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**SCHAUM'S
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**THEORY AND PROBLEMS OF
COMPUTER
GRAPHICS**

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Schaum's Outline of Theory and Problems of
COMPUTER GRAPHICS

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To Qian, Wei, and my teachers

ZHIGANG XIANG

To Sharon, Adam, Sam, and the memory of Gordon S. Kalley

ROY PLASTOCK

PREFACE

We live in a world full of scientific and technological advances. In recent years it has become quite difficult not to notice the proliferation of something called computer graphics. Almost every computer system is set up to allow the user to interact with the system through a graphical user interface, where information on the display screen is conveyed in both textual and graphical forms. Movies and video games are popular showcases of the latest technology for people, both young and old. Watching the TV for a while, the likelihood is that you will see the magic touch of computer graphics in a commercial.

This book is both a self-contained text and a valuable study aid on the fundamental principles of computer graphics. It takes a goal-oriented approach to discuss the important concepts, the underlying mathematics, and the algorithmic aspects of the computerized image synthesis process. It contains hundreds of solved problems that help reinforce one's understanding of the field and exemplify effective problem-solving techniques.

Although the primary audience are college students taking a computer graphics course in a computer science or computer engineering program, any educated person with a desire to look into the inner workings of computer graphics should be able to learn from this concise introduction. The recommended prerequisites are some working knowledge of a computer system, the equivalent of one or two semesters of programming, a basic understanding of data structures and algorithms, and a basic knowledge of linear algebra and analytical geometry.

The field of computer graphics is characterized by rapid changes in how the technology is used in everyday applications and by constant evolution of graphics systems. The life span of graphics hardware seems to be getting shorter and shorter. An industry standard for computer graphics often becomes obsolete before it is finalized. A programming language that is a popular vehicle for graphics applications when a student begins his or her college study is likely to be on its way out by the time he or she graduates.

In this book we try to cover the key ingredients of computer graphics that tend to have a lasting value (only in relative terms, of course). Instead of compiling highly equipment-specific or computing environment-specific information, we strive to provide a good explanation of the fundamental concepts and the relationship between them. We discuss subject matters in the overall framework of computer graphics and emphasize mathematical and/or algorithmic solutions. Algorithms are presented in pseudo-code rather than a particular programming language. Examples are given with specifics to the extent that they can be easily made into working versions on a particular computer system.

We believe that this approach brings unique benefit to a diverse group of readers. First, the book can be read by itself as a general introduction to computer graphics for people who want technical substance but not the burden of implementational overhead. Second, it can be used by instructors and students as a resource book to supplement any comprehensive primary text. Third, it may serve as a stepping-stone for practitioners who want something that is more understandable than their graphics system's programmer's manuals.

The first edition of this book has served its audience well for over a decade. I would like to salute and thank my coauthors for their invaluable groundwork. The current version represents a significant revision to the original, with several chapters replaced to cover new topics, and the remaining material updated throughout the rest of the book. I hope that it can serve our future audience as well for years to come.

Thank you for choosing our book. May you find it stimulating and rewarding.

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