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Index Numbers : Concepts and Applications

INTRODUCTION

Index numbers occupy a place of great prominence in business statistics. Though originally developed for measuring the effect of change in prices, there is hardly any field today where index numbers are not used. They are used to feel the pulse of the economy and they have come to be used as indicators of inflationary or deflationary tendencies. In fact, they are described as *barometers of economic activity*, *i.e.*, if one wants to get an idea as to what is happening to an economy he should look to important indices like the index number of industrial production, agricultural production, business activity, etc.

Index numbers are playing an increasingly significant role in business planning and in the formulation of executive decisions. They are not directly used to prepare forecasts but many of the techniques employed in preparing forecasts utilize index numbers. For example, in correlation analysis either the dependent or independent variable or both may be in the form of index numbers. Many businesses are often reluctant to give out information concerning sales, profits, and the like. They may be induced to release some of the same data in the form of index numbers which permit the absolute value of this restricted information to be concealed. Under such conditions, it is possible to present index numbers indicating whether a firm's profits or sales have increased or decreased over a period of years without revealing the total amount of profits or sales.

An index number* may be described as a specialised average designed to measure the change in the level of a phenomenon with respect to time, geographic location or other characteristics such as income, etc. Thus, when we say that the index number of wholesale prices is 125 for the period Jan., 2010 compared to Jan., 2009 it means there is a net increase in the prices of wholesale commodities to the extent of 25 per cent.

For a proper understanding of the term index number, the following points are worth considering :

(1) *Index numbers are specialised averages.* As explained in the chapter on Measures of Central Tendency, an average is a single figure representing group of figures. However, to obtain an average the items must be comparable, for example, the average weight of men, women and children of a certain locality has no meaning at all. Furthermore, the unit of measurement must be the same for all the items. Thus an average of the weight expressed in kg., lb., etc., has no meaning. However, this is not so with index numbers. Index numbers are used for purposes of comparison in situations where two or more series are expressed in different units or the series are composed of different types of items. For example,

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*An index number is a device which shows by its variation the changes in a magnitude which is not capable of accurate measurement in itself or of direct valuation in practice—Wheldom : *Business Statistics*.

"An index number represents the general level of magnitude of the changes between two or more situations of a number of variables taken as a whole."—*Karmel*

Index numbers are quantitative measures of the general level of growth of prices, production, inventory and other quantities of economic interest.—*Ronold* while constructing a consumer price index the various items are divided into broad heads, namely (*i*) Food, (*ii*) Clothing, (*iii*) Fuel and Lighting, (*iv*) House Rent, and (*v*) Miscellaneous. These items are expressed in different units ; thus under the head 'food' wheat and rice may be quoted per quintal, ghee per kg., etc. Similarly, cloth may be measured in terms of metres. An average of all these items expressed in different units is obtained by using the technique of index numbers.

(2) *Index numbers measure the change in the level of a phenomenon.* Since index numbers are essentially averages they describe in one single figure the increase or decrease in the level of a phenomenon under study. If the index of industrial production is 115 in 2010 compared to 2009, it means that there is net increase in industrial production to the extent of 15%. It should be carefully noted that even where an index is showing a net increase, it may include some items which have actually decreased in value and others which have remained constant.

(3) *Index numbers measure the effect of changes over a period of time.* They are occasionally revised to take into account changes in the economy effected by technology, consumer tastes and spending patterns. Index numbers are most widely used for measuring changes over a period of time. Thus we can find out the net change in agricultural prices from the beginning of First Plan period to the end of the Eighth Plan period. Similarly, we can compare the agricultural production, industrial production, imports, exports, wages, etc., at two different times. However, it should be noted that index numbers not only measure change over a period of time but also compare economic conditions of different locations, different industries, different cities or different countries. But since the basic problems are essentially the same and since most of the important index numbers published by the Government and private research organisations refer to data collected at different times, we shall consider in this chapter index numbers measuring changes relative to time only. However, methods described can be applied to other cases also.

Uses of Index Numbers

Index numbers are indispensable tools of economic and business analysis. Their significance can be best appreciated by the following points :

(1) *They help in framing suitable policies.* Many of the economic and business policies are guided by index numbers. For example, for deciding the increase in dearness allowance of the employees, the employer has to depend primarily upon the cost of living index. If wages and salaries are not adjusted in accordance with the cost of living, very often it leads to strikes and lockouts which in turn cause considerable waste of resources.

Though index numbers are most widely used in the evaluation of business and economic conditions, there are a large number of other fields also where index numbers are useful. For example, sociologists may speak of population indices; psychologists measure intelligence quotients, which are essentially index numbers comparing a person's intelligence score with that of an average for his or her age; health authorities prepare indices to display changes in the adequacy of hospital facilities; and educational research organisations have devised formulae to measure changes in effectiveness of school systems.

(2) *They reveal trends and tendencies.* Since index numbers are most widely used for measuring changes over a period of time, the time series so formed enable us to study the general trend of the phenomenon under study. For example, by examining index numbers of imports for India for the last 8-10 years we can say that our imports are showing an upward tendency, *i.e.*, they are rising year after year. Similarly by examining the index numbers of industrial production, business activity, etc., for the last few years we can conclude about the trend of production and business activity. By examining the trends of the phenomenon under study we can draw very important conclusions as to how much change is taking place due to the effect of seasonality, cyclical forces, irregular forces, etc.

(3) *Index numbers are very useful in deflating.* Index numbers are used to adjust the original data for price changes, or to adjust wage for cost of living changes and thus transform nominal wages into real wages. Moreover, nominal income can be transformed into real income and nominal sales into real sales through appropriate index numbers. This point will be discussed in detail towards the end of the chapter.

Classification of Index Numbers

Index numbers may be classified in terms of what they measure. In economics and business the classifications are : (1) price; (2) quantity; (3) value; and (4) special purpose.*

Only price and quantity index numbers are discussed in detail. The others will be mentioned, but without details of how to construct them since both value and special purpose index numbers do not offer new problems in construction. Since the details of construction of all types of index numbers can be understood if the construction of price index numbers is understood, we shall devote major attention to them.

Problems in the Construction of Index Numbers

Before constructing index numbers a careful thought must be given to the following problems :

1. The purpose of the index. At the very outset the purpose of constructing the index must be very clearly decided—what the index is to measure and why? There is no all purpose index. Every index is of limited and particular use. Thus, a price index that is intended to measure consumers' prices must not include wholesale prices. And if such an index is intended to measure the cost of living of poor families, great care should be taken not to include goods ordinarily used by middle class and upper-income groups. Failure to decide clearly the purpose of the index would lead to confusion and wastage of time with no fruitful results. All other problems such as the base year, the number of commodities to be included, the prices of the commodities, etc., are decided in the light of the purpose for which the index is being constructed.

The problem of the scope of the index, *i.e.,* the field covered by the index, is linked up with the purpose of the index and the data available. The data available, or rather the lack of them, may necessitate the modification of the purpose.

2. Availability and comparability of data. It is needless to say that it is impossible to make appropriate comparisons unless the necessary statistical data can be obtained. Many persons while constructing an index have been frustrated by the fact that essential information was tabulated by countries, whereas actually they needed it by townships, they have run into difficulties because sales data were available only by type of merchandise and not by brand.

The problem of comparability of data used in an index can also be quite troublesome. It is an exceedingly difficult problem to make sure that prices are actually comparable that they really refer to goods and services that are identical in quality. The comparability of statistical data may also be questioned if parts of the data were collected by different agencies. Mistakes in the selection of data that are really not comparable, can also be made at times due to the carelessness of the persons constructing the index.

To summarise it is important to keep in mind that, in so far as is possible, data which are used in the construction of an index number must be comparable in the sense that if one wants to compare prices one is not really of comparing quality. Furthermore, the goods or services to which the prices or quantities refer must adhere to uniform definitions that is, rigorous specifications. How to achieve these goals in practice is a problem that has never been solved entirely to everyone's satisfaction.

* Index numbers may be constructed for a single commodity, called *simple index numbers,* or for a group of commodities called *composite index numbers.*

3. Selection of base period. Whenever index numbers are constructed, a reference is made to some base period. The base period is the period with which comparisons of relative changes are made. It may be a year, month or a day. The index for base period is always taken as 100. Though the selection of the base period would primarily depend upon the object of the index, the following points need careful consideration in the selection of base period:

(i) *The base period should be normal one.* The period that is selected as base should be normal, *i.e.*, it should be free from abnormalities like wars, earthquakes, famines, booms, depressions, etc. However, at times it is really difficult to select a year which is normal in all respects—a year which is normal in one respect may be abnormal in another. To solve this problem an average of a number of years, say, 3 or 4 (preferably covering one complete cycle), may be taken as the base. The process of averaging will reduce the effect of extremes. Thus the average of the period from 2006 to 2010 may be considered normal whereas no individual year in that span may be considered normal.

(ii) *The base period should not be too distant in the past.* Since index numbers are helpful in decision-making and economic policies are often a matter of short period, we should not select a base period that is too distant in the past. For example, for deciding increase in dearness allowance at present, there is no advantage in taking 1995 or 2000 as the base ; the comparison should be with the preceding year or the year after which dearness allowance has not been revised.

(iii) *Fixed base or chain base.* While selecting the base decision has to be made as to whether the base shall remain fixed or not, *i.e.*, whether we have a fixed base or chain base index. In the fixed base method, the year or the period of years to which all other prices are related is constant for all times. On the other hand, in the chain base method the prices of a year are linked with those of the preceding year and not with the fixed year. Naturally the chain base method gives a better picture than what is obtained by fixed base method. However, much would depend upon the purpose of constructing the index.

4. Selection of number of items. Every item cannot be included while constructing an index number and hence one has to select a sample. For example, while constructing a price index it is impossible to include each and every commodity. Hence, it is necessary to decide what commodities to include. The commodities should be selected in such a manner that they are representative of the tastes, habits and customs of the people for whom the index is meant. Thus in a consumer price index for working class, items like colour T.V., motor cars, refrigerators, cosmetics, etc., find no place. A decision must also be made on the number of commodities to be included and their qualities. Here we should note that the larger the number of commodities included, the more representative shall be the index but at the same time the greater shall be the cost and the time taken. The purpose of the index shall help in deciding the number of commodities. Thus, in a general price index a larger number of commodities shall have to be included as compared to a specific purpose index such as the index number of the prices of foodgrains or industrial raw materials.

It is also necessary to decide the grade or quality of the items to be included in the index. Index numbers shall give wrong result if at one time one set of qualities is included and at another time another set. To avoid confusion about qualities it is desirable that as far as possible no standardised or graded items are included so that they can be easily identified after a time lapse.

5. Price quotations. After the commodities have been selected, the next problem is to obtain price quotations for these commodities. It is a well-known fact that prices of many commodities vary from place to place and even from shop to shop in the same market. It is impracticable to obtain price quotations from all the places where a commodity is dealt with. A selection must be made of representative places and persons. These places should be those which are well-known for trading for that particular commodity. After the places from where the price quotations are to be obtained are decided, the next thing is to appoint some person or institution who can supply price quotations as and when required. Great care must be exercised to see that the price reporting agency is unbiased. In order to check the accuracy of price quotations supplied by an agency is that quotations are obtained from more than one agency. If there is some reliable journal or magazine supplying price quotations then it may be utilised.

In order to ensure uniformity the manner in which prices are to be quoted must also be decided. There are two methods of quoting prices : (*i*) money prices, and (*ii*) quantity prices. In the former case prices are quoted per unit of commodity, for example, sugar Rs. 2600 per quintal (100 kg.) and in the latter case prices are quoted per unit of money. Thus, sugar may be quoted as 1/2 kg. for thirteen rupees. The former method is free from confusion and is generally adopted while quoting prices.

A decision must also be made as to whether the wholesale prices or retail prices are required. The choice would depend upon the purpose of the index. Thus in a consumer price index, the wholesale price shall not be representative at all. If the prices of certain commodities are controlled by the government then it is these controlled prices that should be taken into account and not the black-market prices, which may be much higher.

6. Choice of an average. Since index numbers are specialized averages, a decision has to be made as to which particular average (*i.e.*, arithmetic mean, median, mode, geometric mean or harmonic mean) should be used for constructing the index. Median, mode and mean are almost never used in constructing the index numbers. Basically a choice has to be made between arithmetic mean and geometric mean. Theoretically speaking, geometric mean is the best average in the construction of index numbers because of the following reasons : (*i*) in the construction of index numbers we are concerned with ratios or relative changes and the geometric mean gives equal weights to equal ratio of change; (*ii*) geometric mean is less susceptible to major variations as a result of violent fluctuations in the values of the individual items; and (*iii*) index numbers calculated by using this average are reversible and, therefore, base shifting is easily possible. The geometric mean index always satisfies the time reversal test.

Despite theoretical justification for favouring geometric mean, arithmetic mean is more popularly used while constructing index numbers. This is for the reason that arithmetic mean is much more simple to compute than the geometric mean. However, wherever possible, in the interest of greater accuracy geometric mean should be preferred. It is gratifying to note that with the growing use of calculating devices, geometric mean is becoming more popular in constructing index numbers.

7. Selection of appropriate weights. The problem of selecting suitable weights is quite important and at the same time quite difficult to decide. The term 'weight' refers to the relative importance of the different items in the construction of the index. All items are not of equal importance and hence it is necessary to devise some suitable method whereby the varying importance of the different items is taken into account. This is done by allocating weights. Thus we have broadly two types of indices—unweighted indices and weighted indices. In the former case, no specific weights are assigned, whereas in the latter case specific weights are assigned to various items. It may be pointed out here that no index is unweighted in strict sense of the term as weights implicitly enter in unweighted indices because we are giving equal importance to all the items and hence weights are unity. It is, therefore, necessary to adopt some suitable method of weighting so that arbitrary and haphazard weights may not affect the results.

There are two methods of assigning weights: (*i*) implicit, and (*ii*) explicit.

In the implicit weighting, a commodity or its variety is included in the index a number of times. Thus, if wheat is to be given in an index twice as much weight as rice, then two varieties of wheat as against one rice may be included in the series. On the other hand, in case of implicit weighting some outward evidence of importance of the various items in the index is given. When the explicit weights are assigned the questions are: (*i*) By what do we weight? and (*ii*) What types of weight do we use?

(*i*) In order to bring out the economic importance of the commodities involved the weight can be production figures, consumption figures or distribution figures.

(*ii*) Weights are of two types: quantity weights and value weights. A quantity weight, symbolised by q, means the amount of commodity produced, distributed, or consumed in some time period. A value weight, on the other hand, combines price with quantity 'produced, distributed or consumed'. Value is in terms of rupees and is symbolised by $p \times q$ where p stands for the price and q for the quantity.

Now the question is whether to choose quantity weights or value weights. The statistician is not free to choose here. If the aggregative method is used while constructing index, then quantities are used as weights because price times quantity will always give the same units, namely, rupees. On the other hand, in averaging price relatives quantity figures cannot be used. It is for the reason that if we multiply percentages by quantities expressed in different units, we get result in different units; for example, percentage tonnes will give tonnes and percentages multiplied by kg. will give kg. Such figures cannot be used in computation. But if percentages are multiplied by value figures which are always expressed in rupees, we get answer in rupees only. Hence the statistician will use q as a weight in the method of aggregating actual prices and must use $p \times q$ as a weight in the method of averaging price relatives.*

Another problem in connection with weights is that of deciding whether the weights shall be fixed or fluctuating. Since the relative importance of the different items does not remain the same for all times, it is logical to vary the weights from time to time. Such an index would give better results. However, when fluctuating weights are used one must be very careful in interpreting the index because not only changes in prices but also changes in weights are affecting the index.

One of the outstanding problems in index number construction is that of devising a weighting system that will accurately represent the commodities throughout the period covered by the index number. Many systems have been tried, such as getting the average importance of the commodities over a period of years, but no perfect system has yet been developed.

8. Selection of an appropriate formula. A large number of formulae have been devised for constructing the index. The problem very often is that of selecting the most appropriate formula. The choice of the formula would depend not only on the purpose of the index but also on the data available. Prof. Irving Fisher has suggested that an appropriate index is that which satisfies time reversal test and factor reversal test. Theoretically, Fisher's method is considered as 'ideal' for constructing index numbers. However, from a practical point of view there are certain limitations of this index which shall be discussed later. As such, no particular formula can be regarded as the best under all circumstances. On the basis of his knowledge of the characteristics of different formulae a discriminating investigator will choose technical methods adapted to his data and appropriate to his purposes.

None of the above problems is simple to solve in practice and the final index is usually the product of compromise between theoretical standards and the standards attainable with the given data.

METHODS OF CONSTRUCTING INDEX NUMBERS

A large number of formulae have been devised for constructing index numbers. Broadly speaking, they can be grouped under two heads:

(*a*) Unweighted indices; and

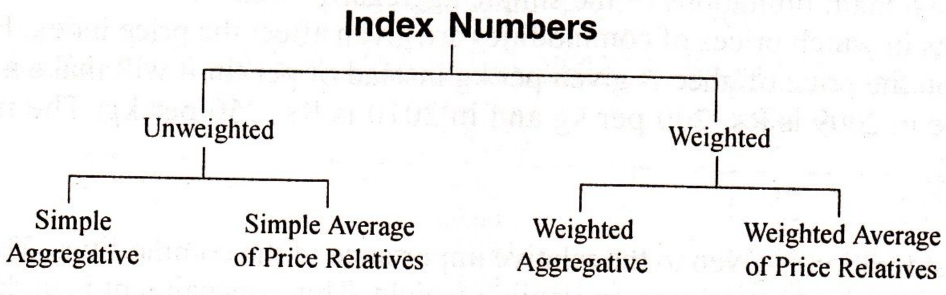
(*b*) Weighted indices.

In the unweighted indices, weights are not expressly assigned whereas in the weighted indices, weights are assigned to the various items. Each of these types may further be divided under two heads:

(*i*) Simple Aggregative; and

(*ii*) Simple Average of Price Relatives.

The following chart illustrates the various methods :



*Sometimes in the absence of actual weights arbitrary magnitudes may have to be used as weights. However, it is unscientific to use these weights and, therefore, they should be only in the crudest forms of index numbers.

A. UNWEIGHTED INDEX NUMBERS

I. Simple Aggregative Method

This is the simplest method of constructing index numbers. When this method is used to construct a price index, the total of current year prices for the various commodities in question is divided by the total of base year prices and the quotient is multiplied by 100. Symbolically,

$$P_{01} = \frac{\Sigma p_1}{\Sigma p_0} \times 100$$

where Σp_1 = Total of current year prices for various commodities, and
Σp_0 = Total of base year prices for various commodities.

This method of constructing the index is very simple and the steps required in computation are :

(*i*) Add the current year prices for various commodities, *i.e.*, obtain Σp_1.

(*ii*) Add the base year prices for the same commodities, *i.e.*, obtain Σp_0.

(*iii*) Divide Σp_1 by Σp_0 and multiply the quotient by 100.

Illustration 1. From the following data construct an index number for 2010 taking 2009 as base :

Commodity and unit	*Price (Rs.) 2009*	*Price (Rs.) 2010*
Butter (kg.)	110.00	120.00
Cheese (kg.)	75.00	80.00
Milk (lt.)	13.00	13.00
Bread (1)	9.00	9.00
Eggs (Doz.)	18.00	20.00
Ghee (1 tin)	850.00	860.00

Solution. CONSTRUCTION OF PRICE INDEX

Commodity	*Price in 2009* p_0	*Price in 2010* p_1
Butter (kg.)	110.00	120.00
Cheese (kg.)	75.00	80.00
Milk (lt.)	13.00	13.00
Bread (1)	9.00	9.00
Eggs (Doz.)	18.00	20.00
Ghee (1 tin)	850.00	860.00
	$\Sigma p_0 = 1075.00$	$\Sigma p_1 = 1102.00$

$$P_{01} = \frac{\Sigma p_1}{\Sigma p_0} \times 100 = \frac{1102}{1075} \times 100 = 102.51$$

This means that as compared to 2009, in 2010 there is a net increase in price of commodities included in the index to the extent of 2.51%.

Limitations of the Method

There are two main limitations of the simple aggregative index :

(1) The units in which prices of commodities are given affect the price index. For example, if in the above illustration the price of ghee is given per kg instead of per tin it will make a difference. Suppose the price of ghee in 2009 is Rs. 200 per kg and in 2010 is Rs. 250 per kg. The index would then be $\frac{250}{200} \times 100 = 125$.

(2) No consideration is given to the relative importance of the commodities. The unit by which each item happens to be priced introduces an implicit weight. This concealment is undesirable and severely restricts the usefulness of an index number arrived at through the method of simple aggregate of actual prices.

II. Simple Average of Relatives Method

When this method is used to construct a price index, price relatives are obtained for the various items included in the index and then an average of these relatives is obtained using any one of the measures of central tendency, *i.e.*, arithmetic mean, median, mode, geometric mean or harmonic mean. When arithmetic mean is used for averaging the relatives, the formula for computing the index is :

$$P_{01} = \frac{\Sigma\left(\frac{p_1}{p_0} \times 100\right)}{N}$$

where N refers to the number of items (commodities) whose price relatives are thus averaged.

Although any measure of central tendency can be used to obtain the overall index, price relatives are generally averaged either by the arithmetic or the geometric mean. When geometric mean is used for averaging the price relatives, the formula for obtaining the index becomes

$$\log P_{01} = \frac{\Sigma \log\left(\frac{p_1}{p_0} \times 100\right)}{N} \quad \text{or} \quad \frac{\Sigma \log P}{N}$$

where $$P = \frac{p_1}{p_0} \times 100$$

or, $$P_{01} = \text{antilog}\left[\frac{\left(\Sigma \log \frac{p_1}{p_0}\right) \times 100}{N}\right] = \text{antilog}\left(\frac{\Sigma \log P}{N}\right)$$

Other measures of central tendency are not in common use for averaging relatives.

Illustration 2. From the data of Illustration 1, compute price index by simple average of price relatives method based on (*a*) arithmetic mean, and (*b*) geometric mean.

Solution. (*a*) PRICE INDEX BASED ON SIMPLE AVERAGE OF PRICE RELATIVES

Commodities	*Price (Rs.) 2009 (p_0)*	*Price (Rs.) 2010 (p_1)*	$\frac{p_1}{p_0} \times 100$
Butter (kg.)	110.00	120.00	109.09
Cheese (kg.)	75.00	80.00	106.67
Milk (lt.)	13.00	13.00	100.00
Bread (1)	9.00	9.00	100.00
Eggs (Doz.)	18.00	20.00	111.11
Ghee (1 tin)	850.00	860.00	101.18
$N = 6$			$\Sigma \frac{p_1}{p_0} \times 100 = 628.05$

$$\text{Price Index or } P_{01} = \frac{\Sigma \frac{p_1}{p_0} \times 100}{N} = \frac{628.05}{6} = 104.67.$$

(*b*) PRICE INDEX BASED ON GEOMETRIC MEAN OF PRICE RELATIVES

Commodities	*Price (Rs.) 2009 (P_0)*	*Price (Rs.) 2010 (P_1)*	*Price Relatives P*	*log P*
Butter (kg.)	110.00	120.00	109.09	2.0378
Cheese (kg.)	75.00	80.00	106.67	2.0280
Milk (lt.)	13.00	13.00	100.00	2.0000
Bread (1)	9.00	9.00	100.00	2.0000
Eggs (doz.)	18.00	20.00	111.11	2.0457
Ghee (1 tin)	850.00	860.00	101.18	2.0051
				$\Sigma \log P = 12.1166$

$$P_{01} = AL\left[\frac{\sum \log P}{N}\right] = AL\left[\frac{12.1166}{6}\right] = AL\ 2.0194 = 104.57$$

Although arithmetic mean and geometric mean have both been used, the arithmetic mean is often preferred because it is easier to compute and much better known. Some economists, notably F.Y. Edgeworth, have preferred to use the median which is not affected by single extreme value. Since the argument is important only when an index is based on a very small number of commodities, it generally does not carry much weight and the median is seldom used in actual practice.

Merits and Limitations of this Method

Merits. This method has the following two advantages over the previous method:

1. Extreme items do not influence the index. Equal importance is given to all the items.
2. The index is not influenced by the units in which prices are quoted or by the absolute level of individual prices. Relatives are pure numbers and are, therefore, independent of the original units. Consequently, index numbers computed by the relative method would be the same regardless of the way in which prices are quoted.

Limitations. Despite these merits this method is not very satisfactory because of the following two reasons :

1. Difficulty is faced with regard to the selection of an appropriate average. The use of the arithmetic mean is considered as questionable sometimes because it has an upward bias. The use of geometric mean involves difficulties of computations. Other averages are almost never used while constructing index numbers.
2. The relatives are assumed to have equal importance. This is again a kind of concealed weighting system that is highly objectionable since economically some relatives are more important than others.

B. WEIGHTED INDEX NUMBERS

The unweighted index numbers discussed so far are not unweighted in the true sense of the term. They assign equal importance to all the items included in the index and as such they are in reality weighted, weights being implicit rather than explicit. As discussed earlier, in case of unweighted indices it is possible to get different results by changing the importance of different items by quoting prices relative to different units. Implicit weighting (or the unweighted index) is far from realistic in most of the cases. Construction of useful index numbers requires a conscious effort to assign to each commodity a weight in accordance with its importance in the total phenomenon that the index is supposed to describe.

Weighted index numbers are of two types:

I. Weighted Aggregative Index Numbers, and

II. Weighted Average of Relative Index Numbers.

I. Weighted Aggregative Index Numbers

These index numbers are of the simple aggregative type with the fundamental difference that weights are assigned to the various items included in the index. There are various methods of assigning weights and consequently a large number of formulae for constructing index numbers have been devised of which some of the more important ones are :

1. Laspeyres method,
2. Paasche method,
3. Dorbish and Bowley's method,
4. Fisher's Ideal method,
5. Marshall-Edgeworth method, and
6. Kelly's method.

All these methods carry the name of persons who have suggested them.

1. Laspeyres Method. In this method the base year quantities are taken as weights. The formula for constructing index is:

$$P_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} \times 100$$

Steps. (*i*) Multiply the base year prices of various commodities with base year weights and obtain $\Sigma p_1 q_0$.

(*ii*) Multiply the base year prices of various commodities with base year weights and obtain $\Sigma p_0 q_0$.

(*iii*) Divide $\Sigma p_1 q_0$ by $\Sigma p_0 q_0$ and multiply the quotient by 100. This gives us the price index.

Laspeyres index attempts to answer the question: "What is the change in aggregate value of the base period list of goods when valued at given period prices?" The index is very widely used in practical work.

2. Paasche Method. In this method the *current year* quantities are taken as weights. The formula for constructing the index is:

$$P_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times 100$$

Steps. (*i*) Multiply the current year prices of various commodities with current year weights and obtain $\Sigma p_1 q_1$.

(*ii*) Multiply the base year prices of various commodities with current year weights and obtain $\Sigma p_0 q_1$.

(*iii*) Divide $\Sigma p_1 q_1$ by $\Sigma p_0 q_1$ and multiply the quotient by 100.

In general this formula answers the question: "What would be the value of the given period list of goods when valued at base period prices?"

Comparison of Laspeyres and Paasche methods. From a practical point of view, Laspeyres index is often preferred to Paasche's for the simple reason that in Laspeyres index weights (q_0) are the base year quantities and do not change from one year to the next. On the other hand, the use of Paasche index requires the continuous use of new quantity weights for each period considered and in most cases these weights are difficult and expensive to obtain.

An interesting property of Laspeyres and Paasche indices is that the former is generally expected to *overestimate*, or to leave an upward bias, whereas the latter tends to *underestimate, i.e.*, show a downward bias. When the prices increase, there is usually a reduction in the consumption of those items for which the increase has been the most pronounced, and, hence, by using base year quantities we will be giving too much weight to the prices that have increased the most and the numerator of the Laspeyres index will be too large. When the prices go down, consumers often shift their preference to those items which have declined the most and hence, by using base period weights in the numerator of the Laspeyres index we shall not be giving sufficient weight to the prices that have gone down the most and the numerator will again be too large. Similarly because people tend to spend less on goods when their prices are rising the use of the Paasche or current weighting produces an index which tends to underestimate the rise in prices, *i.e.*, it has a downward bias. But the above arguments do not imply that Laspeyres index must necessarily be larger than the Paasche index.

Unless drastic changes have taken place between the base year and the given year, the difference between the Laspeyres and Paasche's will generally be small and either could serve as a satisfactory measure. In practice, however, the base year weighted Laspeyres type index remains the most popular for reasons of its practicability. The Paasche type index can only be constructed when up-to-date data for the weights are available. Furthermore, the price index of a given year can be compared only with the base year. For example, let $P_{2006} = 100, P_{2007} = 130$, and $P_{2008} = 140$. Then P_{2007} and P_{2008} are using different weights and cannot be compared with each other. If these indices had been obtained by the Laspeyres formula, they could be compared because in that case the weights are the same base year weights (q_0). For these reasons, in practice the Paasche formula is usually not used and the Laspeyres type index remains most popular for reasons of its practicability.

3. Dorbish and Bowley's Method. Dorbish and Bowley have suggested simple arithmetic mean of the two indices (Laspeyres and Paasche) mentioned above so as to take into account the influence of both the periods, *i.e.*, current as well as base periods. The formula for constructing the index is:

$$P_{01} = \frac{L + P}{2}$$

where L = Laspeyres Index, P = Paasche Index

or

$$P_{01} = \frac{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} + \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}}{2} \times 100$$

4. Fisher's 'Ideal' Method. Prof. Irving Fisher has given a number of formulae for constructing index number and of these he calls one as the 'ideal' index. The Fisher's Ideal Index is given by the formula* :

$$P_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} \times 100$$

or

$$P_{01} = \sqrt{L \times P}$$

It shall be clear from the above formula that Fisher's Ideal Index is the geometric mean of the Laspeyres and Paasche indices.

The above formula is known as 'Ideal' because of the following reasons:

(*i*) It is based on the geometric mean which is theoretically considered to be the best average for constructing index numbers.

(*ii*) It takes into account both current year as well as base year prices and quantities.

(*iii*) It satisfies both the time reversal test as well as the factor reversal test as suggested by Fisher.

(*iv*) It is free from bias. The two formulae (Laspeyres' and Paasche's) that embody the opposing types and weight biases are, in the ideal formula, crossed geometrically, *i.e.*, by an averaging process that of itself has no bias. The result is the complete cancellation of biases of the kinds revealed by time reversal and factor reversal tests.

It is not, however, a practical index to compute because it is excessively laborious. The data, particularly or the Paasche segment of the index, are not readily available. In practice, statisticians will continue to rely upon simple, although perhaps less exact, index number formulae.

5. Marshall-Edgeworth Method. In this method also both the current year as well as base year prices and quantities are considered. The formula for constructing the index is :

$$P_{01} = \frac{\Sigma (q_0 + q_1) p_1}{\Sigma (q_0 + q_1) p_0} \times 100$$

or opening the brackets

---

*For proof see under Tests For Perfection.

$$P_{01} = \frac{\Sigma p_1 q_0 + \Sigma p_1 q_1}{\Sigma p_0 q_0 + \Sigma p_0 q_1} \times 100$$

It is a simple, readily constructed measure, giving a very close approximation to the results obtained by the ideal formula.

6. Kelly's Method. T.L. Kelly has suggested the following formula for constructing index number :

$$P_{01} = \frac{\Sigma p_1 q}{\Sigma p_0 q} \times 100$$

Here weights are the quantities which may refer to some period, not necessarily the base year or current year. Thus, the average quantity of two or more years may be used as weights. If in the Kelly's formula the average of the quantities of two years is used as weights, the formula becomes

$$P_{01} = \frac{\Sigma p_1 q}{\Sigma p_0 q} \times 100$$

where $$q = \frac{q_0 + q_1}{2}$$

Similarly, the average of the quantities of three or more years can be used as weights. The method is known as *fixed weight aggregative index* and is currently in great favour in the construction of index number series. An important advantage of this formula is that like Laspeyres' index it does not demand yearly changes in the weights. Moreover, the base period can be changed without necessitating corresponding change in the weights. This is very important because the construction of appropriate quantity weights for general purpose index usually requires a considerable amount of work. Weights can thus be kept constant until new census (or other survey) data become available to revise the index.

Illustration 3. Construct index numbers of price from the following data by applying :
1. Laspeyres' method,
2. Paasche's method,
3. Bowley's method,
4. Fisher's method, and
5. Marshall-Edgeworth method.

Commodity	*2009*		*2010*	
	Price	*Quantity*	*Price*	*Quantity*
A	2	8	4	6
B	5	10	6	5
C	4	14	5	10
D	2	19	2	13

Solution.

CALCULATION OF VARIOUS INDICES

Commodity	*2009*		*2010*					
	Price p_0	*Quantity* q_0	*Price* p_1	*Quantity* q_1	p_1q_0	p_0q_0	p_1q_1	p_0q_1
A	2	8	4	6	32	16	24	12
B	5	10	6	5	60	50	30	25
C	4	14	5	10	70	56	50	40
D	2	19	2	13	38	38	26	26
					$\Sigma p_1q_0 = 200$	$\Sigma p_0q_0 = 160$	$\Sigma p_1q_1 = 130$	$\Sigma p_0q_1 = 103$

1. *Laspeyres Method :* $$P_{01} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100 = \frac{200}{160} \times 100 = 125.$$

2. *Paasche's Method :* $$P_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times 100 = \frac{130}{103} \times 100 = 126.21.$$

3. *Bowley's Method :* $$P_{01} = \frac{\dfrac{\Sigma p_1 q_0}{\Sigma p_0 q_0} + \dfrac{\Sigma p_1 q_1}{\Sigma p_0 q_1}}{2} \times 100 = \frac{\dfrac{200}{160} + \dfrac{130}{103}}{2} \times 100$$

$$= \frac{1.25 + 1.2621}{2} \times 100 = \frac{2.5121}{2} \times 100 = 125.605$$

or $$P_{01} = \frac{L + P}{2} = \frac{125 + 126.21}{2} = 125.605$$

4. *Fisher's Ideal Method :* $$P_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} \times 100 = \sqrt{\frac{200}{160} \times \frac{130}{103}} \times 100$$

$$= \sqrt{1.578} \times 100 = 1.256 \times 100 = 125.6.$$

5. *Marshall-Edgeworth Method :* $$P_{01} = \frac{\Sigma (q_0 + q_1) p_1}{\Sigma (q_0 + q_1) p_0} \times 100 = \frac{\Sigma p_1 q_0 + \Sigma p_1 q_1}{\Sigma p_0 q_0 + \Sigma p_0 q_1} \times 100$$

$$= \frac{200 + 130}{160 + 103} \times 100 = \frac{330}{263} \times 100 = 125.475$$

II. Weighted Average of Relative Index Numbers

In the weighted aggregative methods discussed above price relatives were not computed. However, like unweighted relatives method it is also possible to compute weighted average of relatives. For purposes of averaging we may use either the arithmetic mean or the geometric mean. The **steps** in the computation of the weighted arithmetic mean of relatives index number are as follows :

(*i*) Express each item of the period for which the index number is being calculated as a percentage of the same item in the base period.

(*ii*) Multiply the percentages as obtained in step (*i*) for each item by the weight which has been assigned to that item.

(*iii*) Add the results obtained from the several multiplications carried out in step (*ii*).

(*iv*) Divide the sum obtained in step (*iii*) by the sum of the weights used. The result is the index number. Symbolically :

$$P_{01} = \frac{\Sigma PV}{\Sigma V}$$

where P = Price relative

V = Value weights, *i.e.*, $p_0 q_0$.

Instead of using arithmetic mean the geometric mean may be used for averaging relatives. The weighted geometric mean of relatives is computed in the same manner as the unweighted geometric mean of relatives index number except that weights are introduced by applying them to the logarithms of the relatives. When this method is used the formula for computing the index is :

$$P_{01} = \frac{\Sigma V . \log P}{\Sigma V}$$

where $P = \dfrac{p_1}{p_0} \times 100$

and V^* = Value weight, *i.e.*, $p_0 q_0$ for each item.

*If current year values are employed, the weights are $p_1 q_1$. If theoretical values are used as weights, the weights are $p_1 q_0$ or $p_0 q_1$.

Steps : (*i*) Obtain percentage relatives for each item.

(*ii*) Find the logarithm of each percentage relative found in step (*i*).

(*iii*) Multiply the logarithms by weights assigned.

(*iv*) Add the results obtained in step (*iii*).

(*v*) Divide the total obtained in step (*iv*) by the sum of the weights.

(*vi*) Find the antilogarithm of the quotient obtained in step (*v*). This is weighted geometric mean of relatives index number.

Illustration 4. From the following data compute price index by applying weighted average of price relatives method using : (*a*) arithmetic mean, and (*b*) geometric mean.

Commodity	p_0 (*Rs.*)	q_0	p_1 (*Rs.*)
Sugar	18.00	20 kg.	20.00
Flour	12.00	40 kg.	14.00
Milk	15.00	10 lt.	16.00

Solution. (*a*) INDEX NUMBER USING WEIGHTED ARITHMETIC MEAN OF PRICE RELATIVES

Commodity	p_0 (*Rs.*)	q_0	p_1 (*Rs.*)	p_0q_0 V	$\frac{p_1}{p_0} \times 100$ P	PV
Sugar	18.00	20 kg.	20.00	360	111.11	39999.6
Flour	12.00	40 kg.	14.00	480	116.67	56001.6
Milk	15.00	10 lt.	16.00	150	106.67	16000.5
				$\Sigma V = 990$		$\Sigma PV = 112001.7$

$$P_{01} = \frac{\Sigma PV}{\Sigma V} = \frac{112001.7}{990} = 113.13$$

This means that there has been a 13.13 per cent increase in price over the base level.

(*b*) INDEX NUMBER USING GEOMETRIC MEANS OF PRICE RELATIVES

Commodity	p_0 (*Rs.*)	q_0	p_1 (*Rs.*)	V	P	*Log P*	*V.Log P*
Sugar	18.00	20 kg.	20.00	360	111.11	2.046	736.56
Flour	12.00	40 kg.	14.00	480	116.67	2.067	992.16
Milk	15.00	10 lt.	16.00	150	106.67	2.028	304.20
				$\Sigma V = 990$			$\Sigma V. \log P = 2032.92$

$$P_{01} = \text{A.L.}\left[\frac{\Sigma V.\log P}{\Sigma V}\right] = \text{A.L.}\left[\frac{2032.92}{990}\right] = \text{A.L. } 2.0535 = 113.11$$

The result obtained by applying the Laspeyres method would come out to be the same as obtained by weighted arithmetic mean of price relatives method (as shown below):

PRICE INDEX BY LASPEYRES' METHOD

Commodity	p_0 (*Rs.*)	q_0	p_1 (*Rs.*)	p_1q_0	p_0q_0
Sugar	18.00	20 kg.	20.00	400	360
Flour	12.00	40 kg.	14.00	560	480
Milk	15.00	10 lt.	16.00	160	150
				$\Sigma p_1q_0 = 1120$	$\Sigma p_0q_0 = 990$

$$P_{01} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100 = \frac{1120}{990} \times 100 = 113.13$$

The answer is almost the same as obtained by weighted arithmetic mean of price relatives method.

Merits of Weighted Average of Price Relatives Method

The following are the special advantages of weighted average of relative indices over weighted aggregative indices :

(1) When different index numbers are constructed by the average of price relatives method, all of which have the same base, they can be combined to form a new index.

(2) When an index is computed by selecting one item from each of the many sub-groups of items, the values of each sub-group may be used as weights. Then only the method of weighted average of relatives is appropriate.

(3) When a new commodity is introduced to replace the one formerly used, the relative for the new item may be spliced to the relative for the old one, using the former value weights.

(4) The price or quantity relatives for each single item in the aggregate are, in effect, themselves a simple index that often yields valuable information for analysis.

Quantity Index Numbers

Price index numbers measure and permit comparison of the price of certain goods, quantity index numbers. On the other hand, measure and permit comparison of the physical volume of goods produced or distributed or consumed. Though price indices are more widely used, production indices are highly significant as indicators of the level of output in the economy or in parts of it.

In constructing quantity index numbers, the problems confronting the statistician are analogous to those involved in price indices. We measure changes in quantities, and when we weigh we use prices or values as weights. Quantity indices can be obtained easily by changing p to q and q to p in the various formulae discussed above.

Thus, when Laspeyres' method is used

$$Q_{01} = \frac{\sum q_1 p_0}{\sum q_0 p_0} \times 100$$

When Paasche's formula is used

$$Q_{01} = \frac{\sum q_1 p_1}{\sum q_0 p_1} \times 100$$

When Fisher's formula is used

$$Q_{01} = \sqrt{\frac{\sum q_1 p_0}{\sum q_0 p_0} \times \frac{\sum q_1 p_1}{\sum q_0 p_1}} \times 100$$

These formulae represent the *quantity index* in which the quantities of the different commodities are weighted by their prices. However, any other suitable weights can be used instead.

Illustration 5. Compute by suitable method the index number of quantity from the data given below:

	2009		*2010*	
Commodity	*Price*	*Value*	*Price*	*Value*
A	8	80	10	110
B	10	90	12	108
C	16	256	20	340

Solution. Since we are given the value and the price we can obtain quantity figure by dividing value by price for each commodity. We can then apply Fisher's method for finding out quantity index.

Commodity	*2009*		*2010*		q_1p_0	q_0p_0	q_1p_1	q_0p_1
	p_0	q_0	p_1	q_1				
A	8	10	10	11	88	80	110	100
B	10	9	12	9	90	90	108	108
C	16	16	20	17	272	256	340	320
					$\Sigma q_1p_0 = 450$	$\Sigma q_0p_0 = 426$	$\Sigma q_1p_1 = 558$	$\Sigma q_0p_1 = 528$

$$\text{Quantity index or } Q_{01} = \sqrt{\frac{\Sigma q_1p_0}{\Sigma q_0p_0} \times \frac{\Sigma q_1p_1}{\Sigma q_0p_1}} \times 100 = \sqrt{\frac{450}{426} \times \frac{558}{528}} \times 100$$

$$= \sqrt{1.116} \times 100 = 1.056 \times 100 = 105.6$$

Volume Index Numbers

The value of single commodity is the product of its price and quantity. Thus a value index V is the sum of the values of a given year divided by the sum of the values of the base year. The formula, therefore, is

$$V = \frac{\sum p_1q_1}{\sum p_0q_0} \times 100$$

where $\sum p_1q_1$ = Total value of all commodities in the given period and $\sum p_0q_0$ = Total value of all commodities in the base period.

Since in most cases the value figures are given, the formula can be stated more simply

$$V = \frac{\Sigma V_1}{\Sigma V_0}$$

in which V stands for value.

In this type of index both price and quantity are variable in the numerator. Weights do not have to be applied, since they are inherent in the value figures. A value index, therefore, is an aggregate of values. It measures the change in actual values between the base and the given periods.

The value index is not in wide use, although because of the unsatisfactory nature of price and quantity indices, it has been occasionally suggested that they be replaced by the value index. The temptation, however, must be resisted, since the concepts of price level and quantity level answer questions that cannot be answered by the value level. Furthermore, an aggregate of values may be viewed as the product of a price level and quantity level. The division of an aggregate of value into its price and quantity factors may be arbitrary, but this need not create any confusion of thought as long as our concepts of the two factors are consistent.

The test of consistency is that the product of the price and quantity indices must produce the value index.

TESTS FOR PERFECTION

Several formulae have been suggested for constructing index numbers and the problem is that of selecting the most appropriate one in a given situation. The following tests are suggested for choosing an appropriate index:

1. Time Reversal Test,
2. Factor Reversal Test, and
3. Circular Test.

1. Time Reversal Test

Prof. Irving Fisher had made a careful study of the various proposals for computing index numbers and has suggested various tests to be applied to any formula to indicate whether or not it is satisfactory. The two most important of these he calls the time reversal test and the factor reversal test.

Time reversal test is a test to determine whether a given method will work both ways in time, forward and backward. In the words of Fisher, "*The test is that the formula for calculating the index number should be such that it will give the same ratio between one point of comparison and the other, no matter which of the two is taken as base.*" In other words, when the data for any two years are treated by the same method, but with the bases reversed, the two index numbers secured should be reciprocals of each other so that their product is unity. Symbolically, the following relation should be satisfied:

$$P_{01} \times P_{10} = 1$$

where P_{01} is the index for time "1" on time "0" as base and P_{10} is the index for time "0" on time "1" as base. If the product is not unity, there is said to be a time bias in the method. Thus, if from 2009 to 2010 the price of wheat increased from Rs. 2220 to Rs. 2960 per quintal, the price in 2010 should be 133.33 per cent of the price in 2009 and the price in 2009 should be 75 per cent of the price in 2010. One figure is the reciprocal of the other; their product (1.333 × 0.75) is unity.

The test is not satisfied by Laspeyres method and the Paasche method as can be seen below :

When Laspeyres method is used :

$$P_{01} = \frac{\sum p_1q_0}{\sum p_0q_0}; \text{ and } P_{10} = \frac{\sum p_0q_1}{\sum p_1q_1}$$

$$P_{01} \times P_{10} = \frac{\sum p_1q_0}{\sum p_0q_0} \times \frac{\sum p_0q_1}{\sum p_1q_1} \neq 1 \text{ and the test is not satisfied.}$$

When Paasche method is used :

$$P_{01} = \frac{\sum p_1q_0}{\sum p_0q_0}; \text{ and } P_{10} = \frac{\sum p_0q_0}{\sum p_1q_0}$$

$$P_{01} \times P_{10} = \frac{\sum p_1q_0}{\sum p_0q_0} \times \frac{\sum p_0q_1}{\sum p_1q_1} \neq 1 \text{ and the test is not satisfied.}$$

There are five methods which do satisfy the test:

(1) The Fisher's Ideal formula.

(2) Simple geometric mean of price relatives.

(3) Aggregate with fixed weights.

(4) The weighted geometric mean of price relatives with fixed weights.

(5) Marshall-Edgeworth method.

Let us now see how Fisher's Ideal formula satisfies the test.

According to Fisher's Method:

$$P_{01} = \sqrt{\frac{\sum p_1q_0}{\sum p_0q_0} \times \frac{\sum p_1q_1}{\sum p_0q_1}}$$

Changing time, *i.e.,* 0 to 1 and 1 to 0

$$P_{10} = \sqrt{\frac{\sum p_0q_1}{\sum p_1q_1} \times \frac{\sum p_0q_0}{\sum p_1q_0}}$$

$$P_{01} \times P_{10} = \sqrt{\frac{\sum p_1q_0}{\sum p_0q_0} \times \frac{\sum p_1q_1}{\sum p_0q_1} \times \frac{\sum p_0q_1}{\sum p_1q_1} \times \frac{\sum p_0q_0}{\sum p_1q_0}} = \sqrt{1} = 1.$$

Since $P_{01} \times P_{10} = 1$, the Fisher's Ideal index satisfies the test.

2. Factor Reversal Test

Another test suggested by Fisher is known as factor reversal test. It holds that the product of price index and the quantity index should be equal to the corresponding value index. In the words of Fisher, "*Just as each formula should permit the interchange of the two times without giving inconsistent results, so it ought to permit interchanging the prices and quantities without giving inconsistent results, i.e., the* *two results multiplied together should give the true value ratio.*" In other words, the test is that the change in price multiplied by the change in quantity should be equal to the total value of given commodity in a given year is the product of the quantity and the price per unit (total value = $p \times q$). The ratio of the total value in one year to the total value in the preceding year is $\frac{p_1 q_1}{p_1 q_0}$. From one year to the next, both price and quantity should double, the price relative would be 200, the quantity relative 200, and the value relative 400. The total value in the second year would be four times the value in the first year. In other words, if p_1 and p_0 represent prices and q_1 and q_0 the quantities in the current year and the base year, respectively, and if P_{01} represents the change in price in the current year and Q_{01} the change in quantity in the current year, then :

$$P_{10} \times Q_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$$

If the product is not equal to the value ratio, there is, with reference to this test, an error in one or both of the index numbers.

The factor reversal test is satisfied *only* by the Fisher's Ideal Index.

Proof.

$$P_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}}$$

Changing p to q and q to p

$$Q_{01} = \sqrt{\frac{\Sigma q_1 p_0}{\Sigma q_0 p_0} \times \frac{\Sigma q_1 p_1}{\Sigma q_0 p_1}}$$

$$P_{01} \times Q_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times \frac{\Sigma q_1 p_0}{\Sigma q_0 p_0} \times \frac{\Sigma q_1 p_1}{\Sigma q_0 p_1}} = \sqrt{\frac{(\Sigma p_1 q_1)^2}{(\Sigma p_0 q_0)^2}} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$$

Since $P_{01} \times Q_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$ the factor reversal test is satisfied by the Fisher's ideal index.

This means, of course, that the formula serves equally well for constructing indices of quantities as for constructing indices of prices, the quantity index being derived by interchanging p and q in the ideal formula. None of the simple or weighted forms of elementary indices—arithmetic mean, harmonic mean, geometric mean—fulfil the requirements of factor reversal test. It is thus obvious that the strong restrictions imposed by the factor reversal test compel its being ignored in the construction of many highly reputable index numbers.

Some of the authorities on the subject argue that there are no logical reasons for claiming that an index number ought to meet these tests. For example, Karmel has pointed out that as far as time reversal test is concerned collection of goods included in P_{01} is different from that included in P_{10} (q_0 as against q_1) and therefore, one could hardly hope for consistent results.

3. Circular Test

Another test of the adequacy of index number formula is what is known as 'circular test'. If in the use of index numbers interest attaches not merely to a comparison of two years, but to the measurement of price changes over a period of years, it is frequently desirable to shift the base. A formula is said to meet this test if, for example, the 2010 index with 2000 as the base is 200, and the 2000 index with 1995 as the base is again 200, then the 2010 index with 1995 as the base must be 400. Clearly, the desirability of this property is that it enables us to adjust the index values from period to period without referring each time to the original base. A test of this shiftability of base is called the circular test.

This test is just an extension of the time reversal test. The test requires that if an index is constructed for the year *a* on base year *b*, and for the year *b* on base year *c*, we ought to get the same result as if we calculated direct an index for *a* on base year *c* without going through *b* as an intermediary.

Symbolically, if there are three years *a, b, c* the circular test will be satisfied if :

$$\frac{P_b}{P_a} \times \frac{P_c}{P_b} \times \frac{P_a}{P_c} = 1$$

The Laspeyres index does not satisfy the test as can be seen from the following:

If the three years are 0, 1, 2; the index by the Laspeyres method will be

$$\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_2 q_1}{\sum p_1 q_1} \times \frac{\sum p_0 q_2}{\sum p_2 q_2}.$$

The product of all these is not equal to 1. Hence the test is not satisfied. Similarly, it can be shown that the Paasche index and Fisher's index do not satisfy the test. However, the simple aggregative method and the fixed weight aggregative method satisfy the test as can be seen from the following :

When test is applied to the simple aggregative method, we will get

$$\frac{\sum p_1}{\sum p_0} \times \frac{\sum p_2}{\sum p_1} \times \frac{\sum p_0}{\sum p_2} = 1.$$

Similarly, when applied to fixed weight aggregative method we will get :

$$\frac{\sum p_1 q}{\sum p_0 q} \times \frac{\sum p_2 q}{\sum p_1 q} \times \frac{\sum p_0 q}{\sum p_2 q} = 1.$$

The circular test (which amounts, in fact, to a modification of the time reversal test) is met when

$$P_{ba} \times P_{cb} \times P_{ca} = 1.$$

An index which satisfies this test has the advantage of reducing the computation every time a change in the base year has to be made. Such index numbers can be adjusted from year to year without referring each time to the original base.

The circular test is not met by the ideal index or by any of the weighted aggregative with changing weights. The test is met by *simple geometric mean of price relatives and the weighted aggregative fixed weights.* The reason why Laspeyres' and Paasche's index numbers and their derivatives, the Marshall-Edgeworth and the Ideal indices, do not meet the circular test is that the weights in these index numbers depend on the periods between which comparisons are being made. If these periods change, the weights change. For example, if the base period is taken as period 2 rather than period 0, the weights in Laspeyres' index are no longer q_0 but q_2.

Karmel has pointed out that although it may seem reasonable to argue that if a price index between periods 0 and 1 has risen to M and between periods 1 and 2 to N, then between periods 0 and 2 it should have risen to MN. A moment's reflection will show that this requirement is not reasonable. An index number has meaning only in terms of the system of weighting adopted, and one may produce many numerically different but quite valid indices for comparing two periods. The weighting system used in P_{02} (Laspeyres) is the same as that in P_{01} (Laspeyres), but different from in P_{12} (Laspeyres). Consequently, the increase in M is an increase in something different from that in which N is the increase. The product MN is, therefore, a mixture, the exact meaning of which is not clear and which could not be expected to equal a direct comparison between periods 0 and 2.

Illustration 6. Show with the help of the following data that the Time and Factor Reversal Tests are satisfied by Fisher's Ideal Formula for index number construction.

Commodity	*Base Year Price (Rs.)*	*Base Year Quantity (kg.)*	*Current Year Price (Rs.)*	*Current Year Quantity (kg.)*
A	6	50	10	56
B	2	100	2	120
C	4	60	6	61
D	8.5	30	12	24
E	8	40	16	22

Solution : COMPUTATIONS FOR TIME REVERSAL TEST AND FACTOR REVERSAL TEST

Commodity	Base year price (Rs.) p_0	Base year quantity (kg.) q_0	Current year price (Rs.) p_1	Current year quantity (kg.) q_1	p_1q_0	p_0q_0	p_1q_1	p_0q_1
A	6	50	10	56	500	300	560	336
B	2	100	2	120	200	200	240	240
C	4	60	6	61	360	240	366	244
D	8.5	30	12	24	360	255	288	204
E	8	40	16	22	640	320	352	176
					$\Sigma p_1q_0 = 2{,}060$	$\Sigma p_0q_0 = 1{,}315$	$\Sigma p_1q_1 = 1{,}806$	$\Sigma p_0q_1 = 1{,}200$

Time reversal test is satisfied when : $P_{01} \times P_{10} = 1$

$$P_{01} = \sqrt{\frac{\Sigma p_1q_0}{\Sigma p_0q_0} \times \frac{\Sigma p_1q_1}{\Sigma p_0q_1}} \text{ and } P_{10} = \sqrt{\frac{\Sigma p_0q_1}{\Sigma p_1q_1} \times \frac{\Sigma p_0q_0}{\Sigma p_1q_0}}$$

Substituting the values $P_{01} \times P_{10} = \sqrt{\frac{1{,}900}{1{,}360} \times \frac{1{,}880}{1{,}344} \times \frac{1{,}344}{1{,}880} \times \frac{1{,}360}{1{,}900}} = \sqrt{1} = 1.$

Hence time reversal test is satisfied.

Factor reversal test is satisfied when : $P_{01} \times Q_{01} = \frac{\Sigma p_1q_1}{\Sigma p_0q_0}$

$$P_{01} = \sqrt{\frac{\Sigma p_1q_0}{\Sigma p_0q_0} \times \frac{\Sigma p_1q_1}{\Sigma p_0q_1}} \text{ and } Q_{01} = \sqrt{\frac{\Sigma q_1p_0}{\Sigma q_0p_0} \times \frac{\Sigma q_1p_1}{\Sigma q_0p_1}}$$

Hence,

$$P_{01} \times Q_{01} = \sqrt{\frac{1{,}900}{1{,}360} \times \frac{1{,}880}{1{,}344} \times \frac{1{,}344}{1{,}360} \times \frac{1{,}880}{1{,}900}}$$

$$= \frac{1{,}880}{1{,}360} \text{ which is also the value of } \frac{\Sigma p_1q_1}{\Sigma p_0q_0}$$

Hence Fisher's Ideal Index satisfies the factor reversal test.

THE CHAIN INDEX NUMBERS

In the fixed base method discussed so far the base remains the same throughout the series of the index. This method, though convenient, has certain limitations. As time elapses conditions which were once important become less significant and it becomes more difficult to compare accurately present conditions with those of a remote period. New items may have to be included and old ones may have to be deleted in order to make the index more representative. In such cases it may be desirable to use the chain base method. When this method is used the comparisons are not made with a fixed base : rather the base changes from year to year. For example, for 2009, 2008 will be the base ; for 2010, 2009 will be the base and so on. If, however, it is desired to associate these relatives to a common base, the results may be chained to obtain chain indices. Thus in its simplest form, the chain index is one in which the figures for each year (or sub-period thereof) are first expressed as percentages of the preceding year. These percentages are then chained together by successive multiplication to form a chain index.

Steps in Constructing Chain Index

(*i*) Express the figures for each year as percentages of the preceding year. The results so obtained are called link relatives.

(*ii*) Chain together these percentages by successive multiplication to form a chain index. Chain index of any year is the average link relative of that year multiplied by chain index of previous year divided by 100. In the form of formula :

$$\text{Chain Index} = \frac{\text{Current year link relative} \times \text{Previous year chain index}}{100}$$

The link relatives obtained in step (*i*) facilitate comparison from one year to another, *i.e.*, between closely situated periods in which the q's are not likely to have changed much. The chain indices obtained in step (*ii*) by a process of chaining binary comparisons facilitate long-term comparisons.

Chain relatives differ from fixed-base relatives in computation. Chain relatives are computed from link relatives whereas fixed based relatives are computed directly from the original data. The results obtained by the two different methods should be the same, but they may differ from each other slightly due to rounding off of decimal places. Since the process of computing chain relatives is quite complicated and the results are same as the fixed-base relatives obtained from the original data, chain relatives should be used when the original data are not available but the link relatives are.

Illustration 7. From the following data of the wholesale prices of wheat for the ten years, construct index numbers taking 2001 as base. Also find the link relatives.

Year	*Price of wheat (Rs. per 10 kg.)*	*Year*	*Price of wheat (Rs. per 10 kg.)*
2001	50	2006	78
2002	60	2007	82
2003	62	2008	84
2004	65	2009	88
2005	70	2010	90

Solution : CONSTRUCTION OF INDEX NUMBERS TAKING 2001 AS BASE

Year	*Price of wheat*	*Index Number (2001 = 100)*	*Year*	*Price of wheat*	*Index Number (2001 = 100)*
2001	50	100	2006	78	$\frac{78}{50} \times 100 = 156$
2002	60	$\frac{60}{50} \times 100 = 120$	2007	82	$\frac{82}{50} \times 100 = 164$
2003	62	$\frac{62}{50} \times 100 = 124$	2008	84	$\frac{84}{50} \times 100 = 168$
2004	65	$\frac{65}{50} \times 100 = 130$	2009	88	$\frac{88}{50} \times 100 = 176$
2005	70	$\frac{70}{50} \times 100 = 140$	2010	90	$\frac{90}{50} \times 100 = 180$

This means that from 2001 to 2002 there was a 20 per cent increase; from 2001 to 2003 there was 24 per cent increase. If we are interested in finding out increase from 2001 to 2002, from 2002 to 2003, from 2003 to 2004, we shall have to compute the chain indices.

CALCULATION OF LINK RELATIVES

Year	*Price of wheat*	*Link relatives*
2001	50	100.0
2002	60	$\frac{60}{50} \times 100 = 120.0$
2003	62	$\frac{62}{60} \times 100 = 103.3$
2004	65	$\frac{65}{62} \times 100 = 104.8$
2005	70	$\frac{70}{65} \times 100 = 107.7$

2006	78	$\frac{78}{70} \times 100 = 111.4$
2007	82	$\frac{82}{78} \times 100 = 105.1$
2008	84	$\frac{84}{82} \times 100 = 102.4$
2009	88	$\frac{88}{84} \times 100 = 104.8$
2010	90	$\frac{90}{88} \times 100 = 102.3$

Illustration 8. Calculate the fixed base index number and chain base index numbers from the following data. Are the two results same? If not, why ?

Commodity	*Price (in Rs. thousand)*				
	2006	*2007*	*2008*	*2009*	*2010*
I	2	3	5	7	8
II	8	10	12	14	18
III	4	5	7	9	12

Solution. Since base year is not specified the first year in order of time, *i.e.*, 2006 is taken as base. As no weights are given the appropriate method for calculating fixed base numbers is the price relatives method.

FIXED BASE INDEX NUMBERS

Commodity	*Price (in Rs. thousand)*				
	2006	*2007*	*2008*	*2009*	*2010*
I	100	$\frac{3}{2} \times 100 = 150$	$\frac{5}{2} \times 100 = 250$	$\frac{7}{2} \times 100 = 350$	$\frac{8}{2} \times 100 = 400$
II	100	$\frac{10}{8} \times 100 = 125$	$\frac{12}{8} \times 100 = 150$	$\frac{14}{8} \times 100 = 175$	$\frac{18}{8} \times 100 = 225$
III	100	$\frac{5}{4} \times 100 = 125$	$\frac{7}{4} \times 100 = 175$	$\frac{9}{4} \times 100 = 225$	$\frac{12}{4} \times 100 = 300$
Total	300	400	575	750	925
Average, *i.e.*, fixed base I.No.	100	133.3	191.7	250.0	308.3

CHAIN BASE INDEX NUMBERS CHAINED TO 2006

Commodity	*Percentage based on preceding year*				
	2006	*2007*	*2008*	*2009*	*2010*
I	100	150	166.7	140.00	114.3
II	100	125	120.0	116.67	128.6
III	100	125	140.0	128.60	133.3
Total of Link Relatives	300	400	426.7	385.3	376.2
Average	100	133.33	142.23	128.43	125.40
Chain indices	100	133.33	189.64	240.55	305.41

On comparison we find that except for first two years, the series of index numbers obtained by fixed base and chain base method are different. It is because when fixed base and chain base index numbers are computed by combining two or more series chain index numbers will be usually different from fixed base index number except for the first two given years.

Conversion of Chain Index to Fixed Base Index

At times, it may be desired to convert chain base index numbers (C.B.I.) into two fixed base index numbers (F.B.I). The following formula is used for this purpose :

$$\text{Current years' F.B.I.} = \frac{\text{Current year's C.B.I.} \times \text{Previous year's F.B.I.}}{100}$$

The following example shall illustrate the procedure :

Illustration 9. From the chain base index numbers given below prepare fixed base index numbers :

Year	2005	2006	2007	2008	2009	2010
Chain Base Index	80	105	102	95	110	120

Solution. CONVERTING CHAIN BASE INDICES TO FIXED BASE INDICES

Year	*Chain Base Index*	*Conversion*	*Fixed Base Index*
2005	80	—	80.0
2006	105	$\frac{105 \times 80}{100}$	84.0
2007	102	$\frac{102 \times 84}{100}$	85.7
2008	95	$\frac{95 \times 85.7}{100}$	81.4
2009	110	$\frac{110 \times 81.4}{100}$	89.5
2010	120	$\frac{120 \times 89.5}{100}$	107.4

Merits and Demerits of the Chain Base Method

Merits. 1. The chain base method has a great significance in practice because in business data we are more often concerned with making comparisons with the previous period and not with any distant past. The link relatives obtained by chain base method serve this purpose.

2. Chain base method permits the introduction of new commodities and the deletion of old ones without necessitating either the recalculation of entire series or other drastic changes. Thus account may readily be taken of basic changes in production, distribution and consumption habits, changes in quality, etc. Because of this flexibility chain index is used in many types of indices such as the consumer price index and the wholesale price index.

3. Weights can be adjusted as frequently as possible. This flexibility is of great significance in many types of index numbers.

4. Index numbers calculated by the chain base method are free to a greater extent from seasonal variations than those obtained by the other method.

However, one *drawback* of the chain index is that while the percentage of previous year figures give accurate comparisons of year-to-year changes, the long-range comparisons of chained percentages are not strictly valid. However, when the index number user wishes to make year-to-year comparisons, as is so often done by the businessman, the percentages of the preceding year provide a flexible and useful tool.

BASE SHIFTING, SPLICING AND DEFLATING THE INDEX NUMBERS

Base Shifting

One of the most frequent operation necessary in the use of index number is changing the base of an index. Such a change is usually referred to as shifting the base. There may be two reasons for this :

1. The previous base has become too old and is almost useless for purposes of comparison. In practice, it is desirable that the base period chosen for comparison purposes be a period of economic stability which is not too far distant in the past.

2. Comparison is to be made with another series of index numbers having a different base. For example, the consumer price index for a certain region is available with 2004 as base (*i.e.*, 2004 = 100). Now suppose an investigator wants to compare cost of living changes in the community with those of another region for which the corresponding index is given with the base year 2010. In such a case, it shall be necessary to shift the base of the first series from 2004 to 2010.

When base period is to be changed, one possibility is to recompute all index numbers using the new base period. A simpler approximate method is to divide all index numbers for the various years corresponding to the old base period by the index number corresponding to the new base period, expressing the results as percentages. These results represent the new index numbers, the index number for the new base period being 100.

Mathematically speaking, this method is strictly applicable only if the index numbers satisfy the circular test. However, for many types of index numbers the method, fortunately, yields results which in practice are close enough to those which would be obtained theoretically.

Illustration 10. The following index numbers of prices (2001 = 100) are given :

Year	*Index*	*Year*	*Index*
2001	100	2006	410
2002	110	2007	400
2003	120	2008	380
2004	200	2009	370
2005	400	2010	340

Shift the base from 2001 to 2010 and recast the index numbers.

Solution.

INDEX NUMBERS WITH 2001 AS BASE (2001 = 100)

Year	*Index Numbers (2001 = 100)*	*Index Numbers (2007 = 100)*	*Year*	*Index Numbers (2001 = 100)*	*Index Numbers (2007= 100)*
2001	100	$\frac{100}{400} \times 100 =.25.0$	2006	410	$\frac{410}{400} \times 100 = 102.5$
2002	110	$\frac{110}{400} \times 100 = 27.5$	2007	400	$\frac{400}{400} \times 100 = 100.00$
2003	120	$\frac{120}{400} \times 100 = 30.0$	2008	380	$\frac{380}{400} \times 100 = 95.00$
2004	200	$\frac{200}{400} \times 100 = 50.0$	2009	370	$\frac{370}{400} \times 100 = 92.50$
2005	400	$\frac{400}{400} \times 100 = 100.0$	2010	340	$\frac{340}{400} \times 100 = 85.00$

The new series with 2007 as base is obtained easily by dividing each entry of the first column by 400, *i.e.*, the index for 2007 and multiplying the ratio by 100.

Thus

$$\text{Index number for 2001} = \frac{\text{Index number for 2001}}{\text{Index number for 2007}} \times 100 = \frac{100}{400} \times 100 = 25.0$$

$$\text{Index number for 2002} = \frac{\text{Index number for 2002}}{\text{Index number for 2007}} \times 100 = \frac{110}{400} \times 100 = 27.5$$

In a similar manner other indices can also be obtained.

It should be carefully noted that the above method of shifting the base will not necessarily coincide with the method in which we start a new with the original data and recompute the whole series with the new base. It all depends on how the index is constructed and what weights are being used. Nevertheless, since it is sometimes impossible to do otherwise in practice, the simple method illustrated above is often employed regardless of whether a complete recomputation of the index would produce the identical results.

Splicing

The problem of combining two or more overlapping series of index numbers into one continuous series is called splicing. The need for splicing arises for securing continuity in comparison. It happens quite often that an index is discontinued because its base has become too old. A new index may be started with same items and some recent year as base. If it is desired to connect the new index number with that of one discontinued the second number would be spliced to the first one with the result that the index would enable comparison with the old base. The process of splicing is very simple and akin to that used in shifting the base as can be seen from the following illustration.

Illustration 11. The index A given was started in 2001 and continued up to 2006, in which year another index *B* was started. Splice the index *B* to index *A* so that a continuous series of index number from 2001 up to-date may be available.

Year	*Index A*	*Index B*	*Year*	*Index A*	*Index B*
2001	100		2006	138	
2002	110		2007	150	100
2003	112		2008		120
—			2009		140
—			2010		130

Solution. INDEX B SPLICED TO INDEX A

Year	*Index A*	*Index B*	*Index B spliced to Index A 2001 as base*
2001	100		
2002	110		
2003	112		
—			
—			
2006	138		
2007	150	100	$\frac{150}{100} \times 100 = 150$
2008		120	$\frac{150}{100} \times 120 = 180$
2009		140	$\frac{150}{100} \times 140 = 210$
2010		130	$\frac{150}{100} \times 130 = 195$

The spliced index now refers to 2001 as base and we can make a continuous comparison of index numbers from 2001 onwards.

In the above case, it is also possible to splice the new index in such a manner that a comparison could be made with 2001 as base. This would be done by multiplying the old index by the ratio $\frac{100}{150}$. Thus, the spliced index for 2001 would be $\frac{100}{150} \times 100 = 66.7$, for 2002, $\frac{110}{150} \times 100 = 73.3$ for 2003, $\frac{112}{150} \times 100 = 74.7$, etc. This process appears to be more useful because a recent year can be kept as a base. However, much would depend upon the object.

It shall be clear from above that splicing is very useful for enabling comparisons between new and old index numbers. However, it should be noted that splicing can give accurate results only where geometric mean has been used in constructing the index numbers becuase in such a case index numbers are reversible. However, because of difficulties of computation, the geometric mean is not very often used in constructing index numbers.

Use of Index Numbers in Deflating

By deflating we mean making allowances for the effect of changing price levels. A rise in price level means a reduction in the purchasing power of money. To take the case of a single commodity, suppose the price of wheat rises from Rs. 1500 per quintal in 2005 to Rs. 3000 per quintal in 2010, it means that in 2010 one can buy 50 kg. of wheat for Rs. 1500 which he was spending on wheat in 2005 or, in other words, the value of rupee is only 50 paise in 2010 as compared to 2005. Thus, the value (or purchasing power) of a rupee is simply the reciprocal of an appropriate price index written as proportion. If prices increase by 60% the price index is 1.60 and what a rupee will buy is only 1/1.60 or 5/8 of what it used to buy. In other words, the purchasing power of the rupee is 5/8 of what it was or approximately 63 paise. Similarly, if prices increase by 25 per cent the price index is 1.25 (125 per cent), and the purchasing power of the rupee is 1/1.25 = 0.80 = 80 paise.

It shall be clear from above that since the value of money goes down with the rising price the workers or the salaried people are interested not so much in money wages as in real wages, *i.e.*, not how much they earn but how much their income or wage will buy.

For calculating real wages we can multiply money wages by a quantity measuring the purchasing power of the rupee, or better we divide the cash wages by an appropriate price index. This process is referred to as deflating. In principle, it appears to be very simple but in practice the main difficulty consists in finding appropriate index to deflate a given set of values or appropriate deflators. The process of deflating can be expressed in the form of formula as:

$$\text{Real Wage} = \frac{\text{Money Wage}}{\text{Price Index}} \times 100$$

$$\text{Real Wage Index No.} = \frac{\text{Index of Money Wage}}{\text{Price Index}}$$

Illustration 12. Following table gives the weekly wages of workers together with the Price Index Numbers. Compute the Index numbers of real income and interpret them.

Year	*Weekly Wages (inRs.)*	*Price Index*	*Year*	*Weekly Wages (in Rs.)*	*Price Index*
2004	300	100	2008	480	350
2005	340	160	2009	570	420
2006	450	280	2010	575	430
2007	460	290			

Solution :

INDEX NUMBER OF REAL INCOME

Year	*Weekly Wages (in Rs.)*	*Price Index*	*Real Wages*	*Real Wage Indices (2004 = 100)*
2004	300	100	$\frac{300}{100} \times 100 = 300.00$	100.00
2005	340	160	$\frac{340}{160} \times 100 = 212.50$	70.83
2006	450	280	$\frac{450}{280} \times 100 = 160.71$	53.57

2007	460	290	$\frac{460}{290} \times 100 = 158.62$	52.87
2008	480	350	$\frac{480}{350} \times 100 = 137.14$	45.71
2009	570	420	$\frac{570}{420} \times 100 = 135.71$	45.24
2010	575	430	$\frac{575}{430} \times 100 = 133.72$	44.57

The index number of real wages has fallen from 100 in 2004 to 44.57 in 2010. In other words, despite the fact that the weekly wage has increased from Rs. 300 in 2004 to Rs. 575 in 2010, the workers are not better off.

The method discussed above is frequently used to deflate individual values, value series or value indices. Its special use is in problems dealing with such diversified things as rupee sales, rupee inventories of manufacturers, wholesalers and retailers' incomes, wages and the like.

CONSUMER PRICE INDEX NUMBERS

Meaning and Need

The consumer price index numbers, also known as cost of living index numbers, are generally intended to represent the average change over time in the prices paid by the ultimate consumer of a specified basket of goods and services. The need for constructing consumer price indices arises because the general index numbers fail to give an exact idea of the effect of the change in the general price level on the cost of living of different classes of people, since a given change in the level of prices affects different classes of people in different manners. Different classes of people consume different types of commodities and even these same type of commodities are not consumed in the same proportion by different classes of people. For example, the consumption pattern of rich, poor and middle class people varies widely. Not only this, the consumption habits of the people of the same class differ from place to place. For example, the mode of expenditure of a lower division clerk living in Delhi may differ widely from that of another clerk of the same category living in, say, Chennai. The consumer price index helps us in determining the effect of rise and fall in prices of different classes of consumers living in different areas. The construction of such an index is of great significance because very often the demand for a higher wage is based on the cost of living index and the wages and salaries in most countries are adjusted in accordance with the consumer price index.

It should be carefully noted that the cost of living index does not measure the actual cost of living nor the fluctuations in the cost of living due to causes other than the change in the price level; its objects is to find out how much the consumers of a particular class have to pay more for a certain basketful of goods and services in a given period compared to the base period. To bring out clearly this fact, the Sixth International Conference of Labour Statisticians recommended that the term 'cost of living index' should be replaced in appropriate circumstances by the terms '*price of living index*', '*cost of living price index*'. At present, the three terms, namely, cost of living index, consumer price index and retail price index, are used in different countries with practically no difference in their connotation.

It should be clearly understood at the very outset that two different indices representing two different geographical areas cannot be used to compare actual living costs of the two areas. A higher index for one area than for another with same period is no indication that living costs are higher in the one than in the other. All it means is that as compared with the base period, prices have risen in one area than in another. But actual costs depend not only on the rise in prices as compared with the base period, but also on the actual cost of living for the base period which will vary for different regions and for different classes of population.

Utility of the Consumer Price Indices

The Consumer Price Indices are of great significance as can be seen from the following :

(1) The most common use of these indices is in wage negotiations and wage contracts. Automatic adjustments of wage or dearness allowance component of wages are governed in many countries by such indices.

(2) At Government level, the index numbers are used for wage policy, price policy, rent control, taxation and general economic policies.

(3) The index numbers are also used to measure changing purchasing power of the currency, real income, etc.

(4) Index numbers are also used for analysing markets for particular kinds of goods and services.

Construction of a Consumer Price Index

The following are the steps in constructing a consumer price index :

(1) *Decision about the class of people for whom the index is meant.* It is absolutely essential to decide clearly the class of people for whom the index is meant, *i.e.,* whether it relates to industrial workers, teachers, officers, etc. The scope of the index must be clearly defined. For example, when we talk of teachers we are referring to primary teachers, middle class teachers, etc., or to all the teachers taken together. Along with the class of people it is also necessary to decide the geographical area covered by the index. Thus in the example taken above it is to be decided whether all the teachers living in Delhi are to be included or those living in a particular locality of Delhi, say, Chandni Chowk area, Karol Bagh, etc.

(2) *Conducting family budget enquiry.* Once the scope of the index is clearly defined the next step is to conduct a family budget enquiry covering the population group for whom the index is to be designed. The object of conducting a family budget enquiry is to determine the amount that an average family of the group included in the index spends on different items of consumption. While conducting such an enquiry, therefore, the quantities of commodities consumed and their prices are taken into account. The consumption pattern can thus be easily ascertained. It is necessary that the family budget enquiry amongst the class of people to whom the index series is applicable should be conducted during the base period. The Sixth International Conference of Labour Statisticians held in Geneva in 1946 suggested that the period of enquiry of the family budget and the base period should be identical as far as possible.

The enquiry is conducted on a random basis. By applying lottery method some families are selected from the total number and their family budgets are scrutinized in detail. The items on which the money is spent are classified into certain well accepted groups, namely :

(*i*) Food,

(*ii*) Clothing,

(*iii*) Fuel and Lighting,

(*iv*) House Rent, and

(*v*) Miscellaneous.

Each of these groups is further divided into sub-groups. For example, the broad group ‘food’ may be divided into wheat, rice, pulses, sugar, etc. The commodities included are those which are generally consumed by people for whom the index is meant. Through family budget enquiry an average budget is prepared which is the standard budget for that class of people. While constructing the index only such commodities should be included as are not subject to wide variations in quality or to wide seasonal alternations in supply and for which regular and comparable quotations of prices can be obtained.

(3) *Obtaining price quotations.* The collection of retail prices is a very important and, at the same time, very tedious and difficult task because such prices may vary from place to place, shop to shop and person to person. Price quotations should be obtained from the localities in which the class of people concerned reside or from where they usually make their purchases. Some of the principles recommended to be observed in the collection of retail price data required for purposes of construction of cost of living indices are described below :

(*a*) The retail price should relate to a fixed list of items and for each item, the quality should be fixed by means of suitable specifications.

(*b*) Retail prices should be those actually charged to consumer for cash sales.

(*c*) Discount should be taken into account if it is automatically given to all customers.

(*d*) In a period of price control or rationing, where illegal prices are charged openly, such prices should be taken into account along with the control prices.

The most difficult problem in practice is to follow principle (*a*), *i.e.,* the problem of keeping the weights assigned and qualities of the basket of goods and services constant with a view to ensuring that only the effect of price change is measured. To conform to uniform qualities, the accepted method is to draw up detailed descriptions or specifications of the items priced for the use of persons furnishing or collecting the price quotations.

Since prices form the most important component of cost of living indices considerable attention has to be paid to the methods of price collection and to the price collection personnel. Prices are collected usually by special agents or through mailed questionnaire or in some cases through published price lists. The greatest reliance can be placed on the price collection through special agents as they visit the selected retail outlets and collect the prices from them. However, these agents should be properly selected and trained and should be given a manual of instructions as well as manual of specifications of items to be priced. Appropriate methods of price verification should be followed such as '*check pricing*' in which price quotations are verified by means of duplicate prices obtained by different agents or '*purchase checking*' in which actual purchases of goods are made.

After quotations have been collected from all retail outlets, an average price for each of the items included in the index has to be worked out. Such averages are first calculated for the base period of the index and later every month if the index is maintained on a monthly basis. The month of averaging the quotations should be such as to yield unbiased estimates of average prices as being paid by the group as a whole. This, of course, will depend upon the method of selection of retail outlets and also the scope of the index.

In order to convert the prices into index numbers the prices or their relatives must be weighted. The need for weighting arises because the relative importance of various items for different classes of people is not the same. For this reason, the cost of living index is always a weighted index. While conducting the family budget enquiry the amount spent on each commodity by an average family is decided and these constitute the weights. Percentages of expenditure on the different items constitute the 'individual *weights' allocated to the corresponding price relative and the percentage expenditure on the five groups* constitute the 'group weight'.

Methods of Constructing the Index

After the above mentioned problems are carefully decided the index may be constructed by applying any of the following methods :

(1) Aggregate Expenditure method or Aggregative methods, and

(2) Family Budget method or the method of Weighted Relatives.

1. Aggregate Expenditure Method. When this method is applied, the quantities of commodities consumed by the particular group in the base year are estimated which constitute the weights. The prices of commodities for various groups for the current year are multiplied by the quantities consumed in the base year and the aggregate expenditure incurred in buying those commodities is obtained. In a similar manner, the prices of the base year are multiplied by quantities of the base year and aggregate expenditure for the base period is obtained. The aggregate expenditure of the current year is divided by the aggregate expenditure of the base year and the quotient is multiplied by 100. Symbolically,

$$\text{Consumer Price Index} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100.$$

This is in fact the Laspeyres method discussed earlier. This method is the most popular method for constructing consumer price index.

2. Family Budget Method. When this method is applied the family budgets of a large number of people for whom the index is meant are carefully studied and the aggregate expenditure of an average family on various items is estimated. These constitute the weights. The weights are thus the value weights obtained by multiplying the price by quantities consumed (*i.e.*, p_0q_0). The price relatives for each commodity are obtained and these price relatives are multiplied by value weights for each item and the product is divided by the sum of the weights. Symbolically,

$$\text{Consumer Price Index} = \frac{\Sigma PV}{V}$$

where,

$$P = \frac{p_1}{p_0} \times 100 \text{ for each item.}$$

V = Value weights, *i.e.*, p_0q_0.

This method is the same as the weighted average of price relatives method discussed earlier.

This should be noted that the answer obtained by applying the aggregate expenditure method and the family budget method shall be the same.*

Illustration 13. Prices per unit of the items forming consumption bundle of an average middle class family in two periods and percentage of total family budget allocated to those items are given in the following table :

	Food	*Rent*	*Clothing*	*Fuel*	*Misc.*
Per cent expenditure	35	15	20	10	20
Price (Rs.) in period 0	1500	500	1000	200	600
Price (Rs.) in period 1	1740	600	1250	250	900

Compute an appropriate index number and comment on the result.

Solution. The appropriate index number here would be the Consumer Price Index number.

CONSTRUCTION OF CONSUMER PRICE INDEX NUMBER

Index of Expenditure	p_0	p_1	$\frac{p_1}{p_0} \times 100$ P	W	PW
Food	1500	1740	116	35	4,060
Rent	500	600	120	15	1,800
Clothing	1000	1250	125	20	2,500
Fuel	200	250	125	10	1,250
Misc.	600	900	150	20	3,000
				$\Sigma W = 100$	$\Sigma PW = 12{,}610$

$$\text{Consumer Price Index} = \frac{\Sigma PW}{\Sigma W} = \frac{12610}{100} = 126.1$$

*The denominator and numerator in both methods are the same as can be seen from the following : $\Sigma p_1 q_0$ of the Laspeyers' method is the same as ΣPV of the family budget method.

$$PV = (p_1\, q_0) \times p_0 q_0 \text{ which is nothing but } p_1 q_0.$$

The denominator in both methods is also the same.

$$\Sigma p_0 q_0 = \Sigma V$$

Thus there has been an increase of 26.1 per cent in the Consumer Price Index in the current year.

Illustration 14. Construct a consumer price index number from the table given below :

Group	*Index for 2003*	*Expenditure*
1. Food	550	46%
2. Clothing	215	10%
3. Fuel and Lighting	220	7%
4. House Rent	150	12%
5. Miscellaneous	275	25%

Solution : CONSTRUCTION OF CONSUMER PRICE INDEX NUMBER

Group	*Index number* *I*	*Expenditure* *V*	*IV*
Food	550	46	25,300
Clothing	215	10	2,150
Fuel and Lighting	220	7	1,540
House Rent	150	12	1,800
Miscellaneous	275	25	6,875
		$\Sigma V = 100$	$\Sigma IV = 37{,}665$

$$\text{Consumer Price Index} = \frac{\Sigma IV}{\Sigma V} = \frac{37665}{100} = 376.65.$$

Precautions while Using Consumer Price Index

Quite often consumer price indices are misinterpreted. Hence while using these indices the following points should be kept in mind :

1. As pointed out earlier, the consumer price index measure changes in the retail prices only in the given period compared to base period—it does not tell us anything about variation in the living standards at two different places. Thus if the cost of living index for working class for Mumbai is 175 and for Delhi 150 for the same period and for the same class of people it does not necessarily mean that living costs are higher in Mumbai compared to Delhi.

2. While constructing the index it is assumed that the quantities of the base year are constant and hold good for current year also. But this assumption does not appear to be very logical because the pattern of consumption goes on changing with change in fashion, introduction of new commodities in the market, etc. It is desirable, therefore, that while constructing the index the current quantities are taken into account. But this is a difficult task. The Sixth International Conference of Labour Statisticians recommended that the pattern of consumption should be examined and the weights adjusted, if necessary, at intervals of not more than ten years to correspond changes in the consumption pattern. The index also does not take into account changes in qualities. Unlike changes in consumption pattern changes in qualities of goods and services are more frequent and when a marked change in the quality of items occurs appropriate adjustments should be made to ensure that the index takes into account change in qualities also. But in practice it is a difficult proposition to follow, and, therefore, constant qualities are assumed at two different dates which again is a shaky assumption.

3. Like any other index the consumer price index is based on a sample. While constructing the index sampling is used at every stage—in the selection of commodities, in obtaining price quotations, selecting families for family budget enquiry, etc. The accuracy of index thus hinges upon the use of sampling methods. The consumption pattern derived upon the use of sampling methods. The consumption pattern derived from the expenditure data of a sample of households covered in

the course of family budget enquiry has to be representative of all the items in the average budget, the localities from which price data are collected have to be representative of all localities from which the population group makes purchases, the retail outlets from which prices are collected have to be representative of all the retail outlets patronised by the population group, etc. However, it is often difficult to ensure perfect representativeness and in the absence of this the index may fail to provide the real picture.

INDEX NUMBER OF INDUSTRIAL PRODUCTION

The index number of industrial production is designed to measure increase or decrease in the level of industrial production in a given period compared to some base period. It should be noted that such an index measures change in the *quantum* of production and not in *values*. For constructing such an index it is necessary to obtain data about the level of industrial output in the base period and the given period. Usually data about production are collected under the following heads :

1. Textile Industries—cotton, woollen, silk, etc.
2. Mining Industries—iron ore, coal, copper, petroleum, etc.
3. Metallurgical Industries—iron and steel, etc.
4. Mechanical Industries—locomotives, ships, aeroplanes, etc.
5. Industries subject to excise duties—sugar, tobacco, match, etc.
6. Miscellaneous—glass, soap, chemical, cement, etc.

The figures of output for the various industries classified above are obtained on a monthly, quarterly or yearly basis. Weights are assigned to various industries on the basis of some criteria such as capital invested, turnover, net output, production, etc. Usually the weights in the index are based on the values of net output of different industries. The index of industrial production is obtained by taking the simple arithmetic mean or geometric mean of the relatives. When simple arithmetic mean is used, the formula for constructing the index becomes :

$$\text{Index of industrial production} = \frac{\Sigma\left(\dfrac{q_1}{q_0}\right)W}{\Sigma W}$$

where q_1 = Quantity produced in the given period

q_0 = Quantity produced in the base period

W = Relative importance of different items.

For determining the relative share of an individual output to total output the concept of value added is most commonly used.

Illustration 15. Construct the index number of business activity in India from the following data :

Item	*Weightage*	*Index*
Industrial production	36	250
Mineral production	7	135
Internal trade	24	200
Financial activity	20	135
Exports and imports	7	325
Shipping activity	6	300

Solution. CONSTRUCTION OF INDEX NUMBER OF BUSINESS ACTIVITY

Item	*Weightage* W	*Index* I	IW
Industrial production	36	250	9,000
Mineral production	7	135	945
Internal trade	24	200	4,800
Financial activity	20	135	2,700
Exports and imports	7	325	2,275
Shipping activity	6	300	1,800
	$\Sigma W = 100$		$\Sigma IW = 21{,}520$

$$\text{Index No. of Business Activity} = \frac{\Sigma IW}{\Sigma W} = \frac{21{,}520}{100} = 215.2.$$

Limitations of Index Numbers

Though the index numbers are of great significance, the reader must also be aware of their limitations so that he avoids errors of interpretation. The chief limitations of index numbers are :

1. Since index numbers are generally based on a sample, it is not possible to take into account each and every item in the construction of the index.

2. While taking the sample, random sampling is seldom used. This is so because to take sample from thousands of commodities and services, the random procedure could be neither practical nor representative. Typically, indices are constructed from samples deliberately selected. This introduces errors and every effort must be made to minimise these errors.

3. It is often difficult to take into account changes in the quality of products. With the passage of time, tastes and habits of people also change with the result that very often old commodities go out of use and new commodities are introduced. In a really typical index, qualities of commodities should remain the same over a period of time because differences in quality would mean differences in prices also. But very often it is not practicable and it makes comparisons over long period less reliable.

4. A large number of methods are designed for constructing index numbers and different methods of computation give different results. Very often the selection of an appropriate formula creates problem and in the interest of comparability, it is necessary to ensure that the same formula is adopted over a period of time for constructing a particular index. There is no such method of constructing index number which is best from every point of view. Index numbers are specialised averages and are subject to the same limitations as that of average.

5. Just like other statistical tools, index numbers can also be manipulated in such a manner as to draw the desired conclusions. Choosing a freak year is a favourite trick of those who use statistics to mislead. A dishonest capitalist could choose a record year of profits as base and so 'prove' subsequent profits to be pitifully low. Similarly, in order to prove that the current prices are intolerably high dishonest trade unionist may choose a year of exceptionally low prices as base.

6. Since in the construction of index numbers a large number of factual questions are involved, lack of adequate and accurate data in most cases becomes a serious limitation of the index itself. In most cases one cannot collect the data himself and therefore one has to rely on published sources. Ordinarily we draw upon many sources of data which are geographically dispersed. Problems of comparability and reliability thus multiply and the chances of spurious results are increased. One mistake may 'bias' the index such as including the price of one commodity for one time period, or the price of a slightly different commodity for another period or taking the manufacturer's price at one time and wholesale price at another time.

MISCELLANEOUS ILLUSTRATIONS

Illustration 16. Compute Laspeyres', Paasche's and Fisher's price index number for 2010, using the following data concerning three commodities :

Commodity	*2009*		*2010*	
	Price (Rs.)	*Quantity (kg.)*	*Price (Rs.)*	*Quantity (kg.)*
A	15	15	22	12
B	20	5	27	4
C	4	10	7	5

Solution : CALCULATION OF VARIOUS INDICES

Commodity	p_0	q_0	p_1	q_1	p_1q_0	p_0q_0	p_1q_1	p_0q_1
A	15	15	22	12	330	225	264	180
B	20	5	27	4	135	100	108	80
C	4	10	7	5	70	40	35	20
					$\Sigma p_1q_0 = 535$	$\Sigma p_0q_0 = 365$	$\Sigma p_1q_1 = 407$	$\Sigma p_0q_1 = 280$

Laspeyres' Index : $P_{01} = \dfrac{\Sigma p_1q_1}{\Sigma p_0q_0} \times 100 = \dfrac{535}{365} \times 100 = 146.58$

Paasche's Index : $P_{01} = \dfrac{\Sigma p_1q_1}{\Sigma p_0q_1} \times 100 = \dfrac{407}{280} \times 100 = 145.36$

Fisher's Index : $\sqrt{L \times P} = \sqrt{146.58 \times 145.36} = 145.97$

Illustration 17. Calculate the index from the following data using Fisher's Ideal formula :

Commodity	*2009 Base Year*		*2010 Current Year*	
	Price	*Quantity*	*Price*	*Quantity*
A	10	50	12	60
B	8	30	9	32
C	5	35	7	40

Solution. COMPUTATION OF FISHER'S IDEAL INDEX

Commodity	*2009 Base Year*		*2010 Current Year*		p_0q_0	p_1q_0	p_0q_1	p_1q_1
	Price p_0	*Quantity* q_0	*Price* p_1	*Quantity* q_1				
A	10	50	12	60	500	600	600	720
B	8	30	9	32	240	270	256	288
C	5	35	7	40	175	245	200	280
				Total	915	1,115	1,056	1,288

Using Fisher's ideal index formula,

$$P_{01} = \sqrt{\frac{\Sigma p_1q_0}{\Sigma p_0q_0} \times \frac{\Sigma p_1q_1}{\Sigma p_0q_1}} \times 100 = \sqrt{\frac{1115}{915} \times \frac{1288}{1056}} \times 100$$

$$= \sqrt{1.4863} \times 100 = 1.2191 \times 100 = 121.91.$$

Illustration 18. The following table gives the weekly wages (in Rs.) of a worker and the general index number of prices during 2002-2010. Prepare the index number to show the changes in the real weekly wages of the worker.

Year	*Weekly Wages (Rs.)*	*Price Index No.*	*Year*	*Weekly Wages (Rs.)*	*Price Index No.*
2002	360	100	2007	640	290
2003	420	104	2008	680	300
2004	500	115	2009	720	320
2005	550	160	2010	750	[illegible]
2006	600	280			

Solution : INDEX NUMBER SHOWING CHANGES IN THE REAL WEEKLY WAGES OF THE WORKER

Year	*Weekly Wages (in Rs.)*	*Price Index*	*Real Weekly Wages*	*Real Weekly Wages Indices No.*
2002	360	100	$\frac{360}{100} \times 100 = 360.00$	100.00
2003	420	104	$\frac{420}{104} \times 100 = 403.85$	112.18
2004	500	115	$\frac{500}{115} \times 100 = 434.78$	120.77
2005	550	160	$\frac{550}{160} \times 100 = 343.75$	95.49
2006	600	280	$\frac{600}{280} \times 100 = 214.29$	59.53
2007	640	290	$\frac{640}{290} \times 100 = 220.69$	61.30
2008	680	300	$\frac{680}{300} \times 100 = 226.67$	62.96
2009	720	320	$\frac{720}{320} \times 100 = 225.00$	62.50
2010	750	330	$\frac{750}{330} \times 100 = 227.27$	63.13

Illustration 19. In 2009 for working class people, wheat was selling at an average price of Rs. 160 per 10 kg., cloth at Rs. 40 per metre, house rent Rs. 10,000 per house and other items at Rs. 100 per unit. By 2010 cost of wheat rose by Rs. 40 per 10 kg., house rent by Rs. 1,500 per house and other items doubled in price. The working class cost of living index for the year 2010 (with 2009 as base) was 160. By how much the cloth rose in price during the period 2009-10 ?

Solution. Let the rise in price of cloth be X.

INDEX NUMBER FOR 2010

Commodity	*Price*	*Index No.*	*Price 2010*	*Index No.*
Wheat	160	100	200	$\frac{200}{160} \times 100 = 125$
Cloth	40	100	X	$\frac{X}{40} \times 100 = 2.5X$
House rent	10,000	100	11,500	$\frac{11,500}{10,000} \times 100 = 115$
Miscellaneous	100	100	200	$\frac{200}{100} \times 100 = 200$
				$440 + 2.5X$

The index for 2010 as given is 160. Therefore, the sum of the index number of the four commodities would be $160 \times 4 = 640$.

Hence, $440 + 2.5X = 640$

$2.5X = 200$ or $X = 80$

Hence the rise in the price of cloth was Rs. 40 (80 – 40) per metre.

Illustration 20. Owing to change in prices the consumer price index of the working class in a certain area rose in a month by one quarter of what it was before to 225. The index of food became 252 from 198, that of clothing from 185 to 205, that of fuel and lighting from 175 to 195 and that of miscellaneous from 138 to 212. The index of rent, however, remained unchanged at 150. It was known that the weight of clothing, rent and fuel and lighting were the same. Find out the exact weight of all the groups.

(*MBA, Delhi Univ., 2005*)

Solution. Suppose the weights of the groups are as follows :

Food X.

Fuel and Lighting Z.

Miscellaneous Y.

Rent Z.

Clothing Z.

Therefore, the index weighted index in the beginning of the month would be :

	Index I	Weight W	IW
Food	198	X	$198X$
Clothing	185	Z	$185Z$
Fuel and Lighting	175	Z	$175Z$
Rent	150	Z	$150Z$
Miscellaneous	138	Y	$138Y$
		$X+Y+3Z=$	$198X+138Y+510Z$

$$\therefore \text{ Index number} = \frac{198X+138Y+510Z}{X+Y+3Z}$$

Similarly, the weighted index at the end of the month would be :

	I	W	IW
Food	252	X	$252X$
Clothing	205	Z	$205Z$
Fuel and Lighting	195	Z	$195Z$
Rent	150	Z	$150Z$
Miscellaneous	212	Y	$212Y$
		$X+Y+3Z=$	$252X+212Y+550Z$

$$\text{Index number} = \frac{252X+212Y+550Z}{X+Y+3Z}$$

The weighted index at the end of the month was 225 (given). This index is a rise from the first index by one quarter. Therefore, the index at the beginning was 4/5 of 225=180.

Hence the weighted index at the beginning of the month was

$$180 = \frac{198X+138Y+510Z}{X+Y+3Z}$$

$$180X+180Y+540Z = 198X+138Y+510Z$$

or $$18X-42Y-30Z=0 \qquad ...(i)$$

Similarly, the weighted index at the end of month was

$$225 = \frac{252X+212Y+550Z}{X+Y+3Z}$$

$$225X+225Y+675Z = 252X+212Y+550Z$$

or $$27X-13Y-125Z=0 \qquad ...(ii)$$

Let the total weight be equal to 100.

Hence $$X+Y+3Z=100 \qquad ...(iii)$$

Multiplying Eqn. (*iii*) by 18 and subtracting from (*i*), we get

$$-60Y-84Z=-1800$$

or $$60Y+84Z=1800 \qquad ...(iv)$$

Multiplying (*iii*) by 27, and subtracting from eqn. (*ii*), we get

$$-40Y-206Z=-2700$$

or $$40Y+206Z=2700 \qquad ...(v)$$

Multiplying Eqn. (*iv*) by 20, and Eq. (*v*) by 30, and subtracting, we get

$$-4500Z=-45000 \text{ or } Z=10$$

Substituting the value of Z in Eqn. (*iv*)

$$60Y + (84 \times 10) = 1800$$

$$60Y = 1800 - 840 = 960 \text{ or } Y = 16$$

Substituting the value of Y and Z in Eqn. (*iii*)

$$X + 16 + (3 \times 10) = 100$$

$$\therefore \quad X = 100 - 16 - 30 = 54$$

Thus the exact weights are :

Food	54
Clothing	10
Fuel and Lighting	10
Rent	10
Miscellaneous	16

Illustration 21. The subgroup indices of the consumer price index number for urban non-manual employees of an industrial centre for a particular year (with base 2005 = 100) were :

Food	200
Clothing	130
Fuel and Lighting	120
Rent	150
Miscellaneous	140

The weights are 60, 8, 7, 10 and 15 respectively. It is proposed to fix dearness allowance in such a way as to compensate fully the rise in the prices of food and house rent.

What should be the dearness allowance expressed as a percentage of wage ?

Solution. Let the income of the consumer be 100 rupees. He spent 60 rupees on food and 10 rupees on house rent in 2005. Since the index of food is 200 and the house rent 150 for the particular year for which the data are given, in order to maintain the same consumption standards regarding two items, he will have to spend Rs. 120 on food and Rs. 15 on house rent. Since the weights of other items are constant, in order to maintain the same standard he will have to spend 120 + 8 + 7 + 15 + 5 = Rs. 155. Hence the dearness allowance should be 55 per cent.

Illustration 22. Given the data :

	Commodities A	B
p_0	1	1
q_0	10	5
p_1	2	X
q_1	5	2

where p and q respectively stand for price and quantity and subscripts stand for time period. Find X, if the ratio between Laspeyres' (L) and Paasche's (P) Index number is :

$$L : P = 28 : 27$$

Solution. Calculate Laspeyres' and Paasche's Indices and equate them to the given ratio in order to determine the value of X.

Commodities	p_0	q_0	p_1	q_1	p_1q_0	p_0q_0	p_1q_1	p_0q_1
A	1	10	2	5	20	10	10	5
B	1	5	X	2	$5X$	5	$2X$	2
					$\Sigma p_1q_0 = 20 + 5X$	$\Sigma p_0q_0 = 15$	$\Sigma p_1q_1 = 10 + 2X$	$\Sigma p_0q_1 = 7$

Laspeyres' Index: $$P_{01} = \frac{\Sigma p_1q_0}{\Sigma p_0q_0} = \frac{20+5X}{15}$$

Paasche's Index : $$P_{01} = \frac{\Sigma p_1q_1}{\Sigma p_0q_1} = \frac{10+2X}{7}$$

$$\frac{\dfrac{20+5X}{15}}{\dfrac{10+2X}{7}} = \frac{28}{27} \text{ or } \frac{20+5X}{15} \times \frac{7}{10+2X} = \frac{28}{27}$$

$$\frac{140+35X}{150+30X} = \frac{28}{27}$$

$$4200 + 840X = 3780 + 945X$$

$$105X = 420 \text{ or } X = 4$$

Note : In order to work with the ratio, 100 has been omitted from the formula.

Illustration 23. An increase of 50% in the cost of a certain consumption article raises the cost of living of a certain family by 5%. What percentage of its cost of living was due to buying that article before the change in the price ?

Solution. Let the cost of the article before rise be 'X'. After increase, it, therefore, was $\frac{150X}{100} = 1.5\,X$, *i.e.*, the rise was $1.5X - X = 0.5X$ which is equal to an increase of 5% in the cost of living of which we call 'Y', *i.e.*, 'Y', after the increase it became $\frac{105Y}{100} = 1.05Y$ or, in other words, the increase was $1.05Y - Y = 0.05Y$.

Hence $0.5X = 0.05Y$

$$X = \frac{0.5Y}{0.5} = 0.1Y = 10\% \text{ of } Y$$

Thus the expenditure on that item was 10% of the cost of living.

Illustration 24. Compute index number from the following data using Fisher's ideal index formula :

Commodity	*Base Year*		*Current Year*	
	Qty.	*Price*	*Qty.*	*Price*
A	12	10	15	12
B	15	7	20	5
C	24	5	20	9
D	5	16	5	14

(*MBA, M.D. Univ., 2005*)

Solution : CALCULATION OF FISHER'S IDEAL INDEX

Commodity	*Base year*		*Current year*					
	q_0	p_0	q_1	p_1	p_1q_0	p_0q_0	p_1q_1	p_0q_1
A	12	10	15	12	144	120	180	150
B	15	7	20	5	75	105	100	140
C	24	5	20	9	216	120	180	100
D	5	16	5	14	70	80	70	80
					$\Sigma p_1q_0 = 505$	$\Sigma p_0q_0 = 425$	$\Sigma p_1q_1 = 530$	$\Sigma p_0q_1 = 470$

$$P_{01} = \sqrt{\frac{\Sigma p_1q_0}{\Sigma p_0q_0} \times \frac{\Sigma p_1q_1}{\Sigma p_0q_1}} \times 100$$

$$= \sqrt{\frac{505}{425} \times \frac{530}{470}} \times 100 = \sqrt{1.3399} \times 100 = 1.1576 \times 100 = 115.76$$

Illustration 25. Compute the chain index numbers with 2006 prices as base, from the following table giving the average wholesale prices of the commodities *A*, *B* and *C* for the years 2006 to 2010.

Average wholesale price (Rs.)

Commodity	*2006*	*2007*	*2008*	*2009*	*2010*
A	20	16	28	35	21
B	25	30	24	36	45
C	20	25	30	24	30

Solution :

COMPUTATION OF CHAIN INDICES

Commodity	2006	2007	2008	2009	2010
		Relatives based on preceding year			
A	100	$\frac{16}{20} \times 100 = 80$	$\frac{28}{16} \times 100 = 175$	$\frac{35}{28} \times 100 = 125$	$\frac{21}{35} \times 100 = 60$
B	100	$\frac{30}{25} \times 100 = 120$	$\frac{24}{30} \times 100 = 80$	$\frac{36}{24} \times 100 = 150$	$\frac{45}{36} \times 100 = 125$
C	100	$\frac{25}{20} \times 100 = 125$	$\frac{30}{25} \times 100 = 120$	$\frac{24}{30} \times 100 = 80$	$\frac{30}{24} \times 100 = 125$
Average of link relatives	300	325	375	355	310
Average of relatives	100	108.33	125	118.33	103.33
Chain index	100	$\frac{108.33 \times 100}{100} = 108.33$	$\frac{125 \times 108.33}{100} = 135.41$	$\frac{118.33 \times 135.41}{100} = 160.23$	$\frac{103.33 \times 160.23}{100} = 165.57$

Illustration 26. Construct the cost of living index number from the following group data :

Group	*Weights*	*Group Index No.*
Food	47	247
Fuel & Lighting	7	293
Clothing	8	289
House Rent	13	100
Misc.	14	236

(*MBA, Vikram Univ., 2005*)

Solution.

CALCULATION OF COST OF LIVING INDEX

Group	*Weights* W	*Group Index* I	*IW*
Food	47	247	11609
Fuel and Lighting	7	293	2051
Clothing	8	289	2312
House Rent	13	100	1300
Misc.	14	236	3304
	$\Sigma W = 89$		$\Sigma IW = 20576$

$$\text{Cost of living index} = \frac{\Sigma IW}{\Sigma W} = \frac{20576}{89} = 231.19.$$

Illustration 27. The table below shows the average wages in rupees of a group of industrial workers during the year 1999 to 2010. The consumer price indices for these years with 1999 as base are also shown :

Year	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Average wage of workers	1119	1133	1144	1157	1175	1184	1189	1194	1197	1213	1228	1245
Consumer Price index (1999 = 100)	100	107.6	106.6	107.6	116.2	118.8	119.8	120.2	119.9	121.7	125.9	129.3

(*a*) Determine the real wages of the workers during the years 1999-2010 as compared with their wages in 1999.

(*b*) Determine the purchasing power of the rupee for the year 2010 as compared to the year 1999. What is the significance of this result ?

Solution. (*i*) For finding the real wages we have to divide average wage of workers by the consumer price index.

Year	*Average wage of workers*	*Consumer Price Index (1999 = 100)*	*Real wages*
1999	1119	100	$\frac{1119}{100} \times 100 = 1119$
2000	1133	107.6	$\frac{1133}{107.6} \times 100 = 1053$
2001	1144	106.6	$\frac{1144}{106.6} \times 100 = 1073$
2002	1157	107.6	$\frac{1157}{107.6} \times 100 = 1075$
2003	1175	116.2	$\frac{1175}{116.2} \times 100 = 1011$
2004	1184	118.8	$\frac{1184}{118.8} \times 100 = 997$
2005	1189	119.8	$\frac{1189}{119.8} \times 100 = 992$
2006	1194	120.2	$\frac{1194}{120.2} \times 100 = 993$
2007	1197	119.9	$\frac{1197}{119.9} \times 100 = 998$
2008	1213	121.7	$\frac{1213}{121.7} \times 100 = 997$
2009	1228	125.9	$\frac{1228}{125.9} \times 100 = 975$
2010	1245	129.3	$\frac{1245}{129.3} \times 100 = 963$

If we divide Re. 1 by the price index of 2010, we get the purchasing power of rupee in 1999. Thus the purchasing power of rupee in 2010 shall be $100/129.3 = 0.77$. This means that the purchasing power of rupee has gone down—in 2010 the rupee could buy only 77 per cent of what it could buy in 1999.

Illustration 28. Calculate Laspeyres' and Paasche's price and quantity indices from the data given below :

Commodity	*2009* Price	*2009* Qty.	*2010* Price	*2010* Qty.
A	4	10	5	12
B	6	8	7	10
C	10	5	12	4
D	3	12	4	15
E	5	7	5	8

Solution. CALCULATION OF LASPEYRES' AND PAASCHE'S PRICE AND QUANTITY INDICES

Commodity	2009 p_0	2009 q_0	2010 p_1	2010 q_1	p_1q_0	p_0q_0	p_1q_1	p_0q_1
A	4	10	5	12	50	40	60	48
B	6	8	7	10	56	48	70	60
C	10	5	12	4	60	50	48	40
D	3	12	4	15	48	36	60	45
E	5	7	5	8	35	35	40	40
					$\Sigma p_1q_0 = 249$	$\Sigma p_0q_0 = 209$	$\Sigma p_1q_1 = 278$	$\Sigma p_0q_1 = 233$

Laspeyres' index $P_{01} = \dfrac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100 = \dfrac{249}{209} \times 100 = 119.14$

$Q_{01} = \dfrac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100 = \dfrac{233}{209} \times 100 = 111.48$

Paasche's index $P_{01} = \dfrac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times 100 = \dfrac{278}{233} \times 100 = 119.31$

$Q_{01} = \dfrac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times 100 = \dfrac{278}{249} \times 100 = 111.65$

Illustration 29. An enquiry into the budgets of middle class families in a village near Hyderabad gave the following information :

Expenses on :	*Food*	*Rent*	*Clothing*	*Education*	*Misc.*
	30%	25%	15%	10%	20%
Price (Rs.) 2009	1800	1000	700	400	700
Price (Rs.) 2010	2000	1200	900	500	1000

Construct cost of living index and comment on the change in the cost of living in 2010 as compared to 2009.

Solution. CONSTRUCTION OF COST OF LIVING INDEX

Expenses on	*2009* p_0	*2010* p_1	$\frac{p_1}{p_0} \times 100$ P	W	PW
Food	1800	2000	111.11	30	3333.30
Rent	1000	1200	120.00	25	3000.00
Clothing	700	900	128.57	15	1928.55
Education	400	500	125.00	10	1250.00
Misc.	700	1000	142.86	20	2857.20
				$\Sigma W = 100$	$\Sigma PW = 12369.05$

$$\text{Cost of Living Index (for 2010)} = \frac{\Sigma PW}{\Sigma W} = \frac{12369.05}{100} = 123.69$$

Illustration 30. For the following data, calculate price index number of 2010 with 2009 as the base year, using : (*a*) Laspeyres' method, (*b*) Fisher's method.

	2009		*2010*	
	Price	*Quantity*	*Price*	*Quantity*
A	20	8	40	6
B	50	10	60	5
C	40	15	50	15
D	20	20	20	25

Solution. CALCULATION OF PRICE INDEX NUMBERS

Commodity	p_0	q_0	p_1	q_1	p_1q_0	p_0q_0	p_1q_1	p_0q_1
A	20	8	40	6	320	160	240	120
B	50	10	60	5	600	500	300	250
C	40	15	50	15	750	600	750	600
D	20	20	20	25	400	400	500	500
					$\Sigma p_1q_0 = 2070$	$\Sigma p_0q_0 = 1660$	$\Sigma p_1q_1 = 1790$	$\Sigma p_0q_1 = 1470$

Laspeyres' Index : $P_{01} = \dfrac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100 = \dfrac{2070}{1660} \times 100 = 124.7$

Paasche's Index : $P_{10} = \dfrac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times 100 = \dfrac{1790}{1470} \times 100 = 121.77$

Illustration 31. *Calculate from the following data, the Fisher's Ideal Index Number for the year 2010 :*

Commodity Selected	2009 *Price (Rs.)*	2009 *Expenditure on quantity consumed (Rs.)*	2010 *Price (Rs.)*	2010 *Expenditure on quantity consumed (Rs.)*
A	8	200	65	1950
B	20	1400	30	1650
C	5	80	20	900
D	10	360	15	300
E	27	2160	10	600

Solution : First find quantity by dividing expenditure by price.

CALCULATION OF FISHER'S IDEAL INDEX

Commodity	p_0	q_0	p_1	q_1	p_1q_0	p_0q_0	p_1q_1	p_0q_1
A	8	25	65	30	1625	200	1950	240
B	20	70	30	55	2100	1400	1650	1100
C	5	16	20	45	320	80	900	225
D	10	36	15	20	540	360	300	200
E	27	80	10	60	800	2160	600	1620
					$\Sigma p_1q_0 = 5385$	$\Sigma p_0q_0 = 4200$	$\Sigma p_1q_1 = 5400$	$\Sigma p_0q_1 = 3385$

$$P_{01} = \sqrt{\frac{\Sigma p_1q_0}{\Sigma p_0q_0} \times \frac{\Sigma p_1q_1}{\Sigma p_0q_1}} \times 100$$

$$= \sqrt{\frac{5385}{4200} \times \frac{5400}{3385}} \times 100 = 1.430 \times 100 = 143.$$

Illustration 32. Construct Fisher's Ideal Index from the following data and show that it satisfies time reversal and factor reversal tests :

Commodity	2009 *Price*	2009 *Value*	2010 *Price*	2010 *Value*
A	10	100	12	144
B	15	75	20	120
C	8	80	10	110
D	20	60	25	50
E	50	500	60	540

Solution.

CALCULATION OF FISHER'S IDEAL INDEX

Commodity	p_0	q_0	p_1	q_1	p_1q_0	p_0q_0	p_1q_1	p_0q_1
A	10	10	12	12	120	100	144	120
B	15	5	20	6	100	75	120	90
C	8	10	10	11	100	80	110	88
D	20	3	25	2	75	60	50	40
E	50	10	60	9	600	500	540	450
					$\Sigma p_1q_0 = 995$	$\Sigma p_0q_0 = 815$	$\Sigma p_1q_1 = 964$	$\Sigma p_0q_1 = 788$

Fisher's Ideal Index or

$$P_{01} = \sqrt{\frac{\Sigma p_1q_0}{\Sigma p_0q_0} \times \frac{\Sigma p_1q_1}{\Sigma p_0q_1}} \times 100$$

$$= \sqrt{\frac{995}{815} \times \frac{964}{788}} \times 100 = \sqrt{1.4935} \times 100 = 1.222 \times 100 = 122.2$$

Time Reversal Test : Time reversal test is satisfied when :

$$P_{01} \times P_{10} = 1$$

$$P_{10} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_1 q_1} \times \frac{\Sigma p_0 q_0}{\Sigma p_1 q_0}} = \sqrt{\frac{788}{964} \times \frac{815}{995}}$$

$$P_{01} \times P_{10} = \sqrt{\frac{995}{815} \times \frac{964}{788} \times \frac{788}{964} \times \frac{815}{995}} = \sqrt{1} = 1$$

Hence time reversal test is satisfied.

Factor Reversal Test : Factor reversal test is satisfied when :

$$P_{01} \times Q_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$$

$$Q_{01} = \sqrt{\frac{\Sigma q_1 p_0}{\Sigma p_0 q_0} \times \frac{\Sigma q_1 p_1}{\Sigma q_0 p_1}} = \sqrt{\frac{788}{815} \times \frac{964}{995}}$$

$$P_{01} \times Q_{01} = \sqrt{\frac{995}{815} \times \frac{964}{788} \times \frac{788}{815} \times \frac{964}{995}} = \frac{964}{815}$$

$\frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$ is also equal to $\frac{964}{815}$. Hence factor reversal test is satisfied by the given data.

Illustration 33. From the data given below, calculate Fisher's Ideal Index and show that it satisfies time reversal test :

	2009		*2010*	
Commodity	*Price*	*Quantity*	*Price*	*Quantity*
A	12	20	14	30
B	14	13	20	15
C	10	12	15	20
D	6	8	4	10
E	8	5	6	5

Solution. CALCULATION OF FISHER'S IDEAL INDEX

	2009		*2010*					
Commodity	p_0	q_0	p_1	q_1	p_1q_0	p_0q_0	p_1q_1	p_0q_1
A	12	20	14	30	280	240	420	360
B	14	13	20	15	260	182	300	210
C	10	12	15	20	180	120	300	200
D	6	8	4	10	32	48	40	60
E	8	5	6	5	30	40	30	40
					$\Sigma p_1q_0 = 782$	$\Sigma p_0q_0 = 630$	$\Sigma p_1q_1 = 1090$	$\Sigma p_0q_1 = 870$

Fisher's Ideal Index or $P_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} \times 100$

$$= \sqrt{\frac{782}{630} \times \frac{1090}{870}} \times 100 = 1.247 \times 100 = 124.7$$

Time Reversal Test : Time Reversal Test is satisfied when

$$P_{01} \times P_{10} = 1$$

$$P_{10} = \sqrt{\frac{\Sigma p_0 q_1}{\Sigma p_1 q_1} \times \frac{\Sigma p_0 q_0}{\Sigma p_1 q_0}} = \sqrt{\frac{870}{1090} \times \frac{630}{782}}$$

$$P_{01} \times P_{10} = \sqrt{\frac{782}{630} \times \frac{1090}{870} \times \frac{870}{1090} \times \frac{630}{782}} = \sqrt{1} = 1.$$

Illustration 34. Calculate the index number by : (*i*) Paasche's method, and (*ii*) Fisher's method.

Commodity	p_1	q_1	p_0	q_0
A	5	14	3	8
B	8	18	6	25
C	3	25	1	40
D	15	36	12	48
E	9	14	7	18
F	7	13	5	19

(*MBA, M.D. Univ., 2006*)

Solution. CALCULATION OF PAASCHE'S AND FISHER'S INDICES

Commodity	p_1	q_1	p_0	q_0	p_1q_1	p_0q_1	p_1q_0	p_0q_0
A	5	14	3	8	70	42	40	24
B	8	18	6	25	144	108	200	150
C	3	25	1	40	75	25	120	40
D	15	36	12	48	540	432	720	576
E	9	14	7	18	126	98	162	126
F	7	13	5	19	91	65	133	95
					$\Sigma p_1q_1 = 1046$	$\Sigma p_0q_1 = 770$	$\Sigma p_1q_0 = 1375$	$\Sigma p_0q_0 = 1011$

(*i*) Paasche's Index : $$P_{01} = \frac{\Sigma p_1q_1}{\Sigma p_0q_1} \times 100 = \frac{1046}{770} \times 100 = 135.84$$

(*ii*) Fisher's Index : $$P_{01} = \sqrt{\frac{\Sigma p_1q_0}{\Sigma p_0q_0} \times \frac{\Sigma p_1q_1}{\Sigma p_0q_1}} \times 100$$

$$= \sqrt{\frac{1375}{1011} \times \frac{1046}{770}} \times 100 = 1.3592 \times 100 = 135.92$$

Illustration 35. Compute the Laspeyres' and Paasche's price index numbers for the year 2010 using the following data concerning four commodities :

	Commodity			
Quantity (kg.)	*A*	*B*	*C*	*D*
in 2009	8	10	15	20
in 2010	6	5	10	15
Price per kg (Rs.)				
in 2009	20	50	40	20
in 2010	40	60	50	20

Solution. CALCULATION OF LASPEYRES' AND PAASCHE'S PRICE INDEX

Commodity	p_0	p_1	q_0	q_1	p_1q_0	p_0q_0	p_1q_1	p_0q_1
A	20	40	8	6	320	160	240	120
B	50	60	10	5	600	500	300	250
C	40	50	15	10	750	600	500	400
D	20	20	20	15	400	400	300	300
					$\Sigma p_1q_0 = 2070$	$\Sigma p_0q_0 = 1660$	$\Sigma p_1q_1 = 1340$	$\Sigma p_0q_1 = 1070$

Laspeyres' Index : $P_{01} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100 = \frac{2070}{1660} \times 100 = 124.7$

Paasche's Index : $P_{01} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_1} \times 100 = \frac{1340}{1070} \times 100 = 125.23$

PROBLEMS

1-A : Answer the following questions, each question carries **one** mark:

(*i*) What are index numbers ?
(*ii*) Give two important uses of index numbers.
(*iii*) Name two important problems that arise while constructing index number.
(*iv*) Give the formula for Fisher's Ideal Index Number.
(*v*) What is time reversal test ?
(*vi*) What is Laspeyres method of constructing index numbers.
(*vii*) What is quantity index ?
(*viii*) What is base shifting ?
(*ix*) Which average is most appropriate for constructing index numbers.
(*x*) Give any two limitations of index numbers.

1-B : Answer the following questions, each question carries **four** marks:

(*i*) Describe the problem faced in the construction of index numbers. *(M.A. Econ., Madras Univ., 20[illegible])*
(*ii*) Distinguish between Time reversal test and Factor reversal test. *(M.B.A., UPTech. Univ., 20[illegible])*
(*iii*) What is Fisher's Ideal Index ? Why is it called ideal ?
(*iv*) Briefly explain the concept of splicing and deflating.
(*v*) What are fixed base and chain base indices ? Explain with suitable example.

2. (*a*) What is an index number ? Describe briefly its applications in business and industry.
(*b*) Discuss briefly the importance and the use of index numbers in business.

3. (*a*) What are the uses of index numbers ? What are the problems in their construction? *(MBA, Vikram Univ., 20[illegible])*
(*b*) What are index numbers ? How are they constructed ? Explain the role of weights in the construction of general pri[ce] index numbers.
(*c*) Explain the nature and uses of index numbers.

4. What is an index number ? Examine the various problems involved in the construction of an index number. Discuss in bri[ef] the uses of an index number.

5. What is an index number ? Explain the terms price relative, quantity relative and value relative with reference to a sin[gle] commodity and deduce the factor reversal property.

6. Describe the steps involved in the computation of Fisher's Ideal Index Number. What are its advantages and disadva[n]tages ?

7. What is Fisher's Ideal Index ? Why is it called ideal ? Show that it satisfies both the time reversal test as well as the fact[or] reversal test. *(MBA, Sukhadia Univ.; MBA, HPU 200[4])*

8. Laspeyres' price index generally shows an upward trend in the price changes while Paasche's method shows a downward trend on them. Elucidate the statement. *(MBA, Delhi Univ., 2005)*

9. (*a*) "Index numbers are signs and guide-posts along the business highway, that indicate to the businessman how he should drive or manage his affairs". Explain the above statement and also point out the relative advantages of the various types of averages as applied to index numbers. Which would you prefer and why ?
(*b*) What is Fisher's Index ? Why is it called Ideal ?

10. Discuss the following statements :
(*i*) Compute
(*i*) "The purpose determines the type of index number to use."
(*ii*) "An index number is a special type of average."
(*iii*) "There is no such thing as unweighted index numbers."
(*iv*) "The choice of a suitable base period is at best a temporary solution." Why ?
(*v*) "Theoretically, geometric mean is the best average in the construction of index numbers but in practice mostly arithmetic mean is used." Why ?

11. (*a*) " Since the value of the base is always 100, it does not make any difference which period is selected as the base o[n] which to construct an index." Comment.

(b) If you are employed to construct a price index for a department store that sells thousands of items (a) how would you decide on which items to include ? (b) how would you define the price? (c) what weights would you use? (d) which formula would you select ?

(a) Define index numbers. Describe the construction of wholesale price index number elucidating the following points :

(i) Selection of commodities,
(ii) Selection of the prices and the market,
(iii) Selection of the base year,
(iv) Selection of the average,
(v) Decision on the system of weighting.

(b) What is an ideal index ? How does the Fisher formula for ideal index satisfy the following two tests :

(i) Time Reversal Test. and
(ii) Factor Reversal Test.

(a) Distinguish clearly between fixed base and chain base index number and point out their relative merits and demerits. *(B.Com., Delhi Univ., 2009)*

(b) Explain Time Reversal Test and Factor Reversal Test with the help of a suitable example. *(MBA, HPU; MBA, Osmania Univ., 2007)*

What are time reversal and factor reversal tests ? Does the following index number formula satisfy these tests ?

$$I = \sqrt{\frac{\Sigma p_x q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_x q_x}{\Sigma p_0 q_x}} \times 100$$

(a) What is an index number ? Discuss its importance in business and industry.

(b) Explain :

(i) Time reversal test,
(ii) Factor reversal test, and
(iii) Circular test as applied to index number.

What do you understand by reversibility of index numbers ? Explain time reversal and factor reversal test in this context.

(a) It is said that index numbers are a specialized type of averages. How far do you agree with this statement. Explain briefly time reversal and factor reversal tests. *(MBA, Osmania Univ., 2004)*

(b) What are the factor reversal and circular tests of consistency in the selection of an appropriate index formula ? Verify whether Fisher's Ideal Index satisfies such tests.

(c) The following are the prices of six different commodities for 2009 and 2010. Compute a price index by (a) simple aggregative method and (b) average of price relative method by using both arithmetic mean as well as geometric mean.

Commodity	*Unit*	*Price in 2009 (Rs.)*	*Price in 2010 (Rs.)*
Wheat	Quintal	1900	2200
Rice	"	1500	2000
Pulses	"	2000	3000
Ghee	1 kilo	120	122
Butter	"	130	136
Potatoes	"	11	12

Construct an appropriate index for purposes of comparison from the following data :

Commodity	*A*		*B*		*C*	
Year	*Price*	*Qty.*	*Price*	*Qty.*	*Price*	*Qty.*
2009	4	50	3	10	2	5
2010	10	40	8	8	4	4

19. The following table gives the per capita income and cost of living index number of a particular community. Deflate the per capita income by taking into account the rise in the cost of living :

Year	*Per capita income*	*Cost of Living Index No. Base*
2003	800	100
2004	900	150
2005	950	180
2006	1020	200
2007	1150	220
2008	1200	250
2009	1500	300
2010	1600	400

20. Calculate Laspeyres' and Paasche's Index number from the following data :

	Base Year		*Current Year*	
Items	*Qty.*	*Price per Kg.*	*Qty.*	*Price per Kg.*
Bread	10	Rs. 22.50	12	Rs. 25.00
Meat	8	Rs. 80.00	9	Rs. 90.00
Tea	2	Rs. 100.00	4	Rs. 120.00

21. Construct from the following data spliced index continuous with index A and a spliced index continuous with index B:

Year	*Index A*	*Index B*
2005	100	
2006	95	
2007	110	
2008	125	110
2009		105
2010		94

22. (*a*) In the construction of a certain consumer price index, the following group index numbers were found. Calculate the consumer price index by using (*i*) the weighted arithmetic mean, and (*ii*) the weighted geometric mean :

Groups	*Index*	*Weights*
Food	300	5
Fuel and Lighting	250	1
Clothing	280	1
House Rent	200	2
Miscellaneous	150	1

(*b*) In calculating a certain cost of living index number, the following weights were used : Food 15, clothing 0, rent 4, fuels and light 2, miscellaneous 1. Calculate the index for a date when the average percentage increases in prices of items in the various groups over the base period were 32, 54, 47, 74 and 58 respectively.

23. Using the following data, show that Fisher's Ideal formula satisfies the Factor Reversal Test :

Commodity	*Price Per Unit (Rs.)*		*Number of Units*	
	Base Period	*Current Period*	*Base Period*	*Current Period*
A	6	10	50	56
B	2	2	100	120
C	4	6	60	60
D	10	12	30	24
E	8	12	40	36

[139.79]

24. Using the food index and the information given below calculate the cost of living index number :

Groups	*Food*	*Clothing*	*Fuel & Light*	*House Rent*	*Miscellaneous*
Index	–	310	220	150	300
Weight	60	5	8	9	18

25. Given below are the data on prices of some consumer goods and the weights attached to the various items. Compute price index numbers for the year 2010 (Base: 2009 = 100) using (*i*) simple average, and (*ii*) weighted average of price relatives.

Item	*Unit*	*Price (Rs.)* 2009	*Price (Rs.)* 2010	*Weight*
Wheat	kg.	10.00	11.00	2
Milk	litre	15.00	16.00	5
Sugar	kg.	16.00	17.00	8
Shoes (per pair)	Rs.	500.00	550.00	1

26. An enquiry into the budgets of the middle-class families of a certain city revealed that on an average the percentage expenses on the different groups were—Food 45, rent 15, clothing 12, fuel and light 8 and miscellaneous 20. The group index numbers for the current year as compared with a fixed base period were respectively 410, 150, 343, 248 and 285. Calculate the consumer price index number for the current year. Mr. *X* was getting Rs. 240 in the base period and Rs. 480 in the current year. State how much he ought to have received as extra allowance to maintain his former standard of living. (*MBA, HPU, 2005*)

27. The following are the group index numbers and the group weights of an average working class family's budget. Construct the cost of living index number by assuming the weight :

Group	*Index number*	*Weight*
Food	152	48
Fuel and Lighting	110	6
Clothing	130	8
House rent	100	12
Miscellaneous	90	15

[129.73]

28. From the chain base index numbers given below prepare fixed base index numbers.

Year :	2006	2007	2008	2009	2010
Chain base Index No. :	80	140	130	110	90

[80, 112, 145.6, 160.16, 144.1]

29. From the following data prepare index number for real wages of workers :

Year :	2005	2006	2007	2008	2009	2010
Wages (Rs.) :	2000	2500	3110	3600	3900	4000
Index number :	100	160	280	300	330	340

30. Calculate the Fisher's ideal index. Does this data satisfy time and factor reversal tests :

Commodity	*Price*	*Qty.*	*Price*	*Qty.*
A	5	4	4	5
B	5	10	6	10
C	8	15	8	20
D	10	8	12	6
E	2	6	2	8

[106.63, yes]

31. The following table gives the average wholesale price of five groups of commodities for the years 2006 to 2010. Compute chain base index numbers.

Commodity	*2006*	*2007*	*2008*	*2009*	*2010*
A	2	3	4	2	7
B	3	6	9	4	3
C	4	12	20	8	22
D	5	7	22	16	18
E	3	8	11	14	12

32. Compute the index numbers of prices from the following data by applying : (*a*) Laspeyres', (*b*) Paasche's (*c*) Fisher's and (*d*) Bowley's method.

	2009		2010	
Commodity	*Price*	*Quantity*	*Price*	*Quantity*
A	3	8	6	9
B	5	9	8	10
C	6	15	7	12
D	4	20	5	15

[(*a*) 135.98; (*b*) 140.19; (*c*) 138.07; (*d*) 138.09]

33. Prepare index numbers (2003 = 100) from the Link relatives given below :

Year :	2004	2005	2006	2007	2008	2009	2010
Link relatives :	105	75	71	105	98	90	90

34. Calculate Laspeyres', Paasche's, and Fisher's Ideal Index from the following data :

Commodity	*Price*	*Value*	*Price*	*Value*
A	10	100	8	96
B	16	96	14	98
C	12	36	10	40
D	15	60	5	25

[P_{01}=73.29; 72.96; 73.12]

35. Prepare price index numbers for 2010 with 2000 as base year from the following data by using (*i*) Laspeyres' (*ii*) Paasche's and (*iii*) Fisher's method. (Correct up to three decimal)

	Article							
	I		*II*		*III*		*IV*	
Year	*P*	*Q*	*P*	*Q*	*P*	*Q*	*P*	*Q*
2000	12.50	9	9.63	4	7.75	6	5.00	5
2010	18.75	9	7.75	6	8.80	10	10	6.507

[*P*: Price; *Q* : Quantity]

With the help of above data prove that the Time Reversal Test is satisfied by Fisher's formula, but not necessarily by the Laspeyres' and Paasche's index numbers.

36. Construct Fisher's Ideal Index No. for the following data and show that it satisfies the time reversal and factor reversal tests :

	Base Year		*Current Year*	
Commodity	*Price*	*Qty.*	*Price*	*Qty.*
A	6	30	15	40
B	5	40	10	55
C	10	25	12	20
D	4	15	3	30
E	2	50	5	28

(MBA, Madurai-Kamaraj Univ., 2006)

37. From the following prices of these groups of commodities for the years 2003 to 2007, find the chain base index numbers chained to 2003 :

Groups	*2003*	*2004*	*2005*	*2006*	*2007*
I	4	6	8	10	12
II	16	20	24	30	36
III	8	10	16	20	24

(MBA, M.K. Univ., 2007)

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