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Statistical Decision Theory

INTRODUCTION

Each one of us is concerned with making decisions of one kind or the other. Some of these are really difficult to make because of the complexity of a decision situation. It is precisely the complexity that has led many decision-makers to analyse the process of decision-making. It is in this context, that we need a decision theory which may be defined as a body of methods helpful to a decision-maker to select wisely, the best course of action from amongst several alternatives. The problem of statistical decision theory is that, given a situation where there are several available alternative courses of action, each of which may lead to a set of mutually exclusive outcomes associated with certain probabilities, which course of action should a decision-maker take? Any business unit may be faced with various types of problem situations such as: how much product to be produced; how much promotional effort to be expanded; how much quantity to be stocked, etc.

There are some elements which are common to all kinds of decision problems. These are :

1. The Decision-maker. The decision maker refers to an individual or a group of individuals responsible for making a choice of an appropriate course of action from the available courses of action.

2. Courses of Action. Courses of action are also called actions, alternatives, acts or strategies. For a problem situation, all possible courses of action should be included.

3. States of Nature. States of nature are sometimes called outcomes or events. The decision-maker must develop an exhaustive list of possible *future* of events. However, decision-maker has no control over the occurrence of specific event.

4. Payoff. It is the effectiveness associated with a particular combination of a course of action and state of nature.

5. Payoff Table. For a given problem, a payoff table lists the states of nature and a set of given courses of action. For each combination of states of nature and course of action, the payoff is calculated. Let different states of nature or outcomes be represented by O_1, O_2, ..O_m and different courses of actions or strategies by S_1, S_2 , ..., S_n. For a given combination of O_1 and S_1, the corresponding payoff is denoted by a_{ij}. Such a payoff table is shown below :

Courses of actions

States of nature	S_1	S_2 ..... S_j ..... S_n
O_1	a_{11}	a_{12} ..... a_{1j} ..... a_{1n}
O_2	a_{21}	a_{22} ..... a_{2j} ..... a_{2n}
⋮	⋮	⋮ ⋮ ⋮
O_i	a_{i1}	a_{i2} ..... a_{ij} ..... a_{in}
⋮	⋮	⋮ ⋮ ⋮
O_m	a_{m1}	a_{m2} ..... a_{mj} ..... a_{mn}

Payoffs may be evaluated in terms of *Profit, Cost,* or *Opportunity Loss.* When such payoffs are evaluated in terms of profit, they are called *Payoffs.* A payoff table shows the relation between all possible outcomes (states of nature), all possible courses of action and the value associated with these.

In the payoff table, the column headings designate the various courses of action out of which the decision-maker may choose while the row headings show the admissible states of nature under which the decision-maker has to take decision. The cell value a_{ij} shows payoff resulting by taking course of action S_j when the state of nature is O_i for all $j = 1, 2, 3, ..., n$ and $i = 1, 2, 3, ..., m$. A payoff table represents the economics of a problem —a problem of revenue and costs. A payoff may be thought of as a conditional value or conditional profits (losses). It is conditional value in the sense that associated with each course of action there is a certain profit (or loss), given that a specific state of nature has occurred. A payoff table thus contains all conditional values of all possible combinations of courses of action and states of nature.

Decision theory models can be classified on the basis of the degree of certainty. We can visualize decision problems with complete certainty on the one hand and complete uncertainty on the other. Therefore, most of the decision situations fall between these two extremes. We can classify the following types of decision-making and each will be discussed in detail :

(*a*) Decision-making under certainty.

(*b*) Decision-making under risk.

(*c*) Decision-making under uncertainty,

(*d*) Decision-making under conflict or theory of games.

(a) Decision-making under Certainty

In this type of deterministic situation, the outcome of a specified decision can be predetermined with certainty.

(b) Decision-making under Risk

There are certain types of decision problems where there can be more than one outcome and it is possible to assign a probability value to each outcome. In other words, the decision-maker knows the probability of occurrence of each state of nature.

(*i*) *Expected monetary value.* The best strategy is selected on the basis of the highest expected monetary value (EMV). The EMV for a course of action is the sum of the products obtained by multiplying the payoff for a given outcome by its probability value.

If $m_1, m_2, ...m_n$ are the payoffs corresponding to the states of nature $S_1, S_2, ...S_n$ respectively, and the corresponding probabilities of $S_1, S_2, ...S_n$ are $P_1, P_2, ... P_n$, then EMV is defined as :

$$\text{EMV} = m_1p_1 + m_2p_2 + ...m_np_n = \Sigma mp.$$

The following points in relation to decision-making under risk may be found relevant :

(*a*) We must be able to construct conditional payoff table in order to compare the EMVs of different courses of action.

(*b*) The choice of the strategy is purely on the basis of EMV. Sometimes, utility may be more appropriate than EMV.

(*c*) The EMV criterion does not take into account the quality of risk. Generally, the decision-maker goes by the trade-off between risk and return.

(*ii*) *Expected opportunity loss.* Another decision criterion of decision theory is called expected opportunity loss (EOL). Opportunity loss represents the amount of profit that is lost because the most profitable course of action is not taken. To calculate EOL, we must find the conditional opportunity loss

(COL). The COL of the optimal act being zero, the COL of any other act is the difference between the payoff of the optimal act and the action taken, and obviously will always be positive. When payoffs are replaced by their corresponding opportunity losses, we get what is known as the Loss Table. If l_{ij} (the element of a loss table) is the opportunity loss resulted by taking an action Sj, when the state of nature O_i then l_{ij} satisfies the following relation :

$$l_{ij} = \max_j p_{ij}\, l - p_{ij}$$

for all $i = 1, 2, \ldots\ldots, n$

$j = 1, 2, \ldots\ldots, m$

Illustration 1. A baker produces a certain type of special pastry at a total average cost of Rs. 3 and sell it at a price of Rs. 5. This pastry is produced over the weekend and is sold during the following week; such pastry being produced but not sold during a week's time are totally spoiled and have to be thrown. According to past experience, the weekly demand for these pastries is never less than 78 or greater than 80. You are required to formulate action space, state space, payoff table and loss table.

Solution. It is clear from the problem given that the manufacturer will not produce less than 78 or more than 80 pastries. Thus, there are three courses of action open to him :

S_1 = produce 78 pastries

S_2 = „ 79 „

S_3 = „ 80 „

The state of nature is the weekly demand for pastries. There are three possible states of nature, *i.e.*,

O_1 = demand is 78 pastries

O_2 = „ 79 „

O_3 = „ 80 „

The uncertainty element in the problem is the weekly demand. The bakery profits are conditioned by the weekly demand. Cell values of payoff table are computed as follows :

a_{11} = payoff when action S_1 is taken but the state of nature is O_1
= Rs. [5 × 78 – 3 × 78] = Rs. 156.

a_{12} = payoff when action S_2 is taken but the state of nature is O_1
= Rs. [5 × 78 – 3 × 79] = Rs. 153.

a_{13} = payoff when action S_3 is taken but the state of nature is O_1
= Rs. [5 × 78 – 3 × 80] = Rs. 150.

Similarly, a_{21} = payoff when action S_1 is taken but the state of nature is O_2
= Rs. [5 × 78 – 3 × 78] = Rs. 156.

a_{22} = Rs. [5 × 79 – 3 × 79] = Rs. 158.

a_{23} = Rs. [5 × 79 – 3 × 80] = Rs. 155.

Similarly, a_{31} = payoff when action S_1 is taken but the state of nature is O_3
= Rs. [5 × 78 – 3 × 78] = Rs. 156.

a_{32} = Rs. [5 × 79 – 3 × 79] = Rs. 158.

a_{33} = Rs. [5 × 80 – 3 × 80] = Rs. 160.

These values are tabulated below :

PAYOFF TABLE

State of nature / *Courses of Action*	S_1	S_2	S_3
O_1	156	156	150
O_2	158	156	155
O_3	156	158	160

To calculate opportunity losses, we first calculate max a_{1k}, max a_{2k} and max a_{3k}.

$$\max a_{1k} = 156, \max a_{2k} = 158, \max a_{3k} = 160$$

$$l_{11} = 156 - 156 = 0, l_{12} = 156 - 153 = 3, l_{13} = 156 - 150 = 6$$

$$l_{21} = 158 - 156 = 2, l_{22} = 158 - 158 = 0, l_{23} = 158 - 153 = 5$$

$$l_{31} = 160 - 156 = 4, l_{32} = 160 - 158 = 2, l_{33} = 130 - 130 = 0$$

The loss table corresponding to payoff table is given below :

LOSS TABLE

State of nature \ Courses of Action	S_1	S_2	S_3
O_1	0	3	6
O_2	2	0	5
O_3	4	2	0

(iii) Expected value of perfect information. The expected value of perfect information (EVPI) is the difference between expected profit (EP) of the optimal decision without perfect information and that with the perfect information. This expected profit with perfect information (EPPI) is called the expected value of payoff under certainty. The perfect prediction reduces the opportunity losses due to uncertainty to zero. The highest payoff in the absence of perfect predictor is *EP* of the optimal action. The difference between *EPPI* and *EP* is called the *expected value of perfect information* (abbreviated *EVPI*). *EVPI* represents the maximum amount of money which a decision-maker could spend to obtain additional information regarding the states of nature.

It may be noted that *EVPI* is always equal to the *EOL* of selecting the optimum action under uncertainty. The identity $EP + EOL = EPPI$ follows from the result $EVPI = EOL$ and $EVPI = EPPI - EP$.

The main objective of preposterior analysis is to determine whether or not it is profitable to gather additional information regarding the states of nature before taking the final action. Additional information may be gathered by conducting survey, by carrying out an experiment or by some other means. The objective of preposterior analysis is fulfilled by computing *EVPI*. If *EVPI* is relatively larger than the cost involved in gathering additional information, it is advisable to gather the additional information regarding the states of nature.

Illustration 2. A businessman wants to construct a hotel. He usually builds 25,50 or 100 bed hotel, depending on whether anticipated demand is low, medium or high. The businessman has been able to find out net profits which are expressed in the table below and the corresponding probabilities are also given below.

Courses of Action / States of nature	S_1 Build 25-bed-hotel	S_2 Build 50-bed-hotel	S_3 Build 100-bed-hotel
O_1 = Low demand	20,000	–10,000	–30,000
O_2 = Medium demand	25,000	25,000	–5,000
O_3 = High demand	30,000	50,000	60,000

States of nature = Demand	O_1	O_2	O_3	Total
Probability	0.2	0.3	0.5	1.00

(*a*) Compute *EP, EPPI* and *EVPI.*

(*b*) A research firm agrees to conduct a survey for Rs. 8000, and provide him with information regarding the states of nature. Should the survey be conducted ?

Solution. *To compute EP,* we have to compute expected payoff each action under uncertainty as follows :

$$E(S_1) = \text{Rs. } [20{,}000 \times 0.2 + 25{,}000 \times 0.3 + 30{,}000 \times 0.5] = \text{Rs. } 26{,}500$$

$$E(S_2) = \text{Rs. } [-10{,}000 \times 0.2 + 30{,}000 \times 0.3 + 50{,}000 \times 0.5] = \text{Rs. } 32{,}000$$

$$E(S_3) = \text{Rs. } [-30{,}000 \times 0.3 + (-5{,}000) \times 0.3 + 60{,}000 \times 0.5] = \text{Rs. } 19{,}500$$

From the above computation it is clear that the highest expected payoff or profit is associated with action S_2. Hence, the highest expected payoff under certainty = EP = Rs. 32,000.

Next, to compute *EPPI* we have to find out the highest payoff for each action under certainty, *i.e.*, under the assumption that the perfect predictor is available. Clearly from the payoff table, when the state of nature is known to the businessman would take action S_1 as a result of nature are known $O_1$to be O_2 and O_3 he correspondingly takes action S_2 and S_3 by which he makes his net profit 30,000 and 60,000 respectively. The highest expected pay off under certainty is computed as $EPPI$ = Rs. [20,000 × 0.2 + 30,000 × 0.3 + 60,000 × 0.5] = Rs. 43,000. The expected value of perfect information = $EVPI = EPPI - EP$ = Rs. [43,000 – 32,000] = Rs. 11,000. The *EVPI* is relatively larger than the expenditure incurred in conducting a survey in order to collect further information regarding the states of nature. Hence, it is advisable to conduct the survey.

(c) Decision-making under Uncertainty

Competitive decision model is one related to the situation of uncertainty, the probabilities of occurrence of the different events (or the states of nature) are not known and the decision-maker has no way of calculating the expected payoffs for his strategies. This, in other words, means that the decision-maker has to act with imperfect information in such a situation. Consequently, there is no single best criterion for selecting a strategy to deal with such a situation but there are different criteria available for selecting a strategy. The following criteria are important in this context :

(*i*) *Maximin decision rule.* Under this rule, the decision-maker is completely pessimistic. He assumes that the situation will always be disadvantageous. As such he selects that strategy which give largest minimum payoffs, *i.e.*, maximum of the minimums.

(*ii*) *Maximax decision rule.* Under this rule, the decision-maker is quite optimistic. He assumes that the situation will always be to his advantage. He, therefore, selects the strategy which yields him the highest possible pay-off, *i.e.*, maximum of the maximums.

(*iii*) *Minimax decision rule.* This rule is based on general insurance against risk. It insures against the maximum possible risk. Under it, one adopts the strategy which causes minimum of the maximum losses. Because of such an attitude, this rule is sometimes also known as "regret rule", for one looks at loss opportunities (losses) as regrets. The minimax rule items from the work of John Von Neumann and Oskar Morgenstern. This rule states that the decision-maker should minimize maximum harm.

(*iv*) *Hurwicz decision rule.* Hurwicz has developed a decision rule basing it on the maximin and maximax rules with an index of optimism (x) and an index of pessimism ($1 - x$). The value of x always lies between zero and one. The decision-maker should assign a value to x somewhere between 0 and 1. The value of x nearer to 1 means the decision-maker is optimistic, and near to 0 reflects a pessimistic

decision-maker and $x = ½$ reflects a neutralist. Largest and smallest values, say V, U respectively be determined for each and every strategy by applying maximin and maximax rules and then the expected value be determined as under :

Expected value $= x.V + (1 - x)\ U$

Then the strategy having the highest expected value as per the above formula given by Hurwicz is selected.

(*v*) *Laplace decision rule.* This rule is based on the assumption (in case of the probabilities are not known) that the probabilities of different states of nature for a given strategy are all equal. Considering these equal probabilities, the expected payoffs will be calculated as per the method above stated and then the strategy with the largest expected payoff is selected.

From the above description, it should become clear that there is no single best rule for decision-making under the situation of uncertainty. There are several models for the purpose. The choice for the selection of a model should be left to the decision-maker who should ultimately decide as per his own skill and experience considering the environment, firm's policy and other relevant factors.

The following example can be illustrated to exhibit how to make decision under uncertainty :

Illustration 3. A Toy Company is bringing out a new type of toy. The company is attempting to decide whether to bring out a full, partial, or minimal product line. The company has three levels of product acceptance. Management will make its decision on the basis of expected profit from the first year of production. The relevant data are shown in the following table :

	Anticipated 1st year Profit (Rs. 000's)		
Product	*Product line*		
Acceptance	*Full*	*Partial*	*Minimal*
Good	80	70	50
Fair	50	45	40
Poor	–25	–10	0

Take optimal decision under each of the following decision criteria :

(*i*) Maximax, (*ii*) Maximin,

(*iii*) Laplace criteria, (*iv*) Minimax regret.

Solution. (*i*) The maximum profit (in Rs. 000's) for each product line is

Full	—	80
partial	—	70
Minimal	—	50

∴ The maximax of maximum payoffs is for full product line Rs. 80,000.

∴ The company should go for full product line under optimistic or maximax criteria.

(*ii*) The minimum profit (in Rs. 000's) for each product line is

Full	—	–25
partial	—	–10
Minimal	—	– 0

∴ The maximum of minimum payoffs is for minimal product line Rs. 0.

∴ The Company should go for minimal product line under pessimistic or maximin criteria.

(*iii*) If chances of Good, Fair or Poor product acceptance are equal *i.e.*, $\frac{1}{3}$, then the expected profit for each product line is

$$\text{Full} = \frac{1}{3}(80{,}000) + \frac{1}{3}(50{,}000) + \frac{1}{3}(-25{,}000)$$

$$= \frac{1}{3}(1{,}05{,}000) = \text{Rs. } 35{,}000$$

$$\text{Partial} = \frac{1}{3}(70{,}000) + \frac{1}{3}(45{,}000) + \frac{1}{3}(-10{,}000)$$

$$= \frac{1}{3}(1{,}05{,}000) = \text{Rs. } 35{,}000$$

$$\text{Minimal} = \frac{1}{3}(50{,}000) + \frac{1}{3}(40{,}000) + \frac{1}{3}(0)$$
$$= \text{Rs. } 30{,}000$$

$\therefore$ The maximum profit is for Full and Partial product line.

$\therefore$ The Company should go for full or partial product line under Laplace criteria.

(*iv*) The conditional opportunity loss table is

Product Acceptance	*Anticipated 1st year Profit (Rs. 000's)*		
	Product line		
	Full	*Partial*	*Minimal*
Good	0	10	30
Fair	0	5	10
Poor	25	10	0
Maximum loss	25	10	30

$\therefore$ The minimum of maximum loss is for partial product line.

$\therefore$ The company should go for partial product line under minimax regret criteria.

(*d*) Decision-making under Conflict (Theory of Games)

The theory of games which is also called decision-making under conflict, dates back to 1944 with the classic work of J. von Neumann and O. Morgenstern entitled *Theory of Games and Economic Behaviour.* Game theory provides a framework for analysing competitive situations in which the competitors (or players) make use of logical processes and techniques in order to determine an optimal strategy for "winning". Since many situations in business involve competition, game theory is of considerable theoretical interest.

A game can be played between two or more individuals or groups of individuals. Business environment being always competitive, the number of problems which lend themselves to this theory are abundant. For example, two firms may be trying to determine how much of advertising to do, where the options in terms of amounts of advertising, and the payoffs (may be in terms of increase in sales or market share) are known.

A *Game* is described by its set of rules. These rules specify clearly what each person, called a player, is allowed or required to do under all possible sets of circumstances. The rules also define the amount of information, if any, each person receives. A game is finite when each player has a finite number of moves and finite number of choices at each move.

Kinds of Games. It is convenient to classify games according to the number of players, *i.e.*, as two persons, three persons, etc. It is also convenient to distinguish between games whose payoffs are zero sum and those whose are not. If the players make payments only to each other, *i.e.*, the loss of one is the gain of the other, the game is said to be *zero sum.* Thus, solitaire is a one-person game and chess is a two-person game. Mathematically speaking, a zero sum game can be represented as:

In an n person game with players $PL_1, PL_2 \ldots, PL_n$ and payoffs PF_i $(i = 1, 2, \ldots n)$ be made to PL_i at the end of the game (PF_i will be negative if PL_i has to pay rather than receive).

Then, if $\Sigma PEi = 0$, the game is *zero-sum.*

$\neq 0$, the game is non-*zero-sum.*

The usual distinction in game theory is between two-person games and games involving three or more persons. The theory of games of three or more persons (n-person games) is largely undeveloped, and it is precisely this limitation that has restricted the application of game theory from many real life applications. Therefore, the discussion of this chapter will be limited to the presentation and analysis of two-person zero-sum games. The underlying assumptions, the rules of the game are given as follows :

1. The players act rationally and intelligently.
2. Each player has available to him a finite set of possible courses of action.
3. The players attempt to maximize gains and minimize losses.
4. All relevant information is known to each player.
5. The players make individual decisions without direct communication.
6. The players simultaneously select their respective courses of action.

Two-Person Zero-Sum Game

Two-person zero-sum games are the games played by two persons, parties, or groups, with directly opposite interest. One persons's gain in the game is exactly equal to another person's loss, and therefore, the sum total of the gains and lossses equals zero. Each person has alternative choices of *strategies* (moves) available to him, and the rules governing choices are known in advance to the players. The outcome of a set of possible choices of strategies is also known to the two players in advance and is expressed in terms of numerical values.

A two-person zero-sum game is conveniently represented by a game matrix as shown below. A game matrix is often referred to as a *payoff matrix,* because the outcome of the alternative choice of strategies are expressed in terms of payoff units.

		Player B's strategies	
		B_1	B_2.....B_n
Player A's Strategies	A_1	a_{11}	a_{12}.....a_{1n}
	A_2	a_{21}	a_{22}.....a_{2n}
	⋮	⋮	⋮ ⋮
	A_m	a_{m1}	a_{m2}.....a_{mn}

For example, payoff a_{12} refers to the strategy A_1 adopted by player A and strategy B_1 by player B.

The above payoff matrix is in relation to player A. It is important to note that only player A's gains are included in the payoff matrix. However, if the payoff matrix represents gains by player A, then player B in turn loses the same amount gained by player A and the sum of the reward is zero (zero-sum).

A game with a Pure Strategy

Let us consider a game with a payoff matrix presented in the following table. This is a two person zero-sum game with two alternative choices of strategies available to player A and three alternative choices of strategies available to player B. If the payoffs matrix represents the per cent market share obtained by player A, then player B loses the market share that A gains.

		Player B		
		B_1	B_2	B_3
Player A	A_1	80	40	75
	A_2	70	35	30

For example, if player A selects strategy A_1 and player B selects strategy B_1, then player A *wins* 80 per cent of the market share while player B loses 80 per cent of the market share.

In this game, player A's objective is to select a strategy which enables him to gain as much as possible. In contrast, player B's objective is to select a strategy which enables him to lose as little as possible. If a single strategy is chosen by players A and B, then it is referred to as a *pure strategy*. Therefore, a pure strategy is a single strategy in a stable solution. If the solution is not stable, then, we cannot have a pure strategy without further analysis. The first step in the pure strategies is to determine the minimax and maximin value. If the two values coincide (equal), we get the pure strategies for both the players to adopt. This equal value will be termed as *saddle point* and this value will be the value of the game.

Illustration 4. Consider the two-person zero-sum game with the following payoff matrix :

		Player B		
		B_1	B_2	B_3
Player A	A_1	5	4	6
	A_2	2	3	7
	A_3	4	3	0

Determine the optimal pure strategies for both the players and find the value of the game.

Solution. Using maximin principle, player A selects that strategy which is maximum of the minimum gains (payoffs), *i.e.*, best of the worst guaranteed gains. Similarly, player B selects that strategy which is minimum of the maximum losses (payoffs), *i.e.*, the best of the worst losses. In fact, if the payoffs matrix contains both gains and losses for each player, either criterion will yield the same result. Minimax and maximin, both select the best of the worst outcomes.

Using the maximum principle, the strategy to be chosen will be determined based on the values of row minima. Similarly, for minimax principle (for opponent will be determined based on the values of the column maxima. This is shown below:

		Player B			
		B_1	B_2	B_3	Row minimum
Player A	A_1	5	4	6	4*
	A_2	2	3	7	2
	A_3	4	3	0	0
Column maximum		5	4*	7	

In this game, player A *will choose strategy* A_1, which yields the minimum pay off 4. Similarly, the best strategy for player B is a strategy which lead to a minimum column maxima. In this case, player B will choose strategy B_2 which has a maximum loss of 4. Both the maximum value of row minima and the minimum value of column maxima are denoted by asterisks in the game matrix. Since the value of the maximin coincides with the value of the minimax, *an equilibrium* or *saddle point* is determined in this game. It is apparent that a saddle point is that point which is minimum in the row and maximum in the column. The amount of payoff at an equilibrium point is also known as the *value of the game*. Hence, the optimal pure strategies for both the players are : Player A must select strategy A_1 and player B must select strategy B_2. The value of the game is 4 which indicates that player A will gain 4 units and player B will lose 4 units.

A Game with a Mixed Strategy

With no pure strategy solution, both players will prefer to alter the strategy selection or play *a mixed strategy*. Consider the payoff matrix as below :

		Player B	
		B_1	B_2
Player A	A_1	65	45
	A_2	50	55

Since this game has no saddle point, therefore, we cannot have pure strategies. In such cases, each player would like to mix up his strategies in a random selection. The random selection plan involves

selecting each strategy a certain per cent of the time, such that the player's *expected* gains (or losses) are equal, regardless of the opponent's selection of strategies. Selection of a strategy, a given per cent of the time is analogous to the selection of a strategy with a given probability. There are various methods to solve such type of games, but only three methods will be discussed here :

Method 1 (Algebraic)

The method for determining the per cent (or probability) to be associated with a given strategy wiil be illustrated for the previous example. Let us begin with player A, he wishes to select strategy A_1 or A_2 according to probabilities such that his expected gains are the same, regardless of player B's selection of strategies B_1 or B_2. If player B selects strategy B_1, the possible payoffs to player A are 65 and 50. If player A selects strategy A_1 with a probability of p and, therefore, selects strategy A_2 with a probability of $(1-p)$, then his expected gains for this game are given by :

$$65p + 50\,(1-p)$$

On the other hand, if player B selects strategy B_2, then player A's expected gains are :

$$45p + 55\,(1-p)$$

Now, in order for player A to be indifferent to which strategy player B selects, he wishes his expected gains to be equal for each of player B's possible moves. Thus, the two equations of expected gains are set equal and solved for p as given below :

$$65p + 50\,(1-p) = 45p + 55\,(1-p)$$

or $$25p = 5$$

Therefore, $$p = 1/5 = 0.2\ ;\ 1-p = 1-0.2 = 0.8.$$

Hence, player A would select strategy A_1 with a probability of 0.2 and strategy A_2 with a probability of 0.8.

Similarly, player B would determine his probabilities q and $(1-q)$ for selecting strategies B_1 and B_2 respectively, by equating his expected losses if player A chooses strategy A_1 to the expected losses if player A chooses strategy A_2 as follows :

$$65q + 45\,(1-q) = 50q + 55\,(1-q)$$

or $$25q = 10$$

Therefore, $$q = 2/5 = 0.4\ ;\ 1-q = 1-0.4 = 0.6.$$

Hence, player B would select strategy B_1 with a probability of 0.4 and strategy B_2 with a probability of 0.6.

The value of the game is determined by substituting the value of p or q in any of the expected value and is calculated as 53.

Method 2 (Calculus Method)

This method is almost similar to the previous method except that instead of equating the two expected values, the expected value for a given player is maximised. To illustrate this method, let us take the same example discussed in the previous method.

Suppose player A selects strategy A_1 with a probability p and obviously selects A_2 with a probability $(1-p)$ and player B selects strategy B_2 with a probability q and obviously selects strategy B_2 with a probability $(1-q)$. Then the expectation is given as below :

$$E\,(p, q) = 65pq + 45p\,(1-q) + 50\,(1-p)\,q + 55\,(1-p)\,(1-q)$$

If expectation is to be maximized, then

$$\frac{\partial E}{\partial p} = \frac{\partial E}{\partial q} = 0$$

$$\frac{\partial E}{\partial p} = 65q + 45(1-q) - 50q - 55(1-q) = 0$$

or $25q = 10$

Therefore, $q = \frac{10}{25} = 0.4,\ 1 - q = 1 - 0.4 = 0.6$

and $$\frac{\partial E}{\partial q} = 65p + 45p + 50(1-p) - 55(1-q) = 0$$

or $25p = 5$

Therefore, $p = \frac{5}{25} = 0.2\ ;\ 1 - p = 1 - 0.2 = 0.8$

To determine the value of the game, substitute the values of p, $1-p$, q and $1-q$ in the expression of expected value. The value of game is found to be 53 as before.

Illustration 5. Consider a rectangular game whose matrix is :

		Player B	
		1	2
Player A	1	1	3
	2	4	2

Find the best strategies and the value of the game.

Solution. Since the matrix has no saddle point, it is desirable for A and B to play with certain frequencies. suppose A plays 1 with frequency $x(0 \le x \le 1)$ and plays 2 with frequency $(1-x)$; and suppose B plays 1 with frequency $y\,(0 \le y \le 1)$ and plays 2 with frequency $(1-y)$. Then, A's mathematical expectation is given by

$$E(x, y) = 1xy + 3x(1-y) + 4y(1-x) + 2(1-x)(1-y)$$

If expectation is to be maximised :

$$\frac{\partial E}{\partial x} = y + 3(1-y) - 4y - 2(1-y) = 0 \qquad ...(1)$$

$$\frac{\partial E}{\partial y} = x + 3x + 4(1-x) - (1-x) = 0 \qquad ...(2)$$

From (1), $y + 3 - 3y - 4y - 2 + 2y = 0$

$4y = 1$ or $y = 1/4$

From (2), $x + 3x + 2 - 4x - 2 + 2x = 0$

$4x = 2$ or $x = 1/2$

Thus, here A should choose strategies 1 and 2 with equal probability and B should choose 1 with a probability 1/4 and choose 2 with a probability 3/4. The value of the game is found to be 5/2 by substituting these values of x and y in $E(x, y)$.

Illustration 6. Consider a game whose matrix is given as :

		Player B	
		1	2
Player A	1	a	b
	2	c	d

Find the optimal strategies for each player and the value of the game.

Solution. Let x and y be the probabilities of adopting strategies 1 and 2 by player A and B, respectively.

Then $$E(x,y) = axy + bx(1-y) + cy(1-x) + d(1-x)(1-y) \qquad ...(i)$$

$$\frac{\partial E}{\partial x} = ay + (b - by) - cy - d(1-y) = 0$$

$$y(a - b - c + d) = d - b$$

$$y = \frac{(d-b)}{(a-b-c+d)}$$

$$\frac{\partial E}{\partial y} = ax - bx + c - cx - d(1-x) = 0$$

$$x(a-b-c+d) = d-c$$

$$x = \frac{(d-c)}{(a-b-c+d)}$$

Substituting in (*i*), we get

$$V = \frac{(ab-bc)}{(a+d)-(b+c)}$$

However, this method of solution can only be used for 2 × 2 matrices ; for larger matrices we need to develop some other solution procedure.

Method 3 (Graphical Method)

The graphical method can be used to solve games where one of the players has only two alternatives and the other has two or more alternatives. Consider the following game :

		Player B	
		B_1	B_2
Player A	A_1	1	7
	A_2	6	2

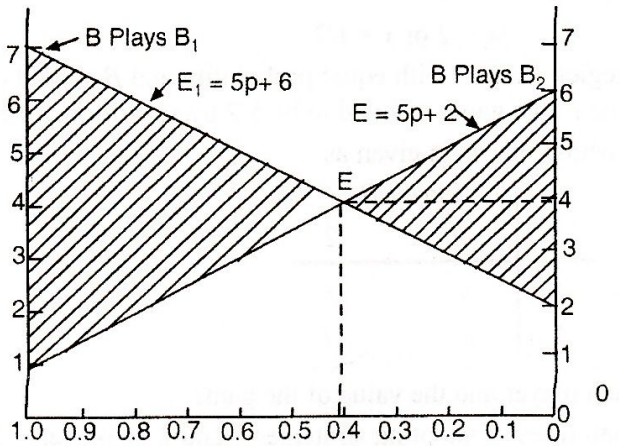
Assume that player B plays strategy B_1 all the time. What will the value of such a game be to player A? The value will depend on what player A does. If player A plays strategy A_1 all the time, the value of the game will be 1. If player A plays strategy A_2 all the time, the value of game will be 6 ; and if he mixes his strategies with a probability p and $(1-p)$, then the expected value of the game will be given by :

$$E_1 = p + 6(1-p) = -5p + 6$$

Similarly, if player B plays strategy B_2 all the time, value of the game to player A will be given by

$$E_2 = 7p + 2(1-p) = 5p + 2$$

The value $E_1 = -5p + 6$ and $E_2 = 5p + 2$ can be represented graphically by straight lines as shown in the figure given below :



The horizontal axis which represents the probability that player A will play strategy A_1 is constructed first, starting with $p = 0$ on the right side up to $p = 1$ on the left side. Every point on the horizontal axis represents the probability mix between strategy A_1 with probability 0.6 and choosing strategy A_2 with probability 0.4.

The vertical axis represents the payoff to player A. Now, two straight lines are plotted, each corresponding to one of player B's alternatives in the following way. For the payoff resulting from player B playing strategy B_1 all the time, a straight line is drawn from 7 on the left side to 2 on the right side. This is the line $E_1 = p + 6(1-p) = -5p + 6$. Next, a straight line is drawn for the situation in which player B plays strategy B_2 all the time; from 1 on the left to 6 on the right, describing the line $E_2 = 7p + 2(1-p) = 5p + 2$. The shaded area represents the entire feasible area of expected payoffs when p varies between 0 and 1. Player A will try to look for a payoff on the upper boundary of this feasible area. Player B will try to establish the payoff at the lower boundary of the feasible area. At point E, no matter what player B does, the payoff to player A is the maximum. However, it is also equal to the minimum payoff, *i.e.*, point E is the lowest on the upper boundary and the highest on the lower boundary.

The probability p can be read off the horizontal axis as 0.4, *i.e.*, player A should play strategy A, 40% of the time, and therefore, strategy A_2 60% of the time. The value of the game is found at point E and can be read off from the vertical axis as $V = 4$.

In a similar manner, it is possible to present player B's situation on an additional graph. It will reveal that player B's strategy is to mix strategy B_1, 50% of the time and strategy B_2, 50% of the time.

Dominance Principle

When any strategy is better than another for all cases, it is said to be dominating the other and it is clear that we can discard the latter one from our consideration. Dominated rows or columns are deleted to reduce the size of the matrix. For example,

		Player B		
		1	2	3
Player A	1	1	7	2
	2	6	2	7
	3	5	1	6

Here, it is clear that each of A's payoffs for strategy 3 is less than that of strategy 2. Thus, strategy 3 is inferior and clearly strategy 2 is a dominating strategy. It appears that we might as well solve the game :

		Player B		
		1	2	3
Player A	1	1	7	2
	2	6	2	7

Also, if we observe, we find B's losses (A's payoffs) are always more for strategy 3 than for strategy 1, so B might as well never play strategy 3. Thus, our game reduces to

		Player B	
		1	2
Player A	1	1	7
	2	6	2

and thus the solution to our original game is the same as the solution to this game (so, we need only to solve for a 2×2 matrix).

Rule of dominance

(*a*) If all elements in a column are greater than or equal to the corresponding elements in another column, then that column is dominated.

(*b*) Similarly, if all elements in a row are less than or equal to the corresponding elements in another row, then that row is dominated.

MISCELLANEOUS ILLUSTRATIONS

Illustration 7. A proprietor of a food-stall has invented a new item of food delicacy which he calls WHIM. He has calculated that the cost of manufacture is Re. 1 per piece and that because of its novelty and quality it would be sold for Rs. 3 per piece. It is, however, perishable, and any goods unsold, at the end of the day are a dead loss. He expects the demand to be variable and has drawn up the following probability distribution expressing his estimates :

No. of pieces demanded :	10	11	12	13	14	15
Probability :	0.07	0.10	0.23	0.38	0.12	0.10

(*i*) Find an expression for his net profit or loss if he manufactures m pieces and only n are demanded. Consider separately the two cases $n \leq m$, and $n > m$.

(*ii*) Assume that he manufactures 12 pieces. Using the results in (*i*) above, find his net profit or loss for each level of demand.

(*iii*) Using the probability distribution, calculate his expected net profit or loss, if he manufactures 12 pieces.

(*iv*) Calculate similarly the expected profit or loss for each of the other levels of manufactures ($10 \leq m \leq 15$).

(*v*) How many pieces should be manufactured so that his net expected profit is maximum ?

Solution. (*i*) The proprietor does not produce more than 15 pieces of WHIM or less than 10 pieces. His profit is determined by the demand (n) and production (m). When the demand is more than the production, his profit shall be

$$\text{Rs. } 3 \times m - \text{Rs. } 1 \times m = \text{Rs. } 2m \qquad (\text{if } n > m)$$

When the production equals or exceeds the demand, his profit shall be

$$\text{Rs. } 3 \times n - \text{Rs. } 1 \times m = [\text{Rs. } 3n - m] \qquad (\text{if } n \leq m)$$

PAYOFF TABLE

Production m / *Demand* n	S_1 *10*	S_2 *11*	S_3 *12*	S_4 *13*	S_5 *14*	S_6 *15*	*Probability*
10	Rs. 20	Rs. 19	Rs. 18	Rs. 17	Rs. 16	Rs. 15	0.7
11	20	22	21	20	19	18	0.10
12	20	22	24	23	22	21	0.23
13	20	22	24	26	25	24	0.38
14	20	22	24	26	28	27	0.12
15	20	22	24	26	28	30	0.10

(*ii*) The S_j column of the payoff table given above, shows the net profit for each level of demand.

(*iii*) If he manufactures 12 pieces, his expected profits will be as follows :

$$0.07 \times 18 + 0.10 \times 21 + 0.23 \times 24 + 0.38 \times 24 + 0.12 \times 24 + 0.10 \times 24 = \text{Rs. } 23.28$$

(*iv*) The expected profits for other levels of manufacture are calculated below :

$$E(S_1) = [0.07 \times 20 + 0.10 \times 20 + 0.23 \times 20 + 0.38 \times 20 + 0.12 \times 20 + 0.10 \times 20] = \text{Rs. } 20$$

$$E(S_2) = [0.07 \times 19 + 0.10 \times 22 + 0.23 \times 22 + 0.38 \times 22 + 0.12 \times 22 + 0.10 \times 22] = \text{Rs. } 21.79$$

$$E(S_6) = \text{Rs. } 23.28 \text{ [Calculated above]}$$

$$E(S_4) = [0.07 \times 17 + 0.10 \times 20 + 0.23 \times 23 + 0.38 \times 26 + 0.12 \times 26 + 0.10 \times 26] = \text{Rs. } 24.08$$

$$E(S_5) = [0.07 \times 16 + 0.10 \times 19 + 0.23 \times 22 + 0.38 \times 25 + 0.12 \times 28 + 0.10 \times 28] = \text{Rs. } 23.74$$

$$E(S_6) = [0.07 \times 15 + 0.10 \times 18 + 0.23 \times 21 + 0.38 \times 24 + 0.12 \times 27 + 0.10 \times 30] = \text{Rs. } 23.04$$

(*v*) From the above calculations it is clear that he should manufacture 13 pieces for maximising his expected profit which is Rs. 24.08.

Illustration 8. Under an employment promotion programme it is proposed to allow sale of newspapers on the buses during off peak hours. The vendor can purchase the newspapers at a special concessional rate of Rs. 1.25 per copy against the selling price of Rs. 1.40. Any unsold copies are, however, a dead loss. A vendor has estimated the following probability distribution for the number of copies demanded :

Number of copies :	15	16	17	18	19	20
Probability :	0.04	0.19	0.33	0.26	0.11	0.07

How many copies should the vendor buy for maximum gain?

Solution. The vendor does not purchase less than 25 copies or more than 20 copies. His profit is determined by the demand (D) and the number of copies purchased (P). When the demand is more than the number of copies purchased by his profit will be :

$$\text{Rs. } 1.40 \times P - \text{Rs. } 1.25 \times P = \text{Rs. } 0.15\ P \quad \ldots(i)$$

When the demand is less or equal to the number of copies purchased by him the profit is :

$$\text{Rs. } 1.40 \times D - \text{Rs. } 1.25 \times P \quad \ldots(ii)$$

From (i) and (ii) we can make the following payoff table :

PAYOFF TABLE

No. of copies purchased P / *Demand D*	S_1 15	S_2 16	S_3 17	S_4 18	S_5 19	S_6 20	Probability
15	Rs. 2.25	Rs. 2.00	Rs. 1.75	Rs. 1.50	Rs. 1.85	Rs. 1.00	0.04
16	2.25	2.40	2.15	1.90	1.65	1.40	0.19
17	2.25	2.40	2.55	2.30	2.05	1.80	0.33
18	2.25	2.40	2.55	2.70	2.45	2.20	0.26
19	2.25	2.40	2.55	2.70	2.85	3.00	0.11
20	2.25	2.40	2.55	2.70	1.85	3.00	0.07

His expected profits are calculated below :

$E(S_1) = [2.25 \times 0.04 + 2.25 \times 0.19 + 2.25 \times 0.33 + 2.25 \times 0.26 + 2.25 \times 0.11 + 2.25 \times 0.07] = \text{Rs. } 2.25$

$E(S_2) = [2.00 \times 0.04 + 2.40 \times 0.19 + 2.40 \times 0.33 + 2.40 \times 0.26 + 2.40 \times 0.11 + 2.40 \times 0.07] = \text{Rs. } 2.38$

$E(S_3) = [1.75 \times 0.04 + 2.15 \times 0.19 + 2.55 \times 0.33 + 2.55 \times 0.26 + 2.55 \times 0.11 + 2.55 \times 0.07] = \text{Rs. } 2.44$

$E(S_4) = [1.50 \times 0.04 + 1.90 \times 0.19 + 2.3 \times 0.33 + 2.70 \times 0.26 + 2.70 \times 0.11 + 2.70 \times 0.07] = \text{Rs. } 2.37$

$E(S_5) = [1.85 \times 0.04 + 1.65 \times 0.19 + 2.05 \times 0.33 + 2.45 \times 0.26 + 2.85 \times 0.11 + 1.85 \times 0.07] = \text{Rs. } 2.144$

$E(S_6) = [1.00 \times 0.04 + 1.40 \times 0.19 + 1.80 \times 0.33 + 2.20 \times 0.26 + 3.00 \times 0.11 + 3.00 \times 0.07] = \text{Rs. } 2.012$

It is clear from the above calculations that the expected profit is maximum in the third course of action. Hence he should order 17 copies in order to maximise his profit.

Illustration 9. The probability of the demand for trucks for hiring on any day in a given district is as follows :

No. of trucks demanded	:	0	1	2	3	4
Probability	:	0.1	0.2	0.3	0.2	0.2

Trucks have a fixed cost of Rs. 90 each day to keep and daily hire charge (net or variable cost of running) is Rs. 200. If the Trucks-hire company owns 4 Trucks, what is its daily expectation? If the company is about to go into business and currently has no Trucks how many Trucks should it keep ?

Solution. Since the fixed cost of keeping each Truck is Rs. 90 a day, whether it is demanded or not and since, when demanded, it fetches a hire charge (net or variable cost of running) of Rs. 200, the payoffs with 4 Trucks are as under :

No. of Trucks demanded	:	0	1	2	3	4
Payoff (with 4 Trucks)	:	$0 - 90 \times 4$ $= -360$	$200 - 90 \times 4$ $= -160$	$400 - 90 \times 4$ $= 40$	$600 - 90 \times 4$ $= 240$	$800 - 90 \times 4$ $= 440$

Now the daily expectation is obtained by taking the sum total of the product of the above payoff and the corresponding probabilities of demand, *i.e.*,

$(-360)(0.1) + (-160)(0.2) + (40)(0.3) + (240)(0.2) + (440)(0.2) = \text{Rs. } 80$

For answering the other part of the question the payoff table is developed as under :

PAYOFF TABLE

Event (demand)	*Probability*	*Expected payoff (Rs.) Decision to Purchase Trucks (courses of action)*				
		0	1	2	3	4
0	0.1	0	–90	–100	–270	–360
1	0.2	0	+110	+20	–70	–160
2	0.3	0	+110	+220	+130	+40
3	0.2	0	+110	+220	+330	+240
4	0.2	0	+110	+210	+330	+440

COMPUTATION OF EMV OF VARIOUS COURSES OF ACTION

Event (demand)	*Probability*	*Expected payoff (Rs.) Decision to Purchase lorries (courses of action)*				
		0	1	2	3	4
0	0.8	0	–9	–18	–27	–36
1	0.2	0	22	4	–14	–32
2	0.3	0	33	66	39	12
3	0.2	0	22	44	66	48
4	0.2	0	22	44	66	88
EMV		0	90	140	130	80

Since EMV for the second course of action is highest (Rs. 140), therefore, the company should buy 2 lorries.

Illustration 10. A retailer purchases cherries every morning at Rs. 50 a case and sells for Rs. 80 a case remaining unsold at the end of the day can be disposed of next day at a salvage value of Rs. 20 per case (thereafter they have no value). Past sales have ranged from 15 to 18 cases per day. The following is the record of sales for past 120 days :

Cases	:	15	16	17	18
No. of days	:	12	24	48	36

Find how many cases the retailer should purchase per day to maximise his profit.

Solution. Here, number of cases of cherries purchased is an act or course of action and daily demand of the cherries is an event or state of nature. Using the information of the data, the various conditional profit (payoff) values for each act-event combination are given by :

Conditional (payoff) profit = (marginal profit) (cases sold) – (marginal loss) (cases unsold)

= (80–50) (cases sold) – (50–20) (cases unsold)

The resulting conditional payoff and corresponding expected payoff are computed in the following table :

Event (demand) per week	*Probability*	*Conditional payoff (Rs.) Act (Purchases per week)*				Expected payoff (*Rs.*) *Act (Purchases per week)*			
	(1)	15 (2)	16 (3)	17 (4)	18 (5)	15 (1) × (2)	16 (1) × (3)	17 (1) × (4)	18 (1) × (5)
15	0.1	450	420	390	360	45	42	39	36
16	0.2	450	480	450	420	90	96	90	84
17	0.4	450	480	510	480	180	192	204	192
18	0.3	450	480	510	540	135	144	153	162
	Expected Monetary Value (EMV) :					450	474	486	474

Since the act 'purchase 17 cases' yields the highest EMV of Rs. 486, the optimal act for the retailer would be to purchase 17 cases of cherries every morning.

Illustration 11. In a small town, there are two discount stores *ABC* and *XYZ*. They are the only stores that handle sundry goods. The total number of customers is equally divided between the two, because the price and quality of goods sold are equal. Both stores have good reputation in the community, and they render equally good customer services. Assume that a gain of customers by *ABC* is a loss to *XYZ*, and *vice versa*. Both stores plan to run annual pre-Diwali sales during the first week of October. Sales are advertised through the local newspaper, radio and television given below. (Figures in the matrix represent a gain or loss of customers). Find the optimal strategies for both stores and the value of game.

		Strategies of *XYZ*		
		Newspaper	Radio	Television
Strategies of *ABC*	Newspaper	30	40	–80
	Radio	0	15	–20
	Television	90	20	50

(MBA, DU, Nov. 2001; MBA, DU, Oct. 2003)

Solution. We observe that every element in the second row is less than the corresponding element in the third row. Applying the dominance principle, we shall delete the second row. Similarly, every element in the first column is greater than the corresponding element in the third column. Therefore, we delete the first column. Thus, the matrix is reduced to

		Strategies of *XYZ*	
		Radio	Television
Strategies of *ABC*	Newspaper	40	–80
	Television	20	50

Since there is no saddle point in this pay-off matrix, both stores will adopt a mixed strategy method. Let x be the probability of *ABC* for adopting Newspaper strategy. Therefore, probability of adopting Television strategy is $(1-x)$. Similarly, for store *XYZ*, let y and $(1-y)$ be the respective probabilities of adopting Radio and Television strategies. Thus,

$$E = 40xy - 80x(1-y) + 20(1-x)y + 50(1-x)(1-y)$$

Differentiating partially with respect to x and equating to zero, we get

$$\frac{\partial E}{\partial x} = 40y - 80(1-y) - 20y - 50(1-x)(1-y) = 0$$

or, $$y = \frac{13}{15} \quad \text{and } 1 - y = \frac{2}{15}$$

Now, differentiating E partially with respect to y, we get

$$\frac{\partial E}{\partial y} = 40x + 80x + 20(1-x) - 50(1-x) = 0$$

or, $$x = \frac{1}{5} \quad \text{and } 1 - x = \frac{4}{5}$$

Substituting these values in the expression E, we get the value of the game, *i.e.*,

$$E = 40 \times \frac{1}{5} \times \frac{13}{15} - 80 \times \frac{1}{5} \times \frac{2}{15} + 20 \times \frac{4}{5} \times \frac{13}{15} + 50 \times \frac{4}{5} \times \frac{2}{15} = 24$$

Hence, the optimal strategies for both stores are that store *ABC* should go for Newspaper 20% times, no radio advertisement and Television should be adopted 80% times. Similarly, store *XYZ* should not go for Newspaper, 87% times Radio and 13% times Television. The value of the game will be 34 in favour of store *ABC*.

Illustration 12. Two breakfast food manufacturers *ABC* and *XYZ* are comparing for an increased market share. The payoff matrix shown in the following table, shows the increase in the share for *ABC* and decrease in market share of *XYZ* :

		XYZ			
		Give Coupons	Decrease price	Maintain present strategy	Increase advertising
ABC	Give coupons	2	–2	4	1
	Decrease price	6	1	12	3
	Maintain present strategy	–3	2	0	6
	Increase advertising	2	–3	7	1

Simplify the problem by the rule of dominance and then find optimal strategies for both the manufacturers and the value of the game.

Solution. Applying the rule of dominance, the payoff matrix can be reduced to

		XYZ	
		Give Coupons	Decrease Price
ABC	Decrease Price	6	1
	Maintain present strategy	–3	2

Since the payoff matrix cannot be further reduced, therefore, there is no saddle point. In this problem, both manufacturers will mix up the strategies in a random fashion. Let x and $(1-x)$ be the probabilities that the manufacturer *ABC* adopts in using strategies, decrease price and maintain present strategy respectively. Similarly, for manufacturer *XYZ*, let y and $(1-y)$ be the probabilities for using strategies, give coupons and decrease price respectively. Therefore, the expectation of manufacturer *ABC* is given by :

$E = 6xy + x(1-y) - 3(1-x)y + 2(1-x)(1-y)$

For maximizing the expectation, the first partial derivative must be equal to zero.

$$\frac{\partial E}{\partial x} = 6y + (1-y) - 3y - 2(1-y) = 0$$

or, $$y = \frac{1}{10}; \text{ and } (1-y) = \frac{9}{10}$$

$$\frac{\partial E}{\partial y} = 6x - x - 3(1-x) - 2(1-x) = 0$$

or, $$x = \frac{1}{2}; \text{ and } (1-x) = \frac{1}{2}$$

To obtain the value of the game, substitute the values of x and y in the expression E.

$$E = 6 \times \frac{1}{2} \times \frac{1}{10} + \frac{1}{2} \times \frac{9}{10} - 3 \times \frac{1}{2} \times \frac{1}{10} + 2 \times \frac{1}{2} \times \frac{9}{10} = 1.5$$

Hence the optimal strategies for both the manufacturers are that manufacturer *ABC* should adopt strategy 'decrease price' 50% times and strategy 'maintain present strategy, 50% times. Similarly, manufacturer *XYZ* should adopt strategy 'give coupons' 10% times and strategy 'decrease price' 90% times. The value of the game would be in favour of manufacturer *ABC* and the increase in markets share would be 1.5.

PROBLEMS

1-A: Answer the following questions, each question carries **one** mark:

(*i*) What is Statistical decision theory ?
(*ii*) What is payoff table ?
(*iii*) What is opportunity loss table ?
(*iv*) What is EVPI ?
(*v*) What is the difference between course of action and state of nature ?
(*vi*) What is decision-making under risk ?
(*vii*) What is decision-making under uncertainty ?
(*viii*) What is the difference between pure strategy and mixed strategy ?
(*ix*) What do you understand by dominance principle ?
(*x*) What is graphical method of a two-person zero-sum game ?

1-B: Answer the following questions, each question carries **four** marks:

(*i*) Explain briefly, the ingredients of a decision problem with suitable examples.
(*ii*) Describe any two methods of decision-making under uncertainty, pointing out their relative merits and demerits.
(*iii*) What is a two-person zero-sum game ? What are its major limitations ?
(*iv*) Differentiate between maximin and minimax principle.
(*v*) List the different steps in decision-making. (*MBA, Madras Univ., Nov. 2003*)
(*vi*) Explain the difference between decision-making under certainty, risk and uncertainty by giving suitable example.
(*vii*) Describe at least two methods of solving a two-person zero-sum game problem.

2. Explain how statistics is useful in the decision-making process of business and management.

3. How is Expected Value calculated? What are the advantages and disadvantages of using Expected Value as a decision criterion ?

4. (*a*) Decision criteria under situation of uncertainty is governed by the attitude of the decision-maker." Explain.
(*b*) Describe some methods which are useful for decision-making under uncertainty. Illustrate each by an example.

5. Explain clearly the following :
(*i*) Course of action (*ii*) State of nature
(*iii*) Payoff table (*iv*) Opportunity loss. (*MBA, HPU, 2002*)

6. Explain the following, giving a suitable example :
(*i*) The minimax principle (*ii*) The maximin principle, (*iii*) The Bayes principle (*iv*) Expected value of perfect information. (*v*) Highest Expected payoffs with information, (*vi*) Highest expected payoffs under uncertainty.

7. Explain the difference between expected opportunity loss and expected value of perfect information.

8. Explain the maximin and minimax regret criteria of decision-making under uncertainty giving suitable examples.

9. What is meant by 'Statistical Decision Theory'. How is it different from other methods used in decision-making? Describe some methods which are useful for decision-making under uncertainty.

10. Explain the following terms :
(*i*) Two-person zero-sum games,
(*ii*) Principle of dominance, and
(*iii*) Pure strategy in game theory.

11. What is game theory? Include in your answer various approaches in solving for strategies and game values.

12. What is two-person zero-sum game? What are its major limitations ?

13. Explain the terms : minimax strategies, saddle point, mixed strategies and principle of dominance.

14. "The primary contribution of the game theory has been its concept rather than its formal application to solving real-life problems." Do you agree? Discuss.

15. (*a*) "Game theory deals with making decisions under conflict caused by opposing interests." Elucidate this statements by giving appropriate examples.
(*b*) Explain the criterion of maximin and minimax regret in the context of decision theory. (*M. Com., DU, 1999*)

16. A baker makes a certain kind of pastry at night and sells it the next day. It is perishable and must be thrown if not sold during the day. The unit cost and price of the pastry are Re.1 and Rs. 3 respectively. According to the past experience, the daily demand (in hundred) and the respective probabilities are :

Demand	:	20	21	22	23	24
Probability	:	0.1	0.2	0.3	0.3	0.1

(*i*) Construct the payoff table.
(*ii*) Construct the loss table.
(*iii*) Determine the maximin and maximax action.
(*iv*) Compute the highest expected payoff with perfect information.

17. A certain product is manufactured at Rs. 50 and sold at Rs. 75 per unit. The product is such that if it is produced but not sold during a weeks' time, it becomes worthless. The weekly sales records in the past are as follows :

Demand per week	:	20	21	22	23
No. of weeks each sales level was recorded	:	200	350	800	150

(*i*) Calculate the expected sales of the month.
(*ii*) Prepare a table of payoff for different possible acts.
(*iii*) Prepare a table of expected payoffs and select the optimal act.

18. A stall agent at a certain railway station sells for Rs. 1.50 a copy of daily newspaper for which it repays Rs. 1.22. Unsold papers are returned for a refund of 50 paise a copy. The daily sales and corresponding probabilities are as follows :

Daily sales	:	500	600	700
Probability	:	0.5	0.3	0.2

(*i*) How many copies should he order each day ?
(*ii*) If unsold copies cannot be returned and are useless, what should be optimal order each day?

19. N. Sombhai & Co., a wholesale dealer in electrical appliances, was offered an agency for selling Godrej refrigerators. The company estimated that his fixed costs in taking up the agency would be Rs. 1,20,000 per year. Contribution per refrigerator sold would be Rs. 2,000. From a potential target audience of 2,000 buyers in the region, the company assessed that its market for the refrigerator sales would be 2%, 4% or 6% of the target audience with probability 0.1, 0.6 and 0.3 respectively. For a fixed cost of Rs. 10,000, the company could get a sample survey of potential buyers conducted. Whether the company should go in for additional information? Support your decision with appropriate reason in terms of EMV.

20. A company is currently involved in negotiations with its union on the upcoming wage contract. With the aid of an outside mediator, the table below was constructed by the management group. The plus points are to be interpreted as proposed wage increases while a minus figure indicates that a wage reduction is proposed. The mediator informs the management group that he has been in touch with the union and they have constructed a table that is comparable to the table developed by management. Both the company and the union must decide on an overall strategy, before negotiation. The management group understands the relationships of company strategies to union strategies in the following table but lacks specific knowledge of game theory to select the best strategy (or strategies) for the company. You have been called in to assist management on this problem. What game value and strategies are available to the opposing groups?

CONDITIONAL COST (Rs.) TO COMPANY

		Union Strategies			
		U_1	U_2	U_3	U_4
Company Strategies	C_1	+0.25	+0.25	+0.35	– 0.02
	C_2	+0.20	+0.16	+0.08	+0.08
	C_3	+0.14	+0.12	+0.15	+0.13
	C_4	+0.30	+0.14	+0.19	0

21. *A* and *B* play a game in which each has three coins, a 5p., a 10p. and a 20p. Each selects a coin without the knowledge of other's choice. If the sum of the coins is an odd amount, *A* wins *B*'s coin; if the sum is even, *B* wins *A*'s coin. Find the best strategy for each player and the value of the game. *(MBA, Delhi Univ., 1997)*

22. A businessman has three alternatives open to him and each of which can be allowed by any of the four possible events. The conditional payoffs for each action event combination are given below :

	Event			
Action	*A*	*B*	*C*	*D*
S_1	8	0	–10	6
S_2	– 4	12	18	–2
S_3	14	6	0	8

(*a*) If he adopts maximin criterion, what act he should choose?

(*b*) If the criterion of choice is minimax regret; what action should be chosen ?

(*c*) If he uses EMV (Expected Monetary Value) as his decision criterion, what action should he choose (assume that events have equal probability of occurrence)?

23. A person has the choice of running a hot snack stall or an ice-cream and cold drink shop at a certain holiday resort during the coming summer season. If the weather during the season is cool and rainy he can expect to make a profit of Rs. 15,000 and if it is warm he can expect to make a profit of only 3,000 by running a hot snack stall. On the other hand, if his choice is to run an ice-cream and cold drink shop he can expect to make a profit of Rs. 18,000, if the weather is warm and only Rs. 3,000 if the weather is cool and rainy. The meteorological authorities predict that there is 40% chance of the weather being warm during the coming season. You are to advice him as to the choice between the two types of stalls. Base clearly your argument on the expectation of the results of the two courses of action and show the result in a tabular form.

24. Vishal who possesses an amount of Rs. 1 lakh is planning to invest it among three companies : equity shares in company A,B and C. The payoff terms of (i) growth in capital and (ii) returns to capital are known for each of the investments under each of the three economic conditions which may prevail, that is, recession, growth and stability. Assuming that Vishal must make his choice among the three portfolios for a period of one year in advance, his expectations of the net earning (in Rs '000) of his Rs. 1 lakh portfolio after one year is represented by the following matrix :

	Recession	*Stability*	*Growth*
Company *A*	–15	6	10
Company *B*	4	7.5	8
Company *C*	6.5	6	5

Determine the optimal strategies for investment and the expected per cent return for the investor under such a policy.

25. A physician purchases a particular vaccine on Monday each week. The vaccine must be used within the following quantities, otherwise it becomes worthless. The vaccine costs Rs. 5 per dose and the physician charges Rs. 10 per dose. In the past 50 weeks, the physician has administered the vaccine in the following quantities :

Dose per week	:	20	25	50	60
Number of weeks	:	5	15	25	5

Determine how many doses the physician should buy every week.

26. Given is the following payoff matrix :

State of nature	*Probability*	*(Decision Rs.)* *Do not expand*	*Expand*	*Expand*
		100 units	*200 units*	*400 units*
High demand	0.4	2500	3500	5000
Medium demand	0.4	2500	3500	1000
Low demand	0.2	2500	1500	1000

What should be the decision, if we use (*i*) *EMV* criterion (*ii*) the maximin criterion, (*iii*) the maximax criterion and (*iv*) minimax regret criterion ?

27. A toy camera manufacturer produces two models (standard and deluxe). In preparation for the heavy Christmas selling season, he must decide how many of each model to produce. Variable cost of standard camera is Rs. 10 and selling price is Rs. 20; variable cost of the deluxe model is Rs. 35. He estimates demand as follows :

Standard model		*Deluxe model*	
Demand	*Probability*	*Demand*	*Probability*
6,000	0.30	2,000	0.20
8,000	0.70	4,000	0.80

Any camera not sold during the season is sold at salvage price of Rs. 5 for the standard camera and Rs. 10 for the deluxe camera. The manufacturer feels that different segments of the market purchase the two different models, thus the probabilities of sales given above are independent. Assuming unlimited production capacity, the two decisions can be made independently. What are the optimal quantities of each model to produce? What are the two optimal EMV's? (*MBA, M.D. Univ., 1997*)

28. Crown Auto is trying to decide about the size of the plant to be built in Noida. Three alternatives of annual capacity, *viz.*, (*i*) 10,000 units (*ii*) 20,000 units and (*iii*) 30,000 units are under consideration. Demand for the product is not known with certainty but the management has estimated the probabilities for 5 different levels of demand. The profit for each size of plant at different levels of demand is as follows :

Level of demand	*Probability*	*Decision (Rs. in lakhs)*		
		10,000 units	*20,000 units*	*30,000 units*
Very high	0.15	– 0	– 6	– 8
High	0.30	1	0	– 2
Moderate	0.25	1	7	5
Low	0.20	1	7	11
Very low	0.10	1	7	11

What plant capacity would you suggest to the management? Also find EVPI.

29. Two leading firms Nirmala Textiles Ltd., and Swati Rayons Ltd., for years have been selling, shirting which is but a small part of both firms total sales. The Marketing Director of Nirmala Textiles raised the question: "What should his firm's strategies be in terms of advertising for the product in question?" The system group of Nirmala Textiles developed the following data for varying degrees of advertising :

(*i*) No advertising, medium advertising and heavy advertising for both firms will result in equal market share.

(*ii*) Nirmala Textiles with no advertising : 40 per cent of the market with medium advertising by Swati Rayons and 28 per cent of the market with heavy advertising by Swati Rayons.

(*iii*) Nirmala Textiles using medium advertising : 70 per cent of the market with no advertising by Swati Rayons and 45 per cent of the market with heavy advertising by Swati Rayons.

(*iv*) Nirmala Textiles using heavy advertising ; 75 per cent of the market with no advertising by Swati Rayons and 52.5 per cent of market with medium advertising by Swati Rayons.

Based upon the above information, answer the marketing director's question.

30. A newspaper boys buys papers for Rs. 1.75 each and sells them for Rs. 1.95 each. He cannot return unsold newspapers. Daily demand has the following distribution :

No. of customers	:	220	221	222	223	224	225	226	227	228
Probability	:	0.22	0.03	0.05	0.05	0.25	0.05	0.20	0.10	0.05

If each day's demand is independent of the previous day's demand, how many newspapers should be ordered each day ?

31. Solve the following two-person zero-sum game :

		Player *B*				
		1	2	3	4	5
Player *A*	1	2	– 4	– 6	– 3	5
	2	– 3	4	– 4	1	0

32. Two firms are competing for business. Whatever firm *A* gains, firm *B* loses. The table below shows advertising strategies of both firms and the utilities to firm *A* for various market shares in percentages.

		Firm *B*		
		Press	Radio	TV
Firm *A*	Press	60	45	40
	Radio	75	75	60
	TV	80	60	70

Find optimal strategies for both firms and expected percentage of market shares of firm *A*.

33. A small industry finds from the past data that the cost of making an item is Rs. 25, the selling price of an item is Rs. 30, if it is sold within a week, and it could be disposed of at Rs. 20 per item at the end of the week.

Weekly Sales :	≤ 3	4	5	6	7	≥ 8
No. of Weeks :	0	10	20	40	30	0

Find the optimum number of items per week the industry should produce.

34. A management is faced with the problem of choosing one of three products for manufacturing. The potential demand for each product may turn out to be good, moderate or poor. The probabilities for each of states of nature were estimated as follows :

Product	*Nature of Demand*		
	Good	*Moderate*	*Poor*
X	0.70	0.20	0.10
Y	0.50	0.30	0.20
Z	0.40	0.50	0.10

The estimated profit or loss under the various types of nature of demand may be taken as :

	Rs.	*Rs.*	*Rs.*
X	30,000	20,000	10,000
Y	60,000	30,000	20,000
Z	40,000	10,000	–15,000

Prepare the expected monetary value table and advise the management about the choice of product.

35. Solve the following game by using the principle of dominance :

		Player *B*					
		I	II	III	IV	V	VI
Player *A*	1	4	2	0	2	1	1
	2	4	3	1	3	2	2
	3	4	3	7	–5	1	2
	4	4	3	4	–1	2	2
	5	5	3	3	–2	2	2

36. Solve the following two-person zero-sum game :

		Player *B*		
		B_1	B_2	B_3
Player *A*	A_1	4	5	8
	A_2	–2	–3	4
	A_3	–6	–4	0
	A_4	6	–5	2

37. A firm makes pastries, which it sells for Rs. 8 per piece in special boxes containing one dozen each. The direct cost of pastries for the firm is Rs. 4.50 per piece. At the end of the week, the stale pastries are sold off for a lesser price of Rs. 2.50 per piece. The overhead expense attributable to the pastry production is Rs. 1.25 per piece. Fresh pastries are sold in special boxes which cost 50 paise each and the stale pastries are sold wrapped in ordinary paper. The probability distribution of demand per week is as under :

Demand (in dozen)	:	0	1	2	3	4	5
Probability	:	0.01	0.14	0.20	0.50	0.10	0.05

Find the optimal production level of pastries per week.

38. Firm X is fighting for its life against the determination of firm Y to drive it out of the industry. Firm X has the choice of increasing its price, leaving it unchanged or lowering it. Firm Y has the same three options. Firm X's gross sales in the event of each of the possible pairs of choice are shown below :

		Firm Y's Pricing Strategy		
		Increase price	*Do not change*	*Reduce price*
Firm X's Pricing Strategy	Increase Price	90	0	110
	Do not change	110	100	90
	Reduce price	120	70	80

Find the optimal strategies for both the firms and also the value of the game.

39. Assume that two firms are competing for market share for a particular product. Each firm is considering what promotional strategy to employ for the coming period. Assume that the following payoff matrix describes the increase in market share for firm A and the decrease in market share for firm B :

		Firm B		
		No Promotion	*Moderate Promotion*	*Heavy Promotion*
Firm A	No. Promotion	3	0	– 3
	Moderate Promotion	2	3	1
	Heavy Promotion	– 4	2	– 1

Determine the optimal strategies for each firm and the value of the game.

40. For the following payoff matrix, find the value of the game and the strategies of players A and B by using graphical method :

		Player B		
		B_1	B_2	B_3
Player A	A_1	3	–1	4
	A_2	6	7	–2

41. The management of a corporation is in the process of deciding whether to agree to negotiate with the striking union, now or to delay. The decision is difficult because the management does not know the union leadership's position. The union leaders may be adamant and insist on their original demands, they may be ready to compromise or they may be ready to yield and accept the original management offer. The matrix of payoffs to management, as management sees it, is (in Rs. 1 million units) given below :

	UNION POSITION		
	B_1 *Adamant*	B_2 *Compromise*	B_3 *Yield*
A_1 *Negotiate*	–2	–1	2
A_2 *Delay*	5	–2	–3

(*i*) Solve management's problem.
(*ii*) What should be the union's strategy.
(*iii*) Discuss the implications of a conclusion to adopt a random strategy.

42. A producer of boats has estimated the following distribution of demand for a particular kind of boats :

No. demanded :	0	1	2	3	4	5	6
Probability :	0.14	0.27	0.27	0.18	0.09	0.04	0.01

Each boat cost him Rs. 70,000 and he sells for Rs. 1,00,000 each. Any left unsold at the end of the season must be disposed for Rs. 60,000 each. How many should he stock so as to maximise his expected payoff ?

43. There are two companies *A* and *B* in a certain city. Both companies have similar reputation and the total number of customers is equally divided between the two companies. Both the companies want to attract more number of customers by using different media of advertisement. By seeing the market trend, the company *A* constructed the following payoff matrix, where the numbers in the matrix indicate a gain or a loss of customers.

		Company B		
		Newspaper	*Radio*	*T.V.*
	Newspaper	40	50	–70
Company A	*Radio*	10	25	–10
	T.V.	100	30	60

Find optimal strategies for both the companies and value of the game.

44. A group of students raise money each year by selling Souvenirs outside the stadium after a cricket match between Teams *A* and *B*. They can buy any of the three different types of Souvenirs from a supplier. Their sales are mostly dependent on which team wins the match. A conditional payoff table is as under :

	Type of Souvenir		
	I	*II*	*III*
Team *A* wins	Rs. 1,200	Rs. 800	Rs. 300
Team *B* wins	Rs. 250	Rs. 700	Rs. 1,100

(*i*) Construct the opportunity loss table.

(*ii*) Which type of Souvenir should the students buy if probability of Team *A*'s winning is 0.6?

45. The conditional payoffs in rupees for each action-event combination are as under :

	Action			
Event	*1*	*2*	*3*	*4*
A	4	–2	7	8
B	0	6	3	5
C	–5	9	2	–3
D	3	1	4	5
E	6	6	3	2

(*i*) Which is the best action in accordance with the maximin criterion?

(*ii*) Which is the best action in accordance with EMV criterion, presuming all events have equal probabilities of occurrence?

46. In a duopolistic market, two competitor compete for profit with promotional effort as their only controllable variables. Each competitor has the option of increasing or decreasing the promotional expenditure or staying at the normal level. The expected increase in profit of competitor 1 under various situations is shown here (in Rs. 10,000 units) :

		Competitor 2		
		Increase	*Normal*	*Decrease*
Competitor 1	*Increase*	–200	–20	30
	Decrease	–50	20	40
	Normal	80	10	50

Assuming a zero-sum game, find the optimal strategy of each competitor and the value of the game.

47. Two companies *A* and *B* are competing for the similar type of product. Their different strategies are given in the following payoff matrix.

		Company *B*		
		B_1	B_2	B_3
Company A	A_1	2	–2	3
	A_2	–3	5	–1

Determine the best strategies for both the companies and also the value of the game.

48. In a recreation beach, two persons, A and B, are interested in starting a refreshment stall. Initially, only three places are under consideration. The following payoff matrix for different strategies of the players is given :

		B's position		
		Entrance	*Centre*	*Exit*
A's Position	*Entrance*	50	30	40
	Centre	70	50	60
	Exit	60	70	50

What is the best strategy for A and B to start the refreshment stall ?

49. A soft drink company calculated the market share of two products against its major competitor having three products and found out the impact of additional advertisement in any one of its product against the other.

		Competitor		
		1	2	3
Company	1	6	7	15
	2	20	12	10

What is the best strategy for the company as well as the competitor ? What is the payoff obtained by the company and the competitor in the long run? Use graphical method to obtain the solution.

50. Two candidates, X and Y, are competing for the councillors seat in a city municipal corporation, and X is attempting to increase his total votes at the expense of Y. The strategies available to each candidate involve personal contacts, newspapers insertions or television advertising. The increase in votes available to X given various combinations of strategies are given below. Assuming two-person zero-sum game, determine the optimal strategies that should be adopted by X during his election campaign. How many votes should X gain by the following optimal strategy ?

		Y		
		Personal Contacts	*Newspaper*	*Television*
X	*Personal contacts*	30,000	20,000	10,000
	Newspaper	60,000	50,000	25,000
	Television	20,000	40,000	30,000

51. A production manager has calculated that for every additional unit sold he makes an additional profit of Rs. 2, but for every unit left unsold, he loses Rs. 1.20. The probability distribution for the demand (in lakh units) of the product per week is given below :

Demand per week	:	20	21	22	23	24	25	26	27
Probability	:	0.24	0.08	0.09	0.17	0.15	0.13	0.09	0.05

Determine the optimal number of units the production manager should store for a week.

52. Assume that a manager sells an article having normally distributed sales with a mean of 50 units daily and a standard deviation in daily sales of 15 units.The manager purchases this article for Rs. 4 per unit and sells it for Rs.9 per unit. If the article is not sold on the selling day, it is worth nothing. Determine the optimal size of the order of the article, the manager should make daily. *(MFC, Delhi Univ., 1996)*

53. A big breeder can either produce 20 or 30 pigs. The total production of his competitors can be either 5,000 or 10,000 pigs. If they produce 5,000 pigs, his profit per pig is Rs. 60; if they produce 10,000 pigs, his profit per pig is only Rs. 45. Construct a payoff table and also state what would the big breeder decide.

54. Two firms A and B are competing for the same type of product. Their different strategies are given in the following payoff matrix :

		Firm B			
		B_1	B_2	B_3	B_4
Firm A	A_1	35	65	25	5
	A_2	30	20	15	0
	A_3	40	50	0	10
	A_4	55	60	10	50

Using the concept of dominance, reduce this game to 2×2 matrix. Also determine their optimal strategies and the value of the game.

55. For the following matrix, find the optimal strategies for A and B and the value of the game :

		Firm B		
		B_1	B_2	B_3
Firm A	A_1	12	10	8
	A_2	14	14	10
	A_3	16	12	15

56. Under an employment promotion scheme, it is proposed to allow sale of newspaper on the buses during off peak hours. A vendor can purchase the newspaper at a concessional rate of Rs. 1.70 per copy and sell it for Rs. 1.90. Copies unsold at the end of the day are, however, a dead loss. The demand probability distribution has been estimated as follows :

Demand	:	160	170	180	190	200	210
Probability	:	0.04	0.19	0.33	0.26	0.11	0.07

How many copies should the vendor order so as to maximise his expected profit ? *(M.Com., DU, 1999)*

57. Consider the following payoff (profit) matrix :

		State of nature				
		N_1	N_2	N_3	N_4	N_5
Strategy	S_1	60	70	–10	0	40
	S_2	30	45	20	35	–15
	S_3	40	35	25	20	30
	S_4	50	–20	35	25	20

No probabilities are known for the occurrence of the state of nature. Compare the solutions obtained by each of the following criteria: (*a*) Maximin, (*b*) Regret, (*c*) Laplace (*d*) Hurwicz. *(MBA, Madras Univ., 1999)*

58. Consider the following pay off (profit) matrix.

		State of Nature				
		N_1	N_2	N_3	N_4	N_5
Strategy	S_1	60	70	–10	0	40
	S_2	30	45	20	35	–15
	S_3	40	35	25	20	30
	S_4	50	–20	35	25	20

Compare the solutions obtained by Minimax (Savage) and Laplace criterion.

59. A company needs to increase its production beyond its existing capacity. It has narrowed the alternatives to two approaches to do so : (*a*) expansion at a cost of Rs. 8 million, or (*b*) modernization at a cost of Rs. 5 million. Both approaches would require the same amount of time for implementation. Management believes that over the required payback period, demand will either be high or moderate. Since high demand is considered to be somewhat less likely than moderate demand, the probability of high demand has been set at 0.35. If the demand is high, expansion would gross an estimated additional high demand has been set at 0.35. If the demand is high, expansion would gross an estimated additional Rs. 12 million but modernization only an additional Rs. 6 million, due to a lower maximum production capability. On the other hand, if the demand is moderate, the comparable figures would be Rs. 7 million for expansion and Rs. 5 million for modernization.

(*a*) Calculate the conditional profit in relation to variance action and outcome combinations and states of nature.
(*b*) If the company wishes to maximize its expected monetary value (*EMV*), should it modernize or expand ?
(*c*) Calculate the *EVPI*.
(*d*) Construct the conditional opportunity loss table and also calculate *EOL*.

60. Suppose an analysis of demand for a product in the last one year (52 weeks) revealed the demand distribution given in the table given below :

Table : Demand Distribution

Quantity demanded	*No. of weeks this quantity was sold*	*Probability*
30	5	0.10
31	10	0.20
32	16	0.30
33	13	0.25
34	5	0.10
35	3	0.05
	52	1.00

Selling price of the product = Rs. 3.00.

Cost price of the product = Rs. 2.00.

Selling price of more than one week old product = Rs. 1.00 (*i.e.*, loss of unsold unit)

(*i*) Construct the conditional profit table.

(*ii*) Determine the optimum number of units of his commodity, to order weekly in order to maximize his profit.

(*iii*) Compute *EPPI* and *EVPI*.

(*iv*) Construct the conditional loss table.

(*v*) Compute *EOL*.

(*vi*) Compare (*iii*) and (*v*).

61. A Company has to decide on marketing one of the following two types of portable transistor radios—Deluxe and Popular. The market forecast for the coming festival season indicates 75% chance that the market will be good, 15% chance it will be fair and 10% chance it will be poor. The payoffs for each strategy corresponding to the different states of nature is given in the following matrix :

States of Nature

		Market Good	*Market Fair*	*Market poor*
	Probability	0.75	0.15	0.10
Strategy			*Pay offs (Rs.)*	
Deluxe Model		35,000	15,000	5,000
Popular Model		50,000	20,000	(–) 3,000

Which strategy the company should choose ?

62. Mr. Ram buys a perishable commodity at Rs. 5 each. The profit per unit is Rs. 5. This perishable commodity he ca keep in his shop for a week and at the end of each week the leftover are sold to a restaurant for Rs. 3 each (a loss of Rs. each). Mr. Ram has the record for past 100 weeks for his weekly sales as given below :

Weekly demand	:	1	2	3	4	5	6	7
Number of weeks	:	5	10	25	30	20	5	5

(*i*) Construct the conditional profit table.

(*ii*) Determine the optimum number of units of his commodity to order weekly in order to maximize his profit.

(*iii*) Compute *EPPI* and *EVPI*.

(*iv*) Construct the conditional loss table.

(*v*) Compute *EOL*.

(*vi*) Compare (*iii*) and (*v*).

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Madras University

MBA—Business Statistics

Time : Three hours Maximum : 75 marks

Part A *(10 × 1 = 10 marks)*

Answer any TEN questions. All questions carry equal marks.

1. What is probability ?
2. What is a decision making ?
3. Define the term research.
4. Define Sample.
5. What is hypothesis ?
6. Define editing.
7. What do you mean by cluster ?
8. What is Univariate ?
9. List out the types of revenues.
10. What is surplus ?
11. Define parametric.
12. What is nominal scale ?

Part B *(5 × 5 = 25 marks)*

Answer any FIVE questions. All questions carry equal marks.

13. State Baye's theorem and explain its applications.
14. What do you understand by decision tree ? Explain, how it will be useful in decision making ?
15. Discuss the scope and objectives of research.
16. List out and explain the different methods of data collection.
17. What do you mean by conjoint analysis ? Explain.
18. Explain different types of surpluses in detail.
19. State few applications of differentiation and integration.

Part C *(4 × 10 = 40 marks)*

Question no. 20 is compulsory. Answer any three questions from Q.21 to 24. All questions carry equal marks.

20. Five white and six red balls are in a bag. Two drawings of three balls are made such that
    (a) the balls are replaced before the second trial, and
    (b) the balls are not replaced before the second trial. Find the probability that the first drawing will give three white and the second drawing will give three red balls in each case.
21. In an experiment of pea-breeding Mendel obtained the following frequencies of seed : 315 round and yellow, 101 wrinkled and yellow, 108 round and green, 32 wrinkled and green. According to his theory of heredity, the numbers should be in proportion 9 : 3 : 3 : 1. Is there any evidence to doubt the theory at 5% level of significance? (Tabulated value : For $df = 3$, chi-square at 5% is 7.82)
22. Discuss different types of research used in management with examples.
23. The following data present the yields in quintals of common ten sub-divisions of equal area of two agricultural plots :

Plot 1 :	6.2	5.7	6.5	6.0	6.3	5.8	5.7	6.0	6.0	5.8
Plot 2 :	5.6	5.9	5.6	5.7	5.8	5.7	6.0	5.5	5.7	5.5

    Test whether two samples taken from two random populations have the same variance.
    (5% point of F for $v_1 = 9$ and $v_2 = 9$ is 3.18)
24. Give an account of report writing by explaining different types of reports

APPENDIX

Statistical Tables

I. Logarithms

II. Antilogarithms

III. Powers, Roots and Reciprocals

IV. Binomial Coefficients

V. Values of e^{-m}

VI. Ordinates (Y) of the Standard Normal Curve at Z

VII. Areas under the Standard Normal Distribution

VIII. Critical Values of χ^2

IX. Critical Values of t

X. 5% Points of F-distribution

XI. 1% Points of F-Distribution

XII. Control Charts Constants

XIII. Random Numbers

I. LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	5 4	9 8	13 12	17 16	21 20	26 24	30 28	34 32	38 36
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0775	4 4	8 7	12 11	16 15	20 18	23 22	27 26	31 29	35 33
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3 3	7 7	11 10	14 14	18 17	21 20	25 24	28 27	32 31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3 3	6 7	10 10	13 13	16 16	19 19	23 22	26 25	29 29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3 3	6 6	9 9	12 12	15 14	19 17	22 20	25 23	28 26
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3 3	6 6	9 8	11 11	14 14	17 17	20 19	23 22	26 25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3 3	6 5	8 8	11 10	14 13	16 16	19 18	22 21	24 23
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	3 3	5 5	8 8	10 10	13 12	15 15	18 17	20 20	23 22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2 2	5 4	7 7	9 9	12 11	14 14	17 16	19 18	21 21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2 2	4 4	7 6	9 8	11 11	13 13	16 15	18 17	20 19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5456	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	6	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	5	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8

II. LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1 2 3	3 4 5	6 7 8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1 2 3	3 4 5	6 7 8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1 2 2	3 4 5	6 7 7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1 2 2	3 4 5	6 6 7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1 2 2	3 4 5	6 6 7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1 2 2	3 4 5	5 6 7
56	7582	7490	7497	7505	7513	7520	7528	7536	7543	7551	1 2 2	3 4 5	5 6 7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1 2 2	3 4 5	5 6 7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1 1 2	3 4 4	5 6 7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1 1 2	3 4 4	5 6 7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1 1 2	3 4 4	5 5 6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1 1 2	3 4 4	5 6 6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1 1 2	3 3 4	5 6 6
63	7996	8000	8007	8014	8021	8028	8035	8041	8048	8055	1 1 2	3 3 4	5 5 6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1 1 2	3 3 4	5 5 6
65	8129	8136	8142	8149	8156	8162	8269	8176	8182	8189	1 1 2	3 3 4	5 5 6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1 1 2	3 3 4	5 5 6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1 1 2	3 3 4	5 5 6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1 1 2	3 3 4	4 5 6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1 1 2	2 3 4	4 5 6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1 1 2	2 3 4	4 5 6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1 1 2	2 3 4	4 5 5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1 1 2	2 3 4	4 5 5
73	8633	8639	8645	8651	8557	8663	8669	8675	8681	8686	1 1 2	2 3 4	4 5 5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1 1 2	2 3 4	4 5 5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1 1 2	2 3 3	4 5 5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1 1 2	2 3 3	4 5 5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1 1 2	2 3 3	4 4 5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1 1 2	2 3 3	4 4 5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1 1 2	2 3 3	4 4 5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1 1 2	2 3 3	4 4 5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1 1 2	2 3 3	4 4 5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1 1 2	2 3 3	4 4 5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1 1 2	2 3 3	4 4 5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1 1 2	2 3 3	4 4 5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1 1 2	2 3 3	4 4 5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1 1 2	2 3 3	4 4 5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0 1 1	2 2 3	3 4 4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0 1 1	2 2 3	3 4 4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0 1 1	2 2 3	3 4 4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0 1 1	2 2 3	3 4 4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0 1 1	2 2 3	3 4 4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0 1 1	2 2 3	3 4 4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0 1 1	2 2 3	3 4 4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0 1 1	2 2 3	3 4 4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0 1 1	2 2 3	3 4 4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0 1 1	2 2 3	3 4 4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0 1 1	2 2 3	3 4 4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0 1 1	2 2 3	3 4 4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0 1 1	2 2 3	3 3 4

I. ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
.00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0 0 1	1 1 1	2 2 2
.01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0 0 1	1 1 1	2 2 2
.02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0 0 1	1 1 1	2 2 2
.03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0 0 1	1 1 1	2 2 2
.04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0 1 1	1 1 2	2 2 2
.05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0 1 1	1 1 2	2 2 2
.06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0 1 1	1 1 2	2 2 2
.07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0 1 1	1 1 2	2 2 2
.08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0 1 1	1 1 2	2 2 3
.09	1230	1233	1236	1139	1242	1245	1247	1250	1253	1256	0 1 1	1 1 2	2 2 3
.10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0 1 1	1 1 2	2 2 3
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0 1 1	1 2 2	2 2 3
.12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0 1 1	1 2 2	2 2 3
.13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0 1 1	1 2 2	2 3 3
.14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0 1 1	1 2 2	2 3 3
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0 1 1	1 2 2	2 3 3
.16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0 1 1	1 2 2	2 3 3
.17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0 1 1	1 2 2	2 3 3
.18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0 1 1	1 2 2	2 3 3
.19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0 1 1	1 2 2	3 3 3
.20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0 1 1	1 2 2	3 3 3
.21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0 1 1	2 2 2	3 3 3
.22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0 1 1	2 2 2	3 3 3
.23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0 1 1	2 2 2	3 3 4
.24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0 1 1	2 2 2	3 3 4
.25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0 1 1	2 2 2	3 3 4
.26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0 1 1	2 2 3	3 3 4
.27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0 1 1	2 2 3	3 3 4
.28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0 1 1	2 2 3	3 4 4
.29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0 1 1	2 2 3	3 4 4
.30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0 1 1	2 2 3	3 4 4
.31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0 1 1	2 2 3	3 4 4
.32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0 1 1	2 2 3	3 4 4
.33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0 1 1	2 2 3	3 4 4
.34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1 1 2	2 3 3	4 4 5
.35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1 1 2	2 3 3	4 4 5
.36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1 1 2	2 3 3	4 4 5
.37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1 1 2	2 3 3	4 4 5
.38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1 1 2	2 3 3	4 4 5
.39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1 1 2	2 3 3	4 5 5
.40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1 1 2	2 3 4	4 5 5
.41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1 1 2	2 3 4	4 5 5
.42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1 1 2	2 3 4	4 5 6
.43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1 1 2	3 3 4	4 5 6
.44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1 1 2	3 3 4	4 5 6
.45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1 1 2	3 3 4	5 5 6
.46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1 1 2	3 3 4	5 5 6
.47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1 1 2	3 3 4	5 5 6
.48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1 1 2	3 4 4	5 6 6
.49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1 1 2	3 4 4	5 6 6

II. ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
.50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
.51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
.52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
.53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
.54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
.55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
.56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
.57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
.58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
.59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
.60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
.61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
.62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
.63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
.64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
.65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
.66	4571	4581	4592	4603	4613	4624	4634	4645	4556	4667	1	2	3	4	5	6	7	9	10
.67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
.68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
.69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
.70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
.71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
.72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
.73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
.74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
.75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
.76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
.77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
.78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
.79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
.80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
.81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
.82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
.83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
.84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
.85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
.86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
.87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
.88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
.89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	6	7	9	11	13	14	16
.90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
.91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
.92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
.93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
.94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
.95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
.96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
.97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
.98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
.99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

III. POWERS, ROOTS AND RECIPROCALS

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\sqrt{10n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$\frac{1}{n}$
1	1	1	1	1	3.162	2.154	4.642	1
2	4	8	1.414	1.260	4.472	2.714	5.848	.5000
3	9	27	1.732	1.442	5.477	3.107	6.694	.3333
4	16	64	2	1.587	6.325	3.420	7.638	.2500
5	25	125	2.236	1.710	7.701	3.684	7.937	.2000
6	36	216	2.449	1.817	7.746	3.915	8.434	.1667
7	49	343	2.646	1.913	8.361	4.121	8.879	.1429
8	64	512	2.828	2.000	8.944	4.309	9.283	.1250
9	81	729	3.000	2.080	9.487	4.481	9.655	.1111
10	100	1000	3.162	2.154	10.0	4.642	10.000	.1000
11	121	1331	3.317	2.224	10.488	4.791	10.323	.09091
12	144	1728	3.464	2.289	10.954	4.932	10.627	.08333
13	169	2197	3.606	2.351	11.402	5.066	10.914	.07692
14	196	2744	3.742	2.410	11.832	5.192	11.187	.07143
15	225	3375	3.873	2.466	12.247	5.313	11.447	.06667
16	256	4096	4.000	2.520	12.649	5.429	11.696	.06250
17	289	5913	4.123	2.571	13.038	5.540	11.935	.05882
18	324	5832	4.243	2.621	13.416	5.646	12.164	.05556
19	361	6859	4.359	2.668	13.784	5.749	12.386	.05263
20	400	8000	4.472	2.714	14.142	5.848	12.599	.0500
21	441	9261	4.583	2.759	14.491	5.944	12.806	1.04762
22	484	10648	4.690	2.802	14.832	6.037	13.006	.04545
23	529	12167	4.796	2.844	15.166	6.127	13.200	.04348
24	576	13824	4.899	2.884	15.492	6.214	13.389	.04167
25	625	15625	5.000	2.924	15.811	6.300	13.572	.0400
26	676	17576	5.099	2.962	16.125	6.383	13.751	.03846
27	729	19683	5.196	3.000	16.432	6.463	13.925	.03704
28	784	21952	5.292	3.037	16.733	6.542	14.095	.03571
29	841	24389	5.385	3.072	17.029	6.619	14.260	.03448
30	900	27000	5.477	3.107	17.321	6.694	14.422	.03333
31	961	29791	5.568	3.141	17.607	6.768	14.581	.03226
32	1024	32768	5.657	3.175	17.889	6.840	14.736	.03125
33	1089	35937	5.745	3.208	18.166	6.910	14.888	.03030
34	1156	39304	5.831	3.240	18.439	6.980	15.037	.02941
35	1225	42875	5.916	3.271	18.708	7.047	15.183	.02857
36	1296	46656	6.000	3.302	18.974	7.114	15.326	.02778
37	1369	50653	6.083	3.332	19.235	7.179	15.467	.02703
38	1444	54872	6.164	3.362	19.494	7.243	15.605	.02632
39	1521	59319	6.245	3.391	19.748	7.306	15.741	.02564
40	1600	64000	6.325	3.420	20.00	7.368	15.874	.0250
41	1681	68921	6.403	3.448	20.248	7.429	16.005	.02439
42	1764	74088	6.481	3.476	20.494	7.489	16.134	.02381
43	1849	79507	6.557	3.503	20.736	7.548	16.261	.02326
44	1936	85184	6.633	3.530	20.976	7.606	16.386	.02273
45	2025	91125	6.708	3.557	21.213	7.663	16.510	.02222
46	2116	97336	6.782	3.583	21.448	7.719	16.631	.02174
47	2209	103823	6.856	3.609	21.679	7.775	16.751	.02128
48	2304	110592	6.928	3.634	21.909	7.830	16.869	.02083
49	2401	117649	7.000	3.659	22.136	7.884	16.985	.02041
50	2500	125000	7.071	3.684	22.361	7.937	17.100	.020

III. POWERS, ROOTS AND RECIPROCALS

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\sqrt{10n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$\frac{1}{n}$
51	2601	132651	7.141	3.708	22.583	7.990	17.213	.01961
52	2704	140608	7.211	3.733	22.804	8.041	17.325	.01923
53	2809	148877	7.280	3.756	23.022	8.093	17.435	.01887
54	2916	157464	7.348	3.780	23.238	8.143	17.544	.01852
55	3025	166375	7.416	3.803	23.452	8.193	17.652	.01818
56	3136	175616	7.483	3.826	23.664	8.243	17.758	.01786
57	3249	185193	7.550	3.849	23.875	8.291	17.863	.01754
58	3364	195112	7.616	3.871	24.083	8.340	17.967	.01724
59	3481	205379	7.681	3.893	24.290	8.387	18.070	.01695
60	3600	216000	7.746	3.915	24.495	8.434	18.171	.01667
61	3721	226981	7.810	3.936	24.698	8.481	18.272	.01639
62	3844	238328	7.874	3.958	24.900	8.527	18.371	.01613
63	3969	250047	7.937	3.979	25.100	8.573	18.469	.01587
64	4096	262144	8.000	4.000	25.298	8.618	18.566	.01562
65	4225	274625	8.062	4.021	25.495	8.662	18.663	.01538
66	4356	287496	8.124	4.041	25.690	8.707	18.758	.01515
67	4489	300763	8.185	4.062	25.884	8.750	18.852	.01493
68	4624	314432	8.246	4.082	26.077	8.794	18.945	.01471
69	4761	328509	8.307	4.102	26.268	8.837	19.038	.01449
70	4900	343000	8.367	4.121	26.458	8.879	19.129	.01429
71	5041	357011	8.426	4.141	26.646	8.921	19.220	.01408
72	5184	373248	8.485	4.160	26.833	8.963	19.310	.01389
73	5329	389017	8.544	4.179	27.019	8.004	19.399	.01370
74	5476	405224	8.602	4.198	27.203	9.045	19.487	.01351
75	5625	421875	8.660	4.217	27.386	9.086	19.574	.01333
76	5776	438976	8.718	4.236	27.568	9.126	19.661	.01316
77	5929	456533	8.775	4.254	27.740	9.166	19.747	.01299
78	6084	474552	8.832	4.273	27.928	9.205	19.832	.01282
79	6241	493039	8.883	4.291	28.107	9.244	19.916	.01266
80	6400	512000	8.944	4.309	28.284	9.283	20.000	.01250
81	6561	531441	9.000	4.327	28.460	9.322	20.083	.01235
82	6724	551368	9.055	4.344	28.636	9.360	20.165	.01220
83	6889	571787	9.110	4.362	28.810	9.398	20.247	.01205
84	7056	592704	9.165	4.380	28.983	9.435	20.328	.01190
85	7225	614125	9.220	4.397	29.155	9.473	20.408	.01176
86	7396	636056	9.274	4.414	29.326	9.510	20.488	.01163
87	7569	658503	9.327	4.431	29.496	9.546	20.567	.01149
88	7744	681472	9.381	4.448	29.665	9.583	20.646	.01136
89	7921	704969	9.434	4.465	29.833	9.619	20.724	.01124
90	8100	729000	9.487	4.481	30.000	9.655	20.801	.01111
91	8281	753571	9.538	4.498	30.166	9.691	20.878	.01099
92	8464	778688	9.592	4.514	30.332	9.726	20.954	.01087
93	8649	804357	9.644	4.531	30.496	9.761	21.029	.01075
94	8836	830584	9.695	4.547	30.659	9.796	21.105	.01064
95	9025	857375	9.747	4.563	30.822	9.830	21.179	.01053
96	9216	884736	9.798	4.579	30.984	9.865	21.253	.01042
97	9409	912673	9.849	4.595	31.145	9.899	21.327	.01031
98	9604	941192	9.899	4.610	31.305	9.933	21.400	.01020
99	9801	970299	9.909	4.626	31.464	9.967	21.472	.01010
100	10000	1000000	10.000	4.642	31.623	10.000	21.544	.0100

IV. BINOMIAL COEFFICIENTS

n	$\binom{n}{0}$	$\binom{n}{1}$	$\binom{n}{2}$	$\binom{n}{3}$	$\binom{n}{4}$	$\binom{n}{5}$	$\binom{n}{6}$	$\binom{n}{7}$	$\binom{n}{8}$	$\binom{n}{9}$	$\binom{n}{10}$
0	1										
1	1	1									
2	1	2	1								
3	1	3	3	1							
4	1	4	6	4	1						
5	1	5	10	10	5	1					
6	1	6	15	20	15	6	1				
7	1	7	21	35	35	21	7	1			
8	1	8	28	56	70	56	28	8	1		
9	1	9	36	84	126	126	84	36	9	1	
10	1	10	45	120	210	252	210	120	45	10	1
11	1	11	55	165	330	462	462	330	165	55	11
12	1	12	66	220	495	792	924	792	495	220	66
13	1	13	78	286	715	1287	1716	1716	1287	715	286
14	1	14	91	364	1001	2002	3003	3432	3003	2002	1001
15	1	15	105	455	1365	3003	5005	6435	6435	3005	3003
16	1	16	120	560	1820	4368	8008	11440	12870	11440	8008
17	1	17	136	680	2380	6188	12376	19448	24310	24310	19448
18	1	18	153	816	3060	8568	18564	31824	43758	48620	43758
19	1	19	171	969	3876	11628	27132	50388	75582	92378	92378
20	1	20	190	1140	4845	15504	38760	77520	125970	167960	184756

V. VALUES OF e^{-m} (For Computing Poisson Probabilities)

$(0 < m < 1)$

m	0	1	2	3	4	5	6	7	8	9
0.0	1.0000	.9900	.9802	.9704	.9608	.9512	.9418	.9324	.9231	.9139
0.1	0.9048	.8958	.8860	.8781	.8694	.8607	.8521	.8437	.8353	.8270
0.2	0.8187	.8106	.8025	.7945	.7866	.7788	.7711	.7634	.7558	.7483
0.3	0.7408	.7334	.7261	.7189	.7118	.7047	.6977	.6907	.6839	.6771
0.4	0.6703	.6636	.6570	.6505	.6440	.6376	.6313	.6250	.6188	.6126
0.5	0.6065	.6005	.5945	.5886	.5827	.5770	.5712	.5655	.5599	.5543
0.6	0.5488	.5434	.5379	.5326	.5278	.5220	.5160	.5117	.5066	.5016
0.7	0.4966	.4916	.4868	.4810	.4771	.4724	.4670	.4630	.4584	.4538
0.8	0.4493	.4449	.4404	.4360	.4317	.4274	.4232	.4190	.4148	.4107
0.9	0.4066	.4025	.3985	.3946	.3906	.3867	.3829	.3791	.3753	.3716

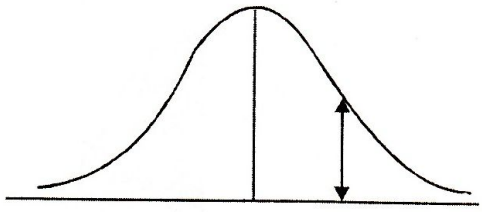
$(m = 1, 2, 3, ..., 10)$

m	1	2	3	4	5	6	7	8	9	10
e^{-m}	.36788	.13534	.04979	.01832	.006737	.002478	.00092	.000335	.000123	.000045

Note : To obtain values of e^{-m} for other values of m, use the laws of exponents.

Example. $e^{-2.35} = (e^{-2.00})\,(e^{-0.35}) = (.13534)\,(.7047) = .095374$

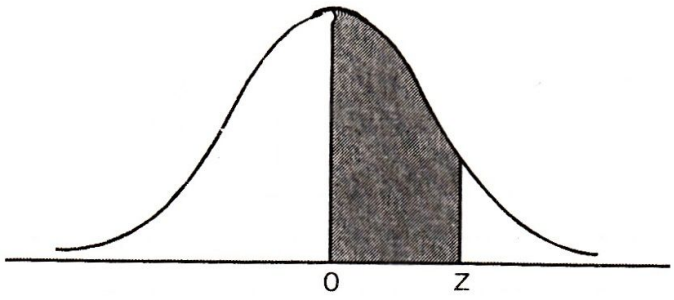
VI. ORDINATES (Y) OF THE STANDARD NORMAL CURVE AT z



z	0	1	2	3	4	5	6	7	8	9
0.0	0.3989	0.3989	0.3989	0.3988	0.3986	0.3984	0.3982	0.3980	0.3977	0.3973
0.1	0.3970	0.3965	0.3961	0.3956	0.3951	0.3945	0.3939	0.3932	0.3925	0.3918
0.2	0.3910	0.3902	0.3894	0.3885	0.3876	0.3867	0.3857	0.3847	0.3836	0.3825
0.3	0.3814	0.3802	0.3790	0.3778	0.3765	0.3752	0.3739	0.3725	0.3712	0.3697
0.4	0.3683	0.3668	0.3653	0.3637	0.3621	0.3605	0.3589	0.3572	0.3555	0.3538
0.5	0.3521	0.3503	0.3485	0.3467	0.3448	0.3429	0.3410	0.3391	0.3372	0.3352
0.6	0.3332	0.3312	0.3292	0.3271	0.3251	0.3230	0.3209	0.3187	0.3166	0.3144
0.7	0.3123	0.3101	0.3079	0.3056	0.3034	0.3011	0.2989	0.2966	0.2943	0.2920
0.8	0.2897	0.2874	0.2850	0.2827	0.2803	0.2780	0.2756	0.2732	0.2709	0.2685
0.9	0.2661	0.2637	0.2613	0.2589	0.2565	0.2541	0.2516	0.2492	0.2468	0.2444
1.0	0.2420	0.2396	0.2371	0.2347	0.2323	0.2299	0.2275	0.2251	0.2227	0.2203
1.1	0.2179	0.2155	0.2131	0.2107	0.2083	0.2059	0.2036	0.2012	0.1989	0.1965
1.2	0.1942	0.1919	0.1895	0.1872	0.1849	0.1826	0.1804	0.1781	0.1758	0.1736
1.3	0.1714	0.1691	0.1669	0.1647	0.1626	0.1604	0.1582	0.1561	0.1539	0.1518
1.4	0.1497	0.1476	0.1450	0.1435	0.1415	0.1394	0.1374	0.1354	0.1334	0.1315
1.5	0.1295	0.1276	0.1257	0.1238	0.1219	0.1200	0.1182	0.1163	0.1145	0.1127
1.6	0.1109	0.1092	0.1074	0.1057	0.1040	0.1023	0.1006	0.0989	0.0923	0.0957
1.7	0.0940	0.0925	0.0909	0.0893	0.0878	0.0863	0.0848	0.0833	0.0818	0.0804
1.8	0.0790	0.0775	0.0761	0.0748	0.0734	0.0721	0.0707	0.0694	0.0681	0.0669
1.9	0.0656	0.0644	0.0632	0.0620	0.0608	0.0596	0.0584	0.0573	0.0562	0.0551
2.0	0.0540	0.0529	0.0519	0.0508	0.0498	0.0488	0.0478	0.0468	0.0459	0.0449
2.1	0.0440	0.0431	0.0422	0.0413	0.0404	0.0396	0.0387	0.0379	0.0371	0.0363
2.2	0.0355	0.0347	0.0339	0.0332	0.0325	0.0317	0.0310	0.0303	0.0297	0.0290
2.3	0.0283	0.0277	0.0270	0.0264	0.0258	0.0252	0.0246	0.0241	0.0235	0.0229
2.4	0.0224	0.0219	0.0213	0.0208	0.0203	0.0198	0.0194	0.0189	0.0184	0.0180
2.5	0.0175	0.0171	0.0167	0.0163	0.0158	0.0154	0.0151	0.0147	0.0143	0.0139
2.6	0.0136	0.0132	0.0129	0.0126	0.0122	0.0119	0.0116	0.0113	0.0110	0.0107
2.7	0.0104	0.0101	0.0099	0.0096	0.0093	0.0091	0.0088	0.0086	0.0084	0.0081
2.8	0.0079	0.0077	0.0075	0.0073	0.0071	0.0069	0.0067	0.0065	0.0063	0.0061
2.9	0.0060	0.0058	0.0056	0.0055	0.0051	0.0051	0.0050	0.0048	0.0047	0.0046
3.0	0.0044	0.0043	0.0042	0.0040	0.0039	0.0038	0.0037	0.0036	0.0035	0.0034
3.1	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026	0.0025	0.0025
3.2	0.0024	0.0023	0.0022	0.0022	0.0021	0.0020	0.0020	0.0019	0.0018	0.0018
3.3	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014	0.0013	0.0013
3.4	0.0012	0.0012	0.0012	0.0011	0.0011	0.0010	0.0010	0.0010	0.0009	0.0009
3.5	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007	0.0007	0.0007	0.0006
3.6	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0004
3.7	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003	0.0003	0.0003	0.0003
3.8	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002	0.0002	0.0002	0.0002
3.9	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.0001

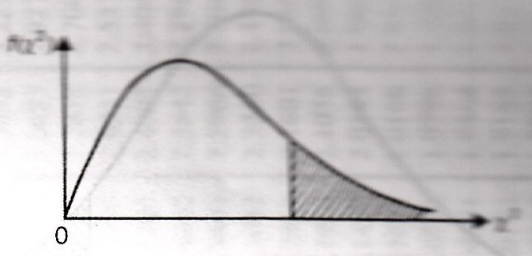
VII. AREAS UNDER THE STANDARD NORMAL DISTRIBUTION

The entries in this table are the probabilities that a standard normal variate is between 0 and Z (the shaded area).



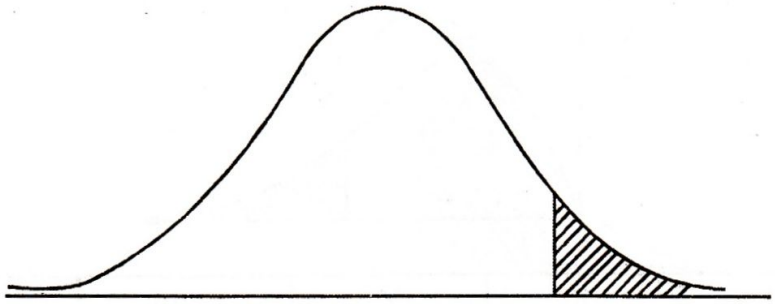
Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1519	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

VIII. CRITICAL VALUES OF CHI-SQUARE (χ^2)

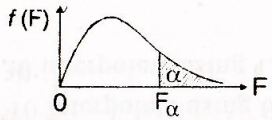


α / *d.f.*	.100	.050	.025	.010	.005	.001
1	2.71	3.84	5.02	6.63	7.88	10.8
2	4.61	5.99	7.38	9.21	10.6	13.8
3	6.25	7.81	9.35	11.3	12.8	16.3
4	7.78	9.49	11.1	13.3	14.9	18.5
5	9.24	11.1	12.8	15.1	16.7	20.5
6	10.6	12.6	14.4	16.8	18.5	22.5
7	12.0	14.1	16.0	18.5	20.3	24.3
8	13.4	15.5	17.5	20.1	22.0	26.1
9	14.7	16.9	19.0	21.7	23.6	27.9
10	16.0	18.3	20.5	23.2	25.2	29.6
11	17.3	19.7	21.9	24.7	26.8	31.3
12	18.5	21.0	23.3	26.2	28.3	32.9
13	19.8	22.4	24.7	27.7	29.8	34.5
14	21.1	23.7	26.1	29.1	31.3	36.1
15	22.3	25.0	27.5	30.6	32.8	37.7
16	23.5	26.3	28.8	32.0	34.3	39.3
17	24.8	27.6	30.2	33.4	35.7	40.8
18	26.0	28.9	31.5	34.8	37.2	42.3
19	27.2	30.1	32.9	36.2	38.6	43.8
20	28.4	31.4	34.2	37.6	40.0	45.3
21	29.6	32.7	35.5	38.9	41.4	46.8
22	30.8	33.9	36.8	40.3	42.8	48.3
23	32.0	35.2	38.1	41.6	44.2	49.7
24	33.2	36.4	39.4	43.0	45.6	51.2
25	34.4	37.7	40.6	44.3	46.9	52.6
26	35.6	38.9	41.9	45.6	48.3	54.1
27	36.7	40.1	43.2	47.0	49.6	55.5
28	37.9	41.3	44.5	48.3	51.0	56.9
29	39.1	42.6	45.7	49.6	52.3	58.3
30	40.3	43.8	47.0	50.9	53.7	59.7
35	46.1	49.8	53.2	57.3	60.3	66.6
40	51.8	55.8	59.3	63.7	66.8	73.4
45	57.5	61.7	65.4	70.0	73.2	80.1
50	63.2	67.5	71.4	76.2	79.5	86.7
55	68.8	73.3	77.4	82.3	85.7	93.2
60	74.4	79.1	83.3	88.4	92.0	99.6
65	80.0	84.8	89.2	94.4	98.1	106.0
70	85.5	90.5	95.0	100.4	104.0	112.3
75	91.1	96.2	100.8	106.4	110.3	118.6
80	96.6	101.9	106.6	112.3	116.3	124.8
85	102.1	107.5	112.4	118.2	122.3	131.0
90	107.6	113.1	118.1	124.1	128.3	137.2
95	113.0	118.8	123.9	130.0	134.2	143.3
100	118.5	124.3	129.6	135.8	140.2	149.4

IX. CRITICAL VALUES OF t



$d.f.$	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
inf.	1.282	1.645	1.960	2.326	2.576



X. 5% POINTS OF FISHER'S F-DISTRIBUTION

n \ m	1	2	3	4	5	6	7	8	9	10	12	15	20	30	60	α
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.95	248.01	250.09	252.20	254.32
2	18.513	19.000	19.164	19.247	19.296	19.330	19.353	19.371	19.385	19.396	19.413	19.420	19.446	19.462	19.479	19.496
3	10.128	9.5521	9.2766	9.1172	9.0135	8.9406	8.8868	8.8452	8.8123	8.7855	8.7446	8.7029	8.6602	8.6166	8.5720	8.5265
4	7.7086	6.9443	6.5914	6.3883	6.2560	6.1631	6.0942	6.0410	5.9988	5.9644	5.9117	5.8578	5.8025	5.7459	5.6878	5.0281
5	6.6079	5.7861	5.4095	5.1922	5.0503	4.9503	4.8753	4.8183	4.7725	4.7351	4.6777	4.6188	4.5581	4.4957	4.4314	4.3650
6	5.9874	5.1433	4.7571	4.5337	4.3874	4.2839	4.2066	4.1468	4.0990	4.0600	3.9999	3.9381	3.8742	3.8082	3.7398	3.6688
7	5.5914	4.7374	4.3468	4.1203	3.9715	3.8660	3.7870	3.7257	3.6767	3.6365	3.5747	3.5108	3.4445	3.3758	3.3043	3.2298
8	5.3177	4.4590	4.0662	3.8378	3.6875	3.5806	3.5005	3.4381	3.3881	3.3472	3.2840	3.2184	3.1503	3.0794	3.0053	2.9276
9	5.1174	4.2565	3.8626	3.6331	3.4817	3.3738	3.2927	3.2296	3.1789	3.1373	3.0729	3.0001	2.9365	2.8637	2.7872	2.7007
10	4.9646	4.1028	3.7083	3.4780	3.3258	3.2172	3.1355	3.0717	3.0204	2.9782	2.9130	2.8450	2.7740	2.6996	2.6211	2.5379
11	4.8443	3.9823	3.5874	3.3567	3.2039	3.0946	3.0123	2.9480	2.8962	2.8536	2.7876	2.7186	2.6464	2.5705	2.4901	2.4045
12	4.7272	3.8853	3.4903	3.2502	3.1059	2.9961	2.9134	2.8486	2.7964	2.7534	2.6866	2.6169	2.5436	2.4663	2.3842	2.2962
13	4.6672	3.8056	3.4105	3.1791	3.0254	2.9153	2.8321	2.7669	2.7144	2.6710	2.6037	2.5331	2.4589	2.3803	2.2966	2.2064
14	4.6001	3.7389	3.3439	3.1122	2.9582	2.8477	2.7642	2.6987	2.6458	2.6021	2.5342	2.4630	2.3879	2.3082	2.2230	2.1307
15	4.5431	3.6823	3.2874	3.0556	2.9013	2.7905	2.7066	2.6408	2.5876	2.5437	2.4753	2.4035	2.3275	2.2468	2.1601	2.0658
16	4.4940	3.6337	3.2389	3.0069	2.8524	2.7413	2.6572	2.5911	2.5377	2.4935	2.4247	2.3522	2.2756	2.1938	2.1058	2.0096
17	4.4513	3.5915	3.1968	2.9647	2.8100	2.6987	2.6143	2.5480	2.4943	2.4499	2.3807	2.3077	2.2304	2.1477	2.0584	1.9604
18	4.4139	3.5546	3.1599	2.9277	2.7729	2.6613	2.5767	2.5102	2.4563	2.4117	2.3421	2.2686	2.1906	2.1071	2.0166	1.9168
19	4.3808	3.5219	3.1274	2.8951	2.7401	2.6283	2.5435	2.4768	2.4227	2.3779	2.3080	2.2341	2.1555	2.0712	1.9796	1.8780
20	4.3513	3.4928	3.0984	2.8661	2.7100	2.5990	2.5140	2.4471	2.3928	2.3479	2.2776	2.2033	2.1242	2.0391	1.9464	1.8432
21	4.3248	3.4668	3.0725	2.8401	2.6848	2.5727	2.4876	2.4205	2.3661	2.3210	2.2504	2.1757	2.0960	2.0102	1.9165	1.8117
22	4.3009	3.4434	3.0491	2.8167	2.6613	2.5491	2.4638	2.3965	2.3419	2.2967	2.2258	2.1508	2.0707	1.9842	1.8895	1.7831
23	4.2793	3.4221	3.0280	2.7955	2.6500	2.5277	2.4422	2.3748	2.3201	2.2747	2.2036	2.1282	2.0476	1.9605	1.8649	1.7570
24	4.2597	3.4028	3.0088	2.7763	2.6207	2.5082	2.4226	2.3551	2.3002	2.2547	2.1834	2.1077	2.0267	1.9390	1.8424	1.7331
25	4.2417	3.3852	2.9912	2.7587	2.6030	2.4904	2.4047	2.3371	2.2821	2.2365	2.1649	2.0889	2.0075	1.9192	1.8217	1.7110
26	4.2252	3.3690	2.9751	2.7426	2.5868	2.4741	2.3883	2.3205	2.2655	2.2197	2.1479	2.0716	1.9898	1.9010	1.8027	1.6906
27	4.2100	3.3541	2.9604	2.7278	2.5719	2.4591	2.3732	2.3053	2.2501	2.2043	2.1323	2.0558	1.9736	1.8842	1.7851	1.6717
28	4.1960	3.3404	2.9467	2.7141	2.5581	2.4453	2.3593	2.2913	2.2360	2.1900	2.1179	2.0411	1.9586	1.8687	1.7689	1.6541
29	4.1830	3.3277	2.9340	2.7014	2.5454	2.4324	2.3463	2.2782	2.2229	2.1768	2.1045	2.0275	1.9446	1.8543	1.7537	1.6377
30	4.1709	3.3158	2.9223	2.6896	2.5336	2.4205	2.3343	2.2662	2.2107	2.1646	2.0921	2.0148	1.9317	1.8409	1.7396	1.6223
40	4.0848	3.2317	2.8387	2.6060	2.4495	2.3359	2.2490	2.1802	2.1240	2.0772	2.0035	1.9245	1.8389	1.7444	1.6373	1.5089
60	4.0012	3.1504	2.7581	2.5252	2.3688	2.2540	2.1665	2.0970	2.0401	1.9926	1.9174	1.8364	1.7480	1.6491	1.5343	1.3893
120	3.9201	3.0718	2.6802	2.4472	2.2900	2.1750	2.0867	2.0164	1.9588	1.9105	1.8337	1.7505	1.6587	1.5543	1.4290	1.2539
∞	3.8415	2.9957	2.6049	2.3719	2.2141	2.0986	2.0096	1.9384	1.8799	1.8307	1.7522	1.6664	1.5705	1.4591	1.3180	1.0000

XI. 1% POINTS OF FISHER'S F-DISTRIBUTION

n \ m	1	2	3	4	5	6	7	8	9	10	12	15	20	30	60	α
1	4052.2	4999.5	5403.3	5624.6	5763.7	5859.0	5928.3	5981.6	6022.5	6055.8	6106.3	6157.3	6208.7	6260.7	6313.0	6366.0
2	98.503	99.000	99.166	99.249	99.299	99.332	99.356	99.374	99.388	99.399	99.416	99.432	99.449	99.466	99.483	99.501
3	34.116	30.817	29.457	28.710	28.237	27.911	27.672	27.489	27.345	27.229	27.052	26.872	26.690	26.505	26.316	26.125
4	21.198	18.000	16.694	15.977	15.522	15.207	14.976	14.799	14.659	14.546	14.374	14.198	14.020	13.838	13.652	13.463
5	16.258	13.274	12.060	11.392	10.967	10.672	10.456	10.289	10.158	10.051	9.8883	9.7222	9.5527	9.3793	9.2020	9.0204
6	13.745	10.925	9.7795	9.1483	8.7459	8.4661	8.2600	8.1016	7.9761	7.8741	7.7183	7.5590	7.3958	7.2285	7.0568	6.8801
7	12.246	9.5466	8.4513	7.8467	7.4604	7.1914	6.9928	6.8401	6.7188	6.6201	6.4691	6.3143	6.1554	5.9921	5.8236	5.6495
8	11.259	8.6491	7.5910	7.0060	6.6318	6.3707	6.1776	6.0289	5.9106	5.8143	5.6668	5.5151	5.3591	5.1981	5.0316	4.8588
9	10.561	8.0215	6.9919	6.4221	6.0569	5.8018	5.6129	5.4671	5.3511	5.2565	5.1114	4.9621	4.8080	4.6486	4.4831	4.3105
10	10.044	7.5594	6.5523	5.9943	5.6363	5.3858	5.2001	5.0567	4.9424	4.8492	4.7059	4.5582	4.4054	4.2469	4.0819	3.9090
11	9.6460	7.2057	6.2167	5.6683	5.3160	5.0692	4.8861	4.7445	4.6315	4.5393	4.3974	4.2509	4.0990	3.9411	3.7761	3.6025
12	9.3302	6.9266	5.9526	5.4119	5.0643	4.8206	4.6395	4.4994	4.3875	4.2961	4.1553	4.0096	3.8584	3.7008	3.5355	3.3608
13	9.0738	6.7010	5.7394	5.2053	4.8616	4.6204	4.4410	4.3021	4.1911	4.1003	3.9603	3.8154	3.6646	3.5070	3.3413	3.1654
14	8.8616	6.5149	5.5639	5.0354	4.6950	4.4558	4.2779	4.1399	4.0297	3.9394	3.8001	3.6557	3.5052	3.3476	3.1813	3.0040
15	8.6831	6.3589	5.4170	4.8932	4.5556	4.3183	4.1415	4.0045	3.8948	3.8049	3.6662	3.5222	3.3719	3.2141	3.0471	2.8684
16	8.5310	6.2262	5.2922	4.7726	4.4374	4.2016	4.0259	3.8896	3.7804	3.6909	3.5527	3.4089	3.2588	3.1007	2.9330	2.7528
17	8.3997	6.1121	5.1850	4.6690	4.3359	4.1015	3.9267	3.7910	3.6822	3.5931	3.4552	3.3117	3.1615	3.0032	2.8348	2.6530
18	8.2854	6.0129	5.0919	4.5790	4.2479	4.0146	3.8406	3.7054	3.5971	3.5082	3.3706	3.2273	3.0771	2.9185	2.7493	2.5660
19	8.1850	5.9259	5.0103	4.5003	4.1708	3.9386	3.7653	3.6305	3.5225	3.4338	3.2965	3.1533	3.0031	2.8442	2.6742	2.4893
20	8.0960	5.8489	4.9382	4.4307	4.1027	3.8714	3.6987	3.5644	3.4567	3.3682	3.2311	3.0880	2.9377	2.7785	2.6077	2.4212
21	8.0166	5.7804	4.8740	4.3688	4.0421	3.8117	3.6396	3.5056	3.3981	3.3098	3.1729	3.0299	2.8796	2.7200	2.5484	2.3603
22	7.9454	5.7190	4.8166	4.3134	3.9880	3.7583	3.5867	3.4530	3.3458	3.2576	3.1209	2.9709	2.8274	2.6675	2.4951	2.3055
23	7.8811	5.6637	4.7649	4.2635	3.9392	3.7102	3.5390	3.4057	3.2986	3.2106	3.0740	2.9311	2.7805	2.6202	2.4471	2.2559
24	7.8229	5.6136	4.7181	4.2184	3.8951	3.6667	3.4959	3.3629	3.2560	3.1681	3.0316	2.8887	2.7380	2.5773	2.4035	2.2107
25	7.7698	5.5680	4.6755	4.1774	3.8550	3.6272	3.4568	3.3239	3.2172	3.1294	2.9931	2.8502	2.6993	2.5383	2.3637	2.1694
26	7.7213	5.5263	4.6366	4.1400	3.8183	3.5911	3.4210	3.2884	3.1818	3.0941	2.9579	2.8150	2.6640	2.5026	2.3273	2.1315
27	7.6767	5.4881	4.6009	4.1056	3.7848	3.5580	3.3882	3.2558	3.1494	3.0618	2.9256	2.7827	2.6316	2.4699	2.2938	2.0965
28	7.6356	5.4529	4.5681	4.0740	3.7539	3.5276	3.3581	3.2259	3.1195	3.0320	2.8959	2.7530	2.6017	2.4397	2.2629	2.0642
29	7.5976	5.4205	4.5378	4.0449	3.7254	3.4995	3.3302	3.1982	3.0920	3.0045	2.8685	2.7256	2.5742	2.4118	2.2344	2.0342
30	7.5625	5.3904	4.5097	4.0179	3.6990	3.4735	3.3045	3.1726	3.0665	2.9791	2.8431	2.7002	2.5487	2.3860	2.2079	2.0062
40	7.3141	5.1785	4.3126	3.8283	3.5138	3.2910	3.1238	2.9930	2.8876	2.8005	2.6648	2.5216	2.3689	2.2034	2.0194	1.8047
60	7.0771	4.9774	4.1259	3.6491	3.3389	3.1187	2.9530	2.8233	2.7185	2.6318	2.4961	2.3523	2.1978	2.0285	1.8363	1.6006
120	6.8510	4.7865	3.9493	3.4796	3.1735	2.9559	2.7918	2.6629	2.5586	2.4721	2.3363	2.1915	2.0346	1.8600	1.6557	1.3805
∞	6.6349	4.6052	3.7816	3.3192	3.0173	2.8020	2.6393	2.5113	2.4073	2.3209	2.1848	2.0385	1.8783	1.6964	1.4730	1.0000

For $m > 10$ interpolate using $60/m$.

For $n > 30$ interpolate using $120/n$.

XII. FACTORS USEFUL IN THE CONSTRUCTION OF CONTROL CHARTS

	Mean-chart								Range-chart				
Sample size	Factors for control limit			Factors for central line	Factors for control limit				Factors for central line	Factors for control limit			
n	A	A_1	A_2	c_2	B_1	B_2	B_3	B_4	d_2	D_1	D_2	D_3	D_4
2	2.121	3.760	1.881	0.6642	0	1.843	0	3.267	1.128	0	3.686	0	3.267
3	1.732	3.394	1.023	0.7236	0	1.858	0	2.566	1.693	0	4.358	0	2.575
4	1.500	2.880	0.729	0.7979	0	1.808	0	2.269	2.059	0	4.698	0	2.282
5	1.342	1.596	0.577	0.8407	0	1.756	0	2.089	2.326	0	4.918	0	2.115
6	1.225	1.410	0.483	0.8686	0.026	1.711	0.030	1.970	2.534	0	5.078	0	2.004
7	1.134	1.277	0.419	0.8882	0.105	1.672	0.118	1.888	2.704	2.205	5.203	0.076	1.924
8	1.061	1.175	0.073	0.9027	0.167	1.638	0.185	1.815	2.847	0.387	5.307	0.136	1.864
9	1.000	1.094	0.037	0.9139	0.219	1.609	0.239	1.761	2.970	0.546	5.394	0.184	1.816
10	0.949	1.028	0.308	0.9227	0.262	1.584	0.284	1.716	3.078	0.687	5.469	0.223	1.777
11	0.905	0.973	0.285	0.9300	0.299	1.561	0.321	1.679	3.173	0.812	5.534	0.256	1.744
12	0.866	0.925	0.256	0.9359	0.331	1.541	0.354	1.646	3.258	0.924	5.592	0.284	1.716
13	0.832	0.883	0.249	0.9410	0.359	1.523	0.382	1.618	3.336	1.026	5.646	0.308	1.692
14	0.802	0.848	0.235	0.9453	0.384	1.507	0.406	1.594	3.407	1.121	5.693	0.329	1.671
15	0.775	0.816	0.223	0.9490	0.406	1.492	0.428	1.572	3.472	1.207	5.737	0.348	1.652
16	0.750	0.788	0.212	0.9523	0.427	1.478	0.448	1.542	3.352	1.285	5.279	0.365	1.636
17	0.728	0.762	0.203	0.9551	0.445	1.465	0.466	1.534	3.588	1.359	5.817	0.379	1.621
18	0.707	0.738	0.816	0.9576	0.461	1.454	0.482	1.518	3.640	1.426	5.854	0.404	1.608
19	0.688	0.717	0.187	0.9599	0.477	1.443	0.497	1.503	3.689	1.490	5.888	0.404	1.596
20	0.671	0.697	0.180	0.9619	0.491	1.433	0.510	1.490	3.735	1.548	5.922	0.414	1.585
21	0.655	0.670	0.173	0.9638	0.504	1.424	0.523	1.447	3.778	1.606	5.950	0.425	1.575
22	0.640	0.662	0.167	0.9655	0.516	1.415	0.534	1.466	3.819	1.659	5.979	0.434	1.566
23	0.626	0.647	0.162	0.9670	0.527	1.407	0.545	1.455	3.858	1.710	6.006	0.443	1.557
24	0.612	0.632	0.157	0.9684	0.538	1.399	0.555	1.445	3.895	1.759	6.031	0.452	1.548
25	0.600	0.319	0.153	0.9696	0.548	1.392	0.565	1.435	3.931	1.804	6.058	0.459	1.541

XIV. RANDOM NUMBERS

58941	72711	39408	91620	27963	96478	21559	19246	88097	44026
02349	71389	45608	60947	60775	73181	43264	56895	04232	59604
89210	44546	27174	27499	53523	63110	57106	20865	91683	80688
11826	91326	29664	01603	23156	89223	43429	95353	44662	59433
69810	17100	35066	00815	01552	06392	31437	70385	45863	75971
81060	33449	68055	83844	90942	74857	52418	68723	47830	63010
56135	80647	51404	06626	10042	93629	37609	57215	08409	81906
57361	65304	93258	56760	63348	24949	11839	29793	37457	59377
24548	56415	61927	64416	29934	00755	09418	14230	62887	92683
66504	02036	02922	63569	17906	38076	32135	19096	96970	75917
45068	05520	56321	22693	35089	07694	04252	23791	60249	83010
99717	01542	72990	43413	59744	44595	71326	91382	45114	20245
05394	61840	83089	09224	78530	33996	49965	04851	18280	14039
38155	42661	02363	67625	34683	95372	74733	63558	09665	22610
04319	04318	99387	86874	12549	38369	54952	91579	26023	81076
18134	90062	10761	54548	49505	52685	63903	13193	33905	66936
32012	42710	34650	73236	66167	21788	03581	40699	10396	81827
78101	44392	53767	15220	66319	72953	14071	59148	95154	72852
23469	42846	94810	16151	08029	50554	03891	38313	34016	18671
35342	56119	97190	43635	84249	61254	80993	55431	90793	62603
65846	18076	12415	30193	42777	85611	57635	51362	79907	77364
22184	33998	87436	37430	45246	11400	20986	43996	73112	88474
83668	66236	79665	88312	93047	12088	86937	70794	01041	74867
90083	70696	13558	98995	58159	04700	90443	13168	31553	67891
97765	27552	49617	51734	20819	70198	67906	00880	82899	66065
49988	13176	94219	88698	41755	56216	66832	17748	04963	54859
78257	86249	46134	51865	09836	73966	65711	41699	11732	17173
30946	22210	79302	40300	08852	27528	84648	79589	95295	72895
19468	76358	69203	02760	28625	70476	76410	32988	10194	94917
30806	80857	84383	78450	26245	91763	73117	33047	03577	62599
42163	68332	98851	50252	56911	62693	73817	98693	18728	94741
39249	51463	95963	07929	66728	47761	81472	44806	15592	71357
88717	29289	77360	09030	39605	87507	85446	51257	89555	75520
16767	57345	42285	56670	88445	85799	76200	21795	38894	58070
77516	96648	51868	48140	13583	94911	13318	64741	64336	95103
87192	66483	55649	36764	86132	12463	28385	94242	32065	45233
74078	64120	04643	14351	71381	26133	68269	65145	28152	39087
94119	20108	78101	81276	00835	63835	87174	42446	08882	27067
62180	27453	18567	55524	86088	00069	59254	24654	77371	26409
56199	05993	71201	78852	65889	32719	13758	23937	90740	16866
04994	09879	70337	11861	69032	51915	23510	32050	52052	24004
21725	43827	78862	67699	01009	07050	73324	06732	27510	33761
24365	37661	18956	50064	39500	17450	18030	63124	48061	59412
14762	69734	89150	93126	17700	94400	76075	08317	27324	72723
28387	99781	52977	01657	92602	41043	05686	15650	29970	95877

Source: Extracted from Table of 105,00 Random Decimal Digits Statement No. 4914, File No. 261-A (Washington D.C. Interstate Commerce Commission, 1949)

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* Dr. S.L. Gupta is an Associate Professor of Marketing at Appeejay School of Marketing. Dr. Gupta possesses 4 years' corporate experience and 7 years' academic experience and specialises in marketing stream. He has been teaching Marketing Management, Marketing Research, Sales and Distribution Management, Consumer Behaviour at various Institutes since 1992. He is accredited management teacher from All India Management Association.

** Ms. Sumitra Pal has extensive industrial and teaching experience in various management schools.

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An Indian Perspective

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APPENDICES (Pp. 62)

Cases • Selected Sources of Information • Suggested Readings • Review Questions.

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- Absolutely new chapter on Current Trends and Tendencies.
- Completely revised Data-Environment, incorporating new data, charts and statistical analysis.
- The book projects a viewpoint which is refreshing as well as thought-provoking.

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About the Author

Manabendra Adhikary (born 1941), Ph.D. (Bloomington, Indiana). A.E.A. Diploma (Boulder, Colorado), M.A. (Delhi School of Economics) is Ex-Dean, Faculty of Management Studies, University of Delhi. Dr. Adhikary is having more than 35 years of teaching, research and consultancy experience at home and abroad.

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