CHAPTER V

INDUCTANCE

Self-inductance. Whenever the magnetic flux linking with a circuit changes, an e.m.f. is induced in the circuit. This e.m.f. is given by

$$e = -N\frac{d\phi}{dt}$$

where e = induced e.m.f.

N = number of turns with which the flux links

$\frac{d\phi}{dt}$ = rate of change of the interlinking flux in webers per second

The negative sign indicates that the direction of the e.m.f. is such as to oppose the change in the flux.

If, now, the change in the flux is due to a change in the current flowing in the circuit itself (by which current the interlinking-magnetic flux is produced) and if, also, the reluctance of the path of the magnetic flux is constant, then

$$\phi = ki$$

where i is the current in the circuit.

Thus,
$$e = -Nk\frac{di}{dt}$$

or
$$e = -Nk\frac{di}{dt} \text{ volts}$$

Since $k = \frac{\phi}{i}$, the above expression can be written

$$e = -\left(N\frac{\phi}{i}\right)\frac{di}{dt} \text{ volts}$$

or
$$e = -L\frac{di}{dt} \text{ volts}$$

where L is the "coefficient of self-induction" or, simply, the "inductance" of the circuit. Obviously L is constant for any given circuit, only if k is constant—i.e. when no magnetic material is present. If i is expressed in amperes, $\frac{di}{dt}$ is in amperes per second, and k is the flux produced by 1 amp flowing in the circuit. Then L is in henrys.

It follows from the above that the inductance of a circuit, in henrys, can be expressed in words as

Number of turns $\times$ flux produced per ampere

(When, as in an a.c. circuit, part of which is wound on a core of magnetic material, the flux per ampere is not constant, a better definition for the inductance is given by the induced voltage divided by the rate of change of current.)

Mutual Inductance. If two coils are close together and unit current flows in one of them, then the number of "linkages" with the other coil, of the magnetic flux due to this current, is called "the coefficient of mutual induction," or simply the "mutual inductance" between the coils. By "linkages" is meant the product of flux and the number of turns on the coil.

If the current i_1 in coil 1 varies, its rate of change being $\frac{di_1}{dt}$, then the e.m.f., e_2, induced in the second coil is given by

$$e_2 = - M \frac{di_1}{dt}$$

where M is the mutual inductance.

If i_1 is in amperes, and M is the number of linkages with coil 2, per ampere in coil 1, then

$$e_2 = - M \frac{di_1}{dt} \text{ volts} \quad . \quad . \quad . \quad (5.1)$$

If a current i_2 flows in coil 2 instead of a current in coil 1, then the e.m.f. induced in coil 1 when the rate of change of current in coil 2 is $\frac{di_2}{dt}$ is given by

$$e_1 = - M \frac{di_2}{dt} \text{ volts}$$

it being assumed that M is the same in each case.

To determine the *direction of the induced e.m.f.* in coil 1, consider the current in coil 2 to be increasing; then a self-induced e.m.f. will be produced in coil 2, the direction of which is in opposition to the direction of the current. Since the same flux which induces this self-induced e.m.f. is also inducing the e.m.f. in coil 1, this latter e.m.f. will also be in a direction opposing that of the current in coil 2. If the circuit of coil 1 is closed, a current will flow, due to the induced e.m.f. and in the same direction. This current reduces the interlinking flux and thus reduces the self-inductance of coil 2. Hence there is a mutual action between the coils.

Mutual inductance, like self-inductance, is measured in henrys.

A mutual inductance of 1 henry exists between two circuits when a rate of change of current of 1 amp per second in one circuit induces an e.m.f. of 1 volt in the other circuit.

Relations Between Self- and Mutual Inductance. Suppose that two coils, having respectively N_1 and N_2 turns, are so close together that the whole of the flux produced by a current in one coil links with the other. Let this flux be ϕ when the current in coil 1 is i_1. Then the self-inductance of coil 1 is $L_1 = N_1 \frac{\phi}{i_1}$ and the mutual inductance is $M = N_2 \frac{\phi}{i_1} = \frac{N_2}{N_1} L_1$.

Similarly, if i_2 flows in coil 2 producing a flux ϕ', its self-inductance $L_2 = \frac{N_2\phi'}{i_2}$ and $M = N_1 \frac{\phi'}{i_2} = \frac{N_1}{N_2} L_2$.

$$\therefore \frac{N_2}{N_1} L_1 = \frac{N_1}{N_2} L_2 = M$$

or

$$M^2 = L_1 L_2$$

$$M = \sqrt{L_1 L_2} \quad . \quad . \quad . \quad . \quad (5.2)$$

As stated above, this relationship is true only when the whole of the flux from one coil links with the other. In practice this condition is not fulfilled, although if the coils are very close together it is very nearly so. The ratio $\frac{M}{\sqrt{L_1 L_2}}$ is called the *coefficient of coupling*, and is of importance, especially in radio work. If this ratio is nearly unity, the circuits are said to be "close coupled," while if it is considerably less than unity they are said to be "loosely coupled."

Self-inductances in Series. If two coils, of self-inductances L_1 and L_2 henrys, are connected in series and the mutual inductance between the coils is M, then if the flux produced by coil 2, linking with coil 1, is in the same direction, at any instant, as the self-produced flux of coil 1, the effective self-inductance of coil 1 is $L_1 + M$. In the same way the effective self-inductance of coil 2 is $L_2 + M$, provided that the self- and mutual fluxes are in the same direction at any instant. Thus the total self-inductance of the circuit is

$$L = L_1 + L_2 + 2M$$

Expressing this generally, to include the case when the mutual and self-induced fluxes are in opposition at any instant, we have

$$L = L_1 + L_2 \pm 2M \quad . \quad . \quad . \quad (5.3)$$

Figs. 5.1 (*a*) and (*b*) show two coils connected in series, and with the directions of current in them such that their magnetic effects are (*a*) cumulative, (*b*) in opposition.

In the first case, $L = L_1 + L_2 + 2M$, and in the second case $L = L_1 + L_2 - 2M$.

1. Inductance of Two Long, Parallel Cylinders. Consider two long, parallel cylinders X and Y, each of radius of cross-section R metres and carrying currents of I amp, in opposite directions as shown in Fig. 5.2 (a). Let the distance between the axes of the cylinders be D metres, and the surrounding medium be air. Suppose,

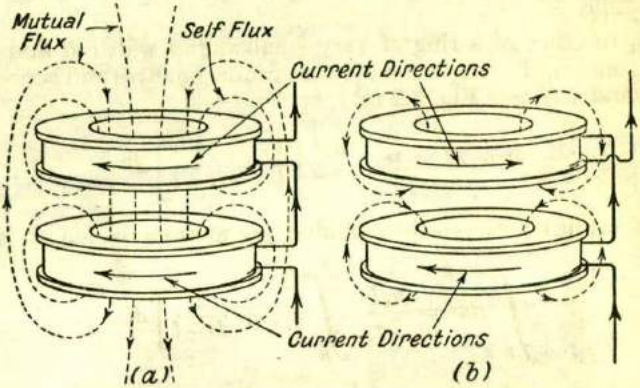

FIG. 5.1 SELF-INDUCTANCES IN SERIES

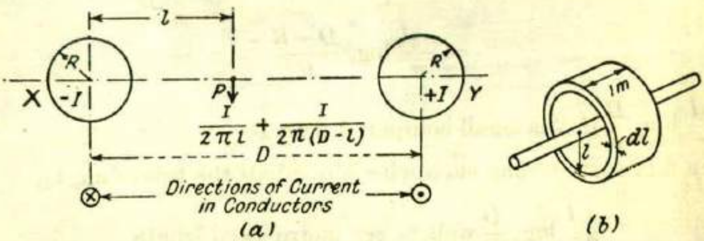

FIG. 5.2 INDUCTANCE OF TWO LONG PARALLEL CYLINDERS

also, that the material of which the cylinders are made is non-magnetic.

The flux, produced by the currents, and to which the inductance is due, is composed of two parts which must be treated separately. These are (a) the flux surrounding the two conductors, and (b) the flux which exists inside the conductors themselves. These will be considered in order.

(a) The magnetic field strength H at a distance r metres from a conductor carrying I amperes is $\frac{I}{2\pi r}$.

Thus, the total magnetizing force at a point P distant l metres from cylinder X and $(D - l)$ metres from Y is $\frac{I}{2\pi l} + \frac{I}{2\pi(D-l)}$, the forces due to the two cylinders being added separately because the currents in them are in opposite directions. The resultant force is downwards, as shown in the figure. Since the medium between the cylinders is air, the flux density B at P is also equal to $\frac{\mu_0 I}{2\pi l} + \frac{\mu_0 I}{2\pi(D-l)}$.

Thus, the flux in a ring of very small radial width dl and axial length 1 metre, the ring being of mean radius l metres and concentric with cylinder X (see Fig. 5.2 (*b*)), is

$$B \times dl \times 1 = \frac{\mu_0 I}{2\pi}\left[\frac{1}{l} + \frac{1}{D-l}\right] dl$$

The total flux between the cylinders per metre axial length is

$$\int_R^{D-R} Bdl = \frac{\mu_0 I}{2\pi} \int_R^{D-R}\left[\frac{1}{l} + \frac{1}{D-l}\right] dl$$

$$= \frac{\mu_0 I}{2\pi}\Big[\log_e l - \log_e (D-l)\Big]_R^{D-R}$$

$$= \frac{\mu_0 I}{\pi} \log_e \frac{D-R}{R}$$

or $\frac{\mu_0 I}{\pi} \log_e \frac{D}{R}$ if R is small compared with D.

The flux surrounding each wire is one-half the total flux, i.e.

$$\frac{\mu_0 I}{2\pi} \log_e \frac{D}{R} \text{ webers per metre axial length}$$

Since inductance in henrys = No. of turns × flux per amp, the inductance of one conductor per metre axial length due to its external flux alone is

$$\frac{\mu_0}{2\pi} \log_e \frac{D}{R} \text{ henrys}$$

(*b*) In considering the flux existing inside each conductor, assume that the current is distributed uniformly over the cross-section of the conductor. This assumption is justified if the supply frequency is low. At high frequencies, the current flows almost entirely in the outside "skin" of the conductor and in this case the flux inside the conductor is negligibly small. The expression for inductance derived below, together with most of the succeeding expressions, gives therefore the "low-frequency" inductance. Slight modifications, due to the negligible internal flux at high frequencies, are required to convert them to expressions for "high-frequency" inductance.

It is convenient to consider the conductor as being made up of a very large number of filaments, all parallel to the axis and each carrying a small fraction of the total conductor current I (see Fig. 5.3).

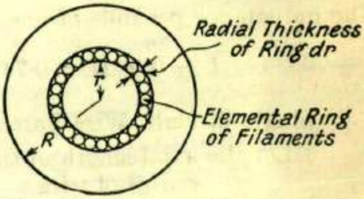

FIG. 5.3. CURRENT DISTRIBUTION IN A CYLINDRICAL CONDUCTOR

Consider an elemental ring of radius r and radial width dr as shown.

Then the current enclosed within this ring is

$$I_r = \frac{\pi r^2}{\pi R^2} \times I = \frac{r^2}{R^2} I$$

and the flux density at radius r within the conductor is

$$B_r = \frac{\mu_0 I_r}{2\pi r} \text{ webers/metre}^2$$

the relative permeability of the conductor material being unity.

The flux in the elemental ring, of axial length 1 metre, radius r, and radial width dr, is $B_r dr \times 1$

$$= \frac{\mu_0 I_r}{2\pi r} dr \text{ webers}$$

This flux does not surround the whole conductor current, but only the current I_r. Referring it to the whole conductor, we have

$$d\phi = \frac{\mu_0 I_r}{2\pi r} dr \times \frac{r^2}{R^2}$$

by multiplying by $\frac{r^2}{R^2}$ —the inverse ratio of the numbers of filaments.

Thus
$$d\phi = \frac{\mu_0 I r^2}{2\pi r R^2} \times \frac{r^2}{R^2} dr = \frac{\mu_0 I r^3}{2\pi R^4} dr$$

The total flux of the inside of the conductor, referred to the whole conductor,

$$= \int_0^R \frac{\mu_0 I r^3}{2\pi R^4} dr = \frac{\mu_0 I R^4}{8\pi R^4} = \frac{\mu_0 I}{8\pi} \text{ webers}$$

For *one conductor* the inductance L is $\dfrac{\text{flux}}{\text{current}}$

$$= \frac{\dfrac{\mu_0 I}{2\pi}\log_e\dfrac{D}{R} + \dfrac{\mu_0 I}{8\pi}}{I} = \frac{\mu_0}{2\pi}\log_e\frac{D}{R} + \frac{\mu_0}{8\pi}$$

$$= \frac{\mu_0}{4\pi}\left(2\log_e\frac{D}{R} + \frac{1}{2}\right) \text{ henrys per metre axial length}$$

The inductance per mile of one conductor is thus

$$L = 0{\cdot}0804 + 0{\cdot}740\log_{10}\frac{D}{R} \text{ millihenrys per mile} \quad (5.4)$$

2. Single Straight Wire Parallel to Earth

Let the axial length of the wire be l metres
radius of wire be R metres
height of wire above earth be H metres

Assume the wire to be of non-magnetic material and that the radius of the wire is small compared with its length. Then, using the method of images, imagine that an exactly similar conductor, running parallel to the overhead one, is embedded in the earth at a depth H metres immediately below the latter. If the embedded conductor carries the same current as the overhead one, but in the opposite direction, then the distribution of the magnetic field will be the same as that of the single overhead conductor existing alone.

The distance between the overhead and imaginary embedded conductor is $2H$. We may, therefore, from the results of the previous paragraph, state the inductance of the overhead conductor as

$$L = 0{\cdot}0804 + 0{\cdot}740\log_{10}\frac{2H}{R} \text{ millihenrys per mile}$$

replacing D by $2H$.

The inductance of a single straight cylindrical conductor, distant from earth and other conductors, is given by

$$L = 2l\left(\log_e\frac{2l}{R} - 0{\cdot}75\right) \times 10^{-7} \text{ henrys}$$

where l = length of wire in metres,
R = radius of wire in metres,

it being assumed that the material of the wire is non-magnetic and that the surrounding medium is also non-magnet

If the wire is of magnetic material the inductance is given by

$$L = 2l\left(\log_e\frac{2l}{R} - 1 + \frac{\mu_r}{4}\right) \times 10^{-7} \text{ henrys} \quad . \quad (5.5)$$

where μ_r = relative permeability of the material of the wire.

3. A Single Circular Turn of Round Wire. The inductance for continuous current and low frequencies is given by Rayleigh and Niven's Formula (Ref. (1)), i.e.—

$$L = \frac{\mu_0 D}{2}\left[\left(1 + \frac{d^2}{8D^2}\right)\log_e \frac{8D}{d} + \frac{d^2}{24D^2} - 1{\cdot}75\right] \text{henrys} \quad . \quad (5.6)$$

where D = mean diameter of the turn in metres,
d = diameter of cross-section of the wire in metres.

It is assumed in this formula that the circle is complete, i.e. there is no gap in it, and that the wire is of non-magnetic material.

If a gap of length g metres is left in the circle, then

$$L = \left(1 - \frac{g}{\pi D}\right)\frac{\mu_0 D}{2}\left[\left(1 + \frac{d^2}{8D^2}\right)\log_e \frac{8D}{d} + \frac{d^2}{24D^2} - 1{\cdot}75\right] \text{henrys} \quad (5.7)$$

If, instead of one circular turn, we have a circular coil of circular cross-section and N turns, the self-inductance of the coil is

$$L = \frac{\mu_0}{2} N^2 D\left[\left(1 + \frac{d^2}{8D^2}\right)\log_e \frac{8D}{d} + \frac{d^2}{24D^2} - 1{\cdot}75\right] \text{henrys} \quad . \quad (5.8)$$

where d is now the diameter of the section of the coil. The formula for a single turn is obviously a special case of the coil when $N = 1$.

For high frequencies the formula, given by Grover (Ref (2)) for a single turn, is

$$L = \frac{\mu_0 D}{2}\left[\left(1 - \frac{d^2}{4D^2}\right)\log_e \frac{8D}{d} - 2\right] \text{henrys} \quad . \quad . \quad (5.9)$$

4. Mutual Inductance Between Two Concentric Circles. The mutual inductance between two concentric circles can be calculated by integration, using the equation

$$H = \frac{idl}{4\pi r^2}\sin\theta$$

Fig. 5.4 (a) shows two concentric circular wires of radii r_1 and r_2, the outer of which carries a current of i amperes. If the flux threading the inner circle, due to the current in an element dl of the outer circle, is calculated, then the total flux threading the inner circle, when i amperes flow in the outer, can be found by integrating over the whole circumference of the latter. The mutual inductance is then given by $\frac{\phi}{i}$, where ϕ is the total flux linking with the inner circle.

It can be shown by integration* that the flux linking with the inner circle when a current of i amperes flows in the outer is

$$\mu_0 r_1 i\left\{\log_e \frac{8r_1}{r_1 - r_2} - 2\right\}$$

* See Drysdale and Jolley, *Electrical Measuring Instruments*, Vol. I.

if $r_1 - r_2$ is small and assuming the medium to be air. Thus, the mutual inductance between the circles is given by

$$M = \mu_0 r_1 \left\{ \log_e \frac{8r_1}{r_1 - r_2} - 2 \right\} \text{ henrys}$$

the radii r_1 and r_2 being expressed in metres.

If, instead of being circles of one turn only, the coils had a number of turns, these turns being assumed to be coincident in space, then

$$M = \mu_0 r_1 N_1 N_2 \left\{ \log_e \frac{8r_1}{r_1 - r_2} - 2 \right\} \text{ henrys} \quad . \quad (5.10)$$

where N_1 = No. of turns on outer coil,
N_2 = No. of turns on inner coil.

The length $r_1 - r_2$ may be expressed as the distance between the circumferences of the coils. Since the assumption that the turns on the coils are coincident in space is not usually justified even as an approximation, a length R called the "geometrical mean distance,"

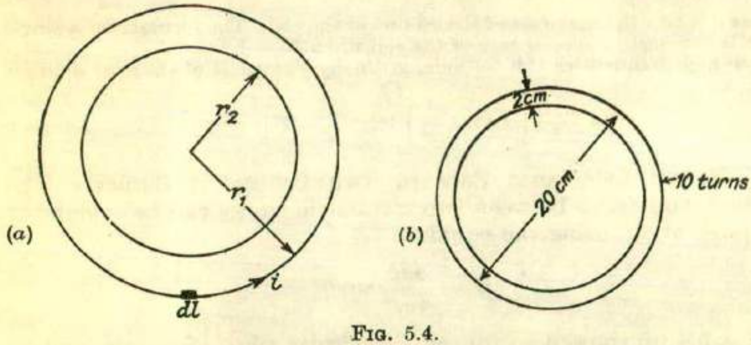

FIG. 5.4.

first introduced by Maxwell, is used instead of $r_1 - r_2$. The mutual inductance is then given by

$$M = \mu_0 N_1 N_2 r_1 \left\{ \log_e \frac{8r_1}{R} - 2 \right\} \text{ henrys} \quad . \quad (5.11)$$

"*Geometrical Mean Distance*" may be defined as follows. Consider a point P external to a circuit. Let d_1, d_2, d_3, etc., be distances from P to various points on the circuit. Then, if an infinite number of these distances be taken, the "geometrical mean distance" R is given by

$$R = \sqrt[n]{d_1 d_2 d_3} \text{ etc., where } n_{\to\infty}$$

or

$$\log_e R = \text{limit}_{n\to\infty} \frac{1}{n} \Sigma \log_e d$$

The factor R is used in many of the formulae for the calculation of both mutual and self-inductance. In the case of self-inductance, R is the g.m.d. of the circuit from itself, or, in the case of a multi-turn coil, of the turns from each other.

To find the self-inductance of the coil of N_1 turns, using Equation (5.11), with the correct value of R we have

$$L = \mu_0 N_1^2 r_1 \left\{ \log_e \frac{8r_1}{0{\cdot}7788\rho} - 2 \right\} \text{ henrys} \quad . \quad . \quad (5.12)$$

The g.m.d. of a circular area from itself is $0{\cdot}7788\ \rho$, where, in this case, ρ is the radius of the cross-section of the coil (assumed circular).

This expression, if compared with the expression given previously (Equation (5.8)) for the self-inductance of a similarly-shaped coil, will be found to give the same result in any particular case, provided ρ is small compared with r.

Example. Calculate the self-inductance of a coil of mean diameter 20 cm. having 10 turns, whose cross-section *is circular* and of radius 1 cm (Fig 5.4 (*b*)).

Using Equation (5.12), we have

(i) $L = 4\pi \times 10^{-7} \times 100 \times 10^{-1} \left[\log_e \frac{8 \times 10^{-1}}{0{\cdot}7788 \times 10^{-2}} - 2 \right]$

$= 0{\cdot}000033080$ henry

(ii) Using Equation (5.8),

$$L = \frac{4\pi \times 10^{-7} \times 100 \times 20 \times 10^{-2}}{2} \left[\left(1 + \frac{4 \times 10^{-4}}{8 \times 4 \times 10^{-2}}\right) \times \log_e \frac{8 \times 20 \times 10^{-2}}{2 \times 10^{-2}} + \frac{4 \times 10^{-4}}{24 \times 4 \times 10^{-2}} - 1{\cdot}75 \right]$$

$$= \frac{4{,}000\pi}{10^9} \left[\frac{801}{800} \times 4{\cdot}3828 + \frac{1}{2{,}400} - \frac{7}{4} \right]$$

$= 0{\cdot}000033150$ henry

Table VIII gives some of the more important geometrical mean distances.

Some exact expressions for the geometrical mean distance in several cases are given by Butterworth (*Dictionary of Applied Physics*, Vol. II, p. 391). For the calculation of geometrical mean distances, see Refs. (3), (4), and (12) at the end of the chapter.

If two circles are coaxial, but not concentric, and if the difference $r_1 - r_2$ between their radii is not small compared with their radii, then the formula (Ref. (11)) is

$$M = \mu_0 \sqrt{r_1 r_2} \left\{ \log_e \frac{8\sqrt{r_1 r_2}}{D_1} \left[1 + \frac{3}{16}\alpha - \frac{15}{1024}\alpha^2 + \frac{35}{128^2}\alpha^3 \ldots \right] - \left[2 + \frac{1}{16}\alpha - \frac{31}{2048}\alpha^2 + \frac{247}{6(128)^2}\alpha^3 \ldots \right] \right\} \text{ henrys} \quad . \quad (5.13)$$

where $\alpha = D_1 \sqrt{\dfrac{1}{r_1 r_2}}$

This equation may be written

$$M = M_o \sqrt{r_1 r_2}$$

where M_o equals $\frac{4\pi}{10^7}$ multiplied by the expression in brackets.

Nottage (Ref. (5)) gives a table of values of M_o for different values of $\frac{D_1}{D_2}$ where D_1 and D_2 are the least and greatest distances between the circles (see Fig. 5.5). M_o varies from zero when $\frac{D_1}{D_2} = 1$, to 50·16 when $\frac{D_1}{D_2} = 0{\cdot}01$.

If the circles have approximately equal radii, and the distance between them is small compared with their radius,

$$M = \mu_0 r_2 \left(\log_e \frac{8r_2}{d} - 2\right) \text{ henrys} \quad . \quad . \quad . \quad (5.14)$$

where d is the distance (in metres) between the circles.

5. **Mutual Inductance Between Two Coaxial Circular Coils of Rectangular Cross-section of Winding.** From the previous paragraph an approximate formula can be derived for the mutual inductance between two coaxial circular coils of rectangular cross-section, i.e.

$$M = N_1 N_2 M_o \text{ henrys} \quad . \quad . \quad . \quad (5.15)$$

where M_o is the mutual inductance between the two central turns of the two coils and can be obtained from Equation (5.13). N_1 and N_2 are the numbers of turns on the two coils.

The accuracy of this formula is of the order of 1 per cent in most practical cases.

TABLE VIII

GEOMETRICAL MEAN DISTANCES

Shape of Circuit	Geometrical Mean Distance (R)	Interpretation of Symbols Used
Line from itself .	$R = 0{\cdot}22313l$	l = length of line
Rectangular area from itself .	$R = 0{\cdot}2235\,(a + b)$ (approx. expression)	a and b = sides of rectangle
Circular area from itself . . .	$R = 0{\cdot}7788r$	r = radius of circle
Annular ring from itself . . .	$\log_e R = \log_e r_1 - \log_e \frac{m}{(m^2-1)^2} + \frac{(3-m^2)}{4(m^2-1)}$	r_1 = external radius r_2 = internal radius $m = \frac{r_1}{r_2}$
Ellipse from itself .	$\log_e R = \log_e \frac{a+b}{2} - 0{\cdot}25$	a and b = semi-axes of ellipse
Two parallel straight lines . . .	$\log_e R = \frac{D^2}{l^2} \log_e D + \frac{1}{2}\left(1 - \frac{D^2}{l^2}\right) \log_e (D^2 + l^2) + \frac{D}{l} \tan^{-1} \frac{l}{D} - \frac{3}{2}$	l = length of lines D = distance between lines

Rayleigh's Formula. This is a more exact formula than the above, since it takes into account the dimensions of the cross-sections of the coil windings to a greater degree.

Referring to Fig. 5.6, let the mutual inductance between a circle of radius r_1 with centre X, passing through point a_1, and a circle of radius r_2, centre Y, passing through O_2, be given by MO_2a_1. There will be, in all, eight such mutual inductances—four referred to coil 2 and four referred to coil 1. a_1, b_1, c_1, and d_1 are the mid-points of the sides

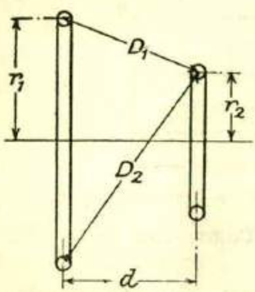

FIG. 5.5

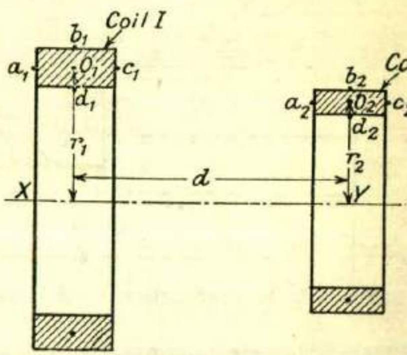

FIG. 5.6. MUTUAL INDUCTANCE BETWEEN COAXIAL COILS

of the section of coil 1, of which section O_1 is the centre point. The points a_2, b_2, c_2, d_2, and O_2 are similarly situated on the section of coil 2.

Then, by Rayleigh's formula, the mutual inductance between the coils is given by

$$M = \frac{1}{6}\Big(MO_1a_2 + MO_1b_2 + MO_1c_2 + MO_1d_2 + MO_2a_1 + MO_2b_1 + MO_2c_1 + MO_2d_1 - 2M_o\Big) \quad . \quad . \quad . \quad . \quad (5.16)$$

where M_o is the mutual inductance between the two central circles of the coils (through points O_1 and O_2). The mutual inductances MO_1a_2, etc., can be calculated as indicated in the previous paragraph.*

If instead of one of the coils being, as above, external to the other and displaced axially from it, one of the coils is inside the other at its centre, the coils still being coaxial, the mutual inductance can be calculated as below. This case refers particularly to the mutual inductance used in ballistic galvanometer work for calibration

* Other methods of calculation of the mutual inductance between two such coils, due to Lyle and Nottage, are given by the latter (Ref. (5)).

purposes, where a small coil is fixed inside a long circular solenoid as in Fig. 5.7.

Let l = length of long solenoid in metres
R = radius of long solenoid in metres
r = radius of internal short solenoid in metres
N_1 = No. of turns on outer solenoid
N_2 = No. of turns on inner solenoid

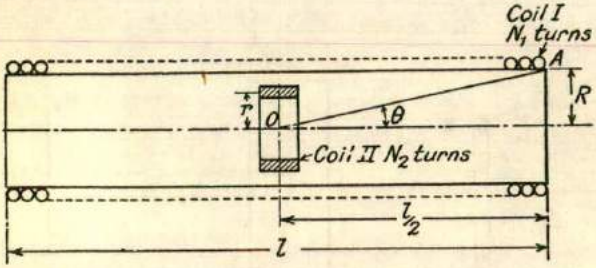

FIG. 5.7. MUTUAL INDUCTANCE BETWEEN CONCENTRIC COILS

If a current of I amp flows in the outer solenoid, the magnetic field strength at its centre is given by

$$H = \frac{N_1 I}{l}\cos\theta$$

$$= \frac{N_1 I}{l}\,\frac{\frac{l}{2}}{\sqrt{R^2 + \frac{l^2}{4}}} = \frac{1}{2}\,\frac{N_1 I}{\sqrt{R^2 + \frac{l^2}{4}}}$$

The value of the flux density at the centre is obtained by multiplying by μ_0. Thus the flux threading the small solenoid is

$$\frac{\mu_0 N_1 I}{2\sqrt{R^2 + \frac{l^2}{4}}} \times \pi r^2 = \phi$$

The mutual inductance is thus $\frac{\phi N_2}{I}$ henrys

or
$$M = \frac{\mu_0 \pi N_1 N_2 r^2}{2\sqrt{R^2 + \frac{l^2}{4}}} \quad . \quad . \quad . \quad (5.17)$$

This is, however, only an approximate expression for the mutual inductance, since the strength of field H only refers to the centre point O of the solenoid and its intensity varies both axially and radially.

Corrections. If the internal coil had negligible axial length, the mutual inductance, corrected for radial variation of field strength, would be obtained by multiplying M (above) by the expression (Ref. (12))

$$1 + \frac{3}{8}\frac{R^2r^2}{\left(R^2+\frac{l^2}{4}\right)^2} + \frac{5}{64}\frac{R^4r^4}{\left(R^2+\frac{l^2}{4}\right)^4}\left(3-\frac{l^2}{R^2}\right)$$

Correction for the axial length of the internal coil is obtained by subtracting a quantity, given by the following expression, from the mutual inductance (Ref. (12))—

$$-\frac{5}{9}\sqrt{\frac{3}{5}}\frac{S}{l}N_1N_2\left\{M_{\left(\frac{l}{2}-\frac{1}{2}\sqrt{\frac{3}{5}}\frac{S}{2}\right)} - M_{\left(\frac{l}{2}+\frac{1}{2}\sqrt{\frac{3}{5}}\frac{S}{2}\right)}\right\}$$

where S = length of internal short coil

and $M_{\left(\frac{l}{2}-\frac{1}{2}\sqrt{\frac{3}{5}}\cdot\frac{S}{2}\right)}$ and $M_{\left(\frac{l}{2}+\frac{1}{2}\sqrt{\frac{3}{5}}\cdot\frac{S}{2}\right)}$ indicate the mutual inductances between two circles, of radii R and r, at distances apart of

$$\left(\frac{l}{2}-\frac{1}{2}\sqrt{\frac{3}{5}}\cdot\frac{S}{2}\right) \text{ and } \left(\frac{l}{2}+\frac{1}{2}\sqrt{\frac{3}{5}}\cdot\frac{S}{2}\right) \text{ respectively.}$$

As the above two corrections (for axial and radial variation of field) tend to neutralize one another, the difference between the final value of the mutual inductance and the approximate value originally obtained is usually quite small—of the order of 1 in 1,000 for usual dimensions of a mutual inductance for ballistic galvanometer calibration.

If both $\frac{l}{R}$ and $\frac{S}{r}$ are equal to $\sqrt{3}$, and if the ratio is small, the corrections become unnecessary (Ref. (17)).

The mutual inductance when the short coil, instead of being situated *within* the long coil at its centre, is situated at the centre but *outside*, is obtained by the same method as above, but R and r are interchanged.

Example. Calculate the mutual inductance between two coaxial circular coils, given that—

Length of long coil	=	80 cm
Radius of long coil	=	4 cm
No. of turns of long coil	=	500
Length of short coil	=	6 cm
Radius of short coil	=	3 cm
No. of turns of short coil	=	150

Small coil placed inside, and at the centre of, the larger coil.

Then $$M = \frac{4\pi^2 \times 500 \times 150 \times 3^2 \times 10^{-4} \times 10^{-7}}{2\sqrt{\left(4^2+\frac{80^2}{4}\right)} \times 10^{-4}}$$

$$= \frac{332}{10^6} \text{ henry, or 332 microhenrys (very nearly)}$$

6. **Self-inductance of Circular Coils of Rectangular Cross-section of Winding.** Consider a single-layer coil of axial length l metres radius of cross-section r metres having N turns, with a current of I amp flowing in it. If $\frac{l}{r}$ is great, the field strength within the coil is $\frac{NI}{l}$. If no magnetic material is present, the flux density within the coil is $\mu_0 \frac{NI}{l}$, the flux inside the solenoid is $\mu_0 \frac{NI}{l} \pi r^2$, and the inductance is

$$L = \frac{\mu_0 \pi N^2 r^2}{l} \text{ henrys (approx.)} \quad . \quad . \quad (5.18)$$

This may also be written

$$L = \frac{S^2}{10^7 l} \text{ (approx.)}$$

where $S =$ the total length of wire on the coil $= 2\pi N r$ metres.

These formulae must be regarded as approximate only, since the expression for the field strength is only true for an infinitely long solenoid. In practice, the *whole* of the flux produced does not link with *all* the turns, and this reduces the inductance of the solenoid. Nagaoka (Ref. (7)) has given the values of a factor by which the above expression may be multiplied in order to take into account the dispersion of the lines of force. This factor varies according to the ratio of length to diameter of the coil.

Equation (5.18) may be written

$$L = \frac{\mu_0 \pi N^2 d^2}{4l}$$

d being the diameter of the coil. Introducing Nagaoka's factor K, we have

$$L = \frac{\mu_0 \pi N^2 d^2}{4l} \times K \quad . \quad . \quad . \quad (5.19)$$

which is considerably more exact than the previous equation (5.18).

If the wire on the solenoid is closely wound, so that adjacent turns are touching, this expression gives results which are sufficiently accurate for most purposes. A correction is necessary if the turns are widely spaced. Fig. 5.8 gives the values of the factor K for different ratios of length to diameter of coil. The curve refers to a single-layer coil or to a coil whose depth of winding is small compared with its diameter.

Coursey (Ref. (8)) has given values of a second factor K_1, for use when the depth of winding on a coil is appreciable. This factor varies with the ratio $\frac{\text{depth of winding}}{\text{mean diameter of coil}}$ and also with the ratio $\frac{\text{length of coil}}{\text{depth of winding}}$. The inductance formula, when K_1 is used, becomes

$$L = (K - K_1)\frac{\mu_0 \pi N^2 d^2}{4l} \text{ henrys} \qquad (5.20)$$

A full table of Nagaoka's factors is given by Nottage (Ref. (5)), where other tables for the calculation of the inductance of special

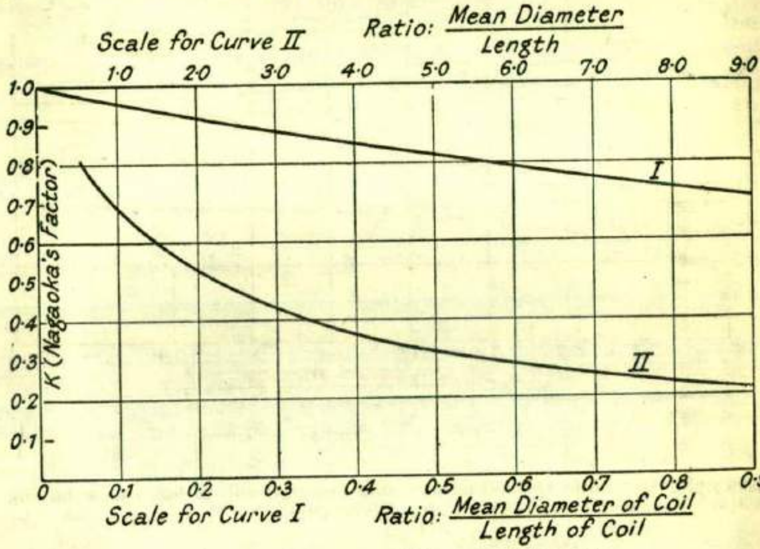

FIG. 5.8. CURVES OF NAGAOKA FACTORS

coils are also given. For Coursey's curves and further tables for such calculations, see Refs. (5), (6), (8), (9), (10).

Fig. 5.9 gives values of Coursey's factor $(K - K_1)$ for various ratios of depth of winding to mean diameter of coil, and of length of coil to depth of winding.

Equations (5.19) and (5.20) are especially suited to long, circular coils whose depth of winding is small compared with their mean diameter. The assumption is made, in these formulae, that the distribution of the current over the cross-section is uniform.

Based on formulae derived by Rayleigh and Niven, Lyle, and Spielrein, Grover (Ref. (9)) gives the formula

$$L = \frac{N^2 dP}{2 \times 10^7} \text{ henrys} \qquad (5.21)$$

d being the mean diameter of the coil. P is a factor depending upon the ratios of the various dimensions of the coil, and the values of P for different coil dimension ratios are given by Grover (*loc. cit.*).

This formula is more suited to the calculation of the inductances of short circular coils of rectangular cross-section whose depth of winding is comparatively large compared with their mean diameter, although it can be used also for the calculation of inductance in the same cases as Equation (5.20) with very little error.

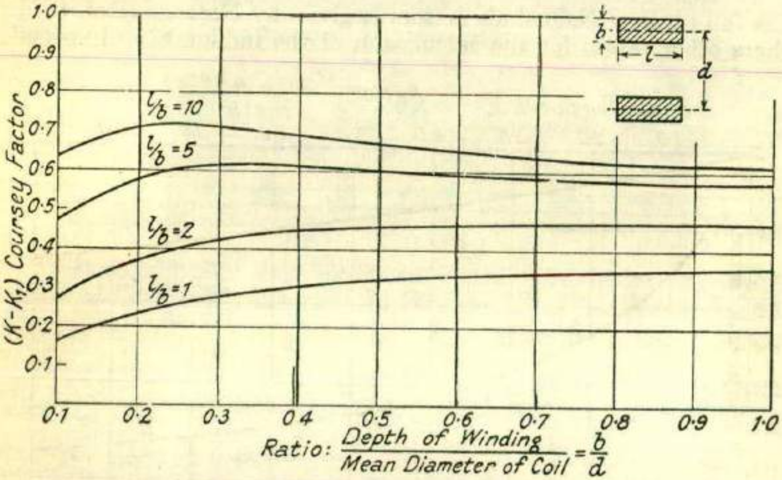

FIG. 5.9. CURVES OF COURSEY FACTORS

Example. Calculate the inductance of a circular coil, of 500 turns, having a rectangular cross-section of winding, given that

Axial length of coil = 10 cm
Mean diameter of coil = 5 cm
Depth of winding of coil = 1 cm

(i) Using the Coursey curve (Fig. 5.9) in conjunction with Equation (5.20),

Ratio $\frac{l}{b} = 10$. Ratio $\frac{b}{d} = \frac{1}{5} = 0{\cdot}2$.

From the curve, $(K - K_1) = 0{\cdot}701$

$$L = \frac{0{\cdot}701 \times 4\pi^2 \times 10^{-7} \times 500^2 \times 5^2 \times 10^{-4}}{4 \times 10 \times 10^{-2}}$$

$$= \frac{4{,}325}{10^6} \text{ henry or 4,325 microhenrys.}$$

(ii) Using Equation (5.21.),

$$\frac{b}{l} = 0{\cdot}1 \quad \frac{b}{d} = 0{\cdot}2$$

From Grover's table, the value of P corresponding to these ratios is 6·92. Hence

$$L = \frac{500^2 \times 5 \times 10^{-2} \times 6{\cdot}92}{2 \times 10^7}$$

$$= \frac{4{,}325}{10^6} \text{ henry, or 4,325 microhenrys, as before}$$

Correction for Thickness of Insulation. As mentioned above, the formulae given take no account of the insulation between turns on the coil. For accurate calculations a correction for this insulation must be applied, although it is usually quite small.

This correction is made by subtracting the quantity $\frac{6{\cdot}283}{10^7}\, dN(A+B)$ henrys (Ref. (5)) from the calculated inductance, where d and N are as above, and A and B are constants depending upon the relative thickness of insulation and number of turns on the coil respectively. Values of these constants are given by Nottage (*lot. cit.*).

(7) **Self-inductance of Flat Coils.** By "flat" coils are meant those whose axial length is small compared with their mean diameter and depth of winding.

Spielrein gave the formula for such flat or "disc" coils of circular form as

$$L = \frac{N^2 dQ}{2 \times 10^7} \text{ henrys} \quad . \quad . \quad . \quad (5.22)$$

where N = No. of turns on the coil

d = mean diameter of the coil in metres

and Q is a factor which can be calculated from the expression

$$Q = \frac{\left(1+\frac{b}{d}\right)^3}{4\left(\frac{b}{d}\right)^2}\left[\begin{array}{l}6{\cdot}96957 - \beta^3\ 30{\cdot}3008 \log_{10}\frac{1}{\beta} + 9{\cdot}08008 \\ + 1{\cdot}48044\beta^5 + 0{\cdot}33045\beta^7 + 0{\cdot}12494\beta^9 + \ldots\end{array}\right]$$

where b = depth of winding

$$\beta = \text{the ratio } \frac{\text{inner radius of coil}}{\text{outer radius of coil}}$$

A table of values of the factor Q are given by Grover (Ref. (9)) for different values of $\frac{b}{d}$. If the axial length of the coil is appreciable Equation (5.21) applies.

To Correct for Insulation Thickness. In the case of a flat spiral wound with metal strip or ribbon of rectangular cross-section, the quantity

$$\frac{12{\cdot}57}{10^7}\, Nr(A_1 + B_1) \text{ henrys (Ref. Grover, } \textit{loc. cit.}\text{)}$$

is added to the calculated inductance.

N = No. of turns on coil

r = mean radius of coil in metres

$$A_1 = \log_e \frac{\nu + 1}{\nu + \tau}$$

$$B_1 = -2\left[\frac{N-1}{N}\delta_{12} + \frac{N-2}{N}\delta_{13} + \frac{N-3}{N}\delta_{14} + \cdots \frac{1}{N}\delta_{1n}\right]$$

$\nu = \frac{w}{D}$, where w = axial length of strip

D = distance between adjacent turns

$\tau = \frac{t}{D}$, where t = thickness of strip

The factors δ_{12}, δ_{13}, etc., are given in tabular form by Grover for different values of τ and ν.

8. **Self-inductance in Other Cases.** (*a*) *Coils Wound on Polygonal Formers*. Grover (Ref. (10)), in a Bureau of Standards paper on this subject, gives a method of calculating the inductance of coils of this general form by obtaining, in each case, the "equivalent radius" of the coil, and then treating it as a circular coil having this radius. The formulae for calculation of the equivalent radii are somewhat complex, and reference should be made to the original paper for information on the subject.

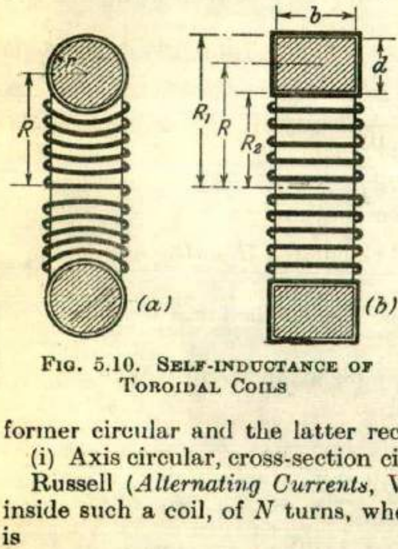

FIG. 5.10. SELF-INDUCTANCE OF TOROIDAL COILS

(*b*) *Toroidal Coils*. These are coils whose axis and cross-section are either both circular or the former circular and the latter rectangular.

(i) Axis circular, cross-section circular (torus) (Fig. 5.10 (*a*)).

Russell (*Alternating Currents*, Vol. I, p. 50) shows that the flux inside such a coil, of N turns, when a current of I amp flows in it, is

$$\phi = \mu_0 N I (R - \sqrt{R^2 - r^2})$$

where R = mean radius of axis of coil in metres

r = radius of the cross-section of the coil in metres

Thus the inductance is given by

$$L = \mu_0 N^2 (R - \sqrt{R^2 - r^2}) \text{ henrys} \quad . \quad . \quad (5.23)$$

(ii) Axis circular, cross-section rectangular (Fig. 5.10 (*b*)).

Again, from Russell's expression for the flux within such a coil, we have

$$\phi = \frac{2NbI}{10^7}\log_e \frac{R + \frac{d}{2}}{R - \frac{d}{2}}$$

where b = the breadth of the coil

d = radial depth of the coil, both in metres

R = mean radius of axis of coil in metres

Thus
$$L = \frac{2bN^2}{10^7}\log_e \frac{R + \frac{d}{2}}{R - \frac{d}{2}}$$

or
$$L = \frac{2bN^2}{10^7}\log_e \frac{R_1}{R_2} \text{ henrys} \quad . \quad . \quad (5.24)$$

where R_1 and R_2 are the outer and inner radii of the ring respectively.

Design of Inductance Coils for Maximum Time-constant. The ratio $\frac{\text{inductance}}{\text{resistance}}$ for any coil is spoken of as its "time-constant." It is usually desirable, in designing inductance coils, to make this ratio as great as possible. This means that the dimensions must be such that the greatest possible inductance is obtained with a given length of wire. Since the resistance of coils increases considerably at high frequencies, compared with the direct-current, or low-frequency, resistance, it is difficult to give rules for the most economical design of coil to suit different frequencies when these are high.

Referring to low-frequency conditions, the maximum inductance for a given length of wire, using a coil of rectangular cross-section, is obtained when the cross-section is square, i.e. when the axial length of the coil is equal to the depth of winding on the coil.

Maxwell showed, also, that with a square-section coil the inductance is maximum when the mean diameter of the coil is made 3·7 times the axial length of the coil, but, as already pointed out on page 93, later work by Shawcross and Wells (Ref. (26)) has shown that 2·95 (or more conveniently 3) is a better value than 3·7 for the ratio of mean diameter to axial length.

The maximum inductance for a given length of wire, if the coil is not limited as to shape, is obtained by making the coil of circular cross-section, with a ratio of $\frac{\text{radius of circular axis of coil}}{\text{radius of cross-section of the coil}}$ = 2·575. (Ref. (25)).

A valuable paper by H. B. Brooks on the design of inductance coils is mentioned in Ref. (27).

Iron-cored Inductors. The formulae for inductance so far considered have all been for coils with air, or non-magnetic, cores. The inductance of iron-cored coils cannot be calculated easily with great accuracy owing to the fact that the permeability of the iron core is not constant, but varies with the magnetizing force producing the flux. If an expression is to be given for the permeability under these conditions it must be some mean value of the different permeabilities occurring at different times throughout the current cycle. The question is further complicated by the fact that the value of the magnetizing force is not the same for all parts of the iron core, even for a given value of current in the coil. Thus the effective value to be assigned to the permeability of the core of such a coil is largely a matter for experimental determination under a given set of conditions.

The foregoing remarks apply especially to coils with open iron cores—say in the form of a straight bar. If the core is nearly closed, having a comparatively narrow air gap, the calculation of inductance can be carried out approximately as follows—

Let S_i = reluctance of iron path $= \dfrac{l_1}{\mu_0 \mu_r A}$

S_a = reluctance of air gap $= \dfrac{l_2}{\mu_0 A}$

(where l_1 and l_2 are the lengths of the iron and air paths respectively and μ_r is the mean relative permeability of the iron)

N = No. of turns on the coil

I = r.m.s. value of the current in the coil

ϕ = r.m.s. value of the flux produced

A = cross-section of the magnetic path

Then,

$$\phi = \frac{NI}{S_i + S_a}$$

Now, obviously the value of the flux per ampere, i.e. $\dfrac{\phi}{I}$, would be constant if $S_i + S_a$ were constant. But, although the reluctance of the air gap S_a is constant whatever the value of the magnetizing force, the iron-path reluctance S_i varies with varying current as pointed out above. If, however, the reluctance S_a is made large compared with S_i, the variation in the latter is negligible, since S_i may then be entirely neglected with very little error.

$$L = \frac{\phi N}{I}$$

$$= \frac{N^2}{S_a} = \frac{4\pi N^2 A}{10^7 l_2} \text{ henrys} \quad . \quad . \quad . \quad (5.25)$$

Under these circumstances, the iron core provides a low-reluctance path for the flux, thus increasing the latter for a given magnetizing force, and hence increasing the inductance of the coil.

Example. A coil of 500 turns is wound on a cylindrical former 10 cm long and 1·5 cm radius. This former is placed on a rectangular iron core of effective cross-section 2 sq. cm, and whose length of magnetic path is 30 cm. The core contains an air gap 0·5 cm long. A current of 0·1 amp r.m.s. flows through the coil. Given that the mean permeability of the iron of the core under these conditions is 1,000, calculate the inductance of the coil.
Reluctance of air gap

$$S_a = \frac{0{\cdot}5 \times 10^{-2}}{\mu_0 \times 2 \times 10^{-4}} = \frac{0{\cdot}25}{\mu_0} \times 10^2$$

Reluctance of iron path

$$S_i = \frac{30 \times 10^{-2}}{1{,}000\, \mu_0 \times 2 \times 10^{-4}} = \frac{0{\cdot}015}{\mu_0} \times 10^2$$

$$\text{Flux} = \phi = \frac{\mu_0 \times 500 \times 0{\cdot}1 \times 10^{-2}}{0{\cdot}25 + 0{\cdot}015} = 2{\cdot}37 \times 10^{-6}\ \text{Wb}$$

$$\text{Inductance} = \frac{\phi \times N}{I} = \frac{2{\cdot}37 \times 10^{-6} \times 500}{0{\cdot}1} = 0{\cdot}0118\ \text{henry}$$

Obviously, if the reluctance of the iron path had been entirely neglected, the calculated inductance would have been some 6 per cent larger than the above value. Thus, uncertainty as to the correct value of the permeability of the iron under working conditions causes a negligible error if the air gap is made comparatively large.

To illustrate the effect of the iron core in increasing the inductance, we will calculate the inductance of the same coil with an air core.

From the approximate equation (5.18) this inductance is

$$\frac{4\pi^2 \times 10^{-7} \times 500^2 \times 1{\cdot}5^2 \times 10^{-4}}{10 \times 10^{-2}} = 0{\cdot}0022 \text{ henry (approx.)}$$

Skin Effect. It was pointed out earlier in the chapter that there is internal flux inside a straight cylindrical conductor which is carrying current. Considering the conductor to be made up of an infinite number of small filaments, parallel to its axis, each carrying a small fraction of the total current, I amp, of the conductor, and assuming the current density to be uniform over the conductor cross-section (an assumption which is really justified only with unidirectional or low-frequency current), we have for the flux density at a radius r within the conductor

$$B_r = \frac{\mu_0 I_r}{2\pi r} \text{ where } I_r = \frac{r^2}{R^2} \cdot I$$

R being the radius of the conductor itself.

$$\therefore B_r = \frac{\mu_0 r^2 I}{2\pi r R^2} = \frac{\mu_0 r I}{2\pi R^2}$$

Thus, $B_r \propto r$. In Fig. 5.11 B_r is shown plotted against radius r. The total flux surrounding the filaments of the conductor (including

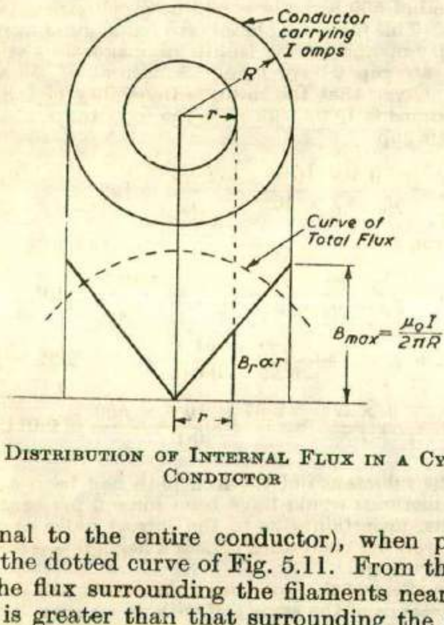

FIG. 5.11. DISTRIBUTION OF INTERNAL FLUX IN A CYLINDRICAL CONDUCTOR

the flux external to the entire conductor), when plotted against radius r, gives the dotted curve of Fig. 5.11. From this curve it can be seen that the flux surrounding the filaments near the centre of the conductor is greater than that surrounding the filaments near its surface. Thus the centre filaments have greater inductance than the surface filaments.

If P is the resistance of one filament and L its inductance, then its impedance is $\sqrt{P^2 + \omega^2 L^2}$, where $\omega = 2\pi \times$ frequency. At low frequencies the term $\omega^2 L^2$ is small compared with P, so that, if a voltage V is applied to the two ends of the conductor, the current carried by any one filament is $\frac{V}{\sqrt{P^2 + \omega^2 L^2}} = \frac{V}{P}$ (very nearly), and the current density within the conductor is uniform over its cross-section. At high frequencies P^2 is small compared with $\omega^2 L^2$ and the current carried by a filament is $\frac{V}{\omega L}$ (very nearly). Under these conditions, then, the difference in inductance between central and surface filaments becomes very important. The central filaments carry only a very small current, due to their greater inductance, and the current in the conductor is almost entirely carried by the surface filaments, i.e. by the outer "skin" of the conductor. Hence the name "*skin effect*" given to this phenomenon.

The effective cross-section of the conductor at high frequencies is therefore only the area of an outer skin, and the resistance of the conductor is increased accordingly. Thus the "*high-frequency-resistance*" of a conductor is higher than its d.c. or low-frequency resistance, the difference depending upon the cross-section of the conductor, the frequency, and upon the permeability and resistivity of the material of the conductor. Since the material used for such conductors is usually non-magnetic, the permeability is almost always unity.

The high-frequency resistance of a conductor is given by

$$R_f = R\left[1 + \frac{1}{12}A^2 - \frac{1}{180}A^4 + \ldots\right] \quad . \quad (5.26)$$

R is the steady-current resistance, and

$$A = \frac{2\pi f l \mu_r}{10^7 R} = \frac{2\pi f l \mu_r}{10^7 \times \dfrac{l \times \rho}{\pi r^2}} = \frac{2\pi^2 r^2 f \mu_r}{10^7 . \rho}$$

where f = frequency

l = length of conductor in metres

r = radius of conductor in metres

ρ = resistivity of conductor material in ohm-metres

μ_r = relative permeabibty of the material of the conductor

The inductance of the entire conductor is slightly reduced by the skin effect, since there is less internal flux.

The high-frequency inductance is given by

$$L = \frac{l}{10^7}\left[M + \frac{1}{2} - \frac{1}{48}A^2 + \frac{13}{8640}A^4 - \ldots\right] \text{henrys} \quad . \quad (5.27)$$

M is a constant which depends upon the position of the return conductor of the circuit. The above two equations are due to Maxwell.

Reduction of Skin Effect. From Equation (5.26) it is obvious that the smaller the term A is made, the less the increase of resistance of the conductor with increasing frequency.

A can be kept small by making the radius r of the conductor as small as is consistent with current-carrying requirements and by using non-magnetic material (so that $\mu_r = 1$). If high-resistivity material can be used, so that ρ is large, this again will reduce A.

Other means which are adopted to reduce the effect are the employment of tubular conductors, or conductors consisting of two parallel discs with a number of parallel high-resistance rods, set at equal distances apart round their circumferences, joining them together, the whole forming a cage or barrel-shaped arrangement. In these cases the internal flux of the conductor is small and the conductors may be thought of as consisting merely of "skins" with hollow interiors.

Stranded conductors are used; these consist of a large number of fine strands, insulated from one another, and woven so that each strand lies as much at the centre of the conductor, and as much at the surface, as every other strand. In such conductors all the strands have the same surrounding flux and therefore have equal inductances.*

Skin Effect in Coils. Consider a cylindrical coil. The flux within such a coil is, of course, axial, and is distributed over the cross-section right up to the outer surface of the winding. Thus, in addition to the internal flux distribution previously considered, as affecting the inductance of the imaginary component filaments of the conductors, we have now a greater inductance of the radially outermost filaments of the coil, as compared with the filaments on the inner surface of the winding. This is due to the fact that they enclose the whole of the coil flux, while the latter only enclose the flux within the winding (i.e. the flux in the core of the coil). A variation of the inductance between the various filaments is also caused by the proximity of other conductors.

Morecroft (Ref. (11)) has carried out a full investigation of the effect in various types of coils, and his work should be consulted for further information on the subject. The effect is usually negligibly small in coils when used at low frequencies for alternating-current measurement purposes.

In addition to the skin effect upon the actual self-inductance of a coil, there is the effect of this current redistribution upon the temperature coefficient of inductance. The extent of the redistribution within a conductor (assuming unity permeability) depends upon its resistivity as well as upon the frequency. Therefore, as the resistivity changes with temperature, the inductance of a coil wound with that conductor varies with temperature to an extent which depends upon frequency. W. H. F. Griffiths† shows how to deter-

* For a number of curves relating to the high-frequency resistance of straight conductors, the reader should consult Morecroft's *Principles of Radio Communication*, Chap. II.

† *The Wireless Engineer*, Vol. XIX, No. 221, pp. 56–63.

mine the more or less narrow band of frequencies in which this augmentation of temperature coefficient becomes appreciable.

Skin Effect in Iron Plates. In iron plates which are carrying alternating magnetic flux, the skin effect is of a different nature from that considered above. It is, in this case, the flux which is forced outwards so as to be carried almost entirely by the outer

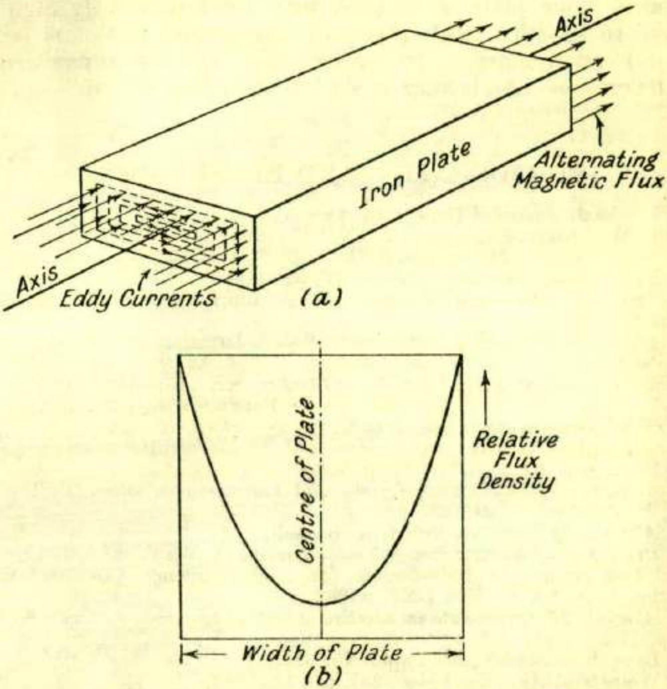

Fig. 5.12. Flux Distribution in Iron Plates

"skin" of the plate instead of being distributed uniformly over the cross-section.

The effect is due to the demagnetizing effect of "eddy currents" induced in the iron plates by the alternating flux. The plate itself acts as the short-circuited secondary winding of a transformer, and eddy currents flow in paths lying in a plane perpendicular to the axis of the plate, as shown in Fig. 5.12 (*a*), the currents being induced by the alternating flux.

The effect will be referred to later in Chapter XIV, on "Eddy Currents."

Obviously the magnitude of the effect depends upon the thickness of the plate and upon the frequency, and it is for this reason that, in order to obtain uniform flux distribution—and hence economical utilization of the iron—in cores of alternating current apparatus, it is necessary to limit the thickness of the laminations to be used, according to the supply frequency.

Fig. 5.12 (*b*) illustrates the diminution of flux density at the centre of an iron plate due to this demagnetizing effect. The figure refers to a fairly thick plate when used with a comparatively high frequency. In practice, with plates of the normal thickness (about 0·014 in.) and commercial frequencies, the variation in flux density over the cross-section is very much less than that shown.

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CHAPTER VI

MEASUREMENT OF INDUCTANCE AND CAPACITANCE

Self-inductance. Several approximate methods of measuring self-inductance are worthy of mention before the more precise methods —most of which are alternating-current bridge methods—are described.

AMMETER AND VOLTMETER METHOD. Inductances of about 50 to 500 millihenrys can be measured by this method. It is suitable for iron-cored coils, since the full normal current to be carried by the coil can be passed through it during the measurement.

A suitable current, of normal frequency, is passed through the coil, and this is measured by an a.c. ammeter while the voltage drop across the coil is measured by a high-resistance voltmeter. The d.c. resistance R of the coil—which will be the same as the a.c. resistance, to a close approximation, if the frequency f is low—must also be measured. Then the inductance L of the coil is given by

$$L = \frac{\sqrt{Z^2 - R^2}}{2\pi f} \text{ henrys}$$

where $Z = \dfrac{\text{voltmeter reading}}{\text{ammeter reading}}$ = the impedance of the coil

Application of the A.C. Potentiometer to the Method. An improvement upon this simple method is the introduction of an alternating-current potentiometer (see Chapter VIII) for the more precise measurement of the current and the voltage drop.

A non-inductive resistance is then connected in series with the coil under test and the voltage drop across this, as well as that across the coil, is measured. The phase of the voltage drop across the coil, as well as its magnitude, is measured.

Let R = the value of the non-inductive resistance

θ = the phase angle between the current and the voltage drop across the coil

V' = the voltage drop across the coil

I = the current (of frequency f)

Then $$V' = I\sqrt{r^2 + (2\pi f L)^2}$$

where r and L are the resistance and inductance of the coil under test.

Also
$$I = \frac{V}{R}$$

where V = the voltage drop across the non-inductive resistance.

$$V' = \frac{V}{R}\sqrt{r^2 + (2\pi fL)^2}$$

or
$$\sqrt{r^2 + (2\pi fL)^2} = \frac{V'R}{V}$$

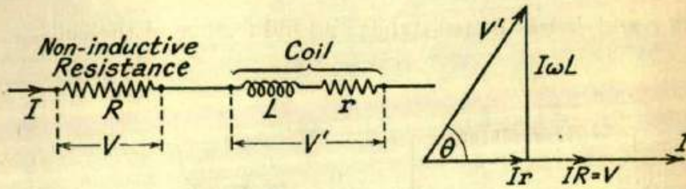


FIG. 6.1 VECTOR DIAGRAM FOR A.C. POTENTIOMETER METHOD

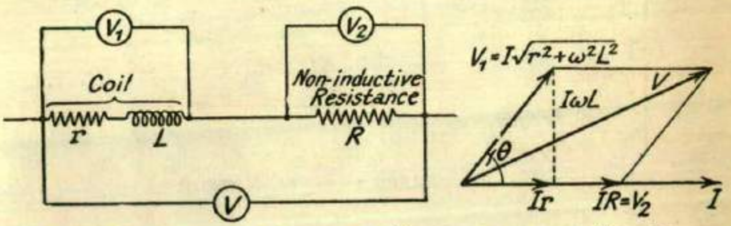


FIG. 6.2. VECTOR DIAGRAM FOR THREE-VOLTMETER METHOD

From the vector diagram of Fig. 6.1,

$$I \times 2\pi fL = V' \sin\theta = I\sqrt{r^2 + (2\pi fL)^2}\sin\theta$$

$$\therefore L = \frac{V'R}{V \times 2\pi f}\sin\theta \quad . \quad . \quad . \quad . \quad (6.1)$$

Similarly,
$$\frac{Ir}{V'} = \cos\theta$$

from which
$$r = \frac{V'R}{V}\cos\theta \quad . \quad . \quad . \quad . \quad (6.2)$$

No measurement of the d.c. resistance is necessary, but the frequency and phase angle θ must be observed.

THREE-VOLTMETER METHOD. The connections of this method are shown in Fig. 6.2. A suitable current is passed through the coil, in series with a non-inductive resistance R, and the voltage drops across both parts of the circuit and across the whole circuit are measured as shown.

From the vector diagram in the figure,

$$V^2 = V_1^2 + V_2^2 + 2V_1V_2 \cos\theta$$

or
$$\cos\theta = \frac{V^2 - V_1^2 - V_2^2}{2V_1V_2}$$

But
$$\cos\theta = \frac{r}{\sqrt{r^2 + (2\pi fL)^2}}$$

where r and L are the resistance and inductance of the coil.

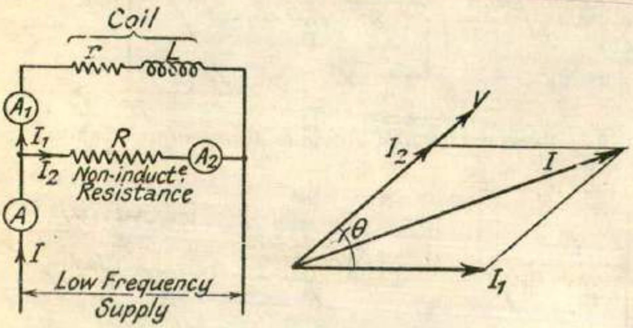


Fig. 6.3. Three-ammeter Method

Thus
$$\sqrt{r^2 + (2\pi fL)^2} = \frac{2rV_1V_2}{V^2 - V_1^2 - V_2^2}$$

or
$$L = \frac{1}{2\pi f}\sqrt{\frac{4r^2V_1^2V_2^2}{(V^2 - V_1^2 - V_2^2)^2} - r^2} \quad . \quad . \quad . \quad (6.3)$$

The resistance r is measured on direct current. Alternatively, since R is known, and $\cos\theta$ having been determined, $r = \frac{V_1R}{V_2}\cos\theta$.

Three-Ammeter Method. The diagram of connections and vectoı diagram for this method are as shown in Fig. 6.3. In this case the non-inductive resistance R, together with an ammeter, is connected in parallel with the coil whose inductance is to be measured.

The theory of the method is exactly similar to that of the three-voltmeter method, but with currents I_1, I_2, and I replacing V_1, V_2, and V.

Thus
$$L = \frac{1}{2\pi f}\sqrt{\frac{4r^2I_1^2I_2^2}{(I^2 - I_1^2 - I_2^2)^2} - r^2} \quad . \quad . \quad . \quad (6.4)$$

ALTERNATING CURRENT BRIDGE METHODS. The best, and most usual methods for the precise measurement of self- and mutual inductance and capacitance are those employing a bridge network with an alternating current supply. The supply may be of commercial frequency—when a vibration galvanometer is used as the detector —or it may be of higher frequency (say 500 to 2,000 c/s), when telephones or thermionic detectors (see Ref. (1)) are employed.

These networks are all, in general, modifications of the original Wheatstone bridge network and their operation is also similar.

In the Wheatstone bridge method of measuring resistance with direct current, the bridge is balanced (i.e. zero galvanometer deflection is obtained) when the voltage drops across the two arms

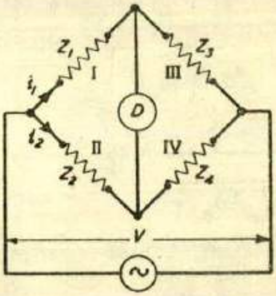


FIG. 6.4. GENERAL FOUR-ARM BRIDGE NETWORK

connecting one of the supply terminals to the two ends of the galvanometer branch of the network are equal in magnitude. With a.c. networks these voltage drops must also be alike in *phase* as well as in magnitude, and for this reason the introduction of inductances or capacitances in other arms of the network is necessary when (say) an inductance to be measured is connected in one of the arms.

The bridge network to be chosen for the measurement of a given self-inductance depends upon the magnitude of that inductance and upon its time-constant—i.e. the ratio of inductance to resistance. In the following the magnitudes of the inductances to the measurement of which the various methods are best suited will be stated.

GENERAL BALANCE EQUATIONS FOR THE 4-ARM BRIDGE. Consider the bridge network shown in Fig. 6.4. Z_1, Z_2, Z_3, Z_4 represent the arm impedances in symbolic notation. At balance, no current flows in the detector D, so the voltage drop across arm I = the voltage drop across arm II,

i.e. $$i_1Z_1 = i_2Z_2$$

Therefore $$\frac{V}{Z_1 + Z_3} \cdot Z_1 = \frac{V}{Z_2 + Z_4} \cdot Z_2$$

or $$Z_1(Z_2 + Z_4) = Z_2(Z_1 + Z_3)$$

giving $$Z_1Z_4 = Z_2Z_3$$

or $$\frac{Z_1}{Z_3} = \frac{Z_2}{Z_4} \quad . \quad . \quad . \quad . \quad (6.5)$$

The solution may be completed for any bridge by expressing the impedance operators in symbolic notation, rearranging and separately equating the real and imaginary parts.

Equation (6.5) will be used as a starting point in the solution of the ensuing bridge networks.

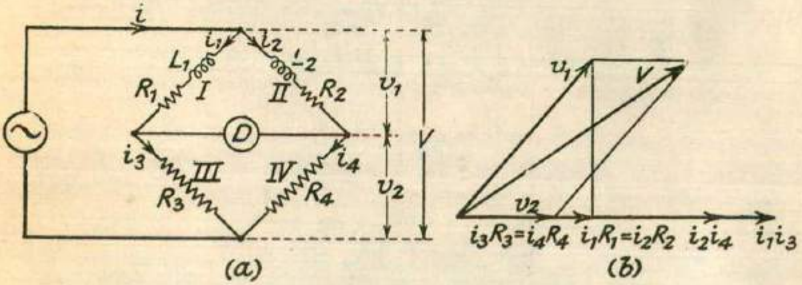


FIG. 6.5. MAXWELL'S METHOD FOR THE MEASUREMENT OF SELF-INDUCTANCE

It can be seen that this result will also be obtained if the source and detector are interchanged.

Maxwell's Method. In this method the unknown inductance is compared with a known self-inductance. The connections for a.c. working, together with the vector diagram, are given in Fig. 6.5.

L_1 = unknown self-inductance of resistor R_1.
L_2 = known self-inductance of resistor R_2.
R_3 and R_4 = non-inductive resistors
D = detector

The resistances R_1, R_2, etc., include, of course, the resistances of the leads and contact resistances in the various arms. It is most convenient to use, for the known inductance L_2, a variable self-inductance of constant resistance, its inductance being of the same order as that of L_1.

The bridge is balanced by varying L_2 and one of the resistors R_3 or R_4. Alternatively, R_3 and R_4 can be kept constant, and the resistance of one of the other two arms can be varied by connecting in the arm an additional resistor.

Theory. Applying the relationship given in Equation (6.5), at balance

$$\frac{R_1 + j\omega L_1}{R_3} = \frac{R_2 + j\omega L_2}{R_4}$$

or

$$R_1R_4 + j\omega L_1R_4 = R_2R_3 + j\omega L_2R_3$$

Equating real and imaginary parts, we have

$$R_1R_4 = R_2R_3$$

or

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

and also,

$$\frac{L_1}{L_2} = \frac{R_3}{R_4}$$

Thus

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} = \frac{L_1}{L_2} \quad . \quad . \quad . \quad . \quad (6.6)$$

The inductances L_1 and L_2 should be placed at a distance from one another, and the leads used in the arms should be carefully twisted together to avoid loops. It should be remembered in this connection that a loop having an enclosed area of 1 sq. ft has an inductance of roughly 1 microhenry.

The vector diagram of Fig. 6.5 (*b*) is for balance conditions, and shows i_1 and i_3 in phase with i_2 and i_4. This is obviously brought about by adjusting the impedances of the various branches so that these currents lag by the same phase angle behind the applied voltage V.

This method is very suitable for the measurement of inductances of medium magnitudes and can be arranged to give results of considerable precision.

MAXWELL'S BRIDGE FOR COMPARING SELF-INDUCTANCE WITH CAPACITANCE. This bridge network, together with the Hay bridge, forms the basis of many commercial bridges for the measurement of self-inductance.

The diagram of connections is given in Fig. 6.6.

L = self-inductance to be measured
R_1 = effective resistance of the inductor
R_2, R_3, R_4 = known non-inductive resistances
C = standard capacitor

At balance,

$$\frac{R_1 + j\omega L}{R_3} = R_2\left[\frac{1}{R_4} + j\omega C\right]$$

giving the balance equations

$$R_1 = \frac{R_2R_3}{R_4} \text{ and } L = CR_2R_3 \quad . \quad . \quad (6.7)$$

The bridge is preferably balanced by varying C and R_4, giving independent settings.

The Q-factor of the inductor is given by $\frac{\omega L}{R_1}$, and at balance

$$Q = \frac{\omega L}{R_1} = \omega C R_4 \quad . \quad . \quad . \quad (6.8)$$

This is made use of in some commercial bridges where C and ω are fixed, and R_4 is variable and calibrated in Q-values. In such circumstances the other balance condition is met by varying either R_2 or R_3, and this variable arm can be calibrated directly in inductance values. The bridge is not suitable for high Q-values

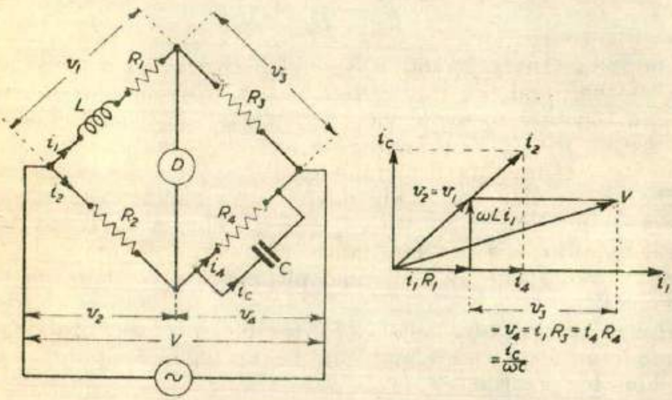


FIG. 6.6. MAXWELL BRIDGE FOR INDUCTANCE MEASUREMENT

because the required value of R_4 for balance becomes impracticably high.

Hay's Bridge. This method of measurement is particularly suited to the measurement of inductances having high Q-values. The diagram of connections and the vector diagram are given in Fig. 6.7. L is the inductance to be measured, R_1 is its resistance, C is a standard capacitor, and R_2, R_3 and R_4 are non-inductive resistances.

At balance,

$$\frac{R_1 + j\omega L_1}{R_3} = \frac{R_2}{\left(R_4 - \frac{j}{\omega C}\right)}$$

from which

$$L = \frac{R_2 R_3 C}{1 + \omega^2 R_4^2 C^2} \quad . \quad . \quad . \quad (6.9)$$

and the effective resistance of the coil is

$$R_1 = \frac{R_2 R_3 R_4 \omega^2 C^2}{1 + \omega^2 R_4^2 C^2} \quad . \quad . \quad . \quad (6.10)$$

The Q-factor of the inductor is given by

$$Q = \frac{\omega L}{R_1} = \frac{1}{\omega C R_4} \quad . \quad . \quad (6.11)$$

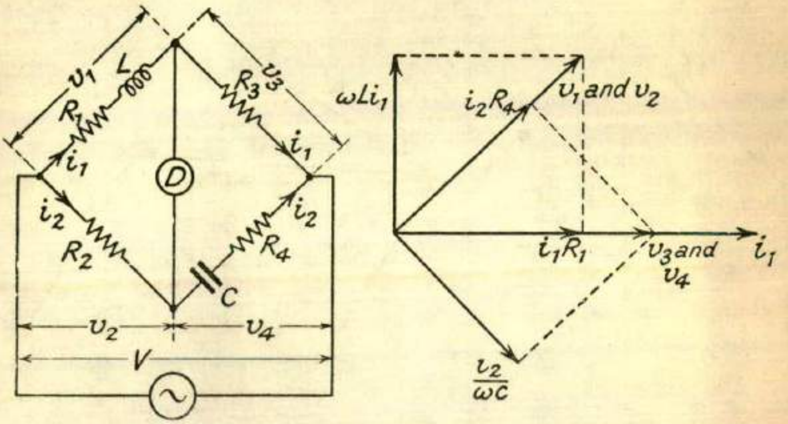


Fig. 6.7. Hay Bridge for the Measurement of Large Self-inductances

Balance may be obtained by varying C and R_4. In commercial bridges C is normally fixed and R_2 varied instead. If the bridge is used for inductors having Q-values greater than 10, $\omega^2 R_4^2 C^2 \ll 1$ and $L \doteqdot R_2 R_3 C$. Hence R_2 can be calibrated in values of inductance with an error less than one per cent for values of Q above 10.

Anderson Bridge. This method requires a standard capacitor in terms of which the self-inductance is expressed. It is actually a modification of Maxwell's method of comparing an inductance with a capacitance. The method is applicable to the precise measurement of inductances over a wide range of values.

Fig. 6.8 gives the diagram of connections and the vector diagram for balanced conditions.

L = self-inductance to be measured

C = standard capacitor

R_1 = resistance of arm 1 (including the resistance of the inductor)

r, R_2, R_3, R_4 = known non-inductive resistances

In the original method a battery and key were used instead of an alternating current supply. R_2, R_3, and R_4 were adjusted to give a balance for steady currents, with the battery key closed. The resistance r was then adjusted (without altering the original resistance settings) to give a balance when the battery key was opened or closed, the two balances being quite independent of one another.

When used with alternating currents, it is still convenient to obtain a preliminary balance for steady currents, using an ordinary galvanometer as detector, the alternating-current balance being then

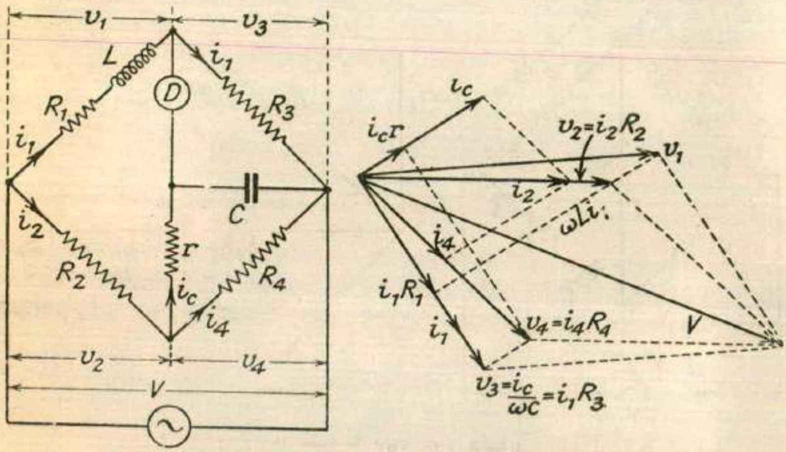


Fig. 6.8. Anderson Bridge for the Measurement of Self-inductance

obtained by varying r. Either telephones or a vibration galvanometer—according to the supply frequency—must be used for the detector when alternating currents are used.

When the bridge is finally balanced the self-inductance is given by

$$L = \frac{C \cdot R_3}{R_4}[r(R_2 + R_4) + R_2R_4] \qquad . \qquad . \quad (6.12)$$

Theory. Assume the capacitor C to be loss-free and the resistances completely non-inductive.

The balance equations can be readily derived by applying the star-delta transformation to the three resistances R_2, r, R_4. The transformed circuit is given in Fig. 6.9, where R_5, R_6 and R_7 are the arms of the corresponding delta.

$$R_5 = \frac{R_2r + R_4r + R_2R_4}{R_4}$$

$$R_6 = \frac{R_2r + R_4r + R_2R_4}{R_2}$$

$$R_7 = \frac{R_2r + R_4r + R_2R_4}{r}$$

Now, R_7 shunts the source and therefore does not affect the balance. It is apparent that the transformed bridge network is that of the Maxwell inductance bridge, whose balance equations have been shown to be

$$L = CR_3R_5 \text{ and } R_1 = R_3\frac{R_5}{R_6}$$

Substituting for R_5 and R_6,

$$L = \frac{CR_3}{R_4}[R_2r + R_4r + R_2R_4]$$

and
$$R_1 = \frac{R_2R_3}{R_4} \quad . \quad . \quad . \quad (6.13)$$

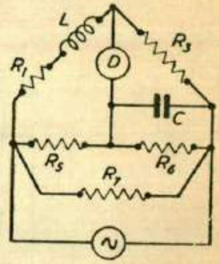


FIG. 6.9. TRANSFORMED ANDERSON BRIDGE NETWORK

If the capacitor is not perfect, but has dielectric loss, the self-inductance value given by the above expression is unaltered, but the value of R_1 is affected.

If the self-inductance of the leads to the coil under test is appreciable, this may be measured by short-circuiting the coil and obtaining a second balance. The actual inductance of the coil may then be obtained by subtraction.

The method can also be used to measure the capacitance of the capacitor C if a calibrated self-inductance is available.

Butterworth's Method. This method is especially suitable for the measurement of small inductances (e.g. a few microhenrys).

The diagram of connections and the vector diagram are given in Fig. 6.10.

L = the self-inductance (of resistance R_1) to be measured
R_2 = a slide wire
C = fixed standard capacitor
R_3, R_4, r = non-inductive resistances.

The resistance balance, or the balance for steady current, can be obtained independently of the inductance balance by adjusting the resistances R_3 and R_4, while the inductance balance is obtained by adjustment of r and the slide wire setting.

At balance

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \quad . \quad . \quad . \quad . \quad (6.14)$$

and
$$L = \frac{(R_2 - r_2)}{R_4}[(R_3 + R_4)r + R_3(R_4 + r_2)]C \quad . \quad (6.15)$$

(Ref. (6)). These conditions may be obtained from the mesh equations or by a method similar to that used with the Anderson bridge but using two transformations.

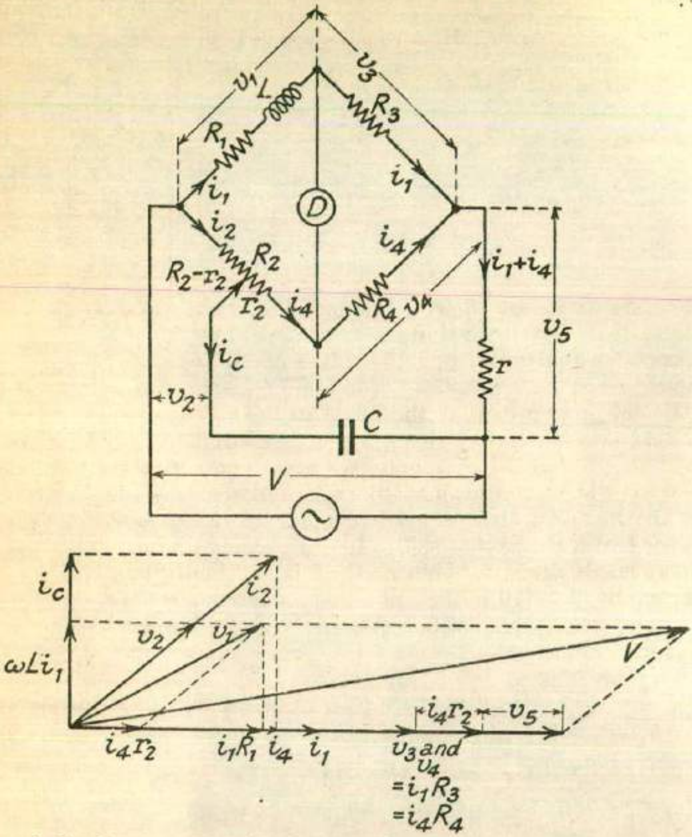


FIG. 6.10. BUTTERWORTH'S METHOD FOR THE MEASUREMENT OF SMALL SELF-INDUCTANCES

To obtain maximum sensitivity, Butterworth has shown that the following relationships should be fulfilled—

$$R_4 = \sqrt{S \, . \, D}, \qquad R_3 = \sqrt{R_1 D \left(\frac{R_1 + S}{R_1 + D}\right)}$$

$$R_2 = \sqrt{R_1 S \left(\frac{R_1 + D}{R_1 + S}\right)}$$

where S is the resistance of the alternator branch (including r) and D is the resistance of the detector branch. The resistance r should be small.

Measurement with Superposed D.C. and A.C. The Hay bridge may be used for the measurement of self-inductance in the case of iron-cored coils in which both direct and alternating currents are flowing. The arrangement of the bridge for such a measurement is as shown in Fig. 6.11.

The direct current, which may be adjusted to the required value by r_1, passes through R_1, L and R_3 only, the capacitors preventing its passage through the other branches. The alternating current is blocked from the d.c. supply by the large choke L_1. This current may be measured by connecting an r.m.s.-reading valve voltmeter, unresponsive to direct current, across R_3; the Moullin valve voltmeter (p. 700) is very suitable for this purpose. The resistor R_3 is

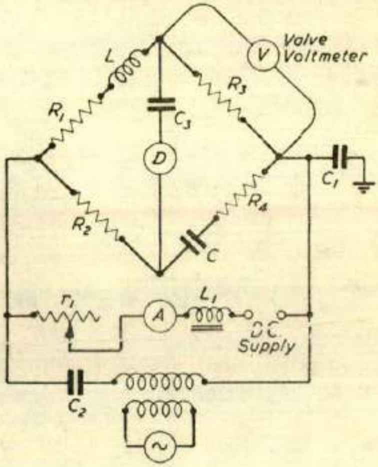


FIG. 6.11. HAY BRIDGE FOR MEASUREMENT WITH SUPERPOSED D.C. AND A.C.

usually low in value as it may be required to dissipate considerable power, and the valve voltmeter will exercise a negligible loading effect; it can be disconnected, however, at balance, to verify this. R_2 is usually between 10,000 and 100,000 ohms. The bridge is usually balanced by C and R_4. The large capacitor C_1 effectively earths one corner of the bridge to alternating current without directly earthing the d.c. supply. This bridge is commonly operated at 50 c/s and a vibration galvanometer is used as detector.

The Owen bridge described later in this chapter (p. 236) may also be used for this class of measurement and has some advantages over the Hay bridge. Full information on the use of the Owen bridge is given in B.S. 933: 1941.

Heaviside-Campbell Bridge. This method employs a standard variable mutual inductor, and can be used for the measurement of self-inductance over a very wide range. It is one of the best methods for general laboratory use. Fig. 6.12 shows the diagram of connections of Heaviside's bridge.

The primary of the mutual inductor is in the supply circuit, and the secondary, of self-inductance L_2 and resistance R_2, forms arm II of the bridge. The inductance to be measured, of self-inductance L_1 and resistance R_1, is placed in arm I of the bridge. R_3 and R_4 are non-inductive resistances.

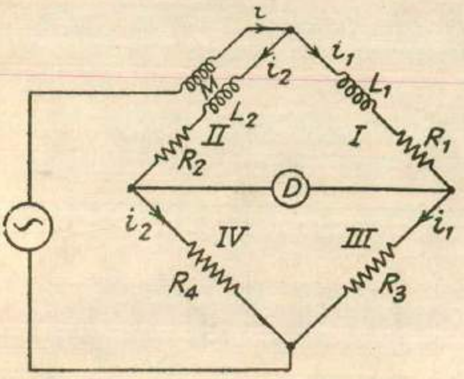


FIG. 6.12. HEAVISIDE BRIDGE

Balance may be obtained by varying the mutual inductance and resistances, R_3 and R_4. At balance,

$$i_2(R_2 + j\omega L_2) + j\omega M i = (R_1 + j\omega L_1)i_1$$

and

$$i_2 R_4 = i_1 R_3$$

Since

$$i = i_1 + i_2$$

$$i_2\,[R_2 + j\omega(L_2 + M)] = i_1\,[R_1 + j\omega(L_1 - M)]$$

Thus

$$\frac{R_2 + j\omega(L_2 + M)}{R_4} = \frac{R_1 + j\omega(L_1 - M)}{R_3}$$

or

$$R_3\,[R_2 + j\omega(L_2 + M)] = R_4\,[R_1 + j\omega(L_1 - M)]$$

Equating real and imaginary quantities,

$$R_2 R_3 = R_1 R_4 \quad . \qquad . \qquad . \quad (6.16)$$

and

$$R_3(L_2 + M) = R_4(L_1 - M) \quad . \qquad . \qquad . \quad (6.17)$$

If the resistances R_3 and R_4 are equal,

$$L_2 + M = L_1 - M$$

or

$$L_1 - L_2 = 2M$$

In *Campbell's Modification* of the bridge (Refs. (10) and (11)), the resistances R_3 and R_4 are made equal. A "balancing coil" of self-inductance equal to the self-inductance L_2 of the mutual inductance secondary coil, and of slightly greater resistance than the latter, is introduced in arm I, in series with the inductance to be measured. A non-inductive resistance box and a "constant-inductance rheostat" are also introduced in arm II. These additions are shown in Fig. 6.13.

Balance is now obtained, by variation of the mutual inductance and the resistance r, with the coil L_1R_1, whose inductance and

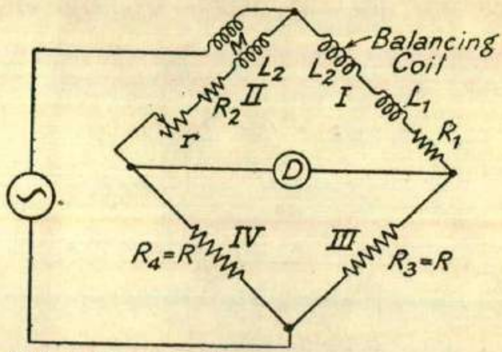


FIG. 6.13. CAMPBELL'S MODIFICATION OF THE HEAVISIDE BRIDGE

resistance are to be measured, in circuit. Suppose the readings of the mutual inductance and resistance r are M_1 and r_1. The coil L_1R_1 is now removed, or short-circuited across its terminals, and balance is again obtained, giving, say, readings M_2 and r_2.

Then
$$L_1 = 2(M_1 - M_2)$$
and
$$R_1 = r_1 - r_2$$

By this method of operation the self-inductance and resistance of the leads are eliminated, and the inductance and resistance of the coil are obtained directly.

The use of a balancing coil in the above arrangement reduces the sensitivity of the bridge. Fig. 6.14 shows a better arrangement, which improves the sensitivity and eliminates the balancing coil. In this arrangement the secondary fixed coil of the mutual inductor is made up of two equal coils L,L, the primary coil reacting with both of them as shown. L_1 is the coil whose self-inductance is to be measured. The resistances R_3 and R_4 are equal ($R_3 = R_4 = R$). When so arranged the bridge is known as the *Heaviside-Campbell Equal Ratio Bridge.*

At balance—obtained by varying the constant-inductance rheo stat r, and the mutual inductances M_1 and M_2—we have the relation ships

$$R_1 = R_2$$

and $$L_1 = 2(M_1 + M_2)$$

where R_1 and R_2 are the total resistances of arms I and II, and $M_1 + M_2$ is the reading of the mutual inductor. With equal ratio arms R_3 and R_4 it is obvious that the magnitude of L_1 which can be measured is limited to twice the range of the mutual inductor.

If L_1 is greater than this value, unequal ratio arms are used with

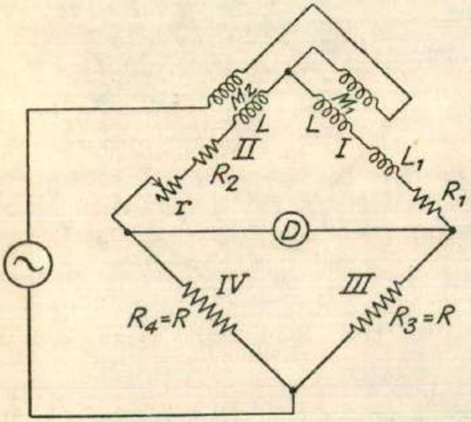


FIG 6.14. HEAVISIDE-CAMPBELL EQUAL RATIO BRIDGE

a balancing coil l, the connections then being as shown in Fig. 6.15. Let the ratio be

$$\frac{R_4}{R_3} = n$$

Then, if the inductance of the balancing coil is made equal to $\frac{L_2}{n}$ (where L_2 = inductance of mutual-inductor secondary coil) the balance conditions are

$$\frac{R_2}{R_1} = n$$

and $$L_1 = (n + 1)M$$

When arranged as described above, the bridge can be used for the measurement of inductances varying from very low values to medium values. D. W. Dye (Ref. (9)) has modified the arrangement in order to make it suitable also for the measurement of large inductances, and, when so modified, the method is a very good one for this purpose.

Kuriyama Method. B. Hague (Ref. (1), 4th Edition, p. 433) has described an interesting method, due to M. Kuriyama, of measuring

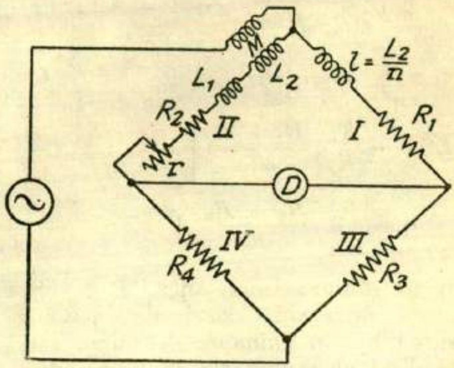


FIG. 6.15. HEAVISIDE-CAMPBELL BRIDGE WITH BALANCING COIL

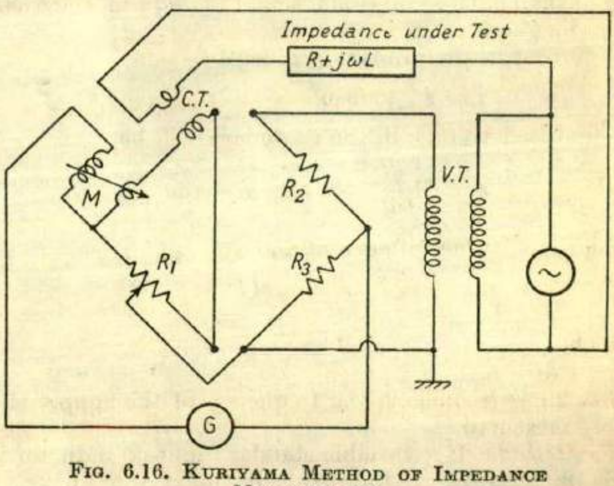


FIG. 6.16. KURIYAMA METHOD OF IMPEDANCE MEASUREMENT

the value of an impedance which is carrying a large current at high voltage. Instrument transformers are used to isolate the bridge from the high-voltage circuit. The connections of the method are given in Fig. 6.16 in which $C.T.$ and $V.T.$ are the current and voltage transformers whose ratios and phase angles (see Chapter XIX) are K_c and β (for $C.T.$) and K_v and γ (for $V.T.$).

If the impedance of the primary of the current transformer is negligible compared with that under test, it can be shown that

$$R \doteqdot \frac{K_v}{K_c} \cdot \frac{R_2 + R_3}{R_3} [R_1 + \omega M(\beta - \gamma)]$$

$$= \frac{K_v}{K_c} \cdot \frac{R_2 + R_3}{R_3} \cdot R_1$$

and also
$$L \doteqdot -\frac{K_v}{K_c} \cdot \frac{R_2 + R_3}{R_3} \left[M - \frac{R_1}{\omega}(\beta - \gamma)\right]$$

$$= -\frac{K_v}{K_c} \cdot \frac{R_2 + R_3}{R_3} . M$$

Measurement of Mutual Inductance. The simplest method of measuring mutual inductance consists in passing an alternating current—measured by an ammeter—through the primary of the mutual inductor and observing the voltage induced in the secondary by means of an electrostatic voltmeter. It is important that the current shall have a purely sinusoidal wave-form, since harmonics may introduce serious errors.

If the current in the primary is given by

$$i = I_{max} \sin \omega t$$

then the induced voltage in the secondary will be

$$e = M \frac{di}{dt} = MI_{max} \omega \cos \omega t$$

or, taking r.m.s. values of current and voltage,

$$E = \omega MI$$

from which
$$M = \frac{E}{\omega I}$$

Since $\omega = 2\pi \times$ frequency, the frequency of the supply should be accurately measured.

Felici's Method. If a variable standard mutual inductor is available this method is an improvement upon the above. It can, however, only be used for the measurement of mutual inductance within the range of the standard inductor. The connections are shown in Fig. 6.17, where M_1 is the mutual inductance to be measured and M is a variable standard mutual inductor.

An alternating current is passed through the two primary coils in series, and the secondary coils are connected in opposition. The standard inductance M is varied until the vibration galvanometer D shows no deflection. Then, the reading of the standard gives the mutual inductance M_1.

Some difficulty may be encountered—especially at high frequencies—in obtaining zero deflection of the detector D. The self-capacitance of the mutual inductors, and eddy currents induced in circuits, or metal parts, in the vicinity, may render it impossible to obtain exact balance, only a minimum deflection being obtainable.

Campbell (Ref. (3)) has discussed these effects very fully and shows how they may be taken into account.

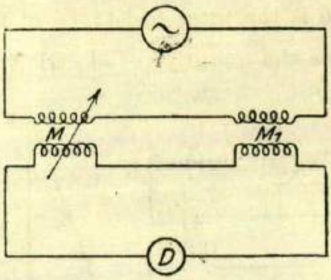


FIG. 6.17. FELICI'S METHOD OF MEASURING MUTUAL INDUCTANCE

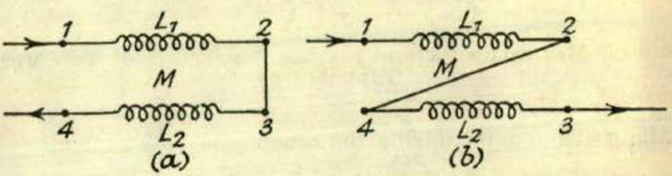


FIG. 6.18. MUTUAL INDUCTANCE CONNECTED AS A SELF-INDUCTANCE

MEASUREMENT OF A MUTUAL INDUCTANCE AS A SELF-INDUCTANCE. It was pointed out in Chapter V that if two coils, of self-inductances L_1 and L_2, are connected in series, and if the mutual inductance between them is M, then the self-inductance of the arrangement is given by $L = L_1 + L_2 \pm 2M$, the choice of sign depending upon the connections and on the relative positions of the two coils.

A simple method of obtaining the mutual inductance M is to measure—by one of the methods already described—the self-inductance of the combination, first with the two coils connected in series, as in Fig. 6.18 (*a*), and then when connected as shown in Fig. 6.18 (*b*). In the first case $L = L_1 + L_2 + 2M$, and in the second $L' = L_1 + L_2 - 2M$, where L and L' are the measured self-inductances. By subtraction,

$$L - L' = 4M \qquad (6.18)$$

or

$$M = \frac{L - L'}{4}$$

MEASUREMENT BY BALLISTIC GALVANOMETER. The secondary winding of the mutual inductor is connected to a ballistic galvanometer, and a current of I amp is passed through the primary winding. Upon reversal of the current I an average e.m.f. of $\frac{2MI}{t}$ volts is induced in the secondary winding, where M is the mutual inductance in henrys and t is the time in seconds taken for the reversal of the current I. The average current in the galvanometer circuit will be $\frac{2MI}{tR}$ amp, where R is the total resistance of the ballistic galvanometer circuit. Thus the quantity of electricity passed through the

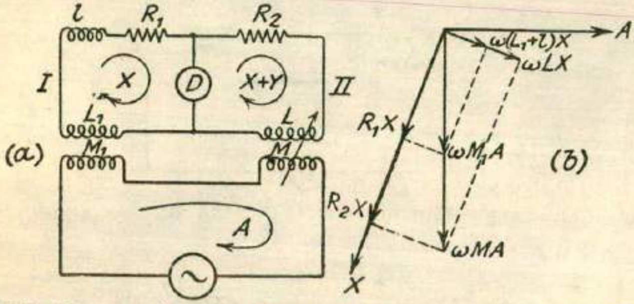


FIG. 6.19. MAXWELL'S METHOD FOR THE COMPARISON OF TWO MUTUAL INDUCTANCES

ballistic galvanometer during the reversal is $\frac{2MI}{R}$ coulombs. Now, the equation giving the deflection of a ballistic galvanometer when a quantity of electricity Q passes through it is

$$Q = \frac{T}{\pi} \cdot K \left(1 + \frac{\lambda}{2}\right) \sin \frac{\theta}{2} \text{ (see Chap. IX)}$$

where T = time in seconds of one complete vibration of the galvanometer moving system

K = the galvanometer constant

θ = the "throw" of the galvanometer

λ = the logarithmic decrement of the galvanometer vibration

Hence, the mutual inductance M is given by

$$M = \frac{RT}{2\pi I} \cdot K \left(1 + \frac{\lambda}{2}\right) \sin \frac{\theta}{2} \qquad . \quad . \quad (6.19)$$

Maxwell's Method. The connections for the comparison of two unequal mutual inductances are shown in Fig. 6.19 (*a*). M_1 is the mutual inductance to be compared with that of the standard variable inductor M. L_1 and L are the self-inductances of their secondary windings. R_1 and R_2 are the total resistances of the two branches, and l is a variable self-inductance inserted in either branch I or branch II to obtain exact balance. The final balance is obtained by successive adjustments of l, R_1, and R_2.

Theory. Using the mesh currents as in Fig. 6.19 (*a*), the mesh equations are—

Mesh I.

$$R_1X + j\omega(L_1 + l)X - DY + j\omega M_1A = 0$$

or

$$X[R_1 + j\omega(L_1 + l)] - DY + j\omega M_1A = 0$$

Mesh II.

$$(R_2 + j\omega L)(X + Y) + DY + j\omega MA = 0$$

or

$$(R_2 + j\omega L)X + (R_2 + D + j\omega L)Y + j\omega MA = 0$$

where D = impedance of the detector circuit.

Since the detector current Y is zero at balance, we have

$$X[R_1 + j\omega(L_1 + l)] + j\omega M_1A = 0 \quad . \quad . \quad . \quad \text{(i)}$$

$$X(R_2 + j\omega L) + j\omega MA = 0 \quad . \quad . \quad . \quad \text{(ii)}$$

Substituting in (i) for A from (ii),

$$X[R_1 + j\omega(L_1 + l)] - M_1\frac{(R_2 + j\omega L)X}{M} = 0$$

or

$$R_1 + j\omega(L_1 + l) = \frac{M_1}{M}(R_2 + j\omega L)$$

Equating real and imaginary quantities, we have

$$R_1 = \frac{M_1}{M}R_2 \text{ or } \frac{R_1}{R_2} = \frac{M_1}{M}$$

and

$$L_1 + l = \frac{M_1}{M}L \text{ or } \frac{L_1 + l}{L} = \frac{M_1}{M}$$

Hence

$$\frac{M_1}{M} = \frac{R_1}{R_2} = \frac{L_1 + l}{L} \quad . \quad . \quad . \quad (6.20)$$

Fig. 6.19 (*b*) gives the vector diagram for the network under balance conditions. Then, since $Y = 0$, the current in both of the branches I and II will be X.

Campbell's Method of Comparing Two Unequal Mutual Inductances. Fig. 6.20 gives the connections of the network for this method. M is the unknown mutual inductor whose primary winding has self-inductance L. M_1 is a standard mutual inductor with self-inductance L_1 in its primary winding. Suppose that M is greater than the maximum value of M_1; then a variable self-inductance, in series with the primary of the unknown, is inserted to make $L + l$ greater than L_1. R_1, R_2, R_3, and R_4 are the resistances of the four arms.

The switches S_1 and S_2 are first thrown on to contact a,a, so as to exclude from the detector circuit the secondaries of the mutual inductors. The bridge is then balanced by varying the resistances and the self-inductance l. Then, as shown in connection with Maxwell's method for the comparison of self-inductances,

$$\frac{R_1}{R_2}=\frac{R_3}{R_4}=\frac{L_1}{L+l} \quad . \quad . \quad . \quad (6.21)$$

The secondaries of the mutual inductors are then connected in series with the detector, and in opposition to one another. This is

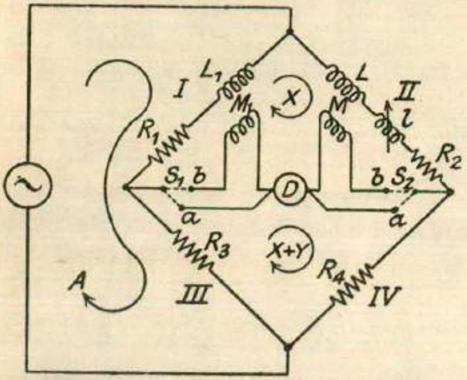


FIG. 6.20. CAMPBELL'S METHOD FOR THE COMPARISON OF TWO MUTUAL INDUCTANCES

done by throwing over switches S_1 and S_2 on to contacts b,b. Balance is then again obtained by adjusting the variable standard M_1, the four branches I, II, III, and IV being left unaltered.

Theory. Taking mesh currents X, $(X + Y)$, and A, as in the figure, and noting that $Y = 0$ at balance, we have—

Mesh I.

$$(R_1 + j\omega L_1)(X - A) + [R_2 + j\omega(l + L)]X - j\omega MX - j\omega M_1(X - A) = 0$$

or $$(R_1 + j\omega L_1 - j\omega M_1)(X - A) + [R_2 + j\omega(l + L) - j\omega M]X = 0$$

Mesh II.

$$R_4X + R_3(X - A) + j\omega MX + j\omega M_1(X - A) = 0$$

or $$(R_4 + j\omega M)X + (R_3 + j\omega M_1)(X - A) = 0$$

Hence

$$(R_1 + j\omega L_1 - j\omega M_1)(X - A) = -X[R_2 + j\omega(l + L) - j\omega M]$$

and $$(R_3 + j\omega M_1)(X - A) = -X(R_4 + j\omega M)$$

By division,

$$\frac{R_1 + j\omega L_1 - j\omega M_1}{R_2 + j\omega M_1} = \frac{R_3 + j\omega(l + L) - j\omega M}{R_4 + j\omega M}$$

Hence

$$R_1R_4 + j\omega L_1R_4 - j\omega M_1R_4 + j\omega MR_1 - \omega^2L_1M$$
$$= R_2R_3 + j\omega R_2(l + L) - j\omega MR_2 + j\omega M_1R_2 - \omega^2M_1(l + L)$$

Equating real and imaginary quantities,

$$R_1R_4 - \omega^2L_1M = R_2R_3 - \omega^2M_1(l + L) \quad . \quad . \quad . \quad \text{(i)}$$
$$L_1R_4 - M_1R_4 + MR_1 = R_2(l + L) - MR_2 + M_1R_2 \quad . \quad . \quad \text{(ii)}$$

Using the conditions for the preliminary balance, viz. $\frac{R_1}{R_2} = \frac{R_3}{R_4} = \frac{L_1}{l + L}$ we have, from Equation (i),

$$L_1M = M_1(l + L) \text{ or } \frac{M}{M_1} = \frac{l + L}{L_1} = \frac{R_2}{R_1} = \frac{R_4}{R_3}$$

and from Equation (ii),

$$M_1(R_4 + R_2) = M(R_1 + R_3) \text{ or } \frac{M}{M_1} = \frac{R_2 + R_4}{R_1 + R_3}$$

Hence, finally, the balance conditions are

$$\frac{M}{M_1} = \frac{l + L}{L_1} = \frac{R_2 + R_4}{R_1 + R_3} = \frac{R_2}{R_1} = \frac{R_4}{R_3} \quad . \quad . \quad (6.22)$$

Heydweiller's Modification of Carey-Foster's Method. The connections of the method are as in Fig. 6.21 (*a*). M is the mutual inductor to be measured, having a self-inductance L in its secondary winding; l is an additional self-inductance which may be necessary to obtain balance of the bridge. C is a standard capacitor. R_3 and R_4 are non-inductive resistors, and R_1 is the total resistance of arm I. The resistance R_2 is made zero in Heydweiller's modification of the original Carey-Foster bridge.

When R_2 is zero the resistor R_4 is obviously connected directly across the supply (neglecting the primary of M). For this reason R_4 is often a non-inductive, oil-cooled standard resistor. Balance is obtained by varying R_3, R_4, and C. The primary of the mutual inductor must be connected so that the voltage induced by it in arm I neutralizes the voltage drop due to the current i_1 in this branch, since, when $R_2 = 0$, the voltage drop in branch I must be zero for balance of the bridge to be obtained.

Theory. At balance,

$$i_1[R_1 + j\omega(l + L)] - j\omega Mi = i_2R_2$$

or, since $i = i_1 + i_2$,

$$i_1[R_1 + j\omega(l + L) - j\omega M] = i_2[R_2 + j\omega M]$$

Also,

$$i_1\left[R_3 - \frac{j}{\omega C}\right] = i_2R_4$$

Therefore
$$\frac{R_1 + j\omega(l + L) - j\omega M}{R_3 - \frac{j}{\omega C}} = \frac{R_2 + j\omega M}{R_4}$$

or
$$R_1R_4 = R_2R_3 + \frac{M}{C}$$

$$\omega(l + L)R_4 - \omega MR_4 = \omega MR_3 - \frac{R_2}{\omega C}$$

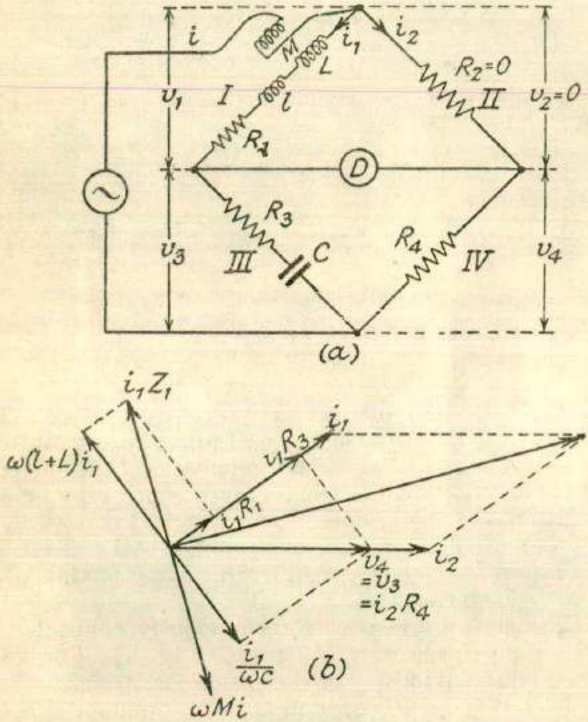


FIG. 6.21. HEYDWEILLER'S MODIFICATION OF CAREY-FOSTER'S METHOD

Thus
$$M = C(R_1R_4 - R_2R_3)$$

and
$$(l + L) = M\left(1 + \frac{R_3}{R_4}\right) - \frac{R_2}{\omega^2 CR_4}$$

When $R_2 = 0$, the expression for $(l + L)$ is made independent of frequency, since the second term is then zero and

$$(l + L) = M\left(1 + \frac{R_3}{R_4}\right) \quad . \quad . \quad . \quad (6.23)$$

Fig. 6.21 (*b*) gives the vector diagram for balance conditions, when $R_2 = 0$. The vector $i_1 Z_1$ representing the voltage drop in the impedance (Z_1) of branch I, is counterbalanced by the vector $\omega M i$, representing the induced voltage in the secondary of the mutual inductance, so that v_1 is zero.

Measurement of Capacitance. Although the commonest, and usually the best, methods of measuring capacitance are the alternating current bridge methods, the apparatus required for such methods may not always be available. Under such circumstances one of the following methods might be used.

AMMETER AND VOLTMETER METHOD. If an alternating voltage of pure sine wave-form is applied to a capacitor of capacitance C farads, a current of ωCV amp will flow, where V is the r.m.s. value of the applied voltage and $\omega = 2\pi \times$ frequency. If the current is measured by a low-reading ammeter and the voltage across the capacitor by an electrostatic voltmeter, the capacitance can be determined in terms of the readings of these instruments and of the frequency.

Instead of measuring the current by an ammeter, a non-inductive resistor of known value may be connected in series with the capacitor, and the voltage drop across this resistor measured by the voltmeter. The current is then given by the voltmeter reading divided by the series resistance.

If the voltage wave-form contains harmonics of appreciable magnitude, a correction may be made for this by multiplying the measured value of the capacitance, obtained as above, by the factor

$$\sqrt{\frac{V_1^2 + V_3^2 + V_5^2 + \ldots}{V_1^2 + 9V_3^2 + 25V_5^2 + \ldots}}$$

where V_1, V_3, V_5, etc., are the values of the various components of the voltage wave-form. It is important, in measuring capacitance, to bear in mind the fact that, since the capacitive reactance is $\frac{1}{2\pi fC}$, the reactance to the harmonics is less than the reactance to the fundamental of the voltage wave, and thus the current wave is not of the same shape as the voltage wave, the harmonics being accentuated (see Chapter XV).

Fleming and Clinton's Commutator Method. In this method the capacitor being measured is alternately charged from a battery and then discharged through a moving-coil galvanometer, by means of a motor-driven commutator. The actual commutator is now rarely used, a high-speed relay operated from an alternating supply being equally effective, and with this modification the method is useful for the direct display of capacitance. The circuit arrangement is shown in Fig. 6.22.

If the times of charge and discharge of the capacitor are small compared with the period of the galvanometer, the latter is continuously deflected.

Let this deflection correspond to a current of I amp in the galvanometer. Then, if Q is the charge (in coulombs) given to the capacitor at each charge, and N is the number of charges per second, the quantity of electricity discharged through the galvanometer is NQ coulombs per second.

Thus the current $I = NQ$

But $Q = CV$, where V is the battery voltage

Therefore $I = NCV$

or

$$C = \frac{I}{NV} \text{ farads} \quad . \quad . \quad . \quad . \quad (6.24)$$

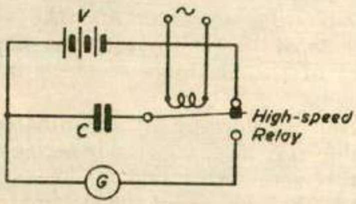


FIG. 6.22. RELAY METHOD FOR CAPACITANCE MEASUREMENTS

Leakage in the capacitor may be detected by connecting the galvanometer in series with the battery to measure the charging current, a short-circuiting wire replacing the galvanometer in the discharge circuit. The capacitance of the capacitor, determined from $\frac{I'}{NV}$, where I' is the charging current, should be the same as the previously determined value if leakage is negligible.

MAXWELL'S COMMUTATOR BRIDGE METHOD has already been described in Chapter II, page 68.

BALLISTIC GALVANOMETER METHOD. In this method the capacitor is charged to a known voltage V by means of a battery, and then discharged through a ballistic galvanometer, the connections being the same as those of Fig. 6.22, except that a key replaces the relay. The quantity of electricity (in coulombs) discharged by the capacitor is then given by

$$Q = \frac{T}{\pi} . K \left(1 + \frac{\lambda}{2}\right) \sin \frac{\theta}{2}$$

Then

$$C = \frac{Q}{V}$$

This method can also be used for the comparison of an unknown capacitance with a standard by comparing the quantities of electricity discharged through the ballistic galvanometer when charged to the same voltage in each case.

With direct-current methods of measurement, the times of charge and of discharge are important in the case of absorptive capacitors. since the measured value of the capacitance will depend to some extent upon these times (see Chapter IV).

A.C. BRIDGE METHODS. *De Sauty Method.* This method is the simplest way of comparing two capacitances. When used on alternating current the connections are as in Fig. 6.23.

C_1 = capacitor whose capacitance is to be measured

C_2 = a standard capacitor

R_1 and R_2 = non-inductive resistors

Balance is obtained by varying either R_1 or R_2.

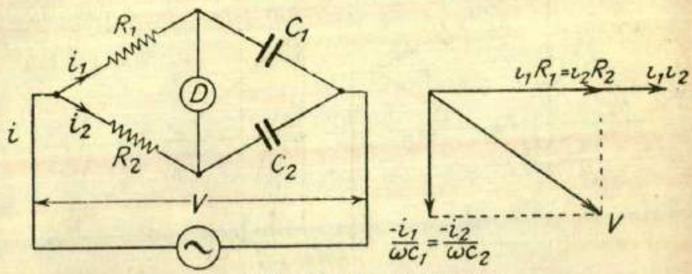


FIG. 6.23. DE SAUTY BRIDGE

At balance, $$i_1R_1 = i_2R_2$$

and $$-\frac{j}{\omega C_1}i_1 = -\frac{j}{\omega C_2}i_2$$

Thus $$\frac{R_1}{R_2} = \frac{C_2}{C_1}$$

or $$C_1 = C_2\frac{R_2}{R_1} \quad . \quad . \quad . \quad . \quad (6.25)$$

For maximum sensitivity, C_2 should be equal to C_1. The advantage of the simplicity of this method is largely nullified by the fact that it is impossible to obtain a perfect balance if the capacitors are not both free from dielectric loss. Only in the case of air capacitors can a perfect balance be obtained.

If two imperfect capacitors are to be compared, the bridge is modified by connecting resistors in series with them, as in Fig. 6.24 (*a*). R_3 and R_4 are the series resistors, while r_1 and r_2 are small resistances representing the loss components of the capacitors. Balance is obtained by variation of the resistances R_1, R_2, R_3, R_4.

At balance,

$$i_1R_1 = i_2R_2$$

$$i_1\left[R_3 + r_1 - \frac{j}{\omega C_1}\right] = i_2\left[R_4 + r_2 - \frac{j}{\omega C_2}\right]$$

from which it follows that

$$\frac{R_1}{R_2} = \frac{R_3 + r_1}{R_4 + r_2} = \frac{C_2}{C_1}$$

The vector diagram of Fig. 6.24 (*b*) shows the relative positions of the vector quantities under balance conditions. The angles δ_1

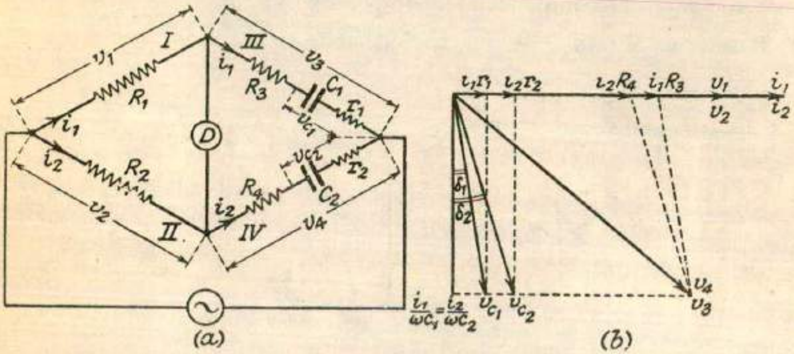


FIG. 6.24. MODIFICATION OF DE SAUTY BRIDGE

and δ_2 are the phase angles of capacitors C_1 and C_2 respectively. Obviously

$$\tan \delta_1 = \frac{r_1}{\frac{1}{\omega C_1}} = r_1\omega C_1$$

and

$$\tan \delta_2 = r_2\omega C_2$$

From the condition

$$\frac{C_2}{C_1} = \frac{R_3 + r_1}{R_4 + r_2}$$

we have

$$C_2r_2 - C_1r_1 = C_1R_3 - C_2R_4$$

or

$$\omega C_2r_2 - \omega C_1r_1 = \omega(C_1R_3 - C_2R_4)$$

i.e.

$$\tan \delta_2 - \tan \delta_1 = \omega(C_1R_3 - C_2R_4)$$

Since

$$\frac{C_2}{C_1} = \frac{R_1}{R_2}$$

$$\tan \delta_2 - \tan \delta_1 = \omega C_1\left(R_3 - \frac{R_1R_4}{R_2}\right) \quad . \quad . \quad (6.26)$$

from which expression the phase angle of one capacitor can be found in terms of the phase angle of the other. This method is due to Grover (Ref. (16)).

In the vector diagram the angles δ_2 and δ_1 are exaggerated for convenience in drawing. These angles are usually small, and thus it is usually a sufficiently good approximation to write

$$\tan(\delta_2 - \delta_1) = \omega C_1\left(R_3 - \frac{R_1 R_4}{R_2}\right) \qquad (6.27)$$

Grover's Series Inductance Method. This method is somewhat similar to the above, inductors being used instead of series resistors.

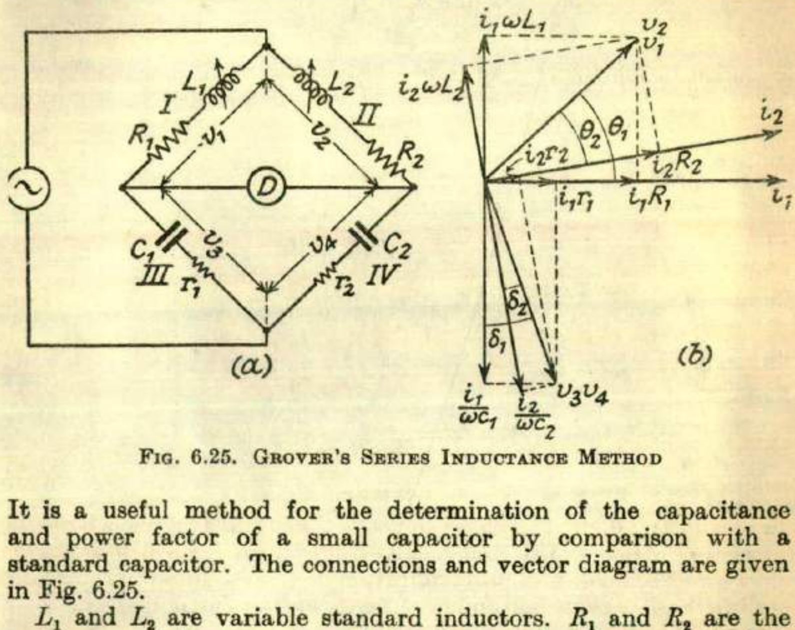


FIG. 6.25. GROVER'S SERIES INDUCTANCE METHOD

It is a useful method for the determination of the capacitance and power factor of a small capacitor by comparison with a standard capacitor. The connections and vector diagram are given in Fig. 6.25.

L_1 and L_2 are variable standard inductors. R_1 and R_2 are the resistances of the arms in which these inductors are situated. Non-inductive resistors may be connected in series with L_1 and L_2, in which case R_1 and R_2 are the resistances of the arms, including the resistances of L_1 and L_2. C_1 is the unknown capacitance. C_2 is that of the standard capacitor, while r_1 and r_2 are resistances representing the loss components of these capacitors.

Balance is obtained by variation of the inductances L_1 and L_2, and of the series resistances in these arms if necessary.

At balance, $$(R_1 + j\omega L_1)i_1 = (R_2 + j\omega L_2)i_2$$

and $$\left(r_1 - \frac{j}{\omega C_1}\right)i_1 = \left(r_2 - \frac{j}{\omega C_2}\right)i_2$$

From which we have

$$\frac{C_1}{C_2} = \frac{R_2}{R_1} + \frac{\omega^2 C_1}{R_1}(L_1 r_2 - L_2 r_1) \quad . \quad . \quad (6.28)$$

In most cases, when L_1 and L_2 are not large, the second term may be neglected, whence

$$\frac{C_1}{C_2} = \frac{R_2}{R_1} \quad . \quad . \quad . \quad . \quad (6.29)$$

The phase angles δ_1 and δ_2 of the two capacitors may be obtained from the expressions

$$\tan \delta_1 = r_1 \omega C_1$$

and

$$\tan \delta_2 = r_2 \omega C_2$$

Substituting the relationship

$$\frac{C_1}{C_2} = \frac{R_2}{R_1}$$

in the first of the balance conditions, which is

$$r_2 - \frac{R_2}{R_1} r_1 = \frac{L_2}{R_1 C_1} - \frac{L_1}{C_2 R_1}$$

we have

$$r_2 - \frac{C_1}{C_2} r_1 = \frac{L_2}{R_1 C_1} - \frac{L_1}{C_2 R_1}$$

whence

$$\omega C_2 r_2 - \omega C_1 r_1 = \omega \left(\frac{L_2}{R_2} - \frac{L_1}{R_1}\right)$$

or

$$\tan \delta_1 - \tan \delta_2 = \omega \left(\frac{L_1}{R_1} - \frac{L_2}{R_2}\right)$$

$$= \tan \theta_1 - \tan \theta_2 \quad . \quad . \quad . \quad (6.30)$$

where θ_1 and θ_2 are the phase angles of the inductance arms I and II.

Owen's Bridge. This bridge, the connections for which are given in Fig. 6.26, is, in fact, a modification of Grover's method just described and enables self-inductance to be measured in terms of a standard capacitance.

The inductor under test, $R + j\omega L$, is connected in series with a variable non-inductive resistor R_1. The standard capacitance is C_s.

At balance,

$$\frac{R + R_1 + j\omega L}{R_2} = \frac{R_3 - \frac{j}{\omega C}}{-\frac{j}{\omega C_s}}$$

from which

$$R = R_2 \frac{C_s}{C} - R_1$$

and

$$L = R_2 R_3 C_s$$

This bridge, which can cover a wide range of inductance measurements with limited apparatus, is convenient to use, since the balance, which is independent of frequency and wave-form, is obtained by successive adjustment of the two resistors R_1 and R_3.

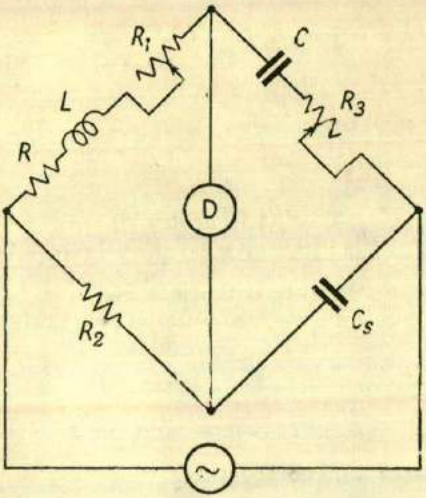


FIG. 6.26. OWEN BRIDGE

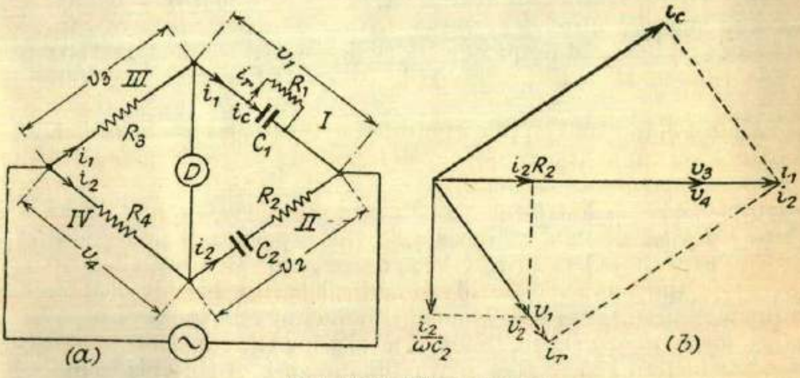


FIG. 6.27. WIEN BRIDGE

Wien's Method. This method is a convenient one when an imperfect capacitor is shunted by a resistance as is the case in cable testing. In Fig. 6.27 (*a*), in which the connections of the bridge are shown, C_1 is the equivalent shunt capacitance of the capacitor, and R_1 the shunt resistance. The capacitor C_2 is a standard air capacitor and R_2, R_3, and R_4, are non-inductive resistors. If the unknown capacitor is not already shunted by a resistance, the resistor R_1 is placed in parallel with it. Balance is obtained by variation of the resistances R_2, R_3, and R_4.

At balance,

$$i_1\left(\frac{R_1}{1 + j\omega C_1 R_1}\right) = i_2\left(R_2 - \frac{j}{\omega C_2}\right)$$

and

$$i_1 R_3 = i_2 R_4$$

Hence it follows that

$$C_1 = \frac{\frac{R_4}{R_3} \times C_2}{1 + \omega^2 R_2^2 C_2^2} \quad . \quad . \quad . \quad (6.31)$$

and

$$R_1 = \frac{R_3(1 + \omega^2 R_2^2 C_2^2)}{\omega^2 R_2 R_4 C_2^2} \quad . \quad . \quad . \quad (6.32)$$

The vector diagram for balance conditions is shown in Fig. 6.27 (*b*).

The Wien bridge is an important frequency bridge. If R_3 and R_4 are fixed resistances having a ratio $\frac{R_4}{R_3} = 2$, and if $C_1 = C_2 = C$, and $R_1 = R_2 = R$, then both balance equations become $\omega CR = 1$. For frequency measurement C is fixed and the two resistors R are variable decade boxes ganged together. Since $\omega = \frac{1}{CR}$, and $\frac{1}{R}$ is a conductance, it is customary in commercial bridges to use conductance boxes, i.e. decade resistance boxes in which resistors are added in parallel instead of series, giving a linear range of conductance and hence of frequency. A typical 4-dial conductance box has decades of $10 \times 0{\cdot}1$, $10 \times 0{\cdot}01$, $10 \times 0{\cdot}001$ and $10 \times 0{\cdot}0001$ mhos.

The most important application of the Wien bridge network is its extensive use as the frequency-determining network in modern resistance–capacitance oscillators.

OTHER BRIDGE METHODS OF MEASURING CAPACITANCE. The *Schering Bridge* method of measuring the capacitance and power factor of capacitors has already been described in Chapter IV.

Some of the methods already described earlier in this chapter for the measurement of self- or mutual inductance in terms of capacitance form convenient methods of measuring the capacitance of a capacitor in terms of self- or mutual inductance, if suitable inductance standards are available. Obviously, such methods may be used either way about without modification, the theory of the method remaining the same. Two such methods are *Anderson's Bridge* and the *Carey-Foster Bridge*.*

* Other methods of measuring both inductance and capacitance are given in Hague's *Alternating Current Bridge Methods* and in the *Dictionary of Applied Physics*, Vol. II, to which works the reader is referred for further information on the subject.

BRIDGE METHODS FOR SPECIAL PURPOSES. *Measurement of the Self-inductance of Alternating-current Resistance Standards.* It is often important that the self-inductance of heavy-current, low-resistance standards for use in alternating-current measurements should be known. The inductance of such standards is usually very small, and its measurement necessitates special methods. Wattmeter methods, using a reflecting-type wattmeter, have been devised but do not compare favourably with alternating-current bridge methods, and the latter are therefore in more general use.

Campbell's Method. The connections of this method (Ref. (13)) are given in Fig. 6.28. The standard resistor R_s has four terminals,

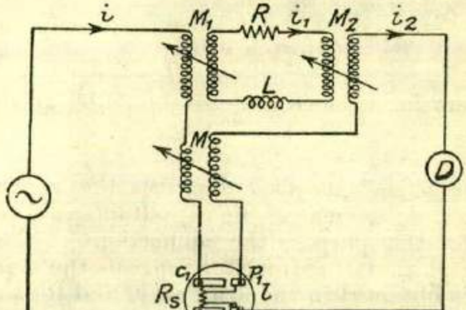


FIG. 6.28. CAMPBELL'S METHOD

c_1,c_2 being its "current terminals," and p_1,p_2 its "potential terminals"—hence the name "four-terminal resistors" given to such standards. The self-inductance of R_s is l. M_1, M_2, and M are three variable mutual inductors, the former two not necessarily being of known values, and the latter being a low-reading standard inductor. R and L are the resistance and inductance of the circuit containing the secondary winding of M_1 and the primary of M_2. D is the detector. Balance is obtained by variation of the three mutual inductances as required. The inductors should be well spaced in order to avoid mutual inductance effects between them.

Theory. At balance the sum of the voltages in the detector circuit is zero.

Thus,
$$(R_s + j\omega l)i + j\omega Mi + j\omega M_2 i_1 = 0$$
or
$$i[R_s + j\omega(l + M)] + j\omega M_2 i_1 = 0$$

Also, in the circuit containing L and R,
$$j\omega M_1 i + i_1(R + j\omega L) = 0$$

Hence
$$\frac{R_s + j\omega(l + M)}{j\omega M_1} = \frac{j\omega M_2}{R + j\omega L}$$
$$RR_s - \omega^2 L(l + M) + j\omega LR_s + j\omega R(l + M) = -\omega^2 M_1 M_2$$

Equating real and imaginary quantities, we have

$$RR_s - \omega^2 L(l + M) = - \omega^2 M_1 M_2$$

$$RR_s = \omega^2[L(l + M) - M_1 M_2]$$

and

$$R(l + M) + LR_s = 0 \quad . \quad . \quad . \quad . \quad (6.33)$$

Thus the self-inductance l can be found if L, R_s, R, and M are known; the values of M_1 and M_2 need not be known.

If M_1 and M are reversed the expression for l becomes

$$\frac{M - l}{R_s} = \frac{L}{R}$$

or

$$l = M - L\frac{R_s}{R}$$

It is necessary for balance that M should be greater than l and $M_1 M_2 > \frac{L^2 R_s}{R}$.

In the same paper Campbell describes the application of this method to the measurement of the capacitance and power factor of capacitors. For this purpose the connections of the network are the same, except that the capacitor replaces the standard resistor R_s. Balance is obtained in the same way, and it can be shown by the method used above that the balance conditions are

$$\frac{1}{C\omega^2} = M + \frac{rL}{R} \quad . \quad . \quad . \quad (6.34)$$

and

$$Rr = \left[M_1 M_2 - \frac{L^2 r}{R}\right]\omega^2 \quad . \quad . \quad . \quad (6.35)$$

where C is the capacitance, and r the series resistance representing the loss component, of the capacitor under test. The power factor $r\omega C$ can be found from the two equations

$$r = \frac{RM_1 M_2 \omega^2}{R^2 + L^2\omega^2} \quad . \quad . \quad . \quad . \quad (6.36)$$

and

$$\frac{1}{C\omega^2} = M + \frac{LM_1 M_2 \omega^2}{R^2 + L^2\omega^2} \quad . \quad . \quad . \quad (6.37)$$

which follow from the balance conditions. If the time constant $\frac{L}{R}$ is small

$$C = \frac{1}{\omega^2 M} \quad . \quad . \quad . \quad . \quad (6.38)$$

to a very close approximation.

Hartshorn's Method for the Measurement of the Self-inductance of Low-resistance Standards. This method, described by L. Hartshorn

(Ref. (18)), is essentially a modification of the Kelvin double bridge (see Chapter VII), and compares the phase angles of two low-resistance standards. To obtain the requisite sensitivity the supply is of telephonic frequency, and telephones are used as the detector. This is justifiable, since the frequency has little effect upon the inductance of such standards. The connections are given in Fig. 6.29.

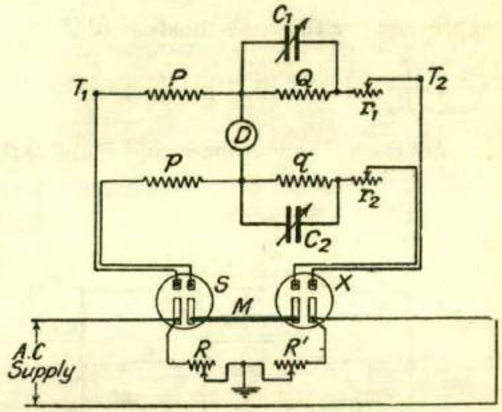


FIG. 6.29. HARTSHORN'S METHOD FOR THE MEASUREMENT OF SELF-INDUCTANCE OF LOW-RESISTANCE STANDARDS

S and X are the two low-resistance standards whose phase angles are to be compared, that of S being known and that of X unknown. Let their resistances be R_s and R_x and their self-inductances L_s and L_x. P, Q, p, and q are shielded non-reactive resistors, and r_1 and r_2 are low resistances—most suitably slide-wires—for fine adjustment of the resistances of the arms. C_1 and C_2 are variable air capacitors, shunting Q and q. The leads from the potential terminals of S and X are run close together to avoid inductive loops. R and R' are variable resistors with their connecting point earthed as shown. If the ratio $\frac{R}{R'}$ is made equal to $\frac{S}{X}\left(=\frac{P}{Q}=\frac{p}{q}\right)$ the potential of the detector D is that of earth, although it is not actually earthed. By this means earth-capacitance effects between the telephone detector D and the operator's head are eliminated. This point will be discussed further in connection with the Wagner earth device (see page 247).

Operation. In operation the following procedure is adopted—

1. Adjust r_1 and C_1, with connections as shown, until balance of the bridge—i.e. silence in the telephones—is obtained.

2. Remove the link M, connecting S and X, and obtain balance again by adjusting r_2 and C_2.

3. Replace the link and obtain balance again by adjusting r_1 and C_1.

4. Repeat the above procedure until balance is obtained with the link either in or out. Let the reading of C_1 for this condition be C_1'.

5. Remove the link, transfer the supply to the points T_1, T_2, and adjust C_1 until balance is again obtained. Let the setting of C_1 now be C_1''.

Then the expression for the time-constant of X is

$$\frac{L_x}{R_x} = \frac{L_s}{R_s} - \frac{1}{2} Q(C' + C'') - \frac{L_P}{P} + \frac{L_Q}{Q} \quad . \quad . \quad (6.39)$$

where L_P and L_Q are the self-inductances, and P and Q the resistances of P and Q.

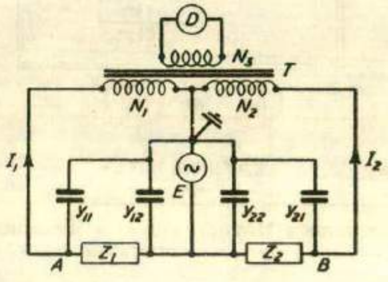


FIG. 6.30. INDUCTIVELY-COUPLED RATIO BRIDGE

The original paper should be consulted for a fuller consideration of the method.

Bridges with Inductively-coupled Ratio Arms. The Wagner earth device (see p. 247), which is used to bring the terminals of the detector branch of a bridge to earth potential, has the disadvantage that its branches must be adjusted with every change of earth capacitance, i.e. with every bridge measurement. The two balances—that of the bridge network itself and that of the Wagner earth—are mutually dependent so that a series of alternate balances must be made to arrive at a final simultaneous balance.

As an alternative method of reducing earth capacitance effects, A. D. Blumlein (Brit. Pat. No. 323037) proposed replacement of the resistive arms of the bridge network by a pair of tightly-coupled inductors. By this means it is possible to bring the three terminals of these two arms to almost the same potential, so that, by connecting the centre terminal to earth, earth capacitance effects at all four corners of the bridge can be eliminated.

A simple form of bridge having inductively-coupled ratios is shown in Fig. 6.30. The ratio transformer T consists of a toroidal core of high-permeability alloy strip having three windings each uniformly distributed over the core. The balance detector, which can be a telephone when the bridge is used in the audio-frequency range, is connected to winding N_3. The two primary windings N_1 and N_2 are arranged so that, with the current directions shown, their fields are in opposition, and when there is no flux in the core, as indicated by silence on the telephone, a condition of ampere-turn balance must exist. Thus

$$I_1N_1 = I_2N_2 \quad . \quad . \quad . \quad . \quad (6.40)$$

Since there is no flux in the core at balance, there is no induced voltage in N_1 and N_2, and the voltage drops across them are simply those due to the winding resistances. The winding resistances can be made extremely small, and the resistive voltage drops can be neglected. It follows that

$$I_1 = \frac{E}{Z_1} \text{ and } I_2 = \frac{E}{Z_2}$$

Hence, by substituting in Equation (6.40),

$$\frac{N_1}{N_2} = \frac{Z_1}{Z_2}$$

It is clear from the form of the result that Z_1 and Z_2 must be similar impedances differing only in magnitude and not in phase angle—a capacitance–resistance combination must be balanced by a capacitance–resistance combination.

The impedances Z_1 and Z_2 are assumed to be associated with stray earth admittances Y_{11}, Y_{12} and Y_{21}, Y_{22} respectively. It can be seen that Y_{12} and Y_{22} shunt the source and do not affect the balance condition. The admittances Y_{11} and Y_{22} shunt N_1 and N_2 respectively, but, since there is no potential difference across these windings at balance, the points A and B are effectively at earth potential and these admittances do not affect the balance condition. Discrimination against earth admittances is virtually complete, and the bridge is very suitable for the measurement of very small capacitances.

The bridge can be arranged to cover a wide range of measurements for a given variable arm, simply by changing the ratio of N_1/N_2 by means of tappings on the windings. A further range extension can be arranged by inserting a transformer having two secondaries with a common earth connection between the source and the bridge. The impedances Z_1 and Z_2 are supplied from tappings on the separate secondaries; thus differing voltages are applied to the two sides of the bridge.

Bridges using inductively-coupled ratio arms have been developed for use up to 100 Mc/s, and a useful survey of these techniques is given in Ref. (56).

Double-ratio bridges having two sets of inductively-coupled ratio arms have been described by H. A. M. Clark and P. B. Vanderlyn (Ref. (42)).

An interesting application of the double-ratio bridge is in a height indicator for aircraft. W. L. Walton and M. E. Pemberton (Ref. (43)) describe a direct-capacitance altimeter which uses the change in capacitance, with height, between the aircraft and the earth as a measure of low altitudes (up to about 200 feet). The sensitivity of the bridge for this purpose can be made sufficient to detect capacitance changes of 1 $\mu\mu$F.

SOURCES OF ERROR IN BRIDGE MEASUREMENTS, AND PRECAUTIONS. Although it is best, in considering the sources of error in a.c. bridge measurements, to treat each particular method separately, the space available here does not permit such consideration. The possible sources of error are considered in general below. For more detailed treatment the reader is referred to Dr. B. Hague's excellent book on *Alternating Current Bridge Methods*, to which work the author has referred for some of the information contained in this chapter.

Stray Field Effects. Errors may be caused by the fact that the various arms of the bridge network may be—unintentionally—either magnetically or electrostatically coupled, due to the "stray" magnetic or electrostatic fields existing round apparatus included in the network. When such effects are present the simple theory of the network—considering each arm as being entirely separate from the other arms except where intentionally coupled together—is no longer quite true. Under these conditions the detector may indicate balance—or zero deflection—when balance conditions have not really been obtained.

In networks containing two or more self- or mutual inductances there may be mutual inductance between two of them in different bridge arms. Usually, stray magnetic fields will be more important than stray electrostatic fields when inductances and resistances only are present. If the bridge contains capacitors the opposite is the case, errors then being caused by inter-capacitance between the various arms. Loops formed by the leads connecting a piece of apparatus to the bridge may also introduce errors owing to their inductance. In inductance measurements the leads should be twisted together to avoid such loops, while in capacitance measurements the leads should be separated from one another to avoid capacitance between them. As already pointed out, it is possible in some cases to eliminate the effects of the leads by making two measurements on the bridge—one with the apparatus under test in circuit, and one with the piece of apparatus short-circuited—or by substituting a variable standard for the unknown and adjusting it to give balance with the same bridge settings as when the unknown was in circuit.

To avoid errors due to magnetic coupling between arms the inductance coils used should be wound astatically—i.e. having no appreciable stray magnetic field—or magnetic screening may be adopted. For such screening a thin sheet of high-permeability material is placed so as to prevent the stray magnetic field from reaching the apparatus in the other arms. The inductance coils should, also, be arranged at some distance from one another and from the bridge. Mutual inductance between two pieces of apparatus can often be detected by altering their relative positions and observing if the bridge settings for balance are altered thereby.

In some cases errors are caused by direct induction effects between the supply to the bridge, and the detector circuit, which may cause the detector to indicate the passage of a current through it when the bridge is, in reality, balanced. Such effects may be eliminated by placing the supply alternator at some distance from the bridge and by supplying the bridge through an "inter-bridge transformer." The latter is a transformer having windings which are very well insulated from one another and which have an earthed metallic screen between them. Such transformers should have a closed magnetic circuit to avoid magnetic leakage, the core often consisting of sheet-steel ring punchings. Usually there are several windings on the transformer to afford a choice in the working voltage.

When telephones are being used, direct induction between apparatus in the bridge arms and the telephone circuit may produce sound in the telephones when the bridge is balanced. The presence of such direct induction may be detected by moving the head, when the bridge is almost balanced, so as to alter the plane of the telephones relative to the bridge. If no difference in sound is detected when the telephones are thus moved about, it may be assumed that such induction effects are negligible. To eliminate such effects when present, the telephones must be disconnected from the bridge and moved

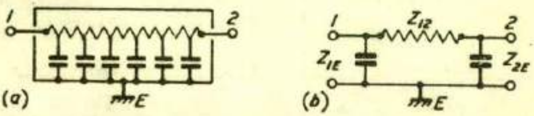


FIG. 6.31. EQUIVALENT CIRCUIT OF A SHIELDED RESISTOR

so that silence is obtained when the bridge is supplied with power from the alternator. Adjustment of the bridge to obtain balance must be carried out with the telephones in this position.

Errors due to electrostatic coupling between the various arms of the bridge and to earth capacitances of the various pieces of apparatus may be guarded against by electrostatic screening. The use of such screens renders these capacitance effects definite in magnitude and independent of the distribution of the apparatus forming tne bridge network. By this means the effect of such inter- and earth-capacitances upon the accuracy of the bridge may be made very small.

A similar purpose is served by the various earthing devices, of which that due to Wagner (Ref. (21)) is commonly used.

Screening of Bridge Components. Surrounding a bridge component with an earthed screen ensures that stray capacitances are to earth. Fig. 6.31 (*a*) shows a resistor surrounded by an earthed screen, and it is apparent that the resistor has a distributed capacitance to earth. It can be shown by line theory (Ref. (1)) that such a resistor may be adequately represented by a π-network of lumped impedances as shown in Fig. 6.31 (*b*). If R, L and C represent the total resistance, self-inductance and earth capacitance respectively, then

$$Z_{12} \doteqdot R + j\omega\left(L + \frac{CR^2}{6}\right) \qquad (6.41)$$

and

$$Z_{1E} = Z_{2E} = \frac{2}{j\omega C}$$

L and C are both very small in a well-designed resistor, and it is apparent that the distributed capacitance C effectively increases the self-inductance of the resistor. This reasoning can be extended to components in series and to components with unsymmetrical shielding, and it is possible to show that any combination of impedances can be represented by a lumped impedance with a different earth admittance at each end. This is very important in bridge networks, because we can thereby lump all earth capacitances, including those due to the source and detector, at the ends of each arm. Earth admittances are usually mainly capacitive and may conveniently be treated as simple capacitances. Fig. 6.32 (*a*) shows the positions of the effective earth capacitances; these can be regrouped as single capacitances appearing at each bridge apex as in Fig. 6.32 (*b*). By the use of the general star–mesh transformation

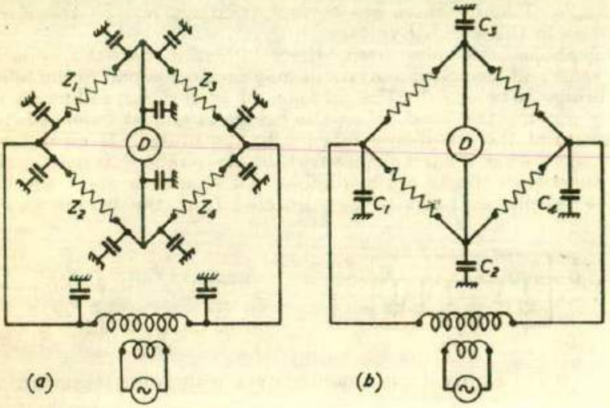


FIG. 6.32. LUMPED EARTH CAPACITANCES IN A BRIDGE NETWORK

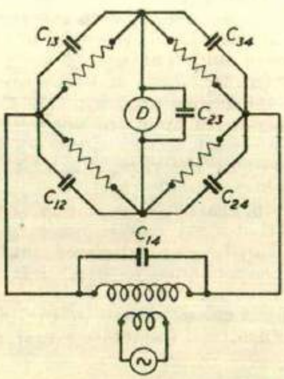


FIG. 6.33. POSITIONS OF THE TRANSFORMED EARTH CAPACITANCES

(Rosen's Theorem, p. 131), the four-arm star of capacitances C_1, C_2, C_3, C_4 can be transformed to six capacitances as shown in Fig. 6.33, and it is clear that those shunting the bridge arms affect the balance. There will therefore be some uncertainty in the actual values of the bridge arms.

If a component is surrounded with a screen connected to one end, the impedance appearing between the two terminals is completely defined, and there is a single capacitance appearing between one terminal and earth. Such component screens are shown in Fig. 6.34 for a resistor and a capacitor. The screen around the resistor will add a small shunting capacitance and alter its impedance; the effect of this is considered on p. 253. An example of the use of screens is shown in the general bridge network in Fig. 6.35. One apex of the bridge is earthed, and it can be seen that, with the screen connections chosen, none of the earth capacitances affects the bridge except that due to the source transformer which shunts arm *I* as shown. It is desirable that this transformer should have a double shield on the secondary, i.e. the secondary winding is provided with an enveloping shield connected to one end, the whole then being enclosed in an earthed outer shield. With such an arrangement the inter-screen capacitance is known, and if the component it shunts is a capacitance, it can

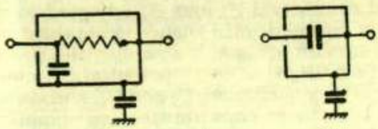


FIG. 6.34. DEFINING COMPONENT SCREENS

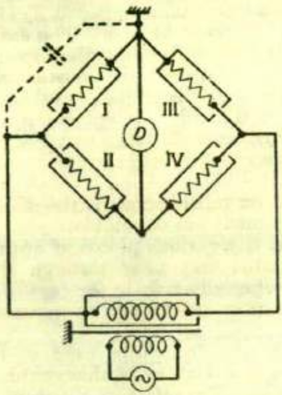


FIG. 6.35. SCREEN CONNECTIONS FOR A FOUR-ARM BRIDGE

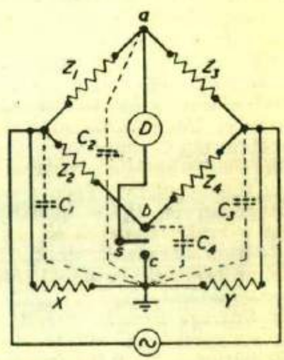


FIG. 6.36. CONNECTIONS OF WAGNER EARTHING DEVICE

be taken into consideration in the calibration; otherwise it will have least effect if this component is a low-value resistor.

When bridges are to be designed for measurements of the highest accuracy, very careful attention must be given to the screening. It is often necessary to provide some components with two screens, one inside the other, and to include all leads in the screening system.

Wagner Earthing Device. If each component in a bridge has a defining screen connected to one end, a very high accuracy in measurement is made possible by the addition of a Wagner earthing arm. This device removes all the earth capacitances from the bridge network.

Fig. 6.36 shows the connections of the device for use in conjunction with the general form of bridge network, in which Z_1, Z_2, Z_3 and Z_4 are the impedances of the bridge arms. X and Y are the two variable impedances of the Wagner earth branch, the centre point of which is earthed as shown. These impedances may consist of variable resistances and capacitances similar to those used in the arms of the bridge proper, but not necessarily of known value. The two impedances X and Y must be capable of forming a balanced bridge with Z_1 and Z_3 or Z_2 and Z_4, and can be a duplicate of either of these pairs of arms.

C_1, C_2, C_3 and C_4 are the stray earth capacitances appearing at the apexes of the bridge. D is the detector.

If the switch s is on contact b, balance of the bridge may be obtained by adjustment of the impedances Z_2 and Z_4. The presence of the earth capacitances will prevent a true balance being obtained, but a point of minimum sound can be obtained.

After adjusting the bridge to give minimum sound, the switch s is thrown to contact c so that the telephones are then connected between a and earth; X and Y are next adjusted until minimum sound is obtained. The telephones are next reconnected to a,b, and Z_2 and Z_4 adjusted to give minimum sound again. The process is repeated until silence is obtained with the switch on b, and silence, or the minimum attainable sound, with the switch on c. Then all three points a, b and c must be at earth potential. Under these conditions no current flows in the earth capacitances C_2 and C_4, and since C_1 and C_3 shunt the Wagner arms X and Y, these capacitances are eliminated from the bridge network Z_1, Z_2, Z_3, and Z_4.

The capacitances C_1 and C_3, shunting X and Y, complicate these arms, and because of this the combination may not give a true balance against Z_2 and Z_4, only a minimum being attainable during the Wagner balance. The existence of these capacitances should be borne in mind when deciding the form of the Wagner arm, which should preferably be composed of resistances and capacitances. Much of C_1 and C_3 will be due to the earth capacitance of the source transformer, and a common cause of difficulty in achieving a good Wagner balance is due to the use of transformers with large and lossy earth capacitance. Poor insulation in the source transformer will cause resistance to appear shunting C_1 and C_3.

In the double-ratio a.c. bridges mentioned on p. 244 some of the disadvantages of the use of the Wagner earth arrangement are overcome.

Leakage Errors. If the insulation between the various pieces of apparatus forming a bridge network is not good, trouble may arise through leakage currents from one arm to another. This is especially true in the case of high-impedance bridges. To avoid this the apparatus used may be mounted on insulating stands.

Eddy Current Errors. Standard resistors and inductors used in bridge networks should be so constructed as to avoid variation of their values due to eddy currents when the frequency is varied. The effective resistance and inductance of a piece of apparatus under test may vary with frequency due to this cause. Large masses of metal in the vicinity of the bridge should be avoided, as the flux produced by eddy currents induced in them may set up troublesome e.m.f.s in parts of the network. In the case of mutual inductances, eddy current effects, if present, cause the induced voltage to lag by some angle less than 90° behind the inducing current, in which case the simple theory of the network does not hold.

Residual Errors. In speaking of the resistors used in the various bridge networks the description "non-inductive" or "non-reactive" has been applied to them, indicating that their inductance and capacitance are both zero. Although resistors for such purposes are constructed so that these quantities are very small, it cannot always be assumed that they are zero. The term "residual" is used to indicate the small inherent inductance or capacitance of a resistance coil. In precise work it is sometimes necessary to take these residuals into account—for which purpose they must either be measured or calculated—in order that errors due to them shall be avoided. The self-capacitance of a coil is usually only important when the coil has many turns and the supply frequency is high. The resistance and inductance of such coils are increased and reduced, respectively, due to this cause by amounts which are proportional to the square of the frequency (Refs. (19), (20)).

Frequency and Wave-form Errors. Some of the bridges previously described are independent of the frequency of the supply in the sense that the balance conditions do not involve the frequency. In such cases, therefore, the frequency of the supply is only important in its effect upon the effective resistance and inductance of the apparatus under test; and the fact that the supply wave-form contains harmonics is only important for the same reason.

In the case of networks in which the balance conditions do involve the frequency, the latter is important and must be carefully measured. The wave-form of the supply is also obviously of importance, since the bridge cannot be balanced both for the fundamental and the harmonics in the wave-form (if any) simultaneously. If telephones are employed in such bridges it will be found impossible to obtain complete silence, only a point of minimum sound being obtainable.

There are two means of circumventing this difficulty. The first is by using some form of "wave filter" such as those described by Campbell (Ref. (14)), and the second is by using a tuned detector, such as a vibration galvanometer, instead of telephones. Such detectors will not respond appreciably to frequencies other than that of the fundamental of the supply.

It is advantageous, if possible, to use in the bridge network such values of the impedances that the frequency term in the expression for the quantity to be measured is reduced to zero.

Apparatus Used in Conjunction with A.C. Bridge Networks.

1. SOURCES OF CURRENT. Before dealing in detail with current sources, it should be pointed out that the current-carrying capacity of the impedances forming the arms of the network should be carefully considered. It may happen that the total impedance of the two arms forming one "side" of the network (such as arms Z_1 and Z_3 in Fig. 6.36) is low compared with the impedances of the other two "sides." In such a case the low-impedance side may take an excessive current, with consequent damage to the apparatus in these arms. The current-carrying capacity of resistance boxes is usually marked under each dial: failing this it is safe to allow ½ watt per coil in the box. Only the highest resistance decade, in a given setting, need be considered.

Electronic oscillators are now almost universally used as bridge sources. They have the advantage that the frequency is constant, easily adjustable, and determinable with accuracy. The wave-form is very close to a pure sine wave, and the power output is sufficient for most bridge measurements.

The simplest form of oscillator depends for its operation upon the fact that a circuit containing inductance L and capacitance C has a natural frequency of oscillation given by $f = \dfrac{1}{2\pi\sqrt{LC}}$, provided that the circuit resistance is small. If such a circuit has an e.m.f. induced in it and is then left to oscillate, the frequency of oscillation of the current in the circuit will be given by the above expression. The decay of the oscillation due to the circuit resistance may be prevented by supplying energy with the aid of a three-electrode valve.

Fig. 6.37 gives the connections of a simple triode valve oscillator. The h.t. anode supply and the 6·3 V heater supply are normally derived from an a.c. mains-operated power supply. The inductor

L_3 and the associated variable capacitor form the tuned circuit. The feedback necessary to maintain oscillation is obtained by amplifying the voltage induced in L_1 from L_3, and feeding the amplified signal back into the tuned circuit. The bridge circuit may be supplied by the coil L_2 coupled to L_3.

While this oscillator is simple to construct and, in fact, is particularly useful at high frequencies, in the audio-frequency range the resistance–capacitance oscillator is to be preferred. Fig. 6.38 gives the circuit diagram of a resistance–capacitance oscillator in

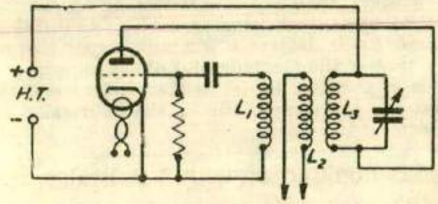


Fig. 6.37. Connections of Triode Valve Oscillator

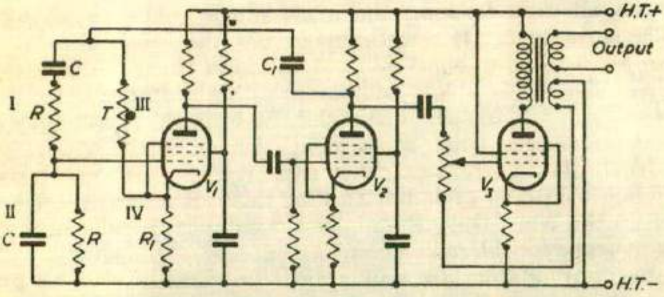


Fig. 6.38. Thermistor-stabilized Wien Bridge Oscillator

which the frequency-determining network is a Wien bridge. The four bridge arms are marked I, II, III and IV. The output signal from the bridge is applied between grid and cathode of V_1, and the two-stage amplifier comprising V_1 and V_2 gives a positive feedback to the bridge through C_1 and causes oscillation. The thermistor (thermally sensitive resistor) T controls the amplitude of oscillation. This element has a high resistance when cold, and the bridge is initially unbalanced. When the oscillations build up, the thermistor heats up until its resistance approaches a value of $2R_1$, at which value the bridge output would be zero at its characteristic frequency. A steady state is reached in which the bridge is just off balance and a constant voltage is maintained across the bridge.

The action of the thermistor is shown in Fig. 6.39. The curve ORT is the voltage/current characteristic for the thermistor T, and the straight line OQU that for the resistor R_1. The combined characteristic is given by the curve OSV, and the voltage across the bridge is given by SP. If the component values are correctly chosen, S should lie at or near the bottom of the dip in the combined characteristic when the voltage across T is twice the voltage across R_1. These voltages are represented by PR and PQ respectively.

This type of oscillator gives an excellent wave-form with stable frequency. Tuning is usually effected by making the capacitors C a pair of ganged variable air capacitors, and the frequency range is changed by selecting differing values for R. A considerable degree of negative feedback is required in the output stage to avoid distortion.

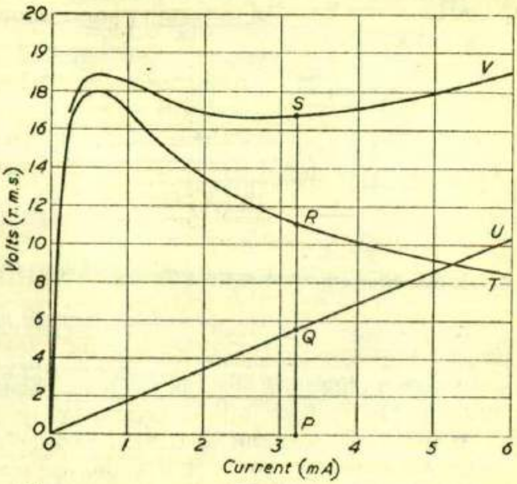


FIG. 6.39. WORKING-POINT FOR THERMISTOR-STABILIZED OSCILLATOR

Oscillators suitable for bridge work are offered by a considerable number of manufacturers. An oscillator made by H. W. Sullivan Ltd. has a frequency range of 40 c/s to 125 kc/s with a power output of 7 watts.

Another interesting oscillator made by H. W. Sullivan Ltd. uses the beat-tone principle and covers a frequency range of 50–170,000 c/s with an accuracy of 0·1 per cent. This oscillator has a frequency stability of a few parts in a million during a period of measurement.

Although, for ordinary bridge measurements of inductance and capacitance, a fixed-frequency oscillator of 1,000 c/s and output of about 1 watt is adequate, for more specialized work continuously variable oscillators are preferable with outputs up to 5 watts. The high power may be necessary on some occasions, but it is a better practice, wherever possible, to limit the power supplied to a bridge and employ an electronic detector-amplifier of an aperiodic type in which balance detection is sensed both orally by telephones, and visually by a pointer galvanometer having a logarithmic deflection (to avoid damage which may be caused by a greatly unbalanced bridge). A logarithmic response may be obtained by shunting the output meter with a metal rectifier.

2. VARIABLE RESISTORS, INDUCTORS, AND CAPACITORS. Fixed standards of resistance, inductance, and capacitance have been described in Chapter II.

Variable Resistors. These are usually in the form of resistance boxes containing a large number of resistance coils which are selected either by plugs or by rotary decade switches. Fig. 6.40 illustrates the various methods of arranging and mounting the

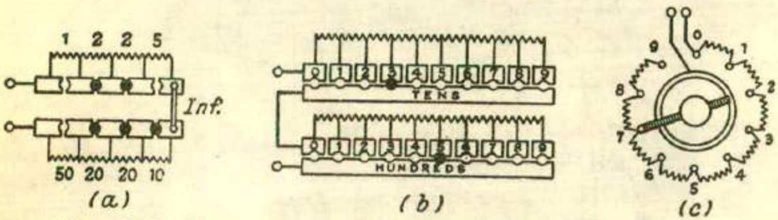


FIG. 6.40. CONSTRUCTION OF VARIOUS TYPES OF VARIABLE RESISTORS

coils in such boxes. Fig. 6.40 (*a*) shows the simplest arrangement, in which the removal of a plug puts a coil in circuit. The reading, as shown, is 56 ohms. An open-circuiting link or plug is usually provided. Fig. 6.40 (*b*) shows the "straight-decade" arrangement. One plug only is necessary for each decade, the reading with plugs as shown being 530 ohms. The dial arrangement, the value of the resistance in which is varied by the rotation of a laminated copper brush or arm, is shown in Fig. 6.40 (*c*).

Although lower contact resistances can be obtained with well-made plugs, these are now falling into disuse in favour of the rotary switch, which is much simpler to use and is easy to protect from dust and moisture by enclosure in the box. Reasonably low, and constant, contact resistances in the region of 1 milliohm or less can readily be obtained with a well-designed instrument switch. There are very many switch designs, but the essential features of a good design are large, well-made bearings and robust contacts and brushes. Phosphor bronze is often used for the brush and brass for the contacts. Some manufacturers use silver-covered contacts, and quite recently the silver contact with a silver–graphite brush has been introduced by Muirhead & Co., Ltd. Normally metal-to-metal contacts require occasional cleaning and greasing, special contact greases (e.g. Elvolube, Gulf Oil, Ltd.) being available for this purpose, but this attention is claimed to be unnecessary with the silver/silver–graphite combination.

The resistances are usually arranged in switched decades, and the boxes may have any number of dials between one and six, with decades ranging from $10 \times 0{\cdot}1$ ohm to $10 \times 10{,}000$ ohms. The accuracy of adjustment is usually between $\pm\, 0{\cdot}05$ per cent and $\pm\, 0{\cdot}1$ per cent, but special high-grade boxes have an accuracy of $\pm\, 0{\cdot}01$ per cent.

The resistance coils are commonly wound on insulated brass bobbins for low-frequency work, and Bakelite or ceramic bobbins or thin ceramic cards for high-frequency use. Manganin wire is used in nearly all high-grade resistance coils. The coils are impregnated with shellac after winding and are annealed by baking for at least

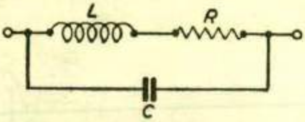


FIG. 6.41. EQUIVALENT CIRCUIT OF A RESISTANCE COIL

10 hours at 140° C. It is possible that the recently introduced nickel–chromium–aluminium alloys with copper or iron, which have resistivities three times that of manganin, may eventually supplant it in high-resistance decades (Ref. (55)).

It is essential that the resistance appearing between the terminals of a resistance box should be virtually non-reactive. The coils can be wound non-inductively by the bifilar winding method discussed previously (see p. 79), but the capacitance between the wires may significantly affect the properties of the coil.

A resistance coil can be represented by the circuit shown in Fig. 6.41, in which R is the d.c. resistance, L the series inductance, and C the effective shunt capacitance; L and C are both small. The admittance of the circuit is

$$Y = \frac{1}{R + j\omega L} + j\omega C$$

and the impedance is

$$Z = \frac{1}{Y} = \frac{R + j\omega L}{1 + j\omega CR - \omega^2 LC}$$

By rationalizing, this reduces to

$$Z = \frac{R + j\omega[L(1 - \omega^2 LC) - CR^2]}{(1 - \omega^2 LC)^2 + \omega^2 C^2 R^2}$$

Since L and C are both small this can be written as

$$Z = \frac{R + j\omega[L(1 - \omega^2 LC) - CR^2]}{1 - \omega^2 C(2L - CR^2)}$$

or $$Z = R[1 + \omega^2 C(2L - CR^2)] + j\omega[L(1 - \omega^2 LC) - CR^2][1 + \omega^2 C(2L - CR^2)]$$

or $$Z = R' + j\omega L' \quad . \quad . \quad . \quad (6.42)$$

L' is termed the residual of the coil and in most cases $\omega^2 LC \leqslant 1$ and the term $\omega^2 C(2L - CR^2) \leqslant 1$. Thus, to a sufficient approximation, $L' = L - CR^2$, and it is either positive or negative depending upon the relative magnitudes of the two terms.

It is apparent from the expression for Z that R' itself changes

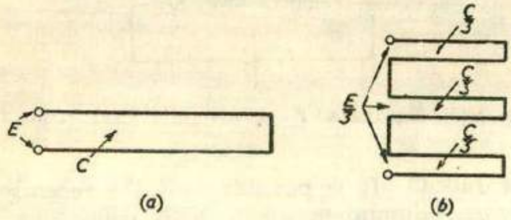


Fig. 6.42. Reduction in Capacitance Current Achieved by Sectionalizing a Non-inductive Resistor

(a) Single bifilar loop: relative capacitance current, 1.
(b) Bifilar loop in three sections: relative capacitance current, 1/9.

with frequency, but in practice the change is small compared with the overall change in impedance due to L'.

The time-constant $\frac{L'}{R'}$ of a resistance coil is a useful guide to its high-frequency performance, and values attained in typical resistance boxes range between 5×10^{-7} to 1×10^{-9} henry per ohm.

If a simple bifilar winding is used the coil capacitance may become prohibitively large. A marked improvement can be obtained by dividing the coil into sections which have not only a smaller capacitance but a smaller p.d. across them. The principle is illustrated in Fig. 6.42, and, in general, winding a given coil in n sections reduces its self-capacitance by a factor $\frac{1}{n^2}$. There are a large number of variations of this principle; in some cases a bifilar winding is not used, the direction of winding being simply reversed at each section.

Since the self-capacitance of single-layer coils wound on bobbins is usually quite small, the reduction of their inductance is often the most important question. The winding of alternate turns of the coil in reverse directions is one method used for this purpose. This method is illustrated in Fig. 6.43 (*a*), the arrangement shown being due to Grover and Curtis (Ref. (17)). The wire is wound on a cylindrical former having an axial slit along the greater part of its length. The wire passes through this slit once in every turn, so as to give reversal of winding direction. The arrow heads show the directions of current in various parts of the winding. Obviously the magnetic effects of adjacent turns neutralize one another. This type of coil has a very small inductance but is somewhat difficult to wind.

Fig. 6.43 (*b*) shows the Chaperon (Ref. (22)) method of winding.

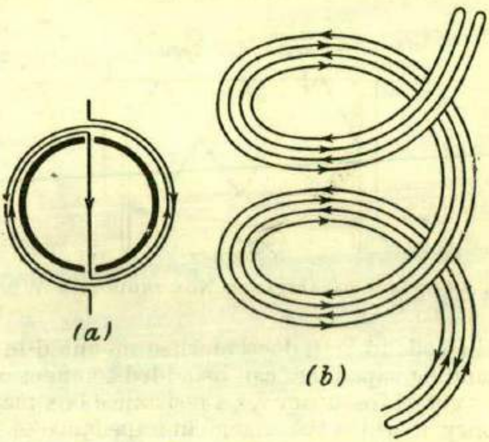


FIG. 6.43. NON-INDUCTIVE WINDINGS

This winding is really an extension of the bifilar principle, the currents in adjacent wires neutralizing one another as regards resultant magnetic field, as shown. Both the inductance and capacitance of coils wound in this way are small.

Modifications of this method have been used in which the winding is divided into a number of sections connected in series.

An important type of winding due to Ayrton and Mather is shown in Fig. 6.44. Here two similar windings are superimposed on a thin card; the windings are in opposite directions and are connected in parallel. The field due to each winding cancels that due to the other, and the effective self-capacitance is very small. This form of winding can give extremely low time-constants and is used extensively in high-frequency resistance boxes, but it is rather difficult to construct as compared with sectionalized coils. Further information on resistance windings may be found in Refs. (1), (17) and (22).

The switchgear and wiring in a resistance box will introduce capacitance which effectively shunts the resistance coils. This is not important with low resistances but can be serious with high-resistance settings. If the box incorporates a $10 \times 10{,}000$ ohm decade, a shunt capacitance of $10\ \mu\mu F$ will, on the maximum setting of 100,000 ohms, give a value of CR^2 of 10^{-1}, and if the self-inductance is negligible, the time-constant $\left(\frac{L'}{R'}\right)$ will be 1×10^{-6}. It is a common practice, therefore, to wind the coils in the high-resistance decades (above 1,000 ohms) in such a manner that the self-inductance increases as the box setting increases. If the coil inductances are correctly chosen the value of the residual given by $L - CR^2$ can be kept very small. It is convenient in practice to

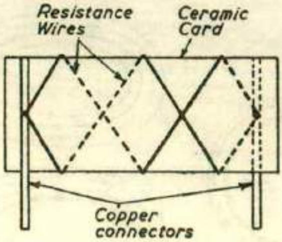


FIG. 6.44. LOW-CAPACITANCE NON-INDUCTIVE WINDING

arrange for the coils to be inductive when mounted in the box, and then small padding capacitors can be added to effect compensation.

The upper limit of frequency for a resistance box may be specified as the frequency at which the change in impedance of the resistance setting is equal to the accuracy in adjustment of the resistance. This limit can be calculated from time-constant $\frac{L'}{R'}$.

Switch decade resistance boxes are manufactured by a number of firms including the Cambridge Instrument Co., Ltd., the Croydon Precision Instrument Co., Muirhead & Co., Ltd., W. G. Pye, Ltd., H. W. Sullivan, Ltd., and H. Tinsley & Co., Ltd.

Some of the Sullivan boxes incorporate a patented pre-set switching device, due to W. H. F. Griffiths, which is employed to minimize the residual inductance and the high-frequency error; the screen capacitance is compensated automatically, and so the effective residual inductance is rendered sensibly independent of screen connection.

Campbell Constant-inductance Rheostat. There are several forms of low-resistance rheostats designed to have a very low and calculable inductance. These are usually constructed on the bifilar principle, and are often of the slide-wire form. Fig. 6.45 (*a*) illustrates the construction of Campbell's constant-inductance rheostat. This is a very useful piece of apparatus for use in certain bridge measurements, since it enables a fine adjustment of the resistance of a bridge arm to be made without altering the inductance settings.

The resistance is increased by moving the sliding contact to the right (Fig. 6.45 (*b*)). Such a movement increases the length of manganin wire in circuit and reduces the length of copper wire in circuit. Whatever the position of the slider, the total length of wire in circuit is always the same, and forms a bifilar loop, thus maintaining the inductance small and constant.

A precision slide wire of this type, made by Muirhead and Co., Ltd., may have any resistance from 0·5 to 10 ohms. One made by the

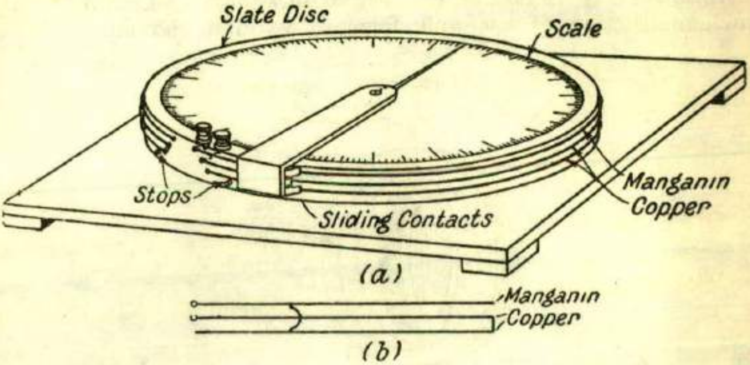


FIG. 6.45. CAMPBELL CONSTANT-INDUCTANCE RHEOSTAT

Cambridge Instrument Co., Ltd. has a total resistance of 0·12 ohms with a residual inductance of 0·2 μH.

A precision decade non-reactive slide resistance designed on the Kelvin–Varley slide principle (see p. 332) is made by H. W. Sullivan, Ltd. It has manganin coils. One type, having a total resistance of 1,000 ohms, has each of its five dials brought out to separate terminals so that it can also be used as a slide resistance of 200, 40, 8 or 1·6 ohms.

Variable Inductors. Such pieces of apparatus should have as high an inductance as possible compared with their resistance, i.e. their time-constant should be great. Their inductance should be continuously variable and should cover as great a range as possible between maximum and minimum settings. In addition, it is highly desirable that the variation of inductance with position of the moving part should obey a straight-line law, and also that the coils should be astatically wound. The inductance for a given position should not, of course, vary with time, and variation of frequency should not cause appreciable variation of inductance.

Most variable inductors are so constructed that they can be used as either self- or mutual inductors. When used as self-inductors the fixed and moving coils are connected in series and the inductance is given by

$$L = L_1 + L_2 \pm 2M$$

where L_1 and L_2 are the self-inductances of the fixed and moving coils respectively, and M is the mutual inductance (variable) between them.

In order to eliminate frequency errors the coils are usually wound with stranded wire and the use of metal parts in the construction is avoided as far as possible.

Ayrton–Perry Inductometer. Fig. 6.46 shows, diagrammatically, the construction of a simple form of variable inductor (self- or

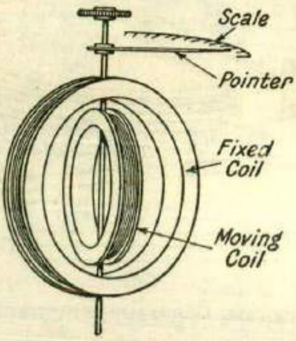


FIG. 6.46. AYRTON–PERRY INDUCTOMETER

mutual) due to Ayrton and Perry. The moving coil is mounted inside the fixed coil and is carried by a spindle which also carries a pointer and handle at the top as shown. Movements of the pointer indicate the variation in the angle between the planes of the coils, but the scale may be graduated to read the inductance directly. When constructed for use in accurate measurements, the coils are wound on mahogany formers whose surfaces are spherical, and great care is necessary in fixing the coils so as to ensure constancy of inductance with time.

This form of inductor can be cheaply and easily constructed for use as a variable self-inductor in cases where the inductance must be variable but not necessarily known, e.g. for use in one arm of the Wagner earth device.

Its disadvantages are that the instrument produces an external magnetic field, which may be troublesome if it is placed near to the bridge network, and that the scale is not linear.

Brooks and Weaver Inductometer. This form of inductor (Ref. (23)) is one of the best forms for general purposes. The coils are wound and connected astatically, the time-constant is high, and the scale is uniform throughout the greater part of the range. It is also fairly easily constructed, and its calibration remains reasonably constant with time even when in continuous use. The current-carrying capacity, also, is high for this type of apparatus.

The construction of the inductor is shown in Fig. 6.47. There are, in all, six link-shaped coils—four fixed and two moving. These are wound with stranded wire. The moving coils have twice as many turns as the fixed. These coils are embedded in ebonite or Bakelite discs which are about 15 in. diameter. Bakelite has the advantage that it has less tendency to warp than ebonite. The top

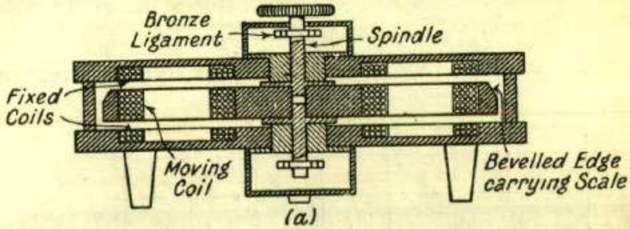


FIG. 6.47. CONSTRUCTION OF BROOKS AND WEAVER INDUCTOMETER

and bottom discs, which are fixed, are about ½ in. thick, and are separated by ebonite or Bakelite pillars. The centre disc is thicker—to carry the larger coils—and is of slightly smaller diameter. It has a bevelled edge, upon which a scale is marked out over 180° of its circumference, this scale being used in conjunction with an index mark on the lower fixed disc. Connections to the moving coils are made through copper or phosphor-bronze ligaments soldered to the two halves of the spindle.

The dimensions of the coils are specially chosen to give a uniform scale, and also to obtain as great an inductance as possible for a given length of wire. The relationships between the various dimensions are given in Fig. 6.48, in terms of the mean radius R of the semicircular ends of the coils. The depth of the moving coils should be the same as their width of winding, i.e. $0{\cdot}78R$, and the depth of the fixed coils $0{\cdot}39R$.

A great advantage of this method of construction is that small variations of the length of gap between (say) the moving disc and the upper fixed disc, due to warping of the former or to wear of the bearings, have no appreciable effect upon the inductance of the instrument, since movement away from the upper fixed disc means movement towards the lower one, thus maintaining the inductance the same within narrow limits.

Fig. 6.49 shows the calibration curve for an inductor of this type when used as a self-inductor (all six coils in series). The instrument had 144 turns on each of its fixed coils and 288 turns on each of its moving coils, the total resistance being 17·5 ohms at 20° C.

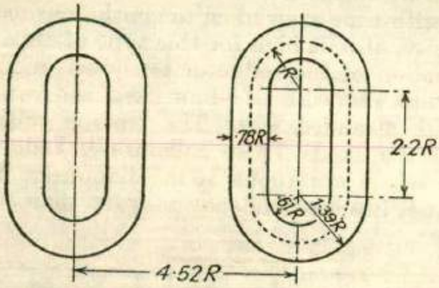


FIG. 6.48. RELATIVE DIMENSIONS OF BROOKS AND WEAVER INDUCTOMETER

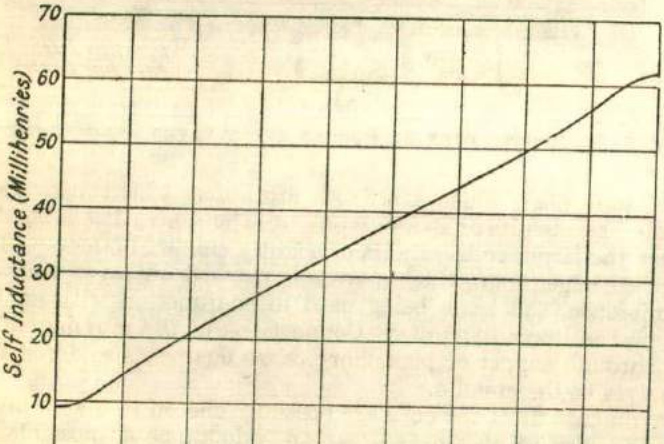


FIG. 6.49. CALIBRATION CURVE OF BROOKS AND WEAVER INDUCTOMETER

Campbell and Butterworth–Tinsley Mutual Inductometers. Both of these instruments are, essentially, variable mutual inductors, such instruments having the advantage that their inductance can be reduced to zero or given negative values, while a variable self-inductance can only be reduced to some minimum value depending upon the self-inductance of the coils and the mutual Inductance between them.

The Campbell instrument, devised by A. Campbell (Ref. (10)) and manufactured by the Cambridge Instrument Co., has an arrangement of coils as shown in Fig. 6.50. *P,P* are two equal coaxial fixed coils forming the primary winding. These are connected in series. The two coils *S,S*, connected in series, form together one of the fixed secondary windings. S_1 is another secondary coil, also fixed, while S_2 is a movable secondary coil. The three secondary windings are connected in series. A link is provided for the purpose of reversing the connections to the moving coil S_2. Coils *S* and S_1 are each divided into ten sections of equal mutual inductance with the primary, and connections are taken from these sections to two dial switches. The mutual inductance of each of the sections of S_1 is,

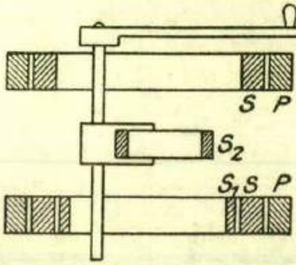


FIG. 6.50. CONSTRUCTION OF CAMPBELL MUTUAL INDUCTOMETER MANUFACTURED BY THE CAMBRIDGE INSTRUMENT CO.

in one form of the instrument, 100 microhenrys, and of coil *S* 1,000 microhenrys, giving a total for the two coils of 11,000 microhenrys. Fine adjustment is obtained by rotation of the secondary moving coil S_2, which is mounted midway between, and parallel to, the fixed coils as shown. An index mark on the handle arm serves the purpose of a pointer, readings being observed on a scale fixed under this arm on the lid of the instrument.

The coils are wound with stranded wire, and marble is used in the best instruments for the coil bobbins and as the framework to which the coils are attached. These instruments have the advantage of high accuracy and simplicity, but possess considerable capacitance, which introduces errors at the higher frequencies.

Butterworth's mutual inductometer (Ref. (7)), manufactured by Messrs. H. Tinsley & Co., is designed so as to eliminate the defect of the Campbell and similar instruments, due to inter-capacitance between the windings. The makers claim that with this type of instrument a correction of only 0·07 per cent is necessary in the case of an instrument calibrated at 50 c/s and used at 1,000 c/s.

This instrument has a fixed primary coil and two sets of three secondary coils, also fixed. There is also a moving secondary coil for fine adjustment. Each set of fixed secondary coils consists of three coils having mutual inductances with the primary in the ratio of 6 : 3 : 1. Connections are made from each set to a commutator which is manipulated as a dial switch. The two dials are marked 1 to 10, the various inductances being obtained by connecting various combinations of the three coils, through the commutator, in series. In some cases one of the coils is reversed to give the required inductance. For example, 8 is obtained by the commutator connecting the 6 and 3 coils in series so that their magnetic effects are cumulative, and the 1 coil is reversed, thus giving the value $8 = 6 + 3 - 1$.

In a common form of the instrument one dial gives 10 millihenrys in steps of 1, while the other dial has a total to 1 millihenry in steps of 0·1, the moving coil giving from − 0·01 to + 0·11 millihenry. The readings in the latter case are observed on a scale placed under the handle arm, as in the Campbell instrument. In this case the total range of the instrument is 11·11 millihenrys.

In the Sullivan–Griffiths variable standard of self- or mutual inductance (Ref. (46)) the formers of both rotor and stator are constructed to have temperature compensation and so to give a high degree of stability to the windings. There are two ranges of self-inductance and one range of mutual inductance; all three are direct-reading in inductance, from a single calibration in terms of self-inductance. Accuracies as high as 0·02 per cent are possible up to frequencies of a few kilocycles per second, but the instrument may be used at frequencies up to 50 kc/s by applying corrections reaching a maximum of 0·3 per cent at this frequency.

The Sullivan–Griffiths decade standards have a number of temperature-compensated coils, each having ten tappings taken to a rotary switch and thus providing a decade of inductance. The coils are all arranged geometrically at mutually zero magnetic coupling. In the standard having a maximum inductance of 1 henry, the finest subdivision on a sixth (continuously variable) dial is 0·05 μH. The decades are adjusted to be direct-reading to an accuracy of 0·03 per cent throughout their entire range and are provided with a calibration of 0·01 per cent.

Variable Capacitors. Variable capacitors may take the form of a subdivided fixed capacitor, various fractions of which can be obtained either by movement of a dial switch or by plugs. If continuous variation of the capacitance is required, as is often the case in a.c. bridge work, a variable air capacitor of the parallel-plate type is used. In some cases a combination of both of the above types may be most useful.

Continuously variable capacitors, having air as dielectric, consist of two sets of plates, usually semicircular—one set fixed and the other moving—arranged so that the moving plates can be rotated in the air gap between the fixed plates as shown in Figs. 6.51 and 6.52. The capacitance is varied by varying the area of the moving plates interleaving with the fixed plates. The plates, which are usually of aluminium or brass, are proportioned so that the

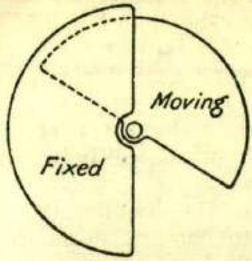


FIG. 6.51.

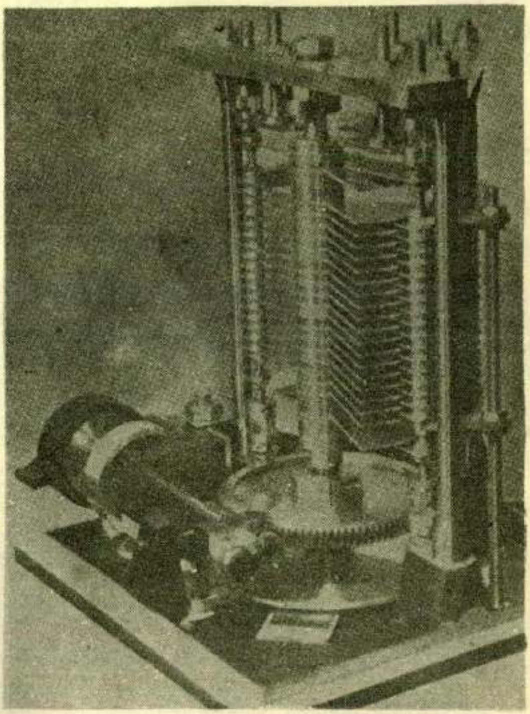


FIG. 6.52. VARIABLE AIR CAPACITOR

capacitance varies in almost exact proportion to the angle turned through. The plates should be made fairly thick, so as to avoid bending, which would alter the calibration, and all corners should be carefully rounded. The bearings must be well fitted, so that the axial distance between the plates shall be definite and constant. The variable air capacitor shown in Fig. 6.52 is of the precision type for use as a laboratory standard or for a.c. bridge measurements. It has a slow-motion, worm-gear device permitting high accuracy in setting and reading. Its temperature coefficient is 30–40 parts in 10^6 per degree centigrade, the power factor being less than 0·0001 at 1,000 c/s, and the residual self-inductance between 0·04 and 0·07 μH, for all settings.

Square-Law Capacitors. Duddell (Ref. (26)) constructed a variable capacitor with the plates shaped so as to give a square law of capacitance variation. Such capacitors are of use in wavemeters for

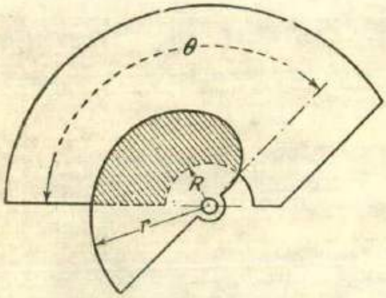


FIG. 6.53. DUDDELL SQUARE-LAW CAPACITOR

radio work where the wavelength is approximately proportional to the square root of the capacitance of the variable capacitor.

The plates were shaped as shown in Fig. 6.53. R is the inner radius of the fixed plates; r is a radius of the moving plates. The law of the curve bounding the moving plates is

$$r^2 = 4K\theta + R^2 \quad . \quad . \quad . \quad . \quad (6.43)$$

K being a constant such that the interleaving area of the plates —shown shaded in the figure—is equal to $K\theta^2$. θ is the angle turned through by the moving plates from their zero position. Then, since the shaded area is proportional to θ^2, it follows that the capacitance also is very nearly proportional to θ^2.

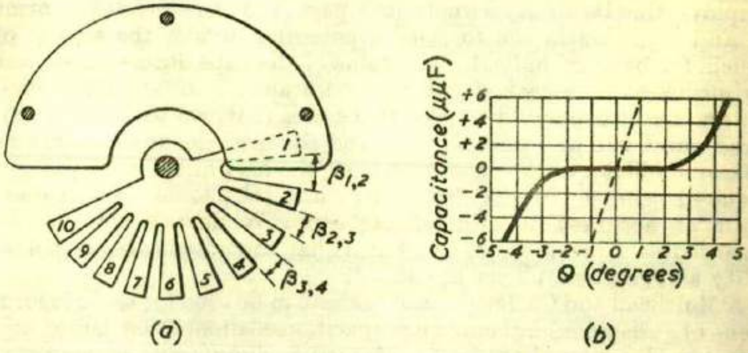
W. H. F. Griffiths* has investigated the laws of variable air capacitors with several different designs of plates. One of these, the Sullivan–Griffiths logarithmic variable air capacitor, covers a wide range of capacitance, the logarithmic scale law giving the same accuracy of scale reading throughout the range for which the law

* *Experimental Wireless and The Wireless Engineer*, Vol. III, No. 28, January, 1926, and Vol. III, No. 39, December, 1926. See also Refs. (44), (49), and particularly Ref. (48) for very exact design of linear-frequency-law capacitors.

holds. This is particularly important for very low values of capacitance.

The Sullivan precision variable air capacitor standard has fused-silica insulation ensuring a high degree of permanence in the calibration. It has a very low power factor, less than 0·00001 for all usual test frequencies and increasing only slightly when the frequency rises above 100 kc/s, a scale accuracy of 1 part in 20,000 (or better), and a temperature coefficient less than 10 parts in 10^6 per degree centigrade.

The Sullivan–Griffiths decade air capacitor (Ref. (57)) is a low-loss, highly stable standard in which all errors due to reading finely-graduated scales are eliminated. The higher precision which can



(*By courtesy of "The Engineer"*)

FIG. 6.54. SULLIVAN–GRIFFITHS DECADE AIR CAPACITOR

be attained in reading a series of decades is obvious. The basic principle is illustrated in Fig. 6.54 (*a*).

Semicircular moving plates of a conventional air capacitor take the form of ten fingers. When each group of fingers is embraced within the fixed plates there is no change in the value of capacitance for small angular movements. The capacitance/angle curve in Fig. 6.54 (*b*) shows how well this condition is met in practice. Special compensating techniques, to overcome the effects of fringing at the edges, have been adopted to achieve this result. The broken curve is for an ordinary variable capacitor having the same range and linear capacitance law. It is clear that precise angular positioning of the plates is not necessary, and ten positions, each located by a click mechanism, provide a decade of capacitance.

The design procedure is rather complicated, and full details are given in Griffiths's paper, Ref. (57). A typical three-dial air capacitor using this principle comprises two air decades and a continuously variable dial. The overall range is 0 — 1,100 $\mu\mu$F with an accuracy of $\pm$ 0·02 per cent or $\pm$ 0·02 $\mu\mu$F.

Self-contained Bridge Networks and Meters. It is of great convenience in a.c. bridge work to have some form of permanently connected bridge. Apart from the saving of time and labour in connecting up the bridge network, such a piece of apparatus, if properly designed, minimizes errors due to inductance and capacitance in the leads, and to leakage effects. A measurement can also be repeated if necessary, with the assurance that the distribution of the bridge will be the same as that employed in the previous measurement.

A number of self-contained a.c. bridges for the measurement of inductance and capacitance have been developed by various manufacturing firms. An interesting example is the "Mufer" capacitance bridge made by the Baldwin Instrument Company. This employs the De Sauty circuit (see page 233) the resistance arms R_1 and R_2 being in the form of a potential divider the setting of which, for balance, indicates the value of the capacitance under test by means of an attached pointer and scale. Two standard capacitors are incorporated to give two scales (0·00005 to 0·016 microfarad and 0·015 to 4 microfarads), and the instrument also contains its own oscillator of the neon-tube type, the only auxiliary apparatus required being a 120 volt dry battery and telephones. The makers claim an accuracy of within 2 per cent. While this may not be regarded as a precision instrument, it has the advantages of portability and great simplicity in use.

A Muirhead and Co. impedance bridge can be used for the measurement of resistance, inductance, capacitance, dissipation factor and Q-factor. It is completely self-contained, including a fixed-frequency oscillator and an amplifier preceding the telephones used as detector; operation is entirely from the a.c. supply mains. Ranges covered are—

Resistance, 0·001 ohm to 1 megohm (accuracy $\pm$ 1 per cent).
Inductance, 1 μH to 1,000 H (accuracy $\pm$ 1 per cent).
Capacitance, 1 $\mu\mu$F to 100 μF (accuracy $\pm$ 1 per cent).
Dissipation factor, 0 to 1·2 (accuracy $\pm$ 15 per cent or $\pm$ 0·005 whichever is the greater).
Q-factor, 0 to 60 on two ranges (accuracy $\pm$ 15 per cent).

Messrs. Evershed and Vignoles make a capacitance meter based, essentially, on the well-known Megger Insulation Tester (see p. 323). The capacitance meter, however, has an electromagnet, instead of a permanent magnet, and an a.c. generator. The armature has a constant-speed centrifugal clutch, and there are two independent windings displaced in phase relationship, the displacement being such that the current in the electromagnet lags 90° behind the voltage across the measuring circuit. Thus, with capacitors present, the currents through the control and deflecting coils are 180° in phase ahead of the field, and by an appropriate arrangement of the connections, they act as if in phase. If the capacitor has imperfect insulation this gives rise to a current component in quadrature with the flux in the electromagnet and hence this current develops no torque.

Another useful instrument is the Avo universal bridge (The Automatic Coil Winder and Electrical Equipment Co., Ltd.). This is a 50-c/s bridge, self-contained, and having 20 calibrated ranges. It covers resistance measurements, in 8 ranges, from 0·5 Ω to 50 MΩ, inductance from 50 mH to 500 H in four ranges and capacitance from 5 μμF to 50 μF in 8 ranges. The accuracy at mid-scale is of the general order of ± 2 per cent.

The Cintel wide-range capacitance bridge (Cinema-Television, Ltd.) is a self-contained equipment including its own oscillator (giving a fixed frequency of 1,592 c/s) and detector. It covers capacitances from 0·002 μμF to 100 μF in 18 steps and the accuracy is ± 1 per cent of full scale on all ranges. It can also be used for high-resistance measurements up to 30,000 MΩ. The balance indicator for the bridge is a dual Electron Ray tube, and a fine balance is obtainable by the slow-motion dials, which have a 50 : 1 step-down ratio. A mutual and self-inductance bridge by the same makers covers a range from 0·001 μH to 30 mH in 12 steps.

The Sullivan–Griffiths direct-reading inductance bridge has a range of 1 μH to 100 μH with an accuracy of 0·1 per cent. It has very small frequency and temperature errors and measures also capacitance from 0·0001 μF to 1 μF, resistance, and iron-cored or air-cored inductance without or with superposed d.c. up to 2 amperes flowing through the coil being measured. This bridge is based upon the use of a standard arm comprising an inductance standard of four decades tapped by rotary switches, which also maintain the resistance of the whole standard arm constant for all values of inductance.

Another Sullivan inductance bridge has been specially designed for the measurement of iron-cored inductances from 10 mH to 1,000 mH without or with superposed d.c. up to 2 amperes. It may be used also for the measurement of air- or dust-cored coils. The network is that of the Owen bridge (see p. 237), but several novel features have been introduced by W. H. F. Griffiths to make the bridge entirely direct-reading for both inductance and resistance.

The Sullivan–Griffiths precision decade capacitance bridge has a wide range (0·1 μμF to 100 μF) and a direct-reading accuracy of 0·01 μμF to 0·01 per cent. The accuracy of direct-reading of power-factor measurement is 0·0001, and both this and the capacitance accuracy are maintained up to high frequencies. The wide range with high accuracy is due to the use of the Griffiths Decade Air Standard of Capacitance described on p. 265.

A range of bridges based on the inductively-coupled ratios described on p. 242 is manufactured by Wayne Kerr Laboratories, Ltd.

Detectors. Electrodynamometer instruments have been used in a modified form as detectors in a.c. bridge measurements. Sumpner (Ref. (27)) introduced an electrodynamometer having an iron core giving very high sensitivity, and Weibel (Ref. (28)) describes several similar instruments designed for the same purpose.

The detectors in most common use for a.c. bridge measurements are, however, the telephone and the vibration galvanometer.

Telephones are widely used as detectors at frequencies of 500 c/s and over, up to 2,000 or 3,000 c/s, and are the most sensitive detectors available for such frequencies. The sensitivity of a telephone varies with the frequency of the supply, since the vibrating diaphragm which produces the sound has certain natural frequencies of vibration at which frequencies resonance is obtained, giving very high sensitivity. Wien (Ref. (29)), when investigating such resonance, found that, for a Bell telephone, resonance was obtained —with consequent highly increased sensitivity—at frequencies of 1,100, 2,800, and 6,500 c/s, and in the case of a Siemens telephone at frequencies of 720, 2,100, and 5,000 c/s.

The sensitivity of the observer's ear must also be taken into account when considering the sensitivity of the telephone as a detector. This varies with frequency. For most people a frequency of 800 c/s is a convenient one, since a note of this frequency is easily distinguished.

In selecting a telephone it is therefore best to choose one which has maximum sensitivity at the frequency at which it is to be used. The resistance of a telephone should match that of the bridge network. The range of resistances obtainable is roughly from 50 ohms to 7,000 or 8,000 ohms, a suitable telephone resistance for bridges of medium impedance being of the order of 200 ohms.

Transformers are sometimes used in conjunction with a low-resistance telephone when the bridge network is of high impedance. The telephone is connected to the transformer secondary (low-voltage side), the primary (high-voltage side) being connected to the branch points of the network to which the detector is usually connected. In this way the voltage applied to the telephone is stepped down and the current stepped up.

Tuned Detectors. To improve the sensitivity of a detector it may be tuned so that resonance—and therefore maximum amplitude of vibration for a given current—is obtained. Such tuned detectors also have the advantage that the response to frequencies other than the fundamental frequency of the supply is very small. Errors due to harmonics in the supply wave-form are thus minimized.

Campbell showed that an ordinary telephone can be tuned by means of a small screw pressing against the diaphragm at an eccentric point.

Amplifiers. Thermionic amplifiers are commonly used to increase the sensitivity in bridge measurements. Several forms are described by Hague (Ref. (1)).

An amplifier-detector made by H. W. Sullivan, Ltd., and operated from 200–250 V a.c. mains, is especially useful in preference to an amplifier and telephones, where measurements are being made under noisy conditions and when the frequency is above or below the audible range. In the input circuit there are two balanced and screened transformers, with different input impedances, either of which may be selected, so that the detector may be used with most a.c. bridges. The linear amplifier, covering a wide range of frequencies, is followed by a bridge-connected metal rectifier and moving-coil microammeter indicating the rectified current. The microammeter is shunted by a metal rectifier network, which causes the meter reading to be approximately proportional to the logarithm of the input voltage. This logarithmic law prevents damage to the instrument when the bridge is considerably out of balance, and simplifies the preliminary bridge-balancing procedure. The sensitivity is from 6 μV to 75 V over a frequency range of 40 c/s to 20 kc/s.

When an a.c. bridge whose balance condition is frequency-dependent is in use, there is usually a substantial unbalance voltage at harmonic frequencies when balance has been obtained at the fundamental frequency. An aperiodic bridge amplifier cannot be used satisfactorily under these conditions and a tuned amplifier is desirable. It is customary to use resistance-capacitance networks as the frequency-selective devices in bridge amplifiers. A three-terminal twin-T network of resistors and capacitors will reject a single frequency, and this frequency may be varied over a wide range by the use of triple-ganged continuously variable resistors. If such a network is shunted across a single valve stage in an amplifier, the stage gives gain only at the frequency to which the twin-T network is adjusted. A selective bridge amplifier using a similar principle has been described by G. H. Rayner (Ref. (58)).

Muirhead and Co. Ltd., make a frequency analyser which is suitable for use as a selective bridge amplifier. This device uses a resistance-capacitance network as the frequency selective element and is tunable in 6 ranges between 30 c/s and 30,000 c/s; an input of 5 μV can be detected.

Vibration Galvanometers are the most widely-used tuned detectors. They are manufactured for various frequencies from 5 cycles per second up to 1,000 cycles, but are most commonly used below 200 cycles per second over which range they are considerably more sensitive than the telephone.

Vibration galvanometers are of two types—

(*a*) Moving-magnet. (*b*) Moving-coil.

The latter type is the more generally used, the moving-magnet type having the disadvantage of being seriously affected by magnetic fields of the resonant frequency, unless adequately screened. The moving-coil galvanometers are not appreciably affected by such fields.

Moving-magnet Type. The galvanometers of this type consist of a suspended system which carries one or more small, permanent magnets, and a light mirror about 2 or 3 mm diameter. The magnets are suspended between the poles of a magnet which is, in some forms, a permanent one, and in others is an electromagnet energized by coils carrying the current to be measured or detected. In the former the current is passed through coils whose magnetic field causes the suspended magnets to oscillate, the permanent magnet acting as the control. The control in other forms is supplied by torsion of the suspension. Air friction is the chief source of damping.

The moving system is tuned to the supply frequency either by altering the tension and length of the suspension or by varying the strength of the permanent magnet field, if such a magnet is included in the instrument.

A beam of light is thrown upon the mirror, and when current is passing through the instrument, the moving system oscillates, producing a band of light on the scale. In adjusting a bridge network to give zero deflection of the galvanometer, this band of light must, of course, be reduced until it again becomes a single spot, of the same diameter as when the supply is switched off. Some practice is necessary in observing when this condition has been attained. It is usually best to switch the galvanometer in and out of circuit and to note if there is any observable difference in the size of the spot in the two cases.

Tuning. To tune the galvanometer, a small current of the supply frequency is passed through it, and the tuning adjustments (variation of the tension and length of suspension or otherwise) are continued until the reflected band of light reaches its maximum length.

It is often helpful, in tuning, to adopt some method such as the following: First vary the frequency of the supply until the instrument shows maximum deflection and note this frequency. Next adjust the galvanometer and again vary the supply frequency to give maximum deflection. Note this frequency and proceed thus by successive steps until maximum deflection is produced by a supply frequency equal to that at which the measurements are to be made.

When used in the bridge network, the galvanometer should be shunted by a variable resistance to protect it against excessive currents when the bridge is out of balance. The shunting can be removed in steps until balance is almost obtained, when the shunt may be entirely removed, so that maximum sensitivity is obtained.

Schering and Schmidt Galvanometer. In this instrument the suspension is a phosphor-bronze strip and carries a light piece of iron and a mirror. This moving system is enclosed in an ebonite tube which can be slipped in between the four poles of two U-shaped magnets as shown (Fig. 6.55). These magnets carry four magnetizing coils, connected in series, through which the alternating current is passed. The two U-shaped magnets themselves fit in between the two poles of another magnet excited by a winding which carries direct current. The resistance of this latter winding is about 20 ohms and it can be supplied from a 10 volt battery. It is for the purpose of polarizing the iron needle of the suspended system. Oscillation of the needle is produced by the distortion of the d.c. magnet field by the superposed alternating field.

The instrument is tuned by variation of the controlling magnetic field by adjustment of the current in the d.c. exciting winding. In vibration galvanometers generally, the smaller the damping, the sharper the resonance curve. If the supply frequency is not absolutely constant it may be convenient to make the tuning curve less sharp by increasing the damping. Provision for this is made, in this

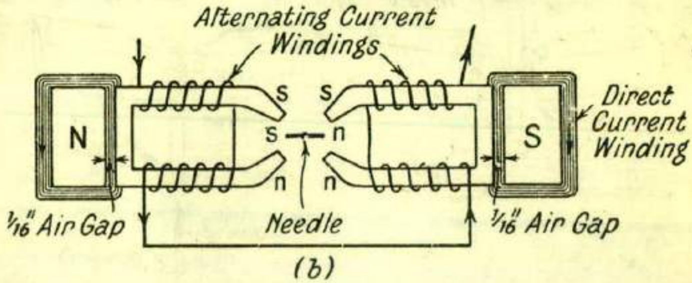


Fig. 6.55. Construction of Schering and Schmidt Galvanometer

instrument, by means of a small piece of copper, adjacent to the moving needle, the position of which can be adjusted by a screw in the suspension piece. Damping is effected due to eddy currents induced in the copper by the moving needle.

Various resistances of the coils carrying the alternating current can be used, a common value being 500 ohms. With a single moving system the frequency range of an instrument of this type is about 25 to 100 cycles per second. The sensitivity, as given by the makers, when a 500 ohm coil is used, varies from 90 mm per microampere at 25 cycles, to 25 mm per microampere at 70 cycles, the scale being distant 1 metre from the instrument.

The Schering instrument is largely used in capacitance bridges at high voltages, its advantages for such work being as follows—

1. High insulation between the alternating current system and the d.c. windings, owing to the fact that a $\frac{1}{16}$ in. air gap is left between the a.c. magnets and the control magnet.

2. It can be tuned from a distance by variation of the d.c. magnet exciting current.

3. The instrument has a very small self-capacitance.

Messrs. H. Tinsley and Co. make an instrument of this type.

Moving-coil Vibration Galvanometers. These galvanometers are of the d'Arsonval type, having a moving coil suspended between the poles of a strong, permanent magnet. The moving system is designed to have a very short, natural period of vibration; and the damping is very small, in order that the resonance curve shall be sharp—i.e. the deflection, for a given current passing through the instrument, is very much reduced by a small departure from the frequency to which the galvanometer is tuned. The alternating current to be detected is passed through the suspended coil, which consists of a

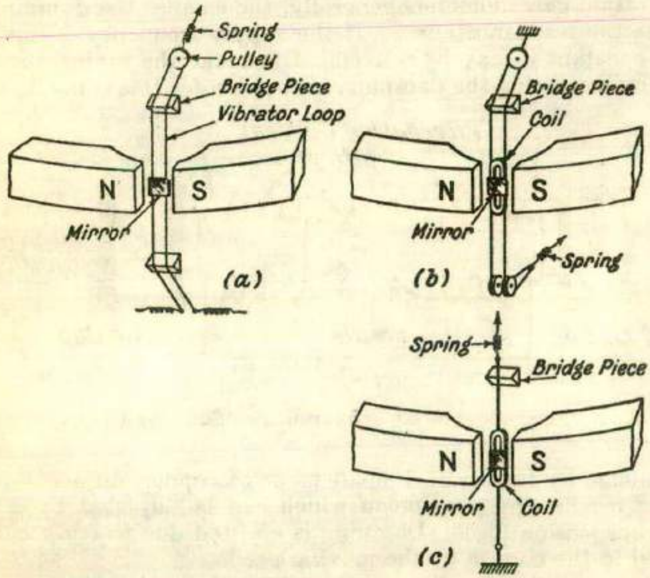


FIG. 6.56. CONSTRUCTION OF MOVING-COIL VIBRATION GALVANOMETERS

few turns—or often of only a single loop—of wire. The moving system carries a small mirror, upon which a beam of light is cast. The system vibrates when an alternating current is passed through the coil, the reflected beam of light from the mirror thus throwing a band of light upon the scale.

These galvanometers are tuned by adjusting the length and tension of the suspended system.

*Duddell Moving-coil Vibration Galvanometer.** In this instrument the moving coil consists of a single loop of fine bronze or platinum-silver wire, this wire passing over a small pulley at the top and being pulled tight by a spring attached to the pulley (Fig. 6.56 (*a*)). The tension of this spring can be adjusted for tuning purposes by

* See Ref. (25).

turning a milled head to which it is attached. The loop of wire is stretched over two ivory bridge pieces, the distance apart of these being adjustable in tuning the instrument. Variation of this distance apart obviously varies the length of the loop which is free to vibrate, and thus varies the natural period of the galvanometer. The galvanometer is roughly tuned by adjustment of the bridge pieces, fine adjustment of the tuning being obtained by varying the tension on the loop.

When a current passes through the loop, a couple, tending to turn the loop about its vertical axis, is produced. When the current reverses, this couple also reverses, thus causing oscillation of the loop when alternating current is passed through it.

This galvanometer can be used for frequencies between 100 and 1,800 cycles per second, the current sensitivity being about 50 mm per microampere, with a scale distance of 1 metre. The effective resistance is about 250 ohms. The sensitivity, if the loop is not too short, is almost inversely proportional to the frequency. In common with moving-coil vibration galvanometers generally, the instrument is not greatly affected by external magnetic fields. It has the disadvantage that the tuning can only be carried out by actually handling the instrument, and is, therefore not very convenient for use in high-voltage work.

Other Moving-coil Vibration Galvanometers. A. Campbell (Refs. (15), (31), (32), (33)) has developed other moving-coil vibration galvanometers. Fig. 6.56 (*b*) shows the construction of his long-range instrument, and Fig. 6.56 (*c*) his short-range pattern. The former instrument has a bifilar suspension carrying a very light coil and a small mirror. The length of the suspension is varied, for tuning purposes, by the movement of a bridge above the coil, and the tension by means of a spiral spring at the bottom of the suspension. The range of frequency covered by such an instrument is from 50 to 1,000 cycles per second, and the sensitivity at 50 cycles is of the order of 60 mm per microampere at 1 metre scale distance, with an effective resistance of about 500 ohms, this sensitivity falling off at the higher frequencies to less than 1 mm per microampere.

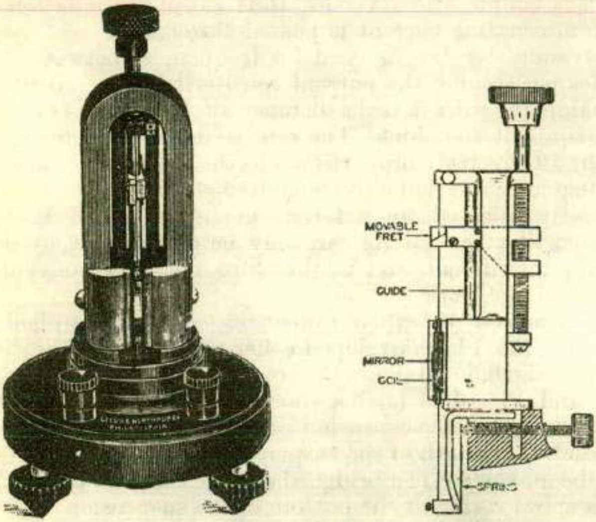
The short-range instrument has a single strip suspension. Tuning is carried out in a similar way to that of the long-range instrument. The frequency range is from 10 cycles to 400 cycles per second, and the sensitivity is very high at the lower frequencies (of the order of 400 mm per microampere at a scale distance of 1 metre when the frequency is 10 cycles per second).

Fig. 6.58 shows the construction and also a resonance curve of a Campbell moving-coil vibration galvanometer, manufactured by the Cambridge Instrument Co.

Fig. 6.57 gives details of the construction of a vibration galvanometer of a similar type, manufactured by the Leeds & Northrup Co. The screw adjustments for variation of the length of and tension on the suspension are clearly shown. The frequency range of this particular instrument is from 50 to 80 cycles per second, the sensitivity being stated by the makers as 40 mm per microampere at a scale distance of 1 metre and a frequency of 60 cycles per second, the resistance being 700 ohms.

To avoid the falling off in sensitivity with increase of frequency, several suspensions are usually provided for use with the same instrument at different frequencies.

Vibration galvanometers, generally, are susceptible to mechanical



(Leeds & Northrup Co.)

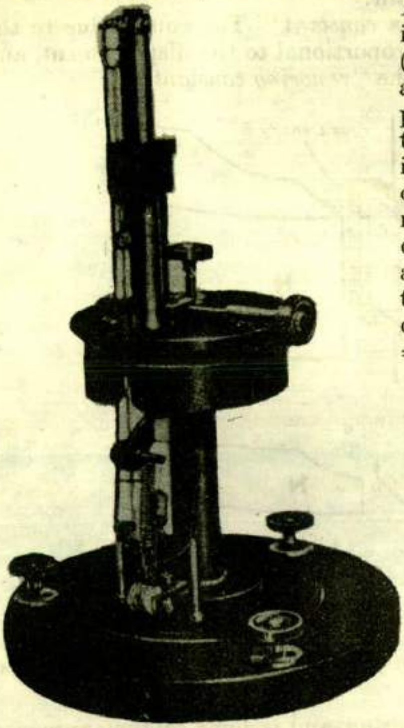
FIG. 6.57. LEEDS AND NORTHRUP MOVING-COIL VIBRATION GALVANOMETER

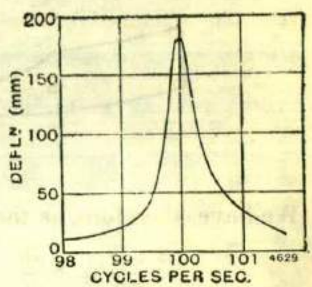
vibrations whose frequency is of the same order as that to which they are tuned. For this reason it is often necessary to provide some form of support which acts as a protection from such vibrations. One method of support which has been found to be satisfactory is to stand the instrument on rubber feet which rest on a heavy block of slate suspended from the ceiling by springs. Underneath the slate may be fitted damping vanes dipping into an oil dash-pot.

Theory of the Vibration Galvanometer with One Degree of Freedom. All the vibration galvanometers described above have only one degree of freedom—i.e. their suspended system only rotates about the axis of suspension. The theory of galvanometers with one degree of freedom was first given by Wenner (Ref. (30)), and the following is based upon his work on the subject.

Considering, first of all, the constants of the galvanometer considered—called by Wenner the "intrinsic constants"—we have

(*a*) The "*displacement constant.*" If the suspended coil is of length l metres (measured along the axis of the suspension), has a width r metres, and N turns, then the couple displacing the coil, when it carries i amperes, and is situated in a magnetic field of flux density B webers per square metre, is $NBilr \cos \theta$ newton-metres (see Fig. 6.59), where θ is the angle (in radians) between the plane of the coil and the direction of the magnetic field. If θ is small, $\cos \theta \fallingdotseq 1$, and the deflecting couple is $NBlir$ newton-metres. Assuming the coil to be rectangular, lr is the area of its plane. Let $lr = A$; then the expression for the couple may be written $NBAi = Gi$. The constant G is called





(*Cambridge Instrument Co., Ltd.*)

FIG. 6.58. CAMPBELL MOVING-COIL VIBRATION GALVANOMETER AND RESONANCE CURVE

the "displacement constant" of the galvanometer, and is equal to NBA.

(*b*) The "*constant of inertia.*" Of the three couples retarding the motion, one is dependent upon the moment of inertia of the suspended system and upon the angular acceleration of this system. This couple may be written $a \frac{d^2\theta}{dt^2}$, where a is the "constant of inertia" or moment of inertia of the system in kilogramme-metres.[2]

(*c*) The "*damping constant.*" Another couple retarding motion is that due to the damping effect of air friction and elastic hysteresis in the suspension. This is usually assumed to be proportional to the angular velocity, and may be written

$$b\,\frac{d\theta}{dt}$$

where b is the "damping constant."

(*d*) The "*control or restoration constant.*" The couple due to the elasticity of the suspension is proportional to the displacement, and may be written $c\theta$, where c is the "*restoring constant.*"

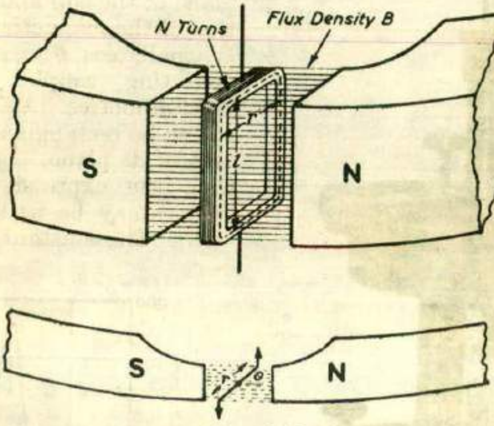


FIG. 6.59.

We have, therefore, as the equation of motion of the system,

$$a\,\frac{d^2\theta}{dt^2} + b\,\frac{d\theta}{dt} + c\theta = Gi \quad . \quad . \quad . \quad . \quad (6.44)$$

Now, if the current i is alternating, and is given by the expression $i = I_{max} \cos \omega t$,

we have $$a\,\frac{d^2\theta}{dt^2} + b\,\frac{d\theta}{dt} + c\theta = GI_{max} \cos \omega t \quad . \quad (6.45)$$

The solution of this differential equation will be in two parts. The expression for θ will be the sum of a Particular Integral and the Complementary Function. The complementary function—obtained by solving the equation

$$a\,\frac{d^2\theta}{dt^2} + b\,\frac{d\theta}{dt} + c\theta = 0$$

—will, in this case, give the expression for the angle θ when the current in the coil is zero—i.e. it will represent the natural free vibration of the coil. As will be shown later, this expression contains a factor of the form e^{-at} so that it represents a vibration of the coil which rapidly dies away when the current is switched on. This is, therefore, the transient part of the solution of the equation of motion.

The particular integral will give an expression for θ which represents the steady vibration of the coil after the current has been switched on for some appreciable time. Proceeding, then, to obtain the complementary function, we have

$$a\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + c\theta = 0$$

The "auxiliary equation" is $am^2 + bm + c = 0$ and the roots of this equation are

$$m_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

and $$m_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Thus $$\theta = Ae^{m_1 t} + Be^{m_2 t} \qquad (6.46)$$

where A and B are constants to be determined from the initial conditions.

In vibration galvanometers the damping is small, and b^2 is less than $4ac$. Thus m_1 and m_2 are imaginary, and may be written

$$m_1 = -k_1 + jk_2$$
$$m_2 = -k_1 - jk_2$$

where $j = \sqrt{-1}$, $k_1 = \frac{b}{2a}$, and $k_2 = \frac{\sqrt{4ac - b^2}}{2a}$

$$\therefore \theta = Ae^{(-k_1 + jk_2)t} + Be^{(-k_1 - jk_2)t}$$
$$= e^{-k_1 t}[Ae^{jk_2 t} + Be^{-jk_2 t}] \qquad (6.47)$$

Since, from trigonometry,

$$e^{jpx} = \cos px + j \sin px$$

and $$e^{-jpx} = \cos px - j \sin px$$

we have

$$\theta = e^{-k_1 t}[A(\cos k_2 t + j \sin k_2 t) + B(\cos k_2 t - j \sin k_2 t)]$$
$$= e^{-k_1 t}[(A + B)\cos k_2 t + j(A - B)\sin k_2 t]$$
$$= e^{-k_1 t}[P \cos k_2 t + Q \sin k_2 t]$$

where $P = A + B$ and $Q = j(A - B)$

i.e. $\theta = e^{-k_1 t}[F \sin (k_2 t + \alpha)]$

where $F = \sqrt{P^2 + Q^2}$ and $\alpha = \tan^{-1} \frac{P}{Q}$

Thus $$\theta = e^{-\frac{b}{2a}t}\left[F \sin\left(\frac{\sqrt{4ac - b^2}}{2a} t + \alpha\right)\right] \tag{6.48}$$

this being the transient portion of the solution, which rapidly decreases in value as t is increased. The constants F and α must be determined from the initial conditions—i.e. they depend upon the position of the coil at the instant corresponding to zero time.

In this expression $\frac{\sqrt{4ac - b^2}}{2a}$ is the angular velocity ω_1, and is equal to $2\pi \times$ the frequency of the vibratory motion.

Thus $$f = \frac{\sqrt{4ac - b^2}}{4\pi a}$$

where f is the "natural frequency" of the vibrating system.

If the damping is negligibly small, $b = 0$, and the "*undamped natural frequency*" is given by

$$f = \frac{\sqrt{4ac}}{4\pi a} = \frac{1}{2\pi}\sqrt{\frac{c}{a}}$$

and obviously depends upon the moment of inertia of the system and upon the controlling forces.

The instrument is critically damped—i.e. it will not vibrate freely —when $f = 0$. This condition is fulfilled when $4ac = b^2$, or when the damping constant $b = 2\sqrt{ac}$.

Proceeding to find the particular integral, we have

$$a\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + c\theta = GI_{max} \cos \omega t$$

or $$\frac{d^2\theta}{dt^2} + \frac{b}{a}\frac{d\theta}{dt} + \frac{c}{a}\theta = \frac{GI_{max}}{a} \cos \omega t$$

Employing the operator D, we have

$$(D^2 + hD + g)\theta = \frac{GI_{max}}{a} \cos \omega t$$

where $h = \frac{b}{a}$ and $g = \frac{c}{a}$

$$\therefore \theta = \frac{\frac{GI_{max}}{a} \cos \omega t}{(D^2 + hD + g)}$$

Multiplying numerator and denominator by $(D^2 - hD + g)$ gives

$$\theta = \frac{\frac{GI_{max}}{a}(D^2 - hD + g)\cos\omega t}{(D^2 + g)^2 - h^2D^2}$$

$$= \frac{\frac{GI_{max}}{a}}{(D^2 + g)^2 - h^2D^2}[-\omega^2\cos\omega t + h\omega\sin\omega t + g\cos\omega t]$$

$$= \frac{GI_{max}}{a}(g - \omega^2)\frac{1}{(D^2 + g)^2 - h^2D^2}\cos\omega t + \frac{GI_{max}}{a}h\omega\frac{1}{(D^2 + g)^2 - h^2D^2}\sin\omega t$$

$$= \frac{\frac{GI_{max}}{a}(g - \omega^2)\cos\omega t}{(g - \omega^2)^2 + h^2\omega^2} + \frac{\frac{GI_{max}}{a}h\omega\sin\omega t}{(g - \omega^2)^2 + h^2\omega^2}$$

Substituting for h and g, we have

$$\theta = \frac{\frac{GI_{max}}{a}\left[\left(\frac{c}{a} - \omega^2\right)\cos\omega t + \frac{b}{a}\omega\sin\omega t\right]}{\left(\frac{c}{a} - \omega^2\right)^2 + \frac{b^2}{a^2}\omega^2}$$

$$= \frac{GI_{max}[(c - \omega^2a)\cos\omega t + b\omega\sin\omega t]}{(c - a\omega^2)^2 + b^2\omega^2}$$

$$= \frac{GI_{max}\sqrt{(c - a\omega^2)^2 + b^2\omega^2}\left[\frac{c - \omega^2a}{\sqrt{(c - a\omega^2)^2 + b^2\omega^2}}\cos\omega t + \frac{b\omega}{\sqrt{(c - a\omega^2)^2 + b^2\omega^2}}\sin\omega t\right]}{(c - a\omega^2)^2 + b^2\omega^2}$$

or

$$\theta = \frac{GI_{max}}{\sqrt{(c - a\omega^2)^2 + b^2\omega^2}}[\cos(\omega t - \beta)] \tag{6.49}$$

where $\beta = \tan^{-1}\frac{b\omega}{c - a\omega^2}$

This obviously represents a steady vibratory motion of amplitude $\frac{GI_{max}}{\sqrt{(c - a\omega^2)^2 + b^2\omega^2}}$ and of frequency $\frac{\omega}{2\pi}$

The complete solution for θ, being the sum of this expression and the expression derived previously as the complementary function, is thus

$$\theta = e^{-\frac{b}{2a}\cdot t}\left[F \sin\left(\frac{\sqrt{4ac - b^2}}{2a}t + \alpha\right)\right] + \frac{GI_{max}}{\sqrt{(c - a\omega^2)^2 + b^2\omega^2}} \cos(\omega t - \beta)] \quad . \quad (6.50)$$

Since the first expression is a transient which usually affects only

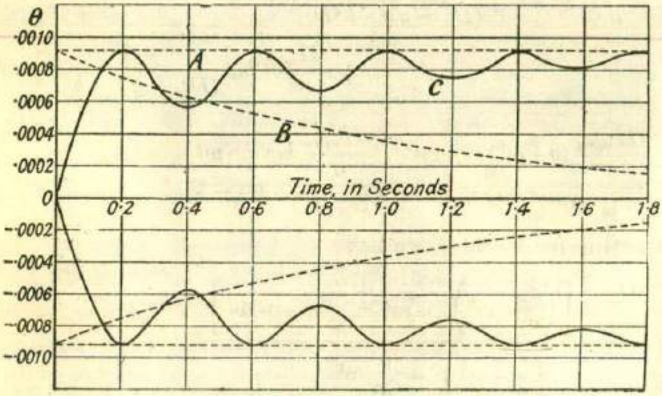


Fig. 6.60

the first few vibrations after switching on, we may neglect it and take as the law of the displacement simply

$$\theta = \frac{GI_{max}}{\sqrt{(c - a\omega^2)^2 + b^2\omega^2}} \cos(\omega t - \beta) \qquad (6.51)$$

Example. Fig. 6.60 shows how the transient term, at the beginning of the vibration period, produces an unsteady state which gradually disappears, giving, finally, a steady vibration.

The curves shown are based on data provided by Campbell* for a vibration galvanometer of the moving-coil type developed by him.

The data are as follows—

Number of turns	40
Mean area of turn. . . .	7×10^{-6} m²
Flux density in gap . . .	0·27 Wb/m²
Effective resistance . . .	1,540 Ω
Resonance frequency . . .	100 c/s
Inertia constant a. . . .	26×10^{-13}
Damping constant b . . .	49×10^{-13}
Restoring constant c . . .	$10{\cdot}4 \times 10^{-7}$
G	$8{\cdot}6 \times 10^{-5}$

* *Dictionary of Applied Physics*, Vol. II, p. 974.

Thus, in the expression for the deflection θ as derived above,

$$\frac{b}{2a} = 0{\cdot}942$$

$$\omega_1 = \frac{\sqrt{4ac - b^2}}{2a} = 632$$

(This expression is equal to $\sqrt{\frac{c}{a}}$ to a very close approximation, and should therefore equal 628 when the instrument is tuned (see page 282), slight errors in the values of the constants probably being responsible for the discrepancy.

$$\beta = 13^\circ\ 11'$$
$$\omega = 628\ (= 2\pi \times 100)$$

When the galvanometer is supplied with a small current at a frequency near resonance, the amplitude of the steady deflection given by

$$\frac{GI_{max}}{\sqrt{(c - a\omega^2)^2 + b^2\omega^2}}$$

is found to be 0·000917.

The value of $\frac{b}{2a}$ being somewhat small, the transient term persists for an appreciable time, taking 2 sec to fall to 0·153 of its initial amplitude. For this reason it is impossible to show, in the figure, the full curves for the two terms in the expression for the deflection θ. Lines passing through successive maximum points of these curves are shown instead. The dotted lines A are the lines passing through the maximum points of the steady deflection curve given by $\theta_1 = 0{\cdot}000917 \cos(\omega t - \beta)$, while the dotted curves B pass through the maximum points on the curve $\theta_2 = e^{-0{\cdot}942t}[F \sin(\omega_1 t + \alpha_1)]$. The full-line curves C pass through the maximum points of the total deflection curve (given by the summation of curves θ_1 and θ_2).

The effect of the transient term is shown by the "beat" effect which gradually dies away as the transient terms disappear. In the figure these beats have been drawn approximately owing to the difficulty of showing the full curves with a scale which is, necessarily, very cramped.

Tuning. In tuning the galvanometer, the object is to make the amplitude $\frac{GI_{max}}{\sqrt{(c - a\omega^2)^2 + b^2\omega^2}}$ as great as possible for a given current I_{max}, which means that $\frac{G}{\sqrt{(c - a\omega^2)^2 + b^2\omega^2}}$ must be made as great as possible. This expression can be increased by increasing the numerator G and by reducing the denominator $\sqrt{(c - a\omega^2)^2 + b^2\omega^2}$.

Since $G = NBA$, it may be made large by using a coil of large area A and with a large number of turns N. G is obviously increased also by increasing the flux density B of the magnetic field in which the coil lies. This latter method is the more important, since increasing the area and number of turns on the coil will increase its moment of inertia so that a will be increased and, thereby, the denominator may be increased.

Considering the denominator: of the three constants a, b, and c contained in it, c is the only one which can usually be varied. The constant c is the control constant and is varied by adjusting the length and tension of the suspension of the moving system, or by variation of the polarizing field of the galvanometer in the case of moving-magnet instruments.

If the supply frequency is fixed—as it usually is in bridge measurements—the tuning process consists of varying c until $c - a\omega^2$ is zero, thus making the denominator of the amplitude expression a minimum. Since $\omega = 2\pi \times$ the supply frequency, we have the condition $c - a(2\pi f)^2 = 0$ ($f =$ supply frequency), which must be satisfied in tuning the instrument.

Thus, c must equal $a(2\pi f)^2$ or $f = \frac{1}{2\pi}\sqrt{\frac{c}{a}}$

It should be noted that this expression for the supply frequency f, in terms of c and a, is the same as the expression for the frequency of the undamped vibration of the galvanometer (see page 278). This means that resonance occurs when the supply frequency is equal to the undamped natural frequency of the galvanometer. The amplitude under resonance conditions is obviously $\frac{GI_{max}}{b\omega}$.

Consider the case of the vibration galvanometer whose constants have already been given.

The constants are—

$$a = 26 \times 10^{-13}$$
$$b = 49 \times 10^{-13}$$
$$c = 10{\cdot}4 \times 10^{-7}$$

and the resonance frequency is given as 100 cycles per second. More exactly the frequency for resonance is $\frac{1}{2\pi}\sqrt{\frac{10{\cdot}4 \times 10^6}{26}}$ or 100·7 cycles per second. The deflection at resonance is

$$\frac{GI_{max}}{b\omega} = \frac{GI_{max}}{\frac{49}{10^{13}} \times 2\pi \times 100{\cdot}7} = GI_{max} \times 32{\cdot}27 \times 10^7$$

The deflection at other frequencies is calculated from the expression $\frac{GI_{max}}{\sqrt{(c - a\omega^2)^2 + b^2\omega^2}}$ since $(c - a\omega^2)$ is zero only at the resonance frequency. A table showing the deflection for a range of frequency from 98·5 to 102 c/s is given opposite, and these values are plotted in Fig. 6.61. The sharpness of the resonance curve of a vibration galvanometer is well illustrated in the curve obtained.

To compare the response of the galvanometer to harmonics in the supply waveform, consider a third harmonic when the supply frequency is that to which the galvanometer is tuned—namely,

TABLE IX

Frequency	ω	Deflection $\times 10^{-7}$
98·5	618·8	$GI_{max} \times$ 2·27
99	622	,, $\times$ 2·93
99·5	625·1	,, $\times$ 4·21
100	628·4	,, $\times$ 7·63
100·5	631·4	,, $\times$ 22·3
100·7	632·6	,, $\times$ 32·3
101	634·6	,, $\times$ 13
101·5	637·7	,, $\times$ 5·78
102	640·9	,, $\times$ 3·57

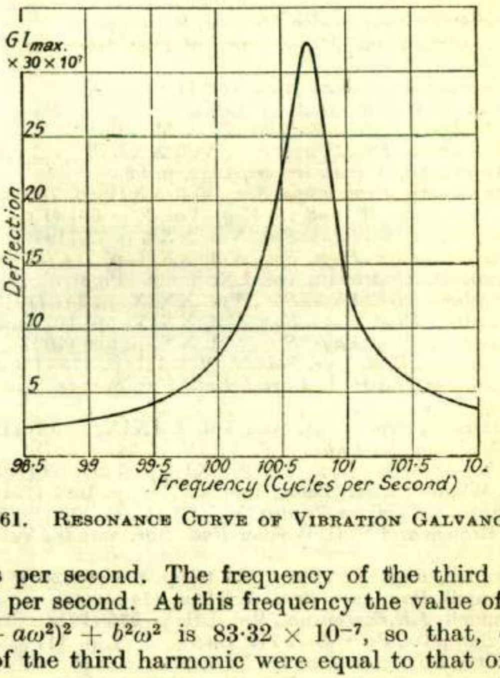


FIG. 6.61. RESONANCE CURVE OF VIBRATION GALVANOMETER

100·7 cycles per second. The frequency of the third harmonic is 302·1 cycles per second. At this frequency the value of the expression $\sqrt{(c - a\omega^2)^2 + b^2\omega^2}$ is $83{\cdot}32 \times 10^{-7}$, so that, even if the amplitude of the third harmonic were equal to that of the fundamental, the amplitude of the deflection would be only $\dfrac{GI_{max} \times 10^7}{83{\cdot}32}$ $= 0{\cdot}012 GI_{max} \times 10^7$. Thus the sensitivity to the fundamental compared with the sensitivity to the third harmonic is $\dfrac{32{\cdot}3}{0{\cdot}012} = 2{,}690$, showing that an entirely negligible error is introduced by the fact that the supply waveform contains harmonics.

The above theory assumes a current, $i = I_{max} \cos \omega t$, flowing through the galvanometer, and therefore refers to the "*current sensitivity*" of such instruments.

The "*voltage sensitivity*" may be determined by considering the case of a given voltage applied to the instrument terminals. In this consideration the voltage *induced* in the coil owing to the fact that, while vibrating, it is cutting through the magnetic field of flux density B must be taken into account. The reader is referred to Hague's *Alternating Current Bridge Methods*, 4th Edition, p. 277, or to the *Dictionary of Applied Physics*, Vol. II, p. 971, for the full theory in this case.

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(53) "Further Notes on the Calibration Permanence and Overall Accuracy of the Series-Gap Precision Variable Air Condenser," W. H. F. Griffiths, *The Wireless Engineer*, Vol. VI, No. 64, pp. 23–30, and No. 65, pp. 77–80.

(54) "Electrical Standards for Research and Industry," a catalogue published by Messrs. H. W. Sullivan, Ltd. in 1959.

(55) "Nickel–Chromium–Aluminium–Copper Resistance Wire," A. H. M. Arnold, *Proc. I.E.E.*, Vol. 103 B, p. 439.

(56) "Radio-frequency Bridges," H. L. Kirke, *Jour. I.E.E.*; Vol. 92, Part III, p. 2 (1945).

(57) "Decade Air Condenser," W. H. F. Griffiths, *Engineer*, Vol. CCII, p. 691 and p. 728 (1956).

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CHAPTER VII

MEASUREMENT OF RESISTANCE

FROM the point of view of measurement, resistances can be classified generally as follows—

(*a*) LOW RESISTANCES. All resistances of the order of 1 ohm and under may be classified thus. In practice such resistances may be met with in the armatures and series windings of large machines, in ammeter shunts, cable lengths, contacts, etc.

(*b*) MEDIUM RESISTANCES. This class includes resistances from about 1 ohm upwards to about 100,000 ohms. In practice the majority of the pieces of electrical apparatus used have resistances which lie between these limits.

(*c*) HIGH RESISTANCES. Resistances of 100,000 ohms and upwards must be so classified.

A classification such as the above is not rigid, but forms a guide as to the method of measurement to be adopted in any particular case.

Measurement of Low Resistance. Methods of measurement which are suitable for medium resistances are in most cases unsuitable for low resistance measurements, chiefly because contact resistances cause serious errors. It is clear that contact resistances of the order (say) of 0·001 ohm—negligible though they may be when a resistance of 100 or more ohms is to be measured—are of great importance when the resistance to be measured is of the order of 0·01 ohm.

Again, it is usually essential, with low resistances, that the two points between which the resistance is to be measured shall be very precisely defined. Thus the methods which are specially adapted to low resistance measurement employ *potential connections* —i.e. connecting leads which form no part of the circuit whose resistance is to be measured, but which connect two points, in this circuit, to the measuring circuit. These two points are spoken of as the *potential terminals*, and serve to fix, definitely, the length of the circuit under test. In the methods used for the precise measurement of low resistance, the "unknown" resistance is compared with a low-resistance standard of the same order as the unknown, and with which it is connected in series. Both resistances are fitted with four terminals—two "current terminals," to be connected to the supply circuit, and two "potential terminals" to be connected to the measuring circuit. This arrangement is shown in Fig. 7.1.

AMMETER AND VOLTMETER METHOD. This method, which is the simplest of all, is in very common use for the measurement of low resistances when an accuracy of the order of 1 per cent is sufficient.

It must be realized, however, that it is, essentially, a comparatively rough method, the accuracy being limited by those of the ammeter and voltmeter used, even if corrections are made for the "shunting" effect of the voltmeter. In Fig. 7.2, R is the resistance to be measured

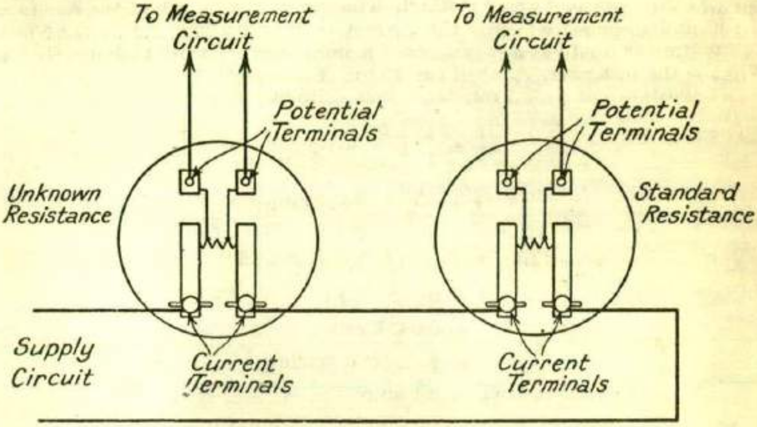


FIG. 7.1. MEASUREMENT OF LOW RESISTANCE

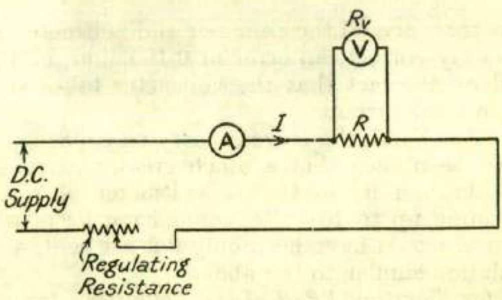


FIG. 7.2. AMMETER AND VOLTMETER METHOD OF RESISTANCE MEASUREMENT

and V is a high-resistance voltmeter of resistance R_v. A current from a steady direct-current supply is passed through R in series with a suitable ammeter.

Then, assuming the current through the unknown resistance to be the same as that measured by the ammeter A, the former is given by

$$R = \frac{\text{voltmeter reading}}{\text{ammeter reading}}$$

If the voltmeter resistance is not very large compared with the resistance to be measured, the voltmeter current will be an appreciable fraction of the current I, measured by the ammeter, and a serious error may be introduced on this account.

Example. A resistance whose actual value is 1 ohm, is to be measured by the ammeter and voltmeter method. The carrying capacity of the resistance is 100 milliamperes, which is the current used in making the measurement. The voltmeter used has a resistance of 5 ohms, and reads up to 100 millivolts. What is the measured value of the 1 ohm resistance?

Let resistance of 1 ohm resistance and voltmeter in parallel $= r$.

Then
$$\frac{1}{r} = \frac{1}{1} + \frac{1}{5} = 1{\cdot}2$$
$$r = \frac{1}{1{\cdot}2} = 0{\cdot}833 \text{ ohm}$$

Voltage drop across the resistance to be measured

$$= 0{\cdot}833 \times 0{\cdot}1$$
$$= 0{\cdot}0833 \text{ volt}$$
$$= \text{voltmeter reading}$$

$$\text{Ammeter reading} = 0{\cdot}1 \text{ amp}$$

$$\text{Thus the measured value of the resistance} = \frac{0{\cdot}0833}{0{\cdot}1} = 0{\cdot}833 \text{ ohm}$$

This means that, even if the ammeter and voltmeter give readings which are exactly correct, an error of 0·166 ohm, or 16·6 per cent, is introduced by the fact that the voltmeter takes an appreciable fraction of the total current.

If, on the other hand, the current-carrying capacity of the 1 ohm resistance had been such that a much greater current could have been passed through it, so that a voltmeter of resistance (say) 500 ohms, reading up to 10 volts, could have been used, then the error introduced would have been only 0·2 per cent, as can be seen from a calculation similar to the above.

Correction for Shunting Effect of the Voltmeter. In general terms, if the actual value of the unknown resistance is R, and its measured value R_m, the voltmeter resistance being R_v, and the ammeter current I, we have—

$$\text{Resistance of voltmeter and } R \text{ in parallel} = \frac{RR_v}{R + R_v}$$

$$\text{Voltage drop across } R = \frac{RR_v I}{R + R_v}$$

$$= \text{voltmeter reading}$$

(assuming the voltmeter to read correctly).

Thus, upon the assumption that the ammeter reading also is exactly correct

$$R_m = \frac{RR_v I}{(R + R_v)I} = \frac{RR_v}{R + R_v}$$

or

$$R = \frac{R_m R_v}{R_v - R_m} \quad . \quad . \quad . \quad . \qquad (7.1)$$

This method is useful in practice in the measurement of such resistances as those armatures, and of joints and contacts when the current-carrying capacity is fairly great and when the results are only required to within the limits of accuracy of the ammeter and voltmeter used.

Potentiometer Method. In the potentiometer method of measuring a low resistance the unknown resistance is compared with a standard resistance of the same order of magnitude.

These standard low resistances are of the type described in Chapter II. The following table gives the resistances and current-carrying capacities of some of a range of standards as manufactured by Messrs. H. Tinsley and Co.

TABLE X

Resistance (ohms)	Current-carrying capacity (amp)
10	1
5	1·4
2	2·2
1	3
0·5	4·5
0·1	22
0·01	150
0·001	700
0·0005	1000
0·0001	2250

The standards are adjusted to within 0·03 per cent of their nominal resistance (0·05 per cent for the last three).

The complete range of these resistance standards is from 10,000 Ω down to 0·0001 Ω. They are suitable for use at frequencies up to 1,000 c/s and their time constants for frequencies up to this value are 1×10^{-6} down to the 0·01 standard. For this and lower resistances the time constant is 3×10^{-6}. The ratings in watts are 10 W over the range 10,000 Ω to 0·2 Ω; 50 W between 0·2 Ω and 0·02 Ω; 200 W between 0·02 Ω and 0·005 Ω; and 500 W between 0·001 Ω and 0·0001 Ω. Below 0·02 Ω water cooling, with a motor-driven stirrer, is used.

The unknown resistance and a standard of the same order of resistance are connected in series as in Fig. 7.1. A steady current is passed through them from a battery of the heavy-current type. The magnitude of this current should be chosen so that a voltage drop of the order of 1 volt is obtained, if possible, across each of the resistances.

The voltage drops across both the unknown resistance and the standard are then measured on the potentiometer (see Chapter VIII), several measurements being made, alternately, and with as small a time interval as possible between the measurements. The mean values of these are taken as the correct voltage drops across the two resistances. By carrying out the measurements in this way the error due to possible variation of the supply current is minimized.

The potential leads to the potentiometer carry no current when the potentiometer is balanced, and thus the current through the two resistances is the same. Then

$$\frac{\text{Resistance of the unknown}}{\text{Resistance of the standard}} = \frac{\text{Voltage drop across the unknown}}{\text{Voltage drop across the standard}}$$

from which the resistance of the unknown is obtained in terms of that of the standard resistance.

Precautions. When used for precise work, resistance standards should be frequently checked against National Physical Laboratory standards, and the most recent calibration of the resistor should be used in calculating the resistance of the unknown. If the standard resistance is subject to appreciable variation with time, it may be necessary to estimate its probable variation from the time of the last calibration, on the assumption that its rate of variation is uniform, and the same as that between the dates of the two preceding calibrations. A standard resistor in which such variation with time is large is, of course, useless for precise work. The temperature of the two resistors, during the test, should be measured, and the resistance of the standard, at the measured temperature, should be obtained from its resistance/temperature curve. The measured resistance of the unknown is that at the measured temperature, and this should be stated, as its temperature coefficient may be large, and hence its resistance may be appreciably different at other temperatures.

In precise work also, a second measurement should be made with the supply reversed (care must be taken to reverse the potential leads to the potentiometer at the same time), and thermo-electric effects should be taken into account as described in the next chapter.

When the necessary precautions are taken, and a good potentiometer and sensitive galvanometer are used, the accuracy obtainable by this method may be within a few parts in 100,000, or within 1 part in 10,000.

KELVIN DOUBLE BRIDGE. This method is one of the best available for the precise measurement of low resistances. It is a development of the Wheatstone bridge by which the errors due to contact and leads resistances are eliminated. The connections of the bridge are shown in Fig. 7.3.

In the figure, X is the low resistance to be measured, and S is a standard resistance of the same order of magnitude. These are connected in series with a low-resistance link r, connecting their adjacent current terminals. A current is passed through them from a battery supply. A regulating resistance and ammeter are connected in the circuit for convenience. Q, M, q, and m are four known, non-inductive resistances, one pair of which (M and m, or Q and q) are variable. These are connected to form two sets of

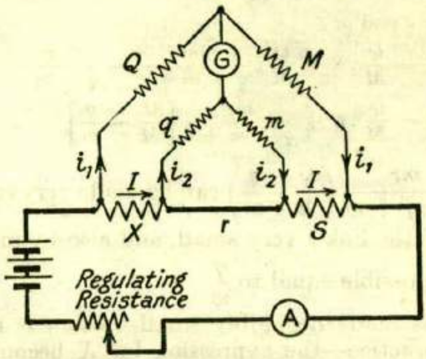


FIG. 7.3 KELVIN DOUBLE BRIDGE METHOD OF MEASURING LOW RESISTANCE

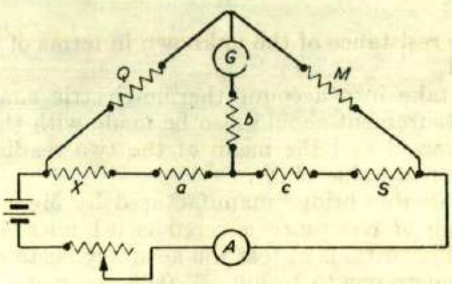


FIG. 7.4. TRANSFORMED KELVIN BRIDGE

ratio arms as shown, a sensitive galvanometer G connecting the dividing points of QM and qm. The ratio $\frac{Q}{M}$ is kept the same as $\frac{q}{m}$, these ratios being varied until zero deflection of the galvanometer is obtained. Then $\frac{X}{S} = \frac{Q}{M} = \frac{q}{m}$, from which X is obtained in terms of S, Q, and M.

Theory. Transform the delta arrangement of resistances q, m and r into an equivalent star (see page 130). The transformed circuit is given in Fig. 7.4, where

$$a = \frac{qr}{q + m + r}, \quad b = \frac{qm}{q + m + r}, \quad c = \frac{mr}{q + m + r}$$

At balance there is no current in the galvanometer so that the resistance b does not enter the balance equation, which becomes that of a Wheatstone bridge. Therefore

$$\frac{Q}{M} = \cdot \frac{X + a}{S + c}$$

or

$$X = \frac{Q}{M} \cdot (S + c) - a$$

Substituting for c and a,

$$X = \frac{Q}{M} \cdot S + \frac{Q}{M} \cdot \frac{mr}{q + m + r} - \frac{qr}{q + m + r}$$

$$= \frac{Q}{M} \cdot S + \frac{mr}{q + m + r}\left[\frac{Q}{M} - \frac{q}{m}\right] \quad . \quad . \quad . \quad (7.2)$$

The term $\frac{mr}{r + q + m}\left(\frac{Q}{M} - \frac{q}{m}\right)$ can be made very small by making the resistance of the link r very small, and also by making the ratio $\frac{Q}{M}$ as nearly as possible equal to $\frac{q}{m}$.

If this term is made negligibly small—which is not difficult to accomplish in practice—the expression for X becomes simply

$$X = \frac{Q}{M} \cdot S$$

which gives the resistance of the unknown in terms of the resistance of the standard.

In order to take into account thermo-electric e.m.f.s (see next chapter), a measurement should also be made with the direction of the current reversed and the mean of the two readings should be taken as the correct value of X.

In a Kelvin double bridge manufactured by Messrs. H. Tinsley Co. the range of resistance covered is 0·1 microhm to 1 ohm. Under specified conditions of test the accuracy is stated as

From 1,000 microhms to 1 ohm . . . 0·05 per cent.

From 100 microhms to 1,000 microhms . . . 0·2 per cent to 0·05 per cent.

From 10 microhms to 100 microhms . . . 0·5 per cent to 0·2 per cent, limited by thermo-e.m.f.s.

There are four internal resistance standards of 1 Ω, 0·1 Ω, 0·01 Ω and 0·001 Ω respectively.

H. W. Sullivan, Ltd., make a precision Kelvin and Wheatstone bridge covering a range of 1 microhm to 1 megohm.

Operation of Kelvin Double Bridge in Practice. The method of operation of the Kelvin double bridge in practice is often somewhat different from that described above, especially when precise measurements of low resistance are to be made.

Instead of varying the ratio arms, keeping the ratio $\frac{Q}{M}$ equal to

$\frac{q}{m}$, to obtain balance of the bridge, the resistances Q, M, q, and m are often made up of resistance coils whose resistances are fixed and are accurately known, together with their temperature coefficients. The ratios $\frac{Q}{M}$ and $\frac{q}{m}$ thus remain fixed during the test, and are made equal to one another, and roughly equal to the ratio $\frac{X}{S}$ assuming this to be known, approximately. If not known, the ratio can easily be determined approximately by measuring X first of all, using a less accurate method such as the ammeter and voltmeter method, or, better, by the potentiometer method.

Adjustment of the bridge to obtain balance is then carried out by shunting either the unknown resistance or the standard by a variable resistance, such as a resistance box. Assuming balance to be obtained by shunting the unknown by a resistance x, let the resistance of X and x in parallel, at balance, be X'. then

$$X' = \frac{Q}{M} S$$

and also

$$\frac{1}{X'} = \frac{1}{X} + \frac{1}{x}$$

from which the value of X can be obtained. As before, measurements are made also with the supply current reversed, and the average value of the two results is taken as the final value. The sensitivity of the bridge can be determined by noting the smallest variation of the shunting resistance x which produces an observable deflection of the galvanometer. The difference in X' for such a variation of x can then easily be calculated, thus giving the sensitivity of the bridge. This method of obtaining fine adjustment, by shunting a low resistance by a resistance box of much greater resistance, will be found a very useful one in electrical measurements generally. It has the advantage, also, that the resistances of the coils in the resistance box need not be known to within any high degree of accuracy, since slight errors in their values introduce negligible errors in the resistance of the combination.

The ratios $\frac{Q}{M}$ and $\frac{q}{m}$ may be made exactly equal by adjustment of the resistances of the leads to be used in connecting up the bridge, since these lead resistances are obviously included in the arms as well as the resistances of the coils themselves. By suitably proportioning the resistances of these copper leads the ratios $\frac{Q}{M}$ and $\frac{q}{m}$ can also be made independent of temperature—if this is necessary —to a very close approximation. The resistances of the leads need only be known with an accuracy of (say) 1 per cent, since they are usually small compared with those of the coils which they connect.

Example.

Resistance of link r = 0·0001 ohm

Resistance of coil in arm Q = $10{\cdot}0027_0$ at 20° C.
(Temperature coefficient 0·00003)

Resistance of coil in arm M = $20{\cdot}0142_5$ at 20° C.
(Temperature coefficient 0·00002)

Resistance of coil in arm q = $10{\cdot}0027_1$ at 20° C.
(Temperature coefficient 0·00003)

Resistance of coil in arm m = $20{\cdot}0067_0$ at 20° C.
(Temperature coefficient 0·000025)

Resistances of copper leads—

In arm Q = 0·0146 at 20° C.
" " M = 0·0660 " "
" " q = $0{\cdot}0061_5$ " "
" " m = 0·0122 " "
} Temperature coefficiont 0·0043

Resistance of standard = $0{\cdot}0100120_2$ at 20° C.
Resistance of "unknown" = 0·005 (about)

To balance the bridge the standard resistance is shunted by a resistance or 18·1 ohms, measurement being made at 20° C.

First, the values of the two ratios $\frac{Q}{M}$ and $\frac{q}{m}$ are calculated for a temperature of 20° C.

Then
$$\frac{Q}{M} = \frac{10{\cdot}0027_0 + 0{\cdot}0146}{20{\cdot}0142_0 + 0{\cdot}0660} = 0{\cdot}49886_5$$
$$\frac{q}{m} = \frac{10{\cdot}0027_1 + 0{\cdot}0061_5}{20{\cdot}0067_0 + 0{\cdot}0122} = 0{\cdot}499970$$

To make these ratios more nearly equal the coil in arm q is shunted by some resistance y, the value of which is found as follows.

Let q' be the shunted value of this coil, then
$$\frac{q' + 0{\cdot}0061_5}{20{\cdot}0189_0} = 0{\cdot}49886_6$$
$$q' = 9{\cdot}9805_8$$

Now,
$$\frac{1}{q'} = \frac{1}{y} + \frac{1}{10{\cdot}0027_1}$$

from which
$$y = 3{,}532 \text{ ohms}$$

Since the two ratios have thus been made exactly equal at 20° C, if the measurement is made at 20° C the value of the unknown resistance is given simply by
$$X = \frac{Q}{M} \cdot S'$$

where S' is the shunted value of the standard. Since the shunt for balance is 18·1 ohms,
$$\frac{1}{S'} = \frac{1}{0{\cdot}0100120_2} + \frac{1}{18{\cdot}1}$$

from which
$$S' = 0{\cdot}010006_5$$
$$\therefore X = 0{\cdot}0.0006_5 \times 0{\cdot}49886_6$$
$$= 0{\cdot}0049919_0 \text{ at } 20° \text{ C.}$$

Consider, now, the effect upon the ratios $\frac{Q}{M}$ and $\frac{q}{m}$ of a rise in temperature of 5° C.

$$\text{Then } \frac{Q}{M} = \frac{10{\cdot}0027_0[1 + 0{\cdot}00003 \times 5] + 0{\cdot}0146[1 + 0{\cdot}0043 \times 5]}{20{\cdot}0142_4[1 + 0{\cdot}00002 \times 5] + 0{\cdot}0660[1 + 0{\cdot}0043 \times 5]}$$

$$= 0{\cdot}49886_8$$

$$\text{and } \frac{q}{m} = \frac{9{\cdot}9805_8[1 + 0{\cdot}00003 \times 5] + 0{\cdot}0061_5[1 + 0{\cdot}0043 \times 5]}{20{\cdot}0067_0[1 + 0{\cdot}00002_5 \times 5] + 0{\cdot}0122[1 + 0{\cdot}0043 \times 5]}$$

$$= 0{\cdot}49887_8$$

Then $\frac{Q}{M} - \frac{q}{m} = -\,0{\cdot}000010$

and the correction term $\frac{mr}{r + q + m}\left(\frac{Q}{M} - \frac{q}{m}\right)$ is equal to

$$-\,\frac{0{\cdot}0001 \times 20{\cdot}0067_0}{10{\cdot}0027_1 + 20{\cdot}0067_0 + 0{\cdot}0001} \times 0{\cdot}00001$$

which is roughly $\frac{2}{3} \times 10^{-9}$ and is therefore entirely negligible.

Sensitivity. Suppose that in the above measurement the smallest change in the value of the shunt across the standard which can be detected is 0·5 ohm. Then, giving this shunt the value 18·6 instead of 18·1, we have

$$\frac{1}{S'} = \frac{1}{0{\cdot}0100120_2} + \frac{1}{18{\cdot}6}$$

from which $S' = 0{\cdot}010006_4$ instead of $0{\cdot}010006_5$, which means a change in S' of 1 part in 100,000, and therefore a change in the value of X of 1 part in 100,000. Thus the sensitivity of the bridge under the above conditions is 1 part in 100,000.

In general, if x is the value of the shunt across S, we have

$$\frac{1}{S'} = \frac{1}{S} + \frac{1}{x}$$

or

$$S' = \frac{Sx}{S + x}$$

Differentiating with respect to x,

$$\frac{dS'}{dx} = S\left[\frac{(S + x) - x}{(S + x)^2}\right] = \frac{S^2}{(S + x)^2} = \frac{\Delta S'}{\Delta x}$$

where $\Delta S'$ is the change in S' for a given change Δx in x.

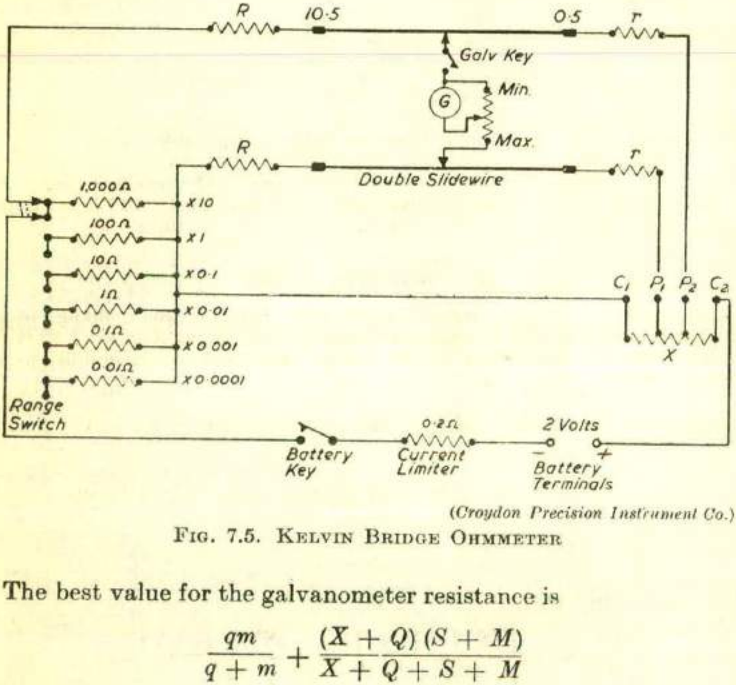
Thus
$$\Delta S' = \frac{S^2}{(S + x)^2} \cdot \Delta x$$

Smith (Ref. (4)) has shown that, if X is changed to $X + \delta X$, the galvanometer current is given by

$$\frac{I\,.\,\delta X}{G + \frac{qm}{q + m} + \left[\frac{(X + Q)\,(S + M)}{X + Q + S + M}\right]}\left[\frac{S + M}{X + Q + S + M}\right]$$

where G is the resistance of the galvanometer. Thus, if Y is the sensitivity of the galvanometer used, in millimetres per micro-ampere, the deflection for a change of δX in the value of X is given in millimetres by

$$D = \frac{YI\delta X \, . \, 10^6 (S + M)}{\left[G + \frac{qm}{q+m} + \frac{(X+Q)(S+M)}{X+Q+S+M}\right](X+Q+S+M)} \quad (7.3)$$



(*Croydon Precision Instrument Co.*)

FIG. 7.5. KELVIN BRIDGE OHMMETER

The best value for the galvanometer resistance is

$$\frac{qm}{q+m} + \frac{(X+Q)(S+M)}{X+Q+S+M}$$

THE KELVIN BRIDGE OHMMETER. This is a modified form of the Kelvin bridge intended for the rapid measurement of the winding resistances in machines and transformers, and for the measurement of contact and earth conductor resistances. The accuracy is of the order of $\pm$ 0·2 per cent, and whilst this is lower than that attainable with the conventional bridge, it is adequate for these particular applications. Balance is obtained by rotating a single dial and the instrument is direct-reading.

Fig. 7.5 gives the circuit diagram of a Kelvin bridge ohmmeter made by the Croydon Precision Instrument Co. The ratios $\frac{Q}{M}$ and $\frac{q}{m}$

of Fig. 7.3 are replaced by a combination of fixed resistors R and r and a double slide-wire, enabling the bridge ratio to be varied continuously between values of 10/1 and 200/1. The double slide-wire is necessary to ensure equality of the inner and outer ratios.

Six standards are provided: these range from 0·01 ohm to 1,000 ohms, one current and potential connection being switched on each standard. The ratio dial is calibrated from 0·5 ohm to 10·5 ohms on the × 1 range using the 100 ohm standard; the scale definition on this range is 0·02 ohm. The overall range of the instrument is from 0·00005 ohm to 105 ohms. Since the current and potential

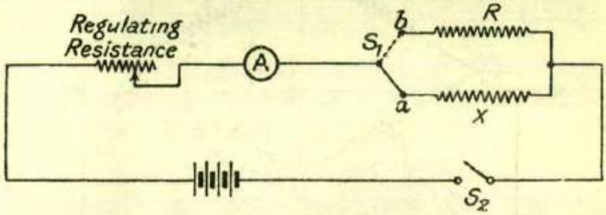


FIG. 7.6. MEASUREMENT OF RESISTANCE BY SUBSTITUTION

leads can affect the calibration, these are provided with the instrument.

Measurement of Medium Resistance. The methods used for such measurements are—

(*a*) Ammeter and voltmeter method.
(*b*) Substitution method.
(*c*) Wheatstone bridge.

(*a*) This method has been considered in the section on low resistance measurements earlier in the chapter.

(*b*) SUBSTITUTION METHOD. The diagram of connections for this method is given in Fig. 7.6. X is the resistance to be measured, while R is a variable known resistance. A battery of ample capacity is used for the supply, since it is important in this method that the supply voltage shall be constant. A is an ammeter of suitable range, or a galvanometer with a shunt which can be varied as required.

With switch S_2 closed, and with switch S_1 on stud *a*, the deflection of the ammeter or galvanometer is observed. S_1 is then thrown on to stud *b* and the variable resistance is adjusted until the same deflection is obtained on the indicating instrument. Then the value of R which produces the same deflection gives the resistance of the unknown directly.

The resistances R and X should be large compared with that of the rest of the circuit. The method is chiefly used—somewhat modified—in the measurement of high resistance. The accuracy of the measurement obviously depends upon the constancy of the supply voltage, of the resistance of the circuit excluding X and R, and upon the sensitivity of the indicating instrument, as well as upon the accuracy with which the resistance R is known.

(*c*) WHEATSTONE BRIDGE. This is the best and commonest method of measuring medium resistances. The general arrangement is shown in Fig. 7.7. P and Q are two known fixed resistances, S being a known variable resistance and R the unknown resistance.

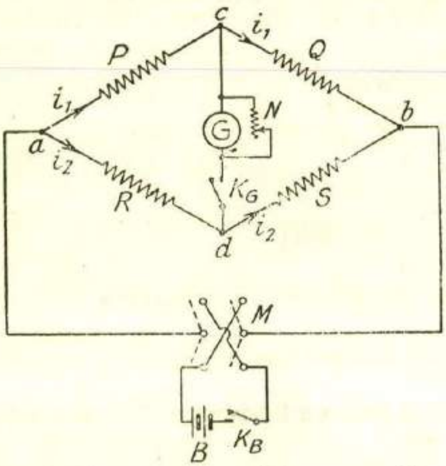


FIG. 7.7. CONNECTIONS OF WHEATSTONE BRIDGE

G is a sensitive D'Arsonval galvanometer shunted by a variable resistance N to avoid excessive deflection of the galvanometer when the bridge is out of balance. This shunt is increased as the bridge approaches balance, so that the shunting is zero—giving full sensitivity of the galvanometer—when balance is almost obtained. B is a battery of two or three cells and M is a reversing switch so that the battery connections to the bridge may be reversed and two separate measurements of the unknown resistance made in order to eliminate thermo-electric errors. K_B and K_G are keys fitted with insulating press-buttons, so that the hand does not come in contact with metal parts of the circuit, thus introducing thermo-electric e.m.f.s. The battery key, K_B, should be closed first, followed by the closing of K_G after a short interval. This avoids a sudden (possibly excessive) galvanometer deflection, due to self-induced e.m.f.s when the unknown resistanee R has appreciable self-inductance.

At balanee—obtained by adjustment of S—the same current i

flows in both of the arms P and Q, since the galvanometer takes no current, and the same current i_2 flows also in arms R and S.

Also, voltage drop across arm P = voltage drop across arm R

and voltage drop across arm Q = voltage drop across arm S

Thus,
$$i_1P = i_2R$$
$$i_1Q = i_2S$$

By division
$$\frac{P}{Q} = \frac{R}{S}$$

or
$$R = \frac{P}{Q} \cdot S \quad . \quad . \quad . \quad . \quad (7.4)$$

from which R is found in terms of P, Q, and S.

The arms P and Q are the "ratio arms" of the bridge and the ratio $\frac{P}{Q}$ may be varied as required to increase the range of the bridge.

In the elementary forms of the Wheatstone bridge the arms P and Q are replaced by a slide-wire of uniform cross-section whose resistance per unit length is constant. A scale is fitted under this slide-wire, and a sliding contact—corresponding to the point c of Fig. 7.7—connects one terminal of the galvanometer to the wire. S is a resistance of the same order of magnitude as the unknown. The sliding contact c is moved until zero deflection of the galvanometer is obtained. Then, since the slide-wire is of uniform cross-section, the ratio $\frac{P}{Q}$ is given by $\frac{\text{length of slide-wire } ac}{\text{length of slide-wire } cb}$, these lengths being obtained from the scale.

As before,
$$R = \frac{P}{Q} \cdot S$$

Wheatstone bridges are normally constructed with either four or five pairs of ratio coils—tens, hundreds, thousands, and ten-thousands, in the bridge containing four pairs—and either four or five decades of resistance coils which constitute the variable arm S. The ratios and decades are usually provided with high-quality instrument switches of the sliding-contact pattern, and although such switches have a low, and constant, contact resistance, plugs are often preferred to switches in bridges of the highest attainable precision. Any uncertainty in contact resistance is thereby eliminated.

Fig. 7.8 shows a precision form of Wheatstone bridge manufactured by H. W. Sullivan, Ltd., which will measure resistance with an accuracy of 0·01 per cent. This bridge incorporates a special circuit arrangement (patented by W. H. F. Griffiths) which enables it to be used for accurate measurement either in absolute units or in the old international units (Ref. (32)). Other manufacturers of precision Wheatstone bridges include H. Tinsley & Co., Ltd., and the Croydon Precision Instrument Co.

Operation of the Bridge. The method of operation can best be illustrated by an example.

Suppose that the actual value of the resistance to be measured is 57·63 ohms and that a four-dial Wheatstone bridge—the dials containing units, tens, hundreds, and thousands—is to be used in conjunction with a galvanometer of ample sensitivity.

The bridge is connected up according to the arrangement shown in Fig. 7.7, care being taken to ensure that all the connections are firmly made and that all the plugs in the bridge blocks are firmly pressed home so that contact resistances may be small and definite. The galvanometer is at first heavily shunted and the ratio arms are made equal (each 10 ohms, say). The battery

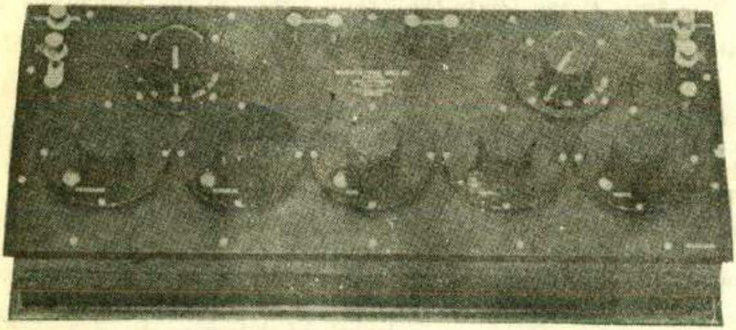


FIG. 7.8. WHEATSTONE BRIDGE

supply switch is closed. Assuming the magnitude of the resistance to be measured to be entirely unknown, first set the variable resistance arm, S, to some small value—say 1 ohm. Depress the key K_B and then lightly press the galvanometer key K_G (immediately raising it again if the galvanometer deflection is excessive). Note the direction of the galvanometer deflection—right or left. Next set the arm S to some high resistance—say 10,000 ohms—and again note the direction of the deflection obtained. If this direction is opposite to the previous one, the unknown resistance has some value between 1 and 10,000 ohms, being nearest in value to the setting of S which gives the smaller deflection. Adjust the resistance S until approximate balance is obtained, and then remove the shunt from the galvanometer, in steps, adjusting S as required, until balance is obtained with the full galvanometer sensitivity. In the case under consideration ($R = 57·63$), it will be found that the galvanometer deflects to the left (say) with S set at 57 and to the right with S set at 58, meaning that R lies between the two. This is the best that can be done with equal ratio arms.

To obtain greater accuracy, make Q 100 ohms, keeping P 10 ohms, at the same time altering S to 570. Then adjust S until approximate balance is obtained again. It will now be found that the balance point is between $S = 576$ and 577. Q should then be made 1,000 ohms (keeping P 10 ohms) and the process repeated. Final balance will be obtained when $S = 5,763$ ohms. Then

$$R = \frac{P}{Q} \cdot S = \frac{10}{1000} \cdot 5,763 = 57{\cdot}63$$

The battery connections are then reversed by the switch M, and the measurement is repeated. If it is found that an alteration of 1 ohm in S disturbs the balance, then the sensitivity is at least 1 part in 5,763. As a check on the balance it should always be ascertained that a slightly smaller value of S causes a galvanometer deflection to the left (say) and that a slight increase in S above the balance point causes a galvanometer deflection to the right.

If, with equal ratio arms at the beginning of the measurement, the unknown resistance does not lie within 1 and 10,000, the ratio arms must be adjusted until an approximate balance is obtained between these limits of S—e.g. if R is greater than 10,000, arm P must be made greater than Q, and if smaller than 1 ohm, Q must be made greater than P.

An accuracy of a few parts in 10,000 is usually obtainable with a good bridge of the type described above.

Best Galvanometer Resistance. The current through the galvano meter for a given change δR in the unknown resistance is given by

$$i_g = \frac{i_2 \cdot \delta R}{G + \left[\dfrac{(R+P)(Q+S)}{R+P+Q+S}\right]} \frac{Q+S}{R+P+Q+S}$$

where G is the galvanometer resistance.

Let $\quad i_2\, \delta R \dfrac{Q+S}{R+P+Q+S} = K$, and $\dfrac{(R+P)(Q+S)}{R+P+Q+S} = A$

Then
$$i_g = \frac{K}{G+A}$$

For given dimensions of the galvanometer coil,

$$\text{Number of turns on coil, } N, \propto \sqrt{G}$$

$$\therefore \text{Deflecting torque} \propto N i_g \propto i_g \sqrt{G}$$

Hence, deflection for a given change δR, and current i_2 is

$$\theta \propto i_g \sqrt{G} \propto \frac{K\sqrt{G}}{G+A}$$

Differentiating we have

$$\frac{d\theta}{dG} \propto \frac{A-G}{2\sqrt{G}(G+A)^2}, \text{ which is zero when } G = A$$

Thus, maximum deflection for a given change in the resistance R, and a given current i_2, is obtained when the galvanometer resistance is given by

$$G = \frac{(R+P)(Q+S)}{R+P+Q+S}$$

Obviously, the sensitivity may be increased, also, by increasing the current i_2. The galvanometer may be of the D'Arsonval type and should be as nearly critically damped as possible.

Precision Measurements with the Wheatstone Bridge. The accuracy in measurement which can be attained with a Wheatstone bridge is determined by the errors in the values of the bridge arms, and by the definition attained in a particular measurement. The balance equation of the bridge shown in Fig. 7.7 is $R = \frac{P}{Q} . S$

Suppose the values of the arms are expressed as

$$P \pm \delta P, Q \pm \delta Q, S \pm \delta S$$

where δP, $\pm \delta Q$, $\pm \delta S$ are the errors in these arms, and are small. Then we may write

$$R \mp \delta R = \frac{P \pm \delta P}{Q \pm \delta Q} (S \pm \delta S)$$

Then the upper limit in the value of R is

$$R + \delta R = \frac{P + \delta P}{Q - \delta Q} (S + \delta S)$$

from which

$$RQ + Q . \delta R - R . \delta Q - \delta Q . \delta R = PS + S . \delta P + P . \delta S + \delta P . \delta S$$

Neglecting the products $\delta Q . \delta R$ and $\delta P . \delta S$ (which are very small) and substituting $RQ = PS$ gives

$$Q . \delta R = S . \delta P + P . \delta S + R . \delta Q$$

Dividing by RQ or PS,

$$\frac{\delta R}{R} = \frac{\delta P}{P} + \frac{\delta S}{S} + \frac{\delta Q}{Q}$$

A similar expression, but of opposite sign, may be obtained by repeating this procedure for $R - \delta R$. Expressing as percentages gives the general error equation

$$\pm \frac{\delta R}{R} \times 100\% \doteq \pm \frac{\delta P}{P} \times 100\% \pm \frac{\delta S}{S} \times 100\% \pm \frac{\delta Q}{Q} \times 100\% \quad . \quad . \quad (7.5)$$

It is apparent from this equation that, if all the resistances at a given setting of a bridge have errors of $\pm$ 0·01 per cent, the error in determination of the unknown R is $\pm$ 0·03 per cent.

The error in the adjustable arm S is increased by an amount equal to half the definition. For example, suppose S is 1,000 ohms and the sensitivity is such that movement of the galvanometer can just be detected when S is moved from 999·9 ohms to 1,000·1 ohms, then the error in S is increased by a determination error of $\pm$ 0·1 ohm or $\pm$ 0·01 per cent, thus increasing the error in R to $\pm$ 0·04 per cent.

The determination error can be reduced by the use of a galvanometer with sufficient sensitivity to permit the least count on the bridge to be interpolated from the galvanometer deflection. Taking a simple example, if a setting of S of 999·9 ohms gives a galvanometer deflection of 14 mm to one side of zero and a setting of 1,000·1 gives 6 mm to the other side, then 20 mm on the galvanometer

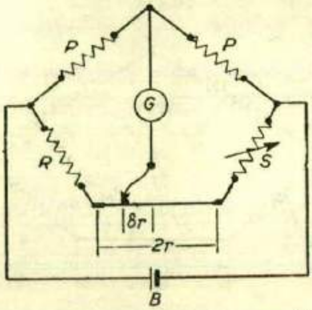


FIG. 7.9. EQUAL-RATIO BRIDGE WITH SLIDE-WIRE

scale corresponds to 0·2 ohm at this setting and the actual value of S is $999{\cdot}9 + \frac{14}{20} \times 0{\cdot}2 = 1{,}000{\cdot}04$ ohms.

Measurements with Equal Ratios. The highest accuracy is normally attained when equal ratios are used, because it is a relatively simple matter to adjust ratios to exact equality. It is therefore usual to extend the definition of precision bridges by the inclusion of a slide-wire.

Fig. 7.9 shows an equal-ratio bridge with a slide-wire included between the adjustable arm S and the unknown R. Suppose the resistance of the slide-wire is $2r$ ohms and balance is obtained with the slider at δr ohms from the mid-point towards R. Then

$$\frac{P}{P} = \frac{R + r - \delta r}{S + r + \delta r}$$

from which

$$S + 2\delta r = R$$

If the slide-wire has an overall resistance of 0·1 ohm it can be calibrated from + 0·1 to − 0·1 ohm, and a definition of 0·001 ohm is readily attainable.

It should be remembered that the precision in measurement of a two-terminal resistance is often limited by resistance variations in the connections.

Reversing Ratios. Bridges of the highest attainable precision are usually provided with some means for reversing the ratios. Suppose one ratio is P and the other $P + \delta P$. Then, with one connection of the ratios,

$$R = \frac{P + \delta P}{P}(S - \delta S)$$

and with the other connection,

$$R = \frac{P}{P + \delta P}(S + \delta S)$$

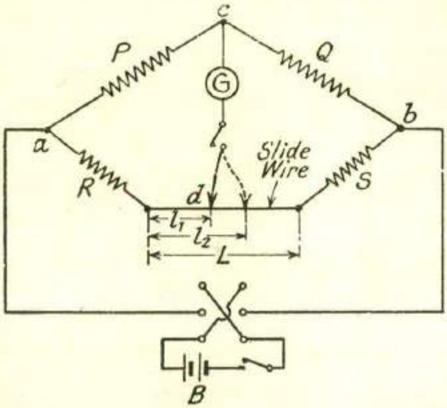


FIG. 7.10. CAREY–FOSTER SLIDE-WIRE BRIDGE

where the bridge settings are $S - \delta S$ and $S + \delta S$ respectively. Taking the mean of the bridge settings S,

$$2S = R\left[\frac{P}{P + \delta P} + \frac{P + \delta P}{P}\right]$$

from which

$$S = R\left[1 + \frac{\delta P^2}{2P^2}\right]$$

If $\dfrac{\delta P}{P}$ is 0·0001, then $\dfrac{\delta P^2}{2P^2} = \dfrac{1}{2} \times 10^{-8}$

which is negligible and the error in the ratios is eliminated.

Carey–Foster Slide-wire Bridge. The connections of this bridge are shown in Fig. 7.10, a slide-wire of length L being included between R and S as shown. This bridge is specially suited to the comparison of two nearly equal resistances.

Resistances P and Q are first adjusted so that the ratio $\frac{P}{Q}$ is approximately equal to the ratio $\frac{R}{S}$. Exact balance is obtained by adjustment of the sliding contact on the slide-wire. Let l_1 be the distance of the sliding contact from the left-hand end of the slide-wire. The resistances R and S are then interchanged and balance again obtained. Let the distance now be l_2.

Then, for the first balance, $$\frac{P}{Q} = \frac{R + l_1 r}{S + (L - l_1)r}$$

where r is the resistance per unit length of the slide-wire.

For the second balance,

$$\frac{P}{Q} = \frac{S + l_2 r}{R + (L - l_2)r}$$

Now, $$\frac{P}{Q} + 1 = \frac{R + l_1 r + S + (L - l_1)r}{S + (L - l_1)r} = \frac{R + S + Lr}{S + (L - l_1)r}$$

also $$\frac{P}{Q} + 1 = \frac{S + l_2 r + R + (L - l_2)r}{R + (L - l_2)r} = \frac{S + R + Lr}{R + (L - l_2)r}$$

Hence $$S + (L - l_1)r = R + (L - l_2)r$$

or $$S - R = (l_1 - l_2)r \quad . \quad . \quad . \quad . \quad (7.6)$$

Thus the difference between S and R is obtained from the resistance per unit length of the slide-wire together with the difference $(l_1 - l_2)$ between the two slide-wire lengths at balance.

The slide-wire is calibrated—i.e. r is obtained—by shunting either S or R by a known resistance and again determining the difference in length $(l_1' - l_2')$.

Suppose that S is known and that S' is its value when shunted by a known resistance; then

$$S - R = (l_1 - l_2)r$$

and $$S' - R = (l_1' - l_2')r$$

$$\frac{S - R}{l_1 - l_2} = \frac{S' - R}{l_1' - l_2'}$$

from which $$R = \frac{S(l_1' - l_2') - S'(l_1 - l_2)}{(l_1' - l_2' - l_1 + l_2)} \quad . \quad . \quad . \quad (7.7)$$

From this expression it can be seen that this method gives a direct comparison between S and R in terms of lengths only, the resistances of P and Q, contact resistances, and the resistances of connecting leads being eliminated.

As it is important that the two resistors R and S shall not be handled or disturbed during the measurement, a special switch is used to effect the interchanging of these two resistors during the test.

Applications of Resistance Measurements by Wheatstone Bridge. Several quite different quantities can be measured through the change which they bring about in the resistance of some form of element adapted to the particular measurement concerned and used in conjunction with a Wheatstone bridge to indicate this change.

Examples in other chapters are the bismuth spiral (see p. 389), the resistance of which increases when it is placed in a magnetic field, the resistance thermometer (see p. 524), and the ionic wind voltmeter (see p. 481).

Some important applications of resistance measurements which do not fall conveniently into the groundwork of other chapters are described below.

Strain Gauge Measurements. For the measurement of stresses and strains in structures and machines the technique of using electrical resistance strain gauges has been built up, particularly during World War II in the aircraft industry. Thin copper-nickel alloy or nickel-chrome wires are used, these having a linear relationship—over the limited range required—between strain and electrical resistance. Nickel-chrome has the higher resistance but is less commonly used because it has some strain hysteresis and also a high temperature coefficient.

These gauges enable accurate measurements of strain to be made in any surface to which they can be cemented so that no slip occurs. They consist of grids of fine wires cemented to a thin paper membrane which is cemented, with nitro-cellulose cement, to the surface under test. The gauge is usually covered with a layer of felt to protect it and to reduce the effect of draughts. Particularly to eliminate electrolytic effects in the gauge, and also to avoid moisture absorption by the matrix, it is coated—when completely dry—with a protective wax such as Di Jell 171.

Care is necessary in preparing the surface to receive the strain gauge and in fixing the gauge itself, and it is important to follow the manufacturer's instructions.

The gauge current may be as high as 25 mA for short periods but 10 mA is recommended for longer tests.

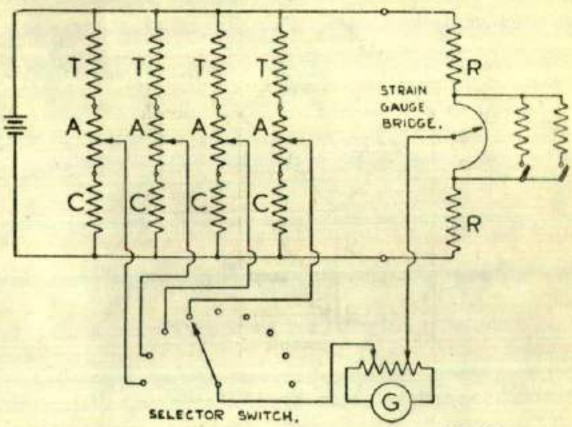
The ratio $\dfrac{\text{Percentage change of resistance}}{\text{Percentage strain}}$ is called the "gauge factor." The resistance change is greater than may be accounted for by the change in the wire dimensions and this factor is commonly about 2·2.

Sample gauges are made up and gauges are rejected if test results show a variation of more than $\pm$ 1 per cent from the mean.

In the measurements of strain, temperature effects are compensated for by using a dummy gauge which is connected in the appropriate arm of a Wheatstone bridge network (see Fig. 7.11). The dummy must be fixed to a piece of the same material as that under test but which is not, of course, under stress. The two gauges must be under the same conditions of ambient temperature.

The arrangement in Fig. 7.11, which is for static measurements, has a number of gauges T, each with its compensating gauge C. Zero balance is obtained by adjustment of the apex resistor A and calibration is by shunting one of the bridge arms R by a high resistance.

Measurement may be either by direct deflection, using a calibrated



(*H. Tinsley & Co., Ltd.*)

Fig. 7.11. Strain Gauge Bridge Network

galvanometer to measure the out-of-balance current, or by a null method through variation of the slide-wire setting. The selector switch shown must be free from thermo-electric effects. The slide-wire is necessitated by the sensitivity required in such measurements, the change in gauge resistance being very small.

In some measurements all four arms of the bridge can be gauges mounted, for example, on the same specimen in tension and compression. Then the zero adjusting and calibrating resistors can be located at a distance from the bridge.

For dynamic measurements, which are usually deflectional and use an oscilloscope or oscillograph, the bridge may be supplied at a carrier frequency from an oscillator, the output being fed into an a.c. amplifier having a flat response over the frequency range to be covered. The arrangement is shown in Fig. 7.12. Capacitance balance is necessary in addition to resistance balance; calibration and zero adjustment are effected by shunting the arms.

Elliott Brothers (London), Ltd., have introduced a "ten-channel strain gauge equipment" to display, on one cathode-ray tube, the strain at ten different points in a test piece.

Fig. 7.13 shows the schematic layout and also the form in which the strain amplitudes and waveforms are displayed. Elliott publication No. G.121A gives a full description of the equipment.

The above is but a brief outline of the strain gauge technique. References (18), (19) and (20) deal with the subject comprehensively.

Electrical Weighing. H. I. Andrews (Ref. (21)) has described the application of the strain gauge principle to the measurement of weight or force in the testing of locomotives or trains under working

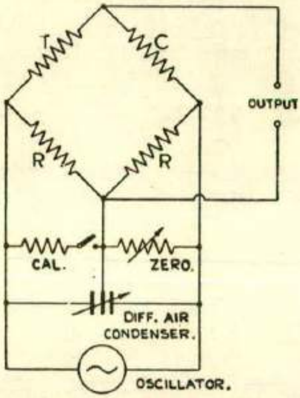


FIG. 7.12.

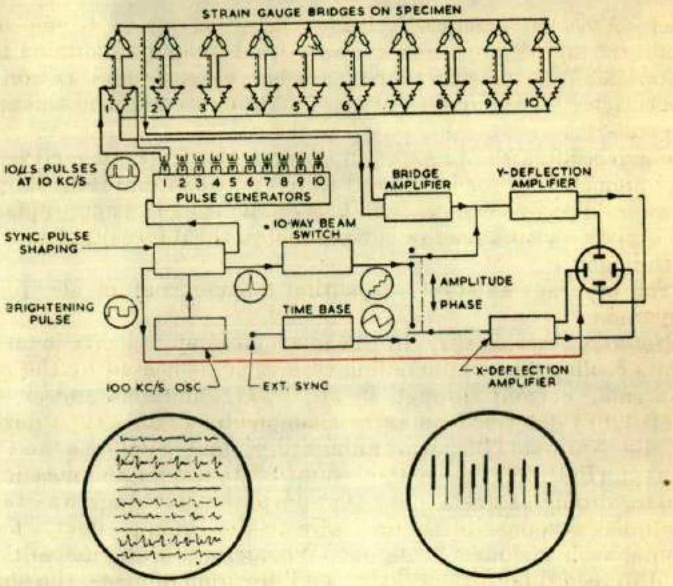
conditions. For this purpose the apparatus must have reliability, rigidity, compactness and simplicity, and must give reasonable accuracy during accelerations of several g with temperature changes up to 30° F.

In its development the aims were to obtain an increased output voltage and to enable robust measuring instruments to be used. The solution adopted was to employ, as the active element, a coil of enamel-insulated constantan wire wound directly on a round bar of nickel-chrome-molybdenum steel, or of mild steel or of duralumin. The coil is included in one arm of a Wheatstone bridge network, and, as an example, with an applied voltage of 230 V and a coil on a mild steel bar stressed to 14 tons/in.^{2}, an output voltage of 0·034 V could be obtained. Constantan wire gives a good strain sensitivity, maintains its calibration indefinitely and has a low temperature coefficient.

When the active coil, of resistance R, forms one arm of a bridge of which the three other arms are each of resistance R also, the output voltage is given by

$$v = \frac{V}{4} \cdot \frac{P}{A} \cdot K \cdot \frac{m}{E}$$

where V is the voltage applied to the bridge, P is the applied load, A is the cross-sectional area of the bar, m and E are Poisson's ratio and Young's modulus, respectively, for the material of the bar, and



(*Elliott Bros. (London) Ltd.*)

FIG. 7.13. TEN-CHANNEL STRAIN GAUGE EQUIPMENT

K is the sensitivity ratio, which is the ratio between the proportional change of resistance of the wire and the circumferential strain in the bar.

The Measurement of Pressure. Another application of similar principles is to the measurement of pressure. Thus, for example, N. Gross and P. H. R. Lane (Ref. (22)) have described an accurate wire-resistance method of measuring pulsating pressures up to 6,000 lb/in.[2] As the pressure-sensitive elements, to replace the normal Bourdon tube types of gauge, which are not sufficiently accurate for rapidly fluctuating pressures, cylindrical steel tubes with a wire winding are used. Two coils of 48 s.w.g. manganin wire are wound side by side on the tube. These are dried and sealed against moisture. The coils are connected in two diagonally opposite arms of a Wheatstone bridge network of which the detector is a mirror galvanometer having a sensitivity of 100 mm/μA at 1 metre, and a periodic time of 0·1 sec. The bridge is supplied from a battery.

The proportionate change $\frac{\Delta R}{R}$ in the resistance of the pressure cell coils is related to the pressure by the expression

$$\frac{\Delta R}{R} = K(D - t)^2(1 - \frac{1}{2}m) \cdot \frac{p}{2EDt}$$

where K is the constant relating electrical strain to mechanical strain, m and E are Poisson's ratio and Young's modulus for the material of the pressure tube, p is the pressure and D and t are, respectively, the diameter of the middle layer and the thickness of the coils.

For recording the pressure pulsations, the light reflected from the galvanometer mirror is allowed to fall on to 35-mm recording paper in an oscilloscope camera, the lens of the camera being replaced by a metal disc with a narrow slit parallel to the direction of movement of the light.

The accuracy is stated as within 0·5 per cent of the full-scale deflection.

Hot-wire Anemometer. In this instrument air velocity is measured by its cooling effect upon a fine wire which is heated by the passage of a small current through it. L. F. G. Simmons and A. Bailey (Ref. (26)) described on early anemometer of this type developed at the National Physical Laboratory, and Simmons and J. A. Beavan (Ref. (27)) gave an account of its use for the measurement and recording of gusts. For this purpose, advantage was taken of the quick response of the hot wire to the cooling effect. Two elements, each included in its own Wheatstone bridge network; were used to record both the velocity and direction of gusts, the detectors in the networks being oscillograph elements.

King's formula (Ref. (28)) for the cooling of fine wires was

$$H = (a + b\sqrt{V})(T - T_a)$$

where H is the rate of heat loss per unit length of a circular wire which is maintained at a temperature T in a steady wind of velocity V, and T_a is the air temperature.

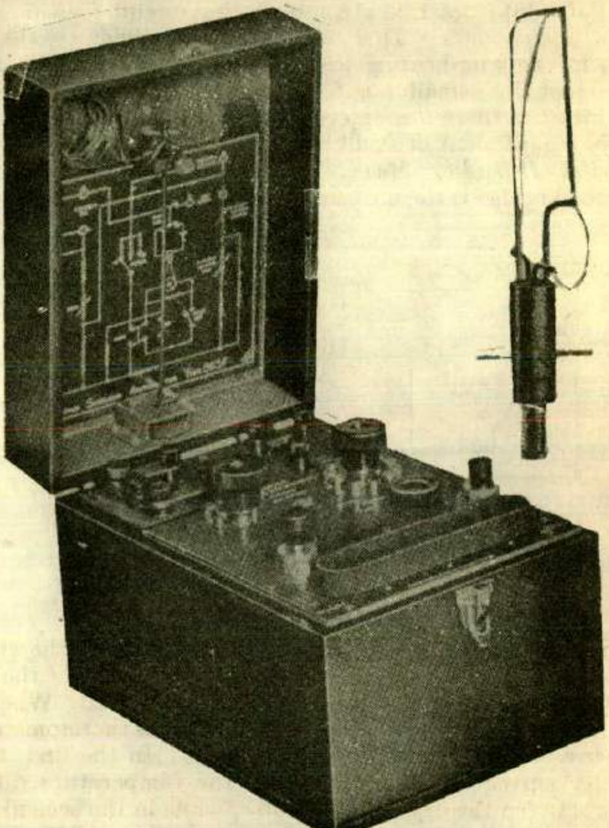
When current i flows in the wire the formula can be written

$$0{\cdot}24\, i^2r = (a + b\sqrt{V})(T - T_a)$$

The left-hand side represents the heat loss in calories per second; r is the resistance per unit length of wire: a and b do not remain constant over an appreciable range of temperature and each is expressible in the form $p + qT$ (Ref. (27)).

Later, Simmons described (Ref. (29)) a shielded form of hot-wire anemometer for low speeds, and an instrument of this type is now manufactured by H. Tinsley & Co.

The hot-wire element, shown in Fig. 7.14, has a fine wire, about 3 cm long, housed in one bore of a small twin-bore silica tube. The



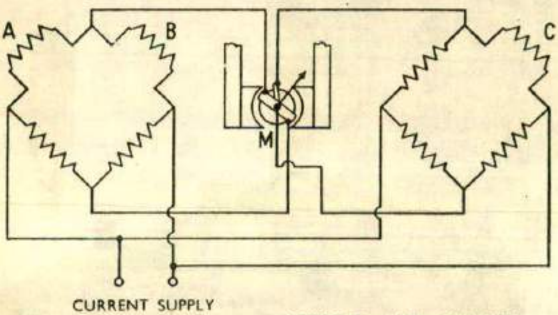
(*H. Tinsley & Co., Ltd.*)

FIG. 7.14. SIMMONS HOT-WIRE ANEMOMETER

other bore contains a thermo-couple for the measurement of the hot-wire temperature. A standard heating current of 0·6 A is used and standardization is carried out by covering the element with a box and then adjusting the controlling resistors until the galvanometer used with the instrument gives a specified deflection. The standing e.m.f. of the thermo-couple is backed off by the zero setting control.

During the air-speed measurements the output from the thermo-couple is read on the galvanometer scale. Calibration of the scale is necessary because the deflection/air-speed relationship is non-linear. The Tinsley instrument has 3 ranges, for 0 to 0·4, 0 to 2·0 and 0 to 5·0 feet per second. L. L. Fox, P. L. Palmer and D. Whittaker (Ref. (30)) describe a compensating circuit for this shielded hot-wire anemometer. This overcomes the difficulty that small changes in the wire-heating current cause large errors in air flow measurement. A small e.m.f. is injected from the heater-wire circuit into the thermo-couple circuit so that the measured e.m.f. is largely independent of small current changes.

Electrical Humidity Meter. Yet another application of the Wheatstone bridge is the measurement of the humidity of the air by



(*Elliott Bros.* (*London*), *Ltd.*)

FIG. 7.15. ELECTRICAL HUMIDITY METER

wet- and dry-bulb thermometers, the latter being of the electrical resistance type. Fig. 7.15 shows the circuit diagram of the Elliott humidity meter. There are two, interconnected, Wheatstone bridges; one contains dry-bulb A and wet-bulb B thermometers, and the other a second dry-bulb thermometer C. In the first, the galvanometer current is proportional to the temperature difference $T - T_1$ between the dry and wet bulbs, while in the second bridge, the galvanometer current is proportional to the dry-bulb temperature T. Instead of actual galvanometers, two coils of a "cross-coil" indicator are used and the pointer reads the psychrometric difference $T - T_1$ corrected by the dry-bulb temperature T, i.e. the instrument reads the *relative humidity*.

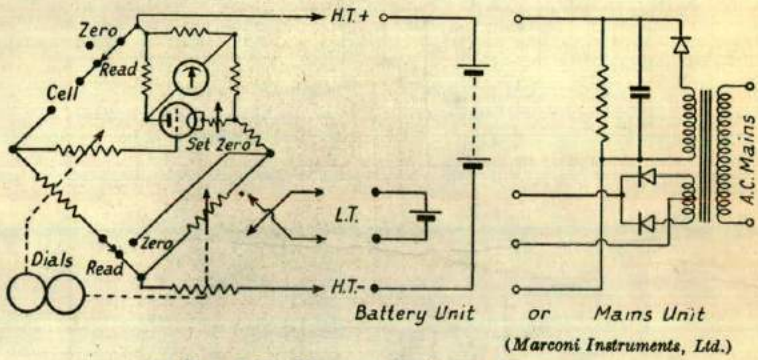
The three thermometers are mounted in a "humidity transmitter" through which the air under test is drawn, first over the two dry bulbs and then over the wet bulb.

The meter is independent of voltage variations in the supply since both bridges are supplied from the same source. A.C. mains can be used with a transformer and rectifier.

Moisture Meter. The principle of the Marconi moisture meter, designed for the measurement of the moisture content of grain and other hygroscopic materials, is shown in Fig. 7.16.

The circuit used (Brit. Patent No. 635674) is an extension of the Wheatstone bridge principle, there being a subsidiary bridge network in the arm opposite to that containing the sample under test. Out-of-balance voltage reduces or increases the resistance of the triode which, in turn, alters the effective resistance of the subsidiary bridge network. This self-balancing feature is included to render the reading of the meter independent of valve characteristics and supply voltage variations.

The sample is contained in a test cell in which a great mechanical



(*Marconi Instruments, Ltd.*)

FIG. 7.16. MARCONI MOISTURE METER

pressure is applied to the material to bring it to a uniform state and eliminate packing errors.

The range of moisture content covered corresponds to equilibrium with atmospheres having relative humidities from about 20 per cent to 90 per cent and the accuracy is $\pm$ 1 per cent m.c.

Marconi Instruments, Ltd., also make a moisture-in-timber meter, though the construction and principle of operation are different from those of the meter just described.

Measurement of High Resistance. When the resistance to be measured is of the order of one or more megohms, the methods of measurement described in the foregoing pages are unsuitable. In such cases the resistance offered to the passage of current along the surface of the insulation is often comparable with the resistance itself, and special methods have to be adopted to take such "surface leakage" into account.

Amongst the high-resistance measurements which are required to be made in practice those of the insulation resistance of cables are very important. The absorption effects in dielectrics have already been mentioned in Chapter IV, and such effects are apt to destroy the value of insulation resistance measurements unless special precautions are taken.

A simple method of measuring insulation resistance is the direct deflection method. A very sensitive moving-coil galvanometer of high resistance (1,000 ohms or more) is connected in series with the resistance to be measured, and to a battery supply. The deflection of the galvanometer gives a measure of the insulation resistance. This method is, however, only sufficient to indicate whether the

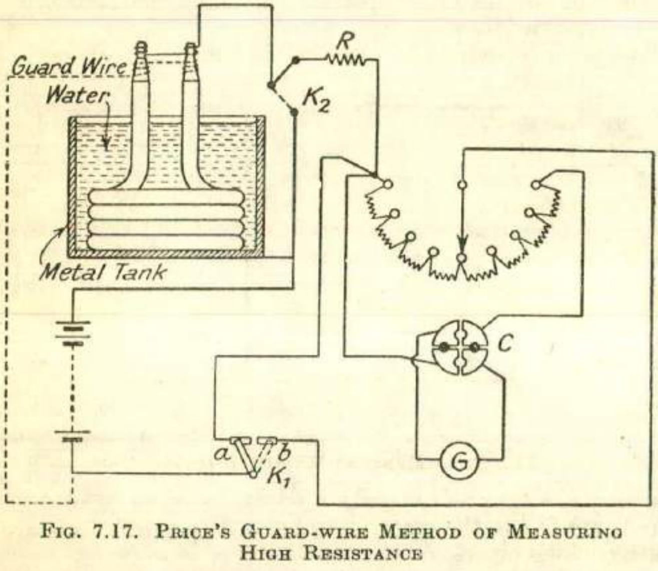


FIG. 7.17. PRICE'S GUARD-WIRE METHOD OF MEASURING HIGH RESISTANCE

insulation is faulty or otherwise, and cannot be regarded as a precise method.

PRICE'S GUARD-WIRE METHOD. Fig. 7.17 gives the connections for a direct deflection method in which the errors due to surface leakage are eliminated by the use of a guard wire. In the figure the resistance to be measured is the insulation resistance of a length of cable. The cable is immersed in water, which is contained in a metal tank, 24 hours being allowed to elapse—the temperature meanwhile being maintained constant—before the test is carried out. This enables the water to soak through any defects which may exist in the insulation, and also allows the insulation to attain the same temperature as the water. The ends of the cable are trimmed as shown, the outer protective covering being removed at these ends for a length of at least 12 in. A bare wire, twisted round the insulation near the end, is connected to the negative pole of the supply battery—the positive pole of which is connected to the metal tank—so that any current which leaks across the insulation surface is taken direct to the battery instead of passing through the galvanometer and increasing its deflection. The galvanometer is shunted as shown, the shunt being of the Ayrton universal type.

The deflection of the galvanometer is observed, and its scale is afterwards calibrated by replacing the insulation resistance by a standard high resistance (usually 1 megohm), the galvanometer shunt being varied, as required, to give a deflection of the same order as before. The galvanometer, which is of the D'Arsonval type, should be very sensitive (at least 1,000 mm per microampere at a scale distance of 1 metre), should have high resistance, and, also, its deflection should be directly proportional to the current flowing through it. The resistance of the universal shunt across the galvanometer may be so chosen that the galvanometer is critically damped, thus saving time in making observations.

The battery should be of about 500 volts, and its e.m.f. should be constant. C is a four-part commutator for reversal of the galvanometer connections. R is a protective resistance of about 100,000 ohms in series with the galvanometer. K_1 is a key which is closed on contact "a" when the battery is first switched on. The galvanometer is thus protected from the sudden initial rush of current which charges the cable—the latter acting, of course, as a capacitor. Contact "b" is sufficiently close to "a" for the circuit to remain closed when the key is being moved over. K_2 is another key for the purpose of discharging the capacitance of the cable.

The galvanometer and its circuit, together with the keys, must be well insulated to prevent leakage currents.

Fig. 7.18 shows the connection diagram for insulation testing when a universal shunt on the Kelvin–Varley slide principle (see p. 332) is used. The resistance of this (Sullivan) four-dial shunt is variable in steps of 1/10,000th part of the whole.

Loss of Charge Method. In this method the insulation resistance to be measured is connected in parallel with a capacitor and electrostatic voltmeter. The capacitor is charged, by means of a battery, to some suitable voltage, and is then allowed to discharge through the resistance, its terminal voltage being observed over a considerable period of time during discharge.

Then, assuming the capacitor to be perfect, if V is its terminal voltage at any time t, Q being the charge, in coulombs, still remaining in the capacitor, and C its capacitance, we have for the current i at time t,

$$i = -\frac{dQ}{dt} = -C\frac{dV}{dt}$$

But $i = \frac{V}{R}$, where R is the resistance to be measured, and througn which the capacitor is discharging.

$$\therefore \frac{V}{R} = -C\frac{dV}{dt}$$

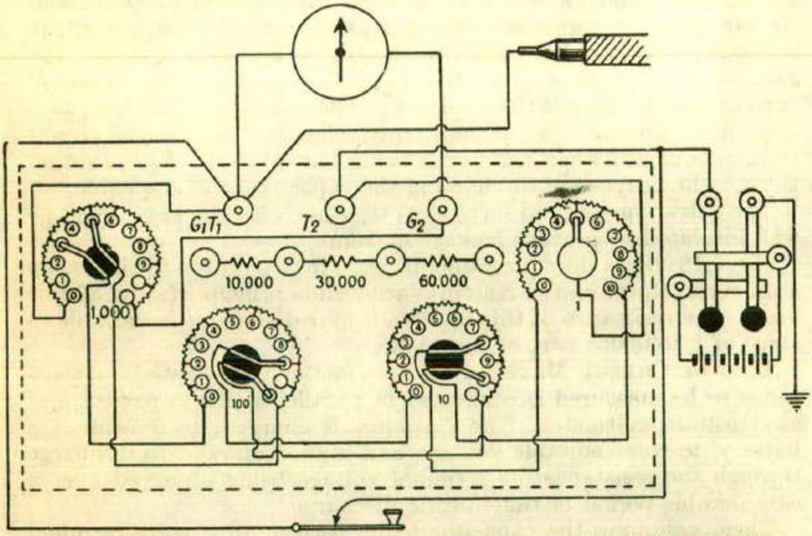
or
$$\frac{V}{R} + C\frac{dV}{dt} = 0$$

Solving this differential equation for V gives $V = Ee^{\frac{-t}{CR}}$, where E is the voltage when $t = 0$ (i.e. the voltage to which the capacitor was originally charged), and e is the base of Napierian logarithms.

Thus
$$\log_e V = \log_e E - \frac{t}{CR}\log_e e = \log_e E - \frac{t}{CR}$$

or
$$R = \frac{t}{C\log_e \frac{E}{V}} = \frac{0{\cdot}4343t}{C\log_{10}\frac{E}{V}} \quad . \quad . \quad . \quad (7.8)$$

R will be given in ohms if t is in seconds and C in farads.



(*H. W. Sullivan, Ltd.*)

FIG. 7.18. FOUR-DIAL SHUNT FOR USE IN INSULATION TESTING

The box contains an independent and standard resistance of 100,000 ohms (between the inner line of terminals), which is used for taking "constants" in direct deflection tests, as also in the fall-of-potential method of localizing faults in a cable, where the potentials are measured by direct deflection, the galvanometer being earthed through this high resistance. In addition, the high resistance is specially subdivided into three sections, 10,000 ohms, 30,000 ohms, and 60,000 ohms, for conveniently observing earth current readings in Schaefer's test for locating breaks and faults in submarine cables.

Example. If $C = 0{\cdot}2$ microfarad, $E = 400$ volts, and the time taken for the capacitor terminal voltage to fall to 250 volts is $1\frac{1}{2}$ min,

$$R = \frac{0{\cdot}4343 \times 90}{\frac{0{\cdot}2}{10^6} \times \log_{10} \frac{400}{250}} = 957{\cdot}6 \times 10^6 \text{ ohms or } 957{\cdot}6 \text{ megohms}$$

If the resistance R is very large, the time for an appreciable fall in voltage is also large. The voltage/time curve will thus be very flat, and, unless great care is taken in measuring the voltages at the beginning and end of a given time t, a serious error may be made in the value of the ratio $\frac{E}{V}$, causing a corresponding error in the measured value of R. More accurate results may thus be obtained by measuring the *change* in the voltage $(E - V)$ directly. Calling this change v, the expression for R then becomes

$$R = \frac{0{\cdot}4343t}{C \log_{10} \frac{E}{E - v}}$$

Laws (Ref. (2)) gives details of a method of measuring this change in voltage, v, using a ballistic galvanometer.

If the insulation resistance of a length of cable is being measured, the cable itself may be used as the capacitor, but its capacitance must be either known or determined.

Several serious difficulties are encountered when this method is used. Owing to absorption effects in the dielectric under test, the current actually flowing through the dielectric is not simply dependent upon its resistance, since an absorption current also flows into the insulation. For this reason observations should be continued for a long period—several hours—if the results of such measurements are to be of value. Again, the insulation resistance of the voltmeter and of the capacitor must be exceedingly high in order that leakage effects shall be negligible.

Effect of the Time of Electrification. As absorption effects are present to a greater or less degree in all insulation resistance measurements, it is important that their influence upon the measured value of the resistance should be considered. In Fig. 4.19 a curve showing the variation of the absorption current in a capacitor with time was given. This current falls away fairly rapidly at first, the decrease thereafter being more gradual. Evershed (Ref. (5)) found, in the case of a length of rubber-covered cable, that the absorption current was some five or six times the true leakage current—dependent upon the resistance only—after a time of application of voltage of 1 min, and was about equal in value to the true leakage current after 7 min. After the voltage had been applied for 6 or 7 hours, the absorption current still formed some 5 to 10 per cent of the total current flowing through the insulation. From these results it is obvious that the insulation resistance as defined by

$$\frac{\text{Applied voltage}}{\text{Current flowing through the insulation}}$$

depends very much upon the time of application of the voltage. In commercial testing a time of application of 1 min is normally specified, but the resistance value so obtained will be very much less than the true value. The magnitude of the insulation resistance itself will influence the effect which the absorption has upon the measured value. If this resistance is low, the absorption current may be negligible compared with the true leakage current, but the former will be of much greater importance in the case of very high resistance insulation.

Effect of Temperature upon Insulation Resistance. The resistance of insulating materials, generally, falls with increase of temperature, the change in resistance being, in some cases, very rapid. For this reason it is important in

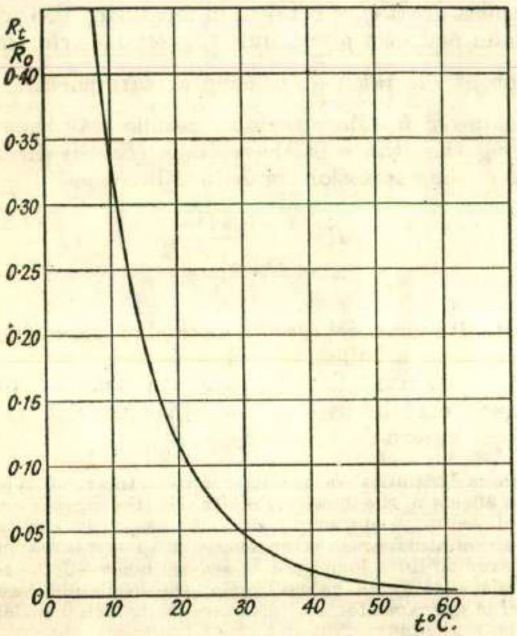


FIG. 7.19. RESISTANCE–TEMPERATURE CURVE FOR RED FIBRE

such measurements that the result should be stated together with the temperature at which the test was carried out.

Koenigsberger and Reichenheim's formula (Ref. (6)) for the variation of the resistance of hard insulating materials with temperature is as follows—

$$R_t = R_0 e^{\frac{-kt}{(273+t)273}}$$

where R_t is the insulation resistance at temperature $t°$ C.

R_0 " " " " 0° C.

k is a constant for any given material.

e is the base of Napierian logarithms.

Fig. 7.19 shows the curve of R_t (expressed as a fraction of R_0) against temperature, for red fibre, for which $k = 8{,}477$. The very great reduction of resistance with increase of temperature can be observed Curtis (Ref. (7)) gives a table of values of the resistivity at 30° C, and of the ratio $\frac{\text{resistivity at } 20° \text{C}}{\text{resistivity at } 30° \text{C}}$ for a number of hard insulating materials. The value of this ratio varies from 1 for India ruby mica to 16·0 for yellow beeswax.

Measurement of Resistance of Specimens of Insulating Materials. Surface and Volume Resistivity. In the above tests the resistance to be measured has been assumed to be that of the insulation of a length of cable. The direct deflection method is often applied to measurements of the resistance of insulating materials, small samples of the material, in sheet form, being used. In such cases it is necessary to distinguish between the "*surface resistivity*" and the "*volume resistivity*," or specific resistance, of the material.

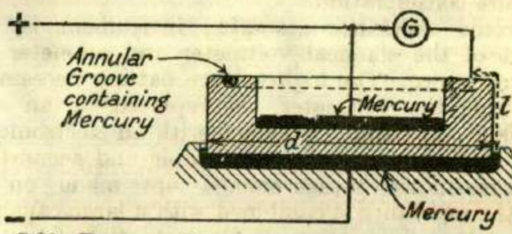


Fig. 7.20. Form of Specimen for Surface Resistivity Measurements

The "*surface resistivity*" is defined as the resistance between opposite edges of a square area of the surface of the material. This quantity depends upon the general condition of the surface and upon the humidity, and is not a constant.

The form of specimen used for such measurements is shown in Fig. 7.20. The sample rests in a pool of mercury to which the negative pole of the battery is connected, an annular groove containing mercury forming the other electrode. There is a pool of mercury also inside the specimen, and a wire from this is taken to the positive side of the battery as shown, being connected so as to form a guard wire to prevent the leakage current which actually passes through the body of the specimen from passing through the galvanometer.

Then, if R is the measured value of the resistance, the surface resistivity is given by $\frac{R \times \pi d}{l}$ where d is the diameter of the specimen and l is the length of the surface path from the annular groove to the outer mercury pool (see figure).

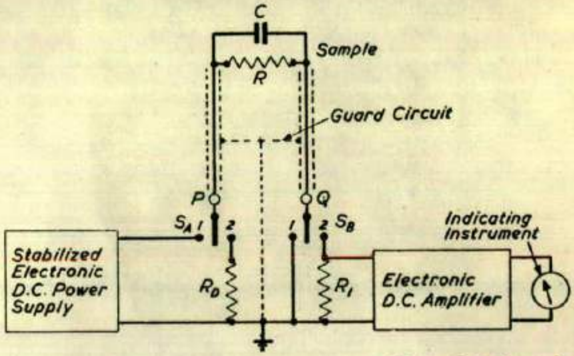
This arrangement and shape of specimen are recommended in the British Electrical and Allied Industries Research Association Report (Ref. (11)). A number of such reports, dealing with the measurement of volume and surface resistivity of insulating materials and the effect of temperature humidity, etc., upon these quantities, have been published (Refs. (8) to (13)). These reports give very full directions for the study of many different forms of insulating materials—vulcanized fibre, hard composite dielectrics, unvarnished textile fabrics, insulating oils, papers, etc., and also lay down the conditions under which the various tests should be made. Recommendations for testing procedures are also given in British Standard specifications dealing with various classes of insulating materials.

Electronic High-resistance Measuring Instruments. In recent years, electronic devices have tended to supplant the galvanometer for high-resistance measurement. A detailed discussion of the electronic techniques which have brought about this change is outside the scope of this book, but the salient features of these devices require consideration.

An electronic resistance-measuring instrument is, in fact, a development of the classical voltmeter and ammeter method of measuring resistance. The high-voltage battery necessary for high resistances, and the voltmeter, are replaced by an a.c., mains-operated, 500 V d.c. source provided with an electronic stabilizing network so that the output is very stable and accurately known. When high-resistance measurements are made on cables or capacitors, the resistance is combined with a large capacitance, and it is apparent that fluctuations in the potential of the supply will cause charging currents to flow through the capacitor. These currents may be considerably greater than the true leakage current in the resistance. The stability of the supply is therefore of paramount importance. Information on electronic stabilizers will be found in Refs. (33), (34).

The ammeter of the classical method is replaced by a sensitive valve-millivoltmeter having a very high input resistance, together with a range of high-value shunt resistors for current measurement. The higher-value resistors are of the high-stability, carbon-film type. If, for example, a voltage of 500 volts is applied to a resistance of 10^{13} ohms, a current of 5×10^{-11} amperes flows, and if this current is passed through a 50×10^{6} ohm resistor, a potential difference of 2·5 millivolts will be developed across it. This potential difference can be measured by the valve millivoltmeter consisting of a d.c. amplifier in which the input valve is of a type having a very low grid current, feeding into a permanent-magnet, moving-coil milliammeter.

Fig. 7.21 shows a schematic diagram of an electronic resistance-measuring equipment. The sample C,R is first connected across the supply for charging, with the switches S_A and S_B in position 1. When the charging period has elapsed, S_B is moved to position 2 and the current through R then flows through R_1. The potential difference across R_1 is then measured by the electronic voltmeter whose output-indicating instrument is calibrated directly in resistance values. Since R_1 can be in the region of 100 megohms, a considerable time interval would be required if the sample had to charge through R_1. The resistance R_D is provided for discharging the sample after test. An important feature is the guard circuit. The insulation resistance between the terminals and leads to the sample is effectively in parallel with the sample, but if they are separately surrounded with an earthed shield as shown dotted, then this resistance is divided into two, shunting the source and R_1 respectively. The resistance shunting the source will not affect the reading, and provided that the insulation resistance shunting R_1 is at least 100 times R_1, no significant error results. R_1 may be as



(Electronic Instruments, Ltd.)

FIG. 7.21. CONNECTIONS OF ELECTRONIC HIGH-RESISTANCE TEST SET

high as 100 megohms but insulation resistances of 10,000 megohms are not difficult to attain.

A practical instrument based on these principles is made by Electronic Instruments, Ltd., and is illustrated in Fig. 7.22. This instrument has alternative internal stabilized d.c. sources of 85 V and 500 V, and with the lower voltage has six resistance ranges covering from 3×10^5 ohms to 2×10^{12} ohms, and with the higher voltage five ranges from 3×10^7 to 2×10^{13} ohms. A charging indicator is incorporated: this gives a delay of 1½ seconds—adequate for capacitances up to 10 μF. The instrument incorporates a guard circuit and either side of the test supply can be earthed. Safety in operation is ensured by restricting the maximum possible current to 2 mA, and both hands are required to depress two widely spaced spring-loaded keys.

Portable Resistance Testing Sets. A portable, and reasonably accurate, form of testing set is often necessary in order that insulation tests may be made on cables and wiring systems after installation. A number of such sets have been developed and are manufactured by various instrument makers. Most of these sets are modifications of the ohmmeter originally designed by Ayrton and Perry. The principle of this instrument is illustrated by Fig. 7.23.

(*Electronic Instruments, Ltd.*)

FIG. 7.22. TWENTY MILLION MEGOHMMETER

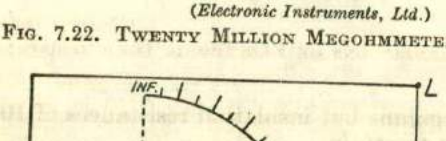


FIG. 7.23. CONNECTIONS OF SIMPLE AYRTON AND PERRY OHMMETER

Two coils, *C* and *P*, are fixed at right angles to one another, and so that their magnetic fields—when current flows through them—both exert a turning moment upon the pivoted magnetic needle *M* to which a pointer is attached. *S*,*S* are the supply terminals, the supply usually being obtained from a small generator, giving about 500 volts, which is turned by hand. P is the voltage coil and is connected, in series with a resistance, across the supply terminals. C is the current coil, connected in series with the resistance, X, to be measured. This coil carries a current which is inversely proportional to the resistance X.

The current in coil P, which is directly proportional to the supply voltage, is fixed, and is independent of the resistance to be measured. The magnetic field of this coil tends to turn the needle in an anti-clockwise direction, while the field of coil C tends to cause clockwise rotation.

The balance position of the needle is such that these two turning moments are equal. If X is infinite, there is no current in C, and the needle sets along the axis of coil P, whereas if the resistance X is very low, the turning moment of C is far greater than that of P, and the needle sets along the axis of C. The scale is graduated in resistance values (usually megohms), the intermediate points between infinity and zero being obtained by calibration.

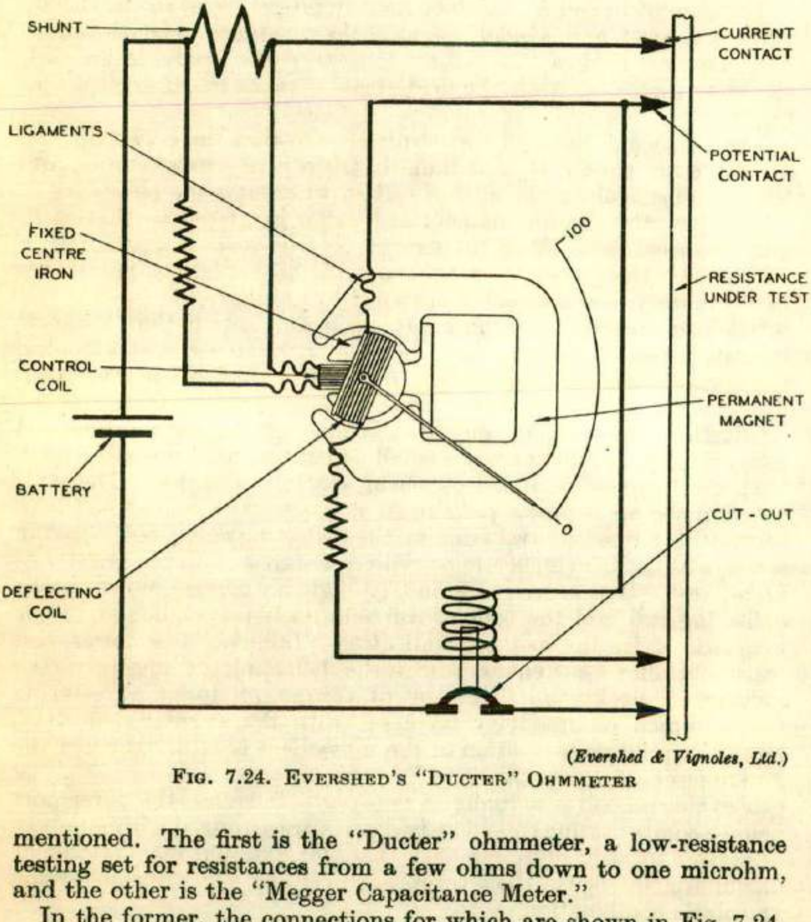
The commonest of the more modern testing sets is the "Megger" insulation tester, manufactured by Messrs. Evershed and Vignoles, the construction and connections of which are shown in Fig. 7.25. The moving system consists of two coils—the "control coil" and the "deflecting coil"—rigidly mounted at an angle to one another and connected, in parallel across a small generator, with polarities such that the torques produced by them are in opposition. The coils move in the air gap of a permanent magnet. The control coil is in series with a fixed control circuit; the deflecting coil is connected in series with a fixed deflecting-circuit resistance and the resistance under test. If this last is infinitely high no current flows in the deflecting coil and the control coil sets itself perpendicular to the magnetic axis, the pointer indicating "Infinity." A lower test resistance allows current to flow in the deflecting coil and turns the movement clockwise The control torque produces a restoring torque which progressively increases with the angular deflection, and the equilibrium position of the movement is attained when the two opposing torques balance.

The control coil is actually in two parts, in series, the outer part being a compensating coil. The two parts are arranged with numbers of turns and radii of action such that, for external magnetic fields of uniform intensity, their torques cancel one another thus giving an astatic combination.

The instrument has a small permanent-magnet d.c. generator (developing 100, 250, 500, 1,000 or 2,500 V in different instruments). This may be hand-driven, through gearing and a centrifugally controlled clutch which slips at a predetermined speed so that a steady voltage can be obtained, or it may be motor-driven.

The "Bridge-Meg" and "Bridge-Megger" testing sets, also made by Evershed and Vignoles, Ltd., can be used for insulation resistance measurements, as a Wheatstone bridge for a wide range of resistance values or for fault localization by the Varley loop method (see p. 515). Variable ratio arms and a four-decade resistance are included for these uses.

Two other instruments made by the same manufacturers should be

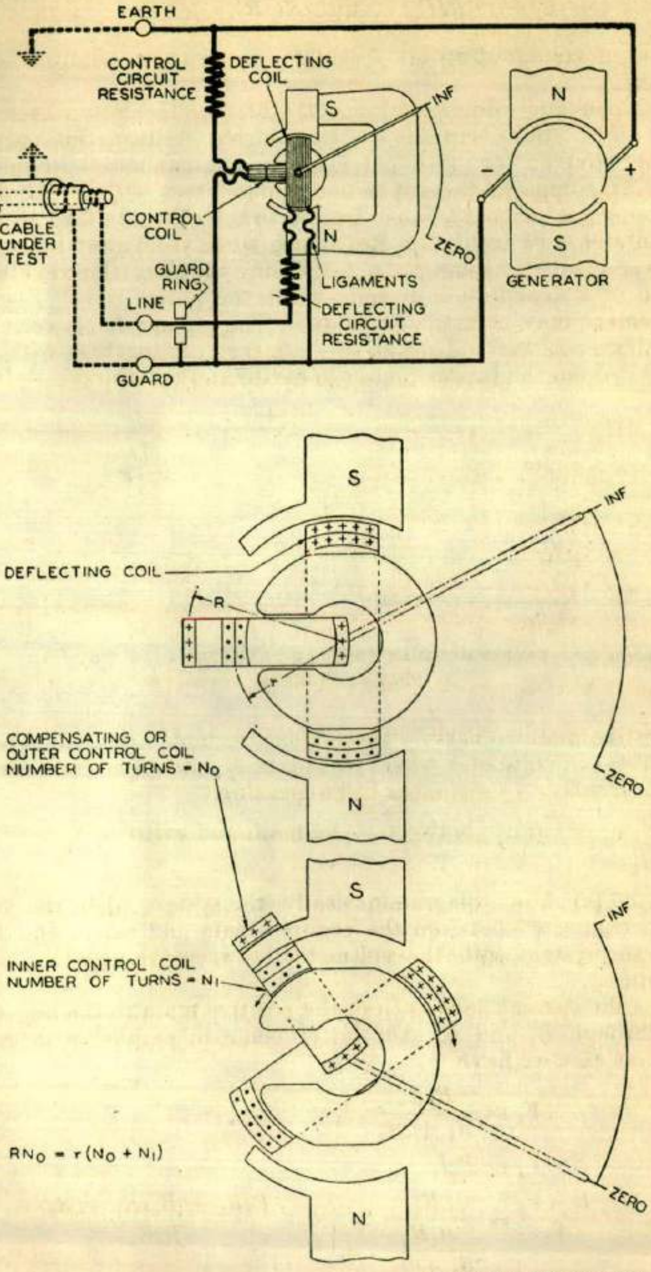


(*Evershed & Vignoles, Ltd.*)

Fig. 7.24. Evershed's "Ducter" Ohmmeter

mentioned. The first is the "Ducter" ohmmeter, a low-resistance testing set for resistances from a few ohms down to one microhm, and the other is the "Megger Capacitance Meter."

In the former, the connections for which are shown in Fig. 7.24, the control coil is connected across a shunt in the main circuit, so that the torque produced by it is proportional to the current in the resistance under test, while the deflecting coil is connected across this resistance and so carries a current proportional to the voltage drop. The position taken up by the moving system is thus dependent on



(*Evershed & Vignoles, Ltd.*)

FIG. 7.25. INTERNAL CONNECTIONS OF THE MEGGER INSULATION TESTER

the ratio of voltage drop and current, i.e. on the resistance being measured.

The Capacitance Meter resembles the Megger Insulation Tester in general form but contains a hand-driven alternating-current generator and an a.c. ratiometer movement calibrated in microfarads. It compares the capacitance under test with a standard capacitance and covers a range from 0 to 0·1 μF up to 0 to 10 μF.

Measurement of Insulation Resistance when the Power is On. It may be necessary, in some cases, to measure the insulation resistance to earth of a distribution system, while the power is on. Such a measurement may be made as follows. The voltage E between the two mains—positive and negative—is measured, together with the voltage V_1 from the positive main to earth, and the voltage V_2 from

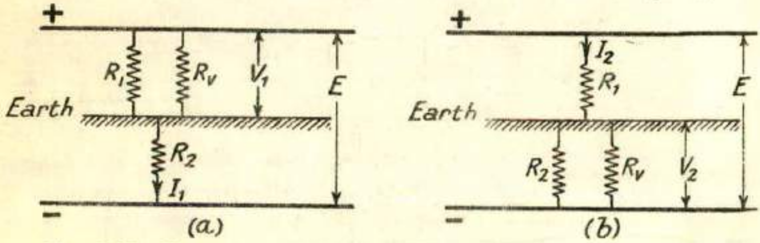


FIG. 7.26. MEASUREMENT OF INSULATION RESISTANCE WHEN THE POWER IS ON

the negative main to earth. These measurements are made with a high-resistance voltmeter whose resistance R_v should be comparable with the insulation resistances to be measured.

Let R_1 = resistance between + ve main and earth
R_2 = " " − ve " "

Fig. 7.26 (*a*) shows, diagrammatically. the system when the voltmeter is connected between the positive main and earth, and Fig. 7.26 (*b*) the system with the voltmeter between the negative main and earth.

If I_1 is the current flowing from the positive main to the negative main, through R_2 and R_1—the latter being in parallel with R_v—in the first case we have

$$V_1 = \frac{R_1R_v}{R_1 + R_v} I_1$$

and $$E - V_1 = R_2I_1$$

Thus, $$\frac{E - V_1}{V_1} = \frac{R_2}{\dfrac{R_1R_v}{R_1 + R_v}} \text{ or } \frac{E}{V_1} = \frac{R_1R_2 + R_v(R_1 + R_2)}{R_1R_v}$$

By similar reasoning, in the second case we have

$$\frac{E}{V_2} = \frac{R_1R_2 + R_v(R_1 + R_2)}{R_2R_v}$$

Hence,
$$\frac{\dfrac{E}{V_1}}{\dfrac{E}{V_2}} = \frac{\dfrac{R_1R_2 + R_v(R_1 + R_2)}{R_1R_v}}{\dfrac{R_1R_2 + R_v(R_1 + R_2)}{R_2R_v}} = \frac{R_2}{R_1} = \frac{V_2}{V_1}$$

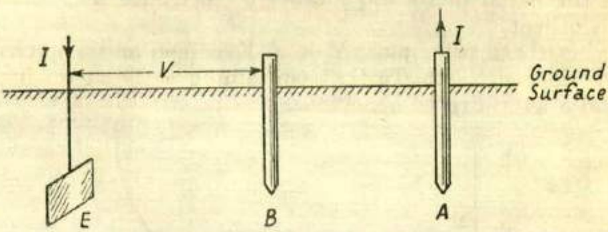


FIG. 7.27

Substituting $R_2 = R_1 \cdot \frac{V_2}{V_1}$ in the expression

$$\frac{E}{V_1} = \frac{R_1R_2 + R_v(R_1 + R_2)}{R_1R_v}$$

we have
$$\frac{E}{V_1} = \frac{R_1 \frac{V_2}{V_1}(R_1 + R_v) + R_1R}{R_1R_v} = \left(\frac{R_1 + R_v}{R_v}\right)\frac{V_2}{V_1} + 1$$

from which
$$R_1 = \left[\frac{E - (V_1 + V_2)}{V_2}\right] R_v \qquad (7.9)$$

Similarly,
$$R_2 = \left[\frac{E - (V_1 + V_2)}{V_1}\right] R_v \qquad (7.10)$$

This method cannot be used if one of the mains is earthed, and is generally only applicable if the insulation resistances to be measured are not more than 1 or 2 megohms.

The Measurement of the Resistance of Earth Connections. The resistance between an earthing plate and the surrounding ground is often an important quantity in distribution systems. It is usually measured by a fall-of-potential method as illustrated by Fig. 7.27. A current is passed through the plate E to an auxiliary electrode A in the earth at a distance away from the plate. A second auxiliary electrode B is inserted between E and A, and the potential difference, V, between E and B is measured for a given current I so that the resistance of the earth connection is $\frac{V}{I}$. The placing of the auxiliary electrodes is, however, important, and G. F. Tagg (Ref. (31)) has called attention to the errors which can arise from incorrect placing of these electrodes, especially when the earth resistance is low. He gives a curve, as in Fig. 7.28, for the resistance measured when B is at various distances from E. The correct value for the resistance of the earth connection is that measured (R_E) when B is at such a distance that the resistance lies on the horizontal part of the curve. Tagg points out that, when the earthing resistance is low, the spacing between the earth plate and auxiliary electrodes may need to be hundreds of feet.

Following these principles, Messrs. Evershed and Vignoles, Ltd., make a "Megger Earth Tester" containing a direct-reading ohmmeter and a hand-driven generator.

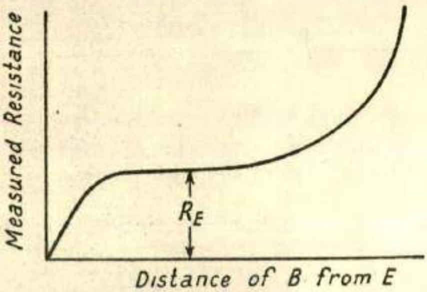


Fig. 7.28

Measurement of Resistance of Electrolytes. Owing to the fact hat a polarization e.m.f. is produced whenever a current passes through an electrolyte, the usual methods of measuring resistance cannot be used to measure the resistance of electrolytes.

Kohlrausch devised a method of measuring their resistivity. The electrolyte is contained in a glass tube having two platinum electrodes dipping into it. This cell is connected in one arm of a Wheatstone bridge network as shown in Fig. 7.29 (a). The bridge is of the slide-wire form, and is supplied from an induction coil, a telephone being used as the detector. The slide-wire may be of special form consisting of a long manganin wire wound spirally in a groove cut in a marble cylinder, the cylinder being stationary and the contact —of hard steel mounted in a manganin rod to avoid thermo-electric e.m.f.s—sliding round the cylinder. The spindle carrying the contact has a thread of the same pitch as that of the groove in which the slide-wire lies.

R is a known resistance of the same order as that of the electrolyte Balance is obtained by adjusting the sliding contact until no sound can be detected in the telephone.

Then, if l_1 and l_2 are the two lengths into which the slide-wire is divided by the sliding contact, the electrolyte resistance X is given by

$$X = \frac{l_1}{l_2} R \quad . \quad . \quad . \quad . \quad (7.11)$$

If the resistivity of the electrolyte is to be measured it is best to use a cylindrical glass tube, of uniform cross-section, supported vertically in a vessel containing the electrolyte, and with its upper end above the surface of the liquid (Fig. 7.29 (*b*)). The electrodes should be of platinum and should be circular, fitting tightly inside the cylinder. The lower electrode may be pierced to allow liquid

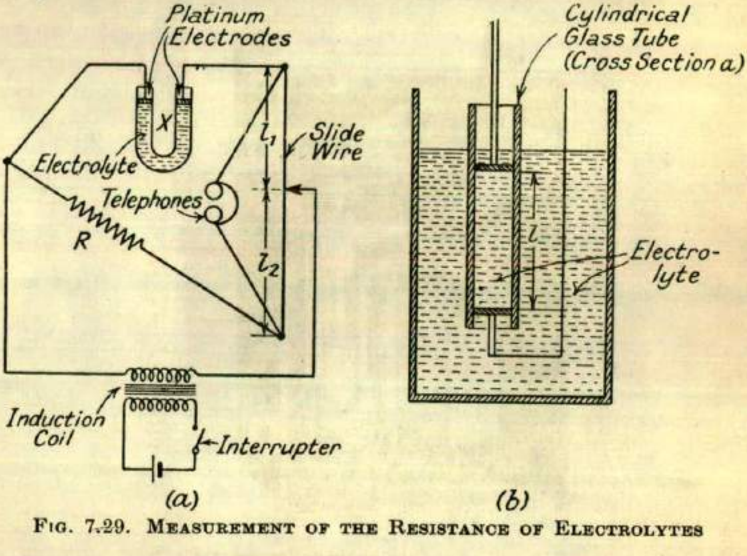


FIG. 7.29. MEASUREMENT OF THE RESISTANCE OF ELECTROLYTES

to flow through it, and the glass tube may be graduated so that the length of the column of electrolyte between the two electrodes can be accurately determined. Then, if a is the cross-sectional area of the column of electrolyte and l is its length, the resistivity is given by $\frac{Xa}{l}$, where X is the measured resistance of the column.

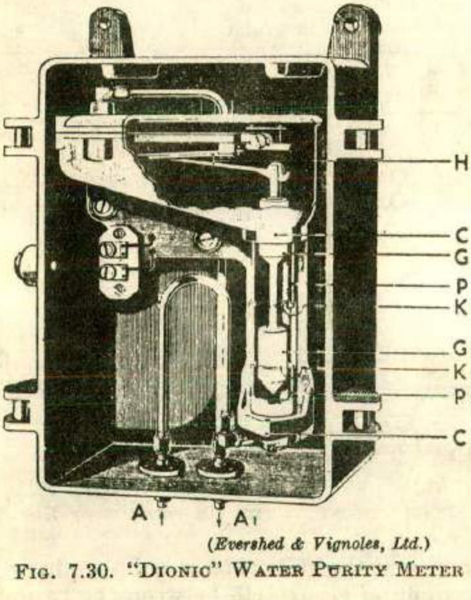
The temperature should be carefully observed when making such measurements, and this temperature stated when the results are given.

Measurement of Water Purity. Closely connected with the measurement of the resistance of electrolytes is that of the purity of water in terms of its electrical conductivity.

The Evershed and Vignoles "Dionic" Water Purity Meters (or Electric Salinometers) are calibrated in conductivity units taking as unity water having a resistance, at 20° C, of 1 megohm between the opposite faces of a centimetre cube. Then, water having a resistance of $\frac{1}{2}$ megohm-cm has a conductivity of 2 and so on. These conductivity measures can be interpreted in terms of water purity from known data concerning the influence of various concentrations of salt upon conductivity.

For example, at 20° C

0·823 grain of common salt per gallon gives conductivity	23 units
5 grains per gallon give conductivity	139 units
7 grains per gallon give conductivity	192 units



(*Evershed & Vignoles, Ltd.*)

FIG. 7.30. "DIONIC" WATER PURITY METER

The apparatus consists of a water conductivity tube (see Fig. 7.30) which is used in conjunction with an indicating (or recording) ohmmeter having a moving system of two rigidly fixed and magnetically opposing coils moving in a permanent-magnet field as in the "Megger Insulation Tester" (see p. 323). The apparatus is, however, operated from a d.c. supply.

Allowance is made in the calibration for the back e.m.f. and there is an automatic device to compensate for changes in temperature over the range 60° to 160° F. This device consists of plungers, G and G_1, which are lowered into the water tubes, P and P_1, by the bimetal levers H. This varies the effective cross-sections of the water columns under test.

These water columns are contained in two insulating tubes, P and P_1, which are separated by a third insulating tube containing two platinum electrodes, K and K_1. The testing current flows upwards through P and downwards through P_1 and the columns of water are of accurately known cross-section. Electrically the columns are in parallel because the electrodes K, K_1 are at the same positive potential, the negative electrode being the gunmetal case C, C which carries the insulating tubes. (This case is of course insulated from the case of the instrument itself and from the water

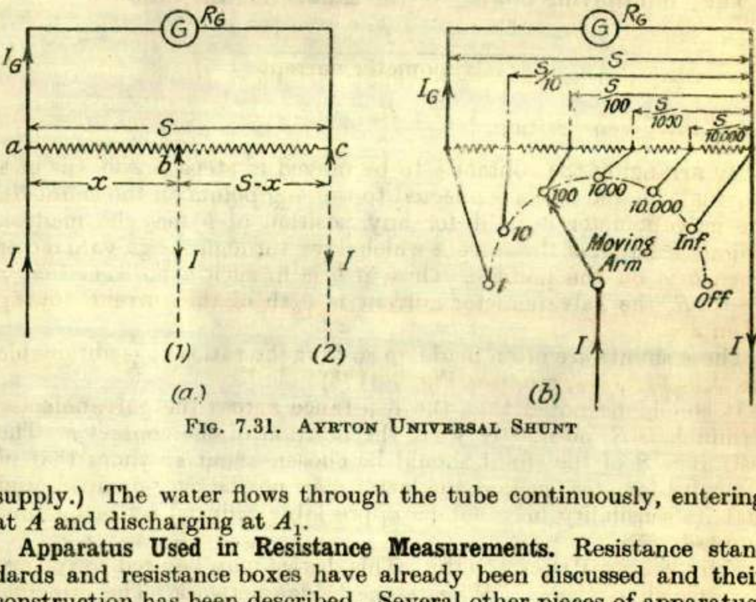


FIG. 7.31. AYRTON UNIVERSAL SHUNT

supply.) The water flows through the tube continuously, entering at A and discharging at A_1.

Apparatus Used in Resistance Measurements. Resistance standards and resistance boxes have already been discussed and their construction has been described. Several other pieces of apparatus, used in connection with resistance measurements, merit description.

AYRTON UNIVERSAL SHUNT. This is an important accessory in galvanometer work. Fig. 7.31 (*a*) shows the connections of such a shunt in diagrammatic form.

The galvanometer G, of resistance R_G, is connected across the outer terminals a,c of the shunt, whose total resistance is S; b is a moving contact and x the resistance between points a,b and depends upon the position of b.

Let I be the current flowing in the main circuit (i.e. into the parallel combination of G and S).

Then, with b in position (1) the galvanometer current is

$$I_G = \frac{x}{x + S - x + R_G} \cdot I = \frac{x}{S + R_G} \cdot I$$

Again, with b in position (2)—when $x = S$—the galvanometer current is

$$I_G' = \frac{S}{S + R_G} . I$$

Thus, in moving b from position (2) to position (1), the galvanometer current is reduced in the ratio $\frac{x}{S}$, this ratio being, therefore independent of the resistance of the galvanometer.

The "multiplying power" of the shunt—i.e. the ratio

$$\frac{I}{\text{Galvanometer current}}$$

—for any given position, is $\frac{S + R_G}{x}$

By arranging the contact b to be moved in steps (by means of a moving arm and studs connected to tapping points on the shunt S), the galvanometer current for any position of b may be made a definite fraction of the current which flows through the galvanometer when b is on the point c. Thus, if b is in such a position that $x = \frac{1}{10} . S$, the galvanometer current is $\frac{1}{10}$th of the current flowing when $x = S$.

These shunts are often made up so that the ratios of $\frac{x}{S}$ obtainable are $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, etc. (see Fig. 7.31 (*b*)).

It should be noted that the resistance across the galvanometer terminals is S, no matter what the position of the contact b. The resistance S of the shunt should be chosen about 10 times that of the galvanometer, so that the latter may not be over-damped, and that its sensibility may not be appreciably reduced by being thus shunted.

Kelvin and Varley Slide. This device may be used to replace a simple slide-wire in a Wheatstone bridge network. The principle is used, also, in the construction of potentiometers and universal shunts. It consists of a slide-wire and a number of resistance coils connected as shown in Fig. 7.32, where the apparatus is inserted in a Wheatstone bridge network.

The lower row of coils consists of eleven coils, each of resistance r ohms. Shunting two of the coils is another row of eleven coils, each of resistance $\frac{r}{5}$ ohms, the shunting connections being made through two sliding contacts which always move together, so that two coils in the bottom row are shunted by the row above, throughout. Again, two of the coils in this second row are shunted in the same way by a slide-wire whose resistance is equal to that of the two coils which it shunts, namely $\frac{2}{5}$ r. Obviously, the number of rows of coils can be increased as far as is justifiable, taking into consideration contact resistance errors, etc.

Since in each case the two coils are shunted by a resistance equal to their own, the resistance of the combination is the same as the resistance of one coil, so that the total effective resistance of each of the rows of coils is in each case that of ten coils only.

The reading of the slide in the figure is 5,346. Thus $\frac{R}{X} = \frac{5,346}{4,654}$.

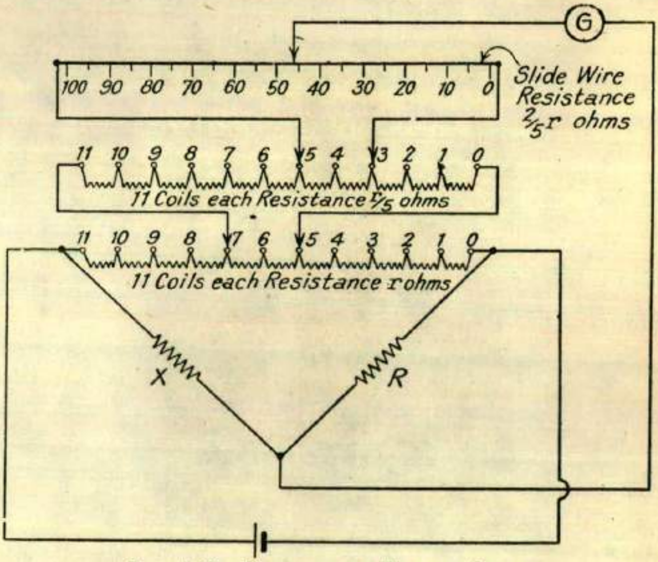


FIG. 7.32. KELVIN AND VARLEY SLIDE

THE D'ARSONVAL GALVANOMETER. This instrument, which is largely used in the various methods of measurement of resistance, and also for potentiometer work, consists essentially of a circular or rectangular coil of many turns of fine wire suspended between the poles of a permanent magnet. There is often a fixed cylindrical iron core inside the coil, the coil sides being situated in the two air gaps between this core and the permanent magnet. The length of the air gaps between the coil and pole faces, and between the coil and core, is usually about $\frac{1}{16}$ in., and the pole faces are shaped so as to give a radial field. The suspension is a single fine strip of phosphor-bronze, and serves as one lead to the coil, the other lead taking the form of a loosely coiled spiral of fine wire leading downwards from the bottom of the coil. The suspension carries a small mirror upon which a beam of light is cast through a glass window in the outer brass case surrounding the instrument. The beam of light is reflected on to a scale—usually at a distance of 1 metre from the mirror—upon which the deflection is measured. A torsion head is provided for adjustment of the coil position and zero setting.

In order to save time in using the galvanometer, damping is provided by winding the coil on a light metal former. The damping is produced by the torque—opposing motion—due to the permanent-magnet field in conjunction with currents which are induced in the metal former when it rotates in this magnetic field.

(*H. Tinsley & Co., Ltd.*)

FIG. 7.33. SUSPENSION-TYPE GALVANOMETER

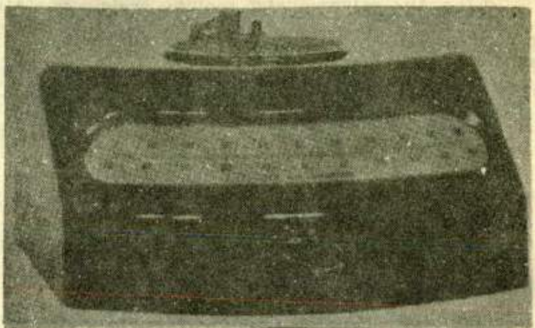
Damping may also be obtained by connecting a fairly low resistance across the galvanometer terminals. The damping then being dependent upon the magnitude of this resistance, it is possible, by suitably adjusting the resistance, to make the damping critical.

Fig. 7.33 shows a portable galvanometer of high sensitivity (up to 1,500 mm/microamp at 1 metre) with a moving coil which is tautly suspended at both ends by silver-gilt suspension strip. This form of suspension is independent of level.

Fig. 7.34 shows a completely self-contained galvanometer with built-in lamp and scale.

The Cambridge Instrument Co. make a very robust form of galvanometer which requires no levelling or clamping. This is the Cambridge "Pot" Galvanometer. It is fitted with a pointer as well as a mirror for use with lamp and scale, and gives a deflection per microampere of 12 millimetres at a scale distance of 1 metre. Its resistance is 50 ohms and its period 1·3 seconds.

K. Copeland, A. C. Downing and A. V. Hill (Ref. (25)) describe a moving-coil galvanometer of extreme sensitivity. Used with photo-electric amplification, the instrument is capable, in a few seconds, of reading to a few thousandths of a microvolt in a 50-Ω circuit. It is stable enough to allow 20-fold to 50-fold magnification under laboratory conditions and the magnified deflection is read on a microammeter. Two of these authors have also described (*Jour. Sci. Insts.*, Vol. 25, pp. 225 and 230) a rapid galvanometer which can record to within 0·02 μV in 0·01 sec.



(Cambridge Instrument Co., Ltd.)
FIG. 7.34. SPOT GALVANOMETER

H. W. Sullivan, Ltd. have introduced a very compact, sensitive, yet robustly suspended galvanometer fitted with shock-absorbing stops which permit overloads of up to 100 times full-scale current without risk of damage to the movement. Sensitivities up to 2,300 mm/μA can be obtained with a periodic time not greater than 2 seconds.

Theory. Let i be the current (assumed constant) flowing through the galvanometer coil. Then the equation of motion of the galvanometer is, from Equation (6.44) (Chapter VI),

$$a \frac{d^2\theta}{dt^2} + b \frac{d\theta}{dt} + c\theta = Gi$$

where θ = the deflection in radians
t = time in seconds
a = moment of inertia of the moving system
b = the damping constant
c = the restoring constant
G = the displacement constant

(see theory of the vibration galvanometer, Chapter VI).

Referring to Equations (6.46) and (6.49) (*loc. cit.*), we have for the solution of the above equation

$$\theta = Ae^{m_1t} + Be^{m_2t} + \frac{Gi}{c} \quad . \quad . \quad . \quad . \quad (7.12)$$

since the current is now constant and equal to i, ω in Equation (6.49) being zero. A and B are constants to be determined from the initial conditions.

Let θ_D be the final steady deflection of the galvanometer. Then $\theta_D = \frac{Gi}{c}$ the expression $Ae^{m_1t} + Be^{m_2t}$ representing a motion which may, or may not, be oscillatory, according to the relative values of the constants a, b, and c.

To determine A and B, suppose that when t is zero, the deflection θ is zero and also $\frac{d\theta}{dt} = 0$ (i.e. the galvanometer moving system is stationary in its zero position).

Then, since when $t = 0$, $\theta = 0$,

$$0 = A + B + \frac{Gi}{c} = A + B + \theta_D$$

Also, since when $t = 0$, $\frac{d\theta}{dt} = 0$,

$$0 = Am_1 + Bm_2$$

Hence,

$$A = \frac{m_2\theta_D}{m_1 - m_2} \text{ and } B = \frac{-m_1\theta_D}{m_1 - m_2}$$

and the equation for θ becomes

$$\theta = \theta_D - \theta_D\left[\frac{m_1}{m_1 - m_2}\,e^{m_2t} - \frac{m_2}{m_1 - m_2}\,e^{m_1t}\right] \quad . \quad . \quad (7.13)$$

Now, $m_1 = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $m_2 = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$

Thus, if $b^2 > 4ac$, both m_1 and m_2 are real and negative. The motion of the galvanometer is thus non-oscillatory, the deflection gradually rising from zero to its maximum value θ_D. The galvanometer under these conditions is said to be "*over-damped.*" If $b^2 < 4ac$, both m_1 and m_2 are imaginary. Then referring to Equation (6.50), the equation of motion is

$$\theta = \theta_D + e^{-\frac{b}{2a}t}F\sin\left(\frac{\sqrt{4ac - b^2}}{2a}t + \alpha\right) \quad . \quad . \quad (7.14)$$

This equation represents an oscillatory motion, the oscillations dying away gradually as the time t is increased, giving finally a steady deflection θ_D.

If f is the frequency of these oscillations, then

$$2\pi f = \frac{\sqrt{4ac - b^2}}{2a}$$

or

$$f = \frac{\sqrt{4ac - b^2}}{4\pi a} \quad . \quad . \quad . \quad . \quad (7.15)$$

The galvanometer, under these conditions, is *under-damped*. If there is no damping $b = 0$ and $T = 2\pi\sqrt{\frac{a}{c}}$ (where T = the periodic time of the oscillations).

If $b^2 = 4ac$, then $m_1 = m_2 = -\frac{b}{2a}$.

In this case of equal roots ($m_1 = m_2$), the general solution for θ takes the form

$$\theta = \theta_D + e^{-\frac{b}{2a}t}[A + Bt] \quad . \quad . \quad . \quad (7.16)$$

To find A and B, let $\theta = 0$ and $\frac{d\theta}{dt} = 0$ when $t = 0$.

Then $$0 = \theta_D + A$$

or $$A = -\theta_D$$

and $$0 = B - \frac{b}{2a} . A$$

or $$B = -\frac{b}{2a} . \theta_D$$

Hence, $$\theta = \theta_D - \theta_D e^{-\frac{bt}{2a}}\left[1 + \frac{b}{2a}t\right] \quad . \quad . \quad . \quad (7.17)$$

Under these conditions the motion of the galvanometer is just non-oscillatory and the damping is said to be "*critical*."

The undamped natural frequency of the instrument is $f = \frac{1}{2\pi}\sqrt{\frac{c}{a}}$ (see p. 278) and, with critical damping, $b^2 = 4ac$ so that $f = \frac{1}{2\pi}\sqrt{\frac{b^2}{4a^2}}$ or $\frac{b}{2a} = 2\pi f$. The equation for the deflection becomes

$$\theta = \theta_D(1 - e^{-2\pi ft} - 2\pi fte^{-2\pi ft})$$

Now, when $t = \frac{1}{f}$, the value of the factor in brackets is

$$1 - e^{-2\pi} - 2\pi e^{-2\pi} = 1 - 0{\cdot}00185 - 0{\cdot}0116 = 0{\cdot}9865$$

Thus, in a time equal to the undamped periodic time, the instrument will, when critically damped, reach a deflection equal to 98·7 per cent of its final deflection when a current I is suddenly applied.

Fig. 7.35 shows the forms of the deflection/time curves in the three cases when the galvanometer is (*a*) over-damped, (*b*) under-damped, (*c*) critically damped. The curves on the left show the rise of the deflection, starting at $\theta = 0$ when $t = 0$, while those on the right show the dying away of the deflection, starting at $\theta = \theta_D$ when $t = 0$.

Influence of the Resistance of the Galvanometer Circuit upon the Damping. In the above, the damping constant b was assumed to be dependent merely upon air friction and elastic hysteresis in the suspension. If the coil is wound upon a metal former, an e.m.f. will be induced in this former when it moves through the magnetic field of the permanent magnet. A current will flow in a closed circuit in the former, and will produce damping even though the galvanometer circuit may be open. The induced e.m.f. is proportional to the angular velocity of the coil $\frac{d\theta}{dt}$ and this additional damping may be taken into account by making the original damping, b, now b'.

If the galvanometer circuit is closed, R being the resistance of this circuit, the current flowing through the galvanometer—neglecting its inductance—will be given by

$$Ri = E - G \cdot \frac{d\theta}{dt}$$

where E is the voltage applied to the galvanometer circuit. The term $G\frac{d\theta}{dt}$ represents a back e.m.f. induced in the coil due to its motion through the magnet field, G being $NBlr$, N being the number of turns on the coil, l its active length, r its breadth and B the flux density of the permanent-magnet field.

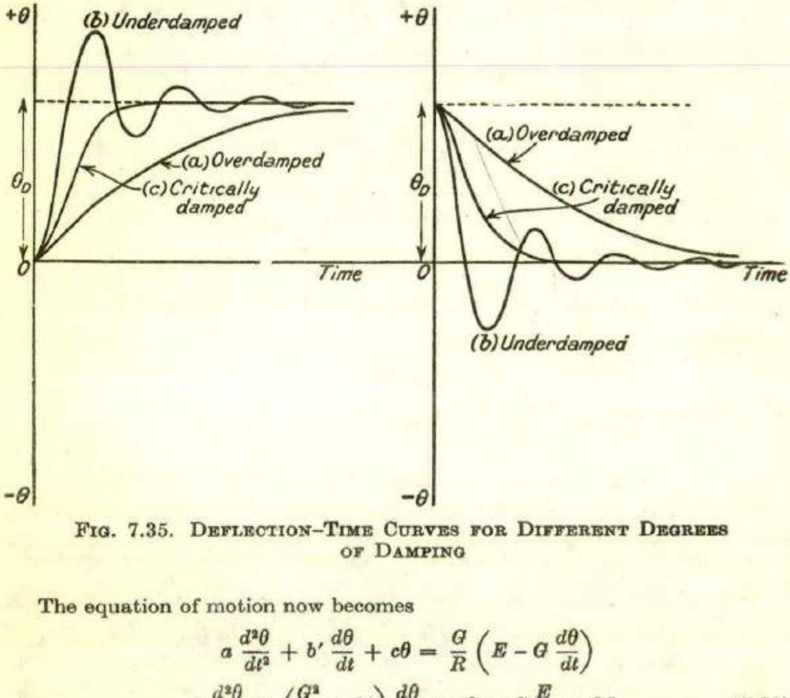


FIG. 7.35. DEFLECTION–TIME CURVES FOR DIFFERENT DEGREES OF DAMPING

The equation of motion now becomes

$$a\frac{d^2\theta}{dt^2} + b'\frac{d\theta}{dt} + c\theta = \frac{G}{R}\left(E - G\frac{d\theta}{dt}\right)$$

or

$$a\frac{d^2\theta}{dt^2} + \left(\frac{G^2}{R} + b'\right)\frac{d\theta}{dt} + c\theta = G\frac{E}{R} = GI \quad . \quad . \quad (7.18)$$

where I is the final value of the galvanometer current when the coil has attained its steady deflection. The effective damping constant is now, therefore, $\left(\frac{G^2}{R} + b'\right)$, and this now replaces b in the original theory.

If b' is small compared with $\frac{G^2}{R}$ the galvanometer is critically damped when $\left(\frac{G^2}{R}\right)^2 = 4ac$, and since G, a, and c are constants for any particular instrument, critical damping may be obtained by variation of R.

It is important, in order to save both time and trouble in using such galvanometers, that the damping shall be properly adjusted, and also that the sensitivity of the galvanometer chosen for a particular measurement shall not greatly exceed that demanded by the work in hand.

Measurement of the Galvanometer Constants. The constants of a D'Arsonval galvanometer can be readily determined provided that the instrument is not fitted with a metal coil-former, or a short-circuited turn, for damping purposes. Four measurements are necessary: these are to determine—

(1) The free period of the galvanometer on open circuit, T.
(2) The external resistance for critical damping, R_s.
(3) The coil resistance, R_c.
(4) The sensitivity S in radians per ampere.

The undamped free period is given by $T = 2\pi\sqrt{\frac{a}{c}}$. If the air damping b' is small compared with $\frac{G^2}{R}$, then $\left(\frac{G^2}{R}\right)^2 = 4ac$, where $R = R_c + R_s$.

The steady-state sensitivity $S = \frac{\theta}{I} = \frac{G}{c}$.

By combining these expressions we obtain the following relationships for the galvanometer constants—

$$G = \frac{T(R_c + R_s)}{\pi S} \text{ newton-metres per amp} \quad (7.19)$$

$$c = \frac{T(R_c + R_s)}{\pi S^2} \text{ newton-metres per rad} \quad (7.20)$$

$$a = \frac{T^3 R}{4\pi^3 S^2} \text{ kilogramme-metre}^2 \quad (7.21)$$

(1) The free period of the galvanometer may be measured by discharging a small charged capacitor through the galvanometer and timing a succession of swings with a stop-watch.

(2) The external critical damping resistance may be measured by connecting a resistance box across the galvanometer, discharging a capacitor through the combination, and adjusting the resistance box until the galvanometer just fails to overshoot when it returns to zero.

(3) The coil resistance is usually measured by Kelvin's false-zero method, since the permissible coil current is too low to permit a normal bridge measurement. Referring to Fig. 7.36, R is a high resistance whose value is chosen so that the galvanometer G gives nearly full-scale deflection with the key K open. One of the arms P or S may be varied until there is no change in the deflection of the galvanometer when K is open or closed, and there is therefore no p.d. between the points A and B. Then the normal Wheatstone Bridge balance equation applies—

$$R_c = S \cdot \frac{Q}{P}$$

(4) The galvanometer sensitivity may be measured by the circuit arrangement given in Fig. 7.37. R_1 is a low resistance of the order of 100 ohms and R_2 is a high resistance of about 1 megohm. The current I is adjusted until the galvanometer gives full-scale deflection,

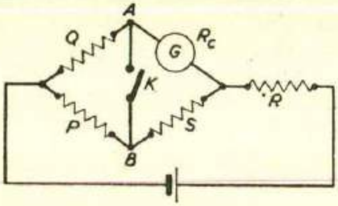


FIG. 7.36. KELVIN'S FALSE-ZERO METHOD

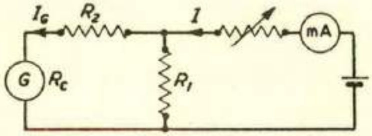


FIG. 7.37. CIRCUIT FOR SENSITIVITY MEASUREMENTS

and if the circuit values are correctly chosen, I may be measured by a milliammeter. The galvanometer current is

$$I_G = \frac{IR_1}{R_1 + R_2 + R_c}$$

and, since $R_2 \gg R_c$ and R_1,

$$I_G = \frac{R_1}{R_2} \cdot I$$

The galvanometer sensitivity is expressed as $\frac{\theta}{I_G}$, where θ is the angular deflection of the coil in radians. If the galvanometer is of the spotlight reflecting type, the angular deflection of the coil is half that of the reflected light beam.

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CHAPTER VIII

POTENTIOMETERS

A POTENTIOMETER is, essentially, a piece of apparatus by means of which e.m.f.s. are compared. If one of two e.m.f.s. is known, the other may be determined by comparison with the known one, and thus the potentiometer is used for the measurement of e.m.f.s by comparison with a standard e.m.f. It may also be applied to the measurement of current and resistance by methods which will be described below.

Potentiometers for Use with Direct Current. The principle of the potentiometer is illustrated by Fig. 8.1, which shows the connections of the most elementary form. A battery B sends a current

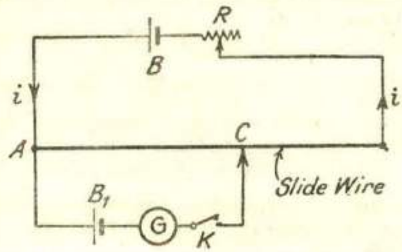


FIG. 8.1. PRINCIPLE OF THE POTENTIOMETER

through a slide-wire of uniform cross-section, R being a regulating resistance to limit the slide-wire current. B_1 is a battery whose e.m.f. is to be measured. This is connected in series with a galvanometer G and a key K, *the polarity of* B_1 *being as shown.*

Suppose that r is the resistance per unit length of the slide-wire, and that i is the current in it when the key K is open. Then, if the length AC is l, the voltage drop across AC is irl.

If the key K is now closed, a current will flow through the galvanometer in the direction A to C if the voltage drop across the length l of the slide-wire is greater than the e.m.f. of battery B_1. (Note that the battery B_1 is connected so as to *oppose* the passage of this current.) If these two e.m.f.s. are equal no current will flow through the galvanometer.

Suppose, now, that the e.m.f.s. of two batteries B_1 and B_2 are to be compared. Then, the first battery B_1 is inserted, as shown in Fig. 8.1, in series with the galvanometer, and the sliding contact C is adjusted until no current flows through the galvanometer. Let the length AC then be l_1. B_1 is then replaced by B_2 and the contact

C again adjusted until no current flows through G. Let the length AC then be l_2.

Then, if $\quad E_1 =$ e.m.f. of battery B_1

$\quad\quad\quad E_2 =$,, ,, ,, B_2

(obviously, both E_1 and E_2 must be less than the e.m.f. of the supply battery B)

we have $\quad E_1 = irl_1$ and $E_2 = irl_2$

so that
$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

A scale is provided in this elementary form of potentiometer, so that l_1 and l_2 may be read off, and the ratio $\frac{l_1}{l_2}$ gives the ratio of the two e.m.f.s as shown above.

If one of the batteries (say B_2) is a standard cell of known voltage, E_2, the e.m.f. of battery B_1, is given by

$$E_1 = \frac{l_1}{l_2} \times E_2$$

Precautions. The supply battery B should be of ample capacity so that the current i in the slide-wire may remain constant throughout the test. A resistance should be connected in series with the galvanometer, or a universal shunt used, for protection during the initial adjustment of the contact C, this resistance being cut out as balance (i.e. zero deflection) is obtained. Such a resistance is also necessary in order that no appreciable current shall be taken from the standard cell—when inserted in the galvanometer branch—during the preliminary adjustment of C. The e.m.f. of a standard cell cannot be relied upon if it is allowed to give any appreciable current.

It should be noted that when the potentiometer is balanced no current is passing through the battery under test, so that the e.m.f. measured is the open-circuit e.m.f. of the battery.

Obviously, in the above elementary form of the potentiometer, the accuracy of measurement depends to a large extent upon the accuracy with which the ratio $\frac{l_1}{l_2}$ can be determined. Assuming the same error in reading l_1 and l_2 to be made, no matter what the length of the slide-wire, the longer the slide-wire the less the percentage error in measurement due to these constant errors in l_1 and l_2. In the modern forms of potentiometer designed for precise measurements, the effect of a very long slide-wire is obtained by connecting a number of resistance coils in series with a comparatively short slide-wire, as described below.

The Crompton Potentiometer. R. E. Crompton first modified the simple slide-wire form of potentiometer described above, the general arrangement of his form of the instrument being shown in Fig. 8.2.

A graduated slide-wire AC is connected in series with fourteen (or more) coils, each of which has a resistance exactly equal to that of the slide-wire (of the order of 10 ohms). There are two moving contacts, P_1 and P_2, sliding over the slide-wire, and the studs of the resistance coils, respectively. B is the supply battery (2 volt), and R_1 and R_2 are two variable resistors, the former consisting of a number of coils for coarse adjustment of the potentiometer current, and the latter taking the form of a slide-wire for fine adjustment.

The galvanometer G is connected in series with a key K, and a multiple circuit switch, by means of which either the standard cell S or the e.m.f. to be measured can be connected in the galvanometer circuit. The terminals to which the apparatus under test is connected are marked positive (+) and negative (–) to avoid the

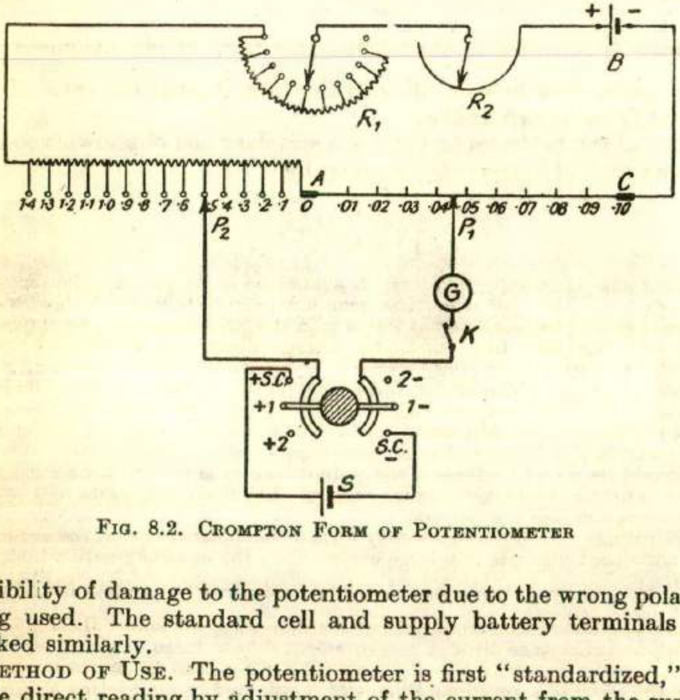


FIG. 8.2. CROMPTON FORM OF POTENTIOMETER

possibility of damage to the potentiometer due to the wrong polarity being used. The standard cell and supply battery terminals are marked similarly.

METHOD OF USE. The potentiometer is first "standardized," i.e. made direct reading by adjustment of the current from the supply battery as follows.

A standard cell—usually of the Weston type, e.m.f. 1·0186 volts—is connected to the terminals marked *S.C.* (*being sure to connect with the correct polarity*). The galvanometer is at first heavily shunted, or is connected in series with a high resistance,* for protection. The potentiometer is then set to read, directly, the e.m.f. of the standard cell—corrected to the room temperature, if necessary. If a Weston cell is used, contact P_2 will be placed on stud 1·0 and

* If the galvanometer is shunted it may be necessary to include a series resistance for the protection of the standard cell.

contact P_1 on 0·0186 on the slide-wire. Resistors R_1 and R_2 are then adjusted until no deflection of the galvanometer is observed with the galvanometer shunt adjusted to give full sensitivity. Leaving the resistors R_1 and R_2 at the settings so obtained, switch over the selector switch to terminals 1, 1, to which the battery whose e.m.f. is to be measured has been connected. Shunting the galvanometer, at first, for protection, adjust P_2 and, finally, P_1, until the potentiometer is again balanced. The reading of the potentiometer will then give the e.m.f. to be measured, directly. If the adjustment of the potentiometer to obtain balance, in this second case, takes any appreciable time, it is advisable to check the initial standardization again. In order to obtain steadiness of the potentiometer current during a test, it is well to allow the

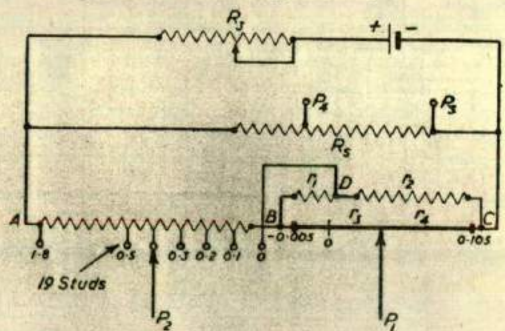


FIG. 8.3. SLIDE-WIRE POTENTIOMETER WITH TRUE ZERO AND INDEPENDENT STANDARDIZING CIRCUIT

current to flow through the potentiometer for a few minutes before making a measurement.

The Modern Form of Slide-wire Potentiometer. There are some obvious disadvantages in the type of potentiometer just considered. It is impossible to arrange for the points P_1 and P_2 to coincide and a true zero reading cannot be obtained. In addition, it is desirable to check the standardization regularly during a series of measurements, and this is inconvenient since it involves resetting the main dials. These two drawbacks are eliminated in the modified potentiometer shown in Fig. 8.3.

The slide-wire *BC* is provided with a shunt resistor which is tapped at *D*, this tapping being made the zero stud on the main dial. When the contact P_1 is in a position such that $\frac{r_1}{r_3} = \frac{r_2}{r_4}$ there is no potential difference between the zero stud and P_1; hence the slide-wire has a true zero. The slider can travel a short distance to the left of the zero position, giving a small negative reading, and movement to the right gives positive readings. The range of the slide-wire is usually from $-0{\cdot}005$ volt to $0{\cdot}105$ volt, and the resistance of the slide-wire and its shunt is slightly greater than that of one step on the main dial.

Standardization in this potentiometer is carried out with the aid of the shunt resistor R_s between A and C. This resistor is tapped at points P_3 and P_4, which are so chosen that a potential difference equal to the e.m.f. of the standard cell (1·01859 volts) exists between them when the potentiometer is correctly standardized. Thus the potentiometer may be standardized independently of the main dial by connecting a galvanometer and standard cell between P_3 and P_4

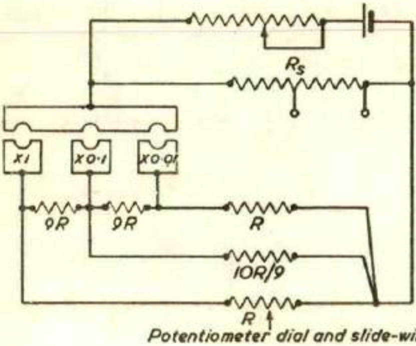


FIG. 8.4. RANGE-CHANGING NETWORK

and adjusting the potentiometer current until balance is achieved This circuit is known as an independent standardizing circuit.

A potentiometer of this type is normally provided with three ranges, $\times$ 1, $\times$ 0·1 and $\times$ 0·01. The range multiplier is shown schematically in Fig. 8.4. The potentiometer resistance is represented by R, and with the resistance values shown, transfer of the range-change plug from $\times$ 1 to $\times$ 0·1 reduces the current in the main dials and slide-wire to one-tenth of its $\times$ 1 value, leaving the total current taken from the battery unchanged. With the plug in the $\times$ 0·01 position the dial and slide-wire current is reduced to one-hundredth of its $\times$ 1 value, again with the total current unchanged. When a range multiplier is provided, the independent standardizing circuit R_s is connected as shown and the potentiometer can be standardized whatever the position of the range multiplier and the main dials.

Constructional Details. All the resistors in a potentiometer, with the exeption of the slide-wires, are usually of manganin because of its high stability, low temperature coefficient and freedom from thermo-electric effect against copper. The accuracy of potentiometers depends upon a very simple principle: if a potential divider is constructed from a number of similar coils all wound from the same batch of wire and annealed together, any subsequent drift appears equally in all the coils, and although the overall resistance value changes, the accuracy of potential division is unimpaired. It is possible to maintain an accuracy of potential division nearly 100 times the overall accuracy of the resistance value of the divider. Potentiometers which realize this accuracy are considered subsequently, but in the case of the simple potentiometer just described the accuracy is limited by the slide-wire.

The most common slide-wire material is platinum–silver alloy and the sliding contact is a copper–silver–gold alloy. The choice of contact and wire material is determined by the necessity to ensure a good potential contact, freedom from thermo-electric e.m.f.s, and minimum wear on the slide-wire. Clearly the slide-wire and its shunt will have different drift characteristics from the main dial resistances, but in a good design the error introduced by a slide-wire is not worse than $\pm$ 0·2 per cent of its full setting. Taking an example, if the potentiometer is on the $\times$ 1 range and the slide-wire covers 0–0·1 volt, the slide-wire introduces a constant error of $\pm$ 0·2 millivolt. At the 1 volt setting this gives an error of $\pm$ 2 in 10^4, and an error of $\pm$ 1 in 10^3 at a setting of 0·2 volt.

Potentiometers are designed for currents between 10 and 50 mA. The larger current is advantageous with slide-wires because a comparatively heavy wire may be used, thus minimizing the effects of wear.

Current Regulator. The current-regulating resistors R_1 and R_2 in Fig. 8.2 are of considerable importance. It must be possible to adjust the current with a definition better than the accuracy of the potentiometer, and the moving contacts of the regulator resistors must not be subject to resistance variations once set in position. This requirement is easily met in R_1, which has a high-quality stud switch. The other resistor R_2 is usually a slide-wire. The regulating resistor R_3 in Fig. 8.3 is a special form of slide-wire wound in a double spiral in a groove cut in an insulated cylinder. Two contacts are fitted, the length of wire included between them being varied by rotation of the cylinder, which is mounted on a spindle having a screw thread cut on it so that rotation causes the cylinder to move up or down in the direction of its axis. A multi-turn slide-wire of this type usually gives sufficient definition in a slide-wire potentiometer of the type shown in Fig. 8.3 and the stud dial can be dispensed with. The multi-turn slide-wire, together with the stud dial, is often used in the higher-precision potentiometers described further on in this work.

Internal Thermo-electric E.M.F.s. It is very important that there shall be no appreciable thermo-electric e.m.f.s. within the potentiometer itself, as such e.m.f.s would obviously affect the readings. The use of manganin resistance wire, which has a low thermo-electric e.m.f. with copper, assists the attainment of this objective. It is desirable also that all parts of the potentiometer shall be at the same temperature; a normal practice is to include all the switch and resistance work in a covered case. ..is has other advantages in that the switches and resistance work are protected from atmospheric fumes, which may cause corrosion and the appearance of small voltaic e.m.f.s at the joints. Potentiometers designed specifically for the measurement of thermocouple e.m.f.s usually have copper terminals.

Leakage. The working parts of the potentiometer are normally mounted on an ebonite or keramot panel, and it is essential that there shall be no surface leakage. The enclosure of the potentiometer previously mentioned helps in that the deposition of moisture on the insulating panel is prevented. Leakage can be troublesome from external circuits if the potentiometer is not at earth potential, and care must be exercised when potentiometers are used to carry out measurements in circuits above earth potential.

THE VERNIER POTENTIOMETER. The limitation in potentiometer performance due to the slide-wire is removed in the Vernier potentiometer. Fig. 8.5 gives a simplified circuit diagram of a modern

Vernier potentiometer. The instrument has two ranges: the normal range of 1·80100 volt down to 10 microvolts; and a lower range of 0·180100 volt down to 1 microvolt. The accuracy of potential division is usually better than 1 part in 10^5 at the 1 volt setting.

There are three measuring dials. The first dial measures up to 1·7 volts (on the $\times$ 1 range), in steps of 0·1 volt; the middle dial has 102 studs and reads up to 0·1 volt in steps of 0·001 volt; while

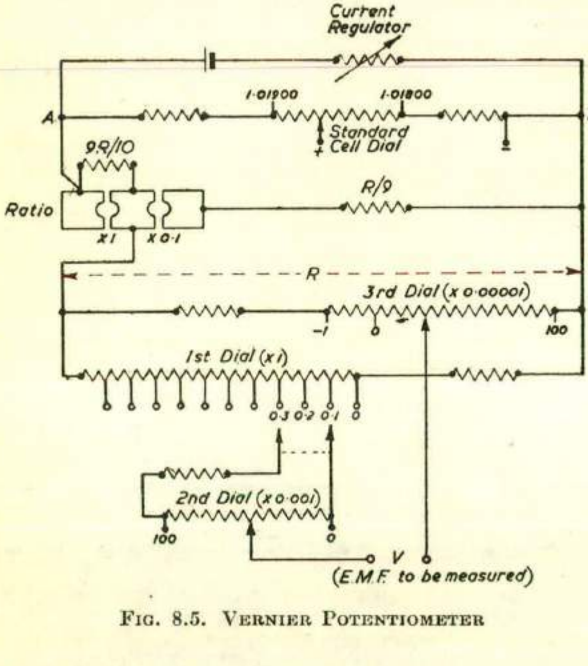


FIG. 8.5. VERNIER POTENTIOMETER

the last dial also has 102 studs and reads from $-$ 0·00001 volt to $+$ 0·001 volt in steps of 0·00001 volt. There is no slide-wire. The resistances of the middle dial shunt two of the coils of the first dial, the moving arm of which carries two contacts spaced two studs apart, as shown. This follows the principle of the Kelvin–Varley slide described in the previous chapter. In actual practice the resistance of the second dial is greater than that between two studs in the main dial, so that the voltage drop across it is greater than 0·1 volt; otherwise the voltage drop in the switch contact resistances and leads would restrict the coverage to less than 0·1 volt. The third dial is obtained from a shunt circuit which permits a true zero and a small negative setting to be obtained.

The lower range of the potentiometer is obtained by moving the plug in the contact blocks marked "ratio" from the position "× 1" to "× 0·1." This inserts a resistance equal to nine-tenths of the total resistance of the working portion of the potentiometer in series with the latter, and at the same time shunts the working portion by a resistance of one-ninth of its total resistance. This reduces the current on the measuring dials to one-tenth of its normal value (10 mA), whilst keeping the battery current the same.

The potentiometer is provided with an independent standardizing circuit *AB* which can be set to any value of standard cell e.m.f. between 1·01800 and 1·01900 in steps of 50 microvolts. Thus, a considerable range of variation in the standard cell temperature can be allowed for. If the full accuracy of the potentiometer is to be attained, it is essential to standardize the potentiometer on the main dials and use the independent standardizing circuit for monitoring purposes.

The method of current regulation used in some of these potentio-

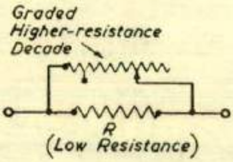


FIG. 8.6. VARIABLE LOW RESISTOR

meters is of particular interest. If a slide-wire is used in the regulator it has to be extremely well made if the variations in contact resistance are not to limit the precision in adjustment of the potentiometer current. This difficulty may be overcome by the use of a shunted dial resistance for the regulator. The principle may be understood by reference to Fig. 8.6. A low resistance *R* is shunted by a series of adjustable high-resistance dials; if, for example, *R* is 0·5 ohm and it is shunted by 12·0 ohms, this is equivalent to a change in the resistance of *R* of 0·02 ohm. The resistance increments in the shunt dials are not equal, being graded to give equal increments in resistance change. It is evident that changes in contact resistance take place in high-resistance circuits and have negligible effect on the total resistance. Regulators constructed on this principle usually have 5 dials giving changes of 10 × 2, 10 × 0·2, 10 × 0·02, 10 × 0·002 and 10 × 0·0002 ohms, but only the lower three dials are of the shunted type. Vernier potentiometers, including some or all of the features discussed, are made by a number of manufacturers, including the Cambridge Instrument Co., Ltd., the Croydon Precision Instrument Co., W. G. Pye, Ltd., and H. Tinsley & Co., Ltd.

THE CONSTANT-RESISTANCE DEFLECTIONAL POTENTIOMETER (Ref. (3)). In this type of potentiometer the resistance of the galvanometer circuit is kept constant for all positions of the moving dial contacts, and provided that the voltage range of the potentiometer is small compared with the voltage of the supply battery, the deflection of the galvanometer is at all times proportional to the out-of-balance e.m.f. This potentiometer is largely used in metallurgical work for the measurement of high temperatures with thermocouples. When cooling curves are being taken, the temperature is changing continuously and it is impossible to balance a conventional potentiometer. In using this potentiometer, however, exact balance is not aimed at; the main dial is adjusted to its nearest setting for the e.m.f. being measured, and the out-of-balance e.m.f. is then read from the calibrated galvanometer scale.

Fig. 8.7 gives a simplified diagram of connections for this

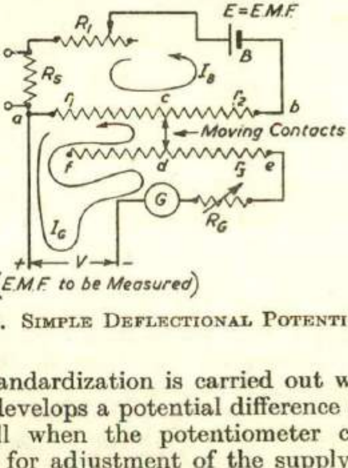


FIG. 8.7. SIMPLE DEFLECTIONAL POTENTIOMETER

potentiometer. Standardization is carried out with the aid of the resistor R_s, which develops a potential difference equal to the e.m.f. of a standard cell when the potentiometer current is correct. R_1 is the rheostat for adjustment of the supply current from the battery B; ab is the resistance representing the main dial of the potentiometer; while ef is a resistance in the galvanometer circuit, by means of which, with the moving contacts c and d, the resistance of the galvanometer circuit is kept constant.

Theory. Let resistance $ac = r_1$, $cb = r_2$, and $de = r_3$. Let the resistance of the galvanometer circuit excluding r_3 be R_G. Let the total resistance of the battery circuit be $R + r_1 + r_2$, where R includes R_1 and R_S. Then, taking mesh currents I_B and I_G as shown, we have, from Kirchhoff's laws,

$$I_B(R + r_1 + r_2) - I_G r_1 = E \quad . \quad . \quad . \quad . \quad (8.1)$$

and

$$I_G(R_G + r_1 + r_3) - I_B r_1 + V = 0 \quad . \quad . \quad . \quad (8.2)$$

Solving, algebraically, for I_G gives

$$I_G = \frac{\dfrac{Er_1}{R + r_1 + r_2} - V}{R_G + r_3 + \dfrac{r_1(R + r_2)}{R + r_1 + r_2}} \quad . \quad . \quad . \quad (8.3)$$

At balance $I_G = 0$, or

$$\frac{Er_1}{R + r_1 + r_2} = V$$

Now $\dfrac{Er_1}{R + r_1 + r_2}$ is $r_1 \times$ potentiometer current at balance, and is the reading on the main dial.

If the e.m.f. V to be measured falls to $(V - dV)$ the galvanometer current —assuming the potentiometer setting to be left the same—is

$$I_G = \frac{\dfrac{Er_1}{R + r_1 + r_2} - (V - dV)}{R_G + r_3 + \dfrac{r_1(R + r_2)}{R + r_1 + r_2}} = \frac{dV}{R_G + r_3 + \dfrac{r_1(R + r_2)}{R + r_1 + r_2}} \quad . \quad (8.4$$

Now, $\left[R_G + r_2 + \dfrac{r_1(R + r_2)}{R + r_1 + r_2}\right]$ is the resistance of the galvanometer circuit including the effect of r_1 being shunted by $(R + r_2)$.

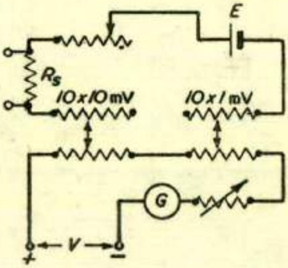


FIG. 8.8. TWO-DIAL DEFLECTIONAL POTENTIOMETER

If the potentiometer is so arranged that r_3 is always equal to r_2 and R is large compared with $(r_1 + r_2)$, then this expression approximates to $(R_g + r_2 + r_1)$, which is constant since $(r_1 + r_2)$ is constant. In practice the potentiometer usually has an overall range of 100 millivolts so that R is 20 times $r_1 + r_2$, and the galvanometer circuit resistance can be considered to be constant.

Thus the galvanometer current is directly proportional to the out-of-balance e.m.f. The galvanometer can be of the pivoted moving-coil type, but greater sensitivity is obtained by the use of a reflecting galvanometer with a ½-metre scale.

Fig. 8.8 shows a simplified diagram of a two-dial version of this potentiometer.

DIESSELHORST OR "THERMOKRAFTFREI" POTENTIOMETER (Ref. (4)). This potentiometer is designed and arranged so as to eliminate, as far as possible, errors due to thermo-electric e.m.f.s, set up at junctions of dissimilar metals, and produced also by the heat from the operator's hand during adjustment of the working parts of the potentiometer. It is, particularly, of use in the measurement of very small e.m.f.s.

Fig. 8.9 gives a diagram of connections of the potentiometer.

B is the supply battery with its rheostat R, while V is the e.m.f. to be measured. S_1 and S_2 are reversing switches. G is the galvanometer, which, with its key K, is in series with the unknown e.m.f., and c, c' are two sliding contacts moving over decades of coils as shown. These contacts are mechanically connected so that a movement of contact c (say) to the left causes an equal movement of c'

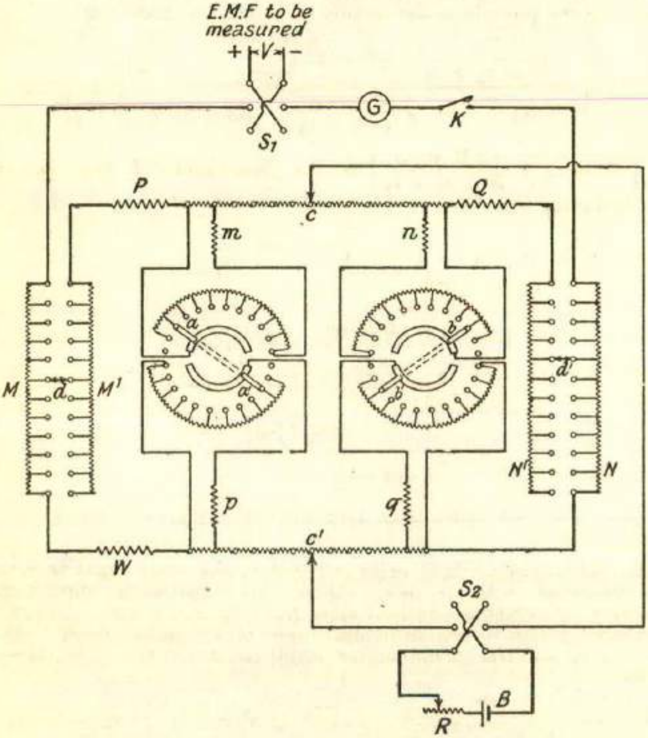


FIG. 8.9. DIESSELHORST POTENTIOMETER

to the right, and thus maintains the resistances of the paths $cPWc'$ and cQc' constant. At d and d' also there are sliders, each having two contacts, sliding over decades M and M' and over N and N'. Decade M is similar to decade M', as is decade N to N', and therefore, again, the resistances of the two paths mentioned above remain constant for all positions of d and d' (see diagram). P, Q, and W are series resistances, while m, n, p, and q are four equal fixed resistances, each connected in series with a variable dial-pattern resistance to form, in all, four variable shunts across coils of the decades over which contacts c and c' slide. The sliding contacts

a and a', and b and b', are mechanically connected, so that they both give the same reading in all positions. The resistances in these dials are arranged so that a variation of contacts a and a' of one stud alters the value of the total resistance of the path $cPWc'$ by an amount equal to one-hundredth of the resistance of one of the coils in decades M or M'. A similar arrangement is employed in the case of the dials over which b and b' slide.

The resistances of the paths $cPWc'$ and cQc' are arranged so that the current in the former path is ten times that in the latter path.

In operation the potentiometer current is first adjusted to its standard value by the use of a standard cell, as in the case of the simpler forms of potentiometer (the standard cell circuit is omitted in the diagram for simplicity). The sliding contacts cc', dd, aa', and bb' are then adjusted until balance is obtained, when the sum of the readings of the five dials gives the value of the unknown e.m.f. (In the diagram the decades over which c and c' slide are shown separately, but in the actual potentiometer they are arranged in dial form similar to the other four dials, with c and c' moving together as do aa' and bb'.) A second measurement with both switches S_1 and S_2 reversed should give the same value to within very narrow limits.

This potentiometer is very largely free from internal thermo-electric effects. Briefly, this is due to the fact that the internal connections of the instrument form a differential arrangement since paths $cPWc'$ and cQc' are in parallel. The thermo-electric e.m.f.s of the two paths tend to neutralize one another. The full theory of the apparatus is given by Diesselhorst (*loc. cit.*) and by Laws (Ref. (1)).

A potentiometer of this type is made by H. Tinsley and Co. It measures e.m.f.s down to 0·1 microvolt, is subdivided to 1 part in 100,000 and has a maximum voltage drop of 0·11111 volt.

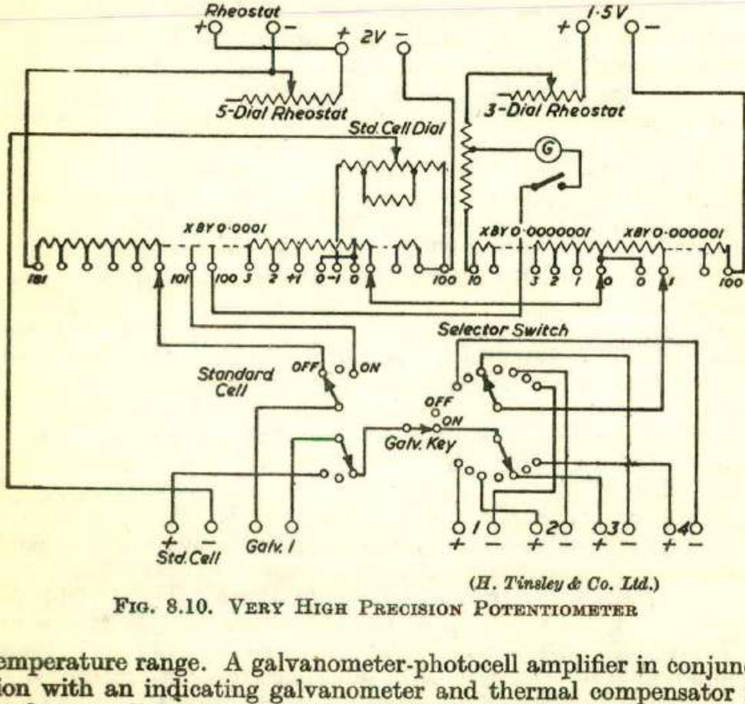
VERY HIGH PRECISION POTENTIOMETER (Fig. 8.10). The accuracy of the vernier potentiometer depends upon the relative stability of a large number of resistances, and the ultimate limitation is the long-term drift of the last dial with respect to the main dial. This difficulty can be overcome to some extent by the use of two potentiometers in series, fed from separate supplies, and having high and low ranges respectively. If the low-range potentiometer is standardized against the other, problems of differential drift are overcome—and an improvement in accuracy is possible. A potentiometer based on this principle is illustrated in Fig. 8.10.

Two series-connected potentiometers are used. The first of these has 181 steps of 0·01 V and 100 steps of 0·0001 V with a 5-dial current regulator. The scale figures of both dials appear in line at one window having a multiplier of 0·0001. The second potentiometer has 100 steps of 0·000001 V and 10 steps of 0·0000001 V with a

3-dial current regulator and a built-in pointer galvanometer for standardizing against the first potentiometer.

The standard cell dial ranges from 1·018539 to 1·018629 in 1 μV steps. The standard cell balance is independent of the potentiometer setting but is made directly against the potentiometer coils, thus avoiding the possibility of error in two different circuits.

Thermal shielding is employed to minimize temperature gradients, and all essential coils are wound from the same reel of wire to preserve the accuracy of the potential division through a wide



(*H. Tinsley & Co. Ltd.*)

FIG. 8.10. VERY HIGH PRECISION POTENTIOMETER

temperature range. A galvanometer-photocell amplifier in conjunction with an indicating galvanometer and thermal compensator is used as a null detector.

Use of the Potentiometer for the Measurement of Resistance, Current, and Voltage. (*a*) RESISTANCE. The measurement of low resistance by the method of comparison with the resistance of a standard, using the potentiometer for the comparison, was discussed in the previous chapter. The connections to the potentiometer when such a measurement is to be made are shown in Fig. 8.11, which is self-explanatory. Care must be taken to ensure that the polarity of the connections is correct, and also that the battery which supplies current to the unknown and standard resistances is insulated from the supply battery of the potentiometer slide-wire. As pointed out in the previous chapter, the ratio $\frac{\text{resistance of unknown}}{\text{resistance of standard}}$ is the same as the ratio of the voltage drops across the two resistances as measured by the potentiometer. Since only the ratio of the voltage drops is required for the determination of the unknown resistance, standardization of the potentiometer by means of a standard cell is not absolutely necessary.

Correction for Thermal e.m.f.s. To take into account any thermal e.m.f. which may be set up at the junctions of dissimilar metals within the two resistances, proceed as follows—

Suppose that the voltage drop across the standard resistance has just been measured, and that the galvanometer is giving no deflection under the balance conditions. Then, first of all determine whether an increase of the slide-wire setting causes the galvanometer to deflect to the left or to the right. Next,

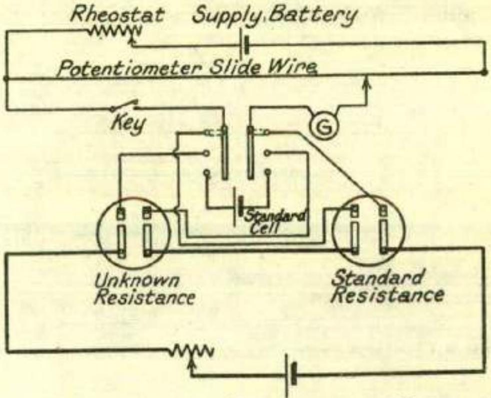


FIG. 8.11. MEASUREMENT OF RESISTANCE BY POTENTIOMETER

open the galvanometer key and set the potentiometer slide-wire or dials to zero, and adjust the galvanometer shunt to give the full sensitivity, as used in balancing the potentiometer during the measurement of the voltage drop across the standard resistance. Break the current circuit of the standard resistance and *immediately* depress the galvanometer key, clamping it down to avoid thermal effects due to the operator's hand. Observe the magnitude and direction of the galvanometer deflection, and calculate the value of the thermal e.m.f. by dividing this deflection by the predetermined value of the deflection per microvolt out-of-balance. Whether this thermal e.m.f. (in microvolts) is to be added to or subtracted from the potentiometer reading obtained when measuring the voltage drop across the standard, depends upon the direction of the deflection produced by the thermal e.m.f. If this direction is the *same* as that produced, initially, by *increase* of the potentiometer setting, the thermal e.m.f. has obviously been "assisting" the potentiometer during the measurement, and hence the value of the thermal e.m.f. should be *added* to the potentiometer reading to give the correct value of the voltage drop to be measured. If the thermal e.m.f. produces a galvanometer deflection in a direction *opposite* to that produced by increasing the potentiometer setting, its value should be *subtracted* from the reading of the potentiometer.

The same procedure should be adopted in the case of the unknown resistance.

It should be noted that these thermo-electric e.m.f.s are usually quite small and, if the measurement is not required to be highly accurate, may be neglected.

(*b*) CURRENT. For the measurement of current by the use of the potentiometer the connections are similar to those of Fig. 8.11, except that the unknown resistance is, of course, omitted, the standard cell and standard resistance only being required. The magnitude of the standard resistance must be so chosen that the voltage drop across it, when the current to be measured is flowing through it, is of the order of 1 volt, and can thus be easily measured on the potentiometer.

The potentiometer is first standardized, and the voltage drop across the standard resistance is then measured. The value of

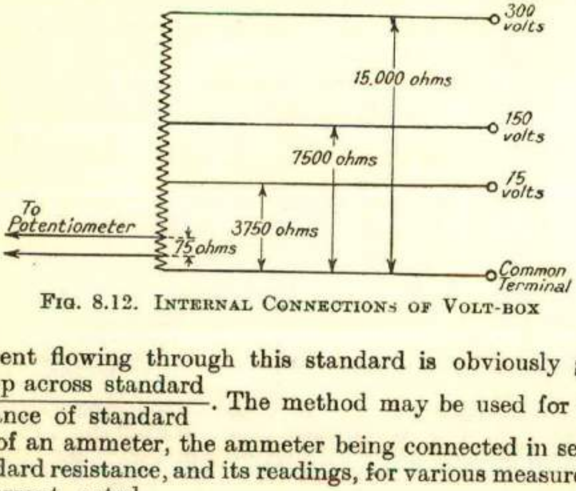


FIG. 8.12. INTERNAL CONNECTIONS OF VOLT-BOX

the current flowing through this standard is obviously given by $\frac{\text{volt drop across standard}}{\text{resistance of standard}}$. The method may be used for the calibration of an ammeter, the ammeter being connected in series with the standard resistance, and its readings, for various measured values of the current, noted.

(*c*) VOLTAGE. The use of the potentiometer for the measurement of e.m.f.s which lie within its range—i.e. are less than 2 volts—has already been described. A "*volt-box*," or "ratio-box," must be used in conjunction with the potentiometer if the voltage to be measured is above 2 volts. The volt-box consists of a high resistance (from 30 to 50 ohms per volt) having a number of tappings, the resistances between the various pairs of tappings being carefully adjusted. Its function is that of a potential divider.

Fig. 8.12 gives a diagram of connections. The leads to the potentiometer are taken from two tapping points, which include between them (say) 75 ohms. If a voltage of the order of 150 volts is to be measured, this voltage is connected between the two terminals marked respectively "common" and "150 volts." Thus, if the measured voltage on the potentiometer is 1·25 volts, the actual voltage applied to the volt-box is $1{\cdot}25 \times (7{,}500/75) = 125$ volts. Instead of being marked "75 volts," "150 volts," etc., most volt-boxes have the markings "multiply by 50," "multiply by 100," etc., opposite the voltage terminals.

Remembering that, at balance, no current flows in the galvanometer circuit—i.e. no current is taken by the leads from the volt-box to the potentiometer, it is obvious that such a piece of apparatus will give exact subdivision of the applied voltage if the resistances between the tapping points are correctly adjusted.

Muirhead and Co., Ltd., make a voltage-dividing resistance box of 2, 3 or 4 dials with a total resistance of 10,000 ohms. The accuracy on d.c. is $\pm$ 0·1 per cent. The maximum permissible input voltage is 200 V r.m.s.

The same manufacturers make a volt-ratio box of 50,000 ohms total resistance for a maximum voltage of 1,000 V.

Voltage Standardizer. Messrs. H. Tinsley & Co. manufacture an instrument called a "voltage standardizer," which can be used on d.c. circuits for the purpose of maintaining at a constant and known value the voltage applied to a test circuit. Thus, in the calibration of substandard wattmeters, the current in the wattmeter current coil will be measured on a potentiometer; the voltage across the voltage coil could be measured through a volt-box on the same potentiometer. It is more convenient and more accurate, however, to utilize the voltage standardizer, which consists of a special form of volt-ratio box, the resistance tappings of which are so adjusted that, when various standard voltages are connected across the "line" terminals, the potential difference across the low-voltage terminals is equal to the e.m.f. of a standard cell. Thus, by connecting a standard cell, through a galvanometer, to these low-voltage terminals and maintaining zero galvanometer deflection, one ensures that the "line" voltage is maintained at the correct value.

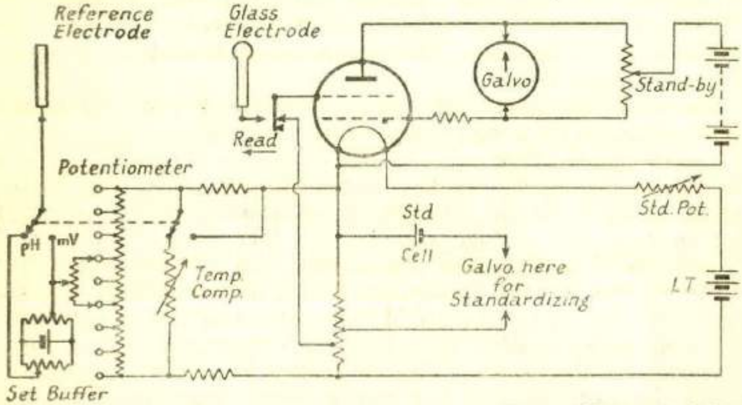
There is a variable dial for correcting for variation of the standard cell voltage with temperature. This instrument was approved by the Electricity Commissioners under the Electricity Supply (Meters) Act, 1936, for use in meter testing.

Potentiometers for Special Purposes. In addition to the applications to the measurement of resistance and current described above, potentiometers are used, in conjunction with auxiliary apparatus, for the measurement of other quantities which are not, essentially, electrical. Such are the measurement of temperature, for which the potentiometer is used together with thermo-couples, and the measurement of the degree of acidity, or alkalinity, of a solution, when a special type of cell, containing the solution under test, is used in conjunction with the potentiometer. The measurement of temperature by potentiometer will be described in Chapter XIII.

For the measurement of acidity of solutions, a special form of cell, with a reference electrode system of calomel and potassium chloride solution, is used. There is a liquid junction between this reference half-cell and the solution under test. An e.m.f. is set up in the cell, and its value depends upon the acidity of the solution.

This e.m.f. is given as

$$E = 0{\cdot}0581p\text{H} + 0{\cdot}2488 \text{ volts at } 20^\circ \text{ C}$$



(*Marconi Instruments, Ltd.*)

FIG. 8.13. FUNCTIONAL DIAGRAM OF pH METER

where pH is a quantity from which the normality of the solution may be obtained. The voltage E may be measured by the potentiometer, and pH—and hence the normality—determined therefrom.

Several direct-reading pH meters and recorders have been developed. Some of these are self-contained and are portable.

The Cambridge Instrument Company make a direct-reading pH indicator which needs standardization against a buffer solution only once a day after which it can be used for routine readings. The pH is read directly on a 120-mm scale covering the pH range 0 to 14. There is temperature compensation fcr the solution temperature over the range 10° to 100° C. The instrument is operated from 200- to 240-V a.c. mains.

The General Electric Co., Ltd., make a battery-operated pH meter covering the same range.

Both the Marconi pH meter (Marconi Instruments, Ltd.) and that made by Muirhead and Co., Ltd., utilize an electrometer valve. In the former, the circuit diagram for which is shown in Fig. 8.13, the e.m.f. from a standardized potentiometer circuit (standardized against a Weston standard cell within the instrument) is applied to the electrometer valve in series opposition to the e.m.f. to be measured. The valve operates a moving-coil balance indicator and the controlling-dial is calibrated to read *p*H directly.

The Muirhead instrument has an electrometer circuit comprising a pentode operating as a d.c. amplifier and taking a very low and constant grid current. It has a linear grid-potential/anode-current characteristic. The anode current is directly proportional to the applied e.m.f., and a backed-off meter in the anode circuit reads *p*H directly. Intrinsically the instrument is a millivoltmeter of variable sensitivity.

These two instruments are calibrated by means of buffer solutions having known values of *p*H, and tablets are supplied from which the required solutions can be made up. They are both battery-operated.

Electronic Instruments, Ltd., make a *p*H meter and also a *p*H transmitter which is mains operated. It is intended for the automatic recording and controlling of *p*H in chemical manufacturing processes.

Potentiometers for Use with Alternating Current. The potentiometer method is an exceedingly useful one for the accurate measurement of alternating currents and voltages, since such measurements are not easily carried out by other methods.

The principle of the alternating-current potentiometer is the same as that of the direct-current instrument, the most important difference in operation being that, whereas in the direct-current potentiometer only the *magnitudes* of the "unknown" e.m.f. and slide-wire voltage drop must be made equal to obtain balance, in the alternating-current instrument the *phases* of these two voltages, as well as their magnitudes, must be equal for balance to be obtained. This condition obviously necessitates modification of the potentiometer as constructed for direct-current work, and means that the operation is somewhat more complicated.

C. V. Drysdale, who played a major part in the development of the a.c. potentiometer, has given the history of the development (Ref. (5)), and has described several types using somewhat different methods.

The frequency and waveform of the current in the slide-wire portion of the potentiometer—i.e. of the supply—must, in all a.c. potentiometers, be exactly the same as those of the voltage to be measured, and for this reason the supply for the instrument must be taken from the same source as the voltage or current to be measured. The various forms differ principally in their method of dealing with the question of phase difference between the slide-wire and "unknown" voltages.

There are two general types: (*a*) those which measure the unknown voltage in polar form, i.e. in terms of its magnitude and relative phase; and (*b*) those measuring the rectangular co-ordinates of the voltage under test. Of the potentiometers described below the Drysdale instrument is of type (*a*), and the Gall and Campbell-Larsen are of type (*b*).

DRYSDALE–TINSLEY A.C. POTENTIOMETER. This instrument consists of a potentiometer of the ordinary direct-current type—the coils in which are non-inductively wound—together with auxiliary apparatus. The auxiliaries include—

(*a*) A Drysdale phase-shifter, or phase-shifting transformer. This consists of a ring-shaped stator within which, fitting closely inside it, is a rotor which carries a winding supplying the potentiometer slide-wire circuit. The stator is wound with either a three-phase or two-phase winding. A rotating field is produced when currents flow

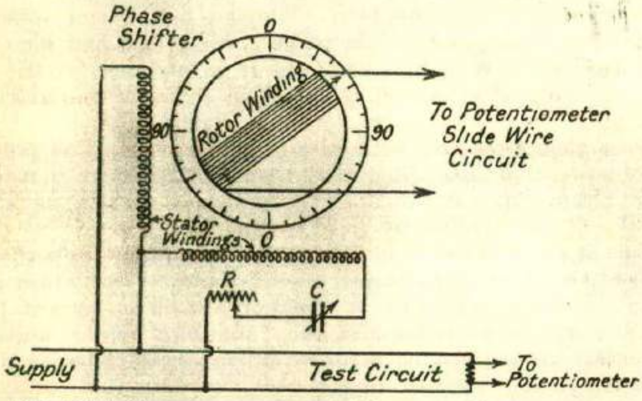


FIG. 8.14. CONNECTIONS OF THE DRYSDALE PHASE-SHIFTER

in the stator winding, and the phase of the secondary, or rotor, current can be changed, relative to the stator supply voltage, by rotating the rotor through any desired angle, the phase displacement of the secondary e.m.f. being equal to the angle through which the rotor is moved from its zero position. The windings are so arranged that this alteration of phase is not accompanied by alteration of the magnitude of the rotor-induced e.m.f. The phase alteration produced is measured on a divided scale fixed to the top of the instrument. Fig. 8.14 shows, diagrammatically, the connections of the phase-shifter arranged for operation from a single-phase supply, using a phase-splitting device consisting of a capacitor and resistor as shown. By successive adjustment of the capacitor and resistor exact quadrature between the currents in the two stator windings may be obtained. This method, using a single-phase supply, forms a very convenient means of supplying the stator windings.

(*b*) A precision-type electro-dynamometer ammeter is required for standardization purposes. To standardize the a.c. potentiometer the slide-wire circuit is switched on to a direct-current supply, and the standard current is obtained in the ordinary way, using a standard cell. This standard current, required to make the potentiometer direct reading, is measured by the precision ammeter which is included in the battery supply circuit of the potentiometer. During operation on alternating current, the ammeter is still included in the supply circuit, and the r.m.s. value of the slide-wire current is maintained at the same value as was required on direct current. This type of ammeter reads correctly on both direct and alternating current, and since the coils of the slide-wire circuit are non-inductively wound, the potentiometer remains direct reading when used

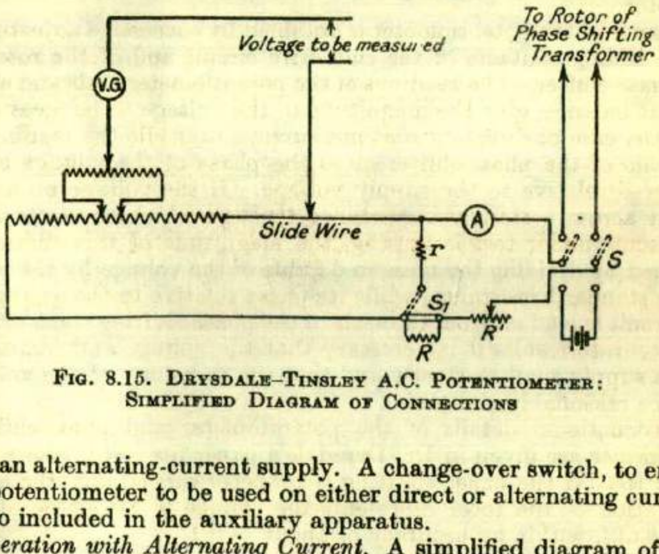


FIG. 8.15. DRYSDALE–TINSLEY A.C. POTENTIOMETER: SIMPLIFIED DIAGRAM OF CONNECTIONS

with an alternating-current supply. A change-over switch, to enable the potentiometer to be used on either direct or alternating current, is also included in the auxiliary apparatus.

Operation with Alternating Current. A simplified diagram of connections of the potentiometer for use with alternating current is given in Fig. 8.15. The Kelvin–Varley slide principle is employed in the slide-wire circuit as shown. *V.G.* is a vibration galvanometer—used as a detector for measurements at commercial frequencies. This must be carefully tuned to give resonance at the frequency of the circuit under test (which is also that of the potentiometer supply, since the two are identical). r is a shunting resistor for the reduction of the range of the potentiometer. When this shunt is put in circuit—by the switch S_1—the resistor R is simultaneously connected in series with the slide-wire circuit in order that the resistance of the working portion of the potentiometer may be maintained constant. R' is a rheostat for adjustment of the slide-wire current. A is the precision ammeter mentioned above. The phase-shifting transformer, whose connections are given in Fig. 8.14, is omitted for clearness.

The potentiometer is first standardized by adjusting rheostat R', and the standard current is noted, the switch S being thrown over to the battery side for this standardization, the vibration galvanometer being replaced by a D'Arsonval galvanometer.

The switch S is then thrown over to the alternating supply side, the standard cell and D'Arsonval galvanometer being previously replaced by the alternating voltage to be measured and the vibration galvanometer, respectively. The stator windings of the phase-shifter are then adjusted to exact quadrature by means of the variable resistor and capacitor, these being adjusted until the alternating current in the slide-wire is constant for all positions of the rotor.

Balance of the potentiometer is obtained by successive adjustment of the sliding contacts of the slide-wire circuit and of the rotor of the phase-shifter. The readings of the potentiometer dials and slide-wire, at balance, give the magnitude of the voltage to be measured, as in the case of direct-current measurements, while the reading on the scale of the phase-shifter gives the phase of the voltage being measured relative to the supply voltage. If the voltage measured is that across a standard resistance through which the current in the circuit under test is flowing, the magnitude of this current is obtained by dividing the measured value of the voltage by the value of the standard resistance, while its phase relative to the voltage of the circuit is read off from the scale of the phase-shifting transformer. For accurate results it is necessary that the voltage and frequency of the supply shall be steady and that the waveform of the voltage shall be reasonably sinusoidal.

Constructional details of the potentiometer and phase-shifting transformer are given in Dr. Drysdale's paper (*loc. cit.*), where it is stated that, if the phase-splitting is properly carried out, the angle of rotation of the rotor represents the change in the phase of the rotor e.m.f. within an accuracy of about $\pm 0{\cdot}1°$.

GALL–TINSLEY A.C. POTENTIOMETER. This potentiometer consists of two separate potentiometer circuits enclosed in a common case. One is called the "in-phase" potentiometer and the other the "quadrature" potentiometer. The slide-wire circuits are supplied with currents which have a phase difference of 90°. On the first of these potentiometers, that component of the "unknown" voltage which is in phase with the current in the slide-wire circuit of the potentiometer is measured. On the other potentiometer the component of the "unknown" voltage in phase with the current in *its* slide-wire circuit is measured. Since the two slide-wire currents are in quadrature, the two measured values are the quadrature components of the unknown voltage. If these measured values are V_1 and V_2 respectively, then the unknown voltage is given by

$V = \sqrt{V_1^2 + V_2^2}$, and its phase difference from the current in the "in-phase" potentiometer slide-wire circuit is given by the angle θ where $\tan \theta = \frac{V_2}{V_1}$.

Fig. 8.16 shows the connections of the potentiometer, simplified somewhat for the sake of clearness. The in-phase and quadrature potentiometers are shown, with their sliding contacts bb' and cc' and rheostats R and R' for current adjustment. The supplies to the

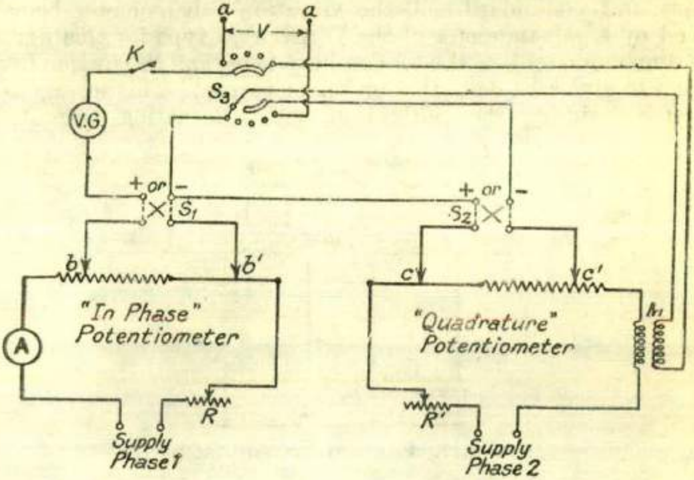


FIG. 8.16. CONNECTIONS OF GALL-TINSLEY A.C. POTENTIOMETER

potentiometer are obtained from a single-phase supply by means of the arrangement shown in Fig. 8.17.

T_1 and T_2 are two step-down transformers which supply about 6 volts to the potentiometer circuits. They also serve to isolate the potentiometers from the line and are usually provided with earthed screens between the windings. The supply to T_2 is obtained through a variable resistor R and variable capacitor C for the purpose of phase splitting. Quadrature is obtained by adjusting C and R.

Referring again to Fig. 8.16, $V.G.$ is a vibration galvanometer (tuned to the supply frequency); K is its key. A is a reflecting dynamometer instrument for maintaining the currents in the two slide-wires at the standard value (50 milliamperes). S_1 and S_2 are two "sign-changing" switches which may be necessary to reverse the direction of the unknown e.m.f. applied to the slide-wires. The necessity of these switches depends on the relative phases of the unknown and slide-wire voltages. S_3 is a selector switch by which the unknown voltages to be measured are placed in circuit. There are four pairs of terminals for the application of such voltages, the connections to only one pair—to which an unknown voltage V is applied—being shown in the figure. This selector switch, when in the position shown in the figure—called the "test position"—allows the current in the quadrature potentiometer slide-wire to be compared with that in the in-phase potentiometer wire, utilizing the mutual inductance M for the purpose.

Operation. The current in the in-phase potentiometer wire is first adjusted to its standard value by means of a direct current supply and a standard cell, the vibration galvanometer being replaced by a galvanometer of the D'Arsonval type for this purpose. The dynamometer is of the torsion-head type, and the torsion head is turned to give zero deflection on direct current. This setting is left untouched during the calibration with alternating current, the

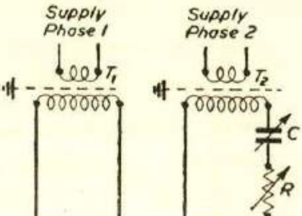


FIG. 8.17. PHASE-SPLITTING CIRCUIT

slide-wire current being adjusted to give zero deflection again. The vibration galvanometer is then placed in circuit and the direct current supply replaced by the alternating supplies.

Now, the magnitude of the current in the quadrature potentiometer wire must be the same as that in the in-phase potentiometer —namely, the standard value of 50 milliamp. These two currents must also be exactly in quadrature. Rheostat R is adjusted until the current in the in-phase potentiometer wire is the standard value (as indicated on A). The selector switch S_3 is then switched on to the test position (shown in Fig. 8.16). Now, the e.m.f. induced in the secondary winding of the mutual inductor M—assuming M to be free from eddy current effects—will lag 90° in phase behind the current in the primary winding, i.e. in the quadrature potentiometer slide-wire. Also, if i is the primary current, then the e.m.f. induced in the secondary is $2\pi \times$ frequency $\times M \times i$, where M is the value of the mutual inductance. Thus, for given values of frequency and mutual inductance, the induced e.m.f., when i has the standard value (50 milliamp), can easily be calculated. E.g., if $f = 50$ cycles per second and $M = 0{\cdot}0318$ henry, the secondary induced e.m.f. is $2\pi \times 50 \times 0{\cdot}0318 \times 0{\cdot}050 = 0{\cdot}5$ volt, when i has the standard value.

The slide-wire of the in-phase potentiometer is thus set to this calculated value of induced e.m.f. in the secondary of M (the slide-wire current being maintained at its standard value), and rheostat R and capacitor C (see Fig. 8.17) are adjusted until exact balance is obtained. For balance, the current in the quadrature potentiometer slide-wire must be both equal to the standard value and also must be exactly 90° out of phase with the current in the in-phase slide-wire. This latter condition follows from the fact that the e.m.f. in the secondary of M lags 90° in phase behind the primary current, and, therefore, for this e.m.f. to be *in phase* with the voltage drop across a portion of the in-phase slide-wire, the current in the primary of M must be in *exact quadrature* with the current in this in-phase slide-wire. Any difference in polarity between the two circuits is corrected for by the sign-changing switches S_1 and S_2.

These adjustments having been made, the unknown voltage is switched in circuit by means of the selector switch S_3. In this position of S_3 the two slide-wire circuits are in series with one another and with the vibration galvanometer. Balance is obtained by adjusting both pairs of sliding contacts (*bb′* and *cc′*) together with the reversal of switches S_1 and S_2, if necessary. At balance, the reading of the slide-wire of the in-phase potentiometer, together with the position of S_1, gives the magnitude and sign of the in-phase component of the unknown voltage, while the reading of the quadrature potentiometer, with the position of S_2, gives the magnitude and sign of the quadrature component.

For example, if both S_1 and S_2 are in the positive position and V_1 and V_2 are the in-phase and quadrature components of the unknown voltage V, then the phase of V is as shown in Fig. 8.18, while its magnitude is $\sqrt{V_1^2 + V_2^2}$.

Errors. The errors which may occur in using this potentiometer may be due to—

(*a*) Slight differences in the reading of the reflecting dynamometer instrument on a.c. as compared with the reading on d.c. Such errors may cause the standard current value on a.c. to be slightly incorrect.

(*b*) Mutual inductance between the various parts of the circuit. An error in the nominal value of the mutual inductance M would cause the current in the quadrature slide-wire circuit to be somewhat different from the standard value.

(*c*) Inaccuracy of the method of measuring the frequency, which again would cause an error in the quadrature slide-wire standard current value.

(*d*) The fact that inter-capacitance, earth capacitance, and mutual inductance effects are present in the slide-wire coils and affect the potential gradient.

(*e*) The existence of harmonics in the supply waveform. Standardization of the potentiometer is upon an r.m.s. current basis, while the potential balances on the slide-wires are dependent upon the fundamental only.

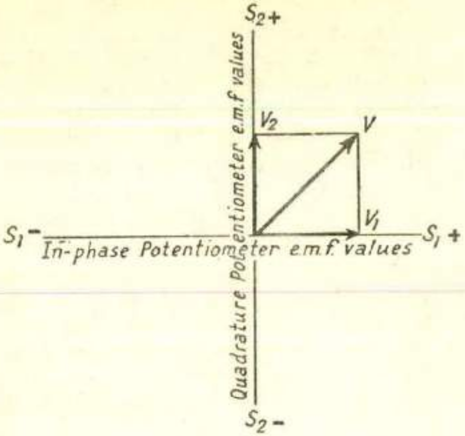


FIG. 8.18

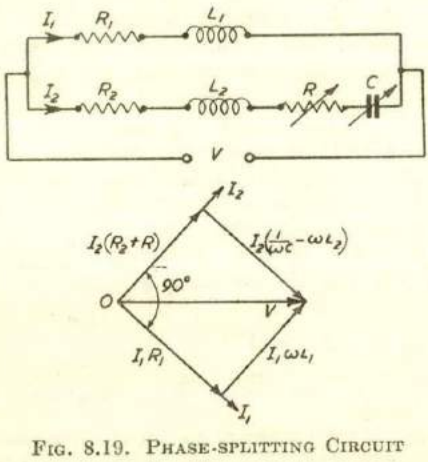


FIG. 8.19. PHASE-SPLITTING CIRCUIT

The action of the phase-splitting circuit may be understood by referring to Fig. 8.19, which gives an equivalent circuit for the two potentiometers. R_1 and L_1 are the equivalent resistance and inductance of the in-phase potentiometer circuit, and R_2 and L_2 those of the quadrature potentiometer. When the potentiometer currents are equal and in quadrature,

$$I_2 = jI_1$$

or
$$\frac{V}{R + R_2 + j\left(\omega L_2 - \frac{1}{\omega C}\right)} = j\,\frac{V}{R_1 + j\omega L_1} \qquad (8.5)$$

Therefore
$$R_1 + j\omega L_1 = j(R + R_2) - \left(\omega L_2 - \frac{1}{\omega C}\right)$$

By separating real and imaginary parts, we obtain the conditions for phase splitting as

$$R_1 + \omega L_2 = \frac{1}{\omega C} \quad . \quad . \quad . \quad . \quad . \quad (8.6)$$

and

$$\omega L_1 - R_2 = R \quad . \quad . \quad . \quad . \quad . \quad (8.7)$$

The phase-splitting is adjusted by means of R and C.

This phase-splitting circuit has been used in later forms of the a.c. potentiometer in place of the original quadrature device

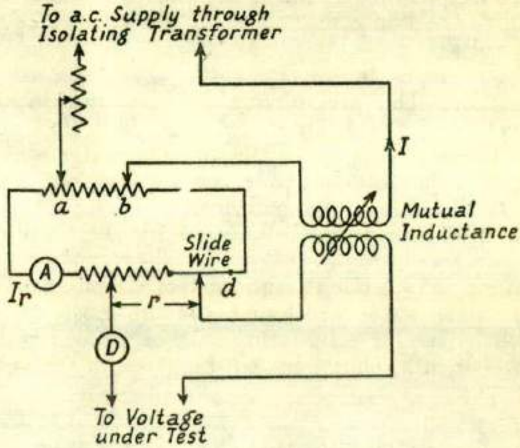


FIG. 8.20

described by D. C. Gall, which used a transformer and variable resistor.

H. Tinsley and Co. also make a precision polar-co-ordinate potentiometer which can be used either as a Drysdale, or as a Gall co-ordinate, potentiometer.

Campbell-Larsen Potentiometer. In this instrument the two rectangular components of the voltage under test are measured in terms of the voltage drop across a slide-wire resistor (for the in-phase component) and the voltage induced in the secondary of a mutual inductor (for the quadrature component). In the original Larsen potentiometer the slide-wire and primary circuit of the mutual inductor were in series and carried the same current, but difficulties in construction of the latter and in the operation of the potentiometer at different frequencies led to Campbell's modification of the instrument (Ref. (10)).

A simplified circuit, as modified, is shown in Fig. 8.20. D is an a.c. detector—either vibration galvanometer or telephones, according to the frequency. While current I passes through the primary of the mutual inductor M, only a portion of this current, namely I_r, passes through the slide-wire circuit. If the resistance between the movable contacts *ab* is S and round the path *adb* is R, then $I_r = I \, . \, S/(R + S)$. The setting of S, by means of a dial resistance, is arranged to be proportional to the frequency at which the test is being carried out, the dial being calibrated directly in terms of frequency. Since $R + S$ is constant in magnitude, $I_r \propto S \propto$ frequency, so that both the voltage drop in the slide-wire $I_r r$ and the voltage in the secondary of the mutual inductor are proportional to frequency. S and M are chosen so that the settings of r and M give the two components of the voltage being measured directly in volts.

A special thermal device (Refs. (9), (11)) is used for the a.c. standardization. The preliminary d.c. standardization utilizes a standard cell, and the reference current, indicated on A, is thus obtained. The a.c. supply to the potentiometer and the voltage under test must be obtained from the same source, but the instrument is isolated through a transformer.

This potentiometer is manufactured by the Cambridge Instrument Co.

Applications of A.C. Potentiometers. Such applications are numerous, as the a.c. potentiometer is the most universal instrument which exists for alternating-current measurements. Only a limited number of applications can be given in the space available here.*

One application—namely the measurement of self-inductance—has already been given in Chapter VI. Others are as follows.

(*a*) Voltmeter Calibration. Low voltages—up to 1·5 volts or thereabouts—can be measured directly. Higher voltages can be measured by using a volt-box (for medium voltages) or two capacitors in series (for high voltages) in conjunction with the potentiometer.

(*b*) Ammeter Calibration. The measurement of various alternating currents required for such calibration may be made by the use of non-inductive standard resistors with the potentiometer, the method being similar to that adopted when the calibration is to be carried out with direct current.

(*c*) Wattmeter and Energy-meter Testing. Fig. 8.21 gives a simplified diagram of the connections for such tests, the arrangement being suited to tests at any power factor. The current coil of the wattmeter is supplied through a step-down transformer, and the voltage coil from the secondary of a variable transformer whose primary is supplied from the rotor of a phase-shifting transformer. The voltage applied to the voltage coil and the current in the current

* The reader should refer to Drysdale's paper (Ref. (5)), to T. Spooner's paper (Ref. (7)), or to D. C. Gall's book (Ref. (9)) for the description of other applications.

coil are measured by the potentiometer, using a volt-box and low-resistance standard as shown. The power factor is varied by rotation of the rotor of the phase-shifter, the reading on the dial of which gives the phase angle between voltage and current. A small mutual inductance M is included to ensure accuracy of measurement at zero power factor (see Drysdale's paper, *loc. cit.*).

Other applications include the measurement of the ratio and phase-angle errors of current transformers, the measurement of core loss and magnetizing current for specimens of sheet steel, the

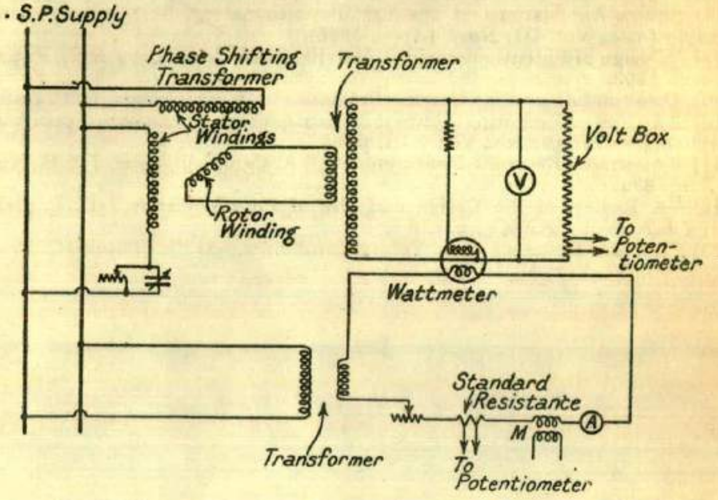


FIG. 8.21. CONNECTIONS FOR WATTMETER TESTING BY A.C. POTENTIOMETER

measurement of alternating magnetic fields and the measurement of capacitance.

A.C. Stabilizer. G. N. Patchett has designed a stabilizer, made by H. Tinsley and Co., to provide a very stable supply, with high purity of waveform, for a.c. measurements such as those with a.c. potentiometers. The requirement for such purposes is a stability of better than $\pm$ 0·1 per cent with a total harmonic distortion of not more than 2 per cent, and the instrument mentioned fulfils this requirement. On normal supply mains a short-term output stability of about 1 in 10,000 can be obtained and the total harmonic distortion is less than 1 per cent.

The output voltage can be varied over the range 215 to 250 volts, and the current ranges are 1 A and 2 A, at 230 V. The performance of the instrument is unaffected by frequency changes over the range 48 to 52 c/s.

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