Ohm's Law, Power, and Energy

Objectives

- *Understand the importance of Ohm's law and how to apply it to a variety of situations.*
- *Be able to plot Ohm's law and understand how to "read" a graphical plot of voltage versus current.*
- *Become aware of the differences between power and energy levels and how to solve for each.*
- *Understand the power and energy flow of a system, including how the flow affects the efficiency of operation.*
- *Become aware of the operation of a variety of fuses and circuit breakers and where each is employed.*

4.1 INTRODUCTION

Now that the three important quantities of an electric circuit have been introduced, this chapter reveals how they are interrelated. The most important equation in the study of electric circuits is introduced, and various other equations that allow us to find power and energy levels are discussed in detail. It is the first chapter where we tie things together and develop a feeling for the way an electric circuit behaves and what affects its response. For the first time, the data provided on the labels of household appliances and the manner in which your electric bill is calculated will have some meaning. It is indeed a chapter that should open your eyes to a wide array of past experiences with electrical systems.

4.2 OHM'S LAW

As mentioned above, the first equation to be described is without question one of the most important to be learned in this field. It is not particularly difficult mathematically, but it is very powerful because it can be applied to any network in any time frame. That is, it is applicable to dc circuits, ac circuits, digital and microwave circuits, and, in fact, any type of applied signal. In addition, it can be applied over a period of time or for instantaneous responses. The equation can be derived directly from the following basic equation for all physical systems:

$$\boxed{\text{Effect} = \frac{\text{cause}}{\text{opposition}}} \tag{4.1}$$

Every conversion of energy from one form to another can be related to this equation. In electric circuits, the *effect* we are trying to establish is the flow of charge, or *current*. The *potential difference*, or voltage, between two points is the *cause* ("pressure"), and the opposition is the *resistance* encountered.

An excellent analogy for the simplest of electrical circuits is the water in a hose connected to a pressure valve, as discussed in Chapter 2. Think of the electrons in the copper wire as the water in the hose, the pressure valve as the applied voltage, and the size of the hose as the factor that determines the resistance. If the pressure valve is closed, the water simply sits in the hose without a general direction, much like the oscillating electrons in a conductor without an applied voltage. When we open the pressure valve, water will flow through the hose much like the electrons in a copper wire when the voltage is applied. In other words, the absence of the "pressure" in one case and the voltage in the other simply results in a system without direction or reaction. The rate at which the water will flow in the hose is a function of the size of the hose. A hose with a very small diameter will limit the rate at which water can flow through the hose, just as a copper wire with a small diameter will have a high resistance and will limit the current.

In summary, therefore, the absence of an applied "pressure" such as voltage in an electric circuit will result in no reaction in the system and no current in the electric circuit. Current is a reaction to the applied voltage and not the factor that gets the system in motion. To continue with the analogy, the greater the pressure at the spigot, the greater is the rate of water flow through the hose, just as applying a higher voltage to the same circuit results in a higher current.

Substituting the terms introduced above into Eq. (4.1) results in

$$\text{Current} = \frac{\text{potential difference}}{\text{resistance}}$$

and

$$\boxed{I = \frac{E}{R}} \qquad \text{(amperes, A)} \qquad \textbf{(4.2)}$$

FIG. 4.1
George Simon Ohm.
Courtesy of the Smithsonian Institution. Photo No. 51,145.

German (Erlangen, Cologne)
(1789–1854)
Physicist and Mathematician
Professor of Physics, University of Cologne

In 1827, developed one of the most important laws of electric circuits: *Ohm's law.* When the law was first introduced, the supporting documentation was considered lacking and foolish, causing him to lose his teaching position and search for a living doing odd jobs and some tutoring. It took some 22 years for his work to be recognized as a major contribution to the field. He was then awarded a chair at the University of Munich and received the Copley Medal of the Royal Society in 1841. His research also extended into the areas of molecular physics, acoustics, and telegraphic communication.

Eq. (4.2) is known as **Ohm's law** in honor of Georg Simon Ohm (Fig. 4.1). The law states that for a fixed resistance, the greater the voltage (or pressure) across a resistor, the greater is the current; and the greater the resistance for the same voltage, the lower is the current. In other words, the current is proportional to the applied voltage and inversely proportional to the resistance.

By simple mathematical manipulations, the voltage and resistance can be found in terms of the other two quantities:

$$\boxed{E = IR} \qquad \text{(volts, V)} \qquad \textbf{(4.3)}$$

and

$$\boxed{R = \frac{E}{I}} \qquad \text{(ohms, } \Omega\text{)} \qquad \textbf{(4.4)}$$

All the quantities of Eq. (4.2) appear in the simple electrical circuit in Fig. 4.2. A resistor has been connected directly across a battery to establish a current through the resistor and supply. Note that

the symbol E is applied to all sources of voltage

and

the symbol V is applied to all voltage drops across components of the network.

Both are measured in volts and can be applied interchangeably in Eqs. (4.2) through (4.4).

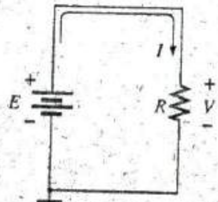


FIG. 4.2
Basic circuit.

Since the battery in Fig. 4.2 is connected directly across the resistor, the voltage V_R across the resistor must be equal to that of the supply. Applying Ohm's law, we obtain

$$I = \frac{V_R}{R} = \frac{E}{R}$$

Note in Fig. 4.2 that the voltage source "pressures" current (conventional current) in a direction that leaves the positive terminal of the supply and returns to the negative terminal of the battery. *This will always be the case for single-source networks.* (The effect of more than one source in the same network is investigated in a later chapter.) Note also that the current enters the positive terminal and leaves the negative terminal for the load resistor R.

For any resistor, in any network, the direction of current through a resistor will define the polarity of the voltage drop across the resistor

as shown in Fig. 4.3 for two directions of current. Polarities as established by current direction become increasingly important in the analyses to follow.

FIG. 4.3
Defining polarities.

EXAMPLE 4.1 Determine the current resulting from the application of a 9 V battery across a network with a resistance of 2.2 Ω.

Solution: Eq. (4.2):

$$I = \frac{V_R}{R} = \frac{E}{R} = \frac{9\text{ V}}{2.2\ \Omega} = \mathbf{4.09\ A}$$

EXAMPLE 4.2 Calculate the resistance of a 60 W bulb if a current of 500 mA results from an applied voltage of 120 V.

Solution: Eq. (4.4):

$$R = \frac{V_R}{I} = \frac{E}{I} = \frac{120\text{ V}}{500 \times 10^{-3}\text{ A}} = \mathbf{240\ \Omega}$$

EXAMPLE 4.3 Calculate the current through the 2 kΩ resistor in Fig. 4.4 if the voltage drop across it is 16 V.

Solution:

$$I = \frac{V}{R} = \frac{16\text{ V}}{2 \times 10^{3}\ \Omega} = \mathbf{8\ mA}$$

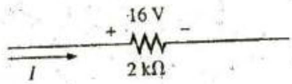


FIG. 4.4
Example 4.3.

EXAMPLE 4.4 Calculate the voltage that must be applied across the soldering iron in Fig. 4.5 to establish a current of 1.5 A through the iron if its internal resistance is 80 Ω.

Solution:

$$E = V_R = IR = (1.5\text{ A})(80\ \Omega) = \mathbf{120\ V}$$

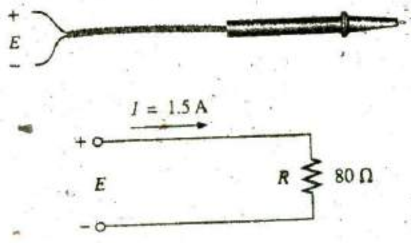


FIG. 4.5
Example 4.4.

In a number of the examples in this chapter, such as Example 4.4, the voltage applied is actually that obtained from an ac outlet in the home, office, or laboratory. This approach was used to provide an opportunity for the student to relate to real-world situations as soon as possible and to demonstrate that a number of the equations derived in this chapter are applicable to ac networks also. Chapter 13 will provide a direct relationship between ac and dc voltages that permits the mathematical substitutions used in this chapter. In other words, don't be concerned that some of the voltages and currents appearing in the examples of this chapter are actually ac voltages, because the equations for dc networks have exactly the same format, and all the solutions will be correct.

4.3 PLOTTING OHM'S LAW

Graphs, characteristics, plots, and the like play an important role in every technical field as modes through which the broad picture of the behavior or response of a system can be conveniently displayed. It is therefore critical to develop the skills necessary both to read data and to plot them in such a manner that they can be interpreted easily.

For most sets of characteristics of electronic devices, the current is represented by the vertical axis (ordinate) and the voltage by the horizontal axis (abscissa), as shown in Fig. 4.6. First note that the vertical axis is in amperes and the horizontal axis is in volts. For some plots, I may be in milliamperes (mA), microamperes (μA), or whatever is appropriate for the range of interest. The same is true for the levels of voltage on the horizontal axis. Note also that the chosen parameters require that the spacing between numerical values of the vertical axis be different from that of the horizontal axis. The linear (straight-line) graph reveals that the resistance is not changing with current or voltage level; rather, it is a fixed quantity throughout. The current direction and the voltage polarity appearing at the top of Fig. 4.6 are the defined direction and polarity for the provided plot. If the current direction is opposite to the defined direction, the region below the horizontal axis is the region of interest for the current I. If the voltage polarity is opposite to that defined, the region to the left of the current axis is the region of interest. For the standard fixed resistor, the first quadrant, or region, of Fig. 4.6 is the only region of interest. However, you will encounter many devices in your electronics courses that use the other quadrants of a graph.

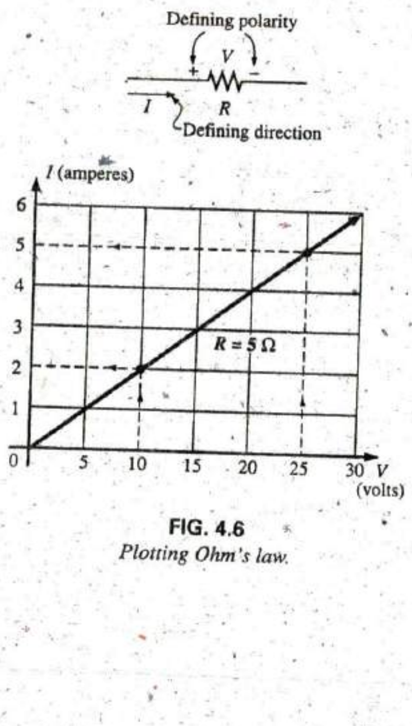


FIG. 4.6
Plotting Ohm's law.

Once a graph such as Fig. 4.6 is developed, the current or voltage at any level can be found from the other quantity by simply using the resulting plot. For instance, at $V = 25$ V, if a vertical line is drawn on Fig. 4.6 to the curve as shown, the resulting current can be found by drawing a horizontal line over to the current axis, where a result of 5 A is obtained. Similarly, at $V = 10$ V, drawing a vertical line to the plot and a horizontal line to the current axis results in a current of 2 A, as determined by Ohm's law.

If the resistance of a plot is unknown, it can be determined at any point on the plot since a straight line indicates a fixed resistance. At any point on the plot, find the resulting current and voltage, and simply substitute into the following equation:

$$\boxed{R_{dc} = \frac{V}{I}} \qquad (4.5)$$

To test Eq. (4.5), consider a point on the plot where $V = 20$ V and $I = 4$ A. The resulting resistance is $R_{dc} = 20\text{ V}/I = 20\text{ V}/4\text{ A} = 5\ \Omega$. For comparison purposes, a 1 Ω and a 10 Ω resistor were plotted on the graph in Fig. 4.7. Note that the lower the resistance, the steeper is the slope (closer to the vertical axis) of the curve.

If we write Ohm's law in the following manner and relate it to the basic straight-line equation

$$\begin{array}{ccccccc} I & = & \frac{1}{R} & \cdot & E & + & 0 \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ y & = & m & \cdot & x & + & b \end{array}$$

we find that the slope is equal to 1 divided by the resistance value, as indicated by the following:

$$\boxed{m = \text{slope} = \frac{\Delta y}{\Delta x} = \frac{\Delta I}{\Delta V} = \frac{1}{R}} \tag{4.6}$$

where Δ signifies a small, finite change in the variable.

Eq. (4.6) reveals that the greater the resistance, the lower is the slope. If written in the following form, Eq. (4.6) can be used to determine the resistance from the linear curve:

$$\boxed{R = \frac{\Delta V}{\Delta I}} \quad \text{(ohms)} \tag{4.7}$$

The equation states that by choosing a particular ΔV (or ΔI), you can obtain the corresponding ΔI (or ΔV, respectively) from the graph, as shown in Fig. 4.8, and then determine the resistance. If the plot is a straight line, Eq. (4.7) will provide the same result no matter where the equation is applied. However, if the plot curves at all, the resistance will change.

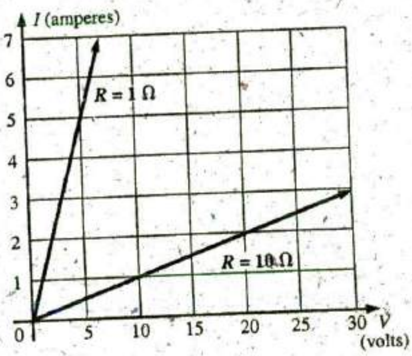


FIG. 4.7
Demonstrating on an I-V plot that the lower the resistance, the steeper is the slope.

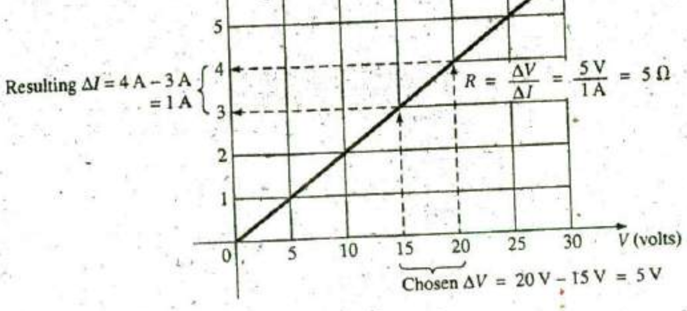


FIG. 4.8
Applying Eq. (4.7).

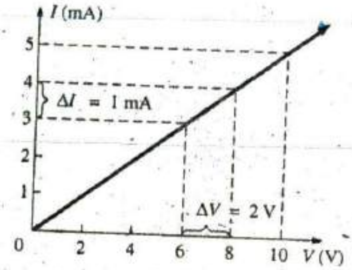


FIG. 4.9
Example 4.5.

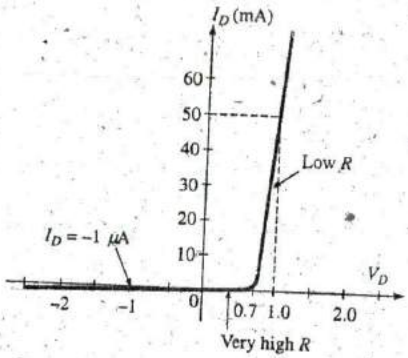


FIG. 4.10
Semiconductor diode characteristics.

EXAMPLE 4.5 Determine the resistance associated with the curve in Fig. 4.9 using Eqs. (4.5) and (4.7), and compare results.

Solution: At $V = 6$ V, $I = 3$ mA, and

$$R_{dc} = \frac{V}{I} = \frac{6\text{ V}}{3\text{ mA}} = \mathbf{2\ k\Omega}$$

For the interval between 6 V and 8 V,

$$R = \frac{\Delta V}{\Delta I} = \frac{2\text{ V}}{1\text{ mA}} = \mathbf{2\ k\Omega}$$

The results are equivalent.

Before leaving the subject, let us first investigate the characteristics of a very important semiconductor device called the **diode,** which will be examined in detail in basic electronics courses. This device ideally acts as a low-resistance path to current in one direction and a high-resistance path to current in the reverse direction, much like a switch that passes current in only one direction. A typical set of characteristics appears in Fig. 4.10. Without any mathematical calculations, the closeness of the characteristic to the voltage axis for negative values of applied voltage indicates that this is the low-conductance (high resistance, switch opened) region. Note that this region extends to approximately 0.7 V positive. However, for values of applied voltage greater than 0.7 V, the vertical rise in the characteristics indicates a high-conductivity (low resistance, switch closed) region. Application of Ohm's law will now verify the above conclusions.

At $V_D = +1$ V,

$$R_{\text{diode}} = \frac{V_D}{I_D} = \frac{1\text{ V}}{50\text{ mA}} = \frac{1\text{ V}}{50 \times 10^{-3}\text{ A}} = 20\ \Omega$$

(a relatively low value for most applications)

At $V_D = -1$ V,

$$R_{\text{diode}} = \frac{V_D}{I_D} = \frac{1\text{V}}{1\ \mu\text{A}} = 1\text{ M}\Omega$$

(which is often represented by an open-circuit equivalent)

4.4 POWER

In general,

the term power is applied to provide an indication of how much work (energy conversion) can be accomplished in a specified amount of time; that is, power is a rate of doing work.

For instance, a large motor has more power than a smaller motor because it has the ability to convert more electrical energy into mechanical energy in the same period of time. Since energy is measured in joules (J) and time in seconds (s), power is measured in joules/second (J/s). The electrical unit of measurement for power is the watt (W), defined by

$$\boxed{1\text{ watt (W)} = 1\text{ joule/second (J/s)}} \tag{4.8}$$

In equation form, power is determined by

$$\boxed{P = \frac{W}{t}} \quad \text{(watts, W, or joules/second, J/s)} \qquad (4.9)$$

with the **energy** (W) measured in joules and the time t in seconds.

The unit of measurement—the watt—is derived from the surname of James Watt (Fig. 4.11), who was instrumental in establishing the standards for power measurements. He introduced the **horsepower** (hp) as a measure of the average power of a strong dray horse over a full working day. It is approximately 50% more than can be expected from the average horse. The horsepower and watt are related in the following manner:

$$\boxed{1 \text{ horsepower} \cong 746 \text{ watts}}$$

The power delivered to, or absorbed by, an electrical device or system can be found in terms of the current and voltage by first substituting Eq. (2.5) into Eq. (4.9):

$$P = \frac{W}{t} = \frac{QV}{t} = V\frac{Q}{t}$$

But

$$I = \frac{Q}{t}$$

so that

$$\boxed{P = VI} \quad \text{(watts, W)} \qquad (4.10)$$

By direct substitution of Ohm's law, the equation for power can be obtained in two other forms:

$$P = VI = V\left(\frac{V}{R}\right)$$

and

$$\boxed{P = \frac{V^2}{R}} \quad \text{(watts, W)} \qquad (4.11)$$

or

$$P = VI = (IR)I$$

and

$$\boxed{P = I^2R} \quad \text{(watts, W)} \qquad (4.12)$$

The result is that the power absorbed by the resistor in Fig. 4.12 can be found directly, depending on the information available. In other words, if the current and resistance are known, it pays to use Eq. (4.12) directly, and if V and I are known, use of Eq. (4.10) is appropriate. It saves having to apply Ohm's law before determining the power.

The power supplied by a battery can be determined by simply inserting the supply voltage into Eq. (4.10) to produce

$$\boxed{P = EI} \quad \text{(watts, W)} \qquad (4.13)$$

The importance of Eq. (4.13) cannot be overstated. It clearly states the following:

The power associated with any supply is not simply a function of the supply voltage. It is determined by the product of the supply voltage and its maximum current rating.

The simplest example is the car battery—a battery that is large, difficult to handle, and relatively heavy. It is only 12 V, a voltage level that

FIG. 4.11
James Watt.
Courtesy of the Smithsonian Institution. Photo No. 30,391.

Scottish (Greenock, Birmingham)
(1736–1819)
Instrument Maker and Inventor
Elected Fellow of the Royal Society of London in 1785

In 1757, at the age of 21, used his innovative talents to design mathematical instruments such as the *quadrant, compass,* and various *scales.* In 1765, introduced the use of a separate *condenser* to increase the efficiency of steam engines. In the following years he received a number of important patents on improved engine design, including a rotary motion for the steam engine (versus the reciprocating action) and a double-action engine, in which the piston pulled as well as pushed in its cyclic motion. Introduced the term horsepower as the average power of a strong dray (small cart) horse over a full working day.

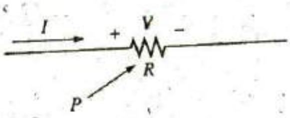


FIG. 4.12
Defining the power to a resistive element.

could be supplied by a battery slightly larger than the small 9 V portable radio battery. However, to provide the **power** necessary to start a car, the battery must be able to supply the high surge current required at starting—a component that requires size and mass. In total, therefore, it is not the voltage or current rating of a supply that determines its power capabilities; it is the product of the two.

Throughout the text, the abbreviation for energy (W) can be distinguished from that for the watt (W) because the one for energy is in italics while the one for watt is in roman. In fact, all variables in the dc section appear in italics, while the units appear in roman.

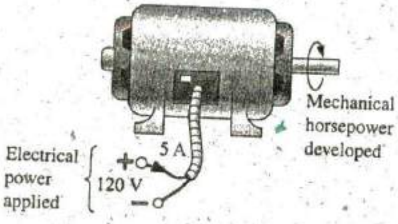


FIG. 4.13
Example 4.6.

EXAMPLE 4.6 Find the power delivered to the dc motor of Fig. 4.13.

Solution: $P = EI = (120\text{ V})(5\text{ A}) = 600\text{ W} = \mathbf{0.6\ kW}$

EXAMPLE 4.7 What is the power dissipated by a 5 Ω resistor if the current is 4 A?

Solution:

$$P = I^2R = (4\text{ A})^2(5\ \Omega) = \mathbf{80\ W}$$

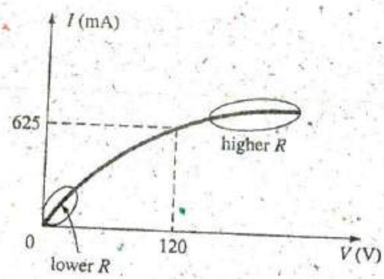


FIG. 4.14
The nonlinear I-V characteristics of a 75 W light bulb (Example 4.8).

EXAMPLE 4.8 The I-V characteristics of a light bulb are provided in Fig. 4.14. Note the nonlinearity of the curve, indicating a wide range in resistance of the bulb with applied voltage. If the rated voltage is 120 V, find the wattage rating of the bulb. Also calculate the resistance of the bulb under rated conditions.

Solution: At 120 V,

$$I = 0.625\text{ A}$$

and

$$P = VI = (120\text{ V})(0.625\text{ A}) = \mathbf{75\ W}$$

At 120 V,

$$R = \frac{V}{I} = \frac{120\text{ V}}{0.625\text{ A}} = \mathbf{192\ \Omega}$$

Sometimes the power is given and the current or voltage must be determined. Through algebraic manipulations, an equation for each variable is derived as follows:

$$P = I^2R \Rightarrow I^2 = \frac{P}{R}$$

and

$$\boxed{I = \sqrt{\frac{P}{R}}} \quad \text{(amperes, A)} \qquad (4.14)$$

$$P = \frac{V^2}{R} \Rightarrow V^2 = PR$$

and

$$\boxed{V = \sqrt{PR}} \quad \text{(volts, V)} \qquad (4.15)$$

EXAMPLE 4.9 Determine the current through a 5 kΩ resistor when the power dissipated by the element is 20 mW.

Solution: Eq. (4.14):

$$I = \sqrt{\frac{P}{R}} = \sqrt{\frac{20 \times 10^{-3}\,\text{W}}{5 \times 10^{3}\,\Omega}} = \sqrt{4 \times 10^{-6}} = 2 \times 10^{-3}\,\text{A}$$
$$= 2\,\text{mA}$$

FIG. 4.15
James Prescott Joule.

British (Salford, Manchester)
(1818–89)
Physicist
Honorary Doctorates from the Universities of Dublin and Oxford

Contributed to the important fundamental *law of conservation of energy* by establishing that various forms of energy, whether electrical, mechanical, or heat, are in the same family and can be exchanged from one form to another. In 1841 introduced *Joule's law,* which stated that the heat developed by electric current in a wire is proportional to the product of the current squared and the resistance of the wire (I^2R). He further determined that the heat emitted was equivalent to the power absorbed, and therefore heat is a form of energy.

4.5 ENERGY

For power, which is the rate of doing work, to produce an energy conversion of any form, it must be *used over a period of time.* For example, a motor may have the horsepower to run a heavy load, but unless the motor is *used* over a period of time, there will be no energy conversion. In addition, the longer the motor is used to drive the load, the greater will be the energy expended.

The **energy** (W) lost or gained by any system is therefore determined by

$$\boxed{W = Pt} \quad \text{(wattseconds, Ws, or joules)} \qquad \textbf{(4.16)}$$

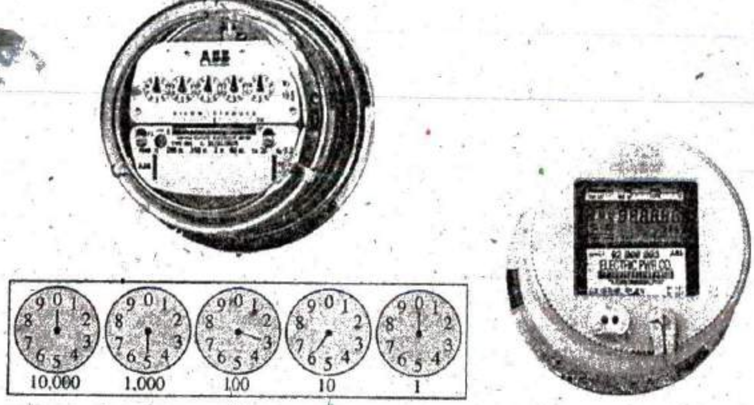
Since power is measured in watts (or joules per second) and time in seconds, the unit of energy is the *wattsecond* or *joule* (note Fig. 4.15). The wattsecond, however, is too small a quantity for most practical purposes, so the *watthour* (Wh) and the *kilowatthour* (kWh) are defined, as follows:

$$\boxed{\text{Energy (Wh)} = \text{power (W)} \times \text{time (h)}} \qquad \textbf{(4.17)}$$

$$\boxed{\text{Energy (kWh)} = \frac{\text{power (W)} \times \text{time (h)}}{1000}} \qquad \textbf{(4.18)}$$

Note that the energy in kilowatthours is simply the energy in watthours divided by 1000. To develop some sense for the kilowatthour energy level, consider that *1 kWh is the energy dissipated by a 100 W bulb in 10 h.*

The **kilowatthour meter** is an instrument for measuring the energy supplied to the residential or commercial user of electricity. It is normally connected directly to the lines at a point just prior to entering the power distribution panel of the building. A typical set of dials is shown in Fig. 4.16, along with a photograph of an analog kilowatthour meter. As indicated, each power of ten below a dial is in kilowatthours. The more rapidly the aluminum disc rotates, the greater is the energy demand. The dials are connected through a set of gears to the rotation of this disc. A solid-state digital meter with an extended range of capabilities also appears in Fig. 4.16.



(a) (b)

FIG. 4.16

Kilowatthour meters: (a) analog; (b) digital.
(Courtesy of ABB Electric Metering Systems.)

EXAMPLE 4.10 For the dial positions in Fig. 4.16(a), calculate the electricity bill if the previous reading was 4650 kWh and the average cost in your area is 11¢ per kilowatthour.

Solution:

$$5360 \text{ kWh} - 4650 \text{ kWh} = 710 \text{ kWh used}$$

$$710 \cancel{\text{kWh}}\left(\frac{11¢}{\cancel{\text{kWh}}}\right) = \mathbf{\$78.10}$$

EXAMPLE 4.11 How much energy (in kilowatthours) is required to light a 60 W bulb continuously for 1 year (365 days)?

Solution:

$$W = \frac{Pt}{1000} = \frac{(60 \text{ W})(24 \text{ h/day})(365 \text{ days})}{1000} = \frac{525{,}600 \text{ Wh}}{1000}$$
$$= \mathbf{525.60 \text{ kWh}}$$

EXAMPLE 4.12 How long can a 340 W plasma TV be on before it uses more than 4 kWh of energy?

Solution:

$$W = \frac{Pt}{1000} \Rightarrow t \text{ (hours)} = \frac{(W)(1000)}{P} = \frac{(4 \text{ kWh})(1000)}{340 \text{ W}} = \mathbf{11.76 \text{ h}}$$

EXAMPLE 4.13 What is the cost of using a 5 hp motor for 2 h if the rate is 11¢ per kilowatthour?

Solution:

$$W \text{ (kilowatthours)} = \frac{Pt}{1000} = \frac{(5 \text{ hp} \times 746 \text{ W/hp})(2 \text{ h})}{1000} = 7.46 \text{ kWh}$$
$$\text{Cost} = (7.46 \text{ kWh})(11¢/\text{kWh}) = \mathbf{82.06¢}$$

EXAMPLE 4.14 What is the total cost of using all of the following at 11¢ per kilowatthour?

A 1200 W toaster for 30 min
Six 50 W bulbs for 4 h
A 500 W washing machine for 45 min
A 4300 W electric clothes dryer for 20 min
An 80 W PC for 6 h

Solution:

$$W = \frac{(1200\text{ W})(\frac{1}{2}\text{ h}) + (6)(50\text{ W})(4\text{ h}) + (500\text{ W})(\frac{3}{4}\text{ h}) + (4300\text{ W})(\frac{1}{3}\text{ h}) + (80\text{ W})(6\text{ h})}{1000}$$

$$= \frac{600\text{ Wh} + 1200\text{ Wh} + 375\text{ Wh} + 1433\text{ Wh} + 480\text{ Wh}}{1000} = \frac{4088\text{ Wh}}{1000}$$

$$W = 4.09\text{ kWh}$$

$$\text{Cost} = (4.09\text{ kWh})(11¢/\text{kWh}) \cong \mathbf{45¢}$$

The chart in Fig. 4.17 shows the national average cost per kilowatthour compared to the kilowatthours used per customer. Note that the cost today is just above the level of 1926, but the average customer uses more than 20 times as much electrical energy in a year. Keep in mind that the chart in Fig. 4.17 is the average cost across the nation. Some states have average rates closer to 7¢ per kilowatthour, whereas others approach 20¢ per kilowatthour.

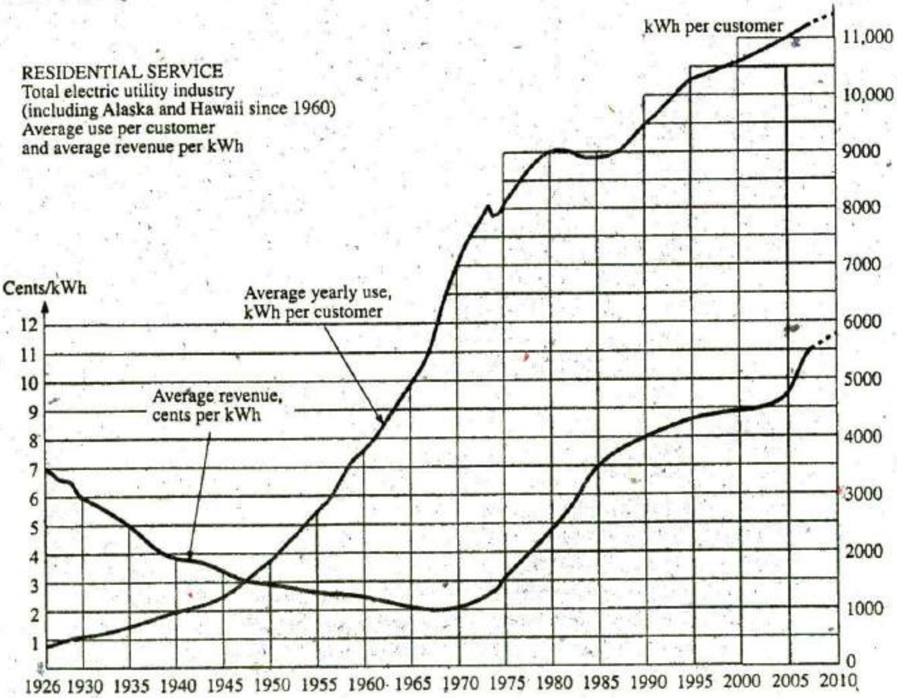


FIG. 4.17

Cost per kWh and average kWh per customer versus time.

(Based on data from Edison Electric Institute.)

Table 4.1 lists some common household appliances with their typical wattage ratings. You might find it interesting to calculate the cost of operating some of these appliances over a period of time, using the chart in Fig. 4.17 to find the cost per kilowatthour.

TABLE 4.1
Typical wattage ratings of some common household items.

Appliance	Wattage Rating	Appliance	Wattage Rating
Air conditioner		Laptop computer:	
(room)	1400	Sleep	<1 (typically
Blow dryer	1300		0.3 to 0.5)
Cellular phone:		Average use	80
Standby	≅35 mW	Microwave oven	1200
Talk	≅4.3 W	Nintendo Wii	19
Clock	2	Radio	70
Clothes dryer		Range (self-cleaning)	12,200
(electric)	4300	Refrigerator	
Coffee maker	900	(automatic defrost)	1800
Dishwasher	1200	Shaver	15
Fan:		Sun lamp	280
Portable	90	Toaster	1200
Window	200	Trash compactor	400
Heater	1500	TV:	
Heating equipment:		Plasma	340
Furnace fan	320	LCD	220
Oil-burner motor	230	VCR/DVD	25
Iron, dry or		Washing machine	500
steam	1000	Water heater	4500
		Xbox 360	187

4.6 EFFICIENCY

A flowchart for the energy levels associated with any system that converts energy from one form to another is provided in Fig. 4.18. Note that the output energy level must always be less than the applied energy due to losses and storage within the system. The best one can hope for is that W_{out} and W_{in} are relatively close in magnitude.

Conservation of energy requires that

Energy input = energy output + energy lost or stored by the system

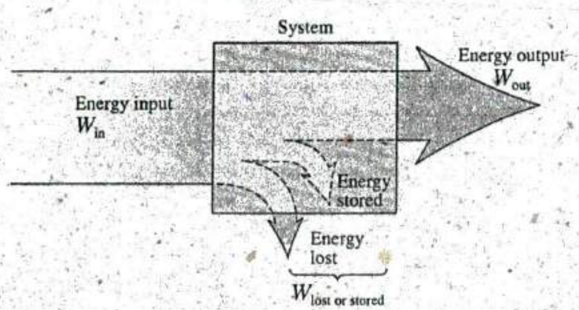


FIG. 4.18
Energy flow through a system.

Dividing both sides of the relationship by t gives

$$\frac{W_{\text{in}}}{t} = \frac{W_{\text{out}}}{t} + \frac{W_{\text{lost or stored by the system}}}{t}$$

Since $P = W/t$, we have the following:

$$\boxed{P_i = P_o + P_{\text{lost or stored}}} \quad \text{(W)} \qquad \textbf{(4.19)}$$

The **efficiency** (η) of the system is then determined by the following equation:

$$\text{Efficiency} = \frac{\text{power output}}{\text{power input}}$$

and

$$\boxed{\eta = \frac{P_o}{P_i}} \quad \text{(decimal number)} \qquad \textbf{(4.20)}$$

where η (the lowercase Greek letter *eta*) is a decimal number. Expressed as a percentage,

$$\boxed{\eta\% = \frac{P_o}{P_i} \times 100\%} \quad \text{(persent)} \qquad \textbf{(4.21)}$$

In terms of the input and output energy, the efficiency in percent is given by

$$\boxed{\eta\% = \frac{W_o}{W_i} \times 100\%} \quad \text{(persent)} \qquad \textbf{(4.22)}$$

The maximum possible efficiency is 100%, which occurs when $P_o = P_i$, or when the power lost or stored in the system is zero. Obviously, the greater the internal losses of the system in generating the necessary output power or energy, the lower is the net efficiency.

EXAMPLE 4.15 A 2 hp motor operates at an efficiency of 75%. What is the power input in watts? If the applied voltage is 220 V, what is the input current?

Solution:

$$\eta\% = \frac{P_o}{P_i} \times 100\%$$

$$0.75 = \frac{(2 \text{ hp})(746 \text{ W/hp})}{P_i}$$

and

$$P_i = \frac{1492 \text{ W}}{0.75} = \mathbf{1989.33 \text{ W}}$$

$$P_i = EI \quad \text{or} \quad I = \frac{P_i}{E} = \frac{1989.33 \text{ W}}{220 \text{ V}} = \mathbf{9.04 \text{ A}}$$

EXAMPLE 4.16 What is the output in horsepower of a motor with an efficiency of 80% and an input current of 8 A at 120 V?

Solution:

$$\eta\% = \frac{P_o}{P_i} \times 100\%$$

$$0.80 = \frac{P_o}{(120\text{ V})(8\text{ A})}$$

and $$P_o = (0.80)(120\text{ V})(8\text{ A}) = 768\text{ W}$$

with $$768\text{ W}\left(\frac{1\text{ hp}}{746\text{ W}}\right) = \mathbf{1.03\text{ hp}}$$

EXAMPLE 4.17 If $\eta = 0.85$, determine the output energy level if the applied energy is 50 J.

Solution:

$$\eta = \frac{W_o}{W_i} \Rightarrow W_o = \eta W_i = (0.85)(50\text{ J}) = \mathbf{42.5\text{ J}}$$

The very basic components of a generating (voltage) system are depicted in Fig. 4.19. The source of mechanical power is a structure such as a paddlewheel that is turned by water rushing over the dam. The gear train ensures that the rotating member of the generator is turning at rated speed. The output voltage must then be fed through a transmission system to the load. For each component of the system, an input and output power have been indicated. The efficiency of each system is given by

$$\eta_1 = \frac{P_{o_1}}{P_{i_1}} \qquad \eta_2 = \frac{P_{o_2}}{P_{i_2}} \qquad \eta_3 = \frac{P_{o_3}}{P_{i_3}}$$

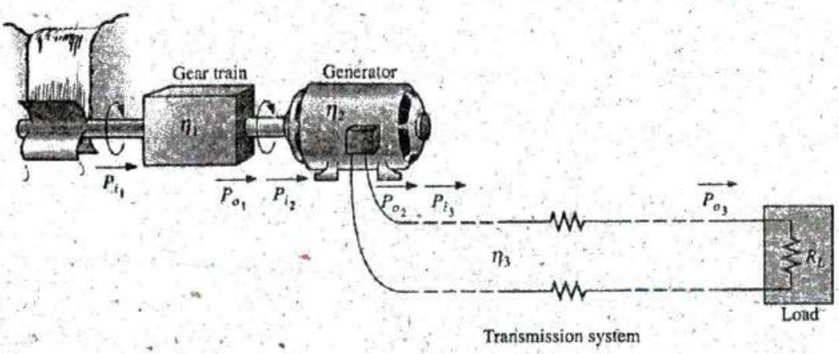


FIG. 4.19
Basic components of a generating system.

If we form the product of these three efficiencies,

$$\eta_1 \cdot \eta_2 \cdot \eta_3 = \frac{P_{o_1}}{P_{i_1}} \cdot \frac{P_{o_2}}{P_{i_2}} \cdot \frac{P_{o_3}}{P_{i_3}} = \frac{P_{o_3}}{P_{i_1}}$$

and substitute the fact that $P_{i_2} = P_{o_1}$ and $P_{i_3} = P_{o_2}$, we find that the quantities indicated above will cancel, resulting in P_{o_3}/P_{i_1}, which is a
... of the efficiency of the ...

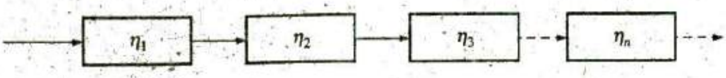


FIG. 4.20
Cascaded system.

In general, for the representative cascaded system in Fig. 4.20,

$$\eta_{\text{total}} = \eta_1 \cdot \eta_2 \cdot \eta_3 \cdots \eta_n \qquad (4.23)$$

EXAMPLE 4.18 Find the overall efficiency of the system in Fig. 4.19 if $\eta_1 = 90\%$, $\eta_2 = 85\%$, and $\eta_3 = 95\%$.

Solution:

$$\eta_T = \eta_1 \cdot \eta_2 \cdot \eta_3 = (0.90)(0.85)(0.95) = 0.727, \text{ or } \mathbf{72.7\%}$$

EXAMPLE 4.19 If the efficiency η_1 drops to 40%, find the new overall efficiency and compare the result with that obtained in Example 4.18.

Solution:

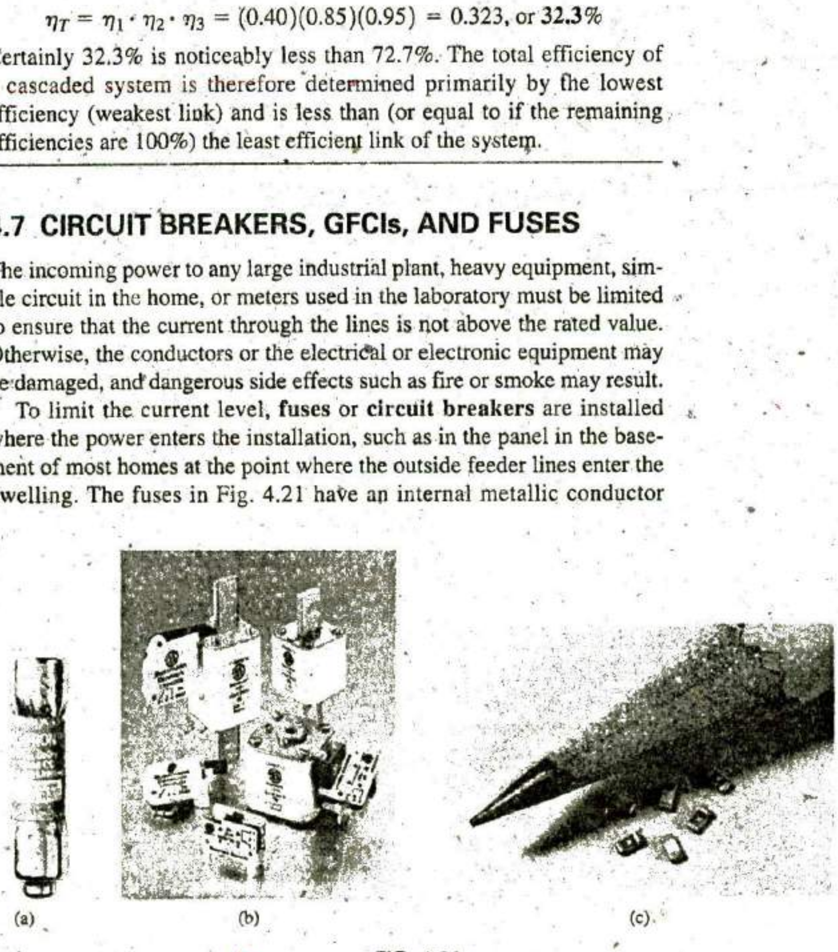
$$\eta_T = \eta_1 \cdot \eta_2 \cdot \eta_3 = (0.40)(0.85)(0.95) = 0.323, \text{ or } \mathbf{32.3\%}$$

Certainly 32.3% is noticeably less than 72.7%. The total efficiency of a cascaded system is therefore determined primarily by the lowest efficiency (weakest link) and is less than (or equal to if the remaining efficiencies are 100%) the least efficient link of the system.

4.7 CIRCUIT BREAKERS, GFCIs, AND FUSES

The incoming power to any large industrial plant, heavy equipment, simple circuit in the home, or meters used in the laboratory must be limited to ensure that the current through the lines is not above the rated value. Otherwise, the conductors or the electrical or electronic equipment may be damaged, and dangerous side effects such as fire or smoke may result.

To limit the current level, **fuses** or **circuit breakers** are installed where the power enters the installation, such as in the panel in the basement of most homes at the point where the outside feeder lines enter the dwelling. The fuses in Fig. 4.21 have an internal metallic conductor



(a) (b) (c)

FIG. 4.21
Fuses: (a) CC-TRON® (0–10 A); (b) Semitron (0–600 A); (c) subminiature surface-mount chip fuses.
(Courtesy of Cooper Bussmann.)

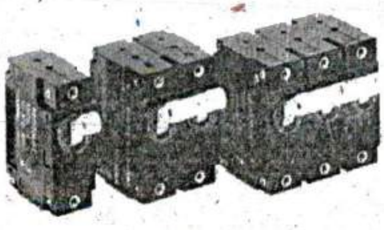


FIG. 4.22
Circuit breakers.
(Reprinted with the permission of Tyco Electronics Corporation, Potter and Brumfield Division.)

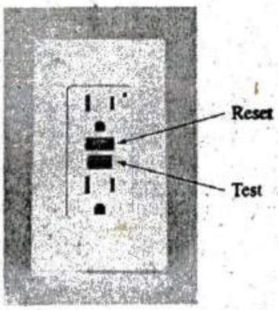


FIG. 4.23
Ground fault circuit interrupter (GFCI): 125 V ac, 60 Hz, 15 A outlet.
(Reprinted with permission of the Leviton Manufacturing Company. Leviton SmartLock™ GFCI.)

FIG. 4.24
A 23 W, 380 mA compact fluorescent lamp (CFL).

through which the current passes; a fuse begins to melt if the current through the system exceeds the rated value printed on the casing. Of course, if the fuse melts through, the current path is broken and the load in its path is protected.

In homes built in recent years, fuses have been replaced by circuit breakers such as those appearing in Fig. 4.22. When the current exceeds rated conditions, an electromagnet in the device will have sufficient strength to draw the connecting metallic link in the breaker out of the circuit and open the current path. When conditions have been corrected, the breaker can be reset and used again.

The most recent National Electrical Code requires that outlets in the bathroom and other sensitive areas be of the ground fault circuit interrupt (GFCI) variety; GFCIs (often abbreviated GFI) are designed to trip more quickly than the standard circuit breaker. The commercial unit in Fig. 4.23 trips in 5 ms. It has been determined that 6 mA is the maximum level that most individuals can be exposed to for a short period of time without the risk of serious injury. A current higher than 11 mA can cause involuntary muscle contractions that could prevent a person from letting go of the conductor and possibly cause him or her to enter a state of shock. Higher currents lasting more than a second can cause the heart to go into fibrillation and possibly cause death in a few minutes. The GFCI is able to react as quickly as it does by sensing the difference between the input and output currents to the outlet; the currents should be the same if everything is working properly. An errant path, such as through an individual, establishes a difference in the two current levels and causes the breaker to trip and disconnect the power source.

4.8 APPLICATIONS

Fluorescent versus Incandescent

A hot topic in the discussion of energy conservation is the growing pressure to switch from incandescent bulbs to fluorescent bulbs such as in Fig. 4.24. Countries throughout the world have set goals for the near future, with some mandating a ban of the use of incandescent bulbs by 2012. Japan is currently at an adoption rate of 80%, and the World Heritage sites of Shirakawa-go and Gokayama were completely stripped of incandescent lighting in 2007 in a bold effort to reduce carbon dioxide emissions near the villages. Germany is at a 50% adoption rate, the United Kingdom at 20%, and the United States at about 6%. Australia announced a complete ban on incandescent lighting by 2009, and Canada by 2012.

This enormous shift in usage is primarily due to the higher energy efficiency of fluorescent bulbs and their longer lifespans. For the same number of lumens (a unit of light measurement) the energy dissipated by an incandescent light can range from approximately four to six times greater than that of a fluorescent bulb. The range is a function of the lumens level. The greater the number of lumens available per bulb, the lower is the ratio. **In other words, the energy saved increases with decrease in the wattage rating of the fluorescent bulb.** Table 4.2 compares the wattage ratings of fluorescent bulbs and incandescent bulbs for the same level of lumens generated. For the same period of time the ratio of the energy used requires that we simply divide the wattage ratings at the same lumens level.

TABLE 4.2
Comparison of the lumens generated by incandescent and fluorescent bulbs.

Incandescents	Lumens	Fluorescents
100 W (950 h)	1675	
	1600	23 W (12,000 h)
	1100	15 W (8000 h)
75 W (1500 h)	1040	
	870	13 W (10,000 h)
60 W (1500 h)	830	
	660	11 W (8000 h)
	580	9 W (8000 h)
40 W (1250 h)	495	
	400	7 W (10,000 h)
25 W (2500 h)	250 250	4 W (8000 h)
15 W (3000 h)	115	

As mentioned, the other positive benefit of fluorescent bulbs is the longevity of the bulbs. A 60 W incandescent bulb will have a rated life of 1500 h, whereas a 13 W fluorescent bulb with an equivalent lumens level is rated to last 10,000 h—a life ratio of 6.67. A 25 W incandescent bulb may have a rated lifetime of 2500 h, but a 4 W fluorescent bulb of similar lumens emission has a rated lifetime of 8000 h—a life ratio of only 3.2. It is interesting to note in Table 4.2 that the lifetime of fluorescent bulbs remains quite high at all wattage ratings, whereas the lifetime of fluorescent bulbs increases substantially with drop in wattage level. Finally, we have to consider the cost level of purchase and use. Currently, a 60 W incandescent bulb can be purchased for about 80¢, whereas a similar lumens rated 13 W fluorescent bulb may cost $2.50—an increase by a factor in excess of 3 : 1. For most people this is an important factor and has without question had an effect on the level of adoption of fluorescent bulbs. However, one must also consider the cost of using the bulbs just described over a period of 1 year. Consider that each is used 5 h/day for 365 days at a cost of 11¢/kWh.

For the incandescent bulb the cost is determined as follows:

$$\text{kWh} = \frac{(5\text{ h})(365\text{ days})(60\text{ W})}{1000} = 109.5\text{ kWh}$$
$$\text{Cost} = (109.5\text{ kWh})(11\text{¢/kWh}) = \mathbf{\$12.05/year}$$

For the fluorescent bulb the cost is determined as follows:

$$\text{kWh} = \frac{(5\text{ h})(365\text{ days})(13\text{ W})}{1000} = 23.73\text{ kWh}$$
$$\text{Cost} = (23.73\text{ kWh})(11\text{¢/kWh}) = \mathbf{\$2.61/year}$$

The cost ratio of fluorescent to incandescent bulbs is about 4.6, which is certainly significant, indicating that the cost of fluorescent lighting is about 22% of that of incandescent lighting. Returning to the initial cost, it is clear that the bulb would pay for itself almost four times over in 1 year.

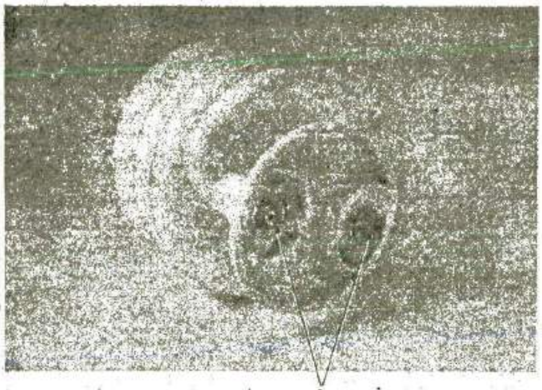
The other positive factor in using lower-wattage bulbs is the savings in carbon dioxide emissions in the process of producing the necessary electrical power. For the two villages in Japan mentioned earlier, where some 700 bulbs were replaced, the savings will be some 24 tons in 1 year. Consider what that number would be if this policy were adopted all over the world.

As with every innovative approach to saving energy, there are some concerns about its worldwide adoption. They include the mercury that is inherent in the design of the fluorescent bulb. Each bulb contains about 5 mg of mercury, an element that can be harmful to the nervous system and brain development. The effects of breaking a bulb are under investigation, and the concern is such that departments of environmental protection in most countries are setting up standards for the clean-up process. In every approach described it is agreed that a room in which breakage occurs should be cleared out and the windows **opened for ventilation.** Then the materials **should not be vacuumed up** but carefully scooped up and put in a small, sealed container. Some agencies go well beyond these two simple measures in the clean-up, but clearly it is a process that must be handled with care. The other concern is how to dispose of the bulbs when they wear out. At the moment this may not be a big problem because the adoption of fluorescent bulbs has just begun, and the bulbs will last for quite a few years. However, there will become a time when the proper disposal will have to be defined. Fortunately, most developed countries are carefully looking at this problem and setting up facilities designed specifically to dispose of this type of appliance. When disposal does reach a higher level, such as the 350 million per year projected in a country such as Japan, it will represent levels that will have to be addressed efficiently and correctly to remove the mercury levels that will result.

Other concerns relate to the light emitted by fluorescent bulbs as compared to that from incandescent bulbs. In general the light emitted by incandescent lights (more red and less blue components) is a closer match to natural light than that from fluorescent bulbs, which emit a bluish tint. However, by applying the proper phosphor to the inside of the bulb, a **white light** can be established that is more comfortable to the normal eye. Another concern is the fact that currently dimmers—a source of energy conservation in many instances—can only be used with specially designed fluorescent bulbs. However, again research is now going on that will probably remove this problem in the near future. One other concern of some importance is the fact that fluorescent bulbs emit ultraviolet (UV) rays (as in the light used in tanning salons), which are not a component of visible light but are a concern to those with skin problems such as those in individuals with lupus; however, once again, the studies are ongoing. For many years fluorescent lights were relegated to ceiling fixtures, where their distance negated most concerns about UV rays, but now they have been brought closer to the consumer. Recall also that the growing of plants in a dark interior space can only be accomplished using fluorescent bulbs because of the UV radiation. Finally, it turns out that, as with all products, you get what you pay for: Cheaper bulbs seem to fail to meet their lifespan guarantee and emit a poorer light spectrum.

The debate could go on for numerous pages, balancing benefits against disadvantages. For instance, consider that the heat generated by incandescent lamps provides some of the heating in large institutions and therefore more heat will have to be supplied if the switch is made to fluorescents. However, in the summer months the cooler fluorescent bulbs will require less cooling, affording a savings at the same location. In the final analysis, it appears that the decision will come down to individuals (unless it is government mandated) and what they are most comfortable with. Be assured, however, that any strong reaction to a proposed switchover will be well documented and should not be a real concern.

The exponential growth of interest in fluorescents in recent years has been primarily due to the introduction of electronic circuitry that can "fire" or "ignite" the bulb in a way that provides a quicker startup and



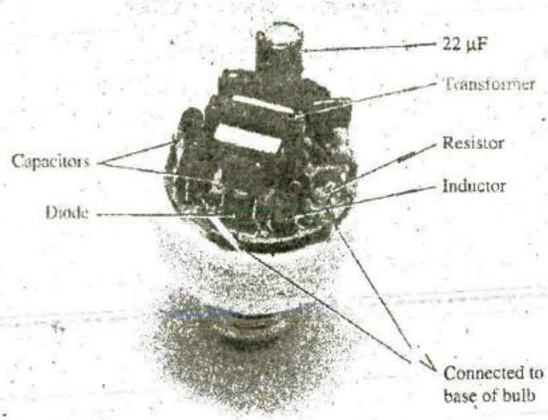


FIGURE 4.25
Internal construction of the CFL of Fig. 4.24.

smaller units. A full description of the older variety of fluorescent bulbs appears in the Application section of Chapter 22, which describes the large size of the ballast transformer and the need for a starter mechanism. A glimpse at the relatively small electronic firing mechanism for a compact fluorescent lamp (CFL) is provided in Fig. 4.25. The bulb section remains completely isolated, with only four leads available to connect to the circuitry, which reduces the possibility of exposure when the bulb is being constructed. The circuitry has been flipped from its position in the base of the bulb. The black and white lead on the edges is connected to the base of the bulb, where it is connected to a 120 V source. Note that the two largest components are the transformer and electrolytic near the center of the printed circuit board (PCB). A number of other elements to be described in the text have been identified.

Microwave Oven

It is probably safe to say that most homes today have a microwave oven (see Fig. 4.26). Most users are not concerned with its operating efficiency. However, it is interesting to learn how the units operate and apply some of the theory presented in this chapter.

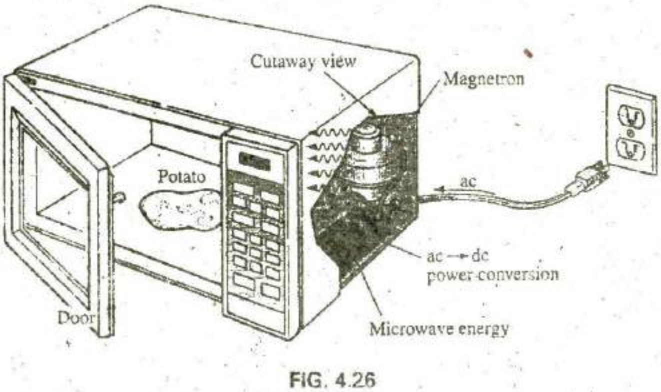


FIG. 4.26
Microwave oven.

First, some general comments. Most microwaves are rated at 500 W to 1200 W at a frequency of 2.45 GHz (almost 2.5 billion cycles per second, compared to the 60 cycles per second for the ac voltage at the typical home outlet—details in Chapter 13). The heating occurs because the water molecules in the food are vibrated at such a high frequency that the friction with neighboring molecules causes the heating effect. Since it is the high frequency of vibration that heats the food, there is no need for the material to be a conductor of electricity. However, any metal placed in the microwave can act as an antenna (especially if it has any points or sharp edges) that will attract the microwave energy and reach very high temperatures. In fact, a browning skillet is now made for microwaves that has some metal embedded in the bottom and sides to attract the microwave energy and raise the temperature at the surface between the food and skillet to give the food a brown color and a crisp texture. Even if the metal did not act as an antenna, it is a good conductor of heat and could get quite hot as it draws heat from the food.

Any container with low moisture content can be used to heat foods in a microwave. Because of this requirement, manufacturers have developed a whole line of microwave cookware that is very low in moisture content. Theoretically, glass and plastic have very little moisture content, but even so, when heated in the oven for a minute or so, they do get warm. It could be the moisture in the air that clings to the surface of each or perhaps the lead used in good crystal. In any case, microwaves should be used only to prepare food. They were not designed to be dryers or evaporators.

The instructions with every microwave specify that the oven should not be turned on when empty. Even though the oven may be empty, microwave energy will be generated and will make every effort to find a channel for absorption. If the oven is empty, the energy might be attracted to the oven itself and could do some damage. To demonstrate that a dry empty glass or plastic container will not attract a significant amount of microwave energy, place two glasses in an oven, one with water and the other empty. After 1 min, you will find the glass with the water quite warm due to the heating effect of the hot water while the other is close to its original temperature. In other words, the water created a heat sink for the majority of the microwave energy, leaving the empty glass as a less attractive path for heat conduction. Dry paper towels and plastic wrap can be used in the oven to cover dishes since they initially have low water molecule content, and paper and plastic are not good conductors of heat. However, it would be very unsafe to place a paper towel in an oven alone because, as said above, the microwave energy will seek an absorbing medium and could set the paper on fire.

The cooking of food by a conventional oven is from the outside in. The same is true for microwave ovens, but they have the additional advantage of being able to penetrate the outside few centimeters of the food, reducing the cooking time substantially. The cooking time with a microwave oven is related to the amount of food in the oven. Two cups of water will take longer to heat than one cup, although it is not a linear relationship, so it will not take twice as long—perhaps 75% to 90% longer. Eventually, if you place enough food in the microwave oven and compare the longer cooking time to that with a conventional oven, you will reach a crossover point at which it would be just as wise to use a conventional oven and get the texture in the food you might prefer.

The basic construction of the microwave oven is depicted in Fig. 4.26. It uses a 120 V ac supply, which is then converted through a high-voltage transformer to one having peak values approaching 5000 V (at substantial current levels)—sufficient warning to leave microwave repair to the local service location. Through the rectifying process briefly described in Chapter 2, a high dc voltage of a few thousand volts is generated that appears across a magnetron. The magnetron, through its very special design (currently the same design as in World War II, when it was invented by the British for use in high-power radar units), generates the required 2.45 GHz signal for the oven. It should be pointed out also that the magnetron has a specific power level of operation that cannot be controlled—once it's on, it's on at a set power level. One may then wonder how the cooking temperature and duration can be controlled. This is accomplished through a controlling network that determines the amount of off and on time during the input cycle of the 120 V supply. Higher temperatures are achieved by setting a high ratio of on to off time, while low temperatures are set by the reverse action.

One unfortunate characteristic of the magnetron is that in the conversion process, it generates a great deal of heat that does not go toward the heating of the food and that must be absorbed by heat sinks or dispersed by a cooling fan. Typical conversion efficiencies are between 55% and 75%. Considering other losses inherent in any operating system, it is reasonable to assume that most microwaves are between 50% and 60% efficient. However, the conventional oven with its continually operating exhaust fan and heating of the oven, cookware, surrounding air, and so on also has significant losses, even if it is less sensitive to the amount of food to be cooked. All in all, convenience is probably the other factor that weighs the heaviest in this discussion. It also leaves the question of how our time is figured into the efficiency equation.

For specific numbers, let us consider the energy associated with baking a 5-oz potato in a 1200 W microwave oven for 5 min if the conversion efficiency is an average value of 55%. First, it is important to realize that when a unit is rated as 1200 W, that is the rated power drawn from the line during the cooking process. If the microwave is plugged into a 120 V outlet, the current drawn is

$$I = P/V = 1200\text{ W}/120\text{ V} = 10.0\text{ A}$$

which is a significant level of current. Next, we can determine the amount of power dedicated solely to the cooking process by using the efficiency level. That is,

$$P_o = \eta P_i = (0.55)(1200\text{ W}) = 600\text{ W}$$

The energy transferred to the potato over a period of 5 min can then be determined from

$$W = Pt = (660\text{ W})(5\text{ min})(60\text{ s}/1\text{ min}) = 198\text{ kJ}$$

which is about half of the energy (nutritional value) derived from eating a 5-oz potato. The number of kilowatthours drawn by the unit is determined from

$$W = Pt/1000 = (1200\text{ W})(6/60\text{ h})/1000 = 0.1\text{ kWh}$$

At a rate of 10¢/kWh we find that we can cook the potato for 1 penny—relatively speaking, pretty cheap. A typical 1550 W toaster oven would take an hour to heat the same potato, using 1.55 kWh and costing 15.5 cents—a significant increase in cost.

Household Wiring

A number of facets of household wiring can be discussed without examining the manner in which they are physically connected. In the following chapters, additional coverage is provided to ensure that you develop a solid fundamental understanding of the overall household wiring system. At the very least you will establish a background that will permit you to answer questions that you should be able to answer as a student of this field.

The one specification that defines the overall system is the maximum current that can be drawn from the power lines since the voltage is fixed at 120 V or 240 V (sometimes 208 V). For most older homes with a heating system other than electric, a 100 A service is the norm. Today, with all the electronic systems becoming commonplace in the home, many people are opting for the 200 A service even if they do not have electric heat. A 100 A service specifies that the maximum current that can be drawn through the power lines into your home is 100 A. Using the line-to-line rated voltage and the full-service current (and assuming all resistive-type loads), we can determine the maximum power that can be delivered using the basic power equation:

$$P = EI = (240\text{ V})(100\text{ A}) = 24{,}000\text{ W} = 24\text{ kW}$$

This rating reveals that the total rating of all the units turned on in the home cannot exceed 24 kW at any one time. If it did, we could expect the main breaker at the top of the power panel to open. Initially, 24 kW may seem like quite a large rating, but when you consider that a self-cleaning electric oven may draw 12.2 kW, a dryer 4.8 kW, a water heater 4.5 kW, and a dishwasher 1.2 kW, we are already at 22.7 kW (if all the units are operating at peak demand), and we have not turned the lights or TV on yet. Obviously, the use of an electric oven alone may strongly suggest considering a 200 A service. However, seldom are all the burners of a stove used at once, and the oven incorporates a thermostat to control the temperature so that it is not on all the time. The same is true for the water heater and dishwasher, so the chances of all the units in a home demanding full service at the same time is very slim. Certainly, a typical home with electric heat that may draw 16 kW just for heating in cold weather must consider a 200 A service. You must also understand that there is some leeway in maximum ratings for safety purposes. In other words, a system designed for a maximum load of 100 A can accept a slightly higher current for short periods of time without significant damage. For the long term, however, the limit should not be exceeded.

Changing the service to 200 A is not simply a matter of changing the panel in the basement—a new, heavier line must be run from the road to the house. In some areas feeder cables are aluminum because of the reduced cost and weight. In other areas, aluminum is not permitted because of its temperature sensitivity (expansion and contraction), and copper must be used. In any event, when aluminum is used, the contractor must be absolutely sure that the connections at both ends are very secure. The National Electric Code specifies that 100 A service must use a #4 AWG copper conductor or #2 aluminum conductor. For 200 A service, a 2/0 copper wire or a 4/0 aluminum conductor must be used, as shown in Fig. 4.27(a). A 100 A or 200 A service must have two lines and a service neutral as shown in Fig. 4.27(b). Note in Fig. 4.27(b) that the lines are coated and insulated from each other, and the service neutral is spread around the inside of the wire coating. At the terminal point, all

FIG. 4.27
200 A service conductors: (a) 4/0 aluminum and 2/0 copper; (b) three-wire 4/0 aluminum service.

the strands of the service neutral are gathered together and securely attached to the panel. It is fairly obvious that the cables of Fig. 4.27(a) are stranded for added flexibility.

Within the system, the incoming power is broken down into a number of circuits with lower current ratings utilizing 15 A, 20 A, 30 A, and 40 A protective breakers. Since the load on each breaker should not exceed 80% of its rating, in a 15 A breaker the maximum current should be limited to 80% of 15 A, or 12 A, with 16 A for a 20 A breaker, 24 A for a 30 A breaker, and 32 A for a 40 A breaker. The result is that a home with 200 A service can theoretically have a maximum of 12 circuits (200 A/16 A = 12.5) utilizing the 16 A maximum current ratings associated with 20 A breakers. However, if they are aware of the loads on each circuit, electricians can install as many circuits as they feel appropriate. The code further specifies that a #14 wire should not carry a current in excess of 15 A, a #12 in excess of 20 A, and a #10 in excess of 30 A. Thus, #12 wire is now the most common for general home wiring to ensure that it can handle any excursions beyond 15 A on the 20 A breaker (the most common breaker size). The #14 wire is often used in conjunction with the #12 wire in areas where it is known that the current levels are limited. The #10 wire is typically used for high-demand appliances such as dryers and ovens.

The circuits themselves are usually broken down into those that provide lighting, outlets, and so on. Some circuits (such as ovens and dryers) require a higher voltage of 240 V, obtained by using two power lines and the neutral. The higher voltage reduces the current requirement for the same power rating, with the net result that the appliance can usually be smaller. For example, the size of an air conditioner with the same cooling ability is measurably smaller when designed for a 240 V line than when designed for 120 V. Most 240 V lines, however, demand a current level that requires 30 A or 40 A breakers and special outlets to ensure that appliances rated at 120 V are not connected to the same outlet. Check the panel in your home and note the number of circuits—in particular, the rating of each breaker and the number of 240 V lines indicated by breakers requiring two slots of the panel. Determine the total of the current ratings of all the breakers in your panel, and explain, using the above information, why the total exceeds your feed level.

For safety sake, grounding is a very important part of the electrical system in your home. The National Electric Code requires that the neutral wire of a system be grounded to an earth-driven rod, a metallic water piping system of 10 ft or more, or a buried metal plate. That ground is then passed on through the electrical circuits of the home for further protection. In a later chapter, the details of the connections and grounding methods are discussed.

4.9 COMPUTER ANALYSIS

Now that a complete circuit has been introduced and examined in detail, we can begin the application of computer methods. As mentioned in Chapter 1, two software packages will be introduced to demonstrate the options available with each and the differences that exist. Each has a broad range of support in the educational and industrial communities. The student version of PSpice (OrCAD Release 16.2 from Cadence Design Systems) has received the most attention, followed by Multisim. Each approach has its own characteristics with procedures that must be followed exactly; otherwise, error messages will appear. Do not assume that you can "force" the system to respond the way you would prefer—every step is well defined, and one error on the input side can yield results of a meaningless nature. At times you may believe that the system is in error because you are absolutely sure you followed every step correctly. In such cases, accept the fact that something was entered incorrectly, and review all your work very carefully. All it takes is a comma instead of a period or a decimal point to generate incorrect results.

Be patient with the learning process; keep notes of specific maneuvers that you learn; and don't be afraid to ask for help when you need it. For each approach, there is always the initial concern about how to start and proceed through the first phases of the analysis. However, be assured that with time and exposure you will work through the required maneuvers at a speed you never would have expected. In time you will be absolutely delighted with the results you can obtain using computer methods.

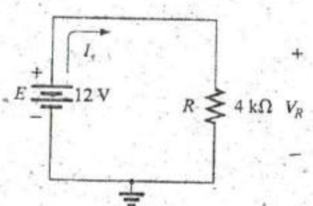


FIG. 4.28
Circuit to be analyzed using PSpice and Multisim.

In this section, Ohm's law is investigated using the software packages PSpice and Multisim to analyze the circuit in Fig. 4.28. Both require that the circuit first be "drawn" on the computer screen and then analyzed (simulated) to obtain the desired results. As mentioned above, the analysis program cannot be changed by the user. The most proficient user is one who can draw the most out of a computer software package.

Although the author feels that there is sufficient material in the text to carry a new student of the material through the programs provided, be aware that this is not a computer text. Rather, it is one whose primary purpose is simply to introduce the different approaches and how they can be applied effectively. Excellent texts and manuals are available that cover the material in a great deal more detail and perhaps at a slower pace. In fact, the quality of the available literature has improved dramatically in recent years.

PSpice

Readers who were familiar with older versions of PSpice will find that the changes in Version 16.2 are primarily in the front end and the simulation process. After executing a few programs, you will find that most of the procedures you learned from older versions will be applicable here also—at least the sequential process has a number of strong similarities.

The installation process for the OrCAD software requires a computer system with DVD capability and the minimum system requirements appearing in Appendix B. After the disk is installed, the question appearing during the license process can be answered by simply replying **port1@host1.** A Yes response to the next few questions followed by selecting **Finish** will install the software—a very simple and direct process.

Once OrCAD Version 16.2 has been installed, you must first open a **Folder** in the **C:** drive for storage of the circuit files that result from the analysis. Be aware, however, that

once the folder has been defined, it does not have to be defined for each new project unless you choose to do so. If you are satisfied with one location (folder) for all your projects, this is a one-time operation that does not have to be repeated with each network.

To establish the **Folder,** simply right-click the mouse on **Start** at the bottom left of the screen to obtain a listing that includes **Explore.** Select **Explore** to obtain the **Start Menu** dialog box and then use the sequence **File-New Folder** to open a new folder. Type in **PSpice** (the author's choice) and left-click to install. Then exit (using the **X** at the top right of the screen). The folder **PSpice** will be used for all the projects you plan to work on in this text.

Our first project can now be initiated by double-clicking on the **OrCAD 16.2 Demo** icon on the screen, or you can use the sequence **Start All Programs-CAPTURE CIS DEMO.** The resulting screen has only a few active keys on the top toolbar. The first keypad at the top left is the **Create document** key (or you can use the sequence **File-New Project**). A **New Project** dialog box opens in which you must enter the **Name** of the project. For our purposes we will choose **PSpice 4-1** as shown in Fig. 4.29 and select **Analog or Mixed A/D** (to be used for all the analyses of this text). Note at the bottom of the dialog box that the **Location** appears as **PSpice** as set above. Click **OK,** and another dialog

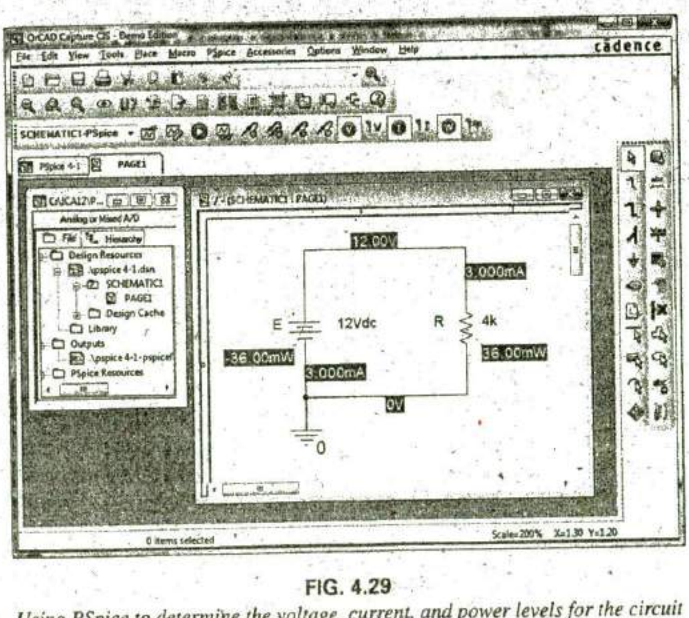


FIG. 4.29

Using PSpice to determine the voltage, current, and power levels for the circuit in Fig. 4.28.

box appears titled **Create PSpice Project.** Select **Create a blank project** (again, for all the analyses to be performed in this text). Click **OK,** and a third toolbar appears at the top of the screen with some of the keys enabled. A **Project Manager Window** appears with **PSpice 4-1** next to an icon and an associated + sign in a small square. Clicking on the + sign will take the listing a step further to **SCHEMATIC1.** Click + again, and **PAGE1** appears; clicking on a − sign reverses the process. Double-clicking on **PAGE1** creates a working window titled **SCHEMATIC1: PAGE1,** revealing that a project can have more than one schematic file and more than one associated page. The width and height of the window can be adjusted by grabbing an edge until you see a double-headed arrow and dragging the border to the desired location. Either window on the screen can be moved by clicking on the top heading to make it dark blue and then dragging it to any location.

Now you are ready to build the simple circuit in Fig. 4.28. Select the **Place a part** key (the key on the right tool bar with a small plus sign and IC structure) to obtain the **Place Part** dialog box. Since this is the first circuit to be constructed, you must ensure that the parts appear in the list of active libraries. Select **Add Library-Browse File,** and select **analog.olb,** and when it appears under the **File name** heading, select **Open.** It will now appear in the **Libraries** listing at the bottom left of the dialog box. Repeat for the **source.olb** and **special.olb** libraries. All three files are required to build the networks appearing in this text. However, it is important to realize that

once the library files have been selected, they will appear in the active listing for each new project without your having to add them each time—a step, such as the Folder step above, that does not have to be repeated with each similar project.

You are now ready to place components on the screen. For the dc voltage source, first select the **Place a part key** and then select **SOURCE** in the library listing. Under **Part List,** a list of available sources appears; select **VDC** for this project. Once **VDC** has been selected, its symbol, label, and value appears on the picture window at the bottom right of the dialog box. Click the icon with the plus sign and IC structure to the left of the **Help** key in the **Place Part** dialog box, and the **VDC** source follows the cursor across the screen. Move it to a convenient location, left-click the mouse, and it will be set in place as shown in Fig. 4.29. Since only one source is required, right-clicking results in a list of options, in which **End Mode** appears at the top. Choosing this option ends the procedure, leaving the source in a red dashed box. If it is red, it is an active mode and can be operated on. Left-clicking puts the source in place and removes the red active status.

One of the most important steps in the procedure is to ensure that a 0 V ground potential is defined for the network so that voltages at any point in the network have a reference point. *The result is a requirement that every network must have a ground defined.* For our purposes, the **0/SOURCE** option will be our choice when the **GND** key is selected. It ensures that one side of the source is defined as 0 V. It is obtained by selecting the ground symbol from the toolbar at the right edge of the screen. A **Place Ground** dialog box appears under which **0/SOURCE** can be selected followed by an **OK** to place on the screen. Finally, you need to add a resistor to the network by selecting the **Place a part** key again and then selecting the **ANALOG** library. Scrolling the options, note that **R** appears and should be selected. Click **OK,** and the resistor appears next to the cursor on the screen. Move it to the desired location and click it in place. Then right-click and **End Mode,** and the resistor has been entered into the schematic's memory. Unfortunately, the resistor ended up in the horizontal position, and the circuit of Fig. 4.28 has the resistor in the vertical position. No problem: Simply select the resistor again to make it red, and right-click. A listing appears in which **Rotate** is an option. It turns the resistor 90° in the counterclockwise direction.

All the required elements are on the screen, but they need to be connected. To accomplish this, select the **Place a wire** key that looks like a step in the right toolbar. The result is a crosshair with the center that should be placed at the point to be connected. Place the crosshair at the top of the voltage source, and left-click it once to connect it to that point. Then draw a line to the end of the next element, and click again when the crosshair is at the correct point. A red line results with a square at each end to confirm that the connection has been made. Then move the crosshair to the other elements, and build the circuit. Once everything is connected, right-clicking provides the **End Mode** option. Do not forget to connect the source to ground as shown in Fig. 4.29.

Now you have all the elements in place, but their labels and values are wrong. To change any parameter, simply double-click on the parameter (the label or the value) to obtain the **Display Properties** dialog box. Type in the correct label or value, click **OK,** and the quantity is changed on the screen. Before selecting **OK,** be sure to check the **Display Format** to specify what will appear on the screen. The labels and values can be moved by simply clicking on the center of the parameter until it is closely surrounded by the four small squares and then dragging it to the new location. Left-clicking again deposits it in its new location.

Finally, you can initiate the analysis process, called **Simulation,** by selecting the **New simulation profile** key in the bottom toolbar of the heading of the screen that resembles a data page with a varying waveform and yellow star in the top right corner. **A New Simulation** dialog box opens that first asks for the **Name** of the simulation. The New Simulation dialog box can also be obtained by using the sequence **PSpice-New Simulation Profile-Bias Point** is entered for a dc solution, and **none** is left in the **Inherit From** request. Then select **Create,** and a **Simulation Settings** dialog box appears in which **Analysis-Analysis Type: Bias Point** is sequentially selected. Click **OK,** and select the **Run PSpice** key (which looks like a green circular key with an arrowhead) or choose **PSpice-Run** from the menu bar. An output window will appear with the dc voltages of the network: **12 V** and **0 V.** The dc currents and power levels can be displayed as shown in Fig. 4.29 by simply selecting the green circular keys with the **I** and **W** in the bottom tool bar at the top of the screen. Individual values can be removed by simply selecting the value and pressing the **Delete** key or the scissors key in the top menu bar. Resulting values can be moved by simply left-clicking the value and dragging it to the desired location.

Note in Fig. 4.29 that the current is 3 mA (as expected) at each point in the network, and the power delivered by the source and dissipated by the resistor is the same, or 36 mW. There are also 12 V across the resistor as required by the configuration.

There is no question that this procedure seems long for such a simple circuit. However, keep in mind that we needed to introduce many new facets of using PSpice that are not discussed again. By the time you finish analyzing your third or fourth network, the procedure will be routine and easy to do.

Multisim

For comparison purposes, Multisim is also used to analyze the circuit in Fig. 4.28. Although there are differences between PSpice and Multisim, such as in initiating the process, constructing the networks, making the measurements, and setting up the simulation procedure, there are sufficient similarities between the two approaches to make it easier to learn one if you are already familiar with the other. The similarities will be obvious only if you make an attempt to learn both. One of the major differences between the two is the option to use actual instruments in Multisim to make the measurements—a positive trait in preparation for the laboratory experience. However, in Multisim, you may not find the extensive list of options available with PSpice. In general, however, both software packages are well prepared to take us through the types of analyses to be encountered in this text. The installation process for **Multisim** is not as direct as for the **OrCAD** demo version because the software package must be purchased to obtain a serial number. In most cases, the **Multisim** package will be available through the local educational facility.

When the **Multisim** icon is selected from the opening window, a screen appears with the heading **Circuit 1-Multisim.** A menu bar appears across the top of the screen, with seven additional toolbars: **Standard, View, Main, Components, Simulation Switch, Simulation,** and **Instruments.** By selecting **View** from the top menu bar followed by **Toolbars,** you can add or delete toolbars. The heading can be changed to **Multisim 4-1** by selecting **File-Save As** to open the **Save As** dialog box. Enter **Multisim 4-1** as the **File name** to obtain the listing of Fig. 4.30.

For the placement of components, **View-Show Grid** was selected so that a grid would appear on the screen. As you place an element, it will automatically be placed in a relationship specific to the grid structure.

To build the circuit in Fig. 4.28, first take the cursor and place it on the battery symbol in the **Component toolbar.** Left-click, and a

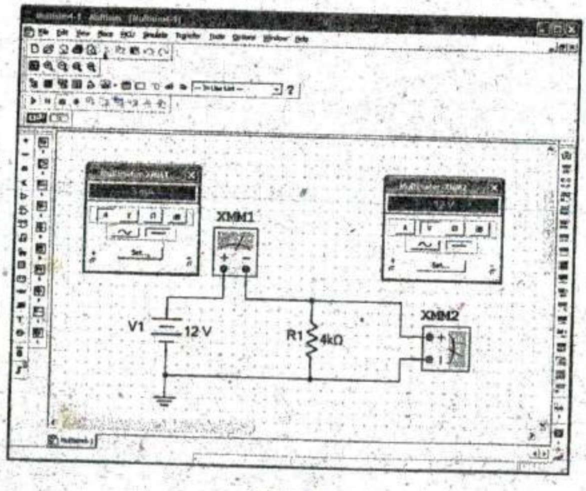


FIG. 4.30
Using Multisim to determine the voltage and current levels for the circuit of Fig. 4.28.

Component dialog box will appear, which provides a list of sources. Under **Component,** select **DC-POWER.** The symbol appears in the adjoining box area. Click **OK.** The battery symbol appears on the screen next to the location of the cursor. Move the cursor to the desired location, and left-click to set the battery symbol in place. The operation is complete. If you want to delete the source, simply left-click on the symbol again to create a dashed rectangle around the source. These rectangles indicate that the source is in the active mode and can be operated on. If you want to delete it, click on the **Delete** key or select the **Scissor** keypad in the **Standard toolbar.** If you want to modify the source, right-click *outside* the rectangle, and you get one list. Right-click *within* the rectangle, and you have a different set of options. At any time, if you want to remove the active state, left-click anywhere on the screen. If you want to move the source, click on the source symbol to create the rectangle, but do not release the mouse. Hold it down and drag the source to the preferred location. When the source is in place, release the mouse. Click again to remove the active state. *From now on, whenever possible, the word "click" means a left-click.* The need for a right click will continue to be spelled out.

For the simple circuit in Fig. 4.30, you need to place a resistor across the source. Select the keypad in the **Components toolbar** that looks like a resistor symbol. A **Select a Component** dialog box opens with a **Family** listing. Selecting **RESISTOR** results in a list of standard values that can be quickly selected for the deposited resistor. However, in this case, you want to use a 4 kΩ resistor, which is not a standard value but can be changed to 4 kΩ by simply changing the value once it has been placed on the screen. Another approach is to add **the Virtual toolbar** (also referred to as the **Basic toolbar**), which provides a list of components for which the value can be set. Selecting the resistor symbol from the **Virtual toolbar** will result in the placement of a resistor with a starting value of 1 kΩ. Once placed on the screen, the value of the resistor can be changed by simply double-clicking on the resistor value to obtain a dialog box that permits the change. The placement of the resistor is exactly the same as that employed for the source above.

In Fig. 4.28, the resistor is in the vertical position, so a rotation must be made. Click on the resistor to obtain the active state, and then right-click within the rectangle. A number of options appear, including **Flip Horizontal, Flip Vertical, 90° Clockwise,** and **90° Counter CW.** To rotate 90° counterclockwise, select that option, and the resistor is automatically rotated 90°.

Finally, you need a ground for all networks. Going back to the **Sources** parts bin, find **GROUND,** which is the fourth option down under **Component.** Select **GROUND** and place it on the screen below the voltage source as shown in Fig. 4.30. Now, before connecting the components together, move the labels and the value of each component to the relative positions shown in Fig. 4.30. Do this by clicking on the label or value to create a small set of squares around the element and then dragging the element to the desired location. Release the mouse, and then click again to set the element in place. To change the label or value, double-click on the label (such as **V1**) to open a **DC_POWER** dialog box. Select **Label** and enter **E** as the **Reference Designation (Ref Des).** Then, before leaving the dialog box, go to **Value** and change the value if necessary. It is very important to realize that you cannot type in the units where the **V** now appears to the right of the value. The suffix is controlled by the scroll keys at the left of the unit of measure. For practice, try the scroll keys, and you will find that you can go from **pV** to **TV.** For now, leave it as **V.** Click **OK,** and both have been changed on the screen. The same process can be applied to the resistive element to obtain the label and value appearing in Fig. 4.30.

Next, you need to tell the system which results should be generated and how they should be displayed. For this example, we use a multimeter to measure both the current and the voltage of the circuit. The **Multimeter** is the first option in the list of instruments appearing in the toolbar to the right of the screen. When selected, it appears on the screen and can be placed anywhere, using the same procedure defined for the components above. Double-click on the meter symbol, and a **Multimeter** dialog box opens in which the function of the meter must be defined. Since the meter **XMM1** will be used as an ammeter, select the letter **A** and the horizontal line to indicate dc level. There is no need to select **Set** for the default values since they have been chosen for the broad range of applications. The dialog meters can be moved to any location by clicking on their heading bar to make it dark blue and then dragging the meter to the preferred position. For the voltmeter, **V** and the horizontal bar were selected as shown in Fig. 4.30. The voltmeter was turned clockwise 90° using the same procedure as described for the resistor above.

Finally, the elements need to be connected. To do this, bring the cursor to one end of an element, say, the top of the voltage source. A small dot and a crosshair appear at the top end of the element. Click once, follow the path you want, and place the crosshair over the positive terminal of the ammeter. Click again, and the wire appears in place.

At this point, you should be aware that the software package has its preferences about how it wants the elements to be connected. That is, you may try to draw it one way, but the computer generation may be a different path. Eventually, you will learn these preferences and can set up the network to your liking. Now continue making the connections appearing in Fig. 4.30, moving elements or adjusting lines as necessary. Be sure that the small dot appears at any point where you want a connection. Its absence suggests that the connection has not been made and the software program has not accepted the entry.

You are now ready to run the program and view the solution. The analysis can be initiated in a number of ways. One option is to select **Simulate** from the top toolbar, followed by **RUN.** Another is to select the **Simulate** key (the green arrow) in the **Simulation toolbar.** The last option, and the one we use the most, utilizes the **OFF/ON, 0/1 Simulation** switch at the top right of the screen. With this last option, the analysis (called **Simulation**) is initiated by clicking the switch into the 1 position. The analysis is performed, and the current and voltage appear on the meter as shown in Fig. 4.30. Note that both provide the expected results.

One of the most important things to learn about applying Multisim:

Always stop or end the simulation (clicking on 0 or choosing OFF) before making any changes in the network. When the simulation is initiated, it stays in that mode until turned off.

There was obviously a great deal of material to learn in this first exercise using Multisim. Be assured, however, that as we continue with more examples, you will find the procedure quite straightforward and actually enjoyable to apply.

PROBLEMS

SECTION 4.2 Ohm's Law

1. What is the voltage across a 220 Ω resistor if the current through it is 5.6 mA?
2. What is the current through a 6.8 Ω resistor if the voltage drop across it is 24 V?
3. How much resistance is required to limit the current to 1.5 mA if the potential drop across the resistor is 24 V?
4. At starting, what is the current drain on a 12 V car battery if the resistance of the starting motor is 40 MΩ ?
5. If the current through a 0.02 MΩ resistor is 3.6 μA, what is the voltage drop across the resistor?
6. If a voltmeter has an internal resistance of 50 kΩ, find the current through the meter when it reads 120 V.
7. If a refrigerator draws 2.2 A at 120 V, what is its resistance?
8. If a clock has an internal resistance of 8 kΩ, find the current through the clock if it is plugged into a 120 V outlet.
9. A washing machine is rated at 4.2 A at 120 V. What is its internal resistance?
10. A CD player draws 125 mA when 4.5 V is applied. What is the internal resistance?
11. The input current to a transistor is 20 μA. If the applied (input) voltage is 24 mV, determine the input resistance of the transistor.
12. The internal resistance of a dc generator is 0.5 Ω. Determine the loss in terminal voltage across this internal resistance if the current is 12 A.

*13. a. If an electric heater draws 9.5 A when connected to a 120 V supply, what is the internal resistance of the heater?
   b. Using the basic relationships of Chapter 2, determine how much energy in joules (J) is converted if the heater is used for 2 h during the day.

14. In a TV camera, a current of 2.4 μA passes through a resistor of 3.3 MΩ. What is the voltage drop across the resistor?

SECTION 4.3 Plotting Ohm's Law

15. a. Plot the curve of *I* (vertical axis) versus *V* (horizontal axis) for a 120 Ω resistor. Use a horizontal scale of 0 to 100 V and a vertical scale of 0 to 1 A.
   b. Using the graph of part (a), find the current at a voltage of 20 V and 50 V.
16. a. Plot the *I-V* curve for a 5 Ω and a 20 Ω resistor on the same graph. Use a horizontal scale of 0 to 40 V and a vertical scale of 0 to 2 A.
   b. Which is the steeper curve? Can you offer any general conclusions based on results?
   c. If the horizontal and vertical scales were interchanged, which would be the steeper curve?
17. a. Plot the *I-V* characteristics of a 1 Ω, 100 Ω, and 1000 Ω resistor on the same graph. Use a horizontal axis of 0 to 100 V and a vertical axis of 0 to 100 A.
   b. Comment on the steepness of a curve with increasing levels of resistance.

*18. Sketch the internal resistance characteristics of a device that has an internal resistance of 20 Ω from 0 to 10 V, an internal resistance of 4 Ω from 10 V to 15 V, and an internal resistance of 1 Ω for any voltage greater than 15 V. Use a horizontal scale that extends from 0 to 20 V and a vertical scale that permits plotting the current for all values of voltage from 0 to 20 V.

*19. a. Plot the *I-V* characteristics of a 2 kΩ, 1 MΩ, and a 100 Ω resistor on the same graph. Use a horizontal axis of 0 to 20 V and a vertical axis of 0 to 10 mA.
   b. Comment on the steepness of the curve with decreasing levels of resistance.
   c. Are the curves linear or nonlinear? Why?

SECTION 4.4 Power

20. If 540 J of energy are absorbed by a resistor in 4 min, what is the power delivered to the resistor in watts?
21. The power to a device is 40 joules per second (J/s). How long will it take to deliver 640 J?
22. a. How many joules of energy does a 2 W nightlight dissipate in 8 h?
   b. How many kilowatthours does it dissipate?
23. How long must a steady current of 1.4 A exist in a resistor that has 3 V across it to dissipate 12 J of energy?
24. What is the power delivered by a 6 V battery if the current drain is 750 mA?
25. The current through a 4 kΩ resistor is 7.2 mA. What is the power delivered to the resistor?
26. The power consumed by a 2.2 kΩ resistor is 240 mW. What is the current level through the resistor?
27. What is the maximum permissible current in a 120 Ω, 2 W resistor? What is the maximum voltage that can be applied across the resistor?
28. The voltage drop across a transistor network is 22 V. If the total resistance is 16.8 kΩ, what is the current level? What is the power delivered? How much energy is dissipated in 1 h?
29. If the power applied to a system is 324 W, what is the voltage across the line if the current is 2.7 A?
30. A 1 W resistor has a resistance of 4.7 MΩ. What is the maximum current level for the resistor? If the wattage rating is increased to 2 W, will the current rating double?
31. A 2.2 kΩ resistor in a stereo system dissipates 42 mW of power. What is the voltage across the resistor?
32. What are the "hot" resistance level and current rating of a 120 V, 100 W bulb?
33. What are the internal resistance and voltage rating of a 450 W automatic washer that draws 3.75 A?
34. A calculator with an internal 3 V battery draws 0.4 mW when fully functional.
   a. What is the current demand from the supply?
   b. If the calculator is rated to operate 500 h on the same battery, what is the ampere-hour rating of the battery?
35. A 20 kΩ resistor has a rating of 100 W. What are the maximum current and the maximum voltage that can be applied to the resistor?
36. What is the total horsepower rating of a series of commercial ceiling fans that draw 30 A at 220 V?

SECTION 4.5 Energy

37. A 10 Ω resistor is connected across a 12 V battery.
    a. How many joules of energy will it dissipate in 1 min?
    b. If the resistor is left connected for 2 min instead of 1 min, will the energy used increase? Will the power dissipation level increase?

38. How much energy in kilowatthours is required to keep a 230 W oil-burner motor running 12 h a week for 5 months? (Use 4 weeks = 1 month.)

39. How long can a 1500 W heater be on before using more than 12 kWh of energy?

40. A 60 W bulb is on for 10 h.
    a. What is the energy used in wattseconds?
    b. What is the energy dissipated in joules?
    c. What is the energy transferred in watthours?
    d. How many kilowatthours of energy were dissipated?
    e. At 11¢/kWh, what was the total cost?

41. a. In 10 h an electrical system converts 1200 kWh of electrical energy into heat. What is the power level of the system?
    b. If the applied voltage is 208 V, what is the current drawn from the supply?
    c. If the efficiency of the system is 82%, how much energy is lost or stored in 10 h?

42. At 11¢/kWh, how long can you play a 250 W color television for $1?

43. The electric bill for a family for a month is $74.
    a. Assuming 31 days in the month, what is the cost per day?
    b. Based on 15-h days, what is the cost per hour?
    c. How many kilowatthours are used per hour if the cost is 11¢/kWh?
    d. How many 60 W lightbulbs (approximate number) could you have on to use up that much energy per hour?
    e. Do you believe the cost of electricity is excessive?

44. How long can you use an Xbox 360 for $1 if it uses 187 W and the cost is 11¢/kWh?

45. The average plasma screen TV draws 339 W of power, whereas the average LCD TV draws 213 W. If each set was used 5 h/day for 365 days, what would be the cost savings for the LCD unit over the year if the cost is 11¢/kWh?

46. The average PC draws 78 W. What is the cost of using the PC for 4 h/day for a month of 31 days if the cost is 11¢/kWh?

*47. a. If a house is supplied with 120 V, 100 A service, find the maximum power capability.
    b. Can the homeowner safely operate the following loads at the same time?
    5 hp motor
    3000 W clothes dryer
    2400 W electric range
    1000 W steam iron
    c. If all the appliances are used for 2 hours, how much energy is converted in kWh?

*48. What is the total cost of using the following at 11¢/kWh?
    a. 1600 W air conditioner for 6 h
    b. 1200 W hair dryer for 15 min
    c. 4800 W clothes dryer for 30 min
    d. 900 W coffee maker for 10 min
    e. 200 W Play Station 3 for 2 h
    f. 50 W stereo for 3.5 h

*49. What is the total cost of using the following at 11¢/kWh?
    a. 200 W fan for 4 h
    b. Six 60 W bulbs for 6 h
    c. 1200 W dryer for 20 min
    d. 175 W desktop computer for 3.5 h
    e. 250 W color television set for 2 h 10 min
    f. 30 W satellite dish for 8 h

SECTION 4.6 Efficiency

50. What is the efficiency of a motor that has an output of 0.5 hp with an input of 340 W?

51. The motor of a power saw is rated 68.5% efficient. If 1.8 hp are required to cut a particular piece of lumber, what is the current drawn from a 120 V supply?

52. What is the efficiency of a dryer motor that delivers 1.2 hp when the input current and voltage are 4 A and 220 V, respectively?

53. A stereo system draws 1.8 A at 120 V. The audio output power is 50 W.
    a. How much power is lost in the form of heat in the system?
    b. What is the efficiency of the system?

54. If an electric motor having an efficiency of 76% and operating off a 220 V line delivers 3.6 hp, what input current does the motor draw?

55. A motor is rated to deliver 2 hp.
    a. If it runs on 110 V and is 90% efficient, how many watts does it draw from the power line?
    b. What is the input current?
    c. What is the input current if the motor is only 70% efficient?

56. An electric motor used in an elevator system has an efficiency of 90%. If the input voltage is 220 V, what is the input current when the motor is delivering 15 hp?

57. The motor used on a conveyor belt is 85% efficient. If the overall efficiency is 75%, what is the efficiency of the conveyor belt assembly?

58. A 2 hp motor drives a sanding belt. If the efficiency of the motor is 87% and that of the sanding belt is 75% due to slippage, what is the overall efficiency of the system?

59. The overall efficiency of two systems in cascade is 78%. If the efficiency of one is 0.9, what is the efficiency, in percent, of the other?

60. a. What is the total efficiency of three systems in cascade with respective efficiencies of 93%, 87%, and 21%?
    b. If the system with the least efficiency (21%) were removed and replaced by one with an efficiency of 80%, what would be the percentage increase in total efficiency?

*61. If the total input and output power of two systems in cascade are 400 W and 128 W, respectively, what is the efficiency of each system if one has twice the efficiency of the other?

SECTION 4.9 Computer Analysis

62. Using PSpice or Multisim, repeat the analysis of the circuit in Fig. 4.28 with $E = 400$ mV and $R = 0.04$ MΩ.

63. Using PSpice or Multisim, repeat the analysis of the circuit in Fig. 4.28, but reverse the polarity of the battery and use $E = 8$ V and $R = 220$ Ω.

GLOSSARY

Circuit breaker A two-terminal device designed to ensure that current levels do not exceed safe levels. If "tripped," it can be reset with a switch or a reset button.

Diode A semiconductor device whose behavior is much like that of a simple switch; that is, it will pass current ideally in only one direction when operating within specified limits.

Efficiency (η) A ratio of output to input power that provides immediate information about the energy-converting characteristics of a system.

Energy (W) A quantity whose change in state is determined by the product of the rate of conversion (P) and the period involved (t). It is measured in joules (J) or wattseconds (Ws).

Fuse A two-terminal device whose sole purpose is to ensure that current levels in a circuit do not exceed safe levels.

Horsepower (hp) Equivalent to 746 watts in the electrical system.

Kilowatthour meter An instrument for measuring kilowatthours of energy supplied to a residential or commercial user of electricity.

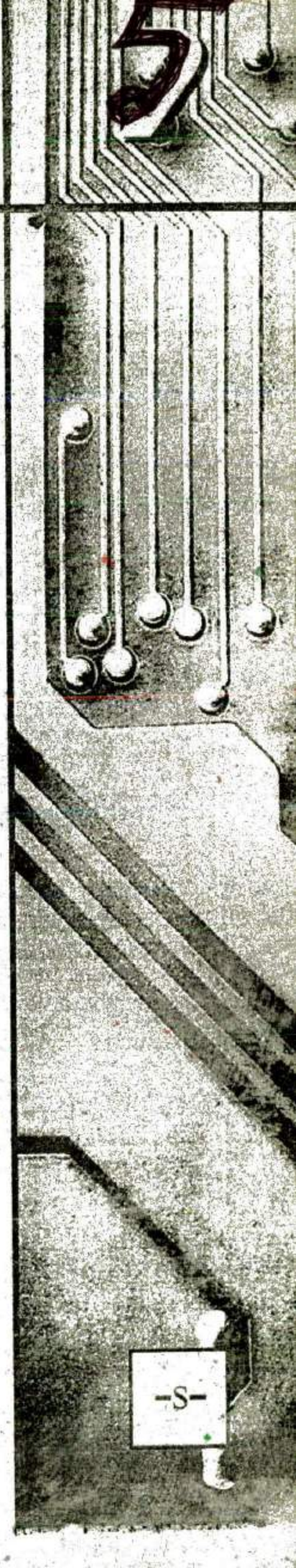
Ohm's law An equation that establishes a relationship among the current, voltage, and resistance of an electrical system.

Power An indication of how much work can be done in a specified amount of time; a *rate* of doing work. It is measured in joules/second (J/s) or watts (W).

Series dc Circuits

Objectives

- *Become familiar with the characteristics of a series circuit and how to solve for the voltage, current, and power to each of the elements.*
- *Develop a clear understanding of Kirchhoff's voltage law and how important it is to the analysis of electric circuits.*
- *Become aware of how an applied voltage will divide among series components and how to properly apply the voltage divider rule.*
- *Understand the use of single- and double-subscript notation to define the voltage levels of a network.*
- *Learn how to use a voltmeter, ammeter, and ohmmeter to measure the important quantities of a network.*



5.1 INTRODUCTION

Two types of current are readily available to the consumer today. One is *direct current* (dc), in which ideally the flow of charge (current) does not change in magnitude (or direction) with time. The other is *sinusoidal alternating current* (ac), in which the flow of charge is continually changing in magnitude (and direction) with time. The next few chapters are an introduction to circuit analysis purely from a dc approach. The methods and concepts are discussed in detail for direct current; when possible, a short discussion suffices to cover any variations we may encounter when we consider ac in the later chapters.

The battery in Fig. 5.1, by virtue of the potential difference between its terminals, has the ability to cause (or "pressure") charge to flow through the simple circuit. The positive terminal attracts the electrons through the wire at the same rate at which electrons are supplied by the negative terminal. As long as the battery is connected in the circuit and maintains its terminal characteristics, the current (dc) through the circuit will not change in magnitude or direction.

If we consider the wire to be an ideal conductor (that is, having no opposition to flow), the potential difference V across the resistor equals the applied voltage of the battery: V (volts) = E (volts).

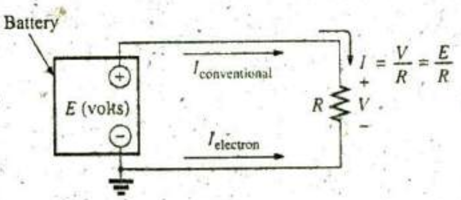


FIG. 5.1
Introducing the basic components of an electric circuit.

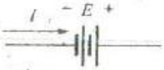


FIG. 5.2
Defining the direction of conventional flow for single-source dc circuits.

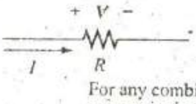


FIG. 5.3
Defining the polarity resulting from a conventional current I through a resistive element.

The current is limited only by the resistor R. The higher the resistance, the less is the current, and conversely, as determined by Ohm's law.

By convention (as discussed in Chapter 2), the direction of **conventional current flow** ($I_{conventional}$) as shown in Fig. 5.1 is opposite to that of **electron flow** ($I_{electron}$). Also, the uniform flow of charge dictates that the direct current I be the same everywhere in the circuit. By following the direction of conventional flow, we notice that there is a rise in potential across the battery (− to +) and a drop in potential across the resistor (+ to −). For single-voltage-source dc circuits, conventional flow always passes from a low potential to a high potential when passing through a voltage source, as shown in Fig. 5.2. However, conventional flow always passes from a high to a low potential when passing through a resistor for any number of voltage sources in the same circuit, as shown in Fig. 5.3.

The circuit in Fig. 5.1 is the simplest possible configuration. This chapter and the following chapters add elements to the system in a very specific manner to introduce a range of concepts that will form a major part of the foundation required to analyze the most complex system. Be aware that the laws, rules, and so on introduced in Chapters 5 and 6 will be used throughout your studies of electrical, electronic, or computer systems. They are not replaced by a more advanced set as you progress to more sophisticated material. It is therefore critical that you understand the concepts thoroughly and are able to apply the various procedures and methods with confidence.

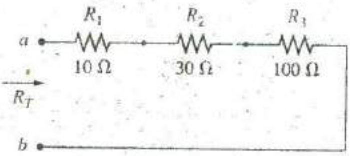


FIG. 5.4
Series connection of resistors.

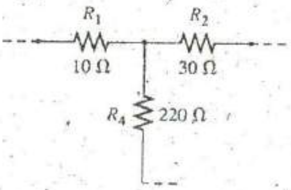


FIG. 5.5
Configuration in which none of the resistors are in series.

5.2 SERIES RESISTORS

Before the series connection is described, first recognize that every fixed resistor has only two terminals to connect in a configuration—it is therefore referred to as a **two-terminal device.** In Fig. 5.4, one terminal of resistor R_2 is connected to resistor R_1 on one side, and the remaining terminal is connected to resistor R_3 on the other side, resulting in one, and only one, connection between adjoining resistors. When connected in this manner, the resistors have established a series connection. If three elements were connected to the same point, as shown in Fig. 5.5, there would not be a series connection between resistors R_1 and R_2.

For resistors in series,

the total resistance of a series configuration is the sum of the resistance levels.

In equation form for any number (N) of resistors,

$$\boxed{R_T = R_1 + R_2 + R_3 + R_4 + \cdots + R_N} \tag{5.1}$$

A result of Eq. (5.1) is that

the more resistors we add in series, the greater is the resistance, no matter what their value.

Further,

the largest resistor in a series combination will have the most impact on the total resistance.

For the configuration in Fig. 5.4, the total resistance is

$$R_T = R_1 + R_2 + R_3$$
$$= 10\ \Omega + 30\ \Omega + 100\ \Omega$$

and $$R_T = \mathbf{140\ \Omega}$$

EXAMPLE 5.1 Determine the total resistance of the series connection in Fig. 5.6. Note that all the resistors appearing in this network are standard values.

Solution: Note in Fig. 5.6 that even though resistor R_3 is on the vertical and resistor R_4 returns at the bottom to terminal b, all the resistors are in series since there are only two resistor leads at each connection point.

Applying Eq. (5.1) gives

$$R_T = R_1 + R_2 + R_3 + R_4$$
$$R_T = 20\ \Omega + 220\ \Omega + 1.2\ \text{k}\Omega + 5.6\ \text{k}\Omega$$

and $$R_T = 7040\ \Omega = \mathbf{7.04\ k\Omega}$$

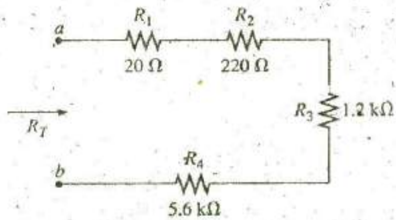


FIG. 5.6
Series connection of resistors for Example 5.1.

For the special case where resistors are the *same value,* Eq. (5.1) can be modified as follows:

$$\boxed{R_T = NR} \tag{5.2}$$

where N is the number of resistors in series of value R.

EXAMPLE 5.2 Find the total resistance of the series resistors in Fig. 5.7. Again, recognize 3.3 kΩ as a standard value.

Solution: Again, don't be concerned about the change in configuration. Neighboring resistors are connected only at one point, satisfying the definition of series elements.

Eq. (5.2): $R_T = NR$
$= (4)(3.3\ \text{k}\Omega) = \mathbf{13.2\ k\Omega}$

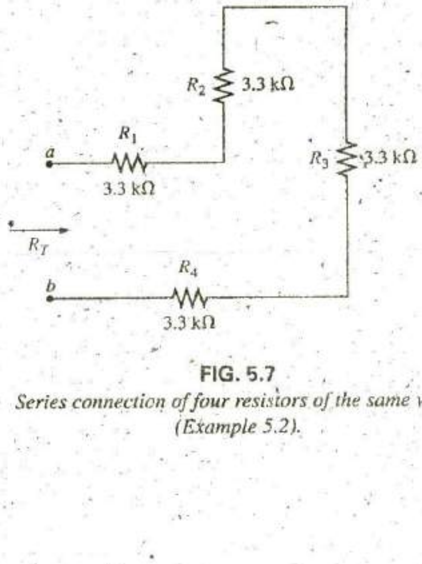


FIG. 5.7
Series connection of four resistors of the same value (Example 5.2).

It is important to realize that since the parameters of Eq. (5.1) can be put in any order,

the total resistance of resistors in series is unaffected by the order in which they are connected.

The result is that the total resistance in Fig. 5.8(a) is the same as in Fig. 5.8(b). Again, note that all the resistors are standard values.

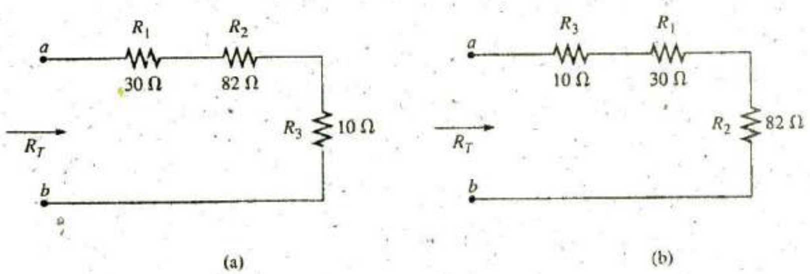


FIG. 5.8
Two series combinations of the same elements with the same total resistance.

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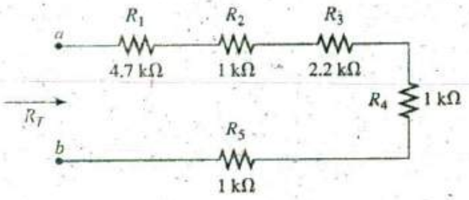

FIG. 5.9
Series combination of resistors for Example 5.3.

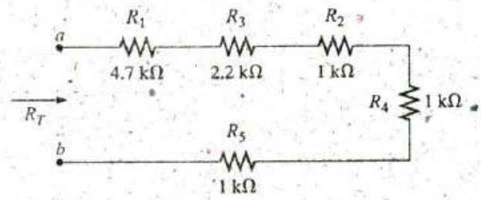

FIG. 5.10
Series circuit of Fig. 5.9 redrawn to permit the use of Eq. (5.2): $R_T = NR$.

EXAMPLE 5.3 Determine the total resistance for the series resistors (standard values) in Fig. 5.9.

Solution: First, the order of the resistors is changed as shown in Fig. 5.10 to permit the use of Eq. (5.2). The total resistance is then

$$R_T = R_1 + R_3 + NR_2$$
$$= 4.7\ \text{k}\Omega + 2.2\ \text{k}\Omega + (3)(1\ \text{k}\Omega) = \mathbf{9.9\ k\Omega}$$

Analogies

Throughout the text, analogies are used to help explain some of the important fundamental relationships in electrical circuits. An analogy is simply a combination of elements of a different type that are helpful in explaining a particular concept, relationship, or equation.

One analogy that works well for the series combination of elements is connecting different lengths of rope together to make the rope longer. Adjoining pieces of rope are connected at only one point, satisfying the definition of series elements. Connecting a third rope to the common point would mean that the sections of rope are no longer in a series.

Another analogy is connecting hoses together to form a longer hose. Again, there is still only one connection point between adjoining sections, resulting in a series connection.

Instrumentation

The total resistance of any configuration can be measured by simply connecting an ohmmeter across the access terminals as shown in Fig. 5.11 for the circuit in Fig. 5.4. *Since there is no polarity associated with resistance,* either lead can be connected to point *a*, with the other lead connected to point *b*. Choose a scale that will exceed the total resistance of the circuit, and remember when you read the response on the meter, if a kilohm scale was selected, the result will be in kilohms. For Fig. 5.11, the 200 Ω scale of our chosen multimeter was used because the total resistance is 140 Ω. If the 2 kΩ scale of our meter were selected, the digital display would read 0.140, and you must recognize that the result is in kilohms.

In the next section, another method for determining the total resistance of a circuit is introduced using Ohm's law.

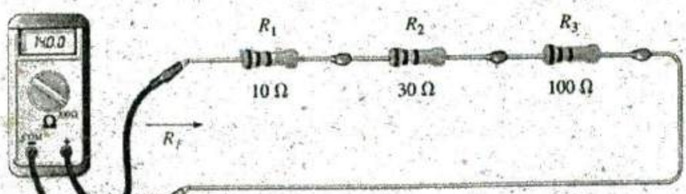

FIG. 5.11
Using an ohmmeter to measure the total resistance of a series circuit.

5.3 SERIES CIRCUITS

If we now take an 8.4 V dc supply and connect it in series with the series resistors in Fig. 5.4, we have the **series circuit** in Fig. 5.12.

A circuit is any combination of elements that will result in a continuous flow of charge, or current, through the configuration.

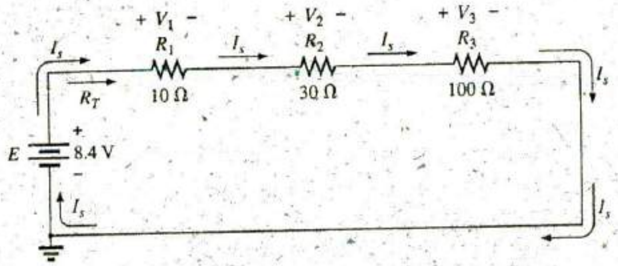

FIG. 5.12
Schematic representation for a dc series circuit.

First, recognize that the *dc supply is also a two-terminal device* with two points to be connected. If we simply ensure that there is only one connection made at each end of the supply to the series combination of resistors, we can be sure that we have established a series circuit.

The manner in which the supply is connected determines the direction of the resulting conventional current. For series dc circuits:

the direction of conventional current in a series dc circuit is such that it leaves the positive terminal of the supply and returns to the negative terminal, as shown in Fig. 5.12.

One of the most important concepts to remember when analyzing series circuits and defining elements that are in series is:

The current is the same at every point in a series circuit.

For the circuit in Fig. 5.12, the above statement dictates that the current is the same through the three resistors and the voltage source. In addition, if you are ever concerned about whether two elements are in series, simply check whether the current is the same through each element.

In any configuration, if two elements are in series, the current must be the same. However, if the current is the same for two adjoining elements, the elements may or may not be in series.

The need for this constraint in the last sentence will be demonstrated in the chapters to follow.

Now that we have a complete circuit and current has been established, the level of current and the voltage across each resistor should be determined. To do this, return to Ohm's law and replace the resistance in the equation by the total resistance of the circuit. That is,

$$\boxed{I_s = \frac{E}{R_T}} \qquad \textbf{(5.3)}$$

with the subscript s used to indicate source current.

It is important to realize that when a dc supply is connected, it does not "see" the individual connection of elements but simply the total

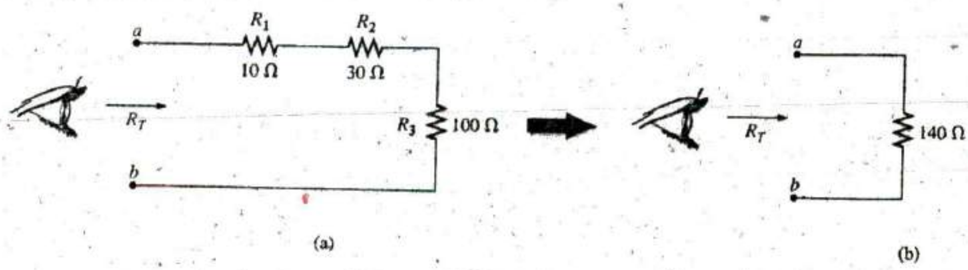

FIG. 5.13
Resistance "seen" at the terminals of a series circuit.

resistance "seen" at the connection terminals, as shown in Fig. 5.13(a). In other words, it reduces the entire configuration to one such as in Fig. 5.13(b) to which Ohm's law can easily be applied.

For the configuration in Fig. 5.12, with the total resistance calculated in the last section, the resulting current is

$$I_s = \frac{E}{R_T} = \frac{8.4\text{ V}}{140\ \Omega} = 0.06\text{ A} = \mathbf{60\ mA}$$

Note that the current I_s at every point or corner of the network is the same. Furthermore, note that the current is also indicated on the current display of the power supply.

Now that we have the current level, we can calculate the voltage across each resistor. First recognize that

the polarity of the voltage across a resistor is determined by the direction of the current.

Current entering a resistor creates a drop in voltage with the polarity indicated in Fig. 5.14(a). Reverse the direction of the current, and the polarity will reverse as shown in Fig. 5.14(b). Change the orientation of the resistor, and the same rules apply as shown in Fig. 5.14(c). Applying the above to the circuit in Fig. 5.12 will result in the polarities appearing in that figure.

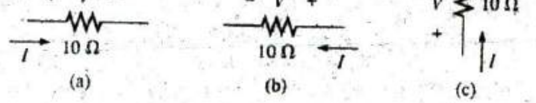

FIG. 5.14
Inserting the polarities across a resistor as determined by the direction of the current.

The magnitude of the voltage drop across each resistor can then be found by applying Ohm's law using only the resistance of each resistor. That is,

$$\boxed{\begin{aligned} V_1 &= I_1R_1 \\ V_2 &= I_2R_2 \\ V_3 &= I_3R_3 \end{aligned}} \tag{5.4}$$

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which for Fig. 5.12 results in

$$V_1 = I_1R_1 = I_sR_1 = (60\text{ mA})(10\ \Omega) = \mathbf{0.6\ V}$$
$$V_2 = I_2R_2 = I_sR_2 = (60\text{ mA})(30\ \Omega) = \mathbf{1.8\ V}$$
$$V_3 = I_3R_3 = I_sR_3 = (60\text{ mA})(100\ \Omega) = \mathbf{6.0\ V}$$

Note that in all the numerical calculations appearing in the text thus far, a unit of measurement has been applied to each calculated quantity. Always remember that a quantity without a unit of measurement is often meaningless.

EXAMPLE 5.4 For the series circuit in Fig. 5.15:

a. Find the total resistance R_T.
b. Calculate the resulting source current I_s.
c. Determine the voltage across each resistor.

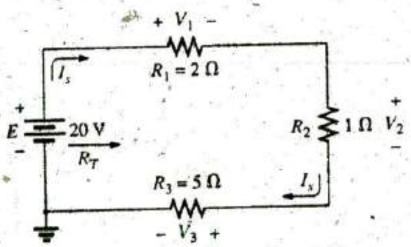

FIG. 5.15
Series circuit to be investigated in Example 5.4.

Solutions:

a. $R_T = R_1 + R_2 + R_3$
$= 2\ \Omega + 1\ \Omega + 5\ \Omega$
$R_T = \mathbf{8\ \Omega}$

b. $I_s = \dfrac{E}{R_T} = \dfrac{20\text{ V}}{8\ \Omega} = \mathbf{2.5\ A}$

c. $V_1 = I_1R_1 = I_sR_1 = (2.5\text{ A})(2\ \Omega) = \mathbf{5\ V}$
$V_2 = I_2R_2 = I_sR_2 = (2.5\text{ A})(1\ \Omega) = \mathbf{2.5\ V}$
$V_3 = I_3R_3 = I_sR_3 = (2.5\text{ A})(5\ \Omega) = \mathbf{12.5\ V}$

EXAMPLE 5.5 For the series circuit in Fig. 5.16:

a. Find the total resistance R_T.
b. Determine the source current I_s and indicate its direction on the circuit.
c. Find the voltage across resistor R_2 and indicate its polarity on the circuit.

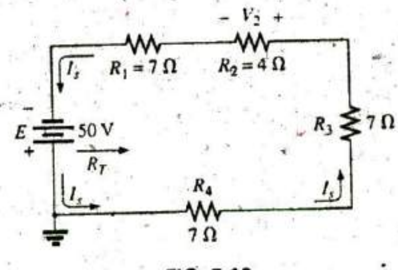

FIG. 5.16
Series circuit to be analyzed in Example 5.5.

Solutions:

a. The elements of the circuit are rearranged as shown in Fig. 5.17.

$$R_T = R_2 + NR_4$$
$$= 4\ \Omega + (3)(7\ \Omega)$$
$$= 4\ \Omega + 21\ \Omega$$
$$R_T = \mathbf{25\ \Omega}$$

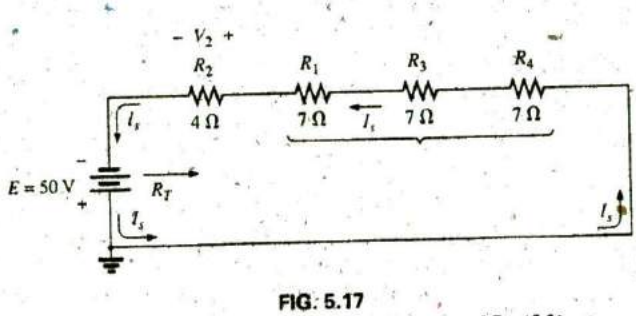

FIG. 5.17
Circuit in Fig. 5.16 redrawn to permit the use of Eq. (5.2).

b. Note that because of the manner in which the dc supply was connected, the current now has a counterclockwise direction as shown in Fig. 5.17:

$$I_s = \frac{E}{R_T} = \frac{50\ \text{V}}{25\ \Omega} = \mathbf{2\ A}$$

c. The direction of the current will define the polarity for V_2 appearing in Fig. 5.17:

$$V_2 = I_2R_2 = I_sR_2 = (2\ \text{A})(4\ \Omega) = \mathbf{8\ V}$$

Examples 5.4 and 5.5 are straightforward, substitution-type problems that are relatively easy to solve with some practice. Example 5.6, however, is another type of problem that requires both a firm grasp of the fundamental laws and equations and an ability to identify which quantity should be determined first. The best preparation for this type of exercise is to work through as many problems of this kind as possible.

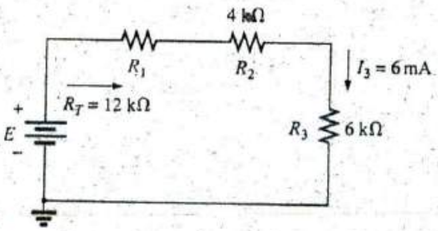

FIG. 5.18
Series circuit to be analyzed in Example 5.6.

EXAMPLE 5.6 Given R_T and I_3, calculate R_1 and E for the circuit in Fig. 5.18.

Solution: Since we are given the total resistance, it seems natural to first write the equation for the total resistance and then insert what we know:

$$R_T = R_1 + R_2 + R_3$$

We find that there is only one unknown, and it can be determined with some simple mathematical manipulations. That is,

$$12\ \text{k}\Omega = R_1 + 4\ \text{k}\Omega + 6\ \text{k}\Omega = R_1 + 10\ \text{k}\Omega$$

and $\quad 12\ \text{k}\Omega - 10\ \text{k}\Omega = R_1$

so that $\quad R_1 = \mathbf{2\ k\Omega}$

The dc voltage can be determined directly from Ohm's law:

$$E = I_sR_T = I_3R_T = (6\ \text{mA})(12\ \text{k}\Omega) = \mathbf{72\ V}$$

Analogies

The analogies used earlier to define the series connection are also excellent for the current of a series circuit. For instance, for the series-connected ropes, the stress on each rope **is the same** as they try to hold the heavy weight. For the water analogy, the flow of water **is the same** through each section of hose as the water is carried to its destination.

Instrumentation

Another important concept to remember is:

The insertion of any meter in a circuit will affect the circuit.

You must use meters that minimize the impact on the response of the circuit. The loading effects of meters are discussed in detail in a later section of this chapter. For now, we will assume that the meters are ideal and do not affect the networks to which they are applied so that we can concentrate on their proper usage.

–S–

Furthermore, it is particularly helpful in the laboratory to realize that

the voltages of a circuit can be measured without disturbing (breaking the connections in) the circuit.

In Fig. 5.19, all the voltages of the circuit in Fig. 5.12 are being measured by voltmeters that were connected without disturbing the original configuration. Note that all the voltmeters are placed **across** the resistive elements. In addition, note that the positive (normally red) lead of the voltmeter is connected to the point of higher potential (positive sign), with the negative (normally black) lead of the voltmeter connected to the point of lower potential (negative sign) for V_1 and V_2. The result is a positive reading on the display. If the leads were reversed, the magnitude would remain the same, but a negative sign would appear as shown for V_3.

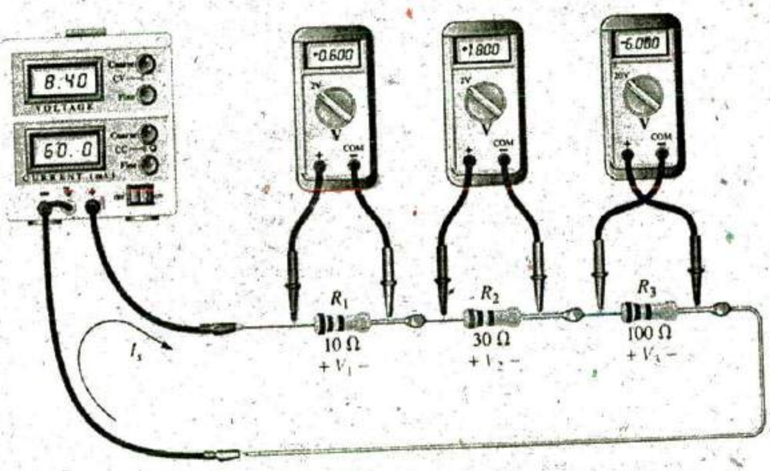

FIG. 5.19

Using voltmeters to measure the voltages across the resistors in Fig. 5.12.

Take special note that the 20 V scale of our meter was used to measure the −6 V level, while the 2 V scale of our meter was used to measure the 0.6 V and 1.8 V levels. The maximum value of the chosen scale must always exceed the maximum value to be measured. In general,

when using a voltmeter, start with a scale that will ensure that the reading is less than the maximum value of the scale. Then work your way down in scales until the reading with the highest level of precision is obtained.

Turning our attention to the current of the circuit, we find that

using an ammeter to measure the current of a circuit requires that the circuit be broken at some point and the meter inserted in series with the branch in which the current is to be determined.

For instance, to measure the current leaving the positive terminal of the supply, the connection to the positive terminal must be removed to create an open circuit between the supply and resistor R_1. The ammeter is then inserted between these two points to form a bridge between the supply and the first resistor, as shown in Fig. 5.20. The ammeter is now in series with the supply and the other elements of the circuit. If each

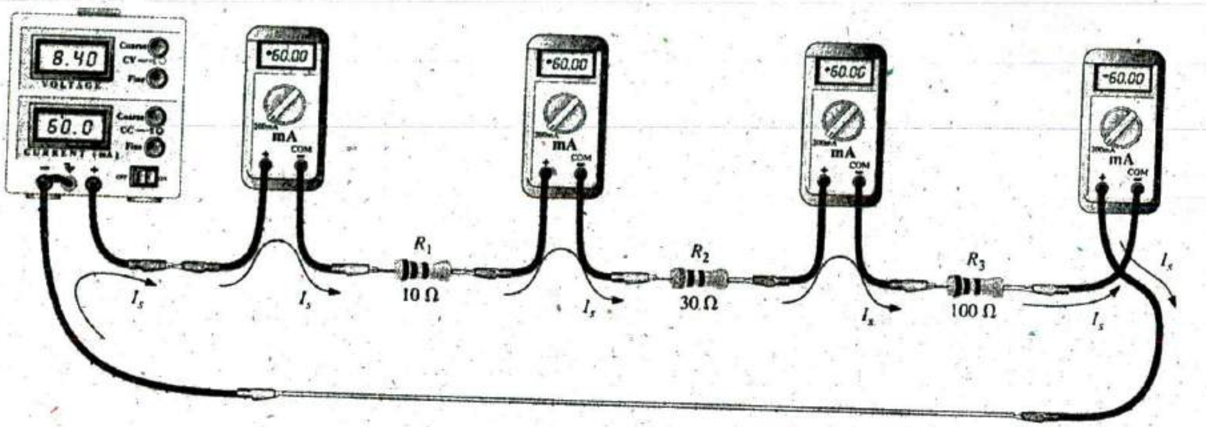

FIG. 5.20
Measuring the current throughout the series circuit in Fig. 5.12.

meter is to provide a positive reading, the connection must be made such that conventional current enters the positive terminal of the meter and leaves the negative terminal. This was done for three of the ammeters, with the ammeter to the right of R_3 connected in the reverse manner. The result is a negative sign for the current. However, also note that the current has the correct magnitude. Since the current is 60 mA, the 200 mA scale of our meter was used for each meter.

As expected, the current at each point in the series circuit is the same using our ideal ammeters.

5.4 POWER DISTRIBUTION IN A SERIES CIRCUIT

In any electrical system, the power applied will equal the power dissipated or absorbed. For any series circuit, such as that in Fig. 5.21,

the power applied by the dc supply must equal that dissipated by the resistive elements.

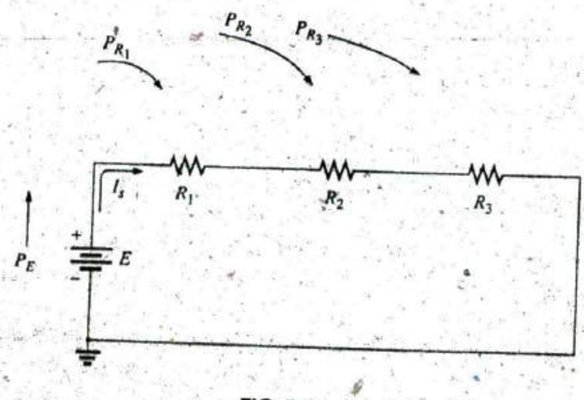

FIG. 5.21
Power distribution in a series circuit.

In equation form,

$$\boxed{P_E = P_{R_1} + P_{R_2} + P_{R_3}} \tag{5.5}$$

The power delivered by the supply can be determined using

$$\boxed{P_E = EI_s} \quad \text{(watts, W)} \tag{5.6}$$

The power dissipated by the resistive elements can be determined by any of the following forms (shown for resistor R_1 only):

$$\boxed{P_1 = V_1I_1 = I_1^2R_1 = \frac{V_1^2}{R_1}} \quad \text{(watts, W)} \tag{5.7}$$

Since the current is the same through series elements, you will find in the following examples that

in a series configuration, maximum power is delivered to the largest resistor.

EXAMPLE 5.7 For the series circuit in Fig. 5.22 (all standard values):

a. Determine the total resistance R_T.
b. Calculate the current I_s.
c. Determine the voltage across each resistor.
d. Find the power supplied by the battery.
e. Determine the power dissipated by each resistor.
f. Comment on whether the total power supplied equals the total power dissipated.

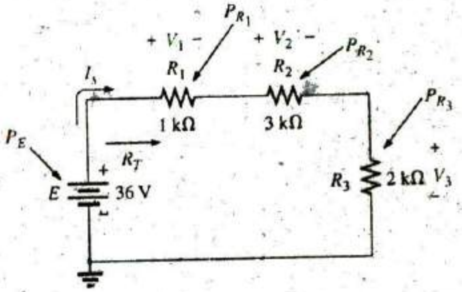

FIG. 5.22
Series circuit to be investigated in Example 5.7.

Solutions:

a. $R_T = R_1 + R_2 + R_3$
$= 1\text{ k}\Omega + 3\text{ k}\Omega + 2\text{ k}\Omega$
$R_T = \mathbf{6\text{ k}\Omega}$

b. $I_s = \dfrac{E}{R_T} = \dfrac{36\text{ V}}{6\text{ k}\Omega} = \mathbf{6\text{ mA}}$

c. $V_1 = I_1R_1 = I_sR_1 = (6\text{ mA})(1\text{ k}\Omega) = \mathbf{6\text{ V}}$
$V_2 = I_2R_2 = I_sR_2 = (6\text{ mA})(3\text{ k}\Omega) = \mathbf{18\text{ V}}$
$V_3 = I_3R_3 = I_sR_3 = (6\text{ mA})(2\text{ k}\Omega) = \mathbf{12\text{ V}}$

d. $P_E = EI_s = (36\text{ V})(6\text{ mA}) = \mathbf{216\text{ mW}}$

e. $P_1 = V_1I_1 = (6\text{ V})(6\text{ mA}) = \mathbf{36\text{ mW}}$
$P_2 = I_2^2R_2 = (6\text{ mA})^2(3\text{ k}\Omega) = \mathbf{108\text{ mW}}$
$P_3 = \dfrac{V_3^2}{R_3} = \dfrac{(12\text{ V})^2}{2\text{ k}\Omega} = \mathbf{72\text{ mW}}$

f. $P_E = P_{R_1} + P_{R_2} + P_{R_3}$
$216\text{ mW} = 36\text{ mW} + 108\text{ mW} + 72\text{ mW} = \mathbf{216\text{ mW}}$ (checks)

5.5 VOLTAGE SOURCES IN SERIES

Voltage sources can be connected in series, as shown in Fig. 5.23, to increase or decrease the total voltage applied to a system. The net voltage is determined by summing the sources with the same polarity and subtracting the total of the sources with the opposite polarity. The net polarity is the polarity of the larger sum.

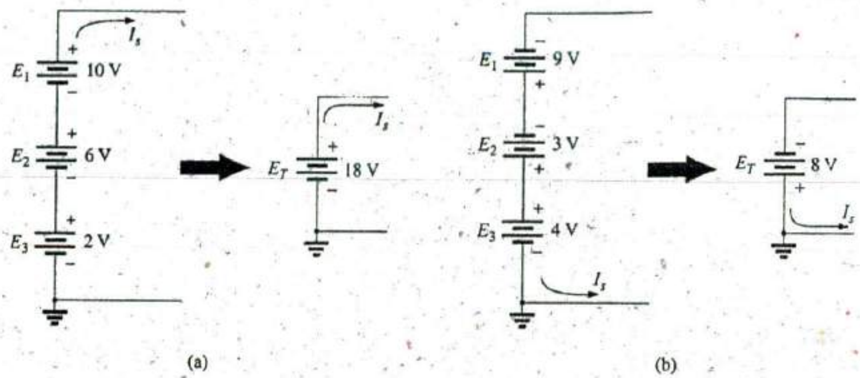

FIG. 5.23
Reducing series dc voltage sources to a single source.

In Fig. 5.23(a), for example, the sources are all "pressuring" current to follow a clockwise path, so the net voltage is

$$E_T = E_1 + E_2 + E_3 = 10\text{ V} + 6\text{ V} + 2\text{ V} = \mathbf{18\text{ V}}$$

as shown in the figure. In Fig. 5.23(b), however, the 4 V source is "pressuring" current in the clockwise direction while the other two are trying to establish current in the counterclockwise direction. In this case, the applied voltage for a counterclockwise direction is greater than that for the clockwise direction. The result is the counterclockwise direction for the current as shown in Fig. 5.23(b). The net effect can be determined by finding the difference in applied voltage between those supplies "pressuring" current in one direction and the total in the other direction. In this case,

$$E_T = E_1 + E_2 + E_3 = 9\text{ V} + 3\text{ V} - 4\text{ V} = \mathbf{8\text{ V}}$$

with the polarity shown in the figure.

Instrumentation

The connection of batteries in series to obtain a higher voltage is common in much of today's portable electronic equipment. For example, in Fig. 5.24(a), four 1.5 V AAA batteries have been connected in series to obtain a source voltage of 6 V. Although the voltage has increased, keep in mind that the maximum current for each AAA battery and for the 6 V supply is still the same. However, the power available has increased by a factor of 4 due to the increase in terminal voltage. Note also, as mentioned in Chapter 2, that the negative end of each battery is connected to the spring and the positive end to the solid contact. In addition, note how the connection is made between batteries using the horizontal connecting tabs.

In general, supplies with only two terminals (+ and −) can be connected as shown for the batteries. A problem arises, however, if the supply has an optional or fixed internal ground connection. In Fig. 5.24(b), two laboratory supplies have been connected in series with both grounds connected. The result is a shorting out of the lower source E_1 (which may damage the supply if the protective fuse does not activate quickly enough) because both grounds are at zero potential. In such cases, the supply E_2 must be left ungrounded (floating), as shown in Fig. 5.24(c), to provide the 60 V terminal voltage. If the laboratory supplies have an internal connection from the negative terminal to ground as a protective

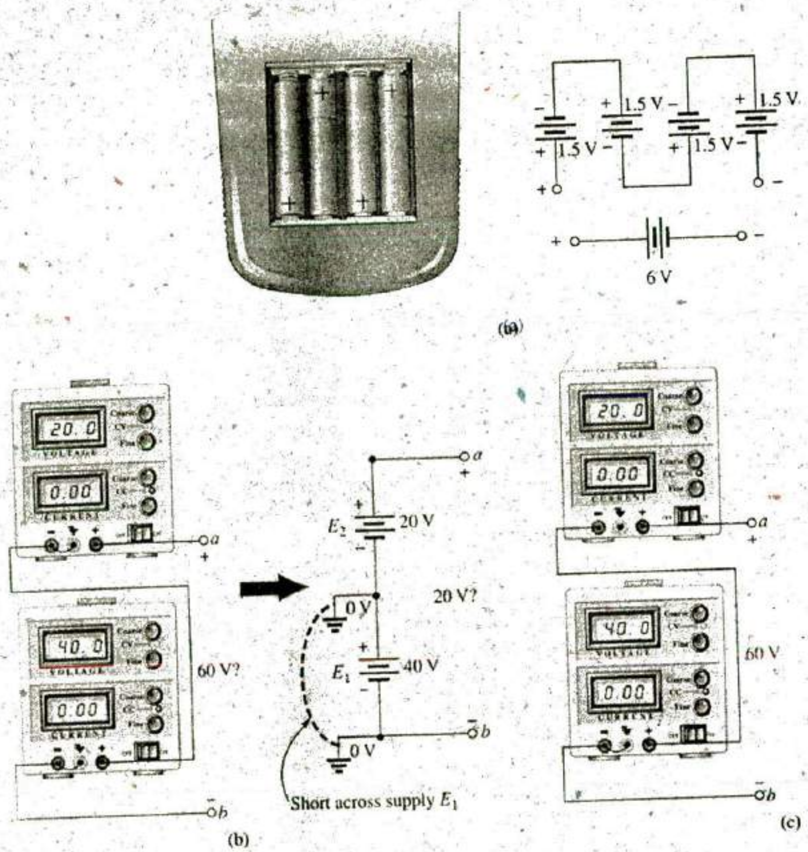

FIG. 5.24
Series connection of dc supplies: (a) four 1.5 V batteries in series to establish a terminal voltage of 6 V; (b) incorrect connections for two series dc supplies; (c) correct connection of two series supplies to establish 60 V at the output terminals.

feature for the users, a series connection of supplies cannot be made. Be aware of this fact, because some educational institutions add an internal ground to the supplies as a protective feature even though the panel still displays the ground connection as an optional feature.

5.6 KIRCHHOFF'S VOLTAGE LAW

The law to be described in this section is one of the most important in this field. It has application not only to dc circuits but also to any type of signal—whether it be ac, digital, and so on. This law is far-reaching and can be very helpful in working out solutions to networks that sometimes leave us lost for a direction of investigation.

The law, called **Kirchhoff's voltage law (KVL),** was developed by Gustav Kirchhoff (Fig. 5.25) in the mid-1800s. It is a cornerstone of the entire field and, in fact, will never be outdated or replaced.

The application of the law requires that we define a closed path of investigation, permitting us to start at one point in the network, travel through the network, and find our way back to the original starting point. The path does not have to be circular, square, or any other defined shape; it must simply provide a way to leave a point and get back to it without leaving the

FIG. 5.25
Gustav Robert Kirchhoff.
Courtesy of the Smithsonian Institution, Photo No. 58,283.

German (Königsberg, Berlin)
(1824–87),
Physicist
Professor of Physics, University of Heidelberg

Although a contributor to a number of areas in the physics domain, he is best known for his work in the electrical area with his definition of the relationships between the currents and voltages of a network in 1847. Did extensive research with German chemist Robert Bunsen (developed the *Bunsen burner*), resulting in the discovery of the important elements of *cesium and rubidium.*

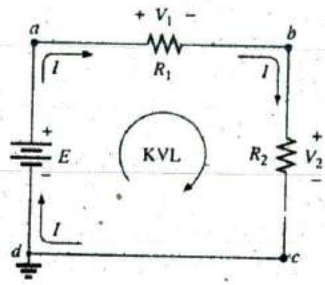

FIG. 5.26
Applying Kirchhoff's voltage law to a series dc circuit.

network. In Fig. 5.26, if we leave point *a* and follow the current, we will end up at point *b*. Continuing, we can pass through points *c* and *d* and eventually return through the voltage source to point *a*, our starting point. The path *abcda* is therefore a closed path, or **closed loop.** The law specifies that

the algebraic sum of the potential rises and drops around a closed path (or closed loop) is zero.

In symbolic form it can be written as

$$\boxed{\Sigma_{\circlearrowright} V = 0} \quad \text{(Kirchhoff's voltage law in symbolic form)} \quad \textbf{(5.8)}$$

where Σ represents summation, $\circlearrowright$ the closed loop, and V the potential drops and rises. The term *algebraic* simply means paying attention to the signs that result in the equations as we add and subtract terms.

The first question that often arises is, Which way should I go around the closed path? Should I always follow the direction of the current? To simplify matters, this text will always try to move in a clockwise direction. By selecting a direction, you eliminate the need to think about which way would be more appropriate. Any direction will work as long as you get back to the starting point.

Another question is, How do I apply a sign to the various voltages as I proceed in a clockwise direction? For a particular voltage, we will assign a positive sign when proceeding from the negative to positive potential—a positive experience such as moving from a negative checking balance to a positive one. The opposite change in potential level results in a negative sign. In Fig. 5.26, as we proceed from point *d* to point *a* across the voltage source, we move from a negative potential (the negative sign) to a positive potential (the positive sign), so a positive sign is given to the source voltage E. As we proceed from point *a* to point *b*, we encounter a positive sign followed by a negative sign, so a drop in potential has occurred, and a negative sign is applied. Continuing from *b* to *c*, we encounter another drop in potential, so another negative sign is applied. We then arrive back at the starting point *d*, and the resulting sum is set equal to zero as defined by Eq. (5.8).

Writing out the sequence with the voltages and the signs results in the following:

$$+E - V_1 - V_2 = 0$$

which can be rewritten as

$$E = V_1 + V_2$$

The result is particularly interesting because it tells us that

the applied voltage of a series dc circuit will equal the sum of the voltage drops of the circuit.

Kirchhoff's voltage law can also be written in the following form:

$$\boxed{\Sigma_{\circlearrowright} V_{\text{rises}} = \Sigma_{\circlearrowright} V_{\text{drops}}} \quad \textbf{(5.9)}$$

revealing that

the sum of the voltage rises around a closed path will always equal the sum of the voltage drops.

To demonstrate that the direction that you take around the loop has no effect on the results, let's take the counterclockwise path and compare results. The resulting sequence appears as

$$-E + V_2 + V_1 = 0$$

yielding the same result of

$$E = V_1 + V_2$$

EXAMPLE 5.8 Use Kirchhoff's voltage law to determine the unknown voltage for the circuit in Fig. 5.27.

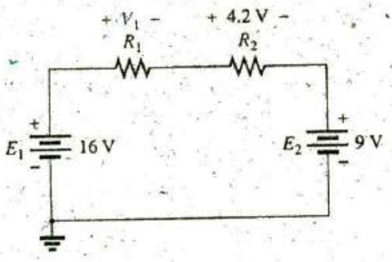

FIG. 5.27
Series circuit to be examined in Example 5.8.

Solution: When applying Kirchhoff's voltage law, be sure to concentrate on the polarities of the voltage rise or drop rather than on the type of element. In other words, do not treat a voltage drop across a resistive element differently from a voltage rise (or drop) across a source. If the polarity dictates that a drop has occurred, that is the important fact, not whether it is a resistive element or source.

Application of Kirchhoff's voltage law to the circuit in Fig. 5.27 in the clockwise direction results in

$$+E_1 - V_1 - V_2 - E_2 = 0$$

and

$$V_1 = E_1 - V_2 - E_2$$
$$= 16\text{ V} - 4.2\text{ V} - 9\text{ V}$$

so

$$V_1 = \mathbf{2.8\text{ V}}$$

The result clearly indicates that you do not need to know the values of the resistors or the current to determine the unknown voltage. Sufficient information was carried by the other voltage levels to determine the unknown.

EXAMPLE 5.9 Determine the unknown voltage for the circuit in Fig. 5.28.

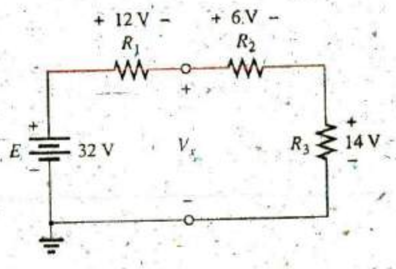

FIG. 5.28
Series dc circuit to be analyzed in Example 5.9.

Solution: In this case, the unknown voltage is not across a single resistive element but between two arbitrary points in the circuit. Simply apply Kirchhoff's voltage law around a path, including the source or resistor R_3. For the clockwise path, including the source, the resulting equation is the following:

$$+E - V_1 - V_x = 0$$

and

$$V_x = E - V_1 = 32\text{ V} - 12\text{ V} = \mathbf{20\text{ V}}$$

For the clockwise path, including resistor R_3, the following results:

$$+V_x - V_2 - V_3 = 0$$

and

$$V_x = V_2 + V_3$$
$$= 6\text{ V} + 14\text{ V}$$

with

$$V_x = \mathbf{20\text{ V}}$$

providing exactly the same solution.

There is no requirement that the followed path have charge flow or current. In Example 5.10, the current is zero everywhere, but Kirchhoff's voltage law can still be applied to determine the voltage between the points of interest. Also, there will be situations where the actual polarity will not be provided. In such cases, simply assume a polarity. If the answer is negative, the magnitude of the result is correct, but the polarity should be reversed.

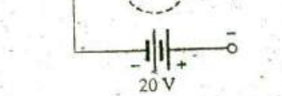

FIG. 5.29
Combination of voltage sources to be examined in Example 5.10.

EXAMPLE 5.10 Using Kirchhoff's voltage law, determine voltages V_1 and V_2 for the network in Fig. 5.29.

Solution: For path 1, starting at point a in a clockwise direction,

$$+25\text{ V} - V_1 + 15\text{ V} = 0$$

and

$$V_1 = \mathbf{40\text{ V}}$$

For path 2, starting at point a in a clockwise direction,

$$-V_2 - 20\text{ V} = 0$$

and $$V_2 = \mathbf{-20\ V}$$

The minus sign in the solution simply indicates that the actual polarities are different from those assumed.

The next example demonstrates that you do not need to know what elements are inside a container when applying Kirchhoff's voltage law. They could all be voltage sources or a mix of sources and resistors. It doesn't matter—simply pay strict attention to the polarities encountered.

Try to find the unknown quantities in the next examples without looking at the solutions. It will help define where you may be having trouble.

Example 5.11 emphasizes the fact that when you are applying Kirchhoff's voltage law, the polarities of the voltage rise or drop are the important parameters, not the type of element involved.

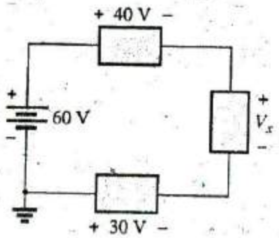

FIG. 5.30
Series configuration to be examined in Example 5.11.

EXAMPLE 5.11 Using Kirchhoff's voltage law, determine the unknown voltage for the circuit in Fig. 5.30.

Solution: Note that in this circuit, there are various polarities across the unknown elements since they can contain any mixture of components. Applying Kirchhoff's voltage law in the clockwise direction results in

$$+60\text{ V} - 40\text{ V} - V_x + 30\text{ V} = 0$$

and $$V_x = 60\text{ V} + 30\text{ V} - 40\text{ V} = 90\text{ V} - 40\text{ V}$$

with $$V_x = \mathbf{50\ V}$$

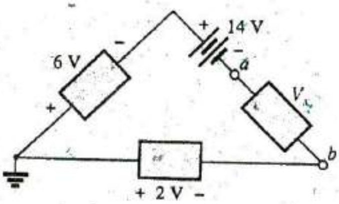

FIG. 5.31
Applying Kirchhoff's voltage law to a circuit in which the polarities have not been provided for one of the voltages (Example 5.12).

EXAMPLE 5.12 Determine the voltage V_x for the circuit in Fig. 5.31. Note that the polarity of V_x was not provided.

Solution: For cases where the polarity is not included, simply make an assumption about the polarity, and apply Kirchhoff's voltage law as before. If the result has a positive sign, the assumed polarity was correct. If the result has a minus sign, the **magnitude is correct,** but the assumed polarity must be reversed. In this case, if we assume point a to be positive and point b to be negative, an application of Kirchhoff's voltage law in the clockwise direction results in

$$-6\text{ V} - 14\text{ V} - V_x + 2\text{ V} = 0$$

and $$V_x = -20\text{ V} + 2\text{ V}$$

so that $$V_x = \mathbf{-18\ V}$$

Since the result is negative, we know that point a should be negative and point b should be positive, but the magnitude of 18 V is correct.

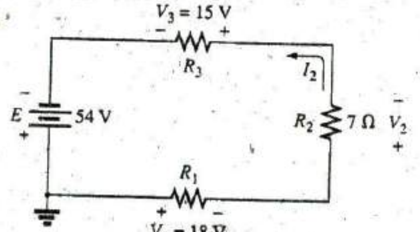

FIG. 5.32
Series configuration to be examined in Example 5.13.

EXAMPLE 5.13 For the series circuit in Fig. 5.32.

a. Determine V_2 using Kirchhoff's voltage law.
b. Determine current I_2.
c. Find R_1 and R_3.

Solutions:

a. Applying Kirchhoff's voltage law in the clockwise direction starting at the negative terminal of the supply results in

$$-E + V_3 + V_2 + V_1 = 0$$

and $$E = V_1 + V_2 + V_3 \text{ (as expected)}$$

so that $$V_2 = E - V_1 - V_3 = 54\text{ V} - 18\text{ V} - 15\text{ V}$$

and $$\mathbf{V_2 = 21\ V}$$

b. $I_2 = \dfrac{V_2}{R_2} = \dfrac{21\text{ V}}{7\ \Omega}$

$\mathbf{I_2 = 3A}$

c. $R_1 = \dfrac{V_1}{I_1} = \dfrac{18\text{ V}}{3\text{ A}} = \mathbf{6\ \Omega}$

with $R_3 = \dfrac{V_3}{I_3} = \dfrac{15\text{ V}}{3\text{ A}} = \mathbf{5\ \Omega}$

EXAMPLE 5.14 Using Kirchhoff's voltage law and Fig. 5.12, verify Eq. (5.1).

Solution: Applying Kirchhoff's voltage law around the closed path:

$$E = V_1 + V_2 + V_3$$

Substituting Ohm's law:

$$I_sR_T = I_1R_1 + I_2R_2 + I_3R_3$$

but $$I_s = I_1 = I_2 = I_3$$

so that $$I_sR_T = I_s(R_1 + R_2 + R_3)$$

and $$\mathbf{R_T = R_1 + R_2 + R_3}$$

which is Eq. (5.1).

5.7 VOLTAGE DIVISION IN A SERIES CIRCUIT

The previous section demonstrated that the sum of the voltages across the resistors of a series circuit will always equal the applied voltage. It cannot be more or less than that value. The next question is, How will a resistor's value affect the voltage across the resistor? It turns out that

the voltage across series resistive elements will divide as the magnitude of the resistance levels.

In other words,

in a series resistive circuit, the larger the resistance, the more of the applied voltage it will capture.

In addition,

the ratio of the voltages across series resistors will be the same as the ratio of their resistance levels.

All of the above statements can best be described by a few simple examples. In Fig. 5.33, all the voltages across the resistive elements are provided. The largest resistor of 6 Ω captures the bulk of the applied voltage, while the smallest resistor, R_3, has the least. In addition, note that since the resistance level of R_1 is six times that of R_3, the voltage across R_1 is six

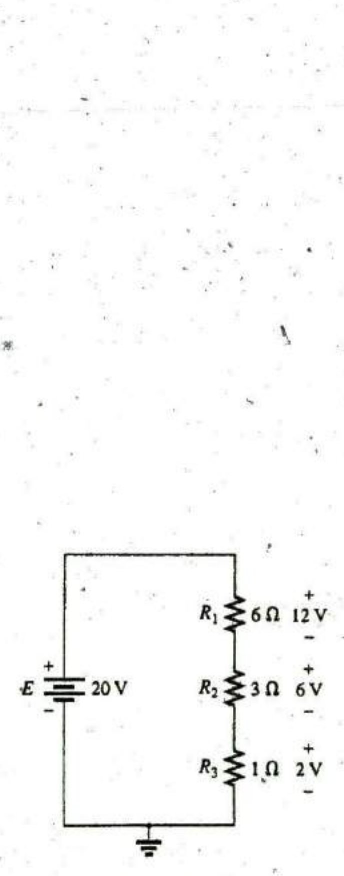

FIG. 5.33
Revealing how the voltage will divide across series resistive elements.

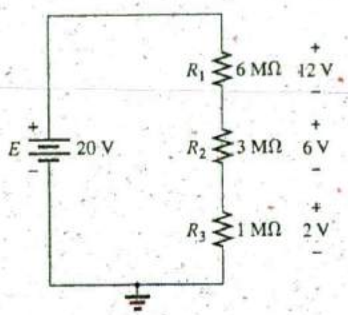

FIG. 5.34
The ratio of the resistive values determines the voltage division of a series dc circuit.

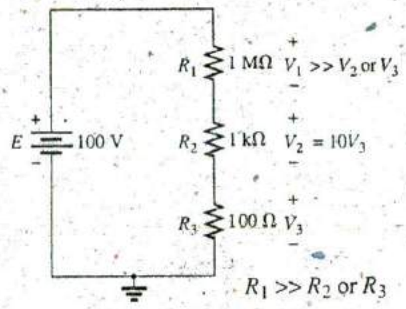

FIG. 5.35
The largest of the series resistive elements will capture the major share of the applied voltage.

times that of R_3. The fact that the resistance level of R_2 is three times that of R_1 results in three times the voltage across R_2. Finally, since R_1 is twice R_2, the voltage across R_1 is twice that of R_2. In general, therefore, the voltage across series resistors will have the same ratio as their resistance levels.

Note that if the resistance levels of all the resistors in Fig. 5.33 are increased by the same amount, as shown in Fig. 5.34, the voltage levels all remain the same. In other words, even though the resistance levels were increased by a factor of 1 million, the voltage ratios remained the same. Clearly, therefore, it is the ratio of resistor values that counts when it comes to voltage division, not the magnitude of the resistors. The current level of the network will be severely affected by this change in resistance level, but the voltage levels remain unaffected.

Based on the above, it should now be clear that when you first encounter a circuit such as that in Fig. 5.35, you will expect that the voltage across the 1 MΩ resistor will be much greater than that across the 1 kΩ or the 100 Ω resistor. In addition, based on a statement above, the voltage across the 1 kΩ resistor will be 10 times as great as that across the 100 Ω resistor since the resistance level is 10 times as much. Certainly, you would expect that very little voltage will be left for the 100 Ω resistor. Note that the current was never mentioned in the above analysis. The distribution of the applied voltage is determined solely by the ratio of the resistance levels. Of course, the magnitude of the resistors will determine the resulting current level.

To continue with the above, since 1 MΩ is 1000 times larger than 1 kΩ, voltage V_1 will be 1000 times larger than V_2. In addition, voltage V_2 will be 10 times larger than V_3. Finally, the voltage across the largest resistor of 1 MΩ will be (10)(1000) = 10,000 times larger than V_3.

Now for some details. The total resistance is

$$\begin{aligned} R_T &= R_1 + R_2 + R_3 \\ &= 1\ \text{M}\Omega + 1\ \text{k}\Omega + 100\ \Omega \\ R_T &= \mathbf{1{,}001{,}100\ \Omega} \end{aligned}$$

The current is

$$I_s = \frac{E}{R_T} = \frac{100\ \text{V}}{1{,}001{,}100\ \Omega} \cong 99.89\ \mu\text{A} \qquad \text{(about } 100\ \mu\text{A)}$$

with

$$\begin{aligned} V_1 &= I_1R_1 = I_sR_1 = (99.89\ \mu\text{A})(1\ \text{M}\Omega) = \mathbf{99.89\ V} \quad \text{(almost the full 100 V)} \\ V_2 &= I_2R_2 = I_sR_2 = (99.89\ \mu\text{A})(1\ \text{k}\Omega) = \mathbf{99.89\ mV} \quad \text{(about 100 mV)} \\ V_3 &= I_3R_3 = I_sR_3 = (99.89\ \mu\text{A})(100\ \Omega) = \mathbf{9.989\ mV} \quad \text{(about 10 mV)} \end{aligned}$$

As illustrated above, the major part of the applied voltage is across the 1 MΩ resistor. The current is in the microampere range due primarily to the large 1 MΩ resistor. Voltage V_2 is about 0.1 V, compared to almost 100 V for V_1. The voltage across R_3 is only about 10 mV, or 0.010 V.

Before making any detailed, lengthy calculations, you should first examine the resistance levels of the series resistors to develop some idea of how the applied voltage will be divided through the circuit. It will reveal, with a minumum amount of effort, what you should expect when performing the calculations (a checking mechanism). It also allows you to speak intelligently about the response of the circuit without having to resort to any calculations.

Voltage Divider Rule (VDR)

The **voltage divider rule (VDR)** permits the determination of the voltage across a series resistor without first having to determine the current of the circuit. The rule itself can be derived by analyzing the simple series circuit in Fig. 5.36.

First, determine the total resistance as follows:

$$R_T = R_1 + R_2$$

Then

$$I_s = I_1 = I_2 = \frac{E}{R_T}$$

Apply Ohm's law to each resistor:

$$V_1 = I_1R_1 = \left(\frac{E}{R_T}\right)R_1 = R_1\frac{E}{R_T}$$

$$V_2 = I_2R_2 = \left(\frac{E}{R_T}\right)R_2 = R_2\frac{E}{R_T}$$

The resulting format for V_1 and V_2 is

$$\boxed{V_x = R_x\frac{E}{R_T}} \qquad \text{(voltage divider rule)} \qquad (5.10)$$

where V_x is the voltage across the resistor R_x, E is the impressed voltage across the series elements, and R_T is the total resistance of the series circuit.

The voltage divider rule states that

the voltage across a resistor in a series circuit is equal to the value of that resistor times the total applied voltage divided by the total resistance of the series configuration.

Although Eq. (5.10) was derived using a series circuit of only two elements, it can be used for series circuits with any number of series resistors.

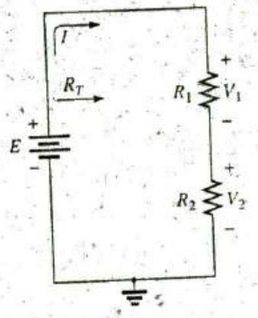

FIG. 5.36
Developing the voltage divider rule.

EXAMPLE 5.15 For the series circuit in Fig. 5.37.

a. Without making any calculations, how much larger would you expect the voltage across R_2 to be compared to that across R_1?
b. Find the voltage V_1 using only the voltage divider rule.
c. Using the conclusion of part (a), determine the voltage across R_2.
d. Use the voltage divider rule to determine the voltage across R_2, and compare your answer to your conclusion in part (c).
e. How does the sum of V_1 and V_2 compare to the applied voltage?

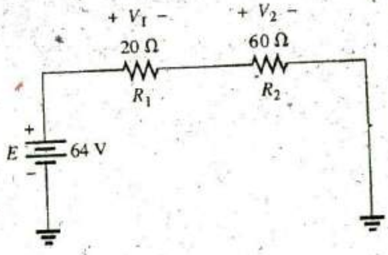

FIG. 5.37
Series circuit to be examined using the voltage divider rule in Example 5.15.

Solutions:

a. Since resistor R_2 is three times R_1, it is expected that $V_2 = 3V_1$.

b. $V_1 = R_1\frac{E}{R_T} = 20\ \Omega\left(\frac{64\ \text{V}}{20\ \Omega + 60\ \Omega}\right) = 20\ \Omega\left(\frac{64\ \text{V}}{80\ \Omega}\right) = \mathbf{16\ V}$

c. $V_2 = 3V_1 = 3(16\ \text{V}) = \mathbf{48\ V}$

d. $V_2 = R_2\frac{E}{R_T} = (60\ \Omega)\left(\frac{64\ \text{V}}{80\ \Omega}\right) = \mathbf{48\ V}$

The results are an exact match.

e. $E = V_1 + V_2$

$64\ \text{V} = 16\ \text{V} + 48\ \text{V} = \mathbf{64\ V}$ (checks)

-S-

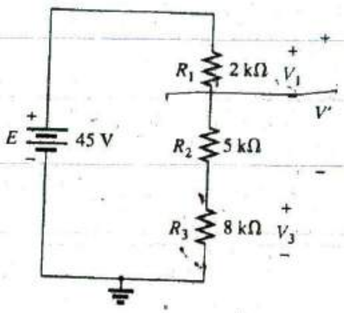

FIG. 5.38
Series circuit to be investigated in Examples 5.16 and 5.17.

EXAMPLE 5.16 Using the voltage divider rule, determine voltages V_1 and V_3 for the series circuit in Fig. 5.38.

Solution:

$$R_T = R_1 + R_2 + R_3$$
$$= 2\text{ k}\Omega + 5\text{ k}\Omega + 8\text{ k}\Omega$$
$$R_T = 15\text{k}\Omega$$
$$V_1 = R_1\frac{E}{R_T} = 2\text{ k}\Omega\left(\frac{45\text{ V}}{15\text{k}\Omega}\right) = \mathbf{6\text{ V}}$$

and

$$V_3 = R_3\frac{E}{R_T} = 8\text{ k}\Omega\left(\frac{45\text{ V}}{15\ \Omega}\right) = \mathbf{24\text{ V}}$$

The voltage divider rule can be extended to the voltage across two or more series elements if the resistance in the numerator of Eq. (5.10) is expanded to include the total resistance of the series resistors across which the voltage is to be found (R'). That is,

$$\boxed{V' = R'\frac{E}{R_T}} \qquad (5.11)$$

EXAMPLE 5.17 Determine the voltage (denoted V') across the series combination of resistors R_1 and R_2 in Fig. 5.38.

Solution: Since the voltage desired is across both R_1 and R_2, the sum of R_1 and R_2 will be substituted as R' in Eq. (5.11). The result is

$$R' = R_1 + R_2 = 2\text{ k}\Omega + 5\text{ k}\Omega = 7\text{ k}\Omega$$

and

$$V' = R'\frac{E}{R_T} = 7\text{ k}\Omega\left(\frac{45\text{ V}}{15\text{ k}\Omega}\right) = \mathbf{21\text{ V}}$$

In the next example you are presented with a problem of the other kind: Given the voltage division, you must determine the required resistor values. In most cases, problems of this kind simply require that you are able to use the basic equations introduced thus far in the text.

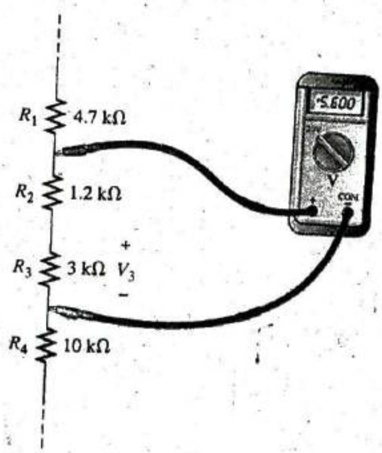

FIG. 5.39
Voltage divider action for Example 5.18.

EXAMPLE 5.18 Given the voltmeter reading in Fig. 5.39, find voltage V_3.

Solution: Even though the rest of the network is not shown and the current level has not been determined, the voltage divider rule can be applied by using the voltmeter reading as the full voltage across the series combination of resistors. That is,

$$V_3 = R_3\frac{(V_{\text{meter}})}{R_3 + R_2} = \frac{3\text{ k}\Omega(5.6\text{ V})}{3\text{ k}\Omega + 1.2\text{ k}\Omega}$$
$$V_3 = \mathbf{4\text{ V}}$$

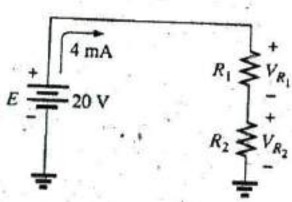

FIG. 5.40
Designing a voltage divider circuit (Example 5.19).

EXAMPLE 5.19 Design the voltage divider circuit in Fig. 5.40 such that the voltage across R_1 will be four times the voltage across R_2; that is $V_{R_1} = 4V_{R_2}$.

Solution: The total resistance is defined by

$$R_T = R_1 + R_2$$

However, if $\quad V_{R_1} = 4V_{R_2}$

then $\quad R_1 = 4R_2$

so that $\quad R_T = R_1 + R_2 = 4R_2 + R_2 = 5R_2$

Applying Ohm's law, we can determine the total resistance of the circuit:

$$R_T = \frac{E}{I_s} = \frac{20\text{ V}}{4\text{ mA}} = 5\text{ k}\Omega$$

so $\quad R_T = 5R_2 = 5\text{ k}\Omega$

and $\quad R_2 = \frac{5\text{ k}\Omega}{5} = \mathbf{1\text{ k}\Omega}$

Then $\quad R_1 = 4R_2 = 4(1\text{ k}\Omega) = \mathbf{4\text{ k}\Omega}$

5.8 INTERCHANGING SERIES ELEMENTS

The elements of a series circuit can be interchanged without affecting the total resistance, current, or power to each element. For instance, the network in Fig. 5.41 can be redrawn as shown in Fig. 5.42 without affecting I or V_2. The total resistance R_T is 35 Ω in both cases, and $I = 70\text{ V}/35\ \Omega = 2\text{ A}$. The voltage $V_2 = IR_2 = (2\text{ A})(5\ \Omega) = 10\text{ V}$ for both configurations.

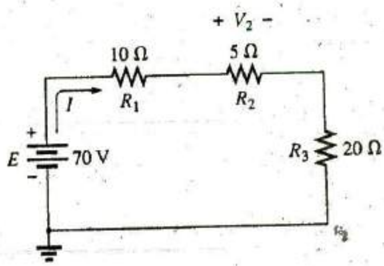

FIG. 5.41
Series dc circuit with elements to be interchanged.

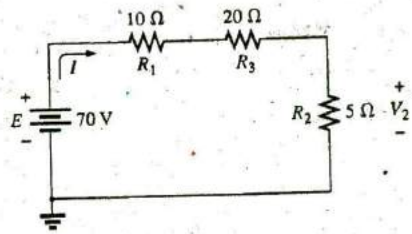

FIG. 5.42
Circuit in Fig. 5.41 with R_2 and R_3 interchanged.

EXAMPLE 5.20 Determine I and the voltage across the 7 Ω resistor for the network in Fig. 5.43.

Solution: The network is redrawn in Fig. 5.44.

$$R_T = (2)(4\ \Omega) + 7\ \Omega = 15\ \Omega$$

$$I = \frac{E}{R_T} = \frac{37.5\text{ V}}{15\ \Omega} = \mathbf{2.5\text{ A}}$$

$$V_{7\Omega} = IR = (2.5\text{ A})(7\ \Omega) = \mathbf{17.5\text{ V}}$$

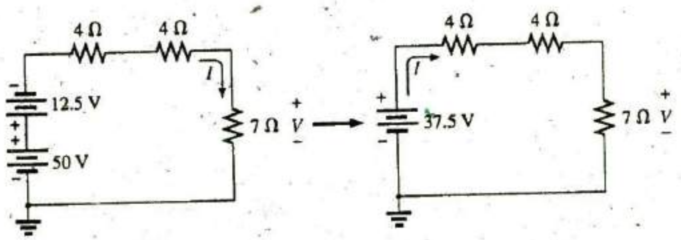

FIG. 5.44
Redrawing the circuit in Fig. 5.43.

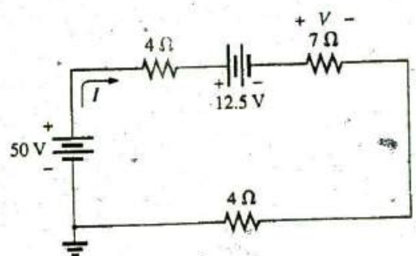

FIG. 5.43
Example 5.20.

5.9 NOTATION

Notation plays an increasingly important role in the analysis to follow. It is important, therefore, that we begin to examine the notation used throughout the industry.

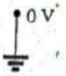

FIG. 5.45
Ground potential.

Voltage Sources and Ground

Except for a few special cases, electrical and electronic systems are grounded for reference and safety purposes. The symbol for the ground connection appears in Fig. 5.45 with its defined potential level—zero volts. A grounded circuit may appear as shown in Fig. 5.46(a), (b), or (c). In any case, it is understood that the negative terminal of the battery and the bottom of the resistor R_2 are at ground potential. Although Fig. 5.46(c) shows no connection between the two grounds, it is recognized that such a connection exists for the continuous flow of charge. If $E = 12$ V, then point a is 12 V positive with respect to ground potential, and 12 V exist across the series combination of resistors R_1 and R_2. If a voltmeter placed from point b to ground reads 4 V, then the voltage across R_2 is 4 V, with the higher potential at point b.

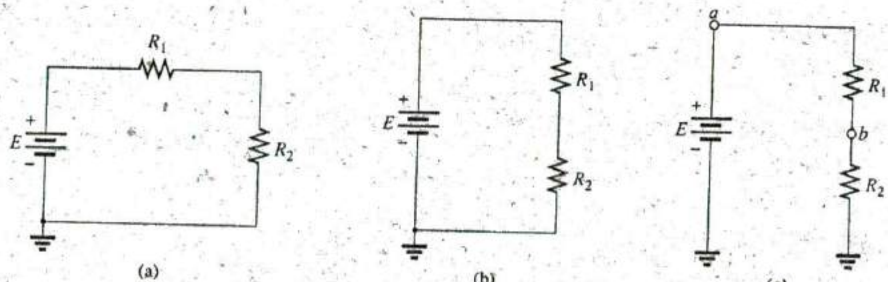

FIG. 5.46
Three ways to sketch the same series dc circuit.

On large schematics where space is at a premium and clarity is important, voltage sources may be indicated as shown in Figs. 5.47(a) and 5.48(a) rather than as illustrated in Figs. 5.47(b) and 5.48(b). In addition, potential levels may be indicated as in Fig. 5.49, to permit a rapid check of the potential levels at various points in a network with respect to ground to ensure that the system is operating properly.

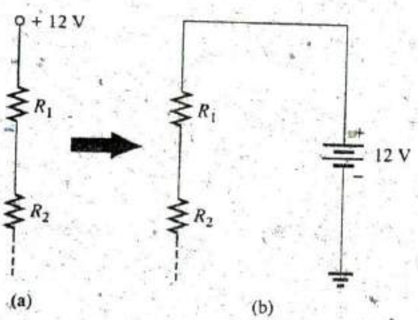

FIG. 5.47
Replacing the special notation for a dc voltage source with the standard symbol.

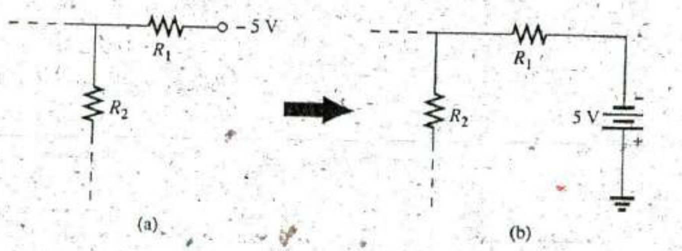

FIG. 5.48
Replacing the notation for a negative dc supply with the standard notation.

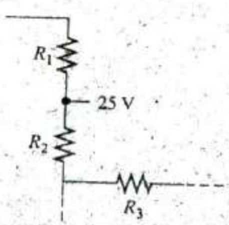

FIG. 5.49
The expected voltage level at a particular point in a network if the system is functioning properly.

Double-Subscript Notation

The fact that voltage is an *across* variable and exists between two points has resulted in a double-subscript notation that defines the first subscript as the higher potential. In Fig. 5.50(a), the two points that define the voltage across the resistor R are denoted by a and b. Since a is the first subscript for V_{ab}, point a must have a higher potential than point b if V_{ab}

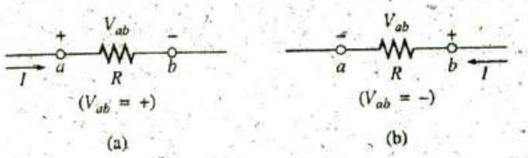

FIG. 5.50
Defining the sign for double-subscript notation.

is to have a positive value. If, in fact, point b is at a higher potential than point a, V_{ab} will have a negative value, as indicated in Fig. 5.50(b).

In summary:

The double-subscript notation V_{ab} specifies point a as the higher potential. If this is not the case, a negative sign must be associated with the magnitude of V_{ab}.

In other words,

the voltage V_{ab} is the voltage at point a with respect to (w.r.t.) point b.

Single-Subscript Notation

If point b of the notation V_{ab} is specified as ground potential (zero volts), then a single-subscript notation can be used that provides the voltage at a point with respect to ground.

In Fig. 5.51, V_a is the voltage from point a to ground. In this case, it is obviously 10 V since it is right across the source voltage E. The voltage V_b is the voltage from point b to ground. Because it is directly across the 4 Ω resistor, $V_b = 4$ V.

In summary:

The single-subscript notation V_a specifies the voltage at point a with respect to ground (zero volts). If the voltage is less than zero volts, a negative sign must be associated with the magnitude of V_a.

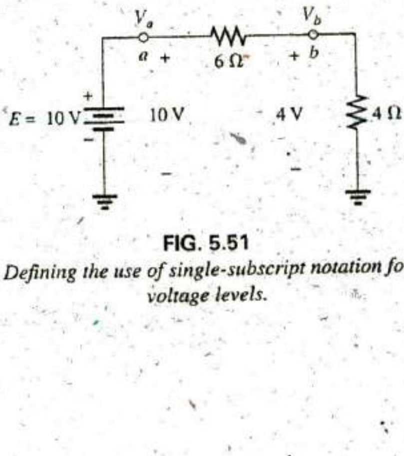

FIG. 5.51
Defining the use of single-subscript notation for voltage levels.

General Comments

A particularly useful relationship can now be established that has extensive applications in the analysis of electronic circuits. For the above notational standards, the following relationship exists:

$$\boxed{V_{ab} = V_a - V_b} \tag{5.12}$$

In other words, if the voltage at points a and b is known with respect to ground, then the voltage V_{ab} can be determined using Eq. (5.12). In Fig. 5.51, for example,

$$\begin{aligned} V_{ab} &= V_a - V_b = 10\text{ V} - 4\text{ V} \\ &= 6\text{ V} \end{aligned}$$

EXAMPLE 5.21 Find the voltage V_{ab} for the conditions in Fig. 5.52.

Solution: Applying Eq. (5.12) gives

$$\begin{aligned} V_{ab} &= V_a - V_b = 16\text{ V} - 20\text{ V} \\ &= \mathbf{-4\text{ V}} \end{aligned}$$

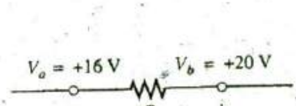

FIG. 5.52
Example 5.21.

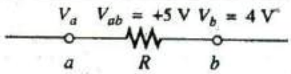

FIG. 5.53
Example 5.22.

Note the negative sign to reflect the fact that point *b* is at a higher potential than point *a*.

EXAMPLE 5.22 Find the voltage V_a for the configuration in Fig. 5.53.

Solution: Applying Eq. (5.12) gives

$$V_{ab} = V_a - V_b$$

and
$$V_a = V_{ab} + V_b = 5\text{ V} + 4\text{ V}$$
$$= \mathbf{9\ V}$$

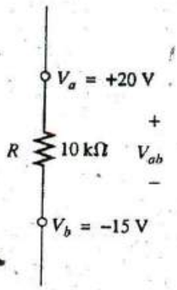

FIG. 5.54
Example 5.23.

EXAMPLE 5.23 Find the voltage V_{ab} for the configuration in Fig. 5.54.

Solution: Applying Eq. (5.12) gives

$$V_{ab} = V_a - V_b = 20\text{ V} - (-15\text{ V}) = 20\text{ V} + 15\text{ V}$$
$$= \mathbf{35\ V}$$

Note in Example 5.23 you must be careful with the signs when applying the equation. The voltage is dropping from a high level of +20 V to a negative voltage of −15 V. As shown in Fig. 5.55, this represents a drop in voltage of 35 V. In some ways it's like going from a positive checking balance of $20 to owing $15; the total expenditure is $35.

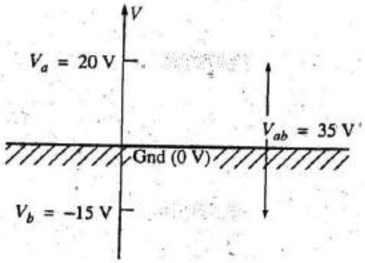

FIG. 5.55
The impact of positive and negative voltages on the total voltage drop.

EXAMPLE 5.24 Find the voltages V_b, V_c, and V_{ac} for the network in Fig. 5.56.

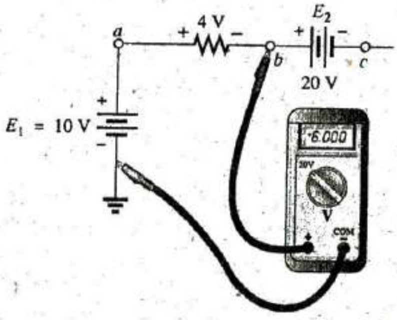

FIG. 5.56
Example 5.24.

Solution: Starting at ground potential (zero volts), we proceed through a rise of 10 V to reach point *a* and then pass through a drop in potential of 4 V to point *b*. The result is that the meter reads

$$V_b = +10\text{ V} - 4\text{ V} = \mathbf{6\ V}$$

as clearly demonstrated by Fig. 5.57.

If we then proceed to point *c*, there is an additional drop of 20 V, resulting in

$$V_c = V_b - 20\text{ V} = 6\text{ V} - 20\text{ V} = \mathbf{-14\ V}$$

as shown in Fig. 5.58:

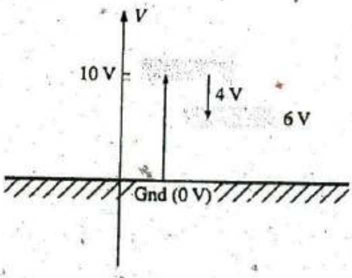

FIG. 5.57
Determining V_b using the defined voltage levels.

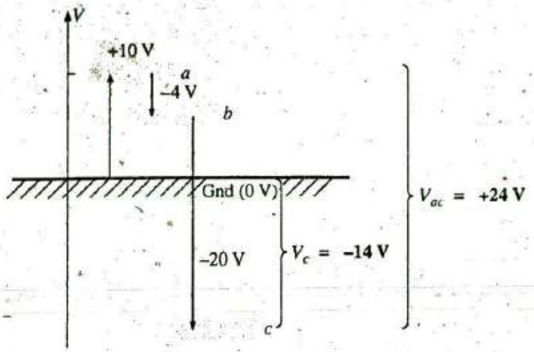

FIG. 5.58
Review of the potential levels for the circuit in Fig. 5.56.

The voltage V_{ac} can be obtainted using Eq. (5.12) or by simply referring to Fig. 5.58:

$$V_{ac} = V_a - V_c = 10\text{ V} - (-14\text{ V}) = \mathbf{24\ V}$$

EXAMPLE 5.25 Determine V_{ab}, V_{cb}, and V_c for the network in Fig. 5.59.

Solution: There are two ways to approach this problem. The first is to sketch the diagram in Fig. 5.60 and note that there is a 54 V drop across the series resistors R_1 and R_2. The current can then be determined using Ohm's law and the voltage levels as follows:

$$I = \frac{54\text{ V}}{45\ \Omega} = 1.2\text{ A}$$
$$V_{ab} = IR_2 = (1.2\text{ A})(25\ \Omega) = \mathbf{30\ V}$$
$$V_{cb} = -IR_1 = -(1.2\text{ A})(20\ \Omega) = \mathbf{-24\ V}$$
$$V_c = E_1 = \mathbf{-19\ V}$$

The other approach is to redraw the network as shown in Fig. 5.61 to clearly establish the aiding effect of E_1 and E_2 and then solve the resulting series circuit:

$$I = \frac{E_1 + E_2}{R_T} = \frac{19\text{ V} + 35\text{ V}}{45\ \Omega} = \frac{54\text{ V}}{45\ \Omega} = 1.2\text{ A}$$

and $\qquad V_{ab} = \mathbf{30\ V} \qquad V_{cb} = \mathbf{-24\ V} \qquad V_c = \mathbf{-19\ V}$

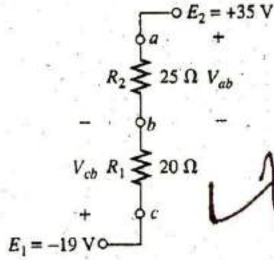

FIG. 5.59
Example 5.25.

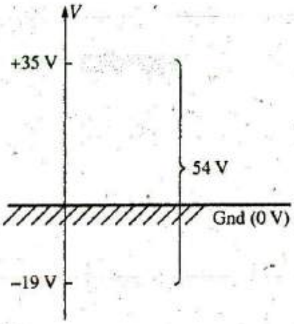

FIG. 5.60
Determining the total voltage drop across the resistive elements in Fig 5.59.

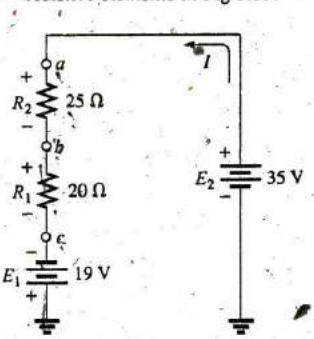

FIG. 5.61
Redrawing the circuit in Fig. 5.59 using standard dc voltage supply symbols.

EXAMPLE 5.26 Using the voltage divider rule, determine the voltages V_1 and V_2 of Fig. 5.62.

Solution: Redrawing the network with the standard battery symbol results in the network in Fig. 5.63. Applying the voltage divider rule gives

$$V_1 = \frac{R_1 E}{R_1 + R_2} = \frac{(4\ \Omega)(24\text{ V})}{4\ \Omega + 2\ \Omega} = \mathbf{16\ V}$$
$$V_2 = \frac{R_2 E}{R_1 + R_2} = \frac{(2\ \Omega)(24\text{ V})}{4\ \Omega + 2\ \Omega} = \mathbf{8\ V}$$

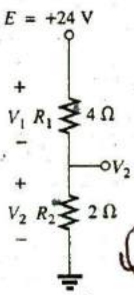

FIG. 5.62
Example 5.26.

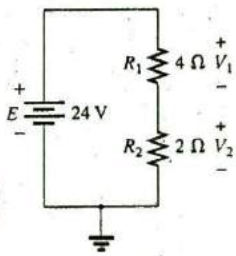

FIG. 5.63
Circuit of Fig. 5.62 redrawn.

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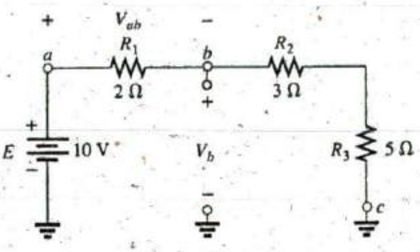

FIG. 5.64
Example 5.27.

EXAMPLE 5.27 For the network in Fig. 5.64.

a. Calculate V_{ab}.
b. Determine V_b.
c. Calculate V_c.

Solutions:

a. Voltage divider rule:

$$V_{ab} = \frac{R_1 E}{R_T} = \frac{(2\ \Omega)(10\ \text{V})}{2\ \Omega + 3\ \Omega + 5\ \Omega} = \mathbf{+2\ V}$$

b. Voltage divider rule:

$$V_b = V_{R_2} + V_{R_3} = \frac{(R_2 + R_3)E}{R_T} = \frac{(3\ \Omega + 5\ \Omega)(10\ \text{V})}{10\ \Omega} = \mathbf{8\ V}$$

or $\quad V_b = V_a - V_{ab} = E - V_{ab} = 10\ \text{V} - 2\ \text{V} = \mathbf{8\ V}$

c. V_c = ground potential = **0 V**

5.10 VOLTAGE REGULATION AND THE INTERNAL RESISTANCE OF VOLTAGE SOURCES

When you use a dc supply such as the generator, battery, or supply in Fig. 5.65, you initially assume that it will provide the desired voltage for any resistive load you may hook up to the supply. In other words, if the battery is labeled 1.5 V or the supply is set at 20 V, you assume that they will provide that voltage no matter what load you may apply. Unfortunately, this is not always the case. For instance, if we apply a 1 kΩ resistor to a dc laboratory supply, it is fairly easy to set the voltage across the resistor to 20 V. However, if we remove the 1 kΩ resistor and replace it with a 100 Ω resistor and don't touch the controls on the supply at all, we may find that the voltage has dropped to 19.14 V. Change the load to a 68 Ω resistor, and the terminal voltage drops to 18.72 V. We discover that the load applied affects the terminal voltage of the supply. In fact, this example points out that

a network should always be connected to a supply before the level of supply voltage is set.

The reason the terminal voltage drops with changes in load (current demand) is that

every practical (real-world) supply has an internal resistance in series with the idealized voltage source

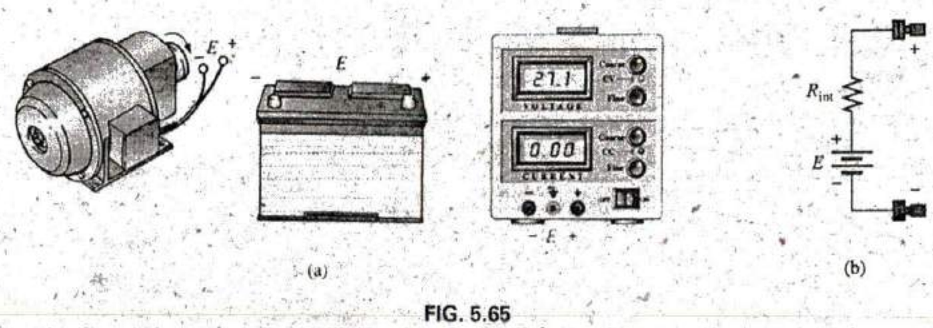

FIG. 5.65
(a) Sources of dc voltage; (b) equivalent circuit.

as shown in Fig. 5.65(b). The resistance level depends on the type of supply, but it is always present. Every year new supplies come out that are less sensitive to the load applied, but even so, some sensitivity still remains.

The supply in Fig. 5.66 helps explain the action that occurred above as we changed the load resistor. Due to the **internal resistance** of the supply, the ideal internal supply must be set to 20.1 V in Fig. 5.66(a) if 20 V are to appear across the 1 kΩ resistor. The internal resistance will capture 0.1 V of the applied voltage. The current in the circuit is determined by simply looking at the load and using Ohm's law; that is, $I_L = V_L/R_L = 20\text{ V}/1\text{ k}\Omega \doteq 20\text{ mA}$, which is a relatively low current.

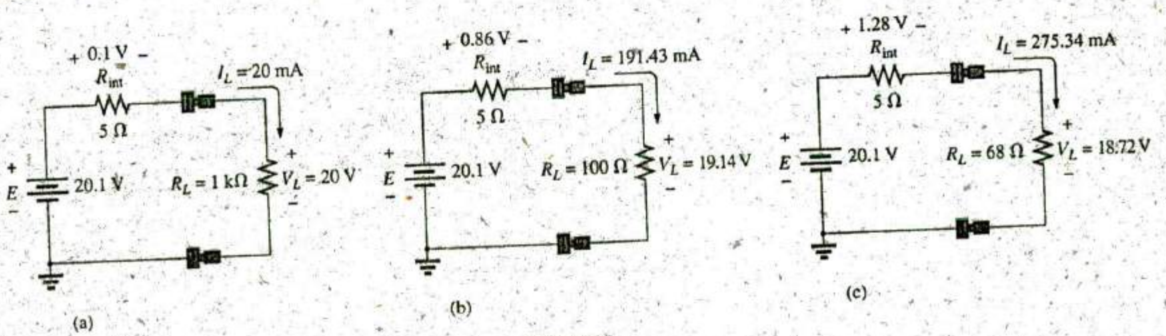

FIG. 5.66
Demonstrating the effect of changing a load on the terminal voltage of a supply.

In Fig. 5.66(b), all the settings of the supply are left untouched, but the 1 kΩ load is replaced by a 100 Ω resistor. The resulting current is now $I_L = E/R_T = 20.1\text{ V}/105\ \Omega = 191.43\text{ mA}$, and the output voltage is $V_L = I_L R = (191.43\text{ mA})(100\ \Omega) = 19.14\text{ V}$, a drop of 0.86 V. In Fig. 5.66(c), a 68 Ω load is applied, and the current increases substantially to 275.34 mA with a terminal voltage of only 18.72 V. This is a drop of 1.28 V from the expected level. Quite obviously, therefore, as the current drawn from the supply increases, the terminal voltage continues to drop.

If we plot the terminal voltage versus current demand from 0 A to 275.34 mA, we obtain the plot in Fig. 5.67. Interestingly enough, it turns out to be a straight line that continues to drop with an increase in current demand. Note, in particular, that the curve begins at a current level of 0 A.

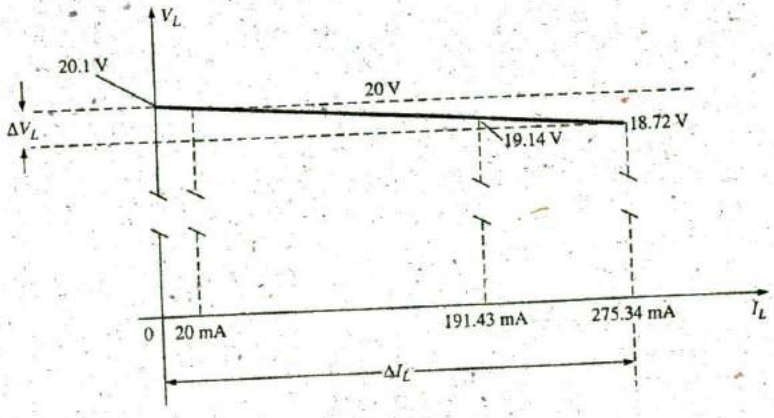

FIG. 5.67
Plotting V_L versus I_L for the supply in Fig. 5.66.

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Under no-load conditions, where the output terminals of the supply are not connected to any load, the current will be 0 A due to the absence of a complete circuit. The output voltage will be the internal ideal supply level of 20.1 V.

The slope of the line is defined by the internal resistance of the supply. That is,

$$\boxed{R_{int} = \frac{\Delta V_L}{\Delta I_L}} \qquad \text{(ohms, } \Omega\text{)} \tag{5.13}$$

which for the plot in Fig. 5.67 results in

$$R_{int} = \frac{\Delta V_L}{\Delta I_L} = \frac{20.1\text{ V} - 18.72\text{ V}}{275.34\text{ mA} - 0\text{ mA}} = \frac{1.38\text{ V}}{275.34\text{ mA}} = 5\ \Omega$$

For supplies of any kind, the plot of particular importance is the output voltage versus current drawn from the supply, as shown in Fig. 5.68(a). Note that the maximum value is achieved under no-load (NL) conditions as defined by Fig. 5.68(b) and the description above. Full-load (FL) conditions are defined by the maximum current the supply can provide on a continuous basis, as shown in Fig. 5.68(c).

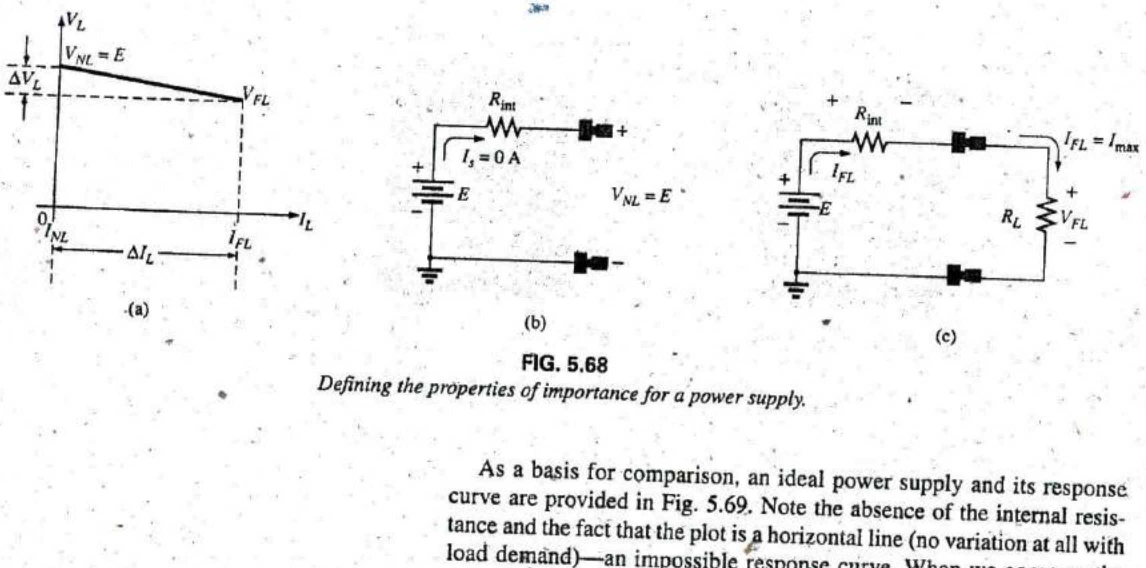


FIG. 5.68
Defining the properties of importance for a power supply.

As a basis for comparison, an ideal power supply and its response curve are provided in Fig. 5.69. Note the absence of the internal resistance and the fact that the plot is a horizontal line (no variation at all with load demand)—an impossible response curve. When we compare the

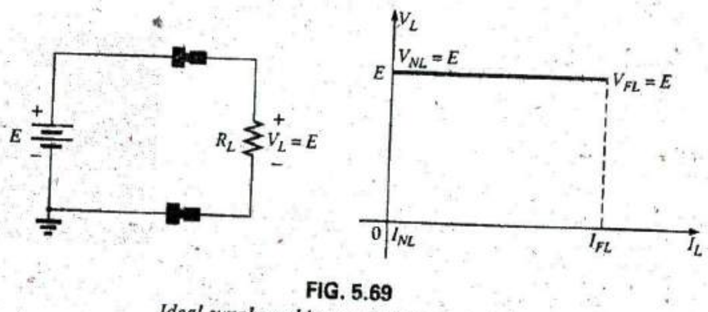


FIG. 5.69
Ideal supply and its terminal characteristics.

curve in Fig. 5.69 with that in Fig. 5.68(a), however, we now realize that the *steeper the slope,* the more sensitive is the supply to the change in load and therefore the *less desirable* it is for many laboratory procedures. In fact,

the larger the internal resistance, the steeper is the drop in voltage with an increase in load demand (current).

To help us anticipate the expected response of a supply, a defining quantity called **voltage regulation** (abbreviated *VR*; often called *load regulation* on specification sheets) was established. The basic equation in terms of the quantities in Fig. 5.68(a) is the following:

$$VR = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100\% \qquad (5.14)$$

The examples to follow demonstrate that

the smaller the voltage or load regulation of a supply, the less will the terminal voltage change with increasing levels of current demand.

For the supply above with a no-load voltage of 20.1 V and a full-load voltage of 18.72 V, at 275.34 mA the voltage regulation is

$$VR = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100\% = \frac{20.1\text{ V} - 18.72\text{ V}}{18.72\text{ V}} \times 100\% \cong \mathbf{7.37\%}$$

which is quite high, revealing that we have a very sensitive supply. Most modern commercial supplies have regulation factors less than 1%, with 0.01% being very typical.

EXAMPLE 5.28

a. Given the characteristics in Fig. 5.70, determine the voltage regulation of the supply.
b. Determine the internal resistance of the supply.
c. Sketch the equivalent circuit for the supply.

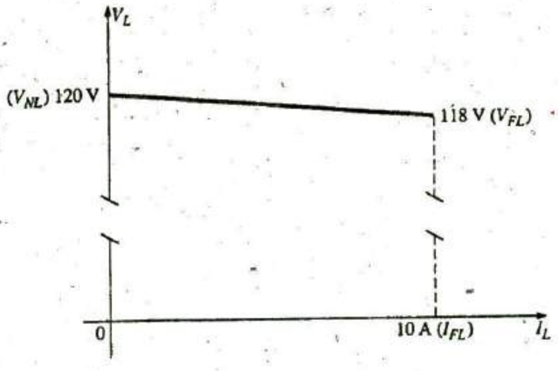


FIG. 5.70
Terminal characteristics for the supply of Example 5.28.

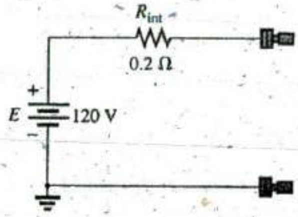


FIG. 5.71
dc supply with the terminal characteristics of Fig. 5.70.

Solutions:

a. $VR = \dfrac{V_{NL} - V_{FL}}{V_{FL}} \times 100\%$

$= \dfrac{120\text{ V} - 118\text{ V}}{118\text{ V}} \times 100\% = \dfrac{2}{118} \times 100\%$

$VR \cong \mathbf{1.7\%}$

b. $R_{int} = \dfrac{\Delta V_L}{\Delta I_L} = \dfrac{120\text{ V} - 118\text{ V}}{10\text{ A} - 0\text{ A}} = \dfrac{2\text{ V}}{10\text{ A}} = \mathbf{0.2\ \Omega}$

c. See Fig. 5.71.

EXAMPLE 5.29 Given a 60 V supply with a voltage regulation of 2%:

a. Determine the terminal voltage of the supply under full-load conditions.
b. If the half-load current is 5 A, determine the internal resistance of the supply.
c. Sketch the curve of the terminal voltage versus load demand and the equivalent circuit for the supply.

Solutions:

a.
$$VR = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100\%$$
$$2\% = \frac{60\text{ V} - V_{FL}}{V_{FL}} \times 100\%$$
$$\frac{2\%}{100\%} = \frac{60\text{ V} - V_{FL}}{V_{FL}}$$
$$0.02V_{FL} = 60\text{ V} - V_{FL}$$
$$1.02V_{FL} = 60\text{ V}$$
$$V_{FL} = \frac{60\text{ V}}{1.02} = \mathbf{58.82\ V}$$

b. $I_{FL} = 10\text{ A}$

$$R_{int} = \frac{\Delta V_L}{\Delta I_L} = \frac{60\text{ V} - 58.82\text{ V}}{10\text{ A} - 0\text{ A}} = \frac{1.18\text{ V}}{10\text{ A}} \cong \mathbf{0.12\ \Omega}$$

c. See Fig. 5.72.

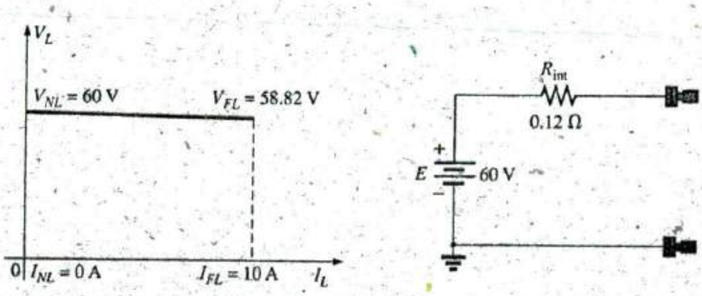


FIG. 5.72
Characteristics and equivalent circuit for the supply of Example 5.29.

5.11 LOADING EFFECTS OF INSTRUMENTS

In the previous section, we learned that power supplies are not the ideal instruments we may have thought they were. The applied load can have an effect on the terminal voltage. Fortunately, since today's supplies have such small load regulation factors, the change in terminal voltage with load can usually be ignored for most applications. If we now turn our attention to the various meters we use in the lab, we again find that they are not totally ideal:

Whenever you apply a meter to a circuit, you change the circuit and the response of the system. Fortunately, however, for most applications, considering the meters to be ideal is a valid approximation as long as certain factors are considered.

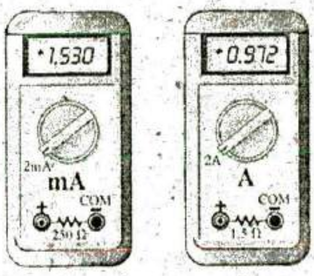
For instance,

any ammeter connected in a series circuit will introduce resistance to the series combination that will affect the current and voltages of the configuration.

The resistance between the terminals of an ammeter is determined by the chosen scale of the ammeter. In general,

for ammeters, the higher the maximum value of the current for a particular scale, the smaller will the internal resistance be.

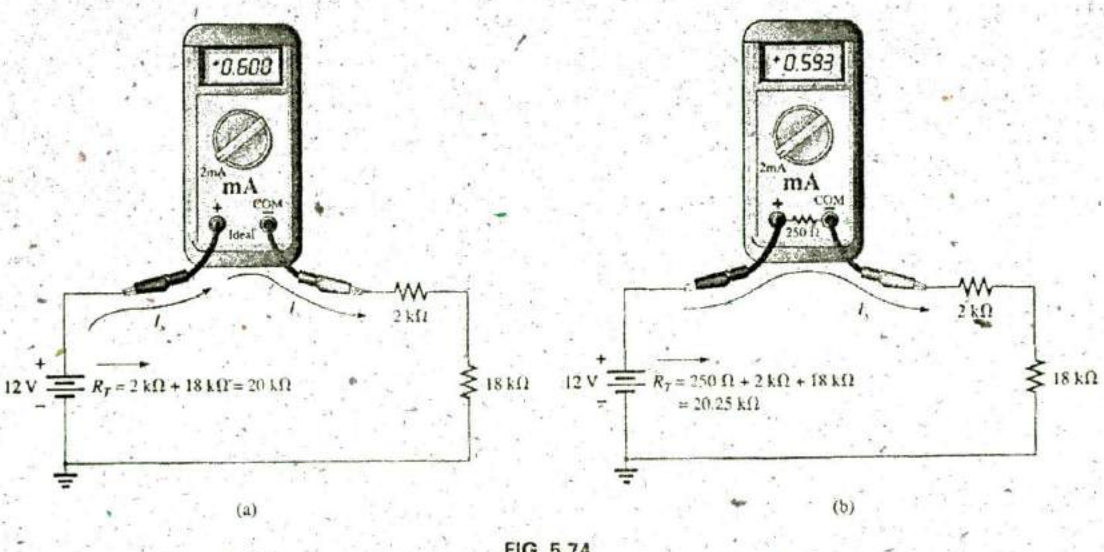
For example, it is not uncommon for the resistance between the terminals of an ammeter to be 250 Ω for a 2 mA scale but only 1.5 Ω for the 2 A scale, as shown in Fig. 5.73(a) and (b). If you are analyzing a circuit in detail, you can include the internal resistance as shown in Fig. 5.73 as a resistor between the two terminals of the meter.



(a) (b)

FIG. 5.73
Including the effects of the internal resistance of an ammeter: (a) 2 mA scale; (b) 2 A scale.

At first reading, such resistance levels at low currents give the impression that ammeters are far from ideal, and that they should be used only to obtain a general idea of the current and should not be expected to provide a true reading. Fortunately, however, when you are reading currents below the 2 mA range, the resistors in series with the ammeter are typically in the kilohm range. For example, in Fig. 5.74(a), for an ideal ammeter, the current



(a) (b)

FIG. 5.74
Applying an ammeter set on the 2 mA scale to a circuit with resistors in the kilohm range: (a) ideal; (b) practical.

displayed is 0.6 mA as determined from $I_s = E/R_T = 12\ \text{V}/20\ \text{k}\Omega = 0.6$ mA. If we now insert a meter with an internal resistance of 250 Ω as shown in Fig. 5.74(b), the additional resistance in the circuit will drop the current to 0.593 mA as determined from $I_s = E/R_T = 12\ \text{V}/20.25\ \text{k}\Omega = 0.593$ mA. Now, certainly the current has dropped from the ideal level, but the difference in results is only about 1%—nothing major, and the measurement can be used for most purposes. If the series resistors were in the same range as the 250 Ω resistors, we would have a different problem, and we would have to look at the results very carefully.

Let us go back to Fig. 5.20 and determine the actual current if each meter on the 2 A scale has an internal resistance of 1.5 Ω. The fact that there are four meters will result in an additional resistance of $(4)(1.5\ \Omega) = 6\ \Omega$ in the circuit, and the current will be $I_s = E/R_T = 8.4\ \text{V}/146\ \Omega \cong 58$ mA, rather than the 60 mA under ideal conditions. This value is still close enough to be considered a helpful reading. However, keep in mind that if we were measuring the current in the circuit, we would use only one ammeter, and the current would be $I_s = E/R_T = 8.4\ \text{V}/141.5\ \Omega \cong 59$ mA, which can certainly be approximated as 60 mA.

In general, therefore, be aware that this internal resistance must be factored in, but for the reasons just described, most readings can be used as an excellent first approximation to the actual current.

It should be added that because of this *insertion problem* with ammeters, and because of the very important fact that the *circuit must be disturbed* to measure a current, ammeters are not used as much as you might initially expect. Rather than break a circuit to insert a meter, the voltage across a resistor is often measured and the current then calculated using Ohm's law. This eliminates the need to worry about the level of the meter resistance and having to disturb the circuit. Another option is to use the clamp-type ammeters introduced in Chapter 2, removing the concerns about insertion loss and disturbing the circuit. Of course, for many practical applications (such as on power supplies), it is convenient to have an ammeter permanently installed so that the current can quickly be read from the display. In such cases, however, the design is such as to compensate for the insertion loss.

In summary, therefore, keep in mind that the insertion of an ammeter will add resistance to the branch and will affect the current and voltage levels. However, in most cases the effect is minimal, and the reading will provide a good first approximation to the actual level.

The loading effect of voltmeters is discussed in detail in the next chapter because loading is not a series effect. In general, however, the results will be similar in many ways to those of the ammeter, but the major difference is that the circuit does not have to be disturbed to apply the meter.

5.12 PROTOBOARDS (BREADBOARDS)

At some point in the design of any electrical/electronic system, a prototype must be built and tested. One of the most effective ways to build a testing model is to use the **protoboard** (in the past most commonly called a **breadboard**) in Fig. 5.75. It permits a direct connection of the power supply and provides a convenient method for holding and connecting the components. There isn't a great deal to learn about the protoboard, but it is important to point out some of its characteristics, including the way the elements are typically connected.

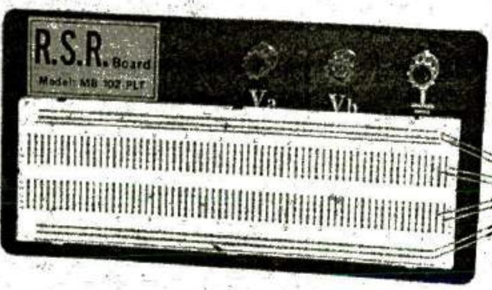


FIG. 5.75
Protoboard with areas of conductivity defined using two different approaches.

The red terminal V_a is connected directly to the positive terminal of the dc power supply, with the black lead V_b connected to the negative terminal and the green terminal used for the ground connection. Under the hole pattern, there are continuous horizontal copper strips under the top and bottom rows, as shown by the copper bands in Fig. 5.75. In the center region, the conductive strips are vertical but do not extend beyond the deep notch running the horizontal length of the board. That's all there is to it, although it will take some practice to make the most effective use of the conductive patterns.

As examples, the network in Fig. 5.12 is connected on the protoboard in the photo in Fig. 5.76 using *two different approaches.* After the dc power supply has been hooked up, a lead is brought down from the positive red terminal to the top conductive strip marked "+." Keep in mind that now the entire strip is connected to the positive terminal of the supply. The negative terminal is connected to the bottom strip marked with a minus sign (−), so that 8.4 V can be read at any point between the top positive strip and the bottom negative strip. A ground connection to the negative terminal of the battery was made at the site of the three terminals. For the user's convenience, kits are available in

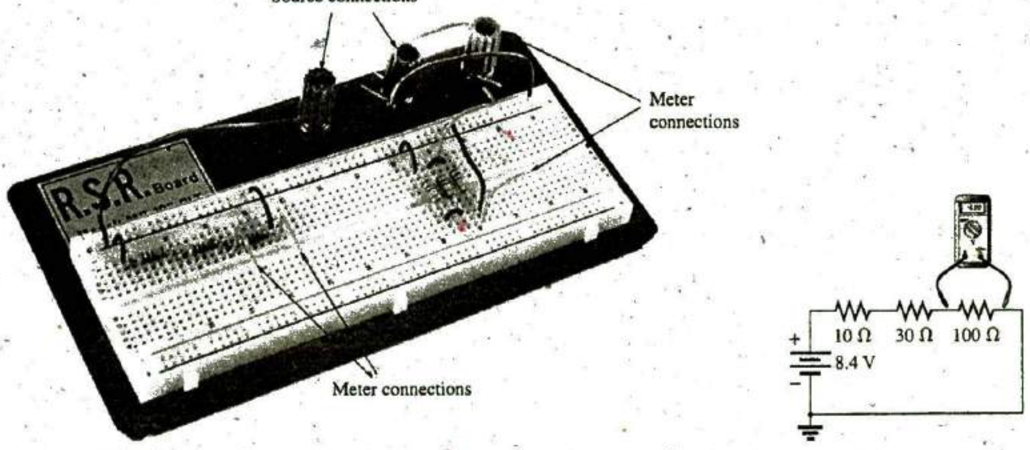


FIG. 5.76
Two setups for the network in Fig. 5.12 on a protoboard with yellow leads added to each configuration to measure voltage V_3 with a voltmeter.

which the length of the wires is color coded. Otherwise, a spool of 24 gage wire is cut to length and the ends are stripped. In general, feel free to use the extra length—everything doesn't have to be at right angles. For most protoboards, 1/4 through 1 W resistors will insert nicely in the board. For clarity, 1/2 W resistors are used in Fig. 5.76. The voltage across any component can be easily read by inserting additional leads as shown in the figure (yellow leads) for the voltage V_3 of each configuration (the yellow wires) and attaching the meter. For any network, the components can be wired in a variety of ways. Note in the configuration on the right that the horizontal break through the center of the board was used to isolate the two terminals of each resistor. Even though there are no set standards, it is important that the arrangement can *easily be understood* by someone else.

Additional setups using the protoboard are in the chapters to follow so that you can become accustomed to the manner in which it is used most effectively. You will probably see the protoboard quite frequently in your laboratory sessions or in an industrial setting.

5.13 APPLICATIONS

Before looking at a few applications, we need to consider a few general characteristics of the series configuration that you should always keep in mind when designing a system. First, and probably the most important, is that

if one element of a series combination of elements should fail, it will disrupt the response of all the series elements. If an open circuit occurs, the current will be zero. If a short circuit results, the voltage will increase across the other elements, and the current will increase in magnitude.

Second, and a thought you should always keep in mind, is that

for the same source voltage, the more elements you place in series, the less is the current and the less is the voltage across all the elements of the series combination.

Last, and a result discussed in detail in this chapter, is that

the current is the same for each element of a series combination, but the voltage across each element is a function of its terminal resistance.

There are other characteristics of importance that you will learn as you investigate possible areas of application, but the above are the most important.

Series Control

One common use of the series configuration is in setting up a system that ensures that everything is in place before full power is applied. In Fig. 5.77, various sensing mechanisms can be tied to series switches, preventing power to the load until all the switches are in the closed or on position. For instance, as shown in Fig. 5.77, one component may test the environment for dangers such as gases, high temperatures, and so on. The next component may be sensitive to the properties of the system to be energized to be sure all components are working. Security is

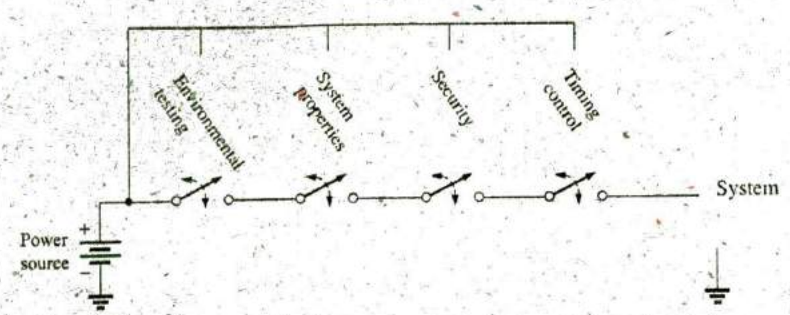


FIG. 5.77
Series control over an operating system.

another factor in the series sequence, and finally a timing mechanism may be present to ensure limited hours of operation or to restrict operating periods. The list is endless, but the fact remains that "all systems must be go" before power reaches the operating system.

Holiday Lights

In recent years, the small blinking holiday lights with 50 to 100 bulbs on a string have become very popular [see Fig. 5.78(a)]. Although holiday lights can be connected in series or parallel (to be described in the next chapter), the smaller blinking light sets are normally connected in series. It is relatively easy to determine if the lights are connected in series. If one wire enters and leaves the bulb casing, they are in series. If two wires enter and leave, they are probably in parallel. *Normally, when bulbs are connected in series, if one burns out (the filament breaks and the circuit opens), all the bulbs go out since the current path has been interrupted.* However, the bulbs in Fig. 5.78(a) are specially designed, as shown in Fig. 5.78(b), to permit current to continue to flow to the other bulbs when the filament burns out. At the base of each bulb, there is a fuse link wrapped around the two posts holding the filament. The fuse link of a soft conducting metal appears to be touching the two vertical posts, but in fact a coating on the posts or fuse link prevents conduction

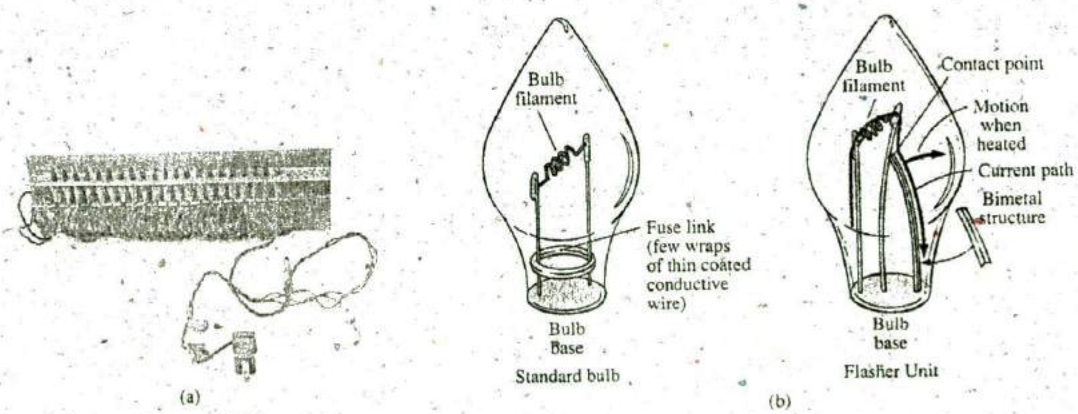


FIG. 5.78
Holiday lights: (a) 50-unit set; (b) bulb construction.

from one to the other under normal operating conditions. If a filament should burn out and create an open circuit between the posts, the current through the bulb and other bulbs would be interrupted if it were not for the fuse link. At the instant a bulb opens up, current through the circuit is zero, and the full 120 V from the outlet appears across the bad bulb. This high voltage from post to post of a single bulb is of sufficient potential difference to establish current through the insulating coatings and spot-weld the fuse link to the two posts. The circuit is again complete, and all the bulbs light except the one with the activated fuse link. Keep in mind, however, that each time a bulb burns out, there is more voltage across the other bulbs of the circuit, making them burn brighter. Eventually, if too many bulbs burn out, the voltage reaches a point where the other bulbs burn out in rapid succession. To prevent this, you must replace burned-out bulbs at the earliest opportunity.

The bulbs in Fig. 5.78(b) are rated 2.5 V at 0.2 A or 200 mA. Since there are 50 bulbs in series, the total voltage across the bulbs will be 50 × 2.5 V or 125 V, which matches the voltage available at the typical home outlet. Since the bulbs are in series, the current through each bulb will be 200 mA. The power rating of each bulb is therefore $P = VI = (2.5\text{ V})(0.2\text{ A}) = 0.5\text{ W}$ with a total wattage demand of $50 \times 0.5\text{ W} = 25\text{ W}$.

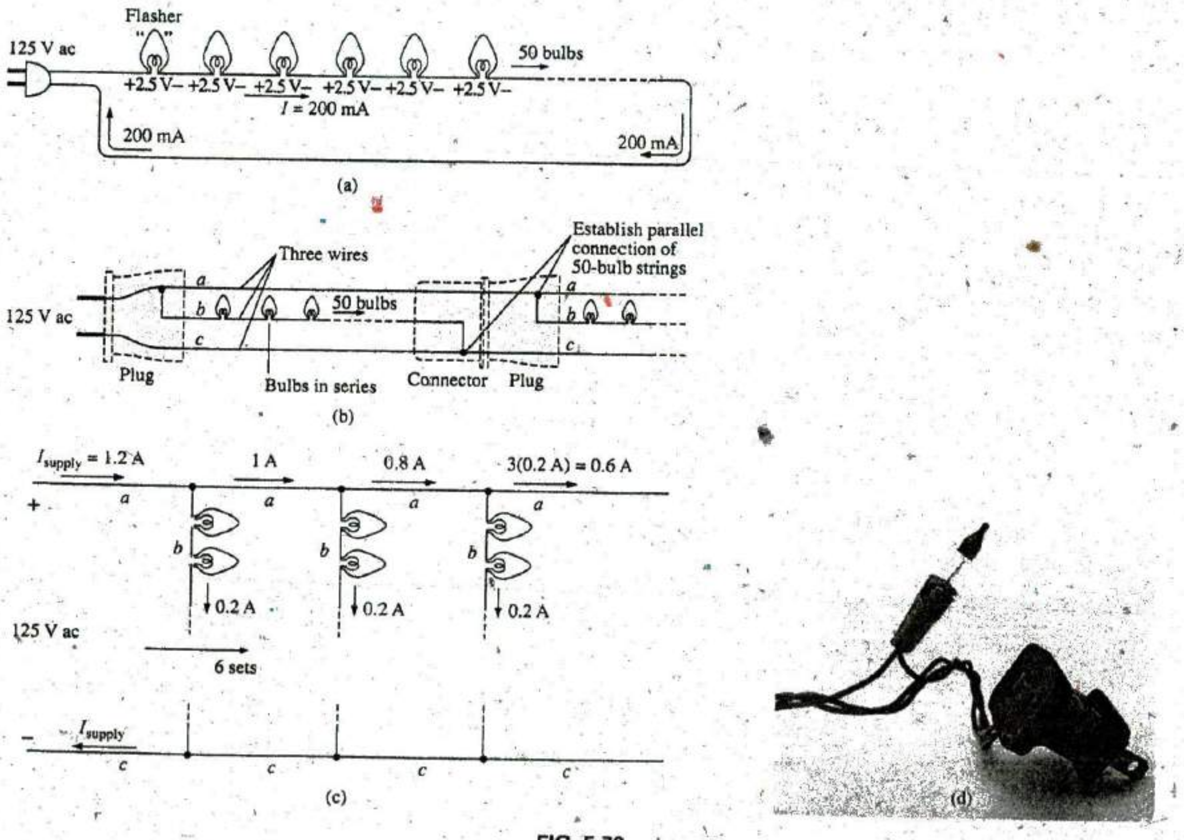


FIG. 5.79
(a) Single-set wiring diagram; (b) special wiring arrangement; (c) redrawn schematic; (d) special plug and flasher unit.

A schematic representation for the set of Fig. 5.78(a) is provided in Fig. 5.79(a). Note that only one flasher unit is required. Since the bulbs are in series, when the flasher unit interrupts the current flow, it turns off all the bulbs. As shown in Fig. 5.78(b), the flasher unit incorporates a bimetal thermal switch that opens when heated by the current to a preset level. As soon as it opens, it begins to cool down and closes again so that current can return to the bulbs. It then heats up again, opens up, and repeats the entire process. The result is an on-and-off action that creates the flashing pattern we are so familiar with. Naturally, in a colder climate (for example, outside in the snow and ice), it initially takes longer to heat up, so the flashing pattern is slow at first, but as the bulbs warm up, the frequency increases.

The manufacturer specifies that no more than six sets should be connected together. How can you connect the sets together, end to end, without reducing the voltage across each bulb and making all the lights dimmer? If you look closely at the wiring, you will find that since the bulbs are connected in series, there is one wire to each bulb with additional wires from plug to plug. Why would they need two additional wires if the bulbs are connected in series? Because when each set is connected together, they are actually in a parallel arrangement (to be discussed in the next chapter). This unique wiring arrangement is shown in Fig. 5.79(b) and redrawn in Fig. 5.79(c). Note that the top line is the hot line to all the connected sets, and the bottom line is the return, neutral, or ground line for all the sets. Inside the plug in Fig. 5.79(d), the hot line and return are connected to each set, with the connections to the metal spades of the plug as shown in Fig. 5.79(b). We will find in the next chapter that the current drawn from the wall outlet for parallel loads is the sum of the current to each branch. The result, as shown in Fig. 5.79(c), is that the current drawn from the supply is $6 \times 200\text{ mA} = 1.2\text{ A}$, and the total wattage for all six sets is the product of the applied voltage and the source current or $(120\text{ V})(1.2\text{ A}) = 144\text{ W}$ with $144\text{ W}/6 = 24\text{ W}$ per set.

Microwave Oven

Series circuits can be very effective in the design of safety equipment. Although we all recognize the usefulness of the microwave oven, it can be quite dangerous if the door is not closed or sealed properly. It is not enough to test the closure at only one point around the door because the door may be bent or distorted from continual use, and leakage can result at some point distant from the test point. One common safety arrangement appears in Fig. 5.80. Note that magnetic switches are located all around the door, with the magnet in the door itself and the magnetic door switch in the main frame. Magnetic switches are simply switches where the magnet draws a magnetic conducting bar between two contacts to complete the circuit—somewhat revealed by the symbol for the device in the circuit diagram in Fig. 5.80. Since the magnetic switches are all in series, they must all be closed to complete the circuit and turn on the power unit. If the door is sufficiently out of shape to prevent a single magnet from getting close enough to the switching mechanism, the circuit will not be complete, and the power cannot be turned on. Within the control unit of the power supply, either the series circuit completes a circuit for operation or a sensing current is established and monitored that controls the system operation.

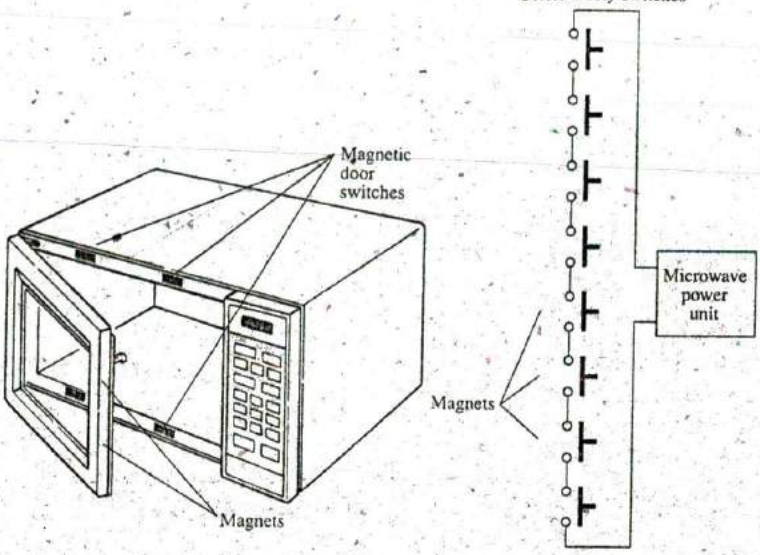


FIG. 5.80
Series safety switches in a microwave oven.

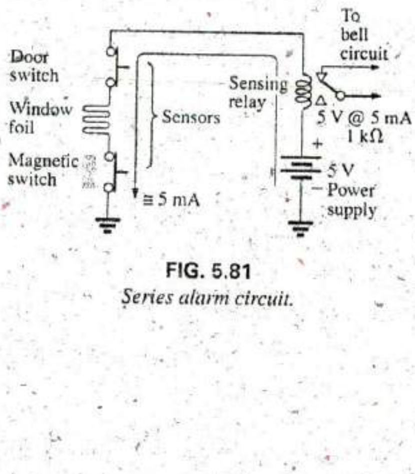


FIG. 5.81
Series alarm circuit.

Series Alarm Circuit

The circuit in Fig. 5.81 is a simple alarm circuit. Note that every element of the design is in a series configuration. The power supply is a 5 V dc supply that can be provided through a design similar to that in Fig. 2.33, a dc battery, or a combination of an ac and a dc supply that ensures that the battery will always be at full charge. If all the sensors are closed, a current of 5 mA results because of the terminal load of the relay of about 1 kΩ. That current energizes the relay and maintains an off position for the alarm. However, if any of the sensors is opened, the current will be interrupted, the relay will let go, and the alarm circuit will be energized. With relatively short wires and a few sensors, the system should work well since the voltage drop across each is minimal. However, since the alarm wire is usually relatively thin, resulting in a measurable resistance level, if the wire to the sensors is too long, a sufficient voltage drop could occur across the line, reducing the voltage across the relay to a point where the alarm fails to operate properly. Thus, wire length is a factor that must be considered if a series configuration is used. Proper sensitivity to the length of the line should remove any concerns about its operation. An improved design is described in Chapter 8.

5.14 COMPUTER ANALYSIS

PSpice

In Section 4.9, the basic procedure for setting up the PSpice folder and running the program were presented. Because of the detail provided in that section, you should review it before proceeding with this example.

Because this is only the second example using PSpice, some detail is provided, but not at the level of Section 4.9.

The circuit to be investigated appears in Fig. 5.82. You will use the **PSpice** folder established in Section 4.9. Double-clicking on the **OrCAD 10.0 DEMO/CAPTURE CIS** icon opens the window. A new project is initiated by selecting the **Create document** key at the top left of the screen (it looks like a page with a star in the upper left corner). The result is the **New Project** dialog box in which **PSpice 5–1** is entered as the **Name.** The **Analog or Mixed A/D** is already selected, and **PSpice** appears as the **Location.** Click **OK,** and the **Create PSpice Project** dialog box appears. Select **Create a blank project,** click **OK,** and the working windows appear. Grab the left edge of the **SCHEMATIC1: PAGE1** window to move it to the right so that you can see both screens. Clicking the + sign in the **Project Manager** window allows you to set the sequence down to **PAGE1.** You can change the name of the **SCHEMATIC1** by selecting it and right-clicking. Choose **Rename** from the list. Enter **PSpice 5–1** in the **Rename Schematic** dialog box. In Fig. 5.83 it was left as **SCHEMATIC1.**

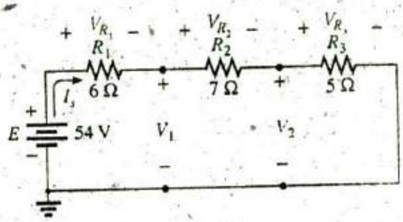


FIG. 5.82
Series dc network to be analyzed using PSpice.

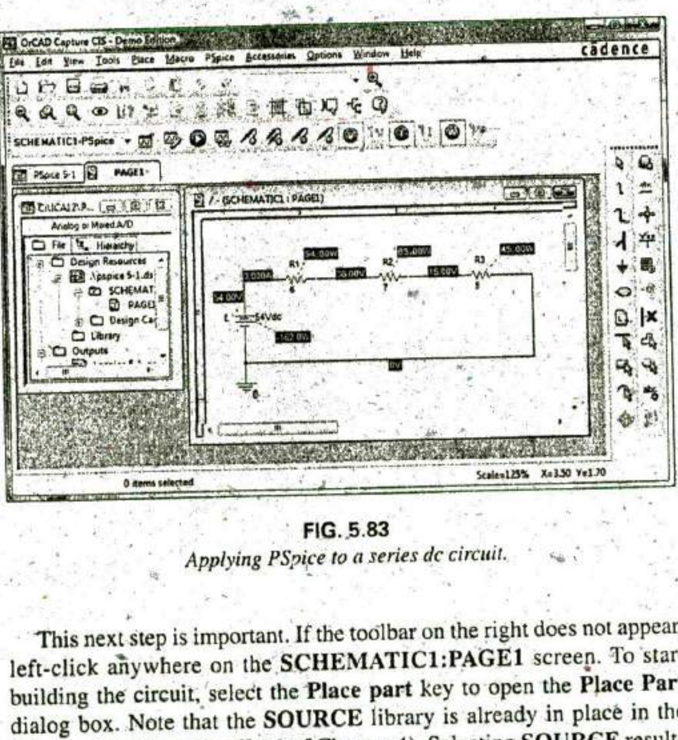


FIG. 5.83
Applying PSpice to a series dc circuit.

This next step is important. If the toolbar on the right does not appear, left-click anywhere on the **SCHEMATIC1:PAGE1** screen. To start building the circuit, select the **Place part** key to open the **Place Part** dialog box. Note that the **SOURCE** library is already in place in the **Library** list (from the efforts of Chapter 4). Selecting **SOURCE** results in the list of sources under **Part List,** and **VDC** can be selected. Click **OK,** and the cursor can put it in place with a single left click. Right-click and select **End Mode** to end the process since the network has only one source. One more left click, and the source is in place. Select the **Place a Part** key again, followed by **ANALOG** library to find the resistor **R.** Once the resistor has been selected, click **OK** to place it next to the cursor on the screen. This time, since three resistors need to be placed, there is no need to go to **End Mode** between depositing each. Simply click one in place, then the next, and finally the third. Then right-click to end the process with **End Mode.** Finally, add a **GND** by selecting the ground key from the right toolbar and selecting **0/SOURCE** in the **Place Ground** dialog box. Click **OK,** and place the ground as shown in Fig. 5.83.

Connect the elements by using the **Place a wire** key to obtain the crosshair on the screen. Start at the top of the voltage source with a left click, and draw the wire, left-clicking it at every 90° turn. When a wire is connected from one element to another, move on to the next connection to be made—there is no need to go **End Mode** between connections. Now set the labels and values by double-clicking on each parameter to obtain a **Display Properties** dialog box. Since the dialog box appears with the quantity of interest in a blue background, type in the desired label or value, followed by **OK.** The network is now complete and ready to be analyzed.

Before simulation, select the **V, I,** and **W** in the toolbar at the top of the window to ensure that the voltages, currents, and power are displayed on the screen. To simulate, select the **New Simulation Profile** key (which appears as a data sheet on the second toolbar down with a star in the top left corner) to obtain the **New Simulation** dialog box. Enter **Bias Point** for a dc solution under **Name,** and hit the **Create** key. A **Simulation Settings-Bias Point** dialog box appears in which **Analysis** is selected and **Bias Point** is found under the **Analysis type** heading. Click **OK,** and then select the **Run PSpice** key (the blue arrow) to initiate the simulation. Exit the resulting screen. The resulting display (Fig. 5.83) shows the current is 3 A for the circuit with 15 V across R_3, and 36 V from a point between R_1 and R_2 to ground. The voltage across R_2 is 36 V − 15 V = 21 V, and the voltage across R_1 is 54 V − 36 V = 18 V. The power supplied or dissipated by each element is also listed.

Multisim

The construction of the network in Fig. 5.84 using Multisim is simply an extension of the procedure outlined in Chapter 4. For each resistive element or meter, the process is repeated. The label for each increases by one as additional resistors or meters are added. Remember from the discussion of Chapter 4 that you should add the meters before connecting the elements together because the meters take space and must be properly oriented. The current is determined by the **XMM1** ammeter and the voltages by **XMM2** through **XMM5.** Of particular importance, note that

in Multisim the meters are connected in exactly the same way they would be placed in an active circuit in the laboratory. Ammeters are in series with the branch in which the current is to be determined, and voltmeters are connected between the two points of interest (across resistors). In addition, for positive readings, ammeters are connected so that conventional current enters the positive terminal, and voltmeters are connected so that the point of higher potential is connected to the positive terminal.

The meter settings are made by double-clicking on the meter symbol on the schematic. In each case, **V** or **I** had to be chosen, but the horizontal line for dc analysis is the same for each. Again, you can select the **Set** key to see what it controls, but the default values of meter input resistance

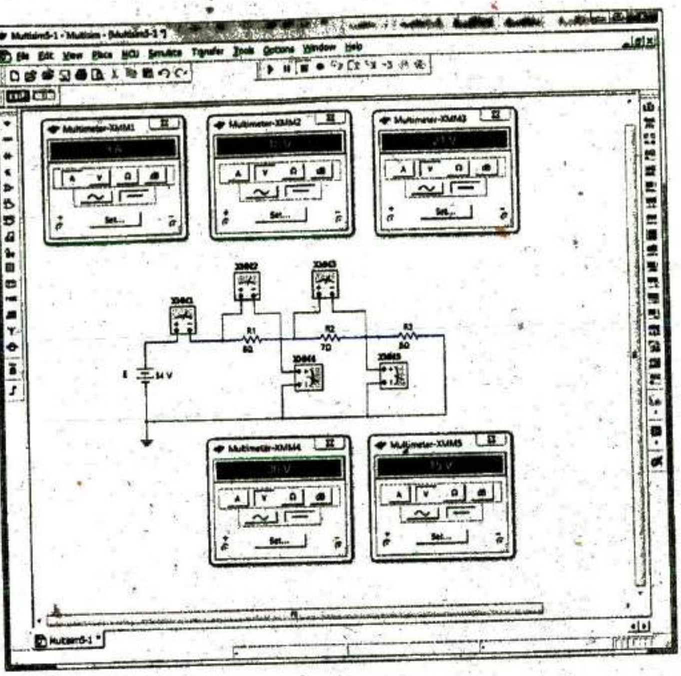


FIG. 5.84
Applying Multisim to a series dc circuit.

levels are fine for all the analyses described in this text. Leave the meters on the screen so that the various voltages and the current level will be displayed after the simulation.

Recall from Chapter 4 that elements can be moved by simply clicking on each schematic symbol and dragging it to the desired location. The same is true for labels and values. Labels and values are set by double-clicking on the label or value and entering your preference. Click **OK,** and they are changed on the schematic. You do not have to first select a special key to connect the elements. Simply bring the cursor to the starting point to generate the small circle and crosshair. Click on the starting point, and follow the desired path to the next connection path. When in the correct location, click again, and the line appears. All connecting lines can make 90° turns. However, you cannot follow a diagonal path from one point to another. To remove any element, label, or line, click on the quantity to obtain the four-square active status, and select the **Delete** key or the scissors key on the top menu bar.

Recall from Chapter 4 that you can initiate simulation through the sequence **Simulate-Run** by selecting the green **Run** key or switching the **Simulate Switch** to the **1** position.

Note from the results that the sum of the voltages measured by XMM2 and XMM4 equals the applied voltage. All the meters are considered ideal, so there is no voltage drop across the XMM1 ammeter. In addition, they do not affect the value of the current measured by XMM1. All the voltmeters have essentially infinite internal resistance, while the ammeters all have zero internal resistance. Of course, the meters can be entered as anything but ideal using the **Set** option. Note also that the sum of the voltages measured by XMM3 and XMM5 equals that measured by XMM4 as required by Kirchhoff's voltage law.

PROBLEMS

SECTION 5.2 Series Resistors

1. For each configuration in Fig. 5.85, find the individual (not combinations of) elements (voltage sources and/or resistors) that are in series. If necessary, use the fact that elements in series have the same current. Simply list those that satisfy the conditions for a series relationship. We will learn more about other combinations later.

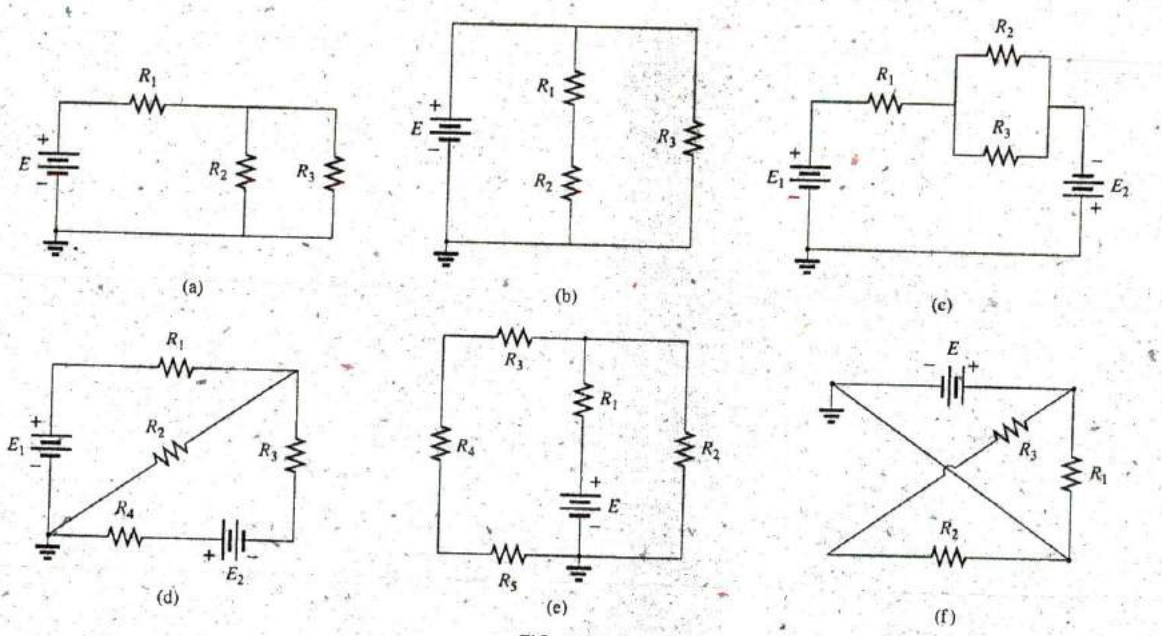


FIG. 5.85
Problem 1.

2. Find the total resistance R_T for each configuration in Fig. 5.86. Note that only standard resistor values were used.

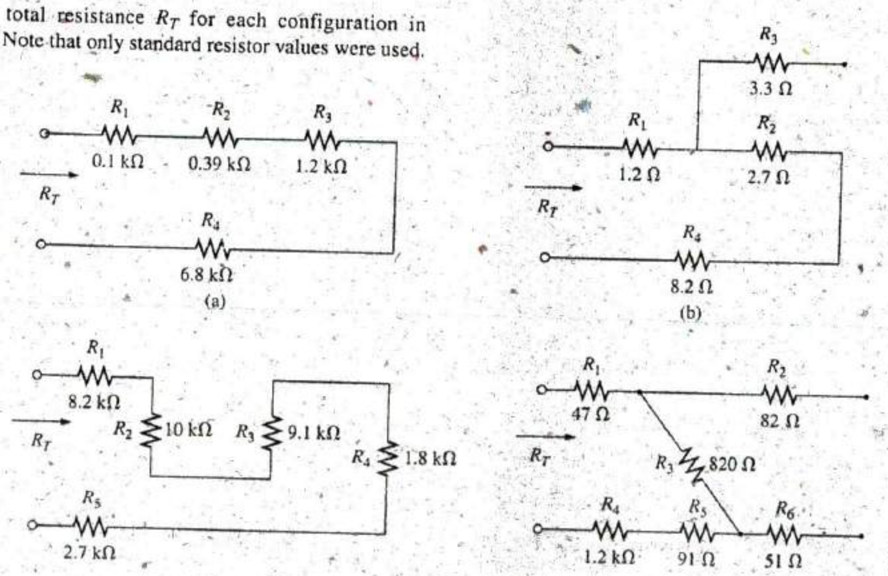


FIG. 5.86
Problem 2.

-S-

3. For each circuit board in Fig. 5.87, find the total resistance between connection tabs 1 and 2.

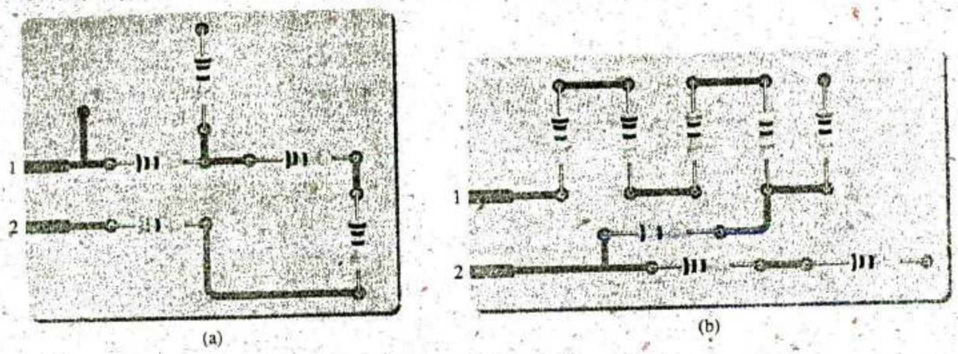


FIG. 5.87
Problem 3.

4. For the circuit in Fig. 5.88, composed of standard values:
   a. Which resistor will have the most impact on the total resistance?
   b. On an approximate basis, which resistors can be ignored when determining the total resistance?
   c. Find the total resistance, and comment on your results for parts (a) and (b).

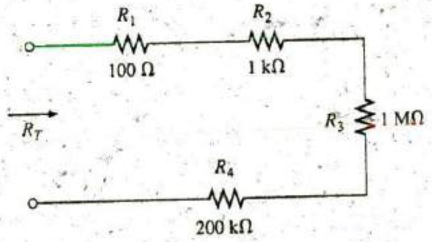


FIG. 5.88
Problem 4.

5. For each configuration in Fig. 5.89, determine the ohmmeter reading.

6. Find the resistance R, given the ohmmeter reading for each configuration of Fig. 5.90.

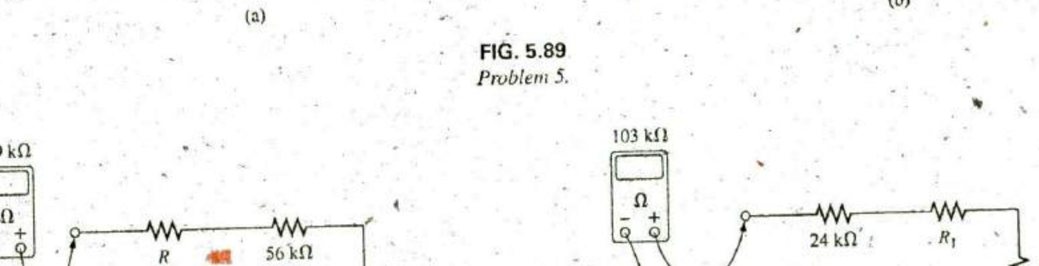


FIG. 5.89
Problem 5.

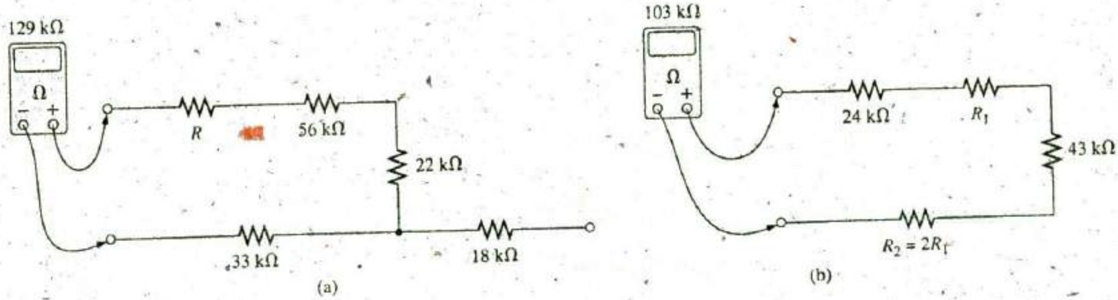


FIG. 5.90
Problem 6.

-S-

7. What is the ohmmeter reading for each configuration in Fig. 5.91?

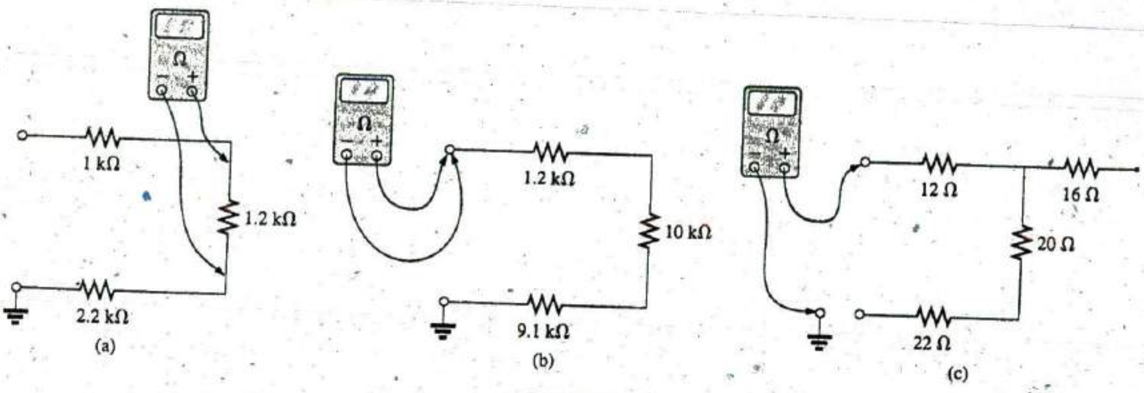


FIG. 5.91
Problem 7.

SECTION 5.3 Series Circuits

8. For the series configuration in Fig. 5.92, constructed of standard values:
   a. Find the total resistance.
   b. Calculate the current.
   c. Find the voltage across each resistive element.
   d. Calculate the power delivered by the source.
   e. Find the power delivered to the 18 Ω resistor.

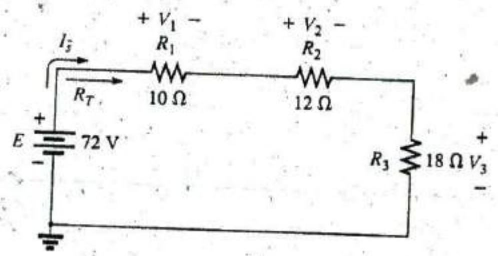


FIG. 5.92
Problem 8.

9. For the series configuration in Fig. 5.93, constructed using standard value resistors:
   a. Without making a single calculation, which resistive element will have the most voltage across it? Which will have the least?
   b. Which resistor will have the most impact on the total resistance and the resulting current? Find the total resistance and the current.
   c. Find the voltage across each element and review your response to part (a).

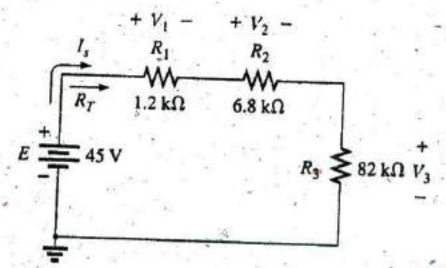


FIG. 5.93
Problem 9.

10. Find the applied voltage necessary to develop the current specified in each circuit in Fig. 5.94.
11. For each network in Fig. 5.95, constructed of standard values, determine:
    a. The current *I*.
    b. The source voltage *E*.
    c. The unknown resistance.
    d. The voltage across each element.
12. For each configuration in Fig. 5.96, what are the readings of the ammeter and the voltmeter?

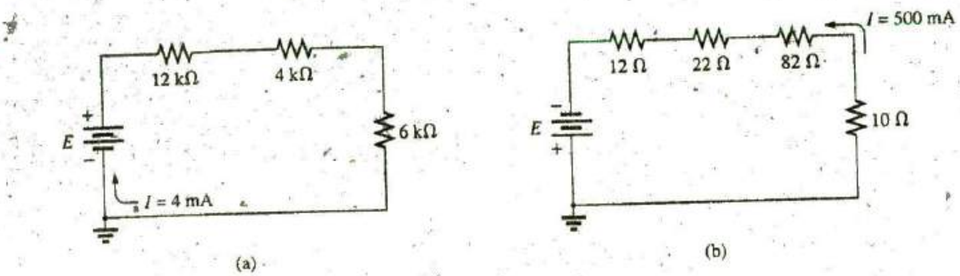


FIG. 5.94
Problem 10.

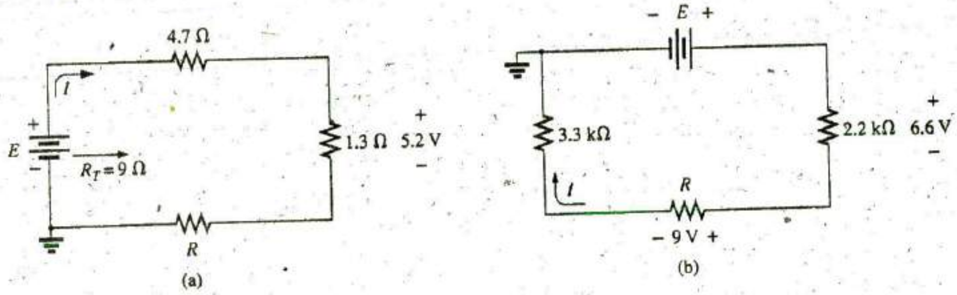


FIG. 5.95
Problem 11.

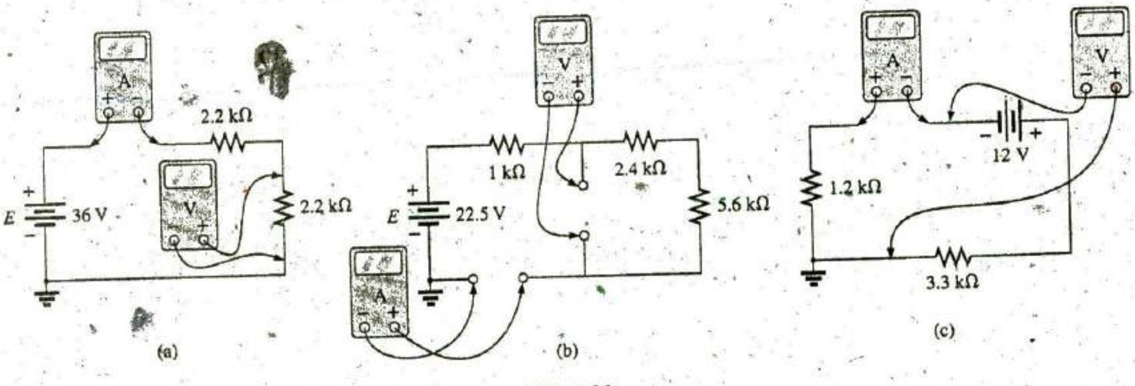


FIG. 5.96
Problem 12.

SECTION 5.4 Power Distribution in a Series Circuit

13. For the circuit in Fig. 5.97, constructed of standard value resistors:
    a. Find the total resistance, current, and voltage across each element.
    b. Find the power delivered to each resistor.
    c. Calculate the total power delivered to all the resistors.
    d. Find the power delivered by the source.
    e. How does the power delivered by the source compare to that delivered to all the resistors?
    f. Which resistor received the most power? Why?
    g. What happened to all the power delivered to the resistors?
    h. If the resistors are available with wattage ratings of 1/2 W, 1 W, 2 W, and 5 W, what minimum wattage rating can be used for each resistor?

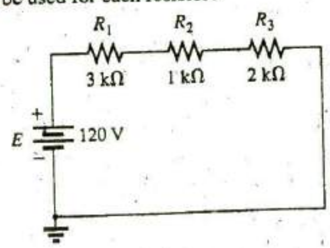


FIG. 5.97
Problem 13.

14. Find the unknown quantities for the circuit of Fig. 5.98 using the information provided.

15. Find the unknown quantities for the circuits in Fig. 5.99 using the information provided.

16. Eight holiday lights are connected in series as shown in Fig. 5.100.
   a. If the set is connected to a 120 V source, what is the current through the bulbs if each bulb has an internal resistance of 28⅛ Ω?
   b. Determine the power delivered to each bulb.
   c. Calculate the voltage drop across each bulb.
   d. If one bulb burns out (that is, the filament opens), what is the effect on the remaining bulbs? Why?

*17. For the conditions specified in Fig. 5.101, determine the unknown resistance.

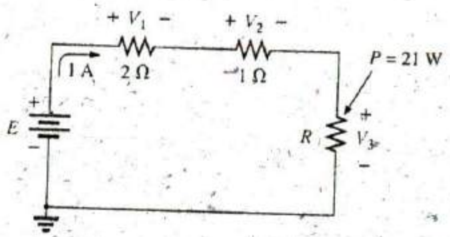


FIG. 5.98
Problem 14.

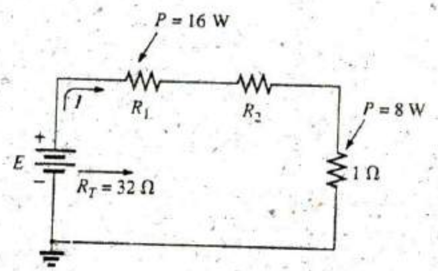


FIG. 5.99
Problem 15.

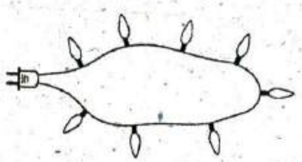


FIG. 5.100
Problem 16.

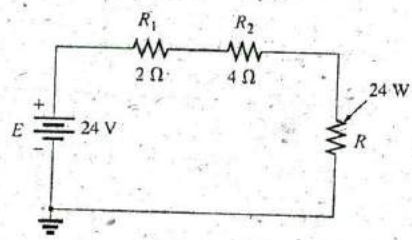


FIG. 5.101
Problem 17.

SECTION 5.5 Voltage Sources in Series

18. Combine the series voltage sources in Fig. 5.102 into a single voltage source between points a and b.

19. Determine the current I and its direction for each network in Fig. 5.103. Before solving for I, redraw each network with a single voltage source.

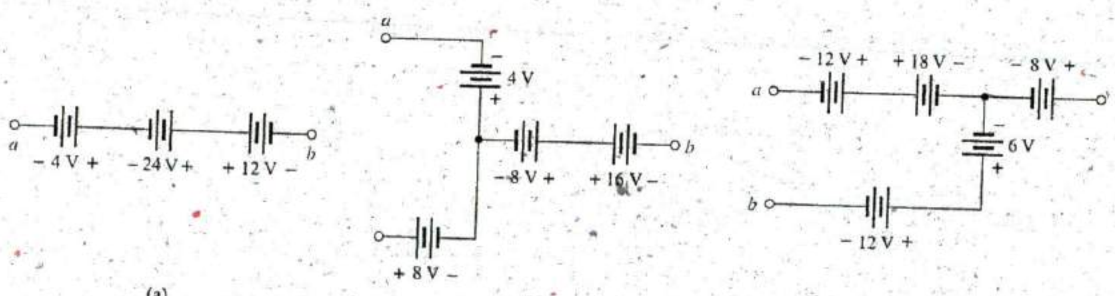


FIG. 5.102
Problem 18.

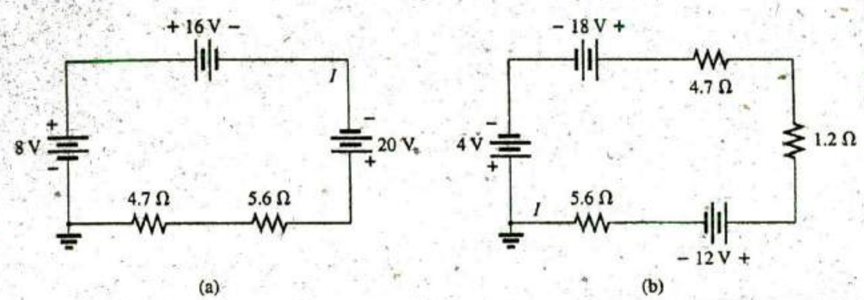


FIG. 5.103
Problem 19.

20. Find the unknown voltage source and resistor for the networks in Fig. 5.104. First combine the series voltage sources into a single source. Indicate the direction of the resulting current.

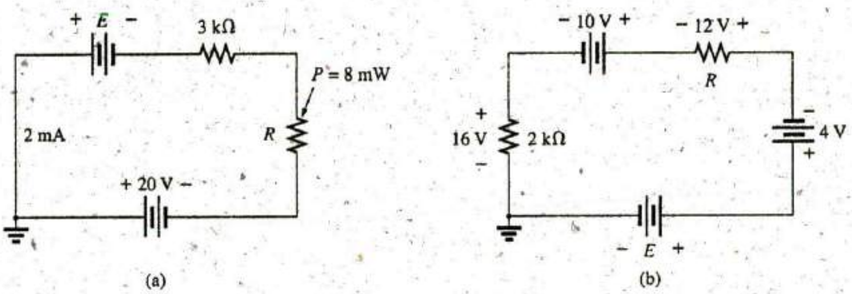


FIG. 5.104
Problem 20.

SECTION 5.6 Kirchhoff's Voltage Law

21. Using Kirchhoff's voltage law, find the unknown voltages for the circuits in Fig. 5.105.

22. **a.** Find the current I for the network of Fig. 5.106.
b. Find the voltage V_2.
c. Find the voltage V_1 using Kirchhoff's voltage law.

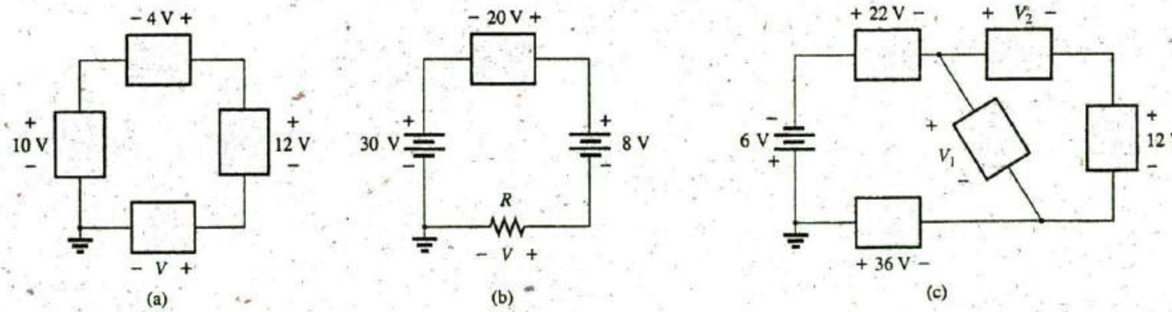


FIG. 5.105
Problem 21.

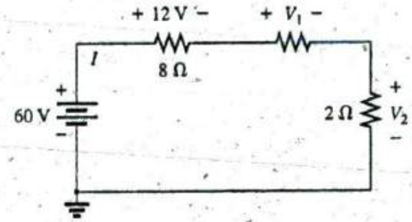


FIG. 5.106
Problem 22.

23. Using Kirchhoff's voltage law, determine the unknown voltages for the series circuits in Fig. 5.107.

24. Using Kirchhoff's voltage law, find the unknown voltages for the configurations in Fig. 5.108.

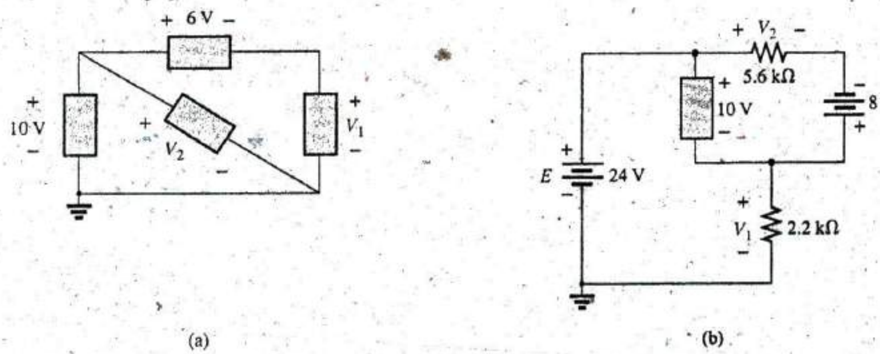


FIG. 5.107
Problem 23.

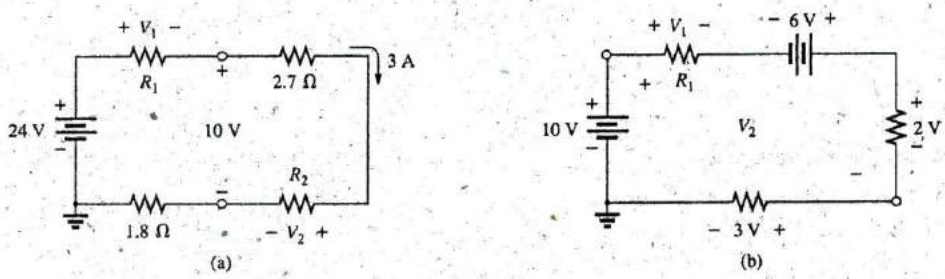


FIG. 5.108
Problem 24.

SECTION 5.7 Voltage Division in a Series Circuit

25. Determine the values of the unknown resistors in Fig. 5.109 using only the provided voltage levels. Do not calculate the current!

26. For the configuration in Fig. 5.110, with standard resistor values:
    a. By inspection, which resistor will receive the largest share of the applied voltage? Why?
    b. How much larger will voltage V_3 be compared to V_2 and V_1?
    c. Find the voltage across the largest resistor using the voltage divider rule.
    d. Find the voltage across the series combination of resistors R_2 and R_3.

SECTION 5.8 Voltage Divider Rule

27. Using the voltage divider rule, find the indicated voltages in Fig. 5.111.

28. Using the voltage divider rule or Kirchhoff's voltage law, determine the unknown voltages for the configurations in Fig. 5.112. Do not calculate the current!

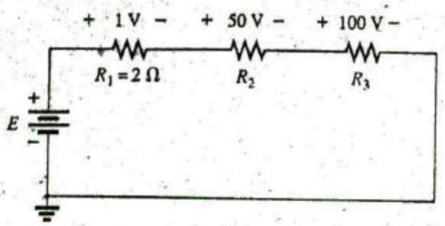


FIG. 5.109
Problem 25.

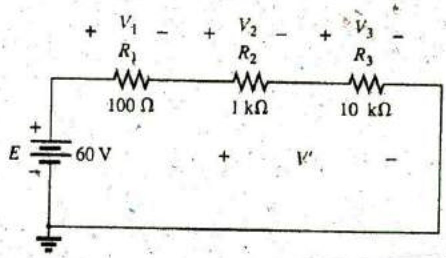


FIG. 5.110
Problem 26.

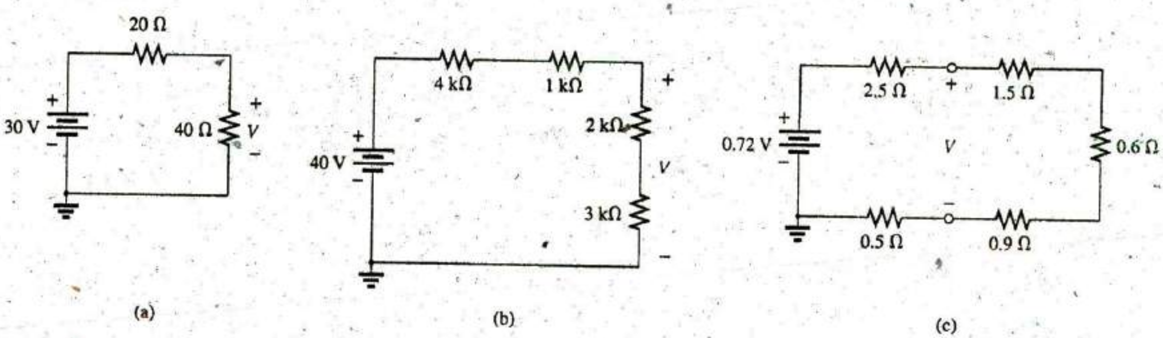


FIG. 5.111
Problem 27.

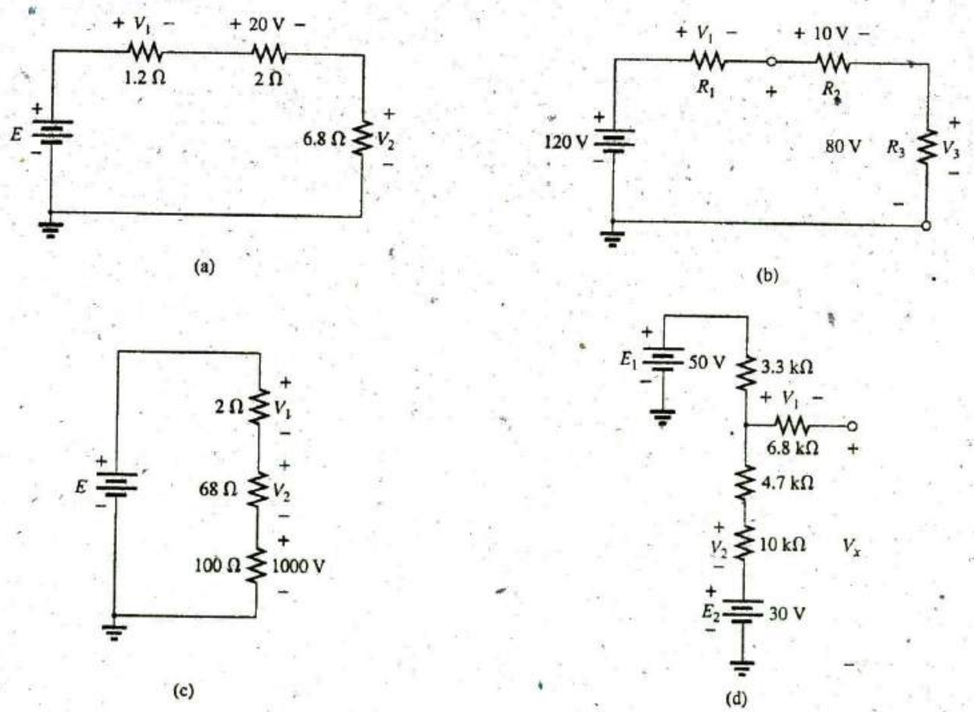


FIG. 5.112
Problem 28.

29. Using the information provided, find the unknown quantities of Fig. 5.113.

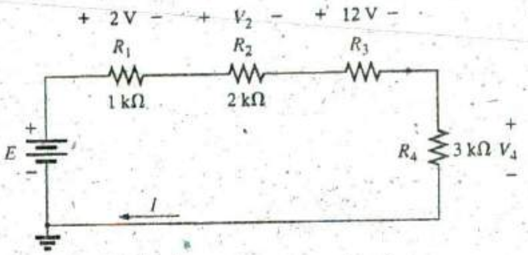


FIG. 5.113
Problem 29.

*30. Using the voltage divider rule, find the unknown resistance for the configurations in Fig. 5.114.

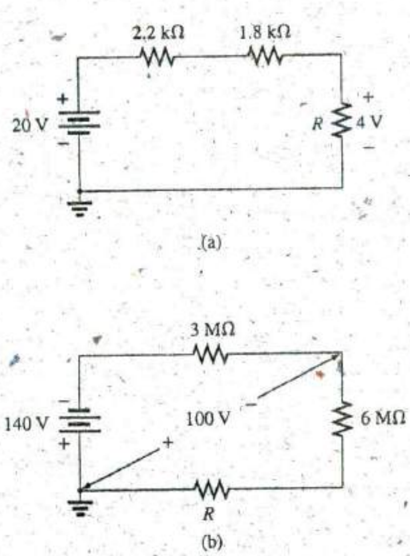


FIG. 5.114
Problem 30.

31. a. Design a voltage divider circuit that will permit the use of an 8 V, 50 mA bulb in an automobile with a 12 V electrical system.
    b. What is the minimum wattage rating of the chosen resistor if 1/4 W, 1/2 W, and 1 W resistors are available?

*32. Design the voltage divider in Fig. 5.115 such that $V_{R_1} = 1/5 V_{R_1}$. That is, find R_1 and R_2.

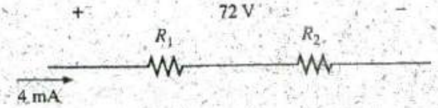


FIG. 5.115
Problem 32.

*33. Find the voltage across each resistor in Fig. 5.116 if $R_1 = 2R_3$ and $R_2 = 7R_3$.

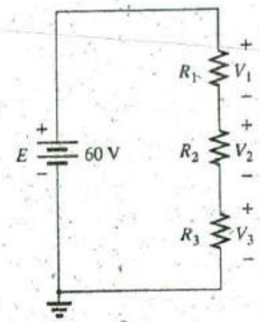


FIG. 5.116
Problem 33.

*34. a. Design the circuit in Fig. 5.117 such that $V_{R_2} = 3V_{R_1}$ and $V_{R_3} = 4V_{R_2}$.
    b. If the current is reduced to 10 μA, what are the new values of R_1, R_2, and R_3? How do they compare to the results of part (a)?

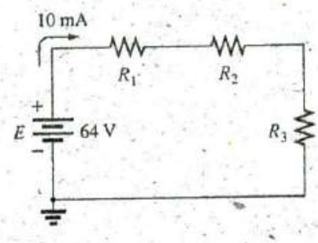


FIG. 5.117
Problem 34.

SECTION 5.10 Notation

35. Determine the voltages V_a, V_b, and V_{ab} for the networks in Fig. 5.118.

36. a. Determine the current I (with direction) and the voltage V (with polarity) for the networks in Fig. 5.119.
    b. Find the voltage V_a.

37. For the network in Fig. 5.120 determine the voltages:
    a. V_a, V_b, V_c, V_d, V_e
    b. V_{ab}, V_{dc}, V_{cb}
    c. V_{ac}, V_{db}

*38. Given the information appearing in Fig. 5.121, find the level of resistance for R_1 and R_3.

39. Determine the values of R_1, R_2, R_3, and R_4 for the voltage divider of Fig. 5.122 if the source current is 16 mA.

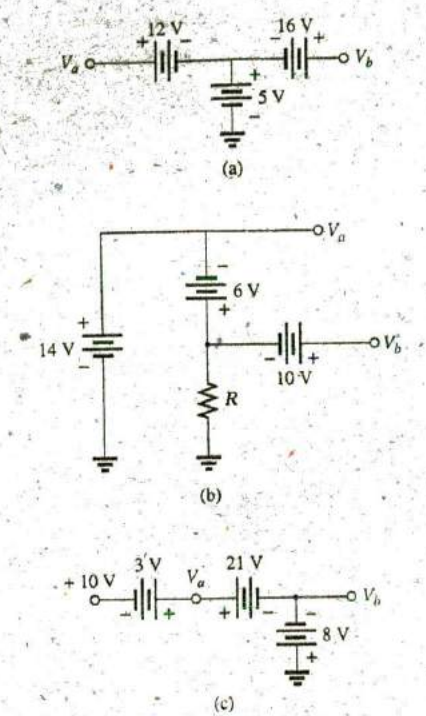


FIG. 5.118
Problem 35.

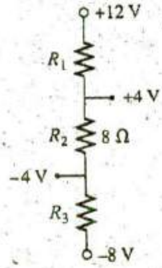


FIG. 5.121
Problem 38.

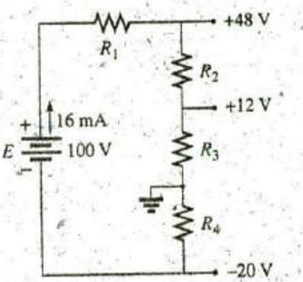


FIG. 5.122
Problem 39.

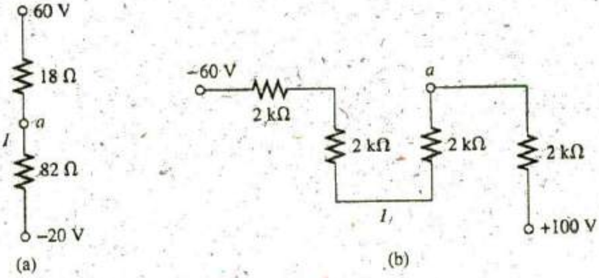


FIG. 5.119
Problem 36.

40. For the network in Fig. 5.123, determine the voltages:
   a. V_a, V_b, V_c, V_d
   b. V_{ab}, V_{cb}, V_{cd}
   c. V_{ad}, V_{ca}

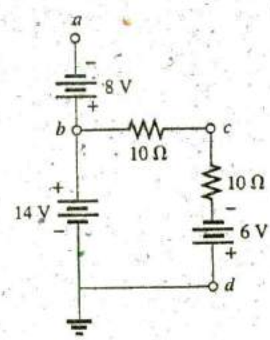


FIG. 5.123
Problem 40.

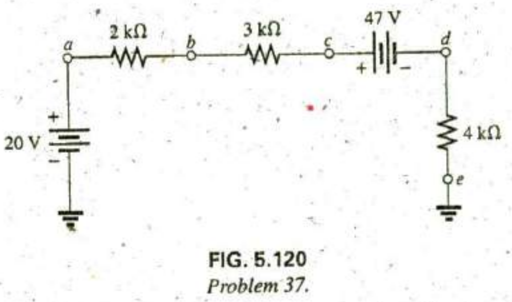


FIG. 5.120
Problem 37.

*41. For the integrated circuit in Fig. 5.124, determine V_0, V_4, V_7, V_{10}, V_{23}, V_{30}, V_{67}, V_{56}, and I (magnitude and direction.)

*42. For the integrated circuit in Fig. 5.125, determine V_0, V_{03}, V_2, V_{23}, V_{12}, and I_i.

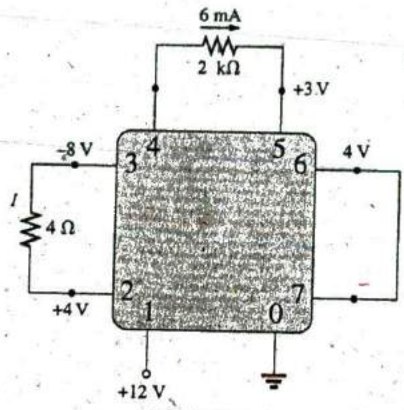


FIG. 5.124
Problem 41.

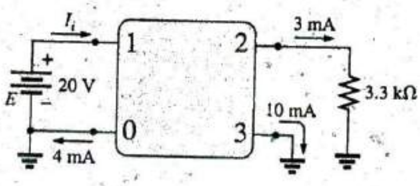


FIG. 5.125
Problem 42.

SECTION 5.9 Voltage Regulation and the Internal Resistance of Voltage Sources

43. **a.** Find the internal resistance of a battery that has a no-load output of 60 V and that supplies a full-load current of 2 A to a load of 28 Ω.
    b. Find the voltage regulation of the supply.

44. **a.** Find the voltage to the load (full-load conditions) for the supply in Fig. 5.126.
    b. Find the voltage regulation of the supply.
    c. How much power is supplied by the source and lost to the internal resistance under full-load conditions?

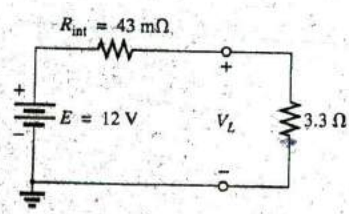


FIG. 5.126
Problem 44.

SECTION 5.10 Loading Effects of Instruments

45. **a.** Determine the current through the circuit in Fig. 5.127.
    b. If an ammeter with an internal resistance of 250 Ω is inserted into the circuit in Fig. 5.127, what effect will it have on the current level?
    c. Is the difference in current level a major concern for most applications?

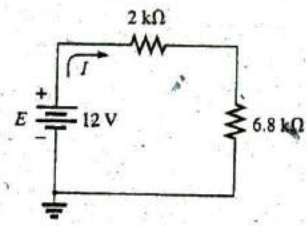


FIG. 5.127
Problem 45.

SECTION 5.11 Computer Analysis

46. Use the computer to verify the results of Example 5.4.
47. Use the computer to verify the results of Example 5.5.
48. Use the computer to verify the results of Example 5.15.

GLOSSARY

Circuit A combination of a number of elements joined at terminal points providing at least one closed path through which charge can flow.

Closed loop Any continuous connection of branches that allows tracing of a path that leaves a point in one direction and returns to that same point from another direction without leaving the circuit.

Internal resistance The inherent resistance found internal to any source of energy.

Kirchhoff's voltage law (KVL) The algebraic sum of the potential rises and drops around a closed loop (or path) is zero.

Protoboard (breadboard) A flat board with a set pattern of conductively connected holes designed to accept 24-gage wire and components with leads of about the same diameter.

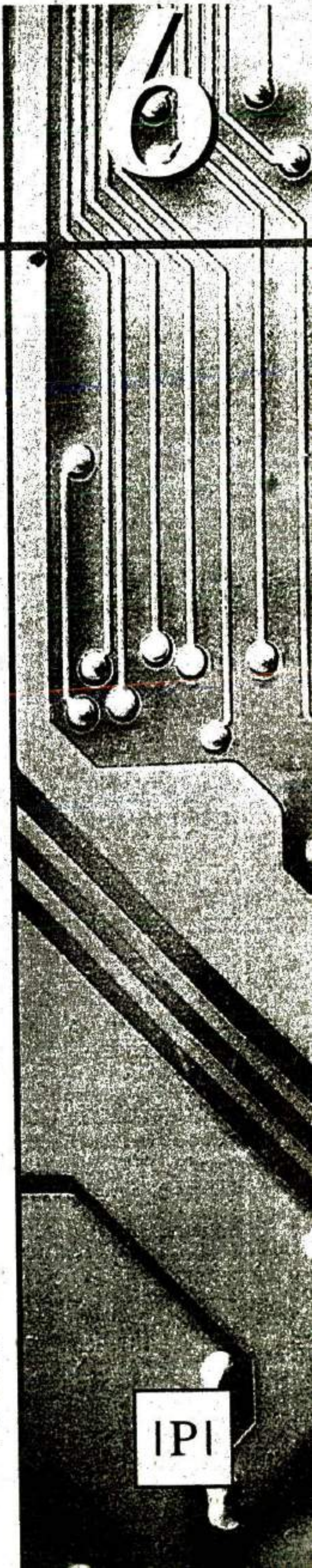
Series circuit A circuit configuration in which the elements have only one point in common and each terminal is not connected to a third, current-carrying element.

Two-terminal device Any element or component with two external terminals for connection to a network configuration.

Voltage divider rule (VDR) A method by which a voltage in a series circuit can be determined without first calculating the current in the circuit.

Voltage regulation (*VR*) A value, given as a percent, that provides an indication of the change in terminal voltage of a supply with a change in load demand.

Parallel dc Circuits



Objectives

- *Become familiar with the characteristics of a parallel network and how to solve for the voltage, current, and power to each element.*
- *Develop a clear understanding of Kirchhoff's current law and its importance to the analysis of electric circuits.*
- *Become aware of how the source current will split between parallel elements and how to properly apply the current divider rule.*
- *Clearly understand the impact of open and short circuits on the behavior of a network.*
- *Learn how to use an ohmmeter, voltmeter, and ammeter to measure the important parameters of a parallel network.*

6.1 INTRODUCTION

Two network configurations, series and parallel, form the framework for some of the most complex network structures. A clear understanding of each will pay enormous dividends as more complex methods and networks are examined. The series connection was discussed in detail in the last chapter. We will now examine the **parallel circuit** and all the methods and laws associated with this important configuration.

6.2 PARALLEL RESISTORS

The term *parallel* is used so often to describe a physical arrangement between two elements that most individuals are aware of its general characteristics.

In general,

two elements, branches, or circuits are in parallel if they have two points in common.

For instance, in Fig. 6.1(a), the two resistors are in parallel because they are connected at points a and b. If both ends were *not* connected as shown, the resistors would not be in parallel. In Fig. 6.1(b), resistors R_1 and R_2 are in parallel because they again have points a and b in common. R_1 is not in parallel with R_3 because they are connected at only one point (b). Further, R_1 and R_3 are not in series because a third connection appears at point b. The same can be said for resistors R_2 and R_3. In Fig. 6.1(c), resistors R_1 and R_2 are in series because they have only one point in common that is not connected elsewhere in the network. Resistors R_1 and R_3 are not in parallel because they have only point a in common. In addition, they are not in series because of the third connection to point a. The same can be said for resistors R_2 and R_3. In a broader context, it can be said that the series combination of resistors R_1 and R_2 is in parallel with resistor R_3 (more will be said about this option in Chapter 7). Furthermore, even though the discussion above was only for resistors, it can be applied to any two-terminal elements such as voltage sources and meters.

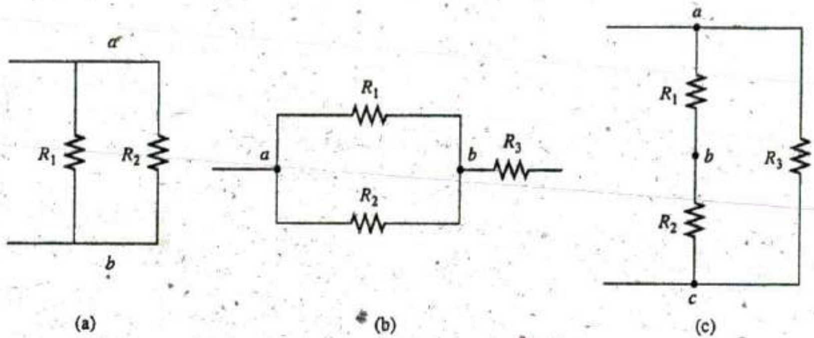


FIG. 6.1
(a) Parallel resistors; (b) R_1 and R_2 are in parallel; (c) R_3 is in parallel with the series combination of R_1 and R_2.

On schematics, the parallel combination can appear in a number of ways, as shown in Fig. 6.2. In each case, the three resistors are in parallel. They all have points a and b in common.

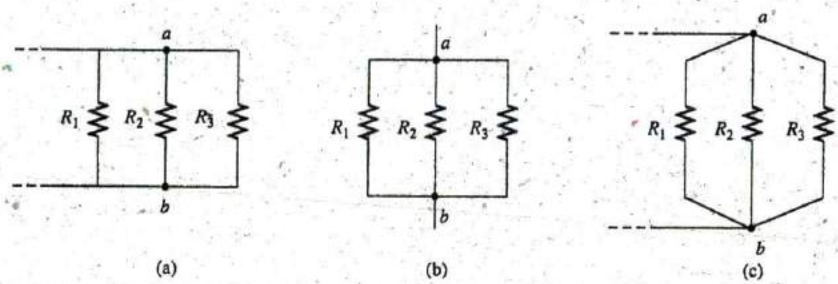


FIG. 6.2
Schematic representations of three parallel resistors.

For resistors in parallel as shown in Fig. 6.3, the total resistance is determined from the following equation:

$$\boxed{\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}} \qquad \textbf{(6.1)}$$

Since $G = 1/R$, the equation can also be written in terms of conductance levels as follows:

$$\boxed{G_T = G_1 + G_2 + G_3 + \cdots + G_N} \qquad \text{(siemens, S)} \qquad \textbf{(6.2)}$$

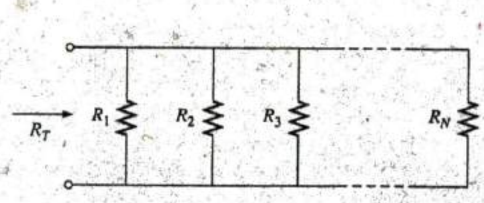


FIG. 6.3
Parallel combination of resistors.

which is an exact match in format with the equation for the total resistance of resistors in series: $R_T = R_1 + R_2 + R_3 + \cdots + R_N$. The result of this duality is that you can go from one equation to the other simply by interchanging R and G.

In general, however, when the total resistance is desired, the following format is applied:

$$\boxed{R_T = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \cdots + \dfrac{1}{R_N}}} \qquad (6.3)$$

Quite obviously, Eq. (6.3) is not as "clean" as the equation for the total resistance of series resistors. You must be careful when dealing with all the divisions into 1. The great feature about the equation, however, is that it can be applied to any number of resistors in parallel.

EXAMPLE 6.1

a. Find the total conductance of the parallel network in Fig. 6.4.
b. Find the total resistance of the same network using the results of part (a) and using Eq. (6.3).

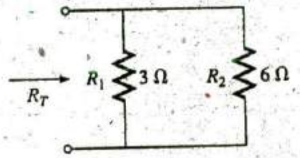


FIG. 6.4
Parallel resistors for Example 6.1.

Solutions:

a. $G_1 = \dfrac{1}{R_1} = \dfrac{1}{3\,\Omega} = 0.333\text{ S}, \qquad G_2 = \dfrac{1}{R_2} = \dfrac{1}{6\,\Omega} = 0.167\text{ S}$
and $G_T = G_1 + G_2 = 0.333\text{ S} + 0.167\text{ S} = \mathbf{0.5\text{ S}}$

b. $R_T = \dfrac{1}{G_T} = \dfrac{1}{0.5\text{ S}} = \mathbf{2\,\Omega}$

Applying Eq. (6.3) gives

$$R_T = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} = \frac{1}{\dfrac{1}{3\,\Omega} + \dfrac{1}{6\,\Omega}}$$
$$= \frac{1}{0.333\text{ S} + 0.167\text{ S}} = \frac{1}{0.5\text{ S}} = \mathbf{2\,\Omega}$$

EXAMPLE 6.2

a. By inspection, which parallel element in Fig. 6.5 has the least conductance? Determine the total conductance of the network and note whether your conclusion was verified.

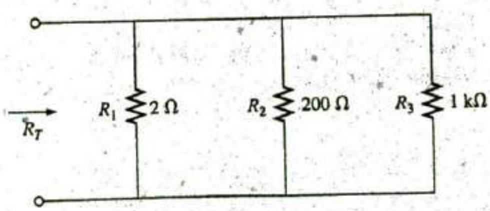


FIG. 6.5
Parallel resistors for Example 6.2.

b. Determine the total resistance from the results of part (a) and by applying Eq. (6.3).

Solutions:

a. Since the 1 kΩ resistor has the largest resistance and therefore the largest opposition to the flow of charge (level of conductivity), it will have the lowest level of conductance:

$$G_1 = \frac{1}{R_1} = \frac{1}{2\,\Omega} = 0.5\text{ S},\ G_2 = \frac{1}{R_2} + \frac{1}{200\,\Omega} = 0.005\text{ S} = 5\text{ mS}$$

$$G_3 = \frac{1}{R_3} = \frac{1}{1\text{ k}\Omega} = \frac{1}{1000\,\Omega} = 0.001\text{ S} = 1\text{ mS}$$

$$G_T = G_1 + G_2 + G_3 = 0.5\text{ S} + 5\text{ mS} + 1\text{ mS}$$
$$= \mathbf{506\text{ mS}}$$

Note the difference in conductance level between the 2 Ω (500 mS) and the 1 kΩ (1 mS) resistor.

b. $R_T = \frac{1}{G_T} = \frac{1}{506\text{ mS}} = \mathbf{1.976\,\Omega}$

Applying Eq. (6 3) gives

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{\frac{1}{2\,\Omega} + \frac{1}{200\,\Omega} + \frac{1}{1\text{ k}\Omega}}$$

$$= \frac{1}{0.5\text{ S} + 0.005\text{ S} + 0.001\text{ S}} = \frac{1}{0.506\text{ S}} = \mathbf{1.98\,\Omega}$$

EXAMPLE 6.3 Find the total resistance of the configuration in Fig. 6.6.

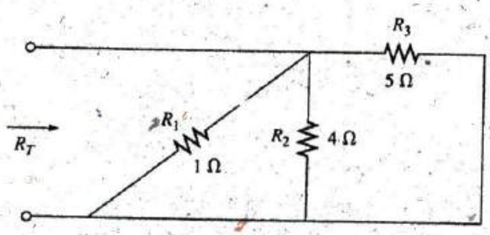


FIG. 6.6
Network to be investigated in Example 6.3.

Solution: First the network is redrawn as shown in Fig. 6.7 to clearly demonstrate that all the resistors are in parallel.

Applying Eq. (6.3) gives

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{\frac{1}{1\,\Omega} + \frac{1}{4\,\Omega} + \frac{1}{5\,\Omega}}$$

$$= \frac{1}{1\text{ S} + 0.25\text{ S} + 0.2\text{ S}} = \frac{1}{1.45\text{ S}} \cong \mathbf{0.69\,\Omega}$$

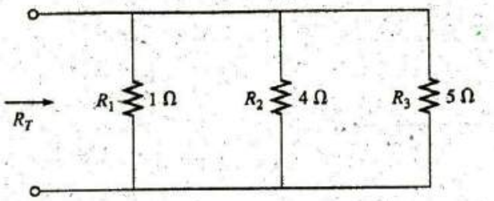


FIG. 6.7
Network in Fig. 6.6 redrawn.

If you review the examples above, you will find that the total resistance is less than the smallest parallel resistor. That is, in Example 6.1, 2 Ω is less than 3 Ω or 6 Ω. In Example 6.2, 1.976 Ω is less than 2 Ω, 100 Ω, or 1 kΩ; and in Example 6.3, 0.69 Ω is less than 1 Ω, 4 Ω, or 5 Ω. In general, therefore,

the total resistance of parallel resistors is always less than the value of the smallest resistor.

This is particularly important when you want a quick estimate of the total resistance of a parallel combination. Simply find the smallest value, and you know that the total resistance will be less than that value. It is also a great check on your calculations. In addition, you will find that

if the smallest resistance of a parallel combination is much smaller than that of the other parallel resistors, the total resistance will be very close to the smallest resistance value.

This fact is obvious in Example 6.2, where the total resistance of 1.976 Ω is very close to the smallest resistance of 2 Ω.

Another interesting characteristic of parallel resistors is demonstrated in Example 6.4.

EXAMPLE 6.4

a. What is the effect of adding another resistor of 100 Ω in parallel with the parallel resistors of Example 6.1 as shown in Fig. 6.8?
b. What is the effect of adding a parallel 1 Ω resistor to the configuration in Fig. 6.8?

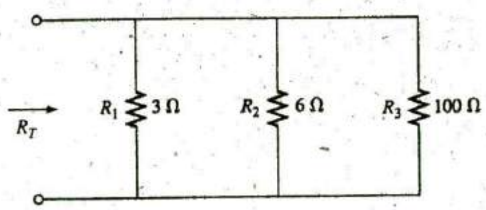


FIG. 6.8
Adding a parallel 100 Ω resistor to the network in Fig. 6.4.

Solutions:

a. Applying Eq. (6.3) gives

$$R_T = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{1}{\frac{1}{3\ \Omega}+\frac{1}{6\ \Omega}+\frac{1}{100\ \Omega}}$$

$$= \frac{1}{0.333\ \text{S} + 0.167\ \text{S} + 0.010\ \text{S}} = \frac{1}{0.510\ \text{S}} = \mathbf{1.96\ \Omega}$$

The parallel combination of the 3 Ω and 6 Ω resistors resulted in a total resistance of 2 Ω in Example 6.1. The effect of adding a resistor in parallel of 100 Ω had little effect on the total resistance because its resistance level is significantly higher (and conductance level significantly less) than that of the other two resistors. The total change in resistance was less than 2%. However, note that the total resistance dropped with the addition of the 100 Ω resistor.

b. Applying Eq. (6.3) gives

$$R_T = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}} = \frac{1}{\frac{1}{3\ \Omega}+\frac{1}{6\ \Omega}+\frac{1}{100\ \Omega}+\frac{1}{1\ \Omega}}$$

$$= \frac{1}{0.333\ \text{S} + 0.167\ \text{S} + 0.010\ \text{S} + 1\ \text{S}} = \frac{1}{0.51\ \text{S}} = \mathbf{0.66\ \Omega}$$

The introduction of the 1 Ω resistor reduced the total resistance from 2 Ω to only 0.66 Ω—a decrease of almost 67%. The fact that the added resistor has a resistance less than that of the other parallel elements and one-third that of the smallest contributed to the significant drop in resistance level.

In part (a) of Example 6.4, the total resistance dropped from 2 Ω to 1.96 Ω. In part (b), it dropped to 0.66 Ω. The results clearly reveal that

the total resistance of parallel resistors will always drop as new resistors are added in parallel, irrespective of their value.

Recall that this is the opposite of what occurs for series resistors, where additional resistors of any value increase the total resistance.

For equal resistors in parallel, the equation for the total resistance becomes significantly easier to apply. For N equal resistors in parallel, Eq. (6.3) becomes

$$R_T = \frac{1}{\frac{1}{R}+\frac{1}{R}+\frac{1}{R}+\cdots+\frac{1}{R_N}}$$

$$= \frac{1}{N\left(\frac{1}{R}\right)} = \frac{1}{\frac{N}{R}}$$

and

$$\boxed{R_T = \frac{R}{N}} \qquad (6.4)$$

In other words,

the total resistance of N parallel resistors of equal value is the resistance of one resistor divided by the number (N) of parallel resistors.

EXAMPLE 6.5 Find the total resistance of the parallel resistors in Fig. 6.9.

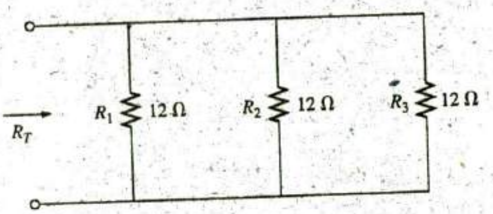


FIG. 6.9
Three equal parallel resistors to be investigated in Example 6.5.

Solution: Applying Eq. (6.4) gives

$$R_T = \frac{R}{N} = \frac{12\ \Omega}{3} = \mathbf{4\ \Omega}$$

EXAMPLE 6.6 Find the total resistance for the configuration in Fig. 6.10.

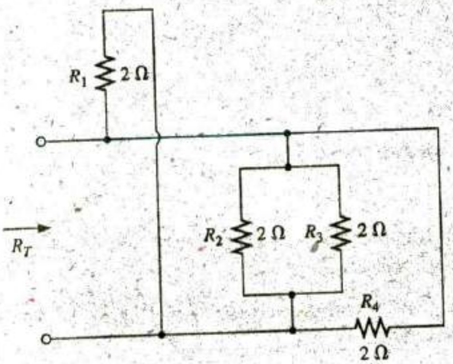


FIG. 6.10
Parallel configuration for Example 6.6.

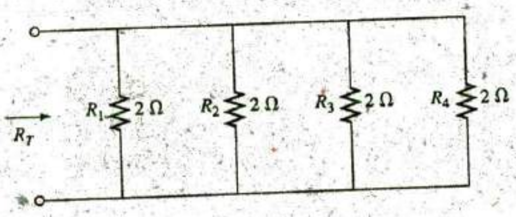


FIG. 6.11
Network in Fig. 6.10 redrawn.

Solution: Redrawing the network results in the parallel network in Fig. 6.11.

Applying Eq. (6.4) gives

$$R_T = \frac{R}{N} = \frac{2\ \Omega}{4} = \mathbf{0.5\ \Omega}$$

Special Case: Two Parallel Resistors

In the vast majority of cases, only two or three parallel resistors will have to be combined. With this in mind, an equation has been derived for two parallel resistors that is easy to apply and removes the need to continually worry about dividing into 1 and possibly misplacing a decimal point. For three parallel resistors, the equation to be derived here can be applied twice, or Eq. (6.3) can be used.

For two parallel resistors, the total resistance is determined by Eq. (6.1):

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

Multiplying the top and bottom of each term of the right side of the equation by the other resistance results in

$$\frac{1}{R_T} = \left(\frac{R_2}{R_2}\right)\frac{1}{R_1} + \left(\frac{R_1}{R_1}\right)\frac{1}{R_2} = \frac{R_2}{R_1R_2} + \frac{R_1}{R_1R_2}$$

$$\frac{1}{R_T} = \frac{R_2 + R_1}{R_1R_2}$$

and

$$\boxed{R_T = \frac{R_1R_2}{R_1 + R_2}} \qquad (6.5)$$

In words, the equation states that

the total resistance of two parallel resistors is simply the product of their values divided by their sum.

EXAMPLE 6.7 Repeat Example 6.1 using Eq. (6.5).

Solution: Eq. (6.5) gives

$$R_T = \frac{R_1R_2}{R_1 + R_2} = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 6\ \Omega} = \frac{18}{9}\ \Omega = \mathbf{2\ \Omega}$$

which matches the earlier solution.

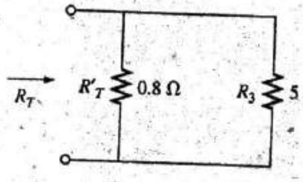


FIG. 6.12
Reduced equivalent in Fig. 6.7.

EXAMPLE 6.8 Determine the total resistance for the parallel combination in Fig. 6.7 using two applications of Eq. (6.5).

Solution: First the 1 Ω and 4 Ω resistors are combined using Eq. (6.5), resulting in the reduced network in Fig. 6.12:

$$\text{Eq. (6.4):}\quad R'_T = \frac{R_1R_2}{R_1 + R_2} = \frac{(1\ \Omega)(4\ \Omega)}{1\ \Omega + 4\ \Omega} = \frac{4}{5}\ \Omega = 0.8\ \Omega$$

Then Eq. (6.5) is applied again using the equivalent value:

$$R_T = \frac{R'_T R_3}{R'_T + R_3} = \frac{(0.8\ \Omega)(5\ \Omega)}{0.8\ \Omega + 5\ \Omega} = \frac{4}{5.8}\ \Omega = \mathbf{0.69\ \Omega}$$

The result matches that obtained in Example 6.3.

Recall that series elements can be interchanged without affecting the magnitude of the total resistance. In parallel networks,

parallel resistors can be interchanged without affecting the total resistance.

The next example demonstrates this and reveals how redrawing a network can often define which operations or equations should be applied.

EXAMPLE 6.9 Determine the total resistance of the parallel elements in Fig. 6.13.

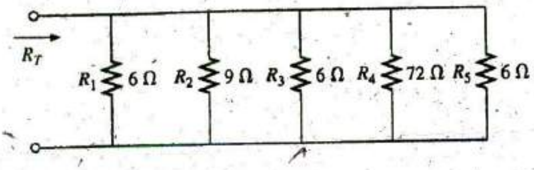


FIG. 6.13
Parallel network for Example 6.9.

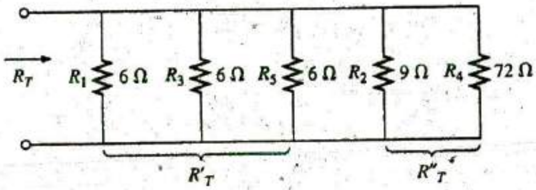


FIG. 6.14
Redrawn network in Fig. 6.13 (Example 6.9).

Solution: The network is redrawn in Fig. 6.14.

Eq. (6.4): $$R'_T = \frac{R}{N} = \frac{6\,\Omega}{3} = 2\,\Omega$$

Eq. (6.5): $$R''_T = \frac{R_2R_4}{R_2 + R_4} = \frac{(9\,\Omega)(72\,\Omega)}{9\,\Omega + 72\,\Omega} = \frac{648}{81}\,\Omega = 8\,\Omega$$

Eq. (6.5): $$R_T = \frac{R'_TR''_T}{R'_T + R''_T} = \frac{(2\,\Omega)(8\,\Omega)}{2\,\Omega + 8\,\Omega} = \frac{16}{10}\,\Omega = \mathbf{1.6\,\Omega}$$

The preceding examples involve direct substitution; that is, once the proper equation has been defined, it is only a matter of plugging in the numbers and performing the required algebraic manipulations. The next two examples have a design orientation, in which specific network parameters are defined and the circuit elements must be determined.

EXAMPLE 6.10 Determine the value of R_2 in Fig. 6.15 to establish a total resistance of 9 kΩ.

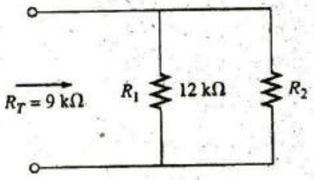


FIG. 6.15
Parallel network for Example 6.10.

Solution:

$$R_T = \frac{R_1R_2}{R_1 + R_2}$$
$$R_T(R_1 + R_2) = R_1R_2$$
$$R_TR_1 + R_TR_2 = R_1R_2$$
$$R_TR_1 = R_1R_2 - R_TR_2$$
$$R_TR_1 = (R_1 - R_T)R_2$$

and $$R_2 = \frac{R_TR_1}{R_1 - R_T}$$

Substituting values gives

$$R_2 = \frac{(9\,\text{k}\Omega)(12\,\text{k}\Omega)}{12\,\text{k}\Omega - 9\,\text{k}\Omega} = \frac{108}{3}\,\text{k}\Omega = \mathbf{36\,k\Omega}$$

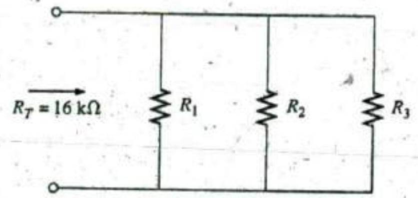


FIG. 6.16
Parallel network for Example 6.11.

EXAMPLE 6.11 Determine the values of R_1, R_2, and R_3 in Fig. 6.16 if $R_2 = 2R_1$, $R_3 = 2R_2$, and the total resistance is 16 kΩ.

Solution: Eq. (6.1) states

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

However, $R_2 = 2R_1$ and $R_3 = 2R_2 = 2(2R_1) = 4R_1$

so that
$$\frac{1}{16\ k\Omega} = \frac{1}{R_1} + \frac{1}{2R_1} + \frac{1}{4R_1}$$

and
$$\frac{1}{16\ k\Omega} = \frac{1}{R_1} + \frac{1}{2}\left(\frac{1}{R_1}\right) + \frac{1}{4}\left(\frac{1}{R_1}\right)$$

or
$$\frac{1}{16\ k\Omega} = 1.75\left(\frac{1}{R_1}\right)$$

resulting in $R_1 = 1.75(16\ k\Omega) = \mathbf{28\ k\Omega}$

so that $R_2 = 2R_1 = 2(28\ k\Omega) = \mathbf{56\ k\Omega}$

and $R_3 = 2R_2 = 2(56\ k\Omega) = \mathbf{112\ k\Omega}$

Analogies

Analogies were effectively used to introduce the concept of series elements. They can also be used to help define a *parallel configuration*. On a ladder, the rungs of the ladder form a parallel configuration. When ropes are tied between a grappling hook and a load, they effectively absorb the stress in a parallel configuration. The cables of a suspended roadway form a parallel configuration. There are numerous other analogies that demonstrate how connections between the same two points permit a distribution of stress between the parallel elements.

Instrumentation

As shown in Fig. 6.17, the total resistance of a parallel combination of resistive elements can be found by simply applying an ohmmeter. There is no polarity to resistance, so either lead of the ohmmeter can be connected to either side of the network. Although there are no supplies in Fig. 6.17, always keep in mind that ohmmeters can never be applied to a "live" circuit. It is not enough to set the supply to 0 V or to turn it off. It may still load down (change the network configuration of) the circuit and change the reading. It is best to remove the supply and apply the

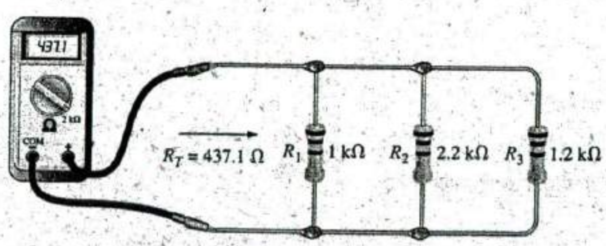


FIG. 6.17
Using an ohmmeter to measure the total resistance of a parallel network.

ohmmeter to the two resulting terminals. Since all the resistors are in the kilohm range, the 20 kΩ scale was chosen first. We then moved down to the 2 kΩ scale for increased precision. Moving down to the 200 Ω scale resulted in an "OL" indication since we were below the measured resistance value.

6.3 PARALLEL CIRCUITS

A **parallel circuit** can now be established by connecting a supply across a set of parallel resistors as shown in Fig. 6.18. The positive terminal of the supply is directly connected to the top of each resistor, while the negative terminal is connected to the bottom of each resistor. Therefore, it should be quite clear that the applied voltage is the same across each resistor. In general,

the voltage is always the same across parallel elements.

Therefore, remember that

if two elements are in parallel, the voltage across them must be the same. However, if the voltage across two neighboring elements is the same, the two elements may or may not be in parallel.

The reason for this qualifying comment in the above statement is discussed in detail in Chapter 7.

For the voltages of the circuit in Fig. 6.18, the result is that

$$\boxed{V_1 = V_2 = E} \tag{6.6}$$

Once the supply has been connected, a source current is established through the supply that passes through the parallel resistors. The current that results is a direct function of the total resistance of the parallel circuit. The smaller the total resistance, the greater is the current, as occurred for series circuits also.

Recall from series circuits that the source does not "see" the parallel combination of elements. It reacts only to the total resistance of the circuit, as shown in Fig. 6.19. The source current can then be determined using Ohm's law:

$$\boxed{I_s = \frac{E}{R_T}} \tag{6.7}$$

Since the voltage is the same across parallel elements, the current through each resistor can also be determined using Ohm's law. That is,

$$\boxed{I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} \quad \text{and} \quad I_2 = \frac{V_2}{R_2} = \frac{E}{R_2}} \tag{6.8}$$

The direction for the currents is dictated by the polarity of the voltage across the resistors. Recall that for a resistor, current enters the positive side of a potential drop and leaves the negative side. The result, as shown in Fig. 6.18, is that the source current enters point a, and currents I_1 and I_2 leave the same point. An excellent analogy for describing the flow of charge through the network of Fig. 6.18 is the flow of water through the parallel pipes of Fig. 6.20. The larger pipe, with less "resistance" to the

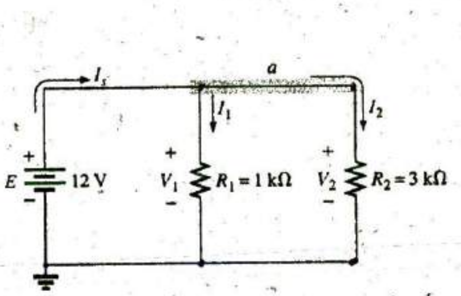


FIG. 6.18
Parallel network.

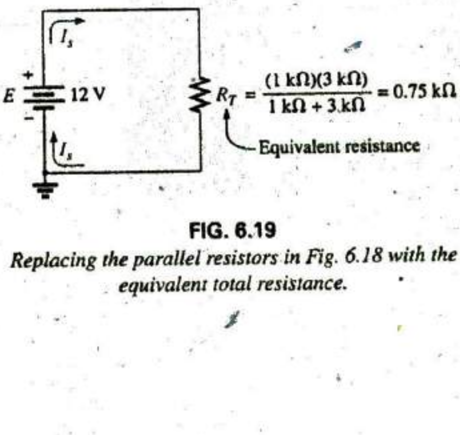


FIG. 6.19
Replacing the parallel resistors in Fig. 6.18 with the equivalent total resistance.

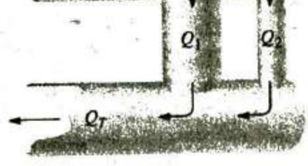


FIG. 6.20
Mechanical analogy for Fig. 6.18.

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flow of water, will have a larger flow of water, through it. The thinner pipe, with its increased "resistance" level, will have less water flowing through it. In any case, the total water entering the pipes at the top Q_T must equal that leaving at the bottom, with $Q_T = Q_1 + Q_2$.

The relationship between the source current and the parallel resistor currents can be derived by simply taking the equation for the total resistance in Eq. (6.1):

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

Multiplying both sides by the applied voltage gives

$$E\left(\frac{1}{R_T}\right) = E\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

resulting in

$$\frac{E}{R_T} = \frac{E}{R_1} + \frac{E}{R_2}$$

Then note that $E/R_1 = I_1$ and $E/R_2 = I_2$ to obtain

$$\boxed{I_s = I_1 + I_2} \qquad (6.9)$$

The result reveals a very important property of parallel circuits:

For single-source parallel networks, the source current (I_s) is always equal to the sum of the individual branch currents.

The duality that exists between series and parallel circuits continues to surface as we proceed through the basic equations for electric circuits. This is fortunate because it provides a way of remembering the characteristics of one using the results of another. For instance, in Fig. 6.21(a), we have a parallel circuit where it is clear that $I_T = I_1 + I_2$. By simply replacing the currents of the equation in Fig. 6.21(a) by a voltage level, as shown in Fig. 6.21(b), we have Kirchhoff's voltage law for a series circuit: $E = V_1 + V_2$. In other words,

for a parallel circuit, the source current equals the sum of the branch currents, while for a series circuit, the applied voltage equals the sum of the voltage drops.

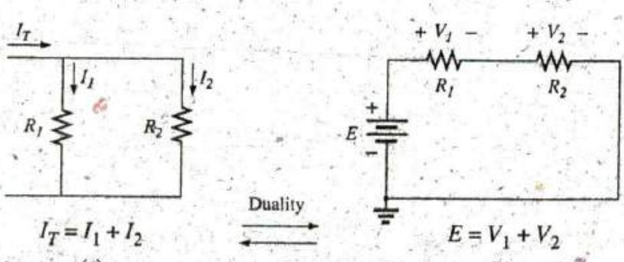

FIG. 6.21
Demonstrating the duality that exists between series and parallel circuits.

EXAMPLE 6.12 For the parallel network in Fig. 6.22:

a. Find the total resistance.
b. Calculate the source current.
c. Determine the current through each parallel branch.
d. Show that Eq. (6.9) is satisfied.

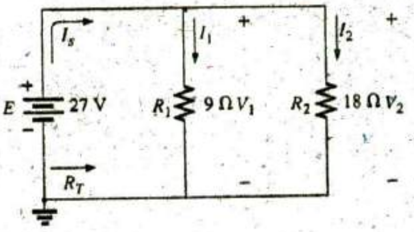

FIG. 6.22
Parallel network for Example 6.12.

Solutions:

a. Using Eq. (6.5) gives

$$R_T = \frac{R_1 R_2}{R_1 + R_2} = \frac{(9\,\Omega)(18\,\Omega)}{9\,\Omega + 18\,\Omega} = \frac{162}{27}\,\Omega = \mathbf{6\,\Omega}$$

b. Applying Ohm's law gives

$$I_s = \frac{E}{R_T} = \frac{27\text{ V}}{6\,\Omega} = \mathbf{4.5\text{ A}}$$

c. Applying Ohm's law gives

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{27\text{ V}}{9\,\Omega} = \mathbf{3\text{ A}}$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{27\text{ V}}{18\,\Omega} = \mathbf{1.5\text{ A}}$$

d. Substituting values from parts (b) and (c) gives

$$I_s = 4.5\text{ A} = I_1 + I_2 = 3\text{ A} + 1.5\text{ A} = 4.5\text{ A} \quad \text{(checks)}$$

EXAMPLE 6.13 For the parallel network in Fig. 6.23.

a. Find the total resistance.
b. Calculate the source current.
c. Determine the current through each branch.

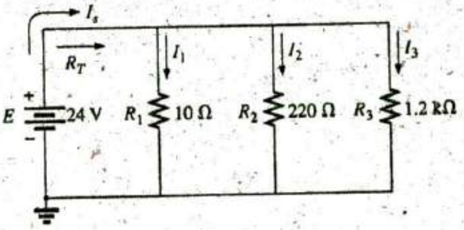

FIG. 6.23
Parallel network for Example 6.13.

Solutions:

a. Applying Eq. (6.3) gives

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{\frac{1}{10\,\Omega} + \frac{1}{220\,\Omega} + \frac{1}{1.2\text{ k}\Omega}}$$

$$= \frac{1}{100 \times 10^{-3} + 4.545 \times 10^{-3} + 0.833 \times 10^{-3}} = \frac{1}{105.38 \times 10^{-3}}$$

$$R_T = \mathbf{9.49\,\Omega}$$

Note that the total resistance is less than that of the smallest parallel resistor, and its magnitude is very close to the resistance of the smallest resistor because the other resistors are larger by a factor greater than 10 : 1.

b. Using Ohm's law gives

$$I_s = \frac{E}{R_T} = \frac{24\text{ V}}{9.49\,\Omega} = \mathbf{2.53\text{ A}}$$

c. Applying Ohm's law gives

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{24\text{ V}}{10\ \Omega} = \mathbf{2.4\ A}$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{24\text{ V}}{220\ \Omega} = \mathbf{0.11\ A}$$

$$I_3 = \frac{V_3}{R_3} = \frac{E}{R_3} = \frac{24\text{ V}}{1.2\text{ k}\Omega} = \mathbf{0.02\ A}$$

A careful examination of the results of Example 6.13 reveals that the larger the parallel resistor, the lower is the branch current. In general, therefore,

for parallel resistors, the greatest current will exist in the branch with the least resistance.

A more powerful statement is that

current always seeks the path of least resistance.

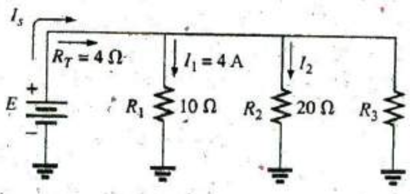

FIG. 6.24

Parallel network for Example 6.14.

EXAMPLE 6.14 Given the information provided in Fig. 6.24:

a. Determine R_3.
b. Find the applied voltage E.
c. Find the source current I_s.
d. Find I_2.

Solutions:

a. Applying Eq. (6.1) gives

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Substituting gives $$\frac{1}{4\ \Omega} = \frac{1}{10\ \Omega} + \frac{1}{20\ \Omega} + \frac{1}{R_3}$$

so that $$0.25\text{ S} = 0.1\text{ S} + 0.05\text{ S} + \frac{1}{R_3}$$

and $$0.25\text{ S} = 0.15\text{ S} + \frac{1}{R_3}$$

with $$\frac{1}{R_3} = 0.1\text{ S}$$

and $$R_3 = \frac{1}{0.1\text{ S}} = \mathbf{10\ \Omega}$$

b. Using Ohm's law gives

$$E = V_1 = I_1R_1 = (4\text{ A})(10\ \Omega) = \mathbf{40\ V}$$

c. $$I_s = \frac{E}{R_T} = \frac{40\text{ V}}{4\ \Omega} = \mathbf{10\ A}$$

d. Applying Ohm's law gives

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{40\text{ V}}{20\ \Omega} = \mathbf{2\ A}$$

Instrumentation

In Fig. 6.25, voltmeters have been connected to verify that the voltage across parallel elements is the same. Note that the positive or red lead of each voltmeter is connected to the high (positive) side of the voltage across each resistor to obtain a positive reading. The 20 V scale was used because the applied voltage exceeded the range of the 2 V scale.

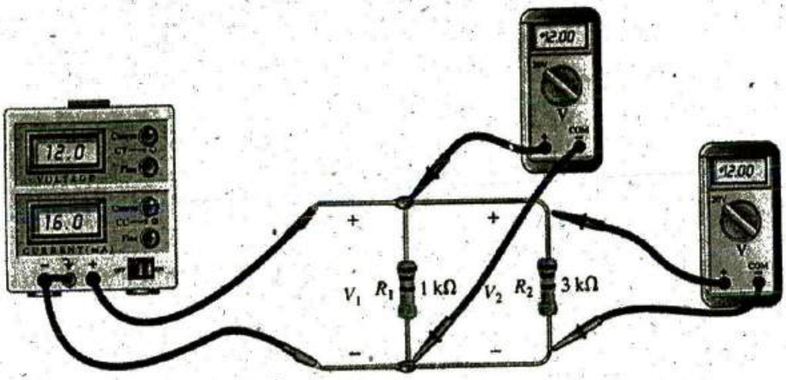

FIG. 6.25
Measuring the voltages of a parallel dc network.

In Fig. 6.26, an ammeter has been hooked up to measure the source current. First, the connection to the supply had to be broken at the positive terminal and the meter inserted as shown. Be sure to use ammeter terminals on your meter for such measurements. The red or positive lead of the meter is connected so that the source current enters that lead and leaves the negative or black lead to ensure a positive reading. The 200 mA scale was used because the source current exceeded the maximum value of the 2 mA scale. For the moment, we assume that the internal resistance of the meter can be ignored. Since the internal resistance of an ammeter on the 200 mA scale is typically only a few ohms, compared to the parallel resistors in the kilohm range, it is an excellent assumption.

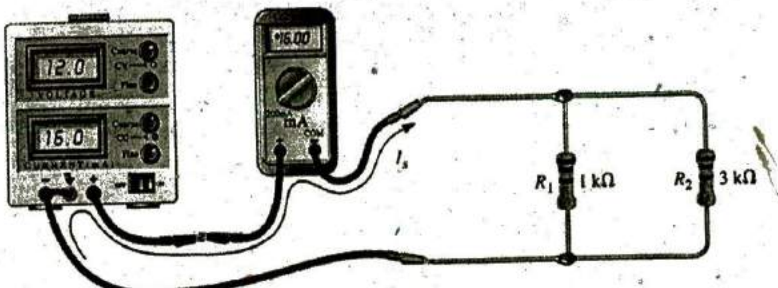

FIG. 6.26
Measuring the source current of a parallel network.

A more difficult measurement is for the current through resistor R_1. This measurement often gives trouble in the laboratory session. First, as shown in Fig. 6.27(a), resistor R_1 must be disconnected from the upper connection point to establish an open circuit. The ammeter is then inserted between the resulting terminals so that the current enters the positive or red terminal, as shown in Fig. 6.27(b). Always remember: When using an ammeter, first establish an open circuit in the branch in which the current is to be measured, and then insert the meter.

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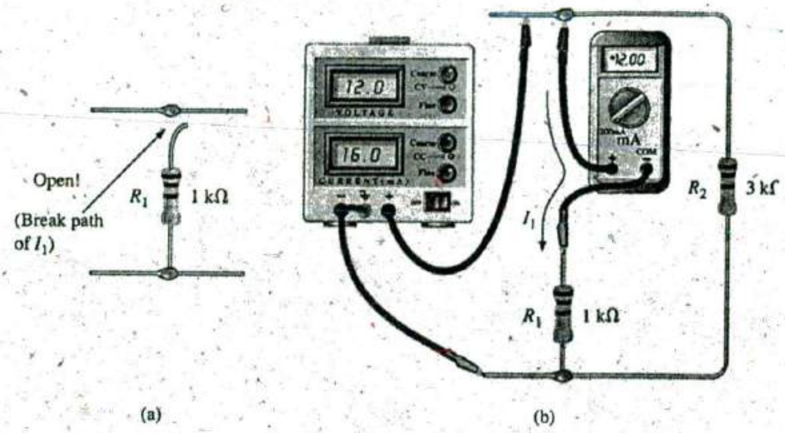


FIG. 6.27
Measuring the current through resistor R_1.

The easiest measurement is for the current through resistor R_2. Break the connection to R_2 above or below the resistor, and insert the ammeter with the current entering the positive or red lead to obtain a positive reading.

6.4 POWER DISTRIBUTION IN A PARALLEL CIRCUIT

Recall from the discussion of series circuits that the power applied to a series resistive circuit equals the power dissipated by the resistive elements. The same is true for parallel resistive networks. In fact,

for any network composed of resistive elements, the power applied by the battery will equal that dissipated by the resistive elements.

For the parallel circuit in Fig. 6.28:

$$\boxed{P_E = P_{R_1} + P_{R_2} + P_{R_3}} \tag{6.10}$$

which is exactly the same as obtained for the series combination.

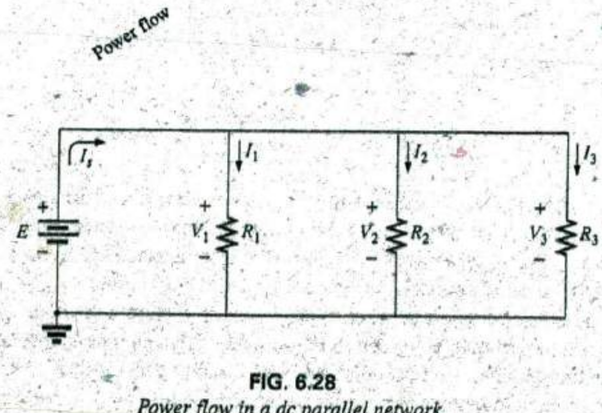


FIG. 6.28
Power flow in a dc parallel network.

The power delivered by the source in the same:

$$\boxed{P_E = EI_s} \qquad \text{(watts, W)} \qquad (6.11)$$

as is the equation for the power to each resistor (shown for R_1 only):

$$\boxed{P_1 = V_1 I_1 = I_1^2 R_1 = \frac{V_1^2}{R_1}} \qquad \text{(watts, W)} \qquad (6.12)$$

In the equation $P = V^2/R$, the voltage across each resistor in a parallel circuit will be the same. The only factor that changes is the resistance in the denominator of the equation. The result is that

in a parallel resistive network, the larger the resistor, the less is the power absorbed.

EXAMPLE 6.15 For the parallel network in Fig. 6.29 (all standard values):

a. Determine the total resistance R_T.
b. Find the source current and the current through each resistor.
c. Calculate the power delivered by the source.
d. Determine the power absorbed by each parallel resistor.
e. Verify Eq. (6.10).

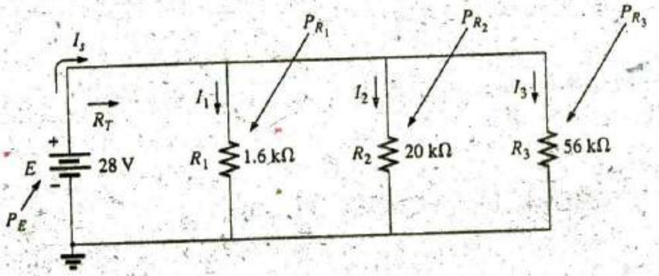


FIG. 6.29
Parallel network for Example 6.15.

Solutions:

a. Without making a single calculation, it should now be apparent from previous examples that the total resistance is less than 1.6 kΩ and very close to this value because of the magnitude of the other resistance levels:

$$R_T = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}} = \frac{1}{\dfrac{1}{1.6\text{ k}\Omega} + \dfrac{1}{20\text{ k}\Omega} + \dfrac{1}{56\text{ k}\Omega}}$$

$$= \frac{1}{625 \times 10^{-6} + 50 \times 10^{-6} + 17.867 \times 10^{-6}} = \frac{1}{692.867 \times 10^{-6}}$$

and $R_T = \mathbf{1.44\text{ k}\Omega}$

b. Applying Ohm's law gives

$$I_s = \frac{E}{R_T} = \frac{28\text{ V}}{1.44\text{ k}\Omega} = \mathbf{19.44\text{ mA}}$$

Recalling that current always seeks the path of least resistance immediately tells us that the current through the 1.6 kΩ resistor will be the largest and the current through the 56 kΩ resistor the smallest.

Applying Ohm's law again gives

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{28\text{ V}}{1.6\text{ k}\Omega} = \mathbf{17.5\text{ mA}}$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{28\text{ V}}{20\text{ k}\Omega} = \mathbf{1.4\text{ mA}}$$

$$I_3 = \frac{V_3}{R_3} = \frac{E}{R_3} = \frac{28\text{ V}}{56\text{ k}\Omega} = \mathbf{0.5\text{ mA}}$$

c. Applying Eq. (6.11) gives

$$P_E = EI_s = (28\text{ V})(19.4\text{ mA}) = \mathbf{543.2\text{ mW}}$$

d. Applying each form of the power equation gives

$$P_1 = V_1 I_1 = EI_1 = (28\text{ V})(17.5\text{ mA}) = \mathbf{490\text{ mW}}$$

$$P_2 = I_2^2 R_2 = (1.4\text{ mA})^2(20\text{ k}\Omega) = \mathbf{39.2\text{ mW}}$$

$$P_3 = \frac{V_3^2}{R_3} = \frac{E^2}{R_3} = \frac{(28\text{ V})^2}{56\text{ k}\Omega} = \mathbf{14\text{ mW}}$$

A review of the results clearly substantiates the fact that the larger the resistor, the less is the power absorbed.

e.

$$P_E = P_{R_1} + P_{R_2} + P_{R_3}$$

$$\mathbf{543.2\text{ mW}} = 490\text{ mW} + 39.2\text{ mW} + 14\text{ mW} = \mathbf{543.2\text{ mW}} \quad \text{(checks)}$$

6.5 KIRCHHOFF'S CURRENT LAW

In the previous chapter, Kirchhoff's voltage law was introduced, providing a very important relationship among the voltages of a closed path. Kirchhoff is also credited with developing the following equally important relationship between the currents of a network, called **Kirchhoff's current law (KCL):**

The algebraic sum of the currents entering and leaving a junction (or region) of a network is zero.

The law can also be stated in the following way:

The sum of the currents entering a junction (or region) of a network must equal the sum of the currents leaving the same junction (or region).

In equation form, the above statement can be written as follows:

$$\boxed{\Sigma I_i = \Sigma I_o} \qquad (6.13)$$

with I_i representing the current entering, or "in," and I_o representing the current leaving, or "out."

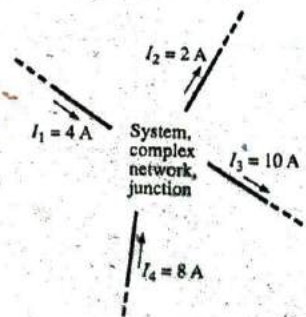


FIG. 6.30
Introducing Kirchhoff's current law.

In Fig. 6.30, for example, the shaded area can enclose an entire system or a complex network, or it can simply provide a connection point (junction) for the displayed currents. In each case, the current entering must equal that leaving, as required by Eq. (6.13):

$$\Sigma I_i = \Sigma I_o$$
$$I_1 + I_4 = I_2 + I_3$$
$$4\text{ A} + 8\text{ A} = 2\text{ A} + 10\text{ A}$$
$$\mathbf{12\text{ A} = 12\text{ A}} \quad \text{(checks)}$$

The most common application of the law will be at a junction of two or more current paths, as shown in Fig. 6.31(a). Some students have difficulty initially determining whether a current is entering or leaving a junction. One approach that may help is to use the water analog in Fig. 6.31(b), where the junction in Fig. 6.31(a) is the small bridge across the stream. Simply relate the current of I_1 to the fluid flow of Q_1, the smaller branch current I_2 to the water flow Q_2, and the larger branch current I_3 to the flow Q_3. The water arriving at the bridge must equal the sum of that leaving the bridge, so that $Q_1 = Q_2 + Q_3$. Since the current I_1 is pointing *at* the junction and the fluid flow Q_1 is *toward* the person on the bridge, both quantities are seen as approaching the junction, and can be considered *entering* the junction. The currents I_2 and I_3 are both leaving the junction, just as Q_2 and Q_3 are leaving the fork in the river. The quantities I_2, I_3, Q_2, and Q_3 are therefore all *leaving* the junction.

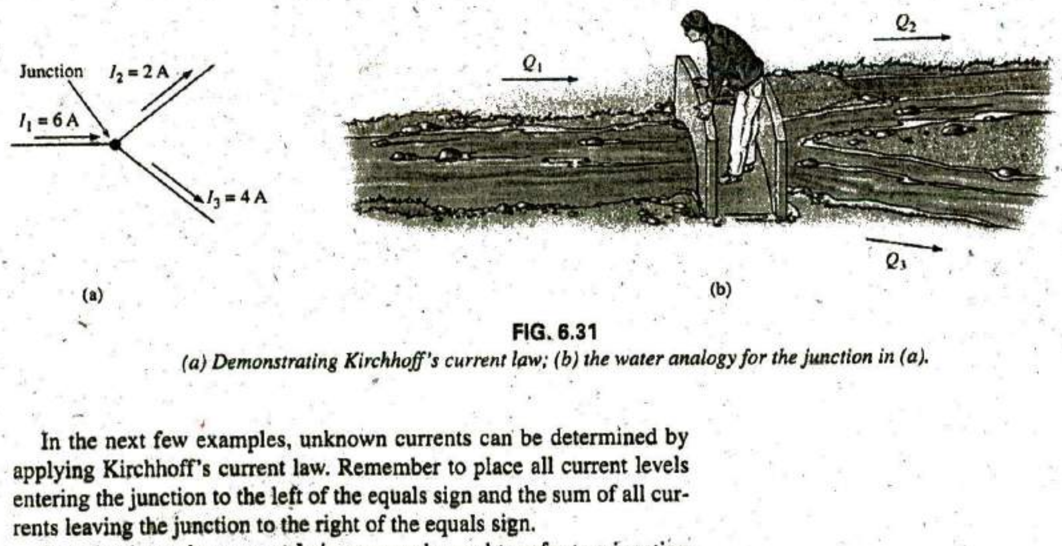


FIG. 6.31
(a) Demonstrating Kirchhoff's current law; (b) the water analogy for the junction in (a).

In the next few examples, unknown currents can be determined by applying Kirchhoff's current law. Remember to place all current levels entering the junction to the left of the equals sign and the sum of all currents leaving the junction to the right of the equals sign.

In technology, the term **node** is commonly used to refer to a junction of two or more branches. Therefore, this term is used frequently in the analyses to follow.

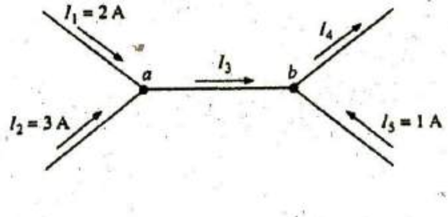


FIG. 6.32
Two-node configuration for Example 6.16.

EXAMPLE 6.16 Determine currents I_3 and I_4 in Fig. 6.32 using Kirchhoff's current law.

Solution: There are two junctions or nodes in Fig. 6.32. Node a has only one unknown, while node b has two unknowns. Since a single equation can be used to solve for only one unknown, we must apply Kirchhoff's current law to node a first.

At node a

$$\Sigma I_i = \Sigma I_o$$
$$I_1 + I_2 = I_3$$
$$2\text{ A} + 3\text{ A} = I_3 = \mathbf{5\text{ A}}$$

At node b, using the result just obtained,

$$\Sigma I_i = \Sigma I_o$$
$$I_3 + I_5 = I_4$$
$$5\text{ A} + 1\text{ A} = I_4 = \mathbf{6\text{ A}}$$

Note that in Fig. 6.32, the width of the blue-shaded regions matches the magnitude of the current in that region.

EXAMPLE 6.17 Determine currents I_1, I_3, I_4, and I_5 for the network in Fig. 6.33.

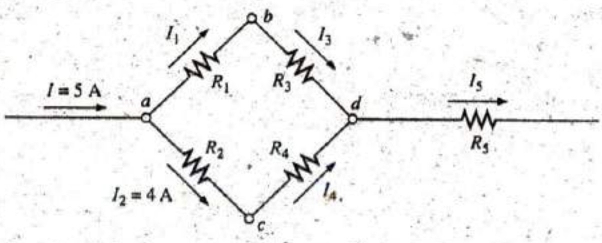


FIG. 6.33
Four-node configuration for Example 6.17.

Solution: In this configuration, four nodes are defined. Nodes a and c have only one unknown current at the junction, so Kirchhoff's current law can be applied at either junction.

At node a

$$\Sigma I_i = \Sigma I_o$$
$$I = I_1 + I_2$$
$$5\text{ A} = I_1 + 4\text{ A}$$

and $$I_1 = 5\text{ A} - 4\text{ A} = \mathbf{1\text{ A}}$$

At node c

$$\Sigma I_i = \Sigma I_o$$
$$I_2 = I_4$$

and $$I_4 = I_2 = \mathbf{4\text{ A}}$$

Using the above results at the other junctions results in the following.

At node b

$$\Sigma I_i = \Sigma I_o$$
$$I_1 = I_3$$

and $$I_3 = I_1 = \mathbf{1\text{ A}}$$

At node d

$$\Sigma I_i = \Sigma I_o$$
$$I_3 + I_4 = I_5$$
$$1\text{ A} + 4\text{ A} = I_5 = \mathbf{5\text{ A}}$$

If we enclose the entire network, we find that the current entering from the far left is $I = 5$ A, while the current leaving from the far right is

$I_5 = 5$ A. The two must be equal since the net current entering any system must equal the net current leaving.

EXAMPLE 6.18 Determine currents I_3 and I_5 in Fig. 6.34 through applications of Kirchhoff's current law.

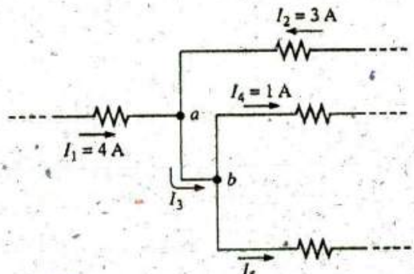


FIG. 6.34
Network for Example 6.18.

Solution: Note first that since node *b* has two unknown quantities (I_3 and I_5), and node *a* has only one, Kirchhoff's current law must first be applied to node *a*. The result is then applied to node *b*.

At node *a*

$$\Sigma I_i = \Sigma I_o$$
$$I_1 + I_2 = I_3$$
$$4\text{ A} + 3\text{ A} = I_3 = \mathbf{7\text{ A}}$$

At node *b*

$$\Sigma I_i = \Sigma I_o$$
$$I_3 = I_4 + I_5$$
$$7\text{ A} = 1\text{ A} + I_5$$

and

$$I_5 = 7\text{ A} - 1\text{ A} = \mathbf{6\text{ A}}$$

EXAMPLE 6.19 For the parallel dc network in Fig. 6.35:

a. Determine the source current I_s.
b. Find the source voltage *E*.
c. Determine R_3.
d. Calculate R_T.

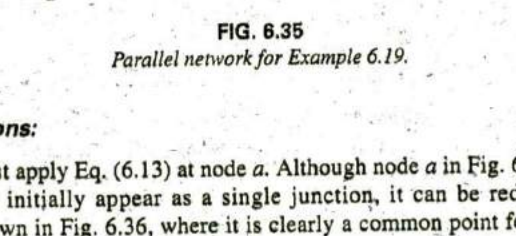


FIG. 6.35
Parallel network for Example 6.19.

Solutions:

a. First apply Eq. (6.13) at node *a*. Although node *a* in Fig. 6.35 may not initially appear as a single junction, it can be redrawn as shown in Fig. 6.36, where it is clearly a common point for all the branches.

The result is

$$\Sigma I_i = \Sigma I_o$$
$$I_s = I_1 + I_2 + I_3$$

Substituting values: $I_s = 8\text{ mA} + 10\text{ mA} + 2\text{ mA} = \mathbf{20\text{ mA}}$

Note in this solution that you do not need to know the resistor values or the voltage applied. The solution is determined solely by the current levels.

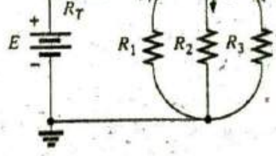


FIG. 6.36
Redrawn network in Fig. 6.35.

b. Applying Ohm's law gives

$$E = V_1 = I_1R_1 = (8\text{ mA})(2\text{ k}\Omega) = \mathbf{16\text{ V}}$$

c. Applying Ohm's law in a different form gives

$$R_3 = \frac{V_3}{I_3} = \frac{E}{I_3} = \frac{16\text{ V}}{2\text{ mA}} = \mathbf{8\text{ k}\Omega}$$

d. Applying Ohm's law again gives

$$R_T = \frac{E}{I_s} = \frac{16\text{ V}}{20\text{ mA}} = \mathbf{0.8\text{ k}\Omega}$$

The application of Kirchhoff's current law is not limited to networks where all the internal connections are known or visible. For instance, all the currents of the integrated circuit in Fig. 6.37 are known except I_1. By treating the entire system (which could contain over a million elements) as a single node, we can apply Kirchhoff's current law as shown in Example 6.20.

Before looking at Example 6.20 in detail, note that the direction of the unknown current I_1 is not provided in Fig. 6.37. On many occasions, this will be true. With so many currents entering or leaving the system, it is difficult to know by inspection which direction should be assigned to I_1. *In such cases, simply make an assumption about the direction and then check out the result. If the result is negative, the wrong direction was assumed. If the result is positive, the correct direction was assumed. In either case, the magnitude of the current will be correct.*

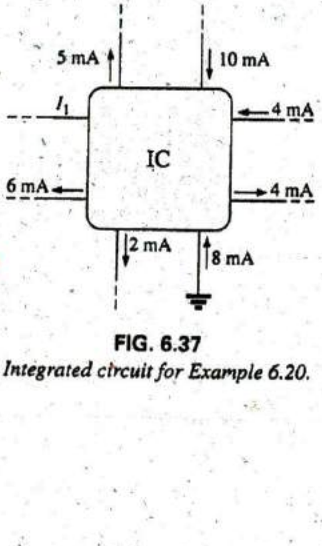


FIG. 6.37
Integrated circuit for Example 6.20.

EXAMPLE 6.20 Determine I_1 for the integrated circuit in Fig. 6.37.

Solution: Assuming that the current I_1 entering the chip results in the following when Kirchhoff's current law is applied, we find

$$\Sigma I_i = \Sigma I_o$$
$$I_1 + 10\text{ mA} + 4\text{ mA} + 8\text{ mA} = 5\text{ mA} + 4\text{ mA} + 2\text{ mA} + 6\text{ mA}$$
$$I_1 + 22\text{ mA} = 17\text{ mA}$$
$$I_1 = 17\text{ mA} - 22\text{ mA} = \mathbf{-5\text{ mA}}$$

We find that the direction for I_1 is *leaving* the IC, although the magnitude of 5 mA is correct.

As we leave this important section, be aware that Kirchhoff's current law will be applied in one form or another throughout the text. *Kirchhoff's laws are unquestionably two of the most important in this field because they are applicable to the most complex configurations in existence today.* They will not be replaced by a more important law or dropped for a more sophisticated approach.

6.6 CURRENT DIVIDER RULE

For series circuits we have the powerful voltage divider rule for finding the voltage across a resistor in a series circuit. We now introduce the equally powerful **current divider rule (CDR)** for finding the current through a resistor in a parallel circuit.

In Section 6.4, it was pointed out that current will always seek the path of least resistance. In Fig. 6.38, for example, the current of 9 A is faced with splitting between the three parallel resistors. Based on the previous sections, it should now be clear without a single calculation that the majority of the current will pass through the smallest resistor of 10 Ω, and the least current will pass through the 1 kΩ resistor. In fact, the current through the 100 Ω resistor will also exceed that through the 1 kΩ resistor. We can take it one step further by recognizing that the resistance of the 100 Ω resistor is 10 times that of the 10 Ω resistor. The result is a current through the 10 Ω resistor that is 10 times that of the 100 Ω resistor. Similarly, the current through the 100 Ω resistor is 10 times that through the 1 kΩ resistor.

In general,

For two parallel elements of equal value, the current will divide equally.

For parallel elements with different values, the smaller the resistance, the greater is the share of input current.

For parallel elements of different values, the current will split with a ratio equal to the inverse of their resistance values.

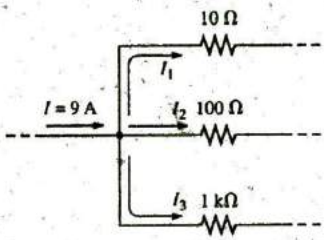


FIG. 6.38
Discussing the manner in which the current will split between three parallel branches of different resistive value.

EXAMPLE 6.21

a. Determine currents I_1 and I_3 for the network in Fig. 6.39.
b. Find the source current I_s.

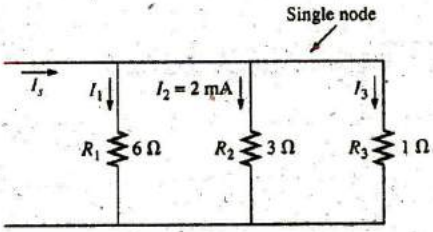


FIG. 6.39
Parallel network for Example 6.21.

Solutions:

a. Since R_1 is twice R_2, the current I_1 must be one-half I_2, and

$$I_1 = \frac{I_2}{2} = \frac{2\text{ mA}}{2} = \mathbf{1\text{ mA}}$$

Since R_2 is three times R_3, the current I_3 must be three times I_2, and

$$I_3 = 3I_2 = 3(2\text{ mA}) = \mathbf{6\text{ mA}}$$

b. Applying Kirchhoff's current law gives

$$\Sigma I_i = \Sigma I_o$$
$$I_s = I_1 + I_2 + I_3$$
$$I_s = 1\text{ mA} + 2\text{ mA} + 6\text{ mA} = \mathbf{9\text{ mA}}$$

Although the above discussions and examples allowed us to determine the relative magnitude of a current based on a known level, they do not provide the magnitude of a current through a branch of a parallel network if only the total entering current is known. The result is a need for the current divider rule, which will be derived using the parallel configuration in Fig. 6.40(a). The current I_T (using the subscript T to indicate the total entering current) splits between the N parallel resistors and then gathers itself together again at the bottom of the configuration. In Fig. 6.40(b), the parallel combination of resistors has been replaced by a single resistor equal to the total resistance of the parallel combination as determined in the previous sections.

The current I_T can then be determined using Ohm's law:

$$I_T = \frac{V}{R_T}$$

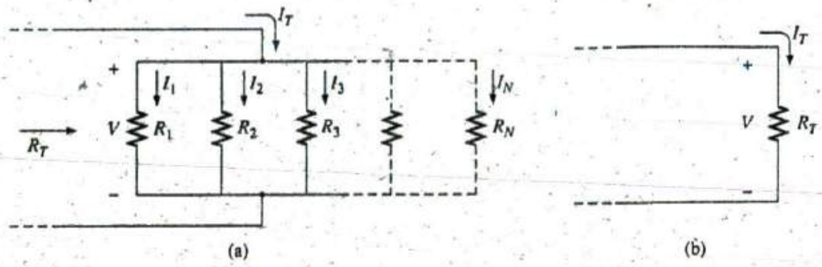


FIG. 6.40
Deriving the current divider rule: (a) parallel network of N parallel resistors; (b) reduced equivalent of part (a).

Since the voltage V is the same across parallel elements, the following is true:

$$V = I_1R_1 = I_2R_2 = I_3R_3 = \cdots = I_xR_x$$

where the product I_xR_x refers to any combination in the series.

Substituting for V in the above equation for I_T, we have

$$I_T = \frac{I_xR_x}{R_T}$$

Solving for I_x, the final result is the **current divider rule:**

$$\boxed{I_x = \frac{R_T}{R_x}I_T} \quad (6.14)$$

which states that

the current through any branch of a parallel resistive network is equal to the total resistance of the parallel network divided by the resistance of the resistor of interest and multiplied by the total current entering the parallel configuration.

Since R_T and I_T are constants, for a particular configuration the larger the value of R_x (in the denominator), the smaller is the value of I_x for that branch, confirming the fact that current always seeks the path of least resistance.

EXAMPLE 6.22 For the parallel network in Fig. 6.41, determine current I_1 using Eq. (6.14).

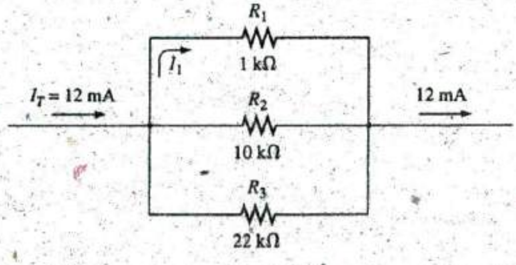


FIG. 6.41
Using the current divider rule to calculate current I_1 in Example 6.22.

Solution: Eq. (6.3):

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$
$$= \frac{1}{\frac{1}{1\,k\Omega} + \frac{1}{10\,k\Omega} + \frac{1}{22\,k\Omega}}$$
$$= \frac{1}{1 \times 10^{-3} + 100 \times 10^{-6} + 45.46 \times 10^{-6}}$$
$$= \frac{1}{1.145 \times 10^{-3}} = \mathbf{873.01\ \Omega}$$

Eq. (6.14): $I_1 = \frac{R_T}{R_1} I_T$

$$= \frac{(873.01\ \Omega)}{1\,k\Omega}(12\text{ mA}) = (0.873)(12\text{ mA}) = \mathbf{10.48\text{ mA}}$$

and the smallest parallel resistor receives the majority of the current.

Note also that

for a parallel network, the current through the smallest resistor will be very close to the total entering current if the other parallel elements of the configuration are much larger in magnitude.

In Example 6.22, the current through R_1 is very close to the total current because R_1 is 10 times less than the next smallest resistance.

Special Case: Two Parallel Resistors

For the case of two parallel resistors as shown in Fig. 6.42, the total resistance is determined by

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

Substituting R_T into Eq. (6.14) for current I_1 results in

$$I_1 = \frac{R_T}{R_1} I_T = \frac{\left(\frac{R_1 R_2}{R_1 + R_2}\right)}{R_1} I_T$$

and

$$\boxed{I_1 = \left(\frac{R_2}{R_1 + R_2}\right) I_T} \qquad \textbf{(6.15a)}$$

Similarly, for I_2,

$$\boxed{I_2 = \left(\frac{R_1}{R_1 + R_2}\right) I_T} \qquad \textbf{(6.15b)}$$

Eq. (6.15) states that

for two parallel resistors, the current through one is equal to the resistance of the other times the total entering current divided by the sum of the two resistances.

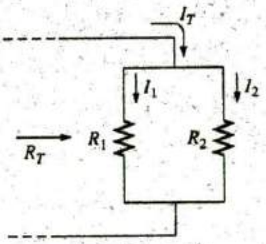


FIG. 6.42
Deriving the current divider rule for the special case of only two parallel resistors.

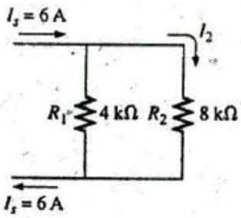


FIG. 6.43
Using the current divider rule to determine current I_2 in Example 6.23.

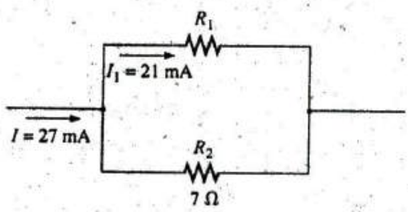


FIG. 6.44
A design-type problem for two parallel resistors (Example 6.24).

Since the combination of two parallel resistors is probably the most common parallel configuration, the simplicity of the format for Eq. (6.15) suggests that it is worth memorizing. Take particular note, however, that the denominator of the equation is simply the sum, not the total resistance, of the combination.

EXAMPLE 6.23 Determine current I_2 for the network in Fig. 6.43 using the current divider rule.

Solution: Using Eq. (6.15b) gives

$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right)I_T$$
$$= \left(\frac{4\text{ k}\Omega}{4\text{ k}\Omega + 8\text{ k}\Omega}\right)6\text{ A} = (0.333)(6\text{ A}) = \mathbf{2\text{ A}}$$

Using Eq. (6.14) gives

$$I_2 = \frac{R_T}{R_2}I_T$$

with $$R_T = 4\text{ k}\Omega \parallel 8\text{ k}\Omega = \frac{(4\text{ k}\Omega)(8\text{ k}\Omega)}{4\text{ k}\Omega + 8\text{ k}\Omega} = 2.667\text{ k}\Omega$$

and $$I_2 = \left(\frac{2.667\text{ k}\Omega}{8\text{ k}\Omega}\right)6\text{ A} = (0.333)(6\text{ A}) = \mathbf{2\text{ A}}$$

matching the above solution.

It would appear that the solution with Eq. 6.15(b) is more direct in Example 6.23. However, keep in mind that Eq. (6.14) is applicable to any parallel configuration, removing the necessity to remember two equations.

Now we present a design-type problem.

EXAMPLE 6.24 Determine resistor R_1 in Fig. 6.44 to implement the division of current shown.

Solution: There are essentially two approaches to this type of problem. One involves the direct substitution of known values into the current divider rule equation followed by a mathematical analysis. The other is the sequential application of the basic laws of electric circuits. First we will use the latter approach.

Applying Kirchhoff's current law gives

$$\Sigma I_i = \Sigma I_o$$
$$I = I_1 + I_2$$
$$27\text{ mA} = 21\text{ mA} + I_2$$

and $$I_2 = 27\text{ mA} - 21\text{ mA} = 6\text{ mA}$$

The voltage V_2: $$V_2 = I_2R_2 = (6\text{ mA})(7\ \Omega) = 42\text{ mV}$$

so that $$V_1 = V_2 = 42\text{ mV}$$

Finally, $$R_1 = \frac{V_1}{I_1} = \frac{42\text{ mV}}{21\text{ mA}} = \mathbf{2\ \Omega}$$

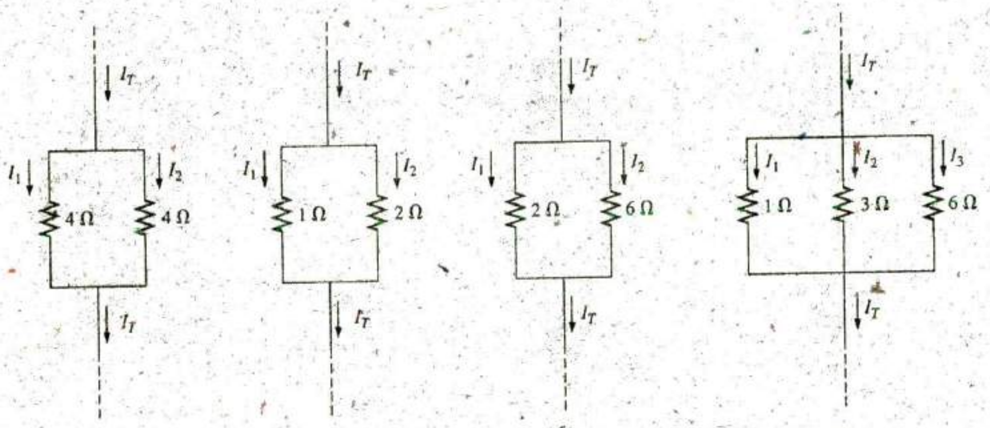
Now for the other approach using the current divider rule:

$$I_1 = \frac{R_2}{R_1 + R_2}I_T$$
$$21\text{ mA} = \left(\frac{7\ \Omega}{R_1 + 7\ \Omega}\right)27\text{ mA}$$

$$(R_1 + 7\ \Omega)(21\ \text{mA}) = (7\ \Omega)(27\ \text{mA})$$
$$(21\ \text{mA})R_1 + 147\ \text{mV} = 189\ \text{mV}$$
$$(21\ \text{mA})R_1 = 189\ \text{mV} - 147\ \text{mV} = 42\ \text{mV}$$

and
$$R_1 = \frac{42\ \text{mV}}{21\ \text{mA}} = 2\ \Omega$$

In summary, therefore, remember that current always seeks the path of least resistance, and the ratio of the resistance values is the inverse of the resulting current levels, as shown in Fig 6.45. The thickness of the blue bands in Fig. 6.45 reflects the relative magnitude of the current in each branch.



$$I_1 = I_2 = \frac{I_T}{2}$$
$$I_1 = 2I_2$$
$$I_1 = \left(\frac{6}{2}\right)I_2 = 3I_2$$
$$I_1 = 6I_3$$
$$I_1 = 3I_2$$
$$I_2 = \left(\frac{6}{3}\right)I_3 = 2I_3$$

FIG. 6.45
Demonstrating how current divides through equal and unequal parallel resistors.

6.7 VOLTAGE SOURCES IN PARALLEL

Because the voltage is the same across parallel elements,

voltage sources can be placed in parallel only if they have the same voltage.

The primary reason for placing two or more batteries or supplies in parallel is to increase the current rating above that of a single supply. For example, in Fig. 6.46, two ideal batteries of 12 V have been placed in

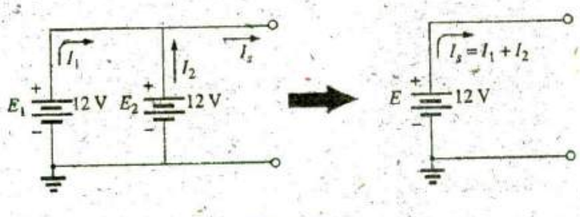


FIG. 6.46
Demonstrating the effect of placing two ideal supplies of the same voltage in parallel.

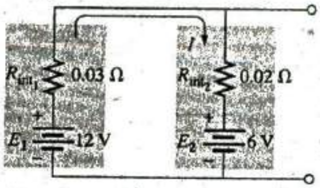


FIG. 6.47
Examining the impact of placing two lead-acid batteries of different terminal voltages in parallel.

parallel. The total source current using Kirchhoff's current law is now the sum of the rated currents of each supply. The resulting power available will be twice that of a single supply if the rated supply current of each is the same. That is,

with $I_1 = I_2 = I$

then $P_T = E(I_1 + I_2) = E(I + I) = E(2I) = 2(EI) = 2P_{(\text{one supply})}$

If for some reason two batteries of different voltages are placed in parallel, both will become ineffective or damaged because the battery with the larger voltage will rapidly discharge through the battery with the smaller terminal voltage. For example, consider two lead-acid batteries of different terminal voltages placed in parallel as shown in Fig 6.47. It makes no sense to talk about placing an ideal 12 V battery in parallel with a 6 V battery because Kirchhoff's voltage law would be violated. However, we can examine the effects if we include the internal resistance levels as shown in Fig. 6.47.

The only current-limiting resistors in the network are the internal resistances, resulting in a very high discharge current for the battery with the larger supply voltage. The resulting current for the case in Fig. 6.47 would be

$$I = \frac{E_1 - E_2}{R_{int_1} + R_{int_2}} = \frac{12\text{ V} - 6\text{ V}}{0.03\ \Omega + 0.02\ \Omega} = \frac{6\text{ V}}{0.05\ \Omega} = 120\text{ A}$$

This value far exceeds the rated drain current of the 12 V battery, resulting in rapid discharge of E_1 and a destructive impact on the smaller supply due to the excessive currents. This type of situation did arise on occasion when some cars still had 6 V batteries. Some people thought, "If I have a 6 V battery, a 12 V battery will work twice as well"—not true!

In general,

it is always recommended that when you are replacing batteries in series or parallel, replace all the batteries.

A fresh battery placed in parallel with an older battery probably has a higher terminal voltage and immediately starts discharging through the older battery. In addition, the available current is less for the older battery, resulting in a higher-than-rated current drain from the newer battery when a load is applied.

6.8 OPEN AND SHORT CIRCUITS

Open circuits and short circuits can often cause more confusion and difficulty in the analysis of a system than standard series or parallel configurations. This will become more obvious in the chapters to follow when we apply some of the methods and theorems.

An **open circuit** is two isolated terminals not connected by an element of any kind, as shown in Fig. 6.48(a). Since a path for conduction does not exist, the current associated with an open circuit must always be zero. The voltage across the open circuit, however, can be any value, as determined by the system it is connected to. In summary, therefore,

an open circuit can have a potential difference (voltage) across its terminals, but the current is always zero amperes.

In Fig. 6.48(b), an open circuit exists between terminals a and b. The voltage across the open-circuit terminals is the supply voltage, but the current is zero due to the absence of a complete circuit.

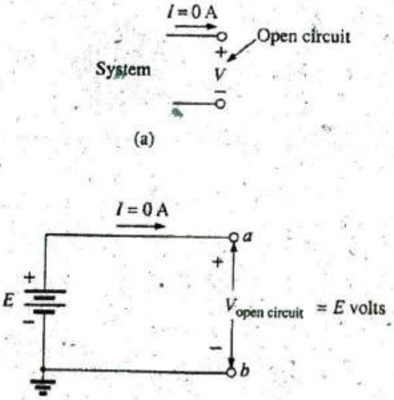


FIG. 6.48
Defining an open circuit.

Some practical examples of open circuits and their impact are provided in Fig. 6.49. In Fig. 6.49(a), the excessive current demanded by the circuit caused a fuse to fail, creating an open circuit that reduced the current to zero amperes. However, it is important to note that *the full applied voltage is now across the open circuit*, so you must be careful when changing the fuse. If there is a main breaker ahead of the fuse, throw it first to remove the possibility of getting a shock. This situation clearly reveals the benefit of circuit breakers: You can reset the breaker without having to get near the hot wires.

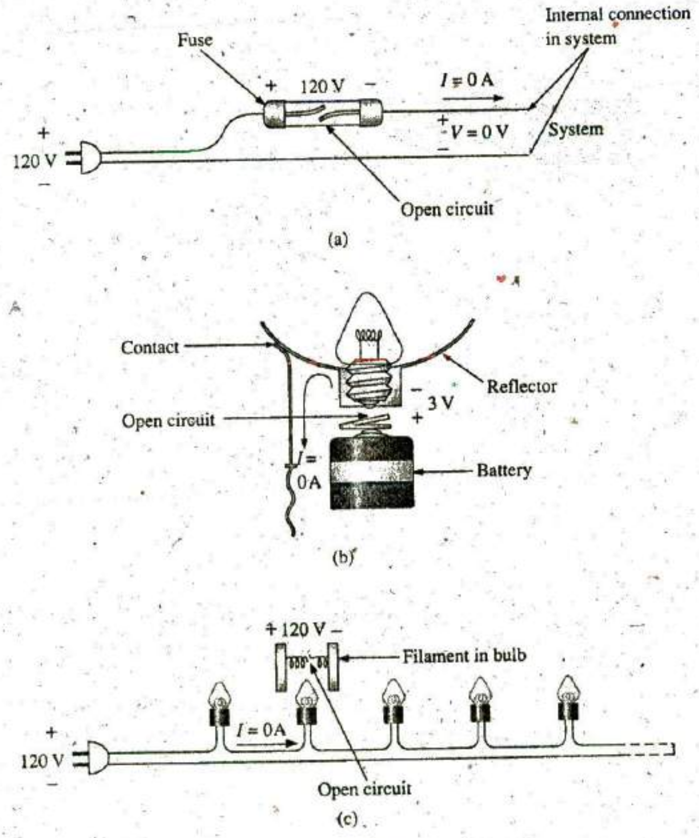


FIG. 6.49

Examples of open circuits.

In Fig. 6.49(b), the pressure plate at the bottom of the bulb cavity in a flashlight was bent when the flashlight was dropped. An open circuit now exists between the contact point of the bulb and the plate connected to the batteries. The current has dropped to zero amperes, but the 3 V provided by the series batteries appears across the open circuit. The situation can be corrected by placing a flat-edge screwdriver under the plate and bending it toward the bulb.

Finally, in Fig. 6.49(c), the filament in a bulb in a series connection has opened due to excessive current or old age, creating an open circuit that knocks out all the bulbs in the series configuration. Again, the current has dropped to zero amperes, but the full 120 V will appear across

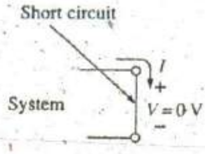


FIG. 6.50
Defining a short circuit.

the contact points of the bad bulb. For situations such as this, *you should remove the plug from the wall before changing the bulb.*

A **short circuit** is a very low resistance, direct connection between two terminals of a network, as shown in Fig. 6.50. The current through the short circuit can be any value, as determined by the system it is connected to, but the voltage across the short circuit is always zero volts because the resistance of the short circuit is assumed to be essentially zero ohms and $V = IR = I(0\ \Omega) = 0$ V.

In summary, therefore,

a short circuit can carry a current of a level determined by the external circuit, but the potential difference (voltage) across its terminals is always zero volts.

In Fig. 6.51(a), the current through the 2 Ω resistor is 5 A. If a short circuit should develop across the 2 Ω resistor, the total resistance of the parallel combination of the 2 Ω resistor and the short (of essentially zero ohms) will be

$$2\ \Omega \parallel 0\ \Omega = \frac{(2\ \Omega)(0\ \Omega)}{2\ \Omega + 0\ \Omega} = 0\ \Omega$$

as indicated in Fig. 6.51(b), and the current will rise to very high levels, as determined by Ohm's law:

$$I = \frac{E}{R} = \frac{10\ \text{V}}{0\ \Omega} \to \infty\ \text{A}$$

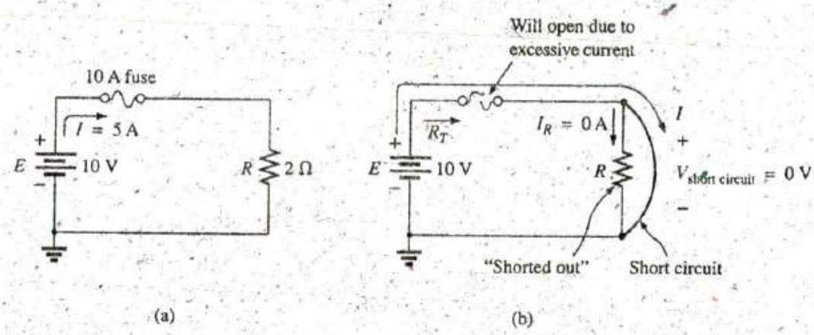


FIG. 6.51
Demonstrating the effect of a short circuit on current levels.

The effect of the 2 Ω resistor has effectively been "shorted out" by the low-resistance connection. The maximum current is now limited only by the circuit breaker or fuse in series with the source.

Some practical examples of short circuits and their impact are provided in Fig. 6.52. In Fig. 6.52(a), a hot (the feed) wire wrapped around a screw became loose and is touching the return connection. A short-circuit connection between the two terminals has been established that could result in a very heavy current and a possible fire hazard. One hopes that the breaker will "pop," and the circuit will be deactivated. Problems such as this are among the reasons aluminum wires (cheaper and lighter than copper) are not permitted in residential or industrial wiring. Aluminum is more sensitive to temperature than copper and will expand and contract due to the heat developed by the current passing

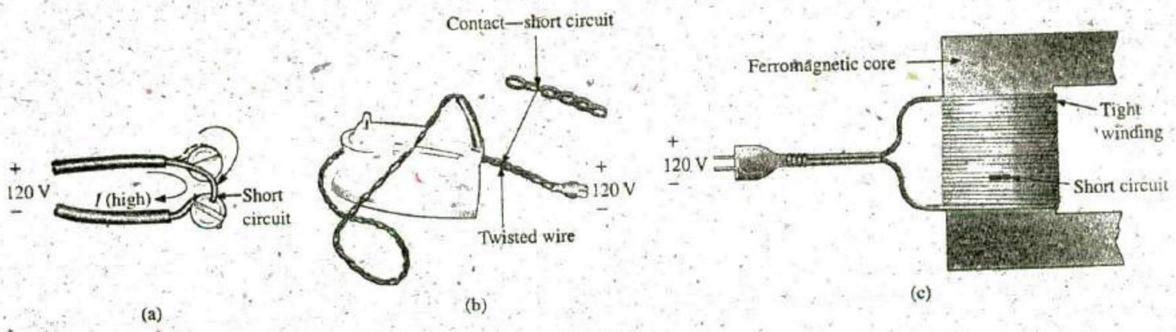


FIG. 6.52
Examples of short circuits.

through the wire. Eventually, this expansion and contraction can loosen the screw, and a wire under some torsional stress from the installation can move and make contact as shown in Fig. 6.52(a). Aluminum is still used in large panels as a bus-bar connection, but it is bolted down.

In Fig. 6.52(b), the wires of an iron have started to twist and crack due to excessive currents or long-term use of the iron. Once the insulation breaks down, the twisting can cause the two wires to touch and establish a short circuit. One can hope that a circuit breaker or fuse will quickly disconnect the circuit. Often, it is not the wire of the iron that causes the problem, but a cheap extension cord with the wrong gage wire. Be aware that you cannot tell the capacity of an extension cord by its outside jacket. It may have a thick orange covering but have a very thin wire inside. Check the gage on the wire the next time you buy an extension cord, and be sure that it is at least #14 gage, with #12 being the better choice for high-current appliances.

Finally, in Fig. 6.52(c), the windings in a transformer or motor for residential or industrial use are illustrated. The windings are wound so tightly together with such a very thin coating of insulation that it is possible with age and use for the insulation to break down and short out the windings. In many cases, shorts can develop, but a short will simply reduce the number of effective windings in the unit. The tool or appliance may still work but with less strength or rotational speed. If you notice such a change in the response, you should check the windings because a short can lead to a dangerous situation. In many cases, the state of the windings can be checked with a simple ohmmeter reading. If a short has occurred, the length of usable wire in the winding has been reduced, and the resistance drops. If you know what the resistance normally is, you can compare and make a judgment.

For the layperson, the terminology *short circuit* or *open circuit* is usually associated with dire situations such as power loss, smoke, or fire. However, in network analysis, both can play an integral role in determining specific parameters of a system. Most often, however, if a short-circuit condition is to be established, it is accomplished with a *jumper*—a lead of negligible resistance to be connected between the points of interest. Establishing an open circuit just requires making sure that the terminals of interest are isolated from each other.

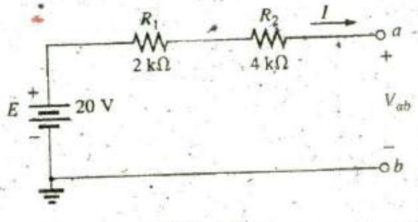


FIG. 6.53
Network for Example 6.25.

EXAMPLE 6.25 Determine voltage V_{ab} for the network in Fig. 6.53.

Solution: The open circuit requires that I be zero amperes. The voltage drop across both resistors is therefore zero volts since

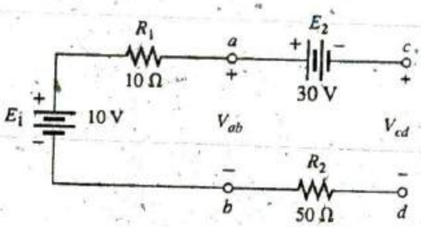


FIG. 6.54
Network for Example 6.26.

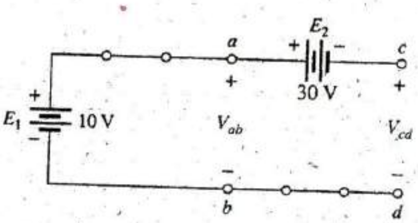


FIG. 6.55
Circuit in Fig. 6.54 redrawn.

$V = IR = (0)R = 0$ V. Applying Kirchhoff's voltage law around the closed loop gives

$$V_{ab} = E = \mathbf{20\ V}$$

EXAMPLE 6.26 Determine voltages V_{ab} and V_{cd} for the network in Fig. 6.54.

Solution: The current through the system is zero amperes due to the open circuit, resulting in a 0 V drop across each resistor. Both resistors can therefore be replaced by short circuits, as shown in Fig. 6.55. Voltage V_{ab} is then directly across the 10 V battery, and

$$V_{ab} = E_1 = \mathbf{10\ V}$$

Voltage V_{cd} requires an application of Kirchhoff's voltage law:

$$+E_1 - E_2 - V_{cd} = 0$$

or
$$V_{cd} = E_1 - E_2 = 10\ \text{V} - 30\ \text{V} = \mathbf{-20\ V}$$

The negative sign in the solution indicates that the actual voltage V_{cd} has the opposite polarity of that appearing in Fig. 6.54.

EXAMPLE 6.27 Determine the unknown voltage and current for each network in Fig. 6.56.

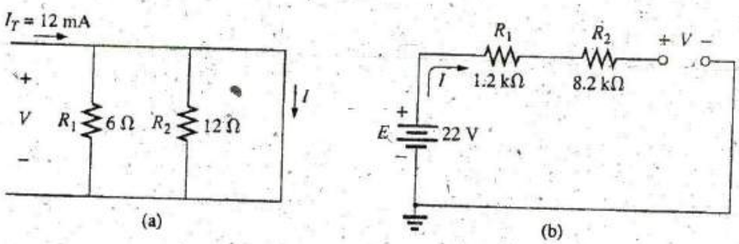


FIG. 6.56
Networks for Example 6.27.

Solution: For the network in Fig. 6.56(a), the current I_T will take the path of least resistance, and since the short-circuit condition at the end of the network is the least-resistance path, all the current will pass through the short circuit. This conclusion can be verified using the current divider rule. The voltage across the network is the same as that across the short circuit and will be zero volts, as shown in Fig. 6.57(a).

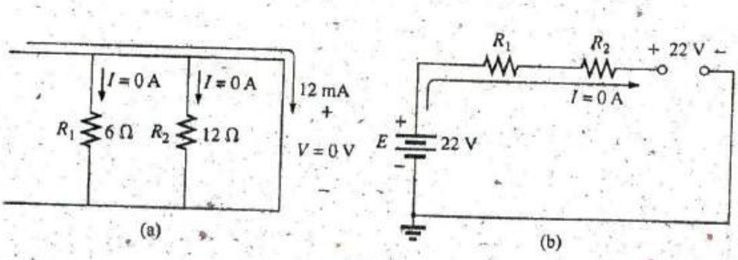


FIG. 6.57
Solutions to Example 6.27.

For the network in Fig. 6.56(b), the open-circuit condition requires that the current be zero amperes. The voltage drops across the resistors must therefore be zero volts, as determined by Ohm's law [$V_R = IR = (0)R = 0$ V], with the resistors acting as a connection from the supply to the open circuit. The result is that the open-circuit voltage is $E = 22$ V, as shown in Fig. 6.57(b).

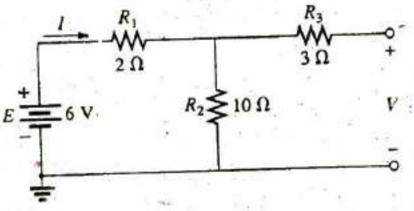


FIG. 6.58
Network for Example 6.28.

EXAMPLE 6.28 Determine V and I for the network in Fig. 6.58 if resistor R_2 is shorted out.

Solution: The redrawn network appears in Fig. 6.59. The current through the 3 Ω resistor is zero due to the open circuit, causing all the current I to pass through the jumper. Since $V_{3\Omega} = IR = (0)R = 0$ V, the voltage V is directly across the short, and

$$V = \mathbf{0\ V}$$

with
$$I = \frac{E}{R_1} = \frac{6\text{ V}}{2\ \Omega} = \mathbf{3\ A}$$

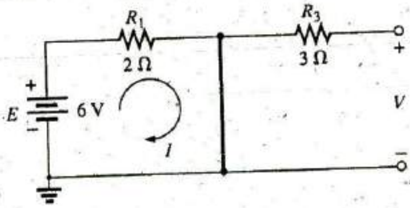


FIG. 6.59
Network in Fig. 6.58 with R_2 replaced by a jumper.

6.9 VOLTMETER LOADING EFFECTS

In previous chapters, we learned that ammeters are not ideal instruments. When you insert an ammeter, you actually introduce an additional resistance in series with the branch in which you are measuring the current. Generally, this is not a serious problem, but it can have a troubling effect on your readings, so it is important to be aware of it.

Voltmeters also have an internal resistance that appears between the two terminals of interest when a measurement is being made. While an ammeter places an additional resistance in series with the branch of interest, a voltmeter places an additional resistance *across* the element, as shown in Fig. 6.60. Since it appears in parallel with the element of interest, *the ideal level for the internal resistance of a voltmeter would be infinite ohms, just as zero ohms would be ideal for an ammeter.* Unfortunately, the internal resistance of any voltmeter is not infinite and changes from one type of meter to another.

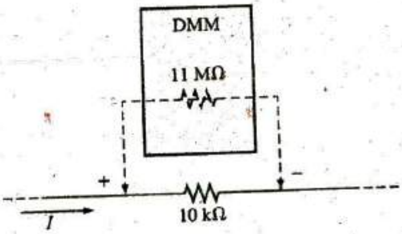


FIG. 6.60
Voltmeter loading.

Most digital meters have a fixed internal resistance level in the megohm range that remains the same *for all its scales.* For example, the meter in Fig. 6.60 has the typical level of 11 MΩ for its internal resistance, no matter which voltage scale is used. When the meter is placed across the 10 kΩ resistor, the total resistance of the combination is

$$R_T = 10\text{ k}\Omega \parallel 11\text{ M}\Omega = \frac{(10^4\ \Omega)(11 \times 10^6\ \Omega)}{10^4\ \Omega + (11 \times 10^6)} = 9.99\text{ k}\Omega$$

and the behavior of the network is not seriously affected. The result, therefore, is that

most digital voltmeters can be used in circuits with resistances up to the high-kilohm range without concern for the effect of the internal resistance on the reading.

However, if the resistances are in the megohm range, you should investigate the effect of the internal resistance.

An analog VOM is a different matter, however, because the internal resistance levels are much lower and the internal resistance levels are a function of the scale used. If a VOM on the 2.5 V scale were placed across the 10 kΩ resistor in Fig. 6.60, the internal resistance might be 50 kΩ, resulting in a combined resistance of

$$R_T = 10\text{ k}\Omega \parallel 50\text{ k}\Omega = \frac{(10^4\,\Omega)(50 \times 10^3\,\Omega)}{10^4\,\Omega + (50 \times 10^3\,\Omega)} = 8.33\text{ k}\Omega$$

and the behavior of the network would be affected because the 10 kΩ resistor would appear as an 8.33 kΩ resistor.

To determine the resistance R_m of any scale of a VOM, simply multiply the **maximum voltage** of the chosen scale by the **ohm/volt (Ω/V) rating** normally appearing at the bottom of the face of the meter. That is,

$$R_m\,(\text{VOM}) = (\text{scale})(\Omega/\text{V rating})$$

For a typical Ω/V rating of 20,000, the 2.5 V scale would have an internal resistance of

$$(2.5\text{ V})(20{,}000\ \Omega/\text{V}) = \mathbf{50\ k\Omega}$$

whereas for the 100 V scale, the internal resistance of the VOM would be

$$(100\text{ V})(20{,}000\ \Omega/\text{V}) = \mathbf{2\ M\Omega}$$

and for the 250 V scale,

$$(250\text{ V})(20{,}000\ \Omega/\text{V}) = \mathbf{5\ M\Omega}$$

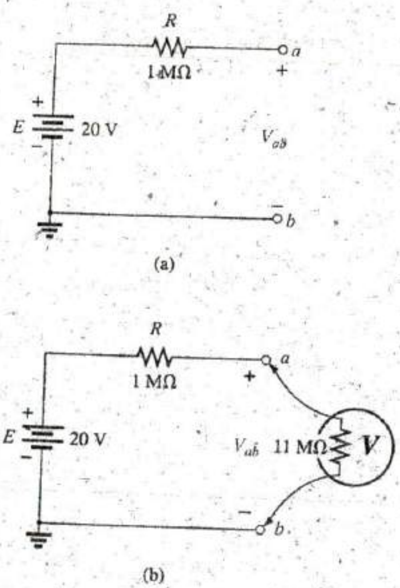


FIG. 6.61
(a) Measuring an open-circuit voltage with a voltmeter; (b) determining the effect of using a digital voltmeter with an internal resistance of 11 MΩ on measuring an open-circuit voltage (Example 6.29).

EXAMPLE 6.29 For the relatively simple circuit in Fig. 6.61(a):

a. What is the open-circuit voltage V_{ab}?
b. What will a DMM indicate if it has an internal resistance of 11 MΩ? Compare your answer to that of part (a).
c. Repeat part (b) for a VOM with an Ω/V rating of 20,000 on the 100 V scale.

Solutions:

a. Due to the open circuit, the current is zero, and the voltage drop across the 1 MΩ resistor is zero volts. The result is that the entire source voltage appears between points *a* and *b*, and

$$V_{ab} = \mathbf{20\ V}$$

b. When the meter is connected as shown in Fig. 6.61(b), a complete circuit has been established, and current can pass through the circuit. The voltmeter reading can be determined using the voltage divider rule as follows:

$$V_{ab} = \frac{(11\text{ M}\Omega)(20\text{ V})}{(11\text{ M}\Omega + 1\text{ M}\Omega)} = \mathbf{18.33\ V}$$

and the reading is affected somewhat.

c. For the VOM, the internal resistance of the meter is

$$R_m = (100\text{ V})\,(20{,}000\ \Omega/\text{V}) = 2\text{ M}\Omega$$

and $$V_{ab} = \frac{(2\text{ M}\Omega)(20\text{ V})}{(2\text{ M}\Omega + 1\text{ M}\Omega)} = \mathbf{13.33\ V}$$

which is considerably below the desired level of 20 V.

6.10 SUMMARY TABLE

Now that the series and parallel configurations have been covered in detail, we will review the salient equations and characteristics of each. The equations for the two configurations have a number of similarities. In fact, the equations for one can often be obtained directly from the other by simply applying the **duality** principle. Duality between equations means that the format for an equation can be applied to two different situations by just changing the variable of interest. For instance, the equation for the total resistance of a series circuit is the sum of the resistances. By changing the resistance parameters to conductance parameters, you can obtain the equation for the total conductance of a parallel network—an easy way to remember the two equations. Similarly, by starting with the total conductance equation, you can easily write the total resistance equation for series circuits by replacing the conductance parameters by resistance parameters. Series and parallel networks share two important dual relationships: (1) between resistance of series circuits and conductance of parallel circuits and (2) between the voltage or current of a series circuit and the current or voltage, respectively, of a parallel circuit. Table 6.1 summarizes this duality.

TABLE 6.1
Summary table.

Series and Parallel Circuits		
Series	**Duality**	**Parallel**
$R_T = R_1 + R_2 + R_3 + \cdots + R_N$	$R \rightleftarrows G$	$G_T = G_1 + G_2 + G_3 + \cdots + G_N$
R_T increases (G_T decreases) if additional resistors are added in series	$R \rightleftarrows G$	G_T increases (R_T decreases) if additional resistors are added in parallel
Special case: two elements $R_T = R_1 + R_2$	$R \rightleftarrows G$	$G_T = G_1 + G_2$
I the same through series elements	$I \rightleftarrows V$	V the same across parallel elements
$E = V_1 + V_2 + V_3$	$E, V \rightleftarrows I$	$I_T = I_1 + I_2 + I_3$
Largest V across largest R	$V \rightleftarrows I$ and $R \rightleftarrows G$	Greatest I through largest G (smallest R)
$V_x = \dfrac{R_x E}{R_T}$	$E, V \rightleftarrows I$ and $R \rightleftarrows G$	$I_x = \dfrac{G_x I_T}{G_T}$
$P = EI_T$	$E \rightleftarrows I$, and $I \rightleftarrows E$	$P = I_T E$
$P = I^2R$	$I \rightleftarrows V$ and $R \rightleftarrows G$	$P = V^2G$
$P = V^2/R$	$V \rightleftarrows I$ and $R \rightleftarrows G$	$P = I^2/G$

The format for the total resistance for a series circuit has the same format as the total conductance of a parallel network, as shown in Table 6.1. All that is required to move back and forth between the series and parallel headings is to interchange the letters R and G. For the special case of two elements, the equations have the same format, but the equation applied for the total resistance of the parallel configuration has changed. In the series configuration, the total resistance increases with each added resistor. For parallel networks, the total conductance increases with each additional conductance. The result is that the total conductance of a series circuit drops with added resistive elements, while the total resistance of parallel networks decreases with added elements.

In a series circuit, the current is the same everywhere. In a parallel network, the voltage is the same across each element. The result is a duality between voltage and current for the two configurations. What is true for one in one configuration is true for the other in the other configuration. In a series circuit, the applied voltage divides between the series elements. In a parallel network, the current divides between parallel elements. For series circuits, the largest resistor captures the largest share of the applied voltage. For parallel networks, the branch with the highest conductance captures the greater share of the incoming current. In addition, for series circuits, the applied voltage equals the sum of the voltage drops across the series elements of the circuit, while the source current for parallel branches equals the sum of the currents through all the parallel branches.

The total power delivered to a series or parallel network is determined by the product of the applied voltage and resulting source current. The power delivered to each element is also the same for each configuration. Duality can be applied again, but the equation $P = EI$ results in the same result as $P = IE$. Also, $P = I^2R$ can be replaced by $P = V^2G$ for parallel elements, but essentially each can be used for each configuration. The duality principle can be very helpful in the learning process. Remember this as you progress through the next few chapters. You will find in the later chapters that this duality can also be applied between two important elements—inductors and capacitors.

6.11 TROUBLESHOOTING TECHNIQUES

The art of *troubleshooting* is not limited solely to electrical or electronic systems. In the broad sense,

troubleshooting is a process by which acquired knowledge and experience are used to localize a problem and offer or implement a solution.

There are many reasons why the simplest electrical circuit might not be operating correctly. A connection may be open; the measuring instruments may need calibration; the power supply may not be on or may have been connected incorrectly to the circuit; an element may not be performing correctly due to earlier damage or poor manufacturing; a fuse may have blown; and so on. Unfortunately, a defined sequence of steps does not exist for identifying the wide range of problems that can surface in an electrical system. It is only through experience and a clear understanding of the basic laws of electric circuits that you can become proficient at quickly locating the cause of an erroneous output.

It should be fairly obvious, however, that the first step in checking a network or identifying a problem area is to have some idea of the expected voltage and current levels. For instance, the circuit in Fig. 6.62 should have a current in the low milliampere range, with the majority of the supply voltage across the 8 kΩ resistor. However, as indicated in Fig. 6.62, $V_{R_1} = V_{R_2} = 0$ V and $V_a = 20$ V. Since $V = IR$, the results immediately suggest that $I = 0$ A and an open circuit exists in the circuit. The fact that $V_a = 20$ V immediately tells us that the connections are true from the ground of the supply to point *a*. The open circuit must therefore exist between R_1 and R_2 or at the ground connection of R_2. An open circuit at either point results in $I = 0$ A and the readings obtained previously. Keep

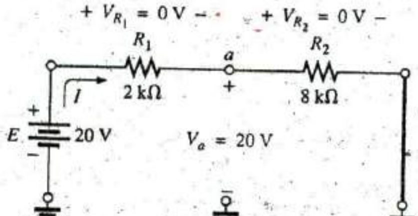


FIG. 6.62
A malfunctioning network.

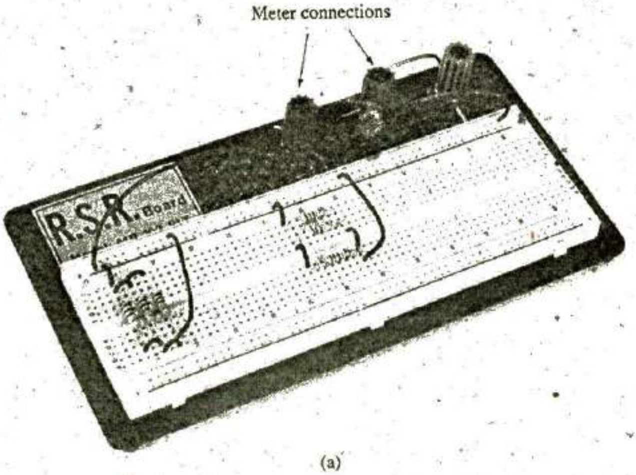
in mind that, even though $I = 0$ A, R_1 does form a connection between the supply and point a. That is, if $I = 0$ A, $V_{R_1} = IR_2 = (0)R_2 = 0$ V, as obtained for a short circuit.

In Fig. 6.62, if $V_{R_1} \cong 20$ V and V_{R_2} is quite small ($\cong 0.08$ V), it first suggests that the circuit is complete, a current does exist, and a problem surrounds the resistor R_2. R_2 is not shorted out since such a condition would result in $V_{R_2} = 0$V. A careful check of the inserted resistor reveals that an 8 Ω resistor was used rather than the 8 kΩ resistor specified—an incorrect reading of the color code. To avoid this, an ohmmeter should be used to check a resistor to validate the color-code reading or to ensure that its value is still in the prescribed range set by the color code.

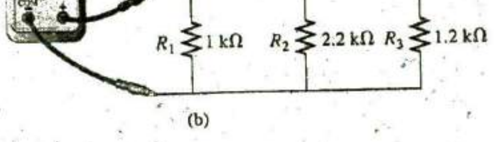
Occasionally, the problem may be difficult to diagnose. You've checked all the elements, and all the connections appear tight. The supply is on and set at the proper level; the meters appear to be functioning correctly. In situations such as this, experience becomes a key factor. Perhaps you can recall when a recent check of a resistor revealed that the internal connection (not externally visible) was a "make or break" situation or that the resistor was damaged earlier by excessive current levels, so its actual resistance was much lower than called for by the color code. Recheck the supply! Perhaps the terminal voltage was set correctly, but the current control knob was left in the zero or minimum position. Is the ground connection stable? The questions that arise may seem endless. However, as you gain experience, you will be able to localize problems more rapidly. Of course, the more complicated the system, the longer is the list of possibilities, but it is often possible to identify a particular area of the system that is behaving improperly before checking individual elements.

6.12 PROTOBOARDS (BREADBOARDS)

In Section 5.12, the protoboard was introduced with the connections for a simple series circuit. To continue the development, the network in Fig. 6.17 was set up on the board in Fig. 6.63(a) using two different techniques. The possibilities are endless, but these two solutions use a fairly straightforward approach.



(a)



(b)

FIG. 6.63

Using a protoboard to set up the circuit in Fig. 6.17.

First, note that the supply lines and ground are established across the length of the board using the horizontal conduction zones at the top and bottom of the board through the connections to the terminals. The network to the left on the board was used to set up the circuit in much the same manner as it appears in the schematic of Fig. 6.63(b). This approach required that the resistors be connected between two vertical conducting strips. If placed perfectly vertical in a single conducting strip, the resistors would have shorted out. Often, setting the network up in a manner that best copies the original can make it easier to check and make measurements. The network to the right in part (a) used the vertical conducting strips to connect the resistors together at each end. Since there wasn't enough room for all three, a connection had to be added from the upper vertical set to the lower set. The resistors are in order R_1, R_2, and R_3 from the top down. For both configurations, the ohmmeter can be connected to the positive lead of the supply terminal and the negative or ground terminal.

Take a moment to review the connections and think of other possibilities. Improvements can often be made, and it can be satisfying to find the most effective setup with the least number of connecting wires.

6.13 APPLICATIONS

One of the most important advantages of the parallel configuration is that

if one branch of the configuration should fail (open circuit), the remaining branches will still have full operating power.

In a home, the parallel connection is used throughout to ensure that if one circuit has a problem and opens the circuit breaker, the remaining circuits still have the full 120 V. The same is true in automobiles, computer systems, industrial plants, and wherever it would be disastrous for one circuit to control the total power distribution.

Another important advantage is that

branches can be added at any time without affecting the behavior of those already in place.

In other words, unlike the series connection, where an additional component reduces the current level and perhaps affects the response of some of the existing components, an additional parallel branch will not affect the current level in the other branches. Of course, the current demand from the supply increases as determined by Kirchhoff's current law, so you must be aware of the limitations of the supply.

The following are some of the most common applications of the parallel configuration.

Car System

As you begin to examine the electrical system of an automobile, the most important thing to understand is that the entire electrical system of a car is run as a *dc system.* Although the generator produces a varying ac signal, rectification converts it to one having an average dc level for charging the battery. In particular, note the use of a filter capacitor in the alternator branch in Fig. 6.64 to smooth out the rectified ac waveform and to provide an improved dc supply. The charged battery must therefore provide the required direct current for the entire electrical system of the car. Thus, the power demand on the battery at any instant is the product

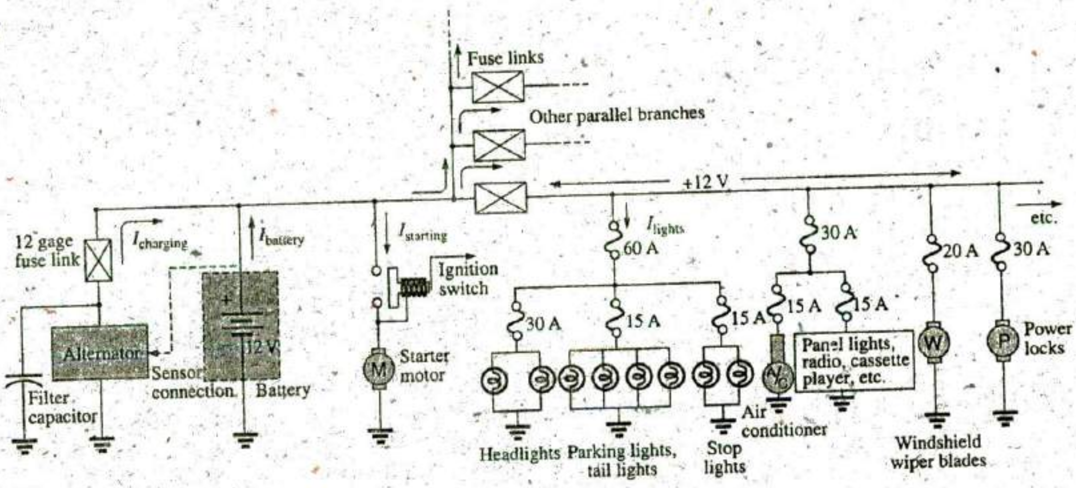


FIG. 6.64
Expanded view of an automobile's electrical system.

of the terminal voltage and the current drain of the total load of every operating system of the car. This certainly places an enormous burden on the battery and its internal chemical reaction and warrants all the battery care we can provide.

Since the electrical system of a car is essentially a parallel system, the total current drain on the battery is the sum of the currents to all the parallel branches of the car connected directly to the battery. In Fig. 6.64, a few branches of the wiring diagram for a car have been sketched to provide some background information on basic wiring, current levels, and fuse configurations. Every automobile has fuse links and fuses, and some also have circuit breakers, to protect the various components of the car and to ensure that a dangerous fire situation does not develop. Except for a few branches that may have series elements, the operating voltage for most components of a car is the terminal voltage of the battery, which we will designate as 12 V even though it will typically vary between 12 V and the charging level of 14.6 V. In other words, each component is connected to the battery at one end and to the ground or chassis of the car at the other end.

Referring to Fig. 6.64, note that the alternator or charging branch of the system is connected directly across the battery to provide the charging current as indicated. Once the car is started, the rotor of the alternator turns, generating an ac varying voltage that then passes through a rectifier network and filter to provide the dc charging voltage for the battery. Charging occurs only when the sensor connected directly to the battery signals that the terminal voltage of the battery is too low. Just to the right of the battery the starter branch was included to demonstrate that there is no fusing action between the battery and starter when the ignition switch is activated. The lack of fusing action is provided because enormous starting currents (hundreds of amperes) flow through the starter to start a car that has not been used for days and/or has been sitting in a cold climate—and high friction occurs between components until the oil starts flowing. The starting level can vary so much that it would be difficult to find the right fuse level, and frequent high currents may damage the fuse link and cause a failure at expected levels of current.

When the ignition switch is activated, the starting relay completes the circuit between the battery and starter, and, it is hoped, the car starts. If a car fails to start, the first thing to check is the connections at the battery, starting relay, and starter to be sure that they are not providing an unexpected open circuit due to vibration, corrosion, or moisture.

Once the car has started, the starting relay opens, and the battery begins to activate the operating components of the car. Although the diagram in Fig. 6.64 does not display the switching mechanism, the entire electrical network of the car, except for the important external lights, is usually disengaged so that the full strength of the battery can be dedicated to the starting process. The lights are included for situations where turning the lights off, even for short periods of time, could create a dangerous situation. If the car is in a safe environment, it is best to leave the lights off when starting, to save the battery an additional 30 A of drain. If the lights are on, they dim because of the starter drain, which may exceed 500 A. Today, batteries are typically rated in cranking (starting) current rather than ampere-hours. Batteries rated with cold cranking ampere ratings between 700 A and 1000 A are typical today.

Separating the alternator from the battery and the battery from the numerous networks of the car are fuse links such as shown in Fig. 6.65. Fuse links are actually wires of a specific gage designed to open at fairly high current levels of 100 A or more. They are included to protect against those situations where there is an unexpected current drawn from the many circuits to which they are connected. That heavy drain can, of course, be from a short circuit in one of the branches, but in such cases the fuse in that branch will probably release. The fuse link is an additional protection for the line if the total current drawn by the parallel-connected branches begins to exceed safe levels. The fuses following the fuse link have the appearance shown in Fig. 6.65(b), where a gap between the legs of the fuse indicates a blown fuse. As shown in Fig. 6.64, the 60 A fuse (often called a *power distribution fuse*) for the lights is a second-tier fuse sensitive to the total drain from the three light circuits. Finally, the third fuse level is for the individual units of a car such as the lights, air conditioner, and power locks. In each case, the fuse rating exceeds the normal load (current level) of the operating component, but the level of each fuse does give some indication of the demand to be expected under normal operating conditions. For instance, headlights typically draw more than 10 A, tail lights more than 5 A, air conditioner about 10 A (when the clutch engages), and power windows 10 A to 20 A, depending on how many are operated at once.

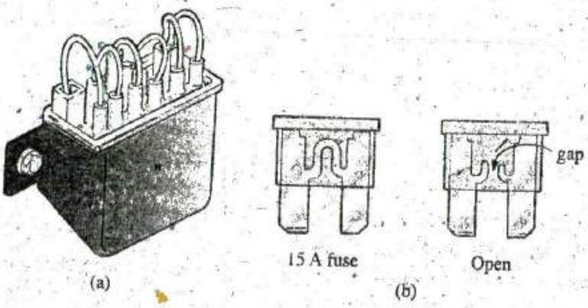


FIG. 6.65
Car fuses: (a) fuse link; (b) plug-in.

Some details for only one section of the total car network are provided in Fig. 6.64. In the same figure, additional parallel paths with their respective fuses have been provided to further reveal the parallel arrangement of all the circuits.

In most vehicles the return path to the battery through the ground connection is through the chassis of the car. That is, there is only one wire to each electrical load, with the other end simply grounded to the chassis. The return to the battery (chassis to negative terminal) is therefore a heavy-gage wire matching that connected to the positive terminal. In some cars constructed of a mixture of materials such as metal, plastic, and rubber, the return path through the metallic chassis may be lost, and two wires must be connected to each electrical load of the car.

House Wiring

In Chapter 4, the basic power levels of importance were discussed for various services to the home. We are now ready to take the next step and examine the actual connection of elements in the home.

First, it is important to realize that except for some very special circumstances, the basic wiring is done in a parallel configuration. Each parallel branch, however, can have a combination of parallel and series elements. Every full branch of the circuit receives the full 120 V or 208 V, with the current determined by the applied load. Figure 6.66(a) provides the detailed wiring of a single circuit having a light bulb and two outlets. Figure 6.66(b) shows the schematic representation. Note that although each load is in parallel with the supply, switches are always connected in series with the load. The power is transmitted to the lamp only when the switch is closed and the full 120 V appears across the bulb. The connection point for the two outlets is in the ceiling box holding the light bulb. Since a switch is not present, both outlets are always "hot" unless the circuit breaker in the main panel is opened. This is important to understand in case you are tempted to change the light fixture by simply turning off the wall switch. True, if you're very careful, you can work with one line at a time (being sure that you don't touch the other line at any time), but it is much safer to throw the circuit breaker on the panel whenever working on a circuit. Note in Fig. 6.66(a) that the *feed* wire (black) into the fixture from the panel is connected to the switch and both outlets at one point. It is not connected directly to the light fixture because the lamp would be on all the time. Power to the light fixture is made available through the switch. The continuous connection to the outlets from the panel ensures that the outlets are "hot" whenever the circuit breaker in the panel is on. Note also how the *return* wire (white) is connected directly to the light switch and outlets to provide a return for each component. There is no need for the white wire to go through the switch since an applied voltage is a two-point connection and the black wire is controlled by the switch.

Proper grounding of the system in total and of the individual loads is one of the most important facets in the installation of any system. There is a tendency at times to be satisfied that the system is working and to pay less attention to proper grounding technique. Always keep in mind that a properly grounded system has a direct path to ground if an undesirable situation should develop. The absence of a direct ground causes the system to determine its own path to ground, and you could be that path if you happened to touch the wrong wire, metal box, metal pipe,

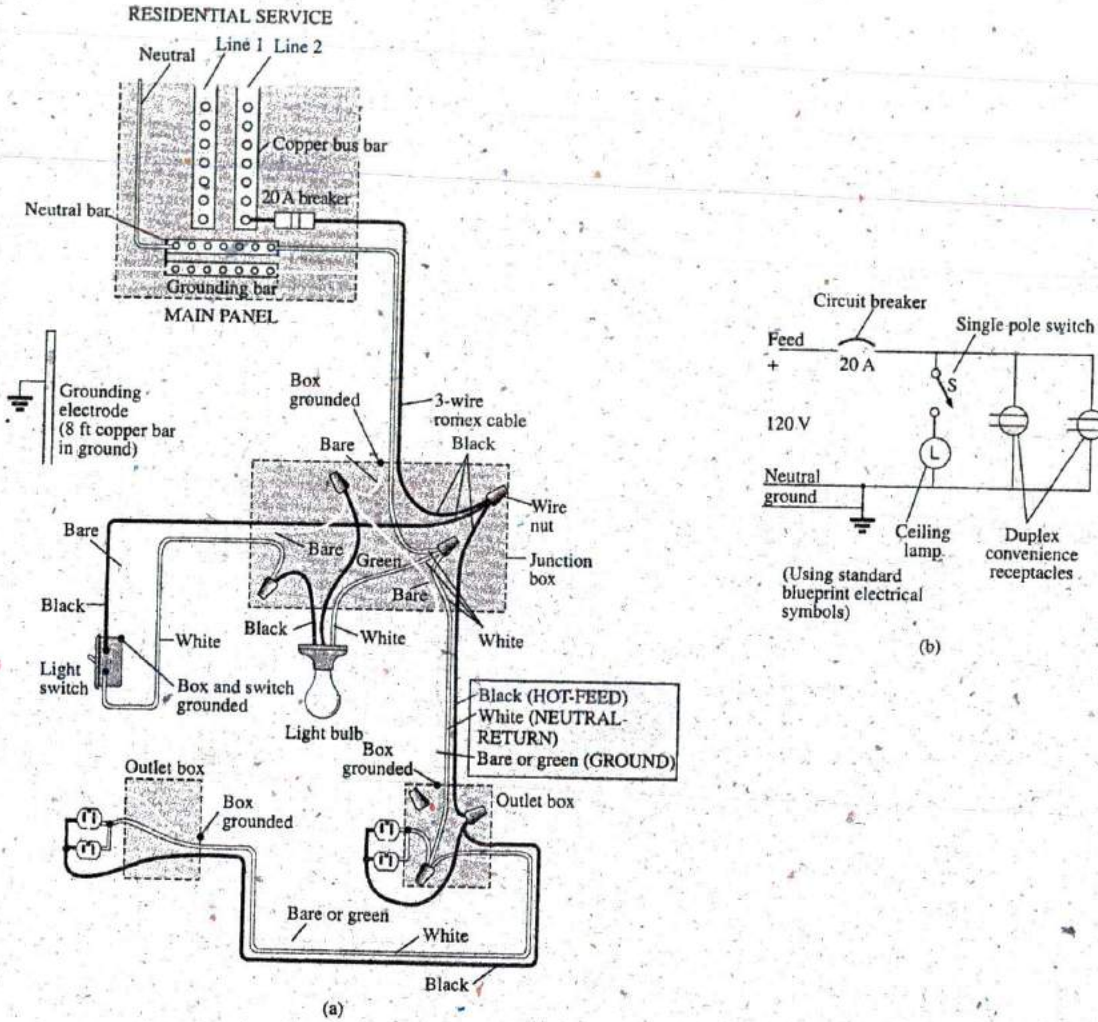


FIG. 6.66
Single phase of house wiring: (a) physical details; (b) schematic representation.

and so on. In Fig. 6.66(a), the connections for the ground wires have been included. For the romex (plastic-coated wire) used in Fig. 6.66(a), the ground wire is provided as a bare copper wire. Note that it is connected to the panel, which in turn is directly connected to the grounded 8 ft copper rod. In addition, note that the ground connection is carried through the entire circuit, including the switch, light fixture, and outlets. It is one continuous connection. If the outlet box, switch box, and housing for the light fixture are made of a conductive material such as metal, the ground will be connected to each. If each is plastic, there is no need for the ground connection. However, the switch, both outlets, and the fixture itself are connected to ground. For the switch and outlets, there is usually a green screw for the ground wire, which is connected to the entire framework of the switch or outlet as shown in Fig. 6.67, including the ground connection of the outlet. For both the switch and the outlet, even the screw or screws used to hold the outside plate in place are grounded since they are screwed into the metal housing of the switch or outlet. When screwed into a metal box, the ground connection can be made by the screws that hold the switch or outlet in the box as shown in Fig. 6.67. *Always pay strict attention to the grounding process whenever installing any electrical equipment.*

On the practical side, whenever hooking up a wire to a screw-type terminal, always wrap the wire around the screw in the clockwise manner so that when you tighten the screw, it grabs the wire and turns it in the same direction. An expanded view of a typical house-wiring arrangement appears in Chapter 15.

Parallel Computer Bus Connections

The internal construction (hardware) of large mainframe computers and personal computers is set up to accept a variety of adapter cards in the slots appearing in Fig. 6.68(a). The primary board (usually the largest), commonly called the *motherboard,* contains most of the functions

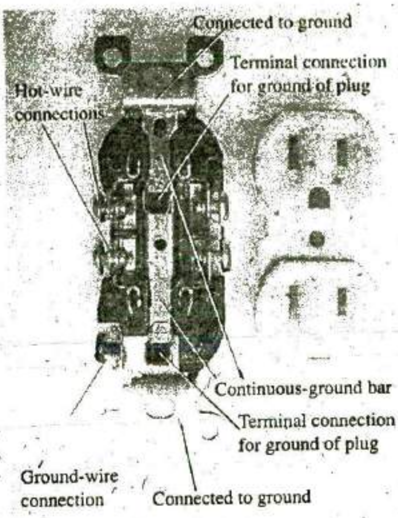


FIG. 6.67
Continuous ground connection in a duplex outlet.

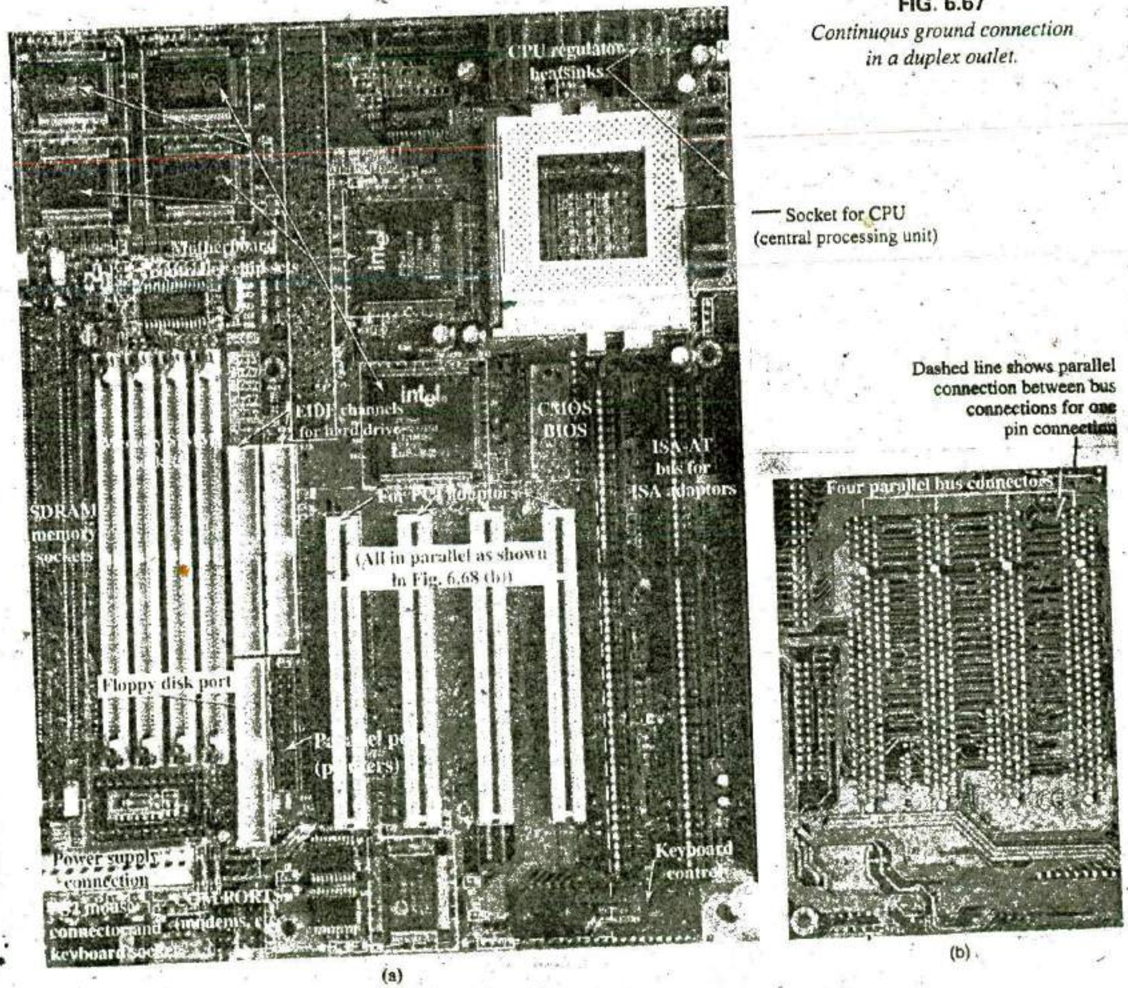


FIG. 6.68
(a) Motherboard for a desktop computer; (b) the printed circuit board connections for the region indicated in part (a).

required for full computer operation. Adapter cards are normally added to expand the memory, set up a network, add peripheral equipment, and so on. For instance, if you decide to add another hard drive to your computer, you can simply insert the card into the proper channel of Fig. 6.68(a). The bus connectors are connected in parallel with common connections to the power supply, address and data buses, control signals, ground, and so on. For instance, if the bottom connection of each bus connector is a ground connection, that ground connection carries through each bus connector and is immediately connected to any adapter card installed. Each card has a slot connector that fits directly into the bus connector without the need for any soldering or construction. The pins of the adapter card are then designed to provide a path between the motherboard and its components to support the desired function. Note in Fig. 6.68(b), which is a back view of the region identified in Fig. 6.68(a), that if you follow the path of the second pin from the top on the far left, you will see that it is connected to the same pin on the other three bus connectors.

Most small laptop computers today have all the options already installed, thereby bypassing the need for bus connectors. Additional memory and other upgrades are added as direct inserts into the motherboard.

6.14 COMPUTER ANALYSIS

PSpice

Parallel dc Network The computer analysis coverage for parallel dc circuits is very similar to that for series dc circuits. However, in this case the voltage is the same across all the parallel elements, and the current through each branch changes with the resistance value. The parallel network to be analyzed will have a wide range of resistor values to demonstrate the effect on the resulting current. The following is a list of abbreviations for any parameter of a network when using PSpice:

$$\begin{aligned}
\mathbf{f} &= 10^{-15}\\
\mathbf{p} &= 10^{-12}\\
\mathbf{n} &= 10^{-9}\\
\mathbf{u} &= 10^{-6}\\
\mathbf{m} &= 10^{-3}\\
\mathbf{k} &= 10^{+3}\\
\mathbf{MEG} &= 10^{+6}\\
\mathbf{G} &= 10^{+9}\\
\mathbf{T} &= 10^{+12}
\end{aligned}$$

In particular, note that **m** (or **M**) is used for "milli" and **MEG** for "megohms." Also, PSpice does not distinguish between upper- and lower-case units, but certain parameters typically use either the upper- or lower-case abbreviation as shown above.

Since the details of setting up a network and going through the simulation process were covered in detail in Sections 4.9 and 5.14 for dc circuits, the coverage here is limited solely to the various steps required. These steps will help you learn how to "draw" a circuit and then run a simulation fairly quickly and easily.

After selecting the **Create document** key (the top left of the screen), the following sequence opens the **Schematic** window: **PSpice 6-1-OK-Create a blank project-OK-PAGE1** (if necessary).

Add the voltage source and resistors as described in detail in earlier sections, but now you need to turn the resistors 90°. You do this by right-clicking before setting a resistor in place. Choose **Rotate** from the list of options, which turns the resistor counterclockwise 90°. It can also be rotated by simultaneously selecting **Ctrl-R.** The resistor can then be placed in position by a left click. An additional benefit of this maneuver is that the remaining resistors to be placed will already be in the vertical position. The values selected for the voltage source and resistors appear in Fig. 6.69.

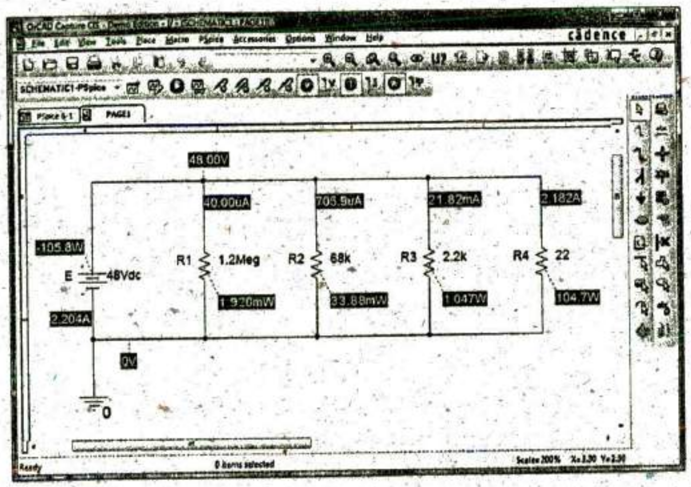


FIG. 6.69
Applying PSpice to a parallel network.

Once the network is complete, you can obtain the simulation and the results through the following sequence: **Select New Simulation Profile key-Bias Point-Create-Analysis-Bias Point-OK-Run PSpice key-Exit(X).**

The result, shown in Fig. 6.69, reveals that the voltage is the same across all the parallel elements, and the current increases significantly with decrease in resistance. The range in resistor values suggests, by inspection, that the total resistance is just less than the smallest resistance of 22 Ω. Using Ohm's law and the source current of 2.204 A results in a total resistance of $R_T = E/I_s = 48\text{ V}/2.204\text{ A} = 21.78\ \Omega$, confirming the above conclusion.

Multisim

Parallel dc Network For comparison purposes with the PSpice approach, the same parallel network in Fig. 6.69 is now analyzed using Multisim. The source and ground are selected and placed as shown in Fig. 6.70 using the procedure defined in previous chapters. For the resistors, choose the resistor symbol in the **BASIC toolbar** listing. However, you must rotate it 90° to match the configuration of Fig. 6.69. You do this by first clicking on the resistor symbol to place it in the active state. (Be sure that the resulting small black squares surround the symbol, label, and value; otherwise, you may have activated only the label or value.) Then right-click inside the rectangle. Select **90° Clockwise,** and the resistor is turned automatically. Unfortunately, there is no continuum here, so the next resistor has to be turned using the same procedure. The

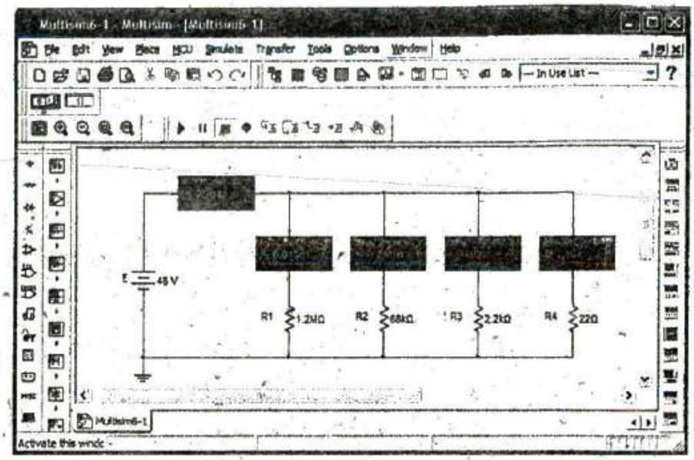


FIG. 6.70
Using the indicators of Multisim to display the currents of a parallel dc network.

values of each resistor are set by double-clicking on the resistor symbol to obtain the dialog box. Remember that the unit of measurement is controlled by the scrolls at the right of the unit of measurement. For Multisim, unlike PSpice, megohm uses capital **M** and milliohm uses lowercase **m.**

This time, rather than use the full meter employed in earlier measurements, let us use the measurement options available in the **Virtual** (also called **BASIC**) **toolbar.** If it is not already available, the toolbar can be obtained through the sequence **View-Toolbars-Virtual.** If the key in the toolbar that looks like a small meter **(Show Measurement Family)** is chosen, it will present four options for the use of an ammeter, four for a voltmeter, and five probes. The four choices for an ammeter simply set the position and the location of the positive and negative connectors. The option **Place Ammeter (Horizontal)** sets the ammeter in the horizontal position as shown in Fig. 6.70 in the top left of the diagram with the plus sign on the left and the minus sign on the right—the same polarity that would result if the current through a resistor in the same position were from left to right. Choosing **Place Ammeter (Vertical)** will result in the ammeters in the vertical sections of the network with the positive connection at the top and negative connection at the bottom, as shown in Fig. 6.70 for the four branches. If you chose **Place Ammeter (Horizontally rotated)** for the source current, it would simply reverse the positions of the positive and negative signs and provide a negative answer for the reading. If **Place Ammeter (Vertically rotated)** was chosen for the vertical branches, the readings would all be correct but with negative signs. Once all the elements are in place and their values set, initiate simulation with the sequence **Simulate-Run.** The results shown in Fig. 6.70 appear.

Note that all the results appear with the meter boxes. All are positive results because the ammeters were all entered with a configuration that would result in conventional current entering the positive current. Also note that, as was true for inserting the meters, the meters are placed in series with the branch in which the current is to be measured.

PROBLEMS

SECTION 6.2 Parallel Resistors

1. For each configuration in Fig. 6.71, find the voltage sources and/or resistors elements (individual elements, not combinations of elements) that are in parallel. Remember that elements in parallel have the same voltage.

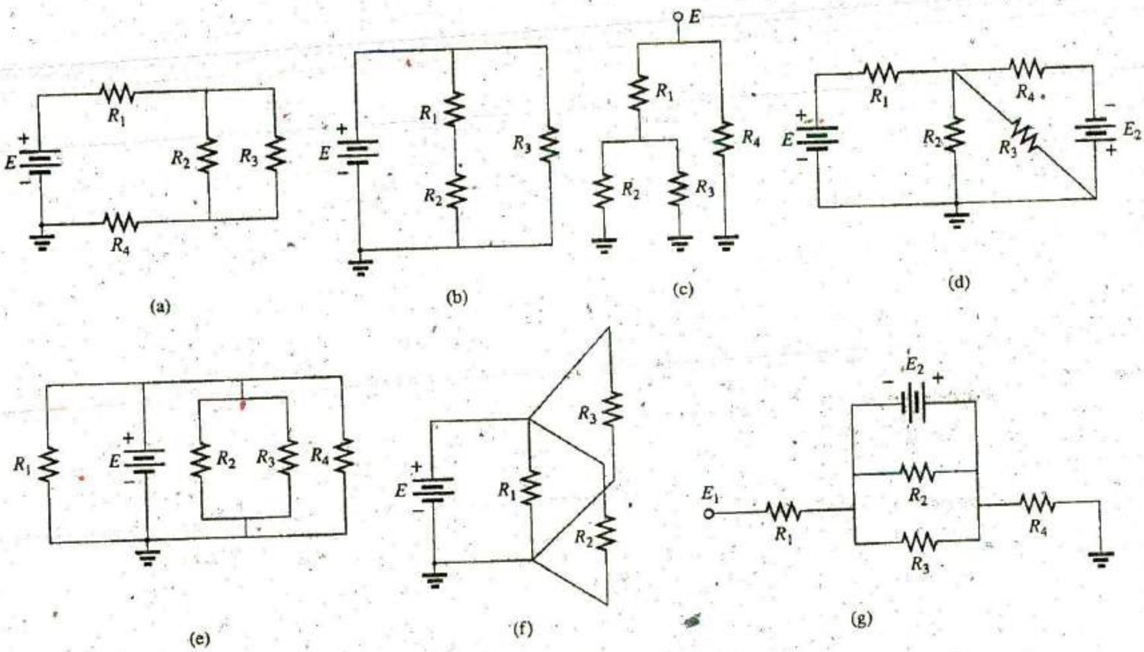


FIG. 6.71
Problem 1.

2. For the network in Fig. 6.72:
   a. Find the elements (voltage sources and/or resistors) that are in parallel.
   b. Find the elements (voltage sources and/or resistors) that are in series.
3. Find the total resistance for each configuration in Fig. 6.73. Note that only standard value resistors were used.
4. For each circuit board in Fig. 6.74, find the total resistance between connection tabs 1 and 2.
5. The total resistance of each of the configurations in Fig. 6.75 is specified. Find the unknown standard value resistance.

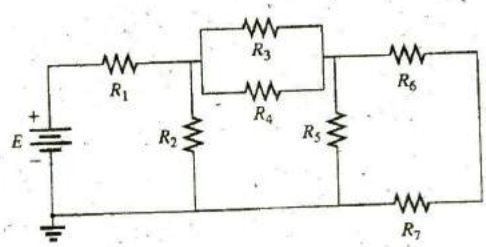


FIG. 6.72
Problem 2.

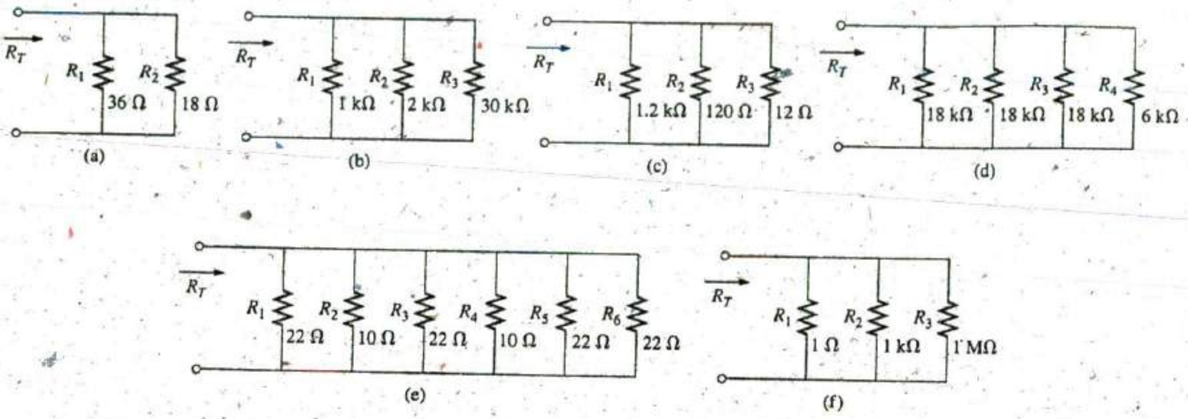


FIG. 6.73
Problem 3.

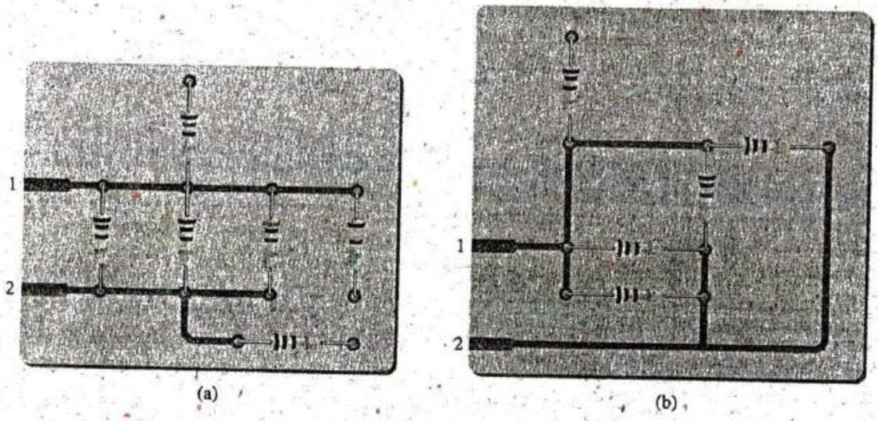


FIG. 6.74
Problem 4.

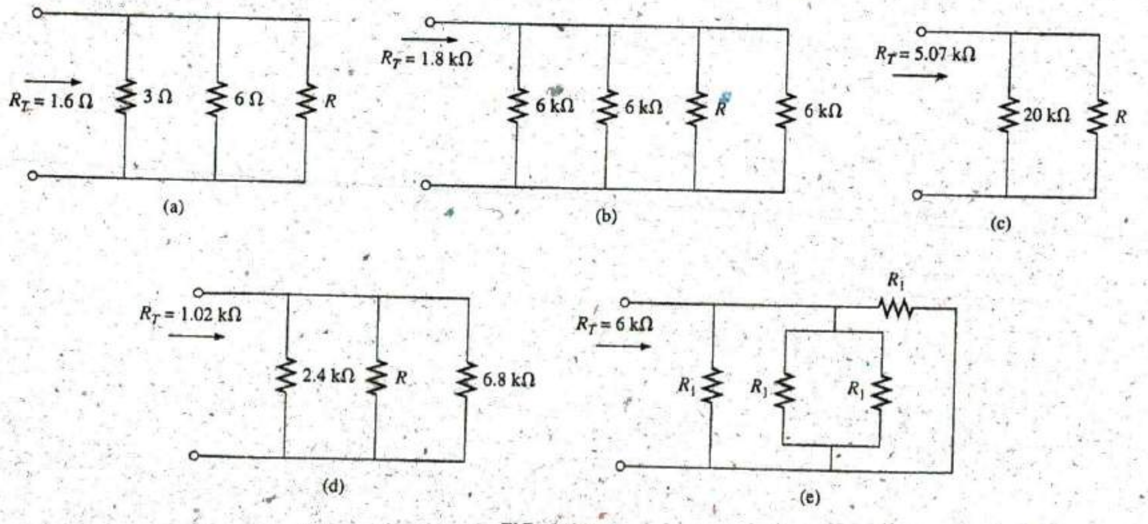


FIG. 6.75
Problem 5.

6. For the parallel network in Fig. 6.76, composed of standard values:
   a. Which resistor has the most impact on the total resistance?
   b. Without making a single calculation, what is an approximate value for the total resistance?
   c. Calculate the total resistance, and comment on your response to part (b).
   d. On an approximate basis, which resistors can be ignored when determining the total reistance?
   e. If we add another parallel resistor of any value to the network, what is the impact on the total resistance?

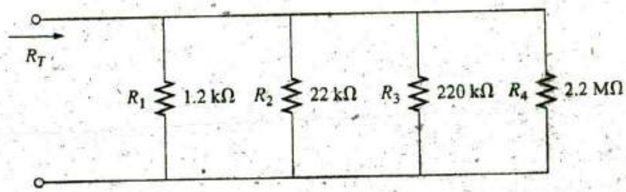


FIG. 6.76
Problem 6.

7. What is the ohmmeter reading for each configuration in Fig. 6.77?

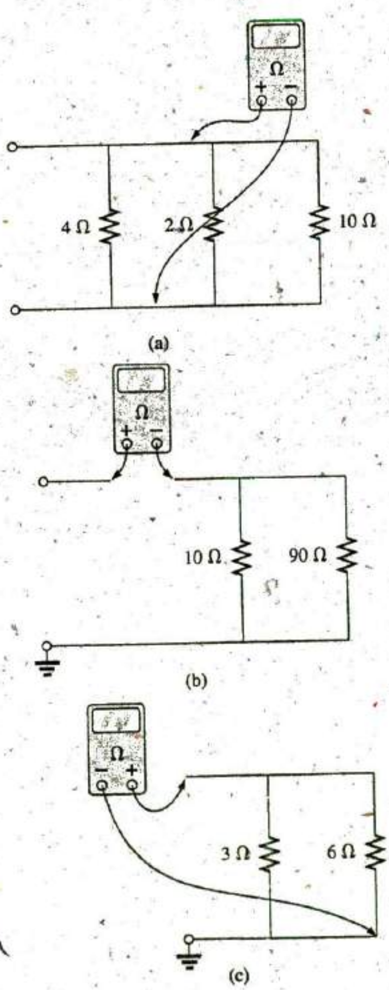


FIG. 6.77
Problem 7.

*8. Determine R_1 for the network in Fig. 6.78.

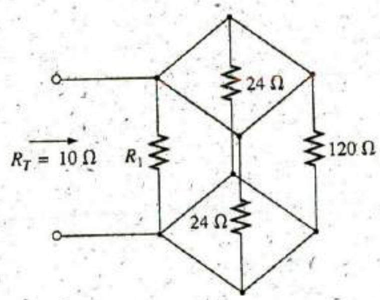


FIG. 6.78
Problem 8.

SECTION 6.3 Parallel Circuits

9. For the parallel network in Fig. 6.79:
   a. Find the total resistance.
   b. What is the voltage across each branch?
   c. Determine the source current and the current through each branch.
   d. Verify that the source current equals the sum of the branch currents.

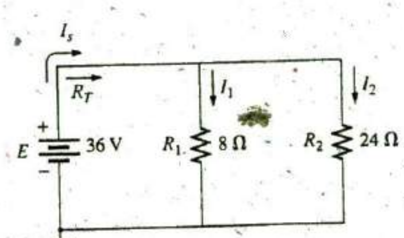


FIG. 6.79
Problem 9.

10. For the network of Fig. 6.80:
  a. Find the current through each branch.
  b. Find the total resistance.
  c. Calculate I_s using the result of part (b).
  d. Find the source current using the result of part (a).
  e. Compare the results of parts (c) and (d).

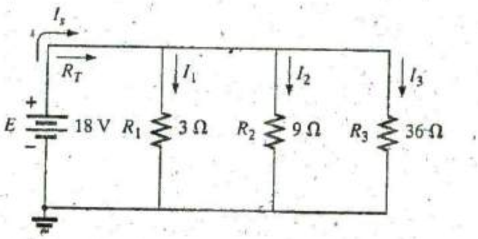


FIG. 6.80
Problem 10.

11. Repeat the analysis of Problem 10 for the network in Fig. 6.81, constructed of standard value resistors.

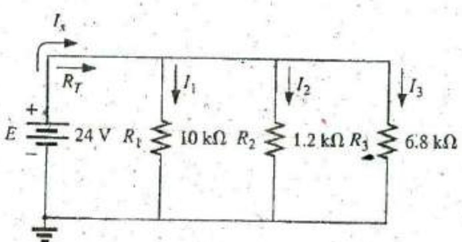


FIG. 6.81
Problem 11.

12. For the parallel network in Fig. 6.82:
  a. Without making a single calculation, make a guess on the total resistance.
  b. Calculate the total resistance, and compare it to your guess in part (a).
  c. Without making a single calculation, which branch will have the most current? Which will have the least?
  d. Calculate the current through each branch, and compare your results to the assumptions of part (c).
  e. Find the source current and test whether it equals the sum of the branch currents.
  f. How does the magnitude of the source current compare to that of the branch currents?

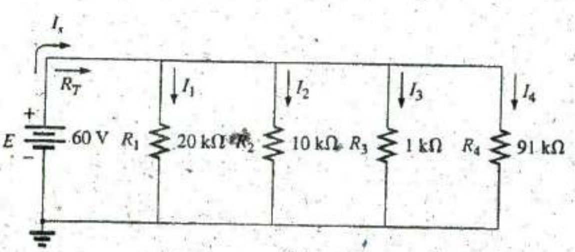


FIG. 6.82
Problem 12.

13. Given the information provided in Fig. 6.83, find:
  a. The resistance R_2.
  b. The supply voltage E.

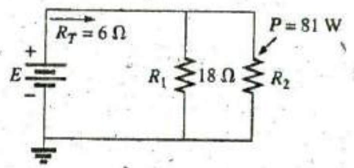


FIG. 6.83
Problem 13.

14. Use the information in Fig. 6.84 to calculate:
  a. The source voltage E.
  b. The resistance R_2.
  c. The current I_1.
  d. The source current I_s.
  e. The power supplied by the source.
  f. The power supplied to the resistors R_1 and R_2.
  g. Compare the power calculated in part (e) to the sum of the power delivered to all the resistors.

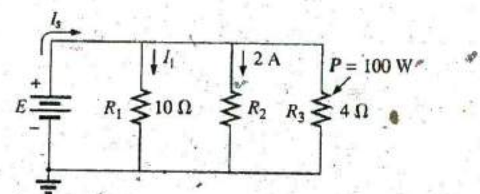


FIG. 6.84
Problem 14.

15. Given the information provided in Fig. 6.85, find the unknown quantities: E, R_1, and I_3.

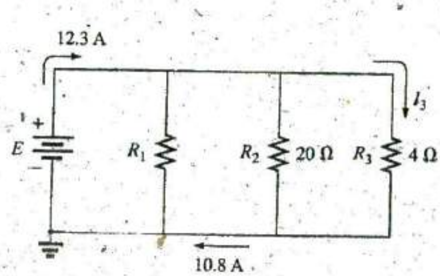


FIG. 6.85
Problem 15.

16. For the network of Fig. 6.86, find:
   a. The voltage V.
   b. The current I_2.
   c. The current I_s.
   d. The power to the 12 kΩ resistor.

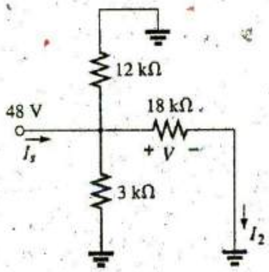


FIG. 6.86
Problem 16.

17. Using the information provided in Fig. 6.87, find:
   a. The resistance R_2.
   b. The resistance R_3.
   c. The current I_s.

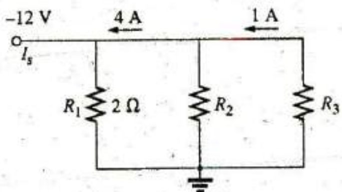


FIG. 6.87
Problem 17.

18. For the network in Fig. 6.81:
   a. Redraw the network and insert ammeters to measure the source current and the current through each branch.
   b. Connect a voltmeter to measure the source voltage and the voltage across resistor R_3. Is there any difference in the connections? Why?

SECTION 6.4 Power Distribution in a Parallel Circuit

19. For the configuration in Fig. 6.88:
   a. Find the total resistance and the current through each branch.
   b. Find the power delivered to each resistor.

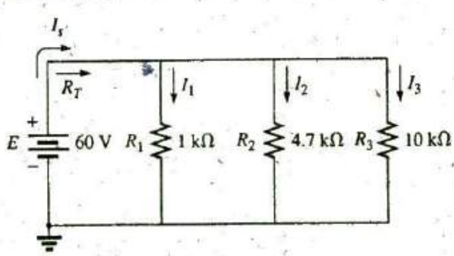


FIG. 6.88
Problem 19.

   c. Calculate the power delivered by the source.
   d. Compare the power delivered by the source to the sum of the powers delivered to the resistors.
   e. Which resistor received the most power? Why?

20. Eight holiday lights are connected in parallel as shown in Fig. 6.89.
   a. If the set is connected to a 120 V source, what is the current through each bulb if each bulb has an internal resistance of 1.8 kΩ?
   b. Determine the total resistance of the network.
   c. Find the current drain from the supply.
   d. What is the power delivered to each bulb?
   e. Using the results of part (d), find the power delivered by the source.
   f. If one bulb burns out (that is, the filament opens up), what is the effect on the remaining bulbs? What is the effect on the source current? Why?

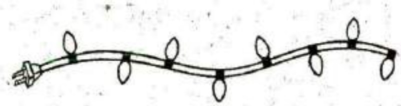


FIG. 6.89
Problem 20.

21. Determine the power delivered by the dc battery in Fig. 6.90.

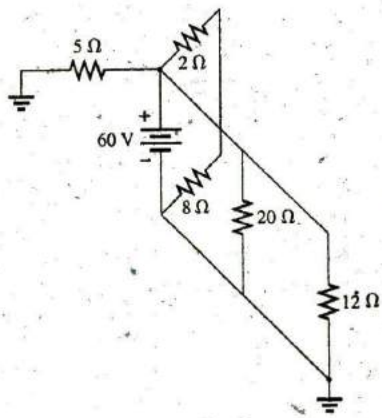


FIG. 6.90
Problem 21.

22. A portion of a residential service to a home is depicted in Fig. 6.91.
   a. Determine the current through each parallel branch of the system.
   b. Calculate the current drawn from the 120 V source. Will the 20 A breaker trip?
   c. What is the total resistance of the network?
   d. Determine the power delivered by the source. How does it compare to the sum of the wattage ratings appearing in Fig. 6.91?

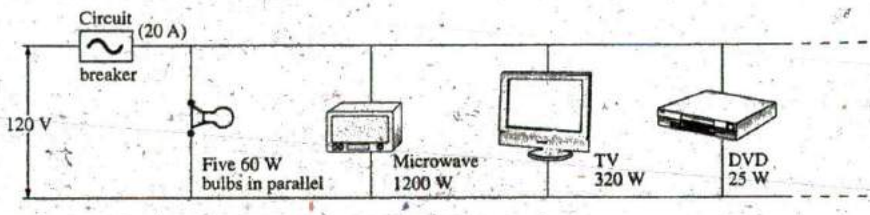


FIG. 6.91
Problem 22.

*23. For the network in Fig. 6.92:

a. Find the current I_1.

b. Calculate the power dissipated by the 4 Ω resistor.

c. Find the current I_2.

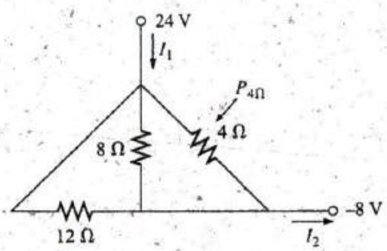


FIG. 6.92
Problem 23.

SECTION 6.5 Kirchhoff's Current Law

24. Using Kirchhoff's current law, determine the unknown currents for the parallel network in Fig. 6.93.

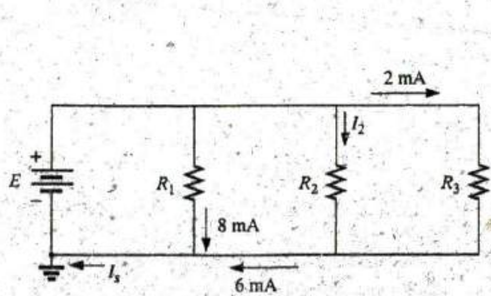


FIG. 6.93
Problem 24.

25. Using Kirchoff's current law, find the unknown currents for the complex configurations in Fig. 6.94.

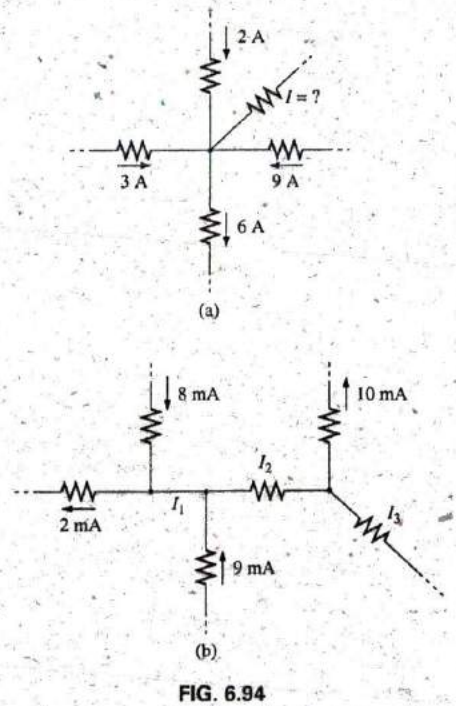


FIG. 6.94
Problem 25.

26. Using Kirchhoff's current law, determine the unknown currents for the networks in Fig. 6.95.

27. Using the information provided in Fig. 6.96, find the branch resistances R_1 and R_3, the total resistance R_T, and the voltage source E.

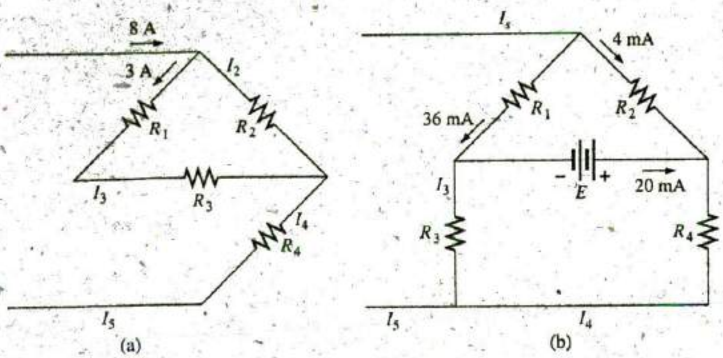


FIG. 6.95
Problem 26.

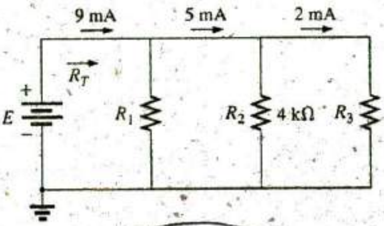


FIG. 6.96
Problem 27.

28. Find the unknown quantities for the networks in Fig. 6.97 using the information provided.

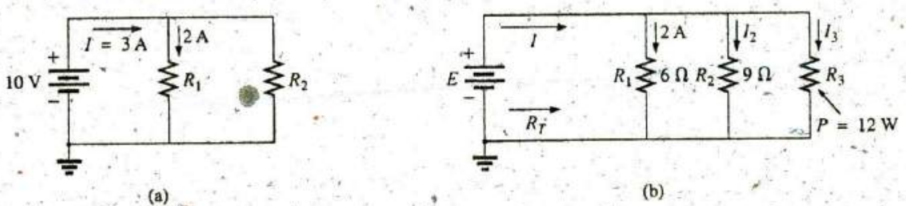


FIG. 6.97
Problem 28.

29. Find the unknown quantities for the networks of Fig. 6.98 using the information provided.

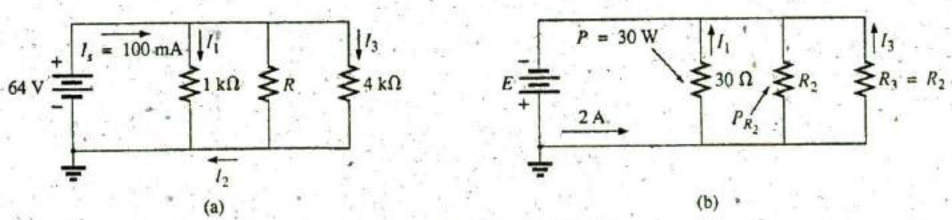


FIG. 6.98
Problem 29.

SECTION 6.6 Current Divider Rule

30. Based solely on the resistor values, determine all the currents for the configuration in Fig. 6.99. Do not use Ohm's law.

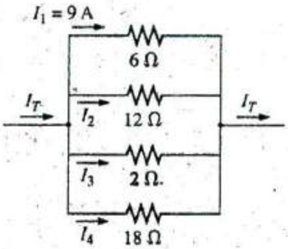


FIG. 6.99
Problem 30.

31. a. Determine one of the unknown currents of Fig. 6.100 using the current divider rule.
    b. Determine the other current using Kirchhoff's current law.

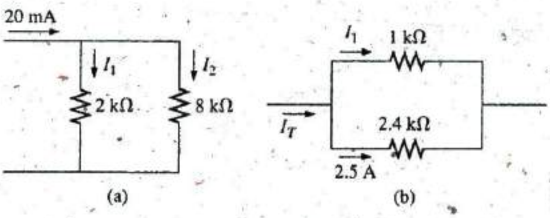


FIG. 6.100
Problem 31.

32. For each network of Fig. 6.101, determine the unknown currents.

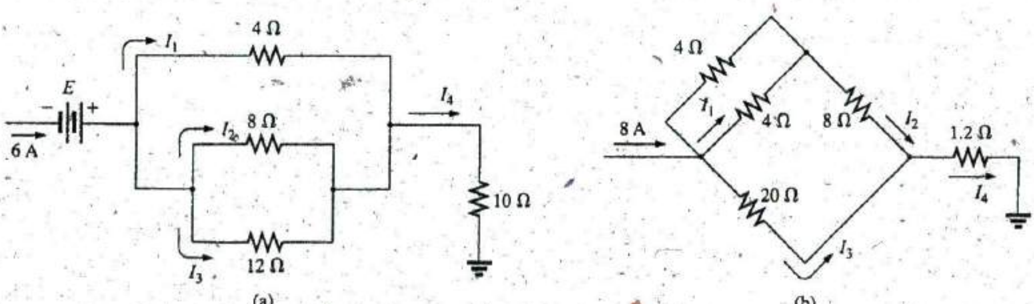


FIG. 6.101
Problem 32.

33. Parts (a) through (e) of this problem should be done by inspection—that is, mentally. The intent is to obtain an approximate solution without a lengthy series of calculations. For the network in Fig. 6.102:
    a. What is the approximate value of I_1, considering the magnitude of the parallel elements?
    b. What is the ratio I_1/I_2? Based on the result of part (a), what is an approximate value of I_2?
    c. What is the ratio I_1/I_3? Based on the result, what is an approximate value of I_3?
    d. What is the ratio I_1/I_4? Based on the result, what is an approximate value of I_4?
    e. What is the effect of the parallel 100 kΩ resistor on the above calculations? How much smaller will the current I_4 be than the current I_1?
    f. Calculate the current through the 1 Ω resistor using the current divider rule. How does it compare to the result of part (a)?
    g. Calculate the current through the 10 Ω resistor. How does it compare to the result of part (b)?
    h. Calculate the current through the 1 kΩ resistor. How does it compare to the result of part (c)?
    i. Calculate the current through the 100 kΩ resistor. How does it compare to the solutions to part (e)?

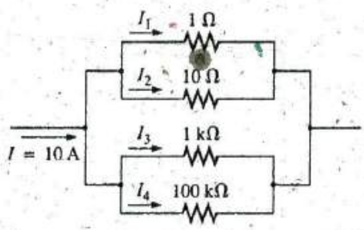


FIG. 6.102
Problem 33.

34. Find the unknown quantities for the networks in Fig. 6.103 using the information provided.

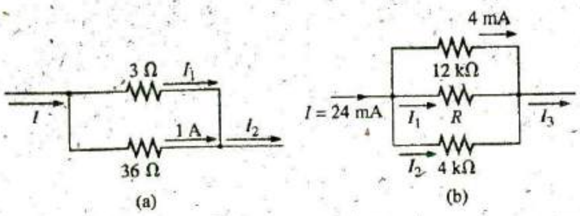


FIG. 6.103
Problem 34.

35. **a.** Find resistance R for the network in Fig. 6.104 that will ensure that $I_1 = 3I_2$.
b. Find I_1 and I_2.

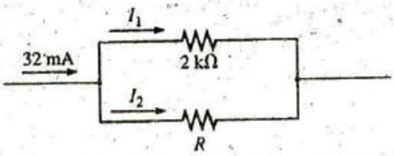


FIG. 6.104
Problem 35.

36. Design the network in Fig. 6.105 such that $I_2 = 2I_1$ and $I_3 = 2I_2$.

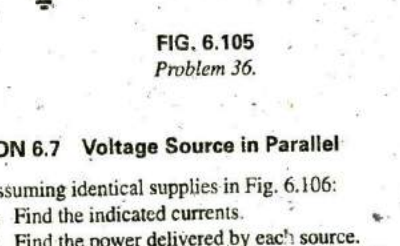


FIG. 6.105
Problem 36.

SECTION 6.7 Voltage Source in Parallel

37. Assuming identical supplies in Fig. 6.106:
a. Find the indicated currents.
b. Find the power delivered by each source.
c. Find the total power delivered by both sources, and compare it to the power delivered to the load R_L.
d. If only source current were available, what would the current drain be to supply the same power to the load? How does the current level compare to the calculated level of part (a)?

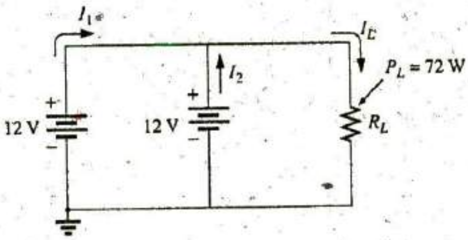


FIG. 6.106
Problem 37.

38. Assuming identical supplies, determine currents I_1, I_2, and I_3 for the configuration in Fig. 6.107.

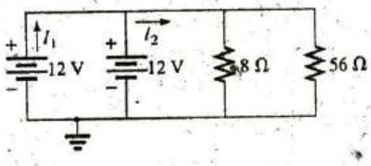


FIG. 6.107
Problem 38.

39. Assuming identical supplies, determine the current I and resistance R for the parallel network in Fig. 6.108.

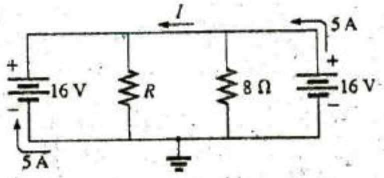


FIG. 6.108
Problem 39.

SECTION 6.8 Open and Short Circuits

40. For the network in Fig. 6.109:
a. Determine I_s and V_L.
b. Determine I_s if R_L is shorted out.
c. Determine V_L if R_L is replaced by an open circuit.

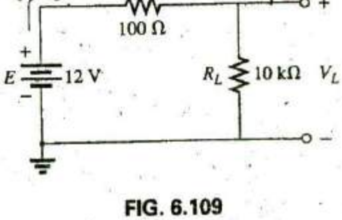


FIG. 6.109
Problem 40.

41. For the network in Fig. 6.110:
 a. Determine the open-circuit voltage V_L.
 b. If the 2.2 kΩ resistor is short circuited, what is the new value of V_L?
 c. Determine V_L if the 4.7 kΩ resistor is replaced by an open circuit.

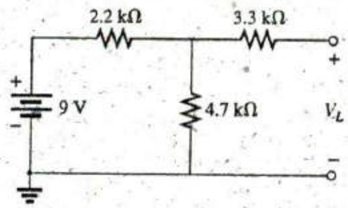


FIG. 6.110
Problem 41.

*42. For the network in Fig. 6.111, determine:
 a. The short-circuit currents I_1 and I_2.
 b. The voltages V_1 and V_2.
 c. The source current I_s.

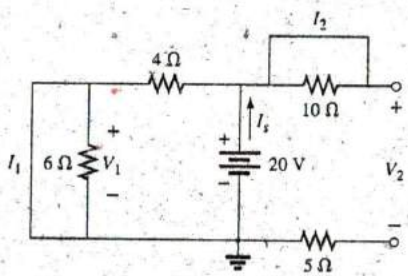


FIG. 6.111
Problem 42.

SECTION 6.9 Voltmeter Loading Effects

43. For the simple series configuration in Fig. 6.112:
 a. Determine voltage V_2.
 b. Determine the reading of a DMM having an internal resistance of 11 MΩ when used to measure V_2.
 c. Repeat part (b) with a VOM having an Ω/V rating of 20,000 using the 20 V scale. Compare the results of parts (b) and (c). Explain any differences.
 d. Repeat parts (a) through (c) with R_1 = 100 kΩ and R_2 = 200 kΩ.
 e. Based on the above, what general conclusions can you make about the use of a DMM or a VOM in the voltmeter mode?

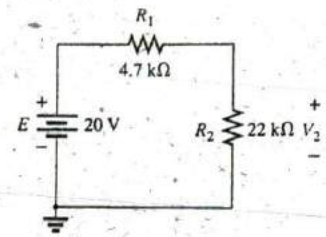


FIG. 6.112
Problem 43.

44. Given the configuration in Fig. 6.113:
 a. What is the voltage between points *a* and *b*?
 b. What will the reading of a DMM be when placed across terminals *a* and *b* if the internal resistance of the meter is 11 MΩ?
 c. Repeat part (b) if a VOM having an Ω/V rating of 20,000 using the 200 V scale is used. What is the reading using the 20 V scale? Is there a difference? Why?

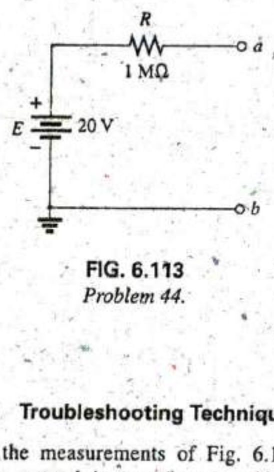


FIG. 6.113
Problem 44.

SECTION 6.10 Troubleshooting Techniques

45. Based on the measurements of Fig. 6.114, determine whether the network is operating correctly. If not, determine why.

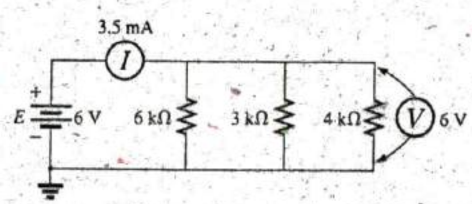


FIG. 6.114
Problem 45.

46. Referring to Fig. 6.115, find the voltage V_{ab} without the meter in place. When the meter is applied to the active network, it reads 8.8 V. If the measured value does not equal the theoretical value, which element or elements may have been connected incorrectly?

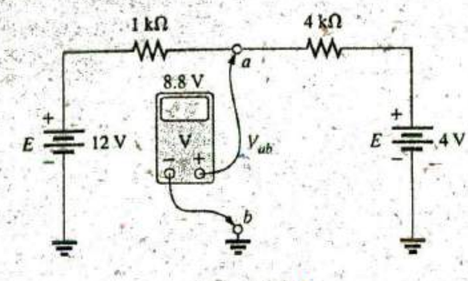


FIG. 6.115
Problem 46.

47. a. The voltage V_a for the network in Fig. 6.116 is −1 V. If it suddenly jumped to 20 V, what could have happened to the circuit structure? Localize the problem area.
    b. If the voltage V_a is 6 V rather than −1 V, explain what is wrong about the network construction.

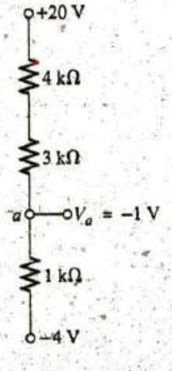


FIG. 6.116
Problem 47.

SECTION 6.14 Computer Analysis

48. Using PSpice or Multisim, verify the results of Example 6.13.
49. Using PSpice or Multisim, determine the solution to Problem 9, and compare your answer to the longhand solution.
50. Using PSpice or Multisim, determine the solution to Problem 11, and compare your answer to the longhand solution.

GLOSSARY

Current divider rule (CDR) A method by which the current through parallel elements can be determined without first finding the voltage across those parallel elements.

Kirchhoff's current law (KCL) The algebraic sum of the currents entering and leaving a node is zero.

Node A junction of two or more branches.

Ohm/volt (Ω/V) rating A rating used to determine both the current sensitivity of the movement and the internal resistance of the meter.

Open circuit The absence of a direct connection between two points in a network.

Parallel circuit A circuit configuration in which the elements have two points in common.

Short circuit A direct connection of low resistive value that can significantly alter the behavior of an element or system.