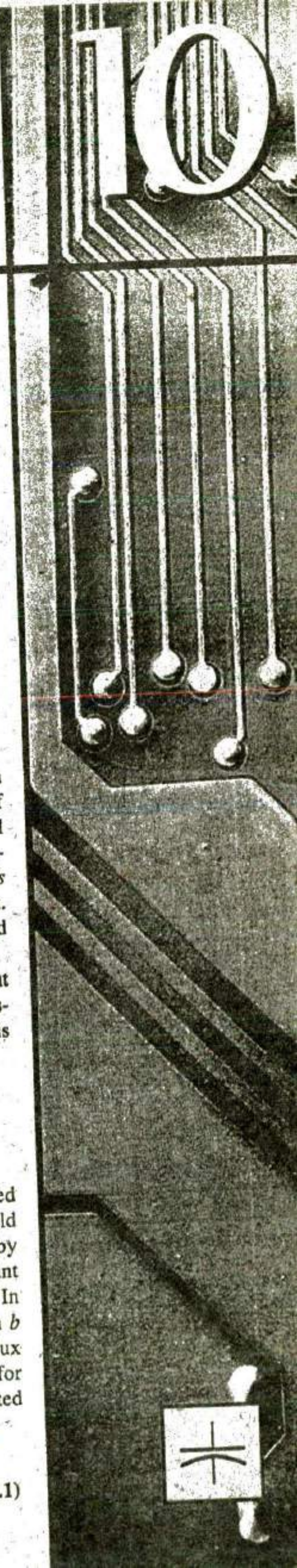
Capacitors



Objectives

- *Become familiar with the basic construction of a capacitor and the factors that affect its ability to store charge on its plates.*
- *Be able to determine the transient (time-varying) response of a capacitive network and plot the resulting voltages and currents.*
- *Understand the impact of combining capacitors in series or parallel and how to read the nameplate data.*
- *Develop some familiarity with the use of computer methods to analyze networks with capacitive elements.*

10.1 INTRODUCTION

Thus far, the resistor has been the only network component appearing in our analyses. In this chapter, we introduce the **capacitor,** which has a significant impact on the types of networks that you will be able to design and analyze. Like the resistor, it is a two-terminal device, but its characteristics are totally different from those of a resistor. In fact, *the capacitor displays its true characteristics only when a change in the voltage or current is made in the network.* All the power delivered to a resistor is dissipated in the form of heat. An ideal capacitor, however, stores the energy delivered to it in a form that can be returned to the system.

Although the basic construction of capacitors is actually quite simple, it is a component that opens the door to all types of practical applications, extending from touch pads to sophisticated control systems. A few applications are introduced and discussed in detail later in this chapter.

10.2 THE ELECTRIC FIELD

Recall from Chapter 2 that a force of attraction or repulsion exists between two charged bodies. We now examine this phenomenon in greater detail by considering the electric field that exists in the region around any charged body. This electric field is represented by **electric flux lines,** which are drawn to indicate the strength of the electric field at any point around the charged body. The denser the lines of flux, the stronger is the electric field. In Fig. 10.1, for example, the electric field strength is stronger in region a than region b because the flux lines are denser in region a than in b. That is, the same number of flux lines pass through each region, but the area A_1 is much smaller than area A_2. The symbol for electric flux is the Greek letter ψ (*psi*). The flux per unit area (flux density) is represented by the capital letter D and is determined by

$$\boxed{D = \frac{\psi}{A}} \qquad \text{(flux/unit area)} \qquad \textbf{(10.1)}$$

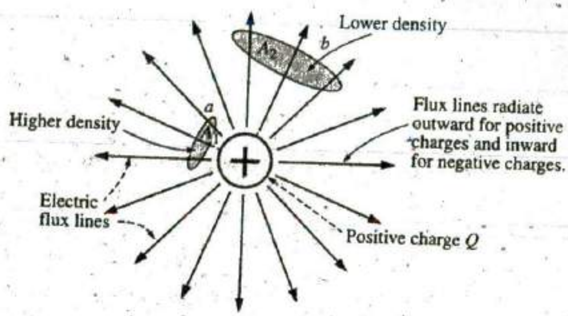


FIG. 10.1
Flux distribution from an isolated positive charge.

The larger the charge Q in coulombs, the greater is the number of flux lines extending or terminating per unit area, independent of the surrounding medium. Twice the charge produces twice the flux per unit area. The two can therefore be equated:

$$\boxed{\psi \equiv Q} \qquad \text{(coulombs, C)} \qquad \textbf{(10.2)}$$

By definition, the **electric field strength** (designated by the capital script letter $\mathscr{E}$) at a point is the force acting on a unit positive charge at that point; that is,

$$\boxed{\mathscr{E} = \frac{F}{Q}} \qquad \text{(newtons/coulomb, N/C)} \qquad \textbf{(10.3)}$$

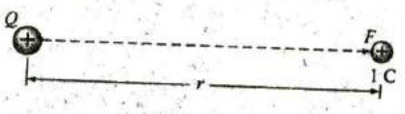


FIG. 10.2
Determining the force on a unit charge r meters from a charge Q of similar polarity.

In Fig. 10.2, the force exerted on a unit (1 coulomb) positive charge by a charge Q, r meters away, can be determined using **Coulomb's law** (Eq. 2.1) as follows:

$$F = k\frac{Q_1Q_2}{r^2} = k\frac{Q(1\text{ C})}{r^2} = \frac{kQ}{r^2} \; (k = 9 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)$$

Substituting the result into Eq. 10.3 for a unit positive charge results in

$$\mathscr{E} = \frac{F}{Q} = \frac{kQ/r^2}{1/\text{C}}$$

and

$$\boxed{\mathscr{E} = \frac{kQ}{r^2}} \qquad \text{(N/C)} \qquad \textbf{(10.4)}$$

The result clearly reveals that the electric field strength is directly related to the size of the charge Q. The greater the charge Q, the greater is the electric field intensity on a unit charge at any point in the neighborhood. However, the distance is a squared term in the denominator. The result is that the greater the distance from the charge Q, the less is the electric field strength, and dramatically so because of the squared term. In Fig. 10.1, the electric field strength at region A_2 is therefore significantly less than at region A_1.

For two charges of similar and opposite polarities, the flux distribution appears as shown in Fig. 10.3. In general,

electric flux lines always extend from a positively charged body to a negatively charged body, always extend or terminate perpendicular to the charged surfaces, and never intersect.

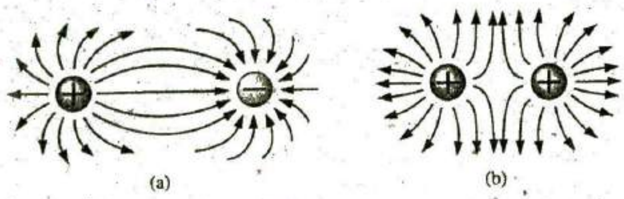


FIG. 10.3
Electric flux distributions: (a) opposite charges; (b) like charges.

Note in Fig. 10.3(a) that the electric flux lines establish the most direct pattern possible from the positive to negative charge. They are evenly distributed and have the shortest distance on the horizontal between the two charges. This pattern is a direct result of the fact that electric flux lines strive to establish the shortest path from one charged body to another. The result is a natural pressure to be as close as possible. If two bodies of the same polarity are in the same vicinity, as shown in Fig. 10.3(b), the result is the direct opposite. The flux lines tend to establish a buffer action between the two with a repulsive action that grows as the two charges are brought closer to one another.

10.3 CAPACITANCE

Thus far, we have examined only isolated positive and negative spherical charges, but the description can be extended to charged surfaces of any shape and size. In Fig. 10.4, for example, two parallel plates of a material such as aluminum (the most commonly used metal in the construction of capacitors) have been connected through a switch and a resistor to a battery. If the parallel plates are initially uncharged and the switch is left open, no net positive or negative charge exists on either plate. The instant the switch is closed, however, electrons are drawn from the upper plate through the resistor to the positive terminal of the battery. There will be a surge of current at first, limited in magnitude by the resistance present. The level of flow then declines, as will be demonstrated in the sections to follow. This action creates a net positive charge on the top plate. Electrons are being repelled by the negative terminal through the lower conductor to the bottom plate at the same rate they are being drawn to the positive terminal. This transfer of electrons continues until the potential difference across the parallel plates is exactly equal to the battery voltage. The final result is a net positive charge on the top plate

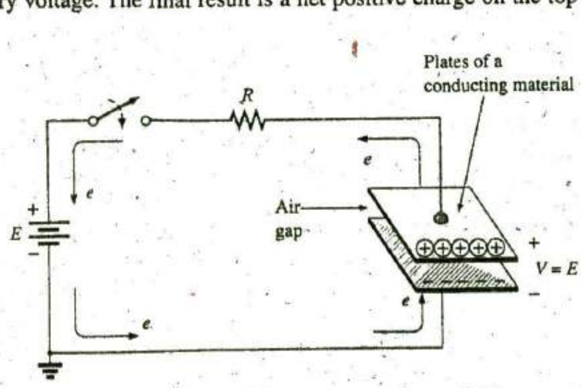


FIG. 10.4
Fundamental charging circuit.

FIG. 10.5
Michael Faraday.
Courtesy of the Smithsonian Institution
Photo No. 51,147

English (London)
(1791–1867)
Chemist and Electrical Experimenter
Honorary Doctorate, Oxford University, 1832

An experimenter with no formal education, he began his research career at the Royal Institute in London as a laboratory assistant. Intrigued by the interaction between electrical and magnetic effects, he discovered *electromagnetic induction,* demonstrating that electrical effects can be generated from a magnetic field (the birth of the generator as we know it today). He also discovered *self-induced currents* and introduced the concept of *lines and fields of magnetic force.* Having received over one hundred academic and scientific honors, he became a Fellow of the Royal Society in 1824 at the young age of 32.

and a negative charge on the bottom plate, very similar in many respects to the two isolated charges in Fig. 10.3(a).

Before continuing, it is important to note that the entire flow of charge is through the battery and resistor—not through the region between the plates. In every sense of the definition, *there is an open circuit between the plates of the capacitor.*

This element, constructed simply of two conducting surfaces separated by the air gap, is called a **capacitor.**

Capacitance is a measure of a capacitor's ability to store charge on its plates—in other words, its storage capacity.

In addition,

the higher the capacitance of a capacitor, the greater is the amount of charge stored on the plates for the same applied voltage.

The unit of measure applied to capacitors is the farad (F), named after an English scientist, Michael Faraday, who did extensive research in the field (Fig. 10.5). In particular,

a capacitor has a capacitance of 1 F if 1 C of charge (6.242×10^{18} electrons) is deposited on the plates by a potential difference of 1 V across its plates.

The farad, however, is generally too large a measure of capacitance for most practical applications, so the microfarad (10^{-6}) or picofarad (10^{-12}) are more commonly encountered.

The relationship connecting the applied voltage, the charge on the plates, and the capacitance level is defined by the following equation:

$$\boxed{C = \frac{Q}{V}} \qquad \begin{aligned} C &= \text{farads (F)} \\ Q &= \text{coulombs (C)} \\ V &= \text{volts (V)} \end{aligned} \qquad \textbf{(10.5)}$$

Eq. (10.5) reveals that for the same voltage (V), the greater the charge (Q) on the plates (in the numerator of the equation), the higher is the capacitance level (C).

If we write the equation in the form

$$\boxed{Q = CV} \qquad \text{(coulombs, C)} \qquad \textbf{(10.6)}$$

it becomes obvious through the product relationship that the higher the capacitance (C) or applied voltage (V), the greater is the charge on the plates.

EXAMPLE 10.1

a. If 82.4×10^{14} electrons are deposited on the negative plate of a capacitor by an applied voltage of 60 V, find the capacitance of the capacitor.
b. If 40 V are applied across a 470 μF capacitor, find the charge on the plates.

Solutions:

a. First find the number of coulombs of charge as follows:

$$82.4 \times 10^{14} \cancel{\text{electrons}}\left(\frac{1\text{ C}}{6.242 \times 10^{18} \cancel{\text{electrons}}}\right) = 1.32\text{ mC}$$

and then

$$C = \frac{Q}{V} = \frac{1.32\text{ mC}}{60\text{ V}} = \mathbf{22\ \mu F} \qquad \text{(a standard value)}$$

b. Applying Eq. (10.6) gives

$$Q = CV = (470\ \mu\text{F})(40\text{ V}) = \mathbf{18.8\ mC}$$

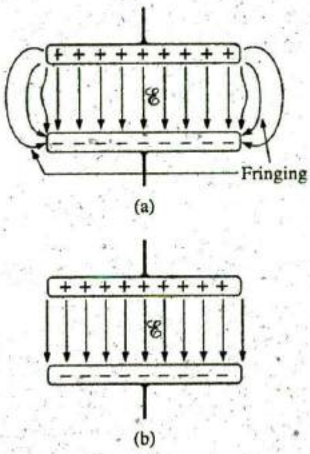


FIG. 10.6
Electric flux distribution between the plates of a capacitor: (a) including fringing; (b) ideal.

A cross-sectional view of the parallel plates in Fig. 10.4 is provided in Fig. 10.6(a). Note the **fringing** that occurs at the edges as the flux lines originating from the points farthest away from the negative plate strive to complete the connection. This fringing, which has the effect of reducing the net capacitance somewhat, can be ignored for most applications. Ideally, and the way we will assume the distribution to be in this text, the electric flux distribution appears as shown in Fig. 10.6(b), where all the flux lines are equally distributed and "fringing" does not occur.

The **electric field strength** between the plates is determined by the voltage across the plates and the distance between the plates as follows:

$$\boxed{\mathscr{E} = \frac{V}{d}} \qquad \begin{aligned} \mathscr{E} &= \text{volts/m (V/m)} \\ V &= \text{volts (V)} \\ d &= \text{meters (m)} \end{aligned} \qquad \mathbf{(10.7)}$$

Note that the distance between the plates is measured in meters, not centimeters or inches.

The equation for the electric field strength is determined by two factors only: *the applied voltage and the distance between the plates.* The charge on the plates does not appear in the equation, nor does the size of the capacitor or the plate material.

Many values of capacitance can be obtained for the same set of parallel plates by the addition of certain insulating materials between the plates. In Fig. 10.7, an insulating material has been placed between a set of parallel plates having a potential difference of V volts across them.

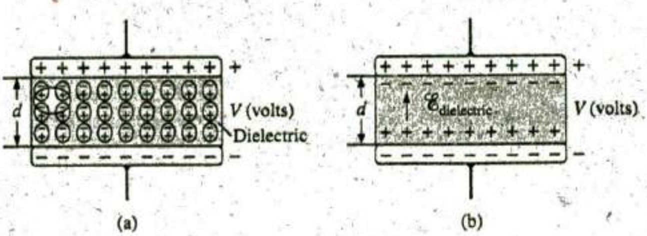


FIG. 10.7
Effect of a dielectric on the field distribution between the plates of a capacitor: (a) alignment of dipoles in the dielectric; (b) electric field components between the plates of a capacitor with a dielectric present.

Since the material is an insulator, the electrons within the insulator are unable to leave the parent atom and travel to the positive plate. The positive components (protons) and negative components (electrons) of each atom do shift, however [as shown in Fig. 10.7(a)], to form *dipoles.*

When the dipoles align themselves as shown in Fig. 10.7(a), the material is *polarized.* A close examination within this polarized material reveals that the positive and negative components of adjoining dipoles are neutralizing the effects of each other [note the oval area in Fig. 10.7(a)]. The layer of positive charge on one surface and the negative charge on the other are not neutralized, however, resulting in the establishment of an electric field within the insulator [$\mathscr{E}_{\text{dielectric}}$; Fig. 10.7(b)].

In Fig. 10.8(a), two plates are separated by an air gap and have layers of charge on the plates as established by the applied voltage and the distance between the plates. The electric field strength is $\mathscr{E}_1$ as defined by Eq. (10.7). In Fig. 10.8(b), a slice of mica is introduced, which, through an alignment of cells within the dielectric, establishes an electric field $\mathscr{E}_2$ that will oppose electric field $\mathscr{E}_1$. The effect is to try to reduce the electric field strength between the plates. However, Eq. (10.7) states that the electric field strength *must be* the value established by the applied voltage and the distance between the plates. This condition is maintained by placing more charge on the plates, thereby increasing the electric field strength between the plates to a level that cancels out the opposing electric field introduced by the mica sheet. The net result is an increase in charge on the plates and an increase in the capacitance level as established by Eq. (10.5).

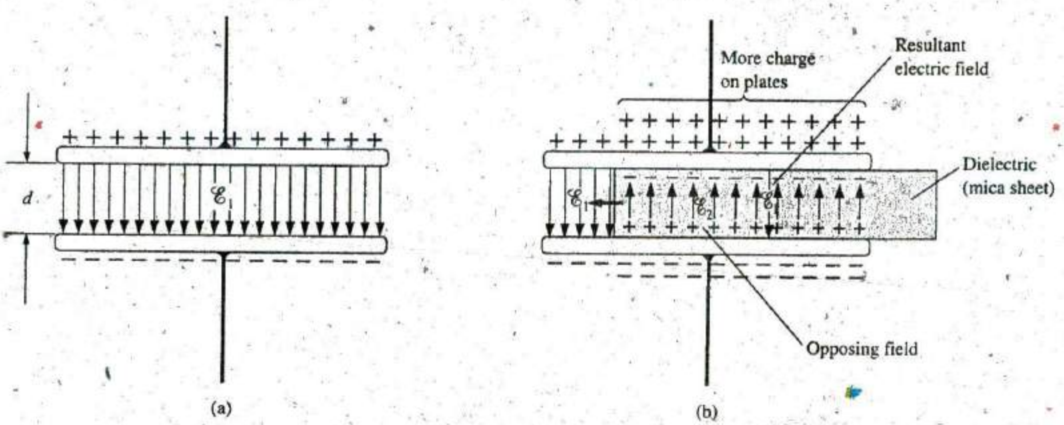


FIG. 10.8
Demonstrating the effect of inserting a dielectric between the plates of a capacitor: (a) air capacitor; (b) dielectric being inserted.

Different materials placed between the plates establish different amounts of additional charge on the plates. All, however, must be insulators and must have the ability to set up an electric field within the structure. A list of common materials appears in Table 10.1 using air as the reference level of 1.* All of these materials are referred to as **dielectrics,** the "di" for *opposing,* and the "electric" from *electric field.* The symbol ϵ_r in Table 10.1 is called the **relative permittivity** (or **dielectric constant**). The term **permittivity** is applied as a measure of how easily a material "permits" the establishment of an electric field in the material. The relative permittivity compares the permittivity of a material to that of air. For instance, Table 10.1 reveals that mica, with a relative permittivity of 5, "permits" the establishment of an opposing electric field in the material five times better than in air. Note the ceramic material at the bottom of the chart with a relative permittivity of 7500—a relative permittivity that makes it a very special dielectric in the manufacture of capacitors.

*Although there is a difference in dielectric characteristics between air and a vacuum, the difference is so small that air is commonly used as the reference level.

TABLE 10.1
Relative permittivity (dielectric constant) ϵ_r of various dielectrics.

Dielectric	ϵ_r (Average Values)
Vacuum	1.0
Air	1.0006
Teflon®	2.0
Paper, paraffined	2.5
Rubber	3.0
Polystyrene	3.0
Oil	4.0
Mica	5.0
Porcelain	6.0
Bakelite®	7.0
Aluminum oxide	7
Glass	7.5
Tantalum oxide	30
Ceramics	20–7500
Barium-strontium titanite (ceramic)	7500.0

Defining ϵ_o as the permittivity of air, we define the relative permittivity of a material with a permittivity ϵ by

$$\boxed{\epsilon_r = \frac{\epsilon}{\epsilon_o}} \quad \text{(dimensionless)} \qquad (10.8)$$

Note that ϵ_r, which (as mentioned previously) is often called the **dielectric constant,** is a dimensionless quantity because it is a ratio of similar quantities. However, permittivity does have the units of farads/meter (F/m) and is 8.85×10^{-12} F/m for air. Although the relative permittivity for the air we breathe is listed as 1.006, a value of 1 is normally used for the relative permittivity of air.

For every dielectric there is a potential that, if applied across the dielectric, will break down the bonds within it and cause current to flow through it. The voltage required per unit length is an indication of its **dielectric strength** and is called the **breakdown voltage.** When breakdown occurs, the capacitor has characteristics very similar to those of a conductor. A typical example of dielectric breakdown is lightning, which occurs when the potential between the clouds and the earth is so high that charge can pass from one to the other through the atmosphere (the dielectric). The average dielectric strengths for various dielectrics are tabulated in volts/mil in Table 10.2 (1 mil = 1/1000 inch).

One of the important parameters of a capacitor is the **maximum working voltage.** It defines the maximum voltage that can be placed across the capacitor on a continuous basis without damaging it or changing its characteristics. For most capacitors, it is the dielectric strength that defines the maximum working voltage.

TABLE 10.2
Dielectric strength of some dielectric materials.

Dielectric	Dielectric Strength (Average Value) in Volts/Mil
Air	75
Barium-strontium titanite (ceramic)	75
Ceramics	75–1000
Porcelain	200
Oil	400
Bakelite®	400
Rubber	700
Paper paraffined	1300
Teflon®	1500
Glass	3000
Mica	5000

10.4 CAPACITORS

Capacitor Construction

We are now aware of the basic components of a capacitor: conductive plates, separation, and dielectric. However, the question remains, How do all these factors interact to determine the capacitance of a capacitor?

Larger plates permit an increased area for the storage of charge, so the area of the plates should be in the numerator of the defining equation. *The smaller the distance between the plates,* the larger is the capacitance, so this factor should appear in the numerator of the equation. Finally, since *higher levels of permittivity* result in higher levels of capacitance, the factor ϵ should appear in the numerator of the defining equation.

The result is the following general equation for capacitance:

$$\boxed{C = \epsilon\frac{A}{d}} \qquad \begin{aligned} C &= \text{farads (F)} \\ \epsilon &= \text{permittivity (F/m)} \\ A &= \text{m}^2 \\ d &= \text{m} \end{aligned} \tag{10.9}$$

If we substitute Eq. (10.8) for the permittivity of the material, we obtain the following equation for the capacitance:

$$\boxed{C = \epsilon_o\epsilon_r\frac{A}{d}} \qquad \text{(farads, F)} \tag{10.10}$$

or if we substitute the known value for the permittivity of air, we obtain the following useful equation:

$$\boxed{C = 8.85 \times 10^{-12}\,\epsilon_r\frac{A}{d}} \qquad \text{(farads, F)} \tag{10.11}$$

It is important to note in Eq. (10.11) that the area of the plates (actually the area of only one plate) is in meters squared (m^2); the distance between the plates is measured in meters; and the numerical value of ϵ_r is simply taken from Table 10.1.

You should also be aware that most capacitors are in the μF, nF, or pF range, not the 1 F or greater range. A 1 F capacitor can be as large as a typical flashlight, requiring that the housing for the system be quite large. Most capacitors in electronic systems are the size of a thumbnail or smaller.

If we form the ratio of the equation for the capacitance of a capacitor with a specific dielectric to that of the same capacitor with air as the dielectric, the following results:

$$\frac{C = \epsilon\dfrac{A}{d}}{C_o = \epsilon_o\dfrac{A}{d}} \Rightarrow \frac{C}{C_o} = \frac{\epsilon}{\epsilon_o} = \epsilon_r$$

and

$$\boxed{C = \epsilon_r C_o} \tag{10.12}$$

The result is that

the capacitance of a capacitor with a dielectric having a relative permittivity of ϵ_r is ϵ_r times the capacitance using air as the dielectric.

The next few examples review the concepts and equations just presented.

EXAMPLE 10.2 In Fig. 10.9, if each air capacitor in the left column is changed to the type appearing in the right column, find the new capacitance level. For each change, the other factors remain the same.

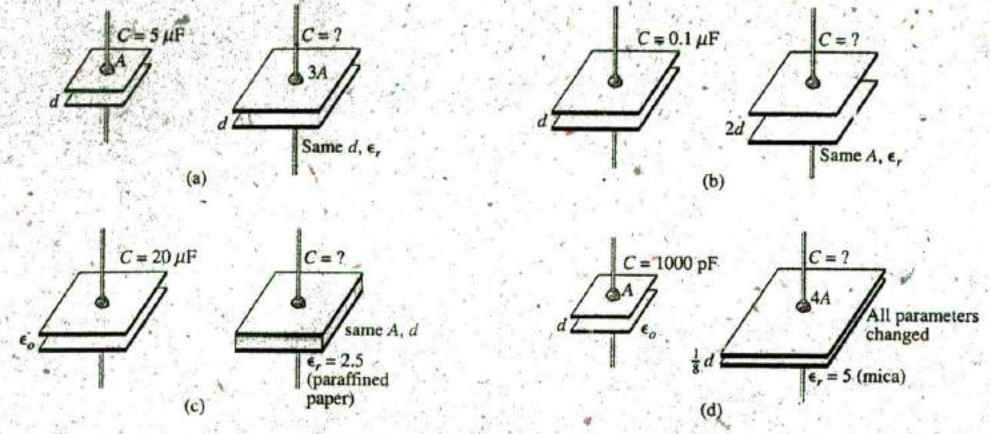


FIG. 10.9
Example 10.2.

Solutions:

a. In Fig. 10.9(a), the area has increased by a factor of three, providing more space for the storage of charge on each plate. Since the area appears in the numerator of the capacitance equation, the capacitance increases by a factor of three. That is,

$$C = 3(C_o) = 3(5\ \mu\text{F}) = \mathbf{15\ \mu F}$$

b. In Fig. 10.9(b), the area stayed the same, but the distance between the plates was increased by a factor of two. Increasing the distance reduces the capacitance level, so the resulting capacitance is one-half of what it was before. That is,

$$C = \frac{1}{2}(0.1\ \mu\text{F}) = \mathbf{0.05\ \mu F}$$

c. In Fig. 10.9(c), the area and the distance between the plates were maintained, but a dielectric of paraffined (waxed) paper was added between the plates. Since the permittivity appears in the numerator of the capacitance equation, the capacitance increases by a factor determined by the relative permittivity. That is,

$$C = \epsilon_r C_o = 2.5(20\ \mu\text{F}) = \mathbf{50\ \mu F}$$

d. In Fig. 10.9(d), a multitude of changes are happening at the same time. However, solving the problem is simply a matter of determining whether the change increases or decreases the capacitance and then placing the multiplying factor in the numerator or denominator of the equation. The increase in area by a factor of four produces a multiplier of four in the numerator, as shown in the equation below. Reducing the distance by a factor of 1/8 will increase the capacitance by its inverse, or a factor of eight. Inserting the mica dielectric increases the capacitance by a factor of five. The result is

$$C = (5)\frac{4}{(1/8)}(C_o) = 160(1000\ \text{pF}) = \mathbf{0.16\ \mu F}$$

In the next example, the dimensions of an air capacitor are provided and the capacitance is to be determined. The example emphasizes the importance of knowing the units of each factor of the equation. Failing to make a conversion to the proper set of units will probably produce a meaningless result, even if the proper equation were used and the mathematics properly executed.

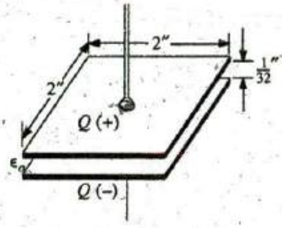


FIG. 10.10
Air capacitor for Example 10.3.

EXAMPLE 10.3 For the capacitor in Fig. 10.10:

a. Find the capacitance.
b. Find the strength of the electric field between the plates if 48 V are applied across the plates.
c. Find the charge on each plate.

Solutions:

a. First, the area and the distance between the plates must be converted to the SI system as required by Eq. (10.11):

$$d = \frac{1}{32}\text{ in.}\left(\frac{1\text{ m}}{39.37\text{ in.}}\right) = 0.794\text{ mm}$$

and $$A = (2\text{ in.})(2\text{ in.})\left(\frac{1\text{ m}}{39.37\text{ in.}}\right)\left(\frac{1\text{ m}}{39.37\text{ in.}}\right) = 2.581 \times 10^{-3}\text{ m}^2$$

Eq. (10.11):

$$C = 8.85 \times 10^{-12}\,\epsilon_r \frac{A}{d} = 8.85 \times 10^{-12}(1)\frac{(2.581 \times 10^{-3}\text{m}^2)}{0.794\text{ mm}} = \mathbf{28.8\text{ pF}}$$

b. The electric field between the plates is determined by Eq. (10.7):

$$\mathscr{E} = \frac{V}{d} = \frac{48\text{ V}}{0.794\text{ mm}} = \mathbf{60.5\text{ kV/m}}$$

c. The charge on the plates is determined by Eq. (10.6):

$$Q = CV = (28.8\text{ pF})(48\text{ V}) = \mathbf{1.38\text{ nC}}$$

In the next example, we will insert a ceramic dielectric between the plates of the air capacitor in Fig. 10.10 and see how it affects the capacitance level, electric field, and charge on the plates.

EXAMPLE 10.4

a. Insert a ceramic dielectric with an ϵ_r of 250 between the plates of the capacitor in Fig. 10.10. Then determine the new level of capacitance. Compare your results to the solution in Example 10.3.
b. Find the resulting electric field strength between the plates, and compare your answer to the result in Example 10.3.
c. Determine the charge on each of the plates, and compare your answer to the result in Example 10.3.

Solutions:

a. From Eq. (10.12), the new capacitance level is

$$C = \epsilon_r C_o = (250)(28.8\text{ pF}) = \mathbf{7200\text{ pF}} = \mathbf{7.2\text{ nF}} = \mathbf{0.0072\ \mu F}$$

which is *significantly higher* than the level in Example 10.3.

b. $\mathscr{E} = \frac{V}{d} = \frac{48 \text{ V}}{0.794 \text{ mm}} = \mathbf{60.5 \text{ kV/m}}$

Since the applied voltage and the distance between the plates did not change, *the electric field between the plates remains the same.*

c. $Q = CV = (7200 \text{ pF})(48 \text{ V}) = \mathbf{345.6 \text{ nC}} = \mathbf{0.35\ \mu C}$.

We now know that the insertion of a dielectric between the plates increases the amount of charge stored on the plates. In Example 10.4, since the relative permittivity increased by a factor of 250, the charge on the plates *increased by the same amount.*

EXAMPLE 10.5 Find the maximum voltage that can be applied across the capacitor in Example 10.4 if the dielectric strength is 80 V/mil.

Solution:

$$d = \frac{1}{32} \text{ in.} \left(\frac{1000 \text{ mils}}{1 \text{ in.}} \right) = 31.25 \text{ mils}$$

and

$$V_{max} = 31.25 \text{ mils} \left(\frac{80 \text{ V}}{\text{mil}} \right) = \mathbf{2.5 \text{ kV}}$$

although the provided working voltage may be only 2 kV to provide a margin of safety.

Types of Capacitors

Capacitors, like resistors, can be listed under two general headings: **fixed** and **variable.** The symbol for the fixed capacitor appears in Fig. 10.11(a). Note that the curved side is normally connected to ground or to the point of lower dc potential. The symbol for variable capacitors appears in Fig. 10.11(b).

FIG. 10.11
Symbols for the capacitor: (a) fixed; (b) variable.

Fixed Capacitors Fixed-type capacitors come in all shapes and sizes. However,

in general, for the same type of construction and dielectric, the larger the required capacitance, the larger is the physical size of the capacitor.

In Fig. 10.12(a), the 10,000 μF electrolytic capacitor is significantly larger than the 1 μF capacitor. However, it is certainly not 10,000 times larger. For the polyester-film type of Fig. 10.12(b), the 2.2 μF capacitor is significantly larger than the 0.01 μF capacitor, but again it is not 220 times larger. The 22 μF tantalum capacitor of Fig. 10.12(c) is about 6 times larger than the 1.5 μF capacitor, even though the capacitance level is about 15 times higher. It is particularly interesting to note that due to the difference in dielectric and construction, the 22 μF tantalum capacitor is significantly smaller than the 2.2 μF polyester-film capacitor and much smaller than 1/5 the size of the 100 μF electrolytic capacitor. The relatively large 10,000 μF electrolytic capacitor is normally used for high-power applications, such as in power supplies and high-output speaker systems. All the others may appear in any commercial electronic system.

FIG. 10.12
Demonstrating that, in general, for each type of construction, the size of a capacitor increases with the capacitance value: (a) electrolytic; (b) polyester-film; (c) tantalum.

The increase in size is due primarily to the effect of area and thickness of the dielectric on the capacitance level. There are a number of ways to increase the area without making the capacitor too large. One is to lay out the plates and the dielectric in long, narrow strips and then roll them all together, as shown in Fig. 10.13(a). The dielectric (remember that it has the characteristics of an insulator) between the conducting strips ensures the strips never touch. Of course, the dielectric must be the type that can be rolled without breaking up. Depending on how the materials are wrapped, the capacitor can be either a cylindrical or a rectangular, box-type shape.

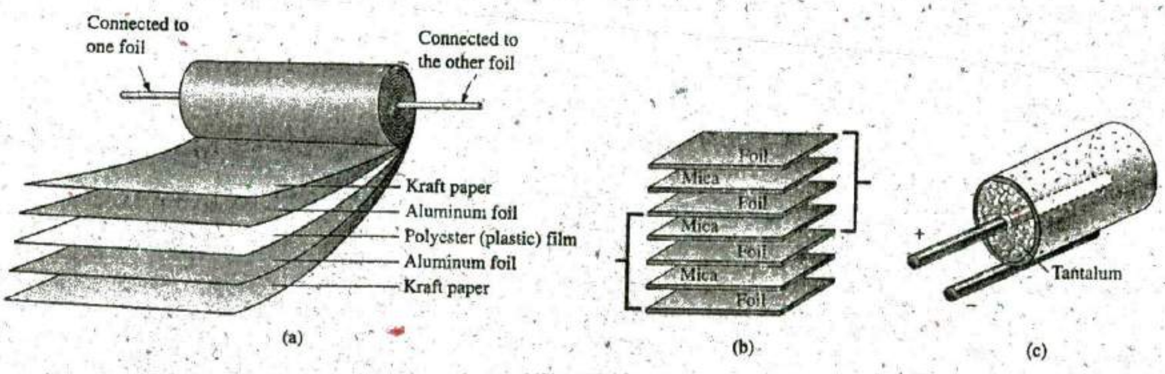


FIG. 10.13
Three ways to increase the area of a capacitor: (a) rolling; (b) stacking; (c) insertion.

A second popular method is to stack the plates and the dielectrics, as shown in Fig. 10.14(b). The area is now a multiple of the number of dielectric layers. This construction is very popular for smaller capacitors.

A third method is to use the dielectric to establish the body shape [a cylinder in Fig. 10.13(c)]. Then simply insert a rod for the positive plate, and coat the surface of the cylinder to form the negative plate, as shown in Fig. 10.13(c). Although the resulting "plates" are not the same in construction or surface area, the effect is to provide a large surface area for storage (the density of electric field lines will be different on the two "plates"), although the resulting distance factor may be larger than desired. Using a dielectric with a high ϵ_r, however, compensates for the increased distance between the plates.

There are other variations of the above to increase the area factor, but the three depicted in Fig. 10.13 are the most popular.

The next controllable factor is the distance between the plates. This factor, however, is very sensitive to how thin the dielectric can be made, with natural concerns because the working voltage (the breakdown voltage) drops as the gap decreases. Some of the thinnest dielectrics are just oxide coatings on one of the conducting surfaces (plates). A very thin polyester material, such as Mylar®, Teflon®, or even paper with a paraffin coating, provides a thin sheet of material than can easily be wrapped for increased areas. Materials such as mica and some ceramic materials can be made only so thin before crumbling or breaking down under stress.

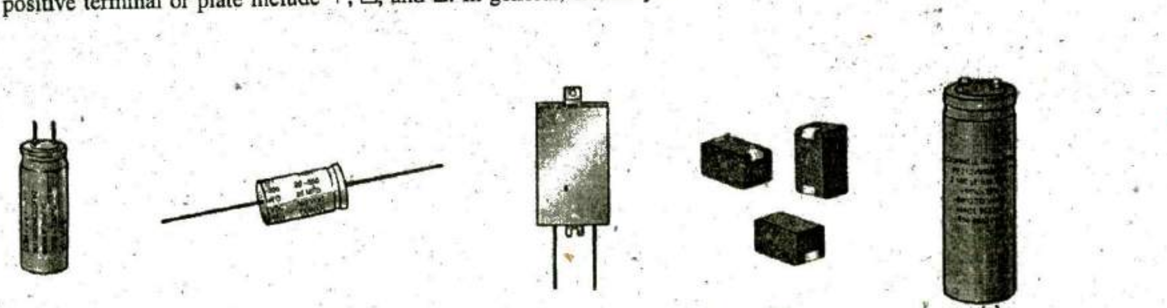
The last factor is the dielectric, for which there is a wide range of possibilities. However, the following factors greatly influence which dielectric is used:

The level of capacitance desired
The resulting size
The possibilities for rolling, stacking, and so on
Temperature sensitivity
Working voltage

The range of relative permittivities is enormous, as shown in Table 10.2, but all the factors listed above must be considered in the construction process.

In general, the most common fixed capacitors are the electrolytic, film, polyester, foil, ceramic, mica, dipped, and oil types.

The **electrolytic capacitors** in Fig. 10.14 are usually easy to identify by their shape and the fact that they usually have a polarity marking on the body (although special-application electrolytics are available that are not polarized). Few capacitors have a polarity marking, but those that do must be connected with the negative terminal connected to ground or to the point of lower potential. The markings often used to denote the positive terminal or plate include +, □, and Δ. In general, electrolytic



(a) (b) (c) (d) (e)

FIG. 10.14
Various types of electrolytic capacitors: (a) miniature radial leads; (b) axial leads; (c) flatpack; (d) surface-mount; (e) screw-in terminals.

capacitors offer some of the highest capacitance values available, although their working voltage levels are limited. Typical values range from 0.1 μF to 15,000 μF, with working voltages from 5 V to 450 V. The basic construction uses the rolling process in Fig. 10.13(a) in which a roll of aluminum foil is coated on one side with aluminum oxide—the aluminum being the positive plate and the oxide the dielectric. A layer of paper or gauze saturated with an electrolyte (a solution or paste that forms the conducting medium between the electrodes of the capacitor) is placed over the aluminum oxide coating of the positive plate. Another layer of aluminum without the oxide coating is then placed over this layer to assume the role of the negative plate. In most cases, the negative plate is connected directly to the aluminum container, which then serves as the negative terminal for external connections. Because of the size of the roll of aluminum foil, the overall size of the electrolytic capacitor is greater than most.

Film, polyester, foil, polypropylene, or **Teflon® capacitors** use a rolling or stacking process to increase the surface area, as shown in Fig. 10.15. The resulting shape can be either round or rectangular, with radial or axial leads. The typical range for such capacitors is 100 pF to 10 μF, with units available up to 100 μF. The name of the unit defines the type of dielectric employed. Working voltages can extend from a few volts to 2000 V, depending on the type of unit.

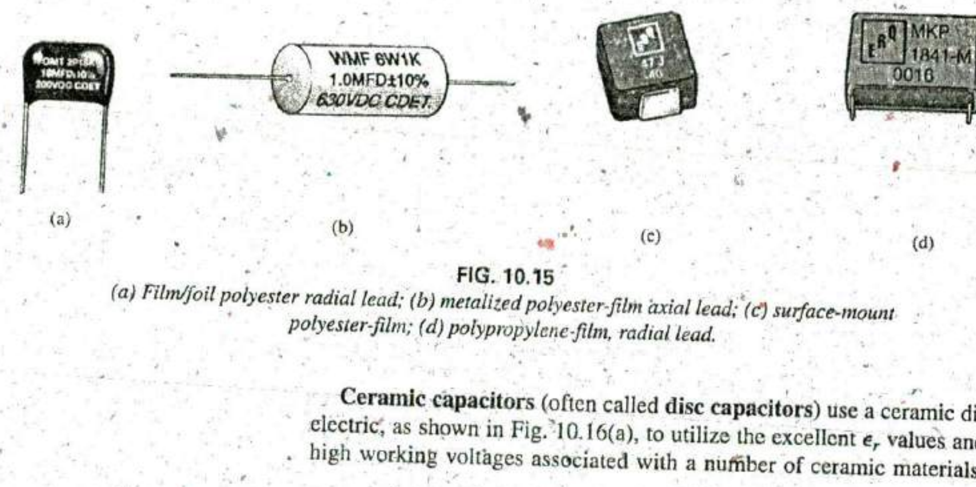


FIG. 10.15
(a) Film/foil polyester radial lead; (b) metalized polyester-film axial lead; (c) surface-mount polyester-film; (d) polypropylene-film, radial lead.

Ceramic capacitors (often called **disc capacitors**) use a ceramic dielectric, as shown in Fig. 10.16(a), to utilize the excellent ϵ_r values and high working voltages associated with a number of ceramic materials.

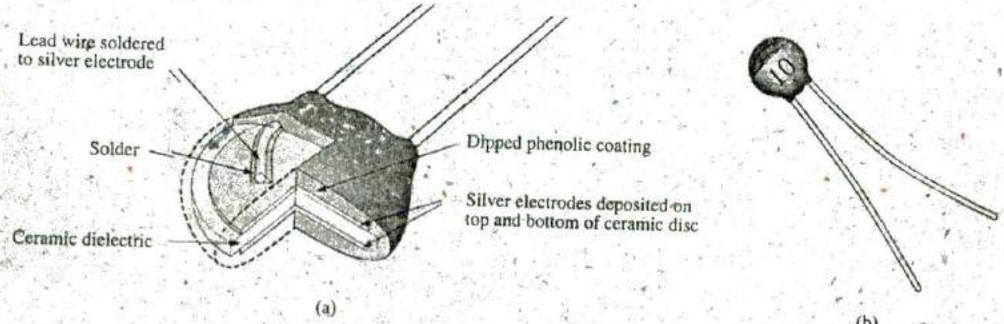


FIG. 10.16
Ceramic (disc) capacitor: (a) construction; (b) appearance.

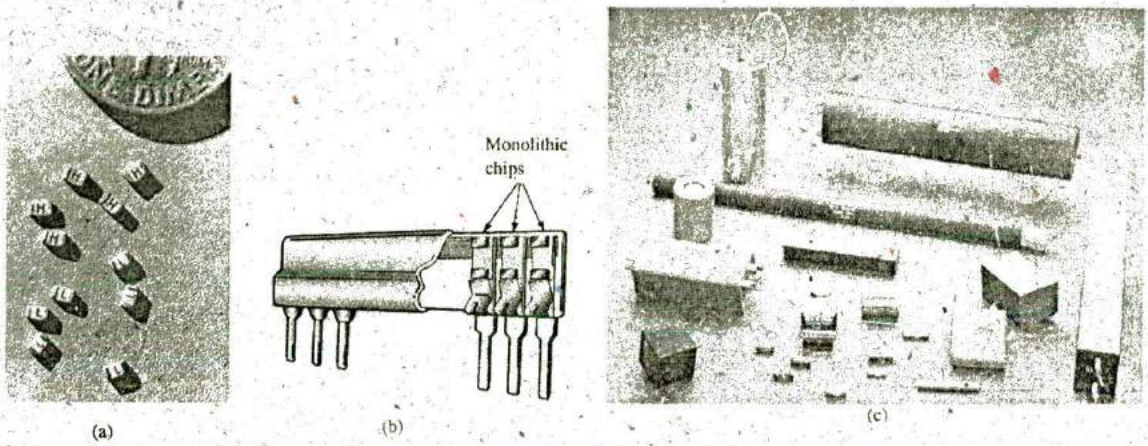


FIG. 10.17
Mica capacitors: (a) and (b) surface-mount monolithic chips; (c) high-voltage/temperature mica paper capacitors.
[(a) and (b) courtesy of Vishay Intertechnology, Inc.; (c) courtesy of Custom Electronics, Inc.]

Stacking can also be applied to increase the surface area. An example of the disc variety appears in Fig. 10.16(b). Ceramic capacitors typically range in value from 10 pF to 0.047 μF, with high working voltages that can reach as high as 10 kV.

Mica capacitors use a mica dielectric that can be monolithic (single chip) or stacked. The relatively small size of monolithic mica chip capacitors is demonstrated in Fig. 10.17(a), with their placement shown in Fig. 10.17(b). A variety of high-voltage mica paper capacitors are displayed in Fig. 10.17(c). Mica capacitors typically range in value from 2 pF to several microfarads, with working voltages up to 20 kV.

Dipped capacitors are made by dipping the dielectric (tantalum or mica) into a conductor in a molten state to form a thin, conductive sheet on the dielectric. Due to the presence of an electrolyte in the manufacturing process, dipped tantalum capacitors require a polarity marking to ensure that the positive plate is always at a higher potential than the negative plate, as shown in Fig. 10.18(a). A series of small positive signs is typically applied to the casing near the positive lead. A group of nonpolarized, mica dipped capacitors are shown in Fig. 10.18(b). They typically range in value from 0.1 μF to 680 μF, but with lower working voltages ranging from 6 V to 50 V.

Most **oil capacitors** such as appearing in Fig. 10.19 are used for industrial applications such as welding, high-voltage power supplies, surge protection, and power-factor correction (Chapter 19). They can provide capacitance levels extending from 0.001 μF all the way up to 10,000 μF, with working voltages up to 150 kV. Internally, there are a number of parallel plates sitting in a bath of oil or oil-impregnated material (the dielectric).

Variable Capacitors All the parameters in Eq. (10.11) can be changed to some degree to create a **variable capacitor.** For example, in Fig. 10.20(a), the capacitance of the variable air capacitor is changed by turning the shaft at the end of the unit. By turning the shaft, you control the amount of common area between the plates: The less common area there is, the lower is the capacitance. In Fig. 10.20(b), we have a much smaller **air trimmer capacitor.** It works under the same principle, but the rotating blades are totally hidden inside the structure. In

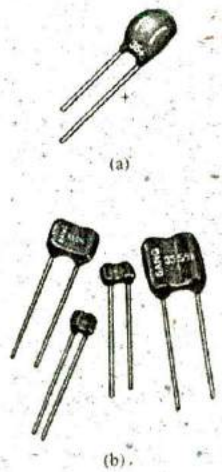
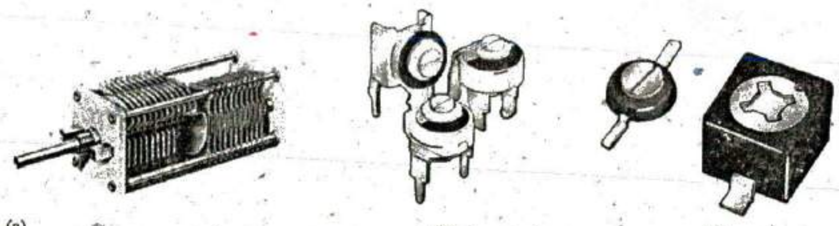


FIG. 10.18
Dipped capacitors: (a) polarized tantalum; (b) nonpolarized mica.

FIG. 10.19
Oil-filled, metallic oval case snubber capacitor (the snubber removes unwanted voltage spikes).



(a) (b) (c)

FIG. 10.20
Variable capacitors: (a) air; (b) air trimmer; (c) ceramic dielectric compression trimmer.
[(a) courtesy of James Millen Manufacturing Co.]

Fig. 10.20(c), the **ceramic trimmer capacitor** permits varying the capacitance by changing the common area as above or by applying pressure to the ceramic plate to reduce the distance between the plates.

Leakage Current and ESR

Although we would like to think of capacitors as ideal elements, unfortunately, this is not the case. There is a dc resistance appearing as R_s in the equivalent model of Fig. 10.21 due to the resistance introduced by the contacts, the leads, or the plate or foil materials. In addition, up to this point, we have assumed that the insulating characteristics of dielectrics prevent any flow of charge between the plates unless the breakdown voltage is exceeded. In reality, however, dielectrics are not perfect insulators, and they do carry a few free electrons in their atomic structure.

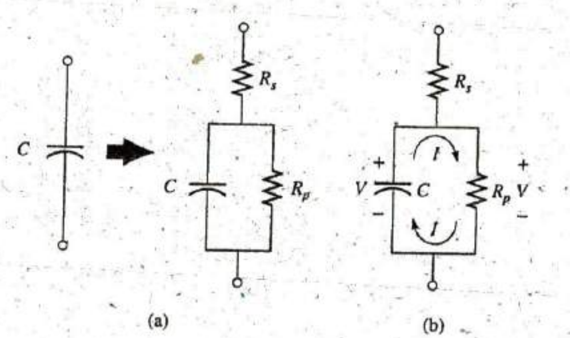


FIG. 10.21
Leakage current: (a) including the leakage resistance in the equivalent model for a capacitor; (b) internal discharge of a capacitor due to the leakage current.

When a voltage is applied across a capacitor, a **leakage current** is established between the plates. This current is usually so small that it can be ignored for the application under investigation. The availability of free electrons to support current flow is represented by a large parallel resistor R_p in the equivalent circuit for a capacitor as shown in Fig. 10.21(a). If we apply 10 V across a capacitor with an internal resistance of 1000 MΩ, the current will be 0.01 μA—a level that can be ignored for most applications.

The real problem associated with leakage currents is not evident until you ask the capacitors to sit in a charged state for long periods of time. As shown in Fig. 10.21(b), the voltage ($V = Q/C$) across a charged capacitor also appears across the parallel leakage resistance and establishes a discharge current through the resistor. In time, the capacitor is totally discharged. Capacitors such as the electrolytic that have high leakage currents (a leakage resistance of 0.5 MΩ is typical) usually have a limited shelf life due to this internal discharge characteristic. Ceramic, tantalum, and mica capacitors typically have unlimited shelf life due to leakage resistances in excess of 1000 MΩ. Thin-film capacitors have lower levels of leakage resistances that result in some concern about shelf life.

There is another quantity of importance when defining the complete capacitive equivalent: the **equivalent series resistance (ESR).** It is a quantity of such importance to the design of switching and linear power supplies that it holds equal weight with the actual capacitance level. It is a frequency-sensitive characteristic that will be examined in Chapter 14 after the concept of frequency response has been introduced in detail. As the name implied, it is included in the equivalent model for the capacitor as a series resistor that includes all the dissipative factors in an actual capacitor that go beyond just the dc resistance.

Temperature Effects: ppm

Every capacitor is temperature sensitive, with the nameplate capacitance level specified at room temperature. Depending on the type of dielectric, increasing or decreasing temperatures can cause either a drop or a rise in capacitance. If temperature is a concern for a particular application, the manufacturer will provide a temperature plot, such as shown in Fig. 10.22, or a **ppm/°C** (parts per million per degree Celsius) rating for the capacitor. Note in Fig. 10.20 the 0% variation from the nominal (nameplate) value at 25°C (room temperature). At 0°C (freezing), it has dropped 20%, while at 100°C (the boiling point of water), it has dropped 70%—a factor to consider for some applications.

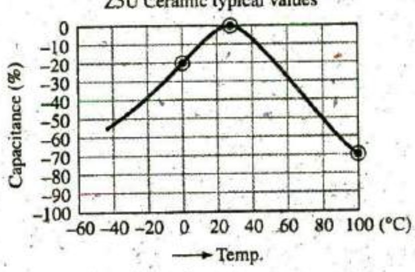


FIG. 10.22
Variation of capacitor value with temperature.

As an example of using the ppm level, consider a 100 μF capacitor with a **temperature coefficient** or **ppm** of −150 ppm/°C. It is important to note the negative sign in front of the ppm value because it reveals that the capacitance will drop with increase in temperature. It takes a moment to fully appreciate a term such as *parts per million.* In equation form, 150 parts per million can be written as

$$\frac{150}{1{,}000{,}000} \times$$

If we then multiply this term by the capacitor value, we can obtain the change in capacitance for each 1°C change in temperature. That is,

$$-\frac{150}{1{,}000{,}000}(100\ \mu\text{F})/°\text{C} = \mathbf{-0.015\ \mu F/°C} = \mathbf{-15{,}000\ pF/°C}$$

If the temperature should rise by 25°C, the capacitance would decrease by

$$-\frac{15{,}000\ \text{pF}}{°\cancel{\text{C}}}(25\ °\cancel{\text{C}}) = \mathbf{-0.38\ \mu F}$$

changing the capacitance level to

$$100\ \mu\text{F} - 0.38\ \mu\text{F} = \mathbf{99.62\ \mu F}$$

Capacitor Labeling

Due to the small size of some capacitors, various marking schemes have been adopted to provide the capacitance level, tolerance, and, if possible, working voltage. In general, however, as pointed out above, *the size of*

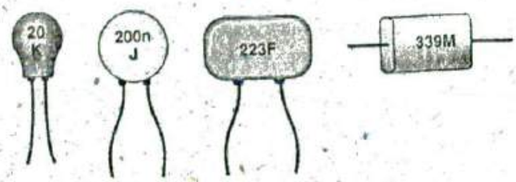


FIG. 10.23
Various marking schemes for small capacitors.

the capacitor is the first indicator of its value. In fact, most marking schemes do not indicate whether it is in μF or pF. It is assumed that you can make that judgment purely from the size. The smaller units are typically in pF and the larger units in μF. Unless indicated by an **n** or **N,** most units are not provided in nF. On larger μF units, the value can often be printed on the jacket with the tolerance and working voltage. However, smaller units need to use some form of abbreviation as shown in Fig. 10.23. For very small units such as those in Fig. 10.23(a) with only two numbers, the value is recognized immediately as being in pF with the **K** an indicator of a $\pm 10\%$ tolerance level. Too often the K is read as a multiplier of 10^3, and the capacitance is read as 20,000 pF or 20 nF rather than the actual 20 pF.

For the unit in Fig. 10.23(b), there was room for a lowercase **n** to represent a multiplier of 10^{-9}, resulting in a value of 200 nF. To avoid unnecessary confusion, the letters used for tolerance do not include **N, U,** or **P,** so the presence of any of these letters in upper- or lowercase normally refers to the multiplier level. The **J** appearing on the unit in Fig. 10.23(b) represents a $\pm 5\%$ tolerance level. For the capacitor in Fig. 10.23(c), the first two numbers are the numerical value of the capacitor, while the third number is the power of the multiplier (or number of zeros to be added to the first two numbers). The question then remains whether the units are μF or pF. With the 223 representing a number of 22,000, the units are certainly not μF because the unit is too small for such a large capacitance. It is a 22,000 pF = 22 nF capacitor. The **F** represents a $\pm 1\%$ tolerance level. Multipliers of 0.01 use an 8 for the third digit, while multipliers of 0.1 use a 9. The capacitor in Fig. 10.23(d) is a $33 \times 0.1 = 3.3\ \mu$F capacitor with a tolerance of $\pm 20\%$ as defined by the capital letter **M.** The capacitance is not 3.3 pF because the unit is too large; again, the factor of size is very helpful in making a judgment about the capacitance level. It should also be noted that **MFD** is sometimes used to signify microfarads.

Measurement and Testing of Capacitors

The capacitance of a capacitor can be read directly using a meter such as the Universal LCR Meter in Fig. 10.24. If you set the meter on **C** for *capacitance,* it will automatically choose the most appropriate unit of measurement for the element, that is, F, μF, nF, or pF. Note the polarity markings on the meter for capacitors that have a specified polarity.

The best check is to use a meter such as the one in Fig. 10.24. However, if it is unavailable, an ohmmeter can be used to determine whether the dielectric is still in good working order or whether it has deteriorated due to age or use (especially for paper and electrolytics). As the dielectric breaks down, the insulating qualities of the material decrease to the point where the resistance between the plates drops to a relatively low

FIG. 10.24
Digital reading capacitance meter.
(Courtesy of B+K Precision.)

level. To use an ohmmeter, be sure that the capacitor is fully discharged by placing a lead directly across its terminals. Then hook up the meter (paying attention to the polarities if the unit is polarized) as shown in Fig. 10.25, and note whether the resistance has dropped to a relatively low value (0 to a few kilohms). If so, the capacitor should be discarded. You may find that the reading changes when the meter is first connected. This change is due to the charging of the capacitor by the internal supply of the ohmmeter. In time the capacitor becomes stable, and the correct reading can be observed. Typically, it should pin at the highest level on the megohm scales or indicate OL on a digital meter.

The above ohmmeter test is not all-inclusive because some capacitors exhibit the breakdown characteristics only when a large voltage is applied. The test, however, does help isolate capacitors in which the dielectric has deteriorated.

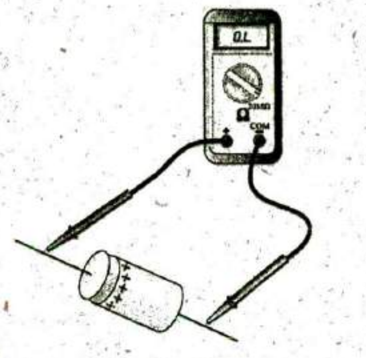


FIG. 10.25
Checking the dielectric of an electrolytic capacitor.

Standard Capacitor Values

The most common capacitors use the same numerical multipliers encountered for resistors.

The vast majority are available with 5%, 10%, or 20% tolerances. There are capacitors available, however, with tolerances of 1%, 2%, or 3%, if you are willing to pay the price. Typical values include 0.1 μF, 0.15 μF, 0.22 μF, 0.33 μF, 0.47 μF, 0.68 μF; and 1 μF, 1.5 μF, 2.2 μF, 3.3 μF, 4.7 μF, 6.8 μF; and 10 pF, 22 pF, 33 pF, 100 pF; and so on.

10.5 TRANSIENTS IN CAPACITIVE NETWORKS: THE CHARGING PHASE

The placement of charge on the plates of a capacitor does not occur instantaneously. Instead, it occurs over a period of time determined by the components of the network. The charging phase—the phase during which charge is deposited on the plates—can be described by reviewing the response of the simple series circuit in Fig. 10.4. The circuit has been redrawn in Fig. 10.26 with the symbol for a fixed capacitor.

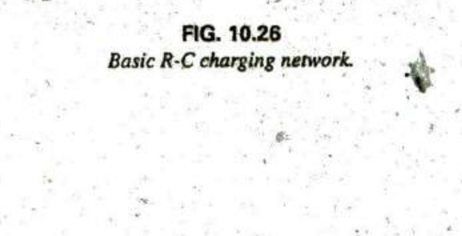


FIG. 10.26
Basic R-C charging network.

Recall that the instant the switch is closed, electrons are drawn from the top plate and deposited on the bottom plate by the battery, resulting in a net positive charge on the top plate and a negative charge on the bottom plate. The transfer of electrons is very rapid at first, slowing down as the potential across the plates approaches the applied voltage of the battery. Eventually, when the voltage across the capacitor equals the applied voltage, the transfer of electrons ceases, and the plates have a net charge determined by $Q = CV_C = CE$. This period of time during which charge is being deposited on the plates is called the **transient period**—a period of time where the voltage or current changes from one steady-state level to another.

Since the voltage across the plates is directly related to the charge on the plates by $V = Q/C$, a plot of the voltage across the capacitor will have the same shape as a plot of the charge on the plates over time. As shown in Fig. 10.27, the voltage across the capacitor is zero volts when the switch is closed ($t = 0$ s). It then builds up very quickly at first since charge is being deposited at a very high rate of speed. As time passes, the charge is deposited at a slower rate, and the change in voltage drops off. The voltage continues to grow, but at a much slower rate. Eventually, as the voltage across the plates approaches the applied voltage, the charging rate is very slow, until finally the voltage across the plates is equal to the applied voltage—the transient phase has passed.

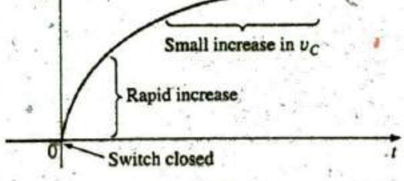


FIG. 10.27
v_C during the charging phase.

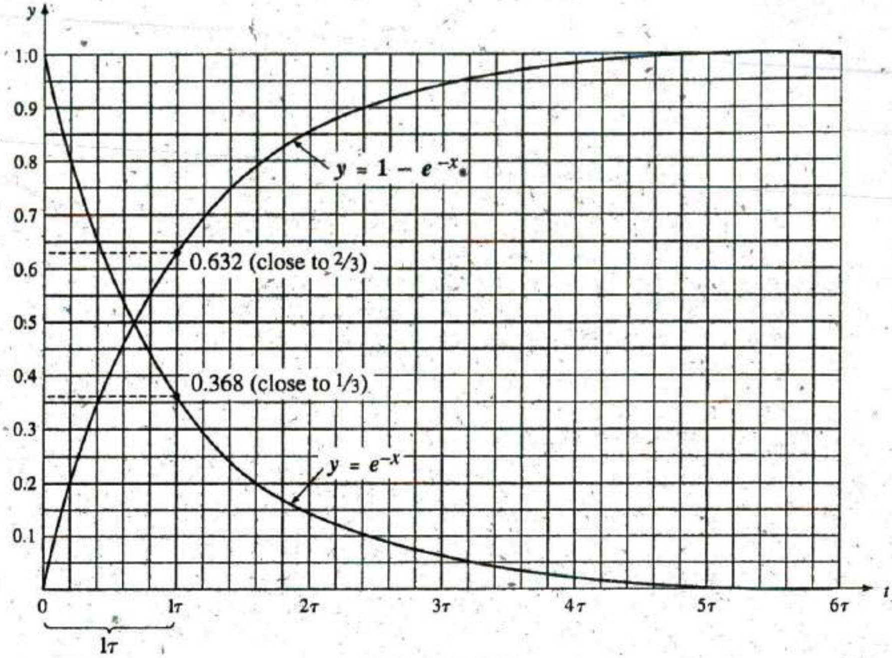


FIG. 10.28
Universal time constant chart.

TABLE 10.3
Selected values of e^{-x}.

$x = 0$	$e^{-x} = e^{-0} = \frac{1}{e^0} = \frac{1}{1} = 1$
$x = 1$	$e^{-1} = \frac{1}{e} = \frac{1}{2.71828\ldots} = 0.3679$
$x = 2$	$e^{-2} = \frac{1}{e^2} = 0.1353$
$x = 5$	$e^{-5} = \frac{1}{e^5} = 0.00674$
$x = 10$	$e^{-10} = \frac{1}{e^{10}} = 0.0000454$
$x = 100$	$e^{-100} = \frac{1}{e^{100}} = 3.72 \times 10^{-44}$

Fortunately, the waveform in Fig. 10.27 from beginning to end can be described using the mathematical function e^{-x}. It is an exponential function that decreases with time, as shown in Fig. 10.28. If we substitute zero for x, we obtain e^{-0}, which by definition is 1, as shown in Table 10.3 and on the plot in Fig. 10.28. Table 10.3 reveals that as x increases, the function e^{-x} decreases in magnitude until it is very close to zero after $x = 5$. As noted in Table 10.3, the exponential factor $e^1 = e = 2.71828$.

A plot of $1 - e^{-x}$ is also provided in Fig. 10.28 since it is a component of the voltage v_C in Fig. 10.27. When e^{-x} is 1, $1 - e^{-x}$ is zero, as shown in Fig. 10.28, and when e^{-x} decreases in magnitude, $1 - e^{-x}$ approaches 1, as shown in the same figure.

You may wonder how this function can help us if it decreases with time and the curve for the voltage across the capacitor increases with time. We simply place the exponential in the proper mathematical form as follows:

$$\boxed{v_C = E(1 - e^{-t/\tau})}_{\text{charging}} \qquad \text{(volts, V)} \qquad \textbf{(10.13)}$$

First note in Eq. (10.13) that the voltage v_C is written in *lowercase (not capital) italic* to point out that it is a function that will change with time—it is not a constant. The exponent of the exponential function is no longer just x, but now is time (t) divided by a constant τ, the Greek letter *tau*. The quantity τ is defined by

$$\boxed{\tau = RC} \qquad \text{(time, s)} \qquad \textbf{(10.14)}$$

The factor τ, called the **time constant** of the network, has the units of time, as shown below using some of the basic equations introduced earlier in this text:

$$\tau = RC = \left(\frac{V}{I}\right)\left(\frac{Q}{V}\right) = \left(\frac{\cancel{V}}{\cancel{Q}/t}\right)\left(\frac{\cancel{Q}}{\cancel{V}}\right) = t \text{ (seconds)}$$

A plot of Eq. (10.13) results in the curve in Fig. 10.29, whose shape is an exact match with that in Fig. 10.27.

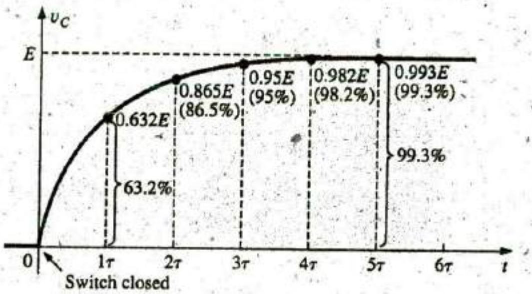


FIG. 10.29
Plotting the equation $v_C = E(1 - e^{-t/\tau})$ versus time (t).

In Eq. (10.13), if we substitute $t = 0$ s, we find that

$$e^{-t/\tau} = e^{-0/\tau} = e^{-0} = \frac{1}{e^0} = \frac{1}{1} = 1$$

and $$v_C = E(1 - e^{-t/\tau}) = E(1 - 1) = \mathbf{0\ V}$$

as appearing in the plot in Fig. 10.29.

It is important to realize at this point that the plot in Fig. 10.29 is not against simply time but against τ, the time constant of the network. If we want to know the voltage across the plates after one time constant, we simply plug $t = 1\tau$ into Eq. (10.13). The result is

$$e^{-t/\tau} = e^{-\tau/\tau} = e^{-1} \cong 0.368$$

and $$v_C = E(1 - e^{-t/\tau}) = E(1 - 0.368) = \mathbf{0.632E}$$

as shown in Fig. 10.29.

At $t = 2\tau$

$$e^{-t/\tau} = e^{-2\tau/\tau} = e^{-2} \cong 0.135$$

and $$v_C = E(1 - e^{-t/\tau}) = E(1 - 0.135) \cong \mathbf{0.865E}$$

as shown in Fig. 10.29.

As the number of time constants increases, the voltage across the capacitor does indeed approach the applied voltage.

At $t = 5\tau$

$$e^{-t/\tau} = e^{-5\tau/\tau} = e^{-5} \cong 0.007$$

and $$v_C = E(1 - e^{-t/\tau}) = E(1 - 0.007) = \mathbf{0.993E} \cong E$$

In fact, we can conclude from the results just obtained that

the voltage across a capacitor in a dc network is essentially equal to the applied voltage after five time constants of the charging phase have passed.

Or, in more general terms,

the transient or charging phase of a capacitor has essentially ended after five time constants.

It is indeed fortunate that the same exponential function can be used to plot the current of the capacitor versus time. When the switch is first closed, the flow of charge or current jumps very quickly to a value limited by the applied voltage and the circuit resistance, as shown in Fig. 10.30. The rate of deposit, and hence the current, then decreases quite rapidly, until eventually charge is not being deposited on the plates and the current drops to zero amperes.

The equation for the current is

$$\boxed{i_C = \frac{E}{R}e^{-t/\tau}}_{\text{charging}} \qquad \text{(amperes, A)} \qquad \textbf{(10.15)}$$

In Fig. 10.26, the current (conventional flow) has the direction shown since electrons flow in the opposite direction.

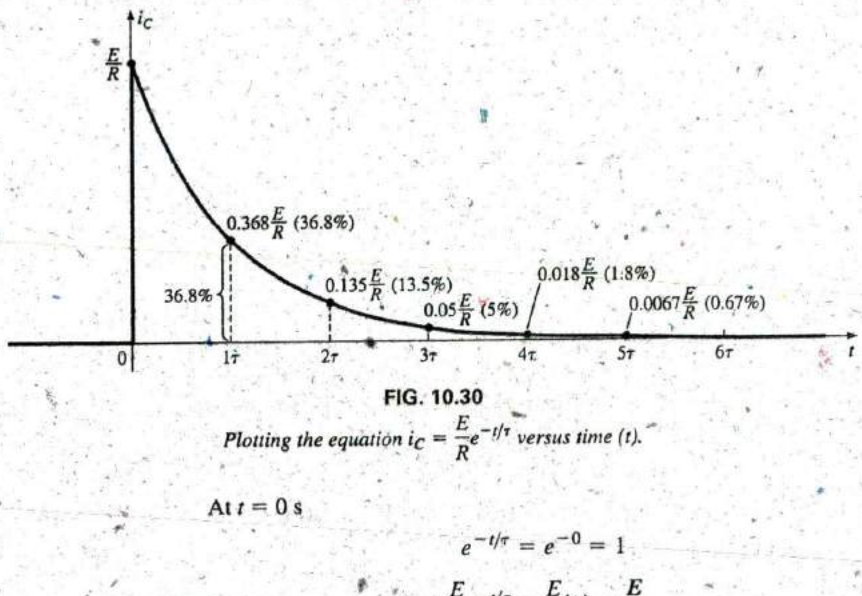


FIG. 10.30

Plotting the equation $i_C = \frac{E}{R}e^{-t/\tau}$ versus time (t).

At $t = 0$ s

$$e^{-t/\tau} = e^{-0} = 1$$

and
$$i_C = \frac{E}{R}e^{-t/\tau} = \frac{E}{R}(1) = \frac{E}{R}$$

At $t = 1\tau$

$$e^{-t/\tau} = e^{-\tau/\tau} = e^{-1} \cong 0.368$$

and
$$i_C = \frac{E}{R}e^{-t/\tau} = \frac{E}{R}(0.368) = \mathbf{0.368}\,\frac{E}{R}$$

In general, Fig. 10.30 clearly reveals that

the current of a capacitive dc network is essentially zero amperes after five time constants of the charging phase have passed.

It is also important to recognize that

during the charging phase, the major change in voltage and current occurs during the first time constant.

The voltage across the capacitor reaches about 63.2% (about 2/3) of its final value, whereas the current drops to 36.8% (about 1/3) of its peak value. During the next time constant, the voltage increases only about 23.3%, whereas the current drops to 13.5%. The first time constant is therefore a very dramatic time for the changing parameters. Between the fourth and fifth time constants, the voltage increases only about 1.2%, whereas the current drops to less than 1% of its peak value.

Returning to Figs. 10.29 and 10.30, note that when the voltage across the capacitor reaches the applied voltage E, the current drops to zero amperes, as reviewed in Fig. 10.31. These conditions match those of an open circuit, permitting the following conclusion:

A capacitor can be replaced by an open-circuit equivalent once the charging phase in a dc network has passed.

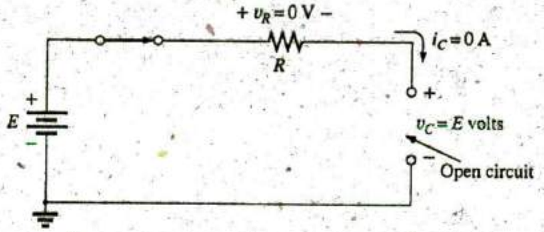


FIG. 10.31
Demonstrating that a capacitor has the characteristics of an open circuit after the charging phase has passed.

This conclusion will be particularly useful when analyzing dc networks that have been on for a long period of time or have passed the transient phase that normally occurs when a system is first turned on.

A similar conclusion can be reached if we consider the instant the switch is closed in the circuit in Fig. 10.26. Referring to Figs. 10.29 and 10.30 again, we find that the current is a peak value at $t = 0$ s, whereas the voltage across the capacitor is 0 V, as shown in the equivalent circuit in Fig. 10.32. The result is that

a capacitor has the characteristics of a short-circuit equivalent at the instant the switch is closed in an uncharged series R-C circuit.

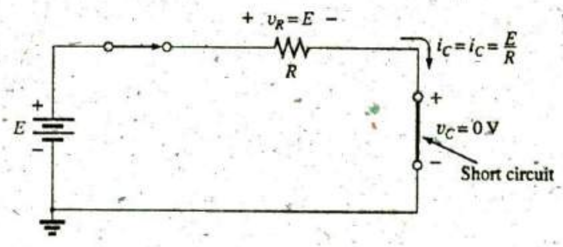


FIG. 10.32
Revealing the short-circuit equivalent for the capacitor that occurs when the switch is first closed.

In Eq. (10.13), the time constant τ will always have some value because some resistance is always present in a capacitive network. In some cases, the value of τ may be very small, but five times that value of τ, no matter how small, must therefore always exist; it cannot be zero. The result is the following very important conclusion:

The voltage across a capacitor cannot change instantaneously.

In fact, we can take this statement a step further by saying that the capacitance of a network is a measure of how much it will oppose a change in voltage in a network. The larger the capacitance, the larger is the time constant, and the longer it will take the voltage across the capacitor to reach the applied value. This can prove very helpful when lightning arresters and surge suppressors are designed to protect equipment from unexpected high surges in voltage.

Since the resistor and the capacitor in Fig. 10.26 are in series, the current through the resistor is the same as that associated with the capacitor. The voltage across the resistor can be determined by using Ohm's law in the following manner:

$$\upsilon_R = i_R R = i_C R$$

so that

$$\upsilon_R = \left(\frac{E}{R}e^{-t/\tau}\right)R$$

and

$$\boxed{\upsilon_R = Ee^{-t/\tau}}_{\text{charging}} \qquad \text{(volts, V)} \qquad (10.16)$$

A plot of the voltage as shown in Fig. 10.33 has the same shape as that for the current because they are related by the constant R. Note, however, that the voltage across the resistor starts at a level of E volts because the voltage across the capacitor is zero volts and Kirchhoff's voltage law must always be satisfied. When the capacitor has reached the applied voltage, the voltage across the resistor must drop to zero volts for the same reason. Always remember that

Kirchhoff's voltage law is applicable at any instant of time for any type of voltage in any type of network.

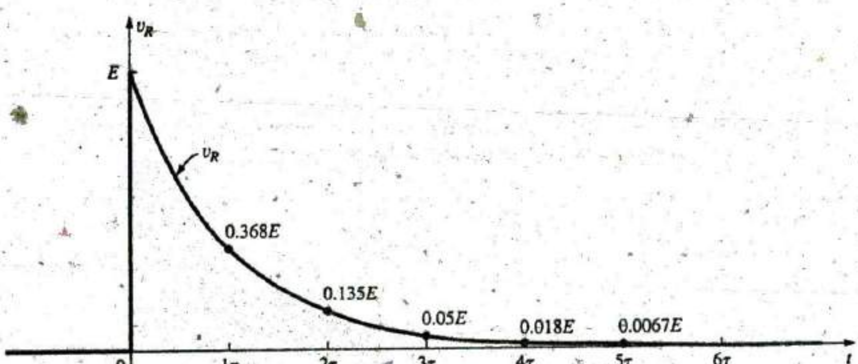


FIG. 10.33
Plotting the equation $\upsilon_R = Ee^{-t/\tau}$ versus time (t).

Using the Calculator to Solve Exponential Functions

Before looking at an example, we will first discuss the use of the TI-89 calculator with exponential functions. The process is actually quite simple for a number such as $e^{-1.2}$. Just select the 2nd function (diamond) key, followed by the function e^x. Then insert the (−) sign from the numerical

FIG. 10.34
Calculator key strokes to determine $e^{-1.2}$.

keyboard (not the mathematical functions), and insert the number 1.2 followed by ENTER to obtain the result of 0.301, as shown in Fig. 10.34. The use of the computer software program Mathcad is demonstrated in a later example.

EXAMPLE 10.6 For the circuit in Fig. 10.35:

a. Find the mathematical expression for the transient behavior of v_C, i_C, and v_R if the switch is closed at $t = 0$ s.
b. Plot the waveform of v_C versus the time constant of the network.
c. Plot the waveform of v_C versus time.
d. Plot the waveforms of i_C and v_R versus the time constant of the network.
e. What is the value of v_C at $t = 20$ ms?
f. On a practical basis, how much time must pass before we can assume that the charging phase has passed?
g. When the charging phase has passed, how much charge is sitting on the plates?
h. If the capacitor has a leakage resistance of 10,000 MΩ, what is the initial leakage current? Once the capacitor is separated from the circuit, how long will it take to totally discharge, assuming a linear (unchanging) discharge rate?

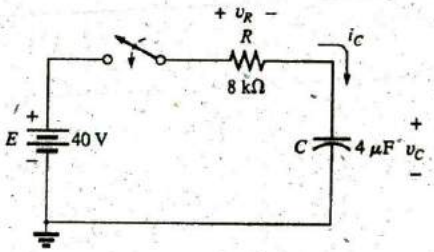


FIG. 10.35
Transient network for Example 10.6.

Solutions:

a. The time constant of the network is

$$\tau = RC = (8\text{ k}\Omega)(4\ \mu\text{F}) = 32\text{ ms}$$

resulting in the following mathematical equations:

$$v_C = E(1 - e^{-t/\tau}) = \mathbf{40\ V(1 - e^{-t/32ms})}$$

$$i_C = \frac{E}{R}e^{-t/\tau} = \frac{40\text{ V}}{8\text{ k}\Omega}e^{-t/32\text{ms}} = \mathbf{5\ mAe^{-t/32\,ms}}$$

$$v_R = Ee^{-t/\tau} = \mathbf{40\ Ve^{-t/32ms}}$$

b. The resulting plot appears in Fig. 10.36.
c. The horizontal scale will now be against time rather than time constants, as shown in Fig. 10.37. The plot points in Fig. 10.37 were taken from Fig. 10.36.

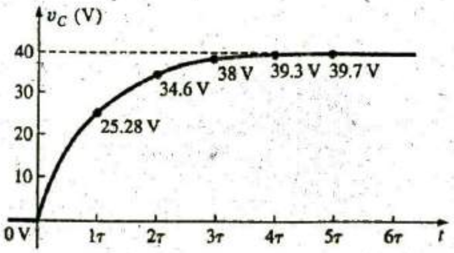


FIG. 10.36
v_C versus time for the charging network in Fig. 10.35.

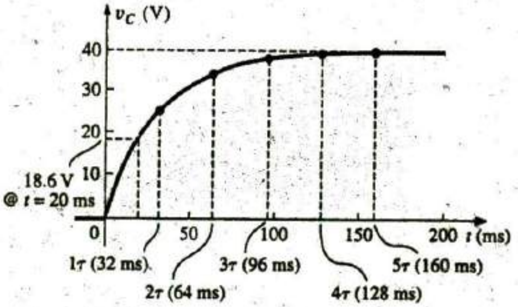


FIG. 10.37
Plotting the waveform in Fig. 10.36 versus time (t).

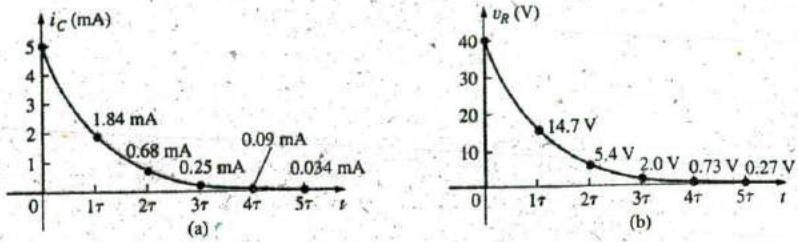


FIG. 10.38

i_C and v_R for the charging network in Fig. 10.36.

d. Both plots appear in Fig. 10.38.

e. Substituting the time $t = 20$ ms results in the following for the exponential part of the equation:

$$e^{t/\tau} = e^{-20\text{ ms}/32\text{ ms}} = e^{-0.625} = 0.535 \qquad \text{(using a calculator)}$$

so that $v_C = 40\text{ V}(1 - e^{t/32\text{ ms}}) = 40\text{ V}(1 - 0.535)$

$$= (40\text{ V})(0.465) = \mathbf{18.6\text{ V}} \qquad \text{(as verified by Fig. 10.37)}$$

f. Assuming a full charge in five time constants results in

$$5\tau = 5(32\text{ ms}) = \mathbf{160\text{ ms}} = \mathbf{0.16\text{ s}}$$

g. Using Eq. (10.6) gives

$$Q = CV = (4\ \mu\text{F})(40\text{ V}) = \mathbf{160\ \mu C}$$

h. Using Ohm's law gives

$$I_{\text{leakage}} = \frac{40\text{ V}}{10{,}000\text{ M}\Omega} = 4\text{ nA}$$

Finally, the basic equation $I = Q/t$ results in

$$t = \frac{Q}{I} = \frac{160\ \mu C}{4\text{ nA}} = (40{,}000\ \cancel{s})\left(\frac{1\ \cancel{\text{min}}}{60\ \cancel{s}}\right)\left(\frac{1\text{ h}}{60\ \cancel{\text{min}}}\right) = \mathbf{11.11\text{ h}}$$

10.6 TRANSIENTS IN CAPACITIVE NETWORKS: THE DISCHARGING PHASE

We now investigate how to discharge a capacitor while exerting some control on how long the discharge time will be. You can, of course, place a lead directly across a capacitor to discharge it very quickly—and possibly cause a visible spark. For larger capacitors such those in TV sets, this procedure should not be attempted because of the high voltages involved—unless, of course, you are trained in the maneuver.

In Fig. 10.39(a), a second contact for the switch was added to the circuit in Fig. 10.26 to permit a controlled discharge of the capacitor. With the switch in position 1, we have the charging network described in the last section. Following the full charging phase, if we move the switch to position 1, the capacitor can be discharged through the resulting circuit in Fig. 10.39(b). In Fig. 10.39(b), the voltage across the capacitor appears directly across the resistor to establish a discharge current. Initially, the current jumps to a relatively high value; then it begins to drop. It drops with time because charge is leaving the plates of the capacitor, which in turn reduces the voltage across the capacitor and thereby the voltage across the resistor and the resulting current.

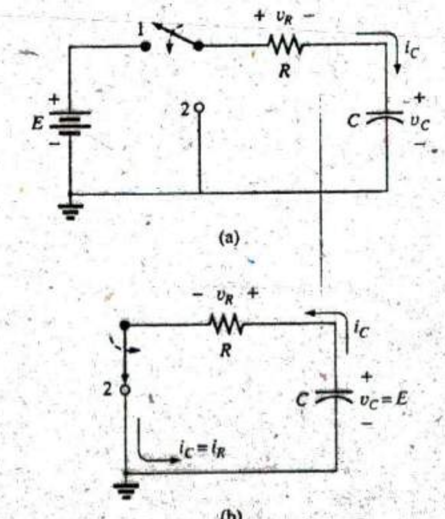


FIG. 10.39

(a) Charging network; (b) discharging configuration.

Before looking at the wave shapes for each quantity of interest, note that current i_C has now reversed direction as shown in Fig. 10.39(b). As shown in parts (a) and (b) in Fig. 10.39, the voltage across the capacitor does not reverse polarity, but the current reverses direction. We will show the reversals on the resulting plots by sketching the waveforms in the negative regions of the graph. In all the waveforms, note that all the mathematical expressions use the same e^{-x} factor appearing during the charging phase.

For the voltage across the capacitor that is decreasing with time, the mathematical expression is

$$\boxed{v_C = Ee^{-t/\tau}}_{\text{discharging}} \tag{10.17}$$

For this circuit, the time constant τ is defined by the same equation as used for the charging phase. That is,

$$\boxed{\tau = RC}_{\text{discharging}} \tag{10.18}$$

Since the current decreases with time, it will have a similar format:

$$\boxed{i_C = \frac{E}{R}e^{-t/\tau}}_{\text{discharging}} \tag{10.19}$$

For the configuration in Fig. 10.39(b), since $v_R = v_C$ (in parallel), the equation for the voltage v_R has the same format:

$$\boxed{v_R = Ee^{-t/\tau}}_{\text{discharging}} \tag{10.20}$$

The complete discharge will occur, for all practical purposes, in five time constants. If the switch is moved between terminals 1 and 2 every five time constants, the wave shapes in Fig. 10.40 will result for v_C, i_C,

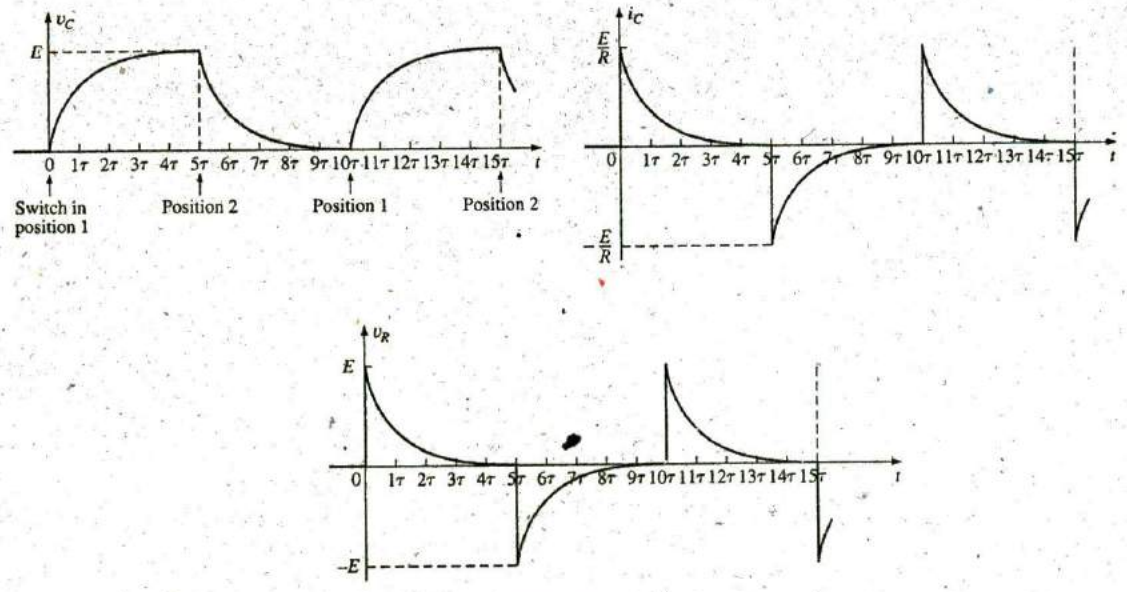


FIG. 10.40
v_C, i_C, and v_R for 5τ switching between contacts in Fig. 10.39(a).

and v_R. For each curve, the current directions and voltage polarities are as defined by the configurations in Fig. 10.39. Note, as pointed out above, that the current reverses direction during the discharge phase.

The discharge rate does not have to equal the charging rate if a different switching arrangement is used. In fact, Example 10.8 will demonstrate how to change the discharge rate.

EXAMPLE 10.7 Using the values in Example 10.6, plot the waveforms for v_C and i_C resulting from switching between contacts 1 and 2 in Fig. 10.39 every five time constants.

Solution: The time constant is the same for the charging and discharging phases. That is,

$$\tau = RC = (8\ \text{k}\Omega)(4\ \mu\text{F}) = 32\ \text{ms}$$

For the discharge phase, the equations are

$$v_C = Ee^{-t/\tau} = \mathbf{40\ Ve^{-t/32\ ms}}$$

$$i_C = -\frac{E}{R}e^{-t/\tau} = \frac{40\ \text{V}}{8\ \text{k}\Omega}e^{-t/32\ \text{ms}} = \mathbf{-5\ mAe^{-t/32\ ms}}$$

$$v_R = v_C = \mathbf{40\ Ve^{-t/32\ ms}}$$

A continuous plot for the charging and discharging phases appears in Fig. 10.41.

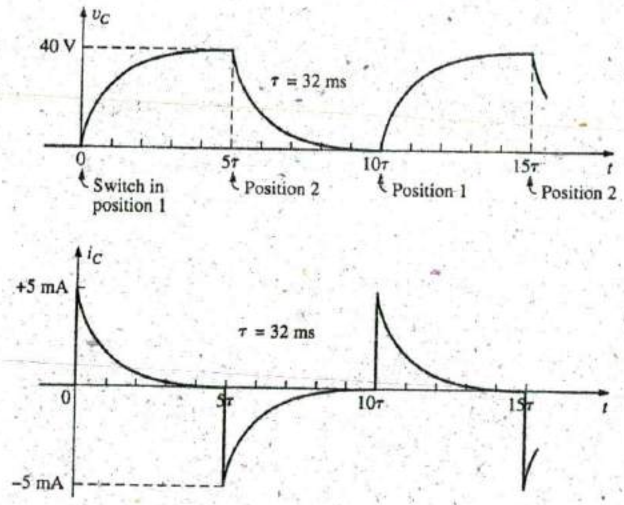


FIG. 10.41
v_C and i_C for the network in Fig. 10.39(a) with the values in Example 10.6.

The Effect of τ on the Response

In Example 10.7, if the value of τ were changed by changing the resistance, the capacitor, or both, *the resulting waveforms would appear the same because they were plotted against the time constant of the network.* If they were plotted against time, there could be a dramatic change in the appearance of the resulting plots. In fact, on an oscilloscope, an instrument designed to display such waveforms, the plots are against time, and the change will be immediately apparent. In Fig. 10.42(a), the waveforms in

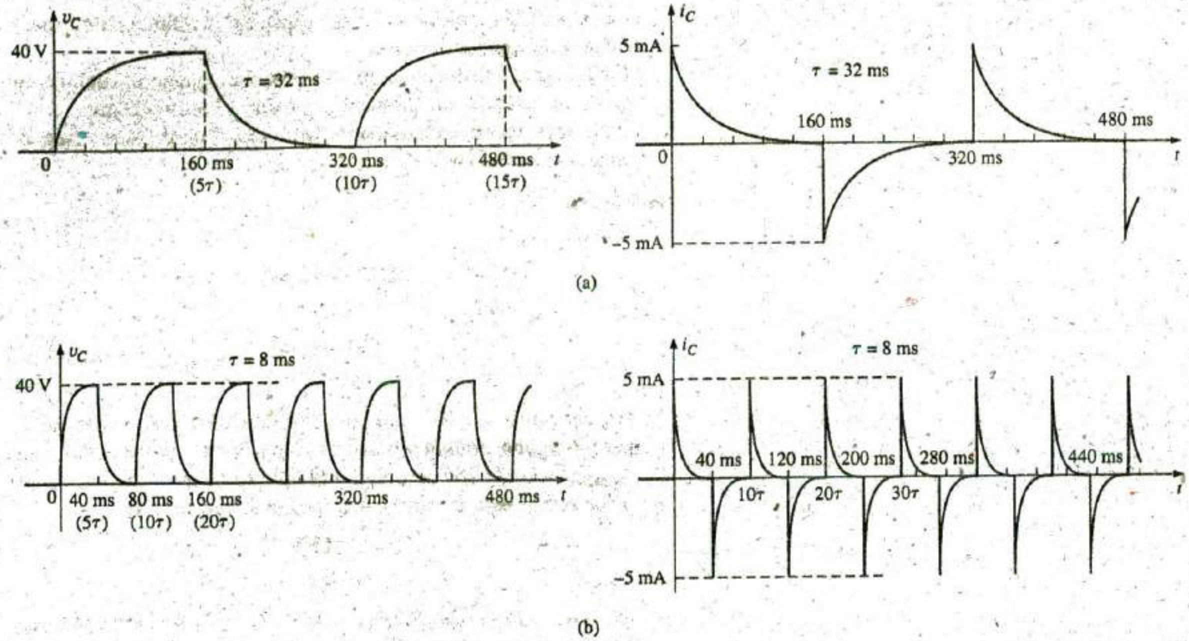


FIG. 10.42

Plotting v_C and i_C versus time in ms: (a) $\tau = 32$ ms; (b) $\tau = 8$ ms.

Fig. 10.41 for v_C and i_C were plotted against time. In Fig. 10.42(b), the capacitance was decreased to 1 μF, which reduces the time constant to 8 ms. Note the dramatic effect on the appearance of the waveform.

For a fixed-resistance network, the effect of increasing the capacitance is clearly demonstrated in Fig. 10.43. The larger the capacitance, and hence the time constant, the longer it takes the capacitor to charge up—there is more charge to be stored. The same effect can be created by holding the capacitance constant and increasing the resistance, but now the longer time is due to the lower currents that are a result of the higher resistance.

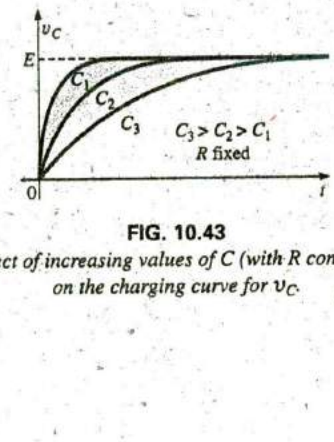


FIG. 10.43

Effect of increasing values of C (with R constant) on the charging curve for v_C.

EXAMPLE 10.8 For the circuit in Fig. 10.44:

a. Find the mathematical expressions for the transient behavior of the voltage v_C and the current i_C if the capacitor was initially uncharged and the switch is thrown into position 1 at $t = 0$ s.
b. Find the mathematical expressions for the voltage v_C and the current i_C if the switch is moved to position 2 at $t = 10$ ms. (Assume

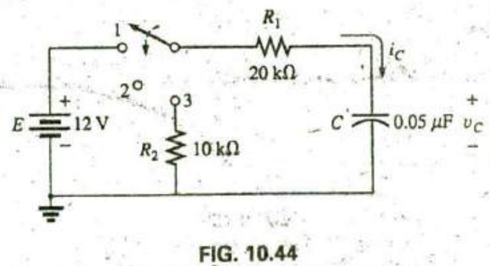


FIG. 10.44

Network to be analyzed in Example 10.8.

that the leakage resistance of the capacitor is infinite ohms; that is, there is no leakage current.)

c. Find the mathematical expressions for the voltage v_C and the current i_C if the switch is thrown into position 3 at $t = 20$ ms.
d. Plot the waveforms obtained in parts (a)–(c) on the same time axis using the defined polarities in Fig. 10.44.

Solutions:

a. *Charging phase:*

$$\tau = R_1C = (20\text{ k}\Omega)(0.05\ \mu\text{F}) = 1\text{ ms}$$
$$v_C = E(1 - e^{-t/\tau}) = \mathbf{12\ V(1 - e^{-t/1\ ms})}$$
$$i_C = \frac{E}{R_1}e^{-t/\tau} = \frac{12\text{ V}}{20\text{ k}\Omega}e^{-t/1\text{ ms}} = \mathbf{0.6\ mAe^{-t/1\ ms}}$$

b. *Storage phase:* At 10 ms, a period of time equal to 10τ has passed, permitting the assumption that the capacitor is fully charged. Since $R_{\text{leakage}} = \infty\ \Omega$, the capacitor will hold its charge indefinitely. The result is that both v_C and i_C will remain at a fixed value:

$$v_C = \mathbf{12\ V}$$
$$i_C = \mathbf{0\ A}$$

c. *Discharge phase* (using 20 ms as the new $t = 0$ s for the equations): The new time constant is

$$\tau' = RC = (R_1 + R_2)C = (20\text{ k}\Omega + 10\text{ k}\Omega)(0.05\ \mu\text{F}) = 1.5\text{ ms}$$
$$v_C = Ee^{-t/\tau'} = \mathbf{12\ Ve^{-t/1.5\ ms}}$$
$$i_C = -\frac{E}{R}e^{-t/\tau'} = -\frac{E}{R_1 + R_2}e^{-t/\tau'}$$
$$= -\frac{12\text{ V}}{20\text{ k}\Omega + 10\text{ k}\Omega}e^{-t/1.5\text{ ms}} = \mathbf{-0.4\ mAe^{-t/1.5\ ms}}$$

d. See Fig. 10.45.

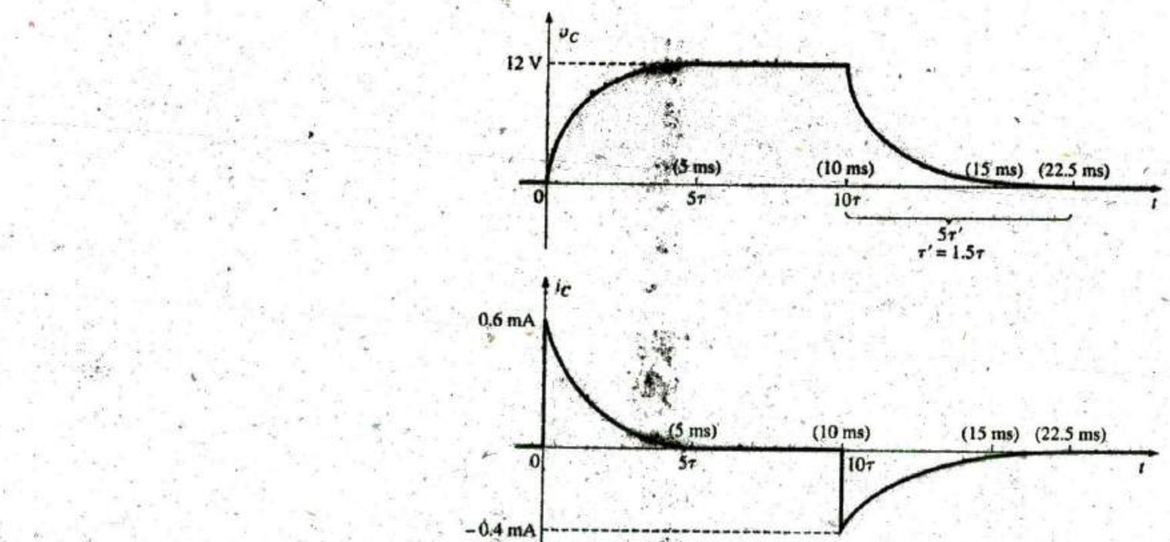


FIG. 10.45
v_C and i_C for the network in Fig. 10.44.

EXAMPLE 10.9 For the network in Fig. 10.46:

a. Find the mathematical expression for the transient behavior of the voltage across the capacitor if the switch is thrown into position 1 at $t = 0$ s.
b. Find the mathematical expression for the transient behavior of the voltage across the capacitor if the switch is moved to position 2 at $t = 1\tau$.
c. Plot the resulting waveform for the voltage v_C as determined by parts (a) and (b).
d. Repeat parts (a)–(c) for the current i_C.

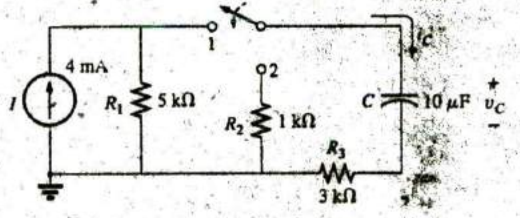


FIG. 10.46
Network to be analyzed in Example 10.9.

Solutions:

a. Converting the current source to a voltage source results in the configuration in Fig. 10.47 for the charging phase.

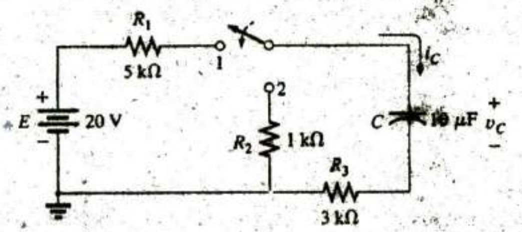


FIG. 10.47
The charging phase for the network in Fig. 10.46.

For the source conversion

$$E = IR = (4\text{ mA})(5\text{ k}\Omega) = 20\text{ V}$$

and $$R_s = R_p = 5\text{ k}\Omega$$

$$\tau = RC = (R_1 + R_3)C = (5\text{ k}\Omega + 3\text{ k}\Omega)(10\ \mu\text{F}) = 80\text{ ms}$$

$$v_C = E(1 - e^{-t/\tau}) = \mathbf{20\ V\ (1 - e^{-t/80\ ms})}$$

b. With the switch in position 2, the network appears as shown in Fig. 10.48. The voltage at 1τ can be found by using the fact that the voltage is 63.2% of its final value of 20 V, so that $0.632(20\text{ V}) = 12.64\text{ V}$. Alternatively, you can substitute into the derived equation as follows:

$$e^{-t/\tau} = e^{-\tau/\tau} = e^{-1} = 0.368$$

and $$v_C = 20\text{ V}(1 - e^{-t/80\text{ ms}}) = 20\text{ V}(1 - 0.368)$$
$$= (20\text{ V})(0.632) = 12.64\text{ V}$$

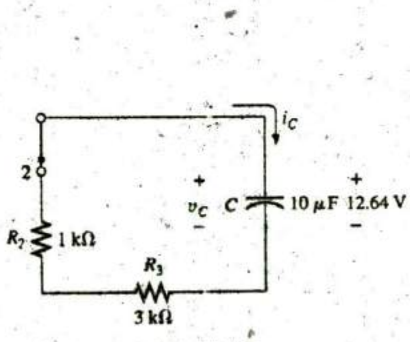


FIG. 10.48
Network in Fig. 10.47 when the switch is moved to position 2 at $t = 1\tau_1$.

Using this voltage as the starting point and substituting into the discharge equation results in

$$\tau' = RC = (R_2 + R_3)C = (1\text{ k}\Omega + 3\text{ k}\Omega)(10\ \mu\text{F}) = 40\text{ ms}$$

$$v_C = Ee^{-t/\tau'} = \mathbf{12.64\ Ve^{-t/40\ ms}}$$

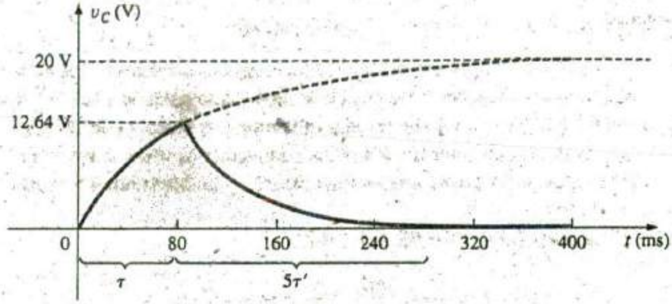


FIG. 10.49
v_C for the network in Fig. 10.47.

c. See Fig. 10.49.
d. The charging equation for the current is

$$i_C = \frac{E}{R}e^{-t/\tau} = \frac{E}{R_1 + R_3}e^{-t/\tau} = \frac{20\text{ V}}{8\text{ k}\Omega}e^{-t/80\text{ ms}} = \mathbf{2.5\text{ mA}e^{-t/80\text{ ms}}}$$

which, at $t = 80$ ms, results in

$$i_C = 2.5\text{ mA}e^{-80\text{ ms}/80\text{ ms}} = 2.5\text{ mA}e^{-1} = (2.5\text{ mA})(0.368) = 0.92\text{ mA}$$

When the switch is moved to position 2, the 12.64 V across the capacitor appears across the resistor to establish a current of 12.64 V/4 kΩ = 3.16 mA. Substituting into the discharge equation with $V_i = 12.64$ V and $\tau' = 40$ ms yields

$$i_C = -\frac{V_i}{R_2 + R_3}e^{-t/\tau'} = -\frac{12.64\text{ V}}{1\text{ k}\Omega + 3\text{ k}\Omega}e^{-t/40\text{ ms}}$$
$$= -\frac{12.64\text{ V}}{4\text{ k}\Omega}e^{-t/40\text{ ms}} = \mathbf{-3.16\text{ mA}e^{-t/40\text{ ms}}}$$

The equation has a minus sign because the direction of the discharge current is opposite to that defined for the current in Fig. 10.48. The resulting plot appears in Fig. 10.50.

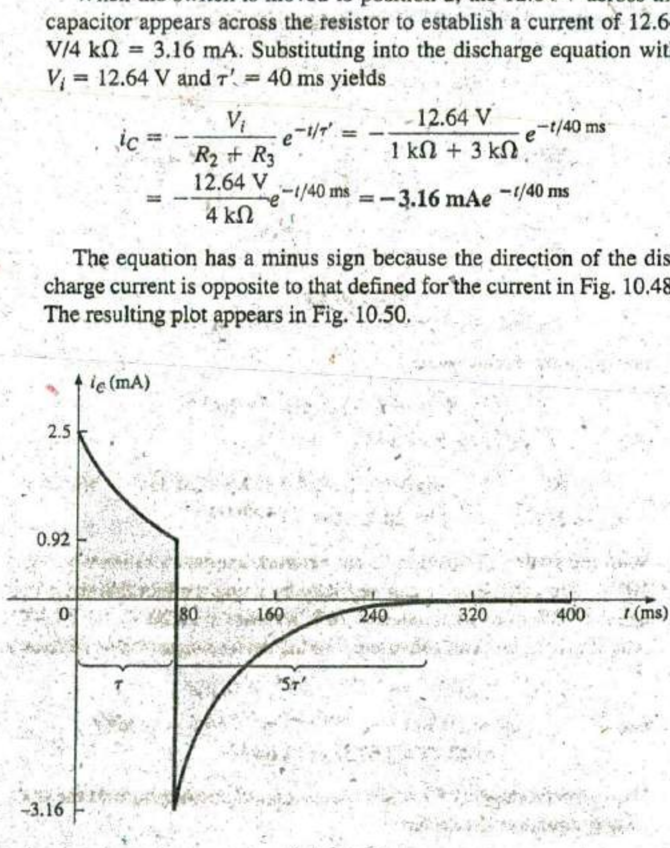


FIG. 10.50
i_C for the network in Fig. 10.47.

10.7 INITIAL CONDITIONS

In all the examples in the previous sections, the capacitor was uncharged before the switch was thrown. We now examine the effect of a charge, and therefore a voltage ($V = Q/C$), on the plates at the instant the switching action takes place. The voltage across the capacitor at this instant is called the **initial value,** as shown for the general waveform in Fig. 10.51.

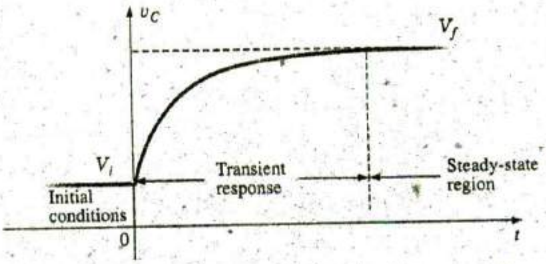


FIG. 10.51
Defining the regions associated with a transient response.

Once the switch is thrown, the transient phase commences until a leveling off occurs after five time constants. This region of relatively fixed value that follows the transient response is called the **steady-state region,** and the resulting value is called the **steady-state** or **final value.** The steady-state value is found by substituting the open-circuit equivalent for the capacitor and finding the voltage across the plates. Using the transient equation developed in the previous section, we can write an equation for the voltage v_C for the entire time interval in Fig. 10.51. That is, for the transient period, the voltage rises from V_i (previously 0 V) to a final value of V_f. Therefore,

$$v_C = E(1 - e^{-t/\tau}) = (V_f - V_i)(1 - e^{-t/\tau})$$

Adding the starting value of V_i to the equation results in

$$v_C = V_i = (V_f - V_i)(1 - e^{-t/\tau})$$

However, by multiplying through and rearranging terms, we obtain

$$v_C = V_i + V_f - V_f e^{-t/\tau} V_i + V_i e^{-t/\tau}$$
$$= V_f V_f e^{-t/\tau} + V_i e^{-t/\tau}$$

We find

$$\boxed{v_C = V_f + (V_i - V_f)e^{-t/\tau}} \tag{10.21}$$

Now that the equation has been developed, it is important to recognize that

Eq. (10.21) is a universal equation for the transient response of a capacitor.

That is, it can be used whether or not the capacitor has an initial value. If the initial value is 0 V as it was in all the previous examples, simply set V_i equal to zero in the equation, and the desired equation results. The final value is the voltage across the capacitor when the open-circuit equivalent is substituted.

EXAMPLE 10.10 The capacitor in Fig. 10.52 has an initial voltage of 4 V.

a. Find the mathematical expression for the voltage across the capacitor once the switch is closed.
b. Find the mathematical expression for the current during the transient period.
c. Sketch the waveform for each from initial value to final value.

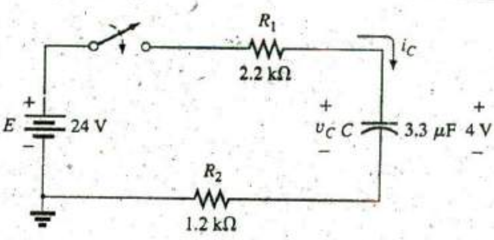


FIG. 10.52
Example 10.10.

Solutions:

a. Substituting the open-circuit equivalent for the capacitor results in a final or steady-state voltage v_C of 24 V.

The time constant is determined by

$$\tau = (R_1 + R_2)C$$
$$= (2.2\ \text{k}\Omega + 1.2\ \text{k}\Omega)(3.3\ \mu\text{F}) = 11.22\ \text{ms}$$

with $\quad 5\tau = 56.1\ \text{ms}$

Applying Eq. (10.21) gives

$$v_C = V_f + (V_i - V_f)e^{-t/\tau} = 24\ \text{V} + (4\ \text{V} - 24\ \text{V})e^{-t/11.22\ \text{ms}}$$

and $\quad \mathbf{v_C = 24\ V - 20\ Ve^{-t/11.22\ ms}}$

b. Since the voltage across the capacitor is constant at 4 V prior to the closing of the switch, the current (whose level is sensitive only to changes in voltage across the capacitor) must have an initial value of 0 mA. At the instant the switch is closed, the voltage across the capacitor cannot change instantaneously, so the voltage across the resistive elements at this instant is the applied voltage less the initial voltage across the capacitor. The resulting peak current is

$$I_m = \frac{E - V_C}{R_1 + R_2} = \frac{24\ \text{V} - 4\ \text{V}}{2.2\ \text{k}\Omega + 1.2\ \text{k}\Omega} = \frac{20\ \text{V}}{3.4\ \text{k}\Omega} = 5.88\ \text{mA}$$

The current then decays (with the same time constant as the voltage v_C) to zero because the capacitor is approaching its open-circuit equivalence.

The equation for i_C is therefore

$$i_C = \mathbf{5.88\ mAe^{-t/11.22\ ms}}$$

c. See Fig. 10.53. The initial and final values of the voltage were drawn first, and then the transient response was included between these levels. For the current, the waveform begins and ends at zero, with the peak value having a sign sensitive to the defined direction of i_C in Fig. 10.52.

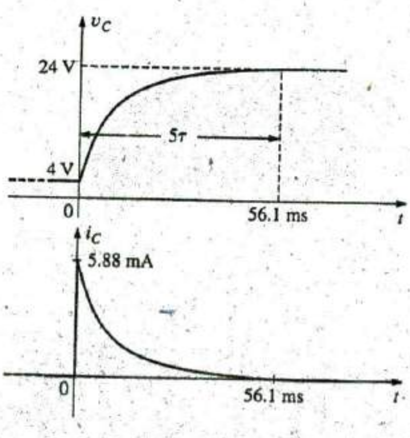


FIG. 10.53
v_C and i_C for the network in Fig. 10.52.

Let us now test the validity of the equation for v_C by substituting $t = 0$ s to reflect the instant the switch is closed. We have

$$e^{-t/\tau} = e^{-0} = 1$$

and $$v_C = 24\text{ V} - 20\text{ V}e^{t/\tau} = 24\text{ V} - 20\text{ V} = 4\text{ V}$$

When $t > 5\tau$,

$$e^{-t/\tau} \cong 0$$

and $$v_C = 24\text{ V} - 20\text{ V}e^{t/\tau} = 24\text{ V} - 0\text{ V} = 24\text{ V}$$

Eq. (10.21) can also be applied to the discharge phase by applying the correct levels of V_i and V_f.

For the discharge pattern in Fig. 10.54, $V_f = 0$ V, and Eq. (10.21) becomes

$$v_C = V_f + (V_i - V_f)e^{-t/\tau} = 0\text{ V} + (V_i - 0\text{ V})e^{-t/\tau}$$

and $$\boxed{v_C = V_i e^{-t/\tau}}_{\text{discharging}} \qquad \textbf{(10.22)}$$

Substituting $V_i = E$ volts results in Eq. (10.17).

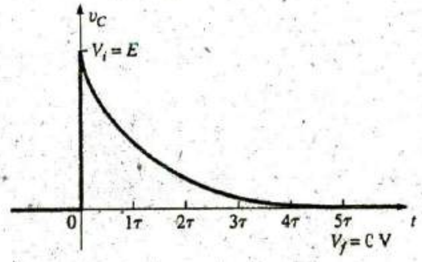


FIG. 10.54
Defining the parameters in Eq. (10.21) for the discharge phase.

10.8 INSTANTANEOUS VALUES

Occasionally, you may need to determine the voltage or current at a particular instant of time that is not an integral multiple of τ, as in the previous sections. For example, if

$$v_C = 20\text{ V}(1 - e^{(-t/2\text{ ms})})$$

the voltage v_C may be required at $t = 5$ ms, which does not correspond to a particular value of τ. Fig. 10.28 reveals that $(1 - e^{t/\tau})$ is approximately 0.93 at $t = 5\text{ ms} = 2.5\tau$, resulting in $v_C = 20(0.93) = 18.6$ V. Additional accuracy can be obtained by substituting $v = 5$ ms into the equation and solving for v_C using a calculator or table to determine $e^{-2.5}$. Thus,

$$\begin{aligned} v_C &= 20\text{ V}(1 - e^{-5\text{ ms}/2\text{ ms}}) = (20\text{ V})(1 - e^{-2.5}) = (20\text{ V})(1 - 0.082) \\ &= (20\text{ V})(0.918) = \mathbf{18.36\text{ V}} \end{aligned}$$

The results are close, but accuracy beyond the tenths place is suspect using Fig. 10.29. The above procedure can also be applied to any other equation introduced in this chapter for currents or other voltages.

Occasionally, you may need to determine the time required to reach a particular voltage or current. The procedure is complicated somewhat by the use of natural logs ($\log_e$, or ln), but today's calculators are equipped to handle the operation with ease.

For example, solving for t in the equation

$$v_C = V_f + (V_i - V_f)e^{-t/\tau}$$

results in

$$\boxed{t = \tau(\log_e)\frac{(V_i - V_f)}{(v_C - V_f)}} \qquad \textbf{(10.23)}$$

For example, suppose that

$$v_C = 20\text{ V}(1 - e^{-t/2\text{ ms}})$$

and the time t to reach 10 V is desired. Since $V_i = 0$ V, and $V_f = 20$ V, we have

$$t = \tau(\log_e)\frac{(V_i - V_f)}{(v_C - V_f)} = (2\text{ ms})(\log_e)\frac{(0\text{ V} - 20\text{ V})}{(10\text{ V} - 20\text{ V})}$$
$$= (2\text{ ms})\left[\log_e\left(\frac{-20\text{ V}}{-10\text{ V}}\right)\right] = (2\text{ ms})(\log_e 2) = (2\text{ ms})(0.693)$$
$$= \mathbf{1.386\text{ ms}}$$

The TI-89 calculator key strokes appear in Fig. 10.55.

2 EE (-) 3 × 2ND LN 2) ENTER **1.39E-3**

FIG. 10.55
Key strokes to determine (2 ms)($\log_e 2$) using the TI-89 calculator.

For the discharge equation,

$$v_C = Ee^{-t/\tau} = V_i\,(e^{-t/\tau}) \qquad \text{with } V_f = 0\text{ V}$$

Using Eq. (10.23) gives

$$t = \tau(\log_e)\frac{(V_i - V_f)}{(v_C - V_f)} = \tau(\log_e)\frac{(V_i - 0\text{ V})}{(v_C - 0\text{ V})}$$

and

$$\boxed{t = \tau\log_e\frac{V_i}{v_C}} \tag{10.24}$$

For the current equation,

$$i_C = \frac{E}{R}e^{-t/\tau} \qquad I_i = \frac{E}{R} \qquad I_f = 0\text{ A}$$

and

$$\boxed{t = \log_e\frac{I_i}{i_C}} \tag{10.25}$$

10.9 THÉVENIN EQUIVALENT: $\tau = R_{Th}C$

You may encounter instances in which the network does not have the simple series form in Fig. 10.26. You then need to find the Thévenin equivalent circuit for the network external to the capacitive element. E_{Th} will be the source voltage E in Eqs. (10.13) through (10.25), and R_{Th} will be the resistance R. The time constant is then $\tau = R_{Th}C$.

EXAMPLE 10.11 For the network in Fig. 10.56:

a. Find the mathematical expression for the transient behavior of the voltage v_C and the current i_C following the closing of the switch (position 1 at $t = 0$ s).

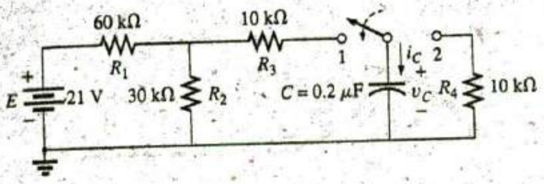


FIG. 10.56
Example 10.11.

b. Find the mathematical expression for the voltage v_C and the current i_C as a function of time if the switch is thrown into position 2 at t = 9 ms.
c. Draw the resultant waveforms of parts (a) and (b) on the same time axis.

Solutions:

a. Applying Thévenin's theorem to the 0.2 μF capacitor, we obtain Fig. 10.57. We have

$$R_{Th} = R_1 \parallel R_2 + R_3 = \frac{(60\text{ k}\Omega)(30\text{ k}\Omega)}{90\text{ k}\Omega} + 10\text{ k}\Omega$$
$$= 20\text{ k}\Omega + 10\text{ k}\Omega = 30\text{ k}\Omega$$
$$E_{Th} = \frac{R_2E}{R_2 + R_1} = \frac{(30\text{ k}\Omega)(21\text{ V})}{30\text{ k}\Omega + 60\text{ k}\Omega} = \frac{1}{3}(21\text{ V}) = 7\text{ V}$$

The resultant Thévenin equivalent circuit with the capacitor replaced is shown in Fig. 10.58.
Using Eq. (10.21) with $V_f = E_{Th}$ and $V_i = 0$ V, we find that

$$v_C = V_f + (V_i - V_f)e^{-t/\tau}$$

becomes

$$v_C = E_{Th} + (0\text{ V} - E_{Th})e^{-t/\tau}$$

or

$$v_C = E_{Th}(1 - e^{-t/\tau})$$

with

$$\tau = RC = (30\text{ k}\Omega)(0.2\text{ }\mu\text{F}) = 6\text{ ms}$$

Therefore,

$$\mathbf{v_C = 7\text{ V}(1 - e^{-t/6\text{ ms}})}$$

For the current i_C:

$$i_C = \frac{E_{Th}}{R}e^{-t/RC} = \frac{7\text{ V}}{30\text{ k}\Omega}e^{-t/6\text{ ms}}$$
$$= \mathbf{0.23\text{ mA}e^{-t/6\text{ ms}}}$$

b. At t = 9 ms,

$$v_C = E_{Th}(1 - e^{-t/\tau}) = 7\text{ V}(1 - e^{-(9\text{ ms}/6\text{ ms})})$$
$$= (7\text{ V})(1 - e^{-1.5}) = (7\text{ V})(1 - 0.223)$$
$$= (7\text{ V})(0.777) = 5.44\text{ V}$$

and $$i_C = \frac{E_{Th}}{R}e^{-t/\tau} = 0.23\text{ mA}e^{-1.5}$$
$$= (0.23 \times 10^{-3})(0.233) = 0.052 \times 10^{-3} = 0.05\text{ mA}$$

Using Eq. (10.21) with $V_f = 0$ V and $V_i = 5.44$ V, we find that

$$v_C = V_f + (V_i - V_f)e^{-t/\tau'}$$

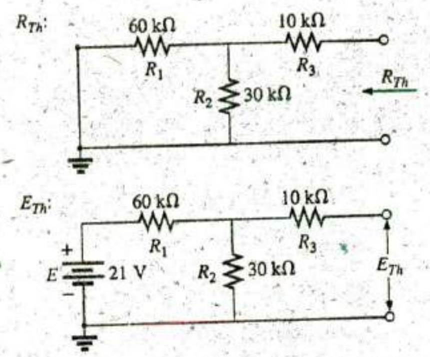


FIG. 10.57
Applying Thévenin's theorem to the network in Fig. 10.56.

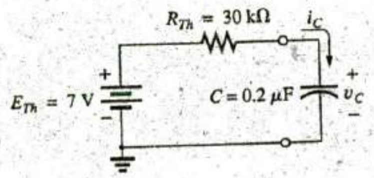


FIG. 10.58
Substituting the Thévenin equivalent for the network in Fig. 10.56.

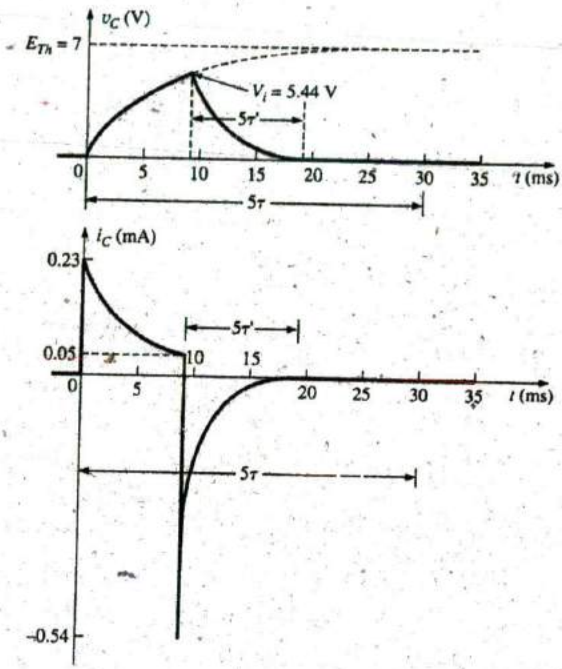


FIG. 10.59
The resulting waveforms for the network in Fig. 10.56.

becomes
$$v_C = 0\text{ V} + (5.44\text{ V} - 0\text{ V})e^{-t/\tau'}$$
$$= 5.44\text{ V}e^{-t/\tau'}$$

with
$$\tau' = R_4C = (10\text{ k}\Omega)(0.2\ \mu\text{F}) = 2\text{ ms}$$

and
$$v_C = \mathbf{5.44\ Ve^{-t/2\,ms}}$$

By Eq. (10.19),

$$I_i = \frac{5.44\text{ V}}{10\text{ k}\Omega} = 0.54\text{ mA}$$

and
$$i_C = I_i e^{-t/\tau} = \mathbf{0.54\ mAe^{-t/2\,ms}}$$

c. See Fig. 10.59.

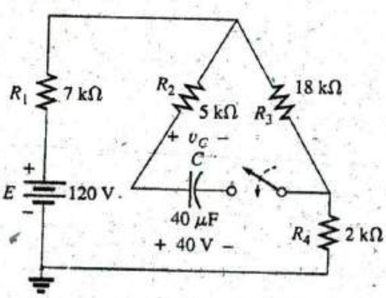


FIG. 10.60
Example 10.12.

EXAMPLE 10.12 The capacitor in Fig. 10.60 is initially charged to 40 V. Find the mathematical expression for v_C after the closing of the switch. Plot the waveform for v_C.

Solution: The network is redrawn in Fig. 10.61.

E_{Th}:

$$E_{Th} = \frac{R_3E}{R_3 + R_1 + R_4} = \frac{(18\text{ k}\Omega)(120\text{ V})}{18\text{ k}\Omega + 7\text{ k}\Omega + 2\text{ k}\Omega} = 80\text{ V}$$

R_{Th}:

$$R_{Th} = 5\text{ k}\Omega + (18\text{ k}\Omega) \parallel (7\text{ k}\Omega + 2\text{ k}\Omega)$$
$$= 5\text{ k}\Omega + 6\text{ k}\Omega = 11\text{ k}\Omega$$

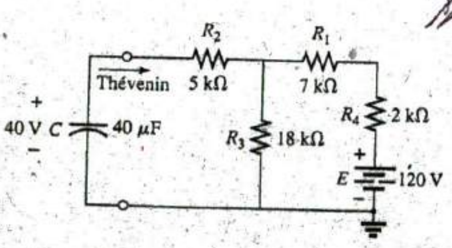


FIG. 10.61
Network in Fig. 10.60 redrawn.

Therefore, $\quad V_i = 40\text{ V} \quad$ and $\quad V_f = 80\text{ V}$

and $\quad \tau = R_{Th}C = (11\text{ k}\Omega)(40\ \mu\text{F}) = 0.44\text{ s}$

Eq. (10.21):
$$\begin{aligned} v_C &= V_f + (V_i - V_f)e^{-t/\tau} \\ &= 80\text{ V} + (40\text{ V} - 80\text{ V})e^{-t/0.44\text{ s}} \end{aligned}$$

and
$$\boldsymbol{v_C = 80\text{ V} - 40\text{ V}e^{-t/0.44\text{ s}}}$$

The waveform appears as in Fig. 10.62.

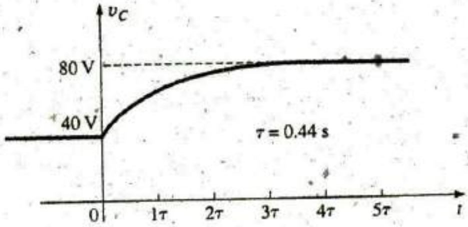


FIG. 10.62
vC for the network in Fig. 10.60.

EXAMPLE 10.13 For the network in Fig. 10.63, find the mathematical expression for the voltage v_C after the closing of the switch (at $t = 0$).

Solution:

$$\begin{aligned} R_{Th} &= R_1 + R_2 = 6\ \Omega + 10\ \Omega = 16\ \Omega \\ E_{Th} &= V_1 + V_2 = IR_1 + 0 \\ &= (20 \times 10^{-3}\text{ A})(6\ \Omega) = 120 \times 10^{-3}\text{ V} = 0.12\text{ V} \end{aligned}$$

and $\quad \tau = R_{Th}C = (16\ \Omega)(500 \times 10^{-6}\text{ F}) = 8\text{ ms}$

so that $\quad \boldsymbol{v_C = 0.12\text{ V}(1 - e^{-t/8\text{ ms}})}$

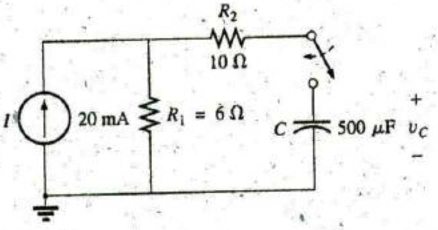


FIG. 10.63
Example 10.13.

10.10 THE CURRENT i_C

There is a very special relationship between the current of a capacitor and the voltage across it. For the resistor, it is defined by Ohm's law: $i_R = v_R/R$. The current through and the voltage across the resistor are related by a constant R—a very simple direct linear relationship. For the capacitor, it is the more complex relationship defined by

$$\boxed{i_C = C\frac{dv_C}{dt}} \qquad \textbf{(10.26)}$$

The factor C reveals that the higher the capacitance, the greater is the resulting current. Intuitively, this relationship makes sense because higher capacitance levels result in increased levels of stored charge, providing a source for increased current levels. The second term, dv_C/dt, is sensitive to the *rate of change* of v_C with time. The function dv_C/dt is called the **derivative** (calculus) of the voltage v_C with respect to time t. The faster the voltage v_C changes with time, the larger will be the factor dv_C/dt and the larger will be the resulting current i_C. That is why the current jumps to its maximum of E/R in a charging circuit the instant the switch is closed. At that instant, if you look at the charging curve for v_C, the voltage is *changing* at its greatest rate. As it approaches its final value, the rate of change decreases, and, as confirmed by Eq. (10.26), the level of current decreases.

Take special note of the following:

The capacitive current is directly related to the rate of change of the voltage across the capacitor, not the levels of voltage involved.

For example, the current of a capacitor will be greater when the voltage changes from 1 V to 10 V in 1 ms than when it changes from 10 V to 100 V in 1 s; in fact, it will be 100 times more.

If the voltage fails to change over time, then.

$$\frac{dv_C}{dt} = 0$$

and

$$i_C = C\frac{dv_C}{dt} = C(0) = 0 \text{ A}$$

In an effort to develop a clearer understanding of Eq. (10.26), let us calculate the **average current** associated with a capacitor for various voltages impressed across the capacitor. The average current is defined by the equation

$$\boxed{i_{C_{av}} = C\frac{\Delta v_C}{\Delta t}} \tag{10.27}$$

where Δ indicates a finite (measurable) change in voltage or time. The instantaneous current can be derived from Eq. (10.27) by letting Δt become vanishingly small; that is,

$$i_{C_{inst}} = \lim_{\Delta t \to 0} C\frac{\Delta v_C}{\Delta t} = C\frac{dv_C}{dt}$$

In the following example, the change in voltage Δv_C will be considered for each slope of the voltage waveform. If the voltage increases with time, the average current is the change in voltage divided by the change in time, with a positive sign. If the voltage decreases with time, the average current is again the change in voltage divided by the change in time, but with a negative sign.

EXAMPLE 10.14 Find the waveform for the average current if the voltage across a 2 μF capacitor is as shown in Fig. 10.64.

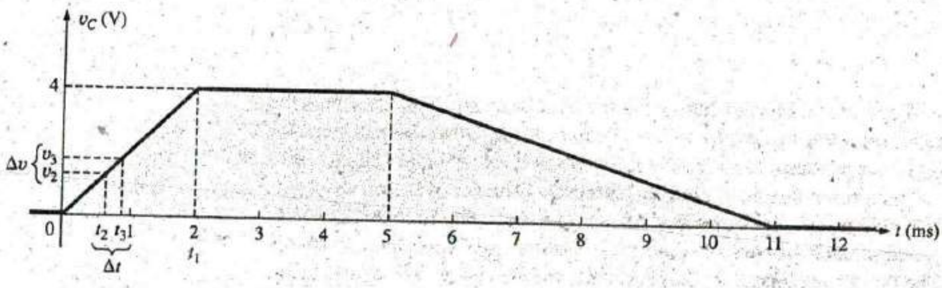


FIG. 10.64
v_C for Example 10.14.

Solutions:

a. From 0 ms to 2 ms, the voltage increases linearly from 0 V to 4 V; the change in voltage $\Delta v = 4\text{ V} - 0 = 4\text{ V}$ (with a positive sign since the voltage increases with time). The change in time $\Delta t = 2\text{ ms} - 0 = 2\text{ ms}$, and

$$i_{C_{av}} = C\frac{\Delta v_C}{\Delta t} = (2 \times 10^{-6}\text{ F})\left(\frac{4\text{ V}}{2 \times 10^{-3}\text{ s}}\right)$$
$$= 4 \times 10^{-3}\text{ A} = \mathbf{4\ mA}$$

b. From 2 ms to 5 ms, the voltage remains constant at 4 V; the change in voltage $\Delta v = 0$. The change in time $\Delta t = 3$ ms, and

$$i_{C_{av}} = C\frac{\Delta v_C}{\Delta t} = C\frac{0}{\Delta t} = \mathbf{0\ mA}$$

c. From 5 ms to 11 ms, the voltage decreases from 4 V to 0 V. The change in voltage Δv is, therefore, 4 V − 0 = 4 V (with a negative sign since the voltage is decreasing with time). The change in time $\Delta t = 11$ ms − 5 ms = 6 ms, and

$$i_{C_{av}} = C\frac{\Delta v_C}{\Delta t} = -(2 \times 10^{-6}\text{ F})\left(\frac{4\text{ V}}{6 \times 10^{-3}\text{ s}}\right)$$
$$= -1.33 \times 10^{-3}\text{ A} = \mathbf{-1.33\ mA}$$

d. From 11 ms on, the voltage remains constant at 0 and $\Delta v = 0$, so $i_{C_{av}} = 0$ mA. The waveform for the average current for the impressed voltage is as shown in Fig. 10.65.

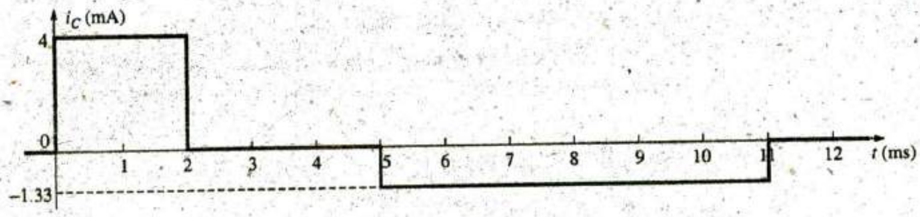


FIG. 10.65
The resulting current i_C for the applied voltage in Fig. 10.64.

Note in Example 10.14 that, in general, the steeper the slope, the greater is the current, and when the voltage fails to change, the current is zero. In addition, the average value is the same as the instantaneous value at any point along the slope over which the average value was found. For example, if the interval Δt is reduced from $0 \rightarrow t_1$ to $t_2 - t_3$, as noted in Fig. 10.64, $\Delta v/\Delta t$ is still the same. In fact, no matter how small the interval Δt, the slope will be the same, and therefore the current $i_{C_{av}}$ will be the same. If we consider the limit as $\Delta t \rightarrow 0$, the slope will still remain the same, and therefore $i_{C_{av}} = i_{C_{inst}}$ at any instant of time between 0 and t_1. The same can be said about any portion of the voltage waveform that has a constant slope.

An important point to be gained from this discussion is that it is not the magnitude of the voltage across a capacitor that determines the current but rather how quickly the voltage *changes* across the capacitor. An applied steady dc voltage of 10,000 V would (ideally) not create any flow of charge (current), but a change in voltage of 1 V in a very brief period of time could create a significant current.

The method described above is only for waveforms with straight-line (linear) segments. For nonlinear (curved) waveforms, a method of calculus (differentiation) must be used.

10.11 CAPACITORS IN SERIES AND IN PARALLEL

Capacitors, like resistors, can be placed in series and in parallel. Increasing levels of capacitance can be obtained by placing capacitors in parallel, while decreasing levels can be obtained by placing capacitors in series.

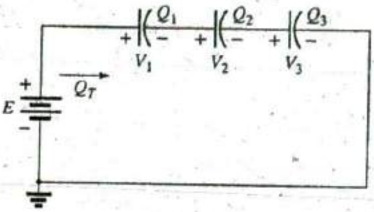


FIG. 10.66
Series capacitors.

For capacitors in series, the charge is the same on each capacitor (Fig. 10.66):

$$\boxed{Q_T = Q_1 = Q_2 = Q_3} \tag{10.28}$$

Applying Kirchhoff's voltage law around the closed loop gives

$$E = V_1 + V_2 + V_3$$

However, $$V = \frac{Q}{C}$$

so that $$\frac{Q_T}{C_T} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3}$$

Using Eq. (10.28) and dividing both sides by Q yields

$$\boxed{\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} \tag{10.29}$$

which is similar to the manner in which we found the total resistance of a parallel resistive circuit. The total capacitance of two capacitors in series is

$$\boxed{C_T = \frac{C_1C_2}{C_1 + C_2}} \tag{10.30}$$

The voltage across each capacitor in Fig. 10.66 can be found by first recognizing that

$$Q_T = Q_1$$

or $$C_TE = C_1V_1$$

Solving for V_1 gives $$V_1 = \frac{C_TE}{C_1}$$

and substituting for C_T gives

$$\boxed{V_1 = \left(\frac{1/C_1}{1/C_1 + 1/C_2 + 1/C_3}\right)E} \tag{10.31}$$

A similar equation results for each capacitor of the network.

For capacitors in parallel, as shown in Fig. 10.67, the voltage is the same across each capacitor, and the total charge is the sum of that on each capacitor:

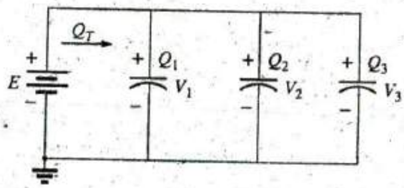


FIG. 10.67
Parallel capacitors.

$$\boxed{Q_T = Q_1 + Q_2 + Q_3} \tag{10.32}$$

However, $$Q = CV$$

Therefore, $$C_TE = C_1V_1 = C_2V_2 = C_3V_3$$

but $$E = V_1 = V_2 = V_3$$

Thus, $$\boxed{C_T = C_1 + C_2 + C_3} \tag{10.33}$$

which is similar to the manner in which the total resistance of a series circuit is found.

EXAMPLE 10.15 For the circuit in Fig. 10.68:

a. Find the total capacitance.
b. Determine the charge on each plate.
c. Find the voltage across each capacitor.

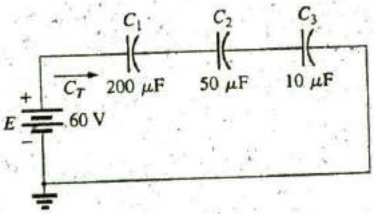


FIG. 10.68
Example 10.15.

Solutions:

a. $$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$
$$= \frac{1}{200 \times 10^{-6}\,\text{F}} + \frac{1}{50 \times 10^{-6}\,\text{F}} + \frac{1}{10 \times 10^{-6}\,\text{F}}$$
$$= 0.005 \times 10^6 + 0.02 \times 10^6 + 0.1 \times 10^6$$
$$= 0.125 \times 10^6$$

and $$C_T = \frac{1}{0.125 \times 10^6} = \mathbf{8\ \mu F}$$

b. $$Q_T = Q_1 = Q_2 = Q_3$$
$$= C_T E = (8 \times 10^{-6}\,\text{F})(60\,\text{V}) = \mathbf{480\ \mu C}$$

c. $$V_1 = \frac{Q_1}{C_1} = \frac{480 \times 10^{-6}\,\text{C}}{200 \times 10^{-6}\,\text{F}} = \mathbf{2.4\ V}$$
$$V_2 = \frac{Q_2}{C_2} = \frac{480 \times 10^{-6}\,\text{C}}{50 \times 10^{-6}\,\text{F}} = \mathbf{9.6\ V}$$
$$V_3 = \frac{Q_3}{C_3} = \frac{480 \times 10^{-6}\,\text{C}}{10 \times 10^{-6}\,\text{F}} = \mathbf{48.0\ V}$$

and $E = V_1 + V_2 + V_3 = 2.4\,\text{V} + 9.6\,\text{V} + 48\,\text{V} = \mathbf{60\ V}$ (checks)

EXAMPLE 10.16 For the network in Fig. 10.69:

a. Find the total capacitance.
b. Determine the charge on each plate.
c. Find the total charge.

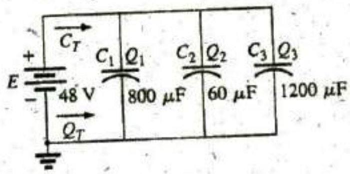


FIG. 10.69
Example 10.16.

Solutions:

a. $C_T = C_1 + C_2 + C_3 = 800\,\mu\text{F} + 60\,\mu\text{F} + 1200\,\mu\text{F} = \mathbf{2060\ \mu F}$

b. $Q_1 = C_1 E = (800 \times 10^{-6}\,\text{F})(48\,\text{V}) = \mathbf{38.4\ mC}$
$Q_2 = C_2 E = (60 \times 10^{-6}\,\text{F})(48\,\text{V}) = \mathbf{2.88\ mC}$
$Q_3 = C_3 E = (1200 \times 10^{-6}\,\text{F})(48\,\text{V}) = \mathbf{57.6\ mC}$

c. $Q_T = Q_1 + Q_2 + Q_3 = 38.4\,\text{mC} + 2.88\,\text{mC} + 57.6\,\text{mC} = \mathbf{98.88\ mC}$

EXAMPLE 10.17 Find the voltage across and the charge on each capacitor for the network in Fig. 10.70.

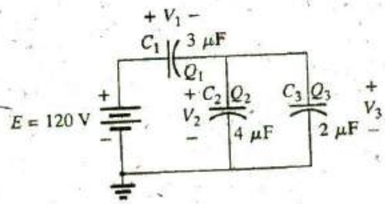


FIG. 10.70
Example 10.17.

Solution:

$$C'_T = C_2 + C_3 = 4\,\mu\text{F} + 2\,\mu\text{F} = 6\,\mu\text{F}$$
$$C_T = \frac{C_1 C'_T}{C_1 + C'_T} = \frac{(3\,\mu\text{F})(6\,\mu\text{F})}{3\,\mu\text{F} + 6\,\mu\text{F}} = 2\,\mu\text{F}$$
$$Q_T = C_T E = (2 \times 10^{-6}\,\text{F})(120\,\text{V}) = \mathbf{240\ \mu C}$$

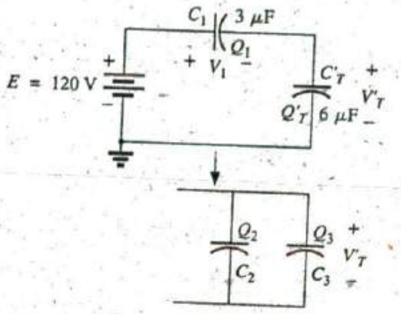


FIG. 10.71
Reduced equivalent for the network in Fig. 10.70.

An equivalent circuit (Fig. 10.71) has

$$Q_T = Q_1 = Q'_T$$

and, therefore,

$$Q_1 = \mathbf{240\ \mu C}$$

and

$$V_1 = \frac{Q_1}{C_1} = \frac{240 \times 10^{-6}\,\text{C}}{3 \times 10^{-6}\,\text{F}} = \mathbf{80\ V}$$

$$Q'_T = 240\ \mu\text{C}$$

Therefore,

$$V'_T = \frac{Q'_T}{C'_T} = \frac{240 \times 10^{-6}\,\text{C}}{6 \times 10^{-6}\,\text{F}} = \mathbf{40\ V}$$

and

$$Q_2 = C_2 V'_T = (4 \times 10^{-6}\,\text{F})(40\ \text{V}) = \mathbf{160\ \mu C}$$
$$Q_3 = C_3 V'_T = (2 \times 10^{-6}\,\text{F})(40\ \text{V}) = \mathbf{80\ \mu C}$$

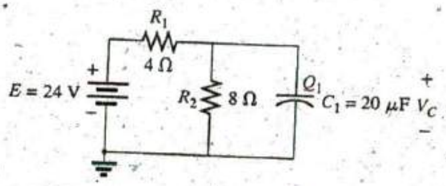


FIG. 10.72
Example 10.18.

EXAMPLE 10.18 Find the voltage across and the charge on capacitor C_1 in Fig. 10.72 after it has charged up to its final value.

Solution: As previously discussed, the capacitor is effectively an open circuit for dc after charging up to its final value (Fig. 10.73).

Therefore,

$$V_C = \frac{(8\ \Omega)(24\ \text{V})}{4\ \Omega + 8\ \Omega} = \mathbf{16\ V}$$

$$Q_1 = C_1 V_C = (20 \times 10^{-6}\,\text{F})(16\ \text{V}) = \mathbf{320\ \mu C}$$

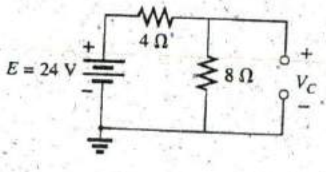


FIG. 10.73
Determining the final (steady-state) value for v_C.

EXAMPLE 10.19 Find the voltage across and the charge on each capacitor of the network in Fig. 10.74(a) after each has charged up to its final value.

Solution: See Fig. 10.74(b). We have

$$V_{C_2} = \frac{(7\ \Omega)(72\ \text{V})}{7\ \Omega + 2\ \Omega} = \mathbf{56\ V}$$

$$V_{C_1} = \frac{(2\ \Omega)(72\ \text{V})}{2\ \Omega + 7\ \Omega} = \mathbf{16\ V}$$

$$Q_1 = C_1 V_{C_1} = (2 \times 10^{-6}\,\text{F})(16\ \text{V}) = \mathbf{32\ \mu C}$$
$$Q_2 = C_2 V_{C_2} = (3 \times 10^{-6}\,\text{F})(56\ \text{V}) = \mathbf{168\ \mu C}$$

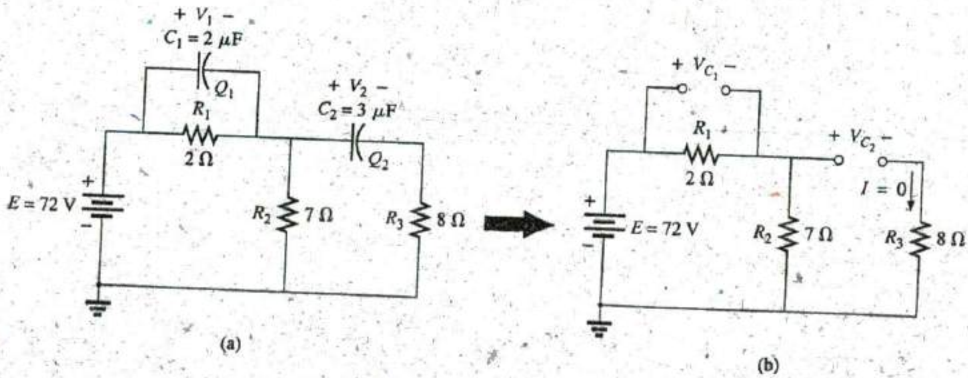


FIG. 10.74
Example 10.19.

10.12 ENERGY STORED BY A CAPACITOR

An ideal capacitor does not dissipate any of the energy supplied to it. It stores the energy in the form of an electric field between the conducting surfaces. A plot of the voltage, current, and power to a capacitor during the charging phase is shown in Fig. 10.75. The power curve can be obtained by finding the product of the voltage and current at selected instants of time and connecting the points obtained. *The energy stored is represented by the shaded area under the power curve.* Using calculus, we can determine the area under the curve:

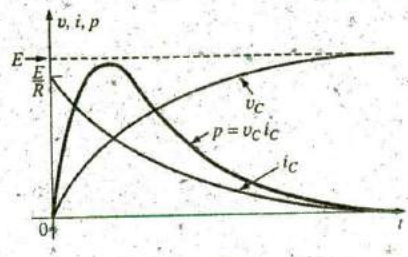


FIG. 10.75
Plotting the power to a capacitive element during the transient phase.

$$W_C = \frac{1}{2} CE^2$$

In general,

$$\boxed{W_C = \frac{1}{2}CV^2} \qquad \text{(J)} \qquad \textbf{(10.34)}$$

where V is the steady-state voltage across the capacitor. In terms of Q and C,

$$W_C = \frac{1}{2}C\left(\frac{Q}{C}\right)^2$$

or

$$\boxed{W_C = \frac{Q^2}{2C}} \qquad \text{(J)} \qquad \textbf{(10.35)}$$

EXAMPLE 10.20 For the network in Fig. 10.74(a), determine the energy stored by each capacitor.

Solution: For C_1:

$$W_C = \frac{1}{2}CV^2$$
$$= \frac{1}{2}(2 \times 10^{-6}\text{ F})(16\text{ V})^2 = (1 \times 10^{-6})(256) = \mathbf{256\ \mu J}$$

For C_2:

$$W_C = \frac{1}{2}CV^2$$
$$= \frac{1}{2}(3 \times 10^{-6}\text{ F})(56\text{ V})^2 = (1.5 \times 10^{-6})(3136) = \mathbf{4704\ \mu J}$$

Due to the squared term, the energy stored increases rapidly with increasing voltages.

10.13 STRAY CAPACITANCES

In addition to the capacitors discussed so far in this chapter, there are **stray capacitances** that exist not through design but simply because two conducting surfaces are relatively close to each other. Two conducting wires in the same network have a capacitive effect between them, as shown in Fig. 10.76(a). In electronic circuits, capacitance levels exist between conducting surfaces of the transistor, as shown in Fig. 10.76(b). In Chapter 11, we will discuss another element, called the *inductor,* which

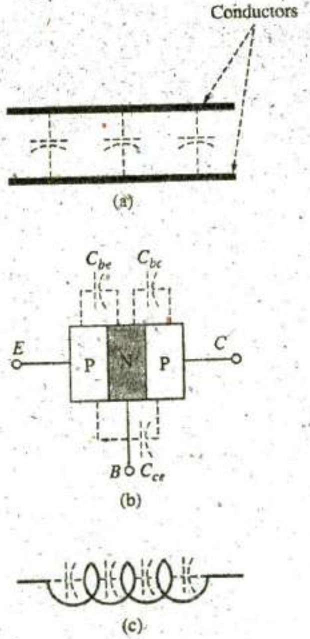


FIG. 10.76
Examples of stray capacitance.

has capacitive effects between the windings [Fig. 10.76(c)]. Stray capacitances can often lead to serious errors in system design if they are not considered carefully.

10.14 APPLICATIONS

This section includes a description of the operation of touch pads and one of the less expensive, throwaway cameras that have become so popular, as well as a discussion of the use of capacitors in the line conditioners (surge protectors) that are used in many homes and throughout the business world. Additional examples of the use of capacitors appear in Chapter 11.

Touch Pad

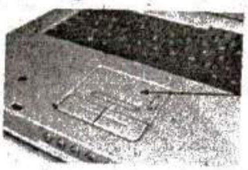


FIG. 10.77
Laptop touch pad.

The touch pad on the computer of Fig. 10.77 is used to control the position of the pointer on the computer screen by providing a link between the position of a finger on the pad to a position on the screen. There are two general approaches to providing this linkage: **capacitance sensing** and **conductance sensing.** Capacitance sensing depends on the charge carried by the human body, while conductance sensing only requires that pressure be applied to a particular position on the pad. In other words, the wearing of gloves or using a pencil will not work with capacitance sensing but is effective with conductance sensing.

There are two methods commonly employed for capacitance testing. One is referred to as the **matrix approach,** and the other is called the **capacitive shunt approach.** The matrix approach requires two sets of parallel conductors separated by a dielectric and perpendicular to each other as shown in Fig. 10.78. Two sets of perpendicular wires are required to permit the determination of the location of the point on the two-dimensional plane—one for the horizontal displacement and the other

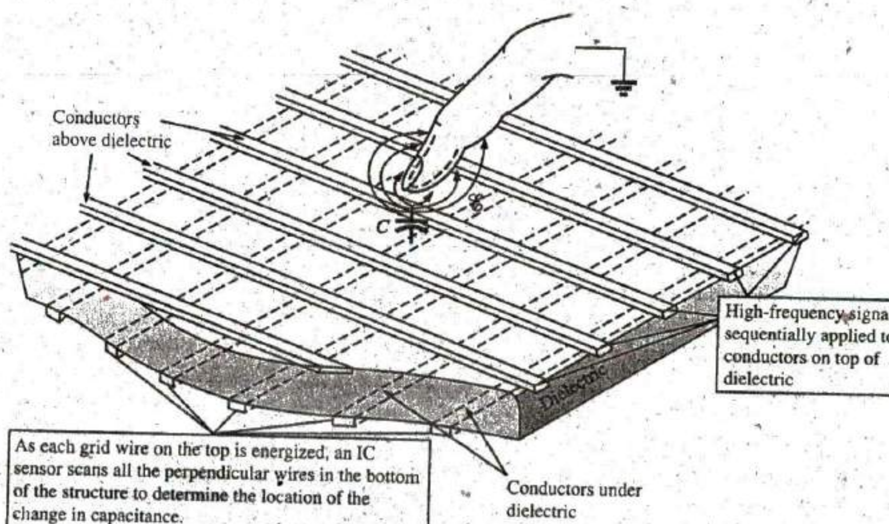


FIG. 10.78
Matrix approach to capacitive sensing in a touch pad.

for the vertical displacement. The result when looking down at the pad is a two-dimensional grid with intersecting points or nodes. Its operation requires the application of a high-frequency signal that will permit the monitoring of the capacitance between each set of wires at each intersection as shown in Fig. 10.78 using ICs connected to each set of wires. When a finger approaches a particular intersection the charge on the finger will change the field distribution at that point by drawing some of the field lines away from the intersection. Some like to think of the finger as applying a **virtual ground** to the point as shown in the figure. Recall from the discussion in Section 10.3 that any change in electric field strength for a fixed capacitor (such as the insertion of a dielectric between the plates of a capacitor) will change the charge on the plates and the level of capacitance determined by $C = Q/V$. The change in capacitance at the intersection will be noted by the ICs. That change in capacitance can then be translated by a **capacitance to digital converter** (CDC) and used to define the location on the screen. Recent experiments have found that this type of sensing is most effective with a soft, delicate touch on the pad rather than hard, firm pressure.

The capacitive shunt approach takes a totally different approach. Rather than establish a grid, a sensor is used to detect changes in capacitive levels. The basic construction for an analog device appears in Fig. 10.79. The sensor has a transmitter and a receiver, both of which are formed on separate printed circuit board (PCB) platforms with a plastic cover over the transmitter to avoid actual contact with the finger. When the excitation signal of 250 kHz is applied to the transmitter platform, an electric field is established between the transmitter and receiver, with a strong fringing effect on the surface of the sensor. If a finger with its negative charge is brought close to the transmitter surface, it will distort the fringing effect by attracting some of the electric field as shown in the figure. The resulting change in total field strength will affect the charge level on the plates of the sensor and therefore the capacitance between the transmitter and receiver. This will be detected by the sensor and provide either the horizontal or vertical position of the contact. The resultant change in capacitance is only in the order of femtofarads, as compared to the picofarads for the sensor, but is still sufficient to be detected by the sensor. The change in capacitance is picked up by a 16-bit Σ-Δ capacitor to digital convertor (CDC) and the

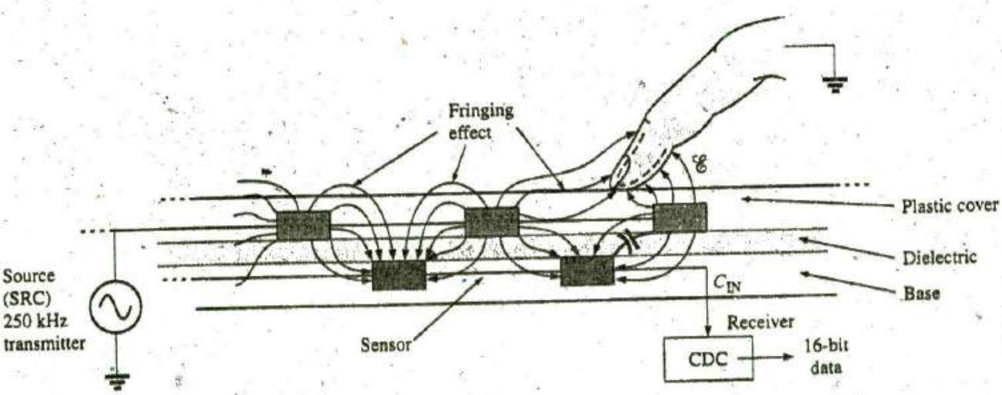
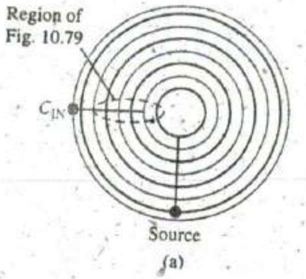


FIG. 10.79
Capacitive shunt approach.



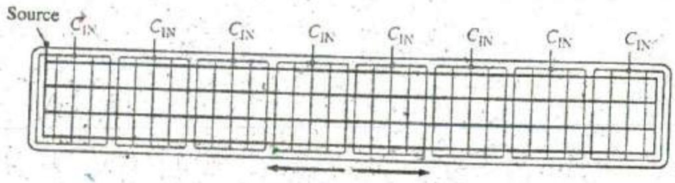


FIG. 10.80
Capacitive shunt sensors: (a) bottom, (b) slice.

results fed into the contoller for the system to which the sensor is connected. The term *shunt* comes from the fact that some of the electric field is "shunted" away from the sensor. The sensors themselves can be made of many different shapes and sizes. For applications such as the circular button for an elevator, the circular pattern of Fig. 10.80(a) may be applied, while for a slide control, it may appear as shown in Fig. 10.80(b). In each case the excitation is applied to the red lines and regions and the capacitance level measured by the C_{IN} blue lines and regions. In other words, a field is established between the red and blue lines throughout the pattern, and touching the pads in any area will reveal a change in capacitance. For a computer touch pad the number of C_{IN} inputs required is one per row and one per column to provide the location in a two-dimensional space.

The last method to be described is the conductance-sensing approach. Basically, it employs two thin metallic conducting surfaces separated by a very thin space. The top surface is usually flexible, while the bottom is fixed and coated with a layer of small conductive nipples. When the top surface is touched, it drops down and touches a nipple, causing the conductance between the two surfaces to increase dramatically in that one location. This change in conductance is then picked up by the ICs on each side of the grid and the location determined for use in setting the position on the screen of the computer. This type of mouse pad permits the use of a pen, pencil, or other nonconductive instrument to set the location on the screen, which is useful in situations in which one may have to wear gloves continually or need to use nonconductive pointing devices because of environmental concerns.

FIG. 10.81
Flash camera: general appearance.

Flash Lamp

The basic circuitry for the flash lamp of the popular, inexpensive, throw-away camera in Fig. 10.81 is provided in Fig. 10.82. The physical circuitry is in Fig. 10.83. The labels added to Fig. 10.83 identify broad areas of the design and some individual components. The major components of the electronic circuitry include a large 160 μF, 330 V, polarized electrolytic capacitor to store the necessary charge for the flash lamp, a flash lamp to generate the required light, a dc battery of 1.5 V, a chopper network to generate a dc voltage in excess of 300 V, and a trigger network to establish a few thousand volts for a very short period of time to fire the flash lamp. There are both a 22 nF capacitor in the trigger network as shown in Figs. 10.82 and 10.83 and a third capacitor of 470 pF in the high-frequency oscillator of the chopper network. In particular, note that

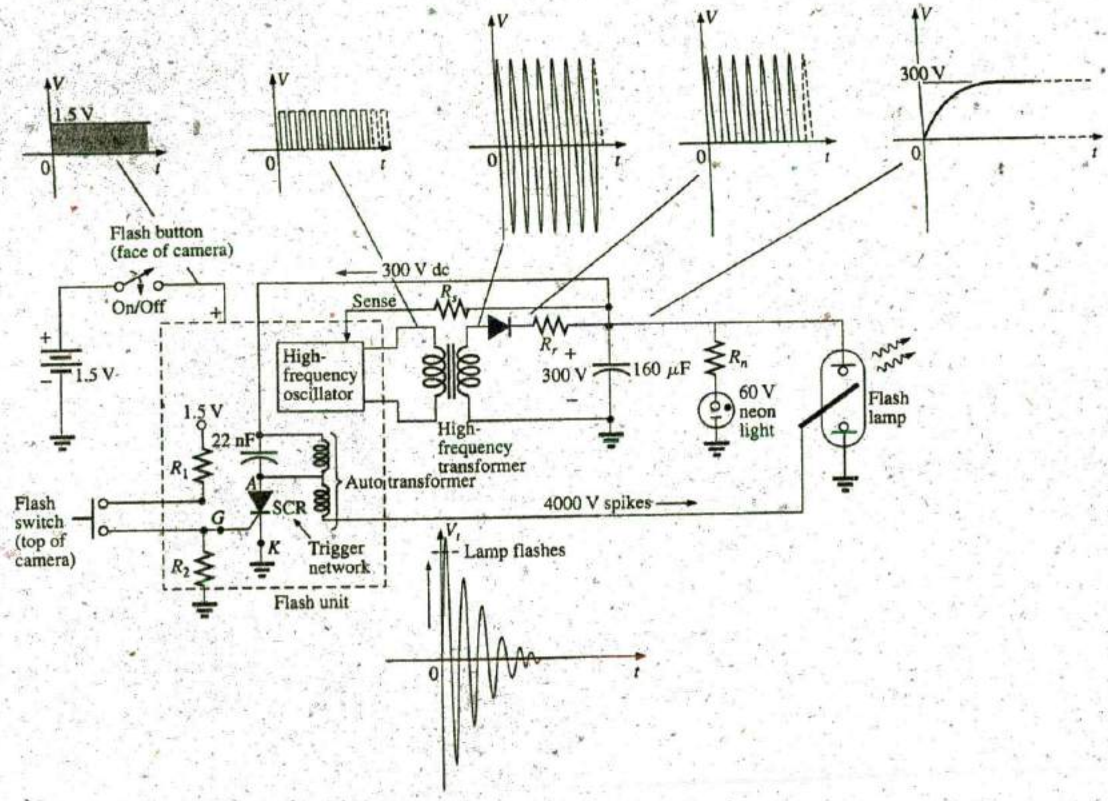
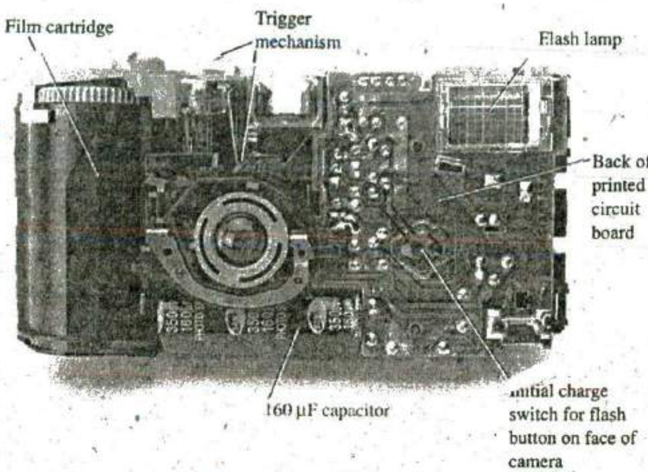


FIG. 10.82
Flash camera: basic circuitry.

the size of each capacitor is directly related to its capacitance level. It should certainly be of some interest that a single source of energy of only 1.5 V dc can be converted to one of a few thousand volts (albeit for a very short period of time) to fire the flash lamp. In fact, that single, small battery has sufficient power for the entire run of film through the camera. Always keep in mind that energy is related to power and time by $W = Pt = (VI)t$. That is, a high level of voltage can be generated for a defined energy level as long as the factors I and t are sufficiently small.

When you first use the camera, you are directed to press the flash button on the face of the camera and wait for the flash-ready light to come on. As soon as the flash button is depressed, the full 1.5 V of the dc battery are applied to an electronic network (a variety of networks can perform the same function) that generates an oscillating waveform of very high frequency (with a high repetitive rate) as shown in Fig. 10.83. The high-frequency transformer then significantly increases the magnitude of the generated voltage and passes it on to a half-wave rectification system (introduced in earlier chapters), resulting in a dc voltage of about 300 V across the 160 μF capacitor to charge the capacitor (as determined by $Q = CV$). Once the 300 V level is reached, the lead marked "sense" in Fig. 10.82 feeds the information back to the oscillator and turns it off until the output dc voltage drops to a low threshold level. When the capacitor is fully charged, the neon light in parallel with the capacitor turns on (labeled "flash-ready lamp" on the camera) to let you know that the camera is ready to use.



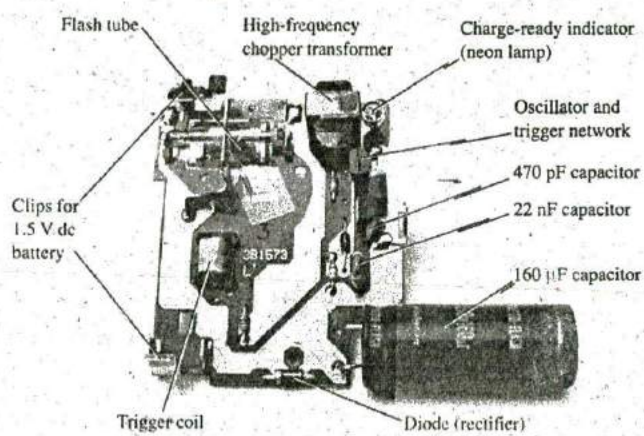


FIG. 10.83
Flash camera: internal construction.

The entire network from the 1.5 V dc level to the final 300 V level is called a *dc-dc converter*. The terminology *chopper network* comes from the fact that the applied dc voltage of 1.5 V was chopped up into one that changes level at a very high frequency so that the transformer can perform its function.

Even though the camera may use a 60 V neon light, the neon light and series resistor R_n must have a full 300 V across the branch before the neon light turns on. Neon lights are simply bulbs with a neon gas that support conduction when the voltage across the terminals reaches a sufficiently high level. There is no filament or hot wire as in a light bulb, but simply conduction through the gaseous medium. For new cameras, the first charging sequence may take 12 s to 15 s. Succeeding charging cycles may only take some 7 s or 8 s because the capacitor still has some residual charge on its plates. If the flash unit is not used, the neon light begins to drain the 300 V dc supply with a drain current in microamperes. As the terminal voltage drops, the neon light eventually turns off. For the unit in Fig. 10.81, it takes about 15 min before the light turns off.

Once off, the neon light no longer drains the capacitor, and the terminal voltage of the capacitor remains fairly constant. Eventually, however, the capacitor discharges due to its own leakage current, and the terminal voltage drops to very low levels. The discharge process is very rapid when the flash unit is used, causing the terminal voltage to drop very quickly ($V = Q/C$) and, through the feedback-sense connection signal, causing the oscillator to start up again and recharge the capacitor. You may have noticed when using a camera of this type that once the camera has its initial charge, you do not need to press the charge button between pictures—it is done automatically. However, if the camera sits for a long period of time, you must depress the charge button, but the charge time is only 3 s or 4 s due to the residual charge on the plates of the capacitor.

The 300 V across the capacitor are insufficient to fire the flash lamp. Additional circuitry, called the *trigger network*, must be incorporated to generate the few thousand volts necessary to fire the flash lamp. The resulting high voltage is one reason that there is a CAUTION note on each camera regarding the high internal voltages generated and the possibility of electrical shock if the camera is opened.

The thousands of volts required to fire the flash lamp require a discussion that introduces elements and concepts beyond the current level of the text. This description is simply a first exposure to some of the interesting possibilities available from the right mix of elements. When the flash switch at the bottom left of Fig. 10.82 is closed, it establishes a connection between the resistors R_1 and R_2. Through a voltage divider action, a dc voltage appears at the gate (G) terminal of the SCR (silicon-controlled rectifier—a device whose state is controlled by the voltage at the gate terminal). This dc voltage turns the SCR "on" and establishes a very low resistance path (like a short circuit) between its anode (A) and cathode (K) terminals. At this point the trigger capacitor, which is connected directly to the 300 V sitting across the capacitor, rapidly charges to 300 V because it now has a direct, low-resistance path to ground through the SCR. Once it reaches 300 V, the charging current in this part of the network drops to 0 A, and the SCR opens up again since it is a device that needs a steady current in the anode circuit to stay on. The capacitor then sits across the parallel coil (with no connection to ground through the SCR) with its full 300 V and begins to quickly discharge through the coil because the only resistance in the circuit affecting the time constant is the resistance of the parallel coil. As a result, a rapidly changing current through the coil generates a high voltage across the coil for reasons to be introduced in Chapter 11.

When the capacitor decays to zero volts, the current through the coil will be zero amperes, but a strong magnetic field has been established around the coil. This strong magnetic field then quickly collapses, establishing a current in the parallel network that recharges the capacitor again. This continual exchange between the two storage elements continues for a period of time, depending on the resistance in the circuit. The more the resistance, the shorter is the "ringing" of the voltage at the output. This action of the energy "flying back" to the other element is the basis for the "flyback" effect that is frequently used to generate high dc voltages such as needed in TVs. In Fig. 10.82, you will find that the trigger coil is connected directly to a second coil to form an autotransformer (a transformer with one end connected). Through transformer action, the high voltage generated across the trigger coil increases further, resulting in the 4000 V necessary to fire the flash lamp. Note in Fig. 10.83 that the 4000 V are applied to a grid that actually lies on the surface of the glass tube of the flash lamp (not internally connected or in contact with the gases). When the trigger voltage is applied, it excites the gases in the lamp, causing a very high current to develop in the bulb for a very short period of time and producing the desired bright light. The current in the lamp is supported by the charge on the 160 μF capacitor, which is dissipated very quickly. The capacitor voltage drops very quickly, the photo lamp shuts down, and the charging process begins again. If the entire process didn't occur as quickly as it does, the lamp would burn out after a single use.

Surge Protector (Line Conditioner)

In recent years we have all become familiar with the surge protector as a safety measure for our computers, TVs, DVD players, and other sensitive instrumentation. In addition to protecting equipment from unexpected surges in voltage and current, most quality units also filter out (remove) electromagnetic interference (EMI) and radio-frequency interference (RFI). EMI encompasses any unwanted disturbances down the power line established by any combination of electromagnetic effects such as those generated by motors on the line, power equipment in the area emitting signals picked up by the power line acting as an antenna, and so on. RFI includes all signals in the air in the audio range and beyond that may also be picked up by power lines inside or outside the house.

The unit in Fig. 10.84 has all the design features expected in a good line conditioner. Figure 10.84 reveals that it can handle the power drawn by six outlets and that it is set up for FAX/MODEM protection. Also note that it has both LED (light-emitting diode) displays, which reveal whether there is fault on the line or whether the line is OK, and an external circuit breaker to reset the system. In addition, when the surge protector is on, a red light is visible at the power switch.

FIG. 10.84
Surge protector: general appearance.

The schematic in Fig. 10.85 does not include all the details of the design, but it does include the major components that appear in most good line conditioners. First note in the photograph in Fig. 10.86 that the outlets are all connected in parallel, with a ground bar used to establish a ground connection for each outlet. The circuit board had to be flipped over to show the components, so it will take some adjustment to relate the position of the elements on the board to the casing. The *feed line* or *hot lead wire* (black in the actual unit) is connected directly from the line to the circuit breaker. The other end of the circuit breaker is connected to the other side of the circuit board. All the large discs that you see are 2 nF capacitors [not all have been included in Fig 10.86 for clarity]. There are quite a few capacitors to handle all the possibilities. For instance, there are capacitors from line to return (black wire to white wire), from line to ground (black to green), and from return to ground (white to ground). Each has two functions. The first and most obvious function is to prevent any spikes in voltage that may come down the line because of external effects such as lightning from reaching the equipment plugged into the unit. Recall from this chapter that the voltage across capacitors cannot change instantaneously and, in fact, acts to squelch any rapid change in voltage across its terminals. The capacitor, therefore, prevents the line to neutral voltage from changing too quickly, and any spike that tries to come down the line has to find another point in the feed circuit to fall across. In this way, the appliances plugged into the surge protector are well protected.

The second function requires some knowledge of the reaction of capacitors to different frequencies and is discussed in more detail in later

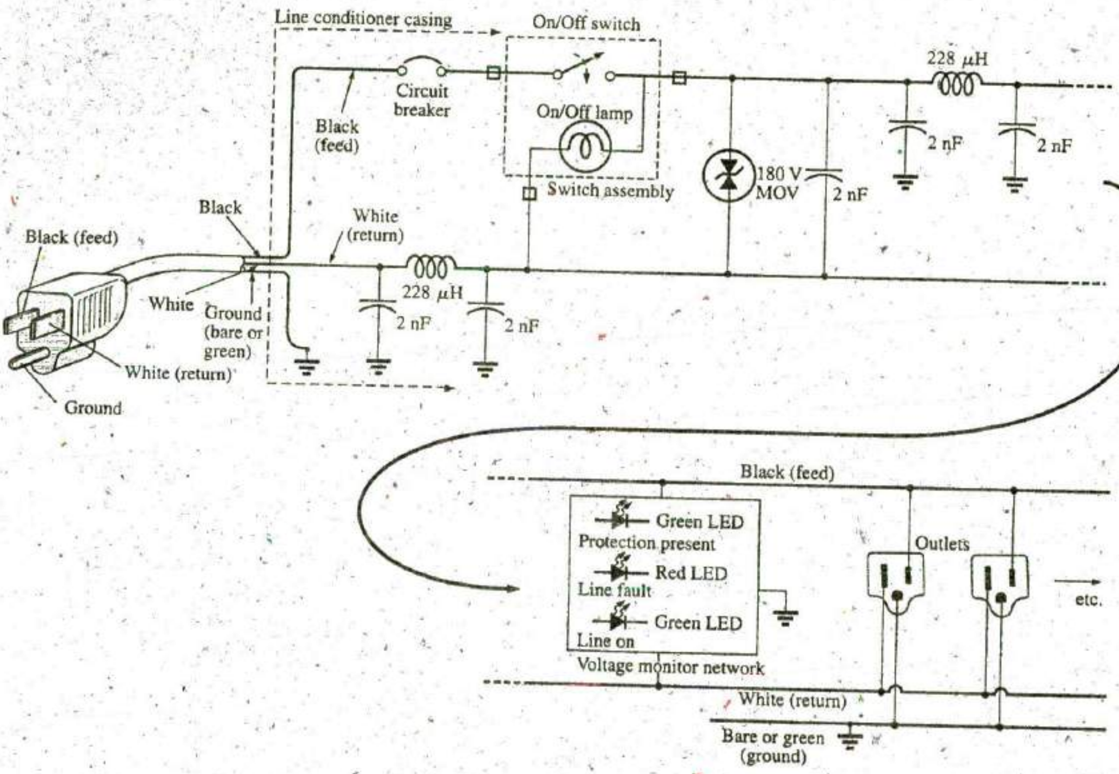


FIG. 10.85
Electrical schematic.

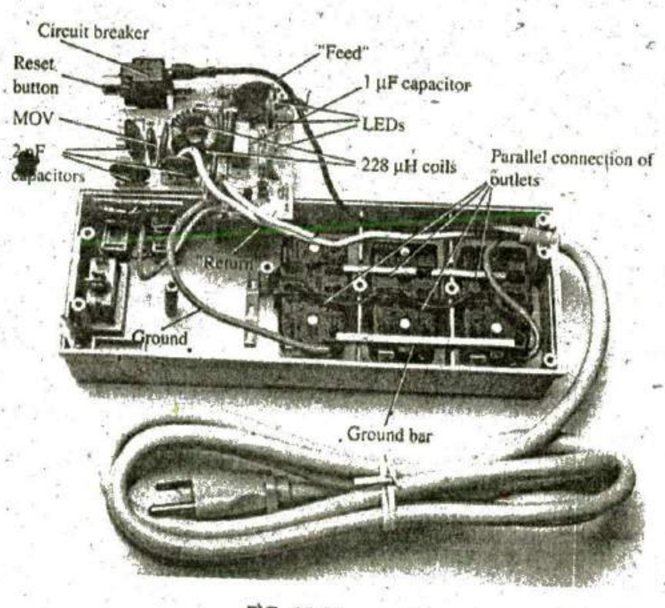


FIG. 10.86
Internal construction of surge protector.

chapters. For the moment, let it suffice to say that the capacitor has a different impedance to different frequencies, thereby preventing undesired frequencies, such as those associated with EMI and RFI disturbances, from affecting the operation of units connected to the line conditioner. The rectangular-shaped capacitor of 1 μF near the center of the board is connected directly across the line to take the brunt of a strong voltage spike down the line. Its larger size is clear evidence that it is designed to absorb a fairly high energy level that may be established by a large voltage—significant current over a period of time that may exceed a few milliseconds.

The large, toroidal-shaped structure in the center of the circuit board in Fig. 10.86 has two coils (Chapter 11) of 228 μH that appear in the line and neutral in Fig. 10.85. Their purpose, like that of the capacitors, is twofold: to block spikes in current from coming down the line and to block unwanted EMI and RFI frequencies from getting to the connected systems. In the next chapter you will find that coils act as "chokes" to quick changes in current; that is, the current through a coil cannot change instantaneously. For increasing frequencies, such as those associated with EMI and RFI disturbances, the reactance of a coil increases and absorbs the undesired signal rather than let it pass down the line. Using a choke in both the line and the neutral makes the conditioner network balanced to ground. In total, capacitors in a line conditioner have the effect of *bypassing* the disturbances, whereas inductors *block* the disturbance.

The smaller disc (blue) between two capacitors and near the circuit breaker is an MOV (metal-oxide varistor), which is the heart of most line conditioners. It is an electronic device whose terminal characteristics change with the voltage applied across its terminals. For the normal range of voltages down the line, its terminal resistance is sufficiently large to be considered an open circuit, and its presence can be ignored. However, if the voltage is too large, its terminal characteristics change from a very large resistance to a very small resistance that can essentially be considered a short circuit. This variation in resistance with applied voltage is the reason for the name *varistor.* For MOVs in North America, where the line voltage is 120 V, the MOVs are 180 V or more. The reason for the 60 V difference is that the 120 V rating is an effective value related to dc voltage levels, whereas the waveform for the voltage at any 120 V outlet has a peak value of about 170 V. A great deal more will be said about this topic in Chapter 13.

Taking a look at the symbol for an MOV in Fig. 10.86, note that it has an arrow in each direction, revealing that the MOV is bidirectional and blocks voltages with either polarity. In general, therefore, for normal operating conditions, the presence of the MOV can be ignored, but if a large spike should appear down the line, exceeding the MOV rating, it acts as a short across the line to protect the connected circuitry. It is a significant improvement to simply putting a fuse in the line because it is voltage sensitive, can react much quicker than a fuse, and displays its low-resistance characteristics for only a short period of time. When the spike has passed, it returns to its normal open-circuit characteristic. If you're wondering where the spike goes if the load is protected by a short circuit, remember that all sources of disturbance, such as lightning, generators, inductive motors (such as in air conditioners, dishwashers, power saws, and so on), have their own "source resistance," and there is always some resistance down the line to absorb the disturbance.

Most line conditioners, as part of their advertising, mention their energy absorption level. The rating of the unit in Fig. 10.84 is 1200 J, which is actually higher than most. Remembering that $W = Pt = EIt$ from the earlier discussion of cameras, we now realize that if a 5000 V spike occurred, we would be left with the product $It = W/E = 1200\ \text{J}/5000\ \text{V} = 240\ \text{mAs}$. Assuming a linear relationship between all quantities, the rated energy level reveals that a current of 100 A could be sustained for $t = 240\ \text{mAs}/100\ \text{A} = 2.4\ \mu\text{s}$, a current of 1000 A for 240 μs, and a current of 10,000 A for 24 μs. Obviously, the higher the power product of E and I, the less is the time element.

The technical specifications of the unit in Fig. 10.84 include an instantaneous response time in the order of picoseconds, with a phone line protection of 5 ns. The unit is rated to dissipate surges up to 6000 V and current spikes up to 96,000 A. It has a very high noise suppression ratio (80 dB; see Chapter 21) at frequencies from 50 kHz to 1000 MHz, and (a credit to the company) it has a lifetime warranty.

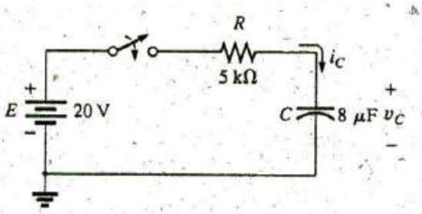


FIG. 10.87
Circuit to be analyzed using PSpice.

10.15 COMPUTER ANALYSIS

PSpice

Transient *RC* Response We now use PSpice to investigate the transient response for the voltage across the capacitor in Fig. 10.87. In all the examples in the text involving a transient response, a switch appeared in series with the source as shown in Fig. 10.88(a). When applying PSpice, we establish this instantaneous change in voltage level by applying a pulse waveform as shown in Fig. 10.88(b) with a pulse width (PW) longer than the period (5τ) of interest for the network.

To obtain a pulse source, start with the sequence **Place part** key-**Libraries-SOURCE-VPULSE-OK.** Once in place, set the label and all the parameters by double-clicking on each to obtain the **Display Properties** dialog box. As you scroll down the list of attributes, you will see the following parameters defined by Fig. 10.89:

V1 is the initial value.
V2 is the pulse level.
TD is the delay time.
TR is the rise time.
TF is the fall time.
PW is the pulse width at the V_2 level.
PER is the period of the waveform.

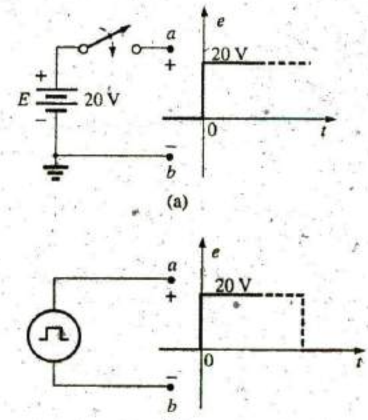


FIG. 10.88
Establishing a switching dc voltage level: (a) series dc voltage-switch combination; (b) PSpice pulse option.

All the parameters have been set as shown on the schematic in Fig. 10.90 for the network in Fig. 10.87. Since a rise and fall time of 0 s is unrealistic from a practical standpoint, 0.1 ms was chosen for each in this example. Further, since $\tau = RC = (5\ \text{k}\Omega) \times (8\ \mu\text{F}) = 20\ \text{ms}$ and $5\tau = 200\ \text{ms}$, a pulse width of 500 ms was selected. The period was simply chosen as twice the pulse width.

Now for the simulation process. First select the **New Simulation Profile** key to obtain the **New Simulation** dialog box in which **PSpice 10-1** is inserted for the **Name** and **Create** is chosen to leave the dialog box. The **Simulation Settings-PSpice 10-1** dialog box results, and under **Analysis,** choose the **Time Domain (Transient)** option under **Analysis type.** Set the **Run to time** at 200 ms so that only the first five time constants will be plotted. Set the **Start saving data after** option at 0 s to ensure that the data are collected immediately. The **Maximum**

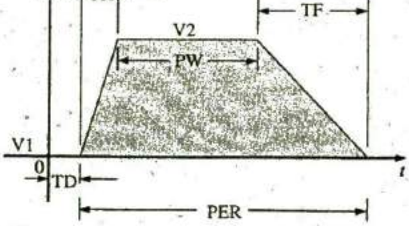


FIG. 10.89
The defining parameters of PSpice VPulse.

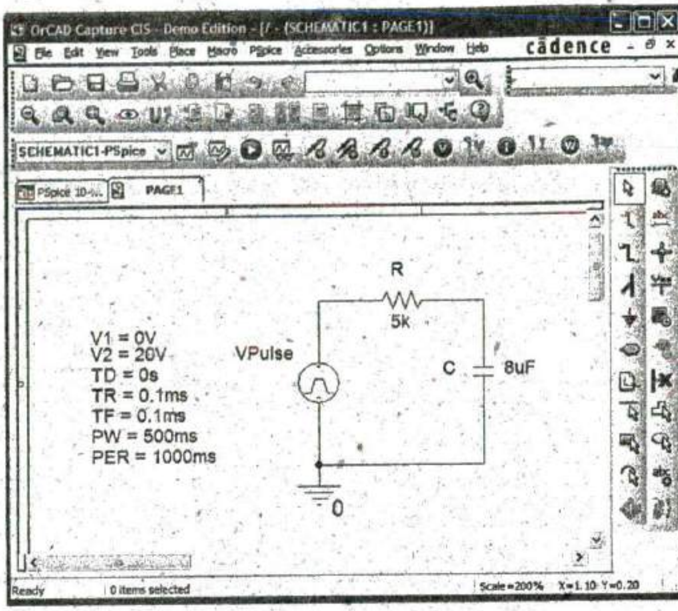


FIG. 10.90

Using PSpice to investigate the transient response of the series R-C circuit in Fig. 10.87.

step size is 1 ms to provide sufficient data points for a good plot. Click **OK,** and you are ready to select the **Run PSpice** key. The result is a graph without a plot (since it has not been defined yet) and an *x*-axis that extends from 0 s to 200 ms as defined above. If the graph fails to appear, check the **Probe Window** in the **Simulation Settings** to ensure that the **Display Probe Window** (with the **after simulation has completed** option selected) is checked, and **Run-PSpice** again. If problems continue and warning messages do not appear, close the screen by selecting the **X** in the top right corner and respond with a **No** to the request to **Save Files in Project.** The graphs should then appear. Finally, if all else seems to fail, try selecting **View Simulation Results** before the **PSpice-Run** sequence. The response will be a **PSpice** dialog box, indicating that the simulation has not been applied and the data are not available. Respond with a **Yes** to perform the simulation, and the graph should appear. To obtain a plot of the voltage across the capacitor versus time, apply the following sequence: **Add Trace** key-**Add Traces** dialog box-**V1(C)-OK.** The plot in Fig. 10.91 results. The color and thickness of the plot and the axis can be changed by placing the cursor on the plot line and right-clicking. Select **Properties** from the list that appears. A **Trace Properties** dialog box appears in which you can change the color and thickness of the line. Since the plot is against a black background, a better printout occurred when yellow was selected and the line was made thicker as shown in Fig. 10.91. For comparison, plot the applied pulse signal also. This is accomplished by going back to **Trace** and selecting **Add Trace** followed by **V(Vpulse:+)** and **OK.** Now both waveforms appear on the same screen as shown in Fig. 10.84. In this case, the plot has a reddish tint so it can be distinguished from the axis and the other plot. Note that it follows the left axis to the top and travels across the screen at 20 V.

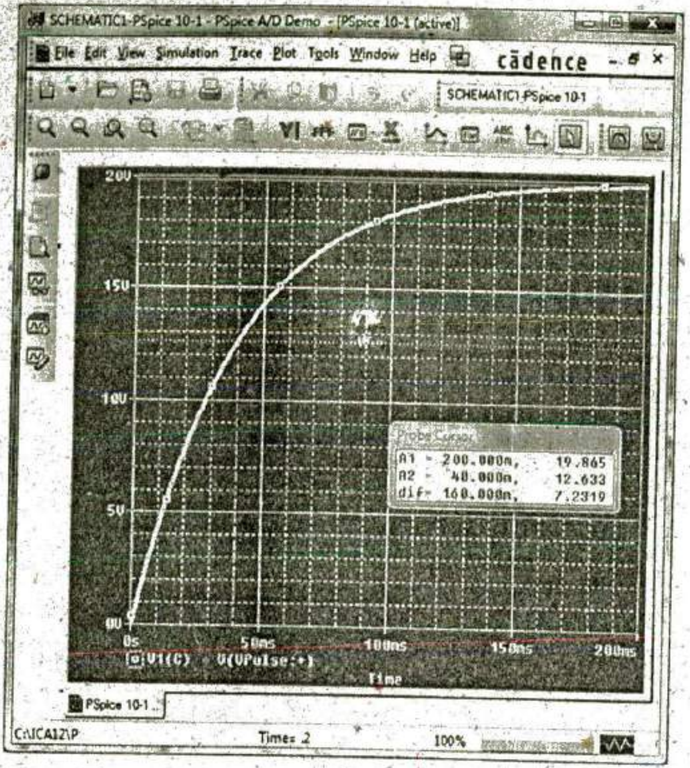


FIG. 10.91

Transient response for the voltage across the capacitor in Fig. 10.87 when VPulse is applied.

If you want the magnitude of either plot at any instant, simply select the **Toggle cursor** key. Then click on **V1(C)** at the bottom left of the screen. A box appears around **V1(C)** that reveals the spacing between the dots of the cursor on the screen. This is important when more than one cursor is used. By moving the cursor to 200 ms, you find that the magnitude (**A1**) is 19.865 V (in the **Probe Cursor** dialog box), clearly showing how close it is to the final value of 20 V. A second cursor can be placed on the screen with a right click and then a click on the same **V1(C)** on the bottom of the screen. The box around **V1(C)** cannot show two boxes, but the spacing and the width of the lines of the box have definitely changed. There is no box around the **Pulse** symbol since it was not selected—although it could have been selected by either cursor. If you now move the second cursor to one time constant of 40 ms, you find that the voltage is 12.659 V as shown in the **Probe Cursor** dialog box. This confirms that the voltage should be 63.2% of its final value of 20 V in one time constant ($0.632 \times 20 \text{ V} = 12.4 \text{ V}$). Two separate plots could have been obtained by going to **Plot-Add Plot to Window** and then using the trace sequence again.

Average Capacitive Current As an exercise in using the pulse source and to verify our analysis of the average current for a purely capacitive network, the description to follow verifies the results of Example 10.14. For the pulse waveform in Fig. 10.64, the parameters of the pulse supply appear in Fig. 10.92. Note that the rise time is now 2 ms, starting at 0 s, and the fall time is 6 ms. The period was set at 15 ms to permit monitoring the current after the pulse had passed.

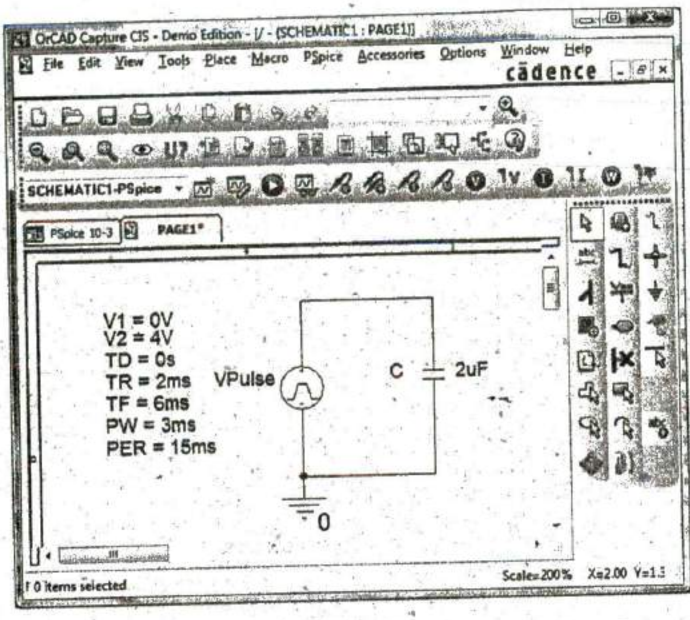


FIG. 10.92
Using PSpice to verify the results in Example 10.14.

Initiate simulation by first selecting the **New Simulation Profile** key to obtain the **New Simulation** dialog box in which **AverageIC** is entered as the **Name.** Choose **Create** to obtain the **Simulation Settings-AverageIC** dialog box. Select **Analysis,** and choose **Time Domain (Transient)** under the **Analysis type** options. Set the **Run to time** to 15 ms to encompass the period of interest, and set the **Start saving data after** at 0 s to ensure data points starting at $t = 0$ s. Select the **Maximum step size** from 15 ms/1000 = 15 μs to ensure 1000 data points for the plot. Click **OK,** and select the **Run PSpice** key. A window appears with a horizontal scale that extends from 0 to 15 ms as defined above. Then select the **Add Trace** key, and choose **I(C)** to appear in the **Trace Expression** below. Click **OK,** and the plot of **I(C)** appears in the bottom of Fig. 10.93. This time it would be nice to see the pulse waveform in the same window but as a separate plot. Therefore, continue with **Plot-Add Plot to Window-Trace-Add Trace-V(Vpulse:+)-OK,** and both plots appear as shown in Fig. 10.93.

Now use the cursors to measure the resulting average current levels. First, select the **I(C)** plot to move the **SEL>>** notation to the lower plot. The **SEL>>** defines which plot for multiplot screens is active. Then select the **Toggle cursor** key, and left-click on the **I(C)** plot to establish the crosshairs of the cursor. Set the value at 1 ms, and the magnitude **A1** is displayed as 4 mA. Right-click on the same plot, and a second cursor results that can be placed at 6 ms to get a response of -1.33 mA **(A2)** as expected from Example 10.14. The plot for **I(C)** was set in the yellow color with a wider line by right-clicking on the curve and choosing **Properties.** You will find after using the **DEMO** version for a while that it informs you that there is a limit of nine files that can be saved under the **File** listing. The result is that any further use of the **DEMO** version requires opening one of the nine files and deleting the contents if you want to run another program. That is, clear the screen and enter the new network.

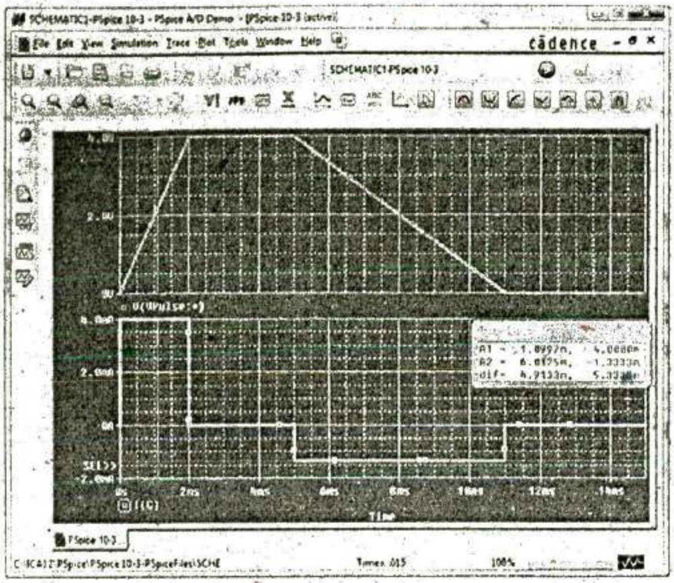


FIG. 10.93
The applied pulse and resulting current for the 2 μF capacitor in Fig. 10.92.

PROBLEMS

SECTION 10.2 The Electric Field

1. a. Find the electric field strength at a point 1 m from a charge of 4 μC.
   b. Find the electric field strength at a point 1 mm [1/1000 the distance of part (a)] from the same charge as part (a) and compare results.
2. The electric field strength is 72 newtons/coulomb (N/C) at a point r meters from a charge of 2 μC. Find the distance r.

SECTIONS 10.3 AND 10.4 Capacitance and Capacitors

3. Find the capacitance of a parallel plate capacitor if 1200 μC of charge are deposited on its plates when 24 V are applied across the plates.
4. How much charge is deposited on the plates of a 0.15 μF capacitor if 45 V are applied across the capacitor?
5. a. Find the electric field strength between the plates of a parallel plate capacitor if 500 mV are applied across the plates and the plates are 1 inch apart.
   b. Repeat part (a) if the distance between the plates is 1/100 inch.
   c. Compare the results of parts (a) and (b). Is the difference in field strength significant?
6. A 6.8 μF parallel plate capacitor has 160 μC of charge on its plates. If the plates are 5 mm apart, find the electric field strength between the plates.
7. Find the capacitance of a parallel plate capacitor if the area of each plate is 0.1 m^2 and the distance between the plates is 0.1 inch. The dielectric is air.
8. Repeat Problem 7 if the dielectric is paraffin-coated paper.
9. Find the distance in mils between the plates of a 2 μF capacitor if the area of each plate is 0.15 m^2 and the dielectric is transformer oil.
10. The capacitance of a capacitor with a dielectric of air is 1360 pF. When a dielectric is inserted between the plates, the capacitance increases to 6.8 nF. Of what material is the dielectric made?
11. The plates of a parallel plate capacitor with a dielectric of Bakelite are 0.2 mm apart and have an area of 0.08 m^2, and 200 V are applied across the plates.
    a. Determine the capacitance.
    b. Find the electric field intensity between the plates.
    c. Find the charge on each plate.
12. A parallel plate air capacitor has a capacitance of 4.7 μF. Find the new capacitance if:
    a. The distance between the plates is doubled (everything else remains the same).
    b. The area of the plates is doubled (everything else remains the same as for the 4.7 μF level).
    c. A dielectric with a relative permittivity of 20 is inserted between the plates (everything else remains the same as for the 4.7 μF level).
    d. A dielectric is inserted with a relative permittivity of 4, and the area is reduced to 1/3 and the distance to 1/4 of their original dimensions.

*13. Find the maximum voltage that can be applied across a parallel plate capacitor of 6800 pF if the area of one plate is 0.02 m^2

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and the dielectric is mica. Assume a linear relationship between the dielectric strength and the thickness of the dielectric.

*14. Find the distance in micrometers between the plates of a parallel plate mica capacitor if the maximum voltage that can be applied across the capacitor is 1200 V. Assume a linear relationship between the breakdown strength and the thickness of the dielectric.

15. A 22 μF capacitor has -200 ppm/°C at room temperature of 20°C. What is the capacitance if the temperature increases to 100°C, the boiling point of water?

16. What is the capacitance of a small teardrop capacitor labeled 40 J? What is the range of expected values as established by the tolerance?

17. A large, flat, mica capacitor is labeled 471F. What are the capacitance and the expected range of values guaranteed by the manufacturer?

18. A small, flat, disc ceramic capacitor is labeled 182K. What are the capacitance level and the expected range of values?

SECTION 10.5 Transients in Capacitive Networks: The Charging Phase

19. For the circuit in Fig. 10.94, composed of standard values:
    a. Determine the time constant of the circuit.
    b. Write the mathematical equation for the voltage v_C following the closing of the switch.
    c. Determine the voltage v_C after one, three, and five time constants.
    d. Write the equations for the current i_C and the voltage v_R.
    e. Sketch the waveforms for v_C and i_C.

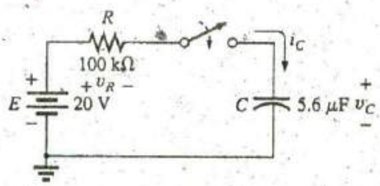


FIG. 10.94
Problems 19 and 20.

20. Repeat Problem 19 for $R = 1$ MΩ, and compare the results.

21. For the circuit in Fig. 10.95, composed of standard values:
    a. Determine the time constant of the circuit.
    b. Write the mathematical equation for the voltage v_C following the closing of the switch.

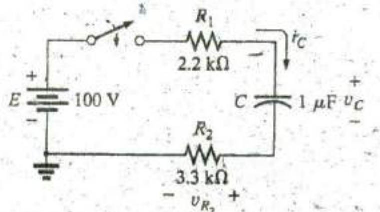


FIG. 10.95
Problem 21.

    c. Determine v_C after one, three, and five time constants.
    d. Write the equations for the current i_C and the voltage v_{R_2}.
    e. Sketch the waveforms for v_C and i_C.

*22. For the circuit in Fig. 10.96, composed of standard values:
    a. Determine the time constant of the circuit.
    b. Write the mathematical equation for the voltage v_C following the closing of the switch.
    c. Write the mathematical expression for the current i_C following the closing of the switch.
    d. Sketch the waveforms of v_C and i_C.

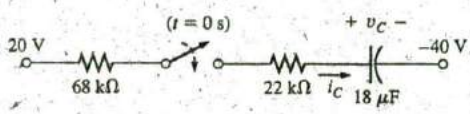


FIG. 10.96
Problem 22.

23. Given the voltage $v_C = 12\text{ V}(1 - e^{-t/100\,\mu s})$:
    a. What is the time constant?
    b. What is the voltage at $t = 50\ \mu$s?
    c. What is the voltage at $t = 1$ ms?

24. The voltage across a 10 μF capacitor in a series R-C circuit is $v_C = 40\text{ mV}(1 - e^{-t/20\text{ ms}})$.
    a. On a practical basis, how much time must pass before the charging phase has passed?
    b. What is the resistance of the circuit?
    c. What is the voltage at $t = 20$ ms?
    d. What is the voltage at 10 time constants?
    e. Under steady-state conditions, how much charge is on the plates?
    f. If the leakage resistance is 1000 MΩ, how long will it take (in hours) for the capacitor to discharge if we assume that the discharge rate is constant throughout the discharge period?

SECTION 10.6 Transients in Capacitive Networks: The Discharging Phase

25. For the R-C circuit in Fig. 10.97, composed of standard values:
    a. Determine the time constant of the circuit when the switch is thrown into position 1.
    b. Find the mathematical expression for the voltage across the capacitor and the current after the switch is thrown into position 1.

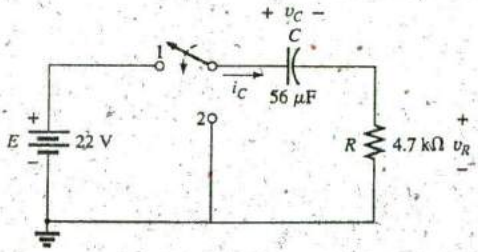


FIG. 10.97
Problem 25.

c. Determine the magnitude of the voltage v_C and the current i_C the instant the switch is thrown into position 2 at $t = 1$ s.
d. Determine the mathematical expression for the voltage v_C and the current i_C for the discharge phase.
e. Plot the waveforms of v_C and i_C for a period of time extending from 0 to 2 s from when the switch was thrown into position 1.

26. For the network in Fig. 10.98, composed of standard values:
a. Write the mathematical expressions for the voltages v_C, and v_{R_1} and the current i_C after the switch is thrown into position 1.
b. Find the values of v_C, v_{R_1}, and i_C when the switch is moved to position 2 at $t = 100$ ms.
c. Write the mathematical expressions for the voltages v_C and v_{R_2} and the current i_C if the switch is moved to position 3 at $t = 200$ ms.
d. Plot the waveforms of v_C, v_{R_2}, and i_C for the time period extending from 0 to 300 ms.

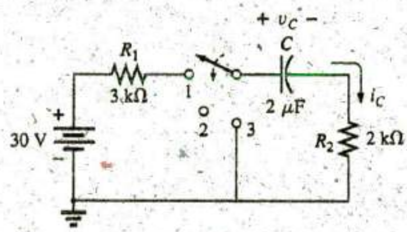


FIG. 10.98
Problem 26.

***27.** For the network in Fig. 10.99, composed of standard values:
a. Find the mathematical expressions for the voltage v_C and the current i_C when the switch is thrown into position 1.
b. Find the mathematical expressions for the voltage v_C and the current i_C if the switch is thrown into position 2 at a time equal to five time constants of the charging circuit.
c. Plot the waveforms of v_C and i_C for a period of time extending from 0 to 30 μs.
d. Plot the waveform of v_R for the same period as in part (a).

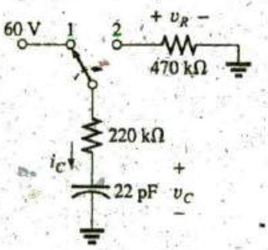


FIG. 10.99
Problem 27.

28. The 1000 μF capacitor in Fig. 10.100 is charged to 12 V in an automobile. To discharge the capacitor before further use, a wire with a resistance of 2 mΩ is placed across the capacitor.

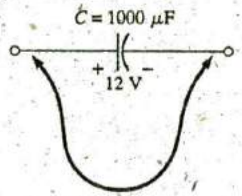


FIG. 10.100
Problem 28.

a. How long will it take to discharge the capacitor?
b. What is the peak value of the current?
c. Based on the answer to part (b), is a spark expected when contact is made with both ends of the capacitor?

SECTION 10.7 Initial Conditions

29. The capacitor in Fig. 10.101 is initially charged to 6 V with the polarity shown.
a. Write the expression for the voltage v_C after the switch is closed.
b. Write the expression for the current i_C after the switch is closed.
c. Plot the results of parts (a) and (b).

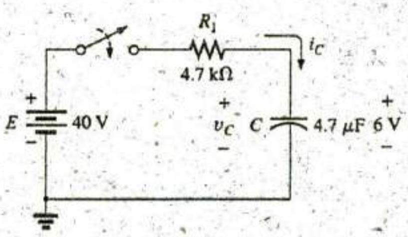


FIG. 10.101
Problem 29.

30. The capacitor in Fig. 10.102 is initially charged to 40 V before the switch is closed. Write the expressions for the voltages v_C and v_R and the current i_C following the closing of the switch. Plot the resulting waveforms.

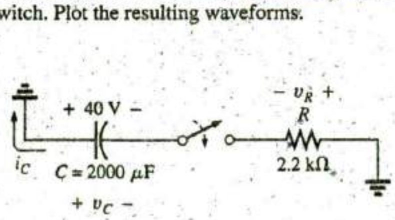


FIG. 10.102
Problem 30.

***31.** The capacitor in Fig. 10.103 is initially charged to 10 V with the polarity shown. Write the expressions for the voltage v_C and the current i_C following the closing of the switch. Plot the resulting waveforms.

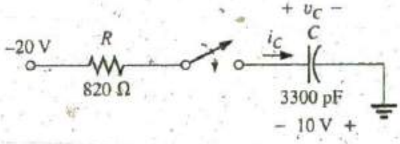


FIG. 10.103
Problem 31.

*32. The capacitor in Fig. 10.104 is initially charged to 8 V with the polarity shown.
   a. Find the mathematical expressions for the voltage v_C and the current i_C when the switch is closed.
   b. Sketch the waveforms of v_C and i_C.

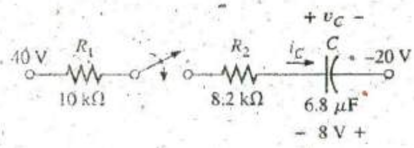


FIG. 10.104
Problem 32.

SECTION 10.8 Instantaneous Values

33. Given the expression $v_C = 140\text{ mV}(1 - e^{-t/2\text{ ms}})$
   a. Determine v_C at $t = 1$ ms.
   b. Determine v_C at $t = 20$ ms.
   c. Find the time t for v_C to reach 100 mV.
   d. Find the time t for v_C to reach 138 mV.

34. For the automobile circuit of Fig. 10.105, V_L must be 8 V before the system is activated. If the switch is closed at t = 0 s, how long will it take for the system to be activated?

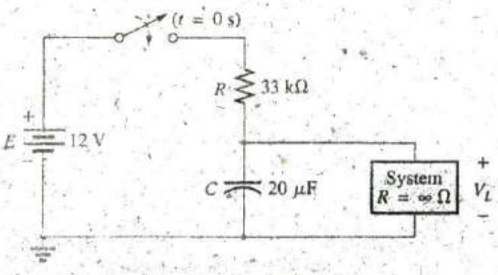


FIG. 10.105
Problem 34.

*35. Design the network in Fig. 10.106 such that the system turns on 10 s after the switch is closed.

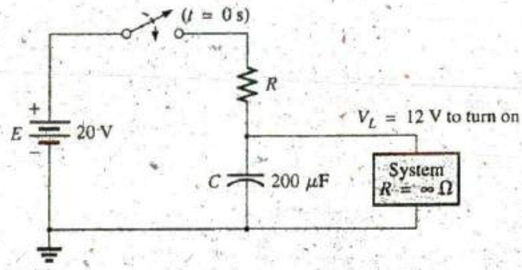


FIG. 10.106
Problem 35.

36. For the circuit in Fig. 10.107:
   a. Find the time required for v_C to reach 48 V following the closing of the switch.
   b. Calculate the current i_C at the instant $v_C = 48$ V.
   c. Determine the power delivered by the source at the instant $t = 2\tau$.

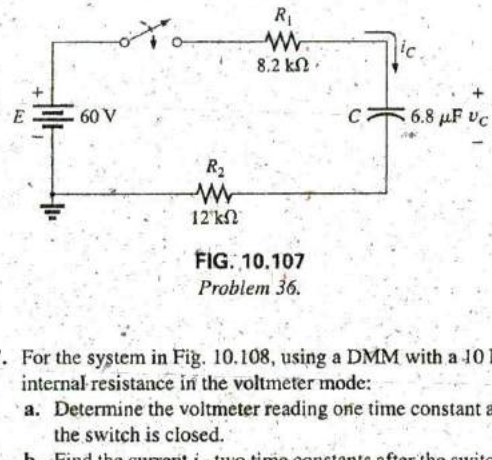


FIG. 10.107
Problem 36.

37. For the system in Fig. 10.108, using a DMM with a 10 MΩ internal resistance in the voltmeter mode:
   a. Determine the voltmeter reading one time constant after the switch is closed.
   b. Find the current i_C two time constants after the switch is closed.
   c. Calculate the time that must pass after the closing of the switch for the voltage v_C to be 50 V.

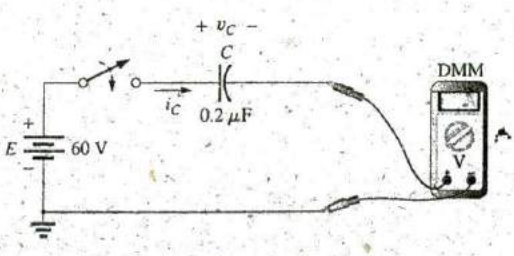


FIG. 10.108
Problem 37.

SECTION 10.9 Thévenin Equivalent: $\tau = R_{Th}C$

38. For the circuit in Fig. 10.109:
    a. Find the mathematical expressions for the transient behavior of the voltage v_C and the current i_C following the closing of the switch.
    b. Sketch the waveforms of v_C and i_C.

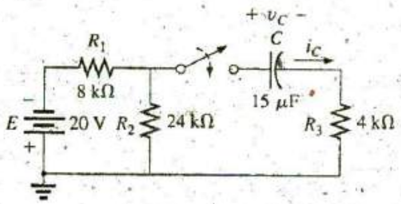


FIG. 10.109
Problem 38.

39. The capacitor in Fig. 10.110 is initially charged to 10 V with the polarity shown.
    a. Write the mathematical expressions for the voltage v_C and the current i_C when the switch is closed.
    b. Sketch the waveforms of v_C and i_C.

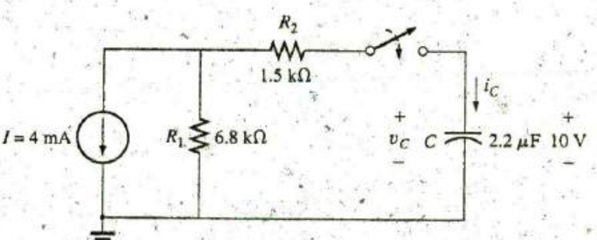


FIG. 10.110
Problem 39.

40. The capacitor in Fig. 10.111 is initially charged to 12 V with the polarity shown.
    a. Write the mathematical expressions for the voltage v_C and the current i_C when the switch is closed.
    b. Sketch the waveforms of v_C and i_C.

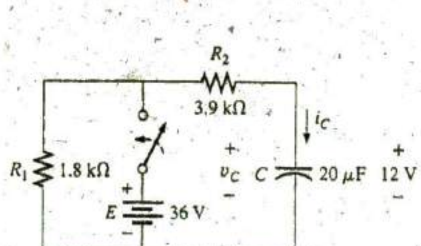


FIG. 10.111
Problem 40.

41. For the circuit in Fig. 10.112:
    a. Find the mathematical expressions for the transient behavior of the voltage v_C and the current i_C following the closing of the switch.
    b. Sketch the waveforms of v_C and i_C.

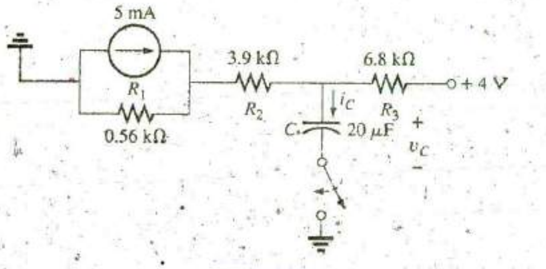


FIG. 10.112
Problem 41.

*42. The capacitor in Fig. 10.113 is initially charged to 8 V with the polarity shown.
    a. Write the mathematical expressions for the voltage v_C and the current i_C when the switch is closed.
    b. Sketch the waveforms of v_C and i_C.

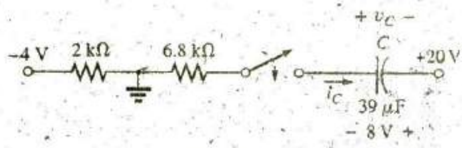


FIG. 10.113
Problem 42.

43. For the system in Fig. 10.114, using a DMM with a 10 MΩ internal resistance in the voltmeter mode:
    a. Determine the voltmeter reading four time constants after the switch is closed.
    b. Find the time that must pass before i_C drops to 3 μA.
    c. Find the time that must pass after the closing of the switch for the voltage across the meter to reach 10 V.

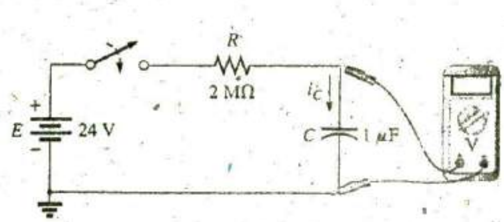


FIG. 10.114
Problem 43.

SECTION 10.10 The Current i_C

44. Find the waveform for the average current if the voltage across the 2 μF capacitor is as shown in Fig. 10.115.

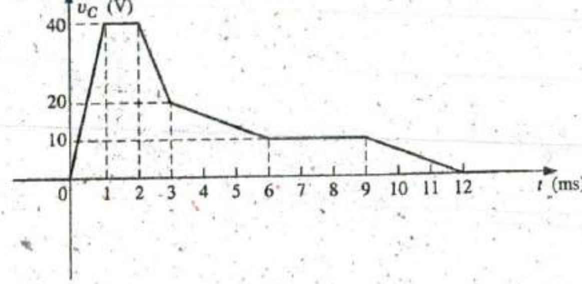


FIG. 10.115
Problem 44.

45. Find the waveform for the average current if the voltage across the 4.7 μF capacitor is as shown in Fig. 10.116.

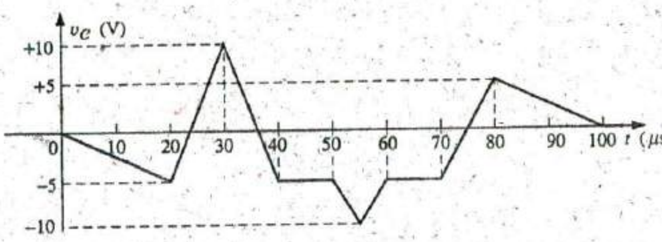


FIG. 10.116
Problem 45.

46. Given the waveform in Fig. 10.117 for the current of a 20 μF capacitor, sketch the waveform of the voltage v_C across the capacitor if $v_C = 0$ V at $t = 0$ s.

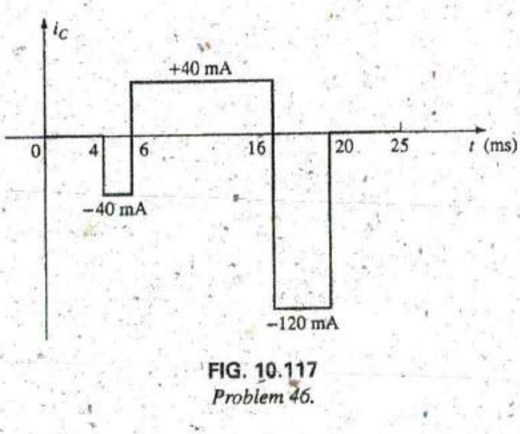


FIG. 10.117
Problem 46.

SECTION 10.11 Capacitors in Series and in Parallel

47. Find the total capacitance C_T for the circuit in Fig. 10.118.

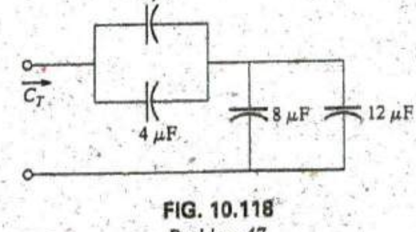


FIG. 10.118
Problem 47.

48. Find the total capacitance C_T for the circuit in Fig. 10.119.

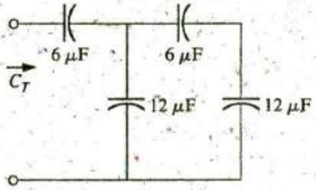


FIG. 10.119
Problem 48.

49. Find the voltage across and the charge on each capacitor for the circuit in Fig. 10.120.

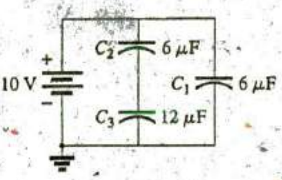


FIG. 10.120
Problem 49.

50. Find the voltage across and the charge on each capacitor for the circuit in Fig. 10.121.

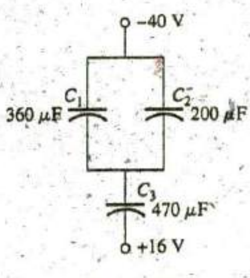


FIG. 10.121
Problem 50.

51. For the configuration in Fig. 10.122, determine the voltage across each capacitor and the charge on each capacitor under steady-state conditions.

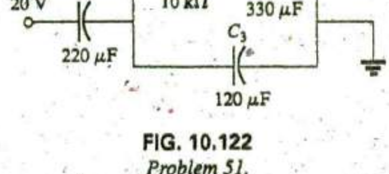


FIG. 10.122
Problem 51.

52. For the configuration in Fig. 10.123, determine the voltage across each capacitor and the charge on each capacitor.

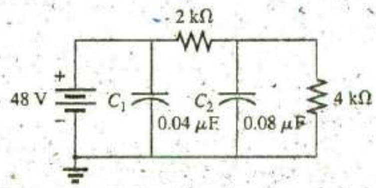


FIG. 10.123
Problem 52.

SECTION 10.12 Energy Stored by a Capacitor

53. Find the energy stored by a 120 pF capacitor with 12 V across its plates.

54. If the energy stored by a 6 μF capacitor is 1200 J, find the charge Q on each plate of the capacitor.

***55.** For the network in Fig. 10.124, determine the energy stored by each capacitor under steady-state conditions.

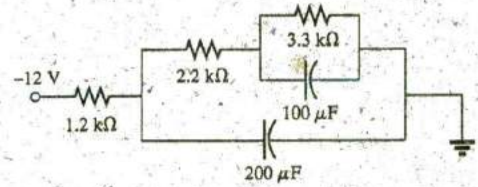


FIG. 10.124
Problem 55.

***56.** An electronic flashgun has a 1000 μF capacitor that is charged to 100 V.

- **a.** How much energy is stored by the capacitor?
- **b.** What is the charge on the capacitor?
- **c.** When the photographer takes a picture, the flash fires for 1/2000 s. What is the average current through the flashtube?
- **d.** Find the power delivered to the flashtube.
- **e.** After a picture is taken, the capacitor has to be recharged by a power supply that delivers a maximum current of 10 mA. How long will it take to charge the capacitor?

SECTION 10.15 Computer Analysis

57. Using PSpice or Multisim, verify the results in Example 10.6.

58. Using the initial condition operator, verify the results in Example 10.8 for the charging phase using PSpice or Multisim.

59. Using PSpice or Multisim, verify the results for v_C during the charging phase in Example 10.11.

60. Using PSpice or Multisim, verify the results in Problem 42.

GLOSSARY

Average current The current defined by a linear (straight line) change in voltage across a capacitor for a specific period of time.

Breakdown voltage Another term for *dielectric strength*, listed below.

Capacitance A measure of a capacitor's ability to store charge; measured in farads (F).

Capacitor A fundamental electrical element having two conducting surfaces separated by an insulating material and having the capacity to store charge on its plates.

Coulomb's law An equation relating the force between two like or unlike charges.

Derivative The instantaneous change in a quantity at a particular instant in time.

Dielectic The insulating material between the plates of a capacitor that can have a pronounced effect on the charge stored on the plates of a capacitor.

Dielectric constant Another term for *relative permittivity*, listed below.

Dielectric strength An indication of the voltage required for unit length to establish conduction in a dielectric.

Electric field strength The force acting on a unit positive charge in the region of interest.

Electric flux lines Lines drawn to indicate the strength and direction of an electric field in a particular region.

Fringing An effect established by flux lines that do not pass directly from one conducting surface to another.

Initial value The steady-state voltage across a capacitor before a transient period begins.

Leakage current The current that results in the total discharge of a capacitor if the capacitor is disconnected from the charging network for a sufficient length of time.

Maximum working voltage That voltage level at which a capacitor can perform its function without concern about breakdown or change in characteristics.

Permittivity A measure of how well a dielectric *permits* the establishment of flux lines within the dielectric.

Relative permittivity The permittivity of a material compared to that of air.

Steady-state region A period of time defined by the fact that the voltage across a capacitor has reached a level that, for all practical purposes, remains constant.

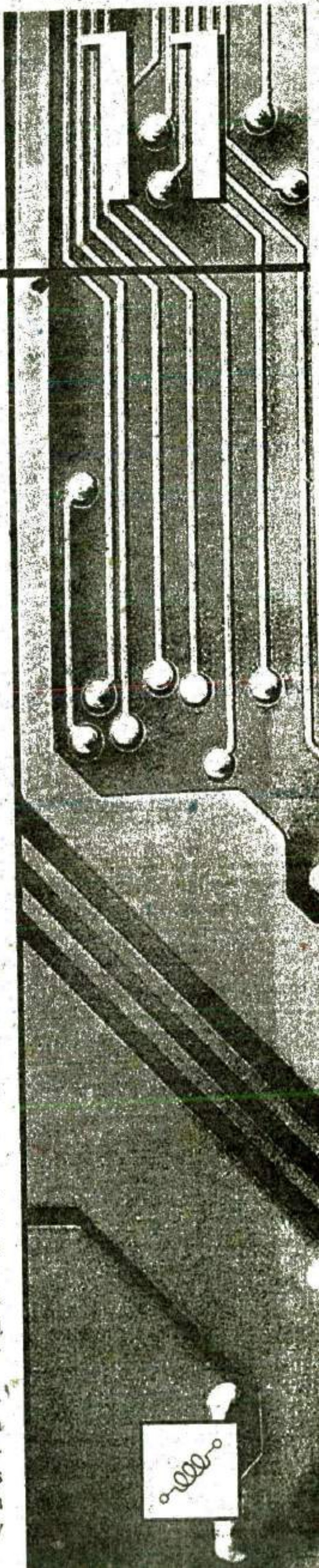
Stray capacitance Capacitances that exist not through design but simply because two conducting surfaces are relatively close to each other.

Temperature coefficient An indication of how much the capacitance value of a capacitor will change with change in temperature.

Time constant A period of time defined by the parameters of the network that defines how long the transient behavior of the voltage or current of a capacitor will last.

Transient period That period of time where the voltage across a capacitor or the current of a capacitor will change in value at a rate determined by the time constant of the network.

Inductors



Objectives

- *Become familiar with the basic construction of an inductor, the factors that affect the strength of the magnetic field established by the element, and how to read the nameplate data.*
- *Be able to determine the transient (time-varying) response of an inductive network and plot the resulting voltages and currents.*
- *Understand the impact of combining inductors in series or parallel.*
- *Develop some familiarity with the use of PSpice or Multisim to analyze networks with inductive elements.*

11.1 INTRODUCTION

Three basic components appear in the majority of electrical/electronic systems in use today. They include the *resistor* and the *capacitor,* which have already been introduced, and the **inductor,** to be examined in detail in this chapter. In many ways, the inductor is the dual of the capacitor; that is, the voltage of one is applicable to the current of the other, and vice versa. In fact, some sections in this chapter parallel those in Chapter 10 on the capacitor. Like the capacitor, *the inductor exhibits its true characteristics only when a change in voltage or current is made in the network.*

Recall from Chapter 10 that a capacitor can be replaced by an open-circuit equivalent under steady-state conditions. You will see in this chapter that an inductor can be replaced by a short-circuit equivalent under steady-state conditions. Finally, you will learn that while resistors dissipate the power delivered to them in the form of heat, ideal capacitors store the energy delivered to them in the form of an electric field. Inductors, in the ideal sense, are like capacitors in that they also store the energy delivered to them—but in the form of a magnetic field.

11.2 MAGNETIC FIELD

Magnetism plays an integral part in almost every electrical device used today in industry, research, or the home. Generators, motors, transformers, circuit breakers, televisions, computers, tape recorders, and telephones all employ magnetic effects to perform a variety of important tasks.

The compass, used by Chinese sailors as early as the second century A.D., relies on a **permanent magnet** for indicating direction. A permanent magnet is made of a material, such as steel or iron, that remains magnetized for long periods of time without the need for an external source of energy.

In 1820, the Danish physicist Hans Christian Oersted discovered that the needle of a compass deflects if brought near a current-carrying conductor. This was the first demonstration that electricity and magnetism were related. In the same year, the French physicist André-Marie Ampère performed experiments in this area and developed what is presently known as **Ampère's circuital law.** In subsequent years, others, such as Michael Faraday, Karl Friedrich Gauss, and James Clerk Maxwell, continued to experiment in this area and developed many

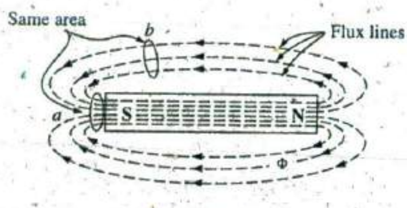


FIG. 11.1
Flux distribution for a permanent magnet.

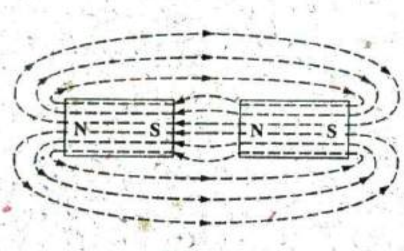


FIG. 11.2
Flux distribution for two adjacent, opposite poles.

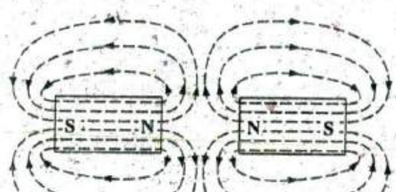


FIG. 11.3
Flux distribution for two adjacent, like poles.

of the basic concepts of **electromagnetism**—magnetic effects induced by the flow of charge, or current.

A magnetic field exists in the region surrounding a permanent magnet, which can be represented by **magnetic flux lines** similar to electric flux lines. Magnetic flux lines, however, do not have origins or terminating points as do electric flux lines but exist in *continuous loops,* as shown in Fig. 11.1.

The magnetic flux lines radiate from the north pole to the south pole, returning to the north pole through the metallic bar. Note the equal spacing between the flux lines within the core and the symmetric distribution outside the magnetic material. These are additional properties of magnetic flux lines in homogeneous materials (that is, materials having uniform structure or composition throughout). It is also important to realize that the continuous magnetic flux line will strive to occupy as small an area as possible. This results in magnetic flux lines of minimum length between the unlike poles, as shown in Fig. 11.2. The strength of a magnetic field in a particular region is directly related to the density of flux lines in that region. In Fig. 11.1, for example, the magnetic field strength at point *a* is twice that at point *b* since twice as many magnetic flux lines are associated with the perpendicular plane at point *a* than at point *b*. Recall from childhood experiments that the strength of permanent magnets is always stronger near the poles.

If unlike poles of two permanent magnets are brought together, the magnets attract, and the flux distribution is as shown in Fig. 11.2. If like poles are brought together, the magnets repel, and the flux distribution is as shown in Fig. 11.3.

If a nonmagnetic material, such as glass or copper, is placed in the flux paths surrounding a permanent magnet, an almost unnoticeable change occurs in the flux distribution (Fig. 11.4). However, if a magnetic material, such as soft iron, is placed in the flux path, the flux lines pass through the soft iron rather than the surrounding air because flux lines pass with greater ease through magnetic materials than through air. This principle is used in shielding sensitive electrical elements and instruments that can be affected by stray magnetic fields (Fig. 11.5).

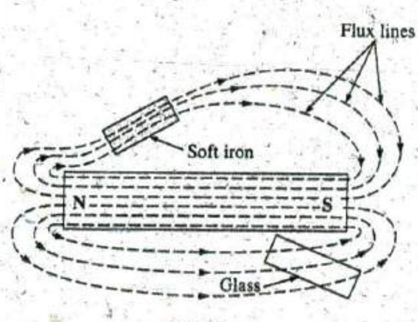


FIG. 11.4
Effect of a ferromagnetic sample on the flux distribution of a permanent magnet.

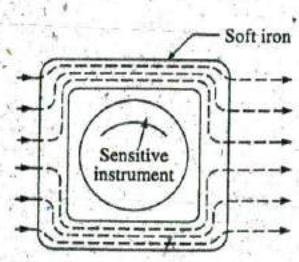


FIG. 11.5
Effect of a magnetic shield on the flux distribution.

A magnetic field (represented by concentric magnetic flux lines, as in Fig. 11.6) is present around every wire that carries an electric current. The direction of the magnetic flux lines can be found simply by placing the thumb of the *right* hand in the direction of *conventional* current flow and noting the direction of the fingers. (This method is commonly called the *right-hand rule.*) If the conductor is wound in a

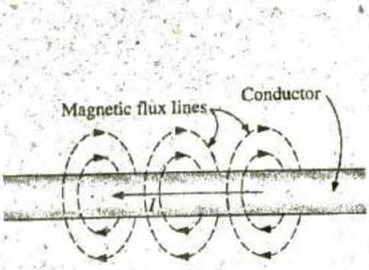


FIG. 11.6
Magnetic flux lines around a current-carrying conductor.

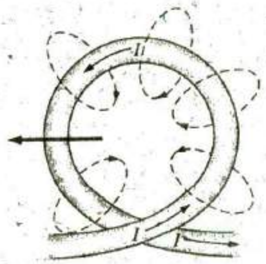


FIG. 11.7
Flux distribution of a single-turn coil.

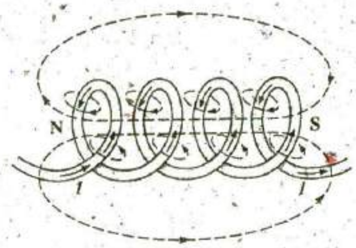


FIG. 11.8
Flux distribution of a current-carrying coil.

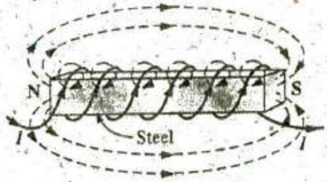


FIG. 11.9
Electromagnet.

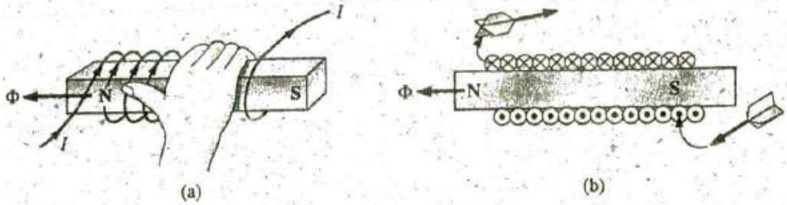


FIG. 11.10
Determining the direction of flux for an electromagnet: (a) method; (b) notation.

single-turn coil (Fig. 11.7), the resulting flux flows in a common direction through the center of the coil. A coil of more than one turn produces a magnetic field that exists in a continuous path through and around the coil (Fig. 11.8).

The flux distribution of the coil is quite similar to that of the permanent magnet. The flux lines leaving the coil from the left and entering to the right simulate a north and a south pole, respectively. The principal difference between the two flux distributions is that the flux lines are more concentrated for the permanent magnet than for the coil. Also, since *the strength of a magnetic field is determined by the density of the flux lines,* the coil has a weaker field strength. The field strength of the coil can be effectively increased by placing certain materials, such as iron, steel, or cobalt, within the coil to increase the flux density within the coil. By increasing the field strength with the addition of the core, we have devised an *electromagnet* (Fig. 11.9) that not only has all the properties of a permanent magnet but also has a field strength that can be varied by changing one of the component values (current, turns, and so on). Of course, current must pass through the coil of the electromagnet for magnetic flux to be developed, whereas there is no need for the coil or current in the permanent magnet. The direction of flux lines can be determined for the electromagnet (or in any core with a wrapping of turns) by placing the fingers of your right hand in the direction of current flow around the core. Your thumb then points in the direction of the north pole of the induced magnetic flux, as demonstrated in Fig. 11.10(a). A cross section of the same electromagnet is in Fig. 11.10(b) to introduce the convention for directions perpendicular to the page. The cross and the dot refer to the tail and the head of the arrow, respectively.

In the SI system of units, magnetic flux is measured in **webers (Wb)** as derived from the surname of Wilhelm Eduard Weber (Fig. 11.11). The applied symbol is the capital Greek letter *phi*, ϕ. The number of flux

FIG. 11.11
Wilhelm Eduard Weber.
Courtesy of the Smithsonian Institution, Photo No. 52,604.

German (Wittenberg, Göttingen)
(1804–91)
Physicist
Professor of Physics, University of Göttingen

An important contributor to the establishment of a system of *absolute units* for the electrical sciences, which was beginning to become a very active area of research and development. Established a definition of electric current in an electromagnetic system based on the magnetic field produced by the current. He was politically active and, in fact, was dismissed from the faculty of the University of Göttingen for protesting the suppression of the constitution by the King of Hanover in 1837. However, he found other faculty positions and eventually returned to Göttingen as director of the astronomical observatory. He received honors from England, France, and Germany, including the Copley Medal of the Royal Society of London.

FIG. 11.12
Nikola Tesla.
Courtesy of the Smithsonian Institution, Photo No. 52,223.

Croatian-American (Smiljan, Paris, Colorado Springs, New York City)
(1856–1943)
Electrical Engineer and Inventor Recipient of the Edison Medal in 1917

Often regarded as one of the most innovative and inventive individuals in the history of the sciences. He was the first to introduce the *alternating-current machine,* removing the need for commutator bars of dc machines. After emigrating to the United States in 1884, he sold a number of his patents on *ac machines, transformers,* and *induction coils* (including the Tesla coil as we know it today) to the Westinghouse Electric Company. Some say that his most important discovery was made at his laboratory in Colorado Springs, where in 1900 he discovered *terrestrial stationary waves.* The range of his discoveries and inventions is too extensive to list here but extends from lighting systems to *polyphase power systems* to a *wireless world broadcasting system.*

FIG. 11.13
Defining the flux density B.

lines per unit area, called the **flux density,** is denoted by the capital letter B and is measured in **teslas** (T) to honor the efforts of Nikola Tesla, a scientist of the late 1800s (Fig. 11.12).

In equation form,

$$\boxed{B = \frac{\Phi}{A}} \qquad \begin{aligned} B &= \text{Wb/m}^2 = \text{teslas (T)} \\ \Phi &= \text{webers (Wb)} \\ A &= \text{m}^2 \end{aligned} \tag{11.1}$$

where Φ is the number of flux lines passing through area A in Fig. 11.13. The flux density at point a in Fig. 11.1 is twice that at point b because twice as many flux lines pass through the same area.

In Eq. (11.1), the equivalence is given by

$$\boxed{1 \text{ tesla} = 1 \text{ T} = 1 \text{ Wb/m}^2} \tag{11.2}$$

which states in words that if 1 weber of magnetic flux passes through an area of 1 square meter, the flux density is 1 tesla.

For the CGS system, magnetic flux is measured in maxwells and the flux density in gauss. For the English system, magnetic flux is measured in lines and the flux density in lines per square inch. The relationship between such systems is defined in Appendix E.

The flux density of an electromagnet is directly related to the number of turns of, and current through, the coil. The product of the two, called the **magnetomotive force,** is measured in **ampere-turns (At)** as defined by

$$\boxed{\mathscr{F} = NI} \qquad \text{(ampere-turns, At)} \tag{11.3}$$

In other words, if you increase the number of turns around a core and/or increase the current through the coil, the magnetic field strength also increases. In many ways, the magnetomotive force for magnetic circuits is similar to the applied voltage in an electric circuit. Increasing either one results in an increase in the desired effect: magnetic flux for magnetic circuits and current for electric circuits.

For the CGS system, the magnetomotive force is measured in gilberts, while for the English system, it is measured in ampere-turns.

Another factor that affects the magnetic field strength is the type of core used. Materials in which magnetic flux lines can readily be set up are said to be **magnetic** and to have a high **permeability.** Again, note the similarity with the word "permit" used to describe permittivity for the dielectrics of capacitors. Similarly, the permeability (represented by the Greek letter *mu,* μ) of a material is a measure of the ease with which magnetic flux lines can be established in the material.

Just as there is a specific value for the permittivity of air, there is a specific number associated with the permeability of air:

$$\boxed{\mu_o = 4\pi \times 10^{-7} \text{ Wb/A} \cdot \text{m}} \tag{11.4}$$

Practically speaking, the permeability of all nonmagnetic materials, such as copper, aluminum, wood, glass, and air, is the same as that for free space. Materials that have permeabilities slightly less than that of free space are said to be **diamagnetic,** and those with permeabilities slightly greater than that of free space are said to be **paramagnetic.** Magnetic materials, such as iron, nickel, steel, cobalt, and alloys of these metals, have permeabilities hundreds and even thousands of times that of free space. Materials with these very high permeabilities are referred to as **ferromagnetic.**

The ratio of the permeability of a material to that of free space is called its **relative permeability**; that is,

$$\boxed{\mu_r = \frac{\mu}{\mu_o}} \qquad (11.5)$$

In general, for ferromagnetic materials, $\mu_r \geq 100$, and for nonmagnetic materials, $\mu_r = 1$.

A table of values for μ to match the provided table for permittivity levels of specific dielectrics would be helpful. Unfortunately, such a table cannot be provided because *relative permeability is a function of the operating conditions.* If you change the magnetomotive force applied, the level of μ can vary between extreme limits. At one level of magnetomotive force, the permeability of a material can be 10 times that at another level.

An instrument designed to measure flux density in milligauss (CGS system) appears in Fig. 11.14. The meter has two sensitivities, 0.5 to 100 milligauss at 60 Hz and 0.2 to 3 milligauss at 60 Hz. It can be used to measure the electric field strength discussed in Chapter 10 on switching to the ELECTRIC setting. The top scale will then provide a reading in kilovolts/meter. (As an aside, the meter of Fig. 11.14 has appeared in TV programs as a device for detecting a "paranormal" response.) Appendix E reveals that $1\ \text{T} = 10^4$ gauss. The magnitude of the reading of 20 milligauss would be equivalent to

$$20\ \text{milligauss}\left(\frac{1\ \text{T}}{10^4\ \text{gauss}}\right) = 2\ \mu\text{T}$$

FIG. 11.14
Milligaussmeter.
(Courtesy of AlphaLab, Inc.)

Although our emphasis in this chapter is to introduce the parameters that affect the nameplate data of an inductor, the use of magnetics has widespread application in the electrical/electronics industry, as shown by a few areas of application in Fig. 11.15.

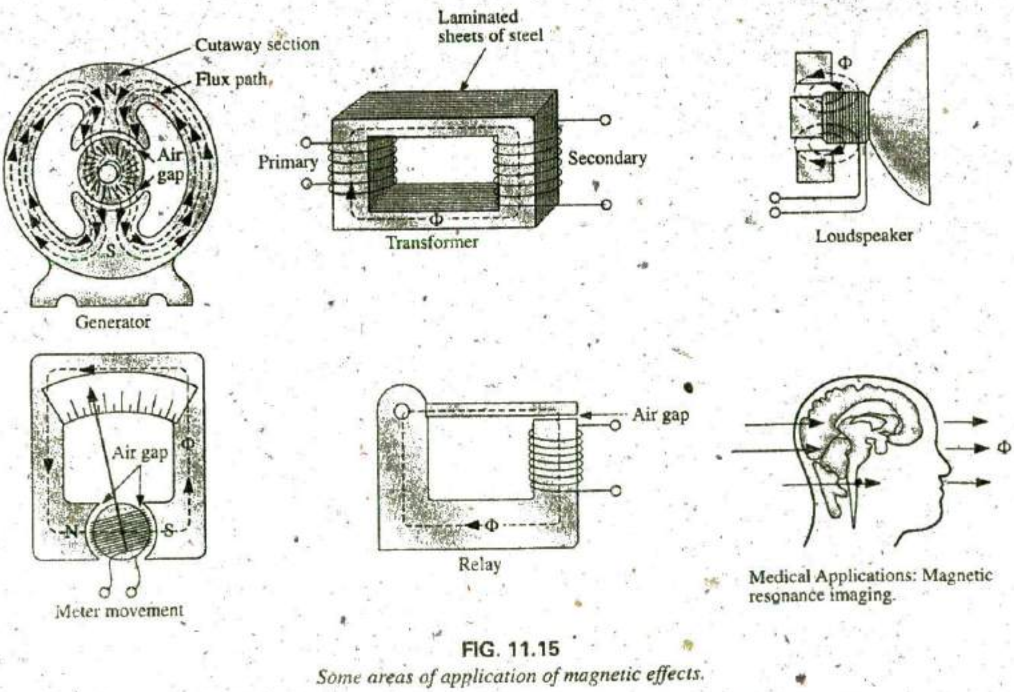


FIG. 11.15
Some areas of application of magnetic effects.

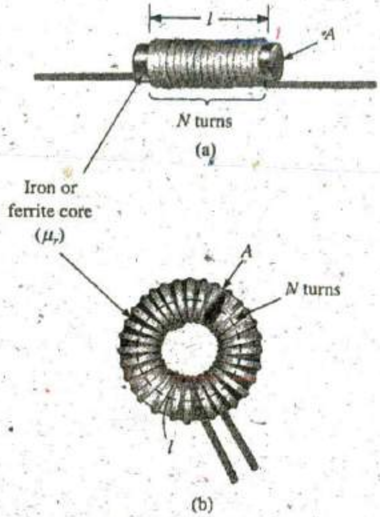


FIG. 11.16
Defining the parameters for Eq. (11.6).

FIG. 11.17
Joseph Henry.
Courtesy of the Smithsonian Institutions, Photo No. 59,054.

American (Albany, NY; Princeton, NJ)
(1797–1878)
Physicist and Mathematician
Professor of Natural Philosophy,
Princeton University

In the early 1800s the title Professor of Natural Philosophy was applied to educators in the sciences. As a student and teacher at the Albany Academy, Henry performed extensive research in the area of electromagnetism. He improved the design of *electromagnets* by insulating the coil wire to permit a tighter wrap on the core. One of his earlier designs was capable of lifting 3600 pounds. In 1832 he discovered and delivered a paper on *self-induction*. This was followed by the construction of an effective *electric telegraph transmitter and receiver* and extensive research on the oscillatory nature of lightning and discharges from a *Leyden jar*. In 1845 he was appointed the first Secretary of the Smithsonian.

11.3 INDUCTANCE

In the previous section, we learned that sending a current through a coil of wire, with or without a core, establishes a magnetic field through and surrounding the unit. This component, of rather simple construction (see Fig. 11.16), is called an **inductor** (often referred to as a **coil**). Its **inductance** level determines the strength of the magnetic field around the coil due to an applied current. The higher the inductance level, the greater is the strength of the magnetic field. In total, therefore,

inductors are designed to set up a strong magnetic field linking the unit, whereas capacitors are designed to set up a strong electric field between the plates.

Inductance is measured in **henries (H),** after the American physicist Joseph Henry (Fig. 11.17). However, just as the farad is too large a unit for most applications, most inductors are of the millihenry (mH) or microhenry (μH) range.

In Chapter 10, 1 farad was defined as a capacitance level that would result in 1 coulomb of charge on the plates due to the application of 1 volt across the plates. For inductors,

1 henry is the inductance level that will establish a voltage of 1 volt across the coil due to a change in current of 1 A/s through the coil.

Inductor Construction

In Chapter 10, we found that capacitance is sensitive to the area of the plates, the distance between the plates, and the dielectric employed. The level of inductance has similar construction sensitivities in that it is dependent on the area within the coil, the length of the unit, and the permeability of the core material. It is also sensitive to the number of turns of wire in the coil as dictated by the following equation and defined in Fig. 11.16 for two of the most popular shapes:

$$\boxed{L = \frac{\mu N^2 A}{l}} \qquad \begin{aligned} \mu &= \text{permeability (Wb/A}\cdot\text{m)} \\ N &= \text{number of turns (t)} \\ A &= \text{m}^2 \\ l &= \text{m} \\ L &= \text{henries (H)} \end{aligned} \qquad \textbf{(11.6)}$$

First note that since the turns are squared in the equation, the number of turns is a big factor. However, also keep in mind that the more turns, the bigger is the unit. If the wire is made too thin to get more windings on the core, the rated current of the inductor is limited. Since higher levels of permeability result in higher levels of magnetic flux, permeability should, and does, appear in the numerator of the equation. Increasing the area of the core or decreasing the length also increases the inductance level.

Substituting $\mu = \mu_r\mu_o$ for the permeability results in the following equation, which is very similar to the equation for the capacitance of a capacitor:

$$L = \frac{\mu_r \mu_o N^2 A}{l}$$

or

$$\boxed{L = 4\pi \times 10^{-7}\frac{\mu_r N^2 A}{l}} \qquad \text{(henries, H)} \qquad \textbf{(11.7)}$$

If we break out the relative permeability as

$$L = \mu_r\left(\frac{\mu_o N^2 A}{l}\right)$$

we obtain the following useful equation:

$$\boxed{L = \mu_r L_o} \tag{11.8}$$

which is very similar to the equation $C = \epsilon_r C_o$. Eq. (11.8) states the following:

The inductance of an inductor with a ferromagnetic core is μ_r times the inductance obtained with an air core.

Although Eq. (11.6) is approximate at best, the equations for the inductance of a wide variety of coils can be found in reference handbooks. Most of the equations are mathematically more complex than Eq. (11.6), but the impact of each factor is the same in each equation.

EXAMPLE 11.1 For the air-core coil in Fig. 11.18:

a. Find the inductance.
b. Find the inductance if a metallic core with $\mu_r = 2000$ is inserted in the coil.

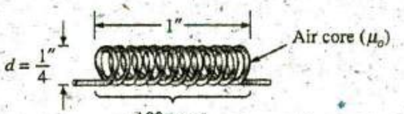


FIG. 11.18
Air-core coil for Example 11.1.

Solutions:

a. $d = \frac{1}{4}\text{in.}\left(\frac{1\text{ m}}{39.37\text{ in.}}\right) = 6.35\text{ mm}$

$$A = \frac{\pi d^2}{4} = \frac{\pi(6.35\text{ mm})^2}{4} = 31.7\ \mu\text{m}^2$$

$$l = 1\text{ in.}\left(\frac{1\text{ m}}{39.37\text{ in.}}\right) = 25.4\text{ mm}$$

$$L = 4\pi \times 10^{-7}\frac{\mu_r N^2 A}{l}$$

$$= 4\pi \times 10^{-7}\frac{(1)(100\text{ t})^2(31.7\ \mu\text{m}^2)}{25.4\text{ mm}} = \mathbf{15.68\ \mu H}$$

b. Eq. (11.8): $L = \mu_r L_o = (2000)(15.68\ \mu\text{H}) = \mathbf{31.36\ mH}$

EXAMPLE 11.2 In Fig. 11.19, if each inductor in the left column is changed to the type appearing in the right column, find the new inductance level. For each change, assume that the other factors remain the same.

Solutions:

a. The only change was the number of turns, but it is a squared factor, resulting in

$$L = (2)^2 L_o = (4)(20\ \mu\text{H}) = \mathbf{80\ \mu H}$$

b. In this case, the area is three times the original size, and the number of turns is 1/2. Since the area is in the numerator, it increases the

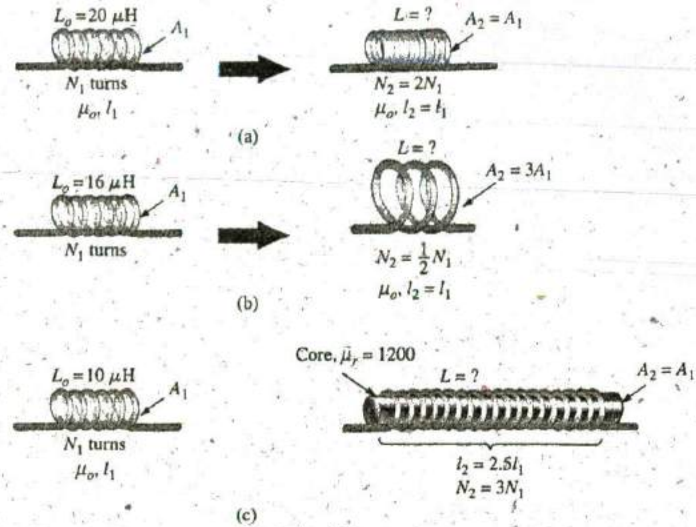


FIG. 11.19
Inductors for Example 11.2.

inductance by a factor of three. The drop in the number of turns reduces the inductance by a factor of $(1/2)^2 = 1/4$. Therefore,

$$L = (3)\left(\frac{1}{4}\right)L_o = \frac{3}{4}(16\ \mu\text{H}) = \mathbf{12\ \mu H}$$

c. Both μ and the number of turns have increased, although the increase in the number of turns is squared. The increased length reduces the inductance. Therefore,

$$L = \frac{(3)^2(1200)}{2.5}L_o = (4.32 \times 10^3)(10\ \mu\text{H}) = \mathbf{43.2\ mH}$$

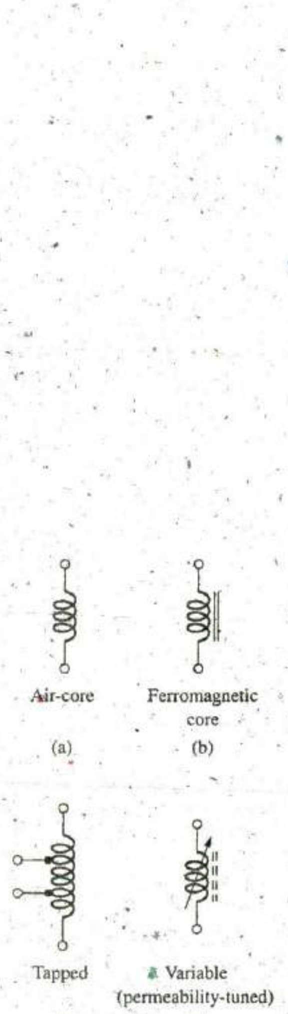


FIG. 11.20
Inductor (coil) symbols.

Types of Inductors

Inductors, like capacitors and resistors, can be categorized under the general headings **fixed** or **variable.** The symbol for a fixed air-core inductor is provided in Fig. 11.20(a), for an inductor with a ferromagnetic core in Fig. 11.20(b), for a tapped coil in Fig. 11.20(c), and for a variable inductor in Fig. 11.20(d).

Fixed Fixed-type inductors come in all shapes and sizes. However,

in general, the size of an inductor is determined primarily by the type of construction, the core used, and the current rating.

In Fig. 11.21(a), the 10 μH and 1 mH coils are about the same size because a thinner wire was used for the 1 mH coil to permit more turns in the same space. The result, however, is a drop in rated current from 10 A to only 1.3 A. If the wire of the 10 μH coil had been used to make the 1 mH coil, the resulting coil would have been many times the size of the 10 μH

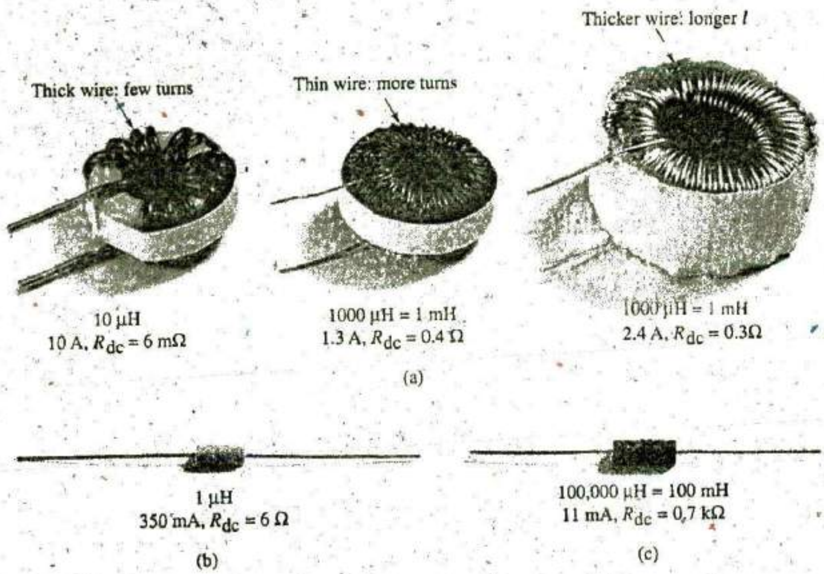


FIG. 11.21
Relative sizes of different types of inductors: (a) toroid, high-current; (b) phenolic (resin or plastic core); (c) ferrite core.

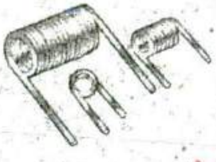
coil. The impact of the wire thickness is clearly revealed by the 1 mH coil at the far right in Fig. 11.21(a), where a thicker wire was used to raise the rated current level from 1.3 A to 2.4 A. Even though the inductance level is the same, the size of the toroid is four to five greater.

The phenolic inductor (using a nonferromagnetic core of resin or plastic) in Fig. 11.21(b) is quite small for its level of inductance. We must assume that it has a high number of turns of very thin wire. Note, however, that the use of a very thin wire has resulted in a relatively low current rating of only 350 mA (0.35 A). The use of a ferrite (ferromagnetic) core in the inductor in Fig. 11.21(c) has resulted in an amazingly high level of inductance for its size. However, the wire is so thin that the current rating is only 11 mA = 0.011 A. Note that for all the inductors, the dc resistance of the inductor increases with a decrease in the thickness of the wire. The 10 μH toroid has a dc resistance of only 6 mΩ, whereas the dc resistance of the 100 mH ferrite inductor is 700 Ω—a price to be paid for the smaller size and high inductance level.

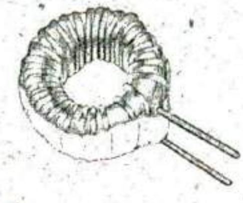
Different types of fixed inductive elements are displayed in Fig. 11.22, including their typical range of values and common areas of application. Based on the earlier discussion of inductor construction, it is fairly easy to identify an inductive element. The shape of a molded film resistor is similar to that of an inductor. However, careful examination of the typical shapes of each reveals some differences, such as the ridges at each end of a resistor that do not appear on most inductors.

Variable A number of variable inductors are depicted in Fig. 11.23. In each case, the inductance is changed by turning the slot at the end of the core to move it in and out of the unit. The farther in the core is, the more the ferromagnetic material is part of the magnetic circuit, and the higher is the magnetic field strength and the inductance level.

Type: Air-core inductors (1–32 turns)
Typical values: 2.5 nH–1 μH
Applications: High-frequency applications

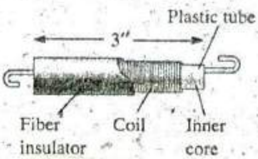


Type: Toroid coil
Typical values: 10μH–30 mH
Applications: Used as a choke in ac power line circuits to filter transient and reduce EMI interference. This coil is found in many electronic appliances.



Type: Hash choke coil
Typical values: 3 μH–1 mH
Applications: Used in ac supply lines that deliver high currents.

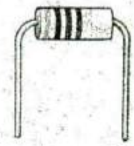
Type: Delay line coil
Typical values: 10 μH–50 μH
Applications: Used in color televisions to correct for timing differences between the color signal and the black-and-white signal.



Type: Common-mode choke coil
Typical values: 0.6 mH–50 mH
Applications: Used in ac line filters, switching power supplies, battery chargers, and other electronic equipment.

Type: RF chokes
Typical values: 10 μH–470 mH
Applications: Used in radio, television, and communication circuits. Found in AM, FM, and UHF circuits.

Type: Molded coils
Typical values: 0.1 μH–100 mH
Applications: Used in a wide variety of circuits such as oscillators, filters, pass-band filters, and others,



Type: Surface-mount inductors
Typical values: 0.01 μH–250 μH
Applications: Found in many electronic circuits that require miniature components on multilayered PCBs (printed circuit boards).

FIG. 11.22
Typical areas of application for inductive elements.

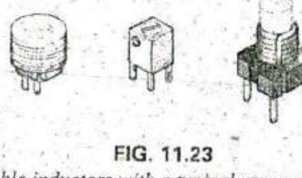


FIG. 11.23
Variable inductors with a typical range of values from 1 μH to 100 μH; commonly used in oscillators and various RF circuits such as CB transceivers, televisions, and radios.

Practical Equivalent Inductors

Inductors, like capacitors, are not ideal. Associated with every inductor is a resistance determined by the resistance of the turns of wire (the thinner the wire, the greater is the resistance for the same material) and by the core losses (radiation and skin effect, eddy current and hysteresis losses—all discussed in more advanced texts). There is also some stray capacitance due to the capacitance between the current-carrying turns of wire of the coil. Recall that capacitance appears whenever there are two conducting surfaces separated by an insulator, such as air, and when those wrappings are fairly tight and are parallel. Both elements are included in the equivalent circuit in Fig. 11.24. For most applications in this text, the capacitance can be ignored, resulting in the equivalent model in Fig. 11.25. The resistance R_l plays an important part in some areas (such as resonance, discussed in Chapter 20)

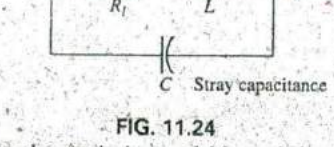


FIG. 11.24
Complete equivalent model for an inductor.

FIG. 11.25
Practical equivalent model for an inductor.

because the resistance can extend from a few ohms to a few hundred ohms, depending on the construction. For this chapter, the inductor is considered an ideal element, and the series resistance is dropped from Fig. 11.25.

Inductor Labeling

Because some inductors are larger in size, their nameplate value can often be printed on the body of the element. However, on smaller units, there may not be enough room to print the actual value, so an abbreviation is used that is fairly easy to understand. First, realize that the *microhenry* (μH) is the fundamental unit of measurement for this marking. Most manuals list the inductance value in μH even if the value must be reported as 470,000 μH rather than as 470 mH. If the label reads 223K, the third number (3) is the power to be applied to the first two. The K is not from *kilo*, representing a power of three, but is used to denote a tolerance of $\pm 10\%$ as described for capacitors. The resulting number of 22,000 is, therefore, in μH, so the 223K unit is a 22,000 μH or 22 mH inductor. The letters J and M indicate a tolerance of $\pm 5\%$ and $\pm 20\%$, respectively.

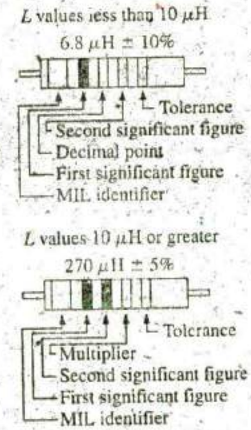
For molded inductors, a color-coding system very similar to that used for resistors is used. The major difference is that *the resulting value is always in μH*, and a wide band at the beginning of the labeling is an MIL ("meets military standards") indicator. Always read the colors in sequence, starting with the band closest to one end as shown in Fig. 11.26.

The standard values for inductors employ the same numerical values and multipliers used with resistors and capacitors. In general, therefore,

Color Code Table

Color[1]	Significant Figure	Multiplier[2]	Inductance Tolerance (%)
Black	0	1	
Brown	1	10	
Red	2	100	
Orange	3	1000	
Yellow	4		
Green	5		
Blue	6		
Violet	7		
Gray	8		
White	9		
None[2]			±20
Silver			±10
Gold	Decimal point		±5

[1] Indicates body color.
[2] The multiplier is the factor by which the two significant figures are multiplied to yield the nominal inductance value.



Cylindrical molded choke coils are marked with five colored bands. A wide silver band, located at one end of the coil, identifies military radio-frequency coils. The next three bands indicate the inductance in microhenries, and the fourth band is the tolerance.

Color coding is in accordance with the color code table, shown on the left. If the first or second band is gold, it represents the decimal point for inductance values less than 10. Then the following two bands are significant figures. For inductance values of 10 or more, the first two bands represent significant figures, and the third is the multiplier.

FIG. 11.26
Molded inductor color coding.

expect to find inductors with the following multipliers: 1 μH, 1.5 μH, 2.2 μH, 3.3 μH, 4.7 μH, 6.8 μH, 10 μH, and so on.

Measurement and Testing of Inductors

The inductance of an inductor can be read directly using a meter such as the Universal LCR Meter (Fig. 11.27), also discussed in Chapter 10 on capacitors. Set the meter to L for inductance, and the meter automatically chooses the most appropriate unit of measurement for the element, that is, H, mH, μH, or pH.

An inductance meter is the best choice, but an ohmmeter can also be used to check whether a short has developed between the windings or whether an open circuit has developed. The open-circuit possibility is easy to check because a reading of infinite ohms or very high resistance results. The short-circuit condition is harder to check because the resistance of many good inductors is relatively small, and the shorting of a few windings may not adversely affect the total resistance. Of course, if you are aware of the typical resistance of the coil, you can compare it to the measured value. A short between the windings and the core can be checked by simply placing one lead of the ohmmeter on one wire (perhaps a terminal) and the other on the core itself. A reading of zero ohms reveals a short between the two that may be due to a breakdown in the insulation jacket around the wire resulting from excessive currents, environmental conditions, or simply old age and cracking.

FIG. 11.27
Digital reading inductance meter.
(Courtesy of B+K Precision.)

11.4 INDUCED VOLTAGE v_L

Before analyzing the response of inductive elements to an applied dc voltage, we must introduce a number of laws and equations that affect the transient response.

The first, referred to as **Faraday's law of electromagnetic induction,** is one of the most important in this field because it enables us to establish ac and dc voltages with a generator. If we move a conductor (any material with conductor characteristics as defined in Chapter 2) through a magnetic field so that it cuts magnetic lines of flux as shown in Fig. 11.28, a voltage is induced across the conductor that can be measured with a sensitive voltmeter. That's all it takes, and, in fact, the faster you move the conductor through the magnetic flux, the greater is the induced voltage. The same effect can be produced if you hold the conductor still and move the magnetic field across the conductor. Note that the direction in which you move the conductor through the field determines the polarity of the induced voltage. Also, if you move the conductor through the field at right angles to the magnetic flux, you generate the maximum induced voltage. Moving the conductor parallel with the magnetic flux lines results in an induced voltage of zero volts since magnetic lines of flux are not crossed.

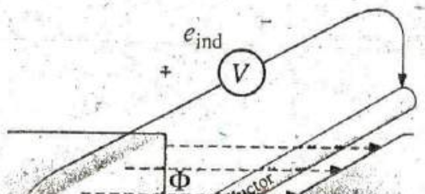


FIG. 11.28
Generating an induced voltage by moving a conductor through a magnetic field.

If we now go a step further and move a coil of N turns through the magnetic field as shown in Fig. 11.29, a voltage will be induced across the coil as determined by **Faraday's law:**

$$\boxed{e = N\frac{d\phi}{dt}} \qquad \text{(volts, V)} \tag{11.9}$$

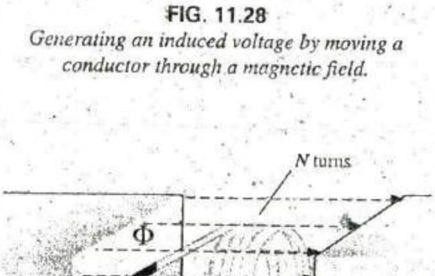


FIG. 11.29
Demonstrating Faraday's law.

The greater the number of turns or the faster the coil is moved through the magnetic flux pattern, the greater is the induced voltage. The term

$d\phi/dt$ is the differential change in magnetic flux through the coil at a particular instant in time. If the magnetic flux passing through a coil remains constant—no matter how strong the magnetic field—the term will be zero, and the induced voltage zero volts. It doesn't matter whether the changing flux is due to moving the magnetic field or moving the coil in the vicinity of a magnetic field. The only requirement is that the flux linking (passing through) the coil changes with time. Before the coil passes through the magnetic poles, the induced voltage is zero because there are no magnetic flux lines passing through the coil. As the coil enters the flux pattern, the number of flux lines cut per instant of time increases until it peaks at the center of the poles. The induced voltage then decreases with time as it leaves the magnetic field.

This important phenomenon can now be applied to the inductor in Fig. 11.30, which is simply an extended version of the coil in Fig. 11.29. In Section 11.2, we found that the magnetic flux linking the coil of N turns with a current I has the distribution shown in Fig. 11.30. If the current through the coil increases in magnitude, the flux linking the coil also increases. We just learned through Faraday's law, however, that a coil in the vicinity of a changing magnetic flux will have a voltage induced across it. The result is that a voltage is induced across the coil in Fig. 11.30 due to the *change in current through the coil.*

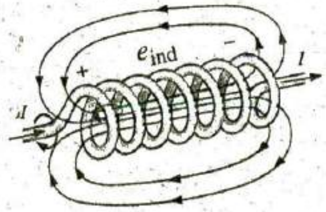


FIG. 11.30
Demonstrating the effect of Lenz's law.

It is very important to note in Fig. 11.30 that the polarity of the induced voltage across the coil is such that it opposes the increasing level of current in the coil. In other words, the changing current through the coil induces a voltage across the coil that is opposing the applied voltage that establishes the increase in current in the first place. The quicker the change in current through the coil, the greater is the opposing induced voltage to squelch the attempt of the current to increase in magnitude. The "choking" action of the coil is the reason inductors or coils are often referred to as **chokes.** This effect is a result of an important law referred to as **Lenz's law,** which states that

an induced effect is always such as to oppose the cause that produced it.

The inductance of a coil is also a measure of the change in flux linking the coil due to a change in current through the coil. That is,

$$\boxed{L = N\frac{d\phi}{di_L}} \qquad \text{(henries, H)} \qquad \textbf{(11.10)}$$

The equation reveals that the greater the number of turns or the greater the change in flux linking the coil due to a particular change in current, the greater is the level of inductance. In other words, coils with smaller levels of inductance generate smaller changes in flux linking the coil for the same change in current through the coil. If the inductance level is very small, there will be almost no change in flux linking the coil, and the induced voltage across the coil will be very small. In fact, if we now write Eq. (11.9) in the form

$$e = N\frac{d\phi}{dt} = \left(N\frac{d\phi}{di_L}\right)\left(\frac{di_L}{dt}\right)$$

and substitute Eq. (11.10), we obtain

$$\boxed{e_L = L\frac{di_L}{dt}} \qquad \text{(volts, V)} \qquad \textbf{(11.11)}$$

which relates the voltage across a coil to the number of turns of the coil and the change in current through the coil.

When induced effects are used in the generation of voltages such as those from dc or ac generators, the symbol e is applied to the induced voltage. However, in network analysis, the voltage induced across an inductor will always have a polarity that opposes the applied voltage (like the voltage across a resistor). Therefore, the following notation is used for the induced voltage across an inductor:

$$\boxed{v_L = L\frac{di_L}{dt}} \qquad \text{(volts, V)} \qquad \textbf{(11.12)}$$

The equation clearly states that

the larger the inductance and/or the more rapid the change in current through a coil, the larger will be the induced voltage across the coil.

If the current through the coil fails to change with time, the induced voltage across the coil will be zero. We will find in the next section that for dc applications, when the transient phase has passed, $di_L/dt = 0$, and the induced voltage across the coil is

$$v_L = L\frac{di_L}{dt} = L(0) = 0\text{ V}$$

The duality that exists between inductive and capacitive elements is now abundantly clear. Simply interchange the voltages and currents of Eq. (11.12), and interchange the inductance and capacitance. The following equation for the current of a capacitor results:

$$v_L = L\frac{di_L}{dt}$$

$$i_C = C\frac{dv_C}{dt}$$

We are now at a point where we have all the background relationships necessary to investigate the transient behavior of inductive elements.

11.5 *R-L* TRANSIENTS: THE STORAGE PHASE

A great number of similarities exist between the analyses of inductive and capacitive networks. That is, what is true for the voltage of a capacitor is also true for the current of an inductor, and what is true for the current of a capacitor can be matched in many ways by the voltage of an inductor. The storage waveforms have the same shape, and time constants are defined for each configuration. Because these concepts are so similar (refer to Section 10.5 on the charging of a capacitor), you have an opportunity to reinforce concepts introduced earlier and still learn more about the behavior of inductive elements.

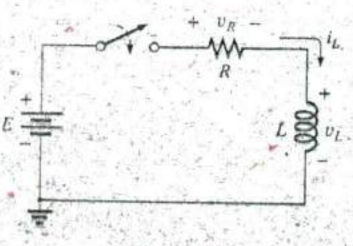


FIG. 11.31
Basic R-L transient network.

The circuit in Fig. 11.31 is used to describe the storage phase. Note that it is the same circuit used to describe the charging phase of capacitors, with a simple replacement of the capacitor by an ideal inductor. Throughout the analysis, it is important to remember that energy is stored in the form of an electric field between the plates of a capacitor. For inductors, on the other hand, energy is stored in the form of a magnetic field linking the coil.

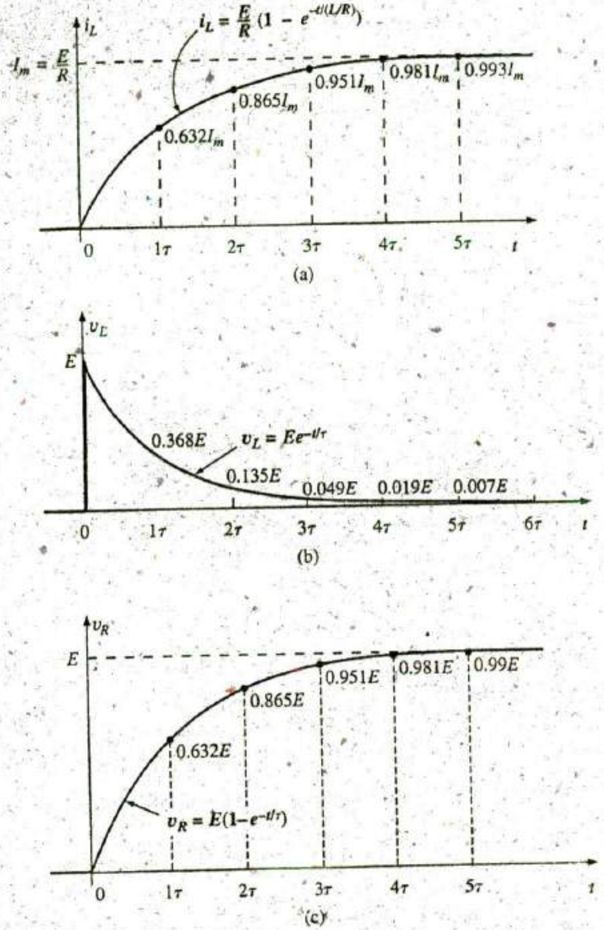


FIG. 11.32

i_L, v_L, and v_R for the circuit in Fig. 11.31 following the closing of the switch.

At the instant the switch is closed, the choking action of the coil prevents an instantaneous change in current through the coil, resulting in $i_L = 0$ A, as shown in Fig. 11.32(a). The absence of a current through the coil and circuit at the instant the switch is closed results in zero volts across the resistor as determined by $v_R = i_R R = i_L R = (0\text{ A})R = 0$ V, as shown in Fig. 11.32(c). Applying Kirchhoff's voltage law around the closed loop results in E volts across the coil at the instant the switch is closed, as shown in Fig. 11.32(b).

Initially, the current increases very rapidly, as shown in Fig. 11.32(a) and then at a much slower rate as it approaches its steady-state value determined by the parameters of the network (E/R). The voltage across the resistor rises at the same rate because $v_R = i_R R = i_L R$. Since the voltage across the coil is sensitive to the rate of change of current through the coil, the voltage will be at or near its maximum value early in the storage phase. Finally, when the current reaches its steady-state value of E/R amperes, the current through the coil ceases to change, and the voltage across the coil drops to zero volts. At any instant of time, the voltage across the coil can be determined using Kirchhoff's voltage law in the following manner: $v_L = E - v_R$.

Because the waveforms for the inductor have the same shape as obtained for capacitive networks, we are familiar with the mathematical format and can feel comfortable calculating the quantities of interest using a calculator or computer.

The equation for the transient response of the current through an inductor is

$$\boxed{i_L = \frac{E}{R}(1 - e^{-t/\tau})} \qquad \text{(amperes, A)} \qquad \textbf{(11.13)}$$

with the time constant now defined by

$$\boxed{\tau = \frac{L}{R}} \qquad \text{(seconds, s)} \qquad \textbf{(11.14)}$$

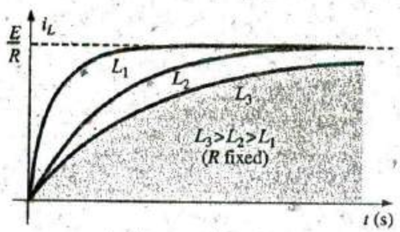


FIG. 11.33
Effect of L on the shape of the i_L storage waveform.

Note that Eq. (11.14) is a ratio of parameters rather than a product as used for capacitive networks, yet the units used are still seconds (for time).

Our experience with the factor $(1 - e^{-t/\tau})$ verifies the level of 63.2% for the inductor current after one time constant, 86.5% after two time constants, and so on. If we keep R constant and increase L, the ratio L/R increases, and the rise time of 5τ increases as shown in Fig. 11.33 for increasing levels of L. The change in transient response is expected because the higher the inductance level, the greater is the choking action on the changing current level, and the longer it will take to reach steady-state conditions.

The equation for the voltage across the coil is

$$\boxed{v_L = Ee^{-t/\tau}} \qquad \text{(volts, V)} \qquad \textbf{(11.15)}$$

and the equation for the voltage across the resistor is

$$\boxed{v_R = E(1 - e^{-t/\tau})} \qquad \text{(volts, V)} \qquad \textbf{(11.16)}$$

As mentioned earlier, the shape of the response curve for the voltage across the resistor must match that of the current i_L since $v_R = i_R R = i_L R$.

Since the waveforms are similar to those obtained for capacitive networks, we will assume that

the storage phase has passed and steady-state conditions have been established once a period of time equal to five time constants has occurred.

In addition, since $\tau = L/R$ will always have some numerical value, even though it may be very small at times, the transient period of 5τ will always have some numerical value. Therefore,

the current cannot change instantaneously in an inductive network.

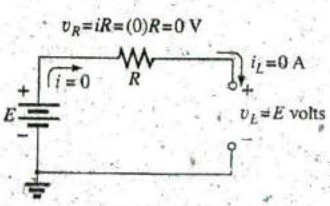


FIG. 11.34
Circuit in Figure 11.31 the instant the switch is closed.

If we examine the conditions that exist at the instant the switch is closed, we find that the voltage across the coil is E volts, although the current is zero amperes as shown in Fig. 11.34. In essence, therefore,

the inductor takes on the characteristics of an open circuit at the instant the switch is closed.

However, if we consider the conditions that exist when steady-state conditions have been established, we find that the voltage across the coil is zero volts and the current is a maximum value of E/R amperes, as shown in Fig. 11.35. In essence, therefore,

the inductor takes on the characteristics of a short circuit when steady-state conditions have been established.

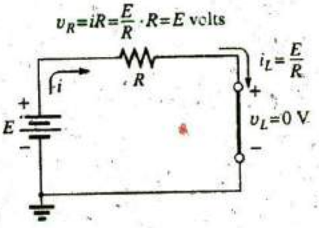


FIG. 11.35
Circuit in Fig. 11.31 under steady-state conditions.

EXAMPLE 11.3 Find the mathematical expressions for the transient behavior of i_L and v_L for the circuit in Fig. 11.36 if the switch is closed at $t = 0$ s. Sketch the resulting curves.

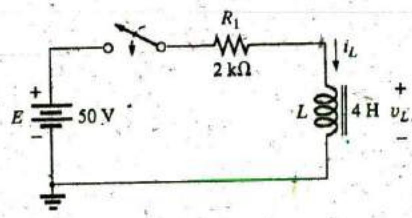


FIG. 11.36
Series R-L circuit for Example 11.3.

Solution: First, we determine the time constant:

$$\tau = \frac{L}{R_1} = \frac{4\text{ H}}{2\text{ k}\Omega} = 2\text{ ms}$$

Then the maximum or steady-state current is

$$I_m = \frac{E}{R_1} = \frac{50\text{ V}}{2\text{ k}\Omega} = 25 \times 10^{-3}\text{A} = 25\text{ mA}$$

Substituting into Eq. (11.13) gives

$$i_L = \mathbf{25\text{ mA}\,(1 - e^{-t/2\text{ ms}})}$$

Using Eq. (11.15) gives

$$v_L = \mathbf{50\text{ V}e^{-t/2\text{ ms}}}$$

The resulting waveforms appear in Fig. 11.37.

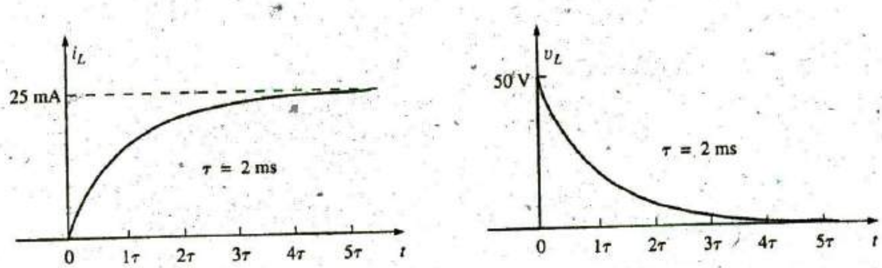


FIG. 11.37
i_L and v_L for the network in Fig. 11.36.

11.6 INITIAL CONDITIONS

This section parallels Section 10.7 on the effect of initial values on the transient phase. Since the current through a coil cannot change instantaneously, the current through a coil begins the transient phase at the initial value established by the network (note Fig. 11.38) before the switch was closed. It then passes through the transient phase until it reaches the steady-state (or final) level after about five time constants. The steady-state level of the inductor current can be found by substituting its short-circuit equivalent (or R_l for the practical equivalent) and finding the resulting current through the element.

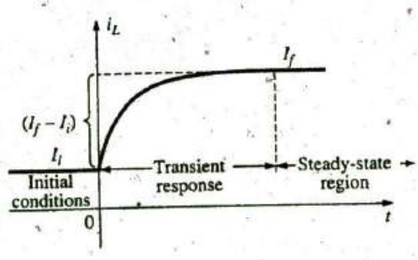


FIG. 11.38
Defining the three phases of a transient waveform.

Using the transient equation developed in the previous section, we can write an equation for the current i_L for the entire time interval in Fig. 11.38; that is,

$$i_L = I_i + (I_f - I_i)(1 - e^{-t/\tau})$$

with $(I_f - I_i)$ representing the total change during the transient phase. However, by multiplying through and rearranging terms as

$$\begin{aligned} i_L &= I_i + I_f - I_f e^{-t/\tau} - I_i + I_i e^{-t/\tau} \\ &= I_f - I_f e^{-t/\tau} + I_i e^{-t/\tau} \end{aligned}$$

we find

$$\boxed{i_L = I_f + (I_i - I_f)e^{-t/\tau}} \qquad (11.17)$$

If you are required to draw the waveform for the current i_L from initial value to final value, start by drawing a line at the initial value and steady-state levels, and then add the transient response (sensitive to the time constant) between the two levels. The following example will clarify the procedure.

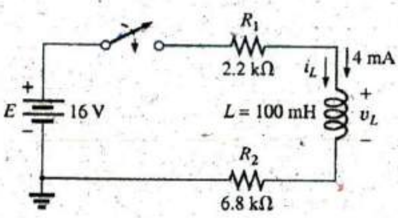


FIG. 11.39
Example 11.4.

EXAMPLE 11.4 The inductor in Fig. 11.39 has an initial current level of 4 mA in the direction shown. (Specific methods to establish the initial current are presented in the sections and problems to follow.)

a. Find the mathematical expression for the current through the coil once the switch is closed.
b. Find the mathematical expression for the voltage across the coil during the same transient period.
c. Sketch the waveform for each from initial value to final value.

Solutions:

a. Substituting the short-circuit equivalent for the inductor results in a final or steady-state current determined by Ohm's law:

$$I_f = \frac{E}{R_1 + R_2} = \frac{16\text{ V}}{2.2\text{ k}\Omega + 6.8\text{ k}\Omega} = \frac{16\text{ V}}{9\text{ k}\Omega} = 1.78\text{ mA}$$

The time constant is determined by

$$\tau = \frac{L}{R_T} = \frac{100\text{ mH}}{2.2\text{ k}\Omega + 6.8\text{ k}\Omega} = \frac{100\text{ mH}}{9\text{ k}\Omega} = 11.11\ \mu\text{s}$$

Applying Eq. (11.17) gives

$$\begin{aligned} i_L &= I_f + (I_i - I_f)e^{-t/\tau} = 1.78\text{ mA} + (4\text{ mA} - 1.78\text{ mA})e^{-t/11.11\ \mu\text{s}} \\ &= \mathbf{1.78\text{ mA} + 2.22\text{ mA}e^{-t/11.11\ \mu\text{s}}} \end{aligned}$$

b. Since the current through the inductor is constant at 4 mA prior to the closing of the switch, the voltage (whose level is sensitive only to changes in current through the coil) must have an initial value of 0 V. At the instant the switch is closed, the current through the coil cannot change instantaneously, so the current through the resistive elements is 4 mA. The resulting peak voltage at $t = 0$ s can then be found using Kirchhoff's voltage law as follows:

$$\begin{aligned} V_m &= E - V_{R_1} - V_{R_2} = 16\text{ V} - (4\text{ mA})(2.2\text{ k}\Omega) - (4\text{ mA})(6.8\text{ k}\Omega) \\ &= 16\text{ V} - 8.8\text{ V} - 27.2\text{ V} = 16\text{ V} - 36\text{ V} = -20\text{ V} \end{aligned}$$

Note the minus sign to indicate that the polarity of the voltage v_L is opposite to the defined polarity of Fig. 11.39.

The voltage then decays (with the same time constant as the current i_L) to zero because the inductor is approaching its short-circuit equivalence.

The equation for v_L is therefore

$$v_L = -20\text{ V}e^{-t/11.11\,\mu s}$$

c. See Fig. 11.40. The initial and final values of the current were drawn first, and then the transient response was included between these levels. For the voltage, the waveform begins and ends at zero, with the peak value having a sign sensitive to the defined polarity of v_L in Fig. 11.39.

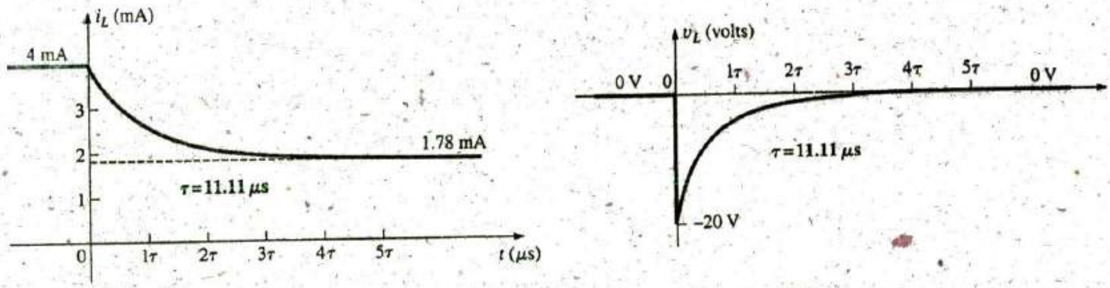


FIG. 11.40
i_L and v_L for the network in Fig. 11.39.

Let us now test the validity of the equation for i_L by substituting t = 0 s to reflect the instant the switch is closed. We have

$$e^{-t/\tau} = e^{-0} = 1$$

and $i_L = 1.78\text{ mA} + 2.22\text{ mA}e^{-t/\tau} = 1.78\text{ mA} + 2.22\text{ mA} = 4\text{ mA}$

When $t > 5\tau$, $\qquad e^{-t/\tau} \cong 0$

and $\quad i_L = 1.78\text{ mA} + 2.22\text{ mA}e^{-t/\tau} = 1.78\text{ mA}$

11.7 *R-L* TRANSIENTS: THE RELEASE PHASE

In the analysis of *R-C* circuits, we found that the capacitor could hold its charge and store energy in the form of an electric field for a period of time determined by the leakage factors. In *R-L* circuits, the energy is stored in the form of a magnetic field established by the current through the coil. Unlike the capacitor, however, an isolated inductor cannot continue to store energy because the absence of a closed path causes the current to drop to zero, releasing the energy stored in the form of a magnetic field. If the series *R-L* circuit in Fig. 11.41 reaches steady-state conditions and the switch is quickly opened, a spark will occur across the contacts due to the rapid change in current from a maximum of E/R to zero amperes. The change in current di/dt of the equation $v_L = L(di/dt)$ establishes a high voltage v_L across the coil that, in conjunction with the applied voltage E, appears across the points of the switch. This is the same mechanism used in the ignition system of a car to ignite the fuel in the cylinder. Some 25,000 V are generated by the rapid decrease in ignition

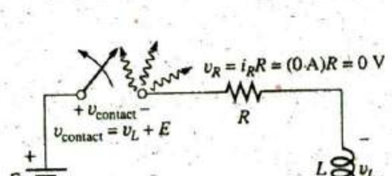


FIG. 11.41
Demonstrating the effect of opening a switch in series with an inductor with a steady-state current.

coil current that occurs when the switch in the system is opened. (In older systems, the "points" in the distributor served as the switch.) This inductive reaction is significant when you consider that the only independent source in a car is a 12 V battery.

If opening the switch to move it to another position causes such a rapid discharge in stored energy, how can the decay phase of an *R-L* circuit be analyzed in much the same manner as for the *R-C* circuit? The solution is to use a network like that in Fig. 11.42(a). When the switch is closed, the voltage across resistor R_2 is E volts, and the *R-L* branch responds in the same manner as described above, with the same waveforms and levels. A Thévenin network of E in parallel with R_2 results in the source as shown in Fig. 11.42(b) since R_2 will be shorted out by the short-circuit replacement of the voltage source E when the Thévenin resistance is determined.

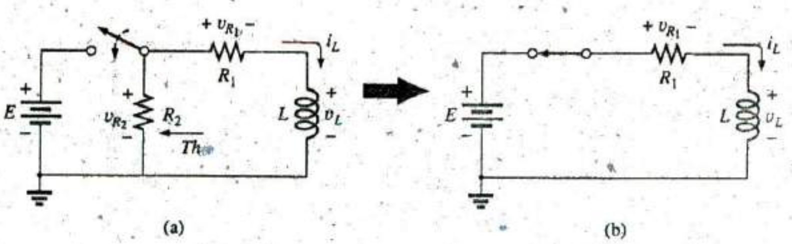


FIG. 11.42
Initiating the storage phase for an inductor by closing the switch.

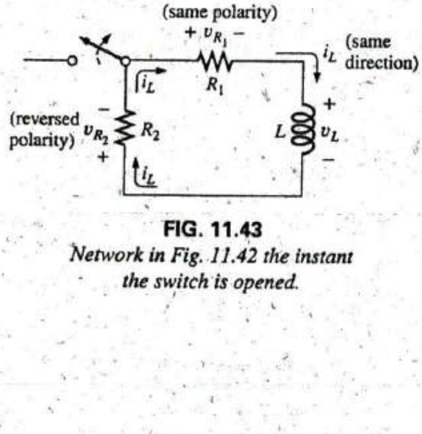


FIG. 11.43
Network in Fig. 11.42 the instant the switch is opened.

After the storage phase has passed and steady-state conditions are established, the switch can be opened without the sparking effect or rapid discharge due to resistor R_2, which provides a complete path for the current i_L. In fact, for clarity the discharge path is isolated in Fig. 11.43. The voltage v_L across the inductor reverses polarity and has a magnitude determined by

$$\boxed{v_L = -(v_{R_1} + v_{R_2})} \qquad \textbf{(11.18)}$$

Recall that the voltage across an inductor can change instantaneously but the current cannot. The result is that the current i_L must maintain the same direction and magnitude, as shown in Fig. 11.43. Therefore, the instant after the switch is opened, i_L is still $I_m = E/R_1$, and

$$\begin{aligned} v_L &= -(v_{R_1} + v_{R_2}) = -(i_1R_1 + i_2R_2) \\ &= -i_L(R_1 + R_2) = -\frac{E}{R_1}(R_1 + R_2) = -\left(\frac{R_1}{R_1} + \frac{R_2}{R_1}\right)E \end{aligned}$$

and
$$\boxed{v_L = -\left(1 + \frac{R_2}{R_1}\right)E} \qquad \text{switch opened} \qquad \textbf{(11.19)}$$

which is bigger than E volts by the ratio R_2/R_1. In other words, when the switch is opened, the voltage across the inductor reverses polarity and drops instantaneously from E to $-[1 + (R_2/R_1)]E$ volts.

As an inductor releases its stored energy, the voltage across the coil decays to zero in the following manner:

$$\boxed{v_L = -V_i e^{-t/\tau'}} \tag{11.20}$$

with $$V_i = \left(1 + \frac{R_2}{R_1}\right)E$$

and $$\tau' = \frac{L}{R_T} = \frac{L}{R_1 + R_2}$$

The current decays from a maximum of $I_m = E/R_1$ to zero.

Using Eq. (11.17) gives

$$I_i = \frac{E}{R_1} \qquad \text{and} \qquad I_f = 0 \text{ A}$$

so that $$i_L = I_f + (I_i - I_f)e^{-t/\tau'} = 0 \text{ A} + \left(\frac{E}{R_1} - 0 \text{ A}\right)e^{-t/\tau'}$$

and $$\boxed{i_L = \frac{E}{R_1}e^{-t/\tau'}} \tag{11.21}$$

with $$\tau' = \frac{L}{R_1 + R_2}$$

The mathematical expression for the voltage across either resistor can then be determined using Ohm's law:

$$v_{R_1} = i_{R_1}R_1 = i_L R_1 = \frac{E}{R_1} R_1 e^{-t/\tau'}$$

and $$\boxed{v_{R_1} = Ee^{-t/\tau'}} \tag{11.22}$$

The voltage v_{R_1} has the same polarity as during the storage phase since the current i_L has the same direction. The voltage v_{R_2} is expressed as follows using the defined polarity of Fig. 11.42:

$$v_{R_2} = -i_{R_2}R_2 = -i_L R_2 = -\frac{E}{R_1} R_2 e^{-t/\tau'}$$

and $$\boxed{v_{R_2} = -\frac{R_2}{R_1} Ee^{-t/\tau'}} \tag{11.23}$$

EXAMPLE 11.5 Resistor R_2 was added to the network in Fig. 11.36 as shown in Fig. 11.44.

a. Find the mathematical expressions for i_L, v_L, v_{R_1}, and v_{R_2} for five time constants of the storage phase.
b. Find the mathematical expressions for i_L, v_L, v_{R_1}, and v_{R_2} if the switch is opened after five time constants of the storage phase.
c. Sketch the waveforms for each voltage and current for both phases covered by this example. Use the defined polarities in Fig. 11.43.

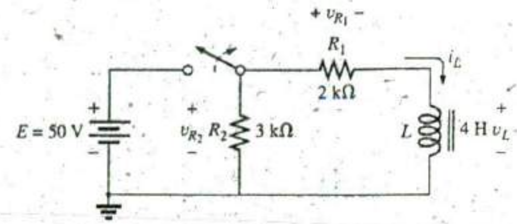


FIG. 11.44
Defined polarities for v_{R_1}, v_{R_2}, v_L, and current direction for i_L for Example 11.5.

Solutions:

a. From Example 11.3:

$$i_L = \mathbf{25\ mA\,(1 - e^{-t/2\ ms})}$$
$$v_L = \mathbf{50\ Ve^{-t/2\ ms}}$$
$$v_{R_1} = i_{R_1}R_1 = i_L R_1 = \left[\frac{E}{R_1}(1 - e^{-t/\tau})\right]R_1 = E(1 - e^{-t/\tau})$$

and
$$v_{R_1} = \mathbf{50\ V(1 - e^{-t/2\ ms})}$$
$$v_{R_2} = E = \mathbf{50\ V}$$

b. $\tau' = \dfrac{L}{R_1 + R_2} = \dfrac{4\ \text{H}}{2\ \text{k}\Omega + 3\ \text{k}\Omega} = \dfrac{4\ \text{H}}{5 \times 10^3\ \Omega}$
$= 0.8 \times 10^{-3}\ \text{s} = 0.8\ \text{ms}$

By Eqs. (11.19) and (11.20):

$$V_i = \left(1 + \frac{R_2}{R_1}\right)E = \left(1 + \frac{3\ \text{k}\Omega}{2\ \text{k}\Omega}\right)(50\ \text{V}) = 125\ \text{V}$$

and
$$v_L = -V_i e^{-t/\tau'} = \mathbf{-125\ Ve^{-t/-0.8\ ms}}$$

By Eq. (11.21):

$$I_m = \frac{E}{R_1} = \frac{50\ \text{V}}{2\ \text{k}\Omega} = 25\ \text{mA}$$

and
$$i_L = I_m e^{-t/\tau'} = \mathbf{25\ mAe^{-t/0.8\ ms}}$$

By Eq. (11.22):

$$v_{R_1} = Ee^{-t/\tau'} = \mathbf{50\ Ve^{-t/0.8\ ms}}$$

By Eq. (11.23):

$$v_{R_2} = -\frac{R_2}{R_1}Ee^{-t/\tau'} = -\frac{3\ \text{k}\Omega}{2\ \text{k}\Omega}(50\ \text{V})e^{-t/\tau'} = \mathbf{-75\ Ve^{-t/0.8\ ms}}$$

c. See Fig. 11.45.

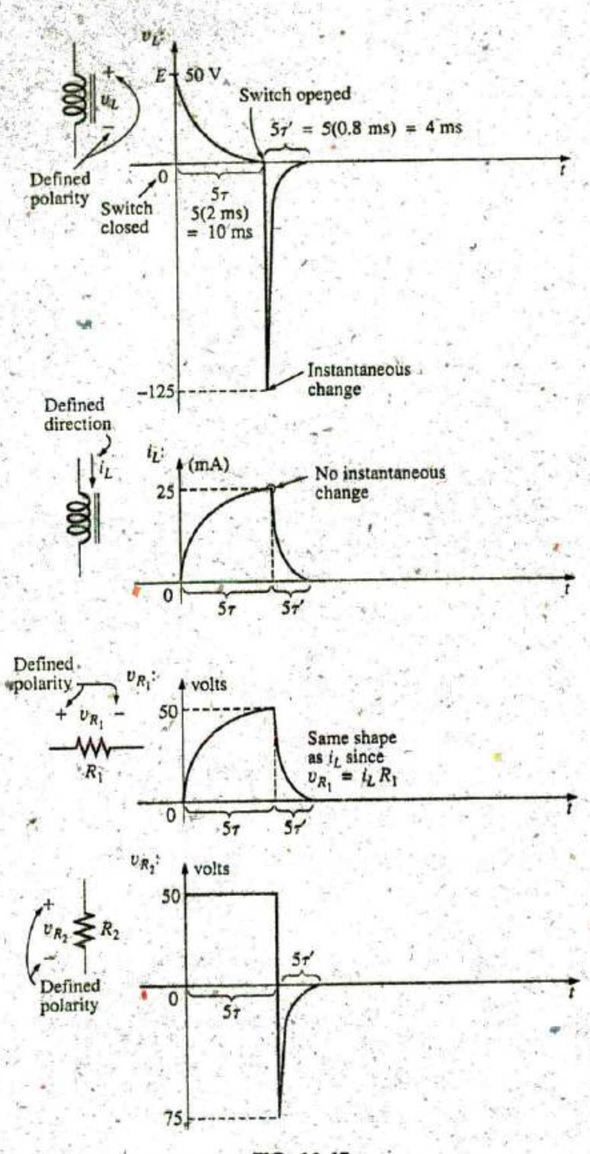


FIG. 11.45
The various voltages and the current for the network in Fig. 11.44.

In the preceding analysis, it was assumed that steady-state conditions were established during the charging phase and $I_m = E/R_1$, with $v_L = 0$ V. However, if the switch in Fig. 11.42 is opened before i_L reaches its maximum value, the equation for the decaying current of Fig. 11.42 must change to

$$\boxed{i_L = I_i e^{-t/\tau'}} \qquad \textbf{(11.24)}$$

where I_i is the starting or initial current. The voltage across the coil is defined by the following:

$$\boxed{v_L = -V_i e^{-t/\tau'}} \tag{11.25}$$

with $$V_i = I_i(R_1 + R_2)$$

11.8 THÉVENIN EQUIVALENT: $\tau = L/R_{Th}$

In Chapter 10 on capacitors, we found that a circuit does not always have the basic form in Fig. 11.31. The solution is to find the Thévenin equivalent circuit before proceeding in the manner described in this chapter. Consider the following example.

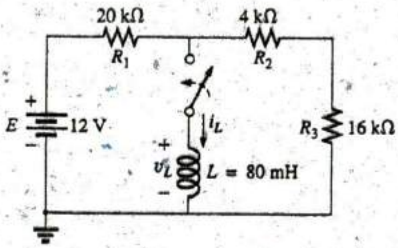


FIG. 11.46
Example 11.6.

EXAMPLE 11.6 For the network in Fig. 11.46:

a. Find the mathematical expression for the transient behavior of the current i_L and the voltage v_L after the closing of the switch (I_i = 0 mA).
b. Draw the resultant waveform for each.

Solutions:

a. Applying Thévenin's theorem to the 80 mH inductor (Fig. 11.47) yields

$$R_{Th} = \frac{R}{N} = \frac{20\text{ k}\Omega}{2} = 10\text{ k}\Omega$$

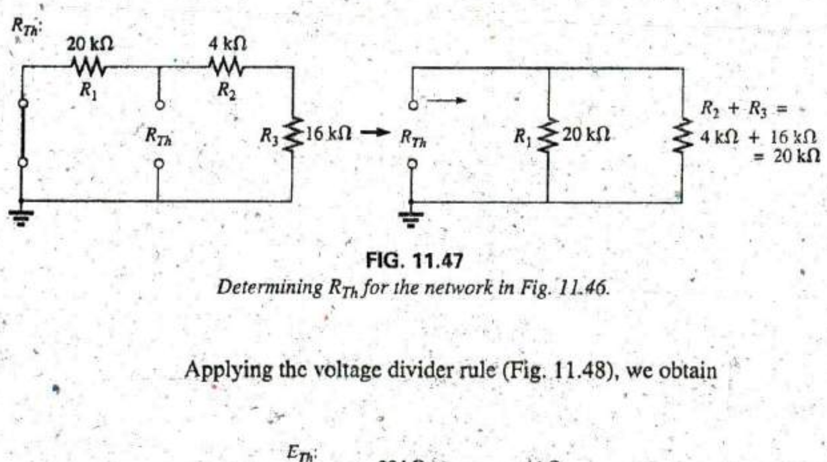


FIG. 11.47
Determining R_{Th} for the network in Fig. 11.46.

Applying the voltage divider rule (Fig. 11.48), we obtain

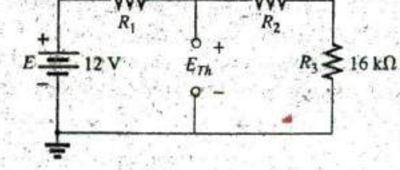


FIG. 11.48
Determining E_{Th} for the network in Fig. 11.46.

$$E_{Th} = \frac{(R_2 + R_3)E}{R_1 + R_2 + R_3}$$
$$= \frac{(4\ \text{k}\Omega + 16\ \text{k}\Omega)(12\ \text{V})}{20\ \text{k}\Omega + 4\ \text{k}\Omega + 16\ \text{k}\Omega} = \frac{(20\ \text{k}\Omega)(12\ \text{V})}{40\ \text{k}\Omega} = 6\ \text{V}$$

The Thévenin equivalent circuit is shown in Fig. 11.49. Using Eq. (11.13) gives

$$i_L = \frac{E_{Th}}{R}(1 - e^{-t/\tau})$$
$$\tau = \frac{L}{R_{Th}} = \frac{80 \times 10^{-3}\ \text{H}}{10 \times 10^3\ \Omega} = 8 \times 10^{-6}\ \text{s} = 8\ \mu\text{s}$$
$$I_m = \frac{E_{Th}}{R_{Th}} = \frac{6\ \text{V}}{10 \times 10^3\ \Omega} = 0.6 \times 10^{-3}\ \text{A} = 0.6\ \text{mA}$$

and $\quad i_L = \mathbf{0.6\ mA\ (1 - e^{-t/8\ \mu s})}$

Using Eq. (11.15) gives

$$v_L = E_{Th}e^{-t/\tau}$$

so that $\quad v_L = \mathbf{6\ V e^{-t/8\ \mu s}}$

b. See Fig. 11.50.

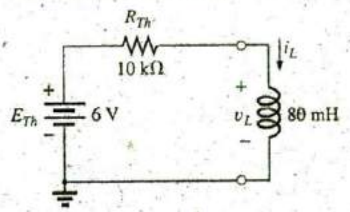


FIG. 11.49
The resulting Thévenin equivalent circuit for the network in Fig. 11.46.

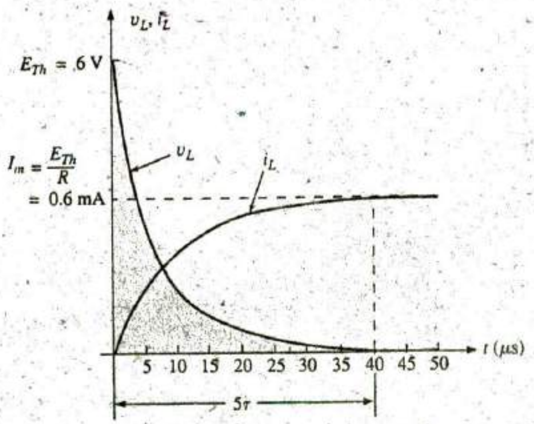


FIG. 11.50
The resulting waveforms for i_L and v_L for the network in Fig. 11.46.

EXAMPLE 11.7 Switch S_1 in Fig. 11.51 has been closed for a long time. At $t = 0$ s, S_1 is opened at the same instant that S_2 is closed to avoid an interruption in current through the coil.

a. Find the initial current through the coil. Pay particular attention to its direction.
b. Find the mathematical expression for the current i_L following the closing of switch S_2.
c. Sketch the waveform for i_L.

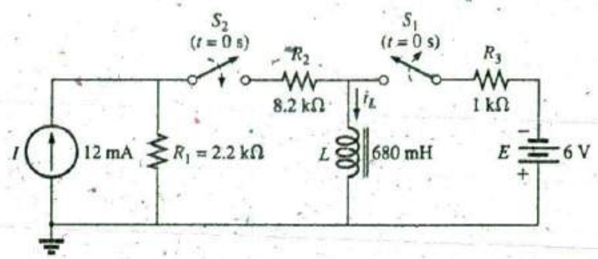


FIG. 11.51
Example 11.7.

Solutions:

a. Using Ohm's law, we find the initial current through the coil:

$$I_i = -\frac{E}{R_3} = -\frac{6\text{ V}}{1\text{ k}\Omega} = -6\text{ mA}$$

b. Applying Thévenin's theorem gives

$$R_{Th} = R_1 + R_2 = 2.2\text{ k}\Omega + 8.2\text{ k}\Omega = 10.4\text{ k}\Omega$$
$$E_{Th} = IR_1 = (12\text{ mA})(2.2\text{ k}\Omega) = 26.4\text{ V}$$

The Thévenin equivalent network appears in Fig. 11.52.

The steady-state current can then be determined by substituting the short-circuit equivalent for the inductor:

$$I_f = \frac{E}{R_{Th}} = \frac{26.4\text{ V}}{10.4\text{ k}\Omega} = 2.54\text{ mA}$$

The time constant is

$$\tau = \frac{L}{R_{Th}} = \frac{680\text{ mH}}{10.4\text{ k}\Omega} = 65.39\ \mu\text{s}$$

Applying Eq. (11.17) gives

$$\begin{aligned} i_L &= I_f + (I_i - I_f)e^{-t/\tau} \\ &= 2.54\text{ mA} + (-6\text{ mA} - 2.54\text{ mA})e^{-t/65.39\ \mu\text{s}} \\ &= \mathbf{2.54\text{ mA} - 8.54\text{ mA}e^{-t/65.39\ \mu s}} \end{aligned}$$

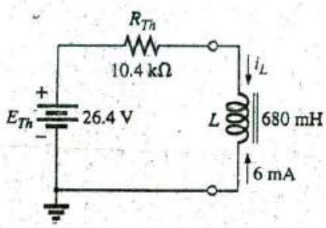


FIG. 11.52
Thévenin equivalent circuit for the network in Fig. 11.51 for $t \geq 0$ s.

c. Note Fig. 11.53.

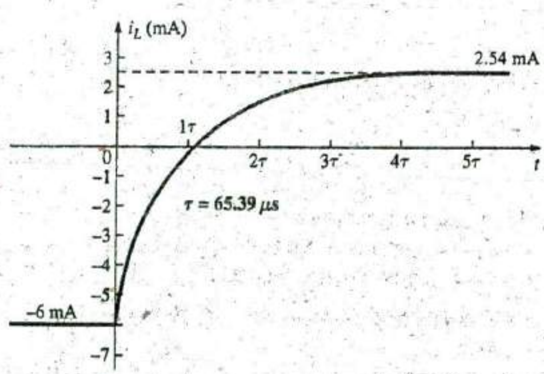


FIG. 11.53
The current i_L for the network in Fig. 11.51.

11.9 INSTANTANEOUS VALUES

The development presented in Section 10.8 for capacitive networks can also be applied to R-L networks to determine instantaneous voltages, currents, and time. The instantaneous values of any voltage or current can be determined by simply inserting t into the equation and using a calculator or table to determine the magnitude of the exponential term.

The similarity between the equations

$$v_C = V_f + (V_i + V_f)e^{-t/\tau}$$

and

$$i_L = I_f + (I_i - I_f)e^{-t/\tau}$$

results in a derivation of the following for t that is identical to that used to obtain Eq. (10.23):

$$\boxed{t = \tau \log_e \frac{(I_i - I_f)}{(i_L - I_f)}} \qquad \text{(seconds, s)} \qquad \textbf{(11.26)}$$

For the other form, the equation $v_C = Ee^{-t/\tau}$ is a close match with $v_L = Ee^{-t/\tau} = V_i e^{-t/\tau}$, permitting a derivation similar to that employed for Eq. (10.23):

$$\boxed{t = \tau \log_e \frac{V_i}{v_L}} \qquad \text{(seconds, s)} \qquad \textbf{(11.27)}$$

For the voltage v_R, $V_i = 0$ V and $V_f = EV$ since $v_R = E(1 - e^{-t/\tau})$. Solving for t yields

$$t = \tau \log_e \left(\frac{E}{E - v_R}\right)$$

or

$$\boxed{t = \tau \log_e \left(\frac{V_f}{V_f - v_R}\right)} \qquad \text{(seconds, s)} \qquad \textbf{(11.28)}$$

11.10 AVERAGE INDUCED VOLTAGE: $v_{L_{av}}$

In an effort to develop some feeling for the impact of the derivative in an equation, the average value was defined for capacitors in Section 10.10, and a number of plots for the current were developed for an applied voltage. For inductors, a similar relationship exists between the induced voltage across a coil and the current through the coil. For inductors, the average induced voltage is defined by

$$\boxed{v_{L_{av}} = L\frac{\Delta i_L}{\Delta t}} \qquad \text{(volts, V)} \qquad \textbf{(11.29)}$$

where Δ indicates a finite (measurable) change in current or time. Eq. (11.12) for the instantaneous voltage across a coil can be derived from Eq. (11.29) by letting V_L become vanishingly small. That is,

$$v_{L_{inst}} = \lim_{\Delta t \to 0} L\frac{\Delta i_L}{\Delta t} = L\frac{di_L}{dt}$$

In the following example, the change in current Δi_L is considered for each slope of the current waveform. *If the current increases with time, the average current is the change in current divided by the change in* *time, with a positive sign. If the current decreases with time, a negative sign is applied.* Note in the example that the faster the current changes with time, the greater is the induced voltage across the coil. When making the necessary calculations, do not forget to multiply by the inductance of the coil. Larger inductances result in increased levels of induced voltage for the same change in current through the coil.

EXAMPLE 11.8 Find the waveform for the average voltage across the coil if the current through a 4 mH coil is as shown in Fig. 11.54.

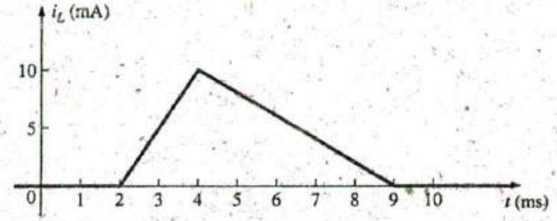


FIG. 11.54
Current i_L to be applied to a 4 mH coil in Example 11.8.

Solutions:

a. *0 to 2 ms:* Since there is no change in current through the coil, there is no voltage induced across the coil. That is,

$$v_L = L\frac{\Delta i}{\Delta t} = L\frac{0}{\Delta t} = \mathbf{0\ V}$$

b. *2 ms to 4 ms:*

$$v_L = L\frac{\Delta i}{\Delta t} = (4 \times 10^{-3}\,\text{H})\left(\frac{10 \times 10^{-3}\,\text{A}}{2 \times 10^{-3}\,\text{s}}\right) = 20 \times 10^{-3}\,\text{V} = \mathbf{20\ mV}$$

c. *4 ms to 9 ms:*

$$v_L = L\frac{\Delta i}{\Delta t} = (-4 \times 10^{-3}\,\text{H})\left(\frac{10 \times 10^{-3}\,\text{A}}{5 \times 10^{-3}\,\text{s}}\right) = -8 \times 10^{-3}\,\text{V} = \mathbf{-8\ mV}$$

d. *9 ms to ∞:*

$$v_L = L\frac{\Delta i}{\Delta t} = L\frac{0}{\Delta t} = \mathbf{0\ V}$$

The waveform for the average voltage across the coil is shown in Fig. 11.55. Note from the curve that

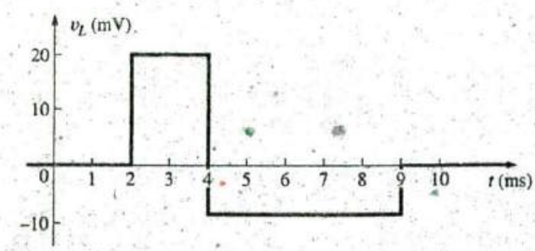


FIG. 11.55
Voltage across a 4 mH coil due to the current in Fig. 11.54.

the voltage across the coil is not determined solely by the magnitude of the change in current through the coil (Δi), but by the rate of change of current through the coil ($\Delta i/\Delta t$).

A similar statement was made for the current of a capacitor due to change in voltage across the capacitor.

A careful examination of Fig. 11.55 also reveals that the area under the positive pulse from 2 ms to 4 ms equals the area under the negative pulse from 4 ms to 9 ms. In Section 11.13, we will find that the area under the curves represents the energy stored or released by the inductor. From 2 ms to 4 ms, the inductor is storing energy, whereas from 4 ms to 9 ms, the inductor is releasing the energy stored. For the full period from 0 ms to 10 ms, energy has been stored and released; there has been no dissipation as experienced for the resistive elements. Over a full cycle, both the ideal capacitor and inductor do not consume energy but store and release it in their respective forms.

11.11 INDUCTORS IN SERIES AND IN PARALLEL

Inductors, like resistors and capacitors, can be placed in series or in parallel. Increasing levels of inductance can be obtained by placing inductors in series, while decreasing levels can be obtained by placing inductors in parallel.

For inductors in series, the total inductance is found in the same manner as the total resistance of resistors in series (Fig. 11.56):

$$\boxed{L_T = L_1 + L_2 + L_3 + \cdots + L_N} \tag{11.30}$$

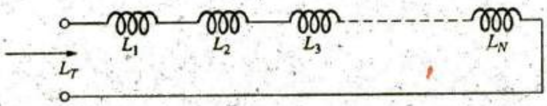


FIG. 11.56
Inductors in series.

For inductors in parallel, the total inductance is found in the same manner as the total resistance of resistors in parallel (Fig. 11.57):

$$\boxed{\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}} \tag{11.31}$$

For two inductors in parallel,

$$\boxed{L_T = \frac{L_1 L_2}{L_1 + L_2}} \tag{11.32}$$

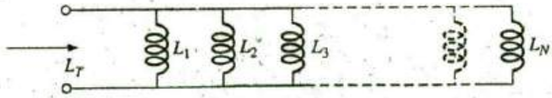


FIG. 11.57
Inductors in parallel.

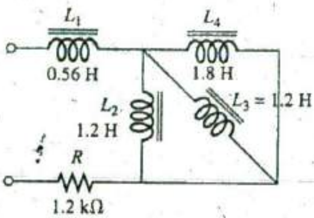


FIG. 11.58
Example 11.9.

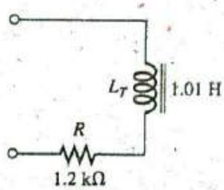


FIG. 11.59
Terminal equivalent of the network in Fig. 11.58.

EXAMPLE 11.9 Reduce the network in Fig. 11.58 to its simplest form.

Solution: Inductors L_2 and L_3 are equal in value and they are in parallel, resulting in an equivalent parallel value of

$$L'_T = \frac{L}{N} = \frac{1.2\text{ H}}{2} = 0.6\text{ H}$$

The resulting 0.6 H is then in parallel with the 1.8 H inductor, and

$$L''_T = \frac{(L'_T)(L_4)}{L'_T + L_4} = \frac{(0.6\text{ H})(1.8\text{ H})}{0.6\text{ H} + 1.8\text{ H}} = 0.45\text{ H}$$

Inductor L_1 is then in series with the equivalent parallel value, and

$$L_T = L_1 + L''_T = 0.56\text{ H} + 0.45\text{ H} = \mathbf{1.01\text{ H}}$$

The reduced equivalent network appears in Fig. 11.59.

11.12 STEADY-STATE CONDITIONS

We found in Section 11.5 that, for all practical purposes, an ideal (ignoring internal resistance and stray capacitances) inductor can be replaced by a short-circuit equivalent once steady-state conditions have been established. Recall that the term *steady state* implies that the voltage and current levels have reached their final resting value and will no longer change unless a change is made in the applied voltage or circuit configuration. For all practical purposes, our assumption is that steady-state conditions have been established after five time constants of the storage or release phase have passed.

For the circuit in Fig. 11.60(a), for example, if we assume that steady-state conditions have been established, the inductor can be removed and replaced by a short-circuit equivalent as shown in Fig. 11.60(b). The short-circuit equivalent shorts out the 3 Ω resistor, and current I_1 is determined by

$$I_1 = \frac{E}{R_1} = \frac{10\text{ V}}{2\,\Omega} = \mathbf{5\text{ A}}$$

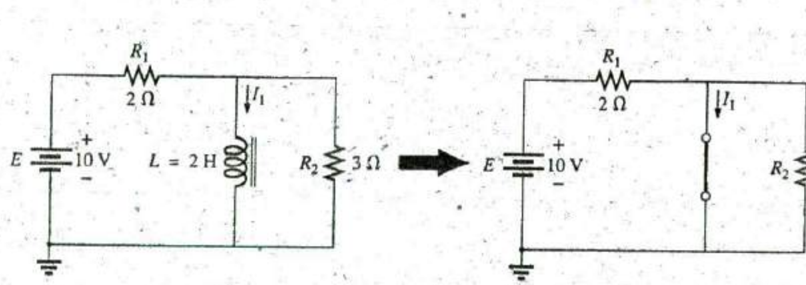


FIG. 11.60
Substituting the short-circuit equivalent for the inductor for $t > 5\tau$.

For the circuit in Fig. 11.61(a), the steady-state equivalent is as shown in Fig. 11.61(b). This time, resistor R_1 is shorted out, and resistors R_2 and R_3 now appear in parallel. The result is

$$I = \frac{E}{R_2 \| R_3} = \frac{21\text{ V}}{2\,\Omega} = \mathbf{10.5\text{ A}}$$

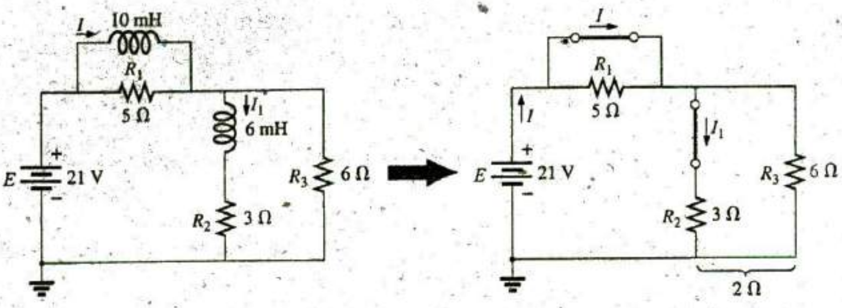


FIG. 11.61
Establishing the equivalent network for $t > 5\tau$.

Applying the current divider rule yields

$$I_1 = \frac{R_3 I}{R_3 + R_2} = \frac{(6\,\Omega)(10.5\text{ A})}{6\,\Omega + 3\,\Omega} = \frac{63}{9}\text{A} = \mathbf{7\text{ A}}$$

In the examples to follow, it is assumed that steady-state conditions have been established.

EXAMPLE 11.10 Find the current I_L and the voltage V_C for the network in Fig. 11.62.

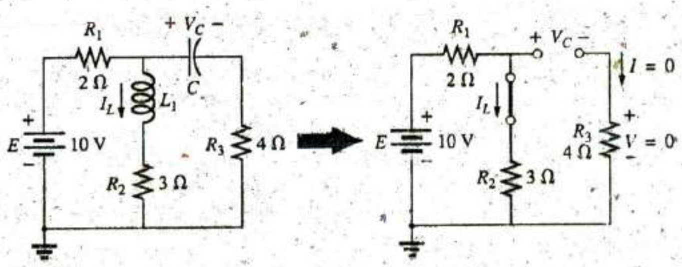


FIG. 11.62
Example 11.10.

Solution:

$$I_L = \frac{E}{R_1 + R_2} = \frac{10\text{ V}}{5\,\Omega} = \mathbf{2\text{ A}}$$

$$V_C = \frac{R_2 E}{R_2 + R_1} = \frac{(3\,\Omega)(10\text{ V})}{3\,\Omega + 2\,\Omega} = \mathbf{6\text{ V}}$$

EXAMPLE 11.11 Find currents I_1 and I_2 and voltages V_1 and V_2 for the network in Fig. 11.63.

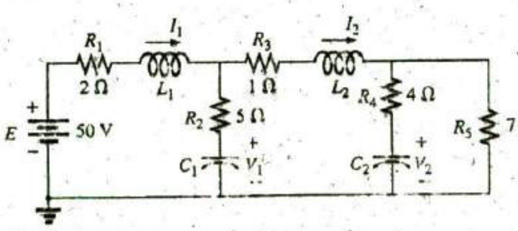


FIG. 11.63
Example 11.11.

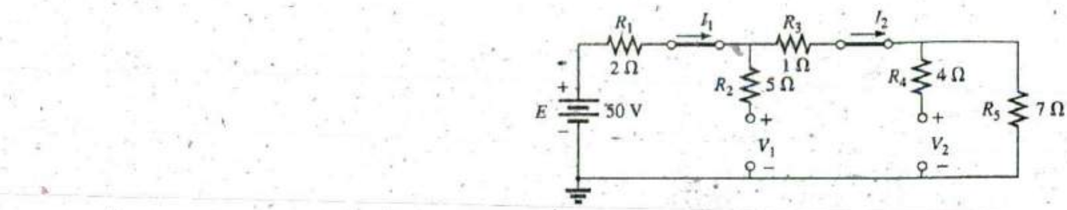


FIG. 11.64
Substituting the short-circuit equivalents for the inductors and the open-circuit equivalents for the capacitor for $t > 5\tau$.

Solution: Note Fig. 11.64.

$$I_1 = I_2 = \frac{E}{R_1 + R_3 + R_5} = \frac{50\text{ V}}{2\ \Omega + 1\ \Omega + 7\ \Omega} = \frac{50\text{ V}}{10\ \Omega} = \mathbf{5\ A}$$

$$V_2 = I_2R_5 = (5\text{ A})(7\ \Omega) = \mathbf{35\ V}$$

Applying the voltage divider rule yields

$$V_1 = \frac{(R_3 + R_5)E}{R_1 + R_3 + R_5} = \frac{(1\ \Omega + 7\ \Omega)(50\text{ V})}{2\ \Omega + 1\ \Omega + 7\ \Omega} = \frac{(8\ \Omega)(50\text{ V})}{10\ \Omega} = \mathbf{40\ V}$$

11.13 ENERGY STORED BY AN INDUCTOR

The ideal inductor, like the ideal capacitor, does not dissipate the electrical energy supplied to it. It stores the energy in the form of a magnetic field. A plot of the voltage, current, and power to an inductor is shown in Fig. 11.65 during the buildup of the magnetic field surrounding the inductor. The energy stored is represented by the shaded area under the power curve. Using calculus, we can show that the evaluation of the area under the curve yields

$$\boxed{W_{\text{stored}} = \frac{1}{2}LI_m^2} \qquad \text{(joules, J)} \qquad \mathbf{(11.33)}$$

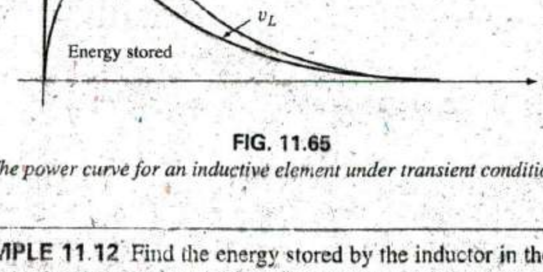


FIG. 11.65
The power curve for an inductive element under transient conditions.

EXAMPLE 11.12 Find the energy stored by the inductor in the circuit in Fig. 11.66 when the current through it has reached its final value.

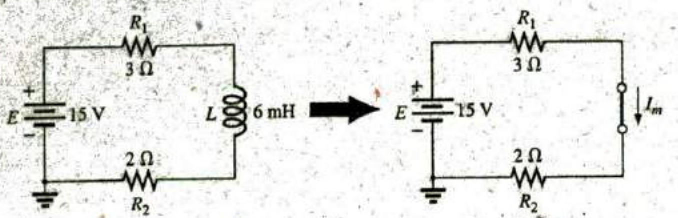


FIG. 11.66
Example 11.12.

Solution:

$$I_m = \frac{E}{R_1 + R_2} = \frac{15\text{ V}}{3\ \Omega + 2\ \Omega} = \frac{15\text{ V}}{5\ \Omega} = 3\text{ A}$$

$$W_{\text{stored}} = \frac{1}{2}LI_m^2 = \frac{1}{2}(6 \times 10^{-3}\text{ H})(3\text{ A})^2 = \frac{54}{2} \times 10^{-3}\text{ J} = \mathbf{27\ mJ}$$

11.14 APPLICATIONS

Camera Flash Lamp

The inductor played an important role in the camera flash lamp circuitry described in the applications section of Chapter 10 on capacitors. For the camera, the inductor was the important component that resulted in the high spike voltage across the trigger coil, which was then magnified by the autotransformer action of the secondary to generate the 4000 V necessary to ignite the flash lamp. Recall that the capacitor in parallel with the trigger coil charged up to 300 V using the low-resistance path provided by the SCR (silicon-controlled rectifier). However, once the capacitor was fully charged, the short-circuit path to ground provided by the SCR was removed, and the capacitor immediately started to discharge through the trigger coil. Since the only resistance in the time constant for the inductive network is the relatively low resistance of the coil itself, the current through the coil grew at a very rapid rate. A significant voltage was then developed across the coil as defined by Eq. (11.12): $v_L = L\,(di_L/dt)$. This voltage was in turn increased by transformer action to the secondary coil of the autotransformer, and the flash lamp was ignited. That high voltage generated across the trigger coil also appears directly across the capacitor of the trigger network. The result is that it begins to charge up again until the generated voltage across the coil drops to zero volts. However, when it does drop, the capacitor again discharges through the coil, establishes another charging current through the coil, and again develops a voltage across the coil. The high-frequency exchange of energy between the coil and capacitor is called *flyback* because of the "flying back" of energy from one storage element to the other. It begins to decay with time because of the resistive elements in the loop. The more resistance, the more quickly it dies out. If the capacitor-inductor pairing is isolated and "tickled" along the way with the application of a dc voltage, the high frequency-generated voltage across the coil can be maintained and put to good use. In fact, it is this flyback effect that is used to generate a steady dc voltage (using rectification to convert the oscillating waveform to one of a steady dc nature) that is commonly used in TVs.

Household Dimmer Switch

Inductors can be found in a wide variety of common electronic circuits in the home. The typical household dimmer uses an inductor to protect the other components and the applied load from "rush" currents—currents that increase at very high rates and often to excessively high levels. This feature is particularly important for dimmers since they are most commonly used to control the light intensity of an incandescent lamp. When a lamp is turned on, the resistance is typically very low, and relatively high currents may flow for short periods of time until the filament of the bulb heats up. The inductor is also effective in blocking high-frequency noise (RFI) generated by the switching action of the triac in the dimmer. A capacitor is also normally included from line to neutral to prevent any voltage spikes from affecting the operation of the dimmer and the applied load (lamp, etc.) and to assist with the suppression of RFI disturbances.

A photograph of one of the most common dimmers is provided in Fig. 11.67(a), with an internal view shown in Fig. 11.67(b). The basic

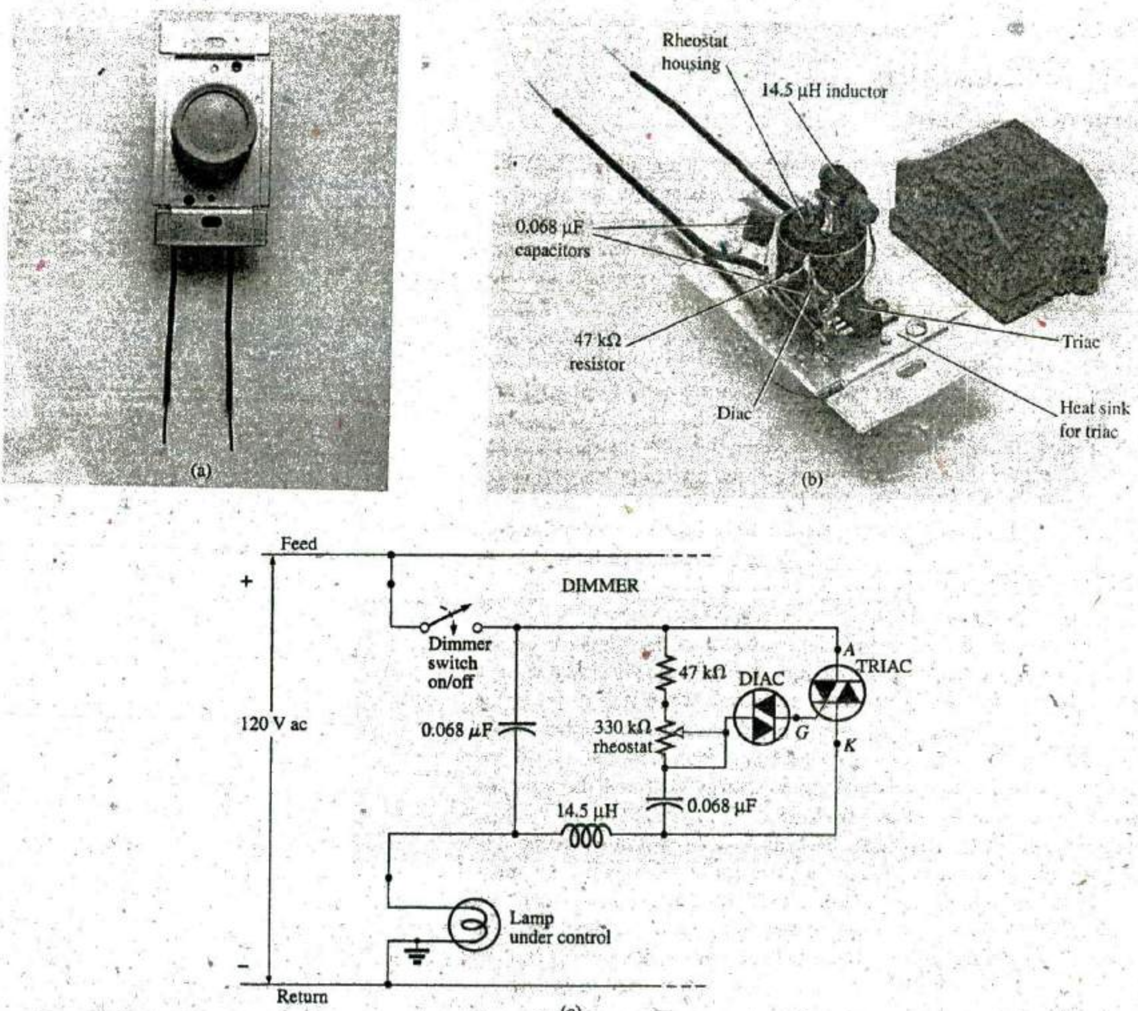


FIG.11.67
Dimmer control: (a) external appearance; (b) internal construction; (c) schematic.

components of most commercially available dimmers appear in the schematic in Fig. 11.67(c). In this design, a 14.5 μH inductor is used in the choking capacity described above, with a 0.068 μF capacitor for the "bypass" operation. Note the size of the inductor with its heavy wire and large ferromagnetic core and the relatively large size of the two 0.068 μF capacitors. Both suggest that they are designed to absorb high-energy disturbances.

The general operation of a dimmer is shown in Fig. 11.68. The controlling network is in series with the lamp and essentially acts as an impedance (like resistance—to be introduced in Chapter 15) that can vary between very low and very high levels. Very low impedance levels resemble a short circuit, so that the majority of the applied voltage appears across the lamp [Fig. 11.68 (a)], and very high impedances approach an open circuit where very little voltage appears across the lamp [Fig. 11.68 (b)]. Intermediate levels of impedance control the terminal voltage of the bulb accordingly. For instance, if the controlling network has a very high impedance (open-circuit equivalent) through half the cycle, as shown in Fig. 11.68(c), the brightness of the bulb will be less than full voltage but not 50% due to the nonlinear relationship between the brightness of a bulb and the applied voltage. A lagging effect is also present in the actual operation of the dimmer, which we will learn about when leading and lagging networks are examined in the ac chapters.

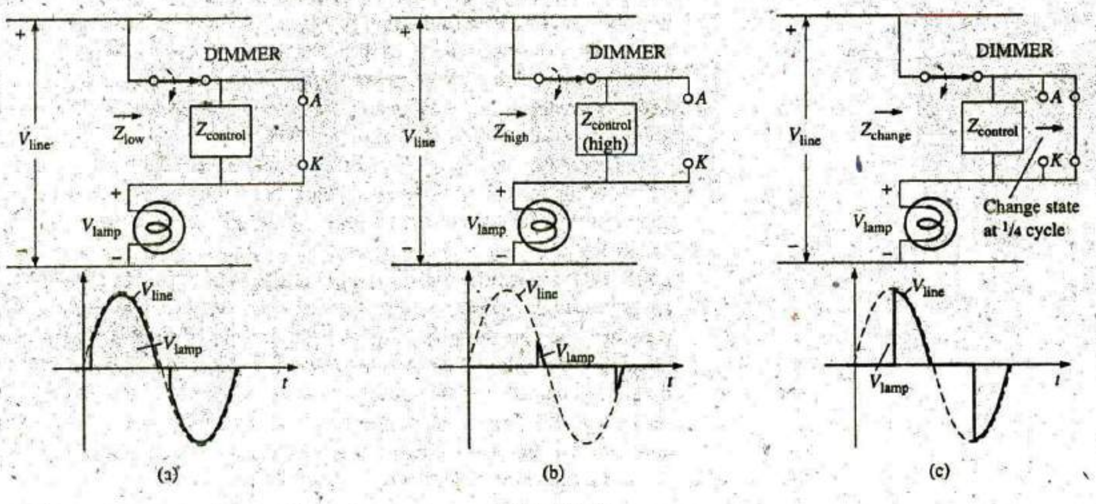


FIG. 11.68

Basic operation of the dimmer in Fig. 11.67: (a) full voltage to the lamp; (b) approaching the cutoff point for the bulb; (c) reduced illumination of the lamp.

The controlling knob, slide, or whatever other method is used on the face of the switch to control the light intensity is connected directly to the rheostat in the branch parallel to the triac. Its setting determines when the voltage across the capacitor reaches a sufficiently high level to turn on the diac (a bidirectional diode) and establish a voltage at the gate (G) of the triac to turn it on. When it does, it establishes a very low resistance path from the anode (A) to the cathode (K), and the applied voltage appears directly across the lamp. When the SCR is off, its terminal resistance between anode and cathode is very high and can be approximated by an open circuit. During this period, the applied voltage does not reach

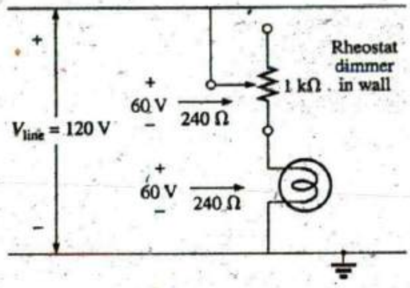


FIG. 11.69
Direct rheostat control of the brightness of a 60 W bulb.

the load (lamp). At this time, the impedance of the parallel branch containing the rheostat, fixed resistor, and capacitor is sufficiently high compared to the load that it can also be ignored, completing the open-circuit equivalent in series with the load. Note the placement of the elements in the photograph in Fig. 11.67(b) and that the metal plate to which the triac is connected is actually a heat sink for the device. The on/off switch is in the same housing as the rheostat. The total design is certainly well planned to maintain a relatively small size for the dimmer.

Since the effort here is to control the amount of power getting to the load, the question is often asked, Why don't we just use a rheostat in series with the lamp? The question is best answered by examining Fig. 11.69, which shows a rather simple network with a rheostat in series with the lamp. At full wattage, a 60 W bulb on a 120 V line theoretically has an internal resistance of $R = V^2/P$ (from the equation $P = V^2/R$) $= (120\text{ V})^2/60\text{ W} = 240\ \Omega$. Although the resistance is sensitive to the applied voltage, we will assume this level for the following calculations.

If we consider the case where the rheostat is set for the same level as the bulb, as shown in Fig. 11.69, there will be 60 V across the rheostat and the bulb. The power to each element is then $P = V^2/R = (60\text{ V})^2/240\ \Omega = 15\text{ W}$. The bulb is certainly quite dim, but the rheostat inside the dimmer switch is dissipating 15 W of power on a continuous basis. When you consider the size of a 2 W potentiometer in your laboratory, you can imagine the size rheostat you would need for 15 W, not to mention the purchase cost, although the biggest concern would probably be all the heat developed in the walls of the house. You would be paying for electric power that was not performing a useful function. Also, if you had four dimmers set at the same level, you would actually be wasting sufficient power to fully light another 60 W bulb.

On occasion, especially when the lights are set very low by the dimmer, a faint "singing" can sometimes be heard from the light bulb. This effect sometimes occurs when the conduction period of the dimmer is very small. The short, repetitive voltage pulse applied to the bulb sets the bulb into a condition similar to a resonance state (Chapter 20). The short pulses are just enough to heat up the filament and its supporting structures, and then the pulses are removed to allow the filament to cool down again for a longer period of time. This repetitive heating and cooling cycle can set the filament in motion, and the "singing" can be heard in a quiet environment. Incidentally, the longer the filament, the louder is the "singing." A further condition for this effect is that the filament must be in the shape of a coil and not a straight wire so that the "slinky" effect can develop.

11.15 COMPUTER ANALYSIS

PSpice

Transient *RL* Response The computer analysis begins with a transient analysis of the network of parallel inductive elements in Fig. 11.70. The inductors are picked up from the **ANALOG** library in the **Place Part** dialog box. As noted in Fig. 11.70, the inductor is displayed with a dot at one end of the coil. The dot is defined by a convention that is used when two or more coils have a mutual inductance, a topic that will be discussed in detail in Chapter 22. In this example, there are no assumed mutual effects, so the dots have no effect on this investigation. However, for this software, the dot is always placed closed to terminal 1 of the

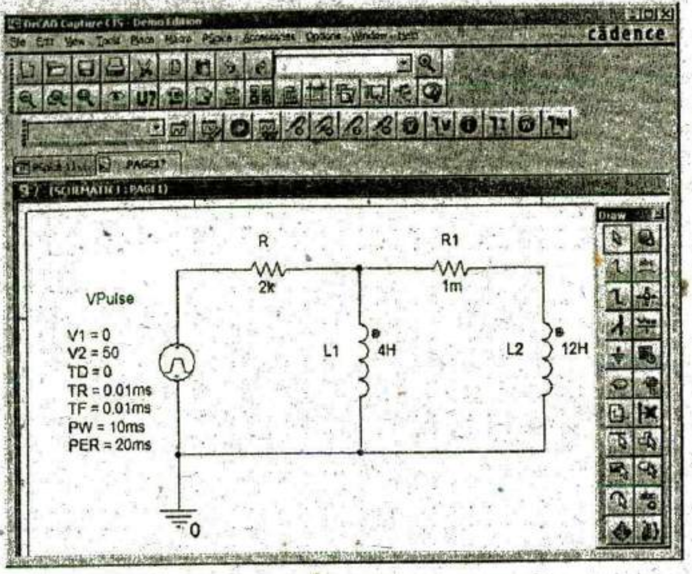


FIG. 11.70
Using PSpice to obtain the transient response of a parallel inductive network due to an applied pulse of 50 V.

inductor. If you bring the pointer controlled by the mouse close to the end of the coil **L1** with the dot, the following will result: **[/L1/1Number:1]**. The number is important because it will define which plot we want to see in the probe response later. When the inductors are placed on the screen, they have to be rotated 270°, which can be accomplished with the **Rotate-Mirror Vertically** sequence.

Also note in Fig. 11.70 the need for a series resistor R_1 within the parallel loop of inductors. In PSpice, inductors must have a series resistor to reflect real-world conditions. The chosen value of 1 mΩ is so small, however, that it will not affect the response of the system. For **VPulse** (obtained from the SOURCE Library), the rise and fall times were selected as 0.01 ms, and the pulse width was chosen as 10 ms because the time constant of the network is $\tau = L_T/R = (4\text{ H} \| 12\text{ H})/2\text{k}\Omega = 1.5$ ms and $5\tau = 7.5$ ms.

The simulation is the same as applied when obtaining the transient response of capacitive networks. In condensed form, the sequence to obtain a plot of the voltage across the coils versus time is as follows: **New SimulationProfile** key-**PSpice 11-1-Create-TimeDomain(Transient)-Run to time:**10ms-**Start saving data after:**0s and **Maximum step size:**5μs-**OK-Run PSpice** key-**Add Trace** key-**V(L2)-OK.** The resulting trace appears in the bottom of Fig. 11.71. A maximum step size of 5 μs was chosen to ensure that it was less than the rise or fall times of 10 μs. Note that the voltage across the coil jumps to the 50 V level almost immediately; then it decays to 0 V in about 8 ms. A plot of the total current through the parallel coils can be obtained through **Plot-Plot to Window-Add Trace** key-**I(R)-OK,** resulting in the trace appearing at the top of Fig. 11.71. When the trace first appeared, the vertical scale extended from 0 A to 30 mA even though the maximum value of i_R was 25 mA. To bring the maximum value to the top of the graph, **Plot** was selected followed by **Axis Settings-Y Axis-User Defined-0A to 25mA-OK.**

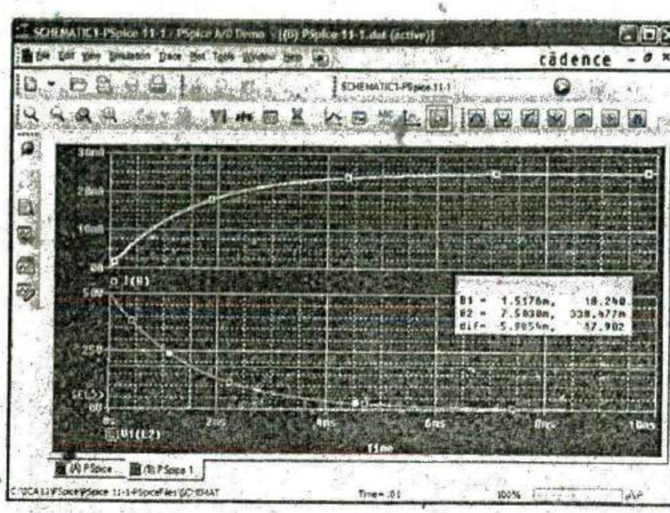


FIG. 11.71
The transient response of v_L and i_R for the network in Fig. 11.70.

For values, the voltage plot was selected, **SEL>>,** followed by the **Toggle cursor** key and a click on the screen to establish the crosshairs. The left-click cursor was set on one time constant of 1.5 ms to reveal a value of 18.24 V for **A1** (about 36.5% of the maximum as defined by the exponential waveform). The right-click cursor was set at 7.5 ms or five time constants, resulting in a relatively low 0.338 V for **A2.**

Transient Response with Initial Conditions The next application verifies the results of Example 11.4, which has an initial condition associated with the inductive element. **VPulse** is again employed with the parameters appearing in Fig. 11.72. Since $\tau = L/R = 100$ mH/$(2.2\ \text{k}\Omega + 6.8\ \text{k}\Omega) = 100$ mH/9 k$\Omega = 11.11\ \mu$s and $5\tau = 55.55\ \mu$s, the pulse width (**PW**) was set to 100 μs. The rise and fall times were set at $100\ \mu\text{s}/1000 = 0.1\ \mu$s.

Setting the initial conditions for the inductor requires a procedure that has not been described as yet. First double-click on the inductor symbol to obtain the **Property Editor** dialog box. Then select **Parts** at the bottom of the dialog box, and select **New Column** to obtain the **Add New Column** dialog box. Under **Name,** enter **IC** (an abbreviation for "initial condition"—not "capacitive current") followed by the initial condition of 4 mA under **Value;** then click **OK.** The **Property Editor** dialog box appears again, but now the initial condition appears as a **New Column** in the horizontal listing dedicated to the inductive element. Now select **Display** to obtain the **Display Properties** dialog box, and under **Display Format** choose **Name and Value** so that both **IC** and **4 mA** appear. Click **OK** to return to the **Property Editor** dialog box. Finally, click on **Apply** and exit the dialog box (**X**). The result is the display in Fig. 11.72 for the inductive element.

Now for the simulation. First select the **New Simulation Profile** key, insert the name **PSpice 11-3,** and follow up with **Create.** Then in the **Simulation Settings** dialog box, select **Time Domain(Transient)** for the **Analysis type** and **General Settings** for the **Options.** The **Run to time** should be 200 μs so that you can see the full effect of the pulse source on the transient response. The **Start saving data after** should remain at 0 s,

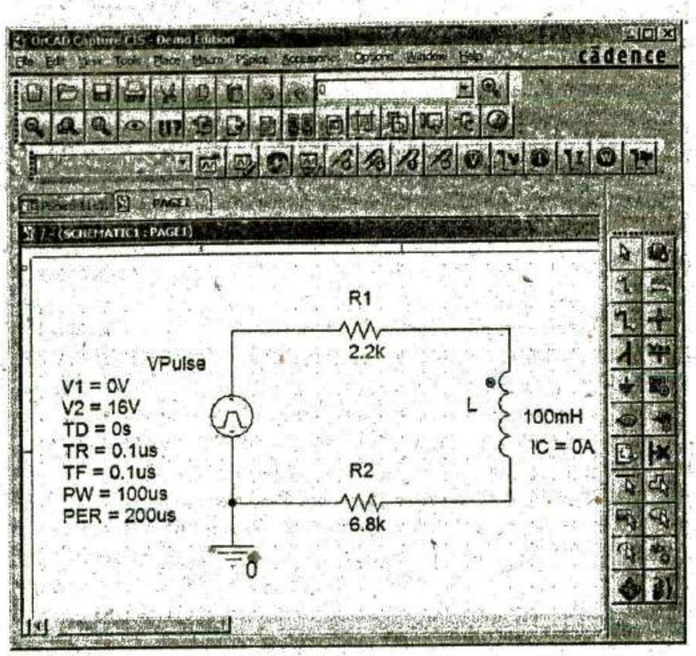


FIG. 11.72
Using PSpice to determine the transient response for a circuit in which the inductive element has an initial condition.

and the **Maximum step size** should be 200 μs/1000 = 200 ns. Click **OK** and then select the **Run PSpice** key. The result is a screen with an x-axis extending from 0 to 200 μs. Selecting **Trace** to get to the **Add Traces** dialog box and then selecting **I(L)** followed by **OK** results in the display in Fig. 11.73. The plot for **I(L)** clearly starts at the initial value of 4 mA and then decays to 1.78 mA as defined by the left-click cursor. The right-click

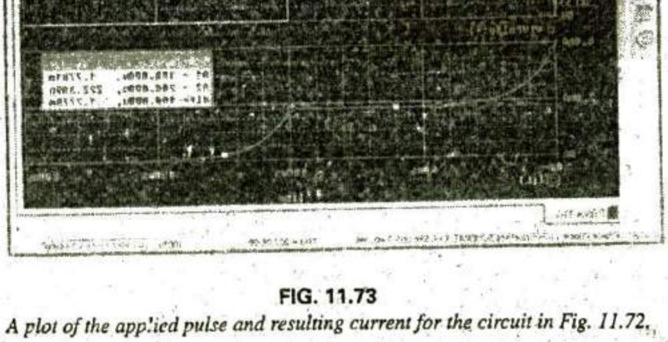


FIG. 11.73
A plot of the applied pulse and resulting current for the circuit in Fig. 11.72.

cursor reveals that the current has dropped to 0.222 μA (essentially 0 A) after the pulse source has dropped to 0 V for 100 μs. The **VPulse** source was placed in the same figure through **Plot-Add Plot to Window-Trace-Add Trace-V(VPulse:+)-OK** to permit a comparison between the applied voltage and the resulting inductor current.

Multisim

The transient response of an R-L network can also be obtained using Multisim. The circuit to be examined appears in Fig. 11.74 with a pulse voltage source to simulate the closing of a switch at $t = 0$ s. The source, **PULSE_VOLTAGE,** is found under **SIGNAL_VOLTAGE SOURCE** Family. When placed on the screen, it appears with a label, an initial voltage, a step voltage, and the time period for each level. All can be changed by double-clicking on the source symbol to obtain the dialog box. As shown in Fig. 11.74, the **Pulsed Value** will be set at 20 V, and the **Delay Time** to 0 s. The **Rise Time** and **Fall Time** will both remain at the default levels of 1 ns. For our analysis we want a **Pulse Width** that is at least twice the 5τ transient period of the circuit. For the chosen values of R and L, $\tau = L/R = 10\text{ mH}/100\ \Omega = 0.1\text{ ms} = 100\ \mu\text{s}$. The transient period of 5τ is therefore 500 μs or 0.5 ms. Thus, a **Pulse Width** of 1 ms would seem appropriate with a **Period** of 2 ms. The result is a frequency of $f = I/T = 1/2\text{ ms} = 500\text{ Hz}$. When the value of the inductor is set at 10 mH using a procedure defined in earlier chapters, an initial value for the current of the inductor can also be set under the heading of **Additional SPICE Simulation Parameters.** In this case, since it is not part of our analysis, it was set at 0 A, as shown in Fig. 11.74. When all have been set and selected, the parameters of the pulse source appear as shown in Fig. 11.74. Next the resistor, inductor, and ground are placed on the screen to complete the circuit.

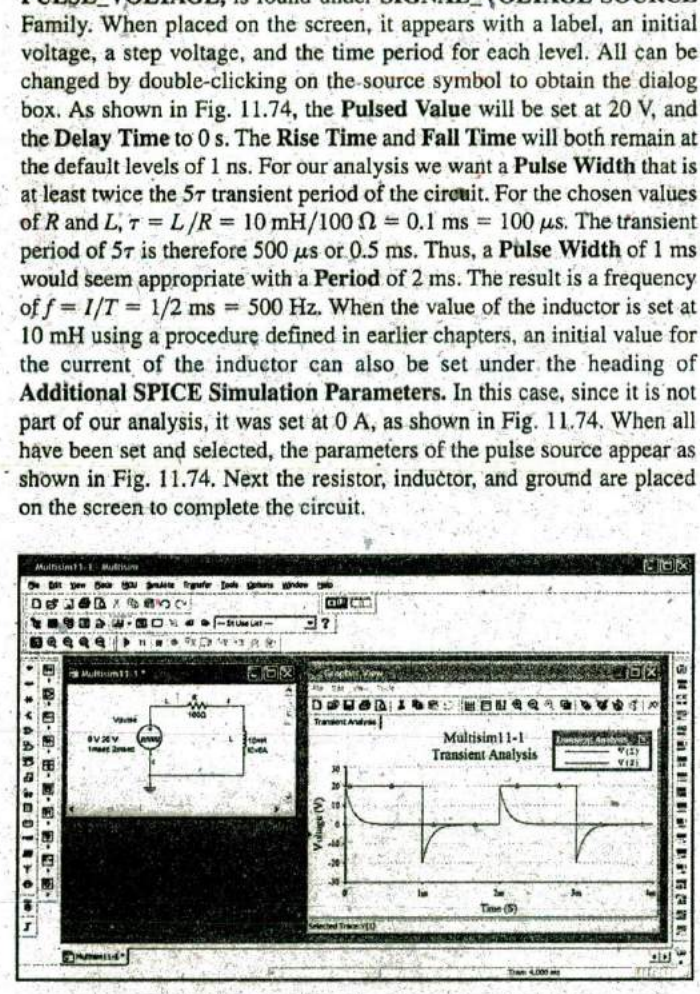


FIG. 11.74
Using Multisim to obtain the transient response for an inductive circuit.

The simulation process is initiated by the following sequence: **Simulate-Analyses-Transient Analysis.** The result is the **Transient Analysis** dialog box in which **Analysis Parameters** is chosen first. Under **Parameters,** use 0 s as the **Start time** and 4 ms (4E-3) as the **End time** so that we get two full cycles of the applied voltage. After enabling the **Maximum time step settings(TMAX),** select the **Minimum number of time points** and set at 1000 to get a reasonably good plot during the rapidly changing transient period. Next, select the **Output variables** section and tell the program which voltage and current levels you are interested in. On the left side of the dialog box is a list of **Variables** that have been defined for the circuit. On the right is a list of **Selected variables for analysis.** In between you see **Add** or **Remove.** To move a variable from the left to the right column, select it in the left column and choose **Add.** It then appears in the right column. To plot both the applied voltage and the voltage across the coil, move **V(1)** and **V(2)** to the right column. Then select **Simulate.** A window titled **Grapher View** appears with the selected plots as shown in Fig. 11.74. Click on the **Show/Hide Grid** key (a red grid on a black axis), and the grid lines appear. Selecting the **Show/Hide Legend** key on the immediate right results in the small **Transient Analysis** dialog box that identifies the color that goes with each nodal voltage. In this case, red is the color of the applied voltage, and blue is the color of the voltage across the coil.

The source voltage appears as expected with its transition to 20 V, 50% duty cycle, and the period of 2 ms. The voltage across the coil jumped immediately to the 20 V level and then began its decay to 0 V in about 0.5 ms as predicted. When the source voltage dropped to zero, the voltage across the coil reversed polarity to maintain the same direction of current in the inductive circuit. Remember that for a coil, the voltage can change instantaneously, but the inductor "chokes" any instantaneous change in current. By reversing its polarity, the voltage across the coil ensures the same polarity of voltage across the resistor and therefore the same direction of current through the coil and circuit.

PROBLEMS

SECTION 11.2 Magnetic Field

1. For the electromagnet in Fig. 11.75:
   a. Find the flux density in Wb/m^2.
   b. What is the flux density in teslas?
   c. What is the applied magnetomotive force?
   d. What would the reading of the meter in Fig. 11.14 read in gauss?

FIG. 11.75
Problem 1.

SECTION 11.3 Inductance

2. For the inductor in Fig. 11.76, find the inductance L in henries.

3. **a.** Repeat Problem 2 with a ferromagnetic core with $\mu_r = 500$.
   b. How is the new inductance related to the old one? How does it relate to the value of μ_r?

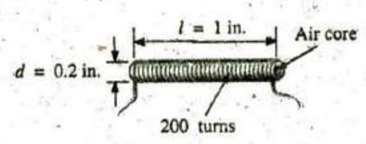


FIG. 11.76
Problems 2 and 3.

4. For the inductor in Fig. 11.77, find the approximate inductance L in henries.

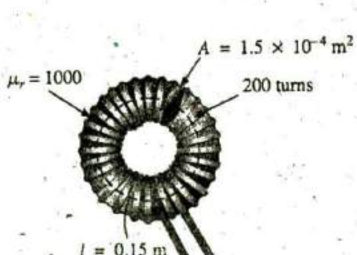


FIG. 11.77
Problem 4.

5. An air-core inductor has a total inductance of 4.7 mH.
   a. What is the inductance if the only change is to increase the number of turns by a factor of three?
   b. What is the inductance if the only change is to increase the length by a factor of three?
   c. What is the inductance if the area is doubled, the length cut in half, and the number of turns doubled?
   d. What is the inductance if the area, length, and number of turns are cut in half and a ferromagnetic core with a μ_r of 1500 is inserted?
6. What are the inductance and the range of expected values for an inductor with the following label?
   a. 392K   b. blue gray black J
   c. 47K   d. brown green red K

SECTION 11.4 Indced Voltage v_L

7. If the flux linking a coil of 50 turns changes at a rate of 120 mW/s, what is the induced voltage across the coil?
8. Determine the rate of change of flux linking a coil if 20 V are induced across a coil of 200 turns.
9. How many turns does a coil have if 42 mV are induced across the coil by a change in flux of 3 mW/s?
10. Find the voltage induced across a coil of 22 mH if the rate of change of current through the coil is:
    a. 1 A/s
    b. 1 mA/ms
    c. 2 mA/10 μs

SECTION 11.5 *R-L* Transients: The Storage Phase

11. For the circuit of Fig. 11.78 composed of standard values:
    a. Determine the time constant.
    b. Write the mathematical expression for the current i_L after the switch is closed.
    c. Repeat part (b) for v_L and v_R.
    d. Determine i_L and v_L at one, three, and five time constants.
    e. Sketch the waveforms of i_L, v_L, and v_R.

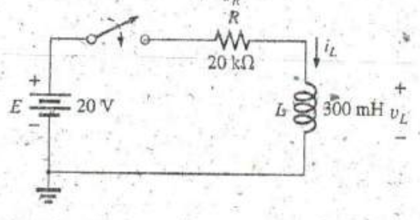


FIG. 11.78
Problem 11.

12. For the circuit in Fig. 11.79 composed of standard values:
    a. Determine τ.
    b. Write the mathematical expression for the current i_L after the switch is closed at $t = 0$ s.
    c. Write the mathematical expression for v_L and v_R after the switch is closed at $t = 0$ s.
    d. Determine i_L and v_L at $t = 1\tau$, 3τ, and 5τ.
    e. Sketch the waveforms of i_L, v_L, and v_R for the storage phase.

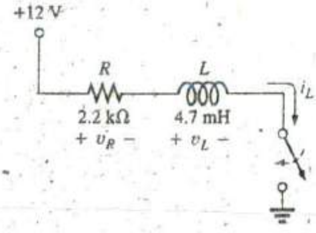


FIG. 11.79
Problem 12.

13. Given a supply of 18 V, use standard values to design a circuit to have the response of Fig. 11.80.

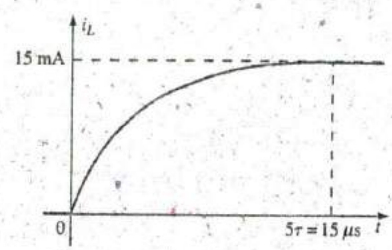


FIG. 11.80
Problem 13.

SECTION 11.6 Initial Conditions

14. For the circuit in Fig. 11.81:
    a. Write the mathematical expressions for the current i_L and the voltage v_L following the closing of the switch. Note the magnitude and the direction of the initial current.
    b. Sketch the waveform of i_L and v_L for the entire period from initial value to steady-state level.

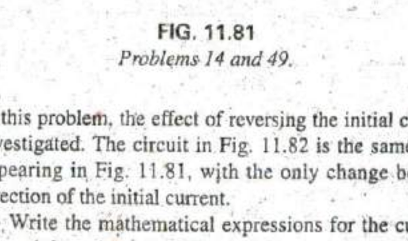


FIG. 11.81
Problems 14 and 49.

15. In this problem, the effect of reversing the initial current is investigated. The circuit in Fig. 11.82 is the same as that appearing in Fig. 11.81, with the only change being the direction of the initial current.
    a. Write the mathematical expressions for the current i_L and the voltage v_L following the closing of the switch. Take careful note of the defined polarity for v_L and the direction for i_L.

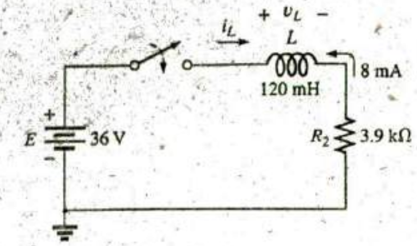


FIG. 11.82
Problem 15.

b. Sketch the waveform of i_L and v_L for the entire period from initial value to steady-state level.
c. Compare the results with those of Problem 14.

16. For the network in Fig. 11.83:
a. Write the mathematical expressions for the current i_L and the voltage v_L following the closing of the switch. Note the magnitude and the direction of the initial current.
b. Sketch the waveform of i_L and v_L for the entire period from initial value to steady-state level.

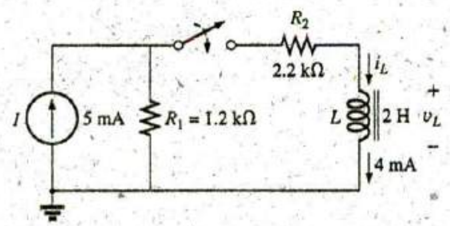


FIG. 11.83
Problem 16.

***17.** For the network in Fig. 11.84:
a. Write the mathematical expressions for the current i_L and the voltage v_L following the closing of the switch. Note the magnitude and direction of the initial current.
b. Sketch the waveform of i_L and v_L for the entire period from initial value to steady-state level.

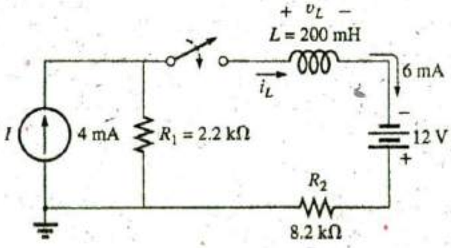


FIG. 11.84
Problem 17.

SECTION 11.7 *R-L* Transients: The Release Phase

18. For the network in Fig. 11.85:
a. Determine the mathematical expressions for the current i_L and the voltage v_L when the switch is closed.
b. Repeat part (a) if the switch is opened after a period of five time constants has passed.
c. Sketch the waveforms of parts (a) and (b) on the same set of axes.

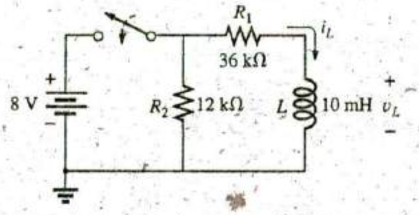


FIG. 11.85
Problem 18.

***19.** For the network in Fig. 11.86:
a. Determine the mathematical expressions for the current i_L and the voltage v_L following the closing of the switch.
b. Repeat part (a) if the switch is opened at $t = 1\ \mu s$.
c. Sketch the waveforms of parts (a) and (b) on the same set of axes.

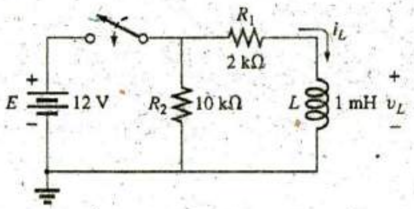


FIG. 11.86
Problem 19.

***20.** For the network in Fig. 11.87:
a. Write the mathematical expression for the current i_L and the voltage v_L following the closing of the switch.
b. Determine the mathematical expressions for i_L and v_L if the switch is opened after a period of five time constants has passed.
c. Sketch the waveforms of i_L and v_L for the time periods defined by parts (a) and (b).
d. Sketch the waveform for the voltage across R_2 for the same period of time encompassed by i_L and v_L. Take careful note of the defined polarities and directions in Fig. 11.87.

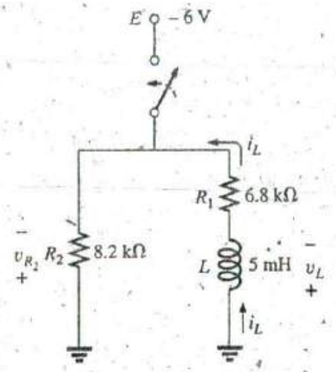


FIG. 11.87
Problem 20.

SECTION 11.8 Thévenin Equivalent: $\tau = L/R_{Th}$

21. For Fig. 11.88:
    a. Determine the mathematical expressions for i_L and v_L following the closing of the switch.
    b. Determine i_L and v_L after one time constant.

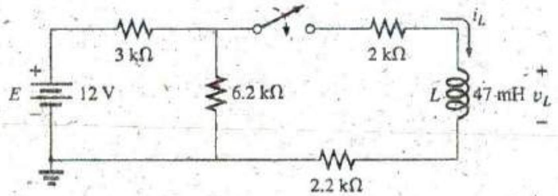


FIG. 11.88
Problems 21 and 50.

22. For Fig. 11.89:
    a. Determine the mathematical expressions for i_L and v_L following the closing of the switch.
    b. Determine i_L and v_L at $t = 100$ ns.

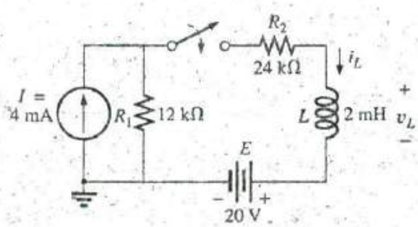


FIG. 11.89
Problem 22.

*23. For Fig. 11.90:
    a. Determine the mathematical expressions for i_L and v_L following the closing of the switch. Note the defined direction for i_L and polarity for v_L.
    b. Calculate i_L and v_L at $t = 10\ \mu s$.
    c. Write the mathematical expressions for the current i_L and the voltage v_L if the switch is opened at $t = 10\ \mu s$.
    d. Sketch the waveforms of i_L and v_L for parts (a) and (c).

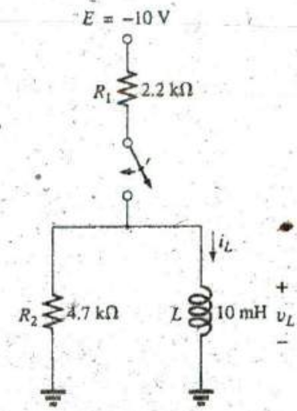


FIG. 11.90
Problem 23.

*24. For the network in Fig. 11.91, the switch is closed at $t = 0$ s.
    a. Determine v_L at $t = 25$ ms.
    b. Find v_L at $t = 1$ ms.
    c. Calculate v_{R_1} at $t = 1\tau$.
    d. Find the time required for the current i_L to reach 100 mA.
    e. What is the effect of the 470 Ω resistor? Explain.

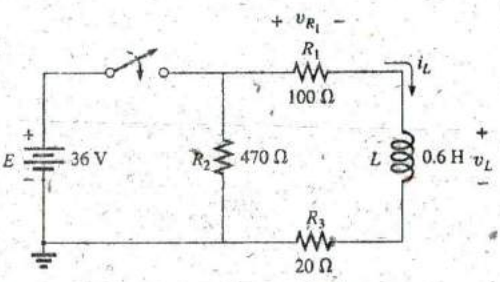


FIG. 11.91
Problem 24.

*25. The switch in Fig. 11.92 has been open for a long time. It is then closed at $t = 0$ s.
    a. Write the mathematical expression for the current i_L and the voltage v_L after the switch is closed.
    b. Sketch the waveform of i_L and v_L from the initial value to the steady-state level.

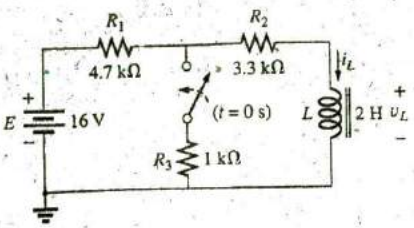


FIG. 11.92
Problem 25.

***26.** **a.** Determine the mathematical expressions for i_L and v_L following the closing of the switch in Fig. 11.93. The steady-state values of i_L and v_L are established before the switch is closed.
b. Determine i_L and v_L after two time constants of the storage phase.
c. Write the mathematical expressions for the current i_L and the voltage v_L if the switch is opened at the instant defined by part (b).
d. Sketch the waveforms of i_L and v_L for parts (a) and (c).

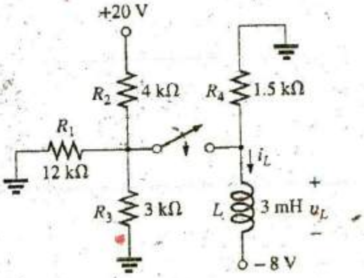


FIG. 11.93
Problem 26.

***27.** The switch for the network in Fig. 11.94 has been closed for about 1 h. It is then opened at the time defined as $t = 0$ s.
a. Determine the time required for the current i_L to drop to 10 μA.
b. Find the voltage v_L at $t = 10\ \mu$s.
c. Calculate v_L at $t = 5\tau$.

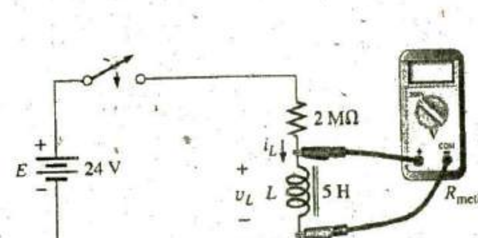


FIG. 11.94
Problem 27.

***28.** The switch in Fig. 11.95 has been closed for a long time. It is then opened at $t = 0$ s.
a. Write the mathematical expression for the current i_L and the voltage v_L after the switch is opened.
b. Sketch the waveform of i_L and v_L from initial value to the steady-state level.

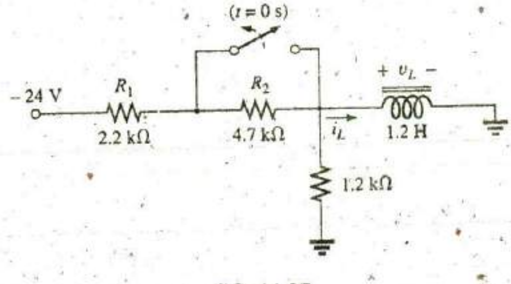


FIG. 11.95
Problem 28.

SECTION 11.9 Instantaneous Values

29. Given $i_L = 100\text{ mA}(1 - e^{-t/20\text{ ms}})$:
a. Determine i_L at $t = 1$ ms.
b. Determine i_L at $t = 100$ ms.
c. Find the time t when i_L will equal 50 mA.
d. Find the time t when i_L will equal 99 mA.

30. **a.** If the measured current for an inductor during the storage phase is 126.4 μA at after a period of one time constant has passed, what is the maximum level of current to be achieved?
b. When the current of part (a) reaches 160 μA, 64.4 μs have passed. Find the time constant of the network.
c. If the circuit's resistance is 500 Ω, what is the value of the series inductor to establish the current of part (a)? Is the resulting inductance a standard value?
d. What is the required supply voltage?

31. The network in Fig. 11.96 employs a DMM with an internal resistance of 10 MΩ in the voltmeter mode. The switch is closed at $t = 0$ s.
a. Find the voltage across the coil the instant after the switch is closed.
b. What is the final value of the current i_L?
c. How much time must pass before i_L reaches 10 μA?
d. What is the voltmeter reading at $t = 12\ \mu$s?

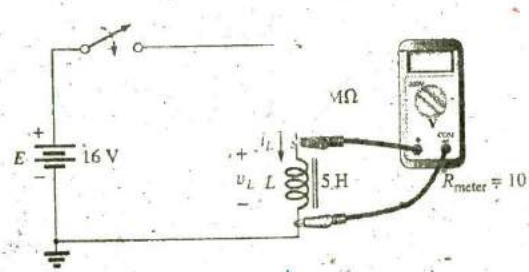


FIG. 11.96
Problem 31.

SECTION 11.10 Average Induced Voltage: $v_{L_{av}}$

32. Find the waveform for the voltage induced across a 200 mH coil if the current through the coil is as shown in Fig. 11.97.

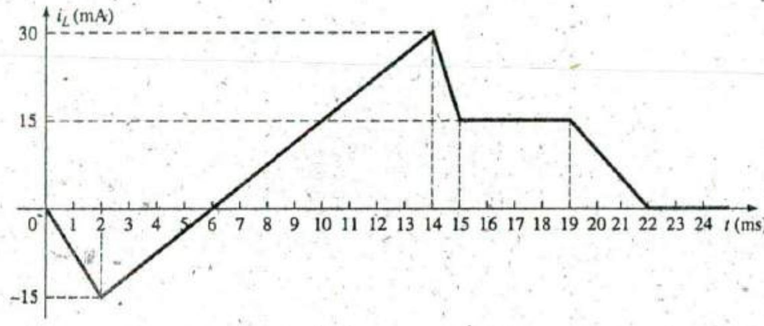


FIG. 11.97
Problem 32.

33. Find the waveform for the voltage induced across a 5 mH coil if the current through the coil is as shown in Fig. 11.98.

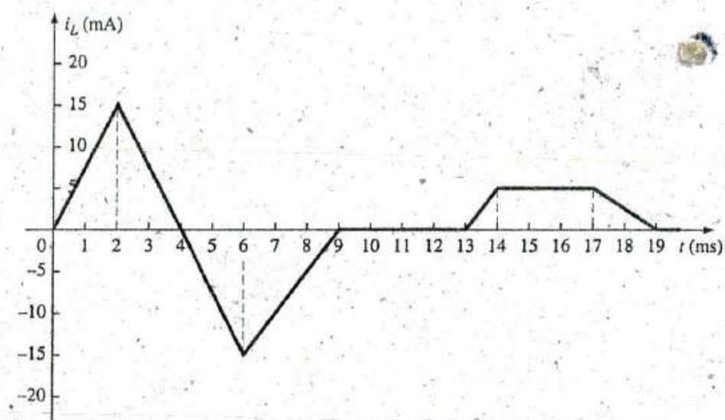


FIG. 11.98
Problem 33.

***34.** Find the waveform for the current of a 10 mH coil if the voltage across the coil follows the pattern in Fig. 11.99. The current i_L is 4 mA at $t = 0^-$ s.

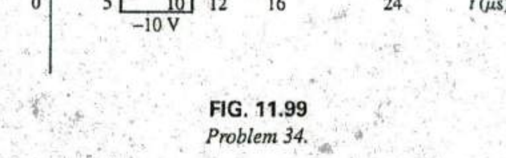


FIG. 11.99
Problem 34.

SECTION 11.11 Inductors in Series and in Parallel

35. Find the total inductance of the circuit of Fig. 11.100.

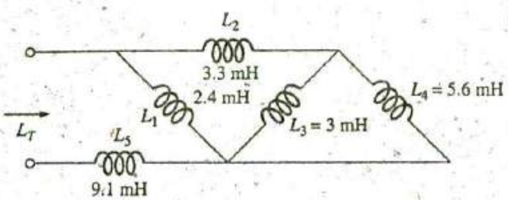


FIG. 11.100
Problem 35.

36. Find the total inductance for the network of Fig. 11.101.

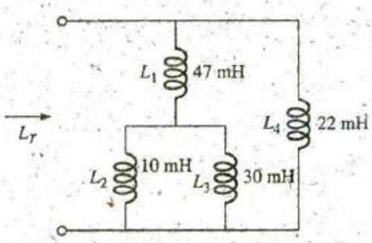


FIG. 11.101
Problem 36.

37. Reduce the network in Fig. 11.102 to the fewest number of components.

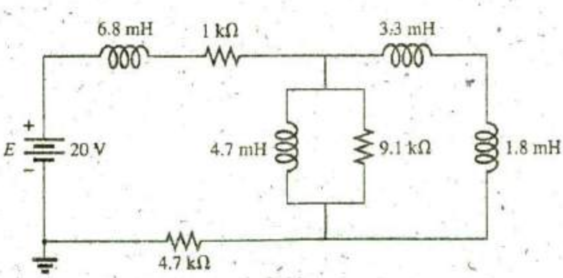


FIG. 11.102
Problem 37.

38. Reduce the network in Fig. 11.103 to the fewest elements.

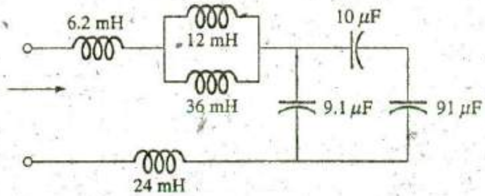


FIG. 11.103
Problem 38.

39. Reduce the network of Fig. 11.104 to the fewest elements.

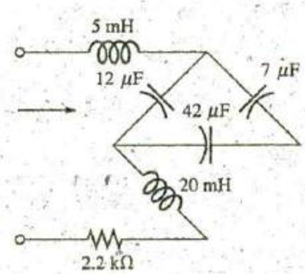


FIG. 11.104
Problem 39.

*40. For the network in Fig. 11.105:

a. Write the mathematical expressions for the voltages v_L and v_R and the current i_L if the switch is closed at $t = 0$ s.
b. Sketch the waveforms of v_L, v_R, and i_L.

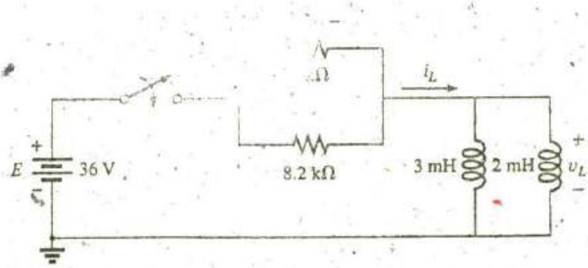


FIG. 11.105
Problem 40.

*41. For the network in Fig. 11.106:

a. Write the mathematical expressions for the voltage v_L and the current i_L if the switch is closed at $t = 0$ s. Take special note of the required v_L.

b. Sketch the waveforms of v_L and i_L.

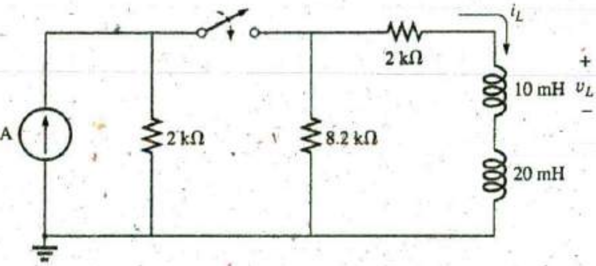


FIG. 11.106
Problem 41.

*42. For the network in Fig. 11.107:

a. Find the mathematical expressions for the voltage v_L and the current i_L following the closing of the switch.

b. Sketch the waveforms of v_L and i_L obtained in part (a).

c. Determine the mathematical expression for the voltage v_{L_3} following the closing of the switch, and sketch the waveform.

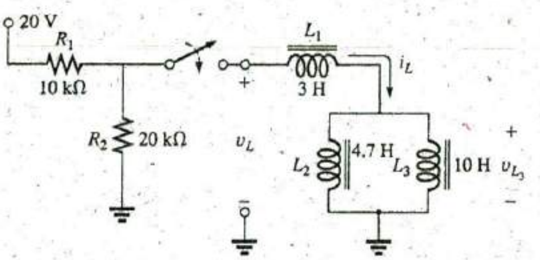


FIG. 11.107
Problem 42.

SECTION 11.12 Steady-State Conditions

43. Find the steady-state currents I_1 and I_2 for the network in Fig. 11.108.

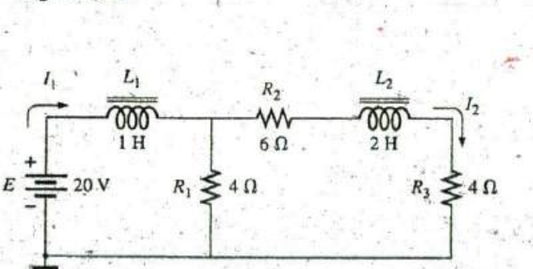


FIG. 11.108
Problem 43.

44. Find the steady-state currents and voltages for the network in Fig. 11.109.

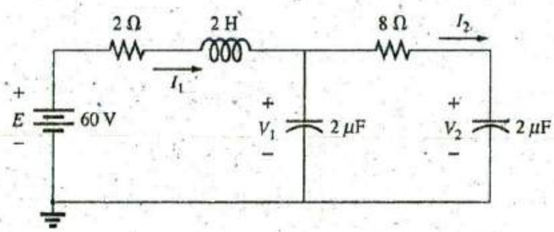


FIG. 11.109
Problem 44.

45. Find the steady-state currents and voltages for the network in Fig. 11.110 after the switch is closed.

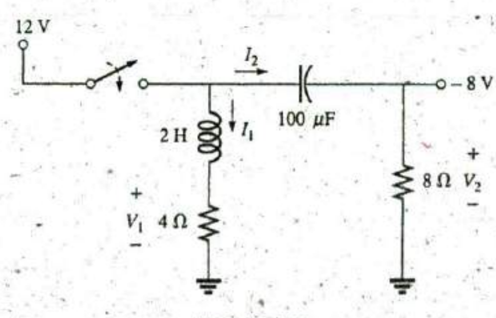


FIG. 11.110
Problem 45.

46. Find the indicated steady-state currents and voltages for the network in Fig. 11.111.

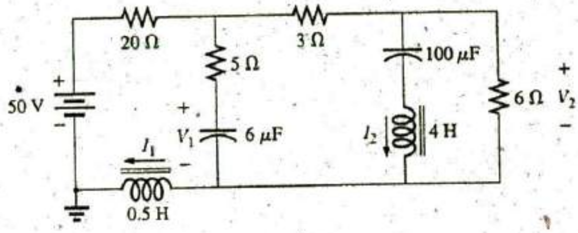


FIG. 11.111
Problem 46.

SECTION 11.15 Computer Analysis

47. Using PSpice or Multisim, verify the results of Example 11.3.
48. Using PSpice or Multisim, verify the results of Example 11.4.
49. Using PSpice or Multisim, find the solution to Problem 14.
50. Using PSpice or Multisim, find the solution to Problem 21.
51. Using PSpice or Multisim, verify the results of Example 11.8.

GLOSSARY

Ampère's circuital law A law establishing the fact that the algebraic sum of the rises and drops of the magnetomotive force (mmf) around a closed loop of a magnetic circuit is equal to zero.

Choke A term often applied to an inductor, due to the ability of an inductor to resist a change in current through it.

Diamagnetic materials Materials that have permeabilities slightly less than that of free space.

Electromagnetism Magnetic effects introduced by the flow of charge, or current.

Faraday's law A law stating the relationship between the voltage induced across a coil and the number of turns in the coil and the rate at which the flux linking the coil is changing.

Ferromagnetic materials Materials having permeabilities hundreds and thousands of times greater than that of free space.

Flux density (B) A measure of the flux per unit area perpendicular to a magnetic flux path. It is measured in teslas (T) or webers per square meter (Wb/m^2).

Inductance (L) A measure of the ability of a coil to oppose any change in current through the coil and to store energy in the form of a magnetic field in the region surrounding the coil.

Inductor (coil) A fundamental element of electrical systems constructed of numerous turns of wire around a ferromagnetic core or an air core.

Lenz's law A law stating that an induced effect is always such as to oppose the cause that produced it.

Magnetic flux lines Lines of a continuous nature that reveal the strength and direction of a magnetic field.

Magnetomotive force (mmf) ($\mathcal{F}$) The "pressure" required to establish magnetic flux in a ferromagnetic material. It is measured in ampere-turns (At).

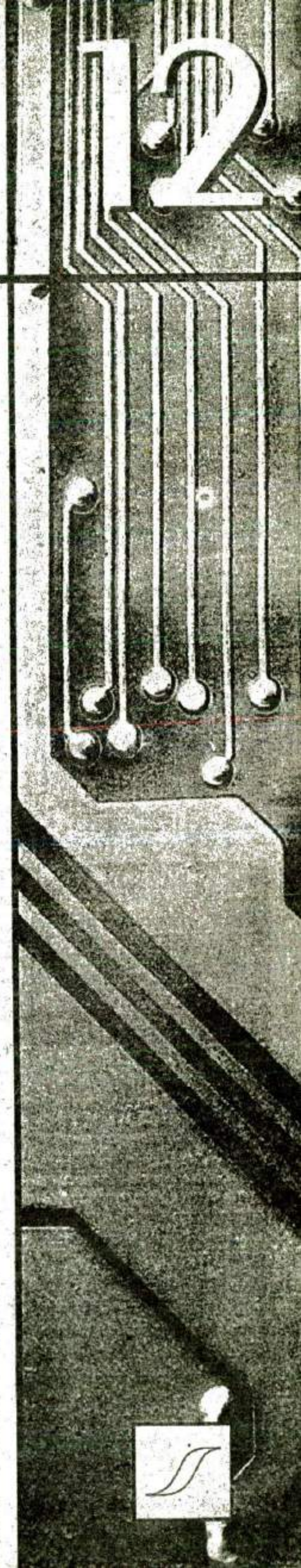
Paramagnetic materials Materials that have permeabilities slightly greater than that of free space.

Permanent magnet A material such as steel or iron that will remain magnetized for long periods of time without the aid of external means.

Permeability (μ) A measure of the ease with which magnetic flux can be established in a material. It is measured in Wb/A · m.

Relative permeability (μ_r) The ratio of the permeability of a material to that of free space.

Magnetic Circuits



Objectives

- *Become aware of the similarities between the analysis of magnetic circuits and electric circuits.*
- *Develop a clear understanding of the important parameters of a magnetic circuit and how to find each quantity for a variety of magnetic circuit configurations.*
- *Begin to appreciate why a clear understanding of magnetic circuit parameters is an important component in the design of electrical/electronic systems.*

12.1 INTRODUCTION

Magnetic and electromagnetic effects play an important role in the design of a wide variety of electrical/electronic systems in use today. Motors, generators, transformers, loudspeakers, relays, medical equipment and movements of all kinds depend on magnetic effects to function properly. The response and characteristics of each have an impact on the current and voltage levels of the system, the efficiency of the design, the resulting size, and many other important considerations.

Fortunately, there is a great deal of similarity between the analyses of electric circuits and magnetic circuits. The magnetic flux of magnetic circuits has properties very similar to the current of electric circuits. As shown in Fig. 11.15, it has a direction and a closed path. The magnitude of the established flux is a direct function of the applied **magnetomotive force,** resulting in a duality with electric circuits, where the resulting current is a function of the magnitude of the applied voltage. The flux established is also inversely related to the structural opposition of the magnetic path in the same way the current in a network is inversely related to the resistance of the network. All of these similarities are used throughout the analysis to clarify the approach.

One of the difficulties associated with studying magnetic circuits is that three different systems of units are commonly used in the industry. The manufacturer, application, and type of component all have an impact on which system is used. To the extent practical, the SI system is applied throughout the chapter. References to the CGS and English systems require the use of Appendix E.

12.2 MAGNETIC FIELD

The magnetic field distribution around a permanent magnet or **electromagnet** was covered in detail in Chapter 11. Recall that flux lines strive to be as short as possible and take the path with the highest permeability. The **flux density** is defined as follows [Eq. (11.1) repeated here for convenience]:

$$\boxed{B = \frac{\Phi}{A}} \qquad \begin{aligned} B &= \mathrm{Wb/m^2} = \text{teslas (T)} \\ \Phi &= \text{webers (Wb)} \\ A &= \mathrm{m}^2 \end{aligned} \tag{12.1}$$

The "pressure" on the system to establish magnetic lines of force is determined by the applied magnetomotive force, which is directly related to the number of turns and current of the magnetizing coil as appearing in the following equation [Eq. (11.3) repeated here for convenience]:

$$\boxed{\mathscr{F} = NI} \qquad \begin{aligned} \mathscr{F} &= \text{ampere-turns (At)} \\ N &= \text{turns (t)} \\ I &= \text{amperes (A)} \end{aligned} \qquad (12.2)$$

The level of magnetic flux established in a ferromagnetic core is a direction function of the permeability of the material. **Ferromagnetic materials** have a very high level of **permeability**, while nonmagnetic materials such as air and wood have very low levels. The ratio of the permeability of the material to that of air is called the **relative permeability** and is defined by the following equation [Eq. (11.5) repeated here for convenience]:

$$\boxed{\mu_r = \frac{\mu}{\mu_o}} \qquad \mu_o = 4\pi \times 10^{-7}\ \text{Wb/A}\cdot\text{m} \qquad (12.3)$$

As mentioned in Chapter 11, the values of μ_r are not provided in a table format because the value is determined by the other quantities of the magnetic circuit. Change the magnetomotive force, and the relative permeability changes.

12.3 RELUCTANCE

The resistance of a material to the flow of charge (current) is determined for electric circuits by the equation

$$R = \rho\frac{l}{A} \qquad (\text{ohms}, \Omega)$$

The **reluctance** of a material to the setting up of magnetic flux lines in the material is determined by the following equation:

$$\boxed{\mathscr{R} = \frac{l}{\mu A}} \qquad (\text{rels, or At/Wb}) \qquad (12.4)$$

where $\mathscr{R}$ is the reluctance, l is the length of the magnetic path, and A is the cross-sectional area. The t in the units At/Wb is the number of turns of the applied winding. More is said about ampere-turns (At) in the next section. Note that the resistance and reluctance are inversely proportional to the area, indicating that an increase in area results in a reduction in each and an *increase* in the desired result: current and flux. For an increase in length, the opposite is true, and the desired effect is reduced. The reluctance, however, is inversely proportional to the permeability, while the resistance is directly proportional to the resistivity. The larger the μ or the smaller the ρ, the smaller are the reluctance and resistance, respectively. Obviously, therefore, materials with high permeability, such as the ferromagnetics, have very small reluctances and result in an increased measure of flux through the core. There is no widely accepted unit for reluctance, although the *rel* and the At/Wb are usually applied.

12.4 OHM'S LAW FOR MAGNETIC CIRCUITS

Recall the equation

$$\text{Effect} = \frac{\text{cause}}{\text{opposition}}$$

appearing in Chapter 4 to introduce Ohm's law for electric circuits. For magnetic circuits, the effect desired is the flux Φ. The cause is the **magnetomotive force (mmf)** $\mathcal{F}$, which is the external force (or "pressure") required to set up the **magnetic flux lines** within the magnetic material. The opposition to the setting up of the flux Φ is the reluctance $\mathcal{R}$.

Substituting, we have

$$\boxed{\Phi = \frac{\mathcal{F}}{\mathcal{R}}} \tag{12.5}$$

Since $\mathcal{F} = NI$, Eq. (12.5) clearly reveals that an increase in the number of turns or the current through the wire in Fig. 12.1 results in an increased "pressure" on the system to establish the flux lines through the core.

Although there is a great deal of similarity between electric and magnetic circuits, you must understand that the flux Φ is not a "flow" variable such as current in an electric circuit. Magnetic flux is established in the core through the alteration of the atomic structure of the core due to external pressure and is not a measure of the flow of some charged particles through the core.

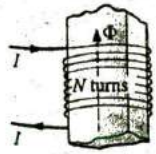


FIG. 12.1
Defining the components of a magnetomotive force.

12.5 MAGNETIZING FORCE

The magnetomotive force per unit length is called the **magnetizing force** (H). In equation form,

$$\boxed{H = \frac{\mathcal{F}}{l}} \qquad \text{(At/m)} \tag{12.6}$$

Substituting for the magnetomotive force results in

$$\boxed{H = \frac{NI}{l}} \qquad \text{(At/m)} \tag{12.7}$$

For the magnetic circuit in Fig. 12.2, if $NI = 40$ At and $l = 0.2$ m, then

$$H = \frac{NI}{l} = \frac{40\text{ At}}{0.2\text{ m}} = 200\text{ At/m}$$

In words, the result indicates that there are 200 At of "pressure" per meter to establish flux in the core.

Note in Fig. 12.2 that the direction of the flux Φ can be determined by placing the fingers of your right hand in the direction of current around the core and noting the direction of the thumb. It is interesting to realize that *the magnetizing force is independent of the type of core material*—it is determined solely by the number of turns, the current, and the length of the core.

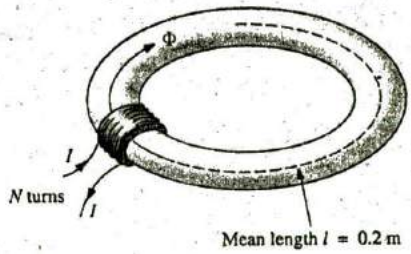


FIG. 12.2
Defining the magnetizing force of a magnetic circuit.

The applied magnetizing force has a pronounced effect on the resulting permeability of a magnetic material. As the magnetizing force increases, the permeability rises to a maximum and then drops to a minimum, as shown in Fig. 12.3 for three commonly employed magnetic materials.

The flux density and the magnetizing force are related by the following equation:

$$\boxed{B = \mu H} \tag{12.8}$$

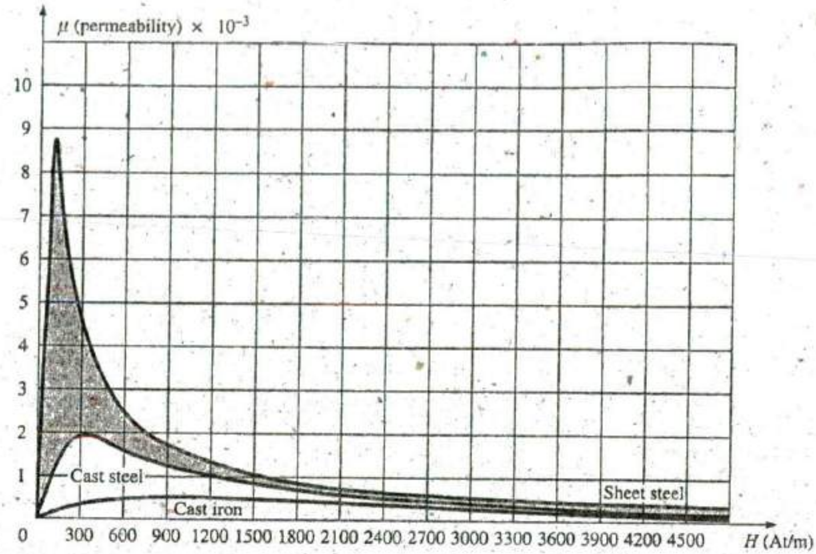


FIG.12.3
Variation of μ with the magnetizing force.

This equation indicates that for a particular magnetizing force, the greater the permeability, the greater is the induced flux density.

Since henries (H) and the magnetizing force (H) use the same capital letter, it must be pointed out that all units of measurement in the text, such as henries, use roman letters, such as H, whereas variables such as the magnetizing force use italic letters, such as H.

FIG.12.4
Series magnetic circuit used to define the hysteresis curve.

12.6 HYSTERESIS

A curve of the flux density B versus the magnetizing force H of a material is of particular importance to the engineer. Curves of this type can usually be found in manuals, descriptive pamphlets, and brochures published by manufacturers of magnetic materials. A typical B-H curve for a ferromagnetic material such as steel can be derived using the setup in Fig. 12.4.

The core is initially unmagnetized, and the current $I = 0$. If the current I is increased to some value above zero, the magnetizing force H increases to a value determined by

$$H\uparrow = \frac{NI\uparrow}{l}$$

The flux ϕ and the flux density B ($B = \phi/A$) also increase with the current I (or H). If the material has no residual magnetism, and the magnetizing force H is increased from zero to some value H_a, the B-H curve follows the path shown in Fig. 12.5 between o and a. If the magnetizing force H is increased until saturation (H_s) occurs, the curve continues as shown in the figure to point b. When saturation occurs, the flux density has, *for all practical purposes*, reached its maximum value. Any further increase in current through the coil increasing $H = NI/l$ results in a very small increase in flux density B.

If the magnetizing force is reduced to zero by letting I decrease to zero, the curve follows the path of the curve between b and c. The flux

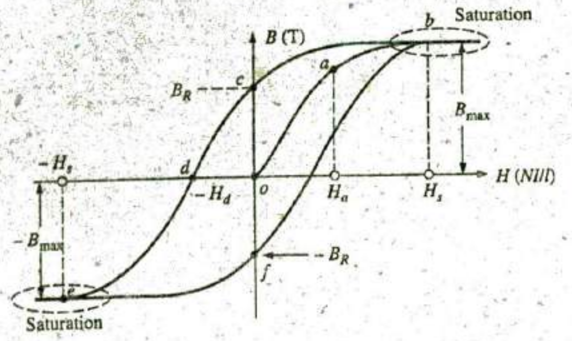


FIG. 12.5
Hysteresis curve.

density B_R, which remains when the magnetizing force is zero, is called the *residual flux density*. It is this residual flux density that makes it possible to create permanent magnets. If the coil is now removed from the core in Fig. 12.4, the core will still have the magnetic properties determined by the residual flux density, a measure of its "retentivity." If the current I is reversed, developing a magnetizing force, $-H$, the flux density B decreases with an increase in I. Eventually, the flux density will be zero when $-H_d$ (the portion of curve from c to d) is reached. The magnetizing force $-H_d$ required to "coerce" the flux density to reduce its level to zero is called the *coercive force*, a measure of the coercivity of the magnetic sample. As the force $-H$ is increased until saturation again occurs and is then reversed and brought back to zero, the path *def* results. If the magnetizing force is increased in the positive direction ($+H$), the curve traces the path shown from f to b. The entire curve represented by *bcdefb* is called the **hysteresis** curve for the ferromagnetic material, from the Greek *hysterein*, meaning "to lag behind." The flux density B *lagged* behind the magnetizing force H during the entire plotting of the curve. When H was zero at c, B was not zero but had only begun to decline. Long after H had passed through zero and had become equal to $-H_d$ did the flux density B finally become equal to zero.

If the entire cycle is repeated, the curve obtained for the same core will be determined by the maximum H applied. Three hysteresis loops for the same material for maximum values of H less than the saturation value are shown in Fig. 12.6. In addition, the saturation curve is repeated for comparison purposes.

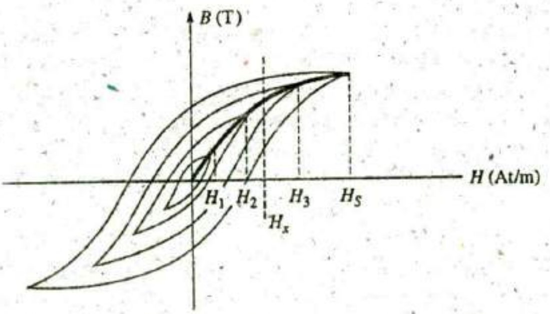


FIG. 12.6
Defining the normal magnetization curve.

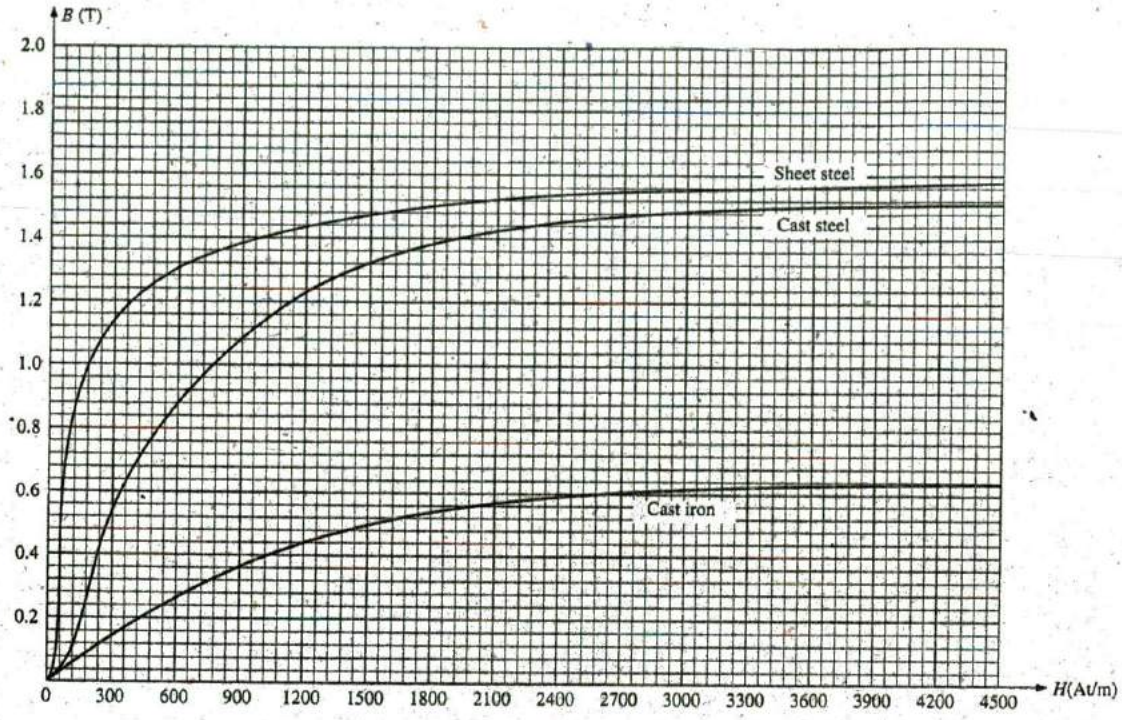


FIG.12.7
Normal magnetization curve for three ferromagnetic materials.

Note from the various curves that for a particular value of H, say, H_x, the value of B can vary widely, as determined by the history of the core. In an effort to assign a particular value of B to each value of H, we compromise by connecting the tips of the hysteresis loops. The resulting curve, shown by the heavy, solid line in Fig. 12.6 and for various materials in Fig. 12.7, is called the *normal magnetization curve.* An expanded view of one region appears in Fig. 12.8.

A comparison of Figs. 12.3 and 12.7 shows that for the same value of H, the value of B is higher in Fig. 12.7 for the materials with the higher μ in Fig. 12.3. This is particularly obvious for low values of H. This correspondence between the two figures must exist since $B = \mu H$. In fact, if in Fig. 12.7 we find μ for each value of H using the equation $\mu = B/H$, we obtain the curves in Fig. 12.3.

It is interesting to note that the hysteresis curves in Fig. 12.6 have a *point symmetry* about the origin; that is, the inverted pattern to the left of the vertical axis is the same as that appearing to the right of the vertical axis. In addition, you will find that a further application of the same magnetizing forces to the sample results in the same plot. For a current I in $H = NI/l$ that moves between positive and negative maximums at a fixed rate, the same B-H curve results during each cycle. Such will be the case when we examine ac (sinusoidal) networks in the later chapters. The reversal of the field (ϕ) due to the changing current direction results in a loss of energy that can best be described by first introducing the *domain theory of magnetism.*

Within each atom, the orbiting electrons (described in Chapter 2) are also spinning as they revolve around the nucleus. The atom, due to its spinning electrons, has a magnetic field associated with it. In nonmagnetic

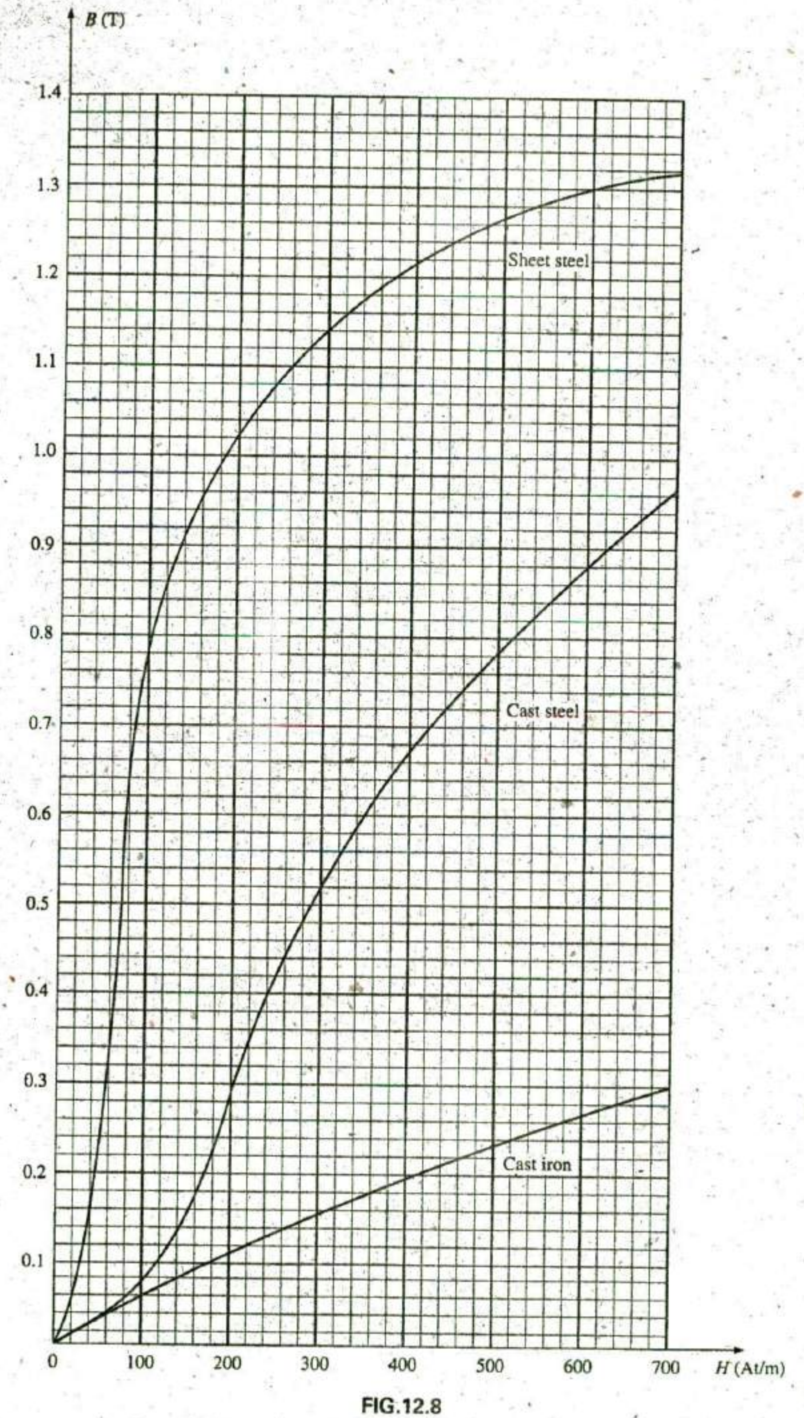
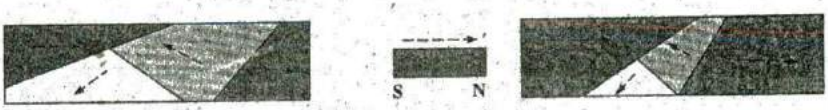


FIG.12.8
Expanded view of Fig. 12.7 for the low magnetizing force region.

materials, the net magnetic field is effectively zero since the magnetic fields due to the atoms of the material oppose each other. In magnetic materials such as iron and steel, however, the magnetic fields of groups of atoms numbering in the order of 10^{12} are aligned, forming very small bar magnets. This group of magnetically aligned atoms is called a **domain.** Each domain is a separate entity; that is, each domain is independent of the surrounding domains. For an unmagnetized sample of magnetic material, these domains appear in a random manner, such as shown in Fig. 12.9(a). The net magnetic field in any one direction is zero.



(a) (b)

FIG.12.9
Demonstrating the domain theory of magnetism.

When an external magnetizing force is applied, the domains that are nearly aligned with the applied field grow at the expense of the less favorably oriented domains, such as shown in Fig. 12.9(b). Eventually, if a sufficiently strong field is applied, all of the domains have the orientation of the applied magnetizing force, and any further increase in external field will not increase the strength of the magnetic flux through the core—a condition referred to as *saturation.* The elasticity of the above is evidenced by the fact that when the magnetizing force is removed, the alignment is lost to some measure, and the flux density drops to B_R. In other words, the removal of the magnetizing force results in the return of a number of misaligned domains within the core. The continued alignment of a number of the domains, however, accounts for our ability to create **permanent magnets.**

At a point just before saturation, the opposing unaligned domains are reduced to small cylinders of various shapes referred to as *bubbles.* These bubbles can be moved within the magnetic sample through the application of a *controlling* magnetic field. These magnetic bubbles form the basis of the recently designed bubble memory system for computers.

TABLE 12.1

	Electric Circuits	Magnetic Circuits
Cause	E	$\mathscr{F}$
Effect	I	Φ
Opposition	R	$\mathscr{R}$

12.7 AMPÈRE'S CIRCUITAL LAW

As mentioned in the introduction to this chapter, there is a broad similarity between the analyses of electric and magnetic circuits. This has already been demonstrated to some extent for the quantities in Table 12.1.

If we apply the "cause" analogy to Kirchhoff's voltage law ($\Sigma_{\circlearrowright} V = 0$), we obtain the following:

$$\boxed{\Sigma_{\circlearrowright} \mathscr{F} = 0} \qquad \text{(for magnetic circuits)} \qquad \textbf{(12.9)}$$

which, in words, states that the algebraic sum of the rises and drops of the mmf around a closed loop of a magnetic circuit is equal to zero; that is, the sum of the rises in mmf equals the sum of the drops in mmf around a closed loop.

Eq. (12.9) is referred to as **Ampère's circuital law.** When it is applied to magnetic circuits, sources of mmf are expressed by the equation

$$\boxed{\mathscr{F} = NI} \qquad \text{(At)} \qquad \textbf{(12.10)}$$

The equation for the mmf drop across a portion of a magnetic circuit can be found by applying the relationships listed in Table 12.1; that is, for electric circuits,

$$V = IR$$

resulting in the following for magnetic circuits:

$$\boxed{\mathscr{F} = \Phi\mathscr{R}} \quad \text{(At)} \qquad \textbf{(12.11)}$$

where ϕ is the flux passing through a section of the magnetic circuit and $\mathscr{R}$ is the reluctance of that section. The reluctance, however, is seldom calculated in the analysis of magnetic circuits. A more practical equation for the mmf drop is

$$\boxed{\mathscr{F} = Hl} \quad \text{(At)} \qquad \textbf{(12.12)}$$

as derived from Eq. (12.6), where H is the magnetizing force on a section of a magnetic circuit and l is the length of the section.

As an example of Eq. (12.9), consider the magnetic circuit appearing in Fig. 12.10 constructed of three different ferromagnetic materials.

Applying Ampère's circuital law, we have

$$\Sigma_C\, \mathscr{F} = 0$$

$$\underbrace{+NI}_{\text{Rise}} - \underbrace{H_{ab}l_{ab}}_{\text{Drop}} - \underbrace{H_{bc}l_{bc}}_{\text{Drop}} - \underbrace{H_{ca}l_{ca}}_{\text{Drop}} = 0$$

$$\underbrace{NI}_{\substack{\text{Impressed}\\ \text{mmf}}} = \underbrace{H_{ab}l_{ab} + H_{bc}l_{bc} + H_{ca}l_{ca}}_{\text{mmf drops}}$$

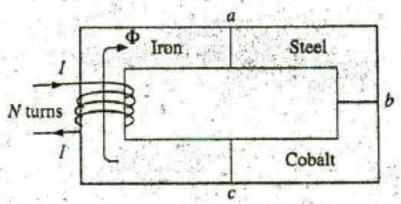


FIG.12.10
Series magnetic circuit of three different materials.

All the terms of the equation are known except the magnetizing force for each portion of the magnetic circuit, which can be found by using the *B-H* curve if the flux density *B* is known.

12.8 FLUX Φ

If we continue to apply the relationships described in the previous section to Kirchhoff's current law, we find that the sum of the fluxes entering a junction is equal to the sum of the fluxes leaving a junction; that is, for the circuit in Fig. 12.11,

$$\Phi_a = \Phi_b + \Phi_c \qquad \text{(at junction } a\text{)}$$

or

$$\Phi_b + \Phi_c = \Phi_a \qquad \text{(at junction } b\text{)}$$

which are equivalent.

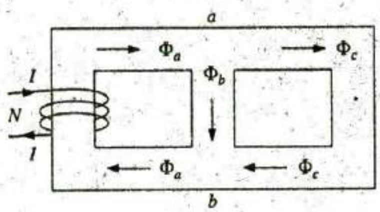


FIG.12.11
Flux distribution of a series-parallel magnetic network.

12.9 SERIES MAGNETIC CIRCUITS: DETERMINING *NI*

We are now in a position to solve a few magnetic circuit problems, which are basically of two types. *In one type, Φ is given, and the impressed mmf NI must be computed.* This is the type of problem encountered in the design of motors, generators, and transformers. *In the other type, NI is given, and the flux Φ of the magnetic circuit must be found.* This type of problem is encountered primarily in the design of magnetic amplifiers and is more difficult since the approach is "hit or miss."

As indicated in earlier discussions, the value of μ varies from point to point along the magnetization curve. This eliminates the possibility of finding the reluctance of each "branch" or the "total reluctance" of a network, as was done for electric circuits, where ρ had a fixed value for any applied current or voltage. If the total reluctance can be determined, Φ can then be determined using the Ohm's law analogy for magnetic circuits.

For magnetic circuits, the level of B or H is determined from the other using the B-H curve, and μ is seldom calculated unless asked for.

An approach frequently used in the analysis of magnetic circuits is the *table* method. Before a problem is analyzed in detail, a table is prepared listing in the far left column the various sections of the magnetic circuit (see Table 12.2). The columns on the right are reserved for the quantities to be found for each section. In this way, when you are solving a problem, you can keep track of what the next step should be and what is required to complete the problem. After a few examples, the usefulness of this method should become clear.

This section considers only *series* magnetic circuits in which the flux ϕ is the same throughout. In each example, the magnitude of the magnetomotive force is to be determined.

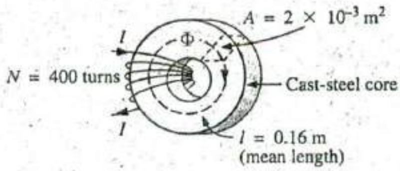


FIG.12.12
Example 12.1.

EXAMPLE 12.1 For the series magnetic circuit in Fig. 12.12:

a. Find the value of I required to develop a magnetic flux of $\Phi = 4 \times 10^{-4}$ Wb.
b. Determine μ and μ_r for the material under these conditions.

Solutions: The magnetic circuit can be represented by the system shown in Fig. 12.13(a). The electric circuit analogy is shown in Fig. 12.13(b). Analogies of this type can be very helpful in the solution of magnetic circuits. Table 12.2 is for part (a) of this problem. The table is fairly trivial for this example, but it does define the quantities to be found.

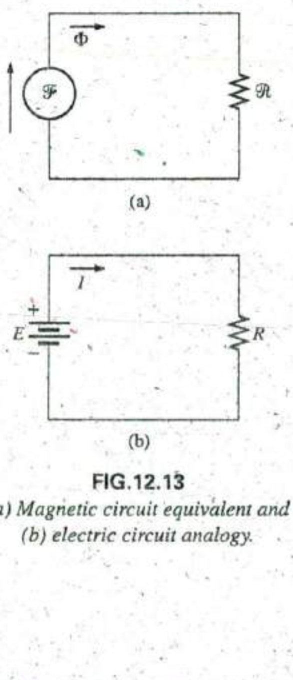


FIG.12.13
(a) Magnetic circuit equivalent and (b) electric circuit analogy.

a. The flux density B is

$$B = \frac{\Phi}{A} = \frac{4 \times 10^{-4}\,\text{Wb}}{2 \times 10^{-3}\,\text{m}^2} = 2 \times 10^{-1}\,\text{T} = 0.2\,\text{T}$$

Using the B-H curves in Fig. 12.8, we can determine the magnetizing force H:

$$H\,(\text{cast steel}) = 170\,\text{At/m}$$

Applying Ampère's circuital law yields

$$NI = Hl$$

and $$I = \frac{Hl}{N} = \frac{(170\,\text{At/m})(0.16\,\text{m})}{400\,\text{t}} = \mathbf{68\,mA}$$

(Recall that t represents turns.)

b. The permeability of the material can be found using Eq. (12.8):

$$\mu = \frac{B}{H} = \frac{0.2\,\text{T}}{170\,\text{At/m}} = \mathbf{1.176 \times 10^{-3}\,Wb/A \cdot m}$$

TABLE 12.2

Section	Φ (Wb)	A (m^2)	B (T)	H (At/m)	l (m)	Hl (At)
One continuous section	4×10^{-4}	2×10^{-3}			0.16	

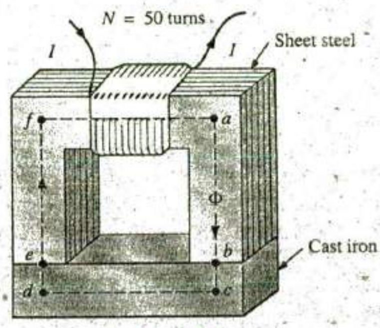
and the relative permeability is

$$\mu_r = \frac{\mu}{\mu_o} = \frac{1.176 \times 10^{-3}}{4\pi \times 10^{-7}} = \mathbf{935.83}$$

EXAMPLE 12.2 The electromagnet in Fig. 12.14 has picked up a section of cast iron. Determine the current I required to establish the indicated flux in the core.

Solution: To be able to use Figs. 12.7 and 12.8, we must first convert to the metric system. However, since the area is the same throughout, we can determine the length for each material rather than work with the individual sections:

$$l_{efab} = 4 \text{ in.} + 4 \text{ in.} + 4 \text{ in.} = 12 \text{ in.}$$
$$l_{bcde} = 0.5 \text{ in.} + 4 \text{ in.} + 0.5 \text{ in.} = 5 \text{ in.}$$
$$12 \text{ in.}\left(\frac{1 \text{ m}}{39.37 \text{ in.}}\right) = 304.8 \times 10^{-3} \text{ m}$$
$$5 \text{ in.}\left(\frac{1 \text{ m}}{39.37 \text{ in.}}\right) = 127 \times 10^{-3} \text{ m}$$
$$1 \text{ in.}^2\left(\frac{1 \text{ m}}{39.37 \text{ in.}}\right)\left(\frac{1 \text{ m}}{39.37 \text{ in.}}\right) = 6.452 \times 10^{-4} \text{ m}^2$$



$l_{ab} = l_{cd} = l_{ef} = l_{fa} = 4$ in.
$l_{bc} = l_{de} = 0.5$ in.
Area (throughout) = 1 in.^{2}
$\Phi = 3.5 \times 10^{-4}$ Wb

FIG. 12.12
Electromagnet for Example 12.2.

The information available from the *efab* and *bcde* specifications of the problem has been inserted in Table 12.3. When the problem has been completed, each space will contain some information. Sufficient data to complete the problem can be found if we fill in each column from left to right. As the various quantities are calculated, they will be placed in a similar table found at the end of the example.

TABLE 12.3

Section	Φ (Wb)	A (m^2)	B (T)	H (At/m)	l (m)	Hl (At)
efab	3.5×10^{-4}	6.452×10^{-4}			304.8×10^{-3}	
bcde	3.5×10^{-4}	6.452×10^{-4}			127×10^{-3}	

The flux density for each section is

$$B = \frac{\Phi}{A} = \frac{3.5 \times 10^{-4} \text{ Wb}}{6.452 \times 10^{-4} \text{ m}^2} = 0.542 \text{ T}$$

and the magnetizing force is

$$H \text{ (sheet steel, Fig. 12.8)} \cong 70 \text{ At/m}$$
$$H \text{ (cast iron, Fig. 12.7)} \cong 1600 \text{ At/m}$$

Note the extreme difference in magnetizing force for each material for the required flux density. In fact, when we apply Ampère's circuital law, we find that the sheet steel section can be ignored with a minimal error in the solution.

Determining Hl for each section yields

$$H_{efab}l_{efab} = (70 \text{ At/m})(304.8 \times 10^{-3} \text{ m}) = 21.34 \text{ At}$$
$$H_{bcde}l_{bcde} = (1600 \text{ At/m})(127 \times 10^{-3} \text{ m}) = 203.2 \text{ At}$$

Inserting the above data in Table 12.3 results in Table 12.4.

TABLE 12.4

Section	Φ (Wb)	A (m^2)	B (T)	H (At/m)	l (m)	Hl (At)
efab	3.5×10^{-4}	6.452×10^{-4}	0.542	70	304.8×10^{-3}	21.34
bcde	3.5×10^{-4}	6.452×10^{-4}	0.542	1600	127×10^{-3}	203.2

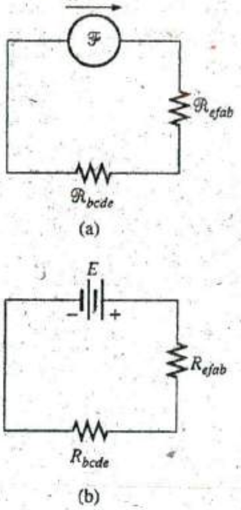


FIG.12.15
(a) Magnetic circuit equivalent and (b) electric circuit analogy for the electromagnet in Fig. 12.14.

The magnetic circuit equivalent and the electric circuit analogy for the system in Fig. 12.14 appear in Fig. 12.15.

Applying Ampère's circuital law, we obtain

$$NI = H_{efab}l_{efab} + H_{bcde}l_{bcde}$$
$$= 21.34 \text{ At} + 203.2 \text{ At} = 224.54 \text{ At}$$

and
$$(50 \text{ t})I = 224.54 \text{ At}$$

so that
$$I = \frac{224.54 \text{ At}}{50 \text{ t}} = \mathbf{4.49\ A}$$

EXAMPLE 12.3 Determine the secondary current I_2 for the transformer in Fig. 12.16 if the resultant clockwise flux in the core is 1.5×10^{-5} Wb.

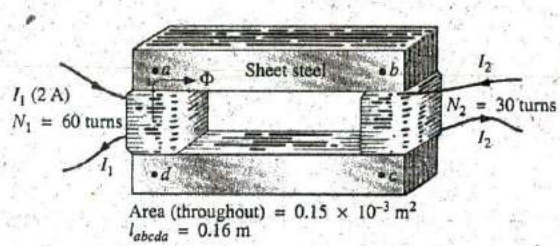


FIG.12.16
Transformer for Example 12.3.

Solution: This is the first example with two magnetizing forces to consider. In the analogies in Fig. 12.17, note that the resulting flux of each is opposing, just as the two sources of voltage are opposing in the electric circuit analogy.

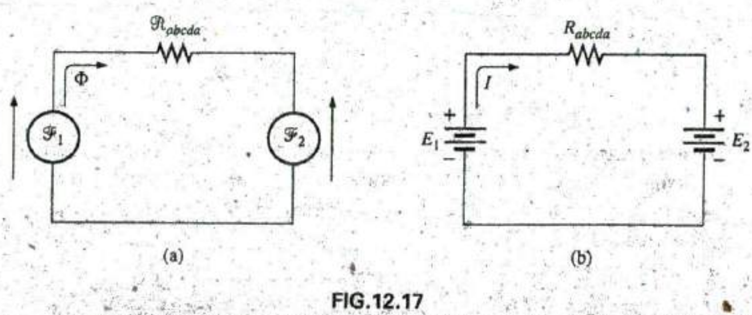


FIG.12.17
(a) Magnetic circuit equivalent and (b) electric circuit analogy for the transformer in Fig. 12.16.

The *abcda* structural data appear in Table 12.5.

TABLE 12.5

Section	Φ (Wb)	A (m²)	B (T)	H (At/m)	l (m)	Hl (At)
abcda	1.5×10^{-5}	0.15×10^{-3}			0.16	

The flux density throughout is

$$B = \frac{\phi}{A} = \frac{1.5 \times 10^{-5}\ \text{Wb}}{0.15 \times 10^{-3}\ \text{m}^2} = 10 \times 10^{-2}\ \text{T} = 0.10\ \text{T}$$

and

$$H\ (\text{from Fig. 12.8}) \cong \frac{1}{5}(100\ \text{At/m}) = 20\ \text{At/m}$$

Applying Ampère's circuital law, we obtain

$$N_1I_1 - N_2I_2 = H_{abcda}l_{abcda}$$
$$(60\ \text{t})(2\ \text{A}) - (30\ \text{t})(I_2) = (20\ \text{At/m})(0.16\text{m})$$
$$120\ \text{At} - (30\ \text{t})I_2 = 3.2\ \text{At}$$

and

$$(30\ \text{t})I_2 = 120\ \text{At} - 3.2\ \text{At}$$

or

$$I_2 = \frac{116.8\ \text{At}}{30\ \text{t}} = \mathbf{3.89\ A}$$

For the analysis of most transformer systems, the equation $N_1I_1 = N_2I_2$ is used. This results in 4 A versus 3.89 A above. This difference is normally ignored, however, and the equation $N_1I_1 = N_2I_2$ considered exact.

Because of the nonlinearity of the B-H curve, *it is not possible to apply superposition to magnetic circuits;* that is, in Example 12.3, we cannot consider the effects of each source independently and then find the total effects by using superposition.

12.10 AIR GAPS

Before continuing with the illustrative examples, let us consider the effects that an air gap has on a magnetic circuit. Note the presence of air gaps in the magnetic circuits of the motor and meter in Fig. 11.15. The spreading of the flux lines outside the common area of the core for the air gap in Fig. 12.18(a) is known as *fringing*. For our purposes, we shall ignore this effect and assume the flux distribution to be as in Fig. 12.18(b).

The flux density of the air gap in Fig. 12.18(b) is given by

$$\boxed{B_g = \frac{\Phi_g}{A_g}} \tag{12.13}$$

where, for our purposes,

$$\Phi_g = \Phi_{\text{core}}$$

and

$$A_g = A_{\text{core}}$$

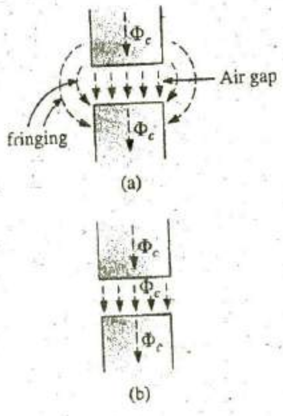


FIG. 12.18
Air gaps: (a) with fringing; (b) ideal.

For most practical applications, the permeability of air is taken to be equal to that of free space. The magnetizing force of the air gap is then determined by

$$\boxed{H_g = \frac{B_g}{\mu_o}} \tag{12.14}$$

and the mmf drop across the air gap is equal to $H_g L_g$. An equation for H_g is as follows:

$$H_g = \frac{B_g}{\mu_o} = \frac{B_g}{4\pi \times 10^{-7}}$$

and

$$\boxed{H_g = (7.96 \times 10^5)B_g} \qquad \text{(At/m)} \tag{12.15}$$

EXAMPLE 12.4 Find the value of I required to establish a magnetic flux of $\phi = 0.75 \times 10^{-4}$ Wb in the series magnetic circuit in Fig. 12.19.

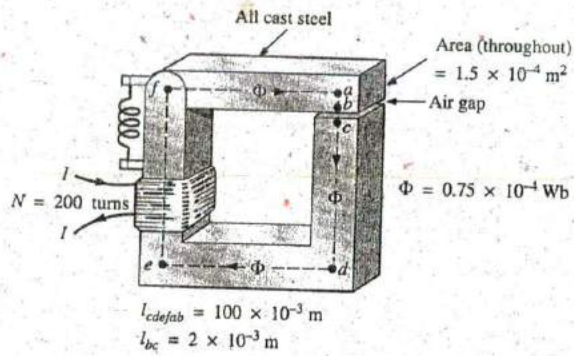


FIG. 12.19
Relay for Example 12.4.

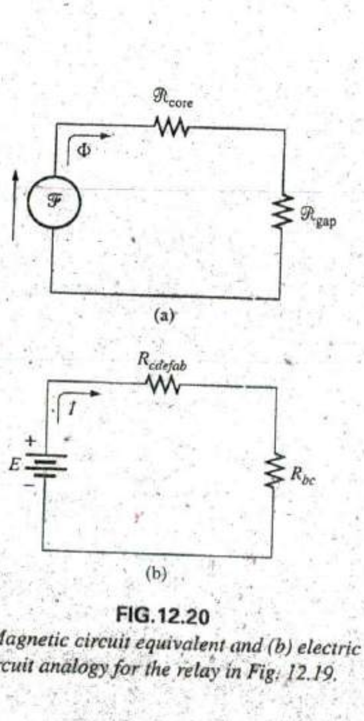


FIG. 12.20
(a) Magnetic circuit equivalent and (b) electric circuit analogy for the relay in Fig. 12.19.

Solution: An equivalent magnetic circuit and its electric circuit analogy are shown in Fig. 12.20.

The flux density for each section is

$$B = \frac{\Phi}{A} = \frac{0.75 \times 10^{-4}\ \text{Wb}}{1.5 \times 10^{-4}\ \text{m}^2} = 0.5\ \text{T}$$

From the B-H curves in Fig. 12.8,

$$H\ (\text{cast steel}) \cong 280\ \text{At/m}$$

Applying Eq. (12.15),

$$H_g = (7.96 \times 10^5)B_g = (7.96 \times 10^5)(0.5\ \text{T}) = 3.98 \times 10^5\ \text{At/m}$$

The mmf drops are

$$H_{core}l_{core} = (280\ \text{At/m})(100 \times 10^{-3}\ \text{m}) = 28\ \text{At}$$
$$H_g l_g = (3.98 \times 10^5\ \text{At/m})(2 \times 10^{-3}\ \text{m}) = 796\ \text{At}$$

Applying Ampère's circuital law, we obtain

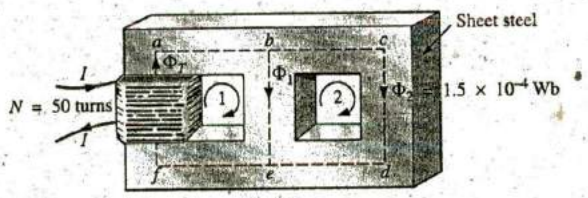
$$NI = H_{core}l_{core} + H_g l_g$$
$$= 28\text{ At} + 796\text{ At}$$
$$(200\text{ t})I = 824\text{ At}$$
$$I = \mathbf{4.12\ A}$$

Note from the above that the air gap requires the biggest share (by far) of the impressed NI because air is nonmagnetic.

12.11 SERIES-PARALLEL MAGNETIC CIRCUITS

As one might expect, the close analogies between electric and magnetic circuits eventually lead to series-parallel magnetic circuits similar in many respects to those encountered in Chapter 7. In fact, the electric circuit analogy will prove helpful in defining the procedure to follow toward a solution.

EXAMPLE 12.5 Determine the current I required to establish a flux of 1.5×10^{-4} Wb in the section of the core indicated in Fig. 12.21.



$l_{bcde} = l_{efab} = 0.2$ m
$l_{be} = 0.05$ m
Cross-sectional area = 6×10^{-4} m² throughout

FIG.12.21
Example 12.5.

Solution: The equivalent magnetic circuit and the electric circuit analogy appear in Fig. 12.22. We have

$$B_2 = \frac{\Phi_2}{A} = \frac{1.5 \times 10^{-4}\text{ Wb}}{6 \times 10^{-4}\text{ m}^2} = 0.25\text{ T}$$

From Fig. 12.8,

$$H_{bcde} \cong 40\text{ At/m}$$

Applying Ampère's circuital law around loop 2 in Figs. 12.21 and 12.22,

$$\Sigma_{\circlearrowright} \mathcal{F} = 0$$
$$H_{be}l_{be} - H_{bcde}l_{bcde} = 0$$
$$H_{be}(0.05\text{ m}) - (40\text{ At/m})(0.2\text{m}) = 0$$
$$H_{be} = \frac{8\text{ At}}{0.05\text{ m}} = 160\text{ At/m}$$

From Fig. 12.8,

$$B_1 \cong 0.97\text{ T}$$

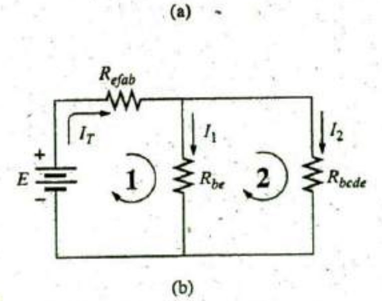


FIG.12.22
(a) Magnetic circuit equivalent and (b) electric circuit analogy for the series-parallel system in Fig. 12.21.

and

$$\Phi_1 = B_1A = (0.97\text{ T})(6 \times 10^{-4}\text{ m}^2) = 5.82 \times 10^{-4}\text{ Wb}$$

The results for *bcde*, *be*, and *efab* are entered in Table 12.6.

TABLE 12.6

Section	Φ (Wb)	A (m²)	B (T)	H (At/m)	l (m)	Hl (At)
bcde	1.5×10^{-4}	6×10^{-4}	0.25	40	0.2	8
be	5.82×10^{-4}	6×10^{-4}	0.97	160	0.05	8
efab		6×10^{-4}			0.2	

Table 12.6 reveals that we must now turn our attention to section *efab:*

$$\Phi_T = \Phi_1 + \Phi_2 = 5.82 \times 10^{-4}\text{ Wb} + 1.5 \times 10^{-4}\text{ Wb}$$
$$= 7.32 \times 10^{-4}\text{ Wb}$$

$$B = \frac{\Phi_T}{A} = \frac{7.32 \times 10^{-4}\text{ Wb}}{6 \times 10^{-4}\text{ m}^2}$$
$$= 1.22\text{ T}$$

From Fig. 12.7,

$$H_{efab} \cong 400\text{ At}$$

Applying Ampère's circuital law, we find

$$+NI - H_{efab}l_{efab} - H_{be}l_{be} = 0$$
$$NI = (400\text{ At/m})(0.2\text{ m}) + (160\text{ At/m})(0.05\text{ m})$$
$$(50\text{ t})I = 80\text{ At} + 8\text{ At}$$
$$I = \frac{88\text{ At}}{50\text{ t}} = \mathbf{1.76\text{ A}}$$

To demonstrate that μ is sensitive to the magnetizing force H, the permeability of each section is determined as follows. For section *bcde*,

$$\mu = \frac{B}{H} = \frac{0.25\text{ T}}{40\text{ At/m}} = 6.25 \times 10^{-3}$$

and
$$\mu_r = \frac{\mu}{\mu_o} = \frac{6.25 \times 10^{-3}}{12.57 \times 10^{-7}} = \mathbf{4972.2}$$

For section *be*,

$$\mu = \frac{B}{H} = \frac{0.97\text{ T}}{160\text{ At/m}} = 6.06 \times 10^{-3}$$

and
$$\mu_r = \frac{\mu}{\mu_o} = \frac{6.06 \times 10^{-3}}{12.57 \times 10^{-7}} = \mathbf{4821}$$

For section *efab*,

$$\mu = \frac{B}{H} = \frac{1.22\text{ T}}{400\text{ At/m}} = 3.05 \times 10^{-3}$$

and
$$\mu_r = \frac{\mu}{\mu_o} = \frac{3.05 \times 10^{-3}}{12.57 \times 10^{-7}} = \mathbf{2426.41}$$

12.12 DETERMINING Φ

The examples of this section are of the second type, where NI is given and the flux Φ must be found. This is a relatively straightforward problem if only one magnetic section is involved. Then

$$H = \frac{NI}{l} \quad H \rightarrow B \qquad (B\text{-}H \text{ curve})$$

and

$$\Phi = BA$$

For magnetic circuits with more than one section, there is no set order of steps that lead to an exact solution for every problem on the first attempt. In general, however, we proceed as follows. We must find the impressed mmf for a *calculated guess* of the flux Φ and then compare this with the specified value of mmf. We can then make adjustments to our guess to bring it closer to the actual value. For most applications, a value within ±5% of the actual Φ or specified NI is acceptable.

We can make a reasonable guess at the value of Φ if we realize that the maximum mmf drop appears across the material with the smallest permeability if the length and area of each material are the same. As shown in Example 12.4, if there is an air gap in the magnetic circuit, there will be a considerable drop in mmf across the gap. As a starting point for problems of this type, therefore, we shall assume that the total mmf (NI) is across the section with the lowest μ or greatest $\mathcal{R}$ (if the other physical dimensions are relatively similar). This assumption gives a value of Φ that will produce a calculated NI greater than the specified value. Then, after considering the results of our original assumption very carefully, we shall *cut* Φ and NI by introducing the effects (reluctance) of the other portions of the magnetic circuit and *try* the new solution. For obvious reasons, this approach is frequently called the *cut and try* method.

EXAMPLE 12.6 Calculate the magnetic flux Φ for the magnetic circuit in Fig. 12.23.

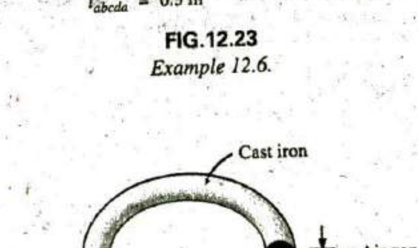


FIG.12.23
Example 12.6.

Solution: By Ampère's circuital law,

$$NI = H_{abcda} l_{abcda}$$

or

$$H_{abcda} = \frac{NI}{l_{abcda}} = \frac{(60\text{ t})(5\text{ A})}{0.3\text{ m}} = \frac{300\text{ At}}{0.3\text{ m}} = 1000\text{ At/m}$$

and

$$B_{abcda} \text{ (from Fig. 12.7)} \cong 0.39\text{ T}$$

Since $B = \Phi/A$, we have

$$\Phi = BA = (0.39\text{ T})(2 \times 10^{-4}\text{ m}^2) = \mathbf{0.78 \times 10^{-4}\ Wb}$$

EXAMPLE 12.7 Find the magnetic flux Φ for the series magnetic circuit in Fig. 12.24 for the specified impressed mmf.

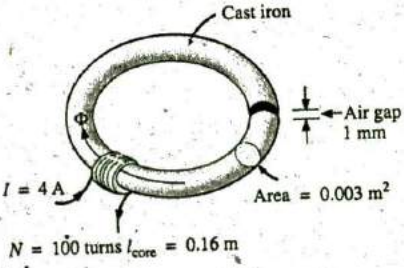


FIG.12.24
Example 12.7.

Solution: Assuming that the total impressed mmf NI is across the air gap, we obtain

$$NI = H_g l_g$$

or

$$H_g = \frac{NI}{l_g} = \frac{400\text{ At}}{0.001\text{ m}} = 4 \times 10^5\text{ At/m}$$

and $$B_g = \mu_o H_g = (4\pi \times 10^{-7})(4 \times 10^5 \text{ At/m}) = 0.503 \text{ T}$$

The flux is given by

$$\begin{aligned}\Phi_g &= \Phi_{core} = B_g A \\ &= (0.503 \text{ T})(0.003 \text{ m}^2) \\ \Phi_{core} &= 1.51 \times 10^{-3} \text{ Wb}\end{aligned}$$

Using this value of Φ, we can find NI. The core and gap data are inserted in Table 12.7.

TABLE 12.7

Section	Φ (Wb)	A (m^2)	B (T)	H (At/m)	l (m)	Hl (At)
Core	1.51×10^{-3}	0.003	0.503	1500 (B-H curve)	0.16	
Gap	1.51×10^{-3}	0.003	0.503	4×10^5	0.001	400

$$H_{core}l_{core} = (1500 \text{ At/m})(0.16 \text{ m}) = 240 \text{ At}$$

Applying Ampère's circuital law results in

$$\begin{aligned}NI &= H_{core}l_{core} + H_g l_g \\ &= 240 \text{ At} + 400 \text{ At} \\ 400 \text{ At} &\neq 640 \text{ At}\end{aligned}$$

Since we neglected the reluctance of all the magnetic paths but the air gap, the calculated value is greater than the specified value. We must therefore reduce this value by including the effect of these reluctances. Since approximately (640 At − 400 At)/640 At = 240 At/640 At ≅ 37.5% of our calculated value is above the desired value, let us reduce Φ by 30% and see how close we come to the impressed mmf of 400 At:

$$\begin{aligned}\Phi &= (1 - 0.3)(1.51 \times 10^{-3} \text{ Wb}) \\ &= 1.057 \times 10^{-3} \text{ Wb}\end{aligned}$$

See Table 12.8. We have

TABLE 12.8

Section	Φ (Wb)	A (m^2)	B (T)	H (At/m)	l (m)	Hl (At)
Core	1.057×10^{-3}	0.003			0.16	
Gap	1.057×10^{-3}	0.003			0.001	

$$B = \frac{\Phi}{A} = \frac{1.057 \times 10^{-3} \text{ Wb}}{0.003 \text{ m}^3} \cong 0.352 \text{ T}$$

$$\begin{aligned}H_g l_g &= (7.96 \times 10^5)B_g l_g \\ &= (7.96 \times 10^5)(0.352 \text{ T})(0.001 \text{ m}) \\ &\cong 280.19 \text{ At}\end{aligned}$$

From the B-H curves,

$$H_{core} \cong 850 \text{ At/m}$$
$$H_{core}l_{core} = (850 \text{ At/m})(0.16 \text{ m}) = 136 \text{ At}$$

Applying Ampère's circuital law yields

$$NI = H_{core}l_{core} + H_g l_g$$
$$= 136\text{ At} + 280.19\text{ At}$$
$$400\text{ At} = \mathbf{416.19\text{ At}} \quad \text{(but within } \pm 5\% \text{ and therefore acceptable)}$$

The solution is, therefore,

$$\Phi \cong \mathbf{1.057 \times 10^{-3}\ Wb}$$

12.13 APPLICATIONS

Speakers and Microphones

Electromagnetic effects are the moving force in the design of speakers such as the one shown in Fig. 12.25. The shape of the pulsating waveform of the input current is determined by the sound to be reproduced by the speaker at a high audio level. As the current peaks and returns to the valleys of the sound pattern, the strength of the electromagnet varies in exactly the same manner. This causes the cone of the speaker to vibrate at a frequency directly proportional to the pulsating input. The higher the pitch of the sound pattern, the higher is the oscillating frequency between the peaks and valleys and the higher is the frequency of vibration of the cone.

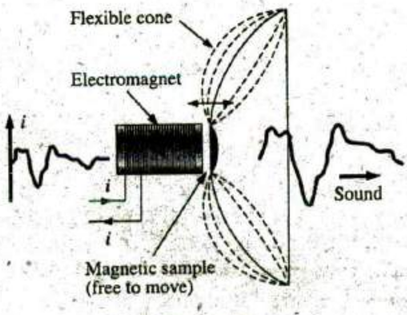


FIG.12.25
Speaker.

A second design used more frequently in more expensive speaker systems appears in Fig. 12.26. In this case, the permanent magnet is fixed, and the input is applied to a movable core within the magnet, as shown in the figure. High peaking currents at the input produce a strong flux pattern in the voice coil, causing it to be drawn well into the flux pattern of the permanent magnet. As occurred for the speaker in Fig. 12.25, the core then vibrates at a rate determined by the input and provides the audible sound.

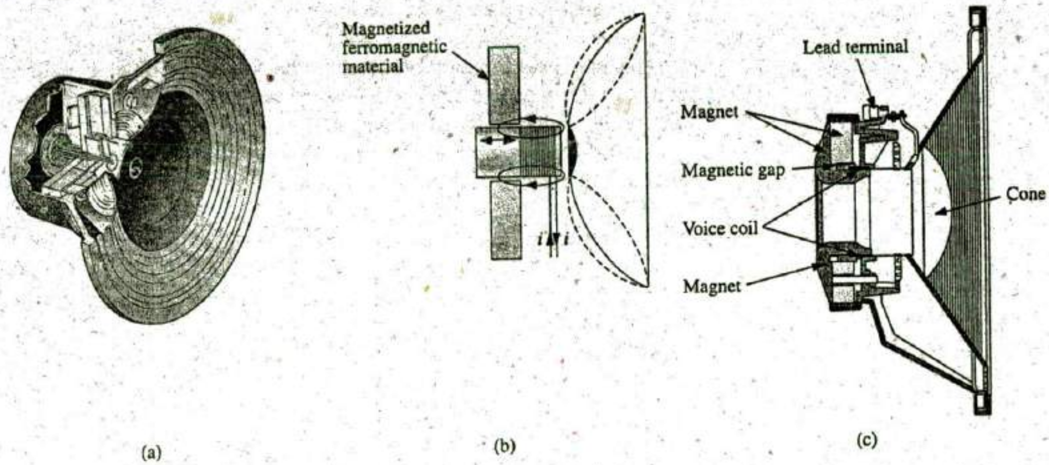


FIG.12.26
Coaxial high-fidelity loudspeaker: (a) construction; (b) basic operation; (c) cross section of actual unit.
(Courtesy of Electro-Voice, Inc.)

Microphones also employ electromagnetic effects. The incoming sound causes the core and attached moving coil to move within the magnetic field of the permanent magnet. Through Faraday's law

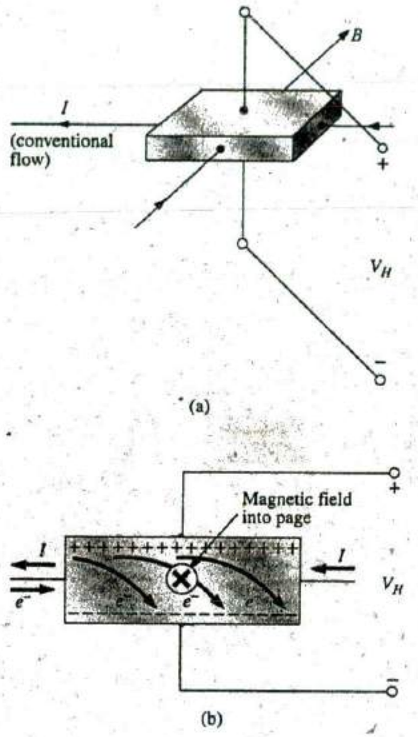


FIG.12.27
Hall effect sensor: (a) orientation of controlling parameters; (b) effect on electron flow.

($e = N\,d\phi/di$), a voltage is induced across the movable coil proportional to the speed with which it is moving through the magnetic field. The resulting induced voltage pattern can then be amplified and reproduced at a much higher audio level through the use of speakers, as described earlier. Microphones of this type are the most frequently employed, although other types that use capacitive, carbon granular, and piezoelectric* effects are available. This particular design is commercially referred to as a *dynamic* microphone.

Hall Effect Sensor

The Hall effect sensor is a semiconductor device that generates an output voltage when exposed to a magnetic field. The basic construction consists of a slab of semiconductor material through which a current is passed, as shown in Fig. 12.27(a). If a magnetic field is applied, as shown in the figure, perpendicular to the direction of the current, a voltage V_H is generated between the two terminals, as indicated in Fig. 12.27(a). The difference in potential is due to the separation of charge established by the Lorentz force first studied by Professor Hendrick Lorentz in the late 1800s. He found that electrons in a magnetic field are subjected to a force proportional to the velocity of the electrons through the field and the strength of the magnetic field. The direction of the force is determined by the left-hand rule. Simply place the index finger of your left hand in the direction of the magnetic field, with the second finger at right angles to the index finger in the direction of conventional current through the semiconductor material, as shown in Fig. 12.27(b). The thumb, if placed at right angles to the index finger, will indicate the direction of the force on the electrons. In Fig. 12.27(b), the force causes the electrons to accumulate in the bottom region of the semiconductor (connected to the negative terminal of the voltage V_H), leaving a net positive charge in the upper region of the material (connected to the positive terminal of V_H). The stronger the current or strength of the magnetic field, the greater is the induced voltage V_H.

In essence, therefore, the Hall effect sensor can reveal the strength of a magnetic field or the level of current through a device if the other determining factor is held fixed. Two applications of the sensor are therefore apparent—to measure the strength of a magnetic field in the vicinity of a sensor (for an applied fixed current) and to measure the level of current through a sensor (with knowledge of the strength of the magnetic field linking the sensor). The gaussmeter in Fig. 11.14 uses a Hall effect sensor. Internal to the meter, a fixed current is passed through the sensor with the voltage V_H indicating the relative strength of the field. Through amplification, calibration, and proper scaling, the meter can display the relative strength in gauss.

The Hall effect sensor has a broad range of applications that are often quite interesting and innovative. The most widespread is as a trigger for an alarm system in large department stores, where theft is often a difficult problem. A magnetic strip attached to the merchandise sounds an alarm when a customer passes through the exit gates without paying for the product. The sensor, control current, and monitoring system are housed in the exit fence and react to the presence of the magnetic

*Piezoelectricity is the generation of a small voltage by exerting pressure across certain crystals.

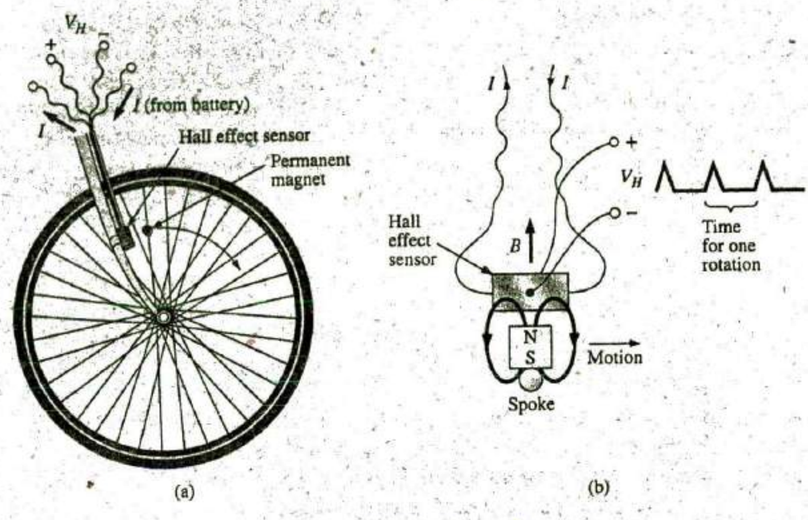


FIG.12.28
Obtaining a speed indication for a bicycle using a Hall effect sensor: (a) mounting the components; (b) Hall effect response.

field as the product leaves the store. When the product is paid for, the cashier removes the strip or demagnetizes the strip by applying a magnetizing force that reduces the residual magnetism in the strip to essentially zero.

The Hall effect sensor is also used to indicate the speed of a bicycle on a digital display conveniently mounted on the handlebars. As shown in Fig. 12.28(a), the sensor is mounted on the frame of the bike, and a small permanent magnet is mounted on a spoke of the front wheel. The magnet must be carefully mounted to be sure that it passes over the proper region of the sensor. When the magnet passes over the sensor, the flux pattern in Fig. 12.28(b) results, and a voltage with a sharp peak is developed by the sensor. For a bicycle with a 26-in.-diameter wheel, the circumference will be about 82 in. Over 1 mi, the number of rotations is

$$5280\ \cancel{\text{ft}}\left(\frac{12\ \cancel{\text{in.}}}{1\ \cancel{\text{ft}}}\right)\left(\frac{1\ \text{rotation}}{82\ \cancel{\text{in.}}}\right) \cong 773\ \text{rotations}$$

If the bicycle is traveling at 20 mph, an output pulse occurs at a rate of 4.29 per second. It is interesting to note that at a speed of 20 mph, the wheel is rotating at more than 4 revolutions per second, and the total number of rotations over 20 mi is 15,460.

Magnetic Reed Switch

One of the most frequently employed switches in alarm systems is the *magnetic reed switch* shown in Fig. 12.29. As shown by the figure, there are two components of the reed switch—a permanent magnet embedded in one unit that is normally connected to the movable element (door, window, and so on) and a reed switch in the other unit that is connected to the electrical control circuit. The reed switch is constructed of two iron-alloy (ferromagnetic) reeds in a hermetically sealed capsule. The cantilevered ends of the two reeds do not touch but are in very close proximity to one another. In the absence of a

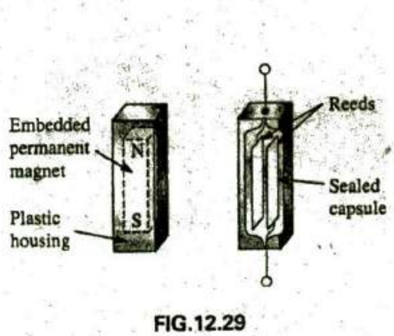


FIG.12.29
Magnetic reed switch.

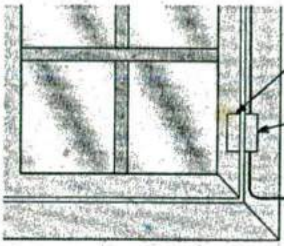


FIG.12.30
Using a magnetic reed switch to monitor the state of a window.

magnetic field, the reeds remain separated. However, if a magnetic field is introduced, the reeds are drawn to each other because flux lines seek the path of least reluctance and, if possible, exercise every alternative to establish the path of least reluctance. It is similar to placing a ferromagnetic bar close to the ends of a U-shaped magnet. The bar is drawn to the poles of the magnet, establishing a magnetic flux path without air gaps and with minimum reluctance. In the open-circuit state, the resistance between reeds is in excess of 100 MΩ, while in the on state it drops to less than 1 Ω.

In Fig. 12.30 a reed switch has been placed on the fixed frame of a window and a magnet on the movable window unit. When the window is closed as shown in Fig. 12.30, the magnet and reed switch are sufficiently close to establish contact between the reeds, and a current is established through the reed switch to the control panel. In the armed state, the alarm system accepts the resulting current flow as a normal secure response. If the window is opened, the magnet leaves the vicinity of the reed switch, and the switch opens. The current through the switch is interrupted, and the alarm reacts appropriately.

One of the distinct advantages of the magnetic reed switch is that the proper operation of any switch can be checked with a portable magnetic element. Simply bring the magnet to the switch and note the output response. There is no need to continually open and close windows and doors. In addition, the reed switch is hermetically enclosed so that oxidation and foreign objects cannot damage it, and the result is a unit that can last indefinitely. Magnetic reed switches are also available in other shapes and sizes, allowing them to be concealed from obvious view. One is a circular variety that can be set into the edge of a door and door jam, resulting in only two small visible disks when the door is open.

Magnetic Resonance Imaging

Magnetic resonance imaging (MRI) provides quality cross-sectional images of the body for medical diagnosis and treatment. MRI does not expose the patient to potentially hazardous X-rays or injected contrast materials such as those used to obtain computerized axial tomography (CAT) scans.

The three major components of an MRI system are a strong magnet, a table for transporting the patient into the circular hole in the magnet, and a control center, as shown in Fig. 12.31. The image is obtained by placing the patient in the tube to a precise depth depending on the cross section to be obtained and applying a strong magnetic field that causes the nuclei of certain atoms in the body to line up. Radio waves of different frequencies are then applied to the patient in the region of interest, and if the frequency of the wave matches the natural frequency of the atom, the nuclei is set into a state of resonance and absorbs energy from the applied signal. When the signal is removed, the nuclei release the acquired energy in the form of weak but detectable signals. The strength and duration of the energy emission vary from one tissue of the body to another. The weak signals are then amplified, digitized, and translated to provide a cross-sectional image such as the one shown in Fig. 12.32. For some patients the claustrophobic feeling they experience while in the circular tube is

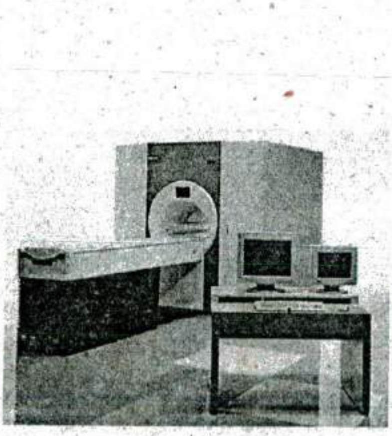


FIG.12.31
Magnetic resonance imaging equipment.
(Courtesy of Siemens Medical Systems, Inc.)

FIG.12.32
Magnetic resonance image.
(Courtesy of Siemens Medical Systems, Inc.)

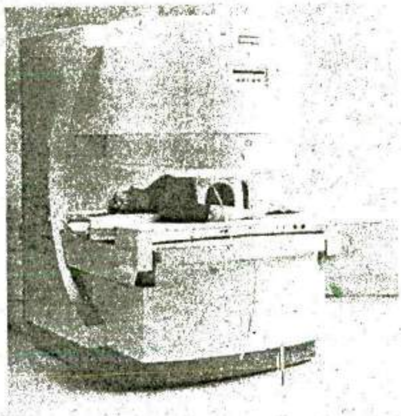


FIG.12.33
Magnetic resonance imaging equipment (open variety).
(Courtesy of Siemens Medical Systems, Inc.)

difficult to contend with. A more open unit has been developed, as shown in Fig. 12.33, that has removed most of this discomfort.

Patients who have metallic implants or pacemakers or those who have worked in industrial environments where minute ferromagnetic particles may have become lodged in open, sensitive areas such as the eyes, nose, and so on, may have to use a CAT scan system because it does not employ magnetic effects. The attending physician is well trained in such areas of concern and will remove any unfounded fears or suggest alternative methods.

PROBLEMS

SECTION 12.2 Magnetic Field

1. Using Appendix E, fill in the blanks in the following table. Indicate the units for each quantity.

	Φ	B
SI	5×10^{-4} Wb	8×10^{-4} T
CGS	______	______
English	______	______

2. Repeat Problem 1 for the following table if area = 2 in.^{2}.

	Φ	B
SI	______	______
CGS	60,000 maxwells	______
English	______	______

3. For the electromagnet in Fig. 12.34:
   a. Find the flux density in the core.
   b. Sketch the magnetic flux lines and indicate their direction.
   c. Indicate the north and south poles of the magnet.

FIG.12.34
Problem 3.

SECTION 12.3 Reluctance

4. Which section of Fig. 12.35—(a), (b), or (c)—has the largest reluctance to the setting up of flux lines through its longest dimension?

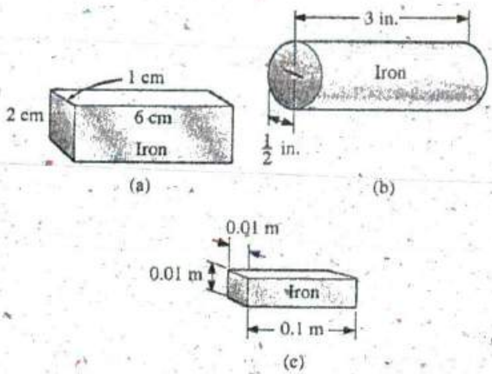


FIG. 12.35
Problem 4.

SECTION 12.4 Ohm's Law for Magnetic Circuits

5. Find the reluctance of a magnetic circuit if a magnetic flux $\Phi = 4.2 \times 10^{-4}$ Wb is established by an impressed mmf of 400 At.

6. Repeat Problem 5 for $\Phi = 72{,}000$ maxwells and an impressed mmf of 120 gilberts.

SECTION 12.5 Magnetizing Force

7. Find the magnetizing force H for Problem 5 in SI units if the magnetic circuit is 6 in. long.

8. If a magnetizing force H of 600 At/m is applied to a magnetic circuit, a flux density B of 1200×10^{-4} Wb/m^2 is established. Find the permeability μ of a material that will produce twice the original flux density for the same magnetizing force.

SECTIONS 12.6–12.9 Hysteresis through Series Magnetic Circuits

9. For the series magnetic circuit in Fig. 12.36, determine the current I necessary to establish the indicated flux.

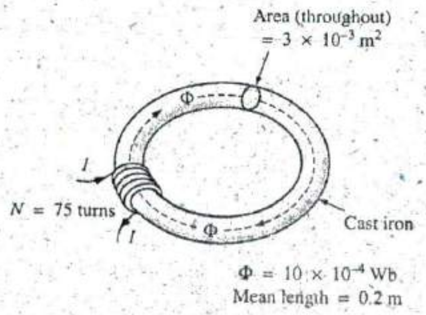


FIG. 12.36
Problem 9.

10. Find the current necessary to establish a flux of $\Phi = 3 \times 10^{-4}$ Wb in the series magnetic circuit in Fig. 12.37.

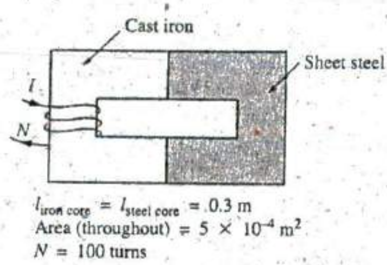


FIG. 12.37
Problem 10.

11. a. Find the number of turns N_1 required to establish a flux $\Phi = 12 \times 10^{-4}$ Wb in the magnetic circuit in Fig. 12.38.
    b. Find the permeability μ of the material.

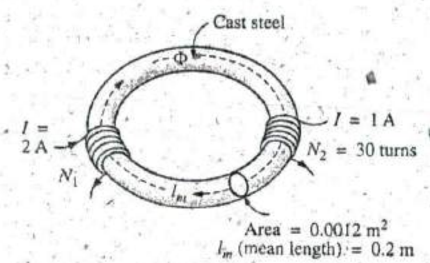


FIG. 12.38
Problem 11.

12. a. Find the mmf (NI) required to establish a flux Φ = 80,000 lines in the magnetic circuit in Fig. 12.39.
    b. Find the permeability of each material.

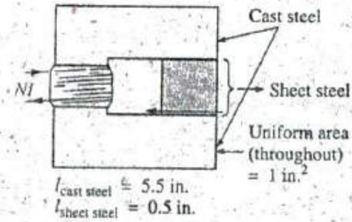


FIG. 12.39
Problem 12.

*13. For the series magnetic circuit in Fig. 12.40 with two impressed sources of magnetic "pressure," determine the current I. Each applied mmf establishes a flux pattern in the clockwise direction.

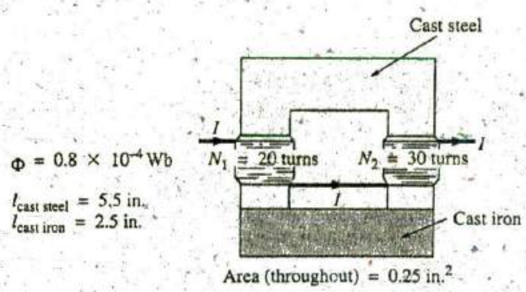


FIG. 12.40
Problem 13.

SECTION 12.10 Air Gaps

14. a. Find the current I required to establish a flux $\Phi = 2.4 \times 10^{-4}$ Wb in the magnetic circuit in Fig. 12.41.
   b. Compare the mmf drop across the air gap to that across the rest of the magnetic circuit. Discuss your results using the value of μ for each material.

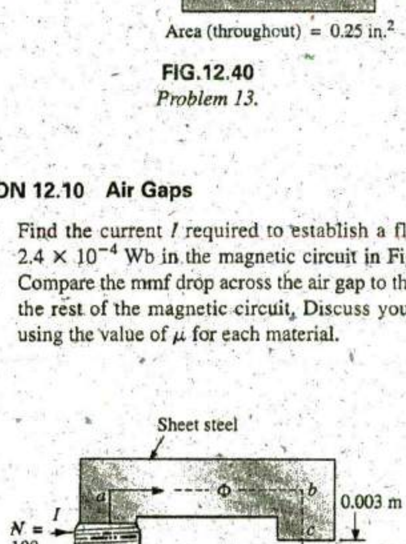


FIG. 12.41
Problem 14.

*15. The force carried by the plunger of the door chime in Fig. 12.42 is determined by

$$f = \frac{1}{2} NI \frac{d\phi}{dx} \qquad \text{(newtons)}$$

where $d\phi/dx$ is the rate of change of flux linking the coil as the core is drawn into the coil. The greatest rate of change of flux occurs when the core is 1/4 to 3/4 the way through. In this region, if Φ changes from 0.5×10^{-4} Wb to 8×10^{-4} Wb, what is the force carried by the plunger?

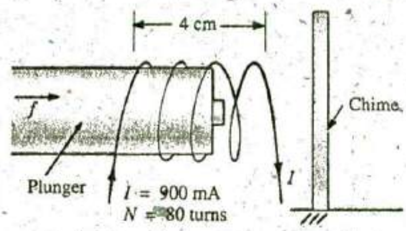


FIG. 12.42
Door chime for Problem 15.

16. Determine the current I_1 required to establish a flux of $\Phi = 2 \times 10^{-4}$ Wb in the magnetic circuit in Fig. 12.43.

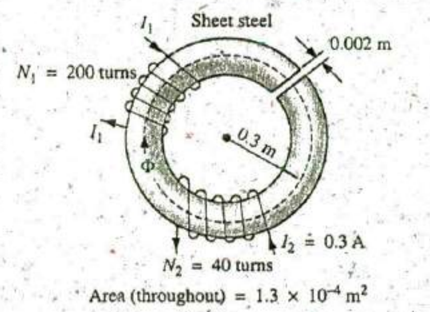


FIG. 12.43
Problem 16.

*17. a. A flux of 0.2×10^{-4} Wb will establish sufficient attractive force for the armature of the relay in Fig. 12.44 to close the contacts. Determine the required current to establish this flux level if we assume that the total mmf drop is across the air gap.
   b. The force exerted on the armature is determined by the equation

$$F(\text{newtons}) = \frac{1}{2} \cdot \frac{B_g^2 A}{\mu_o}$$

where B_g is the flux density within the air gap and A is the common area of the air gap. Find the force in newtons exerted when the flux Φ specified in part (a) is established.

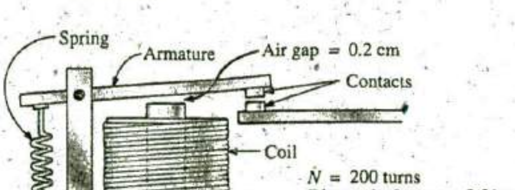
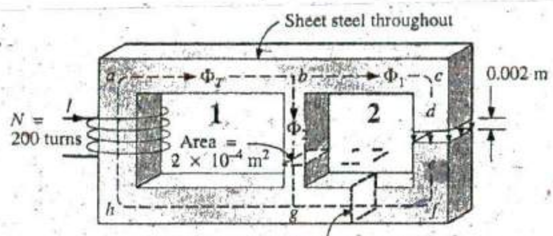


FIG. 12.44
Relay for Problem 17.

SECTION 12.11 Series-Parallel Magnetic Circuits

*18. For the series-parallel magnetic circuit in Fig. 12.45, find the value of I required to establish a flux in the gap of $\Phi_g = 2 \times 10^{-4}$ Wb.



Area for sections other than $bg = 5 \times 10^{-4}$ m²
$l_{ab} = l_{bg} = l_{gh} = l_{ha} = 0.2$ m
$l_{bc} = l_{fg} = 0.1$ m, $l_{cd} = l_{ef} = 0.099$ m

FIG. 12.45
Problem 18.

SECTION 12.12 Determining Φ

19. Find the magnetic flux Φ established in the series magnetic circuit in Fig. 12.46.

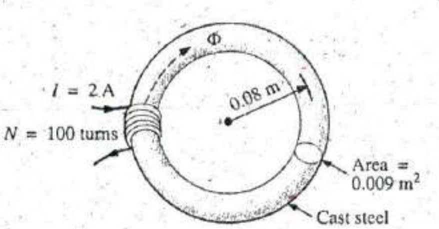
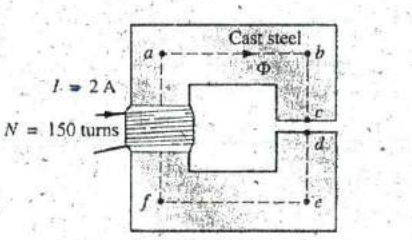


FIG. 12.46
Problem 19.

*20. Determine the magnetic flux Φ established in the series magnetic circuit in Fig. 12.47.



$l_{cd} = 8 \times 10^{-4}$ m
$l_{ab} = l_{be} = l_{ef} = l_{fa} = 0.2$ m
Area (throughout) $= 2 \times 10^{-4}$ m²
$l_{bc} = l_{de}$

FIG. 12.47
Problem 20.

*21. Note how closely the B-H curve of cast steel in Fig. 12.7 matches the curve for the voltage across a capacitor as it charges from zero volts to its final value.

   a. Using the equation for the charging voltage as a guide, write an equation for B as a function of H [$B = f(H)$] for cast steel.
   b. Test the resulting equation at H = 900 At/m, 1800 At/m, and 2700 At/m.
   c. Using the equation of part (a), derive an equation for H in terms of B [$H = f(B)$].
   d. Test the resulting equation at B = 1 T and B = 1.4 T.
   e. Using the result of part (c), perform the analysis of Example 12.1, and compare the results for the current I.

GLOSSARY

Ampère's circuital law A law establishing the fact that the algebraic sum of the rises and drops of the mmf around a closed loop of a magnetic circuit is equal to zero.

Domain A group of magnetically aligned atoms.

Electromagnetism Magnetic effects introduced by the flow of charge or current.

Ferromagnetic materials Materials having permeabilities hundreds and thousands of times greater than that of free space.

Flux density (B) A measure of the flux per unit area perpendicular to a magnetic flux path. It is measured in teslas (T) or webers per square meter (Wb/m²).

Hysteresis The lagging effect between the flux density of a material and the magnetizing force applied.

Magnetic flux lines Lines of a continuous nature that reveal the strength and direction of a magnetic field.

Magnetizing force (H) A measure of the magnetomotive force per unit length of a magnetic circuit.

Magnetomotive force (mmf) ($\mathcal{F}$) The "pressure" required to establish magnetic flux in a ferromagnetic material. It is measured in ampere-turns (At).

Permanent magnet A material such as steel or iron that will remain magnetized for long periods of time without the aid of external means.

Permeability (μ) A measure of the ease with which magnetic flux can be established in a material. It is measured in Wb/Am.

Relative permeability (μ_r) The ratio of the permeability of a material to that of free space.

Reluctance ($\mathcal{R}$) A quantity determined by the physical characteristics of a material that will provide an indication of the "reluctance" of that material to the setting up of magnetic flux lines in the material. It is measured in rels or At/Wb.