7 Process control

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Introduction

Up to now, most of the examples studied have been for plants in a steady state; conditions in the process have not been changing. In practice, no process is in a steady state; processes are always unsteady and require continual control action to keep the product within specification. Process control requires accurate measurement of process parameters, together with some understanding of the ways that changes to the inlet conditions of the process will affect the product. When a process is designed, it is important that it be easy to control; it is best to consider the controllability of a plant at the outset, rather than attempting to design a control system after the rest of the plant has been developed.

This chapter introduces the ideas of process control. Of necessity this is a mathematical subject, but the principles are easy to understand. Once the objectives of a control system have been identified, then it is possible to develop a control model for the process, which identifies the outputs to be controlled and the input variables which can be changed. Depending on how well the process is understood, different types of control model can be developed; if the process can be accurately modelled then it may be possible to produce a **feed-forward** model in which the effect of changes in input variables on the output can be directly predicted; however, if the process is not well-understood, then **feedback** control, in which the change in output conditions with changes in inputs is measured and then used to change the inputs, may be the only one possible. Feedforward control is more efficient and rapid in theory, but requires much better knowledge of the process, and therefore is rarely used in practice. Analysis of process controllability can also help suggest how to design processes to be more easily controlled. As always in food processing, the better understood is a system, the more likely it is to be operated in a profitable manner.

More and more, computers are being used for 'on-line' control; it is now possible to carry out data processing, optimization and the adjustment of process and control parameters in real time. Hierarchical control structures, in which process control and overall process management are closely inte-

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grated, are now becoming common. These structures encompass conventional control techniques at the lowest level of the process, through production control right through to management policy. At the process level the data are processed through software control algorithms and the control signals are fed back to the process. In this sense the computer can be seen as a replacement for traditional control systems. The application of on-line control will undoubtedly continue apace, with huge implications for process efficiency. Nevertheless, the basic principles of process control theory will remain valid, and they will be the focus of this introductory chapter.

Although there are many types of control problem and many different reasons for needing to control a process, two preconditions for any control scheme are that:

1. it must be possible to measure some key indicators; and
2. it must be possible to alter or correct the process behaviour in a predictable and stable way by manipulating one or more inputs.

For example, the composition of a blender product stream might be controlled by altering the flowrate of one of the input streams, or biscuit quality may be controlled by manipulating the heat input to part of the oven. Sometimes it is easy to see which input must be manipulated: the obvious way to control the temperature of a heated vat is to manipulate the heat input. In other cases, such as in controlling a batch dough mixer, it is more difficult to see how to control the process. Measurement is a very important issue for the food industry, but unfortunately there is no space here to discuss such problems: instead the focus will be mainly on issue (2): modelling and controlling processes. Within that framework, the primary emphasis will be on controlling continuous rather than batch or discrete processes.

Some aspects of the material in this chapter are illustrated in the control simulation included on the disk accompanying this text. The example simulates the feedback control of a continuous well-mixed heater, and details of the model and the simulation are given in section 11.12.

Some sections of this chapter (7.4–7.6 in particular) are more mathematical than the others and can be omitted on a first reading. Sections 7.1–7.3 introduce some key concepts of linear systems. Sections 7.7 and 7.8 are concerned respectively with some common controller types and actions (whose principles should be understood) and with a few issues involved in controlling complete processes as opposed to single units.

7.1 What is the control problem?

There are several stages in developing an adequate control strategy for a process. These include:

- defining the main control objectives;
- defining appropriate control structures (identifying potential disturbances and defining what to measure and what to manipulate) at the level of individual units and the whole process;
- specifying the appropriate control laws (relations between the measurement and the magnitude and rate of change of the control variables);
- translating the definition into hardware and software.

7.1.1 Control objectives

Classical control theory has traditionally concentrated on two classes of control objective. The first is to maintain a key parameter constant in the face of disturbances. For example, it might be necessary to maintain the fat content of a milk stream at a constant value despite batch-to-batch variations. There are many problems of this type (called the **regulator** problem) in continuous processes. In extreme cases control may actually be needed to stabilize the plant in the face of disturbances. Sometimes the uncontrolled process output changes because of fluctuations in process inputs, such as the raw materials quality or feedrate, or because of changes in demand for process services such as the steam supply. Some of these fluctuations could be rapid, or high-frequency; others may be much slower, such as variations in the outside temperature. Some may occur as more-or-less random variations about a mean value; others may result from longer-term changes. The food industry is particularly prone to fluctuations in raw material quality and supply.

An obvious first strategy in dealing with this sort of problem is to try to eliminate or minimize those disturbances that **can** be controlled. For example, changes in flow or composition can often be reduced or damped by the judicious use of intermediate storage or buffer tanks.

The second type of control problem, sometimes called the **servo problem**, arises with processes where the conditions **have** to change. In many operations, such as batch mixers and cookers, or batch fermentations, it is necessary to sequence the addition or rate of addition of some components or to adjust an operating parameter – such as mixing speed or heat input – to achieve the desired product quality. The essential problem is to 'steer' the process along a more-or-less defined path towards a final objective, in contrast to the regulator problem, where the aim is to remain within a small region of the desired steady state. The same problem also arises when a change in production volume or quality is called for. It may also arise in a different guise if the process performance changes (for example, because of exchanger fouling). Many processes rely on historical or design information to provide the basis for sequencing: typically – as embodied in many simple programmable logic controllers (PLCs) – valves or motors are switched on or off at predetermined times. This sort of process does not have any built-in mechanism for corrective control action.

It is often assumed that accuracy is synonymous with process control. This is not always true: sometimes all that is needed is to maintain some parameter within a broad band. With a storage tank, for example, it is usually sufficient to ensure that the vessel doesn't overflow or run dry. More generally, means must be found to ensure that a process, or chain of processes, doesn't run out of key inputs during the production cycle. At the process level, this is called **material balancing**.

It will be appreciated, then, that there exists a range of control objectives. From the cases mentioned above it will be clear that it is necessary to consider the dynamic behaviour of the process, and how this is modified by the control system. This presupposes a process model. Linear models usually suffice for regulatory control, but non-linear models may be needed when significant changes in operating conditions are involved, such as in servo control. Here we shall concentrate on some basic ideas of classical theory, which developed around linear **single input single output** (or SISO) systems.

Three complementary methods have traditionally been used. **Time domain methods**, which typically examine the system response to (for example) a step disturbance, focus on the transient behaviour using criteria such as the rise and settling times, degree of oscillation in response, overshoot and offset as the basis for design and tuning. In **frequency domain methods** the system's response and stability are characterized in terms of its frequency response, bandwidth, and gain and phase margins (which measure how close the system is to unstable, highly oscillatory behaviour). The importance of these two methods stems essentially from the use of transfer functions to describe system behaviour, and the ease with which they can be manipulated algebraically. However, a disadvantage is that, in dealing with complex systems, they lead to mathematically high-order functions. **Root locus methods**, which provide a bridge between the two other methods, are the basis of a third set of techniques. Here we shall touch on the first two methods only. First, however, we outline some typical control structures.

7.1.2 Some basic control structures

We can illustrate some of the basic concepts involved in process control by considering the control of a single unit, in this case a continuous mixer–blender (Fig. 7.1(a)). In this example two (liquid) streams with different fat contents X and Y are blended continuously to produce a product stream with fat content Z. We assume that the principal objective is that Z should be controlled as closely as possible. For this example we assume that $X > Y$, so that $Y < Z < X$. In the regulatory problem, the desired value of the outlet fat content is constant. The servo problem corresponds to the case when a new fat product is desired: that is, Z changes. For simplicity, most of the discussion below focuses on the regulatory problem, but we shall see that it is also relevant to the situation where the desired fat content is changed.

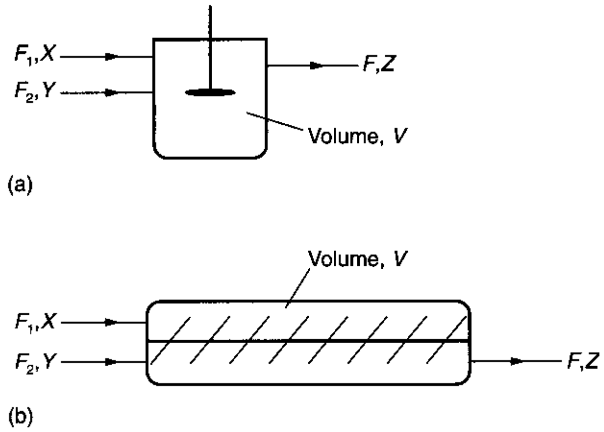


Fig. 7.1 Continuous blenders.

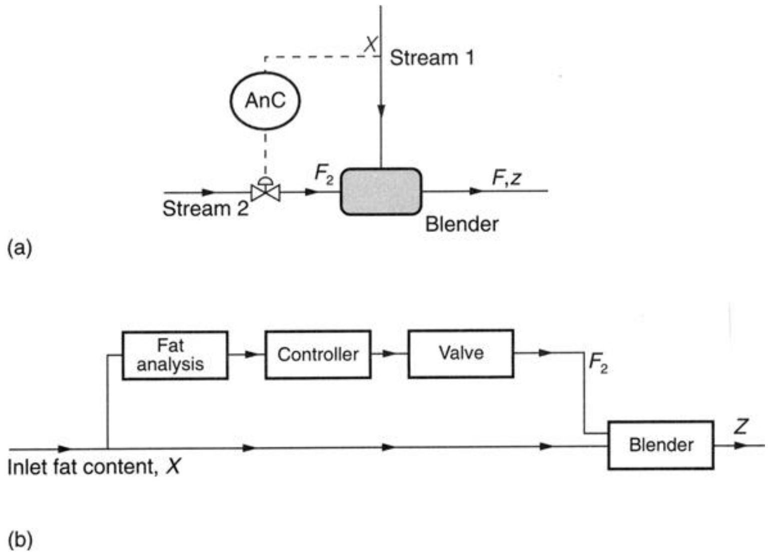


Fig. 7.2 Feedforward control of blender: (a) process flowsheet; (b) block diagram of control loop. AnC: analysis controller.

First we have to devise a control structure: what to measure and what to manipulate. Suppose that all flowrates and fat contents can be measured. The only possible manipulated variables are the two inlet flowrates and the mixer speed or motor power. Changing the mixer speed cannot, of itself, alter the average exit fat content; all it can do is affect the quality of the blend. This leaves us with the flowrates as possible manipulated variables. In theory, if the feed fat contents and flowrates were measurable, one or both of the inlet flows could be altered so as to ensure constant outlet fat content. For example, it would be possible to measure the fat contents X and Y, and one feedrate, say F_1, and then adjust the flow of the other stream F_2 to maintain Z constant. This would be a **feedforward control** scheme. In order to implement it one needs to know how F_2 (the **manipulated variable**) affects the other variables. A simplified version of a feedforward scheme, built around the assumption that significant disturbances would only occur in the fat content of stream 1 (that is, X), is shown in Fig. 7.2(a). Note that, without taking additional measurements, it couldn't be guaranteed that Z was actually at its desired value as, for example, F_1 might deviate from its assumed value, or the actual value of F_2 could be in error. An analogy might be a rally driver who had such confidence in her navigator that she relied only on instructions read from the map.

An alternative scheme would involve measuring the **outlet** fat content Z and then, depending on whether Z was below or above its desired value, adjusting the feedrate of streams F_1 or F_2 to restore Z to its target value. (A similar process could be used independently to control the quality of the blend by manipulating the mixer power.) The principle behind the fat content control scheme is shown for one adjusted flowrate only in Fig. 7.3(a). In practice there would also be a time lapse between realizing that Z was drifting from its desired value and being able to do something about it before it was too late. Assuming that the dynamic problem is not a serious limitation it will be recognized that this scheme – **feedback control** – has the

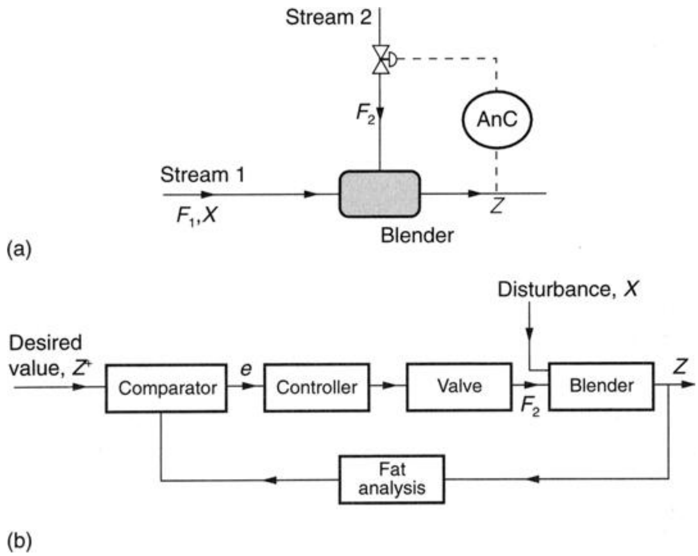


Fig. 7.3 Feedback control of blender: (a) process flowsheet; (b) block diagram of control loop.

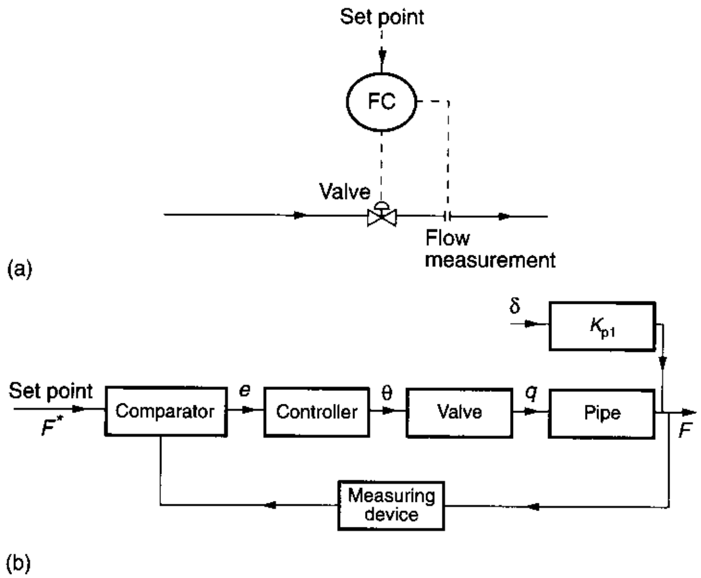


Fig. 7.4 Flow control. FC: flow controller.

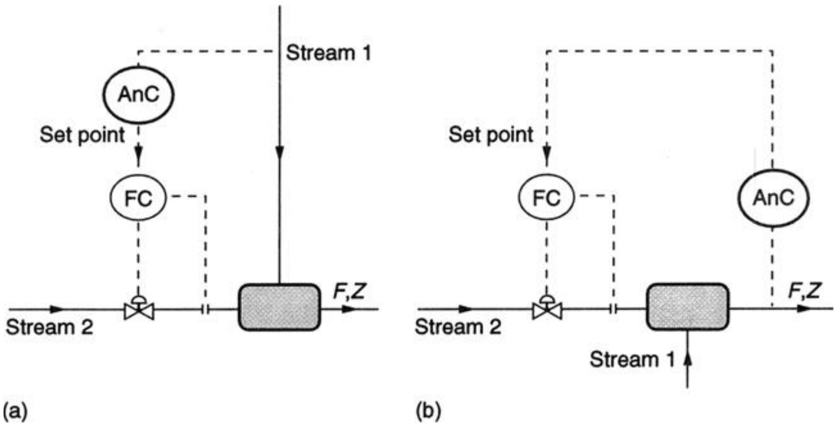


Fig. 7.5 Cascade control: (a) feedforward cascade; (b) feedback cascade.

intuitive advantage that it is based on the actual parameter that it is desired to control. A rough analogy is the strategy we use to adjust the heater setting in the shower: we trim the setting more or less violently in response to the sensation on the top of our head. We soon learn to compensate for the delay between changing the setting and feeling hotter or colder. The full scheme involving the two manipulated streams (that is, changing the flows of stream 1 or 2 depending on whether an increase or decrease in Z was required) would be known as **split-range control**, as different manipulated variables may be used depending on the measured deviation in Z.

In feedforward control (Fig. 7.2) a control signal alters the setting of the valve controlling the flow F_2. As noted above, this involves the assumption that the valve stem moves to precisely the correct position to give the desired flow. Many valves are equipped with positioners to ensure this. A more secure system would employ a **secondary feedback loop**. Figure 7.4 illustrates a flow control loop in which the reading from a measurement device just downstream from the valve is compared with the desired value or set point. Figure 7.5 shows how a secondary loop could modify the set point of the flow control loop in response to changes in the measured fat content X (or, in a feedback system, Z). Such a 'nested' scheme is called **cascade control.**

Sometimes, when two or more streams are blended, it is their flowrates that are subject to change rather than their compositions, and then the desired output consistency will be assured by using **ratio control** to hold the flows in a fixed ratio to each other (Fig. 7.6).

All feedback schemes involve comparison of a measured output with its **desired value** or **set point**, which in the discussion above is assumed constant. If it is desired to change from one output fat content to another or to change the flowrate of a particular stream in some way, this could be achieved by changing the set point either stepwise or in a programmed way. A good design will then ensure that the system output is able to track this change: which, of course, is an example of servo control.

Most of these loops (and, indeed of those used in practice) have a single controlled input and a single output (SISO schemes). However, the fact that more complicated arrangements might be needed can be realized by looking at another feature of the blender: that is, the existence of multiple 'linkages' between the various inputs and some or all of the outputs. As

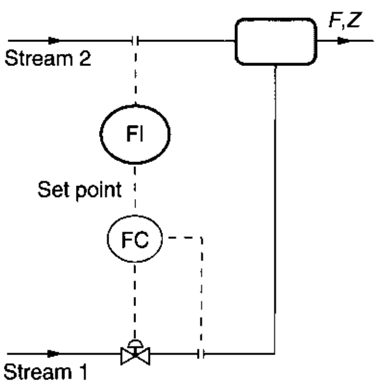


Fig. 7.6 Ratio control. FI: flow indicator.

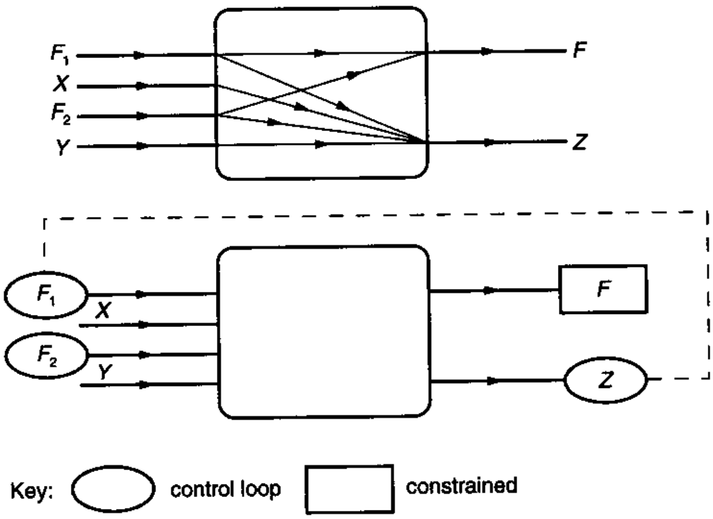


Fig. 7.7 The blender as a multivariable system.

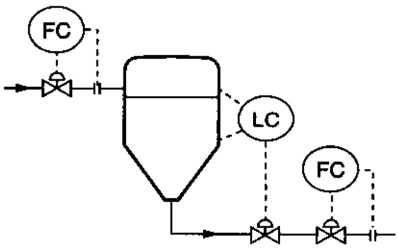


Fig. 7.8 An impossible control scheme. LC: level controller.

shown schematically in Fig. 7.7, changing either of the input flows will lead to a change in both the product flowrate and its composition; however, changes in the inlet composition only affect the composition and **not** the flowrate of the output stream. Where the interactions are weak, this presents no problem, and the whole system can be controlled by a set of independent SISO control loops. In other cases control system design must take account of the multivariable nature of the process in order to ensure that the desired objectives are met.

The ideal design consists of the minimum set of independent control loops. It is tempting to attempt to control everything, whether it is strictly necessary or not. Figure 7.8, showing a continuous liquid buffer tank, illustrates two important considerations. First, any control system must be feasible: the scheme shown, which implies **independent** control of the inlet flow, the level in the separator and the outlet flowrate, is clearly impossible, as there are insufficient degrees of freedom for all three variables to be independent of each other. Controlling the inlet flow and the level means that the outlet flowrate cannot also be varied independently. Second, as noted above, precise control is often **not** necessary. For example, it usually necessary to control the liquid level in a holding tank only within wide margins, as all that is needed is to ensure that the tank doesn't overflow or empty. Recognition of this can provide an important degree of flexibility in a system. It is recommended that, before detailed control of **quality measures** is undertaken, consideration be given first to ensuring adequate **mass balancing** throughout the plant: that is, to ensuring that pumps are always fed and that storage and process vessels don't run dry. Most important of all is to define the control objectives!

From the examples above we can distinguish between the following types of parameter or variable.

- **Disturbances**, such as the inlet concentration X. These fall into two categories: those that are measurable and those that aren't. Note that it would be very dangerous to assume that there was only one disturbance: in the case of the blender, we must ask what **would** happen if the inlet flowrate of milk **did** change?
- **State variables**, which are indicators of the state of the process, and which may include some **measured variables** (such as the outlet concentration Z) or **outputs**. Not all of these variables may be measurable on line; it may be possible to infer some from other readings or by computation. In what follows it will normally be assumed that the measured values provide analogue rather than either/or (open/closed, for example) information.
- **Manipulated or controlled** variables, such as the flowrate F_2.

Note also that in each case there has to be an appropriate **control law**: that is, a defined relation between the measured variable (or realistically its variation from the desired or target value) and the magnitude (and rate) of the change in the manipulated variable. In the example of the shower, the amount by which we change the heater setting in response to the sensation of burning reflects the control law, which we have learned through hard experience. You will see that this law reflects the model of the system: we would respond in one way with a modern, fast-acting shower and in another with an older less powerful type.

Two other features of control systems should also be noted. The first, obvious, point is that the control signals in a plant, such as flowrates, are constrained and not limitless. The second point is that control loops must be integrated into the emergency/alarm system appropriate to the plant. Alarms need to be built in to protect against process or control system failure; control valves should be chosen so that, wherever possible, they fail safe; overrides to shut down process flows in an emergency must also be built in.

7.2 Block diagrams

We can represent the control schemes described above by means of their block diagrams. For linear systems (see below) we shall see that this method of representation is a very powerful tool for control system analysis.

For example, Figs 7.2(b) and 7.3(b) are block diagrams for the feedforward and feedback schemes for blender control. The boxes, or blocks, represent the various components in the control loop. Each block has one or more inputs (such as a flowrate) and an output (for example, a composition or, from a transducer, an electrical signal); the arrowed lines show the direction of signal flow (that is, input → output). Most control systems will have blocks corresponding to the following hardware units:

- the process unit (the blender, holding tank etc.);
- a process sensor;
- a unit (the comparator) where the measured output is compared with the desired value (put in by the operator) to generate an 'error' signal;
- the controller itself, which, in response to the error signal, sends a signal to the final element:
- an actuator (most typically a valve) whose output is the manipulated control variable, usually a flowrate.

In practice the comparator and controller form a single unit, which might nowadays be a computer or programmable logic controller (PLC) device.

The algebra of block diagrams, which is the basis for many techniques for analysing control systems, is developed further in section 7.6.4.

7.3 Process dynamics

Processes and instruments can never react instantaneously to changing inputs. A few types of dynamic behaviour recur very frequently; these can be used to characterize many more complex processes and to explain the elementary principles of regulatory process control.

Two of these – the **first-order lag** and the **dead-time** or **transportation lag** – are particularly important. The difference between them can be grasped qualitatively by comparing the behaviour of a constant-volume well-mixed tank and a pipe (Fig. 7.9).

Consider the effects of a change (assumed a step jump) in the composition of the inlet stream to the two units. The first device is well mixed, so that, instantaneously, the compositions of the output stream and the average composition in the vessel are the same. As we shall see in Chapter 8, well-mixedness implies a very broad spread (from zero to infinity) in the residence times of individual material elements. A change in the composi-

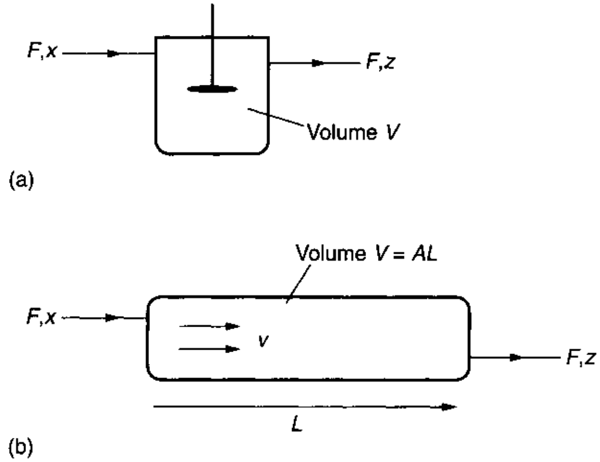


Fig. 7.9 (a) First-order and (b) transportation lags.

tion of the inlet flow will therefore be sensed immediately in the outlet because of the short residence time of some material. However, the full consequences of the inlet change will only be seen some time later. In contrast, material is assumed to be transported without axial mixing along the length of the pipe in Fig. 7.9(b); no change in outlet composition is expected until the transportation lag time has elapsed. We now examine these two systems quantitatively.

7.3.1 First-order systems

As illustrated in Fig. 7.9(a) we assume one input and outlet stream, flowing at a constant rate F through the unit whose volume is V. This could, for example, represent a holding tank for a continuous milk feed whose composition (protein or fat concentration), fluctuates. x and z are the instantaneous concentrations of the species of interest in the inlet stream and the vessel (and in the outlet). The only disturbance considered is a change in inlet concentration x. An instantaneous species balance is, in words:

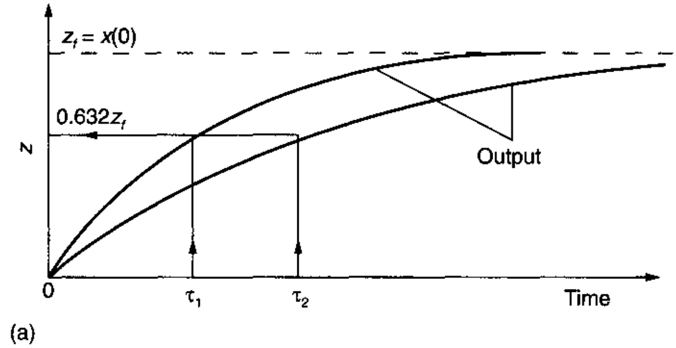
Rate of accumulation = flowrate in – flowrate out

That is,

$$\frac{V\mathrm{d}z}{\mathrm{d}t} = Fx - Fz \tag{7.1}$$

or

$$\frac{\tau\mathrm{d}z}{\mathrm{d}t} + z = x \tag{7.2}$$



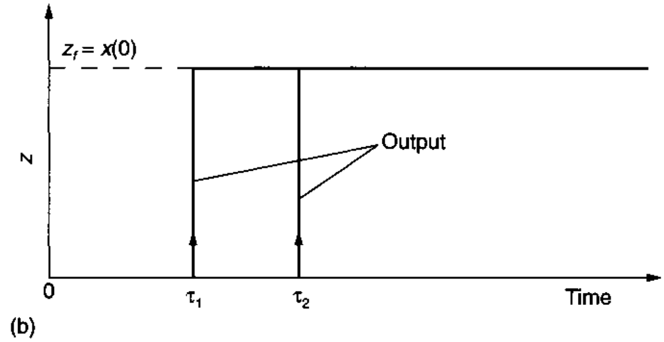


Fig. 7.10 Step response of: (a) a first-order system; (b) dead time. z_f is the final value of the output $z(t)$.

This is a first-order system (as it is described by a first-order linear ordinary differential equation). τ is the system time constant: here it is the mean residence time, V/F.

If the system is subjected to a step change of magnitude $x(0)$ in the inlet concentration, the response (which is readily checked by back-substitution) is

$$z(t) = x(0)[1 - \exp(-t/\tau)] \tag{7.3}$$

where the final value of the outlet concentration is, of course, the same as the inlet, $x(0)$. The response, $z(t)$, is shown in Fig. 7.10(a) for two different values of τ. The larger the system time constant, the slower the response. Note that the effects of the change in inlet concentration are observed immediately in the outlet stream, where the initial rate of change in concentration is $x(0)/\tau$. Two useful results are that z reaches 63.2% of its final value within one time constant and 95% within three time constants.

Many simple processes (such as the flow response of a holding tank or a simple thermocouple) demonstrate, or approximate to, first-order dynamics: that is, they are characterized by a single time constant.

7.3.2 Dead times or transportation lags

Now compare the result above with a **dead time** or **transportation lag** (Fig. 7.9(b)) (sometimes also called the **distance/velocity lag**). A pipeline with turbulent flow is a good approximation to this. Equation (7.1) no longer holds, as the contents are not perfectly mixed. A first approximation to the flow behaviour is that the contents flow at a constant mean horizontal velocity $u = F/A$ through the process unit, just as in the plug flow reactor (Chapter 8, section 8.3.3). A is the pipe cross-sectional area. A change in inlet concentration propagates through the unit with velocity u, appearing unaltered in magnitude at the exit a time $L/u = V/F$ later. Thus, although the system has the same characteristic time constant as the well-mixed process, its behaviour is very different (Fig. 7.10(b)). With a first-order system the first effects of an input change are seen immediately in the outlet stream, although the full effect is not seen until a few time constants have elapsed. With a pure time delay there is no attenuation in the outlet signal, and there is no intimation of a disturbance until one time constant has elapsed. In the case examined here, a step change in inlet concentration would result in the same final exit concentration from both systems.

7.3.3 Series of lags

The step response of a system comprising a first-order lag preceded or followed by a dead time τ_1 would follow the curve given by equation (7.3) but shifted by a time τ_1, as shown in Fig. 7.11(a):

$$z(t) = 0, \qquad t < \tau_1$$
$$z(t) = x(0)\{1 - \exp[-(t - \tau_1)/\tau]\}, \qquad t \geqslant \tau_1 \tag{7.4}$$

This has significant implications for control system design. If a measuring element is placed some distance downstream from the process, the lag between a change occurring and its effects being recognized can have serious consequences for control quality.

It will be obvious intuitively that a system of first-order lags in series would give rise to an increasingly sigmoidal series of responses to a step input to the first unit, as shown qualitatively in Fig. 7.11(b). The response to a step input can never give rise to an oscillatory response.

7.3.4 Second-order lags

Examples of the **second-order** (or **quadratic**) **system** that are often quoted include a damped oscillator and the U-tube manometer. An example of the first would be a load cell. The force on the load cell is resisted by a restoring force from the spring (proportional to its compression) and a viscous damp-

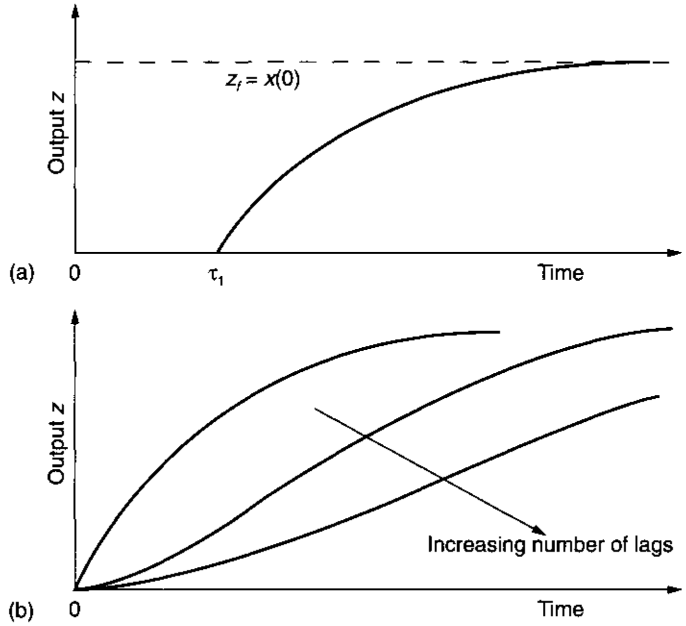


Fig. 7.11 Step response of: (a) first-order lag plus dead time; (b) series of first-order lags.

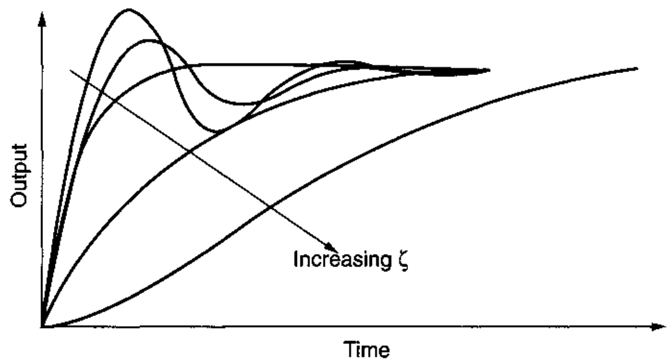


Fig. 7.12 Step response of a second-order system.

ing force, proportional to the rate of compression or movement of the cell. A force balance on both these systems leads to an equation of the form

$$\tau^2 \frac{d^2x}{dt^2} + \zeta \frac{dx}{dt} + x = F(t) \tag{7.5}$$

where x is the deflection (output) and F is the imposed, forcing function.

A feature of this is that the step response, sketched in Fig. 7.12, shows regions of non-oscillatory, or over-damped, response corresponding to $\zeta >$

1, and increasingly oscillatory (or underdamped) behaviour as ζ decreases for $\zeta < 1$. The frequency of this oscillation is related to the two parameters τ and ζ by $\omega\tau = \sqrt{(1 - \zeta^2)}$. In the absence of any viscous damping the system behaves as a simple harmonic oscillator with natural frequency $\omega = 1/\tau$. Two first-order lags in series (for example, one holding tank feeding another) give non-oscillatory behaviour, and exactly correspond to a system with $\zeta > 1$.

In practice, this form of response is far more important than the two examples quoted might suggest, because many closed-loop control systems behave in a qualitatively similar way. The choice of control parameters is often governed by a search for an appropriate compromise between the speed of response (favoured by lower values of ζ and τ) and the extent of oscillatory response that is acceptable.

7.4 Multiple inputs and linearization

All the examples above are based on linear processes. If, in the example used to develop the idea of a first-order lag, the flowrate F was allowed to vary, however, the system would be non-linear because of the product terms $F(t)x(t)$ and $F(t)z(t)$. Many processes are inherently non-linear: for example, the output (pressure signal) from an orifice flowmeter varies with $(\text{flowrate})^2$ (equation (2.14)); the flow through an orifice or valve is, by the same token, proportional to $(\text{pressure drop})^{0.5}$. However, it is always possible to approximate the process by a linearized model to allow the use of the large body of linear theory. This process is illustrated here. *This section and the following example can be omitted on a first reading.*

Consider the well-stirred blender shown in Fig. 7.1(a). We assume that the materials are incompressible, that the volume is maintained constant and that all flows and compositions may vary with time. Instantaneous material balances on the flows of total material and of fat both have the form:

Rate of accumulation = Sum of flows in − Sum of flows out

which, with the constant volume assumption, give

$$F_1(t) + F_2(t) = F(t) \tag{7.6}$$

and

$$V\frac{dZ}{dt} = F_1(t)X(t) + F_2(t)Y(t) - F(t)Z(t) \tag{7.7}$$

$$= F_1X + F_2Y - (F_1 + F_2)Z \tag{7.8}$$

for the total flow and fat respectively. The explicit dependence on time has been suppressed in equation (7.8), where equation (7.6) has been used to eliminate the outlet flow F.

If all of the inlet compositions or flowrates vary with time, equation (7.8) is non-linear. It is often hard to solve, and does not have a general solution. If the only disturbances were due to changes in the fat compositions X and Y, equation (7.8) would be a linear first-order differential equation with constant coefficients (A, B, C), of form

$$\frac{dZ}{dt} = AZ(t) + BX(t) + CY(t)$$

However, the equation can be reduced to an approximate linear form in **all** the variables by working in terms of changes (or deviations) rather than the absolute values of the variables. To do this we write each variable in the form

$$W(t) = W(0) + w(t)$$

where $W(0)$ is the initial (assumed steady) value and $w(t)$ is its deviation from the initial value. For example, we write

$$X(t) = X(0) + x(t)$$

and

$$F_1(t) = F_1(0) + f_1(t)$$

It is also assumed that $w(t)$ is small, so that, where necessary, products of small variables (such as $f_1(t)x(t)$) can be neglected to eliminate non-linear terms. Thus equation (7.8) becomes

$$V\frac{dz}{dt} = [F_1(0) + f_1][X(0) + x] + [F_2(0) + f_2][Y(0) + y] - [F_1(0) + f_1 + F_2(0) + f_2][Z(0) + z] \qquad (7.9)$$

Some of the terms in equation (7.9) cancel because at steady state

$$F_1(0)X(0) + F_2(0)Y(0) = [F_1(0) + F_2(0)]Z(0) \qquad (7.10)$$

Substituting from (7.10) in (7.8) and neglecting the small terms f_1x, f_2y, f_1z, and f_2z gives the general linearized dynamic model:

$$V\frac{dz}{dt} = [X(0) - Z(0)]f_1 + [Y(0) - Z(0)]f_2 + F_1(0)x + F_2(0)y - [F_1(0) + F_2(0)]z \qquad (7.11)$$

which has the linear form

$$V\frac{dz}{dt} = Af_1 + Bf_2 + Cx + Dy - Ez \qquad (7.12)$$

Suppose, for example, that all the inputs are constant except the fat composition of stream 1; equation (7.11) reduces to the simple first-order equation

$$V\frac{dz}{dt} = F_1(0)x - \left[F_1(0) + F_2(0)\right]z \tag{7.13}$$

which can be written in the simpler form (cf. equation (7.2))

$$\tau\frac{dz}{dt} + z = K_p x \tag{7.14}$$

where, as before, τ is the system time constant, here $= V/[F_1(0) + F_2(0)] = V/F(0)$. The constant $K_p = F_1(0)/F(0)$; this is the 'static' gain.

If the system is subjected to a step change of magnitude $x(0)$ in the inlet fat content, the response is as before:

$$z(t) = z(\infty)\left[1 - \exp(-t/\tau)\right] \tag{7.15}$$

where the final value of the outlet fat content $z(\infty) = K_p x(0)$.

The time constant for changes in exit fat content (the blender volume divided by the flowrate) is the same for all possible disturbances in the model. However, changes in inlet flowrate are reflected by instantaneous changes in the outlet flowrate F.

The assumption that fluctuations are small is often justified in the analysis of control systems, on the grounds that a well-designed regulatory control scheme will ensure that deviations are kept small. It is less likely to be generally true, however, when set point changes are introduced, as in the case of so-called servo control, as this often implies significant changes in operation.

EXAMPLE 7.1: THE DYNAMICS OF A HOLDING TANK

A tank, cross-sectional area 1 m^2, is used as a buffer tank. The steady-state flow through the tank is 0.1 m^3 min^{-1}; the output flow is related to the liquid height in the tank by $F_o = 0.1\sqrt{H}$. How would the outlet flow and liquid height respond to a step change in inlet flowrate from 0.1 to 0.12 m^3 min^{-1}?

We first derive the general linearized model for a holding tank, using the symbols defined in Fig. 7.13, with

$$F_o = K\sqrt{H} \tag{7.16}$$

Small changes in flowrate from the steady value f_o are related to small changes in liquid height h by taking the first terms in the Taylor series expansion of equation (7.16):

$$F_o(0) + f_o = K\sqrt{H(0)}\left[1 + 0.5\,h/H(0) + \ldots\right]$$

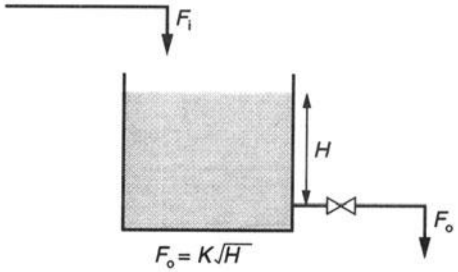


Fig. 7.13 Example 7.1: flow through a holding tank.

That is,

$$f_o = 0.5Kh = h/R_v \tag{7.17}$$

where R_v is the linearized resistance of the outlet valve.

A linearized input–output relation for the flows through the tank then follows directly by substituting for $h = R_v f_o$ (equation (7.17)) into the mass balance:

$$A\frac{dh}{dt} = f_i - f_o \tag{7.18}$$

That is,

$$AR_v\frac{df_o}{dt} + f_o = f_i \tag{7.19a}$$

or

$$\tau\frac{df_o}{dt} + f_o = f_i \tag{7.19b}$$

The system time constant $\tau = AR_v = 2AH(0)/F(0) = 2V/F(0)$. (The factor 2 appears here as a consequence of the non-linear flow/height relation.) Variations in the liquid height follow the equation

$$AR_v\frac{dh}{dt} + h = R_v f_i \tag{7.20}$$

which, not surprisingly, has the same time constant but a different 'static' gain K_p.

Here, $K = 0.1$ and $f_o = 0.05h$, the valve resistance $R_v = 20\,\text{min}\,\text{m}^{-2}$, and the time constant = 20 min. The step change in inlet flow thus results in:

$$f_o = 0.02\left[1 - \exp\left(\frac{-t}{20}\right)\right] \tag{7.21a}$$

and

$$h = 0.4\left[1 - \exp\left(\frac{-t}{20}\right)\right] \quad (7.21b)$$

The liquid height would change by 0.4 m; it would reach 0.25 m within 20 min and be almost steady at its new value within 1 h of the change. If the tank were much smaller or the steady flowrate higher, the speed of response (measured by the time constant) would be correspondingly faster.

7.5 Frequency response

The example above illustrates the ideas underpinning time domain analysis. The principles of frequency response analysis are briefly illustrated here. This section may be omitted on a first reading. We consider the response of a first-order system governed by equation (7.2) or (7.14) to a sinusoidal input $x = x(0)\sin\omega t$. Here ω is measured in radians/time; it is related to the frequency f (cycles/unit time) by $\omega = 2\pi f$. Once the immediate, transient, effects of the disturbance have died down, it can be shown that the output $z(t)$ itself settles down to an oscillatory form given by

$$z(t) = A\sin(\omega t + \phi) \quad (7.22)$$

where the amplitude of $z(t)$ is

$$A = \frac{z(\infty)}{\left(1+\omega^2\tau^2\right)^{0.5}} = \frac{K_p x(0)}{\left(1+\omega^2\tau^2\right)^{0.5}} \quad (7.23a)$$

and the phase shift is:

$$\phi = -\tan^{-1}(\omega\tau) \quad (7.23b)$$

The physical significance of A and ϕ is illustrated in Fig. 7.14.

Note the following.

- The output is also a sine wave with the same frequency as the input.
- The output amplitude is **reduced** from its steady-state or asymptotic value (= $K_p x(0)$) by a factor $1/(1 + \omega^2\tau^2)^{0.5}$: the higher the frequency, or the smaller the system time constant (that is, the faster it is able to respond), the greater is the reduction in the output amplitude, reflecting the combination of resistance and capacity in the system.
- The output signal **lags** the input by an angle ϕ, or a time $\phi/360f = \pi\phi/180\omega$.

The effect of the process dynamics on the magnitude of the outlet is very important. Consider the effect of the holding tank on the outlet flow, for example: here $K_p = 1$. When the frequency of fluctuations in the inlet flow

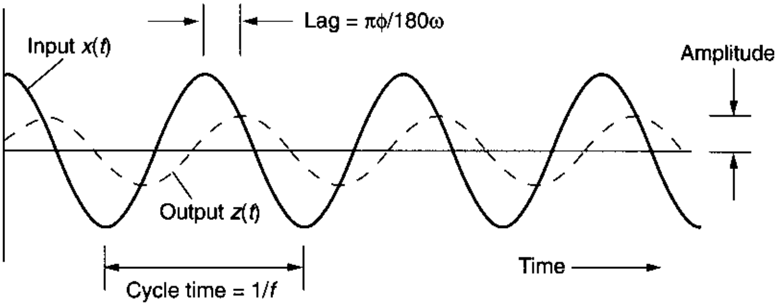


Fig. 7.14 Response to sinusoidal input.

is low in relation to the tank time constant (that is, $\omega \ll 1/\tau$), the magnitude of oscillations in the outlet will be the same as those in the inlet and there will be no time lag between them. If the frequency of the disturbances is high, the tank–valve system exerts a significant damping effect, reducing the magnitude of the variations. For example, if $\omega = 1/\tau$, the outlet flow oscillations are reduced by a factor $1/\sqrt{2}$; if $\omega = 10/\tau$, the amplitude is reduced to approximately one tenth of its inlet value. This is why buffer tanks, if correctly designed, can significantly reduce fluctuations, and effectively decouple one part of a process from the remainder downstream.

It also explains why, if a system is disturbed by, or expected to cope with, quickly varying variables, it is important to ensure that the dynamics of the various process elements (sensors and the like) are also fast. A thermocouple with a time constant of 1 s will have no problem in accurately following the changes of bulk temperature in a large vat of sauce; it may be less satisfactory in coping with a small gas-fired oven.

7.6 Feedforward and feedback control

We now return to a discussion of the control systems introduced earlier in section 7.4. To simplify matters we base the discussion around the blender control problem. This section may also be omitted on a first reading.

7.6.1 Feedforward control

First, consider feedforward control of the blender output, with the single feedforward scheme shown in Fig. 7.2. In order to respond to a variation in X it is necessary to know how X and Z depend on F_2. In other words, we need a model of the process, which has already been derived above:

$$F_1(t) + F_2(t) = F(t) \tag{7.6}$$

and

$$V\frac{dZ}{dt} = F_1(t)X(t) + F_2(t)Y(t) - F(t)Z(t) \tag{7.7}$$

$$= F_1X + F_2Y - (F_1 + F_2)Z \tag{7.8}$$

for the total flow and fat respectively.

The objective of the control scheme is to ensure that the effects of deviations in X are eliminated from Z. It is further assumed that neither F_1 nor Y varies from its steady value $F_1(0)$ or $Y(0)$. How then should F_2 be changed when a change in the inlet fat content X is detected, to ensure that Z remains at its steady value (that is, $dZ/dt = 0$)? The answer from equation (7.8) is that $dZ/dt = 0$ provided F_2 is

$$F_2(t) = \frac{F_1(0)X(t) - F_1(0)Z(0)}{Z(0) - Y(0)}$$

The flowrate of the added stream should always be proportional to the inlet fat content X. It is obvious in this case that the change in flowrate ($= f$) is also proportional to the change in fat content:

$$f = K_c x \tag{7.24}$$

where

$$K_c = \frac{F_1(0)}{Z(0) - Y(0)} \tag{7.25}$$

This is an example of **proportional feedforward** control. Note that the value of K_c, the controller gain, is fixed. If the model is wrong, or the flowrate F_1 is different from the value assumed in calculating K_c, the control system won't respond correctly to changes in inlet fat content, and the controlled value of X will be different from its desired value. The flow must also respond **immediately** to changes in X to achieve the desired objective.

EXAMPLE 7.2: FEEDFORWARD CONTROL

Consider a continuous blender, capacity 100 kg, with the following steady design conditions:

$F_1(0) = 1000\,\text{kg h}^{-1}$
$X(0) = 0.06$ *kg fat/kg milk*
$Y(0) = 0.01$ *kg fat/kg milk*
$Z(0) = 0.05$ *kg fat/kg milk*

The fat content X(t) of the stream inlet F_1 may vary between 0.05 and 0.07 kg/kg. Design a proportional feedforward system to ensure that Z remains constant.

From the steady-state version of the mass balances (equations (7.6) and (7.9)):

$$F_2(0) = 250\,\text{kg}\,\text{h}^{-1} \quad \text{and} \quad F(0) = 1000\,\text{kg}\,\text{h}^{-1}$$

Then from equation (7.25):

$$K_c = \frac{1000}{0.04} = \frac{25\,000\,\text{kg}}{\text{kg fat/kg milk}}$$

Suppose that the inlet fat content jumps instantaneously from 0.06 to 0.061 kg/kg; then F_2 must also change instantaneously by f_2 = (25000) (0.001) = 25 kg h^{-1}, so that F_2 = 275 and F = 1275 kg h^{-1}, to ensure the correct value of Z.

However, if the blender was actually working with an inlet milk flow different from the assumed value of 1000 kg h^{-1}, or the fat content of stream F_2 was not 0.1 kg/kg, or the proportional control constant was not 25000, Z would not be at its desired value, unless the control action was changed to allow for this.

Example 7.2 shows the power of feedforward control. However, for it to be effective every disturbance must be measured and the plant model must be accurate. It is useless, however, in the face of unmeasured plant disturbances, and is sensitive to the accuracy of the plant model.

7.6.2 Feedback control

Some of the key features of any feedback control system can be discussed by first exploring the behaviour of a process whose dynamics are so fast that, in a first analysis, they can be neglected. As noted earlier it is convenient to work in terms of **changes** in key variables rather than their absolute values, not least because it allows us to work with linear models: the object of the control scheme is then, if possible, to reduce the deviation in the measured output to zero. Consider the flow control loop in Fig. 7.4.

In the absence of any control a disturbance δ in the pressure upstream or downstream causes a change f in the uncontrolled flowrate, where

$$f = K_{p1}\delta \tag{7.26}$$

(K_{p1} would be positive for upstream pressure changes and negative for downstream fluctuations.)

With control the effect of the disturbance is compensated by a movement in the valve opening. The measured value f is compared with its desired value (the set point) f^*; the difference between these two, the error signal

$e = f^* - f$, is the input to the controller. The output from the controller causes the valve opening to change to compensate the measured deviation. We assume that a proportional controller is used: that is, a controller whose output $\theta = K_c e$. Further, we assume that the valve itself is linear so that its output flow $q = K_v\theta$. As we are neglecting dynamic effects, the pipe between the valve and measuring point has no effect on the flowrate (that is, with no disturbance, $f = q$; the gain of the pipe or process, $K_p = 1$). Then

$$f = K_{p1}\delta + q \tag{7.27a}$$

$$= K_{p1}\delta + K_v K_c e \tag{7.27b}$$

$$= K_{p1}\delta + K_v K_c\left(f^* - f\right) \tag{7.27c}$$

Rearranging:

$$f = \frac{K_{p1}\delta}{1 + K_v K_c} + K_v K_c \frac{f^*}{1 + K_v K_c} \tag{7.28}$$

This is the 'closed loop' relationship: that is, when the process is controlled, between changes in the flowrate, the disturbance and the set point. Note that the effect of feedback control is to reduce the sensitivity of the output to the input changes. Comparison of equations (7.26) and (7.28) shows that, for a given disturbance, the outlet flow is reduced from its 'open-loop' value (its value without feedback control) by $1/(1 + K_v K_c)$. If the pipe had a static gain K_p this term would become $1/(1 + K_p K_v K_c)$.

At first sight, perhaps the most surprising feature is that it is **not** possible with this control scheme to ensure perfect control. For example, if the system is upset by a disturbance that remains at a finite value, then f must also be finite. If the set point is changed, the output flow f can never exactly equal its desired value! However, the higher the value of the proportional control constant or gain (that is, the more sensitive the control action), the lower the value of f. The larger $K_v K_c$ is the smaller is the effect of a disturbance, as for large $K_v K_c$, $f \approx (K_p/K_v K_c)\delta$; also, the outlet approaches the set point more closely as $f \approx f^*$. The phenomenon whereby the output is always slightly displaced from the desired value is a general feature of all **proportional** feedback control schemes. It is known as **offset**. The reason for offset is that the flow generated by the valve to counteract the effects of disturbances must result from a finite change in the measured variable: if there was no change there would be no control action. One implication of this is that we should look to other forms, apart from proportional control, of control action.

EXAMPLE 7.3: FEEDBACK BLENDER CONTROL

A single feedback control loop, shown in Fig. 7.3, is used to control the same blender as in Example 7.2. What proportional control constant will *ensure that the outlet fat content Z remains within 0.001 kg/kg of its desired value in the face of disturbances in the fat content, X, of stream 1 of up to 0.01 kg/kg?*

The diluent flowrate F_2 is manipulated in response to measured changes in the **outlet** fat concentration Z (Fig. 7.3) by altering the flowrate in direct proportion to the error signal. As before the steady compositions and flowrate are:

$F_1(0) = 1000\,\mathrm{kg\,h^{-1}}$	$F_2(0) = 250\,\mathrm{kg\,h^{-1}}$	$F(0) = 1250\,\mathrm{kg\,h^{-1}}$
$X(0) = 0.06\,\mathrm{kg/kg}$	$Y(0) = 0.01\,\mathrm{kg/kg}$	$Z(0) = 0.05\,\mathrm{kg/kg}$

It is assumed that all process dynamics are very fast, so that even when inputs are changing the process is always at the corresponding steady state.

First we require the closed loop relationship, analogous to equation (7.28), between z, x and f_2 (working with perturbation variables, as before). From the fat balance (equation (7.11)), setting $dz/dt = 0$ because of the quasi-steady state assumption:

$$F(0)z = [Y(0) - Z(0)]f_2 + F_1(0)x$$

That is,

$$z = -0.000032 f_2 + 0.8x \tag{7.29}$$

which is the model for the block marked 'blender' in Fig. 7.3. The flowrate from the combination of controller and valve is proportional to the error signal; that is,

$$f_2 = K_c K_v (z^* - z) \tag{7.30a}$$

so that the closed-loop relationship is:

$$z = -0.000032 K_c K_v (z^* - z) + 0.8x \tag{7.30b}$$

That is,

$$z = -0.000032 K_c K_v (1 + 0.000032 K_c K_v)^{-1} z^* + 0.8(1 + 0.000032 K_c K_v)^{-1} x \tag{7.31}$$

Thus, for constant set point, i.e. $z^* = 0$ and for a change in inlet fat content $x = 0.01$ kg/kg, the condition from equation (7.31) for z to vary by only 0.001 kg/kg is that:

$$1 + 0.000032 K_c K_v = 8$$

That is,

$$K_c K_v = 218750\,\mathrm{kgh^{-1}/(kg/kg)}$$

A higher value will ensure a smaller offset from the ideal, $z = 0$. With this control setting the relation between z and the desired value (corresponding to servo control) is $z = (7/8)z^*$. That is, there would be a steady offset of $0.125z^*$ following a change z^* in the set point.

Note that, as in the first example of proportional feedback control, the term $(1 + K_p K_c K_v)^{-1}$ plays a crucial role in determining the system sensitivity.

7.6.3 Dynamics and control

Now we consider how the process dynamics affects feedback control. Intuitively we might expect the controlled variable to respond to a step change in a disturbance or set point by settling down to the value predicted from the static analysis. This will usually be true, provided the control system is stable (which can only be established from analysis of the dynamics); however, it will also be clear intuitively that the dynamics of the process and the various units in the feedback loop, such as the measuring element, must affect the control behaviour.

Again, we consider the example of the mixer–blender, but with the important difference from Example 7.3 that the dynamics of the blender itself are included. We assume that all other parts of the closed loop have very fast dynamics in comparison with the blender.

Thus the appropriate open-loop model is now a linearized dynamic model for the blender. Again we consider only one disturbance (x) and one manipulated variable (f_2). Then equation (7.6) becomes

$$f_2(t) = f(t) \tag{7.32a}$$

and equation (7.11) is

$$V\frac{dz}{dt} = [Y(0) - Z(0)]f_2 + F_1(0)x - F(0)z \tag{7.32b}$$

$$= Af_2 + F_1(0)x - F(0)z \tag{7.32c}$$

where $A = Y(0) - Z(0)$. As before

$$f_2 = K_c K_v (z^* - z) \tag{7.30a}$$

so that

$$V\frac{dz}{dt} = -[AK_c K_v + F(0)]z(t) + K_c K_v z^*(t) + F_1(0)x(t) \tag{7.33}$$

Equation (7.33), describing the closed loop, has exactly the same first-order form as the open-loop system (equation (7.14)), with the exception that there are two possible 'forcing' functions or inputs, z^* and x. For example, if we wish to examine the response of the system to a step change only in the inlet fat content x (so that $z^* = 0$), equation (7.33) becomes

$$\tau \frac{dz}{dt} + z(t) = Kx(t) \qquad (7.34)$$

where the time constant $\tau = V/[F(0) + AK_cK_v]$ and the closed-loop static gain $K = F_1(0)/[F(0) + AK_cK_v]$. These can be compared with the corresponding open loop values (equation (7.14)) $\tau = V/F(0)$ and $K_p = F_1(0)/F(0)$. Feedback proportional control **reduces** the apparent time constant for the system – that is, speeds up the response – and reduces the ultimate effect of a permanent change on the output, without eliminating it completely, as the offset = $Kx(0)$.

EXAMPLE 7.4

Consider the same blender as in Example 7.3. Compare the open- and closed-loop responses to step and sinusoidal changes in the inlet fat content, X, with the same control setting as in the previous example. As before the steady compositions and flowrates are:

$F_1(0)$ = 1000 kg h^{-1}	$F_2(0)$ = 250 kg h^{-1}	$F(0)$ = 1250 kg h^{-1}
$X(0)$ = 0.06 kg/kg	$Y(0)$ = 0.01 kg/kg	$Z(0)$ = 0.05 kg/kg

The open-loop characteristics are (see equation (7.14)):

$$\tau = \frac{V}{F(0)} = 0.08\,\text{min} \qquad \text{and} \qquad K_p = \frac{F_1(0)}{F(0)} = 0.8$$

As in the previous example, K_cK_v = 218 750 kg h^{-1}/(kg/kg). Note that, in equation (7.33), $A = Z(0) - Y(0)$ = 0.04 kg/kg and AK_cK_v = 8750 kg h^{-1}.

The time constant of the controlled system is (equation (7.34)), τ = 0.01 min, and K = 0.1: both reduced eightfold from the open-loop values. The response of the system to a step change in inlet fat content $x(0)$ = 0.01 kg/kg is, from equation (7.3):

$$\begin{aligned} z(t) &= Kx(0)\{1 - \exp - t/\tau\} \\ &= 0.008\{1 - \exp - 12.5t\} \qquad \text{(open loop)} \\ &= 0.001\{1 - \exp - 100t\} \qquad \text{(closed loop)} \end{aligned}$$

The step responses of the uncontrolled (open loop) and controlled (closed loop) blender are shown in Fig. 7.15.

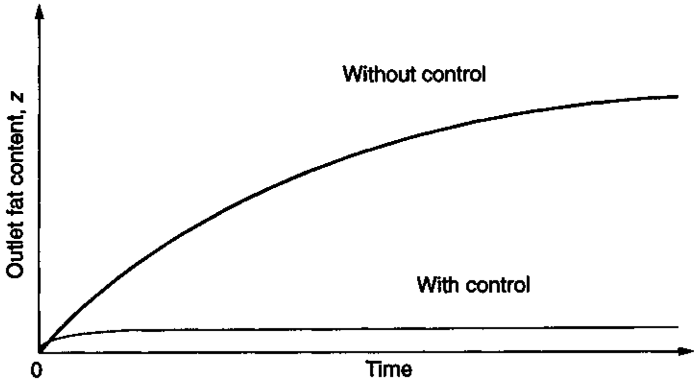


Fig. 7.15 Example 7.4: step response of blender, with and without control.

Note that, as expected, the final value of z is the value calculated from the static analysis. We can also easily obtain the responses of the system to sinusoidal changes in inlet fat content. We assume the same disturbance amplitude (=0.01) as the step disturbance. The amplitude $|z|$ and the phase shift ϕ of the output concentration calculated from equations (7.23a) and (7.23b) (namely $|z| = K_p x(0)/[1 + \omega^2\tau^2]^{0.5}$ and $\phi = -\tan^{-1}(\omega\tau)$) are tabulated below for disturbance frequencies ω = 1 and 1000 rad min^{-1} respectively:

	Frequency, ω			
	1		1000	
	Open loop	Feedback	Open loop	Feedback
$\|z\|$	0.008	0.001	0.0001	0.0001
ϕ	4.6°	0.6°	89.3°	84.2°

When the disturbance varies slowly, the amplitude of the controlled outlet fat content is the same as the steady result following a step change; however, for a rapid disturbance the combined effect of the process dynamics and the feedback loop is the virtual complete elimination of the disturbance, even though it does now lag the input by almost 90°. Note, too, that with this control setting the main improvement over the uncontrolled system is seen at lower frequencies. This is because the blender itself 'irons out' high-frequency disturbances.

Remember that the great advantage of feedback control is that its efficacy does not depend on being able to measure or even identify the principal disturbances. Although the example above was developed on the assumption that there was only one disturbance, the same qualitative behaviour would result if the system was upset by changes in any of the other inputs, such as the flowrate F_1.

Note also that the results above all depend on the assumption of proportional control; below we shall discuss what other forms of controller action are available and how they might be expected to influence process behaviour.

7.6.4 Block diagram representation: the algebra of closed loops

In the examples above, the input–output relationships for the controlled system were found by incorporating equations to represent the proportional control action into the unsteady-state model. Long-winded mathematical derivations for each new problem can be avoided by deriving input–output relationships directly from a block diagram representation of the system. It will be seen that this allows some of the key results above to be generalized.

Block diagrams for various control schemes are shown in some of the diagrams above. The blocks or boxes usually represent a piece of equipment such as the process itself, the controller or a valve. The arrowed lines represent the direction of signal flow, which does not always coincide with the direction of material flow; for example, fluctuations in downstream pressure could act as a disturbance to the flow control system in Fig. 7.4. Nevertheless, this is properly represented as an 'input' ($= \delta$ in Fig. 7.4) to the process, affecting the 'output': the measured flow.

The lines in the block diagram represent the 'signals' (flowrates, temperatures etc.) and their direction. The signals (and the blocks) must obey the rules of dimensionality: we can add two flowrates but not a flowrate and a temperature. Junctions of lines represent addition and subtraction of signals, as shown in Fig. 7.16.

For linear systems each block represents an operator on the input, defined so that output $= G \times$ input, i.e. $\theta = G_1 q$ etc. The **transfer function** G has the dimensions of [output]/[input]. For example, if the input is a flowrate ($\text{kg}\,\text{h}^{-1}$) and the output is a temperature (°C), G has the units $°\text{C}/(\text{kg}\,\text{h}^{-1})$.

The input and output variables (for example x, y and z in Fig. 7.16) are always defined as **deviation** or **perturbation variables**, which are therefore zero at steady state. In the simplest cases the transfer function G is a constant, but it can in fact be any operator, or combination of operators, that obeys the rules of linearity: namely, that if a is a constant the output y corresponding to an input ax [i.e. $G \times (ax)$] is $y = aGx$, and that $G \times (x + z) = Gx + Gz$. For example, the differential operators $\text{d}/\text{d}t$ ($= \text{D}$), $\text{d}^2/\text{d}t^2$ ($= \text{D}^2$) etc. are linear operators. G need not be a scalar quantity: multivariable

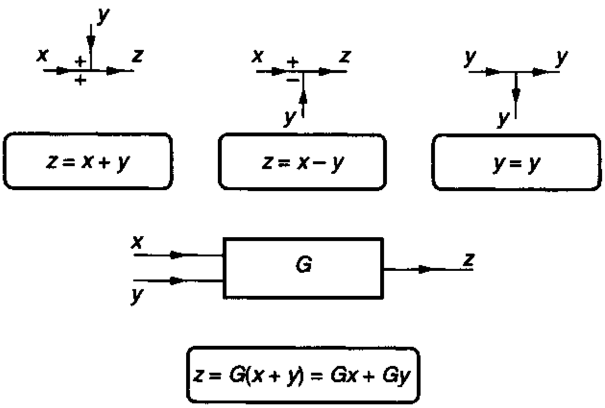


Fig. 7.16 Block diagram and signal flow notation.

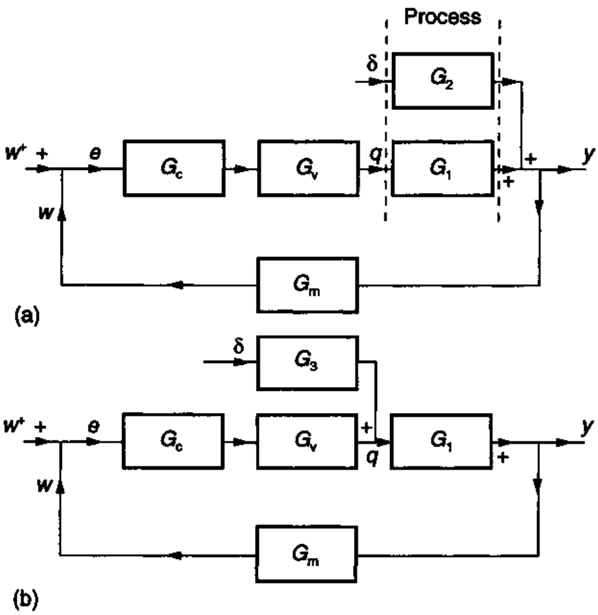


Fig. 7.17 A feedback control system.

systems can be represented by vectors of inputs and outputs linked by transfer function matrices.

Consider Fig. 7.17(a), representing a typical feedback control system. The transfer functions G_1 and G_2 represent the process: they relate changes in the controlled flow q and the disturbance δ to the uncontrolled process output. Without feedback and with $q = 0$ the output would be $y = G_2\delta$. We can easily derive the closed-loop transfer functions between the output, the disturbance and the set point, since

$$y = G_1 q + G_2\delta \tag{7.35a}$$

$$= G_1 G_v G_c e + G_2\delta \tag{7.35b}$$

Now the error e is

$$e = w^* - G_m y \tag{7.36}$$

so that

$$y = G_1 G_v G_c w^* - G_1 G_v G_c G_m y + G_2\delta \tag{7.37}$$

or

$$y = \frac{G_1 G_v G_c w^*}{1 + G_1 G_v G_c G_m} + \frac{G_2\delta}{1 + G_1 G_v G_c G_m} \tag{7.38}$$

That is,

$$y = H_1 w^* + H_2\delta \tag{7.39}$$

This is the **closed-loop transfer function** for the system. If the set point is constant and unchanged, $w^* = 0$ and $y = H_2\delta$. Each of the individual **closed-loop** transfer functions H_1 and H_2 has the same structure:

$$H = \frac{\text{Product of transfer functions between input and output}}{1 + \text{Product of all transfer functions within the loop}}$$

We call the product term in the denominator, $G_1 G_v G_c G_m$, the **system open-loop transfer function**, L.

Thus the closed-loop transfer function H between **any** input x_1 and **any** other signal x_2, such as the controlled output, defined by $x_2 = Hx_1$ is (with negative feedback)

$$H = \frac{G_f}{1 + L} \tag{7.40}$$

where G_f is the product of the transfer functions between the input and the output **in the direction of signal flow** (that is, the forward path transfer function). Positive feedback (generating a signal $e = w^* + w$) produces a closed-loop transfer function of form $G_f/(1 - L)$, which is often unstable, as the control signal reinforces rather than cancels the effect of the disturbance.

Note that the two block diagrams in Fig. 7.17 are exactly equivalent provided that $G_1 G_3 = G_2$.

In general, the larger the value of L the more effective is the control loop in attenuating the effects of a disturbance on the output (as in Fig. 7.17(a) $y = G_2\delta/[1 + L]$), and the closer does the output track the set point, for a given G_2. We can apply the same reasoning – that the smaller the magnitude of $G_2/(1 + L)$ the better – to the dynamic behaviour of a control system. In this case the system frequency response is particularly useful; recall from Example 7.4 that the ratio of the amplitudes or magnitudes of output and input sinusoids is frequency-dependent. The amplitude or modulus of $G_2/(1 + L)$ must be considered as a function of frequency. (For most (proper) systems the amplitude of any transfer function tends to the static gain (K) at low frequencies and towards zero at high frequencies.)

The denominator term $1 + L$, which is called the **characteristic equation**, plays a very important role: the classical methods of stability analysis and control system design are based around this equation.

7.6.5 Feedback and feedback system sensitivity

One important feature of feedback control is its influence on the system sensitivity. Consider a system (Fig. 7.18) with output y and input x; G_f is the forward path transfer function. In the absence of feedback, $y = G_f x$ or $y/x = G_f$.

Differentiation and a little algebra lead to the open-loop result that

$$\left[\frac{\mathrm{d}(y/x)}{(y/x)}\right]_{\mathrm{OL}} = \frac{\mathrm{d}G_f}{G_f} \tag{7.41}$$

With feedback, $y = G_f x/(1 + L) = G_f x/(1 + G_f G)$, where G is the feedback transfer function. Thus

$$\frac{y}{x} = \frac{G_f}{1 + G_f G} \tag{7.42}$$

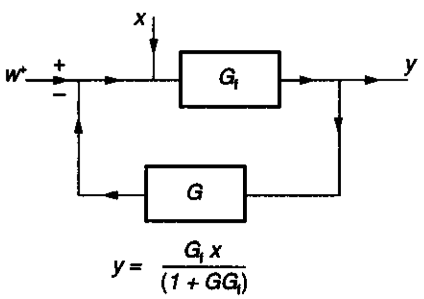


Fig. 7.18 Simple closed loop.

Differentiating with respect to G_f gives the closed-loop result:

$$\left[\frac{d(y/x)}{(y/x)}\right]_{CL} = \frac{dG_f}{G_f(1+G_fG)} \tag{7.43}$$

which $\ll dG_f/G_f$ for $G_fG = L \gg 1$.

The ratios $d(y/x)/(y/x)$ and dG_f/G_f are sensitivity coefficients. Equation (7.43) shows the improvement (reduction) in sensitivity of the relation between x and y – that is, $[d(y/x)/(y/x)]_{CL}$ – because of feedback. The sensitivity of the closed loop is less than the sensitivity of the open loop. Also, the sensitivity of the closed loop to a small change (or modelling error) in the forward path transfer function dG_f/G_f is low. These are most important results: they tell us (what we have already seen above in the discussion of the proportional gain) that the larger L (that is, G_fG) the better in terms of control performance; it also tells us that the result is normally not very sensitive to modelling errors in the process transfer function (unlike feedforward control, which is extremely sensitive to the model).

7.6.6 Feedback cancellation and stability

Figure 7.19 shows a generalized SISO feedback control loop. In the absence of feedback the process output $y = y_p$ is due solely to the effect of the disturbance.

With feedback control the output is the sum of y_p and the compensating output y_c. Ideally, $y_c = -y_p$ as this would ensure that $y = 0$. Now consider how y_c and y_p are related: to do this we can apply the general result for the closed-loop transfer function (equation (7.40)) to give

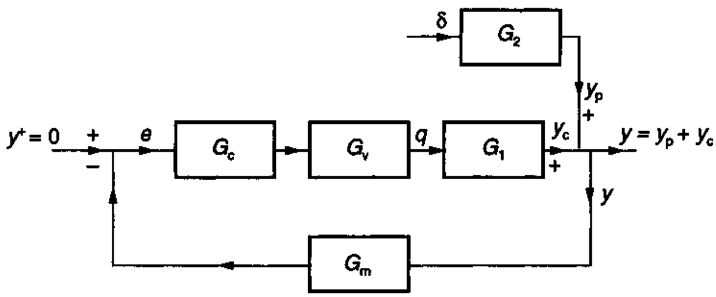


Fig. 7.19 Cancelling signals in feedback control.

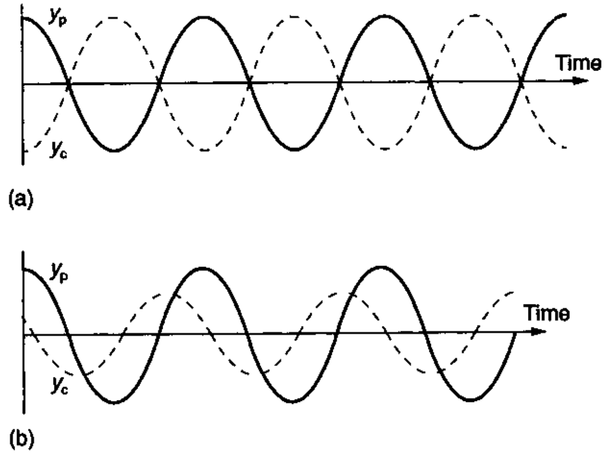


Fig. 7.20 Cancelling effect of controller input: (a) perfect and (b) imperfect cancellation of effect of disturbance.

$$y_c = -\left(\frac{G_1 G_v G_c G_m}{1 + G_1 G_v G_c G_m}\right) y_p \tag{7.44}$$

$$= -\left(\frac{L}{1+L}\right) y_p$$

$$= -\frac{y_p}{1/L + 1} \tag{7.45}$$

For any disturbance $y_c \to -y_p$ provided $L \gg 1$. When the dynamics of all the elements in the loop are fast the transfer functions G_1, G_v etc. are constants (the static gains K_1, K_v etc.) and, theoretically, it is possible to realize the ideal of cancelling the disturbance, provided only that the static open loop gain $K_L = K_1 K_v K_c K_m$ is very large. This is illustrated in Fig. 7.20 for a sinusoidal y_p. The two outputs cancel each other exactly.

However, to be realistic we cannot neglect all the process dynamics. Earlier we saw that the effect of the system dynamics is to change either or both of the amplitude and the phase angle between the input and output signals (a dead time introduces a phase shift only; first-order and higher transfer functions also change the gain). Figure 7.20 also illustrates qualitatively how this can affect the behaviour: the net output y is now longer zero (in fact it is also a sine wave with the same frequency as y_p). Moreover, when the phase shift (which is in general frequency-dependent) reaches 180°, the two signals y_c and y_p are exactly out of phase, and the feedback control reinforces the effects of the disturbance. This situation is potentially unstable and many of the classical control design methods were evolved to ensure that stability is ensured. Although the mathematics involved is beyond the scope of this text we can note that a system in which L has only first-order dynamics (such as a proportional controller and a first-order process) can never become unstable as the maximum phase shift is, as we have seen, 90°. Systems of higher order than 2 or where a dead time is present may, however, become unstable with feedback control. Good design will always attempt to minimize any dead times occurring in the loop.

7.7 Types of controller action

So far, all the discussion has centred on the use of proportional control, partly because we have wanted to avoid unnecessary difficulties with the mathematics. However, we have come across one limitation on proportional action: the existence of offset. High proportional gain leads to lower offset, but it may lead towards instability, or be impractical. In the following section we briefly summarize some other types of control action. The simulation accompanying this book will be found useful in illustrating some of the ideas mentioned below.

7.7.1 On–off control

If the gain K_c of a proportional controller is made very high, its output switches from one extreme to another in response to very small variations in the error signal. Effectively, then, the valve operated by the controller will either be fully open or fully closed according to whether the error signal is negative or positive. (Note that the analyses above did not allow for the constraints to which valves are subject in practice.) In practice, on–off control is implemented by a simple switching relay. It has several advantages: it is cheap; response is usually rapid as the control is either fully on or off; and it is simple (but the relay must be robust to go through many thousands of switching operations). A disadvantage is that the quality of control is usually inferior to that achieved with continuous controllers. It is a form of control that will be familiar to many readers from their domestic heating systems. Anyone who has played with the thermostat on such a system will know that the relay doesn't switch on and off immediately the temperature rises above or drops below the thermostat setting: this would result in continuous high-frequency chattering. Instead there is a small dead zone over which the relay is insensitive.

Figure 7.21 illustrates an on–off controller, together with the effect of the dead band on its behaviour. Figure 7.22 shows how an on–off controller could be used to control the continuous blender that we have examined above. Rather than switch between zero and maximum flow of the control

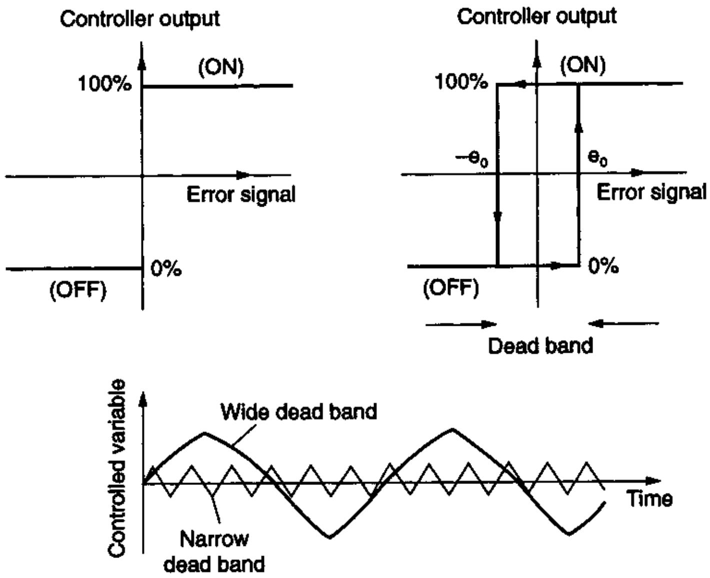


Fig. 7.21 On–off controller: ideal and with dead band.

stream F_2 it is often convenient, as shown here, to maintain a steady 'background' flow with an additional stream as the controlled input.

7.7.2 Integral action: eliminating offset

Although offset can sometimes be effectively eliminated by making K_c as high as possible, in practice the system dynamics often make this impossible, as increasing the controller gain can lead to increasingly oscillatory behaviour and ultimately to instability. It can also lead, as we have seen above, to violent swings in the valve stem position. However, if the controller output signal depends not only on the actual value of the error signal e but on its time **integral**, then offset can be removed, as the signal to the valve will continue to increase so long as the deviation continues. This is known as **integral action**, and is usually implemented in combination with proportional control as a **proportional plus integral** (P + I) controller whose ideal output is

$$v = K_c\left[e + \left(\frac{1}{T_I}\right)\int e\,dt\right] \tag{7.46}$$

T_I is the **integral action time**. (Its inverse is called the **reset rate**.) The smaller its value is the more significant is the integral term. The disadvan-

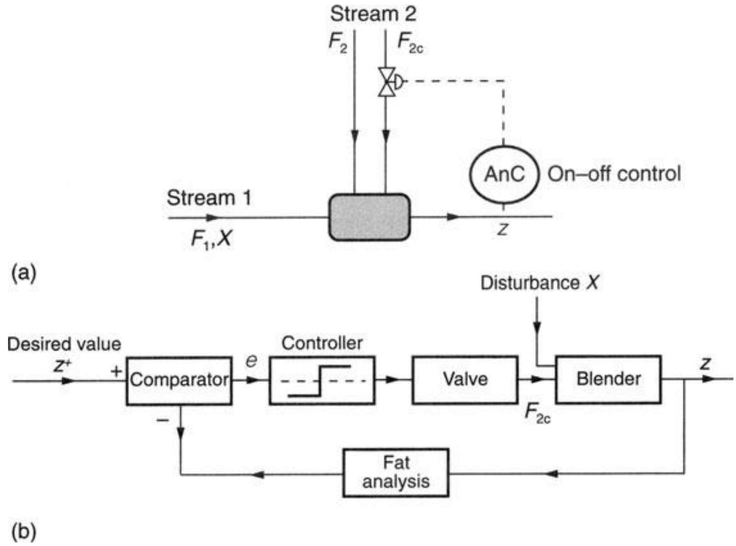


Fig. 7.22 On-off feedback control of blender: (a) process flowsheet; (b) block diagram of control loop.

tage of integral action is that, for a given proportional gain, it tends to make the system response more oscillatory and unstable.

At this stage readers should find it helpful to experiment with the control simulation on the disk accompanying this book and discussed in section 11.12. The example allows the step response of a stirred tank heater (a first-order system) to be examined with and without feedback control. The open loop system has a time constant of 10 min. The controlled behaviour can be examined with proportional or proportional plus integral control; the consequences for control performance and stability of a dead time within the control loop can be examined in some detail.

7.7.3 Derivative action: speeding up response

Another common type of controller action is realized by adding to the controller output a term that is proportional to the **rate of change** of the error signal. The idea is to speed up the control system response to deviations. This type of controller signal is, not unnaturally, called **derivative** action; a simple ideal two-term (P + D) controller would have an instantaneous output:

$$v = K_c\left[e + T_D\left(\frac{de}{dt}\right)\right] \tag{7.47}$$

T_D is the **derivative action time**: the larger its value is the greater is the weight given to the derivative signal. The effect of derivative action is to decrease oscillatory tendencies and to speed up the response (for example, by reducing the settling time following a step disturbance). It does not, however, alter the offset. The derivative action time must be chosen with some care: too high a value can produce an over-sensitive response from the controller, whereby every noisy fluctuation provokes a change in the controller output and in the correcting element.

7.7.4 The three-term controller

The classical **three-term** or P + I + D controller involves contributions, which can be tuned at the controller panel or, nowadays, at the control computer, from all three terms:

$$v = K_c\left[1 + \left(\frac{1}{T_I}\right)\int e\,dt + T_D\left(\frac{de}{dt}\right)\right] \tag{7.48}$$

In practice real controllers are approximations to, rather than exact realizations of, these ideal types. They will incorporate constraints on the control parameters, and hardware or software approximations to the derivative and integral terms. Nonetheless, the main principles outlined above remain valid.

Many methods exist for 'tuning' standard controllers – that is, selecting the most appropriate values of the control parameters – but the techniques used are beyond the scope of this chapter. Where approximate process models are available, probably the most widely used methods are those, like the Ziegler–Nicholls criteria, based on frequency response analysis; details are found in all the standard texts (see further reading section). Root locus methods are also used. Alternatively, in the absence of a process model, an approximate model can be identified experimentally, and used as the basis for controller tuning. In any event, a satisfactory set of controller settings will ensure process stability, while also ensuring a rapid, but not over-oscillatory, response to process disturbances or changes in set point. In practice, the final values of the controller settings are established on line by trial and error around the design values.

Figure 7.23 shows qualitatively how the different control modes influence the dynamic behaviour. The addition of integral action removes the offset associated with proportional control, but at the expense of increased oscillation, for the same proportional control constant. Derivative action will improve the system response over that of the two-term P + I controller. The behaviour of a proportional and a P + I controller can be compared and contrasted by using the simulation with this book. Further comparison will be found in many of the standard textbooks referred to in the bibliography.

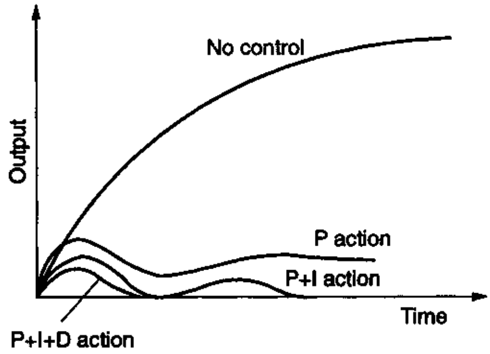


Fig. 7.23 Qualitative step responses of various systems.

7.7.5 Adaptive control

The classical methods for linear systems design can be unsatisfactory for processes with parameters that change with time, such as a heat exchanger subject to fouling, or for non-linear processes, where the assumption that only small changes occur is not valid; at some point the relative insensitivity of feedback loops to process changes is no longer sufficient, and control performance may deteriorate with time. Sometimes, inherent non-linearities can be effectively cancelled by incorporating a compensating non-linear element in the control loop. The correct choice of control valve characteristics (that is, the precise relation between valve lift and flowrate) is a good example. Adaptive controllers respond to changes in the process by automatically adjusting their parameters, such as the proportional gain, so as to compensate for variations in the process characteristics. This is one area where theoretical ideas from a couple of decades ago are now a practicality because of developments in computer hardware and software.

Another area where on-line computation helps is in coping with situations where key process outputs are not measurable but, provided good process models are available, can be inferred using 'soft sensors' from other measurements. Some areas where inferential techniques have been used in control include the control of fermenters and distillation columns (where key concentrations can be inferred from temperatures, flows and pressures). Some of these techniques use mechanistic process models; others are based on 'black box' statistical models, using techniques including neural networks.

7.7.6 Multivariable control

Many processes have several inputs and outputs (MIMO [= multi-input multi-output] in control jargon), and the first problem is to choose the best set of connections between the measurements and the manipulated variables. Figure 7.7 shows the continuous blender (excluding the level) as a MIMO system; note that changes in inlet flowrate affect **both** the output flow and fat content, while changes in the inlet fat contents affect **only** the outlet fat composition. As described earlier, a possible feedback strategy (illustrated schematically) would be to control the product quality by varying the flowrate of one stream. The flow of the other stream could be controlled independently or (not shown) used to control the level of the contents on the blender. In either case, the output flow **cannot** be controlled, as its (average) value is determined by the two input flows. An ideal control system will have the minimum number of single-input single-output loops; these loops should be non-interacting, in the sense that when one loop is active it doesn't influence (or worse, conflict with) another; the response of the system should be fast, direct and stable. Sometimes process interactions are such that this ideal cannot be realized. Often, intuition and a basic understanding of the way the process operates is enough to develop an appropriate control structure, as in the blender example. Another example, showing two schemes to control the temperature and level of a continuous liquid heater, is illustrated in Fig. 7.24. Whether it is possible to control the input flow (L_i) depends on the operations (if any) further upstream, illustrating the point that the whole system and its dynamics must be considered.

Quantitative methods now exist to guide MIMO system design: the simplest of these, such as the relative gain array method, essentially try to establish the control configuration on the basis of the relative sensitivity (that is, the gains) of the various interconnections. Other methods apply frequency response methods to the whole system. Useful rules of thumb in selecting the control configuration include the following.

- Select manipulated variables that have a direct and fast effect (usually implying a high gain) on the controlled variable.

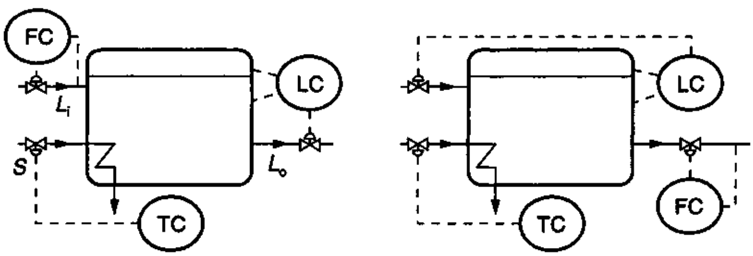


Fig. 7.24 Two configurations for heater control. L_i, L_o, output flows; S, steam rate; Q, heat input; θ, disturbance.

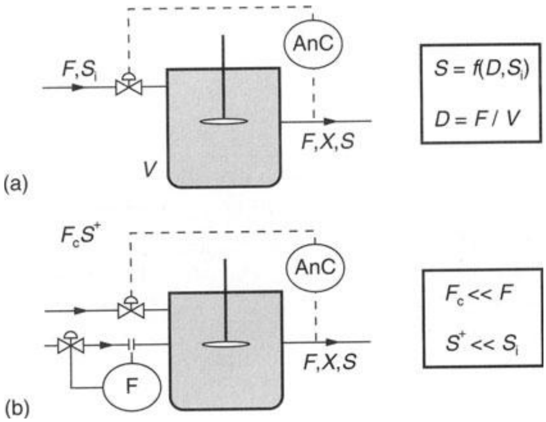


Fig. 7.25 Continuous fermenter: decoupling inputs.

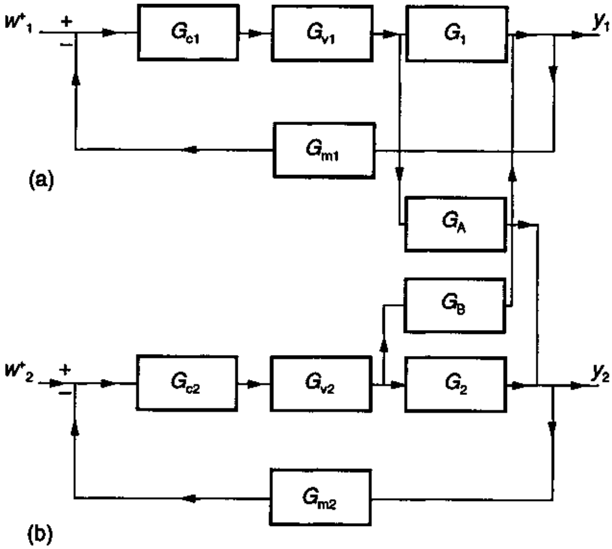


Fig. 7.26 Block diagram of interacting system.

- Where possible, avoid interactions between control loops.
- Minimize time delays within the loops.

Sometimes interactions can be decoupled by careful engineering design. For example, the performance of a continuous fermenter can depend both on the inlet substrate concentration and the feed flowrate (that is, the dilution rate). Scheme (a) in Fig. 7.25, to control the residual substrate concentration by changing the dilution rate via the inlet flowrate, is less than ideal for this reason, as the two variables cannot be changed independently. However, scheme (b), in which a highly concentrated stream of substrate is used as an additional feed (but with very low flowrate), allows almost complete decoupling of the two effects, as concentration and flowrate can be manipulated independently of each other.

Alternatively, whole or partial uncoupling can often be achieved by appropriate control system choice. Figure 7.26 shows a block diagram to illustrate a coupled or interacting system. When the transfer functions G_A and G_B are small the degree of interaction is also small. When it is not possible to eliminate such interactions through engineering design it may nevertheless be possible to reduce the interaction or even eliminate it by appropriate design of cross controllers to 'cancel out' the effects of the interactions G_A and G_B.

7.8 Control system design for complete plants

Most modern processes involve a number of operations in series or which follow in sequence. The best procedure is to consider the process unit by unit, as dynamic considerations suggest that attempts to close the loop around the whole plant (strategy (a) in Fig. 7.27) will be less satisfactory than the alternative (b) of a sequence of separately controlled units.

Of course, the implications for successive units must be taken into account because, as we have seen, only a limited number of independent control schemes is possible for any given unit. Often it is convenient to engineer a degree of uncoupling between successive units by introducing

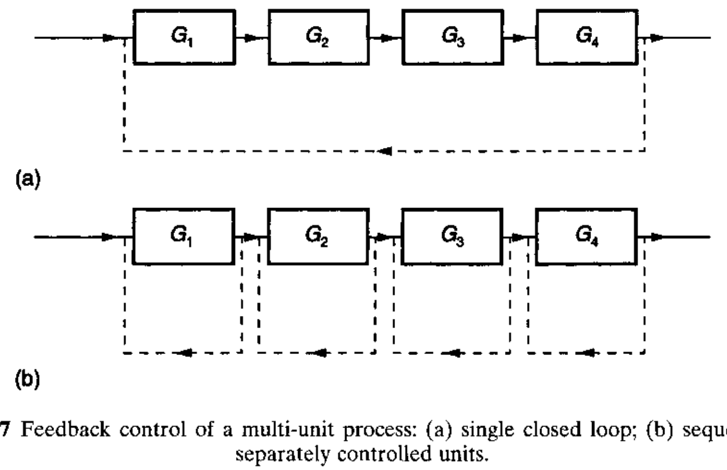


Fig. 7.27 Feedback control of a multi-unit process: (a) single closed loop; (b) sequence of separately controlled units.

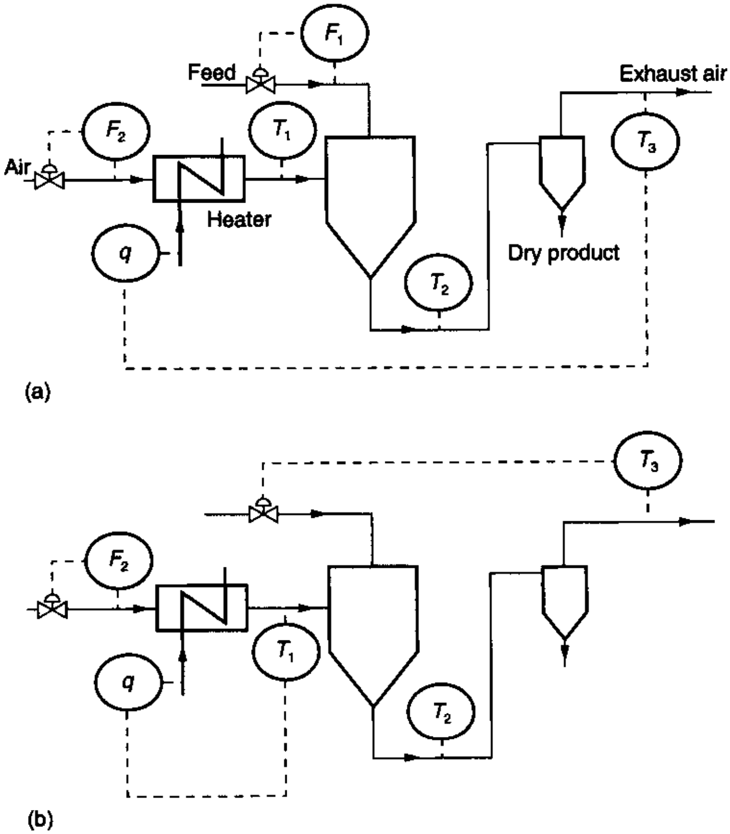


Fig. 7.28 Alternative control schemes for spray drier.

intermediate storage or buffer tanks. An important part of the system design is to ensure that material balance requirements can always be satisfied without process vessels overflowing or running dry, and that suitable alarms and overrides are incorporated into the control structure to cope with unforeseen problems such as failure in the supply of one of the services.

Some other aspects are illustrated by the control of a continuous spray drier (Fig. 7.28). The principle disadvantage of scheme (a) stems from the time delay between the output measured temperature and the control input; with large scale units this would be likely to give rise to poor system performance. Scheme (b) is much better from this point of view, since the time delays are reduced. It is important in any event to ensure that the final measuring element is as close to the feed as practicable. Thus if possible the temperature element would be sited between the drier and the cyclone (i.e. at T_2) rather than after the cyclone at T_3. In practice any system should incorporate an override on the exhaust temperature to shut down the plant or switch to a water feed if this temperature became too high.

Conclusions

Process control, which has been introduced in this chapter, is essentially concerned with making the best use of information in order to ensure that processes work efficiently, that product quality is maintained and that excursions from the desired operating conditions are minimized. Three types of information – all imperfect – might be available: measurements on some of the process inputs and their properties; measurements on some of the process conditions or 'outputs' in the language of control engineering (temperatures, pressures, product flowrates or quality measures such as colour, etc); and, finally, models of how the process is expected to behave. This chapter has introduced some of the methods and ideas underlying the theory and practice of control engineering, in putting these different types of information to best use.

Inevitably, control is concerned with transient behaviour, and this leads to mathematical complications; this chapter has tried as far as possible to avoid unnecessary mathematics, so as to stress the underlying principles. From the first part of the chapter you should have learned about the different types of control objective and their importance. You should also have seen how to represent processes in terms of block diagrams, in which the blocks or boxes represent operations and the directed lines connecting them represent the signal or information flows; later in the chapter the algebra of these diagrams was explained. You should also have seen how the dynamics of processes (even very complex ones) can be classified into a number of simple types or models. Two types of control system – feedforward and feedback – are discussed in some detail: you should understand the differences between them and their relative advantages and disadvantages. You should also understand the main types of controller action, and their significance. It is important to stress that these principles remain valid whether the controller itself is an old-fashioned pneumatic device, or a more up to date programmable logic controller or even a fully-fledged computer control system. Finally, you should also be aware of some of the questions that need to be asked about ways of controlling a complete plant.

Like other chapters, we have not tried to describe the hardware or the technology. Any further study of this field would need to include the rapidly developing technology of computer-based data gathering and process control and of sensor technology.

Further reading

From the large number of texts concerned with process control the following may be found useful:

Buckley, P.S. (1964) *Techniques of Process Control*, John Wiley, New York.
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