Journal of Service Science and Management, 2015, 8, 874-885
Published Online December 2015 in SciRes. http://www.scirp.org/journal/jssm
http://dx.doi.org/10.4236/jssm.2015.86088

A Production Inventory Model of Power Demand and Constant Production Rate Where the Products Have Finite Shelf-Life

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Received 25 November 2015; accepted 26 December 2015; published 29 December 2015

Abstract

A production inventory model has been developed in this paper, basing on constant production rate and market demand, which varies time to time. Seeing the demand pattern the proposed model has been formulated in a power pattern which can be expressed in a linear or exponential form. The model finds the total average optimum inventory cost and optimum time cycle. The model also considers the small amount of decay. Without having backlogs, production starts. Reaching at the desired level of inventories, it stops production. After that due to demands along with the deterioration, it initiates its depletion and after certain periods the inventory gets zero. The model has also been justified with proving the convex property and by giving a numerical example with the sensitivity test.

Keywords

Production Inventory, Shelf-Life, Power Demand, Linear Demand, Exponential Demand and Production Rate

1. Introduction

In last few decades, inventory problems have been studied in a large scale. Inventory is related to stock the items before delivering to the customers. If we visualize our daily needs, we can see that these needs are of two kinds as far as deterioration or decay is concerned. Items like radioactive substances, food grains, fashionable items, pharmaceuticals etc. are one kind, which have finite shelf-life, *i.e.* limited life once in the shelf, or have a sufficient deterioration after a particular time. On the other hand, the items like bricks, steels, heavy woods etc. are

How to cite this paper: Ukil, S.I., Islam, M.E. and Uddin, Md.S. (2015) A Production Inventory Model of Power Demand and Constant Production Rate Where the Products Have Finite Shelf-Life. *Journal of Service Science and Management*, **8**, 874-885. http://dx.doi.org/10.4236/jssm.2015.86088

the other kind which does not have that much of deterioration the one mentioned before. Due to the limited shelf-life and market demand, the stock level or inventory continuously decreases and in this way deterioration occurs. This deterioration affects the inventory seriously and inventory cost increases. To make the inventory cost at optimum level *i.e.* to get the minimum inventory cost, a suitable inventory model is required which suits to meet the actual demand in the market. In minimizing inventory cost this paper proposes an inventory model with power demand, small amount of decay and constant production rate, whereas the existing models very often ignore the production rate; instead those consider the instantaneous replenishment rate. The power demand defines that kind of demand which varies with change of power in the power function.

The organizations give due importance to few parameters which affect the model. Like, in this proposed model power demand has been considered as the market may have a demand of linear type, again shortly it may have the demand of exponential function. The linear demand means that the firms receive demand either in an increasing or decreasing way, but gradually, not suddenly, *i.e.* demand as a linear function. Again, the exponential demand means the demand either in an exponentially increasing way or exponentially decreasing way suddenly. It may be clarified more giving an example, like, e^x is an exponential demand. If the value of x increases, the value of e^x increases geometrically. Again $2x$ can be treated as linear demand as the value of x increases, the value of $2x$ increases slowly. To develop a new model, we have considered that the production rate is constant and greater than demand rate at any time. Production starts when inventory level is zero. Inventory level is highest at $T = t_1$ and after this point, the inventory depletes quickly due to demand only. Demand is considered a type of power pattern in this paper *i.e.* the form of linear or exponential. For unit inventory, amount of decay rate is very small and constant.

Satisfying the convex property and using a numerical example, the paper could justify that the objective of formulating this model is achieved. The objective of the proposed model is to get the optimum inventory cost and optimum time cycle by introducing a time dependent inventory model with constant production rate and power demand. The paper subsequently advances with literature review, assumptions, notations used in the model, development of the model, numerical illustration, sensitivity analysis, conclusion and suggestions for future work in this field.

2. Literature Review

Many researchers have work in the field of inventory problems or in production inventory model to solve the real life problems by building the suitable inventory models. On ground, the business institution faces various types of demand. Demand may be linear, quadratic, exponential, time dependent, level or stock dependent, price dependent etc. Basing on the demand pattern, the firms decide how much to produce and when to produce. Initially, Harris [1] discusses inventory model in 1915. Whitin [2] was the first researcher to develop the inventory model for fashionable goods considering its little decay in the inventory. Ghare and Schrader [3] first pointed out the effect of decay in inventory analysis and discovered the economic order quantity (EOQ) model. They showed the nature of the consumption of the deteriorating items. Sarker *et al.* [4] explained an inventory model where demand was a composite function consisting of a constant component and a variable component proportional to inventory level in a period in which decay was exponential and inventory was positive. Teng *et al.* [5] developed the model with deteriorating items and shortages assuming that the demand function was positive and fluctuating with respect to time. Skouri and Papachristos [6] discussed a continuous review inventory model considering the five costs as deterioration, holding, shortage, opportunity cost due to the lost sales and the replenishment cost due to the linearly dependency on the lot size. Chund and Wee [7] developed an integrated two stages production inventory deteriorating model for the buyer and the supplier on the basis of stock dependent selling rate considering imperfect items and just in time (JIT) multiple deliveries. Applying inventory replenishment policy Mingbao and Wang [8] expressed the inventory model for deteriorating items with trapezoidal type demand rate, where the demand rate is a piecewise linearly functions. Hassan and Bozorgi [9] developed the location of distribution centers with inventory. Chang and Dye [10] expressed an inventory model with deteriorating items, with time varying demand and partial backlogging. Tripathy and Mishra [11] discuss the inventory model with ordering policy for weibull deteriorating items, quadratic demand, and permissible delay in payments. Sarkar *et al*. [12] introduced and inventory model with finite replenishment rate, trade credit policy and price discount offer. Khieng *et al*. [13] presented a production model for the lot-size, order level inventory system with finite production rate and the effect of decay. Ekramol [14] [15] considered various production rates assuming the demand is constant. Mishra *et al*. [16] explained an inventory model for deteriorating items with time dependent demand and time varying holding cost under partial backlogging. Aggarwal [17] developed an order level inventory model for a system constant rate of deterioration. Shiraj [18] discussed the effect of just in time manufacturing system on EOQ model. Sivazlian and Stenfel [19] determined the optimum value of time cycle by using the graphical solution of the equation to obtain the economic order quantity model. Shah and Jaiswal [20] and Dye [21] established an inventory model by considering demand as a function of selling price and three parameters of Weibull rate of deterioration. Billington [22] discussed classic economic production quantity (EPQ) model without backorders or backlogs. Pakkala and Achary [23] established a deterministic inventory model for deteriorating items with two warehouses, while the replenishment rate was finite, demand was uniform and shortage was allowed. Abad [24] discussed regarding optimal pricing and lot sizing under conditions of perish ability and partial backordering. Sing and Pattanak [25]-[27] developed the model for deterioration and time dependent quadratic demand under permissible delay in payment, whereas, we have used the exponential demand with constant production, but ignoring the payment aspect. Amutha and Chandrasekaran [28] formulated the inventory model with deterioration items, quadratic demand and time dependent holding cost. But in our proposed model, we have emphasized on the production rate, power demand and constant holding cost. Ouyang and Cheng [29] explained the inventory model for deteriorating items with exponential declining demand and partial backlogging [30]. Dave and Patel introduced an inventory model for deteriorating Items with time proportional demand, whereas in our proposed model we have included demand in a power pattern.

3. Development of the Model

To develop the proposed model it needs numbers of notations or symbols to clarify the assumptions considered and description explained in this paper. The notations or symbols used in this paper are cited in **Table 1** below.

There may be various types of demand of different types of items in the market. At times it may be linear. Again within vary short span of time it may be changed into exponential type of demand. Taking this type of situation into cognizance, this model is developed. The model is suitable for those kinds of products which have finite shelf-life and ultimately causes the products decay. At the beginning, while time $T = 0$, the production starts with zero inventory with the rate λ which remains constant for entire production cycle. We have considered the demand as a power function which will be in various forms due to the different values of *n*. Say for example, if we consider $n = 1$, the demand function ab^n will be linear and for $n = 2$, it will be exponential. We build up the model by considering the demand as power function in general. Here, demand function in **Figure 1** indicates the linear function as a kind of power function:

Table 1. Notations.

Ser	Notations	Description
1	λ	Production rate.
2	ab^n	Demand rate at time *T* where a,b is any positive integer and $n = 0,1,2,\cdots$ satisfying the condition $\lambda > ab^n$.
3	μ	Very small amount of constant decay rate for unit inventory.
4	$I(\theta)$	Inventory level at instant θ.
5	I_1	Un-decayed inventory at $T = 0$ to t_1.
6	I_2	Un-decayed inventory at $T = t_1$ to T_1.
7	D_1	Deteriorating inventory at $T = 0$ to t_1.
8	D_2	Deteriorating inventory at $T = t_1$ to T_1.
9	Q	Inventory level at time *T* which depicts 0 and Q_1 respectively at $T = 0$ and t_1.
10	$\mathrm{d}\theta$	Vary small portion of instant θ.
11	K_0	Set up cost.
12	h	Average holding cost.
13	$TC(Q_1)$	Total cost in terms of Q_1.
14	Q_1^*	Optimum order quantity.
15	T_1^*	Optimum order interval.

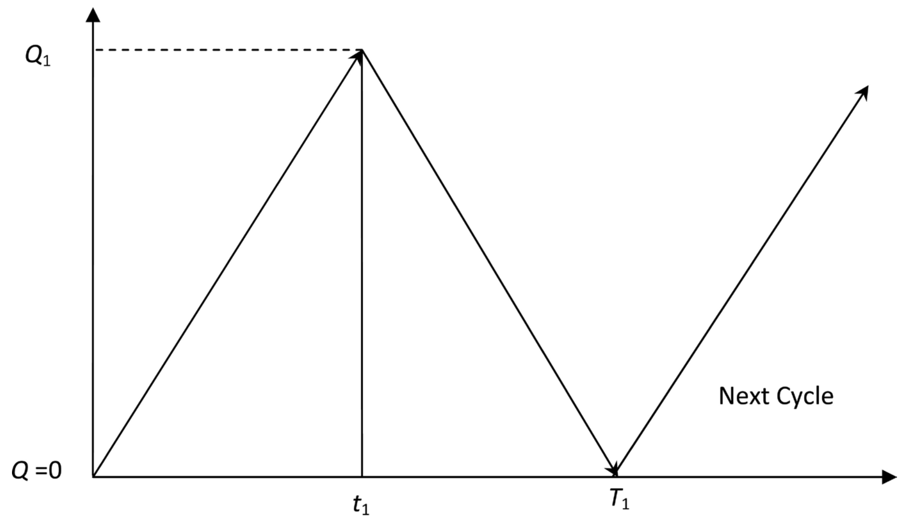

Figure 1. Inventory model with linear demand.

During the period, $T = 0$ to t_1, the inventory increases at the rate of $\mu\lambda - ab^n - \mu I(\theta)$, as ab^n is the demand in the market and $\mu I(\theta)$ is the decay of $I(\theta)$ inventories at instant θ where, μ is the decay of unit inventory in the period. By using the above arguments we can have the following equation:

$$I(\theta + \mathrm{d}\theta) = I(\theta) + (\lambda - ab^n)\mathrm{d}\theta - \mu I(\theta)\mathrm{d}\theta$$

$$\text{or, } I(\theta + \mathrm{d}\theta) - I(\theta) = \left\{(\lambda - ab^n) - \mu I(\theta)\right\}\mathrm{d}\theta$$

$$\text{or, } \operatorname*{Lim}_{\mathrm{d}\theta \to 0} \frac{I(\theta + \mathrm{d}\theta) - I(\theta)}{\mathrm{d}\theta} = (\lambda - ab^n) - \mu I(\theta)$$

$$\text{or, } \frac{\mathrm{d}}{\mathrm{d}\theta} I(\theta) + \mu I(\theta) = \lambda - ab^n.$$

The general solution of the differential equation is, $I(\theta) = \dfrac{\lambda - ab^n}{\mu} + A\mathrm{e}^{-\mu\theta}$.

We now apply the following boundary condition, at $\theta = 0$, we get, $I(\theta) = 0$.

By solving we get, $A = -\dfrac{\lambda - ab^n}{\mu}$.

Therefore,
$$I(\theta) = \frac{\lambda - ab^n}{\mu}\left(1 - \mathrm{e}^{-\mu\theta}\right). \tag{1}$$

Putting another boundary condition, *i.e.* at $\theta = t_1$, $I(\theta) = Q_1$, taking up to first degree of μ ,we get the following equation:

$$\begin{aligned} Q_1 &= \frac{\lambda - ab^n}{\mu}\left(1 - \mathrm{e}^{-\mu\theta}\right) \\ &= \frac{\lambda - ab^n}{\mu}(-\mu t_1) = (\lambda - ab^n)t_1. \end{aligned}$$

Thereby,
$$t_1 = \frac{Q_1}{\lambda - ab^n}. \tag{2}$$

With the help of Equation (1) and by considering up to second degree of μ, the total un-decayed inventory during $\theta = 0$ to t_1 we get,

$$I_1=\int_0^{t_1} I(\theta)\mathrm{d}\theta=\left[\frac{\lambda-ab^n}{\mu}\theta-\frac{\lambda-ab^n}{\mu}\left(\frac{\mathrm{e}^{-\mu\theta}}{-\mu}\right)\right]_0^{t_1}=\left[\frac{\lambda-ab^n}{\mu}\theta+\frac{\lambda-ab^n}{\mu^2}\left(\mathrm{e}^{-\mu\theta}\right)\right]_0^{t_1}$$
$$=\frac{\lambda-ab^n}{\mu}t_1+\frac{\lambda-ab^n}{\mu}\left(\frac{\mathrm{e}^{-\mu t_1}-1}{-\mu}\right)$$
$$=\frac{\lambda-ab^n}{\mu}t_1-\frac{\lambda-ab^n}{\mu}\left(-\mu t_1+\frac{1}{2}\mu^2 t_1^2\right)$$
$$=\frac{1}{2}\left(\lambda-ab^n\right)t_1^2. \tag{3}$$

Considering the decay of the items, we calculate the deteriorating items during the period as below:

$$D_1=\int_0^{t_1}\mu I(\theta)\mathrm{d}\theta=\mu\left[\frac{\lambda-ab^n}{\mu}\theta-\frac{\lambda-ab^n}{\mu}\left(\frac{\mathrm{e}^{-\mu\theta}}{-\mu}\right)\right]_0^{t_1}=\frac{1}{2}\mu\left(\lambda-ab^n\right)t_1^2. \tag{4}$$

Again during $T=t_1$ to T_1, the inventory decreases at the rate of ab^n, as there is no production after time t_1 and inventory reduces due to market demand only. Applying the similar arguments as used before, we get the differential equation as mentioned below:

$$\frac{\mathrm{d}}{\mathrm{d}\theta}I(\theta)+\mu I(\theta)=-ab^n.$$

The general solution of the differential equation is defined below:

$$I(\theta)=\frac{-ab^n}{\mu}+B\mathrm{e}^{-\mu\theta}.$$

Applying the boundary condition at $\theta=T_1$, we get, $I(\theta)=0$.

By solving we get, $B=\frac{ab^n}{\mu}\mathrm{e}^{\mu T_1}$.

Therefore,
$$I(\theta)=-\frac{ab^n}{\mu}+\frac{ab^n}{\mu}\mathrm{e}^{\mu(T_1-\theta)}. \tag{5}$$

Putting another boundary condition, *i.e.* at $\theta=t_1$, $I(\theta)=Q_1$, taking up to first degree of μ ,we get the following equation:

$$Q_1=-\frac{ab^n}{\mu}+\frac{ab^n}{\mu}\mathrm{e}^{\mu(T_1-t_1)}=-\frac{ab^n}{\mu}+\frac{ab^n}{\mu}\left\{1+\mu\left(T_1-t_1\right)\right\}=ab^n\left(T_1-t_1\right).$$

Thereby,
$$t_1=T_1-\frac{Q_1}{ab^n}. \tag{6}$$

Now, with the help of Equation (5) and by considering up to the second degree of μ we get the un-decayed inventory during $T=t_1$ to T_1 as:

$$I_2=\int_{t_1}^{T_1} I(\theta)\mathrm{d}\theta=\left[\frac{\lambda-ab^n}{\mu}\theta-\frac{\lambda-ab^n}{\mu}\left(\frac{\mathrm{e}^{-\mu\theta}}{-\mu}\right)\right]_{t_1}^{T_1}=\frac{\lambda-ab^n}{\mu}\left(T_1-t_1\right)+\frac{\lambda-ab^n}{\mu^2}\left(\mathrm{e}^{-\mu T_1}-\mathrm{e}^{-\mu t_1}\right)$$
$$=\frac{\lambda-ab^n}{\mu}\left(T_1-t_1\right)+\frac{\lambda-ab^n}{\mu^2}\left(-\mu T_1+\frac{1}{2}\mu^2T_1^2+\mu t_1-\frac{1}{2}\mu^2t_1^2\right)$$
$$=\frac{\lambda-ab^n}{\mu}\left(T_1-t_1\right)-\frac{\lambda-ab^n}{\mu}\left(T_1-t_1\right)\left(1-\frac{1}{2}\mu T_1-\frac{1}{2}\mu t_1\right)$$
$$=\frac{1}{2}\mu\left(\lambda-ab^n\right)\left(T_1^2-t_1^2\right). \tag{7}$$

Considering the decay of the items, we calculate the deteriorating items during the period as below:

$$D_2=\int_{t_1}^{T_1}\mu I(\theta)\mathrm{d}\theta=\mu\left[\frac{\lambda-ab^n}{\mu}\theta-\frac{\lambda-ab^n}{\mu}\left(\frac{\mathrm{e}^{-\mu\theta}}{-\mu}\right)\right]_{t_1}^{T_1}=\frac{1}{2}\mu^2\left(\lambda-ab^n\right)\left(T_1^2-t_1^2\right). \tag{8}$$

From Equations (2) and (6) we get,

$$\frac{Q_1}{\lambda-ab^n}=T_1-\frac{Q_1}{ab^n}.$$

Therefore,

$$T_1=\frac{\lambda Q_1}{ab^n\left(\lambda-ab^n\right)}. \tag{9}$$

Total Cost Function: The cost function can be written in the form given below:

$$TC\left(Q_1\right)=\frac{K_0+h\left(I_1+D_1+I_2+D_2\right)}{T_1}. \tag{10}$$

By using Equations (3), (4), (7) and (8) in (10), we get the following result,

$$\begin{aligned}TC\left(Q_1\right)&=\frac{1}{T_1}\left[K_0+\left(\frac{h}{2}+\frac{h\mu}{2}\right)\left(\lambda-ab^n\right)t_1^2+\left(\frac{h\mu}{2}+\frac{h\mu^2}{2}\right)\left(\lambda-ab^n\right)\left(T_1^2-t_1^2\right)\right]\\&=\frac{1}{T_1}\left[K_0+\left(\frac{h}{2}+\frac{h\mu}{2}\right)\left(\lambda-ab^n\right)\left(\frac{Q_1}{\lambda-ab^n}\right)^2+\left(\frac{h\mu}{2}+\frac{h\mu^2}{2}\right)\left(\lambda-ab^n\right)\left\{T_1^2-\left(\frac{Q_1}{\lambda-ab^n}\right)^2\right\}\right]\\&=\frac{1}{T_1}\left[K_0+\left(\frac{h}{2}+\frac{h\mu}{2}\right)\left(\frac{Q_1^2}{\lambda-ab^n}\right)+\left(\frac{h\mu}{2}+\frac{h\mu^2}{2}\right)\left(\lambda-ab^n\right)T_1^2-\left(\frac{h\mu}{2}+\frac{h\mu^2}{2}\right)\left(\frac{Q_1^2}{\lambda-ab^n}\right)\right].\end{aligned}$$

Now using Equations (2) and (9) we get the value as,

$$\begin{aligned}&=\frac{ab^n\left(\lambda-ab^n\right)}{\lambda Q_1}\left[K_0+\left(\frac{h}{2}+\frac{h\mu}{2}\right)\left(\frac{Q_1^2}{\lambda-ab^n}\right)+\left(\frac{h\mu}{2}+\frac{h\mu^2}{2}\right)\left(\lambda-ab^n\right)\left(\frac{\lambda^2Q_1^2}{a^2b^{2n}\left(\lambda-ab^n\right)^2}\right)\right]\\&\quad+\frac{ab^n\left(\lambda-ab^n\right)}{\lambda Q_1}\left[-\left(\frac{h\mu}{2}+\frac{h\mu^2}{2}\right)\left(\frac{Q_1^2}{\lambda-ab^n}\right)\right]\\&=\frac{ab^n\left(\lambda-ab^n\right)}{\lambda Q_1}\left[K_0+\left\{\frac{h}{2}+\frac{\lambda^2}{a^2b^{2n}}\left(\frac{h\mu}{2}+\frac{h\mu^2}{2}\right)-\frac{h\mu^2}{2}\right\}\left(\frac{Q_1^2}{\lambda-ab^n}\right)\right]\\&=\frac{K_0ab^n\left(\lambda-ab^n\right)}{\lambda Q_1}+\frac{hab^n}{2\lambda}\left(1-\mu^2+\frac{\lambda\mu}{a^2b^{2n}}+\frac{\lambda^2\mu^2}{a^2b^{2n}}\right)Q_1.\end{aligned} \tag{11}$$

The objective is now to find out the order quantity Q_1 that minimizes the total inventory cost for the inventory system. The cost depicts Equation (11). In order to determine the optimum order quantity Q_1^* and to verify that Equation (11) is convex in Q_1, we must show that the following two well known properties hold,

1) $\frac{\mathrm{d}}{\mathrm{d}Q_1}TC\left(Q_1\right)=0$ and;

2) $\frac{\mathrm{d}^2}{\mathrm{d}Q_1}TC\left(Q_1\right)>0$.

From the convex property 1) *i.e.* $\frac{\mathrm{d}}{\mathrm{d}Q_1}TC\left(Q_1\right)=0$ we get the equation as below:

$$\frac{K_0 ab^n\left(\lambda-ab^n\right)}{\lambda Q_1^2}=\frac{hab^n}{2\lambda}\left(1-\mu^2+\frac{\lambda\mu}{a^2b^{2n}}+\frac{\lambda^2\mu^2}{a^2b^{2n}}\right)$$

$$\text{or,}\quad Q_1^2=\frac{2K_0a^2b^{2n}\left(\lambda-ab^n\right)}{h\left(a^2b^{2n}-a^2b^{2n}\mu^2+\lambda\mu+\lambda^2\mu^2\right)}.$$

Therefore, the optimum order quantity,

$$Q_1^*=\sqrt{\frac{2K_0a^2b^{2n}\left(\lambda-ab^n\right)}{h\left(a^2b^{2n}-a^2b^{2n}\mu^2+\lambda\mu+\lambda^2\mu^2\right)}}. \tag{12}$$

Now again differentiating the Equation (11) with respect to Q_1, we get,

$$\frac{\mathrm{d}^2}{\mathrm{d}Q_1^2}TC\left(Q_1\right)=\frac{2K_0ab^n\left(\lambda-ab^n\right)}{\lambda Q_1^3}. \tag{13}$$

From Equation (13) we can conclude that the convex property 2) *i.e.* $\frac{\mathrm{d}^2}{\mathrm{d}Q_1^2}TC\left(Q_1\right)>0$, as K_0,a,b,λ,Q_1 and $\lambda-ab^n$ all are positive.

Finally, we can conclude that the equation of total cost function (11) is convex in Q_1. Hence, there is an optimal solution in Q_1 for which the total cost function will be minimal.

4. Numerical Illustration

1) Situation 1: When *n* = 1 (Demand is Linear Function)

In order to give an example with numerical illustration, let us suppose the following parameters, while $n=1$:

$$K_0=100, h=2, a=1, b=2, \lambda=20 \text{ and } \mu=0.01.$$

After putting all the values in Equations (2), (9), (11) and (12) we get the following results:

- Optimum time $t_1^*=2.289$ units at maximum inventory level.
- Optimum order interval $T_1^*=22.894$ units.
- Total average optimum inventory cost $TC^*=8.736$ units and.
- Optimum order quantity $Q_1^*=41.21$ units respectively.

Putting the values of Q_1 arbitrarily either bigger or lesser than Q_1^*, we get the inventory cost gradually increased from the total average optimum inventory cost, which is shown in **Table 2** and **Figure 2** respectively:

It can be mentioned that $n=1$ in the demand function ab^n shows the demand function is a linear form which is depicted in **Figure 1** before.

2) Situation 2: When *n* = 2 (Demand is Exponential Function)

In order to give an example with numerical illustration, let us suppose same parameter we have considered in the first situation as below while $n=2$:

$$K_0=100, h=2, a=1, b=2, \lambda=20 \text{ and } \mu=0.01.$$

After putting all the values in equation no (2), (9), (11) and (12) we get the following results:

- Optimum time $t_1^*=2.482$ units at maximum inventory level.
- Optimum order interval $T_1^*=12.408$ units.
- Total average optimum inventory cost $TC^*=16.12$ units and.
- Optimum order quantity $Q_1^*=39.71$ units respectively.

Putting the values of Q_1 arbitrarily either bigger or lesser than Q_1^*, we get the inventory cost gradually increased from the total average optimum inventory cost, which is shown in **Table 3** and **Figure 3** respectively:

In this case, it is observed that $n=2$ in the demand function ab^n shows the demand function is an exponential form which is depicted in **Figure 4**.

5. Sensitivity Analysis

Now assuming the set up cost K_0 fixed, we study the effects of changes of parameters $\lambda, a, b, h, \mu,$ and n on the optimal length of ordering cycle t_1, optimal time cycle T_1, optimal order quantity, Q_1 and the total average inventory cost TC per unit time in the model. We performed the sensitivity analysis by changing each of the parameters by +50%, +25%, +10%, −10%, −25% and −50% taking one parameter at a time while keeping other parameters unchanged. The details are shown in **Table 4**.

Table 2. Order quantity verses inventory cost for $n = 1$ (when demand is linear).

Inventory (Q_1)	35.210	37.210	39.210	41.210	43.210	45.210	47.210
Total cost (TC)	8.844	8.781	8.747	8.736	8.745	8.773	8.817
Remarks	At a particular inventory level total cost is minimum, before and after this point total cost increases						

Table 3. Order quantity verses inventory cost for $n = 2$ (when demand is exponential).

Inventory (Q_1)	30.71	33.71	36.71	39.71	42.71	45.71	48.71
Total cost (TC)	16.65	16.34	16.17	16.12	16.16	16.28	16.46
Remarks	At a particular inventory level total cost is minimum, before and after this point total cost increases						

Figure 2. Quantity verses total cost.

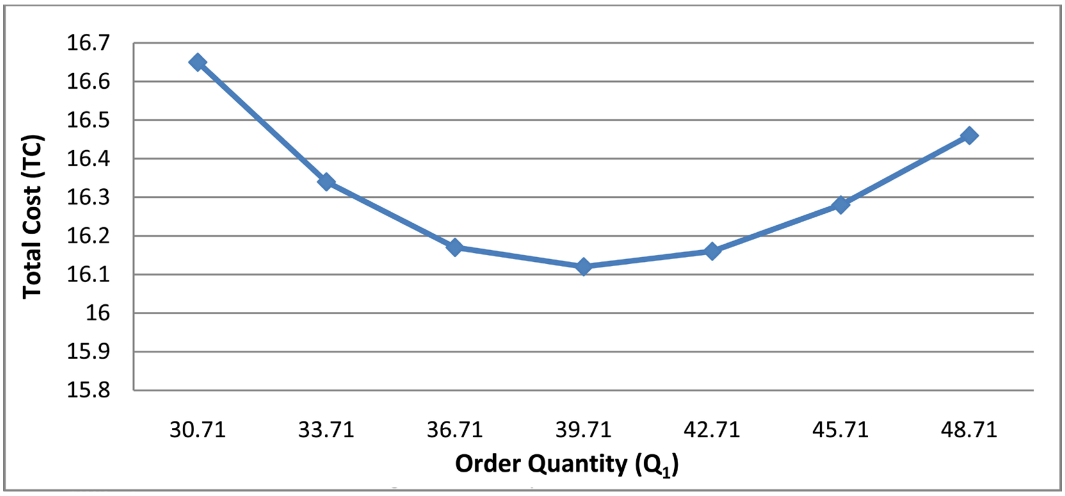

Figure 3. Quantity verses total cost.

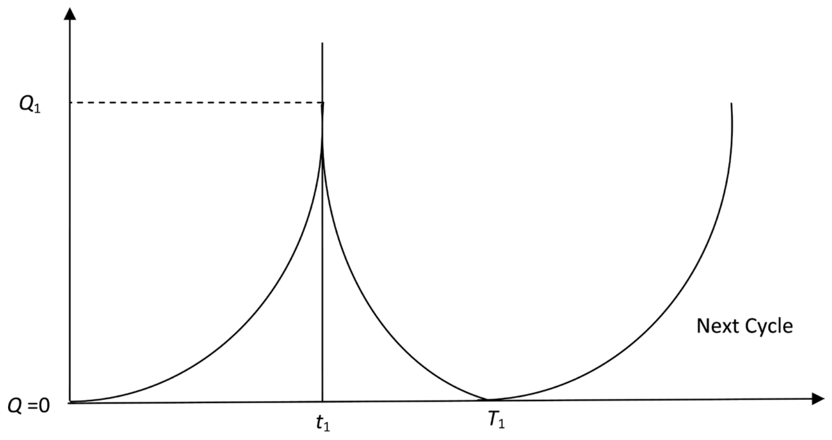

Figure 4. Inventory model with exponential demand.

Table 4. Sensitivity analysis for n = 1 (when demand is linear).

Parameters	Change in %	Value of t_1^*	Value of T_1^*	Value of Q_1^*	Value of TC^*
λ	+50	1.472	22.077	50.512	27.545
	+25	1.792	22.397	46.190	8.019
	+10	2.061	22.666	43.295	8.409
	−10	2.576	23.180	38.980	9.136
	−25	3.170	23.775	35.304	9.937
	−50	5.150	25.756	27.905	12.350
a	+50	2.424	16.161	40.694	12.534
	+25	2.355	18.839	41.054	10.657
	+10	2.315	21.047	41.183	9.509
	−10	2.264	25.159	41.166	7.958
	−25	2.228	29.701	40.888	6.787
	−50	2.169	43.379	39.146	4.860
b	+50	2.424	16.161	40.694	12.534
	+25	2.355	18.839	41.054	10.657
	+10	2.315	21.047	41.183	9.509
	−10	2.264	25.159	41.166	7.958
	−25	2.228	29.701	40.888	6.787
	−50	2.169	43.379	39.146	4.860
h	+50	2.289	22.894	33.648	10.920
	+25	2.289	22.894	36.859	9.828
	+10	2.289	22.894	39.292	9.172
	−10	2.289	22.894	43.434	8.299
	−25	2.289	22.894	47.585	7.644
	−50	2.289	22.894	58.280	6.552
μ	+50	2.289	22.894	40.502	8.890
	+25	2.289	22.894	40.934	8.795
	+10	2.289	22.894	41.073	8.765
	−10	2.289	22.894	41.345	8.707
	−25	2.289	22.894	41.476	8.680
	−50	2.289	22.894	41.855	8.602
n	+50	2.400	16.970	40.833	11.895
	+25	2.339	19.666	41.117	10.193
	+10	2.308	21.533	41.197	9.291
	−10	2.273	24.356	41.190	8.215
	−25	2.249	26.753	41.094	7.497
	−50	2.217	32.357	40.738	6.452

Analyzing the results in the above table we can summarize the following observations:

1) t_1^* and T_1^* decreases while Q_1^* increases in the value of the parameter λ. Here λ is highly sensitive to Q_1^* and moderately sensitive to all other values in the model.

2) t_1^* and TC^* increases and T_1^* decreases while Q_1^* primarily decreases and then again increases with increase in the value of the parameter a,b and n. Here, a and b are highly sensitive to Q_1^*, T_1^* and TC^* and those are moderately sensitive to t_1^*. In the other hand, n is highly sensitive to T_1^* as well as Q_1^* and it is moderately sensitive to t_1^* and TC^*.

3) Q_1^* decreases and TC^* increases while t_1^* and T_1^* remain unchanged with increase in the value of the parameter h. Here h is highly sensitive to Q_1^* and moderately sensitive to TC^*.

4) Q_1^* and TC^* increases while t_1^* and T_1^* remain unchanged with increase in the value of the parameter μ. μ is moderately sensitive to Q_1^* and TC^*.

Analyzing the results in **Table 5** we can summarize the following observations:

Table 5. Sensitivity analysis for $n = 2$ (when demand is exponential).

Parameters	Change in %	Value of			
		t_1^*	T_1^*	Q_1^*	TC^*
λ	+50	1.527	11.453	50.382	14.154
	+25	1.891	11.817	45.387	14.939
	+10	2.206	12.132	42.077	15.582
	−10	2.836	12.762	37.173	16.775
	−25	3.610	13.356	32.991	18.089
	−50	6.618	16.544	24.412	22.034
a	+50	2.836	9.454	37.294	22.568
	+25	2.647	10.588	38.547	19.465
	+10	2.545	11.569	39.256	17.486
	−10	2.421	13.450	40.129	14.713
	−25	2.336	15.571	40.694	12.536
	−50	2.206	22,058	41.210	8.742
b	+50	3.610	8.021	33.119	30.385
	+25	2.888	9.240	36.969	23.305
	+10	2.620	10.823	38.740	18.946
	−10	2.369	14.624	40.481	13.417
	−25	2.237	19.884	41.168	9.707
	−50	2.090	41.795	39.146	4.854
h	+50	2.482	12.408	32.419	20.148
	+25	2.482	12.408	35.514	18.134
	+10	2.482	12.408	37.858	16.925
	−10	2.482	12.408	41.853	15.313
	−25	2.482	12.408	45.848	14.104
	−50	2.482	12.408	56.152	12.089
μ	+50	2.482	12.408	39.526	16.192
	+25	2.482	12.408	39.636	16.147
	+10	2.482	12.408	39.671	16.133
	−10	2.482	12.408	39.739	16.105
	−25	2.482	12.408	39.771	16.092
	−50	2.482	12.408	39.864	16.055
n	+50	3.309	8.272	34.578	28.029
	+25	2.769	9.787	37.733	21.531
	+10	2.577	11.219	39.030	18.138
	−10	2.404	13.806	40.248	14.292
	−25	2.312	16.350	40.833	11.899
	−50	2.206	22.058	41.210	8.742

1) t_1^*, T_1^* and TC^* decreases while Q_1^* increases with increase in the value of the parameter λ. Here λ is highly sensitive to t_1^* as well as Q_1^* and highly sensitive to T_1^* and TC^*.

2) t_1^*, T_1^* and TC^* increases while Q_1^* decreases with increase in the value of the parameter a. Here a is highly sensitive to T_1^* as well as TC^* and moderately sensitive to t_1^* and Q_1^*.

3) t_1^* and TC^* increases while T_1^* and Q_1^* decreases with increase in the value of the parameter b and n. Here b and n are highly sensitive to T_1^* as well as TC^* and moderately sensitive to t_1^* and Q_1^*.

4) Q_1^* decreases and TC^* increases while t_1^* and T_1^* remain unchanged with increase in the value of the parameter h and μ.

6. Conclusion

In the present context of modern age, without the inventory management, the business institution cannot think ahead. By the proper management and thereby developing the suitable inventory model, the institution only can save its production inventory cost. Market demand is always fluctuate. The model is developed considering this demand. Maybe, today the market demand is very high and tomorrow it is low. The inventory model we have proposed in this paper can be suitable to meet both the demands linear or exponential. Because of deterioration, this model also gives the correct result in which the materials have the finite shelf-life. In the proposed model, the production rate and the decay have been considered constant all through. The model develops an algorithm to determine the optimum ordering cost, total average inventory cost and optimum time cycle. The model could establish that with a particular order level the inventory cost is minimal. Here, for n = 1, we got $Q_1^* = 41.210$ units and total average inventory cost $TC^* = 8.736$ units and for n = 2, we got $Q_1^* = 39.705$ units and total average inventory cost $TC^* = 16.119$ units, before and after this point total cost increases sharply.

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