11

Probability

INTRODUCTION

The concept of probability which originated in the seventeenth century has become one of the most fascinating and debatable subjects in recent years. The probability formulae and techniques were developed by Jacob Bernoulli (1654-1705), De Moivre (1667-1754), Thomas Bayes (1702-1761) and Joseph Lagrange (1736-1813). In the nineteenth century, Pierre Simon, Laplace (1749-1827) unified all these early ideas and compiled the first general theory of probability. In fact, volumes have been written on probability and still the intellectual controversy concerning the foundations of probability theory is going on. So many people use the concept of probability in their daily lives without actually being aware of it. For example, we often find people making such statements as: 'It is likely that it may rain', 'We probably will get the contract', 'It is possible that the price of shares may go down further', etc. Though such assertions seem to be quite clear, a careful analysis would reveal that there are considerable difficulties in specifying the meaning of these statements.

In the beginning, the probability theory was successfully applied at the gambling tables. Gradually it was applied in the solution of social, economic, political and business problems. The insurance industry, which emerged in the 19th century, required precise knowledge about the risk of loss in order to calculate premiums. Within a few decades many learning centres were studying probability as a tool for understanding social phenomena. Today the concept of probability has assumed great importance and the mathematical theory of probability has become the basis for statistical applications in both social and decision-making research.

In fact, probability has become a part of our everyday lives. In personal and management decisions, we face uncertainty and use probability theory, whether or not we admit the use of something so sophisticated. We live in a world in which we are unable to forecast the future with complete certainty. Our need to cope with uncertainty leads us to the study and use of probability theory. In many instances, we, as concerned citizens, will have some knowledge about the possible outcomes of a decision. By organizing this information and considering it systematically, we will be able to recognise our assumptions, communicate our reasoning to others, and make a sounder decision than we could by using a shot-in-the-dark approach.

Probability constitutes the foundation of statistical theory and application. Knowledge of probabilistic methods has become increasingly essential in quantitative analysis of business and economic problems. In particular, probability theory is a basic component of the formal theory of decision-making under risk and uncertainty. Probability measures provide the decision-maker with the means for quantifying the uncertainties which affect his choice of appropriate actions. A through understanding of the fundamentals of probability theory will permit a businessman to deal with uncertainty in business situations in such a way that he can assess systematically the risks involved in each alternative, and consequently act to minimize risks.

Over the years, numerous definitions are given and we can classify these into different schools of thought. There are mainly four schools of thought on probability, namely:

1. The classical or *a priori* approach,
2. The relative frequency or empirical approach,
3. The axiomatic approach, and
4. The personalistic approach.

1. The Classical Approach

The classical or *a priori* approach happens to be the earliest. This school of thought assumes that all the possible outcomes of an experiment are mutually exclusive and equally likely. The words "equally likely" convey the notion of equally probable, and mutually exclusively means if one event occurs the other event will not occur, *i.e.*, the classicists believe that each outcome of an experiment has the same chance of appearing as any other and, therefore, can be assigned the same weight (probability) for its occurrence as any other. For example, when we toss a coin the probability of head is equal to the probability of a tail and is equal to 1/2. Similarly, each card drawn at random from well-shuffled deck of playing cards has the same chance to be drawn *i.e.*, 1 in 52 or 1/52, the probability of drawing a heart would be 13/52=1/4, the probability of drawing a black card 36/52=1/2, etc. Thus the classical concept defines the probability of an event as follows: If there are 'a' possible outcomes favourable to the occurrence of an event E, and 'b' possible outcomes unfavourable to the occurrence of E and all these possible outcomes are equally likely and mutually exclusive, then the probability that the event E will occur, denoted by $P(E)$, is

$$P(E) = \frac{a}{a+b} = \frac{\text{Number of outcomes favourable to the occurrence of event } E}{\text{Total number of outcomes}}$$

In the *a priori* method of measurement as well as in all other methods, the probability of an event E is a number such that $0 \leq P(E) \leq 1$, and the sum of the probability that an event will occur and the probability that it will not occur is equal to one.

The classical approach has two interesting characteristics: first, the subjects referred to as fair coins, fair deck of cards, true dice are abstractions in the sense that no real world object exactly possesses the features postulated. If a coin is unbalanced or there is a loaded die, the classical approach of assigning equal probability would offer us nothing but confusion. Secondly, in order to determine probabilities in the above examples, no coins had to be tossed, no cards shuffled, nor dice rolled, *i.e.*, no experimental data were required to be collected. The probability calculations were based entirely upon logical prior (thus, *a priori*) reasoning.

2. Relative Frequency Approach

While the classical theory is useful for solving problems which involve games of chance, it encounters serious difficulties in analysing a wide range of other types of problems. For example, it is inadequate for answering questions such as: What are the probabilities that (a) a man aged 45 will die within the next year, (b) a consumer in a certain metropolitan area will purchase a particular product during the next month, (c) a production process used by a particular firm will produce a defective item.

In none of these situations, it is feasible to establish a set of complete and mutually exclusive outcomes each of which is equally likely to occur. For example, in (a) there are only two possible occurrences, the individual will die during the ensuing year or he will live. The likelihood that he will die is of course, much smaller than he will live. How much smaller? This is the type of question that requires reference to empirical data.

The relative frequency theoreticians agree that the only valid procedure for determining event probabilities is through repetitive experiments. For example, when a coin is tossed, what is the probability that the coin will turn up heads? The relative frequency theorist would actually toss the coin and calculate the proportion of times our coin falls heads. Suppose he tosses the coin 50 times and it falls head 20 times, then the ratio 20/50 is used as the estimate of the probability of heads of this coin. It may be noted that even if the coin is perfect, one may not get exactly 25 heads out of tosses. In other words, it is not possible to obtain the true probability from repeated experiments. However, if the coin were perfect, the estimate would approach the true ratio (probability) as the number of trails increased. Thus, if the coin is tossed 200 times, we may have 85 heads (or 115 tails). The relative frequency becomes 85/200 = 0.425. If we further toss the coin 2,000 times, we may have 980 heads (or 1020 tails); the relative frequency being 980/2000 = 0.49, and so on. Since the probability of an event is determined objectively by repetitive empirical observations, the relative frequency theory is also called the objective or empirical definition of probability.

The ratio of the number of occurrences of an event to the number of possible occurrences in an experiment is referred to as the relative frequency. Two definitions of probability in terms of relative frequency can be given:

(*a*) If an experiment is performed n times under the same conditions and there are 'a' outcomes, $a \leq n$, favouring an event, then an estimate of the probability of that event is the ratio a/n.

(*b*) The estimate of probability of event, a/n approaches a limit, the true probability of the event, when n approaches infinity is given by

$$P(E) = \underset{n \to \infty}{Limit} \frac{a}{n}$$

It may be noted that we can never obtain the probability of an event as given by the above limit. In practice, we can only try to have a close estimate of $P(E)$ based on large n. However, this approach does emphasise that probability involves a long-run concept.

3. The Axiomatic Approach*

The classical approach restricts the calculation of probability to essentially equally likely and mutually exclusively events. The resolution of non-mutually exclusive events of reality into mutually exclusive subevents and the introduction of 'equal likelihood' among events which are essentially not so in reality are questions not clearly treated by the classicists. On the other hand, the empirical or relative frequency approach requires that every question of probabilistic nature be examined experimentally in the laboratory of the mathematician under identical conditions, and that too over a very long period of time, through the process of repeated observations, if estimates of the chances of occurrence of the events under consideration are required.

The axiomatic theory of probability is an honest attempt at constructing a theory of probability, largely free from the inadequacies of both the classical and empirical approaches, in the true mathematical tradition. It is true that the introduction of advanced logic through mathematical abstractions renders the complex real-world situation too idealised (or too simplified) to be of any immediate practical utility. But nonetheless it plays an important role in rendering a reasonable amount of comprehensibility and tractability to the understanding of myriad chance phenomena observed in nature, at least in the initial stages of any scientific inquiry into their structure and composition, where other approaches have at best left them less comprehensible and less tractable. Thus, the primary purpose of the development of an

*For details, see *Probability Theory* by M, Loeve Van Nostrand.

axiomatic theory lies in the fact that it makes available to the inquisitive mind a large body of abstract mathematical concepts, tools and techniques with which to identify, model, study and infer about real-world chance phenomena of interest. For a reasonably complete description of reality will not be 'complete' unless some amount of 'abstraction was made somewhere along the course of the inquiry, and models are nothing but abstractions of reality in some necessary degree.

4. The Personalistic Approach†

Though this approach to probability is relatively recent its application to statistical problems has occurred virtually entirely in the post-world World War II Period, particularly in connection with statistical decision theory. According to the personalistic or subjective concept, the probability of an event is the degree of confidence (or belief) placed in the occurrence of an event by a particular individual based on the evidence available to him. This evidence may consist of relative frequency to data and any other quantitative or qualitative information. According to the degree of belief for its possible occurrence, a subjectivist would assign a weight between 0 and 1 to an event. Thus, if one believes that it is very likely that the event will occur, he will assign it a probability close to one and if he believes that it is unlikely that the event will occur, he will assign a probability close to zero.

This approach is very broad and flexible one, permitting probability assignments to events for which there may be no objective data, or for which there may be a combination of objective and subjective data. The subjective approach grants that different reasonable individuals may differ in their degree of confidence even when offered the same evidence and consequently personal probabilities for the same event may differ in the eyes of different decision-makers.

Though broadly there are four different schools of thought on probability, there is hardly any disagreement on the foundation of probability at the mathematical level as each school defines probability as a ratio or proportion. Each viewpoint has its own merits and depending upon the problem under consideration one may use whichever approach is appropriate and convenient.

Elements of Set Theory

Modern approach to probability theory generally employs set theory and it will be used here for the development of some fundamental concepts and tools.

Sets. A set is any well-defined specified collection of distinct elements or objects. The objects which comprise the set are usually referred to as elements or members of the set and are said to belong to that set or to be contained in it. The set must be well specified or well defined in the sense that it must be possible at least in principle, to specify the set so that one can decide whether any given member does or does not belong to the set. The members of the set are distinct in the sense that repetition of elements is not permitted in specifying the set. The collection of aggregation or totality of elements is referred to simply as a set denoted by S. Thus the following collections are examples of sets :

The students enrolled in a university.

The books in a departmental library.

The employees of a company.

The citizens of India.

The possible outcomes of the roll of a single die.

A set is usualy described in either of the following two ways:

A roster or tabulation method, and the rule or defining property method.

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†The concept was first introduced in 1926 by Frank Ramsey who presented a formal theory of personal probability in his book: *The Foundation of Mathematics and Other Logical Essays* (London: Kegan Paul; New York: Harcourt Brace and World 1931).

Roster or Tabulation Method

The elements are usually enclosed within brackets. For example, the set consisting of the possible outcomes (Tail = T, Head = H) of single toss of a coin may be expressed as:

$$S = \{T, H\}$$

The set of possible outcomes of tossing two coins may be written as:

$$S = \{T, T), (T, H), (H, T), (H, H)\}$$

The order in which the elements of a set are listed is of no importance. It is important, however, that each element be listed only once. Note that in the second example there are four elements in the set, *viz.*, (T, T), (T, H), (H, T) and (H, H).

Rule or Defining Property Method

Sometimes it is helpful to have a brief and exact way to describe sets without listing elements. For example, the set of all university students may be expressed as :

$$S = \{x/x \text{ is a student in the university}\}$$

We read this as "*S* is the set of all *x* such that *x* is a student in the university."

Universal set

The universal set *U* is defined as that set consisting of all the elements under consideration. Thus if *A* is any set and *U* is the universal set, then every element in *A* must be in *U* (since it consists of elements under consideration).

Null Set

A set having no element at all is called a null or an empty set. The symbol used to denote it is a Greek letter ø (Phi).

Subset

If every element of a set *A* is also an element of a set *B*, then *A* is called a subset of *B*. For example, consider the set $A = (3, 5)$ and the set $B = (1, 2, 3, 4, 5)$. We note that every element in the set *A* is also an element of the set *B*. The set *A* is said to be the subset of *B*. Symbolically, we write this as $A \subset B$ read as *A* is contained in *B* or *A* is a subset of *B*.

Equal Sets

Two sets *A* and *B* are said to be equal if and only if every element of *A* is also an element of *B* and *vice versa.* Symbolically, $A = B$ if and ony if $A \subset B$ and $B \subset A$.

Set Operations

We shall now consider certain operations on sets that will result in the formation of new sets.

Intersection of Sets

The intersection of two sets *A* and *B* is the set of elements that are common to both *A* and *B*. Symbolically, the intersection of *A* and *B* is written as $A \cap B = \{x / x \in A \text{ and } x \in B\}$. In the following diagram, the shaded area corresponds to the intersection of sets *A* and *B*. *U* is the universal set.

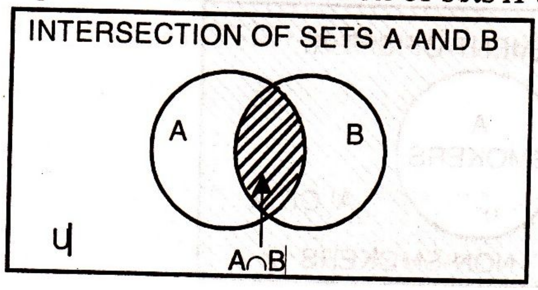


Illustration 1. Consider the sets of numbers :
$U = \{x/x \text{ is positive integer}\}$
$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$B = \{8, 9, 10, 11, 12, 13, 14\}$
Then $A \cap B = \{8, 9, 10\}$.
Since only these elements appear in both A and B.

Disjoint sets

Two sets A and B are called disjoint if they do not intersect. This can be expressed as $A \cap B = ø$ where ø is a null set. When the two sets do not intersect, they are said to be disjoint or mutually exclusive. These sets are shown below in the diagram.

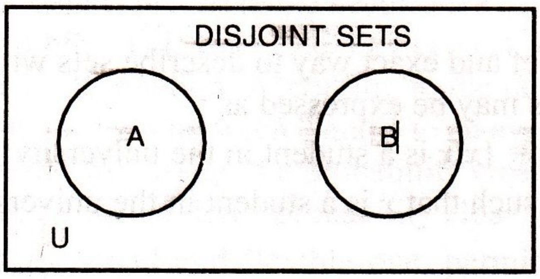


Illustration 2. Consider the sets of numbers :
$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$A = \{1, 2, 3\}$
$B = \{5, 6\}$
Note that A and B do not intersect. This can be expressed as $A \cap B = \phi$.

Union of sets

The union of two sets A and B is the set of elements that belong either to A or B or both. this is expressed as $A \cup B = \{x / x \in A \text{ or } x \in B\}$. The union of two sets sometimes is expressed as the logical sum of the two sets. In the following diagram, the area representing the elements of the set $A \cup B$ has been shaded.

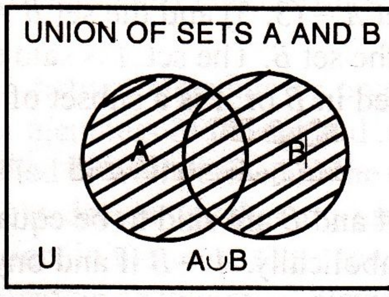
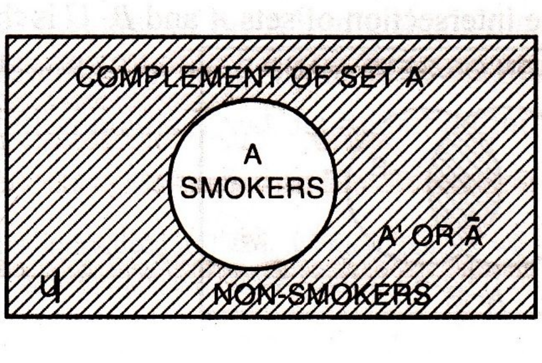


Illustration 3. Consider the set of numbers :
$U = \{x/x \text{ is a positive integer}\}$
$A = \{1, 3, 5\}$
$B = \{3, 4, 5, 6\}$
Then $A \cup B = \{1, 3, 4, 5, 6\}$
Since these elements appear in either A or B or both.

Complement of a Set

If A is a subset of the universal set U, then the complement of A with respect to U is the set of all elements of U that are not in A or the complement of set A is the set of all elements that do not belong to A and is denoted by A' or $\bar{A}$. In symbols, $A' = [x/x \in U \text{ and } x \notin A]$. Suppose we consider the employees of a firm as the universal set. Let all the smokers form a subset. Then all the non-smokers also form a subset which is called the complement of the set constituting smokers. In the following diagram, the area representing the complement of A has been shaded.



Here A and A' do not intersect—that is, $A \cap A' = \phi$. Hence A and A' are mutually exclusive. Another characteristic of this case is that $A \cup A' = U$. Thus A and A' are also completely exhaustive.

Illustration 4. Let $U = \{1, 2, 3, 4\}$
$A = \{1, 2, 3\}$
then $A' = \{4\}$

Difference of Two Sets

The difference of sets A and B is defined as $A - B = (x/x \in A$ and $x \notin B)$. This is shown below as the shaded area.

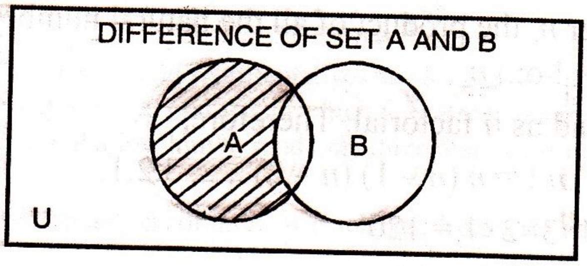


Illustration 5. Let $A = \{1, 3, 5, 7, 9, 11, 13\}$
$B = \{5, 9, 13, 17\}$
then $A - B = \{1, 3, 7, 11\}$

The following illustration will explain the use of different set operations :

Illustration 6. A firm has 231 employees classified by age and job category as follows :

Job	*Category*	*Age Category* A_1	A_2	A_3	A_4	A_5	*Total*
		≤ 20	21–25	26–30	31–35	>35	
B_1	Peons	20	20	15	10	5	70
B_2	Clerks	3	6	3	2	1	15
B_3	Draftsmen	15	30	35	20	10	110
B_4	Salesmen	1	5	10	5	2	23
B_5	Junior Executives	0	1	5	2	0	8
B_6	Executives	0	0	2	2	1	5
	Total	39	62	70	41	19	231

Based on the above table, explain in words the following sets and give the number of employees in each set:

(*a*) $B_1 \cap A_5$
(*b*) $A_2 \cap B_6$
(*c*) $B_4 \cap A_5$
(*d*) $A_1 \cup B_6$
(*e*) $A_3 \cup A_5$
(*f*) $B_2 \cup B_3$
(*g*) A_4'
(*h*) $(A_1 \cup A_2) \cap B_3$
(*i*) $(B_3 \cup B_4) \cap A_5$

Solution. (*a*) $B_1 \cap A_5$ gives us the intersection between peons and age greater than 35, *i.e.*, those peons who are more than the age 35. From the table this gives us the value 5. Hence $B_1 \cap A_5 = 5$.

(*b*) Similarly, $A_2 \cap B_6$ means the intersection between the age group 21–25 and that of executives, *i.e.*, the executives who are in the age group 21–25. From the table this value is 0. Hence $A_2 \cap B_6 = 0$.

(*c*) Similarly, $B_4 \cap A_5 = 2$.

(*d*) $A_1 \cup B_6$ gives the union between age less than 20 and executives, *i.e.*, either in the category of age less than 20 and executives or both, therefore, $A_1 \cup B_6 = 39 + 5 = 44$.

(*e*) Similarly, $A_3 \cup A_5 = 70 + 19 = 89$.

(*f*) Similarly, $B_2 \cup B_3 = 15 + 110 = 125$.

(*g*) A'_4 means not contained in A_4, *i.e.*, all those except the set A_4 containing 41 employees. Hence $A_4 = 231 - 41 = 190$.

(*h*) $(A_1 \cup A_2) \cap B_3$ gives us the union of A_1 and A_2 first then its intersection with B_3, *i.e.*, $(A_1 \cup A_2) \cap B_3 = 15 + 30 = 45$.

(*i*) Similarly, $(B_2 \cup B_4) \cap A_5 = 10 + 2 = 12$.

Counting Techniques

In computing the probability of an event, or the probability of a combination of events, when the total number of possible events is large, it will be convenient to have available some methods for counting the number of such events. In this section, some techniques to facilitate the counting of events will be presented. These are useful for counting number of events comprising the numerator and/or the denominator of a probability.

Factorials

Given the postive integer *n,* the product of all the natural numbers from *n* down through 1 is called *n* factorial and is written as *n* ! or $\underline{|n}$.

The expression *n* ! is read as *n* factorial. Therefore,

$$n! = n(n-1)(n-2) \ldots\ldots 3.2.1. \qquad \ldots(i)$$

For example : $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

From (*i*), we get $n! = n(n-1)!$ and $0! = 1$*

Permutations

A permutation of a number of objects is an arrangement of these objects in a definite order. The number of permutations of a set of *n* objects, taken all together is *n* !. We have, in permutation, *n* spaces to fill. The first space can be filled with any one of the *n* objects and so in *n* ways. After this has been done (in any of the *n* ways), the second space can be filled with any of the remaining $(n-1)$ objects; likewise, the third place can be filled in $(n-2)$ ways, the fourth space in $(n-3)$ ways and so forth. Therefore, the number of ways of filling *n* spaces is

$$n(n-1)(n-2) \ldots\ldots 3.2.1.$$

which is nothing but *n* !.

Denoting this by ${}^nP_n$, we have

${}^nP_n = n!$

Illustration 7. There are four clerks in an office whose tables are arranged in a line against a wall. How many seating arrangements are possible if each clerk can sit at any table?

Solution. $4! = 4.3.2.1 = 24$ ways.

The total number of arrangements of *n* objects taken *r* at a time with $r \le n$ denoted as ${}^nP_r$, is

$${}^nP_r = \frac{n!}{(n-r)!}$$

The permutation of *n* objects taken *r* at a time can also be denoted by the following symbols

$$P(n,r),\ {}_nP_r,\ P_{n,r},\ P^n{}_r$$

Illustration 8. A personnel manager has received requisitions for one typist each from the Production department, Marketing department and Research department. There are seven applicants available from which these three positions may be filled. In how many ways three typists be selected from the seven applicants and assigned to the three different openings?

Solution. There are seven ways to fill the first position after which there are six ways to fill the second position after which there are five ways to fill the third position. This gives $7 \times 6 \times 5 = 210$ ways. The same result can be obtained using the formula

$${}^7P_3 = \frac{7!}{(7-3)!} = \frac{7 \times 6 \times 5 \times 4!}{4!} = 7 \times 6 \times 5 = 210.$$

*Note that $n! = n(n-1)!$ or $(n-1)! = \frac{n!}{n}$

Letting $n = 1$, $0! = 1$.

Combinations

A combination of number of objects is a selection of these objects, considered without regard to their order. The total number of combinations of a set of n objects taken r at a time $(r \leq n)$, usually denoted by ^nC_r and is given by

$$^nC_r = \frac{n!}{r!(n-r)!}$$

This can also be denoted by the following symbols.

$$\binom{n}{r},\ {}_nC_r,\ C_{n,r},\ C^n{}_r$$

Illustration 9. A sales manager has seven field representatives working under him. A local consulting firm at a fee of Rs. 1500 per man, is conducting a three-day seminar on sales to which the sales manager would like to send all the seven of his field representatives. However, his budget will allow him to send only three men. How many different ways are there for him to compose this group of three men?

Solution. The number of possible combinations of three men selected from a set of seven men is

$$^7C_3 = \frac{7!}{4!3!} = \frac{7.6.5.4!}{4!3.2.1} = 35.$$

Note: *It is important to note that in a permutation, order counts; in a combination, order does not count.*

Illustration 10. *Out of 5 mathematicians and 7 statisticians, a committee consisting of 2 mathematicians and 3 statisticians* is to be formed. In how many ways can this be done if (*a*) any mathematician and any statistician can be included (*b*) one particular statistician must be on the committee, (*c*) particular mathematician cannot be on the committee?

Solution. (*a*) 2 mathematicians out of 5 can be selected in $^5C_2$ ways.

3 statisticians out of 7 can be selected in $^7C_3$ ways.

Total number of possible selection = $^5C_2 \times {}^7C_3 = 10 \times 35 = 350$

(*b*) 2 mathematicians out of 5 can be selected in $^5C_2$ ways.

2 additional statisticians out of 6 can be selected in $^6C_2$ ways.

Total number of possible selections = $^5C_2 \times {}^6C_2 = 10 \times 15 = 150.$

(*c*) 2 mathematicians out of 4 can be selected in $^4C_2$ ways.

3 statisticians out of 7 can be selected in $^7C_3$ ways.

Total number of possible selections = $^4C_2 \times {}^7C_3 = 3 \times 35 = 105.$

Random Experiment

A random experiment is a well-defined process of observing a given chance phenomena through a series of trials (finite or infinite) each of which leads to a single outcome.

Observation of chance phenomena is called random experiment so as to distinguish them from experiments under control conditions, for example in a physical laboratory.

Events

And event is a possible outcome of an experiment or a result of a trial or an observation.

Elementary Events

An elementary event or a simple event is a single possible outcome of an experiment. It is thus an event which cannot be further subdivided into a combination of other events.

Compound Events

When two or more events occur in connection with each other, then their simultaneous occurrence is called a compound event. The compound event is an aggregate of simple events.

Mutually Exclusive Events

Two events are said to be mutually exclusive when both cannot happen simultaneously in a single trial or, in other words, the happening of one prevents the happening of the other and *vice versa.* For example, if a single coin is tossed either head can be up or tail can be up, both cannot be up at the same time. Similarly, a person may be either alive or dead at a certain time, he cannot be both alive as well as dead at the same time.

Collectively Exhaustive Events

In the example of fair coin tossing, there are two possible outcomes: head and tail. The list of these outcomes is collectively exhaustive since the result of any toss must be either head or tail. Collective exhaustive events are those which include all possible outcomes. The sum of the probabilities must be one for mutually exclusive and collectively exhaustive events.

Complementary Events

Let A be an event of the number of favourable cases in the experiment, then $\overline{A}$ called the complementary event of A is the number of nonfavourable cases in the experiment. Clearly the events A and $\overline{A}$ are mutually exclusive and collectively exhaustive.

Equally likely Events

Events are said to be equally likely when one does not occur more often than the others. For example, if an unbiased coin or die is thrown, each face may be expected to be observed approximately the same number of times in the long run. Similarly, the cards of a pack of playing cards are so closely alike that we expect each card to appear equally often when a large number of draws are made with replacement. Random number tables are based on this concept.

PROBABILITY LAWS

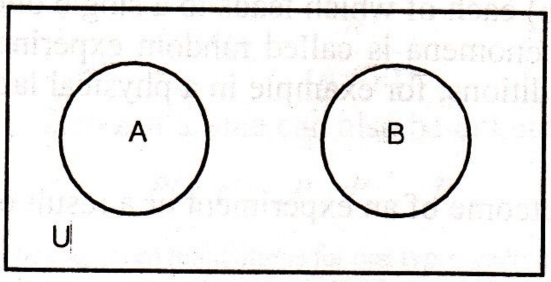
There are several laws that can ease our task of computing probabilities. In this section, we shall discuss two of the fundamental laws of computing probabilities, *viz.*, Addition law and Multiplication law.

Addition Law

The probability of occurrence of either event A or event B of two mutually exclusive (or disjoint sets) events is equal to the sum of their individual probabilities. Symbolically, we may write,

$$P(A \cup B) = P(A) + P(B)$$

DISJOINT EVENTS



Since A and B can be written as a union of simple events in which no simple event of B appears in A, hence, the result follows.

If two events A and B are not mutually exclusive (joint events) then the addition law can be stated as follows:

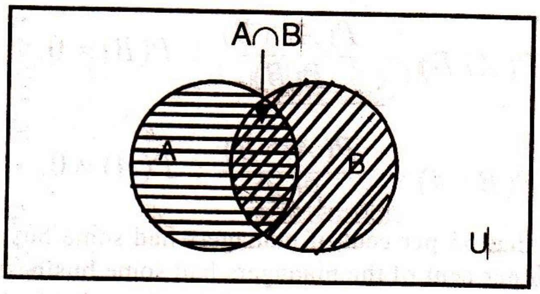
The probability of the occurrence of either event A or event B or both is equal to the probability that event A occurs, plus the probability that event B occurs minus the probability that both events occur. Symbolically, it can be written as

$$P[A \cup B] = P[A] + P[B] - P[A \cap B]$$

Proof. $$P[A \cup B] = \frac{n(A \cup B)}{n(U)}$$

where $n(A \cup B)$ indicates the number of elements belonging to $A \cup B$, and $n(U)$ is the total number of elements in the universal set U.

JOINT EVENTS



$$P(A \cup b) = \frac{n(A) + n(B) - n(A \cap B)}{n(U)}$$ [By adding $n(A)$ and $n(B)$, we count twice $(A \cap B)$. See diagram above.]

$$= \frac{n(A)}{n(U)} + \frac{n(B)}{n(U)} - \frac{n(A \cap B)}{n(U)}$$

$$= P(A) + P(B) - P(A \cap B).$$

Illustration 11. City residents were surveyed recently to determine readership of newspapers available. 50% of the residents read the morning paper, 60% read the evening paper, and 20% read both newspapers. Find the probability that a resident selected reads either the morning or evening paper or both the papers.

Solution. Let A and B represent the events that the resident read morning and evening paper respectively.

Then $$P(A) = 0.50;\ P(B) = 0.60; \text{ and } P(A \cap B) = 0.20$$

The probability that the resident reads either the morning or evening or both the papers is given by :

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.50 + 0.60 - 0.20 = 0.90.$$

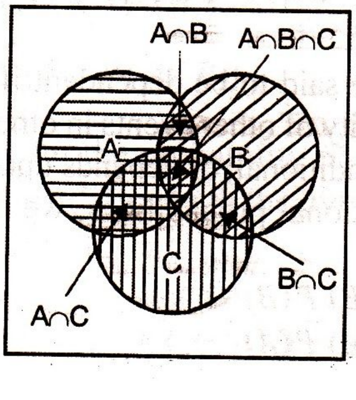
Generalisation

The addition law for mutually exclusive events can be extended to cover any number of events. In particular, if A, B and C are three mutually exclusive events, then the probability that any one of these events will occur is given by

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

When the events are not mutually exclusive, then the formula becomes

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$



Conditional Probability

When we are dealing with probabilities of a subset rather than of the whole set, our attention is focused on the probability of an event in a subset of the whole set. Probabilities associated with the events defined on the subsets are called conditional probabilities. The conditional probability of A, given B, is equal to the probability of $A \cap B$ divided by the probability of B, provided that the probability of B is not zero. Symbolically, we may write this as

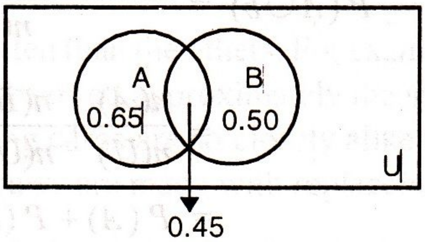
$$P(A/B) = \frac{P(A \cap B)}{P(B)} \; ; \; P(B) \neq 0.$$

Similarly $$P(B/A) = \frac{P(A \cap B)}{P(A)} \; ; \; P(A) \neq 0.$$

Illustration 12. A study showed that 65 per cent of managers had some business education and 50 per cent had some engineering education. Furthermore, 20 per cent of the managers had some business education but no engineering education. What is the probability that a manager has some business education, given that he has some engineering education?

Solution. Let A denote the event that the manager has some business education and B denote that he has some engineering education.

Then $$P(A) = 0.65,\ P(B) = 0.50,\ P(A \cap B) = 0.45$$



Therefore $$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.45}{0.50} = 0.9$$

Hence the required probability that a manager has some business education given that he has engineering education is 0.9.

Multiplication Law

The multiplication law may be stated as follows :

The probability of the joint occurrence of event A and event B is equal to the conditional probability of A given B, times the probability of B.

Symbolically, we write

$$P(A \cap B) = P(A/B) \times P(B)$$

or $$P(B \cap A) = P(B/A) \times P(A)$$

Proof.

$$P(A/B) = \frac{n(A \cap B)}{n(B)} = \frac{n(A \cap B)/n(U)}{n(B)/n(U)} = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = P(A/B) \,.\, P(B).$$

Generalisation. The multiplication law can be extended for more than two events. If we have three events A, B and C which are not mutually exclusive then the formula becomes

$$P(A \cap B \cap C) = P(A) \,.\, P(B/A) \,.\, P(C/A \cap B)$$

For n events $A_1, A_2, \ldots\ldots\ldots, A_n$, the formula becomes

$$P(A_1 \cap A_2 \cap A_3 \cap \ldots\ldots \cap A_n) = P(A_1)\, P(A_2/A_1)\, P(A_3/A_1 \cap A_2 \ldots\ldots) \ldots\ldots\ldots P(A_n/A_1 \cap A_2 \cap \ldots\ldots \cap A_{n-1})$$

Dependent Events. Two events are said to be dependent if the occurrence or non-occurrence of one event in any trial affects the probability of other events in other trials. Thus, in the case of dependent events, the probability of any event is conditional, or depends upon the occurrence or non-occurrence of other events. From definitions of conditional probabilities, we can see that if A and B are dependent events,

$$P(A \cap B) = P(A/B)\, P(B) \quad \ldots(i)$$

or $$P(B \cap A) = P(B/A)\, P(A) \quad \ldots(ii)$$

The order is of no significance in the intersection of two events, since $A \cap B = B \cap A$. Therefore, we get an important property of intersection, *viz.*,

$P(A \cap B) = P(A/B) \cdot P(B) = P(B/A) \cdot P(A)$

Independent Events. Two events are said to be independent, if the probability of the occurrence of one event will not affect the probability of the occurrence of the second event. Independent events are those events whose probabilities are in no way affected by the occurrence of any other event preceding, following or occurring at the same time.

Two events A and B are said to be independent if and only if

$$P(A \cap B) = P(A)\, P(B)$$

which implies from (*i*) and (*ii*), that

$$P(A/B) = P(A)$$

and

$$P(B/A) = P(B)$$

Illustration 13. A candidate is selected for interview of management trainees for 3 companies. For the first company there are 12 candidates, for the second there are 15 candidates and for the third there are 10 candidates. What are the chances of his getting job at least at one of the company?

Solution. The probability that the candidate gets the job at least at one company = 1– probability that the candidates does not get the job in any company.

Probability that the candidate does not get the job in the first company

$$= 1 - \frac{1}{12} = \frac{11}{12}$$

Probability that the candidate does not get the job in the second company

$$= 1 - \frac{1}{15} = \frac{14}{15}$$

Probability that the candidate does not get the job in the third company

$$= 1 - \frac{1}{10} = \frac{9}{10}$$

Since the events are independent, therefore, the probability that the candidate does not get any job in any of the three companies

$$= \frac{11}{12} \times \frac{14}{15} \times \frac{9}{10} = \frac{231}{300} = 0.77$$

Hence the required probability = 1– 0.77 = 0.23.

Bayes' Theorem

It is associated with the name of Thomas Bayes (1702-1761) and is a theorem on probability, concerned with a method of estimating the probabilities of the causes by which an observed event may have been produced. This theorem may be stated as follows :

Let $B_1, B_2, \ldots\ldots, B_n$, be n mutually exclusive events whose union is the universe, and let A be an arbitrary event in the universe, such that $P(A) \neq 0$. Given that $P(A/B_i)$, and $P(B_i)$ $(i = 1, \ldots.., n)$ are known.

$$P(B_j/A) = \frac{P(A/B_j)\, P(B_j)}{\sum_{i=1}^{n} P(B_j)\, P(A/B_j)} \quad \text{for } j = 1, \ldots\ldots, n.$$

This equation is called the formula for the probability of 'Causes', since it enables one to find the probability of a particular B_j, or 'Cause' by which the event A may have been brought about. It is sometimes written in another form as follows :

$$P(B_j/A) = \frac{P(A \cap B_j)}{P(A \cap B_1) + P(A \cap B_2) + \cdots + P(A \cap B_n)}$$

The Bayes' theorem is frequently used as a mechanism for revising the probability of an event after observing information about a process. The initial and revised probabilities are referred to as prior and posterior probabilities respectively.

Illustration 14. In a post office, three clerks are assigned to process incoming mail. The first clerk, B_1, processes 40 per cent, the second clerk, B_2, processes 35 per cent and the third clerk, B_3, processes 25 per cent of the mail. The first clerk has an error rate of 0.04, the second has an error rate of 0.06 and the third has an error rate of 0.03. A mail selected at random from a day's output is found to have an error. The Post Master wishes to know the probability that the mail was processed by the first, second, or third clerk, respectively.

Solution. Let A denote the event that a mail containing an error is selected at random and B_1, B_2 and B_3 be the event that the mail was processed by the first, second and third clerk respectively. Using our usual notation, we want to compute the conditional probabilities :

$$P(B_1 | A),\ P(B_2 | A),\ P(B_3 | A)$$

From the information given, we have

$$P(B_1) = 0.40,\ P(B_2) = 0.35 \text{ and } P(B_3) = 0.25.$$

These probabilities, which can be obtained without additional information are called prior probabilities. We are also given the information that the conditional probabilities observing a record with an error, given that it was processed by one of the three clerks are :

$P(A/B_1) = 0.04$, $P(A/B_2) = 0.06$ and $P(A/B_3) = 0.03$.

From these probabilities, we can calculate joint probabilities :

$P(A \cap B_1) = P(A/B_1)\ P(B_1) = 0.04 \times 0.40 = 0.016$

$P(A \cap B_2) = P(A/B_2)\ P(B_2) = 0.06 \times 0.35 = 0.021$

$P(A \cap B_3) = P(A/B_3)\ P(B_3) = 0.03 \times 0.25 = 0.0075$

Use Bayes' formula to obtain the desired probabilities.

$$P(B_1/A) = \frac{P(A \cap B_1)}{P(A \cap B_1) + P(A \cap B_2) + P(A \cap B_3)}$$

$$= \frac{0.016}{(0.016 + 0.021 + 0.0075)} = \frac{0.016}{0.0445} = 0.36$$

Similarly, $$P(B_2/A) = \frac{P(A \cap B_2)}{0.0445} = \frac{0.021}{0.0445} = 0.47.$$

$$P(B_3/A) = \frac{P(A \cap B_3)}{0.0445} = \frac{0.0075}{0.0445} = 0.17.$$

These probabilities are called posterior probabilities because they were calculated after it was known that the mail was one containing an error.

MISCELLANEOUS ILLUSTRATIONS

Illustration 15. The probability that a contractor will get a plumbing contract is 2/3 and the probability that he will not get an electric contract is 5/9. If the probability of getting at least one contract is 4/5, what is the probability that he will get both?

Solution. Let A and B denote the event that the contractor will get a plumbing and electric contract respectively.

Therefore $$P(A) = \frac{2}{3};\quad P(B) = 1 - \frac{5}{9} = \frac{4}{9};\quad P(A \cup B) = \frac{4}{5}$$

Hence $$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= \frac{2}{3} + \frac{4}{9} - \frac{4}{5} = \frac{14}{45} = 0.31.$$

The required probability that the contractor will get both the contracts is given by 0.31.

Illustration 16. The probability that a management trainee will remain with a company is 0.60. The probability that an employee earns more than Rs. 50,000 per month is 0.50. The probability that an employee is a management trainee who remained with the company or who earns more than Rs. 50,000 per month is 0.70. What is the probability that an employee earns more than Rs. 50,000 per month, given that he is a management trainee who stayed with the company?

Solution. Let A = An employee who earns more than Rs. 50,000 per month.

B = A management trainee who stayed with the company.

Then $P(A) = 0.50; P(B) = 0.60; P(A \cup B) = 0.70$

To get the value of $P(A \cap B)$, we can use the following formula

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Substituting these values, we have

$$0.70 = 0.50 + 0.60 - P(A \cap B) \text{ or } P(A \cap B) = 0.40$$

Therefore, $$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.40}{0.60} = \frac{2}{3} = 0.67.$$

Hence the probability that an employee earns more than Rs. 50,000 per month given he is a management trainee is 0.67.

Illustration 17. The Human Resource department of a company has records which show the following analysis of its 200 engineers.

Age	*Bachelor's degree only*	*Master's degree*	*Total*
Under 30	90	10	100
30 to 40	20	30	50
Over 40	40	10	50
Total	150	50	200

If one engineer is selected at random from the company, find :

(*a*) the probability he has only a bachelor's degree.

(*b*) the probability he has a master's degree, given that he is over 40.

(*c*) the probability he is under 30, given that he has only a bachelor's degree.

Solution. Let us define the events A, B, C and D as follows :

A : An engineer is under 30 years of age.

B : An engineer is over 40 years of age.

C : An engineer has bachelor's degree only.

D : An engineer has a master's degree.

(*a*) The probability of an engineer who has a bachelor's degree only is

$$P(C) = \frac{150}{200} = 0.75.$$

(*b*) The probability of an engineer who has a master's degree, given that he is over 40 years is

$$P(D/B) = \frac{P(D \cap B)}{P(B)} = \frac{10/200}{50/200} = \frac{10}{50} = 0.20$$

(*c*) The probability of an engineer who is under 30 years, given he has only a bachelor's degree is

$$P(A/C) = \frac{P(A \cap C)}{P(C)} = \frac{90/200}{150/200} = \frac{90}{150} = 0.60$$

Illustration 18. An *MBA* applies for a job in two firms X and Y. The probability of his being selected in from X is 0.7 and being rejected at Y is 0.5. The probability of at least one of his applications being rejected is 0.6. What is the probability that he will be selected in one of the firms?

Solution. $P(A) = 0.7; P(\overline{A}) = 1 - 0.7 = 0.3$

$P(B) = 0.5; P(\overline{B}) = 1 - 0.5 = 0.5; P(\overline{A} \cup \overline{B}) = 0.6$

The probability that he will be selected in one of the firms is obtained by using addition rule.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

But $P(A \cap B) = 1 - P(\overline{A} \cup \overline{B}) = 1 - 0.6 = 0.4$

Hence $P(A \cup B) = 0.7 + 0.5 - 0.4 = 0.8.$

The probability of his being selected in one of the firms is 0.8.

Illustration 19. A problem in business statistics is given to five students : A, B, C, D and E. Their chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ and $\frac{1}{6}$. What is the probability that the problem will be solved?

(*MBA, Kumaon Univ., 2000*)

Solution. Let E_1, E_2, E_3, E_4 and E_5 denote the events that the problem is solved by *A*, *B*, *C*, *D* and *E* respectively. Then we have :

$$P(E_1) = \frac{1}{2}; \qquad P(\bar{E}_1) = 1 - P(E_1) = \frac{1}{2}$$

$$P(E_2) = \frac{1}{3}; \qquad P(\bar{E}_2) = 1 - P(E_2) = \frac{2}{3}$$

$$P(E_3) = \frac{1}{4}; \qquad P(\bar{E}_3) = 1 - \frac{1}{4} = \frac{3}{4}$$

$$P(E_4) = \frac{1}{5}; \qquad P(\bar{E}_4) = 1 - \frac{1}{5} = \frac{4}{5}$$

$$P(E_5) = \frac{1}{6}; \qquad P(\bar{E}_5) = 1 - \frac{1}{6} = \frac{5}{6}$$

Since E_1, E_2, E_3, E_4 and E_5 are independent, the probability that all the five students fail to solve the problem is given by

$$P(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3 \cap \bar{E}_4 \cap \bar{E}_5) = P(\bar{E}_1)\,P(\bar{E}_2)\,P(\bar{E}_3)\,P(\bar{E}_4)\,P(\bar{E}_5)$$

$$= \frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\cdot\frac{4}{5}\cdot\frac{5}{6} = \frac{1}{6}.$$

Therefore, the required probability that the problem is solved

$$= 1 - \frac{1}{6} = \frac{5}{6} = 0.83.$$

Illustration 20. The odds that *A* speaks the truth is 3 : 2 and the odds that *B* speaks the truth is 5 : 3. In what percentage of cases are they likely to contradict each other on an identical point? *(MBA, DU, 1999)*

Solution. The probability that *A* speaks the truth = $P(A) = 3/5$

The probability that *B* speaks the truth = $P(B) = 5/8$

The probability that *A* tells a lie = $P(\bar{A}) = 2/5$

The probability that *B* tells a lie = $P(\bar{B}) = 3/8$

The probability that *A* speaks the truth and *B* tells a lie is given by

$$P(A)\,P(\bar{B}) = \frac{3}{5} \times \frac{3}{8} = \frac{9}{40}$$

The probability that *B* speaks the truth and *A* tells a lie is given by :

$$P(B)\,P(\bar{A}) = \frac{5}{8} \times \frac{2}{5} = \frac{10}{40}$$

The required probability = $P(A)\,P(\bar{B}) + P(B)\,P(\bar{A}) = \frac{9}{40} + \frac{10}{40} = \frac{19}{40}$

Hence percentage of cases in which they contradict each other is

$$= \frac{19}{40} \times 100 = 47.5\%$$

Illustration 21. In a certain town, male and female each form 50 per cent of the population. It is known that 20 per cent of the males and 5 per cent of the females are unemployed. A research student studying the employment situation selects an unemployed person at randos. What is the probability that the person so selected is (*a*) male (*b*) female?

Solution. The problem gives us the following probabilities as shown in the table below :

UNEMPLOYMENT DATA

	Unemployed	*Employed*	*Total*
Males	0.100	0.400	0.50
Females	0.025	0.475	0.50
Total	0.125	0.875	1.00

(*a*) $P[\text{Male/Unemployed}] = P(M/U) = \dfrac{p(M \cap U)}{P(U)} = \dfrac{0.10}{0.125} = 0.8$

(*b*) $P[\text{Female/ Unemployed}] = P(F/U) = \dfrac{p(F \cap U)}{P(U)} = \dfrac{0.025}{0.125} = 0.2$

Aliter

This illustration can also be solved by using Bayes' Theorem as shown below :

Given : $P(M) = 0.5, P(F) = 0.5, P(U/M) = 0.2, P(U/F) = 0.05$

(*i*) $$P(M/U) = \frac{P(U/M) \,.\, P(M)}{P(U/M) \,.\, P(M) + P(U/F) \,.\, P(F)}$$

$$= \frac{0.2 \times 0.5}{0.2 \times 0.5 + 0.05 \times 0.5} = \frac{0.10}{0.125} = 0.8$$

Thus the probability that the unemployed person selected being a male is 0.8.

(*ii*) $$P(F/U) = \frac{P(U/F) \,.\, P(F)}{P(U/M) \,.\, P(M) + P(U/F) \,.\, P(F)}$$

$$= \frac{0.05 \times 0.5}{0.2 \times 0.5 + 0.05 \times 0.5} = \frac{0.025}{0.125} = 0.2$$

Thus the probability that the unemployed person selected being a female is 0.2.

Illustration 22. A piece of equipment will function only when all the three components *A*, *B* and *C* are working. The probability of *A* failing during one year is 0.15, that of *B* failing is 0.05 and that of *C* failing is 0.10. What is the probability that the equipment will fail before the end of the year ?

Solution. The probability that component *A* does not fail during the year = 0.85

The probability that component *B* does not fail during the year = 0.95

The probability that component *C* does not fail during the year = 0.90.

Since the events are independent, therefore, the probability that all the three components do not fail = $0.85 \times 0.95 \times 0.90 = 0.727$.

Hence the probability that the equipment will fail before the end of the year

$$= 1 - 0.727 = 0.273$$

Illustration 23. Two computers *A* and *B* are to be marketed. A salesman who is assigned the job of finding customers for them has 60% and 40% chances respectively of succeeding in case of computer *A* and *B*. The computers can be sold independently. Given that he was able to sell at least one computer, what is the probability that computer *A* has been sold ?

(MBA, IGNOU, 2002; MBA, DU, 2002, 2006)

Solution. Let event *A* and *B* denote that the computer *A* and *B* are sold respectively,

Then, $P(A) = 0.60; \;\; P(B) = 0.40$

and $P(A \cap B) = P(A) \,.\, P(B) = 0.60 \times 0.40 = 0.24$ [Independent events]

Probability of selling at least one computer is given by

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.60 + 0.40 - 0.24 = 0.76$$

We are required to find out

$$P(A/A \cup B) = \frac{P(A \cup A \cap B)}{P(A \cup B)} = \frac{P(A)}{P(A \cup B)} = \frac{0.60}{0.76} = 0.7895.$$

Illustration 24. Explain whether or not each of the following claims could be correct :

(*i*) A businessman claims the probability that he will get contract *A* is 0.15 and that he will get contract *B* is 0.20. Furthermore, he claims that the probability of getting *A* or *B* is 0.50.

(*ii*) A market analyst claims that the probability of selling ten million kg. of plastic *A* or five million kg. of plastic *B* is 0.60. He also claims that the probability of selling ten million kg. of *A* and five million pounds of *B* is 0.45.

Solution. (*i*) Let event *A* and *B* denote the probability of getting contract *A* and *B* respectively.

Then, $P(A) = 0.15; \;\; P(B) = 0.20$ and $P(A \cup B) = 0.50$.

Probability of getting both the contracts is

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.15 + 0.20 - 0.50 = -0.15$$

Hence the claim of the businessman is wrong as the probability of getting both the contracts is negative.

(*ii*) Let event A and B denote the selling of ten million of kg. of plastic A and five million kg. of plastic B respectively.

Then $$P(A \cup B) = 0.60 \text{ and } P(A \cap B) = 0.45$$

Therefore $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

or $$P(A) + P(B) = 0.60 + 0.45 = 1.05$$

Since the sum of the probabilities is more than one, hence the claim of the market analyst is wrong.

Illustration 25. In a survey of 100 readers, it was found 40 read magazine A, 15 read magazine B, and 10 read both. What is the probability of a person reading at least one of the magazines ?

Solution. $$P(A) = 0.40, \quad P(B) = 0.15 \quad P(A \cap B) = 0.10$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.40 + 0.15 - 0.10 = 0.45.$$

Therefore, the probability that a person read at least one magazine is 0.45.

Illustration 26. A bag contains 8 red and 5 white balls. The successive drawings of 3 balls are made such that (*i*) balls are replaced before the second trial. (*ii*) The balls are not replaced before the second trial. Find the probability that the first drawing will give 3 white and the second 3 red balls.

Solution. (*a*) *When the balls are replaced.*

Total number of balls in the bag = 13

Total number of red balls = 8

3 balls out of 13 can be drawn in ${}^{13}C_3$ ways.

3 white balls can be drawn out of 5 in ${}^{5}C_3$ ways.

3 red balls can be drawn out of 8 red in ${}^{8}C_3$ ways.

Since balls are replaced before the second draw, the total number of outcomes for both the draws remain the same, *i.e.*, ${}^{13}C_3$.

$$P(A) = \frac{{}^{5}C_3}{{}^{13}C_3} \quad \text{and} \quad P(B) = \frac{{}^{8}C_3}{{}^{13}C_3}$$

$$P(A \cap B) = P(A) \times P(B) = \frac{{}^{5}C_3}{{}^{13}C_3} \times \frac{{}^{8}C_3}{{}^{13}C_3} = \frac{140}{20449} = 0.0068.$$

(*b*) *When balls are not replaced.* When balls are not replaced, there would be no change in $P(A)$. Also the number of outcomes favourable to event B shall be ${}^{8}C_3$ but on the second draw the total number of outcomes would be ${}^{10}C_3$ (since 3 balls drawn are not replaced).

$$P(B/A) = P(B) = \frac{{}^{8}C_3}{{}^{10}C_3}$$

Hence $$P(A \cap B) = P(A) \times P(B) = \frac{{}^{5}C_3}{{}^{13}C_3} \times \frac{{}^{8}C_3}{{}^{10}C_3} = \frac{7}{429} = 0.0163.$$

Illustration 27. A manufacturing firm produces steel pipes in three plants with daily production volumes of 500, 1,000 and 2,000 units respectively. According to past experience it is known that the fractions of defective output produced by the three plants are respectively at random 0.005, 0.008 and 0.010. If a pipe is selected from a day's total production and found to be defective, find out (*i*) from which plant for this defective pipe, the probability is highest. (*ii*) What is the probability that it came from the first plant? *[MBA, IIT, Roorkee, 2004; M.B.A, Hyderabad Univ., 2005, M.B.A, Delhi Univ., 2006]*

Solution.

Let B_1 = Production volume of first plant.

B_2 = Production volume of second plant.

B_3 = Production volume of third plant.

A = a defective item.

$$P(B_1) = \frac{500}{3500} = \frac{1}{7} = 0.1428$$

$$P(B_2) = \frac{1000}{3500} = \frac{2}{7} = 0.2857$$

$$P(B_3) = \frac{2000}{3500} = \frac{4}{7} = 0.5714$$

$P(B_1)$, $P(B_2)$, $P(B_3)$ denotes the probability of selecting a unit from 1st, 2nd, 3rd plant respectively.

Now, calculating the joint probabilities

$$P(B_1 \cap A) = P(B_1) \times P(A/B_1) = 0.1428 \times 0.005 = 0.0007$$
$$P(B_2 \cap A) = P(B_2) \times P(A/B_2) = 0.2857 \times 0.008 = 0.00228$$
$$P(B_3 \cap A) = P(B_3) \times P(A/B_3) = 0.5714 \times 0.01 = 0.0057$$

Using Bayes' theorem, the required probabilities are :

$$P(B_1/A) = \frac{P(B_1 \cap A)}{P(B_1 \cap A) + P(B_2 \cap A) + P(B_3 \cap A)}$$
$$= \frac{0.0007}{0.0007 + 0.00228 + 0.0057} = \frac{0.0007}{0.00868} = 0.081$$

Similarly,
$$P(B_2/A) = \frac{0.00228}{0.00868} = 0.038$$
$$P(B_3/A) = \frac{0.0057}{0.00868} = 0.656$$

(*i*) As $P(B_3/A)$ has the highest probability we can say that it is most likely that the defective pipe has been drawn from the third plant.

(*ii*) The probability that it came from the first plant is given by $P(B_1/A)$ which is 0.081

Illustration 28. A market survey conducted in four cities pertained to preference for brand A soap. The responses are shown below :

	Delhi	*Kolkata*	*Chennai*	*Mumbai*
Yes	45	55	60	50
No	35	45	35	45
No opinion	5	5	5	5

(*i*) What is the probability that a consumer selected at random preferred brand A ?
(*ii*) What is the probability that a consumer preferred brand A and was from Chennai ?
(*iii*) What is the probability that a consumer preferred brand A given that he/she was from Chennai ?
(*iv*) Given that a consumer preferred brand A, what is the probability that he/she was from Mumbai ?

(*MBA, Kumaon Univ., 1999; MBA, Delhi Univ., 2004, 2007*)

Solution.

	Delhi	*Kolkata*	*Chennai*	*Mumbai*	*Total*
Yes	45	55	60	50	210
No	35	45	35	45	160
No opinion	5	5	5	5	20
Total	85	105	100	100	390

Let the event A denote that a consumer selected at random preferred brand A.

(*i*) $$P(A) = \frac{210}{390} = \frac{7}{13} = 0.5385$$

(*ii*) $$(A \cap C) = \frac{60}{390} = \frac{2}{13} = 0.1538$$

(*iii*) $$P(A/C) = \frac{P(A \cap M)}{P(M)} = \frac{60/390}{100/390} = \frac{3}{5} = 0.6$$

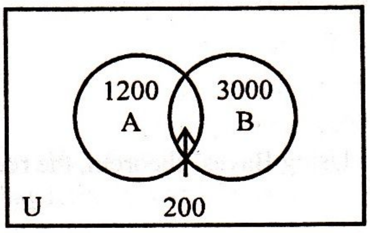
(*iv*) $$P(M/A) = \frac{P(B \cap A)}{P(A)} = \frac{50/390}{210/390} = \frac{5}{21} = 0.238$$

Illustration 29. In a locality, out of 5,000 people residing, 1,200 are above 30 years of age and 3,000 are females. Out of the 1,200 who are 30 years of age 200 are females. Suppose, after a person is chosen you are told that the person is female. What is the probability that she is above 30 years of age ?

(*MBA, Delhi Univ., 2001*)

Solution. Let event A denote that the person is above 30 years of age and event B that the person is a female.

Therefore $P(B) = \dfrac{3000}{5000}$, $P(A \cap B) = \dfrac{200}{5000}$



Hence $P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{200/5000}{3000/5000} = 0.067$

Illustration 30. Two factories manufacture the same machine part. Each part is classified as having either 0, 1, 2 or 3 manufacturing defects. The joint probability distribution for this is given below :

	Number of defects			
	0	1	2	3
Manufacturer A	0.1250	0.0625	0.1875	0.1250
Manufacturer B	0.0625	0.0625	0.1250	0.2500

(*i*) A part is observed to have no defects. What is the probability that it was produced by manufacturer A ?
(*ii*) A part is known to have been produced by manufacturer A. What is the probability that the part has no defects ?
(*iii*) A part is known to have two or more defects. What is the probability that it was manufactured by A ?
(*iv*) A part is known to have one or more defects. What is the probability that it was manufactured by B ?

Solution.

	Number of defects				
	0	1	2	3	
A	0.1250	0.0625	0.1875	0.1250	0.5000
B	0.0625	0.0625	0.1250	0.2500	0.5000
Total	0.1875	0.1250	0.3125	0.3750	1.0000

(*i*) $P(A/\text{No defects}) = \dfrac{P(A \text{ and No defects})}{P(\text{No defects})} = \dfrac{0.1250}{0.1875} = 0.6667$

(*ii*) $P(\text{No defects}/A) = \dfrac{P(\text{No defects and } A)}{P(A)} = \dfrac{0.1250}{0.5000} = 0.2500$

(*iii*) $P(A/2 \text{ or more defects}) = \dfrac{P(A \text{ and 2 or more defects})}{P(2 \text{ or more defects})} = \dfrac{0.3125}{0.6875} = 0.4545$

(*iv*) $P(B/1 \text{ or more defects}) = \dfrac{P(B \text{ and 1 or more defects})}{P(1 \text{ or more defects})} = \dfrac{0.4375}{0.8125} = 0.5385$

Illustration 31. The probability that a new marketing approach will be successful is 0.6. The probability that the expenditure for developing the approach can be kept within the original budget is 0.50. The probability that both of these objectives will be achieved is 0.30.

What is the probability that at least one of these objectives will be achieved. For the two events described above, determine whether the events are independent or dependent. (*MBA, Delhi Univ., 2006*)

Solution : Let A denote the event that the new marketing approach will be successful and the event B denote the event that the expenditure for developing the approach can be kept within the original budget. Therefore, we are given

$$P(A) = 0.6, \; P(B) = 0.5 \quad \text{also} \quad P(A \cap B) = 0.3$$

The probability that at least one of these objectives will be achieved is given by

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.6 + 0.5 - 0.3 = 0.8$$

If events are independent

$$P(A \cap B) = P(A)\,P(B) = 0.6 \times 0.5 = 0.30$$

Which is same as given above. Hence events are independent.

Illustration 32. Of 1000 assembled components, 10 have a working defect and 20 have a structural defect. There is a good reason to assume that no component has both defects. What is the probability that randomly chosen component will have either type of defect ?

Solution. Let event A denote that the component has working defect and event B that the component has structural defect. Therefore, we are given

$$P(A) = \frac{10}{1000} = 0.01, \; P(B) = \frac{20}{1000} = 0.02$$

Also assuming that no component has both defects is given by $P(A \cap B)$. Therefore, the probability that the component will have either type of defect is given as :

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.01 + 0.02 - 0.0 = 0.03.$$

Illustration 33. A study of Job satisfaction was conducted for four occupations. Cabin maker, lawyer, doctor, and systems analyst. Job satisfaction was measured on a scale of 0 – 100. The data obtained are summarized in the following cross tabulation.

Occupation	*Under 50*	*50 – 59*	*60 – 69*	*70 – 79*	*80 – 89*	*Total*
Cabin maker	0	2	4	3	1	10
Lawyer	6	2	1	1	0	10
Doctor	0	5	2	1	2	10
Systems Analyst	2	1	4	3	0	10
Total	8	10	11	8	3	40

(MBA, D.U., 2003)

(*i*) Develop a joint probability table.

(*ii*) What is the probability of one of the participants studied had a satisfaction score in the 80's ?

(*iii*) What is the probability of a satisfaction score in the 80's given the study participant was a doctor ?

(*iv*) What is the probability of one of the participants studied was a lawyer ?

(*v*) What is the probability of one of the participants was a lawyer and received a score under 50 ?

(*vi*) What is the Probability of a satisfaction score under 50 given a person is a lawyer ?

(*vii*) What is the probability of a satisfaction score of 70 or higher ? *(MBA, Delhi Univ., 2003)*

Solution.

(*i*) Joint Probability table is :

Occupation	*Under 50*	*50 – 59*	*60 – 69*	*70 – 79*	*80 – 89*
Cabin maker	0.000	0.050	0.100	0.075	0.250
Lawyer	0.150	0.050	0.025	0.025	0.250
Doctor	0.000	0.125	0.050	0.025	0.250
Systems Analyst	0.050	0.025	0.100	0.075	0.250

(*ii*) $P[\text{Satisfaction score in the 80's}] = \frac{3}{40} = 0.075.$

(*iii*) $P[\text{Satisfaction score in the 80's/Doctor}] = \frac{P[SS\,(80)]}{P[D]} = \frac{2/40}{10/40} = \frac{1}{5} = 0.20.$

(*iv*) $P\,[\text{Lawyer}] = \frac{10}{40} = 0.25.$

(*v*) $P\,[\text{Lawyer and score under 50}] = \frac{P\,(\text{Lawyer} \cap \text{Score Under 50})}{P\,(\text{Score Under 50})} = \frac{6}{40} = 0.15.$

(*vi*) $P\,[\text{Score under 50/Lawyer}] = \frac{P\,[\text{Score under 50} \cap \text{Lawyer}]}{P\,[\text{Lawyer}]} = \frac{6/40}{10/40} = 0.6.$

(*vii*) P [Satisfaction score of 70 or higher]

$= P$ [score of 70 and above] + P [score of 80 and above]

$$= \frac{8}{40} + \frac{3}{40} = \frac{11}{40} = 0.275.$$

Illustration 34. Given the probabilities of three events, A, B and C are $P(A) = 0.35$, $P(B) = 0.45$ and $P(C) = 0.2$. Assuming that A, B and C have occurred, the conditional probabilities of another event, X, occurring are $P(X/A) = 0.8$, $P(X/B) = 0.65$ and $P(X/C) = 0.3$. Find $P(A/X)$, $P(B/X)$ and $P(C/X)$. *(MBA, IGNOU, 2007)*

Solution. We shall make use of Bayes' is theorem to solve this problem.

Given $P(A) = 0.35$, $P(B) = 0.45$, $P(C) = 0.20$. Also $P(X/A) = 0.80$, $P(X/B) = 0.65$, $P(X/C) = 0.30$

$$P(A/X) = \frac{P(A)\ P(X/A)}{P(A)\ P(X/A) + P(X/B)\ P(B) + P(X/C)\ P(C)}$$

$$= \frac{0.35 \times 0.8}{(0.35 \times 0.8) + (0.45 \times 0.65) + (0.2 \times 0.3)}$$

$$= \frac{0.28}{0.28 + 0.29 + 0.06} = \frac{0.28}{0.63} = 0.44$$

$$P(B/X) = \frac{P(B)\ P(X/B)}{P(A)\ P(X/A) + P(B)\ P(X/B) + P(C)\ P(X/C)}$$

$$= \frac{(0.45)\ (0.65)}{(0.35 \times 0.8) + (0.45)\ (0.65) + (0.2)\ (0.3)}$$

$$= \frac{0.29}{0.28 + 0.29 + 0.06} = \frac{0.29}{0.63} = 0.46$$

$$P(C/X) = \frac{P(C)\ P(X/C)}{P(A)\ P(X/A) + P(B)\ P(X/B) + P(C)\ P(X/C)}$$

$$= \frac{(0.2)\ (0.3)}{(0.35 \times 0.8) + (0.45 \times 0.65) + (0.2 \times 0.3)}$$

$$= \frac{0.06}{0.28 + 0.29 + 0.06} = \frac{0.06}{0.63} = 0.09.$$

PROBLEMS

1-A : Answer the following questions, each question carries **one** mark:

(*i*) State the addition law of probability.

(*ii*) What do you mean by the term conditional probability ? *(MBA, Hyderabad Univ., 2006)*

(*iii*) A bag contains 7 red balls and 5 white balls. 2 balls are drawn at random. What is the probability that all of them are red ?

(*iv*) Define the term probability.

(*v*) What are independent events ?

(*vi*) Explain with examples the rule of addition in theory of probability. *(MBA, Madurai-Kamaraj Univ., 2003)*

(*vii*) What is meant by mutually exclusive events ?

(*viii*) Explain Bayes' theorem with the help of a suitable example. *(MBA, Hyderabad Univ., 2005)*

1-B : Answer the following questions, each question carries **four** marks:

(*i*) Three balls are drawn at random from a basket containing 6 blue and 4 red balls. What is the chance that two balls are blue and one is red ?

(*ii*) What is the probability that a leap year selected at random will contain 53 Sundays ? *(M.Com., M.K. Univ., 2008)*

(*iii*) What do you mean by probability ? Explain the importance of probability. *(M.A. Econ., Madras Univ., 2008)*

(*iv*) What are the basic laws of probability ? *(MBA, Madras Univ., 2003)*

(*v*) State and prove the addition law of probability.

2. Explain what do you understand by the term probability. Discuss its importance in managerial decision-making. *(MBA, Delhi Univ., 2007)*

3. Describe briefly the various schools of thought on probability. How does the concept of probability help decision-maker to improve his decisions ?

4. (a) Explain the various approaches to probability. Are they contradictory ?
   (b) Examine critically the different schools of thought on probability.
   (MBA, Kumaon Univ., 2000)
5. Explain with examples the rules of Addition and Multiplication in theory of probability.
6. Give the classical and statistical difinitions of probability and state the relationship, if any, between the two definitions.
7. State and prove the addition and multiplication theorems of probability.
8. (a) Explain with the help of an example the concept of conditional probability.
   (b) Explain the concept of conditional probability and Bayes' theorem. (MBA, IGNOU, 2002)
9. Explain the difference between :
   (i) Simple probability and conditional probability.
   (ii) Independent event and mutually exclusive event.
10. Define independent and mutually exclusive events. Can two events be mutually exclusive and independent simultaneously? Support your answer with an example. (MBA, Delhi Univ., 2007)
11. When are two events said to be independent in the probability sense ? Give examples of dependent and independent events.
12. (a) Explain the concept of probability following the experimental frequency approach.
    (b) What do you understand by conditional probability ? If Prob. $(A + B)$ = Prob. (A) + Prob. (B), are the two events A and B statistically independent ?
13. Write an essay on prior and posterior probabilities and Bayes' theorem and also show how Bayes' theorem can be extended in the case of n events.
14. (a) Make up a realistic problem from your area of interest to illustrate the use of Bayes' theorem.
    (b) State the multiplicative theorem of probability. How is the result modified when the events are independent ?
    (M.Com., Delhi Univ., 2009)
15. The personnel manager of a large manufacturing firm finds that 15 per cent of the firm's employees are junior executives and 25 per cent of the firm's employees are MBAs. He also discovers that 5 per cent of the firm's employees are both junior executives and MBAs. What is the probability of selecting a junior executive if the selection is confined to MBAs ?
    [0.20]
16. A company learned that inventory shortages were associated with a loss of goodwill with a probability 0.10. The company also knew that a loss of goodwill from all causes occurred with a probability of 0.15. What is the probability of an inventory shortage, given a loss of goodwill ?
    [0.67]
17. An article manufactured by a company consists of two parts A and B. In the process of manufacture of part A, 9 out of 100 are likely to be defective. Similarly, 5 out of 100 are likely to be defective in the manufacture of part B. Calculate the probability that the assembled part will not be defective.
    [0.8645]
18. A candidate is selected for interview for three posts. For the first post there are 3 candidates, for the second there are four and for the third there are two candidates. What are his chances of getting at least one post ?
    [0.75]
19. An investment firm purchases 3 stocks for one week trading purposes. It assesses the probability that the stocks will increase in value over the week as 0.8, 0.7 and 0.6 respectively. What is the chance (i) all three stocks will increase and (ii) at least 2 stocks will increase ? (Assume that the movements of these stocks are independent.)
    [(i) 0.336, (ii) 0.788.]
20. A company has two plants to manufacture scooters. Plant 1 manufactures 80% of the scooters and plant 2 manufactures 20%. At Plant 1, 85% scooters are rated as standard quality. At plant 2, only 65% scooters are rated as standard quality.
    (i) What is the probability, that a customer obtains a standard quality scooter if he buys a scooter from the company ?
    (ii) What is the probability, that the scooter came from plant 1, if it is known that the scooter is of standard quality ?
    [(i) 0.81, (ii) 0.84]
21. 10% of the employees of a certain company have been to public school. Of these, 30% hold administrative positions. Of these that have not been to public school, 30% hold administrative positions. If an employee is selected at random from the administrative staff, what is the probability that he was educated in a public school ? (MBA, HPU, 2009)
22. A factory produces a mechanism which consists of three independently manufactured parts. It is known that 1 per cent of part one, 4 per cent of part two and 3 per cent of part three are defective. What is the probability that a complete mechanism is not defective ?
    [0.9218]

23. A manager has two assistants and he bases his decision on information supplied independently by each of them. The probability that he makes a mistake in his thinking is 0.005. The probability that an assistant gives wrong information is 0.3. Assuming that the mistakes made by the manager are independent of the information given by the assistants, find the probability that he reaches a wrong decision. *(MBA, DU, 2001)*

[0.5122]

24. A piece of electronic equipment has two essential parts, A and B. In the past, part A has failed 40% of the time; part B 50% of the time. Parts *A* and *B* operate independently. Assume that both parts must operate to enable the equipment to function. What is the probability that the equipment will function ?

[0.30]

25. Three groups of workers contain 3 men and 1 woman, 2 men and 2 women, and 1 man and 3 women, respectively. One worker is selected at random from each group. What is the probability that the group selected consists of 1 man and 2 women ?

[0.4063]

26. Two sets of candidates are competing for the position on the Board of Directors of a company. The probability is that the first and second sets will win 0.6 and 0.4 respectively. If the first set wins, the probability of introducing a new product is 0.8, and the corresponding probability if the second set wins is 0.3. What is the probability that the new product will be introduced?

[0.60]

27. There are three cars, *A, B* and *C*. Car *A*, contains two males, car *B* contains one male and one female, and car *C* contains two females. If one of these cars is selected at random, and one person is observed to be male, what is the probability that the other person in that car is male ?

[2/3]

28. A salesman has a 60 per cent chance of making a sale to each customer. The behaviour of successive customers is independent. If two customers *A* and *B* enter, what is the probability that the salesman will make a sale to *A* or *B*?

[0.84] *(MBA, DU, 1998)*

29. A factory produces certain types of output by three machines. The respective daily production figures are : Machine $A = 3{,}000$ units; Machine $B = 2{,}500$ units; and Machine $C = 4{,}500$ units. Past experience shows that 1 per cent of the output produced by machine *A* is defective. The corresponding fractions of defectives for the other two machines are 1.2 and 2 per cent respectively. An item is drawn at random from the day's production run and is found to be defective. What is the probability that it comes from the output of (*i*) Machine *A*; (*ii*) Machine *B*; and (*iii*) Machine *C* ?

[(*i*) 0.2; (*ii*) 0.2; (*iii*) 0.6]

30. In a bolt factory machines *A, B* and *C* manufacture respectively 25%, 35% and 40% of the total of their output 5,4 and 2 per cent are defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines *A, B* and *C* ? *(MBA, Delhi Univ., 2010)*

[$A = 0.37$; $B = 0.40$; $C = 0.23$]

31. In a factory manufacturing pens. Machines *X, Y* and *Z* manufacture 30, 30 and 40 per cent of the total production of pens respectively.

Of their output 4, 5 and 10 per cent of the pens are defective. If one pen is selected at random it is found to be defective what is the probability that it is manufactured by machine Z ?

[0.6639] *(MBA, UP Tech. Univ., 200:*

32. The probability that India wins a cricket test match against Pakistan is, given to be 1/3. If India and Pakistan play six te matches, what is the probability that :

(*i*) India will lose all the six test matches ?

(*ii*) India will win at least one test match ? *(MBA, Delhi Univ., 200*

[(*i*) 0.088; (*ii*) 0.912]

33. Three persons *A, B* and *C* are being considered for the appointment as Vice-Chancellor of a university whose chances of be selected for the post are in the proportion 4:2:3 respectively. The probability that *A*, if selected, will introduce democratisat in the University Structure is 0.3 and the corresponding probablities for *B* and *C* doing the same are respectively 0.5 and What is the probability that democratisation would be introduced in the University ? *(MBA, DU, 2(*

[0.511]

34. A husband and wife appear in an interview for two vacancies in the same post. The probability of husband's selection is 1/7 and that of wife's selection is 1/5. What is the probability that
   (*a*) both of them will be selected,
   (*b*) only one of them will be selected, and
   (*c*) none of them will be selected ?
   [(*a*) 1/35, (*b*) 10/35, (*c*) 24/35] (*M.Com., Madurai Kamaraj Univ., 2003*)
35. A hotel gets cars for its guests from three rental agencies, 20 per cent from agency *X*, 40 per cent from agency *Y* and 40 per cent from *Z*. If 14 per cent of the cars from *X*, 4 per cent from *Y* and 8 per cent from *Z*, need tune-ups, what is the probability that car needing a tune-up is delivered to one of the hotel's guests ? (*MBA, DU, 1999*)
   [7.6%]
36. The odds against student *X* solving a Business statistics problem are 8 : 6 and odds in favour of student *Y* solving the same problem are 14:16. (*i*) What is the chance that the problem will be solved if both try ? (*ii*) What is the probability that they both, working independently of each other, solve the problem ? (*iii*) What is the probability that neither solves the problem ?
   [(*i*) 0.6952; (*ii*) 0.2; (*iii*) 0.3048] (*MBA, Delhi Univ., 2004, 2007*)
37. In a certain government office there are 400 employees; there are 150 men, 276 University graduates, 212 married persons, 94 male university graduates, 151 married university graduates, 119 married men, 72 married male university graduates. Find the number of single women who are not university graduates ?
   [54]
38. A lot of vacuum tubes contains 1000 tubes. 10 of which have a defective grid and no other defects and 20 of which have both a defective grid and defective heating element. A tube is drawn at random from the lot and we are told that it has defective grid. What is the probability that it also has a defective heating element ? What model did you use in computing this probability ?
39. Probability that a man will be alive 25 years hence is 0.3 and the probability that his wife will be alive 25 years hence is 0.4. Find the probability that 25 years hence (*i*) both will be alive (*ii*) only the man will be alive (*iii*) only the woman be alive and (*iv*) at least one of them will be alive.
40. A convention begins with an evening lecture, attended by 60% of the delegates. The following morning lecture is attended by 10% of the delegates. Seventy per cent of those attending this session had attended the previous evening session.
   (*i*) What is the probability that a randomly selected delegate attended both the sessions ?
   (*ii*) What is the probability that a delegate who attended the evening session also attended the following morning session ?
   (*iii*) What is the probability that a delegate selected at random attended at least one of the two sessions ?
   (*iv*) Are attendances at the two sessions statistically independent ?
41. Mr. Ram speaks the truth in 3 out of 4 times, while Mr. Shyam speaks the truth in 4 out of 5 times. Find the probability that they will contradict each other in stating the fact.
   [0.35]
42. Two union leaders and 10 directors of a company sit randomly around a round table to decide upon the wage hike as demanded by the union. Find the probability that there will be exactly three directors between the two union leaders.
43. Assume we have three boxes which contain red and black balls as follows :

Box 1	—	3 red and 7 black
Box 2	—	6 red and 4 black
Box 3	—	8 red and 2 black

A ball is drawn from box 1; if it is red, 2nd ball is drawn from box 2. If the 1st ball drawn from box 1 is black, 2nd ball is drawn from box 3.
   (*i*) What is the probability that the two balls are red ?
   (*ii*) What is the probability that one ball is red and another ball is black ?
44. Explain whether or not each of the following claims could be correct :
   (*a*) A supplier claims that the long-run fraction of the resistors he produces which are defective is 0.001. In one lot of 10,000 resistors obtained from the supplier 30 defectives were discovered.
   (*b*) A plant engineer claims the probability that machine will not fail in a one month period is 0.20, the probability that it will fail exactly once is 0.50, the probability that it will fail twice is 0.30 and the probability that it will fail more than twice is 0.30.
   (*c*) A market analyst claims that the probability that sales of less than 4 million pounds in the next year is 0.3, of sales between 4 and 6 million pounds is 0.4 and sales of more than 6 million pounds is 0.2.

45. A production process which turns out transistors has a long-run fraction defective of 0.005. A testing device is used to check each transistor produced. It has been found that the device always indicates that a defective is indeed defective, but for about 1 in every 100 transistors produced it indicates that a good transistor is defective. If the device indicates that a given transistor is defective, what is the probability that it is actually defective ?

46. A certain production process produces items that are 10 per cent defective. Each item is inspected before being supplied to customers but the inspector incorrectly classifies an item 10 per cent of the time. Only items classified as good are supplied. If 820 items in all have been supplied, how many of them are expected to be defective ?
[10]

47. A market research firm is interested in surveying certain attitudes in a small community. There are 1,250 households broken down according to income, ownership of a telephone and ownership of a TV.

	Households with annual income of Rs. 3,00,000 or less		*Household with annual income above Rs. 3,00,000*	
	Telephone Subscriber	*No Telephone*	*Telephone Subscriber*	*No Telephone*
Own TV set	270	200	180	100
No TV set	180	100	120	100

(*i*) What is the probability of obtaining a TV owner in drawing at random ?
(*ii*) If a household has annual income over Rs. 3,00,000 and is a telephone subscriber, what is the probability that he has a TV?
(*iii*) What is the conditional probability of drawing a household that owns a TV given that the household is a telephone subscriber.
(*iv*) Are the events 'ownership of a TV' and 'telephone subscriber' statistically independent ? Comment.
[(*i*) 0.6, (*ii*) 0.6, (*iii*) 0.6, (*iv*) yes]

48. Past surveys show that 40% of the officers at a certain industry own cars. Suppose six officers are selected at random from this industry (with replacement).
(*a*) What is the probability that exactly four will own cars ?
(*b*) What is the probability that at least one will own a car ?
(*c*) What is the theoretical mean of the probability distribution under consideration ?

49. A survey reports that 80% of the population is married and 55% is male. What is the least possible percentage of married men and of married women ?

50. Consider a family with two children. Assuming that each child is as likely to be a boy as it is to be a girl, what is the conditional probability that both children are boys given that (*a*) the elder child is a boy, (*a*) at least one of the children is a boy?

51. A man goes for fishing for the first time. He has three types of bait, only one of which is correct for the type of fish he intends to try. The probability that he will catch a fish if he uses correct bait is 1/3. If he uses the wrong bait, his chances of catching a fish are 1/5.
(*a*) What is the probability that he will catch a fish ?
(*b*) Given the man caught a fish, what is the probability that he used correct type of bait ?

52. If a machine is correctly set up, it will produce 90% acceptable items. If it is incorrectly set up, it will produce 40% acceptable items. Past experience shows that 80% of the set-ups are correctly done. If after a certain set-up the machine produces 2 acceptable items as the first 2 pieces, find the probability that the machine is correctly set up.

53. Consider two events A and B such that $P(A) = 1/8$, $P(A/B) = 1/4$ and $P(B/A) = 1/6$. Examine the following statements and comment on the validity of each of these :
(*i*) A and B are independent.
(*ii*) A and B are mutually exclusive.
(*iii*) Occurrence of A implies that of B.
(*iv*) $P(A/B) = 0.5$.

54. If a pair of dice is thrown, find the probability that the sum is neither 7 nor 11.
[7/9]

55. An investment consultant predicts that the odds against the price of a certain stock will go up during the next week are 2 : 1 and the odds in favour of the price remaining the same are 1 : 3. What is the probability that the price of the stock will go down during the next week ?

56. (*a*) What is the probability that a leap year selected at random will contain either 53 Thursdays or 53 Fridays ?
[3/71]
(*b*) What is the probability that a leap year selected at random will contain 53 sundays?

(*M. Com., Madurai-Kamaraj Univ.*, 2002)

57. A product is assembled from three components X, Y and Z and the probability of these components being defective is 0.01, 0.02 and 0.05. What is the probability that the assembled product will not be defective ? *(MBA, DU, 2002)*
[0.922]

58. According to a survey, the probability that a family owns 2 cars if their annual income is greater than Rs. 15,000 is 7. Of the households surveyed, 50 per cent had income over Rs. 15,000 and 40 per cent had 2 cars. What is the probability that a family has 2 cars and an income over Rs. 15,000 a year ?

59. A box contains 8 red, 3 blue and 9 green balls. If three balls are drawn at random, determine the probability that :
(*i*) all 3 are red ; (*ii*) all 3 are blue ; (*iii*) at least 1 is blue ; (*iv*) 2 are red and 1 green
(*v*) 1 of each colour ; and (*vi*) the balls are drawn in the order red, blue and green colours.

60. A problem in statistics is given to the three students X, Y and Z, whose chances of solving it are 1/3, 1/4, 2/5 respectively. What is the probability that the problem will be solved ?
[7/10]

61. A survey of readership of a certain investment magazine indicates that the proportion of male readers over 40 years is 0.02. The proportion of male readers under 40 is 0.07. What is the probability of a reader being a male ?

62. The cricket team of a University played four matches in Inter-University cricket matches. The captain of the team observed the practice of calling out "Heads" every time when the toss was made. What is the probability of his winning the toss in all the four matches ?
How would the probability be affected if the Captain had made a practice of tossing coin privately before calling out "Head" or "Tail" on each occasion ?

63. A sample of 3 items is selected at random from a box containing 12 items of which 3 are defective. Find the possible number of defective combinations of the said 2 selected items along with probability of a defective combination.

64. In an examination, 30% of the students have failed in Mathematics, 20% of students have failed in Chemistry and 19% have failed in both Mathematics and Chemistry. A student is selected at random.
(*i*) What is the probability that the student has failed in Mathematics if it is known that he has failed in Chemistry ?
(*ii*) What is the probability that the student has failed in Mathematics or in Chemisty ?

65. A company has four production sections, *viz.*, S_1, S_2, S_3 and S_4 which contribute 30%, 20%, 22% and 28% respectively to the total output. It was observed that these sections respectively produced 1%, 2%, 3% and 4% defective units. If a unit is selected at random and found to be defective, what is the probability that the unit so selected has come from either section one or section four ? *(MBA, GGSIPU, 2000; MBA, Delhi Univ., 2004)*

66. A factory has two machines. The empirical evidence has established that Machines (*i*) and (*ii*) produce 30% and 70% of the output respectively. It has also been established that 5% and 1% of the output produced by these machines respectively was defective. A defective item is drawn at random. What is the probability that the defective item was produced by machine (*i*) and (*ii*) ?
[(*i*) 0.682 (*ii*) 0.318]

67. It is believed that in 100 cases of income tax raids, and undisclosed income of more than Rs. 1 lakh is selected. What is the probability that the income tax office will have to make at most 10 raids until the first case of undisclosed income of more than Rs. 1 lakh is detected.

68. A box contains 10 white, 7 black and 3 green balls. 2 balls are drawn at random. Find out the probability that :
(*i*) both are white.
(*ii*) one is white and another is green.
(*iii*) one is black and another is green.
Find the probabilities in case of without replacement.

69. Project VIJAY, NCSO, INDIA sums its operations on 10 computers which may need repairs from time to time during the day. Three of these computers are old, each having a probability of 1/11 of needing repair during the day and seven are new, having corresponding probability of 1/ 21.
Assuming that no computer needs repair twice on the same day, determine the probabilities that on a particular day.
(*i*) just 2 old and no new computers need repair.
(*ii*) if just 2 computers need repair, they are of same type. *(MBA, IGNOU, 2000)*

70. A consignment of 20 picture tubes contain 5 defectives. Two tubes are selected one after another at random. Find the probability that both are defective assuming (*a*) the first is replaced before drawing the second, and (*b*) the first is not replaced.

71. A manager has drafted a scheme for the benefit of employees. To get an idea of the support for the scheme, he random polls literate workers (L) and illiterate workers (I). He polls 30 of each group with the following results :

Opinion For Scheme	*L*	*I*
Strongly Support	9	10
Mildly Support	11	3
Undecided	2	2
Mildly oppose	4	8
Strongly oppose	4	7

(*a*) What is the probability that a literate worker selected randomly from the polled group mildly supports the scheme
(*b*) What is the probability that a worker (literate or illiterate) selected randomly from the polled group strongly or mild supports the scheme ?
(*MBA, IGNOU, 200*

72. Three institutions (*A, B,* and *C*) train students for MBA entrance test. They train in the proportion 25 per cent (*A*), 35 p cent (*B*) and 40 per cent (*C*) of the trained candidates, for *A, B, C*; 5 per cent, 4 per cent and 2 per cent are successful in t entrance test respectively.
A candidate is selected at random and found to be successful in the entrance, find the probability, that he was trained by *B*, or *C*. What is the probability of average success in the MBA entrance ? (*MBA, Bharathidasan Univ., 200*

73. A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he driv one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day the week, the man tossed a fair dice and drove to work if and only if a 6 appeared. Find the probability that he takes a tra on the third day and also the probability that he drives to work in the long run.

74. A machine goes out of order, whenever a component fails. The failure of this part follows a Poisson process with a mean ra of 1 per week. Find the probability that 2 weeks have elapsed since last failure. If there are 5 spare parts of this compone in an inventory and that the next supply is not due in 10 weeks, find the probability that the machine will not be out of ord in the next 10 weeks. (*B.E./ B. Tech., Madras Univ., 200*

75. A manufacturing firm produces pipes in two plants I and II with daily production 1,500 and 2,000 pipes respectively. Th fraction of defective pipes produced by the two plants I and II are 0.006 and 0.008 respectively. If a pipe selected at rando from the day's production is found to be defective, what is the probability that it has come from plant I, plant II ?
(*MBA, Bharathidasan Univ., 200*

76. In a survey of MBA students, the following data were obtained on students' first reason for application to one business school

	Reason for application		
	School quality	*Placement*	*Other*
Enrollment	421	393	76
Status	400	593	46

(*i*) If a student goes full time, what is the probability that the school quality is the first reason for choosing a business school
(*ii*) If a student goes part time, what is the probability that the school quality is the first reason for choosing a busines school ?
(*iii*) Let *A* be the event that a student is full time and let *B* be the event that the student lists the school quality as the first reason for applying. Are events *A* and *B* independent? Justify your answer. (*MBA, Delhi Univ., 2006*)

77. Logic Dynamic Ltd., a computer manufacturing firm, receives shipment of parts from two different suppliers. Supplier *A* supplies the 70% of the total parts and the remaining 30% is supplied by supplier *B*. The historical quality levels of these two suppliers are shown in the following table:

	Good parts (%)	*Defective parts (%)*
Supplier *A*	95	5
Supplier *B*	90	10

(*i*) A part is randomly selected from the firm's inventory, and it is found to be defective, what is the probability that it is supplied by the supplier *A*?
(*ii*) A part is randomly selected from the firm's inventory, and it is found to be good, what is the probability that it is supplied by the supplier *B* ? (*MBA, Delhi Univ., 2009*)

78. A vending machine at MacDonald's fast food restaurant automatically pours soft drinks into cups. The amount of soft drinks dispensed into a cup is normally distributed with mean 9.6 oz and standard deviation 0.6 oz.
(*i*) Estimate the probability that the machine overflow an 10 oz cup.
(*ii*) Estimate the probability that the machine will not overflow an 10 oz cup.
(*iii*) The machine has just been loaded with 745 cups. How many of these do you expect will overflow when served ?
(*MBA, Delhi Univ., 2009*)

79. A manufacturing company has plants in India produces 40% of the total output with 10% defective rate and the average cost of producing $12.50 per unit. If a single unit is found to be defective, what is the probability it has come from India ?

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