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Statistical Quality Control

INTRODUCTION

In this era of ever-growing competition, it has become absolutely necessary for businessmen to keep a continuous watch over the quality of the goods produced. Having once bought the product, etc., a kind of goodwill for the product is developed which leads to increase in sales. However, if the consumers are not happy with the quality of product and the complaints are not given proper attention, it shall be impossible for the manufacturer to continue in the market. Either he would improve the quality or else be forced to quit the market by other producers who could start capturing the market by offering better quality.

Although, the need for maintaining and improving quality standard is growing with increasing competition, the idea of quality control is not a new one. What is new about quality control is the use of statistical techniques which are helpful in maintaining and improving quality standards—hence the term **statistical quality control**. Statistical quality control involves the statistical analysis of the inspection data; such analysis is based on sampling and the principles involved in normal curve. These techniques were developed by *W. A. Shewart,* working for the Bell Telephone Co. in the U.S.A.; the problem he solved was that of checking on the consistency of manufacture of a very large number of components. The idea that statistics might be instrumental in controlling the quality of the manufactured products goes back to 1920's and 1930's, but it was not until the pressure of the production needs developed during the period of World War II that its value was fully appreciated. During that period, the use of this technique spread rapidly in British and American war factories and resulted in great savings. For example, in case of Western Electric Company, the rejects of some items declined to 50 per cent which led to a saving of millions of dollars in overheads. In fact, as a result of the war, it became necessary for many different industries to devote themselves to an all-out production effort, requiring the production of tremendous amounts of war material made to more exacting specifications than ever before and in many cases, made by new methods, with substitute materials; poorly trained help with machines designed for other purposes. It was this exigency which led to the wide acceptance of the statistical quality control. Its success in the war was followed by its continued and expanded use in the post-war period. These days, the statistical quality control is used to some extent in virtually every kind of industry in existence. In fact, it has become an integral and permanent part of management controls.

It is important to distinguish between the unsystematic inspection and supervision which often goes under the name of "quality control", and statistical quality control. The former does not say when or how samples should be taken or how large they should be, ordinarily does not have the advantages that go with graphic presentation and does not enforce a clear objective standard for "take action" or "skip it". The statistical quality control chart makes use of well-thought out, tested rules and avoids the indecision, inconsistency and arbitrariness of haphazard quality control. Statistical quality control is based on the fact that repeated random samples from a fixed population will vary, but in a predictable pattern.

The term 'quality' in statistical quality control is usually related to some measurement made on the items produced, a good quality item having one which conforms to standards specified for the measurement. Quality does not always imply the highest standards of manufacture, for the standard required is often deliberately below the highest possible. It is almost always the consistency of manufacturer which represents the most desirable situation rather than the absolute standard which is maintained.

The need for quality control arises because of the fact that even after the quality standards have been specified, some variation in quality is unavoidable. For example, a machine is producing 1,00,000 bolts per day of 2 cm. length. It is very unlikely that all the screws are exactly 2 cm. in length. If the measuring instrument is sufficiently precise we can detect some screws which are slightly less than 2 cm. and some which are slightly more than 2 cm. This leads to a search of the possible causes of variation in the product. The variation of a quality characteristic can be divided under two heads :

(*i*) *Chance variation, i.e.,* variation which results from many minor causes that behave in a random manner and produce slight differences in product characteristic. For example, slight changes in temperature, pressure, metal hardness and similar factors interact randomly to produce slight variations in product quality. This type of variation is permissible, and indeed inevitable, in manufacturing. There is no way in which it can be completely eliminated—when the variability present in a production process is confined to chance variation, the process is said to be in a state of statistical control.

(*ii*) *Assignable variation, i.e.*, those variations that may be attributed to special non-random causes. Such variations can be result of the several factors such as a change in the character of input such as raw material, improper machine setting, broken or worn parts, mechanical faults in plant, adjustment of a machine by an operator, etc.

Out of these two types of variation, nothing can be done about the former type. However, assignable variation can be detected and corrected. The value of quality control lies in the fact that assignable variations in a process can be quickly detected—in fact these variations are often discoverd before the product becomes defective.

There are two different ways of controlling the quality of a product :

(*i*) Through 100% inspection, *i.e.,* by inspecting each and every item, that is produced; and

(*ii*) Through sampling technique or the use of statistical quality control.

The system of 100% inspection is not very satisfactory because of the following reasons :

(*i*) It is too expensive.

(*ii*) It is not always reliable because it becomes too much a routine for the persons inspecting each and every item and defective pieces may also be labelled 'satisfactory'. Defective pieces may also be passed at times when distraction occurs. For example, even when an inspector is trying to perform his task conscientiously, if someone talks to him or someone else happens to attract his attention, he may at times pass faulty pieces.

(*iii*) The inspection is made at the end of the manufacturing cycle, and hence provides few controls over the manufacturing process.

Thus, we find that even 100% inspection is not infallible. In an effort to find out a more economical and yet practical procedure, greater and greater use is being made of the tool of statistical quality control. *Statistical quality control is simply a statistical method for determining the extent to which quality goals are being met without necessarily checking every item produced and for indicating whether or not the variations which occur are exceeding normal expectations.* The statistical control of quality calls for the application of the theory of sampling and tests of significance.

Quality control methods are applied to two distinct phases of plant operation :

(*i*) The control of a **Process** during manufacture. A process is said to be in a state of statistical control, if the variation is such as would occur in random sampling from some stable population. If this is the case, the variation among the items is attributable to chance and there is no point in seeking special causes for individual cases. But when the process is out of control, it should be possible to locate specific causes for the variation and by removing them to improve the future performance of the process. Statistical quality control may be applied to any repetitive process. Such processes are found not merely in machine production in a factory but also in many management problems. Statistical quality control methods have been used in connection with such diverse problems as the stamping out of bottle caps, errors in the work of accountants, the filling of cartons, complaints received from customers, and airline reservations. The statistical tool applied in process control is the **control charts**. The primary objectives of process control are : (*a*) to keep the manufacturing process in control so that the population of defective units is not excessive, and (*b*) assisting in determining whether a state of control exists.

(*ii*) The inspection of materials to determine their acceptability whether they be in raw, semi-finished or completed state. This is known as **acceptance inspection** or **sampling inspection**. The object of acceptance inspection is to evaluate a definite lot of material that is already in existence and about whose quality, a decision must be made. This is done by inspecting a sample of the material, using definite statistical standards to infer from the quality of the sample whether the whole lot is acceptable. The standards in acceptance inspection are set according to what is required of the product rather than by the inherent capabilities of the process, as in process control. In process control, the population is the infinite number of possible results from the same repetitive process. In sampling inspection, the population is the finite group of items which have been produced, usually referred to as a lot.

Control Charts

A control chart is a statistical device principally used for the study and control of repetitive processes. Walter A. Shewart, its originator, suggested that control chart may serve, firstly, to define the goal of standard for the process that the management might strive to attain; secondly, it may be used as an instrument to attain that goal; and thirdly it may serve as a means of judging whether the goal has been achieved. Thus, it is an instrument to be used in specification, production and inspection.

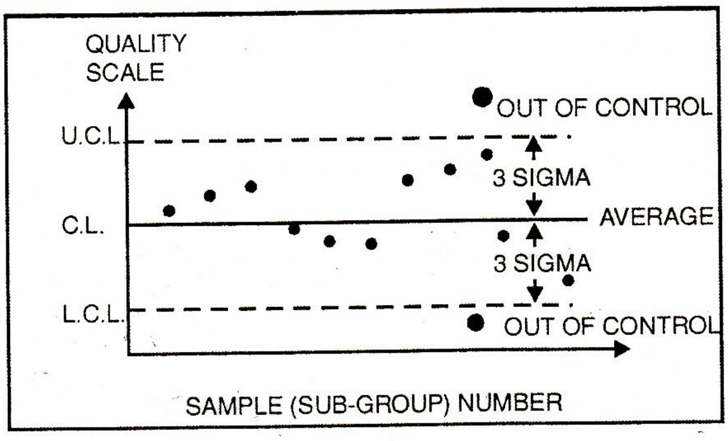
A control chart is essentially a graphic device for presenting data so as to directly reveal the frequency and extent of variations from established standards or goals. Control charts are simple to construct and easy to interpret, and they tell the manager at a glance, whether or not the process is in control, *i.e.*, within the tolerance limits. A control chart consists of three horizontal lines :

(*i*) A central line (CL) to indicate the desired standard or level of the process;

(*ii*) Upper control limit (UCL); and

(*iii*) Lower control limit (LCL).

The outline of a control chart is given below :



From time to time, a sample is taken and the data are plotted on the graph. So long as the sample points fall within the upper and lower control limits, there is nothing to worry as in such a case the variation between the samples is attributed to chance or unknown causes.

It is only when a sample point falls outside the control limits that it is considered to be a danger signal indicating that assignable causes are bringing about variations. Thus, there is no wastage of time and money in an effort to find the reason for random variation but as soon as an assignable cause is apparent, necessary corrective action is taken.

Thus, generally, if all dots are found between the upper and lower control limits, it is assumed that the process is "in control" and only chance causes are present. However, sometimes dots are found arranged in some peculiar way. Although they appear between the control limits, a substantial number of successive dots may be located on the same side of the central line or successive dots may follow a definite path leading towards the upper or lower control limit. Such patterns of dots within control limit should also be considered as danger signals which may indicate a change in the production process. Thus, control charts are not only watched for points falling outside the control limits, they are also scrutinised for unusual patterns suggesting trouble.

The control chart may be "likened to a highway whose control limits are the shoulders on one side and centre line on the other. No car driving along the highway can maintain a perfectly straight path. Unevenness in the road, play in the steering, wheel, gusts of wind, and a host of other factors cause slight variations in the movement of the car. It would hardly be worthwhile to investigate the causes of the small irregularities. However, the moment the car moves outside one of the limits, an assignable cause can be assumed to exist and the investigation should begin. The cause may turn out to be a defect in the steering mechanism, a sleepy driver or some similar correctable factor."

How to set up the Control Limits. The basis of control chart is the setting up of upper and lower control limits. These limits are used as a basis for judging the significance of the quality variations from sample to sample, lot to lot or from time to time. The moment a point falls outside these limits, it is taken to be a danger signal. The control limits serve as a guide for action and, therefore, they are also referred to as **action limits**. Control limits are established by computation based upon :

(*i*) Data covering past and current production records ; and

(*ii*) Statistical formulae whose reliability has been proved in practice.

Although, the nature of the control problem does not permit standardising precise and inflexible rules for computing control limits that will be found suited to all of the various conditions that may be encountered in actual practice, it has been found possible to develop certain general procedures on the basis of experience that will cover a wide range of industrial applications.

In most control problems, it had been found satisfactory to place the control limits above and below the grand average of the statistical measures ($\overline{X}$, σ, R, etc.) that is being plotted at distances of three times a computed value, commonly designated as the "sigma" of the statistical measures, for sub-groups of the size under consideration. These are referred to as *"sigma" limits*.* The logic of drawing 3σ limits is that in case of a normal distribution, $\overline{X} \pm 3\sigma$ covers 99.73 per cent of the items. In other words, occurrence of events beyond the limits of ($\overline{X} \pm 3\sigma$), provided the events lie on a normal curve, is on the whole nearly 3 out of 1,000 events—an extremely remote chance under normal circumstances. Hence, if points fall outside 3 sigma limits they indicate the presence of some assignable causes—all is not due to random causes. It should be noted that if points fall outside 3 sigma limits, there is a good reason for confidence that they point out to some factor contributing to quality variation that can be identified.

* It should be noted that this value of sigma is not the computed standard deviation of the plotted points. In the case of the $\overline{X}$, R and σ charts, it is computed from the individual observed values with sub-groups and the size n of a sub-group.

The selection of standard value (of $\overline{X}$, σ, R, etc.) is probably the most basic problem encountered in setting up a control procedure. The primary aim is not just to get control, but to get control at a satisfactory level. A satisfactory selection depends fundamentally upon the needs of the buyer or user as defined by his specifications. Any questions of cost of production and capability of manufacturing process must, of course, also be taken into account in deciding on a level that will be economical from an average point of view.

Types of Control Charts

Broadly speaking, control charts can be divided under two heads :

(*i*) Control charts of variables, and

(*ii*) Control charts of attributes.

Variables are those quality characteristics of a product which are measurable and can be expressed in specific units of measurement such as diameter of radio knobs which can be measured and expressed in centimetres, tensile strength of cement which can be expressed in specific measures per square inch of space, etc. Attributes, on the other hand, are those product characteristics which are not amenable to measurement. Such characteristics can only be identified by their presence or absence from the product. For example, we may say that plastic is cracked or not cracked, whether the bottles that have been manufactured contain holes or not. Attributes may be judged either by the proportion of units that are defective or by the number of defects per unit. Thus, the data resulting from inspection of a quality characteristic may take any one of the following forms:

(*i*) A record of the actual measurements of the quality characteristics for individual articles or specimens.

(*ii*) A record of number of articles or specimens inspected and of the number found defective.

(*iii*) A record of the number of defects that are found in a sample. When the possible numbers of defects per sample is very large compared with the average number of defects per sample.

For purposes of control data of the first form, listed above may be summarized by taking two statistical measures, the average ($\overline{X}$) and the standard deviation (σ), or the average ($\overline{X}$) and range (R). Data of the second form can be summarized in terms of fraction defective (p), and third form can be summarized in terms of number of defects per unit.

Setting up a Control Procedure

In establishing basic procedures for the operation of a quality control programme, the manufacturer must take the following preliminary steps :

1. Select the quality characteristics that are to be controlled (including the limits of variation).

2. Analyse the production process to determine the kind and location of probable causes of irregularities.

3. Determine how the inspection data are to be collected and recorded, and how they are to be subdivided.

4. Choose the statistical measures that are to be used in the charts.

Depending on the type of inspection data available, any one of the following types of control charts may be used :

1. *Control charts for $\overline{X}$ and σ; and $\overline{X}$ and R*. Such charts are used when measured values of the quality characteristics are at hand.

2. *Control chart for $\overline{X}$ alone.* Control chart for $\overline{X}$ alone is used where experience with control charts for $\overline{X}$ and R, or $\overline{X}$ and σ has demonstrated that instances of lack of control are almost always associated with causes that effect $\overline{X}$ rather than σ or R.

3. *Control chart for σ or R alone.* Control chart for R or σ is used alone where technical reasons render control of $\overline{X}$ unimportant or where control for $\overline{X}$ is known to be unjustifiably expensive.

4. *Control chart for C.* This chart is used where there are circumstances wherein, the inspection consists of determining the number of defects C in a sample. Such is the case, for example, in the examination of finished textiles, materials, wire, etc.

5. *Control chart for p or pn.* Chart for p or pn be used when the records of inspection of testing show merely the number of articles inspected and the number found defective.

These charts are discussed in detail :

$\overline{X}$-Chart. A control chart for sample means, or an $\overline{X}$-chart, is based on the distribution of sample means. It is used to determine if variations in a product dimension are random and to detect assignable variations. The control chart is based on a series of samples or sub-groups of observations drawn randomly from a process over a period of time. The arithmetic means of samples computed and the variation of these means reflect the pattern of variation of the process. The procedure of constructing an $\overline{X}$ - chart is as follows :

1. Obtain the mean of each sample, *i.e.*, $\overline{X}_1, \overline{X}_2, \overline{X}_3$, etc. This is done by dividing the sum of the values included in a sample (ΣX) by the number of observations in the sample (n or sample size).

$$\overline{X} = \frac{\Sigma X}{n}$$

2. Obtain the mean of the sample means, *i.e.*, $\overline{\overline{X}}$. This is done by dividing the sum of the sample means ($\Sigma \overline{X}$) by the number of samples to be included in the chart.

$$\overline{\overline{X}} = \frac{\Sigma \overline{X}}{\text{Number of samples}}$$

3. The control limits are set at

$$UCL = \overline{\overline{X}} + 3\,\sigma_{\overline{X}}$$

$$LCL = \overline{\overline{X}} - 3\,\sigma_{\overline{X}}$$

where, $$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}; \text{ and } \sigma = d/R$$

since, R is a biased estimator of σ and d is the correction factor. The values for d are tabulated in the Appendix at the end of the book.

Therefore, the control limits for $\overline{X}$-chart are :

$$UCL = \overline{\overline{X}} + A_2\,\overline{R}$$

$$LCL = \overline{\overline{X}} - A_2\,\overline{R}$$

Illustration 1. A food company puts mango juice into cans advertised as containing 200 grams of the juice. The weights of the juice drained from cans immediately after filling for 20 samples are taken by a random method (at an interval of every 10 minutes). Each of the samples includes 4 cans. The samples are tabulated in the following table. The weights in the table are given in units of grams in excess of 200 gms. For example, the weight of a juice drained from the first can of the sample is 215 gms which is in excess of 200 gms excess being 15 gms (215 –200=15). Since the unit in the table is gms, the excess is recorded as 15 units in the table. Construct an $\overline{X}$ chart to control the weights of mango juice for the filling.

Sample number	*Weight of each can (4 cans in each sample n = 4)* X			
1	15	12	13	20
2	10	8	8	14
3	8	15	17	10
4	12	17	11	12
5	18	13	15	4
6	20	16	14	20
7	15	19	23	17
8	13	23	14	16
9	9	8	18	5
10	6	10	24	20
11	5	12	20	15
12	3	15	18	18
13	6	18	12	10
14	12	9	15	18
15	15	15	6	16
16	18	17	8	15
17	13	16	5	4
18	10	20	8	10
19	5	15	10	12
20	6	14	12	14

Solution.

CALCULATIONS FOR CHART

Sample number	*Weight of each can (4 cans in each sample, n = 4)* X				*Total weight of 4 cans* ΣX	*Sample Mean* $\overline{X}$	*Sample Range* R
(1)	(2)				(3)		(4)
1	15	12	13	20	60	15.0	8
2	10	8	8	14	40	10.0	6
3	8	15	17	10	50	12.5	9
4	12	17	11	12	52	13.0	6
5	18	13	15	4	50	12.5	14
6	20	16	14	20	70	17.5	6
7	15	19	23	17	74	18.5	8
8	13	23	14	16	66	16.5	10
9	9	8	18	5	40	10.0	13
10	6	10	24	20	60	15.0	18
11	5	12	20	15	52	13.0	15
12	3	15	18	18	54	13.5	15
13	6	18	12	10	46	11.5	12
14	12	9	15	18	54	13.5	9
15	15	15	6	16	52	13.0	10
16	18	17	8	15	58	14.5	10
17	13	16	5	4	38	9.5	12
18	10	20	8	10	48	12.0	12
19	5	15	10	12	42	10.5	10
20	6	14	12	14	46	11.5	8
Total						263.0	211

Calculations :

(1) The mean of each sample $\overline{X}$ is given in column (3). For example, $\overline{X}$ for the first sample is $\frac{60}{4}=15$.

(2) The mean of the sample means $\overline{\overline{X}}$ is obtained from column (3) as follows :

$$\overline{\overline{X}} = \frac{\Sigma \overline{X}}{20} = \frac{263}{20} = 13.15$$

(3) The value of R computed from the values of R is shown in column (4). For example, the values of R for the first sample is computed as follows :

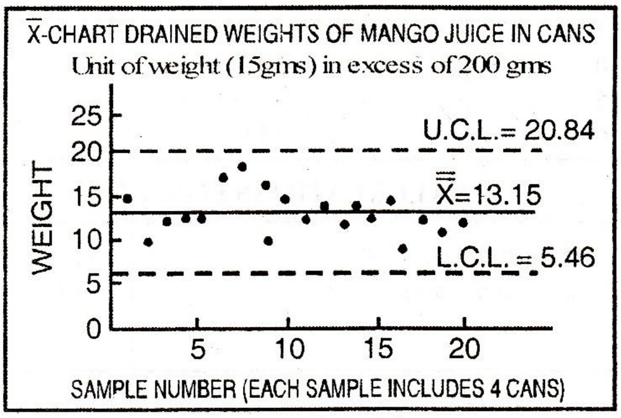
$$R = 20 - 12 = 8$$

(4) The value of $\overline{R}$, *i.e.*, the mean of the values of R is obtained as given below :

$$\overline{R} = \frac{\Sigma R}{20} = \frac{211}{20} = 10.55$$

(5)
$$UCL = \overline{\overline{X}} + A_2 \overline{R}$$
$$= 13.15 + 0.729 \times 10.55 \qquad \text{[the table value of } A_2 \text{ for } n = 4 \text{ is 0.729]}$$
$$= 13.15 + 7.69 = 20.84 \text{ approx.}$$
$$LCL = \overline{\overline{X}} - A_2 \overline{R}$$
$$= 13.15 - 0.729 \times 10.55 = 13.15 - 7.69 = 5.46 \text{ approx.}$$

Note that the values in the above computation are expressed in units of gms in excess of 200 gms. The actual values for the *UCL,* thus is 220.208 and that for *LCL* is 205.56 gms. The control chart for this illustration is given below :



Since, all the points are failing within control limits, the process is in a state of control and hence there is nothing to worry.

Illustration 2. A drilling machine bores holes with a mean diameter of 0.5230 cm and a standard deviation of 0.0032 cm. Calculate the 2-sigma and 3-sigma upper and lower control limits for means of samples 4, and prepare a control chart.

Solution. We have

$$\overline{\overline{X}} = 0.5230 \text{ cm}, \sigma = 0.0032 \text{ cm}, n = 4$$

$$\frac{\sigma}{\sqrt{n}} = \frac{0.0032}{2} = 0.0016$$

2- sigma limits for means of sample of 4:

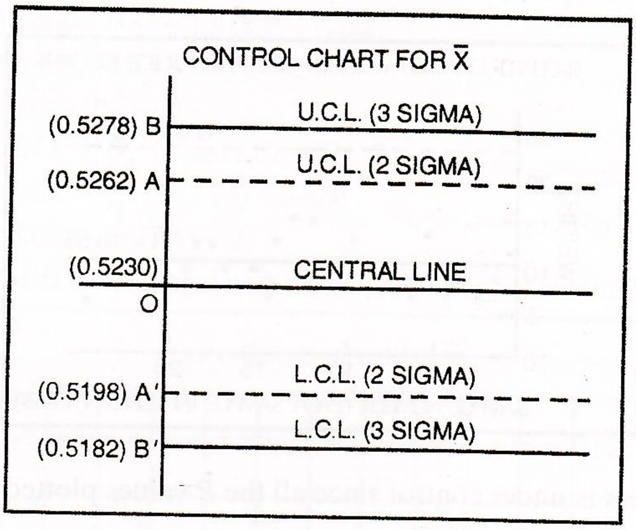
$$UCL = \overline{\overline{X}} + 2\frac{\sigma}{\sqrt{n}}$$
$$= 0.5230 + 2(0.0016) = 0.5262 \text{ cm.}$$
$$\text{Central line} = 0.5230 \text{ cm.}$$
$$LCL = \overline{\overline{X}} - 2(\sigma/\sqrt{n})$$
$$= 0.5230 - 2\,(0.0016) = 0.5198 \text{ cm.}$$

3-sigma limits for means of sample of 4 :

$$UCL = \overline{\overline{X}} + 3\,(\sigma/\sqrt{n})$$
$$= 0.5230 + 3\,(0.0016) = 0.5278 \text{ cm.}$$

Central line = 0.5230 cm.

$$LCL = \overline{\overline{X}} - 3\,(\sigma/\sqrt{n})$$
$$= 0.5230 - 3\,(0.0016) = 0.5182 \text{ cm.}$$



R-Chart

The *R*-chart is used to show the variability or dispersion of the quality produced by a given process. *R*-chart (or σ chart) is the companion chart to the X chart and both are usually required for adequate analysis of the production process under study. The *R*-chart is generally presented along with the $\overline{X}$ chart. The general procedure for constructing the *R*-chart is similar to that of the $\overline{X}$ chart. The required values for constructing the *R*-chart are :

1. The range of each sample, R;
2. The mean of the sample ranges, $\overline{R}$;
3. UCL and LCL

$$UCL_R = \overline{R} + 3\sigma_R, \text{ and}$$
$$LCL_R = \overline{R} - 3\sigma_R$$

where σ_R = The standard error of the range.

The value or σ_R may be estimated by finding the standard deviation of the ranges of the samples included in a chart. In practice, however, it is rather convenient to compute the upper and lower control limits by using the values D_4 and D_2 as provided in Appendix, according to various sample sizes (n = 2 to 20). When the tabulated values are used, the limits may be written as follows :

$$UCL_R = D_4\overline{R}$$
$$LCL_R = D_3\overline{R}.$$

It should be noted that the use of *R*-chart is recommended only for relatively small sample size, rarely more than 12 or 15 units. For the large sample sizes ($n > 12$), the σ chart is to be preferred.

Illustration 3. Prepare an *R*-chart for the data of illustration 1.

Solution. The required values for the chart are :

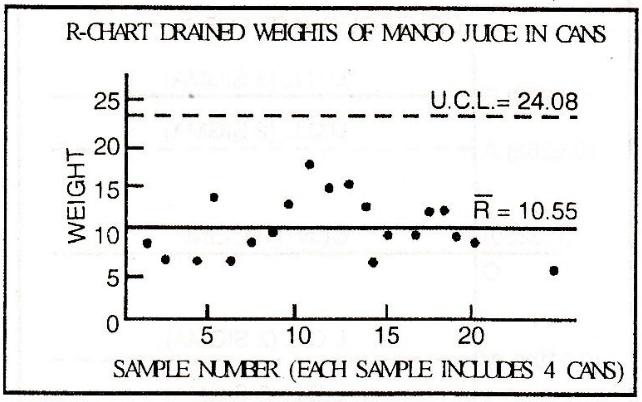
The range of each sample, R (see column 4 of illustration 1.).

The mean of the sample ranges, $\overline{R}$

$$\overline{R} = \frac{211}{20} = 10.55$$

$$UCL_R = D_4\,\overline{R}$$
$$= 2.282\,(10.55) = 24.08 \qquad \text{[table value of } D_4 = 2.282]$$
$$LCL_R = D_3\,\overline{R}$$
$$= 0\,(10.85) = 0.$$

The control chart for R is shown below :



The chart shows that the process is under control since all the R values plotted on the chart are within the two control limits.

The choice between the $\overline{X}$-chart and the R-chart is a managerial problem.

It is better to construct R-chart first. If the R-chart indicates that the dispersion of the quality by the process is out of control, generally, it is better not to construct an $\overline{X}$-chart until the quality dispersion is brought under control.

Illustration 4. The table below gives the (coded) measurements obtained in 20 samples (sub-groups). Construct control charts based on the mean and the range. The values of these statistics are given below for the respective samples :

Sub-group

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	−1	2	1	2	1	1	−1	1	2	−2	0	2	0	0	−1	1	2	2	0	3
	2	0	1	1	−1	−1	−1	1	1	1	1	1	1	0	2	−1	1	0	2	−3
	1	1	0	0	0	2	0	2	−1	−2	−3	−1	−3	−1	1	−2	−1	1	1	−1
	0	0	0	−1	0	0	−2	−1	0	2	2	0	2	0	1	0	0	0	−1	1
	1	1	1	0	−1	−2	1	0	0	1	1	0	1	1	2	2	0	1	1	2
$\overline{X}$:	.6	.8	.6	.4	−.2	0	−.6	.6	.4	0	.2	.4	.2	0	1.0	0	.4	.8	.6	.4
R :	3	1	1	3	2	4	3	3	3	4	5	3	5	2	3	4	3	2	3	6

Solution.

$$\overline{X} = \frac{\overline{X}_1 + \overline{X}_2 + \ldots + \overline{X}_n}{n_1 + n_2 + n_3 + \ldots}$$

$$= \frac{.6 + .8 + .6 + .4 - .2 + 0 - .6 + .6 + .4 + 0 + .2 + .4 + .2 + 0 + 1.0 + 0 + .4 + .8 + .6 + .4}{20}$$

$$= \frac{6.6}{20} = 0.33$$

$$R = \frac{3 + 1 + 1 + 3 + 2 + 4 + 3 + 3 + 3 + 4 + 5 + 3 + 5 + 2 + 3 + 4 + 3 + 2 + 3 + 6}{20}$$

$$= \frac{63}{20} = 3.15$$

From the table* for the sample of size 5, we find that

$$A_2 = 0.577, D_3 = 0 \text{ and } D_4 = 2.115$$

Upper and Lower control limits for $\bar{X}$-chart

$$= \bar{X} \pm A_1 \bar{R} = 0.33 \pm 0.577\,(3.15) = 0.33 \pm 1.818.$$

Lower control limit $= 0.33 - 1.818 = -1.488.$

Upper control limit $= 0.33 + 1.818 = 2.148.$

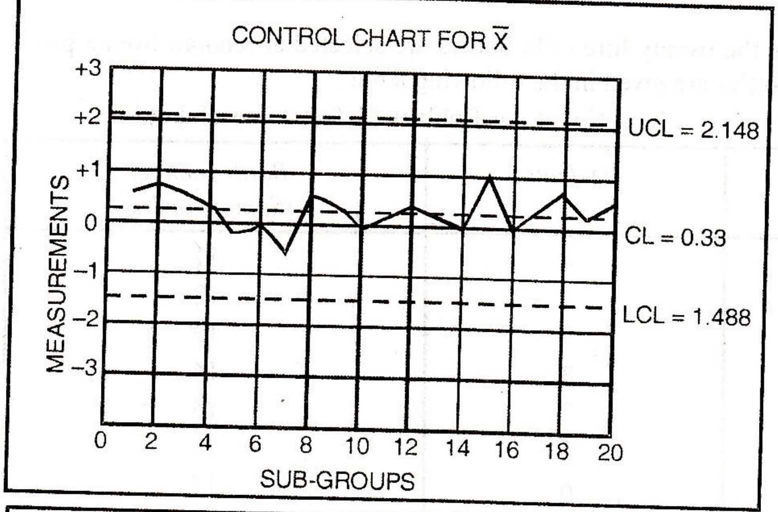
Upper and Lower control limits for R-chart

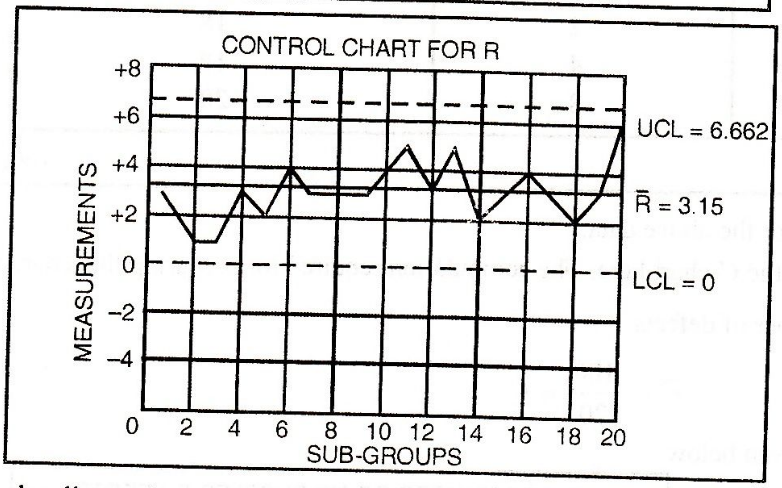
$$UCL = D_4\ \bar{R} \text{ and } LCL = D_3\ \bar{R}$$

$$UCL = 2.115\,(3.15) = 6.662$$

$$LCL = 0\,(3.15) = 0.$$

The following two control charts are prepared with the help of the above control limits:





The fact that in both graphs all sample points are falling within the 3-sigma control limits, can be interpreted as implying that the process is in a state of statistical control or, in other words, that the only kind of variation present is chance variation.

C-Chart

That *C*-chart is designed to control the number of defects per unit. The *C*-chart is based on the Poisson distribution and is very popularly used in statistical work.

Control chart for *C* is used in situations, wherein the opportunity for defects is large while the actual occurrence tends to be small. Such situations are described by the Poisson distribution. This happens, for example, we count the number of imperfections in a piece of cloth, the number of air bubbles in a piece of glass, the number of blemishes in a sheet of paper, etc. Let *C* stand for the number

---

*A_2, D_3 and D_4 are given in the ASTM Manual Table, reproduced and presented at the end of the text. It should be noted that when *n* is 6 or less, $D_4 = 0$ hence, the lower control limits for *R* is taken as zero.

of defects counted in one unit of cloth, paper glass, rolls of wire and $\overline{C}$ for the mean of the defects counted in several (usually 25 or more) such units of cloth, the Central Line of the control chart for C is $\overline{C}$ and the 3-sigma control limits are :

$$\overline{C} \pm 3\sqrt{\overline{C}}.$$

This formula is based on a normal curve approximation of the Poisson distribution. The use of the C-chart is appropriate, if the opportunities for a defect in each production unit are infinite but the probability of a defect at any point is very small and is constant.

Uniform sample size is highly desirable while using the C-chart. Where sample size varies, particularly, if the variation is large, the C-chart becomes difficult to read, and p-chart is the better choice.

Illustration 5. Assume the twenty litre milk bottles are selected at random from a process. The number of air bubbles (defects) observed from the bottles are given in the following table :

[C = No. of Air Bubbles (defects) in each bottle]

Bottle number (Sample order)	*Defects C*	*Bottle number (Sample order)*	*Defects C*
1	4	11	3
2	5	12	5
3	7	13	4
4	3	14	3
5	3	15	4
6	5	16	5
7	6	17	3
8	2	18	7
9	4	19	6
10	8	20	13

Total number of defects = 100

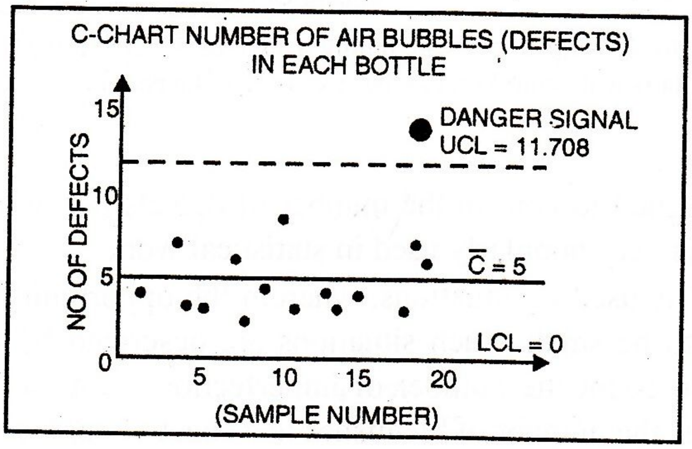
Draw a control chart for the above data.

Solution. We will use the C-chart here. The computation required for preparing this chart are :

$\overline{C}$, *i.e.*, average number of defects

$$\overline{C} = \frac{100}{20} = 5.$$

The control chart is given below :



$$UCL = \overline{C} + 3\sqrt{\overline{C}} = 5 + 3\sqrt{5} = 5 + 3 \times 2.236 = 5 + 6.708 = 11.708.$$

$$LCL = \overline{C} - 3\sqrt{\overline{C}} = 5 - 3\sqrt{5} = 5 - 3 \times 2.236 = 5 - 6.708 = -1.708.$$

The lower control limit will be recorded as zero, since the number of defects cannot be negative.

It is clear from the chart that only one point in respect of last sample falls outside control limits and this is to be treated as danger signal.

Illustration 6. The following table gives the number of errors of alignment observed at final inspection of a certain model of bus. Prepare a C-chart and comment on it.

Bus number	*No. of alignment defects*	*Bus number*	*No. of alignment defects*
1001	6	1011	8
1002	10	1012	6
1003	8	1013	10
1004	7	1014	10
1005	12	1015	6
1006	9	1016	12
1007	5	1017	3
1008	7	1018	11
1009	3	1019	2
1010	4	1020	1

Solution.

$\bar{C}$, *i.e.*, average number of defects $= \frac{140}{20} = 7.$

The control limits and the central line are :

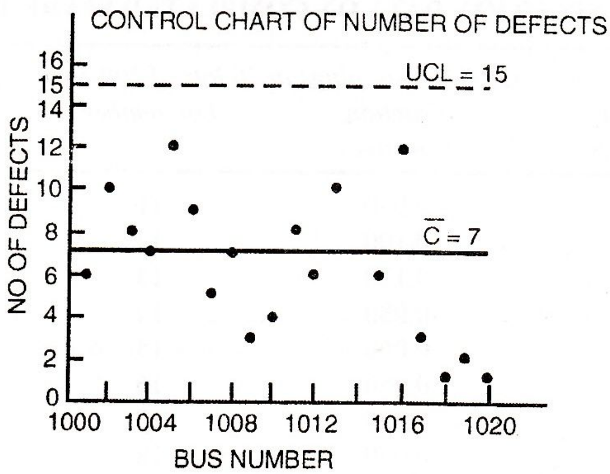
$$UCL = \bar{C} + 3\sqrt{\bar{C}}$$

$$= 7 + 3\sqrt{7} = 7 + (3 \times 2.646) = 14.938 \text{ or } 15.$$

$$\text{Central line} = \bar{C} = 7$$

$$LCL = \bar{C} - 3\sqrt{\bar{C}} = 7 - (3 \times 2.646) = -0.938.$$

Because the number of defects cannot be negative, the lower limit will be taken as zero.



p-Chart

The p-chart is designed to control the percentage or proportion of defectives per sample. A control chart for fraction defective, or p-chart, is based on the distribution of sample proportions. It is assumed that the items are produced by Bernoulli process. This assumption implies that (1) there are only two possible outcomes (acceptable or defective), (2) the outcomes occur randomly, and (3) the probability of either outcome remain unchanged for each trial. Since the number of defectives (c) can be converted into a percentage expressed as a decimal fraction merely by dividing (c) by sample size, the p-chart may be used in lieu of the C-chart. The p-chart has at least two advantages over the C-chart :

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1008	7	1018	11
1009	3	1019	2
1010	4	1020	1

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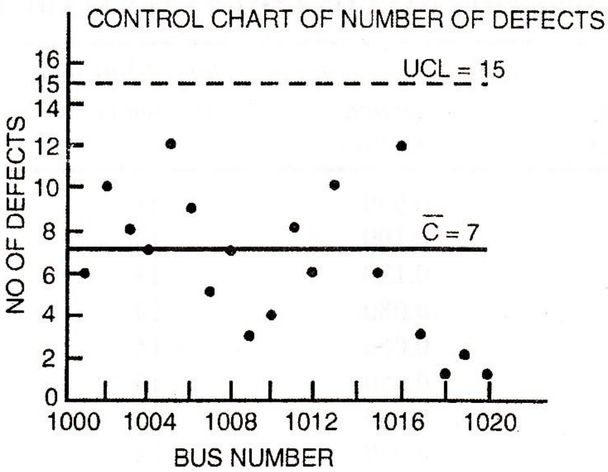

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1. Expressing the defectives as a percentage or fraction of production is more meaningful and more generally understood than would be the statement of the number of defectives. The latter concept must be related in some way to the total number produced.

2. Where the size of the sample varies from sample to sample, the p-chart permits a more straightforward and less clustered up presentation. The p-chart requires, however, that the division c/n be made. This additional computation may be regarded as a slight disadvantage.

The same basic data is used for either chart. When the sample size remains constant from sample to sample, the primary difference lies in the computation of the control limits. The C-chart control limits are set at $\overline{C}$ plus or minus three standard deviations. The p-chart control limits are set at $\overline{p}$ plus or minus three standard errors of the proportion.

This chart has its theoretical basis in the binomial distribution, and generally give best results when the sample size is large, say, at least 50. The steps in constructing the chart are :

(*i*) Compute the average fraction defective (p) by dividing the number of defective by the total number of units inspected.

(*ii*) On the chart, draw a solid horizontal line to present $\overline{p}$.

(*iii*) Determine the upper and lower control limits. The upper and lower control limits are obtained by the average per cent defective plus and minus three times the standard error as follows :

$$UCL = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}\ ; LCL = \overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$$

While constructing the chart, it is generally preferred to express results in terms of 'per cent defective' rather than 'fraction defective'. The per cent defective is 100 p. Any sample point falling outside the control limits is evidence of a possible lack of control in as much as the probability of getting such value by chance is less than 0.003. The following example shall illustrate the procedure :

Illustration 7. In a factory producing spark plugs, the number of defectives found in the inspection of 20 lots of 100 each is given as follows :

INSPECTION DATA ON COMPLETED SPARK PLUGS

(2,000 spark plugs in 20 lots of 100 each)

Lot number	*Number defectives*	*Fraction defectives*	*Lot number*	*Number defectives*	*Fraction defectives*
1	5	0.050	11	4	0.040
2	10	0.100	12	7	0.070
3	12	0.120	13	8	0.080
4	8	0.080	14	2	0.020
5	6	0.060	15	3	0.030
6	5	0.050	16	4	0.040
7	6	0.060	17	5	0.050
8	3	0.030	18	8	0.080
9	3	0.030	19	6	0.060
10	5	0.050	20	10	0.100
				Total = 120	

Construct an appropriate control chart.

Solution. Since we are given fraction defective, the suitable chart will be p-chart.

Calculations for p-chart are :

Average fraction defective,

$$\overline{p} = \frac{120}{2{,}000} = 0.06$$

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

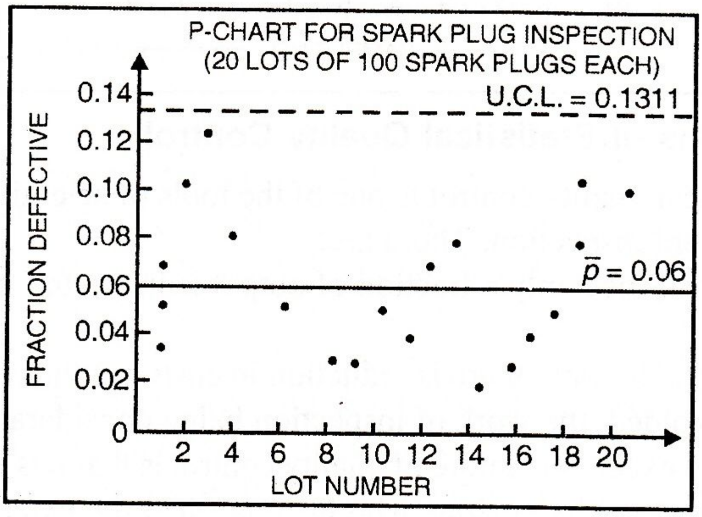
$$= 0.06 + 3\sqrt{\frac{0.06(1-0.06)}{100}} = 0.06 + 3\sqrt{\frac{0.06 \times 0.94}{100}}$$

$$= 0.06 + 3\ (0.0237) = 0.06 + 0.0711 = 0.1311$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.06 - 3\sqrt{\frac{0.06(1-0.06)}{100}}$$

$$= 0.06 - 0.0711 = -0.0111$$

Since the fraction defective cannot be negative, the *LCL* shall be taken as zero here.



The control chart shows that all the points are falling within control limits. Hence, the process is in a state of control.

In order to simplify the work of the person who plots the necessary points on the control charts, the above charts can be modified so that he can directly plot the *number* rather than the fraction or percentage of defectives. Such a chart is called the Control Chart for number of defectives. To obtain such a chart, the central line as well as the control limits are multiplied by *n*. The central line, thus becomes *n* and the control limits are

$$n\bar{p} \pm 3\sqrt{n\bar{p}(1-\bar{p})}$$

Illustration 8. The following data refer to visual defects found at inspection of the first 10 samples of size 100. Use them to obtain upper and lower control limits for percentage defective in samples of 100. Represent the first ten sample results in the chart you prepare to show the central line and control limits.

Sample No.	1	2	3	4	5	6	7	8	9	10	Total
No. of defectives	2	1	1	3	2	3	4	2	2	0	20

Solution. Since there are 20 defective items in 10 samples each of size 100, therefore, $\bar{p}$ = Average fraction defective $= \frac{20}{10 \times 100} = 0.02$.

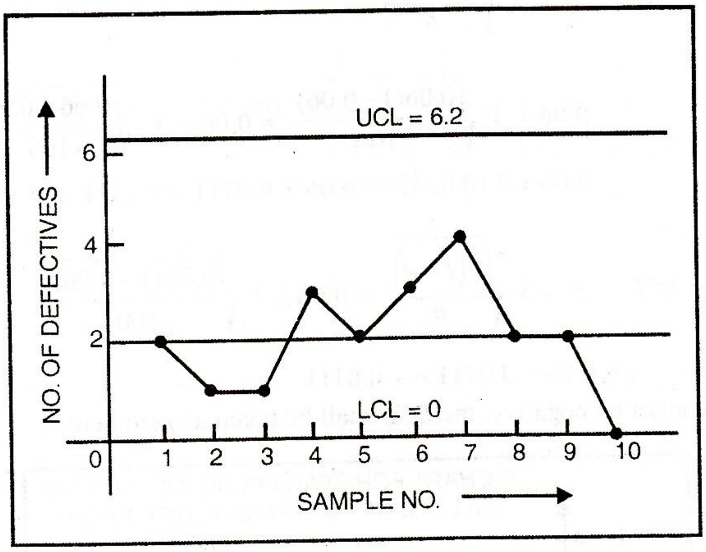
Also, $n = 100$ and $n\bar{p} = 100 \times 0.02 = 2$

and $\sqrt{n\bar{p}(1-\bar{p})} = \sqrt{100 \times 0.02 \times 0.98} = \sqrt{1.96} = 1.4.$

$$UCL = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 2 + 3\ (1.4) = 6.2.$$

Central line $= n\bar{p} = 2.$

$$LCL = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} = 2 - 3\,(1.4) = -2.2.$$



Benefits and Limitations of Statistical Quality Control

Advantages. Statistical quality control is one of the tools of scientific management. It has several advantages over 100 per cent inspection. These are:

(*i*) *Reduction in costs*. Since only a fraction of output is inspected, costs of inspection are greatly reduced.

(*ii*) *Greater efficiency*. Not only there is reduction in costs but the efficiency also goes up because much of the boredom is avoided, the work of inspection being considerably reduced.

(*iii*) *Easy to apply*. An excellent feature of quality control is that it is easy to apply. Once the system is established, it can be operated by persons who have not had the extensive specialized training or a highly mathematical background. It may appear difficult, only because the statistical principles on which it is based are unrecognised or unknown. However, as these principles are actually based on commonsense, the quality control method finds wide application.

(*iv*) *Early detection of faults*. Quality control ensures an early detection of faults and hence a minimum waste of reject production. The moment a sample point falls outside the control limits, it is taken to be a danger signal and necessary corrective action is taken. On the other hand, with 100 per cent production, unwanted variations in quality may be detected at a stage, when a large amount of faulty products have already been produced. Thus, there would be a big wastage. Control chart, on the other hand, provides a graphic picture of how the production is proceeding and to tell management where not to look for trouble.

(*v*) *Adherence to Specifications*. Quality control enables a process to be brought into and held in a state of statistical control, *i.e.*, a state in which variability is the result of chance causes alone. So long as a statistical control continues, specifications can be accurately predicted for the future, which even 100 per cent inspection cannot guarantee. Consequently, it is possible to assess whether the production processes are capable of turning out products which will comply with the given set of specifications.

(*vi*) *The only course*. In certain cases, 100% inspection cannot be carried out without destroying all the products inspected : for example, testing breaking strength of chalks, proofing of ammunition, etc. In such cases, if 100% inspection methods are followed then all the items inspected will be spoiled. In such a case, sampling must be resorted to and with the application of SQC techniques not only that the quality is controlled but also that valid inferences about the total output are drawn from the samples.

(*vii*) *To determine effect of changed process*. With the help of control charts one can easily detect whether or not a change in the production process results is a significant change in quality.

(viii) *Statistical quality control ensures overall coordination.* Statistical quality control provides a basis upon which the difference arising among the various interests in an organisation can be resolved. In some instances, for example, production engineers may set specifications that are so "tight" that the operating staff cannot meet them economically and consequently there is an unnecessary high scrapping rate. In other instances, the specifications may be too loose, and product quality will be sacrificed unnecessarily. In either type of case, the control records provide a valuable aid in solving the problem of getting the operating and engineering forces together on the basis of common understanding. Information on plant capabilities and customer requirements must also be considered in relation to the quality control limits and records of performance and, finally, it should be possible to determine the best practical balance between the cost of quality and the sales value of the product.

SQC has a special role to play in a country like India because of the extraordinary variations encountered in raw materials and in machines. The importance of applying SQC has become greater in our industries in the context of the need for earning foreign exchange by supplying quality goods to successfully compete in the world markets.

Limitations

Despite the great significance of statistical quality control, it should be remembered that it is not a panacea for all quality ills. The techniques of quality control should not be used mechanically rather they should be matched to the process being studied. The application of standard process without adequate study of the process is extremely dangerous, and has in the past led to statistical methods being discredited. Statistical methods applied on a production process are only an information service, and as such must be conditioned by the process to which they are applied. Unless they are used as part of a generally quality awareness, they may only lead to a false sense of security. The responsibility for quality and process decisions rests with the manager incharge of the process and not with the statistician. The charts do not reduce the manager's responsibility.

ACCEPTANCE SAMPLING

The control charts described above cannot be applied to all types of problems. They are useful only to the regulation of the manufacturing process. Another important field of quality control is acceptance sampling. Inspection for acceptance purposes is carried out at many stages in manufacturing. For example, there may be inspection on incoming materials and process inspection at various points in the manufacturing operations, final inspection by a manufacturer of his own product, and ultimately, inspection of the finished product by one or more purchasers. Much of the acceptance inspection is carried out on a sampling basis. The use of sampling inspection by a purchaser to decide whether or not to accept a shipment of product is known as *acceptance sampling.* A sample of the shipment is inspected and if the number of defective items is not more than a stated number known as the *acceptance number,* the shipment is accepted. The standards in acceptance inspection are set according to what is required of the product, rather than by the inherent capabilities of the process, as in the process control. The purpose of acceptance sampling is, therefore, whether to accept or reject a product—it does not attempt to control quality during the manufacturing process, as do the techniques described earlier in the chapter. Sampling inspection may also be referred to as *product control*, because it is designed to provide decision procedures under which, a lot will be accepted or rejected.

Acceptance sampling procedures which were perfected during World War II to meet military needs for quick and accurate inspection of vast supplies of material are now used widely in industry.

A typical application of acceptance is to determine whether a batch of items, called an *inspection lot* or simply a lot, that has been delivered by a supplier, is of acceptable quality. Another application is to a lot that is complete and ready for shipment to customers to make sure that it is of adequate quality. Still another application is to a lot of partly completed material, to determine whether it is of high quality to justify further processing.

Role of Acceptance Sampling

Acceptance sampling is very widely used in practice because of the following reasons :

1. Acceptance sampling is much less expensive than 100 per cent inspection.

2. In many cases it provides better outgoing quality. It is generally agreed that good 100 per cent inspection will remove only about 85 to 95 per cent of the defective material. Very good 100 per cent inspection will remove 99 per cent of the defective items but still not reach 100 per cent. Because of the effect of inspection fatigue involved in 100 per cent inspection, a *good* sampling plan may actually give better quality assurance than 100 per cent inspection. The word '*good*' is *italicised* since many informal sampling plans devised without benefit of knowledge of the laws of change are practically worthless. The result has been widespread use of sampling plans.

3. In modern manufacturing plants, acceptance sampling is used for evaluating the acceptability of incoming lots of raw materials and parts at various stages of manufacture, and final inspection of finished product.

4. Where quality can be tested only by destroying items, as in determining the strength of glass containers, 100 per cent inspection is out of the question and sampling must be used. Of course, there are situations where 100 per cent inspection is not to be put aside ; for example, in testing rifles to be used by soldiers, we cannot risk imperfection in any item and therefore must test each and every rifle.

Since under a sampling inspection plan, a decision is made as to whether to accept a lot or reject a lot on the basis of sample, there is a possibility (1) rejecting a lot as unsatisfactory when it is of acceptable quality, and (2) accepting a lot as satisfactory when in fact it is below the quality level. Hence in any acceptance sampling plan, the producers and the consumers, the sellers and the buyers, are exposed to some risks. These are called *producer's* and *consumer's risks*. The producer's risk is the risk a producer takes that a lot will be rejected by a sampling plan even though it conforms to requirements. This is equivalent to the concept of type I error, or the probability of rejecting a hypothesis when it is in fact true. The consumer's risk is the risk that a lot of certain quality will be accepted by a sampling plan. It is equivalent to type II error which is the probability of accepting a hypothesis when an alternative is true. Before agreeing to an acceptance criterion, the consumers and producers will like to know the risks to which they are exposed, *i.e.*, the probability of rejecting a good lot and the probability of accepting a bad one.

An inspection plan can easily be constructed if the consumers and producers specify these probabilities and also the proportion of defectives above which a lot is considered to be bad and the proportion of defectives below which a lot is considered to be good.

Types of Acceptance Sampling Plans

The following three types of acceptance sampling plans are commonly used :

1. Single Sampling Plan. When the decision whether to accept a lot or reject a lot is made on the basis of only one sample, the acceptance plan is described as single sampling plan. This is the simplest type of sampling plan. In any systematic plan for single sampling three things are specified, namely, (*a*) Number of items N in the lot from which the sample is to be drawn, (*b*) Number of articles n in the random sample drawn from the lot, and (*c*) The acceptance number c. This acceptance number is the maximum allowable number of defective articles in the sample. More than this will cause the rejection of the lot. Thus, a sampling plan may be specified in this way—

$$N = 200, n = 20, c = 1.$$

These numbers may be interpreted as saying, "Take a random sample of 20 from a lot of 200. If the sample contains more than 1 defective, reject the lot ; otherwise accept the lot."

2. Double Sampling Plan. In the single sampling plan discussed above, decision with regard to acceptance or rejection of a lot is based on the evidence of only one sample from the lot. However, double sampling involves the possibility of putting off the decision on the lot until a second sample has been taken. A lot may be accepted at once if the first sample is good enough or rejected at once if the first sample is bad enough. If the first sample is neither good enough nor bad enough, the decision is based on the evidence of the first and second sample combined. In a double sampling plan, 5 things are specified : $n_1, c_1, n_2, n_1 + n_2$ and c_2, where

n_1 = number of pieces in the first sample;

c_1 = acceptance number for the first sample, the maximum number of defectives that will permit the acceptance of the lot on the basis of the first sample;

n_2 = number of pieces in the second sample;

$n_1 + n_2$ = number of pieces in the two samples combined; and

c_2 = acceptance number for the two samples combined, the maximum number of defectives that will permit the acceptance of the lot on the basis of the two samples.

Thus, a double sampling plan may be :

$$N = 500, n_1 = 20, c_1 = 1, n_2 = 60, c_2 = 4.$$

This will be interpreted as follows :

(*i*) Inspect a first sample of 20 from a lot of 500.

(*ii*) Accept the lot on the basis of the first sample, if it contains 1 defective.

(*iii*) Reject the lot on the basis of the first sample, if the sample contains more than 1 defective.

(*iv*) Inspect a second sample of 60, if the first sample contains 2, 3, 4 defectives.

(*v*) Accept the lot on the basis of combined sample of 80, if the combined sample contains 4 or less defectives.

(*vi*) Reject the lot on the basis of combined sample, if the combined sample contains more than 4 defectives.

Advantages of Double Sampling Plan

A double sampling plan has two possible advantages over a single sampling plan :

(*i*) It may reduce the total amount of inspection ; for the first sample taken is less than that called for under a comparable single sampling plan, and, consequently, in all cases in which a lot is accepted or rejected on the first sample, there may be considerable saving in total inspection. It is also possible to reject a lot without completely inspecting the entire sample.

(*ii*) A double sampling plan has the psychological advantage of giving a lot a second chance. To some people, especially the producer, it may seem unfair to reject a lot on the basis of a single sample. Double sampling permits the taking of two samples on which to make a decision.

3. Multiple or Sequential Sampling Plan. Just as double sampling plans may defer the decision on acceptance or rejection until a second sample has been taken, other plans may permit any number of samples before a decision is reached. Plans permitting from three up to an unlimited number of samples are described as multiple or sequential. However, such plans are quite complicated and rarely used in practice.

Selection of a Sampling Plan

All practical sampling plans have an OC curve. The following points, need emphasis regarding the OC curve :

1. There is some chance that good lots will be rejected.
2. There is some chance that bad lots will be accepted.
3. These risks can be calculated by the theory of probability and depend on the number of samples inspected, the acceptance number and the per cent defective in the lots submitted for sample inspection. Given the amount of risks which can be tolerated, a sampling plan can be derived to meet these requirements.
4. The larger the sample used in sample inspection, the nearer the OC curve approaches the ideal. However, beyond a certain point, the added cost in inspecting a large number of parts far exceeds the benefits derived.

In review, the two parameters of an OC curve are the sample size and the acceptance number. The desired quality level (p) and the probability of acceptance (P_x) must be selected so that the proper sampling plan can be designed.

There are four factors which should be decided in a sampling plan :

1. P_1, also known as *AQL* (the Acceptable Quality Level). This is the definition of a good lot.
2. P_2, also known as *RQL* (the Rejectable Quality Level) or *LTPD* (Lot Tolerance Per cent Defective).
3. α, also known as Producer's Risk. This is the probability of rejecting a good lot.
4. β, also known as the Consumer's Risk. This is the probability of accepting a poor lot.

Construction of an OC Curve

An OC curve can be constructed by using either the Poisson distribution of the Thorndike chart. The Poisson distribution can be used in all situations where p is less the 0.10 (or if the pn is less than 5) and the lot size is at least 10 times the size of the sample.

In a situation in which these conditions are not met, the theoretically correct approach is to use the binomial or the hypergeometric distribution can be used without serious loss of accuracy.

To use the Thorndike chart, the following procedure is followed. For each possible value of the lot, fraction defective p, a pn is computed. The Thorndike chart is used to find the probability of C or less defective units. For example, for a lot that is 5 per cent defective ($p = 0.05$) and a sample size of 100 ($n = 100$), *i.e.*, $pn = 5$, the probability of selecting 2 or less defectives is found from the Thorndike chart to be approximately 0.12. If the lot fraction defective is 1 per cent ($p = 0.01$) and the sample size is 100 ($n = 100$), *i.e.*, $pn = 1$, the probability of selecting 2 or less defective is found from the Throndike chart to be approximately 0.92. These results give two plots on the OC curve for the sampling inspection plan where the sample size is 100 and the acceptance number is 2. Other points may be calculated in the same way.

The Operating Characteristic (OC) Curve

In judging various acceptance sampling plans, it is desirable to compare their performance over a range of possible quality levels of submitted product. An excellent picture of this performance is given by the Operating Characteristic curve. Such curves are commonly referred to as OC curves. The OC curve of an acceptance sampling plan shows the ability of the plan to distinguish between good and bad lots. For any given fraction defective p in a submitted lot, the *OC* curve shows the probability p_a that such a lot will be accepted by the given sampling plan or, in other words, the *OC* curve shows the long-run percentage of submitted lots that would be accepted if a great many lot of any stated quality was submitted for inspection. In drawing the OC curve, the following two terms are important.

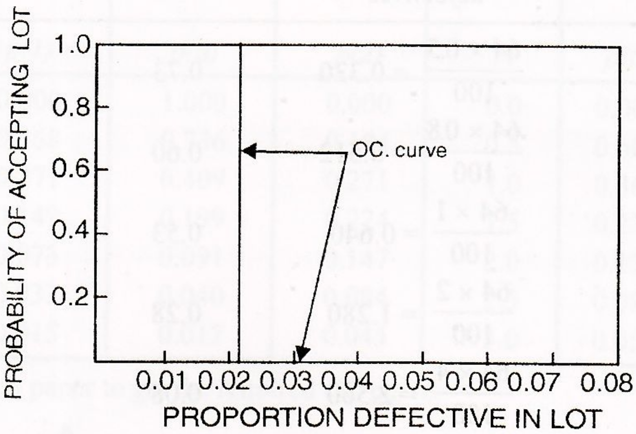
AQL and LTPD

In order to measure the customer's risk, we must define maximum percentage of defective items in lots which the consumer wishes to accept. This is called the *Lot Tolerance Percentage*

Defective or LTPD. Similarly, to measure the producer's risk we define a minimum percentage of defective items in a lot below which the lot should be accepted—this is known as *Acceptable Quality Level* or AQL. The producer's risk is now defined as the probability that a lot having the AQL will be rejected and the consumer's risk as the probability that a lot having LTPD will be accepted. These risks are usually taken as 5% and 10% respectively. The actual levels of the AQL and LTPD must be decided by negotiations between the consumer and the producer.

Shape of an Ideal OC Curve

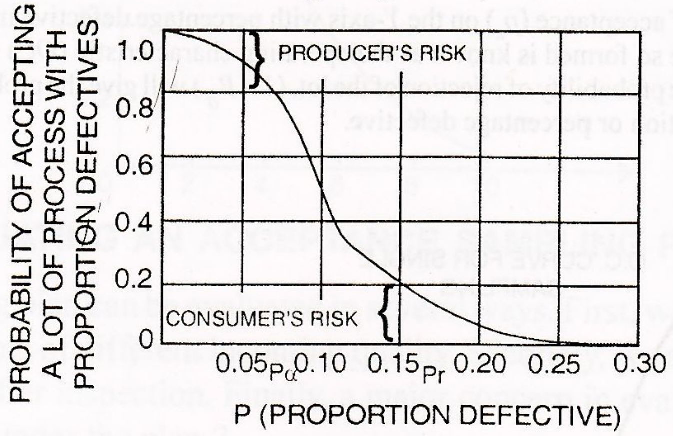
The ideal OC curve would be one for which all good lots are accepted and all bad lots are rejected. Such a curve would look like the following :



No sampling plan can have an *OC* curve of this type. The degree to which an actual *OC* curve approximates the ideal curve depends upon *n* and *c*, *n* representing the sample size and *c* the acceptance number, or the number of defects in the sample which is not to be exceeded.

Shape of a Typical OC Curve

A typical *OC* curve resembles the following diagram :



The points on the horizontal scale represent possible lot or process qualities, and the height of the curve shows the probability that a lot of this quality will be accepted, assuming the specified sampling plan is in use. In the above diagram, it has been assumed that the acceptable and rejectable qualities are measured as proportions of the items that are defective and are $P_a = 0.05$ and are $P_r = 0.15$; from the *OC* curve, the producer's and consumer's risks are seen to be both a little more than 0.10 in this example. (The sampling plan of the above diagram calls for accepting the lot if three or fewer defectives are found in a sample of 40).

The steepness of the *OC* curve depends upon the sample size. The larger the sample, the steeper the curve, and the smaller the zone between the qualities that are almost always accepted and the qualities that are almost always rejected.

The location of the *OC* curve is determined by the maximum number of defective items allowable for acceptance, called the *acceptance number*. If the acceptance number is made large, the curve is shifted to the right. If the acceptance number is made smaller, the curve is shifted to the left.

Illustration 9. For the sampling plan $N = 1{,}200$, $n = 64$ and $C = 1$ determine the probability of acceptance of the following lots: (*i*) 0.5% defective, (*ii*) 0.8% defective, (*iii*) 1% defective, (*iv*) 2% defective, (*v*) 4% defective, (*vi*) 10% defective. Also draw an *OC* curve.

Solution. If the lot contains 0.5% defective, the samples from it will also have an average of 0.5% defective. Hence in a sample of size 64, the average number of defective will be $\frac{64 \times 0.5}{100} = 0.32$. If the sample contains 1 or 0 defective, the lot is to be accepted under the sampling plan. We can obtain the cumulative probability on drawing a sample of 64 containing 0 or 1 defective by using the Poisson approximation to the binomial distribution. The calculation will be as follows :

S. No.	*% defective in the lot*	*Average number of defectives*	*P(0)*	*P(1)*	*P(0) + P(1)* *P(a)*
(*a*)	0.5	$\frac{64 \times 0.5}{100} = 0.320$	0.73	0.23	0.96
(*b*)	0.8	$\frac{64 \times 0.8}{100} = 0.512$	0.60	0.31	0.91
(*c*)	1	$\frac{64 \times 1}{100} = 0.640$	0.53	0.35	0.88
(*d*)	2	$\frac{64 \times 2}{100} = 1.280$	0.28	0.36	0.64
(*e*)	4	$\frac{64 \times 4}{100} = 2.560$	0.08	0.21	0.29
(*f*)	10	$\frac{64 \times 10}{100} = 6.400$	0.002	0.01	0.01

The value 0.96 represents the probability of drawing a sample of 64 with 0 or 1 defective from a lot known to be 0.5% defective. Conversely, we can state that such a sample will enable acceptance of 96 per cent of lots containing 0.5 per cent defectives. In other words, if 1,000 such lots are submitted for inspection under the sampling plan, on an average 960 lots will be accepted and 40 will be rejected.

If we take the probabilities of acceptance (p_a) on the *Y*-axis with percentage defective in the lots submitted on the *X*-axis, and join the various points, the curve so formed is known as the operating characteristic (*OC*) curve of the sampling plan. From the *OC* curve, we can easily obtain the probability of rejection of the lot, $(1 - P_a)$ will give the probability of rejection corresponding to any lot, having a specified proportion or percentage defective.

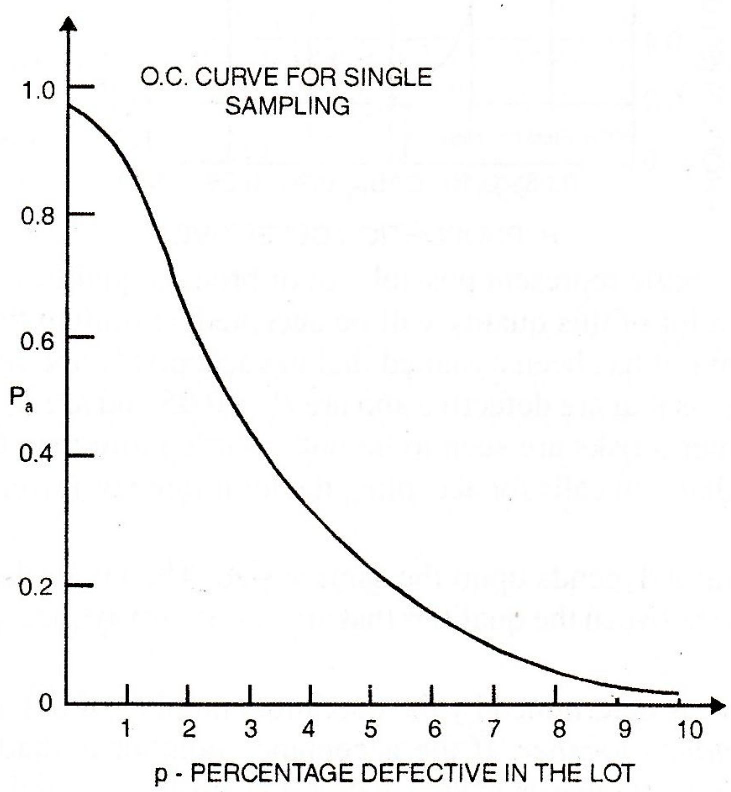


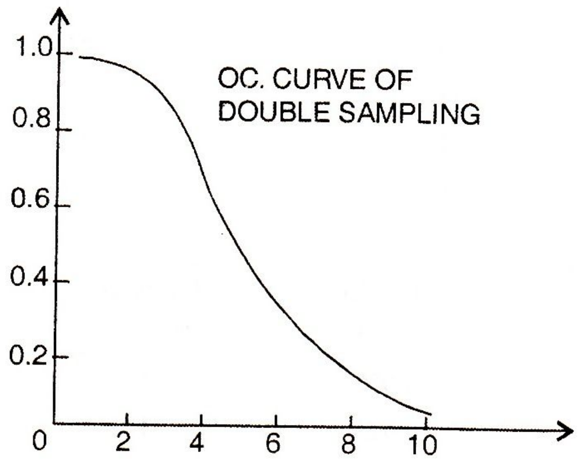
Illustration 10. Draw the *OC* curve of the double sampling plan, given that $N = 1{,}000$, $n_1 = 50$, $c_1 = 1$, $n_2 = 25$, $c_2 = 5$.

Solution. This sampling plan means that a sample of size 50 is drawn, if it gives 0 or 1 defective it is accepted. If it gives 3 or more defective, it is rejected. But, if it gives 2 defectives, then a sample of 25 is drawn. If the total number of defectives is 2, the lot is accepted ; if it is more than 2, the lot is rejected. For various values of percentage defective in lot x, the probability of acceptance shall be obtained as follows :

$$m = \frac{x \times n}{100},\ p(0) = e^{-m},\ p(1) = e^{-m} \times m, \text{ etc.}$$

	Ist sample $n_1 = 50$					*2nd sample* $n_2 = 25$			*Combined Sample*
x	m	$p(0)$	$p(1)$	$p(a)$	$p(2)$	m	$p(0)$	$p(a)$ $p(2) \times p(0)$	$p(a)$
0	0	1.000	0.000	1.000	0.000	0.0	0.000	0.000	1.000
2	1	0.368	0.368	0.736	0.184	0.5	0.606	0.112	0.848
4	2	0.135	0.271	0.409	0.271	1.0	0.368	0.100	0.506
6	3	0.050	0.149	0.199	0.224	1.5	0.273	0.049	0.248
8	4	0.018	0.073	0.091	0.147	2.0	0.135	0.020	0.111
10	5	0.007	0.033	0.040	0.084	2.5	0.082	0.007	0.047
12	6	0.002	0.015	0.017	0.045	3.0	0.050	0.002	0.019

Let us plot these points on the graph paper to get the required *OC* curve.



EVALUATING AN ACCEPTANCE SAMPLING PLAN

An acceptance sampling plan can be evaluated in several ways. First, we may ask how the sampling plan discriminates between lots of different incoming quality. Secondly, we are interested in the average outgoing quality of the lots after inspection. Finally, a major concern in evaluating a plan is cost. What shall be the inspection costs under the plan ?

MISCELLANEOUS ILLUSTRATIONS

Illustration 11. Samples of 100 tubes are drawn randomly from the output of a process that produces several thousand units daily. Sample items are inspected for quality and defective tubes are rejected. The result of a series of 20 samples is shown below :

Sample No.	*No. Inspected*	*No. Defectives*	*Sample No.*	*No. Inspected*	*No. Defectives*
1	100	8	11	100	17
2	100	10	12	100	14
3	100	12	13	100	13
4	100	8	14	100	15
5	100	7	15	100	8
6	100	11	16	100	6
7	100	13	17	100	10
8	100	5	18	100	7
9	100	10	19	100	4
10	100	12	20	100	10

Set up the upper and lower control limits.

Solution. $\bar{p} = \frac{200}{2000} = 0.1$

$$\text{UCL} = 0.1 + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.1 + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{100}} = 0.1 + 3(0.03) = 0.19$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.1 - 3(0.03) = 0.01$$

Illustration 12. A random sample of 200 was taken from daily production of large output of pens and number of defective pens was noted. On the basis of information given below, prepare a control chart for fraction defective. What conclusion do you draw from the control chart ?

Production each day	*No. of defectives*	*Production each day*	*No. of defectives*
1	10	13	8
2	5	14	14
3	10	15	4
4	12	16	10
5	11	17	12
6	9	18	11
7	22	19	26
8	4	20	13
9	12	21	10
10	24	22	9
11	21	23	11
12	15	24	12

Solution.

Production each day	*No. of defectives*	*Fraction defectives*	*Production each day*	*No. of defectives*	*Fraction defectives*
1	10	0.050	13	8	0.040
2	5	0.025	14	14	0.070
3	10	0.050	15	4	0.020
4	12	0.060	16	10	0.050
5	11	0.055	17	12	0.055
6	9	0.045	18	11	0.055
7	22	0.110	19	26	0.130
8	4	0.020	20	13	0.065
9	12	0.060	21	10	0.050
10	24	0.120	22	9	0.045
11	21	0.105	23	11	0.055
12	15	0.075	24	12	0.060

$$\bar{p} = \frac{\text{Total number of defectives}}{\text{Total production}} = \frac{294}{24 \times 200} = 0.061$$

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.01 + 3\sqrt{\frac{0.061\,(1-0.061)}{200}} = 0.061 + 3\,(0.0169) = 0.112$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.061 - 3\,(0.0169) = 0.0103.$$

Since all the points don't fall within control limits, there seems to be something wrong with the production process.

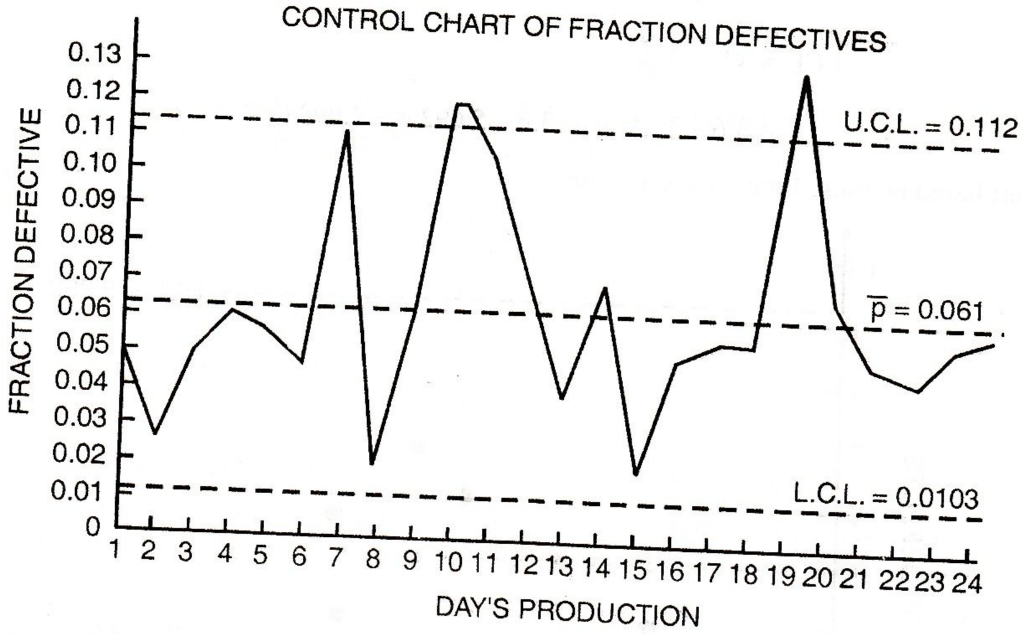


Illustration 13. 20 Television sets were examined for quality control test. The number of defects for each television set are recorded below :

2, 4, 3, 1, 1, 2, 5, 3, 6, 7, 3, 1, 4, 2, 1, 3, 4, 6, 1, 1.

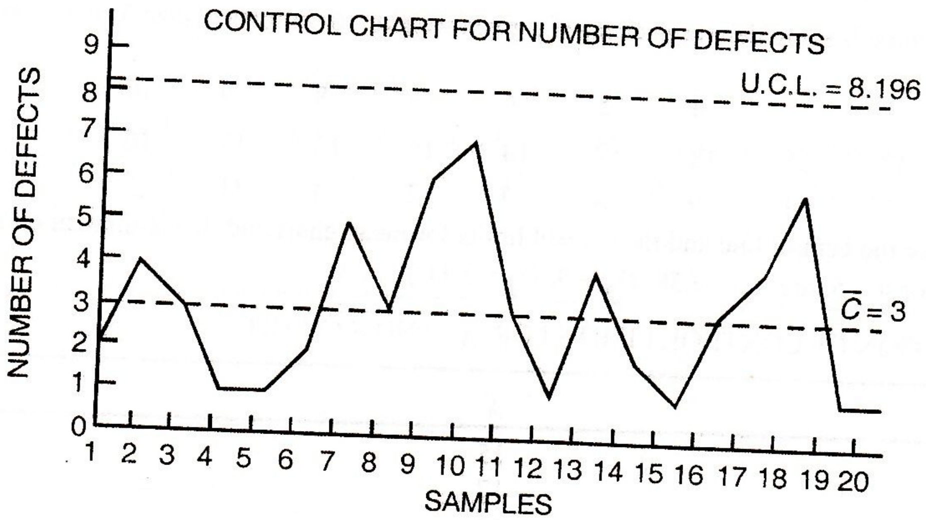
Prepare a C-chart. What conclusion can you draw from it ?

Solution. Total number of defects = 60, Sample size = 20

$$\bar{C} = \frac{60}{20} = 3$$

$$\text{UCL} = \bar{C} + 3 = 3 + 3\sqrt{3} = 3 + 5.196 = 8.196$$

$$\text{LCL} = \sqrt{C} - 3 = 3 - 3\sqrt{3} = 3 - 5.196 = -2.196 \text{ or } 0$$



Since all the points are lying within control limits, the process is in a state of control.

Illustration 14. During an examination of equal length of cloth, the following number of defects are observed :

2, 3, 4, 0, 5, 6, 7, 4, 3, 2.

Draw a control chart for the number of defects and comment whether the process is under control or not.

Solution. Let C denote the number of defects per piece.

$$\Sigma C = 2 + 3 + 4 + 0 + 5 + 6 + 7 + 4 + 3 + 2 = 36.$$

$$\bar{C} = \frac{36}{10} = 3.6$$

Hence central line = 3.6

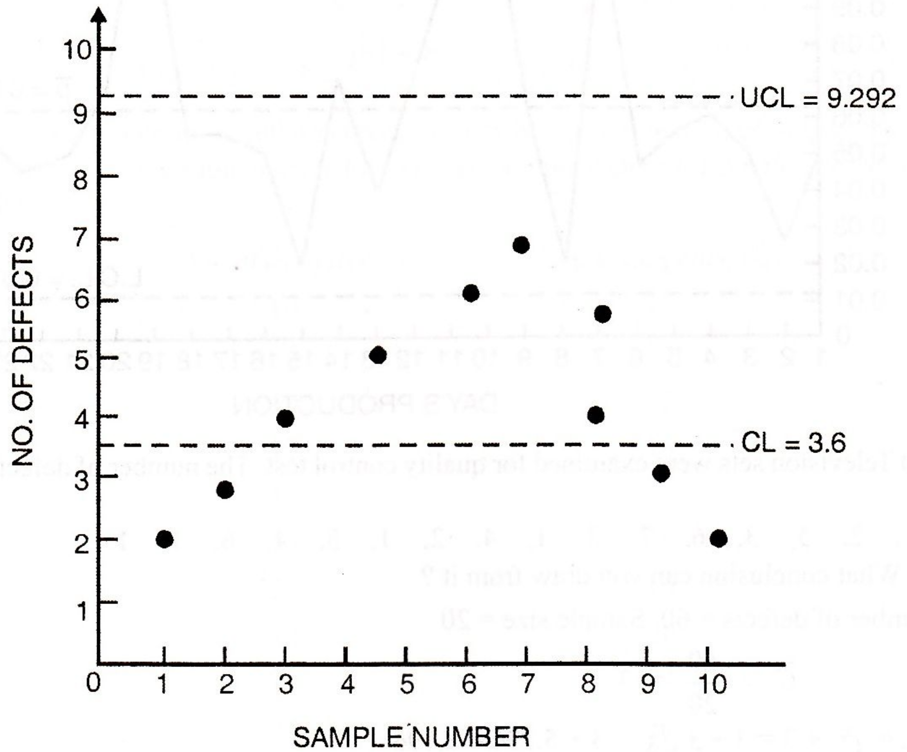
$$\text{UCL} = \bar{C} + 3\sqrt{C}$$

$$= 3.6 + 3\sqrt{3.6} = 3.6 + 5.692 = 9.292$$

$$LCL = \bar{C} - 3\sqrt{\bar{C}}$$

$$= 3.6 - 3\sqrt{3.6} = 3.6 - 5.692 = -2.092 \text{ or } 0$$

The control chart based on these limits is given below :



Since all the points are lying within control limits, the process is in a state of control.

Illustration 15. A machine is set to deliver packets of a given weight. 10 samples of size 5 each were recorded. Below are given relevant data :

Sample No. :	1	2	3	4	5	6	7	8	9	10
Mean ($\bar{X}$) :	15	17	15	18	17	14	18	15	17	16
Range (R) :	7	7	4	9	8	7	12	4	11	5

Calculate the values for the central line and the control limits for mean chart and then comment on the state of control. (Conversion Factors for $n = 5$, are $A_2 = 0.58$, $D_3 = 0$, $D_4 = 2.11$.)

Solution. CALCULATION OF CONTROL LIMITS FOR $\bar{X}$ AND R CHART

Sample No.	$\bar{X}$	R
1	15	7
2	17	7
3	15	4
4	18	9
5	17	8
6	14	7
7	18	12
8	15	4
9	17	11
10	16	5
$n = 10$	$\Sigma\bar{X} = 162$	$\Sigma R = 74$

Control limits for $\bar{X}$ chart

$$UCL = \bar{\bar{X}} + A_2\bar{R}$$

$$LCL = \bar{\bar{X}} - A_2\bar{R}$$

$$\text{Central line} = \bar{\bar{X}} = \frac{\Sigma \bar{X}}{n} = \frac{162}{10} = 16.2$$

$$UCL = 16.2 + 0.58\ (7.4) = 16.2 + 4.292 = 20.492$$

$$LCL = 16.2 - 0.58\ (7.4) = 16.2 - 4.292 = 11.908$$

Control limits for R chart

$$\text{Central line} = \bar{\bar{X}} = \frac{74}{10} = 7.4$$

$$UCL = D_4 \bar{R}$$

$$LCL = D_3 \bar{R}$$

$$UCL = 2.11(7.4) = 15.614$$

$$LCL = 0(7.4) = 0$$

Since all the points are lying within control limits, the process is in a state of control.

Illustration 16. Compute the values for a control chart for C, *i.e.*, number of defectives from the following data pertaining to the number of imperfections in 20 pieces of cloth of equal length in a certain make of polyester and infer whether the process is in a state of control :

2, 3, 5, 8, 12, 2, 3, 4, 6, 5, 6, 10, 4, 6, 5, 7, 4, 9, 7, 3.

Solution. Let C denote the number of defects per piece.

$$\bar{C} = \frac{\Sigma C}{N}$$

$$\Sigma C = 2 + 3 + 5 + 8 + 12 + 2 + 3 + 4 + 6 + 5 + 6 + 10 + 4 + 6 + 5 + 7 + 4 + 9 + 7 + 3 = 111$$

$$\bar{C} = \frac{111}{20} = 5.55$$

$$UCL = \bar{C} + 3\sqrt{\bar{C}} = 5.55 + 3\sqrt{5.55}$$

$$= 5.55 + 7.07 = 12.62.$$

$$LCL = \bar{C} - 3\sqrt{\bar{C}} = 5.55 - 7.07$$

$$= -1.52 \text{ or } 0$$

Since none of the points is falling outside the upper and lower control limits, the process is in a state of control.

Illustration 17. From a transistor production line 20 samples (each sample of 100 transistors) is chosen. The number of defects in each sample are given below :

Sample No.	*No. of defects*	*Sample No.*	*No. of defects*
1	44	11	36
2	48	12	52
3	32	13	35
4	50	14	41
5	29	15	42
6	31	16	30
7	46	17	46
8	52	18	38
9	44	19	26
10	48	20	30

Compute the values for an appropriate control chart and give your comments.

Solution. The appropriate control chart to be used is the C-chart. The computations required for preparing this chart are :

$$\bar{C},\ \textit{i.e.}, \text{ average no. of defects} = \frac{800}{20} = 40$$

$$UCL = \bar{C} + 3\sqrt{\bar{C}} = 40 + 3\sqrt{40}$$

$$= 40 + (3 \times 6.324) = 40 + 18.972 = 58.972$$

$$\text{LCL} = \overline{C} - 3\sqrt{\overline{C}} = 40 - 3\sqrt{40}$$
$$= 40 - (3 \times 6.324) = 40 - 18.972 = 21.028$$

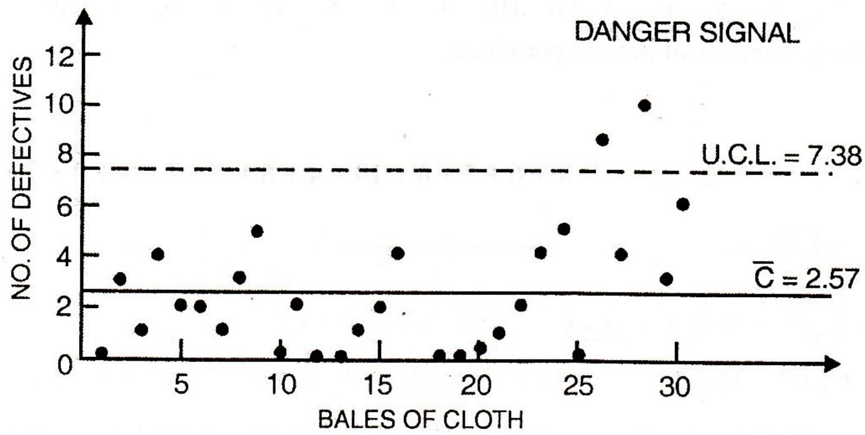
Since all the points are lying within control limits, the process is in a state of control.

Illustration 18. The following are the number of defects noted in the final inspection of 30 bales of woollen cloth :
0, 3, 1, 4, 2, 2, 1, 3, 5, 0, 2, 0, 0, 1, 2, 4, 3, 0, 0, 0, 1, 2, 4, 5, 0, 9, 4, 10, 3 and 6.
Compute the values for an appropriate control chart and give your comments. (*M.Com., Kurukshetra Univ., 1996*)

Solution. The appropriate chart would be a *C*-chart.
Total no. of defects = 87, sample size = 30

$$\overline{C} = \frac{77}{30} = 2.57$$
$$\text{UCL} = \overline{C} + 3\sqrt{\overline{C}} = 2.57 + 3\sqrt{2.57}$$
$$= 2.57 + 4.81 = 7.38$$
$$\text{LCL} = \overline{C} - 3\sqrt{\overline{C}} = 2.57 - 3\sqrt{2.57}$$
$$= 2.57 - 4.81 = -2.24 \text{ or } 0$$

Since lower control limit cannot be negative, we would start with zero.
The control chart is given below :



It is clear from the graph that two points are lying outside the control limits which represent danger signal.

Illustration 19. Samples of 100 tubes are drawn randomly from the output of a process that produces several thousand units daily. Sample tubes are inspected for quality and defective tubes are rejected. The results of 15 samples are shown below :

Sample No.	*No. of defective tubes*	*Sample No.*	*No. of defective tubes*
1	8	9	10
2	10	10	13
3	13	11	18
4	9	12	15
5	8	13	12
6	10	14	14
7	14	15	9
8	6		

On the basis of information given above prepare a control chart for fraction defective.

Solution. CONSTRUCTING CONTROL CHART FOR FRACTION DEFECTIVE

Sample No.	*No. of defectives*	*Fraction defectives*	*Sample No.*	*No. of defectives*	*Fraction defectives*
1	8	0.08	9	10	0.10
2	10	0.10	10	13	0.13
3	13	0.13	11	18	1.18
4	9	0.09	12	15	0.15
5	8	0.08	13	12	0.12
6	10	0.10	14	14	0.14
7	14	0.14	15	9	0.09
8	6	0.06			
			Total	169	

The suitable chart here will be the p-chart.

(i) Average fraction defective

$$\bar{p} = \frac{169}{1500} = 0.113$$

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$= 0.113 + 3\sqrt{\frac{0.113(1-0.113)}{100}} = 0.113 + 0.095 = 0.208.$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$= 0.113 - 3\sqrt{\frac{0.113(1-0.113)}{100}} = 0.113 - 0.095 = 0.018.$$

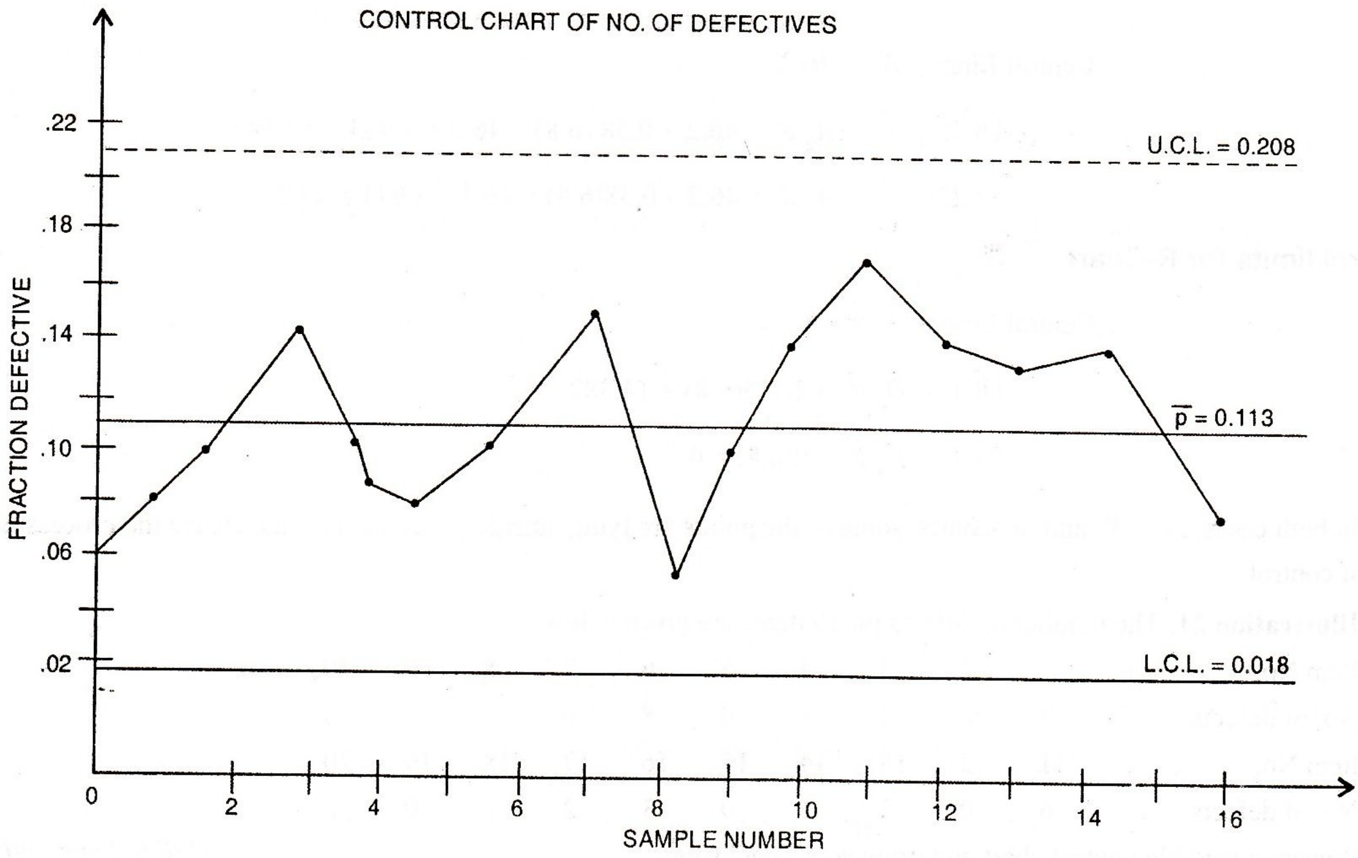


Illustration 20. Ten samples each of size 5 are drawn at regular intervals from a manufacturing process. The sample means ($\bar{X}$) and their average (R) are given below :

Sample No :	1	2	3	4	5	6	7	8	9	10
Mean ($\bar{X}$) :	49	45	48	53	39	47	46	39	51	45
Range (R) :	7	5	7	9	5	8	8	6	7	6

Calculate the control limits in respect of $\bar{X}$ -chart and R-chart.

(You are given : $A_2 = 0.58$, $D_3 = 0$, $D_4 = 2.115$). Comment on the state of control, charts need not be drawn.

Solution. (*a*) Control limits for $\overline{X}$-chart.

Sample No.	*Mean*	*Range*
1	49	7
2	45	5
3	48	7
4	53	9
5	39	5
6	47	8
7	46	8
8	39	6
9	51	7
10	45	6
	$\Sigma \overline{X} = 462$	$\Sigma R = 68$

$$\overline{\overline{X}} = \frac{462}{10} = 46.2, \quad \overline{R} = \frac{68}{10} = 6.8$$

$$\text{Central Line} = \overline{\overline{X}} = 46.2.$$

$$\text{UCL} = \overline{\overline{X}} + A_2\overline{R} = 46.2 + 0.58\ (6.8) = 46.2 + 3.944 = 50.144$$

$$\text{LCL} = \overline{\overline{X}} - A_2\overline{R} = 46.2 - 0.58(6.8) = 46.2 - 3.944 = 42.256$$

Control limits for R-Chart

$$\text{Central Line} = \overline{R} = 6.8$$

$$\text{UCL} = D_4\overline{R} = 2.115(6.8) = 14.382$$

$$\text{LCL} = D_3\overline{R} = 0(6.8) = 0$$

In both cases, *i.e.*, $\overline{X}$ and $\overline{R}$-charts, some of the points are lying outside the control limits. Hence the process is not in a state of control.

Illustration 21. The number of defects on 20 items are given below :

Item No.	:	1	2	3	4	5	6	7	8	9	10
No. of defects	:	2	0	4	1	0	8	0	1	2	0
Item No.	:	11	12	13	14	15	16	17	18	19	20
No. of defects	:	6	0	2	1	0	3	2	1	0	2

Prepare a suitable control chart and draw your conclusion. (*MBA, Pune Univ., 2004*)

Solution. Let C denote the number of defects per item.

$$\overline{C} = \frac{\Sigma C}{n} = \frac{35}{20} = 1.75$$

$$\text{UCL} = \overline{C} + 3\sqrt{\overline{C}}$$

$$= 1.75 + 3\sqrt{1.75}$$

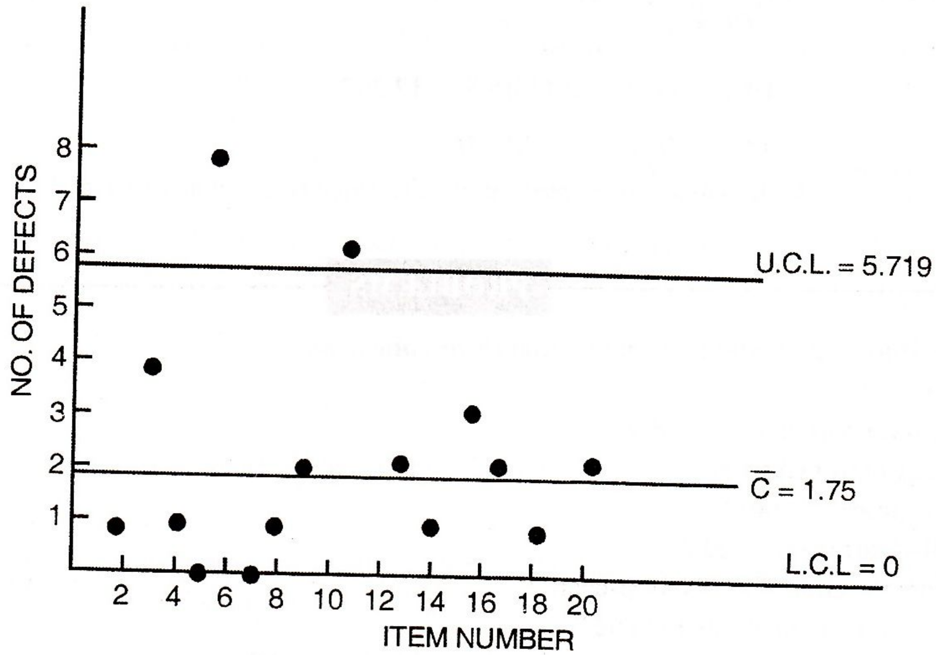
$$= 1.75 + 3 \times 1.323 = 5.719$$

$$\text{LCL} = \overline{C} - 3\sqrt{\overline{C}}$$

$$= 1.75 - 3\sqrt{1.75}$$

$$= 1.75 - 3 \times 1.323 = -2.219 \text{ or } 0$$

The control chart based on these limits is given below :



Since two of the points are lying outside the control limits, the process is not in a state of control.

Illustration 22. The following figures give the number of defectives in 20 samples containing 2000 items :

425,	430,	216,	341,	225,	322,	280,	306,	337,	305,
356,	402,	216,	264,	126,	409,	193,	280,	389,	326.

Find the control limits for the appropriate chart to be used.

Solution. The appropriate chart to be used in this case is a p-chart. Total number of defectives out of 40,000 items in 20 samples is :

$$= 425 + 430 + 216 + \ldots\ldots\ldots\ldots = 6148$$

$$\bar{p} = \frac{6148}{40000} = 0.1537$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}\,(1-\bar{p})}{n}}$$

$$= 0.1537 - 3\sqrt{\frac{0.1537 \times 0.8463}{2000}} = 0.1537 - 0.0244 = 0.1293$$

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}\,(1-\bar{p})}{n}} = 0.1537 + 0.0244 = 0.1781.$$

Illustration 23. You are given the values of sample means ($\bar{X}$) and the range (R) for the samples of size 5 each. Calculate the values for the mean and range control charts and comment on the state of control.

Sample No. :	1	2	3	4	5	6	7	8	9	10
$\bar{X}$:	43	49	37	4	5	37	51	46	43	47
R :	5	6	5	7	7	4	8	6	4	6

You may use the following control chart constants for $n = 5$, $A_2 = 0.58$, $D_3 = 0$, $D_4 = 2.115$. *(M.Com., DU, 2006)*

Solution. Control limits for $\bar{X}$ chart :

$$\text{CL} = \frac{\Sigma\bar{X}}{N} = \frac{442}{10} = 44.2; \quad \bar{X} = \frac{58}{10} = 5.8$$

$$\text{UCL} = \bar{\bar{X}} + A_2\,\bar{R}$$

$$= 44.2 + 0.58(5.8) = 44.2 + 3.364 = 47.564$$

$$\text{LCL} = \bar{\bar{X}} - A_2\,\bar{R}$$

$$= 44.2 - 0.58(5.8) = 44.2 - 3.364 = 40.836$$

Since some of the points are lying outside the control limits, the process is not in a state of control.

Control limits for R chart :

$$CL = \overline{R} = \frac{58}{10} = 5.8$$

$$UCL = D_4 \overline{R} = 2.115(5.8) = 12.267$$

$$LCL = D_3 \overline{R} = 0(5.8) = 0$$

Since none of the points is lying outside the control limits, the range is in a state of control.

PROBLEMS

1-A: Answer the following questions, each question carries **one** mark:

(*i*) What is SQC ?
(*ii*) Give two important uses of SQC.
(*iii*) What is Control Chart ?
(*iv*) What is process control ?
(*v*) Why R-Chart is prepared ?
(*vi*) What are the control limits for p-chart ?
(*vii*) Is 100% inspection totally reliable ?
(*viii*) How do control charts reveal that the process is out of control ?
(*ix*) Distinguish between defects and defectives.
(*x*) What is OC curve ?
(*xi*) Define the terms 'AQL' and 'RQL'.

1-B: Answer the following questions, each question carries **four** marks:

(*i*) Differentiate between control chart of variables and attributes.
(*ii*) Explain the terms 'chance variation' and 'assignable variation' with suitable example.
(*iii*) What is $\overline{X}$-chart ? How are the control limits determined while drawing this chart ?
(*iv*) What is C-chart ? Point out its uses.
(*v*) What is acceptance sampling ? Point out its role in business decision-making.
(*vi*) Distinguish between single sampling and double sampling plans.

2. What is a statistical quality control ? Point out its importance in the industrial world. Also explain the role of control charts.

3. (*a*) Distinguish between the process control and product control.
(*b*) Distinguish between the control limits and tolerance limits.

4. What is a control chart ? Describe how a control chart is constructed and interpreted.

5. Discuss the basic principles underlying control charts. Explain in brief, the construction and use of *p*-chart and *C*-chart.

6. What is control chart ? Explain in brief, the construction and use of mean chart, *p*-chart and range chart.

7. (*a*) What is acceptance sampling ? Point out the role of operating characteristic curve.
(*b*) Critically examine the different types of acceptance sampling plans.

8. (*a*) What do you mean by SQC? Discuss briefly its need and utility in industry. Discuss the causes of variation in quality. ***(MBA, Vikram Univ., 2004)***
(*b*) What are the various types of control charts known to you ? Explain them with examples.

9. "Quality control is attained most efficiently of course, not by the inspection operation itself but by getting at the causes." Comment on the statement. Describe the various devices employed for the maintenance of quality in a uniform flow of manufactured products.

10. Describe control charts for $\overline{X}$ and σ and derive expression for their control limits. What are the advantages of σ-chart over the *R*-chart ?

11. Explain the term "Statistical quality control". How is the process control achieved with the help of control chart ? What are the fundamentals underlying the construction of quality control chart ?

12. (*a*) Describe how a control chart for fraction defective is set ? What modification is needed if varying numbers are inspected on different occasions ?
(*b*) Discuss the role of *C*-chart in statistical quality control.

13. Explain the following terms occurring in sampling inspection plans :
(*a*) A.O.Q.L, (*b*) lot tolerance per cent defectives, (*c*) producer's risk, and (*d*) consumer's risk.

14. (a) Explain what are chance causes and assignable causes of variation in the quality of manufactured product.
(b) Assuming the characteristic variable follows a normal distribution (mean and standard deviation unknown), specify the control limits and the central line for the mean and range charts.

15. (a) Distinguish between process control and product control.
(b) State the different types of acceptance sampling plans.

16. (a) "The control charts make it possible to distinguish between those variations which are due to chance causes and those due to assignable causes".
Explain the terms 'chance causes' and 'assignable causes' and elucidate the statement.
(b) Distinguish between :
(i) Chance causes and assignable causes of variation.
(ii) Defect and defectives.
(iii) Control charts for variables and control charts for attributes.

17. What is the mathematical justification on which the control limits in $\overline{X}$ -charts are set up? What is the purpose of an $\overline{X}$ control chart ?

18. Explain how a control chart helps to control the quality of a manufactured product. Describe the basis of control chart. Distinguish clearly between the charts for variables and charts for attributes.

19. (a) Why $\overline{X}$ and R-charts should be used simultaneously ? Justify with the help of an example.
(b) Explain OC curve. Also explain how various points on the curve are calculated, *i.e.*, show calculations for any point (not for $p = 0$).
(c) Discuss the uses of statistical quality control and control charts.

20. (a) What is an 'OC' curve ? Which OC curve would be called ideal?
(b) Draw OC curve for the following single sampling plan.
$N = 50, n = 10, C = 1$.
(c) Write a short note on C-chart.

21. (a) Explain the construction and function of
(i) $\overline{X}$ -chart (ii) R chart.
(b) State the advantages of quality control.
(c) Explain the construction of double sampling plan.
(d) Differentiate between p-chart and C-chart in context of statistical quality control.

22. (a) Distinguish between random variations and assignable variations. How is the distinction relevant in statistical quality control ? *(M.Com., DU, 1999)*
(b) 25 sub-groups of 5 items each were taken in the measurement of an important dimension of a manufactured part. The mean of the 25 sub-groups was 0.6000 inches and the sum of the ranges of the sub-groups was 0.5 inches. Find the upper and lower control limits for the control chart for $\overline{X}$ and R..

23. The following data are the results of life tests on 15 samples of 6 fluorescent lamps each. The values are in hours.

Sample No.	$\overline{X}$	R	*Sample No.*	$\overline{X}$	R
1	4209	450	9	4420	320
2	4380	390	10	4385	510
3	4560	480	11	4182	490
4	3490	330	12	4260	385
5	3360	460	13	4550	220
6	3450	380	14	3890	490
7	3280	400	15	4280	160
8	3380	440			

(a) Is the process in a state of statistical quality control ?
(b) Assuming assignable causes could be discovered and eliminated, what is your best estimate of the capability of this process ?

24. A plant produces paper for newsprint, and rolls of paper are inspected for defects. The results of the inspection of 25 rolls of paper are given below :

Roll No.	*No. of defects*	*Roll No.*	*No. of defects*
1	10	14	5
2	20	15	4
3	8	16	2
4	12	17	3
5	13	18	6
6	15	19	8
7	25	20	9
8	7	21	15
9	13	22	18
10	18	23	20
11	16	24	10
12	14	25	5
13	6		

Draw control chart for defects and determine whether inspection results indicate stability.

25. Samples of 50 calculators are drawn randomly from the output of a process that produces several thousand units daily. Sampled items are inspected for quality, and faulty calculators are rejected. The result to a series of samples are given below :

Sample results of 15 lots of 50 calculators

Lot No.	*No. inspected*	*No. defectives*	*Lot No.*	*No. inspected*	*No. defectives*
1	50	4	9	50	5
2	50	5	10	50	6
3	50	8	11	50	8
4	50	10	12	50	5
5	50	6	13	50	12
6	50	7	14	50	4
7	50	3	15	50	2
8	50	2			

Draw a control chart and interpret it.

26. The number of defects found in inspecting television set assemblies are as follows for 20 inspection units of five sets each :

Unit :	1	2	3	4	5	6	7	8	9	10
No. of defects :	2	40	38	63	92	45	18	120	45	38
Unit :	11	12	13	14	15	16	17	18	19	20
No. of defects :	40	73	68	90	63	85	56	72	40	50

Set up a control chart to be used for future production.

27. A manufacturer purchases small bolts in cartons that usually contain several thousand bolts. Each shipment consists of a number of cartons. As part of the acceptance procedure for these bolts, 400 bolts are selected at random from each carton and are subjected to visual inspection for certain defects. In a shipment of 10 cartons, the respective percentages of defectives in the sample from each carton are 0.0, 0.5, 0.75, 0.20, 2.250, 0.25 and 1.25. Plot the appropriate control chart and draw your conclusions.

28. A machine is designed to produce ball bearings having a mean diameter of 0.574 cms and a standard deviation of 0.008 cms. To determine whether the machine is in proper working order, a sample of 6 ball bearings is taken every two hours on all the working days (namely Monday to Friday) of the week and the mean diameter is computed from this sample. Design a rule whereby one can be fairly certain that the quality of the products are conforming to required standards. Give a sketch of the control chart.

29. A sample of 200 bolts is drawn at regular intervals from the production line, and each bolt is checked. The number of defective bolts in 20 successive samples are given below :

Sample No.	*Defective bolts*	*Sample No.*	*Defective bolts*
1	3	11	2
2	3	12	3
3	1	13	2
4	3	14	1
5	2	15	1
6	3	16	3
7	2	17	3
8	2	18	3
9	3	19	2
10	3	20	3

Draw a suitable control chart and test whether the process is under control.

30. The following figures give the number of defectives in 20 samples ; each sample containing 2,000 items :
425, 430, 216, 341, 225, 322, 280, 306, 337, 305, 356, 402, 216, 264, 126, 409, 193, 326, 210, 389.
Calculate the values for central line and the control limits for p-chart (Fraction Defective chart). Draw the p-chart and comment if the process can be regarded in control or not.

31. With a view to examine the quality of an engineering product, 10 samples of 200 items each were taken from a day's production and the number of defective items in each sample was recorded as follows :

Sample No.	1	2	3	4	5	6	7	8	9	10
No. of defectives	14	20	36	42	22	18	26	2	12	8

(*i*) Draw a fraction defective quality control chart, showing clearly the upper and lower control limits.
(*ii*) What inference do you draw from this quality control chart ?

32. A large sample of a product gave an "average fraction defective" of 0.068. Calculate for a p-chart the values of control limits (upper, lower and central line), if the size of each sample sub-group is 200.

33. In a glass factory, the task of quality control was done with the help of mean ($\overline{X}$) and standard deviation (σ) charts. 18 samples of 10 items each were chosen and their values ΣX and $\Sigma\sigma$ were found to be 595.1 and 8.28 respectively. Determine 3σ limits for mean and standard deviation charts. You may use the following control factors for your calculations :

n	A_1	B_2	B_3
10	1.03	0.28	1.72

34. In a factory that produces steel tubes, the thickness of walls is to be controlled. Every hour a sample of 6 tubes is taken and after measurements average thickness in centimetres and the range for each sample is noted.

Sample No. :	1	2	3	4	5	6	7	8	9	10
Average thickness :	0.25	0.32	0.42	0.22	0.28	0.10	0.25	0.40	0.06	0.29
Range :	0.25	0.48	0.12	0.12	0.19	0.10	0.06	0.46	0.10	0.32

Draw average and range charts and give your comments whether the process is under control or not.

35. (*a*) Draw an *OC* curve for the following sampling plan, which is used to inspect lots of size 500 items each.
Sample size = 10; Acceptance No. = 1; Rejection No. = 2
(*b*) Describe briefly a multiple acceptance sampling plan.

36. Control on measurements of pitch diameter of thread in aircraft fittings is checked with 5 successive items measured at regular intervals, 5 such samples are given below :

Sample	*Measurement on each item of 5 items per hour*				
1	45	45	44	43	42
2	41	41	44	42	40
3	40	40	42	40	42
4	42	43	42	42	45
5	43	44	47	47	45

(Values are expressed in units of 0.001 inch.)

(i) Construct the $\overline{X}$ and R-charts.

(ii) What inference do you draw from these quality control charts ?

($n = 5, A_1 = 0546, A_2 = 0.577, D_2 = 4.981, D_3 = 0, D_4 = 2.115$)

37. It has been ascertained that when a manufacturing process is under control, the average of the defectives per sample batch of 10 is 12. What limits would you set in a quality control chart based on the examination of defectives in sample batches of 10 ?

38. Process for producing solid state devices such as transistors frequently have a rather high fraction defective. One particular line for making transistors has a long run fraction defective of 0.28 when functioning properly. Every two hours a sample of 50 transistors is examined and the number of defectives in the sample determined. What are the control limits for the p-chart used to control this process ?

39. The Quality Electric Bulbs Ltd. manufactures electric bulbs under an improved process. The Production Engineer takes a random sample of 100 bulbs off the run from each day's output for inspection. The number of defective pieces is determined by applying a high voltage test. Suppose that the process of manufacture when under control admits of a long run fraction defectives of 0.05 ?Determine the control limits on the p-chart.

40. When will you use a control chart for defects ? Plot control chart for the following data pertaining to number of defects in the calculators manufactured by a company :

Calculator No. :	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
No. of Defects :	5	1	0	7	3	6	0	10	2	11	5	8	6	4	1

41. Given below is a record of the number of defects seen in circuit panels used in a computer.

Prepare an appropriate control chart to control the quality of the product. Assess whether the process is in control as per data observed.

If assignable causes have been found to explain outliers, how does this affect the control chart. Show the necessary changes in control limits.

Panel :	1	2	3	4	5	6	7	8	9	10	11	12	13	14
No. of defects :	4	0	1	4	5	3	6	2	2	0	5	10	3	4

42. The following data refers to visual defect found during the inspection of the first 10 samples of size 50 each from a lot of Two-wheelers manufactured by an Automobile Company :

Sample No. :	1	2	3	4	5	6	7	8	9	10
No. of Defectives :	4	3	2	3	4	4	4	1	3	2

Draw the 'p' chart to show that the fraction defectives are under control.

[$UCL = 0.1608$; $LCL = 0$]

43. The following data show the values of sample mean and the range for ten samples of size 5 each. Construct the $\overline{X}$ and Range charts :

Sample No. :	1	2	3	4	5	6	7	8	9	10
Mean ($\overline{X}$) :	11.2	11.8	10.8	11.6	11.0	9.6	10.4	9.6	10.6	10.0
Range (R) :	7	4	8	5	7	4	8	4	7	9

44. The average number of defectives, in 22 sampled lots of 2000 rubber belts each, was found to be 16%. Determine the 3-sigma control limits for the p-chart. ***(Diploma in Mgt., AIMA, 2004)***

45. The following data shows the mean and the range for ten samples of size each. Calculate the values for the central line and control limits for mean-chart and range-chart, and determine whether the process is in control.

Sample No. :	1	2	3	4	5	6	7	8	9	10
Mean :	11.4	12.0	11.0	11.8	11.2	9.8	10.6	9.8	10.8	10.2
Range :	7	4	8	5	7	4	8	4	7	9

(M.Com., DU, 2006)

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