

INDEX

- Acceleration, 396
- Applications, 393
- Area between two curves
 - cartesian, 230
 - polar, 234
- Areas of closed curves, 245
- Areas of plane curves
 - cartesian, 225
 - polar, 234
- Area of
 - cardioid, 236
 - cissoid, 241
 - cycloid, 228
 - ellipse, 227, 231
 - folium of Descartes, 237
 - parabola, 228
 - loop, cartesian eqn., 229
 - loop, polar eqn., 236
- Astroïd, 314
- Auxiliary equation, 363
 - equal roots, 364
 - pair of complex roots, 365
 - real and distinct roots, 364
- Bernoulli's equation, 351
- Beta function, 191, 200
- Binomial differentials, 197
- By parts integration, 38
- Cardioid, 319
- Catenary, 312
- Cauchy's equation, 390
- Centroid, 289 - 296
 - circular arc, 296
 - parabolic lamina, 297
 - quadrant of an ellipse, 298
 - solid hemisphere, 299
- Chainette, 312
- Change of variables, 13
- Cissoïd of Diocles, 317
- Clairaut's equation, 356, 358
- Complementary function, 369, 384
- Complete primitive, 327
- Condition for integrability, 100
- Constant of integration, 3
- Convergent integral, 151
- Cycloid
 - vertex downwards, 311
 - vertex upwards, 310
- Darboux's theorem, 100
- Definite integrals, 2, 92, 94, 113,
 - as the limit of a sum, 92
 - general properties, 128
 - geometrical interpretation, 110
 - lower limit, 2, 93
 - upper limit, 2, 93
- Differential equation
 - definitions 324
 - degree, 325
 - exact, 341
 - first degree, 331
 - first order, 331
 - formation, 325

- geometrical interpretation, 326
 - homogeneous, 336
 - n th order, 384
 - order, 325
 - ordinary, 324
 - partial, 324
 - resolvable into factors, 354
 - second order, 363
 - solution, 326.
 - solvable for x, y , 356
- Delta function, 152
- Divergent integral, 151
- Double integral, 406
application, 416
- Elementary rules of integration, 4
- Equations of second order, 363
special type, 376
- Equiangular spiral, 318
- Eulerian integral, 191
- Evaluation of definite integral, 113
- Evolutes of parabola, ellipse, 315
- Exact equation, 341
- Exponential curves, 316
- Exponential function, 136
- First mean value theorem
of integral calculus, 103, 104
- First principle, 94
- Folium of Descartes, 316
- Fundamental integrals, 5
- Fundamental theorem, 1, 112
- Integral Calculus (main) -31
- Gamma function, 200
- Geometrical interpretation
definite integral, 110
differential eqn., 324
- General laws of integration, 4
- General properties of
definite integrals, 128
- General solution, 327
- Generalized definition, 93
- Homogeneous equation, 336
special form, 337
- Hyperbolic function, 24, 66
- Hypocycloid, 314
- Improper integrals, 150
convergent, divergent,
oscillatory, 151
- Inequalities and limits, 136
- Inferior limit, 93
- Infinite range, 150
- Integrability, 100
necessary and sufficient
condition, 100
- Integrable function, 100
- Integral powers of
tangent and co-tangent, 65
- Integrating factors, 342
- Integrals
definite, 2, 92, 93, 113
Eulerian, 191
improper, 150

- indefinite, 1, 2
- infinite, 150
- involving one parameter, 181
- Riemann, 100
- Integration, 2
 - as the limit of a sum, 91
 - by parts, 38
 - by successive reduction, 181
 - from first principle, 94
 - of infinite series, 159
 - of power series, 159
 - of rational fraction, 79
- Intrinsic equation of
 - cardioid, 268
 - catenary, 267
 - cycloid, 268
- Intrinsic equation to a curve from
 - cartesian eqn., 265
 - pedal eqn., 267
 - polar eqn., 266
- Irrational functions, 165
- Isodine, 404
- Jacobian, 415
- Legendre's equation, 390
- Length of arc of
 - cardioid, 262
 - cycloid, 260
 - evolute, 253
 - loop, 260
 - parabola, 259
 - Length of plane curve from
 - cartesian, 257
 - parametric, 258
 - pedal, 263
 - polar, 261
 - Lemniscate, 320, 321
 - Maçon, 320
 - Limits, 93
 - Line integral, 247
 - Linear equation, 350, 363
 - Logarithmic curve, 316
 - Logarithmic spiral, 318
 - Lower integral, 100
 - Method of substitution
 - definite integral, 115
 - indefinite integral, 18
 - Miscellaneous application, 398
 - Moment of inertia, 289, 300
 - circular plate, 303
 - elliptic lamina, 302
 - rectangular lamina, 302
 - sphere, 304
 - thin uniform rod, 301
 - On some well-known curves, 310
 - Orthogonal trajectories, 393
 - cartesian eqn., 393
 - polar eqn., 394
 - Pappus' theorem, 283
 - Parabolic rule, 249

- Particular integrals, 369, 384
 methods, 370
- Perfect differential, 341
- Primitives and integrals, 107
- Principal value, 151, 155
- Probability curves, 317
- Radius of gyration, 300
- Rational fractions, 79
- Rectification, 257
- Reduction formulæ, 141, 181
 double parameter, 189
 single parameter, 181
 special devices, 195
- Riemann integral, 100
- Rose petal, 321
- Series represented by definite
 integrals, 118
- Separation of variables, 331
- Sign of an area, 243
- Simpson's rule, 247
- Sine spiral, 322
- Singular solution, 327, 358, 359
- Solids of revolution, 274, 281
 volume, 274, 280
- Solution of a differential
 equation, 326
- Some well-known curves, 310
- Special trigonometric functions, 58
- Spiral of Archimedes, 319
- Standard methods of integrations, 7
- Standard integrals, 23, 41, 42, 58
- Strophoid, 317
- Substitution in definite
 integrals, 115
- Superior limits, 93
- Surface-area, 275 - 284
- Summation of series, 118
- Symbolical operation, 370
- Symbolical operators, 369, 370
- Tractrix, 313
- Trial solution, 363
- Triple integrals, 434
 application, 437
- Upper integral, 100
- Upper limit, 2, 93
- Velocity, 396
- Volumes, 274, 276
- Volume and surface-area
 cardioid, 280
 cycloid, 279
 parabola, 278
 paraboloid, 278
- Witch of Agnesi, 318

UNIVERSITY PAPERS
B. A. / B. Sc. (Pass)
CALCUTTA UNIVERSITY
1994

2. (a) Evaluate $\int \frac{\sin x}{\sin x + \cos x} dx$. (b) Prove that $\int_{-\pi/2}^{\pi/2} \sin^5 x dx = 0$.

(c) Find the area of the region bounded by the parabola $y^2 = 4x$ and its latus rectum.

(d) If $B(m, n) = \int_0^1 x^{m-1} (1-x)^{n-1} dx$ ($m, n > 0$),

show that $B\left(\frac{1}{2}, \frac{1}{2}\right) = \pi$.

(e) Evaluate $\iint xy dx dy$ over the region in the positive quadrant for which $x + y \leq 1$.

3. (a) Show that $V = \frac{A}{r} + B$ satisfies the differential equation $\frac{d^2v}{dr^2} + \frac{2}{r} \frac{dv}{dr} = 0$, A, B are constants.

(b) Find the differential equation of all parabolas having their axes parallel to y -axis.

(c) Show that the curve in which the polar subtangent is proportional to the length of radius vector is $Cr = e^{k\theta}$, where k is constant and C is arbitrary.

(d) Show that the curvature is zero at every point on a straight line.

(e) Solve $\frac{dy}{dx} + 1 = e^{x-y}$.

9. (a) Evaluate (i) $\int \frac{\cos 5x + \cos 4x}{1 - 2\cos 3x} dx$;

(ii) $\int \frac{x+1}{\sqrt{4+8x-5x^2}} dx$;

(b) Evaluate $\lim_{n \rightarrow \infty} \sum_{r=1}^{n-1} \frac{1}{n} \sqrt{\frac{n+r}{n-r}}$.

10. (a) If $I_n = \int_0^{\pi/2} \cos^n x \, dx$, n is a positive integer,

show that $I_n = \frac{n-1}{n} I_{n-2}$ and hence find $\int_0^{\pi/2} \sin^6 x \, dx$

(b) Show that the total area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab .

(c) Assuming $\Gamma(n+1) = n!$ (n is a positive integer), and $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$,

find the value of $\int_0^{\pi/2} \sqrt{\sin \theta} \, d\theta \times \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin \theta}}$.

11. (a) Find the length of the arc of the curve

$$x = a (\cos \theta + \theta \sin \theta),$$

$$y = a (\sin \theta - \theta \cos \theta), \text{ from } \theta = 0 \text{ to } \theta = \theta_1.$$

(b) The circle $x^2 + y^2 = a^2$ revolves round x -axis. Find the surface area and the volume of the whole sphere generated.

(c) Show that $\int_0^a \frac{a(x - \sqrt{a^2 - x^2})^2}{(2x^2 - a^2)^2} dx = \frac{1}{\sqrt{2}} \log(\sqrt{2} + 1)$.

12. (a) Find the value of $\iint_R xy(x^2 + y^2) \, dx \, dy$

where $R : [0 \leq x \leq a, 0 \leq y \leq b]$.

(b) Find the intrinsic equation of the catenary $y = c \cosh\left(\frac{x}{c}\right)$

(c) Obtain an approximate value of $\log 2$ by calculating $\int_1^2 \frac{dx}{x}$ by

Simpson's rule dividing the interval into two equal parts.

13. (a) Show that if $l \frac{d^2\theta}{dt^2} + g\theta = 0$ and if $\theta = \alpha$ and $\frac{d\theta}{dt} = 0$ when $t = 0$,

then

$$\theta = \alpha \cos \sqrt{\frac{g}{l}} t.$$

(b) Solve :

$$(i) x^2(x dx + y dy) + 2y(x dy - y dx) = 0;$$

$$(ii) (x^2 + y^2 + 4)x dx + (x^2 - y^2 + 9)y dy = 0;$$

$$(iii) \frac{dy}{dx} + \frac{y}{x} \log y = \frac{y}{x^2} (\log y)^2;$$

$$(iv) xy \left(\frac{dy}{dx} \right)^2 - (x^2 - y^2) \frac{dy}{dx} - xy = 0;$$

14. (a) Find the curve for which the sum of the reciprocals of the radius vector and the polar subtangent is constant.

$$(b) \text{ Solve : (i) } \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + 5y = 10 \sin x;$$

$$(ii) x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = \log x; \quad (iii) \frac{d^2y}{dx^2} + a^2y = \sin ax;$$

$$(iv) (D^2 - 1)y = 2, \quad Dy = 3 \text{ when } y = 1 \text{ and } y = -1 \text{ when } x = 2$$

BURDWAN UNIVERSITY

1993

7. (a) Evaluate the following

$$(i) \int \frac{dx}{(4x - x^2)^{\frac{3}{2}}},$$

$$(ii) \int \frac{dx}{\sin x + \cos x},$$

$$(iii) \int \frac{x^2 + 1}{x^4 + 1} dx,$$

$$(iv) \int \frac{x + \sin x}{1 + \cos x} dx.$$

(b) Evaluate $\int_0^1 x^2 dx$ from the definition of Definite Integral as limit of a sum.

(c) Show that $\int_0^{\frac{\pi}{2}} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x} = \frac{\pi}{2ab}$ [$a > b > 0$].

8. (a) State and prove the Fundamental Theorem of Integral Calculus.

(b) $\int_0^a f(x) dx = \int_0^a f(a-x) dx$. Prove it, and hence show that

$$\int_0^{\frac{\pi}{2}} \log \tan x dx = 0.$$

(c) Evaluate, when possible $\int_0^{\infty} \frac{dx}{x^2 - 1}$.

9. (a) Calculate the area bounded by the curves $y^2 - 4x - 4 = 0$ and $y^2 + 4x - 4 = 0$. (figure necessary).

(b) Find the perimeter of the hypo-cycloid

$$\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = 1.$$

(c) By the method of Integration. Determine the volume of a sphere of radius r .

10. (a) Show that a plane curve for which the length of the normal at a point of the curve is equal to that of the radius vector at that point is either a circle or a rectangular hyperbola.

(b) Solve the following :

(i) $(x^2 + y^2) dy = xy dx,$

(ii) $(2x - y + 1) dx + (2y - x - 1) dy = 0,$

$$(iii) \frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x)e^x \sec y, \quad (iv) \left(\frac{dy}{dx}\right)^2 + 2x \frac{dy}{dx} - 3x^2 = 0,$$

$$(v) (xy \cos xy + \sin xy) dx + x^2 \cos xy dy = 0.$$

11. Solve the following :

$$(i) \frac{d^2 y}{dx^2} - \frac{4dy}{dx} + 4y = x^3 e^{2x},$$

$$(ii) \frac{d^2 y}{dx^2} + y = \sin 2x \text{ when } x=0, y=0 \text{ and } \frac{dy}{dx} = 0,$$

$$(iii) x^2 \frac{d^2 y}{dx^2} + \frac{dy}{dx} + y = \sin \log x^2,$$

$$(iv) \frac{d^2 y}{dx^2} + 4y = 2 \sin x \cos x,$$

$$(v) \frac{d^2 y}{dx^2} = 0 \text{ when } x=0, y=4 \text{ and when } x=\frac{\pi}{2}, y=0.$$

VIDYASAGAR UNIVERSITY

1993

1. (b) (i) If $f(x) = |x-1|$, evaluate $\int_0^2 f(x) dx$.

(ii) Prove that $\int_0^a f(x) dx = \int_0^a f(a-x) dx$.

(iii) Evaluate $\int_{x=0}^{x/2} \int_{y=0}^x \sin(x+y) dx dy$.

(iv) A point $P(x, y)$ moves in xy -plane satisfying $0 \leq x \leq \pi/2$, $0 \leq y \leq \sin x$. Find the area of the region traced out by P .

(v) Use method of integration to evaluate

$$\lim_{n \rightarrow \infty} \left(\frac{1^5 + 2^5 + 3^5 + \dots + n^5}{n^6} \right).$$

(c) (i) Examine whether $\frac{1}{y^2}$ is an integrating factor of the differential equation $y(1 + xy) dx - x dy = 0$.

(ii) Obtain the differential equation whose general solution is $ax + by + c = 0$, a, b, c are arbitrary constants.

(iii) Find the solution of the equation

$$\frac{d^2y}{dx^2} - 2c \frac{dy}{dx} + c^2y = 0, \quad c \text{ is a constant.}$$

7. (a) Evaluate :

$$(i) \int \frac{dx}{(x^2 + 1)(x^2 + x + 1)}; \quad (ii) \int \frac{\cos x \, dx}{\sin x + \cos x}$$

$$(iii) \int \frac{dx}{1 + 3e^x + 2e^{2x}}; \quad (iv) \int x \sqrt{\frac{a-x}{a+x}} \, dx.$$

$$(b) \text{ Evaluate : } \lim_{n \rightarrow \infty} \left[\frac{(n-1)(n+2)(n+3) \dots (2n)}{n^n} \right]^{1/n}$$

$$8. (a) \text{ Evaluate : } \int_0^{\pi/2} \log \sin x \, dx.$$

(b) Evaluate: $\int_R \int xy(x^2 + y^2) dx dy$, where

$$R : [0 \leq x \leq a, 0 \leq y \leq b].$$

9. (a) Show that: $\int_0^{\pi/2} \frac{dx}{\sqrt{\sin x}} \times \int_0^{\pi/2} \sqrt{\sin x} dx = \pi$.

(b) Evaluate: $\int_1^{\infty} \frac{dx}{x(x+1)}$.

(c) If $I_n = \int_0^{\pi/4} \tan^n x dx$, where n is a positive integer,

then show that $I_{(n+1)} + I_{(n-1)} = \frac{1}{n}$.

10. (a) Find the area of the region included between the cardioids

$$r = a(1 + \cos \theta) \text{ and } r = a(1 - \cos \theta).$$

(b) Find the length of the arc of the curve $x = e^{\theta} \sin \theta$, $y = e^{\theta} \cos \theta$ measured from $\theta = 0$ to $\theta = \pi/2$.

11. (a) A curve is such that the segment cut off on the y -axis by the normal at any point (x, y) on it is equal to the distance of the point of contact from the origin. If the curve passes through the point $(0, 1)$, find its equation.

(b) Solve: $p^2 y - p(xy + 1) + x = 0$, $p = \frac{dy}{dx}$.

(c) Solve: $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$.

(d) Solve: $\frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + 3y = 2xe^{3x}$.

(e) Solve: $\frac{dy}{dx} = \frac{x + 2y - 3}{2x + y - 3}$.

(f) Solve: $\frac{d^2y}{dx^2} + 4y = \sin x$, given that

$$y = 1 \text{ and } \frac{dy}{dx} = 1, \text{ when } x = 0.$$

(g) If the gradient of a curve at any point (x, y) is $2y + x$ and it passes through the origin, then find the equation of the curve.

KALYANI UNIVERSITY

1992

4. (a) Show that the integral

$$\int_a^{\infty} \frac{1}{x^n} dx \quad (a > 0) \text{ is convergent if } n > 1.$$

(b) Test the convergence

$$(i) \int_0^1 \frac{dx}{\sqrt{x(1+x^2)}}, \quad (ii) \int_0^1 \frac{dx}{\sqrt{1-x^2}},$$

$$(iii) \int_0^a \frac{dx}{x(\log x)^{3/2}}$$

8. (a) Define Gamma function $\Gamma(x)$. Evaluate $\Gamma(1)$.

(b) Define Beta function $B(m, n)$.

Prove that $B(m, n) = B(m+1, n) + B(m, n+1)$.

(c) Evaluate :

$$(i) \int_0^{\pi/2} \sin^4 x \cos^6 x \, dx, \quad (ii) \int_0^1 x^{5/2} (1-x)^{3/2} \, dx,$$

$$(iii) \int_0^{\infty} e^{-4x} x^{3/2} \, dx.$$

NORTH BENGAL UNIVERSITY

1989

6. Integrate :-

$$(i) \int e^x \frac{x^2 + 1}{(x+1)^2} \, dx, \quad (ii) \int \frac{\sin x \, dx}{\sqrt{1 + \sin x}},$$

$$(iii) \int \frac{dx}{x^2 + 6x + 8}, \quad (iv) \int \frac{x^2 \, dx}{a^4 - x^4},$$

$$(v) \int \frac{dx}{4 + 3 \cos x}.$$

7. (a) State and prove "Fundamental Theorem" of Integral Calculus.

(b) Evaluate :

$$\lim_{n \rightarrow \infty} \left[\frac{1}{\sqrt{(2n-1)}} + \frac{1}{\sqrt{(4n-2^2)}} + \frac{1}{\sqrt{(6n-3^2)}} + \dots + \frac{1}{n} \right].$$

(c) Evaluate :

$$(i) \int_0^{\frac{\pi}{2}} \cos^3 x \, dx, \quad (ii) \int_0^{\frac{\pi}{2}} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x},$$

$$(iii) \int_0^{\pi} \frac{dx}{1 - 2a \cos x + a^2}, \quad 0 < a < 1.$$

8. (a) Find the whole area of $a^2 y^2 = a^2 x^2 - x^4$.

(b) Find the perimeter of $r = a(1 + \cos \theta)$.

(c) Show that the volume generated by the rotation of the conic $x = a \cos \theta$, $y = b \sin \theta$ about the line $x = 2a$ is $4\pi^2 a^2 b$.

9. (a) Find the equation of the curve for which the cartesian subnormal at any point is constant.

(b) Find a curve whose subtangent is of constant length a .

(c) Explain the method of solving the linear first order differential equation

$$\frac{dy}{dx} = Q - Py, \text{ where } P \text{ and } Q \text{ are functions of } x.$$

(d) Solve :

$$(i) (1 + y^2) dx - (\tan^{-1} y - x) dy = 0,$$

$$(ii) \frac{dy}{dx} + \frac{y}{x} = y^2, \quad (iii) \frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 5y = 10 \sin x,$$

$$(iv) l \frac{d^2 \theta}{dt^2} + g\theta = 0, \quad \text{when } \theta = \alpha, \quad t = 0, \quad \frac{d\theta}{dt} = 0.$$

TRIPURA UNIVERSITY

1992

1. (b) (i) Show that $\int e^x [f(x) + f'(x)] dx = e^x f(x)$.

(ii) Show that $\int_{-a}^a f(x) dx = 0$, if $f(x)$ be an odd function

$$= 2 \int_0^a f(x) dx, \text{ if } f(x) \text{ be an even function.}$$

(iii) Show that $\int_0^{\pi/2} \frac{\cos x dx}{\sin x + \cos x} = \frac{\pi}{4}$.

(iv) Find the area enclosed by the ellipse $x = a \cos \varphi$, $y = b \sin \varphi$ and the axes in the first quadrant.

(v) Define definite integral as the limit of a sum.

(c) (i) Solve the equation

$$(x^2 + y^2 - y) dx + x dy = 0.$$

(ii) Eliminate the constants a, b from $y = ax + bx^2$.

(iii) Solve $(D^2 - 5D + 4)y = 0$, $D = \frac{d}{dx}$.

(iv) Find the differential equation of a system of concentric circles having centres at the origin.

(v) Reduce $\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$ into the $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x alone or constants.

7. (a) Integrate the following:

$$(i) \int \sqrt{\frac{1+x}{1-x}} dx; \quad (ii) \int \frac{\log x}{(1+x)^3} dx;$$

$$(iii) \int \frac{\sin x + 2 \cos x}{2 \sin x + \cos x} dx.$$

(b) Obtain the reduction formula of $\int \cos^m x \cos nx \, dx$ and hence find the value of $\int \cos^3 x \cos 5x \, dx$.

8. (a) State the Fundamental Theorem of Integral Calculus.

(b) Evaluate the definite integral $\int_1^2 \frac{2x+5}{3} \, dx$ by the method of limit of a sum

(c) If $I_n = \int_0^{\frac{\pi}{2}} x^n \sin x \, dx$ and $n > 1$, show that

$$I_n + n(n-1)I_{n-2} = \left(\frac{n}{2}\right)^{n-1}$$

$$9. (a) \text{ Show that } \int_0^{\frac{\pi}{2}} \sin^m \theta \cos^n \theta \, d\theta = \frac{\Gamma\left(\frac{m+1}{2}\right)\Gamma\left(\frac{n+1}{2}\right)}{2\Gamma\left(\frac{m+n+2}{2}\right)}$$

where $m > -1$, $n > -1$.

(b) Evaluate $\iint \sqrt{4x^2 - y^2} \, dx \, dy$ over the triangle formed by the straight lines $y = 0$, $x = 1$, $y = x$

10. (a) Find the relation between S and ψ for the parabola $y^2 = 4ax$ with its vertex as the fixed point. (Symbols having usual meanings.)

(b) Find the volume generated by revolving the upper half of the loop of $x(x^2 + y^2) = a(x^2 - y^2)$ about the x -axis.

11. (a) Solve the following :

(i) $(x+y)^2 \frac{dy}{dx} = a^2$;

(ii) $(x+y+1) \, dx - (2x+2y+1) \, dy = 0$;

(iii) $\frac{dy}{dx} + \frac{x}{1-x^2} y = x\sqrt{y}$.

(iv) Solve $x(x+y) = y(x+y)$, $y' = \frac{dy}{dx}$

12. (a) Solve the following ; where $D = \frac{d}{dx}$:

(i) $(D^2 - 4D + 3)y = 2e^{3x}$;

(ii) $(D^2 - D - 2)y = \sin 2x$;

(iii) $(D^2 - 4D + 4)y = x^2$, given $x = 0, y = \frac{3}{8}, Dy = 1$.

(b) A particle starting with velocity u , moves in a straight line with a uniform acceleration f . Find the velocity and distance travelled in any time.

KALYANI UNIVERSITY

1992

4. (a) Show that the integral

$$\int_a^{\infty} \frac{1}{x^n} dx \quad (a > 0) \text{ is convergent if } n > 1.$$

(b) Test the convergence

(i) $\int_0^1 \frac{dx}{\sqrt{x}(1+x^2)}$ (ii) $\int_0^1 \frac{dy}{\sqrt{1-x^2}}$

(iii) $\int_0^{\infty} \frac{dx}{x(\log x)^{3/2}}$

5. (a) Define Gamma function $\Gamma(x)$. Evaluate $\Gamma(1)$.

(b) Define Beta function $B(m, n)$.

Prove that $B(m, n) = B(m+1, n) + B(m, n+1)$.

(c) Evaluate :

(i) $\int_0^{\pi/2} \sin^4 x \cos^6 x dx$. (ii) $\int_0^1 x^{5/2} (1-x)^{3/2} dx$

(iii) $\int_0^{\infty} e^{-4x} x^{3/2} dx$.