Suitability of Distorted Wave Born Approximation Model (DWBA) for Analysis of Heavy-Ion Transfer Reaction

Jennifar Sharmeen, M.S. Hossain, A.H.M. Ruhul Quddus, M.M.A. Zaman

Abstract— Features of distorted wave Born approximation (DWBA) theory for heavy ion induced reactions are investigated. Finite range DWBA theory has been applied and the calculations are done using the computer code DWUACK5. Finite range DWBA theory has been applied and the calculations are done using the computer code DWUCK5. Products of spectroscopic factors, s_1s_2 have been extracted for all the selected transitions. Sufficient amount of spectroscopic information is obtained from the present analysis. Finite range DWBA theory showed its suitability for application to the case of single-particle transfer reactions; where as the two-particle transfer reactions are found to be very much problematic.

Index Terms—Keywords should be taken from the taxonomy (http://www.computer.org/mc/keywords/keywords.htm). Keywords should closely reflect the topic and should optimally characterize the paper. Use about four key words or phrases in alphabetical order, separated by commas (there should not be a period at the end of the index terms)

1 INTRODUCTION

SINCE its inception heavy ion research has developed to a frontier science of nuclear physics in little more than half a decade. The main aim of the research is to synthesize heavy elements. The interaction of heavy-ion projectiles with medium weight nuclei offers interesting possibilities for experimental and theoretical investigations [1, 2]. Since the availability of high quality heavy ion beams a large number of nuclear reactions have been studied using these projectiles. Full-recoil, finite range, DWBA has been associated in the analysis of nuclear transition leading to discrete final states. The exit channel distorting potential and the entrance channel optical model potential are considered to be equal in the conventional DWBA calculations. This approach is insufficient to explain adequately the actual physical phenomena, even for the one-nucleon transfer reaction on closed shell target nuclei which otherwise are known to be the best candidates for application of the theory. Modifications have been proposed for the exit channel distorting potential by increasing the diffuseness of the real part of the potential.

DWBA is important for particle transfer reaction study because it considers a physical phenomenon e.g. distortion of the wave function. If the incident channel energy is such that the Coulomb effects cannot be neglected, then the solution is carried out by the DWBA method. These situations are important for heavy ion interactions and thus for transfer reactions induced by heavy ions. In order to reduce the complexity, in some calculations zero-range approximation is used. But it causes major inaccuracy in the application of DWBA. It is also possible to perform the calculations without this approximation. In that case finite range effects are included in the calculations. The calculations of the present work have been done accordingly.

The DWBA theory has proved to be successful in describing low energy single-nucleon transfer reactions. At energies well above 10 MeV, however, individual nucleon-nucleon interactions are expected to become more important than nucleon-nucleus interactions. An energy dependent effective nucleon-nucleus interaction may play a vital role in such situation. In the past studies heavy ion transfer reactions have been plagued by uncertainties arising from ambiguities in the optical model potential. One important characteristic of heavy ion elastic scattering is the strong absorption of it which has restricted its sensitivity to the extreme surface region of the nucleus. It has been observed that a variety of potentials can provide fits to the data, provided that they have similar values in this critical region. For higher bombarding energies there is evidence that phenomenological optical model potentials can be quite precisely determined, at least for relatively light projectiles and targets.

The present study aims at investigating some important features of heavy-ion induced nuclear reaction through calculating the products of spectroscopic factors s_1s_2 . Different types of reactions e.g. mainly single-nucleon transfer

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ones are chosen for the purpose. Reactions of different nature of varying targets have been considered. These targets include ^{11}B, ^{12}C, ^{40}Ca, ^{58}Ni, ^{208}Pb. Of the targets considered the ^{208}Pb target is an ideal target nucleus for nuclear reaction studies because it has the magic numbers; neutron, N = 126 and proton, Z = 82. So Pb has been used in many light-ion induced reactions. But its interactions with heavier particles is little known. ^{40}Ca is another magic number nucleus. Other targets are also good candidates of shell model nuclei.

2 THE DWUCK FORMALISM

The differential cross-section values necessary for the present investigation for the possible values of angular momenta were calculated using the equation:

$$\frac{d\sigma}{d\Omega} = \frac{\mu_a\mu_b}{(2\pi\eta)^2}\frac{k_b}{k_a}\frac{1}{(2J_A+1)}\frac{1}{(2s_a+1)}\sum_{M_A M_B m_a m_b}|T|^2 \qquad (1)$$

$$= \frac{(2J_B+1)}{(2J_A+1)}\frac{1}{4\pi}\frac{1}{E_a E_b}\frac{k_b}{k_a}\left(\frac{C^2}{AB}\right)^2\frac{1}{(2S_a+1)}\times$$

$$\sum_{jmm_a m_b}\left|\sum_{lsj}\sqrt{2l+1}\,B_{lsj}S_{lsj}^{mm_a m_b}\right|^2$$

Equation (1) has been derived from the angular momentum expression for any specific l, s and j values as given by

$$\frac{d\sigma_{lsj}}{d\Omega} = \frac{2J_B+1}{2J_A+1}\frac{|B_{lsj}|^2}{1.0\times10^4}\frac{\sigma_{DW}^{lsj}}{2j+1}, \qquad m_a \neq m_b \qquad (2)$$

Here E_a and E_b are the centre of mass energies for the entrance and exit channels respectively. In case of a particle transfer reaction, the program DWUCK which is used in the present investigation computes the cross-sections with different normalizations. In the above equation the differential cross section for the reaction A(a,b)B has actually been defined in terms of the transition amplitude, T [3].

$$T=J\int d\vec{r}_b\int d\vec{r}_a\,\chi_f^{(-)*}(\vec{k}_f,\vec{r}_b)\langle bB|V|aA\rangle\chi_i^{(+)}(\vec{k}_i,\vec{r}_a) \qquad (3)$$

where $\chi_f^{(-)}$ and $\chi_i^{(+)}$ are the distorted waves in final and initial channels respectively, $\langle bB|V|aA\rangle$ is the form factor of the reaction and J is the Jacobian of the transformation to the coordinates $\vec{r}_a$ and $\vec{r}_b$, which are the relative coordinates for the systems (a,A) and (b,B).

In absence of spin the distorted wave χ_i for the asymptotic region is given by

$$\chi^{(+)}(\vec{k},\vec{r}) \xrightarrow{r\to\alpha} e^{i\vec{k}\vec{r}} + f(\Theta)\frac{e^{ikr}}{r} \qquad (4)$$

Thus the wave is a sum of the incident plane wave of momentum k, i.e., e^{ikr} and an outgoing scattered wave, $f(\Theta)\frac{e^{ikr}}{r}$. The term $\langle bB|V|aA\rangle$ contains the nuclear structure information as is evident through the following expression:

$$\langle J_B M_B s_b m_b|V|J_A M_A s_a m_a\rangle =$$
$$\sum_{lsj} B_{lsj}\langle J_A jM_A M_B - M_A|J_B M_B\rangle\times$$
$$\langle s_b s\, m_b\, m_a - m_b|s_a m_a\rangle\langle lsmm_a - m_b|jM_B - M_A\rangle\times \qquad (5)$$
$$f_{lsj}(r_a)\delta[r_b - (A/B)r_a]\,i^{-l}\,Y_l^m(ra)^*$$

where B_{lsj} is a measure of the strength of the interaction given by $B_{lsj} = \sqrt{\frac{2s+1}{2s_a+1}}\,A_{lsj}$ and A_{lsj} is the spectroscopic coefficient. The distorted wave $\chi_{m'm}(k,r)$ can be written as

$$\chi_{m'm}(k,r) = \sqrt{\frac{i\pi}{kr}}\sum_{jl} i^L\langle LsMm|JM'\rangle\sqrt{(2l+1)}\;\;\chi_{jL}(k,r)\times$$
$$\langle LsM' - m'm'|JM'\rangle Y_L^{M'-m'}(r)d^l_{\sigma m M'-m'} \qquad (6)$$

where $d^l_{\sigma m}$ are the rotation functions for integral spin.

The transition amplitude can be written by using the definition of $\langle bB|V|aA\rangle$ and in terms of the distorted wave $\chi_{m'm}$

$$T = \sqrt{\frac{4\pi}{k_a k_b}}\sum_{lsj}\langle J_A jM_A M_B - M_A|J_B M_B\rangle\sqrt{(2l+1)B_{lsj}}\times$$
$$\sum_{J_A L_A J_B L_B}\langle L_a s_a\, 0m|J_a m_a\rangle\langle L_b S_b m_a - m - m_b m_a|J_b m_a - m\rangle\times$$
$$\langle J_b sm_s - mm|J_a m_a\rangle(2L_b+1)\langle L_b 100|L_a 0\rangle\times$$
$$\sqrt{(2s_a+1)(2j+1)(2J_b+1)(2L_b+1)}\begin{Bmatrix} L_b & s_b & J_b\\ l & s & j\\ L_a & s_a & J_a\end{Bmatrix}\times$$
$$d^{Lb}_{0m_a - m - m_b}\left(\frac{CB}{A^2}\right)I^{lsj}_{J_a L_a J_b L_b}\,lsj_l^l a^{-1}b^{-1} \qquad (7)$$

The radial integrals are defined as,

$$I^{lsj}_{J_a L_a J_b L_b} = \int dr_c\,\chi_{J_b L_b}\left(k_b,\frac{A}{B}r_a\right)f_{lsj}(r_c)\,\chi_{J_a L_a}(k_a,r_a)$$

If the beta co-efficient is defined as

$$\beta^{mm_a m_b}_{lsj;L_b} = \sum_{J_a L_a J_b} i^{l_a - l_b - l}\langle L_a s_a\, 0m_a|J_a m_a\rangle\times$$
$$\langle l_b s_b m_a - m - m_b|J_b m_a - m\rangle\langle J_b jm_a - m - m_b|J_a m_a\rangle\times$$
$$(2L_b+1)\langle L_b 100|L_a 0\rangle\sqrt{(2s_a+1)(2j+1)(2J_b+1)(2L_b+1)}\times$$
$$\begin{Bmatrix} L_b & s_b & J_b\\ l & s & j\\ L_a & s_a & J_a\end{Bmatrix}\sqrt{(1-|m|)!/(1+|m|)!}\,I^{lsj}_{J_a L_a J_b L_b} \qquad (8)$$

and

$$S^{mm_a m_b}_{lsj} = \sum_{L_b}\beta^{mm_a m_b}_{lsj;L_b}P^{m_a - m m_b}_{Lb}(e) \qquad (9)$$

then the transition amplitude takes the form

$$T=\sqrt{\frac{4\pi}{k_a k_b}}C^2/AB\sum_{lsj}\sqrt{(2L+1)}B_{lsj}\times$$

$$\langle J_A j M_A . M_B - M_A | J_A M_A \rangle S_{lsj}^{m m_a m_b} \quad (10)$$

A description of the radial function and more elaborate discussion on the formulation have been given in our previous paper [5]. The cross section is now given by

$$\sigma_{DW}^{lsj} = \frac{1}{4\pi}\frac{1}{E_a E_b}\frac{k_b}{k_a}\left(\frac{C^2}{AB}\right)^2 \frac{1.0\times10^4\,(2l+1)}{2s_a+1} \times \sum_{m m_a m_b}\left|S_{lsj}^{m m_a m_b}\right|^2, (for\ m_a \neq m_b) \quad (11)$$

which finally takes the form given in equation (1).

3 The DWBA Analysis

In the present work finite range DWBA calculations were done using the code DWUCK5. Here the effective interaction has been assumed to consist of the light ion bonding potential, the Coulomb potential including core corrections, and the real nuclear core-core correction.

$$V(r) = V_R f(X_R) + iV_I f(X_I) + 2\left(\frac{h}{m_{II}C}\right)^2 \times \quad (12)$$

$$V_{so}(1/r)(d/dr) f(X_R)(\vec{L}\cdot\vec{S}) + V_c(r, r_C)$$

The optical model potentials used for the calculation was volume Wood – Saxon for some cases and surface Wood – Saxon type for the rest, as have been described below:

$V(r) = V_R f(X_R) + iV_I f(X_I)$ (Volume Wood – Saxon type potential)

$V(r) = V_{R^*}\, df(dX_R) + iV_I f(X_I)/dX_I$ (Surface Wood – Saxon type potential)

where,

$$f(X_i) = \left[1 + \exp\left(\frac{r - R_{0i}AT^{\frac{1}{3}}}{A_i}\right)\right]^{-1} \quad (13)$$

V_R = Real potential depth, V_I = Imaginary potential depth, A_j = Diffused parameter

and other terms have their usual meaning.

The same set of optical parameters was used for the entrance and exit channels. The magnitude of the absolute cross-sections and consequently of the spectroscopic factors are known to be sensitive to the choice of parameters for the Wood – Saxon potential that generate the bound state in the heavy ion system.

4 Results & Discussions

In the present work investigation of some one-nucleon and two-nucleon transfer reactions induced by heavy ions has been made. The spectroscopic factors were extracted in the work using the following relation between the experimental $\left(\frac{d\sigma(e)}{d\Omega}\right)$ and calculated differential cross section σ_{DW}:

$$\frac{d\sigma(e)}{d\Omega} = s_1 s_2 [2J_t + 1)/(2J_i + 1)]\sigma_{DW} \quad (14)$$

where S_1 and S_2 are the target and residual nucleus spectroscopic factors respectively. And J_i, J_f represent the angular momentum of the corresponding nuclei.

4.1 The $^{11}B(^{12}C,^{11}B)^{12}C$ Reaction at 344.5 MeV

4.1.1 The Ground State in the Residual Nucleus

The $^{11}B(^{12}C,^{11}B)^{12}C$ reaction was initiated by ^{12}C projectiles at incident energy 344.5 MeV. Angular distribution of the reaction products detected in Si surface barrier detector was investigated. The angular range measured covers 3-11.5 degrees at steps of 0.5 degree. The angular resolution was 0.3 degree with uncertainty in angle setting about 0.1 degree. These experimental data on angular distribution were obtained for the present investigation from the work of Jarczyk et al. [5] at the same energy. The potential parameters used in this investigation were collected from the work of Jarczyk et al. [6] who carried out a scattering experiment by ^{12}C projectiles incident on the targets ^{11}B, ^{12}C, ^{27}Al, ^{58}Ni, ^{90}Zr and ^{197}Au nuclei.

In the data, however, there is missing of valuable information at lower angles (Fig. 1). Forgetting these the general trend of variation of the differential cross section with measured angles is followed in between 4 - 1 6 degrees. Beyond this range a maximum in the data can be observed. This is supported by the theoretical calculation, although DWBA overpredicts the differential cross section in this range. Fitting could be done for angular momentum transfer value L = 0, which is agreed upon by the similar observation of Jarczyk et al. [5].

Table I
Spectroscopic factors extracted in the present work together with previously measured ones

Name of the reactions	Configuration of states J^π	Product of spectroscopic factors, S_1S_2		References
		Present values	Reference values	
$^{11}B(^{12}C, ^{11}B)^{12}C_{g.s.}$	$3/2^-$	14.64	2.85	5
$^{11}B(^{12}C, ^{11}B)^{12}C_{4.44.}$	$1/2^-$	6.828	0.79	5
$^{40}Ca(^{13}C, ^{12}B)^{41}Sc_{g.s.}$	$7/2^-$	0.3546	1.21	7
$^{40}Ca(^{13}C, ^{12}B)^{41}Sc_{0.95}$	$7/2^-$	0.711	-	7
$^{40}Ca(^{13}C, ^{12}B)^{41}Sc_{6.0}$	$9/2^-$	2.19	-	7
$^{58}Ni(^{13}C, ^{12}B)^{59}Cu_{0.91}$	$5/2^-$	5.005	2.86	7
$^{208}Pb(^{16}O,^{15}N)^{209}Bi_{g.s.}$	$9/2^-$	3.415	0.75	12
$^{208}Pb(^{16}O,^{15}N)^{209}Bi_{2.822}$	$5/2^-$	1.39	0.74	12
$^{208}Pb(^{16}O,^{15}N)^{209}Bi_{g.s.}$	$9/2^-$	1.11	0.48	12
$^{208}Pb(^{16}O,^{15}N)^{209}Bi_{1.608}$	$13/2^+$	3.97	0.70	12
$^{208}Pb(^{16}O,^{15}N)^{209}Bi_{2.822}$	$5/2^-$	5.17	0.80	12
$^{27}Al(^{6}Li,\alpha)^{29}Si_{g.s.}$	$1/2^+$	0.107	0.117	14
$^{27}Al(^{6}Li,\alpha)^{29}Si_{2.03}$	$5/2^+$	0.043	0.047	14

The experimental value of spectroscopic factors were well reproduced in the work. This value is again supported by

the results of ref [5]. The product of the spectroscopic factors S_1 and S_2 were calculated and is shown in Table 1, together with the results of previous works. The present results agree well with the reference value.

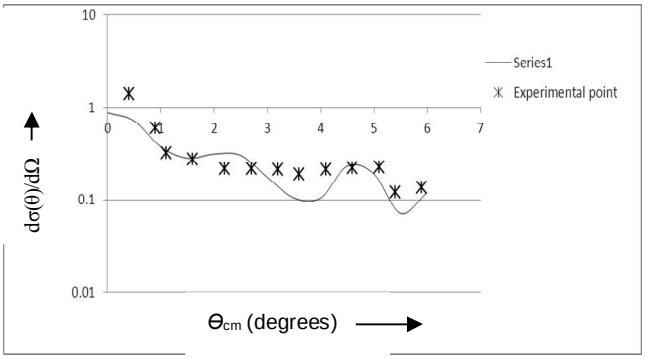

Fig. 1 Angular distributions for the $^{208}Pb(^{16}O,^{15}N)^{209}Bi$ one-proton stripping reaction in the excited state E_x = 2.822 MeV with DWBA calculations

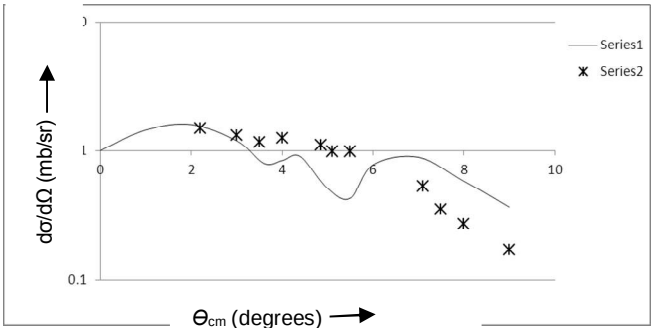

Fig. 2 Angular distributions for one-proton stripping $^{208}Pb(^{12}C,^{11}B)^{209}Bi$ reaction in the ground state with DWBA calculations

4.1.2 The 4.44 MeV State in the Residual Nucleus

As of the previous case of studying g.s. transition the sources of angular distribution data and the potential parameter data for the 4.44 MeV state are respectively due to Jarczyk et al [5] and Jarczyk et al [6]. The obtained potentials were then used to distort waves in the entrance and exit channels in the present DWBA analysis for elastic transfer from which the values of proton spectroscopic factors were extracted [Fig. 2].

The quality of fit is to a large extent similar to the g.s. transition case. In the work positions of maxima and minima are poorly reproduced. At the lower angles the position of the maxima is well reproduced; although at higher angles the fitting was, disturbed by discrepancy in the magnitudes of the maxima and minima. The position of the second maxima is shifted towards the larger angle side at around 12 degrees and the next maxima at the higher energy is not at all reproduced.

The potential binding the proton to the ^{11}B core in the ^{12}C nucleus was assumed to have a Woods-Saxon shape. Such a shape was preferred because of its ability to explain physically real phenomenon. The product of spectroscopic factors S_1 and S_2 were calculated in the work and are shown in Table 1. These values may be compared with those of Jarczyk. et al [5]. The results of the authors are also shown in the table. As against the extracted value of 6.828 in the present investigation the reference value is only 0.79 i.e., about 8.64 times larger. This is the effect of overprediction of DWBA theory in the present work.

4.2 The $^{40}Ca(^{13}C,^{12}B)^{41}Sc$ Reaction at 650 MeV: The g.s., E_x = 0.95 and 6.00 MeV Transitions

The angular distribution data for the transition to the g.s., and the E_x = 0.95 and 6.00 MeV states in the residual nucleus ^{41}Sc obtained through the reaction $^{40}Ca(^{13}C,^{12}B)^{41}Sc$ at incident beam energy 650 MeV have been taken from the work of Braeunig et al. [7]. The work was an investigation carried out using a magnetic spectrometer. Angular distribution of discrete states as well as of structures in the continuum was measured. These data are fitted in the framework of DWBA in the present work, in which only one-step process has been taken into account.

While investigating the response of angular distribution of the outgoing particles ^{12}C with variation of the state of residual nucleus e.g., g.s. and excited state E_x = 0.95 and 6.00 MeV, no remarkable variation could be observed between the states of various degrees of excitation. For both g.s. and excited states the curves were poorly fitted with the experimental points. The relative variation of cross sections for the angles within the range could not be predicted properly by the DWBA theory. Of course, the l-transfer values (l = 3) were agreed upon by the previous workers [7]. As of the other cases the cross section values at higher angles are overpredicted. It is felt that consideration of two-step process would result in a better fitting of the data. The product of the spectroscopic factors S_1 and S_2 were calculated, in the work and is shown in Table 1. These values compare reasonably well with those of Buenerd et al. [8].

4.3 The $^{58}Ni(^{13}C,^{12}B)^{59}Cu$ Reaction at 650 MeV: Optical Potential Sensitivity Investigation

The angular distribution data of the reaction products of Braeunig et al. [7] from proton stripping induced by ^{13}C at 50 MeV/u on ^{12}C, ^{40}Ca, ^{58}Ni were then considered. The necessary optical model parameters are taken from the work of Oertzen et al. [9] and Buenerd et al. [10]. Calculations are done for 3 different potential sets; and the results so obtained are displayed in Fig. 3.

The peak and trough positions in the data are predicted properly; of course, as of the previous cases, the theory again overpredicts the experimental data. Consequently the magnitude of S_1S_2 is found larger in the present work (e.g., 5.005) as against the calculated value 2.86 of Braeunig et al.

Braeunig et al., in their work, found that the experimental values of spectroscopic factors were well reproduced for ^{40}Ca tar-

get, but the cross section gave an absolute value which had a factor of 5-10 times larger for the case under investigation. For the system $^{40}Ca(^{13}C,^{12}C)^{41}Sc$ the response of angular distribution with variation of the state of residual nucleus e.g., g.s. and excited states E_x =0.95 and 6.00 MeV shows that the curves corresponding to the ground states and excited states are well fitted with the experimental points. And the nature of fits is similar. This observation is in support of a similar observation made by Braeunig et al. [7].

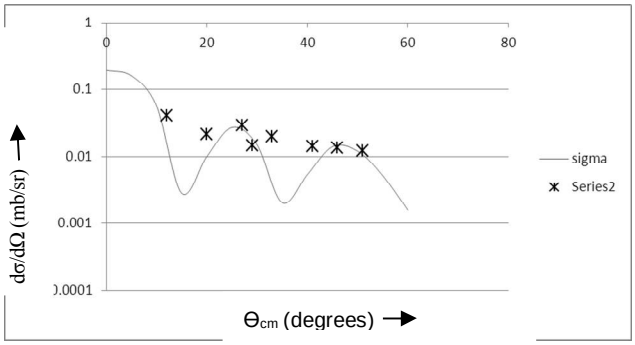

Fig. 3 Angular distribution for two-proton stripping $^{27}Al(^{6}Li,\alpha)^{29}Si$ reaction in the ground state with DWBA calculation

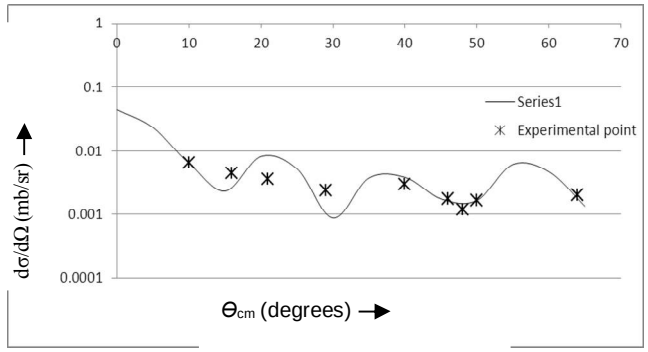

Fig. 4 Angular distribution for two-proton stripping $^{27}Al(^{6}Li,\alpha)^{29}Si$ reaction in the excited state E_x = 2.03 MeV with DWBA calculation

Towards the small angles (upto around 1 degree) the sets didn't show remarkable variation in the cross section; although at large angles it really showed this variation. Of the 3 sets, the Set II showed the best agreement; Set I worst and Set III falling in between. Set I showed higher cross section values compared to the experimental data increasing with angles. Sets II and III differed by imaginary well potential depths only. If comparison is made between these two sets the one having higher values of imaginary potential depth is a better one to predict the variation. Thus it may be commented that increasing imaginary potential depths abruptly would give a better fit; although one may not show adequate importance to the Physics in such a situation. Fernandes et al. [11] also advocated for such sort of arbitrary variation in reaction cross section in single-proton stripping reaction while he studied the ^{28}Si (^{18}O,^{19}F)^{27}Al reaction at 352 MeV.

4.4 The $^{208}Pb(^{16}O,^{15}N)^{209}Bi$ Reaction at 793 MeV: Transition to the g.s. and the 2.822 MeV State in the Residual Nucleus

The angular distribution data for the two states e.g., g.s. and the E_x = 2.822 MeV in ^{209}Bi were collected from the work of Mermaz et al. [12] who conducted an experiment by a 793 MeV ^{16}O beam incident on a Pb target. The outgoing particles were detected through a high resolution energy loss magnetic spectrometer. The energy resolution recorded in the facility was about 200 keV. The data were analyzed adopting DWBA approach for which the necessary optical model potential was taken from the work of Pieper et al. [13]. The potential parameter set gave the best fit to the elastic scattering data of similar situation. A reasonable agreement was obtained between the experimental data and theoretical prediction (Figs. 1 and 2). The maxima and minima positions are found at a larger angle than that is predicted by the DWBA theory. The relative magnitudes of the maxima and minima are well reproduced at most of the locations of the angular range except at 3.3 degrees where a minimum occurs but the DWBA prediction underestimates the actual magnitude.

The product of the spectroscopic factors calculated in the present work are respectively 3.415 and 1.39 for the g.s. and 2.822 MeV excited state as against 0.75 and 0.74 observed in the work of Mermaz et al. [12].

4.5 The $^{208}Pb(^{12}C,^{11}B)^{209}Bi$ Reaction at 480 MeV: Transition to the g.s. and the 1.608 and 2.822 MeV Excited States in the Residual Nucleus

Angular distribution data of the reaction products from the $^{208}Pb(^{12}C,^{11}B)^{209}Bi$ reaction were collected from the work of Mermaz et al. [12] who carried out a proton stripping reaction induced by ^{12}C at 480 MeV/u on ^{208}Pb. The energy resolution was again about 200 keV. DWBA analysis has been carried out in the work using the potential parameter sets of Piepper et al., same for both initial and final scattering states. It is worthwhile to note that this geometry of the form factor parameters best fits the single particle state energies either of ^{209}Pb or ^{209}Bi.'

For the g.s. transition the fit between the theory and experiment is reasonably good (Fig. 2) upto about 4.5 degrees. After this limit the prediction gives larger cross section values than the experimental data. This later trend is common for the other 2 states at E_x = 1.608 and 2.822 MeV (Fig. 1). In all the 3 cases important data at the small angles are missing. For the excited states at 1.608 and 2.822 MeV too the fit is again reasonably good upto around 4.5 degrees. The experimental value of spectroscopic factors was well reproduced for the g.s. transition. For this, the extracted value in the work is 1.11 as against 0.48 extracted by Mermaz et al. The next higher state at 1.608 MeV is populated with spectroscopic factors whose product is 3.97 as against 0.70 reported by Mermaz et al. The calculated value of the product of spectroscopic factors at E_x = 2.822 MeV is 5.1724, which is again 6.3 times larger than that is reported by

Mermaz et al. This ratio is found to increase with excitation energy.

4.6 The $^{27}Al(^{6}Li,\alpha)^{29}Si$ Reaction at 32 MeV: A Test Case of Two-Nucleon Transfer Reaction Induced by Heavy Ions

4.6.1 The Ground State and the 2.03 MeV State in the Residual Nucleus

For this investigation angular distribution data of the reaction products were collected from the work of Dhuga *et al.* [14] who carried out two-proton stripping reaction induced by ^{6}Li at 32 MeV on ^{27}Al. Calculations are done using the potential parameter sets of Ciangaru et al. [15] and Bland et al. [16]. In the calculation different potential parameter sets were used for the incident as well as the outgoing channel. This is the difference between the previously described calculations and this two-nucleon transfer case. Yet no sufficiently good agreement is observed. This observation is substituted by the fact that calculations involving two-particle transfer reactions are more difficult, because at low bombarding energy these reactions occur via multistep processes.

Figs. 3 and 4 show the quality of fit for the g.s. and the excited state at 2.03 MeV. Hardly any structure is observed in the experimental data of the angular distribution. This is true for both the cases. The quality of the fit is not encouraging in any way, although product of spectroscopic factors are found for the g.s. and E_x = 2.03 MeV states respectively as 0.107 and 0.043, but these values compare well with the reference values 0.117 and 0.047 respectively.

5 Conclusion

In the present work investigation of some one-nucleon and two-nucleon transfer reactions induced by heavy ions has been made. The one-nucleon transfer reactions include the $^{11}B(^{12}C,^{11}B)^{12}C$ reaction at 344.5 MeV, the $^{40}Ca(^{13}C,^{12}B)^{41}Sc$ reaction at 650 MeV, the $^{58}Ni(^{13}C,^{12}B)^{59}Cu$ reaction at 0.91 MeV, the $^{208}Pb(^{16}O,^{15}N)^{209}Bi$ reaction at 793 MeV and the $^{208}Pb(^{12}C,^{11}B)^{209}Bi$ reaction at 480 MeV. The two-nucleon transfer reaction includes the $^{27}Al(^{6}Li, \alpha)^{29}Si$ reaction at 32 MeV.

The angular distribution are fairly well reproduced by the DWBA calculations, although the data for the proton transfers show a slight shift in angle relative to the calculations that increase with excitation energy. In addition to it the DWBA theory again overpredicts the experimental data.

DWBA theory failed to reproduce correctly the angular distribution variation for proton transfer measured at low energies. An arbitrary change in the optical potential, however, may improve the situation. A general agreement is obtained between the experimental relative spectroscopic factors and the theoretical values calculated in this work. Of course, the conventional DWBA (exit channel distorting potential taken equal to the entrance channel optical model potential), however, has proven to be inadequate in many applications. The present work also supports this observation.

Cross section magnitudes are found not adequately sensitive to the choice of optical potential at a bombarding energy of 650 MeV. This is in good agreement with 352 MeV results of Fernandes et al. [11]. The relative spectroscopic factors extracted in the work are found to be larger than the reference values. On the whole, it appears that the DWBA theory is able to give the relative magnitudes of the cross sections for proton transfers, provided reasonable choices are made for the various ingredients of the theory. On the other hand, the two-nucleon transfer reactions are very poorly explained by the theory.

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