5

AP Photo/Matt Rourke

National Portrait Gallery, Smithsonian Institution/Art Resource, NY

Education is the key to unlock the golden door of freedom.

—George Washington Carver

George Washington Carver (1859–1943) won international fame for agricultural research. After graduating from Iowa State College, he was appointed a faculty member in the Iowa State Botany Department. Carver took charge of the greenhouse and started a fungus collection that later included more than 20,000 species. This collection brought him professional acclaim in the field of botany.

At the invitation of his friend Booker T. Washington, Carver joined the faculty of the Tuskegee Institute, where he spent the rest of his long and distinguished career. Carver's creative genius accounted for more than 300 inventions from peanuts, 118 inventions from sweet potatoes, and 75 inventions from pecans.

Gathering and analyzing data were important components of Carver's work. Methods you will learn in this course are widely used in research in every field, including agriculture.

For online student resources, visit The Brase/Brase, *Understandable Statistics,* 10th edition web site at **http://www.cengage.com/statistics/brase**

The Binomial Probability Distribution and Related Topics

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PREVIEW QUESTIONS

What is a random variable? How do you compute μ and σ for a discrete random variable? How do you compute μ and σ for linear combinations of independent random variables? (SECTION 5.1)

Many of life's experiences consist of some successes together with some failures. Suppose you make n *attempts to succeed at a certain project. How can you use the binomial probability distribution to compute the probability of* r *successes?* (SECTION 5.2)

How do you compute μ and σ for the binomial distribution? (SECTION 5.3)

How is the binomial distribution related to other probability distributions, such as the geometric and Poisson? (SECTION 5.4)

FOCUS PROBLEM

Personality Preference Types: Introvert or Extrovert?

Isabel Briggs Myers was a pioneer in the study of personality types. Her work has been used successfully in counseling, educational, and industrial settings. In the book *A Guide to the Development and Use of the Myers–Briggs Type Indicators,* by Myers and McCaully, it was reported that based on a very large sample (2282 professors), approximately 45% of all university professors are extroverted.

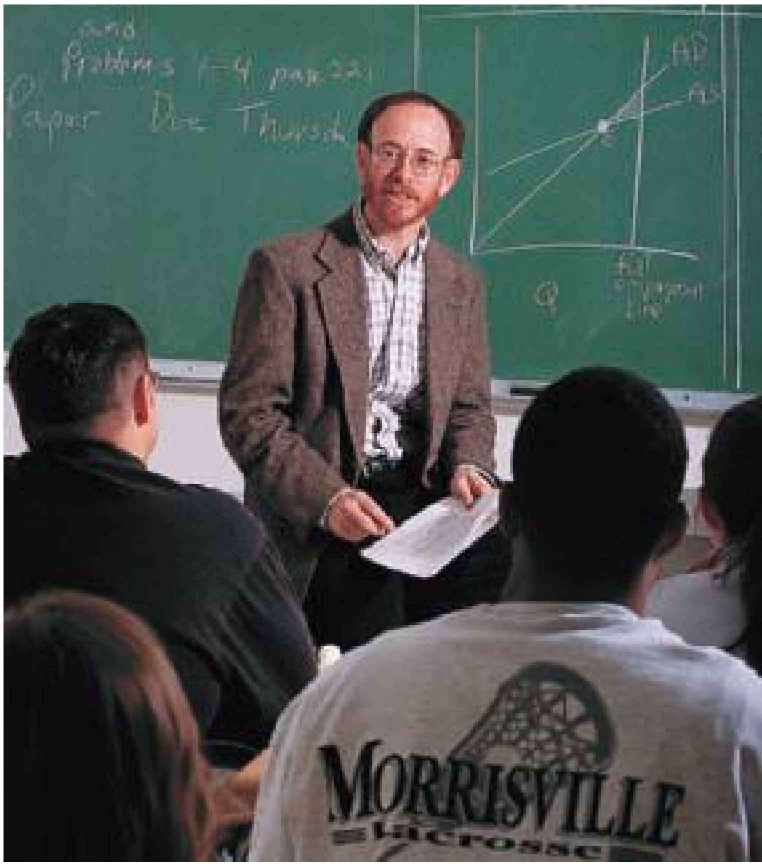
After completing this chapter, you will be able to answer the following questions. Suppose you have classes with six different professors.

(a) What is the probability that all six are extroverts?

(b) What is the probability that none of your professors is an extrovert?

(c) What is the probability that at least two of your professors are extroverts?

(d) In a group of six professors selected at random, what is the *expected number* of extroverts? What is the *standard deviation* of the distribution?



Mitch Wojnarowicz/The Image Works

(e) Suppose you were assigned to write an article for the student newspaper and you were given a quota (by the editor) of interviewing at least three extroverted professors. How many professors selected at random would you need to interview to be at least 90% sure of filling the quota?

(See Problem 24 of Section 5.3.)

COMMENT Both extroverted and introverted professors can be excellent teachers.

SECTION 5.1

Introduction to Random Variables and Probability Distributions

FOCUS POINTS

- Distinguish between discrete and continuous random variables.
- Graph discrete probability distributions.
- Compute μ and σ for a discrete probability distribution.
- Compute μ and σ for a linear function of a random variable x.
- Compute μ and σ for a linear combination of two independent random variables.

Random Variables

For our purposes, we say that a *statistical experiment* or *observation* is any process by which measurements are obtained. For instance, you might count the number of eggs in a robin's nest or measure daily rainfall in inches. It is common practice to use the letter x to represent the quantitative result of an experiment or observation. As such, we call x a variable.

Random Variable

Discrete random variable

Continuous random variable

A quantitative variable x is a **random variable** if the value that x takes on in a given experiment or observation is a chance or random outcome.

A **discrete random variable** can take on only a finite number of values or a countable number of values.

A **continuous random variable** can take on any of the countless number of values in a line interval.

The distinction between discrete and continuous random variables is important because of the different mathematical techniques associated with the two kinds of random variables.

In most of the cases we will consider, a *discrete random variable* will be the result of a count. The number of students in a statistics class is a discrete random variable. Values such as 15, 25, 50, and 250 are all possible. However, 25.5 students is not a possible value for the number of students.

Most of the *continuous random variables* we will see will occur as the result of a measurement on a continuous scale. For example, the air pressure in an automobile tire represents a continuous random variable. The air pressure could, in theory, take on any value from 0 lb/in^2 (psi) to the bursting pressure of the tire. Values such as 20.126 psi, 20.12678 psi, and so forth are possible.

GUIDED EXERCISE 1

Discrete or continuous random variables

Which of the following random variables are discrete and which are continuous?

(a) *Measure* the time it takes a student selected at random to register for the fall term. → Time can take on any value, so this is a continuous random variable.

Continued

GUIDED EXERCISE 1 *continued*

(b) *Count* the number of bad checks drawn on Upright Bank on a day selected at random. → The number of bad checks can be only a whole number such as 0, 1, 2, 3, etc. This is a discrete variable.

(c) *Measure* the amount of gasoline needed to drive your car 200 miles. → We are measuring volume, which can assume any value, so this is a continuous random variable.

(d) Pick a random sample of 50 registered voters in a district and find the number who voted in the last county election. → This is a count, so the variable is discrete.

Probability Distribution of a Discrete Random Variable

A random variable has a probability distribution whether it is discrete or continuous.

Probability distribution

A **probability distribution** is an assignment of probabilities to each distinct value of a discrete random variable or to each interval of values of a continuous random variable.

Features of the probability distribution of a discrete random variable

1. The probability distribution has a probability assigned to *each* distinct value of the random variable.
2. The sum of all the assigned probabilities must be 1.

EXAMPLE 1 DISCRETE PROBABILITY DISTRIBUTION

Dr. Mendoza developed a test to measure boredom tolerance. He administered it to a group of 20,000 adults between the ages of 25 and 35. The possible scores were 0, 1, 2, 3, 4, 5, and 6, with 6 indicating the highest tolerance for boredom. The test results for this group are shown in Table 5-1.

(a) If a subject is chosen at random from this group, the probability that he or she will have a score of 3 is 6000/20,000, or 0.30. In a similar way, we can use relative frequencies to compute the probabilities for the other scores (Table 5-2). These probability assignments make up the probability distribution. Notice that the scores are mutually exclusive: No one subject has two scores. The sum of the probabilities of all the scores is 1.

TABLE 5-1 Boredom Tolerance Test Scores for 20,000 Subjects

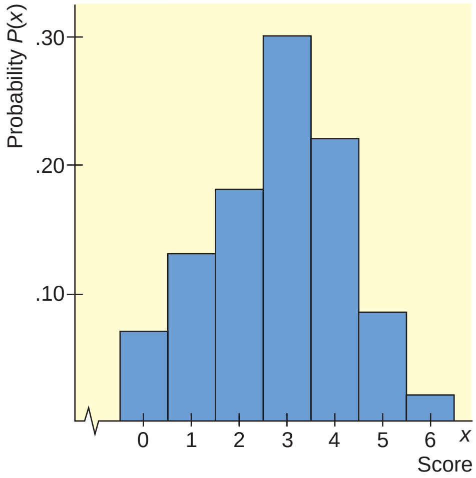
Score	Number of Subjects
0	1400
1	2600
2	3600
3	6000
4	4400
5	1600
6	400

TABLE 5-2 Probability Distribution of Scores on Boredom Tolerance Test

Score x	Probability $P(x)$
0	0.07
1	0.13
2	0.18
3	0.30
4	0.22
5	0.08
6	0.02
	$\Sigma P(x) = 1$

FIGURE 5-1

Graph of the Probability Distribution of Test Scores



(b) The graph of this distribution is simply a relative-frequency histogram (see Figure 5-1) in which the height of the bar over a score represents the probability of that score. Since each bar is one unit wide, the area of the bar over a score equals the height and thus represents the probability of that score. Since the sum of the probabilities is 1, the area under the graph is also 1.

(c) The Topnotch Clothing Company needs to hire someone with a score on the boredom tolerance test of 5 or 6 to operate the fabric press machine. Since the scores 5 and 6 are mutually exclusive, the probability that someone in the group who took the boredom tolerance test made either a 5 or a 6 is the sum

$$P(5 \text{ or } 6) = P(5) + P(6)$$
$$= 0.08 + 0.02 = 0.10$$

Notice that to find $P(5 \text{ or } 6)$, we could have simply added the *areas* of the bars over 5 and over 6. One out of 10 of the group who took the boredom tolerance test would qualify for the position at Topnotch Clothing.

GUIDED EXERCISE 2 Discrete probability distribution

One of the elementary tools of cryptanalysis (the science of code breaking) is to use relative frequencies of occurrence of different letters in the alphabet to break standard English alphabet codes. Large samples of plain text such as newspaper stories generally yield about the same relative frequencies for letters. A sample 1000 letters long yielded the information in Table 5-3.

(a) Use the relative frequencies to compute the omitted probabilities in Table 5-3. → Table 5-4 shows the completion of Table 5-3.

Continued

GUIDED EXERCISE 2 *continued*

TABLE 5-3 Frequencies of Letters in a 1000-Letter Sample

Letter	Freq.	Prob.	Letter	Freq.	Prob.
A	73	——	N	78	0.078
B	9	0.009	O	74	——
C	30	0.030	P	27	0.027
D	44	0.044	Q	3	0.003
E	130	——	R	77	0.077
F	28	0.028	S	63	0.063
G	16	0.016	T	93	0.093
H	35	0.035	U	27	——
I	74	——	V	13	0.013
J	2	0.002	W	16	0.016
K	3	0.003	X	5	0.005
L	35	0.035	Y	19	0.019
M	25	0.025	Z	1	0.001

Source: From *Elementary Cryptanalysis: A Mathematical Approach,* by Abraham Sinkov. Copyright © 1968 and renewed 1996 by Yale University. Used by permission of Random House, Inc. Copyright The Mathematical Association of America 2010. All rights reserved

TABLE 5-4 Entries for Table 5-3

Letter	Relative Frequency	Probability
A	$\frac{73}{1,000}$	0.073
E	$\frac{130}{1,000}$	0.130
I	$\frac{74}{1,000}$	0.074
O	$\frac{74}{1,000}$	0.074
U	$\frac{27}{1,000}$	0.027

(b) Do the probabilities of all the individual letters add up to 1?

Yes.

(c) If a letter is selected at random from a newspaper story, what is the probability that the letter will be a vowel?

If a letter is selected at random,

$$\begin{aligned} P(a, e, i, o, \text{ or } u) &= P(a) + P(e) + P(i) + P(o) + P(u) \\ &= 0.073 + 0.130 + 0.074 + 0.074 + 0.027 \\ &= 0.378 \end{aligned}$$

Mean and standard deviation of a discrete probability distribution

A probability distribution can be thought of as a relative-frequency distribution based on a very large n. As such, it has a mean and standard deviation. If we are referring to the probability distribution of a *population*, then we use the Greek letters μ for the mean and σ for the standard deviation. When we see the Greek letters used, we know the information given is from the *entire population* rather than just a sample. If we have a sample probability distribution, we use $\bar{x}$ (x bar) and s, respectively, for the mean and standard deviation.

The **mean** and the **standard deviation of a discrete population probability distribution** are found by using these formulas:

$\mu = \Sigma xP(x)$; μ is called the expected value of x

$\sigma = \sqrt{\Sigma(x - \mu)^2 P(x)}$; σ is called the standard deviation of x

where x is the value of a random variable,
$P(x)$ is the probability of that variable, and
the sum Σ is taken for all the values of the random variable.

Note: μ is the *population mean* and σ is the underlying *population standard deviation* because the sum Σ is taken over *all* values of the random variable (i.e., the entire sample space).

Expected value

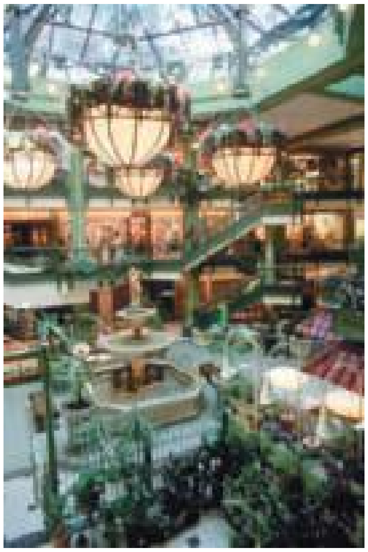
The mean of a probability distribution is often called the *expected value* of the distribution. This terminology reflects the idea that the mean represents a "central point" or "cluster point" for the entire distribution. Of course, the mean or expected value is an average value, and as such, it *need not be a point of the sample space*.

The standard deviation is often represented as a measure of *risk*. A larger standard deviation implies a greater likelihood that the random variable x is different from the expected value μ.

EXAMPLE 2

Expected value, standard deviation

Are we influenced to buy a product by an ad we saw on TV? National Infomercial Marketing Association determined the number of times *buyers* of a product had watched a TV infomercial *before* purchasing the product. The results are shown here:



Kelly-Mooney Photography/Encyclopedia/Corbis

Number of Times Buyers Saw Infomercial	1	2	3	4	5*
Percentage of Buyers	27%	31%	18%	9%	15%

*This category was 5 or more, but will be treated as 5 in this example.

We can treat the information shown as an estimate of the probability distribution because the events are mutually exclusive and the sum of the percentages is 100%. Compute the mean and standard deviation of the distribution.

SOLUTION: We put the data in the first two columns of a computation table and then fill in the other entries (see Table 5-5). The average number of times a buyer views the infomercial before purchase is

$$\mu = \Sigma xP(x) = 2.54 \text{ (sum of column 3)}$$

To find the standard deviation, we take the square root of the sum of column 6:

$$\sigma = \sqrt{\Sigma(x - \mu)^2 P(x)} \approx \sqrt{1.869} \approx 1.37$$

TABLE 5-5 Number of Times Buyers View Infomercial Before Making Purchase

x (number of viewings)	$P(x)$	$xP(x)$	$x - \mu$	$(x - \mu)^2$	$(x - \mu)^2 P(x)$
1	0.27	0.27	−1.54	2.372	0.640
2	0.31	0.62	−0.54	0.292	0.091
3	0.18	0.54	0.46	0.212	0.038
4	0.09	0.36	1.46	2.132	0.192
5	0.15	0.75	2.46	6.052	0.908
		$\mu = \Sigma xP(x) = 2.54$			$\Sigma(x - \mu)^2 P(x) = 1.869$

CALCULATOR NOTE Some calculators, including the TI-84Plus/TI-83Plus/TI-*n*spire (with TI-84Plus keypad) models, accept fractional frequencies. If yours does, you can get μ and σ directly by entering the outcomes in list $\mathbf{L_1}$ with corresponding probabilities in list $\mathbf{L_2}$. Then use **1-Var Stats** $\mathbf{L_1, L_2}$.

GUIDED EXERCISE 3 Expected value

At a carnival, you pay $2.00 to play a coin-flipping game with three fair coins. On each coin one side has the number 0 and the other side has the number 1. You flip the three coins at one time and you win $1.00 for every 1 that appears on top. Are your expected earnings equal to the cost to play? We'll answer this question in several steps.

(a) In this game, the random variable of interest counts the number of 1's that show. What is the sample space for the values of this random variable?

→ The sample space is {0, 1, 2, 3}, since any of these numbers of 1's can appear.

(b) There are eight equally likely outcomes for throwing three coins. They are 000, 001, 010, 011, 100, 101, _______, and _______.

→ 110 and 111.

(c) Complete Table 5-6.

TABLE 5-6

Number of 1's, x	Frequency	$P(x)$	$xP(x)$
0	1	0.125	0
1	3	0.375	_____
2	3	_____	_____
3	_____	_____	_____

→ TABLE 5-7 **Completion of Table 5-6**

Number of 1's, x	Frequency	$P(x)$	$xP(x)$
0	1	0.125	0
1	3	0.375	0.375
2	3	0.375	0.750
3	1	0.125	0.375

(d) The expected value is the sum

$\mu = \Sigma xP(x)$

Sum the appropriate column of Table 5-6 to find this value. Are your expected earnings less than, equal to, or more than the cost of the game?

→ The expected value can be found by summing the last column of Table 5-7. The expected value is $1.50. It cost $2.00 to play the game; the expected value is less than the cost. The carnival is making money. In the long run, the carnival can expect to make an average of about 50 cents per player.

We have seen probability distributions of discrete variables and the formulas to compute the mean and standard deviation of a discrete population probability distribution. Probability distributions of continuous random variables are similar except that the probability assignments are made to intervals of values rather than to specific values of the random variable. We will see an important example of a discrete probability distribution, the binomial distribution, in the next section, and one of a continuous probability distribution in Chapter 6 when we study the normal distribution.

We conclude this section with some useful information about combining random variables.

Linear Functions of a Random Variable

Let a and b be any constants, and let x be a random variable. Then the new random variable $L = a + bx$ is called a *linear function of x*. Using some more advanced mathematics, the following can be proved.

Linear Function of a random variable

Let x be a random variable with mean μ and standard deviation σ. Then the **linear function** $L = a + bx$ has mean, variance, and standard deviation as follows:

$$\mu_L = a + b\mu$$
$$\sigma_L^2 = b^2\sigma^2$$
$$\sigma_L = \sqrt{b^2\sigma^2} = |b|\sigma$$

Linear Combinations of Independent Random Variables

Linear combination of two independent random variables

Suppose we have two random variables x_1 and x_2. These variables are *independent* if any event involving x_1 by itself is *independent* of any event involving x_2 by itself. Sometimes, we want to combine independent random variables and examine the mean and standard deviation of the resulting combination.

Let x_1 and x_2 be independent random variables, and let a and b be any constants. Then the new random variable $W = ax_1 + bx_2$ is called a *linear combination of* x_1 *and* x_2. Using some more advanced mathematics, the following can be proved.

Let x_1 and x_2 be independent random variables with respective means μ_1 and μ_2, and variances σ_1^2 and σ_2^2. For the **linear combination** $W = ax_1 + bx_2$, the mean, variance, and standard deviation are as follows:

$$\mu_W = a\mu_1 + b\mu_2$$
$$\sigma_W^2 = a^2\sigma_1^2 + b^2\sigma_2^2$$
$$\sigma_W = \sqrt{a^2\sigma_1^2 + b^2\sigma_2^2}$$

Note: The formula for the mean of a linear combination of random variables is valid regardless of whether the variables are independent. However, *the formulas for the variance and standard deviation are valid* only if x_1 and x_2 are *independent* random variables. In later work (Chapter 6 on), we will use independent random samples to ensure that the resulting variables (usually means, proportions, etc.) are statistically independent.

EXAMPLE 3 LINEAR COMBINATIONS OF INDEPENDENT RANDOM VARIABLES

Let x_1 and x_2 be independent random variables with respective means $\mu_1 = 75$ and $\mu_2 = 50$, and standard deviations $\sigma_1 = 16$ and $\sigma_2 = 9$.

(a) Let $L = 3 + 2x_1$. Compute the mean, variance, and standard deviation of L.

SOLUTION: L is a linear function of the random variable x_1. Using the formulas with $a = 3$ and $b = 2$, we have

$$\mu_L = 3 + 2\mu_1 = 3 + 2(75) = 153$$
$$\sigma_L^2 = 2^2\sigma_1^2 = 4(16)^2 = 1024$$
$$\sigma_L = |2|\sigma_1 = 2(16) = 32$$

Notice that the variance and standard deviation of the linear function are influenced only by the coefficient of x_1 in the linear function.

(b) Let $W = x_1 + x_2$. Find the mean, variance, and standard deviation of W.

SOLUTION: W is a linear combination of the independent random variables x_1 and x_2. Using the formulas with both a and b equal to 1, we have

$$\mu_W = \mu_1 + \mu_2 = 75 + 50 = 125$$
$$\sigma_W^2 = \sigma_1^2 + \sigma_2^2 = 16^2 + 9^2 = 337$$
$$\sigma_W = \sqrt{\sigma_1^2 + \sigma_2^2} = \sqrt{337} \approx 18.36$$

(c) Let $W = x_1 - x_2$. Find the mean, variance, and standard deviation of W.

SOLUTION: W is a linear combination of the independent random variables x_1 and x_2. Using the formulas with $a = 1$ and $b = -1$, we have

$$\mu_W = \mu_1 - \mu_2 = 75 - 50 = 25$$
$$\sigma_W^2 = 1^2\sigma_1^2 + (-1)^2\sigma_2^2 = 16^2 + 9^2 = 337$$
$$\sigma_W = \sqrt{\sigma_1^2 + \sigma_2^2} = \sqrt{337} \approx 18.36$$

(d) Let $W = 3x_1 - 2x_2$. Find the mean, variance, and standard deviation of W.

SOLUTION: W is a linear combination of the independent random variables x_1 and x_2. Using the formulas with $a = 3$ and $b = -2$, we have

$$\mu_W = 3\mu_1 - 2\mu_2 = 3(75) - 2(50) = 125$$
$$\sigma_W^2 = 3^2\sigma_1^2 + (-2)^2\sigma_2^2 = 9(16^2) + 4(9^2) = 2628$$
$$\sigma_W = \sqrt{2628} \approx 51.26$$

LOOKING FORWARD

Problem 24 of Section 9.1 shows how to find the mean, variance, and standard deviation of a linear combination of two *linearly dependent* random variables.

VIEWPOINT The Rosetta Project

Around 196 B.C., Egyptian priests inscribed a decree on a granite slab affirming the rule of 13-year-old Ptolemy V. The proclamation was in Egyptian hieroglyphics with a translation in a form of ancient Greek. By 1799, the meaning of Egyptian hieroglyphics had been lost for many centuries. However, Napoleon's troops discovered the granite slab (Rosetta Stone). Linguists then used the Rosetta Stone and their knowledge of ancient Greek to unlock the meaning of the Egyptian hieroglyphics.

Linguistic experts say that because of industrialization and globalization, by the year 2100 as many as 90% of the world's languages may be extinct. To help preserve some of these languages for future generations, 1000 translations of the first three chapters of Genesis have been inscribed in tiny text onto 3-inch nickel disks and encased in hardened glass balls that are expected to last at least 1000 years. Why Genesis? Because it is the most translated text in the world. The Rosetta Project is sending the disks to libraries and universities all over the world. It is very difficult to send information into the future. However, if in the year 2500 linguists are using the "Rosetta Disks" to unlock the meaning of a lost language, you may be sure they will use statistical methods of cryptanalysis (see Guided Exercise 2). To find out more about the Rosetta Project, visit the Brase/Brase statistics site at **http://www.cengage.com/statistics/brase** *and find the link to the Rosetta Project site.*

SECTION 5.1 PROBLEMS

1. *Statistical Literacy* Which of the following are continuous variables, and which are discrete?
(a) Number of traffic fatalities per year in the state of Florida
(b) Distance a golf ball travels after being hit with a driver
(c) Time required to drive from home to college on any given day
(d) Number of ships in Pearl Harbor on any given day
(e) Your weight before breakfast each morning

2. *Statistical Literacy* Which of the following are continuous variables, and which are discrete?
(a) Speed of an airplane
(b) Age of a college professor chosen at random
(c) Number of books in the college bookstore
(d) Weight of a football player chosen at random
(e) Number of lightning strikes in Rocky Mountain National Park on a given day

3. *Statistical Literacy* Consider each distribution. Determine if it is a valid probability distribution or not, and explain your answer.

(a)

x	0	1	2
$P(x)$	0.25	0.60	0.15

(b)

x	0	1	2
$P(x)$	0.25	0.60	0.20

4. *Statistical Literacy* Consider the probability distribution of a random variable x. Is the expected value of the distribution necessarily one of the possible values of x? Explain or give an example.

5. *Basic Computation: Expected Value and Standard Deviation* Consider the probability distribution shown in Problem 3(a). Compute the expected value and the standard deviation of the distribution.

6. *Basic Computation: Expected Value* For a fundraiser, 1000 raffle tickets are sold and the winner is chosen at random. There is only one prize, \$500 in cash. You buy one ticket.
(a) What is the probability you will win the prize of \$500?
(b) Your expected earnings can be found by multiplying the value of the prize by the probability you will win the prize. What are your expected earnings?
(c) *Interpretation* If a ticket costs \$2, what is the difference between your "costs" and "expected earnings"? How much are you effectively contributing to the fundraiser?

7. *Critical Thinking: Simulation* We can use the random-number table to simulate outcomes from a given discrete probability distribution. Jose plays basketball and has probability 0.7 of making a free-throw shot. Let x be the random variable that counts the number of successful shots out of 10 attempts. Consider the digits 0 through 9 of the random-number table. Since Jose has a 70% chance of making a shot, assign the digits 0 through 6 to "making a basket from the free throw line" and the digits 7 through 9 to "missing the shot."
(a) Do 70% of the possible digits 0 through 9 represent "making a basket"?
(b) Start at line 2, column 1 of the random-number table. Going across the row, determine the results of 10 "trials." How many free throw shots are successful in this simulation?
(c) Your friend decides to assign the digits 0 through 2 to "missing the shot" and the digits 3 through 9 to "making the basket." Is this assignment valid? Explain. Using this assignment, repeat part (b).

8. *Marketing: Age* What is the age distribution of promotion-sensitive shoppers? A *supermarket super shopper* is defined as a shopper for whom at least 70% of the items purchased were on sale or purchased with a coupon. The following table is based on information taken from *Trends in the United States* (Food Marketing Institute, Washington, D.C.).

Age range, years	18–28	29–39	40–50	51–61	62 and over
Midpoint x	23	34	45	56	67
Percent of super shoppers	7%	44%	24%	14%	11%

For the 62-and-over group, use the midpoint 67 years.

(a) Using the age midpoints x and the percentage of super shoppers, do we have a valid probability distribution? Explain.
(b) Use a histogram to graph the probability distribution of part (a).
(c) Compute the expected age μ of a super shopper.
(d) Compute the standard deviation σ for ages of super shoppers.

9. ***Marketing: Income*** What is the income distribution of super shoppers (see Problem 8). In the following table, income units are in thousands of dollars, and each interval goes up to but does not include the given high value. The midpoints are given to the nearest thousand dollars.

Income range	5–15	15–25	25–35	35–45	45–55	55 or more
Midpoint x	10	20	30	40	50	60
Percent of super shoppers	21%	14%	22%	15%	20%	8%

(a) Using the income midpoints x and the percent of super shoppers, do we have a valid probability distribution? Explain.
(b) Use a histogram to graph the probability distribution of part (a).
(c) Compute the expected income μ of a super shopper.
(d) Compute the standard deviation σ for the income of super shoppers.

10. ***History: Florence Nightingale*** What was the age distribution of nurses in Great Britain at the time of Florence Nightingale? Thanks to Florence Nightingale and the British census of 1851, we have the following information (based on data from the classic text *Notes on Nursing*, by Florence Nightingale). *Note:* In 1851 there were 25,466 nurses in Great Britain. Furthermore, Nightingale made a strict distinction between nurses and domestic servants.

Sean Locke/istockphoto.com

Age range (yr)	20–29	30–39	40–49	50–59	60–69	70–79	80+
Midpoint x	24.5	34.5	44.5	54.5	64.5	74.5	84.5
Percent of nurses	5.7%	9.7%	19.5%	29.2%	25.0%	9.1%	1.8%

(a) Using the age midpoints x and the percent of nurses, do we have a valid probability distribution? Explain.
(b) Use a histogram to graph the probability distribution of part (a).
(c) Find the probability that a British nurse selected at random in 1851 was 60 years of age or older.
(d) Compute the expected age μ of a British nurse contemporary to Florence Nightingale.
(e) Compute the standard deviation σ for ages of nurses shown in the distribution.

11. ***Fishing: Trout*** The following data are based on information taken from *Daily Creel Summary*, published by the Paiute Indian Nation, Pyramid Lake, Nevada. Movie stars and U.S. presidents have fished Pyramid Lake. It is one of the best places in the lower 48 states to catch trophy cutthroat trout. In this

table, x = number of fish caught in a 6-hour period. The percentage data are the percentages of fishermen who catch x fish in a 6-hour period while fishing from shore.

x	0	1	2	3	4 or more
%	44%	36%	15%	4%	1%

(a) Convert the percentages to probabilities and make a histogram of the probability distribution.
(b) Find the probability that a fisherman selected at random fishing from shore catches one or more fish in a 6-hour period.
(c) Find the probability that a fisherman selected at random fishing from shore catches two or more fish in a 6-hour period.
(d) Compute μ, the expected value of the number of fish caught per fisherman in a 6-hour period (round 4 or more to 4).
(e) Compute σ, the standard deviation of the number of fish caught per fisherman in a 6-hour period (round 4 or more to 4).

12. *Criminal Justice: Parole* *USA Today* reported that approximately 25% of all state prison inmates released on parole become repeat offenders while on parole. Suppose the parole board is examining five prisoners up for parole. Let x = number of prisoners out of five on parole who become repeat offenders. The methods of Section 5.2 can be used to compute the probability assignments for the x distribution.

x	0	1	2	3	4	5
$P(x)$	0.237	0.396	0.264	0.088	0.015	0.001

(a) Find the probability that one or more of the five parolees will be repeat offenders. How does this number relate to the probability that none of the parolees will be repeat offenders?
(b) Find the probability that two or more of the five parolees will be repeat offenders.
(c) Find the probability that four or more of the five parolees will be repeat offenders.
(d) Compute μ, the expected number of repeat offenders out of five.
(e) Compute σ, the standard deviation of the number of repeat offenders out of five.

13. *Fundraiser: Hiking Club* The college hiking club is having a fundraiser to buy new equipment for fall and winter outings. The club is selling Chinese fortune cookies at a price of $1 per cookie. Each cookie contains a piece of paper with a different number written on it. A random drawing will determine which number is the winner of a dinner for two at a local Chinese restaurant. The dinner is valued at $35. Since the fortune cookies were donated to the club, we can ignore the cost of the cookies. The club sold 719 cookies before the drawing.
(a) Lisa bought 15 cookies. What is the probability she will win the dinner for two? What is the probability she will not win?
(b) *Interpretation* Lisa's expected earnings can be found by multiplying the value of the dinner by the probability that she will win. What are Lisa's expected earnings? How much did she effectively contribute to the hiking club?

14. *Spring Break: Caribbean Cruise* The college student senate is sponsoring a spring break Caribbean cruise raffle. The proceeds are to be donated to the Samaritan Center for the Homeless. A local travel agency donated the cruise, valued at $2000. The students sold 2852 raffle tickets at $5 per ticket.
(a) Kevin bought six tickets. What is the probability that Kevin will win the spring break cruise to the Caribbean? What is the probability that Kevin will not win the cruise?

(b) ***Interpretation*** Expected earnings can be found by multiplying the value of the cruise by the probability that Kevin will win. What are Kevin's expected earnings? Is this more or less than the amount Kevin paid for the six tickets? How much did Kevin effectively contribute to the Samaritan Center for the Homeless?

15. ***Expected Value: Life Insurance*** Jim is a 60-year-old Anglo male in reasonably good health. He wants to take out a $50,000 term (that is, straight death benefit) life insurance policy until he is 65. The policy will expire on his 65th birthday. The probability of death in a given year is provided by the Vital Statistics Section of the *Statistical Abstract of the United States* (116th Edition).

x = age	60	61	62	63	64
P(death at this age)	0.01191	0.01292	0.01396	0.01503	0.01613

Jim is applying to Big Rock Insurance Company for his term insurance policy.

(a) What is the probability that Jim will die in his 60th year? Using this probability and the $50,000 death benefit, what is the expected cost to Big Rock Insurance?

(b) Repeat part (a) for years 61, 62, 63, and 64. What would be the total expected cost to Big Rock Insurance over the years 60 through 64?

(c) ***Interpretation*** If Big Rock Insurance wants to make a profit of $700 above the expected total cost paid out for Jim's death, how much should it charge for the policy?

(d) ***Interpretation*** If Big Rock Insurance Company charges $5000 for the policy, how much profit does the company expect to make?

16. ***Expected Value: Life Insurance*** Sara is a 60-year-old Anglo female in reasonably good health. She wants to take out a $50,000 term (that is, straight death benefit) life insurance policy until she is 65. The policy will expire on her 65th birthday. The probability of death in a given year is provided by the Vital Statistics Section of the *Statistical Abstract of the United States* (116th Edition).

x = age	60	61	62	63	64
P(death at this age)	0.00756	0.00825	0.00896	0.00965	0.01035

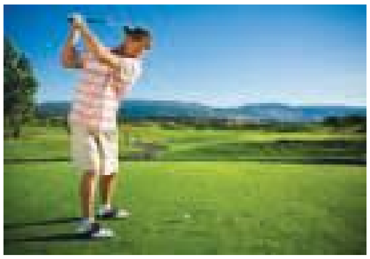
Sara is applying to Big Rock Insurance Company for her term insurance policy.

(a) What is the probability that Sara will die in her 60th year? Using this probability and the $50,000 death benefit, what is the expected cost to Big Rock Insurance?

(b) Repeat part (a) for years 61, 62, 63, and 64. What would be the total expected cost to Big Rock Insurance over the years 60 through 64?

(c) ***Interpretation*** If Big Rock Insurance wants to make a profit of $700 above the expected total cost paid out for Sara's death, how much should it charge for the policy?

(d) ***Interpretation*** If Big Rock Insurance Company charges $5000 for the policy, how much profit does the company expect to make?



Ranplett/iStockphoto.com

17. ***Combination of Random Variables: Golf*** Norb and Gary are entered in a local golf tournament. Both have played the local course many times. Their scores are random variables with the following means and standard deviations.

Norb, $x_1: \mu_1 = 115; \sigma_1 = 12$ Gary, $x_2: \mu_2 = 100; \sigma_2 = 8$

In the tournament, Norb and Gary are not playing together, and we will assume their scores vary independently of each other.

(a) The difference between their scores is $W = x_1 - x_2$. Compute the mean, variance, and standard deviation for the random variable W.

(b) The average of their scores is $W = 0.5x_1 + 0.5x_2$. Compute the mean, variance, and standard deviation for the random variable W.
(c) The tournament rules have a special handicap system for each player. For Norb, the handicap formula is $L = 0.8x_1 - 2$. Compute the mean, variance, and standard deviation for the random variable L.
(d) For Gary, the handicap formula is $L = 0.95x_2 - 5$. Compute the mean, variance, and standard deviation for the random variable L.

18. *Combination of Random Variables: Repair Service* A computer repair shop has two work centers. The first center examines the computer to see what is wrong, and the second center repairs the computer. Let x_1 and x_2 be random variables representing the lengths of time in minutes to examine a computer (x_1) and to repair a computer (x_2). Assume x_1 and x_2 are independent random variables. Long-term history has shown the following times:

Examine computer, $x_1: \mu_1 = 28.1$ minutes; $\sigma_1 = 8.2$ minutes
Repair computer, $x_2: \mu_2 = 90.5$ minutes; $\sigma_2 = 15.2$ minutes

(a) Let $W = x_1 + x_2$ be a random variable representing the total time to examine and repair the computer. Compute the mean, variance, and standard deviation of W.
(b) Suppose it costs \$1.50 per minute to examine the computer and \$2.75 per minute to repair the computer. Then $W = 1.50x_1 + 2.75x_2$ is a random variable representing the service charges (without parts). Compute the mean, variance, and standard deviation of W.
(c) The shop charges a flat rate of \$1.50 per minute to examine the computer, and if no repairs are ordered, there is also an additional \$50 service charge. Let $L = 1.5x_1 + 50$. Compute the mean, variance, and standard deviation of L.

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19. *Combination of Random Variables: Insurance Risk* Insurance companies know the *risk* of insurance is greatly reduced if the company insures not just one person, but many people. How does this work? Let x be a random variable representing the expectation of life in years for a 25-year-old male (i.e., number of years until death). Then the mean and standard deviation of x are $\mu = 50.2$ years and $\sigma = 11.5$ years (Vital Statistics Section of the *Statistical Abstract of the United States,* 116th Edition).

Suppose Big Rock Insurance Company has sold life insurance policies to Joel and David. Both are 25 years old, unrelated, live in different states, and have about the same health record. Let x_1 and x_2 be random variables representing Joel's and David's life expectancies. It is reasonable to assume x_1 and x_2 are independent.

Joel, $x_1: \mu_1 = 50.2; \sigma_1 = 11.5$
David, $x_2: \mu_2 = 50.2; \sigma_2 = 11.5$

If life expectancy can be predicted with more accuracy, Big Rock will have less risk in its insurance business. Risk in this case is measured by σ (larger σ means more risk).
(a) The average life expectancy for Joel and David is $W = 0.5x_1 + 0.5x_2$. Compute the mean, variance, and standard deviation of W.
(b) Compare the mean life expectancy for a single policy (x_1) with that for two policies (W).
(c) Compare the standard deviation of the life expectancy for a single policy (x_1) with that for two policies (W).
(d) The mean life expectancy is the same for a single policy (x_1) as it is for two policies (W), but the standard deviation is smaller for two policies. What happens to the mean life expectancy and the standard deviation when we include more policies issued to people whose life expectancies have the same mean

and standard deviation (i.e., 25-year-old males)? For instance, for three policies, $W = (\mu + \mu + \mu)/3 = \mu$ and $\sigma_W^2 = (1/3)^2\sigma^2 + (1/3)^2\sigma^2 + (1/3)^2\sigma^2 = (1/3)^2(3\sigma^2) = (1/3)\sigma^2$ and $\sigma_W = \frac{1}{\sqrt{3}}\sigma$. Likewise, for n such policies, $W = \mu$ and $\sigma_W^2 = (1/n)\sigma^2$ and $\sigma_W = \frac{1}{\sqrt{n}}\sigma$. Looking at the general result, is it appropriate to say that when we increase the number of policies to n, the risk decreases by a factor of $\sigma_W = \frac{1}{\sqrt{n}}$?

SECTION 5.2

Binomial Probabilities

FOCUS POINTS

- List the defining features of a binomial experiment.
- Compute binomial probabilities using the formula $P(r) = C_{n,r}p^rq^{n-r}$.
- Use the binomial table to find $P(r)$.
- Use the binomial probability distribution to solve real-world applications.

Binomial Experiment

On a TV game show, each contestant has a try at the wheel of fortune. The wheel of fortune is a roulette wheel with 36 slots, one of which is gold. If the ball lands in the gold slot, the contestant wins $50,000. No other slot pays. What is the probability that the game show will have to pay the fortune to three contestants out of 100?

In this problem, the contestant and the game show sponsors are concerned about only two outcomes from the wheel of fortune: The ball lands on the gold, or the ball does not land on the gold. This problem is typical of an entire class of problems that are characterized by the feature that there are exactly two possible outcomes (for each trial) of interest. These problems are called *binomial experiments,* or *Bernoulli experiments,* after the Swiss mathematician Jacob Bernoulli, who studied them extensively in the late 1600s.

Binomial experiments

Features of a binomial experiment

Number of trials, n

Independent trials

Success, S
Failure, F

$P(S) = p$
$P(F) = q = 1 - p$

Number of successes, r

Features of a binomial experiment

1. There is a *fixed number of trials.* We denote this number by the letter n.
2. The n trials are *independent* and repeated under identical conditions.
3. Each trial has only *two outcomes:* success, denoted by S, and failure, denoted by F.
4. For each individual trial, the *probability of success is the same.* We denote the probability of success by p and that of failure by q. Since each trial results in either success or failure, $p + q = 1$ and $q = 1 - p$.
5. The central problem of a binomial experiment is to find the *probability of r successes out of n trials.*

EXAMPLE 4 BINOMIAL EXPERIMENT

Let's see how the wheel of fortune problem meets the criteria of a binomial experiment. We'll take the criteria one at a time.

SOLUTION:

1. Each of the 100 contestants has a trial at the wheel, so there are $n = 100$ trials in this problem.

2. Assuming that the wheel is fair, the *trials are independent,* since the result of one spin of the wheel has no effect on the results of other spins.
3. We are interested in only two outcomes on each spin of the wheel: The ball either lands on the gold, or it does not. Let's call landing on the gold *success* (*S*) and not landing on the gold *failure* (*F*). In general, the assignment of the terms *success* and *failure* to outcomes does not imply good or bad results. These terms are assigned simply for the user's convenience.
4. On each trial the probability p of success (landing on the gold) is 1/36, since there are 36 slots and only one of them is gold. Consequently, the probability of failure is

$$q = 1 - p = 1 - \frac{1}{36} = \frac{35}{36}$$

on each trial.
5. We want to know the probability of 3 successes out of 100 trials, so $r = 3$ in this example. It turns out that the probability the quiz show will have to pay the fortune to 3 contestants out of 100 is about 0.23. Later in this section we'll see how this probability was computed.

Anytime we make selections from a population *without replacement, we do not have independent trials*. However, replacement is often not practical. If the number of trials is quite small with respect to the population, we *almost* have independent trials, and we can say the situation is *closely approximated* by a binomial experiment. For instance, suppose we select 20 tuition bills at random from a collection of 10,000 bills issued at one college and observe if each bill is in error or not. If 600 of the 10,000 bills are in error, then the probability that the first one selected is in error is 600/10,000, or 0.0600. If the first is in error, then the probability that the second is in error is 599/9999, or 0.0599. Even if the first 19 bills selected are in error, the probability that the 20th is also in error is 581/9981, or 0.0582. All these probabilities round to 0.06, and we can say that the independence condition is approximately satisfied.

GUIDED EXERCISE 4 Binomial experiment

Let's analyze the following binomial experiment to determine p, q, n, and r:

According to the *Textbook of Medical Physiology,* 5th Edition, by Arthur Guyton, 9% of the population has blood type B. Suppose we choose 18 people at random from the population and test the blood type of each. What is the probability that three of these people have blood type B? *Note:* Independence is approximated because 18 people is an extremely small sample with respect to the entire population.

(a) In this experiment, we are observing whether or not a person has type B blood. We will say we have a success if the person has type B blood. What is failure?

→ Failure occurs if a person does not have type B blood.

(b) The probability of success is 0.09, since 9% of the population has type B blood. What is the probability of failure, q?

→ The probability of failure is

$$q = 1 - p$$
$$= 1 - 0.09 = 0.91$$

(c) In this experiment, there are n = _______ trials.

→ In this experiment, $n = 18$.

Continued

GUIDED EXERCISE 4 *continued*

(d) We wish to compute the probability of 3 successes out of 18 trials. In this case, r = ________.	In this case, $r = 3$.

Next, we will see how to compute the probability of r successes out of n trials when we have a binomial experiment.

Computing Probabilities for a Binomial Experiment Using the Binomial Distribution Formula

The central problem of a binomial experiment is finding the probability of r successes out of n trials. Now we'll see how to find these probabilities.

A model with three trials

Suppose you are taking a timed final exam. You have three multiple-choice questions left to do. Each question has four suggested answers, and only one of the answers is correct. You have only 5 seconds left to do these three questions, so you decide to mark answers on the answer sheet without even reading the questions. Assuming that your answers are randomly selected, what is the probability that you get zero, one, two, or all three questions correct?

This is a binomial experiment. Each question can be thought of as a trial, so there are $n = 3$ trials. The possible outcomes on each trial are success S, indicating a correct response, or failure F, meaning a wrong answer. The trials are independent—the outcome of any one trial does not affect the outcome of the others.

Probability of success $P(S) = p$

What is the *probability of success* on anyone question? Since you are guessing and there are four answers from which to select, the probability of a correct answer is 0.25. The probability q of a wrong answer is then 0.75. In short, we have a binomial experiment with $n = 3$, $p = 0.25$, and $q = 0.75$.

Now, what are the possible outcomes in terms of success or failure for these three trials? Let's use the notation SSF to mean success on the first question, success on the second, and failure on the third. There are eight possible combinations of S's and F's. They are

SSS *SSF* *SFS* *FSS* *SFF* *FSF* *FFS* *FFF*

To compute the probability of each outcome, we can use the multiplication law because the trials are independent. For instance, the probability of success on the first two questions and failure on the last is

$$P(SSF) = P(S) \cdot P(S) \cdot P(F) = p \cdot p \cdot q = p^2 q = (0.25)^2(0.75) \approx 0.047$$

In a similar fashion, we can compute the probability of each of the eight outcomes. These are shown in Table 5-8, along with the number of successes r associated with each outcome.

TABLE 5-8 Outcomes for a Binomial Experiment with $n = 3$ Trials

Outcome	Probability of Outcome	r (number of successes)
SSS	$P(SSS) = P(S)P(S)P(S) = p^3 = (0.25)^3 \approx 0.016$	3
SSF	$P(SSF) = P(S)P(S)P(F) = p^2q = (0.25)^2(0.75) \approx 0.047$	2
SFS	$P(SFS) = P(S)P(F)P(S) = p^2q = (0.25)^2(0.75) \approx 0.047$	2
FSS	$P(FSS) = P(F)P(S)P(S) = p^2q = (0.25)^2(0.75) \approx 0.047$	2
SFF	$P(SFF) = P(S)P(F)P(F) = pq^2 = (0.25)(0.75)^2 \approx 0.141$	1
FSF	$P(FSF) = P(F)P(S)P(F) = pq^2 = (0.25)(0.75)^2 \approx 0.141$	1
FFS	$P(FFS) = P(F)P(F)P(S) = pq^2 = (0.25)(0.75)^2 \approx 0.141$	1
FFF	$P(FFF) = P(F)P(F)P(F) = q^3 = (0.75)^3 \approx 0.422$	0

Now we can compute the probability of r successes out of three trials for $r = 0$, 1, 2, or 3. Let's compute $P(1)$. The notation $P(1)$ stands for the probability of one success. For three trials, there are three different outcomes that show exactly one success. They are the outcomes *SFF, FSF,* and *FFS*. Since the outcomes are mutually exclusive, we can add the probabilities. So,

$$\begin{aligned} P(1) = P(SFF \text{ or } FSF \text{ or } FFS) &= P(SFF) + P(FSF) + P(FFS) \\ &= pq^2 + pq^2 + pq^2 \\ &= 3pq^2 \\ &= 3(0.25)(0.75)^2 \\ &= 0.422 \end{aligned}$$

In the same way, we can find $P(0)$, $P(2)$, and $P(3)$. These values are shown in Table 5-9.

TABLE 5-9 ***P*(*r*) for *n* = 3 Trials, *p* = 0.25**

r (number of successes)	$P(r)$ (probability of r successes in 3 trials)	$P(r)$ for $p = 0.25$
0	$P(0) = P(FFF) = q^3$	0.422
1	$P(1) = P(SFF) + P(FSF) + P(FFS) = 3pq^2$	0.422
2	$P(2) = P(SSF) + P(SFS) + P(FSS) = 3p^2q$	0.141
3	$P(3) = P(SSS) = p^3$	0.016

We have done quite a bit of work to determine your chances of $r = 0$, 1, 2, or 3 successes on three multiple-choice questions if you are just guessing. Now we see that there is only a small chance (about 0.016) that you will get them all correct.

Table 5-9 can be used as a model for computing the probability of r successes out of only *three* trials. How can we compute the probability of 7 successes out of 10 trials? We can develop a table for $n = 10$, but this would be a tremendous task because there are 1024 possible combinations of successes and failures on 10 trials. Fortunately, mathematicians have given us a direct formula to compute the probability of r successes for any number of trials.

General formula for binomial probability distribution

Formula for the binomial probability distribution

$$P(r) = \frac{n!}{r!(n-r)!} p^r q^{n-r} = C_{n,r} p^r q^{n-r}$$

where n = number of trials

p = probability of success on each trial

$q = 1 - p$ = probability of failure on each trial

r = random variable representing the number of successes out of n trials ($0 \le r \le n$)

! = factorial notation. Recall from Section 4.3 that the factorial symbol $n!$ designates the product of all the integers between 1 and n. For instance, $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$. Special cases are $1! = 1$ and $0! = 1$.

Table for $C_{n,r}$

$C_{n,r} = \dfrac{n!}{r!(n-r)!}$ is the binomial coefficient. Table 2 of Appendix II gives values of $C_{n,r}$ for select n and r. Many calculators have a key designated nCr that gives the value of $C_{n,r}$ directly.

Binomial coefficient $C_{n,r}$

Note: The binomial coefficient $C_{n,r}$ represents the number of combinations of n distinct objects (n = number of trials in this case) taken r at a time (r = number of successes). For more information about $C_{n,r}$, see Section 4.3.

Let's look more carefully at the formula for $P(r)$. There are two main parts. The expression $p^r q^{n-r}$ is the probability of getting one outcome with r successes and $n - r$ failures. The binomial coefficient $C_{n,r}$ counts the number of outcomes that have r successes and $n - r$ failures. For instance, in the case of $n = 3$ trials, we saw in Table 5-8 that the probability of getting an outcome with one success and two failures was pq^2. This is the value of $p^r q^{n-r}$ when $r = 1$ and $n = 3$. We also observed that there were three outcomes with one success and two failures, so $C_{3,1}$ is 3.

Now let's take a look at an application of the binomial distribution formula in Example 5.

EXAMPLE 5 COMPUTE $P(r)$ USING THE BINOMIAL DISTRIBUTION FORMULA

Privacy is a concern for many users of the Internet. One survey showed that 59% of Internet users are somewhat concerned about the confidentiality of their e-mail. Based on this information, what is the probability that for a random sample of 10 Internet users, 6 are concerned about the privacy of their e-mail?

SOLUTION:

(a) This is a binomial experiment with 10 trials. If we assign success to an Internet user being concerned about the privacy of e-mail, the probability of success is 59%. We are interested in the probability of 6 successes. We have

$$n = 10 \quad p = 0.59 \quad q = 0.41 \quad r = 6$$

By the formula,

$$P(6) = C_{10,6}(0.59)^6(0.41)^{10-6}$$

$$= 210(0.59)^6(0.41)^4 \qquad \text{Use Table 2 of Appendix II or a calculator.}$$

$$\approx 210(0.0422)(0.0283) \qquad \text{Use a calculator.}$$

$$\approx 0.25$$

There is a 25% chance that *exactly* 6 of the 10 Internet users are concerned about the privacy of e-mail.

(b) Many calculators have a built-in combinations function. On the TI-84Plus/TI-83Plus/TI-*n*spire (with TI-84Plus keypad) calculators, press the **MATH** key and select **PRB**. The combinations function is designated nCr. Figure 5-2 displays the process for computing $P(6)$ directly on these calculators.

FIGURE 5-2

TI-84Plus/TI-83Plus/TI-*n*spire (with TI-84Plus keypad) Display

```
10 nCr 6*.59^6*.
41^(10-6)
        .2503034245
```

Using a Binomial Distribution Table

In many cases we will be interested in the probability of a range of successes. In such cases, we need to use the addition rule for mutually exclusive events. For instance, for $n = 6$ and $p = 0.50$,

$$P(4 \text{ or fewer successes}) = P(r \leq 4)$$

$$= P(r = 4 \text{ or } 3 \text{ or } 2 \text{ or } 1 \text{ or } 0)$$

$$= P(4) + P(3) + P(2) + P(1) + P(0)$$

It would be a bit of a chore to use the binomial distribution formula to compute all the required probabilities. Table 3 of Appendix II gives values of $P(r)$ for selected p values and values of n through 20. To use the table, find the appropriate section for n, and then use the entries in the columns headed by the p values and the rows headed by the r values.

Table 5-10 is an excerpt from Table 3 of Appendix II showing the section for $n = 6$. Notice that all possible r values between 0 and 6 are given as row headers. The value $p = 0.50$ is one of the column headers. For $n = 6$ and $p = 0.50$, you can find the value of $P(4)$ by looking at the entry in the row headed by 4 and the column headed by 0.50. Notice that $P(4) = 0.234$.

TABLE 5-10 **Excerpt from Table 3 of Appendix II for $n = 6$**

								p						
n	r	.01	.05	.10	. . .	.30	. . .	.50	. . .	.70	. . .	.85	.90	.95
:														
6	0	.941	.735	.531	. . .	.118	. . .	.016	. . .	.001	. . .	.000	.000	.000
	1	.057	.232	.354	. . .	.303	. . .	.094	. . .	.010	. . .	.000	.000	.000
	2	.001	.031	.098	. . .	.324	. . .	.234	. . .	.060	. . .	.006	.001	.000
	3	.000	.002	.015	. . .	.185	. . .	.312	. . .	.185	. . .	.042	.015	.002
	4	.000	.000	.001	. . .	.060	. . .	.234	. . .	.324		.176	.098	.031
	5	.000	.000	.000	. . .	.010	. . .	.094	. . .	.303	. . .	.399	.354	.232
	6	.000	.000	.000	. . .	.001	. . .	.016	. . .	.118	. . .	.377	.531	.735

Likewise, you can find other values of $P(r)$ from the table. In fact, for $n = 6$ and $p = 0.50$,

$$P(r \leq 4) = P(4) + P(3) + P(2) + P(1) + P(0)$$
$$= 0.234 + 0.312 + 0.234 + 0.094 + 0.016 = 0.890$$

Alternatively, to compute $P(r \leq 4)$ for $n = 6$, you can use the fact that the total of all $P(r)$ values for r between 0 and 6 is 1 and the complement rule. Since the complement of the event $r \leq 4$ is the event $r \geq 5$, we have

$$P(r \leq 4) = 1 - P(5) - P(6)$$
$$= 1 - 0.094 - 0.016 = 0.890$$

Note: In Table 3 of Appendix II, probability entries of 0.000 do not mean the probability is exactly zero. Rather, to three digits after the decimal, the probability rounds to 0.000.

CRITICAL THINKING

In Chapter 4, we saw the complement rule of probability. As we saw in the previous discussion, this rule provides a useful strategy to simplify binomial probability computations for a range of successes. For example, in a binomial experiment with $n = 7$ trials, the sample space for the number of successes r is

0 1 2 3 4 5 6 7

Notice that $r = 0$ successes is the complement of $r \geq 1$ successes. By the complement rule,

$$P(r \geq 1) = 1 - P(r = 0)$$

It is faster to compute or look up $P(r = 0)$ and subtract than it is to compute or look up all the probabilities $P(r = 1)$ through $P(r = 7)$.

Likewise, the outcome $r \le 2$ is the complement of the outcome $r \ge 3$, so,

$$P(r \ge 3) = 1 - P(r \le 2) = 1 - P(r = 2) - P(r = 1) - P(r = 0)$$

Before you use the complement rule for computing probabilities, be sure the outcomes used comprise complementary events. Complements can be expressed in several ways. For instance, the complement of the event $r \le 2$ can be expressed as event $r > 2$ or event $r \ge 3$.

Because of rounding in the binomial probability table, probabilities computed by using the addition rule directly might differ slightly from corresponding probabilities computed by using the complement rule.

EXAMPLE 6 Using the binomial distribution table to find $P(r)$

A biologist is studying a new hybrid tomato. It is known that the seeds of this hybrid tomato have probability 0.70 of germinating. The biologist plants six seeds.

(a) What is the probability that *exactly* four seeds will germinate?

SOLUTION: This is a binomial experiment with $n = 6$ trials. Each seed planted represents an independent trial. We'll say germination is success, so the probability for success on each trial is 0.70.

$$n = 6 \quad p = 0.70 \quad q = 0.30 \quad r = 4$$

We wish to find P(4), the probability of exactly four successes.

In Table 3, Appendix II, find the section with $n = 6$ (excerpt is given in Table 5-10). Then find the entry in the column headed by $p = 0.70$ and the row headed by $r = 4$. This entry is 0.324.

$$P(4) = 0.324$$

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(b) What is the probability that *at least* four seeds will germinate?

SOLUTION: In this case, we are interested in the probability of four or more seeds germinating. This means we are to compute $P(r \ge 4)$. Since the events are mutually exclusive, we can use the addition rule

$$P(r \ge 4) = P(r = 4 \quad or \quad r = 5 \quad or \quad r = 6) = P(4) + P(5) + P(6)$$

We already know the value of $P(4)$. We need to find $P(5)$ and $P(6)$.

Use the same part of the table but find the entries in the row headed by the r value 5 and then the r value 6. Be sure to use the column headed by the value of p, 0.70.

$$P(5) = 0.303 \quad \text{and} \quad P(6) = 0.118$$

Now we have all the parts necessary to compute $P(r \ge 4)$.

$$\begin{aligned} P(r \ge 4) &= P(4) + P(5) + P(6) \\ &= 0.324 + 0.303 + 0.118 \\ &= 0.745 \end{aligned}$$

In Guided Exercise 5 you'll practice using the formula for $P(r)$ in one part, and then you'll use Table 3, Appendix II, for $P(r)$ values in the second part.

GUIDED EXERCISE 5 Find $P(r)$

A rarely performed and somewhat risky eye operation is known to be successful in restoring the eyesight of 30% of the patients who undergo the operation. A team of surgeons has developed a new technique for this operation that has been successful in four of six operations. Does it seem likely that the new technique is much better than the old? We'll use the binomial probability distribution to answer this question. We'll compute the probability of at least four successes in six trials for the old technique.

(a) Each operation is a binomial trial. In this case, $n =$ ____, $p =$ ____, $q =$ ____, $r =$ ____.

→ $n = 6, p = 0.30, q = 1 - 0.30 = 0.70, r = 4$

(b) Use your values of n, p, and q, as well as Table 2 of Appendix II (or your calculator), to compute $P(4)$ from the formula:

$$P(r) = C_{n,r}p^r q^{n-r}$$

→
$$P(4) = C_{6,4}(0.30)^4(0.70)^2$$
$$= 15(0.0081)(0.490)$$
$$\approx 0.060$$

(c) Compute the probability of *at least* four successes out of the six trials.

$$P(r \geq 4) = P(r = 4 \text{ or } r = 5 \text{ or } r = 6)$$
$$= P(4) + P(5) + P(6)$$

Use Table 3 of Appendix II to find values of $P(4)$, $P(5)$, and $P(6)$. Then use these values to compute $P(r \geq 4)$.

→ To find $P(4)$, $P(5)$, and $P(6)$ in Table 3, we look in the section labeled $n = 6$. Then we find the column headed by $p = 0.30$. To find $P(4)$, we use the row labeled $r = 4$. For the values of $P(5)$ and $P(6)$, we look in the same column but in the rows headed by $r = 5$ and $r = 6$, respectively.

$$P(r \geq 4) = P(4) + P(5) + P(6)$$
$$= 0.060 + 0.010 + 0.001 = 0.071$$

(d) *Interpetation* Under the older operation technique, the probability that at least four patients out of six regain their eyesight is ______. Does it seem that the new technique is better than the old? Would you encourage the surgeon team to do more work on the new technique?

→ It seems the new technique is better than the old since, by pure chance, the probability of four or more successes out of six trials is only 0.071 for the old technique. This means one of the following two things may be happening:

(i) The new method is no better than the old method, and our surgeons have encountered a rare event (probability 0.071), or

(ii) The new method is in fact better. We think it is worth encouraging the surgeons to do more work on the new technique.

Using Technology to Compute Binomial Probabilities

Some calculators and computer software packages support the binomial distribution. In general, these technologies will provide both the probability $P(r)$ for an exact number of successes r and the cumulative probability $P(r \leq k)$, where k is a specified value less than or equal to the number of trials n. Note that most of the technologies use the letter x instead of r for the random variable denoting the number of successes out of n trials.

TECH NOTES

The software packages Minitab and Excel 2007, as well as the TI-84Plus/TI-83Plus/TI-*n*spire calculators, include built-in binomial probability distribution options. These options give the probability $P(r)$ of a specific number of successes r, as well as the cumulative total probability for r or fewer successes.

TI-84Plus/TI-83Plus/TI-*n*spire (with TI-84Plus keypad) Press the **DISTR** key and scroll to **binompdf** (n, p, r). Enter the number of trials n, the probability of success on a single trial p, and the number of successes r. This gives $P(r)$. For the cumulative probability that there are r or fewer successes, use **binomcdf** (n, p, r).

```
binompdf(6,.3,4)
                 .059535
binomcdf(6,.3,4)
                 .989065
```

$P(r) = 4$ .059535

$P(r \leq 4)$ .989065

Excel 2007 Click the **Insert Function** (f_x). In the dialogue box, select **Statistical** for the category, and then select **Binomdist.** In the next dialogue box, fill in the values *r, n,* and *p*. For *P*(*r*), use false; for *P*(at least *r* successes), use true.

Minitab First, enter the *r* values 0, 1, 2, . . . , *n* in a column. Then use menu choice **Calc ➤ Probability Distribution ➤ Binomial**. In the dialogue box, select Probability for *P*(*r*) or Cumulative for *P*(at least *r* successes). Enter the number of trials *n*, the probability of success *p*, and the column containing the *r* values. A sample printout is shown in Problem 25 at the end of this section.

LOOKING FORWARD

There are several ways to find the probability of *r* successes out of *n* binomial trials. In particular we have used the general formula for the binomial probability distribution, a table of binomial probabilities such as Table 3 of Appendix II, or results from calculator or computer software that is based on the formula. However, depending on the number of trials and the size of the probability of success, the calculations can become tedious, or rounding errors can become an issue (*see* Linking Concepts, Problem 2). In Section 6.6 we will see how to use the *normal* probability distribution to approximate the binomial distribution in the case of sufficiently large numbers of trials *n*. Section 5.4 shows how to use a *Poisson* probability distribution to approximate the binomial distribution when the probability of success on a single trial *p* is very small and the number of trials is 100 or more.

Common expressions and corresponding inequalities

Many times we are asked to compute the probability of a range of successes. For instance, in a binomial experiment with *n* trials, we may be asked to compute the probability of four or more successes. Table 5-11 shows how common English expressions such as "four or more successes" translate to inequalities involving *r*.

Problems 29 and 30 show how to compute conditional probabilities for binomial experiments. In these problems we see how to compute the probability of *r* successes out of *n* binomial trials, given that a certain number of successes will occur.

TABLE 5-11 Common English Expressions and Corresponding Inequalities (consider a binomial experiment with *n* trials and *r* successes)

Expression	Inequality
Four or more successes	$r \geq 4$
At least four successes	That is, $r = 4, 5, 6, \ldots, n$
No fewer than four successes	
Not less than four successes	
Four or fewer successes	$r \leq 4$
At most four successes	That is, $r = 0, 1, 2, 3,$ or 4
No more than four successes	
The number of successes does not exceed four	
More than four successes	$r > 4$
The number of successes exceeds four	That is, $r = 5, 6, 7, \ldots, n$
Fewer than four successes	$r < 4$
The number of successes is not as large as four	That is, $r = 0, 1, 2, 3$

Interpretation Often we are not interested in the probability of a *specific number* of successes out of n binomial trials. Rather, we are interested in a minimum number of successes, a maximum number of successes, or a range of a number of successes. In other words, we are interested in the probability of *at least* a certain number of successes or *no more than* a certain number of successes, or the probability that the number of successes is between two given values.

For instance, suppose engineers have determined that at least 3 of 5 rivets on a bridge connector need to hold. If the probability that a single rivet holds is 0.80, and the performances of the rivets are independent, then the engineers are interested in the probability that *at least* 3 of the rivets hold. Notice that $P(r \geq 3 \text{ rivets hold}) = 0.943$, while $P(r = 3 \text{ rivets hold}) = 0.205$. The probability of *at least* 3 successes is much higher than the probability of *exactly* 3 successes. Safety concerns require that 3 of the rivets hold. However, there is a greater margin of safety if more than 3 rivets hold.

As you consider binomial experiments, determine whether you are interested in a *specific number* of successes or a *range* of successes.

Sampling Without Replacement: Use of the Hypergeometric Probability Distribution

If the population is relatively small and we draw samples without replacement, the assumption of independent trials is not valid and we should not use the binomial distribution.

Hypergeometric probability distribution

The *hypergeometric distribution* is a probability distribution of a random variable that has two outcomes when sampling is done without replacement. This is the distribution that is appropriate when the sample size is so small that sampling without replacement results in trials that are not even approximately independent. A discussion of the hypergeometric distribution can be found in Appendix I.

VIEWPOINT Lies! Lies!! Lies!!! The Psychology of Deceit

This is the title of an intriguing book by C. V. Ford, professor of psychiatry. The book recounts the true story of Floyd "Buzz" Fay, who was falsely convicted of murder on the basis of a failed polygraph examination. During his $2\frac{1}{2}$ years of wrongful imprisonment, Buzz became a polygraph expert. He taught inmates, who freely confessed guilt, how to pass a polygraph examination. (For more information on this topic, see Problem 21.)

SECTION 5.2 PROBLEMS

1. *Statistical Literacy* What does the random variable for a binomial experiment of n trials measure?
2. *Statistical Literacy* What does it mean to say that the trials of an experiment are independent?
3. *Statistical Literacy* For a binomial experiment, how many outcomes are possible for each trial? What are the possible outcomes?
4. *Statistical Literacy* In a binomial experiment, is it possible for the probability of success to change from one trial to the next? Explain.
5. *Interpretation* Suppose you are a hospital manager and have been told that there is no need to worry that respirator monitoring equipment might fail because the probability any one monitor will fail is only 0.01. The hospital has 20 such moni-

tors and they work independently. Should you be more concerned about the probability that *exactly one* of the 20 monitors fails, or that *at least one* fails? Explain.

6. ***Interpretation*** From long experience a landlord knows that the probability an apartment in a complex will not be rented is 0.10. There are 20 apartments in the complex, and the rental status of each apartment is independent of the status of the others. When a minimum of 16 apartment units are rented, the landlord can meet all monthly expenses. Which probability is more relevant to the landlord in terms of being able to meet expenses: the probability that there are *exactly four* unrented units or the probability that there are *four or fewer* unrented units? Explain.

7. ***Critical Thinking*** In an experiment, there are n independent trials. For each trial, there are three outcomes, A, B, and C. For each trial, the probability of outcome A is 0.40; the probability of outcome B is 0.50; and the probability of outcome C is 0.10. Suppose there are 10 trials.
   (a) Can we use the binomial experiment model to determine the probability of four outcomes of type A, five of type B, and one of type C? Explain.
   (b) Can we use the binomial experiment model to determine the probability of four outcomes of type A and six outcomes that are not of type A? Explain. What is the probability of success on each trial?

8. ***Critical Thinking*** In a carnival game, there are six identical boxes, one of which contains a prize. A contestant wins the prize by selecting the box containing it. Before each game, the old prize is removed and another prize is placed at random in one of the six boxes. Is it appropriate to use the binomial probability distribution to find the probability that a contestant who plays the game five times wins exactly twice? Check each of the requirements of a binomial experiment and give the values of n, r, and p.

9. ***Critical Thinking*** According to the college registrar's office, 40% of students enrolled in an introductory statistics class this semester are freshmen, 25% are sophomores, 15% are juniors, and 20% are seniors. You want to determine the probability that in a random sample of five students enrolled in introductory statistics this semester, exactly two are freshmen.
   (a) Describe a trial. Can we model a trial as having only two outcomes? If so, what is success? What is failure? What is the probability of success?
   (b) We are sampling without replacement. If only 30 students are enrolled in introductory statistics this semester, is it appropriate to model 5 trials as independent, with the same probability of success on each trial? Explain. What other probability distribution would be more appropriate in this setting?

10. ***Critical Thinking: Simulation*** Central Eye Clinic advertises that 90% of its patients approved for LASIK surgery to correct vision problems have successful surgeries.
    (a) In the random-number table, assign the digits 0 through 8 to the event "successful surgery" and the digit 9 to the event "unsuccessful surgery." Does this assignment of digits simulate 90% successful outcomes?
    (b) Use the random-digit assignment model of part (a) to simulate the outcomes of 15 trials. Begin at column 1, line 2.
    (c) Your friend assigned the digits 1 through 9 to the event "successful surgery" and the digit 0 to the event "unsuccessful surgery." Does this assignment of digits simulate 90% successful outcomes? Using this digit assignment, repeat part (b).

In each of the following problems, the binomial distribution will be used. Answers may vary slightly depending on whether the binomial distribution formula, the binomial distribution table, or distribution results from a calculator or computer are used. Please answer the following questions and then complete the problem.

What makes up a trial? What is a success? What is a failure?

What are the values of n, p, and q?

11. *Basic Computation: Binomial Distribution* Consider a binomial experiment with $n = 7$ trials where the probability of success on a single trial is $p = 0.30$.
(a) Find $P(r = 0)$.
(b) Find $P(r \geq 1)$ by using the complement rule.

12. *Basic Computation: Binomial Distribution* Consider a binomial experiment with $n = 7$ trials where the probability of success on a single trial is $p = 0.60$.
(a) Find $P(r = 7)$.
(b) Find $P(r \leq 6)$ by using the complement rule.

13. *Basic Computation: Binomial Distribution* Consider a binomial experiment with $n = 6$ trials where the probability of success on a single trial is $p = 0.85$.
(a) Find $P(r \leq 1)$.
(b) *Interpretation* If you conducted the experiment and got fewer than 2 successes, would you be surprised? Why?

14. *Basic Computation: Binomial Distribution* Consider a binomial experiment with $n = 6$ trials where the probability of success on a single trial is $p = 0.20$.
(a) Find $P(0 < r \leq 2)$.
(b) *Interpretation* If you conducted the experiment and got 1 or 2 successes, would you be surprised? Why?

15. *Binomial Probabilities: Coin Flip* A fair quarter is flipped three times. For each of the following probabilities, use the formula for the binomial distribution and a calculator to compute the requested probability. Next, look up the probability in Table 3 of Appendix II and compare the table result with the computed result.
(a) Find the probability of getting exactly three heads.
(b) Find the probability of getting exactly two heads.
(c) Find the probability of getting two or more heads.
(d) Find the probability of getting exactly three tails.

16. *Binomial Probabilities: Multiple-Choice Quiz* Richard has just been given a 10-question multiple-choice quiz in his history class. Each question has five answers, of which only one is correct. Since Richard has not attended class recently, he doesn't know any of the answers. Assuming that Richard guesses on all 10 questions, find the indicated probabilities.
(a) What is the probability that he will answer all questions correctly?
(b) What is the probability that he will answer all questions incorrectly?
(c) What is the probability that he will answer at least one of the questions correctly? Compute this probability two ways. First, use the rule for mutually exclusive events and the probabilities shown in Table 3 of Appendix II. Then use the fact that $P(r \geq 1) = 1 - P(r = 0)$. Compare the two results. Should they be equal? Are they equal? If not, how do you account for the difference?
(d) What is the probability that Richard will answer at least half the questions correctly?

17. *Ecology: Wolves* The following is based on information taken from *The Wolf in the Southwest: The Making of an Endangered Species*, edited by David Brown (University of Arizona Press). Before 1918, approximately 55% of the wolves in the New Mexico and Arizona region were male, and 45% were female. However, cattle ranchers in this area have made a determined effort to exterminate wolves. From 1918 to the present, approximately 70% of wolves in the region are male, and 30% are female. Biologists suspect that male wolves are more likely than females to return to an area where the population has been greatly reduced.
(a) Before 1918, in a random sample of 12 wolves spotted in the region, what was the probability that 6 or more were male? What was the probability that 6 or more were female? What was the probability that fewer than 4 were female?
(b) Answer part (a) for the period from 1918 to the present.

18. ***Sociology: Ethics*** The one-time fling! Have you ever purchased an article of clothing (dress, sports jacket, etc.), worn the item *once* to a party, and then returned the purchase? This is called a *one-time fling*. About 10% of all adults deliberately do a one-time fling and feel no guilt about it (Source: *Are You Normal?,* by Bernice Kanner, St. Martin's Press). In a group of seven adult friends, what is the probability that
(a) no one has done a one-time fling?
(b) at least one person has done a one-time fling?
(c) no more than two people have done a one-time fling?

19. ***Sociology: Mother-in-Law*** Sociologists say that 90% of married women claim that their husband's mother is the biggest bone of contention in their marriages (sex and money are lower-rated areas of contention). (See the source in Problem 18.) Suppose that six married women are having coffee together one morning. What is the probability that
(a) all of them dislike their mother-in-law?
(b) none of them dislike their mother-in-law?
(c) at least four of them dislike their mother-in-law?
(d) no more than three of them dislike their mother-in-law?

20. ***Sociology: Dress Habits*** A research team at Cornell University conducted a study showing that approximately 10% of all businessmen who wear ties wear them so tightly that they actually reduce blood flow to the brain, diminishing cerebral functions (Source: *Chances: Risk and Odds in Everyday Life,* by James Burke). At a board meeting of 20 businessmen, all of whom wear ties, what is the probability that
(a) at least one tie is too tight?
(b) more than two ties are too tight?
(c) no tie is too tight?
(d) at least 18 ties are *not* too tight?

21. ***Psychology: Deceit*** Aldrich Ames is a convicted traitor who leaked American secrets to a foreign power. Yet Ames took routine lie detector tests and each time passed them. How can this be done? Recognizing control questions, employing unusual breathing patterns, biting one's tongue at the right time, pressing one's toes hard to the floor, and counting backward by 7 are countermeasures that are difficult to detect but can change the results of a polygraph examination (Source: *Lies! Lies!! Lies!!! The Psychology of Deceit,* by C. V. Ford, professor of psychiatry, University of Alabama). In fact, it is reported in Professor Ford's book that after only 20 minutes of instruction by "Buzz" Fay (a prison inmate), 85% of those trained were able to pass the polygraph examination even when guilty of a crime. Suppose that a random sample of nine students (in a psychology laboratory) are told a "secret" and then given instructions on how to pass the polygraph examination without revealing their knowledge of the secret. What is the probability that
(a) all the students are able to pass the polygraph examination?
(b) more than half the students are able to pass the polygraph examination?
(c) no more than four of the students are able to pass the polygraph examination?
(d) all the students fail the polygraph examination?

22. ***Hardware Store: Income*** Trevor is interested in purchasing the local hardware/sporting goods store in the small town of Dove Creek, Montana. After examining accounting records for the past several years, he found that the store has been grossing over $850 per day about 60% of the business days it is open. Estimate the probability that the store will gross over $850
(a) at least 3 out of 5 business days.
(b) at least 6 out of 10 business days.
(c) fewer than 5 out of 10 business days.

(d) fewer than 6 out of the next 20 business days. ***Interpretation*** If this actually happened, might it shake your confidence in the statement $p = 0.60$? Might it make you suspect that p is less than 0.60? Explain.

(e) more than 17 out of the next 20 business days. ***Interpretation*** If this actually happened, might you suspect that p is greater than 0.60? Explain.

23. *Psychology: Myers–Briggs* Approximately 75% of all marketing personnel are extroverts, whereas about 60% of all computer programmers are introverts (Source: *A Guide to the Development and Use of the Myers–Briggs Type Indicator,* by Myers and McCaulley).

(a) At a meeting of 15 marketing personnel, what is the probability that 10 or more are extroverts? What is the probability that 5 or more are extroverts? What is the probability that all are extroverts?

(b) In a group of 5 computer programmers, what is the probability that none are introverts? What is the probability that 3 or more are introverts? What is the probability that all are introverts?

24. *Business Ethics: Privacy* Are your finances, buying habits, medical records, and phone calls really private? A real concern for many adults is that computers and the Internet are reducing privacy. A survey conducted by Peter D. Hart Research Associates for the Shell Poll was reported in *USA Today.* According to the survey, 37% of adults are concerned that employers are monitoring phone calls. Use the binomial distribution formula to calculate the probability that

(a) out of five adults, none is concerned that employers are monitoring phone calls.

(b) out of five adults, all are concerned that employers are monitoring phone calls.

(c) out of five adults, exactly three are concerned that employers are monitoring phone calls.

25. *Business Ethics: Privacy* According to the same poll quoted in Problem 24, 53% of adults are concerned that Social Security numbers are used for general identification. For a group of eight adults selected at random, we used Minitab to generate the binomial probability distribution and the cumulative binomial probability distribution (menu selections ➤ **Calc** ➤ **Probability Distributions** ➤ **Binomial**).

Jeff Titcomb/Photographer's Choice/Getty Images

Number	r	P(r)	P(<=r)
	0	0.002381	0.00238
	1	0.021481	0.02386
	2	0.084781	0.10864
	3	0.191208	0.29985
	4	0.269521	0.56937
	5	0.243143	0.81251
	6	0.137091	0.94960
	7	0.044169	0.99377
	8	0.006226	1.00000

Find the probability that out of eight adults selected at random,

(a) at most five are concerned about Social Security numbers being used for identification. Do the problem by adding the probabilities $P(r = 0)$ through $P(r = 5)$. Is this the same as the cumulative probability $P(r \leq 5)$?

(b) more than five are concerned about Social Security numbers being used for identification. First, do the problem by adding the probabilities $P(r = 6)$ through $P(r = 8)$. Then do the problem by subtracting the cumulative probability $P(r \leq 5)$ from 1. Do you get the same results?

26. ***Health Care: Office Visits*** What is the age distribution of patients who make office visits to a doctor or nurse? The following table is based on information taken from the Medical Practice Characteristics section of the *Statistical Abstract of the United States* (116th Edition).

Age group, years	Under 15	15–24	25–44	45–64	65 and older
Percent of office visitors	20%	10%	25%	20%	25%

Suppose you are a district manager of a health management organization (HMO) that is monitoring the office of a local doctor or nurse in general family practice. This morning the office you are monitoring has eight office visits on the schedule. What is the probability that
(a) at least half the patients are under 15 years old? First, explain how this can be modeled as a binomial distribution with 8 trials, where success is visitor age is under 15 years old and the probability of success is 20%.
(b) from 2 to 5 patients are 65 years old or older (include 2 and 5)?
(c) from 2 to 5 patients are 45 years old or older (include 2 and 5)? *Hint:* Success is 45 or older. Use the table to compute the probability of success on a single trial.
(d) all the patients are under 25 years of age?
(e) all the patients are 15 years old or older?

27. ***Binomial Distribution Table: Symmetry*** Study the binomial distribution table (Table 3, Appendix II). Notice that the probability of success on a single trial p ranges from 0.01 to 0.95. Some binomial distribution tables stop at 0.50 because of the symmetry in the table. Let's look for that symmetry. Consider the section of the table for which $n = 5$. Look at the numbers in the columns headed by $p = 0.30$ and $p = 0.70$. Do you detect any similarities? Consider the following probabilities for a binomial experiment with five trials.
(a) Compare P(3 successes), where $p = 0.30$, with P(2 successes), where $p = 0.70$.
(b) Compare P(3 or more successes), where $p = 0.30$, with P(2 or fewer successes), where $p = 0.70$.
(c) Find the value of P(4 successes), where $p = 0.30$. For what value of r is P(r successes) the same using $p = 0.70$?
(d) What column is symmetrical with the one headed by $p = 0.20$?

28. ***Binomial Distribution: Control Charts*** This problem will be referred to in the study of control charts (Section 6.1). In the binomial probability distribution, let the number of trials be $n = 3$, and let the probability of success be $p = 0.0228$. Use a calculator to compute
(a) the probability of two successes.
(b) the probability of three successes.
(c) the probability of two or three successes.

29. ***Expand Your Knowledge: Conditional Probability*** In the western United States, there are many dry land wheat farms that depend on winter snow and spring rain to produce good crops. About 65% of the years, there is enough moisture to produce a good wheat crop, depending on the region (Reference: *Agricultural Statistics*, U.S. Department of Agriculture).
(a) Let r be a random variable that represents the number of good wheat crops in $n = 8$ years. Suppose the Zimmer farm has reason to believe that at least 4 out of 8 years will be good. However, they need at least 6 good years out of 8 to survive financially. Compute the probability that the Zimmers will get at least 6 good years out of 8, given what they believe is true; that is, compute $P(6 \le r \mid 4 \le r)$. See part (d) for a hint.
(b) Let r be a random variable that represents the number of good wheat crops in $n = 10$ years. Suppose the Montoya farm has reason to believe that at least 6 out of 10 years will be good. However, they need at least 8 good years


Tomas Bercic/iStockphoto.com

out of 10 to survive financially. Compute the probability that the Montoyas will get at least 8 good years out of 10, given what they believe is true; that is, compute $P(8 \le r \mid 6 \le r)$.

(c) List at least three other areas besides agriculture to which you think conditional binomial probabilities can be applied.

(d) *Hint for solution:* Review item 6, conditional probability, in the summary of basic probability rules at the end of Section 4.2. Note that

$$P(A \mid B) = \frac{P(A \, and \, B)}{P(B)}$$

and show that in part (a),

$$P(6 \le r \mid 4 \le r) = \frac{P((6 \le r) \, and \, (4 \le r))}{P(4 \le r)} = \frac{P(6 \le r)}{P(4 \le r)}$$

30. ***Conditional Probability: Blood Supply*** Only about 70% of all donated human blood can be used in hospitals. The remaining 30% cannot be used because of various infections in the blood. Suppose a blood bank has 10 newly donated pints of blood. Let r be a binomial random variable that represents the number of "good" pints that can be used.

(a) Based on questionnaires completed by the donors, it is believed that at least 6 of the 10 pints are usable. What is the probability that at least 8 of the pints are usable, given this belief is true? Compute $P(8 \le r \mid 6 \le r)$.

(b) Assuming the belief that at least 6 of the pints are usable is true, what is the probability that all 10 pints can be used? Compute $P(r = 10 \mid 6 \le r)$.

Hint: See Problem 29.

SECTION 5.3

Additional Properties of the Binomial Distribution

FOCUS POINTS

- Make histograms for binomial distributions.
- Compute μ and σ for a binomial distribution.
- Compute the minimum number of trials n needed to achieve a given probability of success $P(r)$.

Graphing a Binomial Distribution

Any probability distribution may be represented in graphic form. How should we graph the binomial distribution? Remember, the binomial distribution tells us the probability of r successes out of n trials. Therefore, we'll place values of r along the horizontal axis and values of $P(r)$ on the vertical axis. The binomial distribution is a *discrete* probability distribution because r can assume only whole-number values such as 0, 1, 2, 3, . . . Therefore, a histogram is an appropriate graph of a binomial distribution.

PROCEDURE

How to graph a binomial distribution

1. Place r values on the horizontal axis.
2. Place $P(r)$ values on the vertical axis.
3. Construct a bar over each r value extending from $r - 0.5$ to $r + 0.5$. The height of the corresponding bar is $P(r)$.

Let's look at an example to see exactly how we'll make these histograms.

EXAMPLE 7 **Graph of a binomial distribution**

A waiter at the Green Spot Restaurant has learned from long experience that the probability that a lone diner will leave a tip is only 0.7. During one lunch hour, the waiter serves six people who are dining by themselves. Make a graph of the binomial probability distribution that shows the probabilities that 0, 1, 2, 3, 4, 5, or all 6 lone diners leave tips.

SOLUTION: This is a binomial experiment with $n = 6$ trials. Success is achieved when the lone diner leaves a tip, so the probability of success is 0.7 and that of failure is 0.3:

$$n = 6 \quad p = 0.7 \quad q = 0.3$$

We want to make a histogram showing the probability of r successes when $r = 0, 1, 2, 3, 4, 5$, or 6. It is easier to make the histogram if we first make a table of r values and the corresponding $P(r)$ values (Table 5-12). We'll use Table 3 of Appendix II to find the $P(r)$ values for $n = 6$ and $p = 0.70$.

FIGURE 5-3

Graph of the Binomial Distribution for $n = 6$ and $p = 0.7$

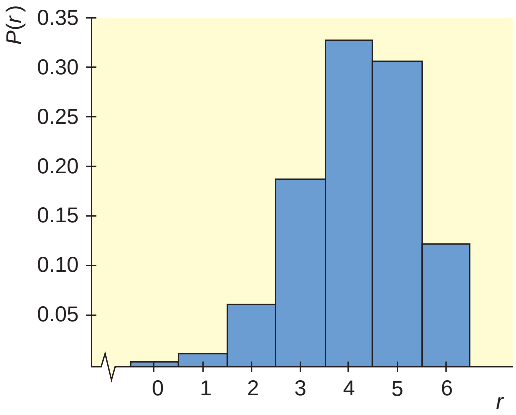


TABLE 5-12 **Binomial Distribution for $n = 6$ and $p = 0.70$**

r	$P(r)$
0	0.001
1	0.010
2	0.060
3	0.185
4	0.324
5	0.303
6	0.118

Mark Richards/PhotoEdit

To construct the histogram, we'll put r values on the horizontal axis and $P(r)$ values on the vertical axis. Our bars will be one unit wide and will be centered over the appropriate r value. The height of the bar over a particular r value tells the probability of that r (see Figure 5-3).

The probability of a particular value of r is given not only by the height of the bar over that r value but also by the *area* of the bar. Each bar is only one unit wide, so its area (area = height times width) equals its height. Since the area of each bar represents the probability of the r value under it, the sum of the areas of the bars must be 1. In this example, the sum turns out to be 1.001. It is not exactly equal to 1 because of rounding error.

Guided Exercise 6 illustrates another binomial distribution with $n = 6$ trials. The graph will be different from that of Figure 5-3 because the probability of success p is different.

GUIDED EXERCISE 6 ***Graph of a binomial distribution***

Jim enjoys playing basketball. He figures that he makes about 50% of the field goals he attempts during a game. Make a histogram showing the probability that Jim will make 0, 1, 2, 3, 4, 5, or 6 shots out of six attempted field goals.

Continued

GUIDED EXERCISE 6 *continued*

(a) This is a binomial experiment with $n =$ ______ trials. In this situation, we'll say success occurs when Jim makes an attempted field goal. What is the value of p?

→ In this example, $n = 6$ and $p = 0.5$.

(b) Use Table 3 of Appendix II to complete Table 5-13 of $P(r)$ values for $n = 6$ and $p = 0.5$.

TABLE 5-13

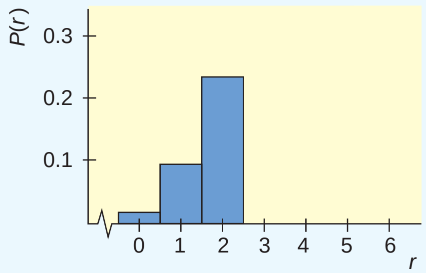
r	$P(r)$
0	0.016
1	0.094
2	0.234
3	_____
4	_____
5	_____
6	_____

→ TABLE 5-14 **Completion of Table 5-13**

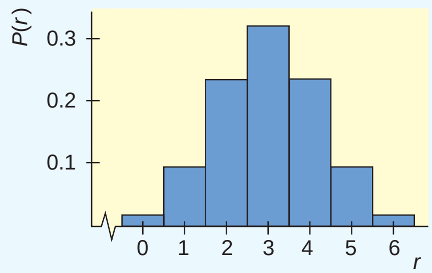
r	$P(r)$
.	.
.	.
.	.
3	0.312
4	0.234
5	0.094
6	0.016

(c) Use the values of $P(r)$ given in Table 5-14 to complete the histogram in Figure 5-4.

FIGURE 5-4 Beginning of Graph of Binomial Distribution for $n = 6$ and $p = 0.5$



→ FIGURE 5-5 Completion of Figure 5-4



(d) The area of the bar over $r = 2$ is 0.234. What is the area of the bar over $r = 4$? How does the probability that Jim makes exactly two field goals out of six compare with the probability that he makes exactly four field goals out of six?

→ The area of the bar over $r = 4$ is also 0.234. Jim is as likely to make two out of six field goals attempted as he is to make four out of six.

In Example 7 and Guided Exercise 6, we see the graphs of two binomial distributions associated with $n = 6$ trials. The two graphs are different because the probability of success p is different in the two cases. In Example 7, $p = 0.7$ and the graph is skewed to the left—that is, the left tail is longer. In Guided Exercise 6, p is equal to 0.5 and the graph is symmetrical—that is, if we fold it in half, the two halves coincide exactly. Whenever *p equals 0.5, the graph of the binomial distribution will be symmetrical no matter how many trials we have.* In Chapter 6, we will see that if the number of trials n is quite large, the binomial distribution is almost symmetrical over the bars containing most of the area even when p is not close to 0.5.

Mean and Standard Deviation of a Binomial Distribution

Two other features that help describe the graph of any distribution are the balance point of the distribution and the spread of the distribution about that balance point. The *balance point* is the mean μ of the distribution, and the *measure of spread* that is most commonly used is the standard deviation σ. The mean μ is the *expected value* of the number of successes.

For the binomial distribution, we can use two special formulas to compute the mean μ and the standard deviation σ. These are easier to use than the general formulas in Section 5.1 for μ and σ of any discrete probability distribution.

PROCEDURE

Mean of a binomial distribution

Standard deviation of a binomial distributions

How to compute μ and σ for a binomial distribution

$\mu = np$ is the **expected number of successes** for the random variable r

$\sigma = \sqrt{npq}$ is the **standard deviation** for the random variable r

where

r is a random variable representing the number of successes in a binomial distribution,
n is the number of trials,
p is the probability of success on a single trial, and
$q = 1 - p$ is the probability of failure on a single trial.

EXAMPLE 8

Compute μ and σ

Let's compute the mean and standard deviation for the distribution of Example 7 that describes that probabilities of lone diners leaving tips at the Green Spot Restaurant.

SOLUTION: In Example 7,

$$n = 6 \quad p = 0.7 \quad q = 0.3$$

For the binomial distribution,

$$\mu = np = 6(0.7) = 4.2$$

The balance point of the distribution is at $\mu = 4.2$. The standard deviation is given by

$$\sigma = \sqrt{npq} = \sqrt{6(0.7)(0.3)} = \sqrt{1.26} \approx 1.12$$

The mean μ is not only the balance point of the distribution; it is also the *expected value* of *r*. Specifically, in Example 7, the waiter can expect 4.2 lone diners out of 6 to leave a tip. (The waiter would probably round the expected value to 4 tippers out of 6.)

GUIDED EXERCISE 7 **Expected value and standard deviation**

When Jim (of Guided Exercise 6) shoots field goals in basketball games, the probability that he makes a shot is only 0.5.

Continued

GUIDED EXERCISE 7 *continued*

(a) The mean of the binomial distribution is the expected value of r successes out of n trials. Out of six throws, what is the expected number of goals Jim will make?

→ The expected value is the mean μ:

$\mu = np = 6(0.5) = 3$

Jim can expect to make three goals out of six tries.

(b) For six trials, what is the standard deviation of the binomial distribution of the number of successful field goals Jim makes?

→ $\sigma = \sqrt{npq} = \sqrt{6(0.5)(0.5)} = \sqrt{1.5} \approx 1.22$

CRITICAL THINKING

Unusual Values

Chebyshev's theorem tells us that no matter what the data distribution looks like, at least 75% of the data will fall within 2 standard deviations of the mean. As we will see in Chapter 6, when the distribution is mound-shaped and symmetrical, about 95% of the data are within 2 standard deviations of the mean. Data values beyond 2 standard deviations from the mean are less common than those closer to the mean.

In fact, one indicator that a data value might be an outlier is that it is more than 2.5 standard deviations from the mean (Source: *Statistics*, by G. Upton and I. Cook, Oxford University Press).

Unusual values

For a binomial distribution, it is unusual for the number of successes r to be higher than $\mu + 2.5\sigma$ or lower than $\mu - 2.5\sigma$.

We can use this indicator to determine whether a specified number of successes out of n trials in a binomial experiment are unusual.

For instance, consider a binomial experiment with 20 trials for which probability of success on a single trial is $p = 0.70$. The expected number of successes is $\mu = 14$, with a standard deviation of $\sigma \approx 2$. A number of successes above 19 or below 9 would be considered unusual. However, such numbers of successes are possible.

Quota Problems: Minimum Number of Trials for a Given Probability

In applications, you do not want to confuse the expected value of r with certain probabilities associated with r. Guided Exercise 8 illustrates this point.

GUIDED EXERCISE 8 *Find the minimum value of n for a given $P(r)$*

A satellite is powered by three solar cells. The probability that any one of these cells will fail is 0.15, and the cells operate or fail independently.

Part I: In this part, we want to find the least number of cells the satellite should have so that the *expected value* of the number of working cells is no smaller than 3. In this situation, n represents the number of cells, r is the number of successful or working cells, p is the probability that a cell will work, q is the probability that a cell will fail, and μ is the expected value, which should be no smaller than 3.

Continued

GUIDED EXERCISE 8 *continued*

(a) What is the value of q? of p?

→ $q = 0.15$, as given in the problem. p must be 0.85, since $p = 1 - q$.

(b) The expected value μ for the number of working cells is given by $\mu = np$. The expected value of the number of working cells should be no smaller than 3, so

$3 \le \mu = np$

From part (a), we know the value of p. Solve the inequality $3 \le np$ for n.

→ $3 \le np$

$3 \le n(0.85)$

$\frac{3}{0.85} \le n$ Divide both sides by 0.85.

$3.53 \le n$

(c) Since n is between 3 and 4, should we round it to 3 or to 4 to make sure that μ is at least 3?

→ n should be at least 3.53. Since we can't have a fraction of a cell, we had best make $n = 4$. For $n = 4$, $\mu = 4(0.85) = 3.4$. This value satisfies the condition that μ be at least 3.

Part II: In this part, we want to find the smallest number of cells the satellite should have to be 97% sure that there will be adequate power—that is, that at least three cells work.

(a) The letter r has been used to denote the number of successes. In this case, r represents the number of working cells. We are trying to find the number n of cells necessary to ensure that (choose the correct statement)

(i) $P(r \ge 3) = 0.97$ or

(ii) $P(r \le 3) = 0.97$

→ $P(r \ge 3) = 0.97$

(b) We need to find a value for n such that

$P(r \ge 3) = 0.97$

Try $n = 4$. Then, $r \ge 3$ means $r = 3$ or 4, so,

$P(r \ge 3) = P(3) + P(4)$

Use Table 3 (Appendix II) with $n = 4$ and $p = 0.85$ to find values of $P(3)$ and $P(4)$. Then, compute $P(r \ge 3)$ for $n = 4$. Will $n = 4$ guarantee that $P(r \ge 3)$ is at least 0.97?

→ $P(3) = 0.368$

$P(4) = 0.522$

$P(r \ge 3) = 0.368 + 0.522 = 0.890$

Thus, $n = 4$ is *not* sufficient to be 97% sure that at least three cells will work. For $n = 4$, the probability that at least three cells will work is only 0.890.

(c) Now try $n = 5$ cells. For $n = 5$,

$P(r \ge 3) = P(3) + P(4) + P(5)$

since r can be 3, 4, or 5. Are $n = 5$ cells adequate? [Be sure to find new values of $P(3)$ and $P(4)$, since we now have $n = 5$.]

→
$$\begin{aligned} P(r \ge 3) &= P(3) + P(4) + P(5) \\ &= 0.138 + 0.392 + 0.444 \\ &= 0.974 \end{aligned}$$

Thus, $n = 5$ cells are required if we want to be 97% sure that at least three cells will work.

In Part I and Part II, we got different values for n. Why? In Part I, we had $n = 4$ and $\mu = 3.4$. This means that if we put up lots of satellites with four cells, we can expect that an *average* of 3.4 cells will work per satellite. But for $n = 4$ cells, there is a probability of only 0.89 that at least three cells will work in any one satellite. In Part II, we are trying to find the number of cells necessary so that the probability is 0.97 that at least three cells will work in any *one* satellite. If we use $n = 5$ cells, then we can satisfy this requirement.

Quota problems

Quotas occur in many aspects of everyday life. The manager of a sales team gives every member of the team a weekly sales quota. In some districts, police have a monthly quota for the number of traffic tickets issued. Nonprofit organizations have recruitment quotas for donations or new volunteers. The basic ideas used to compute quotas also can be used in medical science (how frequently checkups should occur), quality control (how many production flaws should be expected), or risk management (how many bad loans a bank should expect in a certain investment group). In fact, Part II of Guided Exercise 8 is a *quota problem*. To have adequate power, a satellite must have a quota of three working solar cells. Such problems come from many different sources, but they all have one thing in common: They are solved using the binomial probability distribution.

To solve quota problems, it is often helpful to use equivalent formulas for expressing binomial probabilities. These formulas involve the complement rule and the fact that binomial events are independent. Equivalent probabilities will be used in Example 9.

PROCEDURE

How to express binomial probabilities using equivalent formulas

$P(\text{at least one success}) = P(r \geq 1) = 1 - P(0)$

$P(\text{at least two successes}) = P(r \geq 2) = 1 - P(0) - P(1)$

$P(\text{at least three successes}) = P(r \geq 3) = 1 - P(0) - P(1) - P(2)$

$P(\text{at least } m \text{ successes}) = P(r \geq m) = 1 - P(0) - P(1) - \cdots - P(m - 1)$, where $1 \leq m \leq$ number of trials

For a discussion of the mathematics behind these formulas, see Problem 26 at the end of this section.

Example 9 is a quota problem. Junk bonds are sometimes controversial. In some cases, junk bonds have been the salvation of a basically good company that has had a run of bad luck. From another point of view, junk bonds are not much more than a gambler's effort to make money by shady ethics.

The book *Liar's Poker,* by Michael Lewis, is an exciting and sometimes humorous description of his career as a Wall Street bond broker. Most bond brokers, including Mr. Lewis, are ethical people. However, the book does contain an interesting discussion of Michael Milken and shady ethics. In the book, Mr. Lewis says, "If it was a good deal, the brokers kept it for themselves; if it was a bad deal, they'd try to sell it to their customers." In Example 9, we use some binomial probabilities for a brief explanation of what Mr. Lewis's book is talking about.

EXAMPLE 9

Quota

Junk bonds can be profitable as well as risky. Why are investors willing to consider junk bonds? Suppose you can buy junk bonds at a tremendous discount. You try to choose "good" companies with a "good" product. The company should have done well but for some reason did not. Suppose you consider only companies with a 35% estimated risk of default, and your financial investment goal requires four bonds to be "good" bonds in the sense that they will not default before a certain date. Remember, junk bonds that do not default are usually very profitable because they carry a very high rate of return. The other bonds in your investment group can default (or not) without harming your investment plan. Suppose you want to be 95% certain of meeting your goal (quota) of at least four good bonds. How many junk bond issues should you buy to meet this goal?

SOLUTION: Since the probability of default is 35%, the probability of a "good" bond is 65%. Let success S be represented by a good bond. Let n be the number of bonds purchased, and let r be the number of good bonds in this group. We want

$$P(r \geq 4) \geq 0.95$$

This is equivalent to

$$1 - P(0) - P(1) - P(2) - P(3) \geq 0.95$$

Since the probability of success is $p = P(S) = 0.65$, we need to look in the binomial table under $p = 0.65$ and different values of n to find the *smallest value of n* that will satisfy the preceding relation. Table 3 of Appendix II shows that if $n = 10$ when $p = 0.65$, then,

$$1 - P(0) - P(1) - P(2) - P(3) = 1 - 0 - 0 - 0.004 - 0.021 = 0.975$$

The probability 0.975 satisfies the condition of being greater than or equal to 0.95. We see that 10 is the smallest value of n for which the condition

$$P(r \geq 4) \geq 0.95$$

is satisfied. Under the given conditions (a good discount on price, no more than 35% chance of default, and a fundamentally good company), you can be 95% sure of meeting your investment goal with $n = 10$ (carefully selected) junk bond issues.

In this example, we see that by carefully selecting junk bonds, there is a high probability of getting some good bonds that will produce a real profit. What do you do with the other bonds that aren't so good? Perhaps the quote from *Liar's Poker* will suggest what is sometimes attempted.

VIEWPOINT Kodiak Island, Alaska

Kodiak Island is famous for its giant brown bears. The sea surrounding the island is also famous for its king crab. The state of Alaska's, Department of Fish and Game has collected a huge amount of data regarding ocean latitude, ocean longitude, and size of king crab. Of special interest to commercial fishing skippers is the size of crab. Those too small must be returned to the sea. To find locations and sizes of king crab catches near Kodiak Island, visit the Brase/Brase statistics site at **http://www.cengage.com/statistics/brase** *and find the link to the StatLib site hosted by the Department of Statistics at Carnegie Mellon University. Once at StatLib, go to crab data. From this information, it is possible to use methods of this chapter and Chapter 7 to estimate the proportion of legal crab in a sea skipper's catch.*

SECTION 5.3 PROBLEMS

1. *Statistical Literacy* What does the expected value of a binomial distribution with n trials tell you?

2. *Statistical Literacy* Consider two binomial distributions, with n trials each. The first distribution has a higher probability of success on each trial than the second. How does the expected value of the first distribution compare to that of the second?

3. ***Basic Computation: Expected Value and Standard Deviation*** Consider a binomial experiment with $n = 8$ trials and $p = 0.20$.
   (a) Find the expected value and the standard deviation of the distribution.
   (b) ***Interpretation*** Would it be unusual to obtain 5 or more successes? Explain. Confirm your answer by looking at the binomial probability distribution table.

4. ***Basic Computation: Expected Value and Standard Deviation*** Consider a binomial experiment with $n = 20$ trials and $p = 0.40$.
   (a) Find the expected value and the standard deviation of the distribution.
   (b) ***Interpretation*** Would it be unusual to obtain fewer than 3 successes? Explain. Confirm your answer by looking at the binomial probability distribution table.

5. *Critical Thinking* Consider a binomial distribution of 200 trials with expected value 80 and standard deviation of about 6.9. Use the criterion that it is unusual to have data values more than 2.5 standard deviations above the mean or 2.5 standard deviations below the mean to answer the following questions.
   (a) Would it be unusual to have more than 120 successes out of 200 trials? Explain.
   (b) Would it be unusual to have fewer than 40 successes out of 200 trials? Explain.
   (c) Would it be unusual to have from 70 to 90 successes out of 200 trials? Explain.

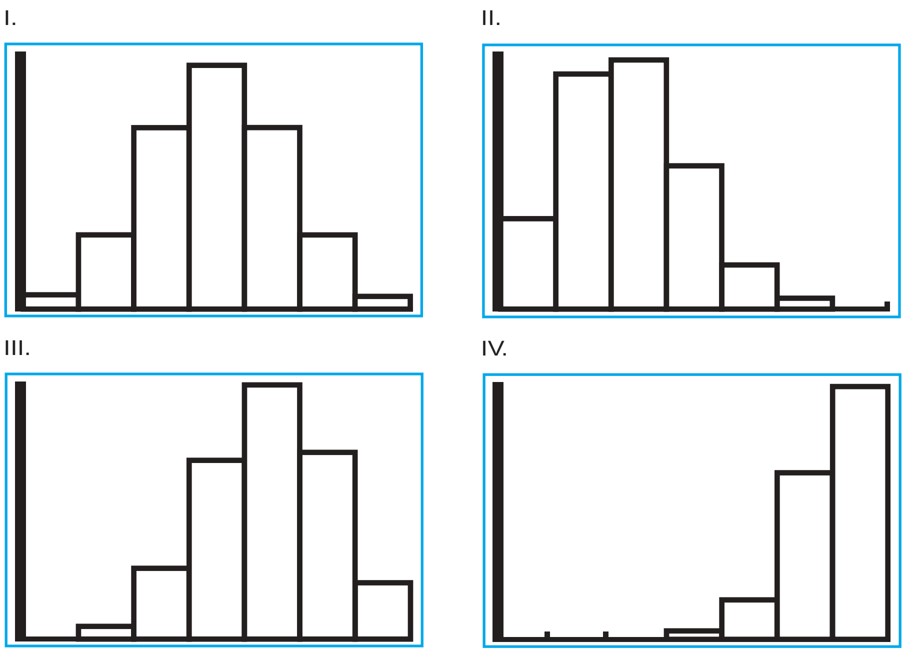
6. *Critical Thinking* Consider a binomial distribution with 10 trials. Look at Table 3 (Appendix II) showing binomial probabilities for various values of p, the probability of success on a single trial.
   (a) For what value of p is the distribution symmetric? What is the expected value of this distribution? Is the distribution centered over this value?
   (b) For small values of p, is the distribution skewed right or left?
   (c) For large values of p, is the distribution skewed right or left?

7. *Binomial Distribution: Histograms* Consider a binomial distribution with $n = 5$ trials. Use the probabilities given in Table 3 of Appendix II to make histograms showing the probabilities of $r = 0, 1, 2, 3, 4,$ and 5 successes for each of the following. Comment on the skewness of each distribution.
   (a) The probability of success is $p = 0.50$.
   (b) The probability of success is $p = 0.25$.
   (c) The probability of success is $p = 0.75$.
   (d) What is the relationship between the distributions shown in parts (b) and (c)?
   (e) If the probability of success is $p = 0.73$, do you expect the distribution to be skewed to the right or to the left? Why?

8. *Binomial Distributions: Histograms* Figure 5-6 shows histograms of several binomial distributions with $n = 6$ trials. Match the given probability of success with the best graph.
   (a) $p = 0.30$ goes with graph ________.
   (b) $p = 0.50$ goes with graph ________.
   (c) $p = 0.65$ goes with graph ________.
   (d) $p = 0.90$ goes with graph ________.
   (e) In general, when the probability of success p is close to 0.5, would you say that the graph is more symmetrical or more skewed? In general, when the probability of success p is close to 1, would you say that the graph is skewed to the right or to the left? What about when p is close to 0?

FIGURE 5-6

Binomial Probability Distributions with $n = 6$ (generated on the TI-84Plus calculator)



9. ***Marketing: Photography*** Does the *kid factor* make a difference? If you are talking photography, the answer may be yes! The following table is based on information from *American Demographics* (Vol. 19, No. 7).

Ages of children in household, years	Under 2	None under 21
Percent of U.S. households that buy photo gear	80%	50%

Let us say you are a market research person who interviews a random sample of 10 households.
(a) Suppose you interview 10 households with children under the age of 2 years. Let r represent the number of such households that buy photo gear. Make a histogram showing the probability distribution of r for $r = 0$ through $r = 10$. Find the mean and standard deviation of this probability distribution.
(b) Suppose that the 10 households are chosen to have no children under 21 years old. Let r represent the number of such households that buy photo gear. Make a histogram showing the probability distribution of r for $r = 0$ through $r = 10$. Find the mean and standard deviation of this probability distribution.
(c) ***Interpretation*** Compare the distributions in parts (a) and (b). You are designing TV ads to sell photo gear. Could you justify featuring ads of parents taking pictures of toddlers? Explain your answer.

10. ***Quality Control: Syringes*** The quality-control inspector of a production plant will reject a batch of syringes if two or more defective syringes are found in a random sample of eight syringes taken from the batch. Suppose the batch contains 1% defective syringes.
(a) Make a histogram showing the probabilities of $r = 0, 1, 2, 3, 4, 5, 6, 7,$ and 8 defective syringes in a random sample of eight syringes.
(b) Find μ. What is the expected number of defective syringes the inspector will find?
(c) What is the probability that the batch will be accepted?
(d) Find σ.

11. ***Private Investigation: Locating People*** Old Friends Information Service is a California company that is in the business of finding addresses of long-lost friends. Old Friends claims to have a 70% success rate (Source: *Wall Street Journal*). Suppose that you have the names of six friends for whom you have no addresses and decide to use Old Friends to track them.
(a) Make a histogram showing the probability of $r = 0$ to 6 friends for whom an address will be found.
(b) Find the mean and standard deviation of this probability distribution. What is the expected number of friends for whom addresses will be found?
(c) ***Quota Problem*** How many names would you have to submit to be 97% sure that at least two addresses will be found?

12. ***Insurance: Auto*** The Mountain States Office of State Farm Insurance Company reports that approximately 85% of all automobile damage liability claims are made by people under 25 years of age. A random sample of five automobile insurance liability claims is under study.
(a) Make a histogram showing the probability that $r = 0$ to 5 claims are made by people under 25 years of age.
(b) Find the mean and standard deviation of this probability distribution. For samples of size 5, what is the expected number of claims made by people under 25 years of age?

13. ***Education: Illiteracy*** *USA Today* reported that about 20% of all people in the United States are illiterate. Suppose you interview seven people at random off a city street.

(a) Make a histogram showing the probability distribution of the number of illiterate people out of the seven people in the sample.
(b) Find the mean and standard deviation of this probability distribution. Find the expected number of people in this sample who are illiterate.
(c) *Quota Problem* How many people would you need to interview to be 98% sure that at least seven of these people can read and write (are not illiterate)?

14. *Rude Drivers: Tailgating* Do you tailgate the car in front of you? About 35% of all drivers will tailgate before passing, thinking they can make the car in front of them go faster (Source: Bernice Kanner, *Are You Normal?*, St. Martin's Press). Suppose that you are driving a considerable distance on a two-lane highway and are passed by 12 vehicles.
(a) Let r be the number of vehicles that tailgate before passing. Make a histogram showing the probability distribution of r for $r = 0$ through $r = 12$.
(b) Compute the expected number of vehicles out of 12 that will tailgate.
(c) Compute the standard deviation of this distribution.

15. *Hype: Improved Products* The *Wall Street Journal* reported that approximately 25% of the people who are told a product is *improved* will believe that it is, in fact, improved. The remaining 75% believe that this is just hype (the same old thing with no real improvement). Suppose a marketing study consists of a random sample of eight people who are given a sales talk about a new, *improved* product.
(a) Make a histogram showing the probability that $r = 0$ to 8 people believe the product is, in fact, improved.
(b) Compute the mean and standard deviation of this probability distribution.
(c) *Quota Problem* How many people are needed in the marketing study to be 99% sure that at least one person believes the product to be improved? *Hint:* Note that $P(r \geq 1) = 0.99$ is equivalent to $1 - P(0) = 0.99$, or $P(0) = 0.01$.

16. *Quota Problem: Archaeology* An archaeological excavation at Burnt Mesa Pueblo showed that about 10% of the flaked stone objects were finished arrow points (Source: *Bandelier Archaeological Excavation Project: Summer 1990 Excavations at Burnt Mesa Pueblo*, edited by Kohler, Washington State University). How many flaked stone objects need to be found to be 90% sure that at least one is a finished arrow point? *Hint:* Use a calculator and note that $P(r \geq 1) \geq 0.90$ is equivalent to $1 - P(0) \geq 0.90$, or $P(0) \geq 0.10$.

17. *Criminal Justice: Parole* *USA Today* reports that about 25% of all prison parolees become repeat offenders. Alice is a social worker whose job is to counsel people on parole. Let us say success means a person does not become a repeat offender. Alice has been given a group of four parolees.
(a) Find the probability $P(r)$ of r successes ranging from 0 to 4.
(b) Make a histogram for the probability distribution of part (a).
(c) What is the expected number of parolees in Alice's group who will not be repeat offenders? What is the standard deviation?
(d) *Quota Problem* How large a group should Alice counsel to be about 98% sure that three or more parolees will not become repeat offenders?

18. *Defense: Radar Stations* The probability that a single radar station will detect an enemy plane is 0.65.
(a) *Quota Problem* How many such stations are required for 98% certainty that an enemy plane flying over will be detected by at least one station?
(b) If four stations are in use, what is the expected number of stations that will detect an enemy plane?

19. *Criminal Justice: Jury Duty* Have you ever tried to get out of jury duty? About 25% of those called will find an excuse (work, poor health, travel out of town, etc.) to avoid jury duty (Source: Bernice Kanner, *Are You Normal?*, St. Martin's Press, New York). If 12 people are called for jury duty,

(a) what is the probability that all 12 will be available to serve on the jury?
(b) what is the probability that 6 or more will *not* be available to serve on the jury?
(c) Find the expected number of those available to serve on the jury. What is the standard deviation?
(d) ***Quota Problem*** How many people n must the jury commissioner contact to be 95.9% sure of finding at least 12 people who are available to serve?

20. ***Public Safety: 911 Calls*** *The Denver Post* reported that a recent audit of Los Angeles 911 calls showed that 85% were not emergencies. Suppose the 911 operators in Los Angeles have just received four calls.
(a) What is the probability that all four calls are, in fact, emergencies?
(b) What is the probability that three or more calls are not emergencies?
(c) ***Quota Problem*** How many calls n would the 911 operators need to answer to be 96% (or more) sure that at least one call is, in fact, an emergency?

21. ***Law Enforcement: Property Crime*** Does crime pay? The *FBI Standard Survey of Crimes* shows that for about 80% of all property crimes (burglary, larceny, car theft, etc.), the criminals are never found and the case is never solved (Source: *True Odds,* by James Walsh, Merrit Publishing). Suppose a neighborhood district in a large city suffers repeated property crimes, not always perpetuated by the same criminals. The police are investigating six property crime cases in this district.
(a) What is the probability that none of the crimes will ever be solved?
(b) What is the probability that at least one crime will be solved?
(c) What is the expected number of crimes that will be solved? What is the standard deviation?
(d) ***Quota Problem*** How many property crimes n must the police investigate before they can be at least 90% sure of solving one or more cases?

22. ***Security: Burglar Alarms*** A large bank vault has several automatic burglar alarms. The probability is 0.55 that a single alarm will detect a burglar.
(a) ***Quota Problem*** How many such alarms should be used for 99% certainty that a burglar trying to enter will be detected by at least one alarm?
(b) Suppose the bank installs nine alarms. What is the expected number of alarms that will detect a burglar?

23. ***Criminal Justice: Convictions*** Innocent until proven guilty? In Japanese criminal trials, about 95% of the defendants are found guilty. In the United States, about 60% of the defendants are found guilty in criminal trials (Source: *The Book of Risks,* by Larry Laudan, John Wiley and Sons). Suppose you are a news reporter following seven criminal trials.
(a) If the trials were in Japan, what is the probability that all the defendants would be found guilty? What is this probability if the trials were in the United States?
(b) Of the seven trials, what is the expected number of guilty verdicts in Japan? What is the expected number in the United States? What is the standard deviation in each case?
(c) ***Quota Problem*** As a U.S. news reporter, how many trials n would you need to cover to be at least 99% sure of two or more convictions? How many trials n would you need if you covered trials in Japan?

24. ***Focus Problem: Personality Types*** We now have the tools to solve the Chapter Focus Problem. In the book *A Guide to the Development and Use of the Myers–Briggs Type Indicators* by Myers and McCaully, it was reported that approximately 45% of all university professors are extroverted. Suppose you have classes with six different professors.
(a) What is the probability that all six are extroverts?
(b) What is the probability that none of your professors is an extrovert?
(c) What is the probability that at least two of your professors are extroverts?

(d) In a group of six professors selected at random, what is the *expected number* of extroverts? What is the *standard deviation* of the distribution?

(e) ***Quota Problem*** Suppose you were assigned to write an article for the student newspaper and you were given a quota (by the editor) of interviewing at least three extroverted professors. How many professors selected at random would you need to interview to be at least 90% sure of filling the quota?

25. ***Quota Problem: Motel Rooms*** The owners of a motel in Florida have noticed that in the long run, about 40% of the people who stop and inquire about a room for the night actually rent a room.

(a) ***Quota Problem*** How many inquiries must the owner answer to be 99% sure of renting at least one room?

(b) If 25 separate inquiries are made about rooms, what is the expected number of inquiries that will result in room rentals?

26. ***Critical Thinking*** Let r be a binomial random variable representing the number of successes out of n trials.

(a) Explain why the sample space for r consists of the set $\{0, 1, 2, \ldots, n\}$ and why the sum of the probabilities of all the entries in the entire sample space must be 1.

(b) Explain why $P(r \geq 1) = 1 - P(0)$.

(c) Explain why $P(r \geq 2) = 1 - P(0) - P(1)$.

(d) Explain why $P(r \geq m) = 1 - P()0 - P(1) - \cdots - P(m - 1)$ for $1 \leq m \leq n$.

SECTION 5.4

The Geometric and Poisson Probability Distributions

FOCUS POINTS

- In many activities, the *first* to succeed wins everything! Use the geometric distribution to compute the probability that the *n*th trial is the first success.
- Use the Poisson distribution to compute the probability of the occurrence of events spread out over time or space.
- Use the Poisson distribution to approximate the binomial distribution when the number of trials is large and the probability of success is small.

In this chapter, we have studied binomial probabilities for the discrete random variable r, the number of successes in n binomial trials. Before we continue in Chapter 6 with continuous random variables, let us examine two other discrete probability distributions, the *geometric* and the *Poisson* probability distributions. These are both related to the binomial distribution.

Geometric Distribution

Geometric probability distribution

Suppose we have an experiment in which we repeat binomial trials until we get our *first success,* and then we stop. Let n be the number of the trial on which we get our *first success*. In this context, n is not a fixed number. In fact, n could be any of the numbers 1, 2, 3, and so on. What is the probability that our first success comes on the nth trial? The answer is given by the *geometric probability distribution*.

Geometric probability distribution

$$P(n) = p(1 - p)^{n-1}$$

where n is the number of the binomial trial on which the *first success* occurs ($n = 1, 2, 3, \ldots$) and p is the probability of success on each trial. *Note:* p must be the same for each trial.

Using some mathematics involving infinite series, it can be shown that the **population mean** and **standard deviation** of the geometric distribution are

$$\mu = \frac{1}{p} \quad \text{and} \quad \sigma = \frac{\sqrt{1-p}}{p}$$

In many real-life situations, we keep on trying until we achieve success. This is true in areas as diverse as diplomacy, military science, real estate sales, general marketing strategies, medical science, engineering, and technology.

To Engineer Is Human: The Role of Failure in Successful Design is a fascinating book by Henry Petroski (a professor of engineering at Duke University). Reviewers for the *Los Angeles Times* describe this book as serious, amusing, probing, and sometimes frightening. The book examines topics such as the collapse of the Tacoma Narrows suspension bridge, the collapse of the Kansas City Hyatt Regency walkway, and the explosion of the space shuttle *Challenger.* Professor Petroski discusses such topics as "success in foreseeing failure" and the "limits of design." What is meant by expressions such as "foreseeing failure" and the "limits of design"? In the next example, we will see how the geometric probability distribution might help us "forecast" failure.

EXAMPLE 10 First success

An automobile assembly plant produces sheet-metal door panels. Each panel moves on an assembly line. As the panel passes a robot, a mechanical arm will perform spot welding at different locations. Each location has a magnetic dot painted where the weld is to be made. The robot is programmed to locate the magnetic dot and perform the weld. However, experience shows that on each trial the robot is only 85% successful at locating the dot. If it cannot locate the magnetic dot, it is programmed to *try again*. The robot will keep trying until it finds the dot (and does the weld) or the door panel passes out of the robot's reach.


ricardo azoury/Istockphoto

(a) What is the probability that the robot's first success will be on attempts $n = 1, 2$, or 3?

SOLUTION: Since the robot will keep trying until it is successful, the geometric distribution is appropriate. In this case, success S means that the robot finds the correct location. The probability of success is $p = P(S) = 0.85$. The probabilities are

n	$P(n) = p(1-p)^{n-1} = 0.85(0.15)^{n-1}$
1	$0.85(0.15)^0 = 0.85$
2	$0.85(0.15)^1 = 0.1275$
3	$0.85(0.15)^2 \approx 0.0191$

(b) The assembly line moves so fast that the robot has a maximum of only three chances before the door panel is out of reach. What is the probability that the robot will be successful before the door panel is out of reach?

SOLUTION: Since $n = 1$ or 2 or 3 is mutually exclusive, then

$$\begin{aligned} P(n = 1 \text{ or } 2 \text{ or } 3) &= P(1) + P(2) + P(3) \\ &\approx 0.85 + 0.1275 + 0.0191 \\ &= 0.9966 \end{aligned}$$

This means that the weld should be correctly located about 99.7% of the time.

(c) *Interpretation* What is the probability that the robot will not be able to locate the correct spot within three tries? If 10,000 panels are made, what is the expected number of defectives? Comment on the meaning of this answer in the context of "forecasting failure" and the "limits of design."

SOLUTION: The probability that the robot will correctly locate the weld is 0.9966, from part (b). Therefore, the probability that it will not do so (after three unsuccessful tries) is $1 - 0.9966 = 0.0034$. If we made 10,000 panels, we would expect (forecast) $(10{,}000)(0.0034) = 34$ defectives. We could reduce this by inspecting every door, but such a solution is most likely too costly. If a defective weld of this type is not considered too dangerous, we can accept an expected 34 failures out of 10,000 panels due to the limits of our production design—that is, the speed of the assembly line and the accuracy of the robot. If this is not acceptable, a new (perhaps more costly) design is needed.

Problem 30 introduces the negative binomial distribution, with additional applications in Problems 31 and 32. Problem 33 explores the proof of the negative binomial distribution formula.

COMMENT The geometric distribution deals with binomial trials that are repeated until we have our *first success* on the nth trial. Suppose we repeat a binomial trial n times until we have k *successes* (not just one). In the literature, the probability distribution for the random variable n is called the *negative binomial distribution*. For more information on this topic, see Problems 30, 31, 32, and 33 at the end of this section.

Poisson Probability Distribution

Poisson probability distribution

If we examine the binomial distribution as the number of trials n gets larger and larger while the probability of success p gets smaller and smaller, we obtain the *Poisson distribution*. Siméon Denis Poisson (1781–1840) was a French mathematician who studied probabilities of rare events that occur infrequently in space, time, volume, and so forth. The Poisson distribution applies to accident rates, arrival times, defect rates, the occurrence of bacteria in the air, and many other areas of everyday life.

As with the binomial distribution, we assume only two outcomes: A particular event occurs (success) or does not occur (failure) during the specified time period or space. The events need to be independent so that one success does not change the probability of another success during the specified interval. We are interested in computing the probability of r occurrences in the given time period, space, volume, or specified interval.

Poisson distribution

Let λ (Greek letter lambda) be the mean number of successes over time, volume, area, and so forth. Let r be the number of successes ($r = 0, 1, 2, 3, \ldots$) in a corresponding interval of time, volume, area, and so forth. Then the probability of r successes in the interval is

$$P(r) = \frac{e^{-\lambda}\lambda^r}{r!}$$

Continued

where e is approximately equal to 2.7183.

Using some mathematics involving infinite series, it can be shown that the **population mean** and **standard deviation** of the Poisson distribution are

$$\mu = \lambda \quad \text{and} \quad \sigma = \sqrt{\lambda}$$

Note: e^x is a key found on most calculators. Simply use 1 as the exponent, and the calculator will display a decimal approximation for e.

There are many applications of the Poisson distribution. For example, if we take the point of view that waiting time can be subdivided into many small intervals, then the actual arrival (of whatever we are waiting for) during any one of the very short intervals could be thought of as an infrequent (or rare) event. This means that the Poisson distribution can be used as a mathematical model to describe the probabilities of arrivals such as cars to a gas station, planes to an airport, calls to a fire station, births of babies, and even fish on a fisherman's line.

EXAMPLE 11 Poisson distribution

Pyramid Lake is located in Nevada on the Paiute Indian Reservation. The lake is described as a lovely jewel in a beautiful desert setting. In addition to its natural beauty, the lake contains some of the world's largest cutthroat trout. Eight- to ten-pound trout are not uncommon, and 12- to 15-pound trophies are taken each season. The Paiute Nation uses highly trained fish biologists to study and maintain this famous fishery. In one of its publications, *Creel Chronicle* (Vol. 3, No. 2), the following information was given about the November catch for boat fishermen.

Richard Rowan/Photo Researchers

Pyramid Lake, Nevada

Total fish per hour = 0.667

Suppose you decide to fish Pyramid Lake for 7 hours during the month of November.

(a) Use the information provided by the fishery biologist in *Creel Chronicle* to find a probability distribution for r, the number of fish (of all sizes) you catch in a period of 7 hours.

SOLUTION: For fish of all sizes, the mean success rate per hour is 0.667.

$$\lambda = 0.667/1\,\text{hour}$$

Since we want to study a *7-hour interval*, we use a little arithmetic to adjust λ to 7 hours. That is, we adjust λ so that it represents the average number of fish expected in a 7-hour period.

$$\lambda = \frac{0.667}{1 \text{ hour}} \cdot \left(\frac{7}{7}\right) = \frac{4.669}{7\,\text{hour}}$$

For convenience, let us use the rounded value $\lambda = 4.7$ for a 7-hour period. Since r is the number of successes (fish caught) in the corresponding 7-hour period and $\lambda = 4.7$ for this period, we use the Poisson distribution to get

$$P(r) = \frac{e^{-\lambda}\lambda^r}{r!} = \frac{e^{-4.7}(4.7)^r}{r!}$$

Recall that $e \approx 2.7183. \ldots$ Most calculators have e^x, y^x, and $n!$ keys (see your calculator manual), so the Poisson distribution is not hard to compute.

(b) What is the probability that in 7 hours you will get 0, 1, 2, or 3 fish of any size?

SOLUTION: Using the result of part (a), we get

$$P(0) = \frac{e^{-4.7}(4.7)^0}{0!} \approx 0.0091 \approx 0.01$$

$$P(1) = \frac{e^{-4.7}(4.7)^1}{1!} \approx 0.0427 \approx 0.04$$

$$P(2) = \frac{e^{-4.7}(4.7)^2}{2!} \approx 0.1005 \approx 0.10$$

$$P(3) = \frac{e^{-4.7}(4.7)^3}{3!} \approx 0.1574 \approx 0.16$$

The probabilities of getting 0, 1, 2, or 3 fish are about 1%, 4%, 10%, and 16%, respectively.

(c) What is the probability that you will get four or more fish in the 7-hour fishing period?

SOLUTION: The sample space of all r values is $r = 0, 1, 2, 3, 4, 5, \ldots$ The probability of the entire sample space is 1, and these events are mutually exclusive. Therefore,

$$1 = P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + \cdots$$

So,

$$\begin{aligned} P(r \geq 4) &= P(4) + P(5) + \cdots = 1 - P(0) - P(1) - P(2) - P(3) \\ &\approx 1 - 0.01 - 0.04 - 0.10 - 0.16 \\ &= 0.69 \end{aligned}$$

There is about a 69% chance that you will catch four or more fish in a 7-hour period.

Use of tables

Table 4 of Appendix II is a table of the Poisson probability distribution for selected values of λ and the number of successes r. Table 5-15 is an excerpt from that table.

To find the value of $P(r = 2)$ when $\lambda = 0.3$, look in the column headed by 0.3 and the row headed by 2. From the table, we see that $P(2) = 0.0333$.

TABLE 5-15 Excerpt from Appendix II, Table 4, "Poisson Probability Distribution"

	λ				
r	0.1	0.2	0.3	0.4	0.5 ....
0	.9048	.8187	.7408	.6703	.6065
1	.0905	.1637	.2222	.2681	.3033
2	.0045	.0164	.0333	.0536	.0758
3	.0002	.0011	.0033	.0072	.0126
4	.0000	.0001	.0003	.0007	.0016

LOOKING FORWARD

In Example 11 we studied the probability distribution of r, the *number* of fish caught. Now, what about the probability distribution of x, the *waiting time* between catching one fish and the next? It turns out that the random variable x has a continuous distribution called an *exponential distribution*. Continuous distributions will be studied in Chapter 6. To learn more about the exponential distribution and waiting time between catching fish, please see Problem 20 at the end of Section 6.1.

TECH NOTES

The TI-84Plus/TI-83Plus/TI-*n*spire calculators have commands for the geometric and Poisson distributions. Excel 2007 and Minitab support the Poisson distribution. All the technologies have both the probability distribution and the cumulative distribution.

TI-84Plus/TI-83Plus/TI-*n*spire (with TI-84Plus keypad) Use the **DISTR** key and scroll to **geometpdf**(p,n) for the probability of first success on trial number n; scroll to **geomecdf**(p,n) for the probability of first success on trial number $\leq n$. Use **poissonpdf**(λ,r) for the probability of r successes. Use **poissoncdf**(λ,r) for the probability of at least r successes. For example, when $\lambda = 0.25$ and $r = 2$, we get the following results.

```
              poissonpdf(.25,2)
P(r = 2)              .0243375245
              poissoncdf(.25,2)
P(r ≤ 2)              .9978385033
```

Excel 2007 Click on the **Insert Function** (f_x). In the dialogue box, select **Statistical** for the category, and then select **Poisson**. Enter the trial number r, and use λ for the mean. False gives the probability $P(r)$ and True gives the cumulative probability P(at least r).

Minitab Put the r values in a column. Then use the menu choice **Calc ➤ Probability Distribution ➤ Poisson**. In the dialogue box, select probability or cumulative, enter the column number containing the r values, and use λ for the mean.

Poisson Approximation to the Binomial Probability Distribution

Poisson approximation to the binomial

In the preceding examples, we have seen how the Poisson distribution can be used over intervals of time, space, area, and so on. However, the Poisson distribution also can be used as a probability distribution for "rare" events. In the binomial distribution, if the number of trials n is large while the probability p of success is quite small, we call the event (success) a "rare" event. Put another way, it can be shown that for most practical purposes, the Poisson distribution will be a very good *approximation to the binomial distribution*, provided the number of trials n is larger than or equal to 100 and $\lambda = np$ is less than 10. As n gets larger and p gets smaller, the approximation becomes better and better.

PROCEDURE

Problems 28 and 29 show how to compute conditional probabilities for binomial experiments using the Poisson approximation for the binomial distribution. In these problems we see how to compute the probability of r successes out of n binomial trials, given that a certain number of successes will occur.

How to approximate binomial probabilities using Poisson probabilities

Suppose you have a binomial distribution with

n = number of trials

r = number of successes

p = probability of success on each trial

Continued

If $n \geq 100$ and $np < 10$, then r has a binomial distribution that is approximated by a Poisson distribution with $\lambda = np$.

$$P(r) \approx \frac{e^{-\lambda}\lambda^r}{r!}$$

Note: $\lambda = np$ is the expected value of the binomial distribution.

EXAMPLE 12 **Poisson approximation to the binomial**

Isabel Briggs Myers was a pioneer in the study of personality types. Today the Myers–Briggs Type Indicator is used in many career counseling programs as well as in many industrial settings where people must work closely together as a team. The 16 personality types are discussed in detail in the book *A Guide to the Development and Use of the Myers–Briggs Type Indicators,* by Myers and McCaulley. Each personality type has its own special contribution in any group activity. One of the more "rare" types is INFJ (introverted, intuitive, feeling, judgmental), which occurs in only about 2.1% of the population. Suppose a high school graduating class has 167 students, and suppose we call "success" the event that a student is of personality type INFJ.

Ariel Skelley/Photographer's Choice/Getty Images

(a) Let r be the number of successes (INFJ students) in the $n = 167$ trials (graduating class). If $p = P(S) = 0.021$, will the Poisson distribution be a good approximation to the binomial?

SOLUTION: Since $n = 167$ is greater than 100 and $\lambda = np = 167(0.021) \approx 3.5$ is less than 10, the Poisson distribution should be a good approximation to the binomial.

(b) Estimate the probability that this graduating class has 0, 1, 2, 3, or 4 people who have the INFJ personality type.

SOLUTION: Since Table 4 (Appendix II) for the Poisson distribution includes the values $\lambda = 3.5$ and $r = 0, 1, 2, 3,$ or 4, we may simply look up the values for $P(r)$, $r = 0, 1, 2, 3, 4$:

$P(r = 0) = 0.0302$ $\quad P(r = 1) = 0.1057$

$P(r = 2) = 0.1850$ $\quad P(r = 3) = 0.2158$

$P(r = 4) = 0.1888$

Since the outcomes $r = 0, 1, 2, 3,$ or 4 successes are mutually exclusive, we can compute the probability of 4 or fewer INFJ types by using the addition rule for mutually exclusive events:

$$\begin{aligned} P(r \leq 4) &= P(0) + P(1) + P(2) + P(3) + P(4) \\ &= 0.0302 + 0.1057 + 0.1850 + 0.2158 + 0.1888 \\ &= 0.7255 \end{aligned}$$

The probability that the graduating class will have four or fewer INFJ personality types is about 0.73.

(c) Estimate the probability that this class has five or more INFJ personality types.

SOLUTION: Because the outcomes of a binomial experiment are all mutually exclusive, we have

$$P(r \leq 4) + P(r \geq 5) = 1$$

or $P(r \geq 5) = 1 - P(r \leq 4) = 1 - 0.7255 = 0.2745$

The probability is approximately 0.27 that there will be five or more INFJ personality types in the graduating class.

Summary

In this section, we have studied two discrete probability distributions. The Poisson distribution gives us the probability of r successes in an interval of time or space. The Poisson distribution also can be used to approximate the binomial distribution when $n \geq 100$ and $np < 10$. The geometric distribution gives us the probability that our first success will occur on the nth trial. In the next guided exercise, we will see situations in which each of these distributions applies.

PROCEDURE

How to identify discrete probability distributions

Distribution	*Conditions and Setting*	*Formulas*
Binomial distribution	1. There are n independent trials, each repeated under identical conditions. 2. Each trial has two outcomes, S = success and F = failure. 3. $P(S) = p$ is the same for each trial, as is $P(F) = q = 1 - p$ 4. The random variable r represents the number of successes out of n trials. $0 \leq r \leq n$	The probability of exactly r successes out of n trials is $P(r) = \frac{n!}{r!(n-r)!} p^r q^{n-r} = C_{n,r} p^r q^{n-r}$ For r, $\mu = np$ and $\sigma = \sqrt{npq}$ Table 3 of Appendix II has $P(r)$ values for selected n and p.
Geometric distribution	1. There are n independent trials, each repeated under identical conditions. 2. Each trial has two outcomes, S = success and F = failure. 3. $P(S) = p$ is the same for each trial, as is $P(F) = q = 1 - p$ 4. The random variable n represents the number of the trial on which the *first success* occurs. $n = 1, 2, 3, \ldots$.	The probability that the first success occurs on the nth trial is $P(n) = pq^{n-1}$ For n, $\mu = \frac{1}{p}$ and $\sigma = \frac{\sqrt{q}}{p}$
Poisson distribution	1. Consider a random process that occurs over time, volume, area, or any other quantity that can (in theory) be subdivided into smaller and smaller intervals. 2. Identify success in the context of the interval (time, volume, area, . . .) you are studying. 3. Based on long-term experience, compute the mean or average number of successes that occur over the interval (time, volume, area, . . .) you are studying. λ = mean number of successes over designated interval 4. The random variable r represents the number of successes that occur over the interval on which you perform the random process. $r = 0, 1, 2, 3, \ldots$.	The probability of r successes in the interval is $P(r) = \frac{e^{-\lambda}\lambda^r}{r!}$ For r, $\mu = \lambda$ and $\sigma = \sqrt{\lambda}$ Table 4 of Appendix II gives $P(r)$ for selected values of λ and r.

Continued

Distribution	*Conditions and Setting*	*Formulas*
Poisson approximation to the binomial distribution	1. There are n independent trials, each repeated under identical conditions. 2. Each trial has two outcomes, S = success and F = failure. 3. $P(S) = p$ is the same for each trial. 4. In addition, $n \geq 100$ and $np < 10$. 5. The random variable r represents the number of successes out of n trials in a binomial distribution.	$\lambda = np$ the expected value of r. The probability of r successes on n trials is $P(r) \approx \frac{e^{-\lambda}\lambda^r}{r!}$ Table 4 of Appendix II gives values of $P(r)$ for selected λ and r.

GUIDED EXERCISE 9 Select appropriate distribution

For each problem, first identify the type of probability distribution needed to solve the problem: binomial, geometric, Poisson, or Poisson approximation to the binomial. Then solve the problem.

(1) Denver, Colorado, is prone to severe hailstorms. Insurance agents claim that a homeowner in Denver can expect to replace his or her roof (due to hail damage) once every 10 years. What is the probability that in 12 years, a homeowner in Denver will need to replace the roof twice because of hail?

(a) Consider the problem stated in part (I). What is success in this case? We are interested in the probability of two successes over a specified time interval of 12 years. Which distribution should we use?

→ Here we can say success is needing to replace a roof because of hail. Because we are interested in the probability of two successes over a time interval, we should use the Poisson distribution.

(b) In part (I), we are told that the average roof replacement is once every 10 years. What is the average number of times the roof needs to be replaced in 12 years? What is λ for the 12-year period?

→ We are given a value of $\lambda = 1$ for 10 years. To compute λ for 12 years, we convert the denominator to 12 years.

$$\lambda = \frac{0.1}{1\,\text{year}} \cdot \frac{12}{12} = \frac{1.2}{12\,\text{year}}$$

For 12 years, we have $\lambda = 1.2$.

(c) To finish part (I), use the Poisson distribution to find the probability of two successes in the 12-year period.

→ We may use Table 4 of Appendix II because $\lambda = 1.2$ and $r = 2$ are values in the table. The table gives $P(r = 2) = 0.2169$. Using the formula or a calculator gives the same result.

$$P(r) = \frac{e^{-\lambda}\lambda^r}{r!}$$

$$P(r = 2) = \frac{e^{-1.2}(1.2)^2}{2!} \approx 0.2169$$

There is about a 21.7% chance the roof will be damaged twice by hail during a 12-year period.

(II) A telephone network substation will keep trying to connect a long-distance call to a trunk line until the fourth attempt has been made. After the fourth unsuccessful attempt, the call number goes into a buffer memory bank, and the caller gets a recorded message to be patient. During

Continued

GUIDED EXERCISE 9 *continued*

peak calling periods, the probability of a call connecting into a trunk line is 65% on each try. What percentage of all calls made during peak times will wind up in the buffer memory bank?

(d) Consider part (II). What is success? We are interested in the probability that a call will wind up in the buffer memory. This will occur if success does not occur before which trial? Since we are looking at the probability of a first success by a specified trial number, which probability distribution do we use?

→ Success is connecting a long-distance call to a trunk line. The call will go into the buffer memory if success is not achieved during the first four attempts. In symbols, the call will go into the buffer if the trial number n of the first success is such that $n \geq 5$. We use the geometric distribution.

(e) What is the probability of success on a single trial? Use this information and the formula for the geometric distribution to compute the probability that the first success occurs on trial 1, 2, 3, or 4. Then use this information to compute $P(n \geq 5)$, where n is the trial number of the first success. What percentage of the calls go to the buffer?

→ Success means the call connects to the trunk line. According to the description in the problem,

$$P(S) = 0.65 = p$$

By the formula for the geometric probability distribution, where n represents the trial number of the first success,

$$P(n) = p(1 - p)^{n-1}$$

Therefore,

$$P(1) = (0.65)(0.35)^0 = 0.65$$
$$P(2) = (0.65)(0.35)^1 = 0.2275$$
$$P(3) = (0.65)(0.35)^2 \approx 0.0796$$
$$P(4) = (0.65)(0.35)^3 \approx 0.0279$$

$$\begin{aligned} P(n \geq 5) &= 1 - P(1) - P(2) - P(3) - P(4) \\ &\approx 1 - 0.65 - 0.2275 - 0.0796 \\ &\quad - 0.0279 \\ &= 0.015 \end{aligned}$$

About 1.5% of the calls go to the buffer.

(III) The murder rate is 3.6 murders per 100,000 inhabitants (Reference: U.S. Department of Justice, Federal Bureau of Investigation). In a community of 1254 people, what is the probability that at least one person will be murdered?

(f) Consider part (III). What is success in this case? What is the value of n? Find p, the probability of success on a single trial, to six places after the decimal.

→ Success is a person being murdered.

$$n = 1254$$
$$p = \frac{3.6}{100,000} = 0.000036$$

(g) Compute np to three decimal places. Is it appropriate to use the Poisson approximation to the binomial? What is the value of λ to three decimal places?

→ $np \approx 0.045$

Yes.

$\lambda = np \approx 0.045$

(h) Estimate $P(r = 0)$ to three decimal places.

→ $$P(r) \approx \frac{e^{-\lambda}\lambda^r}{r!} \approx \frac{e^{-0.045}(0.045)^0}{0!} \approx 0.956$$

Recall that $0! = 1$.

(i) Use the relation $P(r \geq 1) = 1 - P(0)$ to estimate the probability that at least one person will be murdered.

→ $$\begin{aligned} P(r \geq 1) &\approx 1 - 0.956 \\ &\approx 0.044 \end{aligned}$$

VIEWPOINT When Do Cracks Become Breakthroughs?

No one wants to learn by mistakes! However, learning by our successes will not take us beyond the state of the art! Each new idea, technology, social plan, or engineering structure can be considered a new trial. In the meantime, we the laypeople, whose spokesperson is often a poet or writer, will be threatened by both **failures** *and* **successes**. *This is the nature not only of science, technology, and engineering but also of all human endeavors. [For more discussion on this topic, see Problem 18, as well as* To Engineer Is Human: The Role of Failure in Successful Design *by Professor Petroski (Duke University Press).]*

SECTION 5.4 PROBLEMS

1. *Statistical Literacy* For a binomial experiment, what probability distribution is used to find the probability that the *first* success will occur on a specified trial?
2. *Statistical Literacy* When using the Poisson distribution, which parameter of the distribution is used in probability computations? What is the symbol used for this parameter?
3. *Critical Thinking* Suppose we have a binomial experiment with 50 trials, and the probability of success on a single trial is 0.02. Is it appropriate to use the Poisson distribution to approximate the probability of two successes? Explain.
4. *Critical Thinking* Suppose we have a binomial experiment, and the probability of success on a single trial is 0.02. If there are 150 trials, is it appropriate to use the Poisson distribution to approximate the probability of three successes? Explain.
5. *Basic Computation: Geometric Distribution* Given a binomial experiment with probability of success on a single trial $p = 0.40$, find the probability that the first success occurs on trial number $n = 3$.
6. *Basic Computation: Geometric Distribution* Given a binomial experiment with probability of success on a single trial $p = 0.30$, find the probability that the first success occurs on trial number $n = 2$.
7. *Basic Computation: Poisson Distribution* Given a binomial experiment with $n = 200$ trials and probability of success on a single trial $p = 0.04$, find the value of λ and then use the Poisson distribution to estimate the probability of $r = 8$ successes.
8. *Basic Computation: Poisson Distribution* Given a binomial experiment with $n = 150$ trials and probability of success on a single trial $p = 0.06$, find the value of λ and then use the Poisson distribution to estimate the probability of $r \leq 2$ successes.
9. *College: Core Requirement* Susan is taking western civilization this semester on a pass/fail basis. The department teaching the course has a history of passing 77% of the students in western civilization each term. Let $n = 1, 2, 3, \ldots$ represent the number of times a student takes western civilization until the *first* passing grade is received. (Assume the trials are independent.)
   (a) Write out a formula for the probability distribution of the random variable n.
   (b) What is the probability that Susan passes on the first try ($n = 1$)?
   (c) What is the probability that Susan first passes on the second try ($n = 2$)?
   (d) What is the probability that Susan needs three or more tries to pass western civilization?
   (e) What is the expected number of attempts at western civilization Susan must make to have her (first) pass? *Hint:* Use μ for the geometric distribution and round.

10. ***Law: Bar Exam*** Bob is a recent law school graduate who intends to take the state bar exam. According to the National Conference on Bar Examiners, about 57% of all people who take the state bar exam pass (Source: *The Book of Odds* by Shook and Shook, Signet). Let $n = 1, 2, 3, \ldots$ represent the number of times a person takes the bar exam until the *first* pass.
(a) Write out a formula for the probability distribution of the random variable n.
(b) What is the probability that Bob first passes the bar exam on the second try ($n = 2$)?
(c) What is the probability that Bob needs three attempts to pass the bar exam?
(d) What is the probability that Bob needs more than three attempts to pass the bar exam?
(e) What is the expected number of attempts at the state bar exam Bob must make for his (first) pass? *Hint:* Use μ for the geometric distribution and round.

11. ***Sociology: Hawaiians*** On the leeward side of the island of Oahu, in the small village of Nanakuli, about 80% of the residents are of Hawaiian ancestry (Source: *The Honolulu Advertiser*). Let $n = 1, 2, 3, \ldots$ represent the number of people you must meet until you encounter the *first* person of Hawaiian ancestry in the village of Nanakuli.
(a) Write out a formula for the probability distribution of the random variable n.
(b) Compute the probabilities that $n = 1$, $n = 2$, and $n = 3$.
(c) Compute the probability that $n \geq 4$.
(d) In Waikiki, it is estimated that about 4% of the residents are of Hawaiian ancestry. Repeat parts (a), (b), and (c) for Waikiki.

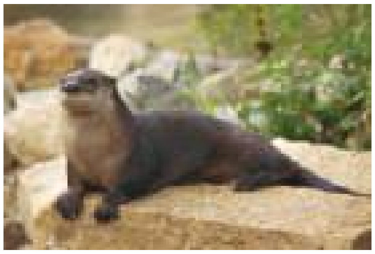
12. ***Agriculture: Apples*** Approximately 3.6% of all (untreated) Jonathan apples had bitter pit in a study conducted by the botanists Ratkowsky and Martin (Source: *Australian Journal of Agricultural Research,* Vol. 25, pp. 783–790). (Bitter pit is a disease of apples resulting in a soggy core, which can be caused either by overwatering the apple tree or by a calcium deficiency in the soil.) Let n be a random variable that represents the first Jonathan apple chosen at random that has bitter pit.
(a) Write out a formula for the probability distribution of the random variable n.
(b) Find the probabilities that $n = 3$, $n = 5$, and $n = 12$.
(c) Find the probability that $n \geq 5$.
(d) What is the expected number of apples that must be examined to find the first one with bitter pit? *Hint:* Use μ for the geometric distribution and round.

13. ***Fishing: Lake Trout*** At Fontaine Lake Camp on Lake Athabasca in northern Canada, history shows that about 30% of the guests catch lake trout over 20 pounds on a 4-day fishing trip (Source: Athabasca Fishing Lodges, Saskatoon, Canada). Let n be a random variable that represents the *first* trip to Fontaine Lake Camp on which a guest catches a lake trout over 20 pounds.
(a) Write out a formula for the probability distribution of the random variable n.
(b) Find the probability that a guest catches a lake trout weighing at least 20 pounds for the *first* time on trip number 3.
(c) Find the probability that it takes more than three trips for a guest to catch a lake trout weighing at least 20 pounds.
(d) What is the expected number of fishing trips that must be taken to catch the first lake trout over 20 pounds? *Hint:* Use μ for the geometric distribution and round.

14. ***Archaeology: Artifacts*** At Burnt Mesa Pueblo, in one of the archaeological excavation sites, the artifact density (number of prehistoric artifacts per 10 liters of sediment) was 1.5 (Source: *Bandelier Archaeological Excavation Project: Summer 1990 Excavations at Burnt Mesa Pueblo and Casa del Rito,* edited by Kohler, Washington State University Department of Anthropology). Suppose you are going to dig up and examine 50 liters of sediment at this site. Let $r = 0, 1, 2, 3, \ldots$ be a random variable that represents the number of prehistoric artifacts found in your 50 liters of sediment.

(a) Explain why the Poisson distribution would be a good choice for the probability distribution of r. What is λ? Write out the formula for the probability distribution of the random variable r.
(b) Compute the probabilities that in your 50 liters of sediment you will find two prehistoric artifacts, three prehistoric artifacts, and four prehistoric artifacts.
(c) Find the probability that you will find three or more prehistoric artifacts in the 50 liters of sediment.
(d) Find the probability that you will find fewer than three prehistoric artifacts in the 50 liters of sediment.

15. ***Ecology: River Otters*** In his doctoral thesis, L. A. Beckel (University of Minnesota, 1982) studied the social behavior of river otters during the mating season. An important role in the bonding process of river otters is very short periods of social grooming. After extensive observations, Dr. Beckel found that one group of river otters under study had a frequency of initiating grooming of approximately 1.7 for every 10 minutes. Suppose that you are observing river otters for 30 minutes. Let $r = 0, 1, 2, \ldots$ be a random variable that represents the number of times (in a 30-minute interval) one otter initiates social grooming of another.



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(a) Explain why the Poisson distribution would be a good choice for the probability distribution of r. What is λ? Write out the formula for the probability distribution of the random variable r.
(b) Find the probabilities that in your 30 minutes of observation, one otter will initiate social grooming four times, five times, and six times.
(c) Find the probability that one otter will initiate social grooming four or more times during the 30-minute observation period.
(d) Find the probability that one otter will initiate social grooming fewer than four times during the 30-minute observation period.

16. ***Law Enforcement: Shoplifting*** *The Denver Post* reported that, on average, a large shopping center has had an incident of shoplifting caught by security once every three hours. The shopping center is open from 10 A.M. to 9 P.M. (11 hours). Let r be the number of shoplifting incidents caught by security in the 11-hour period during which the center is open.
(a) Explain why the Poisson probability distribution would be a good choice for the random variable r. What is λ?
(b) What is the probability that from 10 A.M. to 9 P.M. there will be at least one shoplifting incident caught by security?
(c) What is the probability that from 10 A.M. to 9 P.M. there will be at least three shoplifting incidents caught by security?
(d) What is the probability that from 10 A.M. to 9 P.M. there will be no shoplifting incidents caught by security?

17. ***Vital Statistics: Birthrate*** *USA Today* reported that the U.S. (annual) birthrate is about 16 per 1000 people, and the death rate is about 8 per 1000 people.
(a) Explain why the Poisson probability distribution would be a good choice for the random variable r = number of births (or deaths) for a community of a given population size.
(b) In a community of 1000 people, what is the (annual) probability of 10 births? What is the probability of 10 deaths? What is the probability of 16 births? 16 deaths?
(c) Repeat part (b) for a community of 1500 people. You will need to use a calculator to compute P(10 births) and P(16 births).
(d) Repeat part (b) for a community of 750 people.

18. ***Engineering: Cracks*** Henry Petroski is a professor of civil engineering at Duke University. In his book *To Engineer Is Human: The Role of Failure in Successful Design,* Professor Petroski says that up to 95% of all structural failures, including those of bridges, airplanes, and other commonplace products

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of technology, are believed to be the result of crack growth. In most cases, the cracks grow slowly. It is only when the cracks reach intolerable proportions and still go undetected that catastrophe can occur. In a cement retaining wall, occasional hairline cracks are normal and nothing to worry about. If these cracks are spread out and not too close together, the wall is considered safe. However, if a number of cracks group together in a small region, there may be real trouble. Suppose a given cement retaining wall is considered safe if hairline cracks are evenly spread out and occur on the average of 4.2 cracks per 30-foot section of wall.

(a) Explain why a Poisson probability distribution would be a good choice for the random variable r = number of hairline cracks for a given length of retaining wall.

(b) In a 50-foot section of safe wall, what is the probability of three (evenly spread-out) hairline cracks? What is the probability of three *or more* (evenly spread-out) hairline cracks?

(c) Answer part (b) for a 20-foot section of wall.

(d) Answer part (b) for a 2-foot section of wall. Round λ to the nearest tenth.

(e) Consider your answers to parts (b), (c), and (d). If you had three hairline cracks evenly spread out over a 50-foot section of wall, should this be cause for concern? The probability is low. Could this mean that you are lucky to have so few cracks? On a 20-foot section of wall [part (c)], the probability of three cracks is higher. Does this mean that this distribution of cracks is closer to what we should expect? For part (d), the probability is very small. Could this mean you are not so lucky and have something to worry about? Explain your answers.

19. ***Meteorology: Winter Conditions*** Much of Trail Ridge Road in Rocky Mountain National Park is over 12,000 feet high. Although it is a beautiful drive in summer months, in winter the road is closed because of severe weather conditions. *Winter Wind Studies in Rocky Mountain National Park* by Glidden (published by Rocky Mountain Nature Association) states that sustained gale-force winds (over 32 miles per hour and often over 90 miles per hour) occur on the average of once every 60 hours at a Trail Ridge Road weather station.

(a) Let r = frequency with which gale-force winds occur in a given time interval. Explain why the Poisson probability distribution would be a good choice for the random variable r.

(b) For an interval of 108 hours, what are the probabilities that $r = 2$, 3, and 4? What is the probability that $r < 2$?

(c) For an interval of 180 hours, what are the probabilities that $r = 3$, 4, and 5? What is the probability that $r < 3$?

20. ***Earthquakes: San Andreas Fault*** *USA Today* reported that Parkfield, California, is dubbed the world's earthquake capital because it sits on top of the notorious San Andreas fault. Since 1857, Parkfield has had a major earthquake on the average of once every 22 years.

(a) Explain why a Poisson probability distribution would be a good choice for r = number of earthquakes in a given time interval.

(b) Compute the probability of at least one major earthquake in the next 22 years. Round λ to the nearest hundredth, and use a calculator.

(c) Compute the probability that there will be no major earthquake in the next 22 years. Round λ to the nearest hundredth, and use a calculator.

(d) Compute the probability of at least one major earthquake in the next 50 years. Round λ to the nearest hundredth, and use a calculator.

(e) Compute the probability of no major earthquakes in the next 50 years. Round λ to the nearest hundredth, and use a calculator.

21. **Real Estate: Sales** Jim is a real estate agent who sells large commercial buildings. Because his commission is so large on a single sale, he does not need to sell many buildings to make a good living. History shows that Jim has a record of selling an average of eight large commercial buildings every 275 days.
(a) Explain why a Poisson probability distribution would be a good choice for r = number of buildings sold in a given time interval.
(b) In a 60-day period, what is the probability that Jim will make no sales? one sale? two or more sales?
(c) In a 90-day period, what is the probability that Jim will make no sales? two sales? three or more sales?

22. **Law Enforcement: Burglaries** *The Honolulu Advertiser* stated that in Honolulu there was an average of 661 burglaries per 100,000 households in a given year. In the Kohola Drive neighborhood there are 316 homes. Let r = number of these homes that will be burglarized in a year.
(a) Explain why the Poisson approximation to the binomial would be a good choice for the random variable r. What is n? What is p? What is λ to the nearest tenth?
(b) What is the probability that there will be no burglaries this year in the Kohola Drive neighborhood?
(c) What is the probability that there will be no more than one burglary in the Kohola Drive neighborhood?
(d) What is the probability that there will be two or more burglaries in the Kohola Drive neighborhood?

23. **Criminal Justice: Drunk Drivers** *Harper's Index* reported that the number of (Orange County, California) convicted drunk drivers whose sentence included a tour of the morgue was 569, of which only 1 became a repeat offender.
(a) Suppose that of 1000 newly convicted drunk drivers, all were required to take a tour of the morgue. Let us assume that the probability of a repeat offender is still $p = 1/569$. Explain why the Poisson approximation to the binomial would be a good choice for r = number of repeat offenders out of 1000 convicted drunk drivers who toured the morgue. What is λ to the nearest tenth?
(b) What is the probability that $r = 0$?
(c) What is the probability that $r > 1$?
(d) What is the probability that $r > 2$?
(e) What is the probability that $r > 3$?

24. **Airlines: Lost Bags** *USA Today* reported that for all airlines, the number of lost bags was

May: 6.02 per 1000 passengers   December: 12.78 per 1000 passengers

Note: A passenger could lose more than one bag.
(a) Let r = number of bags lost per 1000 passengers in May. Explain why the Poisson distribution would be a good choice for the random variable r. What is λ to the nearest tenth?
(b) In the month of May, what is the probability that out of 1000 passengers, no bags are lost? that 3 or more bags are lost? that 6 or more bags are lost?
(c) In the month of December, what is the probability that out of 1000 passengers, no bags are lost? that 6 or more bags are lost? that 12 or more bags are lost? (Round λ to the nearest whole number.)

25. **Law Enforcement: Officers Killed** *Chances: Risk and Odds in Everyday Life,* by James Burke, reports that the probability a police officer will be killed in the line of duty is 0.5% (or less).
(a) In a police precinct with 175 officers, let r = number of police officers killed in the line of duty. Explain why the Poisson approximation to the binomial would be a good choice for the random variable r. What is n? What is p? What is λ to the nearest tenth?

(b) What is the probability that no officer in this precinct will be killed in the line of duty?
(c) What is the probability that one or more officers in this precinct will be killed in the line of duty?
(d) What is the probability that two or more officers in this precinct will be killed in the line of duty?

26. ***Business Franchise: Shopping Center*** *Chances: Risk and Odds in Everyday Life,* by James Burke, reports that only 2% of all local franchises are business failures. A Colorado Springs shopping complex has 137 franchises (restaurants, print shops, convenience stores, hair salons, etc.).
(a) Let r be the number of these franchises that are business failures. Explain why a Poisson approximation to the binomial would be appropriate for the random variable r. What is n? What is p? What is λ (rounded to the nearest tenth)?
(b) What is the probability that none of the franchises will be a business failure?
(c) What is the probability that two or more franchises will be business failures?
(d) What is the probability that four or more franchises will be business failures?

27. ***Poisson Approximation to the Binomial: Comparisons***
(a) For $n = 100$, $p = 0.02$, and $r = 2$, compute $P(r)$ using the formula for the binomial distribution and your calculator:

$$P(r) = C_{n,r}p^r(1 - p)^{n-r}$$

(b) For $n = 100$, $p = 0.02$, and $r = 2$, estimate $P(r)$ using the Poisson approximation to the binomial.
(c) Compare the results of parts (a) and (b). Does it appear that the Poisson distribution with $\lambda = np$ provides a good approximation for $P(r = 2)$?
(d) Repeat parts (a) to (c) for $r = 3$.

28. ***Expand Your Knowledge: Conditional Probability*** Pyramid Lake is located in Nevada on the Paiute Indian Reservation. This lake is famous for large cutthroat trout. The mean number of trout (large and small) caught from a boat is 0.667 fish per hour (Reference: *Creel Chronicle,* Vol. 3, No. 2). Suppose you rent a boat and go fishing for 8 hours. Let r be a random variable that represents the number of fish you catch in the 8-hour period.
(a) Explain why a Poisson probability distribution is appropriate for r. What is λ for the 8-hour fishing trip? Round λ to the nearest tenth so that you can use Table 4 of Appendix II for Poisson probabilities.
(b) If you have already caught three trout, what is the probability you will catch a total of seven or more trout? Compute $P(r \geq 7 \mid r \geq 3)$. See Hint below.
(c) If you have already caught four trout, what is the probability you will catch a total of fewer than nine trout? Compute $P(r < 9 \mid r \geq 4)$. See Hint below.
(d) List at least three other areas besides fishing to which you think conditional Poisson probabilities can be applied.

Hint for solution: Review item 6, conditional probability, in the summary of basic probability rules at the end of Section 4.2. Note that

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

and show that in part (b),

$$P(r \geq 7 \mid r \geq 3) = \frac{P((r \geq 7) \text{ and } (r \geq 3))}{P(r \geq 3)} = \frac{P(r \geq 7)}{P(r \geq 3)}$$

29. ***Conditional Probability: Hail Damage*** In western Kansas, the summer density of hailstorms is estimated at about 2.1 storms per 5 square miles. In most cases, a hailstorm damages only a relatively small area in a square mile (Reference: *Agricultural Statistics,* U.S. Department of Agriculture). A crop insurance


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company has insured a tract of 8 square miles of Kansas wheat land against hail damage. Let r be a random variable that represents the number of hailstorms this summer in the 8-square-mile tract.

(a) Explain why a Poisson probability distribution is appropriate for r. What is λ for the 8-square-mile tract of land? Round λ to the nearest tenth so that you can use Table 4 of Appendix II for Poisson probabilities.
(b) If there already have been two hailstorms this summer, what is the probability that there will be a total of four or more hailstorms in this tract of land? Compute $P(r \geq 4 \mid r \geq 2)$.
(c) If there already have been three hailstorms this summer, what is the probability that there will be a total of fewer than six hailstorms? Compute $P(r < 6 \mid r \geq 3)$.

Hint: See Problem 28.

30. ***Expand Your Knowledge: Negative Binomial Distribution*** Suppose you have binomial trials for which the probability of success on each trial is p and the probability of failure is $q = 1 - p$. Let k be a fixed whole number greater than or equal to 1. Let n be the number of the trial on which the kth success occurs. This means that the first $k - 1$ successes occur within the first $n - 1$ trials, while the kth success actually occurs on the nth trial. Now, if we are going to have k successes, we must have at least k trials. So, $n = k, k + 1, k + 2, \ldots$ and n is a random variable. In the literature of mathematical statistics, the probability distribution for n is called the *negative binomial distribution*. The formula for the probability distribution of n is shown in the next display (see Problem 33 for a derivation).

Negative binomial distribution

Negative binomial distribution

Let $k \geq 1$ be a fixed whole number. The probability that the kth success occurs on trial number n is

$$P(n) = C_{n-1,\,k-1}\, p^k q^{n-k}$$

where $$C_{n-1,\,k-1} = \frac{(n-1)!}{(k-1)!(n-k)!}$$

$$n = k, k+1, k+2, \ldots$$

The expected value and standard deviation of this probability distribution are

$$\mu = \frac{k}{p} \quad \text{and} \quad \sigma = \frac{\sqrt{kp}}{p}$$

Note: If $k = 1$, the negative binomial distribution is called the *geometric distribution.*

In eastern Colorado, there are many dry land wheat farms. The success of a spring wheat crop is dependent on sufficient moisture in March and April. Assume that the probability of a successful wheat crop in this region is about 65%. So, the probability of success in a single year is $p = 0.65$, and the probability of failure is $q = 0.35$. The Wagner farm has taken out a loan and needs $k = 4$ successful crops to repay it. Let n be a random variable representing the year in which the fourth successful crop occurs (after the loan was made).

(a) Write out the formula for $P(n)$ in the context of this application.
(b) Compute $P(n = 4)$, $P(n = 5)$, $P(n = 6)$, and $P(n = 7)$.
(c) What is the probability that the Wagners can repay the loan within 4 to 7 years? *Hint:* Compute $P(4 \leq n \leq 7)$.
(d) What is the probability that the Wagners will need to farm for 8 or more years before they can repay the loan? *Hint:* Compute $P(n \geq 8)$.
(e) What are the expected value μ and standard deviation σ of the random variable n? Interpret these values in the context of this application.

31. ***Negative Binomial Distribution: Marketing*** Susan is a sales representative who has a history of making a successful sale from about 80% of her sales contacts. If

she makes 12 successful sales this week, Susan will get a bonus. Let n be a random variable representing the number of contacts needed for Susan to get the 12th sale.

(a) Explain why a negative binomial distribution is appropriate for the random variable n. Write out the formula for $P(n)$ in the context of this application. *Hint:* See Problem 30.

(b) Compute $P(n = 12)$, $P(n = 13)$, and $P(n = 14)$.

(c) What is the probability that Susan will need from 12 to 14 contacts to get the bonus?

(d) What is the probability that Susan will need more than 14 contacts to get the bonus?

(e) What are the expected value μ and standard deviation σ of the random variable n? Interpret these values in the context of this application.

32. ***Negative Binomial Distribution: Type A Blood Donors*** Blood type A occurs in about 41% of the population (Reference: *Laboratory and Diagnostic Tests* by F. Fischbach). A clinic needs 3 pints of type A blood. A donor usually gives a pint of blood. Let n be a random variable representing the number of donors needed to provide 3 pints of type A blood.

(a) Explain why a negative binomial distribution is appropriate for the random variable n. Write out the formula for $P(n)$ in the context of this application. *Hint:* See Problem 30.

(b) Compute $P(n = 3)$, $P(n = 4)$, $P(n = 5)$, and $P(n = 6)$.

(c) What is the probability that the clinic will need from three to six donors to obtain the needed 3 pints of type A blood?

(d) What is the probability that the clinic will need more than six donors to obtain 3 pints of type A blood?

(e) What are the expected value μ and standard deviation σ of the random variable n? Interpret these values in the context of this application.

33. ***Expand Your Knowledge: Brain Teaser*** If you enjoy a little abstract thinking, you may want to derive the formula for the negative binomial probability distribution. Use the notation of Problem 30. Consider two events, A and B.

$$A = \{\text{event that the first } n = 1 \text{ trials contain } k - 1 \text{ successes}\}$$

$$B = \{\text{event that the } n\text{th trial is a success}\}$$

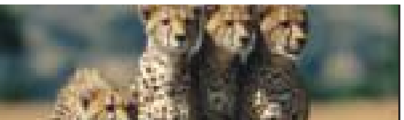
(a) Use the binomial probability distribution to show that the probability of A is $P(A) = C_{n-1,\, k-1}\, p^{k-1} q^{(n-1)-(k-1)}$.

(b) Show that the probability of B is that of a single trial in a binomial experiment, $P(B) = p$.

(c) Why is $P(A \text{ and } B) = P(A) \cdot P(B)$? *Hint:* Binomial trials are independent.

(d) Use parts (a), (b), and (c) to compute and simplify $P(A \text{ and } B)$.

(e) Compare $P(A \text{ and } B)$ with the negative binomial formula and comment on the meaning of your results.



Chapter Review

SUMMARY

This chapter discusses random variables and important probability distributions associated with discrete random variables.

- The value of a *random variable* is determined by chance. Formulas for the mean, variance, and standard deviation of linear functions and linear combinations of independent random variables are given.
- Random variables are either *discrete* or *continuous*.
- A probability distribution of a discrete random variable x consists of all distinct values of x and the corresponding probabilities $P(x)$. For each x, $0 \le P(x) \le 1$ and $\Sigma P(x) = 1$.

- A discrete probability distribution can be displayed visually by a *probability histogram* in which the values of the random variable x are displayed on the horizontal axis, the height of each bar is $P(x)$, and each bar is 1 unit wide.
- For discrete probability distributions,

$$\mu = \Sigma xP(x) \quad \text{and} \quad \sigma = \sqrt{\Sigma(x-\mu)^2 P(x)}$$

- The mean μ is called the *expected value* of the probability distribution.
- A *binomial experiment* consists of a fixed number n of independent trials repeated under identical conditions. There are two outcomes for each trial, called *success* and *failure*. The probability p of success on each trial is the same.
- The number of successes r in a binomial experiment is the random variable for the binomial probability distribution. Probabilities can be computed using a formula or probability distribution outputs from a computer or calculator. Some probabilities can be found in Table 3 of Appendix II.
- For a binomial distribution,

$$\mu = np \quad \text{and} \quad \sigma = \sqrt{npq},$$

where $q = 1 - p$.
- For a binomial experiment, the number of successes is usually within the interval from $\mu - 2.5\sigma$ to $\mu + 2.5\sigma$. A number of successes outside this range of values is unusual but can occur.
- The *geometric probability distribution* is used to find the probability that the first success of a binomial experiment occurs on trial number n.
- The *Poisson distribution* is used to compute the probability of r successes in an interval of time, volume, area, and so forth.
- The *Poisson distribution* can be used to approximate the binomial distribution when $n \geq 100$ and $np < 10$.
- The *hypergeometric distribution* (discussed in Appendix I) is a probability distribution of a random variable that has two outcomes when sampling is done without replacement.

It is important to check the conditions required for the use of each probability distribution.

IMPORTANT WORDS & SYMBOLS

Section 5.1

Random variable 182
  Discrete 182
  Continuous 182
Probability distribution 183
Mean μ of a probability distribution 185
Standard deviation σ of a probability distribution 185
Expected value μ 186
Linear function of a random variable 188
Linear combination of two independent random variables 188

Section 5.2

Binomial experiment 195
Number of trials, n 195
Independent trials 195
Successes and failures in a binomial experiment 195
Probability of success $P(S) = p$ 195
Probability of failure $P(F) = q = 1 - p$ 195
Number of successes, r 195
Binomial coefficient $C_{n,r}$ 198
Binomial probability distribution $P(r) = C_{n,r}p^r q^{n-r}$ 198
Hypergeometric probability distribution (see Appendix I) 204

Section 5.3

Mean for the binomial distribution $\mu = np$ 213
Standard deviation for the binomial distribution $\sigma = \sqrt{npq}$ 213
Quota problem 215

Section 5.4

Geometric probability distribution 222
Poisson probability distribution 224
Poisson approximation to the binomial 227
Negative binomial distribution 238

VIEWPOINT What's Your Type?

Are students and professors really compatible? One way of answering this question is to look at the Myers–Briggs Type Indicators for personality preferences. What is the probability that your professor is introverted and judgmental? What is the probability that you are *extroverted and perceptive? Are most of the leaders in student government extroverted and judgmental? Is it true that members of Phi Beta Kappa have personality types more like those of the professors'? We will consider questions such as these in more detail in Chapter 7 (estimation) and Chapter 8 (hypothesis testing), where we will continue our work with binomial probabilities. In the meantime, you can find many answers regarding careers, probability, and personality types in Applications of the Myers–Briggs Type Indicator in Higher Education, edited by J. Provost and S. Anchors.*

CHAPTER REVIEW PROBLEMS

1. *Statistical Literacy* What are the requirements for a probability distribution?

2. *Statistical Literacy* List the criteria for a binomial experiment. What does the random variable of a binomial experiment measure?

3. *Critical Thinking* For a binomial probability distribution, it is unusual for the number of successes to be less than $\mu - 2.5\sigma$ or greater than $\mu + 2.5\sigma$.
   (a) For a binomial experiment with 10 trials for which the probability of success on a single trial is 0.2, is it unusual to have more than five successes? Explain.
   (b) If you were simply guessing on a multiple-choice exam consisting of 10 questions with 5 possible responses for each question, would you be likely to get more than half of the questions correct? Explain.

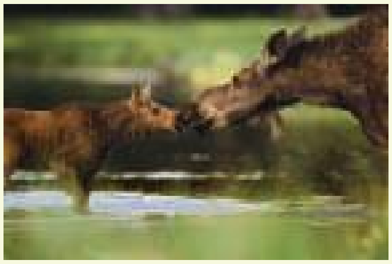
4. *Critical Thinking* Consider a binomial experiment. If the number of trials is increased, what happens to the expected value? to the standard deviation? Explain.

5. *Probability Distribution: Auto Leases* Consumer Banker Association released a report showing the lengths of automobile leases for new automobiles. The results are as follows.

Lease Length in Months	Percent of Leases
13–24	12.7%
25–36	37.1%
37–48	28.5%
49–60	21.5%
More than 60	0.2%

   (a) Use the midpoint of each class, and call the midpoint of the last class 66.5 months, for purposes of computing the expected lease term. Also find the standard deviation of the distribution.
   (b) Sketch a graph of the probability distribution for the duration of new auto leases.

6. *Ecology: Predator and Prey* Isle Royale, an island in Lake Superior, has provided an important study site of wolves and their prey. In the National Park Service Scientific Monograph Series 11, *Wolf Ecology and Prey Relationships on Isle Royale,* Peterson gives results of many wolf–moose studies. Of special interest is the study of the number of moose killed by wolves. In the period from 1958 to 1974, there were 296 moose deaths identified as wolf kills. The age distribution of the kills is as follows.


georgesankar.com/Alamy

Age of Moose in Years	Number Killed by Wolves
Calf (0.5 yr)	112
1–5	53
6–10	73
11–15	56
16–20	2

(a) For each age group, compute the probability that a moose in that age group is killed by a wolf.
(b) Consider all ages in a class equal to the class midpoint. Find the expected age of a moose killed by a wolf and the standard deviation of the ages.

7. ***Insurance: Auto*** State Farm Insurance studies show that in Colorado, 55% of the auto insurance claims submitted for property damage are submitted by males under 25 years of age. Suppose 10 property damage claims involving automobiles are selected at random.
(a) Let r be the number of claims made by males under age 25. Make a histogram for the r-distribution probabilities.
(b) What is the probability that six or more claims are made by males under age 25?
(c) What is the expected number of claims made by males under age 25? What is the standard deviation of the r-probability distribution?

8. ***Quality Control: Pens*** A stationery store has decided to accept a large shipment of ball-point pens if an inspection of 20 randomly selected pens yields no more than two defective pens.
(a) Find the probability that this shipment is accepted if 5% of the total shipment is defective.
(b) Find the probability that this shipment is not accepted if 15% of the total shipment is defective.

9. ***Criminal Justice: Inmates*** According to *Harper's Index*, 50% of all federal inmates are serving time for drug dealing. A random sample of 16 federal inmates is selected.
(a) What is the probability that 12 or more are serving time for drug dealing?
(b) What is the probability that 7 or fewer are serving time for drug dealing?
(c) What is the expected number of inmates serving time for drug dealing?

10. ***Airlines: On-Time Arrivals*** *Consumer Reports* rated airlines and found that 80% of the flights involved in the study arrived on time (that is, within 15 minutes of scheduled arrival time). Assuming that the on-time arrival rate is representative of the entire commercial airline industry, consider a random sample of 200 flights. What is the expected number that will arrive on time? What is the standard deviation of this distribution?

11. ***Agriculture: Grapefruit*** It is estimated that 75% of a grapefruit crop is good; the other 25% have rotten centers that cannot be detected until the grapefruit are cut open. The grapefruit are sold in sacks of 10. Let r be the number of good grapefruit in a sack.
(a) Make a histogram of the probability distribution of r.
(b) What is the probability of getting no more than one bad grapefruit in a sack? What is the probability of getting at least one good grapefruit in a sack?
(c) What is the expected number of good grapefruit in a sack?
(d) What is the standard deviation of the r-probability distribution?

12. ***Restaurants: Reservations*** The Orchard Café has found that about 5% of the diners who make reservations don't show up. If 82 reservations have been made, how many diners can be expected to show up? Find the standard deviation of this distribution.

13. ***College Life: Student Government*** The student government claims that 85% of all students favor an increase in student fees to buy indoor potted plants for the classrooms. A random sample of 12 students produced 2 in favor of the project. What is the probability that 2 or fewer in the sample will favor the project, assuming the student government's claim is correct? ***Interpretation*** Do the data support the student government's claim, or does it seem that the percentage favoring the increase in fees is less than 85%?

14. ***Quota Problem: Financial*** Suppose you are a (junk) bond broker who buys only bonds that have a 50% chance of default. You want a portfolio with at least five bonds that do not default. You can dispose of the other bonds in the

portfolio with no great loss. How many such bonds should you buy if you want to be 94.1% sure that five or more will not default?

15. ***Theater: Coughs*** A person with a cough is a *persona non grata* on airplanes, elevators, or at the theater. In theaters especially, the irritation level rises with each muffled explosion. According to Dr. Brian Carlin, a Pittsburgh pulmonologist, in any large audience you'll hear about 11 coughs per minute (Source: *USA Today*).
(a) Let r = number of coughs in a given time interval. Explain why the Poisson distribution would be a good choice for the probability distribution of r.
(b) Find the probability of three or fewer coughs (in a large auditorium) in a 1-minute period.
(c) Find the probability of at least three coughs (in a large auditorium) in a 30-second period.

16. ***Accident Rate: Small Planes*** Flying over the western states with mountainous terrain in a small aircraft is 40% riskier than flying over similar distances in flatter portions of the nation, according to a General Accounting Office study completed in response to a congressional request. The accident rate for small aircraft in the 11 mountainous western states is 2.4 accidents per 100,000 flight operations (Source: *The Denver Post*).
(a) Let r = number of accidents for a given number of operations. Explain why the Poisson distribution would be a good choice for the probability distribution of r.
(b) Find the probability of no accidents in 100,000 flight operations.
(c) Find the probability of at least 4 accidents in 200,000 flight operations.

17. ***Banking: Loan Defaults*** Records over the past year show that 1 out of 350 loans made by Mammon Bank have defaulted. Find the probability that 2 or more out of 300 loans will default. *Hint:* Is it appropriate to use the Poisson approximation to the binomial distribution?

18. ***Car Theft: Hawaii*** In Hawaii, the rate of motor vehicle theft is 551 thefts per 100,000 vehicles (Reference: U.S. Department of Justice, Federal Bureau of Investigation). A large parking structure in Honolulu has issued 482 parking permits.
(a) What is the probability that none of the vehicles with a permit will eventually be stolen?
(b) What is the probability that at least one of the vehicles with a permit will eventually be stolen?
(c) What is the probability that two or more of the vehicles with a permit will eventually be stolen?

Note: The vehicles may or may not be stolen from the parking structure. *Hint:* Is it appropriate to use the Poisson approximation to the binomial? Explain.

19. ***General: Coin Flip*** An experiment consists of tossing a coin a specified number of times and recording the outcomes.
(a) What is the probability that the *first* head will occur on the second trial? Does this probability change if we toss the coin three times? What if we toss the coin four times? What probability distribution model do we use to compute these probabilities?
(b) What is the probability that the *first* head will occur on the fourth trial? after the fourth trial?

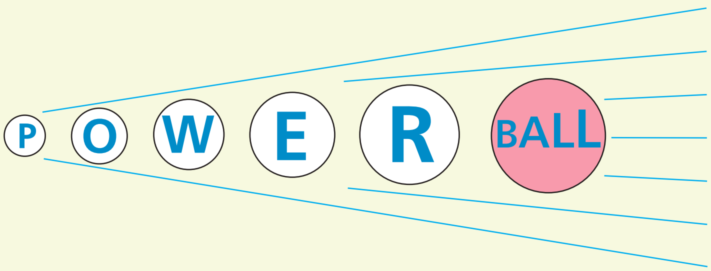
20. ***Testing: CPA Exam*** Cathy is planning to take the Certified Public Accountant Examination (CPA exam). Records kept by the college of business from which she graduated indicate that 83% of the students who graduated pass the CPA exam. Assume that the exam is changed each time it is given. Let $n = 1, 2, 3, \ldots$ represent the number of times a person takes the CPA exam until the *first* pass. (Assume the trials are independent.)
(a) What is the probability that Cathy passes the CPA exam on the first try?
(b) What is the probability that Cathy passes the CPA exam on the second or third try?

DATA HIGHLIGHTS: GROUP PROJECTS

Break into small groups and discuss the following topics. Organize a brief outline in which you summarize the main points of your group discussion.

1. Powerball! Imagine, you could win a jackpot worth at least $10 million. Some jackpots have been worth more than $250 million! Powerball is a multistate lottery. To play Powerball, you purchase a $1 ticket. On the ticket you select five distinct white balls (numbered 1 through 59) and then one red Powerball (numbered 1 through 39). The red Powerball number may be any of the numbers 1 through 39, including any such numbers you selected for the white balls. Every Wednesday and Saturday there is a drawing. If your chosen numbers match those drawn, you win! Figure 5-7 shows all the prizes and the probability of winning each prize and specifies how many numbers on your ticket must match those drawn to win the prize. The Multi-State Lottery Association maintains a web site that displays the results of each drawing, as well as a history of the results of previous drawings. For updated Powerball data, visit the Brase/Brase statistics site at **http://www.cengage.com/statistics/brase** and find the link to the Multi-State Lottery Association.

FIGURE 5-7



Match	Approximate Probability	Prize
5 white balls + Powerball	0.00000000512	Jackpot*
5 white balls	0.000000195	$200,000
4 white balls + Powerball	0.00000138	$10,000
4 white balls	0.0000525	$100
3 white balls + Powerball	0.0000733	$100
3 white balls	0.00279	$7
2 white balls + Powerball	0.00127	$7
1 white ball + Powerball	0.00810	$4
0 white balls + Powerball	0.0162	$3
Overall chance of winning	0.0285	

*The Jackpot will be divided equally (if necessary) among multiple winners and is paid in 30 annual installments or in a reduced lump sum.

(a) Assume the jackpot is $10 million and there will be only one jackpot winner. Figure 5-7 lists the prizes and the probability of winning each prize. What is the probability of *not winning* any prize? Consider all the prizes and their respective probabilities, and the prize of $0 (no win) and its probability. Use all these values to estimate your expected winnings μ if you play one ticket. How much do you effectively contribute to the state in which you purchased the ticket (ignoring the overhead cost of operating Powerball)?

(b) Suppose the jackpot increased to $25 million (and there was to be only one winner). Compute your expected winnings if you buy one ticket. Does the probability of winning the jackpot change because the jackpot is higher?

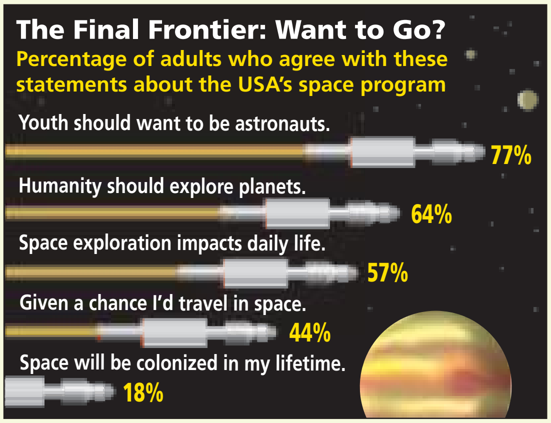
(c) Pretend that you are going to buy 10 Powerball tickets when the jackpot is $10 million. Use the random-number table to select your numbers. Check the Multi-State Lottery Association web site (or any other Powerball site) for the most recent drawing results to see if you would have won a prize.

(d) The probability of winning *any* prize is about 0.0285. Suppose you decide to buy five tickets. Use the binomial distribution to compute the probability of winning (any prize) at least once. *Note:* You will need to use the binomial formula. Carry at least three digits after the decimal.

(e) The probability of winning *any* prize is about 0.0285. Suppose you play Powerball 100 times. Explain why it is appropriate to use the Poisson approximation to the binomial to compute the probability of winning at least one prize. Compute $\lambda = np$. Use the Poisson table to estimate the probability of winning at least one prize.

2. Would you like to travel in space, if given a chance? According to Opinion Research for Space Day Partners, if your answer is yes, you are not alone. Forty-four percent of adults surveyed agreed that they would travel in space if given a chance. Look at Figure 5-8, and use the information presented to answer the following questions.

FIGURE 5-8



Source: Opinion Research for Space Day Partners

(a) According to Figure 5-8, the probability that an adult selected at random agrees with the statement that humanity should explore planets is 64%. Round this probability to 65%, and use this estimate with the binomial distribution table to determine the probability that of 10 adults selected at random, at least half would agree that humanity should explore planets.

(b) Does space exploration have an impact on daily life? Find the probability that of 10 adults selected at random, at least 9 would agree that space exploration does have an impact on daily life. *Hint:* Use the formula for the binomial distribution.

(c) In a room of 35 adults, what is the expected number who would travel in space, given a chance? What is the standard deviation?

(d) What is the probability that the first adult (selected at random) you asked would agree with the statement that space will be colonized in the person's lifetime? *Hint:* Use the geometric distribution.

LINKING CONCEPTS: WRITING PROJECTS

Discuss each of the following topics in class or review the topics on your own. Then write a brief but complete essay in which you summarize the main points. Please include formulas and graphs as appropriate.

1. Discuss what we mean by a binomial experiment. As you can see, a binomial process or binomial experiment involves a lot of assumptions! For example, all

the trials are supposed to be independent and repeated under identical conditions. Is this always true? Can we always be completely certain that the probability of success does not change from one trial to the next? In the real world, there is almost nothing we can be absolutely sure about, so the *theoretical* assumptions of the binomial probability distribution often will not be completely satisfied. Does that mean we cannot use the binomial distribution to solve practical problems? Looking at this chapter, the answer seems to be that we can indeed use the binomial distribution even if not all the assumptions are *exactly* met. We find in practice that the conclusions are sufficiently accurate for our intended application. List three applications of the binomial distribution for which you think, although some of the assumptions are not exactly met, there is adequate reason to apply the binomial distribution anyhow.

2. Why do we need to learn the formula for the binomial probability distribution? Using the formula repeatedly can be very tedious. To cut down on tedious calculations, most people will use a binomial table such as the one found in Appendix II of this book.

   (a) However, there are many applications for which a table in the back of *any* book is not adequate. For instance, compute

   $$P(r = 3) \quad \text{where } n = 5 \text{ and } p = 0.735$$

   Can you find the result in the table? Do the calculation by using the formula. List some other situations in which a table might not be adequate to solve a particular binomial distribution problem.

   (b) The formula itself also has limitations. For instance, consider the difficulty of computing

   $$P(r \geq 285) \quad \text{where } n = 500 \text{ and } p = 0.6$$

   What are some of the difficulties you run into? Consider the calculation of $P(r = 285)$. You will be raising 0.6 and 0.4 to very high powers; this will give you very, very small numbers. Then you need to compute $C_{500,285}$, which is a very, very large number. When combining extremely large and extremely small numbers in the same calculation, most accuracy is lost unless you carry a huge number of significant digits. If this isn't tedious enough, consider the steps you need to compute

   $$P(r \geq 285) = P(r = 285) + P(r = 286) + \cdots + P(r = 500)$$

   Does it seem clear that we need a better way to estimate $P(r \geq 285)$? In Chapter 6, you will learn a much better way to estimate binomial probabilities when the number of trials is large.

3. In Chapter 3, we learned about means and standard deviations. In Section 5.1, we learned that probability distributions also can have a mean and standard deviation. Discuss what is meant by the expected value and standard deviation of a binomial distribution. How does this relate back to the material we learned in Chapter 3 and Section 5.1?

4. In Chapter 2, we looked at the shapes of distributions. Review the concepts of skewness and symmetry; then categorize the following distributions as to skewness or symmetry:

   (a) A binomial distribution with $n = 11$ trials and $p = 0.50$

   (b) A binomial distribution with $n = 11$ trials and $p = 0.10$

   (c) A binomial distribution with $n = 11$ trials and $p = 0.90$

   In general, does it seem true that binomial probability distributions in which the probability of success is close to 0 are skewed right, whereas those with probability of success close to 1 are skewed left?

Using Technology

Binomial Distributions

Although tables of binomial probabilities can be found in most libraries, such tables are often inadequate. Either the value of p (the probability of success on a trial) you are looking for is not in the table, or the value of n (the number of trials) you are looking for is too large for the table. In Chapter 6, we will study the normal approximation to the binomial. This approximation is a great help in many practical applications. Even so, we sometimes use the formula for the binomial probability distribution on a computer or graphing calculator to compute the probability we want.

Applications

The following percentages were obtained over many years of observation by the U.S. Weather Bureau. All data listed are for the month of December.

Location	Long-Term Mean % of Clear Days in Dec.
Juneau, Alaska	18%
Seattle, Washington	24%
Hilo, Hawaii	36%
Honolulu, Hawaii	60%
Las Vegas, Nevada	75%
Phoenix, Arizona	77%

Adapted from *Local Climatological Data,* U.S. Weather Bureau publication, "Normals, Means, and Extremes" Table.

In the locations listed, the month of December is a relatively stable month with respect to weather. Since weather patterns from one day to the next are more or less the same, it is reasonable to use a binomial probability model.

1. Let r be the number of clear days in December. Since December has 31 days, $0 \leq r \leq 31$. Using appropriate computer software or calculators available to you, find the probability $P(r)$ for each of the listed locations when $r = 0, 1, 2, \ldots, 31$.
2. For each location, what is the expected value of the probability distribution? What is the standard deviation?

You may find that using cumulative probabilities and appropriate subtraction of probabilities, rather than addition of probabilities, will make finding the solutions to Applications 3 to 7 easier.

3. Estimate the probability that Juneau will have at most 7 clear days in December.
4. Estimate the probability that Seattle will have from 5 to 10 (including 5 and 10) clear days in December.
5. Estimate the probability that Hilo will have at least 12 clear days in December.
6. Estimate the probability that Phoenix will have 20 or more clear days in December.
7. Estimate the probability that Las Vegas will have from 20 to 25 (including 20 and 25) clear days in December.

Technology Hints

TI-84Plus/TI-83Plus/TI-*n*spire (with TI-84 Plus keypad), Excel 2007, Minitab

The Tech Note in Section 5.2 gives specific instructions for binomial distribution functions on the TI-84Plus/TI-83Plus/TI-*n*spire (with TI-84Plus keypad) calculators, Excel 2007, and Minitab.

SPSS

In SPSS, the function **PDF.BINOM(q,n,p)** gives the probability of q successes out of n trials, where p is the probability of success on a single trial. In the data editor, name a variable r and enter values 0 through n. Name another variable Prob_r. Then use the menu choices **Transform ➤ Compute**. In the dialogue box, use Prob_r for the target variable. In the function group, select **PDF and Noncentral PDF**. In the function box, select **PDF.BINOM(q,n,p)**. Use the variable r for q and appropriate values for n and p. Note that the function **CDF.BINOM(q,n,p)**, from the **CDF and Noncentral CDF** group, gives the cumulative probability of 0 through q successes.