A

Balanced Three-Phase System

A.1 INTRODUCTION

This appendix is intended as a brief review of voltage, current, and power relationships in the three-phase system. The student is expected to have a working knowledge of phasors and complex numbers. A very detailed development of phasors, complex numbers, resonance, single-phase and three-phase, balanced and unbalanced three-phase circuits, and power measurement is available in power-oriented circuits texts such as those listed in the references.

A.2 LETTER DESIGNATIONS FOR VOLTAGES AND CURRENTS

Voltages and currents that are functions of time are expressed in terms of the following equations, where $\omega = 2\pi f$.

$$\begin{aligned} e &= E_{\max}\sin(\omega t + \theta_e) \\ v &= V_{\max}\sin(\omega t + \theta_v) \\ i &= I_{\max}\sin(\omega t + \theta_i) \end{aligned} \tag{A–1}$$

The corresponding root mean square values, also called rms or effective values, are expressed as

$$\begin{aligned} E &= \frac{E_{\max}}{\sqrt{2}} \\ V &= \frac{V_{\max}}{\sqrt{2}} \\ I &= \frac{I_{\max}}{\sqrt{2}} \end{aligned} \tag{A–2}$$

The complex number representations of phasors corresponding to the sinusoidal quantities in equation set (A–1) are expressed as

$$\begin{aligned}\mathbf{E} &= E\angle\theta_e\\ \mathbf{V} &= V\angle\theta_v\\ \mathbf{I} &= I\angle\theta_i\end{aligned} \qquad \text{(A–3)}$$

The letters *e, E,* and **E** are generally used to represent voltage sources, and the letters *v, V,* and **V** are generally used to represent voltage drops or potential differences between two points.[1]

A.3 SERIES-CONNECTED CIRCUIT ELEMENTS

A circuit diagram, phasor diagram, and impedance diagram for the general case of series-connected circuit elements are shown in Figure A.1. The associated voltage, current, and impedance relationships are

$$\mathbf{Z}_s = R + jX_L - jX_C = Z_S\angle\theta_Z \qquad \text{(A–4)}$$

where $X_L = 2\pi fL$ and $X_C = 1/(2\pi fC)$.

$$Z_S = \sqrt{R^2 + (X_L - X_C)^2} \qquad \text{(A–5)}$$

$$\theta_Z = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \qquad \text{(A–6)}$$

$$\begin{aligned}\mathbf{V}_T &= \mathbf{V}_R + \mathbf{V}_L + \mathbf{V}_C\\ \mathbf{V}_T &= \mathbf{I}_T\mathbf{Z}_S = \mathbf{I}_T(R + jX_L - jX_C)\\ \mathbf{I}_T &= \frac{\mathbf{V}_T}{\mathbf{Z}_S} = I_T\angle\theta_i\end{aligned} \qquad \text{(A–7)}$$

The voltage drop across any one of two or more series-connected impedances may be determined by applying the *voltage-divider equation.* Referring to Figure A.2,

$$\mathbf{V}_k = \mathbf{V}_T \cdot \frac{\mathbf{Z}_k}{\mathbf{Z}_S} \qquad \text{(A–8)}$$

where $\mathbf{Z}_S = \mathbf{Z}_1 + \mathbf{Z}_2 + \cdots + \mathbf{Z}_k + \cdots + \mathbf{Z}_n$.

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[1] **Boldfaced type** is used to indicate complex quantities such as phasor current, phasor voltage, impedance, admittance, and complex power.

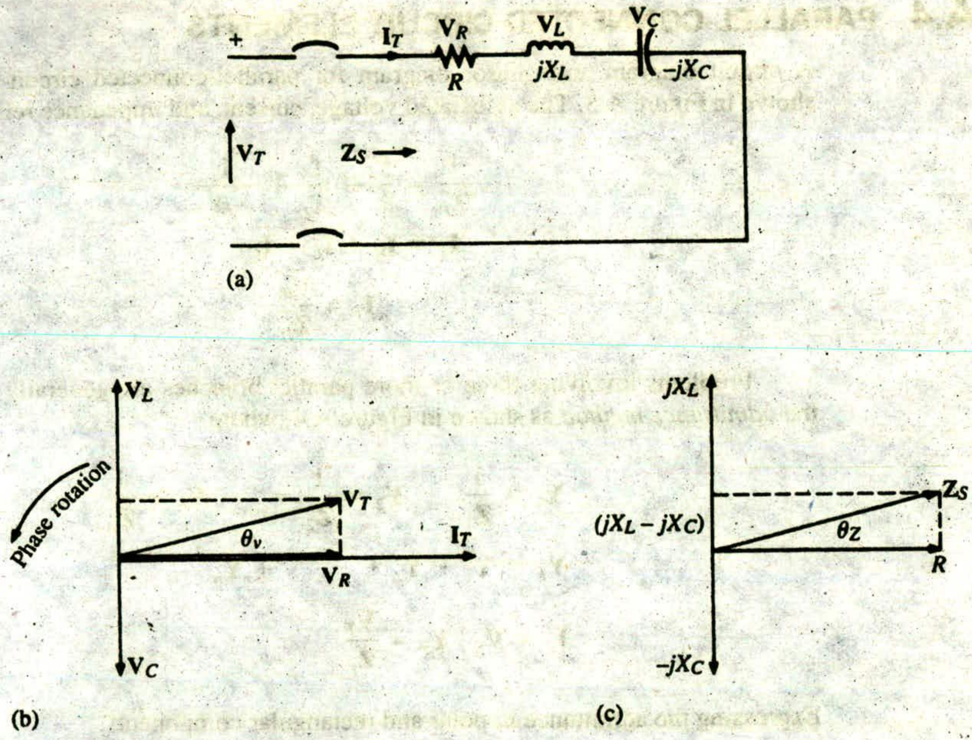
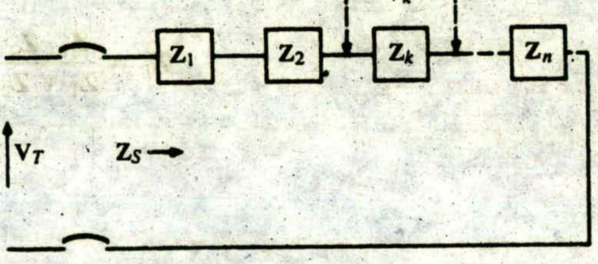


FIGURE A.1
(a) Series circuit; (b) phasor diagram; (c) impedance diagram.

FIGURE A.2
Circuit for voltage-divider equation.



A.4 PARALLEL-CONNECTED CIRCUIT ELEMENTS

A circuit diagram and phasor diagram for parallel-connected circuit elements are shown in Figure A.3. The associated voltage, current, and impedance relationships are

$$\frac{1}{\mathbf{Z}_P} = \frac{1}{R} + \frac{1}{jX_L} + \frac{1}{-jX_C} \tag{A–9}$$

$$\mathbf{I}_T = \mathbf{I}_R + \mathbf{I}_L + \mathbf{I}_C$$

$$\mathbf{I}_T = \frac{\mathbf{V}_T}{\mathbf{Z}_P}$$

Problems involving three or more parallel branches are generally solved using the *admittance method* as shown in Figure A.4, where

$$\mathbf{Y}_1 = \frac{1}{\mathbf{Z}_1} \qquad \mathbf{Y}_2 = \frac{1}{\mathbf{Z}_2} = \qquad \mathbf{Y}_n = \frac{1}{\mathbf{Z}_n} \tag{A–10}$$

$$\mathbf{Y}_P = \mathbf{Y}_1 + \mathbf{Y}_2 + \cdots + \mathbf{Y}_n \tag{A–11}$$

$$\mathbf{I}_T = \mathbf{V}_T \cdot \mathbf{Y}_P = \frac{\mathbf{V}_T}{\mathbf{Z}_P} \tag{A–12}$$

Expressing the admittance in polar and rectangular components,

$$\mathbf{Y} = Y \underline{/\theta_y} = G + jB \tag{A–13}$$

where: Y = admittance in siemens (S)
G = conductance in siemens (S)
B = susceptance, in siemens (S)

The special case for two impedances in parallel reduces to the following much used and easily calculated formula:

$$\mathbf{Z}_P = \frac{\mathbf{Z}_1 \cdot \mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} \tag{A–14}$$

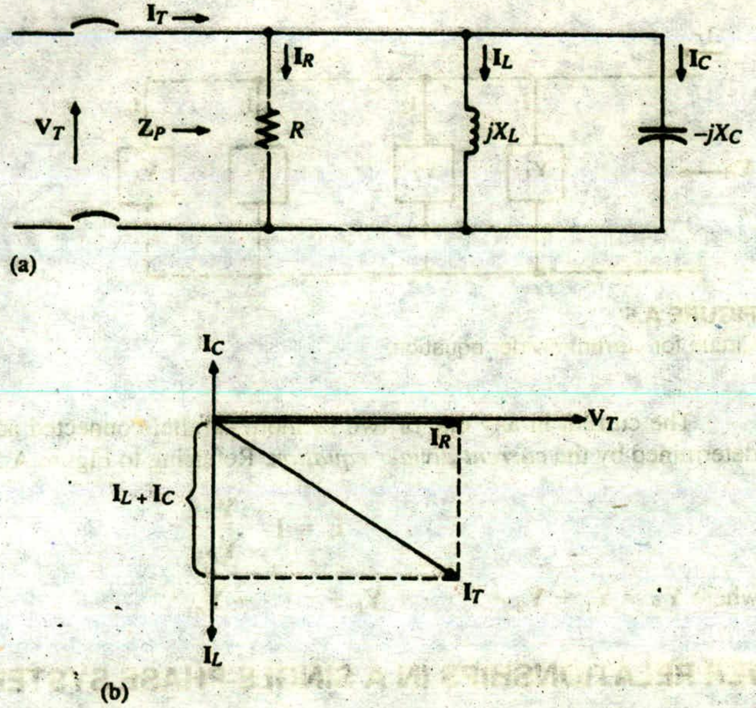
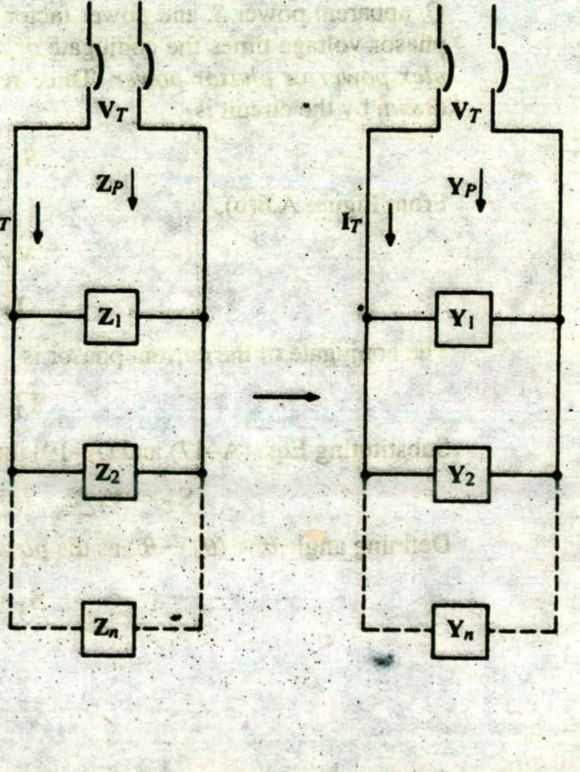


FIGURE A.3
(a) Parallel circuit; (b) phasor diagram.

FIGURE A.4
Impedance-admittance correspondence.



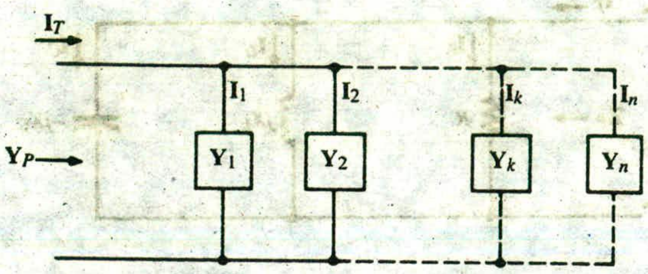


FIGURE A.5
Circuit for current-divider equation.

The current in any one of two or more parallel-connected admittances may be determined by the *current-divider equation.* Referring to Figure A.5,

$$\mathbf{I}_k = \mathbf{I}_T \cdot \frac{\mathbf{Y}_k}{\mathbf{Y}_P} \tag{A–15}$$

where $\mathbf{Y}_P = \mathbf{Y}_1 + \mathbf{Y}_2 + \cdots + \mathbf{Y}_k + \cdots + \mathbf{Y}_n$.

A.5 POWER RELATIONSHIPS IN A SINGLE-PHASE SYSTEM

For the single-phase system shown in Figure A.6(a), the unknown circuit may have any combination of circuit elements in series, parallel, or series–parallel combinations. Regardless of the internal configuration, however, if the line voltage, line current, and corresponding phase angles are known, the active power P, reactive power Q, apparent power S, and power factor F_P can be determined from the product of the phasor voltage times the conjugate of the phasor current; this product is called *complex power* or *phasor power.* Thus, referring to Figure A.6(a), the complex power drawn by the circuit is

$$\mathbf{S}_T = \mathbf{V}_T \cdot \mathbf{I}_T^* \tag{A–16}$$

From Figure A.6(b),

$$\mathbf{V}_T = V_T \underline{/\theta_v} \tag{A–17}$$

$$\mathbf{I}_T = I_T \underline{/\theta_i} \tag{A–18}$$

The conjugate of the current phasor is

$$\mathbf{I}_T^* = I_T \underline{/-\theta_i} \tag{A–19}$$

Substituting Eqs. (A–17) and (A–19) into Eq. (A–16),

$$\mathbf{S}_T = V_T \underline{/\theta_v} \cdot I_T \underline{/-\theta_i} = V_T I_T \underline{/(\theta_v - \theta_i)}$$

Defining angle $\theta = (\theta_v - \theta_i)$ as the *power-factor angle,*

$$\mathbf{S}_T = V_T I_T \underline{/\theta} \tag{A–20}$$

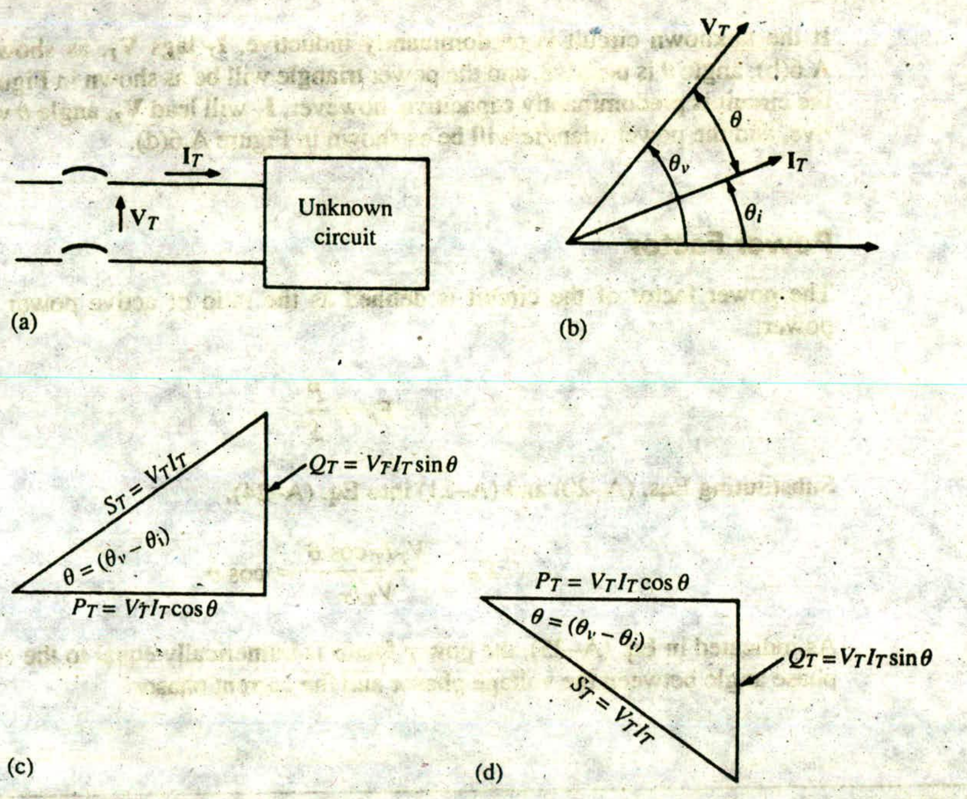


FIGURE A.6
(a) Unknown circuit; (b) phasor diagram; (c) power triangle for lagging power factor; (d) power triangle for leading power factor.

Expressed in rectangular form,

$$\mathbf{S}_T = V_T I_T \cos\theta + jV_T I_T \sin\theta$$

where:

$$\text{Active power (watts)} \quad P_T = V_T I_T \cos\theta \tag{A–21}$$

$$\text{Reactive power (vars)} \quad Q_T = V_T I_T \sin\theta \tag{A–22}$$

Power Triangle

Equations (A–21) and (A–22) represent two legs of the *power triangle* shown in Figure A.6(c). The hypotenuse $V_T I_T$ is the magnitude of the apparent power, and makes an angle θ from the zero-degree line. Thus, the apparent power may be conveniently expressed in terms of the magnitudes of its components:

$$S_T = \sqrt{P_T^2 + Q_T^2} \tag{A–23}$$

If the unknown circuit is predominantly inductive, $\mathbf{I}_T$ lags $\mathbf{V}_T$, as shown in Figure A.6(b), angle θ is positive, and the power triangle will be as shown in Figure A.6(c). If the circuit is predominantly capacitive, however, $\mathbf{I}_T$ will lead $\mathbf{V}_T$, angle θ will be negative, and the power triangle will be as shown in Figure A.6(d).

Power Factor

The power factor of the circuit is defined as the ratio of active power to apparent power:

$$F_P = \frac{P}{S} \tag{A–24}$$

Substituting Eqs. (A–20) and (A–21) into Eq. (A–24),

$$F_P = \frac{V_T I_T \cos\theta}{V_T I_T} = \cos\theta \tag{A–25}$$

As indicated in Eq. (A–25), the power factor is numerically equal to the cosine of the phase angle between the voltage phasor and the current phasor.

EXAMPLE A.1 Assume the current and voltage supplied to a circuit are $125\angle 30^\circ$ A and $460\angle 20^\circ$ V, respectively. Determine (a) apparent power, active power, and reactive power; (b) whether the circuit is predominantly inductive or predominantly capacitive; (c) power factor of the load.

Solution

(a)

$$\mathbf{S} = \mathbf{V}\cdot\mathbf{I}^* = (460\angle 20^\circ)\cdot(125\angle 30^\circ)^* = 460\angle 20^\circ \cdot 125\angle -30^\circ$$

$$\mathbf{S} = 57{,}500\angle -10^\circ = 56{,}626.4 - j9984.8 \text{ VA}$$

Thus,

$$S = \underline{57.5 \text{ kVA}}$$
$$P = \underline{56.6 \text{ kW}}$$
$$Q = \underline{-9.98 \text{ kvar}}$$

(b) The negative reactive power indicates that the load is *predominantly capacitive*. This is also indicated by the given phase angles of current and voltage, which shows the current to be leading the voltage by 10°.

(c)

$$F_P = \cos(-10^\circ) = 0.985 \text{ or } \underline{98.5\% \text{ leading}}$$

A.6 DOUBLE-SUBSCRIPT NOTATION

Double-subscript notation is used in conjunction with assigned letter symbols for voltage in order to assist in circuit analysis and problem solving. The subscripts represent two nodes between which a voltage is measured, and the order of the subscripts indicates the direction of voltage measurement.

Thus, referring to Figure A.7, $\mathbf{V}_{bc}$ is the voltage at node *b* measured with respect to the voltage at node *c*. Voltage $\mathbf{V}_{bc}$ is considered a positive voltage if node *b* has a higher potential than node *c*, and will be considered a negative voltage if node *b* has a lower potential than node *c*.

Applying Ohm's law to impedance $\mathbf{Z}_2$, and noting the assumed direction of current,

$$\mathbf{I}_{bc} = \frac{\mathbf{V}_{bc}}{\mathbf{Z}_2}$$

Note: Voltage measurements from node *c* to node *b* are indicated as $\mathbf{V}_{cb}$. Since the direction of measurement is opposite to that of $\mathbf{V}_{bc}$,

$$\mathbf{V}_{cb} = -\mathbf{V}_{bc}$$

A.7 VOLTAGES IN A WYE-CONNECTED SOURCE

A three-phase, wye-connected system of voltages consists of three AC voltage sources, each equal in magnitude, but displaced from one another by 120 electrical degrees, and connected at a common point as shown in Figure A.8(a). The common point is called the *neutral connection.* The three voltage waves representing the three phase voltages are shown in Figure A.8(b), and the corresponding phasor diagram is shown in Figure A.8(c).

The voltage at the service entrance from terminal *a* to terminal *b* is determined by making a phasor summation of phase voltages while "walking" through the circuit from *a* to *b*. Thus,

$$\mathbf{E}_{a\text{ to }b} = \mathbf{E}_{a\text{ to }a'} + \mathbf{E}_{b'\text{ to }b}$$

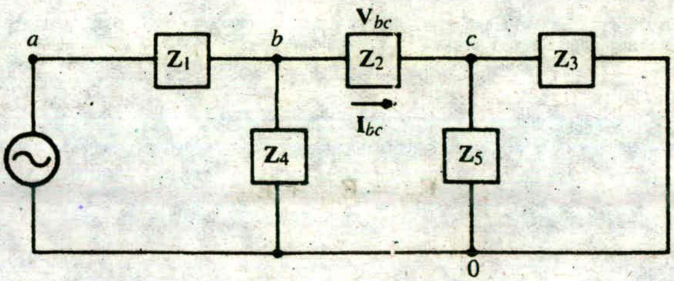


FIGURE A.7
Example of double-subscript notation.

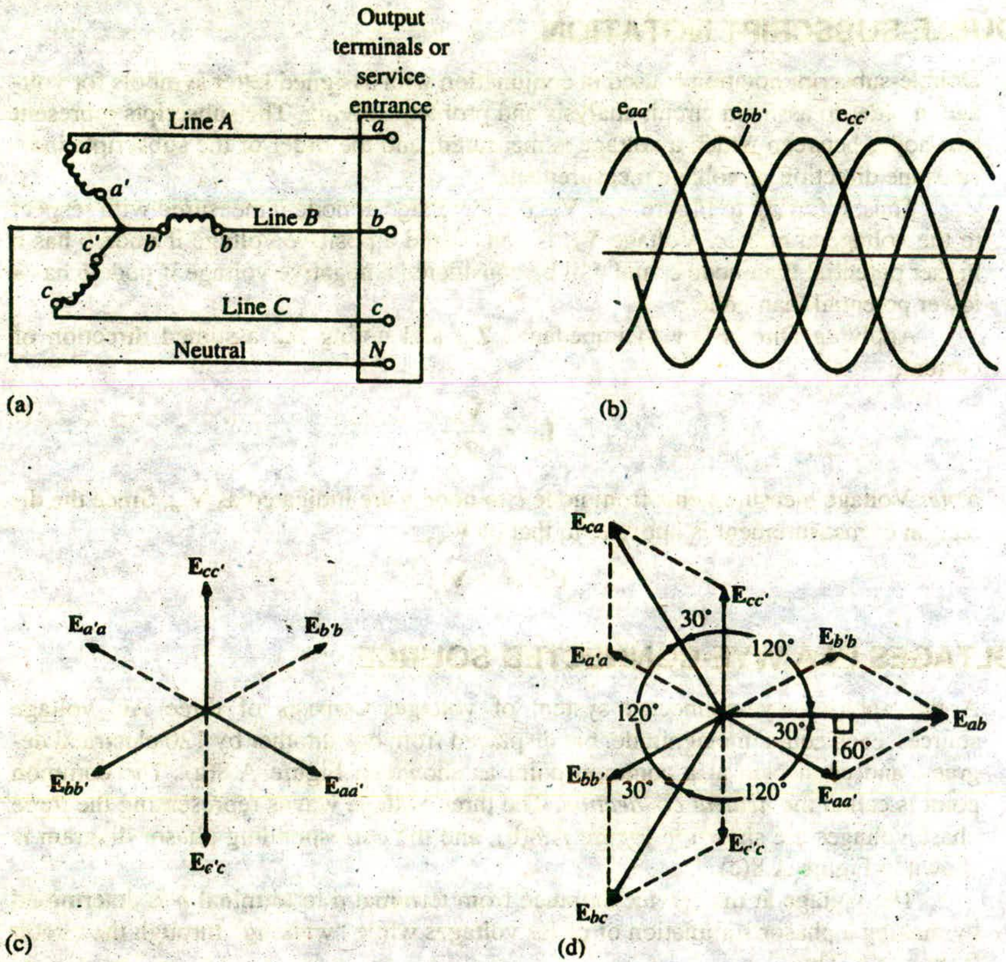


FIGURE A.8
(a) Wye-connected source; (b) voltage waves; (c) phasor diagram of component voltages; (d) graphical addition of voltages.

Or simply,

$$\mathbf{E}_{ab} = \mathbf{E}_{aa'} + \mathbf{E}_{b'b} \tag{A–26}$$

Similarly,

$$\mathbf{E}_{bc} = \mathbf{E}_{bb'} + \mathbf{E}_{c'c} \tag{A–27}$$

$$\mathbf{E}_{ca} = \mathbf{E}_{cc'} + \mathbf{E}_{a'a} \tag{A–28}$$

The voltages between any two line terminals (a, b, *or* c) *are called line-to-line or line voltages, and the voltages between any line terminal and the neutral terminal are called branch voltages or phase voltages.*

A phasor diagram for the graphical addition of voltages in a wye-connected system is shown in Figure A.8(d). From the geometry of the phasor diagram,

$$E_{\text{line}} = \sqrt{3}\,E_{\text{phase}} \tag{A–29}$$

A.8 VOLTAGES IN A DELTA-CONNECTED SOURCE

A three-phase, delta-connected system of voltages, shown in Figure A.9(a), consists of three AC voltage sources $\mathbf{E}_{aa'}$, $\mathbf{E}_{bb'}$, and $\mathbf{E}_{cc'}$, each equal in magnitude, but displaced from each other by 120 electrical degrees. The three voltages sources, called *phases*, are connected in series to form a closed loop, and lines from the three nodes connect the sources to the output terminals.

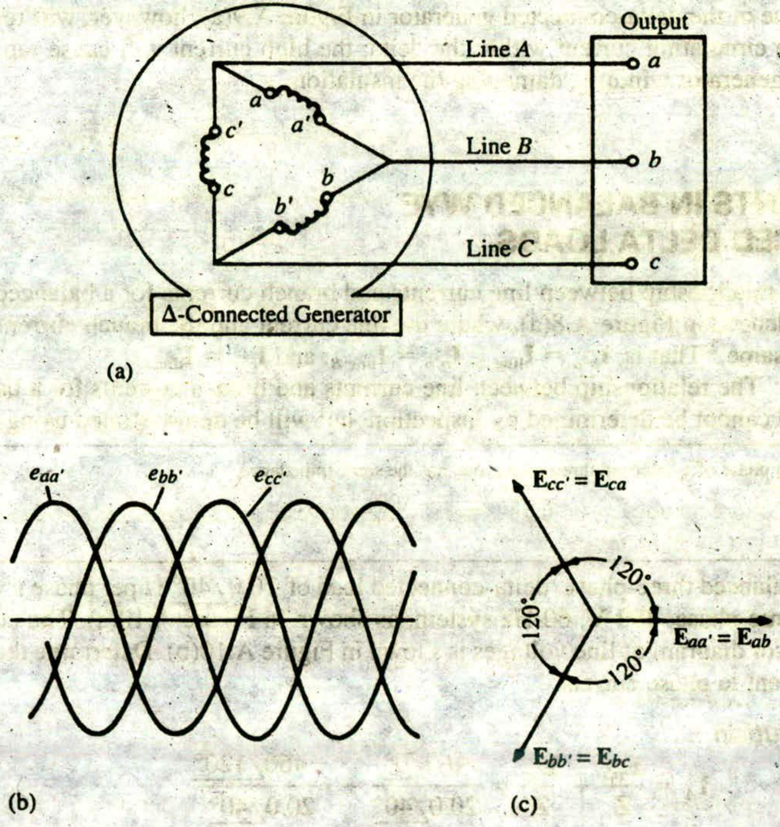


FIGURE A.9
(a) Delta-connected source; (b) voltage waves; (c) phasor diagram.

The voltage waves for the three phase-voltages are shown in Figure A.9(b), and a phasor diagram for the corresponding phasors is shown in Figure A.9(c). *For standardization and convenience in problem solving, phasor* $\mathbf{E}_{ab}$ *is drawn at zero degrees for both wye and delta sources.*

The voltages measured between the output terminals of a generator, or between the service-entrance terminals at a factory, are called *line-to-line voltages,* or simply *line voltages.* Note that for a delta connection, the line voltage is equal to the corresponding phase voltage.

No current circulates in the closed circuit formed by the delta connection because the phasor sum of the three phase-voltages around the loop is equal to zero. This can be determined from the phasor diagram in Figure A.9(c), where the phasor summation is

$$\mathbf{E}_{aa'} + \mathbf{E}_{bb'} + \mathbf{E}_{cc'} = 0$$

Although the resultant voltage around the loop is at all times equal to zero and no current circulates around the closed delta, each of the three phases is still capable of supplying current to external loads. Interchanging the internal connections of any one phase of the delta-connected generator in Figure A.9(a), however, will result in a very high circulating current within the delta; the high current will cause rapid heating of the generator winding, damaging the insulation.

A.9 CURRENTS IN BALANCED WYE AND BALANCED DELTA LOADS

The relationship between line currents and branch currents for a balanced wye load is evidenced in Figure A.8(a), where the line current and the branch current are one and the same.[2] That is, $\mathbf{I}_{a'a} = \mathbf{I}_{\text{line } A}$; $\mathbf{I}_{b'b} = \mathbf{I}_{\text{line } B}$; and $\mathbf{I}_{c'c} = \mathbf{I}_{\text{line } C}$.

The relationship between line currents and branch currents for a balanced delta load cannot be determined by inspection, but will be demonstrated using an example.

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[2] Each phase of a balanced three-phase load has the same impedance.

EXAMPLE A.2

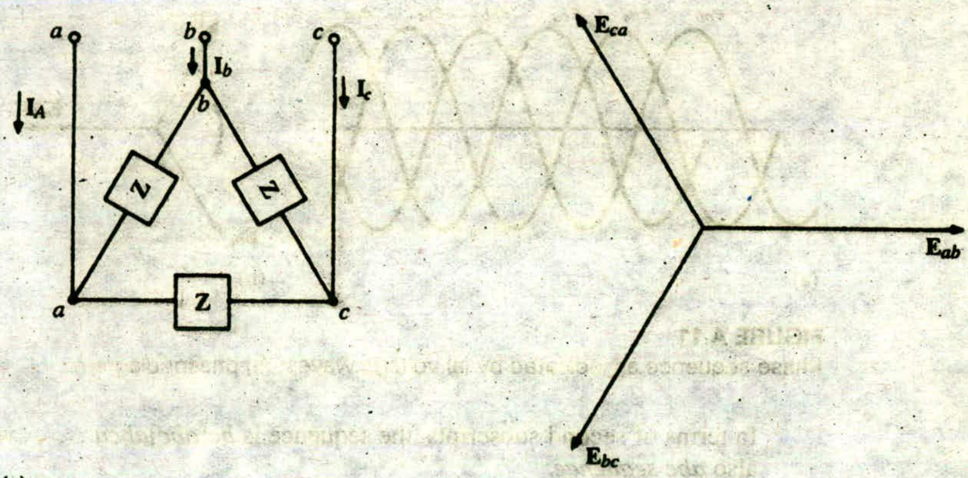
A balanced three-phase, delta-connected load of $20.0\angle 40^\circ\ \Omega$ per phase is connected to a three-phase, 460-V, 60-Hz system, as shown in Figure A.10(a). The corresponding phasor diagram of line voltages is shown in Figure A.10(b). Determine the ratio of line current to phase current.

Solution

$$\mathbf{I}_A = \frac{\mathbf{E}_{ab}}{\mathbf{Z}} + \frac{\mathbf{E}_{ac}}{\mathbf{Z}} = \frac{460\angle 0^\circ}{20.0\angle 40^\circ} + \frac{-460\angle 120^\circ}{20.0\angle 40^\circ}$$

$$\mathbf{I}_A = 23\angle -40^\circ - 23\angle 80^\circ = 17.62 - j14.78 - (3.99 + j22.65)$$

$$\mathbf{I}_A = 13.63 - j37.43 = 39.83\angle -70.0^\circ \text{ A}$$



(a) (b)

FIGURE A.10
Circuit and phasor diagram for Example A.2.

The ratio of the magnitude of the line current to the magnitude of the branch current (also called phase current) is

$$\frac{I_{\text{line}}}{I_{\text{phase}}} = \frac{39.83}{23} = 1.732 = \sqrt{3}$$

The same ratio holds true for all line currents in Figure A.10(a). Thus, for a balanced delta load,

$$I_{\text{line}} = \underline{\sqrt{3} \cdot \mathbf{I}_{\text{phase}}} \qquad \textbf{(A–30)}$$

A.10 PHASE SEQUENCE

Phase sequence is the order or sequence in which the three line-voltages of a three-phase supply reach their maximum positive values.

Phase sequence may be determined from the voltage waves or from a corresponding phasor diagram, as shown in Figures A.11(a) and (b), respectively, where the indicated phase sequence is

$$\mathbf{E}_{ab}, \mathbf{E}_{bc}, \mathbf{E}_{ca}, \mathbf{E}_{ab}, \mathbf{E}_{bc}, \mathbf{E}_{ca}, \ldots$$

For brevity, however, the sequence is generally expressed in terms of only the first subscripts or only the second subscripts:

> In terms of the first subscripts, the sequence is [*abc*]*abcabc* . . . , or simply *abc sequence.*

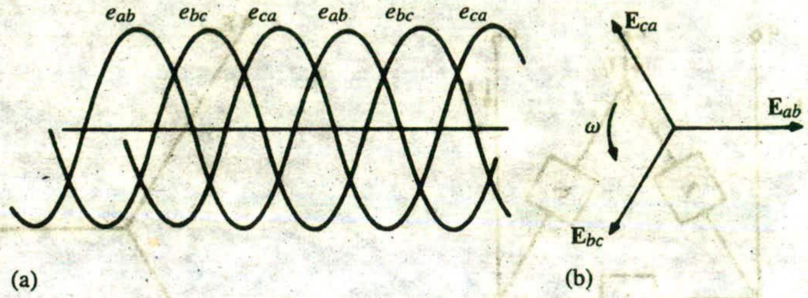


FIGURE A.11
Phase sequence as indicated by (a) voltage waves; (b) phasor diagram.

In terms of second subscripts, the sequence is *bc[abc]abca* . . . , which is also *abc sequence.*

Phase sequence at the load is indicated by reading the letter markings (or number markings) from top to bottom or from left to right as applicable **[1]**. Thus, referring to the circuit in Figure A.12(a), the phase sequence at the motor, reading from top to bottom, is *abc*. Interchanging any two of the three line leads reverses the phase sequence. This is shown in Figures A.12(b), (c), and (d), where interchanging any two line leads

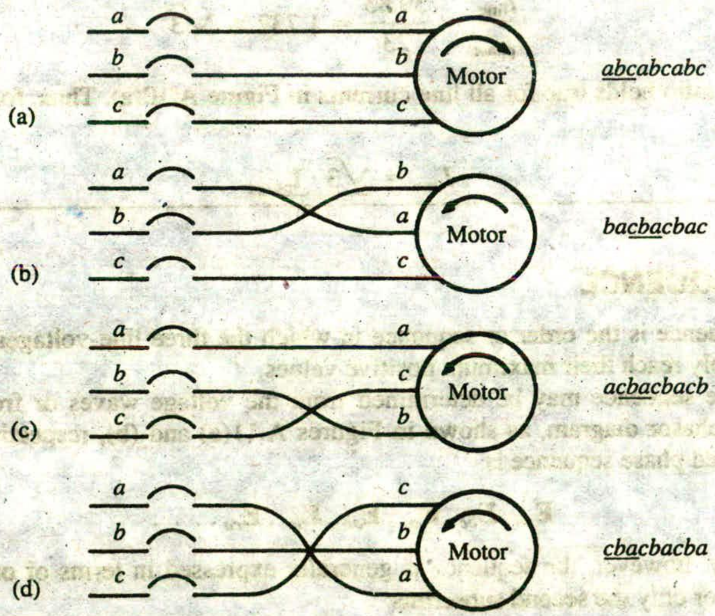


FIGURE A.12
Reversing the phase sequence by interchanging any two line leads.

changes the phase sequence from *abc* to *cba,* and reverses the direction of rotation of the motor. *Note:* There are only two possible phase sequences, *abc* and *cba.*

If the three-phase load has unbalanced impedances, reversing the phase sequence could cause major changes in the magnitudes and phase angles of the three line currents (see Section 21.7 in Reference **[2]**). If a three-phase generator is paralleled with another of opposite phase sequence, both machines may be severely damaged. It is therefore essential that phase sequence be taken into consideration when connecting three-phase loads or when paralleling three-phase generators (see Section 14.7 in Reference **[3]**).

A.11 CALCULATING LINE AND PHASE CURRENTS IN THREE-PHASE CIRCUITS

The procedure for calculating line and phase current in three-phase circuits is the same whether the circuit is wye or delta. Depending on the complexity of the circuit, the current may be determined by using Ohm's law, Kirchhoff's law, and/or loop and node analysis.

Guidelines for Solving Three-Phase Circuit Problems

1. Voltages and currents on the nameplates of electrical apparatus, and in technical literature concerning motors, generators, and other apparatus, are line-to-line rms voltages unless otherwise specified.
2. A universal phasor diagram and a table of load voltages, to be used when solving three-phase circuit problems, are shown in Figure A.13; line and phase voltages apply to wye loads, but only line voltages apply to delta loads. Note also that the voltage between any line and the wye junction (neutral) of a *balanced wye load* is the corresponding phase voltage, even though the junction is not connected to the source neutral.
3. If a wye load is *balanced* (identical impedances per leg) and the three-phase source has balanced voltages, there will be no current in a neutral line connecting the wye junction of the load to the source neutral. Hence, a neutral line connecting the source neutral to the wye junction of a *balanced* load is not necessary and is seldom used. Except for fault conditions (such as opens, shorts, and grounds), *three-phase motors are balanced loads,* and thus neutral lines are not required nor are they supplied for wye-connected motors.
4. Before starting the solution of problems involving multiple loads, an assumed direction of phasor current should be indicated on the diagram for each line and each phase being solved. For convenience and standardization, the direction of current in each line will be assumed to be *from the source to the load.* Once assigned, the assumed direction must not be changed during the solution process.

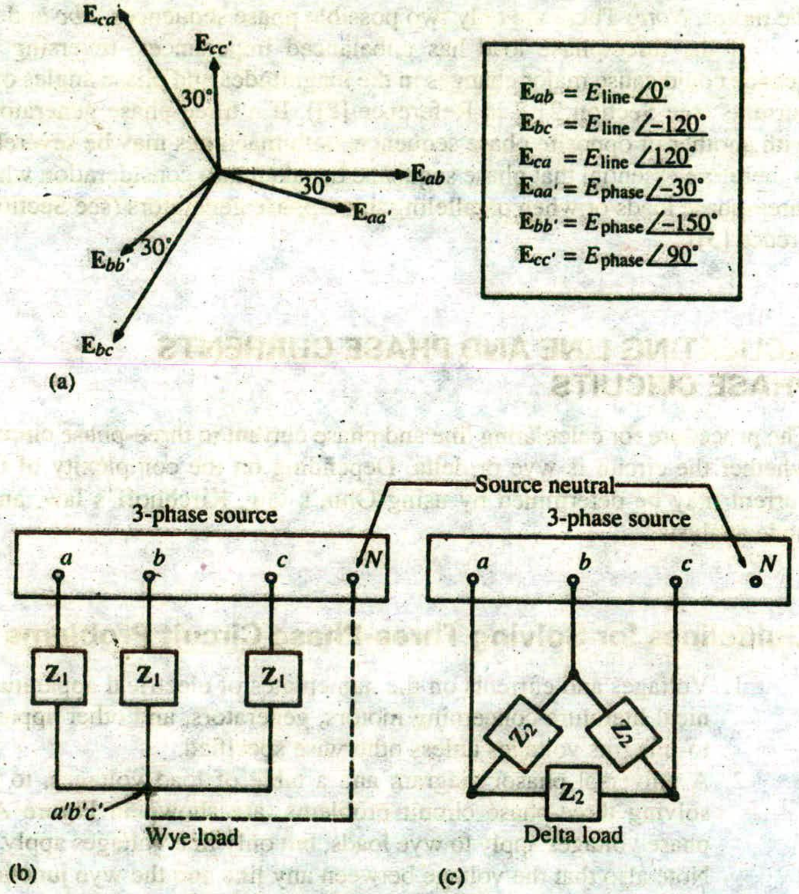


FIGURE A.13
(a) Universal phasor diagram and table of load voltages; (b) wye load; (c) delta load.

EXAMPLE A.3 For the circuit shown in Figure A.14,

$$\mathbf{Z}_1 = 10\angle 30°\ \Omega \qquad \mathbf{Z}_2 = 15\angle 10°\ \Omega \qquad \mathbf{Z}_3 = 20 + j20\ \Omega$$

Determine the ammeter reading.

Solution

Applying Ohm's law and Kirchhoff's current law to line *A*,

$$\mathbf{I}_A = \frac{\mathbf{E}_{aa'}}{\mathbf{Z}_1} + \frac{\mathbf{E}_{ab}}{\mathbf{Z}_2} + \frac{\mathbf{E}_{ac}}{\mathbf{Z}_2} + \frac{\mathbf{E}_{ac}}{\mathbf{Z}_3}$$

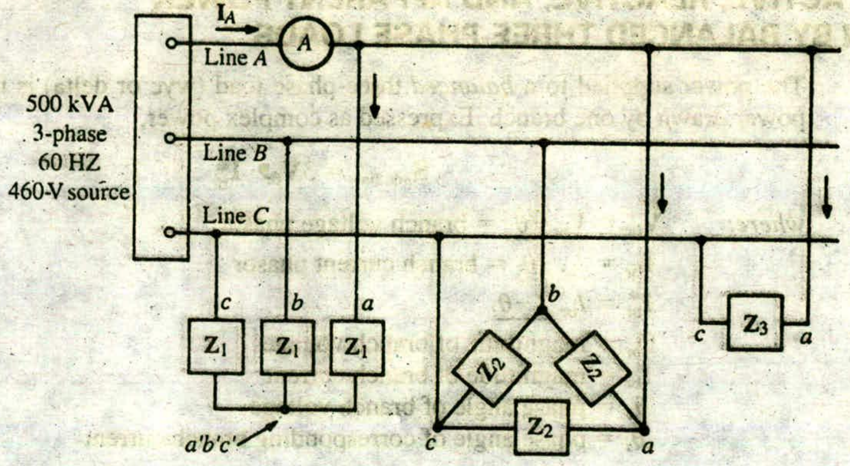


FIGURE A.14
Circuit for Example A.3.

Note that the wye-connected load is balanced. Hence, the wye junction is effectively at potential $a'b'c'$. From the source data, the magnitudes of line and phase voltages at the applicable loads are

$$E_{\text{line}} = 460\text{ V} \qquad E_{\text{phase}} = \frac{460}{\sqrt{3}} = 265.6\text{ V}$$

Using the table of voltages in Figure A.13(a) as a guide,

$$\mathbf{E}_{aa'} = 265.6\underline{/-30^\circ}\text{ V}$$

$$\mathbf{E}_{ab} = 460\underline{/0^\circ}\text{ V}$$

$$\mathbf{E}_{ac} = -\mathbf{E}_{ca} = -460\underline{/120^\circ}\text{ V}$$

Converting $\mathbf{Z}_3$ into polar form, and substituting the corresponding voltages and impedances,

$$\mathbf{Z}_3 = 20 + j20 = 28.28\underline{/45^\circ}$$

$$\mathbf{I}_A = \frac{265.6\underline{/-30^\circ}}{10\underline{/30^\circ}} + \frac{460\underline{/0^\circ}}{15\underline{/10^\circ}} + \frac{-460\underline{/120^\circ}}{15\underline{/10^\circ}} + \frac{-460\underline{/120^\circ}}{28.28\underline{/45^\circ}}$$

$$\mathbf{I}_A = 26.56\underline{/-60^\circ} + 30.67\underline{/-10^\circ} - 30.67\underline{/110^\circ} - 16.27\underline{/75^\circ}$$

$$\mathbf{I}_A = (13.28 - j23) + (30.20 - j5.33) + (10.49 - j28.82) + (-4.21 - j15.72)$$

$$\mathbf{I}_A = 49.76 - j72.87 = 88.23\underline{/-55.67^\circ}$$

The ammeter will read an rms value of 88.2 A.

A.12 ACTIVE, REACTIVE, AND APPARENT POWER DRAWN BY BALANCED THREE-PHASE LOADS

The power supplied to a *balanced* three-phase load (wye or delta) is three times the power drawn by one branch. Expressed as complex power,

$$\mathbf{S}_{3\phi,\,\text{bal}} = 3\mathbf{V}_{\text{br}} \cdot \mathbf{I}^*_{\text{br}} \tag{A–31}$$

where: $\mathbf{V}_{\text{br}} = V_{\text{br}} \angle \theta_v$ = branch voltage phasor
$\mathbf{I}_{\text{br}} = I_{\text{br}} \angle \theta_i$ = branch current phasor
$\mathbf{I}^*_{\text{br}} = I_{\text{br}} \angle -\theta_i$
V_{br} = magnitude of branch voltage
I_{br} = magnitude of branch current
θ_v = phase angle of branch voltage
θ_i = phase angle of corresponding branch current

Expressing Eq. (A–31) in polar form

$$\mathbf{S}_{3\phi,\,\text{bal}} = 3(V_{\text{br}}\angle\theta_v) \cdot (I_{\text{br}}\angle\theta_i)^* = 3V_{\text{br}} \cdot I_{\text{br}}\angle(\theta_v - \theta_i) \tag{A–32}$$

Defining $\theta = (\theta_v - \theta_i)$ as the *power-factor angle,* and expressing Eq. (A–32) in rectangular form,

$$\mathbf{S}_{3\phi,\,\text{bal}} = 3V_{\text{br}} \cdot I_{\text{br}} \cos\theta + j3V_{\text{br}} \cdot I_{\text{br}} \sin\theta \tag{A–33}$$

where: Active power (watts) $= P_{3\phi,\,\text{bal}} = 3V_{\text{br}} \cdot I_{\text{br}} \cos\theta$ **(A–34)**

Reactive power (vars) $= Q_{3\phi,\,\text{bal}} = 3V_{\text{br}} \cdot I_{\text{br}} \sin\theta$ **(A–35)**

Apparent power $= S_{3\phi,\,\text{bal}} = 3V_{\text{br}} \cdot I_{\text{br}}$ **(A–36)**

Three-phase power may be expressed in terms of line voltage and line current by substituting the delta relationship or the wye relationship into Eqs. (A–34), (A–35), and (A–36). As previously shown,

If delta connected: $E_{\text{br}} = E_{\text{line}}$ and $I_{\text{br}} = \dfrac{I_{\text{line}}}{\sqrt{3}}$

If wye connected: $I_{\text{br}} = I_{\text{line}}$ and $E_{\text{br}} = \dfrac{E_{\text{line}}}{\sqrt{3}}$

Making the substitution and simplifying,

$$P_{3\phi,\text{bal}} = \sqrt{3}\, V_{\text{line}} I_{\text{line}} \cos\theta \tag{A–37}$$

$$Q_{3\phi,\text{bal}} = \sqrt{3}\, V_{\text{line}} I_{\text{line}} \sin\theta \tag{A–38}$$

$$S_{3\phi,\text{bal}} = \sqrt{3}\, V_{\text{line}} I_{\text{line}} \tag{A–39}$$

Power Triangle

As evidenced in Eqs. (A–37), (A–38), and (A–39), respectively, the active power and reactive power represent two legs of a right triangle whose hypotenuse is the apparent power. Thus,

$$S_{3\phi,\text{bal}} = \sqrt{P^2_{3\phi,\text{bal}} + Q^2_{3\phi,\text{bal}}} \tag{A–40}$$

The power factor is

$$F_p = \frac{P_{3\phi,\text{bal}}}{S_{3\phi,\text{bal}}} = \cos\theta \tag{A–41}$$

Note: Angle θ is the power-factor angle. It is the angle between the phase voltage and the phase current; it is not the angle between line voltage and line current! Note also that the power factor of a balanced three-phase load is the power factor of one phase. Substituting Eq. (A–41) into Eq. (A–37),

$$P_{3\phi,\text{bal}} = \sqrt{3}\, V_{\text{line}} I_{\text{line}} F_P \tag{A–42}$$

Equation (A–42) is the expression generally used for calculating three-phase power.

EXAMPLE A.4 The phase voltage and phase current at one branch of a balanced delta load are determined to be $460\angle{-120^\circ}$ V and $10\angle{-160^\circ}$ A, respectively.

(a) Using the complex power equation, calculate the three-phase apparent power, active power, reactive power, and power factor.

(b) Is the load inductive or capacitive?

Solution

(a)

$$\mathbf{S}_{\text{br}} = \mathbf{V}_{\text{br}}\mathbf{I}^*_{\text{br}} = 460\angle{-120^\circ} \cdot 10\angle{160^\circ} = 4600\angle{40^\circ}$$

$$\mathbf{S}_{\text{br}} = 3523.80 + j2956.82$$

$$P_{3\phi} = 3P_{\text{br}} = 3 \times 3523.80 = 10{,}571.4 \text{ W} \quad \text{or} \quad \underline{10.6 \text{ kW}}$$

$$Q_{3\phi} = 3Q_{\text{br}} = 3 \times 2956.82 = 8870.46 \text{ var} \quad \text{or} \quad \underline{8.87 \text{ kvar}}$$

$$S_{3\phi} = 3S_{\text{br}} = 3 \times 4600 = 13{,}800 \text{ VA} \quad \text{or} \quad \underline{13.8 \text{ kVA}}$$

$$F_P = \frac{P_{3\phi,\text{bal}}}{S_{3\phi,\text{bal}}} = \frac{10{,}571.4}{13{,}800} = 0.766 \quad \text{or} \quad \underline{76.6\%}$$

(b) The reactive power is positive, indicating a lagging current caused by an inductive load.

A.13 POWER ANALYSIS AND POWER-FACTOR CORRECTION OF BALANCED THREE-PHASE LOADS IN PARALLEL

When data on balanced three-phase loads are expressed in kilovoltamperes, kilowatts, kilovars, power factor, horsepower, and η (efficiency), it is often more convenient to analyze the system on a power basis, as illustrated in the following example.

EXAMPLE A.5 A 440-V, 60-Hz, three-phase source supplies a distribution system containing the following three-phase loads:

Motor 1 Delta-connected induction motor rated at 60 hp and 1775 r/min operating at three-quarters rated load with an efficiency of 90 percent and a power factor of 94 percent.

Motor 2 Wye-connected induction motor rated at 75 hp and 890 r/min, operating at one-half rated load with an efficiency of 88 percent and a power factor of 74 percent.

Resistance Heater Delta connected resistor bank drawing 20 kW.

Determine (a) active power, reactive power, apparent power, and power factor of the system; (b) line current; (c) capacitance and voltage rating of each capacitor of a wye-connected capacitor bank required to correct the system power factor to 1.0 (unity power factor).

Solution

The problem will be solved by constructing a single power diagram that includes the individual power triangles of all loads. Furthermore, since all induction motors operate at a lagging power factor, the power factor angle $\theta = (\theta_v - \theta_i)$ is always positive for induction motors.

Motor 1

$$P_{\text{in}} = \frac{P_{\text{out}}}{\eta} = \frac{60 \times 3/4}{0.90} = 50 \text{ hp}$$

$$P_1 = 50 \times 746 = 37{,}300 \text{ W}$$

$$\theta_1 = \cos^{-1} 0.94 = 19.95°$$

Motor 2

$$P_{\text{in}} = \frac{P_{\text{out}}}{\eta} = \frac{75 \times 1/2}{0.88} = 42.614 \text{ hp}$$

$$P_2 = 42.614 \times 746 = 31{,}790 \text{ W}$$

$$\theta_2 = \cos^{-1} 0.74 = 42.27°$$

Resistance Heater

$$P_3 = 20{,}000 \text{ W} \qquad \theta_3 = 0°$$

The branch current and the branch voltage of a resistor are in phase, angle $\theta_r = (\theta_v - \theta_i) = 0°$, and the reactive power is zero. The individual power triangles are drawn in a common power diagram, as shown in Figure A.15, and the geometry of the individual triangles is used to determine the reactive power drawn by the respective three-phase load. Thus, from Figure A.15,

$$\tan 19.95° = \frac{Q_1}{37{,}300} \qquad \tan 42.27° = \frac{Q_2}{31{,}790}$$

$$Q_1 = 13{,}539 \text{ var} \qquad Q_2 = 28{,}895 \text{ var}$$

(a) The total active power, reactive power, and apparent power drawn by the system are

$$P_{\text{sys}} = P_1 + P_2 + P_3 = 37{,}300 + 31{,}790 + 20{,}000$$

$$= 89{,}090 \text{ W} \quad \Rightarrow \quad \underline{89.1 \text{ kW}}$$

$$Q_{\text{sys}} = Q_1 + Q_2 = 13{,}539 + 28{,}895 = 42{,}434 \text{ var} \quad \Rightarrow \quad \underline{42.4 \text{ kvar}}$$

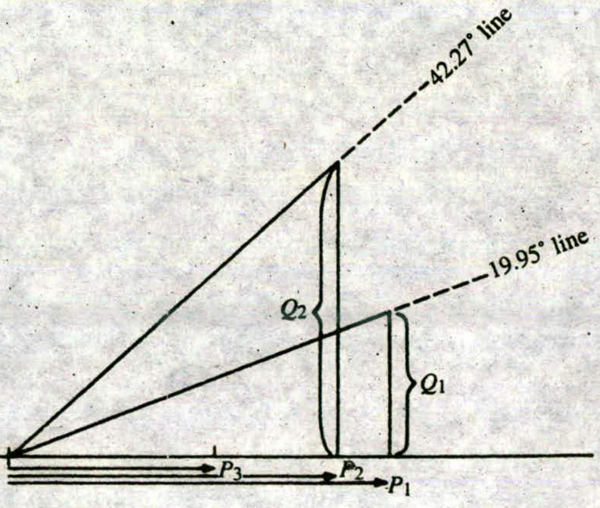
$$S_{\text{sys}} = \sqrt{P_{\text{sys}}^2 + Q_{\text{sys}}^2} = \sqrt{89{,}090^2 + 42{,}434^2} = 98{,}679 \text{ VA} \quad \Rightarrow \quad \underline{98.7 \text{ kVA}}$$

$$F_P = \frac{P_{\text{sys}}}{S_{\text{sys}}} = \frac{89{,}090}{98{,}679} = \underline{0.903}$$

(b) $P_{\text{sys}} = \sqrt{3}\, V_{\text{line}} I_{\text{line}} F_P \quad \Rightarrow \quad 89{,}090 = \sqrt{3} \times 440 \times I_{\text{line}} \times 0.903$

$I_{\text{line}} = \underline{129.5 \text{ A}}$

FIGURE A.15
Power diagram for Example A.5.



(c) To correct the system power factor to unity requires a three-phase capacitor bank with a var rating equal in magnitude to the lagging vars in the system. Thus, the required var rating of the capacitor bank is

$$Q_{3\varnothing} = 42{,}434 \text{ var}$$

$$Q_{\text{br}} = \frac{42{,}434}{3} = 14{,}145 \text{ var}$$

The voltage rating of each capacitor for a wye bank is $440/\sqrt{3} = \underline{254 \text{ V}}$.

$$Q_{\text{br}} = \frac{V_{\text{br}}^2}{X_C} \quad \Rightarrow \quad 14{,}145 = \frac{254^2}{X_C}$$

$$X_C = 4.56\ \Omega$$

$$X_C = \frac{1}{2\pi f C} = \quad \Rightarrow \quad 4.56 = \frac{1}{2\pi 60 C}$$

$$C = \underline{581\ \mu F}$$

SPECIFIC REFERENCES KEYED TO TEXT

1. American Society of Mechanical Engineers. *USA Standard Drafting Practices, Electrical and Electronic Diagrams.* USAS Y14.15-1966.
2. Hubert, C. I. *Electric Circuits AC/DC: An Integrated Approach.* McGraw-Hill, New York, 1982.
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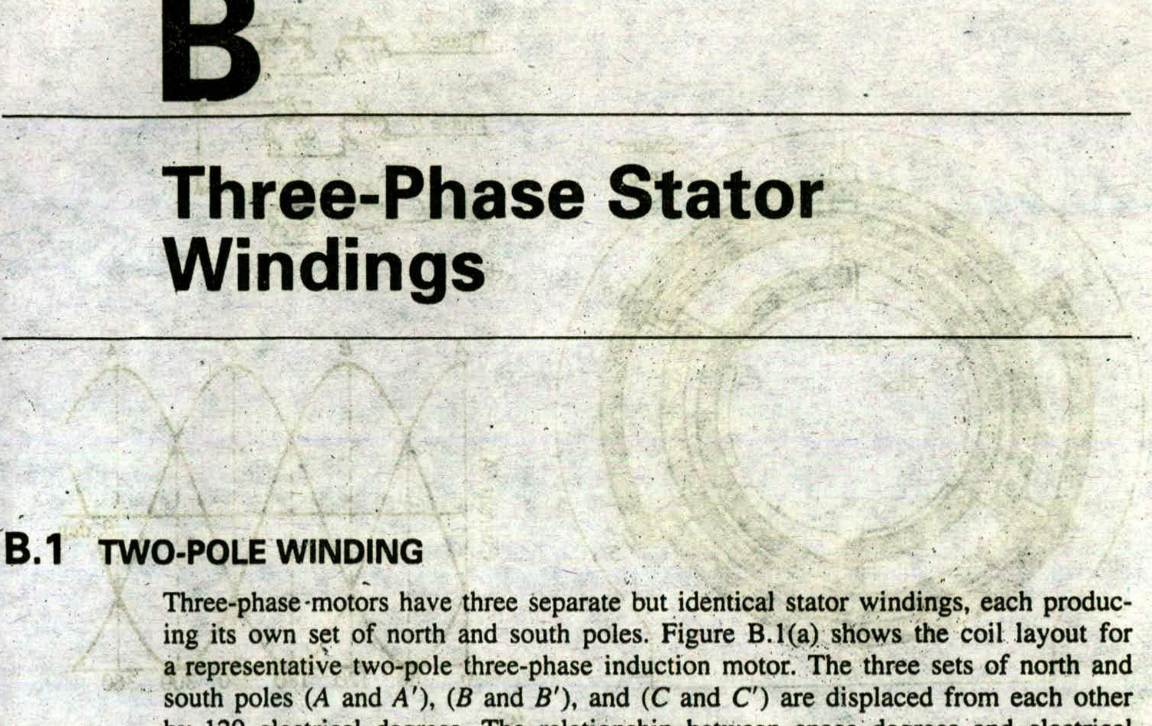

B

Three-Phase Stator Windings

B.1 TWO-POLE WINDING

Three-phase motors have three separate but identical stator windings, each producing its own set of north and south poles. Figure B.1(a) shows the coil layout for a representative two-pole three-phase induction motor. The three sets of north and south poles (A and A'), (B and B'), and (C and C') are displaced from each other by 120 electrical degrees. The relationship between space degrees and electrical degrees is[1]

$$\text{Space deg.} = \frac{2 \times \text{elec. deg.}}{P}$$

Thus, for a two-pole motor, the number of space degrees is equal to the number of electrical degrees; for a four-pole motor the number of space degrees is equal to one-half the number of electrical degrees, etc.

The stator winding for the two-pole motor in Figure B.1(a) has one coil per pole per phase. Coils A, B, and C contribute, in turn, to one magnetic polarity, whereas coils A′, B′, and C′ contribute, in turn, to the opposite magnetic polarity. All coils are wound in the same direction, have one or more turns, and span one-half of the circumference (full pitch). Full pitch is 180 electrical degrees, which is equal to 180 space degrees for a two-pole stator.

The connection diagram for the two-pole stator winding is shown in Figure B.1(b). All coils for a particular phase are connected in series in a manner that will result in alternate north and south poles. Terminal markings *b* and *e* indicate the respective beginning and end of each coil. Thus, when the direction of current through coil A is from *b* to *e*, the direction of current through coil A′ will be from *e* to *b*, causing opposite polarity in the A′ coil.

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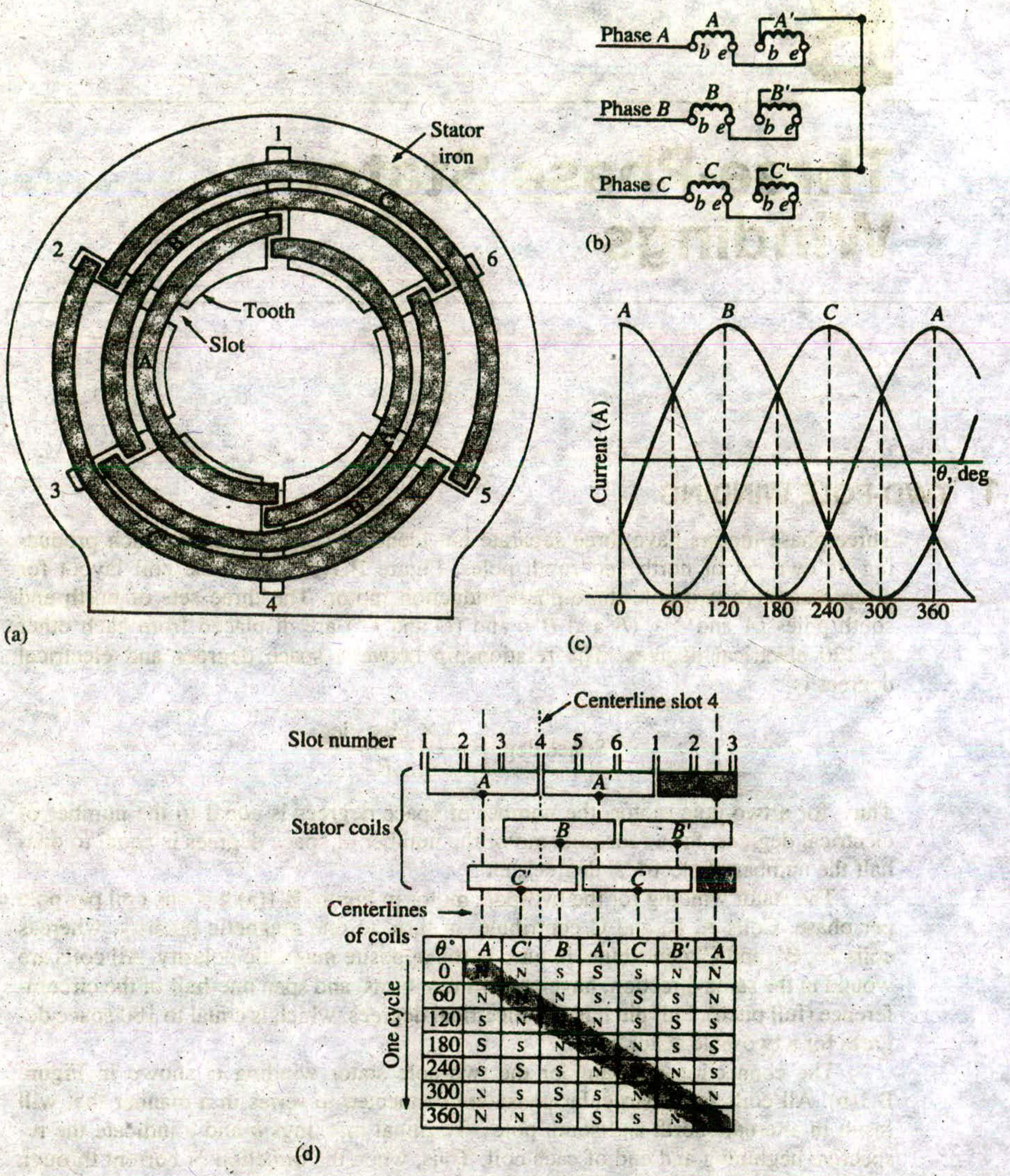
[1] For more information on the relationship between electrical degrees and space degrees, see Section 1.15, Chapter 1.



θ°	A	C'	B	A'	C	B'	A
0	N	N	S	S	S	N	N
60	N	[illegible]	N	S	S	S	N
120	S	N	[illegible]	N	S	S	S
180	S	S	N	[illegible]	N	S	S
240	S	S	S	N	[illegible]	N	S
300	N	S	S	S	N	[illegible]	N
360	N	N	S	S	S	N	[illegible]

FIGURE B.1

(a) Symmetrically spaced stator coils of a two-pole, three-phase induction motor; (b) connection diagram; (c) current waves; (d) developed view of coils in (a).

For the coils shown in Figure B.1(a), a north pole is formed in the unprimed coils when its respective current is positive, and a south pole is formed in the unprimed coils when its respective current is negative. The primed coils have opposite polarity with respect to the corresponding unprimed coils. Thus, when the current in phase A is positive, coil A will be north and coil A′ will be south, etc.

The coil span (pitch) for this stator is three slots: slots 1 and 4 for coil A, slots 3 and 6 for coil B, etc. The top of slot 4 contains one side of coil A and the bottom of slot 4 contains one side of coil A′, the top of slot 5 contains one side of coil C′ and the bottom of slot 5 contains one side of coil C, etc. Note that each coil has two sides, and each slot contains two coil sides. Hence, in effect there are the same number of coils as slots.

The two-pole, three-phase stator shown in Figure B.1(a) is redrawn as a developed view in Figure B.1(d). The developed view was obtained by imagining the coils in Figure B.1(a) to have been removed from the stator iron, while maintaining their relative positions and slot numbers, and then spread flat using slot number 4 as a "hinge."[2] *Note:* There are only two coils per phase; the shaded sections on the extreme right are the same coils repeated. The table appended to Figure B.1(d) shows the relative magnitudes and directions of the flux developed by each coil for the specific phase angles of the three current waves shown in Figure B.1(c); the relative magnitudes of the flux are indicated by the size of the letter (N, S, N, S). The broken lines in Figure B.1(d) connect the centerlines of each coil to the corresponding columns. Reading vertically down, each column shows the respective changes in magnetic polarity that occur within each coil as the three-phase current goes through one cycle, from 0 to 360 electrical degrees.

The shifting of the magnetic poles, with the changing current, is shown as a shaded area in the table. Note that the north pole moves from its "starting" position midway between slots 2 and 3 (at zero electrical degrees), returning to its initial position at 360 electrical degrees. Thus, for the two-pole motor in Figure B.1(a), the rotating flux makes one revolution per cycle (360 electrical degrees) of applied three-phase stator voltage.

B.2 FOUR-POLE WINDING

Figure B.2(a) shows the coil layout for a representative four-pole three-phase induction motor. The three sets of north and south poles are displaced from each other by 120 electrical degrees; in terms of space degrees,

$$\text{Space deg.} = \frac{2 \times \text{elec. deg.}}{P} = \frac{2 \times 180}{4} = 90^\circ$$

Coil sets (A_1 and A_2), (B_1 and B_2), and (C_1 and C_2) contribute in turn to one magnetic polarity, and coil sets ($A_1{}'$ and $A_2{}'$), ($B_1{}'$ and $B_2{}'$), and ($C_1{}'$ and $C_2{}'$) contribute in turn

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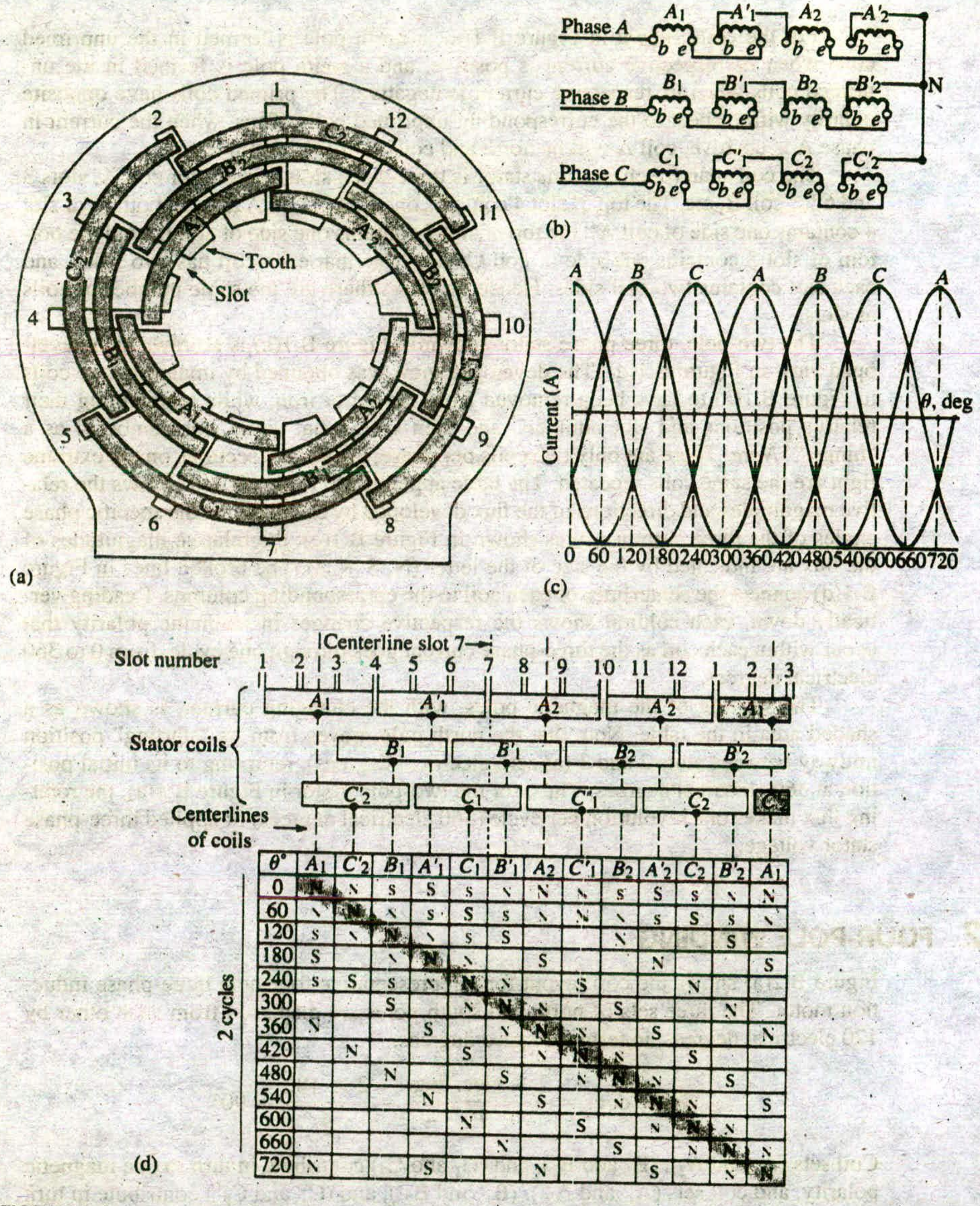
[2] This flat layout of stator coils represents a linear motor. (See Section 7.8, Chapter 7.)



θ°	A_1	C'_2	B_1	A'_1	C_1	B'_1	A_2	C'_1	B_2	A'_2	C_2	B'_2	A_1
0	N	N	S	S	S	N	N	N	S	S	S	N	N
60	N	N	N	S	S	S	N	N	N	S	S	S	N
120	S	N	N	N	S	S			N			S	
180	S		N	N	N		S			N			S
240	S	S	S	N	N	N		S			N		
300			S		N	N	N		S			N	
360	N			S		N	N	N		S			N
420		N			S		N	N	N		S		
480			N			S		N	N	N		S	
540				N			S		N	N	N		S
600					N			S		N	N	N	
660						N			S		N	N	N
720	N			S			N			S		N	N

FIGURE B.2

(a) Symmetrically spaced stator coils of a four-pole, three-phase induction motor; (b) connection diagram; (c) current waves; (d) developed view of coils in (a).

to the opposite magnetic polarity. Each coil of the four-pole stator spans one-quarter of the circumference (90 space degrees), equivalent to 180 electrical degrees.

To obtain alternate north and south poles, the coils for each phase are connected in series, end to end, and beginning to beginning, as shown in Figure B.2(b). As with the previous example for the two-pole motor, a north pole is formed in an unprimed coil when its respective current is positive, and a south pole is formed when its respective current is negative. The primed coils have opposite polarity with respect to the corresponding unprimed coils. Thus, when the current in phase A is positive, coils A_1 and A_2 will be north and coils A_1', and A_2' will be south, etc.

The four-pole three-phase stator shown in Figure B.2(a) is redrawn as a developed view in Figure B.2(d). The developed view was obtained in a manner similar to that used for the two-pole winding, with slot number 7 as the "hinge." *Note:* There are only four coils per phase; the shaded sections on the extreme right are the same coils repeated. The table appended to Figure B.2(d) shows the relative magnitudes and directions of the flux developed by each coil for the specific phase angles of the three current waves in Figure B.2(c); the relative magnitudes are indicated by the size of the letter (N, S, N, S). The broken lines in Figure B.2(d) connect the centerlines of each coil to the corresponding columns. Reading vertically down, each column shows the changes in magnetic polarity that occur within each coil as the three-phase current goes through two cycles, from 0 to 720 electrical degrees. The blank spaces are left as an exercise to be filled in by the student.

The shifting of the magnetic poles, with the changing current, is shown as a shaded area in the table. Note that the north pole moves from its "starting" position midway between slots 2 and 3 at zero electrical degrees to a position midway between slots 8 and 9 at 360 electrical degrees, making only one-half a revolution per cycle of applied voltage. It takes 720 electrical degrees for the rotating flux to make one complete revolution, returning to its starting position midway between slots 2 and 3. This is twice the number of electrical degrees and, hence, twice the time for the rotating flux to make one revolution. Thus, contrasted with the previously discussed two-pole motor, the rotating flux of a four-pole motor revolves at half the speed of the rotating flux of a two-pole motor.

B.3 FULL-PITCH WINDING

The coil span for a full-pitch winding is

$$\delta = \frac{S}{P} \tag{B–1}$$

where:

S = number of stator slots
P = number of poles
δ = full-pitch slots/pole

The coil slot locations for full pitch coils are 1 and $1 + \delta$, 2 and $2 + \delta$, etc.

EXAMPLE B.1 Determine (a) the coil span for the stator of a six pole, three-phase, 54-slot induction motor; (b) list slot locations for several coils.

Solution

(a)

$$\delta = \frac{S}{P} = \frac{54}{6} = \underline{9}$$

(b) Coil slots locations: 1 and 10, 2 and 11, etc.

B.4 FRACTIONAL-PITCH WINDINGS

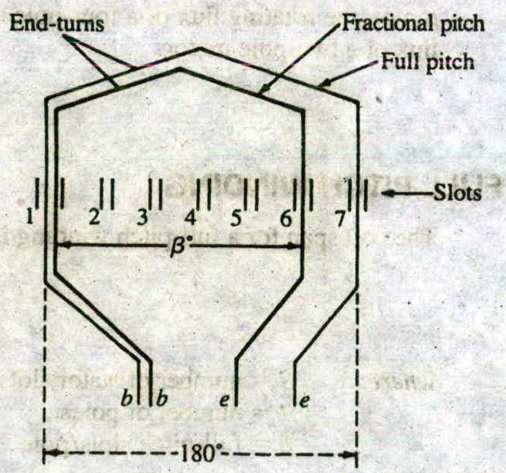
If the stator coils have a span less than full pitch (<180 electrical degrees), the winding is called fractional pitch, short pitch, or chorded [1]. The difference between a full-pitch winding and a fractional-pitch winding is shown in Figure B.3. Assuming the stator has 24 slots and is to be wound for three phases with four poles, the coil span for full pitch will be

$$\delta = \frac{S}{P} = \frac{24}{4} = 6 \text{ slots}$$

Thus, the coil sides for a representative full-pitch coil will be in slots 1 and 7.

The shorter end turns, also called end connections, of a fractional-pitch winding result in a savings in copper, lower resistance, less heat loss in the windings, and higher efficiency than for an equivalent full-pitch winding. Furthermore, the lower leakage reactance of a fractional pitch winding increases the maximum torque that the machine can develop and provides a general overall improvement in machine operation.

FIGURE B.3
Comparison of coils for a full-pitch winding and a fractional-pitch winding.



B.5 DISTRIBUTED WINDINGS

The windings illustrated in Figures B.1 and B.2 are called concentrated windings in that all of the turns per pole per phase are concentrated in one coil. A distributed winding [1] distributes the turns into two or more series connected coils in adjacent slots, as shown in Figure B.4. These are called *pole phase groups.* To accommodate the additional coils, the stator core for a distributed winding has two or more times the amount of slots that would be required if it were housing a concentrated winding. The number of slots per pole phase group may be determined from:

$$S' = \frac{S}{P \times \text{phases}} \tag{B–2}$$

where: S' = number of slots per pole phase group
S = number of slots
P = number of poles

Although the many narrow and shallow slots of the distributed winding cause more flux pulsations per revolution than does the concentrated winding, the pulses are of much lower amplitude and result in a smoother flux distribution. This results in smoother torque, lower amplitudes of vibration, and a better distribution of heat losses in the iron. The voltage produced by a distributed winding is slightly less than that produced by a concentrated winding with the same number of turns. The voltage ratio

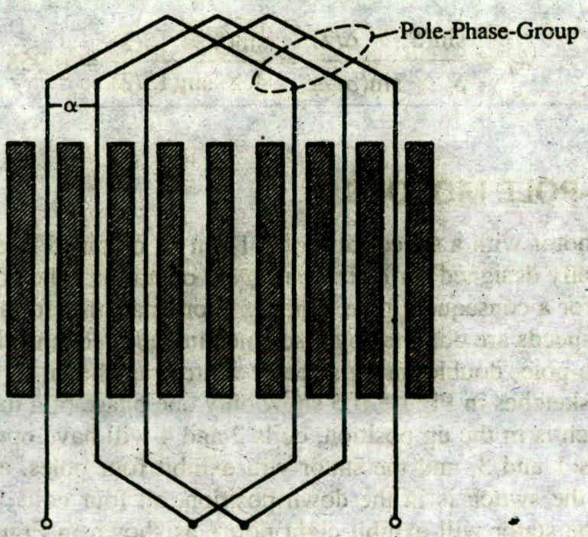


FIGURE B.4
Distributed winding.

of *V*-distributed to *V*-concentrated, called the distribution factor, spread factor, breadth factor, or belt factor, can be calculated from:

$$k_d = \frac{\sin(S' \times (\alpha/2))}{S' \times \sin(\alpha/2)} \tag{B–3}$$

$$\alpha = \frac{P \times 180}{S}$$

where: k_d = distribution factor
α = number of electrical degrees between the centers of adjacent slots
S' = number of slots per pole phase group

EXAMPLE B.2 Given a four-pole, three-phase, 48-slot, full-pitch stator for a 100-hp 460-V motor, determine (a) the coil pitch and a representative span for one coil; (b) the number of slots per pole phase group; (c) the electrical degrees between centers of adjacent slots; (d) the distribution factor.

Solution

(a)

$$\delta = \frac{S}{P} = \frac{48}{4} = \underline{12 \text{ slots}}$$

Span is from slot 1 to slot 13.

(b)

$$S' = \frac{S}{P \times \text{phases}} = \frac{48}{4 \times 3} = \underline{4}$$

(c)

$$\alpha = \frac{P \times 180}{S} = \frac{4 \times 180}{48} = \underline{15^\circ}$$

(d)

$$k_d = \frac{\sin(S' \times \alpha/2)}{S' \times \sin(\alpha/2)} = \frac{\sin(4 \times 15/2)}{4 \times \sin(15/2)} = \underline{0.958}$$

B.6 CONSEQUENT-POLE MOTORS

A two-speed motor with a speed ratio of 2:1 can be obtained from a single winding that is specifically designed for *consequent-pole* operation. The coil pitch of 90 electrical degrees for a consequent pole winding is one-half that for a standard machine. High and low speeds are obtained by disconnecting and reconnecting the windings or by using a two-pole, double-throw selector switch, as shown in Figure B.5(a). For simplicity, all sketches in Figure B.5 show only one phase of a three-phase winding. When the switch is in the up position, coils 2 and 4 will have opposite polarity with respect to coils 1 and 3, and the stator will exhibit four poles, as shown in Figure B.5(b). When the switch is in the down position, all four coils will have the same polarity, and the stator will exhibit eight poles, as shown in Figure B.5(c). Making all four coils north poles forces south poles to form between them; the four south

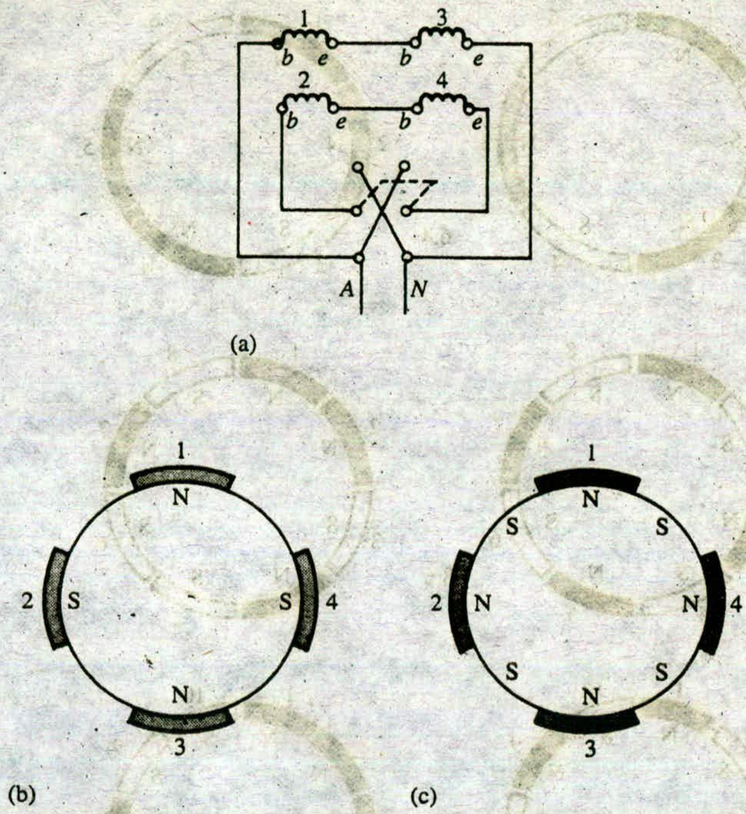


FIGURE B.5
Consequent-pole windings; (a) winding connections to selector switch; (b) coil polarities for four-pole operation; (c) coil polarities for eight-pole operation.

poles in Figure B.5(c) are called consequent poles because they were formed as a consequence of this connection. Note that with respect to the eight-pole connection in Figure B.5(c), the coils have full pitch; the same coils are half-pitch for the four-pole connection.

A more generalized concept of the consequent pole machine is the PAM (pole amplitude modulation) motor **[1],[2]**. The PAM motor is a squirrel-cage induction motor whose *single winding stator* can be connected to provide speed ratios of other than 2:1. Selective switching of coil polarities using a switching arrangement similar to that used for consequent-pole machines are used to obtain the desired number of poles. Representative pole arrangements are shown in Figure B.6. Reversing coils 2, 3, and 4 of the six-pole winding in Figure B.6(a) results in a four-pole winding; reversing coils 2, 3, 4, and 5 in the 8-pole winding in Figure B.6(b) results in a 6-pole winding; reversing coils 2, 5, 6, 8, and 9 of the 10-pole winding in Figure B.6(c) results in a 4-pole winding.

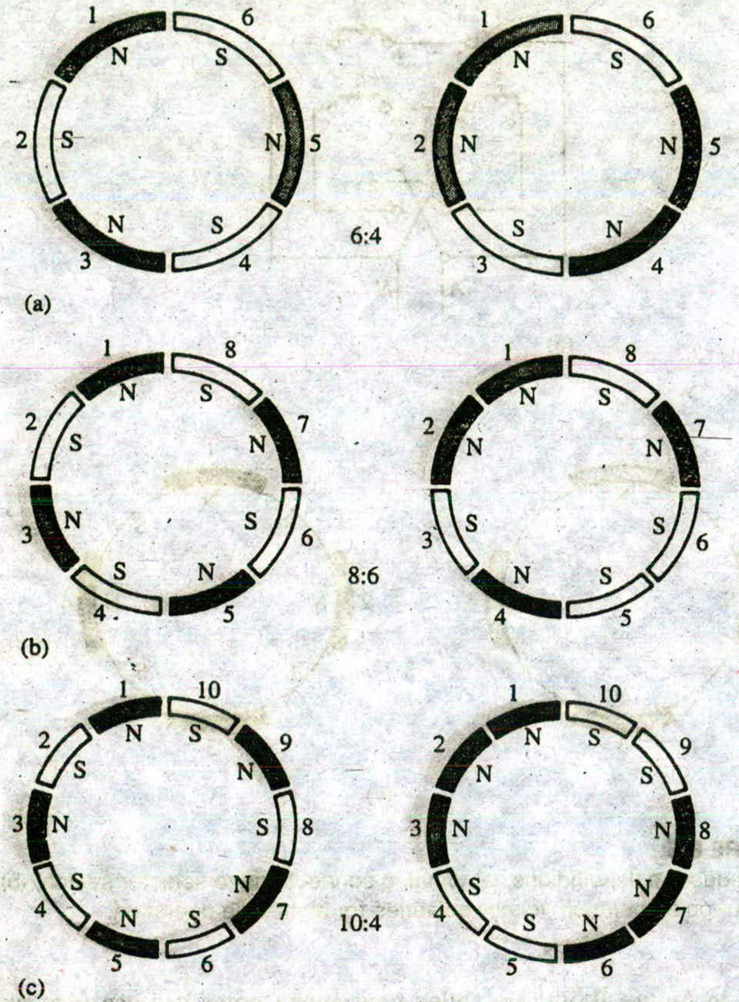


FIGURE B.6
Representative pole arrangements for three representative PAM motors: (a) 6 and 4 poles; (b) 8 and 6 poles; (c) 10 and 4 poles.

Unlike a conventional stator, the PAM stator uses irregular coil groupings that produce space harmonics in the rotating field. Since space harmonics can cause low starting torque, excessive noise, and sharp dips in torque during acceleration, more consideration must be given to the selection of an appropriate winding arrangement. PAM motors, however, have a slightly higher efficiency than a comparable two-speed, two-winding motor; and are smaller, lighter in weight, and generally less expensive than conventional two-speed two-winding motors.

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C

Constant-Horsepower, Constant-Torque, and Variable-Torque Induction Motors

There are three general groups of multispeed squirrel-cage induction motors, each group designed for a specific type of application. They are constant-horsepower, constant-torque, and variable-torque induction motors **[1]**,**[2]**.

C.1 CONSTANT-HORSEPOWER MOTOR

A constant-horsepower, multispeed motor is designed to deliver approximately the same rated horsepower with every synchronous speed connection. Hence, the rated torque for the different speed connections must vary inversely with the synchronous speed. The mathematical relationship involved is the basic power equation $P = Tn/5252$. Thus, for a constant-horsepower, multispeed induction motor,

$$\frac{T_{\mathrm{LO}} n_{\mathrm{LO}}}{5252} = \frac{T_{\mathrm{HI}} n_{\mathrm{HI}}}{5252} \quad \Rightarrow \quad \frac{T_{\mathrm{LO}}}{T_{\mathrm{HI}}} = \frac{n_{\mathrm{HI}}}{n_{\mathrm{LO}}} \qquad \text{(C–1)}$$

For example, the torque that can be delivered at a 900 r/min connection would be twice that at an 1800 r/min connection. Multispeed motors of this type are used for lathes and other machine tools that often require a constant rate of doing work. *It is important to note, however, that the motor will not deliver the same horsepower at all speed connections unless the load demands it.*

C.2 CONSTANT-TORQUE MOTOR

A constant-torque multispeed motor is designed to deliver approximately the same torque with every synchronous speed connection. Hence, the rated horsepower for the different speed connections must vary directly with the synchronous speed. From the basic power equation, $T = 5252P/n$. Hence,

$$\frac{5252P_{\text{LO}}}{n_{\text{LO}}} = \frac{5252P_{\text{HI}}}{n_{\text{HI}}} \quad \Rightarrow \quad \frac{P_{\text{LO}}}{P_{\text{HI}}} = \frac{n_{\text{LO}}}{n_{\text{HI}}} \tag{C-2}$$

For example, the horsepower rating for a 900 r/min connection would be half the horsepower rating at the 1800 r/min connection. Multispeed motors of this type are used for conveyers, compressors, reciprocating pumps, printing presses, and similar loads. *It should be noted, however, that a constant-torque motor will not deliver constant torque unless the load demands it.*

C.3 VARIABLE-TORQUE MOTOR

A variable-torque multispeed motor is designed to have its rated torque vary in direct proportion to the synchronous speed for every speed connection. Hence, its horsepower rating for the different speed connections will be in proportion to the square of the synchronous speed. Expressed mathematically,

$$\frac{T_{\text{LO}}}{T_{\text{HI}}} = \frac{n_{\text{LO}}}{n_{\text{HI}}} \tag{C-3}$$

$$\frac{P_{\text{LO}}}{P_{\text{HI}}} = \frac{n_{\text{LO}}^2}{n_{\text{HI}}^2} \tag{C-4}$$

For example, the torque that can be developed at a 900 r/min connection would be half that at the 1800 r/min connection. Hence, the 900 r/min connection would have one-quarter the horsepower rating of the 1800 r/min connection. Multispeed motors of this type are used for fans, centrifugal pumps, or other loads with similar characteristics. The power requirements for fans and blowers is directly proportional to the cube of the speed. Hence, a lower speed connection requires significantly less power from the motor.

EXAMPLE C.1 A 20-hp, 460-V, 60-Hz, variable-torque induction motor has speeds rated at 1750 and 1150 r/min. What is its horsepower rating at each speed?

Solution

The horsepower rating for the higher speed connection is always the nameplate value. Thus, in this example the horsepower rating for the 1750 r/min connection is 20 hp. The respective synchronous speeds are 1800 r/min and 1200 r/min. The horsepower rating for the 1150 r/min connection is

$$\frac{P_{\text{LO}}}{20} = \frac{1200^2}{1800^2}$$

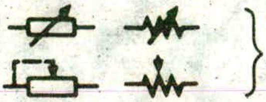
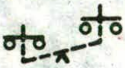
$$P_{\text{LO}} = \underline{8.89\text{ hp}}$$

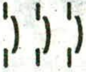
SPECIFIC REFERENCES KEYED TO TEXT

1. Heredos, F. P. Selection and application of multi-speed motors. *IEEE Trans. Industry Applications,* Vol. IA-23, No. 2, Mar./Apr. 1987.
2. National Electrical Manufacturers Association, *Motors and Generators.* Standards Publication No. MG-1-1998, NEMA, Rosslyn, VA, 1999.

D

Selected Graphic Symbols Used in Controller Diagrams

Symbol	Device
	Ground connection
	Fuse
	Resistor
	Rheostat
	Indicating lamp
	Capacitor
	Diode
	Silicon controlled rectifier (SCR)
 or 	Overload heater
	Blowout coil
 or 	Operating coil
	Contact normally open
	Contact normally closed
	Spring-return push button normally open
	Spring-return push button normally closed
	Sustaining-type push button
	Plug-type contact

Symbol	Device
	Mechanical interlock
	Mechanical interlock with fulcrum
	Crossing conductors not connected
	Connected conductors
	Transformer
	Current transformer
	Three-pole circuit breake
	Three-pole power breake for AC circuits rated in excess of 1500 V
	Switch
	Reactor or field winding

		
Bell	Buzzer	Horn or siren

Limit Switches

	Normally open contact
	Normally open contact held closed
	Normally closed contact
	Normally closed contact held open

E

Full-Load Current in Amperes, Direct-Current Motors

The following values of full-load currents* are for motors running at base speed.

	Armature Voltage Rating*					
hp	90 V	120 V	180 V	240 V	500 V	550 V
$\frac{1}{4}$	4.0	3.1	2.0	1.6		
$\frac{1}{3}$	5.2	4.1	2.6	2.0		
$\frac{1}{2}$	6.8	5.4	3.4	2.7		
$\frac{3}{4}$	9.6	7.6	4.8	3.8		
1	12.2	9.5	6.1	4.7		
$1\frac{1}{2}$		13.2	8.3	6.6		
2		17	10.8	8.5		
3		25	16	12.2		
5		40	27	20		
$7\frac{1}{2}$		58		29	13.6	12.2
10		76		38	18	16
15				55	27	24
20				72	34	31
25				89	43	38
30				106	51	46
40				140	67	61
50				173	83	75
60				206	99	90
75				255	123	111
100				341	164	148
125				425	205	185
150				506	246	222
200				675	330	294

*These are average direct-current quantities.

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F

Full-Load Current in Amperes, Single-Phase Alternating-Current Motors

The following values of full-load currents are for motors running at usual speeds and motors with normal torque characteristics. Motors built for especially low speeds or high torques may have higher full-load currents, and multispeed motors will have full-load current varying with speed, in which case the nameplate current ratings shall be used.

To obtain full-load currents of 208- and 200-V motors, increase corresponding 230-V motor full-load currents by 10 and 15 percent, respectively.

The voltages listed are rated motor voltages. The currents listed shall be permitted for system voltage ranges of 110 to 120 and 220 to 240.

hp	115 V	230 V
$\frac{1}{6}$	4.4	2.2
$\frac{1}{4}$	5.8	2.9
$\frac{1}{3}$	7.2	3.6
$\frac{1}{2}$	9.8	4.9
$\frac{3}{4}$	13.8	6.9
1	16	8
$1\frac{1}{2}$	20	10
2	24	12
3	34	17
5	56	28
$7\frac{1}{2}$	80	40
10	100	50

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G

Full-Load Current, Two-Phase Alternating-Current Motors (Four-Wire)

The following values of full-load current are for motors running at speeds usual for belted motors and motors with normal torque characteristics. Motors built for especially low speeds or high torques may require more running current, and multispeed motors will have full-load current varying with speed, in which case the nameplate current rating shall be used. Current in the common conductor of a two-phase, three-wire system will be 1.41 times the value given.

The voltages listed are rated motor voltages. The currents listed shall be permitted for system voltage ranges of 110 to 120, 220 to 240, 440 to 480, and 550 to 600 V.

	Induction-Type Squirrel-Cage and Wound-Rotor (A)				
hp	115 V	230 V	460 V	575 V	2300 V
$\frac{1}{2}$	4	2	1	0.8	
$\frac{3}{4}$	4.8	2.4	1.2	1.0	
1	6.4	3.2	1.6	1.3	
$1\frac{1}{2}$	9	4.5	2.3	1.8	
2	11.8	5.9	3	2.4	
3		8.3	4.2	3.3	
5		13.2	6.6	5.3	
$7\frac{1}{2}$		19	9	8	
10		24	12	10	
15		36	18	14	
20		47	23	19	
25		59	29	24	
30		69	35	28	
40		90	45	36	
50		113	56	45	
60		133	67	53	14
75		166	83	66	18
100		218	109	87	23
125		270	135	108	28
150		312	156	125	32
200		416	208	167	43

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H

Full-Load Current, Three-Phase Alternating-Current Motors

For full-load currents[1] of 208- and 200-V motors, increase the corresponding 230-V motor full-load current by 10 to 15 percent, respectively.

The voltages listed are rated motor voltages. The currents listed shall be permitted for system voltage ranges of 110–120, 220–240, 440–480, and 550–660 V.

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[1] These values of full-load current are for motors running at speeds usual for belted motors and motors with normal torque characteristics. Motors built for especially low speeds or high torques may require more running current, and multispeed motors will have full-load current varying with speed, in which case the nameplate current rating shall be used.

	Induction-Type Squirrel-Cage and Wound-Rotor (A)					Synchronous-Type Unity Power Factor* (A)			
hp	115 V	230 V	460 V	575 V	2300 V	230 V	460 V	575 V	2300 V
½	4	2	1	0.8					
¾	5.6	2.8	1.4	1.1					
1	7.2	3.6	1.8	1.4					
1½	10.4	5.2	2.6	2.1					
2	13.6	6.8	3.4	2.7					
3		9.6	4.8	3.9					
5		15.2	7.6	6.1					
7½		22	11	9					
10		28	14	11					
15		42	21	17					
20		54	27	22					
25		68	34	27		53	26	21	
30		80	40	32		63	32	26	
40		104	52	41		83	41	33	
50		130	65	52		104	52	42	
60		154	77	62	16	123	61	49	12
75		192	96	77	20	155	78	62	15
100		248	124	99	26	202	101	81	20
125		312	156	125	31	253	126	101	25
150		360	180	144	37	302	151	121	30
200		480	240	192	49	400	201	161	40

*For 90 and 80 percent power factor the preceding figures shall be multiplied by 1.1 and 1.25, respectively.

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I

Representative Transformer Impedances for Single-Phase 60-Hz Transformers

Rating (kVA)	Voltage 2400 %R		Voltage 7200 %R	
	%R	%X	%R	%X
10	1.51	1.78	1.60	1.62
50	1.30	2.25	1.29	2.10
100	1.20	2.31	1.20	3.53
250	1.01	4.70	1.00	5.16
500	1.00	4.75	1.00	5.24

J

Unit Conversion Factors

Force: lb × 4.448 = N
Length: ft × 0.3048 = m
Magnetics: Oersteds × 79.577 = A-t/m
Lines × 10^{-8} = Wb
Lines/in.^{2} × 1.55 × 10^{-5} = T
Gausses × 10^4 = T
Power: hp × 746 = W
Rotational speed: r/min × 0.1047 = rad/s
Torque: lb-ft × 0.7376 = N-m

Answers To *Odd-Numbered* Problems

Chapter 1

1. (a) 0.40 Wb, (b) 8 Ω

3. 64.18 V

5. 1.013 T

7. (a) 1499.1 A-t/m, (b) 1.25 T, 0.10 Wb, (c) 663.5, (d) 22486 A-t/Wb

9. (a) 1499.1 A-t/m, (b) 0.48 T, 0.0384 Wb, (c) 254.8, (d) 58557.2 A-t/Wb

11. −47.1%

13. (a) 7.21 V, (b) 7.96 V

15. 65.89 m/s

17. 24 Hz, 89.5 V

19. (a) 474.87 V, (b) 672 cos(28t) V

21. 48.98 W

Chapter 2

1. (a) 121 t, (b) 0.0862 Wb

3. (a) 126 t, 630 t, (b) 3.0 A

5. (a) 2.60 A, (b) 0.460 A, (c) 2.56 A, (d) 220.8 W

7. (a) 15.87 A, (b) 0.173, (c) 1875 var

9. (a) 6600 V, (b) 45.83 A, (c) 1375 A, (d) $0.160 \angle 46^\circ\ \Omega$, (e) 210.1 kW, 217.6 kvar, 302.5 kVA

11. (a) 124.8 V, (b) 624 V, (c) 3.12 A, (d) 1651.05 W, 1031.69 var, 1946.88 VA

13. (a) $8.97 \angle 61.63^\circ\ \Omega$, (b) $0.56 \angle 61.63^\circ\ \Omega$

15. (a) $13.78 \angle 63.21^\circ\ \Omega$, (b) $531.11 \angle 41.95^\circ\ \Omega$, (c) 13.89 A, (d) 7377 V, (e) 0.427 A, (f) $17276.4 \angle -75.5^\circ\ \Omega$

17. (a) $0.0435 \angle 83.5^\circ\ \Omega$, (b) 495.27 V, (c) 3.18 percent

19. (a) 2387.8 V, (b) −0.51 percent, (c) $51.68 \angle -15.21^\circ\ \Omega$

21. (a) 243 V, (b) 5.68 percent, (c) $1341.1\ \angle 48.82^\circ\ \Omega$, (d) $42321\ \angle 75.99^\circ\ \Omega$

23. (a) 0.194 Ω, (b) 0.012 Ω

25. (a) 94,696 A, (b) 3.47 percent

27. 2.84 percent

29. 2.61 percent

31. (a) 3.26 percent, (b) 237.5 V, (c) 464.7 V

33. Plot

35. (a) 345 W, (b) 97.05 percent

37. Plot

39. (a) $R_{eq} = 1.743\ \Omega$, $X_{eq} = 3.233\ \Omega$, $R_{fe} = 11901\ \Omega$, $X_M = 2961.9\ \Omega$, (b) 2.51 percent, (c) 97.4 percent

41. (a) 6572.6 Ω, (b) $R_{PU} = 0.0121$, $X_{PU} = 0.0384$, (c) 98.8 percent, (d) 2.44 percent, (e) 235.6 V, (f) 4712 V

Chapter 3

1. (a) 225 A, (b) $I_{HS} = 44.03$ A, $I_{tr} = 181.1$ A

3. (a) 800 V, (b) 6 A, (c) 2 A, (d) 1600 VA, (e) 3200 VA, (f) 0.015 Wb

5. (a) 1.176, (b) 1.200, (c) 122.4 V

7. (a) 1.100, (b) sketch, (c) 750 A, 681.8 A

9. 10.07

11. (a) $416.67\ \angle -43.95^\circ$ A, (b) $I_A = 234.1\ \angle -44.14^\circ$ A, $I_B = 182.5\ \angle -43.70^\circ$ A

13. $I_A = 38.17\%$, $I_B = 34.99\%$, $I_C = 26.94\%$

15. No, B will overheat.

17. $I_{LS,phase} = I_{LS,line} = 479.2$ A, $I_{HS,phase} = 69.45$ A, $I_{HS,line} = 120.3$ A

19. 69.28 kVA

21. 10,249 A

Chapter 4

1. (a) 1800 r/min, (b) 50 r/min, (c) 0.278

3. (a) 1800 r/min, (b) 0.01388, (c) 25 r/min, (d) 0.833 Hz

5. (a) 20 r/min, (b) 1.0 Hz, 3.6 V

7. (a) 50 r/min, (b) 2.78% (c) 180 r/min

9. (a) 500 r/min, (b) 0.040, (c) 63.10 A, (d) 218.83 lb-ft, (e) 2.0 Hz

11. (a) 22030 W, (b) 26.58 hp, (c) 2203 W, (d) 586 r/min, (e) 238.2 lb-ft, (f) 506 W

13. (a) 706.9 r/min, (b) 945.4 lb-ft, (c) 928.7 lb-ft, (d) 0.805, (e) 1683.8 W

15. (a) 1746.2 r/min, (b) 18.8 lb-ft, (c) 1.23 lb-ft

17. (a) 22.0 hp, 3150.5 r/min, 84.3%

Chapter 5

1. (a) 47.89 lb-ft, (b) 73.68 lb-ft, (c) 36.84 lb-ft

3. (a) 37.7 lb-ft, (b) 29.57 lb-ft, (c) 26.61 lb-ft

5. (a) $11.96\ \angle 37.58^\circ\ \Omega$, (b) $27.77\ \angle -37.58^\circ$ A, (c) 21916.6 W, 16866.9 var, 27655.6 VA, 0.793 lagging, (d) 22.8 A, (e) 861.2 W, (f) 608.7 W, (g) 764.2 W, (h) 20291.2 W, (i) 19682.5 W, (j) 119.04 lb-ft, (k) 25.92 hp, (l) 116.95 lb-ft, (m) 88.23%, (n) sketch.
7. (a) $7.89\ \angle 40.31^\circ\ \Omega$, (b) $33.68\ \angle -40.31^\circ$ A, (c) 20461.9 W, 17356.4 var, 26831.6 VA, 76.26% lagging, (d) 27.11 A, (e) 643.38 W, (f) 421.38 W, (g) 865.3 W, (h) 18963.8 W, (i) 18542.8 W, (j) 148.47 lb-ft, (k) 2280.1 W, (l) 24.37 hp, (m) 88.66%, (n) sketch, (o) LR = 360.54 lb-ft, BD = 342.52 lb-ft, PU = 252.38 lb-ft
9. (a) 810.5 r/min, (b) 392.2 lb-ft
11. No
13. (a) 0.958%, (b) 1782.8 r/min, (c) 215.7 A, (d) 191.9 hp
15. (a) 1169 r/min, (b) 20.1 A, (c) 17.2 hp
17. 26.8 hp
19. (a) 479.2 V, (b) 104.2 hp, (c) 750 r/min, (d) 734.25, (e) 745 lb-ft
21. 0.135 Ω/phase
23. (a) 71 A, (b) $267.3\ \text{A} \le I_{\text{lr}} < 301.2\ \text{A}$
25. (a) 3.17%, (b) 108°C, (c) 5.7 years, (d) 52.8 hp
27. $R_1 = 0.2539\ \Omega$, $X_1 = 0.6836\ \Omega$, $R_2 = 0.1872\ \Omega$, $X_2 = 1.0282\ \Omega$, $X_M = 21.42\ \Omega$
29. $\text{PU}R_1 = 0.0333$, $\text{PU}X_1 = 0.0562$, $\text{PU}R_2 = 0.030$, $\text{PU}X_2 = 0.084$, $\text{PU}X_M = 1.227$, $\text{PU}R_{\text{fe}} = 42.38$
31. $R_1 = 0.1915\ \Omega$, $R_2 = 0.1895\ \Omega$, $X_1 = 0.5745\ \Omega$, $X_2 = 1.3404\ \Omega$, $X_M = 14.92\ \Omega$, fwcor = 405.1 W/phase
33. 28.8 kW
35. 186.53 kW
37. (a) 135.78 A, (b) 441.34 lb-ft, (c) 168 V, (d) 76.38%, (e) 553.7 A, (f) 422.9 A
39. (a) 248.2 A, (b) 595.46 lb-ft, (c) 318.57 A, (d) 1.444
41. (a) 2.358 Ω, (b) 154.4 V, (c) 45.7 lb-ft

Chapter 6

1. (a) 12.34 Ω, (b) Auxiliary $5.53\ \angle -16.83^\circ$ A, Main $20.84\ \angle -46.26^\circ$ A, (c) $25.78\ \angle -40.67^\circ$ A
3. (a) 222.7 μF, (b) $25.6\ \angle -19.7^\circ$ A
5. (a) 11500 μF, (b) 1325 μF, (c) 33.33 hp
7. (a) $I_{\text{line}} = 22.41$ A, $I_{\text{phase}} = 12.94$ A, (b) $I_{\text{line}} = 38.81$ A, $I_A = 12.94$ A, $I_B = 25.87$ A

Chapter 7

1. 30.54% increase
3. (a) 170%, (b) 170%
5. (a) 200, (b) 30 r/s, (c) 26

7. (a) 1000 steps/rev, (b) 210

9. 0.20

Chapter 8

1. 150 r/min

3. 80 poles

5. (a) 5835.6 lb-ft, (b) 161 A, (c) 13669.5 V, (d) −29.2°, (e) 12389.6 lb-ft

7. (a) 496.0 V, (b) −27.8°, 1332.5 lb-ft, (c) 666.2 lb-ft

9. Phasor diagrams

11. (a) plot, (b) sketch, (c) rated: 7280 V/phase, 75% rated: 5500 V/phase, 50% rated: 3680 V/phase

13. (a)−18°, (b) 189,666 W, −26,1053 var, (c) 58.8% leading, (d) 742 lb-ft

15. (a) 0.4931 leading, (b) 542.3 V, (c) −15.57°

17. (a) 78.1% lagging, (b) 0.895 lagging

19. (a) 4084.9 lb-ft, (b) 3063.7 lb-ft, (c) 20,424.4 lb-ft

Chapter 9

1. 1500 r/min

3. (a) 314.46 V, (b) 64.08 Hz

5. (a) 670 hp, (b) 859.1 V, (c) 580.8 A, (d) 499843 W, 338348 var, (e) 82.8% lagging

7. 61.29 Hz, $P_1 = 75.79$ kW, $P_2 = 74.21$ kW

9. (a) 60.15 Hz, (b) $P_A = 313$ kW, $P_B = 373$ kW

11. (a) 24.77 Hz, (b) $P_A = 471.68$ kW, $P_B = 601.87$ kW, $P_C = 626.45$ kW

13. (a) Phasor diagram, (b) $F_P = 76.6\%$ leading, $\delta = -17^\circ$

15. (a) Sketch, (b) 35°, (c) 27°

17. $P_A = 347.76$ kW, $P_B = 115.92$ kW, $Q_A = 143.92$ kvar, $Q_B = 170.09$ kvar

19. (a) Sketch, (b) 59.89 Hz, $P_A = 690.90$ kW, $P_B = 595.45$ kW, $P_C = 563.63$ kW, (c) 73.5% leading

21. 1.449

23. (a) 271.8 V, (b) 65.7°, (c) 274 V, (d) 1.1%, (e) 208 V

25. 7.9%

27. 483 V

29. (a) 1.32 Ω, (b) 1.07

31. (a) 0.5047 Ω, (b) 1.52 Ω, (c) 1.60 Ω

Chapter 10

1. (a) 56.67 Hz, (b) 68 Hz, 228 V

3. (a) 200 V, (b) 280 V

5. 3.57%

7. 36.1%

9. 3005.1 r/min
11. 410.4 r/min
13. 270.7 V
15. 248.5 A
17. 3563.4 r/min
19. (a) 466.6 A, (b) 1268.7 r/min
21. (a) 232.1 V, (b) 12314.5 W, (c) 24.8 lb-ft, (d) 22.5 lb-ft
23. (a) 8674.4 W, (b) 8082.1 W, (c) 17.27 lb-ft
25. (a) 1767.2 W, (b) 3208.8 W, (c) 93.8%
27. (a) 137.0 lb-ft, (b) 23655 W, (c) 144.8 lb-ft, (d) 1.24 Ω, (e) 253.4 lb-ft

Chapter 11

1. (a) 33.2 Ω, (b) 119.7 A, (c) 365.9 W
3. (a) 97.4 Ω, (b) 335.0 W
5. (a) 49.3%, (b) 969.4 r/min
7. (a) 1329.2 A, (b) 1318.8 A, (c) 323109 W, (d) yes
9. (a) 209.6 A, (b) 3.35 A, (c) 206.2 A, (d) 673.3 lb-ft, (e) 0.5347 Ω, (f) 6 turns, (g) 423.3 r/min
11. 700 r/min, 377 r/min
13. (a) 3121 r/min, (b) 5963 W, (c) 13.4 lb-ft
15. (a) 319 r/min, (b) 6.3%, (c) 224 lb-ft
17. 77.5 V
19. 0.442 Ω

Chapter 12

1. (a) 268 V, (b) 7.2%, (c) 7.3 Ω
3. 227.8 V
5. (a) 265 V, (b) 290 V, (c) 16.0%, (d) 73.1 Ω, (e) 315 V
7. (a) 500 A, (b) 5.49 A, (c) 505.49 A, (d) 270 V, (e) 257 V, (f) 2.8%, (g) under, (h) yes
9. (a) 0.001951 Ω, (b) 377.4 W
11. (a) 123.5 V, (b) $I_A = 820$ A, $I_B = 980$ A
13. (a) 562.1 V, (b) $I_A = 483.6$ A, $I_B = 516.4$ A
15. (a) 241.9 V, (b) $I_A = 351.35$ A, $I_B = 756.76$ A, $I_C = 891.89$ A

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