Feedback

Now you can win a prize too !!

Dear Reader,

Reg. *Business Mathematics by* Sancheti & Kapoor

Has it occurred to you that you can do to the students/the future readers a favour by sending your suggestions/comments to improve the book ? In addition, a surprise gift awaits you if you are kind enough to let us have your frank assessment, helpful comments and specific suggestions in detail about the book on a separate sheet as regards the following :

1. Since when have you been using this book ? How many pages of this book have you read so far ?
2. In which section of the book do you find the discussion of the subject more interesting/more inspiring/easier to understand in comparison to other books ?
3. Which sections/chapters in this book do you find tedious and/or confusing ?
4. Which important topics of your syllabus from the point of view of your examination are omitted from the book ?
5. Which chapters of the book do you find irrelevant ?
6. In which chapters of the book is the treatment too elementary or too advanced ? Please support with examination question papers.
7. In which chapters of the book is the treatment not systematic or not organised properly ? Please illustrate.
8. Is there any factual inaccuracy in the book ? Please specify.
9. Which topic in the book is not up-to-date ? Please specify and illustrate.
10. What do you find distinctive or new in the approach of this book or in the treatment of the subject ?
11. Have you come across any misprints/mistakes in the book ? If so, please specify.
12. How would you rate this book : poor/average/outstanding ?
13. What is your assessment of this book as regards presentation of the subject-matter, expression, accuracy and precision and price in relation to other books available on this subject ? Which competing book will you regard as better than this ?
14. Did this book come up to your expectations or were you sorry to read it? Is it worth its price ? Will you recommend it to your students/colleagues/friends ?
15. Any other suggestion/comment you would like to make for the improvement of the book.

Further, you can win a prize for the best criticism on presentation, contents or quality aspect of this book with useful suggestions for improvement. The prize will be awarded each month and will be in the form of our publications as decided by the Editorial Board.

Please feel free to write to us if you have any problem, complaint or grievance regarding our publications or a bright idea to share. We work for you and your success and your Feedback is valuable to us.

Thanking you, .

Yours faithfully

Sultan Chand & Sons

1

Logical Statements and Truth Tables

Structure

Objectives

After studying this chapter, you should be able to understand :

Logical statements, truth tables, negation, compounding, negation of compound statements, tautologies and fallacies, propositions.

Algebra of propositions.

- *Conditional and biconditional statements.*

Arguments and joint denial.

1·0. INTRODUCTION

Logical statements deal with the binary logic used extensively in modern mathematics. Based on Boolean Algebra, the logical statements introduce mathematical logic to the students. There is great relevance of these statements in modern mathematics because the basic philosophy behind the digital computers is based on two digits 1 and 0. The binary codes and logical circuits used in computers employ signals of true or false. The two stable states of an electrical switching circuit are ON and OFF. With these a large number of mathematical operations are performed through suitably designed circuits.

1·1. LOGICAL STATEMENTS

These are assertions in words or symbols which are either true or false but not both. The difference between an ordinary sentence and a logical statement is that it is not possible to say about truth or otherwise of an ordinary sentence whereas true or false is an essential requisite of a logical statement. For example, the expressions : Oh God ! ; How do you run so fast ? are sentences all right but they are not logical statements. The following are some logical statements :

(*i*) The number x is even.

(*ii*) $x^2-1\equiv(x+1)(x-1)$ for all values of x.

(*iii*) The sum of the three angles of a triangle is equal to two right angles.

(*iv*) Economics is a dismal science.

1·2. TRUTH TABLE

It is a table indicating the truth value of a compound statement constituting several statements. The statements are compounded by various connectives. *e.g.*, AND ($\wedge$), OR ($\vee$), NEGATION ($\sim$), etc., each one has a significance and therefore the truth of the compound statement has to be considered taking these into considerations.

A truth table has a number of columns and rows. The number of columns depends upon the number of constituent elements and how involved over their relationships. The initial columns, two or three depending on the constituent statements, deal with the basic truth values. These give all possible combinations of the constituent statements.

The remaining columns are the output columns giving the truth values of the compound statement as a function of certain relationship between constituent statements.

The number of rows in a truth table are determined on the basis of the constituent statements. In case of 2 constituent statements, there are 2^2 or 4 rows and for 3 constituent statements, there are 2^3 or 8 rows.

The truth tables are very useful in finding out the validity of a equivalence relation between functions. For practical purposes, they help in designing and testing the electronic circuits to perform a given operation based on a certain relationship.

1·3. NEGATION

To assert a statement is to say that it is true and to deny it is to say that it is false. Negation is the contradiction of the statement which may be either an assertion or a denial. If p is a statement then $\sim p$ (read as not p) will be its negation. But, if $\sim p$ is a statement then p will be its negation which we can also express as $\sim(\sim p)=p$. The following truth table shows how when p is true, $\sim p$ is false and vice-versa is also there.

Truth Table 1 : p or $\sim p$

p	$\sim p$
T	F
F	T

An electronic circuit showing this type of relationship is as follows :

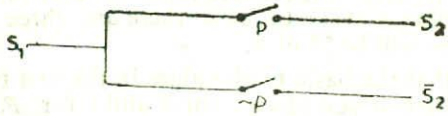

Fig. 1.

The above circuit shows that the electrical impulse passes from S_1 to S_2 in case of p and it will pass to $\bar{S}_2$ (S_2 bar) and not to S_2 in case of $\sim p$.

Double negative is positive which can be verified from the following truth table :

Truth Table 2 : $\sim(\sim p)$

p	$\sim p$	$\sim(\sim p)$
T	F	T
F	T	F

1·4. COMPOUNDING

The method of combining statements is known as compounding ; two or more constituent statements when combined into a joint statement is known as a compound statement. The common connectives used for the purpose are AND ($\wedge$), OR ($\vee$) and NOT ($\sim$). Actually the Boolean Algebra recognises only three operations by which a machine manages all other operations. Other connectives are also converted into these simple operators. The truth value of a compound statement will depend on the truth of the constituent statements. Compounding is done mainly through conjunction and disjunction.

(*i*) **Conjunction.** A joint statement to the effect that each constituent of the statement is true, is compounded by the use of the connective AND ($\wedge$). For example, if p stands for the statement "prices are rising" and q for "the quantity of money is increasing". Then the compound statement $p \wedge q$ indicates that "the prices are rising and the quantity of money is increasing." **A compound statement of two statements will be true only when both the constituent statements are true** and not when either of them is true or when both are not true. The truth value of a compound statement with conjunction will be as follows :

Truth Table 3 : $p \wedge q$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

The first thing to note in the above table is that with 2 statements there are 2^2 or 4 combinations. In case there are three statements, the possible combinations will be 2^3 or 8.

The second is that the basic truth values in the first two columns are 4, 3, 2, 1 in binary digits if you place 1 for T and 0 for F. In case there are three statements then the order will be 7, 6, 5, 4, 3, 2, 1, and 0. However, the order can be reversed there is no special sanctity attached to it

The third is that in the output column, which is the third column in the above table the truth of the compound statement is indicated by T. Wherever the alternative combinations are not in keeping with the relationship, F is written.

An electronic circuit in case of this operation will be in the same series so that the impulse from the initial point S_1 will not pass to the terminal point S_2 if either of the switches are open (*i.e.*, cff). See the circuit below.

Fig. 2.

(*ii*) **Disjunction**. A joint statement asserting that at least one of the constituent statements is true, so that more than one constituent statements can also be true, certainly all are not false is compounded by the use of 'either...or' or simply OR ($\vee$). **A compound statement of two statements will be true if either of them is true or both are true.** If p stands for the optional Maths. and q for the optional Statistics, $p \vee q$ will signify ;

p and q	Maths and Statistics
p and $\sim q$	Maths and not Statistics
q and $\sim p$	Statistics and not Maths.

This relationship will be brought out more clearly by the following truth table which may be compared with the previous one to grasp the difference.

Truth Table 4 : $p \vee q$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

As shown above $p \vee q$ is false only when both p and q are false. This obviously is an inclusive type of disjunction which covers also the situation when both are true.

The plain circuit depicting the relationship is given here under.

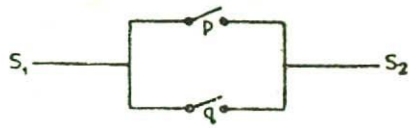

Fig. 3.

The electrical impulse from initial point S_1 will not pass to the terminal point S_2 only when both the switches p and q are open (*i.e.*, off).

The exclusive type of disjunction, which keeps out the situation when all the constituent statements are true, is indicated by $p \vee q$ or $p+q$. In this case the first row of the truth table 4 will have F in the output column. Its circuit will be of the following type :

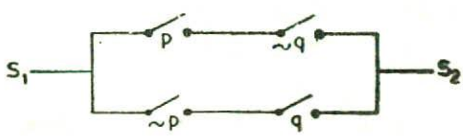

Fig. 4.

The exclusive type of disjunction which conveys the sense of "p or q but not both" is a special case of OR connective. Therefore, unless otherwise stated, $p \vee q$ will be taken in the inclusive sense which will always mean "p or q or both" or "p and/or q."

1·5. NEGATION OF COMPOUND STATEMENT

When a compound statement is negated its connective changes from AND ($\wedge$) to OR ($\vee$) and from OR ($\vee$) to AND ($\wedge$). For example,

$$\sim(p \wedge q) = \sim p \vee \sim q$$
$$\sim(p \vee q) = \sim p \wedge \sim q$$

We can say that the inversion of a function of several terms is obtained by inverting the individual terms and changing the connectives.

This is the famous De Morgan's theorem or law. It is very helpful in simplifying and rearranging the terms of a Boolean function which is sometimes better amenable to certain specialised operations.

The truth of the De Morgan's law can be verified from the following truth table :

Truth Table 5 : $\sim(p \wedge q) = \sim p \vee \sim q$

p (1)	q (2)	$p \wedge q$ (3)	$\sim(p \wedge q)$ (4)	$\sim p$ (5)	$\sim q$ (6)	$\sim p \vee \sim q$ (7)
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

The truth values of columns 4 and 7 are alike which proves the theorem.

The following are some relations based on the above law which can be verified by preparing truth tables :

(*i*) $\sim(p \wedge \sim q) = \sim p \vee \sim\sim q = \sim p \vee q$

(*ii*) $\sim(\sim p \wedge q) = \sim\sim p \vee \sim q = p \vee \sim q$

(*iii*) $\sim(\sim p \vee \sim q) = \sim\sim p \wedge \sim\sim q = p \wedge q$

Example 1. *Let p be the statement "the south-west monsoon is very good this year" and q be the statement "the rivers are rising". Give the verbal translations for (a) and verify the statement (b).*

(*a*) (*i*) $p \vee \sim q$ *and* (*ii*) $\sim(\sim p \vee \sim q)$

(*b*) *the statement* $x > 1 \Leftrightarrow x^2 > 1$ *is false.*

Solution. (*a*) (*i*) The south-west monsoon is very good but the rivers are not rising.

(*ii*) It is not true that the south-west monsoon is not very good or the rivers are not rising. We can also state that the south-west monsoon is very good and the rivers are rising.

(*b*) The statement is false because if $x^2 > 1$ then $x > 1 \vee x < -1$.

Some Illustrations :

Here are some statements along with their symbolic forms using p for practical and q for quiet,

He is practical and quiet,	$p \wedge q$
He is practical and not quiet,	$p \wedge \sim q$
It is false that he is practical and quiet,	$\sim(p \wedge q)$
He is neither quiet nor practical,	$\sim q \vee \sim p$

Here are some statements along with their symbolic forms using p for idealist and q for vocal,

He is neither idealist nor vocal, $\sim p \vee \sim q$
It is not true that he is neither idealist nor vocal, $\sim(\sim p \vee \sim q)$

Here are some verbal expressions for symbolic expressions where p stands for reading 'Patriot', q for reading 'Current', and r for reading 'Reader's Digest'.

$(p \vee q) \wedge \sim r$	He reads Patriot or Current but not Reader's Digest.
$\sim(p \wedge \sim r)$	It is not true that he reads Patriot but not Reader's Digest.
$(p \wedge q) \vee \sim(p \wedge r)$	He reads Patriot and Current or he does not read Patriot and Reader's Digest.
$\sim(\sim p \wedge \sim q)$	It is not true that he reads neither Patriot nor Current.
$p \vee q$	He reads either Patriot or Current.
$\sim \sim p$	It is not true that he does not read Patriot or he reads Patriot.

1·6. TAUTOLOGIES AND FALLACIES

Tautologies or theorems are like axioms which are true for all values. In a truth table of a tautology there will be only T in the last column. For example, $p \vee \sim p = p$, $p \wedge p = p$ and $\sim \sim p = p$ are all tautologies. As against these the fallacies are the contradictions which will never be true and obviously there will be only F in the output column of a truth table. For example, $p \wedge \sim p$ is a contradition, how can a statement and its negative both be true. These truth tables will further reveal this fact.

Truth Table 6 : $p \vee \sim p$

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

Truth Table 7 : $p \wedge \sim p$

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

Since tautology is always true its negation is a fallacy and is always false.

Example 2. *Verify the following statements by constructing truth tables :*

(*i*) $p \vee \sim(p \wedge q)$ *is a tautology.*
(*ii*) $(p \wedge q) \wedge \sim(p \vee q)$ *is a fallacy.*

Solution.

(*i*) **Truth Table 8 :** $p \vee \sim(p \wedge q)$

p	q	$p \wedge q$	$\sim(p \wedge q)$	$p \vee \sim(p \wedge q)$
(1)	(2)	(3)	(4)	(5)
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

Since there is T for all values of p, q in Column 5, it is a tautology.

(ii) **Truth Table 9 :** $(p \wedge q) \wedge \sim(p \vee q)$

p	q	$p \wedge q$	$p \vee q$	$\sim(p \vee q)$	$(p \wedge q) \wedge \sim(p \vee q)$
(1)	(2)	(3)	(4)	(5)	(6)
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

Since there is F for all values of p, q in Column 6, it is a fallacy.

Example 3. *Using truth tables, show that $(p \wedge q) \Rightarrow p$ and $p \Rightarrow (p \vee q)$ are both tautologies, where p, q are any two statements.*

(*C.A., Intermediate December 1981*)

Solution.

Truth Table 10

p	q	$p \wedge q$	$(p \wedge q) \Rightarrow p$	$p \vee q$	$p \Rightarrow (p \vee q)$
(1)	(2)	(3)	(4)	(5)	(6)
T	T	T	T	T	T
T	F	F	T	T	T
F	T	F	T	T	T
F	F	F	T	F	T

Since all the entries in Columns 4 and 6 are T, the given propositions are both tautologies.

1·7 PROPOSITIONS

Compound statements with repetitive use of connectives ($\wedge$, $\vee$, $\sim$, etc.) are called prepositions. For example, if we use p for popular and q for qualified then a few involved propositions will be :

He is popular and qualified.	$p \wedge q$
It is not true that he is popular and qualified.	$\sim(p \wedge q)$
He is neither popular nor qualified.	$\sim p \vee \sim q$
He is popular or he is unpopular and qualified.	$p \vee (\sim p \wedge q)$
It is not true that he is unpopular or unqualified.	$\sim(\sim p \vee \sim q)$

The truth tables for propositions will be constructed in the usual manner, however, it will be safe to procced step by step. We give below truth tables for :

(*i*) $p \wedge (q \vee r)$, and (*ii*) $(p \wedge q) \vee (p \wedge r)$

In the above propositions there are three constituent statements in each and they are equivalent. This will be verified by their respective output truth values in the final columns.

(*i*) **Truth Table 11** : $p\wedge(q\vee r)$

p (1)	q (2)	r (3)	$q\vee r$ (4)	$p\wedge(q\vee r)$ (5)
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

The basic truth values are *TTTT*, *FFFF* in the first column, *TT*, *FF*, *TT*, *FF* in the second column, and *T*, *F*, *T*, *F*, *T*, *F*, *T*, *F* in the third column.

Row-wise you can read the binary values, 7, 6, 5, 4, 3, 2, 1, 0 from first to the eighth row by substituting 1 for *T* and 0 for *F*. In the output column 5 the truth values are based on the truth values of columns 1 and 4 considering the connective AND($\wedge$) between them.

(*ii*) **Truth Table 12** : $(p\wedge q)\vee(p\wedge r)$

p (1)	q (2)	r (3)	$p\wedge q$ (4)	$p\wedge r$ (5)	$(p\wedge q)\vee(p\wedge r)$ (6)
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	F	F
F	T	T	F	F	F
F	T	F	F	F	F
F	F	T	F	F	F
F	F	F	F	F	F

The truth values in column 5 of Truth Table 11 and Column 6 of the Truth Table 12 are alike showing that the two statements are equal. It brings out the distributive property in logical statements.

1·8. ALGEBRA OF PROPOSITIONS

There are certain laws regulating manipulations on propositions. An understanding of these will help in finding out new relations and in establishing equivalence between certain relations. The following are the main laws.

I. *Idempotent law*. This indicates how a statement does not change its truth value by conjunction with another statement in certain circumstances :

(*a*) $p\vee p=p$ (*b*) $p\wedge p=p$

This law shows difference between ordinary algebra where $a+b$ will not be equal to 'a' unless a 0 ; also a . a will be equal to a^2 and not 'a' unless a 1. In logic p or q will be a true statement if there is p only. Also a repeated assertion of p as p and p' is p only in logic.

II. *Associative law.* This indicates that a new statement can be associated with a compound statement with the same connective either from the right or from the left as shown below :

(*a*) $(p \vee q) \vee r = p \vee (q \vee r)$

(*b*) $(p \wedge q) \wedge r = p \wedge (q \wedge r)$

III. *Commutative law.* As per this law order is irrelevant as shown below :

(*a*) $p \vee q = q \vee p$ (*b*) $p \wedge q = q \wedge p$

IV. *Distributive law.* It deals with expansion of a term having different connectives inside and outside the bracket containing a compound statement. Whereas in ordinary algebra the distribution was limited to multiplication over addition, *e.g.*, $a(b+c)=ab+ac$ while $a+(b.c) \neq (a+b).(a+c)$. But, in Boolean algebra it is possible since

(*a*) $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

(*b*) $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$.

Further, the distribution can be from either side, left or right.

V. *Identity law.* Identity elements in Boolean algebra are tautology (t) and fallacy (f) as there were 1 and 0 for multiplication and addition respectively in ordinary algebra. These relationships can be explained as follows :

(*a*) $p \vee t = t$ (*b*) $p \wedge t = p$

(*c*) $p \vee f = p$ (*d*) $p \wedge f = f$

VI. *Complement law.* There are complements in Boolean algebra which perform the same function as done by negative numbers for an additive inverse and reciprocals for a multiplicative inverse in ordinary algebra. These are :

(*a*) $p \vee \sim p \equiv t$ (*b*) $p \wedge \sim p \equiv f$

(*c*) $\sim t \equiv f$ (*d*) $\sim f \equiv t$

VII. *De Morgan's law.* It shows that a statement will not change if we change AND ($\wedge$) by OR ($\vee$) and OR ($\vee$) by AND ($\wedge$) provided that we have the inverses of the constituent statements. This has been shown below :

(*a*) $\sim(p \wedge q) \equiv \sim p \vee \sim q$ (*b*) $\sim(p \vee q) \equiv \sim p \wedge \sim q$

This law helps in stating a Boolean function from the "sum of the products" into the "product of the sums" by taking the complements of the terms.

This law helps simplification as follows :

$$\begin{aligned} \sim(p \vee q) \vee (\sim p \wedge q) &= (\sim p \wedge \sim q) \vee (\sim p \wedge q) \\ &= \sim p \wedge (\sim q \vee q) \\ &= \sim p \wedge t \\ &= \sim p \end{aligned}$$

1·9. CONDITIONAL STATEMENTS

These are statements of the type 'if you read then you will pass' in mathematical terms 'if p then q' or $p \rightarrow q$ or $p \quad q$. Here p is sufficient for q but not essential. There can be q even without p ; in other words, there can be a pass even without reading. Although p was not necessary for q, q is necessary for p. It will not happen that one who reads will fail. **The truth values of p ⇒ q are true when q is true or both q and p are false.** The true table of a conditional statement is as follows :

True Table 13 : $p \rightarrow q$

p	q	$p \quad q$
T	T	T
T	F	F
F	F	T
F	F	T

The table reveals that $p \rightarrow q$ is false only when p is there but q is not there, p is only sufficient cause but not necessary or essential. A conditional statement with connective of implication $(\rightarrow)$ can be expressed through OR and NOT as follows :

$$p \rightarrow q \equiv \sim p \vee q \text{ (i.e., not } p \text{ or } q)$$

Now, the conditional proposition $p \rightarrow q$ has its negation as follows :

$$\sim(p \rightarrow q) \equiv \sim(\sim p \vee q) \equiv \sim\sim p \wedge \sim q \equiv p \wedge \sim q$$

Example 4. *Simplify* $\sim(\sim p \rightarrow \sim q)$.

Solution. $\sim(\sim p \rightarrow \sim q) \equiv \sim(p \vee \sim q) \equiv \sim p \wedge \sim\sim q \equiv \sim p \wedge q$.

In such problems it is always easier to substitute $\sim p \vee q$ for $p \rightarrow q$ or $p \vee \sim q$ for $\sim p \rightarrow \sim q$ as has been done in the above case.

Example 5. *Write the following statements in symbolic form and give their negations* :

(*i*) *If he works hard he will pass the examination.*

(*ii*) *If it rains he will not go for a walk.*

Solution. (*i*) Using p for hard work and q for a pass in examination the statement is $p \rightarrow q$

Its negation is $\quad \sim(p \rightarrow q) = \sim(\sim p \vee q) = p \wedge \sim q$
which in words is : Even if he works hard he may not pass.

(*ii*) Using p for rain and q for walk the symbolic expression is

$$p \rightarrow \sim q$$

Its negation is : $\quad \sim(p \rightarrow \sim q) = \sim(\sim p \vee \sim q) = p \wedge q$
which in words is : Even if it rains he goes for a walk.

Example 6. *In a certain country, it is found that weather follows the following rules* :

If it is fine today, then it is windy tomorrow.

If it is calm today, then it is hot tomorrow.

If it is fine tomorrow, then it is cold tomorrow.

Each day is either hot or cold, wet or fine and calm or windy. Forecast tomorrow's weather if today is fine, calm and cold.

(*C.A. Intermediate December, 0891*)

Solution. The argument is as follows :

(*i*) Fine today $\rightarrow$ windy tomorrow.

If today is fine therefore tomorrow is windy

(*ii*) Calm today $\rightarrow$ hot tomorrow.

If today is calm therefore tomorrow is hot.

(*iii*) Fine tomorrow $\rightarrow$ cold tomorrow

$\equiv \sim$(cold tomorrow) $\rightarrow \sim$(fine tomorrow)

$\equiv$hot tomorrow $\rightarrow$ wet tomorrow.

$\therefore$ The forecast of tomorrow's weather is hot, wet and windy.

Neither converse nor an inverse of a conditional statement is identical, converse inverse and contrapositive of a conditional statement is identical see below :

Converse : $q \rightarrow p \neq p \rightarrow q$

Inverse : $\sim p \rightarrow \sim q \neq p \rightarrow q$

Converse-inverse or contrapositive : $\sim q \rightarrow \sim p \equiv p \rightarrow q$

The validity of the above three statements can be proved by the following truth table :

Truth Table 14 : $p \rightarrow q = \sim q \rightarrow \sim p$

p	q	*Conditional* $p \rightarrow q$	*Converse* $q \rightarrow p$	*Invesse* $\sim p \rightarrow \sim q$	*Converse Inverse* $\sim q \rightarrow \sim p$
(1)	(2)	(3)	(4)	(5)	(6)
T	*T*	*T*	*T*	*T*	*T*
T	*F*	*F*	*T*	*T*	*F*
F	*T*	*T*	*F*	*F*	*T*
F	*F*	*T*	*T*	*T*	*T*

The table reveals that the truth values of $p \rightarrow q$ and $\sim q \rightarrow \sim p$ are alike and truth values of $q \rightarrow p$ and $\sim p \rightarrow \sim q$ are alike. Therefore, a statement of logical implication is not identical with either its converse or its inverse but is identical with its contrapositive or the converse inverse.

Example 7. *Prove by means of a truth table that*

$(p \wedge q) \Rightarrow (p \vee q)$ *is a tautology but* $(p \vee q) \Rightarrow (p \wedge q)$ *is not.*

Solution.

Truth Table 15 : $(p \wedge q) \Rightarrow (p \vee q)$ is not the same as $(p \vee q) \Rightarrow (p \wedge q)$

p	q	$(p \wedge q)$	$(p \vee q)$	$(p \wedge q) \Rightarrow (p \vee q)$	$(p \vee q) \Rightarrow (p \wedge q)$
(1)	(2)	(3)	(4)	(5)	(6)
T	*T*	*T*	*T*	*T*	*T*
T	*F*	*F*	*T*	*T*	*F*
F	*T*	*F*	*T*	*T*	*F*
F	*F*	*F*	*F*	*T*	*T*

$\therefore$ Column 5 shows that $(p \wedge q) \Rightarrow (p \vee q)$ is a tautology and column 6 shows that $(p \vee q) \Rightarrow (p \wedge q)$ is not a tautology.

Example 8. *With the help of a truth table, prove that*

$p \Rightarrow (q \wedge r) \equiv (p \Rightarrow q) \wedge (p \Rightarrow r)$ (*C.A., Entrance, June 1984*)

Solution.

Truth Table 16

p	q	r	$q \wedge r$	$p \Rightarrow (q \Rightarrow r)$	$p \Rightarrow q$	$p \Rightarrow r$	$(p \Rightarrow q) \wedge (p \Rightarrow r)$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
T	*T*	*T*	*T*	*T*	*T*	*T*	*T*
T	*T*	*F*	*F*	*F*	*T*	*F*	*F*
T	*F*	*T*	*F*	*F*	*F*	*T*	*F*
T	*F*	*F*	*F*	*F*	*F*	*F*	*F*
F	*T*	*T*	*T*	*T*	*T*	*T*	*T*
F	*T*	*F*	*F*	*T*	*T*	*T*	*T*
F	*F*	*T*	*F*	*T*	*T*	*T*	*T*
F	*F*	*F*	*F*	*T*	*T*	*T*	*T*

Since the entries in columns No. 5 and 8 are identical, we have

$p \Rightarrow (q \wedge r) \equiv (p \Rightarrow q) \wedge (p \Rightarrow r)$

1·10. BICONDITIONAL STATEMENTS

The conditional statements are of the type "if p then q", *i.e.*, $p \rightarrow q$. The biconditional statements are of the type "if p then q and if q then p" *i.e.*, $p \rightarrow q$ and $q \rightarrow p$ written as $p \leftrightarrow q$. These are also called double implications or equivalent, statements. The essential features of this relationship are :

(*i*) p if and only if q,

(*ii*) q if and only if p,

(*iii*) p is an essential condition for q, and

(*iv*) q is an essential condition for p.

The truth values of p↔q are true when both p and q are true or when both p and q are false.

You can appreciate the difference between conditional and biconditional statements by the following truth table :

Truth Table 17 : $p \rightarrow q \neq p \leftrightarrow q$

p	q	$p \rightarrow q$	$p \leftrightarrow q$
(1)	(2)	(3)	(4)
T	*T*	*T*	*T*
T	*F*	*F*	*F*
F	*T*	*T*	*F*
F	*F*	*T*	*T*

Example 9. *Prove by means of a truth table that*

$p \leftrightarrow q = (p \rightarrow q) \wedge (q \rightarrow p)$

Solution.

Truth Table 18 : $p \leftrightarrow q = (p \to q) \wedge (q \to p)$

p (1)	q (2)	$p \leftrightarrow q$ (3)	$p \to q$ (4)	$q \to p$ (5)	$(p \to q) \wedge (q \to p)$ (6)
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

The like truth values of columns 3 and 6 prove the validity of the statement.

The negation of conditional and biconditional statements are as follows :

(i) $\sim(p \Rightarrow q) = \sim(\sim p \vee q) = p \wedge \sim q$

(ii) $\sim(p \leftrightarrow q) = \sim(\sim p \leftrightarrow \sim q) = p \leftrightarrow \sim q$

$= \sim(\sim q \leftrightarrow \sim p) = q \leftrightarrow \sim p$

Example 10. *Prove by means of a truth table that*

(i) $\sim(p - q) = p \wedge \sim q$

(ii) $\sim(p \leftrightarrow q) = \sim p \leftrightarrow q = p \leftrightarrow \sim q$

Solution.

(i) **Turth Table 19** : $\sim(p \to q) = p \wedge \sim q$

p (1)	q (2)	$p \to q$ (3)	$\sim(p \to q)$ (4)	$\sim q$ (5)	$p \wedge \sim q$ (6)
T	T	T	F	F	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	F	T	F

The truth values of columns 4 and 6 are alike.

$\therefore \quad \sim(p \to q) = p \wedge \sim q.$

(ii) **Turth Table 20** : $\sim(p \leftrightarrow q) = \sim p \leftrightarrow q = p \leftrightarrow \sim q$

p (1)	q (2)	$p \leftrightarrow q$ (3)	$\sim(p \to q)$ (4)	$\sim p$ (5)	$\sim p \leftrightarrow q$ (6)	$\sim q$ (7)	$p \leftrightarrow \sim q$ (8)
T	T	T	F	F	F	F	F
T	F	F	T	F	T	T	T
F	T	F	T	T	T	F	T
F	F	T	F	T	F	T	F

The truth values of columns 4, 6 and 8 being alike prove the equivalence of the three statements.

$\therefore \quad \sim(p \leftrightarrow q) = \sim p \leftrightarrow q = p \leftrightarrow \sim q.$

Example 11. *Prove that* $p \to (q \wedge r) = (p \to q) \wedge (p \to r)$.

Solution.

Truth Table 21 : $p\to(q\wedge r)=(p\to q)\wedge(p\to r)$

p (1)	q (2)	r (3)	$(q\wedge r)$ (4)	$p\to(q\wedge r)$ (5)	$p\to q$ (6)	$p\to r$ (7)	$(p\to q)\wedge(p\to r)$ (8)
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	F	F	T	F
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

The truth values of columns 5 and 8 are alike proving the equivalance of the two statements.

$\therefore \quad p\to(q\wedge r)=(p\to g)\wedge(p\to r).$

Example 12. *Prove that if $p\to q\wedge q\to r$ then $p\to r$.*

Solution.

Truth Table 22 : $[p\to q\wedge q\to r]\Rightarrow[p\to r]$

p (1)	q (2)	r (3)	$(p\to q)$ (4)	$(q\to r)$ (5)	$(p\to r)$ (6)	$[p\to q\wedge q\to r]$ (7)	$[p\to q\wedge q\to r]\Rightarrow[p-r]$ (8)
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

All T's in column 8 show that the statement is a tautology ; in other words, all possible combinations are true. This is called the law of syllogism which is there in logic also.

Example 13. *Use a truth table to prove that $p\to q$ is equivalent to $\sim p\vee q$, where p and q are statements.*

A firm of Chartered Accountants makes the following declaration : An articled clerk from the firm passing the final C.A. Examination in the first attempt will be awarded a prize of Rs. 100 ! Five clerks P, Q, R, S, T appeared for the first time from the firm and only P and O could pass. The firm awards prizes not only to them but to R and S also. Is this action logically justified ? T claims the prize comparing himself with R and S but the firm refuses. Is the refusal logically justified ? How should the statement be worded so that only P and Q will be entitled for the prize ?

Solution. The statement is a conditional statement of the type "if a clerk passes at the C.A. Examination in the first attempt then he shall be awarded a prize of Rs. 100". Symbolically if p then q or $p\to q$. This does not exclude others because the statement will be true even when q is true and p is false. The following truth table will bring it out :

Truth Table 23 : $p \to q \equiv \sim p \vee q$

p	q	$p \to q$	$\sim p$	$\sim p \vee q$
(1)	(2)	(3)	(4)	(5)
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

The truth values of columns 3 and 5 are alike. The truth set of $p \to q = [TT, FT$ and $FF]$. Only in one case where p is true and q is false, the relationship will be invalid, *i.e.*, the statement will be invalid only if the prize is declined to P and Q who have passed in the first attempt but they cannot grudge the prize being given to others who have not passed. To exclude others, the statement should have been of the type "only those who pass the examination in the first attempt, shall be awarded the prize", *i.e.*, "p if and only if q".

Example 14. "*If my brother stands first in the class, I will give him a watch. Either he stood first or I was out of station. I did not give my brother a watch this time. Therefore, I was out of station.*"

Solution. The first statement of promise is of the type $p \to q$,

the second is the possibility of the type $p \vee r$, and

the third is the conclusion of the type $\sim q \to r$.

From the conclusion that he has not given the watch this time implies that he has not stood first $(\sim q \to \sim p)$. But, since there is a possibility of his brother standing first or he being out of station which is true in all cases expect when both are not true, which is not true because as is said he was out of station but this does not prove that his brother stood first. Therefore, from the conclusion it implies that his brother did not stand first.

Negation of a Biconditional Statement. The negation of a biconditional or an equivalent statement is all the more interesting.

$$\sim (p \leftrightarrow q) \equiv \sim p \leftrightarrow q \equiv p \to \sim q$$

Now to understand this type of statements let us take a statement :

He goes aborad if and only if he has a passport, *i.e.*, $(p \leftrightarrow q)$

Its negation can be

(*i*) He goes abroad without obtaining a passport, *i e.*, $p \leftrightarrow \sim q$.

(*ti*) He does not go abroad having obtained the passport, *i.e.*, $\sim p \leftrightarrow q$.

For validity of these statements see Truth Table 20.

1·11. ARGUMENTS

These are assertions of the type that a given set of premises yield a given conclusion as a result thereof. These are expressed as

$$P_1, P_2, \cdots \; P_n \vdash Q \text{ [sign } \vdash \text{ is spoken as turnstile]}$$

or $$P_1 \wedge P_2 \wedge \ldots\ldots \wedge P_n \vdash Q$$

Such an argument is valid when Q is true all the premises P_1, $P_2, \cdots, P_n$ are true If in the same truth table there is a possibility of P_1, P_2. being true but the conclusion is not true, it will be a fallacy.

The validity can also be judged by the relationship $P_1 \wedge P_2 \wedge \ldots \wedge P_n \to Q$ provided it is a tautology, *i.e.*, all the possible combination yield T in the output column.

In arguments the "law of detachment" applies. Under this law if $p \to q$ then both $p \wedge (p \to q) \vdash q$. This can be tested by the following truth table :

Truth Table 24 : $p \wedge (p \to q) \vdash q$

p	q	$p \to q$	$p \wedge (p \to q)$	$p \wedge (p \to q) \vdash q$
(1)	(2)	(3)	(4)	(5)
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

The table reveals in the first row of the fourth column that where q is true, both p and $p \to q$ are true and there is no other situation where both p and $p \to q$ are true but q is not true. The argument is thus valid. The fifth column shows that $[p \wedge (p \to q)] \to q$ is a tautology which more surely proves the validity of the argument.

Look at another argument which looks as if it is valid but it is not.

But the law of detachment does not apply in the argument $p \to q \wedge q \vdash p$ so it is not valid as shown in the following truth table :

Truth Table 25 : $(p \to q) \wedge q \vdash p$

p	q	$p \to q$	$p \to q \wedge q$	$(p \to q) \wedge q \vdash p$
(1)	(2)	(3)	(4)	(5)
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

The above table shows in the first row of column 4 that p is true when $p \to q$ and q are true but again in the third row of the same column it shows that p is false when both $p \to q$ and q are true. This shows that the argument is not valid. Applying the other test we see in the fifth column that $p \to q \wedge q \vdash p$ is not a tautology.

The basic thing to remember in the law of detachment is that $p \to q$ is not the same as $q \to p$. In $p \to q$, even when q is true p need not be true ; p is not a necessary condition for q Therefore, we can detach q but not p in the same manner.

Another law which applies to arguments is the famous "law of syllogism". According to this law, if $p \to q$ and $q \to r \vdash p \to r$. This has been proved earlier.

Example 15. *If he works hard then he will be successful.*

If he is successful then he will be happy.

Therefore, hardwork leads to happiness.

Solution. If we use p for the statement 'he works hard' and q for the statement 'he will be successful' and r for the statement 'he will be happy' then the argument runs as follows :

$$p \rightarrow q \wedge q \rightarrow r \vdash p \rightarrow r$$

which can be proved by a truth table. (See table 22).

Example 16. *If it rains then the crop will be good. It did not rain therefore the crop will not be good.*

Solution. The argument in symbolic from can be stated as :

$$(p \rightarrow q) \wedge \sim p \vdash \sim q$$

or

$$[(p \rightarrow q) \wedge \sim p] \rightarrow \sim q$$

The argument is not valid see the truth table.

Truth Table 26 : $[(p \rightarrow q) \wedge \sim p] \rightarrow \sim q$

p	q	$p \rightarrow q$	$\sim p$	$(p \rightarrow q) \wedge \sim p$	$\sim q$	$[(p \rightarrow q) \wedge \sim p] \rightarrow \sim q$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
T	T	T	F	F	F	T
T	F	F	F	F	T	T
F	T	T	T	T	F	F
F	F	T	T	T	T	T

This third row of column 7 shows that both $p \rightarrow q$ and $\sim p$ are true but $\sim q$ is not true which proves the fallacy. Also the argument $[(p \rightarrow q) \wedge \sim p] \rightarrow \sim q$ is not a tautology as shown by the column 7.

Example 17. *If it rains then the crops are good and crops were not good, therefore, it did not rain.*

Solution. The argument in symbolic form can be stated as follows :

$$(p \rightarrow q) \wedge \sim q \vdash \sim p$$

or

$$[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$$

The argument is valid and can be proved by the following truth table :

Truth Table 27 ; $[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$

p	p	$p \rightarrow q$	$\sim q$	$[(p \rightarrow q) \wedge \sim q]$	$\sim p$	$[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
T	T	T	F	F	F	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

Column 7 proves the validity of the argument, it is a tautology.

1·12. JOINT DENIAL

A new connective ($\downarrow$) called a propositional connective for joint denial, *i.e.*, "neither......nor" can be used to substitute the common connectives $\wedge$, $\vee$, and negation. The connective lengthens the statement but enables substitution of a number of connectives by one only. Since speed is not a problem in computer, the simplification of operations simplifies also the machine design and economises the use of costly hardware. Now $p \downarrow q$ will be read as neither p nor q. The truth table for this is as follows :

Truth Table 28 : $p \downarrow q \equiv \sim p \wedge \sim q$

p (1)	q (2)	$\sim p$ (3)	$\sim q$ (4)	$\sim p \wedge \sim q$ (5)	$p \downarrow q$ (6)
T	T	F	F	F	F
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

The truth values of columns 5 and 6 are alike, The way the joint denial is used for some common connectives has been shown below :

(*i*) $\sim p \equiv p \downarrow p$

(*ii*) $\sim q \equiv q \downarrow q$

(*iii*) $p \wedge q \equiv (\sim p \downarrow \sim q) = (p \downarrow p) \downarrow (q \downarrow q)$ derived from (*i*) and (*ii*)

(*iv*) $p \vee q \equiv \sim (p \downarrow q) = (p \downarrow q) \downarrow (p \downarrow q)$

(*v*) $p \to q \equiv \sim (\sim p \downarrow q)$

$\equiv \sim [(p \sim p) \downarrow q]$

$\equiv \{[(p \downarrow p) \downarrow q] \downarrow [(p \downarrow p) \downarrow q]\}$

The truth tables to prove (*iii*) and (*iv*) above are given below :

Truth Table 29 : $p \wedge q \equiv (p \downarrow p) \downarrow (q \downarrow q)$

p (1)	q (2)	$p \wedge q$ (3)	$p \downarrow q$ (4)	$q \downarrow q$ (5)	$(p \downarrow p) \downarrow (q \downarrow q)$ (6)
T	T	T	F	F	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	F	T	T	F

If may be noticed that $p \downarrow p$ is like $\sim p$ and $q \downarrow q$ is like $\sim q$. The similar truth values of columns 3 and 6 reveal that

$$p \wedge q \equiv (p \downarrow p) \downarrow (q \downarrow q)$$

Truth Table 30 : $p \vee q \equiv (p \downarrow q) \downarrow (p \downarrow q)$

p (1)	q (2)	$p \vee q$ (3)	$p \downarrow q$ (4)	$(p \downarrow q) \downarrow (p \downarrow q)$ (5)
T	T	T	F	T
T	F	T	F	T
F	T	T	F	T
F	F	F	T	F

It may be noticed that $p \downarrow q$ is the negation of $p \vee q$. The similar truth values of columns (3) and (5) prove that $p \vee q \equiv (p \downarrow q) \downarrow (p \downarrow q)$.

EXERCISES

1. Which of the following sentences are logical statements :

(*i*) Taj is at Agra
(*ii*) What do you feel about family planning programme ?
(*iii*) Please mind your business.
(*iv*) Ram is a sincere chap.
(*v*) Where are you going ?

2. Express the following compound statements in words taking p for hard work, q for success and r for job :

(*i*) $p \wedge q \wedge \sim r$
(*ii*) $\sim p \wedge \sim q$
(*iii*) $p \wedge \sim q$
(*iv*) $\sim p \wedge \sim q \wedge r$
(*v*) $\sim (p \vee q) \wedge r$
(*vi*) $p \to q$
(*vii*) $\sim q \to \sim p$

3. Express the following statements through appropriate symbols :

(*i*) It is raining but not pleasant.
(*ii*) It is not raining still it is pleasant.
(*iii*) Either there is a rain or the weather is pleasant.
(*iv*) It is neither raining nor pleasant.
(*v*) It is either raining or not raining and pleasant.
(*vi*) It is not true that it is not raining or not pleasant.

4. State which of the following statements are contrary and contradictory :

(*i*) It is a hot day ; It is a rainy day.
(*ii*) x is an odd number ; x is an even number.
(*iii*) Ram is a truthful person ; Ram is a liar.

5. Construct truth tables for the following and write their truth sets :

(*i*) $\sim (p \vee q)$
(*ii*) $\sim p \wedge q$
(*iii*) $\sim (p \wedge q)$
(*iv*) $p \to [(q \vee r) \wedge \sim (p \leftrightarrow \sim r)]$

6. Write which of these is a tautology or a fallacy :

(*i*) $p \wedge \sim p$
(*ii*) $p \vee \sim p$
(*iii*) $p \vee \sim (p \wedge q)$
(*iv*) $(p \wedge q) \wedge \sim (p \wedge p)$

7. Prove by the use of truth tables if the following identities are true :

(*i*) $p \vee q \equiv \sim (\sim p \wedge \sim q)$
(*ii*) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
(*iii*) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
(*iv*) $p \vee q \equiv (p \vee q) \wedge \sim (p \wedge q)$
(*v*) $p \to (q \wedge r) \equiv (p \to q) \wedge (p \to r)$
(*vi*) If $(p \to q)$ and $(q \to r)$ then $(p \to r)$
(*vii*) $(p \to q) \wedge (q \to p) \equiv (p \leftrightarrow q)$

8. Indicate the relevant law operating in the following propositions

(i) $(p \vee q) \wedge \sim p \equiv \sim p \wedge (p \vee q)$

(ii) $\sim (p \wedge q) \equiv \sim p \vee \sim q$

(iii) $\sim p \wedge (\sim q \vee q) \equiv (\sim p \wedge \sim q) \vee (\sim p \wedge q)$

(iv) $p \wedge t \equiv p$

(v) $\sim p \wedge t \equiv \sim p$

(vi) $f \vee (\sim p \wedge q) \equiv \sim p \wedge q$

(vii) $(\sim p \wedge p) \vee (\sim p \wedge q) \equiv f \vee (\sim p \wedge q)$

(vii) $\sim p \wedge (\sim q \vee q) \equiv \sim p \wedge t$

9. Prove the following equivalence relations by the use of algebra of propositions :

(i) $p \wedge (p \vee q) \equiv p$

(ii) $(p \vee q) \wedge \sim p \equiv \sim p \wedge q$

(iii) $\sim (p \vee q) \vee (\sim p \wedge q) \equiv (\sim p \wedge \sim q) \vee (\sim p \wedge q)$

10. Write the following statements in compound form and then give their negation :

(i) If it is cold he takes tea and not cold drink.

(ii) If he get a high first class he will go for MBA or Chartered Accountancy.

ANSWERS

1. (i) and (iv) only are logical statements.

2. (i) He works hard and was successful in examination but could not get a job.

(ii) He did not work hard and could not succeed.

(iii) He worked hard but could not succeed.

(iv) He did not work hard nor could he succeed but got a job.

(v) It is not true that he worked hard or was successful but got a job.

(vi) If you work hard you will be successful.

(vii) If you are not successful then you have not worked hard.

3. (i) $p \wedge \sim q$ (iv) $\sim p \wedge \sim q$

(ii) $\sim p \wedge q$ (v) $p \vee \sim p \wedge q$

(iii) $p \vee q$ (vi) $\sim (\sim p \vee \sim q)$

4. (i) and (ii) are contrary but not contradictory because although both cannot be true both can be false.

(iii) is contradictory where both can neither be true nor false.

5. Answer for (iv) only is given :

Truth Table : $p \to [(q \vee r) \wedge \sim(p \leftrightarrow \sim r)]$

p	q	r	$q \vee r$	$\sim r$	$(p \leftrightarrow \sim r)$	$\sim(p \leftrightarrow \sim r)$	$(q \vee r) \wedge \sim(p \leftrightarrow \sim r)$	$p \to [(q \vee r) \wedge \sim(p \leftrightarrow \sim r)]$
(1)	(2)	(3)	(4)	(5)	(6)	(7)		(9)
T	T	T	T	F	F	T	T	T
T	T	F	T	T	T	F	F	F
T	F	T	T	F	F	T	T	T
T	F	F	F	T	T	F	F	F
F	T	T	T	F	T	F	F	T
F	T	F	T	T	F	T	T	T
F	F	T	T	F	T	F	F	T
F	F	F	F	T	F	T	F	T

6. (*ii*) and (*iii*) are tautologies.

(*i*) and (*iv*) are fallacies.

8. (*i*) Commutative law.

(*ii*) De Morgan's law.

(*iii*) Distributive law.

(*iv*), (*v*) and (*vi*) Identity law.

(*vii*) and (*viii*) Complement law.

9. (*i*) $p \wedge (p \vee q) \equiv (p \vee f) \wedge (p \vee q)$ Identity law

$\equiv p \vee (f \wedge q)$ Distributive law

$\equiv p \vee f$ Complement law

$\equiv p$ Identity law.

(*ii*) $(p \vee q) \wedge \sim p \equiv \sim p \vee (p \vee q)$ Commutative law

$\equiv (\sim p \wedge p) \vee (\sim p \wedge q)$ Distributive law

$\equiv f \vee (\sim p \wedge q)$ Complement law

$\equiv \sim p \wedge q$ Identity law

(*iii*) $\sim(p \vee q) \vee (\sim p \wedge q) \equiv (\sim p \wedge \sim q) \vee (\sim p \wedge q)$ De Morgan's law

$\equiv \sim p \wedge (\sim q \vee q)$ Distributive law

$\equiv \sim p \wedge t$ Complement law

$\equiv \sim p$ Identity law.

10. (*i*) $p \to (q \wedge \sim r)$ its negation is $\sim[p \to (q \wedge \sim r)]$ which can also be expressed as $p \wedge \sim(q \wedge \sim r) \equiv p \wedge \sim q \vee r$

In words : In cold he takes neither tea nor cold drink.

(*ii*) $p \to (q \vee r)$ its negation is $\sim[p \to (q \vee r)]$ which can also be expressed as

$\sim[\sim p \vee (q \vee r)] \equiv p \wedge \sim(q \vee r)$

$\equiv p \wedge \sim q \wedge \sim r$

In words : In case he gets a high first class he will go neither for MBA nor for Chartered Accountancy.