# 4
# Colour quality evaluation

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## 4.1 Introduction

Instrumental colour measurement systems have been widely used by colour-using industries such as textiles, coatings, plastics, graphic arts and imaging. The most important application is undoubtedly colour quality control by means of colour difference formulae, which are used to quantify colour variations between pairs of specimens. Conventionally, this task was carried out by experienced colourists, but more recently this was replaced by instrumental methods in order to reduce labour costs, save time and apply a more scientific methodology. Some typical colour quality control tasks include:

- setting the magnitude of tolerance for making instrumental pass/fail decisions;
- evaluating fastness grades for assessing change in colour and staining;
- predicting the metameric effect between a pair of specimens;
- determining the change in colour appearance of a single specimen across different illuminants.

All the above tasks rely on the availability of a robust colour difference formula. This has long been eagerly sought by industry. This chapter will briefly review the development of colour difference formulae, including the CIE 2000 colour difference equation, CIEDE2000.[1,2] An example will be given to illustrate the method for establishing a tolerance value for industrial applications. In addition, methods for calculating a metamerism index, relating to changes in illumination of a pair of samples, and a colour inconstancy index, recently proposed by the Colour Measurement Committee (CMC) of the Society of Dyers and Colourists (SDC), will be introduced. Finally, a summary will be given of some new developments.

## 4.2 Colour difference formulae

The development of colour difference formulae can be divided into three stages: before, during and after 1976. Over 20 separate formulae were derived prior to 1976. They can be grouped into three families: those derived to fit MacAdam ellipses[3], Munsell[4] data, and those transformed linearly from CIE tristimulus colour space.[5] Some formulae are still in use today. Some representative formulae from each group are FMC2,[6] ANLAB[7] and HunterLAB,[8] respectively. However, significant progress was really made after the recommendation of CIELAB and CIELUV[5] in 1976.

### 4.2.1 CIE $L^*a^*b^*$ Formula (CIELAB)

In 1976, the CIE recommended two uniform colour spaces: CIELAB (or CIE $L^*a^*b^*$) and CIELUV (or CIE $L^*u^*v^*$) for industries concerned with the subtractive mixture (surface coloration) and additive mixture of colour (e.g. TV), respectively. Although the agreement of these two formulae with the then available experimental data was generally not good, they worked at least equally as well as any of the alternatives. CIELAB has been more widely used than CIELUV, especially in the surface colour industries and so only this is detailed here.

$$\begin{aligned} L^* &= 116\, f(Y/Y_n) - 16 \\ a^* &= 500\, [f(X/X_n) - f(Y/Y_n)] \\ b^* &= 200\, [f(Y/Y_n) - f(Z/Z_n)] \end{aligned} \qquad [4.1]$$

where

$$f(I) = I^{1/3}, \text{ for } I > 0.008856$$

Otherwise,

$$f(I) = 7.787\, I + 16/116$$

where $X$, $Y$, $Z$ and $X_n$, $Y_n$, $Z_n$ are the tristimulus values of the sample and a specific reference white considered. It is common to use the tristimulus values of a CIE standard illuminant or a light source for the $X_n$, $Y_n$, $Z_n$ values.

Correlates of hue and chroma, given in (4.2), are defined by converting the rectangular $a^*$, $b^*$ axes into polar coordinates. The lightness ($L^*$), chroma ($C^*$) and hue ($h_{ab}$) correlates correspond to perceived colour attributes, which are generally much easier to understand when describing colours.

$$\begin{aligned} h_{ab} &= \tan^{-1}(b^*/a^*) \\ C^*_{ab} &= (a^{*2} + b^{*2})^{1/2} \end{aligned} \qquad [4.2]$$

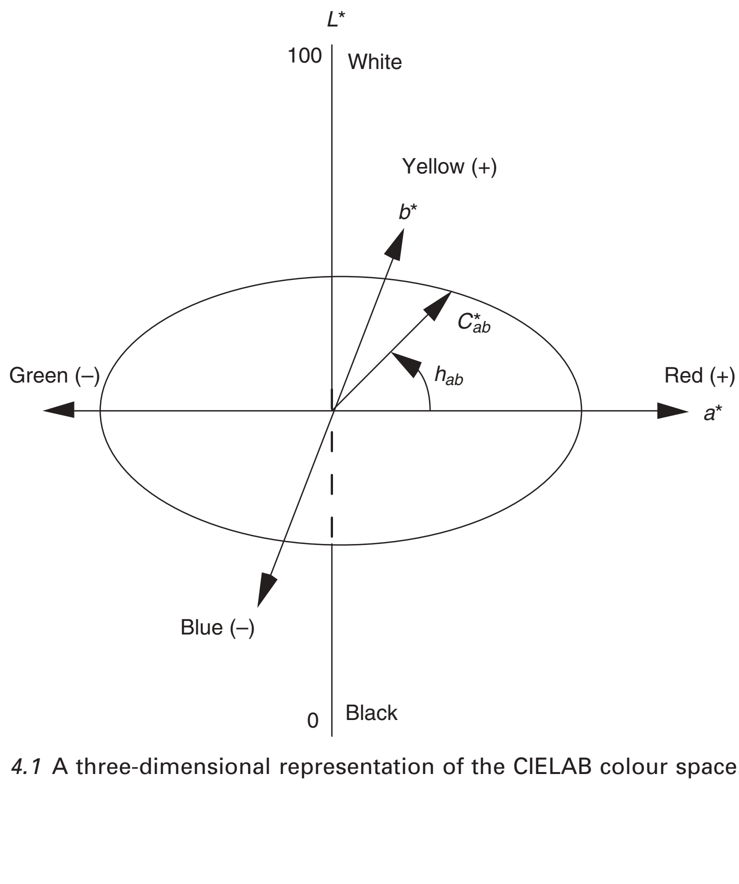
A three-dimensional representation of the CIELAB colour space is shown in Fig. 4.1. The neutral scale is located in the centre of the colour space. The $L^*$ values of 0 and 100 represent a reference black and white, respectively. The $a^*$ and $b^*$ values represent redness–greenness, and yellowness–blueness attributes, respectively. The $C^*_{ab}$ scale is an open-ended scale with a zero origin. (This origin includes all colours in the neutral scale, which do not exhibit hue.) The hue angle, $h_{ab}$, lies between 0° and 360°. Colours are arranged following the sequence of rainbow colours. The four unitary hues (pure red, yellow, green and blue) do not lie exactly at the hue angles of 0°, 90°, 180° and 270°, respectively.

Colour difference, represented by $\Delta E^*_{ab}$, is given in (eqn 4.3) and is calculated as the distance between the standard and sample in the CIELAB colour space.

$$\Delta E^*_{ab} = (\Delta L^{*2} + \Delta a^{*2} + \Delta b^{*2})^{1/2}$$

or

$$\Delta E^*_{ab} = (\Delta L^{*2} + \Delta C^{*\,2}_{ab} + \Delta H^{*\,2}_{ab})^{1/2} \qquad [4.3]$$




*4.1* A three-dimensional representation of the CIELAB colour space.

where

$$\Delta H^*_{ab} = 2\ (C^*_{ab,1}\ C^*_{ab,2})^{1/2}\sin[(h_{ab,2} - h_{ab,1})/2]$$

and subscripts *1* and *2* represent the standard and sample of the pair considered, respectively.

Although the CIELAB colour difference formula is by no means perfect (see later), its colour space is still the most widely used, mainly because it is relatively easy to relate colours as seen with positions on its diagram.

### 4.2.2 Formulae developed after CIELAB

As mentioned previously, the earlier formulae were derived mainly to fit the Munsell and MacAdam data. The viewing conditions applied in these experiments are very different from those used in the surface industries such as textiles and paint. Many sets of experimental results on colour discrimination have been published since 1976 and most of them were conducted using large surface samples viewed under typical industrial viewing conditions. Of these, the important data sets in terms of numbers of observers and sample pairs, and smaller observer variations, are those accumulated by McDonald,[9] Luo and Rigg,[10,11] RIT-Dupont,[12,13] Kim and Nobbs,[14] Witt,[15] Chou *et al.*[16] and Cui *et al.*[17] These data sets were used to develop or to verify more advanced formulae: CMC($k_L$:$k_C$),[18] BFD($k_L$:$k_C$),[19] CIE94[20] and LCD.[14] (In general, one or two of these data sets were used to develop each formula.) Finally, all of these data sets were used to develop the CIEDE2000 formula.[1,2] These more advanced formulae have a common feature: they are all modified versions of CIELAB and have no associated colour space. Only three equations – CMC, CIE94 and CIEDE2000 – are introduced here because they have been adopted by standards organisations such as CIE and ISO.

#### *CMC($k_L$: $k_C$) and JPC79 colour-difference formulae*

McDonald[9] at J.P. Coates company accumulated a comprehensive data set. These visual results were used to derive the JPC79 formula.[21] At a later stage, the formula was further studied by the members from the CMC of the SDC and it was modified to correct some anomalies. The modified formula is named CMC($k_L$: $k_C$)[18] and is the current ISO standard for the textile industry. The formula is given in (eqn 4.4).

$$\Delta E_{CMC} = [(\Delta L^*/k_L S_L)^2 + (\Delta C^*_{ab}/k_C S_C)^2 + (\Delta H^*_{ab}/S_H)^2]^{1/2} \qquad [4.4]$$

where

$$S_L = 0.040975L^*_{ab,1}/(1 + 0.01765L^*_{ab,1})$$

unless

$$L^*_{ab,1} < 16 \text{ when } S_L = 0.511$$

$$S_C = 0.638 + 0.0638\, C^*_{ab,1}/(1 + 0.0131 C^*_{ab,1})$$

$$S_H = S_C(Tf + 1 - f)$$

$$f = [C^{*\,4}_{ab,1}/(C^{*\,4}_{ab,1} + 1900)]^{1/2}$$

$$T = 0.36 + |0.4\,cos(h_{ab,1} + 35°)|$$

*unless*

$$h_{ab,1} \textit{ is between } 164° \textit{ and } 345°$$

*when*

$$T = 0.56 + |0.2\,cos(h_{ab,1} + 168°)|$$

where $\Delta L^*$, $\Delta C^*$ and $\Delta H^*$ are the CIELAB lightness, chroma and hue differences ('batch' minus 'standard'). The $L^*_{ab,1}$, $C^*_{ab,1}$ and $h_{ab,1}$ refer to the 'standard' of a pair of samples. The $k_L$ and $k_C$ parametric factors were included to allow different weights for lightness and chroma, respectively, to be used depending on the circumstances. The best $k_L$ and $k_C$ values have been found to be 2 and 1, respectively, for predicting the acceptability of colour differences for textiles. For predicting the perceptibility of colour differences, $k_L$ and $k_C$ should both equal 1.

A constant $\Delta E$ according to the CMC formula (eqn 4.4) can be considered as an ellipsoid equation in CIELAB $L^*$, $C^*$ and hue polar space with semi-major axes of $k_L S_L$, $k_C S_C$ and $S_H$, respectively. Its chromaticity ellipse points towards the achromatic axis.

### *CIE94($k_L$:$k_C$:$k_H$) colour-difference formula*

Berns *et al.*[12,13] at the Rochester Institute of Technology (RIT) also accumulated visual assessments using glossy acrylic paint pairs. This data set is named RIT–Dupont. A colour difference formula given in (eqn 4.5), having a similar structure to that of CMC($k_L$:$k_C$) but having simpler weighting functions, can fit their data very well. They believed that the CMC formula is over-complicated. (In fitting any formula to a particular set or sets of experimental data, it is always possible to obtain a better fit by making the formula more complex. It is often difficult to judge just how much complexity is justifiable.) The formula was later recommended by the CIE for field trials in 1994[20] and it is thus named the CIE94 colour difference formula.

$$\Delta E_{94} = [(\Delta L^*_{ab}/k_L S_L)^2 + (\Delta C^*_{ab}/k_C S_C)^2 + (\Delta H^*_{ab}/K_H S_H)]^{1/2} \qquad [4.5]$$

where

$$S_L = 1$$

$$S_C = 1 + 0.045C^*_{ab,1}$$

$$S_H = 1 + 0.015C^*_{ab,1}$$

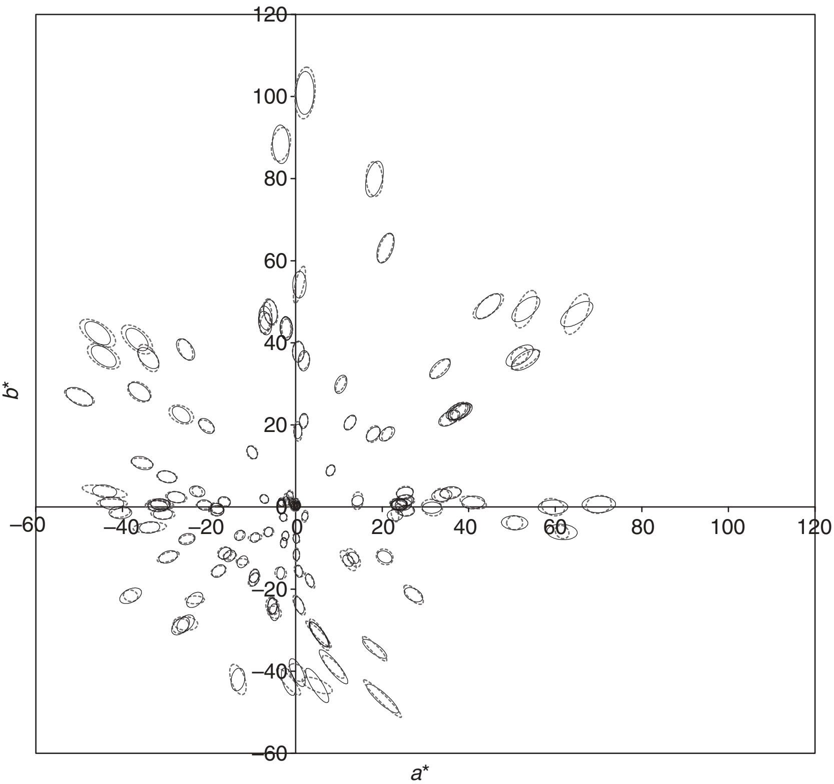
where $C^*_{ab,1}$ refers to the $C^*_{ab}$ of the standard of a pair of samples. In some situations, such as calculating large magnitude colour difference, the geometric mean is suggested rather than using $C^*_{ab,1}$. The $k_L$, $k_C$ and $k_H$ terms are parametric factors accounting for variation in experimental conditions such as luminance level, background, texture and separation. For all applications except for the textile industry, a value of 1 is recommended for all parametric factors. For the textile industry, the $k_L$ factor should be 2 and the $k_C$ and $k_H$ factors should be 1, i.e. CIE94(2:1:1). The parametric factors may be defined by industry groups, depending on the typical viewing conditions for that industry.

### *CIEDE2000($K_L$:$K_C$:$K_H$) colour-difference formula*

After the development of the CIE94 formula, two separate colour difference equations recommended by different organisations co-existed, i.e. CMC($k_L$:$k_C$) by ISO[22] and CIE94 by the CIE.[20] However, these formulae were derived from two main data sets: Luo & Rigg[10,11] and RIT-DuPont.[12,13] Figure 4.2 shows both sets of experimental ellipses – plotted with dashed lines. If the CIELAB formula agreed perfectly with the experimental results, all ellipses should be constant radius circles. Hence, the patterns shown in Fig. 4.2 indicate a poor performance by CIELAB. A clear pattern of ellipses can be seen, i.e. very small ellipses for neutral colours, increasing in size when chroma increases. All ellipses for the blue region point away from the neutral axis, whereas ellipses for all other colour regions generally point towards the neutral point. The latter phenomenon indicates that both the CMC and CIE94 formulae would badly model experimental results in the blue region because the ellipses representing both formulae all point exactly towards neutral. Detailed comparisons of these two formulae reveal there are large discrepancies in predicting lightness differences, and both have errors in predicting colour difference in the grey and blue regions.[23]

With this in mind, a CIE Technical Committee (TC) 1–47 on 'Hue and Lightness Dependent Correction to Industrial Colour Difference Evaluation' led by Alman of DuPont was formed in 1998. It was hoped that a generalised and reliable formula could be achieved.

The members in this TC worked closely together using four selected experimental data sets: Luo and Rigg,[10,11] RIT-Dupont,[12,13] Kim and Nobbs[14] and Witt.[15] A formula named CIEDE2000 – see (eqn 4.6) – was then published.[1,2] It includes five modifications to CIELAB: a lightness weighting




*4.2* RIT-DuPont and BFD experimental chromaticity discrimination ellipses (in ⬭) compared to the corresponding ellipses from the CIEDE2000 equation (in ⬭).

function, $S_L$, a chroma weighting function, $S_C$, a hue weighting function, $S_H$, an interactive term, $R_T$, between chroma and hue differences for improving the performance for blue colours, and a factor, $1 + G$, for re-scaling the CIELAB $a^*$ scale to improve performance with grey colours. The CIE94 and CMC formulae only included the first three corrections and so vary markedly from CIEDE2000 for chromatic differences in the blue and neutral regions. The performance of CIEDE2000 is shown in Fig. 4.2, where solid line ellipses correspond to a constant CIEDE2000 colour difference. It can be seen that these solid line ellipses fit very well to the majority of the experiment ellipses plotted with dashed lines. Melgosa and Huertas[24] also later found that CIEDE2000 is more accurate than CMC at a statistical significance at 95% confidence interval for the combined data set, which includes the above four data sets.

More recent experimental results have been published to verify the performance of CIEDE2000. They all reached the same conclusion that the

CMC, CIE94 and CIEDE2000 out-performed CIELAB by a large margin. However, they all gave a very similar degree of accuracy, except for the blue and near neutral colour regions, for which only CIEDE2000 gave an accurate prediction.

$$\Delta E_{00} = \sqrt{\left(\frac{\Delta L'}{k_L S_L}\right)^2 + \left(\frac{\Delta C'}{k_C S_C}\right)^2 + \left(\frac{\Delta H'}{k_H S_H}\right)^2 + R_T\left(\frac{\Delta C'}{k_C S_C}\frac{\Delta H'}{k_H S_H}\right)} \quad [4.6]$$

where

$$S_L = 1 + \frac{0.015\left(\overline{L'} - 50\right)^2}{\sqrt{20 + \left(\overline{L'} - 50\right)^2}}$$

and

$$S_C = 1 + 0.045\overline{C'}$$

and

$$S_H = 1 + 0.015\overline{C'}T$$

where

$$T = 1 - 0.17\cos(\overline{h'} - 30°) + 0.24\cos(2\overline{h'}) + 0.32\cos(3\overline{h'} + 6°) - 0.20\cos(4\overline{h'} - 63°)$$

and

$$R_T = -\sin(2\Delta\theta)R_C$$

where

$$\Delta\theta = 30\exp\{-[(\overline{h'} - 275°)/25]^2\}$$

and

$$R_c = 2\sqrt{\frac{\overline{C'}^7}{\overline{C'}^7 + 25^7}}$$

and

$$L' = L^*$$

$$a' = (1 + G)a^*$$

$$b' = b^*$$

$$C' = \sqrt{a'^2 + b'^2}$$

$$h' = \tan^{-1}(b'/a')$$

where

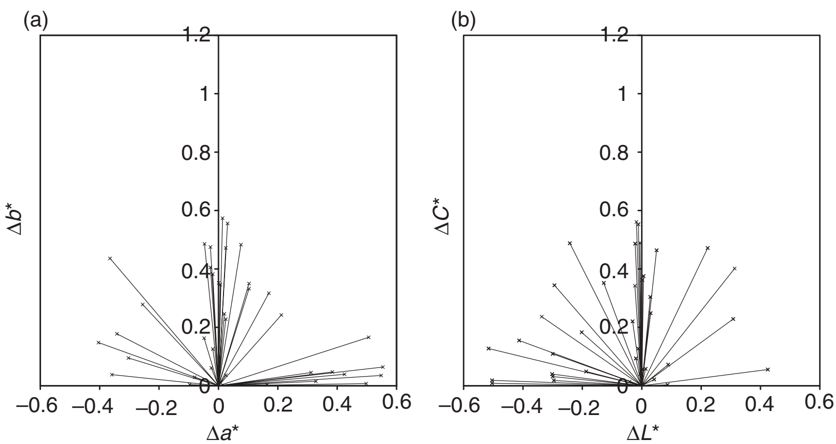
$$G = 0.5\left(1 - \sqrt{\frac{\overline{C^*}^7}{\overline{C^*}^7 + 25^7}}\right)$$

### 4.2.3 Establishing industrial colour tolerance

As mentioned earlier, successful colour quality control is heavily dependent upon the use of a reliable colour difference formula. Furthermore, there is a need to set up a magnitude of tolerance to determine whether a batch is within tolerance (pass) or outside of it (fail) relative to the standard. To obtain a reliable tolerance requires visual data for assessing colour difference pairs in terms of pass or fail.

*Experimental*

A set of paint samples was taken as a worked example here. It includes a grey colour centre having CIELAB values of 51.0, 0.2 and 1.2 for *L**, *a** and *b**, respectively. Each sample had a size of 5 × 10 cm. In total, 38 pairs of samples were selected. Figure 4.3 shows the distribution of sample pairs surrounding the colour centre in the Δ*a** Δ*b** (left) and Δ*L** Δ*C** (right) diagrams. It can be seen that these pairs gave a good coverage for almost all directions. Each sample was measured with a spectrophotometer in terms of CIELAB values under CIE Illuminant D65 and using the CIE 1964 standard colorimetric observer.




*4.3* The sample pair distribution in (a) Δ*a** Δ*b** and (b) Δ*L** Δ*C** diagrams.

Note that it is important to select the sample pairs carefully when determining tolerance. All pairs should have perceptible colour differences around the pass/fail borderline. If their magnitudes are too large, all pairs will be rejected by assessors. Similarly, all pairs will be accepted by assessors for small colour difference magnitudes. The results of these pairs cannot be used in the following data analysis.

These pairs were judged by a panel of ten professional assessors in terms of 'pass' or 'fail'. Each assessor performed their judgements twice for each pair. The assessment was carried out in a viewing cabinet, which included a high quality D65 simulator.

*Measure of fit: wrong decision*

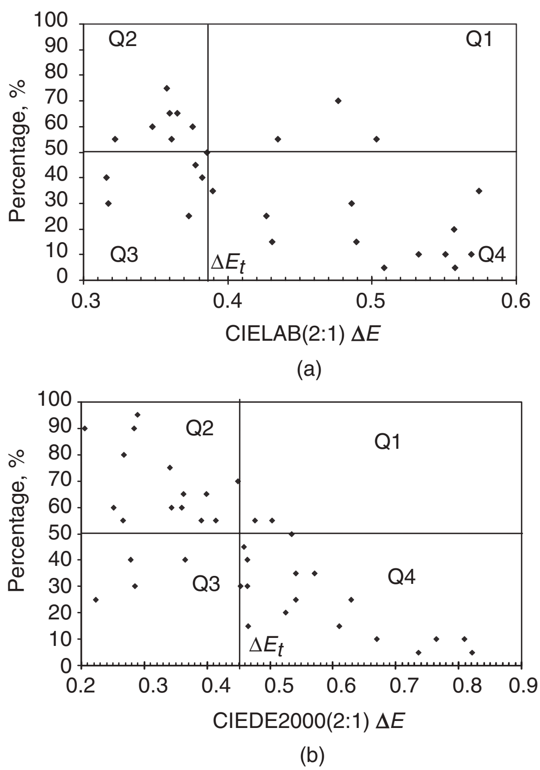
The data accumulated in the above experiment are described in terms of percentage acceptance ($A$%), in which a batch sample is judged as pass against a standard in percentage. For example, an $A$% of 30 indicates that 30% of observers regard the batch as an acceptable match to the standard.

The 'wrong decision' measure[25] was used to indicate the observer accuracy, observer repeatability and colour difference equation's performance. When investigating *observer accuracy*, each individual observer's results were compared with the panel results. For a panel result of 35 $A$% for a batch sample, it is considered as a fail decision because 65% of observers (a majority) reject it. If an individual observer passed it, it will be counted as a wrong decision. Finally, the performance is expressed by $WD$%, which is the number of pairs with the wrong decision divided by the total number of sample pairs. For a perfect agreement, $WD$% should be zero. When examining *observer repeatability*, the $WD$% measure was used to represent the number of wrong decisions made by the two repetitions from a single observer.

The $WD$% measure was also used to indicate the performance of a colour difference formula. An example is given in Fig. 4.4, showing two scatter diagrams which plot visual results in $A$% values against the $\Delta E$ values calculated by CIELAB(2:1) and CIEDE2000(2:1:1), respectively. (A lightness parameter, $k_L$, of two was applied to both formulae.) Each diagram in Fig. 4.4 includes four quadrants (marked Q1 to Q4) divided by the 50 $A$% and colour tolerance ($\Delta E_t$) for $x$- and $y$-axes, respectively. A trend can be found in these diagrams, indicating a decrease of $A$% with an increase of $\Delta E$. This is expected as the samples will be rejected when their colour differences increase. The wrong decision is calculated by the sum of data points in Q1 and Q3, for which the data in Q1 represent small visual differences but large $\Delta E$ values of a colour difference equation. The data in Q3 are also wrong decisions with large visual differences against small instrumental $\Delta E$ values. When calculating $WD$%, the $\Delta E$ value ($x$-axis) will systematically vary from zero to a pre-defined large colour difference (say 10) with a fixed increment (say 0.1). For each small increment, a new *WD*% is calculated and stored. Finally, the minimum *WD*% value represents the tolerance value ($\Delta E_t$). The *WD*% values in Fig. 4.4a and 4.4b are 26% and 18%, respectively, indicating that CIEDE2000(2:1:1) formula out-performed CIELAB(2:1) formula by 8 *WD*% and should be recommended in this application. Note that the $k_L$ value of 2 gives a better performance than $k_L$ of 1 for each formula.

*Observer uncertainty*

As mentioned earlier, ten assessors participated in the experiment and each made their assessments twice. In total, there were 20 observations for each of the 38 pairs studied. Observer uncertainty, including accuracy and repeatability, were analysed for each observer. The results are summarised in Table 4.1 in terms of *WD*%.




*4.4* Method for determining the wrong decision for (a) CIELAB(2:1) and (b) CIEDE2000(2:1:1) colour-difference equations.

*Table 4.1* Summary of the observer uncertainty in WD%

| Assessor | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Mean |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Accuracy | 29 | 26 | 29 | 21 | 34 | 32 | 29 | 13 | 37 | 32 | |
| | 21 | 18 | 26 | 18 | 45 | 40 | 21 | 24 | 40 | 16 | 27 |
| Repeatability | 29 | 32 | 16 | 24 | 37 | 29 | 29 | 16 | 32 | 24 | 28 |

*Table 4.2* Performance (WD%) of formulae for Phase 1 results (38 pairs)

| | | CIEDE2000 | CIELAB | CIE94 | CMC |
|---|---|---|---|---|---|
| $k_L = 1$ | Tolerance $\Delta E_t$ | 0.45 | 0.39 | 0.38 | 0.52 |
| | WD% | 21% | 32% | 32% | 29% |
| $k_L = 2$ | Tolerance $\Delta E_t$ | 0.45 | 0.37 | 0.36 | 0.52 |
| | WD% | 18% | 26% | 26% | 24% |

Table 4.1 results show that the performances of observer accuracy and repeatability are very similar (about 28 $WD\%$). It indicates that assessors could have around one wrong decision in three judgements.

*Testing different colour difference formulae*

The visual results obtained from two phases were used to test four colour difference formulae: CIEDE2000, CMC, CIE94 and CIELAB. Their performances are given in Table 4.2. For each formula, lighting parameters, $k_L$, were set to 1 and 2, respectively.

The results in Table 4.2 show that CIEDE2000 gave the best performance amongst those formulae studied. For all the formulae tested, a $k_L$ of 2 is more suitable than a $k_L$ of 1. Finally, CIEDE2000(2:1:1) with a colour tolerance of 0.45 was the best in this study.

## 4.3 Metamerism

Another application of colour difference formula is in predicting the degree of metamerism between sample pairs. Metamerism occurs when two samples match each other under one set of viewing conditions but fail to match under another. There are four types of metamerism: illuminant, observer, field size and geometric.

Illuminant metamerism is the most important type of metamerism and occurs when two samples appear to match well under one illuminant, but exhibit a large mismatch under a second illuminant. Observer metamerism

*Table 4.3* An example to illustrate the calculation of the metamerism index

| | $\Delta L$ | $\Delta C$ | $\Delta H$ |
|---|---|---|---|
| Reference illuminant (D65) | 1.0 | 1.5 | 2.0 |
| Test illuminant (A) | –1.0 | 1.5 | –2.0 |
| Difference | –2.0 | 0.0 | –4.0 |
| $\Delta E_M = (2^2 + 0^2 + 4^2)^{1/2} = 4.5$ | | | |

occurs when a pair of samples matches for one observer, but fails to match when seen by a second. Field size metamerism arises from a satisfactory match being lost when the field size changes. Geometric metamerism occurs when a mismatch develops due to changes in the illumination and viewing geometry.

The CIE International Lighting Vocabulary[26] defines metamerism as a property of a pair of spectrally different colours having the same tristimulus values under a set of viewing conditions. In practice, it is impossible to achieve precisely identical tristimulus values. However, it is possible to apply some corrections to make the pair under consideration exactly match in the reference illuminant.

In Table 4.3, an example is given of quantifying illuminant metamerism by applying an additive correction. The $\Delta L$, $\Delta C$ and $\Delta H$ are calculated from a colour difference formula (see Section 4.2) under a reference illuminant (e.g. illuminant D65). There are subtracted from the corresponding $\Delta L$, $\Delta C$ and $\Delta H$ under a test illuminant (say illuminant A). Finally, the metamerism index, $\Delta E_m$, is calculated using (eqn 4.7).

$$\Delta E_m = \sqrt{(\Delta L_1 - \Delta L_2)^2 + (\Delta C_1 - \Delta C_2)^2 + (\Delta H_1 - \Delta H_2)^2} \qquad [4.7]$$

where the subscripts 1 and 2 represent the test and reference illuminant, respectively.

In the case of lightness difference, the sample is judged lighter than standard by one unit under illuminant D65, but darker by the same amount under illuminant *A*. Thus, the corrected lightness difference between the two illuminants is two units. The same correction is applied to chroma and hue differences. The resultant metamerism index is therefore 4.5.

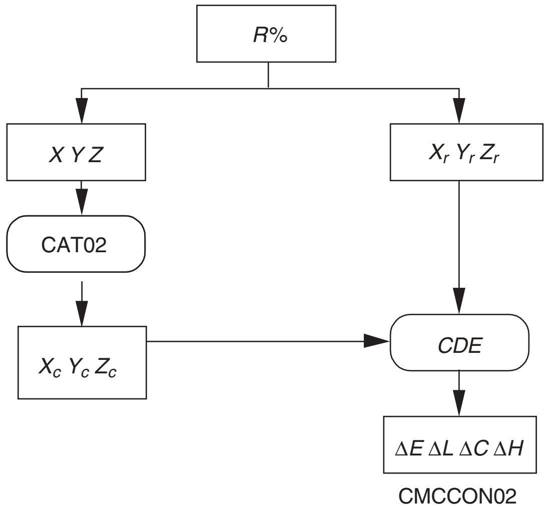
## 4.4 Colour constancy

Another important application of the colour difference formula is in evaluating the degree of colour constancy under different illuminants.

### 4.4.1 Concept of colour constancy

For the majority of industrial colour matching, the aim is to produce samples possessing a spectral reflectance as close as possible to that of the standard. If the reflectances are the same, the match will hold for all illuminants and observers. This type of reproduction is known as a spectral, or non-metameric, match. In many cases it is not possible to obtain a spectral match using the desired set of colourants. If the reflectances are very different, the match might be very good for one illuminant, say D65, and the match may not hold for other illuminants such as illuminant *A* or F11, a tri-narrow band illuminant. This type of match is known as metameric match. The degree of metamerism may be quite small or very marked, depending on the difference between the two reflectance functions in question and on the specific illuminants concerned. A metamerism index was introduced earlier in (eqn 4.7).

Even if the match is only slightly metameric, however, the sample produced may well look a completely different colour under certain other illuminants, depending on the reflectance property of the reference sample. Such samples are said to be non-colour constant. As with metamerism, there may be varying degrees of colour inconstancy, i.e. the severity of colour change to be expected when moving from one illuminant to another. For real samples, visual estimates can be made by observing the sample under different light sources. Using a recipe formulation system, it is possible to calculate alternative recipes using many different sets of colourants. Using a colour inconstancy index, it is possible to determine which recipe gives the most colour constant product.




*4.5* The procedure to calculate the CMCCON02 colour inconstancy index.

Note that, while this problem occurs relatively infrequently (spectral matches to a standard colour are usually requested), it is most important that attention should be paid to inconstancy when it does occur. If the original standard is colour constant, all subsequent spectral matches will also be colour constant. If not, a whole series of non-colour constant, and potentially non-metameric, products will be produced.

### 4.4.2 The structure of a colour inconstancy index

A colour inconstancy index is capable of predicting the magnitude and direction of the change in colour appearance between a sample viewed under a test illuminant (say Illuminant *A*) and the same sample viewed under a reference illuminant (say D65). A colour inconstancy index, named CMCCON02,[27] was recommended by the Colour Measurement Committee (CMC) of the Society of Dyers and Colourists (SDC). The procedure for calculating the CMCCON02 is given in Fig. 4.5. A spectral reflectance function is first obtained by measuring a test specimen with a spectrophotometer. The tristimulus values, $X$, $Y$, $Z$ and $X_r$, $Y_r$, $Z_r$ under illuminants *A* and D65, respectively, are then calculated in the usual way. The CAT02 chromatic adaptation transform is employed to predict the *corresponding colours*, $X_c$, $Y_c$, $Z_c$, under illuminant D65 from the $X$, $Y$, $Z$ values of the sample under Illuminant *A*. Finally, a suitable colour difference equation (CDE) is used to calculate a $\Delta E$ value, together with individual colour difference components ($\Delta L$, $\Delta C$ and $\Delta H$) between $X_c$, $Y_c$, $Z_c$ and $X_r$, $Y_r$, $Z_r$. The magnitude of the $\Delta E$ value indicates the degree of colour inconstancy. The direction of the colour change can be expressed by the individual colour difference components. A $\Delta E$ value of zero indicates complete colour constancy for the specimen being tested.

### 4.4.3 The CAT02 chromatic adaptation transform (CAT02)

The key element of CMCCON02 is the CAT02 chromatic adaptation transform. The computational procedure for CAT02 is given below:

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**Starting data:**
Sample in test illuminant: $X$, $Y$, $Z$
Adopted white in test illuminant: $X_w$, $Y_w$, $Z_w$
Reference white in reference illuminant: $X_{wr}$, $Y_{wr}$, $Z_{wr}$
Luminance of test adapting field (cd/m$^2$): $L_A$

**Transformed data to be obtained:**
Sample corresponding colour in reference illuminant: $X_c$, $Y_c$, $Z_c$

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*Step 1.* For the sample, calculate:

$$\begin{pmatrix} R_w \\ G_w \\ B_w \end{pmatrix} = M_{\text{CAT02}} \begin{pmatrix} X_w \\ Y_w \\ Z_w \end{pmatrix}, \begin{pmatrix} R_{wr} \\ G_{wr} \\ B_{wr} \end{pmatrix} = M_{\text{CAT02}} \begin{pmatrix} X_{wr} \\ Y_{wr} \\ Z_{wr} \end{pmatrix}, \begin{pmatrix} R \\ G \\ B \end{pmatrix} = M_{\text{CAT02}} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}.$$

where

$$M_{\text{CAT02}} = \begin{bmatrix} 0.7328 & 0.4296 & -0.1624 \\ -0.7036 & 1.6975 & 0.0061 \\ 0.0030 & 0.0136 & 0.9834 \end{bmatrix}$$

*Step 2.* Calculate the degree of adaptation, $D$:

$$D = F\left[1 - \left(\frac{1}{3.6}\right) e^{\left(\frac{-La-42}{92}\right)}\right]$$

where $F$ equals 1, 0.9, and 0.8 for average-, dim- and dark-surround conditions, respectively, and where $L_A$ is the luminance of the test adapting field. If $D$ is greater than one or less than zero, truncate it to one or zero, respectively.

For calculating the CMCCON02 index for a physical sample such as textiles, it is recommended that $D$ is set to unity, i.e. it corresponds to a complete adaptation or completely discounts the illuminant.

*Step3.* Calculate $R_c$, $G_c$, $B_c$ from $R$, $G$, $B$ (similarly $R_{wc}$, $G_{wc}$, $B_{wc}$ from $R_w$, $G_w$, $B_w$):

$$R_c = R[D(R_{wr}/R_w) + 1 - D]$$

$$G_c = G[D(G_{wr}/G_w) + 1 - D]$$

$$B_c = B[D(B_{wr}/B_w) + 1 - D]$$

*Step 4.* Calculate for the reference illuminant the corresponding tristimulus values for the sample, $X_c$, $Y_c$, $Z_c$:

$$\begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} = M_{\text{CAT02}}^{-1} \begin{pmatrix} R_c \\ G_c \\ B_c \end{pmatrix},$$

where

$$M_{\mathrm{CAT02}}^{-1} = \begin{bmatrix} 1.096124 & -0.278869 & 0.182745 \\ 0.454369 & 0.473533 & 0.072098 \\ -0.009628 & -0.005698 & 1.015326 \end{bmatrix}$$

Calculation in both the forward and especially the reverse mode are extremely sensitive to rounding off errors, making it essential to employ the full precision implied in $M_{\mathrm{CAT02}}$ and $M^{-1}{}_{\mathrm{CAT02}}$. Some older computer-based colour measurement systems may, therefore, need to employ double precision arithmetic.

## 4.5 Conclusions and future trends

This chapter focused on the field of colour difference applications. Firstly, the author reviewed the development of colour difference formulae. A method for setting tolerance using colour difference formulae for industrial applications was also introduced. Finally, methods for predicting metamerism and colour constancy were described.

It can be concluded that after more than three decades of development of colour difference equations, a robust colour difference formula, CIEDE2000, has been achieved. However, colour difference research is still on-going. Amongst the problems yet to be tackled are:

- Almost all of the recent effort has been spent on modifications to CIELAB. This has resulted in CIEDE2000, which includes five corrections of CIELAB to fit the available experimental data sets. It is highly desirable to derive a formula based upon a new perceptually uniform colour space from a particular colour vision theory. A uniform colour space based upon a colour appearance model, like the CIE colour appearance model CIECAM02,[28] could be an ideal solution.
- All colour difference formulae can only be applied to a limited set of reference viewing conditions, such as those defined by the CIE.[20] It would be extremely useful to derive a parametric colour difference formula capable of taking into account different viewing parameters such as illuminant, size of sample, colour difference magnitudes, separation, background and luminance level.[29,30]
- Almost all of the colour difference formulae were developed only to evaluate colour difference between pairs of large single objects/patches. More and more applications require predicting colour differences between pairs of pictorial images. The current formula does not include the necessary components to consider spatial variations for evaluating such images. There is, therefore, an urgent need to develop a formula for this purpose.[31,32]

## 4.6 References


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