BER Performance Analysis of a UWB Wireless Communication System with Smart Antenna

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Abstract—This paper presents an analytical approach to evaluate bit error rate performance and SINR improvement of an UWB communication system with a smart antenna. The BER performance results are evaluated numerically for various values of antenna parameters like number of antenna elements and number of delays in the tapped delay line. Experimental results demonstrate that there is improvement in receiver sensitivity (SINR) with increase in antenna elements in the smart array but no improvement in the performance with the number of delays in the TDL line.

Index Terms— UWB, Smart antenna, SINR, BER, TDL, Covariance matrix, Interference, AWGN

1 INTRODUCTION

AN antenna becomes smart when the software associated with it is able to compute the complex gain required for achieving the desired antenna radiation pattern. Any shape of the radiation pattern is obtainable with a smart antenna [1-4]. It has been widely used in conventional narrowband and wideband system. Present trend is to use it for ultra-wideband impulse technology to improve their capability [5]. UWB receivers and transmitters are normally of low power capacity. In this paper an analytical approach is presented to evaluate the bit error rate (BER) performance of an ultra-wide band wireless communication system using a smart antenna to investigate the antenna performance in presence of interference. The expression of signal to interference ratio was used to calculate the bit error rate performance of the smart antenna with different parameters like number of antenna elements, delay line and delay period. The minimum power requirement of the receiver along with a frequency response constraint has been attained. The paper is organized as follows. In section 2 system model for UWB smart antenna is given. The expression of the output signal of smart antenna is derived in section 3 Simulation results for smart antenna performance are given in section 4. Conclusions are drawn in section 5.

2 SYSTEM MODEL WITH UWB SMART ANTENNA

The configuration of the smart antenna used for analysis of the UWB wireless communication system is shown in Fig.1. There are *M* elements of antenna array which are omnidirectional and are placed at one wavelength apart at the highest frequency f_h of the UWB signal. Each element has tapped delay line consisting of *K* elements and with time delay of T_0 seconds for each. The continuous time signal received by m_{th} element at the k_{th} tap is given by $x_{mk}(t)$. The output of the 1st. tap behind each element is the signal itself with no delay. Each output of the tapped delay line is multiplied by the variable real weights *w* and then added to produce the array output *y(t)*. Let us assume that X_m and W_m be the column vectors containing the signals and weights at the *k* taps behind element *m* then, [1]

$$X_m = [x_{m1}(t)\, x_{m2}(t) \ldots x_{mk}(t)]^T \tag{1}$$

$$W_m = [w_{m1}\, w_{m2} \ldots w_{mk}]^T \tag{2}$$

The superscript *T* is for transpose. Desired signal, undesired signal or interference and thermal noise are received by the array. We assume that the number of available array elements are greater than the number of sources of interference.

3 SYSTEM ANALYSIS

The signal $x_{mk}(t)$ may be represented as

$$x_{mk}(t) = d_{mk}(t) + i_{mk}(t) + n_{mk}(t), \tag{3}$$

where $d_{mk}(t)$, $i_{mk}(t)$ and $n_{mk}(t)$ are the desired, interference and noise components. By *d(t)* a uniform train of UWB pulses is represented as

$$d(t) = \sum_{n=-\infty}^{\infty} \gamma(t - nT_r), \tag{4}$$

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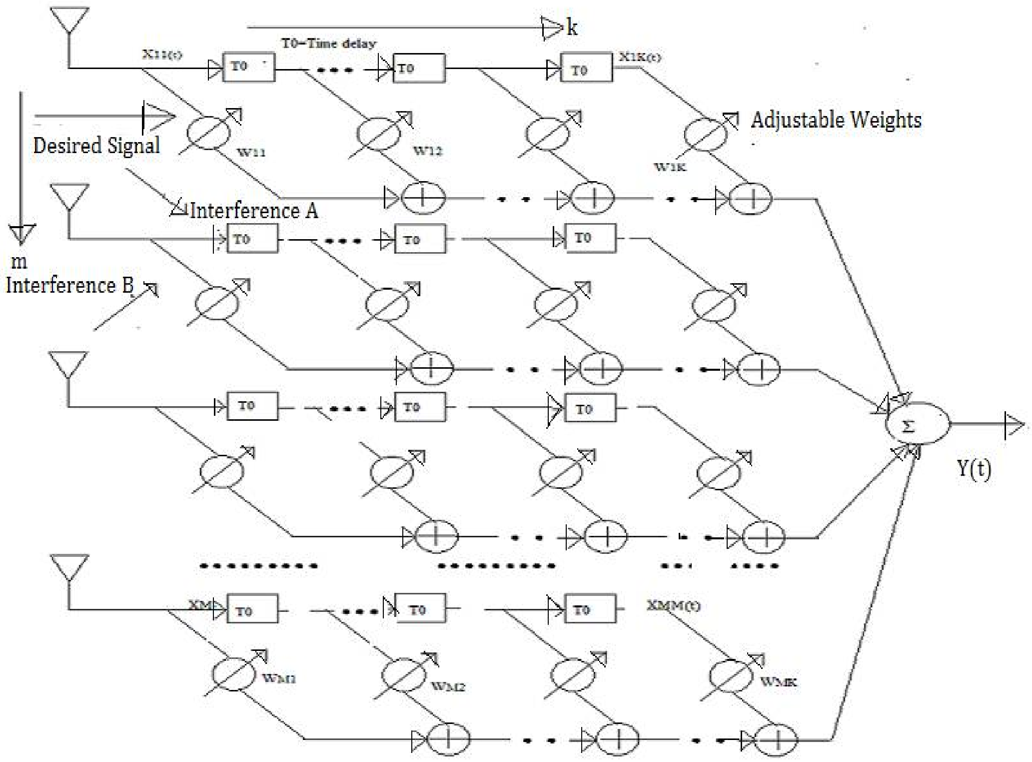

Fig.1 UWB antenna model with different of elements and delays

Where T_r is the pulse repetition interval and $T_r >> T_d$ (delay of the tap delay line) and $\gamma(t)$ is the pulse which in this case is the second derivative of the Gaussian pulse and is given by

$$\gamma(t) = \left(1 - 16\pi\left(\frac{t}{\Delta T}\right)^2\right)\exp\left(-4\pi\left(\frac{t}{\Delta T}\right)^2\right) \quad (5)$$

where ΔT is the nominal duration of the pulse [1] and its frequency domain description is given by

$$\Gamma(\omega) = \frac{\omega^2}{\left(16\pi\Delta f^2\right)^{3/2}}\exp\left(-\frac{\omega^2}{2(16\pi\Delta f^2)}\right) \quad (6)$$

where $\Delta f = 1/\Delta T$ is the effective frequency bandwidth of the pulse . Let X and W are the total signal and weight vectors of the entire array and are given as

$$X = [X_1\, X_2 \ldots X_M]^T \quad (7)$$

$$W = [W_1\, W_2 \ldots W_M]^T \quad (8)$$

The elementary signal vector X_m, the total signal vector X can be decomposed into their constituents in a similar way as in equation (3).

$$X_m = X_{dm} + X_{im} + X_{nm} \quad (9)$$

$$X = X_d + X_i + X_n \,. \quad (10)$$

Let us assume that the desired signal comes at an angle θ_d, $d(t)$ is the waveform on element l, then the desired signal at an arbitrary tap k of element m is given by

$$d_{mk}(t) = d(t - [k-1]T_0 - [m-1]T_d)\,, \quad (11)$$

where T_d is the desired signal spatial propagation delay between elements, with

$$T_d = \frac{d}{c}\sin\theta_d \,, \quad (12)$$

where d is the distance between elements and c is the velocity of light. Similar expression can be written for interference signal $i_{mk}(t)$ as well.

The noise at the input of antenna elements is *awgn* and independent. The noise at any antenna element can be denoted as $n_{j1}(t)$ with a variance of σ^2 given by

$$E[\overset{*}{n}_{ji}(t)\, n_{mi}(t) = \sigma^2\delta_{jm}] \,, \quad (13)$$

where δ_{jm} is the Kronecker delta. The final output of the array is

$$y(t) = W^T X = W^T(X_d + X_i + X_n) \quad (14)$$

The expected power of the array can be obtained by using $y(t)$ as follows:

$$E[y^2(t)] = E[W^T X X^T W] = W^T R W, \quad (15)$$

where $R = [XX^T]$ is the covariance matrix of the received signal. We like to find the most favorable weights of the antenna such that it maintains the specified look direction frequency response of antenna and cancels the interference. To maintain this condition of the array we define constraint matrix C of $MK \times K$ dimension [6]. This constraint matrix makes the weights on j^{th} vertical column of taps to a chosen number f_i which is expressed as $C^T W = F$ where C and F are as

$$C = [I\, I \ldots I]^T \text{ and } F = [f_1 f_2 \ldots f_k] \qquad (16)$$

and I is the $K \times K$ identity matrix and F defines the frequency response of the smart antenna in the look direction. The smart beam problem can be defined as in [6]

$$\min_w W^T R W \qquad (17)$$

subject to

$$C^T W = F\ . \qquad (18)$$

Let us form a cost function as follows by using Lagrange multiplier λ with equation (18) and power equation

$$H(w) = \frac{1}{2} W^T R W + \lambda^T (C^T W - F) \qquad (19)$$

Optimum weight W can be found by taking gradient of H(W) and equating the result to zero [6]. The optimum weight found is

$$W_{opt} = R^{-1} C (C^T R^{-1} C)^{-1} F\ . \qquad (20)$$

The total covariance matrix R is the summation of three covariance matrices.

$$R = R_d + R_i + R_n\ , \qquad (21)$$

where R refers to covariance matrix and the subscripts d, I, n refer to desired signal, interference signal and thermal noise respectively. The autocorrelation function of desired signal is given by

$$R_d(\tau) = E[d(t) d(t-\tau)]\ , \qquad (22)$$

where

$$\tau = (j-k) T_d + (m-n) T_0\ , \qquad (23)$$

T_0 and T_d were defined earlier. j ,k denote the antenna elements of the array and m, n denote the time delays between the taps.

An expression of autocorrelation function of $d(t)$ can also be derived as follows by using Perseval's relation [1]

$$R_d(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Gamma^2(\omega) e^{j\omega t} d\omega$$

$$= \frac{1}{2\pi I_0{}^3} \int_{-\infty}^{\infty} \omega^4 \exp(-\frac{1}{I_0}(\omega^2 - j\omega\tau I_0)) d\omega$$

$$= \frac{\exp\left(-\frac{1}{4} I_0 \tau^2\right)}{8\sqrt{\pi}} \left[\frac{3}{\sqrt{I_0}} - 3\tau^2 \sqrt{I_0} + \tau^4 \frac{(I_0)^{3/2}}{4}\right]\ , \qquad (24)$$

where $I_0 = 16\pi \times \Delta f^2$, $R_d(0) = \frac{3}{32\pi \Delta f}$ gives average power per element for UWB pulses with power spectral density of $\Gamma_d(\omega)$ and $\Gamma(\omega)$ was defined earlier. The interference matrices R_{i1} and R_{i2} for two interferences A and B can be found in the same way by using their individual bandwidth Δf and individual time delays τ (23).

The desired output signal power P_d can be computed as

$$P_d = \frac{1}{2} E\{|d(t)|^2\} = \frac{1}{2} W_{opt}^T R_d W_{opt} \qquad (25)$$

The output interference and noise power are computed from the following equations:

$$P_i = \frac{1}{2} E\{|i(t)|^2\} = \frac{1}{2} W_{opt}^T R_i W_{opt}\ . \qquad (26)$$

$$P_n = \frac{1}{2} E\{|n(t)|^2\} = \frac{1}{2} W_{opt}^T R_n W_{opt}\ . \qquad (27)$$

Using these values of desired output power, the interference power and the thermal noise power we can use the following expression to compute the $SINR$:

$$SINR = \frac{P_d}{P_i + P_n}\ . \qquad (28)$$

Bit error rate depends on the modulation methods applied for transmission of the signals. Let us assume that we use coherent binary PSK modulation for transmission of the signals. For an $AWGN$ channel the BER is given by

$$BER = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right) \qquad (29)$$

where E_b is the transmitted signal energy per bit, $N_0/2$ is the power spectral density of white noise process. We can modify the equation by replacing N_0 with the sum of thermal noise and interference and denoting signal to interference, noise ratio by $SINR$. The equation now looks as follows

$$BER = \frac{1}{2} erfc\left(\frac{SINR}{\sqrt{2}}\right)\ . \qquad (30)$$

The equation (30) can also be expressed in terms of Q-function as well

$$BER = Q(\sqrt{2SINR}) \tag{31}$$

4 RESULTS & DISCUSSION

Analytical results of an UWB wireless communication system based on the analysis presented in sec 3 are depicted here for different system and antenna parameters. Fig-1 shows the smart antenna arrangements with a signal processor. The antenna elements are placed at one wavelength apart. The bandwidth of the signal has a range from 2 GHz to 10 GHz. Time delays are connected with each antenna elements. Delays in seconds (T_0) are calculated taking the inverse of the signal bandwidth. One desired signal and two undesired signals were taken into consideration in the computation. All the signals were assumed to possess equal powers. The noises were uncorrelated and independent. The look direction filter with variable taps were used. One such filter with eight taps is specified by the following vector

$$F^T = [5.0904, -0.0259, -2.9584, -0.0340, 3.6802,$$
$$-0.0080, -1.5831, -0.002]$$

for array with eight number of delay elements. The delay elements were varied from 4 numbers to 14 numbers at an increment of 2 elements. The *F* vector terms have been varied according to the number of delay elements, the values were kept fixed to maintain the output frequency response same. This was done to fulfill the condition of the filter. It is a finite impulse response filter designed for UWB signal with minimum number of taps. Look direction of the array is varied from -90° to 90°. Optimum weights were calculated with 181 positions of look direction.

TABLE 1

SIGNALS USED IN COMPUTATION

Source	Pulse duration (ΔT)	3dB bandwidth (*GHz*)
Desired Signal	0.8 ns	1.25-2.85
Interference-A	0.3 ns	3.25-7.75
Interference-B	0.2 ns	5.0-11.5

Fig-2 depicts *SINR* when one of the interferences *A* is arriving from different directions. At look direction the *BER* coincides at a point for any number of antenna elements (3 dB). This is because the smart antenna can not distinguish between desired signal and interferences at look direction. For other directions it is observed that *SINR* improves with increasing number of antenna elements and the directivity of the antenna also improves. Fig-3 depicts *SINR* when the desired signal is arriving from different directions. It is observed that *SINR* value improves with increase in antenna elements, the directional pattern remains similar. This is because of directional capability of the smart antenna. Although the desired signal is coming from different it seems to the receiver that it is coming from broadside. Fig-4 is a plot showing the effect of receiver power on the bit error probability. There is normal improvement in bit error rate with increase in receiver power. The curves bend downwards more with increase in number of elements which clearly indicates improvement. *BER* as a function of number of delay elements is shown in fig.5. There is no improvement in performance if the number of delays is increased gradually. The plots of *BER* improvement (receiver sensitivity) as a function of number of antenna elements is shown in fig-6. The BER is decreasing with increasing number of antenna elements.

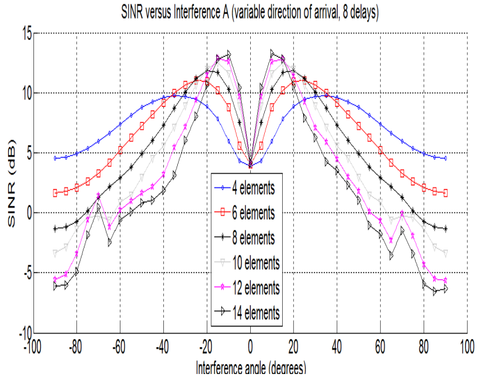

Fig-2: SINR as a function of Interference Angle with antenna elements as parameter

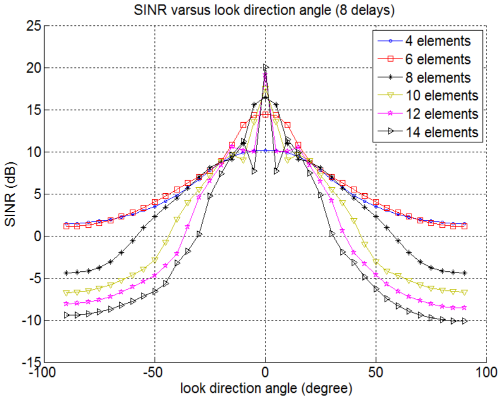

Fig-3 SINR versus look direction angle with antenna elements as parameter

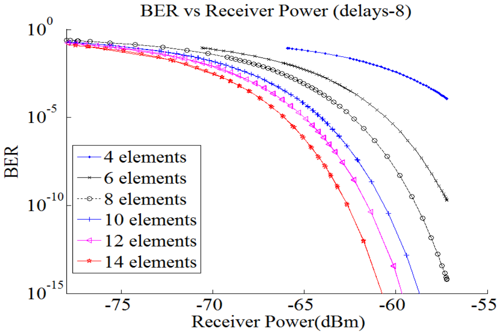

Fig-4 Bit error rate versus receiver power (dBm) with antenna elements as parameter

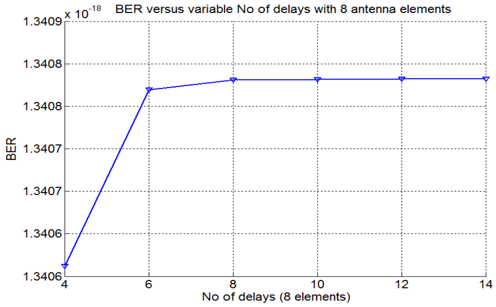

Fig-5 Bit error rate versus number of delay elements in the time delay line with number of antenna elements being fixed at 8.

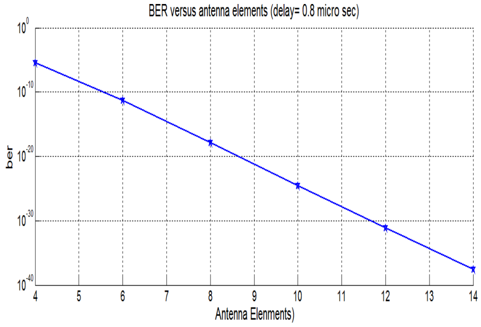

Fig-6 Plot of BER versus number of antenna elements with T0= 0.8 micro -sec. time delay.

5 CONCLUSION

An analytical approach is presented to evaluate the bit error rate performance improvement of a UWB communication system by using smart antenna. Receiver BER performance is evaluated numerically as a function of number of antenna elements as well as a function of delays. It is found that there is BER improvement with increase in no of antenna elements but there is no improvement in BER with increase in number of delays. There is no advantage at all if the number of delays is increased keeping the delay time fixed at the desired signal pulse width duration. The results will find application in the design of UWB communication system.

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