

SCHAUM'S OUTLINE SERIES

THEORY AND PROBLEMS OF

COMPLEX VARIABLES

SI (metric) edition

with an introduction to CONFORMAL MAPPING and its applications

MURRAY R. SPIEGEL

INCLUDING 640 SOLVED PROBLEMS



METRIC EDITIONS

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THEORY AND PROBLEMS

OF

**COMPLEX
VARIABLES**

with an introduction to

**CONFORMAL MAPPING
and its application
SI (METRIC) EDITION**

BY

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Preface

The theory of functions of a complex variable, also called for brevity complex variables or complex analysis, is one of the most beautiful as well as useful branches of mathematics. Although originating in an atmosphere of mystery, suspicion and distrust, as evidenced by the terms "imaginary" and "complex" present in the literature, it was finally placed on a sound foundation in the 19th century through the efforts of Cauchy, Riemann, Weierstrass, Gauss and other great mathematicians.

Today the subject is recognized as an essential part of the mathematical background of engineers, physicists, mathematicians and other scientists. From the theoretical viewpoint this is because many mathematical concepts become clarified and unified when examined in the light of complex variable theory. From the applied viewpoint the theory is of tremendous value in the solution of problems of heat flow, potential theory, fluid mechanics, electromagnetic theory, aerodynamics, elasticity and many other fields of science and engineering.

This book is designed for use as a supplement to all current standard texts or as a textbook for a formal course in complex variable theory and applications. It should also be of considerable value to those taking courses in mathematics, physics, aerodynamics, elasticity or any of the numerous other fields in which complex variable methods are employed.

Each chapter begins with a clear statement of pertinent definitions, principles and theorems together with illustrative and other descriptive material. This is followed by graded sets of solved and supplementary problems. The solved problems serve to illustrate and amplify the theory, bring into sharp focus those fine points without which the student continually feels himself on unsafe ground, and provide the repetition of basic principles so vital to effective learning. Numerous proofs of theorems and derivations of formulae are included among the solved problems. The large number of supplementary problems with answers serve as a complete review of the material in each chapter.

Topics covered include the algebra and geometry of complex numbers, complex differential and integral calculus, infinite series including Taylor and Laurent series, the theory of residues with applications to the evaluation of integrals and series, and conformal mapping with applications drawn from various fields. An added feature is the chapter on special topics which should prove useful as an introduction to some more advanced topics.

Considerably more material has been included here than can be covered in most first courses. This has been done to make the book more flexible, to provide a more useful book of reference and to stimulate further interest in the topics.

I wish to take this opportunity to thank the staff of the Schaum Publishing Company for their splendid cooperation.

M. R. SPIEGEL

Rensselaer Polytechnic Institute
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