2

Flow of Fluids

Fluid flow may be defined as the flow of substances that do not permanently resist distortion.

This definition covers the flow of liquids and gases. Fluid is considered to be a mass of a substance formed by a series of layers. When an attempt is made to change its shape, the layers of fluid slide over one another, until a new shape is attained. At the end, the fluid is relieved off the stresses that it encountered during the flow.

Flow of fluids is observed while handling materials.

(a) **Handling of liquids :** Transportation of materials such as solvents, solutions and suspensions is simpler, cheaper and less troublesome than handling of solids in industrial operations.

(b) **Handling of solids :** Solids are handled in a finely divided state in the form of suspension of fluids, so that transportation becomes easy. This two-phase mixture is known as '*fluidised solids*'.

During the handling process, the behaviour of liquids changes transiently. Such changes have profound influence on heat transfer process, energy losses during pumping, energy changes in pumping etc. Therefore, fluid flow is treated independently.

Flow of fluids is involved in a number of areas of pharmaceutical industries. Some of them are:

Passing of reactants (liquids or gases) into the reaction system.
Transferring of air, nutrient broth into the fermenter.
Bottling of liquid (dosage forms) medicaments into suitable containers.

Transporting of sterile air and sterile water in the production of parenterals.
Mixing of solids and liquids in case of suspensions.
Packing of semisolids in containers.

The nature of flow influences the type of equipment used for handling. Flow characteristics through pipes and channels are relevant. At the same time, the measurement of rate of flow is necessary for the quantification of additives (reactants) into a process.

The subject of fluid flow can be divided into fluid statics and fluid dynamics.

- *Fluid statics* deals with fluids at rest in equilibrium.
- *Fluid dynamics* deals with fluids in motion.

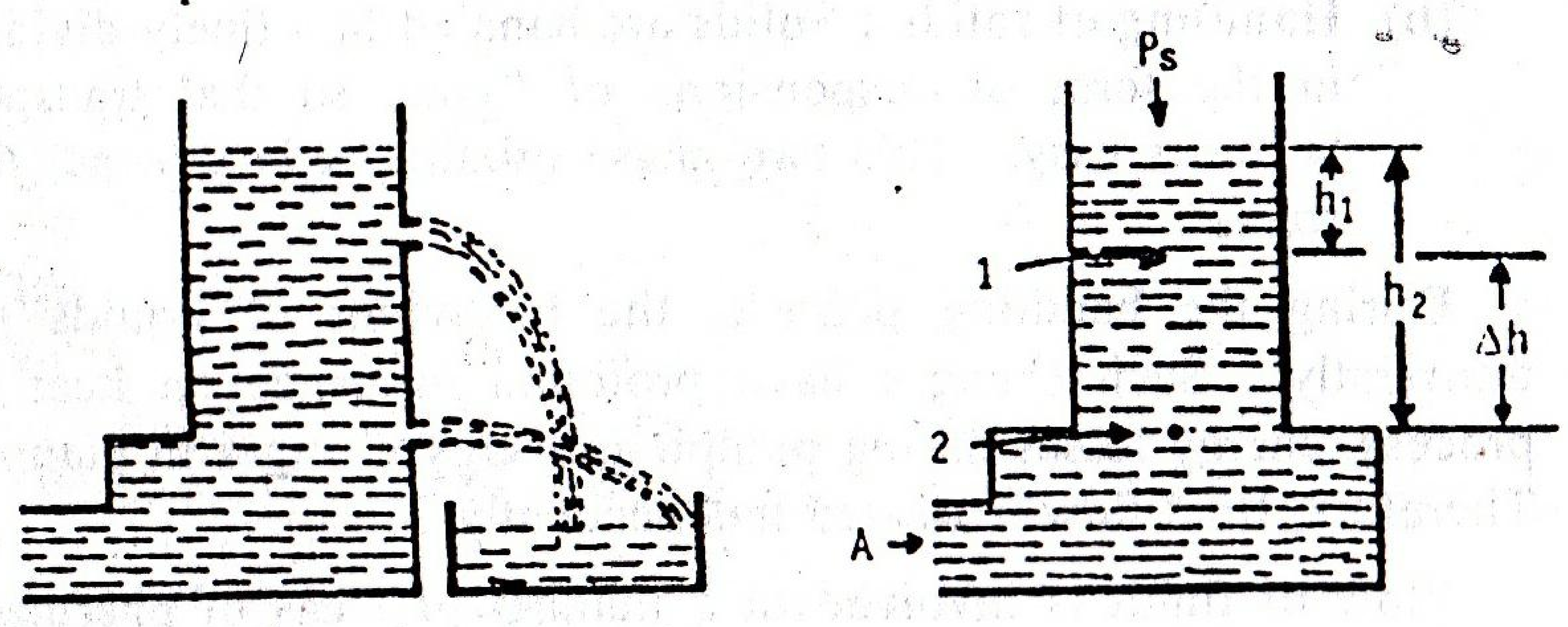
FLUID STATICS

Fluid statics deals with the fluids at rest in equilibrium. The behaviour of a liquid at rest, the nature of pressure it exerts and the variation of pressure at different layers in the liquid are some of the relevant aspects in the pharmaceutical engineering.

Pressure Difference Between Layers of Liquids

Consider a column of liquid as shown in Figure 2-1(a). Two openings are provided to the wall of the vessel at different heights. The rate of flow from these openings is different (Figure 2-1a). This is due to the differences in the pressures exerted at different heights. This behaviour can be quantitatively expressed as follows.

Consider a stationary column of fluid as shown in Figure 2-1(b). The pressure P_s pascals is acting on the surface of the fluid. The stationary column is maintained at constant pressure by applying pressure, P pascals at point A.



(a) (b)

Figure 2-1. Hydrostatic pressure observed at different layers of a stationary column.

Let the cross-section of the column be S metre square and is uniform from top to bottom. The force (further pressure) acting on the liquid at different levels of the liquid column can be determined. The forces (newton) acting on each side (horizontal components) of the point 1 are mutually nullified. The forces in newtons acting below and above the point 1 are evaluated.

Force acting on the liquid at point 1 = force on the surface + force exerted by the liquid above the point 1 (1)

Substituting the force with pressure × area of cross section (S) in equation (1) gives:

Pressure at point 1 × surface area = (Pressure on the surface × surface area) + (mass × acceleration) (2)

$$P_1S = P_sS + \text{volume} \times \text{density} \times \text{acceleration due to gravity}$$
$$= P_sS + \text{height} \times \text{area} \times \text{density} \times \text{acceleration due to gravity}$$

$$P_1S = P_s S + h_1 S\rho g \qquad (3)$$

Since cross sectional surface area is same, equation (3) may be written as:

$$P_1 = P_s + h_1\rho g \qquad (4)$$

Similarly, the pressure acting on the liquid at point 2 may be written as:

$$P_2 = P_s + h_2\rho g \qquad (5)$$

The difference in the pressure may be obtained by subtracting equation (4) from equation (5) as:

$$P_2 - P_1 = g\,(P_s + h_2\rho) - (P_s + h_1\rho)g$$
$$\Delta P = (P_s + h_2\rho - P_s - h_1\rho)g$$
$$\Delta P = (h_2 - h_1)\rho g$$
$$= \Delta h\rho g \qquad (6)$$

Therefore, the pressure difference (ΔP pascals) between any two points can be measured by the distance between those points in a fluid. If the density of fluid (ρ kilogram per metre cube) varies with variation of pressure, an average density could be used. The variation in densities is quite negligible for liquids and gases. Since the difference in the heights (Δh metre) is necessary for the measurement, height can be measured from the bottom of the stationary column.

Applications : The principle of fluid statics is employed in the working of manometers. In such cases, the pressure difference (ΔP) is measured in terms of difference in the heights of the liquid column. It is also applied for quantification of fluid flow as in Bernoulli's theorem.

Manometers

Manometers are the devices used for measuring the pressure difference. Three different manometers are available. These are:

1. Simple
2. Differential
3. Inclined

The principles and applications of these manometers are discussed in the following sections.

Simple manometer : This manometer is the most commonly used. The construction of a simple manometer is shown in Figure 2-2. It consists of a glass U tube filled with a liquid (A) of density, ρ_A kilogram per metre cube. Above liquid A, the arms are filled with liquid B of density ρ_B kilogram per metre cube. The liquids A and B are immiscible and the interface can be seen clearly.

If two different pressures are applied on the two arms, the meniscus of liquid A will be higher in one arm than the other (Figure 2-2). Let the pressure at point 1 is P_1 pascals in left hand-side of the limb. Let the pressure at point 5 is P_2 pascals in the right hand-side of the limb. From the principles of fluid statics, the pressure at point 2 can be written as:

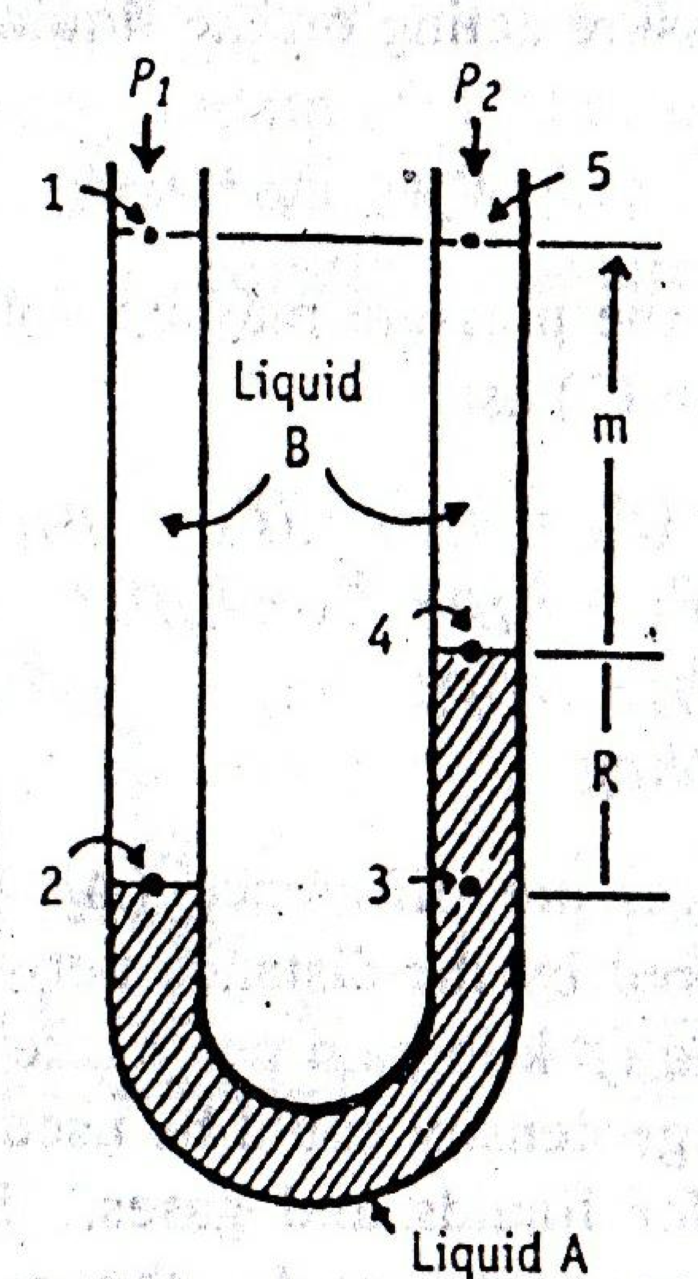


Figure 2-2. The construction of simple manometer.

$$\textit{Pressure at point } 2 = P_1 + (m+R)\rho_B g \qquad (7)$$

where (m+R) = distance from points 3 to 4 + distance from points 4 to 5.

Since, points 2 and 3 are at the same level, the pressure at point 3 may be written as:

$$\textit{Pressure at point } 3 = P_1 + (m+R)\rho_B g \qquad (8)$$

The pressure at point 4 can be written from the right hand-side of the limb (from P_2) as:

$$\textit{Pressure at point } 4 = P_2 + gm\rho_B \qquad (9)$$

In another manner, the pressure at point 4 can be written from point 3 (i.e., from left hand-side) as:

$$\textit{Pressure at point } 4 = P_1 + \rho_B(m + R)g - \rho_A Rg \qquad (10)$$

Equations (9) and (10) represent the pressure at point 4 only. Hence, these equations should be equal. This relationship may be written as:

$$P_1 + g\rho_B(m + R) - \rho_A R\, g = P_2 + gm\rho_B$$

$$P_1 - P_2 = gm\rho_B - \rho_B(m + R)g + \rho_A Rg$$

$$\Delta P = m\rho_B g - m\rho_B g - R\rho_B g + R\rho_A g$$

$$\Delta P = R(\rho_A - \rho_B)g \qquad (11)$$

The important conclusions drawn from equation (11) are:

(1) It is easy to measure R value (metres), i.e., the difference in the levels of liquid A in the two limbs.

(2) The value ΔP pascals is independent of the value of m and also the dimensions of the U tube.

When wide ranges of pressure are applied, the sensitivity of liquids employed in the measurement is important. If the pressure differences are large, mercury (high density, liquid A) can be used as manometric liquid. If the pressure differences are small, liquids such as alcohol, water (for gases) and carbon tetrachloride are used.

Applications: (1) Simple manometer helps in measuring the consumption of gases in the chemical reactions. (2) Manometers are used in conjunction with flow meters for the measurement of flow of fluids. For example, venturi meter and orifice meter are used for the measurement of pressure head using a manometer. Pitot tube measures the velocity head using a manometer.

Differential manometers : Differential manometers find occasional applications. This manometer is suitable for measurement of small pressure differences. It is a sensitive device and useful for measuring even small gas pressures (heads).

The construction of a differential manometer is given in Figure 2-3. The differential manometer is also known as *two-fluid U-tube manometer*. It contains two immiscible liquids A and B having nearly same densities. The U tube consists of enlarged chambers on both limbs. Hence, the meniscus of the liquid in these enlarged chambers does not change appreciably with changes in the reading R.

Using the principle of simple manometers, the pressure difference (ΔP pascals) can be written as:

$$\Delta P = P_1 - P_2 = R(\rho_C - \rho_A)g \qquad (12)$$

Equation (12) indicates that the smaller the difference ($\rho_C - \rho_A$), the larger will be reading on the manometer (R metres) for a given value of ΔP.

Micromanometers based on the liquid column principle are available commercially. They measure the reading with extreme precision and sensitivity. These are free from errors due to capillarity and require no calibration, apart from checking the micrometer scale.

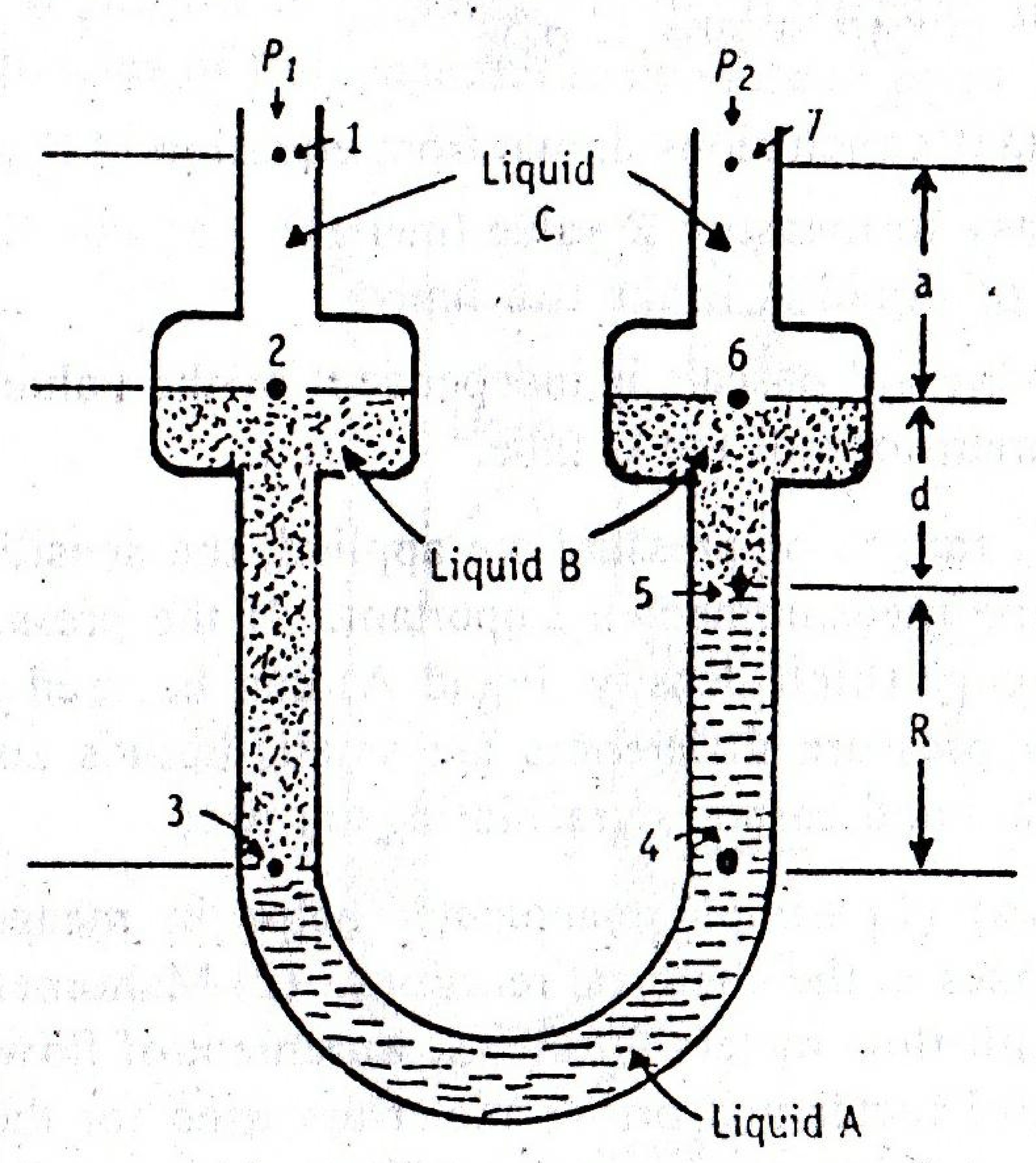


Figure 2-3. Construction of a differential manometer.

Inclined manometers : They have limited applications and, therefore, not discussed here.

FLUID DYNAMICS

Fluid dynamics deals with the study of fluids in motion.

Study of flow properties of liquids is important for pharmacists working in the manufacture of dosage forms, such as simple liquids, gels, ointments, creams and pastes. These systems change their flow behaviour, when exposed to different stress conditions in the following situations.

(1) **Manufacture of dosage forms :** Materials undergo processes such as mixing, flowing through pipes and getting filled in the containers. Flow related changes influence the selection of mixing equipment.

(2) **Handling of drugs for administration :** The syringeability of the medicines, pouring of the liquids, extrusion of ointment from tubes etc., depend on the changes in flow behaviour of dosage forms.

Thus, flow behaviour of liquids is of relevance in pharmacy. Performance of a product depends on the net effect of all the above mentioned processes. Therefore, flow properties are used as important quality control tools to maintain the superiority of the product and to reduce the batch to batch variations.

In general, engineering of fluid flow considers the macroscopic properties. The molecular level interactions are beyond the scope of this book (Refer the book *Physical Pharmaceutics* by C.V.S. Subrahmanyam, Vallabh Prakashan).

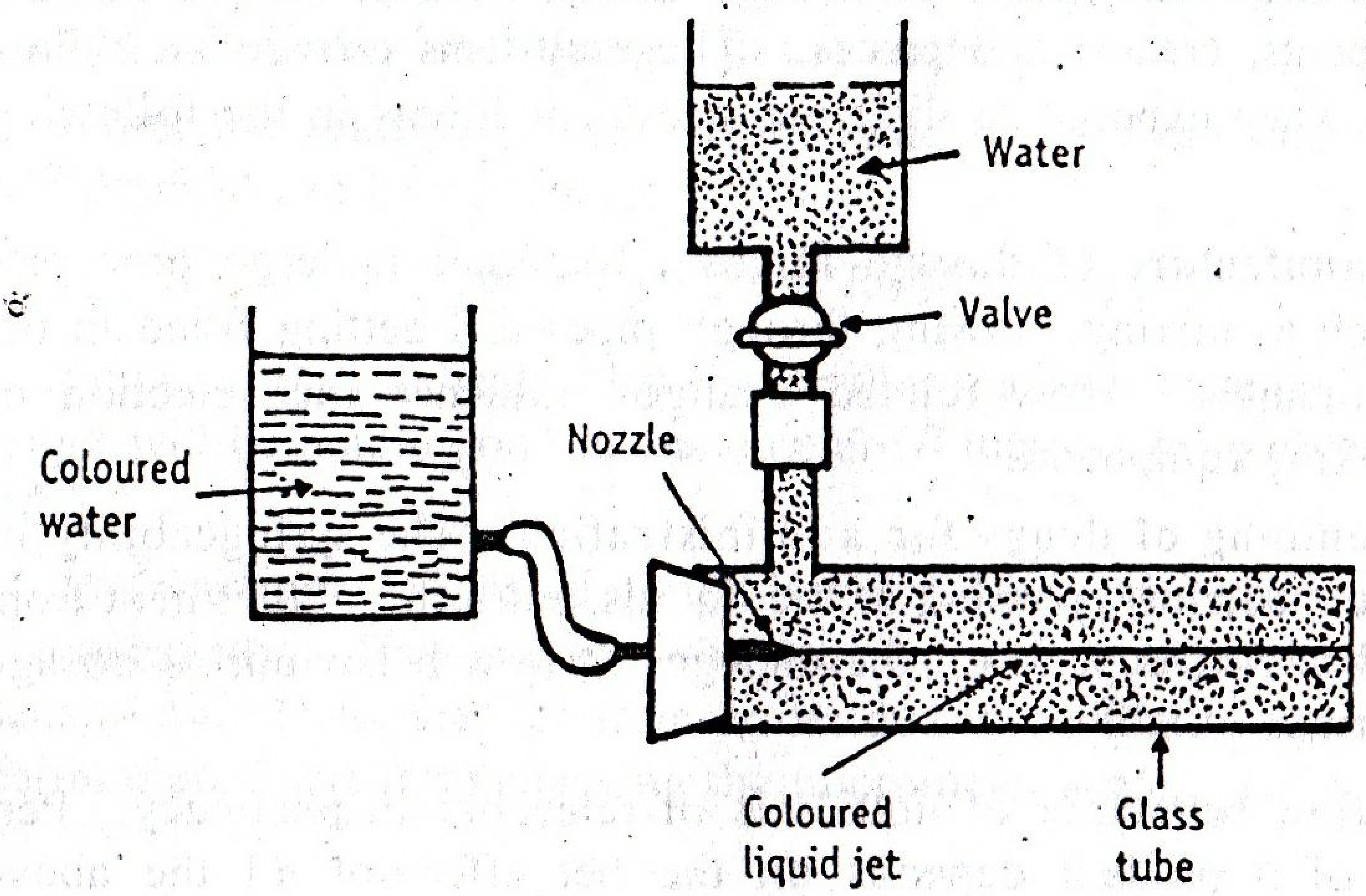
Nature of Fluid Flow–Reynolds Experiment

The flow of fluid through a closed channel (pipeline) can be either viscous or turbulent. These can be observed in the classical Reynolds experiment.

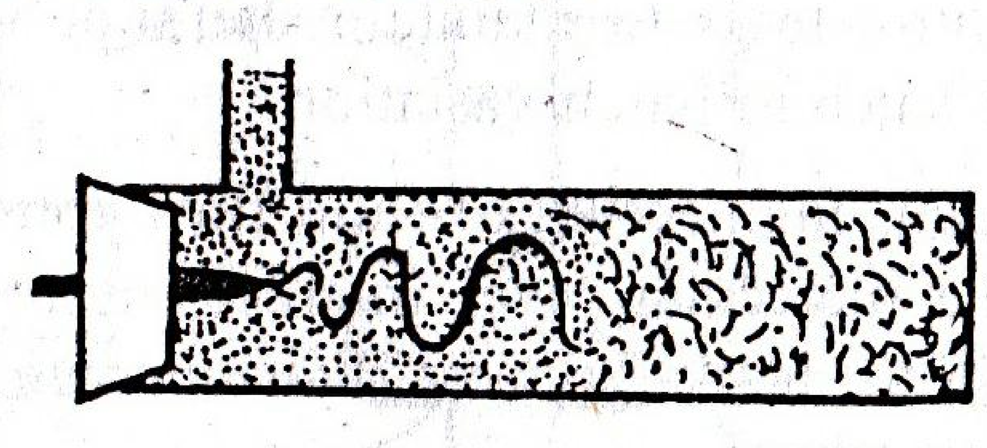
The assembly of the apparatus for the Reynolds experiment is shown in Figure 2-4.

A glass tube is connected to a reservoir of water as shown in Figure 2-4. The rate of flow of water through the tube can be increased or decreased at will through a valve. A reservoir of coloured solution is connected to one end of the glass tube with the help of nozzle(s). Therefore, coloured solution can be introduced into the glass tube as a fine stream. From this experimental setup, the following observations may be made and conclusions can be drawn.

(a) When the velocity of water is low, the thread of coloured water maintains its identity throughout the tube. By introducing similar jets of coloured water at different points in the cross section of the glass tube, it can be noted that no part of the tube exhibits the signs of mixing. In other words, the colour streams are seen as parallel lines. The flow of water is considered to be *viscous or streamline or laminar.*



(a) Laminar or viscous flow.



(b) Turbulent flow.

Figure 2-4. The assembly of the apparatus of the Reynold's experiment.

A *laminar flow* is one in which the fluid particles move in layers or laminar with one layer sliding over the other. Therefore, there is no exchange of fluid particles from one layer to the other. Hence, transfer of lateral momentum to the adjacent layers is not observed.

(b) When the velocity of water is increased (by increasing the flow rate), the threads of coloured water disappear and the entire mass of water gets uniformly coloured. It indicates complete mixing of solution. Then, the flow of water is considered to be *turbulent.*

When the flow attains a certain velocity, it no longer remains steady and eddy currents appear. All the fluid particles are disturbed and get mixed up with each other. Thus, there is a continuous transfer of momentum to adjacent layers. Such a diffused flow is called *turbulent flow.*

(c) The change over of the flow from viscous to turbulent is a critical factor.

Critical velocity is defined as average velocity of any fluid at which viscous flow changes into turbulent flow.

Reynolds number : In Reynolds experiment, the flow conditions are affected by four factors.

Diameter of pipe, m (D)
Average velocity, m/s (u)
Density of liquid, kg/m^3 (ρ)
Viscosity of the fluid, Pa·s (η)

These factors are grouped into a particular expression as given below.

$$\textit{Reynolds number, } Re = \frac{Du\rho}{\eta} \qquad (13)$$

Reynolds number is obtained by the following equation.

$$\text{Reynolds number, } Re = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{\text{mass} \times \text{acceleration of liquid flowing}}{\text{shear stress} \times \text{area}}$$

Inertial forces are due to mass and the velocity of the fluid particles trying to diffuse the fluid particles. The viscous force is the frictional force due to the viscosity of fluid, which makes the motion of fluid in parallel layers.

Therefore, at low velocities, the inertial forces are less when compared to the frictional forces caused by the viscosity. The resulting flow will be viscous in nature, i.e., the particles move in parallel layers. For this reason, laminar flow is sometimes known as *viscous.* On the other hand, when inertial forces are predominant, the fluid layers break up due to the increase in velocity, hence, turbulent flow takes place.

It is a dimensionless group, because the units of factors mutually get cancelled. If $Re < 2000$, the flow is said to be laminar and if $Re > 4000$, the flow is said to be turbulent. If Re lies between 2000 to 4000, the flow changes from laminar to turbulent.

Applications : (1) Reynolds number is used to predict the nature of flow (viscous or turbulent) in a particular set of experimental conditions. (2) The physical stability of suspensions (or emulsions) depends on the rate of settling of particles (or globules). For the study of sedimentation of particles, Stokes' law is used. In this study, rate of sedimentation of particles must not be too rapid to create turbulence. Therefore, type of flow (whether laminar or turbulent) is important. Accordingly Stokes' equation is modified to include Reynolds number. (3) The rate of heat transfer in liquids also depends on the flow, whether viscous or turbulent.

Variation in the Velocity of Flow Across the Cross-Section

When the local velocity of the fluid is plotted against distance from the wall (Figure 2-5), the following conclusions can be drawn.

- The flow of fluid in the middle of the pipe is faster than the fluid nearer to the wall.
- The velocity of fluid approaches zero as the pipe-wall is approached.
- At the actual surface of the pipe-wall, the velocity of the fluid is zero.

Since variations are observed in velocity of flow across the cross section, there should be some means of estimating average velocity.

- In viscous conditions, the average velocity over the whole cross section is 0.5 times the maximum.
- In turbulent flow, the average velocity over the whole cross section is 0.8 times the maximum.

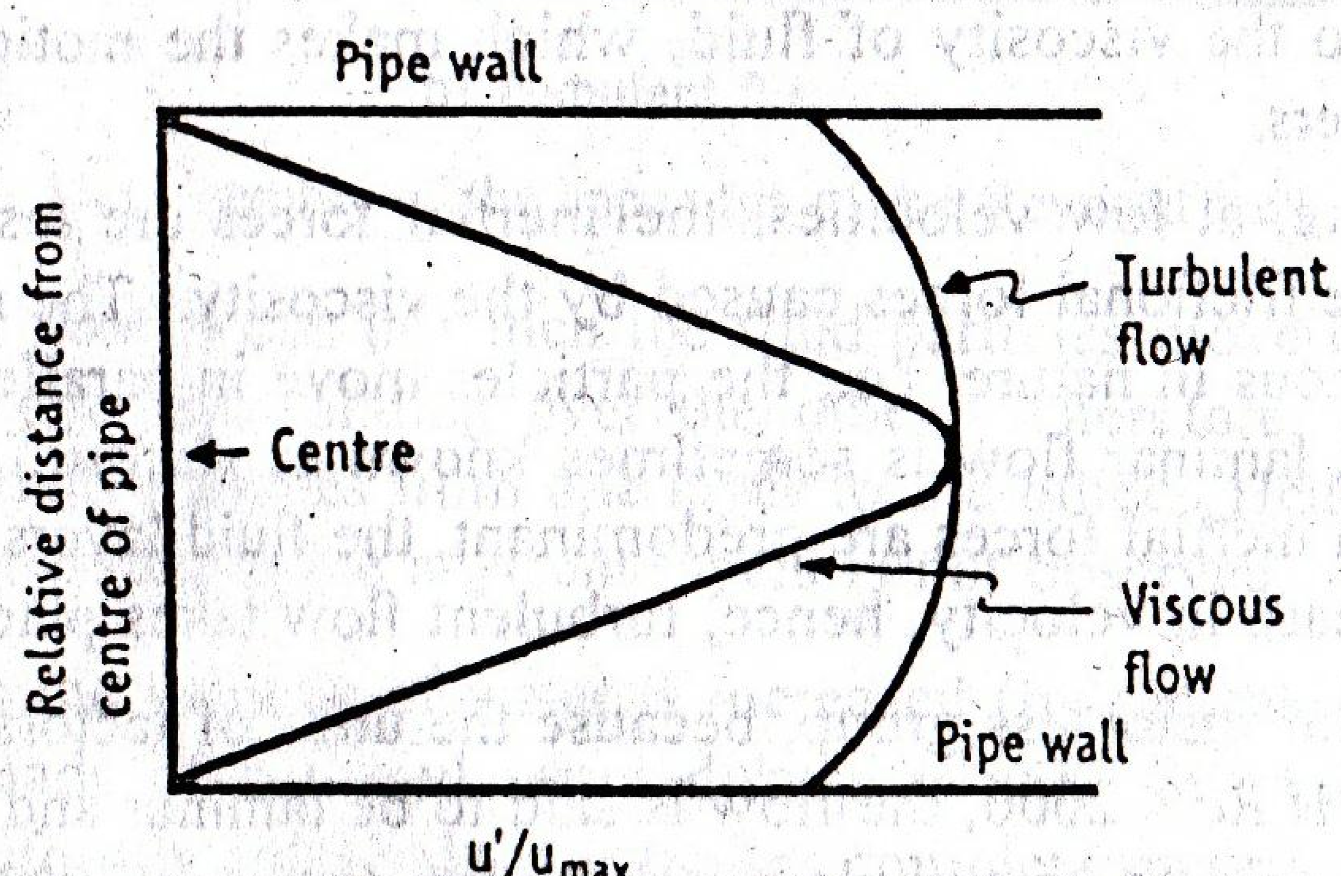


Figure 2-5. Distribution of fluid velocities across the cross section of a pipe

- Since the velocity of the fluid is zero at the wall surface, there should be some layers in viscous flow near the pipe-wall, which acts as a stagnant layer.
- Even if the flow is turbulent at the centre (of the pipe) and viscous at the surface of the wall, a buffer layer exists.
- A gradual transition from one region to another, there exists a boundary layer which is known as buffer layer. This buffer layer oscillates between viscous flow and turbulent flow.

It must be emphasized that boundary layer can never be eliminated. Increasing the velocity of the fluid over the surface will reduce the thickness of the layer, but it will be never eliminated entirely.

Applications : (1) Though the general principles are mentioned above, considerable variation in velocity distribution may be possible with changes in roughness, direction, temperature or cross section of the pipe. Due to these reasons, the shape of curves may be changed. (2) The nature of flow of liquids in pipes determines the rate of heat transfer. The buffer layer in turbulent flow and stagnant layer in viscous flow offer resistance to heat transfer. Further discussion on rate of heat transfer is dealt in Chapter 5 'Flow of Heat'.

BERNOULLI'S THEOREM

When the principle of conservation of energy is applied to the flow of fluids, the resulting equation is called Bernoulli's theorem.

Pumps generally supply energy for conveying liquids from one point to another. Consider such a pump working under isothermal conditions between points A and B, as shown in Figure 2-6.

Bernoulli's theorem states that in a steady state ideal flow of an incompressible fluid, the total energy per unit mass, which consists of pressure energy, kinetic energy and datum energy, at any point of the fluid is constant.

At point A, one kilogram (unit mass) of liquid is assumed to be entering. At this point, liquid experiences pressure energy, kinetic energy and potential energy, which are obtained as follows.

Since liquid is flowing through the pipe at certain pressure, pressure energy in joules may be written as:

$$\textit{Pressure energy} = \frac{P_A}{g\rho_A} \qquad (14)$$

where P_A = Pressure at point A, Pa
g = acceleration due to gravity, m/s
ρ_A = density of the liquid , kg/m^3

Potential energy (datum energy) of a body is defined as the energy possessed by the body by virtue of its position or configuration. The point A is considered at a height of X_A metres above the horizontal datum plane. The potential energy for one kilogram of liquid may be written as:

$$Potential\ energy = X_A \qquad (15)$$

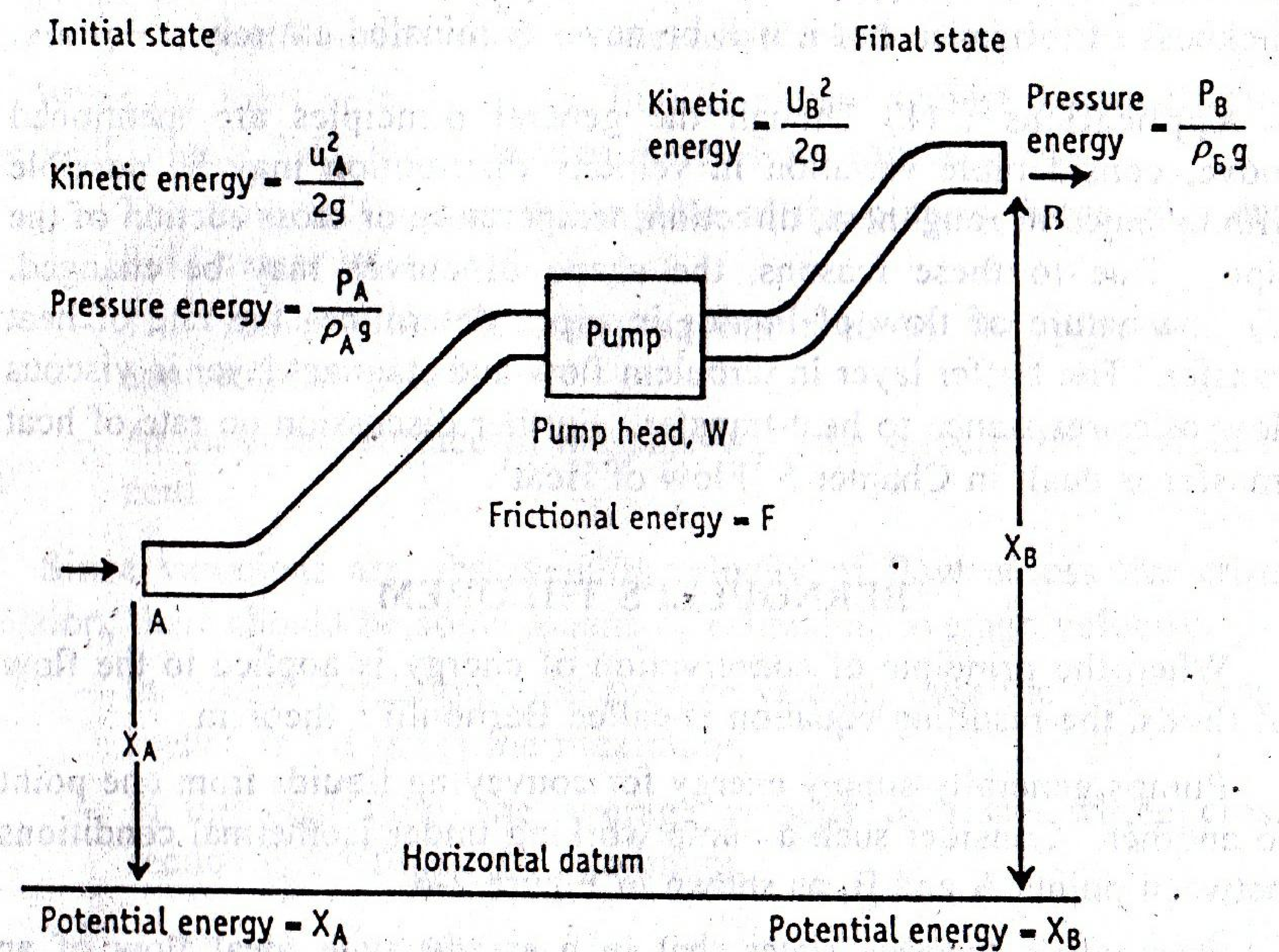


Figure 2-6. Development of Bernoulli's theorem.

Kinetic energy of a body is defined as the energy possessed by the body by virtue of its motion. Since liquid is under motion, the velocity of liquid may be designated as u_A metre per second at point A. The kinetic energy may be expressed as:

$$Kinetic\ energy = \frac{u_A^2}{2g} \qquad (16)$$

The total energy at point A may be summarised by combining equations (14), (15) and (16) as:

Total energy = pressure energy + potential energy + kinetic energy

$$Total\ energy\ at\ point\ A = \frac{P_A}{g\rho_A} + X_A + \frac{u_A^2}{2g} \qquad (17)$$

According to Bernoulli's theorem the total energy at point A is constant. Therefore, equation (17) is:

$$Total\ energy\ at\ point\ A = \frac{P_A}{g\rho_A} + X_A + \frac{u_A^2}{2g} = \text{constant} \qquad (18)$$

After the system reaches the steady state, whenever one kilogram of liquid enters at point A, another kilogram of liquid leaves at point B. Therefore, energy content of one kilogram liquid that is being displaced at point B may be written (Bernoulli's theorem) as:

$$Total\ energy\ at\ point\ B = \frac{P_B}{g\rho_B} + X_B + \frac{u_B^2}{2g} = \text{constant} \qquad (19)$$

where X_B = height from the datum to the pipe, m
u_B = velocity at point B, m/s
P_B = pressure at point B, Pa
ρ_B = density at point B, kg/m^3

If there is no gain or loss of energy, the principle of conservation of energy may be applied to the two points A and B.

INPUT = OUT PUT

Total energy at point A = Total energy at point B

$$\frac{P_A}{g\rho_A} + X_A + \frac{u_A^2}{2g} = \frac{P_B}{g\rho_B} + X_B + \frac{u_B^2}{2g} \qquad (20)$$

Theoretically all kinds of energies involved in fluid flow should be accounted. In the transportation of fluid, the pump has added certain amount of energy, which can be written as:

$$Energy\ added\ by\ the\ pump = +\ wJ \qquad (21)$$

During the transportation of liquid, some energy is converted to heat due to frictional forces and it is inevitable. The energy loss may be written as:

$$Loss\ of\ energy\ due\ to\ friction\ in\ the\ line = FJ \qquad (22)$$

The energy balance between points A and B can be accounted by including equations (21) and (22) in equation (20). This complete equation representing such energy may be written as:

$$\frac{P_A}{g\rho_A} + X_A + \frac{u_A^2}{2g} - F + W = \frac{P_B}{g\rho_B} + X_B + \frac{u_B^2}{2g} \qquad (23)$$

Equation (23) is called *Bernoulli's equation*. Bernoulli's theorem, although derived here over two ends of a system, it is applicable between any two points in a system.

Equation (23) is numerically correct. But it is not correct theoretically, since each of the terms in equation (23) is actually energy term and should be measured in the units of joules per unit mass. In practice, these terms are always referred as the heights and often measured in terms of height of a column of liquid.

Applications: (1) Bernoulli's theorem is applied in the measurement of the rate of fluid flow using orifice meter, venturi meter etc.

(2) Bernoulli's theorem is applied in the working of centrifugal pumps. In these pumps, the kinetic energy is converted into pressure head, which helps in pumping the liquids.

(3) It is easy to measure heights and apply them as energy terms, which is a contribution of Bernoulli's theorem.

Concept of Head (Pressure Head)

The terms in Bernoulli's equation represent energy and supposed to have the units of energy. But numerically these terms give the units of metre. Therefore, energy terms can be measured in the units of metre, which is a unit of height. Hence these terms are known as *heads*.

The *pressure head* is defined as the height of a column of liquid of known density, which is numerically equal to pressure energy term.

The units of height can be obtained by considering the pressure energy term, i.e. equation (14).

$$\textit{Pressure energy} = \frac{P_A}{g\rho_A} = \frac{\text{pressure}}{\text{acceleration due to gravity} \times \text{density}}$$

Consider the units for equation (14).

$$\text{Pressure energy} = \frac{\text{N}}{\text{m}^2} \cdot \frac{1}{(\text{m/s}^2)} \cdot \frac{1}{(\text{kg/m}^3)}$$

$$= \frac{\text{N} \times \text{s}^2 \times \text{m}^3}{\text{m}^2 \times \text{m} \times \text{kg}} = \frac{\text{N}\cdot\text{s}^2}{\text{kg}} \qquad (24)$$

But N (newton) = kg·m/s². Substituting this term in equation (24) gives:

$$\text{Pressure energy} = \frac{\text{kg} \times \text{m} \times \text{s}^2}{\text{s}^2 \times \text{kg}} = \text{m.}$$

Thus, pressure energy is in metre, i.e., as a height. Therefore, the height is termed as head in the discussion of hydraulics. Hence, pressure energy is called pressure head.

In an analogous manner, there are different heads in the Bernoulli's equation (18), namely:

potential heads (X)
velocity heads ($u^2/2g$)
pressure heads ($P/g\rho$)

Similarly, F is known as friction head and W is the head that is added by the pump.

ENERGY LOSSES

Bernoulli's equation includes the term 'loss of energy' in the pipe. According to law of conservation of energy, energy balances have to be properly accounted. Therefore, it is necessary to calculate the energy losses. Fluids experience energy losses in several ways while flowing through a pipe. Some of them are:

1. Friction losses
2. Losses in fittings
3. Enlargement losses
4. Contraction Losses

These are discussed in the following sections.

Friction Losses

During the flow of fluids, frictional forces cause a loss in pressure (ΔP_f pascals). The fluid flow can be either viscous or turbulent, which also influences the losses. In general, the pressure drop (ΔP_f) due to friction in a fluid is:

- directly proportional to the velocity of the fluid (u), m/s
- directly proportional to the density of the fluid (ρ), kg/m³
- directly proportional to the length of the pipe (L), m
- inversely proportional to the diameter of the pipe (D), m

These relationships are proposed in *Fanning equation* for calculating the friction losses, irrespective of the nature of flow (viscous or turbulent).

$$\text{Fanning equation: (viscous or turbulent)} \quad \Delta P_f = \frac{2fu^2L\rho}{D} \qquad (25)$$

where f = friction factor

ΔP_f = pressure drop, Pa

Equation (25) considers the friction losses when the fluid is passing through a straight pipe. The value of f depends on:

- Nature of flow of the fluid (turbulent or viscous).
- Roughness of the inner surface of the pipe. The roughness factors for some conditions of pipes are given below.

Condition of pipe	*Roughness factor*
Smooth brass, copper or lead pipe	0.6
New steel or cast-iron pipe	1.0
Old steel pipe	1.6
Badly rusted cast-iron pipe	2.5

The numerical values reported in the above table give a rough estimate of the magnitude of the effect, which contributes to the friction factors. Frictional loss can be reduced by the addition of soluble, high molecular weight polymers in low concentrations.

In practice, fluids are rarely handled in viscous flow. For viscous flow, *Hagen-Poiseuille equation* could be employed for calculating the pressure drop due to friction.

$$\text{Hagen-Poiseuille equation: (viscous flow)} \quad \Delta P = \frac{32Lu\eta}{D^2} \qquad (26)$$

where η = viscosity of the liquid, Pa·s

ΔP_f = pressure drop, Pa

If the viscosity (η) is known, Hagen-Poiseuille's equation permits the calculation of pressure drop due to friction. However, equation (26) is normally used to calculate the viscosity, by experimentally estimating ΔP.

Friction losses are permanent, since potential and kinetic energies are converted into heat.

Losses in Fittings

Normally, a large number of fittings are included in a pipeline (Figure 2-7). For a liquid passing through a straight pipe, Fanning

equation is applicable for the losses. When fittings are introduced into a straight pipe, they cause disturbances in the flow, which results in additional loss of energy. It is difficult to specify the loss due to each type, because of varying types of fittings.

Losses in fittings may be due to

- change in direction, for example elbow fittings,
- change in the types of fittings, for example, coupling, union, valve or meter.

Losses in fittings, by convention, is expressed in terms of an **equivalent length of straight pipe**, which is given as a certain number of pipe diameters.

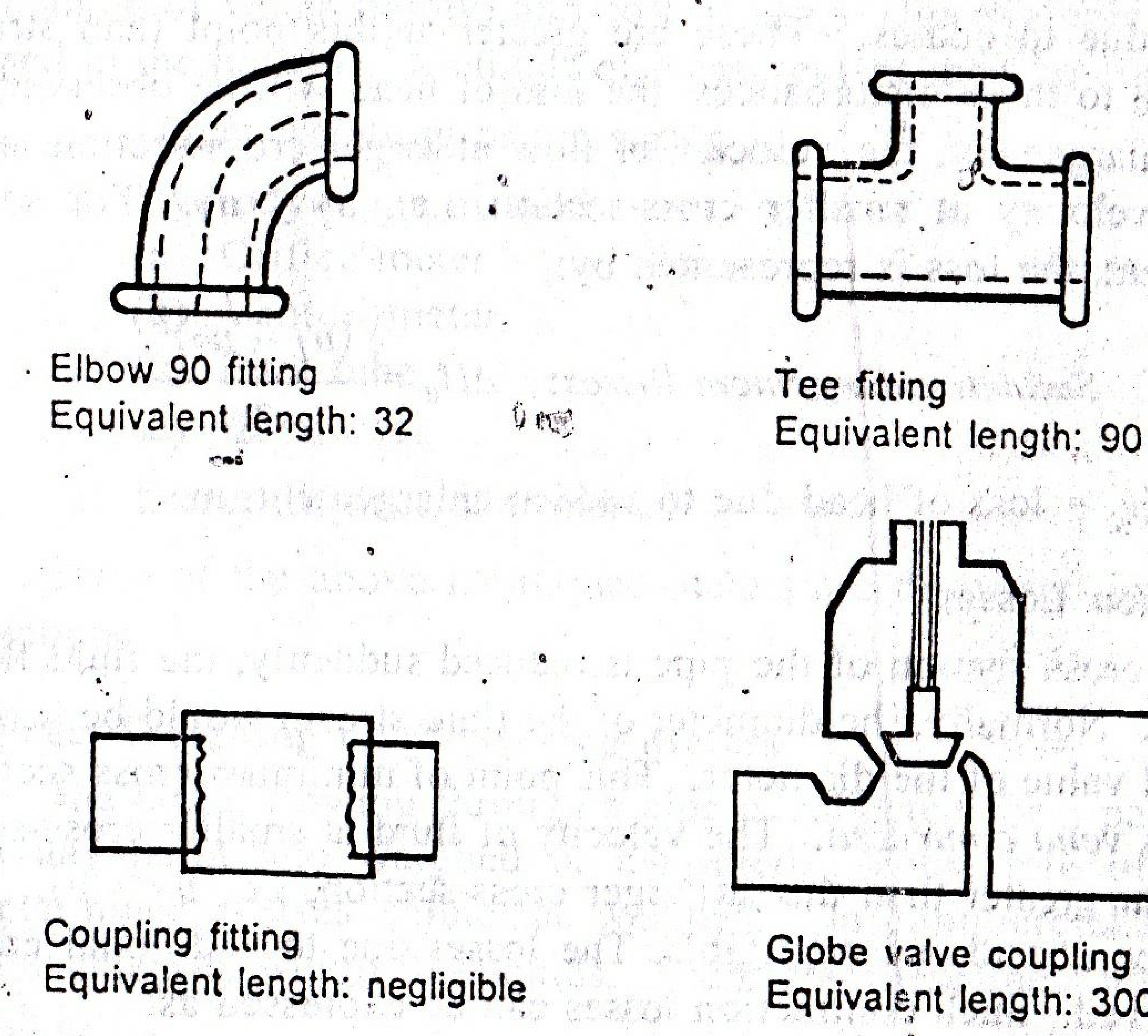


Figure 2-7. Friction losses in fittings of a new screwed pipe.

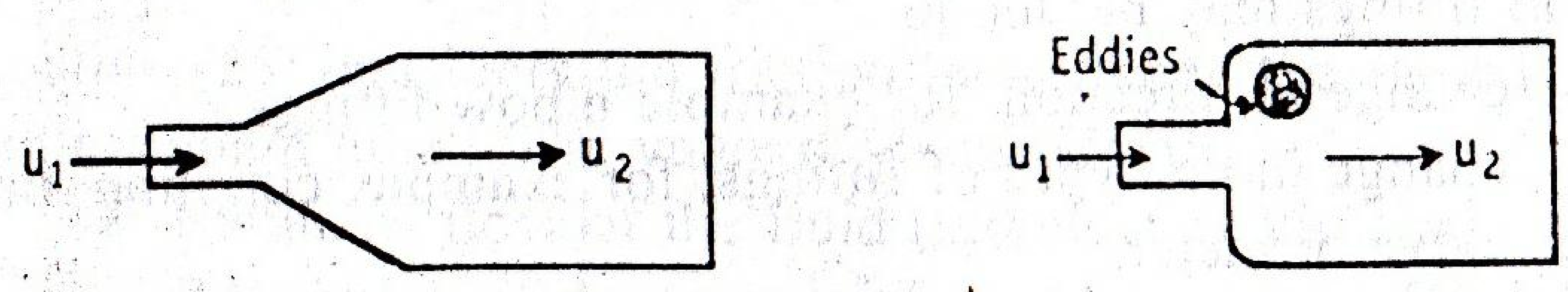
Equivalent lengths for a few screwed fittings are shown in Figure 2-7. For example, a globe valve is fitted in a pipeline having an internal diameter of 50 millimetres. Globe valve has an equivalent length of 300. Then,

Equivalent length of this fitting = 300 × 50 = 15000 mm or 15.0 m.

That means the contribution of a globe valve is equivalent to 15 metres of a straight pipe towards the losses. This length is added to the straight section of the pipe and substituted in the Fanning equation (25) to obtain energy losses due to fittings.

Enlargement Losses

If the cross section of the pipe enlarges *gradually*, the fluid adapts itself to the changed section without any disturbance. Therefore, there is no loss of energy at this point.



(a) Gradual enlargement
No loss of energy

(b) Sudden enlargement
Loss of energy

If the cross section of the pipe changes suddenly, loss of energy is observed due to eddies. These are greater at this point than straight pipe. Due to these disturbances, the loss of head will be observed. In sudden enlargement, the velocity of flow at larger cross-section is less than the velocity at smaller cross-section, i.e., $u_2 < u_1$. For sudden enlargement, the loss is represented by:

$$\textit{Sudden enlargement losses:} \quad \Delta H_e = \frac{(u_1 - u_2)^2}{2g} \qquad (27)$$

where ΔH_e = loss of head due to sudden enlargement, m.

Contraction Losses

If the cross section of the pipe is reduced suddenly, the fluid flow is disturbed. Normally, the diameter of the fluid stream would be less than the initial value of the diameter. This point of minimum cross section is known as *vena contracta*. The velocity of fluid at smaller cross-section will be far greater than that at larger cross-section, i.e., $u_2 > u_1$. Then, u_1 may be considered negligible. The losses due to additional eddying are observed. Such contraction losses can be expressed as:

$$\textit{Sudden contraction losses:} \quad \Delta H_c = \frac{K u_2^2}{2g} \qquad (28)$$

where ΔH_c = loss in head due to sudden contraction, m
K = constant
u_2 = velocity, m/s

(a) Gradual contraction
No loss of energy

(b) Sudden contraction
Loss of energy

The constant K depends on the relative areas of two sections. For example, when the ratio of the areas is 0.5, K is equal to 0.3. For rounded entrance, K value for turbulent flow is about 0.04. For laminar flow, the loss is negligible.

MEASUREMENT OF RATE OF FLOW OF FLUIDS

Whenever fluids are used in a process, it is necessary to measure the rate at which the fluid is flowing through the pipe. This is required for optimization of process parameters in a chemical industry. Measurements are required for the calculation of auditing (for example, town planning of water supply) and cost of usage (for example, cost of water used in the houses). Methods of measurement may be classified as:

1. Direct weighing or measuring
2. Hydrodynamic methods
   (a) Orifice meter
   (b) Venturi meter
   (c) Pitot tube
   (d) Rotameter
3. Direct displacement meter

Some of the above mentioned meters are discussed in the following sections.

Direct Weighing or Measuring Meters

The liquid flowing through a pipe is collected for a particular period at any point and weighed or measured. Thus, rate of flow can be determined. Gases cannot be weighed. On commercial scale, it is not convenient to weigh the liquids. Therefore, these methods are impracticable.

Orifice Meter

Principle : The orifice meter is a thin plate containing a narrow and sharp aperture. When a fluid stream is suddenly allowed to pass through the narrow constriction, the velocity of the fluid at the orifice meter increases compared to the velocity of the fluid in the upstream. This results in corresponding decrease in the pressure head. Bernoulli's theorem provides the basis for correlating the increase in the velocity head with the decrease in the pressure head between two points. The difference in the pressure head (ΔH) may be read from a manometer. If the diameter of the orifice is small compared to the diameter of the pipe, velocity of the fluid at the point before entering the orifice may be considered negligible. In such cases the manometer reading directly gives the velocity of the fluid.

The velocity of the fluid at the thin constriction may be written as:

$$u_o = C_o\sqrt{2g.\Delta H} \qquad (29)$$

where u_o = velocity of fluid at the point of orifice meter, m/s
C_o = a constant
ΔH = difference in head from manometer, m

The ΔH can be measured using a manometer, which is connected to the pipe section between the initial stage and the orifice section. It can be substituted in equation (29) in order to get the velocity of the liquid flowing through the orifice. When cross-section of the pipe is known, the volume of the liquid flowing per hour can be determined.

Construction : The orifice meter is considered to be a thin plate containing a sharp aperture through which a fluid flows. Normally, orifice plate is placed between long straight pipes, so that other fittings do not alter the flow rate that is being measured. Although it is possible to place orifice meter in the side or bottom, for the present discussion, the plate is introduced into the pipe (Figure 2-8). A manometer is connected at points A and B as shown in Figure 2-8.

Working : Orifice meter is referred to as *variable head meter*, i.e., it measures the variation in the pressures across a fixed constriction placed in the path of flow consisting of a constant area.

When fluid stream is allowed to pass through the cross-section of the orifice, the velocity of fluid at point B increases at the expense of pressure head. As a result, the pressure at point A is higher than at point B. Bernoulli's equation provides the basis for correlating the increase in velocity head with the decrease in pressure head. The difference in pressure (ΔH metres) may be read from a manometer, connected to the points A and B as shown in Figure 2-8.

Bernoulli's equation may be applied for two points (A and B) for the given experimental conditions, as given below:

$$\sqrt{u_o^2 - u_A^2} = C_o\sqrt{2g.\Delta H} \qquad (30)$$

where u_o = velocity of fluid at the point of orifice meter, m/s
u_A = velocity of fluid at the point A, i.e., before orifice meter, m/s
C_o = a constant
ΔH = difference in head, m

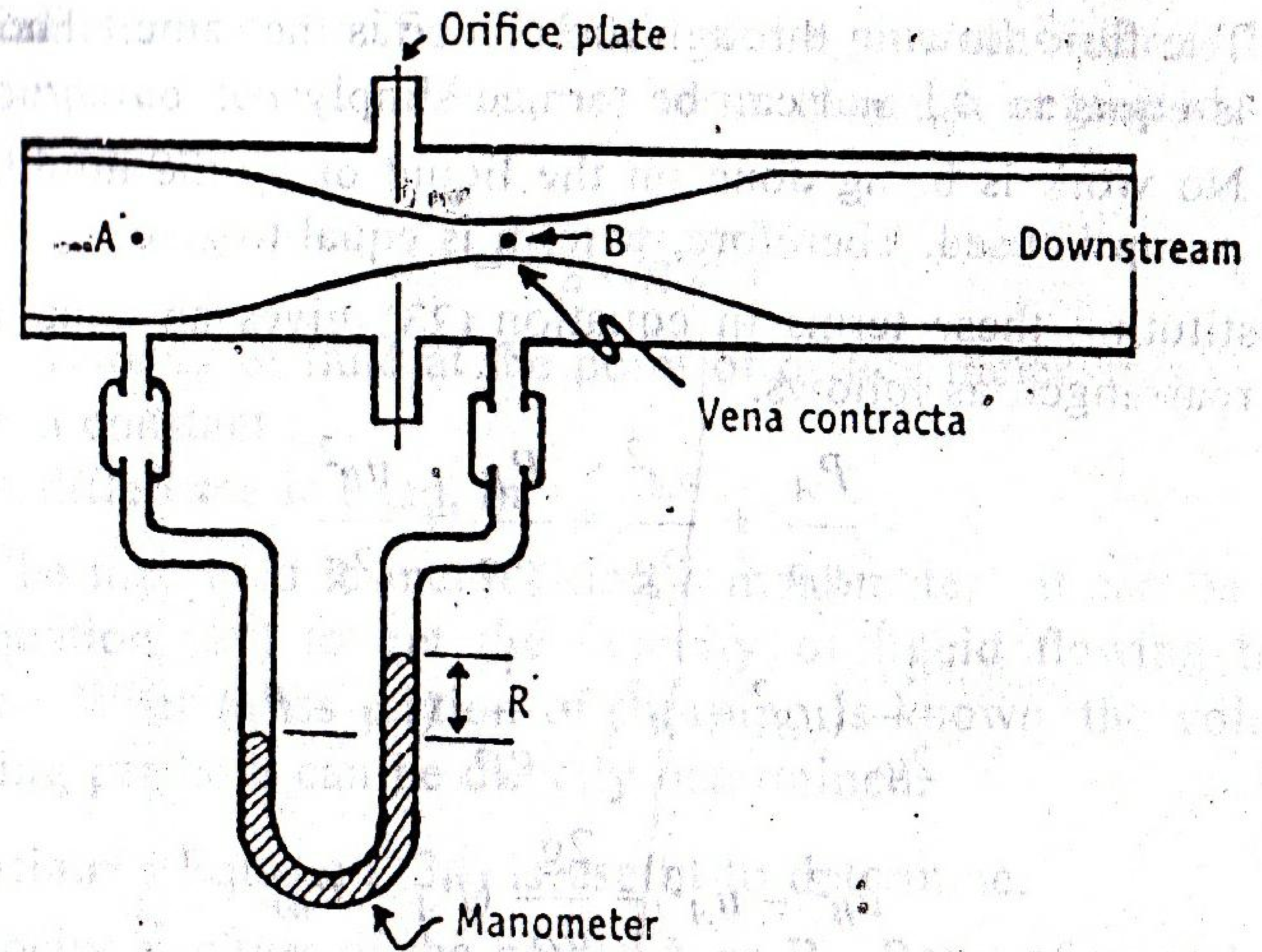


Figure 2-8. Construction and assembly of orifice meter.

If the diameter of the orifice is 1/5th of the pipe diameter or less, u_A is small compared to u_o. Then, u_A term may be neglected. Then equation (30) becomes:

$$u_o = C_o\sqrt{2g.\Delta H} \qquad (29)$$

ΔH can be measured using a manometer. It can be substituted in equation (29) in order to get the velocity of liquid flowing through orifice pipe. When cross-section of the pipe is known, the volume of liquid flowing per hour can be directly determined.

Mathematical treatment : Consider a fluid flowing through a pipe at a certain velocity (u_A). If the edge of orifice is sharp, fluid does not lose the velocity at once, while passing through the orifice. Two points A and B on either side of the orifice meter are chosen and Bernoulli's equation is applied. The Bernoulli's equation is:

$$\frac{P_A}{g\rho_A} + X_A + \frac{u_A^2}{2g} - F + W = \frac{P_B}{g\rho_B} + X_B + \frac{u_B^2}{2g} \qquad (23)$$

In equation (23), the following assumptions may be made based on the working of orifice meter (Figure 2-8).

(1) Let the sections of pipe be horizontal, so that the heights (metres) of the points A and B are same (Figure 2-8). Then, two X terms are identical and get cancelled.

(2) Let the friction losses will not be appreciable and considered negligible. Then, F term becomes zero.

(3) The fluid flowing through the orifice is the same. Therefore, ρ_A is equal to ρ_B, and can be termed simply ρ.

(4) No work is being done on the liquid or by the liquid, since no pump is used. Therefore, w term is equal to zero.

Substituting these terms in equation (23) gives an equation, which can be rearranged as follows.

$$\frac{P_A}{g\rho} + \frac{u_A^2}{2g} = \frac{P_B}{g\rho} + \frac{u_B^2}{2g}$$

$$\frac{1}{2g}(u_B^2 - u_A^2) = \frac{1}{g\rho}(P_A - P_B)$$

$$u_B^2 - u_A^2 = \frac{2g}{g\rho}(P_A - P_B) \qquad (31)$$

$$u_B^2 - u_A^2 = \frac{2g}{g\rho}\Delta P \qquad (32)$$

From the principles of statics, $(\Delta P/g\rho) = \Delta H$ in metres. Introducing this term in equation (32) gives:

$$u_B^2 - u_A^2 = 2g.\Delta H$$

$$\sqrt{u_B^2 - u_A^2} = \sqrt{2g.\Delta H} \qquad (33)$$

Normally, the diameter of the fluid stream would be less than the diameter of the orifice (Figure 2-8). This point of minimum cross section is known as the *vena contracta*. Though the pipe is full of liquid on both sides of the orifice, still the *vena contracta* exists and is surrounded by swirling liquid. Point B is chosen at the *vena contracta*.

In practice, the diameter of the stream at the *vena contracta* is not known, but that of the orifice diameter is known. A constant, C_o, is, therefore, included in equation (33) in order to correct the differences between velocities at orifice and at *vena contracta*. Therefore, equation (33) may be modified in terms of the velocity through the orifice (u_o).

$$\sqrt{u_o^2 - u_A^2} = C_o\sqrt{2g.\Delta H} \qquad (30)$$

The constant C_o also includes some losses due to friction. It depends on the construction (ratio of the orifice diameter to the pipe diameter and position of the orifice taps) and the nature of flow of liquid (Reynolds number).

If the diameter of the orifice is 1/5th of the pipe diameter or less, u_A is small compared to u_o. Then, u_A term may be neglected. Then equation (30) becomes:

$$u_o = C_o\sqrt{2g.\Delta H} \qquad (29)$$

where u_o = velocity of fluid at the point of orifice meter, m/s
C_o = a constant
ΔH = difference in head, m

ΔH can be measured in metre using a manometer. It can be substituted in equation (29) to get the velocity of liquid flowing through orifice pipe. When cross-section of the pipe is known, the volume of liquid flowing per hour can be directly determined.

Applications : Equation (30) is useful to determine:

(1) velocity at either of the points A or B. Ratio of u_o and u_A can be related to ratio of the area of the orifice to the area of the pipe, which is normally known.

(2) volume of the liquid flowing per hour when u_A and cross-section of the pipe are known.

Venturi Meter

Principle : Venturi meter consists of two tapered sections in the pipeline with a gradual constriction (throat) at its centre. When fluid stream is allowed to pass through the narrow throat, the velocity of the fluid increases at the venturi compared to the velocity of the upstream. This results in corresponding decrease in the pressure head. Bernoulli's theorem provides the basis for correlating the increase in the velocity head with the decrease in the pressure head between two points. The difference in the pressure head (ΔH) may be read from a manometer. If the diameter of the venturi is small compared to the diameter of the pipe, velocity of the fluid at the point before entering the venturi may be considered negligible. In such cases, the manometer reading directly gives the velocity of the fluid.

The velocity of the fluid at the narrow constriction (throat) may be written as:

$$u_v = C_v\sqrt{2g.\Delta H} \qquad (34)$$

where u_v = velocity at the throat of the venturi, m/s
C_v = coefficient of the venturi meter
ΔH = difference in head from manometer, m

ΔH can be measured using a manometer. It can be substituted in equation (34) in order to get the velocity of the liquid flowing through the venturi. When the cross-section of the pipe is known, the volume of liquid flowing per hour can be determined.

Construction : A venturi meter consists of two tapered sections inserted in a pipeline (Figure 2-9). Normally, venturi meter is placed between long straight pipes, so that other fittings will not alter the flow rate that is being measured. The upstream cone is normally shorter than the down stream. The tapers are smooth and gradual. Therefore, eddies in the down stream are absent and no power loss is observed. In addition, the cross-section of the high velocity part of the stream is well defined. A manometer is connected at points A and B as shown in Figure 2-9.

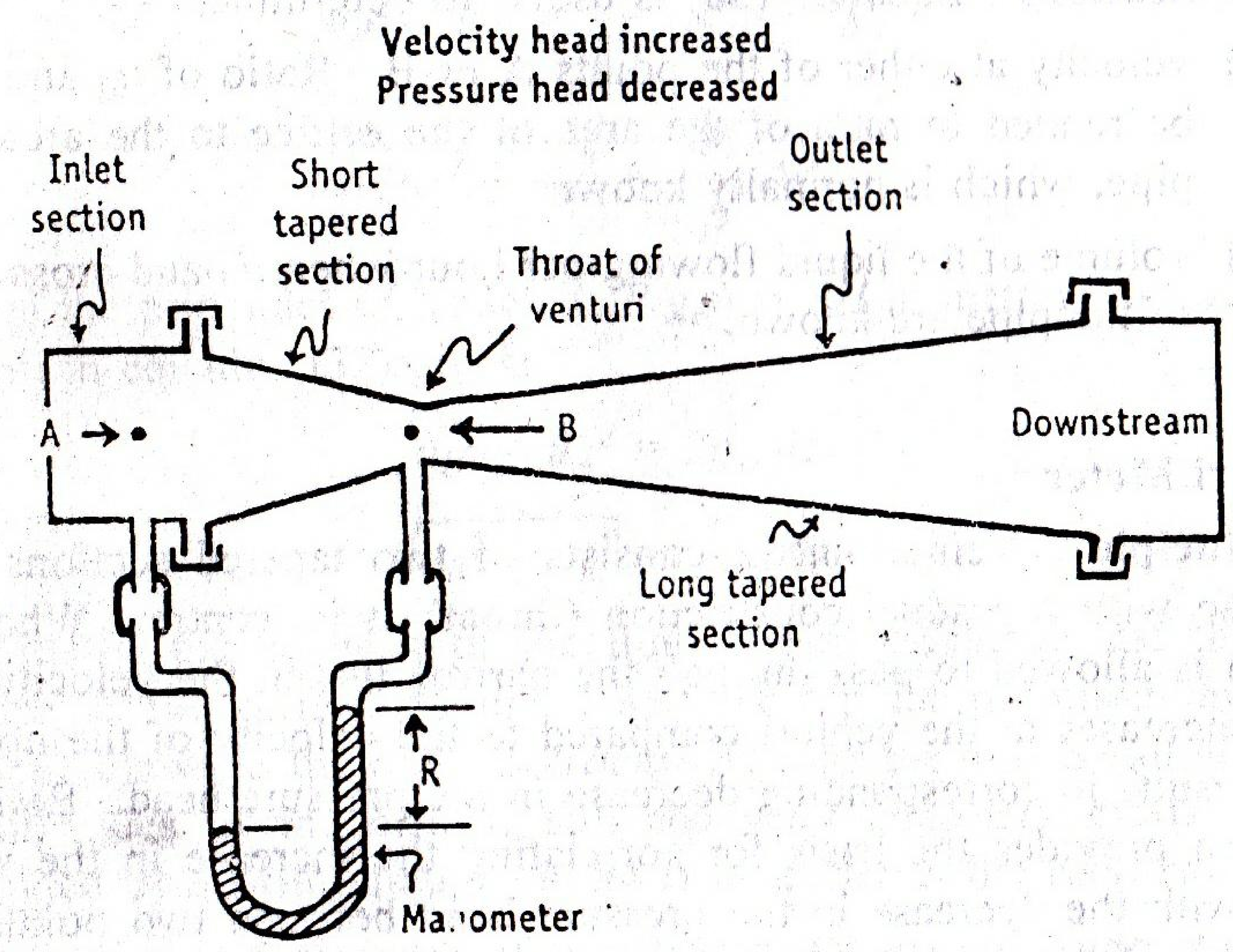


Figure 2-9. Construction of a venturi meter.

Working : Venturi meter is referred to as *variable head meter*, i.e., it measures the variable differential pressure across a fixed constriction placed in the path of flow consisting of a constant area. In a venturi meter, the velocity of the fluid is increased at the throat, due to the constriction. This results in decreased pressure in the up-stream cone. The pressure drop in the upstream cone is utilised to measure the rate of flow using a manometer. Venturi meter nearly confirms the theoretical equations obtained for an orifice meter. On similar lines of equation (30), an equation for venturi meter may be written as:

$$\sqrt{u_v^2 - u_A^2} = C_v\sqrt{2g.\Delta H} \qquad (35)$$

where u_v = velocity at the throat of the venturi, m/s
u_A = velocity at point A (venturi throat), m/s
C_v = coefficient (= 0.98)

If the diameter of the smaller section is one-fifth of the pipe diameter or less, u_A^2 is considered to be small compared to u_v^2. Therefore, u_A^2 term may be disregarded. A simplified form of equation (35) is:

$$u_v = C_v\sqrt{2g.\Delta H} \qquad (34)$$

The difference in pressure (ΔH) can be read in metres directly from the manometer. Thus velocity of the flow may be measured. The value given by the venturi meter is average velocity of the flow. The velocity then decreases and the original pressure is recovered in the down stream cone.

Applications : Venturi meter is commonly used for liquids, especially water. It can also be used for the measurement of gases.

Disadvantages : (1) Venturi meter is expensive. (2) Venturi meter occupies more space. (3) The ratio of throat diameter to pipe diameter cannot be changed.

TABLE 2-1
Differences Between Orifice Meter and Venturi Meter

Orifice meter	*Venturi meter*
(1) Cheap	Expensive
(2) Easy to install	Fabrication is highly technica
(3) Construction can be made at home	It should be purchased from the instrument dealer
(4) Head losses are more	Head losses are insignificant
(5) Power losses are more particularly on fluid that is carried for long periods of time	Power losses are less
(6) Normally used for testing purposes, for example, steam lines etc.,	Used in on-line installation
(7) Greater flexibility	Not flexible, permanent
(8) Reading of the orifice meter is larger under identical conditions	The reading of venturi meter is comparatively smaller under identical conditions

Differences between orifice and venturi meters: The differences between orifice and venturi meters are explained in Table 2-1.

Pitot Tube

Principle : Pitot tube consists of a sensing element with a small constriction compared to the size of the flow channel. When the sensing element is inserted at the centre of the stream, the velocity of flow is increased. This results in decreased pressure head. Tube at right angles to the flow measures pressure head only. The tube that pointed upstream measures pressure head and velocity head. The difference in the above readings indicates the velocity head.

Accordingly to Bernoulli's equation, velocity head of the fluid may be obtained using equation (33).

$$\Delta H_p = \frac{u^2}{2g} \qquad (36)$$

where u = velocity of the flow at the point of insertion, m/s
ΔH_p = difference in head from manometer, m

Therefore, the reading (R) of the manometer measures the velocity head in metres.

Construction : The construction of a pitot tube is shown in Figure 2-10. Pitot tube is also known as *insertion meter*. The size of the sensing element is small compared to the size of the flow channel. The point of measurement may be at the centre of the channel. One tube is perpendicular to the flow direction and the other tube is connected parallel to the direction of flow. The two tubes are connected to the legs of a manometer or a suitable device.

Working : Two tubes are inserted into the pipe in the manner shown in Figure 2-10. Pitot tube is used to measure the *velocity head* of the flow. In this tube, the velocity of fluid is increased at the narrow constriction. This results in decreased pressure. Tube at right angles to the flow measures pressure head only. The tube that points upstream measures pressure head and velocity head. The difference in the above readings indicates velocity head. Therefore, the reading (R) of the manometer measures the velocity head.

As seen in the Bernoulli's equation, velocity head of the fluid (ΔH_p) may be obtained from an equation similar to equation (29), which can be represented as:

$$u^2 = 2g.\Delta H_p$$

This is the theoretical velocity. Actual velocity is given by:

$$u^2 = C_v\sqrt{2g.\Delta H_p} \qquad (36)$$

where C_v = coefficient of pitot tube.

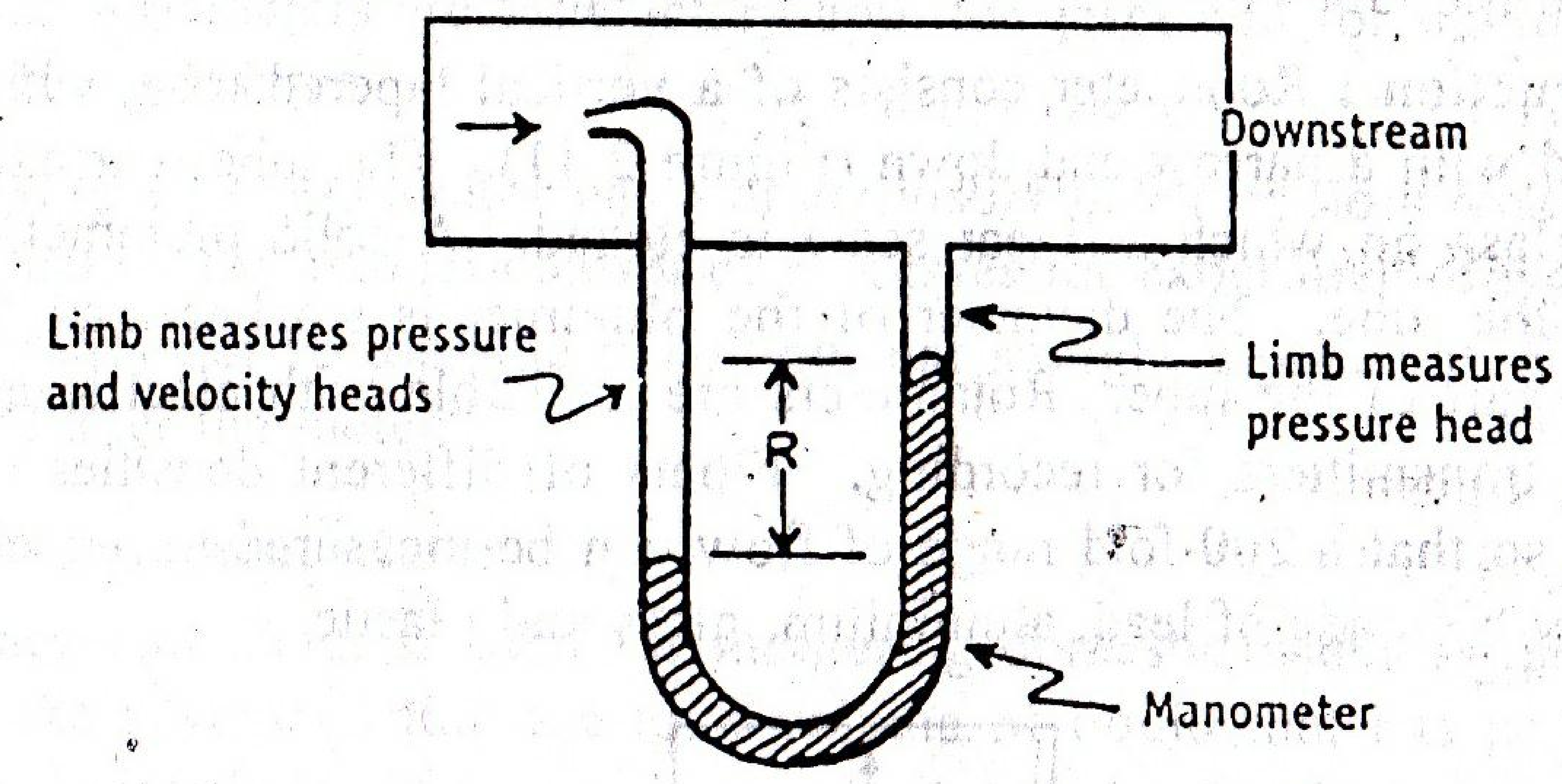


Figure 2-10. Construction and assembly of pitot tube.

Normally, pitot tube measures the velocity at one particular point, i.e., at the point of insertion. The average velocity (across the cross section of the pipe) may be obtained by either ways.

(a) Pitot tube may be inserted at the centre of the pipe and it measures the maximum velocity. Average velocity may be calculated from this maximum by means of calibrated charts.

(b) Adjustable pitot tube may be used to take readings from different points in the cross section. Mean velocity may be found by graphic integration. It is a difficult process.

Advantage : Pitot tube measures the velocity at one point only.

Disadvantages : (1) The pitot tubes themselves cause more disturbance. Eddies within the pressure tube disturb the readings.

(2) They do not give average velocity directly.

(3) For gases, the reading is extremely small. For gases working on low pressure, some form of multiplying gauges must be used.

Rotameter

Rotameter is a variable area meter, i.e., it measures the area of flow, so as to produce a constant head differential. Therefore, rotameters are known as *area meters*.

Principle : Rotameter consists of a vertical, tapered and transparent tube in which a plummet is placed. During the fluid flow through the tube, the plummet rises and falls because of variation in flow. As a result, the area of the annular space between the plummet and the tube varies. The head loss across the annulus is equal to the weight of the plummet. The upper edge of the plummet is used as an index to note the reading on the tapered tube. This value indicates the flow of the fluid.

Construction : Rotameter consists of a vertical tapered tube, which is mounted with a narrow end down (Figure 2-11). The tube is usually made of glass on which a linear scale is etched. A solid plummet is placed in the tube. The diameter of the plummet is smaller than the narrowest part of the tube. Rotameters are available with electric and electronic transmitters for recording. Floats of different densities are available, so that a 200-fold range of flow can be measured accurately. Floats may be made of lead, aluminium, glass and plastic.

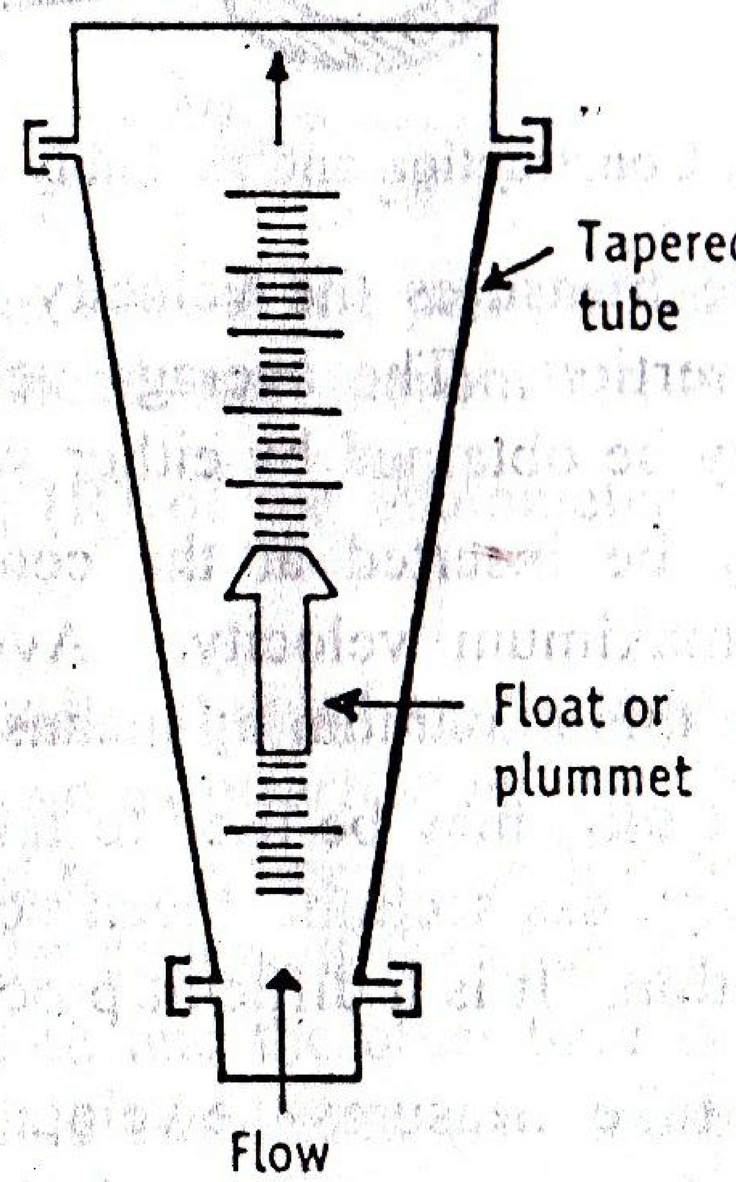


Figure 2-11. The Construction of a rotameter.

Working : As the flow is upward through a tapered tube, the flow of fluid varies. The plummet, which is surrounded by the fluid, rises and falls depending on the rate of flow. The greater the flow rate, the higher the plummet rises in the tube. In rotameters, the pressure drop is constant or nearly constant.

During the fluid flow, the area of the annular space between the plummet and the tube varies. Therefore, the head loss across the annulus is equal to the weight of the plummet. The flow may be read using the upper edge of the plummet as an index. The area is properly calibrated to the flow rate. The reading may be transmitted for recording, integrating and controlling.

Normally, manufacturers supply the necessary data and charts along with the meter while purchasing the instruments.

Uses : Rotameters are extensively used in chemical industries, such as bulk drugs. In the fermenters, the supply of air is controlled through rotameters. Rotameters are satisfactory both for gases and for liquids at high and low pressures.

Advantages : (1) Operator has a direct visual index of flow reading. It is satisfactory for manual control of processes for experimental work.

(2) It does not require the condition that straight pipes should run before and after the meter.

Direct Displacement Meter

Displacement meter is used for measuring domestic water supplies. This has the advantage that the total volume of liquid that has passed can be read directly.

Principle : Displacement meters are used for measuring the flow rate of fluids in small lines. In these meters, a stream of water enters the meter and strikes the moving member (disc as in disc meters or buckets as in current meters). The rate of rotation of the moving member is proportional to the velocity of water passing through the meter. The displacement of moving member is transmitted through a train of gears to the counting dial, which is present in the top of the member.

Advantages : The displacement meters have an advantage over venturi or orifice meters in the sense that the reading represents the total volume of fluid that has passed. This volume divided by a definite period gives the flow rate.

Glossary of Symbols

C_o = Constant for the orifice meter.
C_v = Constant for the pitot tube.
C_V = Constant for the venturi meter.
D = Diameter of the pipe, m
η = Viscosity of the liquid, Pa·s.
f = Friction factor.
g = Acceleration due to gravity, m/s^2.
h_l = Height of the liquid column at a point, m.
ΔH = Loss of head, m.
ΔH_e = Loss of head due to sudden enlargement, m.
ΔH_c = Loss of head due to sudden contraction, m.
ΔHp = Difference in head from manometer in pitot tube, m.

Δh = Height difference between two points, m.
L = Length of pipe, m.
P = Pressure, Pa.
P_s = Pressure on the column of a liquid, Pa.
ΔP_f = Pressure difference due to friction, Pa.
Re = Reynolds number.
ρ = Density of the liquid, kg/m^3.
S = Surface area of the liquid column, m^2.
u = Velocity of the fluid in pipes, m/s.
w = Energy head added by the pipe, J.
X = Height of the liquid pipe from horizontal datum, m.

QUESTION BANK

Each question carries 2 marks

1. Draw a labeled diagram of pitot tube and explain the working of the same.
2. What is the use of a pitot tube? Write its advantages and disadvantages.
3. Write the Fannings equation and explain the terms. What is its importance?
4. Express the Hagen-Poiseuille's relationship. What is its importance?
5. Explain the term 'head'. List the different heads in the Bernoulli's theorem.

What is meant by 'equivalent pipe length? What are its applications?

. What is Reynolds number? Describe its importance.

8. List the advantages and disadvantages of rotameter.
9. How are losses of energy due to enlargement in cross section measured? Give relevant equation and explain the terms.
10. Give Reynolds number and explain the symbols used therein.
11. What is a 'differential head meter'? Name some devices under this category.
12. What is a pressure head? How is it calculated?
13. What are vertical head meters? Describe one such meter.
14. Differentiate between constant pressure and constant area meters. Give examples of devices under each category.

Each question carries 5 marks

1. Describe the types of flow patterns exhibited by liquids in motion.
2. Differentiate between fluid statics and fluid dynamics. Name the fluid flow meter, which gives point velocity.
3. Write Bernoulli's equation and explain the symbols used therein with a labeled diagram.

4. Describe Reynolds classic experiment elucidating different types of flow patterns, when a liquid flows through a closed channel.
5. Explain the characteristics of different types of flow. Add a note on Reynolds number.
6. Compare and contrast the advantages and disadvantages of pitot tube and rotameter.
7. What are the merits and demerits of venturi meter over orifice meter?
8. What is the condition of hydrostatic equilibrium? Obtain the barometric equation.

Each question carries 10 marks

1. Deduce relevant equations for calculation of flow rates using orifice meter.
2. Derive Bernoulli's equation stating the assumptions.
3. Give a neat sketch of two fluid manometers and explain its working principle.
4. Explain the working, principle and construction of venturi meter. Write the expression for the volumetric flow rate of fluid through it.
5. Explain the energy losses that occur when a fluid flows through a pipe with relevant equations.