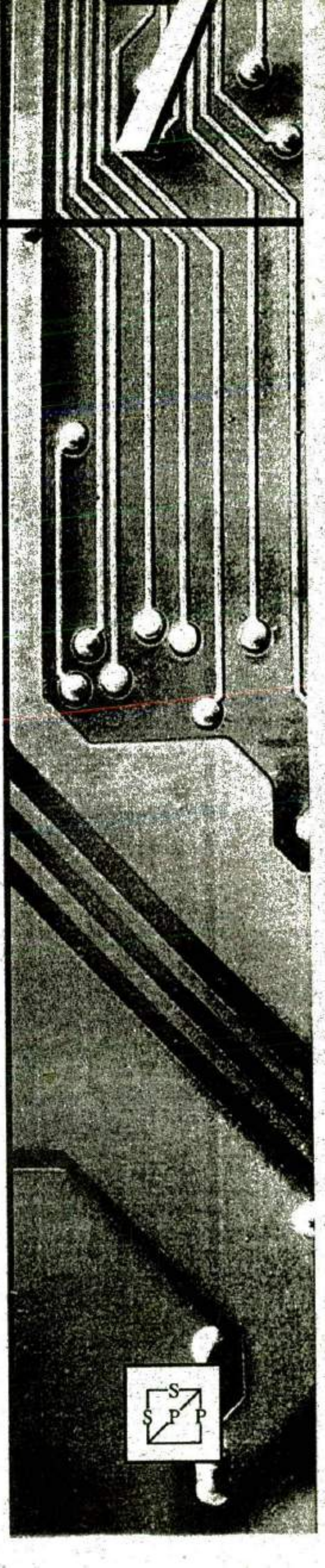
Series-Parallel Circuits



Objectives

- ***Learn about the unique characteristics of series-parallel configurations and how to solve for the voltage, current, or power to any individual element or combination of elements.***
- ***Become familiar with the voltage divider supply and the conditions needed to use it effectively.***
- ***Learn how to use a potentiometer to control the voltage across any given load.***

7.1 INTRODUCTION

Chapters 5 and 6 were dedicated to the fundamentals of series and parallel circuits. In some ways, these chapters may be the most important ones in the text because they form a foundation for all the material to follow. The remaining network configurations cannot be defined by a strict list of conditions because of the variety of configurations that exists. In broad terms, we can look upon the remaining possibilities as either **series-parallel** or **complex.**

A series-parallel configuration is one that is formed by a combination of series and parallel elements.

A complex configuration is one in which none of the elements are in series or parallel.

In this chapter, we examine the series-parallel combination using the basic laws introduced for series and parallel circuits. There are no new laws or rules to learn—simply an approach that permits the analysis of such structures. In the next chapter, we consider complex networks using methods of analysis that allow us to analyze any type of network.

The possibilities for series-parallel configurations are infinite. Therefore, you need to examine each network as a separate entity and define the approach that provides the best path to determining the unknown quantities. In time, you will find similarities between configurations that make it easier to define the best route to a solution, but this occurs only with exposure, practice, and patience. The best preparation for the analysis of series-parallel networks is a firm understanding of the concepts introduced for series and parallel networks. All the rules and laws to be applied in this chapter have already been introduced in the previous two chapters.

7.2 SERIES-PARALLEL NETWORKS

The network in Fig. 7.1 is a series-parallel network. At first, you must be very careful to determine which elements are in series and which are in parallel. For instance, resistors R_1 and R_2 are *not* in series due to resistor R_3 being connected to the common point b between R_1 and R_2. Resistors R_2 and R_4 are *not* in parallel because they are not connected at both ends. They are separated at one end by resistor R_3. The need to be absolutely sure of your definitions from the last two chapters now becomes obvious. In fact, it may be a good idea to refer to those rules as we progress through this chapter.

If we look carefully enough at Fig. 7.1, we do find that the two resistors R_3 and R_4 are in series because they share only point c, and no other element is connected to that point.

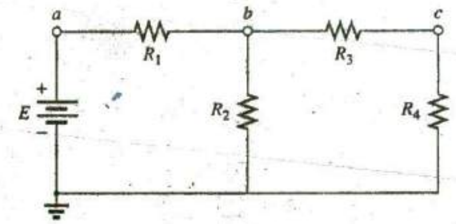


FIG. 7.1
Series-parallel dc network.

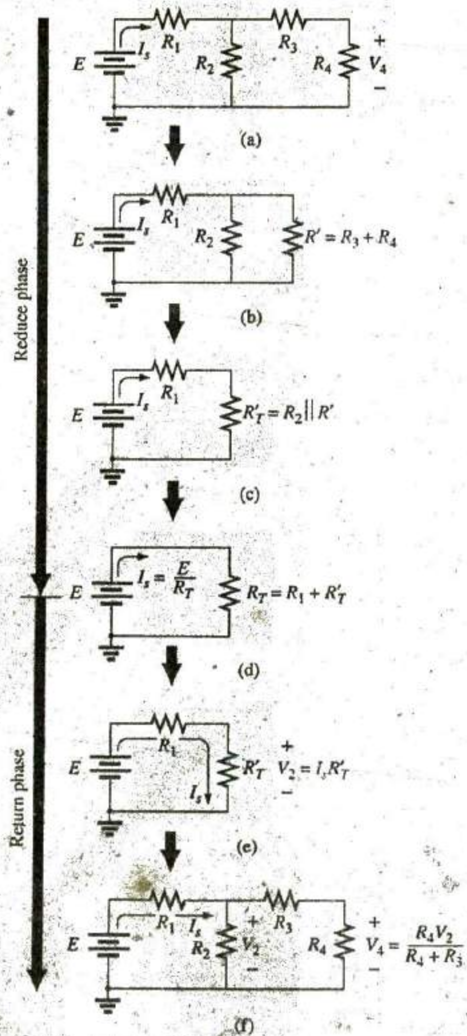


FIG. 7.2
Introducing the reduce and return approach.

Further, the voltage source E and resistor R_1 are in series because they share point a, with no other elements connected to the same point. In the entire configuration, there are no two elements in parallel.

How do we analyze such configurations? The approach is one that requires us to first identify elements that can be combined. Since there are no parallel elements, we must turn to the possibilities with series elements. The voltage source and the series resistor cannot be combined because they are different types of elements. However, resistors R_3 and R_4 can be combined to form a single resistor. The total resistance of the two is their sum as defined by series circuits. The resulting resistance is then in parallel with resistor R_2, and they can be combined using the laws for parallel elements. The process has begun: We are slowly reducing the network to one that will be represented by a single resistor equal to the total resistance "seen" by the source.

The source current can now be determined using Ohm's law, and we can work back through the network to find all the other currents and voltages. The ability to define the first step in the analysis can sometimes be difficult. However, combinations can be made only by using the rules for series or parallel elements, so naturally the first step may simply be to define which elements are in series or parallel. You must then define how to find such things as the total resistance and the source current and proceed with the analysis. In general, the following steps will provide some guidance for the wide variety of possible combinations that you might encounter.

General Approach:

1. ***Take a moment to study the problem "in total" and make a brief mental sketch of the overall approach you plan to use. The result may be time- and energy-saving shortcuts.***
2. ***Examine each region of the network independently before tying them together in series-parallel combinations. This usually simplifies the network and possibly reveals a direct approach toward obtaining one or more desired unknowns. It also eliminates many of the errors that may result due to the lack of a systematic approach.***
3. ***Redraw the network as often as possible with the reduced branches and undisturbed unknown quantities to maintain clarity and provide the reduced networks for the trip back to unknown quantities from the source.***
4. ***When you have a solution, check that it is reasonable by considering the magnitudes of the energy source and the elements in the network. If it does not seem reasonable, either solve the circuit using another approach or review your calculations.***

7.3 REDUCE AND RETURN APPROACH

The network of Fig. 7.1 is redrawn as Fig. 7.2(a). For this discussion, let us assume that voltage V_4 is desired. As described in Section 7.2, first combine the series resistors R_3 and R_4 to form an equivalent resistor R' as shown in Fig. 7.2(b). Resistors R_2 and R' are then in parallel and can be combined to establish an equivalent resistor R'_T as shown in Fig. 7.2(c). Resistors R_1 and R'_T are then in series and can be combined to establish the total resistance of the network as shown in Fig. 7.2(d). The **reduction phase** of the analysis is now complete. The network cannot be put in a simpler form.

We can now proceed with the **return phase** whereby we work our way back to the desired voltage V_4. Due to the resulting series configuration, the source current is also the current through R_1 and R'_T. The voltage across R'_T (and therefore across R_2) can be determined using Ohm's law as shown in Fig. 7.2(e). Finally, the desired voltage V_4 can be determined by an application of the voltage divider rule as shown in Fig. 7.2(f).

The *reduce and return approach* has now been introduced. This process enables you to reduce the network to its simplest form across the source and then determine the source current. In the return phase, you use the resulting source current to work back to the desired unknown. For most single-source series-parallel networks, the above approach provides a viable option toward the solution. In some cases, shortcuts can be applied that save some time and energy. Now for a few examples:

EXAMPLE 7.1 Find current I_3 for the series-parallel network in Fig. 7.3.

Solution: Checking for series and parallel elements, we find that resistors R_2 and R_3 are in parallel. Their total resistance is

$$R' = R_2 \| R_3 = \frac{R_2R_3}{R_2 + R_3} = \frac{(12\ \text{k}\Omega)(6\ \text{k}\Omega)}{12\ \text{k}\Omega + 6\ \text{k}\Omega} = 4\ \text{k}\Omega$$

Replacing the parallel combination with a single equivalent resistance results in the configuration in Fig. 7.4. Resistors R_1 and R' are then in series, resulting in a total resistance of

$$R_T = R_1 + R' = 2\ \text{k}\Omega + 4\ \text{k}\Omega = 6\ \text{k}\Omega$$

The source current is then determined using Ohm's law:

$$I_s = \frac{E}{R_T} = \frac{54\ \text{V}}{6\ \text{k}\Omega} = 9\ \text{mA}$$

In Fig. 7.4, since R_1 and R' are in series, they have the same current I_s. The result is

$$I_1 = I_s = 9\ \text{mA}$$

Returning to Fig. 7.3, we find that I_1 is the total current entering the parallel combination of R_2 and R_3. Applying the current divider rule results in the desired current:

$$I_3 = \left(\frac{R_2}{R_2 + R_3}\right)I_1 = \left(\frac{12\ \text{k}\Omega}{12\ \text{k}\Omega + 6\ \text{k}\Omega}\right)9\ \text{mA} = \mathbf{6\ mA}$$

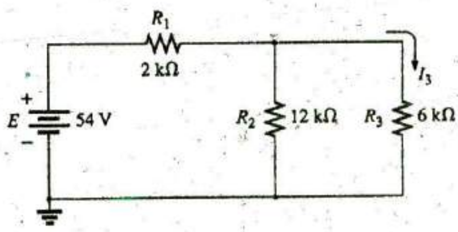


FIG. 7.3
Series-parallel network for Example 7.1.

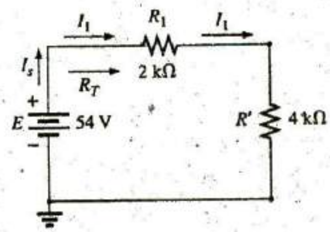


FIG. 7.4
Substituting the parallel equivalent resistance for resistors R_2 and R_3 in Fig. 7.3.

Note in the solution for Example 7.1 that all of the equations used were introduced in the last two chapters—nothing new was introduced except how to approach the problem and use the equations properly.

EXAMPLE 7.2 For the network in Fig. 7.5:

a. Determine currents I_4 and I_s and voltage V_2.
b. Insert the meters to measure current I_4 and voltage V_2.

Solutions:

a. Checking out the network, we find that there are no two resistors in series, and the only parallel combination is resistors R_2

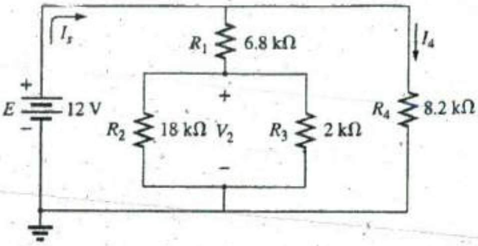


FIG. 7.5
Series-parallel network for Example 7.2.

and R_3. Combining the two parallel resistors results in a total resistance of

$$R' = R_2 \| R_3 = \frac{R_2 R_3}{R_2 + R_3} = \frac{(18\text{ k}\Omega)(2\text{ k}\Omega)}{18\text{ k}\Omega + 2\text{ k}\Omega} = 1.8\text{ k}\Omega$$

Redrawing the network with resistance R' inserted results in the configuration in Fig. 7.6.

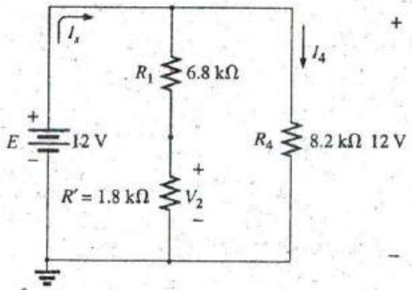


FIG. 7.6
Schematic representation of the network in Fig. 7.5 after substituting the equivalent resistance R′ for the parallel combination of R_2 and R_3.

You may now be tempted to combine the series resistors R_1 and R' and redraw the network. However, a careful examination of Fig. 7.6 reveals that since the two resistive branches are in parallel, the voltage is the same across each branch. That is, the voltage across the series combination of R_1 and R' is 12 V and that across resistor R_4 is 12 V. The result is that I_4 can be determined directly using Ohm's law as follows:

$$I_4 = \frac{V_4}{R_4} = \frac{E}{R_4} = \frac{12\text{ V}}{8.2\text{ k}\Omega} = \mathbf{1.46\text{ mA}}$$

In fact, for the same reason, I_4 could have been determined directly from Fig. 7.5. Because the total voltage across the series combination of R_1 and R'_T is 12 V, the voltage divider rule can be applied to determine voltage V_2 as follows:

$$V_2 = \left(\frac{R'}{R' + R_1}\right)E = \left(\frac{1.8\text{ k}\Omega}{1.8\text{ k}\Omega + 6.8\text{ k}\Omega}\right)12\text{ V} = \mathbf{2.51\text{ V}}$$

The current I_s can be found in one of two ways. Find the total resistance and use Ohm's law, or find the current through the other parallel branch and apply Kirchhoff's current law. Since we already have the current I_4, the latter approach will be applied:

$$I_1 = \frac{E}{R_1 + R'} = \frac{12\text{ V}}{6.8\text{ k}\Omega + 1.8\text{ k}\Omega} = 1.40\text{ mA}$$

and $\quad I_s = I_1 + I_4 = 1.40\text{ mA} + 1.46\text{ mA} = \mathbf{2.86\text{ mA}}$

b. The meters have been properly inserted in Fig. 7.7. Note that the voltmeter is across both resistors since the voltage across parallel elements is the same. In addition, note that the ammeter is in series with resistor R_4, forcing the current through the meter to be the same as that through the series resistor. The power supply is displaying the source current.

Clearly, Example 7.2 revealed how a careful study of a network can

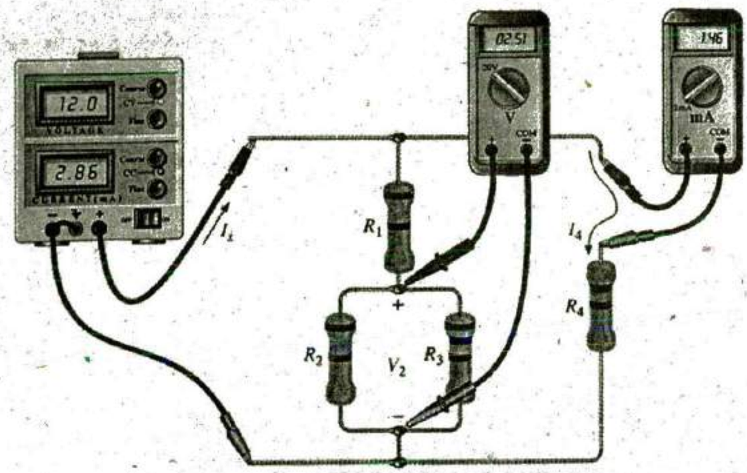


FIG. 7.7
Inserting an ammeter and a voltmeter to measure I_4 and V_2, respectively.

the extra time to sit back and carefully examine a network before trying every equation that seems appropriate.

7.4 BLOCK DIAGRAM APPROACH

In the previous example, we used the reduce and return approach to find the desired unknowns. The direction seemed fairly obvious and the solution relatively easy to understand. However, occasionally the approach is not as obvious, and you may need to look at groups of elements rather than the individual components. Once the grouping of elements reveals the most direct approach, you can examine the impact of the individual components in each group. This grouping of elements is called the *block diagram approach* and is used in the following examples.

In Fig. 7.8, blocks B and C are in parallel (points b and c in common), and the voltage source E is in series with block A (point a in common). The parallel combination of B and C is also in series with A and the voltage source E due to the common points b and c, respectively.

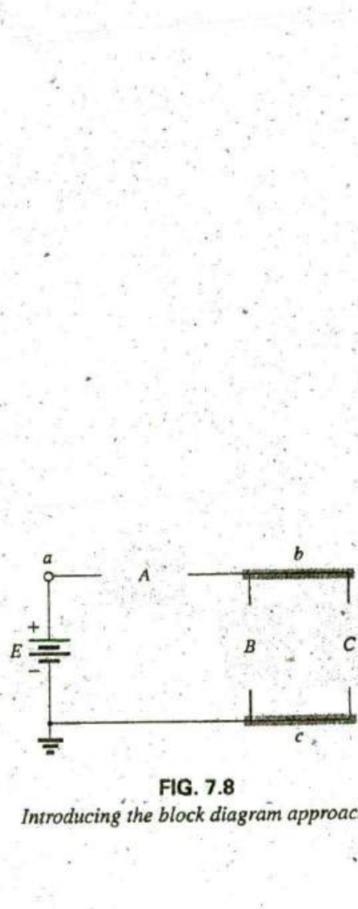


FIG. 7.8
Introducing the block diagram approach.

To ensure that the analysis to follow is as clear and uncluttered as possible, the following notation is used for series and parallel combinations of elements. For series resistors R_1 and R_2, a comma is inserted between their subscript notations, as shown here:

$$R_{1,2} = R_1 + R_2$$

For parallel resistors R_1 and R_2, the parallel symbol is inserted between their subscripted notations, as follows:

$$R_{1\|2} = R_1 \| R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

If each block in Fig. 7.8 were a single resistive element, the network in Fig. 7.9 would result. Note that it is an exact replica of Fig. 7.3 in Example 7.1. Blocks B and C are in parallel, and their combination is in series with block A.

However, as shown in the next example, the same block configuration can result in a totally different network.

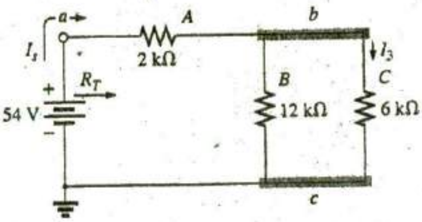


FIG. 7.9
Block diagram format of Fig. 7.3.

EXAMPLE 7.3 Determine all the currents and voltages of the network in Fig. 7.10.

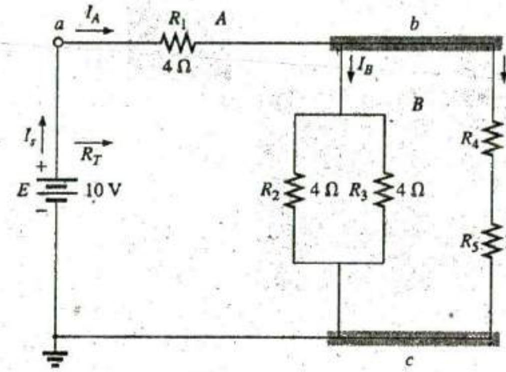


FIG. 7.10
Example 7.3.

Solution: Blocks *A*, *B*, and *C* have the same relative position, but the internal components are different. Note that blocks *B* and *C* are still in parallel, and block *A* is in series with the parallel combination. First, reduce each block into a single element and proceed as described for Example 7.1.

In this case:

$$A: R_A = 4\ \Omega$$

$$B: R_B = R_2 \| R_3 = R_{2\|3} = \frac{R}{N} = \frac{4\ \Omega}{2} = 2\ \Omega$$

$$C: R_C = R_4 + R_5 = R_{4,5} = 0.5\ \Omega + 1.5\ \Omega = 2\ \Omega$$

Blocks *B* and *C* are still in parallel, and

$$R_{B\|C} = \frac{R}{N} = \frac{2\ \Omega}{2} = 1\ \Omega$$

with

$$R_T = R_A + R_{B\|C} = 4\ \Omega + 1\ \Omega = \mathbf{5\ \Omega}$$

(Note the similarity between this equation and that obtained for Example 7.1.)

and

$$I_s = \frac{E}{R_T} = \frac{10\ \text{V}}{5\ \Omega} = \mathbf{2\ A}$$

We can find the currents I_A, I_B, and I_C using the reduction of the network in Fig. 7.10 (recall Step 3) as found in Fig. 7.11. Note that I_A, I_B, and I_C are the same in Figs. 7.10 and Fig. 7.11 and therefore also appear in Fig. 7.11. In other words, the currents I_A, I_B, and I_C in Fig. 7.11 have the same magnitude as the same currents in Fig. 7.10. We have

$$I_A = I_s = \mathbf{2\ A}$$

and

$$I_B = I_C = \frac{I_A}{2} = \frac{I_s}{2} = \frac{2\ \text{A}}{2} = \mathbf{1\ A}$$

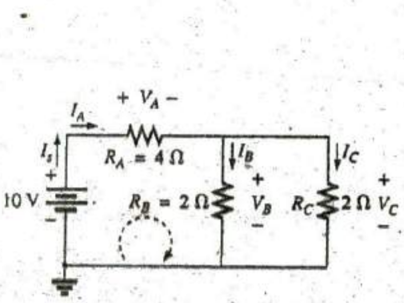


FIG. 7.11
Reduced equivalent of Fig. 7.10.

Returning to the network in Fig. 7.10, we have

$$I_{R_2} = I_{R_3} = \frac{I_B}{2} = \mathbf{0.5\ A}$$

The voltages V_A, V_B, and V_C from either figure are

$$V_A = I_A R_A = (2\ \text{A})(4\ \Omega) = \mathbf{8\ V}$$
$$V_B = I_B R_B = (1\ \text{A})(2\ \Omega) = \mathbf{2\ V}$$
$$V_C = V_B = \mathbf{2\ V}$$

Applying Kirchhoff's voltage law for the loop indicated in Fig. 7.11, we obtain

$$\Sigma_{C} V = E - V_A - V_B = 0$$
$$E = E_A + V_B = 8\ \text{V} + 2\ \text{V}$$

or

$$10\ \text{V} = 10\ \text{V} \quad \text{(checks)}$$

EXAMPLE 7.4 Another possible variation of Fig. 7.8 appears in Fig. 7.12. Determine all the currents and voltages.

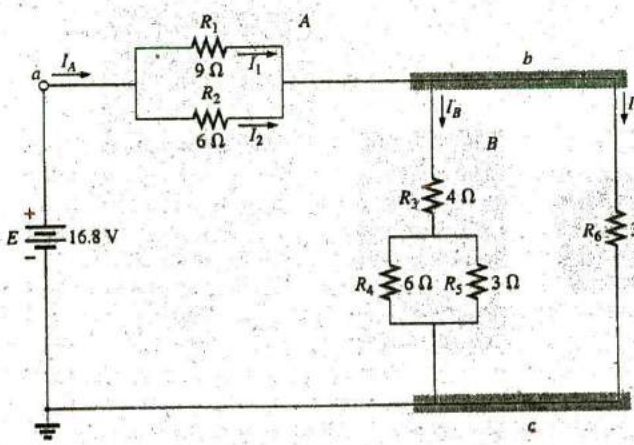


FIG. 7.12
Example 7.4.

Solution:

$$R_A = R_{1\|2} = \frac{(9\ \Omega)(6\ \Omega)}{9\ \Omega + 6\ \Omega} = \frac{54\ \Omega}{15} = 3.6\ \Omega$$

$$R_B = R_3 + R_{4\|5} = 4\ \Omega + \frac{(6\ \Omega)(3\ \Omega)}{6\ \Omega + 3\ \Omega} = 4\ \Omega + 2\ \Omega = 6\ \Omega$$

$$R_C = 3\ \Omega$$

The network in Fig. 7.12 can then be redrawn in reduced form, as shown in Fig. 7.13. Note the similarities between this circuit and the circuits in Figs. 7.9 and 7.11. We have

$$R_T = R_A + R_{B\|C} = 3.6\ \Omega + \frac{(6\ \Omega)(3\ \Omega)}{6\ \Omega + 3\ \Omega}$$
$$= 3.6\ \Omega + 2\ \Omega = 5.6\ \Omega$$

$$I_s = \frac{E}{R_T} = \frac{16.8\ \text{V}}{5.6\ \Omega} = \mathbf{3\ A}$$

$$I_A = I_s = \mathbf{3\ A}$$

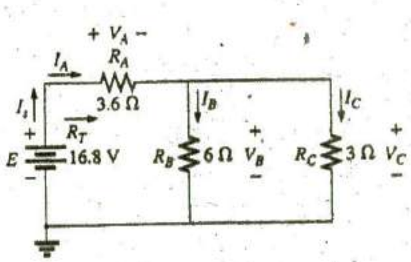


FIG. 7.13
Reduced equivalent of Fig. 7.12.

Applying the current divider rule yields

$$I_B = \frac{R_C I_A}{R_C + R_B} = \frac{(3\ \Omega)(3\ \text{A})}{3\ \Omega + 6\ \Omega} = \frac{9\ \text{A}}{9} = 1\ \text{A}$$

By Kirchhoff's current law,

$$I_C = I_A - I_B = 3\ \text{A} - 1\ \text{A} = \mathbf{2\ A}$$

By Ohm's law,

$$V_A = I_A R_A = (3\ \text{A})(3.6\ \Omega) = \mathbf{10.8\ V}$$
$$V_B = I_B R_B = V_C = I_C R_C = (2\ \text{A})(3\ \Omega) = \mathbf{6\ V}$$

Returning to the original network (Fig. 7.12) and applying the current divider rule gives

$$I_1 = \frac{R_2 I_A}{R_2 + R_1} = \frac{(6\ \Omega)(3\ \text{A})}{6\ \Omega + 9\ \Omega} = \frac{18\ \text{A}}{15} = \mathbf{1.2\ A}$$

By Kirchhoff's current law,

$$I_2 = I_A - I_1 = 3\ \text{A} - 1.2\ \text{A} = \mathbf{1.8\ A}$$

Figs. 7.9, 7.10, and 7.12 are only a few of the infinite variety of configurations that the network can assume starting with the basic arrangement in Fig. 7.8. They were included in our discussion to emphasize the importance of considering each region of the network independently before finding the solution for the network as a whole.

The blocks in Fig. 7.8 can be arranged in a variety of ways. In fact, there is no limit on the number of series-parallel configurations that can appear within a given network. In reverse, the block diagram approach can be used effectively to reduce the apparent complexity of a system by identifying the major series and parallel components of the network. This approach is demonstrated in the next few examples.

7.5 DESCRIPTIVE EXAMPLES

EXAMPLE 7.5 Find the current I_4 and the voltage V_2 for the network in Fig. 7.14 using the block diagram approach.

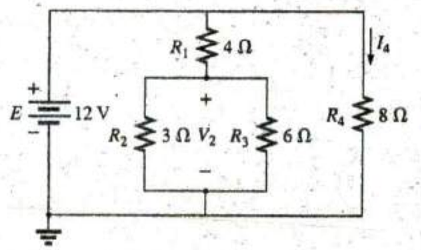


FIG. 7.14
Example 7.5.

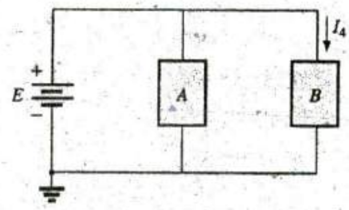


FIG. 7.15
Block diagram of Fig. 7.14.

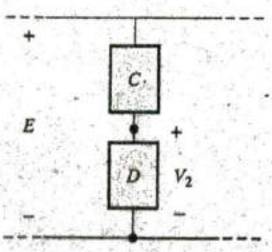


FIG. 7.16
Alternative block diagram for the first parallel branch in Fig. 7.14.

Solution: Note the similarities with the network in Fig. 7.5. In this case, particular unknowns are requested instead of a complete solution. It would, therefore, be a waste of time to find all the currents and voltages of the network. The method used should concentrate on obtaining only the unknowns requested. With the block diagram approach, the network has the basic structure in Fig. 7.15, clearly indicating that the three branches are in parallel and the voltage across A and B is the supply voltage. The current I_4 is now immediately obvious as simply the supply voltage divided by the resultant resistance for B. If desired, block A can be broken down further, as shown in Fig. 7.16, to identify C and D as series elements, with the voltage V_2 capable of being determined using the voltage divider rule once the resistance of C and D is reduced to a single value. This is an example of how making a mental sketch of the approach before applying laws, rules, and so on can help avoid dead ends and frustration.

Applying Ohm's law, we have

$$I_4 = \frac{E}{R_B} = \frac{E}{R_4} = \frac{12\text{ V}}{8\ \Omega} = \mathbf{1.5\ A}$$

Combining the resistors R_2 and R_3 in Fig. 7.14 results in

$$R_D = R_2 \parallel R_3 = 3\ \Omega \parallel 6\ \Omega = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 6\ \Omega} = \frac{18\ \Omega}{9} = 2\ \Omega$$

and, applying the voltage divider rule, we find

$$V_2 = \frac{R_D E}{R_D + R_C} = \frac{(2\ \Omega)(12\text{ V})}{2\ \Omega + 4\ \Omega} = \frac{24\text{ V}}{6} = \mathbf{4\ V}$$

EXAMPLE 7.6 Find the indicated currents and voltages for the network in Fig. 7.17.

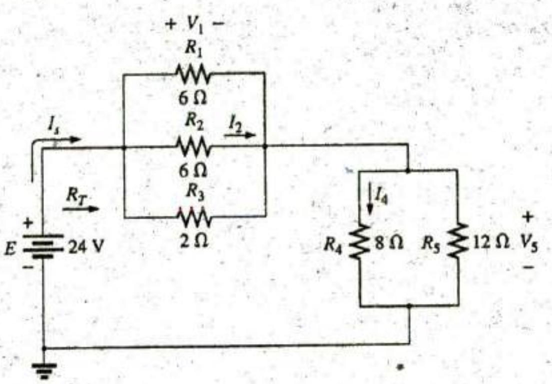


FIG. 7.17
Example 7.6.

Solution: Again, only specific unknowns are requested. When the network is redrawn, be sure to note which unknowns are preserved and which have to be determined using the original configuration. The block diagram of the network may appear as shown in Fig. 7.18, clearly revealing that A and B are in series. Note in this form the number of unknowns that have been preserved. The voltage V_1 is the same across the three parallel branches in Fig. 7.17, and V_5 is the same across R_4 and R_5. The unknown currents I_2 and I_4 are lost since they represent the currents through only one of the parallel branches. However, once V_1 and V_5 are known, you can find the required currents using Ohm's law.

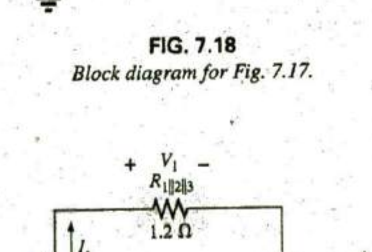


FIG. 7.18
Block diagram for Fig. 7.17.

$$R_{1\|2} = \frac{R}{N} = \frac{6\ \Omega}{2} = 3\ \Omega$$

$$R_A = R_{1\|2\|3} = \frac{(3\ \Omega)(2\ \Omega)}{3\ \Omega + 2\ \Omega} = \frac{6\ \Omega}{5} = 1.2\ \Omega$$

$$R_B = R_{4\|5} = \frac{(8\ \Omega)(12\ \Omega)}{8\ \Omega + 12\ \Omega} = \frac{96\ \Omega}{20} = 4.8\ \Omega$$

The reduced form of Fig. 7.17 then appears as shown in Fig. 7.19, and

$$R_T = R_{1\|2\|3} + R_{4\|5} = 1.2\ \Omega + 4.8\ \Omega = 6\ \Omega$$

$$I_s = \frac{E}{R_T} = \frac{24\text{ V}}{6\ \Omega} = \mathbf{4\ A}$$

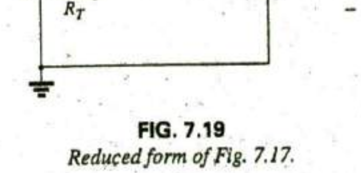


FIG. 7.19
Reduced form of Fig. 7.17.

with $$V_1 = I_s R_{1\|2\|3} = (4\text{ A})(1.2\ \Omega) = \mathbf{4.8\ V}$$
$$V_5 = I_s R_{4\|5} = (4\text{ A})(4.8\ \Omega) = \mathbf{19.2\ V}$$

Applying Ohm's law gives

$$I_4 = \frac{V_5}{R_4} = \frac{19.2\text{ V}}{8\ \Omega} = \mathbf{2.4\ A}$$

$$I_2 = \frac{V_2}{R_2} = \frac{V_1}{R_2} = \frac{4.8\text{ V}}{6\ \Omega} = \mathbf{0.8\ A}$$

The next example demonstrates that unknown voltages do not have to be across elements but can exist between any two points in a network. In addition, the importance of redrawing the network in a more familiar form is clearly revealed by the analysis to follow.

EXAMPLE 7.7

a. Find the voltages V_1, V_3, and V_{ab} for the network in Fig. 7.20.
b. Calculate the source current I_s.

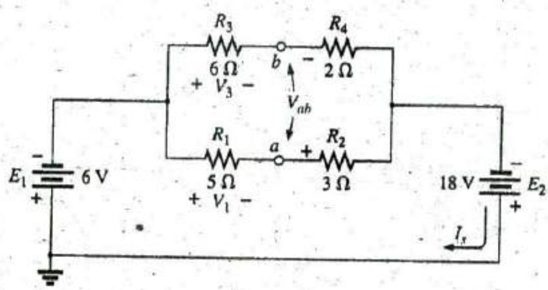


FIG. 7.20
Example 7.7.

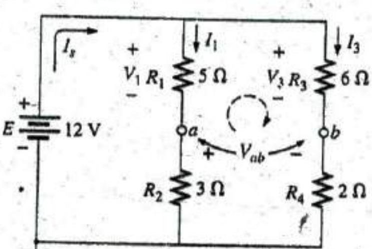


FIG. 7.21
Network in Fig. 7.20 redrawn.

Solutions: This is one of those situations where it may be best to redraw the network before beginning the analysis. Since combining both sources will not affect the unknowns, the network is redrawn as shown in Fig. 7.21, establishing a parallel network with the total source voltage across each parallel branch. The net source voltage is the difference between the two with the polarity of the larger.

a. Note the similarities with Fig. 7.16, permitting the use of the voltage divider rule to determine V_1 and V_3:

$$V_1 = \frac{R_1 E}{R_1 + R_2} = \frac{(5\ \Omega)(12\text{ V})}{5\ \Omega + 3\ \Omega} = \frac{60\text{ V}}{8} = \mathbf{7.5\ V}$$

$$V_3 = \frac{R_3 E}{R_3 + R_4} = \frac{(6\ \Omega)(12\text{ V})}{6\ \Omega + 2\ \Omega} = \frac{72\text{ V}}{8} = \mathbf{9\ V}$$

The open-circuit voltage V_{ab} is determined by applying Kirchhoff's voltage law around the indicated loop in Fig. 7.21 in the clockwise direction starting at terminal a. We have

$$+V_1 - V_3 + V_{ab} = 0$$

and $$V_{ab} = V_3 - V_1 = 9\text{ V} - 7.5\text{ V} = \mathbf{1.5\ V}$$

b. By Ohm's law,

$$I_1 = \frac{V_1}{R_1} = \frac{7.5\text{ V}}{5\,\Omega} = 1.5\text{ A}$$

$$I_3 = \frac{V_3}{R_3} = \frac{9\text{ V}}{6\,\Omega} = 1.5\text{ A}$$

Applying Kirchhoff's current law gives

$$I_s = I_1 + I_3 = 1.5\text{ A} + 1.5\text{ A} = \mathbf{3\text{ A}}$$

EXAMPLE 7.8 For the network in Fig. 7.22, determine the voltages V_1 and V_2 and the current I.

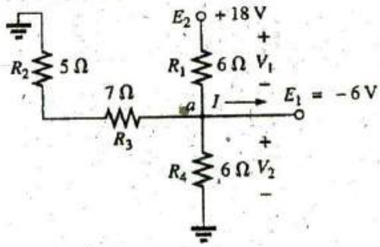


FIG. 7.22
Example 7.8.

Solution: It would indeed be difficult to analyze the network in the form in Fig. 7.22 with the symbolic notation for the sources and the reference or ground connection in the upper left corner of the diagram. However, when the network is redrawn as shown in Fig. 7.23, the unknowns and the relationship between branches become significantly clearer. Note the common connection of the grounds and the replacing of the terminal notation by actual supplies.

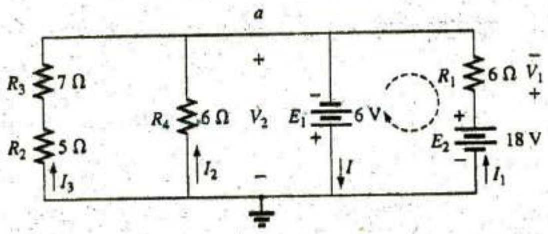


FIG. 7.23
Network in Fig. 7.22 redrawn.

It is now obvious that

$$V_2 = -E_1 = \mathbf{-6\text{ V}}$$

The minus sign simply indicates that the chosen polarity for V_2 in Fig. 7.18 is opposite to that of the actual voltage. Applying Kirchhoff's voltage law to the loop indicated, we obtain

$$-E_1 + V_1 - E_2 = 0$$

and $$V_1 = E_2 + E_1 = 18\text{ V} + 6\text{ V} = \mathbf{24\text{ V}}$$

Applying Kirchhoff's current law to node a yields

$$\begin{aligned} I &= I_1 + I_2 + I_3 \\ &= \frac{V_1}{R_1} + \frac{E_1}{R_4} + \frac{E_1}{R_2 + R_3} \\ &= \frac{24\text{ V}}{6\,\Omega} + \frac{6\text{ V}}{6\,\Omega} + \frac{6\text{ V}}{12\,\Omega} \\ &= 4\text{ A} + 1\text{ A} + 0.5\text{ A} \\ I &= \mathbf{5.5\text{ A}} \end{aligned}$$

The next example is clear evidence that techniques learned in the current chapters will have far-reaching applications and will not be dropped for improved methods. Even though we have not studied the **transistor** yet, the dc levels of a transistor network can be examined using the basic rules and laws introduced in earlier chapters.

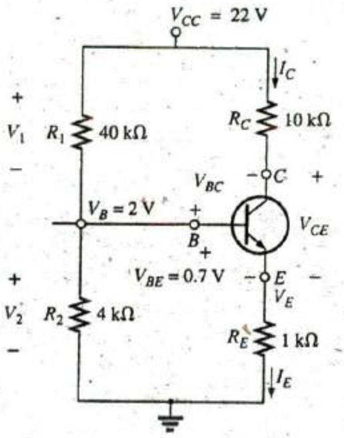


FIG. 7.24
Example 7.9.

EXAMPLE 7.9 For the transistor configuration in Fig. 7.24, in which V_B and V_{BE} have been provided:

a. Determine the voltage V_E and the current I_E.
b. Calculate V_1.
c. Determine V_{BC} using the fact that the approximation $I_C = I_E$ is often applied to transistor networks.
d. Calculate V_{CE} using the information obtained in parts (a) through (c).

Solutions:

a. From Fig. 7.24, we find

$$V_2 = V_B = 2\text{ V}$$

Writing Kirchhoff's voltage law around the lower loop yields

$$V_2 - V_{BE} + V_E = 0$$

or $$V_E = V_2 + V_{BE} = 2\text{ V} - 0.7\text{ V} = \mathbf{1.3\text{ V}}$$

and $$I_E = \frac{V_E}{R_E} = \frac{1.3\text{ V}}{1\text{ k}\Omega} = \mathbf{1.3\text{ mA}}$$

b. Applying Kirchhoff's voltage law to the input side (left region of the network) results in

$$V_2 + V_1 - V_{CC} = 0$$

and $$V_1 = V_{CC} - V_2$$

but $$V_2 = V_B$$

and $$V_1 = V_{CC} - V_2 = 22\text{ V} - 2\text{ V} = \mathbf{20\text{ V}}$$

c. Redrawing the section of the network of immediate interest results in Fig. 7.25, where Kirchhoff's voltage law yields

$$V_C + V_{R_C} - V_{CC} = 0$$

and $$V_C = V_{CC} - V_{R_C} = V_{CC} - I_C R_C$$

but $$I_C = I_E$$

and $$V_C = V_{CC} - I_E R_C = 22\text{ V} - (1.3\text{ mA})(10\text{ k}\Omega) = 9\text{ V}$$

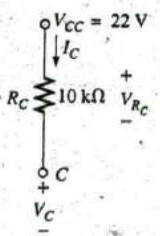


FIG. 7.25
Determining V_C for the network in Fig. 7.24.

Then $$V_{BC} = V_B - V_C = 2\text{ V} - 9\text{ V} = \mathbf{-7\text{ V}}$$

d. $$V_{CE} = V_C - V_E = 9\text{ V} - 1.3\text{ V} = \mathbf{7.7\text{ V}}$$

EXAMPLE 7.10 Calculate the indicated currents and voltage in Fig. 7.26.

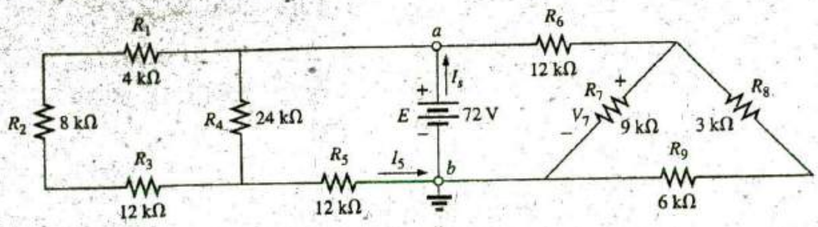


FIG. 7.26
Example 7.10.

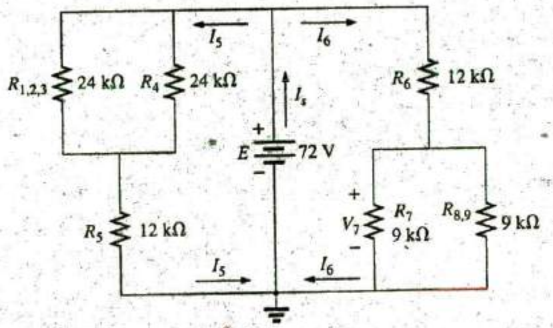


FIG. 7.27
Network in Fig. 7.26 redrawn.

Solution: Redrawing the network after combining series elements yields Fig. 7.27, and

$$I_5 = \frac{E}{R_{(1,2,3)\|4} + R_5} = \frac{72\text{ V}}{12\text{ k}\Omega + 12\text{ k}\Omega} = \frac{72\text{ V}}{24\text{ k}\Omega} = \mathbf{3\text{ mA}}$$

with

$$V_7 = \frac{R_{7\|(8,9)}E}{R_{7\|(8,9)} + R_6} = \frac{(4.5\text{ k}\Omega)(72\text{ V})}{4.5\text{ k}\Omega + 12\text{ k}\Omega} = \frac{324\text{ V}}{16.5} = \mathbf{19.6\text{ V}}$$

$$I_6 = \frac{V_7}{R_{7\|(8,9)}} = \frac{19.6\text{ V}}{4.5\text{ k}\Omega} = \mathbf{4.35\text{ mA}}$$

and $\quad I_s = I_5 + I_6 = 3\text{ mA} + 4.35\text{ mA} = \mathbf{7.35\text{ mA}}$

Since the potential difference between points a and b in Fig. 7.26 is fixed at E volts, the circuit to the right or left is unaffected if the network is reconstructed as shown in Fig. 7.28.

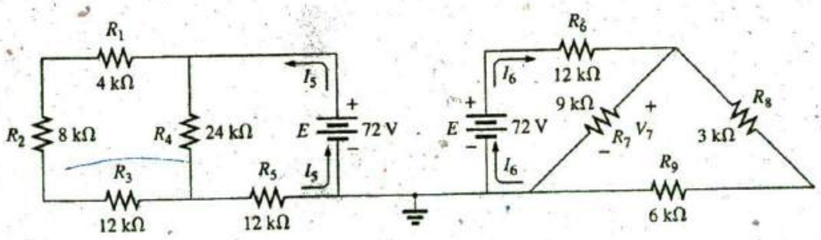


FIG. 7.28
An alternative approach to Example 7.10.

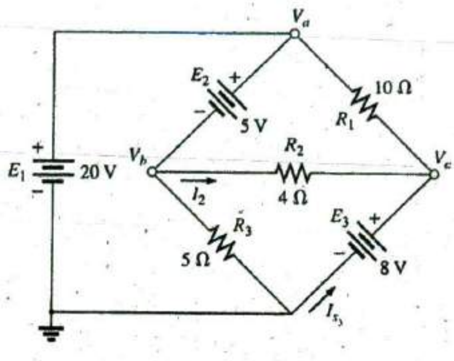


FIG. 7.29
Example 7.11.

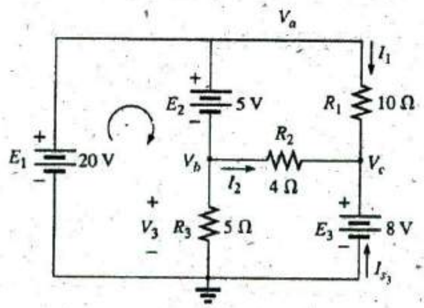


FIG. 7.30
Network in Fig. 7.29 redrawn to better define a path toward the desired unknowns.

We can find each quantity required, except I_s, by analyzing each circuit independently. To find I_s, we must find the source current for each circuit and add it as in the above solution; that is, $I_s = I_5 + I_6$.

EXAMPLE 7.11 For the network in Fig. 7.29:

a. Determine voltages V_a, V_b, and V_c.
b. Find voltages V_{ac} and V_{bc}.
c. Find current I_2.
d. Find the source current I_{s_3}.
e. Insert voltmeters to measure voltages V_a and V_{bc} and current I_{s_3}.

Solutions:

a. The network is redrawn in Fig. 7.30 to clearly indicate the arrangement between elements.

First, note that voltage V_a is directly across voltage source E_1. Therefore,

$$V_a = E_1 = \mathbf{20\ V}$$

The same is true for voltage V_c, which is directly across the voltage source E_3. Therefore,

$$V_c = E_3 = \mathbf{8\ V}$$

To find voltage V_b, which is actually the voltage across R_3, we must apply Kirchhoff's voltage law around loop 1 as follows:

$$+E_1 - E_2 - V_3 = 0$$

and $$V_3 = E_1 - E_2 = 20\ V - 5\ V = 15\ V$$

and $$V_b = V_3 = \mathbf{15\ V}$$

b. Voltage V_{ac}, which is actually the voltage across resistor R_1, can then be determined as follows:

$$V_{ac} = V_a - V_c = 20\ V - 8\ V = \mathbf{12\ V}$$

Similarly, voltage V_{bc}, which is actually the voltage across resistor R_2, can then be determined as follows:

$$V_{bc} = V_b - V_c = 15\ V - 8\ V = \mathbf{7\ V}$$

c. Current I_2 can be determined using Ohm's law:

$$I_2 = \frac{V_2}{R_2} = \frac{V_{bc}}{R_2} = \frac{7\ V}{4\ \Omega} = \mathbf{1.75\ A}$$

d. The source current I_{s_3} can be determined using Kirchhoff's current law at node c:

$$\Sigma I_i = \Sigma I_o$$
$$I_1 + I_2 + I_{s_3} = 0$$

and $$I_{s_3} = -I_1 - I_2 = -\frac{V_1}{R_1} - I_2$$

with $$V_1 = V_{ac} = V_a - V_c = 20\ V - 8\ V = 12\ V$$

so that

$$I_{s_3} = -\frac{12\ V}{10\ \Omega} - 1.75\ A = -1.2\ A - 1.75\ A = \mathbf{-2.95\ A}$$

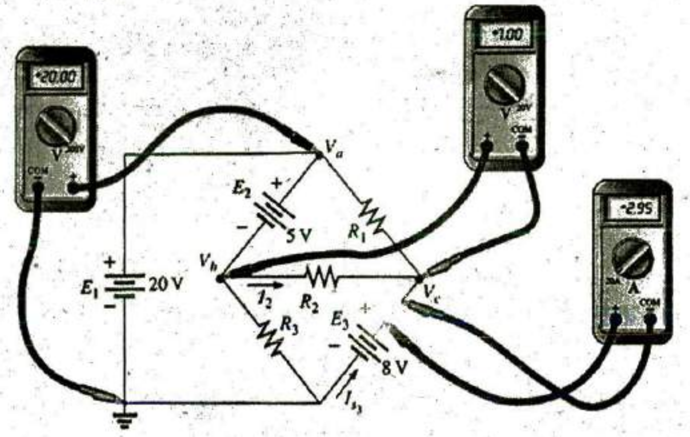


FIG. 7.31
Complex network for Example 7.11.

revealing that current is actually being forced through source E_3 in a direction opposite to that shown in Fig. 7.29.

e. Both voltmeters have a positive reading, as shown in Fig. 7.31, while the ammeter has a negative reading.

7.6 LADDER NETWORKS

A three-section **ladder network** appears in Fig. 7.32. The reason for the terminology is quite obvious for the repetitive structure. Basically two approaches are used to solve networks of this type.

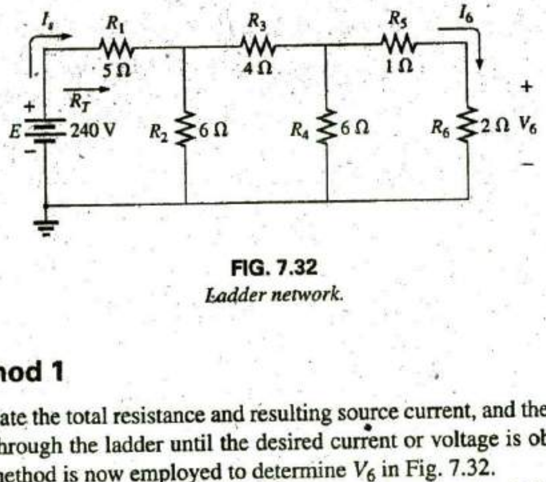


FIG. 7.32
Ladder network.

Method 1

Calculate the total resistance and resulting source current, and then work back through the ladder until the desired current or voltage is obtained. This method is now employed to determine V_6 in Fig. 7.32.

Combining parallel and series elements as shown in Fig. 7.33 results in the reduced network in Fig. 7.34, and

$$R_T = 5\,\Omega + 3\,\Omega = 8\,\Omega$$

$$I_s = \frac{E}{R_T} = \frac{240\text{ V}}{8\,\Omega} = 30\text{ A}$$

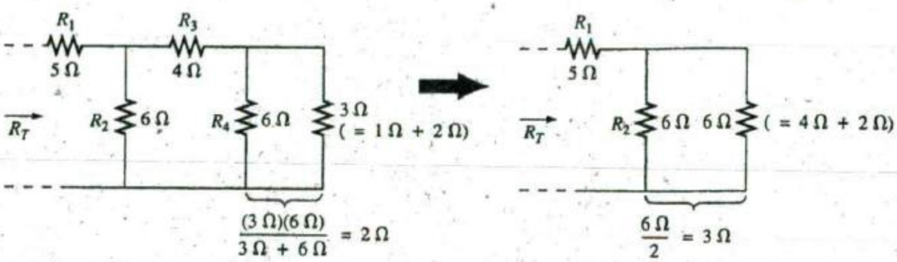


FIG. 7.33
Working back to the source to determine R_T for the network in Fig. 7.32.

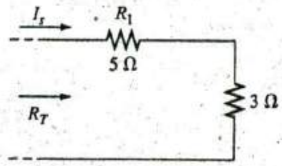


FIG. 7.34
Calculating R_T and I_s.

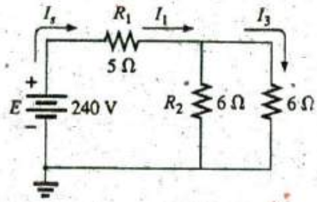


FIG. 7.35
Working back toward I_6.

Working our way back to I_6 (Fig. 7.35), we find that

$$I_1 = I_s$$

and
$$I_3 = \frac{I_s}{2} = \frac{30\text{ A}}{2} = 15\text{ A}$$

and, finally (Fig. 7.36),

$$I_6 = \frac{(6\,\Omega)I_3}{6\,\Omega + 3\,\Omega} = \frac{6}{9}(15\text{ A}) = 10\text{ A}$$

and
$$V_6 = I_6R_6 = (10\text{ A})(2\,\Omega) = \mathbf{20\text{ V}}$$

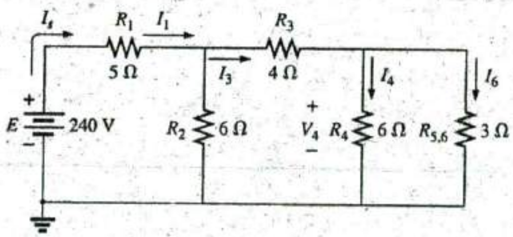


FIG. 7.36
Calculating I_6.

Method 2

Assign a letter symbol to the last branch current and work back through the network to the source, maintaining this assigned current or other current of interest. The desired current can then be found directly. This method can best be described through the analysis of the same network considered in Fig. 7.32, redrawn in Fig. 7.37.

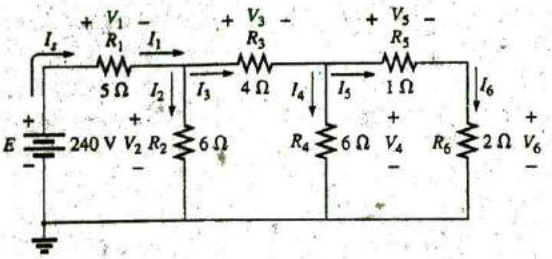


FIG. 7.37
An alternative approach for ladder networks.

The assigned notation for the current through the final branch is I_6:

$$I_6 = \frac{V_4}{R_5 + R_6} = \frac{V_4}{1\,\Omega + 2\,\Omega} = \frac{V_4}{3\,\Omega}$$

or
$$V_4 = (3\,\Omega)I_6$$

so that
$$I_4 = \frac{V_4}{R_4} = \frac{(3\,\Omega)I_6}{6\,\Omega} = 0.5I_6$$

and
$$I_3 = I_4 + I_6 = 0.5I_6 + I_6 = 1.5I_6$$
$$V_3 = I_3R_3 = (1.5I_6)(4\,\Omega) = (6\,\Omega)I_6$$

Also,
$$V_2 = V_3 + V_4 = (6\,\Omega)I_6 + (3\,\Omega)I_6 = (9\,\Omega)I_6$$

so that
$$I_2 = \frac{V_2}{R_2} = \frac{(9\,\Omega)I_6}{6\,\Omega} = 1.5I_6$$

and
$$I_s = I_2 + I_3 = 1.5I_6 + 1.5I_6 = 3I_6$$

with
$$V_1 = I_1R_1 = I_sR_1 = (5\,\Omega)I_s$$

so that
$$E = V_1 + V_2 = (5\,\Omega)I_s + (9\,\Omega)I_6$$
$$= (5\,\Omega)(3I_6) + (9\,\Omega)I_6 = (24\,\Omega)I_6$$

and
$$I_6 = \frac{E}{24\,\Omega} = \frac{240\text{ V}}{24\,\Omega} = 10\text{ A}$$

with
$$V_6 = I_6R_6 = (10\text{ A})(2\,\Omega) = \mathbf{20\text{ V}}$$

as was obtained using method 1.

7.7 VOLTAGE DIVIDER SUPPLY (UNLOADED AND LOADED)

When the term *loaded* is used to describe voltage divider supply, it refers to the application of an element, network, or system to a supply that draws current from the supply. In other words,

the loading down of a system is the process of introducing elements that will draw current from the system. The heavier the current, the greater is the loading effect.

Recall from Section 5.10 that the application of a load can affect the terminal voltage of a supply due to the internal resistance.

No-Load Conditions

Through a voltage divider network such as that in Fig. 7.38, a number of different terminal voltages can be made available from a single supply. Instead of having a single supply of 120 V, we now have terminal voltages of 100 V and 60 V available—a wonderful result for such a simple network. However, there can be disadvantages. One is that the applied

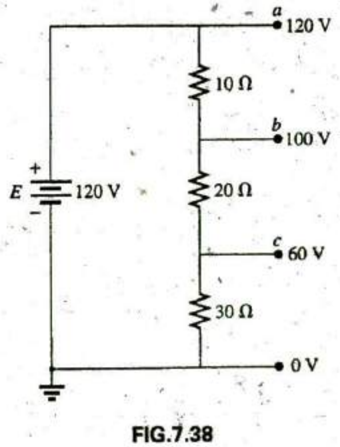


FIG.7.38
Voltage divider supply.

resistive loads can have values too close to those making up the voltage divider network.

In general,

for a voltage divider supply to be effective, the applied resistive loads should be significantly larger than the resistors appearing in the voltage divider network.

To demonstrate the validity of the above statement, let us now examine the effect of applying resistors with values very close to those of the voltage divider network.

Loaded Conditions

In Fig. 7.39, resistors of 20 Ω have been connected to each of the terminal voltages. Note that this value is equal to one of the resistors in the voltage divider network and very close to the other two.

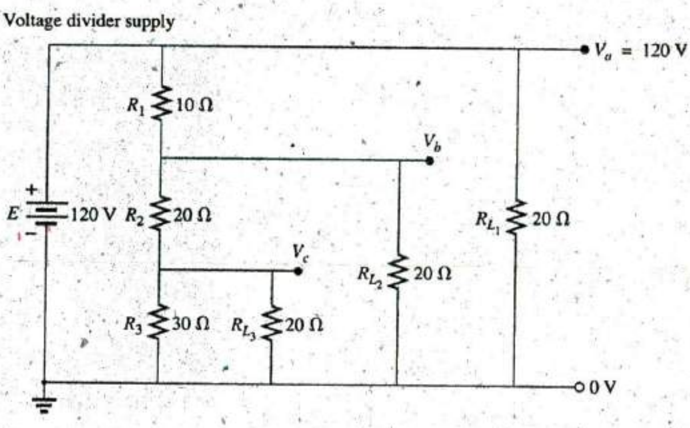


FIG. 7.39
Voltage divider supply with loads equal to the average value of the resistive elements that make up the supply.

Voltage V_a is unaffected by the load R_{L_1} since the load is in parallel with the supply voltage E. The result is $V_a = 120$ V, which is the same as the no-load level. To determine V_b, we must first note that R_3 and R_{L_3} are in parallel and $R'_3 = R_3 \| R_{L_3} = 30\ \Omega \| 20\ \Omega = 12\ \Omega$. The parallel combination gives

$$R'_2 = (R_2 + R'_3) \| R_{L_2} = (20\ \Omega + 12\ \Omega) \| 20\ \Omega$$
$$= 32\ \Omega \| 20\ \Omega = 12.31\ \Omega$$

Applying the voltage divider rule gives

$$V_b = \frac{(12.31\ \Omega)(120\ \text{V})}{12.31\ \Omega + 10\ \Omega} = \mathbf{66.21\ V}$$

versus 100 V under no-load conditions.

Voltage V_c is

$$V_c = \frac{(12\ \Omega)(66.21\ \text{V})}{12\ \Omega + 20\ \Omega} = \mathbf{24.83\ V}$$

versus 60 V under no-load conditions.

The effect of load resistors close in value to the resistor employed in the voltage divider network is, therefore, to decrease significantly some of the terminal voltages.

If the load resistors are changed to the 1 kΩ level, the terminal voltages will all be relatively close to the no-load values. The analysis is similar to the above, with the following results:

$$V_a = \mathbf{120\ V} \qquad V_b = \mathbf{98.88\ V} \qquad V_c = \mathbf{58.63\ V}$$

If we compare current drains established by the applied loads, we find for the network in Fig. 7.39 that

$$I_{L_2} = \frac{V_{L_2}}{R_{L_2}} = \frac{66.21\ \text{V}}{20\ \Omega} = 3.31\ \text{A}$$

and for the 1 kΩ level,

$$I_{L_2} = \frac{98.88\ \text{V}}{1\ \text{k}\Omega} = 98.88\ \text{mA} < 0.1\ \text{A}$$

As demonstrated above, the greater the current drain, the greater is the change in terminal voltage with the application of the load. This is certainly verified by the fact that I_{L_2} is about 33.5 times larger with the 20 Ω loads.

The next example is a design exercise. The voltage and current ratings of each load are provided, along with the terminal ratings of the supply. The required voltage divider resistors must be found.

EXAMPLE 7.12 Determine R_1, R_2, and R_3 for the voltage divider supply in Fig. 7.40. Can 2 W resistors be used in the design?

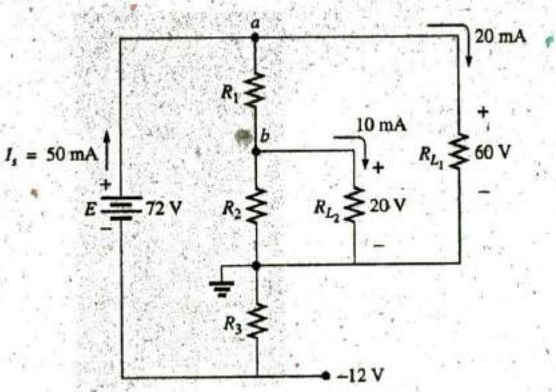


FIG. 7.40
Voltage divider supply for Example 7.12.

Solution: R_3:

$$R_3 = \frac{V_{R_3}}{I_{R_3}} = \frac{V_{R_3}}{I_s} = \frac{12\ \text{V}}{50\ \text{mA}} = \mathbf{240\ \Omega}$$ •24 kΩ

$$P_{R_3} = (I_{R_3})^2 R_3 = (50\ \text{mA})^2\, 240\ \Omega = 0.6\ \text{W} < 2\ \text{W}$$

R_1: Applying Kirchhoff's current law to node a, we have

$$I_s - I_{R_1} - I_{L_1} = 0$$

and $I_{R_1} = I_s - I_{L_1} = 50\text{ mA} - 20\text{ mA} = 30\text{ mA}$

$$R_1 = \frac{V_{R_1}}{I_{R_1}} = \frac{V_{L_1} - V_{L_2}}{I_{R_1}} = \frac{60\text{ V} - 20\text{ V}}{30\text{ mA}} = \frac{40\text{ V}}{30\text{ mA}} = \mathbf{1.33\text{ k}\Omega}$$

$$P_{R_1} = (I_{R_1})^2 R_1 = (30\text{ mA})^2\, 1.33\text{ k}\Omega = 1.197\text{ W} < 2\text{ W}$$

R_2: Applying Kirchhoff's current law at node b, we have

$$I_{R_1} - I_{R_2} - I_{L_2} = 0$$

and $\quad I_{R_2} = I_{R_1} - I_{L_2} = 30\text{ mA} - 10\text{ mA} = 20\text{ mA}$

$$R_2 = \frac{V_{R_2}}{I_{R_2}} = \frac{20\text{ V}}{20\text{ mA}} = \mathbf{1\text{ k}\Omega}$$

$$P_{R_2} = (I_{R_2})^2 R_2 = (20\text{ mA})^2\, 1\text{ k}\Omega = 0.4\text{ W} < 2\text{ W}$$

Since P_{R_1}, P_{R_2}, and P_{R_3} are less than 2 W, 2 W resistors can be used fo the design.

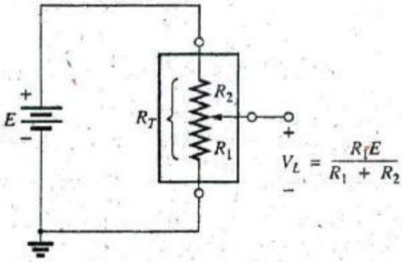


FIG. 7.41
Unloaded potentiometer.

7.8 POTENTIOMETER LOADING

For the unloaded potentiometer in Fig. 7.41, the output voltage is deter mined by the voltage divider rule, with R_T in the figure representing th total resistance of the potentiometer. Too often it is assumed that th voltage across a load connected to the wiper arm is determined solely b the potentiometer and the effect of the load can be ignored. This is defi nitely not the case, as is demonstrated here.

When a load is applied as shown in Fig. 7.42, the output voltage V_L i now a function of the magnitude of the load applied since R_1 is not a shown in Fig. 7.41 but is instead the parallel combination of R_1 and R_L

The output voltage is now

$$\boxed{V_L = \frac{R'E}{R' + R_2} \quad \text{with } R' = R_1 \| R_L} \qquad \textbf{(7.1}$$

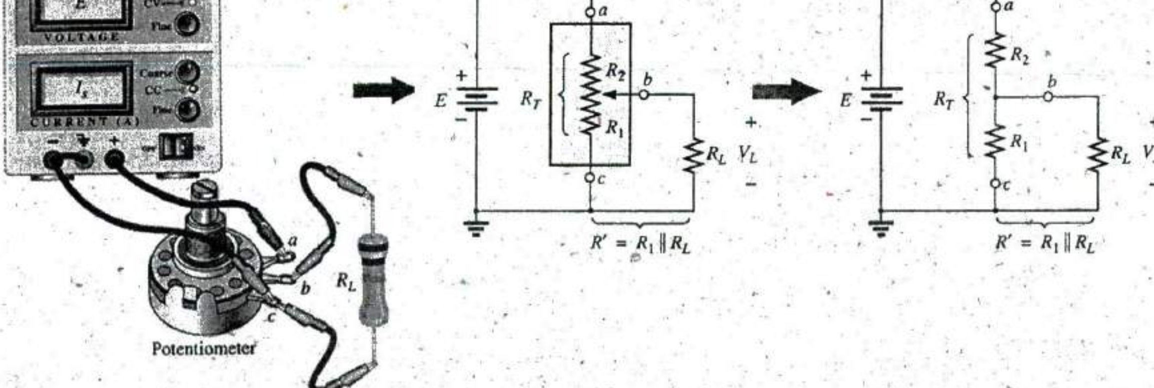


FIG. 7.42
Loaded potentiometer.

If you want to have good control of the output voltage V_L through the controlling dial, knob, screw, or whatever, you must choose a load or potentiometer that satisfies the following relationship:

$$\boxed{R_L \gg R_T} \tag{7.2}$$

In general,

when hooking up a load to a potentiometer, be sure that the load resistance far exceeds the maximum terminal resistance of the potentiometer if good control of the output voltage is desired.

For example, let's disregard Eq. (7.2) and choose a 1 MΩ potentiometer with a 100 Ω load and set the wiper arm to 1/10 the total resistance, as shown in Fig. 7.43. Then

$$R' = 100\text{ k}\Omega \parallel 100\ \Omega = 99.9\ \Omega$$

and
$$V_L = \frac{99.9\ \Omega(10\text{ V})}{99.9\ \Omega + 900\text{ k}\Omega} \cong 0.001\text{ V} = 1\text{ mV}$$

which is extremely small compared to the expected level of 1 V.

In fact, if we move the wiper arm to the midpoint,

$$R' = 500\text{ k}\Omega \parallel 100\ \Omega = 99.98\ \Omega$$

and
$$V_L = \frac{(99.98\ \Omega)(10\text{ V})}{99.98\ \Omega + 500\text{ k}\Omega} \cong 0.002\text{ V} = 2\text{ mV}$$

which is negligible compared to the expected level of 5 V. Even at $R_1 =$ 900 kΩ, V_L is only 0.01 V, or 1/1000 of the available voltage.

Using the reverse situation of $R_T = 100\ \Omega$ and $R_L = 1\text{ M}\Omega$ and the wiper arm at the 1/10 position, as in Fig. 7.44, we find

$$R' = 10\ \Omega \parallel 1\text{ M}\Omega \cong 10\ \Omega$$

and
$$V_L = \frac{10\ \Omega(10\text{ V})}{10\ \Omega + 90\ \Omega} = 1\text{ V}$$

as desired.

For the lower limit (worst-case design) of $R_L = R_T = 100\ \Omega$, as defined by Eq. (7.2) and the halfway position of Fig. 7.42,

$$R' = 50\ \Omega \parallel 100\ \Omega = 33.33\ \Omega$$

and
$$V_L = \frac{33.33\ \Omega(10\text{ V})}{33.33\ \Omega + 50\ \Omega} \cong 4\text{ V}$$

It may not be the ideal level of 5 V, but at least 40% of the voltage E has been achieved at the halfway position rather than the 0.02% obtained with $R_L = 100\ \Omega$ and $R_T = 1\text{ M}\Omega$.

In general, therefore, try to establish a situation for potentiometer control in which Eq. (7.2) is satisfied to the highest degree possible.

Someone might suggest that we make R_T as small as possible to bring the percent result as close to the ideal as possible. Keep in mind, however, that the potentiometer has a power rating, and for networks such as Fig. 7.44, $P_{max} \cong E^2/R_T = (10\text{ V})^2/100\ \Omega = 1\text{ W}$. If R_T is reduced to 10 Ω, $P_{max} = (10\text{ V})^2/10\ \Omega = 10\text{ W}$, which would require a *much larger* unit.

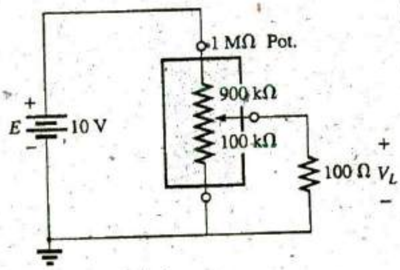


FIG. 7.43
Loaded potentiometer with $R_L \ll R_T$.

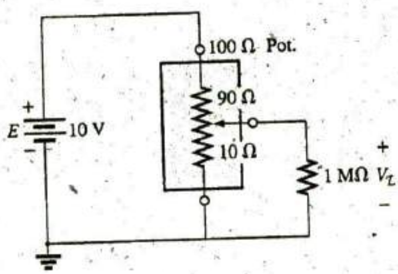


FIG. 7.44
Loaded potentiometer with $R_L \gg R_T$.

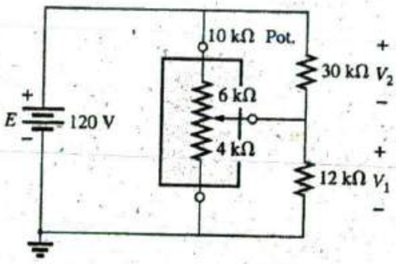


FIG. 7.45
Example 7.13.

EXAMPLE 7.13 Find voltages V_1 and V_2 for the loaded potentiometer of Fig. 7.45.

Solution: Ideal (no load):

$$V_1 = \frac{4\text{ k}\Omega(120\text{ V})}{10\text{ k}\Omega} = \mathbf{48\text{ V}}$$

$$V_2 = \frac{6\text{ k}\Omega(120\text{ V})}{10\text{ k}\Omega} = \mathbf{72\text{ V}}$$

Loaded:

$$R' = 4\text{ k}\Omega \parallel 12\text{ k}\Omega = 3\text{ k}\Omega$$
$$R'' = 6\text{ k}\Omega \parallel 30\text{ k}\Omega = 5\text{ k}\Omega$$
$$V_1 = \frac{3\text{ k}\Omega(120\text{ V})}{8\text{ k}\Omega} = \mathbf{45\text{ V}}$$
$$V_2 = \frac{5\text{ k}\Omega(120\text{ V})}{8\text{ k}\Omega} = \mathbf{75\text{ V}}$$

The ideal and loaded voltage levels are so close that the design can be considered a good one for the applied loads. A slight variation in the position of the wiper arm will establish the ideal voltage levels across the two loads.

7.9 AMMETER, VOLTMETER, AND OHMMETER DESIGN

The designs of this section will use the iron-vane movement of Fig. 7.46 because it is the one that is most frequently used by current instrument manufacturers. It operates using the principle that there is a repulsive force between like magnetic poles. When a current is applied to the coil wrapped around the two vanes, a magnetic field is established within the coil, magnetizing the fixed and moveable vanes. Since both vanes will be magnetized in the same manner, they will have the same polarity, and a force of repulsion will develop between the two vanes. The stronger the applied current, the stronger are the magnetic field and the force of repulsion between

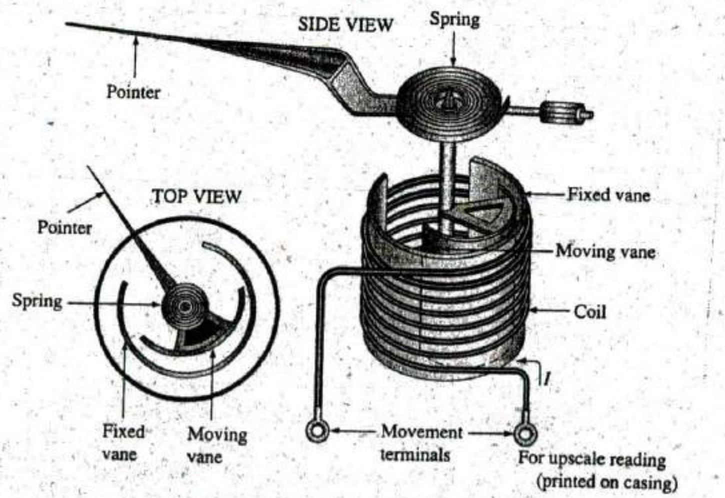


FIG. 7.46
Iron-vane movement.

the vanes. The fixed vane will remain in position, but the moveable vane will rotate and provide a measure of the strength of the applied current.

An iron-vane movement manufactured by the Simpson Company appears in Fig. 7.47(a). Movements of this type are usually rated in terms of current and resistance. The current sensitivity (CS) is the current that will result in a full-scale deflection. The resistance (R_m) is the internal resistance of the movement. The graphic symbol for a movement appears in Fig. 7.47(b) with the current sensitivity and internal resistance for the unit of Fig. 7.47(a).

Movements are usually rated by current and resistance. The specifications of a typical movement may be 1 mA, 50 Ω. The 1 mA is the *current sensitivity (CS)* of the movement, which is the current required for a full-scale deflection. It is denoted by the symbol I_{CS}. The 50 Ω represents the internal resistance (R_m) of the movement. A common notation for the movement and its specifications is provided in Fig. 7.48.

(a)

(b)

FIG. 7.47
Iron-vane movement; (a) photo, (b) symbol and ratings.

The Ammeter

The maximum current that the iron-vane movement can read independently is equal to the current sensitivity of the movement. However, higher currents can be measured if additional circuitry is introduced. This additional circuitry, as shown in Fig. 7.48, results in the basic construction of an ammeter.

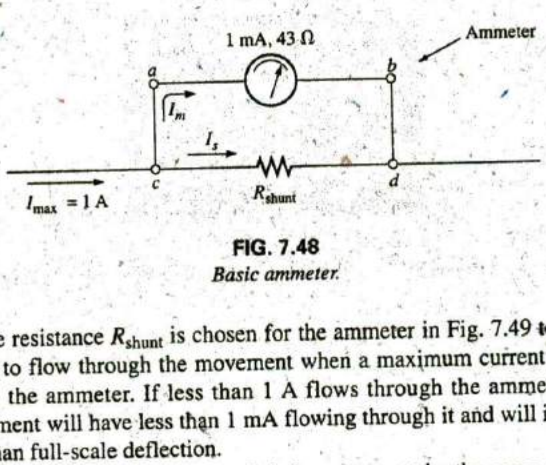


FIG. 7.48
Basic ammeter.

The resistance R_{shunt} is chosen for the ammeter in Fig. 7.49 to allow 1 mA to flow through the movement when a maximum current of 1 A enters the ammeter. If less than 1 A flows through the ammeter, the movement will have less than 1 mA flowing through it and will indicate less than full-scale deflection.

Since the voltage across parallel elements must be the same, the potential drop across *a-b* in Fig. 7.49 must equal that across *c-d*; that is,

$$(1\text{ mA})(43\ \Omega) = R_{shunt}I_s$$

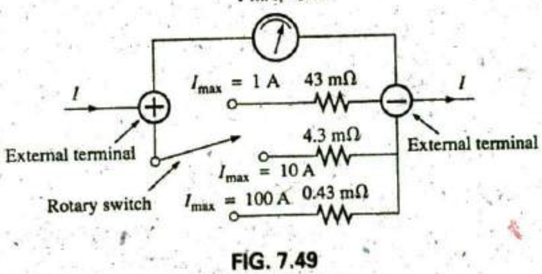


FIG. 7.49
Multirange ammeter.

Also, I_s must equal 1 A − 1 mA = 999 mA if the current is to be limited to 1 mA through the movement (Kirchhoff's current law). Therefore,

$$(1\text{ mA})(43\ \Omega) = R_{\text{shunt}}(999\text{ mA})$$
$$R_{\text{shunt}} = \frac{(1\text{ mA})(43\ \Omega)}{999\text{ mA}}$$
$$\cong 43\text{ m}\Omega \text{ (a standard value)}$$

In general,

$$\boxed{R_{\text{shunt}} = \frac{R_m I_{CS}}{I_{\max} - I_{CS}}} \qquad (7.4)$$

One method of constructing a multirange ammeter is shown in Fig. 7.50, where the rotary switch determines the R_{shunt} to be used for the maximum current indicated on the face of the meter. Most meters use the same scale for various values of maximum current. If you read 375 on the 0–5 mA scale with the switch on the 5 setting, the current is 3.75 mA; on the 50 setting, the current is 37.5 mA; and so on.

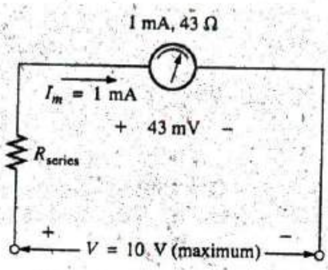


FIG. 7.50
Basic voltmeter.

The Voltmeter

A variation in the additional circuitry permits the use of the iron-vane movement in the design of a voltmeter. The 1 mA, 43 Ω movement can also be rated as a 43 mV (1 mA × 43 Ω), 43 Ω movement, indicating that the maximum voltage that the movement can measure independently is 43 mV. The millivolt rating is sometimes referred to as the *voltage sensitivity (VS)*. The basic construction of the voltmeter is shown in Fig. 7.50.

The R_{series} is adjusted to limit the current through the movement to 1 mA when the maximum voltage is applied across the voltmeter. A lower voltage simply reduces the current in the circuit and thereby the deflection of the movement.

Applying Kirchhoff's voltage law around the closed loop of Fig. 7.50, we obtain

$$[10\text{ V} - (1\text{ mA})(R_{\text{series}})] - 43\text{ mV} = 0$$

or

$$R_{\text{series}} = \frac{10\text{ V} - (43\text{ mV})}{1\text{ mA}} = 9957\ \Omega \cong 10\text{ k}\Omega$$

In general,

$$\boxed{R_{\text{series}} = \frac{V_{\max} - V_{VS}}{I_{CS}}} \qquad (7.5)$$

One method of constructing a multirange voltmeter is shown in Fig. 7.51. If the rotary switch is at 10 V, R_{series} = 10 kΩ; at 50 V, R_{series} = 40 kΩ + 10 kΩ = 50 kΩ; and at 100 V, R_{series} = 50 kΩ + 40 kΩ + 10 kΩ = 100 kΩ.

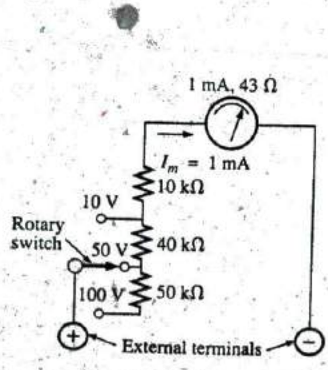


FIG. 7.51
Multirange voltmeter.

The Ohmmeter

In general, ohmmeters are designed to measure resistance in the low, middle, or high range. The most common is the **series ohmmeter,** designed to read resistance levels in the midrange. It uses the series

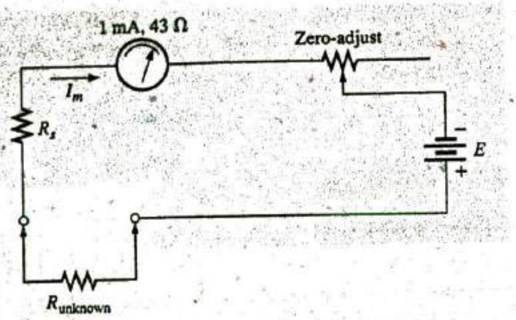


FIG. 7.52
Series ohmmeter.

configuration in Fig. 7.52. The design is quite different from that of the ammeter or voltmeter because it shows a full-scale deflection for zero ohms and no deflection for infinite resistance.

To determine the series resistance R_s, the external terminals are shorted (a direct connection of zero ohms between the two) to simulate zero ohms, and the zero-adjust is set to half its maximum value. The resistance R_s is then adjusted to allow a current equal to the current sensitivity of the movement (1 mA) to flow in the circuit. The zero-adjust is set to half its value so that any variation in the components of the meter that may produce a current more or less than the current sensitivity can be compensated for. The current I_m is

$$I_m\,(\text{full scale}) = I_{CS} = \frac{E}{R_s + R_m + \dfrac{\text{zero-adjust}}{2}} \quad (7.6)$$

and

$$R_s = \frac{E}{I_{CS}} - R_m - \frac{\text{zero-adjust}}{2} \quad (7.7)$$

If an unknown resistance is then placed between the external terminals, the current is reduced, causing a deflection less than full scale. If the terminals are left open, simulating infinite resistance, the pointer does not deflect since the current through the circuit is zero.

An instrument designed to read very low values of resistance and voltage appears in Fig. 7.53. It is capable of reading resistance levels between 10 mΩ (0.01 Ω) and 100 mΩ (0.1 Ω) and voltages between 10 mV and 100 V. Because of its low-range capability, the network design must be a great deal more sophisticated than described above. It uses electronic components that eliminate the inaccuracies introduced by lead and contact resistances. It is similar to the above system in the sense that it is completely portable and does require a dc battery to establish measurement conditions. Special leads are used to limit any introduced resistance levels.

FIG. 7.53
Nanovoltmeter.
(Courtesy of Keithley Instruments.)

The **megohmmeter** (often called a *megger*) is an instrument for measuring very high resistance values. Its primary function is to test the insulation found in power transmission systems, electrical machinery, transformers, and so on. To measure the high-resistance values, a high dc voltage is established by a hand-driven generator. If the shaft is rotated above some set value, the output of the generator is

FIG. 7.54
Megohmmeter.
(Courtesy of AEMC® Instruments, Foxborough, MA.)

fixed at one selectable voltage, typically 250 V, 500 V, or 1000 V—good reason to be careful in its use. A photograph of a commercially available tester is shown in Fig. 7.54. For this instrument, the range is 0 to 5000 MΩ.

7.10 APPLICATIONS

Boosting a Car Battery

Although boosting a car battery may initially appear to be a simple application of parallel networks, it is really a series-parallel operation that is worthy of some investigation. As indicated in Chapter 2, every dc supply has some internal resistance. For the typical 12 V lead-acid car battery, the resistance is quite small—in the milliohm range. In most cases, the low internal resistance ensures that most of the voltage (or power) is delivered to the load and not lost on the internal resistance. In Fig. 7.55, battery #2 has discharged because the lights were left on for 3 hours during a movie. Fortunately, a friend who made sure his own lights were off has a fully charged battery #1 and a good set of 16-ft cables with #6 gage stranded wire and well-designed clips. The investment in a good set of cables with sufficient length and heavy wire is a wise one, particularly if you live in a cold climate. Flexibility, as provided by stranded wire, is also a very desirable characteristic under some conditions. Be sure to check the gage of the wire and not just the thickness of the insulating jacket. You get what you pay for, and the copper is the most expensive part of the cables. Too often the label says "heavy-duty," but the gage number of the wire is too high.

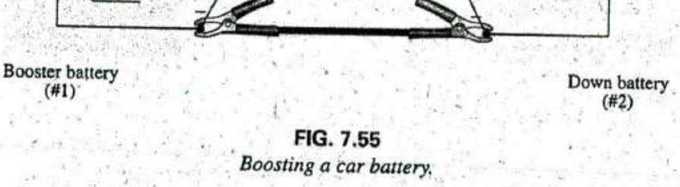


FIG. 7.55
Boosting a car battery.

The proper sequence of events in boosting a car is often a function of to whom you speak or what information you read. For safety's sake, some people recommend that the car with the good battery be turned off when making the connections. This, however, can create an immediate problem if the "dead" battery is in such a bad state that when it is hooked up to the good battery, it immediately drains the good battery to the point that neither car will start. With this in mind, it does make some sense to leave the car running to ensure that the charging process continues until the starting of the disabled car is initiated. *Because accidents do happen, it is strongly recommended that the person making the connections wear the proper type of protective eye equipment. Take sufficient time to be* *sure that you know which are the positive and negative terminals for both cars.* If it's not immediately obvious, keep in mind that the negative or ground side is usually connected to the chassis of the car with a relatively short, heavy wire.

When you are sure which are the positive and negative terminals, first connect one of the red wire clamps of the booster cables to the positive terminal of the discharged battery—all the while being sure that the other red clamp *is not touching the battery or car.* Then connect the other end of the red wire to the positive terminal of the fully charged battery. Next, connect one end of the black cable of the booster cables to the negative terminal of the booster battery, and finally connect the other end of the black cable to the engine block of the stalled vehicle (not the negative post of the dead battery) away from the carburetor, fuel lines, or moving parts of the car. Lastly, have someone maintain a constant idle speed in the car with the good battery as you start the car with the bad battery. After the vehicle starts, remove the cables in the *reverse order* starting with the cable connected to the engine block. Always be careful to ensure that clamps don't touch the battery or chassis of the car or get near any moving parts.

Some people feel that the car with the good battery should charge the bad battery for 5 to 10 minutes before starting the disabled car so the disabled car will be essentially using its own battery in the starting process. Keep in mind that the instant the booster cables are connected, the booster car is making a concerted effort to charge both its own battery and the drained battery. At starting, the good battery is asked to supply a heavy current to start the other car. It's a pretty heavy load to put on a single battery. For the situation in Fig. 7.55, the voltage of battery #2 is less than that of battery #1, and the charging current will flow as shown. The resistance in series with the boosting battery is greater because of the long length of the booster cable to the other car. The current is limited only by the series milliohm resistors of the batteries, but the voltage difference is so small that the starting current will be in safe range for the cables involved. The initial charging current will be $I = (12\text{ V} - 11.7\text{ V})/(20\text{ m}\Omega + 10\text{ m}\Omega) = 0.3\text{ V}/30\text{ m}\Omega = 10\text{ A}$. At starting, the current levels will be as shown in Fig. 7.56 for the resistance levels and battery voltages assumed. At starting, an internal resistance for the starting circuit of $0.1\ \Omega = 100\text{ m}\Omega$ is assumed. Note that the battery of the disabled car has now charged up to 11.8 V with an associated increase in its power level. The presence of two batteries requires that the analysis wait for the methods to be introduced in the next chapter.

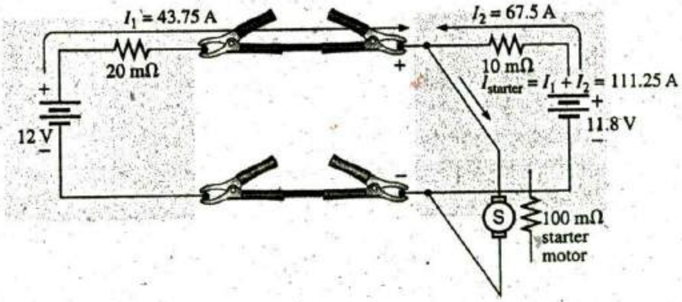


FIG. 7.56
Current levels at starting.

Note also that the current drawn from the starting circuit for the disabled car is over 100 A and that the majority of the starting current is provided by the battery being charged. In essence, therefore, the majority of the starting current is coming from the disabled car. The good battery has provided an initial charge to the bad battery and has provided the additional current necessary to start the car. In total, however, it is the battery of the disabled car that is the primary source of the starting current. For this very reason, the charging action should continue for 5 or 10 minutes before starting the car. If the disabled car is in really bad shape with a voltage level of only 11 V, the resulting levels of current will reverse, with the good battery providing 68.75 A and the bad battery only 37.5 A. Quite obviously, therefore, the worse the condition of the dead battery, the heavier is the drain on the good battery. A point can also be reached where the bad battery is in such bad shape that it cannot accept a good charge or provide its share of the starting current. The result can be continuous cranking of the disabled car without starting and possible damage to the battery of the running car due to the enormous current drain. Once the car is started and the booster cables are removed, the car with the discharged battery will continue to run because the alternator will carry the load (charging the battery and providing the necessary dc voltage) after ignition.

The above discussion was all rather straightforward, but let's investigate what may happen if it is a dark and rainy night, you are rushed, and you hook up the cables incorrectly as shown in Fig. 7.57. The result is two series-aiding batteries and a very low resistance path. The resulting current can then theoretically be extremely high [$I = (12\text{ V} + 11.7\text{ V})/30\text{ m}\Omega = 23.7\text{ V}/30\text{ m}\Omega = 790\text{ A}$], perhaps permanently damaging the electrical system of both cars and, worst of all, causing an explosion that may seriously injure someone. It is therefore very important that you treat the process of boosting a car with great care. Find that flashlight, double-check the connections, and be sure that everyone is clear when you start that car.

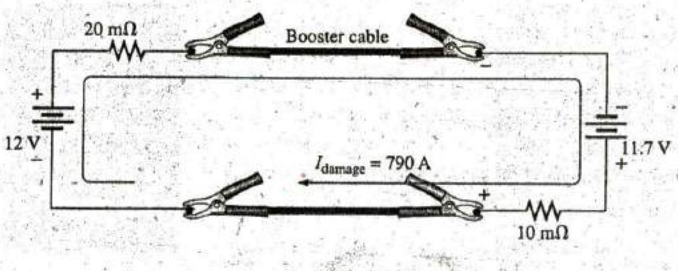


FIG. 7.57
Current levels if the booster battery is improperly connected.

Before leaving the subject, we should point out that getting a boost from a tow truck results in a somewhat different situation: The connections to the battery in the truck are very secure; the cable from the truck is a heavy wire with thick insulation; the clamps are also quite large and make an excellent connection with your battery; and the battery is heavy-duty for this type of expected load. The result is less internal resistance on the supply side and a heavier current from the truck battery. In this case, the truck is really starting the disabled car, which simply reacts to the provided surge of power.

Electronic Circuits

The operation of most electronic systems requires a distribution of dc voltages throughout the design. Although a full explanation of why the dc level is required (since it is an ac signal to be amplified) will have to wait for the introductory courses in electronic circuits, the dc analysis will proceed in much the same manner as described in this chapter. In other words, this chapter and the preceding chapters are sufficient background to perform the dc analysis of the majority of electronic networks you will encounter if given the dc terminal characteristics of the electronic elements. For example, the network in Fig. 7.58 using a transistor will be covered in detail in any introductory electronics course. The dc voltage between the base (*B*) of the transistor and the emitter (*E*) is about 0.7 V under normal operating conditions, and the collector (*C*) is related to the base current by $I_C = \beta I_B = 50I_B$ (β varies from transistor to transistor). Using these facts will enable us to determine all the dc currents and voltages of the network using the laws introduced in this chapter. In general, therefore, be encouraged that you will use the content of this chapter in numerous applications in the courses to follow.

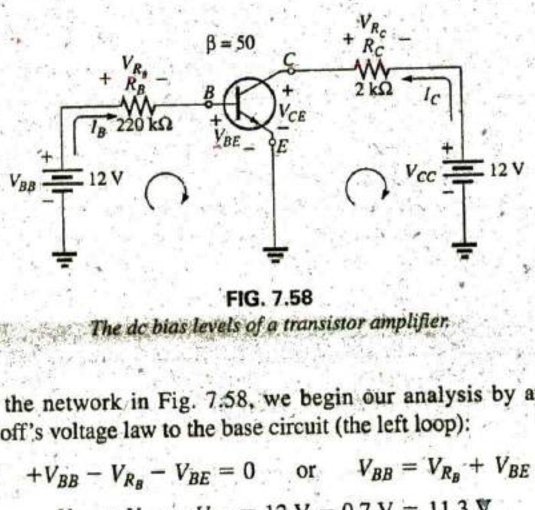


FIG. 7.58
The dc bias levels of a transistor amplifier.

For the network in Fig. 7.58, we begin our analysis by applying Kirchhoff's voltage law to the base circuit (the left loop):

$$+V_{BB} - V_{R_B} - V_{BE} = 0 \quad \text{or} \quad V_{BB} = V_{R_B} + V_{BE}$$

and $$V_{R_B} = V_{BB} - V_{BE} = 12\text{ V} - 0.7\text{ V} = 11.3\text{ V}$$

so that $$V_{R_B} = I_B R_B = 11.3\text{ V}$$

and $$I_B = \frac{V_{R_B}}{R_B} = \frac{11.3\text{ V}}{220\text{ k}\Omega} = \mathbf{51.4\ \mu A}$$

Then $$I_C = \beta I_B = 50I_B = 50(51.4\ \mu\text{A}) = \mathbf{2.57\ mA}$$

For the output circuit (the right loop)

$$+V_{CE} + V_{R_C} - V_{CC} = 0 \quad \text{or} \quad V_{CC} = V_{R_C} + V_{CE}$$

with $$V_{CE} = V_{CC} - V_{R_C} = V_{CC} - I_C R_C = 12\text{ V} - (2.57\text{ mA})(2\text{ k}\Omega)$$
$$= 12\text{ V} - 5.14\text{ V} = \mathbf{6.86\ V}$$

For a typical dc analysis of a transistor, all the currents and voltages of interest are now known: I_B, V_{BE}, I_C, and V_{CE}. All the remaining voltage, current, and power levels for the other elements of the network can now be found using the basic laws applied in this chapter.

The previous example is typical of the type of exercise you will be asked to perform in your first electronics course. For now you only need to be exposed to the device and to understand the reason for the relationships between the various currents and voltages of the device.

7.11 COMPUTER ANALYSIS

PSpice

Voltage Divider Supply We will now use PSpice to verify the results of Example 7.12. The calculated resistor values will be substituted and the voltage and current levels checked to see if they match the handwritten solution.

As shown in Fig. 7.59, the network is drawn as in earlier chapters using only the tools described thus far—in one way, a practice exercise for everything learned about the **Capture CIS Edition.** Note in this case that rotating the first resistor sets everything up for the remaining resistors. Further, it is a nice advantage that you can place one resistor after another without going to the **End Mode** option. Be especially careful with the placement of the ground, and be sure that **0/SOURCE** is used. Note also that resistor R_1 in Fig. 7.59 was entered as 1.333 kΩ rather than 1.33 kΩ as in Example 7.12. When running the program, we found that the computer solutions were not a perfect match to the longhand solution to the level of accuracy desired unless this change was made.

Since all the voltages are to ground, the voltage across R_{L_1} is 60 V; across R_{L_2}, 20 V; and across R_3, −12 V. The currents are also an excellent match with the handwritten solution, with I_E = 50 mA, I_{R_1} = 30 mA, I_{R_2} = 20 mA, I_{R_3} = 50 mA, $I_{R_{L2}}$ = 10 mA, and $I_{R_{L1}}$ = 20 mA. For the display in Fig. 7.59, the **W** option was disabled to permit concentrating on the voltage and current levels. This time, there is an exact match with the longhand solution.

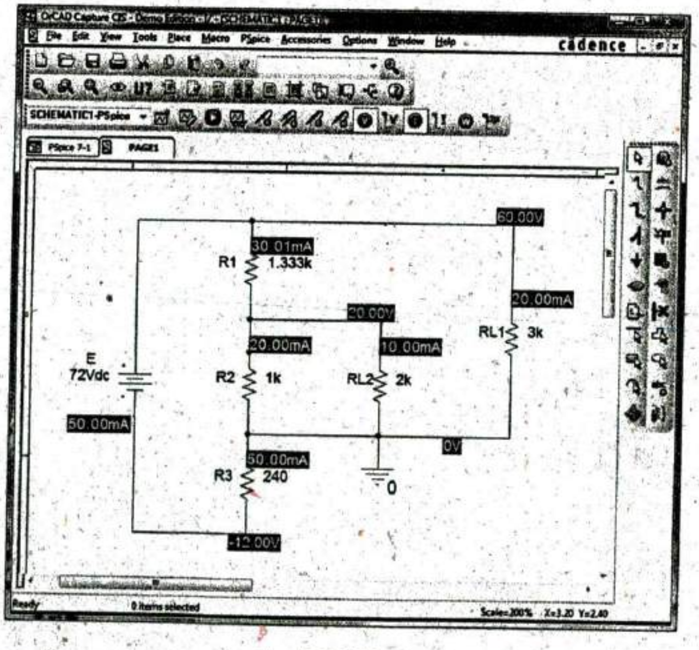


FIG. 7.59
Using PSpice to verify the results of Example 7.12.

PROBLEMS

SECTION 7.2–7.5 Series-Parallel Networks

1. Which elements (individual elements, not combinations of elements) of the networks in Fig. 7.60 are in series? Which are in parallel? As a check on your assumptions, be sure that the elements in series have the same current and that the elements in parallel have the same voltage. Restrict your decisions to single elements, not combinations of elements.

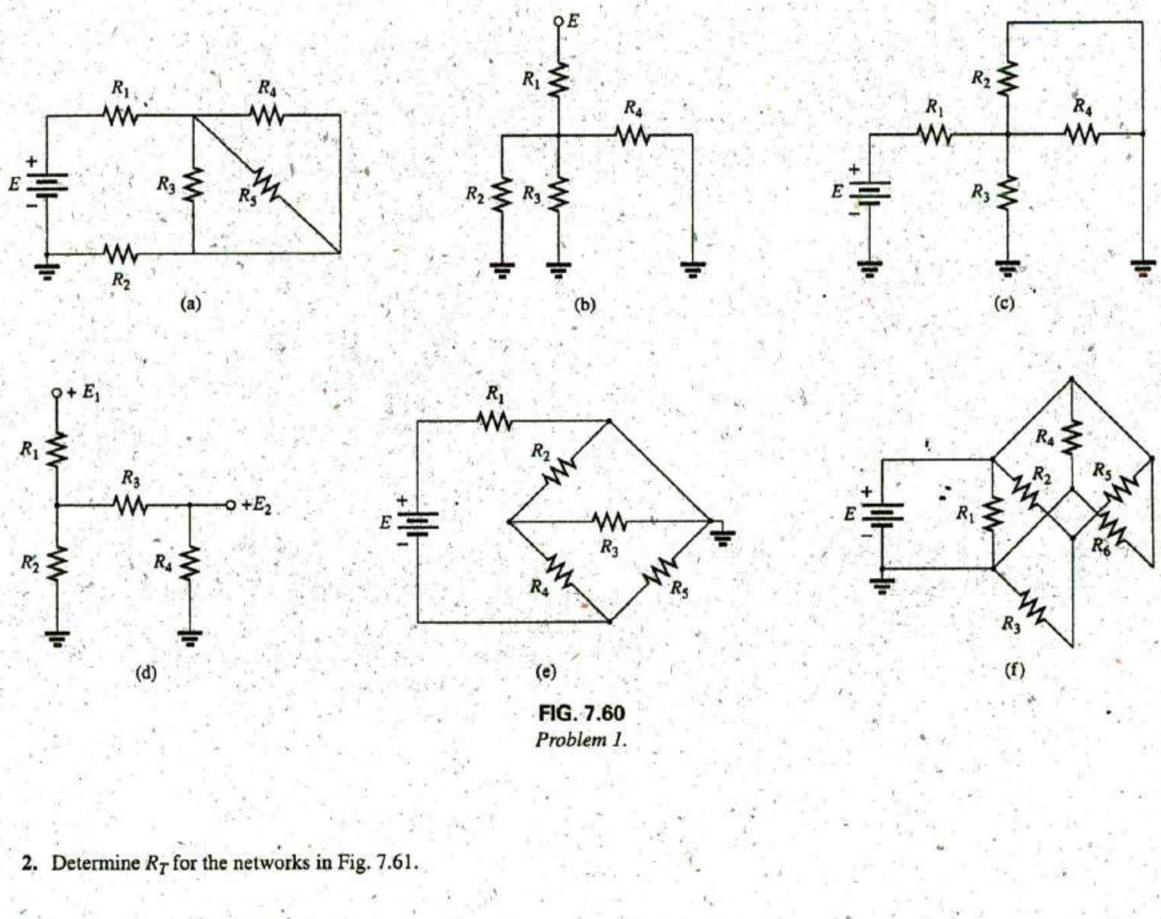


FIG. 7.60
Problem 1.

2. Determine R_T for the networks in Fig. 7.61.

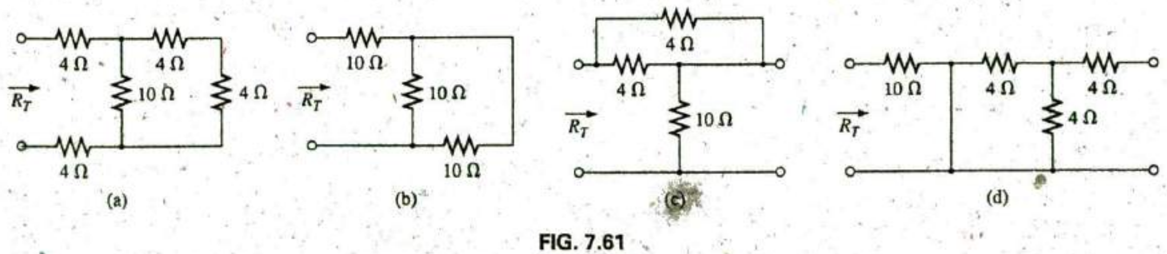


FIG. 7.61
Problem 2.

3. Find the total resistance for the configuration of Fig. 7.62.

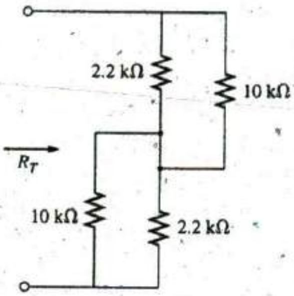


FIG. 7.62
Problem 3.

*4. Find the resistance R_T for the network of Fig. 7.63. Hint! If it was infinite in length, how would the resistance looking into the next vertical 1 Ω resistor compare to the desired resistance R_T?

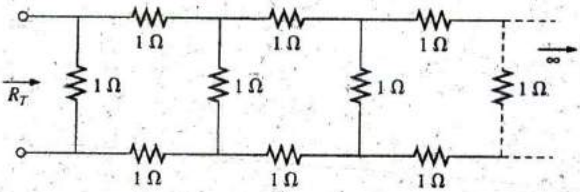


FIG. 7.63
Problem 4.

*5. The total resistance R_T for the network of Fig. 7.64 is 7.2 kΩ. Find the resistance R_1.

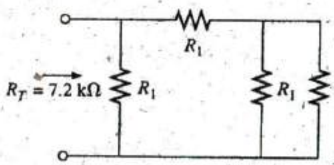


FIG. 7.64
Problem 5.

6. For the network in Fig. 7.65:
   a. Does $I_s = I_5 = I_6$? Explain.
   b. If $I_s = 10$ A and $I_1 = 4$ A, find I_2.
   c. Does $I_1 + I_2 = I_3 + I_4$? Explain.
   d. If $V_2 = 8$ V and $E = 14$ V, find V_3.
   e. If $R_1 = 4\ \Omega$, $R_2 = 2\ \Omega$, $R_3 = 4\ \Omega$, and $R_4 = 6\ \Omega$, what is R_T?
   f. If all the resistors of the configuration are 20 Ω, what is the source current if the applied voltage is 20 V?
   g. Using the values of part (f), find the power delivered by the battery and the power absorbed by the total resistance R_T.

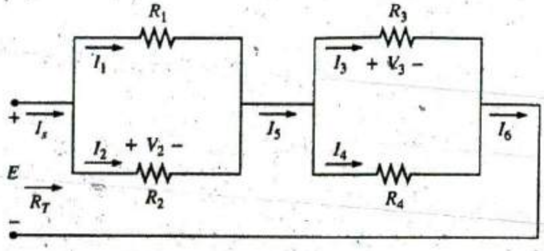


FIG. 7.65
Problem 6.

7. For the network in Fig. 7.66:
   a. Determine R_T.
   b. Find I_s, I_1, and I_2.
   c. Find voltage V_a.

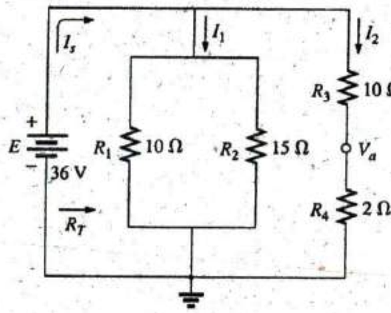


FIG. 7.66
Problem 7.

8. For the network of Fig. 7.67:
   a. Find the voltages V_a and V_b.
   b. Find the currents I_1 and I_s.

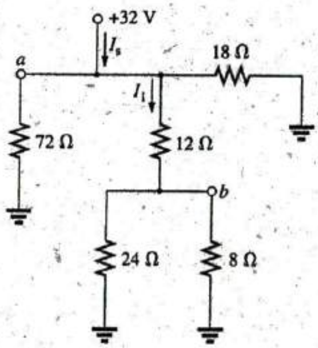


FIG. 7.67
Problem 8.

9. For the network of Fig. 7.68:
   a. Find the voltages V_a, V_b, and V_c.
   b. Find the currents I_1 and I_2.

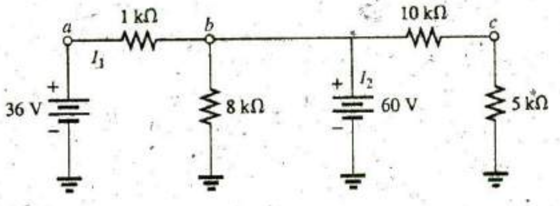


FIG. 7.68
Problem 9.

10. For the circuit board in Fig. 7.69:
    a. Find the total resistance R_T of the configuration.
    b. Find the current drawn from the supply if the applied voltage is 48 V.
    c. Find the reading of the applied voltmeter.

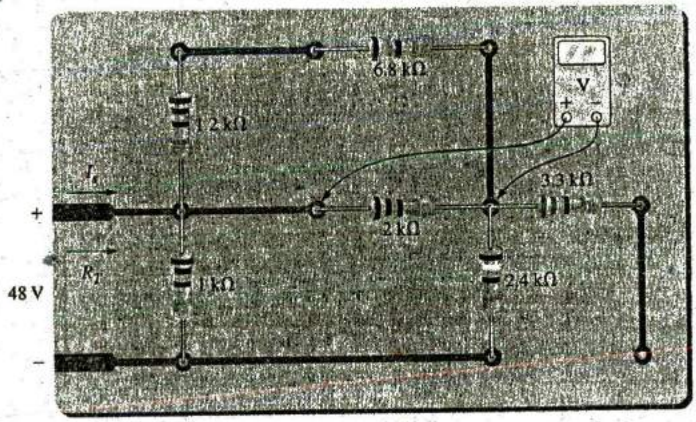


FIG. 7.69
Problem 10.

11. In the network of Fig. 7.70 all the resistors are equal. What are their values?

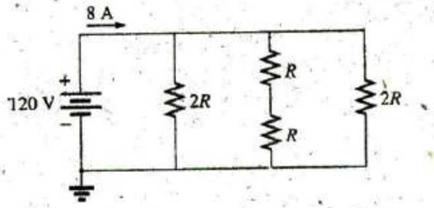


FIG. 7.70
Problem 11.

*12. For the network in Fig. 7.71:
   a. Find currents I_s, I_2, and I_6.
   b. Find voltages V_1 and V_5.
   c. Find the power delivered to the 3 kΩ resistor.

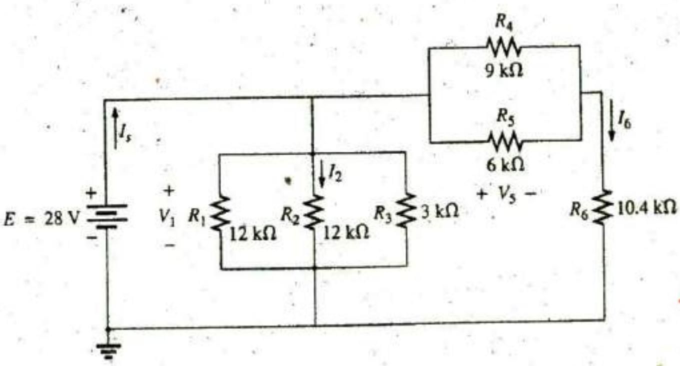


FIG. 7.71
Problem 12.

13. a. Find the magnitude and direction of the currents I, I_1, I_2, and I_3 for the network in Fig. 7.72.
    b. Indicate their direction on Fig. 7.72.

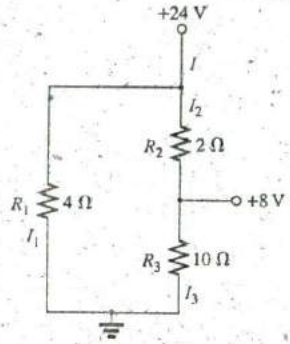


FIG. 7.72
Problem 13.

14. Determine the currents I_1 and I_2 for the network in Fig. 7.73, constructed of standard values.

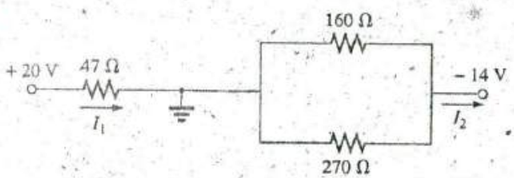


FIG. 7.73
Problem 14.

*15. For the network in Fig. 7.74:
   a. Determine the currents I_s, I_1, I_3, and I_4.
   b. Calculate V_a and V_{bc}.

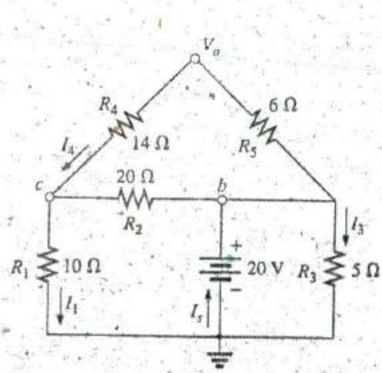


FIG. 7.74
Problem 15.

16. For the network in Fig. 7.75:
   a. Determine the current I_1.
   b. Calculate the currents I_2 and I_3.
   c. Determine the voltage levels V_a and V_b.

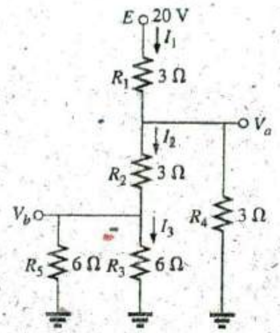


FIG. 7.75
Problem 16.

*17. Determine the dc levels for the transistor network in Fig. 7.76 using the fact that $V_{BE} = 0.7$ V, $V_E = 2$ V, and $I_C = I_E$. That is:
   a. Determine I_E and I_C.
   b. Calculate I_B.
   c. Determine V_B and V_C.
   d. Find V_{CE} and V_{BC}.

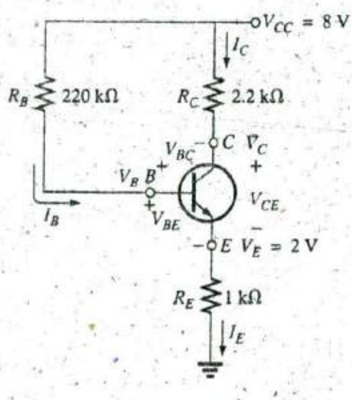


FIG. 7.76
Problem 17.

18. For the network in Fig. 7.77:

a. Determine the current I.

b. Find V_1.

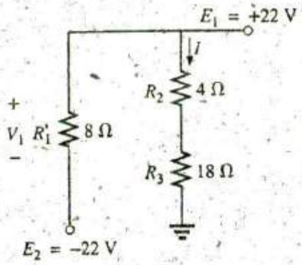


FIG. 7.77
Problem 18.

***19.** For the network in Fig. 7.78:

a. Determine R_T by combining resistive elements.

b. Find V_1 and V_4.

c. Calculate I_3 (with direction).

d. Determine I_s by finding the current through each element and then applying Kirchhoff's current law. Then calculate R_T from $R_T = E/I_s$, and compare the answer with the solution of part (a).

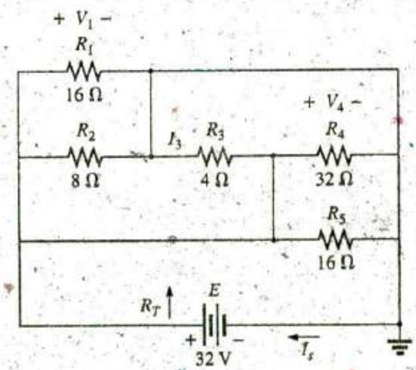


FIG. 7.78
Problem 19.

20. Determine the voltage V_{ab} and the current I for the network of Fig. 7.79. Recall the discussion of short and open circuits in Section 6.8.

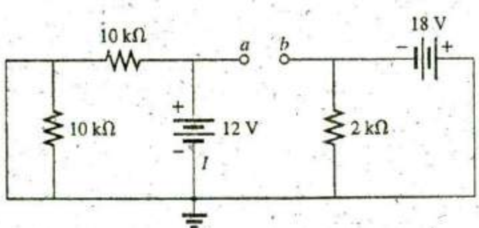


FIG. 7.79
Problem 20.

***21.** For the network of Fig. 7.80:

a. Determine the voltage V_{ab}.

b. Calculate the current I.

c. Find the voltages V_a and V_b.

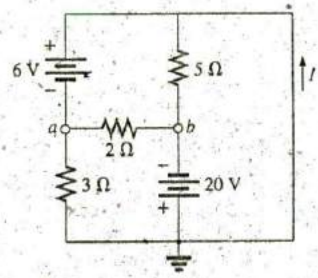


FIG. 7.80
Problem 21.

***22.** For the network in Fig. 7.81:

a. Determine the current I.

b. Calculate the open-circuit voltage V.

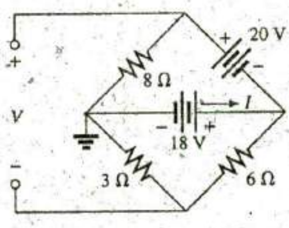


FIG. 7.81
Problem 22.

***23.** For the network in Fig. 7.82, find the resistance R_3 if the current through it is 2 A.

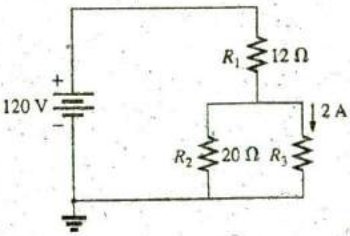


FIG. 7.82
Problem 23.

***24.** If all the resistors of the cube in Fig. 7.83 are 10 Ω, what is the total resistance? (*Hint:* Make some basic assumptions about current division through the cube.)

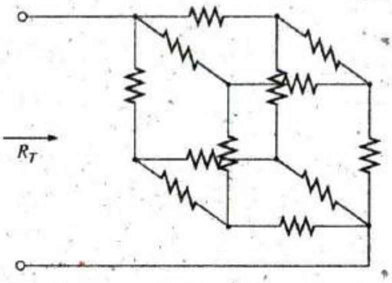


FIG. 7.83
Problem 24.

***25.** Given the voltmeter reading $V = 27$ V in Fig. 7.84:
   a. Is the network operating properly?
   b. If not, what could be the cause of the incorrect reading?

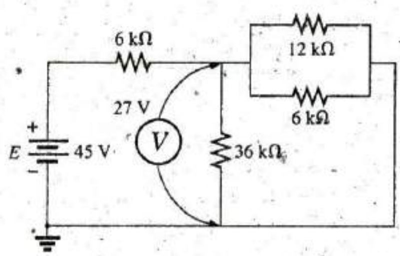


FIG. 7.84
Problem 25.

SECTION 7.6 Ladder Networks

26. For the ladder network in Fig. 7.85:
   a. Find the current I.
   b. Find the current I_7.
   c. Determine the voltages V_3, V_5, and V_7.
   d. Calculate the power delivered to R_7, and compare it to the power delivered by the 240 V supply.

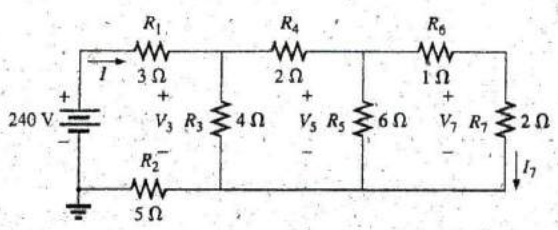


FIG. 7.85
Problem 26.

27. For the ladder network in Fig. 7.86:
   a. Determine R_T.
   b. Calculate I.
   c. Find the power delivered to R_7.

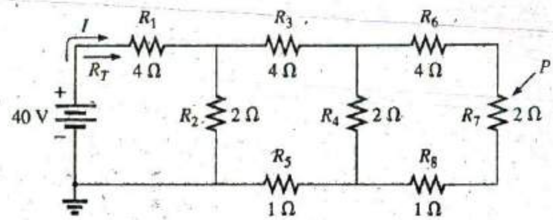


FIG. 7.86
Problem 27.

***28.** Determine the power delivered to the 6 Ω load in Fig. 7.87.

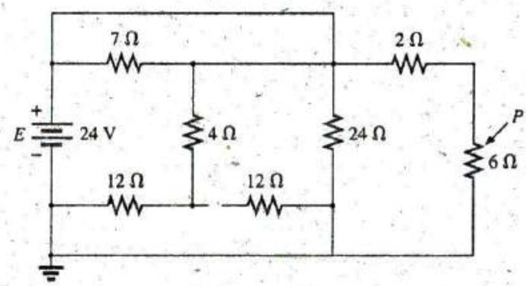


FIG. 7.87
Problem 28.

29. For the multiple ladder configuration in Fig. 7.88:
   a. Determine I.
   b. Calculate I_4.
   c. Find I_6.
   d. Find I_{10}.

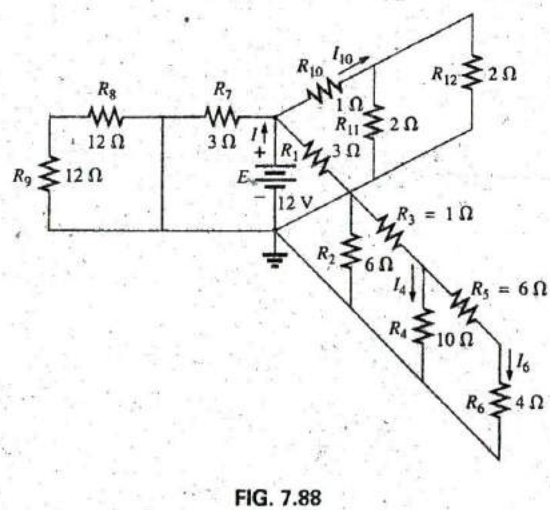


FIG. 7.88
Problem 29.

SECTION 7.7 Voltage Divider Supply (Unloaded and Loaded)

30. Given the voltage-divider supply in Fig. 7.89:
- **a.** Determine the supply voltage E.
- **b.** Find the load resistors R_{L_2} and R_{L_3}.
- **c.** Determine the voltage divider resistors R_1, R_2, and R_3.

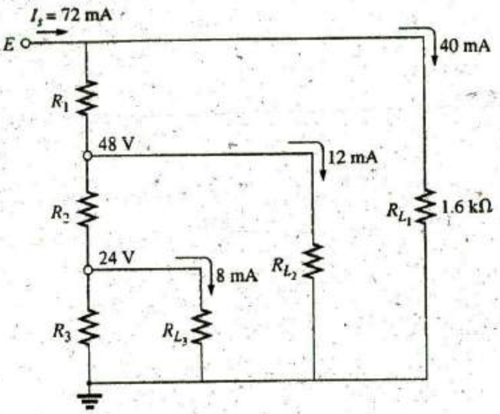


FIG. 7.89
Problem 30.

***31.** Determine the voltage divider supply resistors for the configuration in Fig. 7.90. Also determine the required wattage rating for each resistor, and compare their levels.

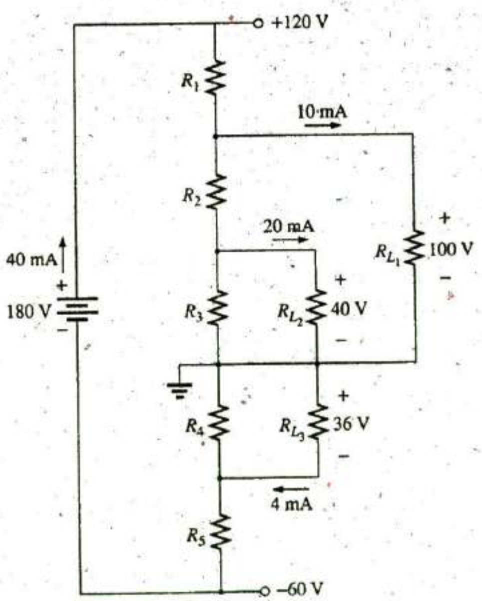


FIG. 7.90
Problem 31.

***32.** A studio lamp requires 40 V at 50 mA to burn brightly. Design a voltage divider arrangement that will work properly off a 120 V source supplying a current of 20 mA. Use resistors as close as possible to standard values, and specify the minimum wattage rating of each.

SECTION 7.8 Potentiometer Loading

***33.** For the system in Fig. 7.91:
- **a.** At first exposure, does the design appear to be a good one?
- **b.** In the absence of the 10 kΩ load, what are the values of R_1 and R_2 to establish 3 V across R_2?
- **c.** Determine the values of R_1 and R_2 to establish $V_{R_L} = 3$ V when the load is applied, and compare them to the results of part (b).

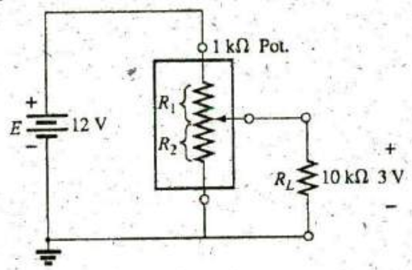


FIG. 7.91
Problem 33.

***34.** For the potentiometer in Fig. 7.92:
- **a.** What are the voltages V_{ab} and V_{bc} with no load applied ($R_{L_1} = R_{L_2} = \infty\ \Omega$)?
- **b.** What are the voltages V_{ab} and V_{bc} with the indicated loads applied?
- **c.** What is the power dissipated by the potentiometer under the loaded conditions in Fig. 7.92?
- **d.** What is the power dissipated by the potentiometer with no loads applied? Compare it to the results of part (c).

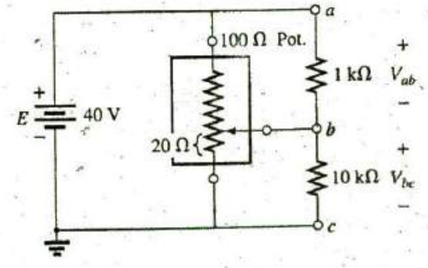


FIG. 7.92
Problem 34.

SECTION 7.9 Ammeter, Voltmeter, and Ohmmeter Design

35. An iron-vane movement is rated 1 mA, 100 Ω.
  a. What is the current sensitivity?
  b. Design a 20 A ammeter using the above movement. Show the circuit and component values.

36. Using a 50 μA, 1000 Ω movement, design a multirange milliammeter having scales of 25 mA, 50 mA, and 100 mA. Show the circuit and component values.

37. An iron-vane movement is rated 50 μA, 1000 Ω.
  a. Design a 15 V dc voltmeter. Show the circuit and component values.
  b. What is the ohm/volt rating of the voltmeter?

38. Using a 1 mA, 1000 Ω movement, design a multirange voltmeter having scales of 5 V, 50 V, and 500 V. Show the circuit and component values.

39. A digital meter has an internal resistance of 10 MΩ on its 0.5 V range. If you had to build a voltmeter with an iron-vane movement, what current sensitivity would you need if the meter were to have the same internal resistance on the same voltage scale?

***40.** **a.** Design a series ohmmeter using a 100 μA, 1000 Ω movement, a zero-adjust with a maximum value of 2 kΩ, a battery of 3 V, and a series resistor whose value is to be determined.
  b. Find the resistance required for full-scale, 3/4-scale, 1/2-scale, and 1/4-scale deflection.
  c. Using the results of part (b), draw the scale to be used with the ohmmeter.

41. Describe the basic construction and operation of the megohmmeter.

***42.** Determine the reading of the ohmmeter for each configuration of Fig. 7.93.

SECTION 7.11 Computer Analysis

43. Using PSpice or Multisim, verify the results of Example 7.2.

44. Using PSpice or Multisim, confirm the solutions of Example 7.5.

45. Using PSpice or Multisim, verify the results of Example 7.10.

46. Using PSpice or Multisim, find voltage V_6 of Fig. 7.32.

47. Using PSpice or Multisim, find voltages V_b and V_c of Fig. 7.40.

GLOSSARY

Complex configuration A network in which none of the elements are in series or parallel.

Iron-vane A movement operating on the principle that there is repulsion between like magnetic poles. The two poles are vanes inside of a fixed coil. One vane is fixed and the other movable with an attached pointer. The higher the applied current, the greater is the deflection of the movable vane and the greater is the deflection of the pointer.

Ladder network A network that consists of a cascaded set of series-parallel combinations and has the appearance of a ladder.

Megohmmeter An instrument for measuring very high resistance levels, such as in the megohm range.

Series ohmmeter A resistance-measuring instrument in which the movement is placed in series with the unknown resistance.

Series-parallel network A network consisting of a combination of both series and parallel branches.

Transistor A three-terminal semiconductor electronic device that can be used for amplification and switching purposes.

Voltage divider supply A series network that can provide a range of voltage levels for an application.

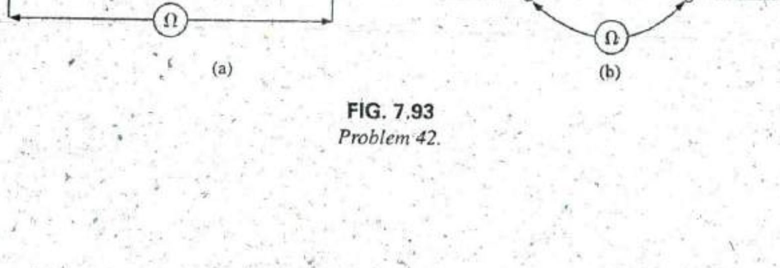
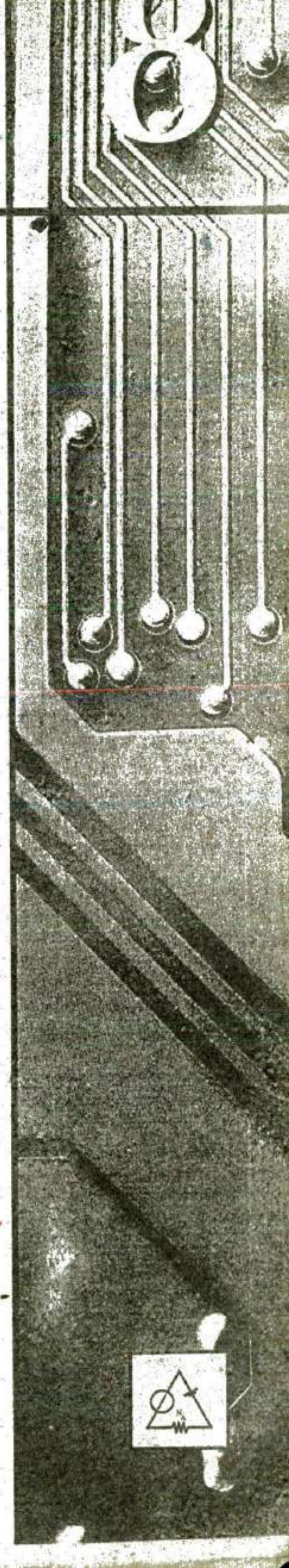


FIG. 7.93
Problem 42.

Methods of Analysis and Selected Topics (dc)



Objectives

- *Become familiar with the terminal characteristics of a current source and how to solve for the voltages and currents of a network using current sources and/or current sources and voltage sources.*
- *Be able to apply branch-current analysis and mesh analysis to find the currents of network with one or more independent paths.*
- *Be able to apply nodal analysis to find all the terminal voltages of any series-parallel network with one or more independent sources.*
- *Become familiar with bridge network configurations and how to perform Δ−Y or Y−Δ conversions.*

8.1 INTRODUCTION

The circuits described in previous chapters had only one source or two or more sources in series or parallel. The step-by-step procedures outlined in those chapters can be applied only if the sources are in series or parallel. There will be an interaction of sources that will not permit the reduction techniques used to find quantities such as the total resistance and the source current.

For such situations, methods of analysis have been developed that allow us to approach, in a systematic manner, networks with any number of sources in any arrangement. To our benefit, the methods to be introduced can also be applied to networks with only *one source* or to networks in which sources are in *series or parallel.*

The methods to be introduced in this chapter include branch-current analysis, mesh analysis, and nodal analysis. Each can be applied to the same network, although usually one is more appropriate than the other. The "best" method cannot be defined by a strict set of rules but can be determined only after developing an understanding of the relative advantages of each.

Before considering the first of the methods, we will examine current sources in detail because they appear throughout the analyses to follow. The chapter concludes with an investigation of a complex network called the *bridge configuration,* followed by the use of Δ-Y and Y-Δ conversions to analyze such configurations.

8.2 CURRENT SOURCES

In previous chapters, the voltage source was the only source appearing in the circuit analysis. This was primarily because voltage sources such as the battery and supply are the most common in our daily lives and in the laboratory environment.

We now turn our attention to a second type of source, called the **current source,** which appears throughout the analyses in this chapter. Although current sources are available as laboratory supplies (introduced in Chapter 2), they appear extensively in the modeling of electronic devices such as the transistor. Their characteristics and their impact on the currents

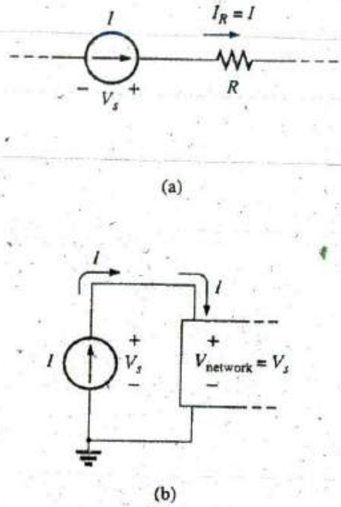


FIG. 8.1
Introducing the current source symbol.

and voltages of a network must therefore be clearly understood if electronic systems are to be properly investigated.

The current source is often described as the *dual* of the voltage source. Just as a battery provides a fixed voltage to a network, a current source establishes a fixed current in the branch where it is located. Further, the current through a battery is a function of the network to which it is applied, just as the voltage across a current source is a function of the connected network. The term *dual* is applied to any two elements in which the traits of one variable can be interchanged with the traits of another. This is certainly true for the current and voltage of the two types of sources.

The symbol for a current source appears in Fig. 8.1(a). The arrow indicates the direction in which it is supplying current to the branch where it is located. The result is a current equal to the source current through the series resistor. In Fig. 8.1(b), we find that the voltage across a current source is determined by the polarity of the voltage drop caused by the current source. For single-source networks, it always has the polarity of Fig. 8.1(b), but for multisource networks it can have either polarity.

In general, therefore,

a current source determines the direction and magnitude of the current in the branch where it is located.

Furthermore,

the magnitude and the polarity of the voltage across a current source are each a function of the network to which the voltage is applied.

A few examples will demonstrate the similarities between solving for the source current of a voltage source and the terminal voltage of a current source. All the rules and laws developed in the previous chapter still apply, so we just have to remember what we are looking for and properly understand the characteristics of each source.

The simplest possible configuration with a current source appears in Example 8.1.

EXAMPLE 8.1 Find the source voltage, the voltage V_1, and current I_1 for the circuit in Fig. 8.2.

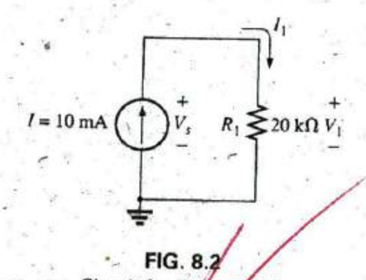


FIG. 8.2
Circuit for Example 8.1.

Solution: Since the current source establishes the current in the branch in which it is located, the current I_1 must equal I, and

$$I_1 = I = \mathbf{10\ mA}$$

The voltage across R_1 is then determined by Ohm's law:

$$V_1 = I_1R_1 = (10\text{ mA})(20\ \Omega) = \mathbf{200\ V}$$

Since resistor R_1 and the current source are in parallel, the voltage across each must be the same, and

$$V_s = V_1 = \mathbf{200\ V}$$

with the polarity shown.

EXAMPLE 8.2 Find the voltage V_s and currents I_1 and I_2 for the network in Fig. 8.3.

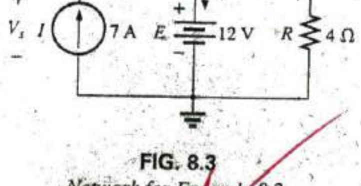


FIG. 8.3
Network for Example 8.2.

Solution: This is an interesting problem because it has both a current source and a voltage source. For each source, the dependent (a function of something else) variable will be determined. That is, for the current source, V_s must be determined, and for the voltage source, I_s must be determined.

Since the current source and voltage source are in parallel,

$$V_s = E = 12\text{ V}$$

Further, since the voltage source and resistor R are in parallel,

$$V_R = E = 12\text{ V}$$

and
$$I_2 = \frac{V_R}{R} = \frac{12\text{ V}}{4\ \Omega} = \mathbf{3\ A}$$

The current I_1 of the voltage source can then be determined by applying Kirchhoff's current law at the top of the network as follows:

$$\Sigma I_i = \Sigma I_o$$
$$I = I_1 + I_2$$

and
$$I_1 = I - I_2 = 7\text{ A} - 3\text{ A} = \mathbf{4\ A}$$

EXAMPLE 8.3 Determine the current I_1 and the voltage V_s for the network in Fig. 8.4.

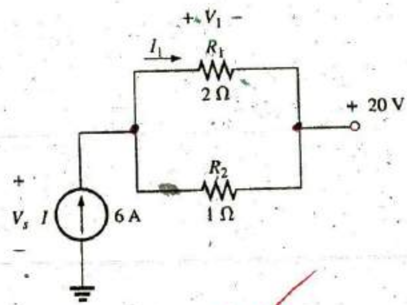


FIG. 8.4
Example 8.3.

Solution: First note that the current in the branch with the current source must be 6 A, no matter what the magnitude of the voltage source to the right. In other words, the currents of the network are defined by I, R_1, and R_2. However, the voltage across the current source is directly affected by the magnitude and polarity of the applied source.

Using the current divider rule gives

$$I_1 = \frac{R_2 I}{R_2 + R_1} = \frac{(1\ \Omega)(6\text{ A})}{1\ \Omega + 2\ \Omega} = \frac{1}{3}(6\text{ A}) = \mathbf{2\ A}$$

The voltage V_1 is given by

$$V_1 = I_1 R_1 = (2\text{ A})(2\ \Omega) = 4\text{ V}$$

Applying Kirchhoff's voltage rule to determine V_s gives

$$+V_s - V_1 - 20\text{ V} = 0$$

and
$$V_s = V_1 + 20\text{ V} = 4\text{ V} + 20\text{ V} = \mathbf{24\ V}$$

In particular, note the polarity of the voltage V_s as determined by the network.

8.3 SOURCE CONVERSIONS

The current source appearing in the previous section is called an *ideal source* due to the absence of any internal resistance. In reality, all sources—whether they are voltage sources or current sources—have some internal resistance in the relative positions shown in Fig. 8.5. For the voltage source, if $R_s = 0\ \Omega$, or if it is so small compared to any series resistors that it can be ignored, then we have an "ideal" voltage source for all practical purposes. For the current source, since the resistor R_P is in parallel, if $R_P = \infty\ \Omega$, or if it is large enough compared to any parallel resistive elements that it can be ignored, then we have an "ideal" current source.

Unfortunately, however, ideal sources *cannot be converted* from one type to another. That is, a voltage source cannot be converted to a

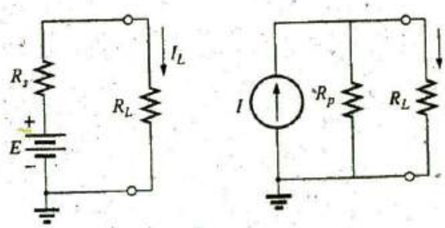


FIG. 8.5
Practical sources: (a) voltage; (b) current.

current source, and vice versa—*the internal resistance must be present.* If the voltage source in Fig. 8.5(a) is to be equivalent to the source in Fig. 8.5(b), any load connected to the sources such as R_L should receive the same current, voltage, and power from each configuration. In other words, if the source were enclosed in a container, the load R_L would not know which source it was connected to.

This type of equivalence is established using the equations appearing in Fig. 8.6. First note that the resistance is the same in each configuration—a nice advantage. For the voltage source equivalent, the voltage is determined by a simple application of Ohm's law to the current source: $E = IR_p$. For the current source equivalent, the current is again determined by applying Ohm's law to the voltage source: $I = E/R_s$. At first glance, it all seems too simple, but Example 8.4 verifies the results.

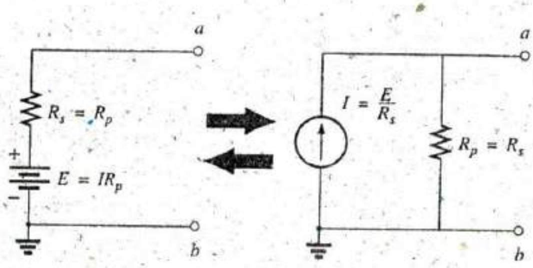


FIG. 8.6
Source conversion.

It is important to realize, however, that

the equivalence between a current source and a voltage source exists only at their external terminals.

The internal characteristics of each are quite different.

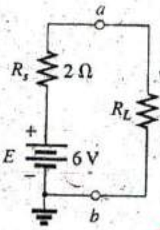


FIG. 8.7
Practical voltage source and load for Example 8.4.

EXAMPLE 8.4 For the circuit in Fig. 8.7:

a. Determine the current I_L.
b. Convert the voltage source to a current source.
c. Using the resulting current source of part (b), calculate the current through the load resistor, and compare your answer to the result of part (a).

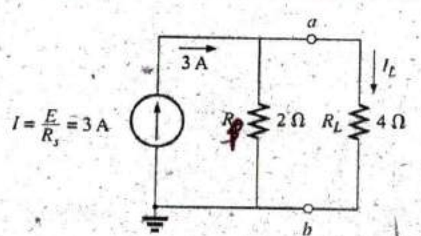


FIG. 8.8
Equivalent current source and load for the voltage source in Fig. 8.7.

Solutions:

a. Applying Ohm's law gives

$$I_L = \frac{E}{R_s + R_L} = \frac{6\text{ V}}{2\ \Omega + 4\ \Omega} = \frac{6\text{ V}}{6\ \Omega} = \mathbf{1\ A}$$

b. Using Ohm's law again gives

$$I = \frac{E}{R_s} = \frac{6\text{ V}}{2\ \Omega} = \mathbf{3\ A}$$

and the equivalent source appears in Fig. 8.8 with the load reapplied.

c. Using the current divider rule gives

$$I_L = \frac{R_p I}{R_p + R_L} = \frac{(2\,\Omega)(3\,\text{A})}{2\,\Omega + 4\,\Omega} = \frac{1}{3}(3\,\text{A}) = \mathbf{1\,A}$$

We find that the current I_L is the same for the voltage source as it was for the equivalent current source—the sources are therefore equivalent.

As demonstrated in Fig. 8.5 and in Example 8.4, note that

a source and its equivalent will establish current in the same direction through the applied load.

In Example 8.4, note that both sources pressure or establish current up through the circuit to establish the same direction for the load current I_L and the same polarity for the voltage V_L.

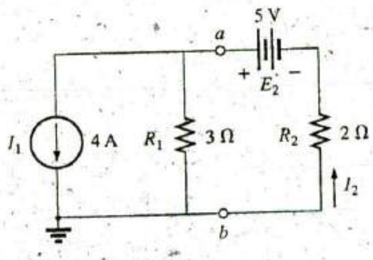


FIG. 8.9
Two-source network for Example 8.5.

EXAMPLE 8.5 Determine current I_2 for the network in Fig. 8.9.

Solution: Although it may appear that the network cannot be solved using methods introduced thus far, one source conversion, as shown in Fig. 8.10, results in a simple series circuit. It does not make sense to convert the voltage source to a current source because you would lose the current I_2 in the redrawn network. Note the polarity for the equivalent voltage source as determined by the current source.

For the source conversion

$$E_1 = I_1 R_1 = (4\,\text{A})(3\,\Omega) = 12\,\text{V}$$

and
$$I_2 = \frac{E_1 + E_2}{R_1 + R_2} = \frac{12\,\text{V} + 5\,\text{V}}{3\,\Omega + 2\,\Omega} = \frac{17\,\text{V}}{5\,\Omega} = \mathbf{3.4\,A}$$

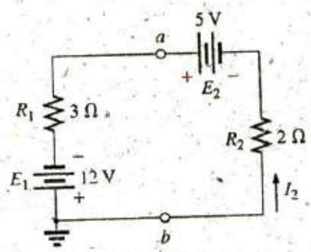


FIG. 8.10
Network in Fig. 8.9 following the conversion of the current source to a voltage source.

8.4 CURRENT SOURCES IN PARALLEL

We found that voltage sources of different terminal voltages cannot be placed in parallel because of a violation of Kirchhoff's voltage law. Similarly,

current sources of different values cannot be placed in series due to a violation of Kirchhoff's current law.

However, current sources can be placed in parallel just as voltage sources can be placed in series. In general,

two or more current sources in parallel can be replaced by a single current source having a magnitude determined by the difference of the sum of the currents in one direction and the sum in the opposite direction. The new parallel internal resistance is the total resistance of the resulting parallel resistive elements.

Consider the following examples.

EXAMPLE 8.6 Reduce the parallel current sources in Fig. 8.11 to a single current source.

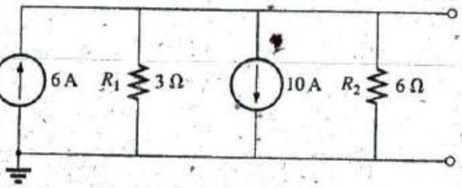


FIG. 8.11
Parallel current sources for Example 8.6.

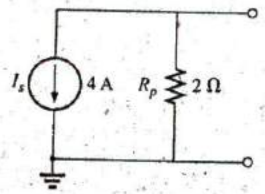


FIG. 8.12
Reduced equivalent for the configuration of Fig. 8.11.

Solution: The net source current is

$$I = 10\text{ A} - 6\text{ A} = \mathbf{4\text{ A}}$$

with the direction being that of the larger source.

The net internal resistance is the parallel combination of resistances, R_1 and R_2:

$$R_p = 3\ \Omega \parallel \Omega = \mathbf{2\ \Omega}$$

The reduced equivalent appears in Fig. 8.12.

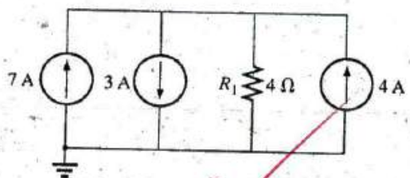


FIG. 8.13
Parallel current sources for Example 8.7.

EXAMPLE 8.7 Reduce the parallel current sources in Fig. 8.13 to a single current source.

Solution: The net current is

$$I = 7\text{ A} + 4\text{ A} - 3\text{ A} = \mathbf{8\text{ A}}$$

with the direction shown in Fig. 8.14. The net internal resistance remains the same.

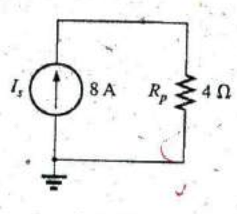


FIG. 8.14
Reduced equivalent for Fig. 8.13.

EXAMPLE 8.8 Reduce the network in Fig. 8.15 to a single current source, and calculate the current through R_L.

Solution: In this example, the voltage source will first be converted to a current source as shown in Fig. 8.16. Combining current sources gives

$$I_s = I_1 + I_2 = 4\text{ A} + 6\text{ A} = \mathbf{10\text{ A}}$$

and

$$R_s = R_1 \parallel R_2 = 8\ \Omega \parallel 24\ \Omega = \mathbf{6\ \Omega}$$

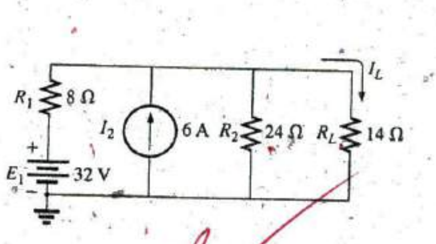


FIG. 8.15
Example 8.8.

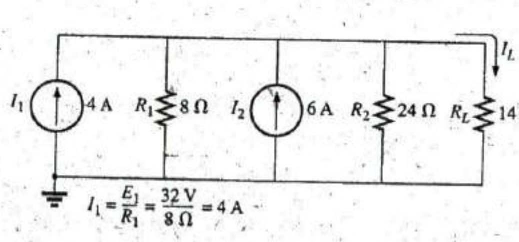


FIG. 8.16
Network in Fig. 8.15 following the conversion of the voltage source to a current source.

Applying the current divider rule to the resulting network in Fig. 8.17 gives

$$I_L = \frac{R_p I_s}{R_p + R_L} = \frac{(6\ \Omega)(10\ \text{A})}{6\ \Omega + 14\ \Omega} = \frac{60\ \text{A}}{20} = \mathbf{3\ A}$$

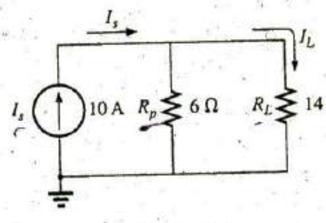


FIG. 8.17
Network in Fig. 8.16 reduced to its simplest form.

8.5 CURRENT SOURCES IN SERIES

The current through any branch of a network can be only single-valued. For the situation indicated at point *a* in Fig. 8.18, we find by application of Kirchhoff's current law that the current leaving that point is greater than that entering—an impossible situation. Therefore,

current sources of different current ratings are not connected in series,

just as voltage sources of different voltage ratings are not connected in parallel.

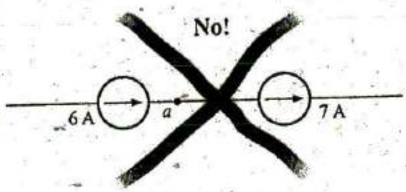


FIG. 8.18
Invalid situation.

8.6 BRANCH-CURRENT ANALYSIS

Before examining the details of the first important method of analysis, let us examine the network in Fig. 8.19 to be sure that you understand the need for these special methods.

Initially, it may appear that we can use the reduce and return approach to work our way back to the source E_1 and calculate the source current I_{s_1}. Unfortunately, however, the series elements R_3 and E_2 cannot be combined because they are different types of elements. A further examination of the network reveals that there are no two like elements that are in series or parallel. No combination of elements can be performed, and it is clear that another approach must be defined.

It must be noted that the network of Fig. 8.19 can be solved if we convert each voltage source to a current source and then combine parallel current sources. However, if a specific quantity of the original network is required, it would require working back using the information determined from the source conversion. Further, there will be complex networks for which source conversions will not permit a solution, so it is important to understand the methods to be described in this chapter.

The first approach to be introduced is called the **branch-current method** because we will define and solve for the currents of each branch of the network. The best way to introduce this method and understand its application is to follow a series of steps, as listed below. Each step is carefully defined in the examples to follow.

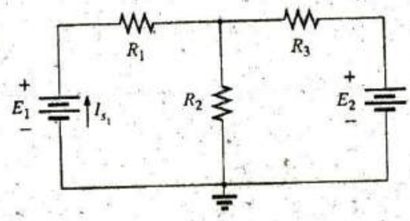


FIG. 8.19
Demonstrating the need for an approach such as branch-current analysis.

Branch-Current Analysis Procedure

1. ***Assign a distinct current of arbitrary direction to each branch of the network.***
2. ***Indicate the polarities for each resistor as determined by the assumed current direction.***
3. ***Apply Kirchhoff's voltage law around each closed, independent loop of the network.***

The best way to determine how many times Kirchhoff's voltage law has to be applied is to determine the number of "windows" in the network. The network in Example 8.9 has a definite similarity to the two-window configuration in Fig. 8.20(a). The result is a need to apply Kirchhoff's voltage law twice. For networks with three windows, as shown in Fig. 8.20(b), three applications of Kirchhoff's voltage law are required, and so on.

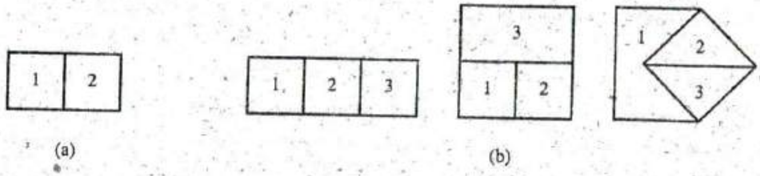


FIG. 8.20
Determining the number of independent closed loops.

4. ***Apply Kirchhoff's current law at the minimum number of nodes that will include all the branch currents of the network.***

The minimum number is one less than the number of independent nodes of the network. For the purposes of this analysis, a **node** is a junction of two or more branches, where a branch is any combination of series elements. Fig. 8.21 defines the number of applications of Kirchhoff's current law for each configuration in Fig. 8.20.

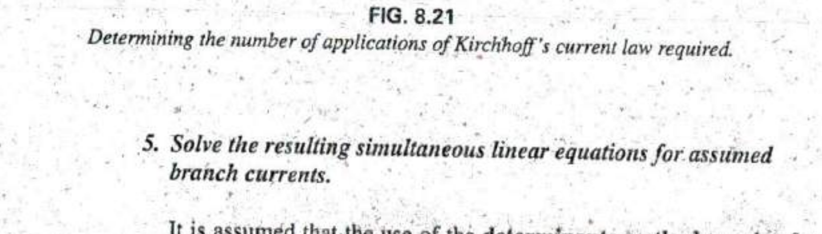


FIG. 8.21
Determining the number of applications of Kirchhoff's current law required.

5. ***Solve the resulting simultaneous linear equations for assumed branch currents.***

It is assumed that the use of the **determinants method** to solve for the currents I_1, I_2, and I_3 is understood and is a part of the student's mathematical background. If not, a detailed explanation of the procedure is provided in Appendix C. Calculators and computer software packages such as MATLAB and Mathcad can find the solutions quickly and accurately.

EXAMPLE 8.9 Apply the branch-current method to the network in Fig. 8.22.

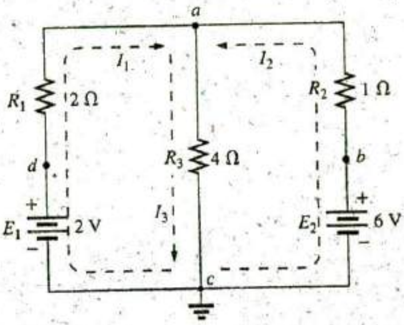


FIG. 8.22
Example 8.9.

Solution 1:

Step 1: Since there are three distinct branches (*cda*, *cba*, *ca*), three currents of arbitrary directions (I_1, I_2, I_3) are chosen, as indicated in Fig. 8.22. The current directions for I_1 and I_2 were chosen to match the "pressure" applied by sources E_1 and E_2, respectively. Since both I_1 and I_2 enter node a, I_3 is leaving.

Step 2: Polarities for each resistor are drawn to agree with assumed current directions, as indicated in Fig. 8.23.

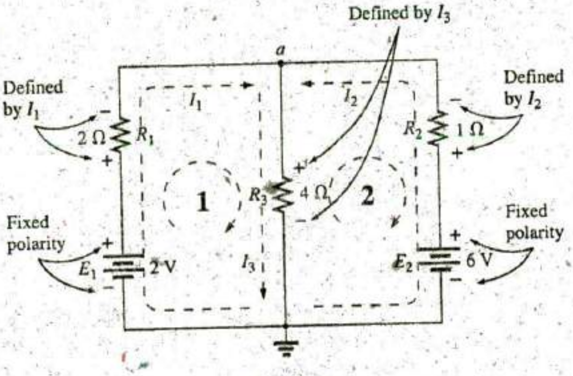


FIG. 8.23
Inserting the polarities across the resistive elements as defined by the chosen branch currents.

Step 3: Kirchhoff's voltage law is applied around each closed loop (1 and 2) in the clockwise direction:

$$\text{loop 1:}\quad \Sigma_{\circlearrowright} V = \underbrace{+E_1}_{\text{Rise in potential}} - \underbrace{V_{R_1} - V_{R_3}}_{\text{Drop in potential}} = 0$$

$$\text{loop 2:}\quad \Sigma_{\circlearrowright} V = \underbrace{+V_{R_3} + V_{R_2}}_{\text{Rise in potential}} \underbrace{- E_2}_{\text{Drop in potential}} = 0$$

and

$$\text{loop 1:}\quad \Sigma_{\circlearrowright} V = \underbrace{+2\text{ V}}_{\substack{\text{Battery}\\\text{potential}}} - \underbrace{(2\ \Omega)I_1}_{\substack{\text{Voltage drop}\\\text{across } 2\ \Omega\\\text{resistor}}} - \underbrace{(4\ \Omega)I_3}_{\substack{\text{Voltage drop}\\\text{across } 4\ \Omega\\\text{resistor}}} = 0$$

$$\text{loop 2:}\quad \Sigma_{\circlearrowright} V = (4\ \Omega)I_3 + (1\ \Omega)I_2 - 6\text{ V} = 0$$

Step 4: Applying Kirchhoff's current law at node a (in a two-node network, the law is applied at only one node) gives

$$I_1 + I_2 = I_3$$

Step 5: There are three equations and three unknowns (units removed for clarity):

$$2 - 2I_1 - 4I_3 = 0 \qquad \text{Rewritten: } 2I_1 + 0 + 4I_3 = 2$$
$$4I_3 + 1I_2 - 6 = 0 \qquad 0 + I_2 + 4I_3 = 6$$
$$I_1 + I_2 = I_3 \qquad I_1 + I_2 - I_3 = 0$$

Using third-order determinants (Appendix C), we have

$$I_1 = \frac{\begin{vmatrix} 2 & 0 & 4 \\ 6 & 1 & 4 \\ 0 & 1 & -1 \end{vmatrix}}{D = \begin{vmatrix} 2 & 0 & 4 \\ 0 & 1 & 4 \\ 1 & 1 & -1 \end{vmatrix}} = -1\ \text{A}$$

A negative sign in front of a branch current indicates only that the actual current is in the direction opposite to that assumed.

$$I_2 = \frac{\begin{vmatrix} 2 & 2 & 4 \\ 0 & 6 & 4 \\ 1 & 0 & -1 \end{vmatrix}}{D} = 2\ \text{A}$$

$$I_3 = \frac{\begin{vmatrix} 2 & 0 & 2 \\ 0 & 1 & 6 \\ 1 & 1 & 0 \end{vmatrix}}{D} = 1\ \text{A}$$

Solution 2: Instead of using third-order determinants as in Solution 1, we can reduce the three equations to two by substituting the third equation in the first and second equations:

$$2 - 2I_1 - 4\overbrace{(I_1 + I_2)}^{I_3} = 0 \qquad 2 - 2I_1 - 4I_1 - 4I_2 = 0$$
$$4\overbrace{(I_1 + I_2)}^{I_3} + I_2 - 6 = 0 \qquad 4I_1 + 4I_2 + I_2 - 6 = 0$$

or

$$-6I_1 - 4I_2 = -2$$
$$+4I_1 + 5I_2 = +6$$

Multiplying through by -1 in the top equation yields

$$6I_1 + 4I_2 = +2$$
$$4I_1 + 5I_2 = +6$$

and using determinants gives

$$I_1 = \frac{\begin{vmatrix} 2 & 4 \\ 6 & 5 \end{vmatrix}}{\begin{vmatrix} 6 & 4 \\ 4 & 5 \end{vmatrix}} = \frac{10 - 24}{30 - 16} = \frac{-14}{14} = -1\ \text{A}$$

TI-89 Solution: The procedure for determining the determinant in Example 8.9 requires some scrolling to obtain the desired math functions, but in time that procedure can be performed quite rapidly. As with any computer or calculator system, it is paramount that you enter all parameters correctly. One error in the sequence negates the entire process. For the TI-89, the entries are shown in Fig. 8.24(a).

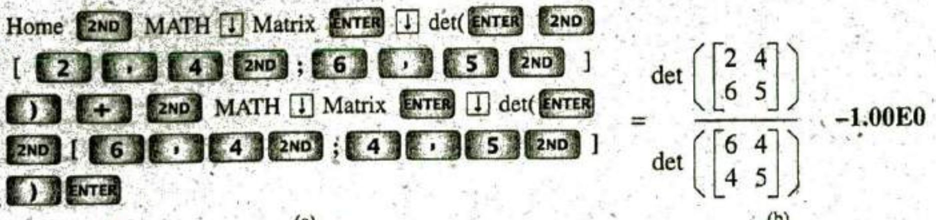


FIG. 8.24
TI-89 solution for the current I_1 of Fig. 8.22.

After you select the last ENTER key, the screen shown in Fig. 8.24(b) appears.

$$I_2 = \frac{\begin{vmatrix} 6 & 2 \\ 4 & 6 \end{vmatrix}}{14} = \frac{36 - 8}{14} = \frac{28}{14} = \mathbf{2\ A}$$

$$I_3 = I_1 + I_2 = -1 + 2 = \mathbf{1\ A}$$

It is now important that the impact of the results obtained be understood. The currents I_1, I_2, and I_3 are the actual currents in the branches in which they were defined. A negative sign in the solution means that the actual current has the opposite direction than initially defined—the magnitude is correct. Once the actual current directions and their magnitudes are inserted in the original network, the various voltages and power levels can be determined. For this example, the actual current directions and their magnitudes have been entered on the original network in Fig. 8.25. Note that the current through the series elements R_1 and E_1 is 1 A; the current through R_3, is 1 A; and the current through the series elements R_2 and E_2 is 2 A. Due to the minus sign in the solution, the direction of I_1 is opposite to that shown in Fig. 8.22. The voltage across any resistor can now be found using Ohm's law, and the power delivered by either source or to any one of the three resistors can be found using the appropriate power equation.

Applying Kirchhoff's voltage law around the loop indicated in Fig. 8.25 gives

$$\Sigma_{\circlearrowright} V = +(4\ \Omega)I_3 + (1\ \Omega)I_2 - 6\ \text{V} = 0$$

or
$$(4\ \Omega)I_3 + (1\ \Omega)I_2 = 6\ \text{V}$$

and
$$(4\ \Omega)(1\ \text{A}) + (1\ \Omega)(2\ \text{A}) = 6\ \text{V}$$
$$4\ \text{V} + 2\ \text{V} = 6\ \text{V}$$
$$6\ \text{V} = 6\ \text{V} \qquad \text{(checks)}$$

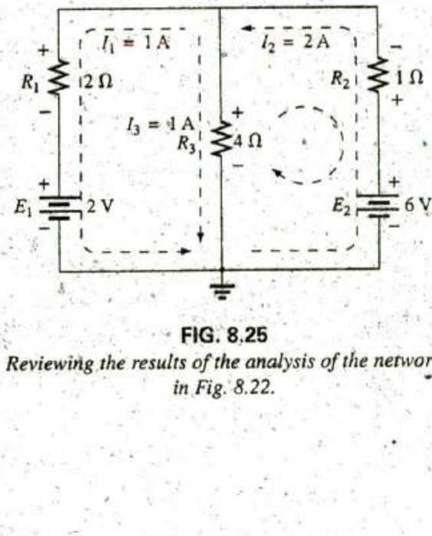


FIG. 8.25
Reviewing the results of the analysis of the network in Fig. 8.22.

EXAMPLE 8.10 Apply branch-current analysis to the network in Fig. 8.26.

Solution: Again, the current directions were chosen to match the "pressure" of each battery. The polarities are then added, and Kirchhoff's voltage law is applied around each closed loop in the clockwise direction. The result is as follows:

$$\text{loop 1:}\quad +15\ \text{V} - (4\ \Omega)I_1 + (10\ \Omega)I_3 - 20\ \text{V} = 0$$
$$\text{loop 2:}\quad +20\ \text{V} - (10\ \Omega)I_3 - (5\ \Omega)I_2 + 40\ \text{V} = 0$$

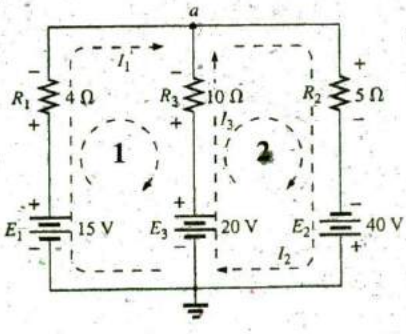


FIG. 8.26
Example 8.10.

Applying Kirchhoff's current law at node a gives

$$I_1 + I_3 = I_2$$

Substituting the third equation into the other two yields (with units removed for clarity)

$$\left.\begin{aligned} 15 - 4I_1 + 10I_3 - 20 &= 0 \\ 20 - 10I_3 - 5(I_1 + I_3) + 40 &= 0 \end{aligned}\right\} \text{Substituting for } I_2 \text{ (since it occurs only once in the two equations)}$$

or

$$\begin{aligned} -4I_1 + 10I_3 &= 5 \\ -5I_1 - 15I_3 &= -60 \end{aligned}$$

Multiplying the lower equation by -1, we have

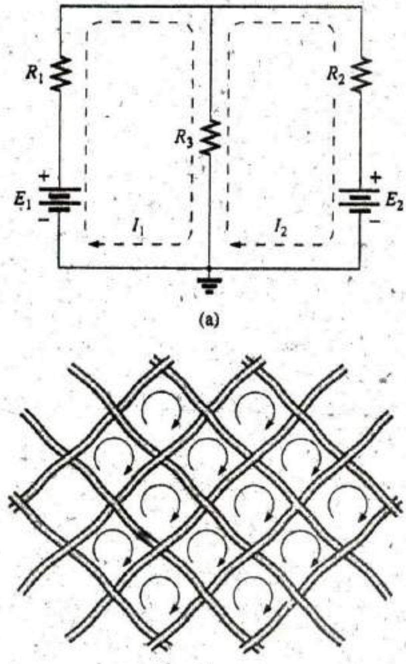
$$\begin{aligned} -4I_1 + 10I_3 &= 5 \\ 5I_1 + 15I_3 &= 60 \end{aligned}$$

$$I_1 = \frac{\begin{vmatrix} 5 & 10 \\ 60 & 15 \end{vmatrix}}{\begin{vmatrix} -4 & 10 \\ 5 & 15 \end{vmatrix}} = \frac{75 - 600}{-60 - 50} = \frac{-525}{-110} = \mathbf{4.77\ A}$$

$$I_3 = \frac{\begin{vmatrix} -4 & 5 \\ 5 & 60 \end{vmatrix}}{-110} = \frac{-240 - 25}{-110} = \frac{-265}{-110} = \mathbf{2.41\ A}$$

$$I_2 = I_1 + I_3 = 4.77\ \text{A} + 2.41\ \text{A} = \mathbf{7.18\ A}$$

revealing that the assumed directions were the actual directions, with I_2 equal to the sum of I_1 and I_3.



(a)

(b)

FIG. 8.27
Defining the mesh (loop) current: (a) "two-window" network; (b) wire mesh fence analogy.

8.7 MESH ANALYSIS (GENERAL APPROACH)

The next method to be described—**mesh analysis**—is actually an extension of the branch-current analysis approach just introduced. By defining a unique array of currents to the network, the information provided by the application of Kirchhoff's current law is already included when we apply Kirchhoff's voltage law. In other words, there is no need to apply step 4 of the branch-current method.

The currents to be defined are called **mesh** or **loop currents.** The two terms are used interchangeably. In Fig. 8.27(a), a network with two "windows" has had two mesh currents defined. Note that each forms a closed "loop" around the inside of each window; these loops are similar to the loops defined in the wire mesh fence in Fig. 8.27(b)—hence the use of the term *mesh* for the loop currents. We will find that

the number of mesh currents required to analyze a network will equal the number of "windows" of the configuration.

The defined mesh currents can initially be a little confusing because it appears that two currents have been defined for resistor R_3. There is no problem with E_1 and R_1, which have only current I_1, or with E_2 and R_2, which have only current I_2. However, defining the current through R_3 may seem a little troublesome. Actually, it is quite straightforward. The current through R_3 is simply the difference between I_1 and I_2, with the direction being that of the larger. This is demonstrated in the examples to follow.

Because mesh currents can result in more than one current through an element, branch-current analysis was introduced first. Branch-current analysis is the straightforward application of the basic laws of electric circuits. Mesh analysis employs a maneuver ("trick," if you prefer) that removes the need to apply Kirchhoff's current law.

Mesh Analysis Procedure

1. ***Assign a distinct current in the clockwise direction to each independent, closed loop of the network. It is not absolutely necessary to choose the clockwise direction for each loop current. In fact, any direction can be chosen for each loop current with no loss in accuracy, as long as the remaining steps are followed properly. However, by choosing the clockwise direction as a standard, we can develop a shorthand method (Section 8.8) for writing the required equations that will save time and possibly prevent some common errors.***

This first step is accomplished most effectively by placing a loop current *within* each "window" of the network, as demonstrated in the previous section, to ensure that they are all independent. A variety of other loop currents can be assigned. In each case, however, be sure that the information carried by any one loop equation is not included in a combination of the other network equations. This is the crux of the terminology *independent*. No matter how you choose your loop currents, the number of loop currents required is always equal to the number of windows of a planar (no-crossovers) network. On occasion, a network may appear to be nonplanar. However, a redrawing of the network may reveal that it is, in fact, planar. This may be true for one or two problems at the end of the chapter.

Before continuing to the next step, let us ensure that the concept of a loop current is clear. For the network in Fig. 8.28, the loop current I_1 is the branch current of the branch containing the 2 Ω resistor and 2 V battery. The current through the 4 Ω resistor is not I_1, however, since there is also a loop current I_2 through it. Since they have opposite directions, $I_{4\Omega}$ equals the difference between the two, $I_1 - I_2$ or $I_2 - I_1$, depending on which you choose to be the defining direction. In other words, *a loop current is a branch current only when it is the only loop current assigned to that branch.*

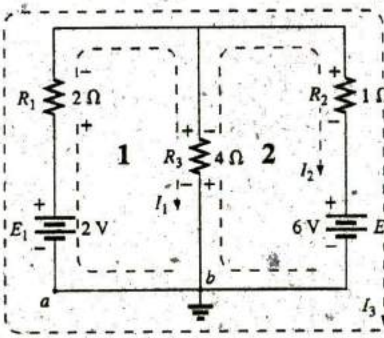


FIG. 8.28
Defining the mesh currents for a "two-window" network.

2. ***Indicate the polarities within each loop for each resistor as determined by the assumed direction of loop current for that loop. Note the requirement that the polarities be placed within each loop. This requires, as shown in Fig. 8.28, that the 4 Ω resistor have two sets of polarities across it.***
3. ***Apply Kirchhoff's voltage law around each closed loop in the clockwise direction. Again, the clockwise direction was chosen to establish uniformity and prepare us for the method to be introduced in the next section.***
   a. ***If a resistor has two or more assumed currents through it, the total current through the resistor is the assumed current of the loop in which Kirchhoff's voltage law is being applied, plus the assumed currents of the other loops passing through in the same direction, minus the assumed currents through in the opposite direction.***
   b. ***The polarity of a voltage source is unaffected by the direction of the assigned loop currents.***
4. ***Solve the resulting simultaneous linear equations for the assumed loop currents.***

EXAMPLE 8.11 Consider the same basic network as in Example 8.9, now appearing as Fig. 8.28.

Solution:

Step 1: Two loop currents (I_1 and I_2) are assigned in the clockwise direction in the windows of the network. A third loop (I_3) could have been included around the entire network, but the information carried by this loop is already included in the other two.

Step 2: Polarities are drawn within each window to agree with assumed current directions. Note that for this case, the polarities across the 4 Ω resistor are the opposite for each loop current.

Step 3: Kirchhoff's voltage law is applied around each loop in the clockwise direction. Keep in mind as this step is performed that the law is concerned only with the magnitude and polarity of the voltages around the closed loop and not with whether a voltage rise or drop is due to a battery or a resistive element. The voltage across each resistor is determined by $V = IR$. For a resistor with more than one current through it, the current is the loop current of the loop being examined plus or minus the other loop currents as determined by their directions. If clockwise applications of Kirchhoff's voltage law are always chosen, the other loop currents are always subtracted from the loop current of the loop being analyzed.

loop 1: $+E_1 - V_1 - V_3 = 0$ (clockwise starting at point a)

$$+2\text{ V} - (2\ \Omega)I_1 - \overbrace{(4\ \Omega)(\underbrace{I_1 - I_2}_{\substack{\text{Total current}\\ \text{through}\\ 4\ \Omega \text{ resistor}}})}^{\substack{\text{Voltage drop across}\\ 4\ \Omega \text{ resistor}}} = 0$$

Subtracted since I_2 is opposite in direction to I_1.

loop 2: $-V_3 - V_2 - E_2 = 0$ (clockwise starting at point b)

$$-(4\ \Omega)(I_2 - I_1) - (1\ \Omega)I_2 - 6\text{ V} = 0$$

Step 4: The equations are then rewritten as follows (without units for clarity):

loop 1: $+2 - 2I_1 - 4I_1 + 4I_2 = 0$

loop 2: $-4I_2 + 4I_1 - 1I_2 - 6 = 0$

and

loop 1: $+2 - 6I_1 + 4I_2 = 0$

loop 2: $-5I_2 + 4I_1 - 6 = 0$

or

loop 1: $-6I_1 + 4I_2 = -2$

loop 2: $+4I_1 - 5I_2 = +6$

Applying determinants results in

$$I_1 = -1\text{ A} \quad \text{and} \quad I_2 = -2\text{ A}$$

The minus signs indicate that the currents have a direction opposite to that indicated by the assumed loop current.

The actual current through the 2 V source and 2 Ω resistor is therefore 1 A in the other direction, and the current through the 6 V source and 1 Ω resistor is 2 A in the opposite direction indicated on the circuit. The current through the 4 Ω resistor is determined by the following equation from the original network:

loop 1: $I_{4\Omega} = I_1 - I_2 = -1\text{ A} - (-2\text{ A}) = -1\text{ A} + 2\text{ A}$

$= \mathbf{1\ A}$ (in the direction of I_1)

The outer loop (I_3) and *one* inner loop (either I_1 or I_2) would also have produced the correct results. This approach, however, often leads to errors since the loop equations may be more difficult to write. The best method of picking these loop currents is the window approach.

EXAMPLE 8.12 Find the current through each branch of the network in Fig. 8.29.

Solution:

Steps 1 and 2: These are as indicated in the circuit. Note that the polarities of the 6 Ω resistor are different for each loop current.

Step 3: Kirchhoff's voltage law is applied around each closed loop in the clockwise direction:

loop 1: $+E_1 - V_1 - V_2 - E_2 = 0$ (clockwise starting at point a)

$$+5\text{ V} - (1\ \Omega)I_1 - (6\ \Omega)(I_1 - I_2) - 10\text{ V} = 0$$

I_2 flows through the 6 Ω resistor in the direction opposite to I_1.

loop 2: $E_2 - V_2 - V_3 = 0$ (clockwise starting at point b)

$$+10\text{ V} - (6\ \Omega)(I_2 - I_1) - (2\ \Omega)I_2 = 0$$

The equations are rewritten as

$$\left.\begin{aligned} 5 - I_1 - 6I_1 + 6I_2 - 10 &= 0 \\ 10 - 6I_2 + 6I_1 - 2I_2 &= 0 \end{aligned}\right\} \begin{aligned} -7I_1 + 6I_2 &= 5 \\ +6I_1 - 8I_2 &= -10 \end{aligned}$$

Step 4:

$$I_1 = \frac{\begin{vmatrix} 5 & 6 \\ -10 & -8 \end{vmatrix}}{\begin{vmatrix} -7 & 6 \\ 6 & -8 \end{vmatrix}} = \frac{-40 + 60}{56 - 36} = \frac{20}{20} = \mathbf{1\ A}$$

$$I_2 = \frac{\begin{vmatrix} -7 & 5 \\ 6 & -10 \end{vmatrix}}{20} = \frac{70 - 30}{20} = \frac{40}{20} = \mathbf{2\ A}$$

Since I_1 and I_2 are positive and flow in opposite directions through the 6 Ω resistor and 10 V source, the total current in this branch is equal to the difference of the two currents in the direction of the larger:

$$I_2 > I_1 \quad (2\text{ A} > 1\text{ A})$$

Therefore,

$$I_{R_2} = I_2 - I_1 = 2\text{ A} - 1\text{ A} = \mathbf{1\ A} \quad \text{in the direction of } I_2$$

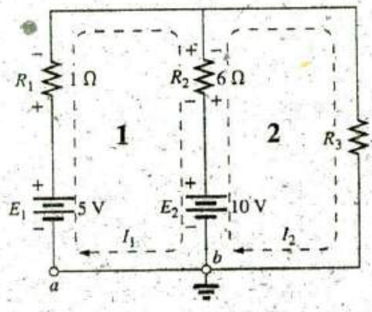


FIG. 8.29
Example 8.12.

It is sometimes impractical to draw all the branches of a circuit at right angles to one another. The next example demonstrates how a portion of a network may appear due to various constraints. The method of analysis is no different with this change in configuration.

EXAMPLE 8.13 Find the branch currents of the networks in Fig. 8.30.

Solution:

Steps 1 and 2: These are as indicated in the circuit.

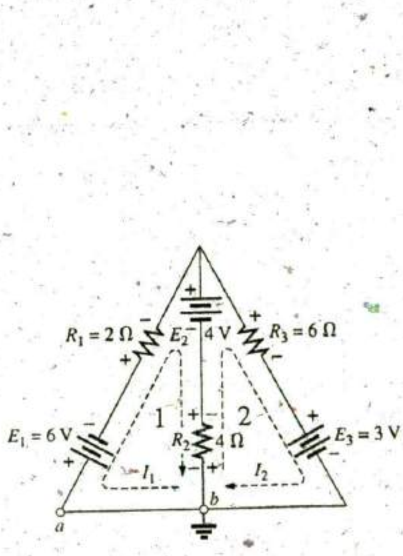


FIG. 8.30
Example 8.13.

Step 3: Kirchhoff's voltage law is applied around each closed loop:

loop 1: $-E_1 - I_1R_1 - E_2 - V_2 = 0$ (clockwise from point *a*)
$-6\text{ V} - (2\ \Omega)I_1 - 4\text{ V} - (4\ \Omega)(I_1 - I_2) = 0$

loop 2: $-V_2 + E_2 - V_3 - E_3 = 0$ (clockwise from point *b*)
$-(4\ \Omega)(I_2 - I_1) + 4\text{ V} - (6\ \Omega)(I_2) - 3\text{ V} = 0$

which are rewritten as

$$\left.\begin{aligned}-10 - 4I_1 - 2I_1 + 4I_2 &= 0\\ +1 + 4I_1 + 4I_2 - 6I_2 &= 0\end{aligned}\right\}\quad \begin{aligned}-6I_1 + 4I_2 &= +10\\ +4I_1 - 10I_2 &= -1\end{aligned}$$

or, by multiplying the top equation by -1, we obtain

$$\begin{aligned}6I_1 - 4I_2 &= -10\\ 4I_1 - 10I_2 &= -1\end{aligned}$$

Step 4: $$I_1 = \frac{\begin{vmatrix}-10 & -4\\ -1 & -10\end{vmatrix}}{\begin{vmatrix}6 & -4\\ 4 & -10\end{vmatrix}} = \frac{100 - 4}{-60 + 16} = \frac{96}{-44} = \mathbf{-2.18\ A}$$

$$I_2 = \frac{\begin{vmatrix}6 & -10\\ 4 & -1\end{vmatrix}}{-44} = \frac{-6 + 40}{-44} = \frac{34}{-44} = \mathbf{-0.77\ A}$$

The current in the 4 Ω resistor and 4 V source for loop 1 is

$$\begin{aligned}I_1 - I_2 &= -2.18\text{ A} - (-0.77\text{ A})\\ &= -2.18\text{ A} + 0.77\text{ A}\\ &= \mathbf{-1.41\ A}\end{aligned}$$

revealing that it is 1.41 A in a direction opposite (due to the minus sign) to I_1 in loop 1.

Supermesh Currents

Occasionally, you will find current sources in a network without a parallel resistance. This removes the possibility of converting the source to a voltage source as required by the given procedure. In such cases, you have a choice of two approaches.

The simplest and most direct approach is to place a resistor in parallel with the current source that has a much higher value than the other resistors of the network. For instance, if most of the resistors of the network are in the 1 to 10 Ω range, choosing a resistor of 100 Ω or higher would provide one level of accuracy for the answer. However, choosing a resistor of 1000 Ω or higher would increase the accuracy of the answer. You will never get the exact answer because the network has been modified by this introduced element. However for most applications, the answer will be sufficiently accurate.

The other choice is to use the **supermesh approach** described in the following steps. Although this approach will provide the exact solution, it does require some practice to become proficient in its use. The procedure is as follows.

Start as before, and assign a mesh current to each independent loop, including the current sources, as if they were resistors or voltage sources. Then mentally (redraw the network if necessary) remove the current sources (replace with open-circuit equivalents), and apply

Kirchhoff's voltage law to all the remaining independent paths of the network using the mesh currents just defined. Any resulting path, including two or more mesh currents, is said to be the path of a **supermesh current.** Then relate the chosen mesh currents of the network to the independent current sources of the network, and solve for the mesh currents. The next example clarifies the definition of supermesh current and the procedure.

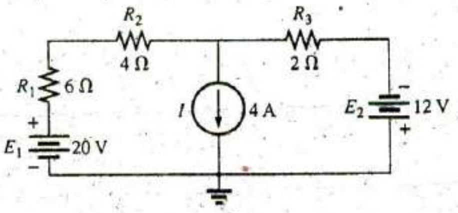


FIG. 8.31
Example 8.14.

EXAMPLE 8.14 Using mesh analysis, determine the currents of the network in Fig. 8.31.

Solution: First, the mesh currents for the network are defined, as shown in Fig. 8.32. Then the current source is mentally removed, as shown in Fig. 8.33, and Kirchhoff's voltage law is applied to the resulting network. The single path now including the effects of two mesh currents is referred to as the path of a *supermesh current.*

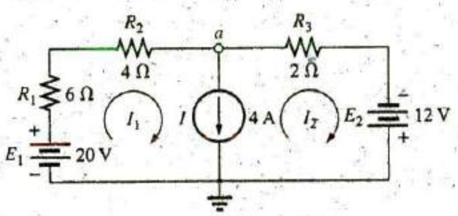


FIG. 8.32
Defining the mesh currents for the network in Fig. 8.31.

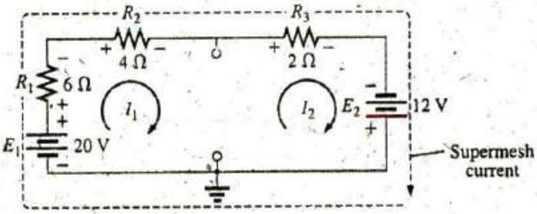


FIG. 8.33
Defining the supermesh current.

Applying Kirchhoff's law gives

$$20\text{ V} - I_1(6\,\Omega) - I_1(4\,\Omega) - I_2(2\,\Omega) + 12\text{ V} = 0$$

or

$$10I_1 + 2I_2 = 32$$

Node *a* is then used to relate the mesh currents and the current source using Kirchhoff's current law:

$$I_1 = I + I_2$$

The result is two equations and two unknowns:

$$\begin{aligned} 10I_1 + 2I_2 &= 32 \\ I_1 - I_2 &= 4 \end{aligned}$$

Applying determinants gives

$$I_1 = \frac{\begin{vmatrix} 32 & 2 \\ 4 & -1 \end{vmatrix}}{\begin{vmatrix} 10 & 2 \\ 1 & -1 \end{vmatrix}} = \frac{(32)(-1) - (2)(4)}{(10)(-1) - (2)(1)} = \frac{40}{12} = \mathbf{3.33\ A}$$

and

$$I_2 = I_1 - I = 3.33\text{ A} - 4\text{ A} = \mathbf{-0.67\ A}$$

In the above analysis, it may appear that when the current source was removed, $I_1 = I_2$. However, the supermesh approach requires that we stick with the original definition of each mesh current and not alter those definitions when current sources are removed.

EXAMPLE 8.15 Using mesh analysis, determine the currents for the network in Fig. 8.34.

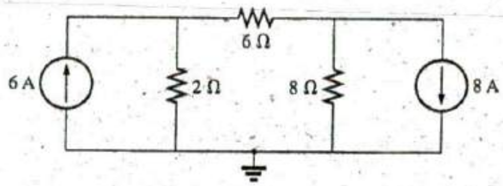


FIG. 8.34
Example 8.15.

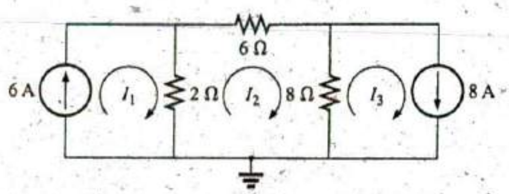


FIG. 8.35
Defining the mesh currents for the network in Fig. 8.34.

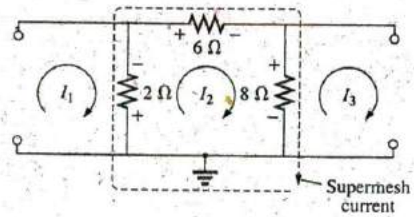


FIG. 8.36
Defining the supermesh current for the network in Fig. 8.34.

Solution: The mesh currents are defined in Fig. 8.35. The current sources are removed, and the single supermesh path is defined in Fig. 8.36.

Applying Kirchhoff's voltage law around the supermesh path gives

$$-V_{2\Omega} - V_{6\Omega} - V_{8\Omega} = 0$$
$$-(I_2 - I_1)2\,\Omega - I_2(6\,\Omega) - (I_2 - I_3)8\,\Omega = 0$$
$$-2I_2 + 2I_1 - 6I_2 - 8I_2 + 8I_3 = 0$$
$$2I_1 - 16I_2 + 8I_3 = 0$$

Introducing the relationship between the mesh currents and the current sources

$$I_1 = 6\text{ A}$$
$$I_3 = 8\text{ A}$$

results in the following solutions:

$$2I_1 - 16I_2 + 8I_3 = 0$$
$$2(6\text{ A}) - 16I_2 + 8(8\text{ A}) = 0$$

and $$I_2 = \frac{76\text{ A}}{16} = \mathbf{4.75\text{ A}}$$

Then $$I_{2\Omega}\downarrow = I_1 - I_2 = 6\text{ A} - 4.75\text{ A} = \mathbf{1.25\text{ A}}$$

and $$I_{8\Omega}\uparrow = I_3 - I_2 = 8\text{ A} - 4.75\text{ A} = \mathbf{3.25\text{ A}}$$

Again, note that you must stick with your original definitions of the various mesh currents when applying Kirchhoff's voltage law around the resulting supermesh paths.

8.8 MESH ANALYSIS (FORMAT APPROACH)

Now that the basis for the mesh-analysis approach has been established, we now examine a technique for writing the mesh equations more rapidly and usually with fewer errors. As an aid in introducing the procedure, the network in Example 8.12 (Fig. 8.29) has been redrawn in Fig. 8.37 with the assigned loop currents. (Note that each loop current has a clockwise direction.)

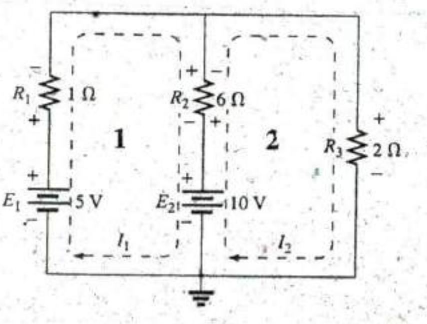


FIG. 8.37
Network in Fig. 8.29 redrawn with assigned loop currents.

The equations obtained are

$$-7I_1 + 6I_2 = 5$$
$$6I_1 - 8I_2 = -10$$

which can also be written as

$$\begin{aligned} 7I_1 - 6I_2 &= -5 \\ 8I_2 - 9I_1 &= 10 \end{aligned}$$

and expanded as

Col. 1	Col. 2	Col. 3
$(1 + 6)I_1 -$	$6I_2$	$= (5 - 10)$
$(2 + 6)I_1 -$	$6I_2$	$= 10$

Note in the above equations that column 1 is composed of a loop current times the sum of the resistors through which that loop current passes. Column 2 is the product of the resistors common to another loop current times that other loop current. Note that in each equation, this column is subtracted from column 1. Column 3 is the *algebraic* sum of the voltage sources through which the loop current of interest passes. A source is assigned a positive sign if the loop current passes from the negative to the positive terminal, and a negative value is assigned if the polarities are reversed. The comments above are correct only for a standard direction of loop current in each window, the one chosen being the clockwise direction.

The above statements can be extended to develop the following *format approach* to mesh analysis.

Mesh Analysis Procedure

1. ***Assign a loop current to each independent, closed loop (as in the previous section) in a clockwise direction.***
2. ***The number of required equations is equal to the number of chosen independent, closed loops. Column 1 of each equation is formed by summing the resistance values of those resistors through which the loop current of interest passes and multiplying the result by that loop current.***
3. ***We must now consider the mutual terms, which, as noted in the examples above, are always subtracted from the first column. A mutual term is simply any resistive element having an additional loop current passing through it. It is possible to have more than one mutual term if the loop current of interest has an element in common with more than one other loop current. This will be demonstrated in an example to follow. Each term is the product of the mutual resistor and the other loop current passing through the same element.***
4. ***The column to the right of the equality sign is the algebraic sum of the voltage sources through which the loop current of interest passes. Positive signs are assigned to those sources of voltage having a polarity such that the loop current passes from the negative to the positive terminal. A negative sign is assigned to those potentials for which the reverse is true.***
5. ***Solve the resulting simultaneous equations for the desired loop currents.***

Before considering a few examples, be aware that since the column to the right of the equals sign is the algebraic sum of the voltage sources in that loop, *the format approach can be applied only to networks in which all current sources have been converted to their equivalent voltage source.*

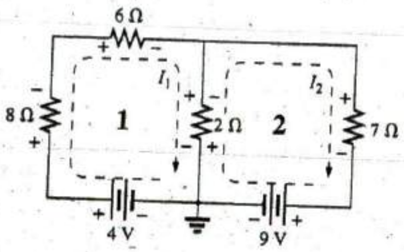


FIG. 8.38
Example 8.16.

EXAMPLE 8.16 Write the mesh equations for the network in Fig. 8.38, and find the current through the 7 Ω resistor.

Solution:

Step 1: As indicated in Fig. 8.38, each assigned loop current has a clockwise direction.

Steps 2 to 4:

$$I_1: \quad (8\ \Omega + 6\ \Omega + 2\ \Omega)I_1 - (2\ \Omega)I_2 = 4\ \text{V}$$
$$I_2: \quad (7\ \Omega + 2\ \Omega)I_2 - (2\ \Omega)I_1 = -9\ \text{V}$$

and

$$16I_1 - 2I_2 = 4$$
$$9I_2 - 2I_1 = -9$$

which, for determinants, are

$$16I_1 - 2I_2 = 4$$
$$-2I_1 + 9I_2 = -9$$

and

$$I_2 = I_{7\Omega} = \frac{\begin{vmatrix} 16 & 4 \\ -2 & -9 \end{vmatrix}}{\begin{vmatrix} 16 & -2 \\ -2 & 9 \end{vmatrix}} = \frac{-144 + 8}{144 - 4} = \frac{-136}{140}$$
$$= -0.97\ \text{A}$$

EXAMPLE 8.17 Write the mesh equations for the network in Fig. 8.39.

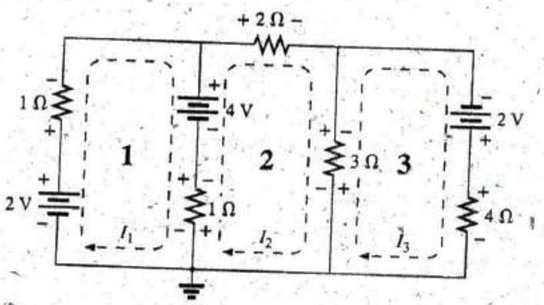


FIG. 8.39
Example 8.17.

Solution: Each window is assigned a loop current in the clockwise direction:

I_1 does not pass through an element mutual with I_3.

$$I_1: \quad (1\ \Omega + 1\ \Omega)I_1 - (1\ \Omega)I_2 + 0 = 2\ \text{V} - 4\ \text{V}$$
$$I_2: \quad (1\ \Omega + 2\ \Omega + 3\ \Omega)I_2 - (1\ \Omega)I_1 - (3\ \Omega)I_3 = 4\ \text{V}$$
$$I_3: \quad (3\ \Omega + 4\ \Omega)I_3 - (3\ \Omega)I_2 + 0 = 2\ \text{V}$$

I_3 does not pass through an element mutual with I_1.

Summing terms yields

$$\begin{aligned} 2I_1 - I_2 + 0 &= -2 \\ 6I_2 - I_1 - 3I_3 &= 4 \\ 7I_3 - 3I_2 + 0 &= 2 \end{aligned}$$

which are rewritten for determinants as

$$\begin{aligned} 2I_1 - I_2 + 0 &= -2 \\ -I_1 + 6I_2 - 3I_3 &= 4 \\ 0 - 3I_2 + 7I_3 &= 2 \end{aligned}$$

Note that the coefficients of the a and b diagonals are equal. This *symmetry* about the c-axis will always be true for equations written using the format approach. It is a check on whether the equations were obtained correctly.

We now consider a network with only one source of voltage to point out that mesh analysis can be used to advantage in other than multisource networks.

EXAMPLE 8.18 Find the current through the 10 Ω resistor of the network in Fig. 8.40.

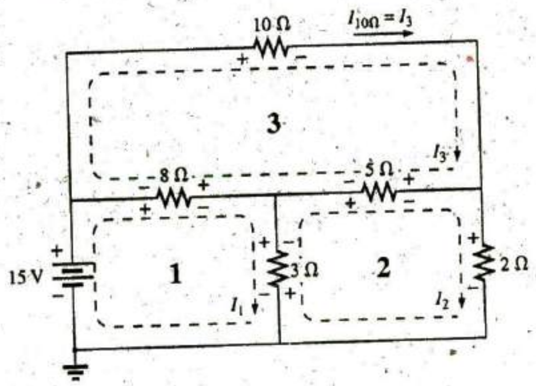


FIG. 8.40
Example 8.18.

Solution:

$$\begin{aligned} I_1&: \quad (8\ \Omega + 3\ \Omega)I_1 - (8\ \Omega)I_3 - (3\ \Omega)I_2 = 15\ \text{V} \\ I_2&: \quad (3\ \Omega + 5\ \Omega + 2\ \Omega)I_2 - (3\ \Omega)I_1 - (5\ \Omega)I_3 = 0 \\ I_3&: \quad (8\ \Omega + 10\ \Omega + 5\ \Omega)I_3 - (8\ \Omega)I_1 - (5\ \Omega)I_2 = 0 \end{aligned}$$

$$\begin{aligned} 11I_1 - 8I_3 - 3I_2 &= 15\ \text{V} \\ 10I_2 - 3I_1 - 5I_3 &= 0 \\ 23I_3 - 8I_1 - 5I_2 &= 0 \end{aligned}$$

or

$$\begin{aligned} 11I_1 - 3I_2 - 8I_3 &= 15\ \text{V} \\ -3I_1 + 10I_2 - 5I_3 &= 0 \\ -8I_1 - 5I_2 + 23I_3 &= 0 \end{aligned}$$

and
$$I_3 = I_{10\Omega} = \frac{\begin{vmatrix} 11 & -3 & 15 \\ -3 & 10 & 0 \\ -8 & -5 & 0 \end{vmatrix}}{\begin{vmatrix} 11 & -3 & -8 \\ -3 & 10 & -5 \\ -8 & -5 & 23 \end{vmatrix}} = 1.22\text{ A}$$

TI-89 Calculator Solution: When the TI-89 calculator is used, the sequence in Fig. 8.41(a) results, which in shorthand form appears as in Fig. 8.41(b). The intermediary 2ND and scrolling steps were not included. This sequence certainly requires some care in entering the data in the required format, but it is still a rather neat, compact format.

Home `2ND` MATH `↓` Matrix `ENTER` `↓` det(`ENTER` `2ND` [`1` `1` `,` `(-)` `3` `,` `1` `5` `2ND` ; `(-)` `3` `,` `1` `0` `,` `0` `2ND` ; `(-)` `8` `,` `(-)` `5` `,` `0` `2ND`] `)` `+` Home `2ND` MATH `↓` Matrix `ENTER` `↓` det(`ENTER` `2ND` [`1` `1` `,` `(-)` `3` `,` `(-)` `8` ; `(-)` `3` `,` `1` `0` `,` `(-)` `5` `2ND` ; `(-)` `8` `,` `(-)` `5` `,` `2` `3` `2ND`] `)` `ENTER` 1.22

(a)

det([11,−3,15;−3,10,0;−8,−5,0])/det([11,−3,−8;−3,10,−5;−8,−5,23]) (ENTER) 1.22

(b)

FIG. 8.41
Using the TI-89 calculator to solve for the current I_3. (a) Key entries; (b) shorthand form.

$$\frac{\det\left(\begin{bmatrix} 11 & -3 & 15 \\ -3 & 10 & 0 \\ -8 & -5 & 0 \end{bmatrix}\right)}{\det\left(\begin{bmatrix} 11 & -3 & -8 \\ -3 & 10 & -5 \\ -8 & -5 & 23 \end{bmatrix}\right)} \qquad 1.22E0$$

FIG. 8.42
The resulting display after properly entering the data for the current I_3.

The resulting display in Fig. 8.42 confirms our solution.

8.9 NODAL ANALYSIS (GENERAL APPROACH)

The methods introduced thus far have all been to find the currents of the network. We now turn our attention to **nodal analysis**—a method that provides the nodal voltages of a network, that is, the voltage from the various **nodes** (junction points) of the network to ground. The method is developed through the use of Kirchhoff's current law in much the same manner as Kirchhoff's voltage law was used to develop the mesh analysis approach.

Although it is not a requirement, we make it a policy to make ground our reference node and assign it a potential level of zero volts. All the other voltage levels are then found with respect to this reference level. For a network of N nodes, by assigning one as our reference node, we have $(N - 1)$ nodes for which the voltage must be determined. In other words,

the number of nodes for which the voltage must be determined using nodal analysis is 1 less than the total number of nodes.

The result of the above is $(N - 1)$ nodal voltages that need to be determined, requiring that $(N - 1)$ independent equations be written to find the nodal voltages. In other words,

the number of equations required to solve for all the nodal voltages of a network is 1 less than the total number of independent nodes.

Since each equation is the result of an application of Kirchhoff's current law, Kirchhoff's current law must be applied $(N - 1)$ times for each network.

Nodal analysis, like mesh analysis, can be applied by a series of carefully defined steps. The examples to follow explain each step in detail.

Nodal Analysis Procedure

1. *Determine the number of nodes within the network.*
2. *Pick a reference node, and label each remaining node with a subscripted value of voltage: V_1, V_2, and so on.*
3. *Apply Kirchhoff's current law at each node except the reference. Assume that all unknown currents leave the node for each application of Kirchhoff's current law. In other words, for each node, don't be influenced by the direction that an unknown current for another node may have had. Each node is to be treated as a separate entity, independent of the application of Kirchhoff's current law to the other nodes.*
4. *Solve the resulting equations for the nodal voltages.*

A few examples clarify the procedure defined by step 3. It initially takes some practice writing the equations for Kirchhoff's current law correctly, but in time the advantage of assuming that all the currents leave a node rather than identifying a specific direction for each branch becomes obvious. (The same type of advantage is associated with assuming that all the mesh currents are clockwise when applying mesh analysis.)

As with mesh and branch-current analysis, a number of networks to be encountered in this section can be solved using a simple source conversion. In Example 8.19, for instance, the network of Fig. 8.43 can be easily solved by converting the voltage source to a current source and combining the parallel current sources. However, as noted for mesh and branch-current analysis, this method can also be applied to more complex networks where a source conversion is not possible.

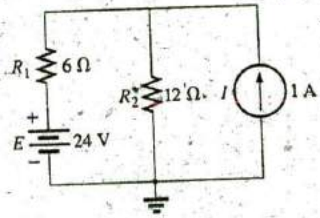


FIG. 8.43
Example 8.19.

EXAMPLE 8.19 Apply nodal analysis to the network in Fig. 8.43.

Solution:

Steps 1 and 2: The network has two nodes, as shown in Fig. 8.44. The lower node is defined as the reference node at ground potential (zero volts), and the other node as V_1, the voltage from node 1 to ground.

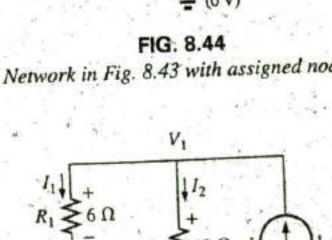


FIG. 8.44
Network in Fig. 8.43 with assigned nodes.

Step 3: I_1 and I_2 are defined as leaving the node in Fig. 8.45, and Kirchhoff's current law is applied as follows:

$$I = I_1 + I_2$$

The current I_2 is related to the nodal voltage V_1 by Ohm's law:

$$I_2 = \frac{V_{R_2}}{R_2} = \frac{V_1}{R_2}$$

The current I_1 is also determined by Ohm's law as follows:

$$I_1 = \frac{V_{R_1}}{R_1}$$

with

$$V_{R_1} = V_1 - E$$

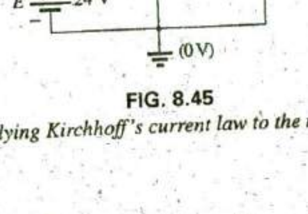


FIG. 8.45
Applying Kirchhoff's current law to the node V_1.

Substituting into the Kirchhoff's current law equation

$$I = \frac{V_1 - E}{R_1} + \frac{V_1}{R_2}$$

and rearranging, we have

$$I = \frac{V_1}{R_1} - \frac{E}{R_1} + \frac{V_1}{R_2} = V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{E}{R_1}$$

or

$$V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{E}{R_1} + 1$$

Substituting numerical values, we obtain

$$V_1\left(\frac{1}{6\,\Omega} + \frac{1}{12\,\Omega}\right) = \frac{24\text{ V}}{6\,\Omega} + 1\text{ A} = 4\text{ A} + 1\text{ A}$$

$$V_1\left(\frac{1}{4\,\Omega}\right) = 5\text{ A}$$

$$V_1 = \mathbf{20\ V}$$

The currents I_1 and I_2 can then be determined by using the preceding equations:

$$I_1 = \frac{V_1 - E}{R_1} = \frac{20\text{ V} - 24\text{ V}}{6\,\Omega} = \frac{-4\text{ V}}{6\,\Omega}$$
$$= \mathbf{-0.67\ A}$$

The minus sign indicates that the current I_1 has a direction opposite to that appearing in Fig. 8.45. In addition,

$$I_2 = \frac{V_1}{R_2} = \frac{20\text{ V}}{12\,\Omega} = \mathbf{1.67\ A}$$

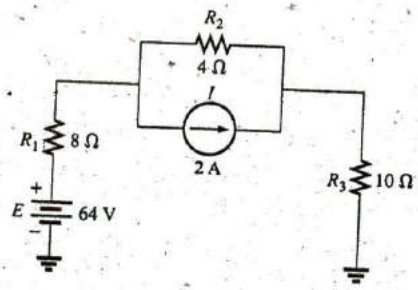


FIG. 8.46
Example 8.20.

EXAMPLE 8.20 Apply nodal analysis to the network in Fig. 8.46.

Solution:

Steps 1 and 2: The network has three nodes, as defined in Fig. 8.47, with the bottom node again defined as the reference node (at ground potential, or zero volts), and the other nodes as V_1 and V_2.

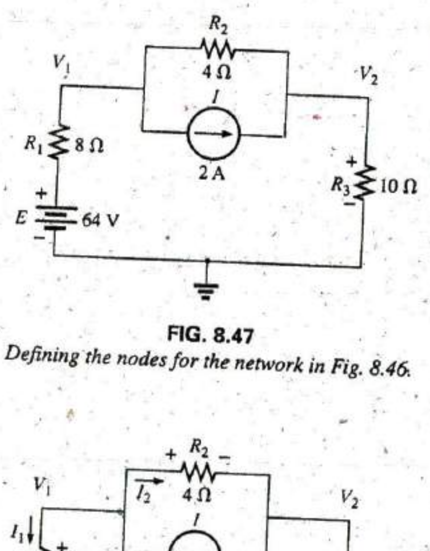


FIG. 8.47
Defining the nodes for the network in Fig. 8.46.

Step 2: For node V_1, the currents are defined as shown in Fig. 8.48, and Kirchhoff's current law is applied:

$$0 = I_1 + I_2 + I$$

with

$$I_1 = \frac{V_1 - E}{R_1}$$

and

$$I_2 = \frac{V_{R_2}}{R_2} = \frac{V_1 - V_2}{R_2}$$

so that

$$\frac{V_1 - E}{R_1} + \frac{V_1 - V_2}{R_2} + I = 0$$

or

$$\frac{V_1}{R_1} - \frac{E}{R_1} + \frac{V_1}{R_2} - \frac{V_2}{R_2} + I = 0$$

and

$$V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - V_2\left(\frac{1}{R_2}\right) = -I + \frac{E}{R_1}$$

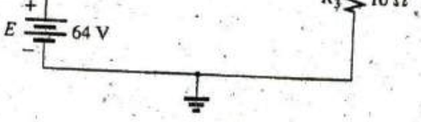


FIG. 8.48
Applying Kirchhoff's current law to node V_1.

Substituting values gives

$$V_1\left(\frac{1}{8\,\Omega}+\frac{1}{4\,\Omega}\right)-V_2\left(\frac{1}{4\,\Omega}\right)=-2\text{ A}+\frac{64\text{ V}}{8\,\Omega}=6\text{ A}$$

For node V_2, the currents are defined as shown in Fig. 8.49, and Kirchhoff's current law is applied:

$$I = I_2 + I_3$$

with

$$I=\frac{V_2-V_1}{R_2}+\frac{V_2}{R_3}$$

or

$$I=\frac{V_2}{R_2}-\frac{V_1}{R_2}+\frac{V_2}{R_3}$$

and

$$V_2\left(\frac{1}{R_2}+\frac{1}{R_3}\right)-V_1\left(\frac{1}{R_2}\right)=I$$

Substituting values gives

$$V_2\left(\frac{1}{4\,\Omega}+\frac{1}{10\,\Omega}\right)-V_1\left(\frac{1}{4\,\Omega}\right)=2\text{ A}$$

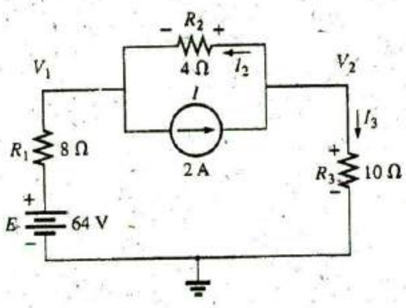


FIG. 8.49
Applying Kirchhoff's current law to node V_2.

Step 4: The result is two equations and two unknowns:

$$V_1\left(\frac{1}{8\,\Omega}+\frac{1}{4\,\Omega}\right)-V_2\left(\frac{1}{4\,\Omega}\right)=6\text{ A}$$
$$-V_1\left(\frac{1}{4\,\Omega}\right)+V_2\left(\frac{1}{4\,\Omega}+\frac{1}{10\,\Omega}\right)=2\text{ A}$$

which become

$$0.375V_1-0.25V_2=6$$
$$-0.25V_1+0.35V_2=2$$

Using determinants, we obtain

$$V_1 = \mathbf{37.82\text{ V}}$$
$$V_2 = \mathbf{32.73\text{ V}}$$

Since E is greater than V_1, the current I_1 flows from ground to V_1 and is equal to

$$I_{R_1}=\frac{E-V_1}{R_1}=\frac{64\text{ V}-37.82\text{ V}}{8\,\Omega}=\mathbf{3.27\text{ A}}$$

The positive value for V_2 results in a current I_{R_3} from node V_2 to ground equal to

$$I_{R_3}=\frac{V_{R_3}}{R_3}=\frac{V_2}{R_3}=\frac{32.73\text{ V}}{10\,\Omega}=\mathbf{3.27\text{ A}}$$

Since V_1 is greater than V_2, the current I_{R_2} flows from V_1 to V_2 and is equal to

$$I_{R_2}=\frac{V_1-V_2}{R_2}=\frac{37.82\text{ V}-32.73\text{ V}}{4\,\Omega}=\mathbf{1.27\text{ A}}$$

The results of $V_1 = 37.82$ V and $V_2 = 32.73$ V confirm the theoretical solution.

EXAMPLE 8.21 Determine the nodal voltages for the network in Fig. 8.50.

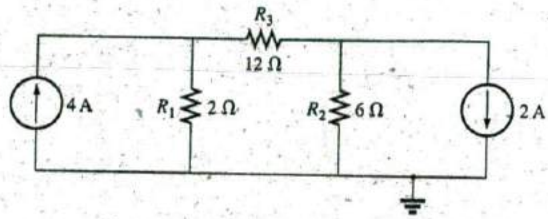


FIG. 8.50
Example 8.21.

Solution:

Steps 1 and 2: As indicated in Fig. 8.51:

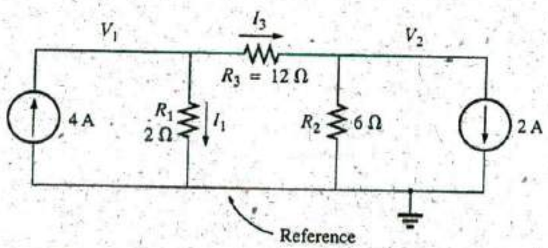


FIG. 8.51
Defining the nodes and applying Kirchhoff's current law to the node V_1.

Step 3: Included in Fig. 8.51 for the node V_1. Applying Kirchhoff's current law gives

$$4\text{ A} = I_1 + I_3$$

and
$$4\text{ A} = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_3} = \frac{V_1}{2\ \Omega} + \frac{V_1 - V_2}{12\ \Omega}$$

Expanding and rearranging gives

$$V_1\left(\frac{1}{2\ \Omega} + \frac{1}{12\ \Omega}\right) - V_2\left(\frac{1}{12\ \Omega}\right) = 4\text{ A}$$

For node V_2, the currents are defined as in Fig. 8.52.

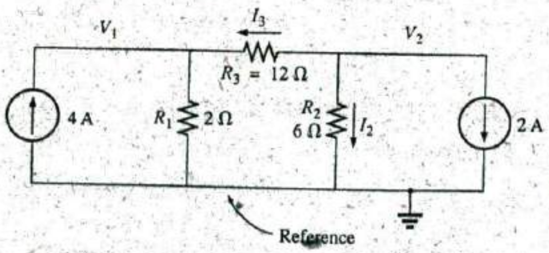


FIG. 8.52
Applying Kirchhoff's current law to the node V_2.

Applying Kirchhoff's current law gives

$$0 = I_3 + I_2 + 2\text{ A}$$

and $\dfrac{V_2 - V_1}{R_3} + \dfrac{V_2}{R_2} + 2\text{ A} = 0 \longrightarrow \dfrac{V_2 - V_1}{12\,\Omega} + \dfrac{V_2}{2\,\Omega} + 2\text{ A} = 0$

Expanding and rearranging gives

$$V_2\left(\frac{1}{12\,\Omega} + \frac{1}{6\,\Omega}\right) - V_1\left(\frac{1}{12\,\Omega}\right) = -2\text{ A}$$

resulting in the following two equations and two unknowns:

$$\left.\begin{aligned} V_1\left(\frac{1}{2\,\Omega} + \frac{1}{12\,\Omega}\right) - V_2\left(\frac{1}{12\,\Omega}\right) &= +4\text{ A} \\ V_2\left(\frac{1}{12\,\Omega} + \frac{1}{6\,\Omega}\right) - V_1\left(\frac{1}{12\,\Omega}\right) &= -2\text{ A} \end{aligned}\right\} \qquad \textbf{(8.1)}$$

producing

$$\left.\begin{aligned} \frac{7}{12}V_1 - \frac{1}{12}V_2 &= +4 \\ -\frac{1}{12}V_1 + \frac{3}{12}V_2 &= -2 \end{aligned}\right\} \quad \begin{aligned} 7V_1 - V_2 &= 48 \\ -1V_1 + 3V_2 &= -24 \end{aligned}$$

and

$$V_1 = \frac{\begin{vmatrix} 48 & -1 \\ -24 & 3 \end{vmatrix}}{\begin{vmatrix} 7 & -1 \\ -1 & 3 \end{vmatrix}} = \frac{120}{20} = +6\text{ V}$$

$$V_2 = \frac{\begin{vmatrix} 7 & 48 \\ -1 & -24 \end{vmatrix}}{20} = \frac{-120}{20} = -6\text{ V}$$

Since V_1 is greater than V_2, the current through R_3 passes from V_1 to V_2. Its value is

$$I_{R_3} = \frac{V_1 - V_2}{R_3} = \frac{6\text{ V} - (-6\text{ V})}{12\,\Omega} = \frac{12\text{ V}}{12\,\Omega} = 1\text{ A}$$

The fact that V_1 is positive results in a current I_{R_1} from V_1 to ground equal to

$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{V_1}{R_1} = \frac{6\text{ V}}{2\,\Omega} = 3\text{ A}$$

Finally, since V_2 is negative, the current I_{R_2} flows from ground to V_2 and is equal to

$$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{V_2}{R_2} = \frac{6\text{ V}}{6\,\Omega} = 1\text{ A}$$

Supernode

Occasionally, you may encounter voltage sources in a network that do not have a series internal resistance that would permit a conversion to a current source. In such cases, you have two options.

The simplest and most direct approach is to *place a resistor in series with the source of a very small value compared to the other resistive* *elements of the network.* For instance, if most of the resistors are 10 Ω or larger, placing a 1 Ω resistor in series with a voltage source provides one level of accuracy for your answer. However, choosing a resistor of 0.1 Ω or less increases the accuracy of your answer. You will never get an exact answer because the network has been modified by the introduced element. However, for most applications, the accuracy will be sufficiently high.

The other approach is to use the **supernode approach** described below. This approach provides an exact solution but requires some practice to become proficient.

Start as usual and assign a nodal voltage to each independent node of the network, including each independent voltage source as if it were a resistor or current source. Then mentally replace the independent voltage sources with short-circuit equivalents, and apply Kirchhoff's current law to the defined nodes of the network. Any node including the effect of elements tied only to *other* nodes is referred to as a *supernode* (since it has an additional number of terms). Finally, relate the defined nodes to the independent voltage sources of the network, and solve for the nodal voltages. The next example clarifies the definition of *supernode.*

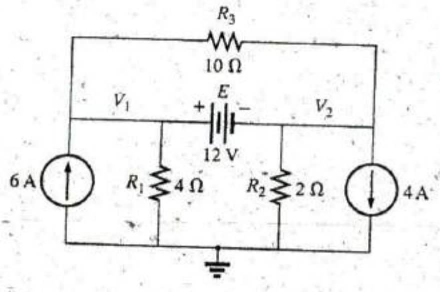


FIG. 8.53
Example 8.22.

EXAMPLE 8.22 Determine the nodal voltages V_1 and V_2 in Fig. 8.53 using the concept of a supernode.

Solution: Replacing the independent voltage source of 12 V with a short-circuit equivalent results in the network in Fig. 8.54. Even though the mental application of a short-circuit equivalent is discussed above, it would be wise in the early stage of development to redraw the network as shown in Fig. 8.54. The result is a single supernode for which Kirchhoff's current law must be applied. Be sure to leave the other defined nodes in place, and use them to define the currents from that region of the network. In particular, note that the current I_3 leaves the supernode at V_1 and then enters the same supernode at V_2. It must therefore appear twice when applying Kirchhoff's current law, as shown below:

$$\Sigma I_i = \Sigma I_o$$

$$6\text{ A} + I_3 = I_1 + I_2 + 4\text{ A} + I_3$$

or

$$I_1 + I_2 = 6\text{ A} - 4\text{ A} = 2\text{ A}$$

Then

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} = 2\text{ A}$$

and

$$\frac{V_1}{4\,\Omega} + \frac{V_2}{2\,\Omega} = 2\text{ A}$$

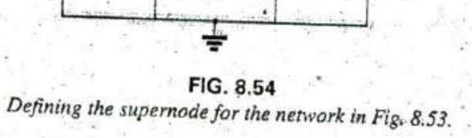


FIG. 8.54
Defining the supernode for the network in Fig. 8.53.

Relating the defined nodal voltages to the independent voltage source, we have

$$V_1 - V_2 = E = 12\text{ V}$$

which results in two equations and two unknowns:

$$\begin{aligned} 0.25V_1 + 0.5V_2 &= 2 \\ V_1 - 1V_2 &= 12 \end{aligned}$$

Substituting gives

$$V_1 = V_2 + 12$$

$$0.25(V_2 + 12) + 0.5V_2 = 2$$

and

$$0.75V_2 = 2 - 3 = -1$$

so that $$V_2 = \frac{-1}{0.75} = \mathbf{-1.33\ V}$$

and $$V_1 = V_2 + 12\ \text{V} = -1.33\ \text{V} + 12\ \text{V} = \mathbf{-10.67\ V}$$

The current of the network can then be determined as follows:

$$I_1\downarrow = \frac{V}{R_1} = \frac{10.67\ \text{V}}{4\ \Omega} = \mathbf{2.67\ A}$$

$$I_2\uparrow = \frac{V_2}{R_2} = \frac{1.33\ \text{V}}{2\ \Omega} = \mathbf{0.67\ A}$$

$$\underset{\rightarrow}{I_3} = \frac{V_1 - V_2}{10\ \Omega} = \frac{10.67\ \text{V} - (-1.33\ \text{V})}{10\ \Omega} = \frac{12\ \Omega}{10\ \Omega} = \mathbf{1.2\ A}$$

A careful examination of the network at the beginning of the analysis would have revealed that the voltage across the resistor R_3 must be 12 V and I_3 must be equal to 1.2 A.

8.10 NODAL ANALYSIS (FORMAT APPROACH)

A close examination of Eq. (8.1) appearing in Example 8.21 reveals that the subscripted voltage at the node in which Kirchhoff's current law is applied is multiplied by the sum of the conductances attached to that node. Note also that the other nodal voltages within the same equation are multiplied by the negative of the conductance between the two nodes. The current sources are represented to the right of the equals sign with a positive sign if they supply current to the node and with a negative sign if they draw current from the node.

These conclusions can be expanded to include networks with any number of nodes. This allows us to write nodal equations rapidly and in a form that is convenient for the use of determinants. A major requirement, however, is that *all voltage sources must first be converted to current sources before the procedure is applied.* Note the parallelism between the following four steps of application and those required for mesh analysis in Section 8.8.

Nodal Analysis Procedure

1. ***Choose a reference node, and assign a subscripted voltage label to the (N − 1) remaining nodes of the network.***
2. ***The number of equations required for a complete solution is equal to the number of subscripted voltages (N − 1). Column 1 of each equation is formed by summing the conductances tied to the node of interest and multiplying the result by that subscripted nodal voltage.***
3. ***We must now consider the mutual terms, which, as noted in the preceding example, are always subtracted from the first column. It is possible to have more than one mutual term if the nodal voltage of current interest has an element in common with more than one other nodal voltage. This is demonstrated in an example to follow. Each mutual term is the product of the mutual conductance and the other nodal voltage, tied to that conductance.***
4. ***The column to the right of the equality sign is the algebraic sum of the current sources tied to the node of interest. A current source is***

assigned a positive sign if it supplies current to a node and a negative sign if it draws current from the node.

5. ***Solve the resulting simultaneous equations for the desired voltages.***

Let us now consider a few examples.

EXAMPLE 8.23 Write the nodal equations for the network in Fig. 8.55.

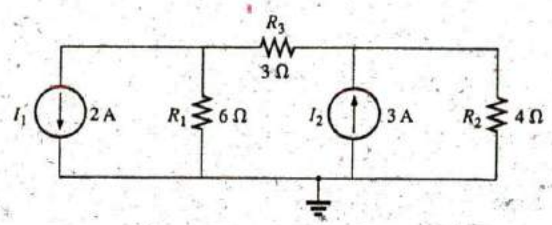


FIG. 8.55
Example 8.23.

Solution:

Step 1: Redraw the figure with assigned subscripted voltages in Fig. 8.56.

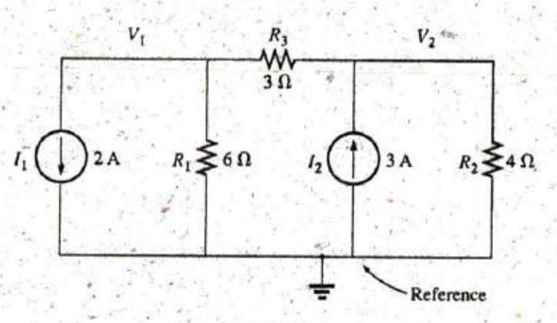


FIG. 8.56
Defining the nodes for the network in Fig. 8.55.

Steps 2 to 4:

Drawing current from node 1

$$V_1\text{:}\quad \underbrace{\left(\frac{1}{6\ \Omega}+\frac{1}{3\ \Omega}\right)}_{\substack{\text{Sum of}\\ \text{conductances}\\ \text{connected}\\ \text{to node 1}}}V_1-\underbrace{\left(\frac{1}{3\ \Omega}\right)}_{\substack{\text{Mutual}\\ \text{conductance}}}V_2=-2\text{ A}$$

Supplying current to node 2

$$V_2\text{:}\quad \underbrace{\left(\frac{1}{4\ \Omega}+\frac{1}{3\ \Omega}\right)}_{\substack{\text{Sum of}\\ \text{conductances}\\ \text{connected}\\ \text{to node 2}}}V_2-\underbrace{\left(\frac{1}{3\ \Omega}\right)}_{\substack{\text{Mutual}\\ \text{conductance}}}V_1=+3\text{ A}$$

and
$$\frac{1}{2}V_1 - \frac{1}{3}V_2 = -2$$
$$-\frac{1}{3}V_1 + \frac{7}{12}V_2 = 3$$

EXAMPLE 8.24 Find the voltage across the 3 Ω resistor in Fig. 8.57 by nodal analysis.

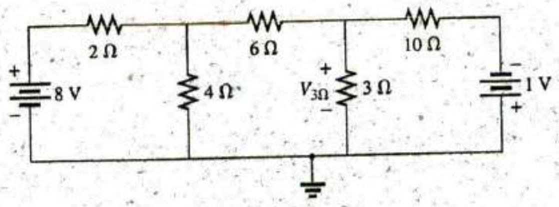


FIG. 8.57
Example 8.24.

Solution: Converting sources and choosing nodes (Fig. 8.58), we have .

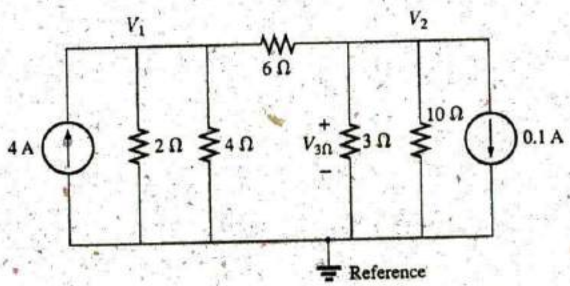


FIG. 8.58
Defining the nodes for the network in Fig. 8.57.

$$\left(\frac{1}{2\,\Omega}+\frac{1}{4\,\Omega}+\frac{1}{6\,\Omega}\right)V_1 - \left(\frac{1}{6\,\Omega}\right)V_2 = +4\text{ A}$$
$$\left(\frac{1}{10\,\Omega}+\frac{1}{3\,\Omega}+\frac{1}{6\,\Omega}\right)V_2 - \left(\frac{1}{6\,\Omega}\right)V_1 = -0.1\text{ A}$$

$$\frac{11}{12}V_1 - \frac{1}{6}V_2 = 4$$
$$-\frac{1}{6}V_1 + \frac{3}{5}V_2 = -0.1$$

resulting in

$$11V_1 - 2V_2 = +48$$
$$-5V_1 + 18V_2 = -3$$

and

$$V_2 = V_{3\Omega} = \frac{\begin{vmatrix} 11 & 48 \\ -5 & -3 \end{vmatrix}}{\begin{vmatrix} 11 & -2 \\ -5 & 18 \end{vmatrix}} = \frac{-33 + 240}{198 - 10} = \frac{207}{188} = \mathbf{1.10\ V}$$

As demonstrated for mesh analysis, nodal analysis can also be a very useful technique for solving networks with only one source.

EXAMPLE 8.25 Using nodal analysis, determine the potential across the 4 Ω resistor in Fig. 8.59.

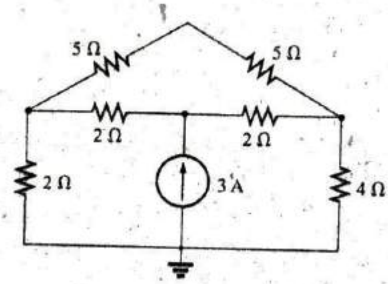


FIG. 8.59
Example 8.25.

Solution: The reference and four subscripted voltage levels were chosen as shown in Fig. 8.60. Remember that for any difference in potential between V_1 and V_3, the current through and the potential drop across each 5 Ω resistor are the same. Therefore, V_4 is simply a mid-voltage level between V_1 and V_3 and is known if V_1 and V_3 are available. We will therefore not include it in a nodal voltage and will redraw the network as shown in Fig. 8.61. Understand, however, that V_4 can be included if desired, although four nodal voltages will result rather than three as in the solution of this problem. We have

$$V_1: \left(\frac{1}{2\,\Omega} + \frac{1}{2\,\Omega} + \frac{1}{10\,\Omega}\right)V_1 - \left(\frac{1}{2\,\Omega}\right)V_2 - \left(\frac{1}{10\,\Omega}\right)V_3 = 0$$

$$V_2: \quad \left(\frac{1}{2\,\Omega} + \frac{1}{2\,\Omega}\right)V_2 - \left(\frac{1}{2\,\Omega}\right)V_1 - \left(\frac{1}{2\,\Omega}\right)V_3 = 3\text{ A}$$

$$V_3: \left(\frac{1}{10\,\Omega} + \frac{1}{2\,\Omega} + \frac{1}{4\,\Omega}\right)V_3 - \left(\frac{1}{2\,\Omega}\right)V_2 - \left(\frac{1}{10\,\Omega}\right)V_1 = 0$$

which are rewritten as

$$\begin{aligned} 1.1V_1 - 0.5V_2 - 0.1V_3 &= 0 \\ V_2 - 0.5V_1 - 0.5V_3 &= 3 \\ 0.85V_3 - 0.5V_2 - 0.1V_1 &= 0 \end{aligned}$$

For determinants, we have

$$\begin{aligned} 1.1V_1 - 0.5V_2 - 0.1V_3 &= 0 \\ -0.5V_1 + 1V_2 - 0.5V_3 &= 3 \\ -0.1V_1 - 0.5V_2 + 0.85V_3 &= 0 \end{aligned}$$

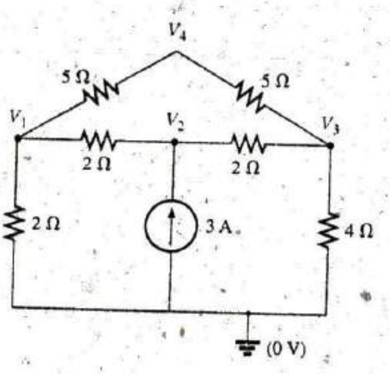


FIG. 8.60
Defining the nodes for the network in Fig. 8.59.

Before continuing, note the symmetry about the major diagonal in the equation above. Recall a similar result for mesh analysis. Examples 8.23 and 8.24 also exhibit this property in the resulting equations. Keep this in mind as a check on future applications of nodal analysis. We have

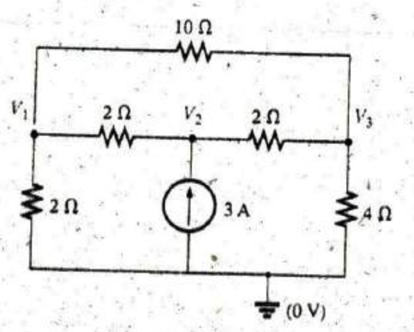


FIG. 8.61
Reducing the number of nodes for the network in Fig. 8.59 by combining the two 5 Ω resistors.

$$V_3 = V_{4\Omega} = \frac{\begin{vmatrix} 1.1 & -0.5 & 0 \\ -0.5 & +1 & 3 \\ -0.1 & -0.5 & 0 \end{vmatrix}}{\begin{vmatrix} 1.1 & -0.5 & -0.1 \\ -0.5 & +1 & -0.5 \\ -0.1 & -0.5 & +0.85 \end{vmatrix}} = \mathbf{4.65\ V}$$

The next example has only one source applied to a ladder network.

EXAMPLE 8.26 Write the nodal equations and find the voltage across the 2 Ω resistor for the network in Fig. 8.62.

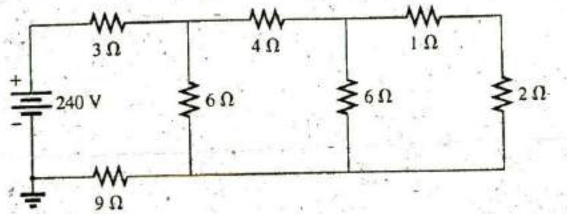


FIG. 8.62
Example 8.26.

Solution: The nodal voltages are chosen as shown in Fig. 8.63. We have

$$V_1: \left(\frac{1}{12\ \Omega} + \frac{1}{6\ \Omega} + \frac{1}{4\ \Omega}\right)V_1 - \left(\frac{1}{4\ \Omega}\right)V_2 + 0 = 20\text{ A}$$

$$V_2: \left(\frac{1}{4\ \Omega} + \frac{1}{6\ \Omega} + \frac{1}{1\ \Omega}\right)V_2 - \left(\frac{1}{4\ \Omega}\right)V_1 - \left(\frac{1}{1\ \Omega}\right)V_3 = 0$$

$$V_3: \left(\frac{1}{1\ \Omega} + \frac{1}{2\ \Omega}\right)V_3 - \left(\frac{1}{1\ \Omega}\right)V_2 + 0 = 0$$

and

$$\begin{aligned} 0.5V_1 - 0.25V_2 + 0 &= 20 \\ -0.25V_1 + \frac{17}{12}V_2 - 1V_3 &= 0 \\ 0 - 1V_2 + 1.5V_3 &= 0 \end{aligned}$$

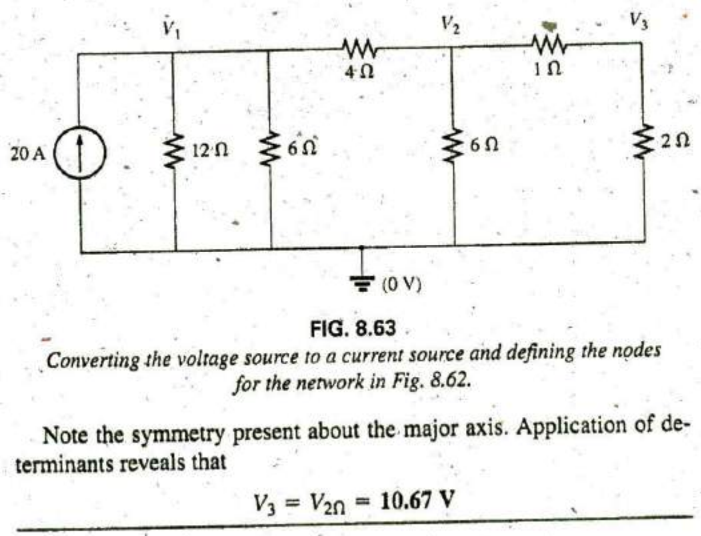


FIG. 8.63
Converting the voltage source to a current source and defining the nodes for the network in Fig. 8.62.

Note the symmetry present about the major axis. Application of determinants reveals that

$$V_3 = V_{2\Omega} = \mathbf{10.67\ V}$$

8.11 BRIDGE NETWORKS

This section introduces the **bridge network,** a configuration that has a multitude of applications. In the following chapters, this type of network is used in both dc and ac meters. Electronics courses introduce these in the discussion of rectifying circuits used in converting a varying signal to one

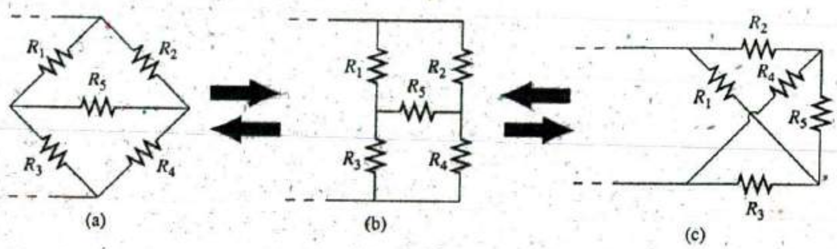


FIG. 8.64
Various formats for a bridge network.

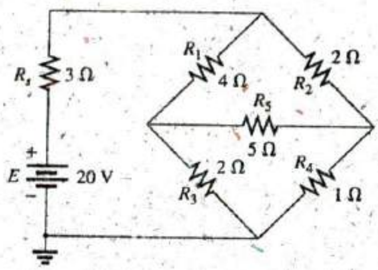


FIG. 8.65
Standard bridge configuration.

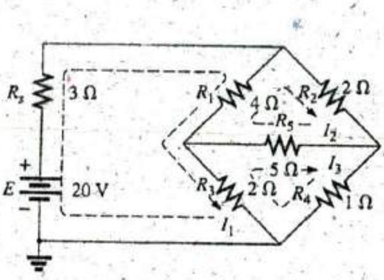


FIG. 8.66
Assigning the mesh currents to the network in Fig. 8.65.

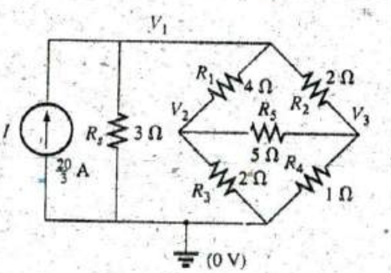


FIG. 8.67
Defining the nodal voltages for the network in Fig. 8.65.

of a steady nature (such as dc). A number of other areas of application also require some knowledge of ac networks; these areas are discussed later.

The bridge network may appear in one of the three forms as indicated in Fig. 8.64. The network in Fig. 8.64(c) is also called a *symmetrical lattice network* if $R_2 = R_3$ and $R_1 = R_4$. Fig. 8.64(c) is an excellent example of how a planar network can be made to appear nonplanar. For the purposes of investigation, let us examine the network in Fig. 8.65 using mesh and nodal analysis.

Mesh analysis (Fig. 8.66) yields

$$(3\ \Omega + 4\ \Omega + 2\ \Omega)I_1 - (4\ \Omega)I_2 - (2\ \Omega)I_3 = 20\ \text{V}$$
$$(4\ \Omega + 5\ \Omega + 2\ \Omega)I_2 - (4\ \Omega)I_1 - (5\ \Omega)I_3 = 0$$
$$(2\ \Omega + 5\ \Omega + 1\ \Omega)I_3 - (2\ \Omega)I_1 - (5\ \Omega)I_2 = 0$$

and

$$9I_1 - 4I_2 - 2I_3 = 20$$
$$-4I_1 + 11I_2 - 5I_3 = 0$$
$$-2I_1 - 5I_2 + 8I_3 = 0$$

with the result that

$$I_1 = \mathbf{4\ A}$$
$$I_2 = \mathbf{2.67\ A}$$
$$I_3 = \mathbf{2.67\ A}$$

The net current through the 5 Ω resistor is

$$I_{5\Omega} = I_2 - I_3 = 2.67\ \text{A} - 2.67\ \text{A} = 0\ \text{A}$$

Nodal analysis (Fig. 8.67) yields

$$\left(\frac{1}{3\ \Omega} + \frac{1}{4\ \Omega} + \frac{1}{2\ \Omega}\right)V_1 - \left(\frac{1}{4\ \Omega}\right)V_2 - \left(\frac{1}{2\ \Omega}\right)V_3 = \frac{20}{3}\ \text{A}$$
$$\left(\frac{1}{4\ \Omega} + \frac{1}{2\ \Omega} + \frac{1}{5\ \Omega}\right)V_2 - \left(\frac{1}{4\ \Omega}\right)V_1 - \left(\frac{1}{5\ \Omega}\right)V_3 = 0$$
$$\left(\frac{1}{5\ \Omega} + \frac{1}{2\ \Omega} + \frac{1}{1\ \Omega}\right)V_3 - \left(\frac{1}{2\ \Omega}\right)V_1 - \left(\frac{1}{5\ \Omega}\right)V_2 = 0$$

and

$$\left(\frac{1}{3\ \Omega} + \frac{1}{4\ \Omega} + \frac{1}{2\ \Omega}\right)V_1 - \left(\frac{1}{4\ \Omega}\right)V_2 - \left(\frac{1}{2\ \Omega}\right)V_3 = 6.67\ \text{A}$$
$$-\left(\frac{1}{4\ \Omega}\right)V_1 + \left(\frac{1}{4\ \Omega} + \frac{1}{2\ \Omega} + \frac{1}{5\ \Omega}\right)V_2 - \left(\frac{1}{5\ \Omega}\right)V_3 = 0$$
$$-\left(\frac{1}{2\ \Omega}\right)V_1 - \left(\frac{1}{5\ \Omega}\right)V_2 + \left(\frac{1}{5\ \Omega} + \frac{1}{2\ \Omega} + \frac{1}{1\ \Omega}\right)V_3 = 0$$

Note the symmetry of the solution.

TI-89 Calculator Solution

With the TI-89 calculator, the top part of the determinant is determined by the sequence in Fig. 8.68 (take note of the calculations within parentheses):

det([6.67,−1/4,−1/2;0,(1/4+1/2+1/5),−1/5;0,−1/5,(1/5+1/2+1/1)]) (ENTER) 10.51E0

FIG. 8.68
TI-89 solution for the numerator of the solution for V_1.

with the bottom of the determinant determined by the sequence in Fig. 8.69.

det([(1/3+1/4+1/2),−1/4,−1/2;−1/4,(1/4+1/2+1/5),−1/5;−1/2,−1/5,(1/5+1/2+1/1)]) (ENTER) 1.31E0

FIG. 8.69
TI-89 solution for the denominator of the equation for V_1.

Finally, the simple division in Fig. 8.70 provides the desired result.

10.51/1.31 (ENTER) 8.02

FIG. 8.70
TI-89 solution for V_1.

and $$V_1 = \mathbf{8.02\ V}$$

Similarly, $V_2 = \mathbf{2.67\ V}$ and $V_3 = \mathbf{2.67\ V}$

and the voltage across the 5 Ω resistor is

$$V_{5\Omega} = V_2 - V_3 = 2.67\text{ A} - 2.67\text{ A} = \mathbf{0\ V}$$

Since $V_{5\Omega} = 0$ V, we can insert a short in place of the bridge arm without affecting the network behavior. (Certainly $V = IR = I \cdot (0) = 0$ V.) In Fig. 8.71, a short circuit has replaced the resistor R_5, and the voltage across R_4 is to be determined. The network is redrawn in Fig. 8.72, and

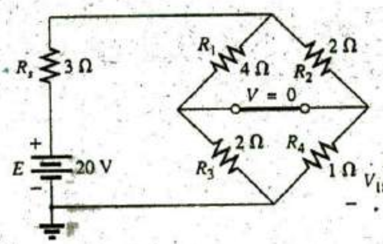


FIG. 8.71
Substituting the short-circuit equivalent for the balance arm of a balanced bridge.

$$V_{1\Omega} = \frac{(2\ \Omega \parallel 1\ \Omega)20\text{ V}}{(2\ \Omega \parallel 1\ \Omega) + (4\ \Omega \parallel 2\ \Omega) + 3\ \Omega} \quad \text{(voltage divider rule)}$$

$$= \frac{\frac{2}{3}(20\text{ V})}{\frac{2}{3} + \frac{8}{6} + 3} = \frac{\frac{2}{3}(20\text{ V})}{\frac{2}{3} + \frac{4}{3} + \frac{9}{3}}$$

$$= \frac{2(20\text{ V})}{2 + 4 + 9} = \frac{40\text{ V}}{15} = \mathbf{2.67\ V}$$

as obtained earlier.

We found through mesh analysis that $I_{5\Omega} = 0$ A, which has as its equivalent an open circuit as shown in Fig. 8.73(a). (Certainly $I = V/R =$

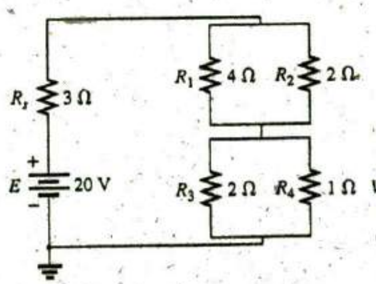


FIG. 8.72
Redrawing the network in Fig. 8.71.

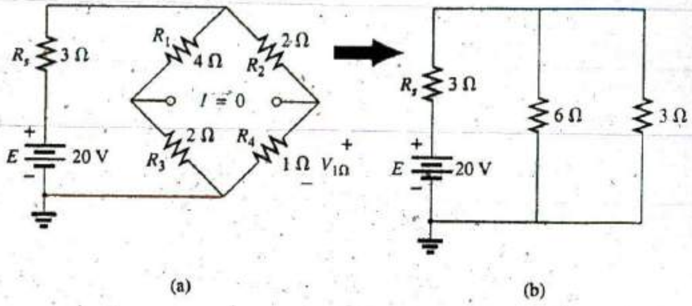


FIG. 8.73
Substituting the open-circuit equivalent for the balance arm of a balanced bridge.

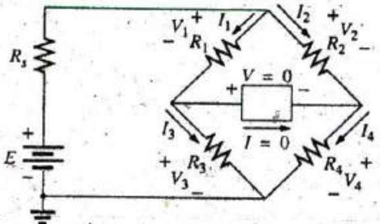


FIG. 8.74
Establishing the balance criteria for a bridge network.

$0/(\infty\ \Omega) = 0$ A.) The voltage across the resistor R_4 is again determined and compared with the result above.

The network is redrawn after combining series elements as shown in Fig. 8.73(b), and

$$V_{3\Omega} = \frac{(6\ \Omega \parallel 3\ \Omega)(20\ \text{V})}{6\ \Omega \parallel 3\ \Omega + 3\ \Omega} = \frac{2\ \Omega(20\ \text{V})}{2\ \Omega + 3\ \Omega} = 8\ \text{V}$$

and

$$V_{1\Omega} = \frac{1\ \Omega(8\ \text{V})}{1\ \Omega + 2\ \Omega} = \frac{8\ \text{V}}{3} = \mathbf{2.67\ V}$$

as above.

The condition $V_{5\Omega} = 0$ V or $I_{5\Omega} = 0$ A exists only for a particular relationship between the resistors of the network. Let us now derive this relationship using the network in Fig. 8.74, in which it is indicated that $I = 0$ A and $V = 0$ V. Note that resistor R_s of the network in Fig. 8.73 does not appear in the following analysis.

The bridge network is said to be *balanced* when the condition of $I = 0$ A or $V = 0$ V exists.

If $V = 0$ V (short circuit between a and b), then

$$V_1 = V_2$$

and

$$I_1R_1 = I_2R_2$$

or

$$I_1 = \frac{I_2R_2}{R_1}$$

In addition, when $V = 0$ V,

$$V_3 = V_4$$

and

$$I_3R_3 = I_4R_4$$

If we set $I = 0$ A, then $I_3 = I_1$ and $I_4 = I_2$, with the result that the above equation becomes

$$I_1R_3 = I_2R_4$$

Substituting for I_1 from above yields

$$\left(\frac{I_2R_2}{R_1}\right)R_3 = I_2R_4$$

or, rearranging, we have

$$\boxed{\frac{R_1}{R_3} = \frac{R_2}{R_4}} \qquad (8.2)$$

This conclusion states that if the ratio of R_1 to R_3 is equal to that of R_2 to R_4, the bridge is balanced, and $I = 0$ A or $V = 0$ V. A method of memorizing this form is indicated in Fig. 8.75.

For the example above, $R_1 = 4\ \Omega$, $R_2 = 2\ \Omega$, $R_3 = 2\ \Omega$, $R_4 = 1\ \Omega$, and

$$\frac{R_1}{R_3} = \frac{R_2}{R_4} \rightarrow \frac{4\ \Omega}{2\ \Omega} = \frac{2\ \Omega}{1\ \Omega} = 2$$

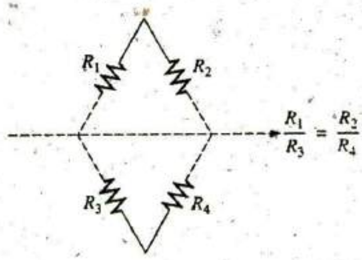


FIG. 8.75
A visual approach to remembering the balance condition.

The emphasis in this section has been on the balanced situation. Understand that if the ratio is not satisfied, there will be a potential drop across the balance arm and a current through it. The methods just described (mesh and nodal analysis) will yield any and all potentials or currents desired, just as they did for the balanced situation.

8.12 Y-Δ (T-π) AND Δ-Y (π-T) CONVERSIONS

Circuit configurations are often encountered in which the resistors do not appear to be in series or parallel. Under these conditions, it may be necessary to convert the circuit from one form to another to solve for any unknown quantities if mesh or nodal analysis is not applied. Two circuit configurations that often account for these difficulties are the **wye (Y)** and **delta (Δ) configurations** depicted in Fig. 8.76(a). They are also referred to as the **tee (T)** and **pi (π)**, respectively, as indicated in Fig. 8.76(b). Note that the pi is actually an inverted delta.

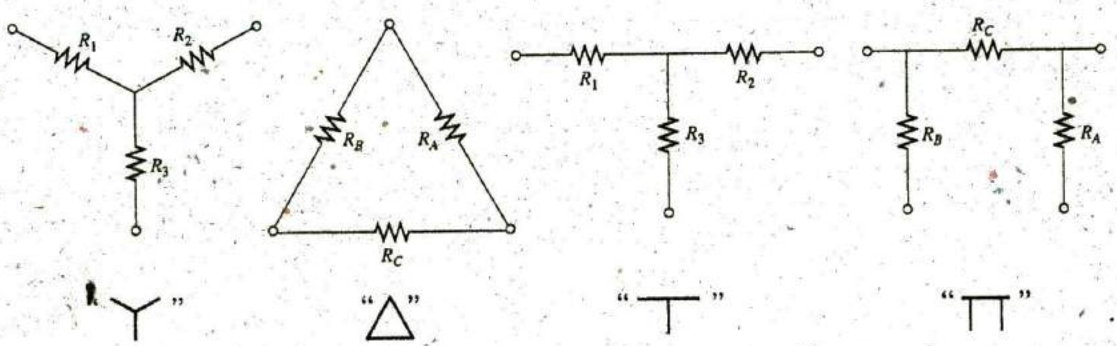


FIG. 8.76
The Y (T) and Δ (π) configurations.

The purpose of this section is to develop the equations for converting from Δ to Y, or vice versa. This type of conversion normally leads to a network that can be solved using techniques such as those described in Chapter 7. In other words, in Fig. 8.77, with terminals *a*, *b*, and *c* held fast, if the wye (Y) configuration were desired *instead of* the inverted delta (Δ) configuration, all that would be necessary is a direct application of the equations to be derived. The phrase *instead of* is emphasized to ensure that it is understood that only one of these configurations is to appear at one time between the indicated terminals.

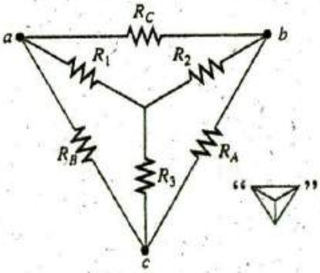


FIG. 8.77
Introducing the concept of Δ-Y or Y-Δ conversions.

It is our purpose (referring to Fig. 8.77) to find some expression for R_1, R_2, and R_3 in terms of R_A, R_B, and R_C, and vice versa, that will ensure that the resistance between any two terminals of the Y configuration will be the same with the Δ configuration inserted in place of the Y configuration (and vice versa). If the two circuits are to be equivalent, the total resistance between any two terminals must be the same. Consider terminals *a-c* in the Δ-Y configurations in Fig. 8.78.

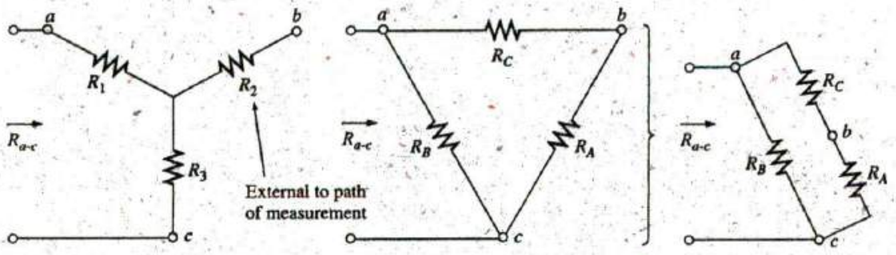


FIG. 8.78
Finding the resistance $R_{a\text{-}c}$ for the Y and Δ configurations.

Let us first assume that we want to convert the Δ (R_A, R_B, R_C) to the Y (R_1, R_2, R_3). This requires that we have a relationship for R_1, R_2, and R_3 in terms of R_A, R_B, and R_C. If the resistance is to be the same between terminals *a-c* for both the Δ and the Y, the following must be true:

$$R_{a\text{-}c}(\text{Y}) = R_{a\text{-}c}(\Delta)$$

so that
$$R_{a\text{-}c} = R_1 + R_3 = \frac{R_B(R_A + R_C)}{R_B + (R_A + R_C)} \quad \textbf{(8.3a)}$$

Using the same approach for *a-b* and *b-c*, we obtain the following relationships:

$$R_{a\text{-}b} = R_1 + R_2 = \frac{R_C(R_A + R_B)}{R_C + (R_A + R_B)} \quad \textbf{(8.3b)}$$

and
$$R_{b\text{-}c} = R_2 + R_3 = \frac{R_A(R_B + R_C)}{R_A + (R_B + R_C)} \quad \textbf{(8.3c)}$$

Subtracting Eq. (8.3a) from Eq. (8.3b), we have

$$(R_1 + R_2) - (R_1 + R_3) = \left(\frac{R_CR_B + R_CR_A}{R_A + R_B + R_C}\right) - \left(\frac{R_BR_A + R_BR_A}{R_A + R_B + R_C}\right)$$

so that
$$R_2 - R_3 = \frac{R_AR_C - R_BR_A}{R_A + R_B + R_C} \quad \textbf{(8.4)}$$

Subtracting Eq. (8.4) from Eq. (8.3c) yields

$$(R_2 + R_3) - (R_2 - R_3) = \left(\frac{R_AR_B + R_AR_C}{R_A + R_B + R_C}\right) - \left(\frac{R_AR_C - R_BR_A}{R_A + R_B + R_C}\right)$$

so that
$$2R_3 = \frac{2R_BR_A}{R_A + R_B + R_C}$$

resulting in the following expression for R_3 in terms of R_A, R_B, and R_C:

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} \tag{8.5a}$$

Following the same procedure for R_1 and R_2, we have

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} \tag{8.5b}$$

and

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} \tag{8.5c}$$

Note that each resistor of the Y is equal to the product of the resistors in the two closest branches of the Δ divided by the sum of the resistors in the Δ.

To obtain the relationships necessary to convert from a Y to a Δ, first divide Eq. (8.5a) by Eq. (8.5b):

$$\frac{R_3}{R_1} = \frac{(R_A R_B)/(R_A + R_B + R_C)}{(R_B R_C)/(R_A + R_B + R_C)} = \frac{R_A}{R_C}$$

or

$$R_A = \frac{R_C R_3}{R_1}$$

Then divide Eq. (8.5a) by Eq. (8.5c):

$$\frac{R_3}{R_2} = \frac{(R_A R_B)/(R_A + R_B + R_C)}{(R_A R_C)/(R_A + R_B + R_C)} = \frac{R_B}{R_C}$$

or

$$R_B = \frac{R_3 R_C}{R_2}$$

Substituting for R_A and R_B in Eq. (8.5c) yields

$$R_2 = \frac{(R_C R_3/R_1)R_C}{(R_3 R_C/R_2) + (R_C R_3/R_1) + R_C}$$

$$= \frac{(R_3/R_1)R_C}{(R_3/R_2) + (R_3/R_1) + 1}$$

Placing these over a common denominator, we obtain

$$R_2 = \frac{(R_3 R_C/R_1)}{(R_1 R_2 + R_1 R_3 + R_2 R_3)/(R_1 R_2)}$$

$$= \frac{R_2 R_3 R_C}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

and

$$R_C = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_3} \tag{8.6a}$$

We follow the same procedure for R_B and R_A:

$$R_A = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1} \tag{8.6b}$$

and $$R_B = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2} \quad (8.6c)$$

Note that the value of each resistor of the Δ is equal to the sum of the possible product combinations of the resistances of the Y divided by the resistance of the Y farthest from the resistor to be determined.

Let us consider what would occur if all the values of a Δ or Y were the same. If $R_A = R_B = R_C$, Eq. (8.5a) would become (using R_A only) the following:

$$R_3 = \frac{R_AR_B}{R_A + R_B + R_C} = \frac{R_AR_A}{R_A + R_A + R_A} = \frac{R_A^2}{3R_A} = \frac{R_A}{3}$$

and, following the same procedure,

$$R_1 = \frac{R_A}{3} \quad R_2 = \frac{R_A}{3}$$

In general, therefore,

$$R_Y = \frac{R_\Delta}{3} \quad (8.7)$$

or

$$R_\Delta = 3R_Y \quad (8.8)$$

which indicates that *for a Y of three equal resistors, the value of each resistor of the Δ is equal to three times the value of any resistor of the Y.* If only two elements of a Y or a Δ are the same, the corresponding Δ or Y of each will also have two equal elements. The converting of equations is left as an exercise for you.

The Y and the Δ often appear as shown in Fig. 8.79. They are then referred to as a **tee (T)** and a **pi** (π) network, respectively. The equations used to convert from one form to the other are exactly the same as those developed for the Y and Δ transformation.

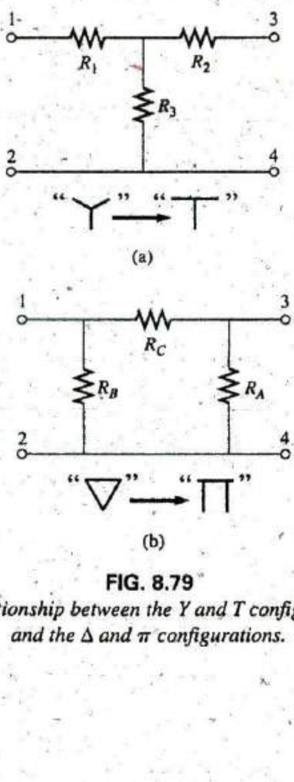


FIG. 8.79
The relationship between the Y and T configurations and the Δ and π configurations.

EXAMPLE 8.27 Convert the Δ in Fig. 8.80 to a Y.

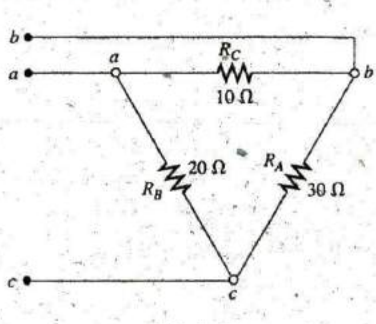


FIG. 8.80
Example 8.27.

Solution:

$$R_1 = \frac{R_BR_C}{R_A + R_B + R_C} = \frac{(20\ \Omega)(10\ \Omega)}{30\ \Omega + 20\ \Omega + 10\ \Omega} = \frac{200\ \Omega}{60} = 3\tfrac{1}{3}\ \Omega$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(30\ \Omega)(10\ \Omega)}{60\ \Omega} = \frac{300\ \Omega}{60} = \mathbf{5\ \Omega}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(20\ \Omega)(30\ \Omega)}{60\ \Omega} = \frac{600\ \Omega}{60} = \mathbf{10\ \Omega}$$

The equivalent network is shown in Fig. 8.81.

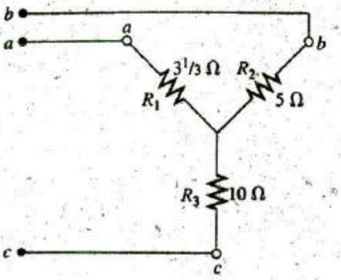


FIG. 8.81
The Y equivalent for the Δ in Fig. 8.80.

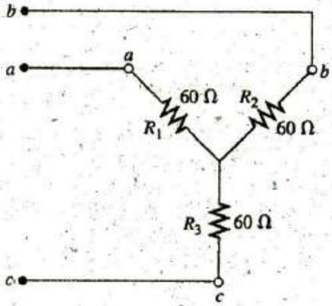


FIG. 8.82
Example 8.28.

EXAMPLE 8.28 Convert the Y in Fig. 8.82 to a Δ.

Solution:

$$R_A = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1}$$
$$= \frac{(60\ \Omega)(60\ \Omega) + (60\ \Omega)(60\ \Omega) + (60\ \Omega)(60\ \Omega)}{60\ \Omega}$$
$$= \frac{3600\ \Omega + 3600\ \Omega + 3600\ \Omega}{60} = \frac{10{,}800\ \Omega}{60}$$
$$R_A = \mathbf{180\ \Omega}$$

However, the three resistors for the Y are equal, permitting the use of Eq. (8.8) and yielding

$$R_\Delta = 3R_Y = 3(60\ \Omega) = 180\ \Omega$$

and

$$R_B = R_C = \mathbf{180\ \Omega}$$

The equivalent network is shown in Fig. 8.83.

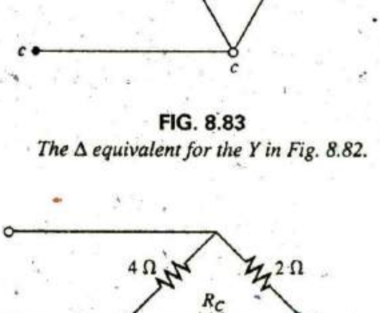


FIG. 8.83
The Δ equivalent for the Y in Fig. 8.82.

EXAMPLE 8.29 Find the total resistance of the network in Fig. 8.84, where $R_A = 3\ \Omega$, $R_B = 3\ \Omega$, and $R_C = 6\ \Omega$.

Solution:

Two resistors of the Δ were equal; therefore, two resistors of the Y will be equal.

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 3\ \Omega + 6\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.5\ \Omega}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{12\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.5\ \Omega}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(3\ \Omega)(3\ \Omega)}{12\ \Omega} = \frac{9\ \Omega}{12} = \mathbf{0.75\ \Omega}$$

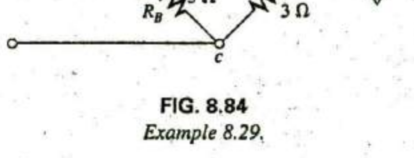


FIG. 8.84
Example 8.29.

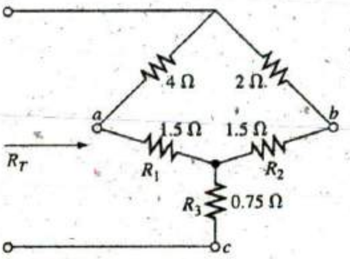


FIG. 8.85
Substituting the Y equivalent for the bottom Δ in Fig. 8.84.

Replacing the Δ by the Y, as shown in Fig. 8.85, yields

$$R_T = 0.75\ \Omega + \frac{(4\ \Omega + 1.5\ \Omega)(2\ \Omega + 1.5\ \Omega)}{(4\ \Omega + 1.5\ \Omega) + (2\ \Omega + 1.5\ \Omega)}$$
$$= 0.75\ \Omega + \frac{(5.5\ \Omega)(3.5\ \Omega)}{5.5\ \Omega + 3.5\ \Omega}$$
$$= 0.75\ \Omega + 2.139\ \Omega$$
$$R_T = \mathbf{2.89\ \Omega}$$

EXAMPLE 8.30 Find the total resistance of the network in Fig. 8.86.

Solutions: Since all the resistors of the Δ or Y are the same, Eqs. (8.7) and (8.8) can be used to convert either form to the other.

a. *Converting the Δ to a Y:* Note: When this is done, the resulting d' of the new Y will be the same as the point d shown in the original figure, only because both systems are "balanced." That is, the resistance in each branch of each system has the same value:

$$R_Y = \frac{R_\Delta}{3} = \frac{6\ \Omega}{3} = 2\ \Omega \qquad \text{(Fig. 8.87)}$$

The network then appears as shown in Fig. 8.88. We have

$$R_T = 2\left[\frac{(2\ \Omega)(9\ \Omega)}{2\ \Omega + 9\ \Omega}\right] = \mathbf{3.27\ \Omega}$$

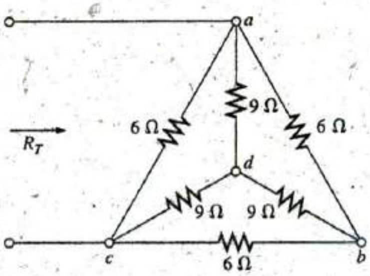


FIG. 8.86
Example 8.30.

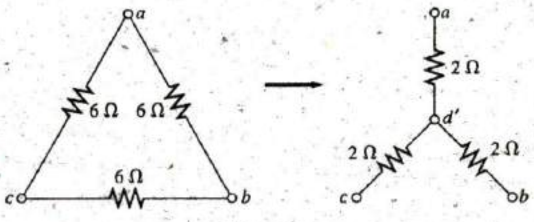


FIG. 8.87
Converting the Δ configuration of Fig. 8.86 to a Y configuration.

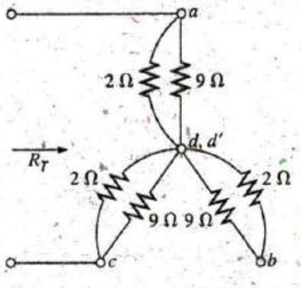


FIG. 8.88
Substituting the Y configuration for the converted Δ into the network in Fig. 8.86.

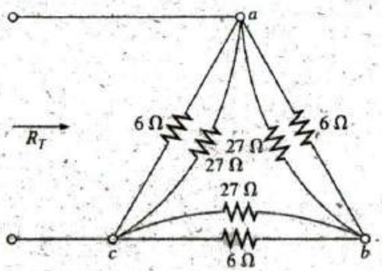


FIG. 8.89
Substituting the converted Y configuration into the network in Fig. 8.86.

b. *Converting the Y to a Δ:*

$$R_\Delta = 3R_Y = (3)(9\ \Omega) = 27\ \Omega \qquad \text{(Fig. 8.89)}$$
$$R'_T = \frac{(6\ \Omega)(27\ \Omega)}{6\ \Omega + 27\ \Omega} = \frac{162\ \Omega}{33} = 4.91\ \Omega$$
$$R_T = \frac{R'_T(R'_T + R'_T)}{R'_T + (R'_T + R'_T)} = \frac{R'_T 2R'_T}{3R'_T} = \frac{2R'_T}{3}$$
$$= \frac{2(4.91\ \Omega)}{3} = \mathbf{3.27\ \Omega}$$

which checks with the previous solution.

8.13 APPLICATIONS

This section discusses the constant-current characteristic in the design of security systems, the bridge circuit in a common residential smoke detector, and the nodal voltages of a digital logic probe.

Constant-Current Alarm Systems

The basic components of an alarm system using a constant-current supply are provided in Fig. 8.90. This design is improved over that provided in Chapter 5 in the sense that it is less sensitive to changes in resistance in the circuit due to heating, humidity, changes in the length of the line to the sensors, and so on. The 1.5 kΩ rheostat (total resistance between points *a* and *b*) is adjusted to ensure a current of 5 mA through the single-series security circuit. The adjustable rheostat is necessary to compensate for variations in the total resistance of the circuit introduced by the resistance of the wire, sensors, sensing relay, and milliammeter. The milliammeter is included to set the rheostat and ensure a current of 5 mA.

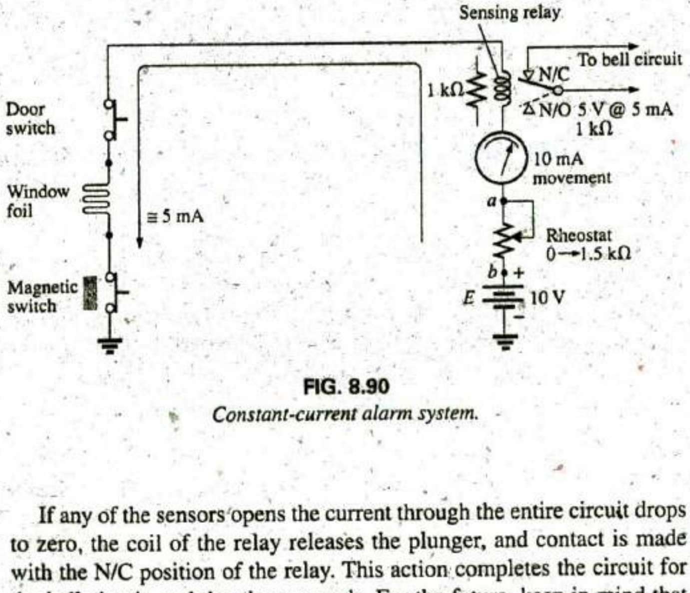


FIG. 8.90
Constant-current alarm system.

If any of the sensors opens the current through the entire circuit drops to zero, the coil of the relay releases the plunger, and contact is made with the N/C position of the relay. This action completes the circuit for the bell circuit, and the alarm sounds. For the future, keep in mind that switch positions for a relay are always shown with no power to the network, resulting in the N/C position in Fig. 8.90. When power is applied, the switch will have the position indicated by the dashed line. That is, various factors, such as a change in resistance of any of the elements due to heating, humidity, and so on, cause the applied voltage to redistribute itself and create a sensitive situation. With an adjusted 5 mA, the loading can change, but the current will always be 5 mA and the chance of tripping reduced. Note that the relay is rated as 5 V at 5 mA, indicating that in the on state the voltage across the relay is 5 V and the current through the relay is 5 mA. Its internal resistance is therefore 5 V/5 mA = 1 kΩ in this state.

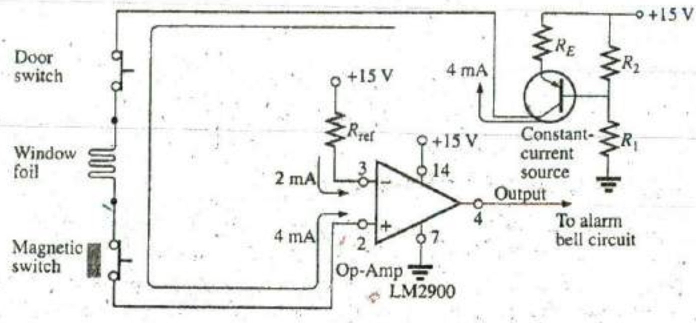


FIG. 8.91
Constant-current alarm system with electronic components.

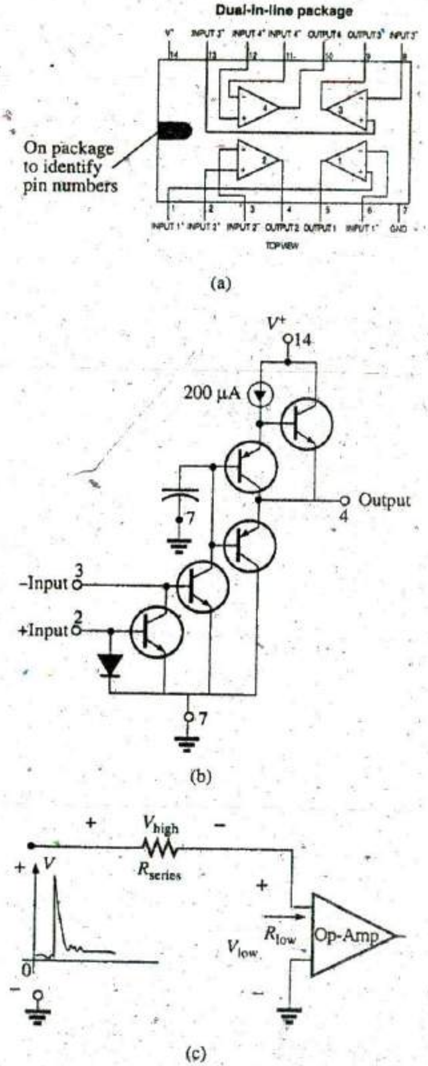


FIG. 8.92
LM2900 operational amplifier: (a) dual-in-line package (DIP); (b) components; (c) impact of low-input impedance.

A more advanced alarm system using a constant current is illustrated in Fig. 8.91. In this case, an electronic system using a single transistor, biasing resistors, and a dc battery are establishing a current of 4 mA through the series sensor circuit connected to the positive side of an operational amplifier (op-amp). Transistors and op-amp devices may be new to you (these are discussed in detail in electronics courses), but for now you just need to know that the transistor in this application is being used not as an amplifier but as part of a design to establish a constant current through the circuit. The op-amp is a very useful component of numerous electronic systems, and it has important terminal characteristics established by a variety of components internal to its design. The LM2900 operational amplifier in Fig. 8.91 is one of four found in the dual-in-line integrated circuit package appearing in Fig. 8.92(a). Pins 2, 3, 4, 7, and 14 were used for the design in Fig. 8.91. Note in Fig. 8.92(b) the number of elements required to establish the desired terminal characteristics—the details of which will be investigated in your electronics courses.

In Fig. 8.91, the designed 15 V dc supply, biasing resistors, and transistor in the upper right corner of the schematic establish a constant 4 mA current through the circuit. It is referred to as a *constant-current source* because the current remains fairly constant at 4 mA even though there may be moderate variations in the total resistance of the series sensor circuit connected to the transistor. Following the 4 mA through the circuit, we find that it enters terminal 2 (positive side of the input) of the op-amp. A second current of 2 mA, called the *reference current,* is established by the 15 V source and resistance R and enters terminal 3 (negative side of the input) of the op-amp. The reference current of 2 mA is necessary to establish a current for the 4 mA current of the network to be compared against. As long as the 4 mA current exists, the operational amplifier provides a "high" output voltage that exceeds 13.5 V, with a typical level of 14.2 V (according to the specification sheet for the op-amp). However, if the sensor current drops from 4 mA to a level below the reference level of 2 mA, the op-amp responds with a "low" output voltage that is typically about 0.1 V. The output of the operational amplifier then signals the alarm circuit about the disturbance. Note from the above that it is not necessary for the sensor current to drop to 0 mA to signal the alarm circuit—just a variation around the reference level that appears unusual.

One very important characteristic of this particular op-amp is that the input impedance to the op-amp is relatively low. This feature is important because you don't want alarm circuits reacting to every voltage spike or turbulence that comes down the line because of external switching action or outside forces such as lightning. In Fig. 8.92(c), for instance, if a high voltage should appear at the input to the series configuration, most of the voltage would be absorbed by the series resistance of the sensor circuit rather than travel across the input terminals of the operational amplifier—thus preventing a false output and an activation of the alarm.

Wheatstone Bridge Smoke Detector

The Wheatstone bridge is a popular network configuration whenever detection of small changes in a quantity is required. In Fig. 8.93(a), the dc

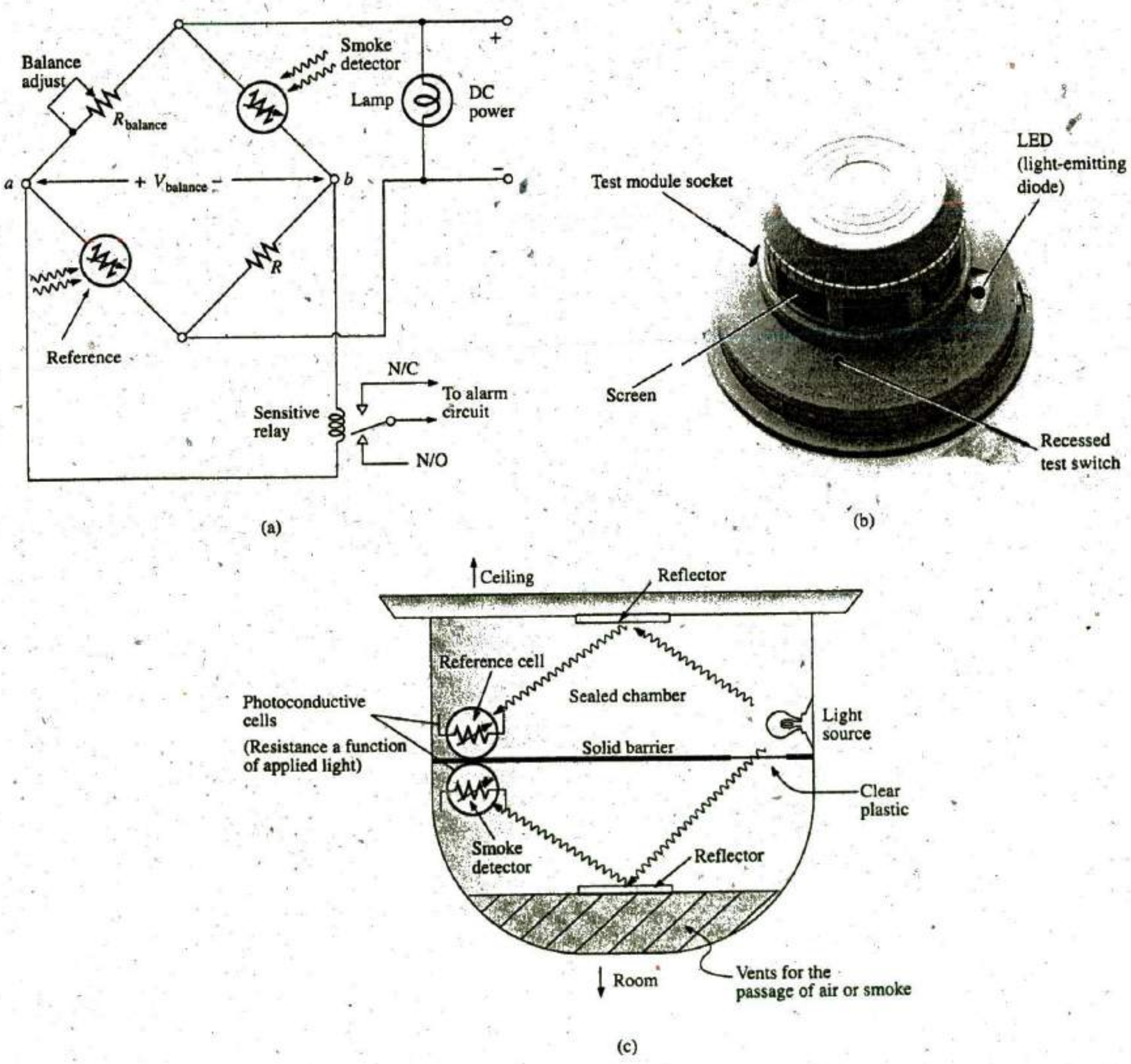


FIG. 8.93

Wheatstone bridge smoke detector: (a) dc bridge configuration; (b) outside appearance; (c) internal construction.

bridge configuration uses a photoelectric device to detect the presence of smoke and to sound the alarm. A photograph of a photoelectric smoke detector appears in Fig. 8.93(b), and the internal construction of the unit is shown in Fig. 8.93(c). First, note that air vents are provided to permit the smoke to enter the chamber below the clear plastic. The clear plastic prevents the smoke from entering the upper chamber but permits the light from the bulb in the upper chamber to bounce off the lower reflector to the semiconductor light sensor (a cadmium photocell) at the left side of the chamber. The clear plastic separation ensures that the light hitting the light sensor in the upper chamber is not affected by the entering smoke. It establishes a reference level to compare against the chamber with the entering smoke. If no smoke is present, the difference in response between the sensor cells will be registered as the normal situation. Of course, if both cells were exactly identical, and if the clear plastic did not cut down on the light, both sensors would establish the same reference level, and their difference would be zero. However, this is seldom the case, so a reference difference is recognized as the sign that smoke is not present. However, once smoke is present, there will be a sharp difference in the sensor reaction from the norm, and the alarm should sound.

In Fig. 8.93(a), we find that the two sensors are located on opposite arms of the bridge. With no smoke present, the balance-adjust rheostat is used to ensure that the voltage V between points a and b is zero volts and the resulting current through the primary of the sensitive relay is zero amperes. Taking a look at the relay, we find that the absence of a voltage from a to b leaves the relay coil unenergized and the switch in the N/O position (recall that the position of a relay switch is always drawn in the unenergized state). An unbalanced situation results in a voltage across the coil and activation of the relay, and the switch moves to the N/C position to complete the alarm circuit and activate the alarm. Relays with two contacts and one movable arm are called *single-pole–double-throw* (SPDT) relays. The dc power is required to set up the balanced situation, energize the parallel bulb so we know that the system is on, and provide the voltage from a to b if an unbalanced situation should develop.

Why do you suppose that only one sensor isn't used, since its resistance would be sensitive to the presence of smoke? The answer is that the smoke detector may generate a false readout if the supply voltage or output light intensity of the bulb should vary. Smoke detectors of the type just described must be used in gas stations, kitchens, dentist offices, and so on, where the range of gas fumes present may set off an ionizing-type smoke detector.

Schematic with Nodal Voltages

When an investigator is presented with a system that is down or not operating properly, one of the first options is to check the system's specified voltages on the schematic. These specified voltage levels are actually the nodal voltages determined in this chapter. *Nodal voltage* is simply a special term for a voltage measured from that point to ground. The technician attaches the negative or lower-potential lead to the ground of the network (often the chassis) and then places the positive or higher-potential lead on the specified points of the network to check the nodal voltages. If they match, it means that section of the system is op-

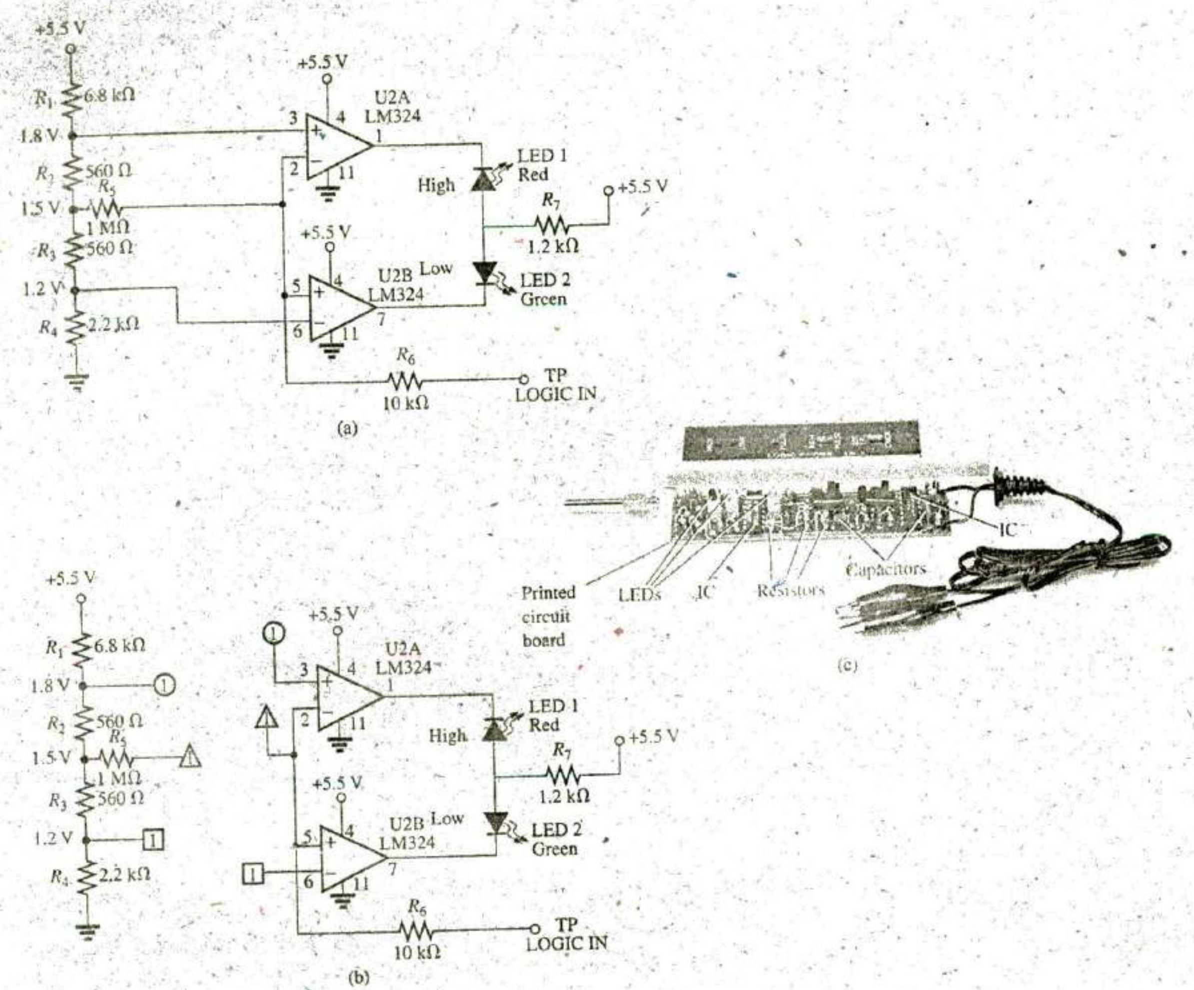


FIG. 8.94

Logic probe: (a) schematic with nodal voltages; (b) network with global connections; (c) photograph of commercially available unit.

erating properly. If one or more fail to match the given values, the problem area can usually be identified. Be aware that a reading of −15.87 V is significantly different from an expected reading of +16 V if the leads have been properly attached. Although the actual numbers seem close, the difference is actually more than 30 V. You must expect some deviation from the given value as shown, but always be very sensitive to the resulting sign of the reading.

The schematic in Fig. 8.94(a) includes the nodal voltages for a logic probe used to measure the input and output states of integrated circuit logic chips. In other words, the probe determines whether the measured voltage is one of two states: high or low (often referred to as "on" or "off" or 1 or 0). If the LOGIC IN terminal of the probe is placed on a chip at a location where the voltage is between 0 and 1.2 V, the voltage is considered to be a low level, and the green LED lights (LEDs are light-emitting semiconductor diodes that emit light when current is passed through them). If the measured voltage is between 1.8 V and 5 V, the reading is considered high, and the red LED lights. Any voltage between 1.2 V and 1.8 V is considered a "floating level" and is an indication that the system being measured is not operating correctly. Note that the reference levels mentioned above are established by the voltage divider network to the left of the schematic. The op-amps used are of such high input impedance that their loading on the voltage divider network can be ignored and the voltage divider network considered a network unto itself. Even though three 5.5 V dc supply voltages are indicated on the diagram, be aware that all three points are connected to the same supply. The other voltages provided (the nodal voltages) are the voltage levels that should be present from that point to ground if the system is working properly.

The op-amps are used to sense the difference between the reference at points 3 and 6 and the voltage picked up in LOGIC IN. Any difference results in an output that lights either the green or the red LED. Be aware, because of the direct connection, that the voltage at point 3 is the same as shown by the nodal voltage to the left, or 1.8 V. Likewise, the voltage at point 6 is 1.2 V for comparison with the voltages at points 5 and 2, which reflect the measured voltage. If the input voltage happened to be 1.0 V, the difference between the voltages at points 5 and 6 would be 0.2 V, which ideally would appear at point 7. This low potential at point 7 would result in a current flowing from the much higher 5.5 V dc supply through the green LED, causing it to light and indicating a low condition. By the way, LEDs, like diodes, permit current through them only in the direction of the arrow in the symbol. Also note that the voltage at point 6 must be higher than that at point 5 for the output to turn on the LED. The same is true for point 2 over point 3, which reveals why the red LED does not light when the 1.0 V level is measured.

Often it is impractical to draw the full network as shown in Fig. 8.94(b) because there are space limitations or because the same voltage divider network is used to supply other parts of the system. In such cases, you should recognize that points having the same shape are connected, and the number in the figure reveals how many connections are made to that point.

A photograph of the outside and inside of a commercially available logic probe is shown in Fig. 8.94(c). Note the increased complexity of system because of the variety of functions that the probe can perform.

8.14 COMPUTER ANALYSIS

PSpice

We will now analyze the bridge network in Fig. 8.67 using PSpice to ensure that it is in the balanced state. The only component that has not been introduced in earlier chapters is the dc current source. To obtain it, first select the **Place a part** key and then the **SOURCE** library. Scrolling the **Part List** results in the option **IDC.** A left click of **IDC** followed by **OK** results in a dc current source whose direction is toward the bottom of the screen. One left click (to make it red, or active) followed by a right click results in a listing having a **Mirror Vertically** option. Selecting that option flips the source and gives it the direction in Fig. 8.67.

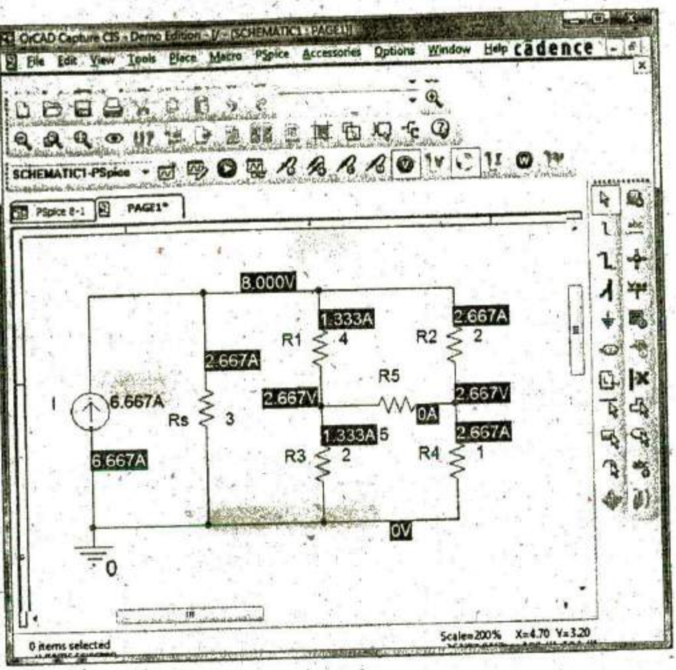


FIG. 8.95
Applying PSpice to the bridge network in Fig. 8.67.

The remaining parts of the PSpice analysis are pretty straightforward, with the results in Fig. 8.95 matching those obtained in the analysis of Fig. 8.67. The voltage across the current source is 8 V positive to ground, and the voltage at either end of the bridge arm is 2.667 V. The voltage across R_5 is obviously 0 V for the level of accuracy displayed, and the current is such a small magnitude compared to the other current levels of the network that it can essentially be considered 0 A. Note also for the balanced bridge that the current through R_1 equals that of R_3, and the current through R_2 equals that of R_4.

Multisim

We will now use Multisim to verify the results in Example 8.18. All the elements of creating the schematic in Fig. 8.96 have been presented in earlier chapters; they are not repeated here to demonstrate how little documentation is now necessary to carry you through a fairly complex network.

For the analysis, both the standard **Multimeter** and meters from the **Show Measurement Family** of the **BASIC toolbar** listing were employed. For the current through the resistor R_5, the **Place Ammeter (Horizontal)** was used, while for the voltage across the resistor R_4, the **Place Voltmeter (Vertical)** was used. The **Multimeter** is reading the voltage across the resistor R_2. In actuality, the ammeter is reading the loop current for the top window, and the voltmeters are showing the nodal voltages of the network.

After simulation, the results displayed are an exact match with those in Example 8.18.

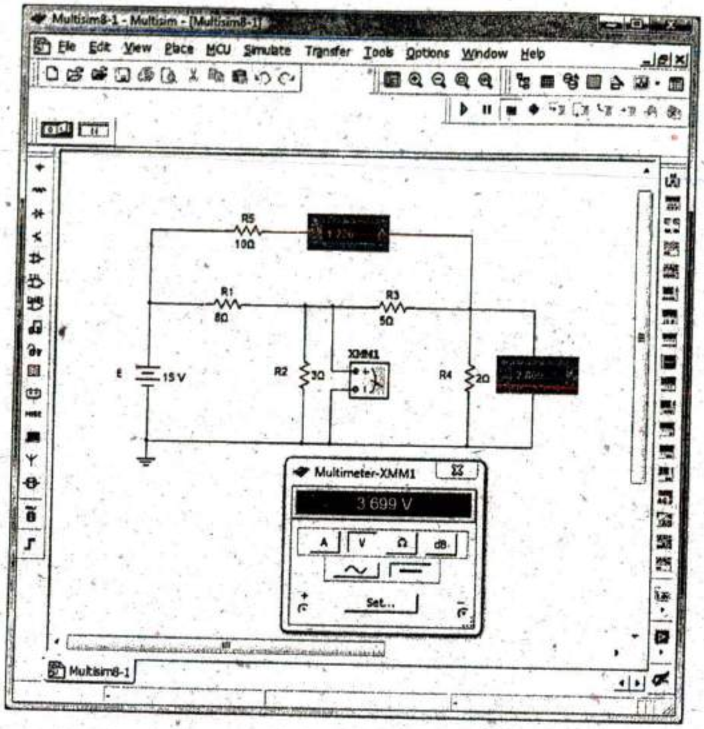


FIG. 8.96
Using Multisim to verify the results in Example 8.18.

PROBLEMS

SECTION 8.2 Current Sources

1. For the network of Fig. 8.97:
   a. Find the currents I_1 and I_2.
   b. Determine the voltage V_s.

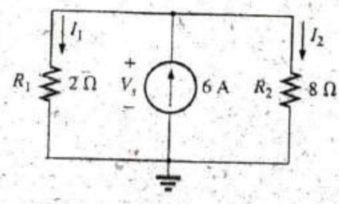


FIG. 8.97
Problem 1.

2. For the network of Fig. 8.98:
   a. Determine the currents I_1 and I_2.
   b. Calculate the voltages V_2 and V_s.

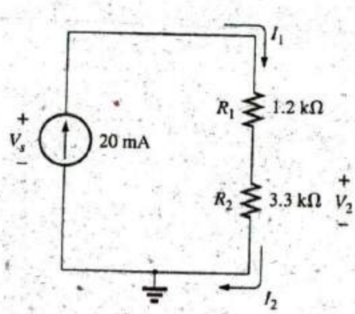


FIG. 8.98
Problem 2.

3. Find voltage V_s (with polarity) across the ideal current source in Fig. 8.99.

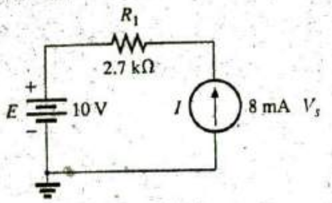


FIG. 8.99
Problem 3.

4. For the network in Fig. 8.100:
   a. Find voltage V_s.
   b. Calculate current I_2.
   c. Find the source current I_s.

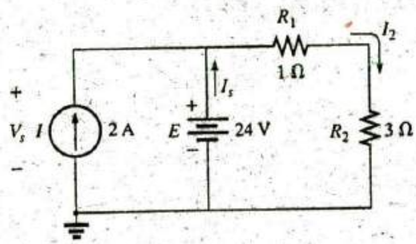


FIG. 8.100
Problem 4.

5. Find the voltage V_3 and the current I_2 for the network in Fig. 8.101.

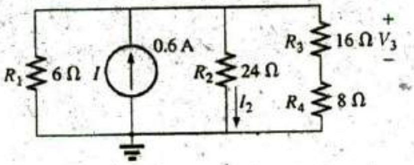


FIG. 8.101
Problem 5.

6. For the network in Fig. 8.102:
   a. Find the currents I_1 and I_s.
   b. Find the voltages V_s and V_3.

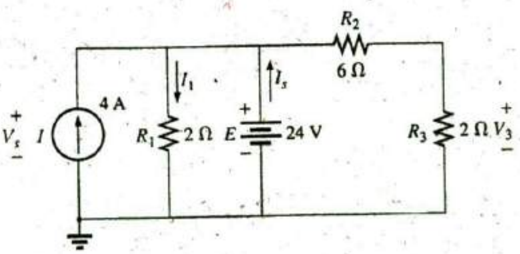


FIG. 8.102
Problem 6.

SECTION 8.3 Source Conversions

7. Convert the voltage sources in Fig. 8.103 to current sources.

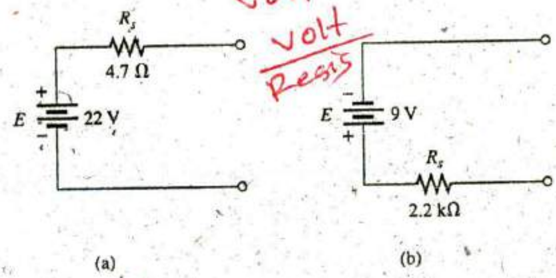


FIG. 8.103
Problem 7.

8. Convert the current sources in Fig. 8.104 to voltage sources.

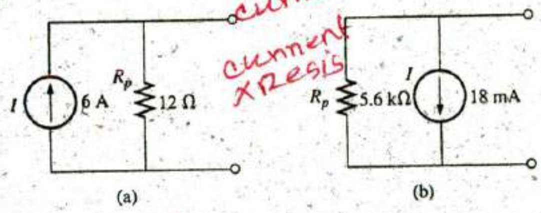


FIG. 8.104
Problem 8.

9. For the network in Fig. 8.105:
   a. Find the current through the 10 Ω resistor. Nothing that the resistance R_L is significantly less than R_p, what was the impact on the current through R_L?
   b. Convert the current source to a voltage source, and recalculate the current through the 10 Ω resistor.

   Did you obtain the same result?

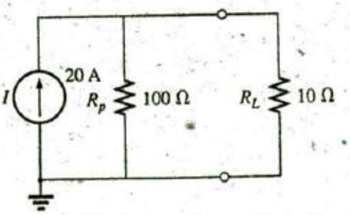


FIG. 8.105
Problem 9.

10. For the configuration of Fig. 8.106:
    a. Convert the current source to a voltage source.
    b. Combine the two series voltage sources into one source.
    c. Calculate the current through the 91 Ω resistor.

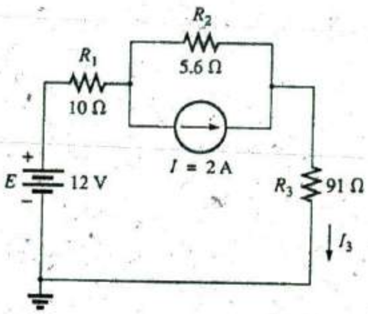


FIG. 8.106
Problem 10.

SECTION 8.4 Current Sources in Parallel

11. For the network in Fig. 8.107:
    a. Replace all the current sources by a single current source.
    b. Find the source voltage V_s.

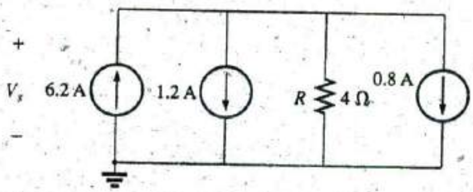


FIG. 8.107
Problem 11.

12. Find the voltage V_s and the current I_1 for the network in Fig. 8.108.

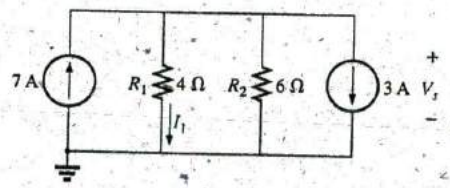


FIG. 8.108
Problem 12.

13. Convert the voltage sources in Fig. 8.109 to current sources.
    a. Find the voltage V_{ab} and the polarity of points a and b.
    b. Find the magnitude and direction of the current I_3.

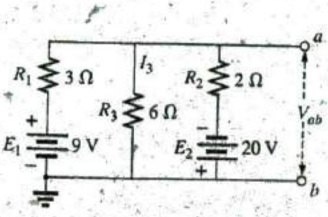


FIG. 8.109
Problems 13 and 37.

14. For the network in Fig. 8.110:
    a. Convert the voltage source to a current source.
    b. Reduce the network to a single current source, and determine the voltage V_1.
    c. Using the results of part (b), determine V_2.
    d. Calculate the current I_2.

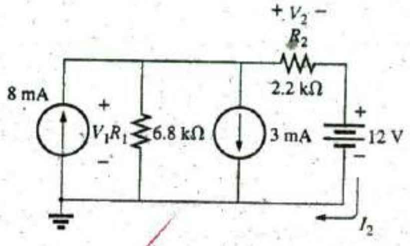


FIG. 8.110
Problem 14.

SECTION 8.6 Branch-Current Analysis

15. a. Using branch-current analysis, find the magnitude and direction of the current through each resistor for the network of Fig. 8.111.
    b. Find the voltage V_a.

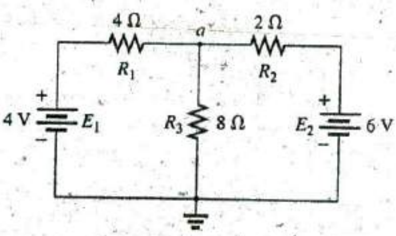


FIG. 8.111
Problems 15, 20, 32, and 70.

16. For the network of Fig. 8.112:
    a. Determine the current through the 12 Ω resistor using branch-current analysis.
    b. Convert the two voltage sources to current sources, and then determine the current through the 12 Ω resistor.
    c. Compare the results of parts (a) and (b).

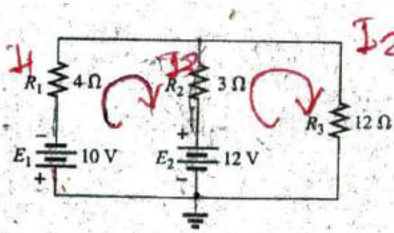


FIG. 8.112
Problems 16, 21, and 33.

*17. Using branch-current analysis, find the current through each resistor for the network of Fig. 8.113. The resistors are all standard values.

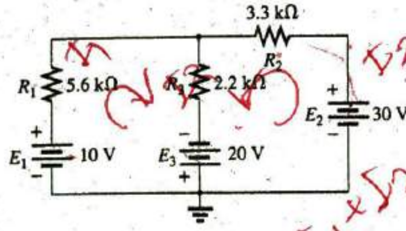


FIG. 8.113
Problems 17, 22, and 34.

*18. a. Using branch-current analysis, find the current through the 9.1 kΩ resistor in Fig. 8.114. Note that all the resistors are standard values.
b. Using the results of part (a), determine the voltage V_a.

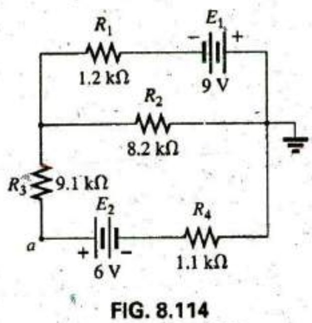


FIG. 8.114
Problems 18 and 23.

*19. For the network in Fig. 8.115:
a. Write the equations necessary to solve for the branch currents.
b. By substitution of Kirchhoff's current law, reduce the set to three equations.
c. Rewrite the equations in a format that can be solved using third-order determinants.
d. Solve for the branch current through the resistor R_3.

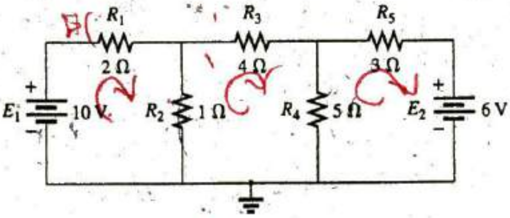


FIG. 8.115
Problems 19, 24, and 35.

SECTION 8.7 Mesh Analysis (General Approach)

20. a. Using the general approach to mesh analysis, determine the current through each resistor of Fig. 8.111.
b. Using the results of part (a), find the voltage V_a.

21. a. Using the general approach to mesh analysis, determine the current through each voltage source in Fig. 8.112.
b. Using the results of part (a), find the power delivered by the source E_2 and to the resistor R_3.

22. a. Using the general approach to mesh analysis, determine the current through each resistor of Fig. 8.113.
b. Using the results of part (a), determine the voltage across the 3.3 kΩ resistor.

23. a. Using the general approach to mesh analysis, determine the current through each resistor of Fig. 8.114.
b. Using the results of part (a), find the voltage V_a.

*24. a. Determine the mesh currents for the network of Fig. 8.115 using the general approach.
b. Through the proper use of Kirchhoff's current law, reduce the resulting set of equations to three.
c. Use determinants to find the three mesh currents.
d. Determine the current through each source, using the results of part (c).

*25. a. Write the mesh equations for the network of Fig. 8.116 using the general approach.
b. Using determinants, calculate the mesh currents.
c. Using the results of part (b), find the current through each source.

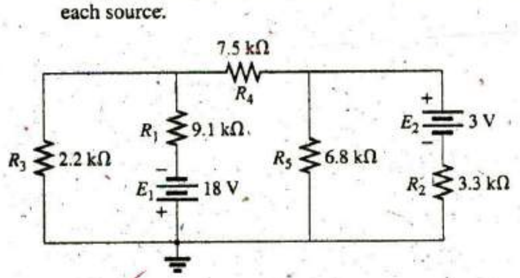


FIG. 8.116
Problems 25 and 36.

*26. a. Write the mesh equations for the network of Fig. 8.117 using the general approach.
b. Using determinants, calculate the mesh currents.
c. Using the results of part (b), calculate the current through the resistor R_5.

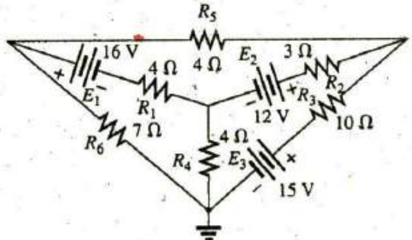


FIG. 8.117
Problem 26.

*27. a. Write the mesh currents for the network of Fig. 8.118 using the general approach.
b. Using determinants, calculate the mesh currents.
c. Using the results of part (b), find the power delivered by the 6 V source.

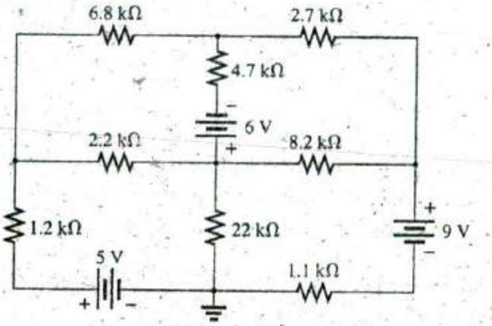


FIG. 8.118
Problems 27, 38, and 71.

***28. a.** Redraw the network of Fig. 8.119 in a manner that will remove the crossover.

b. Write the mesh equations for the network using the general approach.

c. Calculate the mesh currents for the network.

d. Find the total power delivered by the two sources.

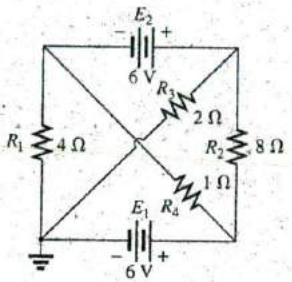


FIG. 8.119
Problem 28.

***29.** For the transistor configuration in Fig. 8.120:

a. Solve for the currents I_B, I_C, and I_E, using the fact that $V_{BE} = 0.7$ V and $V_{CE} = 8$ V.

b. Find the voltages V_B, V_C, and V_E with respect to ground.

c. What is the ratio of output current I_C to input current I_B? [*Note:* In transistor analysis, this ratio is referred to as the *dc beta* of the transistor (β_{dc}).]

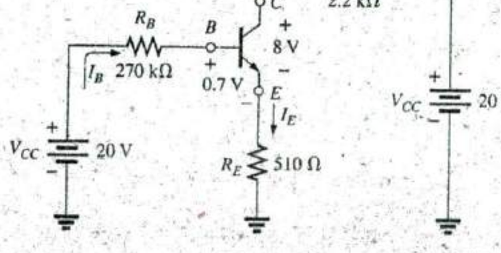


FIG. 8.120
Problem 29.

***30.** Using the supermesh approach, find the current through each element of the network of Fig. 8.121.

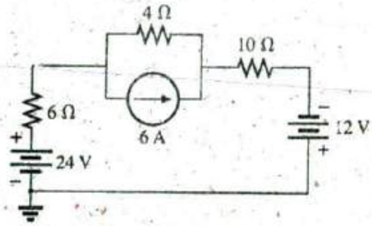


FIG. 8.121
Problem 30.

***31.** Using the supermesh approach, find the current through each element of the network of Fig. 8.122.

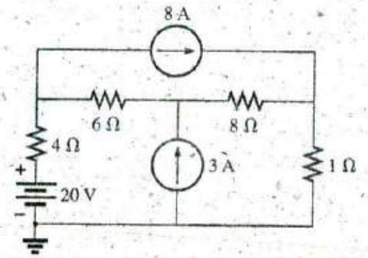


FIG. 8.122
Problem 31.

SECTION 8.8 Mesh Analysis (Format Approach)

32. a. Using the format approach to mesh analysis, write the mesh equations for the network of Fig. 8.111.

b. Solve for the current through the 8 Ω resistor.

33. a. Using the format approach to mesh analysis, write the mesh equations for the network of Fig. 8.112.

b. Solve for the current through the 3 Ω resistor.

34. a. Using the format approach to mesh analysis, write the mesh equations for the network of Fig. 8.113 with three independent sources.

b. Find the current through each source of the network.

***35. a.** Write the mesh equations for the network of Fig. 8.115 using the format approach to mesh analysis.

b. Solve for the three mesh currents, using determinants.

c. Determine the current through the 1 Ω resistor.

***36. a.** Write the mesh equations for the network of Fig. 8.116 using the format approach to mesh analysis.

b. Solve for the three mesh currents, using determinants.

c. Find the current through each source of the network.

37. a. Write the mesh equations for the network of Fig. 8.109 using the format approach.

b. Find the voltage V_{ab} using the result of part (a).

*38. a. Write the mesh equations for the network of Fig. 8.18 using the format approach to mesh analysis.
b. Solve for the four mesh currents using determinants.
c. Find the voltage at the common connection at the center of the diagram.

*39. a. Write the mesh equations for the network of Fig. 8.123 using the format approach to mesh analysis.
b. Use determinants to determine the mesh currents.
c. Find the voltages V_a and V_b.
d. Determine the voltage V_{ab}.

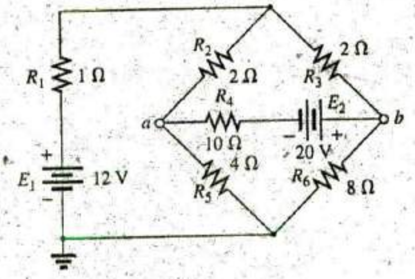


FIG. 8.123
Problems 39 and 56.

SECTION 8.9 Nodal Analysis (General Approach)

40. a. Write the nodal equations using the general approach for the network of Fig. 8.124.
b. Find the nodal voltages using determinants.
c. Use the results of part (a) to find the voltage across the 8 Ω resistor.
d. Use the results of part (a) to find the current through the 2 Ω and 4 Ω resistors.

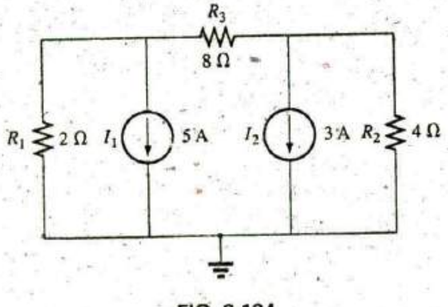


FIG. 8.124
Problems 40 and 51.

41. a. Write the nodal equations using the general approach for the network of Fig. 8.125.
b. Find the nodal voltages using determinants.
c. Using the results of part (a), calculate the current through the 20 Ω resistor.

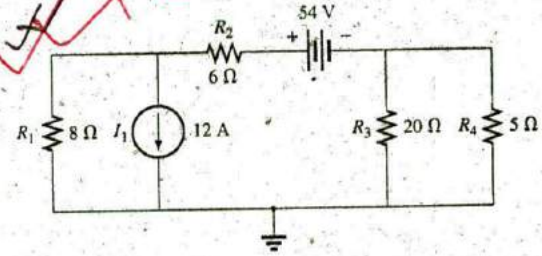


FIG. 8.125
Problems 41 and 52.

42. a. Write the nodal equations using the general approach for the network of Fig. 8.126.
b. Find the nodal voltages using determinants.
c. What is the total power supplied by the current sources?

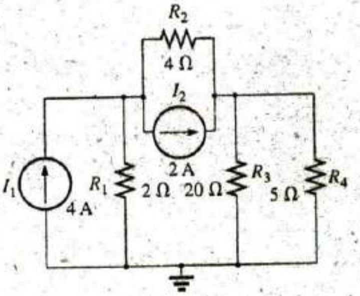


FIG. 8.126
Problem 42.

*43. a. Write the nodal equations for the network of Fig. 8.127.
b. Using determinants, solve for the nodal voltages.
c. Determine the magnitude and polarity of the voltage across each resistor.

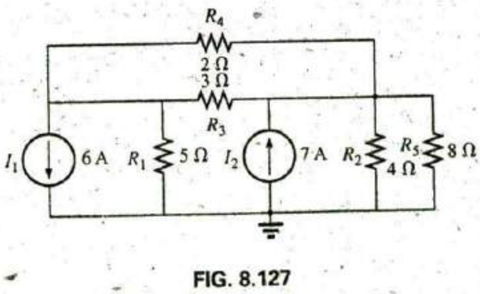


FIG. 8.127
Problem 43.

*44. a. Write the nodal equations for the network of Fig. 8.128 using the general approach.
b. Solve for the nodal voltages using determinants.
c. Find the current through the 6 Ω resistor.

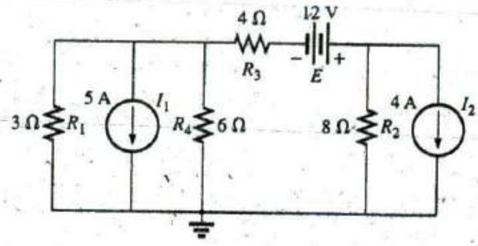


FIG. 8.128
Problem 44.

***45. a.** Write the nodal equations for the network of Fig. 8.129 using the general approach.
b. Solve for the nodal voltages using determinants.
c. Find the voltage across the 5 Ω resistor.

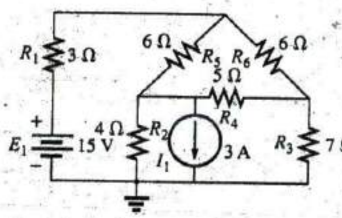


FIG. 8.129
Problem 45.

***46. a.** Write the nodal equations for the network of Fig. 8.130 using the general approach.
b. Solve for the nodal voltages using determinants.
c. Find the voltage across the resistor R_6.

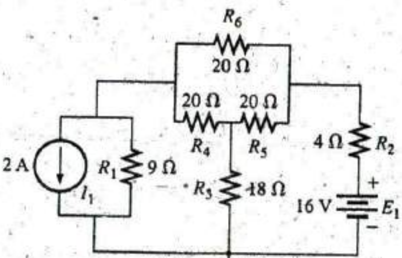


FIG. 8.130
Problems 46 and 53.

***47. a.** Write the nodal equations for the network of Fig. 8.131 using the general approach.
b. Find the nodal voltages using determinants.
c. Determine the current through the 9 Ω resistor.

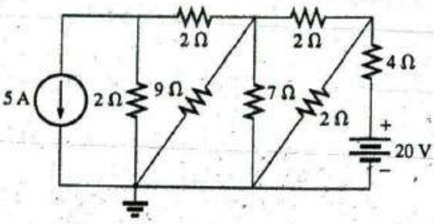


FIG. 8.131
Problems 47, 54, and 72.

***48. a.** Write the nodal equations for the network of Fig. 8.132 using the general approach and find the nodal voltages. Then calculate the current through the 4 Ω resistor.

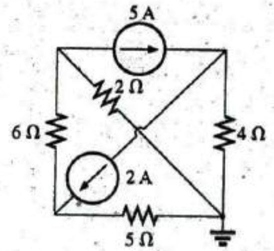


FIG. 8.132
Problems 48 and 55.

***49.** Using the supernode approach, determine the nodal voltages for the network of Fig. 8.133.

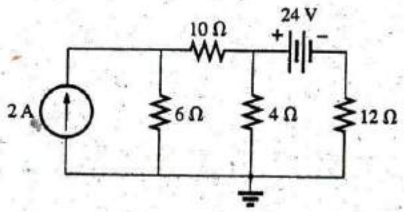


FIG. 8.133
Problem 49.

***50.** Using the supernode approach, determine the nodal voltages for the network of Fig. 8.134.

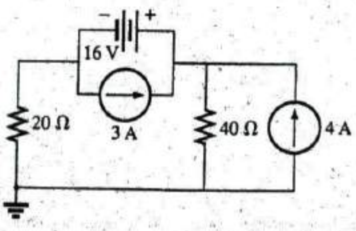


FIG. 8.134
Problem 50.

SECTION 8.10 Nodal Analysis (Format Approach)

51. a. Determine the nodal voltages of Fig. 8.124 using the format approach to nodal analysis.
    b. Then find the voltage across each current source.

52. a. Convert the voltage source of Fig 8.125 to a current source, and then find the nodal voltages using the format approach to nodal analysis.
    b. Use the results of part (a) to find the voltage across the 6 Ω resistor of Fig. 8.125.

*53. a. Convert the voltage source of Fig. 8.130 to a current source, and then apply the format approach to nodal analysis to find the nodal voltages.
    b. Use the results of part (a) to find the current through the 4 Ω resistor.

*54. a. Convert the voltage source of Fig. 8.131 to a current source, and then apply the format approach to nodal analysis to find the nodal voltages.
    b. Use the results of part (a) to find the current through the 9 Ω resistor.

*55. a. Using the format approach, find the nodal voltages of Fig. 8.132 using nodal analysis.
    b. Using the results of part (a), find the current through the 2 Ω resistor.

*56. a. Convert the voltage sources of Fig. 8.123 to current sources, and then find the nodal voltages of the resulting network using the format approach to nodal analysis.
    b. Using the results of part (a), find the voltage between points *a* and *b*.

SECTION 8.11 Bridge Networks

57. For the bridge network in Fig. 8.135:
    a. Write the mesh equations using the format approach.
    b. Determine the current through R_5.
    c. Is the bridge balanced?
    d. Is Eq. (8.2) satisfied?

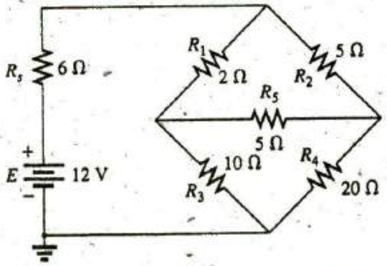


FIG. 8.135
Problems 57 and 58.

58. For the network in Fig. 8.135:
    a. Write the nodal equations using the format approach.
    b. Determine the voltage across R_5.
    c. Is the bridge balanced?
    d. Is Eq. (8.2) satisfied?

59. For the bridge in Fig. 8.136:
    a. Write the mesh equations using the format approach.
    b. Determine the current through R_5.
    c. Is the bridge balanced?
    d. Is Eq. (8.2) satisfied?

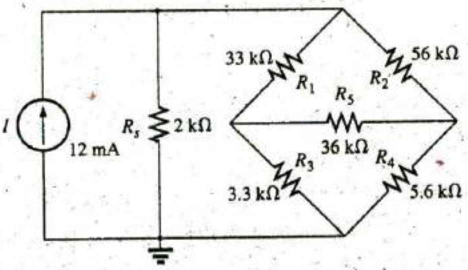


FIG. 8.136
Problems 59 and 60.

60. For the bridge network in Fig. 8.136:
    a. Write the nodal equations using the format approach.
    b. Determine the current across R_5.
    c. Is the bridge balanced?
    d. Is Eq. (8.2) satisfied?

*61. Determine the current through the source resistor R_s in Fig. 8.137 using either mesh or nodal analysis. Explain why you chose one method over the other.

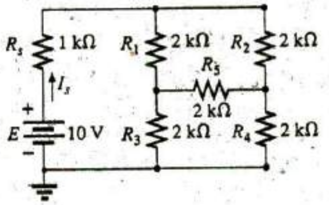


FIG. 8.137
Problem 61.

*62. Repeat Problem 61 for the network of Fig. 8.138.

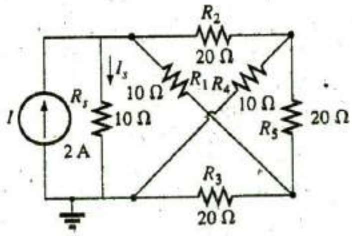


FIG. 8.138
Problem 62.

SECTION 8.12 Y-Δ (T-π) and Δ-Y (π-T) Conversions

63. Using a Δ-Y or Y-Δ conversion, find the current I for the network of Fig. 8.139.

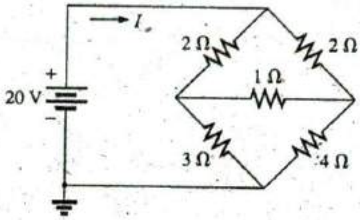


FIG. 8.139
Problem 63.

64. Convert the Δ of 6.8 kΩ resistors in Fig. 8.140 to a T configuration and find the current I.

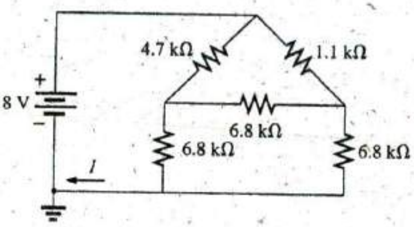


FIG. 8.140
Problem 64.

65. For the network of Fig. 8.141, find the current I without using Y-Δ conversion.

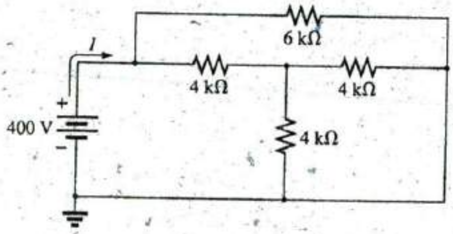


FIG. 8.141
Problem 65.

66. a. Using a Δ-Y or Y-Δ conversion, find the current I in the network of Fig. 8.142.
b. What other method could be used to find the current I?

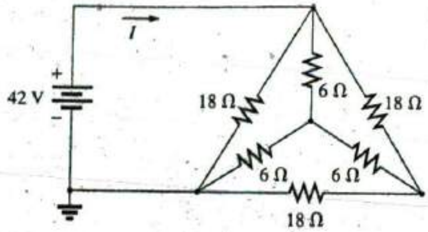


FIG. 8.142
Problem 66.

67. The network of Fig. 8.143 is very similar to the two-source networks solved using mesh or nodal analysis. We will now use a Y-Δ conversion to solve the same network. Find the source current I_{s_1} using a Y-Δ conversion.

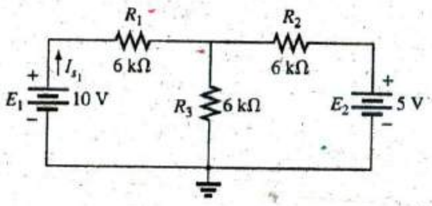


FIG. 8.143
Problem 57.

68. a. Replace the π configuration in Fig. 8.144 (composed of 3 kΩ resistors) with a T configuration.
b. Solve for the source current I_s.

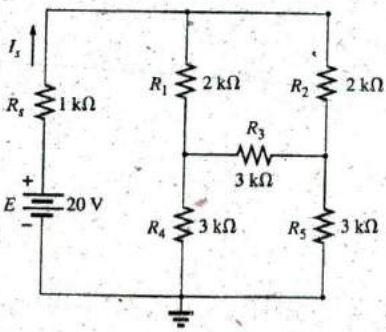


FIG. 8.144
Problem 68.

*69. Using Y-Δ or Δ-Y conversion, determine the total resistance of the network in Fig. 8.145.

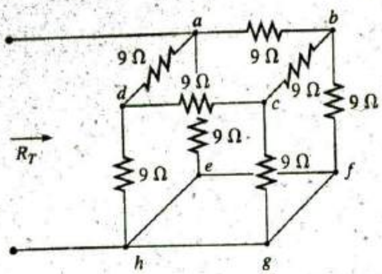


FIG. 8.145
Problem 69.

SECTION 8.14 Computer Analysis

PSpice or Multisim

70. Using schematics, find the current through each element in Fig. 8.111.

*71. Using schematics, find the mesh currents for the network in Fig. 8.118.

*72. Using schematics, determine the nodal voltages for the network in Fig. 8.131.

GLOSSARY

Branch-current method A technique for determining the branch currents of a multiloop network.

Bridge network A network configuration typically having a diamond appearance in which no two elements are in series or parallel.

Current sources Sources that supply a fixed current to a network and have a terminal voltage dependent on the network to which they are applied.

Delta (Δ), pi (π) configuration A network structure that consists of three branches and has the appearance of the Greek letter delta (Δ) or pi (π).

Determinants method A mathematical technique for finding the unknown variables of two or more simultaneous linear equations.

Mesh analysis A technique for determining the mesh (loop) currents of a network that results in a reduced set of equations compared to the branch-current method.

Mesh (loop) current A labeled current assigned to each distinct closed loop of a network that can, individually or in combination with other mesh currents, define all of the branch currents of a network.

Nodal analysis A technique for determining the nodal voltages of a network.

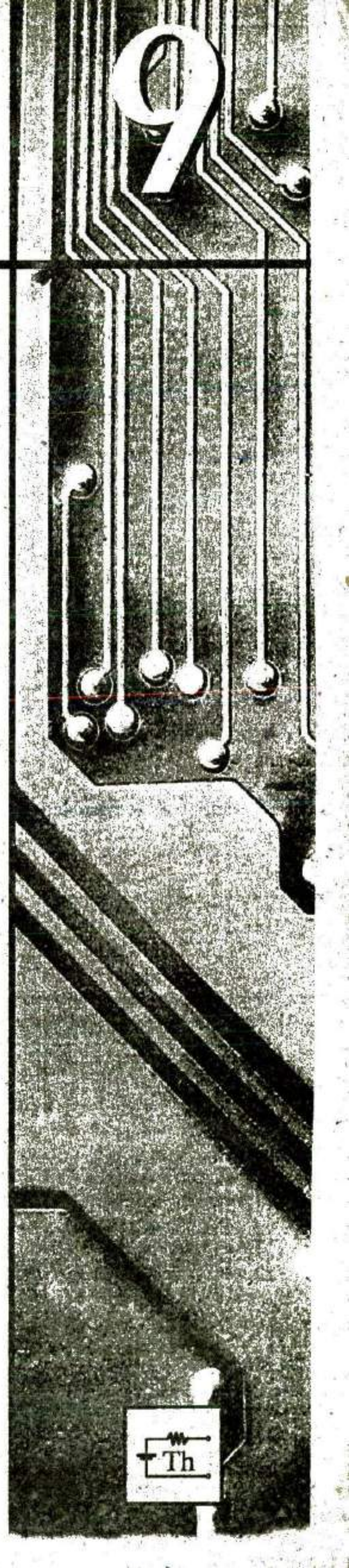
Node A junction of two or more branches in a network.

Supermesh current A current defined in a network with ideal current sources that permits the use of mesh analysis.

Supernode A node defined in a network with ideal voltage sources that permits the use of nodal analysis.

Wye (Y), tee (T) configuration A network structure that consists of three branches and has the appearance of the capital letter Y or T.

Network Theorems



Objectives

- ***Become familiar with the superposition theorem and its unique ability to separate the impact of each source on the quantity of interest.***
- ***Be able to apply Thévenin's theorem to reduce any two-terminal, series-parallel network with any number of sources to a single voltage source and series resistor.***
- ***Become familiar with Norton's theorem and how it can be used to reduce any two-terminal, series-parallel network with any number of sources to a single current source and a parallel resistor.***
- ***Understand how to apply the maximum power transfer theorem to determine the maximum power to a load and to choose a load that will receive maximum power.***
- ***Become aware of the reduction powers of Millman's theorem and the powerful implications of the substitution and reciprocity theorems.***

9.1 INTRODUCTION

This chapter introduces a number of theorems that have application throughout the field of electricity and electronics. Not only can they be used to solve networks such as encountered in the previous chapter, but they also provide an opportunity to determine the impact of a particular source or element on the response of the entire system. In most cases, the network to be analyzed and the mathematics required to find the solution are simplified. All of the theorems appear again in the analysis of ac networks. In fact, the application of each theorem to ac networks is very similar in content to that found in this chapter.

The first theorem to be introduced is the superposition theorem, followed by Thévenin's theorem, Norton's theorem, and the maximum power transfer theorem. The chapter concludes with a brief introduction to Millman's theorem and the substitution and reciprocity theorems.

9.2 SUPERPOSITION THEOREM

The **superposition theorem** is unquestionably one of the most powerful in this field. It has such widespread application that people often apply it without recognizing that their maneuvers are valid only because of this theorem.

In general, the theorem can be used to do the following:

- *Analyze networks such as introduced in the last chapter that have two or more sources that are not in series or parallel.*
- *Reveal the effect of each source on a particular quantity of interest.*
- *For sources of different types (such as dc and ac, which affect the parameters of the network in a different manner) and apply a separate analysis for each type, with the total result simply the algebraic sum of the results.*

The first two areas of application are described in detail in this section. The last are covered in the discussion of the superposition theorem in the ac portion of the text.

The superposition theorem states the following:

The current through, or voltage across, any element of a network is equal to the algebraic sum of the currents or voltages produced independently by each source.

In other words, this theorem allows us to find a solution for a current or voltage using *only one source at a time*. Once we have the solution for each source, we can combine the results to obtain the total solution. The term *algebraic* appears in the above theorem statement because the currents resulting from the sources of the network can have different directions, just as the resulting voltages can have opposite polarities.

If we are to consider the effects of each source, the other sources obviously must be removed. Setting a voltage source to zero volts is like placing a short circuit across its terminals. Therefore,

when removing a voltage source from a network schematic, replace it with a direct connection (short circuit) of zero ohms. Any internal resistance associated with the source must remain in the network.

Setting a current source to zero amperes is like replacing it with an open circuit. Therefore,

when removing a current source from a network schematic, replace it by an open circuit of infinite ohms. Any internal resistance associated with the source must remain in the network.

The above statements are illustrated in Fig. 9.1.

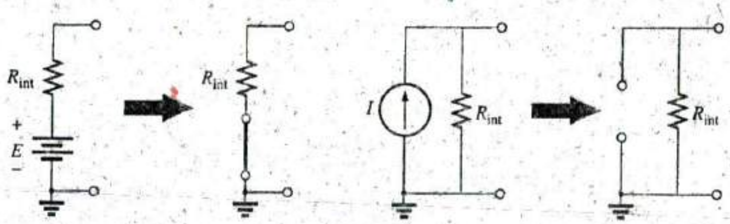


FIG. 9.1
Removing a voltage source and a current source to permit the application of the superposition theorem.

Since the effect of each source will be determined independently, the number of networks to be analyzed will equal the number of sources.

If a particular current of a network is to be determined, the contribution to that current must be determined for *each source*. When the effect of each source has been determined, those currents in the same direction are added, and those having the opposite direction are subtracted; the algebraic sum is being determined. The total result is the direction of the larger sum and the magnitude of the difference.

Similarly, if a particular voltage of a network is to be determined, the contribution to that voltage must be determined for each source. When the effect of each source has been determined, those voltages with the same polarity are added, and those with the opposite polarity are subtracted; the algebraic sum is being determined. The total result has the polarity of the

Superposition cannot be applied to power effects because the power is related to the square of the voltage across a resistor or the current through a resistor. The squared term results in a nonlinear (a curve, not a straight line) relationship between the power and the determining current or voltage. For example, doubling the current through a resistor does not double the power to the resistor (as defined by a linear relationship) but, in fact, increases it by a factor of 4 (due to the squared term). Tripling the current increases the power level by a factor of 9. Example 9.1 demonstrates the differences between a linear and a nonlinear relationship.

A few examples clarify how sources are removed and total solutions obtained.

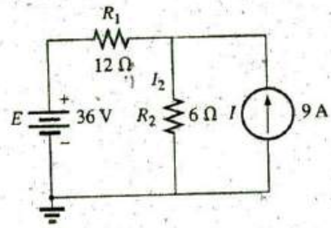


FIG. 9.2
Network to be analyzed in Example 9.1 using the superposition theorem.

EXAMPLE 9.1

a. Using the superposition theorem, determine the current through resistor R_2 for the network in Fig. 9.2.
b. Demonstrate that the superposition theorem is not applicable to power levels.

Solutions:

a. In order to determine the effect of the 36 V voltage source, the current source must be replaced by an open-circuit equivalent as shown in Fig. 9.3. The result is a simple series circuit with a current equal to

$$I'_2 = \frac{E}{R_T} = \frac{E}{R_1 + R_2} = \frac{36\text{ V}}{12\ \Omega + 6\ \Omega} = \frac{36\text{ V}}{18\ \Omega} = 2\text{ A}$$

Examining the effect of the 9 A current source requires replacing the 36 V voltage source by a short-circuit equivalent as shown in Fig. 9.4. The result is a parallel combination of resistors R_1 and R_2. Applying the current divider rule results in

$$I''_2 = \frac{R_1(I)}{R_1 + R_2} = \frac{(12\ \Omega)(9\text{ A})}{12\ \Omega + 6\ \Omega} = 6\text{ A}$$

Since the contribution to current I_2 has the same direction for each source, as shown in Fig. 9.5, the total solution for current I_2 is the sum of the currents established by the two sources. That is,

$$I_2 = I'_2 + I''_2 = 2\text{ A} + 6\text{ A} = \mathbf{8\text{ A}}$$

b. Using Fig. 9.3 and the results obtained, we find the power delivered to the 6 Ω resistor

$$P_1 = (I'_2)^2(R_2) = (2\text{ A})^2(6\ \Omega) = \mathbf{24\text{ W}}$$

Using Fig. 9.4 and the results obtained, we find the power delivered to the 6 Ω resistor

$$P_2 = (I''_2)^2(R_2) = (6\text{ A})^2(6\ \Omega) = \mathbf{216\text{ W}}$$

Using the total results of Fig. 9.5, we obtain the power delivered to the 6 Ω resistor

$$P_T = I_2^2 R_2 = (8\text{ A})^2(6\ \Omega) = \mathbf{384\text{ W}}$$

It is now quite clear that the power delivered to the 6 Ω resistor using the total current of 8 A is not equal to the sum of the power levels due to each source independently. That is,

$$P_1 + P_2 = 24\text{ W} + 216\text{ W} = 240\text{ W} \neq P_T = 348\text{ W}$$

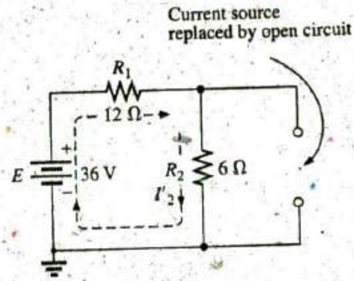


FIG. 9.3
Replacing the 9 A current source in Fig. 9.2 by an open circuit to determine the effect of the 36 V voltage source on current I_2.

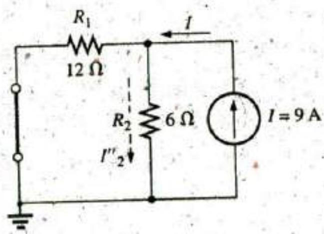


FIG. 9.4
Replacing the 36 V voltage source by a short-circuit equivalent to determine the effect of the 9 A current source on current I_2.

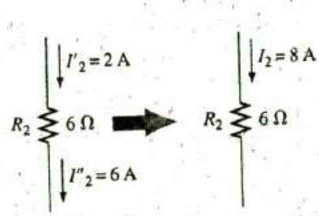


FIG. 9.5
Using the results of Figs. 9.3 and 9.4 to determine current I_2 for the network in Fig. 9.2.

To expand on the above conclusion and further demonstrate what is meant by a *nonlinear relationship*, the power to the 6 Ω resistor versus current through the 6 Ω resistor is plotted in Fig. 9.6. Note that the curve is not a straight line but one whose rise gets steeper with increase in current level.

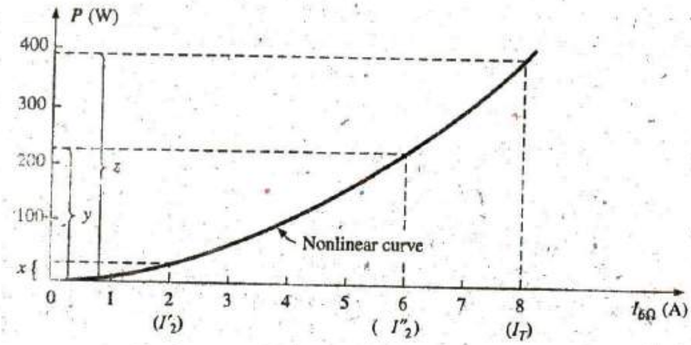


FIG. 9.6
Plotting power delivered to the 6 Ω resistor versus current through the resistor.

Recall from Fig. 9.4 that the power level was 24 W for a current of 2 A developed by the 36 V voltage source, shown in Fig. 9.6. From Fig. 9.5, we found that the current level was 6 A for a power level of 216 W, shown in Fig. 9.6. Using the total current of 8 A, we find that the power level in 384 W, shown in Fig. 9.6. Quite clearly, the sum of power levels due to the 2 A and 6 A current levels does not equal that due to the 8 A level. That is,

$$x + y \neq z$$

Now, the relationship between the voltage across a resistor and the current through a resistor is a linear (straight line) one as shown in Fig. 9.7, with

$$c = a + b$$

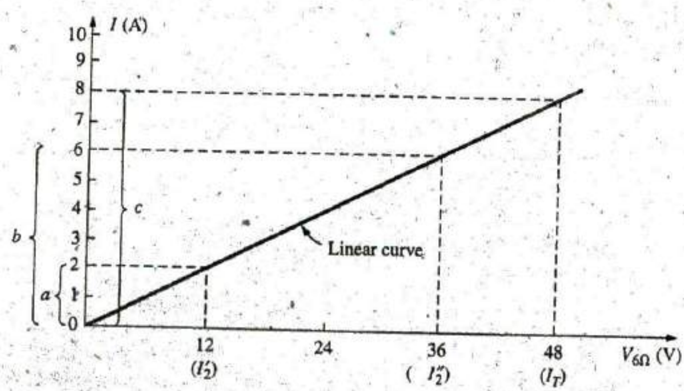


FIG. 9.7
Plotting I versus V for the 6 Ω resistor.

EXAMPLE 9.2 Using the superposition theorem, determine the current through the 12 Ω resistor in Fig. 9.8. Note that this is a two-source network of the type examined in the previous chapter when we applied branch-current analysis and mesh analysis.

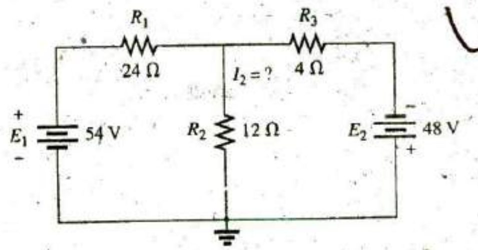


FIG. 9.8
Using the superposition theorem to determine the current through the 12 Ω resistor (Example 9.2).

Solution: Considering the effects of the 54 V source requires replacing the 48 V source by a short-circuit equivalent as shown in Fig. 9.9. The result is that the 12 Ω and 4 Ω resistors are in parallel.

The total resistance seen by the source is therefore

$$R_T = R_1 + R_2 \parallel R_3 = 24\ \Omega + 12\ \Omega \parallel 4\ \Omega = 24\ \Omega + 3\ \Omega = 27\ \Omega$$

and the source current is

$$I_s = \frac{E_1}{R_T} = \frac{54\ \text{V}}{27\ \Omega} = 2\ \text{A}$$

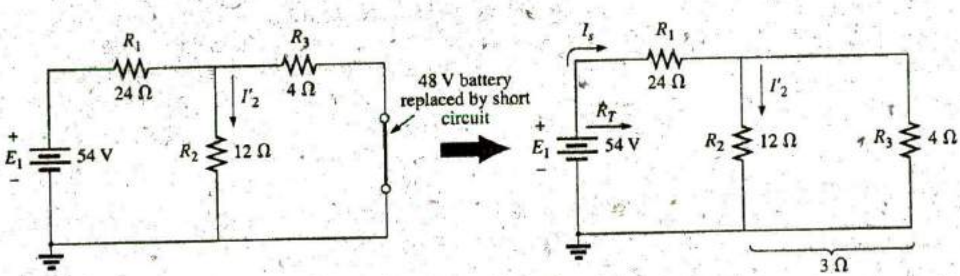


FIG. 9.9
Using the superposition theorem to determine the effect of the 54 V voltage source on current I_2 in Fig. 9.8.

Using the current divider rule results in the contribution to I_2 due to the 54 V source:

$$I'_2 = \frac{R_3 I_s}{R_3 + R_2} = \frac{(4\ \Omega)(2\ \text{A})}{4\ \Omega + 12\ \Omega} = 0.5\ \text{A}$$

If we now replace the 54 V source by a short-circuit equivalent, the network in Fig. 9.10 results. The result is a parallel connection for the 12 Ω and 24 Ω resistors.

Therefore, the total resistance seen by the 48 V source is

$$R_T = R_3 + R_2 \parallel R_1 = 4\ \Omega + 12\ \Omega \parallel 24\ \Omega = 4\ \Omega + 8\ \Omega = 12\ \Omega$$

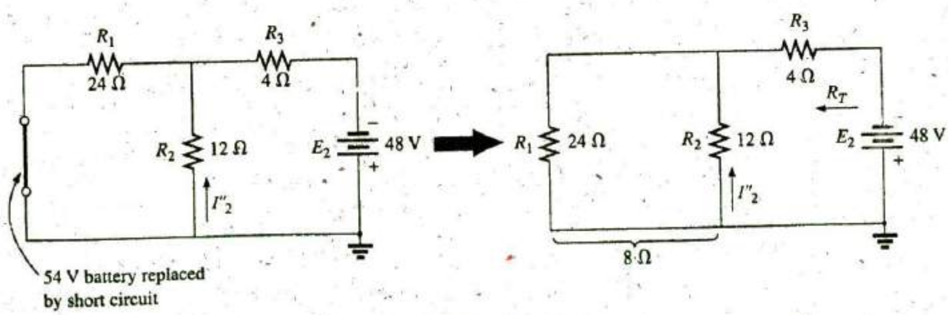


FIG. 9.10
Using the superposition theorem to determine the effect of the 48 V voltage source on current I_2 in Fig. 9.8.

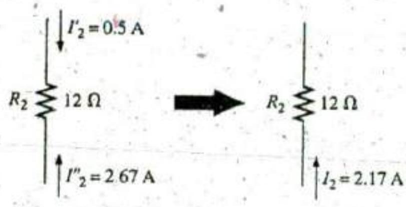


FIG. 9.11
Using the results of Figs. 9.9 and 9.10 to determine current I_2 for the network in Fig. 9.8.

and the source current is

$$I_s = \frac{E_2}{R_T} = \frac{48\text{ V}}{12\ \Omega} = 4\text{ A}$$

Applying the current divider rule results in

$$I''_2 = \frac{R_1(I_s)}{R_1 + R_2} = \frac{(24\ \Omega)(4\text{ A})}{24\ \Omega + 12\ \Omega} = 2.67\text{ A}$$

It is now important to realize that current I_2 due to each source has a different direction, as shown in Fig. 9.11. The net current therefore is the difference of the two and in the direction of the larger as follows:

$$I_2 = I''_2 - I'_2 = 2.67\text{ A} - 0.5\text{ A} = \mathbf{2.17\text{ A}}$$

Using Figs. 9.9 and 9.10 in Example 9.2, we can determine the other currents of the network with little added effort. That is, we can determine all the branch currents of the network, matching an application of the branch-current analysis or mesh analysis approach. In general, therefore, not only can the superposition theorem provide a complete solution for the network, but it also reveals the effect of each source on the desired quantity.

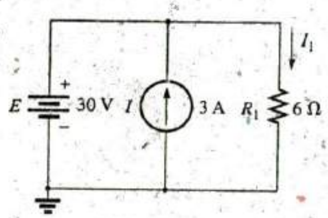


FIG. 9.12
Two-source network to be analyzed using the superposition theorem in Example 9.3.

EXAMPLE 9.3 Using the superposition theorem, determine current I_1 for the network in Fig. 9.12.

Solution: Since two sources are present, there are two networks to be analyzed. First let us determine the effects of the voltage source by setting the current source to zero amperes as shown in Fig. 9.13. Note that the resulting current is defined as I'_1 because it is the current through resistor R_1 due to the voltage source only.

Due to the open circuit, resistor R_1 is in series (and, in fact, in parallel) with the voltage source E. The voltage across the resistor is the applied voltage, and current I'_1 is determined by

$$I'_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{30\text{ V}}{6\ \Omega} = 5\text{ A}$$

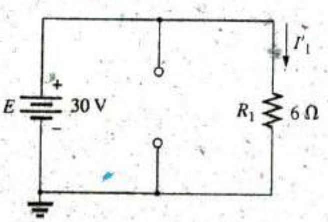


FIG. 9.13
Determining the effect of the 30 V supply on the current I_1 in Fig. 9.12.

Now for the contribution due to the current source. Setting the voltage source to zero volts results in the network in Fig. 9.14, which presents us with an interesting situation. The current source has been replaced with a short-circuit equivalent that is directly across the current source and resistor R_1. Since the source current takes the path of least resistance, it chooses the zero ohm path of the inserted short-circuit equivalent, and the current through R_1 is zero amperes. This is clearly demonstrated by an application of the current divider rule as follows:

$$I''_1 = \frac{R_{sc}I}{R_{sc} + R_1} = \frac{(0\ \Omega)I}{0\ \Omega + 6\ \Omega} = 0\text{ A}$$

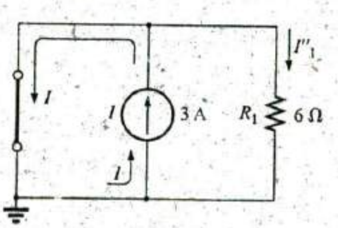


FIG. 9.14
Determining the effect of the 3 A current source on the current I_1 in Fig. 9.12.

Since I'_1 and I''_1 have the same defined direction in Figs. 9.13 and 9.14, the total current is defined by

$$I_1 = I'_1 + I''_1 = 5\text{ A} + 0\text{ A} = \mathbf{5\text{ A}}$$

Although this has been an excellent introduction to the application of the superposition theorem, it should be immediately clear in Fig. 9.12 that the voltage source is in parallel with the current source and load resistor R_1, so the voltage across each must be 30 V. The result is that I_1 must be determined solely by

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{30\text{ V}}{6\ \Omega} = \mathbf{5\ A}$$

EXAMPLE 9.4 Using the principle of superposition, find the current I_2 through the 12 kΩ resistor in Fig. 9.15.

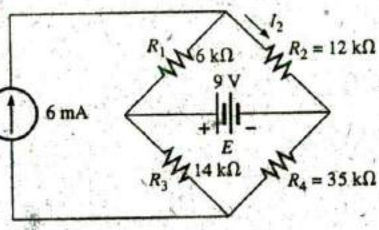


FIG. 9.15
Example 9.4.

Solution: Consider the effect of the 6 mA current source (Fig. 9.16).

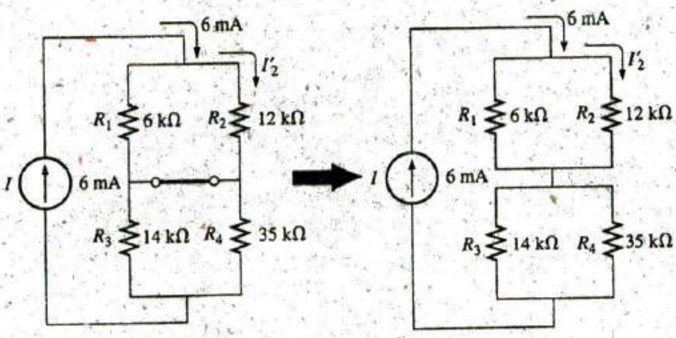


FIG. 9.16
The effect of the current source I on the current I_2.

The current divider rule gives

$$I'_2 = \frac{R_1 I}{R_1 + R_2} = \frac{(6\text{ k}\Omega)(6\text{ mA})}{6\text{ k}\Omega + 12\text{ k}\Omega} = 2\text{ mA}$$

Considering the effect of the 9 V voltage source (Fig. 9.17) gives

$$I''_2 = \frac{E}{R_1 + R_2} = \frac{9\text{ V}}{6\text{ k}\Omega + 12\text{ k}\Omega} = 0.5\text{ mA}$$

Since I'_2 and I''_2 have the same direction through R_2, the desired current is the sum of the two:

$$\begin{aligned} I_2 &= I'_2 + I''_2 \\ &= 2\text{ mA} + 0.5\text{ mA} \\ &= \mathbf{2.5\ mA} \end{aligned}$$

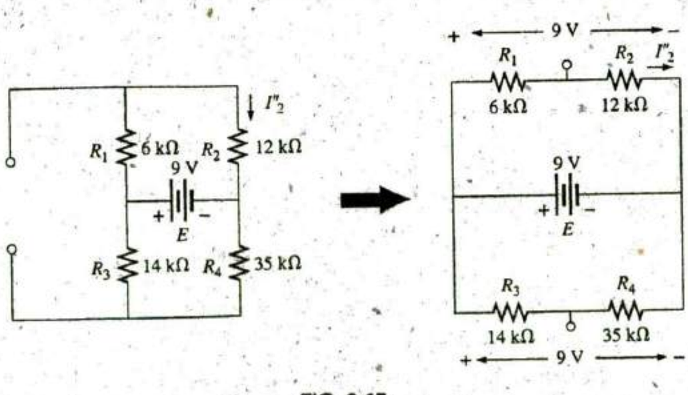


FIG. 9.17
The effect of the voltage source E on the current I_2.

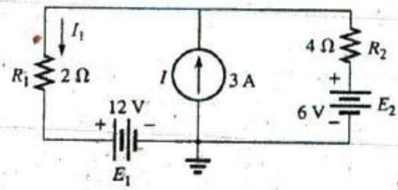


FIG. 9.18
Example 9.5.

EXAMPLE 9.5 Find the current through the 2 Ω resistor of the network in Fig. 9.18. The presence of three sources results in three different networks to be analyzed.

Solution: Consider the effect of the 12 V source (Fig. 9.19):

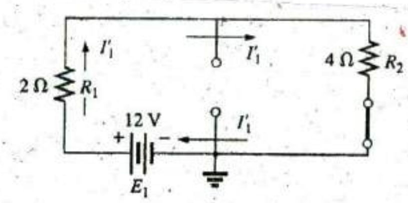


FIG. 9.19
The effect of E_1 on the current I.

$$I'_1 = \frac{E_1}{R_1 + R_2} = \frac{12\text{ V}}{2\ \Omega + 4\ \Omega} = \frac{12\text{ V}}{6\ \Omega} = 2\text{ A}$$

Consider the effect of the 6 V source (Fig. 9.20):

$$I''_1 = \frac{E_2}{R_1 + R_2} = \frac{6\text{ V}}{2\ \Omega + 4\ \Omega} = \frac{6\text{ V}}{6\ \Omega} = 1\text{ A}$$

Consider the effect of the 3 A source (Fig. 9.21): Applying the current divider rule gives

$$I'''_1 = \frac{R_2 I}{R_1 + R_2} = \frac{(4\ \Omega)(3\text{ A})}{2\ \Omega + 4\ \Omega} = \frac{12\text{ A}}{6} = 2\text{ A}$$

The total current through the 2 Ω resistor appears in Fig. 9.22, and

Same direction as I_1 in Fig. 9.18 — Opposite direction to I_1 in Fig. 9.18

$$I_1 = I''_1 + I'''_1 - I'_1$$
$$= 1\text{ A} + 2\text{ A} - 2\text{ A} = 1\text{ A}$$

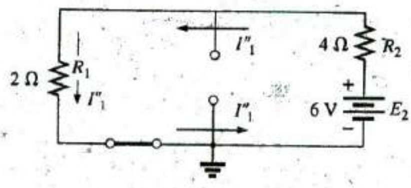


FIG. 9.20
The effect of E_2 on the current I_1.

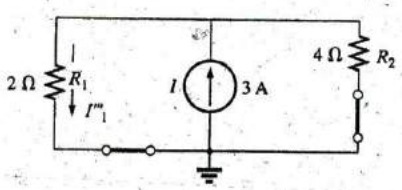


FIG. 9.21
The effect of I on the current I_1.

FIG. 9.22
The resultant current I_1.

9.3 THÉVENIN'S THEOREM

The next theorem to be introduced, **Thévenin's theorem,** is probably one of the most interesting in that it permits the reduction of complex networks to a simpler form for analysis and design.

In general, the theorem can be used to do the following:

- ***Analyze networks with sources that are not in series or parallel.***
- ***Reduce the number of components required to establish the same characteristics at the output terminals.***

- ***Investigate the effect of changing a particular component on the behavior of a network without having to analyze the entire network after each change.***

All three areas of application are demonstrated in the examples to follow.

Thévenin's theorem states the following:

Any two-terminal dc network can be replaced by an equivalent circuit consisting solely of a voltage source and a series resistor as shown in Fig. 9.23.

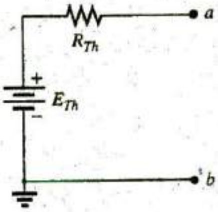


FIG. 9.23
Thévenin equivalent circuit.

The theorem was developed by Commandant Leon-Charles Thévenin in 1883 as described in Fig. 9.24.

To demonstrate the power of the theorem, consider the fairly complex network of Fig. 9.25(a) with its two sources and series-parallel connections. The theorem states that the entire network inside the blue shaded area can be replaced by one voltage source and one resistor as shown in Fig. 9.25(b). If the replacement is done properly, the voltage across, and the current through, the resistor R_L will be the same for each network. The value of R_L can be changed to any value, and the voltage, current, or power to the load resistor is the same for each configuration. Now, this is a very powerful statement—one that is verified in the examples to follow.

The question then is, How can you determine the proper value of Thévenin voltage and resistance? In general, finding the Thévenin *resistance* value is quite straightforward. Finding the Thévenin *voltage* can be more of a challenge and, in fact, may require using the superposition theorem or one of the methods described in Chapter 8.

Fortunately, there are a series of steps that will lead to the proper value of each parameter. Although a few of the steps may seem trivial at first, they can become quite important when the network becomes complex.

Thévenin's Theorem Procedure

Preliminary:

1. ***Remove that portion of the network where the Thévenin equivalent circuit is found. In Fig. 9.25(a), this requires that the load resistor R_L be temporarily removed from the network.***
2. ***Mark the terminals of the remaining two-terminal network. (The importance of this step will become obvious as we progress through some complex networks.)***

R_{Th}:

3. ***Calculate R_{Th} by first setting all sources to zero (voltage sources are replaced by short circuits and current sources by open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.)***

E_{Th}:

4. ***Calculate E_{Th} by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals. (This step is invariably the one that causes most confusion and errors. In all cases, keep in mind that it is the open-circuit potential between the two terminals marked in step 2.)***

FIG. 9.24
Leon-Charles Thévenin.
Courtesy of the Bibliothèque École Polytechnique, Paris, France.

French (Meaux, Paris)
(1857–1927)
Telegraph Engineer, Commandant and Educator
École Polytechnique and École Supérieure de Télégraphie

Although active in the study and design of telegraphic systems (including underground transmission), cylindrical condensers (capacitors), and electromagnetism, he is best known for a theorem first presented in the French *Journal of Physics—Theory and Applications* in 1883. It appeared under the heading of "Sur un nouveau théorème d'électricité dynamique" ("On a new theorem of dynamic electricity") and was originally referred to as the *equivalent generator theorem*. There is some evidence that a similar theorem was introduced by Hermann von Helmholtz in 1853. However, Professor Helmholtz applied the theorem to animal physiology and not to communication or generator systems, and therefore he has not received the credit in this field that he might deserve. In the early 1920s AT&T did some pioneering work using the equivalent circuit and may have initiated the reference to the theorem as simply Thévenin's theorem. In fact, Edward L. Norton, an engineer at AT&T at the time, introduced a current source equivalent of the Thévenin equivalent currently referred to as the Norton equivalent circuit. As an aside, Commandant Thévenin was an avid skier and in fact was commissioner of an international ski competition in Chamonix, France, in 1912.

Conclusion:

5. Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit. This step is indicated by the placement of the resistor R_L between the terminals of the Thévenin equivalent circuit as shown in Fig. 9.25(b).

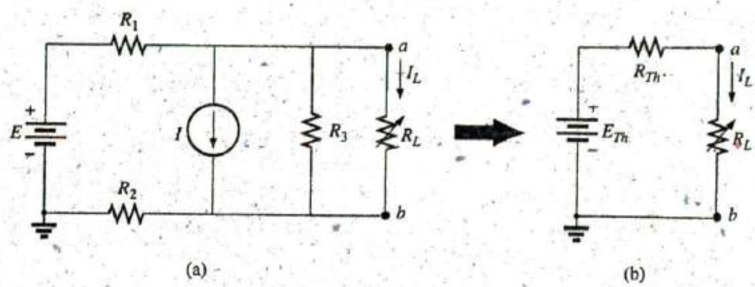


FIG. 9.25
Substituting the Thévenin equivalent circuit for a complex network.

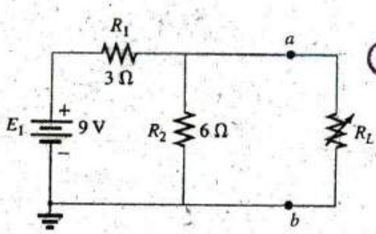


FIG. 9.26
Example 9.6.

EXAMPLE 9.6 Find the Thévenin equivalent circuit for the network in the shaded area of the network in Fig. 9.26. Then find the current through R_L for values of 2 Ω, 10 Ω, and 100 Ω.

Solution:

Steps 1 and 2: These produce the network in Fig. 9.27. Note that the load resistor R_L has been removed and the two "holding" terminals have been defined as *a* and *b*.

Step 3: Replacing the voltage source E_1 with a short-circuit equivalent yields the network in Fig. 9.28(a), where

$$R_{Th} = R_1 \| R_2 = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 6\ \Omega} = \mathbf{2\ \Omega}$$

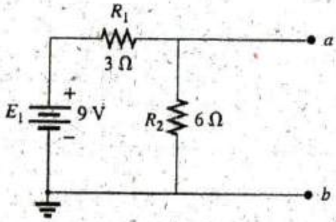


FIG. 9.27
Identifying the terminals of particular importance when applying Thévenin's theorem.

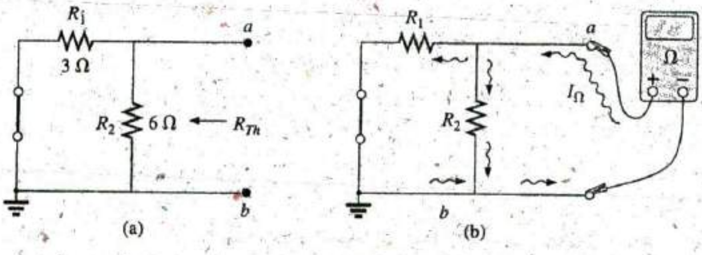


FIG. 9.28
Determining R_{Th} for the network in Fig. 9.27.

The importance of the two marked terminals now begins to surface. They are the two terminals across which the Thévenin resistance is measured. It is no longer the total resistance as seen by the source, as determined in the majority of problems of Chapter 7. If some difficulty develops when determining R_{Th} with regard to whether the resistive elements are in series or parallel, consider recalling that the ohmmeter sends out a trickle current into a resistive combination and senses the level of the resulting voltage to establish the measured resistance level. In Fig. 9.28(b), the trickle current of the ohmmeter approaches the network through terminal a, and when it reaches the junction of R_1 and R_2, it splits as shown. The fact that the trickle current splits and then recombines at the lower node reveals that the resistors are in parallel as far as the ohmmeter reading is concerned. In essence, the path of the sensing current of the ohmmeter has revealed how the resistors are connected to the two terminals of interest and how the Thévenin resistance should be determined. Remember this as you work through the various examples in this section.

Step 4: Replace the voltage source (Fig. 9.29). For this case, the open-circuit voltage E_{Th} is the same as the voltage drop across the 6 Ω resistor. Applying the voltage divider rule gives

$$E_{Th} = \frac{R_2E_1}{R_2 + R_1} = \frac{(6\ \Omega)(9\ \text{V})}{6\ \Omega + 3\ \Omega} = \frac{54\ \text{V}}{9} = \mathbf{6\ V}$$

It is particularly important to recognize that E_{Th} is the open-circuit potential between points a and b. Remember that an open circuit can have any voltage across it, but the current must be zero. In fact, the current through any element in series with the open circuit must be zero also. The use of a voltmeter to measure E_{Th} appears in Fig. 9.30. Note that it is placed directly across the resistor R_2 since E_{Th} and V_{R_2} are in parallel.

Step 5: (Fig. 9.31):

$$I_L = \frac{E_{Th}}{R_{Th} + R_L}$$

$R_L = 2\ \Omega$: $$I_L = \frac{6\ \text{V}}{2\ \Omega + 2\ \Omega} = \mathbf{1.5\ A}$$

$R_L = 10\ \Omega$: $$I_L = \frac{6\ \text{V}}{2\ \Omega + 10\ \Omega} = \mathbf{0.5\ A}$$

$R_L = 100\ \Omega$: $$I_L = \frac{6\ \text{V}}{2\ \Omega + 100\ \Omega} = \mathbf{0.06\ A}$$

If Thévenin's theorem were unavailable, each change in R_L would require that the entire network in Fig. 9.26 be reexamined to find the new value of R_L.

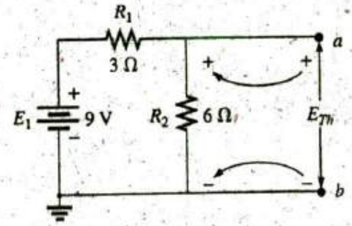


FIG. 9.29
Determining E_{Th} for the network in Fig. 9.27.

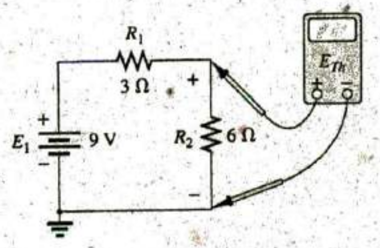


FIG. 9.30
Measuring E_{Th} for the network in Fig. 9.27.

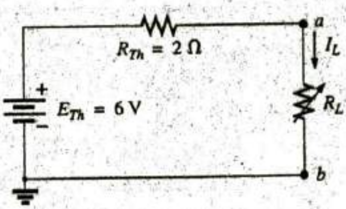


FIG. 9.31
Substituting the Thévenin equivalent circuit for the network external to R_L in Fig. 9.26.

EXAMPLE 9.7 Find the Thévenin equivalent circuit for the network in the shaded area of the network in Fig. 9.32.

Solution:

Steps 1 and 2: See Fig. 9.33.

Step 3: See Fig. 9.34. The current source has been replaced with an open-circuit equivalent and the resistance determined between terminals a and b.

In this case, an ohmmeter connected between terminals a and b sends out a sensing current that flows directly through R_1 and R_2 (at the same level). The result is that R_1 and R_2 are in series and the Thévenin resistance is the sum of the two,

$$R_{Th} = R_1 + R_2 = 4\ \Omega + 2\ \Omega = \mathbf{6\ \Omega}$$

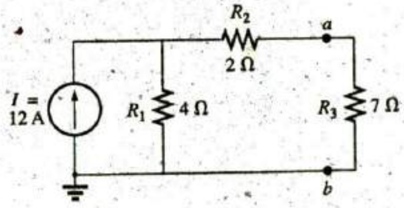


FIG. 9.32
Example 9.7.

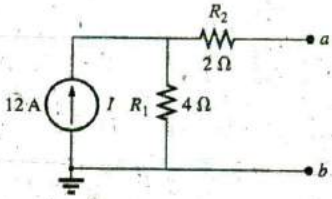


FIG. 9.33
Establishing the terminals of particular interest for the network in Fig. 9.32.

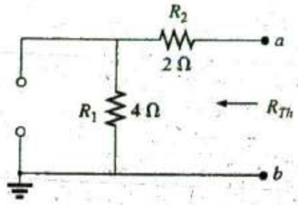


FIG. 9.34
Determining R_{Th} for the network in Fig. 9.33.

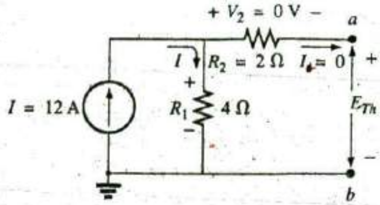


FIG. 9.35
Determining E_{Th} for the network in Fig. 9.33.

Step 4: See Fig. 9.35. In this case, since an open circuit exists between the two marked terminals, the current is zero between these terminals and through the 2 Ω resistor. The voltage drop across R_2 is, therefore,

$$V_2 = I_2R_2 = (0)R_2 = 0\text{ V}$$

and
$$E_{Th} = V_1 = I_1R_1 = IR_1 = (12\text{ A})(4\Omega) = \mathbf{48\text{ V}}$$

Step 5: See Fig. 9.36.

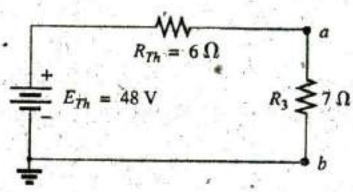


FIG. 9.36
Substituting the Thévenin equivalent circuit in the network external to the resistor R_3 in Fig. 9.32.

EXAMPLE 9.8 Find the Thévenin equivalent circuit for the network in the shaded area of the network in Fig. 9.37. Note in this example that there is no need for the section of the network to be preserved to be at the "end" of the configuration.

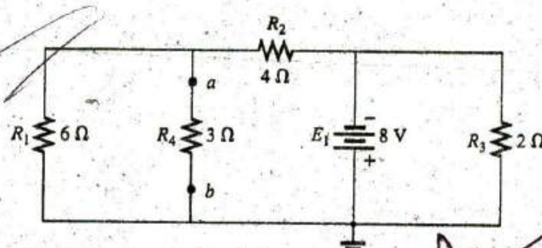


FIG. 9.37
Example 9.8.

Solution:

Steps 1 and 2: See Fig. 9.38.

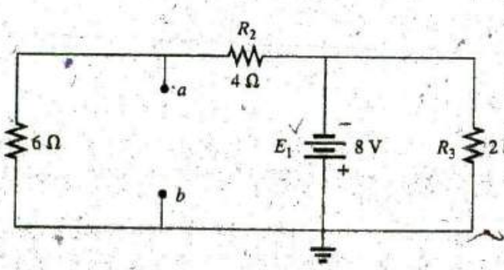


FIG. 9.38
Identifying the terminals of particular interest for the network in Fig. 9.37.

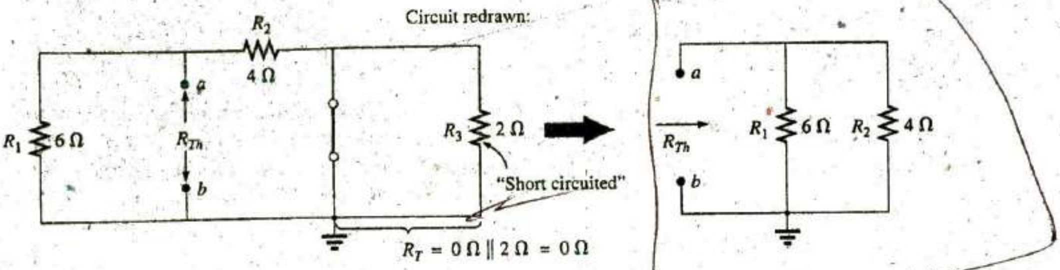


FIG. 9.39
Determining R_{Th} for the network in Fig. 9.38.

Step 3: See Fig. 9.39. Steps 1 and 2 are relatively easy to apply, but now we must be careful to "hold" onto the terminals a and b as the Thévenin resistance and voltage are determined. In Fig. 9.39, all the remaining elements turn out to be in parallel, and the network can be redrawn as shown. We have

$$R_{Th} = R_1 \| R_2 = \frac{(6\ \Omega)(4\ \Omega)}{6\ \Omega + 4\ \Omega} = \frac{24\ \Omega}{10} = \mathbf{2.4\ \Omega}$$

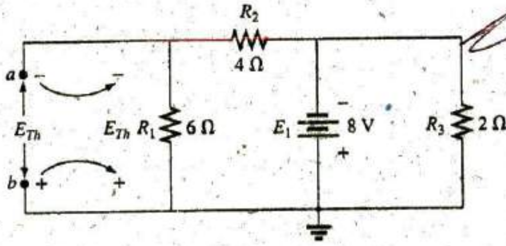


FIG. 9.40
Determining E_{Th} for the network in Fig. 9.38.

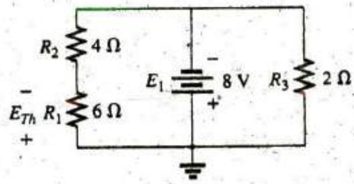


FIG. 9.41
Network of Fig. 9.40 redrawn.

Step 4: See Fig. 9.40. In this case, the network can be redrawn as shown in Fig. 9.41. Since the voltage is the same across parallel elements, the voltage across the series resistors R_1 and R_2 is E_1, or 8 V. Applying the voltage divider rule gives

$$E_{Th} = \frac{R_1E_1}{R_1 + R_2} = \frac{(6\ \Omega)(8\ \text{V})}{6\ \Omega + 4\ \Omega} = \frac{48\ \text{V}}{10} = \mathbf{4.8\ V}$$

Step 5: See Fig. 9.42.

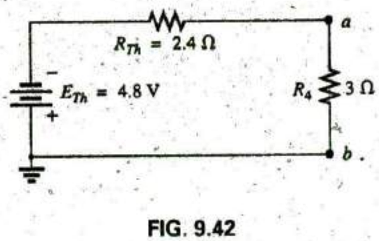


FIG. 9.42
Substituting the Thévenin equivalent circuit for the network external to the resistor R_4 in Fig. 9.37.

The importance of marking the terminals should be obvious from Example 9.8. Note that there is no requirement that the Thévenin voltage have the same polarity as the equivalent circuit originally introduced.

EXAMPLE 9.9 Find the Thévenin equivalent circuit for the network in the shaded area of the bridge network in Fig. 9.43.

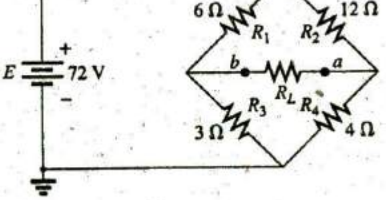


FIG. 9.43
Example 9.9.

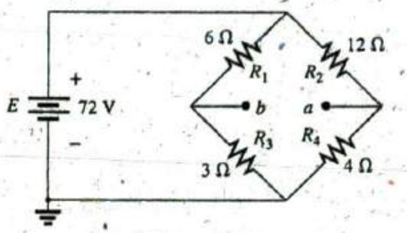


FIG. 9.44
Identifying the terminals of particular interest for the network in Fig. 9.43.

Solution:

Steps 1 and 2: See Fig. 9.44.

Step 3: See Fig. 9.45. In this case, the short-circuit replacement of the voltage source E provides a direct connection between c and c' in Fig. 9.45(a), permitting a "folding" of the network around the horizontal line of a-b to produce the configuration in Fig. 9.45(b).

$$\begin{aligned} R_{Th} = R_{a-b} &= R_1 \parallel R_3 + R_2 \parallel R_4 \\ &= 6\,\Omega \parallel 3\,\Omega + 4\,\Omega \parallel 12\,\Omega \\ &= 2\,\Omega + 3\,\Omega = \mathbf{5\,\Omega} \end{aligned}$$

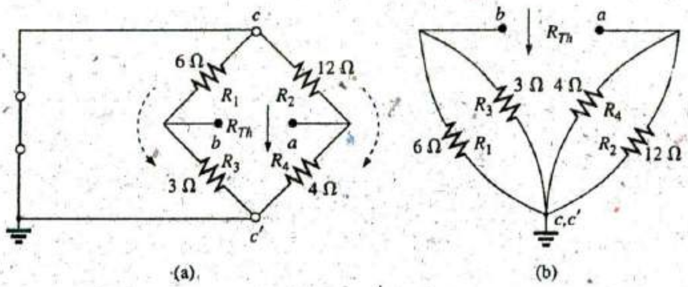


FIG. 9.45
Solving for R_{Th} for the network in Fig. 9.44.

Step 4: The circuit is redrawn in Fig. 9.46. The absence of a direct connection between a and b results in a network with three parallel branches. The voltages V_1 and V_2 can therefore be determined using the voltage divider rule:

$$V_1 = \frac{R_1 E}{R_1 + R_3} = \frac{(6\,\Omega)(72\text{ V})}{6\,\Omega + 3\,\Omega} = \frac{432\text{ V}}{9} = 48\text{ V}$$

$$V_2 = \frac{R_2 E}{R_2 + R_4} = \frac{(12\,\Omega)(72\text{ V})}{12\,\Omega + 4\,\Omega} = \frac{864\text{ V}}{16} = 54\text{ V}$$

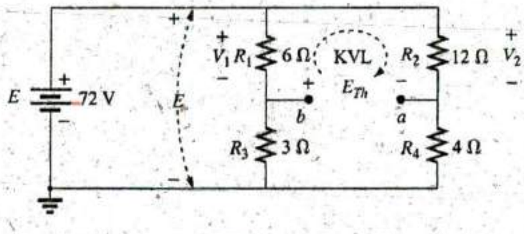


FIG. 9.46
Determining E_{Th} for the network in Fig. 9.44.

Assuming the polarity shown for E_{Th} and applying Kirchhoff's voltage law to the top loop in the clockwise direction results in

$$\Sigma_{\text{C}} V = +E_{Th} + V_1 - V_2 = 0$$

and
$$E_{Th} = V_2 - V_1 = 54\text{ V} - 48\text{ V} = \mathbf{6\text{ V}}$$

Step 5: See Fig. 9.47.

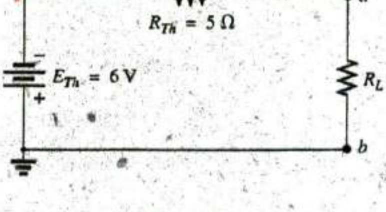


FIG. 9.47
Substituting the Thévenin equivalent circuit for the network external to the resistor R_L in Fig. 9.43.

Thévenin's theorem is not restricted to a single passive element, as shown in the preceding examples, but can be applied across sources, whole branches, portions of networks, or any circuit configuration, as shown in the following example. It is also possible that you may have to use one of the methods previously described, such as mesh analysis or superposition, to find the Thévenin equivalent circuit.

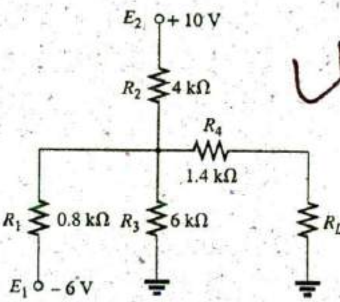


FIG. 9.48
Example 9.10.

EXAMPLE 9.10 (Two sources) Find the Thévenin circuit for the network within the shaded area of Fig. 9.48.

Solution:

Steps 1 and 2: See Fig. 9.49. The network is redrawn.

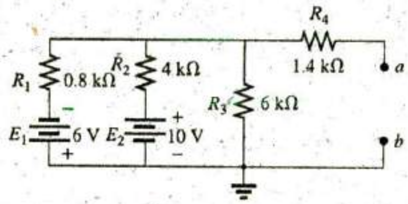


FIG. 9.49
Identifying the terminals of particular interest for the network in Fig. 9.48.

Step 3: See Fig. 9.50.

$$\begin{aligned} R_{Th} &= R_4 + R_1 \| R_2 \| R_3 \\ &= 1.4\text{ k}\Omega + 0.8\text{ k}\Omega \| 4\text{ k}\Omega \| 6\text{ k}\Omega \\ &= 1.4\text{ k}\Omega + 0.8\text{ k}\Omega \| 2.4\text{ k}\Omega \\ &= 1.4\text{ k}\Omega + 0.6\text{ k}\Omega \\ &= \mathbf{2\text{ k}\Omega} \end{aligned}$$

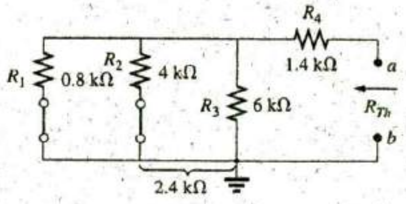


FIG. 9.50
Determining R_{Th} for the network in Fig. 9.49.

Step 4: Applying superposition, we will consider the effects of the voltage source E_1 first. Note Fig. 9.51. The open circuit requires that $V_4 = I_4R_4 = (0)R_4 = 0$ V, and

$$E'_{Th} = V_3$$
$$R'_T = R_2 \| R_3 = 4\text{ k}\Omega \| 6\text{ k}\Omega = 2.4\text{ k}\Omega$$

Applying the voltage divider rule gives

$$V_3 = \frac{R'_T E_1}{R'_T + R_1} = \frac{(2.4\text{ k}\Omega)(6\text{ V})}{2.4\text{ k}\Omega + 0.8\text{ k}\Omega} = \frac{14.4\text{ V}}{3.2} = 4.5\text{ V}$$
$$E'_{Th} = V_3 = 4.5\text{ V}$$

For the source E_2, the network in Fig. 9.52 results. Again, $V_4 = I_4R_4 = (0)R_4 = 0$ V, and

$$E''_{Th} = V_3$$
$$R'_T = R_1 \| R_3 = 0.8\text{ k}\Omega \| 6\text{ k}\Omega = 0.706\text{ k}\Omega$$

and $$V_3 = \frac{R'_T E_2}{R'_T + R_2} = \frac{(0.706\text{ k}\Omega)(10\text{ V})}{0.706\text{ k}\Omega + 4\text{ k}\Omega} = \frac{7.06\text{ V}}{4.706} = 1.5\text{ V}$$
$$E''_{Th} = V_3 = 1.5\text{ V}$$

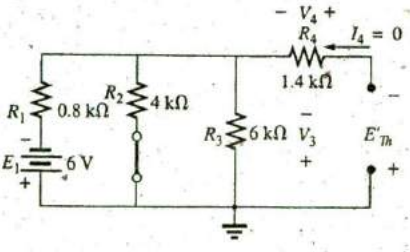


FIG. 9.51
Determining the contribution to E_{Th} from the source E_1 for the network in Fig. 9.49.

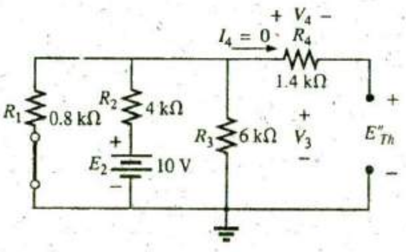


FIG. 9.52
Determining the contribution to E_{Th} from the source E_2 for the network in Fig. 9.49.

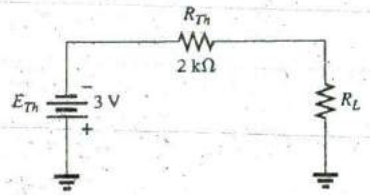


FIG. 9.53

Substituting the Thévenin equivalent circuit for the network external to the resistor R_L in Fig. 9.48.

Since E'_{Th} and E''_{Th} have opposite polarities,

$$\begin{aligned} E_{Th} &= E'_{Th} - E''_{Th} \\ &= 4.5\text{ V} - 1.5\text{ V} \\ &= \mathbf{3\ V} \qquad (\text{polarity of } E'_{Th}) \end{aligned}$$

Step 5: See Fig. 9.53.

Experimental Procedures

Now that the analytical procedure has been described in detail and a sense for the Thévenin impedance and voltage established, it is time to investigate how both quantities can be determined using an experimental procedure.

Even though the Thévenin resistance is usually the easiest to determine analytically, the Thévenin voltage is often the easiest to determine experimentally, and therefore it will be examined first.

Measuring E_{Th} The network of Fig. 9.54(a) has the equivalent Thévenin circuit appearing in Fig. 9.54(b). The open-circuit Thévenin voltage can be determined by simply placing a voltmeter on the output terminals in Fig. 9.54(a) as shown. This is due to the fact that the open circuit in Fig. 9.54(b) dictates that the current through and the voltage across the Thévenin resistance must be zero. The result for Fig. 9.54(b) is that

$$V_{oc} = E_{Th} = \mathbf{4.5\ V}$$

In general, therefore,

the Thévenin voltage is determined by connecting a voltmeter to the output terminals of the network. Be sure the internal resistance of the voltmeter is significantly more than the expected level of R_{Th}.

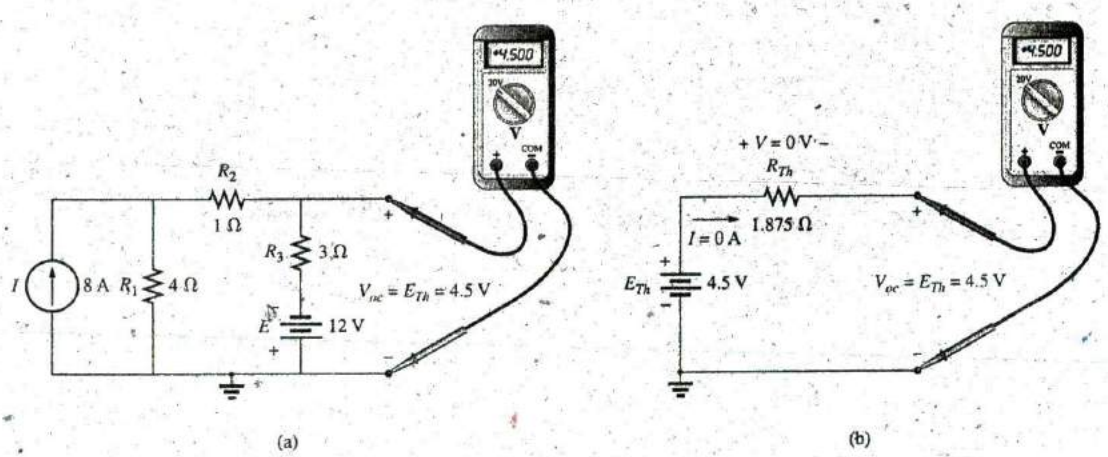


FIG. 9.54

Measuring the Thévenin voltage with a voltmeter: (a) actual network; (b) Thévenin equivalent.

Measuring R_{Th}

Using An Ohmmeter In Fig. 9.55, the sources in Fig. 9.54(a) have been set to zero, and an ohmmeter has been applied to measure the Thévenin resistance. In Fig. 9.54(b), it is clear that if the Thévenin voltage is set to zero volts, the ohmmeter will read the Thévenin resistance directly.

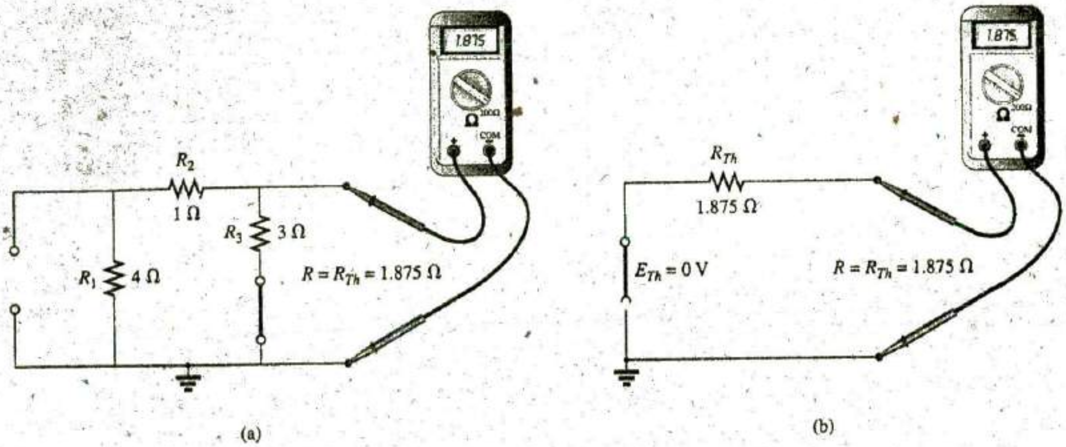


FIG. 9.55
Measuring R_{Th} with an ohmmeter: (a) actual network; (b) Thévenin equivalent.

In general, therefore,

the Thévenin resistance can be measured by setting all the sources to zero and measuring the resistance at the output terminals.

It is important to remember, however, that ohmmeters cannot be used on live circuits, and you cannot set a voltage source by putting a short circuit across it—it causes instant damage. The source must either be set to zero or removed entirely and then replaced by a direct connection. For the current source, the open-circuit condition must be clearly established; otherwise, the measured resistance will be incorrect. For most situations, it is usually best to remove the sources and replace them by the appropriate equivalent.

Using a Potentiometer If we use a potentiometer to measure the Thévenin resistance, the sources can be left as is. For this reason alone, this approach is one of the more popular. In Fig. 9.56(a), a potentiometer has been connected across the output terminals of the network to establish the condition appearing in Fig. 9.56(b) for the Thévenin equivalent. If the resistance of the potentiometer is now adjusted so that the voltage across the potentiometer is one-half the measured Thévenin voltage, the Thévenin resistance must match that of the potentiometer. Recall that for a series circuit, the applied voltage will divide equally across two equal series resistors.

If the potentiometer is then disconnected and the resistance measured with an ohmmeter as shown in Fig. 9.56(c), the ohmmeter displays the Thévenin resistance of the network. In general, therefore,

the Thévenin resistance can be measured by applying a potentiometer to the output terminals and varying the resistance until the output voltage is one-half the measured Thévenin voltage. The resistance of the potentiometer is the Thévenin resistance for the network.

Using the Short-Circuit Current The Thévenin resistance can also be determined by placing a short circuit across the output terminals and finding the current through the short circuit. Since ammeters ideally have zero internal ohms between their terminals, hooking up an ammeter as shown in Fig. 9.57(a) has the effect of both hooking up a short circuit across the

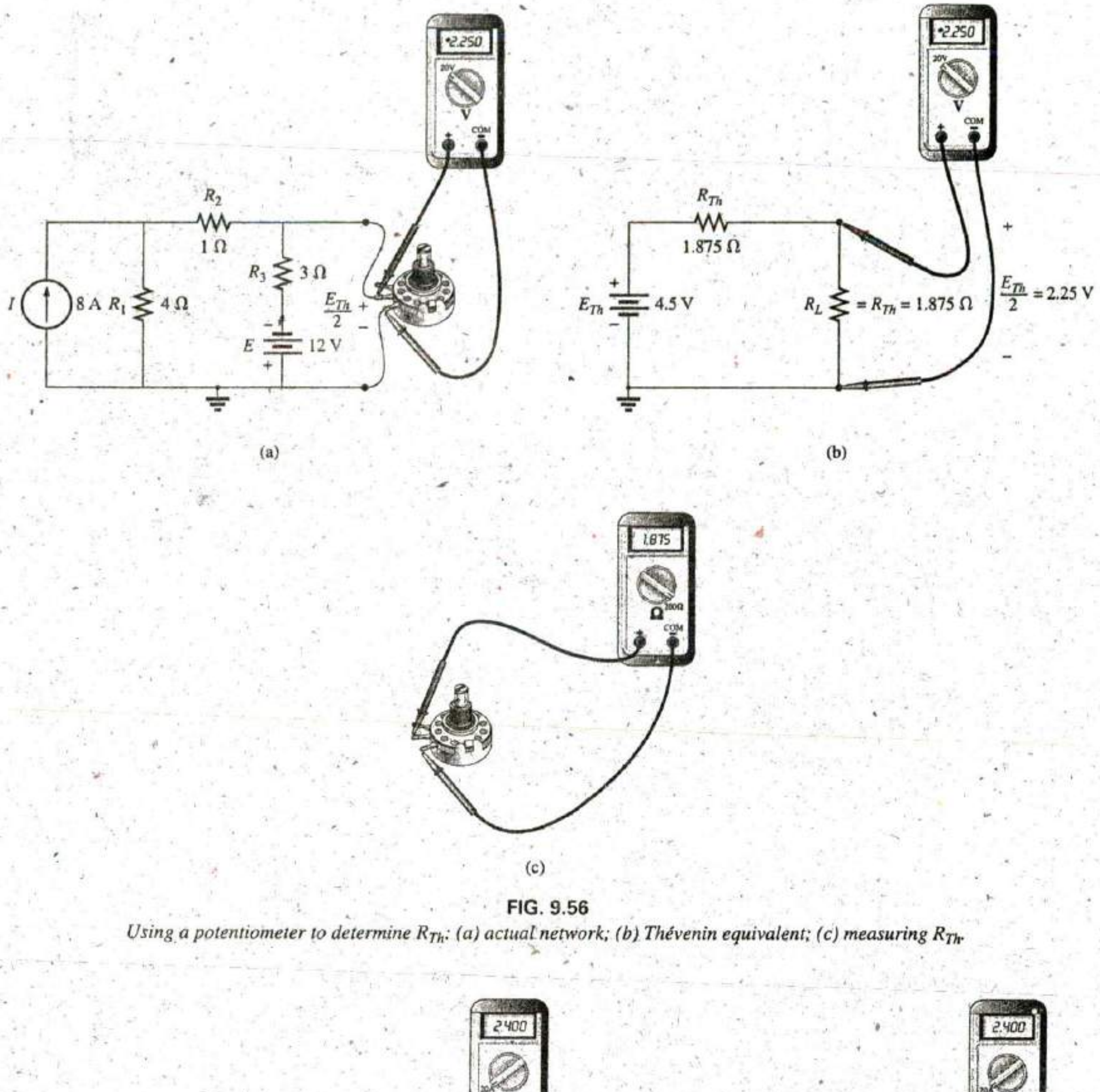


FIG. 9.56

Using a potentiometer to determine R_{Th}: (a) actual network; (b) Thévenin equivalent; (c) measuring R_{Th}.

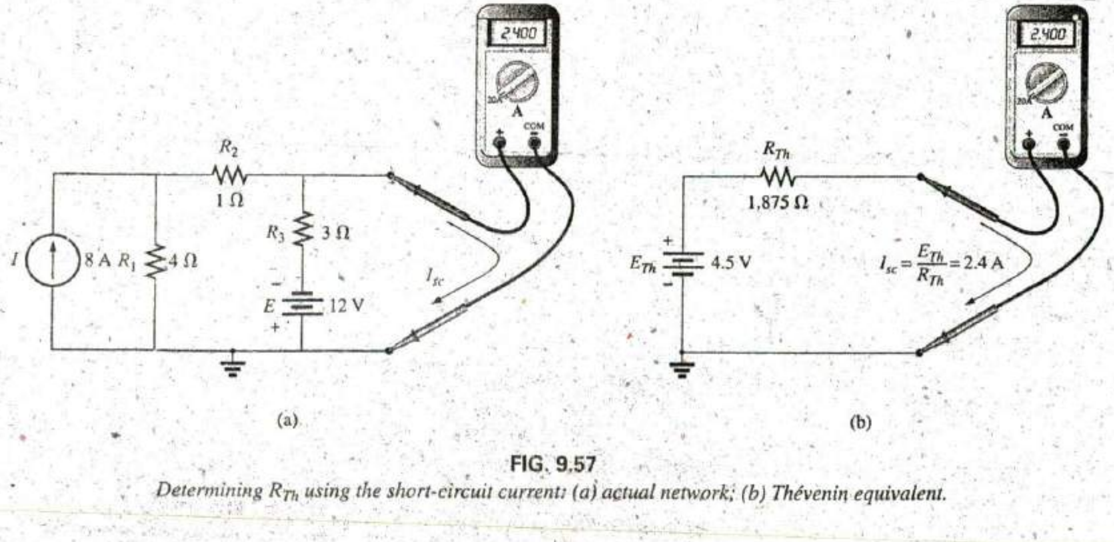


FIG. 9.57

Determining R_{Th} using the short-circuit current: (a) actual network; (b) Thévenin equivalent.

terminals and measuring the resulting current. The same ammeter was connected across the Thévenin equivalent circuit in Fig. 9.57(b).

On a practical level, it is assumed, of course, that the internal resistance of the ammeter is approximately zero ohms in comparison to the other resistors of the network. It is also important to be sure that the resulting current does not exceed the maximum current for the chosen ammeter scale.

In Fig. 9.57(b), since the short-circuit current is

$$I_{sc} = \frac{E_{Th}}{R_{Th}}$$

the Thévenin resistance can be determined by

$$R_{Th} = \frac{E_{Th}}{I_{sc}}$$

In general, therefore,

the Thévenin resistance can be determined by hooking up an ammeter across the output terminals to measure the short-circuit current and then using the open-circuit voltage to calculate the Thévenin resistance in the following manner:

$$\boxed{R_{Th} = \frac{V_{oc}}{I_{sc}}} \qquad \textbf{(9.1)}$$

As a result, we have three ways to measure the Thévenin resistance of a configuration. Because of the concern about setting the sources to zero in the first procedure and the concern about current levels in the last, the second method is often chosen.

9.4 NORTON'S THEOREM

In Section 8.3, we learned that every voltage source with a series internal resistance has a current source equivalent. The current source equivalent can be determined by **Norton's theorem** (Fig. 9.58). It can also be found through the conversions of Section 8.3.

The theorem states the following:

Any two-terminal linear bilateral dc network can be replaced by an equivalent circuit consisting of a current source and a parallel resistor, as shown in Fig. 9.59.

The discussion of Thévenin's theorem with respect to the equivalent circuit can also be applied to the Norton equivalent circuit. The steps leading to the proper values of I_N and R_N are now listed.

Norton's Theorem Procedure

Preliminary:

1. ***Remove that portion of the network across which the Norton equivalent circuit is found.***
2. ***Mark the terminals of the remaining two-terminal network.***

R_N:

3. ***Calculate R_N by first setting all sources to zero (voltage sources are replaced with short circuits and current sources with open circuits) and then finding the resultant resistance between the two***

FIG. 9.58
Edward L. Norton.
Reprinted with the permission of Lucent Technologies, Inc./Bell Labs.

American (Rockland, Maine; Summit, New Jersey)
1898–1983
Electrical Engineer, Scientist, Inventor
Department Head: Bell Laboratories
Fellow: Acoustical Society and Institute of Radio Engineers

Although interested primarily in communications circuit theory and the transmission of data at high speeds over telephone lines, Edward L. Norton is best remembered for development of the dual of Thévenin equivalent circuit, currently referred to as *Norton's equivalent circuit.* In fact, Norton and his associates at AT&T in the early 1920s are recognized as being among the first to perform work applying Thévenin's equivalent circuit and referring to this concept simply as Thévenin's theorem. In 1926, he proposed the equivalent circuit using a current source and parallel resistor to assist in the design of recording instrumentation that was primarily current driven. He began his telephone career in 1922 with the Western Electric Company's Engineering Department, which later became Bell Laboratories. His areas of active research included network theory, acoustical systems, electromagnetic apparatus, and data transmission. A graduate of MIT and Columbia University, he held nineteen patents on his work.

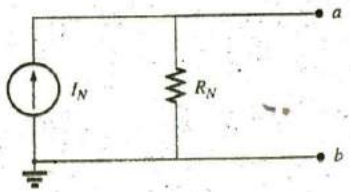


FIG. 9.59
Norton equivalent circuit.

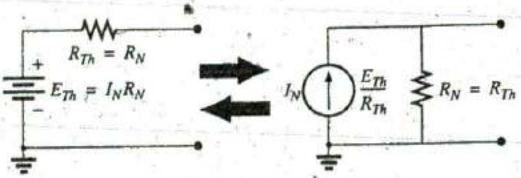


FIG. 9.60
Converting between Thévenin and Norton equivalent circuits.

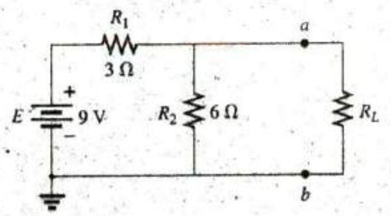


FIG. 9.61
Example 9.11.

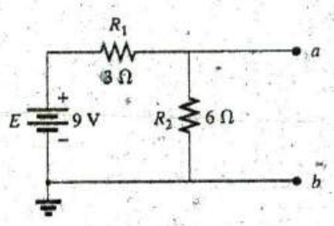


FIG. 9.62
Identifying the terminals of particular interest for the network in Fig. 9.61.

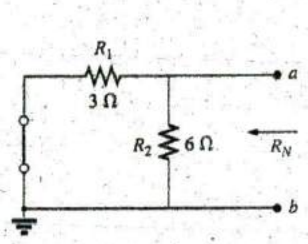


FIG. 9.63
Determining R_N for the network in Fig. 9.62.

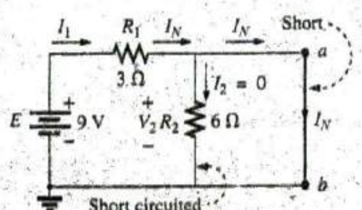


FIG. 9.64
Determining I_N for the network in Fig. 9.62.

marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.) Since $R_N = R_{Th}$, the procedure and value obtained using the approach described for Thévenin's theorem will determine the proper value of R_N.

I_N:

4. ***Calculate I_N by first returning all sources to their original position and then finding the short-circuit current between the marked terminals. It is the same current that would be measured by an ammeter placed between the marked terminals.***

Conclusion:

5. ***Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.***

The Norton and Thévenin equivalent circuits can also be found from each other by using the source transformation discussed earlier in this chapter and reproduced in Fig. 9.60.

EXAMPLE 9.11 Find the Norton equivalent circuit for the network in the shaded area in Fig. 9.61.

Solution:

Steps 1 and 2: See Fig. 9.62.

Step 3: See Fig. 9.63, and

$$R_N = R_1 \parallel R_2 = 3\,\Omega \parallel 6\,\Omega = \frac{(3\,\Omega)(6\,\Omega)}{3\,\Omega + 6\,\Omega} = \frac{18\,\Omega}{9} = \mathbf{2\,\Omega}$$

Step 4: See Fig. 9.64, which clearly indicates that the short-circuit connection between terminals *a* and *b* is in parallel with R_2 and eliminates its effect. I_N is therefore the same as through R_1, and the full battery voltage appears across R_1 since

$$V_2 = I_2 R_2 = (0)6\,\Omega = 0\,\text{V}$$

Therefore,

$$I_N = \frac{E}{R_1} = \frac{9\,\text{V}}{3\,\Omega} = \mathbf{3\,A}$$

Step 5: See Fig. 9.65. This circuit is the same as the first one considered in the development of Thévenin's theorem. A simple conversion indicates that the Thévenin circuits are, in fact, the same (Fig. 9.66).

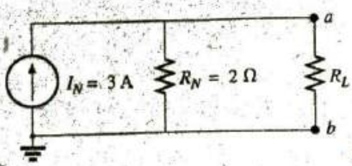


FIG. 9.65
Substituting the Norton equivalent circuit for the network external to the resistor R_L in Fig. 9.61.

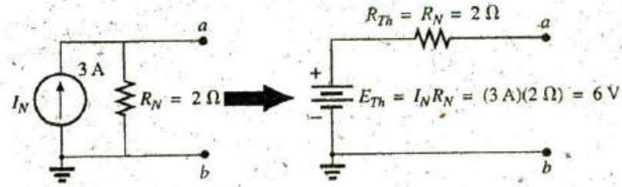


FIG. 9.66
Converting the Norton equivalent circuit in Fig. 9.65 to a Thévenin equivalent circuit.

EXAMPLE 9.12 Find the Norton equivalent circuit for the network external to the 9 Ω resistor in Fig. 9.67.

Solution:

Steps 1 and 2: See Fig. 9.68.

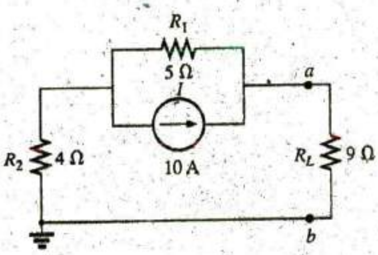


FIG. 9.67
Example 9.12.

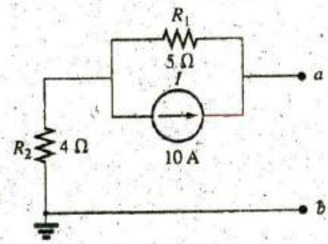


FIG. 9.68
Identifying the terminals of particular interest for the network in Fig. 9.67.

Step 3: See Fig. 9.69, and

$$R_N = R_1 + R_2 = 5\ \Omega + 4\ \Omega = \mathbf{9\ \Omega}$$

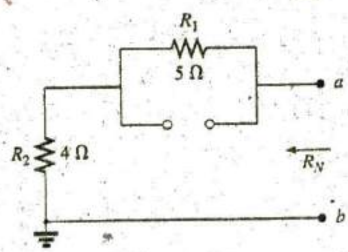


FIG. 9.69
Determining R_N for the network in Fig. 9.68.

Step 4: As shown in Fig. 9.70, the Norton current is the same as the current through the 4 Ω resistor. Applying the current divider rule gives

$$I_N = \frac{R_1 I}{R_1 + R_2} = \frac{(5\ \Omega)(10\text{ A})}{5\ \Omega + 4\ \Omega} = \frac{50\text{ A}}{9} = \mathbf{5.56\ A}$$

Step 5: See Fig. 9.71.

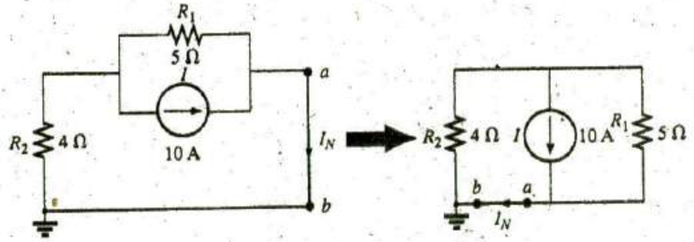


FIG. 9.70
Determining I_N for the network in Fig. 9.68.

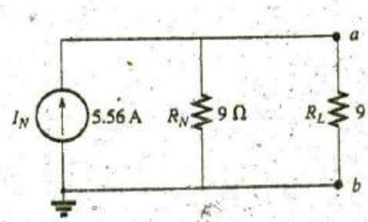


FIG. 9.71
Substituting the Norton equivalent circuit for the network external to the resistor R_L in Fig. 9.67.

EXAMPLE 9.13 (Two sources) Find the Norton equivalent circuit for the portion of the network to the left of *a-b* in Fig. 9.72.

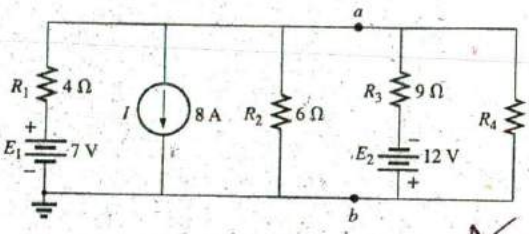


FIG. 9.72
Example 9.13.

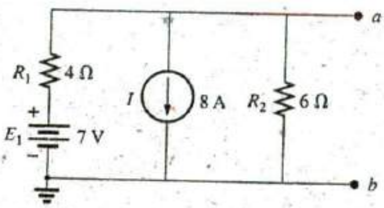


FIG. 9.73
Identifying the terminals of particular interest for the network in Fig. 9.72.

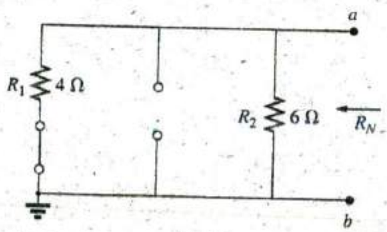


FIG. 9.74
Determining R_N for the network in Fig. 9.73.

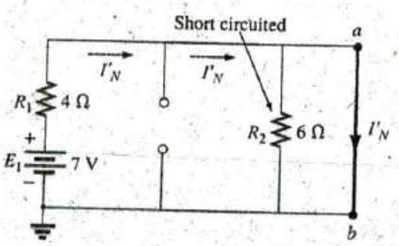


FIG. 9.75
Determining the contribution to I_N from the voltage source E_1.

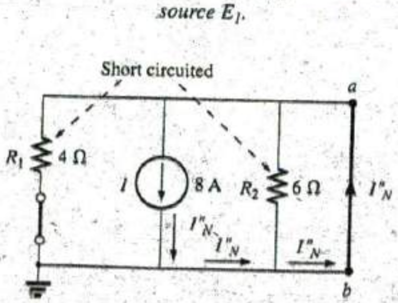


FIG. 9.76
Determining the contribution to I_N from the current source I.

Solution:

Steps 1 and 2: See Fig. 9.73.

Step 3: See Fig. 9.74, and

$$R_N = R_1 \parallel R_2 = 4\ \Omega \parallel 6\ \Omega = \frac{(4\ \Omega)(6\ \Omega)}{4\ \Omega + 6\ \Omega} = \frac{24\ \Omega}{10} = \mathbf{2.4\ \Omega}$$

Step 4: (Using superposition) For the 7 V battery (Fig. 9.75),

$$I'_N = \frac{E_1}{R_1} = \frac{7\ \text{V}}{4\ \Omega} = 1.75\ \text{A}$$

For the 8 A source (Fig. 9.76), we find that both R_1 and R_2 have been "short circuited" by the direct connection between *a* and *b*, and

$$I''_N = I = 8\ \text{A}$$

The result is

$$I_N = I''_N - I'_N = 8\ \text{A} - 1.75\ \text{A} = \mathbf{6.25\ A}$$

Step 5: See Fig. 9.77.

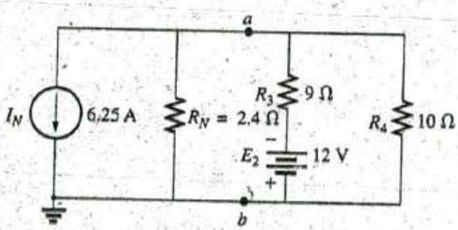


FIG. 9.77
Substituting the Norton equivalent circuit for the network to the left of terminals a-b in Fig. 9.72.

Experimental Procedure

The Norton current is measured in the same way as described for the short-circuit current (I_{sc}) for the Thévenin network. Since the Norton and Thévenin resistances are the same, the same procedures can be followed as described for the Thévenin network.

9.5 MAXIMUM POWER TRANSFER THEOREM

When designing a circuit, it is often important to be able to answer one of the following questions:

What load should be applied to a system to ensure that the load is receiving maximum power from the system?

Conversely:

For a particular load, what conditions should be imposed on the source to ensure that it will deliver the maximum power available?

Even if a load cannot be set at the value that would result in maximum power transfer, it is often helpful to have some idea of the value that will draw maximum power so that you can compare it to the load at hand. For instance, if a design calls for a load of 100 Ω, to ensure that the load receives maximum power, using a resistor of 1 Ω or 1 kΩ results in a power transfer that is much less than the maximum possible. However, using a load of 82 Ω or 120 Ω probably results in a fairly good level of power transfer.

Fortunately, the process of finding the load that will receive maximum power from a particular system is quite straightforward due to the **maximum power transfer theorem,** which states the following:

A load will receive maximum power from a network when its resistance is exactly equal to the Thévenin resistance of the network applied to the load. That is,

$$\boxed{R_L = R_{Th}} \qquad (9.2)$$

In other words, for the Thévenin equivalent circuit in Fig. 9.78, when the load is set equal to the Thévenin resistance, the load will receive maximum power from the network.

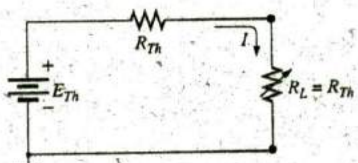


FIG. 9.78
Defining the conditions for maximum power to a load using the Thévenin equivalent circuit.

Using Fig. 9.78 with $R_L = R_{Th}$, we can determine the maximum power delivered to the load by first finding the current:

$$I_L = \frac{E_{Th}}{R_{Th} + R_L} = \frac{E_{Th}}{R_{Th} + R_{Th}} = \frac{E_{Th}}{2R_{Th}}$$

Then we substitute into the power equation:

$$P_L = I_L^2 R_L = \left(\frac{E_{Th}}{2R_{Th}}\right)^2 (R_{Th}) = \frac{E_{Th}^2 R_{Th}}{4R_{Th}^2}$$

and

$$\boxed{P_{L_{max}} = \frac{E_{Th}^2}{4R_{Th}}} \qquad (9.3)$$

To demonstrate that maximum power is indeed transferred to the load under the conditions defined above, consider the Thévenin equivalent circuit in Fig. 9.79.

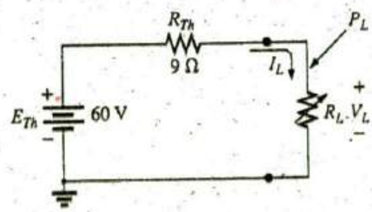


FIG. 9.79
Thévenin equivalent network to be used to validate the maximum power transfer theorem.

Before getting into detail, however, if you were to guess what value of R_L would result in maximum power transfer to R_L, you might think that the smaller the value of R_L, the better it is because the current reaches a maximum when it is squared in the power equation. The problem is, however, that in the equation $P_L = I_L^2 R_L$, the load resistance is a multiplier. As it gets smaller, it forms a smaller product. Then again, you might suggest larger values of R_L because the output voltage increases, and power is determined by $P_L = V_L^2/R_L$. This time, however, the load resistance is in the denominator of the equation and causes the resulting power to decrease. A balance must obviously be made between the load resistance and the resulting current or voltage. The following discussion shows that

maximum power transfer occurs when the load voltage and current are one-half their maximum possible values.

For the circuit in Fig. 9.79, the current through the load is determined by

$$I_L = \frac{E_{Th}}{R_{Th} + R_L} = \frac{60\text{ V}}{9\,\Omega + R_L}$$

The voltage is determined by

$$V_L = \frac{R_L E_{Th}}{R_L + R_{Th}} = \frac{R_L(60\text{ V})}{R_L + R_{Th}}$$

and the power by

$$P_L = I_L^2 R_L = \left(\frac{60\text{ V}}{9\,\Omega + R_L}\right)^2 (R_L) = \frac{3600 R_L}{(9\,\Omega + R_L)^2}$$

If we tabulate the three quantities versus a range of values for R_L from 0.1 Ω to 30 Ω, we obtain the results appearing in Table 9.1. Note in particular that when R_L is equal to the Thévenin resistance of 9 Ω, the power

TABLE 9.1

R_L (Ω)	P_L (W)	I_L (A)	V_L (V)
0.1	4.35	6.60	0.66
0.2	8.51	6.52	1.30
0.5	19.94	6.32	3.16
1	36.00	6.00	6.00
2	59.50	5.46	10.91
3	75.00	5.00	15.00
4	85.21	4.62	18.46
5	91.84	4.29	21.43
6	96.00	4.00	24.00
7	98.44 Increase	3.75 Decrease	26.25 Increase
8	99.65	3.53	28.23
9 (R_{Th})	100.00 (Maximum)	3.33 ($I_{max}/2$)	30.00 ($E_{Th}/2$)
10	99.72	3.16	31.58
11	99.00	3.00	33.00
12	97.96	2.86	34.29
13	96.69	2.73	35.46
14	95.27	2.61	36.52
15	93.75	2.50	37.50
16	92.16	2.40	38.40
17	90.53	2.31	39.23
18	88.89	2.22	40.00
19	87.24	2.14	40.71
20	85.61	2.07	41.38
25	77.86	1.77	44.12
30	71.00	1.54	46.15
40	59.98	1.22	48.98
100	30.30	0.55	55.05
500	6.95 Decrease	0.12 Decrease	58.94 Increase
1000	3.54	0.06	59.47

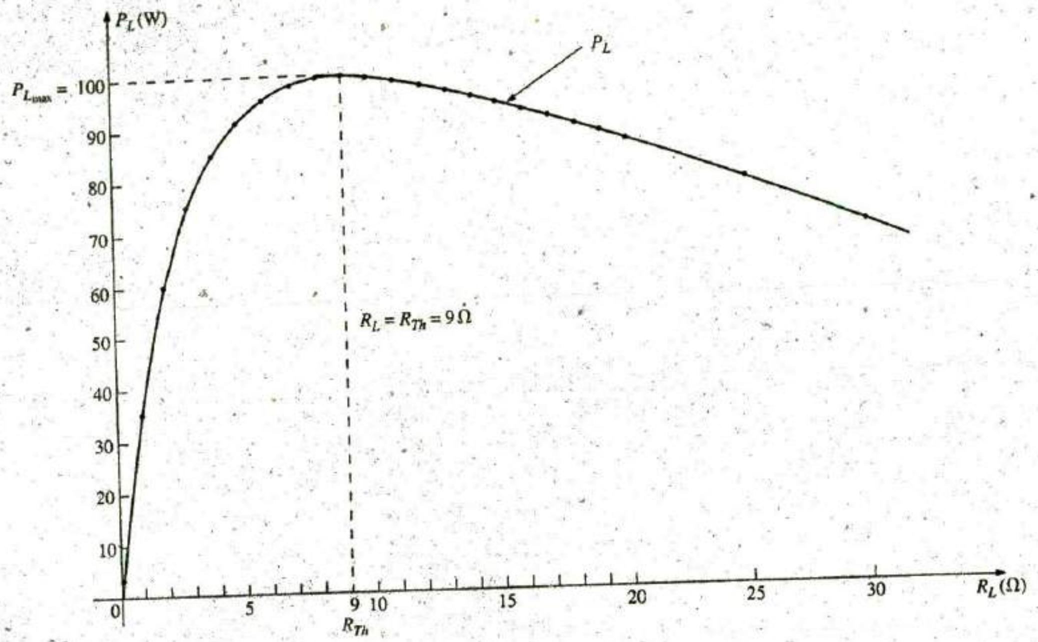


FIG. 9.80
P_L versus R_L for the network in Fig. 9.79.

has a maximum value of 100 W, the current is 3.33 A, or one-half its maximum value of 6.60 A (as would result with a short circuit across the output terminals), and the voltage across the load is 30 V, or one-half its maximum value of 60 V (as would result with an open circuit across its output terminals). As you can see, there is no question that maximum power is transferred to the load when the load equals the Thévenin value.

The power to the load versus the range of resistor values is provided in Fig. 9.80. Note in particular that for values of load resistance less than the Thévenin value, the change is dramatic as it approaches the peak value. However, for values greater than the Thévenin value, the drop is a great deal more gradual. This is important because it tells us the following:

If the load applied is less than the Thévenin resistance, the power to the load will drop off rapidly as it gets smaller. However, if the applied load is greater than the Thévenin resistance, the power to the load will not drop off as rapidly as it increases.

For instance, the power to the load is at least 90 W for the range of about 4.5 Ω to 9 Ω below the peak value, but it is at least the same level for a range of about 9 Ω to 18 Ω above the peak value. The range below the peak is 4.5 Ω, while the range above the peak is almost twice as much at 9 Ω. As mentioned above, if maximum transfer conditions cannot be established, at least we now know from Fig. 9.80 that any resistance relatively close to the Thévenin value results in a strong transfer of power. More distant values such as 1 Ω or 100 Ω result in much lower levels.

It is particularly interesting to plot the power to the load versus load resistance using a log scale, as shown in Fig. 9.81. Logarithms will be discussed in detail in Chapter 21, but for now notice that the spacing

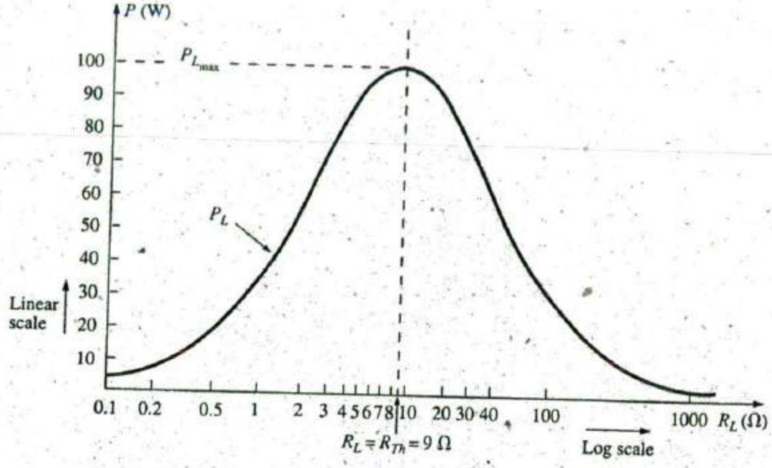


FIG. 9.81
P_L versus R_L for the network in Fig. 9.79.

between values of R_L is not linear, but the distance, between powers of ten (such as 0.1 and 1, 1 and 10, and 10 and 100) are all equal. The advantage of the log scale is that a wide resistance range can be plotted on a relatively small graph.

Note in Fig. 9.81 that a smooth, bell-shaped curve results that is symmetrical about the Thévenin resistance of 9 Ω. At 0.1 Ω, the power has dropped to about the same level as that at 1000 Ω, and at 1 Ω and 100 Ω, the power has dropped to the neighborhood of 30 W.

Although all of the above discussion centers on the power to the load, it is important to remember the following:

The total power delivered by a supply such as E_{Th} is absorbed by both the Thévenin equivalent resistance and the load resistance. Any power delivered by the source that does not get to the load is lost to the Thévenin resistance.

Under maximum power conditions, only half the power delivered by the source gets to the load. Now, that sounds disastrous, but remember that we are starting out with a fixed Thévenin voltage and resistance, and the above simply tells us that we must make the two resistance levels equal if we want maximum power to the load. On an efficiency basis, we are working at only a 50% level, but we are content because *we are getting maximum power out of our system.*

The dc operating efficiency is defined as the ratio of the power delivered to the load (P_L) to the power delivered by the source (P_s). That is,

$$\boxed{\eta\% = \frac{P_L}{P_s} \times 100\%} \qquad \textbf{(9.4)}$$

For the situation where $R_L = R_{Th}$,

$$\eta\% = \frac{I_L^2 R_L}{I_L^2 R_T} \times 100\% = \frac{R_L}{R_T} \times 100\% = \frac{R_{Th}}{R_{Th} + R_{Th}} \times 100\%$$

$$= \frac{R_{Th}}{2R_{Th}} \times 100\% = \frac{1}{2} \times 100\% = \mathbf{50\%}$$

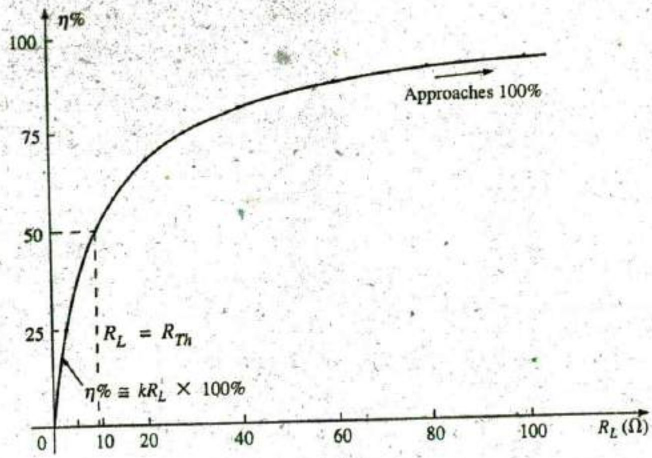


FIG. 9.82
Efficiency of operation versus increasing values of R_L.

For the circuit in Fig. 9.79, if we plot the efficiency of operation versus load resistance, we obtain the plot in Fig. 9.82, which clearly shows that the efficiency continues to rise to a 100% level as R_L gets larger. Note in particular that the efficiency is 50% when $R_L = R_{Th}$.

To ensure that you completely understand the effect of the maximum power transfer theorem and the efficiency criteria, consider the circuit in Fig. 9.83, where the load resistance is set at 100 Ω and the power to the Thévenin resistance and to the load are calculated as follows:

$$I_L = \frac{E_{Th}}{R_{Th} + R_L} = \frac{60\text{ V}}{9\ \Omega + 100\ \Omega} = \frac{60\text{ V}}{109\ \Omega} = 550.5\text{ mA}$$

with $$P_{R_{Th}} = I_L^2 R_{Th} = (550.5\text{ mA})^2(9\ \Omega) \cong \mathbf{2.73\ W}$$

and $$P_L = I_L^2 R_L = (550.5\text{ mA})^2(100\ \Omega) \cong \mathbf{30.3\ W}$$

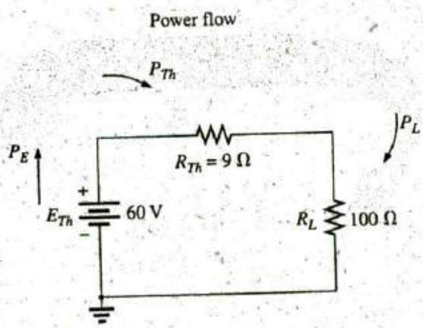


FIG. 9.83
Examining a circuit with high efficiency but a relatively low level of power to the load.

The results clearly show that most of the power supplied by the battery is getting to the load—a desirable attribute on an efficiency basis. However, the power getting to the load is only 30.3 W compared to the 100 W obtained under maximum power conditions. In general, therefore, the following guidelines apply:

If efficiency is the overriding factor, then the load should be much larger than the internal resistance of the supply. If maximum power transfer is desired and efficiency less of a concern, then the conditions dictated by the maximum power transfer theorem should be applied.

A relatively low efficiency of 50% can be tolerated in situations where power levels are relatively low, such as in a wide variety of electronic systems, where maximum power transfer for the given system is usually more important. However, when large power levels are involved, such as at generating plants, efficiencies of 50% cannot be tolerated. In fact, a great deal of expense and research is dedicated to raising power generating and transmission efficiencies a few percentage points. Raising an efficiency level of a 10 MkW power plant from 94% to 95% (a 1% increase) can save 0.1 MkW, or 100 million watts, of power—an enormous saving.

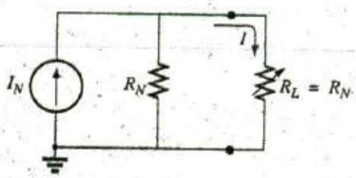


FIG 9.84
Defining the conditions for maximum power to a load using the Norton equivalent circuit.

In all of the above discussions, the effect of changing the load was discussed for a fixed Thévenin resistance. Looking at the situation from a different viewpoint, we can say

if the load resistance is fixed and does not match the applied Thévenin equivalent resistance, then some effort should be made (if possible) to redesign the system so that the Thévenin equivalent resistance is closer to the fixed applied load.

In other words, if a designer faces a situation where the load resistance is fixed, he or she should investigate whether the supply section should be replaced or redesigned to create a closer match of resistance levels to produce higher levels of power to the load.

For the Norton equivalent circuit in Fig. 9.84, maximum power will be delivered to the load when

$$\boxed{R_L = R_N} \qquad (9.5)$$

This result [Eq. (9.5)] will be used to its fullest advantage in the analysis of transistor networks, where the most frequently applied transistor circuit model uses a current source rather than a voltage source.

For the Norton circuit in Fig. 9.84,

$$\boxed{P_{L_{max}} = \frac{I_N^2 R_N}{4}} \quad \text{(W)} \qquad (9.6)$$

EXAMPLE 9.14 A dc generator, battery, and laboratory supply are connected to resistive load R_L in Fig. 9.85.

a. For each, determine the value of R_L for maximum power transfer to R_L.
b. Under maximum power conditions, what are the current level and the power to the load for each configuration?
c. What is the efficiency of operation for each supply in part (b)?
d. If a load of 1 kΩ were applied to the laboratory supply, what would the power delivered to the load be? Compare your answer to the level of part (b). What is the level of efficiency?
e. For each supply, determine the value of R_L for 75% efficiency.

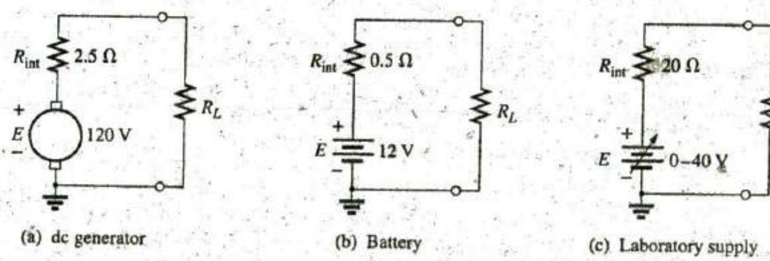


FIG. 9.85
Example 9.14.

Solutions:

a. For the dc generator,

$$R_L = R_{Th} = R_{int} = \mathbf{2.5\ \Omega}$$

For the 12 V car battery,

$$R_L = R_{Th} = R_{int} = \mathbf{0.05\ \Omega}$$

For the dc laboratory supply,

$$R_L = R_{Th} = R_{int} = \mathbf{20\ \Omega}$$

b. For the dc generator,

$$P_{L_{max}} = \frac{E_{Th}^2}{4R_{Th}} = \frac{E^2}{4R_{int}} = \frac{(120\text{ V})^2}{4(2.5\ \Omega)} = \mathbf{1.44\ kW}$$

For the 12 V car battery,

$$P_{L_{max}} = \frac{E_{Th}^2}{4R_{Th}} = \frac{E^2}{4R_{int}} = \frac{(12\text{ V})^2}{4(0.05\ \Omega)} = \mathbf{720\ W}$$

For the dc laboratory supply,

$$P_{L_{max}} = \frac{E_{Th}^2}{4R_{Th}} = \frac{E^2}{4R_{int}} = \frac{(40\text{ V})^2}{4(20\ \Omega)} = \mathbf{20\ W}$$

c. They are all operating under a 50% efficiency level because $R_L = R_{Th}$.

d. The power to the load is determined as follows:

$$I_L = \frac{E}{R_{int} + R_L} = \frac{40\text{ V}}{20\ \Omega + 1000\ \Omega} = \frac{40\text{ V}}{1020\ \Omega} = 39.22\text{ mA}$$

and $\quad P_L = I_L^2 R_L = (39.22\text{ mA})^2(1000\ \Omega) = \mathbf{1.54\ W}$

The power level is significantly less than the 20 W achieved in part (b). The efficiency level is

$$\eta\% = \frac{P_L}{P_s} \times 100\% = \frac{1.54\text{ W}}{EI_s} \times 100\% = \frac{1.54\text{ W}}{(40\text{ V})(39.22\text{ mA})} \times 100\%$$

$$= \frac{1.54\text{ W}}{1.57\text{ W}} \times 100\% = \mathbf{98.09\%}$$

which is markedly higher than achieved under maximum power conditions—albeit at the expense of the power level.

e. For the dc generator,

$$\eta = \frac{P_o}{P_s} = \frac{R_L}{R_{Th} + R_L} \quad (\eta \text{ in decimal form})$$

and

$$\eta = \frac{R_L}{R_{Th} + R_L}$$

$$\eta(R_{Th} + R_L) = R_L$$

$$\eta R_{Th} + \eta R_L = R_L$$

$$R_L(1 - \eta) = \eta R_{Th}$$

and

$$\boxed{R_L = \frac{\eta R_{Th}}{1 - \eta}} \qquad (9.7)$$

$$R_L = \frac{0.75(2.5\ \Omega)}{1 - 0.75} = \mathbf{7.5\ \Omega}$$

For the battery,

$$R_L = \frac{0.75(0.05\ \Omega)}{1 - 0.75} = \mathbf{0.15\ \Omega}$$

For the laboratory supply,

$$R_L = \frac{0.75(20\ \Omega)}{1 - 0.75} = \mathbf{60\ \Omega}$$

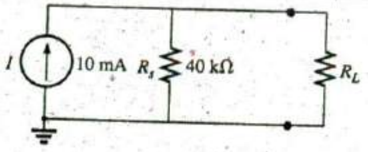


FIG. 9.86
Example 9.15.

EXAMPLE 9.15 The analysis of a transistor network resulted in the reduced equivalent in Fig. 9.86.

a. Find the load resistance that will result in maximum power transfer to the load, and find the maximum power delivered.
b. If the load were changed to 68 kΩ, would you expect a fairly high level of power transfer to the load based on the results of part (a)? What would the new power level be? Is your initial assumption verified?
c. If the load were changed to 8.2 kΩ, would you expect a fairly high level of power transfer to the load based on the results of part (a)? What would the new power level be? Is your initial assumption verified?

Solutions:

a. Replacing the current source by an open-circuit equivalent results in

$$R_{Th} = R_s = 40\ \text{k}\Omega$$

Restoring the current source and finding the open-circuit voltage at the output terminals results in

$$E_{Th} = V_{oc} = IR_s = (10\ \text{mA})(40\ \text{k}\Omega) = 400\ \text{V}$$

For maximum power transfer to the load,

$$R_L = R_{Th} = \mathbf{40\ k\Omega}$$

with a maximum power level of

$$P_{L_{max}} = \frac{E_{Th}^2}{4R_{Th}} = \frac{(400\ \text{V})^2}{4(40\ \text{k}\Omega)} = \mathbf{1\ W}$$

b. Yes, because the 68 kΩ load is greater (note Fig. 9.80) than the 40 kΩ load, but relatively close in magnitude.

$$I_L = \frac{E_{Th}}{R_{Th} + R_L} = \frac{400\ \text{V}}{40\ \text{k}\Omega + 68\ \text{k}\Omega} = \frac{400}{108\ \text{k}\Omega} \cong 3.7\ \text{mA}$$

$$P_L = I_L^2 R_L = (3.7\ \text{mA})^2(68\ \text{k}\Omega \cong \mathbf{0.93\ W}$$

Yes, the power level of 0.93 W compared to the 1 W level of part (a) verifies the assumption.

c. No, 8.2 kΩ is quite a bit less (note Fig. 9.80) than the 40 kΩ value.

$$I_L = \frac{E_{Th}}{R_{Th} + R_L} = \frac{400\ \text{V}}{40\ \text{k}\Omega + 8.2\ \text{k}\Omega} = \frac{400\ \text{V}}{48.2\ \text{k}\Omega} \cong 8.3\ \text{mA}$$

$$P_L = I_L^2 R_L = (8.3\ \text{mA})^2(8.2\ \text{k}\Omega) \cong \mathbf{0.57\ W}$$

Yes, the power level of 0.57 W compared to the 1 W level of part (a) verifies the assumption.

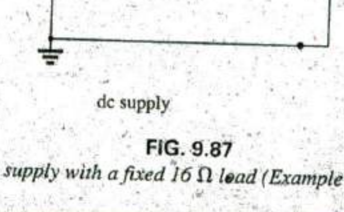


FIG. 9.87
dc supply with a fixed 16 Ω load (Example 9.16).

EXAMPLE 9.16 In Fig. 9.87, a fixed load of 16 Ω is applied to a 48 V supply with an internal resistance of 36 Ω.

a. For the conditions in Fig. 9.87, what is the power delivered to the load and lost to the internal resistance of the supply?
b. If the designer has some control over the internal resistance level of the supply, what value should he or she make it for maximum power to the load? What is the maximum power to the load? How does it compare to the level obtained in part (a)?
c. Without making a single calculation, find the value that would result in more power to the load if the designer could change the internal resistance to 22 Ω or 8.2 Ω. Verify your conclusion by calculating the power to the load for each value.

Solutions:

a. $$I_L = \frac{E}{R_s + R_L} = \frac{48\text{ V}}{36\ \Omega + 16\ \Omega} = \frac{48\text{ V}}{52\ \Omega} = 923.1\text{ mA}$$

$$P_{R_s} = I_L^2 R_s = (923.1\text{ mA})^2(36\ \Omega) = \mathbf{30.68\ W}$$

$$P_L = I_L^2 R_L = (923.1\text{ mA})^2(16\ \Omega) = \mathbf{13.63\ W}$$

b. Be careful here. The quick response is to make the source resistance R_s equal to the load resistance to satisfy the criteria of the maximum power transfer theorem. However, this is a totally different type of problem from what was examined earlier in this section. If the load is fixed, the smaller the source resistance R_s, the more applied voltage will reach the load and the less will be lost in the internal series resistor. In fact, the source resistance should be made as small as possible. If zero ohms were possible for R_s, the voltage across the load would be the full supply voltage, and the power delivered to the load would equal

$$P_L = \frac{V_L^2}{R_L} = \frac{(48\text{ V})^2}{16\ \Omega} = \mathbf{144\ W}$$

which is more than 10 times the value with a source resistance of 36 Ω.

c. Again, forget the impact in Fig. 9.80: The smaller the source resistance, the greater is the power to the fixed 16 Ω load. Therefore, the 8.2 Ω resistance level results in a higher power transfer to the load than the 22 Ω resistor.

For $R_s = 8.2\ \Omega$

$$I_L = \frac{E}{R_s + R_L} = \frac{48\text{ V}}{8.2\ \Omega + 16\ \Omega} = \frac{48\text{ V}}{24.2\ \Omega} = 1.983\text{ A}$$

and $P_L = I_L^2 R_L = (1.983\text{ A})^2(16\ \Omega) \cong \mathbf{62.92\ W}$

For $R_s = 22\ \Omega$

$$I_L = \frac{E}{R_s + R_L} = \frac{48\text{ V}}{22\ \Omega + 16\ \Omega} = \frac{48\text{ V}}{38\ \Omega} = 1.263\text{ A}$$

and $P_L = I_L^2 R_L = (1.263\text{ A})^2(16\ \Omega) \cong \mathbf{25.52\ W}$

EXAMPLE 9.17 Given the network in Fig. 9.88, find the value of R_L for maximum power to the load, and find the maximum power to the load.

Solution: The Thévenin resistance is determined from Fig. 9.89:

$$R_{Th} = R_1 + R_2 + R_3 = 3\ \Omega + 10\ \Omega + 2\ \Omega = 15\ \Omega$$

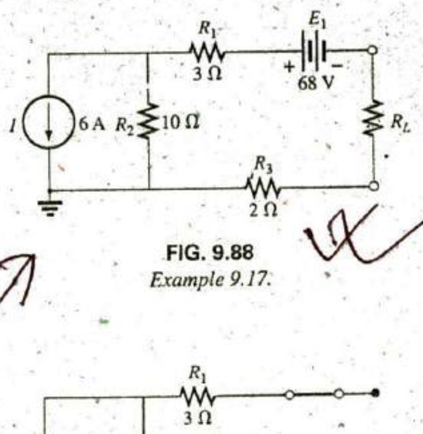


FIG. 9.88
Example 9.17.

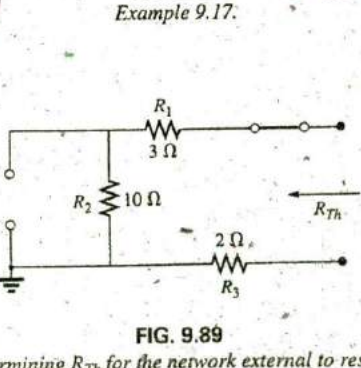


FIG. 9.89
Determining R_{Th} for the network external to resistor R_L in Fig. 9.88.

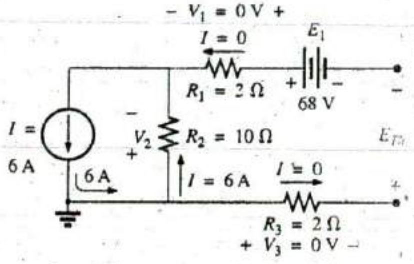


FIG. 9.90
Determining E_{Th} for the network external to resistor R_L in Fig. 9.88.

so that $$R_L = R_{Th} = 15\ \Omega$$

The Thévenin voltage is determined using Fig. 9.90, where

$$V_1 = V_3 = 0\text{ V} \quad\text{and}\quad V_2 = I_2R_2 = IR_2 = (6\text{ A})(10\ \Omega) = 60\text{ V}$$

Applying Kirchhoff's voltage law gives

$$-V_2 - E + E_{Th} = 0$$

and $$E_{Th} = V_2 + E = 60\text{ V} + 68\text{ V} = \mathbf{128\ V}$$

with the maximum power equal to

$$P_{L_{max}} = \frac{E_{Th}^2}{4R_{Th}} = \frac{(128\text{ V})^2}{4(15\text{ k}\Omega)} = \mathbf{273.07\ W}$$

9.6 MILLMAN'S THEOREM

Through the application of **Millman's theorem,** any number of parallel voltage sources can be reduced to one. In Fig. 9.91, for example, the three voltage sources can be reduced to one. This permits finding the current through or voltage across R_L without having to apply a method such as mesh analysis, nodal analysis, superposition, and so on. The theorem can best be described by applying it to the network in Fig. 9.91. Basically, three steps are included in its application.

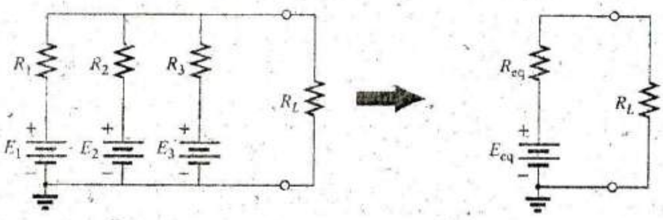


FIG. 9.91
Demonstrating the effect of applying Millman's theorem.

Step 1: Convert all voltage sources to current sources as outlined in Section 8.3. This is performed in Fig. 9.92 for the network in Fig. 9.91.

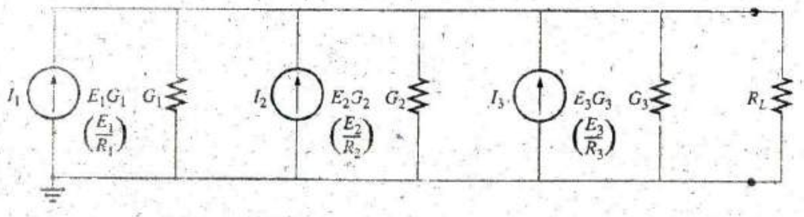


FIG. 9.92
Converting all the sources in Fig. 9.91 to current sources.

Step 2: Combine parallel current sources as described in Section 8.4. The resulting network is shown in Fig. 9.93, where

$$I_T = I_1 + I_2 + I_3 \qquad \text{and} \qquad G_T = G_1 + G_2 + G_3$$

Step 3: Convert the resulting current source to a voltage source, and the desired single-source network is obtained, as shown in Fig. 9.94.

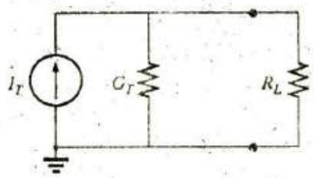


FIG. 9.93
Reducing all the current sources in Fig. 9.92 to a single current source.

In general, Millman's theorem states that for any number of parallel voltage sources,

$$E_{eq} = \frac{I_T}{G_T} = \frac{\pm I_1 \pm I_2 \pm I_3 \pm \cdots \pm I_N}{G_1 + G_2 + G_3 + \cdots + G_N}$$

or

$$E_{eq} = \frac{\pm E_1G_1 \pm E_2G_2 \pm E_3G_3 \pm \cdots \pm E_NG_N}{G_1 + G_2 + G_3 + \cdots + G_N} \tag{9.8}$$

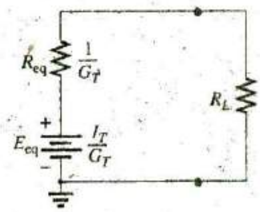


FIG. 9.94
Converting the current source in Fig. 9.93 to a voltage source.

The plus-and-minus signs appear in Eq. (9.8) to include those cases where the sources may not be supplying energy in the same direction. (Note Example 9.18.)

The equivalent resistance is

$$R_{eq} = \frac{1}{G_T} = \frac{1}{G_1 + G_2 + G_3 + \cdots + G_N} \tag{9.9}$$

In terms of the resistance values,

$$E_{eq} = \frac{\pm \dfrac{E_1}{R_1} \pm \dfrac{E_2}{R_2} \pm \dfrac{E_3}{R_3} \pm \cdots \pm \dfrac{E_N}{R_N}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \cdots + \dfrac{1}{R_N}} \tag{9.10}$$

and

$$R_{eq} = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \cdots + \dfrac{1}{R_N}} \tag{9.11}$$

Because of the relatively few direct steps required, you may find it easier to apply each step rather than memorizing and employing Eqs. (9.8) through (9.11).

EXAMPLE 9.18 Using Millman's theorem, find the current through and voltage across the resistor R_L in Fig. 9.95.

Solution: By Eq. (9.10),

$$E_{eq} = \frac{+\dfrac{E_1}{R_1} - \dfrac{E_2}{R_2} + \dfrac{E_3}{R_3}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}$$

The minus sign is used for E_2/R_2 because that supply has the opposite polarity of the other two. The chosen reference direction is therefore that of E_1 and E_3. The total conductance is unaffected by the direction, and

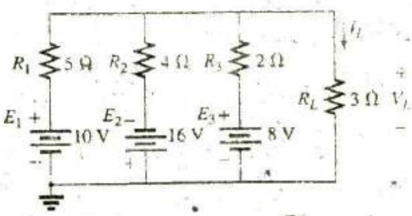


FIG. 9.95
Example 9.18.

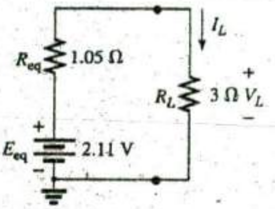


FIG. 9.96
The result of applying Millman's theorem to the network in Fig. 9.95.

$$E_{eq} = \frac{+\dfrac{10\text{ V}}{5\,\Omega} - \dfrac{16\text{ V}}{4\,\Omega} + \dfrac{8\text{ V}}{2\,\Omega}}{\dfrac{1}{5\,\Omega} + \dfrac{1}{4\,\Omega} + \dfrac{1}{2\,\Omega}} = \frac{2\text{ A} - 4\text{ A} + 4\text{ A}}{0.2\text{ S} + 0.25\text{ S} + 0.5\text{ S}}$$

$$= \frac{2\text{ A}}{0.95\text{ S}} = \mathbf{2.11\text{ V}}$$

with $$R_{eq} = \frac{1}{\dfrac{1}{5\,\Omega} + \dfrac{1}{4\,\Omega} + \dfrac{1}{2\,\Omega}} = \frac{1}{0.95\text{ S}} = \mathbf{1.05\,\Omega}$$

The resultant source is shown in Fig. 9.96, and

$$I_L = \frac{2.11\text{ V}}{1.05\,\Omega + 3\,\Omega} = \frac{2.11\text{ V}}{4.05\,\Omega} = \mathbf{0.52\text{ A}}$$

with $$V_L = I_L R_L = (0.52\text{ A})(3\,\Omega) = \mathbf{1.56\text{ V}}$$

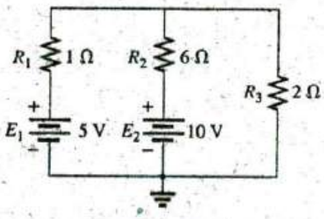


FIG. 9.97
Example 9.19.

EXAMPLE 9.19 Let us now consider the type of problem encountered in the introduction to mesh and nodal analysis in Chapter 8. Mesh analysis was applied to the network of Fig. 9.97 (Example 8.12). Let us now use Millman's theorem to find the current through the 2 Ω resistor and compare the results.

Solutions:

a. Let us first apply each step and, in the (b) solution, Eq. (9.10). Converting sources yields Fig. 9.98. Combining sources and parallel conductance branches (Fig. 9.99) yields

$$I_T = I_1 + I_2 = 5\text{ A} + \frac{5}{3}\text{ A} = \frac{15}{3}\text{ A} + \frac{5}{3}\text{ A} = \frac{20}{3}\text{ A}$$

$$G_T = G_1 + G_2 = 1\text{ S} + \frac{1}{6}\text{ S} = \frac{6}{6}\text{ S} + \frac{1}{6}\text{ S} = \frac{7}{6}\text{ S}$$

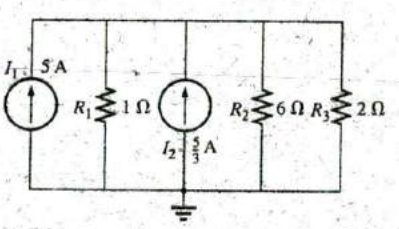


FIG. 9.98
Converting the sources in Fig. 9.97 to current sources.

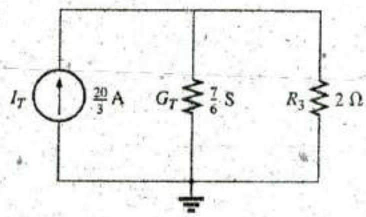


FIG. 9.99
Reducing the current sources in Fig. 9.98 to a single source.

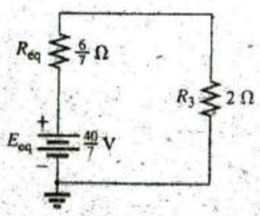


FIG. 9.100
Converting the current source in Fig. 9.99 to a voltage source.

Converting the current source to a voltage source (Fig. 9.100), we obtain

$$E_{eq} = \frac{I_T}{G_T} = \frac{\dfrac{20}{3}\text{ A}}{\dfrac{7}{6}\text{ S}} = \frac{(6)(20)}{(3)(7)}\text{ V} = \frac{40}{7}\text{ V}$$

and $$R_{eq} = \frac{1}{G_T} = \frac{1}{\dfrac{7}{6}\text{ S}} = \frac{6}{7}\,\Omega$$

so that

$$I_{2\Omega} = \frac{E_{eq}}{R_{eq} + R_3} = \frac{\frac{40}{7}\,\text{V}}{\frac{6}{7}\,\Omega + 2\,\Omega} = \frac{\frac{40}{7}\,\text{V}}{\frac{6}{7}\,\Omega + \frac{14}{7}\,\Omega} = \frac{40\,\text{V}}{20\,\Omega} = \mathbf{2\,A}$$

which agrees with the result obtained in Example 8.18.

b. Let us now simply apply the proper equation, Eq. (9.10):

$$E_{eq} = \frac{+\frac{5\,\text{V}}{1\,\Omega} + \frac{10\,\text{V}}{6\,\Omega}}{\frac{1}{1\,\Omega} + \frac{1}{6\,\Omega}} = \frac{\frac{30\,\text{V}}{6\,\Omega} + \frac{10\,\text{V}}{6\,\Omega}}{\frac{6}{6\,\Omega} + \frac{1}{6\,\Omega}} = \mathbf{\frac{40}{7}\,V}$$

and

$$R_{eq} = \frac{1}{\frac{1}{1\,\Omega} + \frac{1}{6\,\Omega}} = \frac{1}{\frac{6}{6\,\Omega} + \frac{1}{6\,\Omega}} = \frac{1}{\frac{7}{6}\,\text{S}} = \mathbf{\frac{6}{7}\,\Omega}$$

which are the same values obtained above.

The dual of Millman's theorem (Fig. 9.91) appears in Fig. 9.101. It can be shown that I_{eq} and R_{eq}, as in Fig. 9.101, are given by

$$\boxed{I_{eq} = \frac{\pm I_1R_1 \pm I_2R_2 \pm I_3R_3}{R_1 + R_2 + R_3}} \quad \textbf{(9.12)}$$

and

$$\boxed{R_{eq} = R_1 + R_2 + R_3} \quad \textbf{(9.13)}$$

The derivation appears as a problem at the end of the chapter.

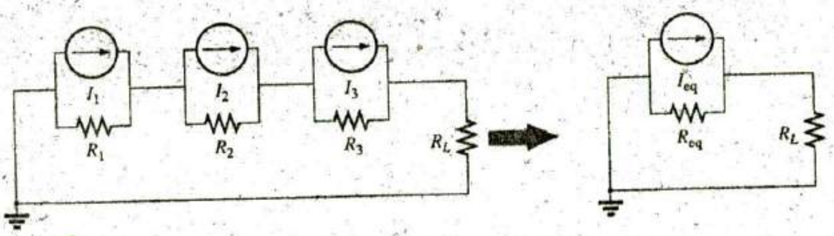


FIG. 9.101
The dual effect of Millman's theorem.

9.7 SUBSTITUTION THEOREM

The **substitution theorem** states the following:

If the voltage across and the current through any branch of a dc bilateral network are known, this branch can be replaced by any combination of elements that will maintain the same voltage across and current through the chosen branch.

More simply, the theorem states that for branch equivalence, the terminal voltage and current must be the same. Consider the circuit in Fig. 9.102, in which the voltage across and current through the branch

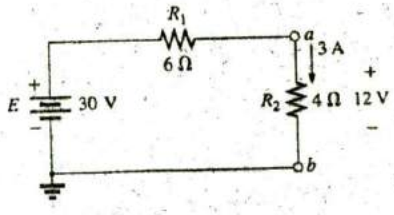


FIG. 9.102
Demonstrating the effect of the substitution theorem.

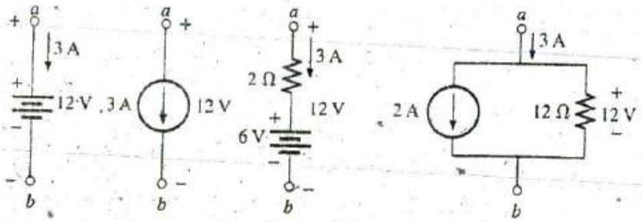


FIG. 9.103
Equivalent branches for the branch a-b in Fig. 9.102.

a-b are determined. Through the use of the substitution theorem, a number of equivalent *a-a'* branches are shown in Fig. 9.103.

Note that for each equivalent, the terminal voltage and current are the same. Also consider that the response of the remainder of the circuit in Fig. 9.102 is unchanged by substituting any one of the equivalent branches. As demonstrated by the single-source equivalents in Fig. 9.103, *a known potential difference and current in a network can be replaced by an ideal voltage source and current source, respectively.*

Understand that this theorem cannot be used to *solve* networks with two or more sources that are not in series or parallel. For it to be applied, a potential difference or current value must be known or found using one of the techniques discussed earlier. One application of the theorem is shown in Fig. 9.104. Note that in the figure the known potential difference V was replaced by a voltage source, permitting the isolation of the portion of the network including R_3, R_4, and R_5. Recall that this was basically the approach used in the analysis of the ladder network as we worked our way back toward the terminal resistance R_5.

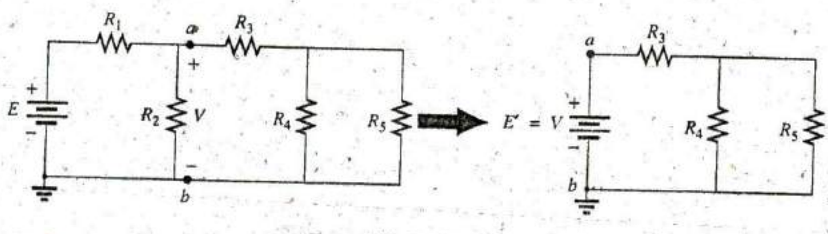


FIG. 9.104
Demonstrating the effect of knowing a voltage at some point in a complex network.

The current source equivalence of the above is shown in Fig. 9.105, where a known current is replaced by an ideal current source, permitting the isolation of R_4 and R_5.

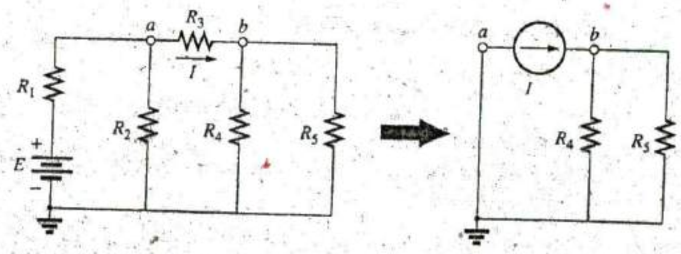


FIG. 9.105
Demonstrating the effect of knowing a current at some point in a complex network.

Recall from the discussion of bridge networks that $V = 0$ and $I = 0$ were replaced by a short circuit and an open circuit, respectively. This substitution is a very specific application of the substitution theorem.

9.8 RECIPROCITY THEOREM

The **reciprocity theorem** is applicable only to single-source networks. It is, therefore, not a theorem used in the analysis of multisource networks described thus far. The theorem states the following:

The current I in any branch of a network due to a single voltage source E anywhere else in the network will equal the current through the branch in which the source was originally located if the source is placed in the branch in which the current I was originally measured.

In other words, the location of the voltage source and the resulting current may be interchanged without a change in current. The theorem requires that the polarity of the voltage source have the same correspondence with the direction of the branch current in each position.

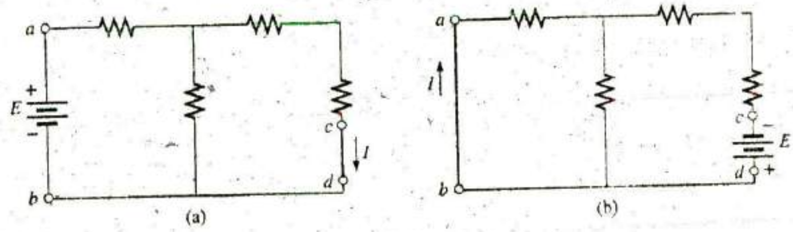


FIG. 9.106
Demonstrating the impact of the reciprocity theorem.

In the representative network in Fig. 9.106(a), the current I due to the voltage source E was determined. If the position of each is interchanged as shown in Fig. 9.106(b), the current I will be the same value as indicated. To demonstrate the validity of this statement and the theorem, consider the network in Fig. 9.107, in which values for the elements of Fig. 9.106(a) have been assigned.

The total resistance is

$$R_T = R_1 + R_2 \| (R_3 + R_4) = 12\,\Omega + 6\,\Omega \| (2\,\Omega + 4\,\Omega)$$
$$= 12\,\Omega + 6\,\Omega \| 6\,\Omega = 12\,\Omega + 3\,\Omega = 15\,\Omega$$

and $$I_s = \frac{E}{R_T} = \frac{45\text{ V}}{15\,\Omega} = 3\text{ A}$$

with $$I = \frac{3\text{ A}}{2} = \mathbf{1.5\text{ A}}$$

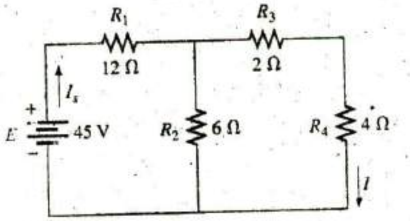


FIG. 9.107
Finding the current I due to a source E.

For the network in Fig. 9.108, which corresponds to that in Fig. 9.106(b), we find

$$R_T = R_4 + R_3 + R_1 \| R_2$$
$$= 4\,\Omega + 2\,\Omega + 12\,\Omega \| 6\,\Omega = 10\,\Omega$$

and $$I_s = \frac{E}{R_T} = \frac{45\text{ V}}{10\,\Omega} = 4.5\text{ A}$$

so that $$I = \frac{(6\,\Omega)(4.5\text{ A})}{12\,\Omega + 6\,\Omega} = \frac{4.5\text{ A}}{3} = \mathbf{1.5\text{ A}}$$

which agrees with the above.

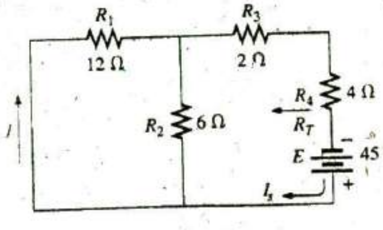


FIG. 9.108
Interchanging the location of E and I of Fig. 9.107 to demonstrate the validity of the reciprocity theorem.

The uniqueness and power of this theorem can best be demonstrated by considering a complex, single-source network such as the one shown in Fig. 9.109.

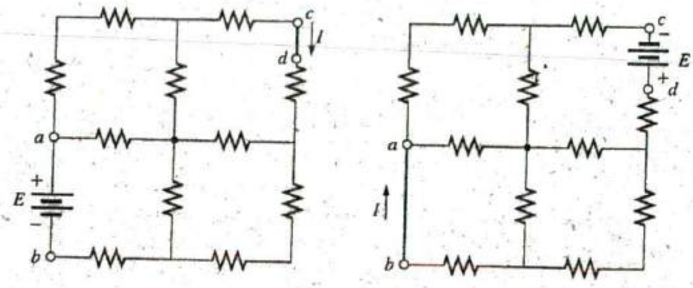


FIG. 9.109
Demonstrating the power and uniqueness of the reciprocity theorem.

9.9 COMPUTER ANALYSIS

Once you understand the mechanics of applying a software package or language, the opportunity to be creative and innovative presents itself. Through years of exposure and trial-and-error experiences, professional programmers develop a catalog of innovative techniques that are not only functional but very interesting and truly artistic in nature. Now that some of the basic operations associated with PSpice have been introduced, a few innovative maneuvers will be made in the examples to follow.

PSpice

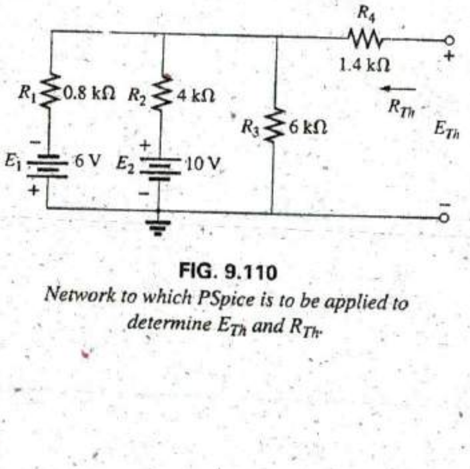


FIG. 9.110
Network to which PSpice is to be applied to determine E_{Th} and R_{Th}.

Thévenin's Theorem The application of Thévenin's theorem requires an interesting maneuver to determine the Thévenin resistance. It is a maneuver, however, that has application beyond Thévenin's theorem whenever a resistance level is required. The network to be analyzed appears in Fig. 9.110 and is the same one analyzed in Example 9.10 (Fig. 9.48).

Since PSpice is not set up to measure resistance levels directly, a 1 A current source can be applied as shown in Fig. 9.111, and Ohm's law can be used to determine the magnitude of the Thévenin resistance in the following manner:

$$|R_{Th}| = \left|\frac{V_s}{I_s}\right| = \left|\frac{V_s}{1\text{ A}}\right| = |V_s| \tag{9.14}$$

In Eq. (9.14), since $I_s = 1$ A, the magnitude of R_{Th} in ohms is the same as the magnitude of the voltage V_s (in volts) across the current source. The result is that when the voltage across the current source is displayed, it can be read as ohms rather than volts.

When PSpice is applied, the network appears as shown in Fig. 9.111. Flip the voltage source E_1 and the current source by right-clicking on the source and choosing the **Mirror Vertically** option. Set both voltage sources to zero through the **Display Properties** dialog box obtained by double-clicking on the source symbol. The result of the **Bias Point** simulation is 2 kV across the current source. The Thévenin resistance is therefore 2 kΩ between the two terminals of the network to the left of the current source (to match the results of Example 9.10). In total, by

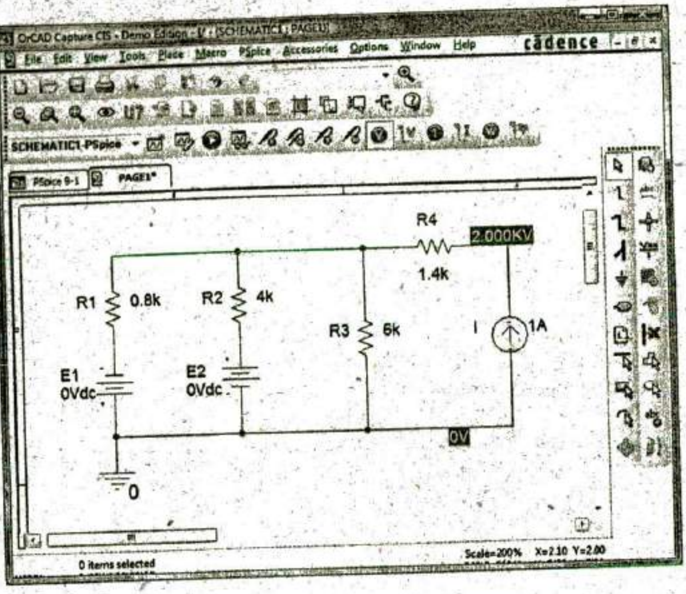


FIG. 9.111

Using PSpice to determine the Thévenin resistance of a network through the application of a 1 A current source.

setting the voltage source to 0 V, we have dictated that the voltage is the same at both ends of the voltage source, replicating the effect of a short-circuit connection between the two points.

For the open-circuit Thévenin voltage between the terminals of interest, the network must be constructed as shown in Fig. 9.112. The

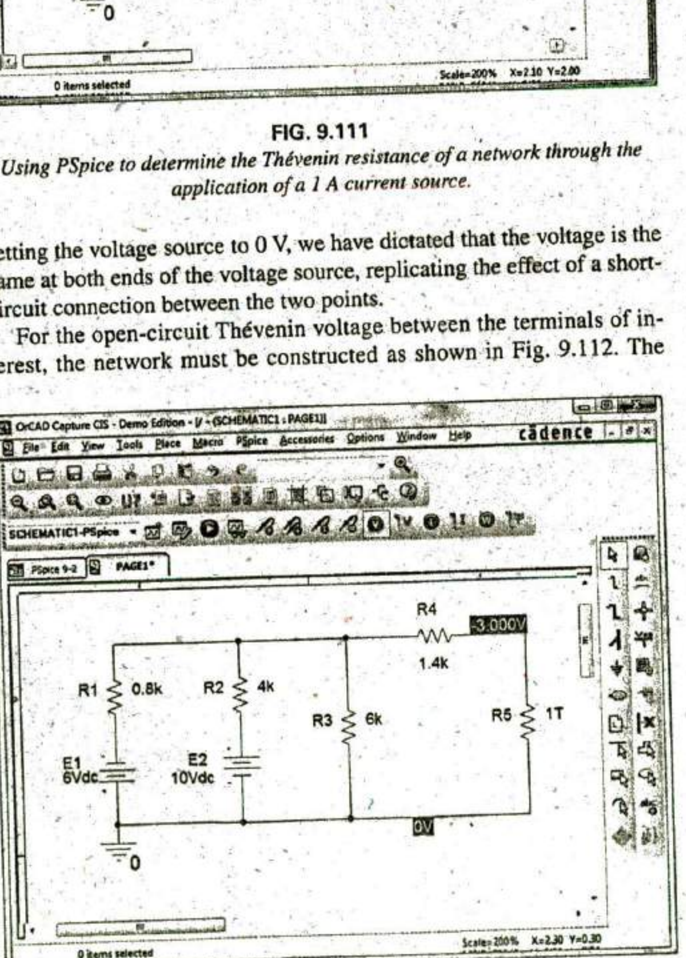


FIG. 9.112

Using PSpice to determine the Thévenin voltage for a network using a very large resistance value to represent the open-circuit condition between the terminals of interest.

resistance of 1 T (= 1 million MΩ) is considered large enough to represent an open circuit to permit an analysis of the network using PSpice. PSpice does not recognize floating nodes and generates an error signal if a connection is not made from the top right node to ground. Both voltage sources are now set on their prescribed values, and a simulation results in 3 V across the 1 T resistor. The open-circuit Thévenin voltage is therefore 3 V, which agrees with the solution in Example 9.10.

Maximum Power Transfer The procedure for plotting a quantity versus a parameter of the network is now introduced. In this case, the output power versus values of load resistance is used to verify that maximum power is delivered to the load when its value equals the series Thévenin resistance. A number of new steps are introduced, but keep in mind that the method has broad application beyond Thévenin's theorem and is therefore well worth the learning process.

The circuit to be analyzed appears in Fig. 9.113. The circuit is constructed in exactly the same manner as described earlier except for the value of the load resistance. Begin the process by starting a **New Project** labeled **PSpice 9-3,** and build the circuit in Fig. 9.113. For the moment, do not set the value of the load resistance.

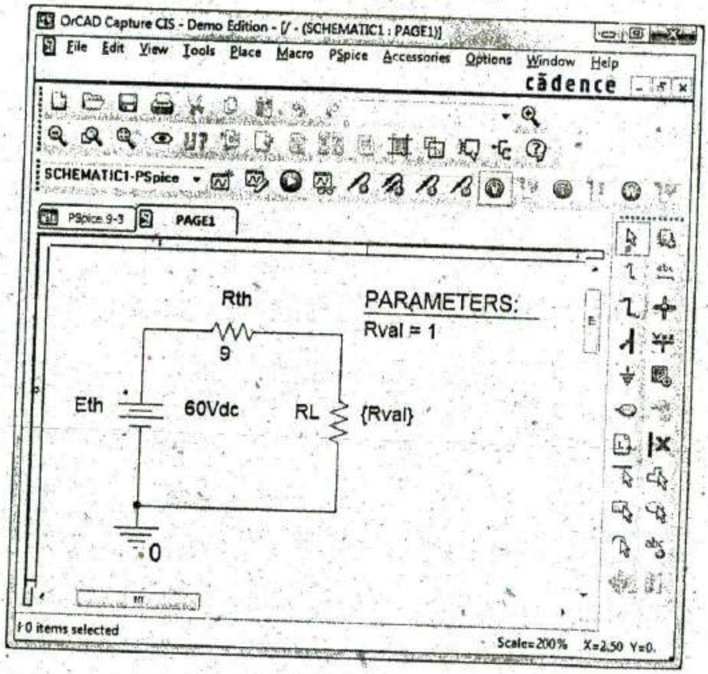


FIG. 9.113
Using PSpice to plot the power to R_L for a range of values for R_L.

The first step is to establish the value of the load resistance as a variable since it will not be assigned a fixed value. Double-click on the value of **RL,** which is initially 1 kΩ, to obtain the **Display Properties** dialog box. For **Value,** type in **{Rval}** and click in place. The brackets (*not* parentheses) are required, but the variable does not have to be called **Rval**—it is the choice of the user. Next select the **Place part** key to obtain the **Place Part** dialog box. If you are not already in the **Libraries** list, choose **Add Library** and add **SPECIAL** to the list. Select the **SPECIAL** library and scroll the **Part List** until **PARAM** appears. Select it; then click **OK** to obtain a rectangular box next to the cursor on the screen. Select a spot near **Rval,** and deposit the rectangle. The result is **PARAMETERS:** as shown in Fig. 9.113.

Next double-click on **PARAMETERS:** to obtain a **Property Editor** dialog box, which should have **SCHEMATIC1:PAGE1** in the second column from the left. Now select the **New Column** option from the top list of choices to obtain the **Add New Column** dialog box. Under **Name,** enter **Rval** and under **Value,** enter 1 followed by an **OK** to leave the dialog box. The result is a return to the **Property Editor** dialog box but with **Rval** and its value (below **Rval**) added to the horizontal list. Now select **Rval/1** by clicking on **Rval** to surround **Rval** by a dashed line and add a black background around the **1.** Choose **Display** to produce the **Display Properties** dialog box, and select **Name and Value** followed by **OK.** Then exit the **Property Editor** dialog box **(X)** to display the screen in Fig. 9.113. Note that now the first value (1 Ω) of **Rval** is displayed.

We are now ready to set up the simulation process. Under **PSpice,** select the **New Simulation Profile** key to open the **New Simulation** dialog box. Enter **DC Sweep** under **Name** followed by **Create.** The **Simulation Settings-DC Sweep** dialog box appears. After selecting **Analysis,** select **DC Sweep** under the **Analysis type** heading. Then leave the **Primary Sweep** under the **Options** heading, and select **Global parameter** under the **Sweep variable.** The **Parameter name** should then be entered as **Rval.** For the **Sweep type,** the **Start value** should be 1 Ω; but if we use 1 Ω, the curve to be generated will start at 1 Ω, leaving a blank from 0 to 1 Ω. The curve will look incomplete. To solve this problem, select 0.001 Ω as the **Start value** (very close to 0 Ω) with an **Increment** of 1 Ω. Enter the **End value** as 30.001 Ω to ensure a calculation at $R_L = 30\ \Omega$. If we used 30 Ω as the end value, the last calculation would be at 29.001 Ω since 29.001 Ω + 1 Ω = 30.001 Ω, which is beyond the range of 30 Ω. The values of **RL** will therefore be 0.001 Ω, 1.001 Ω, 2.001 Ω, . . . 29.001 Ω, 30.001 Ω, and so on, although the plot will look as if the values were 0 Ω, 1 Ω, 2 Ω, 29 Ω, 30 Ω, and so on. Click **OK,** and select **Run** under **PSpice** to obtain the display in Fig. 9.114.

Note that there are no plots on the graph, and that the graph extends to 32 Ω rather than 30 Ω as desired. It did not respond with a plot of power versus **RL** because we have not defined the plot of interest for the computer. To do this, select the **Add Trace** key (the key that has a red curve peaking in the middle of the plot) or **Trace-Add Trace** from the top menu bar. Either choice results in the **Add Traces** dialog box. The most important region of this dialog box is the **Trace Expression** listing at the bottom. The desired trace can be typed in directly, or the quantities of interest can be chosen from the list of **Simulation Output Variables** and deposited in the **Trace Expression** listing. To find the power to **RL** for the chosen range of values for **RL,** select **W(RL)** in the listing; it then appears as the **Trace Expression.** Click **OK,** and the plot in Fig. 9.115 appears. Originally, the plot extended from 0 Ω to 35 Ω. We reduced the range to 0 Ω to 30 Ω by selecting **Plot-Axis Settings-X Axis-User Defined 0 to 30-OK.**

Select the **Toggle cursor** key (which has an arrow set in a blue background), and seven options will open to the right of the key that include **Cursor Peak, Cursor Trough, Cursor Slope, Cursor Min, Cursor Max, Cursor Point,** and **Cursor Search.** Select **Cursor Max,** and the

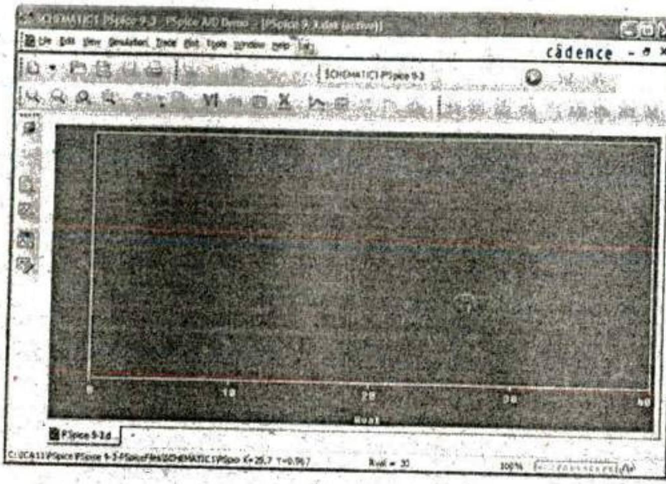


FIG. 9.114
Plot resulting from the dc sweep of R_L for the network in Fig. 9.113 before defining the parameters to be displayed.

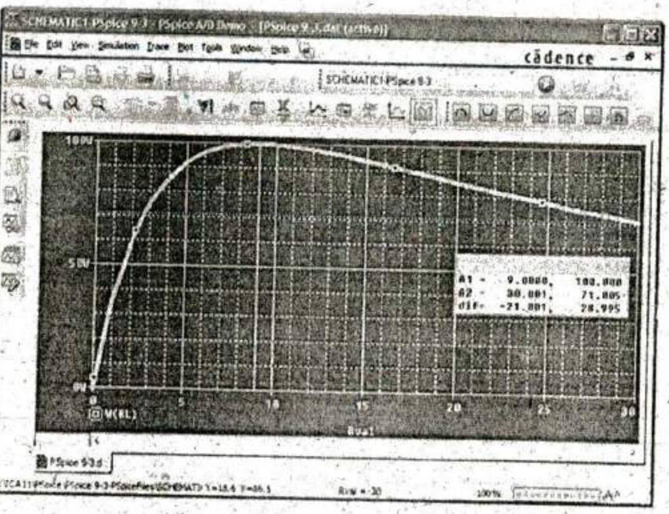


FIG. 9.115
A plot of the power delivered to R_L in Fig. 9.113 for a range of values for R_L extending from 0 Ω to 30 Ω.

Probe Cursor dialog box at the bottom right of the screen will reveal where the peak occurred and the power level at that point. Note that **A1** is 9.001 to reflect a load of 9 Ω, which equals the Thévenin resistance. The maximum power at this point is 100 W, as also indicated to the right of the resistance value. The **Probe Cursor** box can be moved to any position on the screen simply by selecting it and dragging it to the desired position. A second cursor can be generated by right-clicking the mouse on the **Cursor Point** option and moving it to a resistance of 30 Ω. The result is **A2** = 30 Ω, with a power level of 71.005 W, as shown on the plot. Notice also that the plot generated appears as a listing at the bottom left of the screen as **W(RL)**.

Multisim

Superposition Let us now apply superposition to the network in Fig. 9.116, which appeared earlier as Fig. 9.2 in Example 9.1, to permit a comparison of resulting solutions. The current through R_2 is to be determined. With the use of methods described in earlier chapters for the application of Multisim, the network in Fig. 9.117 results, which allows us to determine the effect of the 36 V voltage source. Note in Fig. 9.117 that both the voltage source and current source are present even though we are finding the contribution due solely to the voltage source. Obtain the voltage source by selecting the **Place Source** option at the top of the left toolbar to open the **Select a Component** dialog box. Then select **POWER_SOURCES** followed by **DC_POWER** as described in earlier chapters. You can also obtain the current source from the same dialog box by selecting **SIGNAL_CURRENT** under **Family** followed by **DC_CURRENT** under **Component.** The current source can be flipped vertically by right-clicking the source and selecting **Flip Vertical.** Set the current source to zero by left-clicking the source twice to obtain the **DC_CURRENT** dialog box. After choosing **Value,** set **Current(I)** to 0 A.

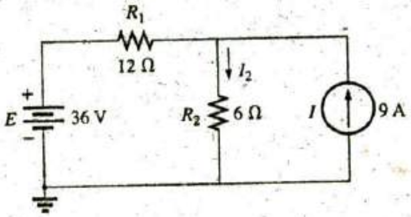


FIG. 9.116
Applying Multisim to determine the current I_2 using superposition.

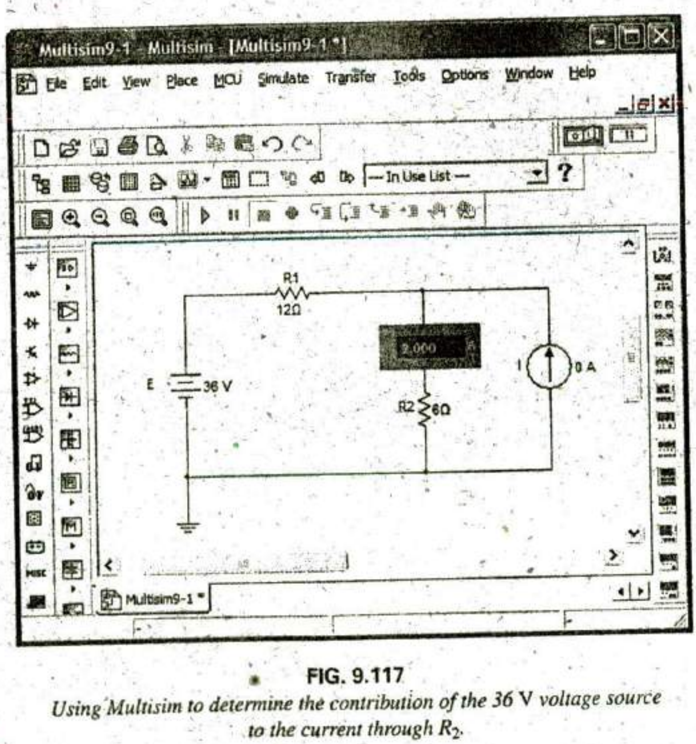


FIG. 9.117
Using Multisim to determine the contribution of the 36 V voltage source to the current through R_2.

Following simulation, the results appear as in Fig. 9.117. The current through the 6 Ω resistor is 2 A due solely to the 36 V voltage source. The positive value for the 2 A reading reveals that the current due to the 36 V source is down through resistor R_2.

For the effects of the current source, the voltage source is set to 0 V as shown in Fig. 9.118. The resulting current is then 6 A through R_2, with the same direction as the contribution due to the voltage source.

The resulting current for the resistor R_2 is the sum of the two currents: $I_T = 2\text{ A} + 6\text{ A} = 8\text{ A}$, as determined in Example 9.1.

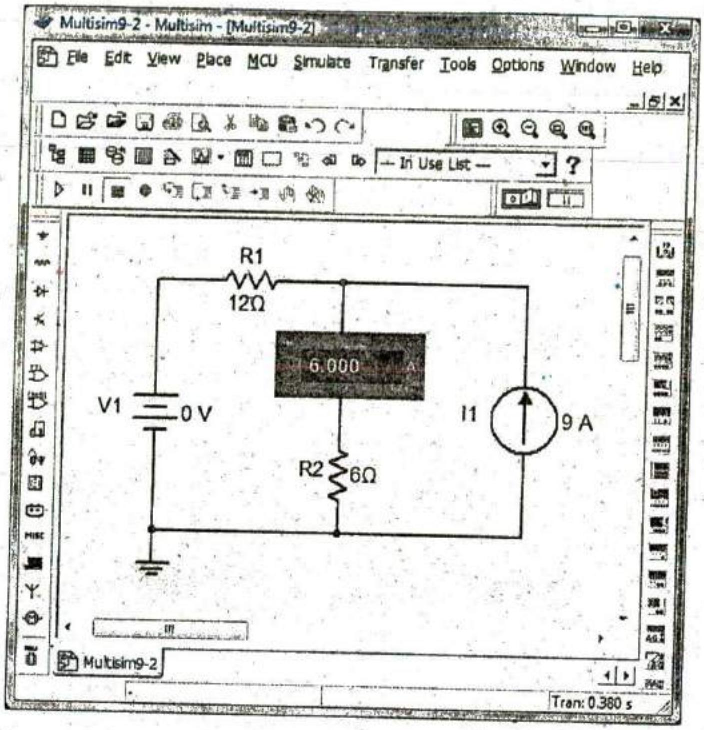


FIG. 9.118
Using Multisim to determine the contribution of the 9 A current source to the current through R_2.

PROBLEMS

SECTION 9.2 Superposition Theorem

1. a. Using the superposition theorem, determine the current through the 12 Ω resistor of Fig. 9.119.
   b. Convert both voltage sources to current sources and recalculate the current to the 12 Ω resistor.
   c. How do the results of parts (a) and (b) compare?

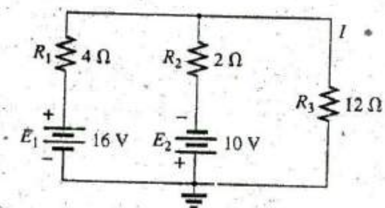


FIG. 9.119
Problem 1.

2. a. Using the superposition theorem, determine the voltage across the 4.7 Ω resistor of Fig. 9.120.
   b. Find the power delivered to the 4.7 Ω resistor due solely to the current source.
   c. Find the power delivered to the 4.7 Ω resistor due solely to the voltage source.
   d. Find the power delivered to the 4.7 Ω resistor using the voltage found in part (a).

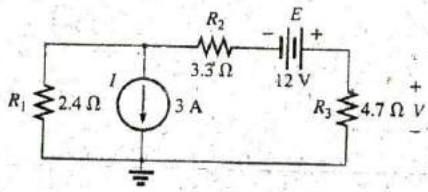


FIG. 9.120
Problem 2.

   e. How do the results of part (d) compare with the sum of the results to parts (b) and (c)? Can the superposition theorem be applied to power levels?

3. Using the superposition theorem, determine the current through the 56 Ω resistor of Fig. 9.121.

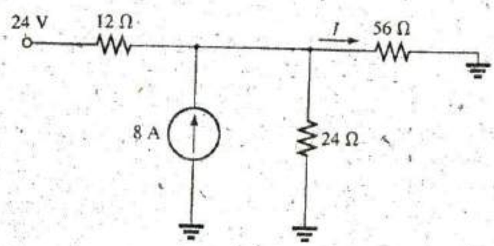


FIG. 9.121
Problem 3.

4. Using superposition, find the current I through the 24 V source in Fig. 9.122.

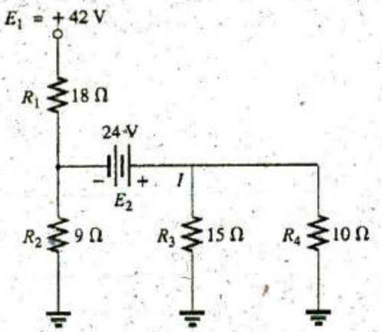


FIG. 9.122
Problem 4.

5. Using superposition, find the voltage V_2 for the network in Fig. 9.123.

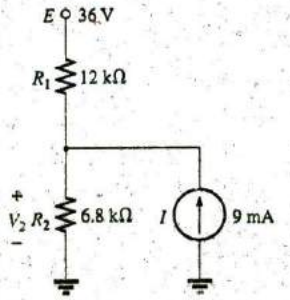


FIG. 9.123
Problem 5.

*6. Using superposition, find the current through R_1 for the network in Fig. 9.124.

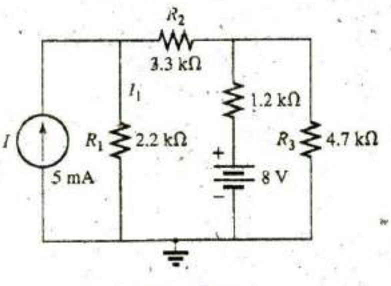


FIG. 9.124
Problem 5.

*7. Using superposition, find the voltage across the 6 A source in Fig. 9.125.

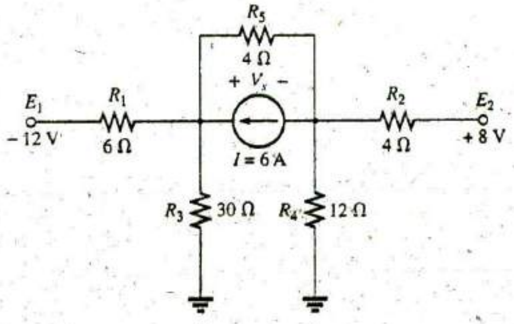


FIG. 9.125
Problem 7.

SECTION 9.3 Thévenin's Theorem

8. a. Find the Thévenin equivalent circuit for the network external to the resistor R in Fig. 9.126.
   b. Find the current through R when R is 2 Ω, 30 Ω, and 100 Ω.

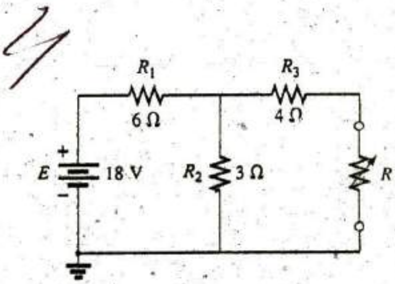


FIG. 9.126
Problem 8.

9. a. Find the Thévenin equivalent circuit for the network external to the resistor R for the network in Fig. 9.127.
   b. Find the power delivered to R when R is 2 kΩ and 100 kΩ.

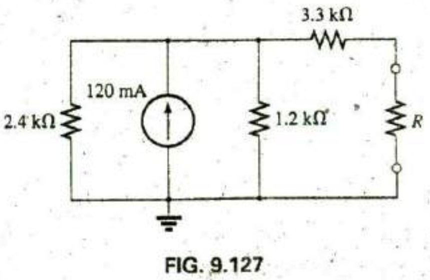


FIG. 9.127
Problem 9.

10. a. Find the Thévenin equivalent circuit for the network external to the resistor R for the network in Fig. 9.128.
    b. Find the power delivered to R when R is 2 Ω and 100 Ω.

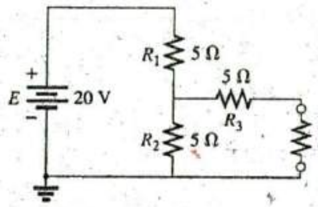


FIG. 9.128
Problem 10.

11. Find the Thévenin equivalent circuit for the network external to the resistor R for the network in Fig. 9.129.

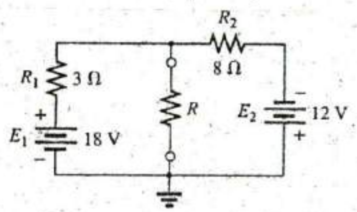


FIG. 9.129
Problem 11.

12. Find the Thévenin equivalent circuit for the network external to the resistor R for the network in Fig. 9.130.

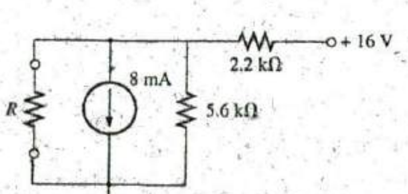


FIG. 9.130
Problem 12.

*13. Find the Thévenin equivalent circuit for the network external to the resistor R in Fig. 9.131.

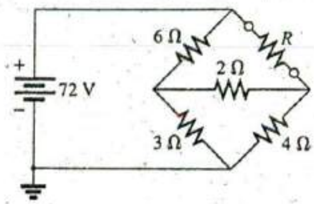


FIG. 9.131
Problem 13.

14. a. Find the Thévenin equivalent circuit for the portions of the network of Fig. 9.132 external to points a and b.
    b. Redraw the network with the Thévenin circuit in place and find the current through the 1.2 kΩ resistor.

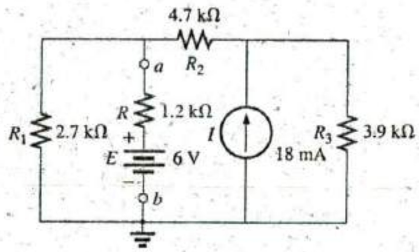


FIG. 9.132
Problem 14.

*15. a. Determine the Thevénin equivalent circuit for the network external to the resistor R in Fig. 9.133.
    b. Find the current through the resistor R if its value is 20 Ω, 50 Ω, and 100 Ω.
    c. Without having the Thévenin equivalent circuit, what would you have to do to find the current through the resistor R for all the values of part (b)?

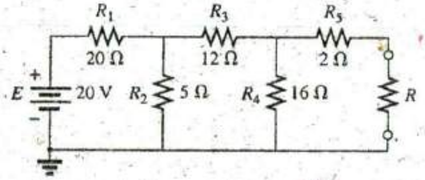


FIG. 9.133
Problem 15.

*16. a. Determine the Thévenin equivalent circuit for the network external to the resistor R in Fig. 9.134.
    b. Find the polarity and magnitude of the voltage across the resistor R if its value is 1.2 kΩ.

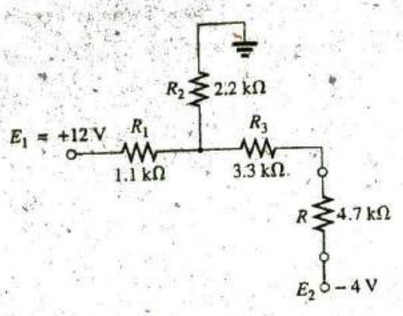


FIG. 9.134
Problem 16.

*17. For the network in Fig. 9.135, find the Thévenin equivalent circuit for the network external to the load resistor R_L.

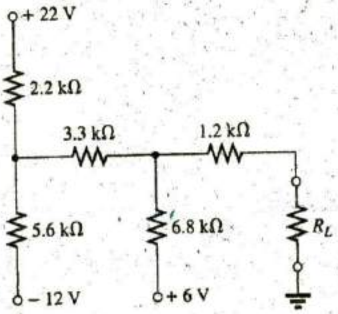


FIG. 9.135
Problem 17.

*18. For the transistor network in Fig. 9.136:
   a. Find the Thévenin equivalent circuit for that portion of the network to the left of the base (*B*) terminal.
   b. Using the fact that $I_C = I_E$ and $V_{CE} = 8$ V, determine the magnitude of I_E.
   c. Using the results of parts (a) and (b), calculate the base current I_B if $V_{BE} = 0.7$ V.
   d. What is the voltage V_C?

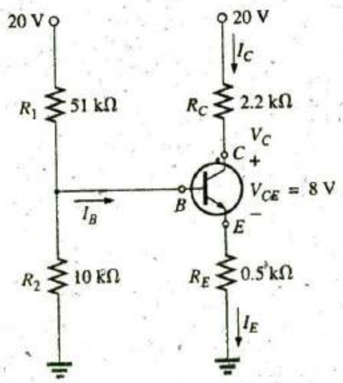


FIG. 9.136
Problem 18.

19. For each vertical set of measurements appearing in Fig. 9.137, determine the Thévenin equivalent circuit.

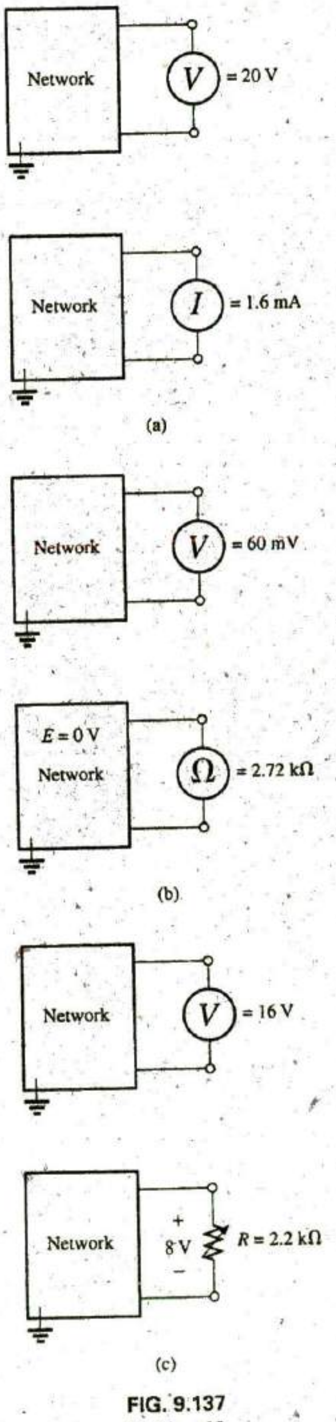


FIG. 9.137
Problem 19.

*20. For the network of Fig. 9.138, find the Thévenin equivalent circuit for the network external to the 300 Ω resistor.

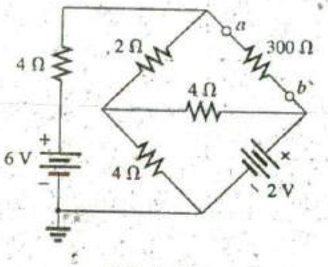


FIG. 9.138
Problem 20.

SECTION 9.4 Norton's Theorem

21. a. Find the Norton equivalent circuit for the network external to the resistor R in Fig. 9.126.
    b. Convert the Norton equivalent circuit to the Thévenin form.
    c. Find the Thévenin equivalent circuit using the Thévenin approach and compare results with part (b).
22. a. Find the Norton equivalent circuit for the network external to the resistor R in Fig. 9.127.
    b. Convert the Norton equivalent circuit to the Thévenin form.
    c. Find the Thévenin equivalent circuit using the Thévenin approach and compare results with part (b).
23. Find the Norton equivalent circuit for the network external to the resistor R in Fig. 9.129.
24. Find the Norton equivalent circuit for the network external to the resistor R in Fig. 9.130.

*25. Find the Norton equivalent circuit for the network external to the resistor R in Fig. 9.131.

*26. Find the Norton equivalent circuit for the network external to the resistor R in Fig. 9.133.

*27. Find the Norton equivalent circuit for the network external to the resistor R in Fig. 9.135.

*28. Find the Norton equivalent circuit for the network external to the 300 Ω resistor in Fig. 9.138.

*29. a. Find the Norton equivalent circuit external to points a and b in Fig. 9.139.
    b. Find the magnitude and polarity of the voltage across the 100 Ω resistor using the results of part (a).

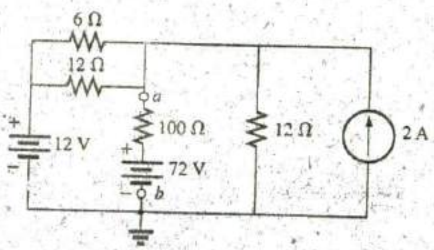


FIG. 9.139
Problem 29.

SECTION 9.5 Maximum Power Transfer Theorem

30. a. Find the value of R for maximum power transfer to R for the network of Fig. 9.126.
    b. Determine the maximum power of R.
31. a. Find the value of R for maximum power transfer to R for the network of Fig. 9.129.
    b. Determine the maximum power of R.
32. a. Find the value of R for maximum power transfer to R for the network of Fig. 9.131.
    b. Determine the maximum power to R.

*33. a. Find the value of R_L in Fig. 9.135 for maximum power transfer to R_L.
    b. Find the maximum power to R_L.

34. a. For the network of Fig. 9.140, determine the value of R for maximum power to R.
    b. Determine the maximum power to R.
    c. Plot a curve of power to R versus R for R ranging from 1/4 to 2 times the value determined in part (a) using an increment of 1/4 the value of R. Does the curve verify the fact that the chosen value of R in part (a) will ensure maximum power transfer?

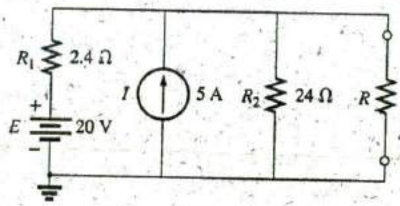


FIG. 9.140
Problem 34.

*35. Find the resistance R_1 in Fig. 9.141 such that the resistor R_4 will receive maximum power. Think!

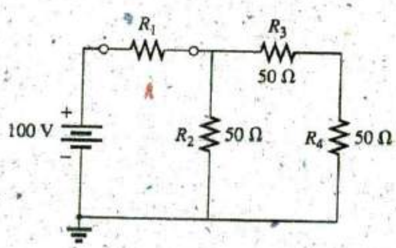


FIG. 9.141
Problem 35.

*36. a. For the network in Fig. 9.142, determine the value of R_2 for maximum power to R_4.
    b. Is there a general statement that can be made about situations such as those presented here and in Problem 35?

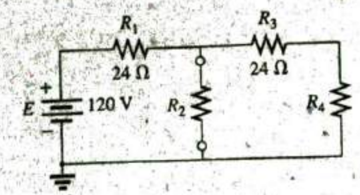


FIG. 9.142
Problem 36.

*37. For the network in Fig. 9.143, determine the level of R that will ensure maximum power to the 100 Ω resistor. Find the maximum power to R_L.

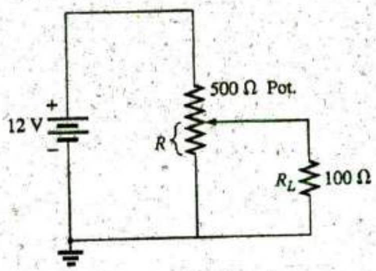


FIG. 9.143
Problem 37.

SECTION 9.6 Millman's Theorem

38. Using Millman's theorem, find the current through and voltage across the resistor R_L in Fig. 9.144.

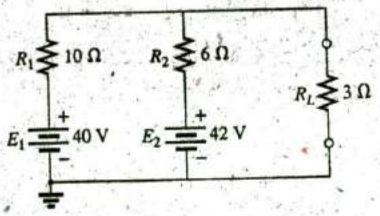


FIG. 9.144
Problem 38.

39. Repeat Problem 38 for the network in Fig. 9.145.

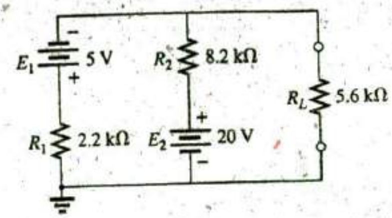


FIG. 9.145
Problem 39.

40. Using Millman's theorem, find the current through and voltage across the resistor R_L in Fig. 9.146.

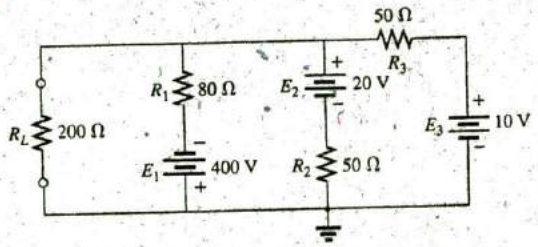


FIG. 9.146
Problem 40.

41. Using the dual of Millman's theorem, find the current through and voltage across the resistor R_L in Fig. 9.147.

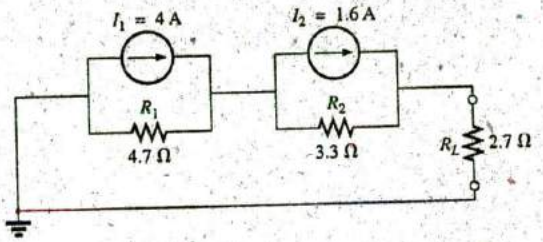


FIG. 9.147
Problem 41.

42. Using the dual of Millman's theorem, find the current through and voltage across the resistor R_L in Fig. 9.148.

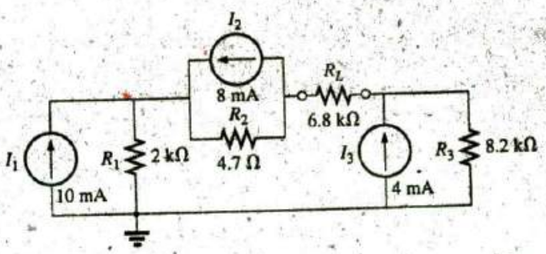


FIG. 9.148
Problem 42.

SECTION 9.7 Substitution Theorem

43. Using the substitution theorem, draw three equivalent branches for the branch *a-b* of the network in Fig. 9.149.

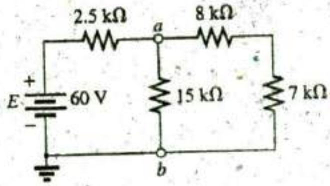


FIG. 9.149
Problem 43.

44. Using the substitution theorem, draw three equivalent branches for the branch *a-b* of the network in Fig. 9.150.

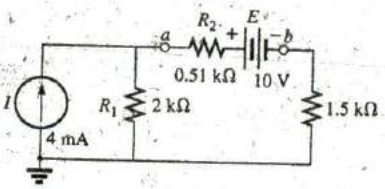


FIG. 9.150
Problem 44.

*45. Using the substitution theorem, draw three equivalent branches for the branch *a-b* of the network of Fig. 9.151.

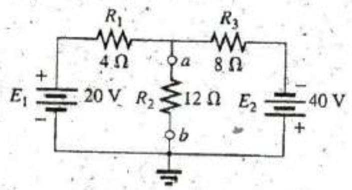


FIG. 9.151
Problem 45.

SECTION 9.8 Reciprocity Theorem

46. a. For the network in Fig. 9.152(a), determine the current *I*.
   b. Repeat part (a) for the network in Fig. 9.152(b).
   c. Is the reciprocity theorem satisfied?

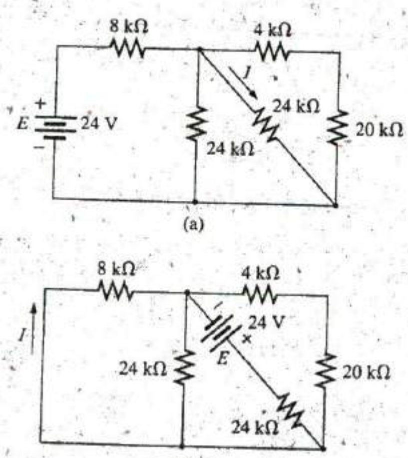


FIG. 9.152
Problem 46.

47. a. For the network of Fig. 9.153(a), determine the current *I*.
   b. Repeat part (a) for the network in Fig. 9.153(b).
   c. Is the reciprocity theorem satisfied?

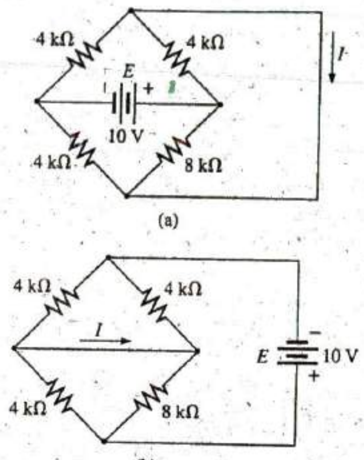


FIG. 9.153
Problem 47.

48. a. Determine the voltage *V* for the network in Fig. 9.154(a).
   b. Repeat part (a) for the network in Fig. 9.154(b).
   c. Is the dual of the reciprocity theorem satisfied?

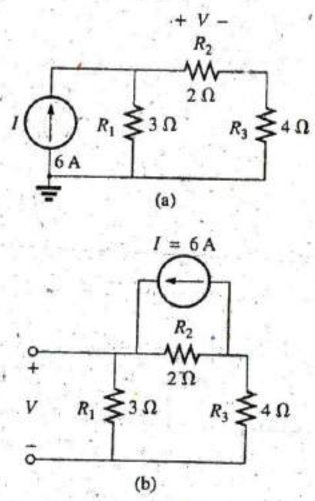


FIG. 9.154
Problem 48.

SECTION 9.9 Computer Analysis

49. Using PSpice or Multisim, determine the voltage V_2 and its components for the network in Fig. 9.123.

50. Using PSpice or Multisim, determine the Thévenin equivalent circuit for the network in Fig. 9.131.

*51. a. Using PSpice, plot the power delivered to the resistor R in Fig. 9.128 for R having values from 1 Ω to 10 Ω.
b. From the plot, determine the value of R resulting in the maximum power to R and the maximum power to R.
c. Compare the results of part (a) to the numerical solution.
d. Plot V_R and I_R versus R, and find the value of each under maximum power conditions.

*52. Change the 300 Ω resistor in Fig. 9.138 to a variable resistor, and using Pspice, plot the power delivered to the resistor versus values of the resistor. Determine the range of resistance by trial and error rather than first performing a longhand calculation. Determine the Norton equivalent circuit from the results. The Norton current can be determined from the maximum power level.

GLOSSARY

Maximum power transfer theorem A theorem used to determine the load resistance necessary to ensure maximum power transfer to the load.

Millman's theorem A method using source conversions that will permit the determination of unknown variables in a multiloop network.

Norton's theorem A theorem that permits the reduction of any two-terminal linear dc network to one having a single current source and parallel resistor.

Reciprocity theorem A theorem that states that for single-source networks, the current in any branch of a network due to a single voltage source in the network will equal the current through the branch in which the source was originally located if the source is placed in the branch in which the current was originally measured.

Substitution theorem A theorem that states that if the voltage across and current through any branch of a dc bilateral network are known, the branch can be replaced by any combination of elements that will maintain the same voltage across and current through the chosen branch.

Superposition theorem A network theorem that permits considering the effects of each source independently. The resulting current and/or voltage is the algebraic sum of the currents and/or voltages developed by each source independently.

Thévenin's theorem A theorem that permits the reduction of any two-terminal, linear dc network to one having a single voltage source and series resistor.