6 Generation with Limited Energy Supply

6.1 INTRODUCTION

The economic operation of a power system requires that expenditures for fuel be minimized over a period of time. When there is no limitation on the fuel supply to any of the plants in the system, the economic dispatch can be carried out with only the present conditions as data in the economic dispatch algorithm. In such a case, the fuel costs are simply the incoming price of fuel with, perhaps, adjustments for fuel handling and maintenance of the plant.

When the energy resource available to a particular plant (be it coal, oil, gas, water, or nuclear fuel) is a limiting factor in the operation of the plant, the entire economic dispatch calculation must be done differently. Each economic dispatch calculation must account for what happened before and what will happen in the future.

This chapter begins the development of solutions to the dispatching problem "over time." The techniques used are an extension of the familiar Lagrange formulation. Concepts involving slack variables and penalty functions are introduced to allow solution under certain conditions.

The example chosen to start with is a fixed fuel supply that must be paid for, whether or not it is consumed. We might have started with a limited fuel supply of natural gas that must be used as boiler fuel because it has been declared as "surplus." The take-or-pay fuel supply contract is probably the simplest of these possibilities.

Alternatively, we might have started directly with the problem of economic scheduling of hydroelectric plants with their stored supply of water or with light-water-moderated nuclear reactors supplying steam to drive turbine generators. Hydroelectric plant scheduling involves the scheduling of water flows, impoundments (storage), and releases into what usually prove to be a rather complicated hydraulic network (namely, the watershed). The treatment of nuclear unit scheduling requires some understanding of the physics involved in the reactor core and is really beyond the scope of this current text (the methods useful for optimizing the unit outputs are, however, quite similar to those used in scheduling other limited energy systems).

6.2 TAKE-OR-PAY FUEL SUPPLY CONTRACT

Assume there are N normally fueled thermal plants plus one turbine generator, fueled under a "*take-or-pay*" *agreement*. We will interpret this type of agreement as being one in which the utility agrees to use a minimum amount of fuel during a period (the "take") or, failing to use this amount, it agrees to pay the minimum charge. This last clause is the "pay" part of the "take-or-pay" contract.

While this unit's cumulative fuel consumption is below the minimum, the system excluding this unit should be scheduled to minimize the total fuel cost, subject to the constraint that the total fuel consumption for the period for this particular unit is equal to the specified amount. Once the specified amount of fuel has been used, the unit should be scheduled normally. Let us consider a special case where the minimum amount of fuel consumption is also the maximum. The system is shown in Figure 6.1. We will consider the operation of the system over $j_{\max}$ time intervals j where $j = 1, \ldots, j_{\max}$, so that

$$P_{1j}, P_{2j}, \ldots, P_{Tj} \qquad \text{(power outputs)}$$

$$F_{1j}, F_{2j}, \ldots, F_{Nj} \qquad \text{(fuel cost rate)}$$

and

$$q_{T1}, q_{T2}, \ldots, q_{Tj} \qquad \text{(take-or-pay fuel input)}$$

are the power outputs, fuel costs, and take-or-pay fuel inputs, where

$P_{ij} \triangleq$ power from i^{th} unit in the j^{th} time interval

$F_{ij} \triangleq$ R/h cost for i^{th} unit during the j^{th} time interval

$q_{Tj} \triangleq$ fuel input for unit T in j^{th} time interval

$F_{Tj} \triangleq$ R/h cost for unit T in j^{th} time interval

$P_{\text{load } j} \triangleq$ total load in the j^{th} time interval

$n_j \triangleq$ Number of hours in the j^{th} time interval

Mathematically, the problem is as follows:

$$\min \sum_{j=1}^{j_{\max}} \left(n_j \sum_{i=1}^{N} F_{ij} \right) + \sum_{j=1}^{j_{\max}} n_j F_{Tj} \tag{6.1}$$

subject to

$$\phi = \sum_{j=1}^{j_{\max}} n_j q_{Tj} - q_{\text{TOT}} = 0 \tag{6.2}$$

and

$$\psi_j = P_{\text{load } j} - \sum_{i=1}^{N} P_{ij} - P_{Tj} = 0 \qquad \text{for } j = 1 \ldots j_{\max} \tag{6.3}$$

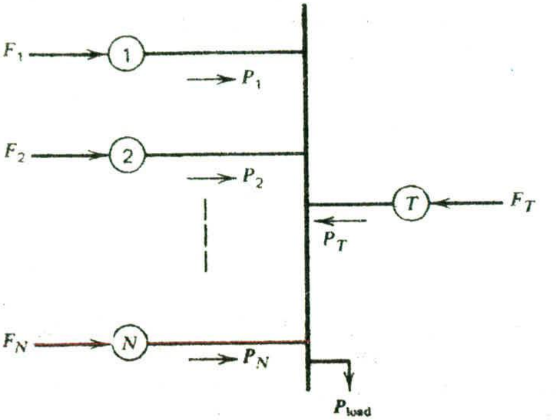


FIG. 6.1 $N + 1$ unit system with take-or-pay fuel supply at unit T.

or, in words,

> We wish to determine the minimum production cost for units 1 to N subject to constraints that ensure that fuel consumption is correct and also subject to the set of constraints to ensure that power supplied is correct each interval.

Note that (for the present) we are ignoring high and low limits on the units themselves. It should also be noted that the term

$$\sum_{j=1}^{j_{max}} n_j F_{Tj}$$

is constant because the total fuel to be used in the "T" plant is fixed. Therefore, the total cost of that fuel will be constant and we can drop this term from the objective function.

The Lagrange function is

$$\mathscr{L} = \sum_{j=1}^{j_{max}} n_j \sum_{i=1}^{N} F_{ij} + \sum_{j=1}^{j_{max}} \lambda_j \left(P_{\text{load}\, j} - \sum_{i=1}^{N} P_{ij} - P_{Tj} \right) + \gamma \left(\sum_{j=1}^{j_{max}} n_j q_{Tj} - q_{\text{TOT}} \right) \quad (6.4)$$

The independent variables are the powers P_{ij} and P_{Tj}, since $F_{ij} = F_i(P_{ij})$ and

$q_{Tj} = q_T(P_{Tj})$. For any given time period, $j = k$,

$$\frac{\partial \mathscr{L}}{\partial P_{ik}} = 0 = n_k \frac{\mathrm{d}F_{ik}}{\mathrm{d}P_{ik}} - \lambda_k \qquad \text{for } i = 1 \ldots N \tag{6.5}$$

and

$$\frac{\partial \mathscr{L}}{\partial P_{Tk}} = -\lambda_k + \gamma n_k \frac{\mathrm{d}q_{Tk}}{\mathrm{d}P_{Tk}} = 0 \tag{6.6}$$

Note that if one analyzes the dimensions of γ, it would be R per unit of q (e.g., R/ft^3, R/bbl, R/ton). As such, γ has the units of a "fuel price" expressed in volume units rather than MBtu as we have used up to now. Because of this, γ is often referred to as a "pseudo-price" or "shadow price." In fact, once it is realized what is happening in this analysis, it becomes obvious that we could solve fuel-limited dispatch problems by simply adjusting the price of the limited fuel(s); thus, the terms "pseudo-price" and "shadow price" are quite meaningful.

Since γ appears unsubscripted in Eq. 6.6, γ would be expected to be a constant value over all the time periods. This is true unless the fuel-limited machine is constrained by fuel-storage limitations. We will encounter such limitations in hydroplant scheduling in Chapter 7. The appendix to Chapter 7 shows when to expect a constant γ and when to expect a discontinuity in γ.

Figure 6.2a shows how the load pattern may look. The solution to a fuel-limited dispatching problem will require dividing the load pattern into time intervals, as in Figure 6.2b, and assuming load to be constant during each interval. Assuming all units are on-line for the period, the optimum dispatch could be done using a simple search procedure for γ, as is shown in Figure 6.3. Note that the procedure shown in Figure 6.3 will only work if the fuel-limited unit does not hit either its high or its low limit in any time interval.

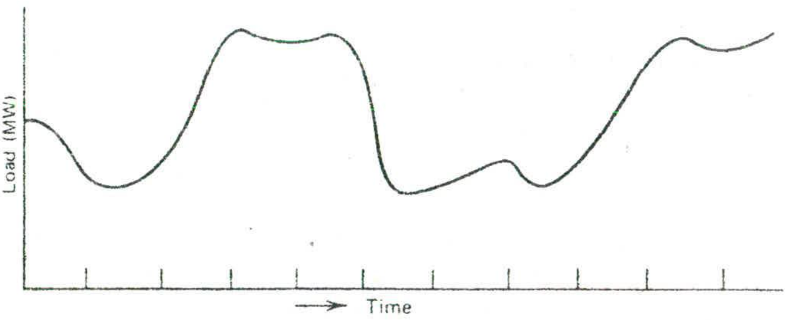


FIG. 6.2a Load pattern.

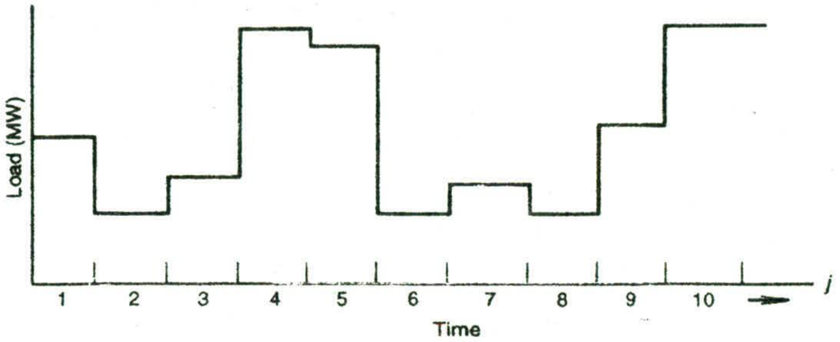


FIG. 6.2b Discrete load pattern.

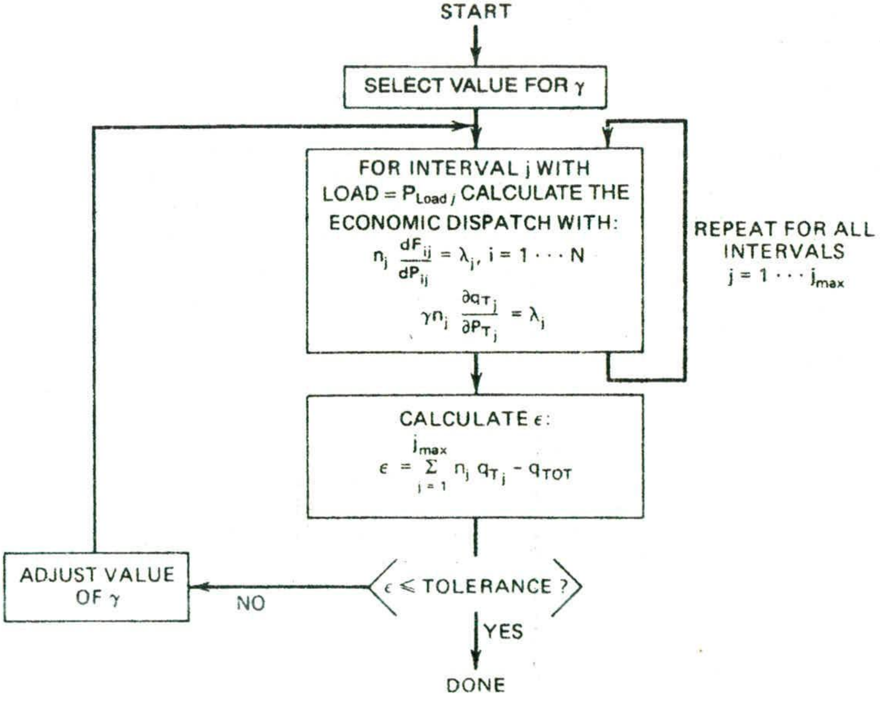


FIG. 6.3 Gamma search method.

6.3 COMPOSITE GENERATION PRODUCTION COST FUNCTION

A useful technique to facilitate the take-or-pay fuel supply contract procedure is to develop a composite generation production cost curve for all the non-fuel-constrained units. For example, suppose there were N non-fuel constrained units to be scheduled with the fuel-constrained unit as shown in Figure 6.4. Then a composite cost curve for units 1, 2, . . . , N can be developed.

$$F_s(P_s) = F_1(P_1) + \ldots + F_N(P_N) \tag{6.7}$$

where

$$P_s = P_1 + \ldots + P_N$$

and

$$\frac{\mathrm{d}F_1}{\mathrm{d}P_1} = \frac{\mathrm{d}F_2}{\mathrm{d}P_2} = \ldots = \frac{\mathrm{d}F_N}{\mathrm{d}P_N} = \lambda$$

If one of the units hits a limit, its output is held constant, as in Chapter 3, Eq. 3.6.

A simple procedure to allow one to generate $F_s(P_s)$ consists of adjusting λ from $\lambda^{\min}$ to $\lambda^{\max}$ in specified increments, where

$$\lambda^{\min} = \min\left(\frac{\mathrm{d}F_i}{\mathrm{d}P_i}, i = 1 \ldots N\right)$$

$$\lambda^{\max} = \max\left(\frac{\mathrm{d}F_i}{\mathrm{d}P_i}, i = 1 \ldots N\right)$$

At each increment, calculate the total fuel consumption and the total power output for all the units. These points represent points on the $F_s(P_s)$ curve. The points may be used directly by assuming $F_s(P_s)$ consists of straight-line segments between the points, or a smooth curve may be fit to the points using a least-squares fitting program. Be aware, however, that such smooth curves may have undesirable properties such as nonconvexity (e.g., the first derivative is not monotonically increasing). The procedure to generate the points on $F_s(P_s)$ is shown in Figure 6.5.

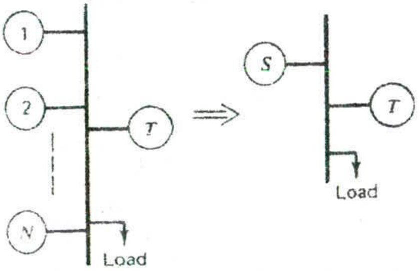


FIG. 6.4 Composite generator unit.

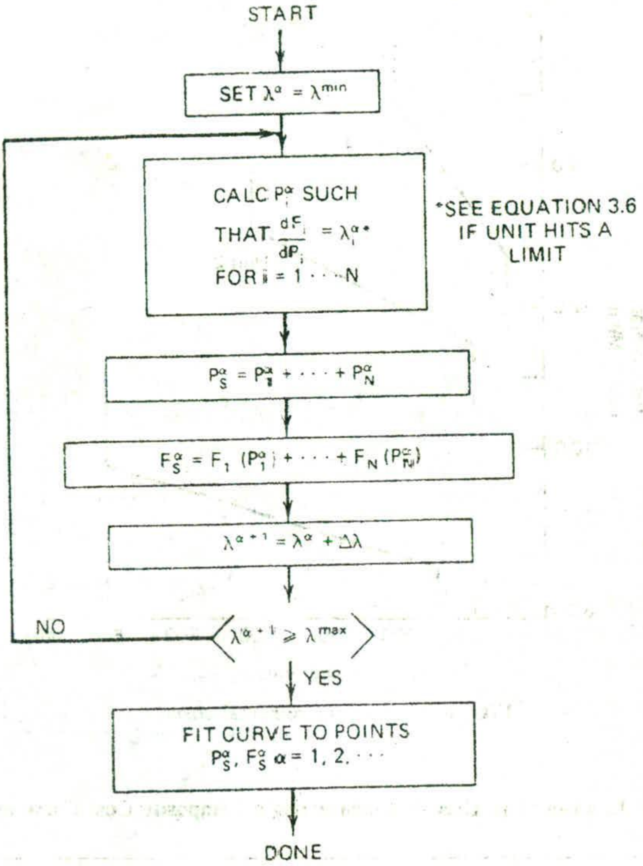


FIG. 6.5 Procedure for obtaining composite cost curve.

EXAMPLE 6A

The three generating units from Example 3A are to be combined into a composite generating unit. The fuel costs assigned to these units will be

Fuel cost for unit 1 = 1.1 R/MBtu

Fuel cost for unit 2 = 1.4 R/MBtu

Fuel cost for unit 3 = 1.5 R/MBtu

Figure 6.6a shows the individual unit incremental costs, which range from 8.3886 to 14.847 R/MWh. A program was written based on Figure 6.5, and λ was stepped from 8.3886 to 14.847

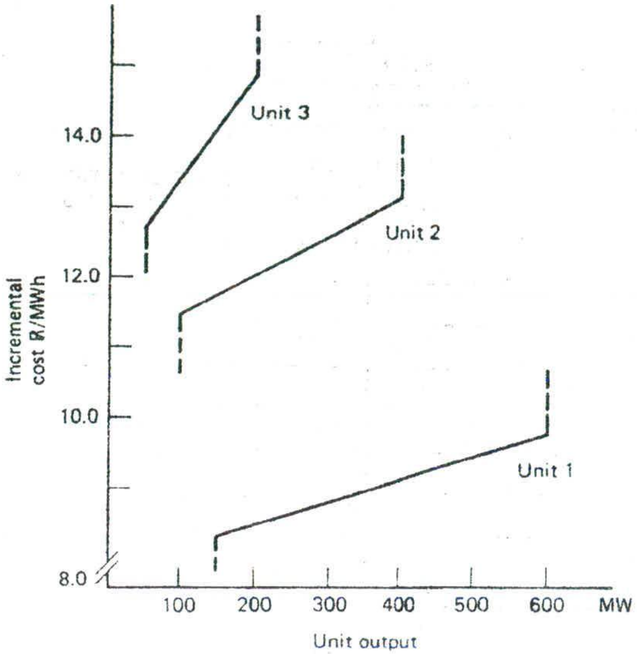


FIG. 6.6a Unit incremental costs.

TABLE 6.1 Lambda Steps Used in Constructing a Composite Cost Curve for Example 6A

Step	λ	P_s	F_s	F_s Approx
1	8.3886	300.0	4077.12	4137.69
2	8.7115	403.4	4960.92	4924.39
3	9.0344	506.7	5878.10	5799.07
4	9.3574	610.1	6828.66	6761.72
5	9.6803	713.5	7812.59	7812.35
6	10.0032	750.0	8168.30	8204.68
7	11.6178	765.6	8348.58	8375.29
8	11.9407	825.0	9048.83	9044.86
9	12.2636	884.5	9768.28	9743.54
10	12.5866	943.9	10506.92	10471.31
11	12.9095	1019.4	11469.56	11436.96
12	13.2324	1088.4	12369.40	12360.58
13	13.5553	1110.67	12668.51	12668.05
14	13.8782	1133.00	12974.84	12979.63
15	14.2012	1155.34	13288.37	13295.30
16	14.5241	1177.67	13609.12	13615.09
17	14.8470	1200.00	13937.00	13938.98

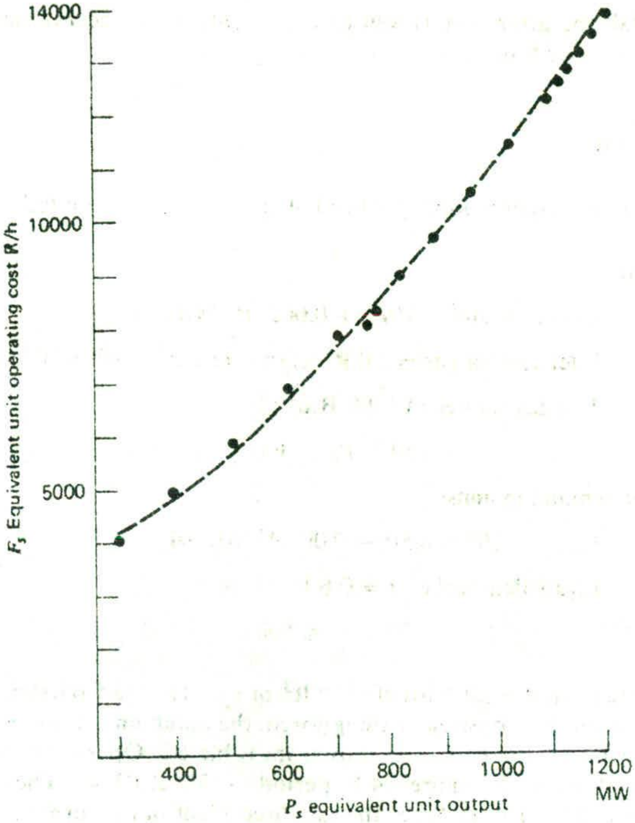


FIG. 6.6b Equivalent unit input/output curve.

At each increment, the three units are dispatched to the same λ and then outputs and generating costs are added as shown in Figure 6.5. The results are given in Table 6.1. The result, called F_s approx in Table 6.1 and shown in Figure 6.6b, was calculated by fitting a second-order polynomial to the P_s and F_s points using a least-squares fitting program. The equivalent unit function is

$$F_s \text{ approx}(P_s) = 2352.65 + 4.7151 P_s + 0.0041168 P_s^2$$

$$(\text{R/h}) \qquad 300 \text{ MW} \le P_s \le 1200 \text{ MW}$$

The reader should be aware that when fitting a polynomial to a set of points, many choices can be made. The preceding function is a good fit to the total operating cost of the three units, but it is not that good at approximating the incremental cost. More-advanced fitting methods should be used if one desires to match total operating cost as well as incremental cost. See Problem 6.2 for an alternative procedure.

EXAMPLE 6B

Find the optimal dispatch for a gas-fired steam plant given the following.

Gas-fired plant:

$$H_T(P_T) = 300 + 6.0P_T + 0.0025P_T^2 \text{ MBtu/h}$$

Fuel cost for gas = 2.0 ₨/ccf (where 1 ccf = 10^3 ft^3)

The gas is rated at 1100 Btu/ft^3

$$50 \leq P_T \leq 400$$

Composite of remaining units:

$$H_s(P_s) = 200 + 8.5P_s + 0.002P_s^2 \text{ MBtu/h}$$

Equivalent fuel cost = 0.6 ₨/MBtu

$$50 \leq P_s \leq 500$$

The gas-fired plant must burn $40 \cdot 10^6$ ft^3 of gas. The load pattern is shown in Table 6.2. If the gas constraints are ignored, the optimum economic schedule for these two plants appears as is shown in Table 6.3. Operating cost of the composite unit over the entire 24-h period is 52,128.03 ₨. The total gas consumption is $21.8 \cdot 10^6$ ft^3. Since the gas-fired plant must burn $40 \cdot 10^6$ ft^3 of gas, the cost will be 2.0 ₨/1000 ft^3 × $40 \cdot 10^6$ ft^3, which is 80,000 ₨ for the gas. Therefore, the total cost will be 132,128.03 ₨. The solution method shown in Figure 6.3 was used with γ values ranging from 0.500 to 0.875. The final value for γ is 0.8742 ₨/ccf with an optimal schedule as shown in Table 6.4. This schedule has a fuel cost for the composite unit of 34,937.47 ₨. Note that the gas unit is run much harder and that it does not hit either limit in the optimal

TABLE 6.2 Load Pattern

Time Period	Load
1. 0000–0400	400 MW
2. 0400–0800	650 MW
3. 0800–1200	800 MW
4. 1200–1600	500 MW
5. 1600–2000	200 MW
6. 2000–2400	300 MW

Where: $n_j = 4$, $j = 1 \ldots 6$.

TABLE 6.3 Optimum Economic Schedule (Gas Constraints Ignored)

Time Period	P_s	P_T
1	350	50
2	500	150
3	500	300
4	450	50
5	150	50
6	250	50

TABLE 6.4 Optimal Schedule (Gas Constraints Met)

Time Period	P_s	P_T
1	197.3	202.6
2	353.2	296.8
3	446.7	353.3
4	259.7	240.3
5	72.6	127.4
6	135.0	165.0

schedule. Further, note that the total cost is now

$$34{,}937.47\ \text{R} + 80{,}000\ \text{R} = 114{,}937.4\ \text{R}$$

so we have lowered the total fuel expense by properly scheduling the gas plant.

6.4 SOLUTION BY GRADIENT SEARCH TECHNIQUES

An alternative solution procedure to the one shown in Figure 6.3 makes use of Eqs. 6.5 and 6.6.

$$n_k \frac{dF_{ik}}{dP_{ik}} = \lambda_k$$

and

$$\lambda_k = \gamma n_k \frac{dq_{Tk}}{dP_{Tk}}$$

then

$$\gamma = \left(\frac{\dfrac{dF_{ik}}{dP_{ik}}}{\dfrac{dq_{Tk}}{dP_{Tk}}} \right) \tag{6.8}$$

For an optimum dispatch, γ will be constant for all hours j, $j = 1 \ldots j_{\max}$.

We can make use of this fact to obtain an optimal schedule using the procedures shown in Figure 6.7a or Figure 6.7b. Both these procedures attempt to adjust fuel-limited generation so that γ will be constant over time. The algorithm shown in Figure 6.7a differs from the algorithm shown in Figure 6.7b in the way the problem is started and in the way various time intervals are

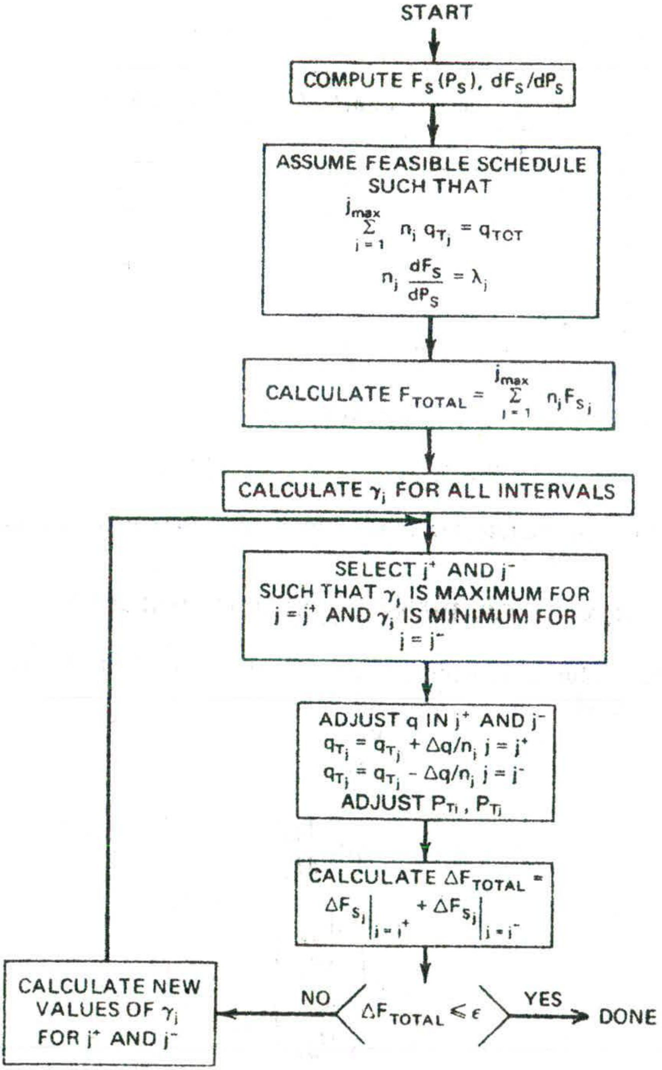


FIG. 6.7a Gradient method based on relaxation technique.

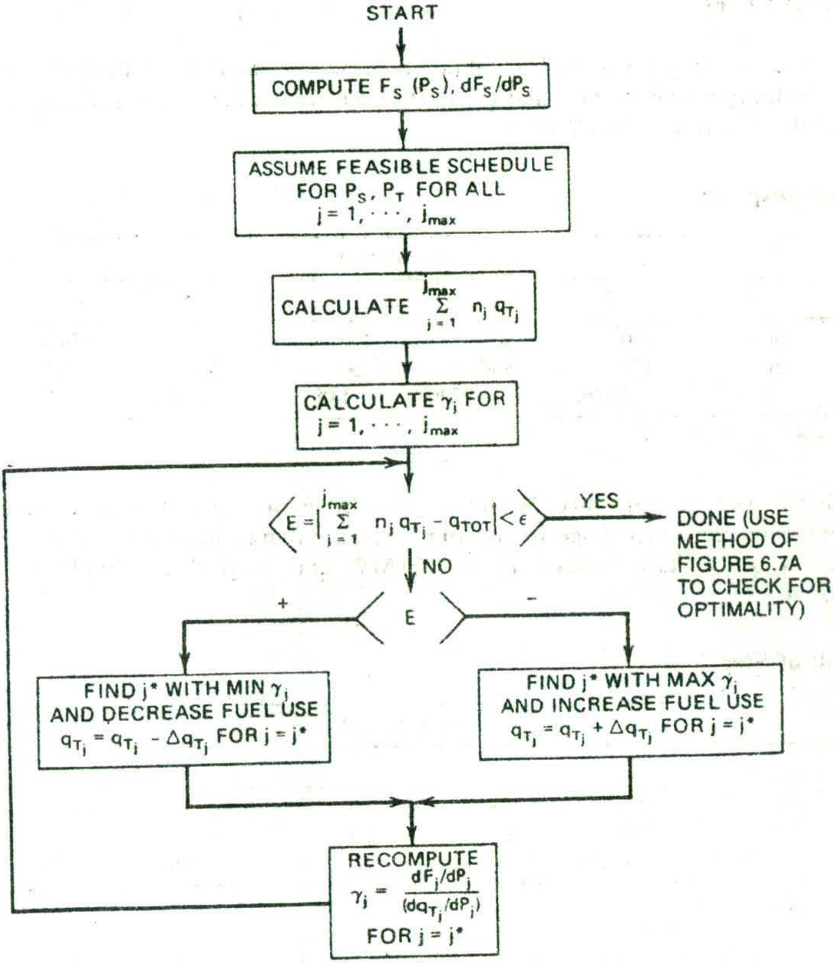


FIG. 6.7b Gradient method based on a simple search.

selected for adjustment. The algorithm in Figure 6.7a requires an initial feasible but not optimal schedule and then finds an optimal schedule by "pairwise" trade-offs of fuel consumption while maintaining problem feasibility. The algorithm in Figure 6.7b does not require an initial feasible fuel usage schedule but achieves this while optimizing. These two methods may be called gradient methods because q_{Tj} is treated as a vector and the γ_j values indicate the gradient of the objective function with respect to q_{Tj}. The method of Figure 6.7b should be followed by that of Figure 6.7a to insure optimality.

EXAMPLE 6C

Use the method of Figure 6.7b to obtain an optimal schedule for the problem given in Example 6B. Assume that the starting schedule is the economic dispatch schedule shown in Example 6B.

Initial Dispatch

	Time Period					
	1	2	3	4	5	6
P_s	350	500	500	450	150	250
P_T	50	150	300	50	50	50
γ	1.0454	1.0266	0.9240	1.0876	0.9610	1.0032

$\sum q_T = 21.84 \cdot 10^6$ ft³.

Since we wish to burn $40.0 \cdot 10^6$ ft³ of gas, the error is negative; therefore, we must increase fuel usage in the time period having maximum γ, that is, period 4. As a start, increase P_T to 150 MW and drop P_s to 350 MW in period 4.

Result of Step 1

	Time Period					
	1	2	3	4	5	6
P_s	350	500	500	350	150	250
P_T	50	150	300	150	50	50
γ	1.0454	1.0266	0.9240	0.9680	0.9610	1.0032

$\sum q_T = 24.2 \cdot 10^6$ ft³.

The error is still negative, so we must increase fuel usage in the period with maximum γ, which is now period 1. Increase P_T to 200 MW and drop P_s to 200 MW in period 1.

Result of Step 2

	Time Period					
	1	2	3	4	5	6
P_s	200	500	500	350	150	250
P_T	200	150	300	150	50	50
γ	0.8769	1.0266	0.9240	0.9680	0.9610	1.0032

$\sum q_T = 27.8 \cdot 10^6$ ft³.

and so on. After 11 steps, the schedule looks like this:

	Time Period					
	1	2	3	4	5	6
P_s	200	350	450	250	75	140
P_T	200	300	350	250	125	160
γ	0.8769	0.8712	0.8772	0.8648	0.8767	0.8794

$\sum q_T = 40.002 \cdot 10^6 \text{ ft}^3$.

which is beginning to look similar to the optimal schedule generated in Example 6A.

6.5 HARD LIMITS AND SLACK VARIABLES

This section takes account of hard limits on the take-or-pay generating unit. The limits are

$$P_T \geq P_{T\min} \tag{6.9}$$

and

$$P_T \leq P_{T\max} \tag{6.10}$$

These may be added to the Lagrangian by the use of two constraint functions and two new variables called *slack variables* (see Appendix 3A). The constraint functions are

$$\psi_{1j} = P_{Tj} - P_{T\max} + S_{1j}^2 \tag{6.11}$$

and

$$\psi_{2j} = P_{T\min} - P_{Tj} + S_{2j}^2 \tag{6.12}$$

where S_{1j} and S_{2j} are slack variables that may take on any real value including zero.

The new Lagrangian then becomes

$$\begin{aligned}\mathscr{L} = \sum_{j=1}^{j_{\max}} n_j \sum_{i=1}^{N} F_{ij} + \sum_{j=1}^{j_{\max}} \lambda_j \left(P_{\text{load } j} - \sum_{i=1}^{N} P_{ij} - P_{Tj} \right) + \gamma \left(\sum_{j=1}^{j_{\max}} n_j q_{Tj} - Q_{\text{TOT}} \right) \\ + \sum_{j=1}^{j_{\max}} \alpha_{1j}(P_{Tj} - P_{T\max} + S_{1j}^2) + \sum_{j=1}^{j_{\max}} \alpha_{2j}(P_{T\min} - P_{Tj} + S_{2j}^2)\end{aligned} \tag{6.13}$$

where α_{1j}, α_{2j} are Lagrange multipliers. Now, the first partial derivatives for the k^{th} period are

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial P_{ik}} &= 0 = n_k \frac{\mathrm{d}F_{ik}}{\mathrm{d}P_{ik}} - \lambda_k \\
\frac{\partial \mathcal{L}}{\partial P_{Tk}} &= 0 = -\lambda_k + \alpha_{1k} - \alpha_{2k} + \gamma n_k \frac{\mathrm{d}q_{T_k}}{\mathrm{d}P_{T_k}} \\
\frac{\partial \mathcal{L}}{\partial S_{1k}} &= 0 = 2\alpha_{1k} S_{1k} \\
\frac{\partial \mathcal{L}}{\partial S_{2k}} &= 0 = 2\alpha_{2k} S_{2k}
\end{aligned} \tag{6.14}$$

As we noted in Appendix 3A, when the constrained variable (P_{T_k} in this case) is within bounds, the new Lagrange multipliers $\alpha_{1k} = \alpha_{2k} = 0$ and S_{1k} and S_{2k} are nonzero. When the variable is limited, one of the slack variables, S_{1k} or S_{2k}, becomes zero and the associated Lagrange multiplier will take on a nonzero value.

Suppose in some interval k, $P_{Tk} = P_{\max}$, then $S_{1k} = 0$ and $\alpha_{1k} \neq 0$. Thus,

$$-\lambda_k + \alpha_{1k} + \gamma n_k \frac{\mathrm{d}q_{Tk}}{\mathrm{d}P_{Tk}} = 0 \tag{6.15}$$

and if

$$\lambda_k > \gamma n_k \frac{\mathrm{d}q_{Tk}}{\mathrm{d}P_{Tk}}$$

the value of α_{1k} will take on the value just sufficient to make the equality true.

EXAMPLE 6D

Repeat Example 6B with the maximum generation on P_T reduced to 300 MW. Note that the optimum schedule in Example 6A gave a $P_T = 353.3$ MW in the third time period. When the limit is reduced to 300 MW, the gas-fired unit will have to burn more fuel in other time periods to meet the $40 \cdot 10^3$ ft^3 gas consumption constraint.

TABLE 6.5 Resulting Optimal Schedule with $P_{T\max} = 300$ MW

Time Period j	P_{sj}	P_{Tj}	λ_j	$\gamma_{nj} \dfrac{\partial q_T}{\partial P_{Tj}}$	α_{1j}
1	183.4	216.6	5.54	5.54	0
2	350.0	300.0	5.94	5.86	0.08
3	500.0	300.0	6.3	5.86	0.44
4	245.4	254.6	5.69	5.69	0
5	59.5	140.5	5.24	5.24	0
6	121.4	178.6	5.39	5.39	0

Table 6.5 shows the resulting optimal schedule where $\gamma = 0.8603$ and total cost = 122,984.83 R.

6.6 FUEL SCHEDULING BY LINEAR PROGRAMMING

Figure 6.8 shows the major elements in the chain making up the delivery system that starts with raw-fuel suppliers and ends up in delivery of electric power to individual customers. The basic elements of the chain are as follows.

The suppliers: These are the coal, oil, and gas companies with which the utility must negotiate contracts to acquire fuel. The contracts are usually written for a long term (10 to 20 yr) and may have stipulations, such as the minimum and maximum limits on the quantity of fuel delivered over a specified time period. The time period may be as long as a year, a month, a week, a day, or even for a period of only a few minutes. Prices may change, subject to the renegotiation provisions of the contracts.

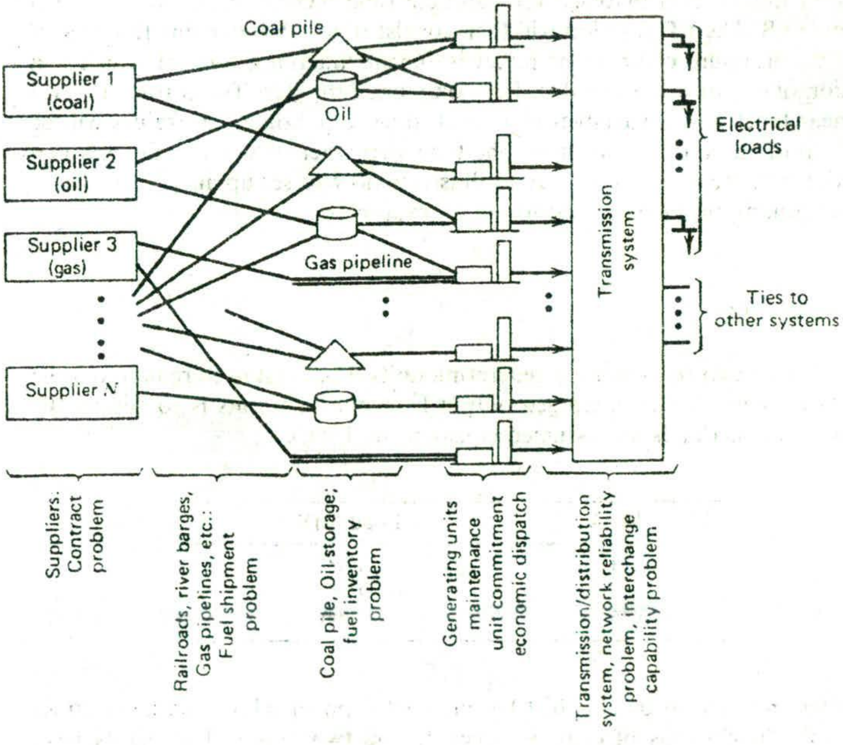


FIG. 6.8 Energy delivery system.

Transportation: Railroads, unit trains, river barges, gas-pipeline companies, and such, all present problems in scheduling of deliveries of fuel.

Inventory: Coal piles, oil storage tanks, underground gas storage facilities. Inventories must be kept at proper levels to forestall fuel shortages when load levels exceed forecast or suppliers or shippers are unable to deliver. Price fluctuations also complicate the decisions on when and how much to add or subtract from inventories.

The remainder of the system—generators, transmission, and loads—are covered in other chapters.

One of the most useful tools for solving large fuel-scheduling problems is linear programming (LP). If the reader is not familiar with LP, an easily understood algorithm is provided in the appendix of this chapter.

Linear programming is an optimization procedure that minimizes a linear objective function with variables that are also subject to linear constraints. Because of this limitation, any nonlinear functions either in the objective or in the constraint equations will have to be approximated by linear or piecewise linear functions.

To solve a fuel-scheduling problem with linear programming, we must break the total time period involved into discrete time increments, as was done in Example 6B. The LP solution will then consist of an objective function that is made up of a sum of linear or piecewise linear functions, each of which is a function of one or more variables from only one time step. The constraints will be linear functions of variables from each time step. Some constraints will be made up of variables drawn from one time step whereas others will span two or more time steps. The best way to illustrate how to set up an LP to solve a fuel-scheduling problem will be to use an example.

EXAMPLE 6E

We are given two coal-burning generating units that must both remain on-line for a 3-wk period. The combined output from the two units is to supply the following loads (loads are assumed constant for 1 wk).

Week	Load (MW)
1	1200
2	1500
3	800

The two units are to be supplied by one coal supplier who is under contract to supply 40,000 tons of coal per week to the two plants. The plants have existing coal inventories at the start of the 3-wk period. We must solve for the following.

1. How should each plant be operated each week?
2. How should the coal deliveries be made up each week?

The data for the problem are as follows.

Coal: Heat value = 11,500 Btu/lb = 23 MBtu/ton (1 ton = 2000 lb)

Coal can all be delivered to one plant or the other or it can be split, some going to one plant, some to the other, as long as the total delivery in each week is equal to 40,000 tons. The coal costs 30 R/ton or 1.3 R/MBtu.

Inventories: Plant 1 has an initial inventory of 70,000 tons; its final inventory is not restricted

Plant 2 has an initial inventory of 70,000 tons; its final inventory is not restricted

Both plants have a maximum coal storage capacity of 200,000 tons of coal.

Generating units:

Unit	Min (MW)	Max (MW)	Heat Input at Min (MBtu/h)	Heat Input at Max (MBtu/h)
1	150	600	1620	5340
2	400	1000	3850	8750

The input versus output function will be approximated by a linear function for each unit:

$$H_1(P_1) = 380.0 + 8.267P_1$$

$$H_2(P_2) = 583.3 + 8.167P_2$$

The unit cost curves are

$$F_1(P_1) = 1.3 \text{ R/MBtu} \times H_1(P_1) = 495.65 + 10.78P_1 \text{ (R/h)}$$

$$F_2(P_2) = 1.3 \text{ R/MBtu} \times H_2(P_2) = 760.8 + 10.65P_2 \text{ (R/h)}$$

The coal consumption q(tons/h) for each unit is

$$q_1(P_1) = \frac{1}{23}\left(\frac{\text{tons}}{\text{MBtu}}\right) \times H_1(P_1) = 16.52 + 0.3594P_1 \text{ tons/h}$$

$$q_2(P_2) = \frac{1}{23}\left(\frac{\text{tons}}{\text{MBtu}}\right) \times H_2(P_2) = 25.36 + 0.3551P_2 \text{ tons/h}$$

To solve this problem with linear programming, assume that the units are to be operated at a constant rate during each week and that the coal deliveries will each take place at the beginning of each week. Therefore, we will set up the problem with 1-wk time periods and the generating unit cost functions and coal consumption functions will be multiplied by 168 h to put them on a "per week" basis; then,

$$\begin{aligned} F_1(P_1) &= 83{,}269.2 + 1811P_1 \text{ R/wk} \\ F_2(P_2) &= 127.814.4 + 1789P_2 \text{ R/wk} \\ q_1(P_1) &= 2775.4 + 60.4P_1 \text{ tons/wk} \\ q_2(P_2) &= 4260.5 + 59.7P_2 \text{ tons/wk} \end{aligned} \tag{6.16}$$

We are now ready to set up the objective function and the constraints for our linear programming solution.

Objective function: To minimize the operating cost over the 3-wk period. The objective function is

$$\begin{aligned} \text{Minimize } Z = F_1[P_1(1)] + F_2[P_2(1)] + F_1[(P_1(2)] + F_2[P_2(2)] \\ + F_1[P_1(3)] + F_2[P_2(2)] \end{aligned} \tag{6.17}$$

where $P_i(j)$ is the power output of the i^{th} unit during the j^{th} week, $j = 1 \ldots 3$.

Constraints: During each time period, the total power delivered from the units must equal the scheduled load to be supplied; then

$$\begin{aligned} P_1(1) + P_2(1) &= 1200 \\ P_1(2) + P_2(2) &= 1500 \\ P_1(3) + P_2(3) &= 800 \end{aligned} \tag{6.18}$$

Similarly, the coal deliveries, D_1 and D_2, made to plant 1 and plant 2, respectively, during each week must sum to 40,000 tons; then

$$\begin{aligned} D_1(1) + D_2(1) &= 40{,}000 \\ D_1(2) + D_2(2) &= 40{,}000 \\ D_1(3) + D_2(3) &= 40{,}000 \end{aligned} \tag{6.19}$$

The volume of coal at each plant at the beginning of each week plus the delivery of coal to that plant minus the coal burned at the plant will give the coal remaining at the beginning of the next week. Letting V_1 and V_2 be the volume of coal in each coal pile at the beginning of the week, respectively, we have the following set of equations governing the two coal piles.

$$\begin{aligned} V_1(1) + D_1(1) - q_1(1) &= V_1(2) \\ V_2(1) + D_2(1) - q_2(1) &= V_2(2) \\ V_1(2) + D_1(2) - q_1(2) &= V_1(3) \\ V_2(2) + D_2(2) - q_2(2) &= V_2(3) \\ V_1(3) + D_1(3) - q_1(3) &= V_1(4) \\ V_2(3) + D_2(3) - q_2(3) &= V_2(4) \end{aligned} \tag{6.20}$$

where $V_i(j)$ is the volume of coal in the i^{th} coal pile at the beginning of the j^{th} week.

To set these equations up for the linear-programming solutions, substitute the $q_1(P_1)$ and $q_2(P_2)$ equations from 6.16 into the equations of 6.20. In addition, all constant terms are placed on the right of the equal sign and all variable terms on the left; this leaves the constraints in the standard form for inclusion in the LP. The result is

$$\begin{aligned} D_1(1) - 60.4P_1(1) - V_1(2) &= 2775.4 - V_1(1) \\ D_2(1) - 59.7P_2(1) - V_2(2) &= 4260.5 - V_2(1) \\ V_1(2) + D_1(2) - 60.4P_1(2) - V_1(3) &= 2775.4 \\ V_2(2) + D_2(2) - 59.7P_2(2) - V_2(3) &= 4260.5 \\ V_1(3) + D_1(3) - 60.4P_1(3) - V_1(4) &= 2775.4 \\ V_2(3) + D_2(3) - 59.7P_2(3) - V_2(4) &= 4260.5 \end{aligned} \tag{6.21}$$

Note: $V_1(1)$ and $V_2(1)$ are constants that will be set when we start the problem.

The constraints from Eqs. 6.18, 6.19, and 6.21 are arranged in a matrix, as shown in Figure 6.9. Each variable is given an upper and lower bound in keeping with the "upper bound" solution shown in the appendix of this chapter. The $P_1(t)$ and $P_2(t)$ variables are given the upper and lower bounds corresponding

Problem Variable	D1(1)	P1(1)	D2(1)	P2(1)	V1(2)	D1(2)	P1(2)	V2(2)	D2(2)	P2(2)	V1(3)	D1(3)	P1(3)	V2(3)	D2(3)	P2(3)	V1(4)	V2(4)	
LP Variable	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}	Constraint Units
Constraint																			
1		1		1															1200
2	1		1																40000
3	1	− 60.4			− 1														2775.4 − $V_1(1)$
4			1	− 59.7				− 1											4260.5 − $V_2(1)$
5							1			1									1500
6						1			1										40000
7					1	1	− 60.4				− 1								2775.4
8								1	1	− 59.7				− 1					4260.5
9													1			1			800
10												1			1				40000
11											1	1	− 60.4				− 1		2775.4
12														1	1	− 59.7		− 1	4260.5
Variable min.	0	150	0	400	0	0	150	0	0	400	0	0	150	0	0	400	0	0	
Variable max.	40000	600	40000	1000	200000	40000	600	200000	40000	1000	200000	40000	600	200000	40000	1000	200000	200000	
	Week 1				Week 2						Week 3						Final Conditions		

FIG. 6.9 Linear-programming constraint matrix for Example 6E.

to the upper and lower limits on the generating units. $D_1(t)$ and $D_2(t)$ are given upper and lower bounds of 40,000 and zero. $V_1(t)$ and $V_2(t)$ are given upper and lower bounds of 200,000 and zero.

Solution: The solution to this problem was carried out with a computer program written to solve the upper bound LP problem using the algorithm shown in the Appendix. The first problem solved had coal storage at the beginning of the first week of

$$V_1(1) = 70{,}000 \text{ tons}$$

$$V_2(1) = 70{,}000 \text{ tons}$$

The solution is:

Time Period	V_1	D_1	P_1	V_2	D_2	P_2
1	70000.0	0	200	70000.0	40000.0	1000
2	55144.6	0	500	46039.5	40000.0	1000
3	22169.2	19013.5	150	22079.0	20986.5	650
4	29347.3					

Optimum cost = 6,913,450.8 Ɍ.

In this case, there are no constraints on the coal deliveries to either plant and the system can run in the most economic manner. Since unit 2 has a lower incremental cost, it is run at its maximum when possible. Furthermore, since no restrictions were placed on the coal pile levels at the end of the third week, the coal deliveries could have been shifted a little from unit 2 to unit 1 with no effect on the generation dispatch.

The next case solved was purposely structured to create a fuel shortage at unit 2. The beginning inventory at plant 2 was set to 50,000 tons, and a requirement was imposed that at the end of the third week the coal pile at unit 2 be no less than 8000 tons. The solution was made by changing the right-hand side of the fourth constraint from $-65{,}739.5$ (i.e., $4260.5 - 70{,}000$) to -45739.5 (i.e., $4260.5 - 50{,}000$) and placing a lower bound on $V_2(4)$ (i.e., variable X_{18}) of 8000. The solution is:

Time Period	V_1	D_1	P_1	V_2	D_2	P_2
1	70000.0	0	200	50000.0	40000.0	1000
2	55144.6	0	500	26039.5	40000.0	1000
3	22169.2	0	300.5276	2079.0	40000.0	499.4724
4	1241.9307			8000.0		

Optimum cost = 6,916,762.4 Ɍ.

Note that this solution requires unit 2 to drop off its generation in order to meet the end-point constraint on its coal pile. In this case, all the coal must be delivered to plant 2 to minimize the overall cost.

The final case was constructed to show the interaction of the fuel deliveries and the economic dispatch of the generating units. In this case, the initial coal piles were set to 10,000 tons and 150,000 tons, respectively. Furthermore, a restriction of 30,000 tons minimum in the coal pile at unit 1 at the end of the third week was imposed.

To obtain the most economic operation of the two units over the 3-wk period, the coal deliveries will have to be adjusted to insure both plants have sufficient coal. The solution was obtained by setting the right-hand side of the third and fourth constraint equations to -7224.6 and -145739.5, respectively, as well as imposing a lower bound of 30,000 on $V_1(4)$ (i.e., variable X_{17}). The solution is:

Time Period	V_1	D_1	P_1	V_2	D_2	P_2
1	10000.0	4855.4	200	150000.0	35144.6	1000
2	0.0	40000.0	500	121184.1	0	1000
3	7024.6	40000.0	150	57223.6	0	650
4	35189.2			14158.1		

Optimum cost = 6.913,450.8 R.

The LP was able to find a solution that allowed the most economic operation of the units while still directing enough coal to unit 1 to allow it to meet its end-point coal pile constraint. Note that, in practice, we would probably not wish to let the coal pile at unit 1 go to zero. This could be prevented by placing an appropriate lower bound on all the volume variables (i.e., X_5, X_8, X_{11}, X_{14}, X_{17}, and X_{18}).

This example has shown how a fuel-management problem can be solved with linear programming. The important factor in being able to solve very large fuel-scheduling problems is to have a linear-programming code capable of solving large problems having perhaps tens of thousands of constraints and as many, or more, problem variables. Using such codes, elaborate fuel-scheduling problems can be optimized out over several years and play a critical role in utility fuel-management decisions.

APPENDIX
Linear Programming

Linear programming is perhaps the most widely applied mathematical programming technique. Simply stated, linear programming seeks to find the optimum value of a linear objective function while meeting a set of linear constraints. That is, we wish to find the optimum set of x values that minimize the following objective function:

$$Z = c_1x_1 + c_2x_2 + \ldots + c_Nx_N$$

subject to a set of linear constraints:

$$\begin{gathered} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1N}x_N \leq b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2N}x_N \leq b_2 \\ \vdots \end{gathered}$$

In addition, the variables themselves may have specified upper and lower limits.

$$x_i^{\min} \leq x_i \leq x_i^{\max} \qquad i = 1 \ldots N$$

There are a variety of solutions to the LP problem. Many of these solutions are tailored to a particular type of problem. This appendix will not try to develop the theory of alternate LP solution methods. Rather, it will present a simple LP algorithm that can be used (or programmed on a computer) to solve the applicable power-system sample problems given in this text.

The algorithm is presented in its simplest form. There are alternative formulations, and these will be indicated when appropriate. If the student has access to a standard LP program, such a standard program may be used to solve any of the problems in this book.

The LP technique presented here is properly called an *upper-bounding dual linear programming algorithm*. The "upper-bounding" part of its name refers to the fact that variable limits are handled implicitly in the algorithm. "Dual" refers to the theory behind the way in which the algorithm operates. For a complete explanation of the primal and dual algorithms, refer to the references cited at the end of this chapter.

In order to proceed in an orderly fashion to solve a dual upper-bound linear programming problem, we must first add what is called a *slack variable* to each constraint. The slack variable is so named because it equals the difference or slack between a constraint and its limit. By placing a slack variable into an inequality constraint, we can transform it into an equality constraint. For example, suppose we are given the following constraint.

$$2x_1 + 3x_2 \leq 15 \tag{6A.1}$$

We can transform this constraint to an equality constraint by adding a slack variable, x_3.

$$2x_1 + 3x_2 + x_3 = 15 \tag{6A.2}$$

If x_1 and x_2 were to be given values such that the sum of the first two terms in Eq. 6A.2 added up to less than 15, we could still satisfy Eq. 6A.2 by setting x_3 to the difference. For example, if $x_1 = 1$ and $x_2 = 3$, then $x_3 = 4$ would satisfy Eq. 6A.2. We can go even further, however, and restrict the values of x_3 so that Eq. 6A.2 still acts as an inequality constraint such as Eq. 6A.1. Note that when the first two terms of Eq. 6A.2 add to exactly 15, x_3 must be set to zero. By restricting x_3 to always be a positive number, we can force Eq. 6A.2 to yield the same effect as Eq. 6A.1. Thus,

$$\left.\begin{array}{c} 2x_1 + 3x_2 + x_3 = 15 \\ 0 \leq x_3 \leq \infty \end{array}\right\} \text{ is equivalent to: } 2x_1 + 3x_2 \leq 15$$

For a "greater than or equal to" constraint, we merely change the bounds on the slack variable:

$$\left.\begin{array}{c} 2x_1 + 3x_2 + x_3 = 15 \\ -\infty \leq x_3 \leq 0 \end{array}\right\} \text{ is equivalent to: } 2x_1 + 3x_2 \geq 15$$

Because of the way the dual upper-bounding algorithm is initialized, we will always require slack variables in every constraint. In the case of an equality constraint, we will add a slack variable and then require its upper and lower bounds to both equal zero.

To solve our linear programming algorithm, we must arrange the objective function and constraints in a tabular form as follows.

$$\begin{array}{llll} a_{11}x_1 + a_{12}x_2 + \ldots + \underbrace{x_{\text{slack}_1}} & & & = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots & + x_{\text{slack}_2} & & = b_2 \\ c_1x_1 + \; c_2x_2 + \ldots & & -Z & = 0 \end{array} \qquad (6A.3)$$

$$\underbrace{\hphantom{x_{\text{slack}_1} \qquad + x_{\text{slack}_2} \qquad -Z}}_{\text{Basis variables}}$$

Because we have added slack variables to each constraint, we automatically have arranged the set of equations into what is called *canonical* form. In canonical form, there is at least one variable in each constraint whose coefficient is zero in all the other constraints. These variables are called the *basis variables*. The entire solution procedure for the linear programming algorithm centers on performing "pivot" operations that can exchange a nonbasis variable for a basis variable. A pivot operation may be shown by using our tableau in Eq. 6A.3. Suppose we wished to exchange variable x_1, a nonbasis variable, for x_{slack_2}, a slack variable. This could be accomplished by "pivoting" on column 1, row 2. To carry out the pivoting operation we execute the following steps.

Pivoting on Column 1, Row 2

Step 1 Multiply row 2 by $1/a_{21}$. That is, each $a_{2j}, j = 1 \ldots N$ in row 2 becomes

$$a'_{2j} = \frac{a_{2j}}{a_{21}} \qquad j = 1 \ldots N$$

and

$$b_2 \text{ becomes } b'_2 = \frac{b_2}{a_{21}}$$

Step 2 For each row i ($i \neq 2$), multiply row 2 by a_{i1} and subtract from row i. That is, each coefficient a_{ij} in row i ($i \neq 2$) becomes

$$a'_{ij} = a_{ij} - a_{i1}a'_{2j} \qquad j = 1 \ldots N$$

and

$$b_i \text{ becomes } b'_i = b_i - a_{i1}b'_2$$

Step 3 Last of all, we also perform the same operations in step 2 on the cost row. That is, each coefficient c_j becomes

$$c'_j = c_j - c_1a'_{2j} \qquad j = 1 \ldots N$$

The result of carrying out the pivot operation will look like this:

$$\begin{aligned} a'_{12}x_2 + \ldots x_{\text{slack}_1} + a'_{1s_2}x_{\text{slack}_2} &= b'_1 \\ x_1 + a'_{22}x_2 + \ldots \quad + a'_{2s_2}x_{\text{slack}_2} &= b'_2 \\ c'_2x_2 + \ldots \quad + c'_{s2}x_{\text{slack}_2} - Z &= Z' \end{aligned}$$

Notice that the new basis for our tableau is formed by variable x_1 and x_{slack_1}, x_{slack_2} no longer has zero coefficients in row 1 or the cost row.

The dual upper-bounding algorithm proceeds in simple steps wherein variables that are in the basis are exchanged for variables out of the basis. When an exchange is made, a pivot operation is carried out at the appropriate row and column. The nonbasis variables are held equal to either their upper or their lower value, while the basis variables are allowed to take any value without respect to their upper or lower bounds. The solution terminates when all the basis variables are within their respective limits.

In order to use the dual upper-bound LP algorithm, follow these rules.

Start:

1. Each variable that has a nonzero coefficient in the cost row (i.e., the objective function) must be set according to the following rule.

$$\text{If } C_j > 0, \quad \text{set } x_j = x_j^{\min}$$

$$\text{If } C_j < 0, \quad \text{set } x_j = x_j^{\max}$$

2. If $C_j = 0$, x_j may be set to any value, but for convenience set it to its minimum also.
3. Add a slack variable to each constraint. Using the x_j values from steps 1 and 2, set the slack variables to make each constraint equal to its limit.

Variable Exchange:

1. Find the basis variable with the greatest violation; this determines the row to be pivoted on. Call this row R. If there are no limit violations among the basis variables, we are done. The most-violated variable leaves the basis and is set equal to the limit that was violated.
2. Select the variable to enter the basis using one of the following column selection procedures.

Column Selection Procedure P1 (Most-violated variable below its minimum)

Given constraint row R, whose basis variable is below its minimum and is the worst violation. Pick column S, so that, $c_S/(-a_{R,S})$ is minimum for all S that meet the following rules:

a. S is not in the current basis.

b. $a_{R,S}$ is not equal to zero.

c. If x_S is at its minimum, then $a_{R,S}$ must be negative and c_S must be positive or zero.

d. If x_S is at its maximum, then $a_{R,S}$ must be positive and c_S must be negative or zero.

Column Selection Procedure P2 (Most-violated variable above its maximum)

Given constraint row R, whose basis variable is above its maximum and is the worst violation. Pick column S, so that, $c_S/a_{R,S}$ is the minimum for all S that meet the following rules:

a. S is not in the current basis.

b. $a_{R,S}$ is not already zero.

c. If x_S is at its minimum, then $a_{R,S}$ must be positive and c_S must be positive or zero.

d. If x_S is at its maximum, then $a_{R,S}$ must be negative and c_S must be negative or zero.

3. When a column has been selected, pivot at the selected row R (from step 1) and column S (from step 2). The pivot column's variable, S, goes into the basis.

If no column fits the column selection criteria, we have an infeasible solution. That is, there are no values for $x_1 \ldots x_N$ that will satisfy all constraints

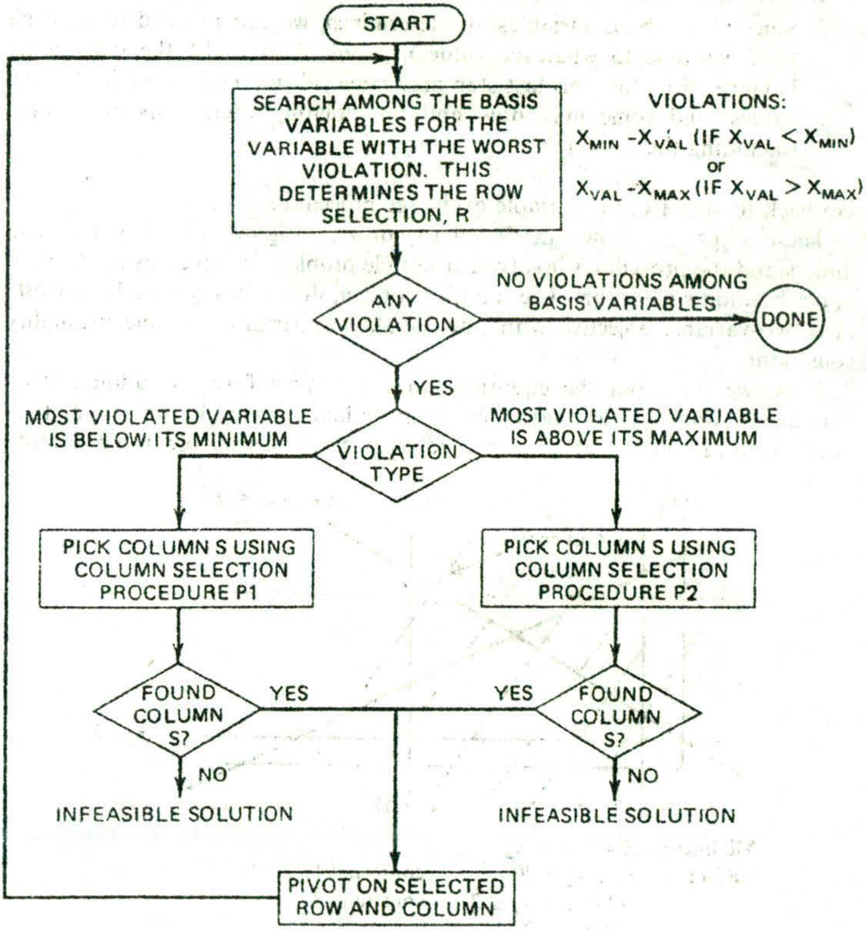


FIG. 6.10 Dual upper-bound linear programming algorithm.

simultaneously. In some problems, the cost coefficient c_S associated with column S will be zero for several different values of S. In such a case, $c_S/a_{R,S}$ will be zero for each such S and none of them will be the minimum. The fact that c_S is zero means that there will be no increase in cost if any of the S values are pivoted into the basis; therefore, the algorithm is indifferent to which one is chosen.

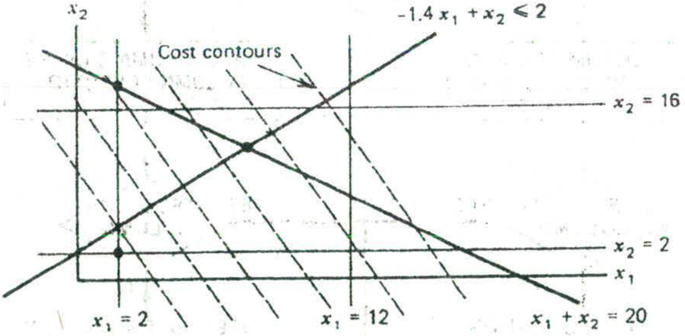
Setting the Variables after Pivoting.

1. All nonbasis variables, except x_S, remain as they were before pivoting.
2. The most violated variable is set to the limit that was violated.
3. Since all nonbasis variables are determined, we can proceed to set each basis variable to whatever value is required to make the constraints balance. Note that this last step may move all the basis variables to new values, and some may now end up violating their respective limits (including the x_S variable).

Go back to step 1 of the variable exchange procedure.

These steps are shown in flowchart form in Figure 6.10. To help you understand the procedures involved, a sample problem is solved using the dual upper-bounding algorithm. The sample problem, shown in Figure 6.11, consists of a two-variable objective with one equality constraint and one inequality constraint.

First, we must put the equations into canonical form by adding slack variables x_3 and x_4. These variables are given limits corresponding to the type of constraint into which they are placed, x_3 is the slack variable in the equality



Minimize: $Z = 2x_1 + x_2$
Subject to: $x_1 + x_2 = 20$ constraint 1
$-1.4x_1 + x_2 \leq 2$ constraint 2
$2 \leq x_1 \leq 12$
$2 \leq x_2 \leq 16$

FIG. 6.11 Sample linear programming problem.

constraint, so its limits are both zero; x_4 is in an inequality constraint, so it is restricted to be a positive number. To start the problem, the objective function must be set to the minimum value it can attain, and the algorithm will then seek the minimum constrained solution by increasing the objective just enough to reach the constrained solution. Thus, we set both x_1 and x_2 at their minimum values since the cost coefficients are both positive. These conditions are shown here:

Constraint 1:	x_1	$+ x_2$	$+ x_3$		$= 20 \leftarrow$ R
Constraint 2:	$-1.4x_1$	$+ x_2$		$+ x_4$	$= 2$
Cost:	$2x_1$	$+ x_2$			$-Z = 0$
					$0 \leq x_3 \leq 0$
					$0 \leq x_4 \leq \infty$
Minimum:	2	2	0	0	
Present value:	2	2	16	2.8	6
Maximum:	12	16	0	∞	
			Basis variable 1 ↑ Worst-violated variable	Basis variable 2	

We can see from these conditions that variable x_3 is the worst-violated variable and that it presently exceeds its maximum limit of zero. Thus, we must use column procedure P2 on constraint number 1. This is summarized as follows:

Using selection procedure P2 on constraint 1:

$i = 1 \quad a_1 > 0 \quad x_1 = x_1^{\min} \quad c_1 > 0 \quad \text{then} \quad \frac{c_1}{a_1} = \frac{2}{1} = 2$

$i = 2 \quad a_2 > 0 \quad x_2 = x_2^{\min} \quad c_1 > 0 \quad \text{then} \quad \frac{c_2}{a_2} = \frac{1}{1} = 1$

min c_i/a_i is 1 at $i = 2$

Pivot at column 2, row 1

To carry out the required pivot operations on column 2, row 1, we need merely subtract the first constraint from the second constraint and from the objective function. This results in:

Constraint 1:	x_1	$+ x_2$	$+ x_3$			$= 20$
Constraint 2:	$-2.4x_1$		$- x_3$	$+ x_4$		$= -18 \leftarrow$ R
Cost:	x_1		$- x_3$		$-Z$	$= -20$
Minimum:	2	2	0	0		
Present value:	2	18	0	−13.2	22	
Maximum:	12	16	0	∞		
		Basis variable 1		Basis variable 2 ↖ Worst-violated variable		

We can see now that the variable with the worst violation is x_4 and that x_4 is below its minimum. Thus, we must use selection procedure P1 as follows:

Using selection procedure P1 on constraint 2:

$i = 1 \quad a_1 < 0 \quad x_1 = x_1^{\min} \qquad c_1 > 0 \quad \text{then } \dfrac{c_1}{-a_1} = \dfrac{1}{-(-2.4)} = 0.4166$

$i = 3 \quad a_3 < 0 \quad x_3 = x_3^{\min} = x_3^{\max} \; c_3 < 0 \quad \text{then } x_3 \text{ is not eligible}$

Pivot at column 1, row 2

After pivoting, this results in:

Constraint 1:		x_2	$+ 0.5833x_3$	$+ 0.4166x_4$		$=$	12.5
Constraint 2:	x_1		$+ 0.4166x_3$	$- 0.4166x_4$		$=$	7.5
Cost:			$- 1.4166x_3$	$+ 0.4166x_4$	$- Z$	$=$	-27.5
Minimum:	2	2	0	0			
Present value:	7.5	12.5	0	0	-27.5		
Maximum:	12	16	0	∞			
	Basis variable 1	**Basis variable 2**					

At this point, we have no violations among the basis variables, so the algorithm can stop at the optimum.

$$\left.\begin{array}{l} x_1 = 7.5 \\ x_2 = 12.5 \end{array}\right\} \text{cost} = 27.5$$

See Figure 6.11 to verify that this is the optimum. The dots in Figure 6.11 show the solution points beginning at the starting point $x_1 = 2$, $x_2 = 2$, cost $= 6.0$, then going to $x_1 = 2$, $x_2 = 18$, cost $= 22.0$, and finally to the optimum $x_1 = 7.5$, $x_2 = 12.5$, cost $= 27.5$.

How does this algorithm work? At each step, two decisions are made.

1. Select the most-violated variable.
2. Select a variable to enter the basis.

The first decision will allow the procedure to eliminate, one after the other, those constraint violations that exist at the start, as well as those that happen during the variable-exchange steps. The second decision (using the column selection procedures) guarantees that the rate of increase in cost, to move the violated variable to its limit, is minimized. Thus, the algorithm starts from a minimum cost, infeasible solution (constraints violated), toward a minimum cost, feasible solution, by minimizing the rate of cost increase at each step.

PROBLEMS

6.1 Three units are on-line all 720 h of a 30-day month. Their characteristics are as follows:

$$H_1 = 225 + 8.47P_1 + 0.0025P_1^2, \ 50 \le P_1 \le 350$$

$$H_2 = 729 + 6.20P_2 + 0.0081P_2^2, \ 50 \le P_2 \le 350$$

$$H_3 = 400 + 7.20P_3 + 0.0025P_3^2, \ 50 \le P_3 \le 450$$

In these equations, the H_i are in MBtu/h and the P_i are in MW.

Fuel costs for units 2 and 3 are 0.60 R/MBtu. Unit 1, however, is operated under a take-or-pay fuel contract where 60,000 tons of coal are to be burned and/or paid for in each 30-day period. This coal costs 12 R/ton delivered and has an average heat content of 12,500 Btu/lb (1 ton = 2000 lb).

The system monthly load-duration curve may be approximated by three steps as follows.

Load (MW)	Duration (h)	Energy (MWh)
800	50	40000
500	550	275000
300	120	36000
Total	720	351000

a. Compute the economic schedule for the month assuming all three units are on-line all the time and that the coal must be consumed. Show the MW loading for each load period, the MWh of each unit, and the value of gamma (the pseudo-fuel cost).

b. What would be the schedule if unit 1 was burning the coal at 12 R/ton with no constraint to use 60,000 tons? Assume the coal may be purchased on the spot market for that price and compute all the data asked for in part a. In addition, calculate the amount of coal required for the unit.

6.2 Refer to Example 6A, where three generating units are combined into a single composite generating unit. Repeat the example, except develop an equivalent incremental cost characteristic using only the incremental characteristics of the three units. Using this composite incremental characteristic plus the zero-load intercept costs of the three units, develop the total cost characteristic of the composite. (Suggestion: Fit the composite incremental cost data points using a linear approximation and a least-squares fitting algorithm.)

6.3 Refer to Problem 3.8, where three generator units have input–output curves specified as a series of straight-line segments. Can you develop a composite input–output curve for the three units? Assume all three units are on-line and that the composite input–output curve has as many linear segments as needed.

6.4 Refer to Example 6E. The first problem solved in Example 6E left the end-point restrictions at zero to 200,000 tons for both coal piles at the end of the 3-wk period. Resolve the first problem [$V_1(1) = 70{,}000$ and $V_2(1) = 70{,}000$] with the added restriction that the final volume of coal at plant 2 at the end of the third week be at least 20,000 tons.

6.5 Refer to Example 6E. In the second case solved with the LP algorithm (starting volumes equal to 70,000 and 50,000 for plant 1 and plant 2, respectively), we restricted the final volume of the coal pile at plant 2 to be 8000 tons. What is the optimum schedule if this final volume restriction is relaxed (i.e., the final coal pile at plant 2 could go to zero)?

6.6 Using the linear programming problem in the text shown in Example 6E, run a linear program to find the following:

1. The coal unloading machinery at plant 2 is going to be taken out for maintenance for one week. During the maintenance work, no coal can be delivered to plant 2. The plant management would like to know if this should be done in week 2 or week 3. The decision will be based on the overall three-week total cost for running both plants.
2. Could the maintenance be done in week 1? If not, why not?

Use as initial conditions those found in the beginning of the sample LP executions found in the text; i.e., $V_1(1) = 70{,}000$ and $V_2(2) = 70{,}000$.

6.7 The "Cut and Shred Paper Company" of northern Minnesota has two power plants. One burns coal and the other burns natural gas supplied by the Texas Gas Company from a pipeline. The paper company has ample supplies of coal from a mine in North Dakota and it purchases gas as take-or-pay contracts for fixed periods of time. For the 8-h time period shown below, the paper company must burn $15 \cdot 10^6$ ft^3 of gas.

The fuel costs to the paper company are

Coal:	0.60 \$/MBtu
Gas:	2.0 \$/ccf (where 1 ccf = 1000 ft^3) the gas is rated at 1100 Btu/ft^3

Input–output characteristics of generators:

Unit 1 (coal unit): $H_1(P_1) = 200 + 8.5P_1 + 0.002P_1^2$ MBtu/h

$50 < P_1 < 500$

Unit 2 (gas unit): $H_2(P_2) = 300 + 6.0P_2 + 0.0025P_2^2$ MBtu/h

$50 < P_2 < 400$

Load (both load periods are 4 h long):

Period	Load (MW)
1	400
2	650

Assume both units are on-line for the entire 8 h.

Find the most economic operation of the paper company power plants, over the 8 h, which meets the gas consumption requirements.

6.8 Repeat the example in the Appendix, replacing the $x_1 + x_2 = 20$ constraint with:

$$x_1 + x_2 < 20$$

Redraw Figure 6.11 and show the admissible, convex region.

6.9 An oil-fired power plant (Figure 6.12) has the following fuel consumption curve.

$$q(\text{bbl/h}) = \begin{cases} 50 + P + 0.005P^2 & \text{for } 100 \le P \le 500 \text{ MW} \\ 0 & \text{for } P = 0 \end{cases}$$

The plant is connected to an oil storage tank with a maximum capacity of 4000 bbl. The tank has an initial volume of oil of 3000 bbl. In addition, there is a pipeline supplying oil to the plant. The pipeline terminates in the same storage tank and must be operated by contract at 500 bbl/h. The oil-fired power plant supplies energy into a system, along with other units. The other units have an equivalent cost curve of

$$F_{eq} = 300 + 6P_{eq} + 0.0025P_{eq}^2$$

$$50 \le P_{eq} \le 700 \text{ MW}$$

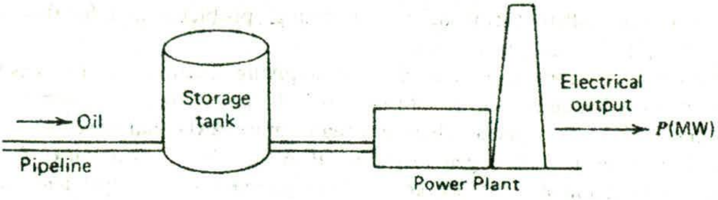


FIG. 6.12 Oil-fired power plant with storage tank for Problem 6.9.

The load to be supplied is given as follows:

Period	Load (MW)
1	400
2	900
3	700

Each time period is 2 h in length. Find the oil-fired plant's schedule using dynamic programming, such that the operating cost on the equivalent plant is minimized and the final volume in the storage tank is 2000 bbl at the end of the third period. When solving, you may use 2000, 3000, and 4000 bbl as the storage volume states for the tank. The q versus P function values you will need are included in the following table.

q(bbl/h)	P(MW)
0	0
200	100.0
250	123.6
500	216.2
750	287.3
1000	347.2
1250	400.0
1500	447.7
1800	500.0

The plant may be shut down for any of the 2-h periods with no start-up or shut-down costs.

FURTHER READING

There has not been a great deal of research work on fuel scheduling as specifically applied to power systems. However, the fuel-scheduling problem for power systems is not really that much different from other "scheduling" problems, and, for this type of problem, a great deal of literature exists.

References 1–4 are representative of efforts in applying scheduling techniques to the power system fuel-scheduling problem. References 5–8 are textbooks on linear programming that the authors have used. There are many more texts that cover LP and its variations. The reader is encouraged to study LP independently of this text if a great deal of use is to be made of LP. Many computing equipment and independent software companies have excellent LP codes that can be used, rather than writing one's own code. Reference 8 is the basis for the algorithm in the appendix to this chapter. References 9–11 give recent techniques used.

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7 Hydrothermal Coordination

7.1 INTRODUCTION

The systematic coordination of the operation of a system of hydroelectric generation plants is usually more complex than the scheduling of an all-thermal generation system. The reason is both simple and important. That is, the hydroelectric plants may very well be coupled both electrically (i.e., they all serve the same load) and hydraulically (i.e., the water outflow from one plant may be a very significant portion of the inflow to one or more other, downstream plants).

No two hydroelectric systems in the world are alike. They are all different. The reasons for the differences are the natural differences in the watersheds, the differences in the manmade storage and release elements used to control the water flows, and the very many different types of natural and manmade constraints imposed on the operation of hydroelectric systems. River systems may be simple with relatively few tributaries (e.g., the Connecticut River), with dams in series (hydraulically) along the river. River systems may encompass thousands of acres, extend over vast multinational areas, and include many tributaries and complex arrangements of storage reservoirs (e.g., the Columbia River basin in the Pacific Northwest).

Reservoirs may be developed with very large storage capacity with a few high-head plants along the river. Alternatively, the river may have been developed with a larger number of dams and reservoirs, each with smaller storage capacity. Water may be intentionally diverted through long raceways that tunnel through an entire mountain range (e.g., the Snowy Mountain scheme in Australia). In European developments, auxiliary reservoirs, control dams, locks, and even separate systems for pumping water back upstream have been added to rivers.

However, the one single aspect of hydroelectric plants that differentiates the coordination of their operation more than any other is the existence of the many, and highly varied, constraints. In many hydrosystems, the generation of power is an adjunct to the control of flood waters or the regular, scheduled release of water for irrigation. Recreation centers may have developed along the shores of a large reservoir so that only small surface water elevation changes are possible. Water release in a river may well have to be controlled so that the river is navigable at all times. Sudden changes, with high-volume releases of water, may be prohibited because the release could result in a large wave traveling downstream with potentially damaging effects. Fish ladders may be needed. Water releases may be dictated by international treaty.

To repeat: all hydrosystems are different.

7.1.1 Long-Range Hydro-Scheduling

The coordination of the operation of hydroelectric plants involves, of course, the scheduling of water releases. The *long-range hydro-scheduling problem* involves the long-range forecasting of water availability and the scheduling of reservoir water releases (i.e., "drawdown") for an interval of time that depends on the reservoir capacities.

Typical long-range scheduling goes anywhere from 1 wk to 1 yr or several years. For hydro schemes with a capacity of impounding water over several seasons, the long-range problem involves meteorological and statistical analyses.

Nearer-term water inflow forecasts might be based on snow melt expectations and near-term weather forecasts. For the long-term drawdown schedule, a basic policy selection must be made. Should the water be used under the assumption that it will be replaced at a rate based on the statistically expected (i.e., mean value) rate, or should the water be released using a "worst-case" prediction. In the first instance, it may well be possible to save a great deal of electric energy production expense by displacing thermal generation with hydro-generation. If, on the other hand, a worst-case policy was selected, the hydroplants would be run so as to minimize the risk of violating any of the hydrological constraints (e.g., running reservoirs too low, not having enough water to navigate a river). Conceivably, such a schedule would hold back water until it became quite likely that even worst-case rainfall (runoff, etc.) would still give ample water to meet the constraints.

Long-range scheduling involves optimizing a policy in the context of unknowns such as load, hydraulic inflows, and unit availabilities (steam and hydro). These unknowns are treated statistically, and long-range scheduling involves optimization of statistical variables. Useful techniques include:

1. Dynamic programming, where the entire long-range operation time period is simulated (e.g., 1 yr) for a given set of conditions.
2. Composite hydraulic simulation models, which can represent several reservoirs.
3. Statistical production cost models.

The problems and techniques of long-range hydro-scheduling are outside the scope of this text, so we will end the discussion at this point and continue with short-range hydro-scheduling.

7.1.2 Short-Range Hydro-Scheduling

Short-range hydro-scheduling (1 day to 1 wk) involves the hour-by-hour scheduling of all generation on a system to achieve minimum production cost for the given time period. In such a scheduling problem, the load, hydraulic inflows, and unit availabilities are assumed known. A set of starting conditions (e.g., reservoir levels) is given, and the optimal hourly schedule that minimizes a desired objective, while meeting hydraulic steam, and electric system constraints, is sought. Part of the hydraulic constraints may involve meeting "end-point" conditions at the end of the scheduling interval in order to conform to a long-range, water-release schedule previously established.

7.2 HYDROELECTRIC PLANT MODELS

To understand the requirements for the operation of hydroelectric plants, one must appreciate the limitations imposed on operation of hydro-resources by flood control, navigation, fisheries, recreation, water supply, and other demands on the water bodies and streams, as well as the characteristics of energy conversion from the potential energy of stored water to electric energy. The amount of energy available in a unit of stored water, say a cubic foot, is equal to the product of the weight of the water stored (in this case, 62.4 lb) times the height (in feet) that the water would fall. One thousand cubic feet of water falling a distance of 42.5 ft has the energy equivalent to 1 kWh. Correspondingly, 42.5 ft^3 of water falling 1000 ft also has the energy equivalent to 1 kWh.

Consider the sketch of a reservoir and hydroelectric plant shown in Figure 7.1. Let us consider some overall aspects of the falling water as it travels from the reservoir through the penstock to the inlet gates, through the hydraulic turbine down the draft tube and out the tailrace at the plant exit. The power that the water can produce is equal to the rate of water flow in cubic feet per

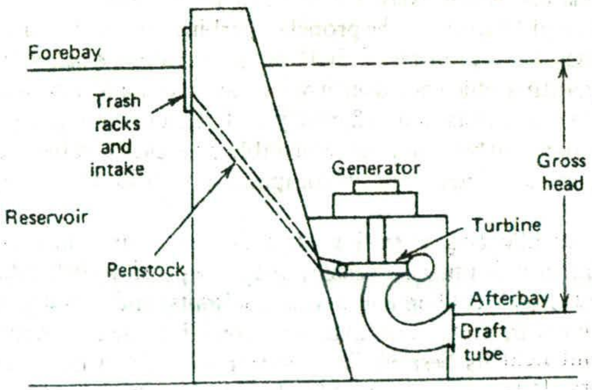

FIG. 7.1 Hydroplant components.

second times a conversion coefficient that takes into account the net head (the distance through which the water falls, less the losses in head caused by the flow) times the conversion efficiency of the turbine generator. A flow of 1 ft^3/sec falling 100 ft has the power equivalent of approximately 8.5 kW. If the flow-caused loss in head was 5%, or 5 ft, then the power equivalent for a flow of 1 ft^3 of water per second with the net drop of 100 − 5, or 95 ft, would have the power equivalent of slightly more than 8 kW (8.5 × 95%). Conversion efficiencies of turbine generators are typically in the range of 85 to 90% at the best efficiency operating point for the turbine generator, so 1 ft^3/sec falling 100 ft would typically develop about 7 kW at most.

Let us return to our description of the hydroelectric plant as illustrated in Figure 7.1. The hydroelectric project consists of a body of water impounded by a dam, the hydroplant, and the exit channel or lower water body. The energy available for conversion to electrical energy of the water impounded by the dam is a function of the gross head; that is, the elevation of the surface of the reservoir less the elevation of the afterbay, or downstream water level below the hydroelectric plant. The head available to the turbine itself is slightly less than the gross head, due to the friction losses in the intake, penstock, and draft tube. This is usually expressed as the *net head* and is equal to the gross head less the flow losses (measured in feet of head). The flow losses can be very significant for low head (10 to 60 ft) plants and for plants with long penstocks (several thousand feet). The water level at the afterbay is influenced by the flow out of the reservoir, including plant release and any spilling of water over the top of the dam or through bypass raceways. During flooding conditions such as spring runoff, the rise in afterbay level can have a significant and adverse effect on the energy and capacity or power capacity of the hydroplant.

The type of turbine used in a hydroelectric plant depends primarily on the design head for the plant. By far the largest number of hydroelectric projects use reaction-type turbines. Only two types of reaction turbines are now in common use. For medium heads (that is, in the range from 60 to 1000 ft), the Francis turbine is used exclusively. For the low-head plants (that is, for design heads in the range of 10 to 60 ft), the propeller turbine is used. The more modern propeller turbines have adjustable pitch blading (called *Kaplan turbines*) to improve the operating efficiency over a wide range of plant net head. Typical turbine performance results in an efficiency at full gate loading of between 85 to 90%. The Francis turbine and the adjustable propeller turbine may operate at 65 to 125% of rated net head as compared to 90 to 110% for the fixed propeller.

Another factor affecting operating efficiency of hydro-units is the MW loading. At light unit loadings, efficiency may drop below 70% (these ranges are often restricted by vibration and cavitation limits) and at full gate may rise to about 87%. If the best use of the hydro-resource is to be obtained, operation of the hydro-unit near its best efficiency gate position and near the designed head is necessary. This means that unit loading and control of reservoir forebay are necessary to make efficient use of hydro-resources. Unit loading should be

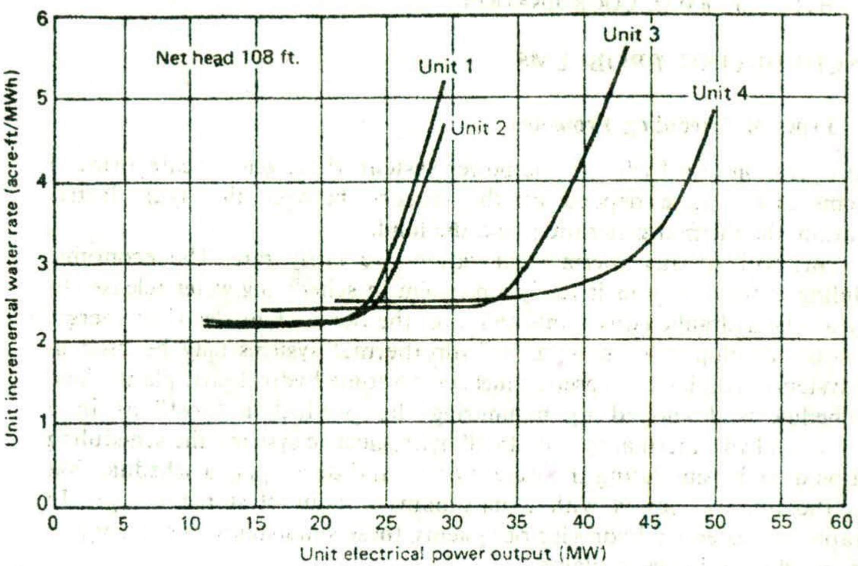

FIG. 7.2 Incremental water rate versus power output.

near best efficiency gate position, and water-release schedules must be coordinated with reservoir inflows to maintain as high a head on the turbines as the limitations on forebay operations will permit.

Typical plant performance for a medium head, four-unit plant in South America is illustrated in Figure 7.2. The incremental "water rate" is expressed in acre-feet per megawatt hour.* The rise in incremental water rate with increasing unit output results primarily from the increased hydraulic losses with the increased flow. A composite curve for multiple unit operation at the plant would reflect the mutual effects of hydraulic losses and rise in afterbay with plant discharge. Very careful attention must be given to the number of units run for a given required output. One unit run at best efficiency will usually use less water than two units run at half that load.

High-head plants (typically over 1000 ft) use impulse or Pelton turbines. In such turbines, the water is directed into spoon-shaped buckets on the wheel by means of one or more water jets located around the outside of the wheel.

In the text that follows, we will assume a characteristic giving the relationship between water flow through the turbine, q, and power output, P(MW), where q is expressed in ft^3/sec or acre-ft/h. Furthermore, we will not be concerned with what type of turbine is being used or the characteristics of the reservoir, other than such limits as the reservoir head or volume and various flows.

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* An *acre-foot* is a common unit of water volume. It is the amount of water that will cover 1 acre to a depth of 1 ft (43,560 ft^3). It also happens to be nearly equal to half a cubic foot per second flow for a day (43,200 ft^3). An acre-foot is equal to $1.2335 \cdot 10^3$ m^3.

7.3 SCHEDULING PROBLEMS

7.3.1 Types of Scheduling Problems

In the operation of a hydroelectric power system, three general categories of problems arise. These depend on the balance between the hydroelectric generation, the thermal generation, and the load.

Systems without any thermal generation are fairly rare. The economic scheduling of these systems is really a problem in scheduling water releases to satisfy all the hydraulic constraints and meet the demand for electrical energy. Techniques developed for scheduling hydrothermal systems may be used in some systems by assigning a pseudo-fuel cost to some hydroelectric plant. Then the schedule is developed by minimizing the production "cost" as in a conventional hydrothermal system. In all hydroelectric systems, the scheduling could be done by simulating the water system and developing a schedule that leaves the reservoir levels with a maximum amount of stored energy. In geographically extensive hydroelectric systems, these simulations must recognize water travel times between plants.

Hydrothermal systems where the hydroelectric system is by far the largest component may be scheduled by economically scheduling the system to produce the minimum cost for the thermal system. These are basically problems in scheduling energy. A simple example is illustrated in the next section where the hydroelectric system cannot produce sufficient energy to meet the expected load.

The largest category of hydrothermal systems include those where there is a closer balance between the hydroelectric and thermal generation resources and those where the hydroelectric system is a small fraction of the total capacity. In these systems, the schedules are usually developed to minimize thermal-generation production costs, recognizing all the diverse hydraulic constraints that may exist. The main portion of this chapter is concerned with systems of this type.

7.3.2 Scheduling Energy

Suppose, as in Figure 7.3, we have two sources of electrical energy to supply a load, one hydro and another steam. The hydroplant can supply the load

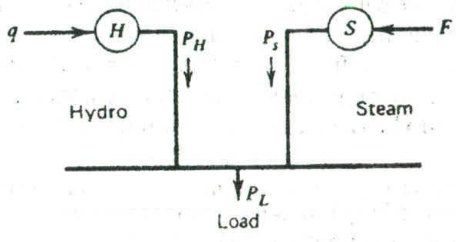


FIG. 7.3 Two-unit hydrothermal system.

by itself for a limited time. That is, for any time period j,

$$P_{Hj}^{\max} \geq P_{\text{load } j} \qquad j = 1 \ldots j_{\max} \tag{7.1}$$

However, the energy available from the hydroplant is insufficient to meet the load.

$$\sum_{j=1}^{j_{\max}} P_{Hj} n_j \leq \sum_{j=1}^{j_{\max}} P_{\text{load } j} n_j \qquad n_j = \text{number of hours in period } j$$

$$\sum_{j=1}^{j_{\max}} n_j = T_{\max} = \text{total interval} \tag{7.2}$$

We would like to use up the entire amount of energy from the hydroplant in such a way that the cost of running the steam plant is minimized. The steam-plant energy required is

$$\underbrace{\sum_{j=1}^{j_{\max}} P_{\text{load } j} n_j}_{\substack{\text{Load} \\ \text{energy}}} - \underbrace{\sum_{j=1}^{j_{\max}} P_{Hj} n_j}_{\substack{\text{Hydro-} \\ \text{energy}}} = \underbrace{E}_{\substack{\text{Steam} \\ \text{energy}}} \tag{7.3}$$

We will not require the steam unit to run for the entire interval of $T_{\max}$ hours. Therefore,

$$\sum_{j=1}^{N_s} P_{Sj} n_j = E \qquad N_s = \text{number of periods the steam plant is run} \tag{7.4}$$

Then

$$\sum_{j=1}^{N_s} n_j \leq T_{\max}$$

the scheduling problem becomes

$$\text{Min } F_T = \sum_{j=1}^{N_s} F(P_{Sj}) n_j \tag{7.5}$$

subject to

$$\sum_{j=1}^{N_s} P_{Sj} n_j - E = 0 \tag{7.6}$$

and the Lagrange function is

$$\mathscr{L} = \sum_{j=1}^{N_s} F(P_{Sj}) n_j + \alpha\left(E - \sum_{j=1}^{N_s} P_{sj} n_j\right) \tag{7.7}$$

Then

$$\frac{\partial \mathscr{L}}{\partial P_{sj}} = \frac{dF(P_{sj})}{dP_{sj}} - \alpha = 0 \qquad \text{for } j = 1 \ldots N_s$$

or

$$\frac{dF(P_{sj})}{dP_{sj}} = \alpha \qquad \text{for } j = 1 \ldots N_s \tag{7.8}$$

This means that the steam plant should be run at constant incremental cost for the entire period it is on. Let this optimum value of steam-generated power be P_s^*, which is the same for all time intervals the steam unit is on. This type of schedule is shown in Figure 7.4.

The total cost over the interval is

$$F_T = \sum_{j=1}^{N_s} F(P_s^*)n_j = F(P_s^*) \sum_{j=1}^{N_s} n_j = F(P_s^*)T_s \tag{7.9}$$

where

$$T_s = \sum_{j=1}^{N_s} n_j = \text{the total run time for the steam plant}$$

Let the steam-plant cost be expressed as

$$F(P_s) = A + BP_s + CP_s^2 \tag{7.10}$$

then

$$F_T = (A + BP_s^* + CP_s^{*2})T_s \tag{7.11}$$

also note that

$$\sum_{j=1}^{N_s} P_{sj}n_j = \sum_{j=1}^{N_s} P_s^*n_j = P_s^*T_s = E \tag{7.12}$$

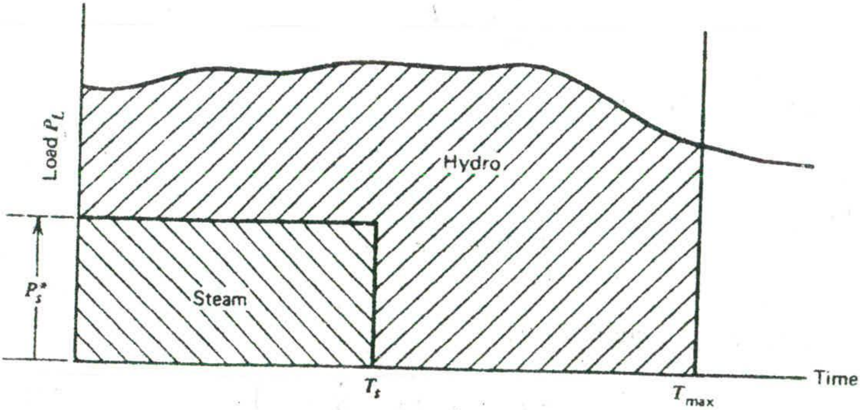


FIG. 7.4 Resulting optimal hydrothermal schedule.

Then

$$T_s = \frac{E}{P_s^*} \tag{7.13}$$

and

$$F_T = (A + BP_s^* + CP_s^{*2})\left(\frac{E}{P_s^*}\right) \tag{7.14}$$

Now we can establish the value of P_s^* by minimizing F_T:

$$\frac{\mathrm{d}F_T}{\mathrm{dP}_s^*} = \frac{-AE}{P_s^{*2}} + CE = 0 \tag{7.15}$$

or

$$P_s^* = \sqrt{A/C} \tag{7.16}$$

which means the unit should be operated at its maximum efficiency point long enough to supply the energy needed, E. Note, if

$$F(P_s) = A + BP_s + CP_s^2 = f_c \times H(P_s) \tag{7.17}$$

where f_c is the fuel cost, then the heat rate is

$$\frac{H(P_s)}{P_s} = \frac{1}{f_c}\left(\frac{A}{P_s} + B + CP_s\right) \tag{7.18}$$

and the heat rate has a minimum when

$$\frac{\mathrm{d}}{\mathrm{d}P_s}\left[\frac{H(P_s)}{P_s}\right] = 0 = \frac{-A}{P_s^2} + C \tag{7.19}$$

giving best efficiency at

$$P_s = \sqrt{A/C} = P_s^* \tag{7.20}$$

EXAMPLE 7A

A hydroplant and a steam plant are to supply a constant load of 90 MW for 1 wk (168 h). The unit characteristics are

Hydroplant: $q = 300 + 15P_H$ acre-ft/h

$0 \le P_H \le 100$ MW

Steam plant: $H_s = 53.25 + 11.27P_s + 0.0213P_s^2$

$12.5 \le P_s \le 50$ MW

Part 1

Let the hydroplant be limited to 10,000 MWh of energy. Solve for T_s^*, the run time of the steam unit. The load is 90 × 168 = 15,120 MWh, requiring 5120 MWh to be generated by the steam plant.

The steam plant's maximum efficiency is at $\sqrt{53.25/0.0213} = 50$ MW. Therefore, the steam plant will need to run for 5120/50 or 102.4 h. The resulting schedule will require the steam plant to run at 50 MW and the hydroplant at 40 MW for the first 102.4 h of the week and the hydroplant at 90 MW for the remainder.

Part 2

Instead of specifying the energy limit on the steam plant, let the limit be on the volume of water that can be drawn from the hydroplants' reservoir in 1 wk. Suppose the maximum drawdown is 250,000 acre-ft, how long should the steam unit run?

To solve this we must account for the plant's q versus P characteristic. A different flow will take place when the hydroplant is operated at 40 MW than when it is operated at 90 MW. In this case,

$$q_1 = [300 + 15(40)] \times T_s \text{ acre-ft}$$

$$q_2 = [300 + 15(90)] \times (168 - T_s) \text{ acre-ft}$$

and

$$q_1 + q_2 = 250{,}000 \text{ acre-ft}$$

Solving for T_s we get 36.27 h.

7.4 THE SHORT-TERM HYDROTHERMAL SCHEDULING PROBLEM

A more general and basic short-term hydrothermal scheduling problem requires that a given amount of water be used in such a way as to minimize the cost of running the thermal units. We will use Figure 7.5 in setting up this problem.

The problem we wish to set up is the general, short-term hydrothermal scheduling problem where the thermal system is represented by an equivalent unit, P_s, as was done in Chapter 6. In this case, there is a single hydroelectric plant, P_H. We assume that the hydroplant is not sufficient to supply all the load demands during the period and that there is a maximum total volume of water that may be discharged throughout the period of $T_{\max}$ hours.

In setting up this problem and the examples that follow, we assume all spillages, s_j, are zero. The only other hydraulic constraint we will impose initially is that the total volume of water discharged must be exactly as

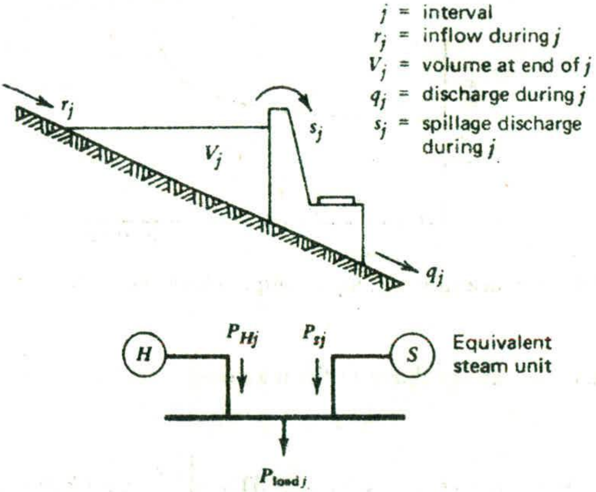


FIG. 7.5 Hydrothermal system with hydraulic constraints.

defined. Therefore, the mathematical scheduling problem may be set up as follows:

Problem:
$$\text{Min } F_T = \sum_{j=1}^{j_{\max}} n_j F_j \tag{7.21}$$

Subject to:
$$\sum_{j=1}^{j_{\max}} n_j q_j = q_{\text{TOT}} \qquad \text{total water discharge}$$

$$P_{\text{load } j} - P_{Hj} - P_{sj} = 0 \qquad \text{load balance for } j = 1 \ldots j_{\max}$$

where

$$n_j = \text{length of } j^{\text{th}} \text{ interval}$$

$$\sum_{j=1}^{j_{\max}} n_j = T_{\max}$$

and the loads are constant in each interval. Other constraints could be imposed, such as:

$$V_j|_{j=0} = V_s \qquad \text{starting volume}$$

$$V_j|_{j=j_{\max}} = V_E \qquad \text{ending volume}$$

$$q_{\min} \leq q_j \leq q_{\max} \qquad \text{flow limits for } j = 1 \ldots j_{\max}$$

$$q_j = Q_j \qquad \text{fixed discharge for a particular hour}$$

Assume constant head operation and assume a q versus P characteristic is available, as shown in Figure 7.6, so that

$$q = q(P_H) \tag{7.22}$$

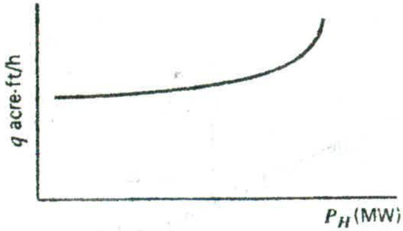


FIG. 7.6 Hydroelectric unit input–output characteristic for constant head.

We now have a similar problem to the take-or-pay fuel problem. The Lagrange function is

$$\mathcal{L} = \sum_{j=1}^{j_{max}} [n_j F(P_{sj}) + \lambda_j(P_{\text{load } j} - P_{Hj} - P_{sj})] + \gamma \left[\sum_{j=1}^{j_{max}} n_j q_j(P_{Hj}) - q_{\text{TOT}} \right] \tag{7.23}$$

and for a specific interval $j = k$,

$$\frac{\partial \mathcal{L}}{\partial P_{sk}} = 0$$

gives

$$n_k \frac{\mathrm{d}F_{sk}}{\mathrm{d}P_{sk}} = \lambda_k \tag{7.24}$$

and

$$\frac{\partial \mathcal{L}}{\partial P_{Hk}} = 0$$

gives

$$\gamma n_k \frac{\mathrm{d}q_k}{\mathrm{d}P_{Hk}} = \lambda_k \tag{7.25}$$

This is solved using the same techniques shown in Chapter 6.

Suppose we add the network losses to the problem. Then at each hour,

$$P_{\text{load } j} + P_{\text{loss } j} - P_{Hj} - P_{sj} = 0 \tag{7.26}$$

and the Lagrange function becomes

$$\mathcal{L} = \sum_{j=1}^{j_{max}} [n_j F(P_{sj}) + \lambda_j(P_{\text{load } j} + P_{\text{loss } j} - P_{Hj} - P_{sj})] + \gamma \left[\sum_{j=1}^{j_{max}} n_j q_j(P_{Hj}) - q_{\text{TOT}} \right] \tag{7.27}$$

with resulting coordination equations (hour k):

$$n_k \frac{dF(P_{sk})}{dP_{sk}} + \lambda_k \frac{\partial P_{\text{loss }k}}{\partial P_{sk}} = \lambda_k \tag{7.28}$$

$$\gamma n_k \frac{dq(P_{Hk})}{dP_{Hk}} + \lambda_k \frac{\partial P_{\text{loss }k}}{\partial P_{Hk}} = \lambda_k \tag{7.29}$$

This gives rise to a more complex scheduling solution requiring three loops, as shown in Figure 7.7. In this solution procedure, ε_1 and ε_2 are the respective tolerances on the load balance and water balance relationships.

Note that this problem ignores volume and hourly discharge rate constraints.

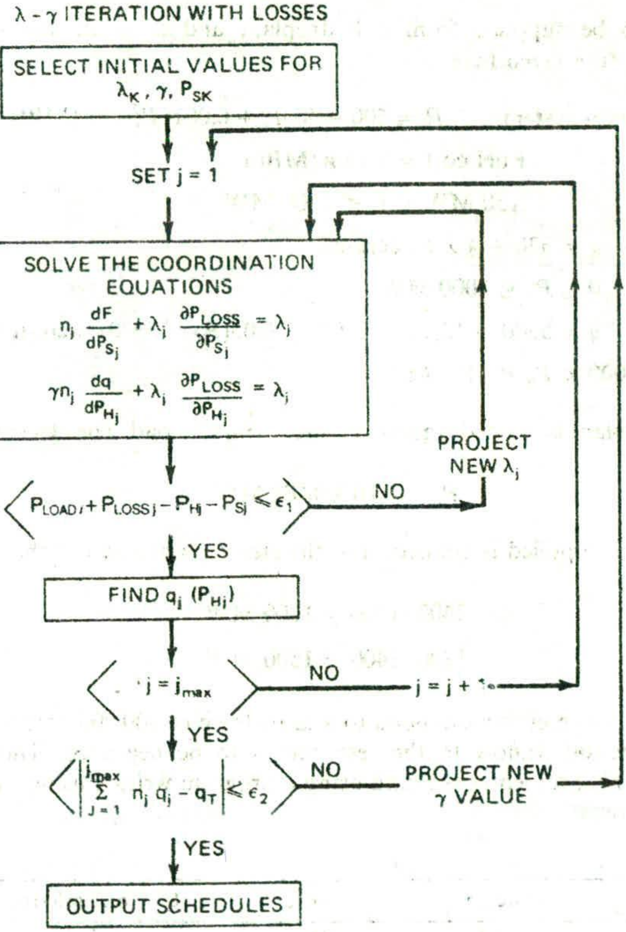


FIG. 7.7 A λ–γ iteration scheme for hydrothermal scheduling.

As a result, the value of γ will be constant over the entire scheduling period as long as the units remain within their respective scheduling ranges. The value of γ would change if a constraint (i.e., $V_j = V_{\max}$, etc.) were encountered. This would require that the scheduling logic recognize such constraints and take appropriate steps to adjust γ so that the constrained variable does not go beyond its limit. The appendix to this chapter gives a proof that γ is constant when no storage constraints are encountered. As usual, in any gradient method, care must be exercised to allow constrained variables to move off their constraints if the solution so dictates.

EXAMPLE 7B

A load is to be supplied from a hydroplant and a steam system whose characteristics are given here.

Equivalent steam system: $H = 500 + 8.0P_s + 0.0016P_s^2$ (MBtu/h)

Fuel cost = 1.15 R/MBtu

$150 \text{ MW} \leq P_s \leq 1500 \text{ MW}$

Hydroplant: $q = 330 + 4.97P_H$ acre-ft/h

$0 \leq P_H \leq 1000 \text{ MW}$

$q = 5300 + 12(P_H - 1000) + 0.05(P_H - 1000)^2$ acre-ft/h

$1000 < P_H < 1100 \text{ MW}$

The hydroplant is located a good distance from the load. The electrical losses are

$$P_{\text{loss}} = 0.00008P_h^2 \text{ MW}$$

The load to be supplied is connected at the steam plant and has the following schedule:

2400–1200 = 1200 MW

1200–2400 = 1500 MW

The hydro-unit's reservoir is limited to a drawdown of 100,000 acre-ft over the entire 24-h period. Inflow to the reservoir is to be neglected. The optimal schedule for this problem was found using a program written using Figure 7.7. The results are:

Time Period	P steam	P hydro	Hydro-Discharge (acre-ft/h)
2400–1200	567.4	668.3	3651.5
1200–2400	685.7	875.6	4681.7

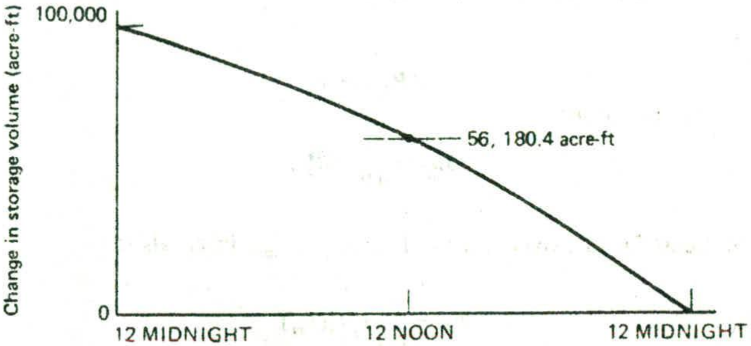


FIG. 7.8 Change in storage volume (=cumulative discharge) versus time for Example 7B.

The optimal value for γ is 2.028378 Ɍ/acre-ft. The storage in the hydroplant's reservoir goes down in time as shown in Figure 7.8. No natural inflows or spillage are assumed to occur.

7.5 SHORT-TERM HYDRO-SCHEDULING: A GRADIENT APPROACH

The following is an outline of a first-order gradient approach, as shown in Figure 6.7a, to the problem of finding the optimum schedule for a hydrothermal power system. We assume a single equivalent thermal unit with a convex input–output curve and a single hydroplant. Let:

j = the interval = $1, 2, 3, \ldots, j_{\max}$

V_j = storage volume at the end of interval j

q_j = discharge rate during interval j

r_j = inflow rate into the storage reservoir during interval j

P_{sj} = steam generation during j^{th} interval.

s_j = spillage discharge rate during interval j

$P_{\text{loss } j}$ = losses, assumed here to be zero

$P_{\text{load } j}$ = received power during the j^{th} interval (load)

P_{Hj} = hydro-generation during the j^{th} hour

Next, we let the discharge from the hydroplant be a function of the hydro-power output only. That is, a constant head characteristic is assumed.

Then,

$$q_j(P_{Hj}) = q_j$$

so that to a first order,*

$$\Delta q_j = \frac{dq_j}{dP_H} \Delta P_{Hj}$$

The total cost for fuel over the $j = 1, 2, 3, \ldots, j_{\max}$ intervals is

$$F_T = \sum_{j=1}^{j_{\max}} n_j F_j(P_{sj})$$

This may be expanded in a Taylor series to give the change in fuel cost for a change in steam-plant schedule.

$$\Delta F_T = \sum_{j=1}^{j_{\max}} n_j [F'_j \Delta P_{sj} + \tfrac{1}{2} F''_j (\Delta P_{sj})^2 + \ldots]$$

To the first order this is

$$\Delta F_T = \sum_{j=1}^{j_{\max}} n_j F'_j \Delta P_{sj}$$

In any given interval, the electrical powers must balance:

$$P_{\text{load } j} - P_{sj} - P_{Hj} = 0$$

so that,

$$\Delta P_{sj} = -\Delta P_{Hj}$$

or

$$\Delta P_{sj} = -\frac{\Delta q_j}{\left(\dfrac{dq_j}{dP_{Hj}}\right)}$$

Therefore,

$$\Delta F_T = -\sum_{j=1}^{j_{\max}} n_j \left[\frac{\left(\dfrac{dF_j}{dP_{sj}}\right)}{\left(\dfrac{dq_j}{dP_{Hj}}\right)}\right] \Delta q_j = -\sum_{j=1}^{j_{\max}} n_j \gamma_j \Delta q_j$$

where

$$\gamma_j = \frac{\left(\dfrac{dF_j}{dP_{sj}}\right)}{\left(\dfrac{dq_j}{dP_{Hj}}\right)}$$

---

* ΔP_s and ΔF designate changes in the quantities P_s and F.

The variables γ_j are the incremental water values in the various intervals and give an indication of how to make the "moves" in the application of the first-order technique. That is, the "steepest descent" to reach minimum fuel cost (or the best period to release a unit of water) is the period with the maximum value of γ. The values of water release, Δq_j, must be chosen to stay within the hydraulic constraints. These may be determined by use of the *hydraulic continuity equation*:

$$V_j = V_{j-1} + (r_j - q_j - s_j)n_j$$

to compute the reservoir storage each interval. We must also observe the storage limits,

$$V_{\min} \le V_j \le V_{\max}$$

We will assume spillage is prohibited so that all $s_j = 0$, even though there may well be circumstances where allowing $s_j > 0$ for some j might reduce the thermal system cost.

The discharge flow may be constrained both in rate and in total. That is,

$$q_{\min} \le q_j \le q_{\max}$$

and

$$\sum_{j=1}^{j_{\max}} n_j q_j = q_{\mathrm{TOT}}$$

The flowchart in Figure 6.7a illustrates the application of this method. Figure 7.9 illustrates a typical trajectory of storage volume versus time and illustrates the special rules that must be followed when constraints are taken. Whenever a constraint is reached (that is, storage V_j is equal to $V_{\min}$ or $V_{\max}$), one must choose intervals in a more restricted manner than as shown in Figure 6.7a. This is summarized here.

1. **No Constraints Reached**
   Select the pair of intervals j^- and j^+ anywhere from $j = 1 \ldots j_{\max}$.

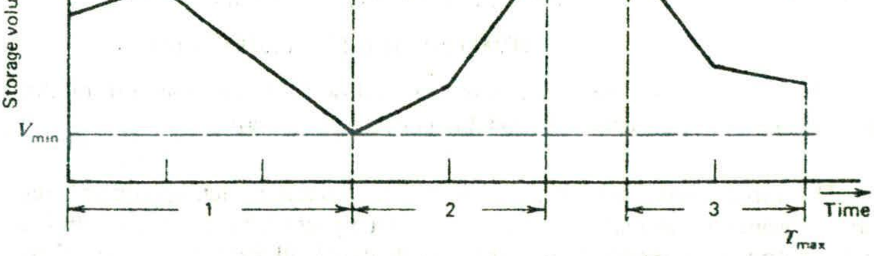


FIG. 7.9 Storage volume trajectory.

2. **A Constraint Is Reached**

   Option A: Choose the j^- and j^+ within one of the subintervals. That is, choose both j^- and j^+ from periods 1, 2, or 3 in Figure 7.9. This will guarantee that the constraint is not violated. For example, choosing a time j^+ within period 1 to increase release, and choosing j^- also in period 1 to decrease release, will mean no net release change at the end of subinterval 1, so the $V_{\min}$ constraint will not be violated.

   Option B: Choose j^- and j^+ from different subintervals so that the constraint is no longer reached. For example, choosing j^+ within period 2 and j^- within period 1 will mean the $V_{\min}$ and $V_{\max}$ limits are no longer reached at all.

Other than these special rules, one can apply the flowchart of Figure 6.7a exactly as shown (while understanding that q is water rather than fuel as in Figure 6.7a).

EXAMPLE 7C

Find an optimal hydro-schedule using the gradient technique of section 7.5. The hydroplant and equivalent steam plant are the same as Example 7B, with the following additions.

Load pattern:	First day	2400–1200 = 1200 MW
		1200–2400 = 1500 MW
	Second day	2400–1200 = 1100 MW
		1200–2400 = 1800 MW
	Third day	2400–1200 = 950 MW
		1200–2400 = 1300 MW

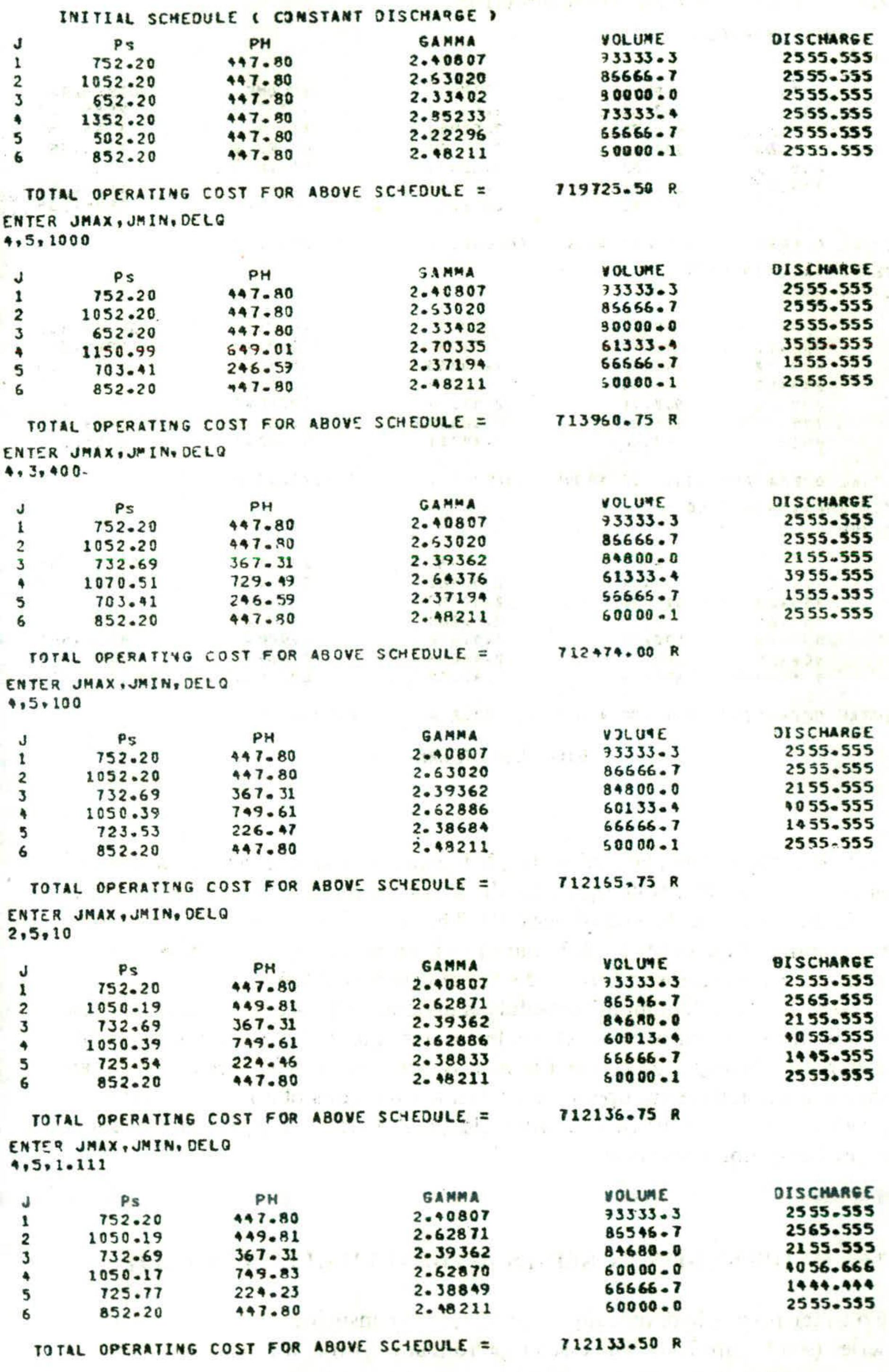
Hydro-reservoir:

1. 100,000 acre-ft at the start.
2. Must have 60,000 acre-ft at the end of schedule.
3. Reservoir volume is limited as follows:

$$60{,}000 \text{ acre-ft} \le V \le 120{,}000 \text{ acre-ft}$$

4. There is a constant inflow into the reservoir of 2000 acre-ft/h over the entire 3-day period

The initial schedule has constant discharge; thereafter, each update or "step" in the gradient calculations was carried out by entering the j^+, j^- and Δq into a computer terminal that then recalculated all period γ values, flows, and so forth. The results of running this program are shown in Figure 7.10.



```
    INITIAL SCHEDULE ( CONSTANT DISCHARGE )
J          Ps            PH                GAMMA              VOLUME          DISCHARGE
1        752.20        447.80             2.40807            93333.3          2555.555
2       1052.20        447.80             2.63020            86666.7          2555.555
3        652.20        447.80             2.33402            80000.0          2555.555
4       1352.20        447.80             2.85233            73333.4          2555.555
5        502.20        447.80             2.22296            66666.7          2555.555
6        852.20        447.80             2.48211            60000.1          2555.555

 TOTAL OPERATING COST FOR ABOVE SCHEDULE =          719725.50  R
ENTER JMAX,JMIN,DELQ
4,5,1000

J          Ps            PH                GAMMA              VOLUME          DISCHARGE
1        752.20        447.80             2.40807            93333.3          2555.555
2       1052.20        447.80             2.63020            86666.7          2555.555
3        652.20        447.80             2.33402            80000.0          2555.555
4       1150.99        649.01             2.70335            61333.4          3555.555
5        703.41        246.59             2.37194            66666.7          1555.555
6        852.20        447.80             2.48211            60000.1          2555.555

 TOTAL OPERATING COST FOR ABOVE SCHEDULE =          713960.75  R
ENTER JMAX,JMIN,DELQ
4,3,400-

J          Ps            PH                GAMMA              VOLUME          DISCHARGE
1        752.20        447.80             2.40807            93333.3          2555.555
2       1052.20        447.80             2.63020            86666.7          2555.555
3        732.69        367.31             2.39362            84800.0          2155.555
4       1070.51        729.49             2.64376            61333.4          3955.555
5        703.41        246.59             2.37194            66666.7          1555.555
6        852.20        447.80             2.48211            60000.1          2555.555

 TOTAL OPERATING COST FOR ABOVE SCHEDULE =          712474.00  R
ENTER JMAX,JMIN,DELQ
4,5,100

J          Ps            PH                GAMMA              VOLUME          DISCHARGE
1        752.20        447.80             2.40807            93333.3          2555.555
2       1052.20        447.80             2.63020            86666.7          2555.555
3        732.69        367.31             2.39362            84800.0          2155.555
4       1050.39        749.61             2.62886            60133.4          4055.555
5        723.53        226.47             2.38684            66666.7          1455.555
6        852.20        447.80             2.48211            60000.1          2555.555

 TOTAL OPERATING COST FOR ABOVE SCHEDULE =          712165.75  R
ENTER JMAX,JMIN,DELQ
2,5,10

J          Ps            PH                GAMMA              VOLUME          DISCHARGE
1        752.20        447.80             2.40807            93333.3          2555.555
2       1050.19        449.81             2.62871            86546.7          2565.555
3        732.69        367.31             2.39362            84680.0          2155.555
4       1050.39        749.61             2.62886            60013.4          4055.555
5        725.54        224.46             2.38833            66666.7          1445.555
6        852.20        447.80             2.48211            60000.1          2555.555

 TOTAL OPERATING COST FOR ABOVE SCHEDULE =          712136.75  R
ENTER JMAX,JMIN,DELQ
4,5,1.111

J          Ps            PH                GAMMA              VOLUME          DISCHARGE
1        752.20        447.80             2.40807            93333.3          2555.555
2       1050.19        449.81             2.62871            86546.7          2565.555
3        732.69        367.31             2.39362            84680.0          2155.555
4       1050.17        749.83             2.62870            60000.0          4056.666
5        725.77        224.23             2.38849            66666.7          1444.444
6        852.20        447.80             2.48211            60000.0          2555.555

 TOTAL OPERATING COST FOR ABOVE SCHEDULE =          712133.50  R
```

FIG. 7.10 Computer printout for Example 7C. *(Continued on next page)*

```
ENTER JMAX,JMIN,DELQ
2,3,800

 J         Ps          PH              GAMMA          VOLUME        DISCHARGE
 1       752.20      447.80           2.40807         93333.3        2555.555
 2       889.22      610.78           2.50953         76945.7        3365.555
 3       893.65      206.35           2.51280         84680.0        1355.555
 4      1050.17      749.83           2.62870         60000.0        4056.666
 5       725.77      224.23           2.38849         65665.7        1444.444
 6       852.20      447.80           2.48211         60000.0        2555.555

  TOTAL OPERATING COST FOR ABOVE SCHEDULE =       711020.75 R
ENTER JMAX,JMIN,DELQ
4,1,750

 J         Ps          PH              GAMMA          VOLUME        DISCHARGE
 1       903.11      296.89           2.51981        102333.3        1805.555
 2       889.22      610.78           2.50953         85946.7        3365.555
 3       893.65      206.35           2.51280         93680.0        1355.555
 4       899.26      900.74           2.51696         60000.0        4806.665
 5       725.77      224.23           2.38849         66666.7        1444.444
 6       852.20      447.90           2.48211         60000.1        2555.555

  TOTAL OPERATING COST FOR ABOVE SCHEDULE =       710040.75 R
ENTER JMAX,JMIN,DELQ
6,5,400

 J         Ps          PH              GAMMA          VOLUME        DISCHARGE
 1       903.11      296.89           2.51981        102333.3        1805.555
 2       889.22      610.78           2.50953         85946.7        3365.555
 3       893.65      206.35           2.51280         93680.0        1355.555
 4       899.26      900.74           2.51696         60000.0        4806.665
 5       806.25      143.75           2.44809         71465.7        1044.444
 6       771.72      528.29           2.42252         60000.1        2955.555

  TOTAL OPERATING COST FOR ABOVE SCHEDULE =       709877.38 R
```

FIG. 7.10 (*Continued*)

Note that the column labeled VOLUME gives the reservoir volume at the *end* of each 12-h period. Note that after the fifth step, the volume schedule reaches its bottom limit at the end of period 4. The subsequent steps require a choice of j^+ and j^- from either $\{1, 2, 3, \text{ and } 4\}$ or from $\{5, 6\}$. (P_s, P_H are MW, gamma is Ɽ/acre-ft, volume is in acre-ft, discharge is in acre-ft/h.)

Note that the "optimum" schedule is undoubtedly located between the last two iterations. If we were to release less water in any of the first four intervals and more during 5 or 6, the thermal system cost would increase. We can theoretically reduce our operating costs a few fractions of an Ɽ by leveling the γ values in each of the two subintervals, $\{1, 2, 3, 4\}$ and $\{5, 6\}$, but the effort is probably not worthwhile.

7.6 HYDRO-UNITS IN SERIES (HYDRAULICALLY COUPLED)

Consider now, a hydraulically coupled system consisting of three reservoirs in series (see Figure 7.11). The discharge from any upstream reservoir is assumed

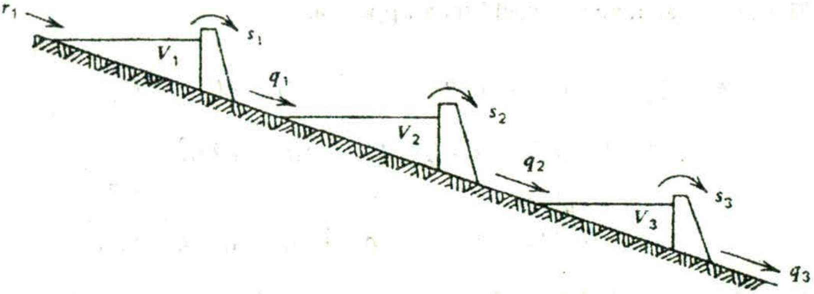


FIG. 7.11 Hydraulically coupled hydroelectric plants.

to flow directly into the succeeding downstream plant with no time lag. The hydraulic continuity equations are

$$V_{1j} = V_{1j-1} + (r_{1j} - s_{1j} - q_{1j})n_j$$
$$V_{2j} = V_{2j-1} + (q_{1j} + s_{1j} - s_{2j} - q_{2j})n_j$$
$$V_{3j} = V_{3j-1} + (q_{2j} + s_{2j} - s_{3j} - q_{3j})n_j$$

where

r_j = inflow
V_j = reservoir volume
s_j = spill rate over the dam's spillway
q_j = hydroplant discharge
n_j = numbers of hours in each scheduling period

The object is to minimize

$$\sum_{j=1}^{j_{\max}} n_j F(P_{sj}) = \text{total cost} \tag{7.30}$$

subject to the following constraints

$$P_{\text{load } j} - P_{sj} - P_{H1j} - P_{H2j} - P_{H3j} = 0$$

and

$$\begin{aligned} V_{1j} - V_{1j-1} - (r_{1j} - s_{1j} - q_{1j})n_j &= 0 \\ V_{2j} - V_{2j-1} - (q_{1j} + s_{1j} - s_{2j} - q_{2j})n_j &= 0 \\ V_{3j} - V_{3j-1} - (q_{2j} + s_{2j} - s_{3j} - q_{3j})n_j &= 0 \end{aligned} \tag{7.31}$$

All equations in set 7.31 must apply for $j = 1 \ldots j_{\max}$.

The Lagrange function would then appear as

$$\begin{aligned}\mathscr{L} = \sum_{j=1}^{j_{\max}} \{&[n_j F(P_{sj}) - \lambda_j(P_{\text{load } j} - P_{sj} - P_{H1j} - P_{H2j} - P_{H3j})]\\ &+ \gamma_{1j}[V_{1j} - V_{1j-1} - (r_{1j} - s_{1j} - q_{1j})n_j]\\ &+ \gamma_{2j}[V_{2j} - V_{2j-1} - (q_{1j} + s_{1j} - s_{2j} - q_{2j})n_j]\\ &+ \gamma_{3j}[V_{3j} - V_{3j-1} - (q_{2j} + s_{2j} - s_{3j} - q_{3j})n_j]\}\end{aligned}$$

Note that we could have included more constraints to take care of reservoir volume limits, end-point volume limits, and so forth, which would have necessitated using the Kuhn–Tucker conditions when limits were reached.

Hydro-scheduling with multiple-coupled plants is a formidable task. Lambda–gamma iteration techniques or gradient techniques can be used; in either case, convergence to the optimal solution can be slow. For these reasons, hydro-scheduling for such systems is often done with dynamic programming (see Section 7.8) or linear programming (see Section 7.9).

7.7 PUMPED-STORAGE HYDROPLANTS

Pumped-storage hydroplants are designed to save fuel costs by serving the peak load (a high fuel-cost load) with hydro-energy and then pumping the water back up into the reservoir at light load periods (a lower cost load). These plants may involve separate pumps and turbines or, more recently, reversible pump turbines. Their operation is illustrated by the two graphs in Figure 7.12. The

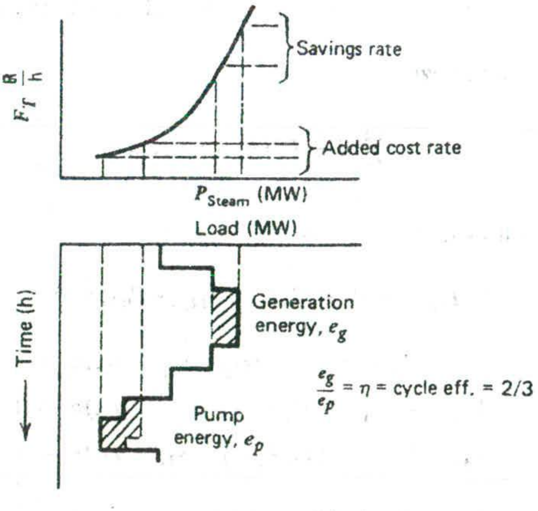


FIG. 7.12 Thermal input–output characteristic and typical daily load cycle.

first is the composite thermal system input–output characteristic and the second is the load cycle.

The pumped-storage plant is operated until the added pumping cost exceeds the savings in thermal costs due to the *peak shaving operations*. Figure 7.12 illustrates the operation on a daily cycle. If

$$\left.\begin{aligned} e_g &= \text{generation, MWh} \\ e_p &= \text{pumping load, MWh} \end{aligned}\right\} \text{for the same volume of water}$$

then the cycle efficiency is

$$\eta = \frac{e_g}{e_p} \qquad (\eta \text{ is typically about } 0.67)$$

Storage reservoirs have limited storage capability and typically provide 4 to 8 or 10 h of continuous operation as a generator. Pumped-storage plants may be operated on a daily or weekly cycle. When operated on a weekly cycle, pumped-storage plants will start the week (say a Monday morning in the United States) with a full reservoir. The plant will then be scheduled over a weekly period to act as a generator during high load hours and to refill the reservoir partially, or completely, during off-peak periods.

Frequently, special interconnection arrangements may facilitate pumping operations if arrangements are made to purchase low-cost, off-peak energy. In some systems, the system operator will require a complete daily refill of the reservoir when there is any concern over the availability of capacity reserves. In those instances, economy is secondary to reliability.

7.7.1 Pumped-Storage Hydro-Scheduling with a λ–γ Iteration

Assume:

1. Constant head hydro-operation.
2. An equivalent steam unit with convex input–output curve.
3. A 24-h operating schedule, each time intervals equals 1 h.
4. In any one interval, the plant is either pumping or generating or idle (idle will be considered as just a limiting case of pumping or generating).
5. Beginning and ending storage reservoir volumes are specified.
6. Pumping can be done continuously over the range of pump capability.
7. Pump and generating ratings are the same.
8. There is a constant cycle efficiency, η.

The problem is set up ignoring reservoir volume constraints to show that the same type of equations can result as those that arose in the conventional hydro-case. Figure 7.13 shows the water flows and equivalent electrical system.

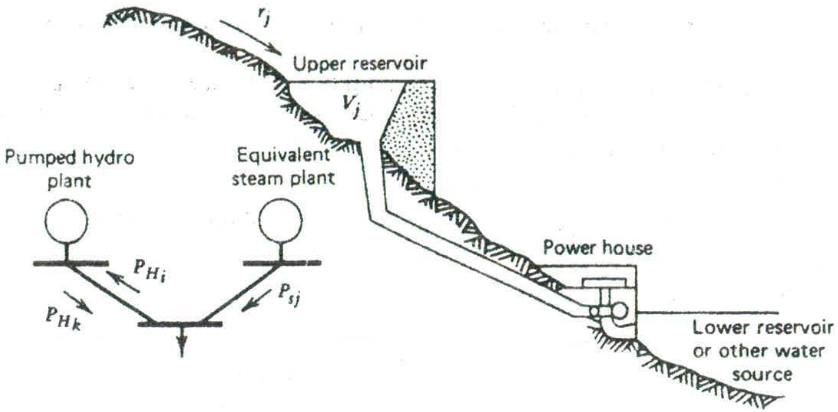


FIG. 7.13 Pumped-storage hydraulic flows and electric system flows.

In some interval, j,

r_j = inflow (acre-ft/h)

V_j = volume at end of interval (acre-ft)

q_j = discharge if generating (acre-ft/h)

or

w_j = pumping rate if pumping (acre-ft/h)

Intervals during the day are classified into two sets:

$\{k\}$ = intervals of generation

$\{i\}$ = intervals of pumping

The reservoir constraints are to be monitored in the computational procedure. The initial and final volumes are

$$V_0 = V_s$$

$$V_{24} = V_e$$

The problem is to minimize the sum of the hourly costs for steam generation over the day while observing the constraints. This total fuel cost for a day is (note that we have dropped n_j here since $n_j = 1$ h):

$$F_T = \sum_{j=1}^{24} F_j(P_{sj})$$

We consider the two sets of time intervals:

1. $\{k\}$: **Generation intervals:** The electrical and hydraulic constraints are

$$P_{\text{load } k} + P_{\text{loss } k} - P_{sk} - P_{Hk} = 0$$
$$V_k - V_{k-1} - r_k + q_k = 0$$

These give rise to a Lagrange function during a generation hour (interval k) of

$$E_k = F_k + \lambda_k(P_{\text{load } k} + P_{\text{loss } k} - P_{sk} - P_{Hk}) + \gamma_k(V_k - V_{k-1} - r_k + q_k) \qquad (7.32)$$

2. $\{i\}$: **Pump intervals:** Similarly, for a typical pumping interval, i,

$$P_{\text{load } i} + P_{\text{loss } i} - P_{si} + P_{Hi} = 0$$
$$V_i - V_{i-1} - r_i - w_i = 0 \qquad (7.33)$$
$$E_i = F_i + \lambda_i(P_{\text{load } i} + P_{\text{loss } i} - P_{si} + P_{Hi}) + \gamma_i(V_i - V_{i-1} - r_i - w_i)$$

Therefore, the total Lagrange function is

$$E = \sum_{\{k\}} E_k + \sum_{\{i\}} E_i + \varepsilon_s(V_0 - V_s) + \varepsilon_e(V_{24} - V_e) \qquad (7.34)$$

where the end-point constraints on the storage have been added.

In this formulation, the hours in which no pumped hydro activity takes place may be considered as pump (or generate) intervals with

$$P_{Hi} = P_{Hk} = 0$$

To find the minimum of $F_T = \sum F_j$, we set the first partial derivatives of E to zero.

1. $\{k\}$: **Generation intervals:**

$$\frac{\partial E}{\partial P_{sk}} = 0 = -\lambda_k\left(1 - \frac{\partial P_{\text{loss}}}{\partial P_{sk}}\right) + \frac{\mathrm{d}F_k}{\mathrm{d}P_{sk}}$$
$$\frac{\partial E}{\partial P_{Hk}} = 0 = -\lambda_k\left(1 - \frac{\partial P_{\text{loss}}}{\partial P_{Hk}}\right) + \gamma_K\frac{\mathrm{d}q_k}{\mathrm{d}P_{Hk}} \qquad (7.35)$$

2. $\{i\}$: **Pump intervals:**

$$\begin{aligned} \frac{\partial E}{\partial P_{si}} &= 0 = -\lambda_i\left(1 - \frac{\partial P_{\text{loss}}}{\partial P_{si}}\right) + \frac{dF_i}{dP_{si}} \\ \frac{\partial E}{\partial P_{Hi}} &= 0 = +\lambda_i\left(1 + \frac{\partial P_{\text{loss}}}{\partial P_{Hk}}\right) - \gamma_i \frac{dw_i}{dP_{Hi}} \end{aligned} \tag{7.36}$$

For the $\partial E/\partial V$, we can consider any interval of the entire day—for instance, the ℓth interval—which is not the first or 24th hour.

$$\frac{\partial E}{\partial V_\ell} = 0 = \gamma_\ell - \gamma_{\ell+1}$$

and for $\ell = 0$ and $= 24$

$$\frac{\partial E}{\partial V_0} = 0 = -\gamma_1 + \varepsilon_s \quad \text{and} \quad \frac{\partial E}{\partial V_{24}} = 0 = \gamma_{24} + \varepsilon_e \tag{7.37}$$

From Eq. 7.37, it may be seen that γ is a constant. Therefore, it is possible to solve the pumped-storage scheduling problem by means of a λ–γ iteration over the time interval chosen. It is necessary to monitor the calculations to prevent a violation of the reservoir constraints, or else to incorporate them in the formulation.

It is also possible to set up the problem of scheduling the pumped-storage hydroplant in a form that is very similar to the gradient technique used for scheduling conventional hydroplants.

7.7.2 Pumped-Storage Scheduling by a Gradient Method

The interval designations and equivalent electrical system are the same as those shown previously. This time, losses will be neglected. Take a 24-h period and start the schedule with no pumped-storage hydro-activity initially. Assume that the steam system is operated each hour such that

$$\frac{dF_j}{dP_{sj}} = \lambda_j \qquad j = 1, 2, 3, \ldots, 24$$

That is, the single, equivalent steam-plant source is realized by generating an economic schedule for the load range covered by the daily load cycle.

Next, assume the pumped-storage plant generates a small amount of power, ΔP_{Hk}, at the peak period k. These changes are shown in Figure 7.14. The change in steam-plant cost is

$$\Delta F_k = \frac{\partial F_k}{\partial P_{sk}} \Delta P_{sk} = -\frac{dF_k}{dP_{sk}} \Delta P_{Hk} \quad \text{or} \quad \Delta F_k = -\lambda_k \Delta P_{Hk} \tag{7.38}$$

which is the savings due to generating ΔP_{Hk}.

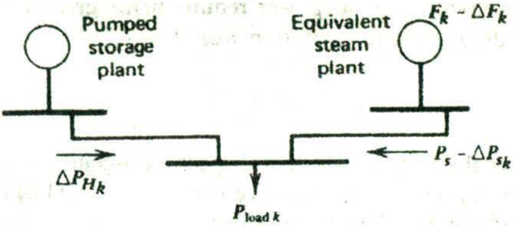


FIG. 7.14 Incremental increase in hydro-generation in hour k.

Next, we assume that the plant will start the day with a given reservoir volume and we wish to end with the same volume. The volume may be measured in terms of the MWh of generation of the plant. The overall operating cycle has an efficiency, η. For instance, if $\eta = 2/3$; 3 MWh of pumping are required to replace 2 MWh of generation water use. Therefore, to replace the water used in generating the ΔP_{Hk} power, we need to pump an amount ($\Delta P_{Hk}/\eta$).

To do this, search for the lowest cost (=lowest load) interval, i, of the day during which to do the pumping: This changes the steam system cost by an amount

$$\Delta F_i = \frac{\partial F_i}{\partial P_{si}} \Delta P_{si} = \frac{dF_i}{dP_{si}} \left(\frac{\Delta P_{Hk}}{\eta} \right) = \frac{\lambda_i}{\eta} \Delta P_{Hk} \tag{7.39}$$

The total cost change over the day is then

$$\begin{aligned} \Delta F_T &= \Delta F_k + \Delta F_i \\ &= \Delta P_{Hk} \left(\frac{\lambda_i}{\eta} - \lambda_k \right) \end{aligned} \tag{7.40}$$

Therefore, the decision to generate in k and replace the water in i is economic if ΔF_T is negative (a decrease in cost); this is true if

$$\lambda_k > \frac{\lambda_i}{\eta} \tag{7.41}$$

There are practical considerations to be observed, such as making certain that the generation and pump powers required are less than or equal to the pump or generation capacity in any interval. The whole cycle may be repeated until:

1. It is no longer possible to find periods k and i such that $\lambda_k = \lambda_i/\eta$.
2. The maximum or minimum storage constraints have been reached.

When implementing this method, it may be necessary also to do pumping in more than one interval to avoid power requirements greater than the unit rating. This can be done; then the criterion would be

$$\lambda_k > (\lambda_i + \lambda_{i'})/\eta$$

Figure 7.15 shows the way in which a single pump-generate step could be made. In this figure, the maximum capacity is taken as 1500 MW, where the pumped-storage unit is generating or pumping.

These procedures assume that commitment of units does not change as a result of the operation of the pumped-storage hydroplant. It does not presume that the equivalent steam-plant characteristics are identical in the 2 h because the same techniques can be used when different thermal characteristics are present in different hours.

Longer cycles may also be considered. For instance, you could start a schedule for a week and perhaps find that you were using the water on the weekday peaks and filling the reservoir on weekends. In the case where a reservoir constraint was reached, you would split the week into two parts

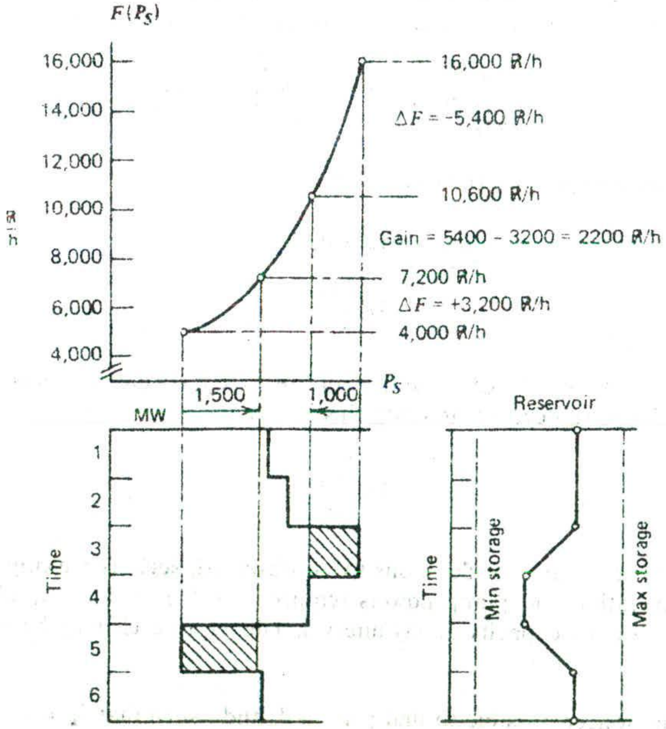


FIG. 7.15 Single step in gradient iteration for a pumped-storage plant. Cycle efficiency is two-thirds. Storage is expressed in equivalent MWh of generation.

and see if you could increase the overall savings by increasing the plant use. Another possibility may be to schedule each day of a week on a daily cycle. Multiple, uncoupled pumped-storage plants could also be scheduled in this fashion. The most reasonable-looking schedules would be developed by running the plants through the scheduling routines in parallel. (Schedule a little on plant 1, then shift to plant 2, etc.) In this way, the plants will all share in the peak shaving. Hydraulically coupled pumped-storage plants and/or pump-back plants combined with conventional hydroplants may be handled similarly.

EXAMPLE 7D

A pumped-storage plant is to operate so as to minimize the operating cost of the steam units to which it is connected. The pumped-storage plant has the following characteristics.

Generating: q positive when generating, P_H is positive and $0 \le P_H \le +300$ MW

$$q(P_H) = 200 + 2P_H \text{ acre-ft } (P_H \text{ in MW})$$

Pumping: q negative when pumping, P_P is negative and $-300 \text{ MW} \le P_P \le 0$

$$q(P_p) = -600 \text{ acre-ft/h} \quad \text{with} \quad P_p = -300 \text{ MW}$$

Operating restriction: The pumped hydroplant will be allowed to operate only at -300 MW when pumping. Cycle efficiency $\eta = 0.6667$ [the efficiency has already been built into the $q(P_H)$ equations].

The equivalent steam system has the cost curve

$$F(P_s) = 3877.5 + 3.9795P_s + 0.00204P_s^2 \text{ R/h } (200 \text{ MW} \le P_s \le 2500 \text{ MW})$$

Find the optimum pump-generate schedule using the gradient method for the following load schedule and reservoir constraint.

Load Schedule (Each Period is 4 h Long)

Period	Load (MW)
1	1600
2	1800
3	1600
4	500
5	500
6	500

The reservoir starts at 8000 acre-ft and must be at 8000 acre-ft at the end of the sixth period.

Initial Schedule

Period	Load (MW)	P_s	λ	Hydropump/Gen. (+ = gen., − = pump)	Reservoir Volume at End of Period
1	1600	1600	10.5	0	8000
2	1800	1800	11.3	0	8000
3	1600	1600	10.5	0	8000
4	500	500	6.02	0	8000
5	500	500	6.02	0	8000
6	500	500	6.02	0	8000

Starting with $k = 2$ and $i = 4$: $\lambda_2 = 11.3$; $\lambda_4 = 6.02$; $\lambda_4/\eta = 9.03$.

Therefore, it will pay to generate as much as possible during the second period as long as the pump can restore the equivalent acre-ft of water during the fourth period. Therefore, the first schedule adjustment will look like the following.

Period	Load (MW)	P_s	λ	Hydropump/ Gen.	Reservoir Volume at End of Period
1	1600	1600	10.5	0	8000
2	1800	1600	10.5	+200	5600
3	1600	1600	10.5	0	5600
4	500	800	7.24	−300	8000
5	500	500	6.02	0	8000
6	500	500	6.02	0	8000

Next, we can choose to generate another 200 MW from the hydroplant during the first period and restore the reservoir during the fifth period.

Period	Load (MW)	P_s	λ	Hydropump/ Gen.	Reservoir Volume at End of Period
1	1600	1400	9.69	+200	5600
2	1800	1600	10.5	+200	3200
3	1600	1600	10.5	0	3200
4	500	800	7.24	−300	5600
5	500	800	7.24	−300	8000
6	500	500	6.02	0	8000

Finally, we can also generate in the third period and replace the water in the sixth period.

Period	Load (MW)	P_s	λ	Hydropump/ Gen.	Reservoir Volume at End of Period
1	1600	1400	9.69	+200	5600
2	1800	1600	10.50	+200	3200
3	1600	1400	9.69	+200	800
4	500	800	7.24	−300	3200
5	500	800	7.24	−300	5600
6	500	800	7.24	−300	8000

A further savings can be realized by "flattening" the steam generation for the first three periods. Note that the costs for the first three periods as shown in the preceding table would be:

Period	P_s	Cost (R)	λ	Hydropump/Gen.
1	1400	53788.80	9.69	+200
2	1600	61868.40	10.50	+200
3	1400	53788.80	9.69	+200
4, 5, 6	800	100400.40	7.24	−300
		269846.40		

If we run the hydroplant at full output during the peak (period 2) and then reduce the amount generated during periods 1 and 3, we will achieve a savings.

Period	P_s	Cost (R)	λ	Hydropump/Gen.
1	1450	55747.50	9.90	+150
2	1500	57747.00	10.10	+300
3	1450	55747.50	9.90	+150
4, 5, 6		100400.40	7.24	−300
		269642.40		

The final reservoir schedule would be:

Period	Reservoir Volume
1	6000
2	2800
3	800
4	3200
5	5600
6	8000

7.8 DYNAMIC-PROGRAMMING SOLUTION TO THE HYDROTHERMAL SCHEDULING PROBLEM

Dynamic programming may be applied to the solution of the hydrothermal scheduling problem. The multiplant, hydraulically coupled systems offer computational difficulties that make it difficult to use that type of system to illustrate the benefits of applying DP to this problem. Instead we will illustrate the application with a single hydroplant operated in conjunction with a thermal system. Figure 7.16 shows a single, equivalent steam plant, P_s, and a hydroplant with storage, P_H, serving a single series of loads, P_L. Time intervals are denoted by j, where j runs between 1 and $j_{\max}$.

Let:

r_j = net inflow rate during period j

V_j = storage volume at the end of period j

q_j = flow rate through the turbine during period j

P_{Hj} = power output during period j

s_j = spillage rate during period j

P_{sj} = steam-plant output

$P_{\text{load } j}$ = load level

F_j = fuel cost rate for period j

Both starting and ending storage volumes, V_0 and $V_{j_{\max}}$, are given, as are the period loads. The steam plant is assumed to be on for the entire period. Its input–output characteristic is

$$F_j = a + bP_{sj} + cP_{sj}^2 \text{ R/h} \tag{7.42}$$

The water use rate characteristic of the hydroelectric plant is

$$q_j = d + gP_{Hj} + hP_{Hj}^2, \text{ acre-ft/h} \quad \text{for } P_{Hj} > 0 \tag{7.43}$$

and

$$= 0 \quad \text{for } P_{Hj} = 0$$

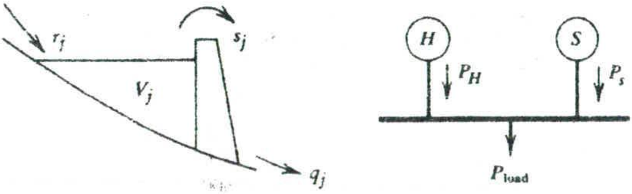


FIG. 7.16 Hydrothermal system model used in dynamic-programming illustration.

The coefficients a through h are constants. We will take the units of water flow rate as acre-ft/h. If each interval, j, is n_j hours long, the volume in storage changes as

$$V_j = V_{j-1} + n_j(r_j - q_j - s_j) \tag{7.44}$$

Spilling water will not be permitted (i.e., all $s_j = 0$).

If V_i and V_k denote two different volume states, and

$$V_{j-1} = V_i$$
$$V_j = V_k$$

then the rate of flow through the hydro-unit during interval j is

$$q_j = \frac{(V_i - V_k)}{n_j} + r_j$$

where q_j must be nonnegative and is limited to some maximum flow rate, $q_{\max}$, which corresponds to the maximum power output of the hydro-unit. The scheduling problem involves finding the minimum cost trajectory (i.e., the volume at each stage). As indicated in Figure 7.17, numerous feasible trajectories may exist.

The DP algorithm is quite simple. Let:

$\{i\}$ = the volume states at the start of the period j

$\{k\}$ = the states at the end of j

$TC_k(j)$ = the total cost from the start of the scheduling period to the end of period j for the reservoir storage state V_k

$PC(i, j-1: k, j)$ = production cost of the thermal system in period j to go from an initial volume of V_i to an end of period volume V_k.

The forward DP algorithm is then,

$$TC_k(0) = 0$$

and

$$TC_k(j) = \min_{\{i\}} [TC_i(j-1) + PC(i, j-1: k, j)] \tag{7.45}$$

We must be given the loads and natural inflows. The discharge rate through the hydro-unit is, of course, fixed by the initial and ending storage levels and this, in turn, establishes the values of P_H and P_s. The computation of the thermal production cost follows directly.

There may well be volume states in the set V_k that are unreachable from

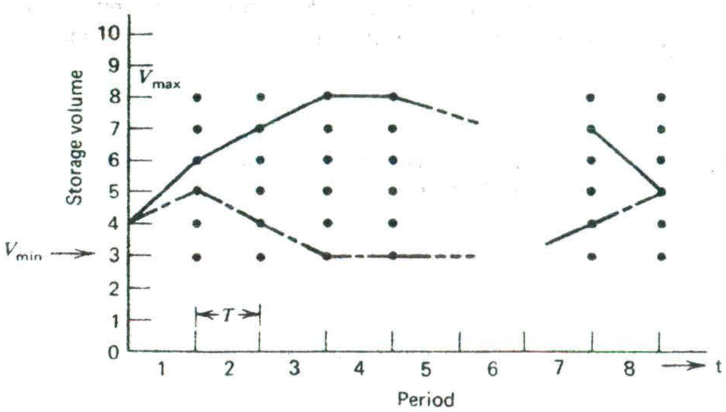


FIG. 7.17 Trajectories for hydroplant operation.

some of the initial volume states V_i because of the operating limits on the hydroplants. There are many variations on the hydraulic constraints that may be incorporated in the DP computation. For example, the discharge rates may be fixed during certain intervals to allow fish ladders to operate or to provide water for irrigation.

Using the volume levels as state variables restricts the number of hydropower output levels that are considered at each stage, since the discharge rate fixes the value of power. If a variable-head plant is considered, it complicates the calculation of the power level as an average head must be used to establish the value of P_H. This is relatively easy to handle.

EXAMPLE 7E

It is, perhaps, better to use a simple numerical example than to attempt to discuss the DP application generally. Let us consider the two-plant case just described with the steam-plant characteristics as shown in Figure 7.18 with $F = 700 + 4.8P_s + P_s^2/2000$, R/h, and $dF/dP_s = 4.8 + P_s/1000$, R/MWh, for P_s in MW and $200 \le P_s \le 1200$. MW. The hydro-unit is a constant-head plant, shown in Figure 7.19, with

$$q = 260 + 10P_H \text{ for } P_H > 0, \qquad q = 0 \text{ for } P_H = 0$$

where P_H is in MW, and

$$0 \le P_H \le 200 \text{ MW}$$

The discharge rate is in acre-ft/h. There is no spillage, and both initial and final

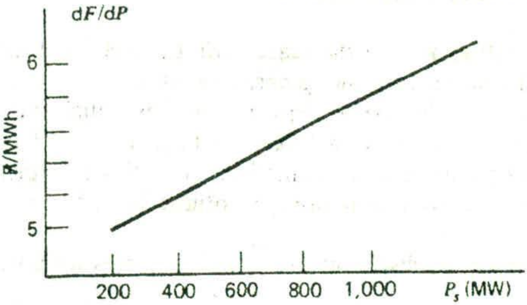


FIG. 7.18 Steam plant incremental cost function.

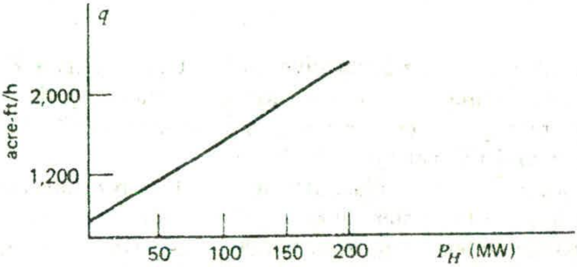


FIG. 7.19 Hydroplant q versus P_H function.

volumes are 10,000 acre-ft. The storage volume limits are 6000 and 18,000 acre-ft. The natural inflow is 1000 acre-ft/h.

The scheduling problem to be examined is for a 24-h day with individual periods taken as 4 h each ($n_j = 4.0$ h). The loads and natural inflows into the storage pond are:

Period j	$P_{\text{load } j}$ (MW)	Inflow Rate $r(j)$ (acre-ft/h)
1	600	1000
2	1000	1000
3	900	1000
4	500	1000
5	400	1000
6	300	1000

Procedure

If this were an actual scheduling problem, we might start the search using a coarse grid on both the time interval and the volume states. This would permit the future refinement of the search for the optimal trajectory after a crude search had established the general neighborhood. Finer grid steps bracketing the range of the coarse steps around the initial optimal trajectory could then be used to establish a better path. The method will work well for problems with convex (concave) functions. For this example, we will limit our efforts to 4-h time steps and storage volume steps that are 2000 acre-ft apart.

During any period, the discharge rate through the hydro-unit is

$$q_j = \frac{(V_{j-1} - V_j)}{4} + 1000 \tag{7.46}$$

The discharge rate must be nonnegative and not greater than 2260 acre-ft/h. For this problem, we may use the equation that relates P_H, the plant output, to the discharge rate, q. In a more general case, we may have to deal with tables that relate P_H, q, and the net hydraulic head.

The DP procedure may be illustrated for the first two intervals as follows. We take the storage volume steps at 6000, 8000, 10,000, . . . , 18,000 acre-ft. The initial set of volume states is limited to 10,000 acre-ft. (In this example, volumes will be expressed in 1000 acre-ft to save space.) The table here summarizes the calculations for $j = 1$; the graph in Figure 7.20 shows the trajectories. We need

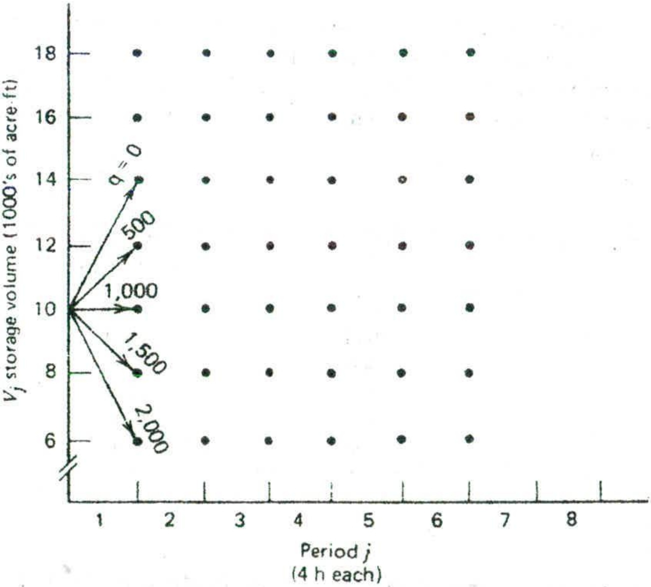


FIG. 7.20 Initial trajectories for DP example.

not compute the data for greater volume states since it is possible to do no more than shut the unit down and allow the natural inflow to increase the amount of water stored.

$j = 1$ $\quad P_L(1) = 600$ MW $\quad \{i\} = 10$

V_k	q	P_H	P_s	$TC_k(j)(\text{R})$
14	0	0	600	15040
12	500	24	576	14523
10	1000	74	526	13453
8	1500	124	476	12392
6	2000	174	426	11342

The tabulation for the second and succeeding intervals is more complex since there are a number of initial volume states to consider. A few are shown in the following table and illustrated in Figure 7.21.

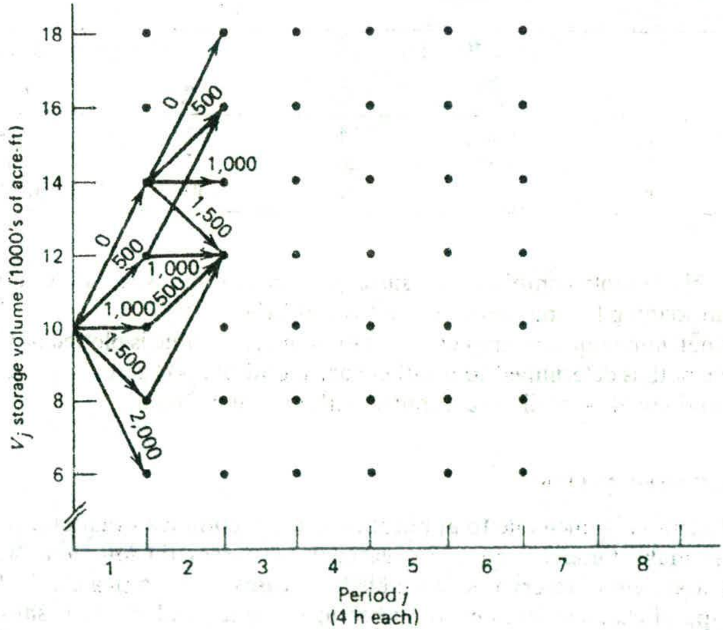


FIG. 7.21 Second-stage trajectories for DP example.

$j = 2$		$P_L = 1000$ MW			$\{i\} = [6, 8, 10, 12, 14]$
V_k	V_i	q	P_H	P_s	$TC_k(j)$(R)
18	14	0	0	1000	39040[a]
16	14	500	24	976	38484[a]
16	12	0	0	1000	38523
14	14	1000	74	926	37334[a]
14	12	500	24	976	37967
14	10	0	0	1000	37453
12	14	1500	124	876	39194[a]
12	12	1000	74	926	39818
12	10	500	24	976	36897
12	8	0	0	1000	36392
⋮			⋱		⋮
6	10	2000	174	826	33477[a]
6	8	1500	124	876	33546
6	6	1000	74	926	33636

[a] Denotes the minimum cost path.

Finally, in the last period, the following combinations:

$j = 6$		$P_L = 300$ MW			$\{i\} = [6, 8, 10, 12, 14]$
V_k	V_i	q	P_H	P_s	$TC_k(j)$(R)
10	10	1000	74	226	82240.61
10	8	500	24	276	82260.21
10	6	0	0	300	81738.46

are the only feasible combinations since the end volume is set at 10 and the minimum loading for the thermal plant is 200 MW.

The final, minimum cost trajectory for the storage volume is plotted in Figure 7.22. This path is determined to a rather coarse grid of 2000 acre-ft by 4-h steps in time and could be easily recomputed with finer increments.

7.8.1 Extension to Other Cases

The DP method is amenable to application in more complex situations. Longer time steps make it useful to compute seasonal *rule curves*, the long-term storage plan for a system of reservoirs. Variable-head cases may be treated. A sketch of the type of characteristics encountered in variable-head plants is shown in Figure 7.23. In this case, the variation in maximum plant output may be as important as the variation in water use rate as the net head varies.

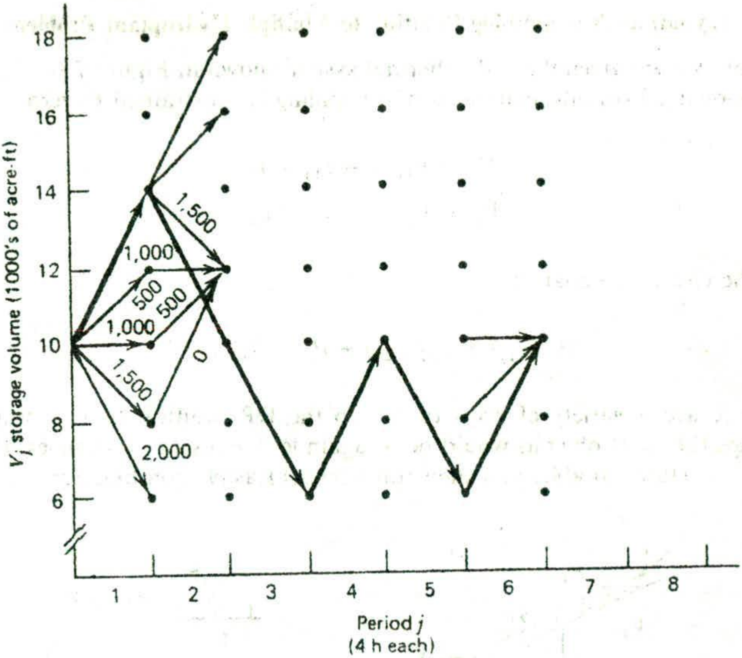
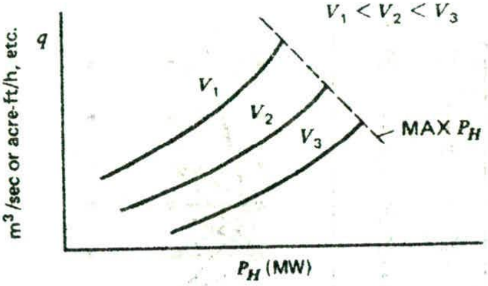


FIG. 7.22 Final trajectory for hydrothermal-scheduling example.



Variable head plant
$q = q\ (P_H, \bar{V})$
$\bar{V}$ = average volume used to represent the effect of the hydraulic head

FIG. 7.23 Input–output characteristic for variable-head hydroelectric plant.

7.8.2 Dynamic-Programming Solution to Multiple Hydroplant Problem

Suppose we are given the hydrothermal system shown in Figure 7.24. We have the following hydraulic equations when spilling is constrained to zero

$$V_{1j} = V_{1j-1} + r_{1j} - q_{1j}$$

$$V_{2j} = V_{2j-1} + q_{1j} - q_{2j}$$

and the electrical equation

$$P_{H1}(q_{1j}) + P_{H2}(q_{2j}) + P_{sj} - P_{\text{load } j} = 0$$

There are a variety of ways to set up the DP solution to this program. Perhaps the most obvious would be to again let the reservoir volumes, V_1 and V_2, be the state variables and then run over all feasible combinations. That is,

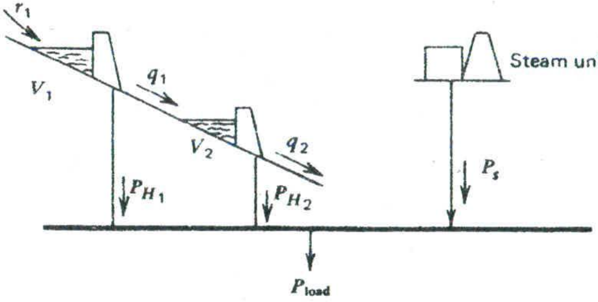


FIG. 7.24 Hydrothermal system with hydraulically coupled hydroelectric plants.

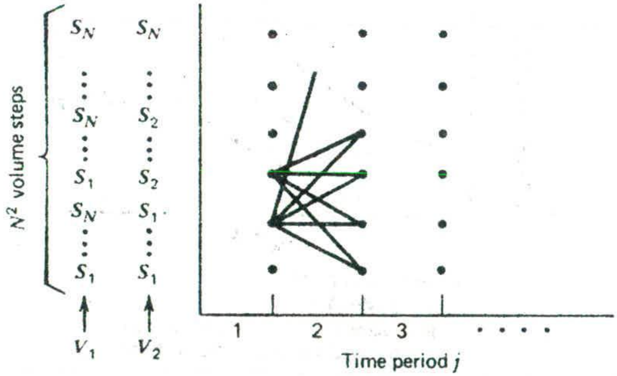


FIG. 7.25 Trajectory combinations for coupled plants.

let V_1 and V_2 both be divided into N volume steps $S_1 \ldots S_2$. Then the DP must consider N^2 steps at each time interval, as shown in Figure 7.25.

This procedure might be a reasonable way to solve the multiple hydroplant scheduling problem if the number of volume steps were kept quite small. However, this is not practical when a realistic schedule is desired. Consider, for example, a reservoir volume that is divided into 10 steps ($N = 10$). If there were only one hydroplant, there would be 10 states at each time period, resulting in a possible 100 paths to be investigated at each stage. If there were two reservoirs with 10 volume steps, there would be 100 states at each time interval with a possibility of 10,000 paths to investigate at each stage.

This dimensionality problem can be overcome through the use of a procedure known as *successive approximation*. In this procedure, one reservoir is scheduled while keeping the other's schedule fixed, alternating from one reservoir to the other until the schedules converge. The steps taken in a successive approximation method appear in Figure 7.26.

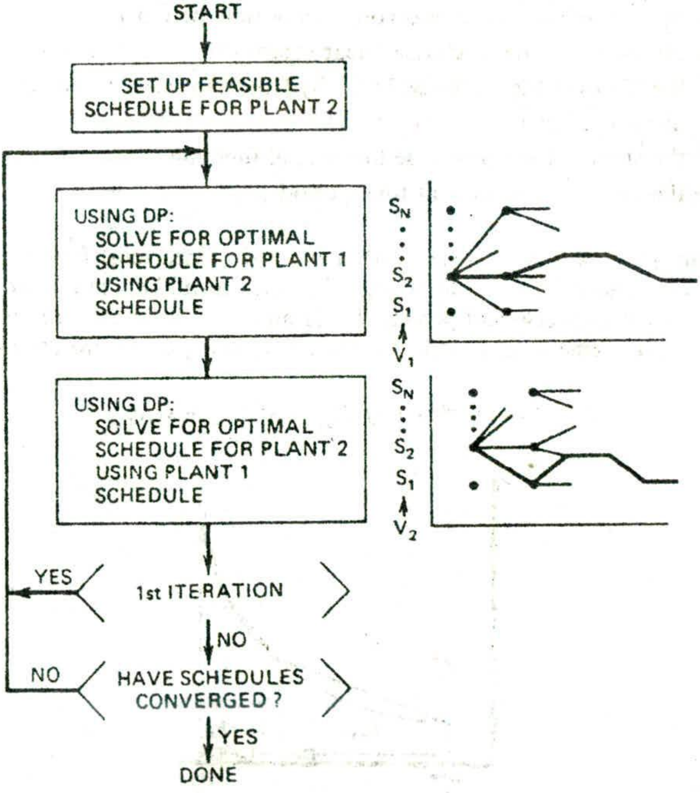


FIG. 7.26 Successive approximation solution.

7.9 HYDRO-SCHEDULING USING LINEAR PROGRAMMING

One of the more useful ways to solve large hydro-scheduling problems is through the use of linear programming. Modern LP codes and computers make this an increasingly useful option. In this section, a simple, single reservoir hydroplant operating in conjunction with a single steam plant, as shown in Figure 7.5, will be modeled using linear programming (see reference 16).

First, we shall show how each of the models needed are expressed as linear models which can be incorporated in an LP. The notation is as follows:

P_{sj} = the steam plant net output at time period j
P_{hj} = the hydroplant net output at time period j
q_j = the turbine discharge at time period j
s_j = the reservoir spill at time period j
V_j = the reservoir volume at time period j
r_j = the net inflow to the reservoir during time period j
sf_k = the slopes of the piecewise linear steam-plant cost function
sh_k = the slopes of the piecewise linear hydroturbine electrical output versus discharge function
sd_k = the slopes of the piecewise linear spill function
$P_{\text{load}\,j}$ = the net electrical load at time period j

The steam plant will be modeled with a piecewise linear cost function, $F(P_j)$, as shown in Figure 7.27. The three segments shown will be represented as P_{sj1}, P_{sj2}, P_{sj3} where each segment power, P_{sjk}, is measured from the start of the k^{th} segment. Each segment has a slope designated sf_1, sf_2, sf_3; then, the cost function itself is

$$F(P_{sj}) = F(P_s^{\min}) + sf_1 P_{sj1} + sf_2 P_{sj2} + sf_3 P_{sj3} \tag{7.47}$$

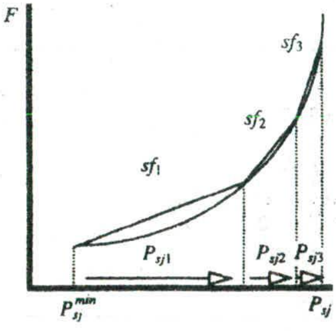


FIG. 7.27 Steam plant piecewise linear cost function.

and

$$0 < P_{sjk} < P_{sjk}^{\max} \qquad \text{for } k = 1, 2, 3 \tag{7.48}$$

and finally

$$P_{sj} = P_s^{\min} + P_{sj1} + P_{sj2} + P_{sj3} \tag{7.49}$$

The hydroturbine discharge versus the net electrical output function is designated $P_h(q_j)$ and is also modeled as a piecewise linear curve. The actual characteristic is usually quite nonlinear, as shown by the dotted line in Figure 7.28. As explained in reference 16, hydroplants are rarely operated close to the low end of this curve, rather they are operated close to their maximum efficiency or full gate flow points. Using the piecewise linear characteristic shown in Figure 7.28, the plant will tend to go to one of these two points.

In this model, the net electrical output is given as a linear sum:

$$P_{hj} = sh_1 q_{j1} + sh_2 q_{j2} \tag{7.50}$$

The spill out of the reservoir is modeled as a function of the reservoir volume and it is assumed that the spill is zero if the volume of water in the reservoir is less than a given limit. This can easily be modeled by the piecewise linear characteristic in Figure 7.29, where the spill is constrained to be zero if the volume of water in the reservoir is less than the first volume segment where

$$s_j = sd_1 V_{j1} + sd_2 V_{j2} + sd_3 V_{j3} \tag{7.53}$$

and

$$0 \leq V_{jk} \leq V_{jk}^{\max} \quad \text{for } k = 1, 2, 3 \tag{7.54}$$

then

$$V_j = V_{j1} + V_{j2} + V_{j3} \tag{7.55}$$

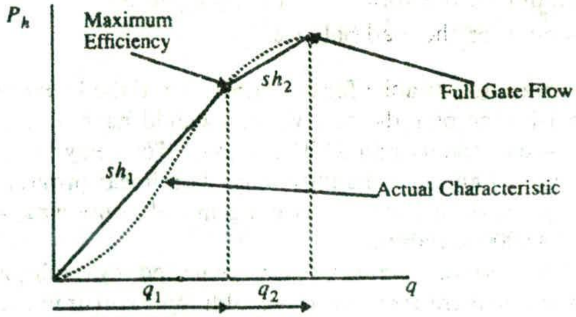


FIG. 7.28 Hydroturbine characteristic.

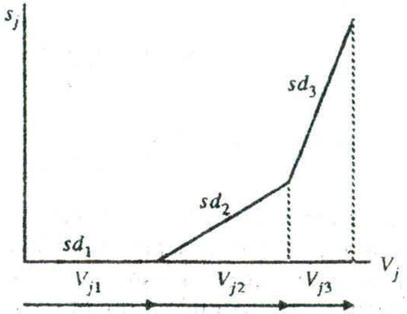


FIG. 7.29 Spill characteristic.

The hydro-scheduling linear program then consists of the following; minimize

$$\sum_{j=1}^{j_{\max}} F(P_{sj})$$

subject to

$$V_j - V_{j-1} - (r_j - s_j - q_j) = 0 \quad \text{for } j = 1 \ldots j_{\max}$$

where

$$P_{hj} = P_h(q_j)$$

$$s_j = s(V_j)$$

and

$$P_{sj} + P_{hj} - P_{\text{load } j} = 0 \qquad \text{for } j = 1 \ldots j_{\max}$$

Note that this simple hydro-scheduling problem will generate eight constraints for each time step:

- Two constraints for the steam-plant characteristic.
- Two constraints for the hydroturbine characteristic.
- Two constraints for the spill characteristic.
- One constraint for the volume continuity equation.
- One constraint for the load balance.

In addition, there are 15 variables for each time step. If the linear program were to be run with 1-h time periods for 1 week, it would have to accommodate a model with 1344 constraints and 2520 variables. This may seem quite large, but is actually well within the capability of modern linear programming codes. Reference 16 reports on a hydro-scheduling model containing about 10,000 constraints and 35,000 variables.

When multiple reservoir/plant models connected by multiple rivers and channels are modeled, there are many more additional constraints and variables needed. Nonetheless, the use of linear programming is common and can be relied upon to give excellent solutions.

APPENDIX
Hydro-Scheduling with Storage Limitations

This appendix expands on the Lagrange equation formulation of the fuel-limited dispatch problem in Chapter 6 and the reservoir-limited hydro-dispatch problem of Chapter 7. The expansion includes generator and reservoir storage limits and provides a proof that the "fuel cost" or "water cost" Lagrange multiplier γ will be constant unless reservoir storage limitations are encountered.

To begin, we will assume that we have a hydro-unit and an equivalent steam unit supplying load as in Figure 7.5. Assume that the scheduling period is broken down into three equal time intervals with load, generation, reservoir inflow, and such, constant within each period. In Chapter 6 (Section 6.2, Eqs. 6.1–6.6) and Chapter 7 (Section 7.4, Eqs. 7.22–7.29) we assumed that the total q was to be fixed at q_{TOT}, that is (see Section 7.4 for definition of variables),

$$q_{\text{TOT}} = \sum_{j=1}^{j_{\max}} n_j q(P_{Hj}) \tag{7A.1}$$

In the case of a storage reservoir with an initial volume V_0, this constraint is equivalent to fixing the final volume in the reservoir. That is,

$$V_0 + n_1[r_1 - q(P_{H1})] = V_1 \tag{7A.2}$$

$$V_1 + n_2[r_2 - q(P_{H2})] = V_2 \tag{7A.3}$$

$$V_2 + n_3[r_3 - q(P_{H3})] = V_3 \tag{7A.4}$$

Substituting Eq. 7A.2 into Eq. 7A.3 and then substituting the result into Eq. 7A.4, we get

$$V_0 + \sum_{j=1}^{3} n_j r_j - \sum_{j=1}^{3} n_j q(P_{Hj}) = V_3 \tag{7A.5}$$

or

$$V_0 + \sum_{j=1}^{3} n_j r_j - q_{\text{TOT}} = V_3 \tag{7A.6}$$

Therefore, fixing q_{TOT} is equivalent to fixing V_3, the final reservoir storage. The optimization problem will be expressed as:

Minimize total steam plant cost:

$$\sum_{j=1}^{3} n_j F_s(P_{sj})$$

Subject to equality constraints:
$$\begin{aligned} &P_{\text{load}\,j} - P_{sj} - P_{Hj} = 0 \quad \text{for } j = 1, 2, 3 \\ &V_0 + n_1 r_1 - n_1 q(P_{H2}) = V_1 \\ &V_1 + n_2 r_2 - n_2 q(P_{H2}) = V_2 \\ &V_2 + n_3 r_3 - n_3 q(P_{H3}) = V_3 \end{aligned}$$

And subject to inequality constraints:
$$\begin{aligned} &V_j > V^{\min} \quad V_j < V^{\max} \\ &P_{sj} > P_s^{\min} \quad P_{sj} < P_s^{\max} \quad \text{for } j = 1, 2, 3 \\ &P_{Hj} = P_H^{\min} \quad P_{Hj} < P_H^{\max} \end{aligned}$$

We can now write a Lagrange equation to solve this problem:

$$\begin{aligned} \mathcal{L} = {} & \sum_{j=1}^{3} n_j F_s(P_{sj}) + \sum_{j=1}^{3} \lambda_j (P_{\text{load}\,j} - P_{sj} - P_{Hj}) \\ & + \gamma_1[-V_0 - n_1 r_1 + n_1 q(P_{H1}) + V_1] \\ & + \gamma_2[-V_1 - n_2 r_2 + n_2 q(P_{H2}) + V_2] \\ & + \gamma_3[-V_2 - n_3 r_3 + n_3 q(P_{H3}) + V_3] \\ & + \sum_{j=1}^{3} \alpha_j^- (V^{\min} - V_j) + \sum_{j=1}^{3} \alpha_j^+ (V_j - V^{\max}) \\ & + \sum_{j=1}^{3} \mu_{sj}^- (P_s^{\min} - P_{sj}) + \sum_{j=1}^{3} \mu_{sj}^+ (P_{sj} - P_2^{\max}) \\ & + \sum_{j=1}^{3} \mu_{Hj}^- (P_H^{\min} - P_{Hj}) + \sum_{j=1}^{3} \mu_{Hj}^+ (P'_{Hj} - P_H^{\max}) \end{aligned} \tag{7A.7}$$

where

n_j, P_{sj}, P_{Hj}, and $q(P_{Hj})$ are as defined in Section 7.4

λ_j, γ_j, α_j^-, α_j^+, μ_{sj}^-, μ_{sj}^+, $\mu_{\mathrm{H}j}^-$, $\mu_{\mathrm{H}j}^+$ are Lagrange multipliers

$V^{\min}$ and $V^{\max}$ are limits on reservoir storage

$P_s^{\min}$, $P_s^{\max}$, and $P_H^{\min}$ are limits on the generator output at the equivalent system and hydroplants, respectively

We can set up the conditions for an optimum using the Kuhn–Tucker equations as shown in Appendix 3A. The first set of conditions are

$$\frac{\partial \mathcal{L}}{\partial P_{sj}} = n_j \frac{\mathrm{d}F_s}{\mathrm{d}P_{sj}} - \lambda_j - \mu_{sj}^- + \mu_{sj}^+ = 0 \tag{7A.8}$$

$$\frac{\partial \mathcal{L}}{\partial P_{Hj}} = -\lambda_j + \gamma_j n_j \frac{\mathrm{d}q(P_{Hj})}{\mathrm{d}P_{Hj}} - \mu_{Hj}^- + \mu_{Hj}^+ = 0 \tag{7A.9}$$

$$\frac{\partial \mathcal{L}}{\partial V_j} = \gamma_j - \gamma_{j+1} - \alpha_j^- + \alpha_j^+ = 0 \tag{7A.10}$$

The second and third set of conditions are just the original equality and inequality constraints. The fourth set of conditions are

$$\alpha_j^-(V^{\min} - V_j) = 0 \qquad \alpha_j^- \geq 0 \tag{7A.11}$$

$$\alpha_j^+(V_j - V^{\max}) = 0 \qquad \alpha_j^+ \geq 0 \tag{7A.12}$$

$$\mu_{sj}^-(P_s^{\min} - P_{sj}) = 0 \qquad \mu_{sj}^- \geq 0 \tag{7A.13}$$

$$\mu_{sj}^+(P_{sj} - P_s^{\max}) = 0 \qquad \mu_{sj}^+ \geq 0 \tag{7A.14}$$

$$\mu_{Hj}^-(P_H^{\min} - P_{Hj}) = 0 \qquad \mu_{Hj}^- \geq 0 \tag{7A.15}$$

$$\mu_{Hj}^+(P_{Hj} - P_H^{\max}) = 0 \qquad \mu_{Hj}^+ \geq 0 \tag{7A.16}$$

If we assume that no generation limits are being hit, then μ_{sj}^-, μ_{sj}^+, μ_{Hj}^-, and μ_{Hj}^+ for $j = 1, 2, 3$ are each equal to zero. The solution in Eqs. 7A.8, 7A.9, and 7A.10 is

$$n_j \frac{dF_s}{dP_{sj}} = \lambda_j \tag{7A.17}$$

$$\gamma_j n_j \frac{dq(P_{Hj})}{dP_{Hj}} = \lambda_j \tag{7A.18}$$

$$\gamma_j - \gamma_{j+1} = \alpha_j^- - \alpha_j^+ \tag{7A.19}$$

Now suppose the following volume-limiting solution exists:

$$V_1 > V^{\min} \quad \text{and} \quad V_1 < V^{\max}$$

then by Eq. 7A.11 and Eq. 7A.12

$$\alpha_1^- = 0 \quad \text{and} \quad \alpha_1^+ = 0$$

and

$$V_2 = V^{\min} \quad \text{and} \quad V_2 < V^{\max}$$

then

$$\alpha_2^- > 0 \qquad \alpha_2^+ = 0$$

Then clearly, from Eq. 7A.19,

$$\gamma_1 - \gamma_2 = \alpha_1^- - \alpha_1^+ = 0$$

so

$$\gamma_1 = \gamma_2$$

and

$$\gamma_2 - \gamma_3 = \alpha_2^- - \alpha_2^+ > 0$$

so

$$\gamma_2 > \gamma_3$$

Thus, we see that γ_j will be constant over time unless a storage volume limit is hit. Further, note that this is true regardless of whether or not generator limits are hit.

PROBLEMS

7.1 Given the following steam-plant and hydroplant characteristics:

Steam plant:

Incremental cost $= 2.0 + 0.002P_s$ R/MWh and $100 \leq P_s \leq 500$ MW

Hydroplant:

Incremental water rate $= 50 + 0.02P_H$ ft^3/sec/MW $0 \leq P_H \leq 500$ MW

Load:

Time Period	P_{load} (MW)
1400-0900	350
0900-1800	700
1800-2400	350

Assume:

- The water input for $P_H = 0$ may also be assumed to be zero, that is

$$q(P_H) = 0 \quad \text{for } P_H = 0$$

- Neglect losses.
- The thermal plant remains on-line for the 24-h period.

Find the optimum schedule of P_s and P_H over the 24-h period that meets the restriction that the total water used is 1250 million ft^3 of water; that is,

$$q_{TOT} = 1.25 \times 10^9 \text{ ft}^3$$

7.2 Assume that the incremental water rate in Problem 7.1 is constant at 60 $\text{ft}^3/\text{sec/MW}$ and that the steam unit is not necessarily on all the time. Further, assume that the thermal cost is

$$F(P_s) = 250 + 2P_s + P_s^2/1000$$

Repeat Problem 7.1 with the same water constraint.

7.3 Gradient Method for Hydrothermal Scheduling

A thermal-generation system has a composite fuel cost characteristic that may be approximated by

$$F = 700 + 4.8P_s + P_s^2/2000, \text{ Ɍ/h}$$

for

$$200 \leq P_s \leq 1200 \text{ MW}$$

The system load may also be supplied by a hydro-unit with the following characteristics:

$$q(P_H) = 0 \quad \text{when } P_H = 0$$

$$q(P_H) = 260 + 10P_H, \text{ acre-ft/h} \quad \text{for } 0 < P_H \leq 200 \text{ MW}$$

$$q(P_H) = 2260 + 10(P_H - 200) + 0.028(P_H - 200)^2 \text{ acre/h} \quad \text{for } 200 < P_H \leq 250 \text{ MW}$$

The system load levels in chronological order are as follows:

Period	P_{load} (MW)
1	600
2	1000
3	900
4	500
5	400
6	500

Each period is 4 h long.

7.3.1 Assume the thermal unit is on-line all the time and find the optimum schedule (the values of P_s and P_H for each period) such that the hydroplant uses 23,500 acre-ft of water. There are no other hydraulic constraints or storage limits, and you may turn the hydro-unit off when it will help.

7.3.2 Now, still assuming the thermal unit is on-line each period, use a gradient method to find the optimum schedule given the following conditions on the hydroelectric plant.

a. There is a constant inflow into the storage reservoir of 1000 acre-ft/h.

b. The storage reservoir limits are

$$V_{\max} = 18{,}000 \text{ acre-ft}$$

and

$$V_{\min} = 6000 \text{ acre-ft}$$

c. The reservoir starts the day with a level of 10,000 acre-ft, and we wish to end the day with 10,500 acre-ft in storage.

7.4 Hydrothermal Scheduling using Dynamic Programming

Repeat Example 7E except the hydroelectric unit's water rate characteristic is now one that reflects a variable head. This characteristic also exhibits a maximum capability that is related to the net head. That is.

$$q = 0 \quad \text{for } P_H = 0$$

$$q = 260 + 10P_H\left(1.1 - \frac{\bar{V}}{100{,}000}\right) \quad \text{acre-ft/h}$$

for

$$0 < P_H \leq 2000\left(0.9 + \frac{\bar{V}}{100{,}000}\right) \text{MW}$$

where

$$\bar{V} = \text{average reservoir volume}$$

For this problem, assume constant rates during a period so that

$$\bar{V} = \tfrac{1}{2}(V_k + V_i)$$

where

V_k = end of period volume

V_i = start of period volume

The required data are

Fossil unit: On-line entire time

$$F = 770 + 5.28P_s + 0.55 \times 10^{-3}P_s^2 \text{ Ꝝ/h}$$

for

$$200 \leq P_s \leq 1200 \text{ MW}$$

Hydro-storage and inflow:

$$r = 1000 \text{ acre-ft/h inflow}$$

$$6000 \leq V \leq 18{,}000 \text{ acre-ft storage limits}$$

$$V = 10{,}000 \text{ acre-ft initially}$$

and

$$V = 10{,}000 \text{ acre-ft at end of period}$$

Load for 4-h periods:

J: Period	P_{load} (MW)
1	600
2	1000
3	900
4	500
5	400
6	300

Find the optimal schedule with storage volumes calculated at least to the nearest 500 acre-ft.

7.5 Pumped-Storage Plant Scheduling Problem

A thermal generation system has a composite fuel-cost characteristic as follows:

$$F = 250 + 1.5P_s + P_s^2/200 \text{ Ꝝ/h}$$

for

$$200 \leq P_s \leq 1200 \text{ MW}$$

In addition, it has a pumped-storage plant with the following characteristics:

1. Maximum output as a generator = 180 MW (the unit may generate between 0 and 180 MW).
2. Pumping load = 200 MW (the unit may only pump at loads of 100 or 200 MW).

3. The cycle efficiency is 70% (that is, for every 70 MWh generated, 100 MWh of pumping energy are required).
4. The reservoir storage capacity is equivalent to 1600 MWh of generation.

The system load level in chronological order is the same as that in Problem 7.3.

a. Assume the reservoir is full at the start of the day and must be full at the end of the day. Schedule the pumped-storage plant to minimize the thermal system costs.

b. Repeat the solution to part a, assuming that the storage capacity of the reservoir is unknown and that it should be at the same level at the end of the day. How large should it be for minimum thermal production cost?

Note: In solving these problems you may assume that the pumped-storage plant may operate for partial time periods. That is, it does not have to stay at a constant output or pumping load for the entire 4-h load period.

7.6 The "Light Up Your Life Power Company" operates one hydro-unit and four thermal-generating units. The on/off schedule of all units, as well as the MW output of the units, is to be determined for the load schedule given below.

Thermal unit data (fuel cost = 1.0 $/MBTU):

Unit No.	Max (MW)	Min (MW)	Incremental Heat Rate (Btu/kWh)	No-load Energy Input (MBtu/hr)	Start Up (MBtu)	Min Up Time (h)	Min Down Time (h)
1	500	70	9950	300	800	4	4
2	250	40	10200	210	380	4	4
3	150	30	11000	120	110	4	8
4	150	30	11000	120	110	4	16

Hydroplant data:

$$Q(P_h) = 1000 + 25P_h \text{ acre-ft/h}$$

where

$$0 < P_h < 200 \text{ MW}$$

min up and down time for the hydroplant is 1 h.

Load data (each time period is 4 h):

Time Period	P_{load} (MW)
1	600
2	800
3	700
4	1150

The starting conditions are: units 1 and 2 are running and have been up for 4 h, units 3, 4, and the hydro-unit are down and have been for 16 h.

Find the schedule of the four thermal units and the hydro-unit that minimizes thermal production cost if the hydro-units starts with a full reservoir and must use 24,000 acre-ft of water over the 16-h period.

7.7 The "Lost Valley Paper Company" of northern Maine operates a very large paper plant and adjoining facilities. All of the power supplied to the paper plant must come from its own hydroplant and a group of thermal-generation facilities that we shall lump into one equivalent generating plant. The operation of the hydro-facility is tightly governed by the Maine Department of Natural Resources.

Hydroplant data:

$$Q(P_h) = 250 + 25P_h \text{ acre-ft/h}$$

and

$$0 < P_h < 500 \text{ MW}$$

Equivalent steam-plant data:

$$F(P_s) = 600 + 5P_s + 0.005P_s^2 \text{ \$/h}$$

and

$$100 < P_s < 1000 \text{ MW}$$

Load data (each period is 4 h):

Time Period	P_{load} (MW)
1	800
2	1000
3	500

The Maine Department of Natural Resources had stated that for the 12-h period above, the hydroplant starts at a full reservoir containing 20,000 acre-ft of water and ends with a reservoir that is empty. Assume that there is no inflow to the reservoir and that both units are on-line for the entire 12 h.

Find the optimum schedule for the hydroplant using dynamic programming. Use only three volume states for this schedule: 0, 10,000, and 20,000 acre-ft.

FURTHER READING

The literature relating to hydrothermal scheduling is extensive. For the reader desiring a more complete guide to these references, we suggest starting with reference 1, which is a bibliography covering 1959 through 1972, prepared by a working group of the Power Engineering Society of IEEE.

References 2 and 3 contain examples of simulation methods applied to the scheduling of large hydroelectric systems. The five-part series of papers by Bernholz and Graham (reference 4) presents a fairly comprehensive package of techniques for optimization of short-range hydrothermal schedules applied to the Ontario Hydro system. Reference 5 is an example of optimal scheduling on the Susquehanna River.

A theoretical development of the hydrothermal scheduling equations is contained in reference 6. This 1964 reference should be reviewed by any reader contemplating undertaking a research project in hydrothermal scheduling methods. It points out clearly the impact of the constraints and their effects on the pseudomarginal value of hydroelectric energy.

Reference 7 illustrates an application of gradient-search methods to the coupled plants in the Ontario system. Reference 8 illustrates the application of dynamic-programming techniques to this type of hydrothermal system in a tutorial fashion. References 9 and 10 contain examples of methods for scheduling pumped-storage hydroelectric plants in a predominantly thermal system. References 11–16 show many recent scheduling techniques.

This short reference list is only a sample. The reader should be aware that a literature search in hydrothermal-scheduling methods is a major undertaking. We suggest the serious student of this topic start with reference 1 and its predecessors and successors.

1. "Description and Bibliography of Major Economy-Security Functions, Parts I, II, and III," IEEE Working Group Report, *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-100, January 1981, pp. 211–235.
2. Bruderell, R. N., Gilbreath, J. H., "Economic Complementary Operation of Hydro Storage and Steam Power in the Integrated TVA System," *AIEE Transactions*, Vol. 78, June 1959, pp. 136–150.
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    b. "Part II. Extension of Basic Theory," *AIEE Transactions*, Vol. 81, Part III, January 1962, pp. 1089–1096.

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8 Production Cost Models

8.1 INTRODUCTION

Production cost models are computational models designed to calculate a generation system's production costs, requirements for energy imports, availability of energy for sales to other systems, and fuel consumption. They are widely used throughout the electric utility industry as an aid in long-range system planning, in fuel budgeting, and in system operation. The primary function of computing future system energy costs is accomplished by using computer models of expected load patterns and simulating the operation of the generation system to meet these loads. Since generating units are not perfectly reliable and future load levels cannot be forecast with certainty, many production cost programs are based on probabilistic models and are used to compute the statistically expected need for emergency energy and capacity supplies or the need for controlled load demand reductions.

The digital simulation of the generation system involves representation of:

1. Generating unit efficiency characteristics (input–output curves, etc.).
2. Fuel costs per unit of energy supplied.
3. System operating policies with regard to scheduling of unit operation and the economic dispatching of groups of units that are on-line.
4. Contracts for the purchases and sales of both energy and power capability.

When hydroelectric plants are a part of the power system, the production cost simulation will involve models of the policies used to operate these plants. The first production cost models were deterministic, in that the status of all units and energy resources was assumed to be known and the load is a single estimate.

Production cost programs involve modeling all of the generation characteristics and many of the controls discussed previously, including fuel costs and supply, economic dispatch, unit commitment and hydrothermal coordination. They also involve modeling the effects of transactions, a subject to be considered in a later chapter. Deterministic programs incorporate the generation scheduling techniques in some sort of simulation model. In the most detailed of these, the on-line unit commitment program might be used in an off-line study mode. These are used in studying issues that are related to system operations such as purchase and sale decisions, transmission access issues and near-term decisions regarding operator-controlled demand management.

Stochastic production cost models are usually used for longer-range studies that do not involve near-term operational considerations. In these problem areas, the risk of sudden, random, generating unit failures and random deviations of the load from the mean forecast are considered as probability distributions. This chapter describes the basic ideas used in the probabilistic production cost models.

It is not possible to delve into all the details involved in a typical modern computer program since these programs may be quite large, with tens of thousands of lines of code and thousands of items of data. Any such discussion would be almost instantly out of date since new problems keep arising. For example, the original purpose of these production cost programs was primarily computation of future system operating costs. In recent years, these models have been used to study such diverse areas as the possible effects of load management, the impact of fuel shortages, issues related to nonutility generation, and the reliability of future systems.

The "universal" block diagram in Figure 8.1 shows the organization of a "typical" energy production cost program. The computation simulates the system operation on a chronological basis with system data input being altered at the start of each interval. These programs must be able to recognize and take into account, in some fashion, the need for scheduled maintenance outages. Logic may be incorporated in this type of program to simulate the maintenance outage allocation procedure actually used, as well as to process maintenance schedules that are input to the program.

Expansion planning and fuel budgeting production cost programs require load models that cover weeks, months, and/or years. The expected load patterns may be modeled by the use of typical, normalized hourly load curves for the various types of days expected in each subinterval (i.e., month or week) or else by the use of load duration or load distribution curves. Load models used in studying operational issues involve the next few hours, days or weeks and are usually chronological load cycles.

A *load duration curve* expresses the period of time (say number of hours) in a fixed interval (day, week, month, or year) that the load is expected to equal or exceed a given megawatt value. It is usually plotted with the load on the vertical axis and the time period on the horizontal axis.

The scheduling of unit maintenance outages may involve time intervals as short as a day or as long as a year. The requirements for economic data such as unit, plant, and system consumption and fuel costs, are usually on a monthly basis. When these time interval requirements conflict, as they often do, the load model must be created in the model for the smallest subinterval involved in the simulation.

Production cost programs may be found in many control centers as part of the overall "application program" structure. These production cost models are usually intended to produce shorter-term computations of production costs (i.e., a few hours to the entire week) in order to facilitate negotiations for energy (or power) interchange or to compute cost savings in order to allocate economic

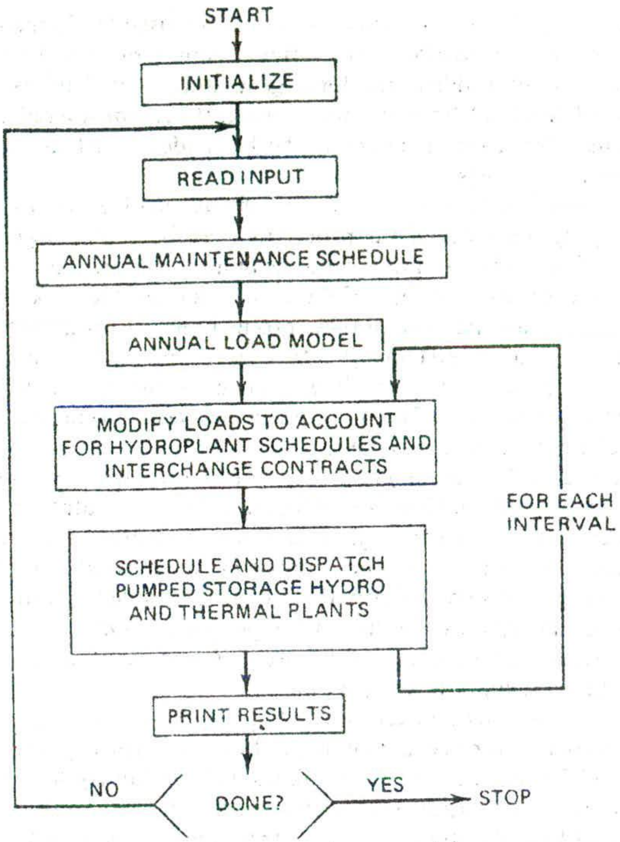

FIG. 8.1 Block diagram for a typical, single area energy production cost program used for planning.

benefits among pooled companies. In either application, the production cost simulation is used to evaluate costs under two or more assumptions. For example, in interchange negotiations, the system operators can evaluate the cost of producing the energy on the system versus the costs of purchasing it.

In U.S. power pools where units owned by several different utilities are dispatched by the control center, it is usually necessary to compute the production cost "savings" due to pooled operation. That is, each seller of energy is paid for the cost of producing the energy sold and may be given one-half the production cost "savings" of the system receiving the energy. One way of determining these savings is to simulate the production costs of each system supplying just its own load. In fact, in at least one U.S. pool this is called "own-load dispatch." These computed production costs can be compared with actual costs to arrive at the charges for transferring energy. The models used are deterministic and typically use the actual load patterns that occurred during the period under study. Scheduling computations frequently are performed with models that are similar to those used for real-time operational control.

Production cost computations are also needed in fuel budgeting. This involves making computations to forecast the needs for future fuel supplies at specific plant sites. Arrangements for fuel supplies vary greatly among utilities. In some instances, the utility may control the mining of coal or the production and transportation of natural gas; in others it may contract for fuel to be delivered to the plant. In many cases, the utility will have made a long-term arrangement with a fuel supplier for the fuel needed for a specific plant. (Examples are "mine-mouth" coal plants or nuclear units.) In still other cases, the utility may have to obtain fuel supplies on the open (i.e., "spot") market at whatever prices are prevailing at that time. In any case, it is necessary to make a computation of the expected fuel supply requirements so that proper arrangements can be made sufficiently in advance of the requirements. This requires a forecast of specific quantities (and large quantities) of fuel at given future dates.

Fuel budgeting models are usually very detailed. Deterministic or probabilistic production cost simulations may be used for this application. In some cases, where the emphasis is on the scheduling of fuel resources, transportation and fuel storage, the production cost computations might be one part of a large linear programming model. In these cases, the loads might be modeled by the expected energy demand in a day, week, month or season. Scheduling of generation would be done using a linear model of the input–output characteristics.

The operating center production cost needs may have a 7-day time horizon. The fuel budgeting time span may encompass 1 to 5 years and might, in the case of the mine-mouth plant studies, extend out to the expected life of the plant. System expansion studies usually encompass a minimum of 10 years and in many cases extend to 30 years into the future. It is this difference in time horizon that makes different models and approaches suitable for different problems.

8.2 USES AND TYPES OF PRODUCTION COST PROGRAMS

Table 8.1 lists the major features that may vary from program to program and indicates, along the horizontal axis, the major program uses of:

1. Long-range planning.
2. Fuel budgeting.
3. Operations planning.
4. Weekly schedules.
5. Allocation of pool savings.

TABLE 8.1 Energy Production Cost Programs

Load Model	Interval Considered	Economic Dispatch Procedure for Thermal Units	Long-Range Planning	Fuel Budgeting	Operations Planning	Weekly Schedules	Allocation of "Pool Savings"
Total energy or load duration	Seasons or years	Block loading[a]	×				
Load duration or load cycles	Months or weeks	Incremental loading	×	×	×		
Load duration or load cycles	Months, weeks or days	Incremental loading with forced outages considered	×	×	×		
Load cycle	Weeks or days	Incremental loading (losses)	×	×	×	×	×

[a] The term "block loading" refers to the scheduling of complete units in economic order without regard to incremental cost. The procedure is illustrated in this section.

Also indicated are the types of programs that have been found useful, so far, in each application. The type of load model used will determine, in part, the suitability of each program type for a given application.

The types of production cost programs shown in Table 8.1, which utilize chronological load patterns (i.e., load cycles) and deterministic scheduling methods, are computer implementations of the economic dispatching techniques and unit commitment methods explored previously. That is, production costs and fuel consumption are computed repetitively, assuming that the load cycles are known for an extended period into the future and that the availability of every unit can be predicted with 100% certainty for each subinterval of that future period.

In models using probabilistic representations of the future loads and generating unit availabilities, the expected values of production costs and fuel consumption are computed without the assumption of a perfectly known future.

There are other types of production cost programs that are known by various names. Some include different ways of categorizing the program, models, or computational methods that are used. For example there are "Monte Carlo," probabilistic simulations that are detailed, deterministic programs with the added feature that unit outages and deviations of loads from those forecast are incorporated by the use of synthetic sampling techniques. Random numbers are generated at regular time intervals and used to develop sample results from the appropriate probability distributions. These numbers determine the status of a unit; operating at full capability, on forced outage, or coming back into a state where it is available, if it was previously unavailable. The magnitude of the load deviation from the magnitude forecast may also be determined by a random number using a "forecast error" probability density. Other programs might combine some of the approximate generation scheduling techniques with load models that separate the week into weekdays and weekend days and consider only 4 wks per year, one for each season. In these (so-called "quick-and-dirty") models, the weekly cost and fuel consumption are multiplied by appropriate scaling factors to compute total seasonal values. On the other end of the complexity scale, there are programs which consider the dispatch of several interconnected areas and utilize power flow constraints caused by the transmission interconnections to restrict interarea interchange levels. Optimal power flow programs could be used in the same fashion.

So far, networks have only been represented in production cost programs by simplified models, such as using penalty factors, using a DC power flow (or equivalent distribution factors based on a DC model) or using a transportation network. AC power flows are useful for security-constrained economic dispatch, unit commitment and purchase–sale analyses. Optimal power flows may be used to study transmission power and VAR flow patterns to develop prices for the use of transmission systems.

In the complex, deterministic programs, the loads may be represented by chronologically arranged load cycle patterns. These patterns consist of hourly (or bi-hourly) loads that might be calculated using typical, daily load cycle patterns for workdays, weekend days and holidays throughout the period. The development of these typical patterns from historical data is an art; using them to develop forecasts of future load cycles is straightforward once the overall load forecast is developed. The earlier load models were load-duration curves and we shall utilize them to explore the various techniques.

8.2.1 Production Costing Using Load-Duration Curves

In representing future loads, sometimes it is satisfactory to specify only the total energy generation for a period. This is satisfactory if only total fuel consumption and production costs are of interest and neither capacity limitations nor chronological effects are important.

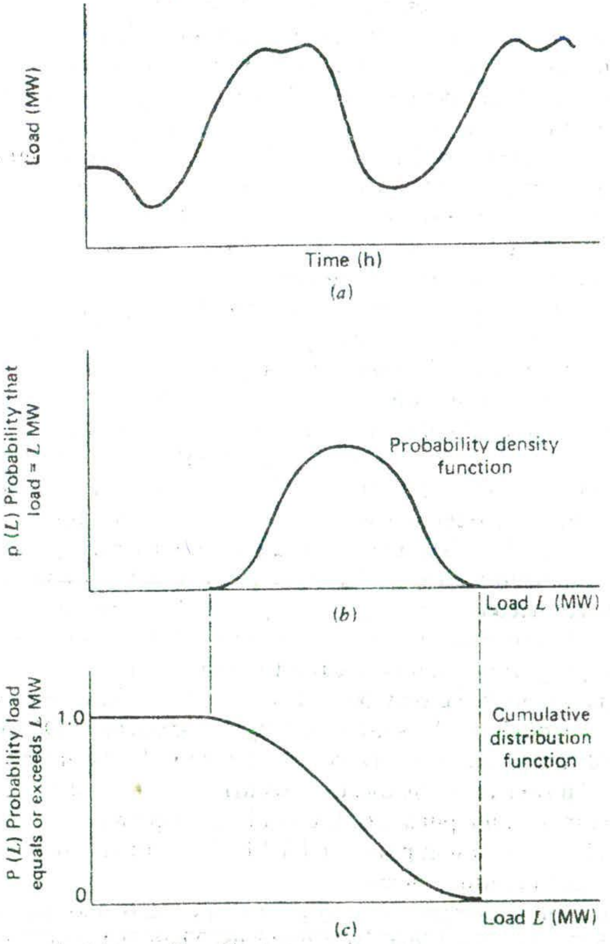

FIG. 8.2 Load probability functions.

Where capacity limitations are of more concern, a *load-duration curve* might be used. Figure 8.2 shows an expected load pattern in (a), a histogram of load for a given time period in (b), and the load-duration curve constructed from it in (c). In practical developments, the density and distribution functions may be developed as histograms where each load level, L, denotes a range of loads. These last two curves are expressed in both hours and per unit probability versus the megawatts of load. Figure 8.3 shows the more conventional representation of a load-duration curve where the probability has been multiplied by the period length to show the number of hours that the load equals, or exceeds, a given level, L (MW). It is conventional in deterministic production cost analyses to show this curve with the load on the vertical axis. In the probabilistic calculations, the form shown on Figure 8.2c is used.

In the simulation of the economic dispatch procedures with this type of load model, thermal units may be *block-loaded*. This means the units (or major segments of a unit) on the system are ordered in some fashion (usually cost) and are assumed to be fully loaded, or loaded up to the limitations of the load-duration curve. Figure 8.4 shows this procedure for a system where the internal peak load is 1700 MW. The units are considered to be loaded in a sequence determined by their average cost at full load in R/MWh. The amount of energy generated by each unit is equal to the area under the load-duration curve between the load levels in megawatts supplied by each unit.

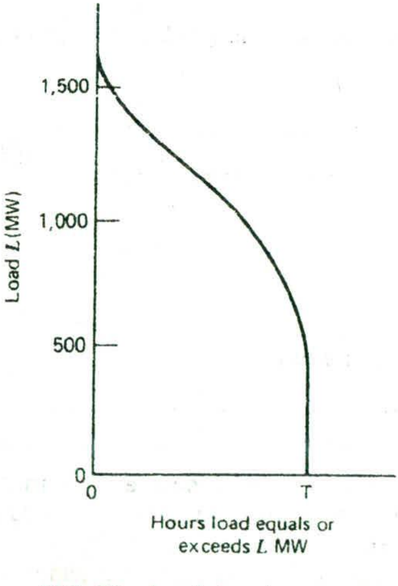

FIG. 8.3 Load-duration curve.

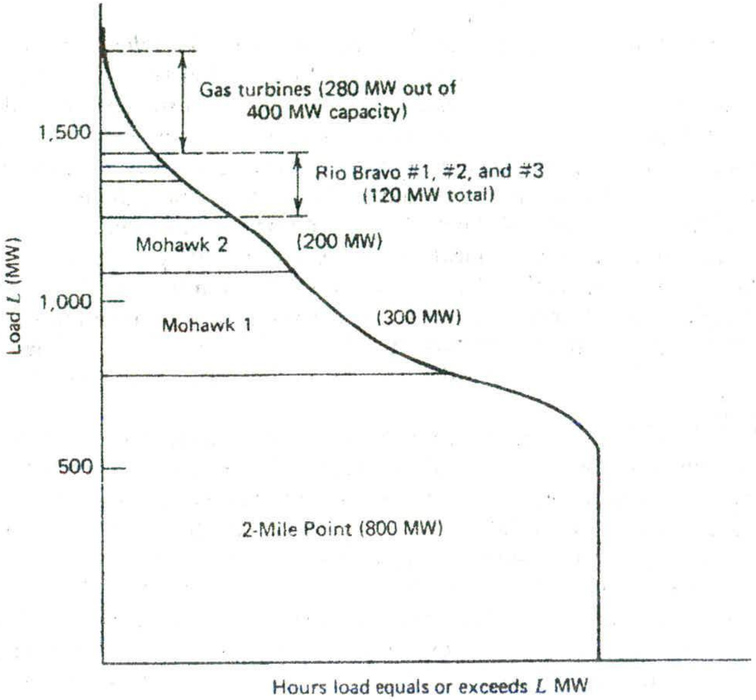

FIG. 8.4 Block-loaded units.

This system consists of three plants plus an array of gas-turbine generating units. These are:

Unit	Maximum capability (MW)
2-Mile Point	800
Mohawk 1	300
Mohawk 2	200
Rio Bravo 1	75
Rio Bravo 2	25
Rio Bravo 3	20
Eight gas turbines (each 50 MW)	400
	Total 1820

Note that in this system, the gas turbines are not used appreciably since the peak load is only 1700 MW and each unit is assumed to be available all the time during the interval.

Besides representing the thermal generating plants, the various production cost programs must also simulate the effects of hydroelectric plants with and without water storage, contracts for energy and capacity purchases and sales, and pumped-storage hydroelectric plants. The action of all these results in a modified load to be served by the array of thermal units. The scheduling of the thermal plants should be simulated to consider the security practices and policies of the power system as well as to simulate, to some appropriate degree, the economic dispatch procedures used on the system to control the unit output levels.

More complex production cost programs used to cover shorter time periods may duplicate the logic and procedures used in the control of the units. The most complex involve the procedures discussed in the previous three chapters on unit commitment and hydrothermal scheduling. These programs will usually use hourly forecasts of energy (i.e., the "hourly, integrated load" forecast) and thermal-generating-unit models that include incremental cost functions, start-up costs, and various other operating constraints.

EXAMPLE 8A

Let us consider the load-duration curve technique for a system of two units. Initially, the random forced outages of the generating units will be neglected. Then, we will incorporate consideration of these outages in order to show their effects on production costs and the ability of the small sample system to serve the load pattern expected. The load consists of the following:

x-Load (MW)	Duration (h)	Energy (MWh)
100	20	2000
80	60	4800
40	20	800
	Total = 100	7600

From these data, we may construct a load-duration curve in tabular and graphic form. The load-duration curve shows the number of hours that the load equals or exceeds a given value.

x-Load (MW)	Exact Duration, $Tp(x)$	$TP_n(k)$ Hours that Load Equals or Exceeds x
0	0	100
20	0	100
40	20	100
60	0	80
80	60	80
100	20	20
100+		0

In this table, $p(x)$ is the load density function: the probability that the load is exactly x MW and $P_n(x)$ is the load distribution function; the probability that the load is equal to, or exceeds, x MW.

The table has been created for uniform load-level steps of 20 MW each. The table also introduces the notation that is useful in regarding the load-duration curve as a form of probability distribution. The load density and distribution functions, $p(x)$ and $P_n(x)$, respectively, are probabilities. Thus, $p(20) = 0$, $p(40) = 20/100 = 0.2$, $p(60) = 0$, and so forth, and $P_n(20) = P_n(40) = 1.0$, $P_n(60) = 0.8$, and so forth. The distribution function, $P_n(x)$, and the density, $p(x)$, are related as follows.

$$P_n(x) = 1 - \int_x^{\infty} p(x)\,dx \tag{8.1}$$

For discrete-density functions (or histograms) in tabular form, it is easiest to construct the distribution by cumulating the probability densities from the highest to the lowest values of the argument (the load levels).

The load-duration curve is shown in Figure 8.5 in a way that is convenient to use for the development of the probabilistic scheduling methods.

The two units of the generating system have the following characteristics.

Unit	Power Output (MW)	Fuel Input (10^6 Btu/h)	Fuel Cost (R/10^6 Btu)	Fuel Cost Rate (R/h)	Incremental Fuel Cost (R/MWh)	Unit Forced Outage Rate (per unit)
1	0	160	1	160	—	
	80	800	1	800	8	0.05
2	0	80	2	160	—	
	40	400	2	800	16	0.10

In this table the fuel cost rate for each unit is a linear function of the power output, P. That is,

$$F(P) = \text{fuel cost at zero output} + \text{incremental cost rate} \times P.$$

In addition to the usual input–output characteristics, forced outage rates are assumed. This rate represents the fraction of time that the unit is not available, due to a failure of some sort, out of the total time that the unit should be available for service. In computing forced outage rates, periods where a unit is on scheduled outage for maintenance are excluded. The unit forced outage rates are initially neglected, and the two units are assumed to be available 100% of the time.

Units are "block-loaded," with unit 1 being used first because of its lower average cost per MWh. The load-duration curve itself may be used to visualize the unit loadings. Figure 8.6 shows the two units block-loaded.

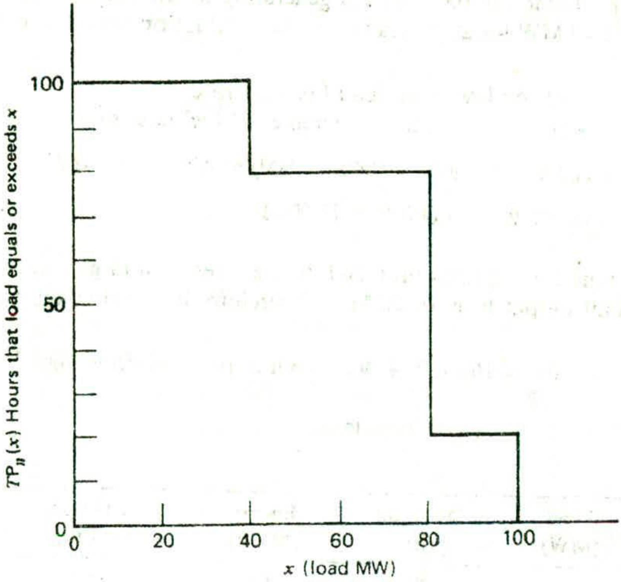

FIG. 8.5 Load-duration curve for Example 8A.

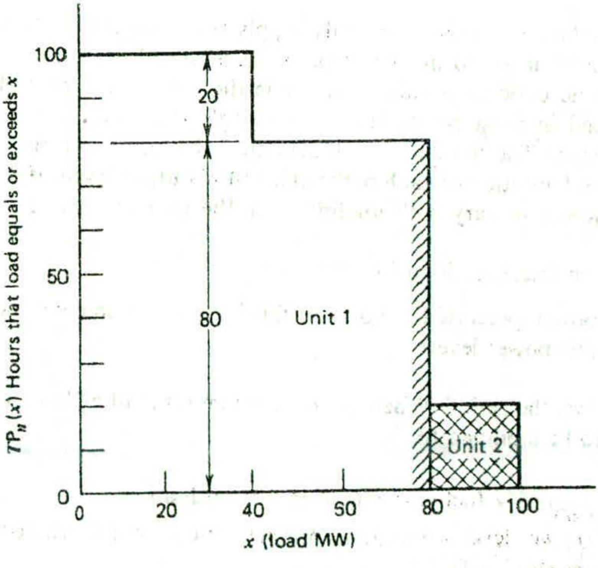

FIG. 8.6 Load-duration curve with block-loaded units.

Unit 1 is on-line for 100 h and is generating at an output level of 80 MW for 80 h and 40 MW for 20 h. Therefore, the production costs for unit 1 for this period:

= hours on line × no load fuel cost rate
+ energy generated × incremental fuel cost rate

= 100 h × 160 Ɍ/h + (6400 + 800) MWh × 8 Ɍ/MWh

= 16,000 Ɍ + 57,600 Ɍ = 73,600 Ɍ

Similarly, unit 2 is required only 20 h in the interval and generates 400 MWh at a constant output level of 20 MW. Therefore, its production costs for this period:

= 20 h × 160 Ɍ/h + 400 MWh × 16 Ɍ/MWh = 9600 Ɍ

These data are summarized as follows.

Unit	Load (MW)	Duration (h)	Energy (MWh)	Fuel Used (10^6 Btu)	Fuel Cost (Ɍ)
1	40	20	800	9600	9600
	80	80	6400	64000	64000
			7200	73600	73600
2	20	20	400	4800	9600
			7600	78400	83200

Note that these two units can easily supply the expected loads. If a third unit were available it would not be used, except as standby reserve.

This same basic approach to compute the production cost of a particular unit is used in most production cost models that represent individual unit characteristics. The simulation will determine the hours that the unit is on-line and the total duration or each of the unit's MW output levels. If the incremental cost is allowed to vary with loading level, the unit cost can be calculated as:

= hours on line × no load fuel cost
+ $\sum$ (power generated × hours at this level × incremental fuel cost rate at this power level)

summed over the period. When nonzero, minimum loading levels are considered, this has to be modified to:

= hours on-line × fuel cost rate at minimum load
+ $\sum$ [(power level − minimum power) × incremental fuel cost rate × hours at this level]

It gets more involved when continuous functions (polynomials, for example) are used to model input–output cost curves.

8.2.2 Outages Considered

Next, let us consider the effects of the random forced outages of these units and compute the expected production costs. This is a situation that contains relatively few possible events so that the expected operation of each unit may be determined by enumeration of all the possible outcomes. For this procedure, it is easiest at this point to utilize the load density rather than the load-distribution function.

EXAMPLE 8B

Load level by load level, the operation and generation of the two units are as follows.

1. Load = 40 MW; duration 20 h

Unit 1:	On-line	20 h
	Operates	0.95 × 20 = 19 h
	Output	40 MW
	Energy	19 × 40 = 760 MWh
Unit 2:	On-line	1 h
	Operates	0.9 × 1 = 0.9 h
	Output	40 MW
	Energy	0.9 × 40 = 36 MWh

Load energy = 800 MWh
Generation = 796 MWh
Unserved energy = 4 MWh
Shortages 40 MW for 0.1 h

2. Load = 80 MW; duration 60 h

Unit 1:	On-line	60 h
	Operates	0.95 × 60 = 57 h
	Output	80 MW
	Energy	57 × 80 = 4560 MWh
Unit 2:	On-line	60 h total
	Operates	0.9 × 3 = 2.7 h
	Output	40 MW
	Energy	2.7 × 40 = 108 MWh

Load energy = 4800 MWh
Generation = 4668 MWh
Unserved energy = 132 MWh
Shortages 80 MW for 0.3 h = 24 MWh
40 MW for 2.7 h = 108 MWh
132 MWh

3. Load = 100 MW; duration 20 h

Unit 1:
On-line 20 h
Operates $0.95 \times 20 = 19$ h
Output 80 MW
Energy $19 \times 80 = 1520$ MWh

Unit 2:
On-line 20 h
Operates as follows:

a. Unit 1 on-line and operating 19 h
Unit 2 operates $0.9 \times 19 = 17.1$ h
Output 20 MW
Energy $17.1 \times 20 = 342$ MWh
Shortage 20 MW for 1.9 h

b. Unit 1 supposedly on-line, but not operating 1 h
Unit 2 operates $0.9 \times 1 = 0.9$ h,
Output 40 MW
Energy $0.9 \times 40 = 36$ MWh
Shortages 100 MW for 0.1 h
60 MW for 0.9 h
Load energy = 2000 MWh
Generation = 1898 MWh
Unserved energy = 102 MWh
Shortages 100 MW for 0.1 h = 10 MWh
60 MW for 0.9 h = 54 MWh
20 MW for 1.9 h = 38 MWh
102 MWh

Because this example is so small, it has been necessary to make an arbitrary assumption concerning the commitment of the second unit. The assumption made is that the second unit will be on-line for any load level that equals or exceeds the capacity of the first unit. Thus, the second unit is on-line for the 60-h duration of the 80 MW load. This assumption agrees with the algorithm developed later in the chapter.

The enumeration of the possible states is not quite complete. We have accounted for the periods when the load is satisfied and the times when there will be a real shortage of capacity. In addition, we need to separate the periods when the load is satisfied into periods where there is excess capability (more generation than load) and periods when the available capacity exactly matches the load (generation equals load). The latter periods are called *zero MW shortage* because there is no reserve capacity in that period. This information is needed in case an additional unit becomes available or emergency capacity needs to be purchased. This additional capacity would need to be operated during the entire period of a zero MW shortage because the occurrence of a real shortage is a random event depending on the failure of an operating generator.

For this example there are two such periods, one during the 40-MW load period and the other during the 80-MW load period. That is, the additional "zero MW shortage" conditions occur during those periods when the load is supplied precisely with no additional available capacity. Therefore, to the shortage events presented previously, we add the following.

Load (MW)	Duration (h)	Unit 1	Unit 2	Zero Reserve Expected Duration
1. 40	20	Out	In	0.05 × 0.9 × 20 = 0.9
2. 80	60	In	Out	0.95 × 0.1 × 60 = 5.7
				6.6 h

These "zero MW shortage" events are of significance, since their total expected duration determines the number of hours that any additional units will be required.

All these events may be presented in an orderly fashion. Since each unit may be either on or off and there are three loads, the total number of possible events is 3 × 2 × 2 = 12. These are summarized along with the consequence of each event in Table 8.2.

Now, having enumerated all the possible operating events, it is possible to compute the expected production costs and shortages. Recall from Example 8A that the operating cost characteristics of the two units are

$$F_1 = 160 + 8P_1, \text{ Ɽ/h}$$

and

$$F_2 = 160 + 16P_2, \text{ Ɽ/h}$$

and the fuel costs are 1 and 2 Ɽ/10^6 Btu, respectively. The calculated operating costs considering forced ourages are computed using the data from Table 8.2.

TABLE 8.2 Summary of All Possible States

			Unit 1		Unit 2		Combined Event	
Load (MW)	Duration (h)	Event No.	Status[a]	Power (MW)	Status[a]	Power (MW)	Duration (h)	Consequence
40	20	1	1	40	1	0	17.1	Load satisfied; unit 2 not required
		2	1	40	0	0	1.9	Same as event no. 1
		3	0	0	1	40	0.9	Load satisfied; 0 MW shortage 0.9 h
		4	0	0	0	0	0.1	40 MW shortage 0.1 h
80	60	5	1	80	1	0	51.3	Load satisfied; unit 2 not required
		6	1	80	0	0	5.7	Load satisfied; 0 MW shortage 5.7 h
		7	0	0	1	40	2.7	40 MW shortage 2.7 h
		8	0	0	0	0	0.3	80 MW shortage 0.3 h
100	20	9	1	80	1	20	17.1	Load satisfied
		10	1	80	0	0	1.9	20 MW shortage 1.9 h
		11	0	0	1	40	0.9	60 MW shortage 0.9 h
		12	0	0	0	0	0.1	100 MW shortage 0.1 h

[a] 1 denotes available and 0 denotes unavailable.

These are:

Unit	Hours On-line	Total Expected Operating Hours	Expected Energy Generation (MWh)	Expected Fuel Use (10^6 Btu)	Expected Production Cost (Ɍ)
1	100	95.0	6840	69920	69920
2	81	72.9	522	10008	20016
Totals			7362	79928	89936

The expected energy generated by unit 1 is the summation over the load levels of the product of the probability that the unit is available, $p = 0.95$, times the load level in MW, times the hours duration of the load level. The expected production costs for unit 1

$$= 95 \text{ h} \times 160 \text{ Ɍ/h} + 6840 \text{ MWh} \times 8 \text{ Ɍ/MWh}$$

and for unit 2

$$= 72.9 \text{ h} \times 160 \text{ Ɍ/h} + 522 \text{ MWh} \times 16 \text{ Ɍ/MWh}$$

Compared to the results of Example 8A, the fuel consumption has increased 1.95% over that found neglecting random forced outages, and the total cost has increased 8.1%. This cost would be increased even more if the unserved energy, 238 MWh, were to be supplied by some high-cost emergency source.

The expected unserved demands and energy may be summarized from the preceding data as shown in Table 8.3. The last column is the distribution of the need for additional capacity, $TP_n(x)$, referred to previously, computed after the two units have been scheduled. Data such as these are computed in probabilistic production cost programs to provide probabilistic measures of the generation system adequacy (i.e., reliability). If costs are assigned to the unsupplied demand and energy (representing replacement costs for emergency purchases of capacity and energy or the economic loss to society as a whole),

TABLE 8.3 Unserved Load

Unserved Demand (MW)	Duration of Shortage (h)	Unserved Energy (MWh)	Duration of Given Shortages or More (h)
0	6.6	0	12.6
20	1.9	38	6.0
40	2.8	112	4.1
60	0.9	54	1.3
80	0.3	24	0.4
100	0.1	10	0.1
Totals	12.6	238	

these data will provide an additional economic measure of the generation system.

This relatively simple example leads to a lengthy series of computations. The results point out the importance of considering random forced outages of generating units when production costs are being computed for prolonged future periods. The small size of this example tends to magnify the expected unserved demand distribution. In order to supply, reliably, a peak demand of 100 MW with a small number of units, the total capacity would be somewhere in the neighborhood of 200 MW. On the other hand, the relatively low forced outage rates of the units used in Example 8B tend to minimize the effects of outages on fuel consumption. Large steam turbine generators of 600 MW capacity, or more, frequently exhibit forced outage rates in excess of 10%.

It should also be fairly obvious at this point that the process of enumerating each possible state in order to compute expected operation, energy generation, and unserved demands, cannot be carried much further without an organized and efficient scheduling method. For N_L load levels and N units, each of which may be on or off, there are $N_L \times 2^N$ possible events to enumerate. The next section will develop the types of procedures that are found in many probabilistic production cost programs.

8.3 PROBABILISTIC PRODUCTION COST PROGRAMS

Until the 1970s, production cost estimates were usually computed on the basis that the total generating capacity is always available, except for scheduled maintenance outages. Operating experience indicates that the forced outage rate of thermal-generating units tends to increase with the unit size. Power system energy production costs are adversely affected by this phenomenon. The frequent long-duration outages of the more efficient base-load units require running the less efficient, more expensive plants at higher than expected capacity factors* and the importation of emergency energy. Some utility systems report the operation of peaking units for more than 150 h each month, when these same units were originally justified under the assumption that they would be run over a few hours per month, if at all.

Two measures of system unreliability (i.e., generation system inadequacy to serve the expected demands) due to random, forced generator failures are:

---

* *Capacity factor* is defined as follows.

$$\frac{\text{MWh generated by the unit}}{\text{(Number of hours in the period of interest)(unit full-load MW capacity)}}$$

Thus, a higher value (close to unity) indicates that a unit was run most of the time at full load. A lower value indicates the unit was loaded below full capacity most of the time or was shut down part of the time.

1. The period of time when the load is greater than available generation capacity.
2. The expected levels of power and energy that must be imported to satisfy the load.

The maximum emergency import power and total energy imported are different dimensions of the same measure. These quantities and the expected shortage duration are useful as sensitive indicators of the need for additional capacity or interconnection capability. Some of these ideas are discussed further in the Appendix.

8.3.1 Probabilistic Production Cost Computations

Production cost programs that recognize unit forced outages and compute the statistically expected energy production cost have been developed and used widely. Mathematical methods based on probability methods make use of probabilistic models of both the load to be served and the energy and capacity resources. The models of the generation need to represent the unavailability of basic energy resources (i.e., hydro-availability), the random forced outages of units, and the effects of contracts for energy sales and/or purchases. The computation may also include the expected cost of emergency energy over the tie lines, which is sometimes referred to as the *cost of unsupplied energy*.

The basic difficulties that were noted when using deterministic approaches to the calculation of systems production cost were:

1. The base-load units of a system are loaded in the models for nearly 100% of an interval.
2. The midrange, or "cycling," units are loaded for periods that depend on their priority rank and the shape of the load-duration curve.
3. For any system with reasonably adequate reserve level, the peaking units have nearly zero capacity factors.

These conditions are, in fact, all violated to a greater or lesser extent whenever random-unit forced outages occur on a real system. The unavailability of thermal-generating units due to unexpected, randomly occurring outages is fairly high for large-sized units. Values of 10% are common for full forced outages. That is, for a full forced outage rate of q, per unit, the particular generating unit is completely unavailable for $100q\%$ of the time it is supposed to be available. Generating units also suffer partial outages where the units must be derated (i.e., run at less than full capacity) for some period of time, due to the forced outage of some system component (e.g., a boiler feed pump or a fan motor). These partial forced outages may reach very significant levels. It is not uncommon to see data reflecting a 25% forced reduction in maximum generating unit capability for 20% of the time it is supposed to be available.

Data on unit outage rates are collected and processed in the United States by the National Electric Reliability Council (NERC). The collection and processing of these data are important and difficult tasks. Performance data of this nature are essential if rational projections of component and system unavailability are to be made.

There are two techniques that have been used to handle the convolution of the load distributions with the capacity–probability density functions of the units: numerical convolutions where discrete values are used to model all of the distributions, and analytical methods that use continuous functional representations. Both techniques may be further divided into approaches that perform the convolutions in different orders. In what will be referred to here as the *unserved load distribution method*, the individual unit probability–capacity densities are convolved with the load distribution in a sequence determined by a fixed economic loading criterion to develop a series of *unserved load distributions*. Unit energy production is the difference between the unserved load energy before the unit is scheduled (i.e., convolved with the previous unserved load distribution) and after it has been scheduled. The load forecast is the initial unserved load distribution. In the *expected cost method*, the unit probability–capacity densities are first convolved with each other in sequence to develop distributions of available capacity and the expected cost curve as a function of the total power generated. This expected cost curve may then be used with the load distribution to produce the expected value of the production cost to serve the given load forecast distribution. We shall explore the numerical convolution techniques.

The analytical methods use orthogonal functions to represent both the load and capacity–probability densities of the units. These are the methods based on the use of *cumulants*. The merit of this analytical method is that it is usually a much more rapid computation. The drawback appears to be the concern over accuracy (as compared with numerical convolution results). The references at the end of this chapter provide a convenient starting point for a further exploration of this approach. The discussions of the numerical convolution techniques which follow should provide a sufficient basis for appreciating the approach, its utility, and its difficulties.

8.3.2 Simulating Economic Scheduling with the Unserved Load Method

In the developments that follow, it is assumed that data are available that describe generating units in the following format.

Maximum Power Output Available (MW)	Probability Unit Is Available to Load to this Power (per unit)	Cost of Generating Maximum Available (R/h)
$C(1) = 0$	$p(1)$	$F(1)$ = minimum cost
$C(2)$	$p(2)$	$F(2)$
$C(3)$	$p(3)$	$F(3)$
⋮	⋮	⋮
$C(n)$ = maximum	$p(n)$	$F(n)$

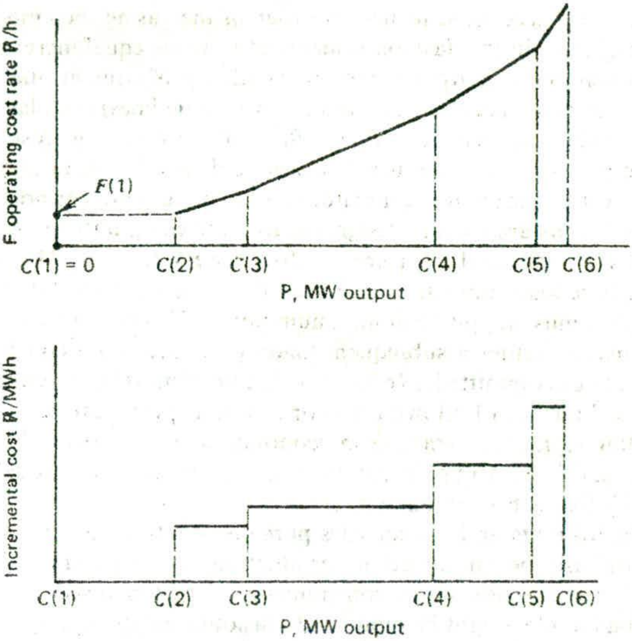

FIG. 8.7 Unit characteristics.

Pictorially, the unit characteristics needed are shown in Figure 8.7.

The probabilistic production cost procedure uses thermal-unit heat rate characteristics (i.e., heat input rate versus electric power output) that are linear segments. This type of heat rate characteristic is essential to the development of an efficient probabilistic computational algorithm since it results in stepped incremental cost curves. This simplies the economic scheduling algorithm since any segment is fully loaded before the next is required. These unit input–output characteristics may have any number of segments so that a unit may be represented with as much detail as is desired. Unit thermal data are converted to cost per hour using fuel costs and other operating costs, as is the case with any economic dispatching technique.

The probabilistic production cost model simulates economic loading procedures and constraints. Fuel budgeting and planning studies utilize suitable approximations in order to permit the probabilistic computation of expected future costs. For instance, unit commitment will usually be approximated using a priority order. The priority list might be computed on the basis of average cost per megawatt-hour at full load with units grouped in blocks by minimum downtime requirements. Within each block of units with similar downtimes, units could be ordered economically by average cost per megawatt-hour at full load.

With unit commitment order established, the various available loading segments can be placed in sequence, in order of increasing incremental costs. The loading of units in this fashion is identical to using equal incremental cost scheduling where input–output curves are made up of straight-line segments. Finally, emergency sources (i.e., tie lines or pseudo tie lines) are placed last on the loading order list. The essential difference between the results of the probabilistic procedure and the usual economic dispatch computations is that all the units will be required if generator forced outages are considered.

"Must-run" units are usually designated in these computations by requiring minimum downtimes equal to or greater than a week (i.e., 7 × 24 = 168 h), or more. These base-load units are committed first. After the must-run units are committed, they must supply their minimum power. The next lowest-cost block of capacity may be either a subsequent loading segment on a committed unit or a new unit to be committed. (Remember that units must be committed before they are loaded further.) Following this or a similar procedure results in a list of unit loading segments, arranged in economic loading order, which is then convenient and efficient to use in the probabilistic production cost calculations and to modify for each scheduling interval.

Storage hydro-units and system sales/purchase contracts for interconnected systems must also be simulated in production cost programs. The exact treatment of each depends on the constraints and costs involved. For example, a monthly load model might be modified to account for storage hydro by *peak shaving*. In the peak-shaving approach, the hydro-unit production is scheduled to serve the peak load levels, ignoring hydraulic constraints (but not the capacity limit) and assuming a single incremental cost curve for the thermal system for the entire scheduling interval. This can be done taking into account both hydro-unit forced outages and hydro-energy availability (i.e., amount of interval energy available versus the probability of its being available). System purchases and sales are often simulated as if they were stored energy systems. Sales (or purchases) from specific units are more difficult to model, and the modeling depends on the details of the contract. For instance, a "pure" unit transaction is made only when the unit is available. Other "less pure" contracts might be made where the transaction might still take place using energy produced by other units under specified conditions.

In the probabilistic production cost approach, the load is modeled in the same way as it was in the previously illustrated load-duration curve approach; as a probability distribution expressed in terms of hours that the load is expected to equal or exceed the value on the horizontal axis. This is a monotonically decreasing function with increasing load and could be converted to a "pure" probability distribution by dividing by the number of hours in the load interval being modeled. This model is illustrated in Figures 8.2, 8.3, 8.5, and 8.6. Therefore, each load-duration curve is treated either as a cumulative probability distribution,

$$P_n(x) \text{ versus } x$$

where P_n = probability of needing x MW, or more; or when expressed in hours, it is $TP_n(x)$, where T is the duration of the particular time interval. Also,

$$P_n(x) = 1 \quad \text{for } x \le 0$$

The load distribution is usually expressed in a table, $TP_n(x)$, which may be fairly short. The table needs to be only as long as the maximum load divided by the uniform MW interval size used in constructing the table. In applying this approach to a digital computer, it is both convenient and computationally efficient to think in terms of regular discrete steps and recursive algorithms. Various load-duration curves for the entire interval to be studied are arranged in the sequence to be used in the scheduling logic. There is no requirement that a single distribution $P_n(x)$ be used for all time periods. In developing the unit commitment schedule, it is necessary to verify not only that the maximum load plus spinning reserve is equal to or less than the sum of the capacities of the committed units, but also that the sum of the minimum loading levels of the committed units is not greater than the minimum load to be served.

A number of different descriptions have been used in the literature to explain this probabilistic procedure of thermal unit scheduling. The following has been found to be the easiest to grasp by someone unfamiliar with this procedure, and is theoretically sound. If there is a segment of capacity with a total of C MW available for scheduling, and if we denote:

q = the probability that C MW are unavailable (i.e., its unavailability)

and

p = 1 − q
= the probability or "availability" of this segment

then after this segment has been scheduled, the probability of needing x MW or more is now $P'_n(x)$. Since the occurrence of loads and unexpected unit outages are statistically independent events, the new probability distribution is a combination of mutually exclusive events with the same measure of need for additional capacity. That is,

$$P'_n(x) = qP_n(x) + pP_n(x + C) \tag{8.2}$$

In words, $qP_n(x)$ is the probability that new capacity C is unavailable times the probability of needing x, or more, MW, and $pP_n(x + C)$ is the probability C is available times the probability $(x + C)$, or more, is needed. These two terms represent two mutually exclusive events, each representing combined events where x MW, or more, remain to be served by the generation system.

This is a recursive computational algorithm, similar to the one used to develop the capacity outage distribution in the Appendix, and will be used in sequence to convolve each unit or loading segment with the distribution of load not served. It should be recognized that the argument of the probability distribution can be negative after load has been supplied and that $P_n(x)$ is zero for x greater than the peak load. Initially, when only the load distribution is used to develop $T\mathrm{P}_n(x)$, $\mathrm{P}_n(x) = 1$ for all $x \leq 0$.

Example 8B provides an introduction to the complexities involved in an enumerative approach to the problem at hand. By extending some of the ideas presented briefly in the Appendix to this chapter, a recursive technique (i.e., algorithm) may be developed to organize the probabilistic production cost calculations.

First, we note that the generation requirements for any generating segment are determined by the knowledge of the distribution $T\mathrm{P}_n(x)$ that exists prior to the dispatch (i.e., scheduling) of the particular generating segment. That is, the value of $T\mathrm{P}_n(0)$ determines the required hours of operation of a new unit. The area under the distribution $T\mathrm{P}_n(x)$ for x between zero and the rating of the unit loading segment determines the requirements for energy production. Assuming the particular generation segment being dispatched is not perfectly reliable (i.e., that it is unavailable for some fraction of the time it is required), there will be a residual distribution of demands that cannot be served by this particular segment because of its forced outage.

Let us represent the forced outage (i.e., unavailability) rate for a generation segment of C MW, and $T\mathrm{P}_n(x)$, the distribution of unserved load prior to scheduling the unit. Assume the unit segment to be scheduled is a complete generating unit with an input–output cost characteristic

$$F = F_0 + F_1 \mathrm{P}, \qquad \text{R/h}$$

for $0 \leq \mathrm{P} \leq C$ MW. The unit will be required $T\mathrm{P}_n(0)$ hours, but on average it will be available only $(1 - \mathrm{q})T\mathrm{P}_n(0)$ hours. The energy required by the load distribution that could be served by the unit is

$$E = T \int_{x=0}^{x=C} \mathrm{P}_n(x)\,\mathrm{d}x$$

or

$$= T \sum_{x=0+}^{x=C} \mathrm{P}_n(x)\,\Delta x$$

for discrete distributions. The unit can only generate $(1 - \mathrm{q})E$ because of its expected unavailability.

These data are sufficient to compute the expected production costs. These costs for this period

$$= F_0 \times (1 - \mathrm{q})T\mathrm{P}_n(0) + (1 - \mathrm{q})EF_1, \qquad \text{R}$$

Having scheduled the unit, there is a residual of unserved demands due to the forced outages of the unit. The recursive algorithm for the distribution of the probabilities of unserved load may be used to develop the new distribution of unserved load after the unit is scheduled. That is,

$$T\mathrm{P}'_n(x) = \mathrm{q}\,T\mathrm{P}_n(x) + (1 - \mathrm{q})T\mathrm{P}_n(x + C) \qquad (8.3)$$

The process may be repeated until all units have been scheduled and a residual distribution remains that gives the final distribution of unserved demand.

Refer to the unit data described in Figure 8.7 and the accompanying text. The minimum load cost, $F(1)$, shown on this figure is associated only with the first loading segment, $C(2)$ to $C(3)$, since the demands of this portion of the unit will determine the maximum hours of operation of the unit.

A general scheduling algorithm may be developed based on these conditions. In this development, we temporarily put aside until the next section some of the practical and theoretical problems associated with scheduling units with multiple steps and nonzero minimum load restrictions. The procedure shown in flowchart form on Figure 8.8 is a method for computing the expected production costs for a single time period, T hours in duration.

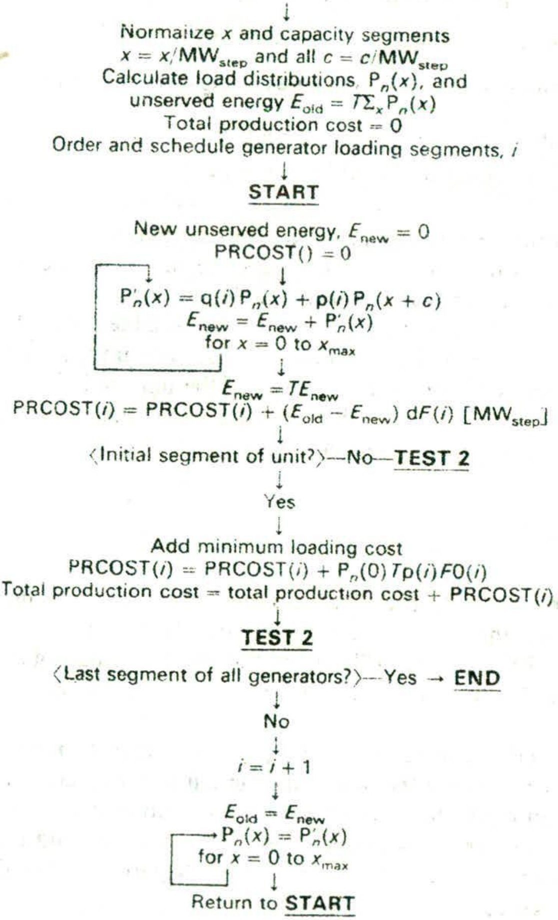

FIG. 8.8 Unserved load method for computing probabilistic production costs.

Besides the terms defined on Figure 8.7 we require the following nomenclature and definitions:

$i = 1, 2, \ldots, i_{\max}$, ordered capacity segments to be scheduled

$c(i) = C(i + 1) - C(i)$, capacity of the i^{th} segment (MW)

$dF(i) = \dfrac{[F(i + 1) - F(i)]}{c(i)}$ incremental cost rate for the i^{th} segment (℞/MWh)

$F0(i)$ = minimum load cost rate for i^{th} segment of unit (℞/h)

$p(i)$ = availability of segment i (per unit)

$q(i) = 1 - p(i)$, unavailability of segment i (per unit)

$x = 0, 1, 2, \ldots, x_{\max}$, equally spaced load levels

MW_{step} = uniform interval for representing load distribution (MW)

$PRCOST(i)$ = production costs for i^{th} segment (℞)

$E, E', E''' \ldots$ = remaining unserved load energy

In this algorithm, the energy generated by any particular loading segment of a generator is computed as the difference in unserved energy before and after the segment is scheduled. Since the incremental cost $[dF(i)]$ of any segment is constant, this is sufficient to determine the added costs due to loading of the unit above its minimum. For initial portions of a unit, $TP_n(0)$ determines the number of hours of operation required of the unit and is used to add the minimum load operating costs. We will illustrate the application of this procedure to the system described in Examples 8A and 8B.

EXAMPLE 8C

The computation of the expected production costs using the method shown in Figure 8.8 and the procedures involved can be illustrated with the data in Example 8A. Initially, we will ignore the forced outage of the two units and then follow this with an extension to incorporate the inclusion of forced outages.

With zero forced outage rates, the analysis of Example 8A is merely repeated in a different format where the load-duration curve is treated as a probability distribution. Figure 8.9 shows the initial load-duration curve in part (a); the modified curve after unit 1 is loaded is shown in part (b), and the final curve after both units are loaded is shown in part (c). Negative values of x represent load that has been served.

The computations involved in the convolutions may be illustrated in tabular

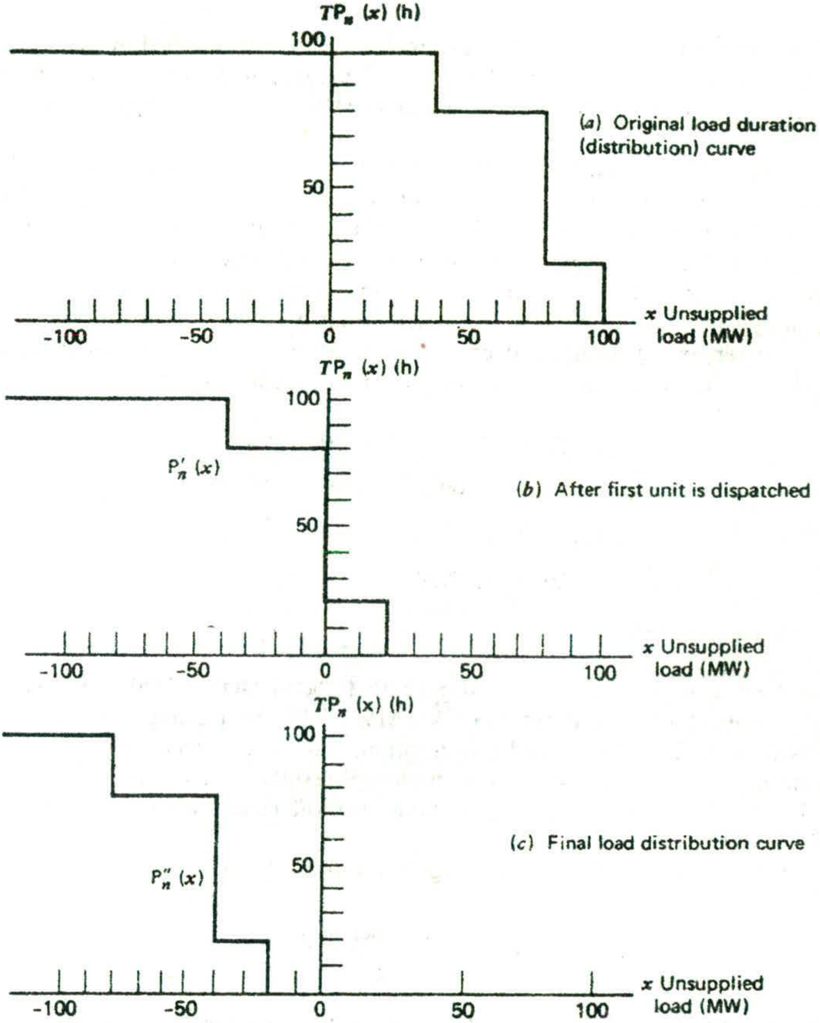

FIG. 8.9 Load-distribution curves redrawn as load probability distributions.

format. In general, in going from the j^{th} distribution to the $(j+1)^{\text{st}}$,

$$P_n^{j+1}(x) = qP_n^j(x) + pP_n^j(x+c)$$

where

$p = 1 - q =$ "innage rate" of unit or segment being loaded
$x + c, x =$ unsupplied load variables (MW)
$c =$ capacity of unit (MW)
$P_n^j(x) =$ probability of needing to supply x or more MW at j^{th} stage

Both sides of the recursive relationship above may be multiplied by the interval duration, T, to convert it to the format illustrated in Figure 8.9. Recall that unit 1 was rated at 80 MW and unit 2 at 40 MW, and for Example 8A all $q = 0$ and all $p = 1$.

Table 8.4 shows the load probability for unserved loads of 0 to 100+ MW. The range of valid MW values need not extend beyond the maximum load nor be less than zero. If you wish to consider the distribution extended to show the served load, $TP_n(x)$ may be extended to negative values. Only the energy for the positive x portion of this distribution represents real load energy. A negative unsupplied energy is, of course, an energy that has been supplied.

The remaining unsupplied energy levels at each step are denoted on the bottom of each column in Table 8.4 and are computed as follows.

$$
\begin{aligned}
E &= 100 \times 20 + 80(80 - 20) + 40 \times (100 - 80) \text{ MWh} \\
&= 20 \text{ h} \times (100 + 100 + 80 + 80 + 20) \text{ MW} \\
&= 7600 \text{ MWh} \\
E' &= 20 \times (20) = 400 \text{ MWh} \\
E'' &= 0
\end{aligned}
$$

Unit 1 was on-line for 100 h and generated 7600 − 400 = 7200 MWh. Unit 2 was on-line for 80 h and generated 400 MWh. The unit loadings, loading levels, durations at those levels, fuel consumption, and production costs can easily be determined using these data. The numerical results are the same as shown in Example 8A. You should be able to duplicate those results using the distributions $P_n(x)$, $P_n'(x)$ and $P_n''(x)$.

Next let us consider forced outage rates for each unit. Let

$$q_i = 0.05 \text{ per unit}$$

TABLE 8.4 Load Probability for Unserved Loads after Scheduling Two Units

x (MW)	$TP_n(x)$ (h)	$TP_n'(x) = TP_n(x + 80)$ (h)	$TP_n''(x) = TP_n'(x + 40)$ (h)
0	100	80	0
20	100	20	
40	100	0	
60	80		
80	80		
100	20		
100+	0	↓	↓
Energy (for $x \geq 0$) =	E	$= E'$	$= E''$

and

$$q_2 = 0.10 \text{ per unit}$$

be the forced outage rates of units 1 and 2, respectively. The recursive equation to obtain $P'_n(x)$ from the original load distribution, omitting the common factor T, is now

$$P'_n(x) = 0.05\,P_n(x) + 0.95\,P_n(x + 80)$$

The original and resultant unserved load distributions are now as follows (Figure 8.10 shows these distributions).

x (MW)	$TP_n(x)$ (h)	$TP'_n(x)$ (h)
0	100	76 + 5 = 81
20	100	19 + 5 = 24
40	100	0 + 5 = 5
60	80	0 + 4 = 4
80	80	0 + 4 = 4
100	20	0 + 1 = 1
100+	0	0
Energy	7600 MWh	760 MWh

These data may be used to compute the loadings, durations, energy produced, fuel consumption, and production cost for unit 1. Unit 1 may be loaded to 80 MW for 80 h and 40 MW for a maximum of 20 h according to the distribution $TP_n(x)$ shown in Figure 8.10. The unit is available only 95% of the time on the average. The loadings, generation, fuel consumption, and

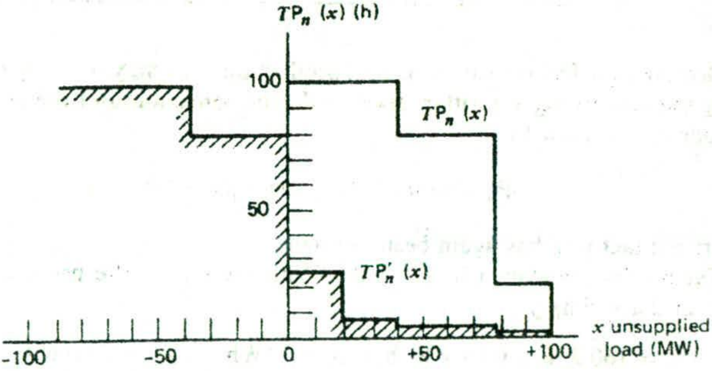

FIG. 8.10 Original and convolved load probability distributions.

fuel cost data for unit 1 are as follows and are identical with those from Example 8A.

Unit 1 Load (MW)	Duration (h)	Energy (MWh)	Fuel Used (10^6 Btu)	Fuel Cost (℞)
40	$0.95 \times 20 = 19$	760	9120	9120
80	$0.95 \times 80 = 76$	6080	60800	60800
		6840	69920	69920

If only production cost and/or fuel consumption are required, without detailed loading profiles, the production costs may be computed using the algorithm developed. That is, the production cost of unit 1:

$$= 160\ \text{℞/h} \times 0.95 \times 100\ \text{h} + 8\ \text{℞/MWh} \times (7600 - 760)\ \text{MWh}$$

$$= 69{,}920\ \text{℞}$$

The detailed loadings and durations for unit 2 may also be computed using the distribution of unserved energy after the unit has been scheduled, $TP'_n(x)$. The unit is required 81 h, is required at zero load for $81 - 24 = 57$ h, may generate 40 MW for 5 h and 20 MW for $24 - 5 = 19$ h. The resulting generation and fuel costs are as follows.

Unit 2 Load (MW)	Duration (h)	Energy (MWh)	Fuel Used (10^6 Btu)	Fuel Cost (℞)
0	51.3	0	4104	8208
20	17.1	342	4104	8208
40	4.5	180	1800	3600
	72.9	522	10008	20016

However, the fuel consumption and production costs may be easily computed using the scheduling algorithm developed. The convolution of the second unit is done in accord with

$$P''_n(x) = 0.1\, P'_n(x) + 0.9\, P'_n(x + 40)$$

where the factor T has again been omitted.

The results are shown in Table 8.5. With these data, the production costs for unit 2 are simply

$$= 160\ \text{℞/h} \times 0.90 \times 81\ \text{h} + 16\ \text{℞/MWh} \times (760 - 238)\ \text{MWh}$$

$$= 20{,}016\ \text{℞}$$

TABLE 8.5 Load Probability for Unserved Loads after Scheduling Unit 1 and Unit 2

x (MW)	$TP'_n(x)$ (h)	$TP''_n(x)$ (h)
0	81	12.6
20	24	6.0
40	5	4.1
60	4	1.3
80	4	0.4
100	1	0.1
100+	0	0
Energy	760 MWh	238 MWh

The final, unserved energy distribution is shown in Figure 8.11. Note that there is still an expected requirement to supply 100 MW. The probability of needing this much capacity is 0.001 per unit (or 0.1%), which is not insignificant.

In order to complete the example, we may compute the cost of supplying the remaining 238 MWh of unsupplied load energy. This must be based on an estimate of the cost of emergency energy supply or the value of unsupplied energy. For this example, let us assume that emergency energy may be purchased (or generated) from a unit with a net heat rate of 12,000 Btu/KWh and a fuel cost of 2 R/MBtu. These are equal to the heat rate and cost associated with unit 2 and are not too far out of line with the costs for energy from the two units previously scheduled. The cost of supplying this 238 MWh is then

$$238 \text{ MWh} \times 12 \text{ MBtu/MWh} \times 2 \text{ R/MBtu} = 5{,}712 \text{ R}$$

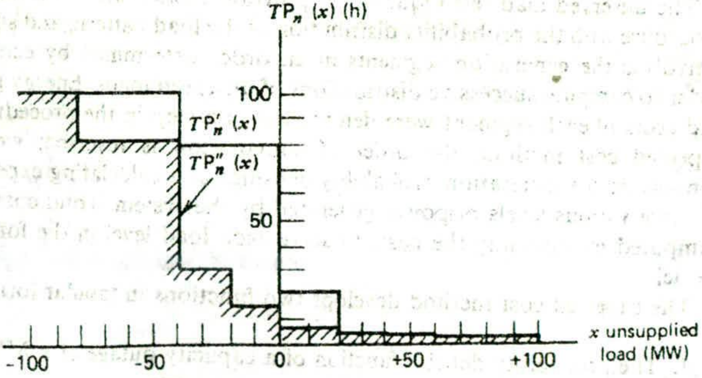

FIG. 8.11 Final distribution of unserved load.

TABLE 8.6 Results of Examples 8A and 8C Compared

	Fuel Used (10^6 Btu)	Fuel Cost (℞)	Unsupplied Energy (MWh)	Cost of Emergency Energy (℞)	Total Cost (℞)
Example 8A	78400	83200	0	0	83200
Example 8D	79928	89936	238	5712	95648
Difference	1528	6736	—	—	12448
% Difference	1.95%	8.1%	—	—	15%

In summary, we may compare the results of Example 8A (computed with forced outages neglected) with the results from Example 8C, where they have been included and an allowance has been made for purchasing emergency energy (see Table 8.6). Ignoring forced outages results in a 1.95% underestimate of fuel consumption, a complete neglect of the need for and costs of emergency energy supplies, and an 8.1% underestimate of the total production costs.

The final unsupplied energy distribution may also be used to provide indexes for the need for additional transmission and/or generation capacity. This is an entire new area, however, and will not be explored here since the primary concern of this text is the operation, scheduling, and cost for power generation.

8.3.3 The Expected Cost Method

The expected cost technique is both an extension of an idea explored earlier in the discussion of hydrothermal scheduling, the system composite cost characteristic, and a variation in the convolution process used in the probabilistic approach. Using a composite system cost characteristic simplifies the computation of the total system production cost to serve a given load pattern. The expected cost per hour is given by the composite cost characteristic as a function of the power level. Calculating the production cost merely involves looking up the cost rates determined by the various load levels in the load model.

The unserved load technique of the previous section starts the convolution procedure with the probability distribution of the load pattern, and successively convolves the generation segments in an order determined by economics in order to compute successive distributions of unserved loads. Energy generation and costs of each segment were determined as a step in the procedure. In the expected cost method, the order of convolution is reversed; we start by convolving the generation probability densities and calculating expected costs to serve various levels of power generated by the system. Total costs are then computed by summing the costs to serve each load level in the forecast load model.

The expected cost method develops two functions in tabular form:

1. The probability density function of a capacity outage of x MW, $P_e(x)$.
2. The expected cost for serving a load of k (MW).

In this method, the function $P_e(x)$ represents the probability that the on-line generating units have an outage of *exactly* x MW. Keep in mind that the variables x and k, defined above, refer to the outage and load magnitudes, respectively. The expected cost rate for serving k MW of load demand is identical in its nature to the composite cost characteristic discussed in an earlier chapter, *except that it is a statistical expectation* that is computed in a fashion that recognizes the probability of random outages of the generation capacity. Thus, any generation being scheduled must serve the load demand, including any capacity shortages due to both random outages of previously scheduled capacity and demand levels in excess of the previously scheduled capacity. Therefore, we require the probability density function of the generation capacity. This function may be computed in a recursive manner, similar to those explored in the appendix of this chapter.

The recursive algorithm for developing a new capacity outage density, $P'_e(x)$, when adding a unit of "c" (MW) is:

$$P'_e(x) = qP_e(x - c) + pP_e(x) \tag{8.4}$$

where

$P_e(x)$ = prior probability of a capacity outage of x MW
c = capacity of generation segment
q = forced outage rate
$p = 1 - q$

and x ranges from zero to the total capacity, s, previously convolved. We need the initial values of this density function (i.e., for $s = 0$) in order to start the recursive computations. With no capacity scheduled these are:

$$P_e(x) = 1.0 \quad \text{for } x = 0$$

and

$$P_e(x) = 0 \quad \text{for all nonzero values of } x$$

We may develop the algorithm for recursive computation of the expected cost function by considering a simplified case where generators are represented by a single straight-line cost characteristic where minimum power level is zero and maximum is given by $c(i)$ MW. The index "i" represents the i^{th} unit, as previously. Let $p(i) = 1 - q(i)$ represent the availability of this unit and $F_i(L)$ the cost rate (R/h) when the unit is generating a power of L MW. When all units have been scheduled, the maximum generation is the value $S = \sum_i c(i)$, the sum of all generator capacities. The load that may be supplied is denoted by k MW, and ranges from zero to S. (Note that there is a significant difference between s, the capacity scheduled previously as part of this computational process, and S, the total capacity of the system.)

Assume that we are in the midst of computing of the expected cost function, EC(k). The capacity scheduled to this point is s MW. The new unit to be scheduled, unit "i," has a capacity of $c(i)$ MW. For any load level below the total capacity previously scheduled, s; that is for,

$$k \leq s$$

the new segment will supply the loads that were not served because of the outages of the previously scheduled segments *within the range of its capabilities*. The generation to be scheduled can only be loaded between zero and the maximum, c. For a given load level, k, the loading of the new segment is:

$$\begin{aligned} L &= k - (s - x), \quad \text{for } 0 \leq [k - (s - x)] \leq c \\ &= 0 \quad \text{for } [k - (s - x)] < 0 \\ &= c \quad \text{for } [k - (s - x)] > c \end{aligned} \tag{8.5}$$

There will be a feasible set of outages $\{x\}$ that must be considered. The increase in the expected cost to serve load level, k, is then,

$$\Delta\text{EC}(k) = \text{p}(i) \textstyle\sum_{\{x\}} \text{P}_e(x) F(L) \quad \text{for } 0 \leq k \leq s \tag{8.6}$$

When the load level k exceeds s,

$$\text{EC}(k) = \text{EC}(s)$$

EXAMPLE 8D

The previous 2-unit case of Examples 8A, 8B, and 8C can be used to illustrate the procedure. Load levels and capacity steps will be taken at 20-MW intervals so that the initial capacity–probability density is:

x (MW)	$\text{P}_e(x)$
0	1.0
Nonzero	0

The first unit is an 80-MW unit with $\text{p}(1) = 0.95$ and $F_1 = 160 + 8\text{P}_1$. The unit loading is

$$L = k - (s - x) = k + x, \quad \text{since } s = 0$$

The first expected cost table is then:

k (MW)	ΔEC(k)(Ɍ/h)	EC(k)(Ɍ/h)
0	$0.95[P_e(0)F_1(0)] = 152$	152
20	$0.95[P_e(0)F_1(20)] = 304$	304
40	$0.95[P_e(0)F_1(40)] = 456$	456
60	$0.95[P_e(0)F_1(60)] = 608$	608
80	$0.95[P_e(0)F_1(80)] = 760$	760
100		760
⋮		⋮

The new value of the dispatched capacity is $s = 80$ and the new outage-probability table is:

x (MW)	$P_e(x)$
0	0.95
20	0
40	0
60	0
80	0.05
100	0
	1.00

The second unit's data are:

$$c = 40 \text{ MW}, \qquad q = 0.10, \qquad p = 0.90, \qquad \text{and} \qquad F_2 = 160 + 16P_2$$

Therefore, $L = k - (s - x) = k + x - 80$, and the second expected cost table is:

k (MW)	ΔEC(k)(Ɍ/h)	EC(k)(Ɍ/h)
0	$0.9[0.5F_2(0)] = 7.2$	$152 + 7.2 = 159.2$
20	$0.9[0.05F_2(20)] = 21.6$	325.6
40	$0.9[0.05F_2(40)] = 36$	492
60	$0.9[0.05F_2(40)] = 36$	644
80	$0.9[0.05F_2(40) + 0.95F_2(0)] = 172.8$	932.8
100	$0.9[0.05F_2(40) + 0.95F_2(20)] = 446.4$	1206.4
120	$0.9[F_2(40)] = 720$	1480
140		1480
160		

The new value of s is 120 MW and the new outage-probability table is:

x (MW)	$P_e(x)$
0	0.855
20	0
40	0.095
60	0
80	0.045
100	0
120	0.005
140	0
	1.000

We could stop at this point. Instead let's add an emergency source (an interconnection, perhaps) that will supply emergency power at a rate of 24 Ɍ/MW or energy at 24 Ɍ/MWh. We assume the source to be perfectly reliable, so that we may represent this source by a large unit with

$$c \geq 120\text{ MW}, \qquad \mathrm{q} = 0, \qquad \mathrm{p} = 1.0, \qquad \text{and} \qquad F = 24(L)$$

where L represents the emergency load. Then

$$L = k + x - S = k + x - 120$$

The final expected cost function computations are:

k (MW)	ΔEC(k)(Ɍ/h)		EC(k)(Ɍ/h)
0	0.005[24(0)]	= 0	159.2
20	0.005[24(20)]	= 2.4	328
40	0.005[24(40)] + 0.045[24(0)]	= 4.8	496.8
60	0.005[24(60)] + 0.045[24(20)]	= 28.8	672.8
80	0.005[24(80)] + 0.045[24(40)] + 0.095[24(0)]	= 52.8	985.6
100	0.005[24(100)] + 0.045[24(60)] + 0.095[24(20)]	= 122.4	1328.8
120	0.005[24(120)] + 0.045[24(80)] + 0.095[24(40)] + 0.855[24(0)]	= 192	1672
140	.	= 672	2152
160	.	= 1152	2632
⋮	⋮	⋮	⋮

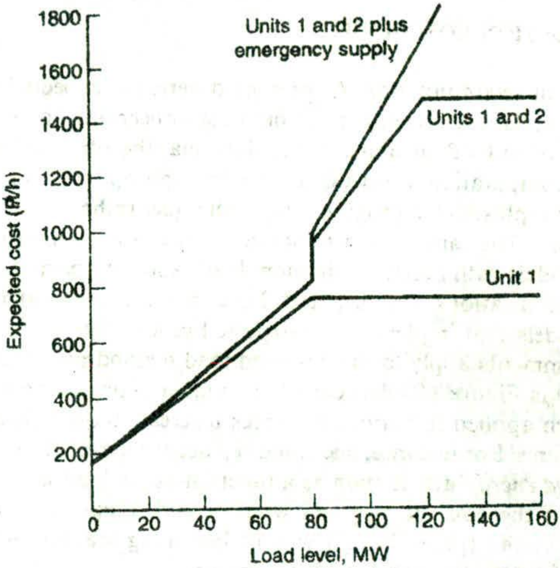

FIG. 8.12 Expected cost versus load level for Example 8D.

Figure 8.12 illustrates the expected cost versus the load level for this simple example.

The calculation of the production cost involves the determination of the expected cost at each load level in Ɍ/h and the duration of that load level. This duration is the probability density function of the load multiplied by the period length in hours, so that we are, in effect, performing the final step in convolving the load and capacity–cost distributions. For Example 8D we may develop the following table. This value agrees with that obtained in Example 8B when the cost of the 238 MWh of emergency energy required is included.

Load (MW)	Duration (h)	Expected Cost Rate (Ɍ/h)	Expected Cost (Ɍ)
40	20	496.8	9936
80	60	985.6	59136
100	20	1328.8	26576
		Total production cost =	95648

A computational flow chart similar to Figure 8.9 could be developed. (We leave this as a potential exercise.) The expected cost method has the merit that the cost rate data remain fixed with a fixed generation system and may be used to compute thermal-unit costs for different load patterns and energy purchases or sales without recomputation. As presented here, the expected cost method suffers from the lack of readily available data concerning the costs and fuel consumption of individual units. These data may be obtained when care is taken in the computational process to save the appropriate information. This involves more sophisticated programming techniques rather than new engineering applications. The same comment applies to the utilization of more realistic generation models with nonzero minimum loads and with partial outage states. All these complications can be, and have been, incorporated in various computer models that implement the expected cost method.

Similar comments apply to the unserved load method presented previously. The flowchart in Figure 8.9 offers clues to a number of programming techniques that have been applied in various instances to create more efficient computational procedures. For instance, one could replace the *unserved load* distribution by an *unserved energy* distribution as a function of the load level. This saves a step or two in the computation and would speed things up quite a bit. But these "tricks of the trade" have a way of becoming less important with the availability of ever-more-rapid small computers.

8.3.4 A Discussion of Some Practical Problems

The examples illustrate the simplicity of the basic computation of the scheduling technique used in this type of probabilistic production cost program where the load is modeled using a discrete tabular format. There are detailed complications, extensions, and exceptions that arise in the practical implementation of any production cost technique. This section reviews the procedures used previously, in the unserved load method, to point out some of these considerations. No attempt is made to describe a complete, detailed program. The intent is to point out some of the practical considerations and discuss some of the approaches that may be used.

First, consider Figure 8.13, which shows the *cumulative load distribution* (i.e.,

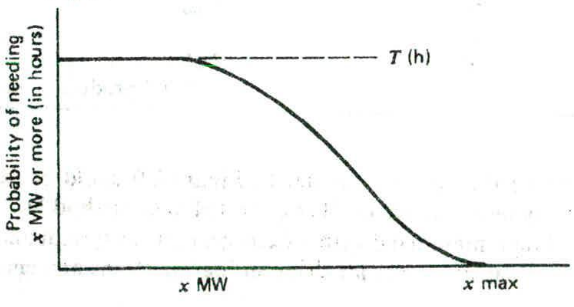

FIG. 8.13 Load probability distribution.

TABLE 8.7 Sample Subinterval Loading Data: Segment Data

Unit Number (i)	No. (j)	P_{min} (MW)	P_{max} (MW)	Cost	Outage Rate (q_{ij}) per Unit	Innage or Availability Rate
3	1	0	20	Ɍ/h	0.05	0.95
1	1	0	20	Ɍ/h	0.02	0.98
4	1	0	40	Ɍ/h	0.02	0.98
1	2	20+	60	Ɍ/MHh	0.05	0.95
3	2	20+	50	Ɍ/MWh	0.05	0.95
4	2	40+	50	Ɍ/MWh	0.05	
⋮	⋮	⋮	⋮	⋮	⋮	⋮

a load-duration curve treated as a cumulative probability distribution) for an interval of T hours. Next, assume an ordered list of loading segments as shown in Table 8.7. Units 3, 1, and 4 are to be committed initially, so that the sum of their capacities at full output equals or exceeds the peak load plus capacity required for spinning reserves. If we assume that two segments for each of these three units, this commitment totals 160 MW. Assume such a table includes all the units available in that subinterval. The cost data for the first three loading segments are the total costs per hour at the minimum loading levels of 20, 20, and 40 MW, respectively, and the remaining cost data are the incremental costs in Ɍ per MWh for the particular segment. Table 8.7 is the ordered list of loading segments where each segment is loaded, generation and cost are computed, and the cumulative load distribution function is convolved with the segment.

There are two problems presented by these data that have not been discussed previously. First, the minimum loading sections of the initially committed units must be loaded at their minimum load points. For instance, the minimum load for unit 4 is 40 MW, which means it cannot satisfy loads less than 40 MW. Second, each unit has more than one loading segment. The loading of a unit's second loading segment, by considering the probability distribution of unserved load after the first segment of a unit has been scheduled, would violate the combinatorial probability rules that have been used to develop the scheduling algorithm, since the unserved load distribution includes events where the first unit was out of service. That is, the loading of a second or later section is not statistically independent of the availability of the previously scheduled sections of the particular unit. Both these concerns require further exploration in order to avoid the commitment of known errors in the procedure.

The situation with block-loaded units (or a nonzero minimum loading limit) is easily handled. Suppose the unserved load distribution prior to loading such a block-loaded segment is $T\,\mathrm{P}_n(x)$ and the unit data are

q = unavailability rate, per unit

$\mathrm{p} = 1 - \mathrm{q}$

c = capacity of segment

By *block-loading* it is meant that the output of this particular segment is limited to exactly c MW. The nonzero minimum loading limit may be handled in a similar fashion.

The convolution of this segment with $T\,\mathrm{P}_n(x)$ now must be handled in parts. For load demands below the minimum output, c, the unit is completely unavailable. For $x \geq c$, the unit may be loaded to c MW output. The algorithm for combining the mutually exclusive events where x, or more, MW of load remain unserved must now be performed in segments, depending on the load. For load levels, x, such that

$$x \geq c$$

the new unserved load distribution is

$$\mathrm{P}'_x(x) = \mathrm{q}\,\mathrm{P}_n(x) + \mathrm{p}\,\mathrm{P}_n(x + c) \tag{8.7}$$

where the period length, T, has been omitted. For some loads, $x < c$, the unit cannot operate to supply the load. Let $\mathrm{p}_n(x)$ denote the probability density of load x. In discrete form,

$$\mathrm{p}_n(x) = \mathrm{P}_n(x) - \mathrm{P}_n(x + \mathrm{MW}_{\mathrm{step}}) \tag{8.8}$$

where $\mathrm{MW}_{\mathrm{step}}$ = uniform interval in tabulation of $\mathrm{P}_n(x)$. For loads equal to or greater than c, the probability of exactly x MW after the unit has been scheduled is

$$\mathrm{p}'_n(x) = \mathrm{q}\,\mathrm{p}_n(x) + \mathrm{p}\,\mathrm{p}_n(x + c) \tag{8.9}$$

For loads less than c (i.e., $0 \leq x \leq c$),

$$\mathrm{p}'_n(x) = \mathrm{p}_n(x) + \mathrm{p}\,\mathrm{p}_n(x + c) \tag{8.10}$$

For convenience in computation, let

$$\mathrm{p}_n(x) = (\mathrm{q} + \mathrm{p})\mathrm{p}_n(x) \tag{8.11}$$

for $0 \leq x < c$. Then for this same load range,

$$\mathrm{p}'_n(x) = \mathrm{q}\mathrm{p}_n(x) + \mathrm{p}\,\mathrm{p}_n(x + c) + \mathrm{p}\,\mathrm{p}_n(x) \tag{8.12}$$

Next, the new unserved load energy distribution may be found by integration of the density function from the maximum load to the load in question. For discrete representations and for $x \geq c$,

$$\mathrm{P}_n'(x) = \mathrm{q}\,\mathrm{P}_n(x) + \mathrm{p}\mathrm{P}_n(x + c) \tag{8.13}$$

For loads less than c; that is, $0 \leq x \leq c$,

$$\mathrm{P}_n'(x) = \mathrm{q}\,\mathrm{P}_n(x) + \mathrm{p}\,\mathrm{P}_n(x + c) + \mathrm{p}[\mathrm{P}_n(x) - \mathrm{P}_n(c)] \tag{8.14}$$

The last term represents those events for loads between x and c, wherein the unit cannot operate. The term $[\mathrm{P}_n(x) - \mathrm{P}_n(c)]$ is the probability density of those loads taken as a whole. The first term, $\mathrm{q}\,\mathrm{P}_n(x)$, resulted from assuming that the unit could supply any load below its maximum.

This format for the block-loaded unit makes it easy to modify the unserved load scheduling algorithm presented previously. The effects of restriction to block-loading a unit may be illustrated using the data from Example 8C.

EXAMPLE 8E

The two-unit system and load distribution of Example 8C will be used with one modification. Instead of allowing the second unit to operate anywhere between 0 and 40 MW output, we will assume its operation is restricted to 40 MW only. The cost of this unit was

$$F_2 = 160 + 16P_2, \qquad \text{R/h}$$

so that for $P_2 = 40$ MW, $F_2 = 800$ R/h.

Recall (see Table 8.5) that after the first 80 MW unit was scheduled, the unserved load distribution was

x (MW)	$T\mathrm{P}_n'(x)$ (h)
0	81
20	24
40	5
60	4
80	4
100	1

With an unserved load energy of 760 MWh. With a restriction to block-loading, the unit is on-line only 5 h. The energy it generates is therefore $5 \times 40 \times 0.9 = 180$ MWh. The new distribution of unserved load after the unit is scheduled is as follows.

x (MW)	$TP'_n(x)$ (h)	$q\,TP'(x) + p\,TP'(x+c)$	$p\,T[P_n(x) - P_n(c)]$	$TP''_n(x)$ (h)
0	81	12.6	$0.9[81 - 5]$	81.0
20	24	6.0	$0.9[24 - 5]$	23.1
40	5	4.1	—	4.1
60	4	1.3	—	1.3
80	4	0.4	—	0.4
100	1	0.1	—	0.1

The unserved load energy is now 580 MWh.

The quantitative significance of the precise treatment of block-loaded units has been magnified by the smallness of this example. In studies of practical-sized systems, block-loading restrictions are frequently ignored by removing the restriction on minimum loadings or are treated in some satisfactory, approximate fashion. For long-range studies, these restrictions usually have minor impact on overall production costs.

The analysis of the effects of the statistical dependence of the multiple-loading segments of a unit is somewhat more complicated. The distribution of unserved load probabilities, $TP_n(x)$, at any point in the scheduling algorithm is independent of the order in which various units are scheduled. Only the generation and hours of operation are dependent on the scheduling order. This may easily be verified by a simple numerical example, or it may be deduced from the recursive relationship presented for $TP_n(x)$.

Suppose we have a second section to be incrementally loaded for some machine at a point in the computations where the distribution of unserved load is $TP_n(x)$. The outage of this second incremental loading section is obviously not statistically independent of the outage of the unit as a whole. Therefore, the effect of the first section must be removed from $TP_n(x)$, prior to determining the loading of the second segment. This is known as *deconvolution*.

For this illustration of one method for handling multiple segments, we will assume:

1. The capacity of the segment extends from C_1 to C_2 where $C_2 > C_1$.
2. The first segment had a capacity of C_1.
3. The outage rates of both segments are equal to q per unit.

In the process of arriving at the distribution $TP_n(x)$, the initial segment of

C_1 MW was convolved in the usual fashion. That is,

$$T\mathrm{P}_n(x) = \mathrm{q}\, T\mathrm{P}'_n(x) + \mathrm{p}\, T\mathrm{P}'_n(x + C_1) \tag{8.15}$$

The distribution $T\mathrm{P}_n(x)$ is independent of the order in which segments are convolved. Only the loading of each segment depends on this order.

Therefore, we may consider that $T\mathrm{P}'_n(x)$ represents an artificial distribution of load probabilities with the initial segment of the unit removed. This pseudo-distribution, $T\mathrm{P}'_n(x)$, must be determined in order to evaluate the loading on the segment between C_1 and C_2. Several techniques may be used to recover $T\mathrm{P}'_n(x)$ from $T\mathrm{P}_n(x)$. The convolution equation may be solved for either $T\mathrm{P}'_n(x)$ or $T\mathrm{P}'_n(x + c)$. The deconvolution process is started at the maximum load if the equation is solved for $T\mathrm{P}'_n(x)$. That is,

$$T\mathrm{P}'_n(x) = \frac{1}{\mathrm{q}}\, T\mathrm{P}_n(x) - \frac{\mathrm{p}}{\mathrm{q}}\, T\mathrm{P}'_n(x + c)$$

and (8.16)

$$T\mathrm{P}'_n(x) = 0 \quad \text{for } x > \text{maximum load}$$

We will use this procedure to illustrate the method because the procedures and algorithms discussed have not preserved the distributions for negative values of unserved load (i.e., already-served loads). As a practical computational matter, it would be better practice to preserve the entire distribution $T\mathrm{P}_n(x)$ and solve the convolution equation for $T\mathrm{P}_n(x + c)$. That is,

$$T\mathrm{P}_n(x + c) = \frac{1}{\mathrm{p}}\, T\mathrm{P}_n(x) - \frac{\mathrm{q}}{\mathrm{p}}\, T\mathrm{P}'_n(x) \tag{8.17}$$

or shifting arguments, by letting $y = x + c$,

$$T\mathrm{P}_n(y) = \frac{1}{\mathrm{p}}\, T\mathrm{P}_n(y - c) - \frac{\mathrm{q}}{\mathrm{p}}\, T\mathrm{P}'_n(y - c) \tag{8.18}$$

In this case, the deconvolution is started at the point at which

$$-y = \text{sum of dispatched generation}$$

since

$$T\mathrm{P}_n(y) = T$$

$$\text{for all } y < -\text{sum of dispatched generation}$$

Even though we will use the first deconvolution equation for illustration, the second should be used in any computer implementation where repeated deconvolutions are to take place. Since $\mathrm{q} \ll \mathrm{p}$, the factors $1/\mathrm{q}$ and p/q in the first formulation will amplify any numerical errors that occur in computing the successive distributions. We use this potentially, numerically treacherous formulation here only as a convenience in illustration.

To return, we obtain the deconvolved distribution $TP'_n(x)$ by removing the effects of the first loading segment. Then the loading of the second segment from C_1 to C_2 is determined using $TP'_n(x)$, and the new, remaining distribution of unserved load is obtained by adding the total unit of C_2 MW to the distribution so that

$$TP''_n(x) = q\, TP'_n(x) + p\, TP'_n(x + C_2) \tag{8.19}$$

EXAMPLE 8F

Assume that in our previous examples, the first unit had a total capacity of 100 MW instead of 80. This last segment might have an incremental cost rate of 20 R/MWh so that it would not be dispatched until after the second unit had been used. Assume the outage rate of 0.05 per unit applies to the entire unit. Let us determine the loading on this second section and the final distribution of unserved load.

The distribution of unserved load from the previous examples is

x (MW)	$TP''_n(x)$ (h)
0	12.6
20	6.0
40	4.1
60	1.3
80	0.4
100	0.1

The deconvolved distribution may be computed starting at $x = 100$ MW using Eq. 8.16 and working up the table. The table was constructed with $c = 80$ MW for the capacity of this unit. The deconvolved distribution is

$$TP'_n(100) = \frac{0.1}{0.05} = 2$$

$$TP'_n(80) = \frac{0.4}{0.05} = 8$$

$$\vdots$$

The new distribution, adding the entire 100 MW unit, is determined using $c = 100$ MW and is

$$TP''_x(x) = 0.05\, TP'_x(x) + 0.95\, TP'_n(x + 100)$$

The results are as follows.

x (MW)	$TP_n(x)$ (h)	$TP'_n(x)$ (h)	$TP''_n(x)$ (h)
0	12.6	100	6.9
20	6.0	82	4.1
40	4.1	82	4.1
60	1.3	26	1.3
80	0.4	8	0.4
100	0.1	2	0.1
Energy	238 MWh		200 MWh

Thus, the second section of the first unit generates 38 MHh.

This computation may be verified by examining the detailed results of Example 8B, where the various load and outage combination events were enumerated. At a load of 100 MW, the second segment of unit 2 would have been loaded to the extent shown by this example. You should be able to identify two periods where the second section would have reduced previous shortages of 0 and 20 MW. This procedure and the example are theoretically correct but computationally tedious. Furthermore, the repeated deconvolution process may lead to numerical round-off errors unless care is taken in any practical implementation.

Approximations are frequently made in treating sequential loading segments. These are usually based on the assumption that the subsequent loading sections are independent of the previously loaded segments. That is, that they are equivalent to new, independent units with ratings that are equal to the capacity increment of the segment. When these types of approximations are made, they are justified on the basis of numerical tests. They generally perform more than adequately for larger systems but should be avoided for small systems.

The two extensions discussed here are only examples of the many extensions and modifications that may be made. When the computations of expected production costs are made as a function of the load to be served, these characteristics may be used as pseudogenerators in scheduling hydroelectric plants, pumped-storage units, or units with limited fuel supplies.

There have been further extensions in the theoretical development as well. It is quite feasible to represent the distribution of available capacity by the use of suitable orthogonal polynomials. Gram–Charlier series are frequently used to model probabilistic phenomena. They are most useful with a reasonably uniform set of generator capacities and outage rates. By representing the expected load distribution also as an analytic function it is possible to develop analytical expressions for unserved energy distributions and expected production costs. Care must be exercised in using these approximations when one or two very large generators are added to systems previously composed of a uniform array of capacities. We will not delve further into this area in this text. The remainder of this chapter is devoted to a further example and problems.

8.4 SAMPLE COMPUTATION AND EXERCISE

The discussion of the probabilistic techniques is more difficult than their performance. We will illustrate the unserved load method further using a three-unit system. The three generating units each may be loaded from 0 MW to their respective ratings. For ease of computation, we assume linear input–output cost curves and only full-forced outage rates (that is, the unit is either completely available or completely unavailable). The unit data are as follows.

Unit No.	Maximum Rating (MW)	Input–Output Cost Curve (Ɍ/h)	Full-Forced Outage Rate (per unit)
1	60	$60 + 3P_1$	0.2
2	50	$70 + 3.5P_2$	0.1
3	20	$80 + 4P_3$	0.1

In these cost curves P_i are in MW. In addition, the system is served over a tie line. Emergency energy is available without limit (MW or MWh) at a cost rate of 5 Ɍ/MWh.

The load model is a distribution curve for a 4-week interval (a 672-h period). That is, the expected load is as shown in Table 8.8. The total load energy is 43,680 MWh.

8.4.1 No Forced Outages

The economic dispatch of these units for each load level is straightforward. The units are to be loaded in the order shown. The sum of the peak load demand

TABLE 8.8 Load Distribution

Load Level (MW)	Hours of Existence	Probability	Hours Load Equals or Exceeds	Probability of Needing Load or More (pu)
30	134.4	0.2	672.0	1.00
50	134.4	0.2	537.6	0.80
70	134.4	0.2	403.2	0.60
80	168.0	0.25	268.8	0.40
100	100.8	0.15	100.8	0.15
	672.0			

(100 MW) and the total capability (130 MW) is 230 MW. Therefore, the probability table of needing capacity will extend eventually from -130 MW to $+100$ MW. It is convenient in digital computer implementation to work in uniform MW steps. For this example, we will use 10 MW.

As each unit is dispatched, the probability distribution of needing x or more MW [i.e., $\mathrm{P}_n(x)$] is modified (i.e., convolved) using the following:

$$T\mathrm{P}_n'(x) = T\mathrm{P}_n(x + c)$$

where

$\mathrm{P}_n'(x)$ and $\mathrm{P}_n(x)$ = new and old distributions, respectively

T = time period, 672 h in this instance

c = capability of unit or segment when it is in state j

Table 8.9 shows initial distribution in the second column. The load energy to be served is

$$E = 672 \sum_{x=0}^{100} \mathrm{P}_n(x)\,\Delta x = 43{,}680 \text{ MWh}$$

With zero-forced outage rate, the 60-MW unit loading results in the $\mathrm{P}_n(x)$ distribution shown in the third column. The resultant load energy to be served is now:

$$E' = (0.15 \times 20 + 0.4 \times 10 + 0.6 \times 10) \times 672 = 8736 \text{ MWh}$$

which means unit 1 generated

$$43{,}680 - 8736 = 34{,}944 \text{ MWh}$$

The unit was on-line for 672 h, and the incremental cost rate was 3 Ɍ/h. Therefore, the cost for unit 1 is

$$\begin{aligned} \text{Total cost} &= \sum_T F(P_t) \times \Delta t = \sum_T (60 + 3P_t)\,\Delta t = \sum_T (60\,\Delta t + 3P_t\,\Delta t) \\ &= 60T + 3 \text{ (MWh generated), since MWh} = \sum_T P_t\,\Delta t \\ &= 60 \text{ Ɍ/h} \times 672 \text{ h} + 34{,}944 \text{ MWh} \times 3 \text{ Ɍ/MWh} = 145{,}152 \text{ Ɍ} \end{aligned}$$

Unit 2 serves the remaining load distribution (third column) and results in the distribution shown in the fourth column. This unit is only on-line for 60% of the interval, so that its cost is

$$0.6 \times 70 \text{ Ɍ/h} \times 672 \text{ h} + 8736 \text{ MWh} \times 3.5 \text{ Ɍ/MWh} = 58{,}800 \text{ Ɍ}$$

The total system cost is 203,952 Ɍ, and unit 3 is not used at all. These results are summarized in Table 8.10.

TABLE 8.9 Three-Unit Example: Zero-Forced Outage Rates

x (MW)	$P_n(x)$ (pu)	$P'_n(x)$ (pu)	$P''_n(x)$ (pu)
−130	↑	↑	↑
−120			
−110			
−100			
−90			
−80			1.0
−70			0.8
−60			0.8
−50			0.6
−40			0.6
−30		1.0	0.4
−20		0.8	0.15
−10		0.8	0.15
0		0.6	0
10		0.6	
20		0.4	
30	1.0	0.15	
40	0.8	0.15	
50	0.8	0	
60	0.6		
70	0.6		
80	0.4		
90	0.15		
100	0.15		
110	0	↓	↓
$E/672$	65	13	0
MWh	43680	8736	0

TABLE 8.10 Summary of Results: Zero-Forced Outage Rates

Unit Number	Capacity (MW)	Outage Rate (pu)	Hours On-Line	Energy Generated (MWh)	Cost (Ɍ)
1	60	0.000	672.0	34944.0	145152.0
2	50	0.000	403.0	8736.0	58800.0
3	20	0.000	0	0	0
4	100	0.000	0	0	0
Total	230			43680.0	203952.0

Average system cost = 4.6692 Ɍ/MWh.

8.4.2 Forced Outages Included

When the forced outage is included, the convolution of the probability distribution is accomplished by

$$P'_n(x) = q\, P_n(x) + p\, P_n(x + c)$$

where

$$q = \text{forced outage rate (pu)}$$

$$p = 1 - q = \text{“innage” rate}$$

Table 8.11 shows the computations for the first unit in the third and fourth columns.

The first unit is on-line 0.8 × 672 = 537.6 h and generates 27,955.2 MWh. (The initial load demand contains 43,680 MWh; the modified distribution in

TABLE 8.11 Three-Unit Example Including Forced Outage Rates

x (MW)	$P_n(x)$ (pu)	$P_n(x+60)$ (pu)	$P'_n(x)$ (pu)	$P'_n(x+50)$ (pu)	$P''_n(x)$ (pu)	$P'''_n(x)$ (pu)
−130	↑	↑	↑	↑		
−120						
−110						
−90						
−80				1.00		
−70				0.84		
−60				0.84		
				0.68		
−40				0.68		
−30		1.0	1.0	0.52		
−20		0.8	0.84	0.32		
−10		0.8	0.84	0.28		
0		0.6	0.68	0.16	0.212	
10		0.6	0.68	0.12	0.176	
20		0.4	0.52	0.12	0.160	
30	1.0	0.15	0.32	0.08	0.104	
40	0.8	0.15	0.28	0.03	0.055	
50	0.8	0	0.16	0.03	0.043	
60	0.6		0.12	0	0.012	
70	0.6		0.12		0.012	
80	0.4		0.08		0.008	
90	0.15		0.03		0.003	
100	0.15		0.03		0.003	
	0	↓	0	↓	0	
$E/672$	65		23.4		5.76	
MWh	43680		15724.8		3870.72	

column 4 contains 15,724.8 MWh.) Therefore, the first unit's cost is

$$60\ \text{Ɍ/h} \times 537.6\ \text{h} + 3\ \text{Ɍ/MWh} \times 27{,}955.2\ \text{MWh} = 116{,}121.6\ \text{Ɍ}$$

The distribution of needed capacity is shown *partially* in the sixth column of Table 8.11. Sufficient data are shown to compute the load energy remaining. (*Load energy* is the portion of the distribution, $P_n^j(x)$, for $x \geq 0$). The unserved load energy after scheduling unit 2 is

$$0.576 \times 10 \times 672 = 3870.72\ \text{MWh}$$

This means unit 2 generated an energy of 15,724.8 − 3870.72 = 11,854.08 MWh at an incremental cost of 3.5 Ɍ/MWh; or 41,489.28 Ɍ. The unit was on-line for 411.264 h at a cost rate of 70 Ɍ/h. This brings the total cost to 70,277.76 Ɍ for unit 2. Note that the operating time (i.e., the "hours on-line") is 0.9 × 0.68 × 672 h. The first factor represents the probability that the unit is available, the second the fraction of the time interval that the load requires unit 2, and the 672-h factor is the length of the interval.

Table 8.12 shows a summary of the results for this three-unit plus tie-line sample exercise when outage rates are included. The third unit and tie line are utilized a substantial amount compared with ignoring forced outages. The total cost for the 4-wk interval increased by almost 5%.

The resulting successive convolutions are shown in Figure 8.14. After the entire 130 MW of generating capacity has been dispatched, the distribution of unserved load is represented by the portion of the lowest curve to the right of the zero MW point (it is shaded).

Table 8.13 shows the distribution of emergency energy delivery over the tie line.

This chapter has only provided an introduction to this area. Practical schemes exist to handle much more complex unit and load models, to incorporate limited energy and pumped-storage units, and to compute generation reliability indices. They are all based on techniques similar to those introduced here.

TABLE 8.12 Results

Unit No.	Capacity (MW)	Outage Rate (pu)	Hours On-Line	Energy Generated (MWh)	Cost (Ɍ)
1	60	0.200	538.0	27955.0	116122.0
2	50	0.100	411.0	11854.0	70278.0
3	20	0.100	128.0	2032.0	18386.0
4	100	0.000	111.0	1839.0	9193.0
Total	230			43680	213979.0

Average system cost = 4.8589 Ɍ/MWh.

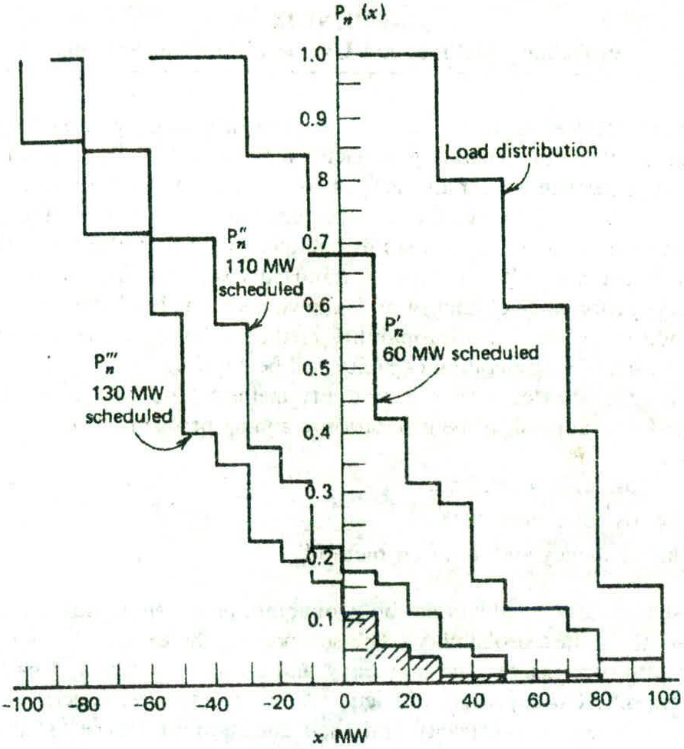

FIG. 8.14 Successive convolutions.

TABLE 8.13 Emergency Energy

Level No.	Loading (MW)	Hours
1	10.0	30.71
2	20.0	11.02
3	30.0	22.04
4	40.0	0.81
5	50.0	4.50
6	60.0	3.02
7	70.0	0.27
8	80.0	2.15
9	100.0	0.20
Total		74.72

APPENDIX

Probability Methods and Uses in Generation Planning

The major application of probability methods in power systems has been in the area of planning generating capacity requirements. This application, no matter what particular technique is used, assigns a probability to the generating capacity available, describes the load demands in some manner, and provides a numerical measure of the probability of failing to supply the expected power or energy demands. By defining a standard *risk level* (i.e., a standard or maximum probability of failure) and allowing system load demands to grow as a function of time, these probability methods may be utilized to calculate the time when new generating capacity will be required.

Three general categories of probability methods and measures have been developed and applied to the generation planning problem. These are:

1. The loss-of-load method.
2. The loss-of-energy method.
3. The frequency and duration method.

The first measures reliability as the probability of meeting peak loads (or its converse, the failure probability). The second uses the expected loss of energy as a reliability measure. The frequency and duration method is based on a somewhat different approach. It calculates the expected frequencies of outages of various amounts of capacity and their corresponding expected durations. These calculated values are then used with appropriate, forecasted loads and reliability standards to establish capacity reserve margins.

The mathematical techniques used are straightforward applications of probability methods. First, to review combined probabilities, let

$P(A)$ = probability that event A occurs

$P(B)$ = probability that event B occurs

$P(A \cap B)$ = joint probability that A and B occur together

$P(A \cup B)$ = probability that either A occurs by itself, or B occurs by itself, or A and B occur together..

Conditional probabilities will be omitted from this discussion. [A *conditional probability* is the probability that A will occur if B already has occurred and may be expressed $P(A/B)$].

A few needed rules from combinatorial probabilities are:

1. If A and B are independent events (i.e., whether A occurs or not has no bearing on B), then the joint probability that A and B occur together is $P(A \cap B) = P(A)P(B)$.

2. If the favorable result of an event is for A or B or both to occur, then the probability of this favorable result is $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
3. If, in rule 2, A and B are "mutually exclusive" events (i.e., if one occurs, the other cannot), then $P(A \cap B) = 0$ and $P(A \cup B) = P(A) + P(B)$.
4. The number of combinations of n things taken r at a time is given by the formula

$$ {}_nC_r = \frac{n!}{r!\,(n-r)!} \tag{8A.1}$$

5. In general, the probability of exactly r occurrences in n trials of an event that has a constant probability of occurrence p is

$$ P_n(r) = {}_nC_r p^r q^{n-r} = \frac{n!}{r!\,(n-r)!} p^r q^{n-r} \tag{8A.2}$$

where $q = 1 - p$.

Rule 5 is a generalized form of the binomial expansion, applying to all terms of the binomial $(p + q)^n$. This distribution has had widespread use in generating-system probability studies. For example, assume that a generation system is composed of four identical units and that each of these units has a probability p of being in service at any randomly chosen time. The probability of being out of service is $q = 1 - p$. Assume that each machine's behavior is independent of the others. Then, a table may be constructed showing the probability of having 4, 3, 2, 1, and none in service.

Number in Service	Probability of Occurrence
4	$P(4) = {}_4C_4 p^4 q^{4-4} = \frac{4!}{4!\,(4-4)!} p^4 = p^4$
3	$P(3) = {}_4C_3 p^3 q^{4-3} = \frac{4!}{3!\,(4-3)!} p^3 q = 4p^3 q$
2	$P(2) = {}_4C_2 p^2 q^{4-2} = \frac{4!}{2!\,(4-2)!} p^2 q^2 = 6p^2 q^2$
1	$P(1) = {}_4C_1 p^1 q^{4-1} = \frac{4!}{1!\,(4-1)!} pq^3 = 4pq^3$
0	$P(0) = {}_4C_0 p^0 q^{4-0} = \frac{4!}{0!\,(4-0)!} q^4 = q^4$

In this table, each of the probabilities is a term of the binomial expansion of the form:

$$ {}_4C_n p^n q^{4-n} $$

where n is the number of units in service.

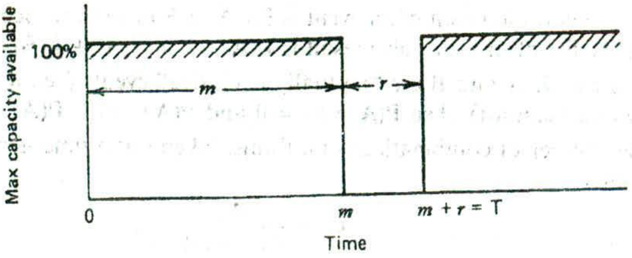

FIG. 8.15 Average availability cycle for a unit with two states.

These relationships assume a long-term average availability cycle, as shown in Figure 8.15 for a given unit. In this long-term average cycle,

m = average time available before failures

r = average repair time

$T = m + r$ = mean time between failures

Using these definitions for the generator taken as a binary state device,

$$\mathrm{p} = \frac{m}{T} = \text{“innage rate” (per unit)}$$

$$\mathrm{q} = 1 - \mathrm{p} = \frac{r}{T} = \text{“outage rate” (per unit)}$$

Generating units may also be considered to be multistate devices when each state is characterized by the maximum available capacity and the probability of existence of that particular state. For instance, a large unit may have a forced reduction in output of, say, 20% of its rating when one boiler feed pump is out of service. This may happen 25% of the total time the unit is supposed to be available. In this case, each unit state (j) can be characterized by

$C(j)$ = maximum capacity available in state (j)

$\mathrm{p}(j)$ = probability that the unit is in state (j)

where

$$\sum_{j=1}^{n} \mathrm{p}(j) = 1.0$$

$C(1) = 0$ (unit down)

$C(n) = 100\%$ capability (unit at full capacity)

In the probabilistic production cost calculations we attach other parameters to a state, such as the incremental cost for loading the unit between $C(j-1)$ and $C(j)$ MW.

The use of reliability techniques based on probability mathematics for generation planning frequently involves the construction of tables that show capacity on outage and the corresponding probability of that much, or more, capacity being on outage. The binomial probability distribution is cumbersome to use in practical computations. We will illustrate the simple numerical convolution using recursive techniques that are useful and efficient in handling units of various capacities and outage rates. The model of the generating capacity to be developed in this case is a table such as the following.

k	O_k Generating Capacity Outage (MW)	Probability of Occurrence of O_k or greater $= P_0(O_k)$
1	0	1.000000
2	15	0.950000
3	25	0.813000
4	35	0.095261
⋮	⋮	⋮

On this table

k = index showing the entry number

O_k = generating capacity outage (MW)

$\mathrm{P}_0(O_k)$ = cumulative probability = probability of the occurrence of an outage of O_k, or larger

This probability is a distribution rather than the density described with the binomial probability. It is a cumulative value rather than an exact probability (i.e., "exact" means probability density function).

Let each machine of the previously discussed hypothetical four-machine system be rated 10 MW, and let $\mathrm{p}(k)$ be the exact probability of occurrence of a particular event characterized by a given outage value. The table started previously may be expanded into Table 8.14. The function $\mathrm{P}(O_k)$ is monotonic, and it should be obvious that the probability of having a zero or larger capacity outage is 1.0.

Since all generators do not have the same capacity or outage rate, the simple relationship for the binomial distribution in Table 8.14 does not hold in the general case. Beside the unit capability, the only other parameter associated with a generator in this technique is the average outage existence rate, q.

A simple recursive algorithm exists to add a unit to an existing outage probability table. Suppose an outage probability table exists that gives

$$\mathrm{P}_0(x) \text{ versus } x$$

TABLE 8.14 Outage Probabilities

k	No. of Machines in Service	MW Outage O_k	$p(k)$ = Exact Probability of Outage O_k	$P(O_k)$ = Probability of Outage O_k, or Larger
1	4	0	p^4	$p^4 + 4p^3q + 6p^2q^2 + 4pq^3 + q^4 \equiv 1.0$
2	3	10	$4p^3q$	$4p^3q + 6p^2q^2 + 4pq^3 + q^4$
3	2	20	$6p^2q^2$	$6p^2q^2 + pq^3 + q^4$
4	1	30	$4pq^3$	$4pq^3 + q^4$
5	0	40	q^4	q^4

Installed capacity = 40 MW.

where

$P_0(x)$ = probability of x MW or more on outage

x = MW outage state

Now suppose you wish to add an "n-state" unit to the table that is described by

$p(j)$ = probability unit is in state j

$C(j)$ = maximum capacity of state j

$C(n)$ = capacity of unit

$O_j = C(n) - C(j)$ = MW outage for state j

Then the new table of outage probabilities may be found by a numerical convolution:

$$P'_0(x) = \sum_{j=1}^{n} p(j) P_0(x - O_j) \tag{8A.3}$$

where

$$P_0(\leq 0) = 1.0$$

This algorithm is an application of the combinational rules for independent, mutually exclusive "events." Each term of the algorithm is made up of (1) the event that the new unit is in state j with an outage O_j MW, and (2) the event that the "old" system has an outage of $(x - O_j)$ MW. The combined event, therefore, has an outage of x MW, or more.

EXAMPLE 8G

Assume we have a generating system consisting of the following machines with their associated outage rate.

MW	Outage Rate
MW	0.02
10	0.02
10	0.02
10	0.02
10	0.02
5	0.02

The exact probability outage table for the first four units could be calculated using the binomial distribution directly and would result in the following table.

MW Outage x	Exact Probability $p(x)$	Cumulative Probability $P_0(x)$
0	0.922368	1.000000
10	0.075295	0.077632
20	0.002305	0.002337
30	0.000032	0.000032
40	0	0

Now, the fifth machine can exist in either of two states: (1) it is in service with a probability of $p = 1 - q = 0.98$ and no additional system capacity is out, or (2) it is out of service with a probability of being in that state of $q = 0.02$, and 5 MW additional capacity is out of service.

The resulting outage-probability table will have additional outages because of the new combinations that have been added. This can be easily overcome by expanding the table developed for four machines to include these new outages. This is shown in Table 8.15, along with an example where the fifth, 5 MW, unit is added to the table.

TABLE 8.15 Adding Fifth Unit

x (MW)	$P_0(x)$	$0.98\ P_0(x)$	$0.02\ P_0(x-5)$	$P'_0(x)$
0	1.000000	0.980000	0.020000	1.000000
5	0.077632	0.076079	0.020000	0.096079
10	0.077632	0.076079	0.001553	0.077632
15	0.002337	0.002290	0.001553	0.003843
20	0.002337	0.002290	0.000047	0.002337
25	0.000032	0.000031	0.000047	0.000078
30	0.000032	0.000031	0	0.000031
35	0	0	0	0
40	0	0	0	0
45	0	0	0	0

The correctness of this approach and the resulting table may be seen by calculating the exact state probabilities for all possible combinations. That is,

MW Out x	Exact Probability $p(x)$
	New machine in service
$0 + 0$	$0.922368 \times 0.98 = 0.903921$
$10 + 0$	$0.075295 \times 0.98 = 0.073789$
$20 + 0$	$0.002305 \times 0.98 = 0.002258$
$30 + 0$	$0.000032 \times 0.98 = 0.000031$
$40 + 0$	$0 \times 0.98 = 0$
	New machine out of service
$0 + 5 = 5$	$0.922368 \times 0.02 = 0.018447$
$10 + 5 = 15$	$0.075295 \times 0.02 = 0.001506$
$20 + 5 = 25$	$0.002305 \times 0.02 = 0.000047$
$30 + 5 = 35$	$0.000032 \times 0.02 = 0$
$40 + 5 = 45$	$0 \times 0.02 = 0$

The exact state probabilities are combined by adding the probabilities for the mutually exclusive events that have identical outages; the results are shown in Table 8.16. Table 8.16 is the capacity model for the five-unit system and is usually assumed to be fixed until new machines are added or a machine is retired, or the model is altered to reflect scheduled maintenance outage.

This model was constructed using maximum capacities and calculating capacity outage probability distributions. Similar techniques may be used to construct available capacity distributions. A similar convolution is used in the probabilistic production cost computations. The form of the distribution is different because we are dealing with a scheduling problem rather than with

TABLE 8.16 Table of Combined Probabilities

MW Outage x	Exact Probability $p(x)$	Cumulative Probability $P'_0(x)$
0	0.903921	1.000000
5	0.018447	0.096079
10	0.073789	0.077632
15	0.001506	0.003843
20	0.002259	0.002337
25	0.000047	0.000078
30	0.000031	0.000031
35	0	0
40	0	0
45	0	0

the static, long-range planning problem. In the present case, we are interested in a distribution of capacity outage probabilities; in the scheduling problem, we require a distribution of unserved load probabilities.

PROBLEMS

8.1 Add another unit to Example 8G (in the Appendix). The new unit should have a capacity of 10 MW and an availability of 90%. That is, its outage rate is 0.10 per unit. Use the recursive algorithm illustrated in the Appendix. How far must the MW outage table be extended?

8.2 If the probability density function of unsupplied load power for a 1-h interval is $p_n(x)$ and the cumulative distribution is

$$P_n(x) = 1 - \int_0^x p_n(y)\,dy$$

demonstrate, using ordinary calculus, that the unsupplied energy is

$$\int_0^{x_{\max}} P_n(y)\,dy$$

where

$x_{\max}$ = maximum load in the 1-h interval

y = dummy variable used to represent the load

Hint: $p_n(x)$ is the probability, or normalized duration, that a load of x MW exists. The energy represented by this load is then $xp_n(x)$. Find the total energy represented by the entire load distribution.

8.3 Complete Table 8.11 for the second unit (i.e., complete the sixth column). Convolve the third unit and determine the data for column 7 [$P_n'''(x)$] and the energy generation of the third unit and its total cost. Find the distribution of energy to be served over the tie line. If this energy costs 5 R/MWh, what is the cost of this emergency supply and the total cost of production for this 4-wk interval?

8.4 Repeat Example 8C to find the minimum cost dispatch assuming that the fuel for unit 2 has been obtained under a take-or-pay contract and is limited to 4500 MBtu. Emergency energy will be purchased at 50 R/MWh. Find the minimum expected system cost including the cost of emergency energy.

8.5 Repeat the calculation of the system in Section 8.4 using the *expected cost method*. Show the development of the characteristic as each unit is scheduled. Plot the expected cost versus the power output. Check the total cost against the results in Section 8.4.

8.6 Repeat the sample computation of Section 8.4, except assume the input–output characteristic of unit 2 with its ratings have changed to the following.

Output (MW)		Input–Output Cost Curve (R/h)	Forced Outage Rate
Section 1	0–50	$70 + 3.5P_2$	0.1
Section 2	50–60	$245 + 4.5(P_2-50)$	0.1

Schedule section 2 of unit 2 after unit 3 and before the emergency energy. Use the techniques of Example 8F and deconvolve section 1 of unit 2 prior to determining the loading on section 2. Repeat the analysis, ignoring the statistical dependence of section 2 on section 1. (That is, schedule a 10-MW "unit" to represent section 2 without deconvolving section 1.)

FURTHER READING

The literature concerning production cost simulations is profuse. A survey of various types of model is contained in reference 1. References 2–4 describe deterministic models designed for long-range planning. Reference 5 provides an entry into the literature of Monte Carlo simulation methods applied to generation planning and production cost computations.

The two texts referred to in references 6 and 7 provide an introduction to the use of probabilistic models and methods for power-generation planning. Reference 8 illustrates the application to a single area. These methods have been extended to consider the effects of transmission interconnections on generation system reliability in references 9–12.

The original probabilistic production cost technique was presented by E. Jamoulle and his associates in a difficult-to-locate Belgian publication (reference 13). The basic methodology has been discussed and illustrated in a number of IEEE papers; references 14–16 are examples.

In many of these articles, the presentation of the probabilistic methodology is couched in a sometimes confusing manner. Where authors such as R. R. Booth and others discuss an "equivalent load distribution," they are referring to the same distribution, $TP_n(x)$, discussed in this chapter. These authors allow the distribution to grow from zero load to some maximum value equal to the sum of the maximum load plus the sum of the capacity on forced outage. We have found this concept difficult to impart and prefer the present presentation. The practical results are identical to those found more commonly in the literature.

The models of approximation using orthogonal expansions to represent capacity distributions have been presented by Stremel and his associates. Reference 17 provides an entry into this literature.

References 15 and 18 lead into the development of the expected production cost method.

References 19–26 contain examples of different approaches to computing probabilistic data and the extension of the methods to different problem areas and generation plant configurations. The last two references are extensions of these techniques to incorporate transmission network. Reference 28 is concerned with unit commitment, but it represents the type of technique that would be useful in shorter-term production cost applications involving transmission-constrained scheduling.

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9 Control of Generation

9.1 INTRODUCTION

So far, this text has concentrated on methods of establishing optimum dispatch and scheduling of generating units. It is important to realize, however, that such optimized dispatching would be useless without a method of control over the generator units. Indeed, the control of generator units was the first problem faced in early power-system design. The methods developed for control of individual generators and eventually control of large interconnections play a vital role in modern energy control centers.

A generator driven by a steam turbine can be represented as a large rotating mass with two opposing torques acting on the rotation. As shown in Figure 9.1, T_{mech}, the mechanical torque, acts to increase rotational speed whereas T_{elec}, the electrical torque, acts to slow it down. When T_{mech} and T_{elec} are equal in magnitude, the rotational speed, ω, will be constant. If the electrical load is increased so that T_{elec} is larger than T_{mech}, the entire rotating system will begin to slow down. Since it would be damaging to let the equipment slow down too far, something must be done to increase the mechanical torque T_{mech} to restore equilibrium; that is, to bring the rotational speed back to an acceptable value and the torques to equality so that the speed is again held constant.

This process must be repeated constantly on a power system because the loads change constantly. Furthermore, because there are many generators supplying power into the transmission system, some means must be provided to allocate the load changes to the gnerators. To accomplish this, a series of control systems are connected to the generator units. A governor on each unit maintains its speed while supplementary control, usually originating at a remote control center, acts to allocate generation. Figure 9.2 shows an overview of the generation control problem.

9.2. GENERATOR MODEL

Before starting, it will be useful for us to define our terms.

ω = rotational speed (rad/sec)

α = rotational acceleration

δ = phase angle of a rotating machine

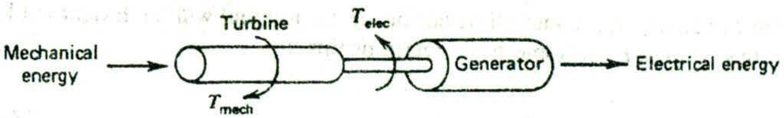


FIG. 9.1 Mechanical and electrical torques in a generating unit.

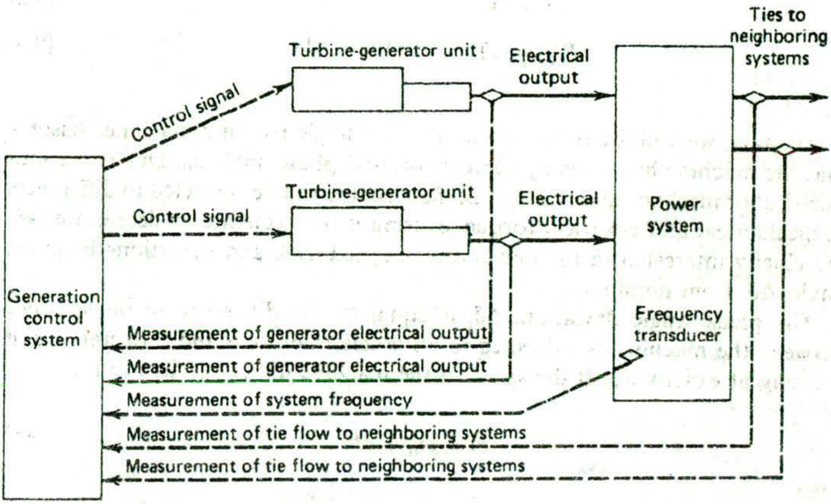


FIG. 9.2 Overview of generation control problem.

T_{net} = net accelerating torque in a machine

T_{mech} = mechanical torque exerted on the machine by the turbine

T_{elec} = electrical torque exerted on the machine by the generator

P_{net} = net accelerating power

P_{mech} = mechanical power input

P_{elec} = electrical power output

I = moment of inertia for the machine

M = angular momentum of the machine

where all quantities (except phase angle) will be in per unit on the machine base, or, in the case of ω, on the standard system frequency base. Thus, for example, M is in per unit power/per unit frequency/sec.

In the development to follow, we are interested in deviations of quantities about steady-state values. All steady-state or nominal values will have a "0" subscript (e.g., ω_0, T_{net_0}), and all deviations from nominal will be designated by a "Δ" (e.g., $\Delta\omega$, ΔT_{net}). Some basic relationships are

$$I\alpha = T_{\text{net}} \tag{9.1}$$

$$M = \omega I \tag{9.2}$$

$$P_{\text{net}} = \omega T_{\text{net}} = \omega(I\alpha) = M\alpha \tag{9.3}$$

To start, we will focus our attention on a single rotating machine. Assume that the machine has a steady speed of ω_0 and phase angle δ_0. Due to various electrical or mechanical disturbances, the machine will be subjected to differences in mechanical and electrical torque, causing it to accelerate or decelerate. We are chiefly interested in the deviations of speed, $\Delta\omega$, and deviations in phase angle, $\Delta\delta$, from nominal.

The phase angle deviation, $\Delta\delta$, is equal to the difference in phase angle between the machine as subjected to an acceleration of α and a reference axis rotating at exactly ω_0. If the speed of the machine under acceleration is

$$\omega = \omega_0 + \alpha t \tag{9.4}$$

then

$$\begin{aligned}\Delta\delta &= \underbrace{\int (\omega_0 + \alpha t)\,\mathrm{d}t}_{\substack{\text{Machine absolute}\\ \text{phase angle}}} - \underbrace{\int \omega_0\,\mathrm{d}t}_{\substack{\text{Phase angle of}\\ \text{reference axis}}}\\ &= \omega_0 t + \tfrac{1}{2}\alpha t^2 - \omega_0 t\\ &= \tfrac{1}{2}\alpha t^2 \end{aligned} \tag{9.5}$$

The deviation from nominal speed, $\Delta\omega$, may then be expressed as

$$\Delta\omega = \alpha t = \frac{\mathrm{d}}{\mathrm{d}t}(\Delta\delta) \tag{9.6}$$

The relationship between phase angle deviation, speed deviation, and net accelerating torque is

$$T_{\text{net}} = I\alpha = I\frac{\mathrm{d}}{\mathrm{d}t}(\Delta\omega) = I\frac{\mathrm{d}^2}{\mathrm{d}t^2}(\Delta\delta) \tag{9.7}$$

Next, we will relate the deviations in mechanical and electrical power to the deviations in rotating speed and mechanical torques. The relationship between net accelerating power and the electrical and mechanical powers is

$$P_{\text{net}} = P_{\text{mech}} - P_{\text{elec}} \tag{9.8}$$

which is written as the sum of the steady-state value and the deviation term,

$$P_{\text{net}} = P_{\text{net}_0} + \Delta P_{\text{net}} \tag{9.9}$$

where

$$P_{\text{net}_0} = P_{\text{mech}_0} - P_{\text{elec}_0}$$

$$\Delta P_{\text{net}} = \Delta P_{\text{mech}} - \Delta P_{\text{elec}}$$

Then

$$P_{\text{net}} = (P_{\text{mech}_0} - P_{\text{elec}_0}) + (\Delta P_{\text{mech}} - \Delta P_{\text{elec}}) \tag{9.10}$$

Similarly for torques,

$$T_{\text{net}} = (T_{\text{mech}_0} - T_{\text{elec}_0}) + (\Delta T_{\text{mech}} - \Delta T_{\text{elec}}) \tag{9.11}$$

Using Eq. 9.3, we can see that

$$P_{\text{net}} = P_{\text{net}_0} + \Delta P_{\text{net}} = (\omega_0 + \Delta\omega)(T_{\text{net}_0} + \Delta T_{\text{net}}) \tag{9.12}$$

Substituting Eqs. 9.10 and 9.11, we obtain

$$\begin{aligned}(P_{\text{mech}_0} - P_{\text{elec}_0}) + (\Delta P_{\text{mech}} - \Delta P_{\text{elec}}) = (\omega_0 + \Delta\omega)[(T_{\text{mech}_0} - T_{\text{elec}_0})\\ + (\Delta T_{\text{mech}} - \Delta T_{\text{elec}})]\end{aligned} \tag{9.13}$$

Assume that the steady-state quantities can be factored out since

$$P_{\text{mech}_0} = P_{\text{elec}_0}$$

and

$$T_{\text{mech}_0} = T_{\text{elec}_0}$$

and further assume that the second-order terms involving products of $\Delta\omega$ with ΔT_{mech} and ΔT_{elec} can be neglected. Then

$$\Delta P_{\text{mech}} - \Delta P_{\text{elec}} = \omega_0(\Delta T_{\text{mech}} - \Delta T_{\text{elec}}) \tag{9.14}$$

As shown in Eq. 9.7, the net torque is related to the speed change as follows:

$$(T_{\text{mech}_0} - T_{\text{elec}_0}) + (\Delta T_{\text{mech}} - \Delta T_{\text{elec}}) = I\frac{d}{dt}(\Delta\omega) \tag{9.15}$$

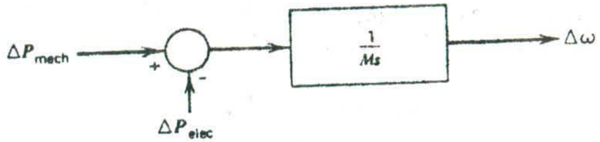


FIG. 9.3 Relationship between mechanical and electrical power and speed change.

then since $T_{mecho} = T_{eleco}$, we can combine Eqs. 9.14 and 9.15 to get

$$\Delta P_{mech} - \Delta P_{elec} = \omega_0 I \frac{d}{dt}(\Delta\omega)$$

$$= M \frac{d}{dt}(\Delta\omega) \tag{9.16}$$

This can be expressed in Laplace transform operator notation as

$$\Delta P_{mech} - \Delta P_{elec} = Ms\,\Delta\omega \tag{9.17}$$

This is shown in block diagram form in Figure 9.3.

The units for M are watts per radian per second per second. We will always use per unit power over per unit speed per second where the per unit refers to the machine rating as the base (see Example 9A).

9.3 LOAD MODEL

The loads on a power system consist of a variety of electrical devices. Some of them are purely resistive, some are motor loads with variable power–frequency characteristics, and others exhibit quite different characteristics. Since motor loads are a dominant part of the electrical load, there is a need to model the effect of a change in frequency on the net load drawn by the system. The relationship between the change in load due to the change in frequency is given by

$$\Delta P_{L(freq)} = D\,\Delta\omega \quad \text{or} \quad D = \frac{\Delta P_{L(freq)}}{\Delta\omega}$$

where D is expressed as percent change in load divided by percent change in frequency. For example, if load changed by 1.5% for a 1% change in frequency, then D would equal 1.5. However, the value of D used in solving for system dynamic response must be changed if the system base MVA is different from the nominal value of load. Suppose the D referred to here was for a net

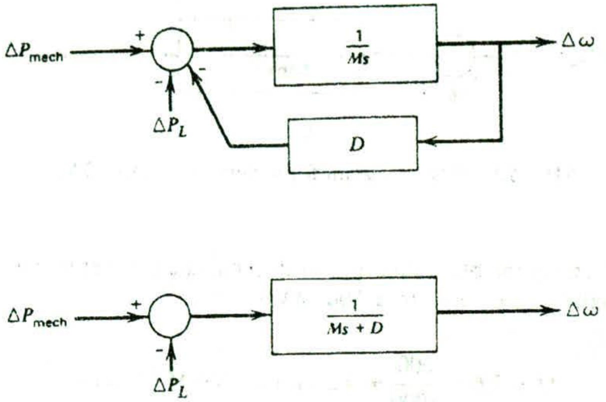


FIG. 9.4 Block diagram of rotating mass and load as seen by prime-mover output.

connected load of 1200 MVA and the entire dynamics problem were to be set up for a 1000-MVA system base. Note that $D = 1.5$ tells us that the load would change by 1.5 pu for 1 pu change in frequency. That is, the load would change by 1.5 × 1200 MVA or 1800 MVA for a 1 pu change in frequency. When expressed on a 1000-MVA base, D becomes

$$D_{1000\text{-MVA base}} = 1.5 \times \left(\frac{1200}{1000}\right) = 1.8$$

The net change in P_{elec} in Figure 9.3 (Eq. 9.15) is

$$\Delta P_{\text{elec}} = \underbrace{\Delta P_L}_{\substack{\text{Nonfrequency-}\\ \text{sensitive load}\\ \text{change}}} + \underbrace{D\,\Delta\omega}_{\substack{\text{Frequency-sensitive}\\ \text{load change}}} \tag{9.18}$$

Including this in the block diagram results in the new block diagram shown in Figure 9.4.

EXAMPLE 9A

We are given an isolated power system with a 600-MVA generating unit having an M of 7.6 pu MW/pu frequency/sec on a machine base. The unit is supplying a load of 400 MVA. The load changes by 2% for a 1% change in frequency.

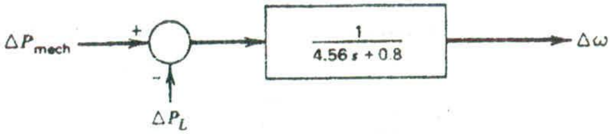


FIG. 9.5 Block diagram for system in Example 9A.

First, we will set up the block diagram of the equivalent generator load system. Everything will be referenced to a 100 MVA base.

$$M = 7.6 \times \frac{600}{1000} = 4.56 \text{ on a 1000-MVA base}$$

$$D = 2 \times \frac{400}{1000} = 0.8 \text{ on a 1000-MVA base}$$

Then the block diagram is as shown in Figure 9.5.

Suppose the load suddenly increases by 10 MVA (or 0.01 pu); that is,

$$\Delta P_L(s) = \frac{0.01}{s}$$

then

$$\Delta\omega(s) = -\frac{0.01}{s}\left(\frac{1}{4.56s + 0.8}\right)$$

or taking the inverse Laplace transform,

$$\Delta\omega(t) = (0.01/0.8)e^{-(0.8/4.56)t} - (0.01/0.8)$$

$$= 0.0125e^{-0.175t} - 0.0125$$

The final value of $\Delta\omega$ is -0.0125 pu, which is a drop of 0.75 Hz on a 60-Hz system.

When two or more generators are connected to a transmission system network , we must take account of the phase angle difference across the network in analyzing frequency changes. However, for the sake of governor analysis, which we are interested in here, we can assume that frequency will be constant over those parts of the network that are tightly interconnected. When making such an assumption, we can then lump the rotating mass of the turbine generators together into an equivalent that is driven by the sum of the individual turbine mechanical outputs. This is illustrated in Figure 9.6 where all turbine generators were lumped into a single equivalent rotating mass, M_{equiv}.

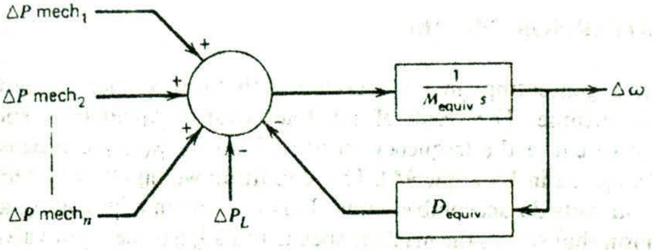


FIG. 9.6 Multi-turbine-generator system equivalent.

Similarly, all individual system loads were lumped into an equivalent load with damping coefficient, D_{equiv}.

9.4 PRIME-MOVER MODEL

The prime mover driving a generator unit may be a steam turbine or a hydroturbine. The models for the prime mover must take account of the steam supply and boiler control system characteristics in the case of a steam turbine, or the penstock characteristics for a hydro turbine. Throughout the remainder of this chapter, only the simplest prime-mover model, the nonreheat turbine, will be used. The models for other more complex prime movers, including hydro turbines, are developed in the references (see Further Reading).

The model for a nonreheat turbine, shown in Figure 9.7, relates the position of the valve that controls emission of steam into the turbine to the power output of the turbine, where

T_{CH} = "charging time" time constant

ΔP_{valve} = per unit change in valve position from nominal

The combined prime-mover–generator-load model for a single generating unit can be built by combining Figure 9.4 and 9.7, as shown in Figure 9.8.

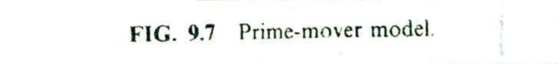


FIG. 9.7 Prime-mover model.

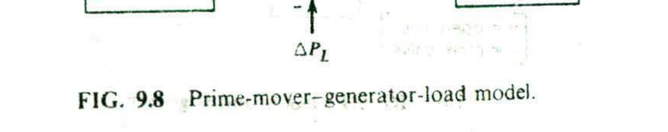


FIG. 9.8 Prime-mover–generator-load model.

9.5 GOVERNOR MODEL

Suppose a generating unit is operated with fixed mechanical power output from the turbine. The result of any load change would be a speed change sufficient to cause the frequency-sensitive load to exactly compensate for the load change (as in Example 9A). This condition would allow system frequency to drift far outside acceptable limits. This is overcome by adding a governing mechanism that senses the machine speed, and adjusts the input valve to change the mechanical power output to compensate for load changes and to restore frequency to nominal value. The earliest such mechanism used rotating "flyballs" to sense speed and to provide mechanical motion in response to speed changes. Modern governors use electronic means to sense speed changes and often use a combination of electronic, mechanical, and hydraulic means to effect the required valve position changes. The simplest governor, called the *isochronous governor*, adjusts the input valve to a point that brings frequency back to nominal value. If we simply connect the output of the speed-sensing mechanism to the valve through a direct linkage, it would never bring the frequency to nominal. To force the frequency error to zero, one must provide what control engineers call reset action. Reset action is accomplished by integrating the frequency (or speed) error, which is the difference between actual speed and desired or reference speed.

We will illustrate such a speed-governing mechanism with the diagram shown in Figure 9.9. The speed-measurement device's output, ω, is compared with a reference, ω_{ref}, to produce an error signal, $\Delta\omega$. The error, $\Delta\omega$, is negated and then amplified by a gain K_G and integrated to produce a control signal, ΔP_{valve}, which causes the main steam supply valve to open (ΔP_{valve} position) when $\Delta\omega$ is negative. If, for example, the machine is running at reference speed and the electrical load increases, ω will fall below ω_{ref} and $\Delta\omega$ will be negative. The action of the gain and integrator will be to open the steam valve, causing the turbine to increase its mechanical output, thereby increasing the electrical

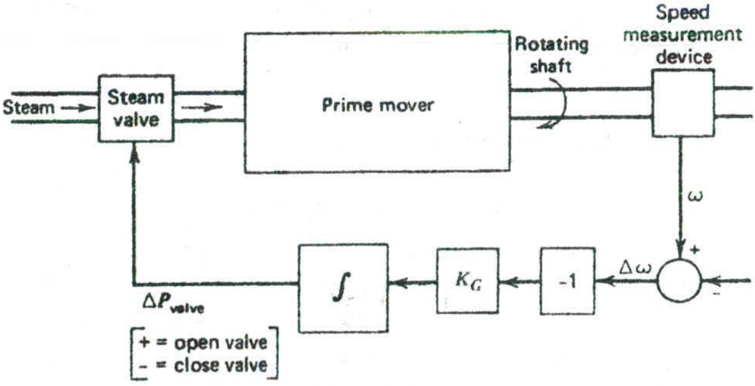


FIG. 9.9 Isochronous governor.

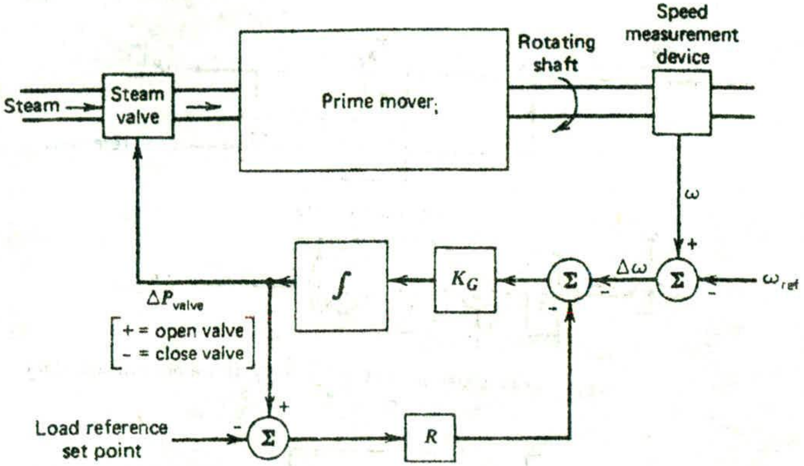


FIG. 9.10 Governor with speed-droop feedback loop.

output of the generator and increasing the speed ω. When ω exactly equals ω_{ref}, the steam valve stays at the new position (further opened) to allow the turbine generator to meet the increased electrical load.

The isochronous (constant speed) governor of Figure 9.9 cannot be used if two or more generators are electrically connected to the same system since each generator would have to have precisely the same speed setting or they would "fight" each other, each trying to pull the system's speed (or frequency) to its own setting. To be able to run two or more generating units in parallel on a generating system, the governors are provided with a feedback signal that causes the speed error to go to zero at different values of generator output.

This can be accomplished by adding a feedback loop around the integrator as shown in Figure 9.10. Note that we have also inserted a new input, called the *load reference*, that we will discuss shortly. The block diagram for this governor is shown in Figure 9.11, where the governor now has a net gain of $1/R$ and a time constant T_G.

The result of adding the feedback loop with gain R is a governor characteristic as shown in Fig. 9.12. The value of R determines the slope of the characteristic. That is, R determines the change on the unit's output for a given change in frequency. Common practice is to set R on each generating unit so that a change from 0 to 100% (i.e., rated) output will result in the same frequency change for each unit. As a result, a change in electrical load on a system will be compensated by generator unit output changes proportional to each unit's rated output.

If two generators with drooping governor characteristics are connected to a power system, there will always be a unique frequency, at which they will share a load change between them. This is illustrated in Figure 9.13, showing two units with drooping characteristics connected to a common load.

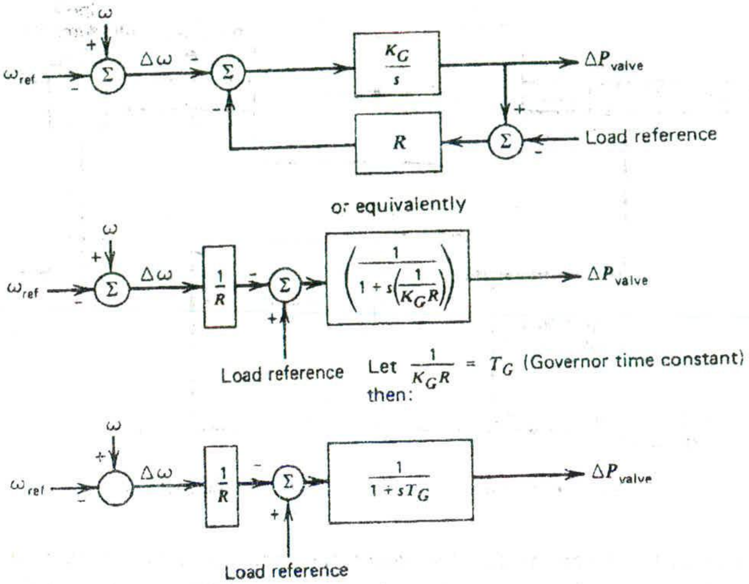


FIG. 9.11 Block diagram of governor with droop.

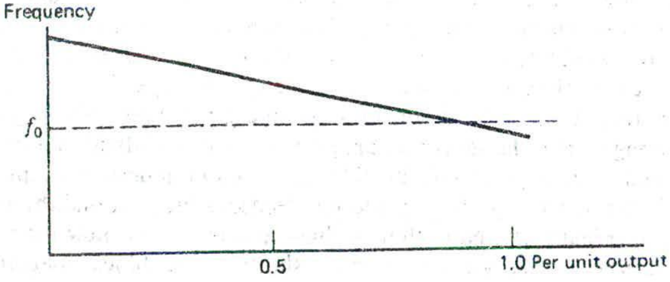


FIG. 9.12 Speed-droop characteristic.

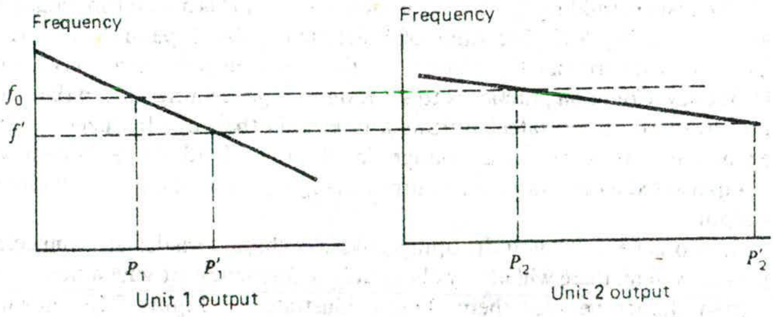


FIG. 9.13 Allocation of unit outputs with governor droop.

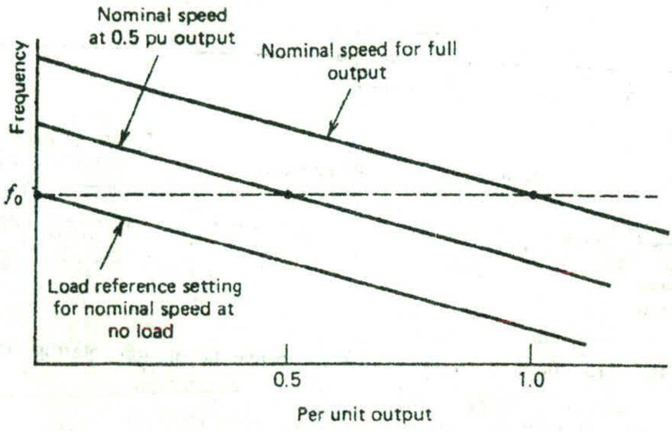


FIG. 9.14 Speed-changer settings.

As shown in Figure 9.13, the two units start at a nominal frequency of f_0. When a load increase, ΔP_L, causes the units to slow down, the governors increase output until the units seek a new, common operating frequency, f'. The amount of load pickup on each unit is proportional to the slope of its droop characteristic. Unit 1 increases its output from P_1 to P_1', unit 2 increases its output from P_2 to P_2' such that the net generation increase, $P_1' - P_1 + P_2' - P_2$, is equal to ΔP_L. Note that the actual frequency sought also depends on the load's frequency characteristic as well.

Figure 9.10 shows an input labeled "load reference set point." By changing the load reference, the generator's governor characteristic can be set to give reference frequency at any desired unit output. This is illustrated in Figure 9.14. *The basic control input to a generating unit as far as generation control is concerned is the load reference set point.* By adjusting this set point on each unit, a desired unit dispatch can be maintained while holding system frequency close to the desired nominal value.

Note that a steady-state change in ΔP_{valve} of 1.0 pu requires a value of R pu change in frequency, $\Delta\omega$. One often hears unit regulation referred to in percent. For instance, a 3% regulation for a unit would indicate that a 100% (1.0 pu) change in valve position (or equivalently a 100% change in unit output) requies a 3% change in frequency. Therefore, R is equal to pu change in frequency divided by pu change in unit output. That is,

$$R = \frac{\Delta\omega}{\Delta P} \text{ pu}$$

At this point, we can construct a block diagram of a governor–prime–mover–rotating mass/load model as shown in Figure 9.15. Suppose that this generator

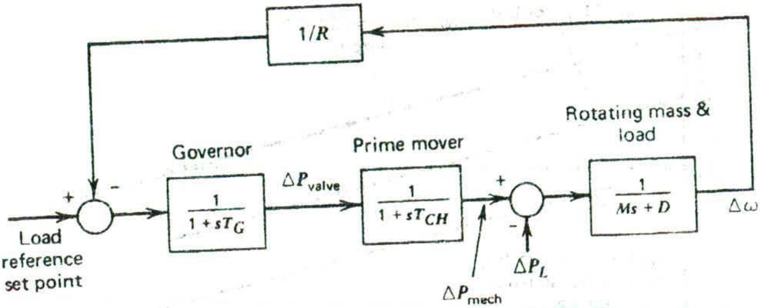


FIG. 9.15 Block diagram of governor, prime mover, and rotating mass.

experiences a step increase in load,

$$\Delta P_L(s) = \frac{\Delta P_L}{s} \tag{9.19}$$

The transfer function relating the load change, ΔP_L, to the frequency change, $\Delta\omega$, is

$$\Delta\omega(s) = \Delta P_L(s)\left[\frac{\dfrac{-1}{Ms+D}}{1+\dfrac{1}{R}\left(\dfrac{1}{1+sT_G}\right)\left(\dfrac{1}{1+sT_{CH}}\right)\left(\dfrac{1}{Ms+D}\right)}\right] \tag{9.20}$$

The steady-state value of $\Delta\omega(s)$ may be found by

$$\Delta\omega \text{ steady state} = \lim_{s\to 0}\,[s\,\Delta\omega(s)]$$

$$= \frac{-\Delta P_L\left(\dfrac{1}{D}\right)}{1+\left(\dfrac{1}{R}\right)\left(\dfrac{1}{D}\right)} = \frac{-\Delta P_L}{\dfrac{1}{R}+D} \tag{9-21}$$

Note that if D were zero, the change in speed would simply be

$$\Delta\omega = -R\,\Delta P_L \tag{9.22}$$

If several generators (each having its own governor and prime mover) were connected to the system, the frequency change would be

$$\Delta\omega = \frac{-\Delta P_L}{\dfrac{1}{R_1}+\dfrac{1}{R_2}+\cdots+\dfrac{1}{R_n}+D} \tag{9.23}$$

9.6 TIE-LINE MODEL

The power flowing across a transmission line can be modeled using the DC load flow method shown in Chapter 4.

$$P_{\text{tie flow}} = \frac{1}{X_{\text{tie}}} (\theta_1 - \theta_2) \tag{9.24}$$

This tie flow is a steady-state quantity. For purposes of analysis here, we will perturb Eq. 9.24 to obtain deviations from nominal flow as a function of deviations in phase angle from nominal.

$$\begin{aligned} P_{\text{tie flow}} + \Delta P_{\text{tie flow}} &= \frac{1}{X_{\text{tie}}} [(\theta_1 + \Delta\theta_1) - (\theta_2 + \Delta\theta_2)] \\ &= \frac{1}{X_{\text{tie}}} (\theta_1 - \theta_2) + \frac{1}{X_{\text{tie}}} (\Delta\theta_1 - \Delta\theta_2) \end{aligned} \tag{9.25}$$

Then

$$\Delta P_{\text{tie flow}} = \frac{1}{X_{\text{tie}}} (\Delta\theta_1 - \Delta\theta_2) \tag{9.26}$$

where $\Delta\theta_1$ and $\Delta\theta_2$ are equivalent to $\Delta\delta_1$ and $\Delta\delta_2$ as defined in Eq. 9.6. Then, using the relationship of Eq. 9.6,

$$\Delta P_{\text{tie flow}} = \frac{T}{s} (\Delta\omega_1 - \Delta\omega_2) \tag{9.27}$$

where $T = 377 \times 1/X_{\text{tie}}$ (for a 60-Hz system).

Note that $\Delta\theta$ must be in radians for ΔP_{tie} to be in per unit megawatts, but $\Delta\omega$ is in per unit speed change. Therefore, we must multiply $\Delta\omega$ by 377 rad/sec (the base frequency in rad/sec at 60 Hz). T may be thought of as the "tie-line stiffness" coefficient.

Suppose now that we have an interconnected power system broken into two areas each having one generator. The areas are connected by a single transmission line. The power flow over the transmission line will appear as a a positive load to one area and an equal but negative load to the other, or vice versa, depending on the direction of flow. The direction of flow will be dictated by the relative phase angle between the areas, which is determined by the relative speed deviations in the areas. A block diagram representing this interconnection can be drawn as in Figure 9.16. Note that the tie power flow was defined as going from area 1 to area 2; therefore, the flow appears as a load to area 1 and a power source (negative load) to area 2. If one assumes that mechanical powers are constant, the rotating masses and tie line exhibit damped oscillatory characteristics known as synchronizing oscillations. (See problem 9.3 at the end of this chapter.)

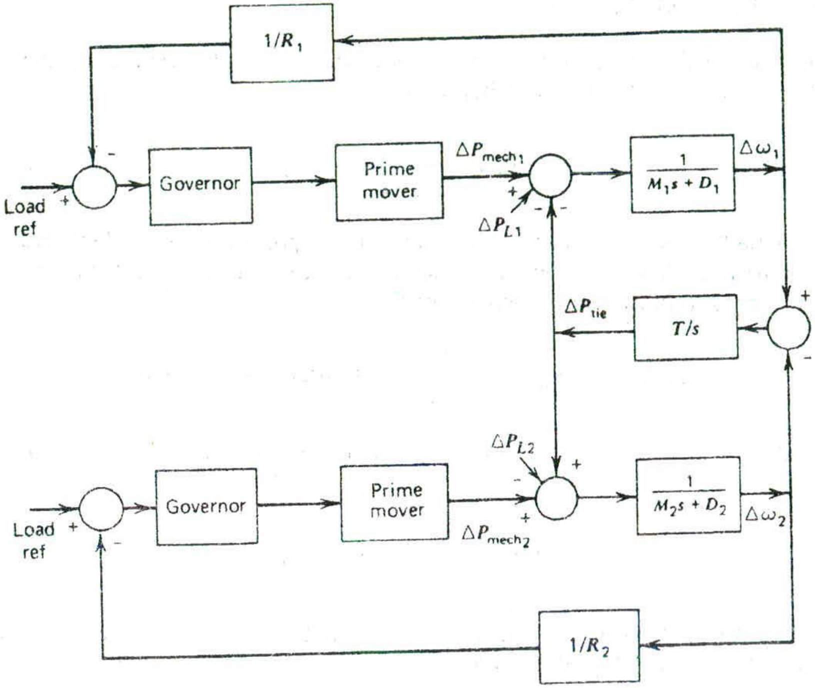


FIG. 9.16 Block diagram of interconnected areas.

It is quite important to analyze the steady-state frequency deviation, tie-flow deviation, and generator outputs for an interconnected area after a load change occurs. Let there be a load change ΔP_{L_1} in area 1. In the steady state, after all synchronizing oscillations have damped out, the frequency will be constant and equal to the same value on both areas. Then

$$\Delta\omega_1 = \Delta\omega_2 = \Delta\omega \quad \text{and} \quad \frac{d(\Delta\omega_1)}{dt} = \frac{d(\Delta\omega_2)}{dt} = 0 \tag{9.28}$$

and

$$\begin{aligned} \Delta P_{\text{mech}_1} - \Delta P_{\text{tie}} - \Delta P_{L_1} &= \Delta\omega D_1 \\ \Delta P_{\text{mech}_2} + \Delta P_{\text{tie}} &= \Delta\omega D_2 \\ \Delta P_{\text{mech}_1} &= \frac{-\Delta\omega}{R_1} \\ \Delta P_{\text{mech}_2} &= \frac{-\Delta\omega}{R_2} \end{aligned} \tag{9.29}$$

By making appropriate substitutions in Eq. 9.29,

$$
\begin{aligned}
-\Delta P_{\text{tie}} - \Delta P_{L_1} &= \Delta\omega\left(\frac{1}{R_1} + D_1\right) \\
+\Delta P_{\text{tie}} &= \Delta\omega\left(\frac{1}{R_2} + D_2\right)
\end{aligned}
\tag{9.30}
$$

or, finally

$$\Delta\omega = \frac{-\Delta P_{L_1}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + D_1 + D_2} \tag{9.31}$$

from which we can derive the change in tie flow:

$$\Delta P_{\text{tie}} = \frac{-\Delta P_{L_1}\left(\dfrac{1}{R_2} + D_2\right)}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + D_1 + D_2} \tag{9.32}$$

Note that the conditions described in Eqs. 9.28 through 9.32 are for the new steady-state conditions after the load change. The new tie flow is determined by the net change in load and generation in each area. We do not need to know the tie stiffness to determine this new tie flow, although the tie stiffness will determine how much difference in phase angle across the tie will result from the new tie flow.

EXAMPLE 9B

You are given two system areas connected by a tie line with the following characteristics.

Area 1	Area 2
$R = 0.01$ pu	$R = 0.02$ pu
$D = 0.8$ pu	$D = 1.0$ pu
Base MVA = 500	Base MVA = 500

A load change of 100 MW (0.2 pu) occurs in area 1. What is the new steady-state frequency and what is the change in tie flow? Assume both areas were at nominal frequency (60 Hz) to begin.

$$\Delta\omega = \frac{-\Delta P_{L_1}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + D_1 + D_2} = \frac{-0.2}{\dfrac{1}{0.01} + \dfrac{1}{0.02} + 0.8 + 1} = -0.00131752 \text{ pu}$$

$$f_{\text{new}} = 60 - 0.00132(60) = 59.92 \text{ Hz}$$

$$\Delta P_{\text{tie}} = \Delta\omega\left(\frac{1}{R_2} + D_2\right) = -0.00131752\left(\frac{1}{0.02} + 1\right) = -0.06719368 \text{ pu}$$

$$= -33.6 \text{ MW}$$

The change in prime-mover power would be

$$\Delta P_{\text{mech}_1} = \frac{-\Delta\omega}{R_1} = -\left(\frac{-0.00131752}{0.01}\right) = 0.13175231 \text{ pu} = 65.876 \text{ MW}$$

$$\Delta P_{\text{mech}_2} = \frac{-\Delta\omega}{R_2} = -\left(\frac{-0.00131752}{0.02}\right) = 0.06587615 \text{ pu} = 32.938 \text{ MW}$$

$$= 98.814 \text{ MW}$$

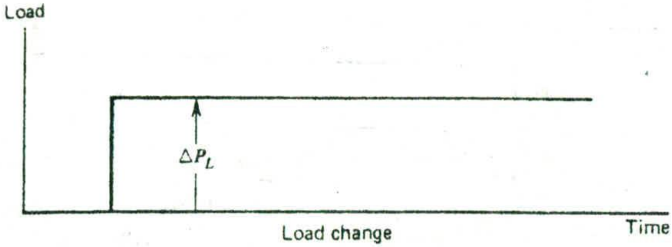
The total changes in generation is 98.814 MA, which is 1.186 MW short of the 100 MW load change. The change in total area load due to frequency drop would be

For area 1 = $\Delta\omega D_1$ = -0.0010540 pu = -0.527 MW

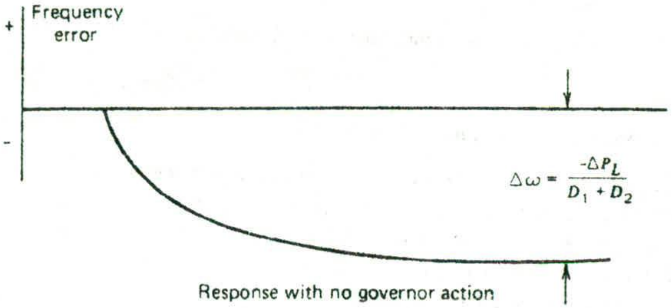
For area 2 = $\Delta\omega D_2$ = -0.00131752 pu = -0.6588 MW

Therefore, the total load change is $=1.186$ MW, which accounts for the difference in total generation change and total load change. (See Problem 9.2 for further variations on this problem.)

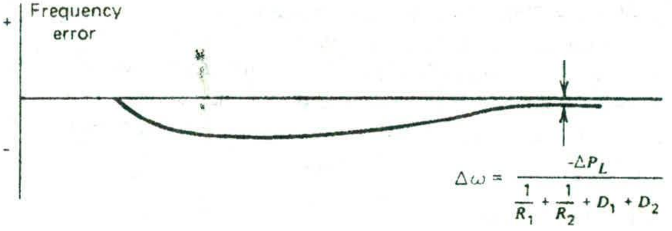
If we were to analyze the dynamics of the two-area systems, we would find that a step change in load would always result in a frequency error. This is illustrated in Figure 9.17, which shows the frequency response of the system to a step-load change. Note that Figure 9.17 only shows the average frequency (omitting any high-frequency oscillations).



Load change



Response with no governor action



Response with governor action

FIG. 9.17 Frequency response to load change.

9.7 GENERATION CONTROL

Automatic generation control (AGC) is the name given to a control system having three major objectives:

1. To hold system frequency at or very close to a specified nominal value (e.g., 60 Hz).
2. To maintain the correct value of interchange power between control areas.
3. To maintain each unit's generation at the most economic value.

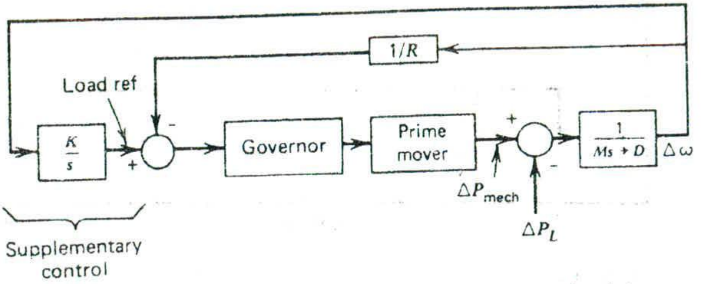


FIG. 9.18 Supplementary control added to generating init.

9.7.1 Supplementary Control Action

To understand each of the three objectives just listed, we may start out assuming that we are studying a single generating unit supplying load to an isolated power system. As shown in Section 9.5, a load change will produce a frequency change with a magnitude that depends on the droop characteristics of the governor and the frequency characteristics of the system load. Once a load change has occurred, a supplementary control must act to restore the frequency to nominal value. This can be accomplished by adding a reset (integral) control to the governor, as shown in Figure 9.18.

The reset control action of the supplementary control will force the frequency error to zero by adjustment of the speed reference set point. For example, the error shown in the bottom diagram of Figure 9.17 would be forced to zero.

9.7.2 Tie-Line Control

When two utilities interconnect their systems, they do so for several reasons. One is to be able to buy and sell power with neighboring systems whose operating costs make such transactions profitable. Further, even if no power is being transmitted over ties to neighboring systems, if one system has a sudden loss of a generating unit, the units throughout all the interconnection will experience a frequency change and can help in restoring frequency.

Interconnections present a very interesting control problem with respect to allocation of generation to meet load. The hypothetical situation in Figure 9.19 will be used to illustrate this problem. Assume both systems in Figure 9.19 have equal generation and load characteristics ($R_1 = R_2$, $D_1 = D_2$) and, further, assume system 1 was sending 100 MW to system 2 under an interchange agreement made between the operators of each system. Now, let system 2 experience a sudden load increase of 30 MW. Since both units have equal generation characteristics, they will both experience a 15 MW increase, and the tie line will experience an increase in flow from 100 MW to 115 MW. Thus, the 30 MW load increase in system 2 will have been satisfied by a 15 MW increase in generation in system 2, plus a 15 MW increase in tie flow into system 2. This

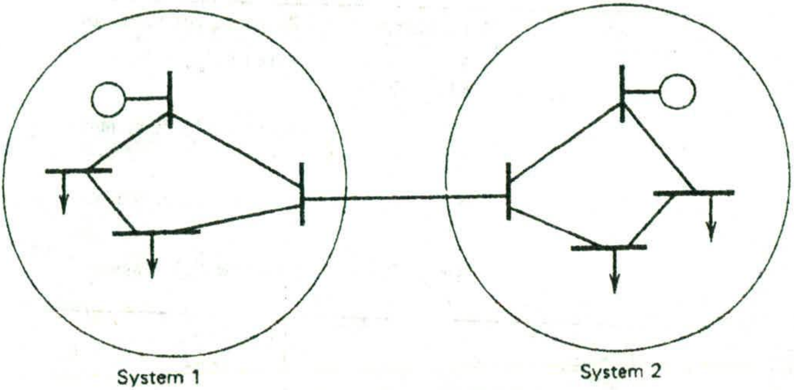


FIG. 9.19 Two-area system.

would be fine, except that system 1 contracted to sell only 100 MW, not 115 MW, and its generating costs have just gone up without anyone to bill the extra cost to. What is needed at this point is a control scheme that recognizes the fact that the 30 MW load increase occurred in system 2 and, therefore, would increase generation in system 2 by 30 MW while restoring frequency to nominal value. It would also restore generation in system 1 to its output before the load increase occurred.

Such a control system must use two pieces of information: the system frequency and the net power flowing in or out over the tie lines. Such a control scheme would, of necessity, have to recognize the following.

1. If frequency decreased and net interchange power leaving the system increased, a load increase has occurred outside the system.
2. If frequency decreased and net interchange power leaving the system decreased, a load increase has occurred inside the system.

This can be extended to cases where frequency increases. We will make the following definitions.

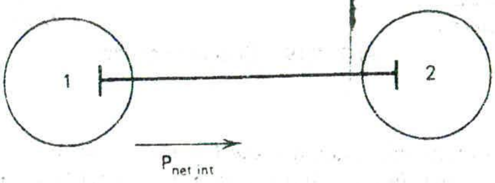
$$P_{\text{net int}} = \text{total actual net interchange}$$
$$(+ \text{ for power leaving the system; } - \text{ for power entering})$$

$$P_{\text{net int sched}} = \text{scheduled or desired value of interchange} \tag{9.33}$$

$$\Delta P_{\text{net int}} = P_{\text{net int}} - P_{\text{net int sched}}$$

Then, a summary of the tie-line frequency control scheme can be given as in the table in Figure 9.20.

$\Delta\omega$	$\Delta P_{\text{net int}}$	Load change	Resulting control action
−	−	ΔP_{L_1} + ΔP_{L_2} 0	Increase P_{gen} in system 1
+	+	ΔP_{L_1} − ΔP_{L_2} 0	Decrease P_{gen} in system 1
−	+	ΔP_{L_1} 0 ΔP_{L_2} +	Increase P_{gen} in system 2
+	−	ΔP_{L_1} 0 ΔP_{L_2} −	Decrease P_{gen} in system 2



ΔP_{L_1} = Load change in area 1
ΔP_{L_2} = Load change in area 2

FIG. 9.20 Tie-line frequency control actions for two-area system.

We define a *control area* to be a part of an interconnected system within which the load and generation will be controlled as per the rules in Figure 9.20. The control area's boundary is simply the tie-line points where power flow is metered. All tie lines crossing the boundary must be metered so that total control area net interchange power can be calculated.

The rules set forth in Figure 9.20 can be implemented by a control mechanism that weighs frequency deviation, $\Delta\omega$, and net interchange power, $\Delta P_{\text{net int}}$. The frequency response and tie flows resulting from a load change, ΔP_{L_1}, in the two-area system of Figure 9.16 are derived in Eqs. 9.28 through 9.32. These results are repeated here.

Load Change	Frequency Change	Change in Net Interchange	
ΔP_{L_1}	$\Delta\omega = \dfrac{-\Delta P_{L_1}}{\dfrac{1}{R_1}+\dfrac{1}{R_2}+D_1+D_2}$	$\Delta P_{\text{net int}_1} = \dfrac{-\Delta P_{L_1}\left(\dfrac{1}{R_2}+D_2\right)}{\dfrac{1}{R_1}+\dfrac{1}{R_2}+D_1+D_2}$	(9.34)

This corresponds to the first row of the table in Figure 9.20; we would therefore require that

$$\Delta P_{\text{gen}_1} = \Delta P_{L_1}$$

$$\Delta P_{\text{gen}_2} = 0$$

The required change in generation, historically called the *area control error* or ACE, represents the shift in the area's generation required to restore frequency and net interchange to their desired values. The equaions for ACE for each area are

$$\begin{aligned} \text{ACE}_1 &= -\Delta P_{\text{net int}_1} - B_1\,\Delta\omega \\ \text{ACE}_2 &= -\Delta P_{\text{net int}_2} - B_2\,\Delta\omega \end{aligned} \tag{9.35}$$

where B_1 and B_2 are called *frequency bias factors*. We can see from Eq. 9.34 that setting bias factors as follows:

$$\begin{aligned} B_1 &= \left(\frac{1}{R_1} + D_1\right) \\ B_2 &= \left(\frac{1}{R_2} + D_2\right) \end{aligned} \tag{9.36}$$

results in

$$\text{ACE}_1 = \left(\frac{+\Delta P_{L_1}\left(\frac{1}{R_2} + D_2\right)}{\frac{1}{R_1} + \frac{1}{R_2} + D_1 + D_2}\right) - \left(\frac{1}{R_1} + D_1\right)\left(\frac{-\Delta P_{L_1}}{\frac{1}{R_1} + \frac{1}{R_2} + D_1 + D_2}\right) = \Delta P_{L_1}$$

$$\text{ACE}_2 = \left(\frac{-\Delta P_{L_1}\left(\frac{1}{R_2} + D_2\right)}{\frac{1}{R_1} + \frac{1}{R_2} + D_1 + D_2}\right) - \left(\frac{1}{R_2} + D_2\right)\left(\frac{-\Delta P_{L_1}}{\frac{1}{R_1} + \frac{1}{R_2} + D_1 + D_2}\right) = 0$$

This control can be carried out using the scheme outlined in Figure 9.21. Note that the values of B_1 and B_2 would have to change each time a unit was committed or decommitted, in order to have the exact values as given in Eq. 9.36. Actually, the integral action of the supplementary controller will guarantee a reset of ACE to zero even when B_1 and B_2 are in error.

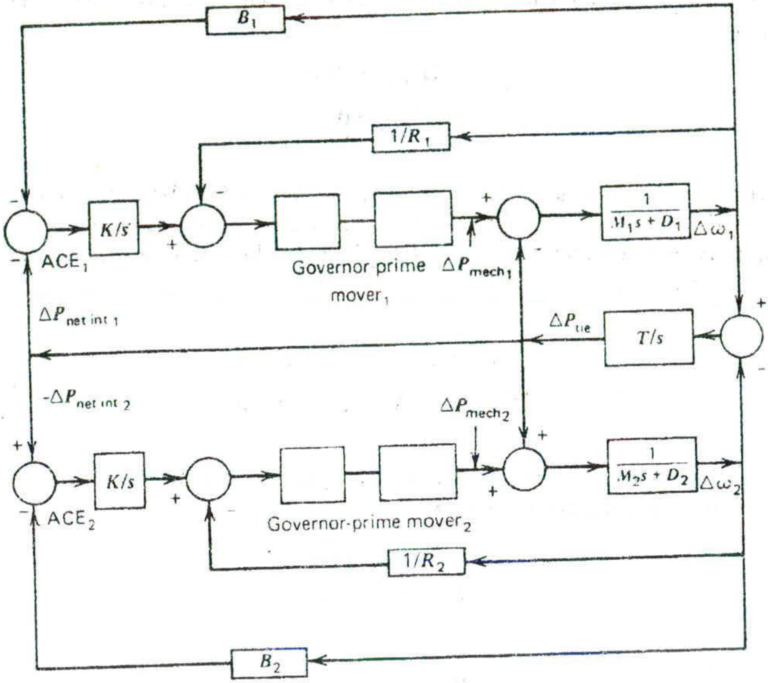


FIG. 9.21 Tie-line bias supplementary control for two areas.

9.7.3 Generation Allocation

If each control area in an interconnected system had a single generating unit, the control system of Figure 9.21 would suffice to provide stable frequency and tie-line interchange. However, power systems consist of control areas with many generating units with outputs that must be set according to economics. That is, we must couple an economic dispatch calculation to the control mechanism so it will know how much of each area's total generation is required from each individual unit.

One must remember that a particular total generation value will not usually exist for a very long time, since the load on a power system varies continually as people and industries use individual electric loads. Therefore, it is impossible to simply specify a total generation, calculate the economic dispatch for each unit, and then give the control mechanism the values of megawatt output for each unit—unless such a calculation can be made very quickly. Until the widespread use of digital computer-based control systems, it was common practice to construct control mechanisms such as we have been describing using analog computers. Although analog computers are not generally proposed for new control-center installations today, there are some in active use. An analog computer can provide the economic dispatch and allocation of generation in an area on an instantaneous basis through the use of function generators set to equal the units' incremental heat rate curves. *B* matrix loss formulas were also incorporated into analog schemes by setting the matrix coefficients on precision potentiometers.

When using digital computers, it is desirable to be able to carry out the economic-dispatch calculations at intervals of one to several minutes. Either the output of the economic dispatch calculation is fed to an analog computer (i.e., a "digitally directed analog" control system) or the output is fed to another program in the computer that executes the control functions (i.e., a "direct digital" control system). Whether the control is analog or digital, the allocation of generation must be made instantly when the required area total generation changes. Since the economic-dispatch calculation is to be executed every few minutes, a means must be provided to indicate how the generation is to be allocated for values of total generation other than that used in the economic-dispatch calculation.

The allocation of individual generator output over a range of total generation values is accomplished using base points and participation factors. The economic-dispatch calculation is executed with a total generation equal to the sum of the present values of unit generation as measured. The result of this calculation is a set of base-point generations, $P_{i_{\text{base}}}$, which is equal to the most economic output for each generator unit. The rate of change of each unit's output with respect to a change in total generation is called the unit's *participation factor*, *pf* (see Section 3.8 and Example 3I in Chapter 3). The base point and participation factors are used as follows

$$P_{i_{\text{des}}} = P_{i_{\text{base}}} + pf_i \times \Delta P_{\text{total}} \tag{9.37}$$

where

$$\Delta P_{\text{total}} = P_{\text{new total}} - \sum_{\substack{\text{all}\\ \text{gen}}} P_{i_{\text{base}}} \tag{9.38}$$

and

$P_{i_{\text{des}}}$ = new desired output from unit i

$P_{i_{\text{base}}}$ = base-point generation for unit i

pf_i = participation factor for unit i

ΔP_{total} = change in total generation

$P_{\text{new total}}$ = new total generation

Note that by definition (e.g., see Eq. 3.35) the participation factors must sum to unity. In a direct digital control scheme, the generation allocation would be made by running a computer code that was programmed to execute according to Eqs. 9.37 and 9.38.

9.7.4 Automatic Generation Control (AGC) Implementation

Modern implementation of automatic generation control (AGC) schemes usually consists of a central location where information pertaining to the system is telemetered. Control actions are determined in a digital computer and then transmitted to the generation units via the same telemetry channels. To implement an AGC system, one would require the following information at the control center.

1. Unit megawatt output for each committed unit.
2. Megawatt flow over each tie line to neighboring systems.
3. System frequency.

The output of the execution of an AGC program must be transmitted to each of the generating units. Present practice is to transmit raised or lower pulses of varying lengths to the unit. Control equipment then changes the unit's load reference set point up or down in proportion to the pulse length. The "length" of the control pulse may be encoded in the bits of a digital word that is transmitted over a digital telemetry channel. The use of digital telemetry is becoming commonplace in modern systems wherein supervisory control (opening and closing substation breakers), telemetry information (measurements of MW, MVAR, MVA voltage, etc.) and control information (unit raise/lower) is all sent via the same channels.

The basic reset control loop for a unit consists of an integrator with gain K as shown in Figure 9.22. The control loop is implemented as shown in Figure 9.23. The P_{des} control input used in Figures 9.22 and 9.23 is a function of system frequency deviation, net interchange error, and each unit's deviation from its scheduled economic output.

The overall control scheme we are going to develop starts with ACE, which is a measure of the error in total generation from total desired generation. ACE is calculated according to Figure 9.24. ACE serves to indicate when total generation must be raised or lowered in a control area. However, ACE is not the only error signal that must "drive" our controller. The individual units

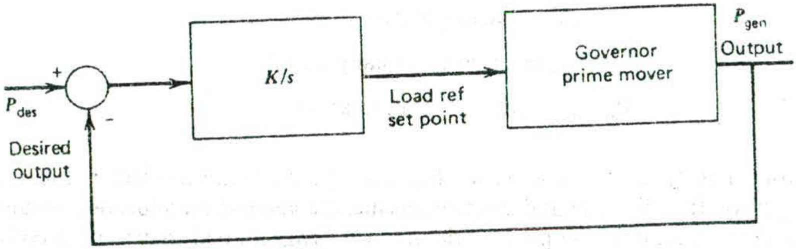


FIG. 9.22 Basic generation control loop.

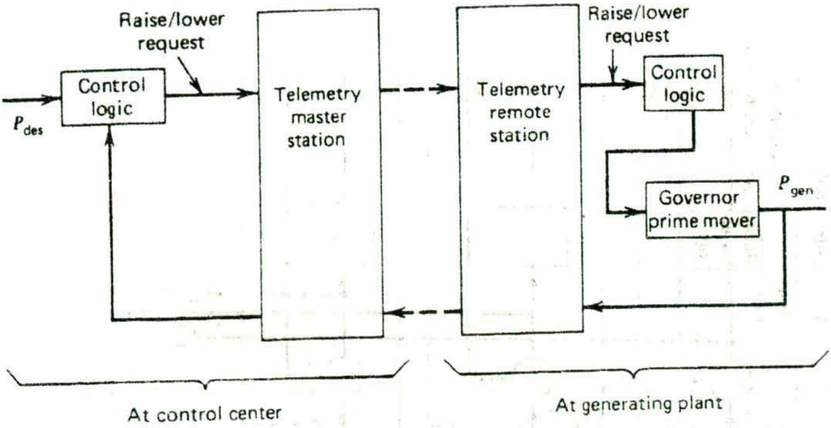


FIG. 9.23 Basic generation control loop via telemetry.

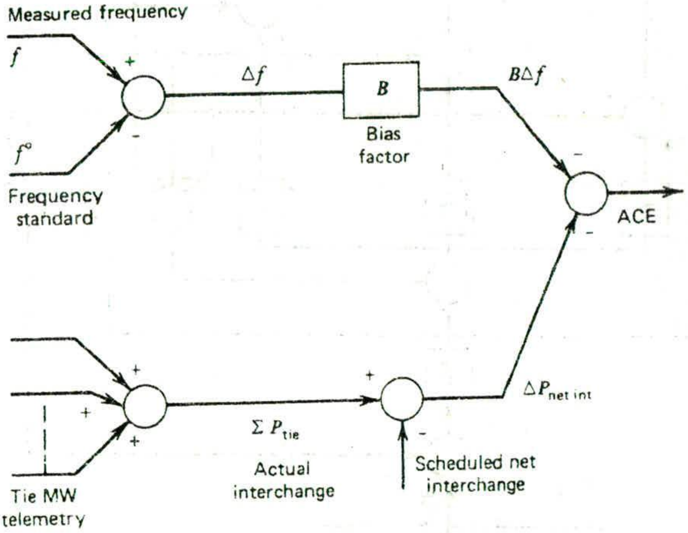


FIG. 9.24 ACE calculation.

may deviate from the economic output as determined by the base point and participation-factor calculation.

The AGC control logic must also be driven by the errors in unit output so as to force the units to obey the economic dispatch. To do this, the sum of the unit output errors is added to ACE to form a composite error signal that drives the entire control system. Such a control system is shown schematically in Figure 9.25, where we have combined the ACE calculation, the generation allocation calculation, and the unit control loop.

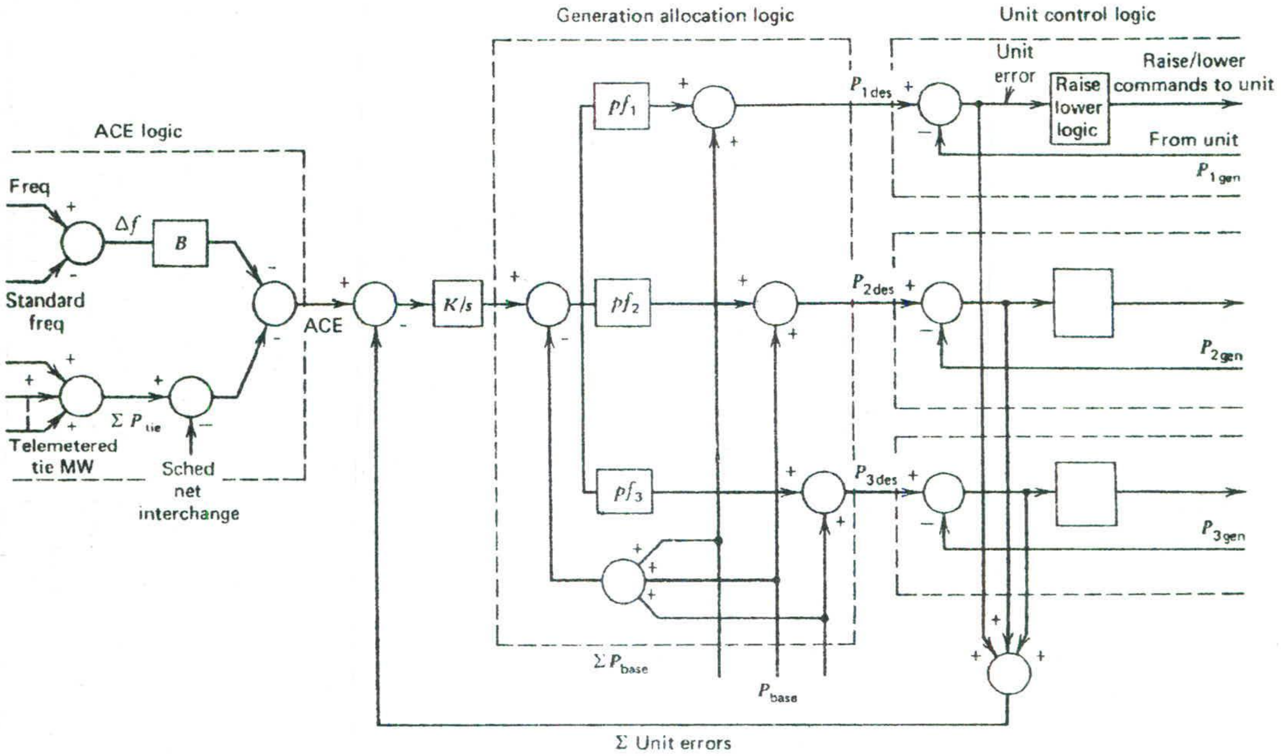


FIG. 9.25 Overview of AGC logic.

Investigation of Figure 9.25 shows an overall control system that will try to drive ACE to zero as well as driving each unit's output to its required economic value. Readers are cautioned that there are many variations to the control execution shown in Figure 9.25. This is especially true of digital implementations of AGC where great sophistication can be programmed into an AGC computer code.

Often the question is asked as to what constitutes "good" AGC design. This is difficult to answer, other than in a general way, since what is "good" for one system may be different in another. Three general criteria can be given.

1. The ACE signal should ideally be kept from becoming too large. Since ACE is directly influenced by random load variations, this criterion can be treated statistically by saying that the standard deviation of ACE should be small.
2. ACE should not be allowed to "drift." This means that the integral of ACE over an appropriate time should be small. "Drift" in ACE has the effect of creating system time errors or what are termed *inadvertent interchange errors.*
3. The amount of control action called for by the AGC should be kept to a minimum. Many of the errors in ACE, for example, are simply random load changes that need not cause control action. Trying to "chase" these random load variations, will only wear out the unit speed-changing hardware.

To achieve the objectives of good AGC, many features are added, as described briefly in the next section.

9.7.5 AGC Features

This section will serve as a simple catalog of some of the features that can be found in most AGC systems.

Assist action: Often the incremental heat rate curves for generating units will give trouble to an AGC when an excessive ACE occurs. If one unit's participation factor is dominant, it will take most of the control action and the other units will remain relatively fixed. Although it is the proper thing to do as far as economics are concerned, the one unit that is taking all the action will not be able to change its output fast enough when a large ACE calls for a large change in generation. The assist logic then comes into action by moving more of the units to correct ACE. When the ACE is corrected, the AGC then restores the units back to economic output.

Filtering of ACE: As indicated earlier, much of the change in ACE may be random noise that need not be "chased" by the generating units. Most

AGC programs use elaborate, adaptive nonlinear filtering schemes to try to filter out random noise from true ACE deviations that need control action.

Telemetry failure logic: Logic must be provided to insure that the AGC will not take wrong action when a telemetered value it is using fails. The usual design is to suspend all AGC action when this condition happens.

Unit control detection: Sometimes a generating unit will not respond to raised lower pulses. For the sake of overall control, the AGC ought to take this into account. Such logic will detect a unit that is not following raised/lower pulses and suspend control to it, thereby causing the AGC to reallocate control action among the other units on control.

Ramp control: Special logic allows the AGC to ramp a unit form one output to another at a specified rate of change in output. This is most useful in bringing units on-line and up to full output.

Rate limiting: All AGC designs must account for the fact that units cannot change their output too rapidly. This is especially true of thermal units where mechanical and thermal stresses are limiting. The AGC must limit the rate of change such units will be called on to undergo during fast load changes.

Unit control modes: Many units in power systems are not under full AGC control. Various special control modes must be provided such as manual, base load, and base load and regulating. For example, base load and regulating units are held at their base load value—but are allowed to move as assist action dictates, and are then restored to base-load value.

PROBLEMS

9.1 Suppose that you are given a single area with three generating units as shown in Figure 9.26.

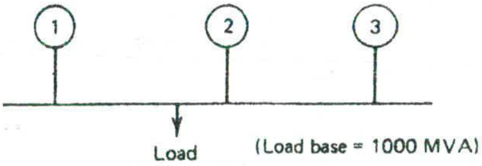


FIG. 9.26 Three-generator system for Problem 9.1.

Unit	Rating (MVA)	Speed Droop R (per unit on unit base)
1	100	0.01
2	500	0.015
3	500	0.015

The units are initially loaded as follows:

$$P_1 = 80 \text{ MW}$$
$$P_2 = 300 \text{ MW}$$
$$P_3 = 400 \text{ MW}$$

Assume $D = 0$; what is the new generation on each unit for a 50-MW load increase? Repeat with $D = 1.0$ pu (i.e., 1.0 pu on load base). Be careful to convert all quantities to a common base when solving.

9.2 Using the values of R and D in each area, for Example 9B, resolve for the 100-MW load change in area 1 under the following conditions:

Area 1: base MVA = 2000 MVA
Area 2: base MVA = 500 MVA

Then solve for a load change of 100 MW occurring in area 2 with R values and D values as in Example 9B and base MVA for each area as before.

9.3 Given the block diagram of two interconnected areas shown in Figure 9.27 (consider the prime-mover output to be constant, i.e., a "blocked" governor):

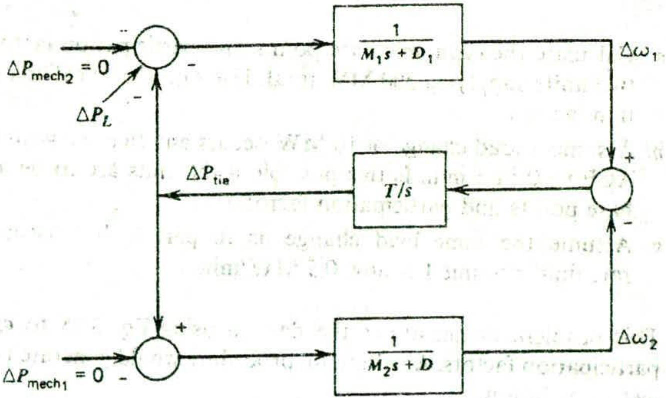


FIG. 9.27 Two-area system for Problem 9.3

a. Derive the transfer functions that relate $\Delta\omega_1(s)$ and $\Delta\omega_2(s)$ to a load change $\Delta P_L(s)$.

b. For the following data (all quantities refer to a 1000-MVA base),

$$M_1 = 3.5 \text{ pu} \qquad D_1 = 1.00$$
$$M_2 = 4.0 \text{ pu} \qquad D_2 = 0.75$$
$$T = 377 \times 0.02 \text{ pu} = 7.54 \text{ pu}$$

calculate the final frequency for load-step change in area 1 of 0.2 pu (i.e., 200 MW). Assume frequency was at nominal and tie flow was 0 pu.

c. Derive the transfer function relating tie flow, $\Delta P_{\text{tie}}(s)$ to $\Delta P_L(s)$. For the data of part b calculate the frequency of oscillation of the tie power flow. What happens to this frequency as tie stiffness increases (i.e. $T \rightarrow \infty$)?

9.4 Given two generating units with data as follows.

Unit 1: Fuel cost: $F_1 = 1.0$ Ɽ/MBtu

$$H_1(P_1) = 500 + 7P_1 + 0.002P_1^2 \text{ MBtu/h}$$
$$150 < P_1 < 600 \quad \text{Rate limit} = 2 \text{ MW/min}$$

Unit 2: Fuel cost: $F_2 = 0.98$ Ɽ/MBtu

$$H_2(P_2) = 200 + 8P_2 + 0.0025P_2^2 \text{ MBtu/h}$$
$$125 \leq P_2 \leq 500 \text{ MW} \quad \text{Rate limit} = 2 \text{ MW/min}$$

a. Calculate the economic base points and participation factors for these two units supplying 500 MW total. Use Eq. 3.35 to calculate participation factors.

b. Assume a load change of 10 MW occurs and that we wish to clear the ACE to 0 in 5 min. Is this possible if the units are to be allocated by base points and participation factors?

c. Assume the same load change as in part b, but assume that the rate limit on unit 1 is now 0.5 MW/min.

This problem demonstrates the flaw in using Eq. 3.35 to calculate the **participation** factors. An alternate procedure would generate participation factors as follows.

Let t be the time in minutes between economic-dispatch calculation executions. Each unit will be assigned a range that must be obeyed in performing the economic dispatch.

$$P_i^{\max} = P_i^0 + t \times \text{rate limit}_i$$
$$P_i^{\min} = P_i^0 - t \times \text{rate limit}_i \qquad (9.39)$$

The range thus defined is simply the maximum and minimum excursion the unit could undergo within t minutes. If one of the limits described is outside the unit's normal economic limits, the economic limit would be used. Participation factors can then be calculated by resolving the economic dispatch at a higher value and enforcing the new limits described previously.

d. Assume $T = 5$ min and that the perturbed economic dispatch is to be resolved for 510 MW. Calculate the new participation factors as

$$pf_i = \frac{P_i^\Delta - P_{i_{\text{base pt}}}}{\Delta P_{\text{total}}}$$

where

$$P_{i_{\text{base pt}}} = \text{base economic solution}$$

$$P_{1_{\text{base}}} + P_{2_{\text{base}}} = 500 \text{ MW}$$

$$P_i^\Delta = \text{perturbed solution}$$

$$P_1^\Delta + P_2^\Delta = 510 \text{ MW}$$

with limits as calculated in Eq. 9.35.

Assume the initial unit generations P_i^0 were the same as the base points found in part a. And assume the rate limits were as in part c (i.e., unit 1 rate lim = 0.5 MW/min, unit 2 rate lim = 2 MW/min). Now check to see if 1part c gives a different result.

9.5 The interconnected systems in the eastern United States and Canada have a total capacity of about 5×10^5 MW. The equivalent inertia and damping constants are approximately

$$M = 8 \text{ pu MW/pu frequency/sec}$$

and

$$D = 1.5$$

both on the system capacity base. It is necessary to correct for time errors every so often. The electrical energy involved is not insignificant.

a. Assume that a time error of 1 sec is to be corrected by deliberately supplying a power unbalance of a constant amount for a period of 1 h. Find the power unbalance required. Express the amount in MWH.

b. Is this energy requirement a function of the power unbalance? Assume a power unbalance is applied to the system of a duration "delta T". During this period, the unbalance of power is constant; after the period it is zero. Does it make any difference if the length of time is long or short? Show the response of the system. The time deviation is the integral of the frequency deviation.

9.6 In Fig. 9.16 assume that system 2 represents a system so large that it is effectively an "infinite bus." M_2 is much greater than M_1 and the frequency deviation in system 2 is zero.

- **a.** Draw the block diagram including the tie line between areas 1 and 2. What is the transfer function for a load change in area 1 and the tie flow?
- **b.** The reactance of the tie is 1 pu on a 1000-MW base. Initially, the tie flow is zero. System 1 has an inertia constant (M_1) of 10 on the same base. Load damping and governor action are neglected. Determine the equation for the tie-line power flow swings for a sudden short in area 1 that causes an instantaneous power drop of 0.02 pu (2%), which is restored instantly. Assume that $\Delta P_{L_i}(s) = -0.02$, and find the frequency of oscillation and maximum angular deviation between areas 1 and 2.

FURTHER READING

The reader should be familiar with the basics of control theory before attempting to read many of the references cited here. A good introduction to automatic generation control is the book *Control of Generation and Power Flow on Interconnected Systems*, by Nathan Cohn (reference 4 in Chapter 1). Other sources of introductory material are contained in references 1–3.

Descriptions of how steam turbine generators are modeled are found in references 4 and 5; reference 6 shows how hydro-units can be modeled. Reference 7 shows the effects to be expected from various prime-mover and governing systems. References 8–10 are representative of advances made in AGC techniques through the late 1960s and early 1970s. Other special interests in AGC design include special-purpose optimal filters (see references 10 and 11), direct digital control schemes (see references 12–15), and control of jointly owned generating units (see reference 16).

Research in control theory toward "optimal control" techniques was used in several papers presented in the late 1960s and early 1970s. As far as is known to the authors, optimal control techniques have not, as of the writing of this text, been utilized successfully in a working AGC system. Reference 17 is representative of the papers using optimal control theory.

Recent research has included an approach that takes the short-term load forecast, economic dispatch, and AGC problems, and approaches them as one overall control problem. References 18 and 19 illustrate this approach. References 20–22 are excellent overviews of more recent work in AGC.

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