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Tests of Hypothesis

INTRODUCTION

A hypothesis is an assumption about the population parameter to be tested based on sample information. The statistical testing of hypothesis is the most important technique in statistical inference. Hypothesis tests are widely used in business and industry for making decisions. It is here that probability and sampling theory plays an ever increasing role in constructing the criteria on which business decisions are made. Very often in practice we are called upon to make decisions about population on the basis of sample information. For example, we may wish to decide on the basis of sample data whether a new medicine is really effective in curing a disease, whether one training procedure is better than another, etc. Such decisions are called *statistical decisions*.

In attempting to reach decisions, it is useful to make assumptions or guesses about the populations involved. Such assumptions, which may or may not be true, are called *statistical hypothesis* and in general are statements about the probability distributions of the population. The hypothesis is made about the value of some parameter, but the only facts available to estimate the true parameter are those provided by a sample. If the sample statistic differs from the hypothesis made about the population parameter, a decision must be made as to whether or not this difference is significant. If it is, the hypothesis is rejected. If not, it must be accepted. Hence, the term "tests of hypothesis".

Now, if θ be the parameter of the population and is the estimate of θ in the random sample drawn from the population, then the difference between θ and $\hat{\theta}$ should be small. In fact, there will be some difference between θ and $\hat{\theta}$ because $\hat{\theta}$ is based on sample observations and is different for different samples. Such a difference is known as difference due to sampling fluctuations. If the difference between θ and $\hat{\theta}$ is large, then the probability that it is exclusively due to sampling fluctuations is small. Difference which is caused because of sampling fluctuations is called insignificant difference and the difference due to some other reasons is known as significant difference. A significant difference arises due to the fact that either the sampling procedure is not purely random or sample is not from the given population.

Procedure of Hypothesis Testing

The general procedure followed in testing hypothesis comprises the following steps :

(1) *Set up a hypothesis*. The first step in hypothesis testing is to establish the hypothesis to be tested. Since statistical hypothesis are usually assumptions about the value of some unknown parameter, the hypothesis specifies a numerical value or range of values for the parameter. The conventional approach to hypothesis testing is not to construct single hypothesis about the population parameter, but rather to set up two different hypothesis. These hypothesis are normally referred to as (*i*) null hypothesis denoted by H_0, and (*ii*) alternative hypothesis denoted by H_1.

The null hypothesis asserts that there is no true difference in the sample statistic and population parameter under consideration (hence the word "null" which means invalid, void or amounting to nothing) and that the difference found is accidental arising out of fluctuations of sampling.

A hypothesis which states that there is no difference between assumed and actual value of the parameter is the null hypothesis and the hypothesis that is different from the null hypothesis is the alternative hypothesis. If the sample information leads us to reject H_0, then we will accept the alternative hypothesis H_1. Thus, the two hypothesis are constructed so that if one is true, the other is false and *vice versa*.

The rejection of the null hypothesis indicates that the differences have statistical significance and the acceptance of the null hypothesis indicates that the differences are due to chance. As against the null hypothesis, the alternative hypothesis specifies those values that the researcher believes to hold true. The alternative hypothesis may embrace the whole range of values rather than single point.

(2) *Set up a suitable significance level.* Having set up a hypothesis, the next step is to select a suitable level of significance. The confidence with which an experimenter rejects or retains null hypothesis depends on the significance level adopted. The level of significance, usually denoted by "α", is generally specified before any samples are drawn, so that results obtained will not influence our choice. Though any level of significance can be adopted, in practice, we either take 5 per cent or 1 per cent level of significance. When we take 5 per cent level of significance then there are about 5 chances out of 100 that we would reject the null hypothesis when it should be accepted, *i.e.*, we are about 95% confident that we have made the right decision. When we test a hypothesis at a 1 per cent level of significance, there is only one chance out of 100 that we would reject the null hypothesis when it should be accepted, *i.e.*, we are about 99% confident that we have made the right decision. When the null hypothesis is rejected at $\alpha = 0.5$, the test result is said to be "significant". When the null hypothesis is rejected at $\alpha = 0.01$, the test result is said to be "highly significant".

(3) *Determination of a suitable test statistic.* The third step is to determine a suitable test statistic and its distribution. Many of the test statistics that we shall encounter will be of the following form :

$$\text{Test statistic} = \frac{\text{Sample statistic} - \text{Hypothesised population parameter}}{\text{Standard error of the sample statistic}}$$

(4) *Determine the critical region.* It is important to specify, before the sample is taken, which values of the test statistic will lead to a rejection of H_0 and which lead to acceptance of H_0. The former is called the *critical region.* The value of α, the level of significance, indicates the importance that one attaches to the consequences associated with incorrectly rejecting H_0. It can be shown that when the level of significance is α, the optimal critical region for a two-sided test consists of that $\alpha/2$ per cent of the area in the right-hand tail of the distribution plus that $\alpha/2$ per cent in the left hand tail. Thus, establishing a critical region is similar to determining a $100\,(1-\alpha)\%$ confidence interval. In general, one uses a level of significance of $\alpha = 0.05$, indicating that one is willing to accept a 5 per cent chance of being wrong to reject H_0.

(5) *Doing computations.* The fifth step in testing hypothesis is the performance of various computations from a random sample of size *n*, necessary for the test statistic obtained in step (3). Then, we need to see whether sample result falls in the critical region or in the acceptance regions.

(6) *Making decisions.* Finally, we may draw statistical conclusions and the management may take decisions. A statistical decision or conclusion comprises either accepting the null hypothesis or rejecting it. The decision will depend on whether the computed value of the test criterion falls in the region of rejection or the region of acceptance. If the hypothesis is being tested at 5 per cent level of significance and the observed set of results has a probability less than 5 per cent, we reject the null hypothesis and the difference between the sample statistic and the hypothetical population parameter is considered to be significant. On the other hand, if the testing statistic falls in the region of non-rejection, the null hypothesis is accepted and the difference between the sample statistic and the hypothetical population parameter is not regarded as significant, *i.e.*, it can be explained by chance variations.

Type I and Type II Errors

When a statistical hypothesis is tested, there are four possible results :

(1) The hypothesis is true but our test rejects it.

(2) The hypothesis is false but our test accepts it.

(3) The hypothesis is true and our test accepts it.

(4) The hypothesis is false and our test rejects it.

Obviously, the first two possibilities lead to errors. If we reject a hypothesis when it should be accepted (possibility No. 1), we say that a *Type I error* has been made. On the other hand, if we accept a hypothesis when it should be rejected (possibility No. 2), we say that a *Type II error* has been made. In either case a wrong decision or error in judgment has occurred.

TWO KINDS OF ERRORS IN HYPOTHESIS TESTING

Condition

Decision	H_0 : True	H_0 : False
Accept H_0	Correct Decision	Type II Error
Reject H_0	Type I Error	Correct Decision

The probability of committing a *type I error* is designated as "α" and is called the *level of significance*. Therefore,

$$\alpha = P_r \text{ [Type I error]}$$
$$= P_r \text{ [Rejecting } H_0/H_0 \text{ is true]}$$

must be the complement of

$$(1-\alpha) = P_r \text{ [Accepting } H_0/H_0 \text{ is true]}.$$

This probability $(1-\alpha)$ corresponds to the concept of 100 $(1-\alpha)$% confidence interval. Our efforts would obviously be to have a small probability of making a type I error. Hence the objective is to construct the test to *minimise* α.

Similarly, the probability of committing a type II error is designated by β. Thus

$$\beta = P_r \text{ [Type II error]}$$
$$= P_r \text{ [Accepting } H_0/H_0 \text{ is false]}$$

and $$(1-\beta) = P_r \text{ [Rejecting } H_0/H_0 \text{ is false]}.$$

This probability $(1-\beta)$ is known as the *power* of a statistical test.

The following table gives the probabilities associated with each of the four cells shown in the previous table :

The decision is :	The null hypothesis is True	False
Accept H_0	$(1-\alpha)$ Confidence level	β
Reject H_0	α	$(1-\beta)$ Power of the test
Sum	1.00	1.00

Note that the probability of each decision outcome is a conditional probability and the elements in the same column sum to 1.0, since the events with which they are associated are complement. However, α and β are not independent of each other, nor are they independent of the sample size *n*. When *n* is fixed, if α is lowered then β normally rises and *vice versa*. If *n* is increased, it is possible for both α and β to decrease. Since, increasing the sample size involves money and time, therefore, one should decide how much additional money and time, he is willing to spare on increasing the sample size in order to reduce the size of α and β.

In order for any tests of hypothesis or rules of decisions to be good, they must be designed so as to minimise errors of decision. However, this is not a simple matter, since for a given sample size, an attempt to decrease one type of error is accompanied in general by an increase in other type of error. The probability of making type I error is fixed in advance by the choice of level of significance employed in the test. We can make the type I error as small as we please, by lowering the level of significance. But, by doing so, we increase the chance of accepting a false hypothesis, *i.e.*, of making a type II error. It follows that it is impossible to minimise both errors simultaneously. In the long run, errors of type I are perhaps more likely to prove serious in research programmes in social sciences than are errors of type II. In practice, one type of error may be more serious than the other and so a compromise should be reached in favour of limitations of the more serious error. The only way to reduce both types of error is to increase the sample size which may or may not be possible.

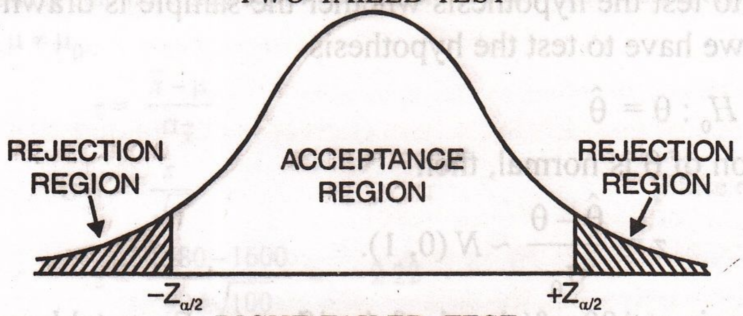
One-Tailed and Two-Tailed Tests

Basically, there are three kinds of problems of tests of hypothesis. They include :

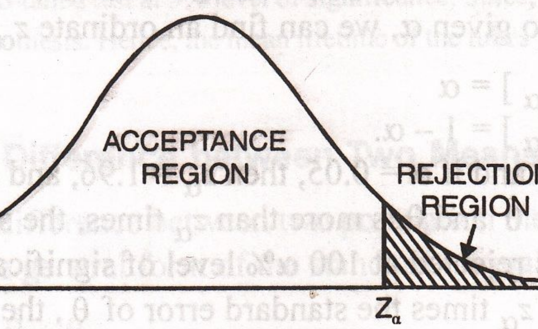
(*i*) two-tailed tests, (*ii*) right-tailed test, and (*iii*) left-tailed test.

Two-tailed test is that where the hypothesis about the population mean is rejected for value of falling into either tail of the sampling distribution. When the hypothesis about population mean is rejected only for value of falling into one of the tails of the sampling distribution, then it is known as one-tailed test. If, it is right tail then it is called right-tailed test or one-sided alternative to the right and if it is on the left tail, then, it is one-sided alternative to the left and called left-tailed test. For example, $H_0 : \mu = 100$ tested against $H_1 : \mu > 100$ or < 100 is one-tailed test since H_1 specifies that μ lies on particular side of 100. The same null hypothesis tested against $H_1 : \mu \neq 100$ is a two-tailed test since μ can be on either side of 100. The following diagrams would make it more clear :

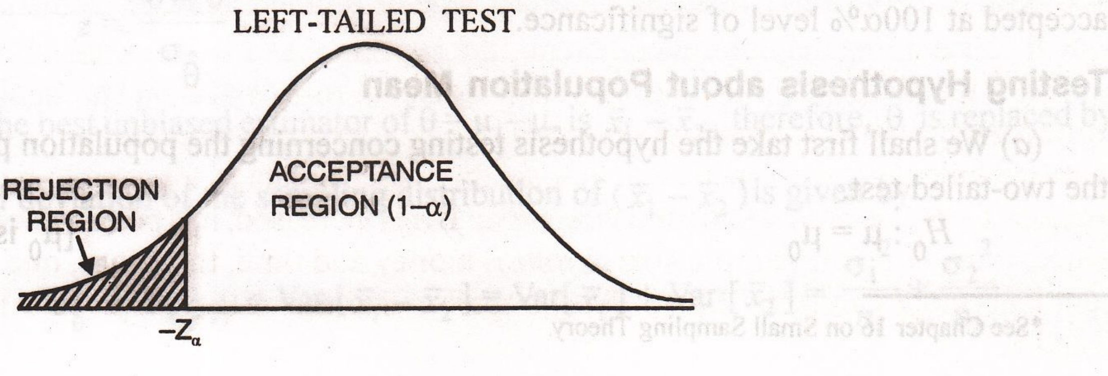
TWO-TAILED TEST



RIGHT-TAILED TEST



LEFT-TAILED TEST



The following table gives critical values of z for both one-tailed and two-tailed tests at various levels of significance. Critical values of z for other levels of significance are found by use of the table of normal curve areas :

Level of Significance	0.10	0.05	0.01	0.005	0.0002
Critical value of z for one-tailed tests	-1.28 or 1.28	-1.645 or 1.645	-2.33 or 2.33	-2.58 or 2.58	-2.88 or 2.88
Critical value of z for two-tailed tests	-1.645 and 1.645	-1.96 and 1.96	-2.58 and 2.58	-2.81 and 2.81	-3.08 and 3.08

Tests of Hypothesis Concerning Large Samples

Though, it is difficult to draw a clear-cut line of demarcation between large and small samples, it is generally agreed that if the size of sample exceeds 30, it should be regarded as a large sample. The tests of significance used for large samples are different from the ones used for small samples* for the reason that the assumptions we make in case of large samples do not hold for small samples. Tests of hypothesis involving large samples are based on the following assumptions :

(1) The sampling distribution of a sample statistic is approximately normal.

(2) Values given by the samples are sufficiently close to the population value and can be used in its place for the standard error of the estimate.

Thus, we have seen that the normal distribution plays a vital role in tests of hypothesis based on large samples (central limit theorem).

Suppose $\hat{\theta}$ is an unbiased estimate of θ, the population parameter. On the basis of $\hat{\theta}$, taken from sample observations, it is to test the hypothesis whether the sample is drawn from a population whose parameter value is θ, *i.e.*, we have to test the hypothesis

$$H_0 : \theta = \hat{\theta}$$

If sampling distribution of θ is normal, then

$$z = \frac{\hat{\theta} - \theta}{\sigma_{\hat{\theta}}} \sim N(0,\ 1).$$

Let us test the hypothesis at 100 α% level of significance. From tables of area under the standard normal curve corresponding to given α, we can find an ordinate z_α such that

$$P_r[\,|z_\alpha| > z_\alpha\,] = \alpha$$
$$P_r[-z_\alpha \le z \le z_\alpha] = 1 - \alpha.$$

If $\alpha = .01$, then $z_\alpha = 2.58$ and if $\alpha = 0.05$, then $z_\alpha = 1.96$, and so on.

If the difference between $\hat{\theta}$ and θ is more than z_α times, the standard error of $\hat{\theta}$, the difference is regarded significant and H_0 is rejected at 100 α% level of significance and if the difference between θ and $\hat{\theta}$ is less than or equal to z_α times the standard error of $\hat{\theta}$, the difference is insignificant and H_0 is accepted at 100α% level of significance.

Testing Hypothesis about Population Mean

(*a*) We shall first take the hypothesis testing concerning the population parameter μ by considering the two-tailed test.

$H_0 : \mu = \mu_0$ [μ_0 is hypothesised value of μ]

*See Chapter 16 on Small Sampling Theory.

Since the best unbiased estimator of μ is the sample mean $\bar{x}$, therefore, we shall focus our attention on the sampling distribution of $\bar{x}$. From central limit theorem, we know

$$\bar{x} \sim N(\mu, \sigma_{\bar{x}})$$

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}}$$

where $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$ [If σ is known.]

$= \dfrac{s}{\sqrt{n}}$ [If σ is unknown for large samples.]

If the calculated value of $z < -z_{\alpha/2}$ or $> z_{\alpha/2}$, the null hypothesis is rejected.

(*b*) If the hypothesis involves a right-tailed test. For example,

$$H_0 : \mu \le \mu_0 \text{ and } H_1 : \mu > \mu_0.$$

For the calculated value $z > z_\alpha$, the null hypothesis is rejected.

(*c*) If the hypothesis involves a left-tailed test, *i.e.*,

$$H_0 : \mu \ge \mu_0 \text{ and } H_1 : \mu < \mu_0$$

then for the value $z < -z_\alpha$, the null hypothesis is rejected.

Illustration 1. The mean lifetime of a sample of 100 light tubes produced by a company is found to be 1,580 hours with standard deviation of 90 hours. Test the hypothesis that the mean lifetime of the tubes produced by the company is 1,600 hours.

Solution. The null hypothesis is that there is no significant difference between the sample mean and hypothetical population mean, *i.e.*, $H_0 : \mu = \mu_0$ and $H_1 : \mu \ne \mu_0$.

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}}$$

where $\sigma_{\bar{x}} = \dfrac{s}{\sqrt{n}}$ [Since σ is unknown for large samples.]

$$z = \frac{1580 - 1600}{90/\sqrt{100}} = -2.22$$

The critical value is $z = \pm\, 1.96$ for a two-tailed test at 5% level of significance. Since, the computed value of $z = -2.22$ falls in the rejection region, we reject the null hypothesis. Hence, the mean lifetime of the tubes produced by the company may not be 1,600 hours.

Testing Hypothesis about the Difference between Two Means

The test statistic for testing the difference between two population means, when the populations are normally distributed, is based on the general form of the standard normal statistic as given below :

$$z = \frac{\hat{\theta} - \theta}{\sigma_{\hat{\theta}}}$$

where $\theta = \mu_1 - \mu_2$. Since, the best unbiased estimator of $\theta = \mu_1 - \mu_2$ is $\bar{x}_1 - \bar{x}_2$, therefore, $\hat{\theta}$ is replaced by $\bar{x}_1 - \bar{x}_2$. $\sigma_{\hat{\theta}}$, the standard deviation of the sampling distribution of $(\bar{x}_1 - \bar{x}_2)$ is given by

$$\sigma^2_{\hat{\theta}} = \sigma^2_{\bar{x}_1 - \bar{x}_2} = \text{Var}\,[\bar{x}_1 - \bar{x}_2] = \text{Var}[\bar{x}_1] + \text{Var}\,[\bar{x}_2] = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

Therefore, the z statistic is given by

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

The null hypothesis is $H_0 : \mu_1 - \mu_2 = 0$

Then, the z statistic is reduced to $z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$

At 5% level of significance, the critical value of z for two-tailed test $= \pm 1.96$. If the computed value of z is greater than $+1.96$ or less than -1.96, then reject H_0, otherwise accept H_0.

In case σ_1^2 and σ_2^2 are not known then for large samples, s_1^2 and s_2^2 can be used instead.

Illustration 2. You are working as a purchase manager for a company. The following information has been supplied to you by two manufacturers of electric bulbs :

	Company A	Company B
Mean life (in hours)	1,300	1,288
Standard deviation (in hours)	82	93
Sample size	100	100

Which brand of bulbs are you going to purchase if you desire to take a risk of 5% ? *(MBA, Kumaun Univ., 2002)*

Solution. Let us take the null hypothesis that there is no significant difference in the quality of the two brands of bulbs, *i.e.*, $H_0 : \mu_1 = \mu_2$

[Since σ_1^2 and σ_2^2 are not known, therefore, can be replaced by s_1^2 and s_2^2.

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{1300 - 1288}{\sqrt{\frac{(82)^2}{100} + \frac{(93)^2}{100}}}$$

$$= \frac{12}{\sqrt{67.24 + 86.49}} = \frac{12}{12.399} = 0.968$$

Since our computed value of $z = 0.968$ is less than critical value of $z = 1.96$ (5% level), we accept the null hypothesis. Hence the quality of two brands of bulbs do not differ significantly.

Test of Hypothesis Concerning Attributes

As distinguished from variables where quantitative measurement of a phenomenon is possible in case of attributes we can only find out the presence or absence of a certain characteristic. For example, in the study of attribute 'employment' a sample may be taken and people classified as employed and unemployed. With such data, the binomial type of problem may be formed. The selection of an individual on sampling may be called 'event', the appearance of an attribute 'A' may be taken as "success" and its non-appearance, as "failure". The sampling distribution of the number of successes, being a binomial probability model would have its mean $\mu = np$ and its standard deviation $\sigma = \sqrt{npq}$.

Then $$z = \frac{x - np}{\sqrt{npq}} \sim N(0, 1).$$

Illustration 3. In 600 throws of six-faced die, odd points appeared 360 times. Would you say that the die is fair at 5% level of significance ?

Solution. Let us take the hypothesis that the die is not biased.

$$p = q = \frac{1}{2},\ n = 600,\ np = 300.$$

Applying the formula ;

$$z = \frac{x - np}{\sqrt{npq}} = \frac{360 - 300}{\sqrt{600 \times \frac{1}{2} \times \frac{1}{2}}} = \frac{60}{12.25} = 4.9.$$

Since, the computed value of z is greater than the table value (1.96 at 5% level of significance), the hypothesis is rejected. Hence, the die does not seem to be fair.

Testing Hypothesis about a Population Proportion

The population parameter of interest is population proportion π. If the sample size is large, then sample proportion p will be approximately normally distributed. Then

$$z = \frac{\hat{\theta} - \theta}{\sigma_{\hat{\theta}}} \sim N(0, 1).$$

The null hypothesis is that there is no significant difference between the sample proportion and population proportion, *i.e.*, $H_0 : p = \pi$

Since the sample proportion p is unbiased estimator of π,

$$\therefore \quad z = \frac{p - \pi}{\sigma_p};\ \text{where } \sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}}.$$

Therefore, the statistic

$$z = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} \sim N(0, 1).$$

If $|z| < z_\alpha$, the null hypothesis is rejected with 100α% level of significance.

Illustration 4. A sales clerk in the departmental store claims that 60% of the shoppers entering the store leave without making a purchase. A random sample of 50 shoppers showed that 35 of them left without buying anything. Are these sample results consistent with the claim of the sales clerk ? Use a level of significance of 0.05.

Solution. The null hypothesis is

$$H_0 : \pi = 0.60.$$

The sample proportion $\quad p = \frac{35}{50} = 0.70.$

Using the z statistic, we have

$$z = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} = \frac{0.70 - 0.60}{\sqrt{(0.6)(0.4)/50}} = 1.45.$$

The critical value of z is 1.64 at 5% level of significance.

Since, the computed value of $z = 1.45$ is less than the critical value of $z = 1.64$, therefore, the null hypothesis cannot be rejected. Hence, based on this sample data, we cannot reject the claim of the sales clerk.

Testing Hypothesis about the Difference Between Two Proportions

Let p_1 and p_2 be the sample proportions obtained in large samples of sizes n_1 and n_2 drawn from respective populations having proportions π_1 and π_2. We can test the null hypothesis that there is no difference between the population proportions, *i.e.*,

$$H_0 : \pi_1 = \pi_2.$$

As shown in the earlier chapter, the sampling distribution of differences in proportion, $p_1 - p_2$ is normally distributed with mean

$$\mu_{p_1 - p_2} = \pi_1 - \pi_2$$

and standard deviation

$$\sigma_{p_1 - p_2} = \sqrt{\frac{\pi_1(1-\pi_1)}{n_1} + \frac{\pi_2(1-\pi_2)}{n_2}}.$$

Therefore, the statistic is

$$z = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sqrt{\frac{\pi_1(1-\pi_1)}{n_1} + \frac{\pi_2(1-\pi_2)}{n_2}}}$$

If the null hypothesis is true, p_1 and p_2 are two independent unbiased estimators of the same parameter $\pi_1 = \pi_2 = \pi$. Thus, our procedure is to pool our observations to obtain the best estimate of the common value π. The pooled estimate of π is the weighted mean of the two sample proportions, *i.e.*,

$$p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

Our test statistic then becomes

$$z = \frac{p_1 - p_2}{\sigma_{p_1 - p_2}}, \quad \text{where } \sigma_{p_1 - p_2} = \sqrt{p(1-p)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

Illustration 5. In a random sample of 100 persons taken from village *A*, 60 are found to be consuming tea. In another sample of 200 persons taken from village *B*, 100 persons are found to be consuming tea. Do the data reveal significant difference between the two villages so far as the habit of taking tea is concerned ? (*MBA, Delhi Univ., 1999*)

Solution. Let us take the hypothesis that there is no significant difference between the two villages so far as the habit of taking tea is concerned, *i.e.*, $\pi_1 = \pi_2$.

We are given :

$$p_1 = \frac{x_1}{n_1} = \frac{60}{100} = 0.6, n_1 = 100.$$

$$p_2 = \frac{x_2}{n_2} = \frac{100}{200} = 0.5, n_2 = 200.$$

The appropriate statistics to be used here is given by

$$z = \frac{p_1 - p_2}{\sqrt{p(1-p)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where

$$p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{60 + 100}{100 + 200} = 0.53$$

$$z = \frac{0.6 - 0.5}{\sqrt{(0.53)(0.47)\left(\frac{1}{100} + \frac{1}{200}\right)}} = \frac{0.1}{\sqrt{(0.53)(0.47)(0.015)}}$$

$$= \frac{0.1}{\sqrt{0.0037}} = \frac{0.1}{0.0608} = 1.64.$$

Since, the computed value of z is less than the critical value of $z = 1.96$ at 5% level of significance, therefore, we accept the hypothesis. Hence, we conclude that there is no significant difference in the habit of taking tea in the two villages A and B.

Illustration 6. Before an increase in excise duty on tea, 400 people out of a sample of 500 people were found to be tea drinkers. After an increase in duty, 400 people were tea drinkers in a sample of 600 people. State, whether there is a significant decrease in the consumption of tea. *(MBA, Delhi Univ., 2002)*

Solution. Let us take the hypothesis that there is no significant decrease in the consumption of tea after the increase in duty, *i.e.*, $\pi_1 = \pi_2$.

We are given

$$p_1 = \frac{x_1}{n_1} = \frac{400}{500} = 0.8,\ n_1 = 500.$$

$$p_2 = \frac{x_2}{n_2} = \frac{400}{600} = 0.667,\ n_2 = 600.$$

The appropriate test statistic to be used here is given by

$$z = \frac{p_1 - p_2}{\sqrt{p(1-p)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where

$$p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{400 + 400}{500 + 600} = 0.73$$

$$z = \frac{0.8 - 0.667}{\sqrt{(0.73)(0.27)\left(\frac{1}{500} + \frac{1}{600}\right)}} = \frac{0.133}{\sqrt{(0.73)(0.27)(0.0037)}}$$

$$= \frac{0.133}{\sqrt{0.00073}} = \frac{0.133}{0.027} = 4.93.$$

Since, the computed value of z is greater than the critical value of $z = 1.96$ at 5% level of significance, therefore, hypothesis is rejected. Hence, there is a significant decrease in the consumption of tea after an increase in duty.

MISCELLANEOUS ILLUSTRATIONS

Illustration 7. From the following data obtained from a sample of 1,000 persons, calculate the standard error of mean :

Weekly Earnings (Rs. hundred) :	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80
No. of persons :	50	100	150	200	200	100	100	100

Is it likely that the sample has come fom the population with an average weekly earnings of Rs. 4,200.

Solution.

CALCULATION OF STANDARD DEVIATION

Weekly Earnings (Rs. hundred)	X	f	$(X-45)/10$ d	fd	fd^2
0–10	5	50	–4	–200	800
10–20	15	100	–3	–300	900
20–30	25	150	–2	–300	600
30–40	35	200	–1	–200	200
40–50	45	200	0	0	0
50–60	55	100	+1	+100	100
60–70	65	100	+2	+200	400
70–80	75	100	+3	+300	900
		$n = 1000$		$\Sigma fd = -400$	$\Sigma fd^2 = 3{,}900$

$$\bar{x} = A + \frac{\Sigma fd}{n} \times i = 45 - \frac{400}{1000} \times 10 = 41$$

$$s = \sqrt{\frac{\Sigma fd^2}{n} - \left(\frac{\Sigma fd}{n}\right)^2} \times i = \sqrt{\frac{3900}{1000} - \left(\frac{-400}{1000}\right)^2} \times 10$$

$$= 1.934 \times 10 = 19.34$$

$$\sigma_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{19.34}{\sqrt{1000}} = \frac{19.34}{31.62} = 0.612$$

Therefore, the standard error of mean is 0.612.

$$\bar{x} = 41, \mu = 42, \sigma_{\bar{x}} = 0.612.$$

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{41-42}{0.612} = -1.634$$

Since, the computed value of z is less than the critical value of $z = \pm 1.96$, it is not significant and hence there is no significant difference between the sample average and the population average weekly earnings and the difference could have arisen due to fluctuations of sampling.

Illustration 8. A sample of 400 managers is found to have a mean height of 171.38 cms. Can it be reasonably regarded as a sample from a large population of mean height 171.17 cms and standard deviation of 3.30 cms ?

Solution. The null hypothesis is that there is no significant difference between the sample mean height and the population mean height.

Given $\bar{x} = 171.38, \mu = 171.17, n = 400$, and $\sigma = 3.30$

Applying the test statistic

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{171.38-171.17}{3.30/\sqrt{400}} = \frac{0.21}{0.16} = 1.31$$

Since, the computed value of $z = 1.31$ is less than critical value of $z = 1.96$ at 5% level of significance, therefore, the null hypothesis is accepted. Hence, there is no significant difference between the sample mean height and population mean height.

Illustration 9. Intelligence test given to two groups of boys and girls gave the following information :

	Mean Score	*S.D.*	*Number*
Girls	75	10	50
Boys	70	12	100

Is the difference in the mean scores of boys and girls statistically significant ? *(MBA, S.V. Univ., 2004; MBA, DU,2005)*

Solution. Let us take the hypothesis that the difference in the mean score of boys and girls is not significant, *i.e.*, $\mu_1 = \mu_2$

We are given $\bar{x}_1 = 75, \bar{x}_2 = 70, s_1^2 = 100, s_2^2 = 144, n_1 = 50, n_2 = 100.$

The appropriate statistic to be used here is given by

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \quad \left[\text{Since } \sigma_1^2 = s_1^2; \sigma_2^2 = s_2^2 \text{ and } \mu_1 = \mu_2\right]$$

$$= \frac{75-70}{\sqrt{\frac{100}{50} + \frac{144}{100}}} = \frac{5}{\sqrt{3.44}} = \frac{5}{1.855} = 2.695$$

Since, the computed value $z = 2.695$ is greater than the critical value of $z = 2.58$ at 1% level of significance, therefore, the hypothesis is rejected. Hence, the difference in the mean score of boys and girls is statistically significant.

Illustration 10. In a survey of buying habits, 400 women shoppers are chosen at random in super market A. Their average weekly food expenditure is Rs. 250 with a standard deviation of Rs. 40. For another group of 400 women shoppers chosen at random in super market B located in another area of the same city, the average weekly food expenditure is Rs. 220 with a standard deviation of Rs. 55.Test at 1% level of significance, whether the average weekly food expenditures of the populations of women shoppers are equal.

Solution. The null hypothesis is that the average weekly food expenditures of the two populations are same, *i.e.*,$\mu_1 = \mu_2$.

Since σ_1^2 and σ_2^2 (the population variances) are not known, we can estimate from the sample variances (provided sample size is large), *i.e.*,

$$\sigma_1^2 = s_1^2, \quad \sigma_2^2 = s_2^2$$

Given : $n_1 = 400, n_2 = 400, \bar{x}_1 = 250, \bar{x}_2 = 220, s_1 = 40, s_2 = 55$

Applying the test statistic

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \quad [\text{since } \sigma_1^2 = s_1^2; \sigma_2^2 = s_2^2]$$

$$= \frac{250-220}{\sqrt{\frac{(40)^2}{400}+\frac{(55)^2}{400}}} = \frac{30\times 20}{\sqrt{4625}} = \frac{600}{68.01} = 8.822$$

Since, the value of z is much greater than 3, the null hypothesis is rejected. Hence, the average weekly expenditure of two populations of women shoppers differ significantly.

Illustration 11. A dice is thrown 49152 times and of these 25145 yielded either 4 or 5 or 6. Is this consistent with the hypothesis that the dice must be unbiased ?

Solution. Let the coming of 4, 5 or 6 be termed as success, then the null hypothesis can be stated as that the dice is unbiased.

Given, $n = 49152$, and p = proportion of success $= \frac{25145}{49152} = 0.512$

The appropriate statistic to be used is

$$z = \frac{p-\pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} = \frac{p-\pi}{\sqrt{\frac{p(1-p)}{n}}}$$

$$= \frac{0.512-0.5}{\sqrt{\frac{(0.512)(0.488)}{49152}}} = \frac{0.012}{0.002} = 6.0$$

Since, the computed value of $z = 6$ is much greater than the critical value of $z = 3$, it is significant, and therefore, null hypothesis is rejected. Hence, the dice is certainly biased.

Illustration 12. An ambulance service claims that it takes, on the average, 8.9 minutes to reach its destination in emergency calls. To check on this claim, the agency which licenses ambulance services has then timed on 50 emergency calls, getting a mean of 9.3 minutes with a standard deviation of 1.8 minutes. At the level of significance of 0.05, does this constitute evidence that the figure claimed is too low ?

Solution. Let us take the hypothesis that there is no significant difference between the figure observed and the figure claimed, *i.e.*, 9.3 and 8.9.

We are given : $\mu = 8.9$, $\bar{x} = 9.3$, $s = 1.8$, $n = 50$.

The appropriate statistic to be used is given by

$$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \qquad [\sigma^2 = s^2 \text{ for large samples}]$$

$$= \frac{9.3-8.9}{1.8/\sqrt{50}} = \frac{0.4}{0.25} = 1.6$$

Since, the computed value of $z = 1.6$ is less than the critical value of $z = 1.96$ at 5% level of significance, therefore, the hypothesis is accepted. Hence, there is no significant difference between the average figure observed and the average figure claimed.

Illustration 13. A coin is tossed 100 times under identical conditions independently yielding 30 heads and 70 tails. Test at 1% level of significance, whether or not the coin is unbiased. State clearly the null hypothesis and the alternative hypothesis.

Solution. Let the null hypothesis be that the coin is unbiased. If p is the probability of getting head, then

$$H_0 : \pi = 0.5 \text{ and } H_1 : \pi \neq 0.5.$$

The appropriate statistic to be used here is z-statistic. We are given :

$p = 0.3$, $\pi = 0.5$ and $n = 100$

$$z = \frac{p-\pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} = \frac{0.3-0.5}{\sqrt{\frac{(0.5)(0.5)}{100}}} = \frac{-0.2\times 10}{\sqrt{0.25}} = -4.$$

Since, the computed value of $z = -4$ is greater than critical values of $z = \pm 2.58$ at 1% level of significance, therefore, we reject the null hypothesis. Hence, the coin is biased.

Illustration 14. A product is produced in two ways. A pilot test on 64 times from each method indicates that the product of Method 1 has sample mean tensile strength 106 lbs and a standard deviation 12 lbs, whereas in Method 2 the corresponding values of mean and standard deviation are 100 lbs and 10 lbs respectively. Greater tensile strength in the product is preferable. Use an appropriate large sample test at 5% level of significance to test whether or not Method 1 is better for processing the product. State clearly the null hypothesis.

(MBA, Delhi Univ., 2003)

Solution. Let the null hypothesis be that there is no significant difference between Method 1 and Method 2, *i.e.*, $H_0 : \mu_1 = \mu_2$

We are given

$$\bar{x}_1 = 106, \bar{x}_2 = 106, s_1 = 12, s_2 = 10, n_1 = n_2 = 64$$

Using the test statistic $z = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$

$$= \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \frac{106-100}{\sqrt{\dfrac{(12)^2}{64} + \dfrac{(10)^2}{64}}} = \frac{6 \times 8}{\sqrt{244}} = 3.07.$$

Since, the computed value of $z = 3.07$ is greater than the critical value of $z = 1.64$ at 5% level of significance, the null hypothesis is rejected. Hence, Method 1 is better than Method 2.

Illustration 15. A company is considering two different television advertisements for the promotion of a new product. Management believes that advertisement *A* is more effective than advertisement *B*. Two test market areas with virtually identical consumer characteristics are selected : advertisement *A* is used in one area and advertisement *B* in the other area. In a random sample of 60 customers who saw advertisement *A*, 18 tried the product. In a random sample of 100 customers who saw advertisement *B*, 22 tried the product. Does this indicate that advertisement *A* is more effective than advertisement *B*, if a 5% level of significance is used ? *(MBA, IGNOU 2002; MBA, Delhi Univ., 2005)*

Solution. Let the null hypothesis be that there is no significant difference in the effectiveness of the two advertisements *A* and *B*, *i.e.*, $H_0 : \pi_1 = \pi_2$.

The appropriate statistic to be used is

$$z = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sigma_{p_1 - p_2}} = \frac{p_1 - p_2}{\sqrt{p(1-p)\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}} \quad [\because \pi_1 = \pi_2]$$

where $p_1 = \dfrac{x_1}{n_1} = \dfrac{18}{60} = 0.30, n_1 = 60$

$$p_2 = \frac{x_2}{n_2} = \frac{22}{100} = 0.22, n_2 = 100$$

and $p = \dfrac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \dfrac{x_1 + x_2}{n_1 + n_2} = \dfrac{40}{160} = 0.25$

$$z = \frac{0.30 - 0.22}{\sqrt{(0.25)(0.75)\left(\dfrac{1}{60} + \dfrac{1}{100}\right)}} = \frac{0.08}{\sqrt{0.005}} = \frac{0.08}{0.071} = 1.13$$

Since, the computed value of $z = 1.13$ is less than the critical value of $z = 1.645$* at 5% level of significance, therefore, the null hypothesis is accepted. Hence, there is no significant difference in the effectiveness of the two advertisements *A* and *B*.

Illustration 16. 500 units from a factory are inspected and 12 are found to be defective, 800 units from another factory are inspected and 12 are found to be defective. Can it be concluded at 5% level of significance that production at second factory is better than in first factory ? *(MBA, Delhi Univ., 2007)*

Solution. Let us, take the null hypothesis that there is no significant difference in the proportion of defective items in the two factories.

$$p_1 = \frac{x_1}{n_1} = \frac{12}{500} = 0.024; \quad p_2 = \frac{x_2}{n_2} = \frac{12}{800} = 0.015$$

$$z = \frac{p_1 - p_2}{\sqrt{p(1-p)\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$$

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* Normally in testing hypothesis, we use two-tailed test and the critical value of *z* at 5% level is 1.96. In this question, one-tailed test has been used and the critical value at 5% is 1.645.

$$p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{12 + 12}{500 + 800} = 0.018$$

$$z = \frac{0.024 - 0.015}{\sqrt{(0.018)\,(0.982)\,(0.00325)}} = \frac{0.009}{0.0076} = 1.184.$$

Since, the computed value of z is less than the critical value of $z = 1.96$ at 5% level of significance, therefore, our null hypothesis holds good. Hence, we cannot conclude that the production in the second factory is better than in the first factory.

Illustration 17. A buyer of electric bulbs bought 100 bulbs each of two famous brands. Upon testing these he found that brand A had a mean life of 1500 hours with a standard deviation of 50 hours whereas brand B had a mean life of 1530 hours with a standard deviation of 60 hours. Can it be concluded at 5 per cent level of significance that the two brands differ significantly in quality of the bulbs.

Solution. Let us, take the null hypothesis that the two brands of bulbs do not differ significantly in quality.

We are given $\bar{x}_1 = 1500$, $\bar{x}_2 = 1530$, $s_1 = 50$, $s_2 = 60$, $n_1 = 100$, $n_2 = 100$.

The appropriate statistic to be used here is given by :

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{1500 - 1530}{\sqrt{\frac{(50)^2}{100} + \frac{(60)^2}{100}}} = -\frac{30}{7.81} = -3.84.$$

Since, the computed value of z is more than the table value of $z = 1.96$ at 5% level of significance, the null hypothesis is rejected. Hence, the two brands of bulbs differ significantly in quality.

Illustration 18. Two types of new cars produced in India are tested for petrol mileage. One group consisting of 36 cars averaged 14 kms. per litre. While the other group consisting of 72 cars averaged 12.5 kms. per litre.

(*a*) What test statistic is appropriate, if

$$\sigma_1^2 = 1.5 \text{ and } \sigma_2^2 = 2.0\ ?$$

(*b*) Test, whether there exists a significant difference in the petrol consumption of these two types of cars (use $\alpha = 0.01$).

(*MBA, IIT Roorkee, 2000*)

Solution. We are given the following information :

$$n_1 = 36 \qquad \bar{x}_1 = 14 \qquad \sigma_1^2 = 1.5$$
$$n_2 = 72 \qquad \bar{x}_2 = 12.5 \qquad \sigma_2^2 = 2.0$$

(*a*) The appropriate test statistic to be used is the test of difference between two means.

(*b*) Let us take the null hypothesis that there is no significant difference in the petrol consumption of the two types of cars, *i.e.*, $H_0 : \mu_1 = \mu_2$.

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{14 - 12.5}{\sqrt{\frac{1.5}{36} + \frac{2}{72}}} = \frac{1.5}{0.264} = 5.68$$

Since, the calculated value of $z = 5.68$ is greater than the critical value of $z = 2.58$ (1% level), the null hypothesis is rejected. Hence, there is a significant difference in the petrol consumption of the two types of cars.

Illustration 19. The Educational Testing Service conducted a study to investigate difference between the scores of male and female students on the Scholastic Aptitude Test. The study identified a random sample of 562 female and 852 male students who had achieved the same high score on the mathematics portion of the test. That is, the female and male students were viewed as having similarly high abilities in mathematics. The verbal scores for the two samples are as given :

Female students : $\bar{x}_1 = 547$; $s_1 = 83$; Male student : $\bar{x}_2 = 525$; $s_2 = 78$

Do the data support the conclusion that given a population of female students and a population of male students with similarly high mathematics abilities, the female students will have a significantly higher verbal ability ? Test at a 5% level of significance. What is your conclusion ?

(*MBA, DU, 2003*)

Solution : Given : $\bar{x}_1 = 547$, $\bar{x}_2 = 525$, $s_1 = 83$, $s_2 = 78$, $n_1 = 562$, $n_2 = 852$

Let us take the null hypothesis be that there is no significant difference between male and female verbal ability, *i.e.*, $H_0 : \mu_1 - \mu_2 \geq 0$ and $H_1 : \mu_1 - \mu_2 < 0$

Using
$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{547 - 525}{\sqrt{\frac{(83)^2}{562} + \frac{(78)^2}{852}}}$$

$$= \frac{22}{\sqrt{\frac{6889}{562}+\frac{6084}{852}}} = \frac{22}{\sqrt{12.258+7.1408}} = \frac{22}{\sqrt{19.3988}} = \frac{22}{4.044} = 4.995$$

The computed value of z is greater than the table value of $z = \pm 1.96$. Therefore, reject the null hypothesis. Hence, there is a significant difference between the male and female verbal ability or female student have higher verbal ability.

Illustration 20. Record of several years of applicants for admission at FMS showed their mean score is 315. An administrator is interested in knowing whether the caliber of recent applicants has changed. For testing this hypothesis the scores of a sample of 100 applicants from the scores of recent applicants is obtained from admission office. The mean for this turned out to be 328. The sample standard deviation is 38, which may also be assumed for the population. Test the hypothesis using 5% level of significance. *(MBA, Delhi Univ., 2009)*

Solution. Let us take the hypothesis that there is no change in the calibre of recent applicants. The appropriate statistic to be used is given by

$$z = \frac{\bar{X} - \mu}{\sigma}\sqrt{n}$$

$$\bar{X} = 315, \mu = 328, \sigma = 38, n = 100$$

Substituting the values

$$z = \frac{315 - 328}{38} \times \sqrt{100} = \frac{13}{38} \times 10 = \frac{130}{38} = 3.421$$

Since the calculated value of z is more than the critical value of $z = 1.96$ at 5% level of significance, the hypothesis is rejected. Hence, there seems to be a change in the cablire of recent applicants.

PROBLEMS

1-A: Answer the following questions, each question carries **one** mark:

(*i*) What is a hypothesis ?

(*ii*) What is Null Hypothesis ? *(MBA, Madurai-Kamaraj Univ., 2003)*

(*iii*) What is standard error ?

(*iv*) Explain clearly the terms "standard error" and "sampling disribution". *(MBA, Madurai-Kamaraj Univ., 2008)*

(*v*) What is type I error ?

(*vi*) What is type II error ?

(*vii*) Differentiate between type I and type II error ?

(*viii*) What are the critical values at 1% and 5% level.

(*ix*) What is degrees of freedom ?

(*x*) What do you understand by large sample ?

1-B : Answer the following questions, each question carries marks **four** : *(M.Com., M.K. Univ., 2003)*

(*i*) State the procedure followed in testing of hypothesis.

(*ii*) Differentiate the following pairs of concepts :

(*i*) Statistic and parameter.

(*ii*) Null and Alternative Hypothesis.

(*iii*) Type I and Type II error.

(*iii*) Explain the different steps in testing hypothesis.

(*iv*) Define the two types of errors in testing a statistical hypothesis. *(MBA, Anna Univ., 2003)*

(*v*) Explain the difference between two proportions in test of hypothesis.

2. What is test of hypothesis ? Discuss various tests of hypothesis for the cases when the size of sample is large.
3. Explain the procedure generally followed in testing of a hypothesis.
4. Describe the various steps involved in testing of hypothesis. What is the role of standard error in testing of hypothesis ? *(M.Com., Delhi Univ.; MBA UP Tech. Univ., 2007)*
5. Define the standard error of a statistic. How is it helpful in testing of hypothesis and decision-making ?
6. What do you understand by null hypothesis and level of significance ? Explain with the help of an example.
7. Write short notes on the following :

(*i*) Type I and Type II error.

(*ii*) In the hypothesis testing process, what is the importance of *null hypothesis* ?

(*iii*) "In every hypothesis testing, the two types of errors are always present."– If this is true then explain what is the use of hypothesis testing ?

8. Explain clearly the procedure of testing hypothesis. Also point out the assumptions in hypothesis testing in large samples. (*M.Phil., Kurukshetra Univ.*)

9. Differentiate the following pairs of concepts :
   (*i*) Statistic and parameter.
   (*ii*) Critical region and acceptance region.
   (*iii*) Null and alternative hypothesis.
   (*iv*) One-tailed and two-tailed test.
   (*v*) Type I and Type II errors.

10. There is always a trade off between Type I and Type II errors. Discuss.

11. Intelligence test on two groups of boys and girls gave the following results :

	Mean	*S.D.*	*Sample size*
Girls :	75	15	150
Boys :	70	20	250

Is there a significant difference in the mean scores obtained by boys and girls ? (*M.Com., Madurai-Kamaraj Univ., 2002; MBA, Kumaun Univ., 2009*)

12. In a sample of 1000 persons from the village of Himachal Pradesh, 660 are found to be consumers of rice and the rest consumers of wheat. Can it be concluded that both the food articles are equally popular ?

13. Random samples of 100 bolts manufactured by machine '*A*' and 50 bolts from machine '*B*' showed 10 and 6 defective bolts respectively. Is there a significant difference in the performance of the two machines ?

14. The mean lifetime of 200 fluorescent light tubes made by a company gave mean lifetime of 1560 hours with a standard deviation of 50 hours. Is it likely that the sample has come from a population with a mean lifetime of 1,500 hours ?

15. A soap manufacturer wanted to know what percentage of the citizens of Mumbai use his soap. He conducted a survey and found that out of 500 persons selected at random for the purpose, only 10% use his soap. He spent Rs. 5 lakh on an advertisement campaign to attract more customers. In order to know the result of his campaign he conducted another survey and found that out of 600 persons 15% are using his soap. Do you think that the expenditure has really increased the percentage of citizens of Mumbai using his soap ?

16. A machine puts out 20 imperfect articles in a sample of 1000. After the machine is overhauled, it puts out 5 imperfect articles in a sample of 300. Has the machine improved ?

17. In North Delhi, out of a random sample of 500 households, 25% declared that they were regular readers of 'Femina'. In South Delhi, the proportion in a sample of 600 was 30%. Is there a significant difference in the two proportions ?

18. A firm found with the help of a sample survey of a city (size of a sample 900) the 3/4ths of the population consumes things produced by them. The firm then advertised the goods in paper and on radio. After one year, a sample of size 1000 reveals that proportions of consumers of the goods produced by the firm is 4/5th. Is this rise significant to indicate that the advertisement was effective ?

19. X is a normally distributed random variable. The variance of X is σ^2 and is known. Construct a test criterion to test the hypothesis that the mean of X is equal to μ_0 (a given constant). Suppose σ^2 was unknown, suggest an unbiased estimator of σ^2 and give (state) the test criterion to be used in this case.

20. A sample of size 400 was drawn and the sample mean was found to be 99. Test whether this sample could have come from a normal population with mean 100 and variance 64 at 5% level of significance.

21. A manufacturer claimed that at least 95% of the equipments which he supplied to a factory conformed to specifications. An examination of a sample of 200 pieces of equipment revealed that 18 were faulty. Test his claim at a significant level (*i*) .05; (*ii*) 0.1.

22. In a certain factory, there are two independent processes manufacturing the same item. The average weight in a sample of 250 items produced from one process is found to be 120 gm with a standard deviation of 12 gm while the corresponding figure in a sample of 400 items from the other process are 124 and 14 gms. Is the difference between the mean weights significant at 1% level of significance ?

23. The mean breaking strength of the cables supplied by a manufacturer is 1800 with a standard deviation 100. By a new technique in the manufacturing process, it is claimed that the breaking strength of the cables has increased. In this claim a sample of 50 cables is tested. It is found that the mean breaking strength is 1850. Can we support the claim at 1% level of significance ?

24. A sample of 400 male students is found to have a mean height of 171.38 cm. Can it be reasonably regarded and a sample from a large population with mean height 171.17 cm and standard deviation 3.30 cms ?

25. Give the requirements for applying Normal distribution to a problem of testing the significance of single mean. Give the null hypothesis H_0 and describe the procedure of testing H_0 against various possible alternative hypothesis H_1 at 5% level of significance.

Given $\bar{x}_1 = 82, \sigma = 10, n = 100$, test the hypothesis that $\mu = 86$.

26. In a random sample of 500 persons from town A, 200 are found to be consumers of wheat. In a sample of 400 from town B, 220 are found to be consumers of wheat. Do these data reveal a significant difference between town A and town B so far as the proportion of wheat consumer is concerned ?

27. A company produces two makes of bulbs, A and B. 200 bulbs of each make were tested and it was found that make A has mean life of 2560 hours and S.D. 90 hours, whereas make B had 2650 hours mean life and S.D. 75 hours. Is there a significant difference between the mean life of two makes ?

28. An equal opportunities committee is conducting an investigation if in comparable jobs, men and women workers are paid identical wages. The following information is obtained on 75 males and 64 females :

Salary	Male	Female
Mean (Rs.)	11,530	10,620
S.D.	780	750

Test at 5% level of significance, whether men and women workers are paid identical wages.

29. In a credit co-operative run by a large company it was found that during the past year, a sample of 300 loans issued showed that 37% of the loans were made to women employees. A similar study carried out 5 years ago showed that the proportion of women employees seeking loans was 32%. Do these data give sufficient evidence to conclude that more women employees are seeking loans in the recent year than before.
Use a 5% significance level for test.

30. Data were collected from two cities as regards the starting stipend paid to new management trainees.
Do the data give evidence that the stipend paid in city B is significantly more than that in city A?
Test at a significance level of 1%.

City	*Monthly Stipend (Means)*	*Sample Standard Deviations*	*Sample Size*
A	Rs. 8,400	Rs. 80	200
B	Rs. 8,600	Rs. 120	175

31. A manufacturer of steel rods considers that the manufacturing is working properly, if the mean length of the rods is 8.6 inches. The standard deviation of these rods always runs about 0.3 inch. The manufacturer would like to see, if the process is working correctly by taking a random sample of size $n = 36$. There is no indication whether or not the rods may be too short or too long.
(*a*) Establish null and alternative hypothesis for this problem.
(*b*) Would you use a one-tailed test or a two-tailed test ?
(*c*) If the random sample yields an average length of 8.7 inches, would you accept null hypothesis or alternative hypothesis?

32. A random sample of 400 villages was taken from Dhanbad and the average population per village was found to be 527 with a standard deviation of 45. Another random sample of 400 villages was taken from Muzaffarpur where the average population per village was found to be 505 with a standard deviation of 50. Using an appropriate test of significance, state clearly if the difference between the two averages is statistically significant at 5% level.

33. You are given the following information relating to purchase of bulbs from two manufacturers A and B :

Manufacturer	*No. of Bulbs bought*	*Mean life*	*S.D.*
A	100	2950 hrs.	100 hrs.
B	100	2970 hrs.	90 hrs.

Is there a significant difference in the mean life of two makes of bulbs ?

34. A man buys 200 electric bulbs of each of two well-known makes taken at random from stock for testing purposes. He finds that 'Make A' has a mean life of 2,500 hours with a standard deviation of 90 hours and 'Make B' has a mean life of 2,650 hours with a standard deviation of 75 hours. Is there a significant difference in the mean life of these two makes at 5% level of significance ?

35. Random samples drawn from two places gave the following data relating to the heights of adult males :

	Place A	*Place B*
Mean height (in inches)	68.50	65.50
Standard deviation (in inches)	2.5	3.0
No. of adult males in sample	1,200	1,500

Test at 5% level, that the mean height is the same for adults in the two places.

36. A stock broker claims that he can predict with 80 per cent accuracy whether a stock's market value will rise or fall during the coming month. As a test, he predicts the outcome of 40 stocks and is correct in 28 of the predictions. Does this evidence support the stock broker's claims ?

37. Two research laboratories have independently produced drugs that provide relief to arthritis patients. The first drug was tested on a group of 100 arthritis patients and produced an average of 8.5 hours of relief with a standard deviation of 2 hours. The second drug was tested on 75 patients, producing an average of 7.8 hours of relief with a standard deviation of 1.5 hours. At a significance level of 1 per cent, does the first drug provide a significantly longer period of relief ?

38. In a simple random sample of 600 men taken from a big city, 450 are found to be smokers. In another simple random sample of 900 men taken from another city, 450 are smokers. Do the data indicate that there is a significant difference in the habit of smoking in the two cities ?

39. An auto company decided to introduce a new six cylinder car whose mean petrol consumption is claimed to be lower than that of the existing auto engine. It was found that the mean petrol consumption for the 50 cars was 14 km per litre with a standard deviation of 3.5 km. per litre. Test for the company at 5% level of significance, whether the claim, the new car petrol consumption is 13.5 km per litre on the average is acceptable.

40. The management of a company claims that the average weekly income of their employees is Rs. 900. The trade union disputes this claim stressing that it is rather less. An independent survey of 150 randomly selected employees estimated the average to be Rs. 856 and the Standard Deviation to be Rs. 364.26. Would you accept the view of the management or the trade union ?

41. Two brands of bulbs are quoted at the same price. A buyer tested a random sample of 100 bulbs of each brand and found the following :

	Mean life (hrs)	*S.D. (hours)*
Brand I	1300	82
Brand II	1248	83

Is there a significant difference in the quality of two brands of bulbs at 5% level of significance ?
[4.45]
(*MBA, Delhi Univ., 2006*)

42. (*a*) In two large populations, there are 30% and 25% fair coloured people, respectively. Is this difference likely to be hidden in samples of 1200 and 900 respectively from the two populations ? (Given, the tabulated value of test statistics at 5% level of significance is 1.96)
(*MBA, IGNOU, 2004*)

(*b*) A filling machine at a soft drink factory is designed to fill bottles of 200 ml with a standard deviation of 10 ml. A random sample of 50 filled bottles was taken and the average volume of soft drink was computed to be 198 ml per bottle. Test the hypothesis that the mean volume of soft drink per bottle is not less than 200 ml at 5% level of significance.
(*MBA, IGNOU, 2007*)

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