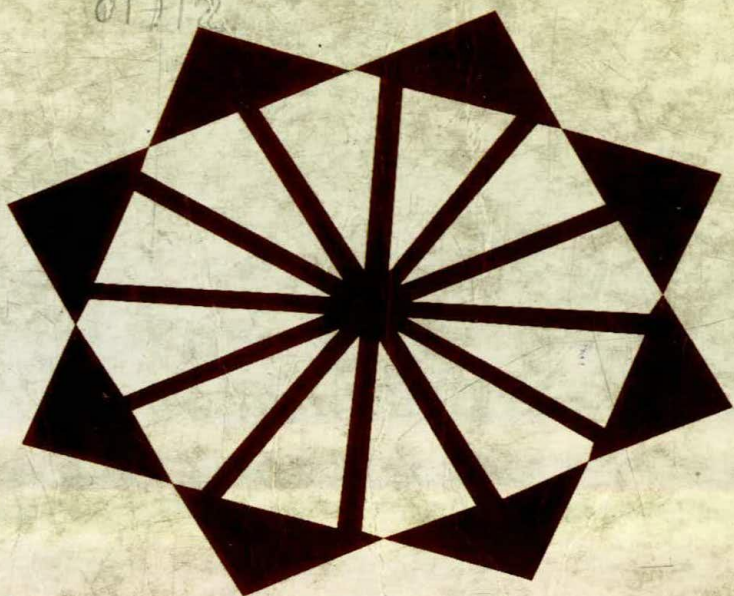


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By

**BHU DEV SHARMA**

M.Sc. Ph. D.,

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# PREFACE

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The book has been written to provide the students with the material in a systematic and easy to understand way.

In a book of this character there will not be found much that is new or original. An attempt has been made to explain all the articles with clarity and to give model solutions of as many examples as were available. To attain this object many treatises on this and allied subject have been consulted. Numerous examples from examination papers of various Indian Universities have been taken. Examples which require special attention from the examination point of view have been marked with an asterisk (\*).

The author is confident that this book, like this other books, will be liked and appreciated.

My thanks are due to the Publishers and Printers for their keen interest in the book.

Every suggestion to improve the book will be gratefully acknowledged.

—*Bhu Dev Sharma*

## PREFACE OF THE NEW EDITION

This edition is practically a reprint of 11th edition. Misprints have been removed as far as possible, University references have been made upto-date.

I am thankful to large number of studentks and teachers who have written to me in admiration of the book.

Suggestions for improvement will be thankfully received.

—*Bhu Dev Sharma*

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