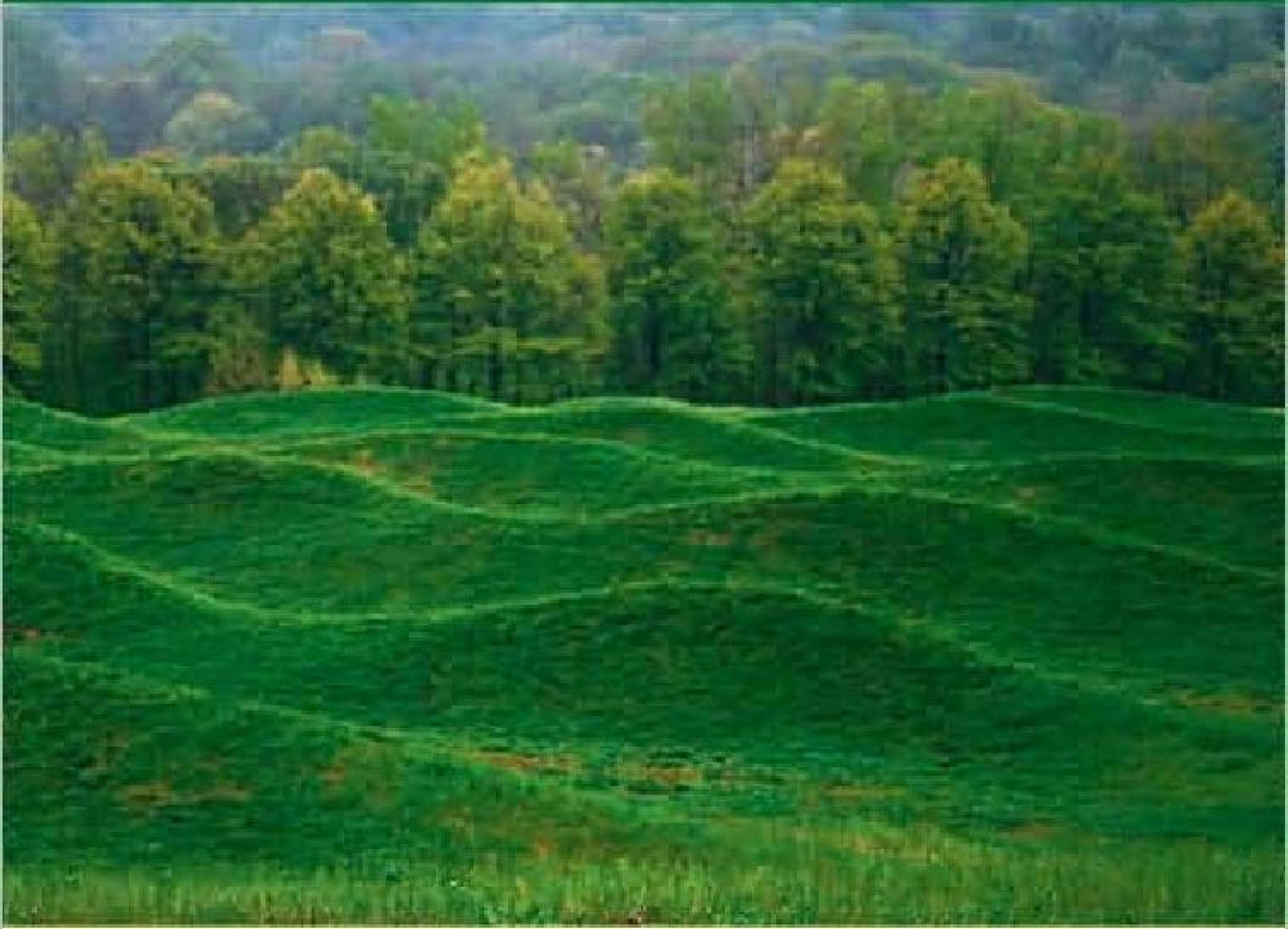
Third Edition

DISCRETE-TIME SIGNAL PROCESSING



Alan V. Oppenheim • Ronald W. Schafer

with a companion website by Mark A. Yoder and Wayne T. Padgett

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THIRD EDITION

Discrete-Time Signal Processing

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To Phyllis, Justine, and Jason

To Dorothy, Bill, Tricia, Ken, and Kate
and in memory of John

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PREFACE

This third edition of *Discrete-Time Signal Processing* is a descendent of our original textbook *Digital Signal Processing* published in 1975. That very successful text appeared at a time when the field was young and just beginning to develop rapidly. At that time the topic was taught only at the graduate level and at only a very few schools. Our 1975 text was designed for such courses. It is still in print and is still used successfully at a number of schools in the United States and internationally.

By the 1980's, the pace of signal processing research, applications and implementation technology made it clear that digital signal processing (DSP) would realize and exceed the potential that had been evident in the 1970's. The burgeoning importance of DSP clearly justified a revision and updating of the original text. In organizing that revision, it was clear that so many changes had occurred both in the field and in the level and style with which the topic was taught, that it was most appropriate to develop a new textbook, strongly based on our original text, while keeping the original text in print. We titled the new book, published in 1989, *Discrete-Time Signal Processing* to emphasize that most of the DSP theory and design techniques discussed in the text apply to discrete-time systems in general, whether analog or digital.

In developing *Discrete-Time Signal Processing* we recognized that the basic principles of DSP were being commonly taught at the undergraduate level, sometimes even as part of a first course on discrete-time linear systems, but more often, at a more advanced level in third-year, fourth-year, or beginning graduate subjects. Therefore, it was appropriate to expand considerably the treatment of such topics as linear systems, sampling, multirate signal processing, applications, and spectral analysis. In addition, more examples were included to emphasize and illustrate important concepts. Consistent with the importance that we placed on well constructed examples and homework problems, that new text contained more than 400 problems.

While the field continued to advance in both theory and applications, the underlying fundamentals remained largely the same, albeit with a refinement of emphasis, understanding and pedagogy. Consequently, the Second Edition of *Discrete-Time Signal Processing* was published in 1999. That new edition was a major revision, with the intent of making the subject of discrete-time signal processing even more accessible to students and practicing engineers, without compromising on the coverage of what we considered to be the essential concepts that define the field.

This third edition of *Discrete-Time Signal Processing* is a major revision of our Second Edition. The new edition is in response to changes in the way the subject is taught and to changes in scope of typical courses at the undergraduate and first-year graduate level. It continues the tradition of emphasizing the accessibility of the topics to students and practicing engineers and focusing on the fundamental principles with broad applicability. A major feature of the new edition is the incorporation and expansion of some of the more advanced topics, the understanding of which are now essential in order to work effectively in the field. Every chapter of the second edition has undergone significant review and changes, one entirely new chapter has been added, and one chapter has been restored and significantly up-dated from the first edition. With this third edition, a closely integrated and highly interactive companion web site has been developed by Professors Mark Yoder and Wayne Padgett of Rose-Hulman Institute of Technology. A more complete discussion of the website is given in the website overview section following this Preface.

As we have continued to teach the subject over the ten years since the second edition, we have routinely created new problems for homework assignments and exams. Consistent with the importance that we have always placed on well constructed examples and homework problems, we have selected over 130 of the best of these to be included in the third edition, which now contains a total of more than 700 homework problems overall. The homework problems from the second edition that do not appear in this new edition are available on the companion web site.

As in the earlier generations of this text, it is assumed that the reader has a background of advanced calculus, along with a good understanding of the elements of complex numbers and complex variables. A background in linear system theory for continuous-time signals, including Laplace and Fourier transforms, as taught in most undergraduate electrical and mechanical engineering curricula, remains a basic prerequisite. It is also now common in most undergraduate curricula to include an early exposure to discrete-time signals and systems, discrete-time Fourier transforms and discrete-time processing of continuous-time signals.

Our experience in teaching discrete-time signal processing at the advanced undergraduate level and the graduate level confirms that it is essential to begin with a careful review of these topics so that students move on to the more advanced topics from a solid base of understanding and a familiarity with a consistent notational framework that is used throughout the course and the accompanying textbook. Most typically in a first exposure to discrete-time signal processing in early undergraduate courses, students learn to carry out many of the mathematical manipulations, but it is in revisiting the topics that they learn to reason more deeply with the underlying concepts. Therefore in this edition we retain the coverage of these fundamentals in the first five chapters, enhanced with new examples and expanded discussion. In later sections of some chapters, some topics such as quantization noise are included that assume a basic background in random signals. A brief review of the essential background for these sections is included in Chapter 2 and in Appendix A.

An important major change in DSP education that has occurred in the past decade or so is the widespread use of sophisticated software packages such as MATLAB, LabVIEW, and Mathematica to provide an interactive, "hands-on" experience for students. The accessibility and ease of use of these software packages provide the opportunity to connect the concepts and mathematics that are the basis for discrete-time signal processing, to applications involving real signals and real-time systems. These software packages are well documented, have excellent technical support, and have excellent user interfaces. These make them easily accessible to students without becoming a distraction from the goal of developing insight into and intuition about the fundamentals. It is now common in many signal processing courses to include projects and exercises to be done using one or several of the available software packages. Of course, this needs to be done carefully in order to maximize the benefit to student learning by emphasizing experimentation with the concepts, parameters, and so on, rather than simple cookbook exercises. It is particularly exciting that with one of these powerful software packages installed, every student's laptop computer becomes a state-of-the-art laboratory for experimenting with discrete-time signal processing concepts and systems.

As teachers, we have consistently looked for the best way to use computer resources to improve the learning environment for our students. We continue to believe in textbooks as the best way to encapsulate knowledge in the most convenient and stable form. Textbooks necessarily evolve on a relatively slow time scale. This ensures a certain stability and provides the time to sort through developments in the field and to test ways of presenting new ideas to students. On the other hand, changes in computer software and hardware technology are on a much faster time scale. Software revisions often occur semi-annually, and hardware speeds continue to increase yearly. This, together with the availability of the world-wide-web, provides the opportunity to more frequently update the interactive and experimental components of the learning environment. For these reasons, providing separate environments for the basic mathematics and basic concepts in the form of the textbook and the hands-on interactive experience primarily through the world-wide-web seems to be a natural path.

With these thoughts in mind, we have created this third edition of *Discrete-Time Signal Processing*, incorporating what we believe to be the fundamental mathematics and concepts of discrete-time signal processing and with tight coupling to a companion website created by our colleagues Mark Yoder and Wayne Padgett of Rose-Hulman Institute of Technology. The website contains a variety of interactive and software resources for learning that both reinforce and expand the impact of the text. This website is described in more detail in the introductory section following this Preface. It is designed to be dynamic and continually changing to rapidly incorporate new resources developed by the authors of the text and by the website authors. The website will be sensitive to the continually changing hardware and software environments that serve as the platform for visualization of abstract concepts and experimentation with real signal processing problems. We are excited by the virtually limitless potential for this companion website environment to significantly improve our ability to teach and our students' ability to learn the subject of discrete-time signal processing.

The material in this book is organized in a way that provides considerable flexibility in its use at both the undergraduate and graduate level. A typical one-semester undergraduate elective might cover in depth Chapter 2, Sections 2.0–2.9; Chapter 3; Chapter 4, Sections 4.0–4.6; Chapter 5, Sections 5.0–5.3; Chapter 6, Sections 6.0–6.5; and Chapter 7, Sections 7.0–7.3 and a brief overview of Sections 7.4–7.6. If students have studied discrete-time signals and systems in a previous signals and systems course, it would be possible to move more quickly through the material of Chapters 2, 3, and 4, thus freeing time for covering Chapter 8. A first-year graduate course or senior elective could augment the above topics with the remaining topics in Chapter 5, a discussion of multirate signal processing (Section 4.7), an exposure to some of the quantization issues introduced in Section 4.8, and perhaps an introduction to noise shaping in A/D and D/A converters as discussed in Section 4.9. A first-year graduate course should also include exposure to some of the quantization issues addressed in Sections 6.6–6.9, a discussion of optimal FIR filters as incorporated in Sections 7.7–7.9, and a thorough treatment of the discrete Fourier transform (Chapter 8) and its computation using the FFT (Chapter 9). The discussion of the DFT can be effectively augmented with many of the examples in Chapter 10. In a two-semester graduate course, the entire text including the new chapters on parametric signal modeling (Chapter 11) and the cepstrum (Chapter 13) can be covered along with a number of additional advanced topics. In all cases, the homework problems at the end of each chapter can be worked with or without the aid of a computer, and problems and projects from the website can be assigned to strengthen the connection between theory and computer implementation of signal processing systems.

We conclude this Preface with a summary of chapter contents highlighting the significant changes in the third edition.

In Chapter 2, we introduce the basic class of discrete-time signals and systems and define basic system properties such as linearity, time invariance, stability, and causality. The primary focus of the book is on linear time-invariant systems because of the rich set of tools available for designing and analyzing this class of systems. In particular, in Chapter 2 we develop the time-domain representation of linear time-invariant systems through the convolution sum and discuss the class of linear time-invariant systems represented by linear constant-coefficient difference equations. In Chapter 6, we develop this class of systems in considerably more detail. Also in Chapter 2 we discuss the frequency-domain representation of discrete-time signals and systems through the discrete-time Fourier transform. The primary focus in Chapter 2 is on the representation of sequences in terms of the discrete-time Fourier transform, i.e., as a linear combination of complex exponentials, and the development of the basic properties of the discrete-time Fourier transform.

In Chapter 3, we develop the z-transform as a generalization of the Fourier transform. This chapter focuses on developing the basic theorems and properties of the z-transform and the development of the partial fraction expansion method for the inverse transform operation. A new section on the unilateral z-transform has been added in this edition. In Chapter 5, the results developed in Chapters 2 and 3 are used extensively in a detailed discussion of the representation and analysis of linear time-invariant systems. While the material in Chapters 2 and 3 might be review for many students, most introductory signals and systems courses will not contain either the depth or breadth of coverage of these chapters. Furthermore, these chapters establish notation that will be used throughout the text. Thus, we recommend that Chapters 2 and 3 be studied as carefully as is necessary for students to feel confident of their grasp of the fundamentals of discrete-time signals and systems.

Chapter 4 is a detailed discussion of the relationship between continuous-time and discrete-time signals when the discrete-time signals are obtained through periodic sampling of continuous-time signals. This includes a development of the Nyquist sampling theorem. In addition, we discuss upsampling and downsampling of discrete-time signals, as used, for example, in multirate signal processing systems and for sampling rate conversion. The chapter concludes with a discussion of some of the practical issues encountered in conversion from continuous time to discrete time including prefiltering to avoid aliasing, modeling the effects of amplitude quantization when the discrete-time signals are represented digitally, and the use of oversampling in simplifying the A/D and D/A conversion processes. This third edition includes new examples of quantization noise simulations, a new discussion of interpolation filters derived from splines, and new discussions of multi-stage interpolation and two-channel multi-rate filter banks.

In Chapter 5 we apply the concepts developed in the previous chapters to a detailed study of the properties of linear time-invariant systems. We define the class of ideal, frequency-selective filters and develop the system function and pole-zero representation for systems described by linear constant-coefficient difference equations, a class of systems whose implementation is considered in detail in Chapter 6. Also in Chapter 5, we define and discuss group delay, phase response and phase distortion, and the relationships between the magnitude response and the phase response of systems, including a discussion of minimum-phase, allpass, and generalized linear phase systems. Third edition changes include a new example of the effects of group delay and attenuation, which is also available on the website for interactive experimentation.

In Chapter 6, we focus specifically on systems described by linear constant-coefficient difference equations and develop their representation in terms of block diagrams and linear signal flow graphs. Much of this chapter is concerned with developing a variety of the important system structures and comparing some of their properties. The importance of this discussion and the variety of filter structures relate to the fact that in a practical implementation of a discrete-time system, the effects of coefficient inaccuracies and arithmetic error can be very dependent on the specific structure used. While these basic issues are similar for digital and discrete-time analog implementations, we illustrate them in this chapter in the context of a digital implementation through a discussion of the effects of coefficient quantization and arithmetic roundoff noise for digital filters. A new section provides a detailed discussion of FIR and IIR lattice filters for implementing linear constant-coefficient difference equations. As discussed in Chapter 6 and later in Chapter 11, this class of filter structures has become extremely important in many applications because of their desirable properties. It is common in discussions of lattice filters in many texts and papers to tie their importance intimately to linear prediction analysis and signal modeling. However the importance of using lattice implementations of FIR and IIR filters is independent of how the difference equation to be implemented is obtained. For example the difference equation might have resulted from the use of filter design techniques as discussed in Chapter 7, the use of parametric signal modeling as discussed in Chapter 11 or any of a variety of other ways in which a difference equation to be implemented arises.

While Chapter 6 is concerned with the representation and implementation of linear constant-coefficient difference equations, Chapter 7 is a discussion of procedures for obtaining the coefficients of this class of difference equations to approximate a desired system response. The design techniques separate into those used for infinite impulse response (IIR) filters and those used for finite impulse response (FIR) filters. New examples of IIR filter design provide added insight into the properties of the different approximation methods. A new example on filter design for interpolation provides a framework for comparing IIR and FIR filters in a practical setting.

In continuous-time linear system theory, the Fourier transform is primarily an analytical tool for representing signals and systems. In contrast, in the discrete-time case, many signal processing systems and algorithms involve the explicit computation of the Fourier transform. While the Fourier transform itself cannot be computed, a sampled version of it, the discrete Fourier transform (DFT), can be computed, and for finite-length signals the DFT is a complete Fourier representation of the signal. In Chapter 8, the DFT is introduced and its properties and relationship to the discrete-time Fourier transform (DTFT) are developed in detail. In this chapter we also provide an introduction to the discrete cosine transform (DCT) which plays a very important role in applications such as audio and video compression.

In Chapter 9, the rich and important variety of algorithms for computing or generating the DFT is introduced and discussed, including the Goertzel algorithm, the fast Fourier transform (FFT) algorithms, and the chirp transform. In this third edition, the basic upsampling and downsampling operations discussed in Chapter 4 are used to provide additional insight into the derivation of FFT algorithms. As also discussed in this chapter, the evolution of technology has considerably altered the important metrics in evaluating the efficiency of signal processing algorithms. At the time of our first book in the 1970's both memory and arithmetic computation (multiplications and also floating point additions) were costly and the efficiency of algorithms was typically judged by how much of these resources were required. Currently it is commonplace to use additional memory to increase speed and to reduce the power requirements in the implementation of signal processing algorithms. In a similar sense, multi-core platforms have in some contexts resulted in favoring parallel implementation of algorithms even at the cost of increased computation. Often the number of cycles of data exchange, communication on a chip, and power requirements are now key metrics in choosing the structure for implementing an algorithm. As discussed in chapter 9, while the FFT is more efficient in terms of the required multiplications than the Goertzel algorithm or the direct computation of the DFT, it is less efficient than either if the dominant metric is communication cycles since direct computation or the Goertzel algorithm can be much more highly parallelized than the FFT.

With the background developed in the earlier chapters and particularly Chapters 2, 3, 5, and 8, we focus in Chapter 10 on Fourier analysis of signals using the DFT. Without a careful understanding of the issues involved and the relationship between the continuous-time Fourier transform, the DTFT, and the DFT, using the DFT for practical signal analysis can often lead to confusion and misinterpretation. We address a number of these issues in Chapter 10. We also consider in some detail the Fourier analysis of signals with time-varying characteristics by means of the time-dependent Fourier transform. New in the third edition is a more detailed discussion of filter bank analysis including an illustration of the MPEG filter bank, new examples of time-dependent Fourier analysis of chirp signals illustrating the effect of window length, and more detailed simulations of quantization noise analysis.

Chapter 11 is an entirely new chapter on the subject of parametric signal modeling. Starting with the basic concept of representing a signal as the output of an LTI system, Chapter 11 shows how the parameters of the signal model can be found by solution of a set of linear equations. Details of the computations involved in setting up and solving the equations are discussed and illustrated by examples. Particular emphasis is on the Levinson–Durbin solution algorithm and the many properties of the solution that are easily derived from the details of the algorithm such as the lattice filter interpretation.

Chapter 12 is concerned with the discrete Hilbert transform. This transform arises in a variety of practical applications, including inverse filtering, complex representations for real bandpass signals, single-sideband modulation techniques, and many others. With the advent of increasingly sophisticated communications systems and the growing richness of methods for efficiently sampling wide-band and multi-band continuous-time signals, a basic understanding of Hilbert transforms is becoming increasingly important. The Hilbert transform also plays an important role in the discussion of the cepstrum in Chapter 13.

Our first book in 1975 and the first edition of this book in 1989 included a detailed treatment of the class of nonlinear techniques referred to as cepstral analysis and homomorphic deconvolution. These techniques have become increasingly important and now have widespread use in applications such as speech coding, speech and speaker recognition, analysis of geophysical and medical imaging data, and in many other applications in which deconvolution is an important theme. Consequently with this edition we reintroduce those topics with expanded discussion and examples. The chapter contains a detailed discussion of the definition and properties of the cepstrum and the variety of ways of computing it including new results on the use of polynomial rooting as a basis for computing the cepstrum. An exposure to the material in Chapter 13 also offers the reader the opportunity to develop new insights into the fundamentals presented in the early chapters, in the context of a set of nonlinear signal analysis techniques with growing importance and that lend themselves to the same type of rich analysis enjoyed by linear techniques. The chapter also includes new examples illustrating the use of homomorphic filtering in deconvolution.

We look forward to using this new edition in our teaching and hope that our colleagues and students will benefit from the many enhancements from the previous editions. Signal processing in general and discrete-time signal processing in particular have a richness in all their dimensions that promises even more exciting developments ahead.

Alan V. Oppenheim
Ronald W. Schafer

The Companion Website

A companion website has been developed for this text by Mark A. Yoder and Wayne T. Padgett of Rose-Hulman Institute of Technology and is accessible at www.pearsonhighered.com/oppenheim. This web companion, which will continuously evolve, is designed to reinforce and enhance the material presented in the textbook by providing visualizations of important concepts and a framework for obtaining "hands-on" experience with using the concepts. It contains six primary elements: *Live Figures*, *Build-a-Figures*, *MATLAB-based homework problems*, *MATLAB-based projects*, *Demos*, and additional *Traditional Homework Problems*, each tying into specific sections and pages in the book.

Live Figures

The Live Figures element reinforces concepts in the text by presenting "live" versions of select figures. With these, the reader is able to interactively investigate how parameters and concepts interoperate using graphics and audio. Live Figures were created with NI LabVIEW signal processing tools. The following three examples provide a glimpse of what is available with this element of the website:

Figure 2.10(a)-(c) in Section 2.3 on page 28 shows the graphical method for computing a discrete-convolution with the result shown in Figure 2.10(d). The corresponding Live Figure allows the user to choose the input signals and manually slide the flipped input signal past the impulse response and see the result being calculated and plotted. Users can quickly explore many different configurations and quickly understand how graphical convolution is applied.

Figure 4.73 on page 231 of Section 4.9.2 shows the power spectral density of the quantization noise and signal after noise shaping. The Live Figure shows the spectrum of the noise and signal as a live audio file plays. The reader can see and hear the noise as the noise shaping is enabled or disabled and as a lowpass filter is applied to remove the noise.

Figure 5.5(a) on page 282 of Section 5.1.2 shows three pulses, each of a different frequency, which enter an LTI system. Figure 5.6 on page 283 shows the output of the LTI system. The associated Live Figure allows students to experiment with the location of the poles and zeros in the system as well as the amplitude, frequency, and position of the pulses to see the effect on the output. These are just three examples of the many web-based Live Figures accessible on the companion website.

Build-a-Figure

The Build-a-Figure element extends the concept of the Live Figure element. It guides the student in recreating selected figures from the text using MATLAB to reinforce the understanding of the basic concepts. Build-a-Figures are not simply step-by-step recipes for constructing a figure. Rather, they assume a basic understanding of MATLAB and introduce new MATLAB commands and techniques as they are needed to create the figures. This not only further reinforces signal processing concepts, but also develops MATLAB skills in the context of signal processing. As an example, Figures 2.3 and 2.5 on pages 12 and 16 in Section 2.1 of the text are plots of several sequences. The corresponding Build-a-Figures introduce the MATLAB plot command techniques for labeling plots, incorporating Greek characters, and including a legend. Later Build-a-Figures use this knowledge as needed in creating plots. The Noise Shaping and Group Delay Build-a-Figures (Figure 4.73, page 231 and Figure 5.5, page 282) have instructions for recreating the Live Figures discussed above. Rather than giving step-by-step instructions, they introduce new MATLAB commands and suggest approaches for recreating the figures with considerable latitude for experimentation.

MATLAB Homework Problems

Through the MATLAB Homework Problems element, the companion website provides a primary mechanism for combining MATLAB with homework exercises. One aspect of this is the use of homework to practice using MATLAB somewhat in the style of Build-a-Figures. These exercises are much like non-MATLAB exercises but with MATLAB used to facilitate certain parts, such as in plotting results. The second avenue is the use of MATLAB to explore and solve problems that cannot be done by mathematical analysis. The MATLAB problems are all classroom tested and tend to be short exercises, comparable to the Basic Problems in the textbook, in which the user is asked to complete straightforward signal processing tasks using MATLAB. These problems are modest in scope as would be typical of one of several problems in a weekly homework assignment. Some problems are directly linked to analytic problems in the text, while others will stand on their own. Many of the problems blend analytic solutions with MATLAB, emphasizing the complementary value of each approach.

MATLAB-Based Projects

The MATLAB-based Projects element contains longer and more sophisticated projects or exercises than the homework problems. The projects explore important concepts from the textbook in greater depth and are relatively extensive. Projects are linked to sections of the text and can be used once that section is understood. For example, the first project is somewhat tutorial in nature and can be used at any stage. It introduces MATLAB and shows how it is used to create and manipulate discrete-time signals and systems. It assumes that the students have some programming experience, but not necessarily in MATLAB. Many of the other projects require some filter design techniques and therefore tie in with Chapter 7 (Filter Design Techniques) or later. They explore topics such as FIR and IIR filter design, filter design for sample rate conversion, testing a "Folk Theorem" about humans not being able to hear phase in a signal, enhancing speech by removing noise, hardware considerations for removing noise, spectral estimation and more. All have been classroom tested and some have led to student publications.

Demos

The Demos are interactive demonstrations that relate to specific chapters. Unlike the Live Figures, they do not tie directly to a given figure. Rather, they illustrate a bigger idea that the student can understand after completing the chapter. For example, one demo shows the importance of using a linear-phase filter when it is essential to preserve the shape of a bandlimited pulse.

Additional Traditional Homework Problems

A sixth important component of the website is a collection of problems that were removed from the second edition to make room for new problems. These problems can be used to supplement the problems in the text. Each of these problems is given in `.pdf` and `.tex` form along with any figures needed to create the problem.

In summary, the companion web site is a rich set of resources which are closely tied to the textbook. The resources range from the Live Figures which reinforce new concepts to the MATLAB-based projects which challenge the students to go beyond the textbook to explore new ideas. This website will continuously evolve as new teaching resources are developed by the authors of the text and by the website authors, Mark Yoder and Wayne Padgett.

THE COVER

In this third edition of Discrete-Time Signal Processing we continue the cover theme of "waves" as a symbol of our book and of signal processing. The cover of the first edition was a colorful rendering of a time-varying spectral waterfall plot. For the second edition, the artist Vivian Berman carried the theme forward resulting in a combination of spectral plots and artistic wave patterns. In considering possibilities for the cover, we were particularly drawn to a striking photograph by Librado Romero in a *New York Times* article (May 7, 2009). This article by Holland Cotter entitled "Storm King Wavefield" was about the artist Maya Lin's new work at the Storm King Art Center.[1] With this suggestion, Kristine Carney at Pearson/Prentice-Hall produced the beautiful cover for this edition.

To us, the grass-covered earthen waves in Ms. Lin's sculpture symbolize much about the field of Signal Processing and suggest the perfect evolution of our covers. As the *New York Times* article states,

> "Like any landscape, it is a work in progress. Vegetation is still coming in, drainage issues are in testing mode, and there are unruly variables: woodchucks have begun converting one wave into an apartment complex."

Change a few words here and there, and it provides an intriguing description of the field of Discrete-Time Signal Processing. It has a beautiful solid framework. Furthermore, new ideas, constraints, and opportunities keep the field fluid and dynamically changing, and there will always be a few "unruly variables." As Mr. Cotter also notes, Ms. Lin's work

> "sharpens your eye to existing harmonies and asymmetries otherwise overlooked."

Even after more than 40 years of living and working in the field of signal processing, we are consistently surprised and delighted by the harmonies, symmetries, and asymmetries that continue to be revealed.

[1]Information about the Storm King Art Center can be found at www.stormking.org and about Ms. Lin and her beautiful art at www.mayalin.com.

Acknowledgments

This third edition of *Discrete-Time Signal Processing* has evolved from the first two editions (1989, 1999) which originated from our first book *Digital Signal Processing* (1975). The influence and impact of the many colleagues, students and friends who have assisted, supported and contributed to those earlier works remain evident in this new edition and we would like to express again our deep appreciation to all whom we have acknowledged more explicitly in those previous editions.

Throughout our careers we both have had the good fortune of having extraordinary mentors. We would each like to acknowledge several who have had such a major impact on our lives and careers.

Al Oppenheim was profoundly guided and influenced as a graduate student and throughout his career by Professor Amar Bose, Professor Thomas Stockham, and Dr. Ben Gold. As a teaching assistant for several years with and as a doctoral student supervised by Professor Bose, Al was significantly influenced by the inspirational teaching, creative research style and extraordinary standards which are characteristic of Professor Bose in everything that he does. Early in his career Al Oppenheim was also extremely fortunate to develop a close collaboration and friendship with both Dr. Ben Gold and Professor Thomas Stockham. The incredible encouragement and role model provided by Ben was significant in shaping Al's style of mentoring and research. Tom Stockham also provided significant mentoring, support and encouragement as well as ongoing friendship and another wonderful role model. The influence of these extraordinary mentors flows throughout this book.

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a friend at a crucial time, a valued colleague, and a wonderfully creative engineer. Jim Flanagan is a giant in the area of speech science and engineering and an inspiration to all who are so lucky as to have worked with him. Not all great teachers carry the title "Professor". He taught Ron and many others the value of careful thought, the value of dedication to a field of learning, and the value of clear and lucid writing and expression. Ron Schafer freely admits appropriating many habits of thought and expression from these great mentors, and does so with confidence that they don't mind at all.

Throughout our academic careers, MIT and Georgia Tech have provided us with a stimulating environment for research and teaching and have provided both encouragement and support for this evolving project. Since 1977 Al Oppenheim has spent several sabbaticals and almost every summer at the Woods Hole Oceanographic Institution (WHOI) and he is deeply appreciative of this special opportunity and association. It was during those periods and in the wonderful WHOI environment that much of the writing of the various editions of this book were carried out.

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Our association with Prentice Hall Inc. began several decades ago with our first book published in 1975 and has continued through all three editions of this book as well as with other books. We feel extremely fortunate to have worked with Prentice Hall. The encouragement and support provided by Marcia Horton and Tom Robbins through this and many other writing projects and by Michael McDonald, Andrew Gilfillan, Scott Disanno, and Clare Romeo with this edition have significantly enhanced the enjoyment of writing and completing this project.

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