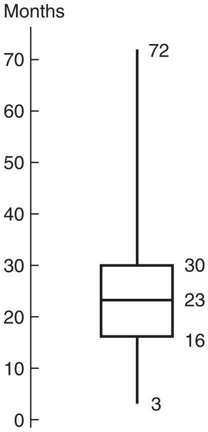
CHAPTER 3

Section 3.3

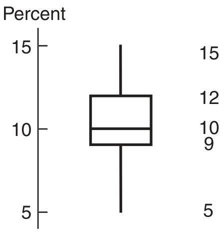
(c) Box-and-Whisker Plot



8. (a) Low = 3; Q_1 = 16; median = 23; Q_3 = 30; high = 72; IQR = 14.
Clerical Staff Length of Employment (months)

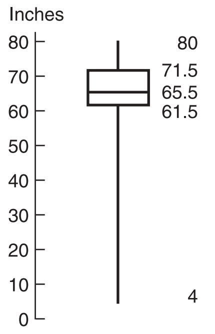


10. (a) Low = 5; Q_1 = 9; median = 10; Q_3 = 12; high = 15; IQR = 3.
(b) First quartile, since it is below Q_1.
High School Dropout Percentage by State



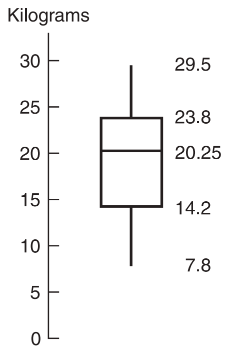
12. (a) Low value = 4; Q_1 = 61.5; median = 65.5; Q_3 = 71.5; high value = 80.
(b) IQR = 10.
(c) Lower limit = 46.5; upper limit = 86.5.

(d) Yes, the value 4 is below the lower limit and is probably an error. Our guess is that one of the students is 4 feet tall and listed height in feet instead of inches. There are no values above the upper limit.
Students' Heights (inches)

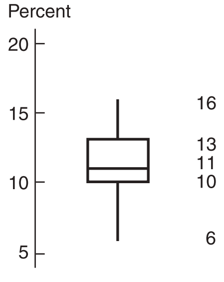


Chapter 3 Review

8. (a) Low = 7.8; Q_1 = 14.2; (kilograms) median = 20.25; Q_3 = 23.8; high = 29.5.
(b) IQR = 9.6 kilograms.
(d) Yes, the lower half shows slightly more spread.
Maize Harvest



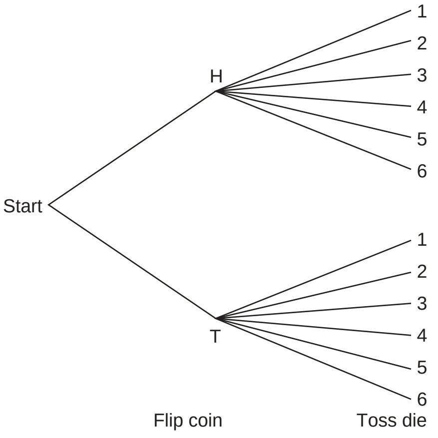
10. (a) Low = 6; Q_1 = 10; median = 11; Q_3 = 13; high = 16; IQR = 3.
Soil Water Content



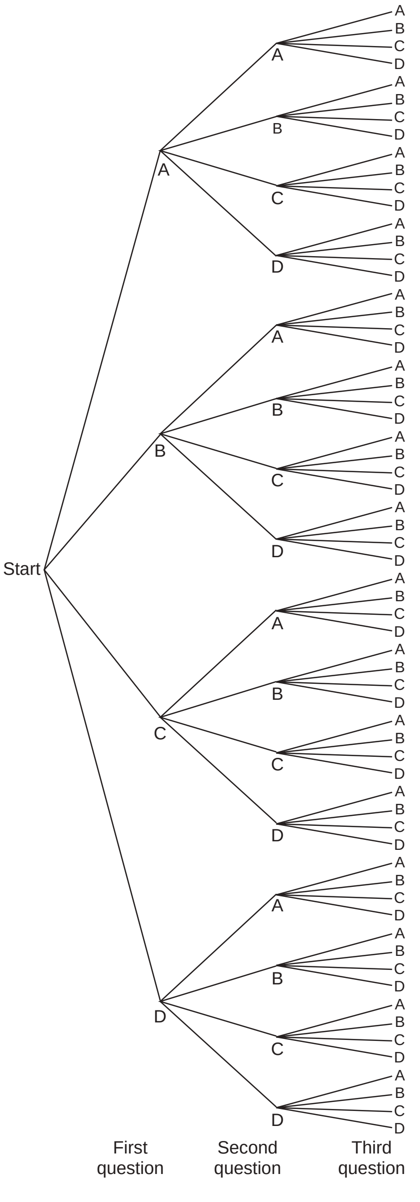
CHAPTER 4

Section 4.3

6. (a) Outcomes of Flipping a Coin and Tossing a Die

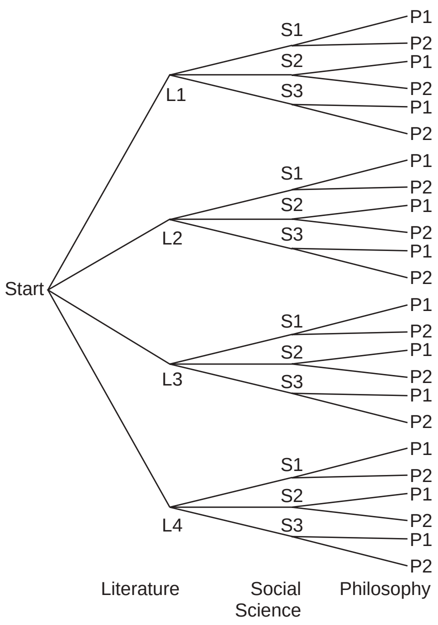


8. (a) Outcomes of Three Multiple-Choice Questions



Chapter 4 Review

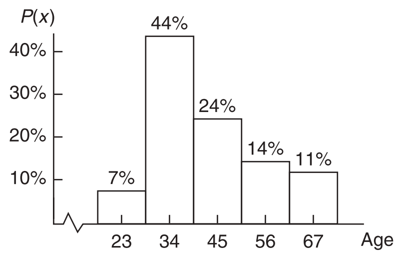
18. Ways to Satisfy Literature, Social Science, and Philosophy Requirements



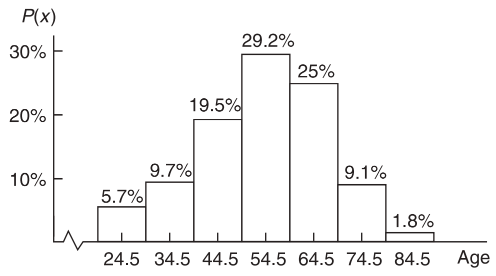
CHAPTER 5

Section 5.1

8. (b) Age of Promotion-Sensitive Shoppers

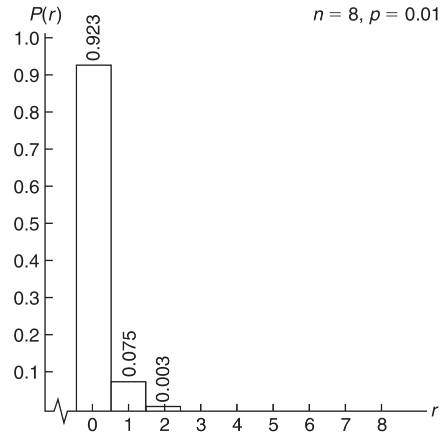


10. (b) Age of Nurses

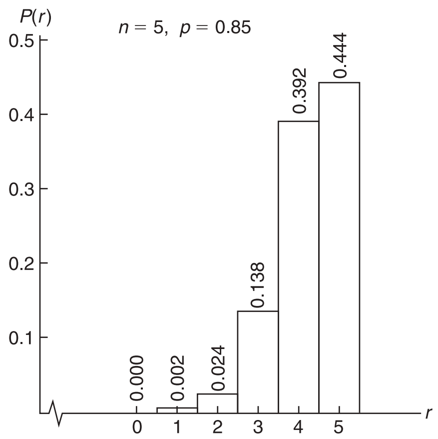


Section 5.3

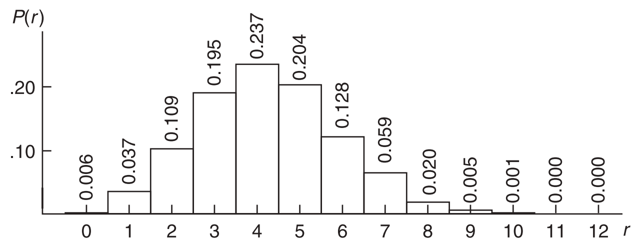
10. (a) Binomial Distribution for Number of Defective Syringes



12. (a) Binomial Distribution for Number of Automobile Damage Claims by People Under Age 25



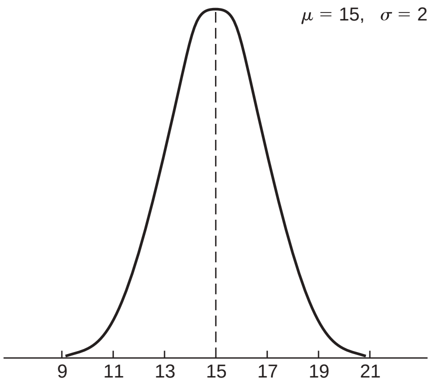
14. (a) Binomial Distribution for Drivers Who Tailgate



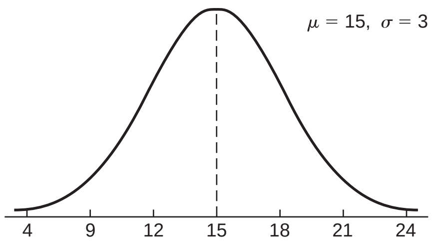
CHAPTER 6

Section 6.1

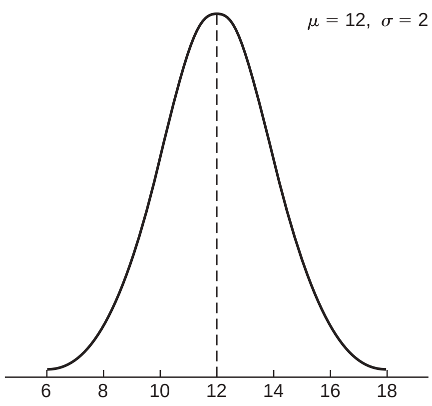
4. (a) Normal Curve



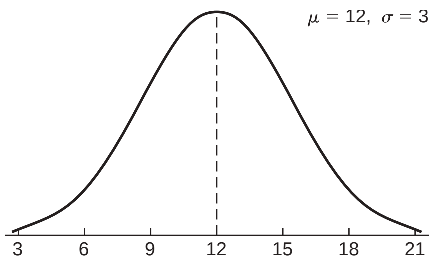
(b) Normal Curve



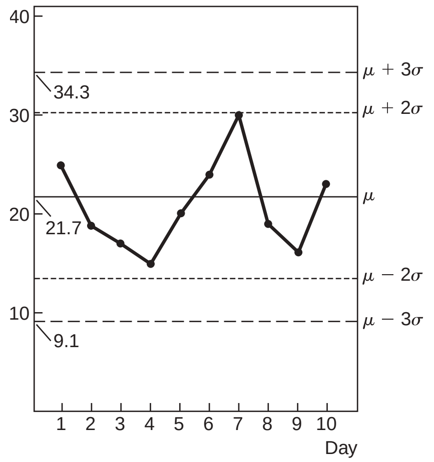
(c) Normal Curve



(d) Normal Curve

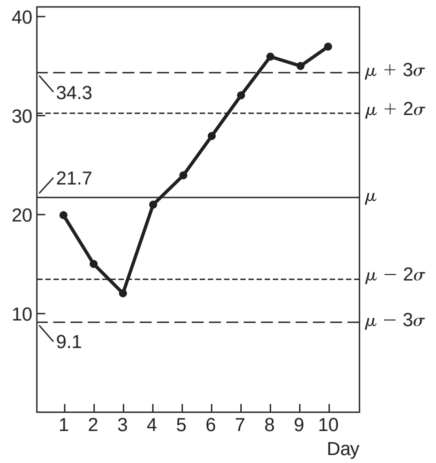


12. (a) Visitors Treated Each Day by YPMS (first period)



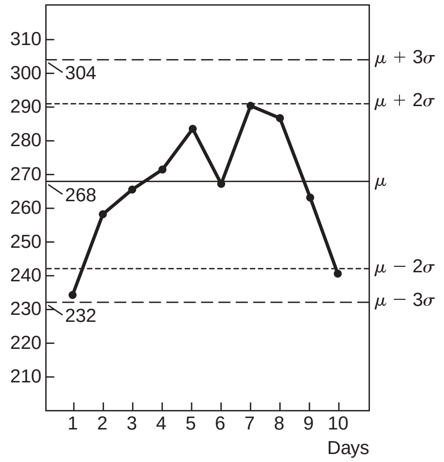
In control.

(b) Visitors Treated Each Day by YPMS (second period)



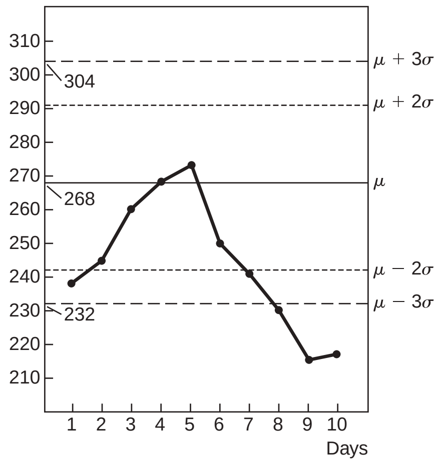
Out-of-control signals I and III are present.

14. (a) Number of Rooms Rented (first period)



In control.

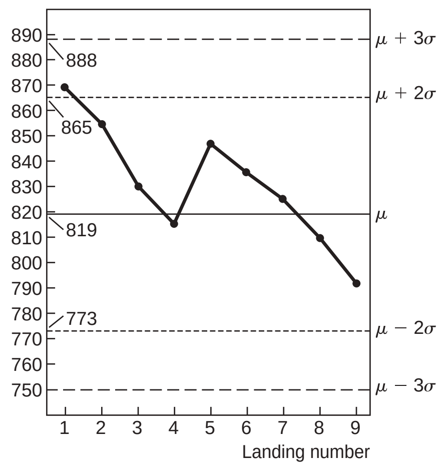
(b) Number of Rooms Rented (second period)



Out-of-control signals I and III are present.

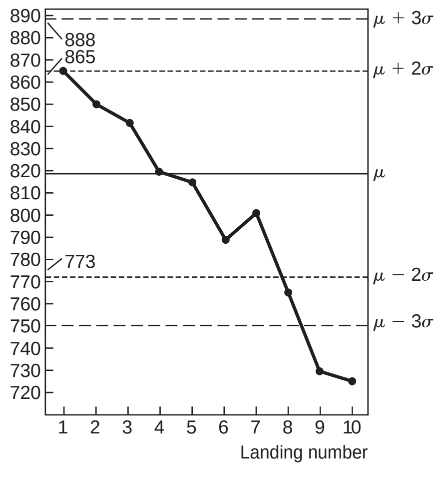
Chapter 6 Review

20. (a) Hydraulic Pressure in Main Cylinder of Landing Gear of Airplanes (psi)—First Data Set



In control.

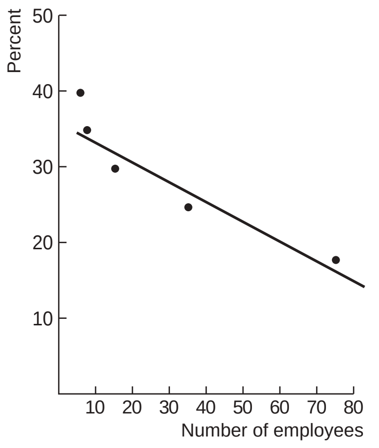
(b) Hydraulic Pressure in Main Cylinder of Landing Gear of Airplanes (psi)—Second Data Set
Out of control signals I and III are present.



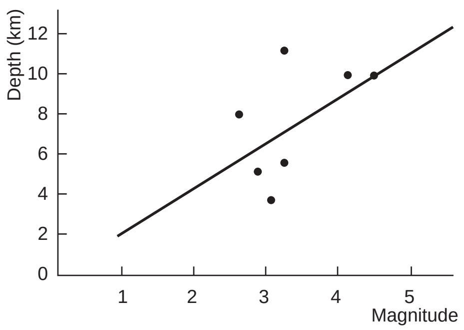
CHAPTER 9

Section 9.1

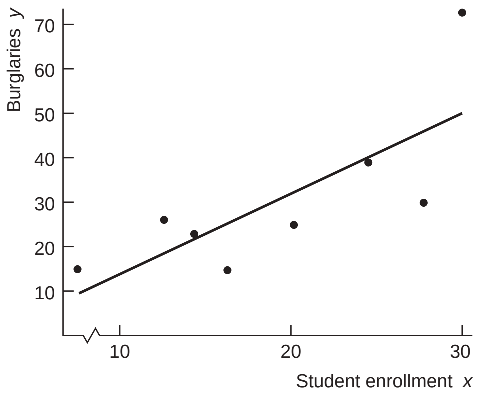
14. (a) Group Health Insurance Plans: Average Number of Employees versus Administrative Costs as a Percentage of Claims



16. (a) Magnitude (Richter Scale) and Depth (km) of Earthquakes

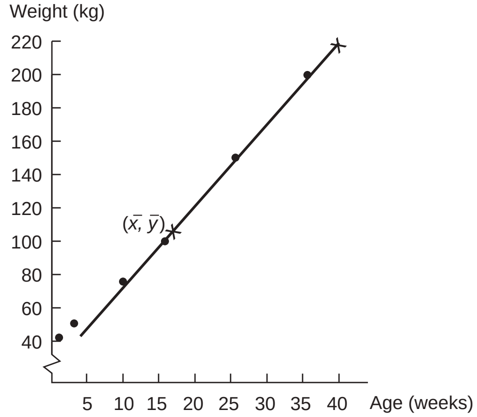


18. (a) Student Enrollment (in thousands) versus Number of Burglaries

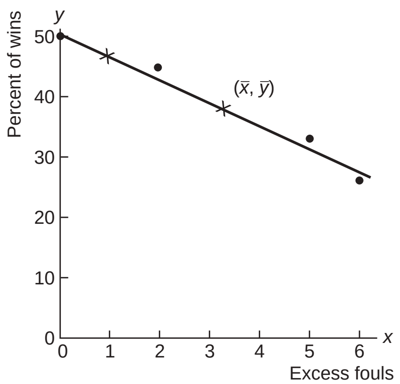


Section 9.2

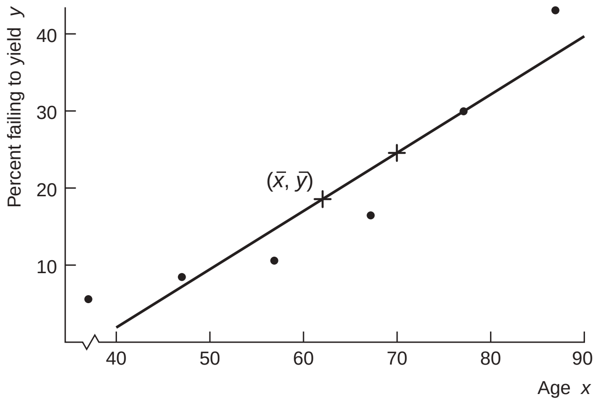
8. (a) Age and Weight of Healthy Calves



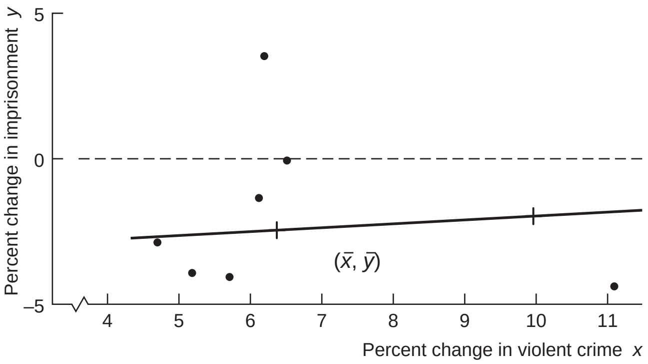
10. (a) Fouls and Basketball Wins



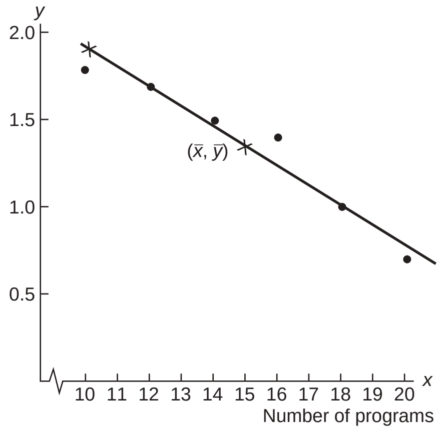
12. (a) Age and Percentage of Fatal Accidents Due to Failure to Yield



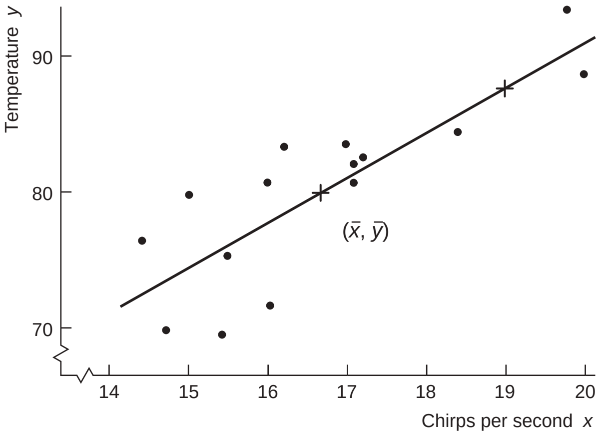
14. (a) Percent Change in Rate of Violent Crime and Percent Change in Rate of Imprisonment in U.S. Population



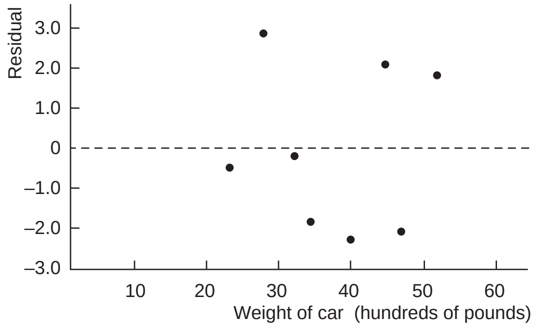
16. (a) Number of Research Programs and Mean Number of Patents per Program



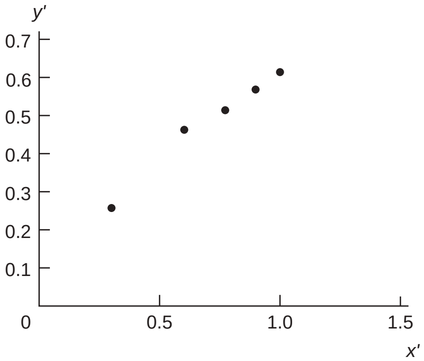
18. (a) Chirps per Second and Temperature (°F)



20. (a) Residuals: 2.9; 2.1; −0.1; −2.1; −0.5; −2.3; −1.9; 1.9.
Residual Plot

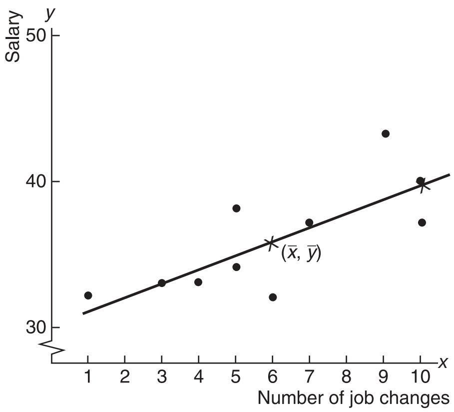


24. (a) Model with $(x'y')$ Data Pairs

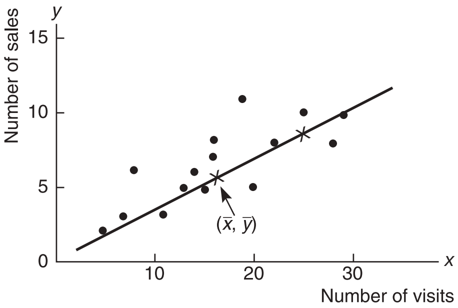


Chapter 9 Review

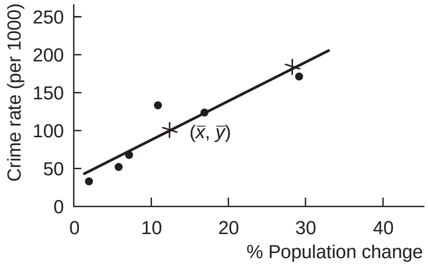
6. (a) Annual Salary (thousands) and Number of Job Changes



8. (a) Number of Insurance Sales and Number of Visits



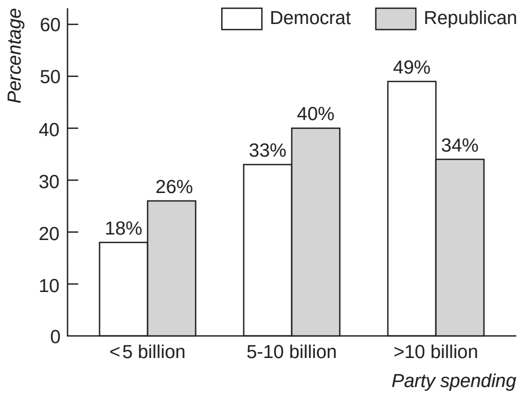
10. (a) Percent Population Change and Crime Rate



CHAPTER 10

Section 10.1

14. (i) Percentage of Each Party Spending Designated Amount



Section 10.5

2. (a) $\alpha = 0.05$; H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$; H_1: Not all the means are equal.
(b–f)

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	*F* Ratio	*P*-value	Test Decision
Between groups	421.033	3	140.344	1.573	> 0.100	Do not reject H_0
Within groups	1516.967	17	89.233			
Total	1938.000	20				

From TI-84, P-value ≈ 0.2327.

4. (a) $\alpha = 0.01$; H_0: $\mu_1 = \mu_2 = \mu_3$; H_1: Not all the means are equal.
(b–f)

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	*F* Ratio	*P*-value	Test Decision
Between groups	215.680	2	107.840	0.816	> 0.100	Do not reject H_0
Within groups	1981.725	15	132.115			
Total	2197.405	17				

From TI-84, P-value ≈ 0.4608.

6. (a) $\alpha = 0.05$; H_0: $\mu_1 = \mu_2 = \mu_3$; H_1: Not all the means are equal.
(b–f)

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	*F* Ratio	*P*-value	Test Decision
Between groups	2.441	2	1.2207	2.95	between 0.050 and 0.100	Do not reject H_0
Within groups	7.448	18	0.4138			
Total	9.890	20				

From TI-84, P-value ≈ 0.0779.

8. (a) $\alpha = 0.05$; H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$; H_1: Not all the means are equal.
(b–f)

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	*F* Ratio	*P*-value	Test Decision
Between groups	18.965	3	6.322	14.910	< 0.001	Reject H_0
Within groups	5.517	13	0.424			
Total	24.482	16				

From TI-84, P-value ≈ 0.0002.

Chapter 10 Review

8. One-way ANOVA. H_0: $\mu_1 = \mu_2 = \mu_3$; H_1: Not all the means are equal.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	*F* Ratio	*P*-value	Test Decision
Between groups	1.002	2	0.501	0.443	> 0.100	Fail to reject H_0
Within groups	10.165	9	1.129			
Total	11.167	11				

TI-84 gives P-value ≈ 0.6651.

Index

FREQUENTLY USED FORMULAS

n = sample size N = population size f = frequency

Chapter 2

Class width $= \dfrac{\text{high} - \text{low}}{\text{number of classes}}$ (increase to next integer)

Class midpoint $= \dfrac{\text{upper limit} + \text{lower limit}}{2}$

Lower boundary = lower boundary of previous class + class width

Chapter 3

Sample mean $\bar{x} = \dfrac{\Sigma x}{n}$

Population mean $\mu = \dfrac{\Sigma x}{N}$

Weighted average $= \dfrac{\Sigma xw}{\Sigma w}$

Range = largest data value − smallest data value

Sample standard deviation $s = \sqrt{\dfrac{\Sigma(x - \bar{x})^2}{n - 1}}$

Computation formula $s = \sqrt{\dfrac{\Sigma x^2 - (\Sigma x)^2/n}{n - 1}}$

Population standard deviation $\sigma = \sqrt{\dfrac{\Sigma(x - \mu)^2}{N}}$

Sample variance s^2

Population variance σ^2

Sample coefficient of variation $CV = \dfrac{s}{\bar{x}} \cdot 100$

Sample mean for grouped data $\bar{x} = \dfrac{\Sigma xf}{n}$

Sample standard deviation for grouped data

$$s = \sqrt{\frac{\Sigma(x - \bar{x})^2 f}{n - 1}} = \sqrt{\frac{\Sigma x^2 f - (\Sigma xf)^2/n}{n - 1}}$$

Chapter 4

Probability of the complement of event A
$P(A^c) = 1 - P(A)$

Multiplication rule for independent events
$P(A \text{ and } B) = P(A) \cdot P(B)$

General multiplication rules
$P(A \text{ and } B) = P(A) \cdot P(B|A)$
$P(A \text{ and } B) = P(B) \cdot P(A|B)$

Addition rule for mutually exclusive events
$P(A \text{ or } B) = P(A) + P(B)$

General addition rule
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Permutation rule $P_{n,r} = \dfrac{n!}{(n - r)!}$

Combination rule $C_{n,r} = \dfrac{n!}{r!(n - r)!}$

Chapter 5

Mean of a discrete probability distribution $\mu = \Sigma x P(x)$

Standard deviation of a discrete probability distribution

$\sigma = \sqrt{\Sigma(x - \mu)^2 P(x)}$

Given $L = a + bx$

$\mu_L = a + b\mu$

$\sigma_L = |b|\sigma$

Given $W = ax_1 + bx_2$ (x_1 and x_2 independent)

$\mu_W = a\mu_1 + b\mu_2$

$\sigma_W = \sqrt{a^2\sigma_1^2 + b^2\sigma_2^2}$

For Binomial Distributions

r = number of successes; p = probability of success;

$q = 1 - p$

Binomial probability distribution $P(r) = C_{n,r}\, p^r q^{n-r}$

Mean $\mu = np$

Standard deviation $\sigma = \sqrt{npq}$

Geometric Probability Distribution

n = number of trial on which first success occurs

$P(n) = p(1 - p)^{n-1}$

Poisson Probability Distribution

r = number of successes

λ = mean number of successes over given interval

$P(r) = \dfrac{e^{-\lambda}\lambda^r}{r!}$

Chapter 6

Raw score $x = z\sigma + \mu$ Standard score $z = \dfrac{x - \mu}{\sigma}$

Mean of $\bar{x}$ distribution $\mu_{\bar{x}} = \mu$

Standard deviation of $\bar{x}$ distribution $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$

Standard score for $\bar{x}$ $z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

Mean of $\hat{p}$ distribution $\mu_{\hat{p}} = p$

Standard deviation of $\hat{p}$ distribution $\sigma_{\hat{p}} = \sqrt{\dfrac{pq}{n}}$; $q = 1 - p$

Chapter 7

Confidence Interval

for μ

$\bar{x} - E < \mu < \bar{x} + E$

where $E = z_c \dfrac{\sigma}{\sqrt{n}}$ when σ is known

$E = t_c \dfrac{s}{\sqrt{n}}$ when σ is unknown

with $d.f. = n - 1$

for p ($np > 5$ and $n(1-p) > 5$)

$\hat{p} - E < p < \hat{p} + E$

where $E = z_c \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

$\hat{p} = \dfrac{r}{n}$

for $\mu_1 - \mu_2$ (independent samples)

$(\bar{x}_1 - \bar{x}_2) - E < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + E$

where $E = z_c \sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$ when σ_1 and σ_2 are known

$E = t_c \sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$ when σ_1 or σ_2 is unknown

with $d.f.$ = smaller of $n_1 - 1$ and $n_2 - 1$

(*Note*: Software uses Satterthwaite's approximation for degrees of freedom $d.f.$)

for difference of proportions $p_1 - p_2$

$(\hat{p}_1 - \hat{p}_2) - E < p_1 - p_2 < (\hat{p}_1 - \hat{p}_2) + E$

where $E = z_c \sqrt{\dfrac{\hat{p}_1\hat{q}_1}{n_1} + \dfrac{\hat{p}_2\hat{q}_2}{n_2}}$

$\hat{p}_1 = r_1/n_1;\ \hat{p}_2 = r_2/n_2$

$\hat{q}_1 = 1 - \hat{p}_1;\ \hat{q}_2 = 1 - \hat{p}_2$

Sample Size for Estimating

means $n = \left(\dfrac{z_c\sigma}{E}\right)^2$

proportions

$n = p(1-p)\left(\dfrac{z_c}{E}\right)^2$ with preliminary estimate for p

$n = \dfrac{1}{4}\left(\dfrac{z_c}{E}\right)^2$ without preliminary estimate for p

Chapter 8

Sample Test Statistics for Tests of Hypotheses

for μ (σ known) $\quad z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

for μ (σ unknown) $\quad t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$; $d.f. = n - 1$

for p ($np > 5$ and $nq > 5$) $\quad z = \dfrac{\hat{p} - p}{\sqrt{pq/n}}$

where $q = 1 - p;\ \hat{p} = r/n$

for paired differences d $\quad t = \dfrac{\bar{d} - \mu_d}{s_d/\sqrt{n}}$; $d.f. = n - 1$

for difference of means, σ_1 and σ_2 known

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

for difference of means, σ_1 or σ_2 unknown

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

$d.f.$ = smaller of $n_1 - 1$ and $n_2 - 1$

(*Note*: Software uses Satterthwaite's approximation for degrees of freedom $d.f.$)

for difference of proportions

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\dfrac{\bar{p}\,\bar{q}}{n_1} + \dfrac{\bar{p}\,\bar{q}}{n_2}}}$$

where $\bar{p} = \dfrac{r_1 + r_2}{n_1 + n_2}$ and $\bar{q} = 1 - \bar{p}$

$\hat{p}_1 = r_1/n_1;\ \hat{p}_2 = r_2/n_2$

Chapter 9

Regression and Correlation

Pearson product-moment correlation coefficient

$$r = \frac{n\Sigma xy - (\Sigma x)(\Sigma y)}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\sqrt{n\Sigma y^2 - (\Sigma y)^2}}$$

Least-squares line $\hat{y} = a + bx$

where $b = \dfrac{n\Sigma xy - (\Sigma x)(\Sigma y)}{n\Sigma x^2 - (\Sigma x)^2}$

$a = \bar{y} - b\bar{x}$

Coefficient of determination $= r^2$

Sample test statistic for r

$t = \dfrac{r\sqrt{n-2}}{\sqrt{1-r^2}}$ with $d.f. = n - 2$

Standard error of estimate $S_e = \sqrt{\dfrac{\Sigma y^2 - a\Sigma y - b\Sigma xy}{n-2}}$

Confidence interval for y

$\hat{y} - E < y < \hat{y} + E$

where $E = t_c S_e \sqrt{1 + \dfrac{1}{n} + \dfrac{n(x - \bar{x})^2}{n\Sigma x^2 - (\Sigma x)^2}}$

with $d.f. = n - 2$

Sample test statistic for slope b

$$t = \frac{b}{S_e}\sqrt{\Sigma x^2 - \frac{1}{n}(\Sigma x)^2} \text{ with } d.f. = n - 2$$

Confidence interval for β

$b - E < \beta < b + E$

$$\text{where } E = \frac{t_c S_e}{\sqrt{\Sigma x^2 - \frac{1}{n}(\Sigma x)^2}} \text{ with } d.f. = n - 2$$

Chapter 10

$$\chi^2 = \Sigma \frac{(O - E)^2}{E} \text{ where}$$

O = observed frequency and

E = expected frequency

For tests of independence and tests of homogeneity

$$E = \frac{(\text{row total})(\text{column total})}{\text{sample size}}$$

For goodness of fit test E = (given percent)(sample size)

Tests of independence $d.f. = (R - 1)(C - 1)$

Test of homogeneity $d.f. = (R - 1)(C - 1)$

Goodness of fit $d.f.$ = (number of categories) $- 1$

Confidence interval for σ^2; $d.f. = n - 1$

$$\frac{(n-1)s^2}{\chi^2_U} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_L}$$

Sample test statistic for σ^2

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} \text{ with } d.f. = n - 1$$

Testing Two Variances

$$\text{Sample test statistic } F = \frac{s_1^2}{s_2^2}$$

where $s_1^2 \geq s_2^2$

$d.f._N = n_1 - 1$; $d.f._D = n_2 - 1$

ANOVA

k = number of groups; N = total sample size

$$SS_{TOT} = \Sigma x_{TOT}^2 - \frac{(\Sigma x_{TOT})^2}{N}$$

$$SS_{BET} = \sum_{\text{all groups}} \left(\frac{(\Sigma x_i)^2}{n_i}\right) - \frac{(\Sigma x_{TOT})^2}{N}$$

$$SS_W = \sum_{\text{all groups}} \left(\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n_i}\right)$$

$$SS_{TOT} = SS_{BET} + SS_W$$

$$MS_{BET} = \frac{SS_{BET}}{d.f._{BET}} \text{ where } d.f._{BET} = k - 1$$

$$MS_W = \frac{SS_W}{d.f._W} \text{ where } d.f._W = N - k$$

$$F = \frac{MS_{BET}}{MS_W} \text{ where } d.f. \text{ numerator} = d.f._{BET} = k - 1;$$

$d.f.$ denominator $= d.f._W = N - k$

Two-Way ANOVA

r = number of rows; c = number of columns

$$\text{Row factor } F: \frac{MS \text{ row factor}}{MS \text{ error}}$$

$$\text{Column factor } F: \frac{MS \text{ column factor}}{MS \text{ error}}$$

$$\text{Interaction } F: \frac{MS \text{ interaction}}{MS \text{ error}}$$

with degrees of freedom for

row factor $= r - 1$ interaction $= (r - 1)(c - 1)$

column factor $= c - 1$ error $= rc(n - 1)$

Chapter 11

Sample test statistic for x = proportion of plus signs to all signs ($n \geq 12$)

$$z = \frac{x - 0.5}{\sqrt{0.25/n}}$$

Sample test statistic for R = sum of ranks

$$z = \frac{R - \mu_R}{\sigma_R} \text{ where } \mu_R = \frac{n_1(n_1 + n_2 + 1)}{2} \text{ and}$$

$$\sigma_R = \sqrt{\frac{n_1 n_2(n_1 + n_2 + 1)}{12}}$$

Spearman rank correlation coefficient

$$r_s = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)} \text{ where } d = x - y$$

Sample test statistic for runs test

R = number of runs in sequence

Procedure for Hypothesis Testing

Use appropriate experimental design and obtain random samples of data (see Sections 1.2 and 1.3).

In the context of the application:

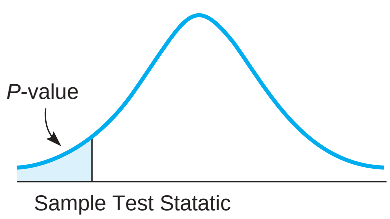
1. State the null hypothesis H_0 and the alternate hypothesis H_1. Set the level of significance α for the test.
2. Determine the appropriate sampling distribution and compute the sample test statistic.
3. Use the type of test (one-tailed or two-tailed) and the sampling distribution to compute the P-value of the corresponding sample test statistic.
4. Conclude the test. If P-value $\leq \alpha$ then reject H_0. If P-value $> \alpha$ then do not reject H_0.
5. Interpret the conclusion in the context of the application.

Finding the *P*-Value Corresponding to a Sample Test Statistic

Use the appropriate sampling distribution as described in procedure displays for each of the various tests.

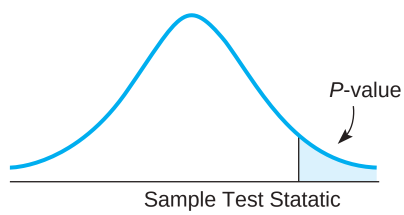
Left-Tailed Test

P-value = area to the left of the sample test statistic



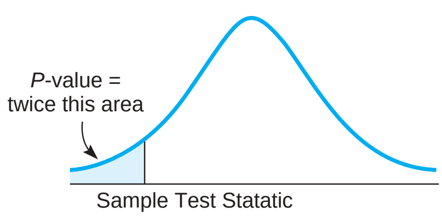
Right-Tailed Test

P-value = area to the right of the sample test statistic

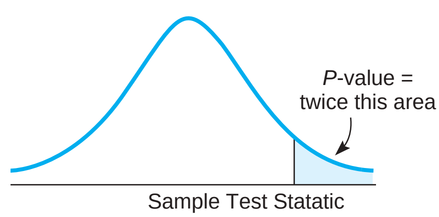


Two-Tailed Test

Sample test statistic lies to *left* of center
P-value = twice area to the left of sample test statistic



Sample test statistic lies to *right* of center
P-value = twice area to the right of sample test statistic



Sampling Distributions for Inferences Regarding μ or p

Parameter	Condition	Sampling Distribution
μ	σ is known and x has a normal distribution or $n \geq 30$	Normal distribution
μ	σ is not known and x has a normal or mound-shaped, symmetric distribution or $n \geq 30$	Student's t distribution with $d.f. = n - 1$
p	$np > 5$ and $n(1 - p) > 5$	Normal distribution