.... **APPENDIX A**

POWER RATIOS AND DECIBELS

In system calculations and measurements involving the use of power ratios, it is customary practice to use a unit called the *decibel*. The decibel, commonly abbreviated as dB, is one tenth of a larger unit, the *bel*.[1] In practice, however we find that for most applications the bel is too large a unit; hence, the wide use of dB as the unit for expressing power ratios. The dB is particularly appropriate for sound measurements because the ear responds to sound in an approximately logarithmic fashion. Thus, equal dB increments are perceived by the ear as equal increments in sound.

Let P denote the power at some point of interest in a system. Let P_0 denote the *reference* power level with which the power P is to be compared. The number of decibels in the power ratio P/P_0 is defined as $10 \log_{10}(P/P_0)$. For example, a power ratio of 2 corresponds to 3 dB, and a power ratio of 10 corresponds to 10 dB.

We may also express the signal power P itself in dB if we divide P by one watt or one milliwatt. In the first case, we express the signal power P in dBW as $10 \log_{10}(P/1\text{W})$, where W is the abbreviation for watt. In the second case, we express the signal power P in dBm as $10 \log_{10}(P/1\text{mW})$, where mW is the abbreviation for milliwatt.

[1]The unit, bel, is named in honor of Alexander Graham Bell. In addition to inventing the telephone, Bell was the first to use logarithmic power measurements in sound and hearing research.

APPENDIX B

BESSEL FUNCTION

A Bessel function of the first kind of order n and argument x, commonly denoted by $J_n(x)$, is defined by

$$J_n(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \exp(jx\sin\theta - jn\theta)\,d\theta \tag{B.1}$$

or, equivalently,

$$J_n(x) = \frac{1}{\pi}\int_{0}^{\pi} \cos(x\sin\theta - n\theta)\,d\theta \tag{B.2}$$

Just as the trigonometric functions can be expanded in power series, so can the Bessel function $J_n(x)$ be expanded in a power series:

$$J_n(x) = \sum_{m=0}^{\infty} \frac{(-1)^m(\frac{1}{2}x)^{n+2m}}{m!(n+m)!} \tag{B.3}$$

Evaluating Eq. B.3 for $n = 0, 1, 2$, for example, we thus have

$$J_0(x) = 1 - \frac{x^2}{2^2} + \frac{x^4}{2^2 \cdot 4^2} - \frac{x^6}{2^2 \cdot 4^2 \cdot 6^2} + \cdots \tag{B.4}$$

$$J_1(x) = \frac{x}{2} - \frac{x^3}{2^2 \cdot 4} + \frac{x^5}{2^2 \cdot 4^2 \cdot 6} - \cdots \tag{B.5}$$

$$J_2(x) = \frac{x^2}{2 \cdot 4} - \frac{x^4}{2^2 \cdot 4 \cdot 6} + \frac{x^6}{2^2 \cdot 4^2 \cdot 6 \cdot 8} - \cdots \tag{B.6}$$

B.1 PROPERTIES OF BESSEL FUNCTION

The Bessel function $J_n(x)$ has the following properties[1]:

1. $$J_n(x) = (-1)^n J_{-n}(x) \tag{B.7}$$

[1]For a mathematical treatment of Bessel functions, see Whittaker and Watson (1927), pp. 355–385.

To prove this relation, we replace θ by $(\pi - \theta)$ in Eq. B.2. Then, noting that $\sin(\pi - \theta) = \sin\theta$, we get

$$J_n(x) = \frac{1}{\pi}\int_0^\pi \cos(x\sin\theta + n\theta - n\pi)\,d\theta$$

$$= \frac{1}{\pi}\int_0^\pi [\cos(n\pi)\cos(x\sin\theta + n\theta) + \sin(n\pi)\sin(x\sin\theta + n\theta)]\,d\theta$$

For integer values of n, we have

$$\cos(n\pi) = (-1)^n$$
$$\sin(n\pi) = 0$$

Therefore,

$$J_n(x) = \frac{(-1)^n}{\pi}\int_0^\pi \cos(x\sin\theta + n\theta)\,d\theta \tag{B.8}$$

From Eq. B.2 we also find that by replacing n with $-n$:

$$J_{-n}(x) = \frac{1}{\pi}\int_0^\pi \cos(x\sin\theta - n\theta)\,d\theta \tag{B.9}$$

The desired result follows immediately from Eqs. B.8 and B.9.

2.
$$J_n(x) = (-1)^n J_n(-x) \tag{B.10}$$

This relation is obtained by replacing x with $-x$ in Eq. B.2, and then using Eq. B.8.

3.
$$J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x}J_n(x) \tag{B.11}$$

This *recurrence formula* is useful in constructing tables of Bessel coefficients. For example, the use of Eq. B.11 for $n = 1$ yields a value for $J_1(x)$ that is in exact agreement with that of Eq. B.5.

4. For small values of x, we have

$$J_n(x) \simeq \frac{x^n}{2^n n!} \tag{B.12}$$

This relation is obtained simply by retaining the first term in the power series of Eq. B.3 and ignoring the higher-order terms. Thus, when x is small, we have

$$J_0(x) \simeq 1$$
$$J_1(x) \simeq \frac{x}{2} \tag{B.13}$$

5. For large values of x, we have

$$J_n(x) \simeq \sqrt{\frac{2}{\pi x}}\cos\left(x - \frac{\pi}{4} - \frac{n\pi}{2}\right) \tag{B.14}$$

This shows that for large values of x, the Bessel function $J_n(x)$ behaves like a sine wave with progressively decreasing amplitude.

6. With x real and fixed, $J_n(x)$ approaches zero as the order n goes to infinity.

7.
$$\sum_{n=-\infty}^{\infty} J_n(x)\exp(jn\phi) = \exp(jx\sin\phi) \tag{B.15}$$

To prove this property, consider the sum $\sum_{n=-\infty}^{\infty} J_n(x)\exp(jn\phi)$ and use the formula of Eq. B.1 for $J_n(x)$ to obtain

$$\sum_{n=-\infty}^{\infty} J_n(x)\exp(jn\phi) = \frac{1}{2\pi}\sum_{n=-\infty}^{\infty}\exp(jn\phi)\int_{-\pi}^{\pi}\exp(jx\sin\theta - jn\theta)\,d\theta$$

Interchanging the order of integration and summation:

$$\sum_{n=-\infty}^{\infty} J_n(x)\exp(jn\phi)$$
$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} d\theta\,\exp(jx\sin\theta)\sum_{n=-\infty}^{\infty}\exp[jn(\phi-\theta)] \tag{B.16}$$

From Example 14 of Chapter 2, we deduce that

$$\delta(\phi-\theta) = \frac{1}{2\pi}\sum_{n=-\infty}^{\infty}\exp[jn(\phi-\theta)], \qquad -\pi \leqslant \phi-\theta \leqslant \pi \tag{B.17}$$

Therefore, substituting Eq. B.17 in B.16, and using the sifting property of a delta function, we get

$$\sum_{n=-\infty}^{\infty} J_n(x)\exp(jn\phi) = \int_{-\pi}^{\pi}\exp(jx\sin\theta)\delta(\phi-\theta)\,d\theta$$
$$= \exp(jx\sin\phi)$$

which is the desired result.

8.
$$\sum_{n=-\infty}^{\infty} J_n^2(x) = 1, \qquad \text{for all } x \tag{B.18}$$

To prove this property, we may proceed as follows. We observe that $J_n(x)$ is real. Hence, multiplying Eq. B.1 by its complex conjugate, and summing over all possible values of n, we get

$$\sum_{n=-\infty}^{\infty} J_n^2(x) = \frac{1}{(2\pi)^2}\sum_{n=-\infty}^{\infty}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\exp(jx\sin\theta$$
$$- jn\theta - jx\sin\phi + jn\phi)\,d\theta\,d\phi$$

Interchanging the order of double integration and summation:

$$\sum_{n=-\infty}^{\infty} J_n^2(x) = \frac{1}{(2\pi)^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} d\theta\,d\phi\,\exp[jx(\sin\theta - \sin\phi)]$$
$$\times \sum_{n=-\infty}^{\infty}\exp[jn(\phi-\theta)] \tag{B.19}$$

Substituting Eq. B.17 in B.19, and using the sifting property of a delta function, we finally get

$$\sum_{n=-\infty}^{\infty} J_n^2(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\theta = 1$$

which is the desired result.

B.2 *TABLE OF VALUES*

Table B.1 gives values of the Bessel function $J_n(x)$ for values of the order n from 0 up to 14, and for values of the argument x in the interval (0.5, 12).

This table may be used to illustrate properties 3 through 8 listed in Section B.1. The reader is invited to do this as an exercise.

TABLE B.1 Bessel Functions

$n \backslash x$	0.5	1	2	3	4	6	8	10	12
	$J_n(x)$								
0	0.9385	0.7652	0.2239	−0.2601	−0.3971	0.1506	0.1717	−0.2459	0.0477
1	0.2423	0.4401	0.5767	0.3391	−0.0660	−0.2767	0.2346	0.0435	−0.2234
2	0.0306	0.1149	0.3528	0.4861	0.3641	−0.2429	−0.1130	0.2546	−0.0849
3	0.0026	0.0196	0.1289	0.3091	0.4302	0.1148	−0.2911	0.0584	0.1951
4	0.0002	0.0025	0.0340	0.1320	0.2811	0.3576	−0.1054	−0.2196	0.1825
5	—	0.0002	0.0070	0.0430	0.1321	0.3621	0.1858	−0.2341	−0.0735
6		—	0.0012	0.0114	0.0491	0.2458	0.3376	−0.0145	−0.2437
7			0.0002	0.0025	0.0152	0.1296	0.3206	0.2167	−0.1703
8			—	0.0005	0.0040	0.0565	0.2235	0.3179	0.0451
9				0.0001	0.0009	0.0212	0.1263	0.2919	0.2304
10				—	0.0002	0.0070	0.0608	0.2075	0.3005
11					—	0.0020	0.0256	0.1231	0.2704
12						0.0005	0.0096	0.0634	0.1953
13						0.0001	0.0033	0.0290	0.1201
14						—	0.0010	0.0120	0.0650

APPENDIX C

SYSTEM NOISE AND CALCULATIONS

The inevitable presence of *noise* in a communication system causes the reliable transmission of electrical signals through the system to be undermined. It is therefore important to know how noise arises in the system. There are many potential sources of noise. The sources of noise may be *external* to the system (e.g., atmospheric noise, galactic noise, man-made noise) or *internal* to the system. The second category includes an important type of noise that arises owing to spontaneous fluctuations of current or voltage in electrical circuits. This type of noise, in one way or another, is present in every communication system and represents a fundamental limitation on the transmission or detection of signals.

In this appendix we briefly discuss the physical sources of noise in electrical circuits and develop quantitative measures for assessing the presence of noise in a system. We finish the discussion by presenting link calculations for line-of-sight propagation through free space.

C.1 ELECTRICAL NOISE

In an electrical circuit, noise is generated owing to various physical phenomena.[1] We have *thermal noise* produced by the random motion of electrons in conducting media, and *shot noise* produced by random fluctuations of current flow in electronic devices. These two are the most common examples of spontaneous fluctuation noise encountered in electrical circuits.

Besides thermal noise and shot noise, transistors exhibit a low-frequency noise phenomenon known as *flicker noise*. The mean-square value of flicker noise is inversely proportional to frequency; hence, it is also referred to as "one-over-f" noise. Yet another type of noise encountered in semiconductor devices is *burst noise,* whose mean-square value falls off as $1/f^2$.

Flicker noise and burst noise are both *nonwhite,* with their degrading effects being observed at low frequencies. Ordinarily, they can be ignored above a few kilohertz. On the other hand, thermal noise and shot noise are both *white* for all practical purposes; hence, their degrading influence on the operation of a communication system extends right across the com-

[1]For a detailed treatment of electrical noise, see Bell (1985), Van Der Ziel (1970) and Robinson (1974).

plete frequency band of interest. A brief discussion of thermal noise and shot noise is therefore in order.

THERMAL NOISE

Thermal noise[2] is an ubiquitous source of noise that arises from thermally induced motion of electrons in conducting media. In a conductor there is a large number of "free" electrons and an equally large number of ions bound by strong molecular forces. The ions vibrate randomly about their normal positions. The free electrons move along randomly oriented paths, owing to continuous collisions with the vibrating ions. The net effect of this random motion is an electric current that is likewise random. However, the mean value of the current is zero since, on the average, there are as many electrons moving in one direction as there are in another.

A thorough analysis of thermal noise requires the use of thermodynamic and quantum mechanical considerations that are beyond the scope of this book. For the purpose of the discussion presented here, it suffices to say that the power spectral density of thermal noise produced by a resistor is given by[3]

$$S_{TN}(f) = \frac{2h|f|}{\exp(h|f|/kT) - 1} \tag{C.1}$$

where T is the *absolute temperature in degrees Kelvin*, k is *Boltzmann's constant*, and h is *Planck's constant*. Note that the power spectral density $S_{TN}(f)$ is measured in watts per hertz. For "low" frequencies defined by

$$f << \frac{kT}{h}$$

we may use the approximation

$$\exp\left(\frac{h|f|}{kT}\right) \simeq 1 + \frac{h|f|}{kT}$$

Correspondingly, we may approximate the formula of Eq. C.1 as follows

$$S_{TN}(f) \simeq 2kT \tag{C.2}$$

[2] Thermal noise was first studied experimentally by Johnson in 1928, and for this reason it is sometimes referred to as "Johnson noise"; see Johnson (1928). Johnson's experiments were confirmed theoretically by Nyquist; see Nyquist (1928).
[3] For a discussion of the physical issues involved in the formulation of Eq. C.1, and for a historical account of the pertinent literature, see Bell (1985).

To develop a feeling for the frequencies for which the use of this approximate formula is justified, we assume operation at a room temperature of 17°C, for which we have $T = 290°K$. Then, using the values of Boltzmann's constant and Planck's constant:

$$k = 1.38 \times 10^{-23} \text{ joules/°K}$$

and

$$h = 6.63 \times 10^{-34} \text{ joule} \cdot \text{second}$$

we find that

$$\frac{kT}{h} = 6 \times 10^{12} \text{ Hz}$$

This upper frequency limit lies in the infrared region that is well above the spectrum of frequencies encountered in conventional electrical communication systems. Therefore, for all practical purposes the use of the approximate formula of Eq. C.2 is perfectly justified.

Thus, given a resistor of R ohms, we find from Eq. C.2 that the mean-square value of the *thermal noise voltage* measured across the terminals of the resistor equals

$$\begin{aligned} E[V_{TN}^2] &= 2RB_N S_{TN}(f) \\ &= 4kTRB_N \text{ volts}^2 \end{aligned} \tag{C.3}$$

where B_N is the bandwidth (in hertz) over which the noise voltage is measured. We may thus model a noisy resistor by the *Thévenin equivalent circuit* consisting of a noise voltage generator with a mean-square value of $E[V_{TN}^2]$ in series with a noiseless resistor, as in Fig. C.1*a*. Alternatively,

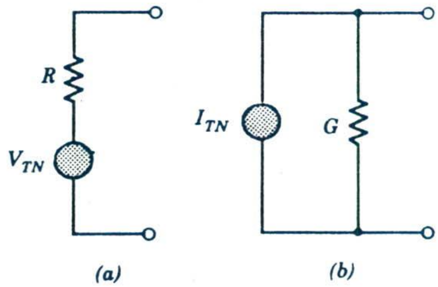

Figure C.1
Models of a noisy resistor. (a) Thévenin equivalent circuit. (b) Norton equivalent circuit.

we may use the *Norton equivalent circuit* consisting of a noise current generator in parallel with a noiseless conductance, as in Fig. C.1*b*. The mean-square value of the noise current generator is

$$\begin{aligned} E[I_{TN}^2] &= \frac{1}{R^2} E[V_{TN}^2] \\ &= 4kTGB_N \text{ amps}^2 \end{aligned} \tag{C.4}$$

where $G = 1/R$ is the conductance.

It is also of interest to note that because the number of electrons in a resistor is very large and their random motions inside the resistor are statistically independent of each other, the central limit theorem indicates that thermal noise is Gaussian-distributed with zero mean. Accordingly, for the band of frequencies encountered in electrical communication systems, we may model thermal noise as *white Gaussian noise* of zero mean.

EXAMPLE 1

Consider a system for which the noise bandwidth $B_N = 10$ kHz, the absolute temperature $T = 290°$K (i.e., room temperature), and the resistance $R = 5$ KΩ. Then, the use of Eq. C.3 yields the mean-square value of the thermal noise voltage to be 0.8×10^{-12} volts squared. That is, the thermal noise voltage produced by the resistor has a root mean-square (rms) value equal to 0.89 μV.

AVAILABLE NOISE POWER

Noise calculations involve the transfer of power, so we find that the use of the *maximum-power transfer theorem* is applicable. This theorem states that *the maximum possible power is transferred from a source of internal resistance R to a load of resistance R_l when $R_l = R$.* Under this matched condition, the power produced by the source is divided equally between the internal resistance of the source and the load resistance, and the power delivered to the load is referred to as the *available power*. Applying the maximum-power transfer theorem to the Thévenin equivalent circuit of Fig. C.1*a* or the Norton equivalent circuit of Fig. C.1*b*, we find that a noisy resistor produces an *available noise power* equal to kTB_N watts.

SHOT NOISE

Shot noise arises in electronic devices because of the *discrete* nature of current flow in the device. The process assumes the existance of an *average* current flow that manifests itself in the form of electrons flowing from the cathode to the plate in *vacuum tubes,* holes and electrons flowing in *semiconductor devices,* and photons emitted in *photodiodes.* Although the average number of particles moving across the device per unit time is assumed to be constant, the process of current flow through the device exhibits *fluctuations* about the average value. The manner in which these fluctuations arise varies from one device to another. In a vacuum-tube device, the fluctuations are produced by the random emission of electrons from the cathode. In a semiconductor device, the cause is the random diffusion of electrons or the random recombination of electrons with holes. In a photodiode, it is the random emission of photons. In all these devices, the physical mechanism that controls current flow through the device has built-in statistical fluctuations about some average value. The shot noise produced by these fluctuations is thus *dependent on the average value of the current.*

Consider for example, a *temperature-limited vacuum diode*, shown in Fig. C.2. It consists of two electrodes enclosed in a vacuum: a *cathode*, which is heated so that it emits electrons; and an *anode* or *plate*, which is maintained at a positive potential with respect to the cathode so that it gathers the electrons. We assume that the cathode-plate potential difference is large enough to cause the electrons emitted thermionically by the heated cathode to be pulled to the plate with such high velocities that space-charge effects are negligible. The plate current is then determined effectively by the rate at which electrons are emitted from the cathode. By considering the plate current as the sum of a succession of current pulses, with each pulse caused by the transit of one electron through the cathode-plate space, we find that the mean-square value of the randomly fluctuating component of the current is given by

$$E[I_{SN}^2] = 2qIB_N \text{ amps}^2 \tag{C.5}$$

where q is the *electron charge* equal to 1.60×10^{-19} coulombs, I is the mean value of the current in amperes, and B_N is the bandwidth of the measuring instrument in hertz. Equation C.5 is called the *Schottky formula*. The typical transit time of an electron from cathode to plate is on the order of 10^{-9} sec. The Schottky formula holds provided that the operating frequency is small compared with the reciprocal of the transit time, so that we may neglect transit time effects.

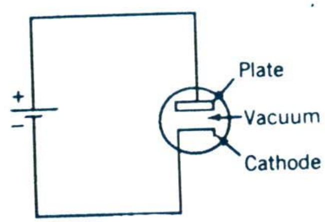

Figure C.2
Temperature-limited vacuum diode.

Another important characteristic of shot noise is that it is Gaussian-distributed with zero mean. This follows from the fact that the number of electrons contributing to the shot noise current is very large, and their random emissions from the cathode are, for practical purposes, statistically independent of each other. Hence, the central limit theorem predicts a Gaussian distribution for shot noise.

The Schottky formula (C.5) also holds for a *semiconductor junction* diode. In this case the mean value I of the current is given by the *diode equation:*

$$I = I_s \exp\left(\frac{qV}{kT}\right) - I_s \tag{C.6}$$

where V is the voltage applied across the diode and I_s is the saturation current; the other constants are as defined previously. Thus, the current I consists of two components that produce statistically independent shot-noise contributions of their own, as shown by

$$\begin{aligned} E[I_{SN}^2] &= 2qI_s \exp\left(\frac{qV}{kT}\right)B_N + 2qI_sB_N \\ &= 2q(I + 2I_s)B_N \end{aligned} \tag{C.7}$$

Figure C.3 shows the *noise model* of a junction diode.[4] The model includes the *dynamic resistance* of the diode, defined by

$$\begin{aligned} r &= \frac{\delta V}{\delta I} \\ &= \frac{kT}{q(I + I_s)} \end{aligned} \tag{C.8}$$

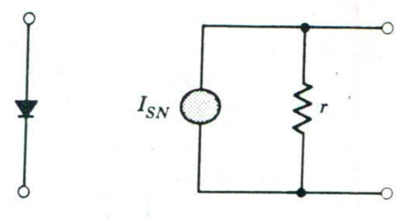

Figure C.3
(a) Junction diode. (b) Shot-noise model.

[4]For details of noise models for semiconductor diodes and transistors, see Robinson (1974), pp. 93–116.

Note, however, that the dynamic resistance r is noiseless since it does not involve power dissipation.

In a *bipolar junction transistor,* shot noise is generated at both the emitter and collector junctions. On the other hand, in a *junction field-effect transistor* the use of an insulated gate structure avoids junction shot noise; nevertheless, shot noise is produced by the flow of gate current. Of course, in both devices thermal noise arises from internal ohmic resistance: *base resistance* in a bipolar transistor and *channel resistance* in a field effect transistor.

C.2 NOISE FIGURE

A convenient measure of the noise performance of a linear two-port device is furnished by the *noise figure,*[5] which lends itself to both circuit analysis and measurement. Consider a linear two-port device connected to a signal source of internal impedance $Z(f) = R(f) + jX(f)$ at the input, as in Fig. C.4. The noise voltage $v(t)$ represents the thermal noise associated with the internal resistance $R(f)$ of the source. The output noise of the device is made up of two contributions, one due to the source and the other due to the device itself. We define *the available output noise power in a band of width B_N centered at frequency f as the maximum average noise power in this band obtainable at the output of the device.* The maximum noise power that the two-port device can deliver to an external load is obtained when the load impedance is the complex conjugate of the output impedance of the device, that is, when the resistance is matched and the reactance is tuned out. We define *the noise figure of the two-port device as the ratio of the total available output noise power (due to the device and the source) per unit bandwidth to the portion thereof due solely to the source.*

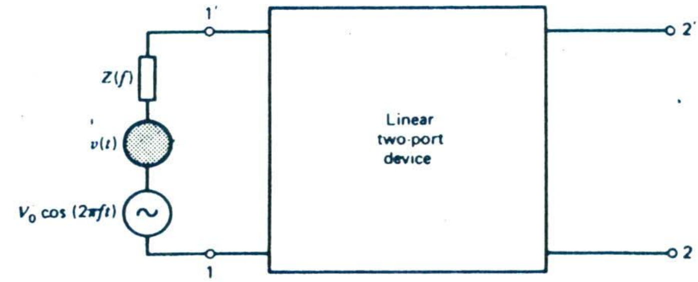

Figure C.4
Linear two-port device.

[5]For a classic paper on noise figure, see Friis (1944).

Let the spectral density of the total available noise power at the device output be $S_{NO}(f)$, and the spectral density of the available noise power due to the source at the device input be $S_{NS}(f)$. Also let $G(f)$ denote the *availabe power gain of the two-port device, defined as the ratio of the available signal power at the output of the device to the available signal power of the source when the signal is a sinusoidal wave of frequency f*. Then we may express the noise figure $F(f)$ of the device as

$$F(f) = \frac{S_{NO}(f)}{G(f)S_{NS}(f)} \tag{C.9}$$

If the device were noise free, $S_{NO}(f) = G(f)S_{NS}(f)$, and the noise figure would then be unity. In a physical device, however, $S_{NO}(f)$ is larger than $G(f)S_{NS}(f)$, so that the noise figure is always larger than unity. The noise figure is commonly expressed in decibels, that is, as $10 \log_{10}F(f)$.

The noise figure may also be expressed in an alternative form. Let $P_S(f)$ denote the available signal power from the source, which is the maximum average signal power that can be obtained. For the case of a source providing a single-frequency signal component with open-circuit voltage $V_0 \cos(2\pi f t)$, the available signal power is obtained when the load connected to the source is $Z^*(f) = R(f) - jX(f)$, yielding the value

$$\begin{aligned} P_S(f) &= \left[\frac{V_0}{2R(f)}\right]^2 R(f) \\ &= \frac{V_0^2}{4R(f)} \end{aligned} \tag{C.10}$$

The available signal power at the output of the device is therefore,

$$P_O(f) = G(f)P_S(f) \tag{C.11}$$

Then, mulitplying both the numerator and denominator of the right side of Eq. C.9 by $P_S(f)B_N$, we have

$$\begin{aligned} F(f) &= \frac{P_S(f)S_{NO}(f)B_N}{G(f)P_S(f)S_{NS}(f)B_N} \\ &= \frac{P_S(f)S_{NO}(f)B_N}{P_O(f)S_{NS}(f)B_N} \\ &= \frac{\rho_S(f)}{\rho_O(f)} \end{aligned} \tag{C.12}$$

where

$$\rho_S(f) = \frac{P_S(f)}{S_{NS}(f)B_N} \tag{C.13}$$

$$\rho_O(f) = \frac{P_O(f)}{S_{NO}(f)B_N} \tag{C.14}$$

We refer to $\rho_S(f)$ as the *available signal-to-noise ratio of the source* and to $\rho_O(f)$ as the *available signal-to-noise ratio at the device output,* both measured in a narrow band of width B_N centered at f. Since the noise figure is always greater than unity, it follows from Eq. C.12 that the signal-to-noise ratio always decreases with amplification, which is a significant result.

The noise figure $F(f)$, as defined herein, is a function of the operating frequency and hence is referred to as the *spot noise figure*. In contrast, we define an *average noise figure* F of a two-port device as the ratio of the total noise power at the device output to the output noise power due solely to the source. That is,

$$F = \frac{\int_{-\infty}^{\infty} S_{NO}(f)\, df}{\int_{-\infty}^{\infty} G(f)S_{NS}(f)\, df} \tag{C.15}$$

It is apparent that in the case of thermal noise in the input circuit with $R(f)$ constant, and constant gain throughout a fixed band with zero gain at other frequencies, the spot noise figure $F(f)$ and the average noise figure F are identical.

C.3 EQUIVALENT NOISE TEMPERATURE

A disadvantage of the noise figure F is that when it is used to compare low-noise devices, the values obtained are all close to unity. This makes the comparison rather difficult. In such cases, it is preferable to use the *equivalent noise temperature*. Consider a linear two-port device with its input resistance matched to the internal resistance of the source as shown in Fig. C.5. In this diagram, we have also included the noise voltage generator associated with the internal resistance R_s of the source. The mean-square value of this noise voltage is $4kTR_sB_N$. Hence, the available noise power at the device input is

$$\mathcal{N}_s = kTB_N \tag{C.16}$$

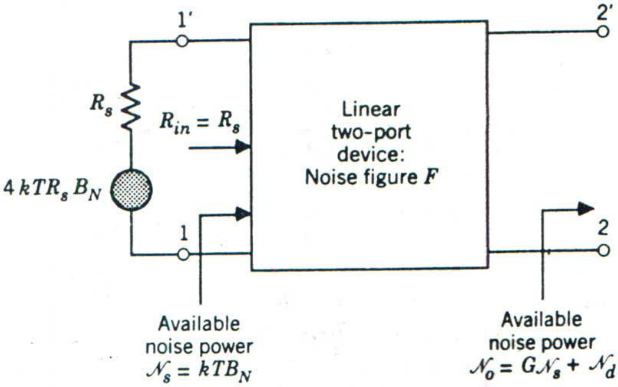

Figure C.5
Linear two-port device matched to the internal resistance of a source connected to the input.

Let $\mathcal{N}_d$ denote the noise power contributed by the two-port device to the total available output noise power $\mathcal{N}_o$. We define $\mathcal{N}_d$ as

$$\mathcal{N}_d = GkT_eB_N \tag{C.17}$$

where G is the available power gain of the device and T_e is its equivalent noise temperature. Then it follows that the total output noise power is

$$\begin{aligned}\mathcal{N}_o &= G\mathcal{N}_s + \mathcal{N}_d \\ &= Gk(T + T_e)\, B_N\end{aligned} \tag{C.18}$$

The noise figure of the device is therefore

$$\begin{aligned}F &= \frac{\mathcal{N}_o}{G\mathcal{N}_s} \\ &= \frac{T + T_e}{T}\end{aligned}$$

Solving for the equivalent noise temperature:

$$T_e = T(F - 1) \tag{C.19}$$

where F is the noise figure of the device measured under matched input conditions, and with the noise source at temperature T. Equation C.19 is the desired relation between the equivalent noise temperature and noise figure of a two-port network.

NOISE SPECTRAL DENSITY

A composite two-port network with equivalent noise temperature T_e (referred to the input) produces the available noise power

$$\mathcal{N}_{av} = kT_e B_N \tag{C.20}$$

Hence, recognizing that $\mathcal{N}_{av} = N_0 B_N$, we find that the noise may be modeled as white Gaussian noise with zero mean and power spectral density $N_0/2$, where

$$N_0 = kT_e \tag{C.21}$$

Note that the power spectral density of the noise so modeled depends only on Boltzmann's constant and the equivalent noise temperature T_e. It is the simplicity of this model that makes the equivalent noise temperature of a composite network such a useful concept.

C.4 CASCADE CONNECTION OF NOISY NETWORKS

Consider next a pair of noisy two-port networks with available power gains G_1 and G_2 that are connected in cascade, as depicted in Fig. C.6. It is assumed that the equivalent noise temperatures of the individual networks are T_1 and T_2, respectively. The total noise power $\mathcal{N}_o$ at the system output is made up of three contributions:

1. *The source noise power $\mathcal{N}_s$ generated at the input of network 1 and amplified by both networks*: The contribution of $\mathcal{N}_s$ to the total noise power $\mathcal{N}_o$ is

$$G_1G_2\mathcal{N}_s = G_1G_2(kTB_N)$$

where we have made use of the expression for $\mathcal{N}_s$ given in Eq. C.16. This contribution is represented by the top paths in Fig. C.6.

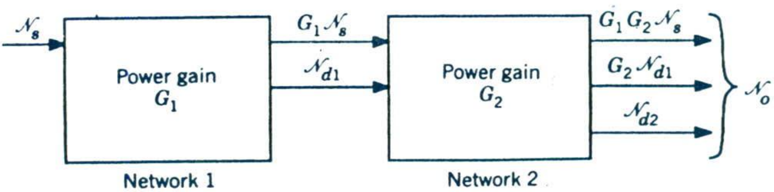

Figure C.6
A cascade of two noisy networks.

2. *The noise power $\mathcal{N}_{d1}$ introduced in network 1 and amplified by network 2*: The contribution to $\mathcal{N}_o$ produced by $\mathcal{N}_{d1}$ is

$$G_2\mathcal{N}_{d1} = G_2(G_1 kT_1 B_N)$$

where we have made use of Eq. C.17, adapted to the situation at hand. This contribution is represented by the middle paths in Fig. C.6.

3. *The noise power $\mathcal{N}_{d2}$ introduced in network 2*: This final contribution to the total noise power $\mathcal{N}_o$ is

$$\mathcal{N}_{d2} = G_2 kT_2 B_N$$

where again we have made use of Eq. C.17. This contribution is represented by the bottom path in Fig. C.6.

Adding these three contributions, we therefore get

$$\begin{aligned}
\mathcal{N}_o &= G_1 G_2 \mathcal{N}_s + G_2 \mathcal{N}_{d1} + \mathcal{N}_{d2} \\
&= G_1 G_2 (kTB_N) + G_2(G_1 kT_1 B_N) + G_2 kT_2 B_N \\
&= G_1 G_2 k\left(T + T_1 + \frac{T_2}{G_1}\right) B_N
\end{aligned}$$

Here we recognize the product $G_1 G_2$ as the overall value of the available power gain of the cascaded pair of networks shown in Fig. C.6. Accordingly, by analogy with Eq. C.18, we may define an equivalent noise temperature T_e for this network as

$$T_e = T_1 + \frac{T_2}{G_1} \tag{C.22}$$

The result of Eq. C.22 may be readily generalized to the cascade connection of any number of noisy two-port networks, as shown by

$$T_e = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \cdots \tag{C.23}$$

where $T_1, T_2, T_3, \ldots$ are the equivalent noise temperatures of the individual networks, and $G_1, G_2, G_3, \ldots$ are their available power gains, respectively. Equation C.23 is known as the *Friis formula*.

Correspondingly, we may express the overall noise figure F of the cascade connection of any number of two-port networks as

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots \tag{C.24}$$

where $F_1, F_2, F_3, \ldots$ are the noise figures of the individual networks.

From Eq. C.23 we see that if the first stage of a cascade connection of noisy two-port networks has a high available power gain, then the overall value of the equivalent noise temperture T_e is practically the same as that of the first stage; Eq. C.24 reveals a similar result formulated in terms of the noise figure. It is for this reason that we find in a low-noise receiver, extra care is taken in the design of the pre-amplifier at the front end of the receiver.

C.5 TELECOMMUNICATION LINK CALCULATIONS

In this section we present signal and noise power calculations for telecommunication links that rely on *line-of-sight propagation* through space. Such calculations are encountered in a *satellite communication system*,[6] for example. In this system, a message signal is transmitted from a ground station via the *uplink* to a synchronous satellite, amplified in a *transponder* therein, and then retransmitted from the satellite via the *downlink* to another ground station. The satellite is positioned in a *geostationary orbit* (around the earth) so that it is always visible to different ground stations located inside the satellite antenna's coverage zones on the earth's surface. In effect, the satellite acts as a powerful *repeater* in the sky. Another important application is that of a deep-space telecommunication system[7] used for the transmission of information between a spacecraft and a ground station. In this application the system is provided with a *tracking* capability such that the spacecraft is always visible to the ground station. For the analysis presented here, we will consider a telecommunication link that is illustrative of space applications. Nevertheless, the results of the analysis are equally applicable to a *line-of-sight radio microwave link* between a transmitting source and a receiver located a known distance apart.

CALCULATION OF RECEIVED SIGNAL POWER

Figure C.7 illustrates a link between a spacecraft and a ground station. The link includes a *transmitting source* (on the spacecraft) with its output radiated through the spacecraft's antenna. At the ground station, a *receiving antenna* is used to collect signal power from the incoming electromagnetic wave and feed it to the *low-noise receiver* through a piece of *waveguide*.

Let the transmitting source radiate a total power P_T. If this power were radiated *isotropically* (i.e., uniformly in all directions), then the *power flux density* at a distance r from the source is $P_T/4\pi r^2$, where $4\pi r^2$ is the surface area of a sphere of radius r. In practice, we use a highly directional *antenna*

[6]For a detailed treatment of link calculations in satellite communications, see Pratt and Bostian (1986), Chapter 4.

[7]For a detailed treatment of link calculations in deep-space communications, see Yuen (1983), Section 1.2.

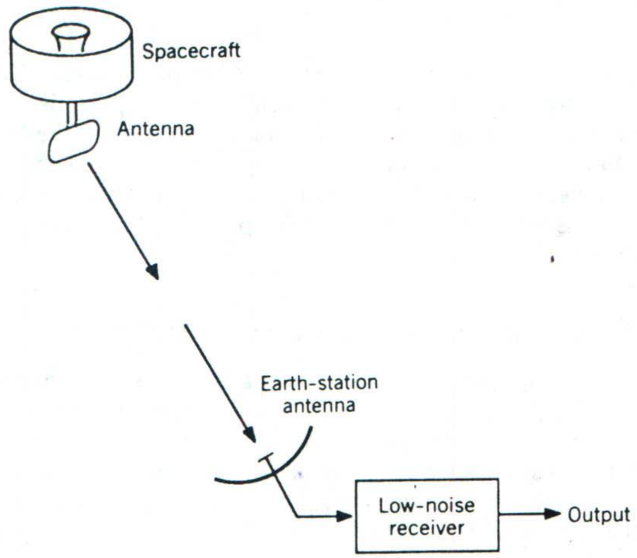

Figure C.7
Space communication link.

so that the transmitted power is radiated primarily along a particular direction of interest. The antenna has a *gain* that is defined as the *ratio of power radiated per unit solid angle in a given direction to the average power radiated per unit solid angle.* Let G_T denote the gain of the transmitting antenna in the direction in which maximum power is radiated; this direction is called the *boresight* of the antenna. The gain G_T is a measure of the increase in power radiated by the antenna over that radiated from an isotropic source. Thus, for a transmitter of total power P_T driving a *lossless* antenna with gain G_T, the power flux density at distance r in the direction of the antenna boresight is given by

$$\mathfrak{F} = \frac{P_T G_T}{4\pi r^2} \tag{C.25}$$

Let A_{eff} denote the *effective aperture area* of the receiving antenna. This area is related to the *physical aperture area* A of the antenna by

$$A_{eff} = \eta A \tag{C.26}$$

where η is the *aperture efficiency*. Typically, η is in the range of 40 to 90%, depending on the type of antenna used. The gain of the receiving antenna G_R is defined in terms of the effective aperture area A_{eff} by

$$G_R = \frac{4\pi A_{eff}}{\lambda^2} \tag{C.27}$$

where λ is the *wavelength* of the transmitted electromagnetic wave; λ is equal to c/f where c is the *speed of propagation* (which is the same as the *speed of light*), and f is the transmitting frequency. Equivalently, we have

$$A_{eff} = \frac{\lambda^2 G_R}{4\pi} \tag{C.28}$$

Hence, given the power flux density $\mathfrak{F}$ at the receiving antenna with effective aperture area A_{eff}, the *received power* is

$$P_R = A_{eff}\mathfrak{F} \tag{C.29}$$

Thus, substituting Eqs. C.25 and C.28 in C.29, we get the desired result:

$$P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi r}\right)^2 \tag{C.30}$$

Equation C.30 is known as the *Friis transmission formula.*

The product $P_T G_T$ in this equation is called the *effective isotropic radiated power* (EIRP). It describes the combination of the transmitting source and antenna in terms of an effective isotropic source with power $P_T G_T$ radiated uniformly in all directions. The term $(4\pi r/\lambda)^2$ is called the *path loss* or *space loss*; it may be viewed as the ratio of received power to transmitted power between two antennas that are separated by a distance r.

From Eq. C.30 we see that for given values of wavelength λ and distance r, the received signal power P_R can be increased by three methods:

1. *The spacecraft-transmitted power P_T is increased.* Typically, P_T is 20 W or less. Even though this transmitted power may appear low, the input power required for its generation represents a substantial fraction of the total power available on the spacecraft. Hence, there is a physical limit on how large a value we can assign to the transmitted power P_T.
2. *The gain G_T of the transmitting antenna is increased.* This will help concentrate the transmitted power more intensely in the direction of the receiving antenna. However, a large value of G_T requires the use of a large antenna. The choice of G_T is therefore limited by size and weight constraints permissible on the spacecraft.
3. *The gain G_R of the receiving antenna is increased.* This will enable the receiver to collect as much of the radiated signal power as possible. Here again, size and weight constraints place a physical limit on the size of the ground-station antenna, although these constraints are far less demanding than those on the spacecraft antenna; we typically have $G_R >> G_T$.

Let the receiving antenna gain and the space loss be expressed in *decibels* (dB). Likewise, let the effective radiated power and the received power be expressed in *decibels relative to 1 watt* (dBW). Then, we may restate the Friis transmission formula in the form:

$$P_R = \text{EIRP} + G_R - L_S \tag{C.31}$$

where

$$\text{EIRP} = 10 \log_{10}(P_T G_T)$$

$$G_R = 10 \log_{10}\left(\frac{4\pi A_{eff}}{\lambda^2}\right)$$

$$L_S = 20 \log_{10}\left(\frac{4\pi r}{\lambda}\right)$$

In Eq. C.30, G_R appears as a power ratio, whereas in Eq. C.31 it is expressed in decibels.

Equation C.31 is idealized in that it does not account for *losses* in the link. To correct for this, it is customary to include a term that represents the combined effect of losses in the atmosphere due to rain attenuation, losses in the transmitting and receiving antennas, and possible loss of gain due to mispointing of the antennas. Let L_0 denote the overall value of this loss expressed in decibels; this term is sometimes called the *system margin*. Then we may modify the expression for the received signal power as

$$P_R = \text{EIRP} + G_R - L_S - L_0 \tag{C.32}$$

Equation C.32 represents a *link power budget* in that it allows the system designer of a telecommunication link to adjust controllable parameters such as the EIRP or the receiving antenna gain G_R and make quick calculations of the received power.

The received power P_R is commonly called the *carrier power*. This is because in a space communication link the method of modulation commonly used for transmitting message signals maintains the envelope of the sinusoidal carrier wave constant; hence, the carrier power is typically equal to the received power.

EXAMPLE 2

A spacecraft is located at a distance of 40,000 km from a ground station on the earth's surface. A transmitting source of frequency 4 GHz radiates a power of 10 W through an antenna with a gain of 20 dB. Assume that the effective aperture area of the receiving antenna is 10 m^2. We want to calculate the received signal power, ignoring losses in the links.

The effective radiated power equals

$$\begin{aligned}\text{EIRP} &= 10\log_{10}(P_T G_T)\\ &= 10\log_{10}P_T + 10\log_{10}G_T\\ &= 10\log_{10}10 + 20\\ &= 30\ \text{dBW}\end{aligned}$$

The speed of propagation equals the speed of light: $c = 3 \times 10^8$ m/s. The wavelength of the transmitted electromagnetic wave is therefore

$$\begin{aligned}\lambda &= \frac{c}{f}\\ &= \frac{3 \times 10^8}{4 \times 10^9}\\ &= 0.075\ \text{m}\end{aligned}$$

Hence, the space loss equals

$$\begin{aligned}L_S &= 20\log_{10}\left(\frac{4\pi r}{\lambda}\right)\\ &= 20\log_{10}\left(\frac{4\pi \times 4 \times 10^7}{0.075}\right)\\ &= 196.5\ \text{dB}\end{aligned}$$

The gain of the receiving antenna equals

$$\begin{aligned}G_R &= 10\log_{10}\left(\frac{4\pi A_{eff}}{\lambda^2}\right)\\ &= 10\log_{10}\left(\frac{4\pi \times 10}{0.075^2}\right)\\ &= 43.5\ \text{dB}\end{aligned}$$

Thus, using Eq. C.32 and ignoring the system margin L_0, we find that the received signal power equals

$$\begin{aligned}P_R &= 30 + 43.5 - 196.5\\ &= -123\ \text{dBW}\end{aligned}$$

Equivalently, we have

$$P_R = 5 \times 10^{-13}\ \text{watts}$$

The low value of power P_R indicates that the received signal at the ground station is extremely weak. The inclusion of system margin L_0 (to account for losses in the link) will make the received signal even weaker.

CALCULATION OF SYSTEM NOISE TEMPERATURE

In a space communication link the receiver has to cope with extremely weak signals, as illustrated in Example 2; the weakness results from transmission over long distances. It is therefore imperative that we keep the available noise power low so as to permit a high quality of communication. From Eq. C.20 we see that the available noise power at the receiver input is kT_eB_N, where k is Boltzmann's constant, T_e is the equivalent noise temperature of the receiver, and B_N is the noise bandwidth. Accordingly, the effect of noise is minimized by adopting a combination of two independent strategies:

1. *The equivalent noise temperature of the receiver is maintained as low as possible.* An effective way of accomplishing this requirement is to employ *cryogenically cooled* parametric amplifiers. An ordinary amplifier converts power from a dc source (e.g., power supply or battery) into power at some signal frequency of interest. A *parametric amplifier*, on the other hand, converts power at one frequency (from a source generally known as the *pump*) into power at another frequency, the signal frequency.[8] With liquid helium cooling at 4°K above absolute zero, parametric amplifiers are able to achieve a noise temperature of 20° to 40°K at 4 GHz. Another way of providing low-noise amplification at microwave frequencies is to use cryogenically cooled maser amplifiers. A *maser* (*m*icrowave *a*mplification by the *s*timulated *e*mission of *r*adiation) performs amplification by a quantum-mechanical process, adding almost no noise to the signal it amplifies. Cryogenically cooled parametric and maser amplifiers, however, are expensive to install and maintain; their use can therefore be justified only in large ground stations. Without physical cooling, a noise temperature from 70 to 200°K can be achieved by using gallium arsenide field effect transistors (GaAsFET) or uncooled parametric amplifiers.
2. *The noise bandwidth is minimized, while still permitting the information-carrying (modulated) signals to pass through unaltered.* The value used for the noise bandwidth B_N should be the band-pass version of the equivalent noise bandwidth. For a rough calculation, we may use instead the overall 3-dB bandwidth of the amplifier in the receiver; the error introduced by so doing is usually small.

[8] For a discussion of parametric amplifiers, see Angelakos and Everhart (1968), pp. 82–93. This reference also presents a description of masers (pp. 93–98).

To calculate the *system noise temperature,* we have to include, in addition to the noise generated in the receiver, two other sources of noise:

1. *Antenna noise due to random radiation picked up by the ground-station antenna.* This random radiation includes that from the atmosphere, hot bodies in the field of view of the antenna, the omnipresent 2.7°K thermal background of the universe, and that portion of the ground seen by the sidelobes of the antenna. Let T_a denote the *antenna noise temperature.* Let L_w denote the *loss factor* in the antenna feed and waveguide, where $L_w \geq 1$. Accordingly, the input noise is attenuated by the factor L_w. That is, the antenna noise temperature referred to the amplifier input equals T_a/L_w.
2. *Noise generated due to resistance in the antenna feed and waveguide:* Let T_{amb} denote the *ambient temperature.* Then the equivalent noise generator that represents resistance in the antenna feed and waveguide has the noise temperature

$$T_l = \left(1 - \frac{1}{L_w}\right)T_{amb} \tag{C.33}$$

Note that when there is no loss, L_w is unity and T_l is zero.

Figure C.8 shows the components responsible for noise in the receiving system. Let T_s denote the *system noise temperature* referred to the receiver input. From Fig. C.8, we see that T_s is given by

$$T_s = \frac{T_a}{L_w} + \left(1 - \frac{1}{L_w}\right)T_{amb} + T_e \tag{C.34}$$

where, as defined previously,

T_a = antenna noise temperature

L_w = loss factor in the antenna feed and waveguide

T_{amb} = ambient temperature of the antenna feed and waveguide

T_e = equivalent noise temperature of the receiver

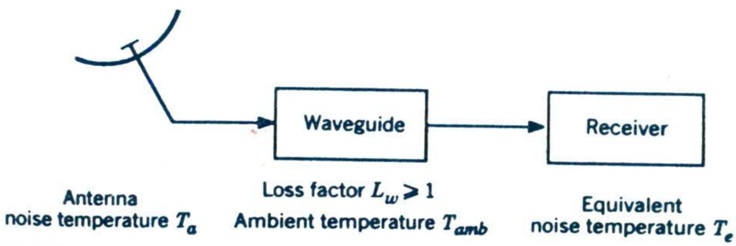

Figure C.8
Components responsible for receiving system noise in a space communication link.

CARRIER-TO-NOISE RATIO

The *carrier-to-noise ratio* (CNR) is defined as the ratio of the carrier power to the available noise power, with both measured at the receiver input. As mentioned previously, the carrier power is the same as the received signal power P_R. The formula for P_R is described by the Friis transmission equation (C.30). To calculate the available noise power at the receiver input, we use the expression kT_sB_N, where k is Boltzmann's constant, T_s is the system noise temperature, and B_N is the noise bandwidth. We therefore have .

$$\mathrm{CNR} = \frac{P_R}{kT_sB_N} \tag{C.35}$$

Here again, the simplicity of this formula stems from the use of noise temperature as the measure of how noisy the system is.

EXAMPLE 3

Consider a receiver with a cryogenically cooled amplifier. The equivalent noise temperature of the receiver is 20°K. The ground station uses a large antenna operating at a frequency of 4 GHz and an elevation of 5°; the antenna noise temperature is estimated to be 50°K. Calculate the system noise temperature, assuming no loss in the antenna feed and waveguide. Hence, calculate the carrier-to-noise ratio, assuming a carrier power of −123 dBW (as in Example 2) and a noise bandwidth of 36 MHz.

From Eq. C.34 we see that with $L_w = 1$ the system noise temperature equals

$$\begin{aligned} T_s &= T_a + T_e \\ &= 50 + 20 \\ &= 70°\mathrm{K} \end{aligned}$$

Hence, the available noise power equals

$$\begin{aligned} kT_sB_N &= 1.38 \times 10^{-23} \times 70 \times 36 \times 10^6 \\ &= 3.48 \times 10^{-14}\ \mathrm{W} \\ &= -134.6\ \mathrm{dBW} \end{aligned}$$

Thus, the use of Eq. C.35 yields the following value for the carrier-to-noise ratio expressed in decibels:

$$\begin{aligned} 10 \log_{10}(\mathrm{CNR}) &= -123 + 134.6 \\ &= 11.6\ \mathrm{dB} \end{aligned}$$

APPENDIX D

TABLES

In Tables 1 through 8 of Appendix D, we present the following material:

1. Summary of properties of the Fourier transform.
2. Short table of Fourier transform pairs.
3. Table of trigonometric identities.
4. Short table of series expansions and summations.
5. Short table of integrals.
6. Values of the error function erf(u) for u in the range 0 to 3.30.
7. List of useful constants.
8. List of recommended unit prefixes.

TABLE 1 Properties of the Fourier Transform

Property	Mathematical Description
1. Linearity	$ag_1(t) + bg_2(t) \rightleftharpoons aG_1(f) + bG_2(f)$ where a and b are constants
2. Time scaling	$g(at) \rightleftharpoons \frac{1}{\lvert a\rvert} G\left(\frac{f}{a}\right)$ where a is a constant
3. Duality	If $g(t) \rightleftharpoons G(f)$, then $G(t) \rightleftharpoons g(-f)$
4. Time shifting	$g(t - t_0) \rightleftharpoons G(f) \exp(-j2\pi f t_0)$
5. Frequency shifting	$\exp(j2\pi f_c t) g(t) \rightleftharpoons G(f - f_c)$
6. Area under $g(t)$	$\int_{-\infty}^{\infty} g(t)\, dt = G(0)$
7. Area under $G(f)$	$g(0) = \int_{-\infty}^{\infty} G(f)\, df$
8. Differentiation in the time domain	$\frac{d}{dt} g(t) \rightleftharpoons j2\pi f G(f)$
9. Integration in the time domain	$\int_{-\infty}^{t} g(\tau)\, d\tau \rightleftharpoons \frac{1}{j2\pi f} G(f) + \frac{G(0)}{2} \delta(f)$
10. Conjugate functions	If $g(t) \rightleftharpoons G(f)$, then $g^*(t) \rightleftharpoons G^*(-f)$
11. Multiplication in the time domain	$g_1(t) g_2(t) \rightleftharpoons \int_{-\infty}^{\infty} G_1(\lambda) G_2(f - \lambda)\, d\lambda$
12. Convolution in the time domain	$\int_{-\infty}^{\infty} g_1(\tau) g_2(t - \tau)\, d\tau \rightleftharpoons G_1(f) G_2(f)$

TABLE 2 Fourier Transform Pairs

Time Function	Fourier Transform
$\mathrm{rect}\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}(fT)$
$\mathrm{sinc}(2Wt)$	$\frac{1}{2W}\,\mathrm{rect}\left(\frac{f}{2W}\right)$
$\exp(-at)u(t), \quad a > 0$	$\frac{1}{a + j2\pi f}$
$\exp(-a\lvert t\rvert), \quad a > 0$	$\frac{2a}{a^2 + (2\pi f)^2}$
$\exp(-\pi t^2)$	$\exp(-\pi f^2)$
$\begin{cases} 1 - \frac{\lvert t\rvert}{T}, & \lvert t\rvert < T \\ 0, & \lvert t\rvert \geq T \end{cases}$	$T\,\mathrm{sinc}^2(fT)$
$\delta(t)$	1
1	$\delta(f)$
$\delta(t - t_0)$	$\exp(-j2\pi f t_0)$
$\exp(j2\pi f_c t)$	$\delta(f - f_c)$
$\cos(2\pi f_c t)$	$\frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$
$\sin(2\pi f_c t)$	$\frac{1}{2j}[\delta(f - f_c) - \delta(f + f_c)]$
$\mathrm{sgn}(t)$	$\frac{1}{j\pi f}$
$\frac{1}{\pi t}$	$-j\,\mathrm{sgn}(f)$
$u(t)$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
$\sum_{i=-\infty}^{\infty} \delta(t - iT_0)$	$\frac{1}{T_0}\sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_0}\right)$

TABLE 3 Trigonometric Identities

$$\exp(\pm j\theta) = \cos\theta \pm j\sin\theta$$

$$\cos\theta = \tfrac{1}{2}[\exp(j\theta) + \exp(-j\theta)]$$

$$\sin\theta = \frac{1}{2j}[\exp(j\theta) - \exp(-j\theta)]$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\cos^2\theta - \sin^2\theta = \cos(2\theta)$$

$$\cos^2\theta = \tfrac{1}{2}[1 + \cos(2\theta)]$$

$$\sin^2\theta = \tfrac{1}{2}[1 - \cos(2\theta)]$$

$$2\sin\theta\cos\theta = \sin(2\theta)$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

$$\sin\alpha\sin\beta = \tfrac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos\alpha\cos\beta = \tfrac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin\alpha\cos\beta = \tfrac{1}{2}[\sin(\alpha - \beta) + \sin(\alpha + \beta)]$$

TABLE 4 Series Expansions and Summations

1. *Expansions*

Taylor series

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \cdots$$

where

$$f^{(n)}(a) = \left.\frac{d^n f(x)}{dx^n}\right|_{x=a}$$

MacLaurin series

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(n)}(0)}{n!}x^n + \cdots$$

where

$$f^{(n)}(0) = \left.\frac{d^n f(x)}{dx^n}\right|_{x=0}$$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots, \qquad |nx| < 1$$

Exponential series

$$\exp x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots$$

Logarithmic series

$$\ln(1+x) = x - \tfrac{1}{2}x^2 + \tfrac{1}{3}x^3 - \cdots$$

Trigonometric series

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \cdots$$

$$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \cdots$$

$$\tan x = x + \tfrac{1}{3}x^3 + \tfrac{2}{15}x^5 + \cdots$$

$$\sin^{-1} x = x + \tfrac{1}{6}x^3 + \tfrac{3}{40}x^5 + \cdots$$

$$\tan^{-1} x = x - \tfrac{1}{3}x^3 + \tfrac{1}{5}x^5 - \cdots, \qquad |x| < 1$$

$$\operatorname{sinc} x = 1 - \frac{1}{3!}(\pi x)^2 + \frac{1}{5!}(\pi x)^4 - \cdots$$

2. *Summations*

Arithmetic series

$$\sum_{n=1}^{N} n = \frac{N(N+1)}{2}$$

Geometric series

$$\sum_{n=0}^{N} r^n = \frac{1 - r^{N+1}}{1 - r}$$

TABLE 5 Integrals

Indefinite integrals

$$\int x\sin(ax)\,dx = \frac{1}{a^2}[\sin(ax) - ax\cos(ax)]$$

$$\int x\cos(ax)\,dx = \frac{1}{a^2}[\cos(ax) + ax\sin(ax)]$$

$$\int x\exp(ax)\,dx = \frac{1}{a^2}\exp(ax)(ax - 1)$$

$$\int x\exp(ax^2)\,dx = \frac{1}{2a}\exp(ax^2)$$

$$\int \exp(ax)\sin(bx)\,dx = \frac{1}{a^2 + b^2}\exp(ax)[a\sin(bx) - b\cos(bx)]$$

$$\int \exp(ax)\cos(bx)\,dx = \frac{1}{a^2 + b^2}\exp(ax)[a\cos(bx) + b\sin(bx)]$$

$$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab}\tan^{-1}\left(\frac{bx}{a}\right)$$

$$\int \frac{x^2\,dx}{a^2 + b^2x^2} = \frac{x}{b^2} - \frac{a}{b^3}\tan^{-1}\left(\frac{bx}{a}\right)$$

Definite integrals

$$\int_0^\infty \frac{x\sin(ax)}{b^2 + x^2}\,dx = \frac{\pi}{2}\exp(-ab), \qquad a > 0,\ b > 0$$

$$\int_0^\infty \frac{\cos(ax)}{b^2 + x^2}\,dx = \frac{\pi}{2b}\exp(-ab), \qquad a > 0,\ b > 0$$

$$\int_0^\infty \frac{\cos(ax)}{(b^2 - x^2)^2}\,dx = \frac{\pi}{4b^3}[\sin(ab) - ab\cos(ab)], \qquad a > 0,\ b > 0$$

$$\int_0^\infty \operatorname{sinc} x\,dx = \int_0^\infty \operatorname{sinc}^2 x\,dx = \tfrac{1}{2}$$

$$\int_0^\infty \exp(-ax^2)\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}, \qquad a > 0$$

$$\int_0^\infty x^2\exp(-ax^2)\,dx = \frac{1}{4a}\sqrt{\frac{\pi}{a}}, \qquad a > 0$$

Integration by parts

$$\int f(t)g'(t)\,dt = f(t)g(t) - \int f'(t)g(t)\,dt$$

where

$$f'(t) = \frac{df(t)}{dt}$$

and

$$g'(t) = \frac{dg(t)}{dt}$$

TABLE 6 Error Function

u	erf(u)	u	erf(u)
0.00	0.00000	1.10	0.88021
0.05	0.05637	1.15	0.89612
0.10	0.11246	1.20	0.91031
0.15	0.16800	1.25	0.92290
0.20	0.22270	1.30	0.93401
0.25	0.27633	1.35	0.94376
0.30	0.32863	1.40	0.95229
0.35	0.37938	1.45	0.95970
0.40	0.42839	1.50	0.96611
0.45	0.47548	1.55	0.97162
0.50	0.52050	1.60	0.97635
0.55	0.56332	1.65	0.98038
0.60	0.60386	1.70	0.98379
0.65	0.64203	1.75	0.98667
0.70	0.67780	1.80	0.98909
0.75	0.71116	1.85	0.99111
0.80	0.74210	1.90	0.99279
0.85	0.77067	1.95	0.99418
0.90	0.79691	2.00	0.99532
0.95	0.82089	2.50	0.99959
1.00	0.84270	3.00	0.99998
1.05	0.86244	3.30	0.999998

The *error function* erf(u) is defined by

$$\mathrm{erf}(u) = \frac{2}{\sqrt{\pi}} \int_0^u \exp(-z^2)\, dz$$

Note that

$$\mathrm{erf}(0) = 0$$
$$\mathrm{erf}(\infty) = 1$$
$$\mathrm{erf}(-u) = -\mathrm{erf}(u)$$

The *complementary error function* erfc(u) is defined by

$$\mathrm{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_u^\infty \exp(-z^2)\, dz$$

These two functions are related by

$$\mathrm{erfc}(u) = 1 - \mathrm{erf}(u)$$

TABLE 7 Useful Constants

Physical Constants	
Boltzmann's constant	$k = 1.38 \times 10^{-23}$ joule/degree Kelvin
Planck's constant	$h = 6.626 \times 10^{-34}$ joule·second
Electron (fundamental) charge	$q = 1.602 \times 10^{-19}$ coulomb
Speed of light in vacuum	$c = 2.998 \times 10^{8}$ meters/second
Standard (absolute) temperature	$T_0 = 273$ degree Kelvin
Thermal voltage	$V_T = 0.026$ volt at room temperature
Thermal energy kT at standard temperature	$kT_0 = 3.77 \times 10^{-21}$ joule
One hertz (Hz) = 1 cycle/second	
One watt (W) = 1 joule/second	
Mathematical Constants	
Base of natural logarithm	$e = 2.7182818$
Logarithm of e to base 2	$\log_2 e = 1.442695$
Logarithm of 2 to base e	$\ln 2 = 0.693147$
Logarithm of 2 to base 10	$\log_{10} 2 = 0.30103$
Pi	$\pi = 3.1415927$

TABLE 8 Recommended Unit Prefixes

Multiples and Submultiples	Prefixes	Symbols
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

GLOSSARY

CONVENTIONS AND NOTATIONS

1. The symbol $|\ \ |$ means the magnitude of the complex quantity contained within.
2. The symbol arg( ) means the phase angle of the complex quantity contained within.
3. The symbol Re[ ] means the "real part of," and Im[ ] means the "imaginary part of."
4. The symbol ln(·) denotes the natural logarithm of the quantity contained within, whereas the logarithm to the base a is denoted by $\log_a(\ )$. The symbol exp( ) denotes the exponential function; for example, $\exp(x)$ denotes e^x, where e is the base of the natural logarithm.
5. The use of an asterisk as superscript denotes complex conjugate; for example, x^* is the complex conjugate of x.
6. The symbol $\rightleftharpoons$ indicates a Fourier transform pair. For example, let $g(t)$ denote a time function and $G(f)$ denote its Fourier transform; we then write $g(t) \rightleftharpoons G(f)$.
7. The symbol F[ ] indicates the Fourier transform operation; for example $F[g(t)] = G(f)$. The symbol $F^{-1}[\ ]$ indicates the inverse Fourier transform operation; for example $F^{-1}[G(f)] = g(t)$
8. The symbol ☆ denotes convolution; for example

$$x(t) \star h(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)\, d\tau$$

9. The symbol $\oplus$ denotes modulo-two addition.
10. The use of subscript p indicates that the pertinent function is periodic; for example, the function $g_p(t)$ is a periodic function of time t.
11. The use of a caret (hat) over a function indicates one of two things:
    (a) The Hilbert transform of a function. For example, the function $\hat{g}(t)$ is the Hilbert transform of $g(t)$.
    (b) The estimate of an unknown parameter. For example, the quantity $\hat{\alpha}(\mathbf{x})$ is an estimate of the unknown parameter α, based on the observation vector $\mathbf{x}$.
12. The use of a tilde over a function indicates the complex envelope of a narrow-band signal. For example, the function $\tilde{g}(t)$ is the complex envelope of the narrow-band signal $g(t)$.
13. The use of subscript + indicates the pre-envelope of a signal. For example, the function $g_+(t)$ is the pre-envelope of the signal $g(t)$. We may thus write $g_+(t) = g(t) + j\hat{g}(t)$, where $\hat{g}(t)$ is the Hilbert transform of $g(t)$.

14. The use of subscripts I and Q denotes the in-phase and quadrature components of a narrow-band signal or a narrow-band random process with respect to the carrier $\cos(2\pi f_c t)$.
15. The term "baseband" refers to the band of frequencies representing the original signal delivered by a source of information.
16. In the context of a probability system, $\mathcal{S}$ denotes a sample space of elementary events, and $\mathcal{E}$ denotes a class of events that is a subset of the sample space $\mathcal{S}$.
17. Random variables are uppercase (e.g., X or $\mathbf{X}$), whereas sample values of random variables are lowercase (e.g., x or $\mathbf{x}$).
18. The symbol $E[\ ]$ means the expected value of the random variable enclosed within.
19. The symbol $\mathrm{Var}[\ ]$ means the variance of the random variable enclosed within.
20. The symbol $\mathrm{Cov}[\ ]$ means the covariance of the two random variables enclosed within.
21. The average probability of symbol error is denoted by P_e. In the case of binary signaling techniques, P_{e0} denotes the conditional probability of error given that symbol 0 was transmitted, and P_{e1} denotes the conditional probability of error given that symbol 1 was transmitted. The a priori probabilities of symbols 0 and 1 are denoted by p and q, respectively.
22. The symbol $\langle\ \rangle$ denotes the time average of the sample function enclosed within.
23. A boldface lowercase letter denotes a vector, and a boldface uppercase letter denotes a matrix.

FUNCTIONS

1. Rectangular function $$\mathrm{rect}(t) = \begin{cases} 1, & -\frac{1}{2} < t < \frac{1}{2} \\ 0, & |t| > \frac{1}{2} \end{cases}$$
2. Unit step function $$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$
3. Signum function $$\mathrm{sgn}(t) = \begin{cases} 1, & t > 0 \\ -1, & t < 0 \end{cases}$$
4. Dirac delta function $$\delta(t) = 0, \quad t \neq 0$$ $$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

   or equivalently $$\int_{-\infty}^{\infty} g(t)\, \delta(t - t_0)\, dt = g(t_0)$$

5. Sinc function $\text{sinc}(x) = \dfrac{\sin(\pi x)}{\pi x}$

6. Sine integral $\text{Si}(u) = \displaystyle\int_0^u \frac{\sin x}{x}\,dx$

7. Error function $\text{erf}(u) = \dfrac{2}{\sqrt{\pi}}\displaystyle\int_0^u \exp(-z^2)\,dz$

   Complementary error function $\text{erfc}(u) = 1 - \text{erf}(u)$

8. Bessel function of the first kind of order n $J_n(x) = \dfrac{1}{2\pi}\displaystyle\int_{-\pi}^{\pi} \exp(jx\sin\theta - jn\theta)\,d\theta$

ABBREVIATIONS

ac: alternating current
ADM: adaptive delta modulation
APK: amplitude-phase keying
AWGN: additive white Gaussian noise
AM: amplitude modulation
ASK: amplitude-shift keying
BPF: band-pass filter
CDMA: code-division multiple access
CNR: carrier-to-noise ratio
CPFSK: continuous-phase frequency-shift keying
CW: continuous wave
dB: decibel
dBm: decibel milliwatt
dBW: decibel watt
dc: direct current
DM: delta modulation
DPCM: differential pulse-code modulation
DPSK: differential phase-shift keying
DSBSC: double-sideband suppressed-carrier
EIRP: effective isotropic radiated power
FDM: frequency-division multiplexing
FDMA: frequency-division multiple access
FET: field-effect transistor
FFT: fast Fourier transform
FIR: finite-duration impulse response
FM: frequency modulation
FMFB: frequency modulator with feedback

FSK: frequency-shift keying
HDTV: high-definition television
Hz: hertz
IF: intermediate frequency
iid: independent and identically distributed
ISI: intersymbol interference
LMS: least mean-square
LPF: low-pass filter
modem: modulator-demodulator
MSK: minimum shift keying
NRZ: nonreturn-to-zero
NTSC: National Television System Committee
PAM: pulse-amplitude modulation
PCM: pulse-code modulation
PDM: pulse-duration modulation
PLL: phase-locked loop
PM: phase modulation
PN: pseudonoise
PPM: pulse-position modulation
PSK: phase-shift keying
QAM: quadrature-amplitude modulation
QPSK: quadriphase-shift keying
rad: radian
RF: radio frequency
rms: root mean-square
RZ: return-to-zero
s: second
SLSC: split-luminance and split-chrominance
SNR: signal-to-noise ratio
$(\mathrm{SNR})_C$: channel signal-to-noise ratio
$(\mathrm{SNR})_O$: output signal-to-noise ratio
SSB: single sideband
TDM: time-division multiplexing
TDMA: time-division multiple access
TV: television
V: volt
VCO: voltage-controlled oscillator
VLSI: very large-scale integration
VSB: vestigial sideband
WSS: wide-sense stationary
W: watt

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ANSWERS TO EXERCISES

Many of the exercises have their answers integrated into them. The answers to the remaining exercises are presented below on a chapter-by-chapter basis.

CHAPTER 2

Exercise 1: The amplitude spectra have a form similar to the amplitude of a sinc function. In case (a), the main lobe will be one-quarter the height and 2.5 times the width of that in case (b).
Exercise 4: $g(t) = 1/(1 - j2\pi t) = 1/[1 + (2\pi t)^2] + j2\pi t/[1 + (2\pi t)^2]$
Exercise 5: $G(f) = (1/2j)\,[1/(1 + j2\pi(f - f_c)) - 1/(1 + j2\pi(f + f_c))]$
Exercise 9: On the left side of the relation, $g_2(t)$ is replaced by $g_2^*(t)$.
Exercise 10: On the right side of the relation, $G_2(f)$ is replaced by $G_2^*(f)$. The result of changes in Exercise 10 will be the same as those in Exercise 9, as shown by $\int_{-\infty}^{\infty} g_1(t)g_2^*(t)\,dt = \int_{-\infty}^{\infty} G_1(f)G_2^*(f)\,df$
Exercise 11: Bandwidth $= 1/T$.
Exercise 12: Bandwidth $= a/2\pi$.
Exercise 15: $\cos(2\pi f_c t)u(t) \rightleftharpoons f/[j2\pi(f^2 - f_c^2)] + \frac{1}{4}[\delta(f - f_c) + \delta(f + f_c)]$

CHAPTER 3

Exercise 1: The system is noncausal because $h(t)$ is nonzero for negative time. The system, however, is stable because $\int_{-\infty}^{\infty} |h(t)|\,dt = \int_{-\infty}^{0} \exp(at)\,dt = 1/a$.
Exercise 4: The modification may arise through the use of a phase-inverting amplifier or transformer.
Exercise 5: $[1/H_c(f)] = \sum_{m=-M}^{M} C_m \exp(-j2\pi fm\,\Delta\tau)$ where M is some finite integer.
Exercise 6: In theory, the delay τ_0 would have to be infinitely large for the ideal low-pass filter to be causal.
Exercise 8: In-phase component is $m(t)$, and quadrature component is zero.
Exercise 9: $y(0) = \int_{-\infty}^{\infty} H(f)X(f)\,df = \int_{-\infty}^{\infty} \mathrm{Re}[H(f)X(f)]df$; $\tilde{y}(0) = \int_{-\infty}^{\infty} \tilde{H}(f)\tilde{X}(f)\,df$; $y(0) = \mathrm{Re}[\tilde{y}(0)]$. Hence, $\mathrm{Re}[H(f)X(f)] = \mathrm{Re}[\tilde{H}(f)\tilde{X}(f)]$. The product $H(f)X(f)$ occupies the band $f_c - W \leq f \leq f_c + W$ for positive frequencies, and its image for negative frequencies. On the other hand, the product $\tilde{H}(f)\tilde{X}(f)$ occupies the band $-W \leq f \leq W$. With $\tilde{X}(0) = 2X(f_c)$, it follows that we must have $\tilde{H}(0) = H(f_c)$.
Exercise 10: Phase delay $\tau_p = \beta(f)/f$, where $\beta(f)$ is the phase. If $\beta(f)$ is constant for all f, the phase delay becomes inversely proportional to frequency, and phase distortion results.

CHAPTER 4

Exercise 4: $R_g(0) = \int_{-\infty}^{\infty} \Psi_g(f)\,df$; $\Psi_g(0) = \int_{-\infty}^{\infty} R_g(\tau)\,d\tau$
Exercise 10: *Direct procedure:* Use the formula for the autocorrelation function of the energy signal $g(t)$. *Indirect procedure:* 1. Compute the Fourier transform $G(f)$

of the signal $g(t)$. 2. Compute the energy spectral density $\Psi_g(f) = |G(f)|^2$. 3. Compute the inverse Fourier transform of $\Psi_g(f)$.

Exercise 11: *Direct procedure:* Use the formula for the autocorrelation function of the power signal $g(t)$. *Indirect procedure:* 1. Compute the Fourier transform $G_T(f)$ of the truncated signal $g_T(t)$ for large T. 2. Compute $|G_T(f)|^2$. 3. Compute the inverse Fourier transform of $|G_T(f)|^2$.

Exercise 15:

$$R_{g_p}(\tau) = \begin{cases} \dfrac{A^2}{2}\left(1 + \dfrac{2\tau}{T_0}\right) & -\dfrac{T_0}{4} \leq \tau \leq 0 \\ \dfrac{A^2}{2}\left(1 - \dfrac{2\tau}{T_0}\right) & 0 \leq \tau \leq \dfrac{T_0}{4} \\ 0, & \text{for the remainder of the period} \end{cases}$$

$$S_{g_p}(f) = \frac{A^2}{16} \sum_{n=-\infty}^{\infty} \operatorname{sinc}^2\left(\frac{n}{4}\right) \delta\left(f - \frac{n}{T_0}\right)$$

Exercise 16: $R_N(0) = N_0/4RC$
Exercise 17: $R_N(0) = (N_0/2) \int_{-\infty}^{\infty} df/[1 + (2\pi fRC)^2] = N_0/4RC$
Exercise 18: $B_N = 1/(4RC) = \pi B/2$

CHAPTER 5

Exercise 1: Constant 1
Exercise 2: Number of representation levels = 512
Exercise 3: Code word length = 9; Bit rate = 75.6 Mb/s; Bandwidth = 75.6 MHz
Exercise 4: 768 kb/s.
Exercise 5: 1. Four-level Gray coding. 2. On-off signaling, polar signaling, and bipolar signaling. 3. Return-to-zero signaling, and Manchester coding.
Exercise 6: $\delta^2/3$.
Exercise 7:

System Level	Number of Voice Channels	Number of Picturephone Channels	Number of Television Channels
T1	24		
T2	96	1	
T3	672	7	1
T4	4032	42	6

CHAPTER 6

Exercise 2: 6.312 MHz.
Exercise 3:

b_k		0	0	1	0	1	1	0
a_k	0	0	0	1	1	0	1	1

Polar representation of a_k, volts	-1	-1	-1	$+1$	$+1$	-1	$+1$	$+1$
c_k, volts		-2	-2	0	$+2$	0	0	$+2$
$\lvert c_k \rvert$		2	2	0	2	0	0	2
$\hat{b}_k$		0	0	1	0	1	1	0

Exercise 4: The duobinary technique has correlated digits, while the other two methods have independent digits.

Exercise 5:

b_k			0	0	1	0	1	1	0
a_k	1	1	1	1	0	1	1	0	1
Polar representation of a_k, volts	$+1$	$+1$	$+1$	$+1$	-1	$+1$	$+1$	-1	$+1$
c_k, volts			0	0	-2	0	$+2$	-2	0
$\lvert c_k \rvert$, volts			0	0	2	0	2	2	0
$\hat{b}_k$			0	0	1	0	1	1	0

Exercise 6:

b_k			0	0	1	0	1	1	0
a_k	0	0	0	0	1	0	0	1	0
Polar representation of a_k, volts	-1	-1	-1	-1	$+1$	-1	-1	$+1$	-1
c_k, volts			0	0	$+2$	0	-2	$+2$	0
$\lvert c_k \rvert$, volts			0	0	2	0	2	2	0
$\hat{b}_k$			0	0	1	0	1	1	0

CHAPTER 7

Exercise 11: The low- and high-frequency ends of the amplitude response of the sideband shaping filter are the opposite of those in Fig. 7.24.
Exercise 13: The wide band 90° phase shifter (Hilbert transformer) of Fig. 7.20 is replaced by a filter whose transfer function is defined by Eq. 7.64.
Exercise 14: $f_l = f_o - f_c$
Exercise 15: $f_i(t) = 5 + 2\cos(4\pi t)$, hertz
Exercise 16: The input of the phase modulator has a triangular waveform. The output is the same as the FM wave shown in Fig. 7.37*b*.
Exercise 17: $\Delta f = 0.2$ kHz; $\beta = 0.2$. The carrier frequency f_c plays no role in these two calculations.
Exercise 18: The lower side-frequency component of a narrow-band FM wave is shifted in phase by 180° compared to that of a standard amplitude-modulated wave.
Exercise 19: A standard AM contains a carrier of fixed amplitude and two side-frequencies. A wideband FM contains a carrier of varying amplitude and an infinite number of side-frequencies.
Exercise 20: For $\beta = 0.2$, the amplitude spectrum of the FM wave consists essentially of a carrier at f_c and a pair of side-frequencies at $f_c \pm f_m$.
Exercise 21: $B = 130$ kHz
Exercise 22: Mid-band frequency of the filter is $2f_1$; Bandwidth of the filter is $2B_1$; where f_1 is the carrier frequency of $s_1(t)$ and B_1 is its bandwidth.
Exercise 23: $\Delta f = 60$ kHz; $\beta = 12$. Separation of adjacent side-frequencies is 5 kHz

CHAPTER 8

Exercise 1: The probability of getting one head and one tail is 1/2 if no distinction is made as to which coin turns up head.
Exercise 2: The probability that the two dice add up to 6 is 5/36.
Exercise 3: $P(A_0|B_1) = pp_0/[pp_0 + (1-p)p_1]$;
$P(A_1|B_0) = pp_1/[(1-p)p_0 + pp_1]$.
Exercise 4: (a) $P(B_0) = \frac{1}{2}$; (b) $P(B_1) = \frac{1}{2}$; (c) $P(A_0|B_0) = 1 - 10^{-4}$;
(d) $P(A_1|B_1) = 1 - 10^{-4}$
Exercise 5: $P(|X - m_X| < \varepsilon) = 1 - (\sigma_X^2/\varepsilon^2)$
Exercise 7: $P(-3 < X \leq 13) = \text{erf}(1/\sqrt{2})$
Exercise 8: Mean-square value $= A^2$
Exercise 9: First, $\sigma(f)$ is nonnegative for all f. Second, the total area under the curve of $\sigma(f)$ is one.
Exercise 11: As N becomes large, the power spectral density of the PN sequence assumes the same form as that of the corresponding random binary wave.
Exercise 12: $S_Y(f) = 4\sin^2(\pi f T)S_X(f) \simeq 4\pi^2 f^2 T^2 S_X(f)$. That is, for low-frequency inputs, the comb filter acts as a differentiator.
Exercise 14: $f_Y(x) = 1/(2\sqrt{\pi N_0 W})\exp(-x^2/4N_0 W)$
Exercise 15: (a) For the narrow-band random process $X(t)$, $R_X(\tau) = 2N_0 W\,\text{sinc}(2W\tau)\cos(2\pi f_c \tau)$. (b) For the in-phase component $X_I(t)$ and quadrature component $X_Q(t)$: $R_{X_I}(\tau) = R_{X_Q}(\tau) = 2N_0 W\,\text{sinc}(2W\tau)$.

CHAPTER 9

Exercise 1: (d) When using DSBSC modulation and coherent detection, the translated signal sidebands add coherently, whereas the translated noise sidebands add incoherently.
Exercise 4: $(SNR)_{O,AM} = 2\rho k_a^2 P$.
Exercise 5: 11.4 dB
Exercise 7: The values of threshold channel signal-to-noise ratio are (a) $(SNR)_C \geq$ 17.8 dB; (b) $(SNR)_C \geq 20.8$ dB. The corresponding values of average transmitted power P_c are (a) $P_c \geq 4.5$ mW; (b) $P_c \geq 9$ mW.
Exercise 8: The power spectral density of de-emphasized noise varies with frequency as $f^2/[1 + (f^2/f_0^2)]$, where f_0 is the cutoff frequency of the de-emphasis filter. Thus, starting at the initial value of zero at zero frequency, the de-emphasized noise power spectral density increases with frequency, reaches a peak, and then decreases with frequency.

CHAPTER 10

Exercise 2: (a) The matched filter output reaches its peak at time $t = t_0 + T$. (b) The maximum value of the output is proportional to signal energy.
Exercise 6: (a) $\lambda = \frac{1}{2}E$; (b) $\lambda = (N_0/2)\ ln\ (2) + \frac{1}{2}E$;
(c) $\lambda = -(N_0/2)\ ln\ (2) + \frac{1}{2}E$
Exercise 8: (b) 1.5 dB; (c) 0.68 dB.

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