3 Introduction to heat transfer

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Introduction

The importance of heat transfer in food processing is obvious; the changes in food safety and palatability on heating are so marked that many food processes are built around heat transfer. The physical principles which underlie these processes are introduced here. Two modes of heat transfer are discussed: thermal conduction through a stationary medium, and thermal convection through a moving fluid. In the latter case, mixing of hot and cold fluids is the main mechanism of heat transfer, which is usually more rapid than thermal conduction.

Heat transfer operations are usually carried out within some type of specifically designed heat exchanger, such as shell-and-tube or plate exchangers. These units must be designed using equations which predict the heat transfer rate. Some of these equations will use the dimensionless form introduced in Chapter 2; they aim to predict the heat transfer coefficient which relates the heat flux in a given situation to the temperature difference. Various equations are given here to illustrate simple models for heat transfer and to allow simple design calculations to be made. The rate of heat transfer can be related to the power input to the system and thus to the pressure drop through it; models will be developed to take account of these factors.

It is very rare that a single heat transfer process governs the heating of a food material: usually heating or cooling occurs as a result of a combination of processes. For example, in canning, steam condensing on the outside of the can gives rapid heat transfer; but the heat must then be conducted slowly into the food. The slowest process will control the overall heat transfer rate, and identification of this process can simplify the design of process plant. For example, the slowest process in the cooling of solids is usually that of conduction of heat within the solid, so there is no point in enhancing heat transfer between the outside of the solids and its surroundings, beyond the point where it is no longer critical. This key point is often missed in industrial practice. This chapter develops two ways of analysing the problem: first, equations for calculating the overall combined effect of sequential heat

transfer resistances are developed, and secondly, a dimensionless group which shows the relative significance of internal and external heat transfer is derived. These principles are developed further in Chapter 9.

3.1 Heat conduction

3.1.1 Fourier's law

The importance of heat transfer to food processing requires no emphasis when both heating and chilling are such common operations. In addition, freezing and sterilization are operations involving heat transfer. Initially we shall consider the transfer of heat solely by conduction. Inevitably, heat conduction occurs from some hot region of space to some colder neighbouring part. The physical law governing heat conduction within a solid or a stationary fluid is most easily expressed by considering a long, thin metal rod, as depicted in Fig. 3.1. In this one-dimensional situation, position is characterized by the distance x from the left-hand end of the rod. Suppose we begin with the left-hand end relatively hot, so that temperature varies along the rod as shown in Fig. 3.1. This is for an initial time, $t = 0$. If the rod is left for an infinite time ($t = \infty$), a uniform temperature profile (shown by the broken line) will be reached. In general, the rate at which heat flows past any plane along the rod is found to be proportional to the local temperature gradient (dT/dx). This leads to **Fourier's law**:

$$q = -\lambda \frac{dT}{dx} \qquad (3.1)$$

where q is the rate at which heat flows through unit area per unit time; its units are $J\,m^{-2}s^{-1}$ or $W\,m^{-2}$ (i.e. those of a heat flux); dT/dx is the local 'driving force' for heat transfer (units $K\,m^{-1}$); and λ is a constant of proportionality, called the **thermal conductivity**, with units $W\,m^{-1}K^{-1}$.

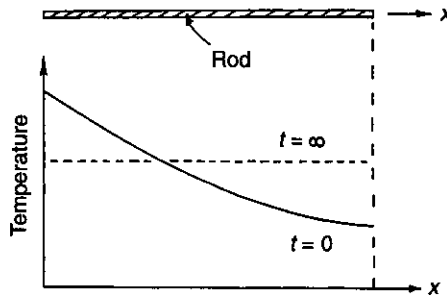**Fig. 3.1** Initial and final temperature in a long heat-conducting rod.

Table 3.1 Values of thermal conductivity, λ, at 20 °C

Substance	λ (W m^{-1}K^{-1})	Substance	λ (W m^{-1}K^{-1})
Silver	41	Mercury	8.8
Copper	386	Water	0.60
Aluminium	200	Ethanol	0.18
Iron	73	Olive oil	0.17
Stainless steel	~42	Benzene	0.15
Ice	2.0	Toluene	0.15
Glass (window)	0.8	Hexane	0.12
Brick (masonry)	~0.7	CCl$_2$F$_2$	0.07
Wood	~0.3	CO$_2$	0.015
Concrete (dry)	0.13	H$_2$	0.18
Charcoal	0.05	O$_2$	0.026
Asbestos	0.11	N$_2$	0.025
Cork	0.045	Air	0.025
Carbohydrate	0.58	NH$_3$	0.024
Apple (75% water)	0.51	H$_2$O vapour	0.023
Chicken meat	0.49	Banana (75% water)	0.48
Muscle	0.41	Human skin	0.37
Protein	0.20	Fat	0.18

Figure 3.1 shows that dT/dx is negative and yet heat flows in a 'positive' direction; this is why the minus sign is included in equation (3.1). This can be compared with the Hagen–Poiseuille equation (2.37) relating volumetric flowrate to pressure gradient for laminar flow; again the minus sign appears. Table 3.1 lists values of λ for various substances at room temperature and pressure. It is seen that λ varies immensely for solids, but not much for liquids, except for liquid metals and water, which are somewhat anomalous. As for gases, λ for H$_2$ and He is large, because these have small molecular masses and sizes. Thus a larger heavy molecule like CO$_2$ has a smaller λ than air. The kinetic theory of gases provides a good theoretical basis for predicting λ for a pure gas and also mixtures of gases. Also, λ for a gas is independent of pressure, but increases with temperature. The value of λ for N$_2$ at 100 °C is 0.031 W m^{-1}K^{-1}, only slightly larger than that listed above. Thus if heat is being conducted from 100 to 20 °C in N$_2$, it is sufficiently accurate to use a mean λ of $(0.025 + 0.031)/2 = 0.028$ W m^{-1}K^{-1}. Some foods, such as potato, have λ close to that of their major constituents (in this case water and carbohydrate). Otherwise, linear rules exist for predicting λ for heterogeneous mixtures, with the contribution from each component being proportional to its volume fraction, as well as its value of λ. For solids, liquids and gases the thermal conductivity increases with temperature. This is often not very significant for pure substances experiencing small changes in temperature. However, because (for example) starch solution gels at

around 60°C, the associated change in its thermal conductivity can be appreciable.

3.1.2 Steady-state heat conduction

Most thermal processes in the food industry are non-steady-state ones (because temperature changes with time), with, for example, the need to freeze or sterilize a food. Unsteady-state processes are very complex mathematically; before dealing with them, we shall consider the simpler case of constant temperatures, which do not vary with time. We now consider the conduction of heat through an infinitely wide slab of material, which has two parallel faces a distance L apart, as shown in Fig. 3.2. This geometry is of relevance in both the thawing of meats and the heating of packet foods. Suppose that the upper face is held at a fixed temperature T_1, whereas the cooler, lower face is always at T_2, with $T_1 > T_2$. The temperature within the material will eventually reach a **steady** distribution, which does not change with time; this is shown in Fig. 3.3, where x is now the distance from the upper face. Heat flows from the hot face to the cold one at a rate given by equation (3.1). Thus the heat flux is
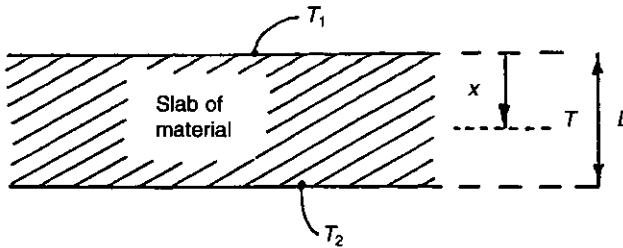

Fig. 3.2 A large slab of material with two parallel faces, distance L apart and at temperatures T_1 and T_2, with $T_1 > T_2$.
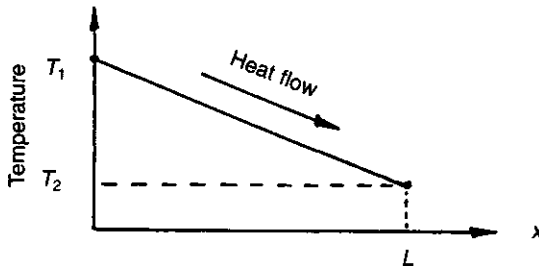

Fig. 3.3 Steady-state variation of temperature within the material of Fig. 3.2.

$$q = -\lambda \frac{dT}{dx} = \frac{\lambda(T_1 - T_2)}{L}$$

In this situation q does not vary with x; otherwise heat would be accumulating or being lost somewhere, which is not possible here in the steady state. Because q is not a function of x, equation (3.1) requires that dT/dx is independent of x, i.e. q is a constant, so that temperature must vary linearly with x from T_1 at the top face to T_2 at the lower side. It is now convenient to introduce two new parameters. The first is a **heat transfer coefficient** h defined by

$$q = h(T_1 - T_2) \qquad (3.2)$$

Equation (3.2) asserts that in this situation the heat flux q is proportional to the applied temperature difference, or overall driving force, $(T_1 - T_2)$, for heat transfer. Hence h is simply a constant of proportionality and has units of $W\,m^{-2}K^{-1}$. Comparison of the two preceding equations gives

$$h = \frac{\lambda}{L} \qquad (3.3)$$

The heat transfer coefficient turns out to be a very convenient way of expressing measured heat fluxes as a function of an applied temperature difference.

The second new parameter is a dimensionless group, which in general is equal to hd/λ, where d is a characteristic dimension for the particular problem. Here one can only take $d = L$, so that this group, called the **Nusselt number**, is given by

$$Nu = \frac{hL}{\lambda}$$

Use of equation (3.3) yields $Nu = 1$ for this general problem.

EXAMPLE 3.1

A room in a house has one external wall, 6m × 3m. The temperature in the room is 20°C; outside it is –5°C. Calculate the rate of energy loss through this external wall, given that it is 0.3m thick and made of brick (without a cavity) with a thermal conductivity of 0.7Wm⁻¹K⁻¹.

$$\text{Rate of heat loss} = \lambda \times \text{Area} \times \frac{\Delta T}{\text{thickness}}$$

$$= 0.7 \times 6 \times 3 \times \frac{25}{0.3} = 1.05\text{kW}$$

A similar analysis will now be carried out for the radial flow of heat in a very long pipe with thick walls. The pipe is shown in Fig. 3.4. The inside wall is held at a constant temperature T_1 (for example, by passing a mixture of steam and boiling water through the pipe), while the outer wall is held at a lower, but constant temperature T_2. Heat thus flows radially outwards. Figure 3.4 shows an imaginary surface of radius r, with $r_1 < r < r_2$. The rate at which heat flows past this surface is $Q = 2\pi rqL$ for length L of the pipe or of the imaginary surface. Again, Q cannot be a function of r in the steady state, so substitution into equation (3.1) leads to

$$q = \frac{Q}{2\pi rL} = -\lambda \frac{dT}{dr}$$

or

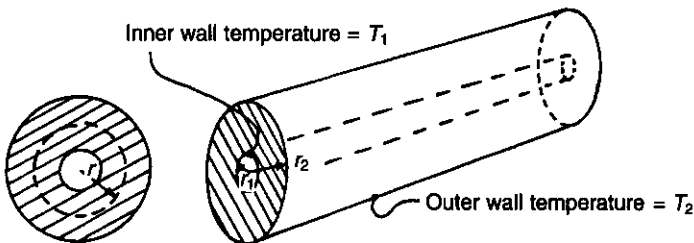$$\frac{Q}{2\pi L} \int_{r_1}^{r_2} \frac{dr}{r} = -\lambda \int_{T_1}^{T_2} dT$$

That is,

$$\frac{Q}{2\pi L} \ln\left(\frac{r_2}{r_1}\right) = \lambda\left(T_1 - T_2\right) \tag{3.4}$$

This enables Q to be calculated. Alternatively, the variation of T with r within the pipe's walls could have been calculated, using in addition

$$\frac{Q}{2\pi L} \ln\left(\frac{r}{r_1}\right) = \lambda\left(T_1 - T\right)$$

The above definition of $Q = 2\pi rqL$ and equation (3.4) lead to

$$\frac{Q}{2\pi L} = rq = \frac{\lambda\left(T_1 - T_2\right)}{\ln\left(\frac{r_2}{r_1}\right)} = \text{a constant}$$

Fig. 3.4 Long pipe with thick walls and length L, and the inside wall maintained hotter than the outside wall. Inner and outer radii are r_1 and r_2.

so that the product qr is a constant. This makes it now impossible to define a unique heat transfer coefficient, $h = q/\Delta T$, where $\Delta T = T_1 - T_2$ and is the applied temperature difference (the driving force) leading to heat transfer. In fact, because qr is constant, we have to specify r in order to obtain a value for h. Two possibilities arise. If we choose $r = r_1$, then h referred to this inner surface will be

$$h_1 = \frac{\lambda}{r_1 \ln\left(\dfrac{r_2}{r_1}\right)}$$

Alternatively, if we choose $r = r_2$, then the heat transfer coefficient with respect to the outer surface of the pipe would be

$$h_2 = \frac{\lambda}{r_2 \ln\left(\dfrac{r_2}{r_1}\right)}$$
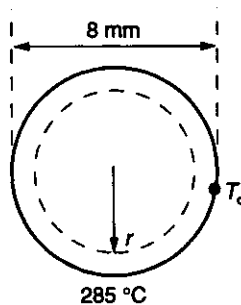
As for the Nusselt number, we have:

at $r = r_1; Nu = \dfrac{h_1 r_1}{\lambda} = \dfrac{1}{\ln\left(r_2/r_1\right)}$

at $r = r_2; Nu = \dfrac{h_2 r_2}{\lambda} = \dfrac{1}{\ln\left(r_2/r_1\right)}$

So although h depends on whether $r = r_1$ or r_2, the corresponding Nu does not. Herein lies a major importance of the Nusselt number.

EXAMPLE 3.2

In a nuclear reactor, the fuel elements may be considered to consist of long cylindrical rods of uranium oxide of diameter 8 mm surrounded by a

285 °C

very thin layer of aluminium cladding. The elements are cooled by boiling water at 285 °C with a heat transfer coefficient of 35 kW m²K⁻¹. If heat is generated uniformly throughout the uranium oxide at a rate of 760 MW m⁻³, find the temperature of the cladding and the maximum temperature in the uranium oxide. The thermal conductivity of uranium oxide may be taken to be 2.3 W m⁻¹K⁻¹.

Consider unit length of rod.
Rate of heat transfer from cylinder to water

$$= \pi \times 8 \times 10^{-3} \times 35 \times 10^{3}(T_c - 285)$$

$$= \frac{\pi}{4}(8 \times 10^{-3})^2 \times 760 \times 10^6$$

$$= 38\,202\,\text{W}$$

$$T_c - 285 = 43.4 \Rightarrow T_c = 328.4\,°\text{C}$$

Heat flux across imaginary cylinder of radius r (see diagram)

$$= \frac{\pi r^2 \times 760 \times 10^6}{2\pi r}$$

$$= 380 \times 10^6 r = -\lambda \frac{dT}{dr}$$

Therefore

$$\frac{380 \times 10^6}{2.3} \int_{4 \times 10^3}^{0} r\,dr = -\int_{328.4}^{T_{max}} dT$$

$$\frac{380 \times 10^6}{2.3} \times \frac{(4 \times 10^{-3})^2}{2} = T_{max} - 328.4$$

$$1321.7 = T_{max} - 328.4$$

$$T_{max} = 1650.1\,°\text{C}$$

As a final example of steady-state heat conduction, consider a hollow sphere with thick walls; a cross-section is shown in Fig. 3.5. Again the inner and outer radii are r_1 and r_2, where the temperatures are held at T_1 and T_2, respectively. Assume $T_1 > T_2$, so that heat flows radially outwards past any imaginary sphere of radius r, where $r_1 < r < r_2$. The total rate at which heat passes this surface of radius r is

$$Q = 4\pi r^2 q$$

where q is the heat flux (in W m⁻²) at r. In the steady state Q is independent of r, or else, for example, there might be a local accumulation of heat. In this case equation (3.1) becomes
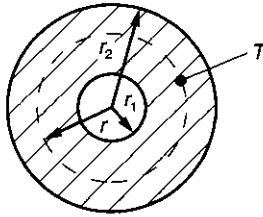
Fig. 3.5 Cross-section of a spherical shell with thick walls. The inner surface is held at a constant temperature T_1, while the outer surface is maintained at T_2.

$$q = \frac{Q}{4\pi r^2} = -\lambda \frac{\mathrm{d}T}{\mathrm{d}r} \qquad (3.5)$$

or

$$\frac{Q}{4\pi}\int_{r_1}^{r_2} \frac{\mathrm{d}r}{r^2} = -\lambda \int_{T_1}^{T_2} \mathrm{d}T$$

or

$$\frac{Q}{4\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \lambda\left(T_1 - T_2\right) \qquad (3.6)$$

Equation (3.5) shows that q is proportional to $1/r^2$, which again creates problems when defining h. Thus

$$h = \frac{q}{\Delta T} = \frac{Q/\left(4\pi r^2\right)}{T_1 - T_2} \qquad (3.7)$$

so that h is a function of r and once again there is not one unique h for this problem. The most interesting case is for $r_2 \to \infty$, corresponding to heat conduction from a sphere of radius r_1 into an infinite stagnant medium. In this case equation (3.6) gives

$$\frac{Q}{4\pi r_1} = \lambda\left(T_1 - T_2\right)$$

for $T = T_2$ at $r_2 = \infty$. We can thus obtain h referred to $r = r_1$ from equation (3.7) as

$$h_1 = \frac{Q/\left(4\pi r_1^2\right)}{T_1 - T_2} = \frac{\lambda}{r_1}$$

When defining a Nusselt number it is traditional to use the diameter of the sphere as the characteristic dimension. In this case

$$Nu = \frac{h(2r_1)}{\lambda} = 2 \qquad (3.8)$$

EXAMPLE 3.3

Estimate the heat transfer coefficient h *for a sphere of diameter* d = 25 mm *surrounded by stagnant air, using* $\lambda = 0.025\,W m^{-1} K^{-1}$.

Here $Nu = 2 = \dfrac{hd}{\lambda}$

Therefore

$$h = \frac{2\lambda}{d} = \frac{2 \times 0.025}{25 \times 10^{-3}} = 2.0\,W m^{-2} K^{-1}$$

EXAMPLE 3.4

Estimate h *for a sphere of diameter* d = 1 mm *in stagnant air.*

Because $h \propto 1/d$, from equation (3.8):

$$h = 2.0 \times 25/1 = 50\,W m^{-2} K^{-1}$$

Note that, because $h \propto 1/d$, h becomes large for small spheres.

3.1.3   *Problems of thawing and freezing foods*

The problems are best illustrated by considering water (e.g. on a pond) freezing, with the air above the ice inevitably at a temperature below the melting point of ice. The situation is depicted in Fig. 3.6, where the interface between the ice and water is at T_{mp}, the melting point of ice (0 °C). If A is the area of the pond's surface, the heat conducted through the ice to the air is $A\rho h_{fi}dx$ when the thickness of the ice increases by an amount dx. Here ρ is the density of ice and h_{fi} is its latent heat of fusion. The situation is similar to that shown in Fig. 3.3, and once again dT/dx is constant between the upper and lower surfaces of the ice. Substitution into Equation (3.1) yields

$$q = -\frac{A\rho h_{fi}dx}{A dt} = -\lambda \frac{(T_{mp} - T_A)}{x}$$

or

$$\frac{dx}{dt} = \frac{\lambda(T_{mp} - T_a)}{\rho h_{fi} x}$$
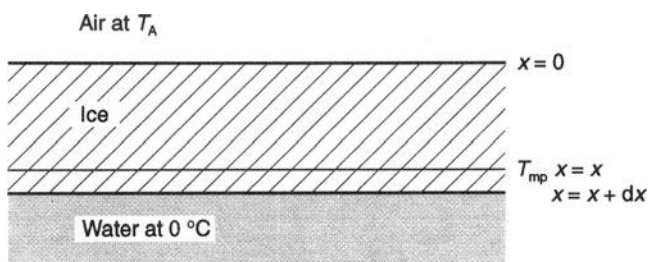
Fig. 3.6 Ice forming on a pond. At a time t the thickness of the ice is x, but at a later time $(t + dt)$ the thickness has grown to $x + dx$.

If $x = x_0$ at $t = 0$, then after a time t the thickness grows to

$$x^2 = x_0^2 + \frac{2\lambda\left(T_{mp} - T_A\right)}{\rho h_{fi}} \cdot t$$

which is obtained by integrating the previous equation. It is worthwhile stressing that the above treatment has assumed: (a) only the removal of latent heat is significant in this problem; (b) all the physical properties (λ, ρ, h_{fi}) of ice can be taken not to vary with temperature; (c) the freezing front is at a constant temperature; and (d) the system is in effect in a steady state. It is worthwhile inserting the following numerical values: $T_A = -10\,°C$, $\rho = 920\,kg\,m^{-3}$, $\lambda = 2.0\,W\,m^{-1}\,K^{-1}$, $x_0 = 0$ and $h_{fi} = 334\,kJ\,kg^{-1}$. In this case the first 10 mm of ice takes 768 s (12.8 min) to form, the second 10 mm a further 38.4 min, and so on.

As shown below, this approach can simply be applied to foods.

EXAMPLE 3.5

It is intended to freeze a hamburger (of thickness 0.012 m), which is 60wt% water, using air at −40 °C on both sides. Estimate the freezing time using the above approach. Will it be an over- or underestimate?

Data

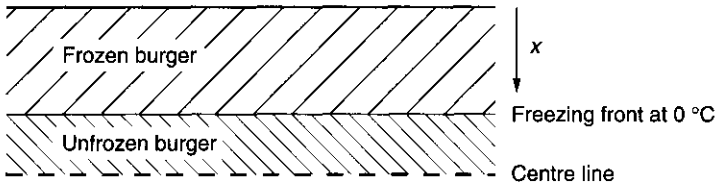Latent heat of fusion of water: 334 kJ kg^{-1}
Thermal conductivity of burger: 0.18 W m^{-1} K^{-1}
Density of burger: 870 kg m^{-3}

Since the burger has two sides, it is only necessary for the freezing front from each side to move 0.006 m into the food.

$$q = -\lambda \frac{40}{x} = -h_{fi} \times 0.6\rho \frac{dx}{dt}$$

$$\frac{40\lambda}{0.6h_{fi}\rho} = x \frac{dx}{dt} \quad \text{or} \quad x^2 = \frac{80\lambda t}{0.6h_{fi}\rho}$$

The freezing time is $\dfrac{0.6 \times 334 \times 10^3 \times 870 \times (0.012/2)^2}{80 \times 0.18}$

or 436 s. This is an underestimate because heat transfer through the air has been ignored. This is dealt with below in section 3.3.4. There are other errors, such as the sensible heat (i.e. that associated with a change of temperature, in contrast with latent heats, which are transferred without a body changing temperature) being ignored. Thus a greater amount of heat than that calculated above must be conducted through the burger to reduce its temperature from above zero to almost −40 °C. A more general version of the above treatment is given in section 3.3.4.

3.1.4   Unsteady-state heat conduction

Previous problems have involved temperatures that are constant with time, but vary in space. If unsteady-state processes (those that vary with time) are considered, time derivatives must be taken into account, as well as spatial ones. Consider the long thin metal rod of Fig. 3.1 with a temperature profile that changes with time. The rod is shown in Fig. 3.7. The heat flux from left to right at position x is q $(= -\lambda dT/dx)$. Let the heat flux in the same, positive, direction at $x + dx$ be $q + dq$. The increment between x and $x + dx$ has a volume $A dx$ and mass $A\rho dx$, where A is the constant cross-sectional area and ρ is density. The rate at which the heat content, or enthalpy, of this increment of length decreases is

$$-A \rho \, dx c_P \frac{\partial T}{\partial t} = A \, dq$$

Here c_P is the heat capacity of the rod per unit mass, so c_P has units $J kg^{-1} K^{-1}$. Note that q is the heat flux away from the origin, so that if dq is
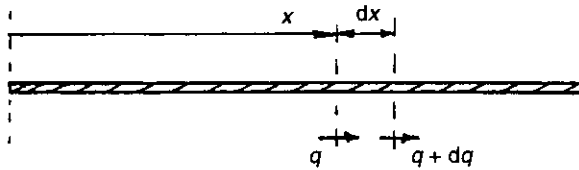
Fig. 3.7 A long thin rod of cross-sectional area A, where the temperature varies both with position x and with time.

positive, the increment cools. Thus the signs in the above equation are correct and

$$-\rho\, c_P \left(\frac{\partial T}{\partial t}\right) dx = dq = d\left(-\lambda \frac{\partial T}{\partial x}\right)$$

Here, as the temperature is a function both of distance and time, the notation for partial differentiation is needed and leads to:

$$\frac{\partial T}{\partial t} = \frac{1}{\rho c_P} \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right)$$

If λ is independent of temperature, this becomes

$$\frac{\partial T}{\partial t} = \frac{\lambda}{\rho c_P} \frac{\partial^2 T}{\partial x^2} = \alpha \frac{\partial^2 T}{\partial x^2} \tag{3.9}$$

Here $\alpha\ (= \lambda/\rho c_P)$ is called the **thermal diffusivity** of the rod. Its units are $m^2 s^{-1}$, which also are those of kinematic viscosity and the diffusion coefficient.

EXAMPLE 3.6

Estimate the thermal diffusivity for air and water, respectively, both at 20 °C, given $\rho = 1.29\,kg\,m^{-3}$ for air and $c_P = 1005\,J\,kg^{-1}K^{-1}$ for air and $c_P = 4.187\,J\,kg^{-1}K^{-1}$ for water. Use the values of λ in Table 3.1.
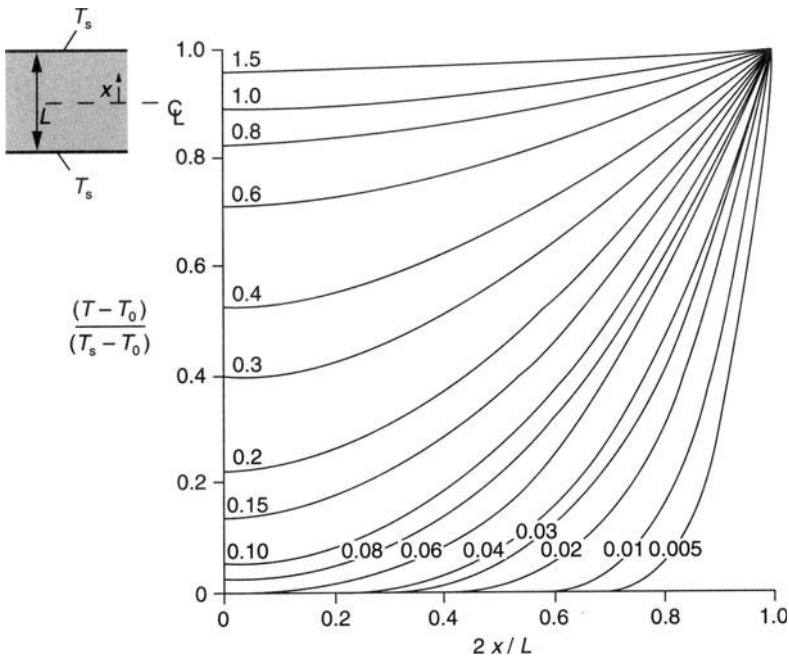
For air:

$$\alpha = \frac{\lambda}{\rho c_P} = \frac{0.025}{1.29 \times 1005} = 1.9 \times 10^{-5}\, m^2\, s^{-1}$$

For water:

$$\alpha = \frac{0.60}{10^3 \times 4.187 \times 10^3} = 1.4 \times 10^{-7}\, m^2\, s^{-1}$$

The difference between the two values of α should be noted.

Fig. 3.8 Temperature distributions at various times within a slab of initial temperature T_0 and surface temperatures of T_s. The numbers on the curves correspond to the values of $4\alpha t/L^2$.

Equation (3.9) presents difficulties as far as obtaining simple analytical solutions is concerned; in fact they only exist for a few problems. In general, solutions can be obtained for equation (3.9), provided the boundary conditions are specified, using computerized techniques. We shall just present graphical solutions to a restricted number of problems. Consider first the slab in Fig. 3.2. Suppose the entire material initially has a uniform temperature T_0, but at time $t = 0$ the two faces are suddenly raised to a temperature T_s, which is then maintained at all subsequent times. Figure 3.8 is a plot of $(T - T_0)/(T_s - T_0)$ against $2x/L$ for various values of $4\alpha t/L^2$. It is clear from Fig. 3.8 that when $4\alpha t/L^2 = 2.0$, the material is uniformly heated. This corresponds to a heating time of $L^2/2\alpha$ for the slab. Similar plots are given in Fig. 3.9 for a solid sphere which is initially at uniform temperature T_0. From a time $t = 0$ the surface of the sphere is held at a temperature T_s so that heat flows radially inwards. The curves in Fig. 3.9 are again of the fractional temperature rise $(T - T_0)/(T_s - T_0)$ against dimensionless radius r/r_s, where r_s is the sphere's radius, for increasing values of $\alpha t/r_s^2$. Figure 3.9 shows that at a time when $\alpha t/r_s^2 = 0.5$, the sphere is close to being at a uniform temperature T_s. Thus $r_s^2/2\alpha$ is a characteristic heating time, proportional to $1/\alpha$. Something more accurate is presented in the following example.
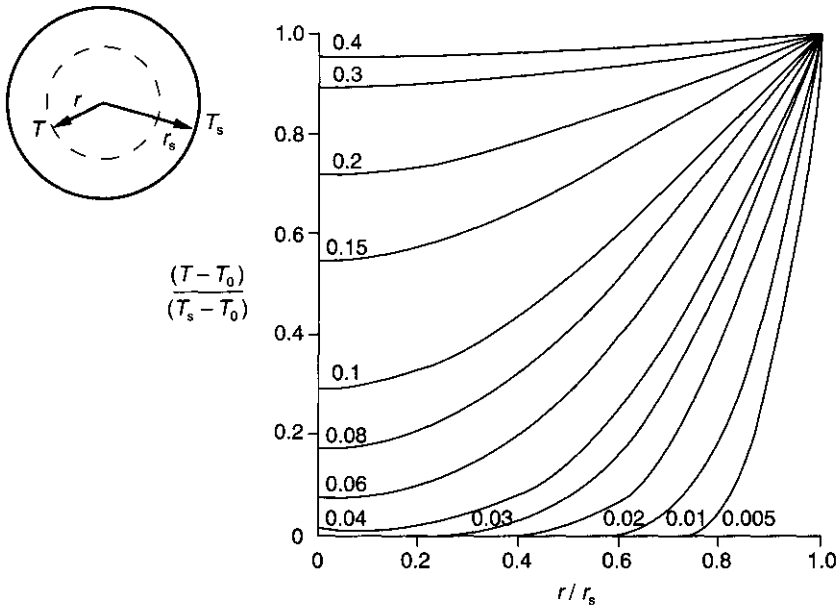
Fig. 3.9 Temperature profiles at various times t in a solid sphere of radius r_s, with initial temperature T_0 and constant surface temperature T_s. The numbers on the curves are the values of $\alpha t/r_s^2$, where $\alpha = \lambda/\rho c_P$ is the thermal diffusivity of the solid. The graph thus gives information on the temperature T at a distance r from the centre of symmetry.

EXAMPLE 3.7

A potato has the following properties:

$$\lambda = 0.50\,W\,m^{-1}\,K^{-1}$$
$$\rho = 1100\,kg\,m^{-3}$$
$$c_P = 3.5\,kJ\,kg^{-1}\,K^{-1}$$

It may be considered to be boiled when its centre reaches 85 °C, when immersed in water boiling at 100 °C. Estimate the time taken to boil a potato (initially at 20 °C) of radius 25 mm. Assume the potato is spherical.

When the centre has reached 85 °C:

$$\frac{T - T_0}{T_s - T_0} = \frac{85 - 20}{100 - 20} = 0.8125$$

From Fig. 3.9 the ratio $(T - T_0)/(T_s - T_0) = 0.8125$ at $r = 0$ when $\alpha t/r_s^2 = 0.24$. Thus the boiling time is

$$t = \frac{0.24\,r_s^2}{\alpha} = 0.24 \times \left(25 \times 10^{-3}\right)^2 \times \frac{1100 \times 3.5 \times 10^3}{0.50}$$
$$= 1155\,s = 19.3\,min$$
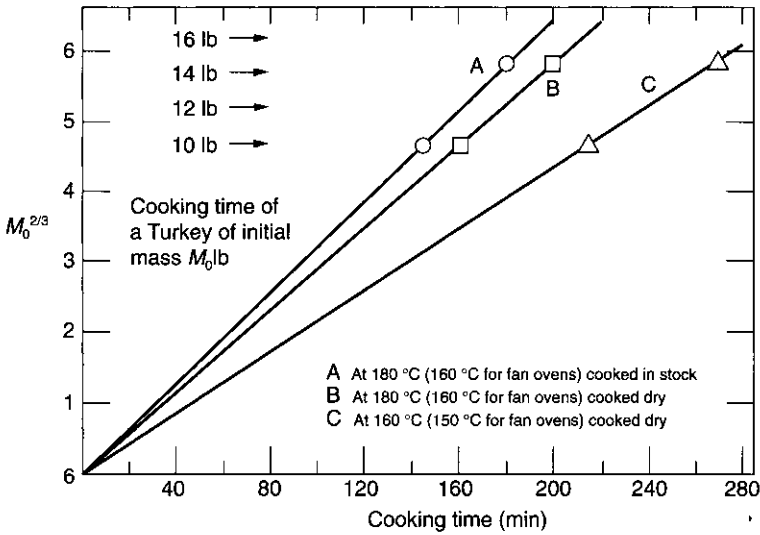
Fig. 3.10 Plots of $M_0^{2/3}$ (where M_0 is the initial mass of a turkey) against the cooking time determined experimentally. Conditions for cooking schemes A, B and C are given on the graph.

Charts such as Figs 3.8 and 3.9 are useful in calculating the cooking or freezing times of foods with simple geometries. The complex shapes of real foods are often difficult to approximate; computer simulations are thus necessary to predict their process times.

The above discussion can be taken a stage further. Consider the roasting of a turkey. It turns out that the optimal cooking time is the minimum time required to heat the centre of the bird to 70°C. This gives a reasonably tender piece of meat with an exterior that is slightly overcooked, i.e. browned. If a turkey is assumed to be spherical, it will have an initial mass equal to $M_0 = 4\pi r_m^3\rho/3$ for an initial density ρ and mean radius r_m. As a result $r_m = (3M_0/4\pi\rho)^{1/3}$. The above discussion indicates that $\alpha t_r/r_m^2$ will be a constant, if t_r is the time for the temperature (at the centre of the turkey) to reach 70°C with a fixed oven temperature. Thus a graph of r_m^2 against roasting time t_r should be a straight-line plot through the origin. This is seen to be the case in Fig. 3.10, which is actually a plot of $M_0^{2/3}$ (proportional to r_m^2) against cooking time. In each of the three cooking situations a straight-line plot through the origin is obtained. Such charts are obviously useful for predicting cooking times.

3.2   Heat transfer in flowing systems

Conduction of heat is slower than convective heat transfer, where heat is mainly transferred by the movement of a fluid. Convective processes cannot

be modelled analytically in the way conduction can; it is accordingly neces-
sary to construct more empirical models and also to devise correlations to
predict heat transfer rates. First we investigate the result of dimensional
analysis. This approach will be seen to identify some relevant parameters,
which in fact are dimensionless groups. Afterwards in sections 3.2.2 and
3.2.3 two oversimple models (the film model and the Reynolds analogy) are
described, because they bring out the relevant dimensionless groups, but
without being quantitatively precise.

3.2.1 Dimensional analysis

Here we consider one fluid flowing, so that the transfer of heat will be by
convection, as well as by conduction. Nevertheless, it is possible to use the
heat transfer coefficient, defined by $q = h\Delta T$ in equation (3.2). We begin by
considering the situation in Fig. 3.11. Here a cold fluid is passed through a
thin-walled pipe whose walls are maintained uniformly hot: for example, by
having steam condense on the exterior surface. Heat thus flows from the
pipe to the fluid inside it. It is possible to define a local heat transfer
coefficient $h = q/\Delta T$ for a point along the system. Here q is the local heat flux
and $\Delta T = (T_w - T)$ is the difference between the wall and fluid temperatures
at the particular point. This parameter h is likely to depend on the following
seven quantities:

d = pipe's internal diameter (m)
v = mean velocity of the fluid in the pipe (m s^{-1})
λ = thermal conductivity of the fluid (W m^{-1} K^{-1})
ρ = density of the fluid (kg m^{-3})
c_P = specific heat capacity of the fluid (J kg^{-1} K^{-1})
μ = viscosity of the fluid (N s m^{-2})
T = temperature of the fluid (K)

It has been assumed that the pipe's length does not matter. Four of the
above (λ, ρ, c_P and μ) are physical properties of the fluid being heated.
Dimensional analysis, as discussed in Chapter 2, leads to
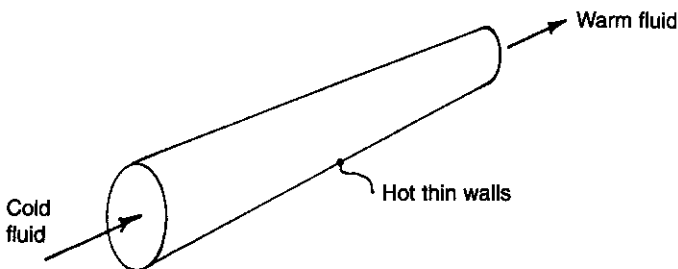
Fig. 3.11 A fluid being heated by passing it through a pipe whose walls are maintained hot.

$$Nu = f\left(Re,\ Pr\right) \qquad\qquad (3.10)$$

where

$$Nu = \frac{hd}{\lambda}$$

$$Re = \frac{\rho v d}{\mu}$$

$$Pr = \frac{\mu c_P}{\lambda}$$

The three dimensionless groups are the **Nusselt**, **Reynolds** and **Prandtl numbers**. They incorporate h and the above seven variables. The Nusselt number gives h, the Reynolds number characterizes the nature of the fluid flow (laminar or turbulent), and the Prandtl number is determined entirely by the physical properties of the fluid being heated.

As a digression it is seen that

$$Pr = \frac{\mu}{\rho} \cdot \frac{\rho c_P}{\lambda} = \frac{\nu}{\alpha}$$

where $\nu = \mu/\rho$ is the kinematic viscosity of the fluid. Both ν and α have dimensions of $m^2 s^{-1}$. It turns out that Pr for all gases is close to unity. However, for liquids Pr varies considerably, as seen from Table 3.2, where the effects of μ and λ are clear. Note that food materials with high viscosities, such as soups, will tend to have Pr greater than that of water.

It is common to introduce another dimensionless group, the **Stanton number**. This is given by

$$St = \frac{h}{\rho v c_P} = \frac{hd}{\lambda} \cdot \frac{\lambda}{\mu c_P} \frac{\mu}{\rho v d} = \frac{Nu}{Pr.Re}$$

Thus equation (3.10) could be re-written as

$$St = f\left(Re,\ Pr\right) \qquad\qquad (3.11)$$

Table 3.2 Values of Prandtl number for various liquids

Liquid	Pr	Comment
Mercury	0.0027	μ is low, λ high
Water	6	
Kerosene	127	
Glycerine	17000	μ is high, λ low
Liquid polymers	$>10^4$	
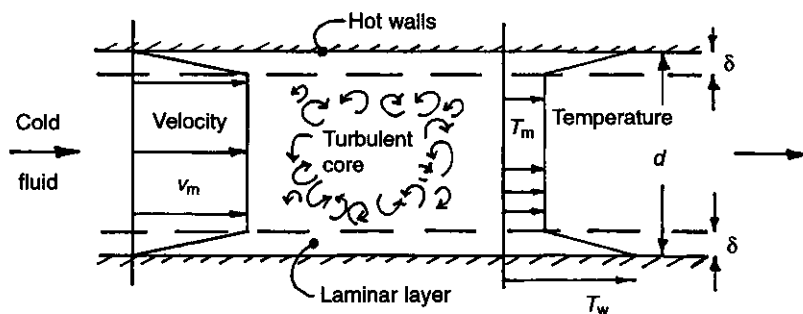
Fig. 3.12 The film model of turbulent flow through a pipe. The fluid has two regions: a turbulent core and a laminar layer adjacent to the walls. The velocity and temperature profiles across the pipe are as shown, v_m and T_m being their mean values in the middle of the fluid.

No new information is obtained by introducing St, only algebraic convenience. The dimensionless group St crops up quite naturally in a variety of situations and so is sometimes used as an alternative to the Nusselt number.

3.2.2 Film model

We now consider various theories of heat transfer to the fluid in Fig. 3.11 when it is in turbulent flow: that is, $Re > 4000$. It should be stressed that any model, such as the film model, oversimplifies a situation in order that some progress can be made in the analysis of a difficult problem. Thus sections 3.2.2 and 3.2.3 must not be taken too literally; they are intended only as simplified pictures of a complex situation. The **film model** is depicted in Fig. 3.12. The assumption is a crude one and is that two regions exist in the flowing fluid: a laminar region next to the walls, and a turbulent well-mixed core for the fluid. The temperature and velocity profiles in a radial direction within the fluid are also shown in Fig. 3.12. In this model all transfer of momentum and heat occurs across a single layer of thickness δ, where $\delta \ll d$, the internal diameter of the pipe. The velocity gradient at the wall is v_m/δ, so that the shear stress in the fluid adjacent to the wall, assuming a Newtonian fluid, is

$$\tau_w = \frac{\mu v_m}{\delta}$$

From Fig. 3.11 the heat flux from the wall to the liquid is

$$q = \frac{\lambda\left(T_w - T_m\right)}{\delta}$$

so that the heat transfer coefficient is

$$h = \frac{q}{T_w - T_m} = \frac{\lambda}{\delta} = \frac{\lambda \tau_w}{\mu v_m}$$

One can now derive the Nusselt number:

$$Nu = \frac{hd}{\lambda} = \frac{\tau_w d}{\mu v_m} = \frac{1}{2} \frac{\tau_w}{\left(\frac{1}{2} \rho v_m^2\right)} \cdot \frac{\rho v_m d}{\mu}$$

Remembering the friction factor c_f to be defined by $\tau_w = c_f(\rho v_m^2/2)$, this gives

$$Nu = \frac{c_f Re}{2} \tag{3.12}$$

The friction factor is normally a function of Reynolds number only. In fact, for $4000 < Re < 10^5$ (turbulent flow within a pipe) the Blasius relation

$$c_f = 0.079 Re^{-1/4} \tag{3.13}$$

holds, so that equation (3.12) becomes

$$Nu = 0.040 Re^{3/4} \tag{3.14}$$

This is a particular from of equation (3.10), and is interesting in that it predicts that Nu is independent of Pr. The very simple assumptions in the film model should be noted, especially of there being heat transfer by simple conduction across a boundary layer or film. The picture in Fig. 3.12 of high temperatures in the fluid adjacent to the wall is a useful one: for example, in the study of fouling (see sections 3.3.3 and 8.3).

   Equation (3.14) enables us to estimate Nu. If $4000 < Re < 10^5$, then from equation (3.13) $0.01 > c_f > 0.0044$ and $20 < Nu < 220$. The above model gives $h = \lambda/\delta$, so that $Nu = hd/\lambda = d/\delta$. Thus the Nusselt number is the ratio of the two distances shown in Fig. 3.11. Thus, according to this model, heat is conducted across a thin layer which has thickness roughly equal to 1/10th to 1/100th of the pipe's radius. If $Nu = 200$ and the fluid is water flowing through a pipe 30 mm internal diameter, then $h = 200\lambda/d = 200 \times 0.6/30 \times 10^{-3} = 4000\,W\,m^{-2}K^{-1}$. Alternatively,

$$Nu = \frac{h}{\lambda/d}$$

is the ratio of two heat transfer coefficients: the actual h for the transfer of heat (mainly by convection) from the walls of the pipe to the fluid; and λ/d, the heat transfer coefficient (see equation (3.3)) for conduction across a film of constant thickness d. Finally, equation (3.12) can be recast to yield

$$St\,Pr = \frac{c_f}{2} \tag{3.15}$$

Also, it should be mentioned that the film model provides a way of visualizing heat transfer situations in a simple way. Thus it is common to think of

there being a film, across which there is a linear temperature profile. Of couse, this is not strictly true, but it helps to think about problems generally.

3.2.3 Reynolds analogy

An alternative approach to the film model is the **Reynolds analogy**, which takes a quite different, but simple, view as to what is important in a turbulent fluid flowing close to a containing wall. Here the path of a tiny fluid element is as shown in Fig. 3.13. The eddy stays in the bulk of the fluid and occasionally transfers very rapidly to the wall, where it remains for a period before suddenly transferring back to the body of the moving fluid. Let m be the mass of eddies arriving at unit area of the wall per unit time. Because there is no net mass transfer, m is also the mass of fluid returning to the bulk from unit area of the wall per unit time. On moving from the bulk to the wall the eddies bring a momentum in the x direction equal to mv_m (v_m = mean bulk velocity of the fluid) and also a heat or enthalpy of mc_pT_m (T_m = mean temperature of fluid of heat capacity c_p). The eddies returning to the bulk do so with the properties of the fluid adjacent to the wall. They thus bring x-wise momentum equal to mv_w, where v_w is the fluid velocity at the wall. We shall take $v_w = 0$. In addition, the eddies arriving in the bulk fluid from the wall have heat mc_pT_w, where T_w is the wall's temperature. Thus

Net momentum flux to the wall = $mv_m = \tau_w$
Net heat flux from the wall = $mc_p(T_w - T_m) = q$

and

$$\frac{\tau_w}{v_m} = \frac{q}{c_P\left(T_w - T_m\right)}$$

Cold
fluid

Hot wall
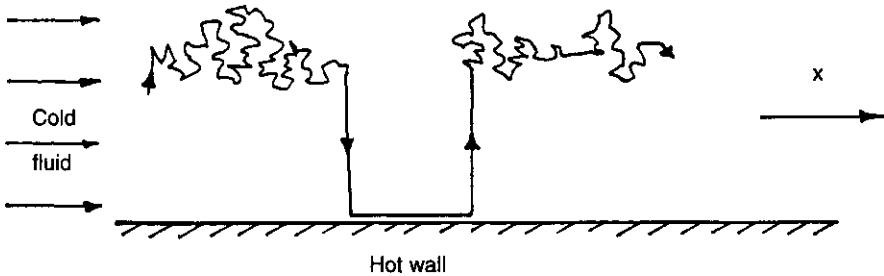
Fig. 3.13 Assumed typical path of a fluid element inside a pipe or over a general surface, which is hot enough to be transferring heat to the fluid.

But

$$h = \frac{q}{\left(T_w - T_m\right)}$$

so

$$\frac{\tau_w}{v_m} = \frac{h}{c_P}$$

and

$$\frac{\tau_w}{\rho v_m^2} = \frac{h}{\rho c_P v_m}$$

or

$$St = \frac{c_f}{2} \tag{3.16}$$

Equation (3.16) is the prediction of this model for turbulent flow and should be compared with equation (3.15) from the film model. Of course, when Pr = 1 for a gas, there is no difference between equations (3.15) and (3.16). However, for a typical liquid with Pr = up to 10, the difference is important. In fact, h predicted by the Reynolds analogy is up to ten times larger than that given by the film model. For viscous food fluids, the Reynolds analogy and also the film model may predict h poorly, because the flow is probably not turbulent. Nevertheless, note that so far we have examined two over-simplified models of heat transfer in a turbulent fluid. The film model assumes that heat conduction in the fluid adjacent to the wall is the controlling factor. In the other model, the Reynolds analogy, the random motion of eddies is all important. We now ask the question: which model is nearer the truth?

3.2.4   The j-factor analogy

By way of a recap, we have had so far that

$$St = f\left(Re, \; Pr\right) \tag{3.11}$$

from dimensional analysis. The film model gave

$$St \, Pr = \frac{c_f}{2} \tag{3.15}$$

whereas the Reynolds analogy resulted in

$$St = \frac{c_f}{2} \tag{3.16}$$

These last two equations are of the general form $St Pr^n = c_f/2$, where n is either unity or zero. The *j*-factor analogy puts the truth somewhere between equations (3.15) and (3.16), so that

$$St\, Pr^{2/3} = \frac{c_f}{2} \qquad (3.17)$$

Thus the Prandtl number now appears raised to the intermediate power of 2/3. This result of what is called the *j*-factor analogy derives partly from theoretical treatment: for example, of heat transfer from a plane surface into a region of laminar flow. However, as seen below, it does give an acceptable description of experimental measurements within their considerable experimental errors. Thus we have a simple result in equation (3.17), which derives from both theory and experiment. This is a common situation, whereby a simple result has been arrived at from a mixture of theory and experiment. This approach has also borne fruit in analysis of mass transfer, as noted in Chapter 4.

3.2.5 Experimental measurements

Measurements have been made of heat transfer rates to a turbulent fluid flowing in pipes. For smooth, straight, clean circular pipes the result of Dittus–Boelter (Kay and Nedderman, 1985) is

$$Nu = 0.023\, Re^{0.8}\, Pr^{0.4} \qquad (3.18)$$

It has to be stressed that the reproducibility and precision of the measurements are not good; there is an uncertainty of 30% in the magnitude of Nu predicted by equation (3.18), which holds for $Re > 5000$. Equation (3.17) for the *j*-factor analogy can be recast as

$$Nu = 0.040\, Re^{3/4}\, Pr^{1/3}$$

and predictions of Nu from these two equations give values of Nu in very satisfactory agreement. The difference between $Pr^{0.4}$ and $Pr^{1/3}$ is of no consequence for liquids of $Pr < 10$, because $10^{0.4}/10^{1/3} = 1.17$ and 17% is well within experimental accuracy. Thus in practice one would either use an experimental correlation like equation (3.18), if one is available or use the *j*-factor analogy. The film model and Reynolds analogy are only of historical interest. The values of Nu for turbulent flow are much higher than those for **laminar** flow in a pipe, for which the semi-theoretical result

$$Nu = 1.86 \left(Re.\, Pr.\, d/L\right)^{1/3} \left(\mu/\mu_w\right)^{0.14} \qquad (3.19)$$

of Sieder and Tate (Kay and Nedderman, 1985) holds. Here d/L is the ratio of internal diameter to length for the pipe; the term arises because the

steady state is established slowly for laminar flow. Indeed the fluid must flow quite a distance (equal to around 25 internal diameters of the pipe) before a parabolic velocity profile is established. The final term in equation (3.19), μ/μ_w, is the ratio of the viscosity on the centreline of the fluid to that at the walls. The difference arises because the fluid might well be hotter at the walls, so that $\mu > \mu_w$. Generally speaking Nu and h are some ten times larger for turbulent than for laminar flow. For this reason laminar flow is avoided in heat transfer devices whenever possible. However, it was noted above that some liquid foods, such as soups, are very viscous fluids, so that they are unlikely to be in turbulent flow. In these cases, laminar flow is unavoidable. Moreover, not only are food fluids viscous, they can be sensitive to shear.

EXAMPLE 3.8

Air at 40°C flows with a mean velocity of 15ms^{-1} inside a smooth tube of internal diameter 2.4cm. Calculate the Reynolds number for the flow and confirm that the flow is turbulent. Calculate the heat transfer coefficient using Blasius's law ($c_f = 0.079\,Re^{-0.25}$) and:

(a) Reynolds analogy ($St = \frac{1}{2}c_f$)
(b) the film model ($St\,Pr = \frac{1}{2}c_f$)
(c) the j-factor correlation ($St\,Pr^{2/3} = \frac{1}{2}c_f$)
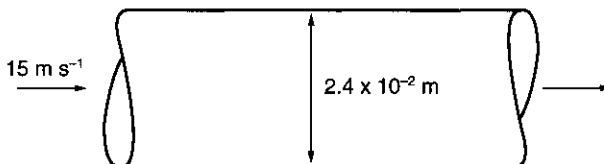*(d) the Dittus–Boelter correlation ($Nu = 0.023Re^{0.8}Pr^{0.4}$)*

Data for air at 40°C

$$\rho = 1.13\,kg\,m^{-3}$$
$$\mu = 1.90 \times 10^{-5}\,N\,s\,m^{-2}$$
$$c_P = 1006\,J\,kg^{-1}K^{-1}$$
$$\lambda = 0.027\,W\,m^{-1}K^{-1}$$

$$Pr = \frac{\mu c_P}{\lambda} = \frac{1.90\times10^{-5}\times1006}{0.027} = 0.708$$

$$Re = \frac{15\times1.13\times2.4\times10^{-2}}{1.9\times10^{-5}} = 21\,410.5$$

(N.B. $Re > 2000$, so it is turbulent.)

$$c_f = 0.079\,Re^{-1/4} = 6.531\times10^{-3}$$

15 m s^{-1} 2.4 x 10^{-2} m

(a) Reynolds analogy

$$St = c_f/2 = \frac{h}{\rho v c_P}$$

Therefore

$$h = \frac{6.531 \times 10^{-3}}{2} \times 1.13 \times 15 \times 1006$$

$$= 55.7 \text{ W m}^{-2} \text{ K}^{-1}$$

(b) Film model

$$St\, Pr = \frac{1}{2} c_f$$

Therefore

$$h = \frac{55.7}{0.708}$$

$$= 78.7 \text{ W m}^{-2} \text{ K}^{-1}$$

(c) j-factor

$$St\, Pr^{2/3} = \frac{1}{2} c_f$$

Therefore

$$h = \frac{55.7}{(0.708)^{2/3}}$$

$$= 70.1 \text{ W m}^{-2} \text{ K}^{-1}$$

(d) Dittus–Boelter

$$Nu = 0.023\, Re^{0.8}\, Pr^{0.4} = 58.4$$

Therefore

$$h = 58.4 \times \frac{\lambda}{d} = 58.4 \times \frac{0.027}{2.4 \times 10^{-2}}$$

$$= 65.7 \text{ W m}^{-2} \text{ K}^{-1}$$

EXAMPLE 3.9

Using the data for air in Example 3.8, derive the Nusselt number for laminar flow inside a pipe with Re = 400, d/L = 10⁻² *and* μ/μ_w = 1.

Equation (3.19) gives

$$Nu = 1.86(Re.\ Pr.\ d/L)^{1/3}$$

Therefore

$$Nu = 1.86(400 \times 0.708 \times 10^{-2})^{1/3} = 1.3$$

Note that Nu is much lower for laminar flow than for turbulent flows. Of course, Pr for a liquid can be larger than for a gas, which also affects Nu.

3.2.6   Other geometries

The above discussion related primarily to flow inside a circular pipe. However, pressure drops and hence c_f have been measured for flow in rough pipes, around bends, in coiled or non-circular pipes, and so on. If c_f is known from measuring the pressure drop in one of these situations, then St and h can be predicted by using the j-factor analogy, equation (3.17). This is a particularly useful feature for internal flows. As for a fluid flowing over the exterior of an object, such as a sphere or a cylinder, remember that the above theories made an analogy between the transfer of heat and momentum. Normally one measures a drag force when a fluid flows over an object and in fact the drag force has two components: form drag and skin friction drag. It is the latter skin frictional drag that is related to heat transfer and so has to be isolated. Some useful correlations are as follows:

1. For flow over a flat plate:

$$Nu = 0.66\,Re^{1/2}\,Pr^{1/3} \qquad \text{for } Re < 10^5$$

$$Nu = 0.036\,Re^{0.8}\,Pr^{1/3} \qquad \text{for } Re > 10^5$$

2. For turbulent flow across the outside of a cylinder:

$$Nu = 0.38\,Re^{0.57}\,Pr^{1/3}$$

3. For flow over a sphere:

$$Nu = 2.0 + 0.7\,Re^{1/2}\,Pr^{1/3}$$

This last result clearly contains two terms added together. The first term is equation (3.8) for heat conduction with $Re = 0$ (no flow), whereas the second term is due to convection with turbulence. The correlation, for a

solid sphere with fluid flowing over its surface, is most useful for predicting heat transfer rates in typical foods containing two phases of a solid and a liquid.
4. For flow in a plate heat exchanger: these exchangers have complex geometries, but are often made from stacks of flat plates, with alternately hot and cold fluids in between the plates and their heat transfer correlations are not generally published. Equations such as

$$Nu = 0.068 \, Re^{0.7} \, Pr^{0.4}$$

can be used, but only with caution, and for turbulent flow. Nevertheless, an equation of the type $Nu = l \, Re^m \, Pr^n$ is of the right form for a plate heat exchanger, with the constants l, m and n depending on the particular geometry.

3.3   Heat exchange: more practical aspects

3.3.1   Overall heat transfer coefficients

Consider the situation depicted in Fig. 3.14, which is a modification of Fig. 3.11. Cold liquid flows inside the thin-walled tube, whose walls are heated by hot air blown over the external surface. Thus heat flows first from the hot air to the pipe, followed by heat conduction through the pipe's wall and then finally there is heat transfer to the liquid flowing inside the tube. So far we have developed expressions to predict h for the transfer of heat from the hot air to the tube and also from the tube to the colder liquid flowing internally. Figure 3.15 shows the rough shape of the plot of temperature against radial distance from the axis of the system. Of course, Fig. 3.15 refers to one distance along the length of the tube. There the temperatures of the bulk of the liquid, of the incident air and the wall are T_L, T_A and T_W, respectively. At this axial distance, one can write the local heat flux (per unit area of wall) as
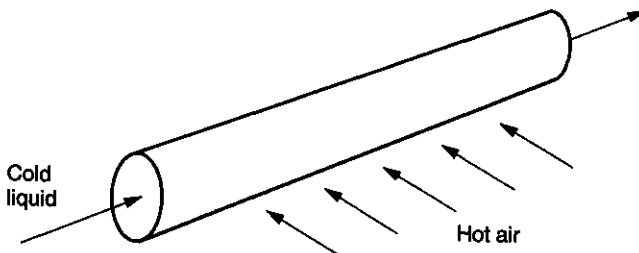

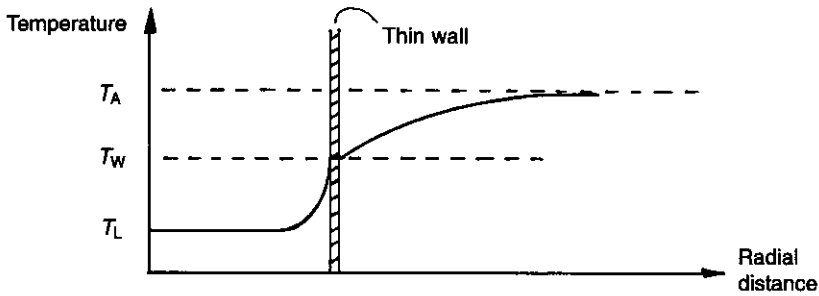**Fig. 3.14** A long thin-walled tube inside which flows a liquid and over which flows much hotter air.

Fig. 3.15 Radial temperature profile at some arbitrary point along the tube in Fig. 3.14.

$$q = h_A\left(T_A - T_W\right) = h_L\left(T_W - T_L\right) \qquad (3.20)$$

Here h_A and h_L are called the air-side and liquid-side heat transfer coefficients, respectively. In addition, an overall heat transfer coefficient, U, can be defined using

$$q = U\left(T_A - T_L\right)$$

Thus U gives q in terms of the overall temperature difference between the incident air and the liquid at this axial distance. Clearly T_L and T_W both increase with distance along the pipe. Equation (3.20) gives

$$T_A - T_W = \frac{q}{h_A}$$

and

$$T_W - T_L = \frac{q}{h_L}$$

which on addition, to eliminate the unknown T_W, yield

$$T_A - T_L = q\left(\frac{1}{h_A} + \frac{1}{h_L}\right) \equiv \frac{q}{U}$$

Hence

$$\frac{1}{U} = \frac{1}{h_A} + \frac{1}{h_L} \qquad (3.21)$$

so that heat transfer coefficients in this case add by their reciprocals. If $h_L \gg h_A$ then $U \approx h_A$ and heat transfer is air-side-controlled, so $T_L \approx T_W$. Alternatively, if $h_A \gg h_L$ then $U \approx h_L$; that is, heat transfer is liquid-side-controlled, with $T_A \approx T_W$.

EXAMPLE 3.10

An oil-cooler consists of 100 thin-walled tubes each of 15mm internal diameter operated in parallel. The oil flows through the shell side of the heat exchanger (that is, around the outside of the tubes) and the water flows inside the tubes. The shell-side heat transfer coefficient is $1.2kWm^{-2}K^{-1}$, whereas that on the water side is given by Nu = $0.023Re^{0.8}Pr^{0.4}$. Find the water-side heat transfer coefficient and hence the overall heat transfer coefficient when the flowrate of water is (a) $10kgs^{-1}$, (b) $20kgs^{-1}$.

Data for water

$$\mu = 1 \times 10^{-3}\,N\,s\,m^{-2}$$
$$\lambda = 0.61\,W\,m^{-1}\,K^{-1}$$
$$c_P = 4190\,J\,kg^{-1}\,K^{-1}$$

(a) Flowrate $10\,kg\,s^{-1}$

Water-side: $Nu = 0.023\,Re^{0.8}\,Pr^{0.4}$

$$10\text{ kg s}^{-1} = \frac{\pi d^2}{4} \times 100 \times v\rho \Rightarrow v = \frac{40}{\pi(15\times10^{-3})^2 \times 10^3} = 0.566\text{ m s}^{-1}$$

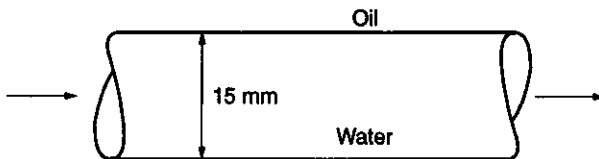$$Re = \frac{\rho vd}{\mu} = 10^3 \times \frac{0.566 \times 15 \times 10^{-3}}{10^{-3}} = 8488$$

$$Pr = \frac{\mu c_P}{\lambda} = \frac{10^{-3} \times 4190}{0.61} = 6.87$$

$$Nu = 0.023(8488)^{0.8} \times 6.87^{0.4} = 69.1 = \frac{h_w d}{\lambda}$$

Therefore

$$h_w = \frac{69.1 \times 0.61}{15 \times 10^{-3}} = 2810\text{ W m}^{-2}\text{ K}^{-1}$$

$$\frac{1}{U} = \frac{1}{2810} + \frac{1}{1200} \Rightarrow U = 841\text{W m}^{-2}\text{ K}^{-1}$$

(b)   Flowrate 20 kg s^{-1}

$$Nu = 69.1 \times 2^{0.8} = 120.3$$

$$h_W = \frac{120.3 \times 0.61}{15 \times 10^{-3}} = 4893 \text{ W m}^{-2} \text{ K}^{-1}$$

$$\frac{1}{U} = \frac{1}{4893} + \frac{1}{1200} \Rightarrow U = 964 \text{ W m}^{-2} \text{ K}^{-1}$$

3.3.2 Cylindrical pipe with thick wall

Suppose the wall in Figs 3.14 and 3.15 is thick enough for the temperature difference between its inner and outer surfaces to be significant. As in Fig. 3.4 the inner and outer radii of the walls are r_1 and r_2, where the temperatures are T_1 and T_2, respectively. Equation (3.4), after adjustment for heat flowing radially inwards (Q is negative), rather than outwards, gives

$$T_2 - T_1 = \frac{Q}{2\pi\lambda L} \cdot \ln\left(\frac{r_2}{r_1}\right)$$

where Q is the overall rate at which heat is transferred in a pipe of length L. But from above

$$T_A - T_2 = \frac{Q}{2\pi r_2 h_A L}$$

$$T_1 - T_L = \frac{Q}{2\pi r_1 h_L L}$$

Addition of the above three equations yields

$$T_A - T_L = \frac{Q}{2\pi L}\left[\frac{1}{r_1 h_L} + \frac{1}{r_2 h_A} + \frac{\ln(r_2/r_1)}{\lambda}\right]$$

As for an overall heat transfer coefficient, again there is a choice, because one has either

$$Q = 2\pi r_1 U_1 (T_A - T_B) L$$

or

$$Q = 2\pi r_2 U_2 (T_A - T_B) L$$

depending on whether the inner or outer radius is used as the reference surface. Thus

$$\frac{1}{r_1 U_1} = \frac{1}{r_2 U_2} = \frac{1}{r_1 h_L} + \frac{1}{r_2 h_A} + \frac{\ln(r_2/r_1)}{\lambda} \tag{3.22}$$

Here U_1 is the overall heat transfer coefficient with respect to the inside surface of the pipe, and U_2 is referred to the outer surface. Equation (3.22) could be used for a thin-walled tube of radius r_1, but with lagging from r_1 to r_2.

EXAMPLE 3.11

Water flows at 2kgs⁻¹ along a circular pipe of 50mm internal diameter, which is lagged with a 10mm thickness of ceramic fibre insulation (λ = 0.2Wm⁻²K⁻¹). The heat transfer coefficient at the outer surface is 50Wm⁻²K⁻¹. Using the Dittus–Boelter correlation (Nu = 0.023Re⁰·⁸Pr⁰·⁴) for the inner surface, calculate the overall heat transfer coefficient: (a) based on the outer surface area; (b) based on the inner surface area.
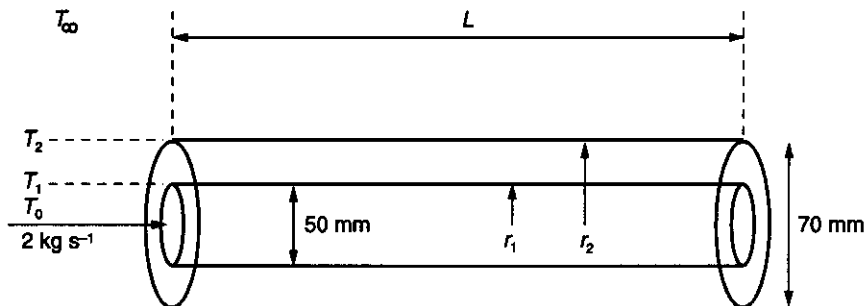
Data for water

$$\mu = 1 \times 10^{-3}\,\mathrm{N\,s\,m^{-2}}$$
$$\lambda = 0.61\,\mathrm{W\,m^{-1}\,K^{-1}}$$
$$c_P = 4190\,\mathrm{J\,kg^{-1}\,K^{-1}}$$

(a)   Outer surface area

$$Q = \frac{2\pi\lambda L (T_2 - T_1)}{\ln(r_2/r_1)} \tag{1}$$

$$2\,\mathrm{kg\,s^{-1}} = \frac{\pi d_1^2}{4} v\rho \text{ for } d_1 = 2r_1$$

$$\Rightarrow v = \frac{2 \times 4}{\pi(50 \times 10^{-3})^2 \times 1000} = 1.019\,\mathrm{m\,s^{-1}}$$

$$Pr = \frac{\mu c_P}{\lambda} = \frac{10^{-3} \times 4190}{0.61} = 6.869$$

$$Re = \frac{d_1 v \rho}{\mu} = \frac{50 \times 10^{-3} \times 1.019 \times 1000}{10^{-3}} = 5.095 \times 10^4$$

$$Nu = 0.023 \times (5.095 \times 10^4)^{0.8} \times 6.869^{0.4} = 289.9 = \frac{h_w d_1}{\lambda}$$

$$h_W = \frac{\lambda Nu}{d_1} = \frac{289.9 \times 0.61}{50 \times 10^{-3}} = 3536$$

$$Q = \pi d_2 L U_2 (T_\infty - T_0) \Rightarrow T_\infty - T_0 = \frac{Q}{\pi d_2 L U_2}$$

$$Q = 50 \, \pi d_2 L (T_\infty - T_2) \Rightarrow T_\infty - T_2 = \frac{Q}{50 \pi d_2 L} \qquad (2)$$

$$(1) \Rightarrow T_2 - T_1 = \frac{Q \ln(d_2/d_1)}{2\pi \lambda L} \qquad (3)$$

$$Q = \pi d_1 L \times 3536 (T_1 - T_0) \Rightarrow T_1 - T_0 = \frac{Q}{3536 \pi d_1 L} \qquad (4)$$

Add (2) + (3) + (4):

$$(T_\infty - T_0) = Q \left[\frac{1}{50\pi d_2 L} + \frac{\ln(d_2/d_1)}{2\pi\lambda L} + \frac{1}{3536\pi d_1 L} \right]$$

Therefore

$$\frac{1}{\pi d_2 L U_2} = \frac{1}{50\pi d_2 L} + \frac{\ln(d_2/d_1)}{2\pi\lambda L} + \frac{1}{3536\pi d_1 L}$$

$$\frac{1}{d_2 U_2} = \left\{ \frac{1}{50 \times 70 \times 10^{-3}} + \frac{\ln(7/5)}{2 \times 0.2} + \frac{1}{3536 \times 50 \times 10^{-3}} \right\}$$

$$= \frac{1}{3.5} + 0.8412 + \frac{1}{176.8} = 1.1326$$

Therefore

$$U_2 = \frac{1}{70 \times 10^{-3} \times 1.1326} = 12.6 \text{ W m}^{-2} \text{ K}^{-1}$$

(b)   Inner surface area

Write:

$$Q = \pi d_1 L U_1 (T_\infty - T_0)$$

So

$$d_1 U_1 = d_2 U_2$$

Therefore

$$U_1 = \frac{d_2 U_2}{d_1} = \frac{7}{5} \times 12.6 = 17.7 \text{ W m}^{-2} \text{ K}^{-1}$$

3.3.3 Fouling

The deposition of solids in heat exchangers can be a severe problem in food processing (see section 9.3). Even the effect of thin layers on heat transfer can be considerable. Consider the thin-walled tube of Fig. 3.14, for which equations (3.20) and (3.21) apply. Suppose there is a very thin layer of grease applied to the inner or outer surface of the tube. If the thickness of the grease is $\delta = 0.01 \text{ mm} = 10 \mu\text{m}$, equation (3.3) gives the heat transfer coefficient for conduction through the grease as λ/δ. Thus equation (3.21) becomes

$$\frac{1}{U} = \frac{1}{h_A} + \frac{1}{h_L} + \frac{\delta}{\lambda}$$

Typical values are $h_A = 50 \text{ W m}^{-2}\text{K}^{-1}$ and $h_L = 3600 \text{ W m}^{-2}\text{K}^{-1}$, indicating that for clean surfaces the resistance to heat transfer is almost all on the gas side. This would give $U \approx h_A$ and $T_W \approx T_L$. Grease has a typical value of $\lambda = 0.1 \text{ W m}^{-1}\text{K}^{-1}$, so that now

$$\frac{1}{U} = \frac{1}{50} + \frac{1}{3600} + \frac{10^{-5}}{0.1}$$

This gives $U = 49.07 \text{ W m}^{-2}\text{K}^{-1}$ with the grease, but $U = 49.32 \text{ W m}^{-2}\text{K}^{-1}$ with totally clean surfaces. The change in U in this case is not dramatic, because $U \approx h_A$. Nevertheless, the resistance to heat transfer across this very thin layer of grease is comparable to that on the liquid side and is consequently important. In general, fouling arises by corrosion and also the deposition of algae; in food preparation, surfaces also scale by solid deposits from proteins, sugars and fats undergoing heterogeneous reactions. The thicknesses of these surface deposits are normally much in excess of the $10 \mu\text{m}$ assumed here. For numerical examples with fouling, see Examples 3.14 and 3.15 below, where $\delta/\lambda = F$ is called the fouling factor.

3.3.4 Freezing times

Having dealt with resistances to heat transfer in series, it is now possible to take Example 3.5 and calculate more realistically the time to freeze an
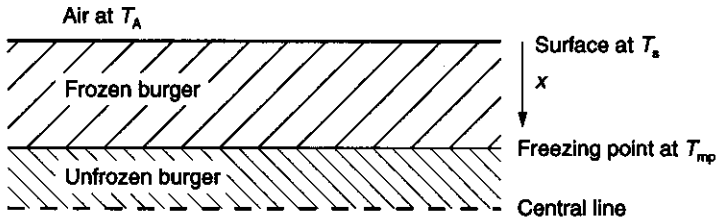
Fig. 3.16 Freezing of a beef burger: schematic diagram.

object, but now including h, the heat transfer coefficient between the cold air and the exterior surface of the burger. The situation is shown in Fig. 3.16.

Again, the same assumptions will be made:

1. The heat being transferred is mainly the latent heat associated with freezing, so that sensible heats (i.e. those associated with merely temperature changes and equal to $\int c_p dT$) are negligible.
2. Heat transfer is one-dimensional in the burger and is through a frozen layer of thickness x from a freezing front at T_{mp} to the surface temperature T_s.
3. All physical properties are constant.

Considering unit area of the burger's outer surface, the rates of heat arrival and loss are equal. Therefore

$$q = h\left(T_s - T_A\right) = \frac{\lambda}{x}\left(T_{mp} - T_s\right)$$

Thus

$$T_s - T_A = \frac{q}{h}$$

$$T_{mp} - T_s = \frac{q}{\left(\lambda/x\right)}$$

Addition of the last two equations eliminates T_s and yields

$$T_{mp} - T_A = \Delta T = q\left\{\frac{1}{h} + \frac{x}{\lambda}\right\}$$

where ΔT is the temperature difference, as defined above.

But the heat flux q is related to the rate of freezing; so

$$q = \theta_w h_{fi} \rho_s \frac{dx}{dt}$$

where θ_w is the fraction of water (by mass) in the burger of density ρ_s. Also

$$\frac{dx}{dt}\left(\frac{1}{h}+\frac{x}{\lambda}\right)=\frac{\Delta T}{\theta_w h_{fi}\rho_s}$$

Integration with the boundary condition that $x = 0$ for $t = 0$ gives

$$\frac{x}{h}+\frac{x^2}{2\lambda}=\frac{\Delta T t}{\theta_w h_{fi}\rho_s}$$

Thus the freezing time can be estimated. Likewise the treatment can be modified to deduce the time to thaw an object.

3.3.5   Unsteady-state heat transfer revisited

Consider a particle (such as a potato), when it is rapidly immersed into a hotter fluid such as boiling water. The situation is shown in Fig. 3.17. The water is kept boiling at a temperature T_w which exceeds T_0, the initial temperature of the potato. In this situation it is usual to define a dimensionless group, the **Biot number**, where

$$Bi = \frac{\text{internal resistance to heat conduction inside potato}}{\text{surface resistance to heat transfer from}}$$

the boiling water to the potato

$$=\frac{V/A}{\lambda}\bigg/\frac{1}{h}=\frac{Vh}{A\lambda}$$
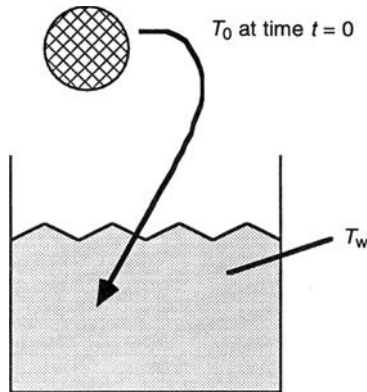
Fig. 3.17 A potato, initially at temperature T_0, being immersed in boiling water at temperature T_w.

where

$$\frac{V}{A} = \frac{\text{volume of potato}}{\text{surface area of potato}} = L = \text{a characteristic length}$$
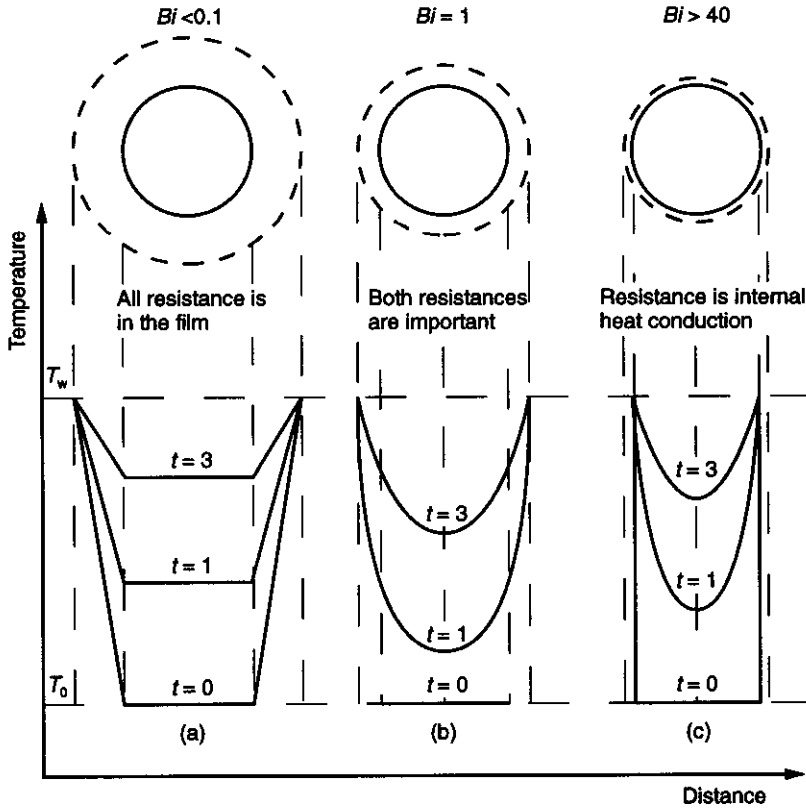
and h is the heat transfer coefficient from the boiling liquid to the surface of the potato. If the potato is spherical, then

$$L = \frac{V}{A} = \frac{\pi d^3/6}{\pi d^2} = \frac{d}{6}$$

where d is the diameter. Thus the Biot number becomes

$$Bi = \frac{hL}{\lambda} = \frac{hd}{6\lambda}$$

It can now be seen that if Bi is large $(Bi > 40)$, then the dominant resistance to the transfer of heat from the boiling water to the colder potato

Fig. 3.18 Plots of temperature along a line through the centre of the potato (see text for description) for increasing times ($t = 0$, 1, 3 arbitrary units), as well as for the cases of small, intermediate and large Biot numbers. The film around the potato is also shown.

is internal: that is, associated with heat conduction from the surface of the potato towards its colder centre. This is shown in Fig. 3.18, which is a plot of temperature versus distance through the centre of symmetry for increasing times. Thus part (c) of Fig. 3.18 shows a thin external film, and the surface temperature of the potato is always the temperature of the boiling water. This situation, of $Bi \gg 1$, has already been described in Fig. 3.9. It is worth noting that alternative definitions of Bi exist, so that care must be exercised to ascertain the precise definition. Thus Bi is often written as $Bi = hd/2\lambda$, so that the characteristic length is taken here to be $d/2$: that is, the radius, rather than $d/6$ as used above. Care always has to be taken to ascertain this characteristic distance; thus elsewhere in other chapters $d/2$ is used occasionally instead of $d/6$. This lack of consistency is common in the literature and must be accepted as a fact of life.

EXAMPLE 3.12

Consider the potato in Example 3.7, but with a diameter of 20mm. Evaluate Bi *when immersed in vigorously boiling water, such that the transfer of heat from the water to the potato is governed by* Nu *= 250. Given also that the thermal conductivity of water is 0.6 Wm⁻¹ K⁻¹, derive and comment on the value of the Biot number.*

$$Bi = \frac{dh}{6\lambda} = \frac{Nu.\lambda_w}{6\lambda} = \frac{250 \times 0.6}{6 \times 0.5} = 50$$

As *Bi* is the ratio of the resistances to internal and external heat transfer, the conclusion is that there is in effect no temperature gradient in the liquid adjacent to the potato. However, there are temperature gradients in the potato.

The other extreme of small Biot number is described in part (a) of Fig. 3.18. Here, at a given time, the potato has the same internal temperature, independent of position within it; also, the external film is thick. This situation of Bi approaching zero is easy to analyse mathematically. The total rate of heat transfer is

$$hA\left(T_w - T\right) = V\rho c_P \frac{dT}{dt}$$

where T is the temperature of the potato. Thus, using $L = V/A = d/6$:

$$\frac{dT}{dt} = \frac{hA}{V\rho c_P}\left(T_w - T\right) = \frac{h}{L\rho c_P}\left(T_w - T\right)$$

which leads to

$$\frac{T_w - T}{T_w - T_o} = \exp\left(-\frac{ht}{L\rho c_P}\right)$$

Finally, for the intermediate case, $Bi \approx 1$, one has the time-dependent heat conduction equation (3.9), but modified for this three-dimensional situation, where position can be defined as a radius r. Also, an extra equation couples the heat fluxes at the surface of the potato:

$$hA\left(T_w - T_s\right) = \lambda A\left(\frac{\partial T}{\partial r}\right)_{\text{surface}}$$

Here T_s is the surface temperature of the potato. In this general case, the solution gives temperature as a function of r and t and is usually expressed on charts.

3.3.6   Concentric tube heat exchanger

This simplest heat exchanger for the transfer of heat from one flowing fluid to another is shown in Fig. 3.19. It is useful to analyse this simple idealization. Hot fluid flows from left to right within the central tube of diameter d. The flowrate is $W \text{kg s}^{-1}$ and its specific heat capacity is $C_P \text{J kg}^{-1}\text{K}^{-1}$. Initially its temperature is T_1, which falls to T_2, but becomes T at a distance x (see Fig. 3.19) along the system. Its temperature is taken to be $T + \delta T$ at a distance $x + \delta x$. As for the outer concentric tube, cold fluid flows within it from right to left, so that this fluid is heated to a temperature θ_1 from θ_2. This fluid in the outer shell has flowrate $w \text{kg s}^{-1}$ and heat capacity $c_P \text{J kg}^{-1}\text{K}^{-1}$; its temperatures at x and $x + \delta x$ are θ and $\theta + \delta\theta$, respectively. The whole system is lagged. The theory below can be made to cover the case of co-current flow of the two fluids simply by reversing the sign of w. Consider the increment of length δx in Fig. 3.19; the overall heat transfer coefficient is U. Thus within this increment of length δx the rate of heat transfer from the inner to the outer fluid is

$$\pi d \delta x U\left(T - \theta\right)$$

This is the rate at which heat is lost by the inner fluid, i.e. $-WC_P\delta T$, or alternatively it equals the rate with which heat is gained by the outer fluid, i.e. $-wc_P\delta\theta$. Hence
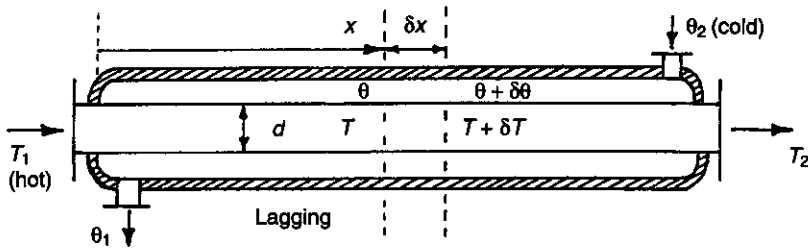

Fig. 3.19 A lagged heat exchanger with a concentric tube and outer shell for counter-current flow of two fluids.

$$\frac{dT}{dx} = -\frac{\pi U d}{W C_P}(T - \theta)$$

$$\frac{d\theta}{dx} = -\frac{\pi U d}{w c_P}(T - \theta)$$

Now introduce $\Delta T = T - \theta$, which is the local temperature difference: that is, the local driving force leading to the transfer of heat. Subtraction of the second from the first equation above gives

$$\frac{d(\Delta T)}{dx} = -\pi U d\left(\frac{1}{W C_P} - \frac{1}{w c_P}\right)\Delta T$$

$$-\int_{\Delta T_1}^{\Delta T_2} \frac{d(\Delta T)}{\Delta T} = \pi U d\left(\frac{1}{W C_P} - \frac{1}{w c_P}\right)\int_0^L dx$$

where L is the overall length of the heat exchanger and ΔT_1 and ΔT_2 are the differences in temperature of the two fluids at the left- and right-hand ends, respectively. Thus

$$\Delta T_1 = T_1 - \theta_1 \qquad \text{at} \qquad x = 0$$
$$\Delta T_2 = T_2 - \theta_2 \qquad \text{at} \qquad x = L$$

and the above leads to

$$\ln\left(\frac{\Delta T_1}{\Delta T_2}\right) = \pi U d L\left(\frac{1}{W C_P} - \frac{1}{w c_P}\right)$$

$$= A U\left(\frac{1}{W C_P} - \frac{1}{w c_P}\right) \tag{3.23}$$

where $A = L\pi d$ is the total surface area available for heat transfer. Once the above heat exchanger is operating in the steady state, the overall rate with which heat is transferred from the inner to the outer fluid is

$$Q = W C_P\left(T_1 - T_2\right) = w c_P\left(\theta_1 - \theta_2\right)$$

Hence

$$\Delta T_1 - \Delta T_2 = T_1 - \theta_1 - T_2 + \theta_2$$

$$= Q\left(\frac{1}{W C_P} - \frac{1}{w c_P}\right) \tag{3.24}$$

Equations (3.23) and (3.24) give

$$\ln\left(\frac{\Delta T_1}{\Delta T_2}\right) = \frac{A U\left(\Delta T_1 - \Delta T_2\right)}{Q}$$

or

$$Q = \frac{A U \left(\Delta T_1 - \Delta T_2 \right)}{\ln \left(\dfrac{\Delta T_1}{\Delta T_2} \right)} \qquad (3.25)$$

Equation (3.25) is of the form Q = area for heat transfer × heat transfer coefficient × mean temperature difference between the two fluids. This latter quantity is seen from equation (3.25) to be the logarithmic mean (ΔT_{1m}) of the ΔT at the two ends of the heat exchanger. Equation (3.25) is thus

$$Q = A U \Delta T_{1m} \qquad (3.26)$$

with

$$\Delta T_{1m} = \frac{\Delta T_1 - \Delta T_2}{\ln \left(\Delta T_1 / \Delta T_2 \right)}$$

Equation (3.26) is a general result applicable to both co- and counter-current flows. This is because the result is independent of the sign of w. Also, an identical result is obtained for the case shown in Fig. 3.10, when the wall temperature was maintained constant: for example, by having steam condense on the outer surface. A special case arises in the above theory when $WC_P = wc_P$. Here $T_1 - T_2 = \theta_1 - \theta_2$, so that $\Delta T_1 = \Delta T_2$. Here the logarithmic mean of ΔT_1 and ΔT_2 is ΔT_1, so that $Q = UA\Delta T_1$. A useful result is that the logarithmic mean of ΔT_1 and ΔT_2 in general lies between their arithmetic and geometric means.

EXAMPLE 3.13

Air flows at 0.002 kg s⁻¹ through a 15 mm diameter tube of length 1.2 m. The tube's wall is maintained at 100 °C and the air's inlet temperature is 25 °C. Using the Dittus–Boelter correlation (Nu = 0.023 Re⁰·⁸ Pr⁰·⁴), calculate the heat transfer coefficient and the temperature of the air leaving the tube. Why are these values unaffected by the thermal expansion of the air?

Data for air (assume constant)

$$c_P = 1005 \, \text{J kg}^{-1} \text{K}^{-1}$$
$$\mu \;= 1.8 \times 10^{-5} \, \text{Pa s}$$
$$Pr = 0.74$$

$$\text{Total mass flow rate} = w = \frac{\pi d^2 v \rho}{4}$$
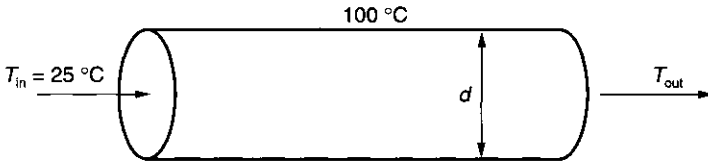
Therefore

$$Re = \frac{\rho v d}{\mu} = \frac{4w}{\pi d^2} \cdot \frac{d}{\mu} = \frac{4w}{\pi D \mu}$$

Hence Re is proportional to w only and so is not affected by any thermal expansion of the fluid. Thus

$$Re = \frac{4 \times 0.002}{\pi \times 15 \times 10^{-3} \times 1.8 \times 10^{-5}} = 9431$$

$$Nu = 0.023 Re^{0.8} Pr^{0.4} = 0.023 \times 9431^{0.8} \times 0.74^{0.4} = 30.8$$

Therefore

$$h = \frac{Nu.\lambda}{d} = \frac{Nu}{d} \frac{\mu c_P}{Pr} = \frac{30.8 \times 1.8 \times 10^{-5} \times 1005}{15 \times 10^{-3} \times 0.74} = 50.3 \ \text{W m}^{-2} \ \text{K}^{-1}$$

$$w(T_{out} - T_{in})c_P = \pi d L h \left(\frac{(100 - 25) - (100 - T_{out})}{\ln\left(\dfrac{100 - 25}{100 - T_{out}}\right)} \right)$$

After cancelling, this gives:

$$\ln\left(\frac{100 - 25}{100 - T_{out}}\right) = \frac{\pi d L}{w c_P} \cdot \frac{Nu \lambda}{d}$$

Therefore T_{out} is independent of pressure and so is unaffected by thermal expansion. Now because

$$\lambda = \mu c_P / Pr$$

$$\ln\left(\frac{100 - 25}{100 - T_{out}}\right) = \frac{\pi \times 1.2 \times 30.8 \times 1.8 \times 10^{-5}}{0.002 \times 0.74} = 1.414$$

Therefore

$$\frac{75}{100 - T_{out}} = 4.112$$

$$T_{out} = 81.76\,^\circ\text{C} = 81.8\,^\circ\text{C}$$

EXAMPLE 3.14

Water flows at 60°C with a mass flowrate of 10kgs⁻¹ along a thin-walled circular pipe of 50mm internal diameter, which is surrounded by air at 20°C. The heat transfer coefficient at the outer surface is 200Wm⁻²K⁻¹, whereas at the inner surface, heat transfer is determined by the Dittus–Boelter correlation (Nu = 0.023Re⁰·⁸Pr⁰·⁴).

(a) Assuming that the tube wall is sufficiently thin for conduction to be neglected, calculate the overall heat transfer coefficient and hence the heat flux from the water.

(b) During use, a thin scale is deposited on the inner tube wall. If the fouling factor for the deposit is $0.001\,m^2\,KW^{-1}$, calculate the percentage reduction in the heat flux.
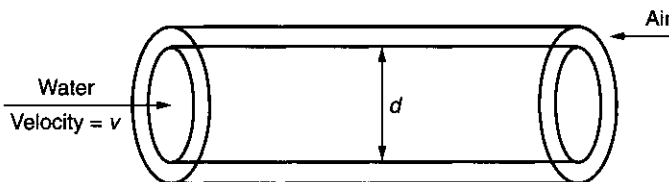
Data for water

$$\mu = 1.5 \times 10^{-3}\,Pa\,s$$
$$c_P = 4187\,J\,kg^{-1}K^{-1}$$
$$\lambda = 0.6\,W\,m^{-1}K^{-1}$$

(a) $10\,kg\,s^{-1} = \dfrac{\pi d^2}{4}\,v\rho$

Therefore

$$v = \frac{4 \times 10}{\pi \times (5 \times 10^{-2})^2 \times 1000} = 5.093\,m\,s^{-1}$$

$$Re = \frac{\rho vd}{\mu} = \frac{1000 \times 5.093 \times 50 \times 10^{-3}}{1.5 \times 10^{-3}} = 169765$$

$$Pr = \frac{\mu c_P}{\lambda} = \frac{1.5 \times 10^{-3} \times 4187}{0.6} = 10.47$$

$$Nu = 0.023 \times 169765^{0.8} \times 10.47^{0.4} = 898.6 = \frac{hd}{\lambda}$$

$$h = \frac{898.6 \times 0.6}{50 \times 10^{-3}} = 10784\, W\,m^{-2}\,K^{-1}$$

$$\frac{1}{U} = \frac{1}{h_{outer}} + \frac{1}{h_{inner}} = \frac{1}{200} + \frac{1}{10784}$$

$$U = 196.4\, W\,m^{-2}\,K^{-1}$$

Therefore

$$q = U(60 - 20) = 7.854\, kW\,m^{-2}$$

(b) Percentage reduction due to fouling

$$\frac{1}{U} = \frac{1}{h_{outer}} + \frac{1}{h_{inner}} + F$$

where F is the fouling factor

$$\frac{1}{U} = \frac{1}{200} + \frac{1}{10784} + 10^{-3} \Rightarrow U = 164.1\, W\,m^{-2}\,K^{-1}$$
$$q = 40 \times 164.1 = 6.565\, kW\,m^{-2}$$

Percentage reduction in the heat flux, q

$$= 16.41\%$$

EXAMPLE 3.15

A corrosive drink flows at 0.4 kg s⁻¹ and 100 °C into a counter-current heat exchanger, where it loses heat to a flow of cooling water. The water flows at 0.3 kg s⁻¹ and enters the exchanger at 20 °C. When newly installed, the heat exchanger has an overall heat transfer coefficient of 300 W m⁻² K⁻¹ and the drink is cooled to 50 °C. Calculate the outlet water temperature and hence the surface area of the heat exchanger.

Corrosion occurs during operation, such that the overall heat transfer coefficient decreases to 200 W m⁻² K⁻¹. If the flowrates and inlet temperatures are unaltered, calculate the outlet temperature of the drink.
Specific heat capacity (c_P) data:
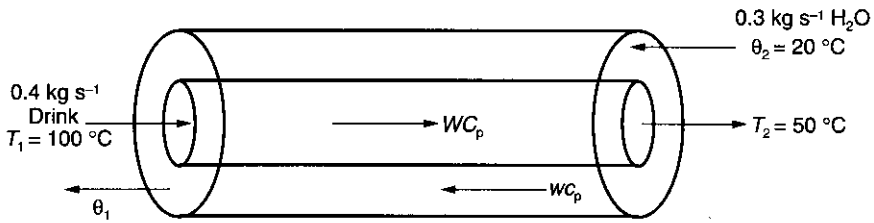Water: $c_P = 4180\, J\,kg^{-1}\,K^{-1}$
Drink: $C_P = 3900\, J\,kg^{-1}\,K^{-1}$

Overall heat transfer rate $= Q = WC_P(T_1 - T_2) = wc_P(\theta_1 - \theta_2)$

$$Q = 0.4 \times 3900 \times (100 - 50) = 78\,kW = 0.3 \times 4180(\theta_1 - 20)$$

Therefore

$$\theta_1 = 82.2\,°C$$

$$\Delta T_1 = 100 - 82.2 = 17.8\,°C; \quad \Delta T_2 = 50 - 20 = 30\,°C$$

$$\Delta T_{lm} = \frac{30 - 17.8}{\ln\left(\dfrac{30}{17.8}\right)} = 23.4\,°C$$

$$Q = Ah\Delta T_{lm}$$

Therefore

$$A = \frac{78 \times 10^3}{300 \times 23.4} = 11.1\,m^2$$

After corrosion:

$$wc_P(\theta_1 - 20) = WC_P(100 - T_2) = Q$$

$$0.3 \times 4180(\theta_1 - 20) = 0.4 \times 3900(100 - T_2)$$

$$\theta_1 - 20 = 1.244(100 - T_2)$$

$$\theta_1 = 144.4 - 1.244\,T_2$$

$$WC_P(100 - T_2) = wc_P(\theta_1 - 20) = Q$$

$$100 - T_2 = \frac{Q}{WC_P}; \quad \theta_1 - 20 = \frac{Q}{wc_P}$$

$$(100 - T_2) - (\theta_1 - 20) = \Delta T_1 - \Delta T_2$$

$$= \left(\frac{1}{0.4 \times 3900} - \frac{1}{0.3 \times 4180}\right)\frac{UA(\Delta T_1 - \Delta T_2)}{\ln\left(\dfrac{\Delta T_1}{\Delta T_2}\right)}$$

Therefore after cancelling $(\Delta T_1 - \Delta T_2)$

$$\ln\left(\frac{\Delta T_1}{\Delta T_2}\right) = \left(\frac{1}{0.4 \times 3900} - \frac{1}{0.3 \times 4180}\right)200 \times 11.1 = -0.3472$$

Therefore

$$\frac{\Delta T_1}{\Delta T_2} = 0.7066 = \frac{100 - \theta_1}{T_2 - 20} = \frac{-44.4 + 1.244\,T_2}{T_2 - 20}$$

$$0.7066\,T_2 - 20 \times 0.7066 = -44.4 + 1.244\,T_2$$

$$T_2 = 56.3\,^{\circ}\text{C}$$

3.3.7   Equipment for heat transfer

Section 9.2 examines more fully the types of heat transfer equipment used in the food industry. In practice heat exchangers are more complicated than that shown in Fig. 3.19. A typical but possibly simple one used in the chemical industry is shown in Fig. 3.20. This achieves a large increase in the area for heat transfer, which is the major difficulty with the heat exchanger in Fig. 3.19. The fluid on the shell side of Fig. 3.20 has a flow affected by baffles. These support the long tubes and also generate turbulence on the shell side. Alternatively, the two fluids can be separated by plates, rather than flowing through tubes; plate heat exchangers are widely used in pasteurization and sterilization plants. Figure 3.21 shows one of the more complex heat exchangers used in the food industry. This is a scraped-surface heat exchanger and is a modification of that in Fig. 3.19. There is an outer concentric tube with one fluid flowing inside it. The inner region, through which the other fluid flows from end to end, has a rotating shaft (500–700 rpm) with blades attached. Thus the surface of the inner pipe shown in Fig. 3.21 is maintained free of deposits. A heat exchanger such as that in Fig. 3.21 can handle fluids containing solid particles up to 25 mm in size. Although the physical operation of these exchangers is complex, the principles on which they operate are the same as those which govern the simpler systems, such as that shown in Fig. 3.19.

Also, complicated heat exchangers can be analysed in a manner analogous to that for the concentric tube heat exchanger shown in Fig. 3.19. Thus the overall rate of heat transfer can be expressed in general as
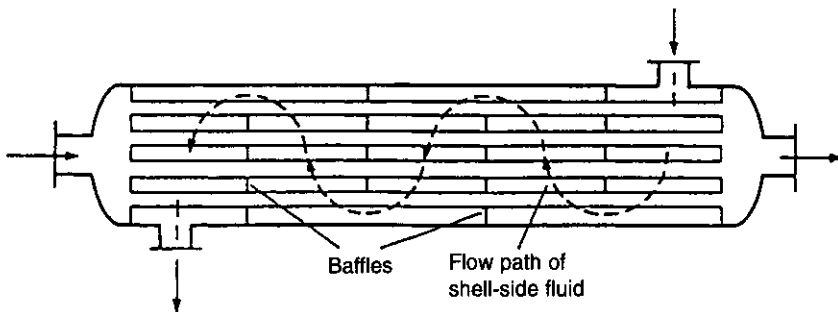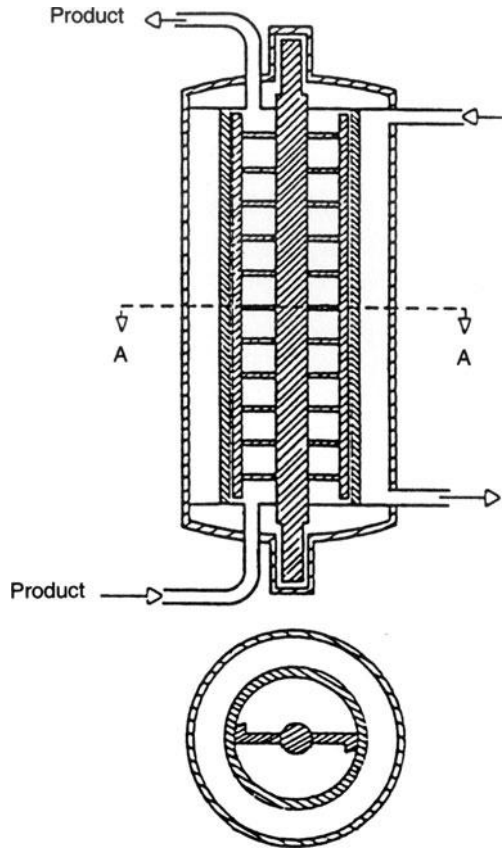

Baffles   Flow path of
           shell-side fluid

Fig. 3.20 A single-pass shell and tube heat exchanger.

Fig. 3.21 Schematic diagram of a scraped-surface heat exchanger, together with a plan view of the cross-section A–A.

$$Q = AUY\Delta T_{\mathrm{lm}}$$

where Y is a correction factor introduced into equation (3.26). Values of Y are tabulated (for example, in Kay and Nedderman, 1985) for various configurations, such as multipass heat exchangers.

As a final comment, it should be stressed that the chemical industry usually insists (for economic reasons) that a local driving force for heat transfer (ΔT) between two fluids should be at least 10°C. In the food industry it is rarely possible to have ΔT as large as this, otherwise several problems arise, such as fouling from one of the fluids undergoing chemical change. The fact that $\Delta T < 10\,°C$ can accordingly result in heat exchangers being relatively large and expensive. In addition, the use of $\Delta T < 10\,°C$ has consequences when networks of heat exchangers are employed. These matters are discussed further in Chapter 9.

Conclusions

The two mechanisms of heat transfer which are most relevant to the thermal operations in the food industry are conduction and convection. Heat is transferred through the walls of a container or vessel and within solid foods or stagnant fluids by conduction. You should understand the basic laws of conduction, how to define the Nusselt number and why it provides a convenient dimensionless group to·describe the process. You should also understand how the basic laws of conduction can be applied both to steady (i.e. time-invariant) and unsteady state processes, which are typical of many food process operations such as canning, freezing, baking, etc.

You should also understand how the process of convection operates in the transfer of heat in flowing systems. Under these conditions heat transfer will depend on the Reynolds number(s) of the moving phase(s) and their physical properties, as measured by a new dimensionless group, the Prandtl number. You should understand how the heat transfer coefficient is defined and some of the approaches to its prediction when heat transfer occurs between a flowing gas or liquid and a solid surface.

In practice, for example in heat transfer across the walls in a heat exchanger or between a hot gas and a biscuit, various heat transfer processes operate in series. You should understand how the processes of convection and conduction can be combined to give an overall heat transfer coefficient. The equations given here to calculate heat transfer coefficients again only apply to particular situations, but the principle of combining the various steps is general. The more complex case of radiative heat transfer is not dealt with here, although many of the key concepts remain valid; radiation can be combined with other processes as described here. The classical problem of the transient heating of a solid by convection from a surrounding fluid was also analysed, and from this you should appreciate the significance of the Biot number which can be used as a measure of the relative importance of the external (convective) and internal (conductive) processes. The idea of a limiting resistance, corresponding to the slowest or rate-controlling process, is a very important one which will also be used in the next chapter.

Some of the basic principles of heat exchanger operation and design were also introduced and, as a result, you should know how to calculate the heat transfer area for a given operation. This particular discussion is taken further in Chapter 9, and is also illustrated by a realistic example in the disk accompanying this book.

Further reading

As with fluid mechanics, all libraries contain many texts on introductory heat transfer. Amongst these are:

Carslaw, H.S. and Jaeger, J.C. (1986) *Conduction of Heat in Solids*, Clarendon Press, Oxford.
Coulson, J.M., Richardson, J.F., Backhurst, J.R. and Harker, J.H. (1977) *Chemical Engineering: Vol 1*, Pergamon, Oxford.
Hallström, B., Skjöldebrand, C. and Trägårdh, C. (1988) *Heat Transfer and Food Products*, Elsevier, London.
Kay, J.M. and Nedderman, R.M. (1985) *Fluid Mechanics and Transfer Processes*, Cambridge University Press, Cambridge.
Levenspiel, O. (1984) *Engineering Flow and Heat Exchange*, Plenum, New York.
McAdams, W.H. (1954) *Heat Transmission*, McGraw-Hill, New York.