

# CHAPTER 1 Game Theory and Strategic Behavior

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**PREVIEW**

The distinguishing characteristic of oligopolistic markets is that managers must consider the effects of their decisions on other firms and must also anticipate how managers of those other firms will respond. Basically, life in an oligopoly can be interpreted as a high-stakes game where the objective is to earn economic profits by outguessing your rivals. In fact, important insights into oligopolistic markets have been achieved by using a method of analysis called game theory.

The first section of this chapter introduces basic concepts of game theory. The second illustrates how game theory can be used to understand and predict the behavior of managers in oligopolies. The final section of the chapter considers strategies that are used to prevent entry in a market.

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**INTRODUCTION TO GAME THEORY**

Game theory was developed in the 1950s by mathematician John von Neumann and economist Oskar Morgenstern. The technique was designed to evaluate situations where individuals and organizations can have conflicting objectives. It can be used to analyze a broad range of activities, including dating and mating strategies, parlor games, legal and political negotiations, and economic behavior. For example, in wage negotiations between unions and firms, a primary objective of the management team is to keep the total wage bill as small as possible, while union negotiators want to maximize wage payments. In peace talks between two nations at war, each country wants to arrange a settlement that is to its advantage. During courtship, both men and women may adopt complicated and sometimes devious strategies based on their individual objectives. In all of these situations, game theory could be used to analyze the bargaining process between the two parties.

Over time, game theory has evolved to become a very broad and complex subject. Consequently, only the most basic elements can be presented here. However, even these fundamental concepts can provide valuable insights into business behavior. In this section, the nomenclature and structure of game theory are introduced. Although the political and courtship applications of the techniques probably are more interesting, the discussion is limited to the use of game theory in an economic setting.

### ✓ The Payoff Matrix

At the heart of game theory are the concepts of strategies and payoffs (A *strategy* is a course of action taken by one of the participants in a game, and the *payoff* is the result or outcome of the strategy.) Consider the simple example of two children engaged in a penny-flipping game. One child tosses the coin and the other calls it. If a head or tail is correctly predicted, then the caller gets the penny. If not, the other person wins the coin. In this situation, the strategy is the caller's choice of a head or a tail, and the payoff or outcome is that either the caller wins a penny or the coin tosser wins the coin.

In this simple game, the coin tosser is a passive participant, and the outcome depends on the caller's choice and the result of a random flip of the coin. In more complicated games, the outcome may depend not only on the choice made by one person, but also on the strategies selected by one or more other participants.

TABLE 11.1 Payoff Matrix (millions)

		Firm 2	
		No Price Change	Price Increase
Firm 1	No Price Change	10, 10	100, -30
	Price Increase	-20, 30	140, 35

Consider a market with two competing firms whose objective is to increase their profits by price changes. Assume that each firm has two possible strategies—it can either maintain its price at the present level or it can increase its price. In this game, there are four possible combinations of strategies—both of the firms increase their prices, neither firm increases its price, firm 1 increases its price but firm 2 does not, and firm 2 increases price but the other firm does not.

The results for each of the four combinations of strategies are shown in Table 11.1. The first number in each cell is the profit for firm 1, and the second value is the corresponding profit for firm 2. Table 11.1 is referred to as a *payoff matrix* because it shows the outcomes or payoffs that result from each combination of strategies adopted by the two participants in the game.

### Nash Equilibrium

In discussing models of perfect competition and monopoly in chapter 9, the focus was on determining equilibrium conditions. These conditions were the rates of output that allowed the firms to maximize profits. Because the equilibriums represented the best the firms could do, there was no reason to change. For both of these market structures, the profit-maximizing equilibrium was the rate of output where marginal revenue equaled marginal cost. Actions of the other firms in the market were irrelevant because they had an insignificant impact on the firm (perfect competition) or because there were no other firms (monopoly).

But the payoffs shown in Table 11.1 depend not only on the pricing alternative chosen by each firm, but also on the strategy selected by the other firm. If firm 1 decides not to raise prices, it will have \$10 million in profits if firm 2 also does not change its price, and \$100 million if firm 2 implements a price increase. If firm 1 raises prices, profits will be \$-20 million if the other firm makes no change and \$140 million if firm 2 does increase its price.

The objective of each firm is to do the best it can based on what the other firm does. Faced with the payoff matrix of Table 11.1, is there an equilibrium result? Suppose that firm 1 does not change its price. In this situation, the optimal strategy for firm 2 is also no price change because it will earn \$10 million in profit, compared to a loss of \$30 million if it increases prices. Note that if firm 2 does not change prices, the best strategy for firm 1 is to hold the line on prices.

Thus, for the payoff matrix of Table 11.1, no price changes is an equilibrium, because neither firm can benefit by increasing its price if the other firm does not. This result is referred to as a Nash equilibrium (for mathematician John Nash). A *Nash equilibrium* is defined as a set of strategies such that none of the participants in the game can improve their payoff, given the strategies of the other participants. For the game de-

TABLE 11.2 Dominant Strategy (millions)

		Firm 2	
		No Price Change	Price Increase
Firm 1	No Price Change	10, 10	100, -30
	Price Increase	-20, 30	140, 25

scribed in Table 11.1, firm 1's choice not to raise prices is optimal if firm 2 doesn't raise prices and vice versa. In chapter 10, the equilibrium for a Cournot duopoly was determined. That result also was a Nash equilibrium because each firm was producing the optimal rate of output based on the output for the other firm. Consequently, neither firm had any incentive to change.

A limitation of the concept of Nash equilibrium is that there can be more than one equilibrium. For example, in Table 11.1, if firm 1 were to increase its price, then firm 2 would earn more profit by increasing its price than by leaving its price unchanged. Similarly, if firm 2 were to initiate a price increase, firm 1 would earn \$40 million more by matching the increase. Thus, both firms increasing their price is also a Nash equilibrium. The actual outcome of the game depends on which action occurs first. Because no price change is the initial condition, the expected outcome of this game would be that prices would not change. For some games, there may be no Nash equilibrium. In these cases, participants may continuously switch from one strategy to another.

### Key Concepts

- A strategy is a course of action that can be used by a player in a game.
- The payoff matrix indicates the outcomes of all the possible combinations of strategies in a game.
- A Nash equilibrium is a set of strategies such that none of the players in a game can improve their payoff, given the strategies of the other participants.

### Dominant Strategies

For the payoffs in Table 11.1, the optimal strategy for each firm depends on the strategy selected by the other firm. But in some situations, one firm's best strategy may not depend on the choice made by other participants in the game. In this case, that firm has a *dominant strategy*. Consider the payoff matrix shown in Table 11.2.

If firm 1 does not change prices, firm 2's best strategy is to also make no price adjustment. But, based on the profit numbers from Table 11.2, if firm 1 increases its price, firm 2 is still better off with no price change because profit will be \$30 million, compared to \$25 million if it increases price. Hence, firm 2's dominant strategy is to hold prices at the existing level, regardless of what firm 1 does. When one player has a dominant strategy, the game will always have a Nash equilibrium because that player will use that strategy and the other will respond with its best alternative. For the payoffs in Table 11.2, firm 1's best response to no price change by firm 2 is also not to change its price.

In the analysis of games, the first step is to determine if any participant has a dominant strategy. If such a strategy exists, then the outcome of the game should be easily determined, because the player will use the dominant strategy, and other participants will adopt their best response. If there is no dominant strategy, the next step is to search for other Nash equilibriums. If there are none, it may still be possible to analyze the game using other techniques.

## Case Study

### Indiana Jones and the Holy Grail

Among the most popular movies of all time are the Indiana Jones sagas. In *Indiana Jones and the Last Crusade*, it is revealed that the hero is a great adventurer but not an astute student of game theory. Near the end of the movie, Jones, his father, and the Nazis are at the site of the Holy Grail. Because the pair refuse to help, the Nazis shoot the father, knowing that he can be saved only by taking a drink from the sacred cup.

Indiana Jones makes his way to the Grail but finds that it is located among hundreds of other chalices. Drinking from the right cup brings eternal life, but a sip from any other causes instant death. The Nazi leader impatiently drinks from the wrong cup and dies, while Indiana Jones makes a well-reasoned (but risky) choice of a chalice, which does turn out to be the Holy Grail. His father also drinks from the cup and is healed of his mortal wound.

Jones should be applauded for his courage, but given a failing mark in economics for not recognizing he had a dominant choice. The best strategy was to give the drink to his father without tasting it first. If the cup was the Holy Grail, his father would be saved. Hence, he would have been as well off as by trying it himself and then giving the life-giving fluid to his father. However, if Jones drank first and it was not the Holy Grail, both would die—Indiana from the liquid and his father from the wound. But if the wrong cup were given to his father first, he would die, but Indiana Jones would be spared.

Giving the cup to his father first is no worse than tasting it first if the Holy Grail is selected and better if it is not. Thus, this was the dominant strategy and should have been selected. ■

SOURCE: This example is taken from A. Dixit and B. Nalebuff, *Thinking Strategically* (New York: Norton, 1991), pp. 59–60.

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### Dominated Strategies

Many games do not have a dominant strategy for any player. In fact, dominance is the exception rather than the rule. However, it may be possible to simplify a game by eliminating dominated strategies. A *dominated strategy* is an alternative that yields a lower payoff than some other strategy, no matter what the other players in the game do.

TABLE 11.3 Dominated Strategy

		Defensive Strategy		
		Defense Against Run	Linebackers Back	Blitz
Offensive Strategy	Run	2	6	14
	Pass	8	7	10

Consider a single play in a football game in which the goal of the offense is to gain as many yards as possible and the goal of the defense is to minimize the yards gained. Assume there is time for just two more plays, and the team with the ball wants to use the first play to get as close to the goal line as possible to try a field goal. The offense has two strategies—to run or to pass. The options for the defense are to defend against the run, to protect against the pass by dropping back their linebackers, or to defend against a pass by using a quarterback blitz. The outcomes of these offensive and defensive strategies are shown in Table 11.3. Each of the numbers in the cells shows yards gained by the offense.

Neither team has a dominant strategy. For the offense, this can be seen by noting that the run gains more ground against the blitz but less against the other two defenses. For the defense, there is no strategy that will always give up less yardage than the others. However, the defense does have one strategy that is dominated. Table 11.3 shows that the blitz yields more ground against both the run and the pass than either of the other two defenses. The implication is that the defense should either counter the run or drop back its linebackers, but should not blitz.

Whenever there is a dominated strategy, the game can be simplified because the dominated strategy should always be avoided. Thus, by reducing the number of viable options, it may be easier to identify an equilibrium or to use other techniques to analyze the outcome.

### Maximin Strategies

Thus far, the analysis of market structures has assumed that managerial decisions focus on maximizing profits. But in highly competitive situations, such as an oligopoly, von Neumann and Morgenstern suggested that decision makers might adopt a risk-averse strategy of assuring that the worst possible outcome is as beneficial as possible, regardless of what other decision makers do. This decision rule is referred to as a *maximin strategy* because it specifies that each player in the game will select the option that maximizes the minimum possible profit (or other desirable outcome).

Consider the following example. The two firms in a duopoly are each thinking about introducing a new product. The profit outcomes for the four possible combinations of strategies are shown by the payoff matrix in Table 11.4. If the firms are trying to maximize profit, the matrix has two Nash equilibriums—the two cases where one firm introduces a new product but the other does not. For example, if firm 1 markets the new product, firm 2 will earn \$2 million if it follows suit, but profit will be \$3 million if it does not. If firm 2 does not introduce a new product, the first firm will earn \$2 million more if it markets the new product than if it does not. Hence, the bottom left-hand cell of the matrix is a Nash equilibrium. Using similar logic, the top right-hand cell is also an equilibrium.

**TABLE 11.4 Maximin Decision (Millions)**

		<i>Firm 2</i>		<i>Firm 1</i> Minimum
		<i>No New Product</i>	<i>New Product</i>	
<i>Firm 1</i>	<i>No New Product</i>	4, 4	3, 6	3
	<i>New Product</i>	6, 3	2, 2	2
<i>Firm 2</i> Minimum		3	2	

But the maximin decision criterion is not a pure profit-maximizing strategy. Rather, it is designed to avoid highly unfavorable outcomes. In applying this principle, each firm first determines the minimum profit that could result from each strategy it could choose. As shown in the table, for firm 1, this is \$3 million if the firm does not introduce the new product and \$2 million if it does. The numbers are the same for firm 2.

The second step is to select the maximum of the minimums. The result is that neither firm should introduce a new product because they will be guaranteed a profit of at least \$3 million by adopting this strategy. Note that the maximin outcome is not one of the two Nash equilibriums. The reason is that loss avoidance rather than profit maximization was the criterion used for decision making.

## Case Study

### Texaco and Pennzoil

In January 1984, the Pennzoil Corporation offered to buy 40 percent of Getty Oil's stock for \$128.50 per share. Getty's board of directors agreed, but then Texaco stepped in and offered \$128 per share for 100 percent of the Getty stock. The Getty directors reversed their approval of Pennzoil's bid and sold the company to Texaco.

Pennzoil promptly sued Texaco for breach of contract. The case was tried in Texas, and in 1985, a jury awarded Pennzoil \$10 billion in damages. Texaco immediately appealed the verdict, and by the fall of 1987, the case was on its final appeal before the U.S. Supreme Court. Before a decision was announced by the Court, Pennzoil made an offer, which was accepted by Texaco, and the suit was settled in December 1987 for \$3 billion.

With some oversimplification, Pennzoil's offer to settle a \$10 billion lawsuit for \$3 billion can be viewed as a maximin problem. That is, the company wanted to maximize the minimum possible payment from Texaco. Assume there were just two possible outcomes from the Supreme Court—either the \$10 billion jury award would be affirmed or it would be overturned and nothing would be awarded. Also assume that Pennzoil's managers had just two options—to wait for the Court's decision or to make an offer they believed Texaco would accept.

If Pennzoil waited for the Supreme Court decision, the outcome would be \$10 billion if the jury verdict was affirmed and zero if it was not. By settling out of court, the minimum payment would be \$3 billion. Thus, Pennzoil's maximin decision strategy was to make the settlement offer.

The use of the maximin strategy was a logical choice by Pennzoil. If the Supreme Court reversed the jury award and the firm received nothing, Pennzoil's managers would have been criticized for not reaching a settlement. Indeed, they could have been sued for failing to act in the best interest of the firm's stockholders. By offering to settle, they were guaranteed at least \$3 billion and avoided the potential criticism and/or litigation. ■

### Mixed Strategies

In all of the games discussed, it has been assumed that each participant selects one course of action. This approach is called a *pure strategy*. But in many games, a pure strategy would be a very poor choice. For example, think about the duel between a baseball pitcher and hitter. If the pitcher throws all curves or all fastballs, a major league batter would have a good chance of getting a hit. To be effective, the pitcher must keep the hitter off balance by throwing a mixture of curves and fastballs. This approach is referred to as a *mixed strategy*.

Table 11.5 depicts the payoffs for the pitcher and hitter. Low percentages favor the pitcher, and high values are the objective of the batter. Note that a batter is more successful when he can anticipate the pitch, and the pitcher is more likely to succeed if he can fool the hitter.

If the pitcher throws all fastballs, the batter will look only for this pitch and will hit .400. Any pitcher who throws only curve balls will also be facing a .400 hitter. Clearly, either of these two pure strategies would be a disaster for the pitcher. The best strategy is to throw a mixture of fastballs and curves. Similarly, the hitter cannot succeed by always anticipating one pitch. Any batter who always goes to the plate looking for a curve will find his average hovering around .200, because pitchers will throw him fastballs, and one who anticipates only fastballs will do no better because he will receive a steady diet of curves.

For the payoff matrix shown in Table 11.5, there are no pure strategies that result in a Nash equilibrium. However, there are equilibrium mixed strategies. If the hitter randomly alternates between anticipating a fastball and a curve on a 50-50 basis and

		<i>Pitcher</i>	
		<i>Throws Fastball</i>	<i>Throws Curve</i>
<i>Hitter</i>	<i>Anticipates Fastball</i>	40%	20%
	<i>Anticipates Curve</i>	20%	40%



the pitcher randomly throws a mixture of 50 percent fastballs and 50 percent curves, the hitter's batting average will be .300. This is because each of the four payoffs in the matrix will occur 25 percent of the time. Thus, the expected frequency of base hits will be  $.25(40\%) + .25(20\%) + .25(20\%) + .25(40\%) = 30\%$ .

Suppose the batter adopts a strategy other than anticipating that fastballs and curves are equally likely. If the pitcher continues to throw a random assortment of 50 percent of each pitch, the hitter will guess wrong more often than right, and his average will drop below .300. Thus, given the pitcher's strategy, the best approach for the batter is to anticipate a random 50-50 assortment.

In contrast, if the batter continues to assume that fastballs and curves are equally likely and the pitcher throws a different mix, the hitter's average will still be .300. For example, suppose the pitcher throws 60 percent fastballs and 40 percent curves. The probability of each payoff in Table 11.5 will now be

Expect fastball and fastball thrown:	$.50 \times .60 = .30$
Expect fastball and curveball thrown:	$.50 \times .40 = .20$
Expect curveball and fastball thrown:	$.50 \times .60 = .30$
Expect curveball and curveball thrown:	$.50 \times .40 = .20$

Multiplying these probabilities by the payoffs for each of the cells in the table, the expected frequency of hits will be  $.30(40\%) + .20(20\%) + .30(30\%) + .20(40\%) = 30\%$ . Surprisingly, this result does not depend on the mixture of fastballs and curves used by the pitcher. As long as the batter continues to anticipate that curves and fastballs are equally likely, the hitter's average will be .300 regardless of what strategy the pitcher selects. Thus, the game has many Nash equilibriums. When mixed strategies are allowed, every game with a finite number of players and a finite number of strategies has at least one equilibrium.

Mixed strategies can be important in many settings. Sometimes they are used to reduce costs. Consider the Internal Revenue Service and its tax audit procedure. It would be extremely expensive to audit every income tax return, so the IRS chooses individuals at random (after using some preselection criteria) for audits. Although the vast majority of returns are not audited, the possibility of an audit encourages tax compliance among the general population.

### Key Concepts

- If a player has a best option, regardless of what other players do, this is referred to as a dominant strategy.
- A dominated strategy is an alternative that always has a lower payoff than some other strategy.
- The maximin criterion involves maximizing the minimum desirable outcome of a game.
- A pure strategy involves always making the same choice, while mixed strategy requires randomly mixing different alternatives.

→ minimax

		Suspect 2		Person 1 Maximum
		Don't Confess	Confess	
Suspect 1	Don't Confess	0, 0	15, 5	15
	Confess	5, 15	5, 5	5
Person 2 Maximum		15	5	

## GAME THEORY AND OLIGOPOLY

Game theory can be used to analyze specific situations faced by managers in oligopolistic markets. This section considers four specific applications. The first is noncooperative games, which is used to illustrate how firms in an oligopoly can find themselves in a situation that is nonoptimal for all participants. The discussion of cooperative games shows how businesses can work together to avoid such situations, and the evaluation of repeated games presents methods for dealing with cheaters. Finally, sequential games are discussed to illustrate advantages that can come from being the first to act in a business setting.

### Noncooperative Games: The Prisoner's Dilemma

A game is considered *noncooperative* if it is not possible to negotiate with other participants and enter into some form of binding agreement. When U.S. firms in oligopolistic markets use pricing strategies to compete for profits, they usually are engaged in noncooperative games because they are legally prohibited by antitrust laws from coordinating their prices.

In some cases, noncooperative games can result in outcomes that are undesirable for the participants and also for society. One example is the *Prisoner's Dilemma*. This model takes its name from the story of two people who were jailed for a crime they allegedly committed. The two suspects are separated and interrogated by the police. Each is told that if she does not confess and the other person does, she will be convicted and put in jail for 15 years. But if she does confess and implicates her friend, then the jail sentence will only be five years. However, because the evidence is circumstantial, if neither confesses, it will be impossible to get a conviction and neither person will go to jail. The payoff matrix for this game is shown in Table 11.6.

Because the two prisoners are interrogated separately, they have no idea whether the other person will confess or not. Hence, this is an example of a noncooperative game. If the suspects are risk averse, they may adopt a minimax decision criterion. This is similar to the maximin strategy discussed earlier, except that now the outcomes are undesirable. Consequently, the minimax approach involves minimizing the maximum jail sentence they could receive.

For suspect 1, the strategy of not confessing involves a maximum jail term of 15 years, while a confession would result in no more than five years in jail. Thus, the maximum sentence is minimized by confessing. The same is true for suspect 2, which implies

		Firm 2	
		Low-Level Advertising	High-Level Advertising
Firm 1	Low-Level Advertising	\$30, \$30	\$10, 40
	High-Level Advertising	\$40, \$10	\$20, \$20

— maximin

that the minimax solution is for both people to confess. But note that if neither person confesses, neither will go to jail. Although not confessing is the best joint strategy, because they are involved in a noncooperative game and cannot influence what the other person does, they end up with a decidedly nonoptimal outcome.

The Prisoner's Dilemma model can be used to explain a number of interesting phenomena in business. One example is the resource waste from advertising in oligopolistic markets. For simplicity, assume that there are just two firms and that managers can choose a high level of advertising or a low level. Also assume that the managers are risk averse and that their objective is to maximize the minimum profit earned by their firms. That is, they use the maximin rule (maximin because the outcome is profit, which is a desirable outcome). Should managers select the low or the high level of advertising?

The payoff matrix for the four possible combinations of advertising strategy is shown in Table 11.7. If firm 1 advertises at a low level, its profit will be \$30 million if the second firm also advertises at a low level and \$10 million otherwise. Thus, minimum profit from low-level advertising is \$10 million. Table 11.7 shows that a high level of advertising will guarantee firm 1 at least \$20 million in profit. Hence, the maximin strategy for firm 1 is to advertise extensively. Similar logic suggests that firm 2 will also choose the high advertising option.

Both firms will opt for high-level advertising, and the result is a profit of \$20 million for each company. However, note that a joint decision not to advertise would have been more profitable for each firm because profit would have been \$30 million. But neither firm dares to select this choice because of the possibility that the other might select the high-advertising strategy and leave it with only \$10 million in profit. The result is that both firms earn less profit and waste resources on mutually offsetting advertising. Although much simplified, this analysis provides insight into the dilemma faced by oligopolists.

The Prisoner's Dilemma is applicable to many game situations. One of the most important is the arms race between the Soviet Union and the United States. Each country spent trillions of dollars on missiles and acquired the capability to destroy the other nation many times over. These expenditures came at the expense of other programs, such as education, medical research, and housing. But neither country was willing to reduce its spending for fear the other would gain a military advantage.

### Cooperative Games: Enforcing a Cartel

In *cooperative games* it is possible to negotiate and enforce agreements that bind the participants in the game to a particular strategy. For example, the two suspects mentioned earlier were in a noncooperative game because they had no way of knowing

what the other person was going to do. If they had been allowed to jointly decide on their strategies and had some means of assuring that the other did not renege, neither would have confessed, and they could have avoided spending any time in jail. In the advertising version of the Prisoner's Dilemma, if the firms could have signed a binding contract pledging that they would use the low advertising strategy, they each would have earned \$10 million more in profit. With the arms race, years were spent trying to negotiate an arms limitation treaty. But, because there was no satisfactory way to verify compliance by the other side, the buildups continued.

The experience of the cigarette industry during the 1960s is an interesting example of how a noncooperative game was turned, by government policy, into a cooperative game. Historically, the major cigarette manufacturers had spent large sums on TV advertising to promote their products. Much of this advertising was mutually offsetting, so the firms probably were in the Prisoner's Dilemma just described. They would have been better off if they had reduced their advertising expenditures, but no one firm could afford to do so unless there was some assurance that their lead would be followed by the other firms.

In 1968, the federal government banned the advertising of cigarettes on television. Initially, the companies fought the ban, but it soon became apparent that any lost sales were more than made up by the savings in advertising. In fact, there is some evidence that sales actually increased because stations were no longer required by the Federal Communications Commission to air antismoking ads that warned of the dangers of cigarettes. With its 1968 ban, the government did for cigarette manufacturers what they were unable to do for themselves—enforce a reduction in advertising.

#### Key Concepts

- In noncooperative games, it is not possible to negotiate and enforce agreements.
- The Prisoner's Dilemma model illustrates that the result of a game can be undesirable for all of the participants.
- In a cooperative game, agreements can be enforced. Such games may be a way to avoid the Prisoner's Dilemma.

### Repeated Games: Dealing with Cheaters

Even if enforced agreement is not possible, firms may be able to escape the Prisoner's Dilemma if the action is a *repeated game*; that is, if it is played many times. Consider the advertising example shown in Table 11.7. If the game is played once, neither firm will adopt a low advertising level because the other could select high advertising, capture most of the profit, and the game would be over. Even if each firm agrees to hold down advertising, unless there is some way to enforce this agreement, the high-advertising equilibrium is likely to occur.

But if the advertising decision is made repeatedly, the outcome may change. A firm that reneges on an agreement and heavily advertises the first time the game is played will find that the other firm will respond by increasing its advertising in the second period. Thus, the cheater's advantage will be temporary, and profits for both firms will be

low for the second period. In addition, the fact that cheating occurred once should cause the other firm to be more cautious in the future. With repeated games, reputations are important in determining the outcome.

What is the optimal strategy for firms playing noncooperative repeated games? A general answer depends on the nature of the game, but an experiment by Robert Axelrod provides interesting insights.<sup>1</sup> He proposed a tournament of two-person repeated Prisoner's Dilemma games with high and low prices as the two alternatives. He invited game theorists from all over the world to submit computer programs that embodied strategies for playing. Programs were paired against each other, and the game was repeated a large number of times.

The result was surprising. The winning strategy was an approach described as "tit-for-tat." Basically, it was that each firm should mimic its rival's behavior from the previous period. If one firm cheats and cuts prices, the other firm responds by cutting prices in the next period. If one firm cooperates by raising prices, the other firm also raises prices. The experiment was later repeated with a larger number of contestants, but tit-for-tat again proved to be the optimal strategy.

In the real world, the advantage of tit-for-tat is that it embodies four principles that are important in any good strategy. First, it is simple, reducing the chance that it will be misunderstood. Second, tit-for-tat never initiates cheating, which could cause a breakdown in cooperation. Third, it never rewards cheating by allowing such behavior to go unpunished. Finally, tit-for-tat is forgiving because it allows cooperation to be quickly restored.

Unfortunately, tit-for-tat can break down if the players know for sure how many times the game will be repeated. The explanation is quite simple. Although cooperation can increase profits, a firm can gain even more if it cheats the last time the game is played, because there is no opportunity for retaliation. But the same is true for the other player, so both firms should cheat on the last play. If cheating will occur on the last round, there is no reason to cooperate on the next to the last round, and so on. The result may be that each play of the game degenerates into a Prisoner's Dilemma.

### Sequential Games: The Advantage of Being First

In the games described so far, it has been implicitly assumed that both players reveal their strategies simultaneously. In a *sequential game*, one of the players acts first and then the other responds. Entry into a new market is an example of a sequential game. The new firm decides whether or not to enter, and the existing firm then decides whether to ignore the new firm or try to prevent entry.

For sequential games, there may be an advantage to the player who acts first. Think about two firms contemplating the introduction of nearly identical new products. The first firm to get the product to the market is more likely to be successful because it can develop brand loyalties and may be able to associate the product with the firm in the minds of consumers. Also, if consumers invest time in learning to use the product of the first firm, users will be less willing to retool and use a similar product from some other

<sup>1</sup>R. Axelrod, *The Evolution of Cooperation* (New York: Basic Books, 1984).

		Firm 2	
		No New Product	Introduce New Product
Firm 1	No New Product	\$2, \$2	-\$5, \$10
	Introduce New Product	\$10, -\$5	-\$7, -\$7

supplier. Word processing and spreadsheet software are good examples. Users who are proficient in one program are unlikely to switch to other programs unless they offer significant advantages.

The advantage of moving first is shown by Table 11.8. Each firm is faced with the decision of whether or not to introduce a new product. Assume that the firms use the maximin criterion. If the firms must announce their decisions independently and simultaneously, the maximin criterion specifies that neither should introduce a new product and that each firm will earn \$2 million.

Now assume that firm 1 has research and development advantages that give it the option of introducing its product first. With firm 1 already in the market, firm 2's optimal strategy is to stay out because it will lose only \$5 million, compared to \$7 million if it enters. Consequently, firm 1 will earn \$10 million as the only supplier. Clearly, firm 1 has benefitted by being the first mover in the market.

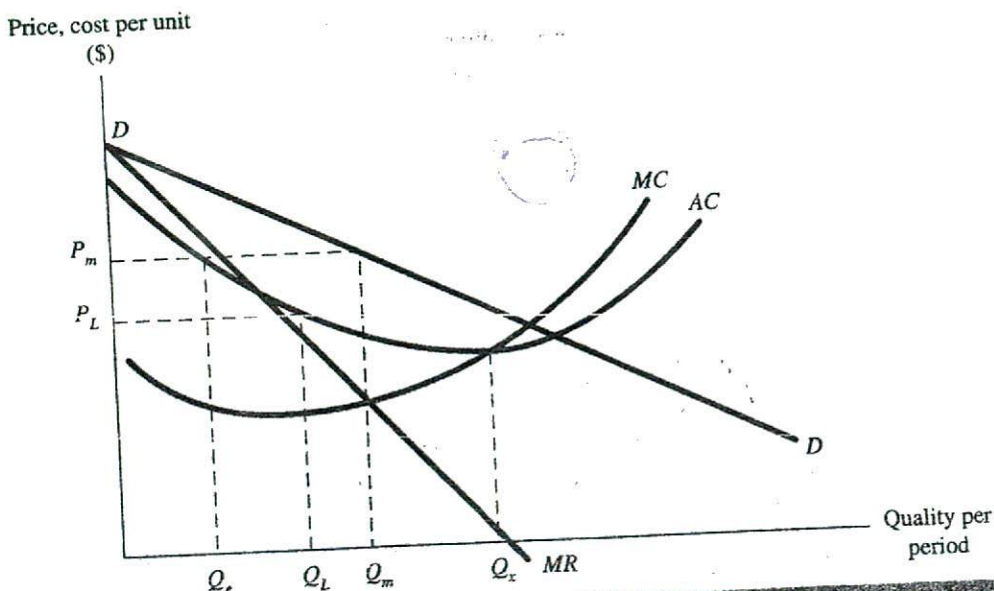
### Key Concepts

- The optimal strategy in a repeated game may be different from the optimal strategy for a game that is only played once.
- A tit-for-tat strategy of mimicking the last choice of the other player may be effective in repeated games.
- In sequential games where one player selects a strategy and the other responds, there is often advantage to being first.

## STRATEGIC BEHAVIOR

In the previous chapter, barriers to entry were identified as probably the most important determinant of market structures. But the four traditional barriers to entry described in that chapter (e.g., economies of scale, product differentiation, control over scarce inputs, and legal factors) are primarily the result of basic conditions that exist in each market. As such, the number of firms in an industry would be determined by how those conditions affect entry in each industry. Over time, markets where entry was not difficult would have many sellers, and those with significant barriers to entry would be dominated by a small number of firms.

But this passive view of barriers to entry is too simplistic. Businesses are run by managers, and these individuals will react aggressively if they believe that entry could significantly affect the profitability of their firms. These reactions may take the form of



strategic behavior designed to deter entry. Although it is not possible to describe all of the options available to managers, this section considers some of the strategic responses that may be used to thwart entry.

### Present versus Future Profits: Limit Pricing

Based on the discussion in chapter 1, it is assumed that long-run profit maximization is the goal of managers. This objective is achieved by maximizing the present value of profits over some planning horizon. To this point, however, the analysis of managerial pricing decisions has focused on maximizing profit in a single period. But the price that maximizes profits in one period may not be consistent with long-run profit maximization. In this section, long-run pricing strategies are considered.

Bain suggested that setting prices to limit entry describes the pricing practice of many firms.<sup>2</sup> His model assumes that monopolists or firms in an oligopoly pursue a pricing strategy designed to prevent new firms from entering the industry. The approach is illustrated by Figure 11.1. Consider a monopolist facing the market demand curve,  $DD$ , and assume that increasing returns to scale provide a cost advantage for large firms. This cost advantage is depicted in Figure 11.1 by an average cost curve that is downward sloping to the output rate  $Q_x$ .

To maximize profit in a single period, the monopolist should increase production until marginal revenue equals marginal cost. This implies an output rate of  $Q_m$  and a price of  $P_m$ . At that price and output rate, the monopolist will earn economic profit for the period. But unless there are substantial barriers to entry, the lure of profit will cause

<sup>2</sup>J. S. Bain, "A Note on Pricing in Monopoly and Oligopoly," *American Economic Review* (March 1949): 448-464.

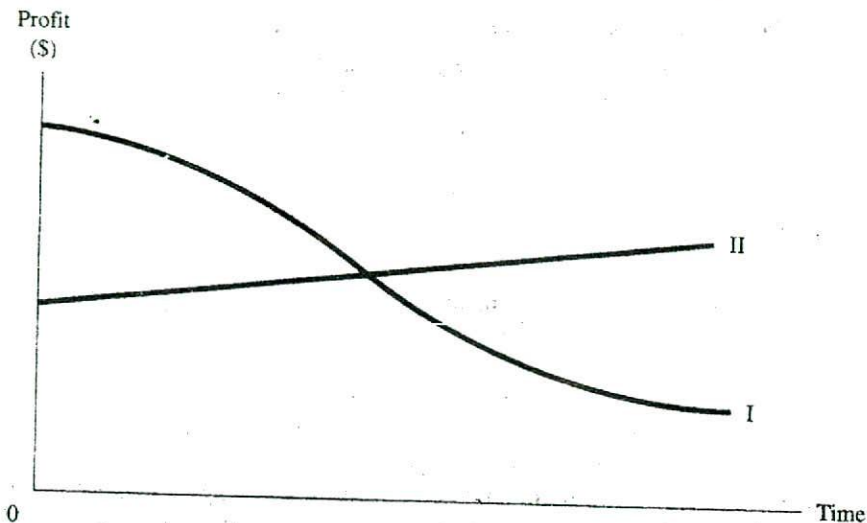


FIGURE 11.1 Profit Stream with Alternative Pricing Strategies

new firms to enter the industry. A potential barrier to entry is the cost advantage of the large firms. That is, because of scale economies, the monopolist producing at  $Q_m$  will have lower costs than a new firm that will produce at output rates of less than  $Q_m$ . However, by setting the profit-maximizing price, the monopolist makes it possible for new firms to enter on a relatively small scale and still earn at least a normal profit. For example, suppose a potential entrant believes that the monopolist will not reduce its price if a new firm enters the market. Thus if the price is  $P_m$ , a new entrant could earn a normal profit by producing at an output rate as low as  $Q_e$ . This can be seen by observing that at the output rate  $Q_e$ , the price,  $P_m$ , equals average cost.

Although a single new entrant may have little effect on the rate of profit earned by the monopolist, if the managers of the monopoly continue to use a pricing strategy that maximizes short-run profit, it is likely that additional firms will enter the market. Over time, the increased competition will force the firm to reduce its price and will also reduce the market share and economic profit of the firm.

Alternatively, the firm could utilize a pricing strategy designed to prevent entry. This approach would require that in each period, managers set a price below the level that maximizes short-run profit. For example, as shown in Figure 11.1, the price might be set at  $P_L$ . At this price, new firms entering the industry and producing at output rate less than  $Q_L$  could not earn a normal profit. Thus, the pricing strategy establishes a barrier to small-scale entry. Although new firms still may enter the market, the size requirement makes entry more difficult and therefore less likely than if the short-run profit-maximizing price,  $P_m$ , had been set.

Using *entry-limiting pricing*, managers alter the firm's profit stream, as shown in Figure 11.2. Economic profit resulting from setting the profit-maximizing price in each period is depicted by the profit stream labeled I. Note that economic profit is substantial in the initial period but declines as new firms enter the market. In contrast, entry-limiting



TABLE 11.9 Alternative Profit Streams

Period	Profit Stream I (millions of dollars)	Profit Stream II (millions of dollars)
1	80	30
2	40	50
3	20	50

pricing generates a relatively constant profit stream over the planning horizon. In fact, if the market is growing, profit may actually increase over time. This is shown by the profit stream labeled II.

In contrast to entry-limiting pricing is Stigler's *open oligopoly model*.<sup>3</sup> Stigler observed that the objective of managers is to maximize the present value of profit. In some cases, this may be achieved by setting a price designed to deter entry. But in other circumstances, profits will be maximized in the long run by setting a high price that provides substantial economic profit in initial periods and then allowing profit to decline as new firms enter the market. In its extreme form, the open oligopoly model might involve setting the profit-maximizing price in each period. In this case, the profit stream would be as shown by I in Figure 11.2.

The optimal long-run pricing strategy is the one that maximizes the present value of profit. Examining the two profit streams in Figure 11.2, it is not obvious which should be chosen. Profit stream I has a high initial profit, but earnings decline rapidly over time. In contrast, profit stream II has less initial profit but shows a slight increase over time.

Basically, the optimal strategy depends on the discount rate used by managers to determine the present value of profit. A high discount rate gives less weight to profits in the future. Thus, a high discount rate would cause the present value of profit stream I to be relatively greater. Conversely, a low discount rate would be more favorable to the entry-limiting approach associated with profit stream II.

The importance of the discount rate in selecting a pricing strategy can be illustrated by a simple example. Suppose that the two profit streams involve only three periods and the annual profit is as shown in Table 11.9. The first profit stream shows rapidly decreasing profit and is consistent with the open oligopoly model. Profit stream II has constant profit, as might result from entry-limiting pricing.

Assume that the discount rate is 20 percent and profit is received at the end of the period. In this case, the present value of profit stream I is \$106 million, which is greater than the present value of II, which is \$105 million. But if the discount rate is 10 percent, the result is reversed. Now the present value of II is \$124 million, whereas that of I is only \$121 million.

Clearly, the appropriate long-run strategy depends on how managers perceive future profits. Managers with a short planning horizon and those who view short-term profits as paramount would be more likely to behave in a manner consistent with the open oligopoly model. Conversely, those who have a longer time horizon and use a lower discount rate would be more likely to pursue entry-limiting pricing.

<sup>3</sup>G. J. Stigler, *The Theory of Price* (New York: Macmillan, 1987), chapter 13.

## Case Study

### Saudi Arabia and World Oil Prices

The Organization of Petroleum Exporting Countries (OPEC) is a cartel consisting of 13 of the world's most important oil-producing nations. Oil ministers from each member of the alliance meet on a regular basis, and their primary objective is to influence world oil prices by establishing maximum production quotas for the cartel and for each country. When oil prices are low, reductions in OPEC production can significantly increase world oil prices. Conversely, if OPEC members increase their supply, prices tend to decline.

But the history of OPEC suggests that agreements on production levels have often been difficult to formulate. Although there are exceptions, countries such as Iraq and Iran have usually advocated lower production quotas, which would result in high world oil prices. In contrast, Saudi Arabia has traditionally supported more moderate prices. It is sometimes alleged that the Saudi position is politically motivated by that nation's ties to the United States, which is a large importer of oil. Although political considerations may be a factor, the limit-pricing model can also be used to explain Saudi policy.

Saudi Arabia is the world's largest petroleum producer. Its known reserves of 162 billion barrels represent one-fourth of all known oil resources, and the present rate of production can be maintained for over 50 years. In normal times (the Persian Gulf War was an exception), Saudi oil revenues are sufficient to sustain a high standard of living for Saudi citizens. In contrast, some other Middle East oil producers, notably Iran and Iraq, have an immediate need for increased revenues to rebuild their economies, which have been ravaged by war and embargoes.

By supporting the advocates of high prices, Saudi Arabia could increase its short-term oil revenues. But Saudi policymakers must also consider the long-term effects of high prices. Economic theory predicts that higher prices will increase quantity supplied because they will allow high-cost oil to be recovered that was not profitable at lower prices.

The problem with higher prices is that oil is just one part of the global energy market. As petroleum prices increase, consumers will shift to other forms of energy, such as nuclear, natural gas, and coal. Higher oil prices may also allow other forms of energy, such as solar, geothermal, and nuclear fusion, to be economically viable. In addition, consumers will respond to higher prices by finding ways to conserve on energy use. Thus, the long-term effect of higher petroleum prices may be reduced demand for oil because of substitution. If large capital expenditures have been made to develop other energy sources, markets for oil may be permanently lost, even if prices decline.

Saudi Arabia's oil reserves are intended to provide the bulk of that nation's export revenues for many years. Thus, Saudi decision makers must adopt pricing policies that will preserve the future demand for petroleum. A limit-pricing approach, which makes it less profitable to substitute other energy forms for oil, is consistent with this objective. In contrast, for countries such as Iran and Iraq, which have critical needs for increased short-term oil revenues, the open oligopoly approach is a rational pricing strategy. ■

### The Value of a Bad Reputation: Price Retaliation

The purpose of limit pricing is to reduce the likelihood of entry by keeping prices at a low level over a long period of time. Another strategic response to the threat of entry is to retaliate by reducing prices when entry actually does occur or if it appears imminent. When the perceived danger has diminished, prices can be increased to whatever level management views as appropriate for market conditions.

An interesting example of price retaliation involved General Foods and Proctor & Gamble. In the early 1970s, General Foods, with its Maxwell House brand, had a 43 percent market share of the noninstant (ground) coffee market in the eastern United States. During the same period, Proctor & Gamble's Folger's brand was the leading seller in the West, but was not distributed in most areas of the East.

In 1971, Proctor & Gamble began to advertise and distribute Folger's in selected eastern cities. This effort was initiated in General Food's Youngstown, Ohio, sales district, which included the cities of Cleveland and Pittsburgh. General Foods' response to the new entrant was to increase its advertising and cut prices for Maxwell House coffee in this region. At times, the price was actually less than the price of producing the coffee. Profit figures indicate the impact of the price cuts. Profits as a percent of sales dropped from a preentry level of plus 30 percent in 1971 to a negative 30 percent in 1974. When Proctor & Gamble reduced its promotional activities for Folger's coffee in the area, the price of Maxwell House coffee was increased, and General Foods' profit rates quickly returned to their previous level.

General Foods also responded to entry in its eastern markets by aggressively reducing prices in midwestern cities where Maxwell House and Folger's were both being marketed. When Proctor & Gamble moved into the Youngstown region, General Foods cut prices and increased its advertising in Kansas City. When Proctor & Gamble became less aggressive in Youngstown, prices and promotional efforts for Maxwell House were allowed to return to prior levels in the Midwest.

Reducing prices every time entry occurs or appears likely to occur would be a costly proposition for the existing firms in a market. But a few applications of this strong medicine can have a significant preventative effect. If a firm establishes a consistent pattern of reacting to entry by drastically reducing prices, then potential rivals may become convinced that they will face the same response and decide not to compete. Thus, by firmly establishing a reputation for dealing harshly with all new entrants, the firm may create an effective barrier to entry.

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#### Example Evaluating Price Retaliation

Wild Tides is a water park in Southern Georgia. The firm has fixed costs of \$500,000 per year, a variable cost per customer of \$5, receives an average of \$15 in revenue per admission, and has 100,000 customers each year.

At present, the firm is the only water park in the area, but Wild Tides management has learned that another facility is being planned for a nearby community. After the competing park opens, management expects that to get 100,000 patrons to Wild Tides each year, prices would have to be reduced and that the average revenue would drop to \$12.50.

Management believes that if it immediately reduces prices and keeps them at a lower level for 2 years, it can prevent the other water park from ever opening. The necessary price cut would reduce average revenue to \$8 but would increase attendance to

120,000 customers per year. After two years, prices would be raised to their previous level and revenues would again average \$15 for 100,000 patrons.

Assume that Wild Tides uses a discount rate of 10 percent and a planning horizon of 10 years, and that profits are received at the end of each year. Also assume that if prices are reduced for 2 years, there will be no entry for the remaining 8 years of the planning period. Should the firm cut prices?

**Solution** The optimal strategy can be determined by computing the present value of profits for each alternative. If prices are not reduced and entry occurs, revenues will be \$1,250,000 per year and variable costs will be \$500,000 per year. Subtracting out the variable cost and the \$500,000 fixed cost, profit per year will be \$250,000. Using a 10 percent discount rate and a 10-year planning horizon, the present value of profits would be \$1,536,142.

With price cutting to prevent entry, revenues will be \$960,000 (i.e.,  $\$8 \times 120,000$ ) and variable costs will be \$600,000 (i.e.,  $\$5 \times 120,000$ ). Subtracting the \$500,000 fixed cost, profit in each of the first two years will be  $-\$140,000$ . For the next 8 years of the planning period, revenue per customer will be \$15 for each of the 100,000 attendees and profits will be \$500,000 per year. When these amounts are discounted, the present value of profits is \$1,961,540. Because price retaliation gives a greater present value of profits, it is the optimal strategy.

### Key Concepts

- Limit pricing involves setting lower prices, which make small-scale entry difficult.
- A firm that always retaliates with price reductions when faced with a new entrant may use its reputation to deter entry.
- In evaluating limit pricing and price retaliation, the present value of profits resulting from alternative strategies should be considered.

### Establishing Commitment: Capacity Expansion

The threat of price retaliation against new entrants may not be credible if existing firms are unable to produce enough output to meet extra demand resulting from lower prices. In a rapidly growing market in particular, a new entrant may be able to survive by serving new customers that the existing firms cannot supply with their present production capacity. A strategic response by established firms to prevent this from occurring would be to invest in additional capacity. Once this investment has been made, it becomes a sunk cost and places existing firms in a position to expand their production at relatively low cost. The existence of excess capacity provides a strong signal that the established firms can (and probably will) reduce prices as a strategic response to entry in their market.

Investment in excess capacity reduces the profits earned by an existing firm. Hence, this investment will be undertaken only if management believes that the certain and immediate loss of profit from making the investment is less than the expected future profit loss resulting from entry. Table 11.10 illustrates this trade-off. Suppose a monopolist must choose between building a small plant and a large plant and that a second firm must decide whether to enter the market. Profit outcomes for each firm for each of four possible scenarios are shown in the table.

	<i>New Entrant Profit</i>	<i>Monopolist Profit</i>
Small plant		
Entry	\$4 million	\$4 million
No entry	\$0	\$12 million
Large plant		
Entry	-\$4 million	-\$4 million
No entry	\$0	\$8 million

If the monopolist builds the small plant, the competitor will enter because its profit will be \$4 million, as opposed to zero if entry does not occur. But the larger plant will give the existing firm the ability to reduce its prices and still meet total market demand. Consequently, the new firm will not be able to cover its costs and will experience a loss of \$4 million. Thus, if the large plant is built, the better strategy for the new firm is to stay out of the market.

For the monopolist the better strategy is to construct the small plant, restrict output, and continue as the only supplier. But this option is not viable because it will induce entry, and the monopolist's profit will be only \$4 million. But if management is confident that the large plant will deter entry, its construction is the best strategy for the monopolist because profit will be \$8 million.

### Preemptive Action: Market Saturation

Just as the total amount of productive capacity can affect the rate of entry, the geographic location of that capacity can also cause barriers to entry. When costs of transporting a good are high relative to its value, consumers who are not close to a production facility may be required to pay substantially higher prices to have the good delivered to their location. Thus, firms that locate closer to those consumers will have a cost advantage and should be able to attract those customers.

This situation is depicted in Figure 11.3a. A monopolist has a production facility at point *E*, as shown. Although the monopolist may be able to reduce prices and prevent entry near point *E*, high transportation costs may allow new entrants located at point

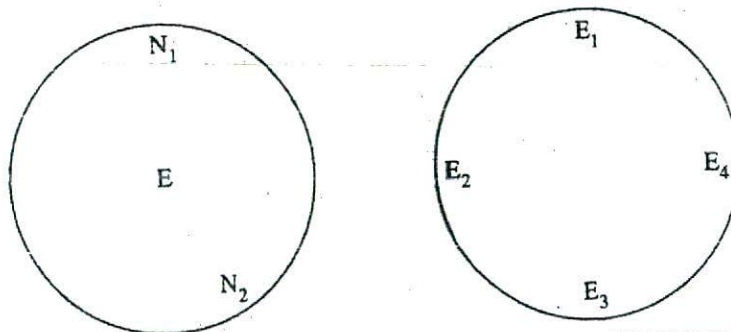


FIGURE 11.3 Market Saturation

$N_1$  to underprice the monopolist and capture the demand of customers who are located near  $N_1$ . Having successfully entered on the geographic fringe of the market, the new supplier may, over time, be able to expand its position and challenge the existing firm in other areas, such as  $N_2$ .

One entry-detering strategy for the existing firm would be to disperse its production facilities, as shown in Figure 11.3b. By the existing firm spreading its plants throughout the market area, the opportunity for the new entrant to take advantage of high transportation costs is greatly reduced. Although the monopolist may lose some of the benefits of economies of scale from this dispersion, the ability to prevent or reduce the likelihood of entry may cause management to decide that geographic market saturation is a viable long-run strategy.

The analysis of geographic saturation can also be applied to product characteristics. Now consider the circles of Figure 11.3 as representing possible characteristics of a product such as automobiles. Cars may be small, medium-sized, or large; convertible or hardtop; loaded with accessories or spartan; high performance or fuel-efficient. Let point  $E$  in Figure 11.3a represent a monopolist producing a single type of car. For example, in the early days of the U.S. automobile industry, Henry Ford only produced the Model T and allowed his customers to select any color they wanted as long as they wanted black. Initially, Ford experienced great success in the market, but eventually the company's market share declined as its competitors introduced models with new features that were not available on the Model T. This entry is shown in Figure 11.3a, by the points  $N_1$ , and  $N_2$  on the fringes of the diagram.

Ford's dominance of the market might have been extended if it had opted for the strategy illustrated by Figure 11.3b of filling the market characteristic space with a variety of models. If Ford had provided more options at the outset, new firms would have had more difficulty carving out their positions in the market. However, another manufacturer first recognized the potential of producing many different varieties of automobiles and responded to saturate the product space. Cadillacs were targeted for the luxury market, Chevrolets for the low-price buyer, and Buicks, Oldsmobiles, and Pontiacs for the middle-price range. Each make was available in several models, and options could be selected to meet the specific wants of individual customers. As a result, Ford lost market share and General Motors for many years was the dominant automobile manufacturer in the United States.

## Case Study

### Brand Proliferation in the Cereal Industry

A perusal of grocery store shelves indicates the large number of brands that are available from each of the three major cereal manufacturers—General Mills, General Foods, and Kellogg's. For example, just a partial list of the cereals produced by Kellogg's includes

Corn Pops  
Frosted Flakes  
Mini Buns  
Just Right  
Corn Flakes  
Fruit Loops  
Crispix  
Special K

Frosted Bran  
All Bran  
Nut and Honey Os  
Apple Jacks  
Mueslix  
Raisin Bran  
Rice Krispies  
Product 19

Kenmei Rice Bran  
Double Dip Crunch  
Low Fat Granola  
Smacks  
Bran Flakes  
Raisin Squares  
Cracklin' Oat Bran  
Frosted Mini-Wheats

The cereal industry is of particular interest because an antitrust case was filed by the Federal Trade Commission alleging that the major manufacturers had attempted to prevent entry into the market by engaging in the "proliferation of brands." In this case, the issue involved not only filling the product market space with cereals brands of every conceivable type, but also saturating shelf space.

In modern supermarkets, there is competition for scarce shelf space. Only a certain amount of area in a store is allocated for cereals. Thus, if more of that space is used for the brands of the major manufacturers, then less will be available for cereals from smaller companies. Because they had filled the shelf space with their products, General Foods, Kellogg's, and General Mills were accused of saturating the market and creating a barrier to entry. Because they had so many different brands, the FTC also argued that product market saturation constituted a barrier to entry.

The case attracted considerable attention from politicians, who felt that pursuing this issue was a waste of money. Ultimately, the FTC dropped the case. Still, the episode illustrates the concept of market saturation. ■

### Key Concepts

- Investment in additional capacity may act as a barrier to entry because it allows a firm to increase production at low variable cost.
- Firms may be able to deter entry by saturating the geographic market with production facilities or the product market with brands or models.

## SUMMARY

Game theory is a technique used to analyze situations where individuals or organizations have conflicting objectives. Payoff matrices indicate the outcomes of all the possible combinations of strategies in a game. A pure strategy involves always selecting one course of action, while a mixed strategy involves one participant randomly using different strategies. A Nash equilibrium is defined as a situation where none of the players can improve their payoff, given the strategies of the other players. A game may have more than one Nash equilibrium. If mixed strategies are allowed, every game has at least one equilibrium.

If a player has a strategy that is best, regardless of what other players do, this is called a dominant strategy, and it should always be used. Dominated strategies always yield a lower payoff than some other strategy. The analysis of games can be simplified by elimi-

nating dominated strategies. Risk-averse decision makers may use the maximin criterion, which involves maximizing the minimum profit or other desirable outcome. If the outcome is undesirable, the minimax strategy would be the corresponding choice. This criterion specifies that the strategy to be chosen should minimize the maximum undesirable outcome.

A game is noncooperative if it is not possible to negotiate with other players and enter into a binding agreement on the strategies to be employed. Noncooperative games may result in undesirable equilibriums. One example, the Prisoner's Dilemma, can be used to explain many phenomena, such as over advertising by oligopolists. The Prisoner's Dilemma outcome may be avoided in cooperative games where agreements can be enforced.

A study by Axelrod determined that the most effective strategy in repeated games may be tit-for-tat, which involves always selecting the choice used by the other player in the previous round. In sequential games, where one player acts first and then the other responds, there may be an advantage to being first.

Firms often engage in strategic behavior to deter new entry. One approach is to keep prices at a low level, which makes small-scale entry more difficult. Price retaliation when entry occurs or if it appears likely is another strategy. A firm that gains a reputation for aggressive price cutting may be able to effectively discourage entry. In evaluating both of these strategies compared to a policy of setting the monopoly price and letting entry occur, the present value of profits for each option should be compared.

The threat to cut prices must be credible to be effective. A firm that invests in capacity expansion signals potential entrants that it is able to meet the demand that would result from lower prices. Once the capacity is in place, variable costs of increasing production are low. Firms may also be able to deter entry by situating their production facilities to saturate the geographic space. Market saturation may also involve filling the product space with many different brands or models.

### Discussion Questions

- 11-1. Consider the Cournot model discussed in chapter 10. Is the solution to that problem a Nash equilibrium? Why or why not?
- 11-2. If one participant in a two-person game has a dominant strategy, will that person always receive a greater payoff from the game than the player who does not have a dominant strategy? Explain.
- 11-3. Reread the Indiana Jones case study. How might ethical or personal values have affected the choice made by Jones?
- 11-4. Under what conditions might a maximin strategy be a rational criterion for a manager to use in decision making?
- 11-5. Suppose a baseball pitcher adopted a strategy of throwing a fastball, then a curve, then a fastball, and so on. Is this mixed strategy? Would it be effective? Why or why not?
- 11-6. How could the Prisoner's Dilemma model be used to explain a price war between duopolists?
- 11-7. Would a manager who was two years from retirement be more likely to favor a limit pricing or an open oligopoly model? Explain.
- 11-8. How could the retaliation model be used to explain the tactics used by Israel and other nations in dealing with terrorists?
- 11-9. In general, how could a firm evaluate whether an investment in excess capacity to deter entry would be profitable?



- 11-10. How would brand loyalty for a product affect the success of a product market saturation strategy?

### Problems

- 11-1. Two firms can either reduce their prices or keep them at the present level. If firm A cuts prices, it will earn \$10 million in profit if firm B also cuts prices, and \$20 million if firm B does not change prices. If firm A makes no price change, it will earn \$0 if firm B reduces prices and \$5 million if firm B makes no price change. The outcomes for firm B are the same as for firm A.
- Develop the payoff matrix for this game.
  - Does the game have a Nash equilibrium?
  - Does either firm have a dominant strategy? Explain.
- 11-2. Two firms produce a homogeneous product for which variable costs are zero. The market demand for the product is given by  $P = 100 - 4Q_T$ , where  $Q_T$  is total output of both firms. Determine the Nash equilibrium output and price.
- 11-3. Consider the mixed strategy game involving the pitcher and batter described on pages 377–378. If the hitter considers a curve and a fastball to be equally likely, prove that any mixed strategy is optimal for the pitcher. Hint: Set up the problem in terms of the probability,  $P_p$ , that the pitcher will throw a fastball.
- 11-4. Two manufacturers of the same product must independently decide whether to build a new production facility. The profit payoff matrix is shown here.
- If they both use the maximin decision criterion, what will be the outcome?
  - Does either firm have a dominant strategy? Explain.

HW

		<i>Firm 2</i>	
		<i>Don't Build</i>	<i>Build</i>
<i>Firm 1</i>	<i>Don't Build</i>	5, 5	0, 10
	<i>Build</i>	10, 0	4, 4

- 11-5. A single decision maker has four strategies, each with four undesirable outcomes (i.e., bigger numbers are worse), as shown here.
- If he is highly risk averse, which strategy will he choose? Explain.
  - If he is risk neutral and knows that each outcome is equally likely, which strategy will he choose? Explain.

		<i>Outcomes</i>			
		<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>
<i>Strategies</i>	<i>A</i>	10	20	30	30
	<i>B</i>	50	10	100	8
	<i>C</i>	60	40	40	40
	<i>D</i>	200	0	40	50

- 11-6. The manager of a firm has four strategies with four possible desirable outcomes for each, as shown here.

- a. If there are any dominated strategies, rewrite the matrix to show the relevant choices faced by the decision maker.
- b. Which strategy would a maximin decision maker select? Explain.

		Outcomes			
		1	2	3	4
Strategies	A	10	20	70	30
	B	50	10	30	8
	C	60	40	40	40
	D	5	15	60	25

- 11-7. A firm has three strategies and is subject to four states of nature as shown following. The outcomes are profits earned by the firm.

		States of Nature			
		A	B	C	D
Strategies	1	5	3	7	11
	2	7	2	14	13
	3	6	4	8	12
	4	1	5	15	20

- a. Does the firm have a dominant or a dominated strategy? If so, which strategy is dominant or dominated? Explain.
- b. Which of the four strategies is the firm's maximin strategy? Explain.
- c. If the outcomes were undesirable, what would be the minimax strategy? Explain.
- d. If the four states of nature are equally likely, what is the expected profit of each strategy?

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11-8. If Joe does not confess, he will not go to prison if Sally does not confess, but he will go to prison for 15 years if she does confess. If Joe does confess, he will go to prison for 3 years regardless of what Sally does. The same outcomes hold for Sally depending on what she does and what Joe does.

- a. Write the payoff matrix for this problem.
- b. Does either person have a dominated strategy? Explain.
- c. What is the minimax solution to this problem?

11-9. If firm 1 does not advertise, it earns a profit of \$10 million if firm 2 does not advertise and a profit of \$4 million if firm 2 does advertise. If firm 1 does advertise, it earns \$20 million if firm 2 does not advertise, and \$6 million if firm 2 does advertise. The same outcomes hold for firm 2 depending on what firm 1 does.

- a. Write the payoff matrix for this problem.
- b. If managers are risk averse and have no information about the strategy of the other firm, what will be the strategy for each firm?

11-10. Phyz, Inc. is considering introduction of a new soft drink. Sparkle Corporation may introduce a similar drink. If Phyz does not market the drink, its profit will be \$100 million if Sparkle does not enter the market, and \$80 million if Sparkle does introduce a drink. If Phyz enters, profits will be \$120 million if Sparkle does not, and \$90 million if Sparkle introduces a drink.

- a. Set up the pay all matrix. If the choices are made independently and simultaneously, what should Phyzz's highly risk-averse managers do?
- b. If Sparkle moves first to introduce the drink, what will be Phyzz's optimal strategy? Explain.
- 11-11. Green Vista golf course charges \$20 per round and has 200,000 golfers play the course each year. Fixed costs for the enterprise are \$1 million, and the variable cost is \$5 per round. If a competing course opens in another part of the city, the price would have to be reduced to \$16 to keep the number of rounds at 200,000. Management believes that entry could be permanently prevented if the price were reduced to \$12 for 3 years. At this price, rounds played would increase to 220,000. Management uses a 6-year planning horizon for decision making. Assume all amounts are received or incurred at the end of the year.
- a. If the discount rate is 5 percent, what is the optimal strategy for the firm?
- b. How would a higher discount rate affect the optimal pricing strategy? What about a longer planning horizon?
- 11-12. The manager of the McRod Corporation is considering pricing strategies for its fishing equipment. One option is to make entry difficult by charging relatively low prices. A profit stream for the next 5 years using this option is shown below. The second option is to set higher prices and allow entry to occur. The 5-year profit stream associated with this pricing strategy is also shown. Assume that profits are received at the end of the year and that the objective of the firm is profit maximization over the planning horizon.

Year	Profit	
	Limit Entry	Allow Entry
1	100	130
2	100	110
3	100	90
4	100	70
5	100	50

- Which pricing strategies should be selected if the manager uses
- a. A discount rate of 5 percent and a 5-year planning horizon?
- b. A discount rate of 10 percent and a 5-year planning horizon?
- c. A discount rate of 5 percent and a 3-year planning horizon?
- d. A discount rate of 10 percent and a 3-year planning horizon?

- 11-13. A manager must select between an open oligopoly and an entry limiting strategy. The profit streams for the two strategies are as shown below. Assume profits are received at the end of the period.

Year	Limit Entry	Open Entry
1	100	120
2	100	100
3	100	70

- a. If the manager uses a discount rate of 10 percent, which is the optimal strategy? Explain.
- b. What discount rate would make the manager indifferent between the two strategies?

# Integrating Study IV CASE

## Pacific Copper

Pacific Copper, a family-owned business, produces copper that is purchased by other firms to make wire, tubing, and sheets. The copper is produced in 1,000-pound ingots and is identifiable as having been produced by Pacific Copper only by the firm's name stamped on each ingot.

### PART I

Pacific operates the only copper mine and smelter in the South Pacific region. Because imports are limited by high transportation costs, the firm is essentially a monopoly with respect to the sale of copper ingots. The only real source of competition comes from scrap copper that has been melted back into ingot form. However, this scrap copper is considered inferior by buyers and sells for a substantially lower price.

Pacific Copper sells to approximately 200 firms in the region. Individual purchases are typically made by experienced buyers, but orders tend to be small and frequent to allow buyers to keep their inventory costs down. Although Pacific maintains a published list of prices, it is not uncommon for preferred customers to be secretly quoted a lower price or better credit terms.

Management estimates that demand for the firm's product is given by the equation

$$P = 3,000 - 0.10 Q$$

where  $P$  is the price per ton and  $Q$  is the number of tons sold per year. Regression analysis suggests that the firm's average and marginal cost equations are

$$AC = 2,400 - 0.10 Q + 0.000002 Q^2$$

and

$$MC = 2,400 - 0.20 Q + 0.000006 Q^2$$

where  $AC$  and  $MC$  are average and marginal costs per ton.

Periodically, Pacific advertises in local business publications. Advertising costs \$1,000 per unit, and the marginal effect of additional units of advertising on profit per period is estimated to be

$$\frac{\Delta\pi}{\Delta A} = 5,000 - 1,000A$$

where  $A$  is units of advertising per period.

**Requirements**

1. If the objective of Pacific management is short-run profit maximization, what will be the optimal price and rate of output? How much profit is being earned?
2. Suppose that copper ore deposits are discovered on nearby islands, and various area entrepreneurs are considering the establishment of competing smelters to produce ingots. What factors should be considered by Pacific's managers in adopting a long-run pricing strategy? Be specific.

**PART II**

After 2 years, eight new firms enter the market. At the present time, Pacific has a 50 percent share of copper ingot sales and the rest of the market is shared equally by the eight new firms. During a recreational outing in Alaska, the managers of the nine copper-producing firms decide to collude and set the price of ingot at the monopoly level.

**Requirement**

Is the price-fixing plan likely to be successful? Why or why not? Be specific.

PART  
**V**  
Pricing Decisions

CHAPTER 12

*Pricing of Goods and Services*

CHAPTER 13

*Pricing and Employment of Inputs*

INTEGRATING CASE

*Study V: Northern Lumber Products, Inc.*

1.  $\frac{1}{x^2} = x^{-2}$   
2.  $\frac{d}{dx} x^{-2} = -2x^{-3}$   
3.  $= -2x^{-3}$   
4.  $= -\frac{2}{x^3}$

$\frac{d}{dx} \frac{1}{x^2} = -\frac{2}{x^3}$

$\frac{d}{dx} \frac{1}{x^2} = -\frac{2}{x^3}$

2

# CHAPTER

# 12

## Pricing of Goods and Services

- **Preview**
- **Pricing of Multiple Products**
  - Products with Interdependent Demands
  - Joint Products
  - Fully Distributed Versus Incremental Cost Pricing
  - Ramsey Pricing
  - Intermediate Products (Transfer Pricing)
- **Price Discrimination**
  - Necessary Conditions for Price Discrimination
  - Types of Price Discrimination
- **Product Bundling**
- **Peak-Load Pricing**
- **Cost-Plus or Markup Pricing**
  - Mechanics of Cost-Plus Pricing
  - Evaluation of Cost-Plus Pricing
  - Cost-Plus Pricing and Economic Theory
- **Summary**
- **Discussion Questions**
- **Problems**



## PREVIEW

Of the decisions made by a manager, none is more critical to the success of a firm than setting the price of output. The immediate and obvious effect of pricing choices is reflected in short-run profits. But prices set today can also have an important impact on future profits. Indeed, pricing decisions frequently are a major factor in determining a firm's long-term success or failure.

The material presented in the previous chapters makes the pricing decision appear deceptively simple. If the firm has some control over price, the rule is to produce until marginal revenue equals marginal cost and charge the price indicated by the demand curve for that quantity. As always, economic theory involves simplifications of reality. A large corporation might produce several hundred products that are sold in many different markets. Sometimes, the price set for one of these products can affect the demand for other products sold by the firm. For example, the price the Gillette Corporation sets for razors may affect the quantity of blades demanded that fit these razors, and vice versa. Similarly, production decisions relating to one product may affect the manufacturing or marketing costs of other products.

This chapter provides a broader perspective for pricing decisions. The first section discusses firms with multiple products and considers both demand and production interdependencies. The second and third sections examine price discrimination and product bundling. Section Four considers the advantages of peak load pricing. Finally, the fifth section introduces cost-plus, or markup, pricing and demonstrates that this practice is consistent with profit maximization.

## PRICING OF MULTIPLE PRODUCTS

Procter & Gamble began in 1837 as a partnership selling soap to residents of Cincinnati, Ohio. Today, the firm has annual sales of over \$30 billion and sells hundreds of products throughout the world. Procter & Gamble's most popular brands are shown in Table 12.1.

<i>Laundry and Cleaning Products</i>	<i>Personal Care Products</i>	<i>Food Products</i>
Cascade	Bounty	Crisco
Cheer	Camay	Crush
Comet	Charmin	Duncan Hines
Dash	Crest	Folger's
Downy	Head & Shoulders	Hires
Ivory Liquid	Ivory Soap	Jif
Mr. Clean	Luvs	Pringle's
Spic and Span	Pampers	
Tide	Pepto-Bismol	
	Scope	
	Sure	

Some of these products are unrelated. For example, the demand for Pringle's Potato Chips is unlikely to be affected by the price of Tide. Similarly, production costs of Pringle's are independent of the amount of Tide produced. However, demand and production of other Procter & Gamble brands are interrelated. Clearly, Luvs and Pampers would be considered substitutes by consumers of disposable diapers. As such, the price of Luvs affects the demand for Pampers and vice versa. Also, the two competing brands share the same production facilities. Thus, if pricing decisions are based partially on costs, prices will be dependent on how costs are allocated between the two products.

When firms produce several products, managers must consider the interrelationships between those products. Pricing techniques for multiproduct firms are considered in this section.

### Products with Interdependent Demands

Products with interdependent demands are either substitutes or complements. For substitutes, such as Luvs and Pampers, a price increase for one good tends to increase the demand for the other. However, the magnitude of the increase also depends on the number of substitutes available from other suppliers. For goods that are complements, a price increase tends to reduce the demand for the other good. Options that can be added to new cars are a good example of complements. Sales of antilock brakes, power windows, and stereo systems by an automobile manufacturer are dependent on the number of vehicles sold by the company. If the price of the basic automobile is increased, vehicle sales will decline, and the demand for options will also decrease.

Correct pricing decisions require that demand interdependencies be taken into account. Instead of setting each price in isolation, the impact of each price on the demand for other products produced by the firm must be considered. The basic objective of the manager should be to determine prices that maximize total profit for the firm rather than only profit earned by individual products.

When demands are interrelated, insight into managerial decisions can be gained by considering the marginal revenue equations for the products. Consider a firm that produces only two goods,  $X$  and  $Y$ . Assume that sales of  $X$  have an impact on the demand for  $Y$  and vice versa. In terms of marginal revenue, this assumption can be stated mathematically as

$$MR_X = \frac{dTR_X}{dQ_X} + \frac{dTR_Y}{dQ_X} \quad (12-1)$$

and

$$MR_Y = \frac{dTR_Y}{dQ_Y} + \frac{dTR_X}{dQ_Y} \quad (12-2)$$

Equation (12-1) indicates that marginal revenue associated with changes in the quantity of  $X$  can be separated into two components. The first,  $dTR_X/dQ_X$ , represents the change in revenues for good  $X$  resulting from a one-unit increase in sales of good  $X$ . The second,  $dTR_Y/dQ_X$ , reflects the demand interdependency. It indicates the change in revenue from the sale of good  $Y$  caused by a one-unit increase in sales of good  $X$ . Equation (12-2) has a similar interpretation in terms of a one-unit increase in sales of good  $Y$ .

The signs of the interdependency terms,  $dTR_Y/dQ_X$  and  $dTR_X/dQ_Y$ , depend on the nature of the relationship between  $X$  and  $Y$ . If the two goods are complements, both terms will be positive, and increased sales of one good will stimulate sales for the other. Conversely, if the two goods are substitutes, the two terms will be negative because additional sales of one good reduce sales of the other.

Clearly, the firm must consider demand interdependencies in order to make optimal pricing and output decisions. Assume that goods  $X$  and  $Y$  are complements. In determining the profit-maximizing rate of output for good  $X$ , if the effect of sales of  $X$  on the demand for  $Y$  is not considered, output of  $X$  would be increased only until  $dTR_X/dQ_X$  equals the marginal cost of producing  $X$ . But, as can be seen from equation (12-1),  $dTR_X/dQ_X$  understates the actual incremental revenue generated by selling an additional unit of  $X$ . Specifically, revenue also is affected by  $dTR_Y/dQ_X$ , which is positive if the two goods are complements. Thus, when demand interdependence is taken into account, profit maximization requires a greater rate of output for good  $X$ . In fact, output of  $X$  should be increased until

$$\frac{dTR_X}{dQ_X} + \frac{dTR_Y}{dQ_X} = MC_X$$

where  $MC_X$  is the additional cost incurred by the firm in producing an additional unit of good  $X$ . Similarly, for goods that are substitutes, it can easily be shown that ignoring the demand interdependency will cause too many units of output to be produced.

## Case Study

### Turkey Prices at Thanksgiving

For most American families, roast turkey is an important part of their Thanksgiving festivities. Although pumpkin pie, potatoes and gravy, and cranberry sauce are a traditional part of the meal, it is the size and taste of the turkey that determine the quality of the dining experience.

For many years, most families ate turkey only at Thanksgiving and Christmas. Today, turkey is recognized as a highly nutritious and relatively inexpensive meat and is eaten throughout the year. Turkey consumption has risen from 5.5 pounds per capita during the 1950s to about 20 pounds per person in the late 1990s. However, demand for turkey still increases dramatically in November and December as families prepare for the holidays.

Economic theory predicts that when the demand for a product increases, the price should also increase. This is exactly what happens at the wholesale level. In early November, turkey producers raise their prices in anticipation of increased purchases by grocery stores and restaurants. For example, in 1997, the average wholesale price of turkey was 72 cents per pound. But during the fourth quarter of that year, the wholesale price averaged 75 cents.

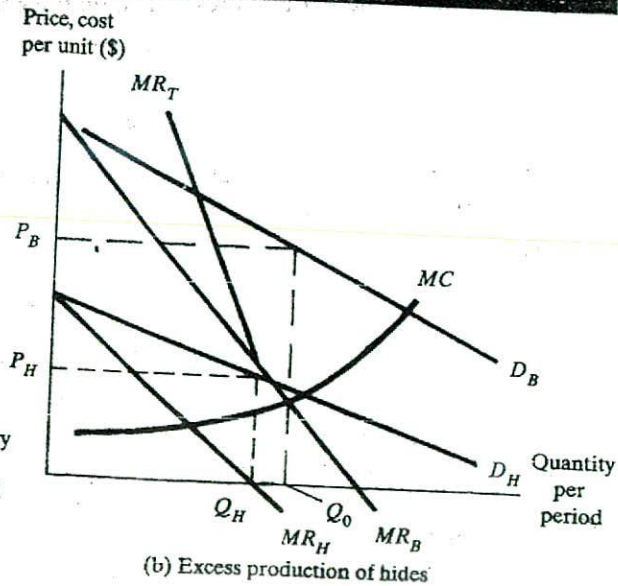
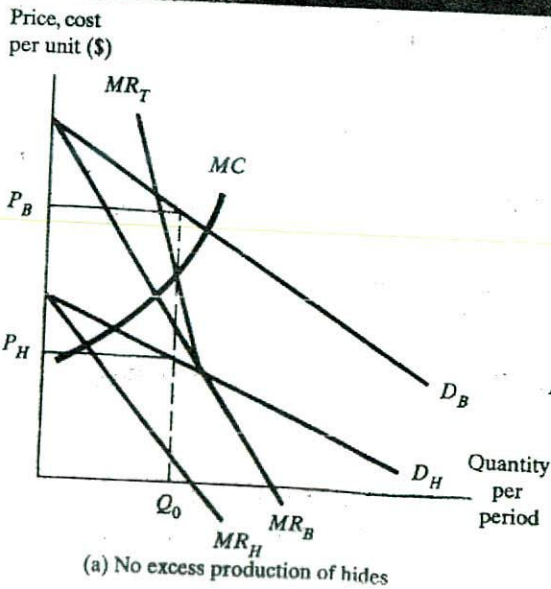
Higher wholesale prices for turkey should result in a higher price at the retail level, but this is not the case. Typically, the price that shoppers pay for turkey is actually

less around Thanksgiving and Christmas than during the rest of the year. The reason is that turkey is often used as a loss leader to entice customers into the store. Retailers assume that any losses they experience on turkey can be more than recouped as shoppers buy all of the other items that they need for the holiday season. Basically, the store owners recognize the interdependence between the demand for turkey and the other products they sell. Because of this complementary relationship, turkey prices are kept low.

### Joint Products

Products can be related in production as well as demand. One type of production interdependency exists when goods are jointly produced in fixed proportions. The process of producing beef and hides in a slaughterhouse is a good example of fixed proportions in production. Each carcass provides a certain amount of meat and one hide. There is little that the slaughterhouse can do to alter the proportions of the two products.

When goods are produced in fixed proportions, they should be thought of as a "product package." Because there is no way to produce one part of this package without also producing the other part, there is no conceptual basis for allocating total production costs between the two goods. These costs have meaning only in terms of the product package. This idea is shown by Figure 12.1a. Note that the figure identifies two demand curves, one for hides and one for beef. Although the two goods are produced together, their demands are independent. However, there is a single marginal cost curve for both products. This reflects the fixed proportions of production, that is, the marginal cost is the cost of supplying one more unit of the product package.



Where goods are jointly produced, pricing decisions should take this interdependency into account. Figure 12.1a indicates how profit-maximizing prices and quantities are determined.  $MR_B$  and  $MR_H$  are the marginal revenue curves for beef and hides. But when an additional animal is processed at a slaughterhouse, both the beef and the hide become available for sale. Hence, the marginal revenue associated with sale of a unit of the product package is the sum of the marginal revenues. This sum is represented by the line  $MR_T$  in Figure 12.1a.  $MR_T$  is determined by adding  $MR_H$  and  $MR_B$  for each rate of output. Graphically, it is the vertical sum of the marginal revenue curves of the two products.

The profit-maximizing rate of output,  $Q_0$ , is determined by the intersection of  $MR_T$  and  $MC$ . The profit-maximizing prices,  $P_H$  and  $P_B$ , are specified by the demand curves for each good at output rate  $Q_0$ .

In Figure 12.1a, note that both  $MR_H$  and  $MR_B$  are positive at  $Q_0$ . In contrast, Figure 12.1b was drawn so that  $MR_H$  is negative at the profit-maximizing quantity. The implication is that sale of an extra hide reduces the revenues received from that product. Clearly, in this situation,  $Q_0$  cannot be the optimal rate of output for hides. A firm should never produce at an output rate where marginal revenue is negative. But a problem arises because the products are jointly produced. Thus, cutting back on production of hides would mean reduced supplies of beef for which marginal revenue is positive.

In cases such as that shown in Figure 12.1b, the profit-maximizing choice for beef is to sell  $Q_H$  at a price of  $P_H$ . Although  $Q_0$  hides will be produced, sales should not be made beyond the point where marginal revenue is negative. Thus, only  $Q_H$  should be sold and the price set at  $P_H$ . Unless costs of disposal or storage are high, the excess hides should be withheld from the market.

### Example Calculating the Profit-Maximizing Prices for Joint Products

A rancher sells hides and beef. The two goods are assumed to be jointly produced in fixed proportions. The marginal cost equation for the beef-hide product package is given by

$$MC = 30 + 5Q$$

The demand and marginal revenue equations for the two products are

BEEF	HIDES
$P = 60 - 1Q$	$P = 80 - 2Q$
$MR = 60 - 2Q$	$MR = 80 - 4Q$

What prices should be charged for beef and hides? How many units of the product package should be produced?

**Solution** Summing the two marginal revenue equations gives

$$MR_T = 140 - 6Q$$

The optimal quantity is determined by equating  $MR_T$  and  $MC$  and solving for  $Q$ . Thus

$$140 - 6Q = 30 + 5Q$$

and, hence,  $Q = 10$ .

Substituting  $Q = 10$  into the demand curves yields a price of \$50 for beef and \$60 for hides. However, before concluding that these prices maximize profits, the marginal revenue at this output rate should be computed for each product to assure that neither

is negative. Substituting  $Q = 10$  into the two marginal revenue equations gives \$40 for each good. Because both marginal revenues are positive, the prices just given maximize profits. If marginal revenue for either product is negative, the quantity sold of that product should be reduced to the point where marginal revenue equals zero.

### Key Concepts

- If a firm produces two goods that are substitutes, the optimal rate of output for each good is less than the rate that would maximize profit if there were no demand interdependence.
- If a firm produces complementary goods, the optimal rates of output are greater than if there were no interdependence.
- For products produced jointly in fixed proportions, output should be increased until the sum of the marginal revenues equals the marginal cost of the product package.

### Fully Distributed versus Incremental Cost Pricing

Some costs are clearly related to the provision of a particular product or service. For example, meters on homes are there for the sole purpose of measuring the amount of electricity used in the house. No one could seriously challenge an accounting system that assigned the costs of those meters to residential electricity customers. However, other costs may not be clearly attributable to a particular product or service. High-voltage transmission lines fit into this category. Electricity for residential users is transmitted to urban areas over such lines. These same facilities are also used to serve industrial and commercial customers. If any two of the three classes of customers stopped using electricity, the same high-voltage transmission lines would still be required to continue serving the remaining customers.

For an electric utility, expenses associated with high-voltage transmission are referred to as *common costs*. Because the facilities are necessary to provide service to each class of customers, any allocation of these common costs is essentially arbitrary. In fact, the concept of allocating common costs is really a contradiction in terms. It involves allocating costs that already have been determined to be conceptually unassignable to any specific product or service.

Despite the problems, many businesses make extensive use of a practice called *fully distributed cost pricing*. This approach allocates a portion of the firm's common costs to each product or service. That is, all common costs are distributed among the products and services of the firm. Then the price of each is set so that it covers the designated portion of common costs plus costs that are directly related to the provision of the product or service.

As mentioned previously, any assignment of common costs must be arbitrary. The real problem with this method of pricing is that the choice of allocation scheme may have an important effect on the price set and, hence, the quantity demanded of the goods and services provided by the firm. A scheme that allocates a small portion of common costs to a product will result in a lower price and greater quantity demanded for that product than will a method that apportions a larger fraction of such costs to the product.

Consider the following example. A firm provides two services, temporary secretarial help and data processing. The firm has \$10 million of common costs that must be paid even if neither service is provided. Provision of the first service is very labor intensive, and 80 percent of all labor costs involve the secretarial workers. In contrast, data processing is capital intensive, and 80 percent of all capital costs involve this service.

The firm's management decides to base prices on fully distributed costs. Two allocation schemes are proposed. The first is to apportion common costs on the basis of labor costs resulting from each service. The second approach allocates common costs in proportion to the amount of capital investment that can be attributed directly to supplying each service.

Note how the alternative methods would affect the price set for each service. An allocation based on labor costs would result in 80 percent, or \$8 million of common costs, being assigned to temporary secretarial help. Thus, the price set for this service would have to be relatively high to cover the common costs. But the data processing service, with its relatively smaller labor expense, would be priced lower because only 20 percent, or \$2 million of common cost, would be allocated to that service. Conversely, an apportionment based on investment would have the opposite effect. Because data processing is capital intensive, a large fraction of common costs would be assigned to that service, and hence the price set for data processing would be higher and that of temporary secretarial assistance lower than with the first allocation scheme.

Although a firm must recover its common costs, it is not necessary that the price of each product be high enough to cover an arbitrarily apportioned share of common costs. Proper pricing does require, however, that prices at least cover the incremental cost of producing each good. Incremental costs are additional costs that would not be incurred if the product were not produced. As long as the price of a product exceeds its incremental costs, the firm can increase total profit by supplying that product. Hence decisions should be based on an evaluation of incremental costs.

The contrast between fully distributed and incremental costs in pricing can be illustrated by considering railroad passenger service. Suppose a railroad has a route that carries passengers between San Francisco and San Diego. The managers are considering an intermediate stop in Los Angeles. The rails, locomotives, and passenger cars already exist, so the primary additional expense would be the energy cost of transporting extra passengers and the establishment of terminal facilities in Los Angeles.

Assume that competition from bus travel between San Francisco and Los Angeles limits the fare that can be charged by the railroad for the trip. Specifically, the managers of the railroad believe that the fare cannot be greater than \$30. Should the new service be offered?

One member of the management team argues that the decision should be based on fully distributed costs and that the San Francisco-Los Angeles service should not be offered unless fares will cover direct costs plus a share of the common costs. For a railroad, the primary common costs would include the rails between the two cities and the engines and cars. Based on the common cost allocation method used by the railroad, it is determined that the fare would have to be at least \$40. Thus, with fully distributed costs as a standard for pricing, the service would not be offered because the fare would be higher than for bus transportation.

Using incremental costs to evaluate and price the service is advocated by another member of the management team. This person's analysis suggests that with the rail lines in place and trains already operating between San Francisco and San Diego, the additional expense

of transporting a passenger from San Francisco to Los Angeles would be only \$15. The implication is that the service should be offered and priced between \$15 and \$30 per trip.

Clearly, the decision regarding the service should be based on incremental analysis. Both the railroad and its patrons could benefit from adding the stop in Los Angeles. If the ticket price is set higher than \$15, the total profits of the firm would increase. Travelers between San Francisco and Los Angeles would benefit because an alternative form of transportation becomes available. Even the customers going from San Francisco to San Diego could benefit. Although the San Francisco–Los Angeles fare would not be high enough to cover its full share of common costs, any price in excess of \$15 could provide a contribution to common costs. Thus, the price of traveling from San Francisco to San Diego could be reduced.

Where common costs are involved, all of a firm's products and services cannot be priced at their incremental cost. In aggregate, prices must be set high enough to allow the firm to recover its common costs. However, the example demonstrates that it is not necessary for each product to cover an arbitrarily determined share of those costs. As long as the price of a new product exceeds its incremental cost, total profit can be increased by providing the product.

The choice between incremental and fully distributed costs is far from academic. Frequently, poor choices are made by managers who insist that every price must cover fully distributed costs. Clearly, the proper approach by managers attempting to maximize profit is to make decisions based on incremental costs.

## *Case Study*

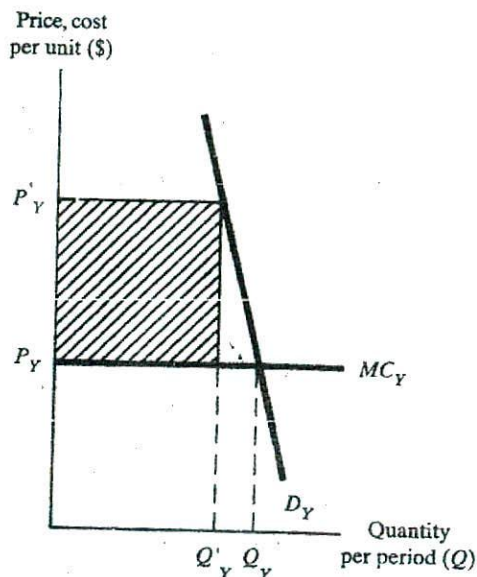
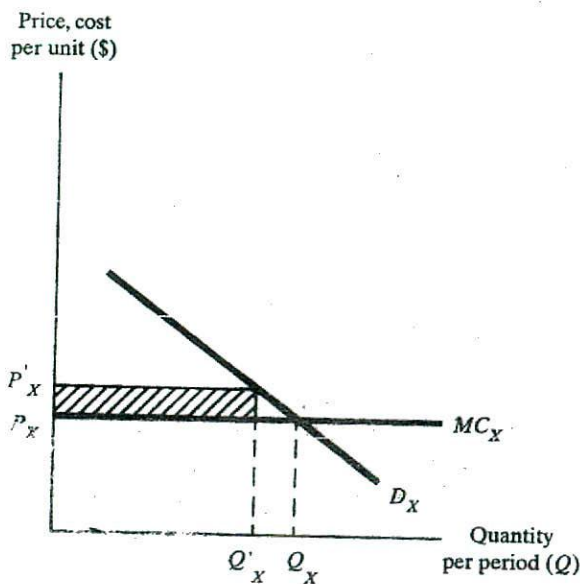
### What Constitutes Unfair Competition?

The Louisville and Nashville Railroad filed a petition with the Interstate Commerce Commission requesting permission to lower its freight rates for a particular route from \$11.86 to \$5.11 per ton. The firm's objective was to be able to meet competition from a company hauling freight by truck and barge. The railroad had substantial costs that could not be clearly allocated to specific routes. Fully distributed costs for the route in question were \$7.59 per ton, but incremental costs were only \$4.69. Because the truck–barge operation had few fixed costs, its fully distributed and incremental costs were nearly equal—about \$5.19 per ton.

The railroad argued that a rate of \$5.11 should be allowed because the \$4.69 in incremental costs would be covered. The competing truck–barge company contended that the proposed rail rates were unfair because they were less than the \$7.59 per ton that represented the firm's fully distributed costs. In this case, the Interstate Commerce Commission based its decision on fully distributed costs and rejected the railroad's proposal for a rate reduction.\* For many years, such decisions by the ICC made it difficult for railroads to compete with other modes of transportation. But since the early 1980s, railroad shipping rates have been deregulated, giving the firms more latitude in price setting. ■

\*Ingot Molds, Pa., to Steelton, Ky., 326 ICC 77 (1965).





### Ramsey Pricing

No product should be supplied by a firm unless its incremental revenues are expected to exceed its incremental cost. If there are common costs, managers must also decide which products will be priced above incremental cost and how much above. For unregulated, profit-maximizing firms, the rules presented in this and other chapters can be used to make such decisions. For regulated firms limited to a maximum rate of profit and for nonprofit enterprises expected to just cover their costs, some other approach may be necessary. One such method is Ramsey pricing.<sup>1</sup>

Assume that an enterprise produces two products,  $X$  and  $Y$ , that the demand for good  $X$  is more elastic than demand for good  $Y$ , and that marginal costs are constant. If  $X$  and  $Y$  are priced at marginal cost ( $P_x$  and  $P_y$ , respectively), then, as shown in Figure 12.2, quantity demanded will be  $Q_x$  for  $X$  and  $Q_y$  for  $Y$ . However, because none of the enterprise's common costs are included in marginal costs, total revenue will be less than total cost.

For the enterprise to cover its total costs, at least one of the two goods must be priced above marginal cost. It was shown in chapter 9 that any deviation from marginal cost pricing results in allocative inefficiency. If efficiency is the objective, there is a need for a "second-best" pricing scheme that allows the enterprise to at least break even while minimizing the adverse effect on resource allocation.

<sup>1</sup>The name is in recognition of pioneering work by economist Frank Ramsey. See F. Ramsey, "A Contribution to the Theory of Taxation," *The Economic Journal* 37 (March 1927): 27-61.

If an enterprise is providing several goods, Ramsey pricing suggests guidelines for the price that should be charged for each good. Because all prices cannot equal marginal costs, the question is how far to set the price of each good above or below marginal cost for that good. In its most simple form, Ramsey pricing requires that price deviations from marginal costs be inversely related to the elasticity of demand. That is, for goods with very elastic demand, the price should be set close to marginal cost. Conversely, for goods with relatively inelastic demand, the price should deviate more from marginal cost. In terms of Figure 12.2, the price of  $X$  should be closer to its marginal cost than the price of good  $Y$ . For example, a price of  $P'_x$  results in quantity demanded of  $Q'_x$ , and  $P'_y$  corresponds to  $Q'_y$ . The shaded area in each panel is the amount that each good contributes to the recovery of the enterprise's common costs. Note that good  $Y$  makes a much greater contribution than good  $X$ .

The rationale for the Ramsey rule is easy to understand. If demand is elastic, increasing the price causes a substantial reduction in quantity demanded. But if demand is highly inelastic, large changes in price will result in little change in the quantity demanded. In the extreme, if the demand were totally inelastic (a vertical demand curve), there would be no change in quantity demanded as price increased. Hence, if deviations from marginal cost pricing are greatest for those goods with inelastic demand, the resource misallocation will be minimized.

Ramsey pricing is sometimes criticized because the largest deviations from marginal cost pricing are imposed on those with the fewest alternatives (i.e., least elastic demand). Although this is true, the more relevant point is that there really is no alternative to using some variant of Ramsey pricing. There is a limit to the contribution to common costs that can be obtained from the sale of goods with elastic demand. If large price increases are imposed on such goods, consumption will decline substantially as buyers shift to alternative goods. The net result is that the firm will obtain little contribution to fixed costs from goods with elastic demand.

Many nonprofit enterprises receive subsidies from tax revenues or charitable contributions. Where such subsidies are provided, revenues from sales do not need to cover the firm's total costs. However, the concept of Ramsey pricing may still be relevant for such enterprises. Even with a subsidy, prices set equal to marginal costs may not allow the enterprise to cover its total costs. However, by using the Ramsey principle, managers could set prices that would recover the necessary amount while minimizing the adverse effect on resource allocation.

In some situations, if prices are equated to marginal costs, the revenues of an enterprise (sales plus subsidies) might exceed its total costs. This could occur if the subsidy were very large or if decreasing returns to scale caused marginal costs to be greater than average total costs. In this case, the organization would be earning economic profit. However, such an outcome is not generally consistent with the notion of a nonprofit enterprise. Thus, there would be a need to reduce prices until total revenue just equals total cost. As before, this should be done in a manner that minimizes changes in consumption patterns in comparison to marginal cost pricing.

The Ramsey pricing rule can be used to achieve this result. The deviations of prices below marginal costs should be inversely proportional to the elasticity of demand. Thus prices of goods for which demand is elastic should be priced near their marginal cost. Conversely, where demand is inelastic, prices should be lower in relation to marginal cost.

### Key Concepts

- A firm's common costs are those that cannot be assigned to any single product or service.
- The use of fully distributed costs can lead to poor pricing decisions. A product can be profitably produced if its price exceeds incremental costs of supplying the product.
- Ramsey pricing is a second-best alternative that can be used when marginal cost pricing is not feasible.
- A simple version of Ramsey pricing specifies that price deviations from marginal cost should be inversely related to the elasticity of demand.

### Example Using Ramsey Pricing

Consider an enterprise that supplies two goods,  $X$  and  $Y$ . For ease of exposition, assume that the marginal cost of providing each is constant and equal to \$10. However, also assume that the firm has common costs of \$99 per period and that these costs must be recovered. Further, suppose that demand elasticities are  $-0.1$  for  $Y$  and  $-1.0$  for  $X$ , and that if prices are equated to marginal costs, 10 units of each product will be sold each period. However, note that if prices equal marginal costs, the firm will incur a loss of \$99 per period. What prices for  $X$  and  $Y$  would allow the firm to recover its fixed and marginal costs while minimizing the adverse effect on resource allocation?

**Solution** The pricing problem is how to increase prices to recover the \$99 in common costs while minimizing the changes in consumption patterns in comparison to those with marginal cost pricing. The Ramsey principle suggests that since demand is more inelastic, product  $Y$  should be priced higher in relation to marginal cost than should product  $X$ . One simple formulation of Ramsey pricing uses the inverse elasticity rule. This specifies that departures from marginal cost should be inversely proportional to elasticity of demand. In this example, since elasticity for  $Y$  is one-tenth that of  $X$ , the deviation of the price of  $Y$  from its marginal cost should be 10 times the deviation of the price of  $X$  from its marginal cost.

Using the inverse elasticity rule, the solution is to price product  $Y$  at \$20 and  $X$  at \$11. Note that a 100 percent increase in the price of  $Y$  will decrease quantity demanded by only 10 percent (because the demand elasticity equals  $-0.1$ ), to nine units. The 10 percent increase in the price of  $X$  will decrease quantity demanded by 10 percent (because elasticity equals  $-1.0$ ), also to nine units. Now each unit of  $Y$  sold makes a \$10 contribution to common costs for a total of \$90. Each unit of  $X$  sold makes a \$1 contribution for a total of \$9. Together, this pricing approach allows the enterprise to recover its common costs of \$99. This objective is achieved with minimal impact on the pattern of consumer demand. For both goods, the reduction in quantity demanded is only one unit as compared to marginal cost pricing.

### Intermediate Products (Transfer Pricing)

Vertical integration is common in modern economic systems. A firm is said to be vertically integrated when it operates at more than one stage of the production process. For

example, some steel producers mine coal and iron, transport the ore on boats owned by the firm, use the coal as an energy source to transform the iron into steel ingots, shape the steel ingots into finished products, and distribute those finished products to consumers. The fabricated steel products received by the consumers are final goods. In contrast, the coal, iron ore, steel ingots, and undelivered steel products are referred to as *intermediate goods*. That is, they are goods or materials that will be needed as inputs at a later stage of the firm's operations.

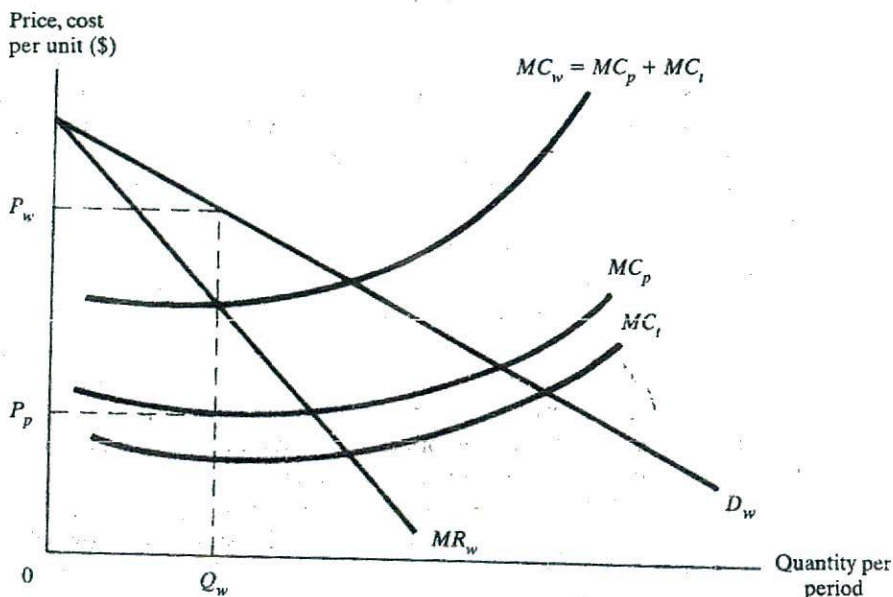
By taking advantage of scale economies, avoiding possible supply disruptions, and bringing complementary aspects of the production process together, a vertically integrated company may be more efficient than several small firms, each operating at a single stage of the production process. However, greater size resulting from vertical integration can cause control problems. Top management may find it difficult to become familiar with each stage of the operation or a cumbersome bureaucracy may develop that makes it difficult to implement decisions.

One method of dealing with such problems is to organize vertically integrated firms into semiautonomous divisions. Each of these divisions has its own function and its own management. Each management team is rewarded based on the performance of the division. In some cases, the evaluation is based on the profit earned by the unit. In an integrated firm, determining the amount of profit that should be credited to a division producing an intermediate product is a difficult task. The problem is that if the unit's function is to provide an input for the next stage of the production process, revenue will depend on the price that is charged for the intermediate good. A high price will increase profits of the unit at the earlier stage of production, whereas a low price will make the later production stage appear more profitable.

A more serious problem is that an incorrect price set for an intermediate good can affect the total profit earned by the firm. Specifically, if the decision makers in each division attempt to maximize profits for their units, the total profit of the firm might be reduced. Thus, it is important that prices of intermediate products be set to maximize overall profits rather than division profit. This objective may require that top-level management be involved in pricing intermediate goods.

The following discussion provides guidelines for the pricing of intermediate products, sometimes referred to as transfer pricing. For simplicity, it is assumed that there are only two stages of production. In the first, rolls of paper are manufactured as an intermediate product. In the second, paper is cut into writing tablets and sold to final consumers. Two alternative situations are considered. The first occurs when an external market exists for the intermediate good. That is, the division making rolls of paper can sell its output to buyers outside the firm, and the unit requiring the paper has alternative sources of supply. The second case is when there is no external market and paper can be bought and sold only between the two divisions of the firm. In this case, there is no market-determined price for the rolls of paper.

**External Market** Assume that the writing tablet division of the integrated firm has the option of obtaining paper from the paper manufacturing division or from independent suppliers. Similarly, the paper manufacturing unit can sell to the writing tablet division or to other buyers. Also assume that the external market is perfectly competitive. This implies that the two units can buy or sell as much as they want at the market-determined price.



With a perfectly competitive external market, there really is no price decision to be made. The paper manufacturing unit, like a competitive firm, faces a horizontal demand curve at the market-determined price. If the managers of that unit attempt to maximize profits, they will increase production until price equals marginal cost. If the paper manufacturing division tries to charge a price in excess of the market price, the writing tablet unit should buy paper from independent suppliers. If the final product division is unwilling to pay the market price, the paper rolls can be sold on the open market.

Where an external market exists, it is not necessary that the output of the paper manufacturing division equal the input demand for the tablet unit. If there is an excess supply of paper rolls, it can be sold to other firms. Similarly, if the firm's internal supply of paper is insufficient, the tablet division can buy from other producers.

**No External Market** If no external market exists or if the divisions are not allowed to trade with other firms, a conflict may arise regarding the proper price to be charged for paper. The paper manufacturing unit may benefit from a higher price, while the division that makes tablets may benefit from a lower price. However, the goal of top management is to determine the price for paper that results in maximum profit for the combined firm.

The optimal price of both the intermediate and the final good can be determined using Figure 12.3. The demand and marginal revenue curves for writing tablets are  $D_w$  and  $MR_w$ , respectively. The marginal cost of producing the paper necessary to make a writing tablet is  $MC_p$ , while the marginal cost involved in transforming the paper into a tablet is  $MC_t$ . Hence, from the perspective of the firm, the marginal cost of each additional tablet is the sum of  $MC_p$  and  $MC_t$ , which is designated as  $MC_w$ . Thus, for the combined firm, the profit-maximizing choice is to produce where  $MR_w = MC_w$ , or at an output rate of  $Q_w$  per period. The corresponding price would be  $P_w$ .

Because  $Q_w$  is the profit-maximizing output of writing tablets for the combined firm, the price set for the intermediate product (paper) must cause the managers of the writing tablet division to produce  $Q_w$  tablets and the managers of the paper division to supply an amount of paper consistent with producing  $Q_w$  tablets.

The solution is for top management to require the paper unit to set its price equal to the marginal cost of producing paper. This directive will cause the tablet division to view  $MC_w = MC_p + MC_t$  as its marginal cost curve and select  $Q_w$  as the profit-maximizing quantity and  $P_w$  as the price. At the same time, by setting a price of  $P_p$  for paper, the paper division will supply the exact amount of product necessary to produce  $Q_w$  tablets.

### Key Concepts

- If there is a perfectly competitive external market for an intermediate good, market forces will cause the price of the good to approach marginal cost, and no managerial pricing decision will be necessary.
- In the absence of an external market, profit maximization requires that the price of each intermediate good be set equal to its marginal cost.

## PRICE DISCRIMINATION

Frequently, the same book is sold at a much lower price in South America and Europe than in the United States. This practice is an example of price discrimination and is designed to increase the total profit of the book publisher. From an economic perspective, price discrimination occurs when price differences between consumers or markets do not reflect variations in the cost of supplying the product.<sup>2</sup>

Sometimes price discrimination involves charging a uniform price when costs differ. Consider the firm that advertises an all-you-can-eat buffet for \$9.95. The first customer is a jockey who does little more than pick at the salads. The second is an NFL defensive tackle, who requires 10,000 calories a day just to maintain his body weight. The uniform price of \$9.95 constitutes price discrimination because the cost of serving the two customers differs markedly.

More commonly, price discrimination occurs when prices differ even though costs are essentially the same. Physicians' services are an example. For a given treatment, there is no reason to believe that costs depend on the income of the recipient. Yet high-income patients are sometimes charged more than the poor for the same services.

### Necessary Conditions for Price Discrimination

Three conditions must be met before a firm can successfully practice price discrimination. First, the firm must have at least some control over price. Obviously, a price taker in a perfectly competitive market is not in a position to engage in price discrimination. Second, it must be possible to group different markets in terms of the price elasticity of demand in each. It will be shown later in the section that firms can increase total profit by charging relatively higher prices in markets where demand is less elastic.

<sup>2</sup>The legal interpretation of price discrimination differs somewhat from the economist's definition. The legal status of price discrimination is discussed in chapter 19.

Finally, the firm's markets must be separable, meaning that products cannot be purchased in one market and then resold in another. Suppose that a firm has identified two markets and charges a high price in the first and a lower price in the second. If the two markets are not separable, price discrimination cannot be successful. Either consumers will go to the low-priced market and make their purchases, or enterprising individuals will buy in the low-priced market and resell at a price below that established by the firm in the high-priced market. In either case, the price differential between the markets will disappear as prices decline in the first market and increase in the second.

## Case Study

### Price Discrimination and the Airlines

Finding the lowest airfare can be a bewildering experience. On any given day, there are tens of thousands of different fares available. With a full 150-seat aircraft flying between two U.S. cities, it would not be uncommon for the passengers to have paid 20 different fares for their seats. In some cases, these differences at least partially reflect amenities associated with higher price tickets. For example, first-class passengers have more leg room and better meals. But in other cases, different prices are charged for the same travel experience. During the summer of 1998, a regular coach round-trip ticket from Chicago to San Francisco cost as much as \$900, but promotional pricing by the airlines allowed some passengers to make the same journey for only \$300. Once on board the plane, service was identical—same cramped seats, same bland meals, and same inconvenient rest rooms.

For many years, the airlines have used what they call yield management to increase their profits. This practice involves both price discrimination and marketing. The price discrimination component is based on variations in price elasticities for different types of customers. Typically, business flyers have less elastic demands because they must meet with suppliers and customers at specific times and in specific locations. Often, these trips are made on relatively short notice. Airlines take advantage of this situation by setting higher prices for tickets that do not require advance purchase. In contrast, vacation travelers often choose between many destinations (including some that do not involve air travel) and plan their trips far in advance. Because these discretionary travel demands are more price sensitive, airlines advertise some seats at lower prices if passengers are willing to buy their tickets 7 to 30 days in advance.

The marketing aspect of yield management strategies involves determining how many low-priced seats to offer. Although airlines are required to set aside at least some seats at the promotional price, they have considerable latitude in determining exactly how many they will allocate to each flight. Flights that usually are full will not have many low-cost seats, while on those that have a history of excess capacity, airlines will offer many such seats in an attempt to draw additional customers. Determining the most profitable mix of seat prices is a complex and ongoing process for the airlines. Computers are used to continuously reevaluate and alter the optimum composition of prices based on the latest information. It is possible for a potential customer to call a travel agent on a Tuesday and be told that there are no promotional fares available on a flight and for another person to call the agent on Wednesday and obtain the low-cost fare on the same flight.

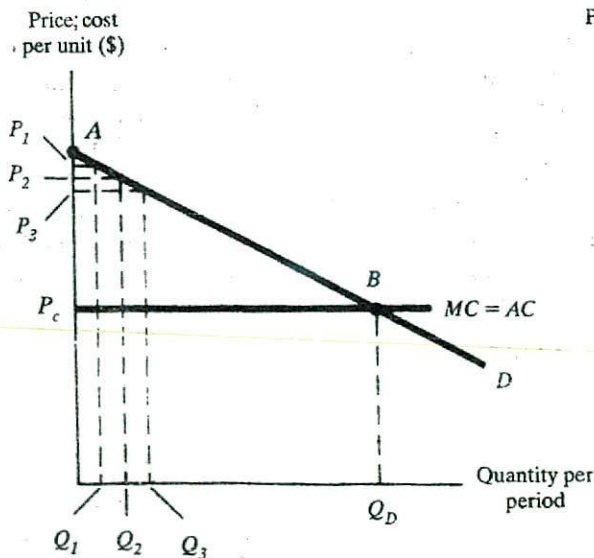
Although airline pricing practices have the appearance of price discrimination, there are other factors that should be considered. Many low-cost fares involve restrictions. The ticket may have to be purchased in advance, a Saturday night stay at the location may be required, and the ticket could be nonrefundable. A purchaser of a regular coach-fare ticket does not face these constraints. Thus, it could be argued that the tickets represent different services and that the higher price for the regular fare reflects the additional convenience associated with that ticket. ■

### Types of Price Discrimination

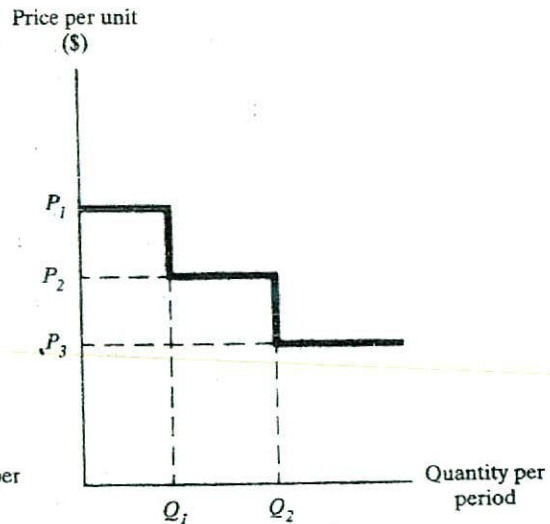
There are many forms of price discrimination, but the standard method of classification identifies three types or degrees of discrimination. Their common characteristic is that they allow the firm to capture part of the consumer surplus that would have resulted from uniform pricing.

**First-Degree Discrimination** Figure 12.4a shows the demand curve faced by a monopolist. The curve indicates the maximum price that can be obtained for successive units of output. For example, the first unit,  $Q_1$ , could command a maximum price of  $P_1$ , the second could be sold for a maximum of  $P_2$ , and so on. To simplify the discussion, it is assumed that marginal cost is constant and equal to average cost.

First-degree price discrimination involves charging the maximum price possible for each unit of output. Thus, the consumer who attaches the greatest value to the product is identified and charged a price of  $P_1$ . Similarly, the consumers willing to pay  $P_2$  for the second unit and  $P_3$  for the third are identified and required to pay  $P_2$  and  $P_3$ , respectively.



(a) First degree price discrimination



(b) Second degree price discrimination



With first-degree price discrimination, the profit-maximizing output rate is where the marginal cost and demand curves intersect. In Figure 12.4a, this occurs at  $Q_D$ . At this point, the maximum price that can be obtained for the product is just equal to the marginal cost of production. Any attempt to sell more than  $Q_D$  units would reduce profits because price would have to be less than marginal cost. Conversely, any rate of output less than  $Q_D$  would not maximize profits because the additional units could be sold (as shown by the demand curve) at prices greater than the marginal cost.

First-degree discrimination is the most extreme form of price discrimination and the most profitable pricing scheme for the firm. Because buyers are charged the maximum price for each unit of output, no consumer surplus remains. As defined in chapter 9, consumer surplus is the difference between the price a consumer is willing to pay and the actual price charged for the good or service. The maximum consumer surplus results when there is no price discrimination, and price is set equal to marginal cost. In Figure 12.4a, this maximum consumer surplus is shown as the area of the triangle  $AP_cB$ . In contrast with first-degree price discrimination, there is no consumer surplus because  $AP_cB$  is captured by the firm as economic profit.

First-degree discrimination is not common because it requires that the seller have complete knowledge of the market demand curve and also of the willingness of individual consumers to pay for the product. In addition, the seller must be able to segment the market so that resale between consumers cannot take place. These requirements are seldom met in actual market situations. However, one possible case involves the sale of Treasury bonds by the federal government. In selling these bonds, the government requires each prospective buyer to submit a sealed bid. Those conducting the auction determine a minimum bid. All the bids that exceed the minimum are accepted and the bidders are obligated to buy at the price they indicated in their bid. Thus, through this process, the government attempts to extract the maximum price that each buyer is willing to pay.

**Second-Degree Discrimination** Second-degree price discrimination is an imperfect form of first-degree discrimination. Instead of setting different prices for each unit, it involves pricing based on the quantities of output purchased by individual consumers. This is illustrated by Figure 12.4b. For each buyer, the first  $Q_1$  units purchased are priced at  $P_1$ , the next  $Q_2 - Q_1$  units are priced at  $P_2$ , and all additional units are priced at  $P_3$ .

In most cases, second-degree price discrimination involves goods and services whose consumption is metered. Electricity is an example. Many electric utilities in the United States use a declining-block tariff in pricing electricity. A typical tariff might specify the following monthly rates for blocks of usage:

First 100 kilowatt-hours	\$0.12 per kwh
Next 300 kilowatt-hours	\$0.10 per kwh
All additional kilowatt-hours	\$0.08 per kwh

It should be observed that just because different prices are charged for different blocks of consumption, it does not necessarily imply second-degree price discrimination. The high rate for the first 100 kilowatt-hours may be intended to recover the fixed costs of serving a customer, such as billing and metering. As such, the \$0.12 for the first 100 kilowatt-hours may not involve price discrimination. But if kilowatt-hours beyond 300 cost the utility the same amount to provide as those in the second block, then price discrimination is being practiced.

In addition to electricity, second-degree price discrimination is often used in setting rates for water, gas, and time-share computer usage. It is also practiced by fast-food establishments. For example, a seller may offer soft drinks for \$1.00, with refills available at \$0.50. This pricing policy reflects the fact that the second drink is less valuable to the customer and will be purchased only at a lower price.

**Third-Degree Discrimination** The most common type of price discrimination is third-degree discrimination. It involves separating consumers or markets in terms of their price elasticity of demand. This segmentation can be based on several factors. Often, third-degree price discrimination occurs in markets that are geographically separated. The practice of selling books at a lower price outside the United States is an example. Evidently, buyers in other countries have greater elasticities of demand than do U.S. buyers. At the same time, costs of collecting and shipping books make it unprofitable for other firms to buy in foreign countries and resell in the United States.

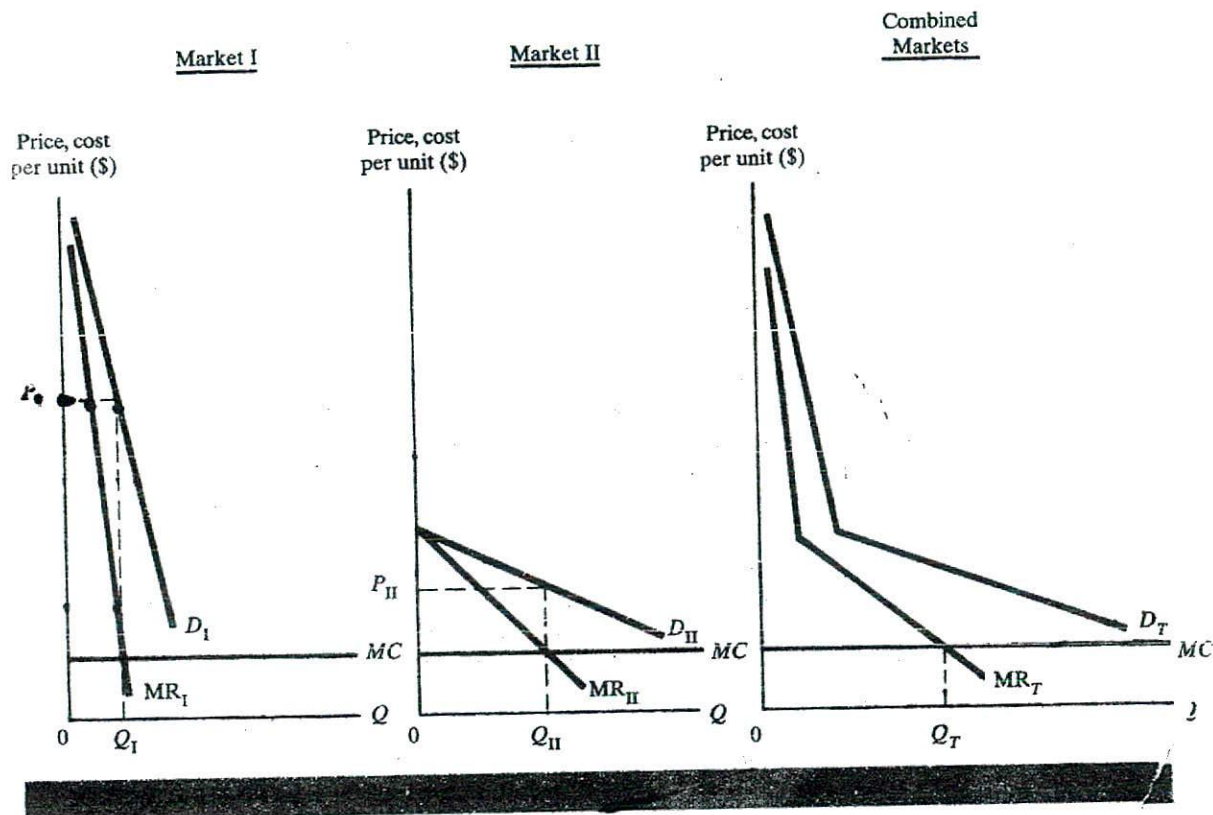
Discrimination can also be based on the nature of use. Telephone customers are classified as either residential or business customers. The monthly charge for a phone located in a business usually is somewhat higher than for a telephone used in a home. The explanation is that business demand is less elastic than residential demand. An individual without a telephone may be able to use a pay phone or go to a neighbor's house to make a call, but for many businesses a telephone is an absolute necessity.

Finally, markets can be segmented based on personal characteristics of consumers. Age is a common basis for price discrimination. Most movie theaters charge a lower price for children than they do for adults. But there is no difference in the cost of providing service to the two groups—one seat is required for each patron regardless of age. The reduced price for children is based on differing demand elasticities. The assumption is that, with less money to spend, a child's demand for movies is more price sensitive than an adult's.

Figure 12.5 is used to show how profit-maximizing prices and quantities are determined with third-degree price discrimination. Consider a firm selling in two markets, I and II. To simplify the discussion, it is assumed that marginal costs are equal and constant in both markets. Demand is assumed to be less elastic in market I than in market II. The marginal revenue curves for the individual markets are  $MR_I$  and  $MR_{II}$ , respectively.

The combined demand curve for the two markets is also shown in Figure 12.5. It represents the sum of the demands in each of the markets at each price. The combined marginal revenue curve is also shown and was computed as the sum of marginal revenues in each market. For the firm, the optimal total output is at  $Q_T$ , the point where  $MR_T = MC$ . Once the profit-maximizing total output has been determined, the next step is to determine the output rates and prices in each market.

Because marginal costs are constant, the decision rule for allocating output is that the marginal revenue should be equal in the two markets. That is, the extra revenue obtained from selling an additional unit in the first market should be the same as that received from selling one more unit in the second market. If the two are not equal, the firm could increase its revenue and profit by allocating additional output to the market with greater marginal revenue. In Figure 12.5, equating marginal revenue to marginal cost means that  $Q_I$  units of output should be sold at a price of  $P_I$  in market I and  $Q_{II}$  units at a price of  $P_{II}$  in market II.



Note that a higher price is charged in market I, where demand is relatively less elastic. This relationship is true in general and is easily explained. Where demand is less elastic, consumers are less sensitive to price, meaning that relatively high prices can be charged. Conversely, in markets where demand is more elastic, the quantity demanded is more sensitive to price. Hence the profit-maximizing price is lower.

### Key Concepts

- Price discrimination occurs when variation in prices for a product in different markets does not reflect variation in costs.
- The three criteria for successful price discrimination are market power, separable markets, and variation in demand elasticity among markets.
- First-degree price discrimination consists of charging the maximum price for each unit. In second-degree discrimination, prices are based on the quantity purchased by consumers.
- Third-degree discrimination does not require charging a higher price in markets with less elastic demand. It is based on the basis of geography, use of the product, or personal characteristics of consumers of the product.

**Example Third-Degree Price Discrimination and Profits**

A firm sells in two markets and has constant marginal costs of production equal to \$2 per unit. The demand and marginal revenue equations for the two markets are as follows:

$$\begin{array}{ll} \text{MARKET I} & \text{MARKET II} \\ P_I = 14 - 2Q_I & P_{II} = 10 - Q_{II} \\ MR_I = 14 - 4Q_I & MR_{II} = 10 - 2Q_{II} \end{array}$$

Using third-degree price discrimination, what are the profit-maximizing prices and quantities in each market? Show that greater profits result from price discrimination than would be obtained if a uniform price were used.

**Solution** With price discrimination, the condition for profit maximization is

$$MR_I = MR_{II} = MC$$

Because marginal cost is equal to \$2, the optimal quantities are the solutions to the equations:

$$MR_I = 14 - 4Q_I = 2 \quad \text{which implies that } Q_I = 3$$

and

$$MR_{II} = 10 - 2Q_{II} = 2 \quad \text{which implies that } Q_{II} = 4$$

Optimal prices are obtained by substituting the profit-maximizing quantities into the demand equations. Thus  $P_I = 8$  and  $P_{II} = 6$ .

Profits in each market are equal to total revenue ( $P \cdot Q$ ) minus total costs ( $MC \cdot Q$ ). Hence

$$\text{profit}_I = \$24 - \$6 = \$18 \quad \text{and} \quad \text{profit}_{II} = \$24 - \$8 = \$16$$

Hence combined profit for the two markets is \$34.

To compute profits in the absence of price discrimination, the combined demand and marginal revenue equations must be computed. The first step is to express the demand equations in terms of quantities. Thus

$$Q_I = 7 - \frac{P}{2} \quad \text{and} \quad Q_{II} = 10 - P$$

Note that the subscript has been dropped from price because the same price is to be charged in each market. Adding the two demand curves gives

$$Q_T = 17 - \frac{3}{2}P$$

The corresponding marginal revenue equation is calculated by solving for  $P$  and using the principle that the marginal revenue function for a linear demand curve has the same intercept and twice the slope. Thus

$$P = 11\frac{1}{3} - \frac{2}{3}Q_T$$

and

$$MR_T = 11\frac{1}{3} - \frac{4}{3}Q_T$$

Equating marginal revenue to marginal cost gives

$$11\frac{1}{3} - \frac{4}{3}Q_T = 2$$

which implies that  $Q = 7$ . Substituting  $Q = 7$  into the combined demand equation yields  $P = 6\frac{2}{3}$ . Hence profit without price discrimination is

$$P \cdot Q - MC \cdot Q = \$46\frac{2}{3} - \$14 = \$32\frac{2}{3}$$

But price discrimination resulted in total profit of \$34. Thus it has been shown that profit can be increased by the use of price discrimination.

## PRODUCT BUNDLING

In the movie *Five Easy Pieces*, actor Jack Nicholson enters a diner and asks for toast and coffee. The waitress curtly informs him that toast is not available, even though the restaurant has both bread and a toaster. To obtain his toast, Nicholson is forced to order a chicken salad sandwich without the chicken, mayonnaise, and lettuce. Although a little unusual, this is a form of product bundling. *Bundling* is the practice of selling two or more products together for a single price. When the products are only available as a package, the pricing strategy is referred to as *pure bundling*. If at least some products can also be purchased separately, then the firm is using *mixed bundling*.

Bundling is a common practice. College and professional sports teams offer season ticket packages that include seats for popular games that are likely to be sold out, together with seats for other games that have less fan appeal. Cultural arts series are often marketed in the same way. Many restaurants offer complete meals that include appetizers and dessert. Car manufacturers provide vehicles with features such as air conditioning, antilock brakes, cassette decks, and airbags as standard equipment at "no extra" price. Computer companies often include certain software, such as an operating system and a word processor, with the machines that they sell.

Why is bundling such a common strategy? One reason is that firms can reduce their production and marketing costs by packaging goods and services in this way. For example, General Motors can customize its facilities to manufacture automobiles with a limited number of option packages. Successful college football teams such as Notre Dame can reduce their ticket sale costs because they have many season ticket holders. The waiters and waitresses at restaurants that offer only complete meals are spared much of the time required for customers to make decisions about each individual component of the meal.

But product bundling can be profitable even where there are no cost savings. Like price discrimination, bundling allows firms to increase their profits by extracting additional consumer surplus. However, in some situations, bundling may be preferable to price discrimination because it requires less information about tastes and preferences of consumers.

A simple example will be used to illustrate how bundling can increase profits. Because of their extremely high production costs, television series rarely make money for their producers when they are first seen on TV. The real payoff comes if the program

lasts two or three years so that there are enough episodes produced to allow them to be sold to individual stations, which rely on reruns to meet their programming needs.

Consider a firm that has acquired the rights to 50 episodes each of two popular programs—*Seinfeld* and *Star Trek*. Suppose stations in two different cities of similar size are contemplating the distributor's offerings and that the maximum prices they will pay, often referred to as the *reservation price*, for a 50-episode series are as shown here:

	<i>Seinfeld</i>	<i>Star Trek</i>
Memphis, Tenn.	\$500,000	\$300,000
Seattle, Wash.	\$300,000	\$500,000

The numbers were set up so that preferences of the two stations would be negatively correlated. Presumably because of local tastes, the station in Memphis has a higher reservation price for *Seinfeld* than for *Star Trek*. In contrast, the Seattle station attaches more value to *Star Trek*. Assume that the same prices must be offered in both cities.

Because preferences of the two stations are negatively correlated, the distributor can increase its profit by bundling. If the firm prices the two series separately, the maximum amount it could charge (and still sell to both stations) would be \$300,000 for each, and total revenue would be \$1,200,000. Now suppose the firm bundles the series and sets a price of \$800,000 for the two. Each station will buy the package because the price does not exceed the sum of the reservation prices of the stations for the two series. But total revenue to the distributor will be \$1,600,000. By bundling, the firm earns an additional \$400,000.

If demands for the series had not been negatively correlated, bundling would not have been advantageous. For example, assume that the reservation prices for the two stations were as follows:

	<i>Seinfeld</i>	<i>Star Trek</i>
Memphis, Tenn.	\$300,000	\$400,000
Seattle, Wash.	\$400,000	\$500,000

In this case, the firm could charge a maximum of \$300,000 for *Seinfeld* and \$400,000 for *Star Trek* and earn a total of \$1,400,000. But the highest package price that could be charged is \$700,000 per station, and this would generate the same amount of revenue. For bundling to be profitable, there must be at least some consumers whose preferences are negatively correlated with others. If there are consumers who have a high reservation price for one good but place a low value on the second, there must be other consumers whose reservation price for the second good is higher than for the first.

### Key Concepts

- Pure bundling involves selling two or more products together for the same price.
- If reservation prices are negatively correlated, bundling can increase profits by capturing part of the consumer surplus.

## PEAK-LOAD PRICING

A firm selling in many markets at the same time can increase its profit by using price discrimination. Similarly, a firm that uses the same facility to supply several markets at

different points in time can increase total profits by the use of peak-load pricing. Basically, peak-load pricing involves charging a higher price for consumers who require service during periods of peak demand and a lower price for those who consume during low or off-peak periods.

Pricing of long-distance telephone calls is a good example. Most long-distance calls are for business purposes and are placed on weekday afternoons. As a result, the switching facilities and lines provided by telephone companies are designed to meet demand during this peak period. In contrast, fewer calls are made late at night and on weekends. But these off-peak calls use the same facilities necessary for peak-period calls. To induce consumers to shift their calling patterns to periods of low demand, phone companies offer substantial discounts for late-night and weekend long-distance calls. If successful, costly additions to capacity can be postponed and existing facilities used more efficiently. As a result, both the company and its customers can benefit. The firm will have reduced costs and hence increased profit. Off-peak customers will pay lower rates. Even peak-period customers may benefit in the long run because the firm's facilities will be more efficiently utilized.

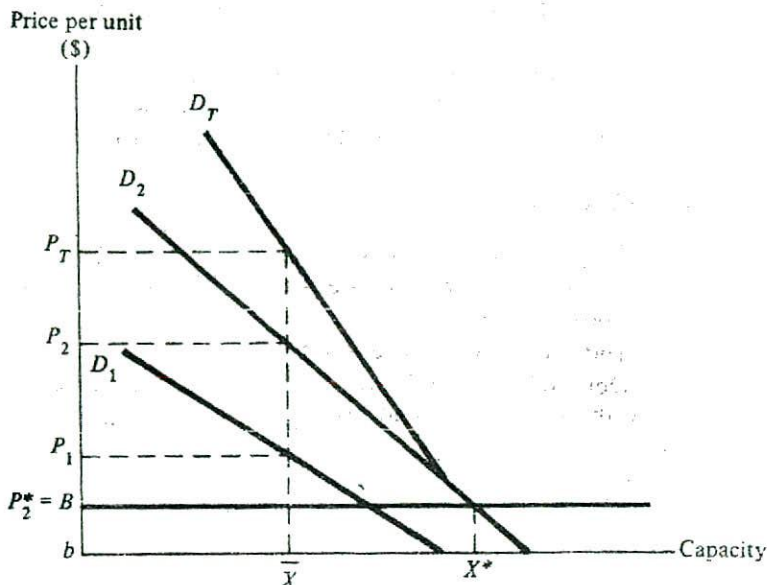
The fundamental principle of peak-load pricing is that those who impose the greatest demand on a firm for production capacity should be those who pay for most of that capacity. The traditional theory of peak-load pricing is discussed here.

Peak-load pricing may be appropriate if three conditions are met in producing a good or service. First, the product cannot be storable. For example, in the case of long-distance telephone calls, the service involves direct communication between two or more people. Calls cannot be stored for use at a later time. A busy executive in Portland would not consider a recorded message from an associate in New York to be an acceptable substitute for a telephone call received during business hours. Second, the same facilities must be used to provide the service during different periods of time. Again using long-distance telephone service as an example, this condition is met because calls placed at different times use the same lines and switching equipment. The third condition is that there must be variation in demand characteristics at different periods of time. Long-distance calls also meet this requirement. Demand is greater during business hours than at other times. In addition, demand for business calls at any given time is usually less elastic than is the demand for personal calls.

To illustrate the concept of peak-load pricing, assume that demand for telephone calls during a day can be divided into two periods of equal length. Period 1 is a time of low demand and extends from 7 P.M. to 7 A.M. Period 2 is from 7 A.M. to 7 P.M. and is the time of peak demand. Let demand for telephone calls in period 2 be greater at all prices than demand in period 1. That is, at any given price the quantity demanded is greater in period 2 than in period 1.

Also assume that only labor and capacity costs are incurred in providing telephone service. Let  $b$  represent labor cost per unit of service, where  $b$  is assumed not to vary by the period or by the demand for service. Similarly, let  $B$  represent the rental cost of a unit of capacity, with  $B$  assumed constant with respect to the amount of capacity purchased. That is, the first unit of capacity costs  $\$B$ , and all additional units cost the firm  $\$B$ .

Let the price paid for telephone service be the sum of the labor cost and the capacity cost (which includes a reasonable return on capital). Because the labor cost per unit of service is assumed to be constant, it can be subtracted from the price in each period without altering the results of the analysis. Figure 12.6 depicts the two demand curves,  $D_1$  and  $D_2$ , after the labor cost has been subtracted. As a result, the vertical scale



starts with  $b$  (the labor cost) instead of zero. Because labor costs have been netted out, Figure 12.6 can be considered a graph of the demand for telephone capacity. Hence, each point on the two demand curves indicates the amount of capacity that the firm's customers demand at a given price of capacity.

The  $D_T$  curve in Figure 12.6 is the result of vertically summing the two individual demand curves (i.e., adding the vertical distances above the horizontal axis at each point along the horizontal axis). Remember that the demands are noncompeting and hence the same capacity is used to provide service in each of the two periods. The individual demand curves can be interpreted as indicating the willingness of consumers to pay for capacity during each period. For example, consumers in period 1 will pay  $P_1$  per unit for  $\bar{X}$  units of capacity and consumers in period 2 will pay  $P_2$  for  $\bar{X}$  units. Because the capacity is usable to serve consumers in each period, the total value or demand for  $\bar{X}$  units of capacity is  $P_T = P_1 + P_2$ .

Efficient resource allocation suggests that capacity should be added until the value of the last unit is just equal to the cost of obtaining it. The cost of each unit of capacity is  $B$ . The total value of capacity is read from the  $D_T$  curve and indicates that at a cost of  $B$ ,  $X^*$  units of capacity should be employed. The peak-load pricing problem is to determine who should pay for the  $X^*$  units of capacity. Notice that even if the cost of capacity were zero, less than  $X^*$  units of capacity would be required to meet the demand in period 1. Thus, the selection of  $X^*$  units provides no benefits to consumers in period 1. The implication is that they should not be required to pay for capacity. Users in that period would properly be charged only the labor cost,  $b$ , per unit of telephone service consumed.

In contrast, because period 2 users attach a positive value to  $X^*$  units of capacity, they should be assessed the cost. Specifically, the value of capacity in period 2 is  $P_2^* = B$ . By assigning this cost to period 2 users, the marginal cost of capacity can be recovered. Thus, the total price of telephone calls to period 2 users should be  $P_2^* + b$  and to period 1 users,  $b$ .



Notice the basic principles involved in this scheme of peak-load pricing. Because the off-peak users place no value on the marginal units of capacity, they pay only the labor cost; the capacity charge is paid by the users who require the capacity. In general, peak-load pricing charges all or most capacity costs to peak-period users and charges off-peak users a price based on the noncapacity costs of serving them.

### Key Concepts

- Peak-load pricing can be used to reduce costs and increase profits if
  1. The same facilities are used to provide a product or service at different periods of time.
  2. The product or service is not storable.
  3. Demand characteristics vary from period to period.
- The theory of peak-load pricing suggests that peak-period users should pay most capacity costs, while off-peak users may be required to pay only variable costs.

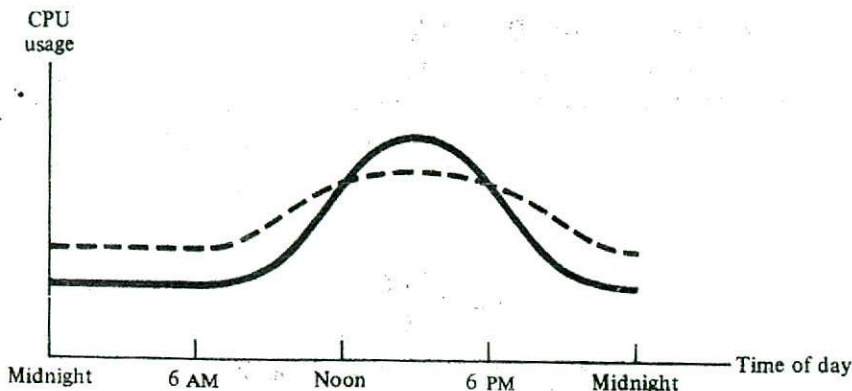
## Case Study

### Peak-Load Pricing of Computer Time

The central processing unit (CPU) is the heart and brain of a computer. Input/output devices transmit information to and from the computer, and storage devices such as on-line disks maintain files of information, but it is the computer's CPU that manages the entire operation and processes the data.

The three criteria for successful peak-load pricing are met by CPU time at large computational facilities. First, usually a computer has only a single CPU, but this equipment is in use constantly. Thus, the same facility is used to provide the service at different periods of time. Second, CPU time not used is lost forever—clearly, the service is not storable. Finally, although a computational center may be able to provide the same service at 3 A.M. as at 3 P.M., late-night service is often not as desirable to the customer. As a result, demand for CPU time is usually much greater during the day than it is at night. The figure depicts demand for CPU time at a hypothetical computer facility during a 24-hour period. The solid line illustrates the usage pattern when the price per second of CPU time does not change with the time of day. Note that demand is very low during the early morning hours, begins to increase around 7 A.M., peaks in the afternoon, and declines in the evening. The result is that the facility is very busy at some times and virtually idle during other periods.

To encourage the use of the computer during off-peak hours, many computational facilities have implemented peak-load pricing. Although the specifics differ among centers, these pricing policies specify relatively high prices during periods of peak demand and much lower prices during off-peak early-morning hours. For example, at a university computer facility, CPU time is priced as follows:



The weekend rates also apply to holidays. Note that CPU time at peak hours (weekdays from 1 to 5 P.M.) is 30 times more costly than during off-peak hours (week-day evenings from 1 to 8 A.M. and weekends and holidays from 5 P.M. to 8 A.M.). As a result, computer usage has declined during peak periods and is much greater during the off-peak hours. This change, resulting from peak-load pricing, is depicted by the dashed line in the figure. By reducing demand at peak periods, the facility may be able to postpone purchases of additional computing capacity. ■

<i>Day</i>	<i>Time Period</i>	<i>Cost per Second</i>
Monday–Friday	8 A.M.–1 P.M.	\$0.03
Monday–Friday	1 P.M.–5 P.M.	0.06
Monday–Friday	5 P.M.–1 A.M.	0.01
Monday–Friday	1 A.M.–8 A.M.	0.002
Saturday–Sunday	8 A.M.–5 P.M.	0.01
Saturday–Sunday	5 P.M.–8 A.M.	0.002

## COST-PLUS OR MARKUP PRICING

The traditional assumption of economic theory is that firms attempt to maximize profits and that this objective is accomplished by increasing production until marginal revenue equals marginal cost and then charging a price determined by the demand curve. In actual practice, however, many firms use cost-plus pricing. Basically, this approach involves setting prices that cover the cost of purchasing or producing a product plus enough profit to allow the firm to earn its target rate of return.

The remainder of this section considers why cost-plus pricing is so popular and how it is implemented. The section also examines the apparent conflict between the cost-plus and marginal revenue equals marginal cost pricing rules.

### Mechanics of Cost-Plus Pricing

Two steps are involved in cost-plus pricing. First, the cost of acquiring or producing the good or service must be determined. The total cost has a variable and a fixed component. In either case, costs are computed on an average basis. That is,

$$AC = AVC + AFC \quad (12-3)$$

where

$$AVC = \frac{TVC}{Q} \quad \text{and} \quad AFC = \frac{TFC}{Q}$$

and  $AC$  is average total costs,  $AVC$  is average variable cost,  $TVC$  is total variable cost,  $AFC$  is average fixed cost,  $TFC$  is total fixed cost, and  $Q$  is the number of units produced. But there is a problem in making the computations. With cost-plus pricing, quantity is used to calculate the price, but quantity is determined by price. This problem is avoided by using an assumed quantity. Typically, this rate of output is based on some percentage of the firm's capacity. For example, for many years General Motors used cost-plus pricing and computed average costs on the assumption that sales would be 80 percent of capacity.

The second step in cost-plus pricing is to determine the markup over costs. As mentioned previously, the overall objective is to set prices that allow the firm to earn its targeted rate of return. Thus if that return requires  $\$X$  of total profit, the markup over costs on each unit of output will be  $X/Q$ . Hence, the price will be

$$P = AVC + AFC + \frac{X}{Q} \quad (12-4)$$

#### Example Cost-Plus Pricing of Automobiles

An automobile manufacturer estimates that total variable costs will be \$500 million and total fixed costs will be \$1 billion in the next year. In setting prices, it is assumed that sales will be 80 percent of the firm's 125,000-vehicle-per-year capacity, or 100,000 units. The target rate of return is 10 percent, which is to be earned on an investment of \$2 billion. If prices are set on a cost-plus basis, what price should be charged for each automobile?

**Solution** A target return of 10 percent on \$2 billion requires that the firm earn a total profit of \$200 million. Computations are based on assumed sales of 100,000 vehicles. Thus, average fixed cost, average variable cost, and profit per unit are

$$AFC = \frac{\$1 \text{ billion}}{100,000} = \$10,000$$

$$AVC = \frac{\$500 \text{ million}}{100,000} = \$5,000$$

$$\frac{X}{Q} = \frac{\$200 \text{ million}}{100,000} = \$2,000$$

Hence, if profit of \$2,000 per vehicle is added to the \$15,000 in average fixed and variable costs, the price will be \$17,000 per car.

### Evaluation of Cost-Plus Pricing

Cost-plus pricing has some important advantages that help explain its appeal. First, it may contribute to price stability. This is a desirable result because price changes can be expensive and may provoke undesirable reactions by competitors. Second, the formula used in cost-plus pricing is simple and easy to use. As will be discussed later, the necessary information is less than for marginal revenue equals marginal cost pricing. Finally, cost-plus pricing provides a clear justification for price changes. A firm desiring to increase its price can point to cost increases as the reason.

However, cost-plus pricing has been criticized on a number of points. One alleged problem is that it is based on costs and does not take demand conditions into account. This shortcoming is compounded by the fact that the cost data used may be the wrong costs. Instead of using incremental or opportunity costs, cost-plus prices often rely on historical or accounting data. In addition, most applications of the procedure are based on fully distributing common costs to the various goods produced by the firm. Problems with prices based on fully distributed costs were discussed earlier in the chapter.

### Cost-Plus Pricing and Economic Theory

At first glance, cost-plus pricing would appear to be inconsistent with economic theory that assumes profit maximization. Moreover, the wide use of cost-plus pricing seems to make analysis based on the marginal revenue equals marginal cost decision rule largely irrelevant. However, the conflict may be more apparent than real. There is reason to believe that use of cost-plus pricing is simply a tool used by businesses in pursuing the goal of long-run profit maximization. As such, cost-plus prices can be shown to be related to, although not identical to, prices based on marginal revenues and marginal costs.

A comparison of the two approaches to pricing starts with a consideration of costs. Although it is true that cost-plus pricing is based on average rather than marginal costs, frequently long-run marginal and average costs are not greatly different. This is especially true in many retailing activities. Thus, use of average costs as a basis for price determination may be considered a reasonable approximation of marginal cost decision making.

The second step in the comparison involves the target rate of return and the resulting markup. How does a manager determine whether the target should be 10 percent or 20 percent? Basically, the decision involves management's perception of demand elasticity and competitive conditions. An example would be grocery stores. The intense competition among these firms holds down profits. As a result, the typical markup for most food items is only about 12 percent over cost.

If the markup over cost is based on demand conditions, cost-plus pricing may not be inconsistent with profit maximization. This can be shown mathematically. Marginal revenue is the derivative of total revenue with respect to quantity. Thus

$$MR = \frac{d(TR)}{dQ} = \frac{d(PQ)}{dQ} = P + \frac{dP}{dQ}Q$$

But  $P + \frac{dP}{dQ}Q$  can be rewritten as  $P\left(1 + \frac{dP}{dQ}\frac{Q}{P}\right)$ . Note that  $(dP/dQ)(Q/P)$  is  $1/E_p$ , where  $E_p$  is price elasticity of demand. Thus

$$MR = P\left(1 + \frac{1}{E_p}\right) \quad (12-5)$$

Profit maximization requires that  $MR = MC$ . As a simplifying assumption, let  $MC = AC$ . Thus, the profit-maximizing price is the solution to

$$P\left(1 + \frac{1}{E_p}\right) = AC$$

which can be written as

$$P\left(\frac{E_p + 1}{E_p}\right) = AC$$

Solving for  $P$  yields

$$P = AC\left(\frac{E_p}{E_p + 1}\right) \quad (12-6)$$

Equation (12-6) can be interpreted as a cost-plus or markup pricing scheme. That is, the price is based on a markup over average costs. The markup,  $E_p/(1 + E_p)$ , is a function of the price elasticity of demand. As demand becomes more elastic, the markup becomes smaller.<sup>3</sup> For example, if  $E_p = -1.5$ , the markup is 3.0. But for  $E_p = -4.0$ , the markup is only 1.33 times average cost.

Thus, cost-plus pricing may simply be the mechanism by which managers pursue profit maximization. Most managers have limited information on demand and costs. Obtaining the additional information necessary to generate accurate estimates of marginal costs and revenues may be prohibitively expensive. Hence, cost-plus pricing may be the most rational approach in maximizing profits.

### Key Concepts

- Cost-plus pricing is widely used by managers and involves a markup over the average cost of acquiring or producing a product.
- Cost-plus pricing may stabilize prices and provide a justification for price changes.
- The markup used in cost-plus pricing is determined by demand elasticities and competition. Markups are lower where demand is more elastic and competition is intense.
- Cost-plus pricing may simply represent the decision rule used by managers in pursuit of profit maximization.

## SUMMARY

Modern corporations may produce hundreds of different products. When the demand for products is interrelated, the firm should take this interdependence into account.

<sup>3</sup>If  $E_p = -1.0$ , the markup is undefined, and if  $-1 < E_p < 0$ , the markup becomes negative. However, these cases are irrelevant because the profit-maximizing firm will never operate on the inelastic portion of its demand curve. The reason is clear. As long as marginal revenue is positive, demand is elastic. But the profit-maximizing firm produces where marginal revenue equals marginal cost. Because marginal cost is positive, marginal revenue is also positive, and demand is elastic at the profit-maximizing rate of output.

When a firm produces complementary goods, the output rate of each good should be greater than if no demand interrelationship existed. For substitutes, output rates should be less than if the goods were independent.

Some goods are jointly produced in fixed proportions. Profit maximization requires that the two goods be considered as a product package. Thus, the rate of output should be increased until marginal cost equals the sum of marginal revenues obtained from selling an additional unit of the product package.

Not all costs are clearly attributable to a particular good or service. Attempts to allocate such common costs are arbitrary. Decisions based on fully distributed costs can result in inefficient resource allocation; incremental costs are a better guide. A good or service can be profitably produced if its price exceeds the incremental cost of supplying it.

If an enterprise has common costs, marginal cost pricing may not be feasible. Ramsey pricing is a second-best alternative that allows the firm to recover its costs while minimizing adverse effects on allocative efficiency. A simple version of Ramsey pricing specifies that deviations from marginal costs should be inversely proportional to the demand elasticities of the goods or services.

Vertical integration causes intermediate goods to be transferred from one division of a firm to another. However, profit maximization for the combined firm requires that those intermediate products be correctly priced. When a perfectly competitive external market exists, market forces will cause price to equal marginal cost. Hence, the intermediate product will be appropriately priced. If there is no external market, management should set the price of the intermediate good at marginal cost.

Price discrimination occurs when variation in price for a product sold in different markets does not correspond to differences in costs. The three conditions for successful price discrimination are market power, variation in the elasticity of demand, and separable markets. First-degree price discrimination involves charging each consumer the maximum amount that he or she is willing to pay. Second-degree discrimination occurs when prices vary depending on the amount purchased. Third-degree discrimination separates markets in terms of elasticity of demand. The segmentation may be based on location, use, or personal characteristics. Usually, higher prices are charged when demand is less elastic.

Product bundling is a pricing scheme that allows firms to capture part of the consumer surplus. Pure bundling involves the selling of two or more products only as a package. If the reservation prices of consumers are negatively correlated, firms can use bundling to increase their profits.

Peak-load pricing can reduce costs and increase profits. The practice is appropriate when three conditions are met. First, the same facilities must be used to provide a product or service at different times. Second, the product or service must not be storable. Third, demand characteristics of consumers must vary from period to period. With peak-load pricing, peak-period consumers will pay most of the capacity costs, while off-peak users will be charged a price based on variable costs.

Cost-plus, or markup, pricing is widely used by firms. This practice involves setting price at average cost plus a markup designed to provide a target rate of return. Advantages of cost-plus pricing include stable prices, a simple formula for pricing, and a clear justification for price changes. Problems include the use of historical and fully distributed costs rather than marginal costs and an apparent failure to take demand conditions into account in price setting.

On closer investigation, it can be shown that markups are related to demand elasticity and competition. Less elastic demand and lack of competition are associated with high markups over cost, while the reverse is true for markets characterized by more elastic demand and intense competition. Faced with a lack of information about demand and costs, it may be that cost-plus pricing is simply the method by which managers pursue profit maximization.

### Discussion Questions

- 12-1. Macmillan Manufacturing produces razor blades and razors. Propose a pricing strategy that would allow the firm to maximize its profit on the two goods. Explain.
- 12-2. Why should goods produced in fixed proportions be regarded as a product package in developing production and pricing strategies?
- 12-3. Should a sheep-ranching operation consider lamb and wool as a product package? Why or why not?
- 12-4. What is meant by the statement that "the assignment of common costs must, by definition, be arbitrary"?
- 12-5. Generating equipment is used to provide electric power to both residential and industrial customers. Assume that total consumption by residential users is twice that of industrial consumers. In setting prices, would it be appropriate to assign two-thirds of the cost of the generating equipment to the residential users? Why or why not?
- 12-6. The managers of a firm are considering offering a new service. The proposed price of the new service would be greater than its incremental cost but would not cover fully distributed costs. As a user of services already provided by the firm, should you favor or oppose the new service? Explain.
- 12-7. How does the presence or absence of external markets affect the role played by top management in pricing intermediate products produced by a vertically integrated firm?
- 12-8. If an intermediate product is available from a perfectly competitive industry, why would a vertically integrated firm produce the product internally? That is, what is the advantage of vertical integration in this case?
- 12-9. A city has only one furniture store. Is it likely that the store could successfully practice price discrimination? Why or why not?
- 12-10. How is bundling similar to price discrimination? Which requires more information about consumer preferences?
- 12-11. Are the three conditions necessary for peak-load pricing met in the case of movies shown in theaters? Explain.
- 12-12. How can peak-load pricing improve resource allocation?
- 12-13. How can cost-plus pricing be reconciled to the "marginal revenue equals marginal cost" rule of economic theory?

### Problems

- 12-1. A small firm traps rabbits for their fur and feet. Each rabbit yields one pelt and two feet (only the hind feet are used to make good-luck charms). The demand for pelts is given by

$$P_P = 2.00 - 0.001Q_P$$

and the demand for rabbit's feet is given by

$$P_F = 1.60 - 0.001Q_F$$

The marginal cost of trapping and processing each rabbit is \$0.60.

- a. What are the profit-maximizing prices and quantities of pelts and rabbit's feet?
  - b. If the demand for rabbit's feet is  $P_F = 1.00 - 0.001Q_F$ , what are the profit-maximizing prices and rates of output?
- 12-2. Mike's Shear Shop provides 4,000 haircuts each month at an average price of \$10 per haircut. The common costs of operating the store are \$24,000 per month. The business is considering hiring a photographer who would take pictures of customers after they had their hair cut. The price of the photographs would be \$5.00, and it is estimated that 2,000 customers would purchase the service each month. The total extra cost of the photographic service is  $2,000 + 2Q$  per month, where  $Q$  is the number of photographs sold.
- a. If the decision is to be based on incremental revenue and incremental cost, should the service be offered? Explain.
  - b. If the decision is to be made on the basis of fully distributed costs and if the \$24,000 in monthly common costs are to be apportioned based on the revenues from haircuts and photos sold each month, should the new service be offered? Why or why not?
- 12-3. Write-Right, a vertically integrated firm, produces both paper and writing tablets. The demand for tablets is given by

$$P_T = 1.00 - 0.001Q$$

where  $Q$  is the quantity of tablets. The marginal cost of producing the paper necessary for each tablet is

$$MC_P = 0.20 + 0.001Q$$

It costs the firm \$0.10 to make the paper into a writing tablet. If there is no external market for the paper, what transfer price should top management set for the paper?

- 12-4. A firm has found a way of using first-degree price discrimination. Demand for its product is given by

$$P = 20 - 2Q$$

Marginal cost is constant and equal to \$6.

- a. With first-degree discrimination, what will be the profit-maximizing rate of output? How much economic profit will the firm earn?
  - b. What will be the profit-maximizing rate of output if the firm does not discriminate and sets one price for all customers? How much economic profit will the firm earn in this case?
- 12-5. Smith Distributing sells videocassettes in two separable markets. The marginal cost of each cassette is \$2. For the first market, demand is given by

$$Q_1 = 20 - 5P_1$$

The demand equation for the second market is



$$Q_2 = 20 - 2P_2$$

- a. If the firm uses third-degree price discrimination, what will be the profit-maximizing price and quantity in each market? How much economic profit will the firm earn?
- b. If the firm charges the same price in both markets, what will be the profit-maximizing price and total quantity? How much economic profit will the firm earn?
- 12-6. Global motors sells its automobiles in both the United States and Japan. Due to trade restrictions, a vehicle sold in one country cannot be resold in the other. The demand functions for the two countries are

$$\text{U.S.} \quad P = 30,000 - 0.40Q$$

$$\text{Japan} \quad P = 20,000 - 0.20Q$$

- The firm's total cost function is  $TC = 10,000,000 + 12,000Q$ . What price should Global charge in each country in order to maximize profit? What will be the total profit?
- 12-7. The demand for a firm's product is given by  $P = 200 - 10Q$ . Marginal cost is constant and equal to \$10.
- a. Using first-degree price discrimination, what is the profit-maximizing output and price? How much profit will the firm earn?
- b. What is the profit-maximizing output and price if the firm sets a uniform price to all buyers? How much profit will the firm earn?
- 12-8. A monopolist sells in two markets and the marginal cost is \$2 in each market.
- a. If demand is given by  $P = 21 - 3Q$  in the first market, what is the profit-maximizing price and output?
- b. If demand is given by  $Q = 21 - P$  in the second market, what is the profit-maximizing price and output?
- c. Which market has the more elastic demand? Explain.
- 12-9. A firm produces two products, *A* and *B*. Demands are independent and marginal costs are zero. The products are sold to three consumers, and each must be charged the same price.
- a. For the following reservation prices, can profits be increased by bundling? Explain. What is the maximum profit?

	Good A	Good B
Consumer 1	\$50	\$40
Consumer 2	\$60	\$35
Consumer 3	\$70	\$50

- b. For the reservation prices shown here, what is the profit-maximizing price strategy? Explain. What is the maximum profit?

	Good A	Good B
Consumer 1	\$50	\$60
Consumer 2	\$60	\$60
Consumer 3	\$70	\$50

- 12-10. The manager of a sporting goods store uses cost-plus pricing to determine the profit-maximizing price of bicycles. The cost of a bicycle to the store is \$80. The manager estimates that the price elasticity of demand is  $-3.0$ . What is the profit-maximizing price?
- 12-11. Grasscutter Inc. makes a product used to trim lawns. The firm has fixed costs of \$100,000 per year. Management expects to sell 2,000 units per year, and at that rate of output, total variable costs will be \$50,000. The firm uses cost-plus pricing to earn a target rate of return on an investment of \$200,000. If the price is set at \$100, what is the target rate of return?

### Problems Requiring Calculus

- 12-12. The House of Music sells low-cost CD players and speakers. The total revenue equation for sales of the two products is given by

$$TR = 200Q_C - 6Q_C^2 + 100Q_S - 4Q_S^2 + Q_CQ_S$$

where  $Q_C$  and  $Q_S$  are quantities of CD players and speakers, respectively. The marginal cost of CD players is \$20 and the marginal cost of speakers is \$10.

- Are the two goods substitutes or complements?
  - What is the profit-maximizing rate of output for each good?
  - What would be the profit-maximizing rate of output if there were no demand interdependence between the two goods?
- 12-13. Culture Extravaganza produces ballets in Boston and New York. Monthly total revenues are given by

$$TR_B = 1,000Q_B^{0.5}$$

and

$$TR_N = 2,000Q_N^{0.5}$$

where  $Q_B$  is the monthly number of Boston patrons and  $Q_N$  is the monthly number of New York ballet attendees. Salaries of the performers are based on attendance, and the firm estimates that the marginal cost is \$10 per attendee in each city.

- If Culture Extravaganza attempts to practice third-degree price discrimination, what will be the profit-maximizing prices and rates of output in each city?
  - Will the firm earn more profit using price discrimination than if a uniform price is set? Explain.
- 12-14. A firm produces two types of calculators,  $x$  and  $y$ . The revenue and cost equations are shown below with  $Q_x$  and  $Q_y$  measured in thousands of calculators per year.

$$\text{Total revenue} = 2Q_x + 3Q_y$$

$$\text{Total cost} = Q_x^2 - 2Q_xQ_y - 2Q_y^2 + 6Q_x + 14Q_y + 5$$

- To maximize profit, how many of each type of calculator should the firm produce?
- What is the maximum profit the firm can earn?

# CHAPTER

# Pricing and Employment of Input

- **Preview**
- **Input Pricing and Employment**
  - Market Structure I: Monopolist—Perfect Competitor
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**PREVIEW**

Individuals earn income by selling resources such as capital and labor to firms. The firm uses these resources to produce the goods and services demanded by consumers. Of critical importance to both the individual and the firm is the price of a unit of the resource. The analysis presumes that the distribution of resources among individuals is a given. That is, the ownership of land, capital resources, and human resources (i.e., the education, training, and experience embodied in labor) has already been determined. Given this initial distribution, the level and distribution of income are determined by the price of each resource unit: These include the wage rate per hour of labor, the rent per acre of land, and the price per unit of capital. Clearly, input prices are also important to the firm because they are a critical factor in determining the amount and mix of resources employed. This chapter focuses on the determination of those input prices and how firms decide how much of an input to employ.

The price of a productive input is determined by supply and demand, just as is the price of output. However, the firm's demand for an input is a derived demand. Firms demand capital, labor, and land not because they have value as such, but because those resources can be used to produce goods and services that have value to consumers. Thus, the demand for inputs is dependent on the demand for the goods and services those inputs are used to produce; hence the use of the term *derived demand*.

The first part of this chapter focuses on the determination of input prices and the amount of an input to be employed. Two different market structures are considered. In the second section, the correspondence between output and input decisions is outlined. The next two sections include an introduction to the concept of economic rent and a discussion of the reasons for significant income differentials among workers. The role of labor unions in input markets is analyzed in the fifth section. Both the economics of collective bargaining and the objectives of labor unions are considered. Minimum wage laws are analyzed in the final section.

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**INPUT PRICING AND EMPLOYMENT**

The analytical framework for studying the demand for productive inputs is based on the theory of production developed in chapter 6. Recall that efficient production requires that the ratio of marginal product to input price be equal for all inputs. For example, if capital and labor are the only inputs, the efficiency condition is

$$\frac{MP_K}{r} = \frac{MP_L}{w} \quad (13-1)$$

where  $MP_K$  and  $MP_L$  are the marginal products of capital and labor and  $r$  and  $w$  are the prices of those inputs.

If one input, say capital, is held constant, it can be shown that the ratio of input price to marginal product is equal to marginal cost. Thus for labor

$$\frac{w}{MP_L} = MC \quad (13-2)$$

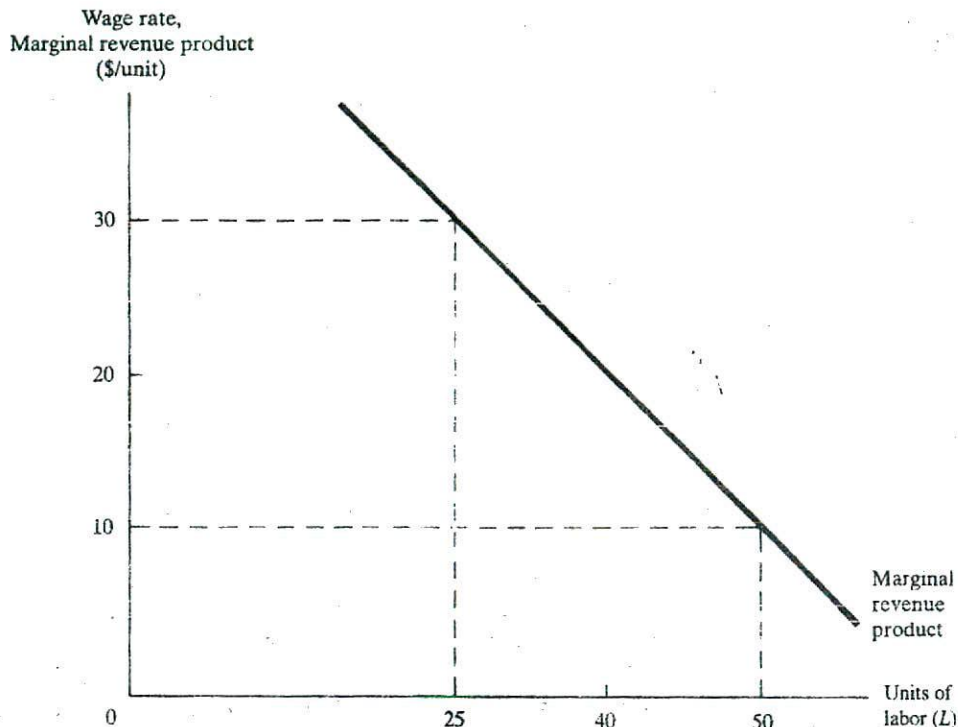


FIGURE 13.1 Marginal Revenue Product Function for Labor

Recall that the profit-maximizing firm must increase output, and therefore the employment of inputs, to the point where marginal cost equals marginal revenue. Thus, in equilibrium, the ratio of input price to marginal product (i.e., marginal cost) must equal marginal revenue. That is,

$$\frac{w}{MP_L} = MR \quad (13-3)$$

Multiplying both sides of equation (13-3) by  $MP_L$  yields

$$w = MR \cdot MP_L \quad (13-4)$$

or

$$w = MRP_L \quad (13-5)$$

which indicates that the profit-maximizing firms should hire an input until the price of the input equals the marginal revenue product ( $MRP$ ) of that input.

The  $MRP$  function is the firm's input demand function. Consider the  $MRP$  function shown in Figure 13.1. It can be used to determine the rate of labor input that will be hired at any wage rate. For example, if the wage is \$10 per unit, 50 units of labor are employed; at \$30 per unit, only 25 units will be employed.

The input demand function (i.e., the marginal revenue product or  $MRP$  function) is determined by multiplying marginal product by marginal revenue. Because of the law

of diminishing marginal returns, the marginal product function will be decreasing, at least in the range that is relevant for the firm. Further, the firm's marginal revenue function will either be horizontal, as in the case of the perfectly competitive firm, or downward sloping. Thus, because the input demand function is found by multiplying the marginal revenue function by the marginal product function, the input demand function must be downward sloping.

The firm sells goods and/or services in the product market and buys inputs in the factor market. The structure of both of these markets will influence the price of the input and the amount employed. The circumstance of the firm on the output side of the market (i.e., whether the firm is a perfect competitor, oligopolist, or monopolist) will affect the firm's input demand function, whereas the market structure on the supply side will influence the input supply curve facing the firm. In this section, input pricing and employment decisions are considered for the following types of firms: (1) a firm that is a monopolist in the product market and a perfect competitor in the input market and (2) a firm that is a monopolist in the product market and a monopsonist in the input market.

The key difference in analyzing the effect of differing conditions in the product market is that price and marginal revenue are constant for the perfect competitor. In contrast, the demand curve facing the monopolist is downward sloping, which implies that the marginal revenue function also has a negative slope.

A perfect competitor in an input market is one of many buyers of the input. No one buyer has any influence on the price of the input. This implies that the input supply function is horizontal (i.e., perfectly elastic), meaning that any firm can buy as much of that input as desired without influencing the price. Thus, the input price facing a perfect competitor in an input market is constant. In contrast, the input supply function facing the monopsonist or single buyer of an input is upward sloping. Additional inputs are obtainable only at higher prices. For example, a large firm in a small town may employ a substantial share of the local labor force. Thus, as a near-monopsonist, any significant increase in the firm's employment of labor would require an increase in the wage rate. Examples of the supply curves for a perfect competitor and a monopsonist are shown in Figure 13.2.

### Key Concepts

- The demand for a productive input is derived from the demand for the goods and services that input is used to produce.
- The demand function for an input is derived from the condition for efficient production. In general, an input should be employed until the input price equals the marginal revenue product of the input.
- Firms that are perfectly competitive in input markets face a horizontal input supply function. Monopsonists face an upward-sloping input supply function.

### Market Structure I: Monopolist (Product Market) — Perfect Competitor (Input Market)

If the firm is a monopolist, or at least has some degree of market power, it faces a downward-sloping demand function for its output. Thus, the marginal revenue curve lies below the demand function. In this case, the firm's demand for labor, as shown by

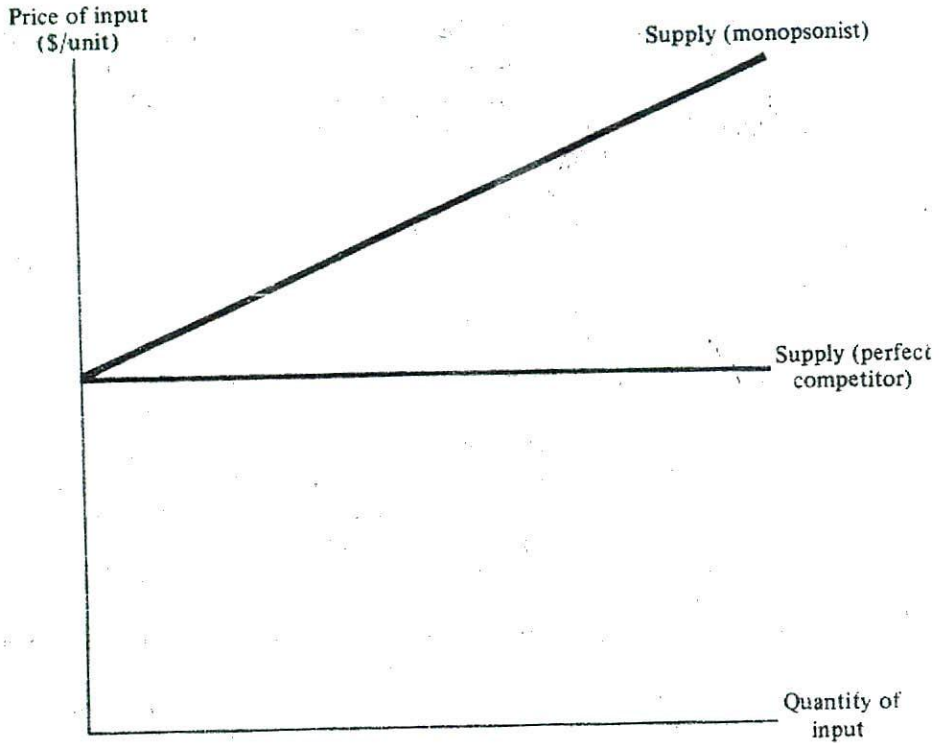


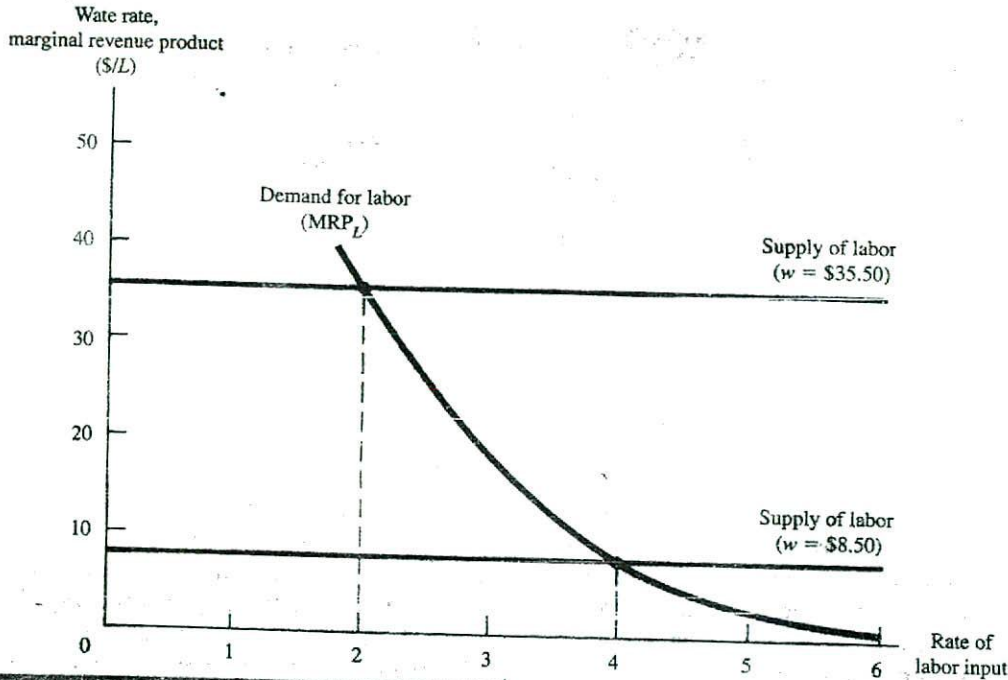
FIGURE 13.2 Input Market: Functions for a Firm

the marginal revenue product function, will decline more rapidly than if price equaled marginal revenue. Because the firm in this example is a perfect competitor in the input market, the supply function is horizontal.

Table 13.1 shows hypothetical values for a firm's total product, marginal product, and marginal revenue. The resulting marginal revenue product or input demand function is shown in the right-hand column of the table. This demand is computed by multi-

TABLE 13.1 Total Product, Marginal Product, and Marginal Revenue for a Monopsonist

Rate of Labor Input	Total Product	Marginal Product	Output Price	Total Revenue	Marginal Revenue	Marginal Revenue Product
0	0	—	—	—	—	—
1	10	10	\$5.00	\$ 50.00	\$5.00	\$50.00
2	19	9	4.50	85.50	3.94	35.50
3	26	7	4.00	104.00	2.64	18.50
4	30	4	3.75	112.50	2.13	8.50
5	32	2	3.60	115.20	1.35	2.70
6	33	1	3.50	115.50	0.30	0.30



plying each entry for marginal product by the corresponding marginal revenue per unit entry.<sup>1</sup> As  $MRP_L$  is the net addition to the firm's total revenue associated with an additional unit of labor,  $MRP_L$  also could be determined as the change in total revenue [in column (5)] associated with a one-unit change in labor.

The input demand function and the supply function are shown in Figure 13.3. If the market price of input is \$8.50, the firm will employ four units of labor because at that input rate, the marginal revenue product is just equal to the wage rate. If the price of labor increased to \$35.50, the firm would employ only two units.

### Market Structure II: Monopolist (Product Market) – Monopsonist (Input Market)

Now consider a hypothetical firm that not only is a monopolist in the product market but also is the only buyer of labor in the area. An electric utility company with a large generating facility near a small town could be an example of such a firm. The company may have a franchise as the only seller of electricity in the region and could also be the only important employer of labor. Therefore, the firm is both a monopolist and a monopsonist. If this firm's output rate is to increase, it must hire more labor. But because the firm faces an upward-sloping supply curve, hiring more of an input will require that the price of that input be increased for all its workers.

<sup>1</sup>Recall that marginal revenue is defined as the change in total revenue divided by a one-unit change in output. When the rate of labor input increases from 0 to 1, output increases from 0 to 10, and total revenue increases from 0 to 50. Therefore, marginal revenue per unit over that interval averages  $\$50/10$  or \$5.00.



**TABLE 13.2 Labor Supply, Total Expenditure, and Marginal Expenditure on Labor for a Monopsonist**

Rate of Labor Input	Labor Price Per Unit	Total Expenditure on Labor	Marginal Expenditure on Labor
0	—	\$ 0	—
1	\$ 5.50	5.50	\$ 5.50
2	8.00	16.00	10.50
3	11.50	34.50	18.50
4	15.00	60.00	25.50
5	18.50	92.50	32.50
6	22.00	132.00	39.50

A labor supply schedule facing the firm is shown in the first two columns of Table 13.2. Also shown is the firm's marginal expenditure on labor function, which is defined as the change in total expenditure on labor associated with a one-unit change in the rate of labor input. Note that to hire more labor, the firm must offer a higher wage rate, and this higher rate must be paid to all inputs hired. Thus, the marginal expenditure on input function lies above the supply curve, reflecting both the higher price necessary to attract new workers and the higher price that must be paid to those already working.

The relevant functions for determining the optimal labor input rate are the marginal revenue product function (from Table 13.1 on page 438) and the marginal expenditure on labor function (from Table 13.2). These data are summarized in Table 13.3. To maximize profit, the firm will employ labor until the marginal revenue product equals the marginal expenditure on labor. In Table 13.3 the marginal revenue product equals marginal expenditure when the rate of labor inputs is three, which is the profit-maximizing input rate. This result is also shown in Figure 13.4, where the  $MRP_L$  function intersects the marginal expenditure function at point *c*, which corresponds to  $L = 3$ . Note that the wage rate paid labor, \$11.50, which is determined by the labor supply function, is less than the marginal revenue product of labor. A monopsonist is able to hire workers at a wage less than the value of their contribution to output. This has led to claims that firms with monopsony power exploit labor.

**TABLE 13.3 Marginal Revenue Product and Marginal Expenditure on Labor**

Rate of Labor Input	Marginal Revenue Product	Labor Price Per Unit	Marginal Expenditure on Labor
0	—	—	—
1	\$50.00	\$ 5.50	\$ 5.50
2	35.50	8.00	10.50
3	18.50	11.50	18.50
4	8.50	15.00	25.50
5	2.70	18.50	32.50
6	0.30	22.00	39.50

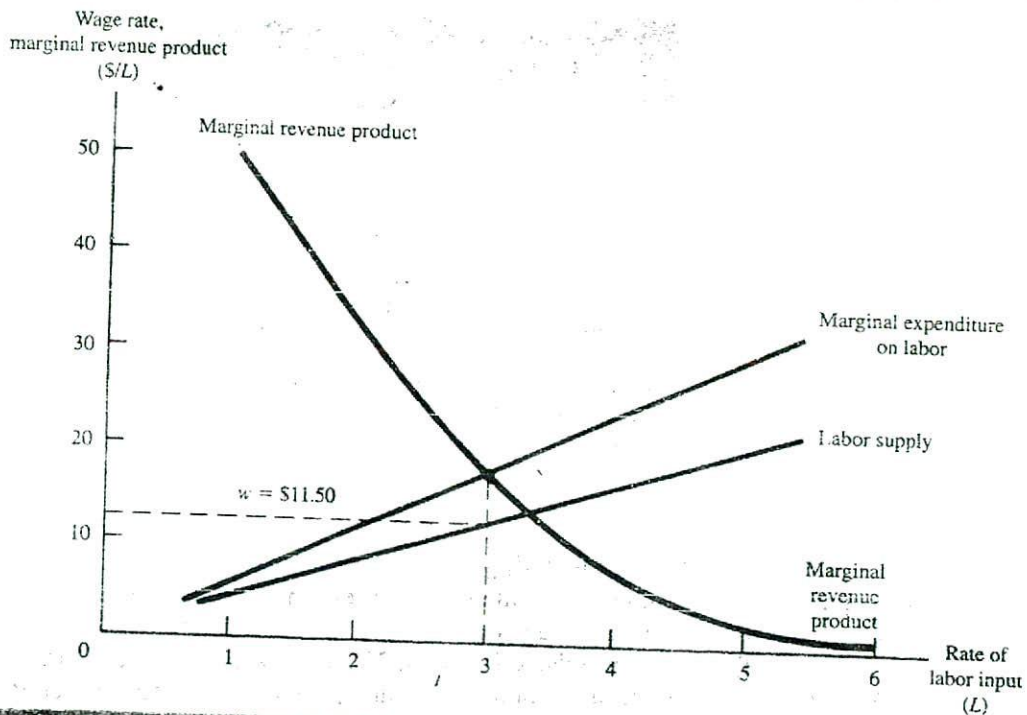


FIGURE 13.4 Determination of the Wage Rate and Optimal Employment of Labor by Monopsonist

## Case Study

### Economic Profits, Monopsony, and the Professional Sports Industry

Admission prices to most professional sports events are relatively expensive. One reason is that most major-league teams have monopoly power in the market they serve. For example, in baseball, only in the metropolitan areas of Chicago, New York, San Francisco, and Los Angeles are there two teams; in all other cities there is only one provider of major-league baseball entertainment. While there may be substitutes, such as high school, college, and semiprofessional teams, judging by attendance and ticket prices, these products are considered inferior to the major-league brand of baseball.

Not only are most major-league sports franchises monopoly sellers of baseball entertainment, they have traditionally been monopsony buyers of the services of the players. By using a system of drafting new players, each team was given the exclusive right to negotiate with the players it selected. The draft reduced the competition for the player's services and held wages down. To maintain this monopsony right, the league also used a reserve clause. Every player was required to sign a standard contract that contained a provision reserving the team's right to that player's services during the next

<i>Year</i>	<i>Average Salary</i>
1970	\$ 29,307
1975	44,676
1980	143,751
1985	371,157
1990	597,357
1995	1,110,766

*Source:* Bureau of Labor Statistics, *Compensation and Working Conditions*, 2, No. 5 (Washington, D.C.: U.S. Government Printing Office, Fall 1997).

season. The combination of the draft and the reserve clause effectively tied a player to one team forever. He could be traded to another team but was not free to shop around among other teams for a better salary.

In baseball, this system existed for about 70 years, but has now been substantially weakened, largely as the result of one owner failing to pay a bonus to a player. In 1974, Jim "Catfish" Hunter, an outstanding pitcher for the Oakland Athletics, claimed that he had not been paid one-half of the \$100,000 specified by his contract. He took the matter to arbitration, as required under baseball rules, and argued that he should be paid the remaining \$50,000 and that he should be declared a free agent, thus allowing him to negotiate with any and all of the other teams. He won on both issues and subsequently signed a long-term contract with the New York Yankees for \$750,000 per year—more than seven times his 1974 salary. The increase indicates that Oakland had been paying him substantially less than his marginal revenue product!

This development was not lost on other players. In 1975, Andy Messersmith of the Los Angeles Dodgers and Dave McNally of the Baltimore Orioles both played without signing a contract. Then they went to arbitration, arguing that the reserve clause in their contract for the prior year was nonbinding, as it only applied to the 1975 season. The arbitrator agreed, and both were declared free agents who could negotiate with any team.

After Messersmith and McNally signed contracts similar to that obtained by Hunter, other players realized the effect of the reserve clause in holding down salaries and demanded that a new contract system be developed. Negotiations between the players' association and management broke down over this issue, and a 17-day strike occurred during spring training in 1977. Both sides finally agreed to an arrangement wherein a player was tied to a team for six years (later reduced to five years), after which he automatically became a free agent. As shown in the table, the result was a tremendous escalation in players' salaries; by 1992, the average exceeded \$1,000,000 per year. Baseball owners were still charging monopoly prices, but they were now sharing the economic profits with the players. ■

**Key Concepts**

- A firm that is a perfect competitor in an input market faces a horizontal input supply function. Profit maximization for such firms requires that an input be increased until its marginal revenue product equals the price of the input.
- A firm that has market power in input markets faces an upward-sloping input supply function. The determination of the profit-maximizing price-quantity combination for the input is made by equating the firm's marginal revenue product and marginal expenditure on input functions.

**THE CORRESPONDENCE BETWEEN OUTPUT AND INPUT DECISIONS**

The managerial decision about how much to produce is made simultaneously with the decision about how much input to employ. Suppose that the capital stock is fixed ( $\bar{K}$ ) and the firm is jointly deciding the rate of output ( $Q$ ) and the amount of labor to employ ( $L$ ). Given the production function

$$Q = f(\bar{K}, L) \quad (13-6)$$

if either  $Q$  or  $L$  is specified, the other variable is determined. Thus, in a technical sense, the firm has only one decision to make.

This can be shown in another way. Suppose that the firm is a perfect competitor in both the output and input markets. Profit maximization requires that the firm increase output until the price of output ( $P$ ) equals marginal cost ( $MC$ ), and efficient input resource utilization requires that labor be hired until the marginal revenue product ( $MRP$ ) equals the unit price of labor ( $w$ ). It can be shown that these two efficiency conditions,

$$P = MC \quad (13-7)$$

and

$$MRP_L = w \quad (13-8)$$

are equivalent; that is, if one is met, so is the other.

Recall that for the perfect competitor, the marginal revenue product is equal to marginal product multiplied by output price ( $P$ ). Thus equation (13-8) can be rewritten as

$$MP_L \cdot P = w$$

Now divide both sides of this equation by the marginal product of labor ( $MP_L$ ), yielding

$$P = \frac{w}{MP_L} \quad (13-9)$$

The ratio of the wage rate to marginal product is simply marginal cost, so the condition specified by equation (13-9) is identical to that in equation (13-7). Thus, the rule  $P = MC$  implies that  $MRP_L = w_L$ , and vice versa. Although it is not demonstrated here, the profit maximization rules for a monopolist,  $MR = MC$  and  $MRP_L = w$ , are also equivalent.

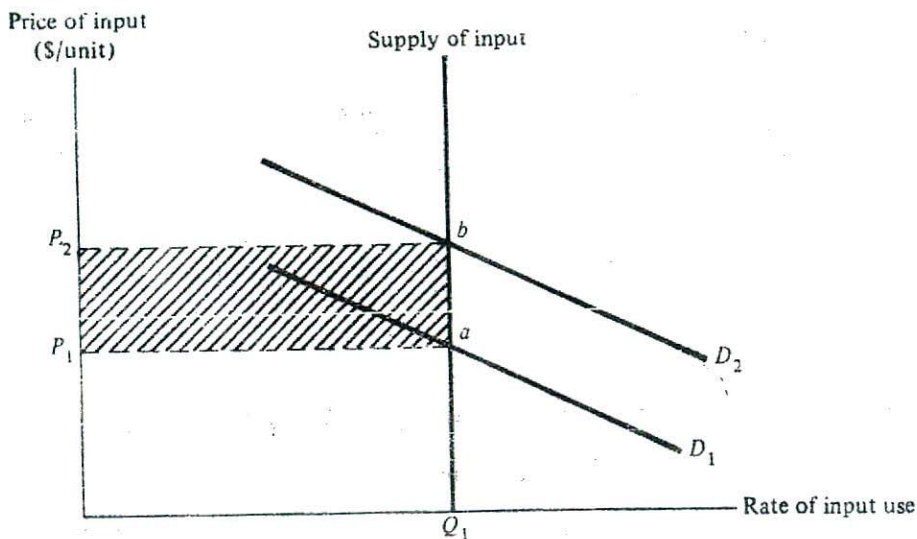


FIGURE 13.5 Determination of the Input Price (Rent) When Supply Is Perfectly Inelastic

### Key Concepts

- The decision to produce a given rate of output implies a decision to hire a certain rate of capital and labor and vice versa.
- The condition for a profit-maximizing output rate (i.e.,  $MR = MC$ ) is equivalent to the condition for a profit-maximizing input rate (i.e.,  $MRP = \text{input price}$ ).

## ECONOMIC RENT

Rent is defined as a payment to any factor of production that has a relatively fixed supply. The term is often used to describe a return to land, and reflects the notion that there is a fixed amount of land available.<sup>2</sup> As shown in Figure 13.5, when the supply of an input is fixed, the supply function is a vertical line. As a result, the price of that input or its rent is entirely determined by demand. If the demand curve is  $D_1$ , the price will be  $P_1$ . If the demand curve increases to  $D_2$ , the price will increase to  $P_2$ . Note that regardless of the change in price, the amount of the input supplied is unchanged.

*Economic rent* is a payment to a factor of production in excess of the minimum amount necessary to induce that factor into employment. Suppose that  $Q_1$  in Figure 13.5 represents the number of undeveloped acres in a particular tract of land. Assume that the cost of developing and marketing this land is equal to  $P_1$ . If the demand func-

<sup>2</sup>Although in a strict sense the total amount of land in the world is fixed, from an economic perspective land is not fixed in supply. The relevant dimensions of the land input are the amount and quality of land in use. Both of these dimensions of the land input are variable. There is much land that is not used for any economic purpose. Periodically, some of this land is developed for agricultural or other uses, and thus the supply of land actually is increased in an economic sense.

tion is below  $D_1$ , the land will not be used because the market price would be less than the cost of supplying it. Only if the price is at least  $P_1$  will the land be offered for use. Thus,  $P_1$  is said to be the *reservation price* for this input. At this price, the land is employed but there is no economic rent. If demand increased to  $D_2$ , the price would increase to  $P_2$ , and the rent would be the shaded rectangle defined by  $P_1P_2ba$ .

Only when the market price of an input is above the reservation price is there an economic rent paid. Because this return is over and above the amount necessary to employ the input, economic rent is sometimes said to be *unearned*. If part or all of this rent were taxed away, it would not change the resulting resource allocation.

## Case Study

### Economic Rent in Professional Baseball

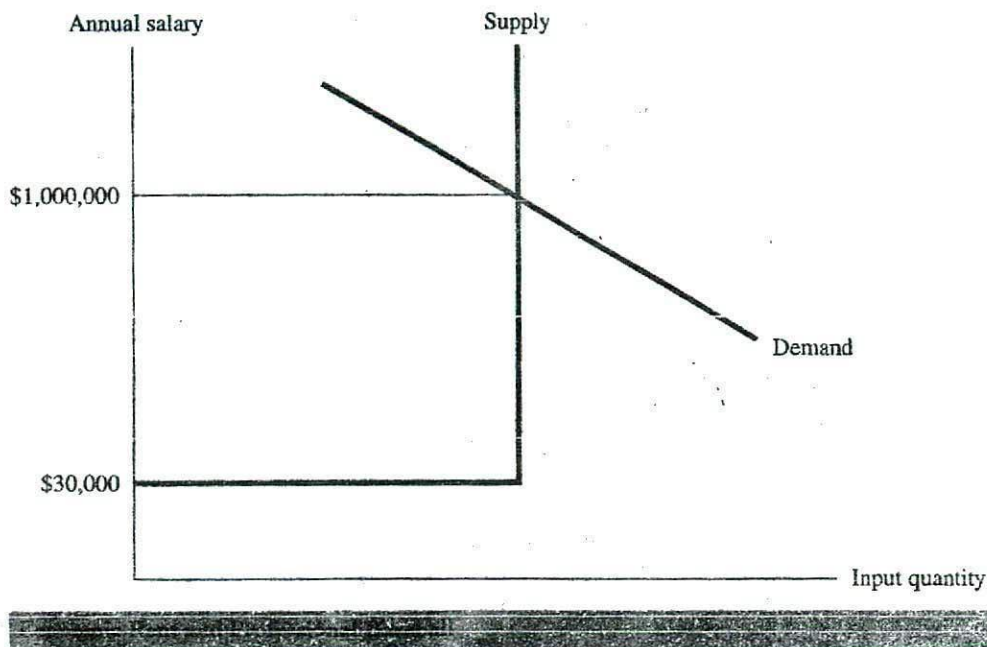
The previous case study discussed how the loss of monopsony power by the owners of major-league baseball teams allowed player salaries to increase.

The All-Salary Team (i.e., the highest paid players) for 1997 included the following:

Player	Team	Contract Years	Average per year
Albert Belle	Chicago White Sox	1997-01	\$11,000,000
Ken Griffey, Jr.	Seattle	1997-00	8,500,000
Roger Clemens	Toronto	1997-99	8,250,000
John Smoltz	Atlanta	1997-00	7,750,000
Mike Piazza	Los Angeles	1997-98	7,500,000
Barry Bonds	San Francisco	1993-98	7,291,667

Source: Bureau of Labor Statistics, *Compensation and Working Conditions*, 2, No. 5 (Washington, DC: U.S. Government Printing Office, Fall 1997).

Certainly, a large part of these salaries represent economic rent. The following conceptual model explains how that rent would be determined. Suppose that there are a fixed number of athletes capable of playing major-league baseball and that each will do something other than play baseball unless the salary is at least \$30,000 per year. Hypothetical demand-and-supply functions for these athletes are shown in the case figure. Note the unusual shape of the supply function. For these athletes, the supply function is horizontal at a price of \$30,000 and then is vertical. This reflects the requirement that the typical athlete must have this much in salary per year to attract him to play baseball; otherwise, he would take another job. The \$30,000 represents the opportunity cost of playing baseball and is the player's reservation price. Any return above that level is an economic rent. The intersection of demand and supply occurs at \$1,000,000, and the athletes are each paid this wage rate. Of this amount, \$970,000 is economic rent and \$30,000 represents the payment necessary to attract these workers away from other jobs. ■



## WAGE AND INCOME DIFFERENTIALS

There are large variations among the wage rates paid to different kinds of labor. Many unskilled workers are paid the federal minimum wage rate and others apparently have so little to offer employers that they are unable to find employment even at this low wage rate. In contrast, it is not unusual for senior lawyers to be paid \$200 or more per hour, skilled surgeons may be paid several thousand dollars for an operation that may take less than an hour, and the salaries of many professional athletes are thousands of dollars per game. Why are there such large differentials in wage rates? Are they the result of market imperfections, such as a lack of information about prices and availability of competing services, or are there economic reasons that explain this phenomenon? Actually, there are important forces on both the demand and supply sides of the input market that explain most of the differences. Those economic forces are considered in this section.

### Demand-Side Considerations

All persons are not created equal. Differences in aptitude, intellectual ability, strength, and other individual attributes mean that some workers are more productive than others. The more productive workers add more to output and to the revenue of the firm. That is, their marginal revenue product is higher than for other workers. Therefore, they are able to command higher wage rates.

Some workers, such as salespersons and assembly line workers, are paid in direct proportion to their output. Thus, their compensation is tied directly to their marginal revenue product, and the more-productive workers will automatically receive higher wages than will their less-productive counterparts. Even where pay is not tied directly to output in a formal sense, firms generally pay higher wages and salaries to their more-productive workers.

Of course, some firms have plantwide or even industrywide wage rate schedules that prescribe that all workers in a certain job category be paid the same rate. Thus, one might find differences in productivity among workers who are paid the same wage rate. Even in these cases, it is expected that the more productive workers will earn more in the long run. These workers will be the first to be asked to work overtime at premium rates, and the better workers will tend to be promoted to higher-paying supervisory positions. Furthermore, the less-productive workers will be terminated if their contribution to output is significantly below that of their counterparts. Therefore, even in firms where there might be a fixed wage rate for all workers in a category, productive workers generally will earn more than will their less-productive associates.

### Supply-Side Considerations

Adjustments in the supply of workers will also result in differential wage rates among jobs. Consider two jobs, *A* and *B*, where the skill and training requirements are much different. For example, job *A* may be a clerk in a retail store that requires a high school education, whereas job *B* requires a college degree in electrical engineering. Clearly, it will be more costly for a person to prepare for job *B*. The cost of attending college for four or more years, including the opportunity cost of the lost income during that period, is substantial. That investment in education will not be made unless there is an expectation of higher earnings than can be earned in job *A*.

The wage adjustment for this required training is made on the supply side of the market. Considerably fewer high school graduates would attend college if they did not expect to increase their earning power. That is, fewer workers will make the necessary investment unless there is the prospect of higher earnings than can be earned in comparable jobs that do not require this training. Thus, the supply function will be such that for any given wage, far fewer workers will offer their labor for sale for job *B* than for job *A*.

Other factors that result in supply-side adjustments in wage rates include risk of death or injury, working conditions, job content, and hours of work. Jobs that involve health risks, poor working conditions, unusual working hours, and/or tedious or strenuous work generally must pay a higher wage rate than jobs with more desirable characteristics. For example, holding other factors constant, a job that required heavy lifting in a very dirty place on the graveyard shift (midnight to 8 A.M.) would probably have to pay a premium wage rate to attract applicants. As in the preceding example, the wage adjustment for these job characteristics is made on the supply side of the market. At any given wage rate, more labor will be supplied to a firm offering a job with desirable characteristics than to a firm offering a less-desirable job. Managers are aware of these factors and offer higher wages for less-desirable jobs.



### Key Concepts

- Economic rent is defined as the return to a factor of production in excess of the payment necessary to keep the factor in its current employment.
- Wage differentials among workers reflect demand-side considerations, such as greater productivity, and supply-side forces, including training requirements, health risks, and working conditions.

## Case Study

### The Premium for Accepting Risk

The typical answer to the question "What value do you place on your life?" would be that it is of infinite value. Few people would offer to give up their life in return for a monetary gain except under the most extraordinary circumstances. Indeed, one of the arguments for a military draft is that military service is so risky that it is not possible to attract adequate volunteers by offering higher pay and benefits. However, the success of the all-volunteer military in the United States has disproved that notion.

Reflection on individual behavior, particularly with respect to job selection, suggests that people will take all kinds of risks to obtain economic and other kinds of rewards. Some recreational thrills, such as skydiving, hang-gliding, and skiing, can only be obtained by assuming a significant risk of injury. Many drivers exceed the speed limit to conserve on time. In fact, even driving at the speed limit is risky. To eliminate all deaths from vehicle accidents might require speed limits below 10 mph and vehicles built like Army tanks. For most people, the opportunity cost of eliminating traffic deaths is simply too high, and they are willing to take their chances at or above current speed limits. Furthermore, people regularly accept work in occupations that involve significant risk of injury. For example, those who repair the steam systems of nuclear power plants often are paid as much as \$100 for a few minutes of work. The reason is because of the exposure to radiation that increases the risk of cancer and other diseases.

Many civilian occupations are more risky than being in military service during a war. During the Vietnam War, an average of about 7,000 U.S. servicemen were killed each year. At that time, there were some 3 million men and women in the military forces. Thus, the death rate was about 2.33 per 1,000 employed in that field. Thaler and Rosen studied death rates and income differentials for a number of high-risk jobs. Several of these jobs, including guards and lumberjacks, had annual death rates higher than that for a military force at war! Jobs such as firefighter and police officer, where risk is often publicized, are actually less risky than a number of other jobs. Annual death rates per 1,000 employees for some of these occupations are shown in the following table.

<i>Occupation</i>	<i>Annual Death Rate Per Thousand Workers</i>	<i>Occupation</i>	<i>Annual Death Rate Per Thousand Workers</i>
Fire fighters	0.44	Mine workers	1.76
Police officers	0.78	Lumberjacks	2.56
Electricians	0.93	Guards	2.67
Crane operators	1.47		

SOURCE: R. Thaler and S. Rosen, "The Value of Saving a Life: Evidence from the Labor Market," in *Household Production and Consumption*, ed. N. E. Terlecky (New York: Columbia University Press—National Bureau of Economic Research, 1975), 265–298.

Thaler and Rosen also analyzed the incremental annual wage associated with these jobs. This increment measures the trade-off people are willing to make between risk and income. On average, it was found that about \$700 in additional annual wages are needed to offset the additional risk associated with the incidence of one more death per 1,000 workers per year. The death rate for lumberjacks (2.56) suggests that the average worker would have to be paid about \$1,800 dollars per year in additional annual wages (i.e.,  $2.56 \times \$700$ ) to accept the additional risk compared to a job where the risk of death is near zero. ■

## LABOR UNIONS

About 10 percent of nonagricultural workers in the United States are members of labor unions. Where workers are organized, the nature of management-worker negotiations is different than for firms without labor unions. The union may act as a monopolist, that is, as a single seller of labor to the firm.

In this section two topics related to labor unions are considered. First, the economics of collective bargaining between firms and labor unions are discussed. The second considers alternative objectives that might influence labor union behavior.

### Labor Unions and Collective Bargaining

Labor unions represent workers in negotiations with management concerning wage rates, fringe benefits, and working conditions. Where the workers of a firm are members of a union, wages are not set unilaterally by management in response to market conditions. Rather, they are determined by negotiations between management and union representatives. Of course, general labor market conditions, including wage rates for comparable workers (both union and nonunion) in other firms and industries, influence these negotiations.

In some industries, collective bargaining is done on an industrywide basis. Representatives from the major companies comprise an industry bargaining team and the labor union has a bargaining team that represents the workers. The result is similar to a

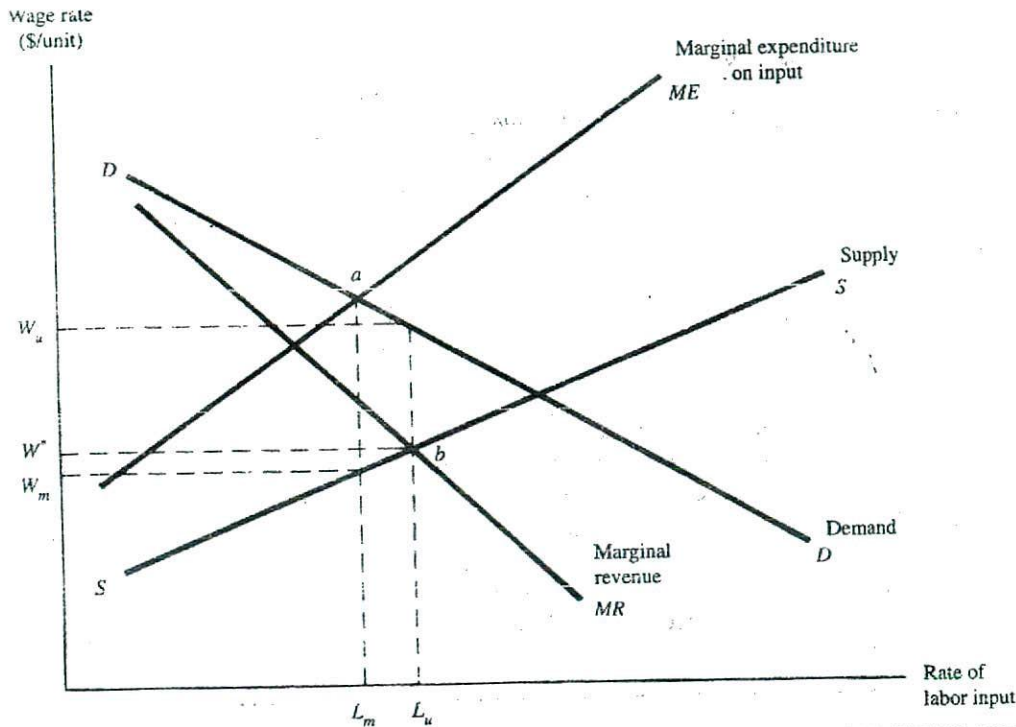


FIGURE 13.6 Wage Rate and Employment Determination for a Bilateral Monopoly

bilateral monopoly situation, where a single seller faces a single buyer in a market. The industry negotiating team acts as a monopsonist or the sole buyer of labor in the industry, and the union acts as a monopolist or the sole seller of labor in the industry.

The relevant labor demand and supply functions for such a situation are shown in Figure 13.6. Consider the union as a seller of labor. Industry demand for this labor is shown by the demand curve  $DD$ , the marginal revenue product of labor, and  $MR$  is the marginal revenue curve associated with the labor demand curve. The supply function for labor is shown by line  $SS$ , and the marginal expenditure for labor is shown by  $ME$ .

If it could act without restraint, the industry bargaining team would maximize profit by equating the labor demand curve and the marginal expenditure curve at point  $a$  and would hire  $L_m$  units of labor. At this input rate, the marginal revenue product is equal to the marginal expenditure on labor. The price or wage rate is determined by the point on the supply curve corresponding to the quantity  $L_m$ , that is,  $W_m$ . Note that this wage is less than the marginal revenue product, meaning that workers are not paid their entire marginal contribution to output.

In contrast, assume that a labor union acting as a monopoly seller of labor wants to maximize the total surplus paid to industry workers. In Figure 13.6, the area below the wage rate but above the supply curve can be thought of as a surplus paid to workers over and above the amount necessary to have them offer their labor for sale. In a sense, it is a form of economic rent. For example, workers would offer  $L_u$  units of labor for sale at a wage of  $W^*$ . That is,  $W^*$  corresponds to that point on the labor supply curve where  $L_u$  units of la-

bor are offered for sale. If they are paid a wage above  $W^*$ , they are earning a surplus. The total surplus accruing to all workers is maximized by equating the marginal revenue function and the supply curve at point  $b$  to determine the quantity hired of  $L_u$  and a wage rate of  $W_u$ . The wage rate  $W_u$  is that point on the industry labor demand curve corresponding to quantity  $L_u$ . Not surprisingly, the union approach results in a higher wage rate and more workers employed than occurs if the management objective is followed.

The exact result of the bargaining process is indeterminate. If management has much more bargaining power than the union, the wage rate may end up being close to  $W_m$ . Conversely, if the relative power lies with the labor union, the negotiations may result in a wage rate closer to  $W_u$ .

One factor determining the outcome is the negotiating skill of those at the bargaining table. However, other forces will also affect the outcome of the negotiations. For example, if the economy is in a recession and the firms have built up large inventories, management may be in a position to weather a prolonged strike. In such a period, labor's position may be weak because of limited availability of alternative jobs for workers should there be a strike. In contrast, in a period of strong demand for industry output, a strike may mean the loss of otherwise large profits. This potential for large profits may also make it difficult for management to plead poverty at the bargaining table.

### Example Wage Rate Determination—The Bilateral Monopoly Case

Teams representing the United Steelworkers of America and management teams representing all major steel producers in the United States are negotiating a new labor contract. Assume that all workers and all firms are represented by these teams. The industry demand for labor is  $w = 200 - 2L$  and the associated marginal revenue function is

$$MR = 200 - 4L$$

The supply curve for labor and marginal expenditure or labor function are

$$w = 8 + 4L \text{ and } ME = 8 + 8L$$

where  $w$  is the daily wage rate and  $L$  is the number of workers employed in thousands. Determine the wage rate and level of employment that will result from these negotiations.

**Solution** Because virtually all firms and workers are represented by the two teams, this is a bilateral monopoly problem. As such, the solution will depend on the relative bargaining power of the two groups and their negotiating skills, and a unique solution does not exist. However, two extreme solutions can be determined.

First, assume that management is able to dominate the negotiations and dictate the terms of the contract. The amount of labor hired is determined by equating the labor demand and marginal expenditure functions

$$200 - 2L = 8 + 8L$$

and solving for  $L$ :

$$L = 19.2$$

The wage rate is found by substituting  $L = 19.2$  into the labor supply function

$$w = 8 + 4(L) = 8 + 4(19.2)$$

or

$$w = 84.80$$

If the union is able to dictate terms of the negotiations, the quantity of labor hired would be found by equating the marginal revenue and labor supply functions and solving for  $L$ , that is,

$$200 - 4L = 8 + 4L$$

or

$$L = 24$$

The wage rate would be determined by substituting  $L = 24$  into the labor demand function:

$$w = 200 - 2L = 200 - 2(24)$$

or

$$w = 152$$

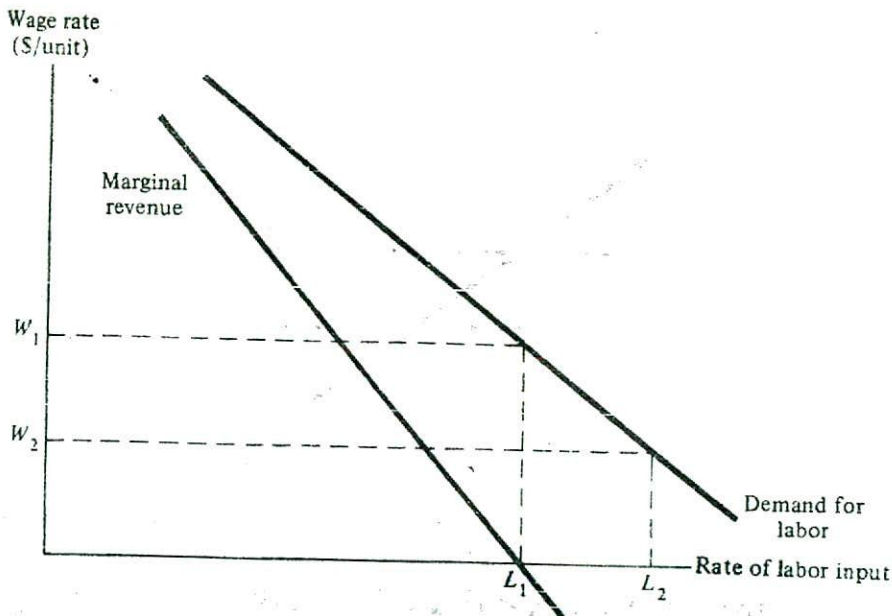
The union solution, of course, has both a higher wage rate and a greater employment than the management solution. In general, neither side would be able to dictate the terms of the agreement, and therefore the actual solution would lie somewhere between these two extremes.

### Union Objectives

In the bilateral monopoly example, it is assumed that the goal of the union is to maximize the net surplus to its members. This surplus is maximized where the marginal revenue curve intersects the supply curve. Note that this approach is analogous to the monopolist maximizing profit by equating marginal revenue and marginal cost. In a sense, the labor union is a monopolist in this labor market and the supply function serves as the marginal cost curve.

Obviously, the union may have other objectives. For example, it may want to have all union members fully employed or it may want to maximize the aggregate income of the membership. To analyze the implications of these alternative goals, consider the demand for labor function and the associated marginal revenue function shown in Figure 13.7. Suppose that there are  $L_2$  workers in the union. To keep all these workers employed requires that the firm bargain for a wage rate of  $W_2$ , the wage rate associated with  $L_2$  on the demand curve.

Conversely, if the objective is maximizing aggregate wages, the union would want to move to  $L_1$ , the point where the  $MR$  curve intersects the horizontal axis. The wage rate associated with this point is  $W_1$ , again determined by the demand curve. In this case, however, there is an unemployment problem, as  $L_2 - L_1$  members of the union will not be employed. Which members are to be unemployed? Will it be those with the least seniority or those with the fewest skills? This is a difficult problem for union leaders to resolve, but it is clear that some union leaders have bargained for and obtained wage agreements that have led to unemployment for a significant number of the union members. For example, shortly after organizing as the United Mine Workers, coal miners bargained for and won much higher wage rates, but at the cost of a significantly higher unemployment rate for mine workers.



There are still other objectives the union leaders may have, such as maintaining their positions of leadership and power in the union. Usually, objectives of union leaders are stated in vague terms such as “higher wages,” “better working conditions,” or “more job security,” and it is very difficult to identify the true objectives of the union. The final result of collective bargaining may reveal more about the true objectives than would any statement by union leaders.

### Key Concepts

- Collective bargaining can be thought of as a bilateral monopoly situation where the firm acts as a monopsonist or the sole buyer of labor and the union acts as a monopolist or the sole seller of labor.
- The outcome of collective bargaining will depend not only on the relevant demand and supply functions but also on the relative bargaining skill and strength of management and union leaders.
- Alternative labor union objectives include full employment of all members of the union, maximization of the net surplus to workers, and maximization of aggregate wages paid to members.

## MINIMUM WAGE LAWS

Managers in the United States and in many other countries work in an environment where the government has set a minimum hourly wage rate for most workers. Such laws generally require that workers be paid not less than a specified rate per hour. Although

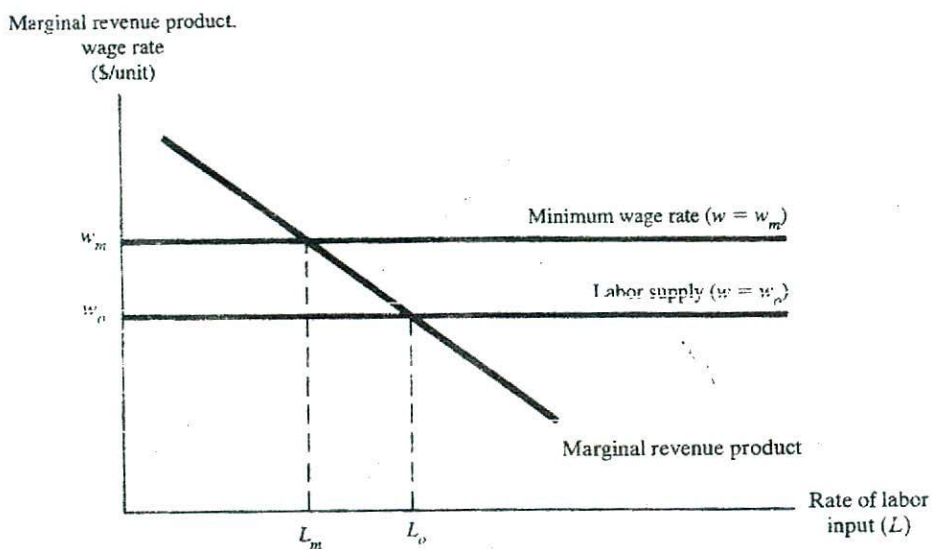


FIGURE 13.8 Managerial Response to a Minimum Wage Law

most economists argue that such laws reduce total employment, especially for young workers with limited job skills, these laws have persisted for decades. The appropriate managerial response to a minimum wage law is the focus of this section.

Consider a firm that operates in product and input markets that are both perfectly competitive. The firm's labor demand ( $MRP_L$ ) function and the supply curve for labor are shown in Figure 13.8. Initially, assume that the wage rate ( $w_0$ ) has been set in the market, and the firm's only decision is the amount of labor to hire. This amount is determined by the intersection of the demand function with the horizontal supply curve and results in a labor input rate of  $L_0$  units per period.

— If a government-mandated minimum wage rate is imposed at any wage rate less than  $w_0$ , it has no effect on the amount of labor employed or the market wage rate. However, if a minimum wage rate higher than  $w_0$ , say  $w_m$ , is legislated, that rate effectively becomes the supply function of labor facing the firm. The firm uses the same profit-maximizing rule (i.e., employ labor until  $MRP_L = w_m$ ), but now employs only  $L_m$  units. Employment by the firm has been reduced by  $L_0 - L_m$  units per period. While those workers who have kept their jobs are better off because their wage rate has increased, other workers are worse off because they are no longer employed.

Labor unions have generally supported increases in the minimum wage. This is somewhat curious because the vast majority of union workers earn substantially more than the minimum wage. One explanation is that skilled workers are a substitute for unskilled workers. Therefore, the cross-price elasticity between the quantity of skilled union labor employed and the wage rate for unskilled labor is positive. If the price of unskilled work increases because of a higher minimum wage, the demand for skilled workers should increase, resulting in higher wages for them. Thus one possible result of a higher minimum wage is more unemployed low-wage workers and more employed skilled workers who will be paid a higher wage than before.

**Key Concepts**

- Minimum wage laws generally require that firms pay workers at least a specified wage rate per hour.
- For firms that are perfect competitors in the labor market, a minimum wage rate that is set above the market wage rate will reduce the amount of labor the firm will employ.

**SUMMARY**

Input prices are determined by the interaction of supply and demand. The demand for inputs is derived from the demand for the output that those inputs are used to produce. The marginal revenue product function, computed by multiplying marginal revenue and marginal product, is the firm's input demand function.

Determining input prices is important because these prices are used in deciding on the optimal mix of labor, capital, and natural resources to be employed by the firm. The approach to input price determination depends on the structure of both the input and output markets. A perfect competitor in an input market faces a constant input price. Thus, the only decision made by the firm is the quantity of the input to be employed. A monopsonist has market power in the input market, and thus faces an upward-sloping input supply function. The input price and quantity employed by the monopsonist are determined by the intersection of the input demand function, with the firm's marginal expenditure on input function.

There is a unique correspondence between output and input decisions. A decision to produce a given rate of output implies a decision to hire certain rates of capital and labor inputs and vice versa. The condition for a profit-maximizing output rate (i.e., marginal cost equals marginal revenue) is equivalent to the condition for a profit-maximizing input rate (i.e., marginal revenue product equals input price).

Economic rent is a return to a factor of production in excess of the minimum payment necessary to keep that factor in its present employment. Wage differentials among workers reflect both demand-side forces, such as differences in productivity, and supply-side factors, such as training requirements, health risks, and working conditions.

Collective bargaining refers to the process of determining wage rates and other working conditions through negotiation between labor union representatives and management from one or several firms. In some cases, collective bargaining can be thought of as a bilateral monopoly situation where the firm acts as a monopsonist, the sole buyer of labor, and the union acts as a monopolist, the sole seller of labor. The outcome of collective bargaining will depend not only on the relevant demand-and-supply functions but also on the relative bargaining skill and strength of management and union leaders. Objectives of labor unions include full employment of all members of the union, maximization of aggregate wages paid to members, and maximization of the net surplus to workers.

Minimum wage laws require that firms pay labor at least a specified wage per hour. If this wage is above that prevailing in a perfectly competitive labor market, the firm will reduce the amount of labor hired, and unemployment will result.



### Discussion Questions

- 13-1. What determines the mix of labor and capital used by the firm to produce output? Explain.
- 13-2. Explain why the term *derived demand* is used to describe the demand for factors of production.
- 13-3. Provide an intuitive explanation for the profit-maximizing rule that the firm should hire an input until the marginal revenue product of the input is equal to the additional cost of the input.
- 13-4. Why is the input supply function horizontal for a perfect competitor in the input market and upward sloping for a monopsonist?
- 13-5. Why is there no unique input price and quantity of input hired in the bilateral monopoly case? What factors will play a role in determining that price and quantity?
- 13-6. Given that a firm always should be producing efficiently, explain why it is impossible for management to separate the decisions about the rate of output to be produced and the rates of capital and labor to be employed?
- 13-7. The annual salary of many players in the National Basketball Association is more than \$1,000,000. Is it "right" for these athletes to make many times more than the average American worker? If that average salary were reduced by one-half, how many players do you think would leave the NBA for other occupations? Explain.
- 13-8. For many years the United States and other countries used a draft system to maintain required personnel levels in their military forces. During most of the period when the draft existed, earnings in military service, including such fringe benefits as housing and meals, were substantially less than in comparable civilian occupations. Supporters of the draft system argued that military service was sufficiently dangerous that regardless of the wage rate, there would not be adequate volunteers. Do you agree or disagree with this position? Explain.
- 13-9. The leader of a major labor union claims that the union's objective is full employment for all members. Despite an unemployment rate of 15 percent for workers in this union, labor negotiators are demanding a 6 percent increase in wage rates for each of the next three years. Workers in other industries have been receiving wage increases of 3 or 4 percent per year. Are the actions of the union negotiators consistent with the stated objective of the union? Explain.

### Problems

- 13-1. Suppose that the demand and supply functions for unskilled labor in an economy are

$$\text{DEMAND: } L_d = 10 - 0.5w$$

$$\text{SUPPLY: } L_s = 6 + 0.9w$$

where  $L$  is millions of workers and  $w$  is the hourly wage rate.

- a. Determine the equilibrium wage rate and number of workers employed.
- b. If the U.S. Congress sets a minimum wage of \$5.15 per hour, what will be the wage rate and the quantity of labor employed? Compared to your answer for part (a), how many workers will lose their jobs?
- c. If the minimum wage rate is set at \$6.00, what will be the wage rate, the number of workers employed, and the number unemployed? Compare these answers to those in part (a).

- 13-2. Suppose the demand for centers on NBA basketball teams is given by the equation

$$L_d = 400,000 - 2w$$

while the supply of centers is given by the function

$$L_s = \begin{cases} 32 & \text{if } w \geq \$40,000 \\ 0 & \text{if } w < \$40,000 \end{cases}$$

where  $L$  is number of players and  $w$  is the annual salary.

- Graph the demand and supply functions.
- Determine the equilibrium wage rate for a center in the NBA.
- Suppose a sudden drop in attendance and number of television viewers shifted the demand function to

$$L_d^* = 300,000 - 3w$$

How would the annual salary change? Would the number of workers playing professional basketball be any different than before the demand curve shifted?

- Based on the supply function, what is your best estimate of the opportunity cost of playing center in the NBA?
- 13-3 The labor demand and supply functions for unskilled labor are

$$\text{Demand: } L = 20 - 0.6w$$

$$\text{Supply: } L = 4 + 1.4w$$

where  $L$  is millions of workers and  $w$  is the hourly wage rate.

- In the absence of any minimum wage legislation, what is the equilibrium wage rate and level of employment?
  - If a government-mandated minimum wage rate is set at \$7.00 per hour, how much labor will be employed? What wage rate will workers be paid?
  - If a government-mandated minimum wage rate is set at \$10.00 per hour how much labor will be employed? What wage rate will workers be paid?
- 13-4. Consider a firm in a perfectly competitive output market where the price of the product is \$10 per unit. Given the following information on the total product of labor function, determine the firm's demand schedule for labor (i.e.,  $MRP_L$ ) by completing the table.

Quantity of Labor	Total Product	Marginal Product	Marginal Revenue Product
0	0	—	—
1	21	_____	_____
2	30	_____	_____
3	38	_____	_____
4	42	_____	_____
5	43	_____	_____
6	40	_____	_____

Suppose this firm is a monopsonist that faces the following labor supply schedule. Use these data to determine the firm's total and marginal expenditure on labor schedules.

Wage Rate	Quantity of Labor Supplied	Total Expenditure on Labor	Marginal Expenditure on Labor
20	0	—	—
22	1	_____	_____
25	2	_____	_____
30	3	_____	_____
35	4	_____	_____
40	5	_____	_____
50	6	_____	_____

Use these data to determine how much labor the firm should employ. What will the wage rate be? Explain.

- 13-5. Demonstrate that, for a monopolist, the profit-maximizing condition that marginal revenue equals marginal cost (i.e.,  $MR = MC$ ) implies that the marginal revenue product of capital must equal the price of capital (i.e.,  $MRP_K = r$ ).
- 13-6. A team representing management of all firms in the automatic widget industry is currently negotiating a new three-year contract with the leaders of the United Widget Workers of America (UWW) labor union. The industry demand function for labor (i.e., the marginal revenue product of labor) is

$$MRP_L = 20 - 2L$$

the marginal revenue function associated with the demand curve is

$$MR = 20 - 4L$$

and the labor supply and marginal expenditure on input functions facing the industry are

$$w = 5 + 2L$$

$$ME = 5 + 4L$$

where  $L$  is the number of workers in thousands and  $w$  is the hourly wage rate.

- If the management team can dominate the negotiations and dictate the terms of the agreement, what wage rate and level of employment will be determined?
  - If the labor union team can dominate the negotiations and dictate the terms of the agreement, what wage rate and level of employment will be determined?
- 13-7. The supply of apartments in a city is fixed at 2,000 units; no new units can be built because of shortage of water. The demand for these units is given by  $Q_d = 5,000 - 5P$  where  $P$  is the monthly price of a unit and  $Q$  is number of units. The reservation price for each unit is \$300 per month.
- Determine the equilibrium monthly price of an apartment.
  - What is the economic rent per unit and the total economic rent?
  - A government rent-control ordinance is passed, limiting the monthly price to \$400 per month. Determine the number of units demanded at this price. How might apartment owners respond to a situation when the quantity demanded exceeds the quantity supplied, but when rent cannot be increased?
- 13-8. Smith owns the only water-producing well in the county. The well produces 100 units of water per period at zero cost. Demand for water in the county is given

- by  $Q = 200 - 2P$  where  $P$  is the price per unit and  $Q$  is the number of units demanded. Smith's reservation price is \$25 per unit.
- What is the equilibrium price of water?
  - What is the economic rent per unit and the total economic rent?
  - The county commission passes a law limiting the price per unit of water to \$40. What is the economic rent per unit and the total economic rent now?
  - The county commission passes a law limiting the price per unit of water to \$20. How much water will be sold?
- 13-9. Assume that all jobs are identical except for the risk of being killed at work. If a job with zero risk of death pays \$20,000 per year, determine the annual wage in each of the jobs listed here. (Use the information provided on page 451 regarding the additional amount necessary to compensate for the risk of being killed.)

<i>Occupation</i>	<i>Annual Death Rate Per Thousand Workers</i>
Police officer	0.78
Crane operator	1.47
Mine worker	1.76
Guard	2.67

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### Problems Requiring Calculus

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- 13-10. The market demand-and-supply functions for Rhubarb Field Dolls are

$$Q_D = 2,000 - 250P$$

and

$$Q_S = 600 + 100P$$

where  $Q_D$  is quantity demanded,  $Q_S$  is quantity supplied, and  $P$  is the price.

Adamco Inc. manufactures these dolls using the following production function:

$$Q = K^{0.5}L^{0.5}$$

where  $Q$  is the rate of output,  $K$  is capital, and  $L$  is labor. Adamco is the largest employer in a small town and faces the following labor supply function:

$$w = 0.5L_S$$

where  $w$  is the wage rate.

If the capital input is fixed at 225, determine the profit-maximizing labor input, rate of output, and wage rate.

# Integrating Case Study V

## CASE V

### Northern Lumber Products, Inc.

#### PART I

Northern Lumber operates a large lumber-processing mill in a small town in Washington state. It is one of the larger lumber producers in the region and has some market power in the sale of that product. A recent consulting study has indicated that the price elasticity of demand for the firm's product is about  $-3.0$ . Also, Northern is the dominant employer in the local labor market, and effectively can be considered as a monopsonist in the purchase of labor. The firm's labor demand (i.e., marginal revenue product) function is:

$$MRP = 1,000 - 2L$$

where  $L$  is the number of workers. Because of its size relative to the labor supply in the area, Northern faces an upward sloping labor supply function,

$$w = 50 + 0.025L$$

where  $w$  is the daily wage rate.

Once the firm determines the optimal rate of labor input and the wage rate, the rate of output is determined. The firm uses a cost-plus pricing formula that includes the price elasticity of demand as a determinant in setting product price. The same study indicated that average cost is about \$300 per unit (1,000 board feet) of lumber.

#### Requirements

1. Determine the amount of labor that the firm should employ in order to maximize profit.
2. Determine the wage rate the firm will have to pay.
3. What price will the firm charge per unit of output?

#### PART II

Another lumber producer may locate a plant in the same area. If it does, there will be more competition for labor and the labor supply function facing Northern will shift to

$$w = 50 + 0.04L$$

Because of increased competition in the market for lumber, Northern's demand function will become more elastic with the price elasticity of demand decreasing to  $-4.0$ .

**Requirements**

1. Determine the amount of labor that the firm should employ in order to maximize profit under this new labor market condition.
2. Determine the new wage rate that the firm will have to pay.

PART  
**VI**  
Risk and Capital Budgeting

CHAPTER 14  
*Risk and Decision Making*

CHAPTER 15  
*Capital Budgeting*

INTEGRATED CASE  
*Study VI: Bentley Enterprises, Inc.*

# CHAPTER

# 14

## Risk and Decision Making

- **Preview**
- **The Concept of Risk**
- **Risk and Decision Making**
  - Risk-Return Evaluation Statistics
  - Risk Preference
- **Risk Management**
  - Insurance
  - Gambling and Insuring: A Contradiction?
  - Adjusting the Discount Rate
  - Diversification
  - Hedging
- **Decision Tree Analysis**
- **Summary**
- **Discussion Questions**
- **Problems**



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**PREVIEW**

Before introducing new products, managers undertake consumer surveys, assess product lines of their competitors, and closely check estimated production costs. These actions are taken to increase the information base on which the decision will be made. Still, there are few sure things in the world of management. Many spectacular failures have occurred despite management having taken all possible steps to ensure success. It is simply not possible to predict consumer behavior or changes in production technology with complete accuracy. Every decision carries with it the prospect that something will go wrong and that instead of earning large profits and increasing shareholder value as expected, losses and decline in shareholder value will be incurred. Despite all of the modern techniques, learning to make decisions where there are risk and uncertainty is the essence of modern management.

In marketing goods and services, firms often try to reduce the risk faced by potential buyers. For example, the Ford Motor Company offers a three-year or 36,000-mile warranty on most important parts of its cars and trucks. The Mauna Kea Beach Hotel in Hawaii offers its guests one free night of lodging if it rains for 30 minutes on any day that the guest is at the hotel. These actions reduce the risk to the customer, but they increase risk for the firm.

Managerial decisions are made in different risk environments. In the case of decision making under certainty, all relevant dimensions of the decision are known. For example, in chapter 8 a linear programming problem was formulated in terms of deciding how much of each of two products should be produced given a specified profit rate for each and specified processing time required for each product on each of these machines. All of the relevant information was assumed to be known with certainty. The result was a unique solution that maximized the firm's profit. That is, there was only one relevant outcome of this decision.

In decision making under uncertainty, there are several, perhaps many, outcomes of a decision, but the probability of each of those outcomes occurring is unknown. For example, a firm may be considering the production of an altogether new product. Because consumers have had no experience with the product, there is no way to estimate the potential demand for it. The marketing department may undertake various market surveys in an attempt to estimate how many people might buy the product, but it is doubtful that much confidence can be placed in such efforts. There may be no way to provide anything but some rough guesses about demand.

Finally, decision making under risk applies when all significant outcomes of a decision are known as is the probability of each outcome occurring. For example, consider the drilling of a well in an already proven oil field. Suppose that 100 wells have been completed, of which 10 are producing 2,000 to 3,000 barrels per day (BPD), 40 are producing 5,000 to 10,000 BPD, and the remaining 50 are producing 10,000 to 15,000 BPD. This kind of historical experience allows the decision maker some basis for assessing the probability of success of the new well.

The focus of this chapter is on decision making under risk. The objective will be to develop guidelines for making rational decisions given the decision maker's attitude about (i.e., preference for) risk. First, the concept of risk is formally defined, and then the principles of probability developed in chapter 2 are used to quantitatively measure risk. Next, the role of risk in making decisions is analyzed; basic to this is an under-

standing of the concept of the manager's preference for risk. Then methods for managing risk are developed and applied. Finally, the use of decision trees is explained.

## THE CONCEPT OF RISK

The analysis of risk is based largely on the concepts of probability and the probability distributions that were developed in chapter 2. First, terms such as *strategy*, *state of nature*, and *outcome* need to be defined. A strategy refers to one of several alternative plans or courses of action that could be implemented in order to achieve a managerial goal. For example, a manager might be considering three strategies designed to increase profits: (1) build a new, more efficient plant that will allow production at lower cost; (2) develop a new marketing program designed to increase sales volume; or (3) redesign the product in order to allow lower-cost production and increased sales due to greater consumer acceptance. A state of nature refers to some condition that may exist in the future that will have a significant effect on the success of any strategy. In making the decision about building a new plant, an important state of nature is the economic climate expected to prevail for the next few years. In this context, the possible states of nature might be (1) recession, (2) normal business conditions, or (3) economic boom. An outcome specifies the gain or loss (usually measured in dollars) associated with a particular combination of strategy and state of nature. For instance, the outcomes associated with the decision to build a new plant might be the present value of all future net profits. Finally, a payoff matrix shows the outcome associated with each combination of strategy and state of nature. An example of a payoff matrix is shown in Table 14.1. For example, if a new plant is built and a recession occurs, the loss is estimated to be \$40 million. Conversely, the combination of a new marketing program and normal business conditions would yield an estimated profit of \$35 million. If the probabilities of each state of nature are known, then the combination of those probabilities and the payoffs for each strategy constitute a probability distribution.

Risk refers to the amount of variability among the outcomes associated with a particular strategy. Where there is only one probable outcome of a decision, there is said to be little risk; where there are many possible outcomes with substantially different dollar returns, there is said to be substantial risk. For example, a manager with \$1 million to invest may be faced with two alternatives. She could buy a one-year Treasury bill yielding 6 percent interest. At the end of a year, the \$1 million investment will return \$1,060,000. The only risk associated with this investment is that the federal government might be unable to pay its debts. This is so unlikely that there is essentially no risk associated with this investment (i.e., there is only one outcome). Indeed, Treasury bills are often referred to as a riskless or risk-free investment.

Strategy	State of Nature (economic conditions)		
	Recession	Normal	Boom
New plant	\$-40	\$25	\$40
New marketing program	-20	35	70
New product design	-15	30	60

The second alternative is the drilling of an oil well in an unproven field. If oil is found, the well will immediately be worth \$50 million, the present value of all net profits from selling this oil. If oil is not found, the well will be worth nothing. This is a risky investment because there is great variability in the range of possible outcomes. In general, the greater the variation in outcomes, the greater is the risk. Thus, the definition of risk is based on the variability in the outcomes of a particular decision.

### Key Concepts

- Risk exists when there is a range of possible outcomes associated with a decision and the probabilities of those outcomes occurring are known. If those probabilities are not known, the decision maker is said to face uncertainty.
- A strategy is a plan or course of action designed to achieve a management goal.
- A state of nature refers to a condition that may exist in the future that will have a significant effect on the success of a strategy.
- An outcome is the gain or loss associated with a particular combination of strategy and state of nature.
- A payoff matrix lists the outcomes associated with each strategy-state of nature combination.

## RISK AND DECISION MAKING

Having defined risk and reviewed some of the related terminology, the task now is to develop quantitative measures of return and risk and to show how they are applied in decision making. Because individuals have different preferences concerning risk taking, it also is important that such preferences be identified and their effect on decisions evaluated. Obviously, individuals respond to the same risky choices in different ways. For example, a few people are willing to risk death or serious injury in their recreational pursuits (e.g., hang gliding), while others prefer a game of Monopoly. One reason for these preferences is their attitude about risk. Rational decision making requires that the expected return be determined and the risk be measured, and that there be information about the manager's preference for risk.

### Risk-Return Evaluation Statistics

Recall from chapter 2 that three statistics were developed to describe a probability distribution. Given a set of outcomes,  $X_i$ , and the probability of each occurring,  $P_i$ , these statistics are

Expected value or mean:

$$\mu = \sum_{i=1}^n P_i(X_i) \quad (14-1)$$

Standard deviation:

$$\sigma = \sqrt{\sum_{i=1}^n P_i(X_i - \mu)^2} \quad (14-2)$$

Coefficient of variation:

$$v = \frac{\sigma}{\mu} \quad (14-3)$$

These statistics have a direct application in measuring the expected return and risk associated with any business decision for which a set of outcomes and their probabilities have been determined. The expected value, the standard deviation, and the coefficient of variation will be referred to as risk - return evaluation statistics.

Suppose that two investments, I and II, are being considered. Both investments require an initial cash outlay of \$100 and have a life of five years. The dollar return on each depends on the rate of inflation over the five-year period. Of course, the inflation rate is not known with certainty, but suppose that the collective judgment of economists is that the probability of no inflation is 0.20, the probability of moderate inflation is 0.50, and the probability of rapid inflation is 0.30. The outcomes are defined as the present value of net profits for the next five years. These outcomes for each state of nature (i.e., rate of inflation) for each investment are shown in Table 14.2.

Analysis of these investments can be made by calculating and comparing the three evaluation statistics for each alternative. The expected value ( $\mu$ ) is an estimate of the expected dollar return for the investment. Because risk has been defined in terms of the variability in outcomes, the standard deviation ( $\sigma$ ) is a measure of risk associated with the investment. The larger  $\sigma$  is, the greater the risk. Risk per dollar of expected return is measured by the coefficient of variation ( $v$ ).

The evaluation statistics for each investment alternative are computed as follows:

#### INVESTMENT I

$$\mu_1 = \sum_{i=1}^n P_i X_i = 0.2(100) + 0.5(200) + 0.3(400) = 240$$

$$\sigma_1 = \sqrt{\sum_{i=1}^n P_i (X_i - \mu)^2} = \sqrt{0.2(100 - 240)^2 + 0.5(200 - 240)^2 + 0.3(400 - 240)^2} = 111.36$$

$$v_1 = \frac{\sigma_1}{\mu_1} = \frac{111.36}{240} = 0.46$$

State of Nature	Probability ( $P_i$ )	Outcome ( $X_i$ )
Investment I		
No inflation	0.20	100
Moderate inflation	0.50	200
Rapid inflation	0.30	400
Investment II		
No inflation	0.20	150
Moderate inflation	0.50	200
Rapid inflation	0.30	250

## INVESTMENT II

$$\mu_{II} = 0.2(150) + 0.5(200) + 0.3(250) = 205$$

$$\sigma_{II} = \sqrt{0.2(150 - 205)^2 + 0.5(200 - 205)^2 + 0.3(250 - 205)^2} = 35.00$$

$$v_{II} = \frac{35.00}{205} = 0.17$$

The expected return for investment I of \$240 is higher than for II (\$205), but I is a riskier investment because  $\sigma_I = 111.36$  is greater than  $\sigma_{II} = 35$ . Also, risk per dollar of expected returns for I ( $v_I = 0.46$ ) is higher than for II ( $v_{II} = 0.17$ ). Which is the better investment? The choice is not clear. It depends on the investor's attitude about taking risks. A young entrepreneur may well prefer I, whereas an older worker investing a few dollars in a retirement account where risk ought to be minimized might prefer II. The entrepreneur is in a better position to absorb a loss if it occurs and is probably accustomed to investing in risky ventures.

**Key Concept**

- Given a probability distribution for the outcomes of a business decision, the statistics of that distribution can be used to evaluate return and risk:
  - The expected value or mean is a measure of expected return.
  - The standard deviation is a measure of risk.
  - The coefficient of variation is a measure of risk per dollar of return.

**Risk Preference**

Generally, higher returns are associated with higher risk. Indeed, the essence of economics is making choices where there is a trade-off. Virtually all investment choices require giving up expected returns for higher risk or taking more risk in the expectation of a greater expected return. Thus, any decision will reflect the manager's attitude or preference for risk, and these preferences differ substantially among individuals. For example, some investors are so averse to taking risk that they keep all their assets in bank deposits insured by the federal government. Others are willing to risk all they own and can borrow to finance extremely risky ventures such as drilling oil wells or buying contracts in the fast-moving commodities market. There must be a reward for taking risk, and this comes in the form of higher returns. The following example will help to explain the concepts of risk and risk preference.

Consider the following gamble: "I will toss a coin 10 times. If a head appears each time, you must pay me \$1,000. If tails appear one or more times, I will pay you a dollar." Most people would reject this offer even though it is a fair game. That is, if the game were played many times, the amount lost would equal the amount won. However, the nature of the game is such that on any one trial the player could lose a large amount and stand to win only a small amount. The probability distribution for the return or payoff for each outcome of the game is shown in Table 14.3. The probability of a head appearing on 10 consecutive tosses of a coin is given by  $(\frac{1}{2})^{10}$  or 0.001. The probability that there is at least one tail in 10 tosses is 1 minus the probability that there would be 10 heads or  $(1.0 - 0.001) = 0.999$ . These probabilities multiplied by their associated payoffs yield an expected value of zero; therefore, it is a fair game.

**TABLE 14.3** Probability Distribution and Expected Value  
for Coin-Tossing Example

Outcome	Probability	Payoff	Probability × Payoff
10 heads	0.001	\$-1,000	\$-1.00
At least one tail	0.999	+ 1	+1.00
		Expected payoff =	0

To understand why most people would reject this gamble, one must understand the concept of preference for risk. It is assumed that individuals can associate satisfaction with money. More money usually means more satisfaction, although for many people each additional dollar brings less satisfaction than the previous dollar did. Economists use the term *utility* as a measure of satisfaction. Although an individual's satisfaction is virtually impossible to measure, it will be assumed that this can be done for a hypothetical individual. This assumption will make it easier to develop several important concepts. One relationship between utility and money is shown in Figure 14.1a. The curve shows that relationship just suggested; that is, utility increases as money increases but at a continually decreasing rate. Thus, the function is concave to the horizontal axis. A person having a concave utility function is said to be *risk averse*.

Suppose that this person has \$10,000 and is offered the opportunity to bet \$5,000 on the toss of a coin. If a head is tossed, she wins \$5,000; if a tail is tossed, she loses \$5,000. She now faces the choice of having \$10,000 with certainty if she chooses not to bet (this option is called the *certain prospect*) or of having \$15,000 with probability 0.5 or \$5,000 with probability 0.5. The latter option is called the *uncertain prospect* or the gamble. The expected monetary value of the uncertain prospect is \$10,000, that is,

$$\mu = 0.5(\$5,000) + 0.5(\$15,000) = \$10,000$$

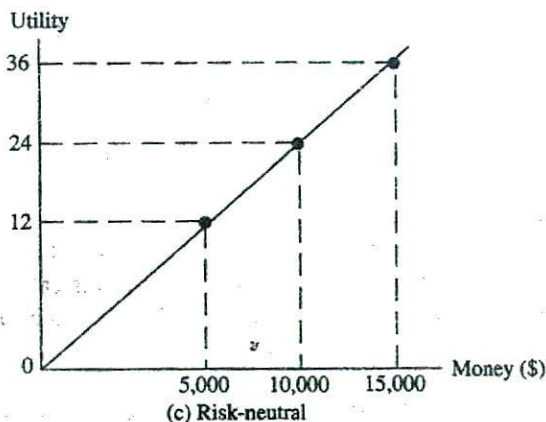
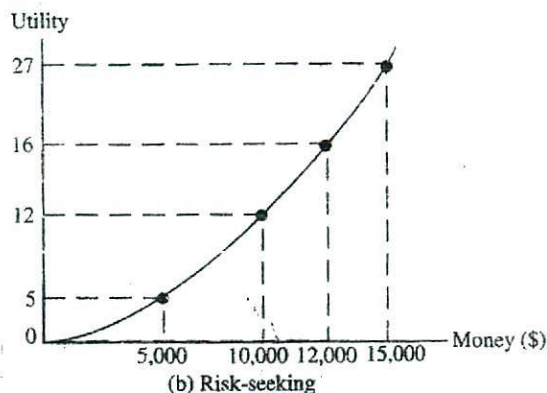
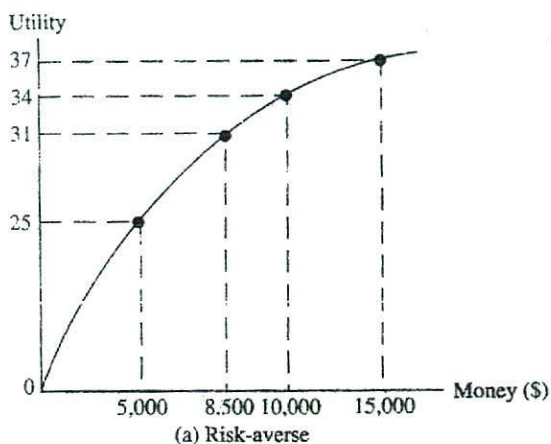
which is the same as the dollar value of the certain prospect.

Now, consider the analysis in terms of the utility associated with each dollar payoff. For a risk-averse person, the dollar payoffs are translated into utility by using a function like that shown in Figure 14.1a. The certain prospect (i.e., having \$10,000) is associated with a utility of about 34. The expected value in utility of the uncertain prospect is determined as follows. A \$5,000 payoff corresponds to a utility value of 25, and a \$15,000 outcome is associated with a utility of 37. Thus, by substituting these utility values for the payoffs, the expected value in terms of utility is computed as

$$\mu = 0.5(25) + 0.5(37) = 31$$

This is less than the utility (34) associated with certain prospect. Thus, this utility-maximizing person prefers the certain prospect and chooses not to accept the wager.

Note that an amount of \$8,500 with certainty would also yield a utility level of 31. This amount is said to be the *certainty equivalent* of the gamble. The individual would pay up to \$1,500 (i.e., the difference between the expected value of the gamble and the certainty equivalent) in order to avoid the gamble. This suggests a formal definition of risk aversion. If the certainty equivalent of the gamble is less than the expected dollar value of the gamble the person is said to be risk averse.



Next, consider a risk-seeking person whose utility function has a continually increasing slope as shown in Figure 14.1b. Here, each additional dollar increases utility more than did the previous dollar. The utility level associated with the certain prospect of \$10,000 is 12. The expected utility for the uncertain prospect is 16; that is,

$$\mu = 0.5(5) + 0.5(27) = 16$$

where the utility levels associated with \$5,000 and \$10,000 are 5 and 27, respectively. This expected value of utility for the uncertain prospect is greater than the utility (12) associated with the certain prospect. Thus, this person would prefer the uncertain

prospect even though the expected dollar returns are the same. The certainty equivalent of the gamble is \$12,000, determined by the amount of money associated with a utility level of 16 on the utility function. In this case, the individual would pay up to \$2,000 to take the gamble.

In general, if the certainty equivalent of the gamble is greater than the expected dollar value of the gamble, the individual is said to be *risk seeking*.

Finally, in the risk-neutral case depicted in Figure 14.1c, the expected value of the utility associated with an uncertain prospect is equal to that of a certain prospect where both have equal expected dollar values. The expected utility associated with the uncertain prospect is 24; that is,  $0.5(12) + 0.5(36) = 24$ . The utility associated with the certain prospect of \$10,000 also is 24. Thus, the certainty equivalent of the gamble (\$10,000) equals the expected value of the gamble; this person is said to be *risk neutral*.

The concept of the preference for risk can be formalized in the following way. At any time, an individual has a given level of wealth,  $W$ , which is known with certainty. The utility of that wealth is also known with certainty if the utility function is specified. Assume the initial level of wealth is \$100, and that the utility function for this person is given by the natural logarithm of wealth, i.e.,

$$U = \ln W$$

This is simply a mathematical function used to describe this individual's transformation of wealth into utility or satisfaction. Note that the utility of the initial wealth of \$100 is 4.61 (i.e.,  $\ln(100) = 4.61$ ). Now assume the individual is faced with a gamble that has the following probability distribution:

<u>Probability (<math>P_i</math>)</u>	<u>Payoff (<math>X_i</math>)</u>
0.5	-80
0.5	+80

This is a fair game because the expected outcome is zero.

After the game is played, the individual's wealth is either \$20 or \$180, depending on which outcome occurs. That is, either \$80 is won or \$80 is lost from the initial wealth of \$100. The expected wealth if the game is played is \$100, i.e.,

$$E(W) = 0.5(20) + 0.5(180) = 100$$

which has a utility of 4.61, that is, the *utility of expected wealth* is

$$U[E(W)] = \ln[E(W)] = \ln(100) = 4.61$$

The *expected utility of wealth* is determined as

$$\begin{aligned} E[U(W)] &= 0.5[U(20)] + 0.5[U(180)] \\ &= 0.5[\ln(20)] + 0.5[\ln(180)] \\ &= 4.10 \end{aligned}$$

Given these probabilities and the outcomes, the expected wealth is \$100. The utility of expected wealth is simply the utility generated if this person had \$100. In contrast, after the game is played the person actually will have either \$20 or \$180 in wealth. Each of these amounts of wealth has a level of utility (i.e.,  $\ln(20) = 3.00$  and  $\ln(180) = 5.19$ ).



By multiplying each of these utility levels by the appropriate probability, the expected utility of wealth (4.10) is determined.

Now, the preference for risk can be formally defined. An individual is said to be risk averse if, when faced with a gamble, the utility of expected wealth is greater than the expected utility of wealth, that is if

$$U[E(W)] > E[U(W)] \Rightarrow \text{Risk Aversion}$$

Clearly, the individual in the preceding example is risk averse by this definition.

An individual is said to be risk seeking if the utility of expected wealth is less than the expected utility of wealth, that is

$$U[E(W)] < E[U(W)] \Rightarrow \text{Risk Seeking}$$

Finally, if the utility of expected wealth is equal to the expected utility of wealth, the individual is said to be risk neutral, that is,

$$U[E(W)] = E[U(W)] \Rightarrow \text{Risk Neutrality}$$

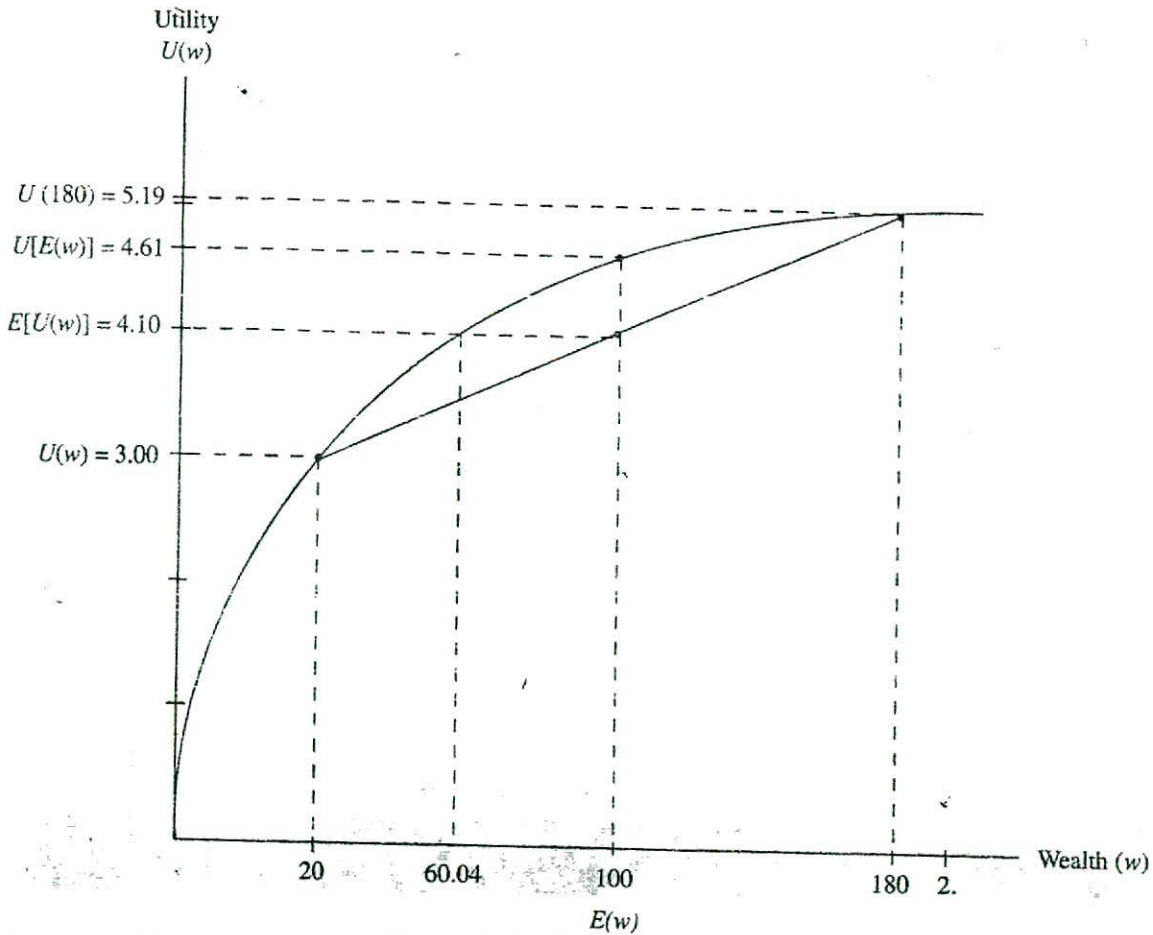
These principles can be demonstrated with reference to the utility function  $U = \ln W$  shown in Figure 14.2. Note that the utility is shown at the initial wealth level (\$100) and at \$20 and \$180, the levels of wealth after the game is played. The notion that  $U[E(W)] > E[U(W)]$  is another way of saying that this person would prefer wealth of \$100 with certainty to a gamble that had an expected wealth of \$100. The risk associated with the gamble has reduced utility.

Notice that the expected utility of wealth of 4.10 could also be achieved by having a certain wealth level of \$60.04, the certainty equivalent of the gamble. This risk-averse individual would be willing to pay up to \$39.96 (i.e., the difference between expected wealth given the gamble and the certainty equivalent of the gamble) to avoid taking the gamble. This difference is called the risk premium. The fact that risk-averse persons are willing to pay up to the level of the risk premium to avoid gambles (such as the risk of loss from automobile accidents, home fires, etc.) creates a market for insurance. As we will see, because some people are risk seekers in some circumstances (i.e., they have negative risk premiums), a market for gambling also exists. That is, firms will respond to risk-averse persons' willingness to pay positive premiums to avoid risk by providing insurance on automobiles, homes, and lives. Other firms respond to the willingness of risk-seeking individuals to pay negative premiums for taking risk by providing various gambling games.

These gambles can be either explicit or implicit. An explicit gamble is one where the individual seeks out a risk decision such as a bet on a horse race or the purchase of common stock. An implicit gamble occurs when the risk is incidental to the primary activity. For example, a home owner faces the risk that her house might burn down. This gamble is not sought out but rather is implicit in the decision to purchase the home. While the risks arise in different ways, they are dealt with in exactly the same manner by the rational decision maker.

Consider another example, involving a person who has initial wealth of \$200 and a utility function given by

$$U = W + \frac{W^3}{10,000}$$

FIGURE 14-1 Utility-Wealth Function for  $U = \ln W$ 

Assume that this person faces a gamble where he will win \$100 with probability 0.5 or lose \$100 with probability 0.5. The probability distribution is shown here:

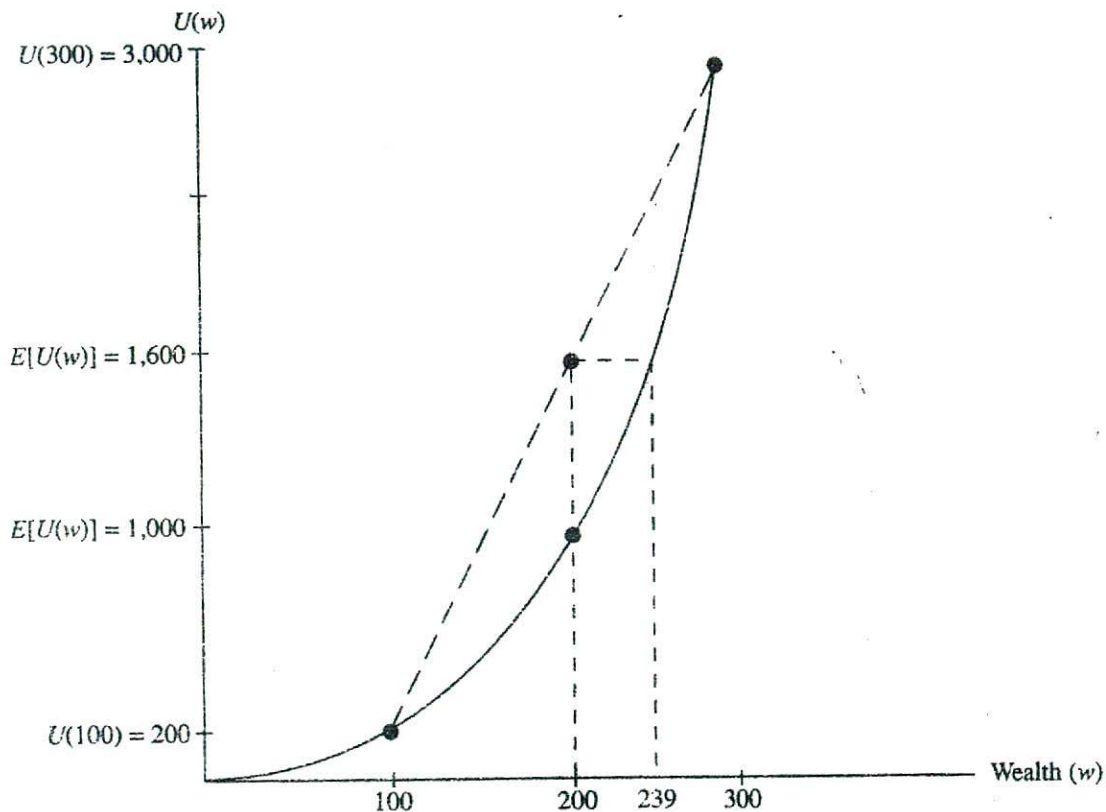
Probability	Outcome ( $X_i$ )	Wealth ( $W_i$ )	$U(W_i)$
0.5	+100	300	3,000
0.5	-100	100	200

Note that this is a fair game because the expected value (in dollars) is zero. Thus, the expected wealth is \$200, the original amount. The utility of expected wealth is 1,000, that is,

$$U[E(W)] = U(200) = 200 + \frac{(200)^2}{10,000} = 1,000$$

But, the expected utility of wealth is

$$E[U(W)] = 0.5[U(300)] + 0.5[U(100)]$$



$$= 0.5(3,000) + 0.5(200) = 1,600$$

Because the expected utility of wealth (1,600) is greater than the utility of expected wealth (1,000), this person is clearly a risk seeker and can be expected to take the gamble.

This example is depicted graphically in Figure 14.3. Note that expected utility of wealth of 1,600 also could be achieved by a certain wealth level of \$239. Thus, the certainty equivalent of the gamble is \$239. Therefore, the risk premium is negative (i.e., the expected wealth, given the gamble of \$200, is less than the certainty equivalent wealth of \$239). This means that this risk-seeking person would pay up to \$39 to take the gamble.

There are many people who are risk seekers, especially when small amounts of money are involved; they are willing to pay a premium for a gambling opportunity. The success of casinos in Las Vegas, Atlantic City, and other places is clear evidence that a market for gambling exists.

## Case Study

### The State Lottery

In recent years, a number of states have developed lotteries in order to generate more revenue for government programs, especially education. In a typical state lottery, of each dollar wagered, about \$0.05 is used for administrative expense, \$0.45 is used for state government programs, and \$0.50 goes into the pool for prizes for the winners.

Consider a lottery program that generates \$1 million a week by selling 1 million tickets at \$1 each. A typical distribution of the prize pool of \$0.5 million is shown here.

Prize Value ( $X_i$ )	Number of Prizes	Probability of Winning ( $P_i$ )	Total Prize Value
\$150,000	1	0.000001	\$150,000
75,000	2	0.000002	150,000
25,000	4	0.000004	100,000
5,000	10	0.000010	50,000
2,000	25	0.000025	50,000
			\$500,000

This is not a fair game (i.e., one where the expected value of the prize equals the amount wagered) because the expected value is not zero. The expected return on a \$1 ticket is only \$0.50, that is,

$$\mu = \sum_i P_i X_i = 0.000001(150,000) + 0.000002(75,000) + 0.000004(25,000) + 0.000010(5,000) + 0.000025(2,000) + 0.999958(0) = \$0.50$$

Note that the certainty equivalent (the \$1 cost of a ticket) is greater than the expected value of the gamble. Because millions of people are willing to play these games regularly implies that their behavior should be characterized as risk seeking, at least when the wagers are for a small amount of money.

Another interesting dimension of these games is that the large prizes often are paid in the form of an annuity. That is, the \$150,000 first prize in the preceding example may reflect the present value of the award, but more often it is advertised as a prize worth more than \$300,000 consisting of 20 annual payments of \$15,000 each. At a discount rate of 8 percent, this annuity has a present value of about \$150,000.

Supporters of the state lottery concept argue that it is a relatively painless way to generate revenue for important state programs. Unlike a tax that one is forced to pay, purchasers of lottery tickets voluntarily contribute to the government. Further, it is argued that if people understand that the expected return on a \$1 ticket is only \$0.50, one must conclude that the fun and the excitement associated with playing are greater than the expected loss.

Some critics of state lotteries claim that people do not really understand the probabilities of winning and that the present value of the large prizes is really less than the amount advertised. Further, there is good evidence that the largest share of the revenue from these programs comes from low-income people; for example, it is estimated that about 75 percent is paid by those earning less than \$15,000 per year. ■

### Key Concepts

- In general, investments that offer higher expected returns also involve greater risk.
- Behavior is said to be
  - Risk averse if the certainty equivalent of a gamble is less than the expected value of the gamble.
  - Risk neutral if the certainty equivalent of a gamble is equal to the expected value of the gamble.
  - Risk seeking if the certainty equivalent of a gamble is greater than the expected value of the gamble.
- Equivalently, behavior is
  - Risk averse if the utility of expected wealth is greater than the expected utility of wealth.
  - Risk neutral if the utility of expected wealth is equal to the expected utility of wealth.
  - Risk seeking if the utility of expected wealth is less than the expected utility of wealth.

## RISK MANAGEMENT

Decision makers manage risk in four fundamental ways. First, insurance can be purchased to protect against losses associated with fire; natural disasters such as flooding; theft; and accidents. Second, hedging by buying various contracts in the financial markets provides another form of insurance against financial risks such as a crop failure for an agricultural enterprise, stock market decline, and currency fluctuations. Third, the firm can reduce risk by diversifying its activity so that a decline in one market may be offset by better conditions in another. Finally, the differential risk associated with various actions being contemplated by the firm can be adjusted for by adding a risk premium to the interest rate used to discount projected cash flows and/or using certainty equivalents as the basis for comparison.

### Insurance

Firms and individuals buy insurance to protect against the financial loss associated with a variety of risks, including fire, theft, floods, earthquakes, and death. The market for insurance exists because people are risk averse. Consider a manager who knows that his fireworks manufacturing firm will earn a net profit of \$100,000 each year but also faces a prob-

TABLE 14.4 Probability Distribution and Expected Value of Profit

Event	Outcome	Probability	Outcome $\times$ Probability
Fire	\$ 20,000	0.5	\$10,000
No fire	100,000	0.5	50,000
			$\mu = \$60,000$

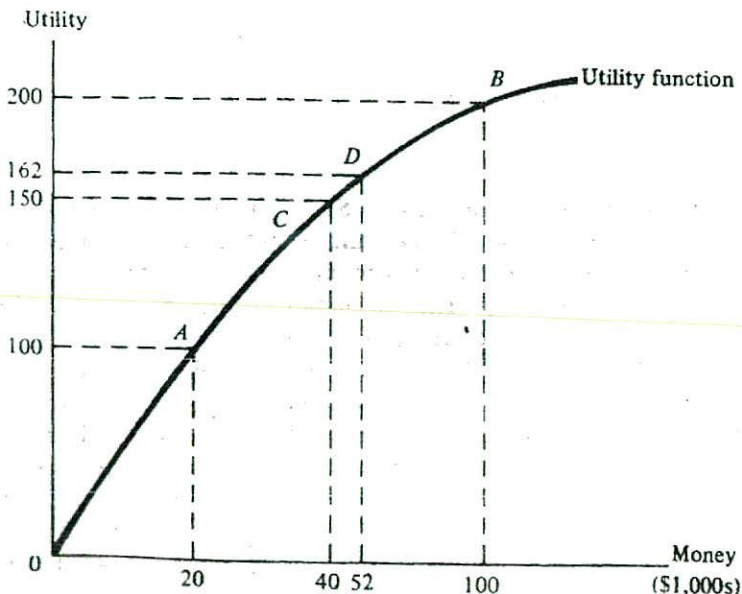
ability of 0.5 that the plant will burn down.<sup>1</sup> Assume that if the plant burns down, it will cost \$80,000 to replace. Now there are two possible outcomes: (1) the plant burns down, so net returns are \$100,000 - \$80,000 = \$20,000; or (2) the plant does not burn down, so the net return is \$100,000. Each outcome has a 0.5 probability of occurrence. The expected monetary value is \$60,000, as shown in Table 14.4. Essentially, the manager is facing a gamble.

Suppose that manager's utility function is similar to that shown in Figure 14.4. Point A corresponds to the outcome associated with the plant burning down (i.e., a dollar payoff of \$20,000 and utility = 100) and point B corresponds to the "no fire" outcome, when the dollar payoff is \$100,000 and the utility is 200. Thus, the expected utility for this uncertain prospect is 150. That is,

$$\mu = 0.5(100) + 0.5(200) = 150$$

Now the certainty equivalent is \$40,000 (i.e., \$40,000 with certainty generates a utility level of 150, as shown by point C in Figure 14.4). So the decision maker would be indifferent between a certain prospect of an income of \$40,000 and the gamble just described, having an expected dollar value of \$60,000.

FIGURE 14.4 Utility-Money Function for a Risk-Averse Manager



<sup>1</sup> Obviously, this is an unrealistic probability. It is used simply for ease of calculation.

This implies that the profit-maximizing (but risk-averse) manager would pay up to \$60,000 for a fire insurance policy on the factory, because having this policy would guarantee a certain outcome of \$40,000 whether or not the plant burned down. That is, the manager would know with certainty that the firm would net \$40,000 (i.e., \$100,000 in profit less the \$60,000 insurance premium). If there is a fire, the insurance company will rebuild the factory at no cost to the firm.

Now consider the situation from the insurance company's perspective. The expected payout by the insurance company is determined by multiplying the probability of a fire by the dollar loss, that is,  $0.5 \times \$80,000 = \$40,000$ . This claim plus, say, 20 percent for operating expenses and profit, suggests that a fire insurance policy for the plant could be offered for about \$48,000. The result is that the manager of the fireworks company could actually have a certain outcome of \$52,000 (i.e., the \$100,000 profit less the \$48,000 insurance premium). This outcome, shown as point *D* in Figure 14.4, has a utility level of 162 and is preferred to the utility level 150 that corresponds to the uncertain prospect. Thus, in this example, the manager can choose between the certain prospect of having a utility level of 162 or an uncertain prospect with an expected utility of 150. Clearly, the rational manager would opt for the certain prospect by purchasing insurance.

Although utility is impossible to measure, the best evidence that most people are risk averse and rational is the existence of a large number of insurance companies and the variety of risks that they insure against. For example, private firms have issued policies that insure pianists' hands and dancers' legs. Ski resorts can buy insurance to protect against a lack of snow, and insurance companies paid more than \$150 million in claims when rocket malfunctions sent two communications satellites into the oblivion of outer space.

## *Case Study*

### Preference for Risk Among Physicians

Jury awards and out-of-court settlements of \$1 million or more are not uncommon in malpractice suits against medical doctors and hospitals. Because one mistake could mean financial ruin, virtually all doctors carry insurance against the risk of malpractice claims. Some in the so-called high-risk specialties, such as obstetrics and orthopedic surgery, face premiums in excess of \$100,000 per year.

It is estimated that medical doctors pay about \$8 billion each year for insurance protection against malpractice claims. Of this amount, only about 35 percent, or \$2.8 billion, is paid out to injured people. The remaining 65 percent (\$5.2 billion) goes for insurance company operating expenses and profits. A large part of the expenses are legal fees to hire lawyers to defend doctors who are sued and payments to the many expert witnesses (including economists) who testify in malpractice suits. Thus, the average physician pays an insurance premium of about \$3 for every dollar of loss expected. This suggests risk-averse behavior on the part of physicians.

Consider a hypothetical physician whose annual income is \$100,000 after all expenses except malpractice insurance. Assume that the award for any successful malpractice suit would be \$100,000 and that the probability of that occurring is 0.10. Thus, the expected loss in any one year is \$10,000 (i.e.,  $0.10 \times \$100,000$ ). Suppose that the cost of an insurance policy against this risk is \$30,000.

If the doctor buys an insurance policy, risk has been eliminated, and net income would be \$70,000, that is, \$100,000 less the \$30,000 insurance premium. If insurance is not purchased, income will be (1) \$100,000 with probability 0.9; or (2) zero, in the case where an injured patient wins a lawsuit with probability 0.10. Thus the expected value of the risky alternative is \$90,000, that is,

$$\mu = 0.9(\$100,000) + 0.1(0) = \$90,000$$

If the decision were based solely on the expected value of dollar returns, the physician would not buy the insurance. But assume that the doctor is risk averse, as indicated by the utility function

$$U = 40I - 0.2I^2$$

where  $I$  is income in thousands. Again, this is simply a mathematical relationship that allows income to be translated into utility for this person. The utility associated with the certain income of \$70,000 is 1,820, that is,

$$U = 40(70) - 0.2(70)^2 = 1,820$$

which is higher than the expected utility ( $\mu$ ) associated with not having insurance. The latter is determined in the following way. First, the probability distribution is outlined and the two income possibilities (\$100,000 and zero) are converted into utility.

<i>Event</i>	<i>Probability</i>	<i>Income</i>	<i>Utility</i>
No suit	0.9	\$100,000	2,000
Suit	0.1	0	0

Then the expected value (in terms of utility) is computed:

$$\mu = 0.9(2,000) + 0.1(0) = 1,800$$

Thus, this doctor is willing to spend \$30,000 each year for insurance against possible claims having an expected value of only \$10,000. Clearly, this is risk-averse behavior.

That virtually all doctors have malpractice insurance suggests that they are risk averse. However, there may be another explanation. Hospitals can be held responsible for a doctor's malpractice. As a result, almost all hospitals require that doctors have malpractice insurance before they are allowed to use the hospital. This reduces the risk of loss for the owners of the hospital. Because having hospital privileges is essential for most medical practices, one would find that even risk-seeking physicians would have insurance. Thus, the high proportion of doctors who have malpractice insurance could be the result of risk-averse hospital administrators forcing their risk preferences on physicians. ■



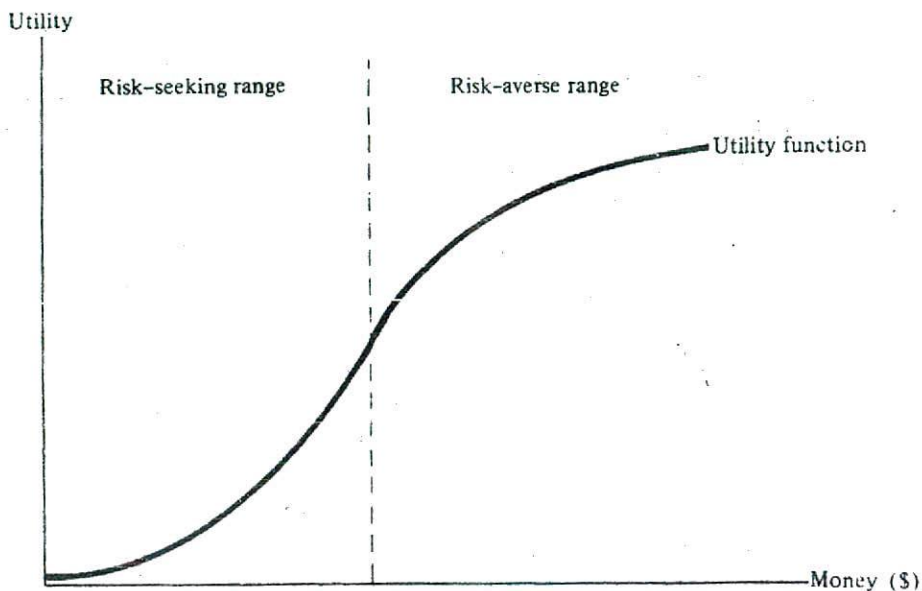


FIGURE 14.5 Friedman-Savage Utility Function

### Gambling and Insuring: A Contradiction?

Individual behavior is generally defined as either risk averse, risk neutral, or risk seeking. Thus, one might expect to find risk-averse individuals spending their time reviewing their insurance coverage and to find risk-seeking persons in casinos in Las Vegas or Atlantic City. But if a survey were made, it probably would show that many people both buy insurance and engage in gambling games of one sort or another. For example, it is probably true that most people who engage in gambling in Las Vegas drove there in insured cars and live in insured homes. This appears to be a contradiction, as it suggests that people are risk seeking and risk averse at the same time. Actually, there is no contradiction, but a complete explanation is beyond the scope of this book. Suffice it to say that this behavior depends on the type of gambling games available and the nature and cost of insurance that can be purchased.

One rather simple explanation for this phenomenon is offered by Friedman and Savage,<sup>2</sup> who argue that the utility function may look as shown in Figure 14.5. It describes the person who is risk seeking when small amounts of money are involved and risk averse when larger amounts may be at risk. A person having such a utility function would engage in gambling games for relatively small amounts of money, but would insure against large losses, such as those associated with a house fire or automobile accident. A utility function of this type is a sufficient condition for a rational person to gamble and have insurance at the same time.

<sup>2</sup>M. Friedman and L. Savage, "The Utility Analysis of Choices Involving Risk," *Journal of Political Economy* 56 (1948):279-304.

**Example Risk Preference and Decision Making**

Consider the problem facing a manager who must choose between two investments, *A* and *B*, each having an initial cost of \$10. The probability distributions for the payoffs for each investment are shown in the following table. Each payoff represents the present value of all future net profits.

<i>A</i>		<i>B</i>	
Probability ( $P_i$ )	Payoff ( $X_i$ )	Probability ( $P_i$ )	Payoff ( $X_i$ )
0.10	\$-20	0.20	\$10
0.50	20	0.40	20
0.40	50	0.40	30

The decision maker's utility function is  $U = 100X - X^2$ , where  $X$  is the dollar payoff.

1. Would you characterize this decision maker as risk seeking, risk neutral, or risk averse? Explain.
2. If the objective is to maximize expected net present value, which investment is the better choice? (For the moment, disregard risk.)
3. Evaluate the risk associated with the dollar returns for each investment.
4. If the objective is utility maximization, which investment should be chosen?

**Solution**

1. Based on the utility function, it is seen that the decision maker is risk averse. This can be shown in the following way. Select any two dollar payoffs, say, \$20 and \$40, and arbitrarily assign them probabilities of occurrence, say, 0.4 and 0.6, respectively. The expected value of these payoffs is

$$\mu(\text{dollars}) = 0.4(20) + 0.6(40) = \$32$$

From the utility function, the utilities associated with \$20 and \$40 are 1,600 and 2,400 respectively. Thus the expected utility value is

$$\mu(\text{utility}) = 0.4(1,600) + 0.6(2,400) = 2,080$$

But if this person were offered \$32 with certainty (i.e., a certain prospect having the same value as the expected dollar value of the investments), the utility would be 2,176. Because the utility is higher for the certain prospect, the person is risk averse.

2. The expected returns for each investment are

$$\mu_A = 0.10(-20) + 0.50(20) + 0.40(50) = 28$$

$$\mu_B = 0.20(10) + 0.40(20) + 0.40(30) = 22$$

Thus, *A* is the preferred choice because it has the highest expected value.

3. The evaluation statistics for risk and risk per dollar of expected return for both investments are

**Investment A:**

$$\sigma_A = \sqrt{0.10(-20 - 28)^2 + 0.50(20 - 28)^2 + 0.40(50 - 28)^2} = 21.4$$

$$v_A = \frac{\sigma_A}{\mu_A} = \frac{21.4}{28} = 0.76$$

**Investment B:**

$$\sigma_B = \sqrt{0.20(10 - 22)^2 + 0.40(20 - 22)^2 + 0.40(20 - 22)^2} = 7.5$$

$$v_B = \frac{\sigma_B}{\mu_B} = \frac{7.5}{22} = 0.34$$

The risk is greater for A ( $\sigma_A = 21.4$  compared to  $\sigma_B = 7.5$ ), and the risk per dollar of expected return is also greater for A ( $v_A = 0.76$  compared to  $v_B = 0.34$ ).

4. To make the decision, assuming the goal is utility maximization, the payoffs must be transformed from dollars into utility using the utility function  $U = 100X - X^2$ . For example, the utility associated with a net payoff of  $-20$  is

$$U = 100(-20) - 20^2 = -2,400$$

Repeating this for each outcome will yield two new probability distributions where the payoffs are in terms of utility.

Probability ( $P_i$ )	Payoff ( $U_i$ )	Probability ( $P_i$ )	Payoff ( $U_i$ )
0.10	-2,400	0.20	1,900
0.50	1,600	0.40	1,600
0.40	2,500	0.40	2,100

The expected utility returns are

$$\mu_A = 0.10(-2,400) + 0.50(1,600) + 0.40(2,500) = 1,560$$

$$\mu_B = 0.20(900) + 0.40(1,600) + 0.40(2,100) = 1,660$$

Hence, in terms of utility, investment B is the preferred alternative.

**Key Concepts**

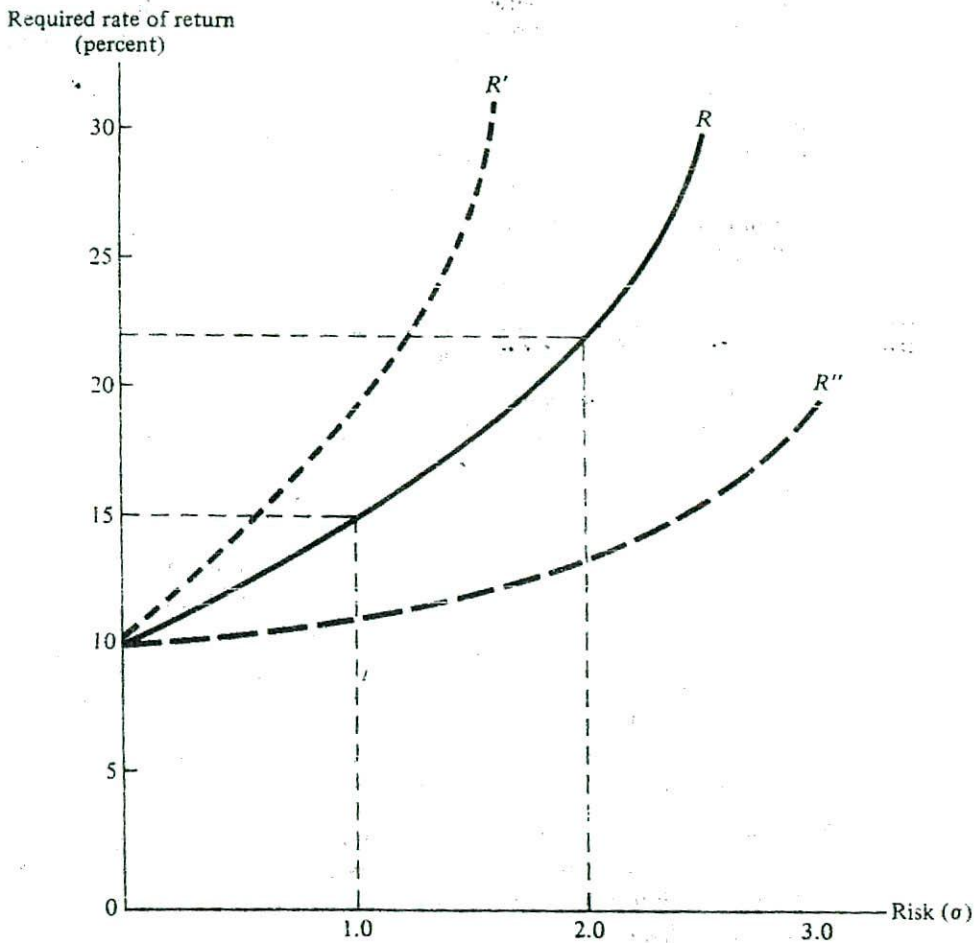
- The market for insurance exists because many people are risk averse. The market for gambling exists because some people are risk seekers.
- Some people exhibit risk-seeking behavior and risk-averse behavior at the same time. One explanation for this is that they are risk seekers when relatively small amounts of money are involved but are risk averse when large amounts are at stake.

**Adjusting the Discount Rate**

Another way to adjust for risk is to use a risk-adjusted discount rate in determining the present value of the future profits associated with an investment. Given the stream of future profits,  $\pi_t$ , the basic evaluation framework

$$PV = \sum_{t=1}^n \frac{\pi_t}{(1+r)^t} \quad (14-4)$$

is modified by using an appropriate risk-adjusted interest or discount rate for  $r$ . Most investors are willing to accept greater risk only if there is the promise of greater returns when compared to an investment with less risk. For example, suppose that the typical



return on an insured bank certificate of deposit is 10 percent per year. Clearly, no rational investor will invest in very risky ventures such as drilling for oil or mining for gold unless the expected return is considerably higher than 10 percent per year.

Suppose that the line  $R$  in Figure 14.6 shows all combinations of risk and return for which a hypothetical investor is indifferent. That is, this function shows the willingness of this investor to trade off risk against return. Clearly, the shape of this function will vary for different individuals, depending on their preference for risk. A very risk-averse person might have a trade-off function similar to the dashed line  $R'$ —any increase in risk must carry with it a significant increase in return. Conversely, another individual's trade-off function might be described by line  $R''$ . Here only a small increase in the rate of return is required to compensate for a rather large increase in risk.

Assume that the rate of return associated with a riskless investment is 10 percent. Recall that a riskless investment would have a standard deviation of zero. For the investor with trade-off function  $R$ , if the risk increases to, say,  $\sigma = 1.0$ , a 15 percent rate of return is

required. The difference between this 15 percent return and the riskless rate of 10 percent is referred to as the *risk premium*. If  $\sigma = 2.0$ , the trade-off function  $R$  indicates that a 22 percent return is required for this person. Thus, the risk premium is 12 percentage points. In evaluating investments, these differential discount rates would be used to evaluate the present value of future profits. That is, net cash flows for a high-risk investment would be discounted using a higher discount rate than would be used for a low-risk alternative.

It should be emphasized that there is no equation or table that relates risk and the discount rate. Clearly, there is a positive relationship between these two factors, but the relationship between them is strictly judgmental and must be made by individual decision makers.

### Example Using Risk-Adjusted Discount Rates

Suppose that management at Showmax Theaters must decide whether to expand by building a few large theaters in large cities or building a number of minitheaters in small cities and towns. Each of the alternatives would require an initial investment of \$2 million. Although the large theaters have a greater expected return, this option has greater risk because there is more competition in the larger cities. In contrast, there is less potential for profit in the small markets, but in many of them there is little or no competition. The expected value of the net profits in each of the next 10 years is \$600,000 per year in the large markets and \$500,000 per year in the small markets.

The value of  $\sigma$  is estimated to be 1.5 in the large markets and 0.5 in the small cities. Management has a risk – return trade-off function similar to that shown by curve  $R$  in Figure 14.6. This means that a rate of about 17 percent would be used to discount cash flows in the large-city alternative and a rate of about 12 percent would be used to discount cash flows in the small-city alternative.

**Solution** The net present value of an investment is determined by subtracting the initial cost of the investment from the present value of future profits. Note that the appropriate risk-adjusted discount rate is used in each case.

#### Large-City Alternative:

$$\begin{aligned} NPV &= \sum_{t=1}^{10} \frac{600,000}{(1 + 0.17)^t} - \$2,000,000 \\ &= \$2,795,162 - \$2,000,000 = \$795,162 \end{aligned}$$

#### Small-City Alternative:

$$\begin{aligned} NPV &= \sum_{t=1}^{10} \frac{500,000}{(1 + 0.12)^t} - \$2,000,000 \\ &= \$2,825,112 - \$2,000,000 = \$825,112 \end{aligned}$$

Although the nondiscounted cash flows each year are greater for the large-city alternative, when adjusted for risk, the present value of the cash flows is greater for the small-city alternative. Thus, building minitheaters in smaller cities is the preferred investment.

### Diversification

The firm can reduce risk by diversification of its assets. That is, it can invest in a variety of activities so that a decline in one area of the business might be offset by an expansion

sion of another. For example, farmers generally plant several different crops as protection against a single crop failure or an unusually low price on one crop. A firm providing hotel services might decide to enter the movie business, reasoning that if for some reason travel declined and people spent more time in their home cities, they might go to more movies. The products are also complementary; the firm could show its own movies in its hotel rooms.

The principle of risk reduction through diversification is most easily understood by thinking in terms of buying two securities,  $A$  and  $B$ , which have the same expected rate of return of  $r_A = r_B = 0.10$  or 10 percent, and the same level of risk, i.e.,  $\sigma_A = \sigma_B = 0.20$ . Assume that 50 percent of the portfolio is invested in asset  $A$  (i.e.,  $w = 0.5$ ) and 50 percent is invested in asset  $B$  (i.e.,  $w = 0.50$ ). Thus, the weighted average return on the portfolio is

$$\begin{aligned} r_p &= w_A r_A + w_B r_B \\ &= 0.5(0.10) + 0.5(0.10) = 0.10 \end{aligned} \quad (14-5)$$

or 10 percent.

By investing in both assets, the expected return is no different than had only one of the assets been selected. The advantage of diversification is that risk can be reduced with no reduction in expected rate of return. Recall that the risk on  $A$  and  $B$  was 0.20, or 20 percent. The risk on a portfolio of two assets is given by

$$\sigma_p = \sqrt{w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B \sigma_{AB} \sigma_A \sigma_B} \quad (14-6)$$

where  $\sigma_{AB}$  is the correlation between the returns on the two investments (i.e., the degree to which the returns move together over time). If the returns are perfectly correlated, where  $\sigma_{AB} = +1.0$ , then the risk on the portfolio is

$$\sigma_p = \sqrt{(0.5)^2(0.20)^2 + (0.5)^2(0.20)^2 + 2(0.5)(0.5)(1.0)(0.20)(0.20)} = 0.20$$

Note there has been no reduction in the level of risk that would have been faced by investing in only one of the assets. But if the correlation is less than perfect, then risk will be reduced. To reduce risk, the returns on the two assets have to be less than perfectly correlated. For any given value of  $\sigma_{AB}$ , the risk for this portfolio is

$$\sigma_p = \sqrt{0.02 + 0.02\sigma_{AB}}$$

Now, the maximum reduction in risk is achieved when there is perfect negative correlation. That is, when  $\sigma_{AB} = -1.0$ . Now, the risk on the portfolio is zero, i.e.,

$$\sigma_p = \sqrt{0.02 + 0.02(-1)} = 0$$

Here risk has been completely eliminated. The portfolio risk for various other values of  $\sigma_{AB}$  is given below:

$\sigma_{AB} = +0.50$	$\sigma_p = 0.17$
$\sigma_{AB} = +0.25$	$\sigma_p = 0.16$
$\sigma_{AB} = 0$	$\sigma_p = 0.14$
$\sigma_{AB} = -0.25$	$\sigma_p = 0.12$
$\sigma_{AB} = -0.50$	$\sigma_p = 0.10$

This is an important result and was an instrumental part of the work in finance that led to the Nobel Prize being awarded to Harry Markowitz, Merton Miller, and William Sharpe in 1992. The essential point is that by developing an efficient portfolio of assets, the risk associated with a given level of return can be reduced or, alternatively, the return on a given level of risk can be increased compared to a nonefficient (i.e., a nondiversified portfolio).

Thus, one way for the firm or individual to reduce risk is through diversification of assets. Individuals can invest in a variety of companies; firms can invest in several different product lines or have stores in various parts of the country or world. If the portfolio of assets is structured correctly, risk can be reduced with no reduction in expected return.

### Hedging

It is possible for firms to insure or hedge against adverse price movements by using futures contracts. These contracts involve entering into a contract to buy or sell a specified quantity of a good of a given quality at a specified price in the future. For example, an agricultural producer in eastern Kansas plants a wheat crop many months prior to its being ready to harvest. At the time of planting, there are a variety of possible risks, including having a poor crop due to bad weather conditions and having the price decline. It is possible to purchase insurance against crop failure, thus eliminating that risk. Further, the producer can eliminate most price risk by entering into what is called a forward contract (i.e., essentially, a contract to sell the wheat crop at a specified price at the time of harvest regardless of what the market price is at that time), or a futures contract. The latter is a contract traded in such places as the Chicago Board of Trade and the Chicago Mercantile Exchange, where the producer buys a contract that specifies the right to sell a given amount of wheat at a specified price in the future.

Contracts in these markets trade continuously, and the value of the contract will go up or down in inverse relationship to the price of wheat. If the cash price of wheat increases, the right to sell at a specified price becomes worth less, and if the cash price goes down, that right to sell becomes more valuable. The producer ultimately will sell wheat at the current cash price, but if that price has gone down, the producer will make a profit on the futures contract that will offset the loss associated with the decline in the cash price. If the cash price goes up, the value of the futures contract declines so the additional profit from selling at the higher cash price is offset by the decline in the value of the futures contract. If set up correctly, all price risk to the producer is virtually eliminated.

For example, assume the cash price is \$4.00 per bushel and the producer plants 500 acres in September and expects a yield of 100 bushels per acre for a total expected crop of 50,000 bushels. The farmer is confident about production costs and has decided that an acceptable profit can be earned if the crop could be sold at about the current cash price of \$4.00 per bushel. Of course, output will not be available until the following August. In the futures market, assume the price of wheat for the August contract is \$4.00 per bushel, and the producer buys a contract to sell 500,000 bushel of wheat at that price. This brokerage cost of these contracts would be about \$1,000 or about \$0.02 per bushel. Essentially, the producer has paid \$1,000 to guarantee a sale of the crop at \$4.00 per bushel for a total of \$200,000.<sup>3</sup> Should the price of wheat fall to \$3.00 per bushel the

<sup>3</sup>There are some additional considerations that complicate these transactions, including the need to maintain a cash reserve with the broker in the event that the value of the contract moves in the wrong direction.

farmer will sell the crop at the cash price for \$150,000. The value of the contract will have increased by \$50,000 and the farmer will sell the contract and capture that profit. Thus, total revenue will be \$200,000 on the two transactions. Should the cash price rise to \$5.00 by August, the crop will sell at that price for revenue of \$250,000, but the futures contract will be sold at a loss of \$50,000, again providing net revenue of \$200,000.

By using the futures market, the producer has paid someone else to assume this price risk. That risk has been transferred to a speculator, who is betting on being able to predict future movement in the price of wheat and make a profit. The wheat producer in our example can sell wheat at a net price of \$3.98 per bushel (the \$4.00 contract price less the \$0.02 per bushel commission) with certainty or opt for the risky alternative where the actual selling price is unknown until the time of sale. Some firms use hedging to reduce risk, while others do not. Part of this is explained by their different preferences for risk. Another part may be explained by ignorance of the ways to reduce risk through this mechanism.

There are futures markets in a wide variety of commodities (e.g., soybeans, cattle, sugar, oil, gold, lumber, etc.) as well as for a variety of foreign currencies and common stock indices such as the S&P 500 and the Nasdaq indexes. A fairly recent development is a futures market in electricity (e.g., the California Oregon Border Electricity contract).

#### Key Concept

- Risk can be managed by
  - Purchasing insurance to cover losses in case of fire, theft, flood, or loss of a key manager.
  - Adjusting the rate used to discount future cash flows to their present value.
  - Diversification of assets.
  - Purchase of forward and/or futures contracts that guarantee a specified price.

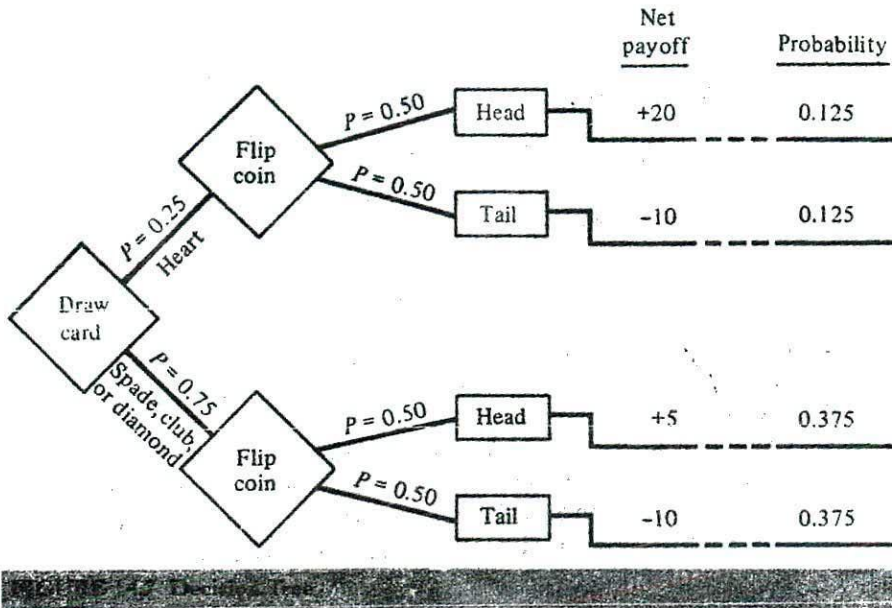
## DECISION TREE ANALYSIS

The previous discussion of risk began with a set of outcomes for each investment and the probability of each outcome occurring. However, some strategic decisions involve a sequence of decisions, states of nature, and possibly even subsequent decisions. Given this complexity, how are the alternative strategies to be evaluated? One approach is to use a decision tree that traces the sequence of events and/or decisions that lead to each outcome. Such a diagram shows two or more branches at each point where a decision or event (i.e., a state of nature) leads to the various outcomes. These branches are similar to those on a tree. Understanding the concept of a decision tree will help in understanding how the data in a probability distribution are determined.

To illustrate the concept, consider the following gambling game. An initial bet of \$10 is made. A card is drawn from an ordinary deck of playing cards and then a coin is tossed. If the player draws a heart and then tosses a head, he receives \$30, for a net payoff of \$20. If a heart is not drawn, but a head is tossed, the player is paid \$15, for a net payoff of \$5. For all other outcomes, the player loses his initial \$10 bet.

When drawing the card, the relevant probabilities are 0.25 that a heart is drawn (there are 13 hearts in a deck of 52 cards, so  $13/52 = 0.25$ ), and the probability that a spade, club, or diamond is drawn is 0.75. When tossing a coin, the probability of a head and the probability of a tail are both equal to 0.5.

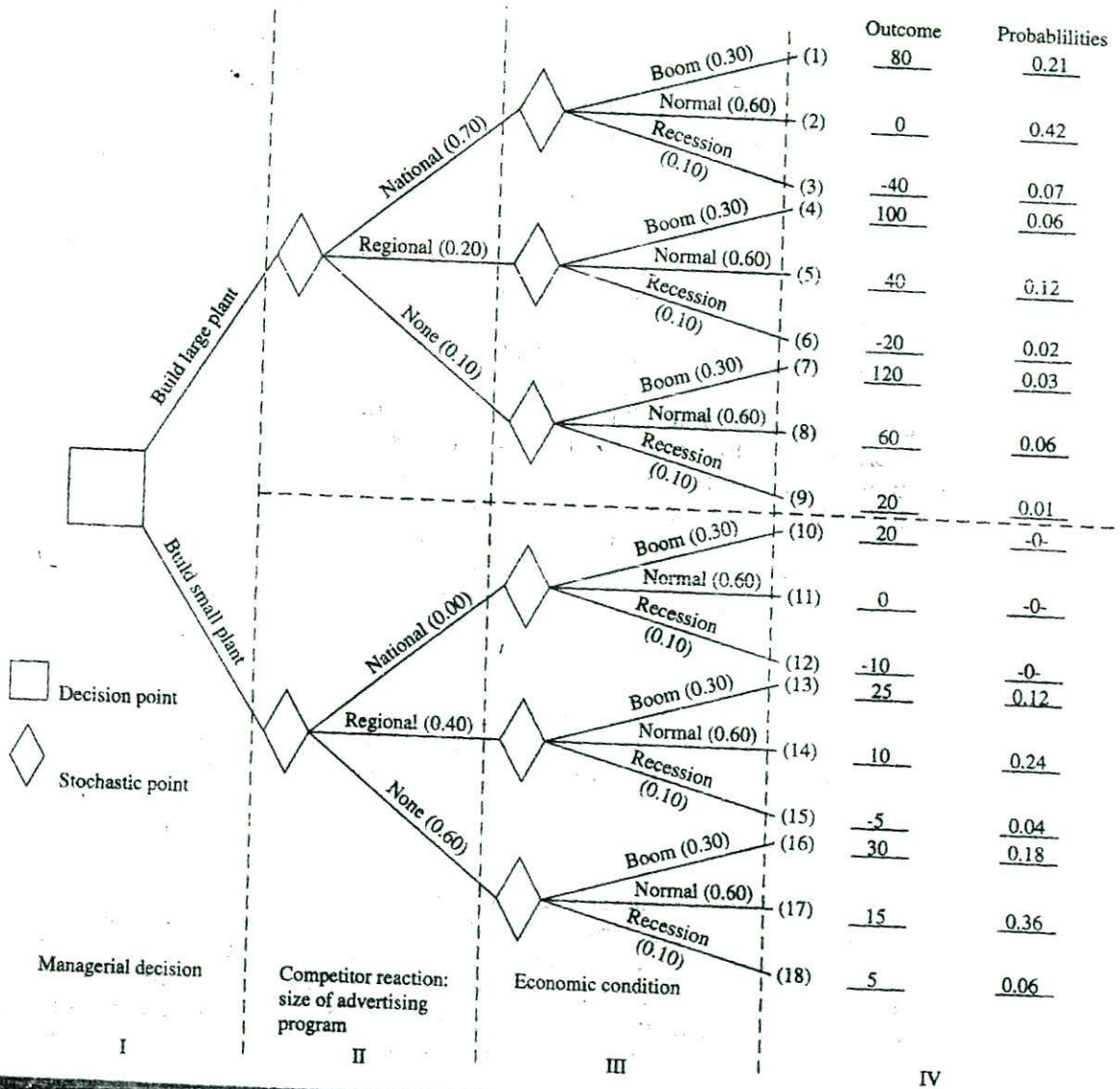




The game can be analyzed using the decision tree shown in Figure 14.7. The probability of the outcome at each step in the process is shown on the branches of the tree. The probability of each final outcome is equal to the product of the probabilities along the branches leading to that outcome. For example, the probability of drawing a heart and then tossing a head for a net payoff of \$20 is  $0.25 \times 0.50 = 0.125$ . By repeating this for each path along the decision tree, the probability of each outcome is determined. By associating the net payoff for each outcome with its respective probability, the probability distribution is determined. Then the usual evaluation statistics that describe expected return, risk, and risk per dollar of expected return can be computed.

The decision tree approach can be directly applied to managerial decision making. Suppose a firm is considering entering a new market. This entry would require building either a large or small plant. This decision is shown in part I of Figure 14.8. A square is used at each branch to show decisions. Note that there are no probabilities associated with such decisions. A diamond is shown at those branches associated with the various states of nature that may occur. A probability must be assigned to each of these branches.

In this example, there are two stochastic elements associated with each decision (the term *stochastic* refers to an outcome that is determined by chance): (1) the reaction of a major competitor in the business and (2) the economic conditions that will prevail. Suppose it is learned that the competitor may respond to the new plant with a new national or regional advertising program or with no new advertising program. Assume that the probability of each occurring will depend on the size of plant built, as shown in Table 14.5. The probabilities for each alternative competitive reaction are entered on the appropriate branches of part II of Figure 14.8. Note that the probabilities depend on the size of plant built. If a large plant is built, the probability is high that the competitor will respond with a major advertising program. Conversely, building a small plant would result in a regional program or no program at all.



Plant Size	Probability That Competitor Responds with:		
	National Program	Regional Program	No New Program
Large plant	0.70	0.20	0.10
Small plant	0.00	0.40	0.60

Suppose that the possible economic conditions and their probabilities are recession (0.10), normal business conditions (0.60), and boom conditions (0.30). These states of nature and their respective probabilities are shown in part III of the decision tree. Finally, the outcome for each branch is the present value of profits under the states of nature (i.e., competitor's reaction and economic conditions) along that branch.

Note that there is a payoff (i.e., profit) associated with each of the 18 possible combinations of decisions and stochastic events. Each combination consists of a plant size, the competitor's reaction, and an economic condition. These payoffs have been listed in part IV of Figure 14.8 together with the probability of that sequence occurring. The probabilities are found by multiplying the probability along each of the branches leading to the outcome. For example, consider the top branch of the decision tree. The probability that the competitor responds with a national advertising program is 0.70, and the probability of an economic boom is 0.30. The product of these probabilities is 0.21, and this is shown as the first entry in the probability distribution on the right-hand side of the figure.

There are two distinct probability distributions outlined in Figure 14.8. The first, consisting of the payoff-probability pairs (1) through (9), corresponds to the decision to build the large plant. Pairs (10) through (18) correspond to the decision to build the small plant. Note that the probabilities sum to one for each distribution.

Finally, based on these probability distributions, the evaluation statistics,  $\mu$ ,  $\sigma$ , and  $\nu$  for each investment alternative can be computed. For example, consider the computation of these statistics for the large-plant alternative:

$$\mu = 0.21(80) + 0.42(0) + 0.07(-40) + 0.06(100) + 0.12(40) + 0.02(-20) + 0.03(120) + 0.06(60) + 0.01(20) = \$31.8 \text{ million}$$

$$\sigma = \sqrt{0.21(80 - 31.8)^2 + 0.42(0 - 31.8)^2 + \dots + 0.01(20 - 31.8)^2} = 43.6$$

$$\nu = \frac{\sigma}{\mu} = \frac{43.6}{31.8} = 1.4$$

These statistics for both plant sizes are summarized in Table 14.6. As usual, the decision will not be an easy one for management. The expected return or expected net present value of all future profits increases with the scale of the plant, but so does risk, as measured by  $\sigma$  and  $\nu$ . For example, the expected return on the large plant, \$31.8 million, is higher than the expected return for the small plant, but the risk per dollar of return (1.4) is also higher. The manager's preference for risk will be a fundamental factor in making the decision.

	Alternative Plant Size	
	Large (dollar figures in millions)	Small
Expected return ( $\mu$ )	\$31.8	\$16.3
Risk ( $\sigma$ )	\$43.6	\$8.9
Risk per dollar of expected return ( $\nu$ )	1.4	0.55

Clearly, decision makers must keep the problem in proper perspective. One way is to include only those alternatives and outcomes that are relevant and significant in terms of both the probability of occurrence and the net payoff. Consider including the possibility of terrorist actions as a possible state of nature when evaluating a decision. For military planners, this state of nature is both relevant and significant. However, for most business decisions such a possibility probably is not relevant, even though the payoff associated with this outcome could be a large negative number. That is, the firm may incur huge losses in the event of such an attack, but this outcome is sufficiently improbable that it is not relevant for most business decisions.

Even with careful attention to including only relevant decisions and stochastic factors, decision trees can easily become very large and complex. The total number of outcomes is equal to the product of the number of decisions or stochastic alternatives along each branch on the tree. For example, a decision to build a new plant might involve six different plant-size alternatives, five different ways a competitor might react, and three different economic environments. Hence the number of outcomes in this case would be 90 (i.e.,  $6 \times 5 \times 3$ ). One could easily imagine decision trees with thousands of possible outcomes. Whereas a computer would have no problem computing the probabilities and evaluation statistics for each of a large number of intricate decision trees, the manager's capacity to evaluate the options is limited.

### Key Concepts

- A decision tree traces sequences of strategies and states of nature to arrive at a set of outcomes. The probability of any outcome is found by multiplying the probabilities on each branch leading to that outcome.
- There is a probability distribution for each choice included in a decision tree.

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## SUMMARY

A decision maker faces risk when there are several outcomes associated with a decision and the probabilities of those outcomes are known. The greater the variation in those outcomes, the greater the risk. The set of outcomes and their associated probabilities comprise a probability distribution. If the probabilities of the outcomes are not known, the decision maker faces uncertainty.

The evaluation of a decision where risk is present is made by determining the expected value ( $\mu$ ), the standard deviation ( $\sigma$ ), and the coefficient of variation ( $v$ ) of the probability distribution for the investment. The expected value estimates the expected return, the standard deviation measures risk, and the coefficient of variation measures risk per dollar of expected return. In general, decisions that promise higher expected returns also carry greater risk.

Behavior is said to be risk averse if the certainty equivalent of a gamble is greater than the expected dollar value of the gamble. In contrast, behavior is risk seeking if the certainty equivalent of a gamble is less than the expected dollar value of the gamble. A market for insurance exists because many people are risk averse. A market for gambling exists because there are risk-seeking individuals. Some people buy insurance and

also engage in gambling games because they are risk seekers when small amounts of money are involved and risk averse when larger amounts are at stake.

A utility function indicates the amount of utility or satisfaction a person receives from wealth or income. It can be used to determine the individual's preference for risk. Risk aversion is indicated by a utility function that increases but at a decreasing rate. A risk-neutral person has a utility function that is linear, and the utility function for a risk-seeking person increases at an increasing rate.

Risk can be managed by using insurance, diversifying, and hedging with various financial instruments. Risk can also be managed by using a risk adjusted interest rate in discounting cash flows. This involves adding a risk premium to the rate used to discount future profits. The risk premium increases as the risk associated with the decision increases. An alternative method for adjusting for risk is to evaluate the certainty equivalent of a risky decision.

A decision tree traces the sequence of strategies and states of nature to determine the set of outcomes associated with a decision. The probability of any outcome is found by multiplying the probabilities on each branch leading to that outcome. There is a probability distribution for each choice included in a decision tree.

### Discussion Questions

- 14-1. Explain how the principle of risk aversion can be used to explain why the owner of a business would buy insurance on the life of a key employee.
- 14-2. Assume the risk-free interest rate (e.g., the rate on 91-day Treasury bills) is 5 percent. What interest rate would you use to discount the projected profits for each of the following investments? Explain. (Hint: What risk premium would you add to the risk-free rate in each case?)
  - a. A new motel in your city.
  - b. An exploratory oil well in an unproven oil field.
  - c. A videotape rental store in your home town.
- 14-3. One manager has said: "Because an individual's utility cannot be measured, there is no reason to consider it when making decisions." Do you agree or disagree? Explain.
- 14-4. Provide two examples of decisions made by administrators at your college or university where a decision tree could have been used.
- 14-5. In what areas can a firm use insurance to reduce risk?
- 14-6. Firm *A* sells one product to a single group of consumers, whereas firm *B* sells a variety of products to different groups of consumers. Which firm is taking more risk? Explain.
- 14-7. What are two examples of decision making under risk and two examples of decision making under uncertainty?
- 14-8. Consider a gamble that involved winning or losing an amount equal to your entire initial wealth. Would you take this gamble if the probability of each outcome was 0.5? What does this imply about your preference for risk? Explain.

### Problems

- 14-1. Do each of the following distributions meet the requirements for a probability distribution? Why or why not?

<table style="width: 100%; border-collapse: collapse;"> <tr> <th style="text-align: left;">(a) <math>P_i</math></th> <th style="text-align: left;"><math>X_i</math></th> </tr> <tr> <td>-0.10</td> <td>10</td> </tr> <tr> <td>-0.20</td> <td>15</td> </tr> <tr> <td>0.30</td> <td>20</td> </tr> <tr> <td>0.50</td> <td>40</td> </tr> <tr> <td>0.50</td> <td>50</td> </tr> </table>	(a) $P_i$	$X_i$	-0.10	10	-0.20	15	0.30	20	0.50	40	0.50	50	<table style="width: 100%; border-collapse: collapse;"> <tr> <th style="text-align: left;">(b) <math>P_i</math></th> <th style="text-align: left;"><math>X_i</math></th> </tr> <tr> <td>0.30</td> <td>-40</td> </tr> <tr> <td>0.30</td> <td>-50</td> </tr> <tr> <td>0.40</td> <td>-200</td> </tr> </table>	(b) $P_i$	$X_i$	0.30	-40	0.30	-50	0.40	-200	<table style="width: 100%; border-collapse: collapse;"> <tr> <th style="text-align: left;">(c) <math>P_i</math></th> <th style="text-align: left;"><math>X_i</math></th> </tr> <tr> <td>0.20</td> <td>4</td> </tr> <tr> <td>0.40</td> <td>8</td> </tr> <tr> <td>0.30</td> <td>0</td> </tr> <tr> <td>0.15</td> <td>12</td> </tr> </table>	(c) $P_i$	$X_i$	0.20	4	0.40	8	0.30	0	0.15	12
(a) $P_i$	$X_i$																															
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0.15	12																															

- 14-2. For each of the following probability distributions, calculate the expected value ( $\mu$ ), standard deviation ( $\sigma$ ), and coefficient of variation ( $v$ ).

<table style="width: 100%; border-collapse: collapse;"> <tr> <th style="text-align: left;">(a) <math>P_i</math></th> <th style="text-align: left;"><math>X_i</math></th> </tr> <tr> <td>0.8</td> <td>20</td> </tr> <tr> <td>0.2</td> <td>-5</td> </tr> </table>	(a) $P_i$	$X_i$	0.8	20	0.2	-5	<table style="width: 100%; border-collapse: collapse;"> <tr> <th style="text-align: left;">(b) <math>P_i</math></th> <th style="text-align: left;"><math>Y_i</math></th> </tr> <tr> <td>0.1</td> <td>10</td> </tr> <tr> <td>0.2</td> <td>20</td> </tr> <tr> <td>0.4</td> <td>15</td> </tr> <tr> <td>0.3</td> <td>30</td> </tr> </table>	(b) $P_i$	$Y_i$	0.1	10	0.2	20	0.4	15	0.3	30
(a) $P_i$	$X_i$																
0.8	20																
0.2	-5																
(b) $P_i$	$Y_i$																
0.1	10																
0.2	20																
0.4	15																
0.3	30																

- 14-3. For each of the following probability distributions, where  $P_i$  is probability and  $X_i$  and  $Y_i$  are outcomes, determine the expected values ( $u_x$  and  $u_y$ ), the standard deviations ( $\sigma_x$  and  $\sigma_y$ ), and the coefficients of variation ( $v_x$  and  $v_y$ ).

<table style="width: 100%; border-collapse: collapse;"> <tr> <th style="text-align: left;">(a) <math>P_i</math></th> <th style="text-align: left;"><math>X_i</math></th> <th style="text-align: left;"><math>Y_i</math></th> </tr> <tr> <td>0.20</td> <td>-5</td> <td>15</td> </tr> <tr> <td>0.40</td> <td>0</td> <td>20</td> </tr> <tr> <td>0.10</td> <td>10</td> <td>30</td> </tr> <tr> <td>0.30</td> <td>20</td> <td>50</td> </tr> </table>	(a) $P_i$	$X_i$	$Y_i$	0.20	-5	15	0.40	0	20	0.10	10	30	0.30	20	50	<table style="width: 100%; border-collapse: collapse;"> <tr> <th style="text-align: left;">(b) <math>P_i</math></th> <th style="text-align: left;"><math>X_i</math></th> <th style="text-align: left;"><math>Y_i</math></th> </tr> <tr> <td>0.10</td> <td>0</td> <td>50</td> </tr> <tr> <td>0.20</td> <td>10</td> <td>40</td> </tr> <tr> <td>0.60</td> <td>20</td> <td>30</td> </tr> <tr> <td>0.10</td> <td>30</td> <td>20</td> </tr> </table>	(b) $P_i$	$X_i$	$Y_i$	0.10	0	50	0.20	10	40	0.60	20	30	0.10	30	20
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0.10	0	50																													
0.20	10	40																													
0.60	20	30																													
0.10	30	20																													

- 14-4. Given the following probability distribution for returns on an investment in the common stock of two firms:

Probability	Percentage Returns	
	Amalgamated Gold	First National Bank
0.2	-20	-5
0.4	-10	4
0.3	20	6
0.1	40	8

Compute the expected value, standard deviation, and coefficient of variation for each investment.

- 14-5. Given the following probability distribution for the percentage returns on the common stock of General Motors and S&P 500 Index, determine the expected return and standard deviation for each.

Probability	Percentage Returns	
	General Motors	S&P Index
0.2	-10	0
0.3	10	5
0.5	20	10

- 14-6. A gamble consists of a choice between two games. The first game requires rolling an ordinary die and receiving \$10 times the number shown. The second game in-

volves flipping a coin and receiving \$70 if it comes up heads and nothing if it comes up tails. Which game would be selected by a person with utility function  $U = \ln W$ ? What choice would be made if the utility function is  $U = W^2$ ?

- 14-7. Consider a person with initial wealth of \$1,000 who has a utility function

$$U = \ln W$$

where  $W$  is wealth. Suppose this person is offered a gamble that involves winning \$100 with probability 0.50 and losing \$100 with probability 0.50.

- Compute the level of utility at the initial wealth.
  - Given that the gamble is accepted, compute the expected wealth, the utility of expected wealth, and the expected utility of wealth.
  - Will this person take the gamble? Explain.
  - What is the certainty equivalent of the gamble.
- 14-8. A sales representative for the Rapid Vacuum Cleaner Company knows from past experience that she will sell a vacuum cleaner in two of every five homes in which she gives a demonstration. In a typical day she will give demonstrations in 10 homes. If it costs \$15 to give each demonstration (this includes all implicit and explicit costs) and if her commission is \$60 for each cleaner sold, determine expected net profit per day.
- 14-9. The Lac DuFlambeau Corporation manufactures a broad line of fishing tackle and accessories. The firm has surplus cash and is considering the acquisition of a firm that manufactures fly rods. A broker identifies two firms, Flyrite and Perfect Rod, that could be purchased for the same amount. The probability distributions for the present value of all future profits for each of these acquisition candidates are as follows:

Flyrite		Perfect Rod	
Probability	Outcome	Probability	Outcome
0.40	0	0.20	100,000
0.20	200,000	0.50	200,000
0.40	500,000	0.30	250,000

- Make a complete investment analysis of each investment (i.e., compute the expected present value of all future profits, a measure of risk, and a measure of risk per dollar of expected returns).
  - Which of the two firms should be acquired? Why?
- 14-10. For each of the following functions relating utility ( $U$ ) and money ( $M$ ), determine if the function implies risk-averse, risk-seeking, or risk-neutral behavior. Explain.
- $U = M + 0.25M^2$
  - $U = 10M$
  - $U = 500M - 2M^2$  (relevant only for  $M \leq 250$ )
- 14-11. A gambler has initial wealth of \$200, a utility function  $U = U^2$ , and is offered a gamble involving a bet of \$100. The payoffs are \$10, \$50, and \$200, with probability 0.4, 0.4, and 0.2, respectively.
- What is the investor's utility if the gamble is not taken?

- b. If the gamble is taken, what is the expected wealth? What is the utility of expected wealth and the expected utility of wealth?
- c. What is the certainty equivalent of the gamble?
- d. Will this person take the gamble if he is a wealth maximizer? Explain.
- e. Will this person take the gamble if he is a utility maximizer? Explain.
- f. What is the most this person would bet in order to take the gamble?
- 14-12. United Steel, Inc., has experienced losses for several years. The production department has determined the only hope for reversing the negative trend in profits is to build a new plant outside the United States. Two alternative locations have been identified. Production in country *A* would be very efficient and low-cost, but the government is unstable. In the event of a revolution, United's assets might be appropriated. Country *B* has a much more stable government, but production costs are considerably higher than in *A*.

The following data show the probability distribution for the present value of all future profits for each alternative.

Country A		Country B	
Probability	Profits (millions)	Probability	Profits (millions)
0.40	\$ 0	0.40	\$10
0.20	20	0.40	20
0.40	60	0.20	30

- a. Make a complete analysis of both alternatives. Which alternative should be chosen? Why?
- b. If the collective utility function of the board of directors is

$$U = 200X - X^2$$

where  $X$  is measured in millions of dollars of profit, which alternative should be selected? Explain.

- 14-13. Management at Unique Publishing has been paying an annual premium of \$3,000 for fire insurance on their \$100,000 plant. Net profit for Unique consistently is \$100,000 per year after deducting the insurance premium. Any uninsured loss due to fire would be an expense in computing net income. A study has shown that the probability of fire during a year is only 0.004. (Assume that any fire would destroy the plant.) A consultant suggests that Unique cancel its fire insurance. Evaluate this recommendation for each of the following assumptions:
- a. The sole objective of management is to maximize profit (i.e., no consideration is given to risk).
- b. The sole management objective is maximizing utility and the relevant utility function is  $U = 100\pi - 0.5\pi^2$ , where  $\pi$  is net profit in thousands of dollars.
- c. Given the utility function from part (b), compute and compare the expected utility of profit and the utility of expected profit. What does this tell you about the risk preferences of management at Unique Publishing?
- 14-14. An individual has a total wealth of \$1,000 including a home valued at \$250. The probability that the home will be totally destroyed by fire in any given year is 0.2. An insurance policy that would cover 100 percent of any fire loss is available for \$60. Assume the individual's objective is utility maximization.



- a. If the individual's utility function is  $U = 2W$ , should the insurance policy be purchased? Explain.
- b. If the individual's utility function is  $U = \sqrt{W}$ , should the insurance policy be purchased? Explain.
- c. If the insurance company's expenses are 20 percent of claims paid out, what is the company's expected profit on this policy?
- 14-15. State University is considering an evening MBA program. Dean Stephens determines that a minimum enrollment of 100 in a graduate program is necessary for a breakeven operation. As the only MBA program in the state, enrollment probably would be 125 students. Southwestern, a private university, is the only other school in the area that offers graduate programs. The dean thinks the probability is about 0.67 that Southwestern would respond to the State program with their own MBA program. If it did, enrollment probably would only be 70 students in the State program. However, if State offered both MBA and MPA (Master of Public Administration) programs, Dean Stephens thinks that enrollment would be 175 with no competition and 90 if Southwestern offers a program. Because of similarities in course requirements, the dean thinks that the MBA and MPA programs could be run as one graduate program. The dean believes that offering both degrees would reduce the probability that Southwestern would enter the MBA degree market to about 0.2.
- a. Construct a decision tree that shows each possible outcome and the probability of its occurring.
- b. Determine the expected number of students in State's MBA program if offered by itself and as a combined MBA-MPA program, where both degrees would be offered.
- 14-16. The manager of the student union building has to decide between introducing a low-cost pizza outlet or a gourmet sandwich shop in the basement of the building. The university's enrollment may increase (with probability = 0.8) or decrease, and there may be a tuition increase (probability = 0.6) or decrease. An enrollment increase would be good for sales of either type of food unit. A tuition increase would have a negative effect on sales but would affect the sandwich shop more than the pizza stand. Annual profits for each alternative and state of nature are shown below.

**Pizza Stand Profit:**

		Tuition	
		Up	Down
Enrollment	Up	\$100	\$200
	Down	50	90

**Gourmet Sandwich Shop:**

		Tuition	
		Up	Down
Enrollment	Up	\$60	\$300
	Down	40	160

- a. Set up the decision tree for this problem.  
 b. Make a complete investment evaluation of the two alternatives. Which should be selected? Explain.
- 14-17. Acme Manufacturing is considering three alternatives, *A*, *B*, and *C*, for a new plant. Cost data for each of these plants are shown here. The cost of shipping one unit of output to the market will vary, because each plant would be at a different location.

	<i>A</i>	<i>B</i>	<i>C</i>
Annual fixed cost	\$100,000	\$200,000	\$300,000
Production cost per unit	20	18	15
Shipping cost per unit	5	6	6

The quantity sold will depend on economic conditions for the next year, as indicated here:

<i>Economic Condition</i>	<i>Probability</i>	<i>Quantity Sold</i>
Normal	0.8	100,000
Recession	0.2	80,000

The per-unit price of the product will depend entirely on the price set by Zenith Steel, the primary competitor in the market. It is thought that the probability of various prices being set will depend on the plant built by Acme, as shown here:

<i>Acme Builds Plant</i>	<i>Probability That Zenith Responds with a Price of:</i>		
	<i>\$20</i>	<i>\$25</i>	<i>\$30</i>
<i>A</i>	0.10	0.40	0.50
<i>B</i>	0.30	0.30	0.40
<i>C</i>	0.60	0.20	0.20

Because of Zenith's dominance in the market, whatever price it sets will be followed by Acme.

- a. Construct a decision tree showing the first-year net profit for Acme for each combination of plant size, economic condition, and pricing reaction.  
 b. Determine the expected first-year profit for each plant alternative. Ignoring risk, which plant should be built?
- 14-18. Sharp Products, a major paper recycling firm, must replace its processing equipment. The only alternatives are the Century Processor, a very efficient but somewhat unreliable machine, and the Sureshot Processor, a less efficient but almost repair-free piece of equipment. The two machines have equal initial costs and have a three-year life. Due to rapid technological change, the machines will have no salvage value at the end of that period. Sharp's engineering department has estimated the expected net cash flows associated with each machine over their three-year lives.

<i>Machine</i>	<i>Expected Net Cash Flows for Year</i>		
	<i>1</i>	<i>2</i>	<i>3</i>
Century	300	400	400
Sureshot	250	350	450

The management at Sharp uses a 14 percent discount rate for most equipment purchases. However, because of the unreliability of the Century Processor, a discount rate of 18 percent is appropriate for that machine. Which machine should be purchased? Explain. (Assume that all cash flows occur at the end of each year.)

- 14-19. A firm has two investment options, with the expected return and risk for each shown in the table below.

	<i>Dynotronics</i>	<i>Macrophone</i>
Expected return ( $r_i$ )	0.12	0.09
Risk ( $\sigma_i$ )	0.11	0.07

If the firm puts 50 percent of its investment funds into each, what is the portfolio rate of return and the portfolio risk if:

- The correlation coefficient for the returns on the two investments is 0.50?
  - The correlation coefficient for the returns on the two investments is  $-0.50$ ?
- 14-20. An investor is offered the opportunity to buy shares of stock in one of two companies, both of which have a 20-year life. Company A pays an annual dividend of \$6.70 per share and has a  $\sigma = 1.0$ . Company B pays an annual dividend of \$10 per share and has a  $\sigma = 3$ . The investor uses the following formula to determine the decimal equivalent of the risk-adjusted rate to use in discounting future dividends:

$$r = 0.10 + 0.02\sigma$$

The investor can buy shares in either company at \$55 per share.

- Which shares if any should be purchased? Explain.
- What is the maximum price that this investor would pay for a share of each stock?

### Computer Problems

The following problems can be solved by using the TOOLS program (downloadable from [www.prenhall.com/petersen](http://www.prenhall.com/petersen)) or by using other computer software.

- 14-21. High-Risk Strategies, an international commodity trading company, is faced with four alternative strategies for trading wheat. The profit for each strategy will depend critically on the weather conditions (i.e., the state of nature) prevailing during the next 12 months. The set of profit outcomes for each strategy-state of nature combination is outlined here:

<i>Weather Conditions (probability)</i>	<i>Strategy</i>			
	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
Hot-dry (0.10)	\$ 2,800,000	\$1,700,000	\$-2,410,000	\$-3,625,000
Warm-wet (0.20)	7,400,000	1,920,000	-3,440,000	-5,100,000
Warm-dry (0.30)	4,520,000	1,800,000	2,140,000	1,460,000
Cool-wet (0.20)	-2,900,000	1,400,000	6,240,000	7,800,000
Cool-dry (0.20)	-2,100,000	1,100,000	5,110,000	6,125,000

Evaluate each alternative and determine which strategy should be implemented.

- 14-22. United Fabricated Products is faced with the decision to build a new plant near the site of a proposed new steel mill. Three plant sizes are under consideration—

small, medium, and large. Unfortunately, if United is to build the plant, it must make the commitment now before it is certain that the steel mill actually will be built. If United delays the decision, its option on the only available site will expire, and a major competitor, Walters Steel Products, will exercise its subordinated option on the land. The best estimate available suggests that the probability the mill will be built is 0.75. The costs are: small plant, \$12.1 million; medium plant, \$18.0 million; and large plant, \$25.7 million.

The profit outcomes for each possible outcome and the initial cost of each plant site are shown here:

Steel Mill	Present Value of Profits (millions) for Each Plant Size		
	Small	Medium	Large
Built	\$14.6	\$29.4	\$45.7
Not built	-3.4	-11.5	-26.5

Should United build a plant and, if so, which size? Explain.

- 14-23. Secure Money Managers limits its investments to five securities. The annual estimated rate of return on each of these for each state of the economy is shown here. The probabilities of the latter are: recession, 0.12; moderate economic growth, 0.65; and rapid economic growth, 0.23. Compute the expected return, risk, and risk per unit of return for each security.

Economic Condition	Security				
	International Meats	United Trucking	Outland Steel	Southern Telephone	HAL Inc.
Recession	11.4	-14.0	-9.5	11.1	5.0
Moderate growth	11.9	9.7	11.0	12.0	13.0
Rapid growth	12.2	21.6	16.2	12.4	14.2

- 14-24. Chi-Town Promotions must select a rock group for its next concert tour. The financial and marketing vice-presidents have estimated the following probability distributions for profits for each of four groups. Which should be selected?

Electric Banana		Zinc Dirigible		The Grandpas and the Grandmas		Hug	
Prob.	Profit	Prob.	Profit	Prob.	Profit	Prob.	Profit
0.73	\$2,450,000	0.42	\$-5,400,000	0.22	\$1,705,000	0.12	\$-3,825,000
0.17	-1,400,000	0.21	1,650,000	0.23	2,124,000	0.23	-1,024,000
0.10	200,000	0.13	4,800,000	0.55	1,925,000	0.29	2,950,200
		0.24	6,450,000			0.36	8,100,200

# CHAPTER

# 15

## Capital Budgeting

- **Preview**
- **Maximization of Shareholder Value and Capital Budgeting**
- **The Capital Budgeting Process**
  - Projecting the Cash Flows
  - Evaluating the Capital Project
  - Capital Rationing and the Profitability Ratio
  - Linear Programming and Capital Rationing
- **The Cost of Capital**
  - Cost of Debt Capital
  - Cost of Equity Capital
  - The Composite Cost of Capital
- **Mergers and Acquisitions**
  - Types of Mergers
  - Merger Incentives
  - Merger Procedures
- **Summary**
- **Discussion Questions**
- **Problems**

**PREVIEW**

Most of the principles of managerial economics covered thus far have focused on short-run decisions. For example, determining the profit-maximizing price and output rate are important short-run decisions. Of equal importance are decisions about the nature of the firm in the long run. What new items should be added to or eliminated from the product line? Should old capital equipment be replaced? Should a new plant be built or should a competitor be acquired to broaden the product line or increase the number of retail outlets?

Decisions such as these are absolutely crucial to the long-run profitability of the firm. Recall that the objective is to make those decisions that will maximize shareholder value, which has been defined as the present value of all future profits or net cash flows. To meet this objective, assets must be continually deployed and redeployed to capture new profit opportunities. For example, General Motors' entry into the credit card market and the development of cable television services by several of the regional telephone companies (e.g., NYNEX and Bell Atlantic) represent significant new resource commitments by these firms.

In this chapter, the firm's long-run decisions are considered. First, the nature of capital budgeting decisions and the process of making long-run investments are considered. Next, alternative evaluation techniques for evaluating capital projects are developed and applied. Several special topics are considered, including capital budgeting in an inflationary environment and allocating limited capital funds under capital rationing. Methods for estimating the cost of capital funds are reviewed, including the Capital Asset Pricing Model (CAPM), which has proven to be one of the fundamental developments in the modern theory of finance. Finally, mergers and acquisitions are discussed. This topic is a special case of capital budgeting. For example, a firm may decide to enter the computer business by building a new plant, staffing it with new production personnel, and developing a marketing force. Alternatively, the firm may seek to acquire (or merge with) an existing firm that already is in the computer market. Consideration of either alternative requires the application of capital budgeting principles.

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**MAXIMIZATION OF SHAREHOLDER VALUE  
AND CAPITAL BUDGETING**

Capital projects are those that are expected to generate returns for more than one year. Capital budgeting refers to the process of planning capital projects, raising funds, and efficiently allocating resources to those capital projects. Examples of capital projects include new factories, machines, automobiles and trucks, and computers. Outlays for research and development and advertising programs are also capital expenditures if the returns on those projects will flow for more than one year.

It is useful to categorize capital projects in the following way:

1. **COST REDUCTION:** Investments in training, machinery, or other capital assets that reduce the cost of producing output.
2. **OUTPUT EXPANSION:** Investments that accommodate increased output in response to actual or expected increases in demand.<sup>1</sup>

<sup>1</sup>The same investments may result in both output expansion and cost reduction simultaneously. This certainly would be true if there are increasing returns to scale in production. In that case, an increase in output due to a capital expenditure necessarily would reduce the average cost of output.

### 3. EXPANSION BY DEVELOPING NEW PRODUCTS AND/OR MARKETS:

Expenditures for the development and production of new products and/or the development of new markets by adding sales staff or by opening new outlets.

### 4. GOVERNMENT REGULATION: Expenditures made to meet government safety, environmental, and other rules.

Capital projects typically are very costly. Indeed, many firms seek external financing to implement a capital spending program. Furthermore, most capital spending projects are not easily reversed. For example, once a new manufacturing plant is built, it may have no other use than the one intended. Thus, it would be difficult to sell if a change in market conditions rendered it unnecessary. For these reasons, capital planning decisions may determine the course for the firm for many years to come. The problems associated with a poor pricing decision or incorrect estimate for one production run may be rectified quickly. Generally, this is not true for a major capital project. If capital spending decisions are poorly made, the firm's existence may be threatened.

Capital budgeting requires information on sales, production costs, advertising, and availability of funds, and therefore generally involves all areas of management. Furthermore, because of its critical long-run importance, the capital budgeting process usually is reviewed on a continuous basis by the top management of the firm.

The principle underlying capital budgeting decisions is that expenditures are made until the marginal return on the last dollar invested equals the marginal cost of capital. The cost of capital is the rate that must be paid on money raised externally by the firm (e.g., by borrowing or selling stock) or the opportunity cost (i.e., the foregone return) on funds the business has that it would have invested elsewhere or that the owners could have spent on consumption.

Recall that the value of the firm is equal to the sum of all future net profits reduced to their present value using an appropriate discount rate, that is,

$$\text{Value} = \sum_{t=1}^n \frac{R_t - C_t}{(1+r)^t} \quad (15-1)$$

where  $R_t$  is revenue in the  $t$ th period,  $C_t$  is cost, and  $r$  is the discount rate, which is the firm's opportunity cost of the funds used to make the investment. The basic capital budgeting question is: Will the capital expenditure, which itself will increase the firm's costs, increase revenue and/or reduce other costs sufficiently to increase the value of the firm? If the rate of return on the project is greater than the marginal cost of capital, the value of the firm will increase, and therefore the proposed capital project should be implemented.

Suppose that management has identified and evaluated each of the capital projects shown in Table 15.1. The projects are listed in decreasing order of rate of return. The schedule can be used to determine the quantity of capital demanded by the firm depending on the cost of capital. Thus, these data represent the firm's demand for capital function, which is shown in Figure 15.1.

Using the marginal revenue/marginal cost principle, the firm will invest as long as the return on an investment is equal to or greater than the cost of capital. For example, if the cost of capital is 20 percent, none of the investment projects would be implemented because that cost exceeds the highest return (18 percent) available on a capital project. Thus, if the firm borrowed \$4.5 million at 20 percent interest to implement the first capital project, the annual interest of \$900,000 would exceed the average annual

TABLE 15.1 List of Hypothetical Capital Expenditures

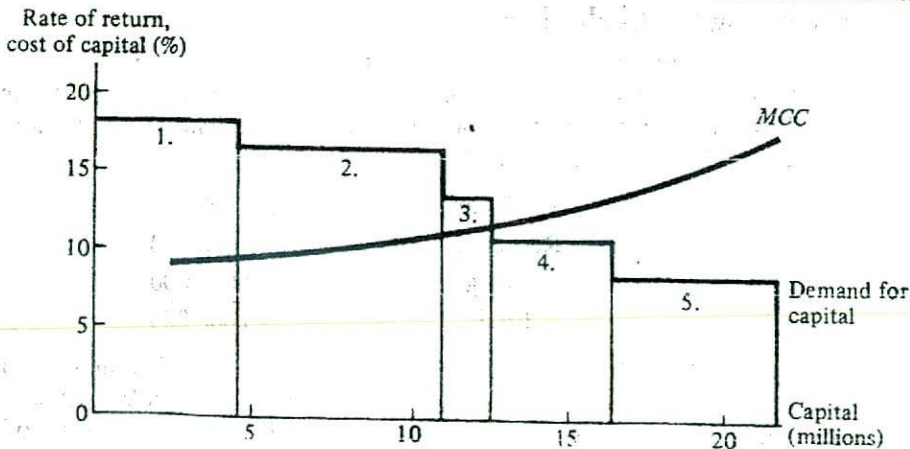
Project Description	Type of Capital Expenditure	Cost (millions)	Rate Return (%)
1. Replacement of obsolete equipment on an assembly line	Cost reduction	\$4.5	18
2. Opening of three new stores on the East Coast	Market expansion	6.2	17
3. Formal training programs for all production employees	Cost reduction	1.5	14
4. Replacement of outdated heating and air-conditioning system	Cost reduction	3.9	12
5. New production facility	Output expansion	5.1	10

return (\$810,000) on the project, and both profit and the value of the firm would be reduced. However, if the cost of capital was 15 percent, both projects 1 and 2 would be implemented because their rates of return are 18 and 17 percent, respectively. In this case, the firm would spend \$10.7 million on these two capital items. If the cost of capital was lower, the firm would make even more capital outlays.

The marginal cost of capital function (*MCC* in Figure 15.1) shows the cost to the firm of obtaining various amounts of capital. Determining the firm's cost of capital is discussed later in the chapter. The cost is shown as a gradually rising function because most firms are required to pay a higher cost to obtain increasing amounts of capital. For example, if the firm is borrowing those funds, the more that is borrowed, the greater is the risk that the firm will be unable to repay the lender. To be compensated for taking that additional risk, the lender must earn a higher return, which is accomplished by increasing the interest rate charged to the firm.

Figure 15.1 indicates that the firm would make investments 1, 2, and 3 because the rate of return exceeds the cost of capital for these projects. Note that the rate of return on investment 3 is 14 percent, and the rate of return on investment 4 is 12 percent.

FIGURE 15.1 Marginal Cost of Capital Function





However, because the firm's cost of capital is now more than 12 percent, investment 3 is made but 4 is not (i.e., in the figure, the *MCC* curve lies above the demand for capital schedule to the right of project 3).

### Key Concepts

- Capital budgeting refers to the process of planning capital projects, raising funds, and efficiently allocating those funds to capital projects.
- Capital expenditures are made in order to reduce cost, increase output, expand into new products or markets, and/or meet government regulations.
- In general, capital expenditures are made until the rate of return on the last dollar invested equals the marginal cost of capital.
- The demand for capital function shows the amount of capital spending that will be made at each cost of capital.

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## THE CAPITAL BUDGETING PROCESS

In most firms, capital budgeting is a continuous process, with proposals being made regularly in all areas of the organization. Typically, each level of the organization has authority to make capital expenditures up to some dollar limit. Proposals in excess of that limit must be screened and approved by higher levels of management to ensure that the projects are consistent with the overall plan for the firm. For example, the production line foreman may have authority to make capital expenditure decisions up to a level of \$25,000. Proposals above that amount must be approved by the plant manager, who may be able to authorize expenditures up to \$250,000 but must have approval from the division vice-president for projects that cost more than that amount. Thus, the larger the dollar cost of the proposal, the more screening steps it must go through.

The first step in this evaluation is to determine the cost of the project. Next, the net cash flows are estimated. Finally, evaluation techniques are used to compare cash inflows to the cost of the project. In the following, it is assumed the cost of the project has already been determined so that the focus is on projecting and evaluating the cash flows.

### Projecting the Cash Flows

Suppose that a firm is considering the addition of a new product line. It is estimated that the cost of new production machinery, reorganization of the production line, and additional working capital for inventory and accounts receivable will require an initial investment of \$20 million. The breakdown of these costs is as follows:

Machinery	\$15,000,000
Reorganization	2,000,000
Working Capital	3,000,000
Total	<u>\$20,000,000</u>

Sales revenues from this product are projected to be \$12 million the first year, but because the new product is a substitute for one of the firm's existing products, the incremental sales revenue will be only \$10 million. That is, the new product will add

\$12 million to sales, but sales of an existing product will fall by \$2 million, yielding a net or incremental sales increase of \$10 million. The marketing staff estimates that sales revenue will increase 10 percent each year. The research and development department has indicated that in 5 years, it can develop an entirely new product to replace the one under consideration. Therefore, management has decided to assume a 5-year life for this product and that any evaluation of the proposed project be limited to a 5-year period.

The production and engineering departments of the firm have developed the following incremental production cost estimates. Variable costs will be 40 percent of revenue and additional fixed costs will be \$100,000 per year. The finance department reports that for depreciation purposes, the new machinery has a 5-year life and that the straight-line depreciation method will be used. That is, the annual depreciation charge against income is one-fifth of the initial cost of the machinery, or \$3 million per year:

$$\text{annual depreciation charge} = \frac{\text{cost}}{\text{years}} = \frac{\$15,000,000}{5} = \$3,000,000$$

The financial vice-president also indicates that the combined federal and state marginal income tax rate for the firm is expected to be 40 percent. Note that depreciation is a noncash expense that is included as an expense in computing income taxes but is then added back in to determine the net cash flow. The only reason it is considered is because of its effect on the firm's income tax liability. The salvage value of the equipment at the end of the 5 years is estimated to be \$4 million. Also, at the end of the 5-year life of this project, the \$3 million that was needed for additional working capital is recovered. To simplify the analysis, it is assumed that all revenues and expenses occur at the end of each year.

The detailed cash flow projection for each year is shown in Table 15.2. Note that there has been no consideration given to the costs of financing this project. The definition of cash flow when making capital budgeting decisions is the after-tax cash flow, assuming that the firm has no debt. The reason for this is that when using an appropriate evaluation technique, future cash flows are discounted at a rate equal to the firm's cost

TABLE 15.2 Annual Cash Flow for Proposed Capital Investment

	Year				
	1	2	3	4	5
Sales	\$10,000,000	\$11,000,000	\$12,100,000	\$13,310,000	\$14,641,000
Less: Variable costs	4,000,000	4,400,000	4,840,000	5,324,000	5,856,400
Fixed costs	100,000	100,000	100,000	100,000	100,000
Depreciation	3,000,000	3,000,000	3,000,000	3,000,000	3,000,000
Profit before tax	\$ 2,900,000	\$ 3,500,000	\$ 4,160,000	\$ 4,886,000	\$ 5,684,600
Less: Income tax	1,160,000	1,400,000	1,664,000	1,954,400	2,273,840
Profit after tax	\$ 1,740,000	\$ 2,100,000	\$ 2,496,000	\$ 2,931,600	\$ 3,410,760
Plus: Depreciation	3,000,000	3,000,000	3,000,000	3,000,000	3,000,000
Net cash flow	\$ 4,740,000	\$ 5,100,000	\$ 5,496,000	\$ 5,931,600	\$ 6,410,760
					Plus: Salvage value of machinery
					\$ 4,000,000
					Recapture of working capital
					3,000,000
					Net cash flow (year 5)
					\$13,410,760

of capital. This automatically accounts for the financing costs. To include such costs as a subtraction from the net cash flows would effectively count them twice, and thus would be incorrect.

### Evaluating the Capital Project

The remaining problem in this example is to rationally compare the projected net cash flows to the initial \$20 million investment that is required so that the proposed project can be accepted or rejected. In order to maximize the value of the firm, the evaluation technique should

1. Consider all relevant cash flows.
2. Discount cash flows using the firm's opportunity cost of capital.
3. Select the one project from a set of mutually exclusive projects that maximizes the value of the firm.<sup>2</sup>
4. Allow each project to be evaluated independently of all others being considered.

Three evaluation techniques will be considered: payback period, internal rate of return, and net present value. As will be shown, the payback period method, although still used in some circles, fails to meet any of the four criteria just outlined and will be dismissed as a viable method. Under most circumstances, the internal rate of return method yields value-maximizing results, but it fails to meet criterion (2) and sometimes (3) and (4). Thus, it is inferior to the net present value method, which meets all four criteria and will always guarantee that the project or projects selected for implementation will maximize the value of the firm.

**Payback Period** Under this method, the number of years that it takes for the net cash flows (undiscounted) to equal the cost of project is defined as the payback period. The decision rule is to select the project that has the shortest payback period. Consider the projected net cash flows for two proposed capital projects, *A* and *B*:

Project	Initial Cost	1	2	3	4	5	Payback Period
A	1,000	500	500	1,000	1,000	1,000	2
B	1,000	250	250	500	5,000	10,000	3

Note that for project *A*, the sum of the cash flows equals the cost of the project in 2 years, whereas it takes 3 years in the case of project *B*. Thus, using the payback period rule, project *A* would be selected. However, even the most casual analysis would indicate that project *B* should be preferred because it has much larger cash flows in periods four and five. The net present value will be higher for *B* than for *A* for any normal discount rate. The payback period method fails because it does not consider all cash flows and does not discount those flows to their present value. Obviously, this method is a poor evaluation technique that warrants no further consideration.

<sup>2</sup>Mutually exclusive means that the selection of one project from a set of alternative projects precludes the others from being implemented. For example, a firm may be considering alternative design proposals for a new plant where only one will be selected.

**Internal Rate of Return** The implicit rate of return on a capital expenditure can be measured using the internal rate of return (*IRR*) evaluation method. The internal rate of return is the discount rate that equates the present value of the cash flows with the initial investment cost. Define  $A_t$  as net cash flow in year  $t$ ,  $C$  as the initial cost of the project, and  $n$  as the life of project. Now the *IRR* is determined by setting

$$\sum_{t=1}^n \left[ \frac{A_t}{(1+r^*)^t} \right] = C \quad (15-2)$$

and solving this equation for  $r^*$ , the internal rate of return. The decision rule is that if  $r^*$  is greater than the cost of capital, the investment should be made. That is, profits, and therefore the value of the firm, will be increased by making this investment because the firm is using capital that costs, say, 12 percent to earn a return greater than 12 percent.

Clearly, solving equation (15-2) for  $r^*$  by hand is difficult when  $n$  is greater than 1. Fortunately, financial calculators and easily used computer programs are available that can solve *IRR* problems quickly. In their absence, trial-and-error methods can also be used. To illustrate the trial-and-error approach, arbitrarily select a discount rate and evaluate the present value of the cash flows. If the present value is higher than the cost, increase the discount rate and repeat the process. If the present value is lower than the cost, reduce the discount rate. By gradually adjusting the discount rate, this iterative process will ultimately lead to the *IRR*.<sup>3</sup>

Again using the data from the example in Table 15.2, the *IRR* method would require setting up the equation

$$\frac{4,740,000}{1+r^*} + \frac{5,100,000}{(1+r^*)^2} + \frac{5,496,000}{(1+r^*)^3} + \frac{5,931,600}{(1+r^*)^4} + \frac{13,410,760}{(1+r^*)^5} + = \$20,000,000$$

and solving for  $r^*$ , which yields  $r^* = 0.179$ , or 17.9 percent. Because this rate exceeds the cost of capital (12 percent), the investment should be made.

**Net Present Value** The net present value (*NPV*) method of evaluation consists of comparing the present value of all net cash flows (appropriately discounted using the firm's cost of capital as the discount rate) to the initial investment cost. If the present value of the cash flows exceeds the cost, the proposal meets the evaluation criterion; that is, the value of the firm will be increased by making the investment. Equivalently, if  $NPV > 0$  the implicit rate of return on the capital expenditure exceeds the firm's cost of capital, and thus future profits will be higher if the investment is made. Using the *NPV* method, the rate of return on the investment is not determined explicitly. Therefore, it is referred to as an *implicit rate of return*. If the net present value is negative, the implicit return is less than the cost of capital.

Formally, the *NPV* rule is that if

$$NPV = \sum_{t=1}^n \left[ \frac{A_t}{(1+r)^t} \right] - C > 0 \quad (15-3)$$

<sup>3</sup>There are special cases where it is possible that two or more discount rates will equate the present value of the cash flows to the cost. That is, multiple values of the internal rate of return are obtained. This can occur in unusual situations where cash flows vary from positive to negative from one period to another. Such cases are so special as not to be of concern in this text. Interested readers are referred to T. Copeland and J. Weston, *Financial Theory and Corporate Policy*, 3rd ed. (Reading, MA: Addison-Wesley, 1988).

the capital expenditure should be made. For the example shown in Table 15.2, suppose that the firm's cost of capital is 12 percent. The net cash flows at the end of each year are given in the table. Note that at the end of the project's life (i.e., 5 years), the cash inflow from the salvage value of the machinery and the recapture of dollars tied up in inventory and accounts receivable are included. Based on a cost of \$20,000,000, the *NPV* calculation would be

$$NPV = \left[ \frac{4,740,000}{1.12} + \frac{5,100,000}{(1.12)^2} + \frac{5,496,000}{(1.12)^3} + \frac{5,931,600}{(1.12)^4} + \frac{13,410,760}{(1.12)^5} \right] - 20,000,000$$

$$= 23,589,040 - 20,000,000 = 3,589,040$$

As the *NPV* > 0, the proposal passes the test. This means that the return on the new product line exceeds the firm's cost of capital, and the investment should be made because it will increase the value of the firm.

**Comparison of *NPV* and *IRR* Evaluation Methods** Generally, when evaluating a single project, the *IRR* and *NPV* methods will yield consistent results. That is, if the net present value is positive, the internal rate of return will be greater than the firm's cost of capital, and vice versa. This is because for net present value to be positive, the implicit rate of return on the project must be greater than the discount rate. Therefore, for any single project, the two approaches usually give the same accept/reject signal.

However, in the case of two or more mutually exclusive projects (i.e., where only one of the investments will be made), the two evaluation techniques can result in contradictory signals about which investment will add more to the value of the firm. Consider the cash flow and evaluation criteria values for investments *A* and *B* in Table 15.3. Project *A* has a lower net present value but a higher internal rate of return. The reason for this inconsistency is that there is a difference in the implied reinvestment rate for the annual cash flows each year. That is, the cash returns that flow from the capital project each year are reinvested, but the rate earned on those reinvested dollars is not known. Therefore, an assumed rate must be used. In the *NPV* approach, it is implicitly assumed that the cash flows are reinvested at an interest rate equal to the firm's cost of

**TABLE 15.3** Cost, Cash Flow, and Evaluation Criteria Data for Two Hypothetical Capital Investments

	Project A	Project B
Initial cost	\$1,000	\$1,000
Net cash flows (year)		
1	450	-300
2	450	0
3	450	500
4	450	600
5	450	2,000
Evaluation criteria		
<i>NPV</i> (12% discount rate)	622	675
<i>IRR</i>	34.9	24.2

capital. Under the *IRR* method, the implicit reinvestment rate is the computed rate of return on the capital investment being considered. This is an optimistic assumption because the rate of return on the firm's investment opportunities in future years may not be as high as for the project being considered. Thus, the *NPV* method offers the more conservative approach.

The *NPV* method meets all of the four criteria presented previously and will always select that project or projects that maximize the value of the firm, whereas under some circumstances, the *IRR* technique will not. For this reason, the *NPV* approach is preferred.

Essentially, the value of the firm will be maximized by aggressively developing capital project proposals, carefully estimating their initial cost and future cash flows, discounting those flows using the firm's opportunity cost of funds, and then implementing all proposals having a positive net present value.

## Case Study

### Capital Budgeting in the Real World

Surveys have been made concerning the procedures used by managers to evaluate proposed capital projects. A summary of the results from one survey on the primary and secondary methods used is shown here:

	<i>Percentage of Firms Using Method</i>	
	<i>1959</i>	<i>1985</i>
Primary evaluation method		
Net present value	7	21
Internal rate of return	—	49
Payback period	42	19
Other	51	11
Secondary evaluation method		
Net present value	1	24
Internal rate of return	1	15
Payback period	15	35
Other	82	26

Although most firms now use the net present value and internal rate of return techniques, a significant number still rely on the payback and other methods that do not use discounted cash flow analysis. However, when compared to the results obtained some 26 years earlier, there has been a marked increase in the number of firms using the better evaluation approaches. In 1959, less than 10 percent of the firms surveyed were using discounted cash flow analysis (i.e., either *NPV* or *IRR*). By 1985, about 70 percent were using one or the other as their primary method and 39 percent were using one of them as their secondary method.

Competitive pressures will ensure that managers will adopt state-of-the-art techniques in all phases of their business operations. To do otherwise will result in lower

profits and perhaps even losses. The firm that is not efficient in all areas, including capital budgeting, is not likely to be successful in a competitive business environment.

SOURCE: Suk H. Kim, Trevor Crick, and Seung H. Kim, "Do Executives Practice What Academics Preach?" *Management Accounting* (November 1986): 42-52.

### Key Concepts

- The internal rate of return is that discount rate that equates the present value of all future net cash flows to the cost of an investment. If the internal rate of return on an investment exceeds the cost of capital, the investment will increase profits.
- Using the net present value evaluation technique, if the present value of all future cash flows (discounted by the firm's cost of capital) exceeds the initial cost of the project, the investment should be made.
- The use of the *NPV* technique will always lead to investment decisions that maximize the value of the firm.
- Generally, the *NPV* and *IRR* methods yield consistent accept/reject signals. However, when comparing two mutually exclusive projects, the relative ranking of the projects can be different using the two methods because the *IRR* technique assumes that future cash flows can be reinvested at the internal rate of return for the project being evaluated.

### Capital Rationing and the Profitability Ratio

Up to this point, it has been assumed that the firm would make all capital expenditures that meet the criteria that  $NPV > 0$  or that  $IRR > \text{cost of capital}$ . However, in some cases, the total amount of money the firm has and that can be obtained by borrowing or by selling stock is less than the total that would be spent if all projects were undertaken. In this case, choices must be made among those projects that meet the evaluation criteria. That is, available capital funds must be rationed among these competing projects.

There are several reasons why a business may be subject to capital rationing. First, the sheer number of capital proposals, if they all were implemented, may exceed management's ability to develop and manage them. Clearly, there is a limit to the scope and number of projects that can be effectively monitored at any given time. Second, internal funds are limited, and management may prefer not to take the risk associated with additional borrowing and/or the possible reduction in control associated with the sale of additional shares of common stock. Finally, an operating unit of a larger firm may arbitrarily be assigned a maximum capital expenditure budget for the year.

Strict adherence to the *NPV* evaluation approach (i.e., selecting the project with the greatest *NPV*) can lead to nonoptimal decisions if capital is rationed. Suppose that

<i>Cost</i>	<i>Project A</i>	<i>Project B</i>	<i>Project C</i>
	<i>\$20,000</i>	<i>\$10,000</i>	<i>\$10,000</i>
	<i>Net cash flows per year</i>		
Period 1	\$9,000	\$4,700	\$4,700
Period 2	9,000	4,700	4,700
Period 3	9,000	4,700	4,700
<i>NPV (12% discount rate)</i>	1,616	1,289	1,289

the firm's capital budget was limited to \$20,000, and it was considering the three investments shown in Table 15.4. Further, assume that the firm's policy is to rank projects by their *NPV* and then implement projects beginning at the top of the list and continuing down until the available funds are exhausted. This approach would be sub-optimal because only project *A* would be selected, thus adding \$1,616 to the value of the firm. But if projects *B* and *C* were selected instead of *A*, the sum of the net present values would be \$2,578. That is, the combination of investments *B* and *C* would add more to profit than would investment *A*. The problem here is that the basic *NPV* approach does not compare the relative magnitudes of the net present value and the initial cost of the project.

Consider another set of capital projects where each has a positive *NPV* but where capital rationing restricts choice to some subset of the entire array of projects. Under these conditions, the problem reduces to finding all combinations of projects that meet the capital constraint, summing the *NPVs* for each combination, and selecting the subset that has the highest sum of the individual *NPVs*. Initially, consider the set of individual projects *A* through *F* in Table 15.5, and assume that total capital spending is limited to \$1 million.

<i>Project</i>	<i>PV of Cash Flows</i>	<i>Initial Cost</i>	<i>NPV</i>	<i>Profitability Ratio (R<sub>p</sub>)</i>
A	300,000	250,000	50,000	1.20
B	510,000	500,000	10,000	1.02
C	790,000	750,000	40,000	1.05
D	600,000	500,000	100,000	1.20
E	280,000	250,000	30,000	1.12
F	310,000	250,000	60,000	1.24
G	1,020,000	800,000	220,000	1.28



The subsets of projects that meet the \$1 million capital budget, their cost, and total NPV for each set are listed here:

<i>Combination</i>	<i>Cost</i>	$\Sigma$ <i>NPV</i>
A, B, E	\$1,000,000	\$ 90,000
A, B, F	1,000,000	120,000
A, C	1,000,000	90,000
A, D, F	1,000,000	210,000
B, D	1,000,000	110,000
B, E, F	1,000,000	100,000
C, E	1,000,000	70,000
C, F	1,000,000	100,000
D, E, F	1,000,000	190,000

Clearly, the combination *A, D, F*, which yields a combined NPV of \$210,000, is the choice that maximizes the value of the firm.

What if another project (*G*) were added to the list that had an initial cost of \$800,000 and a net present value of \$220,000? Note that implementation of this project would preclude investing in any of the others. As its NPV of \$220,000 is higher than for any other feasible combination of projects, *G* should be implemented and the remaining \$200,000 in the capital budget left in the firm's bank account.

An equivalent but less cumbersome approach uses the concept of the profitability ratio ( $R_p$ ) to rank projects. The profitability ratio is computed as one plus the present value of all future cash flows divided by the initial cost of the project; that is,

$$R_p = 1 + \frac{NPV}{COST} \quad (15-4)$$

If the firm ranked the projects *A* through *F* in Table 15.5 by the profitability ratio, projects *F, A*, and *D* would be identified directly as those to be implemented. That is, *F* has the highest ratio (1.24); *A* and *D* are next, both having a ratio of 1.20. The total cost of these three projects is \$1 million, which exhausts the capital budget. When project *G* is included, it has the highest profitability ratio (i.e., 1.28) and has an NPV greater than any other feasible combination of the other projects. Thus, it should be selected even though it does not exhaust the available budget of \$1 million. The profitability ratio approach is more efficient than the alternative of determining the cumulative net present value of each feasible combination of projects.

### Linear Programming and Capital Rationing

The linear programming tool developed in chapter 8 can be used to solve more complex capital budgeting problems. Consider a firm that has developed five capital projects, each of which requires an investment outlay in each of the first 2 years and has a positive net present value as shown in Table 15.6.

To implement all five projects would require \$112 in year 1 and \$42 in year 2. But the firm's capital budget constraints for years 1 and 2 are \$55 and \$30, respectively. That is, capital rationing applies—the firm does not have the resources to undertake all the projects. Because of the separate budget limitation in each of 2 initial years, the straight-

Project	Outlay in Year		Net Present Value
	1 ( $C_{1j}$ )	2 ( $C_{2j}$ )	( $V_j$ )
1	24	5	30
2	45	8	40
3	10	5	35
4	8	4	25
5	25	20	42

forward method used earlier will not work. However, the problem can be set up as a linear program and solved.

Define  $X_j$  as the proportion of project  $j$  undertaken. The objective function is

$$\text{Maximize } V = 30X_1 + 40X_2 + 35X_3 + 25X_4 + 42X_5.$$

(That is, maximize the sum of the net present values of all projects undertaken.) Subject to:

$$(1) 24X_1 + 45X_2 + 10X_3 + 8X_4 + 25X_5 \leq 55$$

$$(2) 5X_1 + 8X_2 + 5X_3 + 4X_4 + 20X_5 \leq 30$$

(i.e., these constraints require that the amount spent on all projects in each year not exceed the available budget),

$$(3)-(7) \quad X_j \leq 1 \text{ for } j = 1, \dots, J$$

(i.e., these constraints require that no more than 100 percent of a project's cost be undertaken) and the nonnegativity constraints

$$(8)-(12) \quad X_j \geq 0, j = 1, \dots, 5$$

The solution is

$$X_1 = 0.6056, X_2 = 0.0, X_3 = 1.0, X_4 = 1.0, X_5 = 0.8986$$

The interpretation is that the firm would fully fund projects 3 and 4 and would fund 60.56 percent of project 1 and 89.56 percent of project 2. Partially funding a capital project may be interpreted as essentially extending the original investment period to more than 2 years. Alternatively, the firm might adopt some sort of decision rule that would allow the capital budget limitation to be raised for projects where  $X_j$  was less than 1.<sup>4</sup>

Solving the dual problem yields a shadow price for each constraint. The shadow price for constraints 1 and 2 are 1.0986 and 0.72, respectively. This means that a \$1 increase in the budget for the first year would increase net present value by \$1.10, and a \$1 budget increase in year 2 would increase net present value by \$0.72. Clearly, the value of this firm could be increased if it could shift part of its budget from year 2 to year 1.

<sup>4</sup>By using the integer programming variant, all values of  $X_j$  would be forced to zero or one.

This might be accomplished by changing the timing of the firm's financing plans (i.e., borrowing, sale of stock, etc.). For example, a one-dollar shift would increase the value of the objective function by \$0.38 (i.e., \$1.10 - \$0.72).

#### Key Concepts

- If the number of different capital projects exceeds the funds available, the firm must engage in capital rationing. In this case, the firm must select that combination of projects that maximizes the total of the net present values.
- The profitability ratio, computed as one plus the net present value divided by initial cost, can be used to rank alternative investments when the firm is subject to capital rationing.
- Linear programming can be used to solve capital budgeting problems where capital rationing applies.

## THE COST OF CAPITAL

The use of the net present value and internal rate of return methods requires that future cash flows be discounted by the firm's cost of the funds used to pay for the costs of the project. The cost of such funds is referred to as the cost of capital. In general, the cost of capital is the return required by investors in the debt and equity securities of the firm. Basically, there are three sources of funds to the firm for capital spending: retained earnings, debt, and equity. In the following, the cost of debt financing and the cost of equity financing through the sale of common stock are considered. The cost of using retained earnings for capital spending is approximately the same as the cost of common stock, so the cost of retained earnings is not discussed explicitly.<sup>5</sup>

### Cost of Debt Capital

There is little controversy about the cost of capital raised by borrowing from banks or by selling bonds. That cost is the net or after-tax interest rate paid on that debt. Because interest, unlike dividends on stock, is deductible from income when computing income taxes, it is the after-tax cost that is important. For a given interest rate ( $i$ ) and marginal tax rate ( $t$ ), the after-tax cost of debt ( $r_d$ ) is given by

$$r_d = i(1 - t) \quad (15-5)$$

For example, if the firm borrows at a 10 percent interest rate and faces a 40 percent marginal tax rate, its after-tax cost of debt capital is

$$r_d = 0.10(1 - 0.40) = 0.06$$

or 6 percent.<sup>6</sup>

<sup>5</sup>A naive manager might view the cost of retained earnings as being zero. This clearly is not the case, if for no other reason than that there is a significant opportunity cost to the owners of the business, who could use those funds for personal consumption or for investment in some other business.

<sup>6</sup>Note that in the equations, the decimal equivalent of the interest rate is used. That is, a 10 percent rate of interest appears as 0.10 in an equation.

Two important considerations should be mentioned. First, if the firm is not earning profits, the pretax and after-tax interest rates will be the same because the marginal tax rate is zero. Second, the concern is with the marginal cost of capital and not the average cost of capital for the firm. For example, the fact that the average cost of debt in the firm's balance sheet is 9 percent is irrelevant. The only important consideration is the cost of raising new capital. This is because the investment decision requires comparing the rate of return on new projects (i.e., the marginal return) with the cost of acquiring additional capital (i.e., the marginal cost of capital).

### Cost of Equity Capital

In general, determining the cost of equity capital from retained earnings or sale of common stock is more complicated and more controversial than determining the cost of debt. In the following discussion, three approaches to estimating this cost are presented. Note that dividends are not deductible from income when computing income taxes, so the firm's tax rate does not play a direct role in determining the cost of equity capital.

**Method I: The Risk-Free Rate Plus Risk Premium** Generally, investment in common stock is considered to be riskier than investment in bonds. The firm has a contractual obligation to make interest and principal payments to bondholders, and such payments must be made before dividends can be paid to stockholders. If profits rise and fall, it is likely that dividends also will fluctuate. However, except in the case of severe financial problems, bondholders will be paid. Thus, it is thought that investors will demand a return on equity ( $r_e$ ) composed of a risk-free return ( $r_f$ ), usually considered to be the rate of return on long-term government bonds, plus a premium for accepting additional risk. This premium reflects the two sources of risk. First, there is the risk associated with investing in the securities of a private company as opposed to buying federal government securities. Second, there is the additional risk associated with buying stock rather than bonds of a business. The premiums associated with the two types of risk are labeled  $e_1$  and  $e_2$ . Thus, the cost of equity capital is

$$r_e = r_f + e_1 + e_2 \quad (15-6)$$

One way to measure the first type of risk ( $e_1$ ) is to use the difference between the rate of interest on the firm's bonds ( $r_d$ ) and the rate of return on government bonds ( $r_f$ ). This difference should increase as risk of default increases. For  $e_2$  a rule of thumb is used to approximate the risk of buying the common stock of a firm rather than bonds. Necessarily, this is based on judgment rather than any formula or equation. A typical approach used by financial analysts is to assume that the return on a firm's common stock should be about 3 to 5 percentage points greater than on its debt. Using the midpoint of this range (i.e., 4 percent) as an estimate of  $e_2$ , the total risk premium ( $e$ ) would be calculated as

$$e = e_1 + e_2$$

or

$$e = (r_d - r_f) + 0.04$$

For example, suppose that the risk-free rate is 8 percent and the firm's bonds are yielding 10 percent. Therefore, the total risk premium would be 6 percentage points, that is,

$$e = (0.10 - 0.08) + 0.04 = 0.06$$

and the firm's cost of equity capital would be

$$r_e = r_f + e = 0.08 + 0.06 = 0.14$$

or 14 percent.

**Method II: Discounted Cash Flow** Method I includes risk but fails to consider the possibility of growth in dividends and the value of common shares over time. An alternative approach is the discounted cash flow method. Just as the value of the firm is the present value of all future profits, the value of a share of common stock is the present value of all future dividends using the investor's required rate of return ( $r_e$ ) as the discount rate. That is, one share of stock entitles the owner to receive a series of payments (i.e., dividends) roughly equivalent to an annuity. Thus, the value of that share should equal the present value of the annuity. If the current dividend per share ( $D_0$ ) is expected to remain constant, the value or price ( $P$ ) of a share will be

$$P = \sum_{t=1}^{\infty} \frac{D_0}{(1 + r_e)^t}$$

or

$$P = D_0 \left[ \sum_{t=1}^{\infty} \frac{1}{(1 + r_e)^t} \right] \quad (15-7)$$

It can be shown that the bracketed term reduces to

$$\frac{1}{r_e}$$

so that the value of a share of stock is simply the dividend rate divided by the required rate of return, that is,

$$P = \frac{D_0}{r_e}$$

However, if the dividend is expected to increase over time at an annual rate of  $g$ , it can be shown that the price per share will be given by

$$P = \frac{D_0}{r_e - g} \quad (15-8)$$

Solving equation (15-7) for  $r_e$  yields an equation for the cost of equity capital:

$$r_e = \frac{D_0}{P} + g \quad (15-9)$$

That is, the return required by the investor is equal to the current dividend yield on the common stock ( $D_0/P$ ) plus an expected growth rate for dividend payments. The esti-

mate of that growth rate might be the historic rate of growth for the firm or, perhaps, the consensus growth rate being used by financial analysts who study the firm.<sup>7</sup>

For example, suppose that a firm is paying a dividend of \$8 per share on common stock that sells for \$100 per share and that there is agreement among financial analysts that the growth rate of dividends of this firm will be 6 percent per year. Using this approach, the firm's cost of equity capital is estimated to be 14 percent. That is,

$$r_e = \frac{8}{100} + 0.06 = 0.14 \text{ or } 14\%$$

Obviously, variations in the price of the stock will change the firm's cost of capital. For example, if investors bid up the price of this stock from \$100 to \$120 per share, the cost of capital will fall to 12.7 percent. That is,

$$r_e = \frac{8}{120} + 0.06 = 0.127 \text{ or } 12.7\%$$

**Method III: Capital Asset Pricing Model (CAPM)** The capital asset pricing model, widely used in the world of quantitative finance, emphasizes not only the risk differential between common stock and government bonds but also the risk differential among stocks.<sup>8</sup> The risk differential between stocks and government bonds is estimated by  $(r_m - r_f)$ , where  $r_m$  is the return on the average common stock and  $r_f$  is the rate on risk-free U.S. government securities.

Relative risk among stocks is measured using the beta coefficient,  $\beta$ , as a risk index. The beta coefficient is the ratio of variability in return on a given stock to variability in return for all stocks. Return on a stock for a period of time, typically one year, is the dividend plus the change in the value of the stock during the period. Using  $k_i^s$  as the total return in the  $i$ th period on an investment in one share of common stock of company  $S$  and  $k_i^m$  as the total return on all common stocks (or a representative sample) in the  $i$ th period, the beta coefficient is determined by estimating the parameters of the following regression equation:

$$k_i^s = a + \beta k_i^m$$

The estimated value of  $\beta$  is the beta coefficient.

A stock with average risk will have a beta value of 1.0, meaning that the returns on that stock vary in proportion to returns on all stocks. A higher-risk stock might have a beta of 2.0, meaning that the variation in return on that stock is twice that of the average stock. For example, if  $\beta = 2$ , when the return on an average stock increase 10 percent, the return on this risky stock increase by 20 percent. Conversely, a beta of 0.5 would be associated with a low-risk stock where return varied only one-half as much as for the average stock.

<sup>7</sup>Most firms that have issued common stock that is actively traded in organized markets are followed by one or more analysts in banks, brokerage companies, mutual funds, and other financial institutions. Their forecasts are readily available, and managers usually make it a point to keep track of what these analysts are predicting for their firm. For some large firms, there may be hundreds of analysts who follow each company.

<sup>8</sup>The theory underlying the development of the capital asset pricing model was an integral part of the revolution that transformed the world's financial markets in the 1970s and 1980s. William Sharpe, Harry Markowitz, and Merton Miller shared the 1992 Nobel Prize for their pioneering work in this area.

Using the capital asset pricing model, the cost of equity capital is the risk-free rate ( $r_f$ ) plus a weighted risk component, that is,

$$r_e = r_f + (r_m - r_f)\beta \quad (15-10)$$

In this model, the overall risk premium for common stock is measured by  $(r_m - r_f)$  and the beta weight ( $\beta$ ) then adjusts for the risk associated with the specific firm in question.

For example, suppose that the risk-free rate ( $r_f$ ) is 8 percent and the average return on common stock is 11 percent. For a firm having a beta of 1.0 (i.e., the risk for the firm is the same as that for the market average), the cost of equity capital is

$$r_e = 0.08 + 1.0(0.11 - 0.08) = 0.11 \text{ or } 11 \text{ percent}$$

Conversely, the cost of capital for a firm having a beta of 2.0 would be significantly higher. That is,

$$r_e = 0.08 + 2.0(0.11 - 0.08) = 0.14 \text{ or } 14 \text{ percent}$$

Estimates of beta for many publicly traded companies are available from various financial service companies. For example, the Value Line Investment Survey reports estimated betas for several thousand firms.<sup>9</sup> Recent estimates for some major U.S. corporations are listed here.

American Telephone & Telegraph (0.85)	BankAmerica Corp. (1.25)
Consolidated Edison (0.80)	Delta Air Lines (1.10)
Ford Motor Corp. (1.05)	General Mills (1.00)
Nike (0.95)	Pepsico (0.95)
Sears, Roebuck (1.05)	Zenith Electronics (0.85)

Note that American Telephone and Consolidated Edison offer below-average risk, while Nike, BankAmerica, and Zenith carry significantly above-average risk.

### The Composite Cost of Capital

Many firms attempt to maintain a constant or target capital structure. For example, the management of a manufacturing business may prefer a capital structure that is 30 percent debt and 70 percent equity. In contrast, managers of an electric utility company may prefer a 60 percent debt and 40 percent equity structure. In either case, capital would be raised periodically both by incurring debt and by selling stock. The differences in capital structure reflect the preference for risk on the part of owners and managers and the nature of the business. A capital structure heavily weighted toward debt implies greater risk because of the greater interest and principal payments that are required. Public utilities tend to have a higher percentage of debt than most other firms because they have a monopoly position, and usually the product they sell is a necessity. This means that the firm will have a reasonably stable and dependable flow of revenue and profit, and this offsets part of the high risk associated with a large proportion of debt in their capital structure.

<sup>9</sup>Value Line Publishing Company, New York.

A firm with a target capital structure may maintain two separate ledgers—a list of capital projects and a list of financing plans (borrowing, sale of stock, etc.). A particular financing option, say selling bonds having a cost of 10 percent, is not tied to one capital project. Rather, the firm uses an overall or composite cost of capital as the evaluation criterion for each capital project.

This composite cost of capital ( $r_c$ ) is a weighted average of the after-tax cost of debt ( $r_d$ ) and equity ( $r_e$ ) capital. The weights are the proportions of debt ( $w_d$ ) and equity ( $w_e$ ) in the firm's capital structure. That is,

$$r_c = w_d r_d + w_e r_e \quad (15-11)$$

For example, suppose that a firm's target capitalization structure is 40 percent debt and 60 percent equity. Over the next twelve months it plans to raise \$100 million by selling \$40 million in bonds at a cost of 8 percent and issuing \$60 million of stock at \$60 per share at a cost of 12.5 percent. Thus, the firm's weighted or composite cost of capital would be 10.7 percent. That is,

$$r_c = (0.40)(8) + (0.60)(12.5)$$

$$r_c = 10.7$$

The composite rate of 10.7 percent would be used as the cost of capital to evaluate all proposed capital expenditures.

### Key Concepts

- The cost of capital is the return required by investors in the debt and equity securities of the firm.
- The cost of debt capital is the after-tax interest rate on the firm's bonds or borrowing.
- Three methods of determining the cost of equity capital are
  1. The risk-free rate plus a risk premium.
  2. Discounted cash flow.
  3. The capital asset pricing model.
- The composite cost of capital is a weighted average of the cost of debt and equity where the weights are the proportions of debt and equity in the firm's target capital structure.

## Case Study

### Measuring the Cost of Equity Capital

Having several ways to determine the cost of equity capital can lead to different cost estimates, depending on the method used. The differences can be significant and may make the difference in a decision to make or not make a capital expenditure. Also, a public utility such as a gas or electric company generally has to offer evidence on the



cost of capital in hearings about the rates the companies may charge its customers. Obviously, in such cases, the company may want to document a high cost of capital to justify higher rates. In contrast, the staff of the regulatory body, who represent the interests of the consumer, may use lower estimates of the cost to keep the rates down.

Although many cases could be cited, a good example is an application for a rate increase by South Central Bell Telephone Company before the Tennessee Public Service Commission. A financial expert hired by the telephone company used the discounted cash flow method to estimate the cost of equity capital at 14 percent. A member of the commission staff used the same technique to estimate the cost at about 11 percent. The explanation for the discrepancy was that the two experts differed in their opinions about the growth rate of dividends.

Clearly, the choice of a growth rate is important. The difference between an 11 percent and a 14 percent cost of equity capital translated into \$20 million in annual profits for the firm. In its decision, the Commission opted for a rate of just over 11 percent. ■

SOURCE: *Public Utilities Reports*, 4th Series, vol. 22, pp. 257-280.

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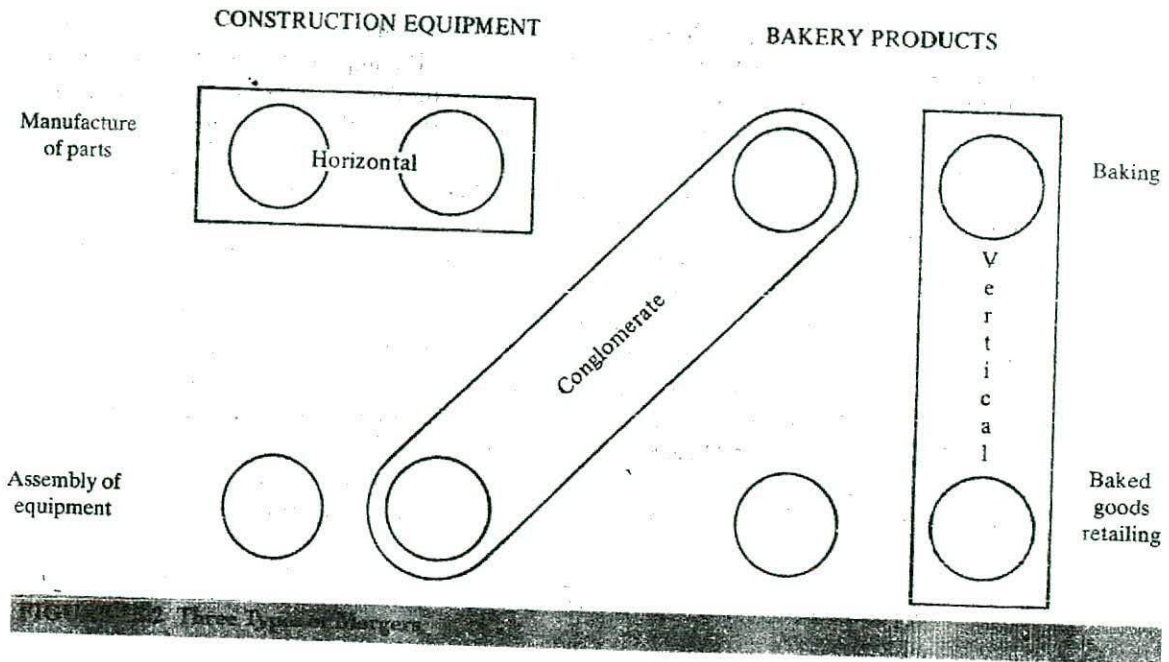
## MERGERS AND ACQUISITIONS

The growth of the firm can occur either internally or externally. When the firm builds a new plant, it is growing internally. For example, General Motors' construction of a new plant is an example of internal growth. Alternatively, the firm may grow by purchasing the assets of another firm (i.e., growth by acquisition) or by agreeing to join with that other firm under single ownership (i.e., growth by merger). Mergers and acquisitions are special cases of capital budgeting.

### Types of Mergers

Mergers are typically divided into three categories: horizontal, vertical, and conglomerate. *Horizontal mergers* involve firms that directly compete for sales of similar products or services. A merger of General Motors and Ford would be a horizontal merger. *Vertical mergers* occur when firms that had been operating at different stages in the production and distribution of a product combine to form a single firm. Vertical mergers may consist of a firm acquiring a seller of its output, such as IBM purchasing a chain of computer stores that sell the firm's computers. Sometimes this is referred to as *downstream* integration. Another type of vertical merger is the acquisition by a firm of a supplier of resources or components needed to make its product. An example would be IBM purchasing a manufacturer of computer chips or disk drives. Mergers with suppliers often are referred to as *upstream* integration. Finally, *conglomerate mergers* involve the joining of firms producing unrelated products. An example would be a merger between Delta Airlines and the Levi Strauss Corporation.

Figure 15.2 depicts the three basic categories of mergers. It shows two industries, construction equipment and bakery products. A horizontal merger is represented by the joining of two manufacturers of parts. The combination of a bakery and baked



goods retailer would be a vertical merger. The merging of an equipment assembly firm and a bakery would constitute a conglomerate merger.

## Case Study

### The Poker Game for Gulf Oil

In March 1984, Standard Oil of California (Socal) and Gulf Corporation signed a preliminary agreement to merge. The terms were that Socal would pay \$80 per share—a total of over \$13 billion—for Gulf's stock. The result was a merger between the nation's 9th and 10th largest industrial corporations. Together, the two firms had 1983 assets of \$45 billion and sales of nearly \$60 billion.

The Socal-Gulf merger emerged as the product of a concerted effort by Gulf to avoid a hostile takeover by an investment group headed by T. Boone Pickens, Jr., the chairman of Mesa Petroleum. In late 1983, Pickens and his colleagues purchased 13 percent of Gulf's stock with the avowed purpose of restructuring the firm in order to increase profits. In response, Gulf's management convened a special shareholders' meeting to change corporate rules in order to make a takeover more difficult. About the same time, Gulf announced that it was considering the acquisition of Superior Oil Co. Although never completed, this merger would have increased Gulf's size and made a takeover more difficult because of the higher cost of the combined firm.

In February 1984, the Pickens group announced that it would pay Gulf's stockholders \$65 per share for 13.5 million shares of stock. If completed, this purchase would have raised the group's stock ownership to over 20 percent, and might have enabled it to replace some of Gulf's directors with individuals loyal to Pickens at the firm's annual stockholder meeting in May.

Gulf's public reaction to the offer by the Pickens group was to label it as "unfair and inadequate." Privately, the company's management began to contact other firms that might be interested in acquiring Gulf. By March, the list of prospective suitors had been pared to three: Socal, Atlantic-Richfield, and an investment firm, Kohlberg, Kravis, Roberts, and Co. Representatives of each of these firms were invited to come to Gulf's headquarters in Pittsburgh and make a careful examination of the firm's records. Following this investigation, each firm submitted a sealed offer for Gulf Oil.

Socal's bid of \$80 per share was higher than Atlantic-Richfield's \$72 but less than the \$87.50 offered by Kohlberg, Kravis, Roberts, and Co. However, the Socal bid was accepted because the firm had access to the \$13 billion necessary to complete the purchase, whereas the Kohlberg firm would have required several months to assemble the necessary funds.

Lest one feel sorry for the unsuccessful effort of T. Boone Pickens and his associates, it should be noted that Socal's offer enabled them to earn a capital gain of \$760 million on the Gulf stock that they owned. ■

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### **Merger Incentives**

Some acquisitions take place because managers embark on the creation of a vast empire over which they can preside. The experience of James Ling, former president of Ling-Temco-Vaught, a conglomerate involved in the defense industry, is a case in point. In 1960, LTV ranked as the 335th largest firm in the United States on the basis of assets. After a flurry of acquisitions, the corporation had climbed to 22nd place by 1968. Mr. Ling's merger efforts were well rewarded in the stock market. In 1963, the price of LTV common stock was \$9 per share. By 1967, the price reached a high of \$169.50 per share. Unfortunately, the bubble finally burst, as cutbacks in defense spending after the Vietnam War and the prospect of legal actions against the firm caused the share price to plummet to about \$7 in 1970.

Although ego and empire building sometimes play a role in merger activity, usually mergers take place because managers believe that the value of the combined firm is greater than that of its constituent parts. Among the possible sources of this synergy are the following.

**Increased Market Power** A merger between two competing firms can result in increased market power for the combined firm. In some cases, this increased market power stems from the elimination of an aggressive competitor. Consider an industry dominated by a few large firms. If one firm aggressively reduces prices, the other firms may be forced to respond with price reductions of their own. The result of this price competition will generally be reduced profit levels. However, one firm may acquire the

aggressive rival by merger. By eliminating the source of active competition, the result may be higher prices, and hence increased profit, for all firms in the industry, albeit at the cost of reduced consumer surplus to buyers of the product.

In other situations, a horizontal merger may increase market power by eliminating excess capacity in an industry. Consider the tobacco industry at the turn of the century. In 1890, five cigarette manufacturers formed a cartel to stabilize prices. In 1898, the cartel expanded its activities into chewing tobacco by purchasing five other manufacturers. One year later, production facilities of 30 other chewing tobacco manufacturers were purchased and immediately closed. The result was a classic example of monopoly in action. By restricting output, the cartel was able to increase prices and profits.

**Technical Economies of Scale** If technological conditions in an industry result in increasing returns to scale, firms with large production facilities will be able to produce at lower average cost per unit than will smaller firms. Thus, small firms that merge may be able to reduce per-unit costs by expanding the scale of their operation. Such technical economies of scale may result from increased specialization of labor, greater use of automated production techniques, and/or sharing of overhead and management expenses.

**Pecuniary Economies of Scale** Pecuniary economies of scale are cost savings that result from increased monopsony power of large firms. That is, large buyers may be able to obtain lower prices for inputs that they purchase. Consider small farming operations seeking to ship fruits and vegetables. If the firms merge to form a single corporation, the resulting firm may require a significant proportion of total trucking capacity in its region. Because the single corporation has greater market power, it is in a better position to negotiate favorable rates with truckers than would many small firms acting independently. The same principle applies in capital and advertising markets. As a major borrower, a firm such as AT&T may be able to obtain lower interest rates than those offered to smaller borrowers. Similarly, with aggregate expenditures in excess of \$1 billion, Procter & Gamble is able to obtain volume discounts for advertising time and space. Firms acquired by AT&T or Procter & Gamble also would be able to buy at lower prices. These cost savings provide an incentive to merge.

**Reduced Transaction Costs** Vertical mergers may reduce transaction costs for the acquiring firm. In the absence of vertical integration, purchasing inputs from suppliers or selling to distributors can involve substantial transaction costs. First, suitable trading partners must be located. Changing prices or needs may dictate periodic reassessment of the suitability of these partners. Next, the terms of transactions with these partners must be established. Because the interests of the trading partners may differ, skill and time may be required to reach agreement. When all parties are satisfied, there is still the problem of monitoring performance. Dissatisfaction by one party may result in a termination of the agreement or costly litigation.

The difficulty of negotiating and maintaining agreements with trading partners is compounded by the incomplete flow of information. Each firm will use caution to avoid disclosing facts that might allow competitors an advantage or permit the trading partner to better its relative position. Even the attempt to be completely open may be partially thwarted by differences in training and procedures of each firm.

Vertical integration reduces transaction costs by establishing permanent linkages among the different stages of production. Thus, there is less need to search for suppliers

and distributors, and the details of interaction between divisions can be dictated by the management of the firm. Similarly, a common set of procedures can be specified for use throughout the firm. Being vertically integrated also reduces the risks associated with dependence on only one or two firms for the supply of important inputs.

**Risk Spreading** Conglomerate mergers may be a means of spreading risk. By diversifying its product line, managers can reduce fluctuations in profit rates. Consider a hypothetical merger between Holiday Inn and the Twentieth-Century Fox Corporation. The hotel and resort business is adversely affected by gasoline shortages and increasing travel costs. In contrast, motion picture producers tend to benefit from conditions that keep people close to home, where they are more likely to attend movies. Thus, a merger between these two firms could produce a conglomerate with smaller fluctuations (i.e., a lower variance) in profits than for either of the firms operating independently. This reduced variability in profit should imply a lower beta value and, hence, a reduced cost of capital for the combined firm.

In this example, there may also be complementarity in the two product lines. Holiday Inn could show Twentieth-Century Fox movies in its motel rooms and Twentieth-Century might include references to Holiday Inns in its movies.

**Valuation Discrepancies** There may be a substantial discrepancy between the acquisition price of a firm and its value to a potential merger partner. In some cases, an acquiring firm may be able to purchase production capacity at a much lower cost than it could build comparable capacity. In other circumstances, resources may be available at a bargain price. The merger between Gulf and Socal discussed in the case study on pages 525–26 is an excellent example. In early 1984, the exploration cost of crude oil was about \$12 per barrel. But by acquiring Gulf, Socal increased its oil reserves by over 1.5 billion barrels at a cost of about \$5 per barrel.

### Key Concepts

- Horizontal mergers involve the combination of two or more firms that produce and sell similar products or services.
- A vertical merger occurs when a firm acquires a seller of its product or a supplier of materials used to manufacture the product.
- Conglomerate mergers combine two firms producing unrelated products or services.
- Horizontal mergers may result in increased market power by eliminating an aggressive competitor or by taking excess capacity from the market.
- Mergers may reduce costs as a result of
  - Technical economies of scale
  - Pecuniary economies of scale
  - Reduced transaction costs
  - Valuation discrepancies
- A merger may be a way of reducing fluctuations in profits, thus reducing risk.

## Merger Procedures

Typically, three basic steps are involved in a merger. First, a suitable acquisition candidate must be identified. Second, the value of the target firm to the acquiring company must be determined. Finally, the managers and/or stockholders of the target firm must consider and approve the offer.

**Identification of a Merger Partner** Several factors must be considered in selecting an acquisition target. One is the size of the prospective merger partner. Normally, a merger with a small firm is easier than with a larger firm because fewer dollars are involved. For example, the acquisition of Gulf cost Socal over \$13 billion. To consummate the merger, Socal had to establish a multibillion-dollar line of credit with banks around the country. A number of other firms were interested in Gulf, but were unable to arrange the necessary financing. Possible antitrust problems also favor the acquisition of small firms. If the participants in a horizontal merger control a substantial share of the market, the combination may be prohibited by the courts. Antitrust law and its application to mergers are considered in chapter 19.

The expected reaction of the owners and managers of the target firm is another factor to be considered in selecting a merger partner. Although hostile takeovers have become more frequent in recent years, this type of acquisition is more difficult than if both parties favor the action. Again, the Socal-Gulf merger is a good example. Gulf's management strongly opposed a takeover by Mesa Petroleum. The firm issued public statements advising shareholders not to tender shares to Mesa. At one point, Gulf's managers went so far as to change the corporation's rules to make a takeover more difficult. In contrast, when Socal expressed interest in a merger, Gulf cooperated in every way. Thus, the time and dollar costs for an acquisition by Socal were much less than they would have been for Mesa Petroleum.

The extent that resources and facilities of a target firm complement those of the acquiring firm is a third factor in selecting a merger partner. A firm needing a stable cash flow to finance expansion or modernization of its facilities may seek a merger partner with substantial cash reserves. A firm that perceives that it has an important gap in its product line will be attracted to a producer of a product that fills that gap. Similarly, firms requiring specific resources are likely to select the owners of those resources as targets for acquisition.

Finally, acquiring firms tend to select merger partners that appear to be good bargains. It may be possible to purchase a firm at a price substantially below its potential value if its owner wants to retire or if the firm has encountered financial difficulties because of poor management or unfavorable economic conditions.

**Determination of the Value of the Target Firm** Once the merger partner has been identified, the value of the firm to the acquiring company must be determined. Conventional analysis suggests that the value of an enterprise is equal to the present value of future profits. That is, annual profits over some planning horizon are estimated and then discounted back to their present value. This amount would represent the maximum price that should be paid for the firm.

Mergers take place because of an anticipated synergism between the two firms. Thus the value of the target firm to the acquiring firm may be considerably greater than the present value of its profits as an independent entity. Hence, an acquiring firm

may be willing to pay a premium price. Fundamentally, the maximum price should still be based on an analysis of the present value of profits. However, in this circumstance, it is incremental profit to the acquiring firm that is the relevant consideration. That is, management should estimate its profit over the planning horizon with and without the merger. The discounted value of the difference is the maximum price that should be paid.

For example, assume that the present value of all future profits is estimated to be \$400 million for Acme Inc. and \$200 million for Peabody and Co. if the two firms continue to operate independently. Also assume that if Acme acquires Peabody, the combined business could take advantage of economies of scale and that the present value of Acme's profit (including the acquisition of Peabody) would be \$800 million. Thus, the present value of the incremental profit for Acme would increase by \$400 million as a result of the merger. If Peabody had 5 million shares of stock outstanding prior to the merger, Acme could bid as much as \$80 for each share (i.e., \$400 million  $\div$  5 million shares) of Peabody's stock.

Theoretically, the value of the firm as an independent entity should be approximated by the market value of its outstanding stock. For example, with the present value of profit equal to \$200 million, the market value of Peabody's stock should be about \$40 per share (i.e., \$200 million  $\div$  5 million shares). If this is true, the likelihood of a merger will depend on the amount of synergism associated with the merger. If the merged firm is no more valuable than its constituent parts, there is no incentive for the acquiring firm to offer a price greater than the current market price for the target. In that case, owners and managers of the prospective merger partner will not benefit, and the combination probably will not take place. In contrast, if there are substantial synergistic benefits to the merger, a premium over the market value of the target firm can be offered. Because the benefit to managers and owners of the target firm can be significant, the merger offer may be viewed favorably. In the example, owners of Peabody's stock could receive as much as \$80 per share for stock that was selling for \$40.

**Presentation of the Offer** Once the maximum value of the target firm has been determined, the next step is to formulate a plan to facilitate the acquisition. If the management of the target firm is favorably disposed to the merger, the procedure should be relatively straightforward. Managers from both firms will meet to determine the details of the arrangement. Once an agreement has been reached, they will make a joint statement indicating the terms of the merger and urging acceptance by the acquired firm's stockholders.

The offer for the shares of stockholders may be in cash or it may be structured as an exchange of stock. A cash offer usually includes a premium over the current market price of the acquired firm's stock. Offers to exchange stock specify that shareholders of the target firm will receive a specified number of shares of the acquiring firm's stock for each share that they hold. The actual number of shares depends on the relative stock price of the two firms and the extent of synergistic benefits of the merger. Consider the hypothetical acquisition of Peabody by Acme. Suppose that Acme's stock is currently selling for \$80 per share, while that of Peabody is at \$40 per share. If these market prices accurately reflect the values of the firms as independent corporations, one share of Peabody stock would be worth as much as two Acme shares. But if combining the two firms increases their joint value, Acme will be willing to offer a more favorable ex-

change rate. For example, Acme may be willing to offer as much as one share of its stock for every Peabody share. The greater the synergistic benefits, the more attractive the rate of exchange can be made to Peabody's stockholders.

If the management of the target firm opposes the merger, the endeavor is more complicated. In this case, the acquiring firm must make its appeal directly to the target firm's stockholders through a tender offer. Such an offer typically indicates that a certain price will be paid for a specified number of shares of the acquiring firm's stock exchanged for that of the target firm. Frequently, the offer has a fixed termination date and is valid only if a minimum number of shares are submitted, or tendered, to the acquiring firm. For example, suppose that Peabody's management opposed the Acme takeover. Acme responds with a tender offer to purchase Peabody's stock at \$60 per share. However, the offer is to be valid for only 60 days and is conditional on at least 51 percent of Peabody's stock being tendered to Acme. If Acme obtains the necessary 51 percent, the voting rights associated with this majority interest can be used to elect a new board of directors who will favor the merger. If sufficient shares are not tendered, the offer may be withdrawn and the shares returned to their original owners.

### Key Concepts

- Prospective merger partners are evaluated in terms of size, anticipated reaction to a merger proposal, price relative to value, and extent of complementary resources.
- The maximum value of a target firm is the present value of the additional profits that would be earned by the acquiring firm after the merger.
- An acquiring firm may offer cash or an exchange of stock for the shares of a target firm.

## Case Study

### The White Knight and Other Antimerger Strategies

Hostile takeovers can be extremely bitter and costly. As a means of thwarting such actions, several defenses can be used by managers of potential target firms. One option is to identify another firm, referred to as a white knight, that agrees to purchase the target firm to prevent its acquisition by a less favorable merger partner. A good example is Gulf, which used Socal and Atlantic Richfield as white knights to ward off unwelcome advances by Mesa Petroleum.

Another strategy is the sale of previously authorized but unissued shares to a sympathetic firm, financial institution, or group of investors. By having more shares outstanding, it is more difficult to obtain the number of shares required for a takeover. A related tactic is to initiate a merger with another firm. The advantage of this approach is that the resulting larger firm may be more difficult to acquire and perhaps less attractive with the new addition.



Then there is the Pac-Man strategy. It requires that the firm that is the target of a hostile take-over retaliate by using a tender offer of its own for the shares of the aggressor. That is, the target firm becomes the hunter instead of the hunted, as with the Pac-Man video game. Using the earlier example, if Acme had offered to pay \$60 for Peabody shares, Peabody might respond with an offer to buy Acme shares for \$90.

A firm subject to an unwanted takeover may swallow what is called a *poison pill* to reduce its attraction to the other firm. One form of poison pill is to borrow money to buy its own stock. As the firm increases its debt and reduces its equity, it becomes a more risky business. That is, the likelihood of insolvency increases, and it becomes a less desirable merger candidate.

Finally, managers may protect themselves against the consequences of a hostile takeover by the inclusion of a golden parachute provision in their contracts. Such provisions are simply short-term employment contracts that become effective when there is a change of corporate control. They provide displaced managers with compensation in the event that they are replaced as a result of a merger. ■

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## SUMMARY

Capital projects are those investments that are expected to generate returns for more than one year. Capital expenditures are made to reduce costs, increase output, expand into new products or markets, and/or meet government regulations. Capital budgeting is the process of planning capital projects, raising funds, and efficiently allocating those funds to capital projects. In general, the firm continues to make capital expenditures until the rate of return on the last dollar invested equals the marginal cost of capital.

The capital budgeting process consists of two phases. First, the net cash flows are estimated, and second, evaluation techniques are used to compare those cash flows to the initial cost of the project. The payback method determines the number of years necessary for the cumulative net cash flows to equal the cost of the project. Because it fails to account for all cash flows and does not discount these cash flows at the firm's opportunity cost of capital, the payback method is an inferior evaluation technique. The net present value (*NPV*) and internal rate of return (*IRR*) concepts are the two most commonly used evaluation methods. Using the *NPV* method, a capital project is determined to be profitable (i.e., would increase the value of the firm) if the present value of all future cash flows exceeds the initial cost of the project. In contrast, the internal rate of return is the discount rate that equates all future cash flows and the project's cost. An investment should be made if the *IRR* exceeds the firm's cost of capital.

Generally, the *NPV* and *IRR* methods yield consistent accept or reject signals for proposed capital projects. However, the *IRR* method has several problems. For example, it sometimes results in multiple solutions for the internal rate of return. Also, it discounts future cash flows at the rate of return for the project being evaluated and not at the firm's opportunity cost of capital. Because of this, if the firm is subject to capital ra-

tioning (i.e., where the firm has more feasible capital projects than it can finance from both internal and external sources), the *IRR* method may not select that project that maximizes the value of the firm.

The profitability ratio is defined as one plus the ratio of net present value to initial project cost. It is used to rank the relative profitability of projects where the firm is subject to capital rationing. Linear programming can be used to determine which capital projects to implement when capital rationing applies.

The cost of capital is the return required by investors in the debt and equity securities of the firm. The cost of debt is the net (after-tax) interest rate paid on that debt. Alternative methods for estimating the cost of equity capital include (1) the risk-free rate plus a risk premium, (2) discounted cash flow, and (3) the capital asset pricing model. The last method uses the beta coefficient (a measure of the variability in return on a given stock relative to variability in return on the average stock) to measure relative risk among the common stock of different companies.

As firms often use both debt and equity financing sources, a weighted cost of capital measure is used. The composite cost of capital is a weighted average of the cost of debt and the cost of equity capital. The weights are the proportions of debt and equity in the firm's target capital structure.

A merger is defined as the joining of two or more firms under single ownership. Usually, mergers take place because management believes that the value of the combined firm is greater than the sum of the individual values. Horizontal mergers involve firms that compete directly for sales of similar goods or services. Vertical mergers occur when firms that had been operating at different stages in the production and distribution process combine to form a single firm. A conglomerate merger combines essentially unrelated firms.

From the standpoint of the acquiring firm, a merger may be a means of increasing its market power by eliminating competitors. Other incentives to merge include cost reductions resulting from economies of scale, reduced transaction costs, and valuation discrepancies. A merger also may be a way of smoothing fluctuations in profits.

Several factors affect the choice of a target for acquisition. A merger with a very large firm may present financial problems and raise antitrust issues. If management of the target firm is expected to oppose the merger, the acquisition is likely to be more costly and difficult than if the proposal is accepted by those managers. The best merger partners are those that have complementary resources and can be purchased at a cost that is low relative to their value to the acquiring firm.

The maximum price that one firm would be willing to pay for another is the present value of the additional profits that are estimated to result from the merger. An acquiring firm may offer cash or an exchange of its stock for that of the target firm.

### Discussion Questions

- 15-1. What is the basic principle underlying the approach used by a profit-maximizing manager in making capital budgeting decisions?
- 15-2. Why is the marginal cost of capital schedule or function upward sloping for most firms?
- 15-3. Contrast the net present value and internal rate of return approaches in the evaluation of capital projects. In general, if the *NPV* is greater than zero, what does this imply about the internal rate of return?

- 15-4. Why is it that the net present value and internal rate of return methods can yield contradictory results when evaluating two mutually exclusive investments? Develop an example of two investments in which this would happen.
- 15-5. What is the relationship between a firm's marginal income tax rate and the net cost of debt capital to the firm?
- 15-6. Some analysts claim that the cost of debt capital is lower than the cost of equity capital for most firms. If this is true, why don't firms rely exclusively on debt financing and not sell any additional common stock? Explain.
- 15-7. One firm in an industry may have 50 percent debt while another may have no debt. Why do some firms in the same industry have substantially different capital structures? Why do most firms in some industries (e.g., public utilities) have a large debt component in their capital structures while most firms in some other industries have relatively little debt?
- 15-8. What is synergy as it applies to mergers? How does it affect the amount that one firm would be willing to pay for another?
- 15-9. Give at least one example (other than those used in the chapter) of each of the following types of mergers:
- Horizontal
  - Conglomerate
  - Vertical
- 15-10. How could a conglomerate merger be used to reduce risk?
- 15-11. Explain how a vertical merger between an electric utility and a coal company could reduce transaction costs for the combined firm.
- 15-12. What are three strategies a firm might use to avoid being acquired by another company?
- 15-13. Suppose that the capital market has correctly valued the price of a firm's stock. Why would an acquiring firm be willing to pay more than the market price? What would determine the premium over the market price that the firm would be willing to pay?

## Problems

- 15-1. Staff members of the financial analysis department of Global Electronics have determined the required investment and the rate of return on each of the following capital projects.

<i>Capital Projects</i>	<i>Required Investment (millions)</i>	<i>Internal Rate of Return (%)</i>
A	5.2	12.9
B	8.6	15.2
C	3.4	10.0
D	5.1	14.8
E	11.2	19.0
F	6.5	7.9

The firm's marginal cost of capital is given by the function

$$r = 8 + 0.10C$$

where  $r$  is rate of return (in percent) and  $C$  is millions of dollars of capital raised for investment.

- Graph the firm's marginal cost of capital function and the firm's capital demand function.
- Which capital projects should be implemented? What should be the firm's total capital investment?
- If a general tightening in the financial markets shifts the firm's marginal cost of capital function to

$$r = 8 + 0.35C$$

determine which projects should be implemented and the total amount spent on capital items.

- 15-2. Tarnutzer Construction Company must replace its front-end loader. The initial cost and annual net cash flows for the two models under consideration are shown in the table. Given the heavy use of the machines, they will be completely worn out at the end of 5 years and have no salvage value. The firm's cost of capital is 12 percent.

	Model	
	Heavyduty	Sure Shovel
Initial cost	\$10,000	\$10,000
Net cash flows (year)		
1	5,000	-4,000
2	5,000	0
3	5,000	6,600
4	5,000	6,600
5	5,000	20,000

Use both the *IRR* and *NPV* methods to evaluate these capital proposals. Which should be purchased? Explain. (Except for the initial cost, assume that all cash flows are received or paid at the end of each year.)

- 15-3. Top management at Transworld, Inc., a large multinational conglomerate, uses the net present value method for evaluating capital expenditure proposals. The firm's cost of capital is 16 percent. Currently, the following eight proposals are under review:

Proposal	Initial Cost (millions)	Net Present Value (millions)
1	\$11.6	\$3.6
2	9.4	-1.3
3	8.7	2.2
4	13.2	2.4
5	14.0	2.0
6	7.5	1.4
7	6.5	1.7
8	9.5	-2.4

- If the corporation has a capital spending budget of \$100 million for the coming year, which capital projects should be implemented? Explain.

b. If the capital spending budget is limited to \$47.5 million, which projects should be undertaken? Explain.

- 15-4. The board of directors of Alder Enterprises wants to maintain a capital structure that is 30 percent debt and 70 percent equity (i.e., retained earnings and common stock). For the current year, the firm expects to earn \$2 million after taxes (the firm's marginal tax rate is 40 percent) on sales of \$15 million. Company policy is to pay out 50 percent of net after-tax income in dividends to the holders of the firm's one million shares of common stock. Management and outside analysts project a growth rate of 8 percent for sales, profits, and dividends. The market value of the firm's common stock is \$20 per share.

The average dividend yield on all common stocks is 14 percent, and the interest rate on U.S. government securities is 12 percent. Both management and the firm's investment advisers, Saunders & Wennergren, agree that Alder could sell bonds at an interest rate of about 14 percent. The estimated beta coefficient for Alder is about 1.5.

- Determine the cost of debt capital for Alder Enterprises.
  - Determine the cost of equity capital using each of the three methods described in the chapter.
  - Determine the composite cost of capital for the firm using the capital asset pricing model to compute the cost of equity capital.
- 15-5. Quickstor, Inc., builds and manages storage units that are rented to individuals and firms. Typically, the firm builds a number (usually 200 to 500) of these garage-like units at sites on the periphery of growing urban areas. Most of the time the storage units are demolished and the site cleared for housing and/or commercial development within 5 to 10 years.

Management is deciding whether to build a 300-unit storage center between Oklahoma City and Norman, Oklahoma. Currently, similar units in the area are being rented for \$85 per month. Each unit would contain 400 square feet and the construction cost per square foot, including site development costs, would be \$8. The operating costs of the project (i.e., insurance, management, property taxes, electricity) are estimated at \$50 per unit per month. Both the rental rate and operating costs are expected to remain constant for the foreseeable future. Quickstor's cost of capital is 10 percent and its marginal income tax rate is 30 percent.

Because of the growth of the University of Oklahoma in Norman, it is likely that the storage units will be demolished after 10 years to make room for student apartments. Demolition costs will be insignificant, and there would be no salvage value for any of the materials. The entire cost of the project will be depreciated in 10 years, using the straight-line method. There is no alternative use for the land for the next 10 years.

- Given that Quickstor's objective is profit maximization, should this project be implemented? Use both the *NPV* and *IRR* evaluation criteria.
  - If your answer to part (a) is that the project should not be implemented, determine the maximum initial investment that could be made so that the project would be profitable.
- 15-6. The head of the marketing department (who has had no training in capital budgeting), has proposed a new direct telephone advertising program that would require an initial investment of \$100,000 in equipment and \$50,000 per year in ad-

ditional labor costs. It is estimated that the program would be effective for 5 years and would increase revenue in each of those years by \$75,000 over the current level. After 5 years, the equipment would be obsolete with zero salvage value and the advertising program ineffective because of the entrance of new competition. She argues that the 4-year payback period makes the proposal a "can't miss" proposition. Do you have any comments on this evaluation procedure? Use the *NPV* method to evaluate this proposal. (Note: The firm's cost of capital is 20 percent, it uses straight-line depreciation, and the marginal tax rate is 50 percent.)

- 15-7. The engineering department of McDougal Steel has developed the following capital project evaluations:

<i>Project</i>	<i>Present Value of All Future Net Cash Flows</i>	<i>Initial Cost</i>
A	\$190,000	\$200,000
B	210,000	150,000
C	600,000	500,000
D	490,000	400,000
E	350,000	300,000

- Assuming an unlimited capital budget, which projects should be implemented? Why?
  - Due to a change in top management philosophy that precludes additional external financing of any kind, the department is faced with a capital budget constraint of \$500,000. Which projects should be implemented? Explain.
  - Five minutes before the department head is to take the results from part (b) to the corporate capital budgeting meeting, Smith comes in with a revolutionary new capital item that has a cost of \$450,000 and an *NPV* of \$120,000. Assuming Smith's analysis is accurate, what should the department head do? Why?
- 15-8. Given the following information on three major corporations, determine the cost of debt and the cost of equity capital for each firm using the three methods described in the chapter (i.e., the risk-free rate plus risk premium, the discounted cash flow, and the capital asset pricing methods).

<i>Firm</i>	<i>Tax Rate</i>	<i>Rate on Long-Term Bonds</i>	<i>Price of Common Stock</i>	<i>Beta</i>	<i>Annual Dividend</i>	<i>Dividend Growth Rate</i>
Chrysler	39%	8.9%	32	1.20	1.60	14%
Johnson & Johnson	36	7.9	59	1.15	0.88	15
Ohio Edison	38	8.5	22	0.80	1.50	2

Assume that the risk-free interest rate is 5 percent and that the average return in the stock market is 12 percent.

- 15-9. Determine the cost of equity capital (using each of the three methods described in the chapter) and the cost of debt capital for each of the following firms. The risk-free rate is 5 percent and the average return on common stock is 12 percent.

<i>Firm</i>	<i>Tax Rate</i>	<i>Rate on Long-Term Bonds</i>	<i>Price of Common Stock</i>	<i>Beta</i>	<i>Annual Dividend</i>	<i>Dividend Growth Rate</i>
Minnesota P&L	34%	7.9%	\$31	0.70	\$2.04	4%
Exxon	38	8.6	63	0.70	3.00	6
Oracle	34	9.2	47	1.5	0	20

- 15-10. World Airways plans to expand its fleet of planes and maintenance facilities in a number of international locations. Management has identified the following six projects, each of which requires investment in each of 2 years. The firm faces a limited capital budget of \$200 million in year 1 and \$300 million in year 2.

<i>Project</i>	<i>Present Value of Outlay in Year (millions)</i>		<i>Net Present Value (millions)</i>
	$1(C_{1j})$	$2(C_{2j})$	
1	40	50	70
2	80	70	100
3	50	60	50
4	40	50	40
5	50	80	90
6	30	50	35

Set up the linear programming problem to determine which capital projects to implement.

- 15-11. The common stock of Throckmorton Machinery is currently selling for \$40 per share, while the price of McKnight Equipment stock is \$10 per share. Top managers at McKnight are planning an offer for the shares of Throckmorton. They believe that by applying their considerable managerial talents, the value of Throckmorton could be increased 25 percent by a merger with McKnight.
- If a cash offer is to be made to owners of Throckmorton common stock, what is the maximum price that McKnight's managers should offer?
  - If the acquisition is to be made by an exchange of stock, what are the most favorable terms that McKnight should offer?
  - What will probably happen to the share price of Throckmorton's stock when the offer is announced? Explain.
- 15-12. International Steel consistently earns a profit of \$100 million per year, and it has 10 million shares of common stock outstanding that have a market value of \$150 per share. East Asia Steel earns a profit of \$20 million per year, and it has 5 million shares of stock outstanding with a market value of \$40 per share. International's management sees the acquisition of East Asia Steel as a way to expand in the Asian market, thus increasing its profits.
- What is the maximum cash price that International should pay if there is no synergy in the acquisition (i.e., if the total profit of the combined firm simply would be equal to the sum of their current profits)?
  - What is the maximum cash price if there was synergy that increased the combined profit by 50 percent?
  - Assuming no synergy, what is the most favorable stock exchange that International could offer?

- d. Assuming the synergy described in part (b), what is the most favorable stock exchange that International could offer?

### Computer Problems

The following problems can be solved by using the TOOLS program (downloadable from [www.prenhall.com/petersen](http://www.prenhall.com/petersen)) or by using other computer software.

- 15-13. International Communications offers discount long-distance telephone service to businesses throughout the world. To meet the improved service offered by its competitors, National must upgrade its entire switching system. Three alternative systems are available. The initial cost and estimated net year-end cash flows over the 10-year lives of the systems are shown here. Use both the net present value (NPV) and internal rate of return (IRR) criteria to determine which system should be selected. The firm's cost of capital is 13.4 percent.

System	Interstate	Scrambler	Regent
<b>Initial Cost</b>	<b>\$300,000</b>	<b>\$135,000</b>	<b>\$130,000</b>
Net profits (year)			
1	45,620	-40,000	5,200
2	51,900	-22,000	8,100
3	55,800	-6,900	10,500
4	60,100	5,500	8,100
5	61,000	30,200	10,500
6	60,000	40,800	8,100
7	55,900	60,400	10,500
8	52,000	80,200	88,100
9	50,600	90,000	105,000
10	40,200	75,400	115,200

- 15-14. Margaret Vangilder manages the annual bazaar at the Pacific United Church. The most recent bazaar resulted in profits of \$10,000, but shoppers had to be turned away because the church recreation room is too small. Given population and income growth in the area, profits could be expected to increase by 10 percent each year if the church could be expanded. A local contractor estimates that such an expansion would cost \$35,000. The market interest rate is 10.125 percent, and the church will be torn down at the end of 10 years to make room for a football stadium. Should the investment be made?
- 15-15. The Sloan Corporation is considering two projects with the following cash flows:

Project	End of Year					
	0	1	2	3	4	5
X	\$-100,000	\$125,000	0	0	0	0
Y	-100,000	0	0	0	0	\$228,000

- a. Compute the net present value for each project using discount rates of 5, 7, 9, 11, 13, and 15 percent. Graph the net present value of each project as a function of the discount rate.



- b. Determine the internal rate of return for each project. For what range of discount rates is the ranking based on the internal rate of return criterion consistent with the ranking based on net present value?
- 15-16. A temporary dam being planned by the federal government has an initial cost of \$60 million and will have net cash benefits extending over a 12-year period as shown.

<i>Year</i>	<i>Net Cash Flow (millions)</i>	<i>Year</i>	<i>Net Cash Flow (millions)</i>
1	0	7	15
2	0	8	15
3	5	9	20
4	5	10	20
5	10	11	25
6	10	12	25

- a. Assume a discount rate of 10 percent. Compute the net present value and the internal rate of return for the project.
- b. Repeat part (a) using discount rates of 5 and 15 percent. Does the choice of the discount rate significantly affect the evaluation of the project? Explain.
- 15-17. A new technique has been proposed for surfacing roads to prevent potholes. It can be used for a maximum of 10 years, but its effectiveness decreases somewhat over time. The initial cost to resurface the road is \$90 million, and the future net cash benefits are shown below.

<i>Year</i>	<i>Net Cash Benefits (millions)</i>	<i>Year</i>	<i>Net Cash Benefits (millions)</i>
1	0	6	16
2	20	7	16
3	20	8	16
4	20	9	16
5	20	10	16

- a. Assume a discount rate of 10 percent and that all benefits accrue at the end of the year. Compute the net present value and the internal rate of return for the proposal.
- b. Repeat part (a) using discount rates of 5 and 15 percent.

# Integrating Case Study VI

## CASE

### Bentley Enterprises, Inc.

Bentley Enterprises is a manufacturer and nationwide distributor of specialty food products, such as ice cream, candy, and bakery goods. The firm's strategic planning group has developed a proposal for locating a number of "shaved ice" retail outlets in the parking lots of shopping centers. The product consists of crushed ice in a paper cone that is mixed with one or more flavors. Management decides that the product can be sold for \$0.80 each. Two alternatives have been proposed. In the first, called the "national" proposal, 500 stands would be located at shopping centers of various sizes across the United States. The second proposal, referred to as the "limited" plan, would place fewer stands (200) in the large malls of major urban areas.

The initial cost of the "igloo-like" structures, including the necessary equipment, is projected to be \$55,000 each. Because of their specialized nature, the buildings will have no value at the end of the life of the project. The average payment of a 4-year lease for each parking lot space would be \$25,000. The entire lease payment would be paid at the beginning of the project. Thus, it is a one-time outlay that would pay for space for 4 years in advance.

At this time, the only competition in this field comes from a few "mom-and-pop" type operations scattered around the country. However, management at Bentley is concerned that Unique Foods, Inc., its major competitor, might respond to the Bentley plan with a similar program. Over the past 20 years, Unique has had a tendency to watch the actions of Bentley and then attempt to duplicate its new programs. In fact, for new products introduced on a national basis, Unique has responded with a similar program about 60 percent of the time. For programs developed on a more limited basis by Bentley, Unique has come up with an almost identical program in 30 percent of the cases.

Average daily volume for each unit for the first year of operation will depend on (1) whether the national or limited program is undertaken and (2) whether Unique responds with a similar program. Estimates of the volume under each combination of events are outlined below. The average volume is higher under the limited program because of the large number of shoppers in the large malls. The stands would be open 365 days a year.

*First-Year Average Daily Volume (units sold)*

<i>Unique</i>	<i>Program</i>	
	<i>National</i>	<i>Limited</i>
Responds	250	300
Doesn't respond	400	500

Because of the novelty nature of the product, management is assuming a "worst-case" scenario that there will be no demand for the product after the 4th year. During this period, both price and units sold are expected to remain constant.

Although the profit associated with any one unit cannot be estimated with accuracy, management thinks that annual profit for the average unit can be determined based on the following operating profit data for eight experimental units that were in operation for 1 year at random locations throughout the country. The data does not include the cost of the structure or the lease payments for parking lot space.

<i>Unit</i>	<i>Annual Revenue (thousands)</i>	<i>Annual Operating Profit (thousands)</i>
1	\$80,000	\$18,000
2	120,000	28,000
3	60,000	11,000
4	90,000	16,000
5	110,000	20,000
6	100,000	25,000
7	70,000	12,000
8	60,000	14,000

The management decision rule at Bentley for evaluating projects such as this is that a project should be implemented only if shareholder value is increased (i.e., if the present value of all future operating profits exceeds the initial project cost). The only alternative for the firm would be to invest in a 4-year bank certificate of deposit that would yield 10 percent interest. Management thinks that the risky nature of the "shaved ice" stands implies a risk premium of four percentage points over the rate available on the bank deposit. Management also insists on having a quantitative measure of the risk associated with each alternative. (Note: A general rule at Bentley is to assume that all annual revenues are received and operating costs incurred on the last day of each year.)

### Requirement

What action should the firm take? Should either of the "shaved ice" stand proposals be implemented or should the firm simply invest its money in a certificate of deposit? Be sure to evaluate risk. ■