Perishable (s, S) Inventory System with Postponed Demands

Mohammad Ekramol Islam

Abstract—This paper analyzes an (s, S) Inventory system where arrivals of customers form a Poisson process. When inventory level reaches zero due to demands, further demands are sent to a pool which has capacity $M (< \infty)$. Service to the pooled customers will be provided after replenishment against the order placed on reaching that level s. Further they are served only if the inventory level is at least *s*+1. The lead-time is exponentially distributed and the life time of each item is assumed to have exponential distribution with the rate $\theta > 0$. The joint probability distributions of the number of customers in the pool and the Inventory level are obtained in the steady state case. Some measures of the system performance in the steady state are derived and relevant numerical illustrations are provided.

Index Terms— Perishable inventory, Postponed demand, Lead-time

1 INTRODUCTION

Inventory models play an important role in Operations Research and Management Science. Usually, the analysis of inventory systems is carried out without considering the effects of deterioration. In several existing models, it is assumed that products have the infinite shelf-life. But in a number of practical situations, a certain amount of decay or waste is experienced on the stocked items. For example, this arises in certain food products subjected to deterioration or radioactive materials where the decay is present.

The deteriorations of items in the inventory system occur due to one or many factors viz. the storage condition, the weather condition including the nature of the particular product under study. Some items in the inventory system may deteriorate whereas others can be stored for an indefinite period without deterioration. The deterioration is usually a function of the total amount of inventory on hand. Hence it is necessary to study inventory system where the concept of deterioration arises.

Goel and Aggarwal[1] studied an inventory model by considering demand as a function of selling price and different rates of deterioration. Nahmias[2] provided a reference list of 77 periodicals and books dealing with ordering policies for perishable inventories. Raafat[3] presented a complete and up-to-date survey of literature for the deteriorating inventory models.

We consider, In this paper, a perishable (s, S) inventory system with postponed demands. We assume that customers arrive to the system according to a Poisson process with rate $\lambda > 0.$ When inventory level depletes to s due to demands or decay or service to a pooled customer, an order for replenishment is placed. The leadtime is exponentially distributed with parameter γ. When inventory level reaches zero, the incoming customers are sent to a pool of capacity M. Any demand that takes place when the pool is full and inventory level is zero, is assumed to be lost. After replenishment, as long as the inventory level is greater than s, the pooled customers are selected according to an exponentially distributed time lag, with rate depending on the number in the pool. We also assume that the lifetime of each item has exponential distribution with rate $\theta > 0.$ Under the above consideration, we built up the model and studied some characteristics. The diagram of the model is shown in **Fig. 1**.

It is also figured out that arrival rate, replenishment rate and inventory carrying cost have the significant impact on the cost function of the model. A real life example of the present model can be a restaurant where food and service are both required to satisfy customers' demand. Customers have a waiting room (pool) to wait during the service leadtime. If the situation is like that the capacity of the waiting room is full and an external customer arrives on the system for a particular item which is available, the system owner must try to satisfy his demand rather than satisfying the pool customers; otherwise, the demand must be lost. In the mean time, the waiting customers generally can have some entertainment facilities to avoid boringness.

An (s, S) inventory system with random life- times and positive leadtime was considered by Kalpakam and Sapna[4]. Berman and Kim[5,6] analyzed a problem in which customers arrive at a service facility according to Poisson process with service times exponentially distributed and each customer demands one item of the inventory; both zero lead time and positive lead time cases were discussed. Krishnamoorthy and Mohammad Ekramol Islam[7]

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considered an (s,S) inventory system with postponed demands where they assumed that customers arrive to the system according to a Poisson process. When inventory level depletes to s due to demands or decay or service to a pooled customer, an order for replenishment is placed. The leadtime is exponentially distributed with parameter γ. When inventory level reaches zero, the incoming customers are sent to a pool of capacity M. From the pool, customers will be picked up for satisfying demands. The concept of postponed customers in queueing model has been considered by Deepak et. al[8]. Artalejo et al[9] made a numerical analysis of (s,S) Inventory system with repeated attempts and Sivakumar and Arivarignan[10] also considered an inventory system with postponed demands. In that article they considered a continuous review perishable inventory system in which the demands arrive according to Morkovian Arrival Process (MAP). Berman and Sapna[11,12] studied an inventory control problem at a service facility, which uses one item of inventory for service provided. They assumed Poisson arrivals, arbitrarily distributed service times and zero leadtime and analyzed the system with the restriction that the waiting space is finite.

Arivarignan, Elango and Arumugam[13] considered a perishable inventory management system at a service facility, with arrival of customers forming a Poisson process. Each customer requires a single item and it is delivered through a service of random duration having the exponential distribution.

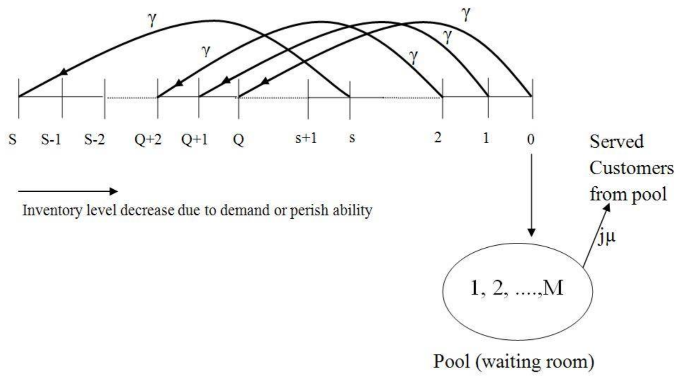

Fig.1 Diagram of the inventory model.

2 Assumptions Considered in Building Up the Model

i. Initially the inventory level is S.

ii. Inter arrival times of demands are exponentially distributed with parameter λ.

iii. Lead time is exponentially distributed with parameter γ.

iv. Life –time of each item has negative exponential distribution with parameter $\theta > 0$.

v. Replenishment quantity $Q = S - s$.

vi. Demand that arrives when the inventory level is 0, enters the pool with rate λ until the pool capacity is full i.e. M. Beyond M, the demand is lost, provided inventory level is also zero

vii. When the inventory level $I(t) > s$, demands from the pooled customers also can be met with the rate $j\mu$ (*j* is the number of customers present in the pool), but when $I(t) \leq s$ only external demands will be met and pooled customers have to wait until the next replenishment.

3 Notations Used in The Model

$I(t) =$ Inventory level at time t

$N(t) =$ Number of customer in the pool

$\{I(t), N(t)\} := \{(i, j) | 0 \leq i \leq S; 0 \leq j \leq M\}$

$E_1 = \{0,1,2,..........,S\}$

$E_2 = \{0,1,2,...........,M\}$

$E = E_1 \times E_2$

$e_{M+1} = (1,1,1,.......1)'$; an $(M+1)$-component column vector of 1.

4 The Proposed Model

It can be verified that $\{(I(t), N(t)), t \geq 0\}$ is a Markov process on the state space E. The infinitesimal generator of the process $A = \left(a(i,j,k,l)\right), (i,j),(k,l) \in E$, can be expressed as:-

$$a\left((i,j),(k,l)\right) = \begin{cases} \lambda & : k = i-1;\ i = s+1,\ldots,S \\ & : j = 0,1,\ldots,M;\ l = j \\ j\mu & : k = i-1;\ i = s+1,\ldots S \\ & : l = j-1;\ j = 1,2,\ldots,M \\ i\theta & : k = i-1;\ i = 1,2,\ldots,S \\ & : l = j;\ j = 0,1,\ldots,M \\ -(\lambda + i\theta + j\mu) & \\ & : k = i;\ i = s+1,\ldots S \\ & : l = j;\ j = 0,1,\ldots,M \\ \lambda & : k = i = 0 \\ & : l = j+1;\ j = 0,1,\ldots,M-1 \\ \gamma & : k = i+Q;\ i = 0,1,\ldots,s \\ & : l = j;\ j = 0,1,\ldots,M \\ -(\lambda + \gamma + i\theta) & : k = i,\ i = 0,1,\ldots,s \\ & : l = j;\ j = 0,1,\ldots,M \\ \lambda & : k = i-1;\ i = 1,2,\ldots,s \\ & : l = j;\ j = 0,1,\ldots,M \end{cases}$$

Define:

$$A_{ik} = \left(a\left((i,j),(k,l)\right)\right)_{j,l\in E_2},\ i,k \in E_1$$

The infinitesimal generator A can be conveniently expressed as a partitioned matrix

$$A = \left(\left(A_{ik}\right)\right)$$

where A_{ik} is a $(M+1)\times(M+1)$ sub-matrix which is given by

$$A_{ik} = \begin{cases} A & \text{if } k=i;\ i=0 \\ B & \text{if } k=i+Q;\ i=0,1,\ldots,s \\ A_i & \text{if } k=i-1;\ i=s+1,\ldots,S \\ B_i & \text{if } k=i;\ i=s+1,\ldots,S \\ C_i & \text{if } k=i;\ i=1,2,\ldots,s \\ D_i & \text{if } k=i-1;\ i=1,2,\ldots,s \end{cases}$$

with

$$A = \left(a_{ij}\right)_{(M+1)\times(M+1)} = \begin{cases} (i,j)\to(i,j+1) \text{ is } \lambda \\ \quad \forall\, i=0;\ j=M-1,\ldots,0 \\ (i,j)\to(i,j) \text{ is } -\gamma \\ \quad \forall\, i=0;\ j=M \\ (i,j)\to(i,j) \text{ is } -(\lambda+\gamma) \\ \quad \forall\, i=0;\ j=M-1,\ldots,0 \\ \text{other elements are } 0 \end{cases} \tag{1}$$

$$B = \operatorname{diag}(\gamma\ldots\gamma) \tag{2}$$

$$A_i = \left(a_{ij}\right)_{(M+1)\times(M+1)} = \begin{cases} (i,j)\to(i,j+1) \text{ is } (\lambda+iQ) \\ \quad \forall\, i=s+1,\ldots,S;\ j=M,\ldots,0 \\ (i,j)\to(i-1,j-1) \text{ is } j\mu \\ \quad \forall\, i=s+1,\ldots,S;\ j=M,\ldots,1 \\ \text{other elements are } 0 \end{cases}$$

(3)

$$B_i = \left(a_{ij}\right)_{(M+1)\times(M+1)} = \begin{cases} (i,j)\to(i,j) \text{ is } (\lambda+i\theta+j\mu) \\ \quad \forall\, i=s+1,\ldots,S;\ j=M,\ldots,1 \\ (i,j)\to(i,j) \text{ is } -\lambda \\ \quad \forall\, i=s+1,\ldots,S;\ j=0 \\ \text{other elements are } 0 \end{cases}$$

(4)

$$C_i = \operatorname{diag}\left(-(\lambda+\gamma+i\theta),\ldots,-(\lambda+\gamma+i\theta)\right) \tag{5}$$

$$D_i = \operatorname{diag}\left(-(\lambda+i\theta),\ldots,-(\lambda+i\theta)\right) \tag{6}$$

So, we can write the partitioned matrix as follows:

$$\tilde{A} = \left(a_{ij}\right)_{(S+1)\times(S+1)} = \begin{cases} (i\to i) \text{ is } A \quad \forall\, i=0 \\ (i\to i) \text{ is } C_i \quad \forall\, i=1,\ldots,s \\ (i\to i) \text{ is } B_i \quad \forall\, i=s+1,\ldots,S \\ (i\to i+Q) \text{ is } B \\ \quad \forall\, i=0,\ldots,s \\ (i\to i-1) \text{ is } D_i \\ \quad \forall\, i=1,\ldots,s \\ (i\to i-1) \text{ is } A_i \\ \quad \forall\, i=s+1,\ldots,S \\ \text{other elements are } 0 \text{ matrix} \end{cases} \tag{7}$$

5 STEADT STATE ANALYSIS

It can be seen from the structure of the matrix A that the state space E is irreducible. Let the limiting distribution be denoted by $\Pi^{(i,j)}$:

$$\Pi^{(i,j)} = \underset{t\to\infty}{\mathrm{Lt}} \Pr\left[I(t), N(t) = (i,j)\right], \quad (i,j) \in E$$

$$\text{write } \Pi = \left(\Pi^{(S)}, \Pi^{(S-1)}, \ldots\ldots\ldots\ldots\ldots, \Pi^{(1)}, \Pi^{(0)}\right) \text{and}$$

$$\Pi^{(K)} = \left(\Pi^{(K,M)}, \Pi^{(K,M-1)}, \ldots\ldots\ldots\ldots\ldots, \Pi^{(K,1)}, \Pi^{(K,0)}\right) \quad \text{for } K = 0,1,\ldots\ldots,S$$

The limiting distribution exists, satisfies the following equations:

$$\Pi\tilde{A} = 0 \qquad \text{and} \qquad \sum \Pi^{(i,j)} = 1 \quad (8.\ a,b)$$

The first equation of the above yields a set of equations which can be represented as a general form in the following manner:

$$\Pi^{(i+1)}A_{i+1} + \Pi^{(i)}B_i + \Pi^{(i-Q)}B = 0 \qquad :i = Q,\ldots\ldots,S-1$$
$$\Pi^{(i+1)}A_{i+1} + \Pi^{(i)}B_i = 0 \qquad :i = s+1,\ldots\ldots,Q-1$$
$$\Pi^{(i+1)}A_{i+1} + \Pi^{(i)}C_i = 0 \qquad :i = s$$
$$\Pi^{(i+1)}D_{i+1} + \Pi^{(i)}C_i = 0 \qquad :i = 1,2,\ldots\ldots\ldots,s-1$$
$$\Pi^{(i+1)}D_{i+1} + \Pi^{(i)}A = 0 \qquad :i = 0$$
$$\Pi^{(S)}B_S + \Pi^{(s)}B = 0$$
$$(9.\text{a - f})$$

The solution of the above equations (except the last one) can conveniently be expressed as

$$\Pi^{(i)} = \Pi^{(0)}\beta_i \qquad :i = 0,1,\ldots\ldots\ldots,S \qquad (10)$$

where

$$\beta_i = \begin{cases} I & \text{if } i = 0 \\ -AD_1^{-1} & \text{if } i = 1 \\ (-1)^i AD_1^{-1}C_1D_2^{-1}\ldots C_iD_i^{-1} & \text{if } i = 2,3,\ldots\ldots,s \\ -\beta_{i-1}C_{i-1}A_i^{-1} & \text{if } i = s+1 \\ -\beta_{i-1}B_{i-1}A_i^{-1} & \text{if } i = s+2,\ldots.,Q \\ -\beta_{i-1}B_{i-1}A_i^{-1} - \beta_{i-(Q+1)}BA_i^{-1} & \text{if } i = Q+1,\ldots,S \end{cases}$$

To compute $\Pi^{(0)}$, we can use the following equation

$$\Pi^{(S)}B_S + \Pi^{(s)}B = 0 \quad \text{and} \quad \sum \Pi^{(K)}e_{M+1} = 1 \quad (11.\text{a,b})$$

which yield, respectively,

$$\Pi^{(0)}\left(\beta_S B_S + \beta_s B\right) = 0 \;\text{ and }\; \Pi^{(0)}\left(I + \sum \beta_i\right)e_{M+1} = 1 \quad (12)$$

6 SYSTEM CHARACTERISTICS OF THE MODEL

a) Average inventory carried to the system level

Let μ_1 denote the average inventory level in the steady state. Then we have:

$$\mu_1 = \sum_{i=1}^{S} i \sum_{j=0}^{M} \Pi^{(i,j)} \qquad (13)$$

b) Mean Re-order rate

Suppose μ_2 is the mean re-order rate. Then we have:-

$$\mu_2 = \lambda \sum_{j=0}^{M} \Pi^{(s+1,j)} + \sum_{j=1}^{M} j\mu\Pi^{(s+1,j)} + (s+1)\theta \sum_{j=0}^{M} \Pi^{(s+1,j)}$$

(14)

c) Mean number of perished items

The mean number of perished items μ_3 is

$$\mu_3 = \sum_{i=1}^{S} i\theta \sum_{j=0}^{M} \Pi^{(i,j)} \qquad (15)$$

d) The average number of customers lost to the system μ_5 is

$$\mu_4 = \lambda \Pi^{(0,M)} \qquad (16)$$

e) Mean number of customers in the pool

The expected number of customers in the pool μ_4 is

$$\mu_5 = j \sum_{i=0}^{S} \Pi^{(i,j)} \qquad (17)$$

f) The probability that the external demands will be satisfied immediately after its arrival, μ_6 is,

$$\mu_6 = \sum_{i=1}^{S} \sum_{j=0}^{M} \Pi^{(i,j)} \qquad (18)$$

g) The probability of the external demands on it's arrival at the pool is

$$\mu_7 = \sum_{j=0}^{M-1} \Pi^{(0,j)} \qquad (19)$$

h) The probability that an pooled customer will be served is

$$\mu_8 = \sum_{i=s+1}^{S} \sum_{j=1}^{M} \Pi^{(i,j)} \qquad (20)$$

7 COST FUNCTION

Define

$C_1 =$ Inventory holding cost per unit per unit time

$C_2 =$ Cost of re-order of the system

$C_3 =$ Cost of items perished in the system

$C_4 =$ Cost of customers lost in the system

$C_5 =$ Holding cost of customers in the pool per unit per unit time

So the total expected cost of the system is

$$E(TC) = C_1\mu_1 + C_2\mu_2 + C_3\mu_3 + C_4\mu_5 .$$

$$E(TC(s,S,M)) = C_1\left(\sum_{i=1}^{S} i \sum_{j=0}^{M} \Pi^{(i,j)}\right) + C_2\left(\lambda \sum_{j=0}^{M} \Pi^{(s+1,j)} + \sum_{j=0}^{M} j\mu\Pi^{(s+1,j)} + (s+1)\theta \sum_{j=0}^{M} \Pi^{s+1,j}\right) + C_3\left(\sum_{i=1}^{S} i\theta \sum_{j=0}^{M} \Pi^{(i,j)}\right) + C_4\left(\lambda\Pi^{(0,M)}\right) + C_5\left(\sum_{i=j}^{M} j \sum_{i=0}^{S} \Pi^{(i,j)}\right) \quad (21)$$

Since Analytic expressions are not available for the stationary probabilities that establishing convexity is impossible .Even when the analytic expressions for these probabilities are available, the expression for cost function may be too unwieldy and hence not analytically tractable.

8 NUMERICAL ILLUSTRATIONS OF THE MODEL

Since analytical expressions are impossible to arrive at, by giving values to the underlying parameters we provide some numerical illustrations. Consider a distributor that maintaining an inventory system of goods. If the total cost for the system is TC , expressed in any unit of monetary value (i.e., taka, dollars etc.), order level(maximum capacity of the system) is $S = 6$ unit, at the beginning of the process with the demand rate $\lambda = 0.3$. Suppose the items are perishable with the rate of decay $\theta = 0.1$, replenishment rate is $\gamma = 0.6$ and an order will be placed when re-order level $s = 2$ that is $Q = 4$, the order quantity. It is also considered that the system has a pool capacity $M = 3$ and from the pool customers will be served at the service rate $\mu = 0.2$. For this particular model, the following costs are also involved i.e., $C_1 =$ Holding/Inventory carrying cost per unit per unit time $= 1$ unit, $C_2 =$ Cost of re-order of the system is=2, $C_3 =$ Cost due to decay/perish of items=3, $C_4 =$ Loss to the system due to customer not joining the system=2 unit and $C_5 =$ Holding cost of customers in the pool per unit per unit time is =1 (may be in taka or dollar). On the basis of the above mentioned specifications we have calculated the values of various system characteristics described in section 6 of the paper and the results are prescribed in table-1 and steady state probabilities of the model are given Appendix I

Table 1

PERFORMANCE MEASURE

μ_1	2.818910
μ_2	0.223892
μ_3	0.281891
μ_4	0.0009928
μ_5	0.168838
μ_6	0.932154
μ_7	0.0645367
μ_8	0.0792327
$E(TC)$	3.691357433

9 SENSITIVITY ANALYSIS

The sensitividy analysis was performed on the basis of arrival rates versus total cost, different rates versus total cost and different costs versus total cost. These analyses are shown in **Fig. 2**, **Fig. 3**, **Fig. 4** and **Fig. 5**, respectively.

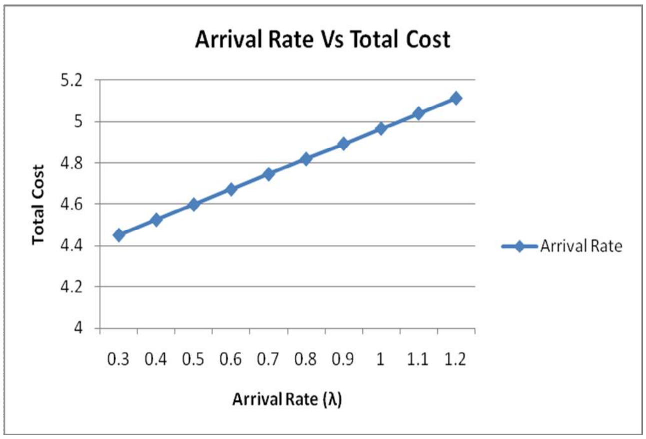

Fig. 2 Arrival rates versus total cost.

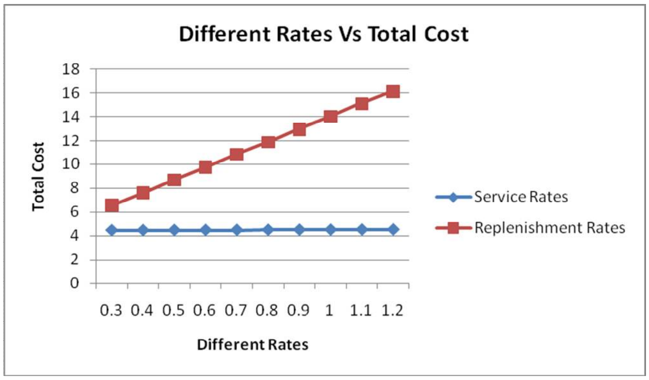

Fig. 3 Different rates versus total cost.

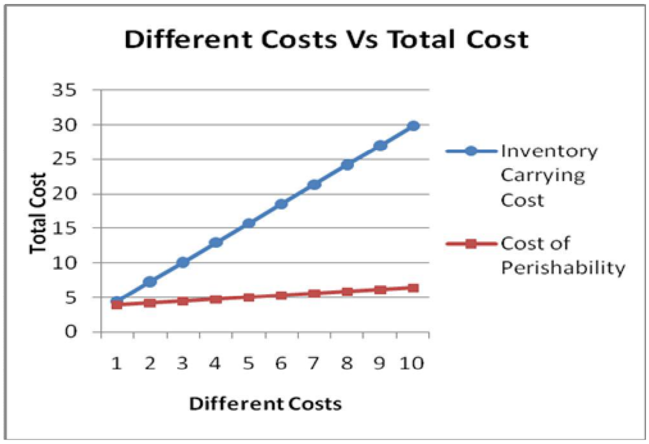

Fig. 4 Different costs versus total cost.

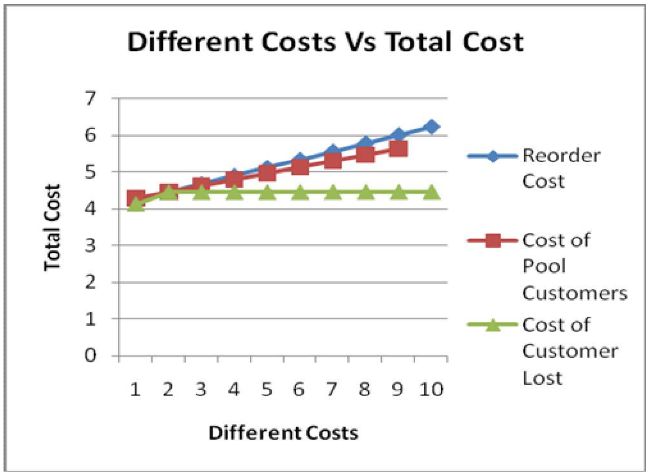

Fig. 5 Different costs versus total cost.

From the graph1, it is observed that if the arrival rate is increased then the total cost is increased in a significant amount. The result is obvious as the rate is increase it has impact on higher re-ordering, lost sales and also increase the cost of carrying pool and buffer customers. Hence arrival rate is vital to this system.

From the graph 2, it is observed that service rate has a little impact. The reason behind that is increasing service rate is simply imply less per unit customers carrying cost to the pool and buffer. But it is observed that if the Lead time rate is increased then the total cost is increased in a significant amount. The result is obvious as the rate is increased, from the observation it seems that lead time rate is very sensitive to the system.

From the graph 3 & 4, it is indicated that inventory carrying cost to the system has a great impact as it increases; total cost is increased drastically. Hence it must be scrutinized properly for selection. It is also indicated that the cost of number of units are perished has a moderate impact as it increase; total cost is increased in a significant manner. Hence it also to be scrutinized properly for selection per unit cost for perishable item.

10 Conclusion

Generally, inventory models with perish ability concept is more complex to analysis than the models where non-perishable items are tackled. The model becomes more complex if the perishable rate is depends upon the amount of items kept to the stock but sound more realistic. In this present paper, we tackled the same and explored some valuable properties for the model which have a great importance for the management to minimize the total cost.

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APPENDIX I

$\Pi^{(0,3)}$	0.003309446	$\Pi^{(3,1)}$	0.021691676
$\Pi^{(0,2)}$	0.006261941	$\Pi^{(3,0)}$	0.334850022
$\Pi^{(0,1)}$	0.016810447	$\Pi^{(4,3)}$	0.00168273
$\Pi^{(0,0)}$	0.041464317	$\Pi^{(4,2)}$	0.004915656
$\Pi^{(1,3)}$	0.000267713	$\Pi^{(4,1)}$	0.023283499
$\Pi^{(1,2)}$	0.001481532	$\Pi^{(4,0)}$	0.137226505
$\Pi^{(1,1)}$	0.006725268	$\Pi^{(5,3)}$	0.000252347
$\Pi^{(1,0)}$	0.093294714	$\Pi^{(5,2)}$	0.001873237
$\Pi^{(2,3)}$	0.000535426	$\Pi^{(5,1)}$	0.01118548
$\Pi^{(2,2)}$	0.002963064	$\Pi^{(5,0)}$	0.017565248
$\Pi^{(2,1)}$	0.013450537	$\Pi^{(6,3)}$	0.000214072
$\Pi^{(2,0)}$	0.186589429	$\Pi^{(6,2)}$	0.00136715
$\Pi^{(3,3)}$	0.000981615	$\Pi^{(6,1)}$	0.007334646
$\Pi^{(3,2)}$	0.00445067	$\Pi^{(6,0)}$	0.057971594