INDEX

UNIVERSITY PAPERS
B. A. / B. Sc. (Pass)
CALCUTTA UNIVERSITY
1994

2. (*a*) Evaluate $\int \frac{\sin x}{\sin x + \cos x}\, dx$. (*b*) Prove that $\int_{-\pi/2}^{\pi/2} \sin^5 x\, dx = 0$.

(*c*) Find the area of the region bounded by the parabola $y^2 = 4x$ and its latus rectum.

(*d*) If $B(m, n) = \int_0^1 x^{m-1}(1-x)^{n-1}\, dx\ (m, n > 0)$,

show that $B\left(\frac{1}{2}, \frac{1}{2}\right) = \pi$.

(*e*) Evaluate $\iint xy\, dx\, dy$ over the region in the positive quadrant for which $x + y \leq 1$.

3. (*a*) Show that $V = \frac{A}{r} + B$ satisfies the differential equation $\frac{d^2v}{dr^2} + \frac{2}{r}\frac{dv}{dr} = 0$, A, B are constants.

(*b*) Find the differential equation of all parabolas having their axes parallel to y-axis.

(*c*) Show that the curve in which the polar subtangent is proportional to the length of radius vector is $Cr = e^{\theta k}$, where k is constant and C is arbitrary.

(*d*) Show that the curvature is zero at every point on a straight line.

(*e*) Solve $\frac{dy}{dx} + 1 = e^{x-y}$.

9. (*a*) Evaluate (*i*) $\int \frac{\cos 5x + \cos 4x}{1 - 2\cos 3x}\, dx$;

(*ii*) $\int \frac{x+1}{\sqrt{4 + 8x - 5x^2}}\, dx$;

(b) Evaluate $\underset{n \to \infty}{Lt} \sum_{r=1}^{n-1} \frac{1}{n} \sqrt{\frac{n+r}{n-r}}$.

10. (a) If $I_n = \int_0^{\pi/2} \cos^n x \, dx$, n is a positive integer,

show that $I_n = \frac{n-1}{n} I_{n-2}$ and hence find $\int_0^{\pi/2} \sin^6 x \, dx$

(b) Show that the total area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab.

(c) Assuming $\Gamma(n+1) = \lfloor n$ (n is a positive integer), and $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$,

find the value of $\int_0^{\pi/2} \sqrt{\sin\theta} \, d\theta \times \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}}$.

11. (a) Find the length of the arc of the curve

$x = a(\cos\theta + \theta\sin\theta)$,

$y = a(\sin\theta - \theta\cos\theta)$, from $\theta = 0$ to $\theta = \theta_1$.

(b) The circle $x^2 + y^2 = a^2$ revolves round x-axis. Find the surface area and the volume of the whole sphere generated.

(c) Show that $\int_0^{a} \frac{a\left(x - \sqrt{a^2 - x^2}\right)^2}{(2x^2 - a^2)^2} dx = \frac{1}{\sqrt{2}} \log(\sqrt{2} + 1)$.

12. (a) Find the value of $\iint_R xy(x^2 + y^2) \, dx \, dy$

where $R : [0 \le x \le a, 0 \le y \le b]$.

(b) Find the intrinsic equation of the catenary $y = c \cosh\left(\frac{x}{c}\right)$

(c) Obtain an approximate value of log 2 by calculating $\int_1^2 \frac{dx}{x}$ by Simpson's rule dividing the interval into two equal parts.

13. (a) Show that if $l\frac{d^2\theta}{dt^2} + g\theta = 0$ and if $\theta = \alpha$ and $\frac{d\theta}{dt} = 0$ when $t = 0$, then

$$\theta = \alpha \cos \sqrt{\frac{g}{l}}\, t.$$

(b) Solve :

(i) $x^2(x\,dx + y\,dy) + 2y(x\,dy - y\,dx) = 0$;

(ii) $(x^2 + y^2 + 4)\,x\,dx + (x^2 - y^2 + 9)\,y\,dy = 0$;

(iii) $\frac{dy}{dx} + \frac{y}{x}\log y = \frac{y}{x^2}(\log y)^2$;

(iv) $xy\left(\frac{dy}{dx}\right)^2 - (x^2 - y^2)\frac{dy}{dx} - xy = 0$;

14. (a) Find the curve for which the sum of the reciprocals of the radius vector and the polar subtangent is constant.

(b) Solve : (i) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 10\sin x$;

(ii) $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$; (iii) $\frac{d^2y}{dx^2} + a^2y = \sin ax$;

(iv) $(D^2 - 1)\,y = 2$, $Dy = 3$ when $y = 1$ and $y = -1$ when $x = 2$

BURDWAN UNIVERSITY

1993

7. (a) Evaluate the following

(i) $\int \frac{dx}{(4x - x^2)^{\frac{3}{2}}}$, (ii) $\int \frac{dx}{\sin x + \cos x}$,

(iii) $\int \frac{x^2 + 1}{x^4 + 1}\,dx$, (iv) $\int \frac{x + \sin x}{1 + \cos x}\,dx$.

(b) Evaluate $\int_0^1 x^2\,dx$ from the definition of Definite Integral as limit of a sum.

(c) Show that $\int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} = \frac{\pi}{2ab}$ $[a > b > 0]$.

8. (a) State and prove the Fundamental Theorem of Integral Calculus.

(b) $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$. Prove it, and hence show that

$$\int_0^{\pi/2} \log \tan x\,dx = 0.$$

(c) Evaluate, when possible $\int_0^{\infty} \frac{dx}{x^2 - 1}$.

9. (a) Calculate the area bounded by the curves $y^2 - 4x - 4 = 0$ and $y^2 + 4x - 4 = 0$. (figure necessary).

(b) Find the perimeter of the hypo-cycloid

$$\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = 1.$$

(c) By the method of Integration. Determine the volume of a sphere of radius r.

10. (a) Show that a plane curve for which the length of the normal at a point of the curve is equal to that of the radius vector at that point is either a circle or a rectangular hyperbola.

(b) Solve the following :

(i) $(x^2 + y^2)\,dy = xy\,dx$,

(ii) $(2x - y + 1)\,dx + (2y - x - 1)\,dy = 0$,

(iii) $\frac{dy}{dx}-\frac{\tan y}{1+x}=(1+x)e^{x}\sec y$, (iv) $\left(\frac{dy}{dx}\right)^2+2x\frac{dy}{dx}-3x^2=0$,

(v) $(xy\cos xy+\sin xy)\,dx+x^2\cos xy\,dy=0$.

11. Solve the following :

(i) $\frac{d^2y}{dx^2}-\frac{4dy}{dx}+4y=x^3e^{2x}$,

(ii) $\frac{d^2y}{dx^2}+y=\sin 2x$ when $x=0, y=0$ and $\frac{dy}{dx}=0$,

(iii) $x^2\frac{d^2y}{dx^2}+\frac{dy}{dx}+y=\sin\log x^2$,

(iv) $\frac{d^2y}{dx^2}+4y=2\sin x\cos x$,

(v) $\frac{d^2y}{dx^2}=0$ when $x=0$, $y=4$ and when $x=\frac{\pi}{2}$, $y=0$.

VIDYASAGAR UNIVERSITY

1993

1. (b) (i) If $f(x)=|x-1|$, evaluate $\int_0^2 f(x)dx$.

(ii) Prove that $\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$.

(iii) Evaluate $\int_{x=0}^{\pi/2}\int_{y=0}^{\pi}\sin(x+y)\,dx\,dy$.

(*iv*) A point $P(x, y)$ moves in xy- plane satisfying $0 \le x \le \pi/2$, $0 \le y \le \sin x$. Find the area of the region traced out by P.

(*v*) Use method of integration to evaluate

$$\lim_{n \to \infty} \left(\frac{1^5 + 2^5 + 3^5 + \cdots + n^5}{n^6} \right).$$

(*c*) (*i*) Examine whether $\frac{1}{y^2}$ is an integrating factor of the differential equation $y(1 + xy)\,dx - x\,dy = 0$.

(*ii*) Obtain the differential equation whose general solution is $ax + by + c = 0$, a, b, c are arbitrary constants.

(*iii*) Find the solution of the equation

$$\frac{d^2y}{dx^2} - 2c\frac{dy}{dx} + c^2y = 0,\ c \text{ is a constant.}$$

7. (*a*) Evaluate :

(*i*) $\int \frac{dx}{(x^2+1)(x^2+x+1)}$; (*ii*) $\int \frac{\cos x\,dx}{\sin x + \cos x}$,

(*iii*) $\int \frac{dx}{1 + 3e^x + 2e^{2x}}$; (*iv*) $x\sqrt{\frac{a-x}{a+x}}\,dx$.

(*b*) Evaluate : $\lim_{n \to \infty} \left[\frac{(n+1)(n+2)(n+3)\cdots(2n)}{n^n} \right]^{1/n}$

8. (*a*) Evaluate : $\int_0^{\pi/2} \log \sin x\,dx$.

(*b*) Evaluate : $\iint_R xy(x^2 + y^2)\,dxdy$, where

$$R : [0 \le x \le a,\ 0 \le y \le b].$$

9. (*a*) Show that : $\int_0^{\pi/2} \frac{dx}{\sqrt{\sin x}} \times \int_0^{\pi/2} \sqrt{\sin x}\,dx = \pi$.

(*b*) Evaluate : $\int_1^{\infty} \frac{dx}{x(x+1)}$.

(*c*) If $I_n = \int_0^{\pi/4} \tan^n x\,dx$, where n is a positive integer,

then show that $I_{(n+1)} + I_{(n-1)} = \frac{1}{n}$.

10. (*a*) Find the area of the region included between the cardioides

$$r = a(1 + \cos\theta) \text{ and } r = a(1 - \cos\theta).$$

(*b*) Find the length of the arc of the curve $x = e^{\theta}\sin\theta$, $y = e^{\theta}\cos\theta$ measured from $\theta = 0$ to $\theta = \pi/2$.

11. (*a*) . curve is such that the segment cut off on the *y*-axis by the normal at any point (x, y) on it is equal to the distance of the point of contact form the origin. If the curve passes through the point $(0, 1)$, find its equation.

(*b*) Solve : $p^2 y - p(xy + 1) + x = 0$, $p = \frac{dy}{dx}$.

(*c*) Solve : $\frac{dy}{dx} + x\sin 2y = x^3\cos^2 y$.

(*d*) Solve : $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 2xe^{3x}$.

(e) Solve : $\dfrac{dy}{dx} = \dfrac{x + 2y - 3}{2x + y - 3}$.

(f) Solve : $\dfrac{d^2y}{dx^2} + 4y = \sin x$, given that

$$y = 1 \text{ and } \frac{dy}{dx} = 1, \text{ when } x = 0.$$

(g) If the gradient of a curve at any point (x, y) is $2y + x$ and it passes through the origin, then find the equation of the curve.

KALYANI UNIVERSITY

1992

4. (a) Show that the integral

$$\int_a^{\infty} \frac{1}{x^n}\, dx \quad (a > 0) \quad \text{is convergent if } n > 1.$$

(b) Test the convergence

(i) $\displaystyle\int_0^1 \frac{dx}{\sqrt{x(1 + x^2)}}$, (ii) $\displaystyle\int_0^1 \frac{dx}{\sqrt{1 - x^2}}$,

(iii) $\displaystyle\int_0^{1/2} \frac{dx}{x(\log x)^2}$

8. (a) Define Gamma function $\Gamma(x)$. Evaluate $\Gamma(1)$.

(b) Define Beta function $B(m, n)$.

Prove that $B(m, n) = B(m + 1, n) + B(m, n + 1)$.

(c) Evaluate :

(i) $\int_0^{\pi/2} \sin^4 x \cos^6 x \, dx$, (ii) $\int_0^1 x^{5/2}(1-x)^{3/2} \, dx$,

(iii) $\int_0^\infty e^{-4x} x^{3/2} \, dx$.

NORTH BENGAL UNIVERSITY

1989

6. Integrate :-

(i) $\int e^x \frac{x^2+1}{(x+1)^2} \, dx$, (ii) $\frac{\sin x \, dx}{\sqrt{1+\sin x}}$,

(iii) $\int \frac{dx}{x^2+6x+8}$, (iv) $\int \frac{x^2 \, dx}{a^4-x^4}$,

(v) $\int \frac{dx}{4+3\cos x}$

7. (a) State and prove "Fundamental Theorem" of Integral Calculus.

(b) Evaluate :

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{(2n-1)}}+\frac{1}{\sqrt{(4n-2^2)}}+\frac{1}{\sqrt{(6n-3^2)}}+\cdots+\frac{1}{n}\right].$$

(c) Evaluate :

(i) $\int_0^{\frac{\pi}{2}} \cos^3 x \, dx$, (ii) $\int_0^{\frac{\pi}{2}} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$,

(iii) $\int_0^{\pi} \frac{dx}{1 - 2a \cos x + a^2}$, $0 < a < 1$.

8. (a) Find the whole area of $a^2y^2 = a^2x^2 - x^4$.

(b) Find the perimeter of $r = a(1 + \cos\theta)$.

(c) Show that the volume generated by the rotation of the conic $x = a\cos\theta$, $y = b\sin\theta$ about the line $x = 2a$ is $4\pi^2 a^2 b$.

9. (a) Find the equation of the curve for which the cartesian subnormal at any point is constant.

(b) Find a curve whose subtangent is of constant length a.

(c) Explain the method of solving the linear first order differential equation

$\frac{dy}{dx} = Q - Py$, where P and Q are functions of x.

(d) Solve :

(i) $(1 + y^2)\,dx - (\tan^{-1} y - x)\,dy = 0$,

(ii) $\frac{dy}{dx} + \frac{y}{x} = y^2$, (iii) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 10 \sin x$,

(iv) $l\frac{d^2\theta}{dt^2} + g\theta = 0$, when $\theta = \alpha$, $t = 0$, $\frac{d\theta}{dt} = 0$.

TRIPURA UNIVERSITY

1992

1. (*b*) (*i*) Show that $\int e^x [f(x) + f'(x)]\, dx = e^x f(x)$.

(*ii*) Show that $\int_{-a}^{a} f(x)\, dx = 0$, if $f(x)$ be an odd function

$= 2\int_{0}^{a} f(x)\, dx$, if $f(x)$ be an even function.

(*iii*) Show that $\int_0^{\pi/2} \frac{\cos x\, dx}{\sin x + \cos x} = \frac{\pi}{4}$.

(*iv*) Find the area enclosed by the ellipse $x = a\cos\varphi$, $y = b\cos\varphi$ and the axes in the first quadrant.

(*v*) Define definite integral as the limit of a sum.

(*c*) (*i*) Solve the equation

$$(x^2 + y^2 - y)\, dx + x\, dy = 0.$$

(*ii*) Eliminate the constants a, b from $y = ax + bx^2$.

(*iii*) Solve $(D^2 - 5D + 4)\, y = 0$, $D = \frac{d}{dx}$.

(*iv*) Find the differential equation of a system of concentric circles having centres at the origin

(*v*) Reduce $\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$ into the $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x alone or constants

7. (*a*) Integrate the following :

(*i*) $\int \sqrt{\frac{1 + x}{1 - x}}\, dx$; (*ii*) $\int \frac{\log x}{(1 + x)^3}\, dx$;

(*iii*) $\int \frac{\sin x + 2\cos x}{2\sin x + \cos x}\, dx$.

(b) Obtain the reduction formula of $\int \cos^m x \cos nx\, dx$ and hence find the value of $\int \cos^3 x \cos 5x\, dx$.

8. (a) State the Fundamental Theorem of Integral Calculus.

(b) Evaluate the definite integral $\int_1^2 \frac{2x+5}{3}\, dx$ by the method of limit of a sum.

(c) If $I_n = \int_0^{\frac{\pi}{2}} x^n \sin x\, dx$ and $n > 1$, show that

$$I_n + n(n-1) I_{n-2} = \left(\frac{\pi}{2}\right)^{n-1}$$

9. (a) Show that $\int_0^{\frac{\pi}{2}} \sin^m\theta \cos^n\theta\, d\theta = \frac{\Gamma\left(\frac{m+1}{2}\right)\Gamma\left(\frac{n+1}{2}\right)}{2\Gamma\left(\frac{m+n+2}{2}\right)}$

where $m > -1$, $n > -1$.

(b) Evaluate $\iint \sqrt{4x^2 - y^2}\, dx\, dy$ over the triangle formed by the straight lines $y = 0$, $x = 1$, $y = x$

10. (a) Find the relation between S and ψ for the parabola $y^2 = 4ax$ with its vertex as the fixed point. (Symbols having usual meanings.)

(b) Find the volume generated by revolving the upper half of the loop of $x(x^2 + y^2) = a(x^2 - y^2)$ about the x-axis.

11. (a) Solve the following :

(i) $(x+y)^2 \frac{dy}{dx} = a^2$;

(ii) $(x + y + 1)\, dx - (2x + 2y + 1)\, dy = 0$;

(iii) $\frac{dy}{dx} + \frac{x}{1-x^2}\, y = x\sqrt{y}$.

(b) Solve $p(p + x) = y(x + y)$, $p = \frac{dy}{dx}$

12. (*a*) Solve the following ; where $D = \frac{d}{dx}$:

(*i*) $(D^2 - 4D + 3)y = 2e^{3x}$;

(*ii*) $(D^2 - D - 2)y = \sin 2x$;

(*iii*) $(D^2 - 4D + 4)y = x^2$, given $x = 0$, $y = \frac{3}{8}$, $Dy = 1$.

(*b*) A particle starting with velocity *u*, moves in a straight line with a uniform acceleration *f*. Find the velocity and distance travelled in any time.

KALYANI UNIVERSITY

1992

4. (*a*) Show that the integral

$$\int_a^\infty \frac{1}{x^n}\,dx \quad (a > 0) \text{ is convergent if } n > 1.$$

(*b*) Test the convergence

(i) $\int_0^1 \frac{dx}{\sqrt{x}(1+x^2)}$ (ii) $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

(iii) $\int_0^a \frac{dx}{x(\log x)^{3/2}}$

5. (*a*) Define Gamma function $\Gamma(x)$. Evaluate $\Gamma(1)$.

(*b*) Define Beta function $B(m, n)$.

Prove that $B(m, n) = B(m+1, n) + B(m, n+1)$.

(c) Evaluate :

(i) $\int_0^{\pi/2} \sin^4 x \cos^6 x\,dx$. (ii) $\int_0^1 x^{5/2}(1-x)^{3/2}\,dx$

(iii) $\int_0^\infty e^{-4x} x^{3/2}\,dx$