

SINUSOIDAL ALTERNATING WAVEFORMS

OBJECTIVES

- Become familiar with the characteristics of a sinusoidal waveform, including its general format, average value, and effective value.
- Be able to determine the phase relationship between two sinusoidal waveforms of the same frequency.
- Understand how to calculate the average and effective values of any waveform.
- Become familiar with the use of instruments designed to measure ac quantities.

13.1 INTRODUCTION

The analysis thus far has been limited to dc networks—networks in which the currents or voltages are fixed in magnitude except for transient effects. We now turn our attention to the analysis of networks in which the magnitude of the source varies in a set manner. Of particular interest is the time-varying voltage that is commercially available in large quantities and is commonly called the *ac voltage*. (The letters *ac* are an abbreviation for *alternating current*.) To be absolutely rigorous, the terminology *ac voltage* or *ac current* is not sufficient to describe the type of signal we will be analyzing. Each waveform in Fig. 13.1 is an **alternating waveform** available from commercial supplies. The term *alternating* indicates only that the waveform alternates between two prescribed levels in a set time sequence. To be absolutely correct, the term *sinusoidal*, *square-wave*, or *triangular* must also be applied.

The pattern of particular interest is the **sinusoidal ac voltage** in Fig. 13.1. Since this type of signal is encountered in the vast majority of instances, the abbreviated phrases *ac voltage* and *ac current* are commonly applied without confusion. For the other patterns in Fig. 13.1, the descriptive term is always present, but frequently the *ac* abbreviation is dropped, resulting in the designation *square-wave* or *triangular* waveforms.

One of the important reasons for concentrating on the sinusoidal ac voltage is that it is the voltage generated by utilities throughout the world. Other reasons include its application throughout electrical, electronic, communication, and industrial systems. In addition, the chapters to follow will reveal that the waveform itself has a number of characteristics that result in a unique response when it is applied to basic electrical elements. The wide range of theorems and methods introduced for dc networks will also be applied to sinusoidal ac systems. Although the application of sinusoidal signals raises the required math level, once the

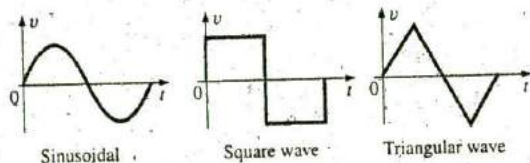


FIG. 13.1
Alternating waveforms.



notation given in Chapter 14 is understood, most of the concepts introduced in the dc chapters can be applied to ac networks with a minimum of added difficulty.

13.2 SINUSOIDAL ac VOLTAGE CHARACTERISTICS AND DEFINITIONS

Generation

Sinusoidal ac voltages are available from a variety of sources. The most common source is the typical home outlet, which provides an ac voltage that originates at a power plant. Most power plants are fueled by water power, oil, gas, or nuclear fusion. In each case, an **ac generator** (also called an *alternator*), as shown in Fig. 13.2(a), is the primary component in the energy-conversion process. The power to the shaft developed by one of the energy sources listed turns a *rotor* (constructed of alternating magnetic poles) inside a set of windings, housed in the *stator* (the stationary part of the dynamo) and induces a voltage across the windings of the stator, as defined by Faraday's law:

$$e = N \frac{d\phi}{dt}$$

Through proper design of the generator, a sinusoidal ac voltage is developed that can be transformed to higher levels for distribution through the power lines to the consumer. For isolated locations where power lines have not been installed, portable ac generators [Fig. 13.2 (b)] are available that run on gasoline. As in the larger power plants, however, an ac generator is an integral part of the design.

In an effort to conserve our natural resources and reduce pollution, wind power, solar energy, and fuel cells are receiving increasing interest from various districts of the world that have such energy sources available in level and duration that make the conversion process viable. The turning propellers of the wind-power station [Fig. 13.2 (c)] are connected directly to the shaft of an ac generator to provide the ac voltage described above. Through light energy absorbed in the form of *photons*,

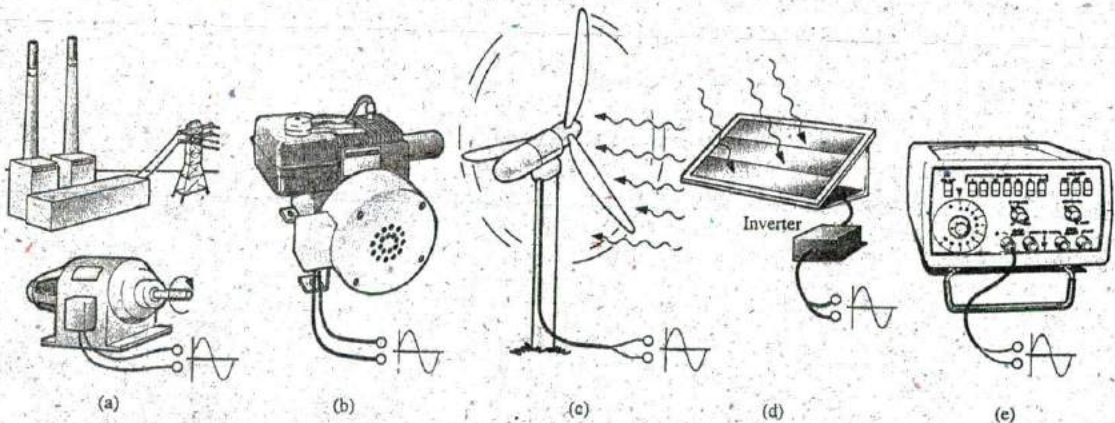


FIG. 13.2

Various sources of ac power: (a) generating plant; (b) portable ac generator; (c) wind-power station; (d) solar panel; (e) function generator.

solar cells [Fig. 13.2 (d)] can generate dc voltages. Through an electronic package called an *inverter*, the dc voltage can be converted to one of a sinusoidal nature. Boats, recreational vehicles (RVs), and so on, make frequent use of the inversion process in isolated areas.

Sinusoidal ac voltages with characteristics that can be controlled by the user are available from **function generators**, such as the one in Fig. 13.2(e). By setting the various switches and controlling the position of the knobs on the face of the instrument, you can make available sinusoidal voltages of different peak values and different repetition rates. The function generator plays an integral role in the investigation of the variety of theorems, methods of analysis, and topics to be introduced in the chapters that follow.

Definitions

The sinusoidal waveform in Fig. 13.3 with its additional notation will now be used as a model in defining a few basic terms. These terms, however, can be applied to any alternating waveform. It is important to remember, as you proceed through the various definitions, that the vertical scaling is in volts or amperes and the horizontal scaling is in units of time.

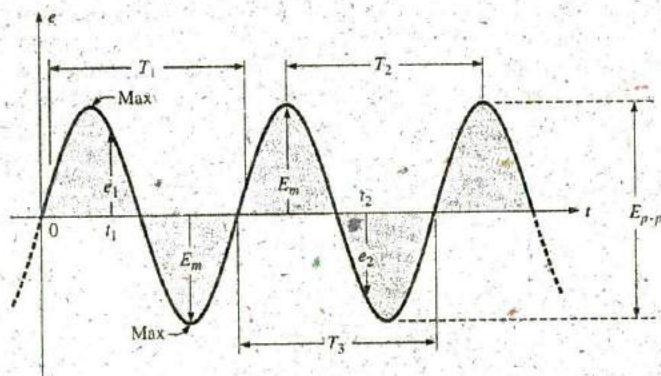


FIG. 13.3

Important parameters for a sinusoidal voltage.

Waveform: The path traced by a quantity, such as the voltage in Fig. 13.3, plotted as a function of some variable, such as time (as above), position, degrees, radians, temperature, and so on.

Instantaneous value: The magnitude of a waveform at any instant of time; denoted by lowercase letters (e_1 , e_2 in Fig. 13.3).

Peak amplitude: The maximum value of a waveform as measured from its *average*, or *mean*, value, denoted by uppercase letters [such as E_m (Fig. 13.3) for sources of voltage and V_m for the voltage drop across a load]. For the waveform in Fig. 13.3, the average value is zero volts, and E_m is as defined by the figure.

Peak value: The maximum instantaneous value of a function as measured from the zero volt level. For the waveform in Fig. 13.3, the peak amplitude and peak value are the same since the average value of the function is zero volts.

Peak-to-peak value: Denoted by E_{p-p} or V_{p-p} (as shown in Fig. 13.3), the full voltage between positive and negative peaks of the waveform, that is, the sum of the magnitude of the positive and negative peaks.



Periodic waveform: A waveform that continually repeats itself after the same time interval. The waveform in Fig. 13.3 is a periodic waveform.

Period (T): The time of a periodic waveform.

Cycle: The portion of a waveform contained in one period of time. The cycles within T_1 , T_2 , and T_3 in Fig. 13.3 may appear different in Fig. 13.4, but they are all bounded by one period of time and therefore satisfy the definition of a cycle.

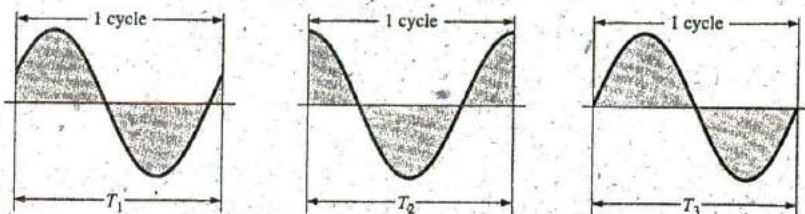


FIG. 13.4

Defining the cycle and period of a sinusoidal waveform.

Frequency (f): The number of cycles that occur in 1 s. The frequency of the waveform in Fig. 13.5(a) is 1 cycle per second, and for Fig. 13.5(b), $2\frac{1}{2}$ cycles per second. If a waveform of similar shape had a period of 0.5 s [Fig. 13.5(c)], the frequency would be 2 cycles per second.



FIG. 13.6

Heinrich Rudolph Hertz,
Courtesy of the Smithsonian
Institution, Photo No. 66,606.

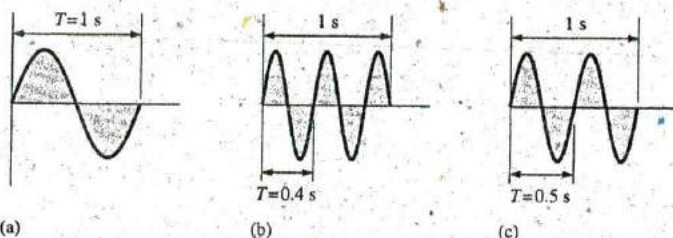


FIG. 13.5

Demonstrating the effect of a changing frequency on the period of a sinusoidal waveform.

The unit of measure for frequency is the *hertz* (Hz), where

$$1 \text{ hertz (Hz)} = 1 \text{ cycle per second (cps)} \quad (13.1)$$

The unit hertz is derived from the surname of Heinrich Rudolph Hertz (Fig. 13.6), who did original research in the area of alternating currents and voltages and their effect on the basic R , L , and C elements. The frequency standard for North America is 60 Hz, whereas for Europe it is predominantly 50 Hz.

As with all standards, any variation from the norm will cause difficulties. In 1993, Berlin, Germany, received all its power from plants generating ac voltages whose output frequency was varying between 50.03 Hz and 51 Hz. The result was that clocks were gaining as much as 4 minutes a day. Alarms went off too soon, VCRs clicked off before the end of the program, and so on, requiring that clocks be continually reset. In 1994, however, when power was linked with the rest of Europe, the precise standard of 50 Hz was reestablished and everyone was on time again.

German (Hamburg, Berlin, Karlsruhe)

(1857–94)

Physicist

Professor of Physics, Karlsruhe Polytechnic and
University of Bonn

Spurred on by the earlier predictions of the English physicist James Clerk Maxwell, Heinrich Hertz produced *electromagnetic waves* in his laboratory at the Karlsruhe Polytechnic while in his early 30s. The rudimentary *transmitter* and *receiver* were in essence the first to broadcast and receive radio waves. He was able to measure the *wavelength* of the electromagnetic waves and confirmed that the *velocity of propagation* is in the same order of magnitude as that of light. In addition, he demonstrated that the *reflective* and *refractive* properties of electromagnetic waves are the same as those for heat and light waves. It was indeed unfortunate that such an ingenious, industrious individual should pass away at the very early age of 37 due to a bone disease.



EXAMPLE 13.1 For the sinusoidal waveform in Fig. 13.7.

- What is the peak value?
- What is the instantaneous value at 0.3 s and 0.6 s?
- What is the peak-to-peak value of the waveform?
- What is the period of the waveform?
- How many cycles are shown?
- What is the frequency of the waveform?

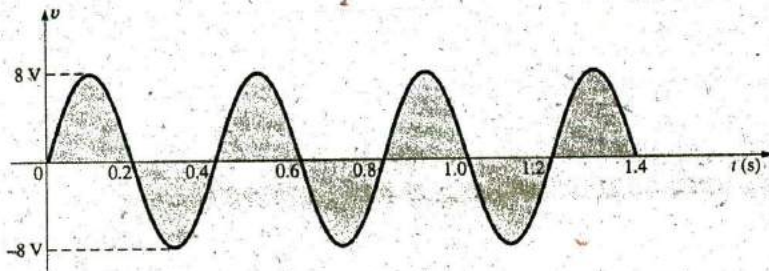


FIG. 13.7
Example 13.1.

Solutions:

- 8 V.
- At 0.3 s, -8 V; at 0.6 s, 0 V.
- 16 V.
- 0.4 s.
- 3.5 cycles.
- 2.5 cps, or 2.5 Hz.

13.3 FREQUENCY SPECTRUM

Using a log scale (described in detail in Chapter 20), we can scale a frequency spectrum from 1 Hz to 1000 GHz on the same axis, as shown in Fig. 13.8. A number of terms in the various portions of the spectrum are probably familiar to you from everyday experiences. Note that the audio range (human ear) extends from only 15 Hz to 20 kHz, but the transmission of radio signals can occur between 3 kHz and 300 GHz. The uniform process of defining the intervals of the radio-frequency spectrum from VLF to EHF is quite evident from the length of the bars in the figure (although keep in mind that it is a log scale, so the frequencies encompassed within each segment are quite different). Other frequencies of particular interest (TV, CB, microwave, and so on) are also included for reference purposes. Although it is numerically easy to talk about frequencies in the megahertz and gigahertz range, keep in mind that a frequency of 100 MHz, for instance, represents a sinusoidal waveform that passes through 100,000,000 cycles in only 1 s—an incredible number when we compare it to the 60 Hz of our conventional power sources. The Intel® Core 2 Extreme processor can run at speeds of 3 GHz. Imagine a product able to handle 3 billion instructions per second—an incredible achievement.

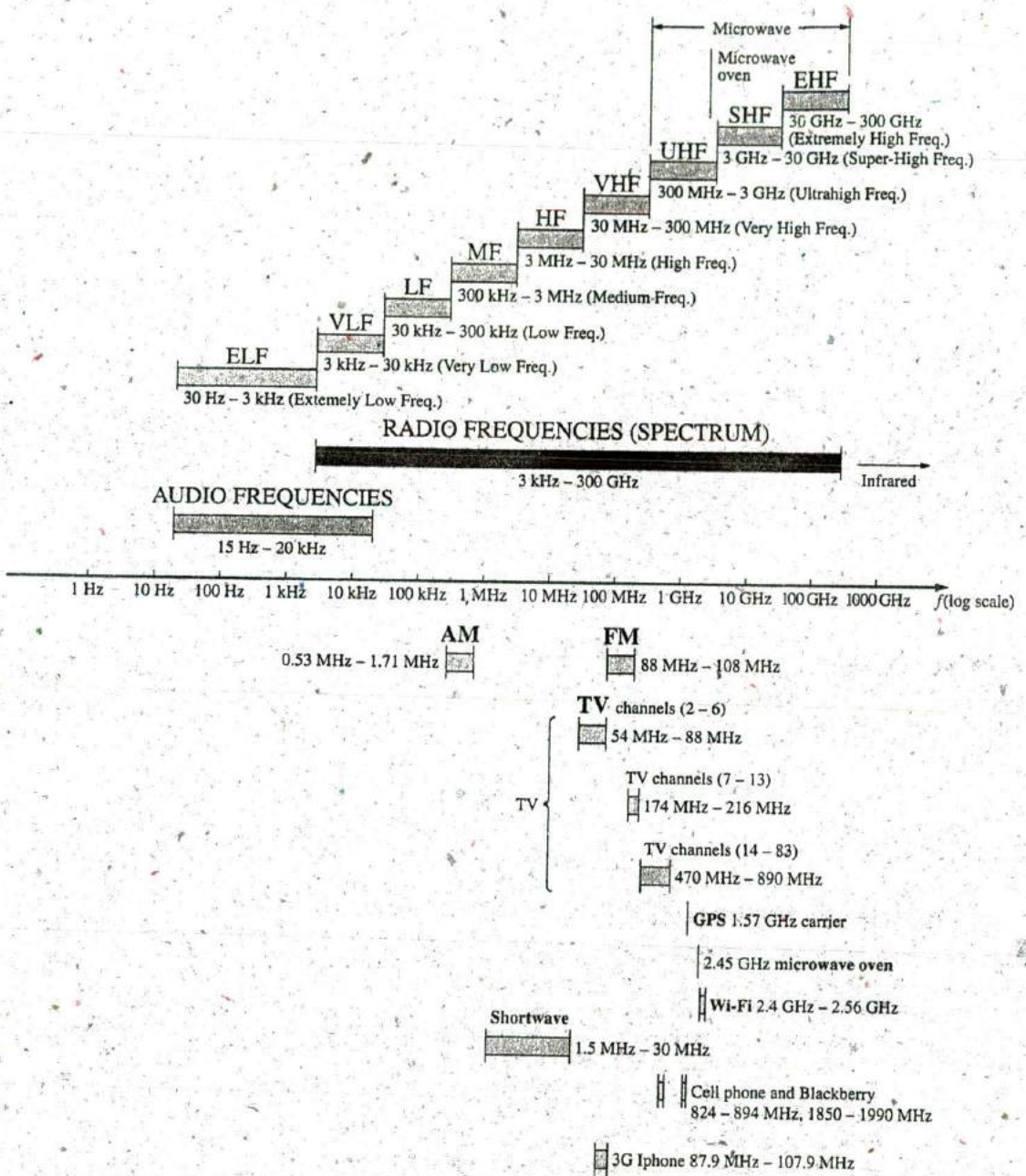


FIG. 13.8
Areas of application for specific frequency bands.

Since the frequency is inversely related to the period—that is, as one increases, the other decreases by an equal amount—the two can be related by the following equation:

$$\boxed{f = \frac{1}{T}} \quad \begin{array}{l} f = \text{Hz} \\ T = \text{seconds (s)} \end{array} \quad (13.2)$$

or

$$\boxed{T = \frac{1}{f}} \quad (13.3)$$

EXAMPLE 13.2 Find the period of periodic waveform with a frequency of

- 60 Hz.
- 1000 Hz.

Solutions:

a. $T = \frac{1}{f} = \frac{1}{60 \text{ Hz}} \cong 0.01667 \text{ s}$ or **16.67 ms**
 (a recurring value since 60 Hz is so prevalent)

b. $T = \frac{1}{f} = \frac{1}{1000 \text{ Hz}} = 10^{-3} \text{ s} = 1 \text{ ms}$

EXAMPLE 13.3 Determine the frequency of the waveform in Fig. 13.9.

Solution: From the figure, $T = (25 \text{ ms} - 5 \text{ ms})$ or $(35 \text{ ms} - 15 \text{ ms}) = 20 \text{ ms}$, and

$$f = \frac{1}{T} = \frac{1}{20 \times 10^{-3} \text{ s}} = 50 \text{ Hz}$$

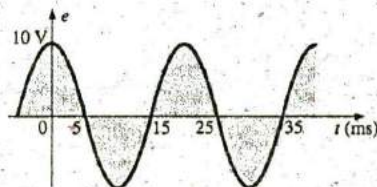


FIG. 13.9
Example 13.3.

In Fig. 13.10, the seismogram resulting from a seismometer near an earthquake is displayed. Prior to the disturbance, the waveform has a relatively steady level, but as the event is about to occur, the frequency begins

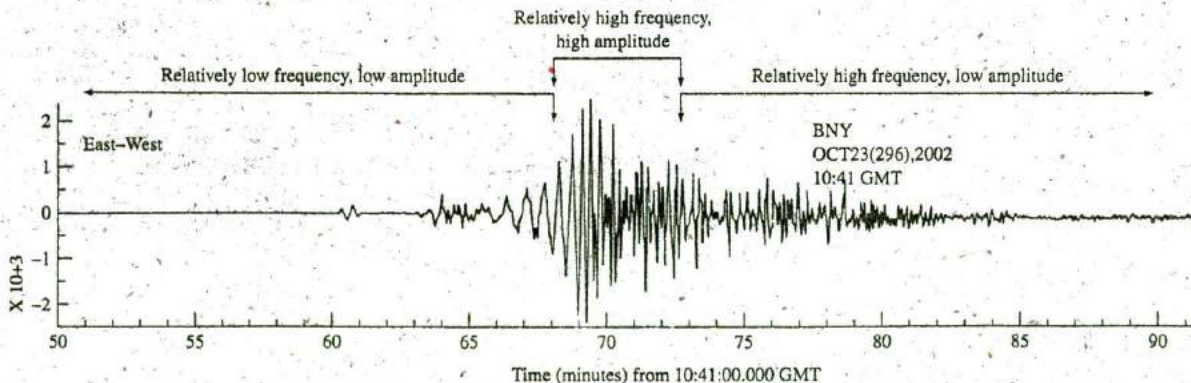


FIG. 13.10

Seismogram from station BNY (Binghamton University) in New York due to magnitude 6.7 earthquake in Central Alaska that occurred at 63.62°N, 148.04°W, with a depth of 10 km, on Wednesday, October 23, 2002.

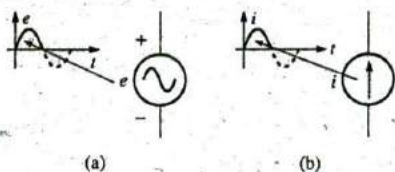


FIG. 13.11

(a) Sinusoidal ac voltage sources; (b) sinusoidal current sources.

to increase along with the amplitude. Finally, the earthquake occurs, and the frequency and the amplitude increase dramatically. In other words, the relative frequencies can be determined simply by looking at the tightness of the waveform and the associated period. The change in amplitude is immediately obvious from the resulting waveform. The fact that the earthquake lasts for only a few minutes is clear from the horizontal scale.

Defined Polarities and Direction

You may be wondering how a polarity for a voltage or a direction for a current can be established if the waveform moves back and forth from the positive to the negative region. For a period of time, a voltage has one polarity, while for the next equal period it reverses. To take care of this problem, a positive sign is applied if the voltage is above the axis, as shown in Fig. 13.11(a). For a current source, the direction in the symbol corresponds with the positive region of the waveform, as shown in Fig. 13.11(b).

For any quantity that will not change with time, an uppercase letter such as V or I is used. For expressions that are time dependent or that represent a particular instant of time, a lowercase letter such as e or i is used.

The need for defining polarities and current direction becomes quite obvious when we consider multisource ac networks. Note in the last sentence the absence of the term *sinusoidal* before the phrase *ac networks*. This phrase will be used to an increasing degree as we progress; *sinusoidal* is to be understood unless otherwise indicated.

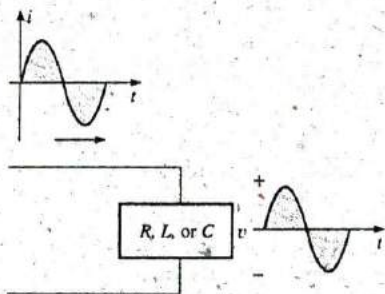


FIG. 13.12

The sine wave is the only alternating waveform whose shape is not altered by the response characteristics of a pure resistor, inductor, or capacitor.

13.4 THE SINUSOIDAL WAVEFORM

The terms defined in the previous section can be applied to any type of periodic waveform, whether smooth or discontinuous. The sinusoidal waveform is of particular importance, however, since it lends itself readily to the mathematics and the physical phenomena associated with electric circuits. Consider the power of the following statement:

The sinusoidal waveform is the only alternating waveform whose shape is unaffected by the response characteristics of R , L , and C elements.

In other words, if the voltage across (or current through) a resistor, inductor, or capacitor is sinusoidal in nature, the resulting current (or voltage, respectively) for each will also have sinusoidal characteristics, as shown in Fig. 13.12. If any other alternating waveform such as a square wave or a triangular wave were applied, such would not be the case.

The unit of measurement for the horizontal axis can be **time** (as appearing in the figures thus far), **degrees**, or **radians**. The term **radian** can be defined as follows: If we mark off a portion of the circumference of a circle by a length equal to the radius of the circle, as shown in Fig. 13.13, the angle resulting is called **1 radian**. The result is

$$1 \text{ rad} \approx 57.296^\circ \approx 57.3^\circ \quad (13.4)$$

where 57.3° is the usual approximation applied.

One full circle has 2π radians, as shown in Fig. 13.14. That is,

$$2\pi \text{ rad} = 360^\circ \quad (13.5)$$

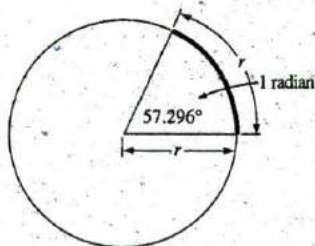


FIG. 13.13

Defining the radian.

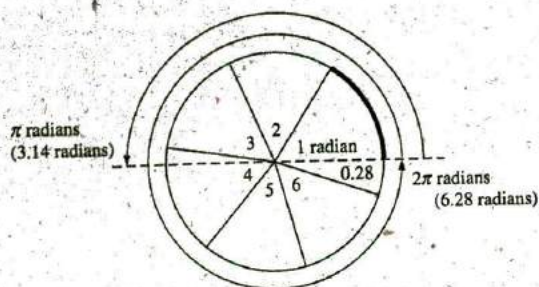


FIG. 13.14

There are 2π radians in one full circle of 360° .

so that

$$2\pi = 2(3.142) = 6.28$$

and

$$2\pi(57.3^\circ) = 6.28(57.3^\circ) = 359.84^\circ \approx 360^\circ$$

A number of electrical formulas contain a multiplier of π . For this reason, it is sometimes preferable to measure angles in radians rather than in degrees.

The quantity π is the ratio of the circumference of a circle to its diameter.

π has been determined to an extended number of places, primarily in an attempt to see if a repetitive sequence of numbers appears. It does not. A sampling of the effort appears below:

$$\pi = 3.14159\ 26535\ 89793\ 23846\ 26433\ \dots$$

Although the approximation $\pi \approx 3.14$ is often applied, all the calculations in the text use the π function as provided on all scientific calculators.

For 180° and 360° , the two units of measurement are related as shown in Fig. 13.14. The conversions equations between the two are the following:

$$\boxed{\text{Radians} = \left(\frac{\pi}{180^\circ}\right) \times (\text{degrees})} \quad (13.6)$$

$$\boxed{\text{Degrees} = \left(\frac{180^\circ}{\pi}\right) \times (\text{radians})} \quad (13.7)$$

Applying these equations, we find

$$90^\circ: \text{Radians} = \frac{\pi}{180^\circ}(90^\circ) = \frac{\pi}{2} \text{ rad}$$

$$30^\circ: \text{Radians} = \frac{\pi}{180^\circ}(30^\circ) = \frac{\pi}{6} \text{ rad}$$

$$\frac{\pi}{3} \text{ rad}: \text{Degrees} = \frac{180^\circ}{\pi} \left(\frac{\pi}{3}\right) = 60^\circ$$

$$\frac{3\pi}{2} \text{ rad}: \text{Degrees} = \frac{180^\circ}{\pi} \left(\frac{3\pi}{2}\right) = 270^\circ$$

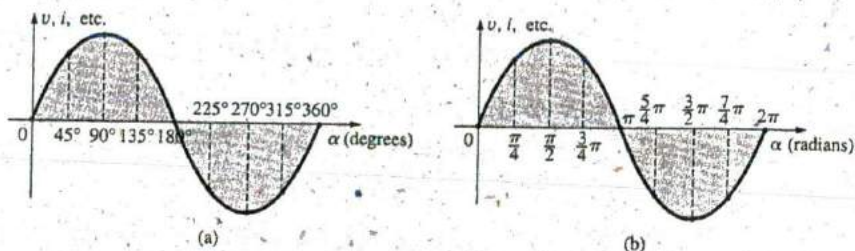


FIG. 13.15

Plotting a sine wave versus (a) degrees and (b) radians.

For comparison purposes, two sinusoidal voltages are plotted in Fig. 13.15 using degrees and radians as the units of measurement for the horizontal axis.

It is of particular interest that the sinusoidal waveform can be derived from the length of the *vertical projection* of a radius vector rotating in a uniform circular motion about a fixed point. Starting as shown in Fig. 13.16(a) and plotting the amplitude (above and below zero) on the coordinates drawn to the right [Figs. 13.16(b) through (i)], we will trace a complete sinusoidal waveform after the radius vector has completed a 360° rotation about the center.

The velocity with which the radius vector rotates about the center, called the **angular velocity**, can be determined from the following equation:

$$\text{Angular velocity} = \frac{\text{distance (degrees or radians)}}{\text{time (seconds)}} \quad (13.8)$$

Substituting into Eq. (13.8) and assigning the lowercase Greek letter *omega* (ω) to the angular velocity, we have

$$\omega = \frac{\alpha}{t} \quad (13.9)$$

and

$$\alpha = \omega t \quad (13.10)$$

Since ω is typically provided in radians per second, the angle α obtained using Eq. (13.10) is usually in radians. If α is required in degrees, Eq. (13.7) must be applied. The importance of remembering the above will become obvious in the examples to follow.

In Fig. 13.16, the time required to complete one revolution is equal to the period (T) of the sinusoidal waveform in Fig. 13.16(i). The radians subtended in this time interval are 2π . Substituting, we have

$$\omega = \frac{2\pi}{T} \quad (\text{rad/s}) \quad (13.11)$$

In words, this equation states that the smaller the period of the sinusoidal waveform of Fig. 13.16(i), or the smaller the time interval before one complete cycle is generated, the greater must be the angular velocity of the rotating radius vector. Certainly this statement agrees with what we have learned thus far. We can now go one step further and apply the

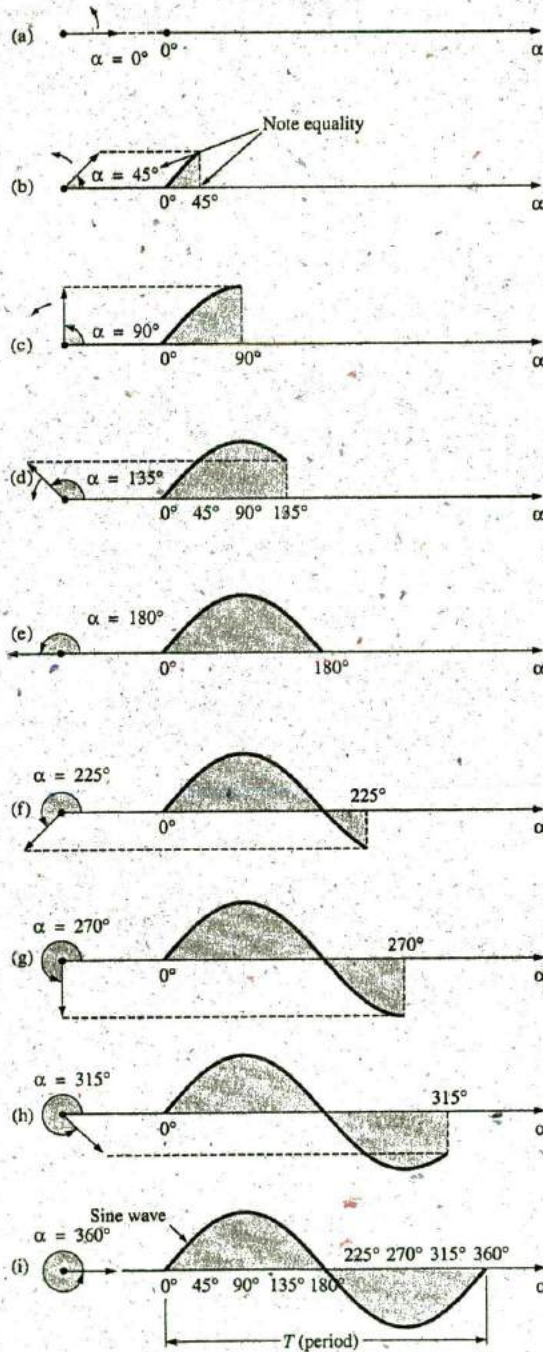


FIG. 13.16

Generating a sinusoidal waveform through the vertical projection of a rotating vector.

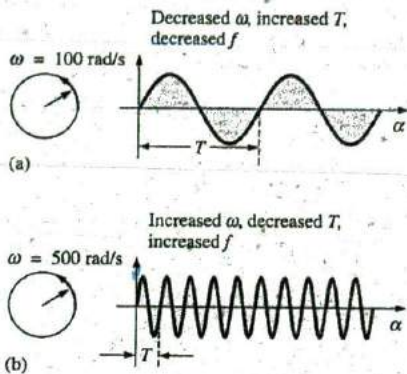


FIG. 13.17
Demonstrating the effect of ω on the frequency and period.

fact that the frequency of the generated waveform is inversely related to the period of the waveform; that is, $f = 1/T$. Thus,

$$\omega = 2\pi f \quad (\text{rad/s}) \quad (13.12)$$

This equation states that the higher the frequency of the generated sinusoidal waveform, the higher must be the angular velocity. Eqs. (13.11) and (13.12) are verified somewhat by Fig. 13.17, where for the same radius vector, $\omega = 100$ rad/s and 500 rad/s.

EXAMPLE 13.4 Determine the angular velocity of a sine wave having a frequency of 60 Hz.

Solution:

$$\omega = 2\pi f = (2\pi)(60 \text{ Hz}) \cong 377 \text{ rad/s}$$

(a recurring value due to 60 Hz predominance).

EXAMPLE 13.5 Determine the frequency and period of the sine wave in Fig. 13.17(b).

Solution: Since $\omega = 2\pi/T$,

$$T = \frac{2\pi}{\omega} = \frac{2\pi \text{ rad}}{500 \text{ rad/s}} = \frac{2\pi \text{ rad}}{500 \text{ rad/s}} = 12.57 \text{ ms}$$

and

$$f = \frac{1}{T} = \frac{1}{12.57 \times 10^{-3} \text{ s}} = 79.58 \text{ Hz}$$

EXAMPLE 13.6 Given $\omega = 200$ rad/s; determine how long it will take the sinusoidal waveform to pass through an angle of 90° .

Solution: Eq. (13.10): $\alpha = \omega t$, and

$$t = \frac{\alpha}{\omega}$$

However, α must be substituted as $\pi/2$ ($= 90^\circ$) since ω is in radians per second:

$$t = \frac{\alpha}{\omega} = \frac{\pi/2 \text{ rad}}{200 \text{ rad/s}} = \frac{\pi}{400} \text{ s} = 7.85 \text{ ms}$$

EXAMPLE 13.7 Find the angle through which a sinusoidal waveform of 60 Hz will pass in a period of 5 ms.

Solution: Eq. (13.11): $\alpha = \omega t$, or

$$\alpha = 2\pi ft = (2\pi)(60 \text{ Hz})(5 \times 10^{-3} \text{ s}) = 1.885 \text{ rad}$$

If not careful, you might be tempted to interpret the answer as 1.885° . However,

$$\alpha(^{\circ}) = \frac{180^{\circ}}{\pi \text{ rad}}(1.885 \text{ rad}) = 108^{\circ}$$

13.5 GENERAL FORMAT FOR THE SINUSOIDAL VOLTAGE OR CURRENT

The basic mathematical format for the sinusoidal waveform is

$$A_m \sin \alpha \quad (13.13)$$

where A_m is the peak value of the waveform and α is the unit of measure for the horizontal axis, as shown in Fig. 13.18.

The equation $\alpha = \omega t$ states that the angle α through which the rotating vector in Fig. 13.16 will pass is determined by the angular velocity of the rotating vector and the length of time the vector rotates. For example, for a particular angular velocity (fixed ω), the longer the radius vector is permitted to rotate (that is, the greater the value of t), the greater is the number of degrees or radians through which the vector will pass. Relating this statement to the sinusoidal waveform, we have that, for a particular angular velocity, the longer the time, the greater is the number of cycles shown. For a fixed time interval, the greater is the angular velocity, the greater is the number of cycles generated.

Due to Eq. (13.10), the general format of a sine wave can also be written

$$A_m \sin \omega t \quad (13.14)$$

with ωt as the horizontal unit of measure.

For electrical quantities such as current and voltage, the general format is

$$\begin{aligned} i &= I_m \sin \omega t = I_m \sin \alpha \\ e &= E_m \sin \omega t = E_m \sin \alpha \end{aligned}$$

where the capital letters with the subscript m represent the amplitude, and the lowercase letters i and e represent the instantaneous value of current and voltage, respectively, at any time t . This format is particularly important because it presents the sinusoidal voltage or current as a function of time, which is the horizontal scale for the oscilloscope. Recall that the horizontal sensitivity of a scope is in time per division, not degrees per centimeter.

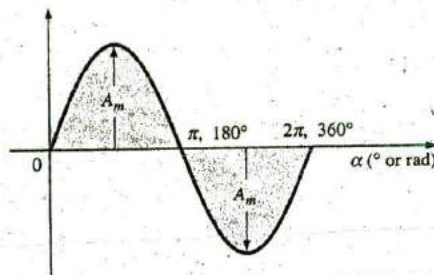


FIG. 13.18
Basic sinusoidal function.

EXAMPLE 13.8 Given $e = 5 \sin \alpha$, determine e at $\alpha = 40^\circ$ and $\alpha = 0.8\pi$.

Solution: For $\alpha = 40^\circ$,

$$e = 5 \sin 40^\circ = 5(0.6428) = 3.21 \text{ V}$$

For $\alpha = 0.8\pi$,

$$\alpha(^\circ) = \frac{180^\circ}{\pi}(0.8\pi) = 144^\circ$$

and

$$e = 5 \sin 144^\circ = 5(0.5878) = 2.94 \text{ V}$$

The angle at which a particular voltage level is attained can be determined by rearranging the equation

$$e = E_m \sin \alpha$$

in the following manner:

$$\sin \alpha = \frac{e}{E_m}$$

which can be written

$$\alpha = \sin^{-1} \frac{e}{E_m} \quad (13.15)$$

Similarly, for a particular current level,

$$\alpha = \sin^{-1} \frac{i}{I_m} \quad (13.16)$$

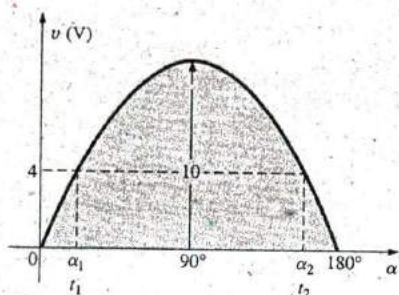


FIG. 13.19
Example 13.9.

EXAMPLE 13.9

- Determine the angle at which the magnitude of the sinusoidal function $v = 10 \sin 377t$ is 4 V.
- Determine the time at which the magnitude is attained.

Solutions:

- Eq. (13.15):

$$\alpha_1 = \sin^{-1} \frac{v}{E_m} = \sin^{-1} \frac{4 \text{ V}}{10 \text{ V}} = \sin^{-1} 0.4 = 23.58^\circ$$

However, Fig. 13.19 reveals that the magnitude of 4 V (positive) will be attained at two points between 0° and 180° . The second intersection is determined by

$$\alpha_2 = 180^\circ - 23.578^\circ = 156.42^\circ$$

In general, therefore, keep in mind that Eqs. (13.15) and (13.16) will provide an angle with a magnitude between 0° and 90° .

- Eq. (13.10): $\alpha = \omega t$, and so $t = \alpha / \omega$. However, α must be in radians. Thus,

$$\alpha(\text{rad}) = \frac{\pi}{180^\circ}(23.578^\circ) = 0.412 \text{ rad}$$

$$\text{and } t_1 = \frac{\alpha}{\omega} = \frac{0.412 \text{ rad}}{377 \text{ rad/s}} = 1.09 \text{ ms}$$

For the second intersection,

$$\alpha(\text{rad}) = \frac{\pi}{180^\circ}(156.422^\circ) = 2.73 \text{ rad}$$

$$t_2 = \frac{\alpha}{\omega} = \frac{2.73 \text{ rad}}{377 \text{ rad/s}} = 7.24 \text{ ms}$$

Calculator Operations

Both \sin and \sin^{-1} are available on all scientific calculators. You can also use them to work with the angle in degrees or radians without having to convert from one form to the other. That is, if the angle is in radians and the mode setting is for radians, you can enter the radian measure directly.

To set the DEGREE mode, proceed as outlined in Fig. 13.20(a) using the TI-89 calculator. The magnitude of the voltage e at 40° can then be found using the sequence in Fig. 13.20(b).

HOME ENTER MODE 1 Angle DEGREE ENTER ENTER

(a)

5 x 2ND SIN 4 0) ENTER 3.21

(b)

FIG. 13.20

(a) Setting the DEGREE mode; (b) evaluating $5 \sin 40^\circ$.

After establishing the RADIAN mode, the sequence in Fig. 13.21 determines the voltage at 0.8π .

5 x 2ND SIN 0 . 8 2ND π) ENTER 2.94

FIG. 13.21

Finding $e = 5 \sin 0.8\pi$ using the calculator in the RADIAN mode.

Finally, the angle in degrees for α_1 in part (a) of Example 13.9 can be determined by the sequence in Fig. 13.22 with the mode set in degrees, whereas the angle in radians for part (a) of Example 13.9 can be determined by the sequence in Fig. 13.23 with the mode set in radians.

2ND SIN⁻¹ 4 + 1 0) ENTER 23.60

FIG. 13.22

Finding $\alpha_1 = \sin^{-1}(4/10)$ using the calculator in the DEGREE mode.

2ND SIN⁻¹ 4 + 1 0) ENTER 0.41

FIG. 13.23

Finding $\alpha_1 = \sin^{-1}(4/10)$ using the calculator in the RADIAN mode.

The sinusoidal waveform can also be plotted against *time* on the horizontal axis. The time period for each interval can be determined from $t = \alpha/\omega$, but the most direct route is simply to find the period T from $T = 1/f$ and break it up into the required intervals. This latter technique is demonstrated in Example 13.10.

Before reviewing the example, take special note of the relative simplicity of the mathematical equation that can represent a sinusoidal waveform. Any alternating waveform whose characteristics differ from those of the sine wave cannot be represented by a single term, but may require two, four, six, or perhaps an infinite number of terms to be represented accurately.

EXAMPLE 13.10 Sketch $e = 10 \sin 314t$ with the abscissa

- angle (α) in degrees.
- angle (α) in radians.
- time (t) in seconds.

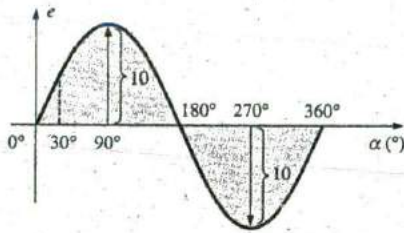


FIG. 13.24

Example 13.10, horizontal axis in degrees.

Solutions:

- a. See Fig. 13.24. (Note that no calculations are required.)
- b. See Fig. 13.25. (Once the relationship between degrees and radians is understood, no calculations are required.)
- c. See Fig. 13.26.

$$360^\circ: T = \frac{2\pi}{\omega} = \frac{2\pi}{314} = 20 \text{ ms}$$

$$180^\circ: \frac{T}{2} = \frac{20 \text{ ms}}{2} = 10 \text{ ms}$$

$$90^\circ: \frac{T}{4} = \frac{20 \text{ ms}}{4} = 5 \text{ ms}$$

$$30^\circ: \frac{T}{12} = \frac{20 \text{ ms}}{12} = 1.67 \text{ ms}$$

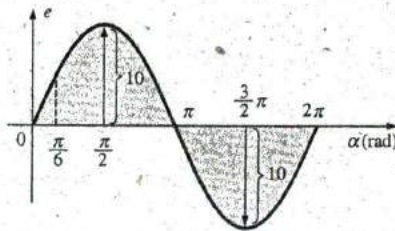


FIG. 13.25

Example 13.10, horizontal axis in radians.

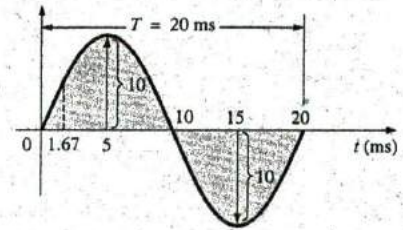


FIG. 13.26

Example 13.10, horizontal axis in milliseconds.

EXAMPLE 13.11 Given $i = 6 \times 10^{-3} \sin 1000t$, determine i at $t = 2 \text{ ms}$.

Solution:

$$\alpha = \omega t = 1000t = (1000 \text{ rad/s})(2 \times 10^{-3} \text{ s}) = 2 \text{ rad}$$

$$\alpha(^\circ) = \frac{180^\circ}{\pi \text{ rad}}(2 \text{ rad}) = 114.59^\circ$$

$$i = (6 \times 10^{-3})(\sin 114.59^\circ) = (6 \text{ mA})(0.9093) = \mathbf{5.46 \text{ mA}}$$

13.6 PHASE RELATIONS

Thus far, we have considered only sine waves that have maxima at $\pi/2$ and $3\pi/2$, with a zero value at $0, \pi$, and 2π , as shown in Fig. 13.25. If the waveform is shifted to the right or left of 0° , the expression becomes

$$A_m \sin(\omega t \pm \theta) \tag{13.17}$$

where θ is the angle in degrees or radians that the waveform has been shifted.

If the waveform passes through the horizontal axis with a *positive-going* (increasing with time) slope *before* 0° , as shown in Fig. 13.27, the expression is

$$A_m \sin(\omega t + \theta) \tag{13.18}$$

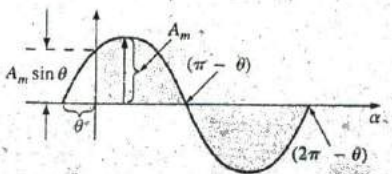


FIG. 13.27

Defining the phase shift for a sinusoidal function that crosses the horizontal axis with a positive slope before 0° .

At $\omega t = \alpha = 0^\circ$, the magnitude is determined by $A_m \sin \theta$. If the waveform passes through the horizontal axis with a positive-going slope after 0° , as shown in Fig. 13.28, the expression is

$$A_m \sin(\omega t - \theta) \quad (13.19)$$

Finally, at $\omega t = \alpha = 0^\circ$, the magnitude is $A_m \sin(-\theta)$, which, by a trigonometric identity, is $-A_m \sin \theta$.

If the waveform crosses the horizontal axis with a positive-going slope $90^\circ (\pi/2)$ sooner, as shown in Fig. 13.29, it is called a *cosine wave*; that is,

$$\sin(\omega t + 90^\circ) = \sin\left(\omega t + \frac{\pi}{2}\right) = \cos \omega t \quad (13.20)$$

or

$$\sin \omega t = \cos(\omega t - 90^\circ) = \cos\left(\omega t - \frac{\pi}{2}\right) \quad (13.21)$$

The terms **leading** and **lagging** are used to indicate the relationship between two sinusoidal waveforms of the *same frequency* plotted on the same set of axes. In Fig. 13.29, the cosine curve is said to *lead* the sine curve by 90° , and the sine curve is said to *lag* the cosine curve by 90° . The 90° is referred to as the phase angle between the two waveforms. In language commonly applied, the waveforms are *out of phase* by 90° . Note that the phase angle between the two waveforms is measured between those two points on the horizontal axis through which each passes with the *same slope*. If both waveforms cross the axis at the same point with the *same slope*, they are *in phase*.

The geometric relationship between various forms of the sine and cosine functions can be derived from Fig. 13.30. For instance, starting at the $+\sin \alpha$ position, we find that $+\cos \alpha$ is an additional 90° in the counterclockwise direction. Therefore, $\cos \alpha = \sin(\alpha + 90^\circ)$. For $-\sin \alpha$ we must travel 180° in the counterclockwise (or clockwise) direction so that $-\sin \alpha = \sin(\alpha \pm 180^\circ)$, and so on, as listed below:

$$\begin{aligned} \cos \alpha &= \sin(\alpha + 90^\circ) \\ \sin \alpha &= \cos(\alpha - 90^\circ) \\ -\sin \alpha &= \sin(\alpha \pm 180^\circ) \\ -\cos \alpha &= \sin(\alpha + 270^\circ) = \sin(\alpha - 90^\circ) \\ &\text{etc.} \end{aligned} \quad (13.22)$$

In addition, note that

$$\begin{aligned} \sin(-\alpha) &= -\sin \alpha \\ \cos(-\alpha) &= \cos \alpha \end{aligned} \quad (13.23)$$

If a sinusoidal expression appears as

$$e = -E_m \sin \omega t$$

the negative sign is associated with the sine portion of the expression, not the peak value E_m . In other words, the expression, if not for convenience, would be written

$$e = E_m(-\sin \omega t)$$

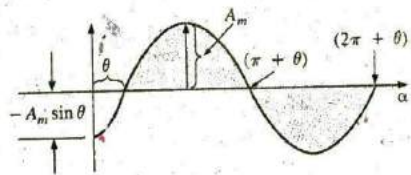


FIG. 13.28
Defining the phase shift for a sinusoidal function that crosses the horizontal axis with a positive slope after 0° .

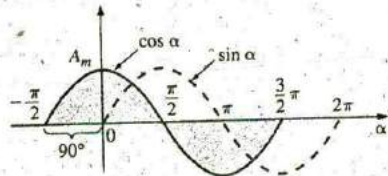


FIG. 13.29
Phase relationship between a sine wave and a cosine wave.

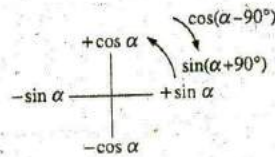


FIG. 13.30
Graphic tool for finding the relationship between specific sine and cosine functions.



Since $-\sin \omega t = \sin(\omega t \pm 180^\circ)$

the expression can also be written

$$e = E_m \sin(\omega t \pm 180^\circ)$$

revealing that a negative sign can be replaced by a 180° change in phase angle (+ or -); that is,

$$e = -E_m \sin \omega t = E_m \sin(\omega t + 180^\circ) = E_m \sin(\omega t - 180^\circ)$$

A plot of each will clearly show their equivalence. There are, therefore, two correct mathematical representations for the functions.

The **phase relationship** between two waveforms indicates which one leads or lags the other and by how many degrees or radians.

EXAMPLE 13.12 What is the phase relationship between the sinusoidal waveforms of each of the following sets?

- $v = 10 \sin(\omega t + 30^\circ)$
 $i = 5 \sin(\omega t + 70^\circ)$
- $i = 15 \sin(\omega t + 60^\circ)$
 $v = 10 \sin(\omega t - 20^\circ)$
- $i = 2 \cos(\omega t + 10^\circ)$
 $v = 3 \sin(\omega t - 10^\circ)$
- $i = -\sin(\omega t + 30^\circ)$
 $v = 2 \sin(\omega t + 10^\circ)$
- $i = -2 \cos(\omega t - 60^\circ)$
 $v = 3 \sin(\omega t - 150^\circ)$

Solutions:

- See Fig. 13.31.
 i leads v by 40° , or v lags i by 40° .

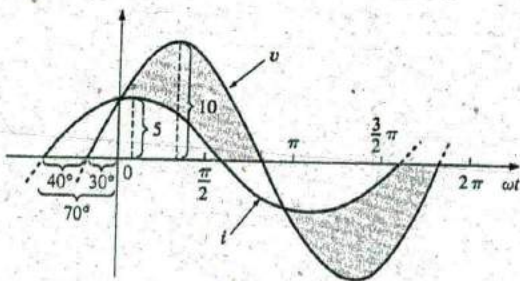


FIG. 13.31

Example 13.12(a): i leads v by 40° .

- See Fig. 13.32.
 i leads v by 80° , or v lags i by 80° .

- See Fig. 13.33.

$$\begin{aligned} i &= 2 \cos(\omega t + 10^\circ) = 2 \sin(\omega t + 10^\circ + 90^\circ) \\ &= 2 \sin(\omega t + 100^\circ) \end{aligned}$$

i leads v by 110° , or v lags i by 110° .

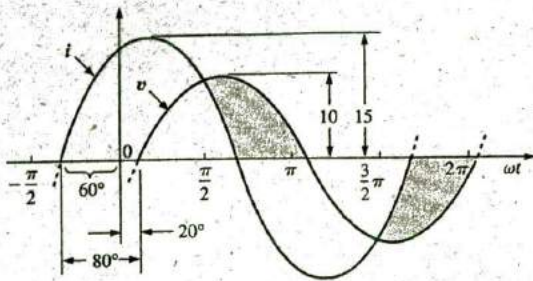


FIG. 13.32

Example 13.12(b): *i* leads *v* by 80° .

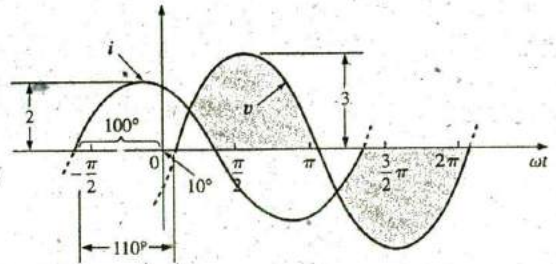


FIG. 13.33

Example 13.12(c): *i* leads *v* by 110° .

d. See Fig. 13.34.

$$\begin{aligned}
 -\sin(\omega t + 30^\circ) &= \sin(\omega t + 30^\circ - 180^\circ) \\
 &= \sin(\omega t - 150^\circ)
 \end{aligned}$$

Note

v leads *i* by 160° , or *i* lags *v* by 160° .

Or using

$$\begin{aligned}
 -\sin(\omega t + 30^\circ) &= \sin(\omega t + 30^\circ + 180^\circ) \\
 &= \sin(\omega t + 210^\circ)
 \end{aligned}$$

Note

i leads *v* by 200° , or *v* lags *i* by 200° .

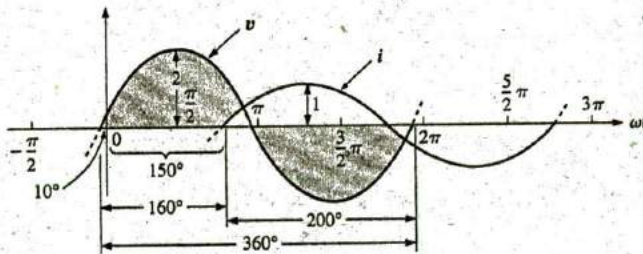


FIG. 13.34

Example 13.12(d): *v* leads *i* by 160° .

e. See Fig. 13.35.

$$\begin{aligned}
 i &= -2 \cos(\omega t - 60^\circ) = 2 \cos(\omega t - 60^\circ - 180^\circ) \\
 &= 2 \cos(\omega t - 240^\circ)
 \end{aligned}$$

By choice

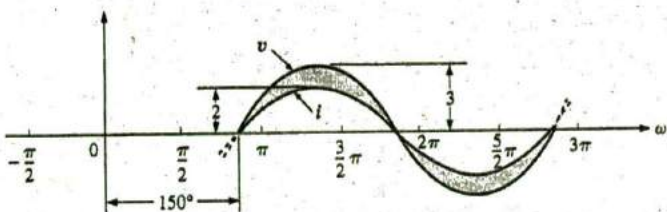


FIG. 13.35

Example 13.12(e): *v* and *i* are in phase.



However, $\cos \alpha = \sin(\alpha + 90^\circ)$

so that $2 \cos(\omega t - 240^\circ) = 2 \sin(\omega t - 240^\circ + 90^\circ)$
 $= 2 \sin(\omega t - 150^\circ)$

v and i are in phase.

Function Generators

Function generators are an important component of the typical laboratory setting. The generator of Fig. 13.36 can generate six different outputs: sine, triangular, and square wave, ramp, +pulse, and -pulse, with frequencies extending from 0.5 Hz to 4 MHz. However, as shown in the output listing, it has a maximum amplitude of $20 V_{p-p}$. A number of other characteristics are included to demonstrate how the text will cover each in some detail.



MAIN OUTPUT	
Frequency range	0.5 Hz to 4 MHz in six ranges
Waveforms	Six waveforms (sine, square, triangle, ramp, +pulse, -pulse)
Amplitude	$20 V_{p-p}$ into an open ($10 V_{p-p}$ in to 50Ω)
Attenuator	0 dB, -20 dB (+2%)—Chapter 21
Output impedance	50Ω (+2%)—Chapter 26
Distortion	<1%, 1 Hz to 100 kHz
Rise/fall time	<60 ns—Chapter 24
SYNC OUTPUT	
Rise time	<40 ns—Chapter 24
Waveforms	Square, pulse—Chapter 24
SWEEP	
Mode	Linear/log sweep—Chapter 21
Rate	From 10 ms to 5 s continuously variable
Sweep output	$10 V_{p-p}$ (open)
Output impedance	$1 k\Omega$ +2%—Chapter 26

FIG. 13.36

Function generator.
 (Courtesy B+K Precision.)

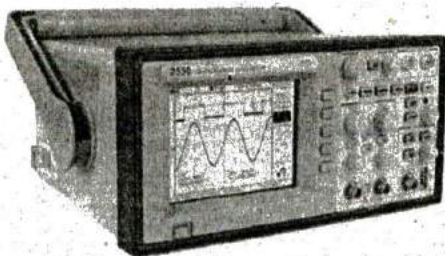


FIG. 13.37

Digital storage oscilloscope.
 (Courtesy B+K Precision.)

The Oscilloscope

The oscilloscope of Fig. 13.37 is an instrument that will display the sinusoidal alternating waveform in a way that will permit the reviewing of all of the waveform's characteristics. In some ways, the screen and the dials give an oscilloscope the appearance of a small TV, but remember that *it can display only what you feed into it*. You can't turn it on and ask for a sine wave, a square wave, and so on; it must be connected to a source or an active circuit to pick up the desired waveform.

The screen has a standard appearance, with 10 horizontal divisions and 8 vertical divisions. The distance between divisions is 1 cm on the vertical

and horizontal scales, providing you with an excellent opportunity to become aware of the length of 1 cm. The vertical scale is set to display voltage levels, whereas the horizontal scale is always in units of time. The vertical sensitivity control sets the voltage level for each division, whereas the horizontal sensitivity control sets the time associated with each division. In other words, if the vertical sensitivity is set at 1 V/div., each division displays a 1 V swing, so that a total vertical swing of 8 divisions represents 8 V peak-to-peak. If the horizontal control is set on 10 $\mu\text{s}/\text{div.}$, 4 divisions equal a time period of 40 μs . Remember, the oscilloscope display presents a sinusoidal voltage versus time, not degrees or radians. Further, the vertical scale is always a voltage sensitivity, never units of amperes:

The oscilloscope of Fig. 13.37 is a digital storage scope, where *storage* indicates that it can store waveform in digital form. The digital storage scope (DSO) is the standard for most laboratories today. At the input to the scope, an analog-to-digital converter (ADC) will convert the analog signal into digital at the rate of 250 MSa/s, or 250,000,000 samples per second—an enormous number—capable of picking up any distortion in the waveform.

EXAMPLE 13.13 Find the period, frequency, and peak value of the sinusoidal waveform appearing on the screen of the oscilloscope in Fig. 13.38. Note the sensitivities provided in the figure.

Solution: One cycle spans 4 divisions. Therefore, the period is

$$T = 4 \text{ div.} \left(\frac{50 \mu\text{s}}{\text{div.}} \right) = 200 \mu\text{s}$$

and the frequency is

$$f = \frac{1}{T} = \frac{1}{200 \times 10^{-6} \text{ s}} = 5 \text{ kHz}$$

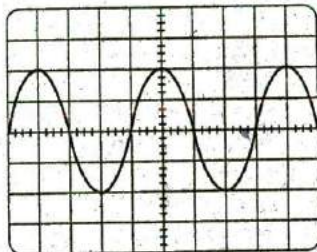
The vertical height above the horizontal axis encompasses 2 divisions. Therefore,

$$V_m = 2 \text{ div.} \left(\frac{0.1 \text{ V}}{\text{div.}} \right) = 0.2 \text{ V}$$

An oscilloscope can also be used to make phase measurements between two sinusoidal waveforms. Virtually all laboratory oscilloscopes today have the dual-trace option, that is, the ability to show two waveforms at the same time. It is important to remember, however, that both waveforms will and must have the same frequency. The hookup procedure for using an oscilloscope to measure phase angles is covered in detail in Section 15.13. However, the equation for determining the phase angle can be introduced using Fig. 13.39.

First, note that each sinusoidal function *has the same frequency*, permitting the use of either waveform to determine the period. For the waveform chosen in Fig. 13.39, the period encompasses 5 divisions at 0.2 ms/div. The phase shift between the waveforms (irrespective of which is leading or lagging) is 2 divisions. Since the full period represents a cycle of 360°, the following ratio [from which Eq. (13.24) can be derived] can be formed:

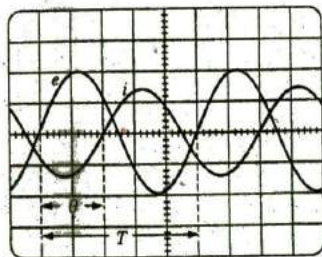
$$\frac{360^\circ}{T \text{ (no. of div.)}} = \frac{\theta}{\text{phase shift (no. of div.)}}$$



Vertical sensitivity = 0.1 V/div.
Horizontal sensitivity = 50 $\mu\text{s}/\text{div.}$

FIG. 13.38

Example 13.13.



Vertical sensitivity = 2 V/div.
Horizontal sensitivity = 0.2 ms/div.

FIG. 13.39

Finding the phase angle between waveforms using a dual-trace oscilloscope.



and

$$\theta = \frac{\text{phase shift (no. of div.)}}{T \text{ (no. of div.)}} \times 360^\circ \quad (13.24)$$

Substituting into Eq. (13.24) results in

$$\theta = \frac{(2 \text{ div.})}{(5 \text{ div.})} \times 360^\circ = 144^\circ$$

and e leads i by 144° .

13.7 AVERAGE VALUE

Even though the concept of the **average value** is an important one in most technical fields, its true meaning is often misunderstood. In Fig. 13.40(a), for example, the average height of the sand may be required to determine the volume of sand available. The average height of the sand is that height obtained if the distance from one end to the other is maintained while the sand is leveled off, as shown in Fig. 13.40(b). The area under the mound in Fig. 13.40(a) then equals the area under the rectangular shape in Fig. 13.40(b) as determined by $A = b \times h$. Of course, the depth (into the page) of the sand must be the same for Fig. 13.40(a) and (b) for the preceding conclusions to have any meaning.

In Fig. 13.40, the distance was measured from one end to the other. In Fig. 13.41(a), the distance extends beyond the end of the original pile in Fig. 13.40. The situation could be one where a landscaper wants to know the average height of the sand if it is spread out over a distance such as defined in Fig. 13.41(a). The result of an increased distance is shown in Fig. 13.41(b). The average height has decreased compared to Fig. 13.40. Quite obviously, therefore, the longer the distance, the lower is the average value.

If the distance parameter includes a depression, as shown in Fig. 13.42(a), some of the sand will be used to fill the depression, resulting in

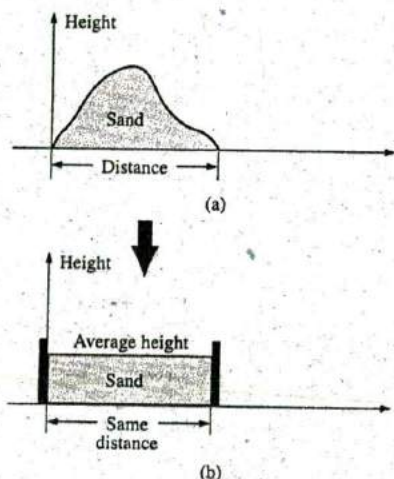


FIG. 13.40
Defining average value.

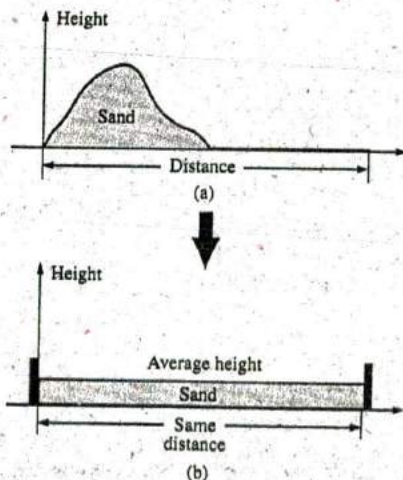


FIG. 13.41
Effect of distance (length) on average value.

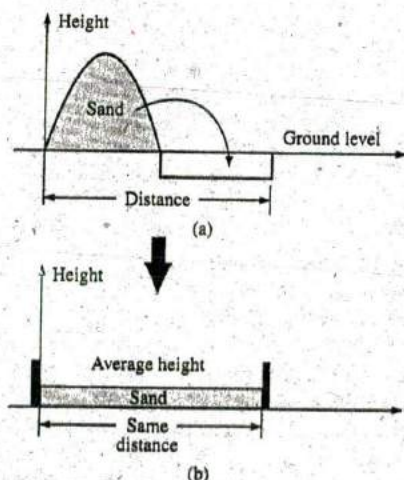


FIG. 13.42
Effect of depressions (negative excursions) on average value.

an even lower average value for the landscaper, as shown in Fig. 13.42(b). For a sinusoidal waveform, the depression would have the same shape as the mound of sand (over one full cycle), resulting in an average value at ground level (or zero volts for a sinusoidal voltage over one full period).

After traveling a considerable distance by car, some drivers like to calculate their average speed for the entire trip. This is usually done by dividing the miles traveled by the hours required to drive that distance. For example, if a person traveled 225 mi in 5 h, the average speed was 225 mi/5 h, or 45 mi/h. This same distance may have been traveled at various speeds for various intervals of time, as shown in Fig. 13.43.

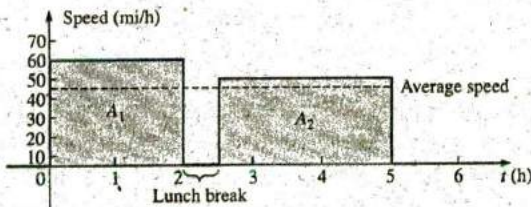


FIG. 13.43

Plotting speed versus time for an automobile excursion.

By finding the total area under the curve for the 5 h and then dividing the area by 5 h (the total time for the trip), we obtain the same result of 45 mi/h; that is,

$$\text{Average speed} = \frac{\text{area under curve}}{\text{length of curve}} \quad (13.25)$$

$$\begin{aligned} \text{Average speed} &= \frac{A_1 + A_2}{5 \text{ h}} = \frac{(60 \text{ mi/h})(2 \text{ h}) + (50 \text{ mi/h})(2.5 \text{ h})}{5 \text{ h}} \\ &= \frac{225}{5} \text{ mi/h} = 45 \text{ mi/h} \end{aligned}$$

Eq. (13.25) can be extended to include any variable quantity, such as current or voltage, if we let G denote the average value, as follows:

$$G \text{ (average value)} = \frac{\text{algebraic sum of areas}}{\text{length of curve}} \quad (13.26)$$

The algebraic sum of the areas must be determined since some area contributions are from below the horizontal axis. Areas above the axis are assigned a positive sign and those below it a negative sign. A positive average value is then above the axis, and a negative value is below it.

The average value of any current or voltage is the value indicated on a dc meter. In other words, over a complete cycle, the average value is the equivalent dc value. In the analysis of electronic circuits to be considered in a later course, both dc and ac sources of voltage will be applied to the same network. You will then need to know or determine the dc (or average value) and ac components of the voltage or current in various parts of the system.



EXAMPLE 13.14 Determine the average value of the waveforms in Fig. 13.44.

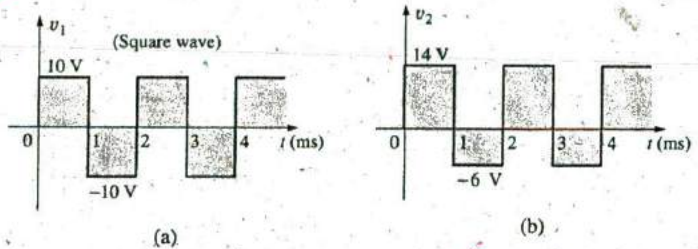


FIG. 13.44
Example 13.14.

Solutions:

- a. By inspection, the area above the axis equals the area below over one cycle, resulting in an average value of zero volts. Using Eq. (13.26) gives

$$G = \frac{(10 \text{ V})(1 \text{ ms}) - (10 \text{ V})(1 \text{ ms})}{2 \text{ ms}} = \frac{0}{2 \text{ ms}} = 0 \text{ V}$$

- b. Using Eq. (13.26) gives

$$G = \frac{(14 \text{ V})(1 \text{ ms}) - (6 \text{ V})(1 \text{ ms})}{2 \text{ ms}} = \frac{14 \text{ V} - 6 \text{ V}}{2} = \frac{8 \text{ V}}{2} = 4 \text{ V}$$

as shown in Fig. 13.45.

In reality, the waveform in Fig. 13.44(b) is simply the square wave in Fig. 13.44(a) with a dc shift of 4 V; that is,

$$v_2 = v_1 + 4 \text{ V}$$

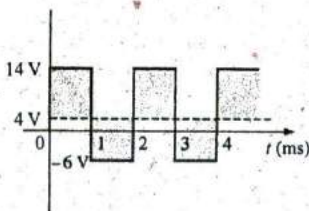


FIG. 13.45
Defining the average value for the waveform in Fig. 13.44(b).

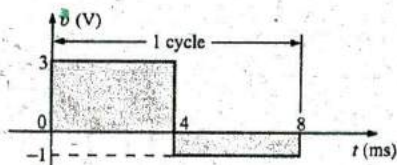


FIG. 13.46
Example 13.15(a).

EXAMPLE 13.15 Find the average values of the following waveforms over one full cycle:

- a. Fig. 13.46.
b. Fig. 13.47.

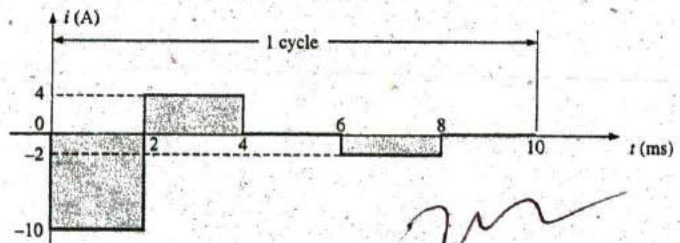


FIG. 13.47
Example 13.15(b).

Solutions:

a. $G = \frac{+(3 \text{ V})(4 \text{ ms}) - (1 \text{ V})(4 \text{ ms})}{8 \text{ ms}} = \frac{12 \text{ V} - 4 \text{ V}}{8} = 1 \text{ V}$

Note Fig. 13.48.

$$\begin{aligned}
 b. \ G &= \frac{-(10\text{ V})(2\text{ ms}) - (4\text{ V})(2\text{ ms}) - (2\text{ V})(2\text{ ms})}{10\text{ ms}} \\
 &= \frac{-20\text{ V} + 8\text{ V} - 4\text{ V}}{10} = \frac{-16\text{ V}}{10} = -1.6\text{ V}
 \end{aligned}$$

Note Fig. 13.49.

We found the areas under the curves in Example 13.15 by using a simple geometric formula. If we encounter a sine wave or any other unusual shape, however, we must find the area by some other means. We can obtain a good approximation of the area by attempting to reproduce the original wave shape using a number of small rectangles or other familiar shapes, the area of which we already know through simple geometric formulas. For example,

the area of the positive (or negative) pulse of a sine wave is $2A_m$.

Approximating this waveform by two triangles (Fig. 13.50), we obtain (using $\text{area} = 1/2 \text{ base} \times \text{height}$ for the area of a triangle) a rough idea of the actual area:

$$\text{Area shaded} = 2 \left(\frac{1}{2} bh \right) = 2 \left[\left(\frac{1}{2} \right) \left(\frac{\pi}{2} \right) (A_m) \right] = \frac{\pi}{2} A_m \cong 1.58 A_m$$

A closer approximation may be a rectangle with two similar triangles (Fig. 13.51):

$$\text{Area} = A_m \frac{\pi}{3} + 2 \left(\frac{1}{2} bh \right) = A_m \frac{\pi}{3} + \frac{\pi}{3} A_m = \frac{2}{3} \pi A_m = 2.094 A_m$$

which is certainly close to the actual area. If an infinite number of forms is used, an exact answer of $2A_m$ can be obtained. For irregular waveforms, this method can be especially useful if data such as the average value are desired.

The procedure of calculus that gives the exact solution $2A_m$ is known as *integration*. Integration is presented here only to make the method recognizable to you; it is not necessary to be proficient in its use to continue with this text. It is a useful mathematical tool, however, and should be learned. Finding the area under the positive pulse of a sine wave using integration, we have

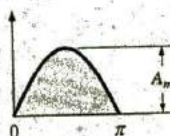
$$\text{Area} = \int_0^{\pi} A_m \sin \alpha \, d\alpha$$

where \int is the sign of integration, 0 and π are the limits of integration, $A_m \sin \alpha$ is the function to be integrated, and $d\alpha$ indicates that we are integrating with respect to α .

Integrating, we obtain

$$\begin{aligned}
 \text{Area} &= A_m [-\cos \alpha]_0^{\pi} \\
 &= -A_m (\cos \pi - \cos 0^\circ) \\
 &= -A_m [-1 - (+1)] = -A_m (-2)
 \end{aligned}$$

Area = $2A_m$



(13.27)

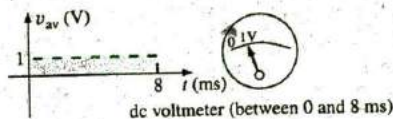


FIG. 13.48

The response of a dc meter to the waveform in Fig. 13.46.

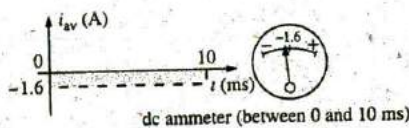


FIG. 13.49

The response of a dc meter to the waveform in Fig. 13.47.

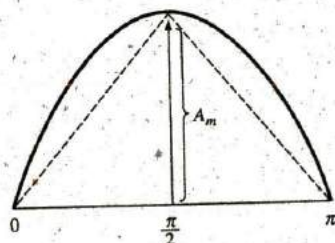


FIG. 13.50

Approximating the shape of the positive pulse of a sinusoidal waveform with two right triangles.

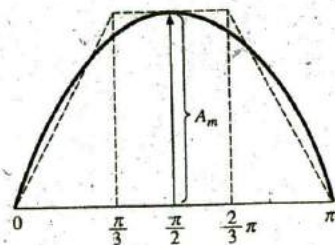


FIG. 13.51

A better approximation for the shape of the positive pulse of a sinusoidal waveform.

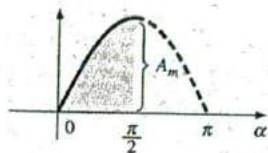


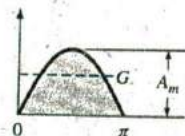
FIG. 13.52

Finding the average value of one-half the positive pulse of a sinusoidal waveform.

$$G = \frac{2A_m}{\pi}$$

and

$$G = \frac{2A_m}{\pi} = 0.637A_m$$



(13.28)

For the waveform in Fig. 13.52,

$$G = \frac{(2A_m/2)}{\pi/2} = \frac{2A_m}{\pi} \quad (\text{The average is the same as for a full pulse.})$$

EXAMPLE 13.16 Determine the average value of the sinusoidal waveform in Fig. 13.53.

Solution: By inspection it is fairly obvious that

the average value of a pure sinusoidal waveform over one full cycle is zero.

Eq. (13.26):

$$G = \frac{+2A_m - 2A_m}{2\pi} = 0 \text{ V}$$

EXAMPLE 13.17 Determine the average value of the waveform in Fig. 13.54.

Solution: The peak-to-peak value of the sinusoidal function is $16 \text{ mV} + 2 \text{ mV} = 18 \text{ mV}$. The peak amplitude of the sinusoidal waveform is, therefore, $18 \text{ mV}/2 = 9 \text{ mV}$. Counting down 9 mV from 2 mV (or 9 mV up from -16 mV) results in an average or dc level of -7 mV , as noted by the dashed line in Fig. 13.54.

EXAMPLE 13.18 Determine the average value of the waveform in Fig. 13.55.

Solution:

$$G = \frac{2A_m + 0}{2\pi} = \frac{2(10 \text{ V})}{2\pi} \cong 3.18 \text{ V}$$

EXAMPLE 13.19 For the waveform in Fig. 13.56, determine whether the average value is positive or negative, and determine its approximate value.

Solution: From the appearance of the waveform, the average value is positive and in the vicinity of 2 mV . Occasionally, judgments of this type will have to be made.

Instrumentation

The dc level or average value of any waveform can be found using a digital multimeter (DMM) or an oscilloscope. For purely dc circuits, set the

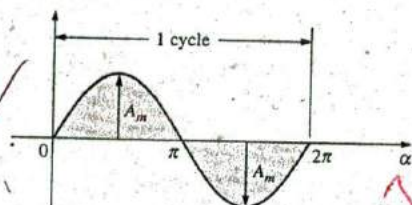


FIG. 13.53

Example 13.16.

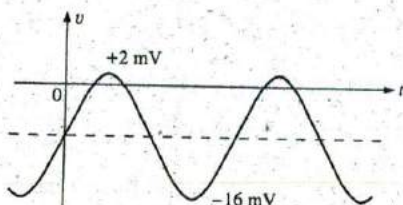


FIG. 13.54

Example 13.17.

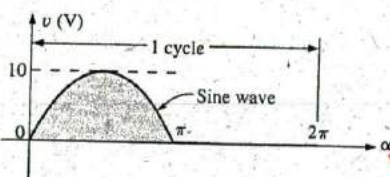


FIG. 13.55

Example 13.18.

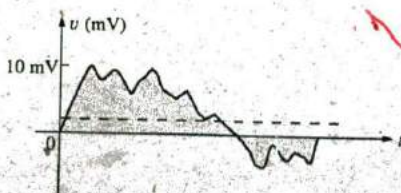


FIG. 13.56

Example 13.19.

DMM on dc, and read the voltage or current levels. Oscilloscopes are limited to voltage levels using the sequence of steps listed below:

1. First choose GND from the DC-GND-AC option list associated with each vertical channel. The GND option blocks any signal to which the oscilloscope probe may be connected from entering the oscilloscope and responds with just a horizontal line. Set the resulting line in the middle of the vertical axis on the horizontal axis, as shown in Fig. 13.57(a).
2. Apply the oscilloscope probe to the voltage to be measured (if not already connected), and switch to the DC option. If a dc voltage is present, the horizontal line shifts up or down, as demonstrated in Fig. 13.57(b). Multiplying the shift by the vertical sensitivity results in the dc voltage. An upward shift is a positive voltage (higher potential at the red or positive lead of the oscilloscope), while a downward shift is a negative voltage (lower potential at the red or positive lead of the oscilloscope).

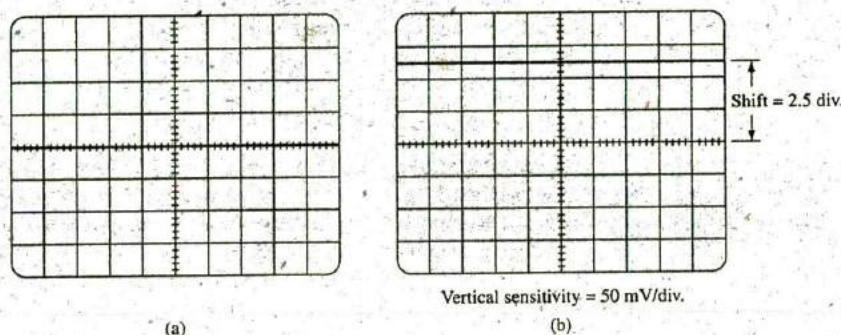


FIG. 13.57

Using the oscilloscope to measure dc voltages; (a) setting the GND condition; (b) the vertical shift resulting from a dc voltage when shifted to the DC option.

In general,

$$V_{dc} = (\text{vertical shift in div.}) \times (\text{vertical sensitivity in V/div.}) \quad (13.29)$$

For the waveform in Fig. 13.57(b),

$$V_{dc} = (2.5 \text{ div.})(50 \text{ mV/div.}) = 125 \text{ mV}$$

The oscilloscope can also be used to measure the dc or average level of any waveform using the following sequence:

1. Using the GND option, reset the horizontal line to the middle of the screen.
2. Switch to AC (all dc components of the signal to which the probe is connected will be blocked from entering the oscilloscope—only the alternating, or changing, components are displayed). Note the location of some definitive point on the waveform, such as the bottom of the half-wave rectified waveform of Fig. 13.58(a); that is, note its position on the vertical scale. For the future, whenever you use the AC option, keep in mind that the computer will distribute the waveform above and below the horizontal axis such that the average value is zero; that is, the area above the axis will equal the area below.

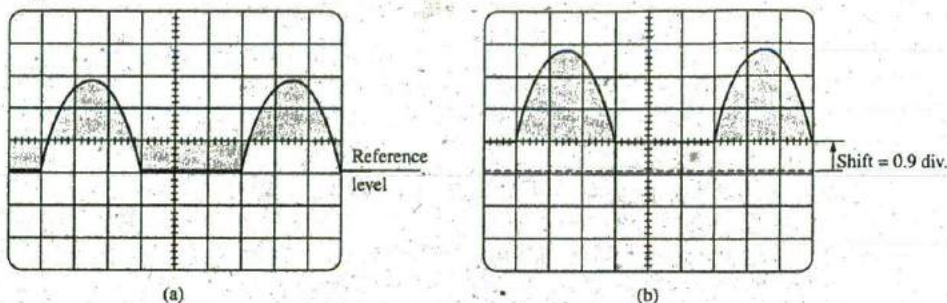


FIG. 13.58

Determining the average value of a nonsinusoidal waveform using the oscilloscope:
 (a) vertical channel on the ac mode; (b) vertical channel on the dc mode.

- Then switch to DC (to permit both the dc and the ac components of the waveform to enter the oscilloscope), and note the shift in the chosen level of part 2, as shown in Fig. 13.58(b). Eq. (13.29) can then be used to determine the dc or average value of the waveform. For the waveform in Fig. 13.58(b), the average value is about

$$V_{av} = V_{dc} = (0.9 \text{ div.})(5 \text{ V/div.}) = 4.5 \text{ V}$$

The procedure outlined above can be applied to any alternating waveform such as the one in Fig. 13.56. In some cases the average value may require moving the starting position of the waveform under the AC option to a different region of the screen or choosing a higher voltage scale. By choosing the appropriate scale, you can enable DMMs to read the average or dc level of any waveform.

13.8 EFFECTIVE (rms) VALUES

This section begins to relate dc and ac quantities with respect to the power delivered to a load. It will help us determine the amplitude of a sinusoidal ac current required to deliver the same power as a particular dc current. The question frequently arises, How is it possible for a sinusoidal ac quantity to deliver a net power if, over a full cycle, the net current in any one direction is zero (average value = 0)? It would almost appear that the power delivered during the positive portion of the sinusoidal waveform is withdrawn during the negative portion, and since the two are equal in magnitude, the net power delivered is zero. However, understand that regardless of *direction*, current of any magnitude through a resistor delivers power to that resistor. In other words, during the positive or negative portions of a sinusoidal ac current, power is being delivered at *each instant of time* to the resistor. The power delivered at each instant, of course, varies with the magnitude of the sinusoidal ac current, but there will be a net flow during either the positive or the negative pulses with a net flow over the full cycle. The net power flow equals twice that delivered by either the positive or the negative regions of sinusoidal quantity.

A fixed relationship between ac and dc voltages and currents can be derived from the experimental setup shown in Fig. 13.59. A resistor in a water bath is connected by switches to a dc and an ac supply. If switch 1 is closed, a dc current I , determined by the resistance R and battery voltage E , is established through the resistor R . The temperature reached by

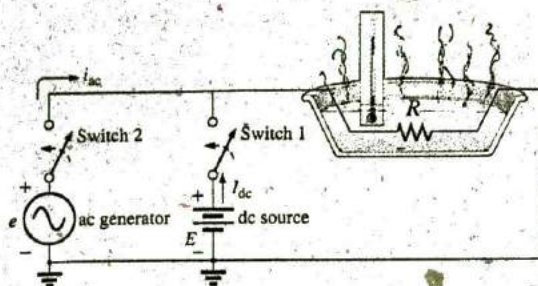


FIG. 13.59

An experimental setup to establish a relationship between dc and ac quantities.

the water is determined by the dc power dissipated in the form of heat by the resistor.

If switch 2 is closed and switch 1 left open, the ac current through the resistor has a peak value of I_m . The temperature reached by the water is now determined by the ac power dissipated in the form of heat by the resistor. The ac input is varied until the temperature is the same as that reached with the dc input. When this is accomplished, the average electrical power delivered to the resistor R by the ac source is the same as that delivered by the dc source.

The power delivered by the ac supply at any instant of time is

$$P_{ac} = (i_{ac})^2 R = (I_m \sin \omega t)^2 R = (I_m^2 \sin^2 \omega t) R$$

However,

$$\sin^2 \omega t = \frac{1}{2}(1 - \cos 2\omega t) \quad (\text{trigonometric identity})$$

Therefore,

$$P_{ac} = I_m^2 R \left[\frac{1}{2}(1 - \cos 2\omega t) \right]$$

and

$$P_{ac} = \frac{I_m^2 R}{2} - \frac{I_m^2 R}{2} \cos 2\omega t \quad (13.30)$$

The average power delivered by the ac source is just the first term, since the average value of a cosine wave is zero even though the wave may have twice the frequency of the original input current waveform. Equating the average power delivered by the ac generator to that delivered by the dc source,

$$P_{av(ac)} = P_{dc}$$

$$\frac{I_m^2 R}{2} = I_{dc}^2 R$$

and

$$I_{dc} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

which, in words, states that

the equivalent dc value of a sinusoidal current or voltage is $1/\sqrt{2}$ or 0.707 of its peak value.



The equivalent dc value is called the **rms** or **effective value** of the sinusoidal quantity.

As a simple numerical example, it requires an ac current with a peak value of $\sqrt{2}(10) = 14.14$ A to deliver the same power to the resistor in Fig. 13.59 as a dc current of 10 A. The effective value of any quantity plotted as a function of time can be found by using the following equation derived from the experiment just described:

$$\text{Calculus format: } I_{\text{rms}} = \sqrt{\frac{\int_0^T i^2(t) dt}{T}} \quad (13.31)$$

$$\text{which means } I_{\text{rms}} = \sqrt{\frac{\text{area}(i^2(t))}{T}} \quad (13.32)$$

In words, Eqs. (13.31) and (13.32) state that to find the rms value, the function $i(t)$ must first be squared. After $i(t)$ is squared, the area under the curve is found by integration. It is then divided by T , the length of the cycle or the period of the waveform, to obtain the average or *mean* value of the squared waveform. The final step is to take the *square root* of the mean value. This procedure is the source for the other designation for the effective value, the **root-mean-square (rms) value**. In fact, since *rms* is the most commonly used term in the educational and industrial communities, it is used throughout this text.

The relationship between the peak value and the rms value is the same for voltages, resulting in the following set of relationships for the examples and text material to follow:

$$\begin{aligned} I_{\text{rms}} &= \frac{1}{\sqrt{2}} I_m = 0.707 I_m \\ E_{\text{rms}} &= \frac{1}{\sqrt{2}} E_m = 0.707 E_m \end{aligned} \quad (13.33)$$

Similarly,

$$\begin{aligned} I_m &= \sqrt{2} I_{\text{rms}} = 1.414 I_{\text{rms}} \\ E_m &= \sqrt{2} E_{\text{rms}} = 1.414 E_{\text{rms}} \end{aligned} \quad (13.34)$$

EXAMPLE 13.20 Find the rms values of the sinusoidal waveform in each part in Fig. 13.60.

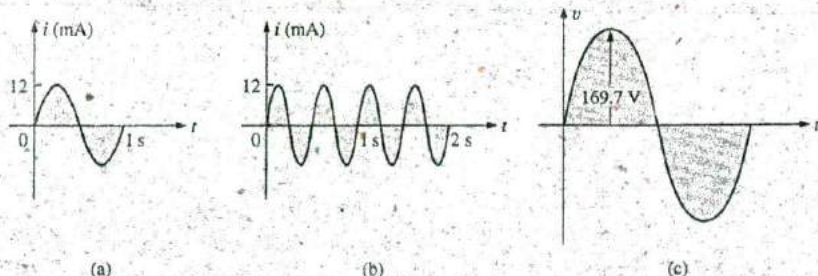


FIG. 13.60
Example 13.20.

Solution: For part (a), $I_{\text{rms}} = 0.707(12 \times 10^{-3} \text{ A}) = 8.48 \text{ mA}$. For part (b), again $I_{\text{rms}} = 8.48 \text{ mA}$. Note that frequency did not change the effective value in (b) compared to (a). For part (c), $V_{\text{rms}} = 0.707(169.73 \text{ V}) \approx 120 \text{ V}$, the same as available from a home outlet.

EXAMPLE 13.21 The 120 V dc source in Fig. 13.61(a) delivers 3.6 W to the load. Determine the peak value of the applied voltage (E_m) and the current (I_m) if the ac source [Fig. 13.61(b)] is to deliver the same power to the load.

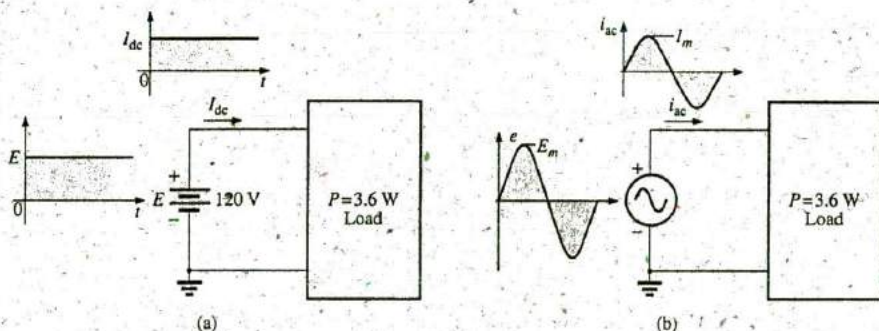


FIG. 13.61
Example 13.21.

Solution:

$$P_{\text{dc}} = V_{\text{dc}} I_{\text{dc}}$$

and
$$I_{\text{dc}} = \frac{P_{\text{dc}}}{V_{\text{dc}}} = \frac{3.6 \text{ W}}{120 \text{ V}} = 30 \text{ mA}$$

$$I_m = \sqrt{2} I_{\text{dc}} = (1.414)(30 \text{ mA}) = 42.42 \text{ mA}$$

$$E_m = \sqrt{2} E_{\text{dc}} = (1.414)(120 \text{ V}) = 169.68 \text{ V}$$

EXAMPLE 13.22 Find the rms value of the waveform in Fig. 13.62.

Solution: v^2 (Fig. 13.63):

$$V_{\text{rms}} = \sqrt{\frac{(9)(4) + (1)(4)}{8}} = \sqrt{\frac{40}{8}} = 2.24 \text{ V}$$

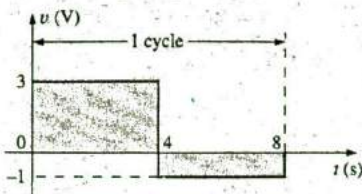


FIG. 13.62
Example 13.22.

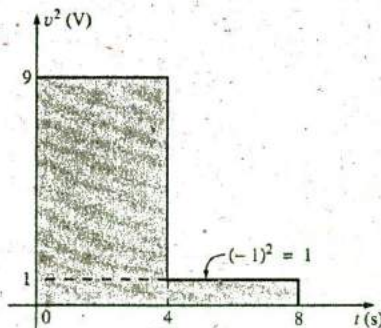


FIG. 13.63
The squared waveform of Fig. 13.62.



EXAMPLE 13.23 Calculate the rms value of the voltage in Fig. 13.64.

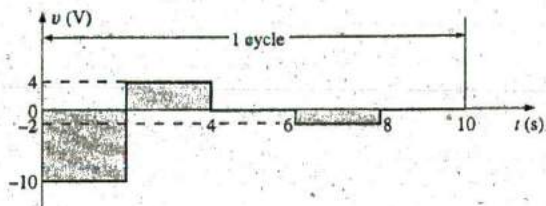


FIG. 13.64
Example 13.23.

Solution: v^2 (Fig. 13.65):

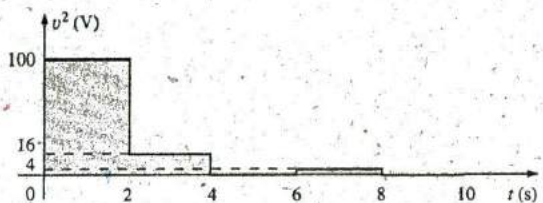


FIG. 13.65
The squared waveform of Fig. 13.64.

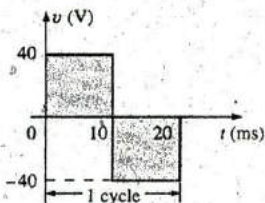


FIG. 13.66
Example 13.24.

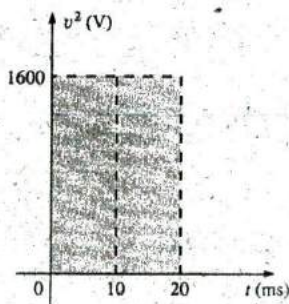


FIG. 13.67
The squared waveform of Fig. 13.66.

EXAMPLE 13.24 Determine the average and rms values of the square wave in Fig. 13.66.

Solution: By inspection, the average value is zero.

v^2 (Fig. 13.67):

$$\begin{aligned} V_{\text{rms}} &= \sqrt{\frac{(1600)(10 \times 10^{-3}) + (1600)(10 \times 10^{-3})}{20 \times 10^{-3}}} \\ &= \sqrt{\frac{(32,000 \times 10^{-3})}{20 \times 10^{-3}}} = \sqrt{1600} = 40 \text{ V} \end{aligned}$$

(the maximum value of the waveform in Fig. 13.66).

The waveforms appearing in these examples are the same as those used in the examples on the average value. It may prove interesting to compare the rms and average values of these waveforms.

The rms values of sinusoidal quantities such as voltage or current are represented by E and I . These symbols are the same as those used for dc voltages and currents. To avoid confusion, the peak value of a waveform always has a subscript m associated with it: $I_m \sin \omega t$. *Caution:* When finding the rms value of the positive pulse of a sine wave, note that the squared area is *not* simply $(2A_m)^2 = 4A_m^2$; it must be found by a completely new integration. This is always true for any waveform that is not rectangular.

A unique situation arises if a waveform has both a dc and an ac component that may be due to a source, such as the one in Fig. 13.68. The combination appears frequently in the analysis of electronic networks where both dc and ac levels are present in the same system.

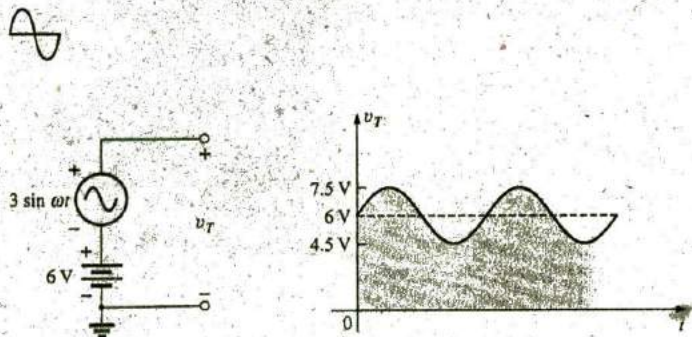


FIG. 13.68

Generation and display of a waveform having a dc and an ac component.

The question arises, What is the rms value of the voltage v_T ? You may be tempted to assume that it is the sum of the rms values of each component of the waveform; that is, $V_{T_{\text{rms}}} = 0.7071(1.5 \text{ V}) + 6 \text{ V} = 1.06 \text{ V} + 6 \text{ V} = 7.06 \text{ V}$. However, the rms value is actually determined by

$$V_{\text{rms}} = \sqrt{V_{\text{dc}}^2 + V_{\text{ac(rms)}}^2} \quad (13.35)$$

which for the waveform in Fig. 13.68 is

$$V_{\text{rms}} = \sqrt{(6 \text{ V})^2 + (1.06 \text{ V})^2} = \sqrt{37.124} \text{ V} \cong 6.1 \text{ V}$$

This result is noticeably less than the solution of 7.06 V.

True rms Meters

Throughout this section, the rms value of a variety of waveforms was determined to help ensure that the concept is correctly understood. However, to use a meter to measure the rms value of the same waveforms would require a specially designed meter. Too often, the face of a meter will read **True rms Multimeter** or such. However, in most cases the meter is only designed to read the rms value of periodic signals with no dc level and have a symmetry about the zero axis. Most multimeters are ac coupled (the dc component of the signal is blocked by a capacitor at the input terminals), so only the ac portion is measured. For such cases one may be able to first determine the rms value of the ac portion of the waveform and then use the dc section of the meter to measure the dc level. Then Eq. (13.35) can be used to determine the correct rms value.

The problem, however, is that many waveforms are not symmetric about the zero axis—How is an rms reading obtained? In general, the rms value of any waveform is a measure of the “heating” potential of the applied waveform, as discussed earlier in this section. A direct result is the development of meters that use a thermal converter calibrated to display the proper rms value. A drawback of this approach, however, is that the meter will draw power from the circuit during the heating process, and the results have a low precision standard. A better approach that is commonly used uses an analog-to-digital converter (ADC) mentioned earlier to digitize the signal, so that the rms value then can be determined to a high degree of accuracy. One such meter appears in Fig. 13.69, which samples the input signal at 1.4 MHz, or 1,400,000 samples per second—certainly sufficient for a wide variety of signals. This meter will run the sampling rate at all times, even when making dc measurements, so both the dc and ac content of a waveform can be displayed at the same time.



FIG. 13.69

True rms multimeter.
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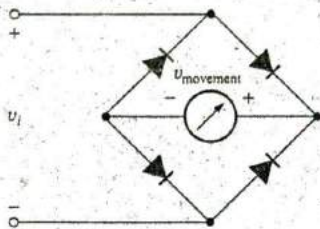
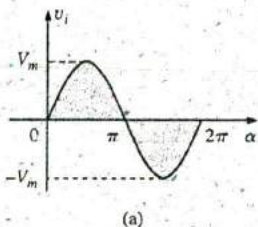
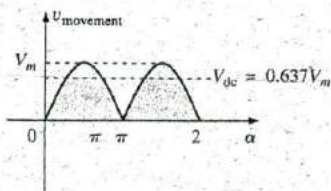


FIG. 13.70

Full-wave bridge rectifier.



(a)



(b)

FIG. 13.71

(a) Sinusoidal input; (b) full-wave rectified signal.

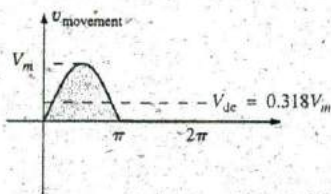


FIG. 13.72

Half-wave rectified signal.

13.9 ac METERS AND INSTRUMENTS

Iron-Vane or d'Arsonval Movement

If an average reading movement such as the iron-vane movement used in the VOM of Fig. 2.29 is used to measure an ac current or voltage, the level indicated by the movement must be multiplied by a **calibration factor**. In other words, if the movement of any voltmeter or ammeter is reading the average value, that level must be multiplied by a specific constant, or calibration factor, to indicate the rms level. For ac waveforms, the signal must first be converted to one having an average value over the time period. Recall that it is zero over a full period for a sinusoidal waveform. This is usually accomplished for sinusoidal waveforms using a bridge rectifier such as in Fig. 13.70. The conversion process, involving four diodes in a bridge configuration, is well documented in most electronic texts.

Fundamentally, conduction is permitted through the diodes in such a manner as to convert the sinusoidal input of Fig. 13.71(a) to one having the appearance of Fig. 13.71(b). The negative portion of the input has been effectively "flipped over" by the bridge configuration. The resulting waveform in Fig. 13.71(b) is called a **full-wave rectified waveform**.

The zero average value in Fig. 13.71(a) has been replaced by a pattern having an average value determined by

$$G = \frac{2V_m + 2V_m}{2\pi} = \frac{4V_m}{2\pi} = \frac{2V_m}{\pi} = 0.637V_m$$

The movement of the pointer is therefore directly related to the peak value of the signal by the factor 0.637.

Forming the ratio between the rms and dc levels results in

$$\frac{V_{rms}}{V_{dc}} = \frac{0.707V_m}{0.637V_m} \cong 1.11$$

revealing that the scale indication is 1.11 times the dc level measured by the movement; that is,

$$\boxed{\text{Meter indication} = 1.11 (\text{dc or average value})} \quad \text{full-wave (13.36)}$$

Some ac meters use a half-wave rectifier arrangement that results in the waveform in Fig. 13.72, which has half the average value in Fig. 13.71(b) over one full cycle. The result is

$$\boxed{\text{Meter indication} = 2.22 (\text{dc or average value})} \quad \text{half-wave (13.37)}$$



Electrodynamometer Movement

The electrodynamicometer movement is a movement that has the distinct advantage of being able to read the turn rms value of any current, voltage, or power measurement without additional circuitry. The basic construction appears in Fig. 13.73, which shows two fixed coils and a rotating coil. The two fixed coils establish a field similar to that established by the permanent magnet in an iron-vane movement. However, in this case, the same current that establishes the field in the fixed coils will also establish the field in the movable coil. The result is opposing polarities between the rotating and fixed coils that will establish a torque on the movable coil and cause it to rotate and provide a reading using the attached pointer. Removing the excitation force will allow the attached spring to bring the pointer back to the rest position. Although the electrodynamicometer movement would be very effective in reading the rms value of any voltage or current, it is used almost exclusively in dc/ac wattmeters for any shape of input. It can also be used for phase shift measurements, harmonic analysis, and frequency measurements, although improving digital electronic technology is the new direction for these areas of application.

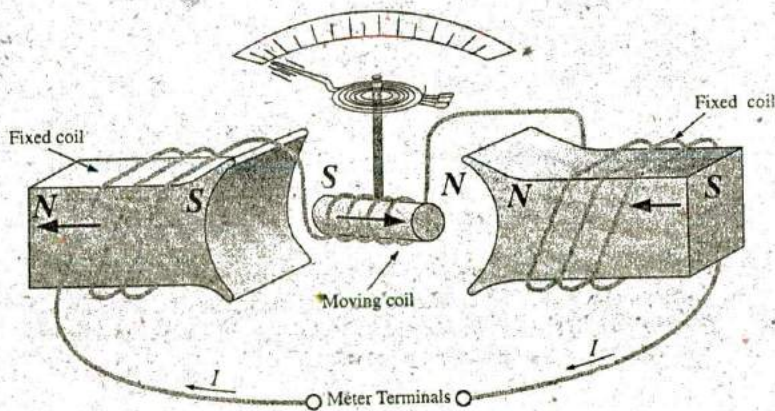


FIG. 13.73
Electrodynamometer movement.

EXAMPLE 13.25 Determine the reading of each meter for each situation in Fig. 13.74(a) and (b).

Solution: For Fig. 13.74(a), situation (1): By Eq. (13.36),

$$\text{Meter indication} = 1.11(20 \text{ V}) = 22.2 \text{ V}$$

For Fig. 13.74(a), situation (2):

$$V_{\text{rms}} = 0.707V_m = (0.707)(20 \text{ V}) = 14.14 \text{ V}$$

For Fig. 13.74(b), situation (1):

$$V_{\text{rms}} = V_{\text{dc}} = 25 \text{ V}$$

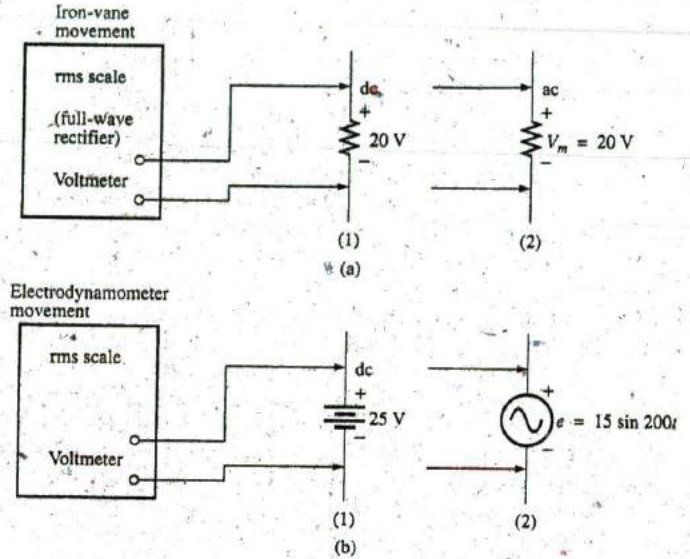


FIG. 13.74

Example 13.25.

For Fig. 13.74(b), situation (2):

$$V_{\text{rms}} = 0.707V_m = 0.707(15\text{ V}) \cong 10.6\text{ V}$$

Frequency Counter

For frequency measurements, the **frequency counter** in Fig. 13.75 provides a digital readout of sine, square, and triangular waves from 0.1 Hz to 2.4 GHz. The temperature-compensated, crystal-controlled time base is stable to ± 1 part per million per year.

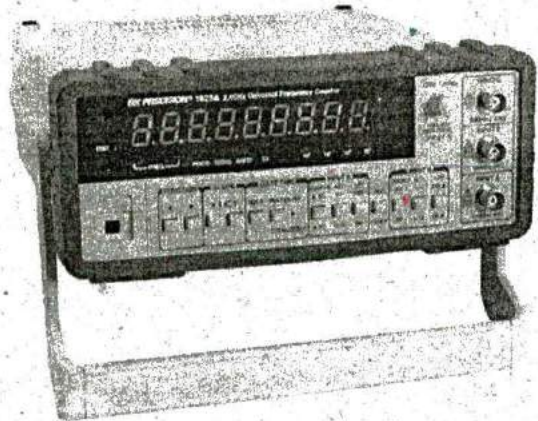


FIG. 13.75

Frequency counter, 2.4 GHz multifunction instrument.
(Courtesy of B+K Precision.)

Clamp-on Meters

The AEMC® **Clamp Meter** in Fig. 13.76 is an instrument that can measure alternating current in the ampere range without having to open the circuit. The loop is opened by squeezing the "trigger"; then it is placed around the current-carrying conductor. Through transformer action, the level of current in rms units appears on the appropriate scale. The Model 501 is auto-ranging (that is, each scale changes automatically) and can measure dc or ac currents up to 400 mA. Through the use of additional leads, it can also be used as a voltmeter (up to 400 V, dc or ac) and an ohmmeter (from zero to 400 Ω).

Impedance Measurements

Before we leave the subject of ac meters and instrumentation, you should understand that

an ohmmeter cannot be used to measure the ac reactance or impedance of an element or system even though reactance and impedance are measured in ohms.

Recall that ohmmeters cannot be used on energized networks—the power must be shut off or disconnected. For an inductor, if the ac power is removed, the reactance of the coil is simply the dc resistance of the windings because the applicable frequency will be 0 Hz. For a capacitor, if the ac power is removed, the reactance of the capacitor is simply the leakage resistance of the capacitor. In general, therefore, always keep in mind that *ohmmeters can read only the dc resistance of an element or network, and only after the applied power has been removed.*

13.10 APPLICATIONS

(120 V at 60 Hz) versus (220 V at 50 Hz)

In North and South America, the most common available ac supply is 120 V at 60 Hz; in Western and Central Europe, Africa, Asia, and Australia, 220 V at 50 Hz is the most common. Japan is unique in that the eastern part of the country uses 100 V at 50 Hz, whereas most of the western part uses 100 V at 60 Hz. It is 220 V at 50 Hz. The choices of rms value and frequency were obviously made carefully because they have such an important impact on the design and operation of so many systems.

The fact that the frequency difference is only 10 Hz reveals that there was agreement on the general frequency range that should be used for power generation and distribution. History suggests that the question of frequency selection originally focused on the frequency that would not exhibit *flicker in the incandescent lamps* available in those days. Technically, however, there really wouldn't be a noticeable difference between 50 and 60 cycles per second based on this criterion. Another important factor in the early design stages was the effect of frequency on the size of transformers, which play a major role in power generation and distribution. Working through the fundamental equations for transformer design, you will find that *the size of a transformer is inversely proportional to frequency*. The result is that transformers operating at 50 Hz must be larger (on a purely mathematical basis about 17% larger) than those operating at 60 Hz. You will therefore find that transformers designed for the international market, where they can operate on 50 Hz or 60 Hz, are designed around the 50 Hz frequency. On the other side of the coin, however,



FIG. 13.76

Clamp-on ammeter and voltmeter.
(Courtesy of AEMC® Instruments, Foxborough, MA.)

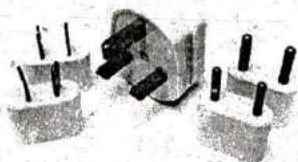


FIG. 13.77
Variety of plugs for a 220 V, 50 Hz connection.

higher frequencies result in increased concerns about arcing, increased losses in the transformer core due to eddy current and hysteresis losses, and skin effect phenomena. Somewhere in the discussion we may wonder about the fact that 60 Hz is an exact multiple of 60 seconds in a minute and 60 minutes in an hour. On the other side of the coin, however, a 60 Hz signal has a period of 16.67 ms (an awkward number), but the period of a 50 Hz signal is exactly 20 ms. Since accurate timing is such a critical part of our technological design, was this a significant motive in the final choice? There is also the question about whether the 50 Hz is a result of the close affinity of this value to the metric system. Keep in mind that powers of ten are all-powerful in the metric system, with 100 cm in a meter, 100°C the boiling point of water, and so on. Note that 50 Hz is exactly half of this special number. All in all, it would seem that both sides have an argument that is worth defending. However, in the final analysis, we must also wonder whether the difference is simply political in nature.

The difference in voltage between the Americas and Europe is a different matter entirely, in the sense that the difference is close to 100%. Again, however, there are valid arguments for both sides. There is no question that larger voltages such as 220 V raise safety issues beyond those raised by voltages of 120 V. However, when higher voltages are supplied, there is less current in the wire for the same power demand, permitting the use of smaller conductors—a real money saver. In addition, motors and some appliances can be smaller in size. Higher voltages, however, also bring back the concern about arcing effects, insulation requirements, and, due to real safety concerns, higher installation costs. In general, however, international travelers are prepared for most situations if they have a transformer that can convert from their home level to that of the country they plan to visit. Most equipment (not clocks, of course) can run quite well on 50 Hz or 60 Hz for most travel periods. For any unit not operating at its design frequency, it simply has to “work a little harder” to perform the given task. The major problem for the traveler is not the transformer itself but the wide variety of plugs used from one country to another. Each country has its own design for the “female” plug in the wall. For a three-week tour, this could mean as many as 6 to 10 different plugs of the type shown in Fig. 13.77. For a 120 V, 60 Hz supply, the plug is quite standard in appearance with its two spade leads (and possible ground connection).

In any event, both the 120 V at 60 Hz and the 220 V at 50 Hz are obviously meeting the needs of the consumer. It is a debate that could go on at length without an ultimate victor.

Safety Concerns (High Voltages and dc versus ac)

Be aware that any “live” network should be treated with a calculated level of respect. Electricity in its various forms is not to be feared but used with some awareness of its potentially dangerous side effects. It is common knowledge that electricity and water do not mix (never use extension cords or plug in TVs or radios in the bathroom) because a full 120 V in a layer of water of any height (from a shallow puddle to a full bath) can be lethal. However, other effects of dc and ac voltages are less known. In general, as the voltage and current increase, your concern about safety should increase exponentially. For instance, under dry conditions, most human beings can survive a 120 V ac shock such as obtained when changing a light bulb, turning on a switch, and so on. Most electricians have experienced such a jolt many times in their careers. However, ask an electrician to relate how it feels to hit 220 V, and the response (if he or she has been unfortunate to have had such an experience) will be totally different. How often have you

heard of a back-hoe operator hitting a 220 V line and having a fatal heart attack? Remember, the operator is sitting in a metal container on a damp ground, which provides an excellent path for the resulting current to flow from the line to ground. If only for a short period of time, with the best environment (rubber-sole shoes, and so on), in a situation where you can quickly escape the situation, most human beings can also survive a 220 V shock. However, as mentioned above, it is one you will not quickly forget. For voltages beyond 220 V rms, the chances of survival go down exponentially with increase in voltage. It takes only about 10 mA of steady current through the heart to put it in defibrillation. In general, therefore, always be sure that the power is disconnected when working on the repair of electrical equipment. Don't assume that throwing a wall switch will disconnect the power. Throw the main circuit breaker and test the lines with a voltmeter before working on the system. Since voltage is a two-point phenomenon, be sure to work with only one line at a time—accidents happen!

You should also be aware that the reaction to dc voltages is quite different from that to ac voltages. You have probably seen in movies or comic strips that people are often unable to let go of a *hot* wire. This is evidence of the most important difference between the two types of voltages. As mentioned above, if you happen to touch a "hot" 120 V ac line, you will probably get a good sting, but *you can let go*. If it happens to be a "hot" 120 V dc line, you will probably not be able to let go, and you could die. Time plays an important role when this happens, because the longer you are subjected to the dc voltage, the more the resistance in the body decreases, until a fatal current can be established. The reason that we can let go of an ac line is best demonstrated by carefully examining the 120 V rms, 60 Hz voltage in Fig. 13.78. Since the voltage is oscillating, there is a period when the voltage is near zero or less than, say, 20 V, and is reversing in direction. Although this time interval is very short, it appears every 8.3 ms and provides a window for you to *let go*.

Now that we are aware of the additional dangers of dc voltages, it is important to mention that under the wrong conditions, dc voltages as low as 12 V, such as from a car battery, can be quite dangerous. If you happen to be working on a car under wet conditions, or if you are sweating badly for some reason or, worse yet, wearing a wedding ring that may have moisture and body salt underneath, touching the positive terminal may initiate the process whereby the body resistance begins to drop, and serious injury could take place. It is one of the reasons you seldom see a professional electrician wearing any rings or jewelry—it is just not worth the risk.

Before leaving this topic of safety concerns, you should also be aware of the dangers of high-frequency supplies. We are all aware of what 2.45 GHz at 120 V can do to a meat product in a microwave oven, and it is therefore very important that the seal around the oven be as tight as possible. However, don't ever assume that anything is absolutely perfect in design—so don't make it a habit to view the cooking process in the microwave 6 in. from the door on a continuing basis. Find something else to do, and check the food only when the cooking process is complete. If you ever visit the Empire State Building, you will notice that you are unable to get close to the antenna on the dome due to the high-frequency signals being emitted with a great deal of power. Also note the large KEEP OUT signs near radio transmission towers for local radio stations. Standing within 10 ft of an AM transmitter working at 540 kHz would bring on disaster. Simply holding (do not try!) a fluorescent bulb near the tower could make it light up due to the excitation of the molecules inside the bulb.

In total, therefore, treat any situation with high ac voltages or currents, high-energy dc levels, and high frequencies with added care.

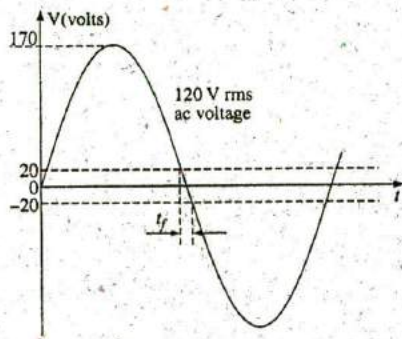


FIG. 13.78

Interval of time when sinusoidal voltage is near zero volts.



13.11 COMPUTER ANALYSIS

PSpice

OrCAD Capture offers a variety of ac voltage and current sources. However, for the purposes of this text, the voltage source **VSIN** and the current source **ISIN** are the most appropriate because they have a list of attributes that cover current areas of interest. Under the library **SOURCE**, a number of others are listed, but they don't have the full range of the above, or they are dedicated to only one type of analysis. On occasion, **ISRC** is used because it has an arrow symbol like that appearing in the text, and it can be used for dc, ac, and some transient analyses. The symbol for **ISIN** is a sine wave that utilizes the plus-and-minus sign (\pm) to indicate direction. The sources **VAC**, **IAC**, **VSRC**, and **ISRC** are fine if the magnitude and the phase of a specific quantity are desired or if a transient plot against frequency is desired. However, they will not provide a transient response against time even if the frequency and the transient information are provided for the simulation.

For all of the sinusoidal sources, the magnitude (**VAMPL**) is the peak value of the waveform, not the rms value. This becomes clear when a plot of a quantity is desired and the magnitude calculated by PSpice is the peak value of the transient response. However, for a purely steady-state ac response, the magnitude provided can be the rms value and the output read as the rms value. Only when a plot is desired will it be clear that PSpice is accepting every ac magnitude as the peak value of the waveform. Of course, the phase angle is the same whether the magnitude is the peak or the rms value.

Before examining the mechanics of getting the various sources, remember that

Transient Analysis provides an ac or a dc output versus time, while AC Sweep is used to obtain a plot versus frequency.

To obtain any of the sources listed above, apply the following sequence: **Place part key-Place Part dialog box-Source**-(enter type of source). Once you select the source, the ac source **VSIN** appears on the schematic with **OFF**, **VAMPL**, and **FREQ**. Always specify **VOFF** as 0 V (unless a specific value is part of the analysis), and provide a value for the amplitude and frequency. Enter the remaining quantities of **PHASE**, **AC**, **DC**, **DF**, and **TD** by double-clicking on the source symbol to obtain the **Property Editor**, although **PHASE**, **DF** (damping factor), and **TD** (time delay) do have a default of 0 s. To add a phase angle, click on **PHASE**, enter the phase angle in the box below, and then select **Apply**. If you want to display a factor such as a phase angle of 60°, click on **PHASE** followed by **Display** to obtain the **Display Properties** dialog box. Then choose **Name and Value** followed by **OK** and **Apply**, and leave the **Properties Editor** dialog box (X) to see **PHASE=60** next to the **VSIN** source. The next chapter includes the use of the ac source in a simple circuit.

Multisim

For Multisim, the ac voltage source is available from three sources—the **Place Source** key pad in the **Components** toolbar, the **Show Power Source Family** in the **Virtual** or **BASIC** toolbar, and the **Function Generator**. The major difference among the options is that the phase angle cannot be set using the **Function Generator**.

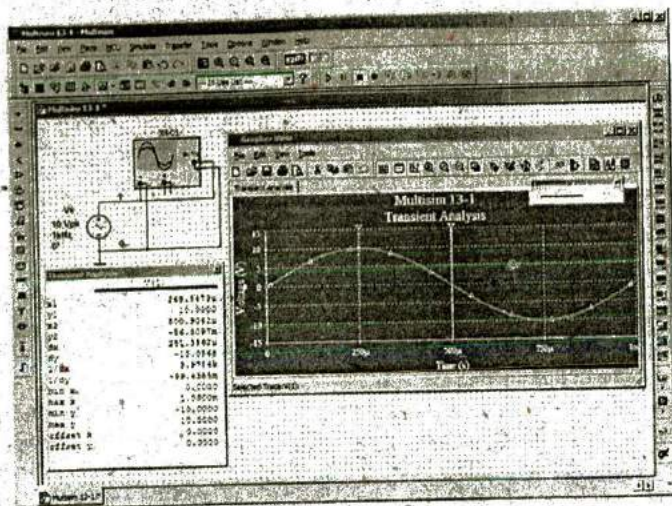


FIG. 13.79

Using the oscilloscope to display the sinusoidal ac voltage source available in the Multisim Sources tool bin.

Using the **Place Source** option, select **SIGNAL_VOLTAGE_SOURCES** group under the **Family** heading, followed by **AC_VOLTAGE**. When selected and placed, it displays the default values for the amplitude, frequency, and phase. All the parameters of the source can be changed by double-clicking on the source symbol to obtain the dialog box. The listing clearly indicates that the set voltage is the peak value. Note that the unit of measurement is controlled by the scrolls to the right of the default label and cannot be set by typing in the desired unit of measurement. The label can be changed by switching the **Label** heading and inserting the desired label. After all the changes have been made in the dialog box, click **OK**, and all the changes appear next to the ac voltage source symbol. In Fig. 13.79, the label was changed to **Vs** and the amplitude to **10 V**, while the frequency and phase angle were left with their default values. It is particularly important to realize that

for any frequency analysis (that is, where the frequency will change), the AC Magnitude of the ac source must be set under Analysis Setup in the SIGNAL_VOLTAGE_SOURCES dialog box. Failure to do so will create results linked to the default values rather than the value set under the Value heading.

To view the sinusoidal voltage set in Fig. 13.79, select an oscilloscope from the **Instrument** toolbar at the right of the screen. When hooking up the oscilloscope, do not worry about overlapping wires. Connections are shown by small, solid dots. It is the fourth option down and has the appearance shown in Fig. 13.79 when selected. Note that it is a dual-channel oscilloscope with an **A** channel and a **B** channel. It has a ground (**G**) connection and a trigger (**T**) connection. The connections for viewing the ac voltage source on the **A** channel are provided in Fig. 13.79. Note that the trigger control is also connected to the **A** channel for sync control. The screen appearing in Fig. 13.79 can be displayed by double-clicking on the oscilloscope symbol on the screen. It has all the major controls of a typical laboratory oscilloscope. When you select **Simulate-Run** or select **1** on the **Simulate Switch**, the ac voltage appears on the screen. Changing the **Time**



base to $100 \mu\text{s}/\text{div}$. results in the display of Fig. 13.79 since there are 10 divisions across the screen and $10(100 \mu\text{s}) = 1 \text{ ms}$ (the period of the applied signal). Changes in the **Time base** are made by clicking on the default value to obtain the scrolls in the same box. For a single waveform like that in Fig. 3.79, be sure to select **Sing.** (for Singular) in the bottom right of the scope. It is important to remember, however, that

changes in the oscilloscope setting or any network should not be made until the simulation is ended by disabling the Simulate-Run option or placing the Simulate switch in the 0 mode.

To stop the simulation, there are three options: choose **Simulate-Stop** from the top toolbar on the screen; select the red square to the right of the green arrow; or click the switch back to the 0 position.

The options within the time base are set by the scroll bars and cannot be changed—again they match those typically available on a laboratory oscilloscope. The vertical sensitivity of the A channel was automatically set by the program at $5 \text{ V}/\text{div}$. to result in two vertical boxes for the peak value as shown in Fig. 13.79. Note the **AC** and **DC** keypads below Channel A. Since there is no dc component in the applied signal, either one results in the same display. The **Trigger** control is set on the positive transition at a level of 0 V. The **T1** and **T2** refer to the cursor positions on the horizontal time axis. By clicking on the small green triangle at the top of the green line at the far left edge of the screen and dragging the triangle, you can move the vertical green line to any position along the axis. In Fig. 13.79, it was moved to the peak value of the waveform at one-quarter of the total period or $0.25 \text{ ms} = 250 \mu\text{s}$. Note the value of **T1** ($250 \mu\text{s}$) and the corresponding value of **VA1** ($9.995 \text{ V} \approx 10.0 \text{ V}$). By selecting the other cursor with a yellow triangle at the top to one-half the total period or $0.5 \text{ ms} = 500 \mu\text{s}$, we find that the value at **T2** ($500 \mu\text{s}$) is 0.008 pV (**VA2**), which is essentially 0 V for a waveform with a peak value of 10 V. The accuracy is controlled by the number of data points called for in the simulation setup. The more data points, the higher is the likelihood of a higher degree of accuracy for the desired quantity. However, an increased number of data points also extends the running time of the simulation. The third line provides the difference between **T2** and **T1** as $250 \mu\text{s}$ and difference between their magnitudes (**VA2-VA1**) as -9.995 V , with the negative sign appearing because **VA1** is greater than **VA2**.

As mentioned above, you can also obtain an ac voltage from the **Function Generator** appearing as the second option down on the **Instrument** toolbar. Its symbol appears in Fig. 13.80 with positive, negative, and ground connections. Double-click on the generator graphic symbol, and the **Function Generator** dialog box appears in which selections can be made. For this example, the sinusoidal waveform is chosen. To set the frequency, click on the unit of measurement to produce a list of options. For this case, kHz was chosen and the 1 left as is. The **Amplitude** (peak value) is set as $V_p = 10 \text{ V}$ and the **Offset** at 0 V. Note that there is no option to set the phase angle as was possible for the source above. Double-clicking on the oscilloscope generates the **Oscilloscope-XSCI** dialog box in which a **Timebase** of $100 \mu\text{s}/\text{div}$. can be set again with a vertical sensitivity of $5 \text{ V}/\text{div}$. Select **1** on the **Simulate** switch, and the waveform of Fig. 13.80 appears. Choosing **Sing.** under **Trigger** results in a fixed display. Set the **Simulate** switch on 0 to end the simulation. Placing the cursors in the same position shows that the waveforms for Figs. 13.79 and 13.80 are the same.

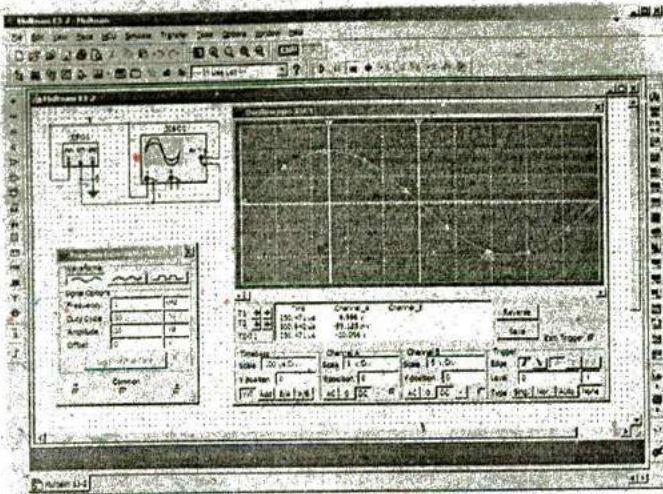


FIG. 13.80

Using the function generator to place a sinusoidal ac voltage waveform on the screen of the oscilloscope.

For most of the Multisim analyses to appear in this text, the **AC_VOLTAGE** under **Place Source** will be employed. However, with such a limited introduction to Multisim, it seemed appropriate to introduce the use of the **Function Generator** because of its close linkage to the laboratory experience.*

PROBLEMS

SECTION 13.2 Sinusoidal ac Voltage Characteristics and Definitions

- For the sinusoidal waveform in Fig. 13.81:
 - What is the peak value?
 - What is the instantaneous value at 15 ms and at 20 ms?
 - What is the peak-to-peak value of the waveform?
 - What is the period of the waveform?
 - How many cycles are shown?

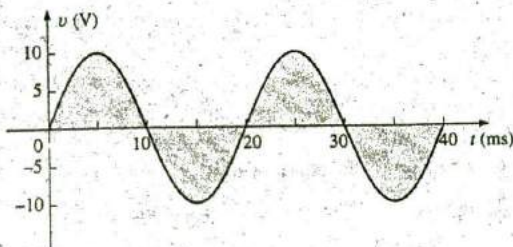


FIG. 13.81
Problem 1.

- For the sinusoidal signal in Fig. 13.82:
 - What is the peak value?
 - What is the instantaneous value at 1 μ s and at 7 μ s.

- What is the peak-to-peak value of the waveform?
- What is the period of the waveform?
- How many cycles are shown?

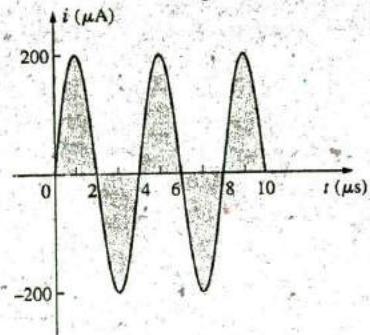


FIG. 13.82
Problem 2.

- For the periodic square-wave waveform in Fig. 13.83:
 - What is the peak value?
 - What is the instantaneous value at 1.5 ms and at 5.1 ms?
 - What is the peak-to-peak value of the waveform?
 - What is the period of the waveform?
 - How many cycles are shown?

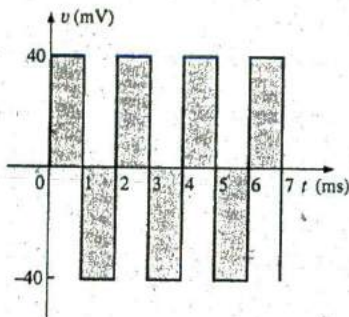
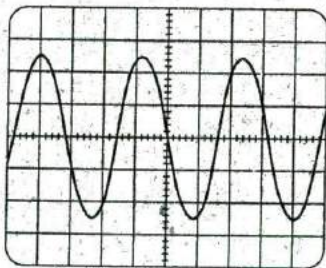


FIG. 13.83
Problem 3.

SECTION 13.3 Frequency Spectrum

- Find the period of a periodic waveform whose frequency is
 - 200 Hz.
 - 40 MHz.
 - 20 kHz.
 - 1 Hz.
- Find the frequency of a repeating waveform whose period is
 - 1 s.
 - $\frac{1}{16}$ s.
 - 40 ms.
 - 25 μ s.
- If a periodic waveform has a frequency of 1 kHz, how long (in seconds) will it take to complete five cycles?
- Find the period of a sinusoidal waveform that completes 80 cycles in 24 ms.
- What is the frequency of a periodic waveform that completes 42 cycles in 6 s?
- For the oscilloscope pattern of Fig. 13.84:
 - Determine the peak amplitude.
 - Find the period.
 - Calculate the frequency.
 Redraw the oscilloscope pattern if a +20 mV dc level were added to the input waveform.



Vertical sensitivity = 50 mV/div.
Horizontal sensitivity = 10 μ s/div.

FIG. 13.84
Problem 9.

SECTION 13.4 The Sinusoidal Waveform

- Convert the following degrees to radians:
 - 40°
 - 60°
 - 135°
 - 170°

- Convert the following radians to degrees:

a. $\frac{\pi}{3}$ b. 1.2π
c. $\frac{1}{16}\pi$ d. 0.6π

- Find the angular velocity of a waveform with a period of
 - 1.8 s.
 - 0.3 ms.
 - 8 μ s.
 - 4×10^{-6} s.
- Find the angular velocity of a waveform with a frequency of
 - 100 Hz.
 - 0.25 kHz.
 - 2 kHz.
 - 0.004 MHz.
- Find the frequency and period of sine waves having an angular velocity of
 - 754 rad/s.
 - 12 rad/s.
 - 6000 rad/s.
 - 0.16 rad/s.
- Given $f = 60$ Hz, determine how long it will take the sinusoidal waveform to pass through an angle of 60°.
- If a sinusoidal waveform passes through an angle of 30° in 5 ms, determine the angular velocity of the waveform.

SECTION 13.5 General Format for the Sinusoidal Voltage or Current

- Find the amplitude and frequency of the following waves:
 - $20 \sin 377t$
 - $12 \sin 2\pi 120t$
 - $10^6 \sin 10,000t$
 - $-8 \sin 10,058t$
- Sketch $6 \sin 754t$ with the abscissa
 - angle in degrees.
 - angle in radians.
 - time in seconds.
- Sketch $-8 \sin 2\pi 80t$ with the abscissa
 - angle in degrees.
 - angle in radians.
 - time in seconds.
- If $e = 300 \sin 157t$, how long (in seconds) does it take this waveform to complete 1/2 cycle?
- Given $i = 0.5 \sin \alpha$, determine i at $\alpha = 72^\circ$.
- Given $v = 20 \sin \alpha$, determine v at $\alpha = 1.2\pi$.
- Given $v = 30 \times 10^{-3} \sin \alpha$, determine the angles at which v will be 6 mV.
- If $v = 40$ V at $\alpha = 30$ and $t = 1$ ms, determine the mathematical expression for the sinusoidal voltage.

SECTION 13.6 Phase Relations

- Sketch $\sin(377t + 60^\circ)$ with the abscissa
 - angle in degrees.
 - angle in radians.
 - time in seconds.
- Sketch the following waveforms:
 - $50 \sin(\omega t + 0^\circ)$
 - $5 \sin(\omega t + 120^\circ)$
 - $2 \cos(\omega t + 10^\circ)$
 - $-20 \sin(\omega t + 10^\circ)$

27. Write the analytical expression for the waveforms of Fig. 13.85 with the phase angle in degrees.

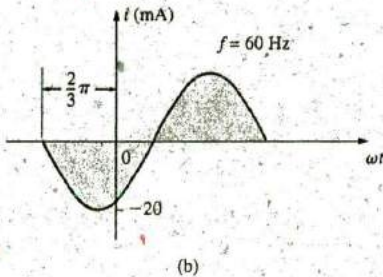
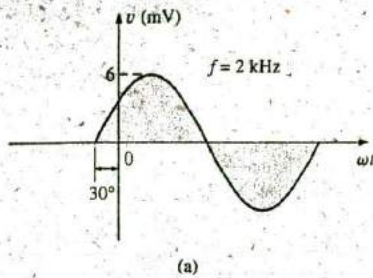


FIG. 13.85
Problem 27.

28. Write the analytical expression for the waveform of Fig. 13.86 with the phase angle in degrees.

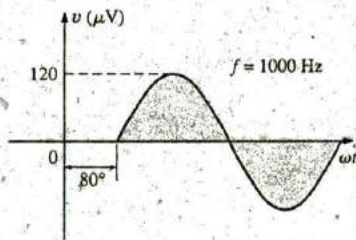


FIG. 13.86
Problem 28.

29. Write the analytical expression for the waveform of Fig. 13.87 with the phase angle in degrees.

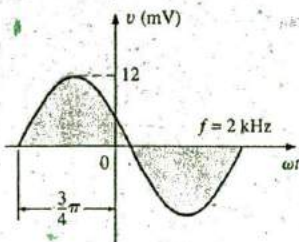


FIG. 13.87
Problem 29.

30. Write the analytical expression for the waveform of Fig. 13.88 with the phase angle in radians.

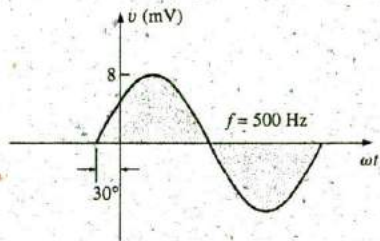


FIG. 13.88
Problem 30.

31. Find the phase relationship between the following waveforms:

$$v = 25 \sin(\omega t + 80^\circ)$$

$$i = 4 \sin(\omega t - 10^\circ)$$

32. Find the phase relationship between the following waveforms:

$$v = 0.2 \sin(\omega t - 60^\circ)$$

$$i = 0.1 \sin(\omega t - 20^\circ)$$

- *33. Find the phase relationship between the following waveforms:

$$v = 2 \cos(\omega t - 30^\circ)$$

$$i = 5 \sin(\omega t + 60^\circ)$$

- *34. Find the phase relationship between the following waveforms:

$$v = -4 \cos(\omega t + 90^\circ)$$

$$i = -2 \sin(\omega t + 10^\circ)$$

- *35. The sinusoidal voltage $v = 160 \sin(2\pi 1000t + 60^\circ)$ is plotted in Fig. 13.89. Determine the time t_1 when the waveform crosses the axis.

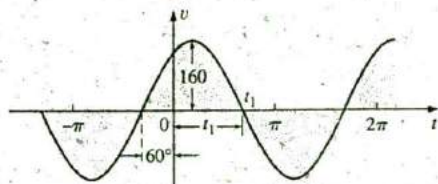


FIG. 13.89
Problem 35.

- *36. The sinusoidal current $i = 20 \times 10^{-3} \sin(50,000t - 40^\circ)$ is plotted in Fig. 13.90. Determine the time t_1 when the waveform crosses the axis.

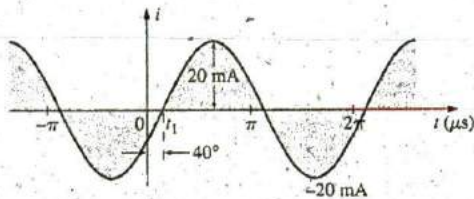
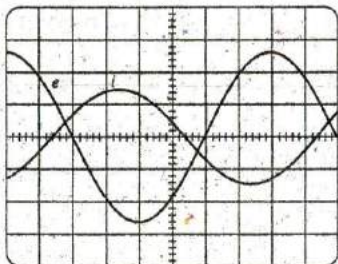


FIG. 13.90
Problem 36.

37. For the waveform of Fig. 13.89, find the time when the waveform has its peak value.
38. For the oscilloscope display in Fig. 13.91:
- Determine the period of the waveform.
 - Determine the frequency of each waveform.
 - Find the rms value of each waveform.
 - Determine the phase shift between the two waveforms and determine which leads and which lags.



Vertical sensitivity = 0.5 V/div.
Horizontal sensitivity = 1 ms/div.

FIG. 13.91
Problem 38.

41. Find the average value of the periodic waveform of Fig. 13.94 over one full cycle.

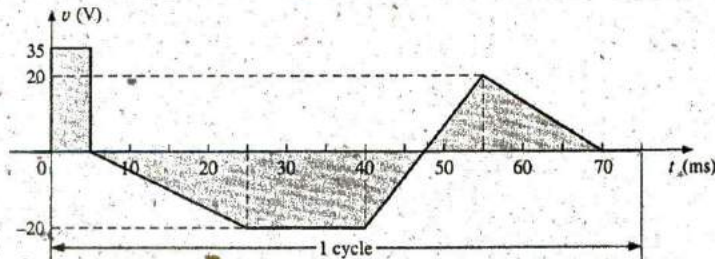


FIG. 13.94
Problem 41.

SECTION 13.7 Average Value

39. Find the average value of the periodic waveform in Fig. 13.92.

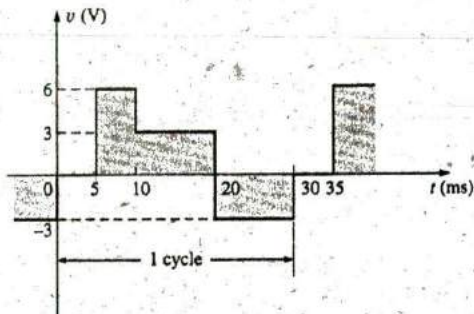


FIG. 13.92
Problem 39.

40. Find the average value of the periodic waveform in Fig. 13.93 over one full cycle.

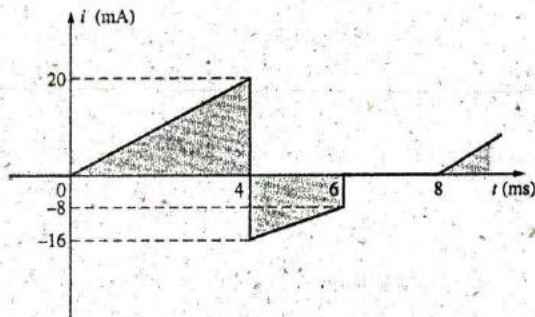


FIG. 13.93
Problem 40.

42. Find the average value of the periodic waveform of Fig. 13.95 over one full cycle.

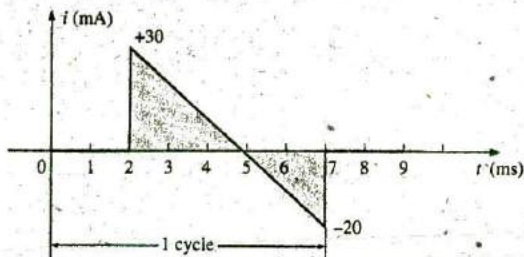


FIG. 13.95
Problem 42.

43. Find the average value of the periodic function of Fig. 13.96:

- By inspection.
- Through calculations.
- Compare the results of parts (a) and (b).

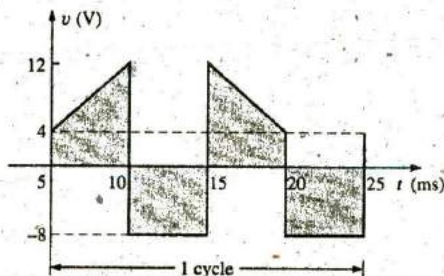


FIG. 13.96
Problem 43.

44. Find the average value of the periodic waveform in Fig. 13.97.

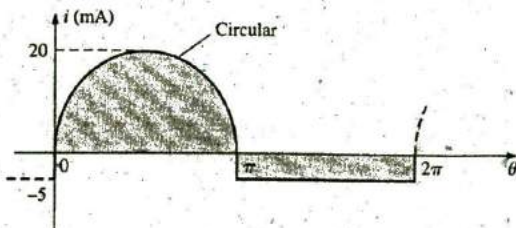
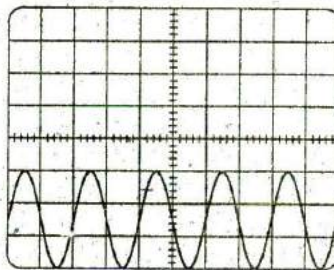


FIG. 13.97
Problem 44.

45. For the waveform in Fig. 13.98:

- Determine the period.
- Find the frequency.

- Determine the average value.
- Sketch the resulting oscilloscope display if the vertical channel is switched from dc to ac.

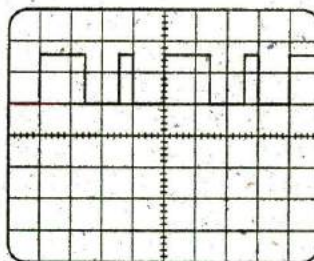


Vertical sensitivity = 10 mV/div.
Horizontal sensitivity = 0.2 ms/div.

FIG. 13.98
Problem 45.

- *46. For the waveform in Fig. 13.99:

- Determine the period.
- Find the frequency.
- Determine the average value.
- Sketch the resulting oscilloscope display if the vertical channel is switched from dc to ac.



Vertical sensitivity = 10 mV/div.
Horizontal sensitivity = 10 μs/div.

FIG. 13.99
Problem 46.

SECTION 13.8 Effective (rms) Values

47. Find the rms values of the following sinusoidal waveforms:

- $v = 120 \sin(377t + 60^\circ)$
- $i = 6 \times 10^{-3} \sin(2\pi 1000t)$
- $v = 8 \times 10^{-6} \sin(2\pi 5000t + 30^\circ)$

48. Write the sinusoidal expressions for voltages and currents having the following rms values at a frequency of 60 Hz, with zero phase shift:

- 4.8 V
- 50 mA
- 2 kV



49. Find the rms value of the periodic waveform in Fig. 13.100 over one full cycle.

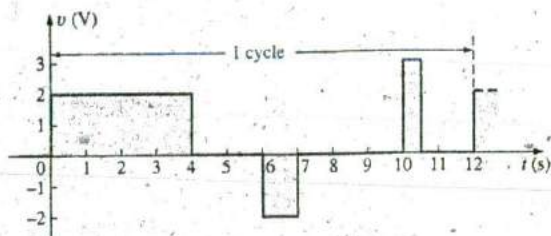


FIG. 13.100
Problem 49.

50. Find the rms value of the periodic waveform in Fig. 13.101 over one full cycle.

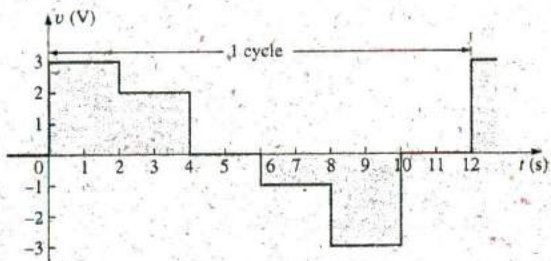


FIG. 13.101
Problem 50.

51. What are the average and rms values of the square wave in Fig. 13.102?

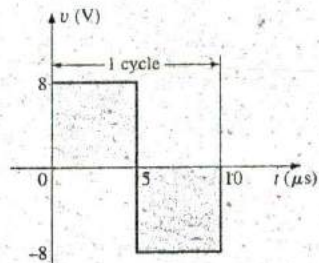
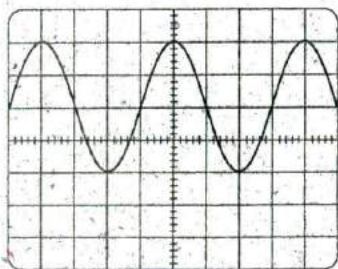


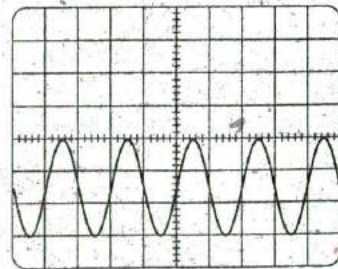
FIG. 13.102
Problem 51.

- *52. For each waveform in Fig. 13.103, determine the period, frequency, average value, and rms value.



Vertical sensitivity = 20 mV/div.
Horizontal sensitivity = 10 μ s/div.

(a)



Vertical sensitivity = 0.2 V/div.
Horizontal sensitivity = 50 μ s/div.

(b)

FIG. 13.103
Problem 52.



- *53. For the waveform of Fig. 13.104:
- Carefully sketch the squared waveform. Note that the equation for the sloping line must first be determined.
 - Using some basic area equations and the approximate approach, find the approximate area under the squared curve.
 - Determine the rms value of the original waveform.
 - Find the average value of the original waveform.
 - How does the average value of the waveform compare to the rms value?

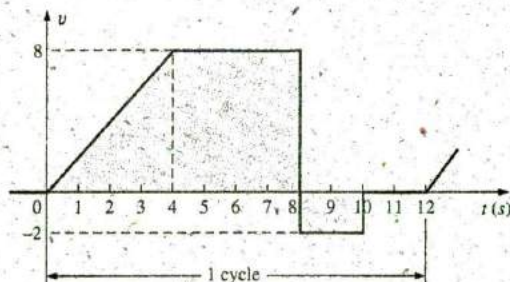
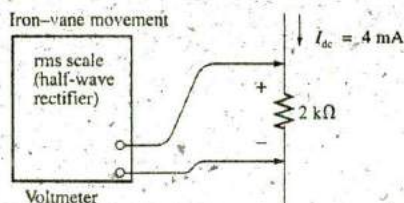


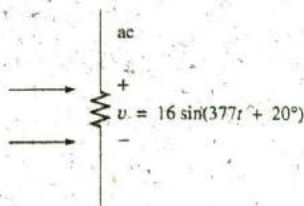
FIG. 13.104
Problem 53.

SECTION 13.9 ac Meters and Instruments

54. Determine the reading of the meter for each situation in Fig. 13.105.



(a)



(b)

FIG. 13.105
Problem 54.

GLOSSARY

- Alternating waveform** A waveform that oscillates above and below a defined reference level.
- Angular velocity** The velocity with which a radius vector projecting a sinusoidal function rotates about its center.
- Average value** The level of a waveform defined by the condition that the area enclosed by the curve above this level is exactly equal to the area enclosed by the curve below this level.
- Calibration factor** A multiplying factor used to convert from one meter indication to another.
- Clamp Meter[®]** A clamp-type instrument that will permit noninvasive current measurements and that can be used as a conventional voltmeter or ohmmeter.
- Cycle** A portion of a waveform contained in one period of time.
- Effective value** The equivalent dc value of any alternating voltage or current.
- Electrodynamometer meters** Instruments that can measure both ac and dc quantities without a change in internal circuitry.
- Frequency (f)** The number of cycles of a periodic waveform that occur in 1 s.
- Frequency counter** An instrument that will provide a digital display of the frequency or period of a periodic time-varying signal.
- Instantaneous value** The magnitude of a waveform at any instant of time, denoted by lowercase letters.
- Lagging waveform** A waveform that crosses the time axis at a point in time later than another waveform of the same frequency.
- Leading waveform** A waveform that crosses the time axis at a point in time ahead of another waveform of the same frequency.
- Oscilloscope** An instrument that will display, through the use of a cathode-ray tube, the characteristics of a time-varying signal.
- Peak amplitude** The maximum value of a waveform as measured from its average, or mean, value, denoted by uppercase letters.
- Peak-to-peak value** The magnitude of the total swing of a signal from positive to negative peaks. The sum of the absolute values of the positive and negative peak values.
- Peak value** The maximum value of a waveform, denoted by uppercase letters.
- Period (T)** The time interval necessary for one cycle of a periodic waveform.
- Periodic waveform** A waveform that continually repeats itself after a defined time interval.
- Phase relationship** An indication of which of two waveforms leads or lags the other, and by how many degrees or radians.
- Radian (rad)** A unit of measure used to define a particular segment of a circle. One radian is approximately equal to 57.3° ; 2π rad are equal to 360° .
- Root-mean-square (rms) value** The root-mean-square or effective value of a waveform.
- Sinusoidal ac waveform** An alternating waveform of unique characteristics that oscillates with equal amplitude above and below a given axis.
- VOM** A multimeter with the capability to measure resistance and both ac and dc levels of current and voltage.
- Waveform** The path traced by a quantity, plotted as a function of some variable such as position, time, degrees, temperature, and so on.

THE BASIC ELEMENTS AND PHASORS

14

OBJECTIVES

- Become familiar with the response of a resistor, an inductor, and a capacitor to the application of a sinusoidal voltage or current.
- Learn how to apply the phasor format to add and subtract sinusoidal waveforms.
- Understand how to calculate the real power to resistive elements and the reactive power to inductive and capacitive elements.
- Become aware of the differences between the frequency response of ideal and practical elements.
- Become proficient in the use of a calculator to work with complex numbers.

14.1 INTRODUCTION

The response of the basic R , L , and C elements to a sinusoidal voltage and current are examined in this chapter, with special note of how frequency affects the "opposing" characteristic of each element. Phasor notation is then introduced to establish a method of analysis that permits a direct correspondence with a number of the methods, theorems, and concepts introduced in the dc chapters.

14.2 DERIVATIVE

To understand the response of the basic R , L , and C elements to a sinusoidal signal, you need to examine the concept of the **derivative** in some detail. You do not have to become proficient in the mathematical technique but simply understand the impact of a relationship defined by a derivative.

Recall from Section 10.10 that the derivative dx/dt is defined as the rate of change of x with respect to time. If x fails to change at a particular instant, $dx = 0$, and the derivative is zero. For the sinusoidal waveform, dx/dt is zero only at the positive and negative peaks ($\omega t = \pi/2$ and $3/2\pi$ in Fig. 14.1), since x fails to change at these instants of time. The derivative dx/dt is actually the slope of the graph at any instant of time.

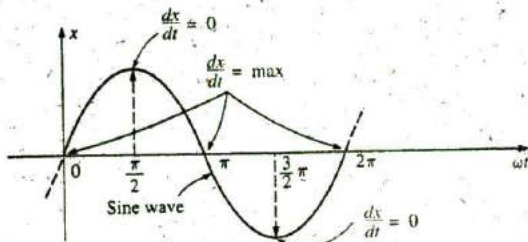


FIG. 14.1

Defining those points in a sinusoidal waveform that have maximum and minimum derivatives.



A close examination of the sinusoidal waveform will also indicate that the greatest change in x occurs at the instants $\omega t = 0, \pi,$ and 2π . The derivative is therefore a maximum at these points. At 0 and 2π , x increases at its greatest rate, and the derivative is given a positive sign since x increases with time. At π , dx/dt decreases at the same rate as it increases at 0 and 2π , but the derivative is given a negative sign since x decreases with time. Since the rate of change at $0, \pi,$ and 2π is the same, the magnitude of the derivative at these points is the same also. For various values of ωt between these maxima and minima, the derivative will exist and have values from the minimum to the maximum inclusive. A plot of the derivative in Fig. 14.2 shows that

the derivative of a sine wave is a cosine wave.

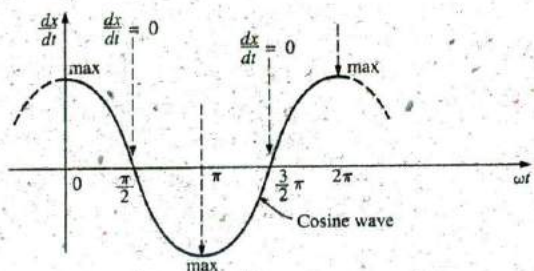


FIG. 14.2

Derivative of the sine wave of Fig. 14.1.

The peak value of the cosine wave is directly related to the frequency of the original waveform. The higher the frequency, the steeper is the slope at the horizontal axis and the greater is the value of dx/dt , as shown in Fig. 14.3 for two different frequencies.

Note in Fig. 14.3 that even though both waveforms (x_1 and x_2) have the same peak value, the sinusoidal function with the higher

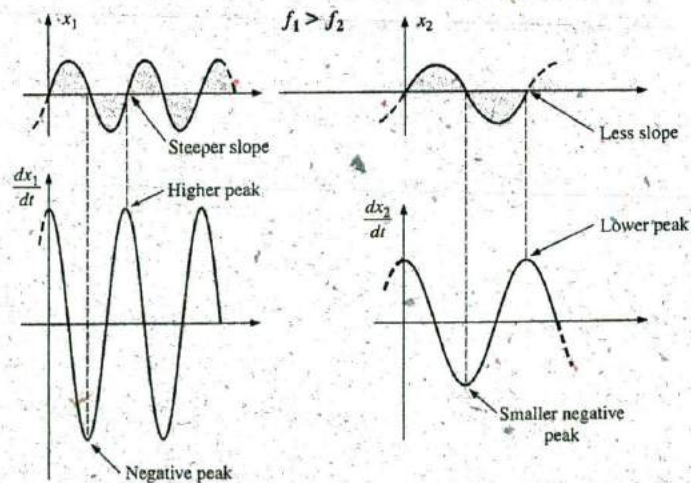


FIG. 14.3

Effect of frequency on the peak value of the derivative.

frequency produces the larger peak value for the derivative. In addition, note that

the derivative of a sine wave has the same period and frequency as the original sinusoidal waveform.

For the sinusoidal voltage

$$e(t) = E_m \sin(\omega t \pm \theta)$$

the derivative can be found directly by differentiation (calculus) to produce the following:

$$\begin{aligned} \frac{d}{dt} e(t) &= \omega E_m \cos(\omega t \pm \theta) \\ &= 2\pi f E_m \cos(\omega t \pm \theta) \end{aligned} \quad (14.1)$$

The mechanics of the differentiation process are not discussed or investigated here, nor are they required to continue with the text. Note, however, that the peak value of the derivative, $2\pi f E_m$, is a function of the frequency of $e(t)$, and the derivative of a sine wave is a cosine wave.

14.3 RESPONSE OF BASIC R, L, AND C ELEMENTS TO A SINUSOIDAL VOLTAGE OR CURRENT

Now that we are familiar with the characteristics of the derivative of a sinusoidal function, we can investigate the response of the basic elements R, L, and C to a sinusoidal voltage or current.

Resistor

For power-line frequencies and frequencies up to a few hundred kilohertz, resistance is, for all practical purposes, unaffected by the frequency of the applied sinusoidal voltage or current. For this frequency region, the resistor R in Fig. 14.4 can be treated as a constant, and Ohm's law can be applied as follows. For $v = V_m \sin \omega t$,

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$

where

$$I_m = \frac{V_m}{R} \quad (14.2)$$

In addition, for a given i ,

$$v = iR = (I_m \sin \omega t)R = I_m R \sin \omega t = V_m \sin \omega t$$

where

$$V_m = I_m R \quad (14.3)$$

A plot of v and i in Fig. 14.5 reveals that

for a purely resistive element, the voltage across and the current through the element are in phase, with their peak values related by Ohm's law.

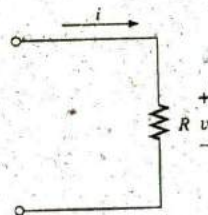


FIG. 14.4

Determining the sinusoidal response for a resistive element.

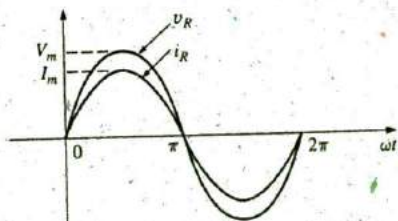


FIG. 14.5

The voltage and current of a resistive element are in phase.

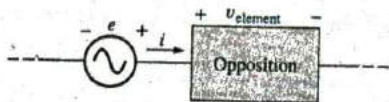


FIG. 14.6

Defining the opposition of an element to the flow of charge through the element.

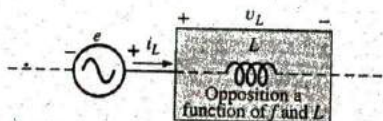


FIG. 14.7

Defining the parameters that determine the opposition of an inductive element to the flow of charge.

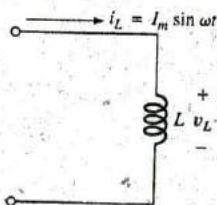


FIG. 14.8

Investigating the sinusoidal response of an inductive element.

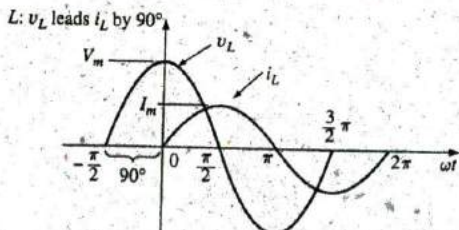


FIG. 14.9

For a pure inductor, the voltage across the coil leads the current through the coil by 90° .

Inductor

For the series configuration in Fig. 14.6, the voltage v_{element} of the boxed-in element opposes the source e and thereby reduces the magnitude of the current i . The magnitude of the voltage across the element is determined by the opposition of the element to the flow of charge, or current i . For a resistive element, we have found that the opposition is its resistance and that v_{element} and i are determined by $v_{\text{element}} = iR$.

We found in Chapter 11 that the voltage across an inductor is directly related to the rate of change of current through the coil. Consequently, the higher the frequency, the greater is the rate of change of current through the coil, and the greater is the magnitude of the voltage. In addition, we found in the same chapter that the inductance of a coil determines the rate of change of the flux linking a coil for a particular change in current through the coil. The higher the inductance, the greater is the rate of change of the flux linkages, and the greater is the resulting voltage across the coil.

The inductive voltage, therefore, is directly related to the frequency (or, more specifically, the angular velocity of the sinusoidal ac current through the coil) and the inductance of the coil. For increasing values of f and L in Fig. 14.7, the magnitude of v_L increases as described above.

Using the similarities between Figs. 14.6 and 14.7, we find that increasing levels of v_L are directly related to increasing levels of opposition in Fig. 14.6. Since v_L increases with both ω ($= 2\pi f$) and L , the opposition of an inductive element is as defined in Fig. 14.7.

We will now verify some of the preceding conclusions using a more mathematical approach and then define a few important quantities to be used in the sections and chapters to follow.

For the inductor in Fig. 14.8, we recall from Chapter 11 that

$$v_L = L \frac{di_L}{dt}$$

and, applying differentiation,

$$\frac{di_L}{dt} = \frac{d}{dt}(I_m \sin \omega t) = \omega I_m \cos \omega t$$

Therefore,
$$v_L = L \frac{di_L}{dt} = L(\omega I_m \cos \omega t) = \omega L I_m \cos \omega t$$

or
$$v_L = V_m \sin(\omega t + 90^\circ)$$

where
$$V_m = \omega L I_m$$

Note that the peak value of v_L is directly related to ω ($= 2\pi f$) and L as predicted in the discussion above.

A plot of v_L and i_L in Fig. 14.9 reveals that

for an inductor, v_L leads i_L by 90° , or i_L lags v_L by 90° .

If a phase angle is included in the sinusoidal expression for i_L , such as

$$i_L = I_m \sin(\omega t \pm \theta)$$

then

$$v_L = \omega L I_m \sin(\omega t \pm \theta + 90^\circ)$$

The opposition established by an inductor in a sinusoidal ac network can now be found by applying Eq. (4.1):

$$\text{Effect} = \frac{\text{cause}}{\text{opposition}}$$

which, for our purposes, can be written

$$\text{Opposition} = \frac{\text{cause}}{\text{effect}}$$

Substituting values, we have

$$\text{Opposition} = \frac{V_m}{I_m} = \frac{\omega L I_m}{I_m} = \omega L$$

revealing that the opposition established by an inductor in an ac sinusoidal network is directly related to the product of the angular velocity ($\omega = 2\pi f$) and the inductance, verifying our earlier conclusions.

The quantity ωL , called the **reactance** (from the word *reaction*) of an inductor, is symbolically represented by X_L and is measured in ohms; that is,

$$\boxed{X_L = \omega L} \quad (\text{ohms, } \Omega) \quad (14.4)$$

In an Ohm's law format, its magnitude can be determined from

$$\boxed{X_L = \frac{V_m}{I_m}} \quad (\text{ohms, } \Omega) \quad (14.5)$$

Inductive reactance is the opposition to the flow of current, which results in the continual interchange of energy between the source and the magnetic field of the inductor. In other words, inductive reactance, unlike resistance (which dissipates energy in the form of heat), does not dissipate electrical energy (ignoring the effects of the internal resistance of the inductor.)

Capacitor

Let us now return to the series configuration in Fig. 14.6 and insert the capacitor as the element of interest. For the capacitor, however, we will determine i for a particular voltage across the element. When this approach reaches its conclusion, we will know the relationship between the voltage and current and can determine the opposing voltage (v_{element}) for any sinusoidal current i .

Our investigation of the inductor revealed that the inductive voltage across a coil opposes the instantaneous change in current through the coil. For capacitive networks, the voltage across the capacitor is limited by the rate at which charge can be deposited on, or released by, the plates of the capacitor during the charging and discharging phases, respectively. In other words, an instantaneous change in voltage across a capacitor is opposed by the fact that there is an element of time required to deposit charge on (or release charge from) the plates of a capacitor, and $V = Q/C$.

Since capacitance is a measure of the rate at which a capacitor will store charge on its plates,

for a particular change in voltage across the capacitor, the greater the value of capacitance, the greater is the resulting capacitive current.

In addition, the fundamental equation relating the voltage across a capacitor to the current of a capacitor [$i = C(dv/dt)$], indicates that

for a particular capacitance, the greater the rate of change of voltage across the capacitor, the greater is the capacitive current.

Certainly, an increase in frequency corresponds to an increase in the rate of change of voltage across the capacitor and to an increase in the current of the capacitor.

The current of a capacitor is therefore directly related to the frequency (or, again more specifically, the angular velocity) and the capacitance of the capacitor. An increase in either quantity results in an increase in the current of the capacitor. For the basic configuration in Fig. 14.10, however, we are interested in determining the opposition of the capacitor as related to the resistance of a resistor and ωL for the inductor. Since an increase in current corresponds to a decrease in opposition, and i_C is proportional to ω and C , the opposition of a capacitor is inversely related to $\omega (= 2\pi f)$ and C .

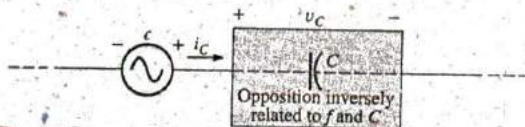


FIG. 14.10

Defining the parameters that determine the opposition of a capacitive element to the flow of charge.

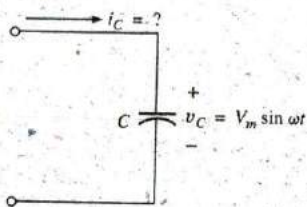


FIG. 14.11

Investigating the sinusoidal response of a capacitive element.

We will now verify, as we did for the inductor, some of the above conclusions using a more mathematical approach.

For the capacitor of Fig. 14.11, we recall from Chapter 11 that

$$i_C = C \frac{dv_C}{dt}$$

and, applying differentiation, we obtain

$$\frac{dv_C}{dt} = \frac{d}{dt}(V_m \sin \omega t) = \omega V_m \cos \omega t$$

Therefore,

$$i_C = C \frac{dv_C}{dt} = C(\omega V_m \cos \omega t) = \omega C V_m \cos \omega t$$

or

$$i_C = I_m \sin(\omega t + 90^\circ)$$

where

$$I_m = \omega C V_m$$

Note that the peak value of i_C is directly related to $\omega (= 2\pi f)$ and C , as predicted in the discussion above.

A plot of v_C and i_C in Fig. 14.12 reveals that

for a capacitor, i_C leads v_C by 90° , or v_C lags i_C by 90° .*

If a phase angle is included in the sinusoidal expression for v_C , such as

$$v_C = V_m \sin(\omega t \pm \theta)$$

then

$$i_C = \omega C V_m \sin(\omega t \pm \theta + 90^\circ)$$

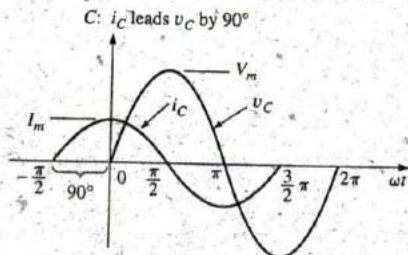


FIG. 14.12

The current of a purely capacitive element leads the voltage across the element by 90° .

*A mnemonic phrase sometimes used to remember the phase relationship between the voltage and current of a coil and capacitor is "ELI the ICE man." Note that the L (inductor) has the E before the I (e leads i by 90°), and the C (capacitor) has the I before the E (i leads e by 90°).

Applying

$$\text{Opposition} = \frac{\text{cause}}{\text{effect}}$$

and substituting values, we obtain

$$\text{Opposition} = \frac{V_m}{I_m} = \frac{V_m}{\omega C V_m} = \frac{1}{\omega C}$$

which agrees with the results obtained above.

The quantity $1/\omega C$, called the **reactance** of a capacitor, is symbolically represented by X_C and is measured in ohms; that is,

$$X_C = \frac{1}{\omega C} \quad (\text{ohms, } \Omega) \quad (14.6)$$

In an Ohm's law format, its magnitude can be determined from

$$X_C = \frac{V_m}{I_m} \quad (\text{ohms, } \Omega) \quad (14.7)$$

Capacitive reactance is the opposition to the flow of charge, which results in the continual interchange of energy between the source and the electric field of the capacitor. Like the inductor, the capacitor does *not* dissipate energy in any form (ignoring the effects of the leakage resistance).

In the circuits just considered, the current was given in the inductive circuit and the voltage in the capacitive circuit. This was done to avoid the use of integration in finding the unknown quantities. In the inductive circuit,

$$\bar{v}_L = L \frac{di_L}{dt}$$

but

$$i_L = \frac{1}{L} \int v_L dt \quad (14.8)$$

In the capacitive circuit,

$$i_C = C \frac{dv_C}{dt}$$

but

$$v_C = \frac{1}{C} \int i_C dt \quad (14.9)$$

Soon, we shall consider a method of analyzing ac circuits that will permit us to solve for an unknown quantity with sinusoidal input without having to use direct integration or differentiation.

It is possible to determine whether a network with one or more elements is predominantly capacitive or inductive by noting the phase relationship between the input voltage and current.

If the source current leads the applied voltage, the network is predominantly capacitive, and if the applied voltage leads the source current, it is predominantly inductive.

Since we now have an equation for the reactance of an inductor or capacitor, we do not need to use derivatives or integration in the examples

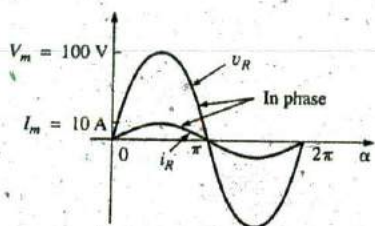


FIG. 14.13
Example 14.1(a).

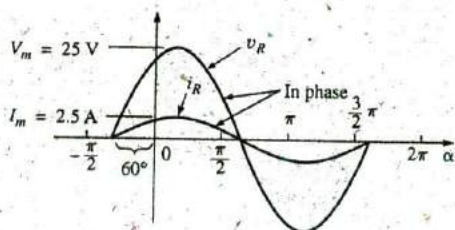


FIG. 14.14
Example 14.1(b).

to be considered. Simply applying Ohm's law, $I_m = E_m/X_L$ (or X_C), and keeping in mind the phase relationship between the voltage and current for each element will be sufficient to complete the examples.

EXAMPLE 14.1 The voltage across a resistor is indicated. Find the sinusoidal expression for the current if the resistor is 10Ω . Sketch the curves for v and i .

- $v = 100 \sin 377t$
- $v = 25 \sin(377t + 60^\circ)$

Solutions:

a. Eq. (14.2): $I_m = \frac{V_m}{R} = \frac{100 \text{ V}}{10 \Omega} = 10 \text{ A}$

(v and i are in phase), resulting in

$$i = 10 \sin 377t$$

The curves are sketched in Fig. 14.13.

b. Eq. (14.2): $I_m = \frac{V_m}{R} = \frac{25 \text{ V}}{10 \Omega} = 2.5 \text{ A}$

(v and i are in phase), resulting in

$$i = 2.5 \sin(377t + 60^\circ)$$

The curves are sketched in Fig. 14.14.

EXAMPLE 14.2 The current through a 5Ω resistor is given. Find the sinusoidal expression for the voltage across the resistor for $i = 40 \sin(377t + 30^\circ)$.

Solution: Eq. (14.3): $V_m = I_m R = (40 \text{ A})(5 \Omega) = 200 \text{ V}$

(v and i are in phase), resulting in

$$v = 200 \sin(377t + 30^\circ)$$

EXAMPLE 14.3 The current through a 0.1 H coil is provided. Find the sinusoidal expression for the voltage across the coil. Sketch the v and i curves.

- $i = 10 \sin 377t$
- $i = 7 \sin(377t - 70^\circ)$

Solutions:

a. Eq. (14.4): $X_L = \omega L = (377 \text{ rad/s})(0.1 \text{ H}) = 37.7 \Omega$

Eq. (14.5): $V_m = I_m X_L = (10 \text{ A})(37.7 \Omega) = 377 \text{ V}$

and we know that for a coil v leads i by 90° . Therefore,

$$v = 377 \sin(377t + 90^\circ)$$

The curves are sketched in Fig. 14.15.

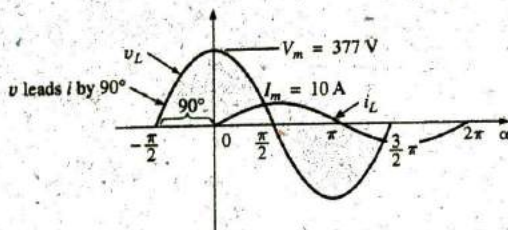


FIG. 14.15
Example 14.3(a).

b. X_L remains at 37.7Ω .

$$V_m = I_m X_L = (7 \text{ A})(37.7 \Omega) = 263.9 \text{ V}$$

and we know that for a coil v leads i by 90° . Therefore,

$$v = 263.9 \sin(377t - 70^\circ + 90^\circ)$$

and

$$v = 263.9 \sin(377t + 20^\circ)$$

The curves are sketched in Fig. 14.16.

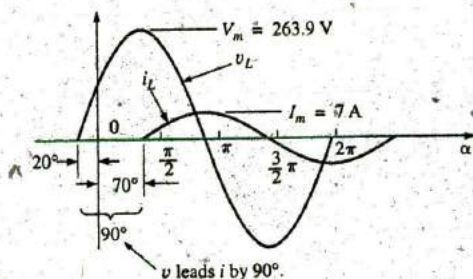


FIG. 14.16
Example 14.3(b).

EXAMPLE 14.4 The voltage across a 0.5 H coil is provided below. What is the sinusoidal expression for the current?

$$v = 100 \sin 20t$$

Solution:

$$X_L = \omega L = (20 \text{ rad/s})(0.5 \text{ H}) = 10 \Omega$$

$$I_m = \frac{V_m}{X_L} = \frac{100 \text{ V}}{10 \Omega} = 10 \text{ A}$$

and we know the i lags v by 90° . Therefore,

$$i = 10 \sin(20t - 90^\circ)$$

EXAMPLE 14.5 The voltage across a $1 \mu\text{F}$ capacitor is provided below. What is the sinusoidal expression for the current? Sketch the v and i curves.

$$v = 30 \sin 400t$$

Solution:

$$\text{Eq. (14.6): } X_C = \frac{1}{\omega C} = \frac{1}{(400 \text{ rad/s})(1 \times 10^{-6} \text{ F})} = \frac{10^6 \Omega}{400} = 2500 \Omega$$

$$\text{Eq. (14.7): } I_m = \frac{V_m}{X_C} = \frac{30 \text{ V}}{2500 \Omega} = 0.0120 \text{ A} = 12 \text{ mA}$$

and we know that for a capacitor i leads v by 90° . Therefore,

$$i = 12 \times 10^{-3} \sin(400t + 90^\circ)$$

The curves are sketched in Fig. 14.17.

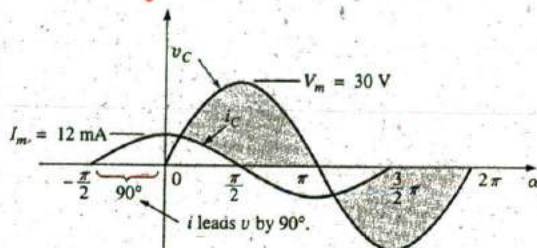


FIG. 14.17

Example 14.5.

EXAMPLE 14.6 The current through a $100 \mu\text{F}$ capacitor is given. Find the sinusoidal expression for the voltage across the capacitor.

$$i = 40 \sin(500t + 60^\circ)$$

Solution:

$$X_C = \frac{1}{\omega C} = \frac{1}{(500 \text{ rad/s})(100 \times 10^{-6} \text{ F})} = \frac{10^6 \Omega}{5 \times 10^4} = \frac{10^2 \Omega}{5} = 20 \Omega$$

$$V_M = I_M X_C = (40 \text{ A})(20 \Omega) = 800 \text{ V}$$

and we know that for a capacitor, v lags i by 90° . Therefore,

$$v = 800 \sin(500t + 60^\circ - 90^\circ)$$

and

$$v = 800 \sin(500t - 30^\circ)$$

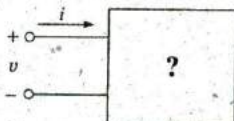


FIG. 14.18

Example 14.7.

EXAMPLE 14.7 For the following pairs of voltages and currents, determine whether the element involved is a capacitor, an inductor, or a resistor. Determine the value of C , L , or R if sufficient data are provided (Fig. 14.18):

- $v = 100 \sin(\omega t + 40^\circ)$
 $i = 20 \sin(\omega t + 40^\circ)$
- $v = 1000 \sin(377t + 10^\circ)$
 $i = 5 \sin(377t - 80^\circ)$
- $v = 500 \sin(157t + 30^\circ)$
 $i = 1 \sin(157t + 120^\circ)$
- $v = 50 \cos(\omega t + 20^\circ)$
 $i = 5 \sin(\omega t + 110^\circ)$

Solutions:

a. Since v and i are *in phase*, the element is a *resistor*, and

$$R = \frac{V_m}{I_m} = \frac{100 \text{ V}}{20 \text{ A}} = 5 \Omega$$

b. Since v *leads* i by 90° , the element is an *inductor*, and

$$X_L = \frac{V_m}{I_m} = \frac{1000 \text{ V}}{5 \text{ A}} = 200 \Omega$$

so that $X_L = \omega L = 200 \Omega$, or

$$L = \frac{200 \Omega}{\omega} = \frac{200 \Omega}{377 \text{ rad/s}} = 0.53 \text{ H}$$

c. Since i leads v by 90° , the element is a capacitor, and

$$X_C = \frac{V_m}{I_m} = \frac{500 \text{ V}}{1 \text{ A}} = 500 \Omega$$

so that $X_C = \frac{1}{\omega C} = 500 \Omega$ or

$$C = \frac{1}{\omega 500 \Omega} = \frac{1}{(157 \text{ rad/s})(500 \Omega)} = 12.74 \text{ F}$$

d. $v = 50 \cos(\omega t + 20^\circ) = 50 \sin(\omega t + 20^\circ + 90^\circ)$
 $= 50 \sin(\omega t + 110^\circ)$

Since v and i are in phase, the element is a resistor, and

$$R = \frac{V_m}{I_m} = \frac{50 \text{ V}}{5 \text{ A}} = 10 \Omega$$

14.4 FREQUENCY RESPONSE OF THE BASIC ELEMENTS

Thus far, each description has been for a set frequency, resulting in a fixed level of impedance for each of the basic elements. We must now investigate how a change in frequency affects the impedance level of the basic elements. It is an important consideration because most signals other than those provided by a power plant contain a variety of frequency levels. The last section made it quite clear that the reactance of an inductor or a capacitor is sensitive to the applied frequency. However, the question is, How will these reactance levels change if we steadily increase the frequency from a very low level to a much higher level?

Although we would like to think of every element as ideal, it is important to realize that every commercial element available today will *not* respond in an ideal fashion for the full range of possible frequencies. That is, each element is such that for a particular range of frequencies, it performs in an essentially ideal manner. However, there is always a range of frequencies in which the performance varies from the ideal. Fortunately, the designer is aware of these limitations and will take them into account in the design.

The discussion begins with a look at the response of the *ideal elements*—a response that will be assumed for the remaining chapters of this text and one that can be assumed for any initial investigation of a network. This discussion is followed by a look at the factors that cause an element to deviate from an ideal response as frequency levels become too low or high.

Ideal Response

Resistor R For an ideal resistor, you can assume that frequency will have absolutely no effect on the impedance level, as shown by the response in Fig. 14.19. Note that at 5 kHz or 20 kHz, the resistance of the resistor remains at 22Ω ; there is no change whatsoever. For the rest of the analyses in this text, the resistance level remains as the nameplate value, no matter what frequency is applied.

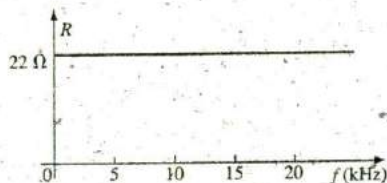


FIG. 14.19
 R versus f for the range of interest.

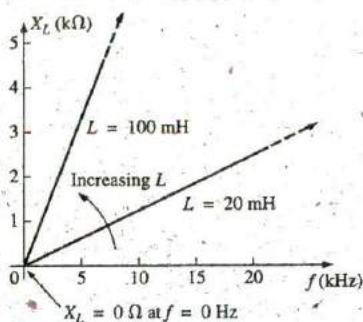


FIG. 14.20
X_L versus frequency.

Inductor L For the ideal inductor, the equation for the reactance can be written as follows to isolate the frequency term in the equation. The result is a constant times the frequency variable that changes as we move down the horizontal axis of a plot:

$$X_L = \omega L = 2\pi fL = (2\pi L)f = kf \quad \text{with } k = 2\pi L$$

The resulting equation can be compared directly with the equation for a straight line:

$$y = mx + b = kf + 0 = kf$$

where $b = 0$ and the slope is k or $2\pi L$. X_L is the y variable, and f is the x variable, as shown in Fig. 14.20. Since the inductance determines the slope of the curve, the higher the inductance, the steeper is the straight-line plot, as shown in Fig. 14.20 for two levels of inductance.

In particular, note that at $f = 0$ Hz, the reactance of each plot is zero ohms, as determined by substituting $f = 0$ Hz into the basic equation for the reactance of an inductor:

$$X_L = 2\pi fL = 2\pi(0 \text{ Hz})L = 0 \Omega$$

Since a reactance of zero ohms corresponds with the characteristics of a short circuit, we can conclude that

at a frequency of 0 Hz, an inductor takes on the characteristics of a short circuit, as shown in Fig. 14.21.

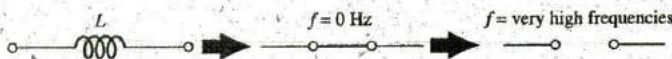


FIG. 14.21

Effect of low and high frequencies on the circuit model of an inductor.

As shown in Fig. 14.21, as the frequency increases, the reactance increases, until it reaches an extremely high level at very high frequencies. The result is that

at very high frequencies, the characteristics of an inductor approach those of an open circuit, as shown in Fig. 14.21.

The inductor, therefore, is capable of handling impedance levels that cover the entire range, from zero ohms to infinite ohms, changing at a steady rate determined by the inductance level. The higher the inductance, the faster it approaches the open-circuit equivalent.

Capacitor C For the capacitor, the equation for the reactance

$$X_C = \frac{1}{2\pi fC}$$

can be written as

$$X_C f = \frac{1}{2\pi C} = k \quad (\text{a constant})$$

which matches the basic format for a hyperbola:

$$yx = k$$

where X_C is the y variable, f the x variable, and k a constant equal to $1/(2\pi C)$.

Hyperbolas have the shape appearing in Fig. 14.22 for two levels of capacitance. Note that the higher the capacitance, the closer the curve approaches the vertical and horizontal axes at low and high frequencies.

At or near 0 Hz, the reactance of any capacitor is extremely high, as determined by the basic equation for capacitance:

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(0 \text{ Hz})C} \Rightarrow \infty \Omega$$

The result is that

at or near 0 Hz, the characteristics of a capacitor approach those of an open circuit, as shown in Fig. 14.23.

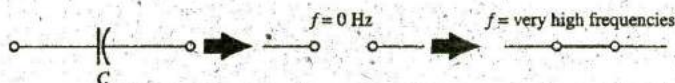


FIG. 14.23

Effect of low and high frequencies on the circuit model of a capacitor.

As the frequency increases, the reactance approaches a value of zero ohms. The result is that

at very high frequencies, a capacitor takes on the characteristics of a short circuit, as shown in Fig. 14.23.

It is important to note in Fig. 14.22 that the reactance drops very rapidly as the frequency increases. It is not a gradual drop as encountered for the rise in inductive reactance. In addition, the reactance sits at a fairly low level for a broad range of frequencies. In general, therefore, recognize that for capacitive elements, the change in reactance level can be dramatic with a relatively small change in frequency level.

Finally, recognize the following:

As frequency increases, the reactance of an inductive element increases, while that of a capacitor decreases, with one approaching an open-circuit equivalent as the other approaches a short-circuit equivalent.

Practical Response

Resistor R In the manufacturing process, every resistive element inherits some stray capacitance levels and lead inductances. For most applications, the levels are so low that their effects can be ignored. However, as the frequency extends beyond a few megahertz, it may be necessary to be aware of their effects. For instance, a number of carbon composition resistors have the frequency response appearing in Fig. 14.24. The 100 Ω resistor is essentially stable up to about 300 MHz, whereas the 100 k Ω resistor starts to drop off at about 15 MHz. In general, therefore, this type of carbon composition resistor has the ideal characteristics of Fig. 14.19 for frequencies up to about 15 MHz. For frequencies of 100 Hz, 1 kHz, 150 kHz, and so on, the resistor can be considered ideal.

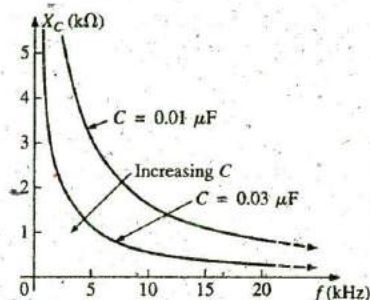


FIG. 14.22
 X_C versus frequency.

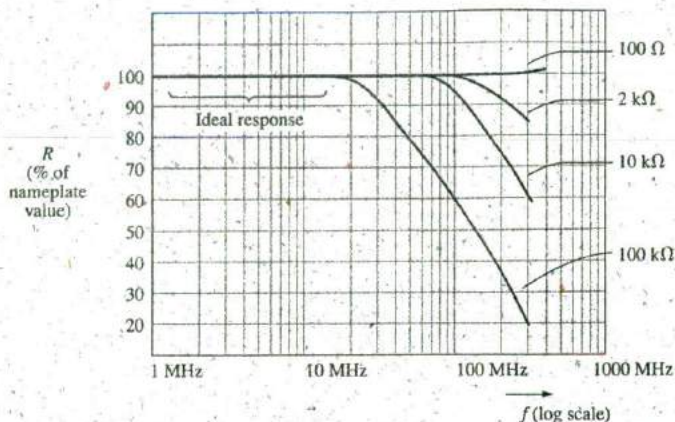


FIG. 14.24

Typical resistance-versus-frequency curves for carbon composition resistors.

The horizontal scale of Fig. 14.24 is a log scale that starts at 1 MHz rather than zero as applied to the vertical scale. Logarithms are discussed in detail in Chapter 21, which describes why the scale cannot start at zero and the fact that the major intervals are separated by powers of 10. For now, simply note that log scales permit the display of a range of frequencies not possible with a linear scale such as was used for the vertical scale of Fig. 14.24. Imagine trying to draw a linear scale from 1 MHz to 1000 MHz using a linear scale. It would be an impossible task unless the horizontal length of the plot was enormous. As indicated above, a great deal more will be said about log scales in Chapter 21.

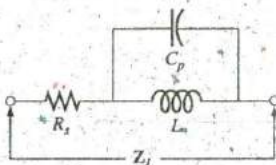


FIG. 14.25

Practical equivalent for an inductor.

Inductor L In reality, inductance can be affected by frequency, temperature, and current. A true equivalent for an inductor appears in Fig. 14.25. The series resistance R_s represents the copper losses (resistance of the many turns of thin copper wire); the eddy current losses (losses due to small circular currents in the core when an ac voltage is applied); and the hysteresis losses (losses due to core losses created by the rapidly reversing field in the core). The capacitance C_p is the stray capacitance that exists between the windings of the inductor.

For most inductors, the construction is usually such that the larger the inductance, the lower is the frequency at which the parasitic elements become important. That is, for inductors in the millihenry range (which is very typical), frequencies approaching 100 kHz can have an effect on the ideal characteristics of the element. For inductors in the microhenry range, a frequency of 1 MHz may introduce negative effects. This is not to suggest that the inductors lose their effect at these frequencies but rather that they can no longer be considered ideal (purely inductive elements).

Fig. 14.26 is a plot of the magnitude of the impedance Z_L of Fig. 14.25 versus frequency. Note that up to about 2 MHz, the impedance increases almost linearly with frequency, clearly suggesting that the 100 μ H inductor is essentially ideal. However, above 2 MHz, all the factors contributing to R_s start to increase, while the reactance due to the capacitive element C_p is more pronounced. The dropping level of capacitive reactance begins to have a shorting effect across the windings of the

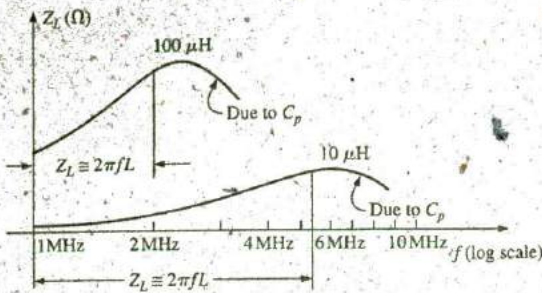


FIG. 14.26

Z_L versus frequency for the practical inductor equivalent of Fig. 14.25.

inductor and reduces the overall inductive effect. Eventually, if the frequency continues to increase, the capacitive effects overcome the inductive effects, and the element actually begins to behave in a capacitive fashion. Note the similarities of this region with the curves in Fig. 14.22. Also, note that decreasing levels of inductance (available with fewer turns and therefore lower levels of C_p) do not demonstrate the degrading effect until higher frequencies are applied.

In general, therefore, the frequency of application for a coil becomes important at increasing frequencies. Inductors lose their ideal characteristics and, in fact, begin to act as capacitive elements with increasing losses at very high frequencies.

Capacitor C The capacitor, like the inductor, is not ideal for the full frequency range. In fact, a transition point exists where the characteristics of a capacitor actually take on those of an inductor. The equivalent model for an inductor appearing in Fig. 14.27(a) is an expanded version of that appearing in Fig. 10.21. An inductor L_s was added to reflect the inductance present due to the capacitor leads and any inductance introduced by the design of the capacitor. The inductance of the leads is typically about $0.05 \mu\text{H}$ per centimeter, which is about $0.2 \mu\text{H}$ for a capacitor with 2 cm leads at each end—a level of inductance that can be important at very high frequencies.

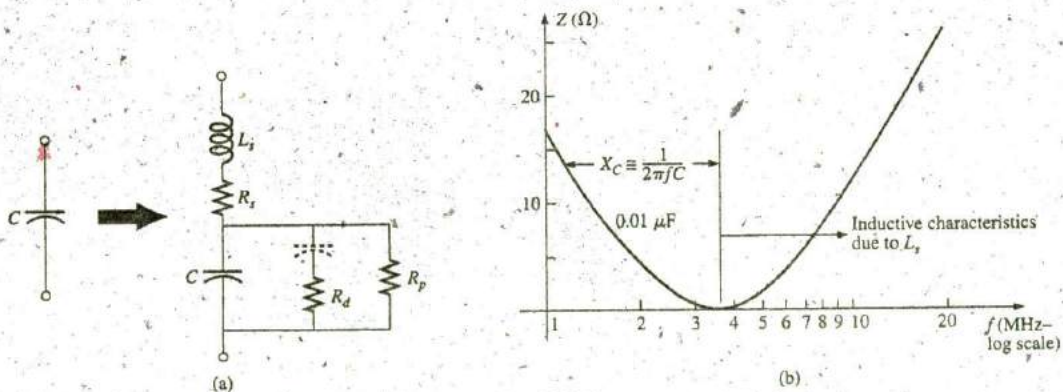


FIG. 14.27

Practical equivalent for a capacitor; (a) network; (b) response.

The resistance R_d reflects the energy lost due to molecular friction within the dielectric as the atoms continually realign themselves in the dielectric due to the applied alternating ac voltage. Of interest, however, the relative permittivity decreases with increasing frequencies but eventually undergoes a complete turnaround and begins to increase at very high frequencies. Notice the capacitor included in series with R_d to reflect the fact that this loss is not present under dc conditions. The capacitor assumes its open-circuit state for dc applications.

The resistance R_p , as introduced earlier, is defined by the resistivity of the dielectric (typically $10^{12} \Omega$ or greater) and the case resistance and will determine the level of leakage current if the capacitor is left to discharge. Depending on the capacitor, the discharge time can extend from a few seconds for some electrolytics to hours (paper) or days (polystyrene), revealing that electrolytics typically have much lower levels of R_p than most other capacitors.

The effect of all the elements on the actual response of a $0.01 \mu\text{F}$ metallized film capacitor with 2 cm leads is provided in Fig. 14.27(b), where the response is almost ideal for the low and mid-frequency range but then at about 3.7 MHz begins to show an inductive response due to L_s .

In general, therefore, the frequency of application is important for capacitive elements because when the frequency increases to a certain level, the element takes on inductive characteristics. Also, the frequency of application defines the type of capacitor (or inductor) that is applied: Electrolytics are limited to frequencies to perhaps 10 kHz, while ceramic or mica can handle frequencies higher than 10 MHz.

The expected temperature range of operation can have an important impact on the type of capacitor chosen for a particular application. Electrolytics, tantalum, and some high- k ceramic capacitors are very sensitive to colder temperatures. In fact, most electrolytics lose 20% of their room-temperature capacitance at 0°C (freezing). Higher temperatures (up to 100°C or 212°F) seem to have less impact in general than colder temperatures, but high- k ceramics can lose up to 30% of their capacitance level at 100°C compared to room temperature. With experience, you will learn the type of capacitor to use for each application and only be concerned when you encounter very high frequencies, extreme temperatures, or very high currents or voltages.

ESR The term *equivalent series resistance* (ESR) was introduced in Chapter 10, where it was noted that the topic would surface again after the concept of frequency response was introduced. In the simplest of terms, the ESR as appearing in the simplistic model of Fig. 14.28(a) is the actual dissipative factor one can expect when using a capacitor at various frequencies. For dc conditions it is essentially the dc resistance of the capacitor appearing as R_s in Fig. 14.27(a). However, for any ac application the level of dissipation will be a function of the levels of R_p and R_d and the frequency applied.

Although space does not permit a detailed derivation here, the ESR for a capacitor is defined by the following equation:

$$\text{ESR} = R_s + \frac{1}{\omega^2 C^2 R_p} + \frac{1}{\omega C^2 R_d}$$

Note that the first term is simply the dc resistance and is not a function of frequency. However, the next two terms are a function of frequency in

the denominator, revealing that they will increase very quickly as the frequency drops. The result is the valid concern about levels of ESR at low frequencies. At high frequencies, the second two terms will die off quickly, leaving only the dc resistance. In general, therefore, keep in mind that

the level of ESR or equivalent series resistance is frequency sensitive and considerably greater at low frequencies than just the dc resistance. At very high frequencies, it approaches the dc level.

It is such an important factor in some designs that instruments have been developed primarily to measure this quantity. One such instrument appears in Fig. 14.28(b).

There are some general rules about the level of ESR associated with various capacitors. For all applications, the lower the ESR, the better. Electrolytic capacitors typically have much higher levels of ESR than film, ceramic, or paper capacitors. A standard electrolytic 22 μF capacitor may have an ESR between 5 and 30 Ω , while a standard ceramic may have only 10 to 100 m Ω , a significant difference. Electrolytics, however, because of their other characteristics, are still very popular in power supply design—it is simply a matter of balancing the ESR level with other important factors.

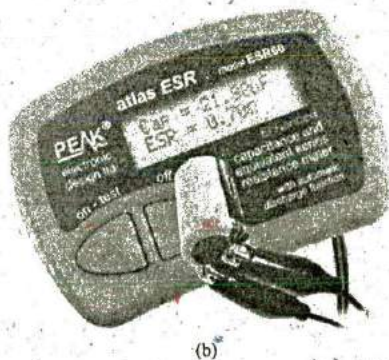
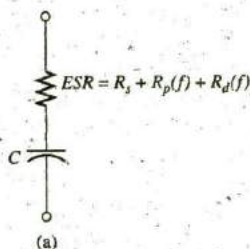


FIG. 14.28

ESR. (a) Impact on equivalent model;
(b) Measuring instrument.

[Part (b) courtesy of Peak Electronic Design Limited.]

EXAMPLE 14.8 At what frequency will the reactance of a 200 mH inductor match the resistance level of a 5 k Ω resistor?

Solution: The resistance remains constant at 5 k Ω for the frequency range of the inductor. Therefore,

$$\begin{aligned} R &= 5000 \Omega = X_L = 2\pi fL = 2\pi Lf \\ &= 2\pi(200 \times 10^{-3} \text{ H})f = 1.257f \end{aligned}$$

and
$$f = \frac{5000 \text{ Hz}}{1.257} \approx 3.98 \text{ kHz}$$

EXAMPLE 14.9 At what frequency will an inductor of 5 mH have the same reactance as a capacitor of 0.1 μF ?

Solution:

$$\begin{aligned} X_L &= X_C \\ 2\pi fL &= \frac{1}{2\pi fC} \\ f^2 &= \frac{1}{4\pi^2 LC} \end{aligned}$$

and

$$\begin{aligned} f &= \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(5 \times 10^{-3} \text{ H})(0.1 \times 10^{-6} \text{ F})}} \\ &= \frac{1}{2\pi\sqrt{5 \times 10^{-10}}} = \frac{1}{(2\pi)(2.236 \times 10^{-5})} = \frac{10^5 \text{ Hz}}{14.05} \approx 7.12 \text{ kHz} \end{aligned}$$

14.5 AVERAGE POWER AND POWER FACTOR

A common question is, How can a sinusoidal voltage or current deliver power to a load if it seems to be delivering power during one part of its cycle and taking it back during the negative part of the sinusoidal cycle? The equal oscillations above and below the axis seem to suggest that over one full cycle there is no net transfer of power or energy. However, as mentioned in the last chapter, there is a net transfer of power over one full cycle because power is delivered to the load at each instant of the applied voltage or current (except when either is crossing the axis) no matter what the direction is of the current or polarity of the voltage.

To demonstrate this, consider the relatively simple configuration in Fig. 14.29, where an 8 V peak sinusoidal voltage is applied across a 2 Ω resistor. When the voltage is at its positive peak, the power delivered at that instant is 32 W, as shown in the figure. At the midpoint of 4 V, the instantaneous power delivered drops to 8 W; when the voltage crosses the axis, it drops to 0 W. Note, however, that when the applied voltage is at its negative peak, the current may reverse, but at that instant, 32 W is still being delivered to the resistor.

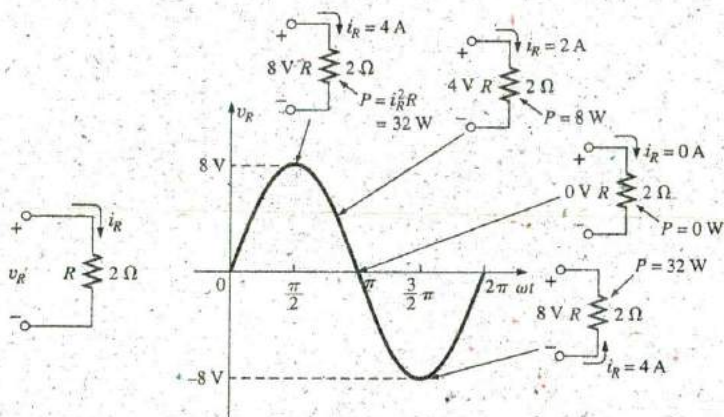


FIG. 14.29

Demonstrating that power is delivered at every instant of a sinusoidal voltage waveform (except $v_R = 0$ V).

In total, therefore,

even though the current through and the voltage across reverse direction and polarity, respectively, power is delivered to the resistive load at each instant of time.

If we plot the power delivered over a full cycle, we obtain the curve in Fig. 14.30. Note that the applied voltage and resulting current are in phase and have twice the frequency of the power curve. For one full cycle of the applied voltage having a period T , the power level peaks for each pulse of the sinusoidal waveform.

The fact that the power curve is always above the horizontal axis reveals that power is being delivered to the load at each instant of time of the applied sinusoidal voltage.

Any portion of the power curve below the axis reveals that power is being returned to the source. The average value of the power curve

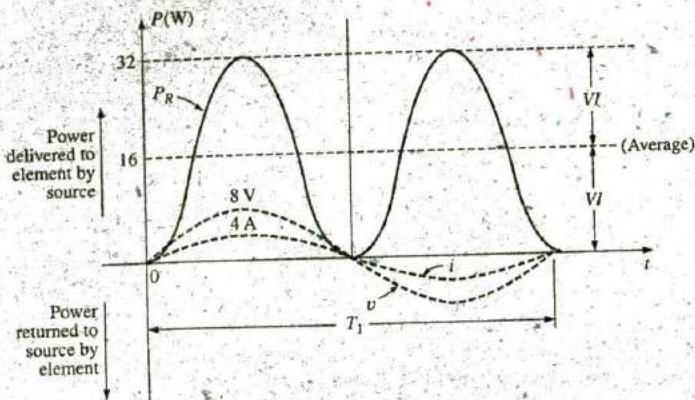


FIG. 14.30

Power versus time for a purely resistive load.

occurs at a level equal to $V_m I_m / 2$, as shown in Fig. 14.30. This power level is called the **average** or **real power** level. It establishes a particular level of power transfer for the full cycle, so that we do not have to determine the level of power to apply to a quantity that varies in a sinusoidal nature.

If we substitute the equation for the peak value in terms of the rms value as

$$P_{av} = \frac{V_m I_m}{2} = \frac{(\sqrt{2} V_{rms})(\sqrt{2} I_{rms})}{2} = \frac{2 V_{rms} I_{rms}}{2}$$

we find that the average or real power delivered to a resistor takes on the following very convenient form:

$$P_{av} = V_{rms} I_{rms} \quad (14.10)$$

Note that the power equation is exactly the same when applied to dc networks as long as we work with rms values.

The above analysis was for a purely resistive load. If the sinusoidal voltage is applied to a network with a combination of R , L , and C components, the instantaneous equation for the power levels is more complex. However, if we are careful in developing the general equation and examine the results, we find some general conclusions that will be very helpful in the analysis to follow.

In Fig. 14.31, a voltage with an initial phase angle is applied to a network with any combination of elements that results in a current with the indicated phase angle.

The power delivered at each instant of time is then defined by

$$\begin{aligned} p &= vi = V_m \sin(\omega t + \theta_v) I_m \sin(\omega t + \theta_i) \\ &= V_m I_m \sin(\omega t + \theta_v) \sin(\omega t + \theta_i) \end{aligned}$$

Using the trigonometric identity

$$\sin A \sin B = \frac{\cos(A - B) - \cos(A + B)}{2}$$

we see that the function $\sin(\omega t + \theta_v) \sin(\omega t + \theta_i)$ becomes

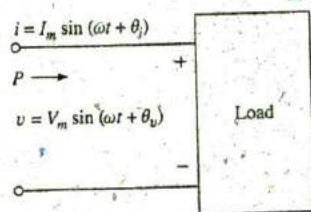


FIG. 14.31

Determining the power delivered in a sinusoidal ac network.

$$\begin{aligned} & \sin(\omega t + \theta_v) \sin(\omega t + \theta_i) \\ &= \frac{\cos[(\omega t + \theta_v) - (\omega t + \theta_i)] - \cos[(\omega t + \theta_v) + (\omega t + \theta_i)]}{2} \\ &= \frac{\cos(\theta_v - \theta_i) - \cos(2\omega t + \theta_v + \theta_i)}{2} \end{aligned}$$

so that

$$p = \underbrace{\left[\frac{V_m I_m}{2} \cos(\theta_v - \theta_i) \right]}_{\text{Fixed value}} - \underbrace{\left[\frac{V_m I_m}{2} \cos(2\omega t + \theta_v + \theta_i) \right]}_{\text{Time-varying (function of } t \text{)}}$$

A plot of v , i , and p on the same set of axes is shown in Fig. 14.32.

Note that the second factor in the preceding equation is a cosine wave with an amplitude of $V_m I_m / 2$ and with a frequency twice that of the voltage or current. The average value of this term is zero over one cycle, producing no net transfer of energy in any one direction.

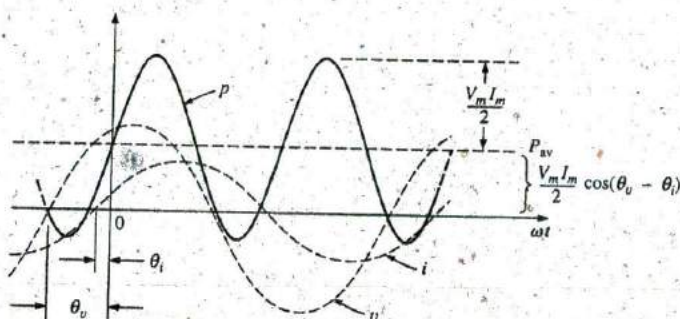


FIG. 14.32

Defining the average power for a sinusoidal ac network.

The first term in the preceding equation, however, has a constant magnitude (no time dependence) and therefore provides some net transfer of energy. This term is referred to as the **average power** or **real power** as introduced earlier. The angle $(\theta_v - \theta_i)$ is the phase angle between v and i . Since $\cos(-\alpha) = \cos \alpha$,

the magnitude of average power delivered is independent of whether v leads i or i leads v .

Defining θ as equal to $|\theta_v - \theta_i|$, where $|\cdot|$ indicates that only the magnitude is important and the sign is immaterial, we have

$$P = \frac{V_m I_m}{2} \cos \theta \quad (\text{watts, W}) \quad (14.11)$$

where P is the average power in watts. This equation can also be written

$$P = \left(\frac{V_m}{\sqrt{2}} \right) \left(\frac{I_m}{\sqrt{2}} \right) \cos \theta$$

or, since

$$V_{\text{eff}} = \frac{V_m}{\sqrt{2}} \quad \text{and} \quad I_{\text{eff}} = \frac{I_m}{\sqrt{2}}$$

Eq. (14.11) becomes

$$P = V_{\text{rms}} I_{\text{rms}} \cos \theta \quad (14.12)$$

Let us now apply Eqs. (14.11) and (14.12) to the basic R , L , and C elements.

Resistor

In a purely resistive circuit, since v and i are in phase, $|\theta_v - \theta_i| = \theta = 0^\circ$, and $\cos \theta = \cos 0^\circ = 1$, so that

$$P = \frac{V_m I_m}{2} = V_{\text{rms}} I_{\text{rms}} \quad (\text{W}) \quad (14.13)$$

or, since $I_{\text{rms}} = \frac{V_{\text{rms}}}{R}$

$$P = \frac{V_{\text{rms}}^2}{R} = I_{\text{rms}}^2 R \quad (\text{W}) \quad (14.14)$$

Inductor

In a purely inductive circuit, since v leads i by 90° , $|\theta_v - \theta_i| = \theta = |-90^\circ| = 90^\circ$. Therefore,

$$P = \frac{V_m I_m}{2} \cos 90^\circ = \frac{V_m I_m}{2} (0) = 0 \text{ W}$$

The average power or power dissipated by the ideal inductor (no associated resistance) is zero watts.

Capacitor

In a purely capacitive circuit, since i leads v by 90° , $|\theta_v - \theta_i| = \theta = |-90^\circ| = 90^\circ$. Therefore,

$$P = \frac{V_m I_m}{2} \cos(90^\circ) = \frac{V_m I_m}{2} (0) = 0 \text{ W}$$

The average power or power dissipated by the ideal capacitor (no associated resistance) is zero watts.

EXAMPLE 14.10 Find the average power dissipated in a network whose input current and voltage are the following:

$$i = 5 \sin(\omega t + 40^\circ)$$

$$v = 10 \sin(\omega t + 40^\circ)$$

Solution: Since v and i are in phase, the circuit appears to be purely resistive at the input terminals. Therefore,

$$P = \frac{V_m I_m}{2} = \frac{(10 \text{ V})(5 \text{ A})}{2} = 25 \text{ W}$$

$$\text{or} \quad R = \frac{V_m}{I_m} = \frac{10 \text{ V}}{5 \text{ A}} = 2 \Omega$$

$$\text{and} \quad P = \frac{V_{\text{rms}}^2}{R} = \frac{[(0.707)(10 \text{ V})]^2}{2} = 25 \text{ W}$$

$$\text{or} \quad P = I_{\text{rms}}^2 R = [(0.707)(5 \text{ A})]^2 (2) = 25 \text{ W}$$

For the following example, the circuit consists of a combination of resistances and reactances producing phase angles between the input current and voltage different from 0° or 90° .

EXAMPLE 14.11 Determine the average power delivered to networks having the following input voltage and current:

- $v = 100 \sin(\omega t + 40^\circ)$
 $i = 20 \sin(\omega t + 70^\circ)$
- $v = 150 \sin(\omega t - 70^\circ)$
 $i = 3 \sin(\omega t - 50^\circ)$

Solutions:

- $V_m = 100$, $\theta_v = 40^\circ$
 $I_m = 20$ A, $\theta_i = 70^\circ$
 $\theta = |\theta_v - \theta_i| = |40^\circ - 70^\circ| = |-30^\circ| = 30^\circ$

and

$$P = \frac{V_m I_m}{2} \cos \theta = \frac{(100 \text{ V})(20 \text{ A})}{2} \cos(30^\circ) = (1000 \text{ W})(0.866) = 866 \text{ W}$$

- $V_m = 150$ V, $\theta_v = -70^\circ$
 $I_m = 3$ A, $\theta_i = -50^\circ$
 $\theta = |\theta_v - \theta_i| = |-70^\circ - (-50^\circ)| = |-70^\circ + 50^\circ| = |-20^\circ| = 20^\circ$

and

$$P = \frac{V_m I_m}{2} \cos \theta = \frac{(150 \text{ V})(3 \text{ A})}{2} \cos(20^\circ) = (225 \text{ W})(0.9397) = 211.43 \text{ W}$$

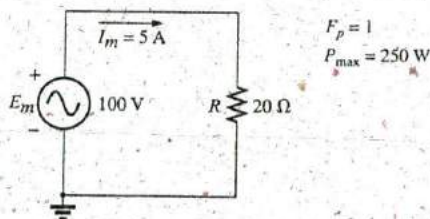


FIG. 14.33

Purely resistive load with $F_p = 1$.

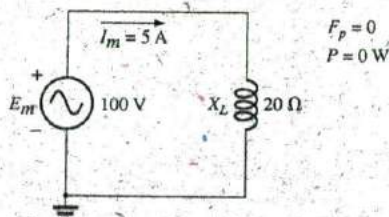


FIG. 14.34

Purely inductive load with $F_p = 0$.

Power Factor

In the equation $P = (V_m I_m / 2) \cos \theta$, the factor that has significant control over the delivered power level is the $\cos \theta$. No matter how large the voltage or current, if $\cos \theta = 0$, the power is zero; if $\cos \theta = 1$, the power delivered is a maximum. Since it has such control, the expression was given the name **power factor** and is defined by

$$\text{Power factor} = F_p = \cos \theta \quad (14.15)$$

For a purely resistive load such as the one shown in Fig. 14.33, the phase angle between v and i is 0° and $F_p = \cos \theta = \cos 0^\circ = 1$. The power delivered is a maximum of $(V_m I_m / 2) \cos \theta = ((100 \text{ V})(5 \text{ A}) / 2)(1) = 250 \text{ W}$.

For a purely reactive load (inductive or capacitive) such as the one shown in Fig. 14.34, the phase angle between v and i is 90° and $F_p = \cos \theta = \cos 90^\circ = 0$. The power delivered is then the minimum value of zero watts, even though the current has the same peak value as that encountered in Fig. 14.33.

For situations where the load is a combination of resistive and reactive elements, the power factor varies between 0 and 1. The more resistive the total impedance, the closer is the power factor to 1; the more reactive the total impedance, the closer is the power factor to 0.

In terms of the average power and the terminal voltage and current,

$$F_p = \cos \theta = \frac{P}{V_{\text{rms}} I_{\text{rms}}} \quad (14.16)$$

The terms *leading* and *lagging* are often written in conjunction with the power factor. They are defined by the current through the load. If the current leads the voltage across a load, the load has a **leading power factor**. If the current lags the voltage across the load, the load has a **lagging power factor**. In other words,

capacitive networks have leading power factors, and inductive networks have lagging power factors.

The importance of the power factor to power distribution systems is examined in Chapter 19. In fact, one section is devoted to power-factor correction.

EXAMPLE 14.12 Determine the power factors of the following loads, and indicate whether they are leading or lagging:

- Fig. 14.35
- Fig. 14.36
- Fig. 14.37

Solutions:

- $F_p = \cos \theta = \cos |40^\circ - (-20^\circ)| = \cos 60^\circ = 0.5$ **leading**
- $F_p = \cos \theta = \cos |80^\circ - 30^\circ| = \cos 50^\circ = 0.64$ **lagging**
- $F_p = \cos \theta = \frac{P}{V_{\text{eff}} I_{\text{eff}}} = \frac{100 \text{ W}}{(20 \text{ V})(5 \text{ A})} = \frac{100 \text{ W}}{100 \text{ W}} = 1$

The load is resistive, and F_p is neither leading nor lagging.

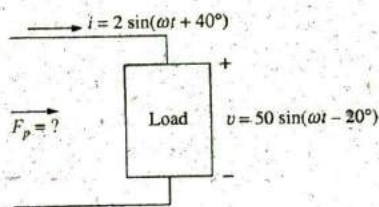


FIG. 14.35
Example 14.12(a).

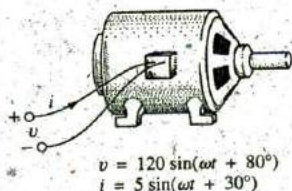


FIG. 14.36
Example 14.12(b).

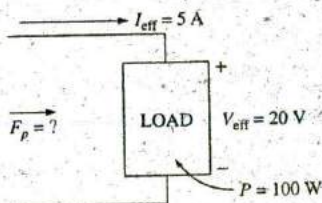


FIG. 14.37
Example 14.12(c).

14.6 COMPLEX NUMBERS

In our analysis of dc networks, we found it necessary to determine the algebraic sum of voltages and currents. Since the same will also be true for ac networks, the question arises, How do we determine the algebraic sum of two or more voltages (or currents) that are varying sinusoidally? Although one solution would be to find the algebraic sum on a point-to-point basis (as shown in Section 14.12), this would be a long and tedious process in which accuracy would be directly related to the scale used.

It is the purpose of this chapter to introduce a system of **complex numbers** that, when related to the sinusoidal ac waveform, results in a technique for finding the algebraic sum of sinusoidal waveforms that is quick, direct, and accurate. In the following chapters, the technique is extended to permit the analysis of sinusoidal ac networks in a manner very similar to that applied to dc networks. The methods and theorems as described for dc networks can then be applied to sinusoidal ac networks with little difficulty.

A **complex number** represents a point in a two-dimensional plane located with reference to two distinct axes. This point can also determine a radius vector drawn from the origin to the point. The horizontal axis is called the *real axis*, while the vertical axis is called the *imaginary axis*. Both are labeled in Fig. 14.38. Every number from zero to $\pm\infty$ can be represented by some point along the real axis. Prior to the development of

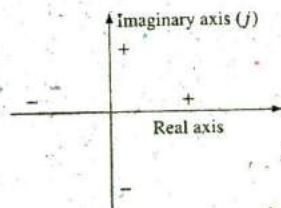


FIG. 14.38
Defining the real and imaginary axes of a complex plane.

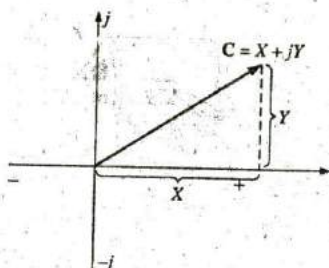


FIG. 14.39
Defining the rectangular form.

14.7 RECTANGULAR FORM

The format for the rectangular form is

$$\boxed{C = X + jY} \quad (14.17)$$

as shown in Fig. 14.39. The letter C was chosen from the word "complex." The **boldface** notation is for any number with magnitude and direction. The *italic* is for magnitude only.

EXAMPLE 14.13 Sketch the following complex numbers in the complex plane:

- a. $C = 3 + j4$ b. $C = 0 - j6$ c. $C = -10 - j20$

Solutions:

- a. See Fig. 14.40. b. See Fig. 14.41. c. See Fig. 14.42.

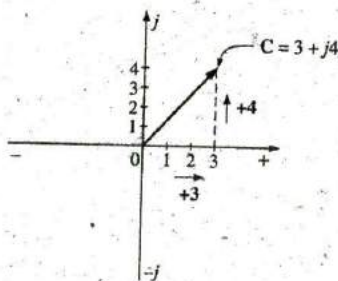


FIG. 14.40
Example 14.13(a).

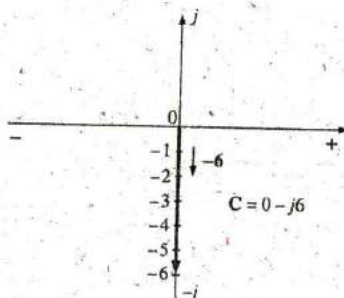


FIG. 14.41
Example 14.13(b).

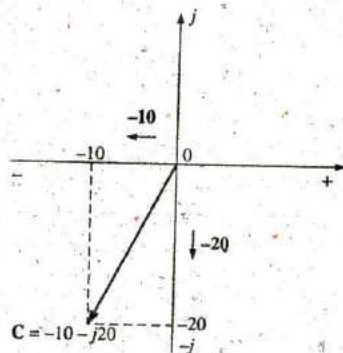


FIG. 14.42
Example 14.13(c).

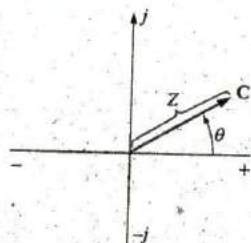


FIG. 14.43
Defining the polar form.

14.8 POLAR FORM

The format for the polar form is

$$\boxed{C = Z \angle \theta} \quad (14.18)$$

with the letter Z chosen from the sequence X, Y, Z .

Z indicates magnitude only, and θ is *always measured counterclockwise (CCW) from the positive real axis*, as shown in Fig. 14.43. Angles measured in the clockwise direction from the positive real axis must have a negative sign associated with them.

A negative sign in front of the polar form has the effect shown in Fig. 14.44. Note that it results in a complex number directly opposite the complex number with a positive sign.

$$\boxed{-C = -Z \angle \theta = Z \angle \theta \pm 180^\circ} \quad (14.19)$$

EXAMPLE 14.14 Sketch the following complex numbers in the complex plane:

- $C = 5 \angle 30^\circ$
- $C = 7 \angle -120^\circ$
- $C = -4.2 \angle 60^\circ$

Solutions:

- See Fig. 14.45.
- See Fig. 14.46.
- See Fig. 14.47.

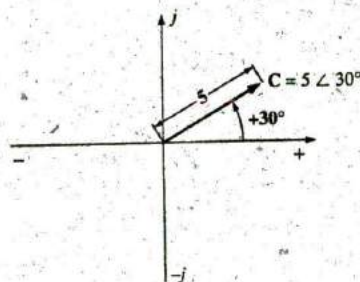


FIG. 14.45
Example 14.14(a).

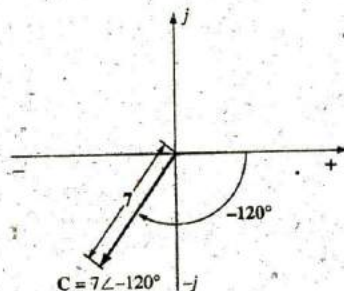


FIG. 14.46
Example 14.14(b).

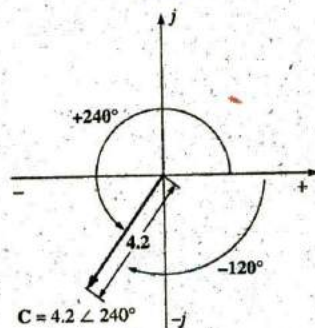


FIG. 14.47
Example 14.14(c).

$$C = -4.2 \angle 60^\circ = 4.2 \angle 60^\circ + 180^\circ = 4.2 \angle 240^\circ$$

14.9 CONVERSION BETWEEN FORMS

The two forms are related by the following equations, as illustrated in Fig. 14.48.

Rectangular to Polar

$$\boxed{Z = \sqrt{X^2 + Y^2}} \quad (14.20)$$

$$\boxed{\theta = \tan^{-1} \frac{Y}{X}} \quad (14.21)$$

Polar to Rectangular

$$\boxed{X = Z \cos \theta} \quad (14.22)$$

$$\boxed{Y = Z \sin \theta} \quad (14.23)$$

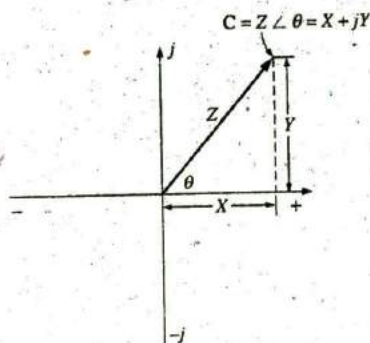


FIG. 14.48
Conversion between forms.

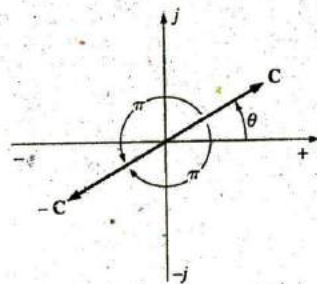


FIG. 14.44
Demonstrating the effect of a negative sign on the polar form.

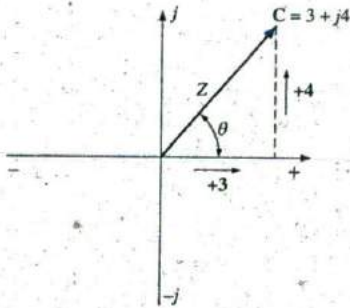


FIG. 14.49
Example 14.15.

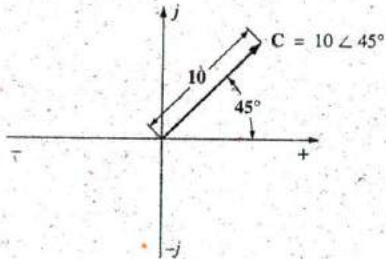


FIG. 14.50
Example 14.16.

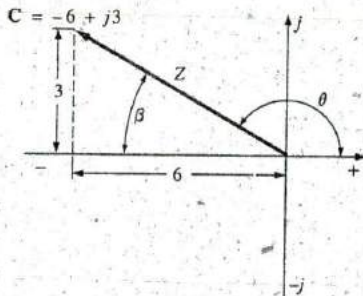


FIG. 14.51
Example 14.17.

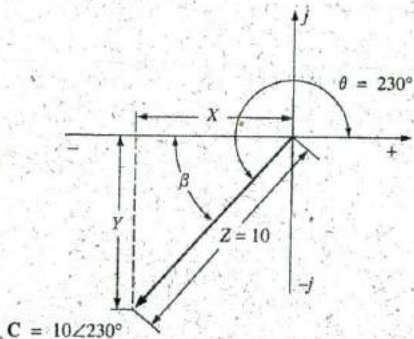


FIG. 14.52
Example 14.18.

EXAMPLE 14.15 Convert the following from rectangular to polar form:

$$C = 3 + j4 \quad (\text{Fig. 14.49})$$

Solution:

$$Z = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$$

$$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.13^\circ$$

and

$$C = 5 \angle 53.13^\circ$$

EXAMPLE 14.16 Convert the following from polar to rectangular form:

$$C = 10 \angle 45^\circ \quad (\text{Fig. 14.50})$$

Solution:

$$X = 10 \cos 45^\circ = (10)(0.707) = 7.07$$

$$Y = 10 \sin 45^\circ = (10)(0.707) = 7.07$$

and

$$C = 7.07 + j7.07$$

If the complex number should appear in the second, third, or fourth quadrant, simply convert it in that quadrant, and carefully determine the proper angle to be associated with the magnitude of the vector.

EXAMPLE 14.17 Convert the following from rectangular to polar form:

$$C = -6 + j3 \quad (\text{Fig. 14.51})$$

Solution:

$$Z = \sqrt{(6)^2 + (3)^2} = \sqrt{45} = 6.71$$

$$\beta = \tan^{-1}\left(\frac{3}{6}\right) = 26.57^\circ$$

$$\theta = 180^\circ - 26.57^\circ = 153.43^\circ$$

and

$$C = 6.71 \angle 153.43^\circ$$

EXAMPLE 14.18 Convert the following from polar to rectangular form:

$$C = 10 \angle 230^\circ \quad (\text{Fig. 14.52})$$

Solution:

$$X = Z \cos \beta = 10 \cos(230^\circ - 180^\circ) = 10 \cos 50^\circ \\ = (10)(0.6428) = 6.428$$

$$Y = Z \sin \beta = 10 \sin 50^\circ = (10)(0.7660) = 7.660$$

and

$$C = -6.43 - j7.66$$

14.10 MATHEMATICAL OPERATIONS WITH COMPLEX NUMBERS

Complex numbers lend themselves readily to the basic mathematical operations of addition, subtraction, multiplication, and division. A few basic rules and definitions must be understood before considering these operations.

Let us first examine the symbol j associated with imaginary numbers. By definition,

$$j = \sqrt{-1} \quad (14.24)$$

Thus,
$$j^2 = -1 \quad (14.25)$$

and with
$$j^3 = j^2j = -1j = -j$$

with
$$j^4 = j^2j^2 = (-1)(-1) = +1$$

$$j^5 = j$$

and so on. Further,

$$\frac{1}{j} = (1)\left(\frac{1}{j}\right) = \left(\frac{j}{j}\right)\left(\frac{1}{j}\right) = \frac{j}{j^2} = \frac{j}{-1}$$

and
$$\frac{1}{j} = -j \quad (14.26)$$

Complex Conjugate

The **conjugate** or **complex conjugate** of a complex number can be found by simply changing the sign of the imaginary part in the rectangular form or by using the negative of the angle of the polar form. For example, the conjugate of

$$C = 2 + j3$$

is
$$2 - j3$$

as shown in Fig. 14.53. The conjugate of

$$C = 2 \angle 30^\circ$$

is
$$2 \angle -30^\circ$$

as shown in Fig. 14.54.

Reciprocal

The **reciprocal** of a complex number is 1 divided by the complex number. For example, the reciprocal of

$$C = X + jY$$

is
$$\frac{1}{X + jY}$$

and that of $Z \angle \theta$ is

$$\frac{1}{Z \angle \theta}$$

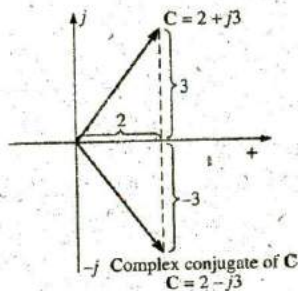


FIG. 14.53

Defining the complex conjugate of a complex number in rectangular form.

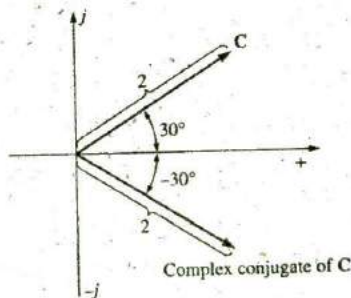


FIG. 14.54

Defining the complex conjugate of a complex number in polar form.

We are now prepared to consider the four basic operations of *addition*, *subtraction*, *multiplication*, and *division* with complex numbers.

Addition

To add two or more complex numbers, add the real and imaginary parts separately. For example, if

$$C_1 = \pm X_1 \pm jY_1 \quad \text{and} \quad C_2 = \pm X_2 \pm jY_2$$

$$\text{then} \quad \boxed{C_1 + C_2 = (\pm X_1 \pm X_2) + j(\pm Y_1 \pm Y_2)} \quad (14.27)$$

There is really no need to memorize the equation. Simply set one above the other and consider the real and imaginary parts separately, as shown in Example 14.19.

EXAMPLE 14.19

- a. Add $C_1 = 2 + j4$ and $C_2 = 3 + j1$.
 b. Add $C_1 = 3 + j6$ and $C_2 = -6 + j3$.

Solutions:

- a. By Eq. (14.27),

$$C_1 + C_2 = (2 + 3) + j(4 + 1) = 5 + j5$$

Note Fig. 14.55. An alternative method is

$$\begin{array}{r} 2 + j4 \\ 3 + j1 \\ \downarrow \downarrow \\ 5 + j5 \end{array}$$

- b. By Eq. (14.27),

$$C_1 + C_2 = (3 - 6) + j(6 + 3) = -3 + j9$$

Note Fig. 14.56. An alternative method is

$$\begin{array}{r} 3 + j6 \\ -6 + j3 \\ \downarrow \downarrow \\ -3 + j9 \end{array}$$

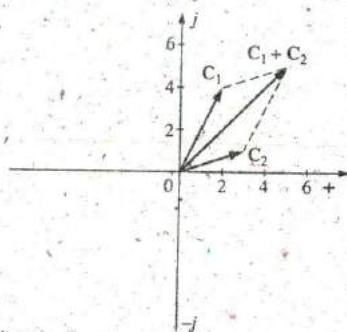


FIG. 14.55
Example 14.19(a).

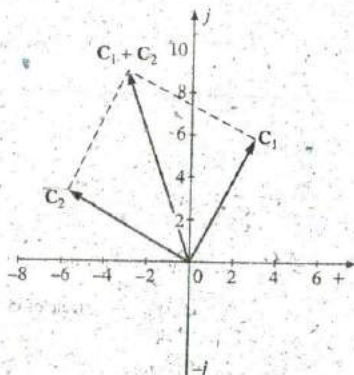


FIG. 14.56
Example 14.19(b).

Subtraction

In subtraction, the real and imaginary parts are again considered separately. For example, if

$$C_1 = \pm X_1 \pm jY_1 \quad \text{and} \quad C_2 = \pm X_2 \pm jY_2$$

$$\text{then} \quad \boxed{C_1 - C_2 = [\pm X_1 - (\pm X_2)] + j[\pm Y_1 - (\pm Y_2)]} \quad (14.28)$$

Again, there is no need to memorize the equation if the alternative method of Example 14.20 is used.

EXAMPLE 14.20

- Subtract $C_2 = 1 + j4$ from $C_1 = 4 + j6$.
- Subtract $C_2 = -2 + j5$ from $C_1 = +3 + j3$.

Solutions:

- By Eq. (14.28),

$$C_1 - C_2 = (4 - 1) + j(6 - 4) = 3 + j2$$

Note Fig. 14.57. An alternative method is

$$\begin{array}{r} 4 + j6 \\ -(1 + j4) \\ \hline \downarrow \downarrow \\ 3 + j2 \end{array}$$

- By Eq. (14.28),

$$C_1 - C_2 = [3 - (-2)] + j(3 - 5) = 5 - j2$$

Note Fig. 14.58. An alternative method is

$$\begin{array}{r} 3 + j3 \\ -(-2 + j5) \\ \hline \downarrow \downarrow \\ 5 - j2 \end{array}$$

Addition or subtraction cannot be performed in polar form unless the complex numbers have the same angle θ or unless they differ only by multiples of 180° .

EXAMPLE 14.21

- $2 \angle 45^\circ + 3 \angle 45^\circ = 5 \angle 45^\circ$. Note Fig. 14.59.
- $2 \angle 0^\circ - 4 \angle 180^\circ = 6 \angle 0^\circ$. Note Fig. 14.60.

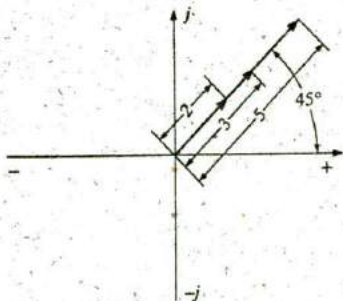


FIG. 14.59
Example 14.21(a).

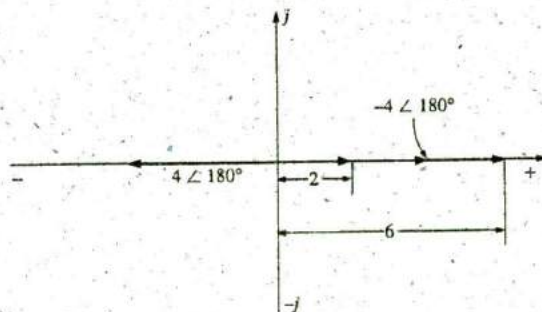


FIG. 14.60
Example 14.21(b).

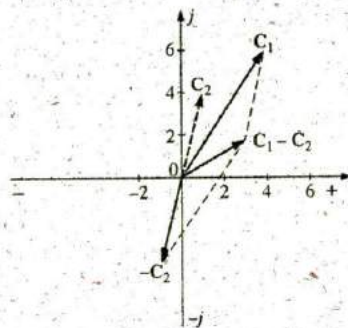


FIG. 14.57
Example 14.20(a).

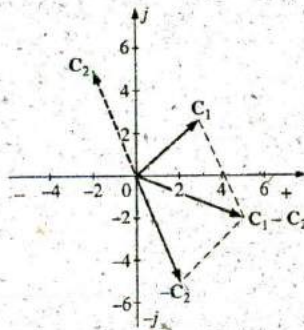


FIG. 14.58
Example 14.20(b).

Multiplication

To multiply two complex numbers in *rectangular* form, multiply the real and imaginary parts of one in turn by the real and imaginary parts of the other. For example, if

$$C_1 = X_1 + jY_1 \quad \text{and} \quad C_2 = X_2 + jY_2$$

$$\begin{aligned} \text{then } C_1 \cdot C_2 &= \begin{array}{r} X_1 + jY_1 \\ \cdot \quad X_2 + jY_2 \\ \hline X_1X_2 + jY_1X_2 \\ \quad + jX_1Y_2 + j^2Y_1Y_2 \\ \hline X_1X_2 + j(Y_1X_2 + X_1Y_2) + Y_1Y_2(-1) \end{array} \end{aligned}$$

$$\text{and} \quad \boxed{C_1 \cdot C_2 = (X_1X_2 - Y_1Y_2) + j(Y_1X_2 + X_1Y_2)} \quad (14.29)$$

In Example 14.22(b), we obtain a solution without resorting to memorizing Eq. (14.29). Simply carry along the j factor when multiplying each part of one vector with the real and imaginary parts of the other.

EXAMPLE 14.22

a. Find $C_1 \cdot C_2$ if

$$C_1 = 2 + j3 \quad \text{and} \quad C_2 = 5 + j10$$

b. Find $C_1 \cdot C_2$ if

$$C_1 = -2 - j3 \quad \text{and} \quad C_2 = +4 - j6$$

Solutions:

a. Using the format above, we have

$$\begin{aligned} C_1 \cdot C_2 &= [(2)(5) - (3)(10)] + j[(3)(5) + (2)(10)] \\ &= -20 + j35 \end{aligned}$$

b. Without using the format, we obtain

$$\begin{array}{r} -2 - j3 \\ +4 - j6 \\ \hline -8 - j12 \\ \quad + j12 + j^2 18 \\ \hline -8 + j(-12 + 12) - 18 \end{array}$$

$$\text{and} \quad C_1 \cdot C_2 = -26 = 26 \angle 180^\circ$$

In *polar* form, the magnitudes are multiplied and the angles added algebraically. For example, for

$$C_1 = Z_1 \angle \theta_1 \quad \text{and} \quad C_2 = Z_2 \angle \theta_2$$

we write

$$\boxed{C_1 \cdot C_2 = Z_1 Z_2 \angle \theta_1 + \theta_2} \quad (14.30)$$

EXAMPLE 14.23

 a. Find $C_1 \cdot C_2$ if

$$C_1 = 5 \angle 20^\circ \quad \text{and} \quad C_2 = 10 \angle 30^\circ$$

 b. Find $C_1 \cdot C_2$ if

$$C_1 = 2 \angle -40^\circ \quad \text{and} \quad C_2 = 7 \angle +120^\circ$$

Solutions:

a. $C_1 \cdot C_2 = (5 \angle 20^\circ)(10 \angle 30^\circ) = (5)(10) \angle 20^\circ + 30^\circ = 50 \angle 50^\circ$

b. $C_1 \cdot C_2 = (2 \angle -40^\circ)(7 \angle +120^\circ) = (2)(7) \angle -40^\circ + 120^\circ$
 $= 14 \angle +80^\circ$

To multiply a complex number in rectangular form by a real number requires that both the real part and the imaginary part be multiplied by the real number. For example,

$$(10)(2 + j3) = 20 + j30$$

and $50 \angle 0^\circ(0 + j6) = j300 = 300 \angle 90^\circ$

Division

To divide two complex numbers in *rectangular* form, multiply the numerator and denominator by the conjugate of the denominator and the resulting real and imaginary parts collected. That is, if

$$C_1 = X_1 + jY_1 \quad \text{and} \quad C_2 = X_2 + jY_2$$

then

$$\frac{C_1}{C_2} = \frac{(X_1 + jY_1)(X_2 - jY_2)}{(X_2 + jY_2)(X_2 - jY_2)}$$

$$= \frac{(X_1X_2 + Y_1Y_2) + j(X_2Y_1 - X_1Y_2)}{X_2^2 + Y_2^2}$$

and

$$\frac{C_1}{C_2} = \frac{X_1X_2 + Y_1Y_2}{X_2^2 + Y_2^2} + j \frac{X_2Y_1 - X_1Y_2}{X_2^2 + Y_2^2} \quad (14.31)$$

The equation does not have to be memorized if the steps above used to obtain it are employed. That is, first multiply the numerator by the complex conjugate of the denominator and separate the real and imaginary terms. Then divide each term by the sum of each term of the denominator squared.

EXAMPLE 14.24

 a. Find C_1/C_2 if $C_1 = 1 + j4$ and $C_2 = 4 + j5$.

 b. Find C_1/C_2 if $C_1 = -4 - j8$ and $C_2 = +6 - j1$.

Solutions:

a. By Eq. (14.31),

$$\frac{C_1}{C_2} = \frac{(1)(4) + (4)(5)}{4^2 + 5^2} + j \frac{(4)(4) - (1)(5)}{4^2 + 5^2}$$

$$= \frac{24}{41} + \frac{j11}{41} \cong 0.59 + j0.27$$



- b. Using an alternative method, we obtain

$$\begin{array}{r} -4 - j8 \\ +6 + j1 \\ \hline -24 - j48 \\ -j4 - j^2 8 \\ \hline -24 - j52 + 8 = -16 - j52 \\ +6 - j1 \\ +6 + j1 \\ \hline 36 + j6 \\ -j6 - j^2 1 \\ \hline 36 + 0 + 1 = 37 \end{array}$$

and $\frac{C_1}{C_2} = \frac{-16}{37} - \frac{j52}{37} = -0.43 - j1.41$

To divide a complex number in rectangular form by a real number, both the real part and the imaginary part must be divided by the real number. For example,

$$\frac{8 + j10}{2} = 4 + j5$$

and $\frac{6.8 - j0}{2} = 3.4 - j0 = 3.4 \angle 0^\circ$

In *polar* form, division is accomplished by dividing the magnitude of the numerator by the magnitude of the denominator and subtracting the angle of the denominator from that of the numerator. That is, for

$$C_1 = Z_1 \angle \theta_1 \quad \text{and} \quad C_2 = Z_2 \angle \theta_2$$

we write

$$\frac{C_1}{C_2} = \frac{Z_1}{Z_2} \angle \theta_1 - \theta_2 \quad (14.32)$$

EXAMPLE 14.25

- a. Find C_1/C_2 if $C_1 = 15 \angle 10^\circ$ and $C_2 = 2 \angle 7^\circ$.
 b. Find C_1/C_2 if $C_1 = 8 \angle 120^\circ$ and $C_2 = 16 \angle -50^\circ$.

Solutions:

a. $\frac{C_1}{C_2} = \frac{15 \angle 10^\circ}{2 \angle 7^\circ} = \frac{15}{2} \angle 10^\circ - 7^\circ = 7.5 \angle 3^\circ$

b. $\frac{C_1}{C_2} = \frac{8 \angle 120^\circ}{16 \angle -50^\circ} = \frac{8}{16} \angle 120^\circ - (-50^\circ) = 0.5 \angle 170^\circ$

We obtain the *reciprocal* in the rectangular form by multiplying the numerator and denominator by the complex conjugate of the denominator:

$$\frac{1}{X + jY} = \left(\frac{1}{X + jY} \right) \left(\frac{X - jY}{X - jY} \right) = \frac{X - jY}{X^2 + Y^2}$$

and

$$\frac{1}{X + jY} = \frac{X}{X^2 + Y^2} - j \frac{Y}{X^2 + Y^2} \quad (14.33)$$

In polar form, the reciprocal is

$$\frac{1}{Z \angle \theta} = \frac{1}{Z} \angle -\theta \quad (14.34)$$

A concluding example using the four basic operations follows.

EXAMPLE 14.26 Perform the following operations, leaving the answer in polar or rectangular form:

$$\begin{aligned} \text{a. } \frac{(2 + j3) + (4 + j6)}{(7 + j7) - (3 - j3)} &= \frac{(2 + 4) + j(3 + 6)}{(7 - 3) + j(7 + 3)} \\ &= \frac{(6 + j9)(4 - j10)}{(4 + j10)(4 - j10)} \\ &= \frac{[(6)(4) + (9)(10)] + j[(4)(9) - (6)(10)]}{4^2 + 10^2} \\ &= \frac{114 - j24}{116} = 0.98 - j0.21 \end{aligned}$$

$$\begin{aligned} \text{b. } \frac{(50 \angle 30^\circ)(5 + j5)}{10 \angle -20^\circ} &= \frac{(50 \angle 30^\circ)(7.07 \angle 45^\circ)}{10 \angle -20^\circ} = \frac{353.5 \angle 75^\circ}{10 \angle -20^\circ} \\ &= 35.35 \angle 75^\circ - (-20^\circ) = 35.35 \angle 95^\circ \end{aligned}$$

$$\begin{aligned} \text{c. } \frac{(2 \angle 20^\circ)^2(3 + j4)}{8 - j6} &= \frac{(2 \angle 20^\circ)(2 \angle 20^\circ)(5 \angle 53.13^\circ)}{10 \angle -36.87^\circ} \\ &= \frac{(4 \angle 40^\circ)(5 \angle 53.13^\circ)}{10 \angle -36.87^\circ} = \frac{20 \angle 93.13^\circ}{10 \angle -36.87^\circ} \\ &= 2 \angle 93.13^\circ - (-36.87^\circ) = 2.0 \angle 130^\circ \end{aligned}$$

$$\begin{aligned} \text{d. } 3 \angle 27^\circ - 6 \angle -40^\circ &= (2.673 + j1.362) - (4.596 - j3.857) \\ &= (2.673 - 4.596) + j(1.362 + 3.857) \\ &= -1.92 + j5.22 \end{aligned}$$

14.11 CALCULATOR METHODS WITH COMPLEX NUMBERS

The process of converting from one form to another or working through lengthy operations with complex numbers can be time-consuming and often frustrating if one lost minus sign or decimal point invalidates the solution. Fortunately, technologists of today have calculators and computer methods that make the process measurably easier with higher degrees of reliability and accuracy.

Calculators

The TI-89 calculator in Fig. 14.61 is only one of numerous calculators that can convert from one form to another and perform lengthy

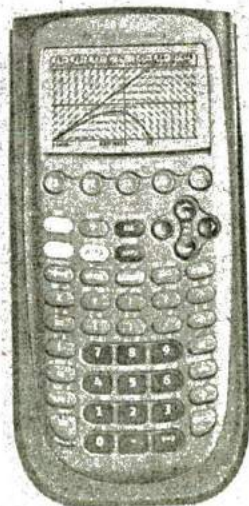


FIG. 14.61

TI-89 scientific calculator.
(Courtesy of Texas Instruments, Inc.)

calculations with complex numbers in a concise, neat form. Not all of the details of using a specific calculator are included here because each has its own format and sequence of steps. However, the basic operations with the TI-89 are included primarily to demonstrate the ease with which the conversions can be made and the format for more complex operations.

There are different routes to perform the conversions and operations below, but these instructions give you one approach that is fairly direct and straightforward. Since most operations are in the DEGREE rather than RADIAN mode, the sequence in Fig. 14.62 shows how to set the DEGREE mode for the operations to follow. A similar sequence sets the RADIAN mode if required.

MODE \downarrow Angle \rightarrow \downarrow DEGREE ENTER ENTER

FIG. 14.62

Setting the DEGREE mode on the TI-89 calculator.

Rectangular to Polar Conversion The sequence in Fig. 14.63 provides a detailed listing of the steps needed to convert from rectangular to polar form. In the examples to follow, the scrolling steps are not listed to simplify the sequence.

In the sequence in Fig. 14.63, an up scroll is chosen after Matrix because that is a more direct path to Vector ops. A down scroll generates the same result, but it requires going through the whole listing. The sequence seems quite long for such a simple conversion, but with practice you will be able to perform the scrolling steps quite rapidly. Always be sure the input data are entered correctly, such as including the i after the y component. Any incorrect entry will result in an error listing.

(3 + 5 2ND i) 2ND MATH \downarrow Matrix \rightarrow
 \uparrow Vector ops \rightarrow \downarrow \rightarrow Polar ENTER ENTER 5.83E0 \angle 59.0E0

FIG. 14.63

Converting $3 + j5$ to the polar form using the TI-89 calculator.

Polar to Rectangular Conversion The sequence in Fig. 14.64 is a detailed listing of the steps needed to convert from polar to rectangular form. Note in the format that the brackets must surround the polar form. Also, the degree sign must be included with the angle to perform the calculation. The answer is displayed in the engineering notation selected.

(5 2ND \angle 53 2ND \cdot 1 2ND MATH \downarrow
 Angle \rightarrow \rightarrow ENTER) 2ND MATH \downarrow Matrix \rightarrow \uparrow Vector ops \rightarrow \downarrow
 Rect: ENTER ENTER 3.00E0+4.00E0i

FIG. 14.64

Converting $5\angle 53.1^\circ$ to the rectangular form using the TI-89 calculator.

Mathematical Operations Mathematical operations are performed in the natural order of operations, but you must remember to select the format for the solution. For instance, if the sequence in Fig. 14.65 did not include the polar designation, the answer would be in rectangular

$$(10 \angle 50^\circ) \times (2 \angle 20^\circ)$$

► Polar **ENTER** 20.00E0 \angle 70.00E0

FIG. 14.65

Performing the operation $(10 \angle 50^\circ)(2 \angle 20^\circ)$.

form even though both quantities in the calculation are in polar form. In the rest of the examples, the scrolling required to obtain mathematical functions is not included to minimize the length of the sequence.

For the product of mixed complex numbers, the sequence of Fig. 14.66 results. Again, the polar form was selected for the solution.

$$(5 \angle 53.1^\circ) \times (2 + j2)$$

► Polar **ENTER** **ENTER** 14.14E0 \angle 98.10E0

FIG. 14.66

Performing the operation $(5 \angle 53.1^\circ)(2 + j2)$.

Finally, Example 14.26(c) is entered as shown by the sequence in Fig. 14.67. Note that the results exactly match those obtained earlier.

$$(2 \angle 20^\circ) \wedge 2 \times (3 + j4)$$

+ $(8 - j6)$ ► Polar **ENTER** **ENTER** 2.00E0 \angle 130.0E0

FIG. 14.67

Verifying the results of Example 14.26(c).

14.12 PHASORS

As noted earlier in this chapter, the addition of sinusoidal voltages and currents is frequently required in the analysis of ac circuits. One lengthy but valid method of performing this operation is to place both sinusoidal waveforms on the same set of axes and add algebraically the magnitudes of each at every point along the abscissa, as shown for $c = a + b$ in Fig. 14.68. This, however, can be a long and tedious process

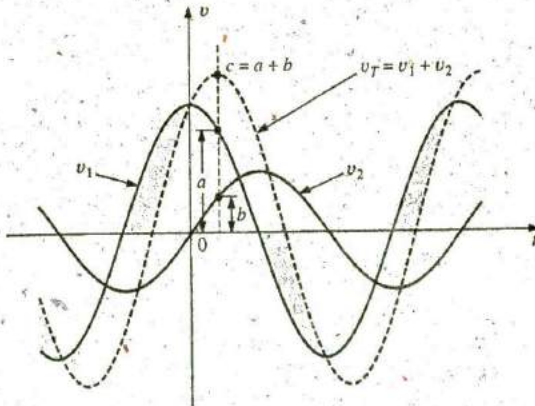


FIG. 14.68

Adding two sinusoidal waveforms on a point-by-point basis.

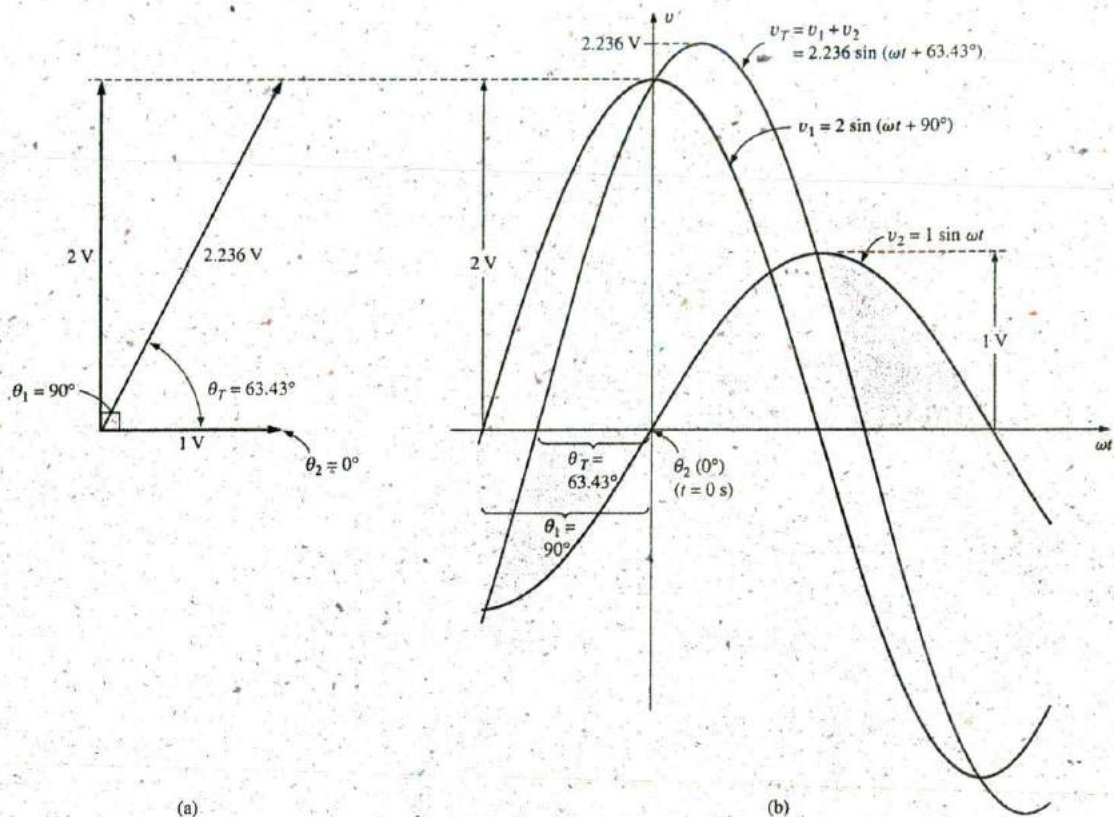


FIG. 14.69

(a) The phasor representation of the sinusoidal waveforms of part (b);
 (b) finding the sum of two sinusoidal waveforms of v_1 and v_2 .

with limited accuracy. A shorter method uses the rotating radius vector first appearing in Fig. 13.16. This *radius vector*, having a *constant magnitude* (length) with *one end fixed at the origin*, is called a **phasor** when applied to electric circuits. During its rotational development of the sine wave, the phasor will, at the instant $t = 0$, have the positions shown in Fig. 14.69(a) for each waveform in Fig. 14.69(b).

Note in Fig. 14.69(b) that v_2 passes through the horizontal axis at $t = 0$ s, requiring that the radius vector in Fig. 14.69(a) be on the horizontal axis to ensure a vertical projection of zero volts at $t = 0$ s. Its length in Fig. 14.69(a) is equal to the peak value of the sinusoid as required by the radius vector in Fig. 13.16. The other sinusoid has passed through 90° of its rotation by the time $t = 0$ s is reached and therefore has its maximum vertical projection as shown in Fig. 14.69(a). Since the vertical projection is a maximum, the peak value of the sinusoid that it generates is also attained at $t = 0$ s, as shown in Fig. 14.69(b). Note also that $v_T = v_1$ at $t = 0$ s since $v_2 = 0$ V at this instant.

It can be shown [see Fig. 14.69(a)] using the vector algebra described in Section 14.10 that

$$1 \text{ V} \angle 0^\circ + 2 \text{ V} \angle 90^\circ = 2.236 \text{ V} \angle 63.43^\circ$$

In other words, if we convert v_1 and v_2 to the phasor form using

$$v = V_m \sin(\omega t \pm \theta) \Rightarrow V_m \angle \pm \theta$$

and add them using complex number algebra, we can find the phasor form for v_T with very little difficulty. It can then be converted to the time domain and plotted on the same set of axes, as shown in Fig. 14.69(b). Fig. 14.69(a), showing the magnitudes and relative positions of the various phasors, is called a **phasor diagram**. It is actually a "snapshot" of the rotating radius vectors at $t = 0$ s.

Therefore, if the addition of two sinusoids is required, you should first convert them to the phasor domain and find the sum using complex algebra. You can then convert the result to the time domain.

The case of two sinusoidal functions having phase angles different from 0° and 90° appears in Fig. 14.70. Note again that the vertical height of the functions in Fig. 14.70(b) at $t = 0$ s is determined by the rotational positions of the radius vectors in Fig. 14.70(a).

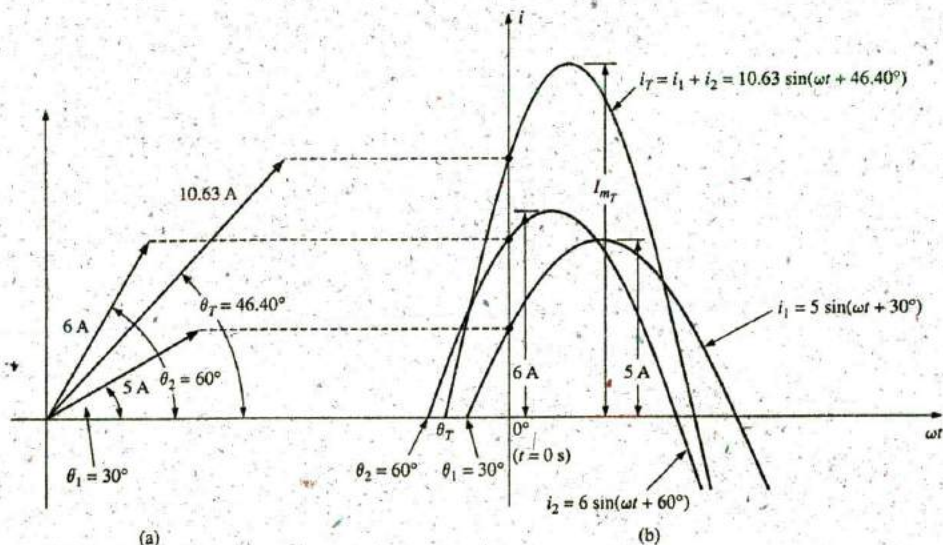


FIG. 14.70

Adding two sinusoidal currents with phase angles other than 90° .

Since the rms, rather than the peak, values are used almost exclusively in the analysis of ac circuits, the phasor will now be redefined for the purposes of practicality and uniformity as having a magnitude equal to the *rms value* of the sine wave it represents. The angle associated with the phasor will remain as previously described—the phase angle.

In general, for all of the analyses to follow, the phasor form of a sinusoidal voltage or current will be

$$V = V \angle \theta \quad \text{and} \quad I = I \angle \theta$$

where V and I are rms values and θ is the phase angle. It should be pointed out that in phasor notation, the sine wave is always the reference, and the frequency is not represented.

Phasor algebra for sinusoidal quantities is applicable only for waveforms having the same frequency.



FIG. 14.71

Charles Proteus Steinmetz,
Courtesy of the Hall of History
Foundation, Schenectady, New York

German-American (Breslau, Germany; Yonkers and
Schenectady, NY, USA)
(1865–1923)

Mathematician, Scientist, Engineer, Inventor,
Professor of Electrical Engineering and
Electrophysics, Union College
Department Head, General Electric Co.

Although the holder of some 200 patents and recognized worldwide for his contributions to the study of hysteresis losses and electrical transients, Charles Proteus Steinmetz is best recognized for his contribution to the study of ac networks. His "Symbolic Method of Alternating-current Calculations" provided an approach to the analysis of ac networks that removed a great deal of the confusion and frustration experienced by engineers of that day as they made the transition from dc to ac systems. His approach (on which the phasor notation of this text is premised) permitted a direct analysis of ac systems using many of the theorems and methods of analysis developed for dc systems. In 1897 he authored the epic work *Theory and Calculation of Alternating Current Phenomena*, which became the authoritative guide for practicing engineers. Dr. Steinmetz was fondly referred to as "The Doctor" at General Electric Company where he worked for some 30 years in a number of important capacities. His recognition as a multi-talented genius is supported by the fact that he maintained active friendships with such individuals as Albert Einstein, Guglielmo Marconi, and Thomas A. Edison, to name just a few. He was President of the American Institute of Electrical Engineers (AIEE) and the National Association of Corporation Schools and actively supported his local community (Schenectady) as president of the Board of Education and the Commission on Parks and City Planning.

The use of phasor notation in the analysis of ac networks was first introduced by Charles Proteus Steinmetz in 1897 (Fig. 14.71).

EXAMPLE 14.27 Convert the following from the time to the phasor domain:

Time Domain	Phasor Domain
a. $\sqrt{2}(50) \sin \omega t$	$50 \angle 0^\circ$
b. $69.6 \sin(\omega t + 72^\circ)$	$(0.707)(69.6) \angle 72^\circ = 49.21 \angle 72^\circ$
c. $45 \cos \omega t$	$(0.707)(45) \angle 90^\circ = 31.82 \angle 90^\circ$

EXAMPLE 14.28 Write the sinusoidal expression for the following phasors if the frequency is 60 Hz:

Phasor Domain	Time Domain
a. $I = 10 \angle 30^\circ$	$i = \sqrt{2}(10) \sin(2\pi 60t + 30^\circ)$ and $i = 14.14 \sin(377t + 30^\circ)$
b. $V = 115 \angle -70^\circ$	$v = \sqrt{2}(115) \sin(377t - 70^\circ)$ and $v = 162.6 \sin(377t - 70^\circ)$

EXAMPLE 14.29 Find the input voltage of the circuit in Fig. 14.72 if

$$\left. \begin{aligned} v_a &= 50 \sin(377t + 30^\circ) \\ v_b &= 30 \sin(377t + 60^\circ) \end{aligned} \right\} f = 60 \text{ Hz}$$

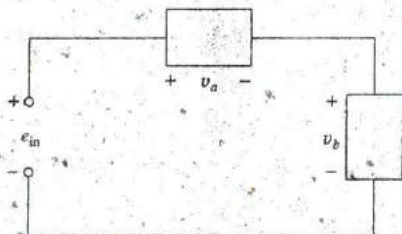


FIG. 14.72

Example 14.29.

Solution: Applying Kirchhoff's voltage law, we have

$$e_{in} = v_a + v_b$$

Converting from the time to the phasor domain yields

$$v_a = 50 \sin(377t + 30^\circ) \Rightarrow V_a = 35.35 \text{ V } \angle 30^\circ$$

$$v_b = 30 \sin(377t + 60^\circ) \Rightarrow V_b = 21.21 \text{ V } \angle 60^\circ$$

Converting from polar to rectangular form for addition yields

$$V_a = 35.35 \text{ V } \angle 30^\circ = 30.61 \text{ V} + j 17.68 \text{ V}$$

$$V_b = 21.21 \text{ V } \angle 60^\circ = 10.61 \text{ V} + j 18.37 \text{ V}$$

Then

$$\begin{aligned} \mathbf{E}_{in} &= \mathbf{V}_a + \mathbf{V}_b = (30.61 \text{ V} + j 17.68 \text{ V}) + (10.61 \text{ V} + j 18.37 \text{ V}) \\ &= 41.22 \text{ V} + j 36.05 \text{ V} \end{aligned}$$

Converting from rectangular to polar form, we have

$$\mathbf{E}_{in} = 41.22 \text{ V} + j 36.05 \text{ V} = 54.76 \text{ V} \angle 41.17^\circ$$

Converting from the phasor to the time domain, we obtain

$$\mathbf{E}_{in} = 54.76 \text{ V} \angle 41.17^\circ \Rightarrow e_{in} = \sqrt{2}(54.76)\sin(377t + 41.17^\circ)$$

and
$$e_{in} = 77.43 \sin(377t + 41.17^\circ)$$

A plot of the three waveforms is shown in Fig. 14.73. Note that at each instant of time, the sum of the two waveforms does in fact add up to e_{in} . At $t = 0$ ($\omega t = 0$), e_{in} is the sum of the two positive values, while at a value of ωt , almost midway between $\pi/2$ and π , the sum of the positive value of v_a and the negative value of v_b results in $e_{in} = 0$.

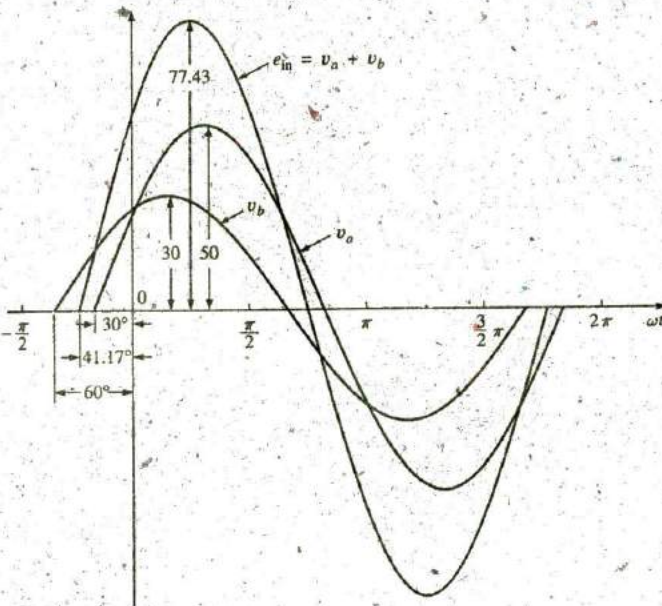


FIG. 14.73
Solution to Example 14.29.

EXAMPLE 14.30 Determine the current i_2 for the network in Fig. 14.74.

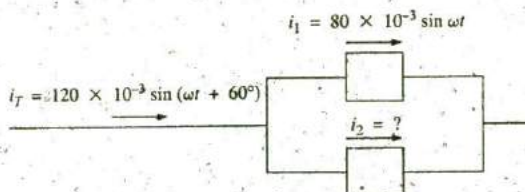


FIG. 14.74
Example 14.30.

Solution: Applying Kirchoff's current law, we obtain

$$i_T = i_1 + i_2 \quad \text{or} \quad i_2 = i_T - i_1$$

Converting from the time to the phasor domain yields

$$i_T = 120 \times 10^{-3} \sin(\omega t + 60^\circ) \Rightarrow 84.84 \text{ mA} \angle 60^\circ$$

$$i_1 = 80 \times 10^{-3} \sin \omega t \Rightarrow 56.56 \text{ mA} \angle 0^\circ$$

Converting from polar to rectangular form for subtraction yields

$$\mathbf{I}_T = 84.84 \text{ mA} \angle 60^\circ = 42.42 \text{ mA} + j 73.47 \text{ mA}$$

$$\mathbf{I}_1 = 56.56 \text{ mA} \angle 0^\circ = 56.56 \text{ mA} + j 0$$

Then

$$\begin{aligned} \mathbf{I}_2 &= \mathbf{I}_T - \mathbf{I}_1 \\ &= (42.42 \text{ mA} + j 73.47 \text{ mA}) - (56.56 \text{ mA} + j 0) \end{aligned}$$

and $\mathbf{I}_2 = -14.14 \text{ mA} + j 73.47 \text{ mA}$

Converting from rectangular to polar form, we have

$$\mathbf{I}_2 = 74.82 \text{ mA} \angle 100.89^\circ$$

Converting from the phasor to the time domain, we have

$$\mathbf{I}_2 = 74.82 \text{ mA} \angle 100.89^\circ \Rightarrow$$

$$i_2 = \sqrt{2}(74.82 \times 10^{-3}) \sin(\omega t + 100.89^\circ)$$

and $i_2 = 105.8 \times 10^{-3} \sin(\omega t + 100.89^\circ)$

A plot of the three waveforms appears in Fig. 14.75. The waveforms clearly indicate that $i_T = i_1 + i_2$.

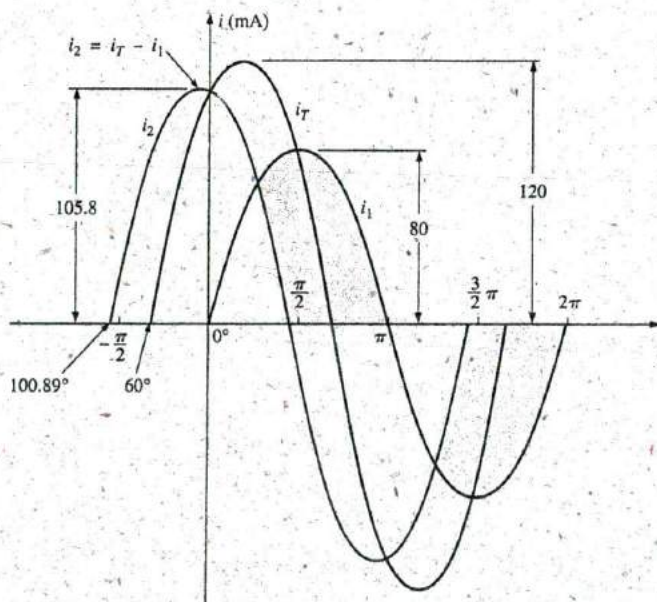


FIG. 14.75

Solution to Example 14.30.

14.13 COMPUTER ANALYSIS

PSpice

Capacitors and the ac Response The simplest of ac capacitive circuits, is now analyzed to introduce the process of setting up an ac source and running an ac transient simulation. The ac source in Fig. 14.76 is obtained through **Place part** key-SOURCE-VSIN-OK. Change the name or value of any parameter by double-clicking on the parameter on the display or by double-clicking on the source symbol to get the **Property Editor** dialog box. Within the dialog box, set the values appearing in Fig. 14.76. This is done by scrolling across the dialog box and selecting the desired quantity from the top listing. When selected, a black box will appear under the quantity of interest. Click on the black box, and it will turn white. Enter the value followed by **Display-Name and Value** if you want the quantity and its value to appear on the screen. If you do not want it to appear on the screen, do not use the **Display** option. Once each quantity is set, the most important step of all must be applied, which is to select the **Apply** key. If you forget to apply the changes, none will be used in the analysis.

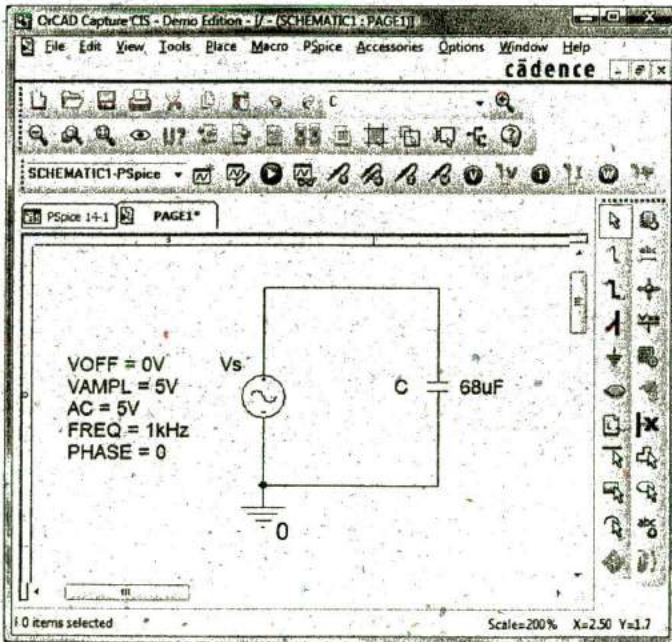


FIG. 14.76

Using PSpice to analyze the response of a capacitor to a sinusoidal ac signal.

The simulation process is initiated by selecting the **New Simulation Profile**. Under **New Simulation**, enter **PSpice 14-1** for the **Name** followed by **Create**. In the **Simulation Settings** dialog box, select **Analysis** and choose **Time Domain(Transient)** under **Analysis type**. Set the **Run to time** at 3 ms to permit a display of three cycles of the sinusoidal waveforms ($T = 1/f = 1/1000 \text{ Hz} = 1 \text{ ms}$). Leave the **Start saving data after** at 0 s, and set the **Maximum step size** at $3 \text{ ms}/1000 = 3 \mu\text{s}$. Clicking **OK** and then selecting the **Run PSpice** icon results in a plot having a horizontal axis that extends from 0 to 3 ms.

Now you must tell the computer which waveforms you are interested in. First, take a look at the applied ac source by selecting **Trace-Add Trace-V(Vs: +)** followed by **OK**. The result is the sweeping ac voltage in the bottom region of the screen in Fig. 14.77. Note that it has a peak value of 5 V, and three cycles appear in the 3 ms time frame. The current for the capacitor can be added by selecting **Trace-Add Trace** and choosing **I(C)** followed by **OK**. The resulting waveform for **I(C)** appears at a 90° phase shift from the applied voltage, with the current leading the voltage (the current has already peaked as the voltage crosses the 0 V axis). Since the peak value of each plot is in the same magnitude range, the 5 appearing on the vertical scale can be used for both. A theoretical analysis results in $X_C = 2.34\Omega$, and the peak value of $I_C = E/X_C = 5\text{ V}/2.34 = 2.136\text{ A}$, as shown in Fig. 14.77.

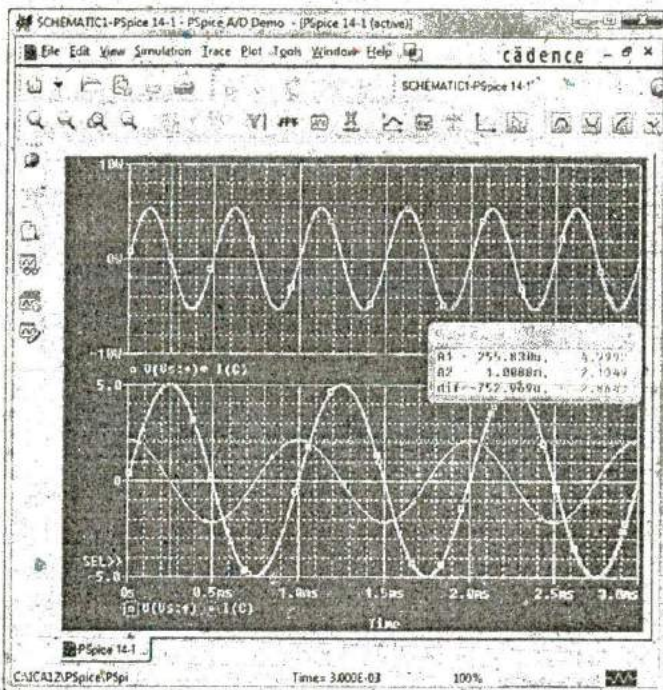


FIG. 14.77

A plot of the voltage, current, and power for the capacitor in Fig. 14.76.

For practice, let us obtain the curve for the power delivered to the capacitor over the same time period. First select **Plot-Add Plot to Window-Trace-Add Trace** to obtain the **Add Traces** dialog box. Then choose **V(Vs: +)**, follow it with a * for multiplication obtained from the **Function** listing on the right side of the **Add Traces** dialog box, and finish by selecting **I(C)**. The result is the expression **V(Vs: +)*I(C)** of the power format: $p = vi$. Click **OK**, and the power plot at the top of Fig. 14.77 appears. Note that over the full three cycles, the area above the axis equals the area below—there is no net transfer of power over the 3 ms period. Note also that the power curve is sinusoidal (which is quite interesting) with a frequency twice that of the applied signal. Using the cursor control, we can determine that the maximum power (peak value of the sinusoidal waveform) is 5.34 W.

The cursors, in fact, have been added to the lower curves to show the peak value of the applied sinusoid and the resulting current.

After selecting the **Toggle cursor** icon, left-click to surround the symbol to the left of **V(Vs: +)** at the bottom of the plot with a dashed line to show that the cursor is providing the levels of that quantity. When placed at $\frac{1}{4}$ of the total period of $250 \mu\text{s}$ (**A1**), the peak value is exactly 5 V as shown in the **Probe Cursor** dialog box. Placing the cursor over the symbol next to **I(C)** at the bottom of the plot and right-clicking assigns the right cursor to the current. Placing it at exactly 1 ms (**A2**) results in a peak value of 2.136 A to match the solution above. To further distinguish between the voltage and current waveforms, the color and the width of the lines of the traces were changed. Place the cursor right on the plot line and right-click. The **Properties** option appears. When **Properties** is selected, a **Trace Properties** dialog box appears in which the yellow color can be selected and the width widened to improve the visibility on the black background. Note that yellow was chosen for **Vs** and green for **I(C)**. Note also that the axis and the grid have been changed to a more visible color using the same procedure.

Multisim

Since PSpice reviewed the response of a capacitive element to an ac voltage, Multisim repeats the analysis for an inductive element. The ac voltage source was derived from the **Place Source** parts bin as described in Chapter 13 with the values appearing in Fig. 14.78 set in the **AC-Voltage** dialog box.

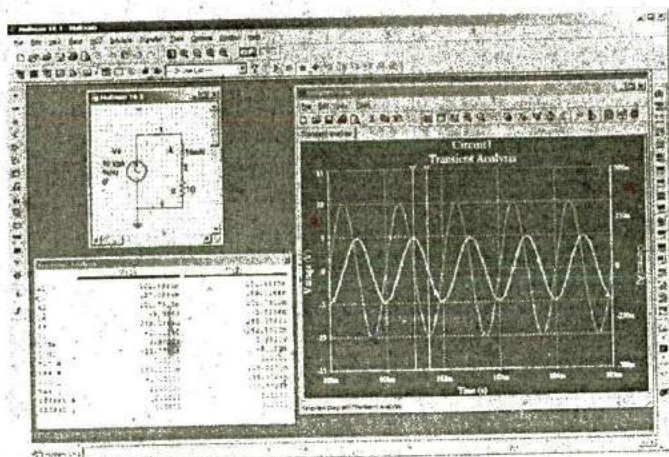


FIG. 14.78

Using Multisim to review the response of an inductive element to a sinusoidal ac signal.

Once the circuit has been constructed, the sequence **Simulate-Analyses-Transient Analysis** results in a **Transient Analysis** dialog box in which the **Start time** is set at 0 s and the **End time** at 105 ms. The 105 ms was set as the **End time** to give the network 100 ms to settle down in its steady-state mode and 5 ms for five cycles in the output display. The **Minimum number of time points** was set at 10,000 to ensure a good display for the rapidly changing waveforms.

Next the **Output** heading was chosen within the dialog box, and the source voltage $V(1)$ and source current $I(VS)$ were moved from the **Variables in Circuit** to **Selected variables for analysis** using the **Add** option. Choosing **Simulate** results in a waveform that extends from 0 s to 105 ms. Even though we plan to save only the response that occurs after 100 ms, the computer is unaware of our interest, and it plots the response for the entire period. This is corrected by selecting the **Properties** keypad in the toolbar at the top of the graph (it looks like a tag and pencil) to obtain the **Graph Properties** dialog box. Selecting **Bottom Axis** permits setting the **Range** from a **Minimum of 0.100 s = 100 ms** to a **Maximum of 0.105 s = 105 ms**. Click **OK**, and the time period of Fig. 14.78 is displayed. The grid structure is added by selecting the **Show Grid** keypad, and the color associated with each curve is displayed if we choose the **Show Legend** key next to it.

It is clear from the plot that the scale for the source current has to be improved for us to be able to clearly read its peak and negative values. This is done by first clicking on the $I(VS)$ curve to set the **Selected Trace** at the bottom of the graph as $I(VS)$. A right click, and one can choose the **Trace Properties** option to obtain the **Graph Properties** dialog box. Under **Y-Vertical Axis**, select **Right Axis** to establish the right axis as the scale to be used for the source current. Then select **Right Axis** and insert the **Label: Current(A)**, select **Enabled** under the **Axis** heading, and finally choose **Pen Size** as 1. The **Scale** is **Linear** and of range $-500E-3$ to $500E-3$ (-500 mA to 500 mA), with **Total Ticks** of 8 and **Minor Ticks** of 2. The result is the plot of Fig. 14.78. The right axis can now be improved by selecting **Graph Properties** again, followed by **Left Axis**, whereby the **Current(A)** can be deleted. We can now see that the source current has a peak value of about 160 mA. For more detail on the waveforms, select the **Show Cursors** key pad to obtain the **Transient Analysis** dialog box with box $V(1)$ and $I(VS)$ listed with the same color headings as used on the graph. Clicking on one of the cursors and moving it horizontally to the maximum value of the current will result in $x1 = 101.0$ ms with $y1$ at 158.91 mA. Actually, the **max y** appears below at 159.07 mA, which could have been obtained if we had increased the number of data points. Moving the other cursor to find the minimum value of current will result in $x2 = 101.24$ ms with $y2$ at 2.51 mA (the closest to the value of 1 obtainable with this data level setting). The maximum value of $V(1)$ appears below as 10 V, which it should be, and the distance between the maximum value of $I(VS)$ and its minimum value is $dx = 242.91 \mu\text{s}$ which is very close to 0.25 ms, or one fourth of the period of the applied signal.

PROBLEMS

SECTION 14.2 Derivative

- Plot the following waveform versus time showing one clear, complete cycle. Then determine the derivative of the waveform using Eq. (14.1), and sketch one complete cycle of the derivative directly under the original waveform. Compare the magnitude of the derivative at various points versus the slope of the original sinusoidal function.

$$v = 1 \sin 6.28t$$

- Repeat Problem 1 for the following sinusoidal function, and compare results. In particular, determine the frequency of the waveforms of Problems 1 and 2, and compare the magnitude of the derivative.

$$v = 1 \sin 314.2t$$

- What is the derivative of each of the following sinusoidal expressions?
 - $10 \sin 377t$
 - $0.6 \sin(200t + 20^\circ)$
 - $\sqrt{2} 20 \sin(157t - 20^\circ)$
 - $-200 \sin(t + 180^\circ)$

SECTION 14.3 Response of Basic R , L , and C Elements to a Sinusoidal Voltage or Current

- The voltage across a $3\ \Omega$ resistor is as indicated. Find the sinusoidal expression for the current. In addition, sketch the v and i sinusoidal waveforms on the same axis.
 - $150 \sin 200t$
 - $30 \sin(377t + 20^\circ)$
 - $6 \cos(\omega t + 10^\circ)$
 - $-12 \sin(\omega t + 40^\circ)$
- The current through a $7\ \text{k}\Omega$ resistor is as indicated. Find the sinusoidal expression for the voltage. In addition, sketch the v and i sinusoidal waveforms on the same axis.
 - $0.1 \sin 1000t$
 - $2 \times 10^{-3} \sin(400t - 120^\circ)$
- Determine the inductive reactance (in ohms) of a $2\ \text{mH}$ coil for
 - dc
 and for the following frequencies:
 - $60\ \text{Hz}$
 - $4\ \text{kHz}$
 - $1.2\ \text{MHz}$
- Determine the closest standard value inductance that has a reactance of
 - $2\ \text{k}\Omega$ at $f = 14.47\ \text{kHz}$.
 - $40\ \text{k}\Omega$ at $f = 5.3\ \text{kHz}$.
- Determine the frequency at which a $1\ \text{mH}$ inductance has the following inductive reactances:
 - $10\ \Omega$
 - $4\ \text{k}\Omega$
 - $12\ \text{k}\Omega$
- The current through a $20\ \Omega$ inductive reactance is given. What is the sinusoidal expression for the voltage? Sketch the v and i sinusoidal waveforms on the same axis.
 - $i = 5 \sin \omega t$
 - $i = 40 \times 10^{-3} \sin(\omega t + 60^\circ)$
 - $i = -6 \sin(\omega t - 30^\circ)$
- The current through a $0.1\ \text{H}$ coil is given. What is the sinusoidal expression for the voltage?
 - $10 \sin 100t$
 - $5 \times 10^{-6} \sin(400t + 20^\circ)$
- The voltage across a $50\ \Omega$ inductive reactance is given. What is the sinusoidal expression for the current? Sketch the v and i sinusoidal waveforms on the same set of axes.
 - $120 \sin \omega t$
 - $30 \sin(\omega t + 20^\circ)$
- The voltage across a $0.2\ \text{H}$ coil is given. What is the sinusoidal expression for the current?
 - $1.5 \sin 60t$
 - $16 \times 10^{-3} \sin(10t + 2^\circ)$
- Determine the capacitive reactance (in ohms) of a $5\ \mu\text{F}$ capacitor for
 - dc
 and for the following frequencies:
 - $60\ \text{Hz}$
 - $2\ \text{kHz}$
 - $2\ \text{MHz}$
- Determine the closest standard value capacitance that has a reactance of
 - $60\ \Omega$ at $f = 265\ \text{Hz}$.
 - $1.2\ \text{k}\Omega$ at $34\ \text{kHz}$.
- Determine the frequency at which a $3.9\ \mu\text{F}$ capacitor has the following capacitive reactances:
 - $10\ \Omega$
 - $1.2\ \text{k}\Omega$
 - $0.1\ \Omega$
 - $2000\ \Omega$
- The voltage across a $2.5\ \Omega$ capacitive reactance is given. What is the sinusoidal expression for the current? Sketch the v and i sinusoidal waveforms on the same set of axes.
 - $120 \sin \omega t$
 - $4 \times 10^{-3} \sin(\omega t + 40^\circ)$
- The voltage across a $1\ \mu\text{F}$ capacitor is given. What is the sinusoidal expression for the current?
 - $30 \sin 200t$
 - $60 \times 10^{-3} \sin 377t$
- The current through a $10\ \Omega$ capacitive reactance is given. Write the sinusoidal expression for the voltage. Sketch the v and i sinusoidal waveforms on the same set of axes.
 - $i = 50 \times 10^{-3} \sin \omega t$
 - $i = 2 \times 10^{-6} \sin(\omega t + 60^\circ)$
- The current through a $0.56\ \mu\text{F}$ capacitor is given. What is the sinusoidal expression for the voltage?
 - $0.20 \sin 300t$
 - $8 \times 10^{-3} \sin(377t - 30^\circ)$
- For the following pairs of voltages and currents, indicate whether the element involved is a capacitor, an inductor, or a resistor, and find the value of C , L , or R if sufficient data are given:
 - $v = 550 \sin(377t + 50^\circ)$
 $i = 11 \sin(377t - 40^\circ)$
 - $v = 36 \sin(754t + 80^\circ)$
 $i = 4 \sin(754t - 170^\circ)$
 - $v = 10.5 \sin(\omega t - 13^\circ)$
 $i = 1.5 \sin(\omega t - 13^\circ)$
- Repeat Problem 20 for the following pairs of voltages and currents:
 - $v = 2000 \sin \omega t$
 $i = 5 \cos \omega t$
 - $v = 80 \sin(157t + 150^\circ)$
 $i = 2 \sin(157t + 60^\circ)$
 - $v = 35 \sin(\omega t - 20^\circ)$
 $i = 7 \cos(\omega t - 110^\circ)$

SECTION 14.4 Frequency Response of the Basic Elements

- Plot X_L versus frequency for a $3\ \text{mH}$ coil using a frequency range of zero to $100\ \text{kHz}$ on a linear scale.
- Plot X_C versus frequency for a $1\ \mu\text{F}$ capacitor using a frequency range of zero to $10\ \text{kHz}$ on a linear scale.
- At what frequency will the reactance of a $1\ \mu\text{F}$ capacitor equal the resistance of a $2\ \text{k}\Omega$ resistor?
- The reactance of a coil equals the resistance of a $10\ \text{k}\Omega$ resistor at a frequency of $5\ \text{kHz}$. Determine the inductance of the coil.



26. Determine the frequency at which a $1 \mu\text{F}$ capacitor and a 10 mH inductor will have the same reactance.
27. Determine the capacitance required to establish a capacitive reactance that will match that of a 2 mH coil at a frequency of 50 kHz .

SECTION 14.5 Average Power and Power Factor

- *28. Find the average power loss and power factor for each of the circuits whose input current and voltage are as follows:
- $v = 60 \sin(\omega t + 30^\circ)$
 $i = 15 \sin(\omega t + 60^\circ)$
 - $v = -50 \sin(\omega t - 20^\circ)$
 $i = -2 \sin(\omega t - 20^\circ)$
 - $v = 50 \sin(\omega t + 80^\circ)$
 $i = 3 \cos(\omega t - 20^\circ)$
 - $v = 75 \sin(\omega t - 5^\circ)$
 $i = 0.08 \sin(\omega t + 35^\circ)$
29. If the current through and voltage across an element are $i = 8 \sin(\omega t + 40^\circ)$ and $v = 48 \sin(\omega t + 40^\circ)$, respectively, compute the power by $I^2 R$, $(V_m I_m / 2) \cos \theta$, and $VI \cos \theta$, and compare answers.
30. A circuit dissipates 100 W (average power) at 150 V (effective input voltage) and 2 A (effective input current). What is the power factor? Repeat if the power is 0 W ; 300 W .
- *31. The power factor of a circuit is 0.5 lagging. The power delivered in watts is 500 . If the input voltage is $50 \sin(\omega t + 10^\circ)$, find the sinusoidal expression for the input current.
32. In Fig. 14.79, $e = 34 \sin(2\pi 60t + 20^\circ)$.
- What is the sinusoidal expression for the current?
 - Find the power loss in the circuit.
 - How long (in seconds) does it take the current to complete six cycles?

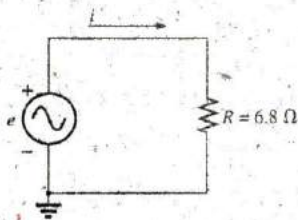


FIG. 14.79
Problem 32.

33. In Fig. 14.80, $e = 128 \sin(1000t + 60^\circ)$.
- Find the sinusoidal expression for i .

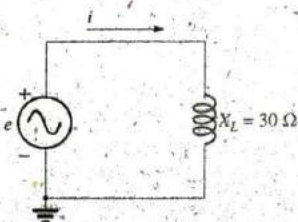


FIG. 14.80
Problem 33.

- Find the value of the inductance L . What is its probable standard value in mH ?
 - Find the average power loss by the inductor.
34. In Fig. 14.81, $i = 30 \times 10^{-3} \sin(2\pi 500t - 20^\circ)$.
- Find the sinusoidal expression for e .
 - Find the value of the capacitance C in microfarads. What is its probable standard value in μF ?
 - Find the average power loss in the capacitor.

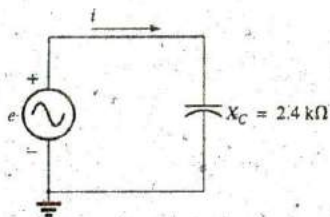


FIG. 14.81
Problem 34.

- *35. For the network in Fig. 14.82 and the applied signal:
- Determine the sinusoidal expressions for i_1 and i_2 .
 - Find the sinusoidal expression for i_s by combining the two parallel capacitors.

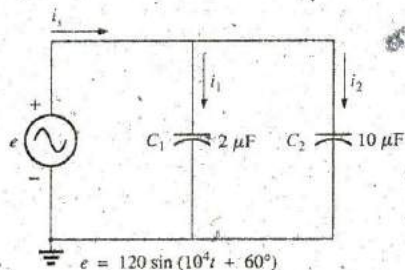


FIG. 14.82
Problem 35.

- *36. For the network in Fig. 14.83 and the applied source:
- Determine the sinusoidal expression for the source voltage v_s .
 - Find the sinusoidal expression for the currents i_1 and i_2 .

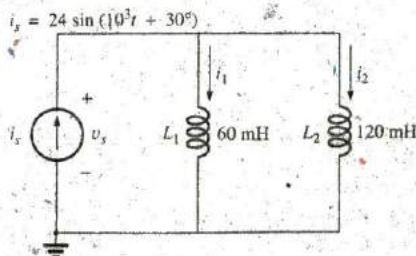


FIG. 14.83
Problem 36.

SECTION 14.9 Conversion between Forms

37. Convert the following from rectangular to polar form:

- a. $4 + j3$
- b. $2 + j2$
- c. $4 + j12$
- d. $1000 + j50$
- e. $-1000 + j4000$
- f. $-0.4 + j0.8$

*38. Convert the following from rectangular to polar form:

- a. $-8 - j16$
- b. $+8 - j4$
- c. $0.02 - j0.003$
- d. $-6 \times 10^{-3} - j6 \times 10^{-3}$
- e. $200 + j0.02$
- f. $-1000 + j20$

39. Convert the following from polar to rectangular form:

- a. $6 \angle 40^\circ$
- b. $12 \angle 120^\circ$
- c. $2000 \angle -90^\circ$
- d. $0.0064 \angle +200^\circ$
- e. $48 \angle 2^\circ$
- f. $5 \times 10^{-4} \angle -20^\circ$

40. Convert the following from polar to rectangular form:

- a. $42 \angle 0.15^\circ$
- b. $2002 \angle -60^\circ$
- c. $0.006 \angle -120^\circ$
- d. $8 \times 10^{-3} \angle -220^\circ$
- e. $15 \angle +180^\circ$
- f. $1.2 \angle -89.9^\circ$

SECTION 14.10 Mathematical Operations with Complex Numbers

41. Perform the following additions in rectangular form:

- a. $(4.2 + j6.8) + (7.6 + j0.2)$
- b. $(142 + j7) + (9.8 + j42) + (0.1 + j0.9)$
- c. $(4 \times 10^{-6} + j76) + (7.2 \times 10^{-7} - j5)$

42. Perform the following subtractions in rectangular form:

- a. $(9.8 + j6.2) - (4.6 + j4.6)$
- b. $(167 + j243) - (-42.3 - j68)$
- c. $(-36.0 + j78) - (-4 - j6) + (10.8 - j72)$

43. Perform the following operations with polar numbers, and leave the answer in polar form:

- a. $6 \angle 20^\circ + 8 \angle 80^\circ$
- b. $42 \angle 45^\circ + 62 \angle 60^\circ - 70 \angle 120^\circ$
- c. $20 \angle -120^\circ - 40 \angle -150^\circ + 8 \angle -210^\circ + 8 \angle +240^\circ$

44. Perform the following multiplications in rectangular form:

- a. $(2 + j3)(6 + j8)$
- b. $(7.8 + j1)(4 + j2)(7 + j6)$
- c. $(400 - j200)(-0.01 - j0.5)(-1 + j3)$

45. Perform the following multiplications in polar form:

- a. $(2 \angle 60^\circ)(4 \angle -40^\circ)$
- b. $(6.9 \angle 8^\circ)(7.2 \angle -72^\circ)$
- c. $(0.002 \angle 120^\circ)(0.5 \angle 200^\circ)(40 \angle +80^\circ)$

46. Perform the following divisions in polar form:

- a. $(42 \angle 10^\circ)/(7 \angle 60^\circ)$
- b. $(0.006 \angle 120^\circ)/(30 \angle +60^\circ)$
- c. $(4360 \angle -20^\circ)/(40 \angle -210^\circ)$

47. Perform the following divisions, and leave the answer in rectangular form:

- a. $(8 + j8)/(2 + j2)$
- b. $(8 + j42)/(-6 - j4)$
- c. $(-4.5 - j6)/(0.1 - j0.8)$

*48. Perform the following operations, and express your answer in rectangular form:

- a. $\frac{(4 + j3) + (6 - j8)}{(3 + j3) - (2 + j3)}$
- b. $\frac{8 \angle 60^\circ}{\frac{(2 \angle 0^\circ) + (100 + j400)}{(6 \angle 20^\circ)(120 \angle -40^\circ)(3 + j8)}}$
- c. $\frac{2 \angle -30^\circ}{2 \angle -30^\circ}$

*49. Perform the following operations, and express your answer in polar form:

- a. $\frac{(0.4 \angle 60^\circ)^2(300 \angle 40^\circ)}{3 + j9}$
- b. $\left(\frac{1}{(0.02 \angle 10^\circ)^2}\right) \left(\frac{2}{j}\right) \left(\frac{1}{6^2 - j\sqrt{900}}\right)$

*50. a. Determine a solution for x and y if

$$(x + jy) + (3x + jy) - j7 = 16 \angle 0^\circ$$

b. Determine x if

$$(10 \angle 20^\circ)(x \angle -60^\circ) = 30.64 - j25.72$$

*51. a. Determine a solution for x and y if

$$(5x + j10)(2 - jy) = 90 - j70$$

b. Determine θ if

$$\frac{80 \angle 0^\circ}{20 \angle \theta} = 3.464 - j2$$

SECTION 14.12 Phasors

52. Express the following in phasor form:

- a. $\sqrt{2}(160)\sin(\omega t + 30^\circ)$
- b. $\sqrt{2}(25 \times 10^{-3})\sin(157t - 40^\circ)$
- c. $100 \sin(\omega t - 90^\circ)$

*53. Express the following in phasor form:

- a. $20 \sin(377t - 180^\circ)$
- b. $6 \times 10^{-6} \cos \omega t$
- c. $3.6 \times 10^{-6} \cos(754t - 20^\circ)$

54. Express the following phasor currents and voltages as sine waves if the frequency is 60 Hz:

- a. $I = 40 \text{ A } \angle 20^\circ$
- b. $V = 120 \text{ V } \angle 10^\circ$
- c. $I = 8 \times 10^{-3} \text{ A } \angle -110^\circ$
- d. $V = \frac{6000}{\sqrt{2}} \text{ V } \angle -180^\circ$

55. For the system in Fig. 14.84, find the sinusoidal expression for the unknown voltage v_a if

$$e_{in} = 60 \sin(377t + 45^\circ)$$

$$v_b = 20 \sin(377t - 45^\circ)$$

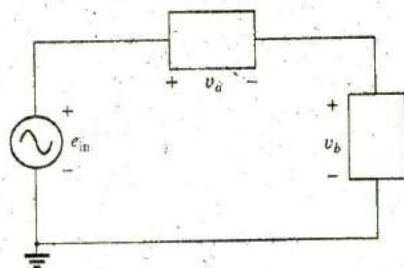


FIG. 14.84
Problem 55.

56. For the system in Fig. 14.85, find the sinusoidal expression for the unknown current i_1 if

$$i_s = 20 \times 10^{-6} \sin(\omega t + 60^\circ)$$

$$i_2 = 6 \times 10^{-6} \sin(\omega t - 30^\circ)$$

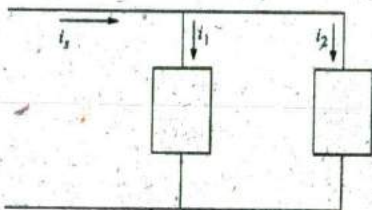


FIG. 14.85
Problem 56.

57. Find the sinusoidal expression for the voltage v_a for the system in Fig. 14.86 if

$$e_{in} = 120 \sin(\omega t + 30^\circ)$$

$$v_b = 30 \sin(\omega t + 60^\circ)$$

$$v_c = 40 \sin(\omega t + 120^\circ)$$

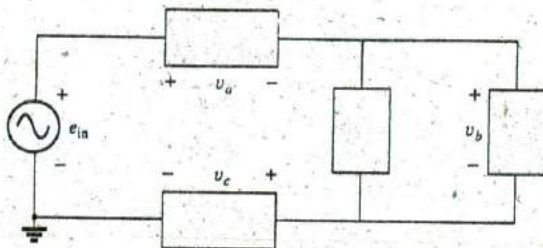


FIG. 14.86
Problem 57.

- *58. Find the sinusoidal expression for the current i_1 for the system in Fig. 14.87 if

$$i_s = 18 \times 10^{-3} \sin(377t + 180^\circ)$$

$$i_2 = 8 \times 10^{-3} \sin(377t - 180^\circ)$$

$$i_3 = 2i_2$$

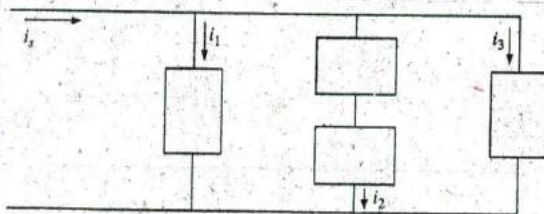


FIG. 14.87
Problem 58.

SECTION 14.13 Computer Analysis

PSpice or Multisim

59. Plot i_c and v_c versus time for the network in Fig. 14.76 for two cycles if the frequency is 0.2 kHz.
60. Plot the magnitude and phase angle of the current i_c versus frequency (100 Hz to 100 kHz) for the network in Fig. 14.76.
- *61. Plot the total impedance of the configuration in Fig. 14.27(a) versus frequency (100 kHz to 100 MHz) for the following parameter values: $C = 0.1 \mu\text{F}$, $L_s = 0.2 \mu\text{H}$, $R_s = 2 \text{ M}\Omega$, and $R_p = 100 \text{ M}\Omega$. For what frequency range is the capacitor "capacitive"?

GLOSSARY

Average or real power The power delivered to and dissipated by the load over a full cycle.

Complex conjugate A complex number defined by simply changing the sign of an imaginary component of a complex number in the rectangular form.

Complex number A number that represents a point in a two-dimensional plane located with reference to two distinct axes. It defines a vector drawn from the origin to that point.

Derivative The instantaneous rate of change of a function with respect to time or another variable.

Leading and lagging power factors An indication of whether a network is primarily capacitive or inductive in nature. Leading power factors are associated with capacitive networks and lagging power factors with inductive networks.

Phasor A radius vector that has a constant magnitude at a fixed angle from the positive real axis and that represents a sinusoidal voltage or current in the vector domain.

Phasor diagram A "snapshot" of the phasors that represent a number of sinusoidal waveforms at $t = 0$.

Polar form A method of defining a point in a complex plane that includes a single magnitude to represent the distance from the origin and an angle to reflect the counterclockwise distance from the positive real axis.

Power factor (F_p) An indication of how reactive or resistive an electrical system is. The higher the power factor, the greater is the resistive component.

Reactance The opposition of an inductor or a capacitor to the flow of charge that results in the continual exchange of energy between the circuit and magnetic field of an inductor or the electric field of a capacitor.

Reciprocal A format defined by 1 divided by the complex number.

Rectangular form A method of defining a point in a complex plane that includes the magnitude of the real component and the magnitude of the imaginary component, the latter component being defined by an associated letter j .

SERIES AND PARALLEL AC CIRCUITS

OBJECTIVES

- Become familiar with the characteristics of series and parallel ac networks and be able to find current, voltage, and power levels for each element.
- Be able to find the total impedance of any series or parallel ac network and sketch the impedance and admittance diagram of each.
- Develop confidence in applying Kirchhoff's current and voltage laws to any series or parallel configuration.
- Be able to apply the voltage divider rule or current divider rule to any ac network.
- Become adept at finding the frequency response of a series or parallel combination of elements.

15.1 INTRODUCTION

In this chapter, phasor algebra is used to develop a quick, direct method for solving both series and parallel ac circuits. The close relationship that exists between this method for solving for unknown quantities and the approach used for dc circuits will become apparent after a few simple examples are considered. Once this association is established, many of the rules (current divider rule, voltage divider rule, and so on) for dc circuits can be readily applied to ac circuits.

SERIES ac CIRCUITS

15.2 IMPEDANCE AND THE PHASOR DIAGRAM

Resistive Elements

In Chapter 14, we found, for the purely resistive circuit in Fig. 15.1, that v and i were in phase, and the magnitude

$$I_m = \frac{V_m}{R} \quad \text{or} \quad V_m = I_m R$$

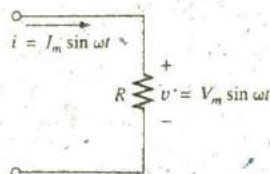


FIG. 15.1
Resistive ac circuit.

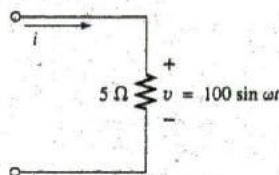


FIG. 15.2
Example 15.1.

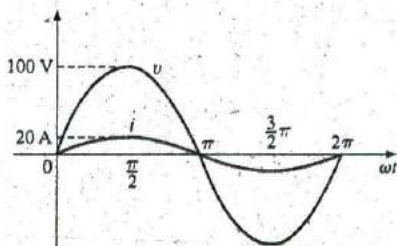


FIG. 15.3
Waveforms for Example 15.1.

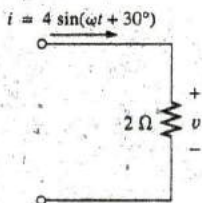


FIG. 15.4
Example 15.2.

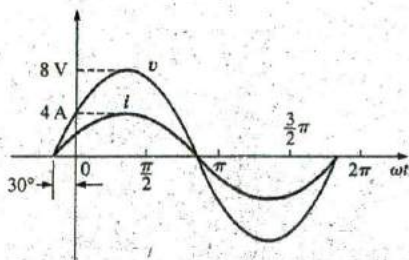


FIG. 15.5
Waveforms for Example 15.2.

In phasor form,

$$v = V_m \sin \omega t \Rightarrow \mathbf{V} = V \angle 0^\circ$$

where $V = 0.707V_m$.

Applying Ohm's law and using phasor algebra, we have

$$\mathbf{I} = \frac{\mathbf{V} \angle 0^\circ}{R \angle \theta_R} = \frac{V}{R} \angle 0^\circ - \theta_R$$

Since i and v are in phase, the angle associated with i also must be 0° . To satisfy this condition, θ_R must equal 0° . Substituting $\theta_R = 0^\circ$, we find

$$\mathbf{I} = \frac{V \angle 0^\circ}{R \angle 0^\circ} = \frac{V}{R} \angle 0^\circ - 0^\circ = \frac{V}{R} \angle 0^\circ$$

so that in the time domain,

$$i = \sqrt{2} \left(\frac{V}{R} \right) \sin \omega t$$

We use the fact that $\theta_R = 0^\circ$ in the following polar format to ensure the proper phase relationship between the voltage and current of a resistor:

$$\mathbf{Z}_R = R \angle 0^\circ \quad (15.1)$$

The boldface roman quantity \mathbf{Z}_R , having both magnitude and an associated angle, is referred to as the *impedance* of a resistive element. It is measured in ohms and is a measure of how much the element will "impede" the flow of charge through the network. The above format will prove to be a useful "tool" when the networks become more complex and phase relationships become less obvious. It is important to realize, however, that \mathbf{Z}_R is *not* a phasor, even though the format $R \angle 0^\circ$ is very similar to the phasor notation for sinusoidal currents and voltages. The term *phasor* is reserved for quantities that vary with time, and R and its associated angle of 0° are fixed, nonvarying quantities.

EXAMPLE 15.1 Using complex algebra, find the current i for the circuit in Fig. 15.2. Sketch the waveforms of v and i .

Solution: Note Fig. 15.3:

$$v = 100 \sin \omega t \Rightarrow \text{phasor form } \mathbf{V} = 70.71 \text{ V } \angle 0^\circ$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_R} = \frac{V \angle \theta}{R \angle 0^\circ} = \frac{70.71 \text{ V } \angle 0^\circ}{5 \Omega \angle 0^\circ} = 14.14 \text{ A } \angle 0^\circ$$

and
$$i = \sqrt{2}(14.14) \sin \omega t = 20 \sin \omega t$$

EXAMPLE 15.2 Using complex algebra, find the voltage v for the circuit in Fig. 15.4. Sketch the waveforms of v and i .

Solution: Note Fig. 15.5:

$$i = 4 \sin(\omega t + 30^\circ) \Rightarrow \text{phasor form } \mathbf{I} = 2.828 \text{ A } \angle 30^\circ$$

$$\mathbf{V} = \mathbf{I} \mathbf{Z}_R = (I \angle \theta)(R \angle 0^\circ) = (2.828 \text{ A } \angle 30^\circ)(2 \Omega \angle 0^\circ) = 5.656 \text{ V } \angle 30^\circ$$

and
$$v = \sqrt{2}(5.656) \sin(\omega t + 30^\circ) = 8.0 \sin(\omega t + 30^\circ)$$

It is often helpful in the analysis of networks to have a **phasor diagram**, which shows at a glance the *magnitudes* and *phase relations* among the various quantities within the network. For example, the phasor diagrams of the circuits considered in the two preceding examples would be as shown in Fig. 15.6. In both cases, it is immediately obvious that v and i are in phase since they both have the same phase angle.

Inductive Reactance

We learned in Chapter 13 that for the pure inductor in Fig. 15.7, the voltage leads the current by 90° and that the reactance of the coil X_L is determined by ωL . We have

$$v = V_m \sin \omega t \Rightarrow \text{phasor form } \mathbf{V} = V \angle 0^\circ$$

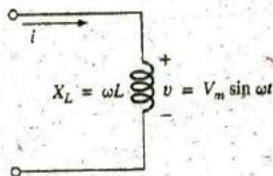


FIG. 15.7
Inductive ac circuit.

By Ohm's law,

$$\mathbf{I} = \frac{V \angle 0^\circ}{X_L \angle \theta_L} = \frac{V}{X_L} \angle 0^\circ - \theta_L$$

Since v leads i by 90° , i must have an angle of -90° associated with it. To satisfy this condition, θ_L must equal $+90^\circ$. Substituting $\theta_L = 90^\circ$, we obtain

$$\mathbf{I} = \frac{V \angle 0^\circ}{X_L \angle 90^\circ} = \frac{V}{X_L} \angle 0^\circ - 90^\circ = \frac{V}{X_L} \angle -90^\circ$$

so that in the time domain,

$$i = \sqrt{2} \left(\frac{V}{X_L} \right) \sin(\omega t - 90^\circ)$$

We use the fact that $\theta_L = 90^\circ$ in the following polar format for inductive reactance to ensure the proper phase relationship between the voltage and current of an inductor:

$$\mathbf{Z}_L = X_L \angle 90^\circ \quad (15.2)$$

The boldface roman quantity \mathbf{Z}_L , having both magnitude and an associated angle, is referred to as the *impedance* of an inductive element. It is measured in ohms and is a measure of how much the inductive element "controls or impedes" the level of current through the network (always keep in mind that inductive elements are storage devices and do not dissipate like resistors). The above format, like that defined for the resistive element, will prove to be a useful tool in the analysis of ac networks. Again, be aware that \mathbf{Z}_L is not a phasor quantity, for the same reasons indicated for a resistive element.

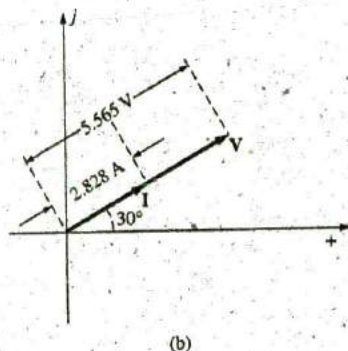
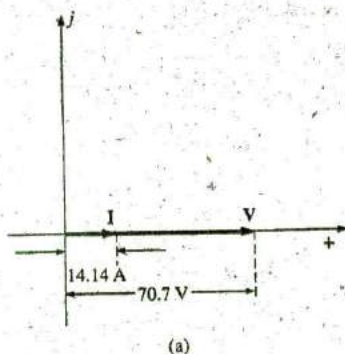


FIG. 15.6
Phasor diagrams for Examples 15.1 and 15.2.

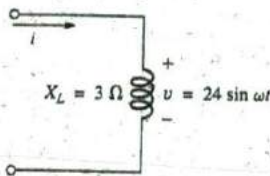


FIG. 15.8
Example 15.3.

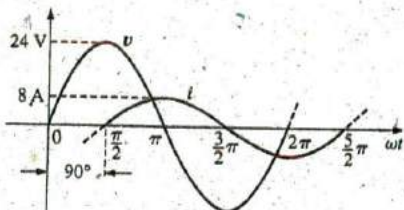


FIG. 15.9
Waveforms for Example 15.3.

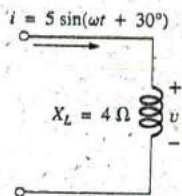


FIG. 15.10
Example 15.4.

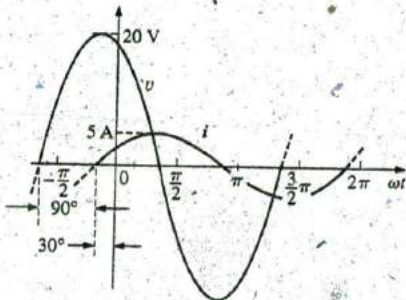


FIG. 15.11
Waveforms for Example 15.4.

EXAMPLE 15.3 Using complex algebra, find the current i for the circuit in Fig. 15.8. Sketch the v and i curves.

Solution: Note Fig. 15.9:

$$v = 24 \sin \omega t \Rightarrow \text{phasor form } \mathbf{V} = 16.968 \text{ V } \angle 0^\circ$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_L} = \frac{V \angle \theta}{X_L \angle 90^\circ} = \frac{16.968 \text{ V } \angle 0^\circ}{3 \Omega \angle 90^\circ} = 5.656 \text{ A } \angle -90^\circ$$

$$\text{and } i = \sqrt{2}(5.656) \sin(\omega t - 90^\circ) = 8.0 \sin(\omega t - 90^\circ)$$

EXAMPLE 15.4 Using complex algebra, find the voltage v for the circuit in Fig. 15.10. Sketch the v and i curves.

Solution: Note Fig. 15.11:

$$i = 5 \sin(\omega t + 30^\circ) \Rightarrow \text{phasor form } \mathbf{I} = 3.535 \text{ A } \angle 30^\circ$$

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_L = (\mathbf{I} \angle \theta)(X_L \angle 90^\circ) = (3.535 \text{ A } \angle 30^\circ)(4 \Omega \angle 90^\circ) = 14.140 \text{ V } \angle 120^\circ$$

$$\text{and } v = \sqrt{2}(14.140) \sin(\omega t + 120^\circ) = 20 \sin(\omega t + 120^\circ)$$

The phasor diagrams for the two circuits of the two preceding examples are shown in Fig. 15.12. Both indicate quite clearly that the voltage leads the current by 90° .

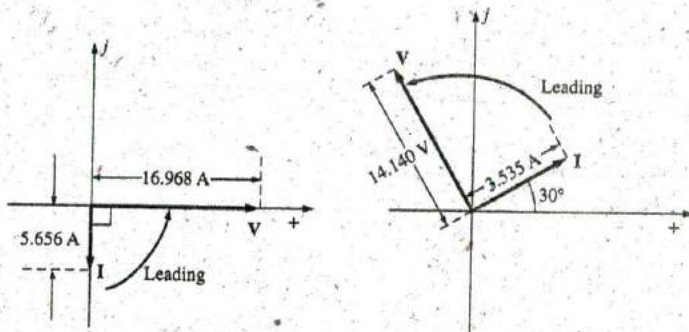


FIG. 15.12
Phasor diagrams for Examples 15.3 and 15.4.

Capacitive Reactance

We learned in Chapter 13 that for the pure capacitor in Fig. 15.13, the current leads the voltage by 90° and that the reactance of the capacitor X_C is determined by $1/\omega C$. We have

$$v = V_m \sin \omega t \Rightarrow \text{phasor form } \mathbf{V} = V \angle 0^\circ$$

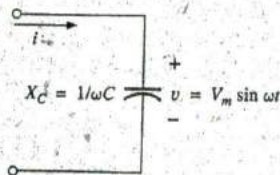


FIG. 15.13
Capacitive ac circuit.

Applying Ohm's law and using phasor algebra, we find

$$\mathbf{I} = \frac{V \angle 0^\circ}{X_C \angle \theta_C} = \frac{V}{X_C} \angle 0^\circ - \theta_C$$

Since i leads v by 90° , i must have an angle of $+90^\circ$ associated with it. To satisfy this condition, θ_C must equal -90° . Substituting $\theta_C = -90^\circ$ yields

$$\mathbf{I} = \frac{V \angle 0^\circ}{X_C \angle -90^\circ} = \frac{V}{X_C} \angle 0^\circ - (-90^\circ) = \frac{V}{X_C} \angle 90^\circ$$

so, in the time domain,

$$i = \sqrt{2} \left(\frac{V}{X_C} \right) \sin(\omega t + 90^\circ)$$

We use the fact that $\theta_C = -90^\circ$, in the following polar format for capacitive reactance to ensure the proper phase relationship between the voltage and current of a capacitor:

$$\boxed{\mathbf{Z}_C = X_C \angle -90^\circ} \quad (15.3)$$

The boldface roman quantity \mathbf{Z}_C , having both magnitude and an associated angle, is referred to as the *impedance* of a capacitive element. It is measured in ohms and is a measure of how much the capacitive element "controls or impedes" the level of current through the network (always keep in mind that capacitive elements are storage devices and do not dissipate like resistors). The above format, like that defined for the resistive element, will prove a very useful tool in the analysis of ac networks. Again, be aware that \mathbf{Z}_C is not a phasor quantity, for the same reasons indicated for a resistive element.

EXAMPLE 15.5 Using complex algebra, find the current i for the circuit in Fig. 15.14. Sketch the v and i curves.

Solution: Note Fig. 15.15:

$$v = 15 \sin \omega t \Rightarrow \text{phasor notation } \mathbf{V} = 10.605 \text{ V } \angle 0^\circ$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_C} = \frac{V \angle \theta}{X_C \angle -90^\circ} = \frac{10.605 \text{ V } \angle 0^\circ}{2 \Omega \angle -90^\circ} = 5.303 \text{ A } \angle 90^\circ$$

and $i = \sqrt{2}(5.303) \sin(\omega t + 90^\circ) = 7.5 \sin(\omega t + 90^\circ)$

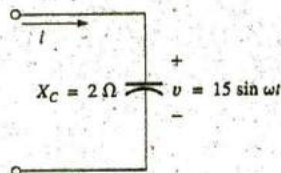


FIG. 15.14
Example 15.5.

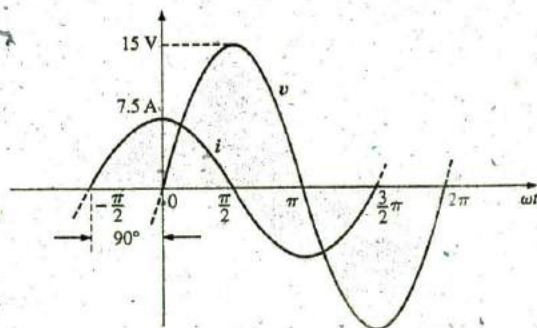


FIG. 15.15
Waveforms for Example 15.5.

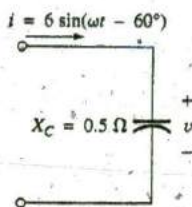


FIG. 15.16
Example 15.6.

EXAMPLE 15.6 Using complex algebra, find the voltage v for the circuit in Fig. 15.16. Sketch the v and i curves.

Solution: Note Fig. 15.17:

$$i = 6 \sin(\omega t - 60^\circ) \Rightarrow \text{phasor notation } \mathbf{I} = 4.242 \text{ A } \angle -60^\circ$$

$$\mathbf{V} = \mathbf{I}Z_C = (\mathbf{I} \angle \theta)(X_C \angle -90^\circ) = (4.242 \text{ A } \angle -60^\circ)(0.5 \Omega \angle -90^\circ) \\ = 2.121 \text{ V } \angle -150^\circ$$

and

$$v = \sqrt{2}(2.121) \sin(\omega t - 150^\circ) = 3.0 \sin(\omega t - 150^\circ)$$

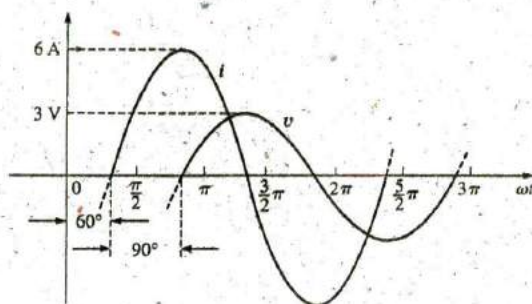
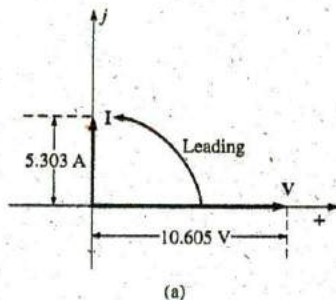


FIG. 15.17
Waveforms for Example 15.6.

The phasor diagrams for the two circuits of the two preceding examples are shown in Fig. 15.18. Both indicate quite clearly that the current i leads the voltage v by 90° .

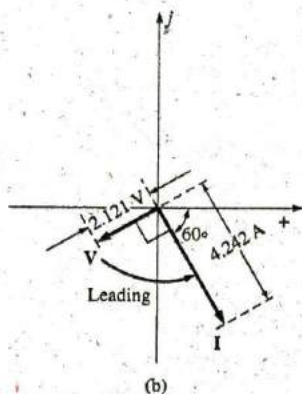


FIG. 15.18
Phasor diagrams for Examples 15.5 and 15.6.

Impedance Diagram

Now that an angle is associated with resistance, inductive reactance, and capacitive reactance, each can be placed on a complex plane diagram, as shown in Fig. 15.19. For any network, the resistance will *always* appear on the positive real axis, the inductive reactance on the positive imaginary axis, and the capacitive reactance on the negative imaginary axis.

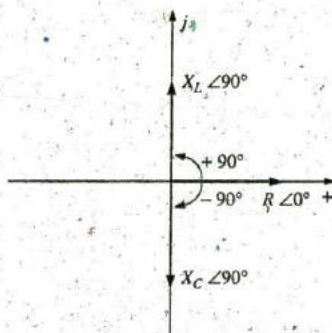


FIG. 15.19
Impedance diagram.

The result is an **impedance diagram** that can reflect the individual and total impedance levels of an ac network.

We will find in the rest of this text that networks combining different types of elements will have total impedances that extend from -90° to $+90^\circ$. If the total impedance has an angle of 0° , it is said to be resistive in nature. If it is closer to 90° , it is inductive in nature. If it is closer to -90° , it is capacitive in nature.

Of course, for single-element networks, the angle associated with the impedance will be the same as that of the resistive or reactive element, as revealed by Eqs. (15.1) through (15.3). It is important to remember that impedance, like resistance or reactance, is not a phasor quantity representing a time-varying function with a particular phase shift. It is simply an operating tool that is extremely useful in determining the magnitude and angle of quantities in a sinusoidal ac network.

Once the total impedance of a network is determined, its magnitude will define the resulting current level (through Ohm's law), whereas its angle will reveal whether the network is primarily inductive or capacitive or simply resistive.

For any configuration (series, parallel, series-parallel, and so on), the angle associated with the total impedance is the angle by which the applied voltage leads the source current. For inductive networks, θ_T will be positive, whereas for capacitive networks, θ_T will be negative.

15.3 SERIES CONFIGURATION

The overall properties of series ac circuits (Fig. 15.20) are the same as those for dc circuits. For instance, the total impedance of a system is the sum of the individual impedances:

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 + \cdots + \mathbf{Z}_N \quad (15.4)$$

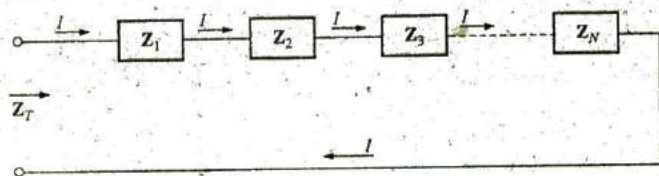


FIG. 15.20
Series impedances.

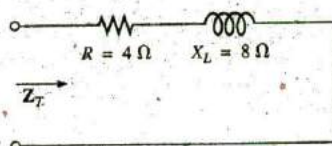


FIG. 15.21
Example 15.7.

EXAMPLE 15.7 Draw the impedance diagram for the circuit in Fig. 15.21, and find the total impedance.

Solution: As indicated by Fig. 15.22, the input impedance can be found graphically from the impedance diagram by properly scaling the real and imaginary axes and finding the length of the resultant vector \mathbf{Z}_T and angle θ_T . Or, by using vector algebra, we obtain

$$\begin{aligned} \mathbf{Z}_T &= \mathbf{Z}_1 + \mathbf{Z}_2 \\ &= R \angle 0^\circ + X_L \angle 90^\circ \\ &= R + jX_L = 4 \Omega + j8 \Omega \\ \mathbf{Z}_T &= 8.94 \Omega \angle 63.43^\circ \end{aligned}$$

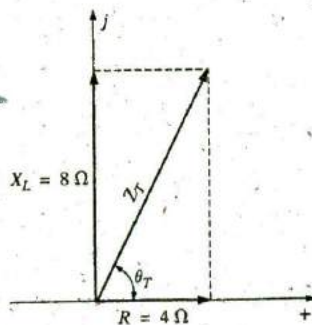


FIG. 15.22
Impedance diagram for Example 15.7.

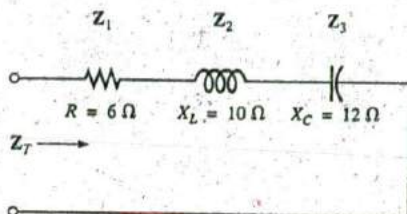


FIG. 15.23
Example 15.8

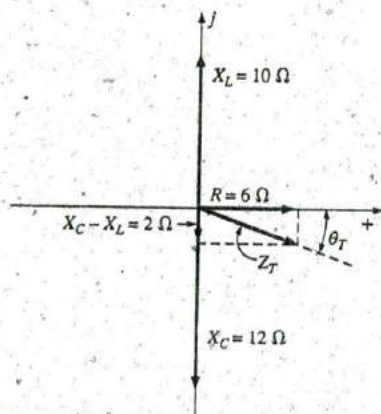


FIG. 15.24
Impedance diagram for Example 15.8.

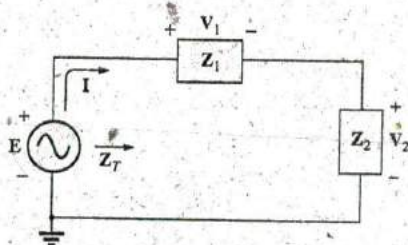


FIG. 15.25
Series ac circuit.

EXAMPLE 15.8 Determine the input impedance to the series network in Fig. 15.23. Draw the impedance diagram.

Solution:

$$\begin{aligned} Z_T &= Z_1 + Z_2 + Z_3 \\ &= R \angle 0^\circ + X_L \angle 90^\circ + X_C \angle -90^\circ \\ &= R + jX_L - jX_C \\ &= R + j(X_L - X_C) = 6 \Omega + j(10 \Omega - 12 \Omega) = 6 \Omega - j2 \Omega \\ Z_T &= 6.32 \Omega \angle -18.43^\circ \end{aligned}$$

The impedance diagram appears in Fig. 15.24. Note that in this example, series inductive and capacitive reactances are in direct opposition. For the circuit in Fig. 15.23, if the inductive reactance were equal to the capacitive reactance, the input impedance would be purely resistive. We will have more to say about this particular condition in a later chapter.

For the representative **series ac configuration** in Fig. 15.25 having two impedances, *the current is the same through each element* (as it was for the series dc circuits) and is determined by Ohm's law:

$$Z_T = Z_1 + Z_2$$

and

$$I = \frac{E}{Z_T} \quad (15.5)$$

The voltage across each element can then be found by another application of Ohm's law:

$$V_1 = IZ_1 \quad (15.6a)$$

$$V_2 = IZ_2 \quad (15.6b)$$

Kirchhoff's voltage law can then be applied in the same manner as it is employed for dc circuits. However, keep in mind that we are now dealing with the algebraic manipulation of quantities that have both magnitude and direction. We have

$$E - V_1 - V_2 = 0$$

or

$$E = V_1 + V_2 \quad (15.7)$$

The power to the circuit can be determined by

$$P = EI \cos \theta_T \quad (15.8)$$

where θ_T is the phase angle between E and I .

Now that a general approach has been introduced, the simplest of series configurations will be investigated in detail to further emphasize the similarities in the analysis of dc circuits. In many of the circuits to be considered, $3 + j4 = 5 \angle 53.13^\circ$ and $4 + j3 = 5 \angle 36.87^\circ$ are used quite frequently to ensure that the approach is as clear as possible and not lost in mathematical complexity. Of course, the problems at the end of the chapter will provide plenty of experience with random values.

R-L

Refer to Fig. 15.26.

Phasor Notation

$$e = 141.4 \sin \omega t \Rightarrow E = 100 \text{ V} \angle 0^\circ$$

Note Fig. 15.27.

 Z_T

$$Z_T = Z_1 + Z_2 = 3 \Omega \angle 0^\circ + 4 \Omega \angle 90^\circ = 3 \Omega + j 4 \Omega$$

and
$$Z_T = 5 \Omega \angle 53.13^\circ$$

Impedance diagram: See Fig. 15.28.

 I

$$I = \frac{E}{Z_T} = \frac{100 \text{ V} \angle 0^\circ}{5 \Omega \angle 53.13^\circ} = 20 \text{ A} \angle -53.13^\circ$$

 V_R and V_L
Ohm's law:

$$V_R = IZ_R = (20 \text{ A} \angle -53.13^\circ)(3 \Omega \angle 0^\circ) = 60 \text{ V} \angle -53.13^\circ$$

$$V_L = IZ_L = (20 \text{ A} \angle -53.13^\circ)(4 \Omega \angle 90^\circ) = 80 \text{ V} \angle 36.87^\circ$$

Kirchhoff's voltage law:

$$\sum_C V = E - V_R - V_L = 0$$

or
$$E = V_R + V_L$$

In rectangular form,

$$V_R = 60 \text{ V} \angle -53.13^\circ = 36 \text{ V} - j 48 \text{ V}$$

$$V_L = 80 \text{ V} \angle +36.87^\circ = 64 \text{ V} + j 48 \text{ V}$$

and

$$E = V_R + V_L = (36 \text{ V} - j 48 \text{ V}) + (64 \text{ V} + j 48 \text{ V}) = 100 \text{ V} + j 0 = 100 \text{ V} \angle 0^\circ$$

as applied.

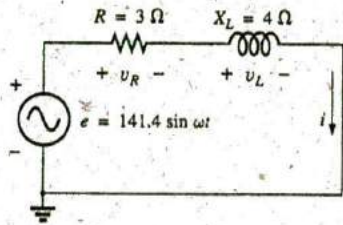
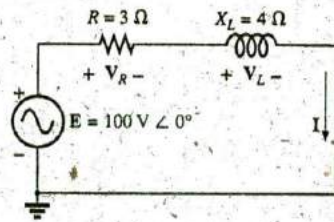
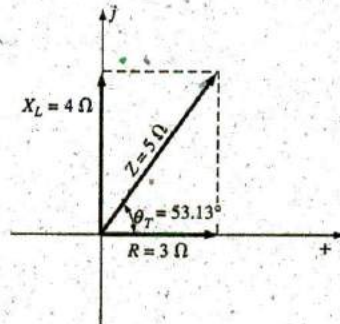
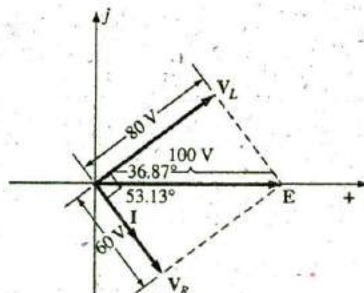
Phasor diagram: Note that for the phasor diagram in Fig. 15.29, I is in phase with the voltage across the resistor and lags the voltage across the inductor by 90° .

Power: The total power in watts delivered to the circuit is

$$\begin{aligned} P_T &= EI \cos \theta_T \\ &= (100 \text{ V})(20 \text{ A}) \cos 53.13^\circ = (2000 \text{ W})(0.6) \\ &= 1200 \text{ W} \end{aligned}$$

 where E and I are effective values and θ_T is the phase angle between E and I , or

$$\begin{aligned} P_T &= I^2 R \\ &= (20 \text{ A})^2 (3 \Omega) = (400)(3) \\ &= 1200 \text{ W} \end{aligned}$$


FIG. 15.26
Series R-L circuit.

FIG. 15.27
Applying phasor notation to the network in Fig. 15.26.

FIG. 15.28
Impedance diagram for the series R-L circuit in Fig. 15.26.

FIG. 15.29
Phasor diagram for the series R-L circuit in Fig. 15.26.

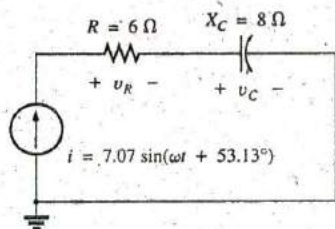


FIG. 15.30
Series R-C ac circuit.

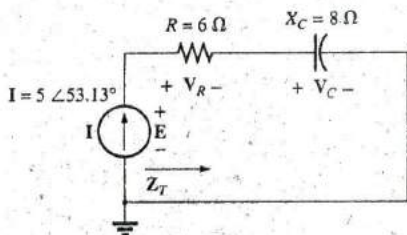


FIG. 15.31
Applying phasor notation to the circuit in
Fig. 15.30.

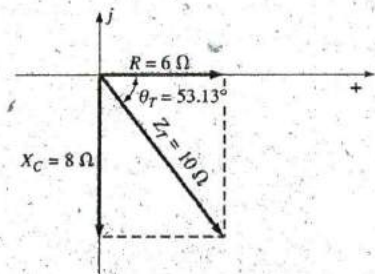


FIG. 15.32
Impedance diagram for the series R-C circuit in
Fig. 15.30.

where I is the effective value, or, finally,

$$\begin{aligned} P_T &= P_R + P_L = V_R I \cos \theta_R + V_L I \cos \theta_L \\ &= (60 \text{ V})(20 \text{ A}) \cos 0^\circ + (80 \text{ V})(20 \text{ A}) \cos 90^\circ \\ &= 1200 \text{ W} + 0 \\ &= 1200 \text{ W} \end{aligned}$$

where θ_R is the phase angle between V_R and I , and θ_L is the phase angle between V_L and I .

Power factor: The power factor F_p of the circuit is $\cos 53.13^\circ = 0.6$ lagging, where 53.13° is the phase angle between E and I .

If we write the basic power equation $P = EI \cos \theta$ as

$$\cos \theta = \frac{P}{EI}$$

where E and I are the input quantities and P is the power delivered to the network, and then perform the following substitutions from the basic series ac circuit as

$$\cos \theta = \frac{P}{EI} = \frac{I^2 R}{EI} = \frac{IR}{E} = \frac{R}{E/I} = \frac{R}{Z_T}$$

we find

$$F_p = \cos \theta_T = \frac{R}{Z_T} \quad (15.9)$$

Reference to Fig. 15.28 also indicates that θ is the impedance angle θ_T as written in Eq. (15.9), further supporting the fact that the impedance angle θ_T is also the phase angle between the input voltage and current for a series ac circuit. To determine the power factor, it is necessary only to form the ratio of the total resistance to the magnitude of the input impedance. For the case at hand,

$$F_p = \cos \theta = \frac{R}{Z_T} = \frac{3 \Omega}{5 \Omega} = 0.6 \text{ lagging}$$

as found above.

R-C

Refer to Fig. 15.30.

Phasor Notation

$$i = 7.07 \sin(\omega t + 53.13^\circ) \Rightarrow \mathbf{I} = 5 \text{ A } \angle 53.13^\circ$$

Note Fig. 15.31.

Z_T

$$Z_T = Z_1 + Z_2 = 6 \Omega \angle 0^\circ + 8 \Omega \angle -90^\circ = 6 \Omega - j 8 \Omega$$

and

$$Z_T = 10 \Omega \angle -53.13^\circ$$

Impedance diagram: As shown in Fig. 15.32.

E

$$E = \mathbf{I} Z_T = (5 \text{ A } \angle 53.13^\circ)(10 \Omega \angle -53.13^\circ) = 50 \text{ V } \angle 0^\circ$$

V_R and V_C

$$\begin{aligned} V_R &= IZ_R = (I \angle \theta)(R \angle 0^\circ) = (5 \text{ A} \angle 53.13^\circ)(6 \Omega \angle 0^\circ) \\ &= 30 \text{ V} \angle 53.13^\circ \end{aligned}$$

$$\begin{aligned} V_C &= IZ_C = (I \angle \theta)(X_C \angle -90^\circ) = (5 \text{ A} \angle 53.13^\circ)(8 \Omega \angle -90^\circ) \\ &= 40 \text{ V} \angle -36.87^\circ \end{aligned}$$

Kirchhoff's voltage law:

$$\sum_C V = E - V_R - V_C = 0$$

or

$$E = V_R + V_C$$

which can be verified by vector algebra as demonstrated for the R - L circuit.

Phasor diagram: Note on the phasor diagram in Fig. 15.33 that the current I is in phase with the voltage across the resistor and leads the voltage across the capacitor by 90° .

Time domain: In the time domain,

$$e = \sqrt{2}(50) \sin \omega t = 70.70 \sin \omega t$$

$$v_R = \sqrt{2}(30) \sin(\omega t + 53.13^\circ) = 42.42 \sin(\omega t + 53.13^\circ)$$

$$v_C = \sqrt{2}(40) \sin(\omega t - 36.87^\circ) = 56.56 \sin(\omega t - 36.87^\circ)$$

A plot of all of the voltages and the current of the circuit appears in Fig. 15.34. Note again that i and v_R are in phase and that v_C lags i by 90° .

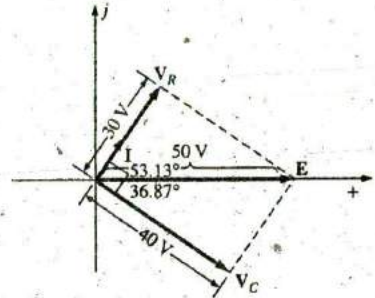


FIG. 15.33

Phasor diagram for the series R - C circuit in Fig. 15.30.

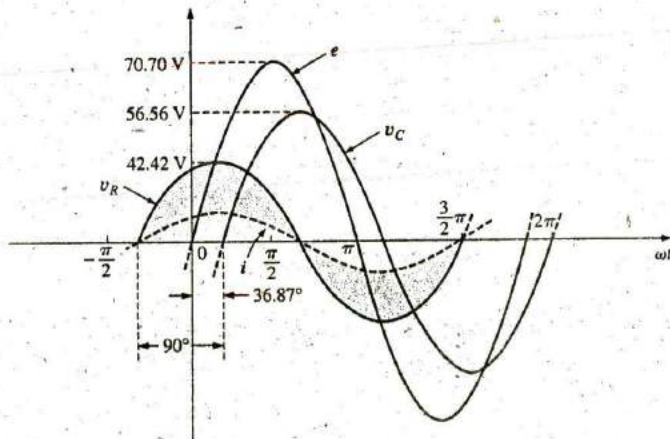


FIG. 15.34

Waveforms for the series R - C circuit in Fig. 15.30.

Power: The total power in watts delivered to the circuit is

$$\begin{aligned} P_T &= EI \cos \theta_T = (50 \text{ V})(5 \text{ A}) \cos 53.13^\circ \\ &= (250)(0.6) = 150 \text{ W} \end{aligned}$$

or

$$\begin{aligned} P_T &= I^2 R = (5 \text{ A})^2 (6 \Omega) = (25)(6) \\ &= 150 \text{ W} \end{aligned}$$

or, finally,

$$\begin{aligned} P_T &= P_R + P_C = V_R I \cos \theta_R + V_C I \cos \theta_C \\ &= (30 \text{ V})(5 \text{ A}) \cos 0^\circ + (40 \text{ V})(5 \text{ A}) \cos 90^\circ \\ &= 150 \text{ W} + 0 \\ &= 150 \text{ W} \end{aligned}$$

Power factor: The power factor of the circuit is

$$F_p = \cos \theta = \cos 53.13^\circ = 0.6 \text{ leading}$$

Using Eq. (15.9), we obtain

$$\begin{aligned} F_p &= \cos \theta = \frac{R}{Z_T} = \frac{6 \Omega}{10 \Omega} \\ &= 0.6 \text{ leading} \end{aligned}$$

as determined above.

R-L-C

Refer to Fig. 15.35.

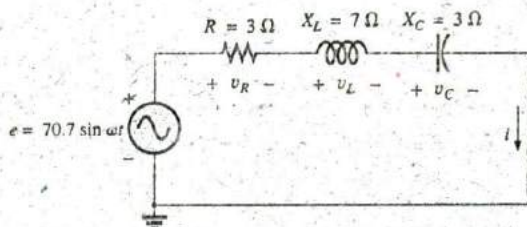


FIG. 15.35

Series R-L-C ac circuit.

Phasor Notation As shown in Fig. 15.36.

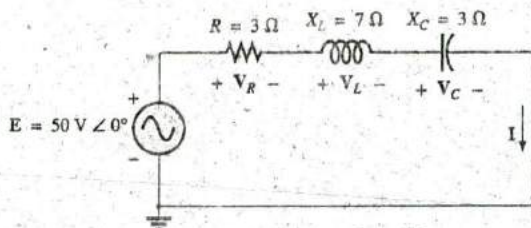


FIG. 15.36

Applying phasor notation to the circuit in Fig. 15.35.

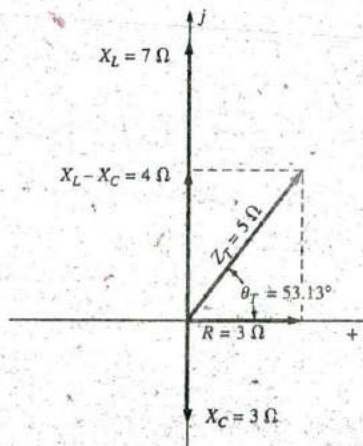


FIG. 15.37

Impedance diagram for the series R-L-C circuit in Fig. 15.35.

Z_T

$$\begin{aligned} Z_T &= Z_1 + Z_2 + Z_3 = R \angle 0^\circ + X_L \angle 90^\circ + X_C \angle -90^\circ \\ &= 3 \Omega + j7 \Omega - j3 \Omega = 3 \Omega + j4 \Omega \end{aligned}$$

and

$$Z_T = 5 \Omega \angle 53.13^\circ$$

Impedance diagram: As shown in Fig. 15.37.

$$I = \frac{E}{Z_T} = \frac{50 \text{ V } \angle 0^\circ}{5 \Omega \angle 53.13^\circ} = 10 \text{ A } \angle -53.13^\circ$$

V_R , V_L , and V_C

$$V_R = IZ_R = (I \angle \theta)(R \angle 0^\circ) = (10 \text{ A} \angle -53.13^\circ)(3 \Omega \angle 0^\circ) \\ = 30 \text{ V} \angle -53.13^\circ$$

$$V_L = IZ_L = (I \angle \theta)(X_L \angle 90^\circ) = (10 \text{ A} \angle -53.13^\circ)(7 \Omega \angle 90^\circ) \\ = 70 \text{ V} \angle 36.87^\circ$$

$$V_C = IZ_C = (I \angle \theta)(X_C \angle -90^\circ) = (10 \text{ A} \angle -53.13^\circ)(3 \Omega \angle -90^\circ) \\ = 30 \text{ V} \angle -143.13^\circ$$

Kirchhoff's voltage law:

$$\sum_C V = E - V_R - V_L - V_C = 0$$

or

$$E = V_R + V_L + V_C$$

which can also be verified through vector algebra.

Phasor diagram: The phasor diagram in Fig. 15.38 indicates that the current I is in phase with the voltage across the resistor, lags the voltage across the inductor by 90° , and leads the voltage across the capacitor by 90° .

Time domain:

$$i = \sqrt{2}(10) \sin(\omega t - 53.13^\circ) = 14.14 \sin(\omega t - 53.13^\circ)$$

$$v_R = \sqrt{2}(30) \sin(\omega t - 53.13^\circ) = 42.42 \sin(\omega t - 53.13^\circ)$$

$$v_L = \sqrt{2}(70) \sin(\omega t + 36.87^\circ) = 98.98 \sin(\omega t + 36.87^\circ)$$

$$v_C = \sqrt{2}(30) \sin(\omega t - 143.13^\circ) = 42.42 \sin(\omega t - 143.13^\circ)$$

A plot of all the voltages and the current of the circuit appears in Fig. 15.39.

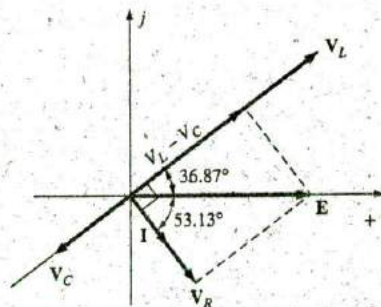


FIG. 15.38

Phasor diagram for the series R-L-C circuit in Fig. 15.35.

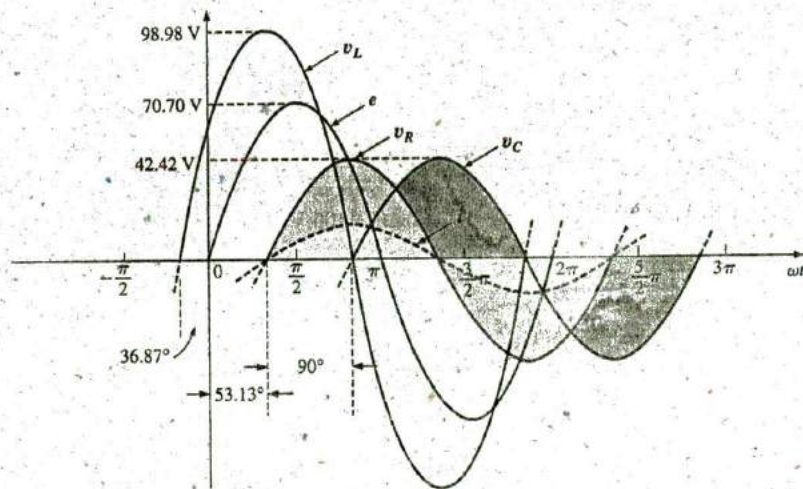


FIG. 15.39

Waveforms for the series R-L-C circuit in Fig. 15.35.

Power: The total power in watts delivered to the circuit is

$$P_T = EI \cos \theta_T = (50 \text{ V})(10 \text{ A}) \cos 53.13^\circ = (500)(0.6) = 300 \text{ W}$$

or
$$P_T = I^2 R = (10 \text{ A})^2 (3 \Omega) = (100)(3) = 300 \text{ W}$$

or

$$\begin{aligned}
 P_T &= P_R + P_L + P_C \\
 &= V_R I \cos \theta_R + V_L I \cos \theta_L + V_C I \cos \theta_C \\
 &= (30 \text{ V})(10 \text{ A}) \cos 0^\circ + (70 \text{ V})(10 \text{ A}) \cos 90^\circ + (30 \text{ V})(10 \text{ A}) \cos 90^\circ \\
 &= (30 \text{ V})(10 \text{ A}) + 0 + 0 = \mathbf{300 \text{ W}}
 \end{aligned}$$

Power factor: The power factor of the circuit is

$$F_p = \cos \theta_T = \cos 53.13^\circ = \mathbf{0.6 \text{ lagging}}$$

Using Eq. (15.9), we obtain

$$F_p = \cos \theta = \frac{R}{Z_T} = \frac{3 \Omega}{5 \Omega} = \mathbf{0.6 \text{ lagging}}$$

15.4 VOLTAGE DIVIDER RULE

The basic format for the **voltage divider rule** in ac circuits is exactly the same as that for dc circuits:

$$\boxed{V_x = \frac{Z_x E}{Z_T}} \quad (15.10)$$

where V_x is the voltage across one or more elements in a series that have total impedance Z_x , E is the total voltage appearing across the series circuit, and Z_T is the total impedance of the series circuit.

EXAMPLE 15.9 Using the voltage divider rule, find the voltage across each element of the circuit in Fig. 15.40.

Solution:

$$\begin{aligned}
 V_C &= \frac{Z_C E}{Z_C + Z_R} = \frac{(4 \Omega \angle -90^\circ)(100 \text{ V} \angle 0^\circ)}{4 \Omega \angle -90^\circ + 3 \Omega \angle 0^\circ} = \frac{400 \angle -90^\circ}{3 - j4} \\
 &= \frac{400 \angle -90^\circ}{5 \angle -53.13^\circ} = \mathbf{80 \text{ V} \angle -36.87^\circ}
 \end{aligned}$$

$$\begin{aligned}
 V_R &= \frac{Z_R E}{Z_C + Z_R} = \frac{(3 \Omega \angle 0^\circ)(100 \text{ V} \angle 0^\circ)}{5 \Omega \angle -53.13^\circ} = \frac{300 \angle 0^\circ}{5 \angle -53.13^\circ} \\
 &= \mathbf{60 \text{ V} \angle +53.13^\circ}
 \end{aligned}$$

EXAMPLE 15.10 Using the voltage divider rule, find the unknown voltages V_R , V_L , V_C , and V_1 for the circuit in Fig. 15.41.

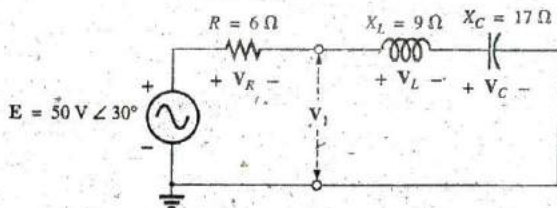


FIG. 15.41
Example 15.10.

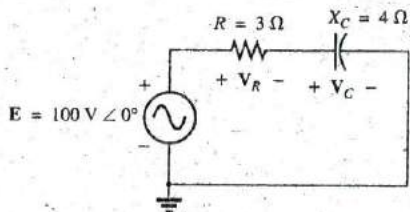


FIG. 15.40
Example 15.9.

Solution:

$$V_R = \frac{Z_R E}{Z_R + Z_L + Z_C} = \frac{(6 \Omega \angle 0^\circ)(50 \text{ V} \angle 30^\circ)}{6 \Omega \angle 0^\circ + 9 \Omega \angle 90^\circ + 17 \Omega \angle -90^\circ}$$

$$= \frac{300 \angle 30^\circ}{6 + j9 - j17} = \frac{300 \angle 30^\circ}{6 - j8}$$

$$= \frac{300 \angle 30^\circ}{10 \angle -53.13^\circ} = 30 \text{ V} \angle 83.13^\circ$$

Calculator The above calculation provides an excellent opportunity to demonstrate the power of today's calculators. For the TI-89 calculator, the sequence of steps to calculate V_R are shown in Fig. 15.42.



FIG. 15.42

Using the TI-89 calculator to determine V_R in Example 15.10.

$$V_L = \frac{Z_L E}{Z_T} = \frac{(9 \Omega \angle 90^\circ)(50 \text{ V} \angle 30^\circ)}{10 \Omega \angle -53.13^\circ} = \frac{450 \text{ V} \angle 120^\circ}{10 \angle -53.13^\circ}$$

$$= 45 \text{ V} \angle 173.13^\circ$$

$$V_C = \frac{Z_C E}{Z_T} = \frac{(17 \Omega \angle -90^\circ)(50 \text{ V} \angle 30^\circ)}{10 \Omega \angle -53.13^\circ} = \frac{850 \text{ V} \angle -60^\circ}{10 \angle -53^\circ}$$

$$= 85 \text{ V} \angle -6.87^\circ$$

$$V_i = \frac{(Z_L + Z_C)E}{Z_T} = \frac{(9 \Omega \angle 90^\circ + 17 \Omega \angle -90^\circ)(50 \text{ V} \angle 30^\circ)}{10 \Omega \angle -53.13^\circ}$$

$$= \frac{(8 \angle -90^\circ)(50 \angle 30^\circ)}{10 \angle -53.13^\circ}$$

$$= \frac{400 \angle -60^\circ}{10 \angle -53.13^\circ} = 40 \text{ V} \angle -6.87^\circ$$

EXAMPLE 15.11 For the circuit in Fig. 15.43:

- Calculate I , V_R , V_L , and V_C in phasor form.
- Calculate the total power factor.
- Calculate the average power delivered to the circuit.

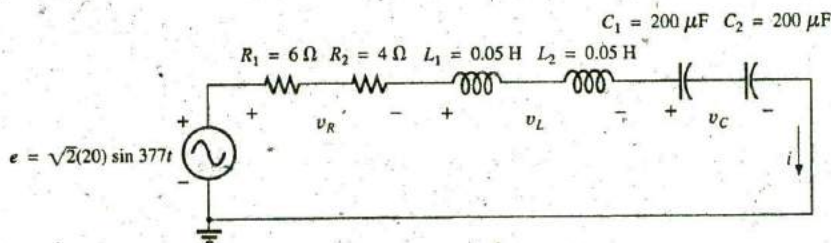


FIG. 15.43
Example 15.11.

- Draw the phasor diagram.
- Obtain the phasor sum of V_R , V_L , and V_C , and show that it equals the input voltage E .
- Find V_R and V_C using the voltage divider rule.

Solutions:

- Combining common elements and finding the reactance of the inductor and capacitor, we obtain

$$R_T = 6 \Omega + 4 \Omega = 10 \Omega$$

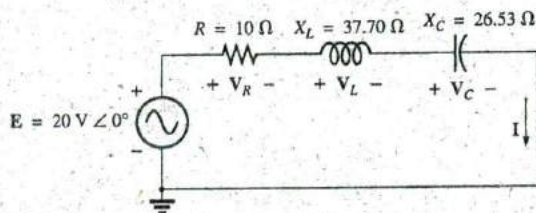
$$L_T = 0.05 \text{ H} + 0.05 \text{ H} = 0.1 \text{ H}$$

$$C_T = \frac{200 \mu\text{F}}{2} = 100 \mu\text{F}$$

$$X_L = \omega L = (377 \text{ rad/s})(0.1 \text{ H}) = 37.70 \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(377 \text{ rad/s})(100 \times 10^{-6} \text{ F})} = \frac{10^6}{37,700} = 26.53 \Omega$$

Redrawing the circuit using phasor notation results in Fig. 15.44.

**FIG. 15.44**

Applying phasor notation to the circuit in Fig. 15.43.

For the circuit in Fig. 15.44,

$$\begin{aligned} Z_T &= R \angle 0^\circ + X_L \angle 90^\circ + X_C \angle -90^\circ \\ &= 10 \Omega + j 37.70 \Omega - j 26.53 \Omega \\ &= 10 \Omega + j 11.17 \Omega = 15 \Omega \angle 48.16^\circ \end{aligned}$$

The current I is

$$I = \frac{E}{Z_T} = \frac{20 \text{ V} \angle 0^\circ}{15 \Omega \angle 48.16^\circ} = 1.33 \text{ A} \angle -48.16^\circ$$

The voltage across the resistor, inductor, and capacitor can be found using Ohm's law:

$$\begin{aligned} V_R = IZ_R &= (I \angle \theta)(R \angle 0^\circ) = (1.33 \text{ A} \angle -48.16^\circ)(10 \Omega \angle 0^\circ) \\ &= 13.30 \text{ V} \angle -48.16^\circ \end{aligned}$$

$$\begin{aligned} V_L = IZ_L &= (I \angle \theta)(X_L \angle 90^\circ) = (1.33 \text{ A} \angle -48.16^\circ)(37.70 \Omega \angle 90^\circ) \\ &= 50.14 \text{ V} \angle 41.84^\circ \end{aligned}$$

$$\begin{aligned} V_C = IZ_C &= (I \angle \theta)(X_C \angle -90^\circ) = (1.33 \text{ A} \angle -48.16^\circ)(26.53 \Omega \angle -90^\circ) \\ &= 35.28 \text{ V} \angle -138.16^\circ \end{aligned}$$

- The total power factor, determined by the angle between the applied voltage E and the resulting current I , is 48.16° :

$$F_p = \cos \theta = \cos 48.16^\circ = 0.667 \text{ lagging}$$

or

$$F_p = \cos \theta = \frac{R}{Z_T} = \frac{10 \Omega}{15 \Omega} = 0.667 \text{ lagging}$$

c. The total power in watts delivered to the circuit is

$$P_T = EI \cos \theta = (20 \text{ V})(1.33 \text{ A})(0.667) = 17.74 \text{ W}$$

d. The phasor diagram appears in Fig. 15.45.

e. The phasor sum of V_R , V_L , and V_C is

$$E = V_R + V_L + V_C = 13.30 \text{ V} \angle -48.16^\circ + 50.14 \text{ V} \angle 41.84^\circ + 35.28 \text{ V} \angle -138.16^\circ$$

$$E = 13.30 \text{ V} \angle -48.16^\circ + 14.86 \text{ V} \angle 41.84^\circ$$

Therefore,

$$E = \sqrt{(13.30 \text{ V})^2 + (14.86 \text{ V})^2} = 20 \text{ V}$$

and $\theta_E = 0^\circ$ (from phasor diagram)

and $E = 20 \angle 0^\circ$

$$f. V_R = \frac{Z_R E}{Z_T} = \frac{(10 \Omega \angle 0^\circ)(20 \text{ V} \angle 0^\circ)}{15 \Omega \angle 48.16^\circ} = \frac{200 \text{ V} \angle 0^\circ}{15 \angle 48.16^\circ} = 13.3 \text{ V} \angle -48.16^\circ$$

$$V_C = \frac{Z_C E}{Z_T} = \frac{(26.5 \Omega \angle -90^\circ)(20 \text{ V} \angle 0^\circ)}{15 \Omega \angle 48.16^\circ} = \frac{530.6 \text{ V} \angle -90^\circ}{15 \angle 48.16^\circ} = 35.37 \text{ V} \angle -138.16^\circ$$

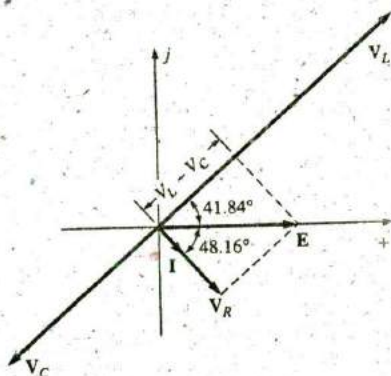


FIG. 15.45

Phasor diagram for the circuit in Fig. 15.43.

15.5 FREQUENCY RESPONSE FOR SERIES ac CIRCUITS

Thus far, the analysis has been for a fixed frequency, resulting in a fixed value for the reactance of an inductor or a capacitor. We now examine how the response of a series circuit changes as the frequency changes. We assume ideal elements throughout the discussion, so that the response of each element will be as shown in Fig. 15.46. Each response in Fig. 15.46 was discussed in detail in Chapter 14.

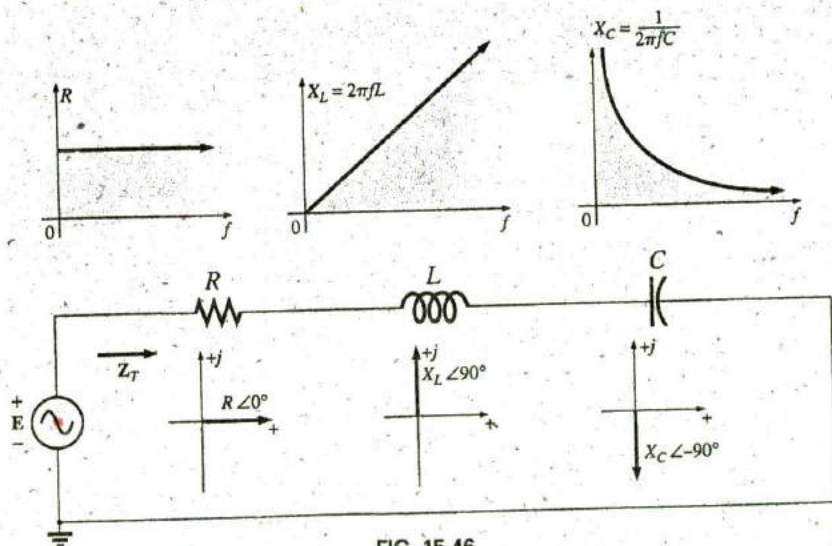


FIG. 15.46

Reviewing the frequency response of the basic elements.

When considering elements in series, remember that the total impedance is the sum of the individual elements and that the reactance of an inductor is in direct opposition to that of a capacitor. For Fig. 15.46, we are first aware that the resistance will remain fixed for the full range of frequencies: It will always be there, but, more important, its magnitude will not change. The inductor, however, will provide increasing levels of impedance as the frequency increases, while the capacitor will provide lower levels of impedance.

We are also aware from Chapter 14 that the inductor has a short-circuit equivalence at $f = 0$ Hz or very low frequencies, while the capacitor is nearly an open circuit for the same frequency range. For very high frequencies, the capacitor approaches the short-circuit equivalence, and the inductor approaches the open-circuit equivalence.

In general, therefore, if we encounter a series R - L - C circuit at very low frequencies, we can assume that the capacitor, with its very large impedance, will be the predominant factor. If the circuit is just an R - L series circuit, the impedance may be determined primarily by the resistive element since the reactance of the inductor is so small. As the frequency increases, the reactance of the coil increases to the point where it totally outshadows the impedance of the resistor. For an R - L - C combination, as the frequency increases, the reactance of the capacitor begins to approach a short-circuit equivalence, and the total impedance will be determined primarily by the inductive element. At very high frequencies, for an R - C series circuit, the total impedance eventually approaches that of the resistor since the impedance of the capacitor is dropping off so quickly.

In total, therefore,

when encountering a series ac circuit of any combination of elements, always use the idealized response of each element to establish some feeling for how the circuit will respond as the frequency changes.

Once you have a logical, overall sense for what the response will be, you can concentrate on working out the details.

Series R - C ac Circuit

As an example of establishing the frequency response of a circuit, consider the series R - C circuit in Fig. 15.47. As noted next to the source, the frequency range of interest is from 0 to 20 kHz. A great deal of detail is provided for this particular combination, so that obtaining the response of a series R - L or R - L - C combination will be quite straightforward.

Since the resistance remains fixed at $5\text{ k}\Omega$ for the full frequency range, and the total impedance is the sum of the impedances, it is immediately obvious that the lowest possible impedance is $5\text{ k}\Omega$. The highest

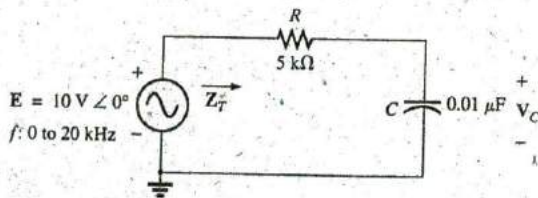


FIG. 15.47

Determining the frequency response of a series R - C circuit.

impedance, however, is dependent on the capacitive element since its impedance at very low frequencies is extremely high. At very low frequencies we can conclude, without a single calculation, that the impedance is determined primarily by the impedance of the capacitor. At the highest frequencies, we can assume that the reactance of the capacitor has dropped to such low levels that the impedance of the combination will approach that of the resistance.

The frequency at which the reactance of the capacitor drops to that of the resistor can be determined by setting the reactance of the capacitor equal to that of the resistor as follows:

$$X_C = \frac{1}{2\pi f_1 C} = R$$

Solving for the frequency yields

$$f_1 = \frac{1}{2\pi RC} \tag{15.11}$$

This significant point appears in the frequency plots in Fig. 15.48. Substituting values, we find that it occurs at

$$f_1 = \frac{1}{2\pi RC} = \frac{1}{2\pi(5 \text{ k}\Omega)(0.01 \text{ }\mu\text{F})} \cong 3.18 \text{ kHz}$$

We now know that for frequencies greater than f_1 , $R > X_C$ and that for frequencies less than f_1 , $X_C > R$, as shown in Fig. 15.48.

Now for the details. The total impedance is determined by the following equation:

$$Z_T = R - jX_C$$

and

$$Z_T = Z_T \angle \theta_T = \sqrt{R^2 + X_C^2} \angle -\tan^{-1} \frac{X_C}{R} \tag{15.12}$$

The magnitude and angle of the total impedance can now be found at any frequency of interest by simply substituting into Eq. (15.12). The presence of the capacitor suggests that we start from a low frequency (100 Hz) and then open the spacing until we reach the upper limit of interest (20 kHz).

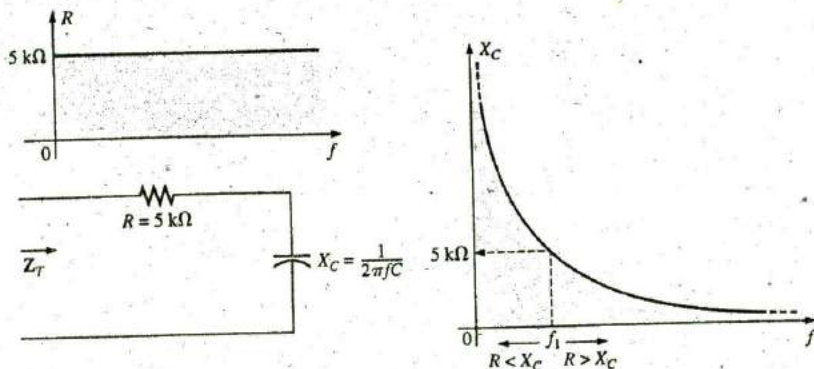


FIG. 15.48

The frequency response for the individual elements of a series R-C circuit.

$f = 100 \text{ Hz}$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(100 \text{ Hz})(0.01 \mu\text{F})} = 159.16 \text{ k}\Omega$$

$$\text{and } Z_T = \sqrt{R^2 + X_C^2} = \sqrt{(5 \text{ k}\Omega)^2 + (159.16 \text{ k}\Omega)^2} = 159.24 \text{ k}\Omega$$

$$\text{with } \theta_T = -\tan^{-1} \frac{X_C}{R} = -\tan^{-1} \frac{159.16 \text{ k}\Omega}{5 \text{ k}\Omega} = -\tan^{-1} 31.83 \\ = -88.2^\circ$$

$$\text{and } Z_T = 159.24 \text{ k}\Omega \angle -88.2^\circ$$

which compares very closely with $Z_C = 159.16 \text{ k}\Omega \angle -90^\circ$ if the circuit were purely capacitive ($R = 0 \Omega$). Our assumption that the circuit is primarily capacitive at low frequencies is therefore confirmed.

 $f = 1 \text{ kHz}$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(1 \text{ kHz})(0.01 \mu\text{F})} = 15.92 \text{ k}\Omega$$

$$\text{and } Z_T = \sqrt{R^2 + X_C^2} = \sqrt{(5 \text{ k}\Omega)^2 + (15.92 \text{ k}\Omega)^2} = 16.69 \text{ k}\Omega$$

$$\text{with } \theta_T = -\tan^{-1} \frac{X_C}{R} = -\tan^{-1} \frac{15.92 \text{ k}\Omega}{5 \text{ k}\Omega} \\ = -\tan^{-1} 3.18 = -72.54^\circ$$

$$\text{and } Z_T = 16.69 \text{ k}\Omega \angle -72.54^\circ$$

A noticeable drop in the magnitude has occurred, and the impedance angle has dropped almost 17° from the purely capacitive level.

Continuing, we obtain

$$f = .5 \text{ kHz: } Z_T = 5.93 \text{ k}\Omega \angle -32.48^\circ$$

$$f = 10 \text{ kHz: } Z_T = 5.25 \text{ k}\Omega \angle -17.66^\circ$$

$$f = 15 \text{ kHz: } Z_T = 5.11 \text{ k}\Omega \angle -11.98^\circ$$

$$f = 20 \text{ kHz: } Z_T = 5.06 \text{ k}\Omega \angle -9.04^\circ$$

Note how close the magnitude of Z_T at $f = 20 \text{ kHz}$ is to the resistance level of $5 \text{ k}\Omega$. In addition, note how the phase angle is approaching that associated with a pure resistive network (0°).

A plot of Z_T versus frequency in Fig. 15.49 completely supports our assumption based on the curves in Fig. 15.48. The plot of θ_T versus frequency in Fig. 15.50 further suggests that the total impedance made a transition from one of a capacitive nature ($\theta_T = -90^\circ$) to one with resistive characteristics ($\theta_T = 0^\circ$).

Applying the voltage divider rule to determine the voltage across the capacitor in phasor form yields

$$V_C = \frac{Z_C E}{Z_R + Z_C} \\ = \frac{(X_C \angle -90^\circ)(E \angle 0^\circ)}{R - jX_C} = \frac{X_C E \angle -90^\circ}{R - jX_C} \\ = \frac{X_C E \angle -90^\circ}{\sqrt{R^2 + X_C^2} \angle -\tan^{-1} X_C/R}$$

$$\text{or } V_C = V_C \angle \theta_C = \frac{X_C E}{\sqrt{R^2 + X_C^2}} \angle -90^\circ + \tan^{-1}(X_C/R)$$

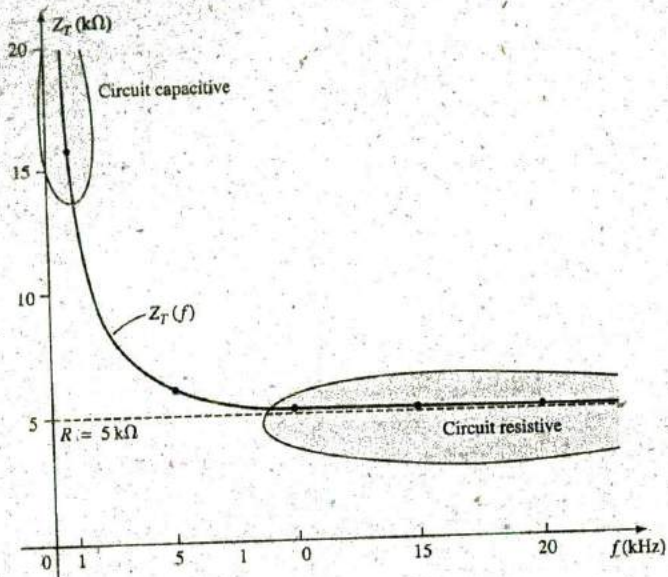


FIG. 15.49

The magnitude of the input impedance versus frequency for the circuit in Fig. 15.47.

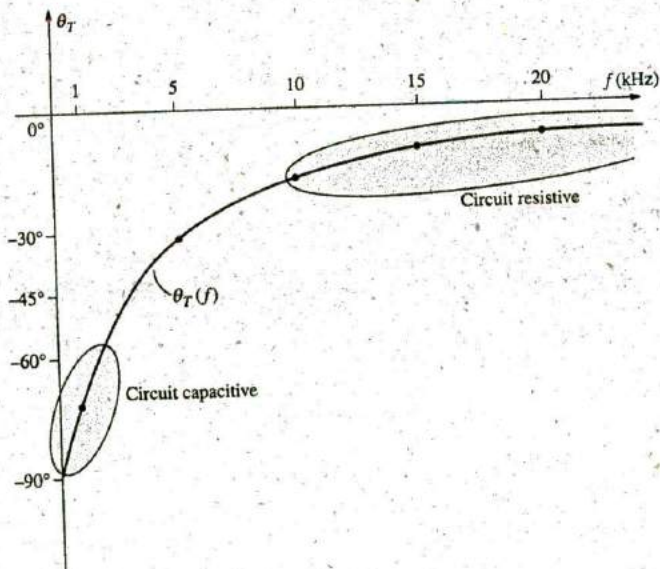


FIG. 15.50

The phase angle of the input impedance versus frequency for the circuit in Fig. 15.47.

The magnitude of V_C is therefore determined by

$$V_C = \frac{X_C E}{\sqrt{R^2 + X_C^2}} \quad (15.13)$$

and the phase angle θ_C by which V_C leads E is given by

$$\theta_C = -90^\circ + \tan^{-1} \frac{X_C}{R} = -\tan^{-1} \frac{R}{X_C} \quad (15.14)$$

To determine the frequency response, X_C must be calculated for each frequency of interest and inserted into Eqs. (15.13) and (15.14).

To begin our analysis, we should consider the case of $f = 0$ Hz (dc conditions).

$f = 0$ Hz

$$X_C = \frac{1}{2\pi(0)C} = \frac{1}{0} \Rightarrow \text{very large value}$$

Applying the open-circuit equivalent for the capacitor based on the above calculation results in the following:

$$V_C = E = 10 \text{ V} \angle 0^\circ$$

If we apply Eq. (15.13), we find

$$X_C^2 \gg R^2$$

and

$$\sqrt{R^2 + X_C^2} \cong \sqrt{X_C^2} = X_C$$

and

$$V_C = \frac{X_C E}{\sqrt{R^2 + X_C^2}} = \frac{X_C E}{X_C} = E$$

with

$$\theta_C = -\tan^{-1} \frac{R}{X_C} = -\tan^{-1} 0 = 0^\circ$$

verifying the above conclusions.

$f = 1$ kHz Applying Eq. (15.13) gives

$$X_C = \frac{1}{2\pi f C} = \frac{1}{(2\pi)(1 \times 10^3 \text{ Hz})(0.01 \times 10^{-6} \text{ F})} \cong 15.92 \text{ k}\Omega$$

$$\sqrt{R^2 + X_C^2} = \sqrt{(5 \text{ k}\Omega)^2 + (15.92 \text{ k}\Omega)^2} \cong 16.69 \text{ k}\Omega$$

$$\text{and } V_C = \frac{X_C E}{\sqrt{R^2 + X_C^2}} = \frac{(15.92 \text{ k}\Omega)(10)}{16.69 \text{ k}\Omega} = 9.54 \text{ V}$$

Applying Eq. (15.14) gives

$$\theta_C = -\tan^{-1} \frac{R}{X_C} = -\tan^{-1} \frac{5 \text{ k}\Omega}{15.9 \text{ k}\Omega}$$

$$= -\tan^{-1} 0.314 = -17.46^\circ$$

and

$$V_C = 9.83 \text{ V} \angle -17.46^\circ$$

As expected, the high reactance of the capacitor at low frequencies has resulted in the major part of the applied voltage appearing across the capacitor.

If we plot the phasor diagrams for $f = 0$ Hz and $f = 1$ kHz, as shown in Fig. 15.51, we find that V_C is beginning a clockwise rotation with an increase in frequency that will increase the angle θ_C and decrease the phase

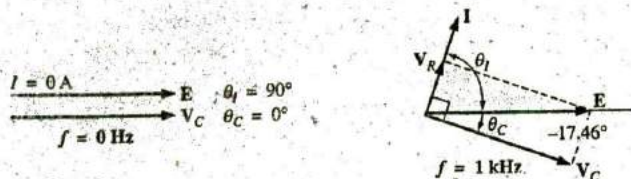


FIG. 15.51

The phasor diagram for the circuit in Fig. 15.47 for $f = 0$ Hz and 1 kHz.

angle between **I** and **E**. Recall that for a purely capacitive network, **I** leads **E** by 90° . As the frequency increases, therefore, the capacitive reactance is decreasing, and eventually $R \gg X_C$ with $\theta_C = -90^\circ$, and the angle between **I** and **E** will approach 0° . Keep in mind as we proceed through the other frequencies that θ_C is the phase angle between V_C and **E** and that the magnitude of the angle by which **I** leads **E** is determined by

$$|\theta_I| = 90^\circ - |\theta_C| \quad (15.15)$$

$f = 5$ kHz - Applying Eq. (15.13) gives

$$X_C = \frac{1}{2\pi fC} = \frac{1}{(2\pi)(5 \times 10^3 \text{ Hz})(0.01 \times 10^{-6} \text{ F})} \cong 3.18 \text{ k}\Omega$$

Note the dramatic drop in X_C from 1 kHz to 5 kHz. In fact, X_C is now less than the resistance R of the network, and the phase angle determined by $\tan^{-1}(X_C/R)$ must be less than 45° . Here,

$$V_C = \frac{X_C E}{\sqrt{R^2 + X_C^2}} = \frac{(3.18 \text{ k}\Omega)(10 \text{ V})}{\sqrt{(5 \text{ k}\Omega)^2 + (3.18 \text{ k}\Omega)^2}} = 5.37 \text{ V}$$

$$\begin{aligned} \text{with } \theta_C &= -\tan^{-1} \frac{R}{X_C} = -\tan^{-1} \frac{5 \text{ k}\Omega}{3.2 \text{ k}\Omega} \\ &= -\tan^{-1} 1.56 = -57.38^\circ \end{aligned}$$

$f = 10$ kHz

$$X_C \cong 1.59 \text{ k}\Omega \quad V_C = 3.03 \text{ V} \quad \theta_C = -72.34^\circ$$

$f = 15$ kHz

$$X_C \cong 1.06 \text{ k}\Omega \quad V_C = 2.07 \text{ V} \quad \theta_C = -78.02^\circ$$

$f = 20$ kHz

$$X_C \cong 795.78 \Omega \quad V_C = 1.57 \text{ V} \quad \theta_C = -80.96^\circ$$

The phasor diagrams for $f = 5$ kHz and $f = 20$ kHz appear in Fig. 15.52 to show the continuing rotation of the V_C vector.

Note also from Figs. 15.51 and 15.52 that the vector V_R and the current **I** have grown in magnitude with the reduction in the capacitive reactance.

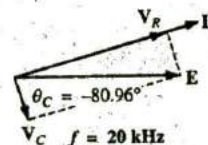
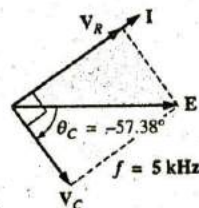


FIG. 15.52

The phasor diagram for the circuit in Fig. 15.47 for $f = 5$ kHz and 20 kHz.

Eventually, at very high frequencies, X_C will approach zero ohms and the short-circuit equivalent can be applied, resulting in $V_C \cong 0$ V and $\theta_C \cong -90^\circ$ and producing the phasor diagram in Fig. 15.53. The network is then resistive, the phase angle between I and E is essentially zero degrees, and V_R and I are their maximum values.

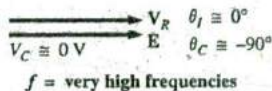


FIG. 15.53

The phasor diagram for the circuit in Fig. 15.47 at very high frequencies.

A plot of V_C versus frequency appears in Fig. 15.54. At low frequencies, $X_C \gg R$, and V_C is very close to E in magnitude. As the applied frequency increases, X_C decreases in magnitude along with V_C as V_R captures more of the applied voltage. A plot of θ_C versus frequency is provided in Fig. 15.55. At low frequencies, the phase angle between V_C and E is very small since $V_C \cong E$. Recall that if two phasors are equal, they must have the same angle. As the applied frequency increases, the network becomes more resistive, and the phase angle between V_C and E approaches 90° . Keep in mind that, at high frequencies, I and E are approaching an in-phase situation, and the angle between V_C and E will approach that between V_C and I , which we know must be 90° (I_C leading V_C).

A plot of V_R versus frequency approaches E volts from zero volts with an increase in frequency, but remember that $V_R \neq E - V_C$ due to the vector relationship. The phase angle between I and E could be plotted directly from Fig. 15.55 using Eq. (15.15).

In Chapter 21, the analysis of this section is extended to a much wider frequency range using a log axis for frequency. It will be demonstrated

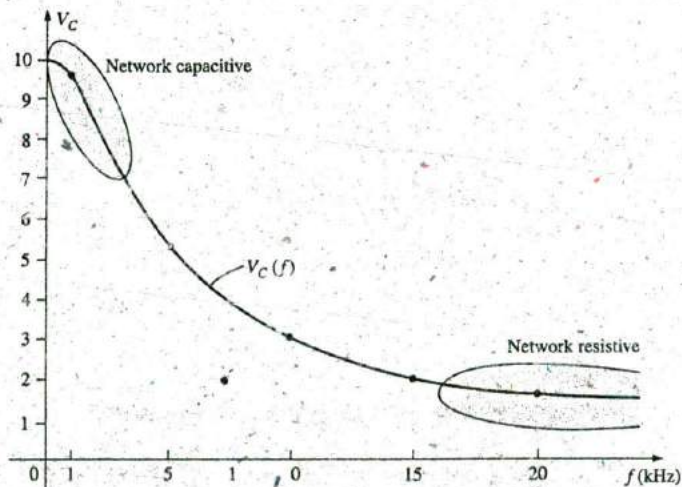
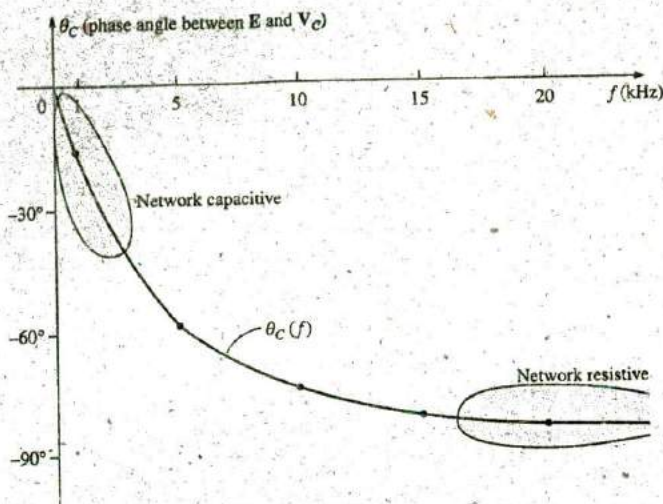


FIG. 15.54

The magnitude of the voltage V_C versus frequency for the circuit in Fig. 15.47.


FIG. 15.55

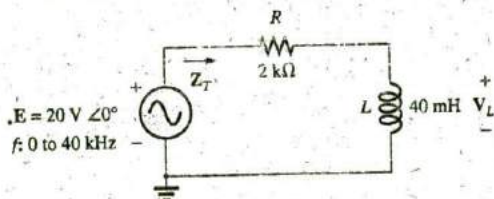
The phase angle between E and V_C versus frequency for the circuit in Fig. 15.47.

that an R - C circuit such as that in Fig. 15.47 can be used as a filter to determine which frequencies will have the greatest impact on the stage to follow. From our current analysis, it is obvious that any network connected across the capacitor will receive the greatest potential level at low frequencies and be effectively "shorted out" at very high frequencies.

The analysis of a series R - L circuit proceeds in much the same manner, except that X_L and V_L increases with frequency and the angle between I and E approaches 90° (voltage leading the current) rather than 0° . If V_L is plotted versus frequency, V_L will approach E , as demonstrated in Example 15.12, and X_L will eventually attain a level at which the open-circuit equivalent is appropriate.

EXAMPLE 15.12 For the series R - L circuit in Fig. 15.56:

- Determine the frequency at which $X_L = R$.
- Develop a mental image of the change in total impedance with frequency without doing any calculations.
- Find the total impedance at $f = 100$ Hz and 40 kHz, and compare your answer with the assumptions of part (b).
- Plot the curve of V_L versus frequency.
- Find the phase angle of the total impedance at $f = 40$ kHz. Can the circuit be considered inductive at this frequency? Why?


FIG. 15.56

Circuit for Example 15.12.

Solutions:

a. $X_L = 2\pi f_1 L = R$

and $f_1 = \frac{R}{2\pi L} = \frac{2 \text{ k}\Omega}{2\pi(40 \text{ mH})} = 7957.7 \text{ Hz}$

- b. At low frequencies, $R > X_L$ and the impedance will be very close to that of the resistor, or $2 \text{ k}\Omega$. As the frequency increases, X_L increases to a point where it is the predominant factor. The result is that the curve starts almost horizontal at $2 \text{ k}\Omega$ and then increases linearly to very high levels.

c. $Z_T = R + jX_L = Z_T \angle \theta_T = \sqrt{R^2 + X_L^2} \angle \tan^{-1} \frac{X_L}{R}$

At $f = 100 \text{ Hz}$:

$$X_L = 2\pi fL = 2\pi(100 \text{ Hz})(40 \text{ mH}) = 25.13 \Omega$$

and
$$Z_T = \sqrt{R^2 + X_L^2} = \sqrt{(2 \text{ k}\Omega)^2 + (25.13 \Omega)^2} = 2000.16 \Omega \approx R$$

At $f = 40 \text{ kHz}$:

$$X_L = 2\pi fL = 2\pi(40 \text{ kHz})(40 \text{ mH}) \approx 10.05 \text{ k}\Omega$$

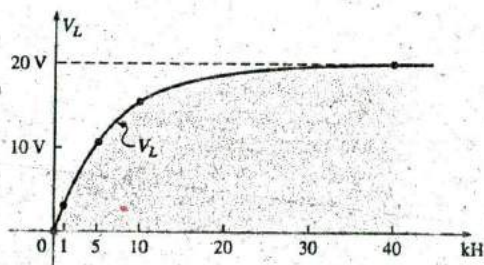
and
$$Z_T = \sqrt{R^2 + X_L^2} = \sqrt{(2 \text{ k}\Omega)^2 + (10.05 \text{ k}\Omega)^2} = 10.25 \text{ k}\Omega \approx X_L$$

Both calculations support the conclusions of part (b).

- d. Applying the voltage divider rule gives

$$V_L = \frac{Z_L E}{Z_T}$$

From part (c), we know that at 100 Hz , $Z_T \approx R$, so that $V_R \approx 20 \text{ V}$ and $V_L \approx 0 \text{ V}$. Part (c) revealed that at 40 kHz , $Z_T \approx X_L$, so that $V_L \approx 20 \text{ V}$ and $V_R \approx 0 \text{ V}$. The result is two plot points for the curve in Fig. 15.57.

**FIG. 15.57**Plotting V_L versus f for the series R - L circuit in Fig. 15.56.

At 1 kHz $X_L = 2\pi fL \approx 0.25 \text{ k}\Omega$

and
$$V_L = \frac{(0.25 \text{ k}\Omega \angle 90^\circ)(20 \text{ V} \angle 0^\circ)}{2 \text{ k}\Omega + j0.25 \text{ k}\Omega} = 2.48 \text{ V} \angle 82.87^\circ$$

At 5 kHz $X_L = 2\pi fL \approx 1.26 \text{ k}\Omega$

and
$$V_L = \frac{(1.26 \text{ k}\Omega \angle 90^\circ)(20 \text{ V} \angle 0^\circ)}{2 \text{ k}\Omega + j1.26 \text{ k}\Omega} = 10.68 \text{ V} \angle 57.79^\circ$$

At 10 kHz $X_L = 2\pi fL \cong 2.5 \text{ k}\Omega$

$$\text{and } V_L = \frac{(2.5 \text{ k}\Omega \angle 90^\circ)(20 \text{ V} \angle 0^\circ)}{2.5 \text{ k}\Omega + j 2.5 \text{ k}\Omega} = 15.63 \text{ V} \angle 38.66^\circ$$

The complete plot appears in Fig. 15.57.

$$e. \theta_T = \tan^{-1} \frac{X_L}{R} = \tan^{-1} \frac{10.05 \text{ k}\Omega}{2 \text{ k}\Omega} = 78.75^\circ$$

The angle θ_T is closing in on the 90° of a purely inductive network. Therefore, the network can be considered quite inductive at a frequency of 40 kHz.

15.6 SUMMARY: SERIES ac CIRCUITS

The following is a review of important conclusions that can be derived from the discussion and examples of the previous sections. The list is not all-inclusive, but it does emphasize some of the conclusions that should be carried forward in the future analysis of ac systems.

For series ac circuits with reactive elements:

1. The total impedance will be frequency dependent.
2. The impedance of any one element can be greater than the total impedance of the network.
3. The inductive and capacitive reactances are always in direct opposition on an impedance diagram.
4. Depending on the frequency applied, the same circuit can be either predominantly inductive or predominantly capacitive.
5. At lower frequencies, the capacitive elements will usually have the most impact on the total impedance, while at high frequencies, the inductive elements will usually have the most impact.
6. The magnitude of the voltage across any one element can be greater than the applied voltage.
7. The magnitude of the voltage across an element compared to the other elements of the circuit is directly related to the magnitude of its impedance; that is, the larger the impedance of an element, the larger is the magnitude of the voltage across the element.
8. The voltages across a coil or capacitor are always in direct opposition on a phasor diagram.
9. The current is always in phase with the voltage across the resistive elements, lags the voltage across all the inductive elements by 90° , and leads the voltage across all the capacitive elements by 90° .
10. The larger the resistive element of a circuit compared to the net reactive impedance, the closer is the power factor to unity.

PARALLEL ac CIRCUITS

15.7 ADMITTANCE AND SUSCEPTANCE

The discussion for parallel ac circuits is very similar to that for dc circuits. In dc circuits, conductance (G) was defined as being equal to $1/R$. The total conductance of a parallel circuit was then found by adding the conductance of each branch. The total resistance R_T is simply $1/G_T$.

In ac circuits, we define admittance (Y) as being equal to $1/Z$. The unit of measure for admittance as defined by the SI system is *siemens*,

which has the symbol S . Admittance is a measure of how well an ac circuit will *admit*, or allow, current to flow in the circuit. The larger its value, therefore, the heavier is the current flow for the same applied potential. The total admittance of a circuit can also be found by finding the sum of the parallel admittances. The total impedance Z_T of the circuit is then $1/Y_T$; that is, for the network in Fig. 15.58,

$$Y_T = Y_1 + Y_2 + Y_3 + \cdots + Y_N \quad (15.16)$$

and

$$Z_T = \frac{1}{Y_T} \quad (15.17)$$

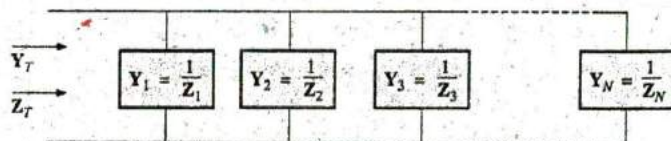


FIG. 15.58
Parallel ac network.

or, since $Z = 1/Y$,

$$\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \cdots + \frac{1}{Z_N} \quad (15.18)$$

and

$$Z_T = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \cdots + \frac{1}{Z_N}} \quad (15.19)$$

matching Eq. (6.3) for dc networks.

For two impedances in parallel,

$$\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

If the manipulations used in Chapter 6 to find the total resistance of two parallel resistors are now applied, the following similar equation results:

$$Z_T = \frac{Z_1 Z_2}{Z_1 + Z_2} \quad (15.20)$$

For N parallel equal impedances (Z_1), the total impedance is determined by

$$Z_T = \frac{Z_1}{N} \quad (15.21)$$

For three parallel impedances,

$$Z_T = \frac{Z_1 Z_2 Z_3}{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3} \quad (15.22)$$

As pointed out in the introduction to this section, conductance is the reciprocal of resistance, and

$$Y_R = \frac{1}{Z_R} = \frac{1}{R \angle 0^\circ} = G \angle 0^\circ \quad (15.23)$$

The reciprocal of reactance ($1/X$) is called **susceptance** and is a measure of how *susceptible* an element is to the passage of current through it. Susceptance is also measured in *siemens* and is represented by the capital letter B .

For the inductor,

$$Y_L = \frac{1}{Z_L} = \frac{1}{X_L \angle 90^\circ} = \frac{1}{X_L} \angle -90^\circ \quad (15.24)$$

Defining

$$B_L = \frac{1}{X_L} \quad (\text{siemens, S}) \quad (15.25)$$

we have

$$Y_L = B_L \angle -90^\circ \quad (15.26)$$

Note that for inductance, an increase in frequency or inductance will result in a decrease in susceptance or, correspondingly, in admittance.

For the capacitor,

$$Y_C = \frac{1}{Z_C} = \frac{1}{X_C \angle -90^\circ} = \frac{1}{X_C} \angle 90^\circ \quad (15.27)$$

Defining

$$B_C = \frac{1}{X_C} \quad (\text{siemens, S}) \quad (15.28)$$

we have

$$Y_C = B_C \angle 90^\circ \quad (15.29)$$

For the capacitor, therefore, an increase in frequency or capacitance will result in an increase in its susceptibility.

For parallel ac circuits, the **admittance diagram** is used with the three admittances, represented as shown in Fig. 15.59.

Note in Fig. 15.59 that the conductance (like resistance) is on the positive real axis, whereas inductive and capacitive susceptances are in direct opposition on the imaginary axis.

For any configuration (series, parallel, series-parallel, and so on), the angle associated with the total admittance is the angle by which the source current leads the applied voltage. For inductive networks, θ_T is negative, whereas for capacitive networks, θ_T is positive.

For parallel ac networks, the components of the configuration and the desired quantities determine whether to use an impedance or admittance approach. If the total impedance is requested, the most direct route may be to use impedance parameters. However, sometimes using admittance parameters can also be very efficient, as demonstrated in some of the examples in the rest of the text. In general, use the approach with which you are more comfortable. Naturally, if the format of the desired quantity is spelled out, it is usually best to work with those parameters.

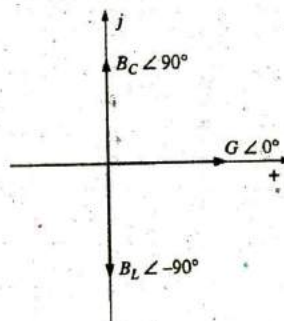


FIG. 15.59
Admittance diagram.

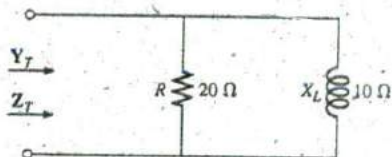


FIG. 15.60
Example 15.13.

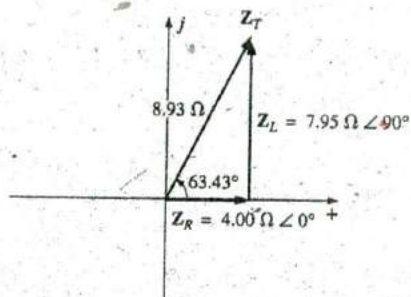


FIG. 15.61
Impedance diagram for the network in Fig. 15.60.

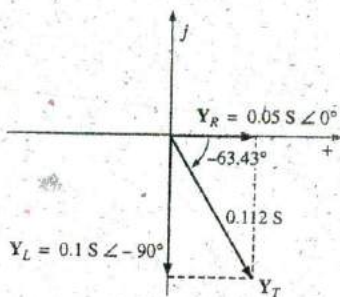


FIG. 15.62
Admittance diagram for the network in Fig. 15.60.

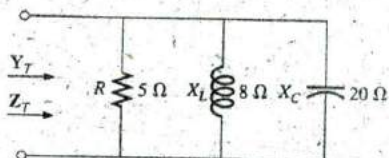


FIG. 15.63
Example 15.14.

EXAMPLE 15.13 For the network in Fig. 15.60:

- Calculate the input impedance.
- Draw the impedance diagram.
- Find the admittance of each parallel branch.
- Determine the input admittance and draw the admittance diagram.

Solutions:

$$\begin{aligned} \text{a. } Z_T &= \frac{Z_R Z_L}{Z_R + Z_L} = \frac{(20 \Omega \angle 0^\circ)(10 \Omega \angle 90^\circ)}{20 \Omega + j10 \Omega} \\ &= \frac{200 \Omega \angle 90^\circ}{22.361 \angle 26.57^\circ} = 8.93 \Omega \angle 63.43^\circ \\ &= 4.00 \Omega + j7.95 \Omega = R_T + jX_{LT} \end{aligned}$$

b. The impedance diagram appears in Fig. 15.61.

$$\begin{aligned} \text{c. } Y_R &= G \angle 0^\circ = \frac{1}{R} \angle 0^\circ = \frac{1}{20 \Omega} \angle 0^\circ = 0.05 \text{ S} \angle 0^\circ \\ &= 0.05 \text{ S} + j0 \end{aligned}$$

$$\begin{aligned} Y_L &= B_L \angle -90^\circ = \frac{1}{X_L} \angle -90^\circ = \frac{1}{10 \Omega} \angle -90^\circ \\ &= 0.1 \text{ S} \angle -90^\circ = 0 - j0.1 \text{ S} \end{aligned}$$

$$\begin{aligned} \text{d. } Y_T &= Y_R + Y_L = (0.05 \text{ S} + j0) + (0 - j0.1 \text{ S}) \\ &= 0.05 \text{ S} - j0.1 \text{ S} = G - jB_L \end{aligned}$$

The admittance diagram appears in Fig. 15.62.

EXAMPLE 15.14 Repeat Example 15.13 for the parallel network in Fig. 15.63.

Solutions:

$$\begin{aligned} \text{a. } Z_T &= \frac{1}{\frac{1}{Z_R} + \frac{1}{Z_L} + \frac{1}{Z_C}} \\ &= \frac{1}{\frac{1}{5 \Omega \angle 0^\circ} + \frac{1}{8 \Omega \angle 90^\circ} + \frac{1}{20 \Omega \angle -90^\circ}} \\ &= \frac{1}{0.2 \text{ S} \angle 0^\circ + 0.125 \text{ S} \angle -90^\circ + 0.05 \text{ S} \angle 90^\circ} \\ &= \frac{1}{0.2 \text{ S} - j0.075 \text{ S}} = \frac{1}{0.2136 \text{ S} \angle -20.56^\circ} \\ &= 4.68 \Omega \angle 20.56^\circ \end{aligned}$$

OR

$$\begin{aligned} Z_T &= \frac{Z_R Z_L Z_C}{Z_R Z_L + Z_L Z_C + Z_R Z_C} \\ &= \frac{(5 \Omega \angle 0^\circ)(8 \Omega \angle 90^\circ)(20 \Omega \angle -90^\circ)}{(5 \Omega \angle 0^\circ)(8 \Omega \angle 90^\circ) + (8 \Omega \angle 90^\circ)(20 \Omega \angle -90^\circ) + (5 \Omega \angle 0^\circ)(20 \Omega \angle -90^\circ)} \\ &= \frac{800 \Omega \angle 0^\circ}{40 \angle 90^\circ + 160 \angle 0^\circ + 100 \angle -90^\circ} \end{aligned}$$

$$\begin{aligned}
 &= \frac{800 \Omega}{160 + j40 - j100} = \frac{800 \Omega}{160 - j60} \\
 &= \frac{800 \Omega}{170.88 \angle -20.56^\circ} \\
 &= 4.68 \Omega \angle 20.56^\circ = 4.38 \Omega + j1.64 \Omega
 \end{aligned}$$

b. The impedance diagram appears in Fig. 15.64.

$$\begin{aligned}
 c. \ Y_R &= G \angle 0^\circ = \frac{1}{R} \angle 0^\circ = \frac{1}{5 \Omega} \angle 0^\circ \\
 &= 0.2 \text{ S} \angle 0^\circ = 0.2 \text{ S} + j0
 \end{aligned}$$

$$\begin{aligned}
 Y_L &= B_L \angle -90^\circ = \frac{1}{X_L} \angle -90^\circ = \frac{1}{8 \Omega} \angle -90^\circ \\
 &= 0.125 \text{ S} \angle -90^\circ = 0 - j0.125 \text{ S}
 \end{aligned}$$

$$\begin{aligned}
 Y_C &= B_C \angle 90^\circ = \frac{1}{X_C} \angle 90^\circ = \frac{1}{20 \Omega} \angle 90^\circ \\
 &= 0.050 \text{ S} \angle +90^\circ = 0 + j0.050 \text{ S}
 \end{aligned}$$

$$\begin{aligned}
 d. \ Y_T &= Y_R + Y_L + Y_C \\
 &= (0.2 \text{ S} + j0) + (0 - j0.125 \text{ S}) + (0 + j0.050 \text{ S}) \\
 &= 0.2 \text{ S} - j0.075 \text{ S} = 0.214 \text{ S} \angle -20.56^\circ
 \end{aligned}$$

The admittance diagram appears in Fig. 15.65.

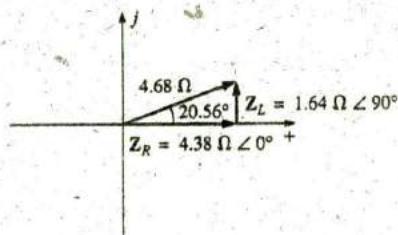


FIG. 15.64

Impedance diagram for the network in Fig. 15.63.

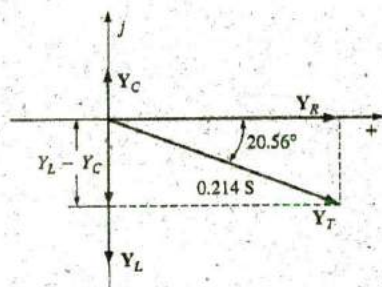


FIG. 15.65

Admittance diagram for the network in Fig. 15.63.

On many occasions, the inverse relationship $Y_T = 1/Z_T$ or $Z_T = 1/Y_T$ will require that we divide the number 1 by a complex number having a real and an imaginary part. This division, if not performed in the polar form, requires that we multiply the numerator and denominator by the conjugate of the denominator, as follows:

$$\begin{aligned}
 Y_T &= \frac{1}{Z_T} = \frac{1}{4 \Omega + j6 \Omega} = \left(\frac{1}{4 \Omega + j6 \Omega} \right) \left(\frac{(4 \Omega - j6 \Omega)}{(4 \Omega - j6 \Omega)} \right) \\
 &= \frac{4 - j6}{4^2 + 6^2}
 \end{aligned}$$

$$\text{and} \quad Y_T = \frac{4}{52} \text{ S} - j \frac{6}{52} \text{ S}$$

To avoid this laborious task each time we want to find the reciprocal of a complex number in rectangular form, a format can be developed using the following complex number, which is symbolic of any impedance or admittance in the first or fourth quadrant:

$$\frac{1}{a_1 \pm jb_1} = \left(\frac{1}{a_1 \pm jb_1} \right) \left(\frac{a_1 \mp jb_1}{a_1 \mp jb_1} \right) = \frac{a_1 \mp jb_1}{a_1^2 + b_1^2}$$

$$\text{or} \quad \boxed{\frac{1}{a_1 \pm jb_1} = \frac{a_1}{a_1^2 + b_1^2} \mp j \frac{b_1}{a_1^2 + b_1^2}} \quad (15.30)$$

Note that the denominator is simply the sum of the squares of each term. The sign is inverted between the real and imaginary parts. A few examples will develop some familiarity with the use of this equation.

EXAMPLE 15.15 Find the admittance of each set of series elements in Fig. 15.66.

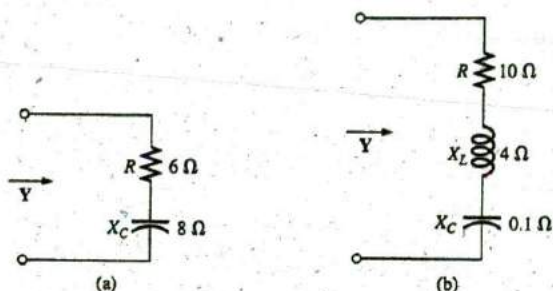


FIG. 15.66
Example 15.15.

Solutions:

$$a. Z = R - jX_C = 6 \Omega - j8 \Omega$$

Eq. (15.30):

$$Y = \frac{1}{6 \Omega - j8 \Omega} = \frac{6}{(6)^2 + (8)^2} + j \frac{8}{(6)^2 + (8)^2}$$

$$= \frac{6}{100} \text{ S} + j \frac{8}{100} \text{ S}$$

$$b. Z = 10 \Omega + j4 \Omega + (-j0.1 \Omega) = 10 \Omega + j3.9 \Omega$$

Eq. (15.30):

$$Y = \frac{1}{Z} = \frac{1}{10 \Omega + j3.9 \Omega} = \frac{10}{(10)^2 + (3.9)^2} - j \frac{3.9}{(10)^2 + (3.9)^2}$$

$$= \frac{10}{115.21} - j \frac{3.9}{115.21} = 0.087 \text{ S} - j0.034 \text{ S}$$

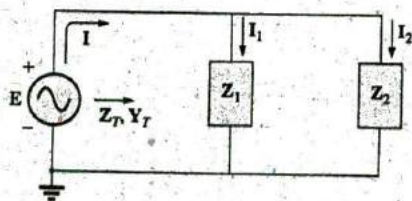


FIG. 15.67
Parallel ac network.

15.8 PARALLEL ac NETWORKS

For the representative parallel ac network in Fig. 15.67, the total impedance or admittance is determined as described in the previous section, and the source current is determined by Ohm's law as follows:

$$I = \frac{E}{Z_T} = EY_T \quad (15.31)$$

Since the voltage is the same across parallel elements, the current through each branch can then be found through another application of Ohm's law:

$$I_1 = \frac{E}{Z_1} = EY_1 \quad (15.32)$$

$$I_2 = \frac{E}{Z_2} = EY_2 \quad (15.33)$$

Kirchhoff's current law can then be applied in the same manner as used for dc networks. However, keep in mind that we are now dealing with the algebraic manipulation of quantities that have both magnitude and direction. We have

$$I - I_1 - I_2 = 0$$

or
$$I = I_1 + I_2 \quad (15.34)$$

The power to the network can be determined by

$$P = EI \cos \theta_T \quad (15.35)$$

where θ_T is the phase angle between E and I .

Let us now look at a few examples carried out in great detail for the first exposure.

R-L

Refer to Fig. 15.68.

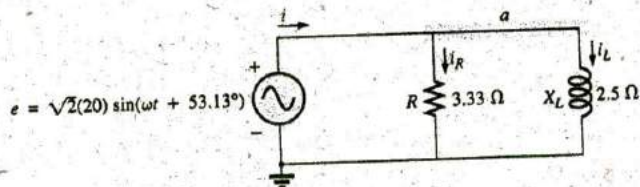


FIG. 15.68
Parallel R-L network.

Phasor Notation As shown in Fig. 15.69.

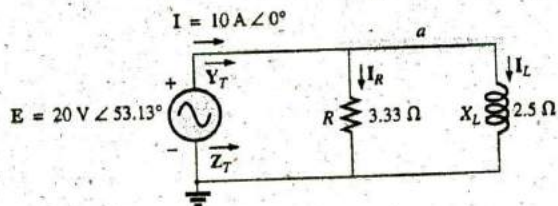


FIG. 15.69
Applying phasor notation to the network in Fig. 15.68.

$$\begin{aligned} Y_T &= Y_R + Y_L \\ &= G \angle 0^\circ + B_L \angle -90^\circ = \frac{1}{3.33 \Omega} \angle 0^\circ + \frac{1}{2.5 \Omega} \angle -90^\circ \\ &= 0.3 \text{ S} \angle 0^\circ + 0.4 \text{ S} \angle -90^\circ = 0.3 \text{ S} - j 0.4 \text{ S} \\ &= 0.5 \text{ S} \angle -53.13^\circ \\ Z_T &= \frac{1}{Y_T} = \frac{1}{0.5 \text{ S} \angle -53.13^\circ} = 2 \Omega \angle 53.13^\circ \end{aligned}$$

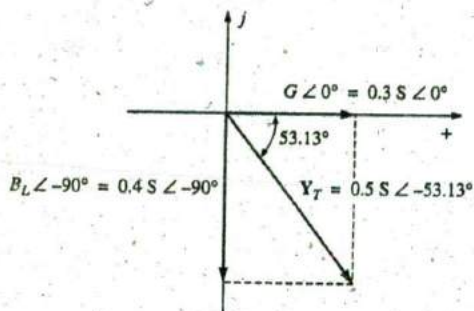


FIG. 15.70

Admittance diagram for the parallel R-L network in Fig. 15.68.

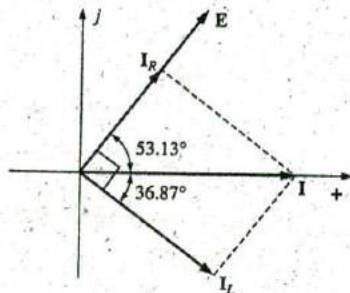


FIG. 15.71

Phasor diagram for the parallel R-L network in Fig. 15.68.

Admittance diagram: As shown in Fig. 15.70.

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \mathbf{E} \mathbf{Y}_T = (20 \text{ V } \angle 53.13^\circ)(0.5 \text{ S } \angle -53.13^\circ) = 10 \text{ A } \angle 0^\circ$$

$$\begin{aligned} \mathbf{I}_R &= \frac{\mathbf{E} \angle \theta}{R \angle 0^\circ} = (E \angle \theta)(G \angle 0^\circ) \\ &= (20 \text{ V } \angle 53.13^\circ)(0.3 \text{ S } \angle 0^\circ) = 6 \text{ A } \angle 53.13^\circ \end{aligned}$$

$$\begin{aligned} \mathbf{I}_L &= \frac{\mathbf{E} \angle \theta}{X_L \angle 90^\circ} = (E \angle \theta)(B_L \angle -90^\circ) \\ &= (20 \text{ V } \angle 53.13^\circ)(0.4 \text{ S } \angle -90^\circ) \\ &= 8 \text{ A } \angle -36.87^\circ \end{aligned}$$

Kirchhoff's current law: At node a,

$$\mathbf{I} - \mathbf{I}_R - \mathbf{I}_L = 0$$

or

$$\mathbf{I} = \mathbf{I}_R + \mathbf{I}_L$$

$$10 \text{ A } \angle 0^\circ = 6 \text{ A } \angle 53.13^\circ + 8 \text{ A } \angle -36.87^\circ$$

$$10 \text{ A } \angle 0^\circ = (3.60 \text{ A} + j 4.80 \text{ A}) + (6.40 \text{ A} - j 4.80 \text{ A}) = 10 \text{ A} + j 0$$

and

$$10 \text{ A } \angle 0^\circ = 10 \text{ A } \angle 0^\circ \quad (\text{checks})$$

Phasor diagram: The phasor diagram in Fig. 15.71 indicates that the applied voltage \mathbf{E} is in phase with the current \mathbf{I}_R and leads the current \mathbf{I}_L by 90° .

Power: The total power in watts delivered to the circuit is

$$\begin{aligned} P_T &= EI \cos \theta_T \\ &= (20 \text{ V})(10 \text{ A}) \cos 53.13^\circ = (200 \text{ W})(0.6) \\ &= 120 \text{ W} \end{aligned}$$

$$\text{or} \quad P_T = I^2 R = \frac{V_R^2}{R} = V_R^2 G = (20 \text{ V})^2(0.3 \text{ S}) = 120 \text{ W}$$

or, finally,

$$\begin{aligned} P_T &= P_R + P_L = EI_R \cos \theta_R + EI_L \cos \theta_L \\ &= (20 \text{ V})(6 \text{ A}) \cos 0^\circ + (20 \text{ V})(8 \text{ A}) \cos 90^\circ = 120 \text{ W} + 0 \\ &= 120 \text{ W} \end{aligned}$$

Power factor: The power factor of the circuit is

$$F_p = \cos \theta_T = \cos 53.13^\circ = 0.6 \text{ lagging}$$

or, through an analysis similar to that used for a series ac circuit,

$$\cos \theta_T = \frac{P}{EI} = \frac{E^2/R}{EI} = \frac{EG}{I} = \frac{G}{I/V} = \frac{G}{Y_T}$$

and

$$F_p = \cos \theta_T = \frac{G}{Y_T} \quad (15.36)$$

where G and Y_T are the magnitudes of the total conductance and admittance of the parallel network. For this case,

$$F_p = \cos \theta_T = \frac{0.3 \text{ S}}{0.5 \text{ S}} = 0.6 \text{ lagging}$$

Impedance approach: The current \mathbf{I} can also be found by first finding the total impedance of the network:

$$\begin{aligned} Z_T &= \frac{Z_R Z_L}{Z_R + Z_L} = \frac{(3.33 \Omega \angle 0^\circ)(2.5 \Omega \angle 90^\circ)}{3.33 \Omega \angle 0^\circ + 2.5 \Omega \angle 90^\circ} \\ &= \frac{8.325 \angle 90^\circ}{4.164 \angle 36.87^\circ} = 2 \Omega \angle 53.13^\circ \end{aligned}$$

Then, using Ohm's law, we obtain

$$I = \frac{E}{Z_T} = \frac{20 \text{ V} \angle 53.13^\circ}{2 \Omega \angle 53.13^\circ} = 10 \text{ A} \angle 0^\circ$$

R-C

Refer to Fig. 15.72.

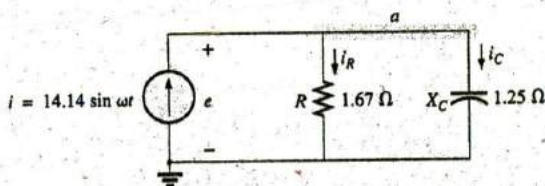


FIG. 15.72
Parallel R-C network.

Phasor Notation As shown in Fig. 15.73.

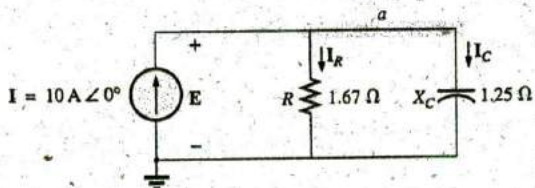


FIG. 15.73
Applying phasor notation to the network in Fig. 15.72.

$$\begin{aligned} Y_T &= Y_R + Y_C = G \angle 0^\circ + B_C \angle 90^\circ = \frac{1}{1.67 \Omega} \angle 0^\circ + \frac{1}{1.25 \Omega} \angle 90^\circ \\ &= 0.6 \text{ S} \angle 0^\circ + 0.8 \text{ S} \angle 90^\circ = 0.6 \text{ S} + j0.8 \text{ S} = 1.0 \text{ S} \angle 53.13^\circ \end{aligned}$$

$$Z_T = \frac{1}{Y_T} = \frac{1}{1.0 \text{ S} \angle 53.13^\circ} = 1 \Omega \angle -53.13^\circ$$

Admittance diagram: As shown in Fig. 15.74.

$$E = I Z_T = \frac{I}{Y_T} = \frac{10 \text{ A} \angle 0^\circ}{1 \text{ S} \angle 53.13^\circ} = 10 \text{ V} \angle -53.13^\circ$$

$$\begin{aligned} I_R &= (E \angle \theta)(G \angle 0^\circ) \\ &= (10 \text{ V} \angle -53.13^\circ)(0.6 \text{ S} \angle 0^\circ) = 6 \text{ A} \angle -53.13^\circ \end{aligned}$$

$$\begin{aligned} I_C &= (E \angle \theta)(B_C \angle 90^\circ) \\ &= (10 \text{ V} \angle -53.13^\circ)(0.8 \text{ S} \angle 90^\circ) = 8 \text{ A} \angle 36.87^\circ \end{aligned}$$

Kirchhoff's current law: At node a,

$$I - I_R - I_C = 0$$

or

$$I = I_R + I_C$$

which can also be verified (as for the R-L network) through vector algebra.

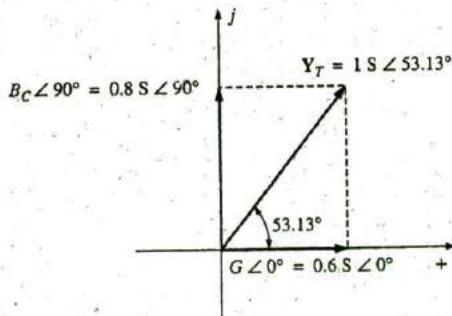


FIG. 15.74
Admittance diagram for the parallel R-C network in Fig. 15.72.

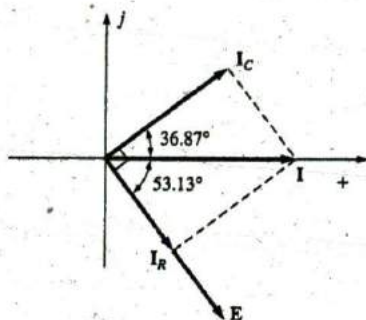


FIG. 15.75
Phasor diagram for the parallel R-C network
in Fig. 15.74.

Phasor diagram: The phasor diagram in Fig. 15.75 indicates that E is in phase with the current through the resistor I_R and lags the capacitive current I_C by 90° .

Time domain:

$$e = \sqrt{2}(10) \sin(\omega t - 53.13^\circ) = 14.14 \sin(\omega t - 53.13^\circ)$$

$$i_R = \sqrt{2}(6) \sin(\omega t - 53.13^\circ) = 8.48 \sin(\omega t - 53.13^\circ)$$

$$i_C = \sqrt{2}(8) \sin(\omega t + 36.87^\circ) = 11.31 \sin(\omega t + 36.87^\circ)$$

A plot of all of the currents and the voltage appears in Fig. 15.76. Note that e and i_R are in phase and e lags i_C by 90° .

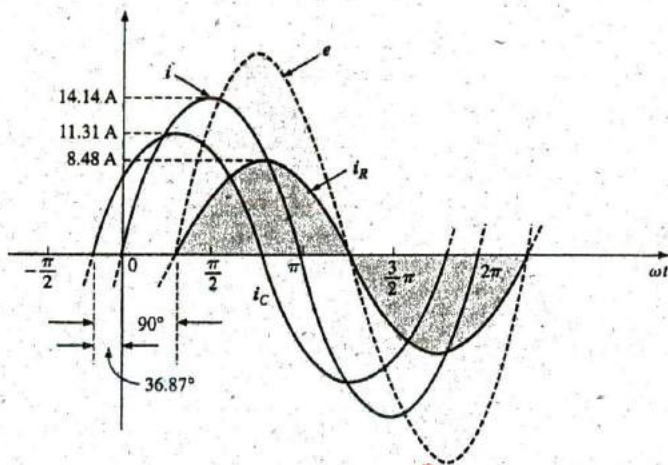


FIG. 15.76

Waveforms for the parallel R-C network in Fig. 15.72.

Power:

$$P_T = EI \cos \theta = (10 \text{ V})(10 \text{ A}) \cos 53.13^\circ = (10)^2(0.6) = 60 \text{ W}$$

or

$$P_T = E^2 G = (10 \text{ V})^2(0.6 \text{ S}) = 60 \text{ W}$$

or, finally,

$$\begin{aligned} P_T &= P_R + P_C = EI_R \cos \theta_R + EI_C \cos \theta_C \\ &= (10 \text{ V})(6 \text{ A}) \cos 0^\circ + (10 \text{ V})(8 \text{ A}) \cos 90^\circ \\ &= 60 \text{ W} \end{aligned}$$

Power factor: The power factor of the circuit is

$$F_p = \cos 53.13^\circ = 0.6 \text{ leading}$$

Using Eq. (15.36), we have

$$F_p = \cos \theta_T = \frac{G}{Y_T} = \frac{0.6 \text{ S}}{1.0 \text{ S}} = 0.6 \text{ leading}$$

Impedance approach: The voltage E can also be found by first finding the total impedance of the circuit:

$$\begin{aligned} Z_T &= \frac{Z_R Z_C}{Z_R + Z_C} = \frac{(1.67 \Omega \angle 0^\circ)(1.25 \Omega \angle -90^\circ)}{1.67 \Omega \angle 0^\circ + 1.25 \Omega \angle -90^\circ} \\ &= \frac{2.09 \angle -90^\circ}{2.09 \angle -36.81^\circ} = 1 \Omega \angle -53.19^\circ \end{aligned}$$

and then, using Ohm's law, we find

$$E = IZ_T = (10 \text{ A } \angle 0^\circ)(1 \Omega \angle -53.19^\circ) = 10 \text{ V } \angle -53.19^\circ$$

R-L-C

Refer to Fig. 15.77.

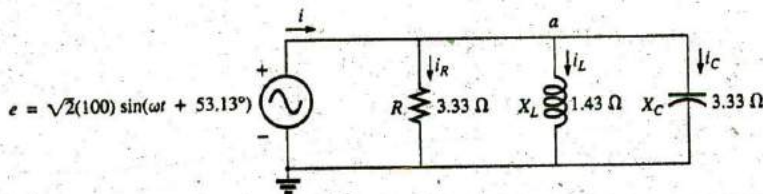


FIG. 15.77
Parallel R-L-C ac network.

Phasor notation: As shown in Fig. 15.78.

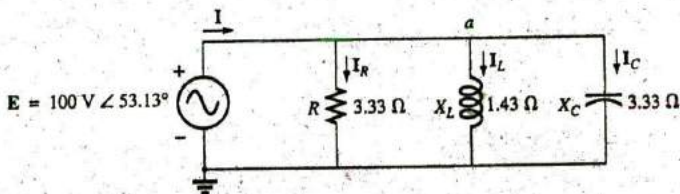


FIG. 15.78
Applying phasor notation to the network in Fig. 15.77.

$$\begin{aligned} Y_T &= Y_R + Y_L + Y_C = G \angle 0^\circ + B_L \angle -90^\circ + B_C \angle 90^\circ \\ &= \frac{1}{3.33 \Omega} \angle 0^\circ + \frac{1}{1.43 \Omega} \angle -90^\circ + \frac{1}{3.33 \Omega} \angle 90^\circ \\ &= 0.3 \text{ S } \angle 0^\circ + 0.7 \text{ S } \angle -90^\circ + 0.3 \text{ S } \angle 90^\circ \\ &= 0.3 \text{ S} - j0.7 \text{ S} + j0.3 \text{ S} \\ &= 0.3 \text{ S} - j0.4 \text{ S} = 0.5 \text{ S } \angle -53.13^\circ \end{aligned}$$

$$Z_T = \frac{1}{Y_T} = \frac{1}{0.5 \text{ S } \angle -53.13^\circ} = 2 \Omega \angle 53.13^\circ$$

Admittance diagram: As shown in Fig. 15.79.

$$I = \frac{E}{Z_T} = EY_T = (100 \text{ V } \angle 53.13^\circ)(0.5 \text{ S } \angle -53.13^\circ) = 50 \text{ A } \angle 0^\circ$$

$$I_R = (E \angle \theta)(G \angle 0^\circ) = (100 \text{ V } \angle 53.13^\circ)(0.3 \text{ S } \angle 0^\circ) = 30 \text{ A } \angle 53.13^\circ$$

$$I_L = (E \angle \theta)(B_L \angle -90^\circ) = (100 \text{ V } \angle 53.13^\circ)(0.7 \text{ S } \angle -90^\circ) = 70 \text{ A } \angle -36.87^\circ$$

$$I_C = (E \angle \theta)(B_C \angle 90^\circ) = (100 \text{ V } \angle 53.13^\circ)(0.3 \text{ S } \angle +90^\circ) = 30 \text{ A } \angle 143.13^\circ$$

Kirchhoff's current law: At node a,

$$I - I_R - I_L - I_C = 0$$

$$I = I_R + I_L + I_C$$

or

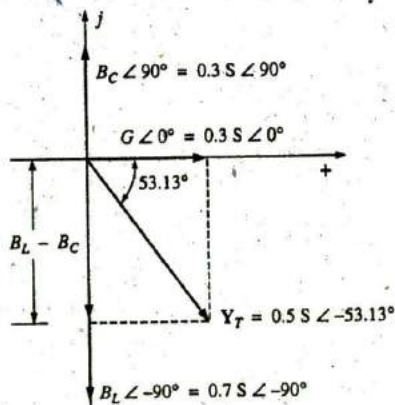


FIG. 15.79
Admittance diagram for the parallel R-L-C network in Fig. 15.77.

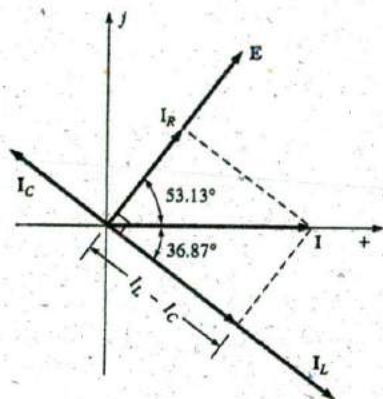


FIG. 15.80

Phasor diagram for the parallel R-L-C network in Fig. 15.77.

Phasor diagram: The phasor diagram in Fig. 15.80 indicates that the impressed voltage E is in phase with the current I_R through the resistor, leads the current I_L through the inductor by 90° , and lags the current I_C of the capacitor by 90° .

Time domain:

$$i = \sqrt{2}(50) \sin \omega t = 70.70 \sin \omega t$$

$$i_R = \sqrt{2}(30) \sin(\omega t + 53.13^\circ) = 42.42 \sin(\omega t + 53.13^\circ)$$

$$i_L = \sqrt{2}(70) \sin(\omega t - 36.87^\circ) = 98.98 \sin(\omega t - 36.87^\circ)$$

$$i_C = \sqrt{2}(30) \sin(\omega t + 143.13^\circ) = 42.42 \sin(\omega t + 143.13^\circ)$$

A plot of all of the currents and the impressed voltage appears in Fig. 15.81.

Power: The total power in watts delivered to the circuit is

$$\begin{aligned} P_T &= EI \cos \theta = (100 \text{ V})(50 \text{ A}) \cos 53.13^\circ = (5000)(0.6) \\ &= 3000 \text{ W} \end{aligned}$$

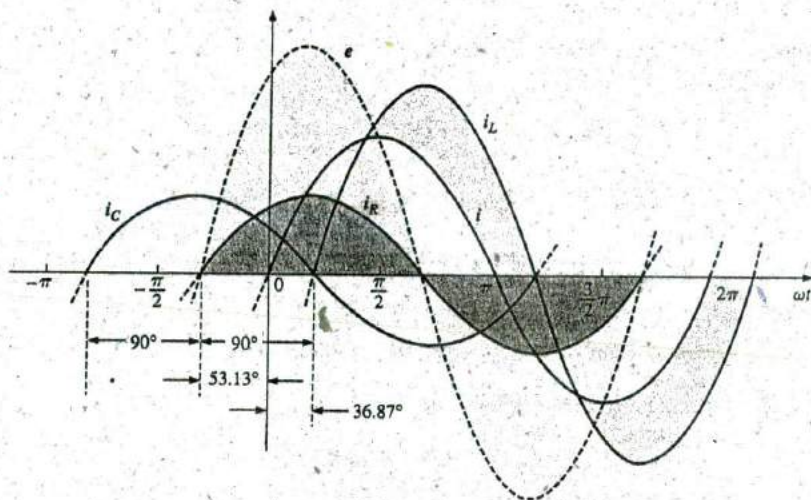


FIG. 15.81

Waveforms for the parallel R-L-C network in Fig. 15.77.

$$\text{or } P_T = E^2 G = (100 \text{ V})^2 (0.3 \text{ S}) = 3000 \text{ W}$$

or, finally,

$$\begin{aligned} P_T &= P_R + P_L + P_C \\ &= EI_R \cos \theta_R + EI_L \cos \theta_L + EI_C \cos \theta_C \\ &= (100 \text{ V})(30 \text{ A}) \cos 0^\circ + (100 \text{ V})(70 \text{ A}) \cos 90^\circ \\ &\quad + (100 \text{ V})(30 \text{ A}) \cos 90^\circ \\ &= 3000 \text{ W} + 0 + 0 \\ &= 3000 \text{ W} \end{aligned}$$

Power factor: The power factor of the circuit is

$$F_p = \cos \theta_T = \cos 53.13^\circ = 0.6 \text{ lagging}$$

Using Eq. (15.36), we obtain -

$$F_p = \cos \theta_T = \frac{G}{Y_T} = \frac{0.3 \text{ S}}{0.5 \text{ S}} = 0.6 \text{ lagging}$$

Impedance approach: The input current I can also be determined by first finding the total impedance in the following manner:

$$Z_T = \frac{Z_R Z_L Z_C}{Z_R Z_L + Z_L Z_C + Z_R Z_C} = 2 \Omega \angle 53.13^\circ$$

and, applying Ohm's law, we obtain

$$I = \frac{E}{Z_T} = \frac{100 \text{ V} \angle 53.13^\circ}{2 \Omega \angle 53.13^\circ} = 50 \text{ A} \angle 0^\circ$$

15.9 CURRENT DIVIDER RULE

The basic format for the **current divider rule** in ac circuits is exactly the same as that for dc circuits; that is, for two parallel branches with impedances Z_1 and Z_2 as shown in Fig. 15.82,

$$I_1 = \frac{Z_2 I_T}{Z_1 + Z_2} \quad \text{or} \quad I_2 = \frac{Z_1 I_T}{Z_1 + Z_2} \quad (15.37)$$

EXAMPLE 15.16 Using the current divider rule, find the current through each impedance in Fig. 15.83.

Solution:

$$I_R = \frac{Z_L I_T}{Z_R + Z_L} = \frac{(4 \Omega \angle 90^\circ)(20 \text{ A} \angle 0^\circ)}{3 \Omega \angle 0^\circ + 4 \Omega \angle 90^\circ} = \frac{80 \text{ A} \angle 90^\circ}{5 \angle 53.13^\circ} = 16 \text{ A} \angle 36.87^\circ$$

$$I_L = \frac{Z_R I_T}{Z_R + Z_L} = \frac{(3 \Omega \angle 0^\circ)(20 \text{ A} \angle 0^\circ)}{5 \Omega \angle 53.13^\circ} = \frac{60 \text{ A} \angle 0^\circ}{5 \angle 53.13^\circ} = 12 \text{ A} \angle -53.13^\circ$$

EXAMPLE 15.17 Using the current divider rule, find the current through each parallel branch in Fig. 15.84.

Solution:

$$I_{R-L} = \frac{Z_C I_T}{Z_C + Z_{R-L}} = \frac{(2 \Omega \angle -90^\circ)(5 \text{ A} \angle 30^\circ)}{-j2 \Omega + 1 \Omega + j8 \Omega} = \frac{10 \text{ A} \angle -60^\circ}{1 + j6} = \frac{10 \text{ A} \angle -60^\circ}{6.083 \angle 80.54^\circ} \approx 1.64 \text{ A} \angle -140.54^\circ$$

$$I_C = \frac{Z_{R-L} I_T}{Z_{R-L} + Z_C} = \frac{(1 \Omega + j8 \Omega)(5 \text{ A} \angle 30^\circ)}{6.08 \angle 80.54^\circ} = \frac{(8.06 \angle 82.87^\circ)(5 \text{ A} \angle 30^\circ)}{6.08 \angle 80.54^\circ} = \frac{40.30 \text{ A} \angle 112.87^\circ}{6.083 \angle 80.54^\circ} = 6.63 \text{ A} \angle 32.33^\circ$$

15.10 FREQUENCY RESPONSE OF PARALLEL ELEMENTS

Recall that for elements in series, the total impedance is the direct sum of the impedances of each element, and the largest real or imaginary component has the most impact on the total impedance. For parallel elements, it

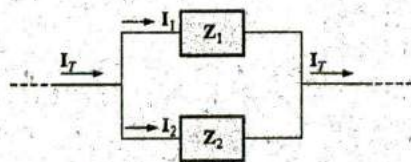


FIG. 15.82

Applying the current divider rule.

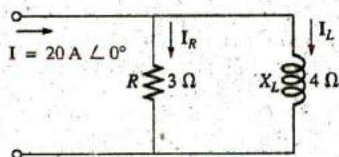


FIG. 15.83

Example 15.16.

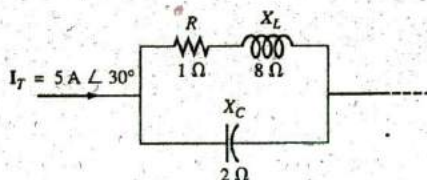


FIG. 15.84

Example 15.17.

is important to remember that *the smallest parallel resistor or the smallest parallel reactance will have the most impact on the real or imaginary component, respectively, of the total impedance.*

In Fig. 15.85, the frequency response has been included for each element of a parallel R - L - C combination. At very low frequencies, the impedance of the coil will be less than that of the resistor or capacitor, resulting in an inductive network in which the reactance of the inductor will have the most impact on the total impedance. As the frequency increases, the impedance of the inductor will increase, while the impedance of the capacitor will decrease. Depending on the components chosen, it is possible that the reactance of the capacitor will drop to a point where it will equal the impedance of the coil before either one reaches the resistance level.

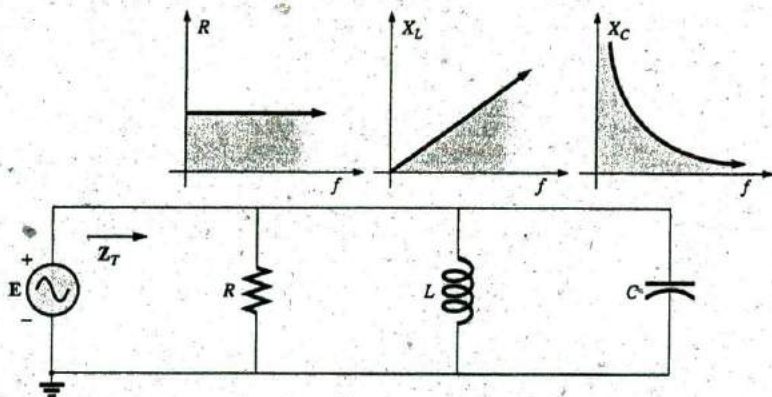


FIG. 15.85

Frequency response for parallel R - L - C elements.

Therefore, it is impossible to make too many broad statements about the effect of each element as the frequency increases. In general, however, for very low frequencies, we can assume that a parallel R - L - C network will be inductive as described above, and at very high frequencies it will be capacitive since X_C will drop to very low levels. In between, a point will result at which X_L will equal X_C and where X_L or X_C will equal R . The frequencies at which these events occur, however, depend on the elements chosen and the frequency range of interest. In general, however, keep in mind that the smaller the resistance or reactance, the greater is its impact on the total impedance of a parallel system.

Let us now note the impact of frequency on the total impedance and inductive current for the parallel R - L network in Fig. 15.86 for a frequency range of zero through 40 kHz.

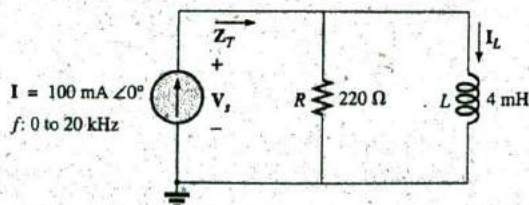


FIG. 15.86

Determining the frequency response of a parallel R - L network.

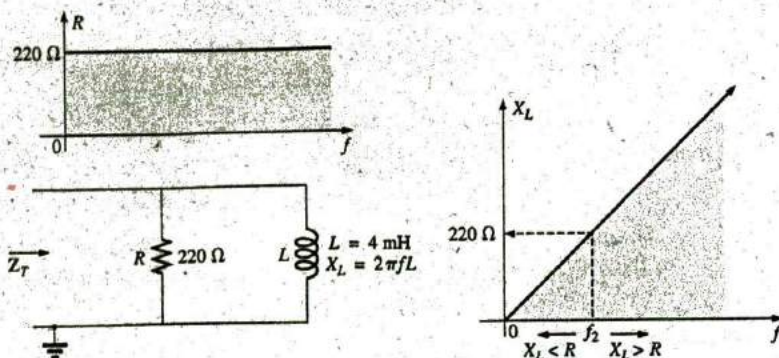


FIG. 15.87

The frequency response of the individual elements of a parallel R-L network.

Z_T Before getting into specifics, let us first develop a "sense" for the impact of frequency on the network in Fig. 15.86 by noting the impedance-versus-frequency curves of the individual elements, as shown in Fig. 15.87. The fact that the elements are now in parallel requires that we consider their characteristics in a different manner than occurred for the series R-C circuit in Section 15.5. Recall that for parallel elements, the element with the smallest impedance will have the greatest impact on the total impedance at that frequency. In Fig. 15.87, for example, X_L is very small at low frequencies compared to R , establishing X_L as the predominant factor in this frequency range. In other words, at low frequencies the network will be primarily inductive, and the angle associated with the total impedance will be close to 90° , as with a pure inductor. As the frequency increases, X_L increases until it equals the impedance of the resistor (220Ω). The frequency at which this situation occurs can be determined in the following manner:

$$X_L = 2\pi f_2 L = R$$

and

$$f_2 = \frac{R}{2\pi L} \quad (15.38)$$

which for the network in Fig. 15.86 is

$$\begin{aligned} f_2 &= \frac{R}{2\pi L} = \frac{220 \Omega}{2\pi(4 \times 10^{-3} \text{ H})} \\ &\cong 8.75 \text{ kHz} \end{aligned}$$

which falls within the frequency range of interest.

For frequencies less than f_2 , $X_L < R$, and for frequencies greater than f_2 , $X_L > R$, as shown in Fig. 15.87. A general equation for the total impedance in vector form can be developed in the following manner:

$$\begin{aligned} Z_T &= \frac{Z_R Z_L}{Z_R + Z_L} \\ &= \frac{(R \angle 0^\circ)(X_L \angle 90^\circ)}{R + jX_L} = \frac{RX_L \angle 90^\circ}{\sqrt{R^2 + X_L^2} \angle \tan^{-1} X_L/R} \end{aligned}$$

and

$$Z_T = \frac{RX_L}{\sqrt{R^2 + X_L^2}} \angle 90^\circ - \tan^{-1} X_L/R$$

so that

$$Z_T = \frac{RX_L}{\sqrt{R^2 + X_L^2}} \quad (15.39)$$

and

$$\theta_T = 90^\circ - \tan^{-1} \frac{X_L}{R} = \tan^{-1} \frac{R}{X_L} \quad (15.40)$$

The magnitude and angle of the total impedance can now be found at any frequency of interest simply by substituting Eqs. (15.39) and (15.40).

 $f = 1 \text{ kHz}$

$$X_L = 2\pi fL = 2\pi(1 \text{ kHz})(4 \times 10^{-3} \text{ H}) = 25.12 \Omega$$

and

$$Z_T = \frac{RX_L}{\sqrt{R^2 + X_L^2}} = \frac{(220 \Omega)(25.12 \Omega)}{\sqrt{(220 \Omega)^2 + (25.12 \Omega)^2}} = 24.96 \Omega$$

with

$$\begin{aligned} \theta_T &= \tan^{-1} \frac{R}{X_L} = \tan^{-1} \frac{220 \Omega}{25.12 \Omega} \\ &= \tan^{-1} 8.76 = 83.49^\circ \end{aligned}$$

and

$$Z_T = 24.96 \Omega \angle 83.49^\circ$$

This value compares very closely with $X_L = 25.12 \Omega \angle 90^\circ$, which it would be if the network were purely inductive ($R = \infty \Omega$). Our assumption that the network is primarily inductive at low frequencies is therefore confirmed.

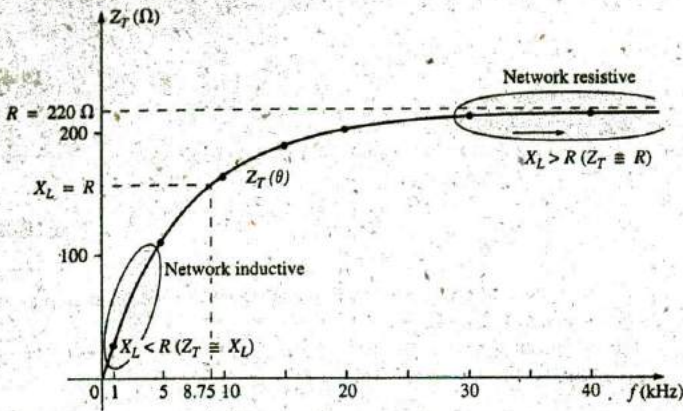
Continuing, we obtain

$$\begin{aligned} f = 5 \text{ kHz: } & Z_T = 109.1 \Omega \angle 60.23^\circ \\ f = 10 \text{ kHz: } & Z_T = 165.5 \Omega \angle 41.21^\circ \\ f = 15 \text{ kHz: } & Z_T = 189.99 \Omega \angle 30.28^\circ \\ f = 20 \text{ kHz: } & Z_T = 201.53 \Omega \angle 23.65^\circ \\ f = 30 \text{ kHz: } & Z_T = 211.19 \Omega \angle 16.27^\circ \\ f = 40 \text{ kHz: } & Z_T = 214.91 \Omega \angle 12.35^\circ \end{aligned}$$

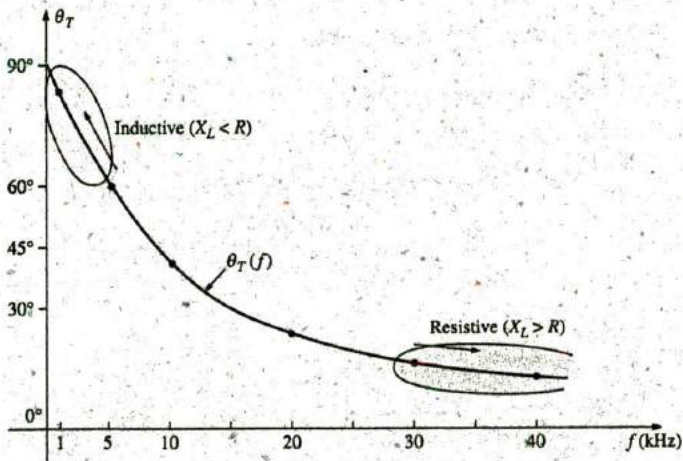
At $f = 40 \text{ kHz}$, note how closely the magnitude of Z_T has approached the resistance level of 220Ω and how the associated angle with the total impedance is approaching zero degrees. The result is a network with terminal characteristics that are becoming more and more resistive as the frequency increases, which further confirms the earlier conclusions developed by the curves in Fig. 15.87.

Plots of Z_T versus frequency in Fig. 15.88 and θ_T in Fig. 15.89 clearly reveal the transition from an inductive network to one that has resistive characteristics. Note that the transition frequency of 8.75 kHz occurs right in the middle of the "knee" of the curves for both Z_T and θ_T .

A review of Figs. 15.49 and 15.88 reveals that a series R - C and a parallel R - L network will have an impedance level that approaches the resistance of the network at high frequencies. The capacitive circuit approaches the level from above, whereas the inductive network does the same from below. For the series R - L circuit and the parallel R - C network, the total impedance will begin at the resistance level and then display the characteristics of the reactive elements at high frequencies.


FIG. 15.88

The magnitude of the input impedance versus frequency for the network in Fig. 15.86.


FIG. 15.89

The phase angle of the input impedance versus frequency for the network in Fig. 15.86.

I_L Applying the current divider rule to the network in Fig. 15.86 results in the following:

$$\begin{aligned} I_L &= \frac{Z_R I}{Z_R + Z_L} \\ &= \frac{(R \angle 0^\circ)(I \angle 0^\circ)}{R + jX_L} = \frac{RI \angle 0^\circ}{\sqrt{R^2 + X_L^2} \angle \tan^{-1} X_L/R} \end{aligned}$$

$$\text{and } I_L = I_L \angle \theta_L = \frac{RI}{\sqrt{R^2 + X_L^2}} \angle -\tan^{-1} X_L/R$$

The magnitude of I_L is therefore determined by

$$I_L = \frac{RI}{\sqrt{R^2 + X_L^2}} \quad (15.41)$$

and the phase angle θ_L , by which I_L leads I_s , is given by

$$\theta_L = -\tan^{-1} \frac{X_L}{R} \quad (15.42)$$

Because θ_L is always negative, the magnitude of θ_L is, in actuality, the angle by which I_L lags I_s .

To begin our analysis, let us first consider the case of $f = 0$ Hz (dc conditions).

$f = 0$ Hz

$$X_L = 2\pi fL = 2\pi(0 \text{ Hz})L = 0 \Omega$$

Applying the short-circuit equivalent for the inductor in Fig. 15.86 results in

$$I_L = I = 100 \text{ mA} \angle 0^\circ$$

as appearing in Figs. 15.90 and 15.91.

$f = 1$ kHz Applying Eq. (15.41) gives

$$X_L = 2\pi fL = 2\pi(1 \text{ kHz})(4 \text{ mH}) = 25.12 \Omega$$

$$\text{and } \sqrt{R^2 + X_L^2} = \sqrt{(220 \Omega)^2 + (25.12 \Omega)^2} = 221.43 \Omega$$

$$\text{and } I_L = \frac{RI}{\sqrt{R^2 + X_L^2}} = \frac{(220 \Omega)(100 \text{ mA})}{221.43 \Omega} = 99.35 \text{ mA}$$

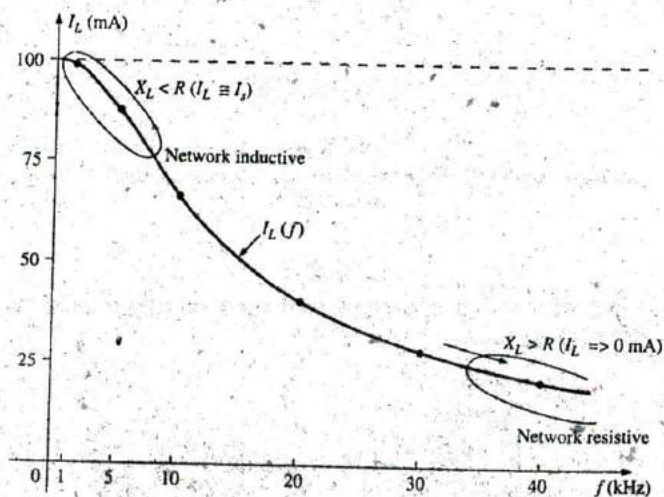
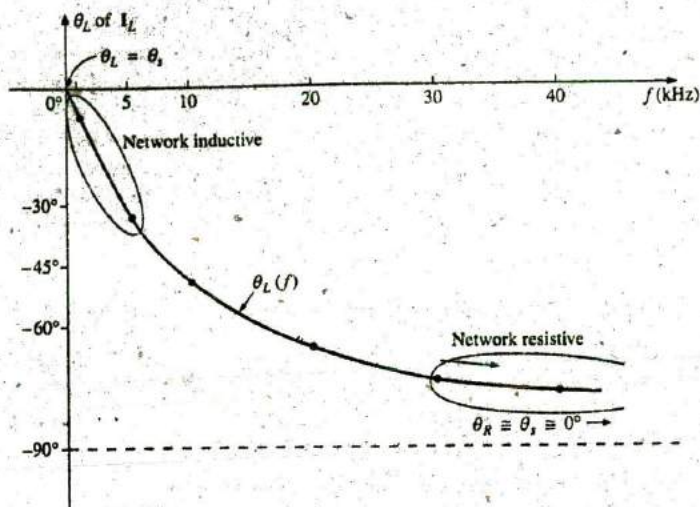


FIG. 15.90

The magnitude of the current I_L versus frequency for the parallel R-L network in Fig. 15.86.


FIG. 15.91

The phase angle of the current I_L versus frequency for the parallel R-L network in Fig. 15.86.

with

$$\theta_L = \tan^{-1} \frac{X_L}{R} = -\tan^{-1} \frac{25.12 \Omega}{220 \Omega} = -\tan^{-1} 0.114 = -6.51^\circ$$

and $I_L = 99.35 \text{ mA} \angle -6.51^\circ$

The result is a current I_L that is still very close to the source current I in both magnitude and phase.

Continuing, we obtain

$$f = 5 \text{ kHz: } I_L = 86.84 \text{ mA} \angle -29.72^\circ$$

$$f = 10 \text{ kHz: } I_L = 65.88 \text{ mA} \angle -48.79^\circ$$

$$f = 15 \text{ kHz: } I_L = 50.43 \text{ mA} \angle -59.72^\circ$$

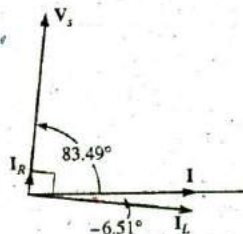
$$f = 20 \text{ kHz: } I_L = 40.11 \text{ mA} \angle -66.35^\circ$$

$$f = 30 \text{ kHz: } I_L = 28.02 \text{ mA} \angle -73.73^\circ$$

$$f = 40 \text{ kHz: } I_L = 21.38 \text{ mA} \angle -77.65^\circ$$

The plot of the magnitude of I_L versus frequency is provided in Fig. 15.90 and reveals that the current through the coil dropped from its maximum of 100 mA to almost 20 mA at 40 kHz. As the reactance of the coil increased with frequency, more of the source current chose the lower-resistance path of the resistor. The magnitude of the phase angle between I_L and I is approaching 90° with an increase in frequency, as shown in Fig. 15.91, leaving its initial value of zero degrees at $f = 0 \text{ Hz}$ far behind.

At $f = 1 \text{ kHz}$, the phasor diagram of the network appears as shown in Fig. 15.92. First note that the magnitude and the phase angle of I_L are very close to those of I . Since the voltage across a coil must lead the current through a coil by 90° , the voltage V_L appears as shown. The voltage across a resistor is in phase with the current through the resistor, resulting in the direction of I_R shown in Fig. 15.92. Of course, at this frequency $R > X_L$, and the current I_R is relatively small in magnitude.


FIG. 15.92

The phasor diagram for the parallel R-L network in Fig. 15.86 at $f = 1 \text{ kHz}$.

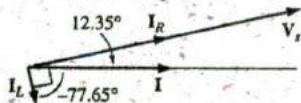


FIG. 15.93

The phasor diagram for the parallel R-L network in Fig. 15.86 at $f = 40$ kHz.

At $f = 40$ kHz, the phasor diagram changes to that appearing in Fig. 15.93. Note that now I_R and I are close in magnitude and phase because $X_L > R$. The magnitude of I_L has dropped to very low levels, and the phase angle associated with I_L is approaching -90° . The network is now more "resistive" compared to its "inductive" characteristics at low frequencies.

The analysis of a parallel R-C or R-L-C network would proceed in much the same manner, with the inductive impedance predominating at low frequencies and the capacitive reactance predominating at high frequencies.

15.11 SUMMARY: PARALLEL ac NETWORKS

The following is a review of important conclusions that can be derived from the discussion and examples of the previous sections. The list is not all-inclusive, but it does emphasize some of the conclusions that should be carried forward in the future analysis of ac systems.

For parallel ac networks with reactive elements:

1. The total admittance or impedance will be frequency dependent.
2. Depending on the frequency applied, the same network can be either predominantly inductive or predominantly capacitive.
3. The magnitude of the current through any one branch can be greater than the source current.
4. The inductive and capacitive susceptances are in direct opposition on an admittance diagram.
5. At lower frequencies, the inductive elements will usually have the most impact on the total impedance, while at high frequencies, the capacitive elements will usually have the most impact.
6. The impedance of any one element can be less than the total impedance (recall that for dc circuits, the total resistance must always be less than the smallest parallel resistor).
7. The magnitude of the current through an element, compared to the other elements of the network, is directly related to the magnitude of its impedance; that is, the smaller the impedance of an element, the larger is the magnitude of the current through the element.
8. The current through a coil is always in direct opposition with the current through a capacitor on a phasor diagram.
9. The applied voltage is always in phase with the current through the resistive elements, leads the voltage across all the inductive elements by 90° , and lags the current through all capacitive elements by 90° .
10. The smaller the resistive element of a network compared to the net reactive susceptance, the closer is the power factor to unity.

15.12 EQUIVALENT CIRCUITS

In a series ac circuit, the total impedance of two or more elements in series is often equivalent to an impedance that can be achieved with fewer elements of different values, the elements and their values being determined by the frequency applied. This is also true for parallel circuits. For the circuit in Fig. 15.94(a),

$$\begin{aligned} Z_T &= \frac{Z_C Z_L}{Z_C + Z_L} = \frac{(5 \Omega \angle -90^\circ)(10 \Omega \angle 90^\circ)}{5 \Omega \angle -90^\circ + 10 \Omega \angle 90^\circ} = \frac{50 \angle 0^\circ}{5 \angle 90^\circ} \\ &= 10 \Omega \angle -90^\circ \end{aligned}$$

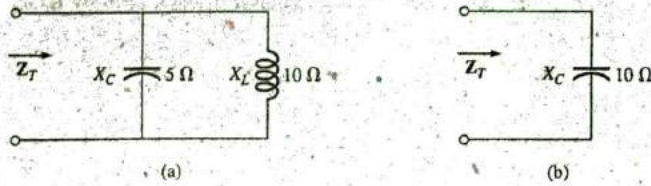


FIG. 15.94

Defining the equivalence between two networks at a specific frequency.

The total impedance at the frequency applied is equivalent to a capacitor with a reactance of 10Ω , as shown in Fig. 15.94(b). Always keep in mind that this equivalence is true only at the applied frequency. If the frequency changes, the reactance of each element changes, and the equivalent circuit changes—perhaps from capacitive to inductive in the above example.

Another interesting development appears if the impedance of a parallel circuit, such as the one in Fig. 15.95(a), is found in rectangular form. In this case,

$$\begin{aligned} Z_T &= \frac{Z_L Z_R}{Z_L + Z_R} = \frac{(4 \Omega \angle 90^\circ)(3 \Omega \angle 0^\circ)}{4 \Omega \angle 90^\circ + 3 \Omega \angle 0^\circ} \\ &= \frac{12 \angle 90^\circ}{5 \angle 53.13^\circ} = 2.40 \Omega \angle 36.87^\circ \\ &= 1.92 \Omega + j1.44 \Omega \end{aligned}$$

which is the impedance of a series circuit with a resistor of 1.92Ω and an inductive reactance of 1.44Ω , as shown in Fig. 15.95(b).

The current I will be the same in each circuit in Fig. 15.94 or Fig. 15.95 if the same input voltage E is applied. For a parallel circuit of one resistive element and one reactive element, the series circuit with the same input impedance will always be composed of one resistive and one reactive element. The impedance of each element of the series circuit will be different from that of the parallel circuit, but the reactive elements will always be of the same type; that is, an R - L circuit and an R - C parallel circuit will have an equivalent R - L and R - C series circuit, respectively. The same is true when converting from a series to a parallel circuit. In the discussion to follow, keep in mind that

the term equivalent refers only to the fact that for the same applied potential, the same impedance and input current will result.

To formulate the equivalence between the series and parallel circuits, the equivalent series circuit for a resistor and reactance in parallel can be found by determining the total impedance of the circuit in rectangular form; that is, for the circuit in Fig. 15.96(a),

$$Y_p = \frac{1}{R_p} + \frac{1}{\pm jX_p} = \frac{1}{R_p} \mp j \frac{1}{X_p}$$

and

$$\begin{aligned} Z_p &= \frac{1}{Y_p} = \frac{1}{\frac{1}{R_p} \mp j \frac{1}{X_p}} \\ &= \frac{1/R_p}{(1/R_p)^2 + (1/X_p)^2} \pm j \frac{1/X_p}{(1/R_p)^2 + (1/X_p)^2} \end{aligned}$$

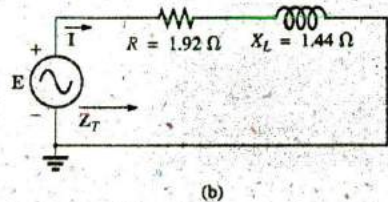
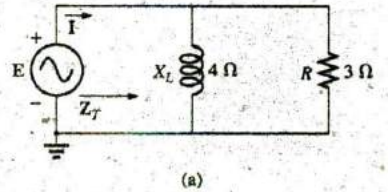


FIG. 15.95

Finding the series equivalent circuit for a parallel R - L network.

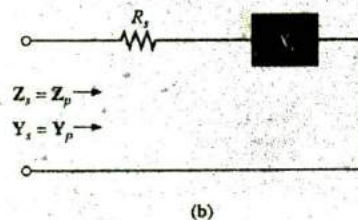
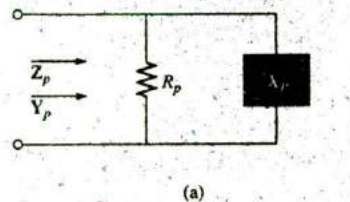


FIG. 15.96

Defining the parameters of equivalent series and parallel networks.

Multiplying the numerator and denominator of each term by $R_p^2 X_p^2$ results in

$$\begin{aligned} Z_p &= \frac{R_p X_p^2}{X_p^2 + R_p^2} \pm j \frac{R_p^2 X_p}{X_p^2 + R_p^2} \\ &= R_s \pm jX_s \quad [\text{Fig. 15.96(b)}] \end{aligned}$$

and

$$R_s = \frac{R_p X_p^2}{X_p^2 + R_p^2} \quad (15.43)$$

with

$$X_s = \frac{R_p^2 X_p}{X_p^2 + R_p^2} \quad (15.44)$$

For the network in Fig. 15.95,

$$R_s = \frac{R_p X_p^2}{X_p^2 + R_p^2} = \frac{(3 \Omega)(4 \Omega)^2}{(4 \Omega)^2 + (3 \Omega)^2} = \frac{48 \Omega}{25} = 1.92 \Omega$$

and

$$X_s = \frac{R_p^2 X_p}{X_p^2 + R_p^2} = \frac{(3 \Omega)^2(4 \Omega)}{(4 \Omega)^2 + (3 \Omega)^2} = \frac{36 \Omega}{25} = 1.44 \Omega$$

which agrees with the previous result.

The equivalent parallel circuit for a circuit with a resistor and reactance in series can be found by finding the total admittance of the system in rectangular form; that is, for the circuit in Fig. 15.96(b),

$$\begin{aligned} Z_s &= R_s \pm jX_s \\ Y_s &= \frac{1}{Z_s} = \frac{1}{R_s \pm jX_s} = \frac{R_s}{R_s^2 + X_s^2} \mp j \frac{X_s}{R_s^2 + X_s^2} \\ &= G_p \mp jB_p = \frac{1}{R_p} \mp j \frac{1}{X_p} \quad [\text{Fig. 15.96(a)}] \end{aligned}$$

or

$$R_p = \frac{R_s^2 + X_s^2}{R_s} \quad (15.45)$$

with

$$X_p = \frac{R_s^2 + X_s^2}{X_s} \quad (15.46)$$

For the above example,

$$R_p = \frac{R_s^2 + X_s^2}{R_s} = \frac{(1.92 \Omega)^2 + (1.44 \Omega)^2}{1.92 \Omega} = \frac{5.76 \Omega}{1.92} = 3.0 \Omega$$

and

$$X_p = \frac{R_s^2 + X_s^2}{X_s} = \frac{5.76 \Omega}{1.44} = 4.0 \Omega$$

as shown in Fig. 15.95 (a).

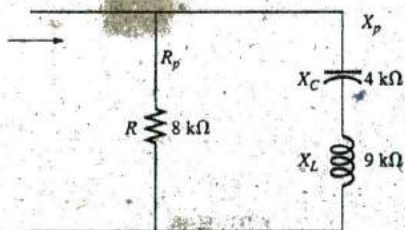


FIG. 15.97
Example 15.18.

EXAMPLE 15.18 Determine the series equivalent circuit for the network in Fig. 15.97.

Solution:

$$R_p = 8 \text{ k}\Omega$$

$$X_p(\text{resultant}) = |X_L - X_C| = |9 \text{ k}\Omega - 4 \text{ k}\Omega|$$

$$= 5 \text{ k}\Omega$$

and

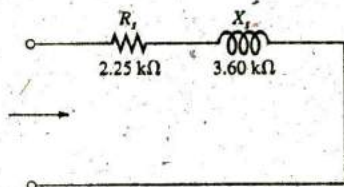
$$R_s = \frac{R_p X_p^2}{X_p^2 + R_p^2} = \frac{(8 \text{ k}\Omega)(5 \text{ k}\Omega)^2}{(5 \text{ k}\Omega)^2 + (8 \text{ k}\Omega)^2} = \frac{200 \text{ k}\Omega}{89} = 2.25 \text{ k}\Omega$$

with

$$X_s = \frac{R_p^2 X_p}{X_p^2 + R_p^2} = \frac{(8 \text{ k}\Omega)^2(5 \text{ k}\Omega)}{(5 \text{ k}\Omega)^2 + (8 \text{ k}\Omega)^2} = \frac{320 \text{ k}\Omega}{89}$$

$$= 3.60 \text{ k}\Omega \quad (\text{inductive})$$

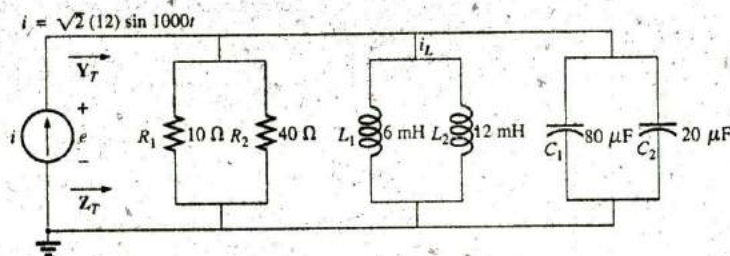
The equivalent series circuit appears in Fig. 15.98.

**FIG. 15.98**

The equivalent series circuit for the parallel network in Fig. 15.97.

EXAMPLE 15.19 For the network in Fig. 15.99:

- Determine Y_T and Z_T .
- Sketch the admittance diagram.
- Find E and I_L .
- Compute the power factor of the network and the power delivered to the network.
- Determine the equivalent series circuit as far as the terminal characteristics of the network are concerned.
- Using the equivalent circuit developed in part (e), calculate E , and compare it with the result of part (c).
- Determine the power delivered to the network, and compare it with the solution of part (d).
- Determine the equivalent parallel network from the equivalent series circuit, and calculate the total admittance Y_T . Compare the result with the solution of part (a).

**FIG. 15.99**

Example 15.19.

Solutions:

- Combining common elements and finding the reactance of the inductor and capacitor, we obtain

$$R_T = 10 \Omega \parallel 40 \Omega = 8 \Omega$$

$$L_T = 6 \text{ mH} \parallel 12 \text{ mH} = 4 \text{ mH}$$

$$C_T = 80 \mu\text{F} + 20 \mu\text{F} = 100 \mu\text{F}$$

$$X_L = \omega L = (1000 \text{ rad/s})(4 \text{ mH}) = 4 \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(1000 \text{ rad/s})(100 \mu\text{F})} = 10 \Omega$$

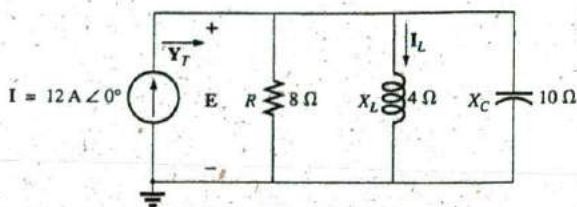


FIG. 15.100

Applying phasor notation to the network in Fig. 15.99.

The network is redrawn in Fig. 15.100 with phasor notation. The total admittance is

$$\begin{aligned} Y_T &= Y_R + Y_L + Y_C \\ &= G \angle 0^\circ + B_L \angle -90^\circ + B_C \angle +90^\circ \\ &= \frac{1}{8 \Omega} \angle 0^\circ + \frac{1}{4 \Omega} \angle -90^\circ + \frac{1}{10 \Omega} \angle +90^\circ \\ &= 0.125 \text{ S} \angle 0^\circ + 0.25 \text{ S} \angle -90^\circ + 0.1 \text{ S} \angle +90^\circ \\ &= 0.125 \text{ S} - j 0.25 \text{ S} + j 0.1 \text{ S} \\ &= 0.125 \text{ S} - j 0.15 \text{ S} = 0.195 \text{ S} \angle -50.194^\circ \end{aligned}$$

$$Z_T = \frac{1}{Y_T} = \frac{1}{0.195 \text{ S}} \angle -50.194^\circ = 5.13 \Omega \angle 50.19^\circ$$

b. See Fig. 15.101.

$$c. \mathbf{E} = \mathbf{I} Z_T = \frac{\mathbf{I}}{Y_T} = \frac{12 \text{ A} \angle 0^\circ}{0.195 \text{ S} \angle -50.194^\circ} = 61.54 \text{ V} \angle 50.19^\circ$$

$$\mathbf{I}_L = \frac{\mathbf{V}_L}{\mathbf{Z}_L} = \frac{\mathbf{E}}{\mathbf{Z}_L} = \frac{61.538 \text{ V} \angle 50.194^\circ}{4 \Omega \angle 90^\circ} = 15.39 \text{ A} \angle -39.81^\circ$$

$$d. F_p = \cos \theta = \frac{G}{Y_T} = \frac{0.125 \text{ S}}{0.195 \text{ S}} = 0.641 \text{ lagging (E leads I)}$$

$$P = EI \cos \theta = (61.538 \text{ V})(12 \text{ A}) \cos 50.194^\circ = 472.75 \text{ W}$$

$$\begin{aligned} e. \mathbf{Z}_T &= \frac{1}{Y_T} = \frac{1}{0.195 \text{ S} \angle -50.194^\circ} = 5.128 \Omega \angle +50.194^\circ \\ &= 3.28 \Omega + j 3.94 \Omega \\ &= R + jX_L \end{aligned}$$

$$X_L = 3.94 \Omega = \omega L$$

$$L = \frac{3.94 \Omega}{\omega} = \frac{3.94 \Omega}{1000 \text{ rad/s}} = 3.94 \text{ mH}$$

The series equivalent circuit appears in Fig. 15.102.

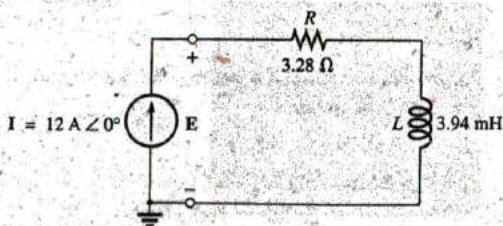


FIG. 15.102

Series equivalent circuit for the parallel R-L-C network in Fig. 15.99 with $\omega = 1000 \text{ rad/s}$.

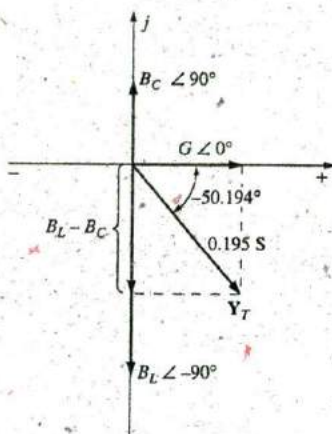


FIG. 15.101

Admittance diagram for the parallel R-L-C network in Fig. 15.99.

$$\begin{aligned}
 \text{f. } E &= IZ_T = (12 \text{ A } \angle 0^\circ)(5.128 \Omega \angle 50.194^\circ) \\
 &= 61.54 \text{ V } \angle 50.194^\circ \quad (\text{as above}) \\
 \text{g. } P &= I^2 R = (12 \text{ A})^2 (3.28 \Omega) = 472.32 \text{ W} \quad (\text{as above}) \\
 \text{h. } R_p &= \frac{R_s^2 + X_s^2}{R_s} = \frac{(3.28 \Omega)^2 + (3.94 \Omega)^2}{3.28 \Omega} = 8 \Omega \\
 X_p &= \frac{R_s^2 + X_s^2}{X_s} = \frac{(3.28 \Omega)^2 + (3.94 \Omega)^2}{3.94 \Omega} = 6.67 \Omega
 \end{aligned}$$

The parallel equivalent circuit appears in Fig. 15.103.

$$\begin{aligned}
 Y_T &= G \angle 0^\circ + B_L \angle -90^\circ = \frac{1}{8 \Omega} \angle 0^\circ + \frac{1}{6.675 \Omega} \angle -90^\circ \\
 &= 0.125 \text{ S } \angle 0^\circ + 0.15 \text{ S } \angle -90^\circ \\
 &= 0.125 \text{ S} - j0.15 \text{ S} = 0.195 \text{ S } \angle 50.194^\circ \quad (\text{as above})
 \end{aligned}$$

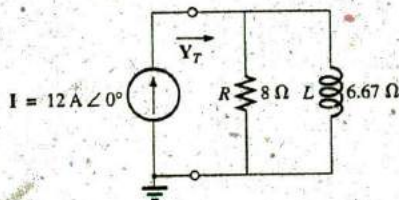


FIG. 15.103

Parallel equivalent of the circuit in Fig. 15.102.

15.13 PHASE MEASUREMENTS

Measuring the phase angle between quantities is one of the most important functions that an oscilloscope can perform. It is an operation that must be performed carefully, however, or you may obtain the incorrect result or damage the equipment. Whenever you are using the dual-trace capability of an oscilloscope, the most important thing to remember is that

both channels of a dual-trace oscilloscope must be connected to the same ground.

Measuring Z_T and θ_T

For ac parallel networks restricted to resistive loads, the total impedance can be found in the same manner as described for dc circuits: Simply remove the source and place an ohmmeter across the network terminals. However,

for parallel ac networks with reactive elements, the total impedance cannot be measured with an ohmmeter.

An experimental procedure must be defined that permits determining the magnitude and the angle of the terminal impedance.

The phase angle between the applied voltage and the resulting source current is one of the most important because (a) it is also the phase angle associated with the total impedance; (b) it provides an instant indication of whether a network is resistive or reactive; (c) it reveals whether a network is inductive or capacitive; and (d) it can be used to find the power delivered to the network.

In Fig. 15.104, a resistor has been added to the configuration between the source and the network to permit measuring the current and finding the phase angle between the applied voltage and the source current.

At the frequency of interest, the applied voltage establishes a voltage across the sensing resistor that can be displayed by one channel of the dual-trace oscilloscope. In Fig. 15.104, channel 1 is displaying the applied voltage and channel 2 the voltage across the sensing resistor. Sensitivities for each channel are chosen to establish the waveforms

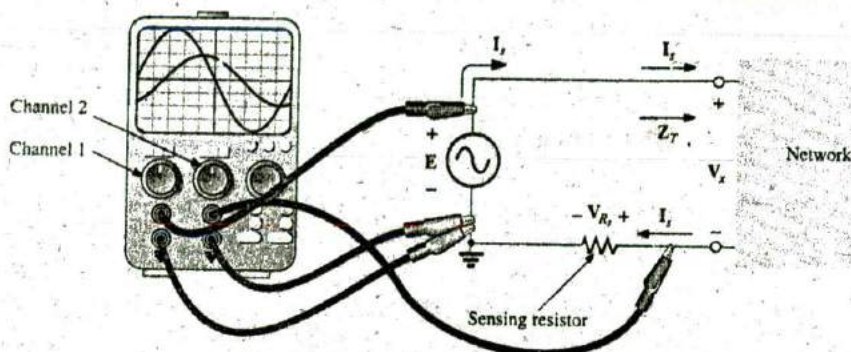


FIG. 15.104

Using an oscilloscope to measure Z_T and θ_T .

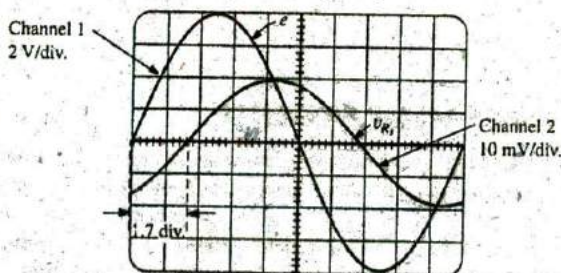


FIG. 15.105

e and v_{R_s} for the configuration in Fig. 15.104.

appearing on the screen in Fig. 15.105. As emphasized above, note that both channels have the same ground connection. In fact, the need for a common ground connection is the only reason that the sensing resistor was not connected to the positive side of the supply. Since oscilloscopes display only voltages versus time, the peak value of the source current must be found using Ohm's law. *Since the voltage across a resistor and the current through the resistor are in phase, the phase angle between the two voltages will be the same as that between the applied voltage and the resulting source current.*

Using the sensitivities, we find that the peak value of the applied voltage is

$$E_m = (4 \text{ div.})(2 \text{ V/div.}) = 8 \text{ V}$$

while the peak value of the voltage across the sensing resistor is

$$V_{R_s(\text{peak})} = (2 \text{ div.})(10 \text{ mV/div.}) = 20 \text{ mV}$$

Using Ohm's law, we find that the peak value of the current is

$$I_s(\text{peak}) = \frac{V_{R_s(\text{peak})}}{R_s} = \frac{20 \text{ mV}}{10 \Omega} = 2 \text{ mA}$$

The sensing resistor is chosen small enough so that the voltage across the sensing resistor is small enough to permit the approximation $V_x = E - V_{R_s} \cong E$. The magnitude of the input impedance is then

$$Z_T = \frac{V_x}{I_s} \cong \frac{E}{I_s} = \frac{8 \text{ V}}{2 \text{ mA}} = 4 \text{ k}\Omega$$

For the chosen horizontal sensitivity, each waveform in Fig. 15.105 has a period T defined by ten horizontal divisions, and the phase angle between the two waveforms is 1.7 divisions. Using the fact that each period of a sinusoidal waveform encompasses 360° , we can set up the following ratios to determine the phase angle θ :

$$\frac{10 \text{ div.}}{360^\circ} = \frac{1.7 \text{ div.}}{\theta}$$

and

$$\theta = \left(\frac{1.7}{10}\right)360^\circ = 61.2^\circ$$

In general,

$$\theta = \frac{(\text{div. for } \theta)}{(\text{div. for } T)} \times 360^\circ \quad (15.47)$$

Therefore, the total impedance is

$$Z_T = 4 \text{ k}\Omega \angle 61.2^\circ = 1.93 \text{ k}\Omega + j3.51 \text{ k}\Omega = R + jX_L$$

which is equivalent to the series combination of a $1.93 \text{ k}\Omega$ resistor and an inductor with a reactance of $3.51 \text{ k}\Omega$ (at the frequency of interest).

Measuring the Phase Angle between Various Voltages

In Fig. 15.106, an oscilloscope is being used to find the phase relationship between the applied voltage and the voltage across the inductor. Note again that each channel shares the same ground connection. The resulting pattern appears in Fig. 15.107 with the chosen sensitivities. This time, both channels have the same sensitivity, resulting in the following peak values for the voltages:

$$E_m = (3 \text{ div.})(2 \text{ V/div.}) = 6 \text{ V}$$

$$V_{L(\text{peak})} = (1.6 \text{ div.})(2 \text{ V/div.}) = 3.2 \text{ V}$$

The phase angle is determined using Eq. 15.45:

$$\theta = \frac{(1 \text{ div.})}{(8 \text{ div.})} \times 360^\circ$$

$$\theta = 45^\circ$$

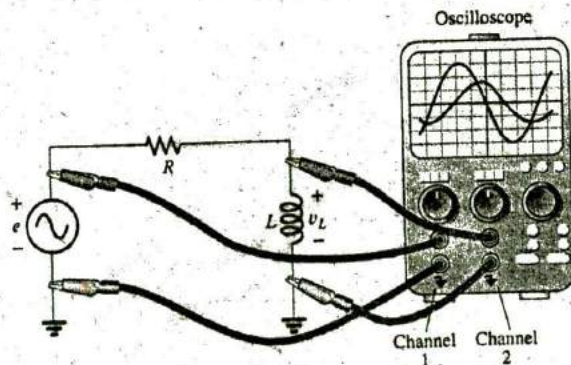


FIG. 15.106

Determining the phase relationship between e and v_L .

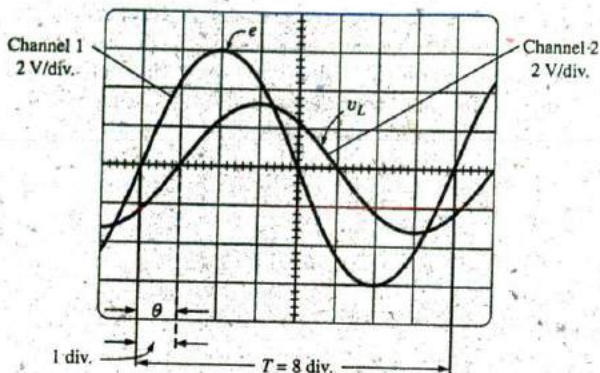


FIG. 15.107

Determining the phase angle between e and v_L for the configuration in Fig. 15.106.

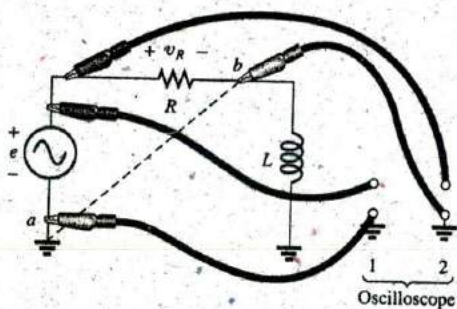


FIG. 15.108

An improper phase-measurement connection.

If the phase relationship between e and v_R is desired, the oscilloscope cannot be connected as shown in Fig. 15.108. The grounds of each channel are internally connected in the oscilloscope, forcing point b to have the same potential as point a . The result would be a direct connection between points a and b that would short out the inductive element. If the inductive element is the predominant factor in controlling the level of the current, the current in the circuit could rise to dangerous levels and damage the oscilloscope or supply. The easiest way to find the phase relationship between e and v_R would be to simply interchange the positions of the resistor and the inductor and proceed as before.

For the parallel network in Fig. 15.109, the phase relationship between two of the branch currents, i_R and i_L , can be determined using a sensing resistor, as shown in the figure. The value of the sensing resistor is chosen small enough in comparison to the value of the series inductive reactance to ensure that it will not affect the general response of the network. Channel 1 displays the voltage v_R , and channel 2 the voltage v_{R_s} . Since v_R is in phase with i_R , and v_{R_s} is in phase with i_L , the phase relationship between v_R and v_{R_s} will be the same as between i_R and i_L . The peak value of each current can be found through a simple application of Ohm's law.

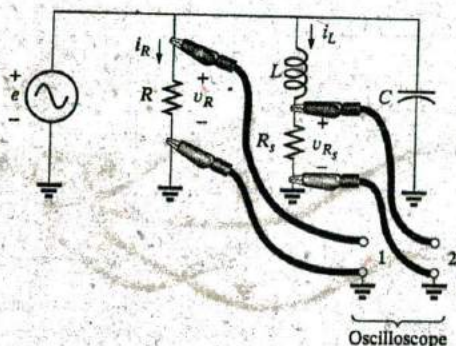


FIG. 15.109

Determining the phase relationship between i_R and i_L .

15.14 APPLICATIONS

Home Wiring

An expanded view of house wiring is provided in Fig. 15.110 to permit a discussion of the entire system. The house panel has been included with the "feed" and the important grounding mechanism. In addition, a number of typical circuits found in the home have been included to provide a sense for the manner in which the total power is distributed.

First note how the copper bars in the panel are laid out to provide both 120 V and 240 V. Between any one bar and ground is the single-phase 120 V supply. However, the bars have been arranged so that 240 V can be obtained between two vertical adjacent bars using a double-gang circuit breaker. When time permits, examine your own panel (but do not remove the cover), and note the dual circuit breaker arrangement for the 240 V supply.

For appliances such as fixtures and heaters that have a metal casing, the ground wire is connected to the metal casing to provide a direct path to ground path for a "shorting" or errant current as described in Section 6.8. For outlets that do not have a conductive casing, the ground lead is connected to a point on the outlet that distributes to all important points of the outlet.

Note the series arrangement between the thermostat and the heater but the parallel arrangement between heaters on the same circuit. In addition,

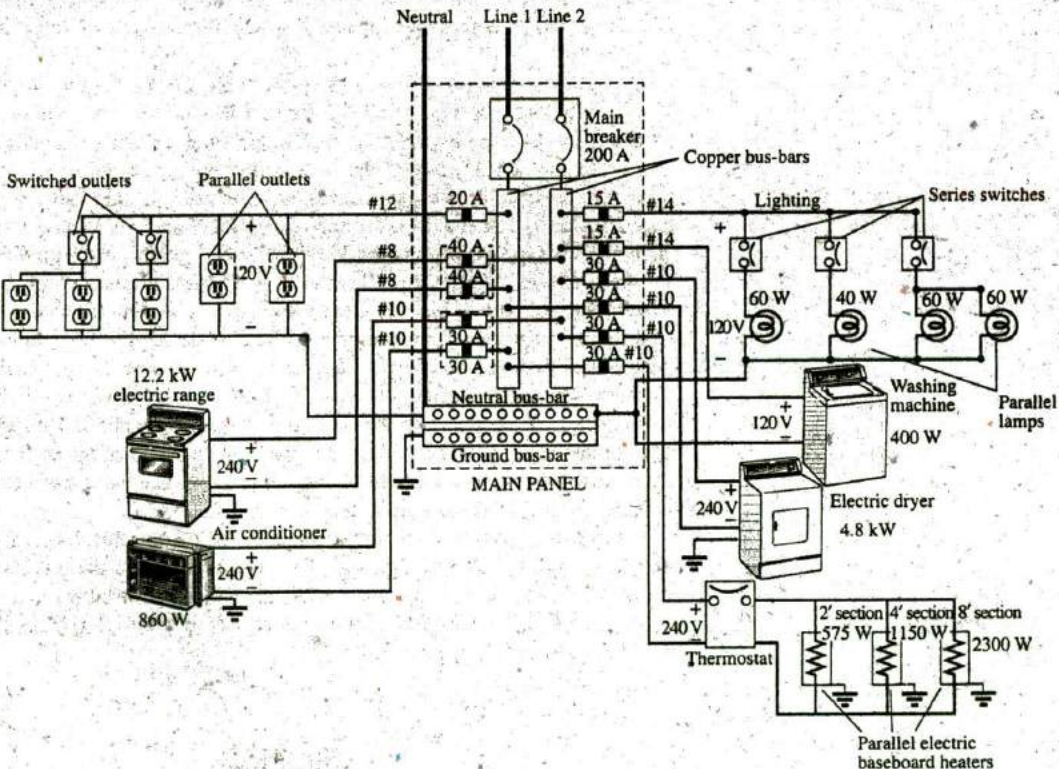


FIG. 15.110
Home wiring diagram.

note the series connection of switches to lights in the upper-right corner but the parallel connection of lights and outlets. Due to high current demand, the air conditioner, heaters, and electric stove have 30 A breakers. Keep in mind that the total current does not equal the product of the two (or 60 A) since each breaker is in a line and the same current will flow through each breaker.

In general, you now have a surface understanding of the general wiring in your home. You may not be a qualified, licensed electrician, but at least you should now be able to converse with some intelligence about the system.

Speaker Systems

The best reproduction of sound is obtained by using different speakers for the low-, mid-, and high-frequency regions. Although the typical audio range for the human ear is from about 100 Hz to 20 kHz, speakers are available from 20 Hz to 40 kHz. For the low-frequency range usually extending from about 20 Hz to 300 Hz, a speaker referred to as a *woofer* is used. Of the three speakers, it is normally the largest. The mid-range speaker is typically smaller in size and covers the range from about 100 Hz to 5 kHz. The *tweeter*, as it is normally called, is usually the smallest of the three speakers and typically covers the range from about 2 kHz to 25 kHz. There is an overlap of frequencies to ensure that frequencies aren't lost in those regions where the response of one drops off and the other takes over. A great deal more about the range of each speaker and their dB response (a term you may have heard when discussing speaker response) is covered in detail in Chapter 21.

One popular method for hooking up the three speakers is the *crossover* configuration in Fig. 15.111. Note that it is nothing more than a parallel network with a speaker in each branch and full applied voltage across each branch. The added elements (inductors and capacitors) were carefully chosen to set the range of response for each speaker. Note that each speaker is labeled with an impedance level and associated frequency. This type of information is typical when purchasing a quality speaker. It immediately identifies the type of speaker and reveals at which frequency it will have its maximum response. A detailed analysis of the same network will be included in Section 21.15. For now, however, it should prove interesting to determine the total impedance of each branch at specific frequencies to see if indeed the response of one will far outweigh the response of the other two. Since an amplifier with an output impedance of $8\ \Omega$ is to be used, maximum transfer of power (see Section 18.5 for ac networks) to the speaker results when the impedance of the branch is equal to or very close to $8\ \Omega$.

Let us begin by examining the response of the frequencies to be carried primarily by the mid-range speaker since it represents the greatest portion of the human hearing range. Since the mid-range speaker branch is rated at $8\ \Omega$ at 1.4 kHz, let us test the effect of applying 1.4 kHz to all branches of the crossover network.

For the mid-range speaker:

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(1.4\ \text{kHz})(47\ \mu\text{F})} = 2.42\ \Omega$$

$$X_L = 2\pi fL = 2\pi(1.4\ \text{kHz})(270\ \mu\text{H}) = 2.78\ \Omega$$

$$R = 8\ \Omega$$

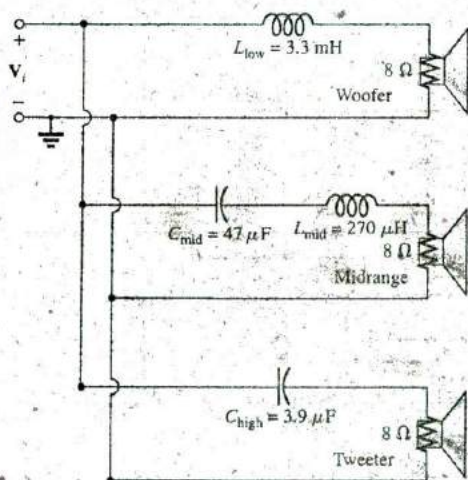
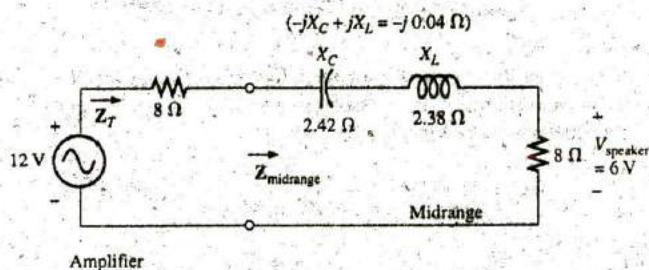


FIG. 15.111
Crossover speaker system.

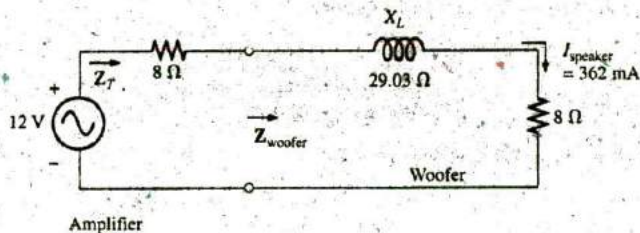
$$\begin{aligned} \text{and } Z_{\text{mid-range}} &= R + j(X_L - X_C) = 8 \Omega + j(2.78 \Omega - 2.42 \Omega) \\ &= 8 \Omega + j0.36 \Omega \\ &= 8.008 \Omega \angle -2.58^\circ \approx 8 \Omega \angle 0^\circ = R \end{aligned}$$

In Fig. 15.112(a), the amplifier with the output impedance of 8Ω has been applied across the mid-range speaker at a frequency of 1.4 kHz . Since the total reactance offered by the two series reactive elements is so small compared to the 8Ω resistance of the speaker, we can essentially replace the series combination of the coil and capacitor by a short circuit of 0Ω . We are then left with a situation where the load impedance is an exact match with the output impedance of the amplifier, and maximum power will be delivered to the speaker. Because of the equal series impedances, each will capture half the applied voltage or 6 V . The power to the speaker is then $V^2/R = (6 \text{ V})^2/8 \Omega = 4.5 \text{ W}$.

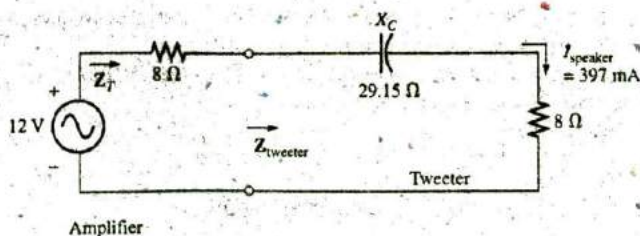
At a frequency of 1.4 kHz , we would expect the woofer and tweeter to have minimum impact on the generated sound. We will now test the validity of this statement by determining the impedance of each branch at 1.4 kHz .



(a)



(b)



(c)

FIG. 15.112

Crossover network: (a) mid-range speaker at 1.4 kHz ; (b) woofer at 1.4 kHz ; (c) tweeter.

For the woofer,

$$X_L = 2\pi fL = 2\pi(1.4 \text{ kHz})(3.3 \text{ mH}) = 29.03 \Omega$$

$$\text{and } Z_{\text{woofer}} = R + jX_L = 8 \Omega + j29.03 \Omega \\ = 30.11 \Omega \angle 74.59^\circ$$

which is a poor match with the output impedance of the amplifier. The resulting network is shown in Fig. 15.112(b).

The total load on the source of 12 V is

$$Z_T = 8 \Omega + 8 \Omega + j29.03 \Omega = 16 \Omega + j29.03 \Omega \\ = 33.15 \Omega \angle 61.14^\circ$$

and the current is

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{12 \text{ V} \angle 0^\circ}{33.15 \Omega \angle 61.14^\circ} \\ = 362 \text{ mA} \angle -61.14^\circ$$

The power to the 8 Ω speaker is then

$$P_{\text{woofer}} = I^2 R = (362 \text{ mA})^2 (8 \Omega) = 1.05 \text{ W}$$

or about 1 W.

Consequently, the sound generated by the mid-range speaker far outweighs the response of the woofer (as it should).

For the tweeter in Fig. 15.112,

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(1.4 \text{ kHz})(3.9 \mu\text{F})} = 29.15 \Omega$$

$$\text{and } Z_{\text{tweeter}} = R - jX_C = 8 \Omega - j29.15 \Omega \\ = 30.33 \Omega \angle -74.65^\circ$$

which, as for the woofer, is a poor match with the output impedance of the amplifier. The current is given as

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{12 \text{ V} \angle 0^\circ}{30.23 \Omega \angle -74.65^\circ} \\ = 397 \text{ mA} \angle 74.65^\circ$$

The power to the 8 Ω speaker is then

$$P_{\text{tweeter}} = I^2 R = (397 \text{ mA})^2 (8 \Omega) = 1.26 \text{ W}$$

or about 1.3 W.

Consequently, the sound generated by the mid-range speaker far outweighs the response of the tweeter also.

All in all, the mid-range speaker predominates at a frequency of 1.4 kHz for the crossover network in Fig. 15.111.

Let us now determine the impedance of the tweeter at 20 kHz and the impact of the woofer at this frequency.

For the tweeter,

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(20 \text{ kHz})(3.9 \mu\text{F})} = 2.04 \Omega$$

$$\text{with } Z_{\text{tweeter}} = 8 \Omega - j2.04 \Omega = 8.26 \Omega \angle -14.31^\circ$$

Even though the magnitude of the impedance of the branch is not exactly 8 Ω , it is very close, and the speaker will receive a high level of power (actually 4.43 W).

For the woofer,

$$X_L = 2\pi fL = 2\pi(20 \text{ kHz})(3.3 \text{ mH}) = 414.69 \Omega$$

with $Z_{\text{woofer}} = 8 \Omega - j414.69 \Omega = 414.77 \Omega \angle 88.9^\circ$

which is a terrible match with the output impedance of the amplifier. Therefore, the speaker will receive a very low level of power ($6.69 \text{ mW} \approx 0.007 \text{ W}$).

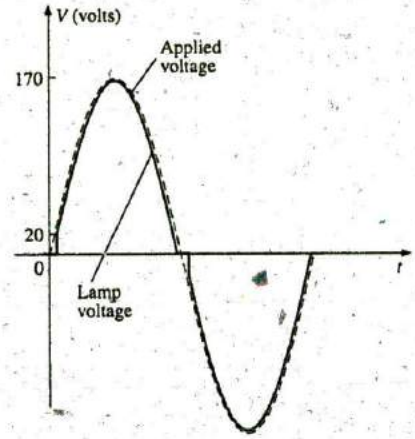
For all the calculations, note that the capacitive elements predominate at low frequencies and the inductive elements at high frequencies. For the low frequencies, the reactance of the coil is quite small, permitting a full transfer of power to the speaker. For the high-frequency tweeter, the reactance of the capacitor is quite small, providing a direct path for power flow to the speaker.

Phase-Shift Power Control

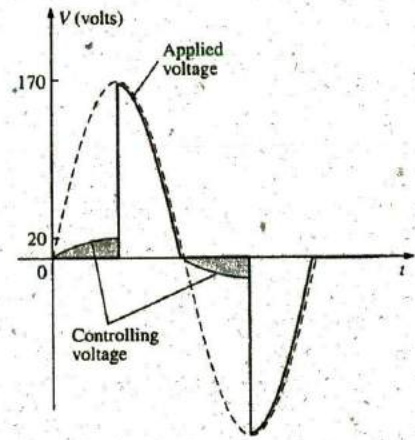
In Chapter 11, the internal structure of a light dimmer was examined and its basic operation described. We can now turn our attention to how the power flow to the bulb is controlled.

If the dimmer were composed of simply resistive elements, all the voltages of the network would be in phase, as shown in Fig. 15.113(a). If we assume that 20 V are required to turn on the triac in Fig. 11.68, the power will be distributed to the bulb for the period highlighted by the blue area of Fig. 15.113(a). For this situation, the bulb is close to full brightness since the applied voltage is available to the bulb for almost the entire cycle. To reduce the power to the bulb (and therefore reduce its brightness), the controlling voltage would need a lower peak voltage, as shown in Fig. 15.113(b). In fact, the waveform in Fig. 15.113(b) is such that the turn-on voltage is not reached until the peak value occurs. In this case, power is delivered to the bulb for only half the cycle, and the brightness of the bulb is reduced. The problem with using only resistive elements in a dimmer now becomes apparent: The bulb can be made no dimmer than the situation depicted by Fig. 15.113(b). Any further reduction in the controlling voltage would reduce its peak value below the trigger level, and the bulb would never turn on.

This dilemma can be resolved by using a series combination of elements such as shown in Fig. 15.114(a) from the dimmer in Fig. 11.68. Note that the controlling voltage is the voltage across the capacitor, while the full line voltage of 120 V rms; 170 V peak, is across the entire branch. To describe the behavior of the network, let us examine the case



(a)



(b)

FIG. 15.113

Light dimmer: (a) with purely resistive elements; (b) half-cycle power distribution.

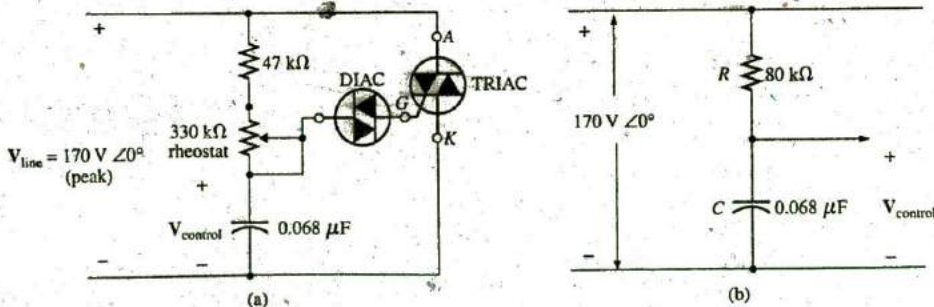


FIG. 15.114

Light dimmer: (a) from Fig. 11.68; (b) with rheostat set at 33 kΩ.

defined by setting the potentiometer (used as a rheostat) to 1/10 its maximum value, or 33 k Ω . Combining the 33 k Ω with the fixed resistance of 47 k Ω results in a total resistance of 80 k Ω and the equivalent network in Fig. 15.114(b).

At 60 Hz, the reactance of the capacitor is

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(60 \text{ Hz})(62 \mu\text{F})} = 42.78 \text{ k}\Omega$$

Applying the voltage divider rule gives

$$\begin{aligned} V_{\text{control}} &= \frac{Z_C V_s}{Z_R + Z_C} \\ &= \frac{(42.78 \text{ k}\Omega \angle -90^\circ)(V_s \angle 0^\circ)}{80 \text{ k}\Omega - j 42.78 \text{ k}\Omega} = \frac{42.78 \text{ k}\Omega V_s \angle -90^\circ}{90.72 \text{ k}\Omega \angle -28.14^\circ} \\ &= 0.472 V_s \angle -61.86^\circ \end{aligned}$$

Using a peak value of 170 V gives

$$\begin{aligned} V_{\text{control}} &= 0.472(170 \text{ V}) \angle -61.86^\circ \\ &= 80.24 \text{ V} \angle -61.86^\circ \end{aligned}$$

producing the waveform in Fig. 15.115(a). The result is a waveform with a phase shift of 61.86° (lagging the applied line voltage) and a relatively high peak value. The high peak value results in a quick transition to the 20 V turn-on level, and power is distributed to the bulb for the major portion of the applied signal. Recall from the discussion in Chapter 11 that the response in the negative region is a replica of that achieved in the positive region. If we reduced the potentiometer resistance further, the phase angle would be reduced, and the bulb would burn brighter. The situation is now very similar to that described for the response in Fig. 15.113(a). In other words, nothing has been gained thus far by using the capacitive element in the control network. However, let us now increase the potentiometer resistance to 200 k Ω and note the effect on the controlling voltage.

That is,

$$R_T = 200 \text{ k}\Omega + 47 \text{ k}\Omega = 247 \text{ k}\Omega$$

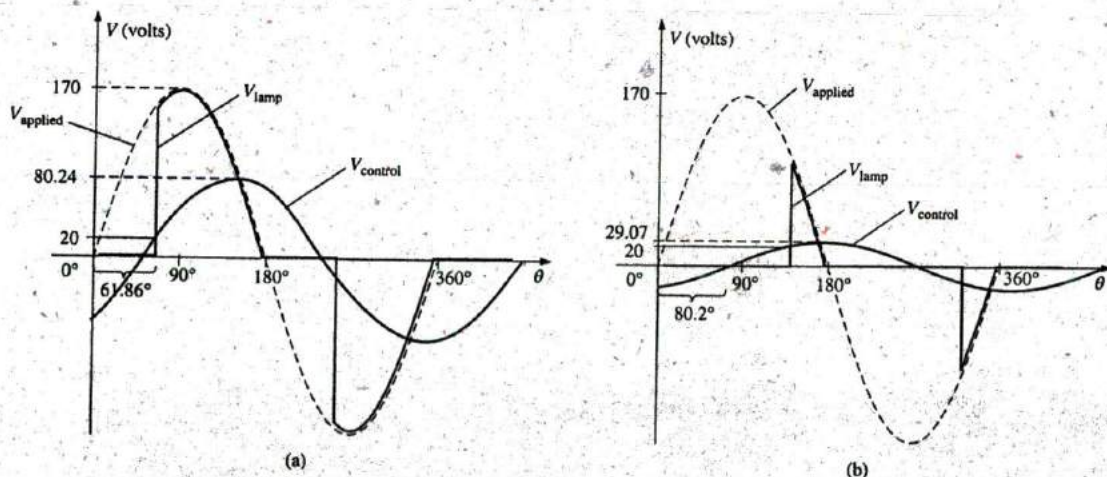


FIG. 15.115

Light dimmer in Fig. 11.68; (a) rheostat set at 33 k Ω ; (b) rheostat set at 200 k Ω .

$$\begin{aligned}
 V_{\text{control}} &= \frac{Z_C V_s}{Z_R + Z_C} \\
 &= \frac{(42.78 \text{ k}\Omega \angle -90^\circ)(V_s \angle 0^\circ)}{247 \text{ k}\Omega - j42.78 \text{ k}\Omega} = \frac{42.78 \text{ k}\Omega V_s \angle -90^\circ}{250.78 \text{ k}\Omega \angle -9.8^\circ} \\
 &= 0.171 V_s \angle -80.2^\circ
 \end{aligned}$$

and using a peak value of 170 V, we have

$$\begin{aligned}
 V_{\text{control}} &= 0.171(170 \text{ V}) \angle -80.2^\circ \\
 &= 29.07 \text{ V} \angle -80.2^\circ
 \end{aligned}$$

The peak value has been substantially reduced to only 29.07 V, and the phase-shift angle has increased to 80.2°. The result, as depicted by Fig. 15.115(b), is that the firing potential of 20 V is not reached until near the end of the positive region of the applied voltage. Power is delivered to the bulb for only a very short period of time, causing the bulb to be quite dim, significantly dimmer than obtained from the response in Fig. 15.113(b).

A conduction angle less than 90° is therefore possible due only to the phase shift introduced by the series *R-C* combination. Thus, it is possible to construct a network of some significance with a rather simple pair of elements.

15.15 COMPUTER ANALYSIS

PSpice

Series *R-L-C* Circuit The *R-L-C* network in Fig. 15.35 is now analyzed using OrCAD Capture. Since the inductive and capacitive reactances cannot be entered onto the screen, the associated inductive and capacitive levels were first determined as follows:

$$X_L = 2\pi fL \Rightarrow L = \frac{X_L}{2\pi f} = \frac{7 \Omega}{2\pi(1 \text{ kHz})} = 1.114 \text{ mH}$$

$$X_C = \frac{1}{2\pi fC} \Rightarrow C = \frac{1}{2\pi fX_C} = \frac{1}{2\pi(1 \text{ kHz})3 \Omega} = 53.05 \mu\text{F}$$

Enter the values into the schematic as shown in Fig. 15.116. For the ac source, the sequence is **Place part icon-SOURCE-VSIN-OK** with **VOFF** set at 0 V, **VAMPL** set at 70.7 V (the peak value of the applied sinusoidal source in Fig. 15.35), and **FREQ** = 1 kHz. Double-click on the source symbol and the **Property Editor** appears, confirming the above choices and showing that **DF** = 0 s, **PHASE** = 0°, and **TD** = 0 s as set by the default levels. You are now ready to do an analysis of the circuit for the fixed frequency of 1 kHz.

The simulation process is initiated by first selecting the **New Simulation Profile** icon and inserting **PSpice 15-1** as the **Name** followed by **Create**. The **Simulation Settings** dialog appears and since you are continuing to plot the results against time, select the **Time Domain (Transient)** option under **Analysis type**. Since the period of each cycle of the applied source is 1 ms, set the **Run to time** at 5 ms so that five cycles appear. Leave the **Start saving data after** at 0 s even though there will be an oscillatory period for the reactive elements before the circuit settles down. Set the **Maximum step size** at 5 ms/1000 = 5 μs. Finally, select **OK** followed by the **Run PSpice** key. The result is a blank screen with an x-axis extending from 0 s to 5 ms.

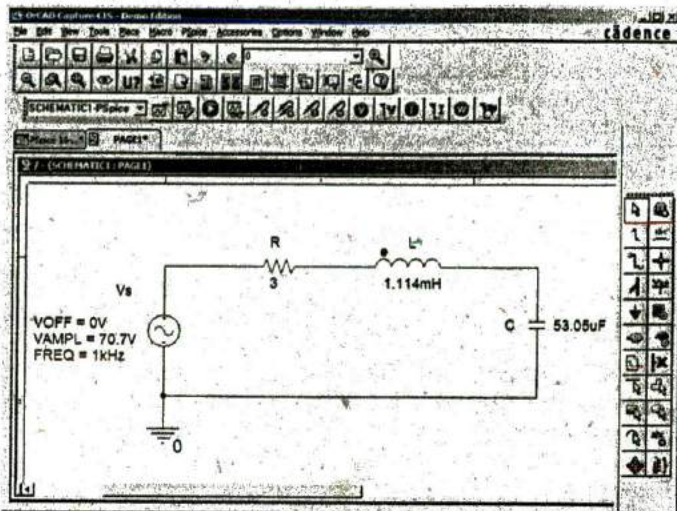


FIG. 15.116

Using PSpice to analyze a series R-L-C ac circuit.

The first quantity of interest is the current through the circuit, so select Trace-Add-Trace followed by $I(R)$ and OK. The resulting plot in Fig. 15.117 clearly shows that there is a period of storing and discharging of the reactive elements before a steady-state level is established. It

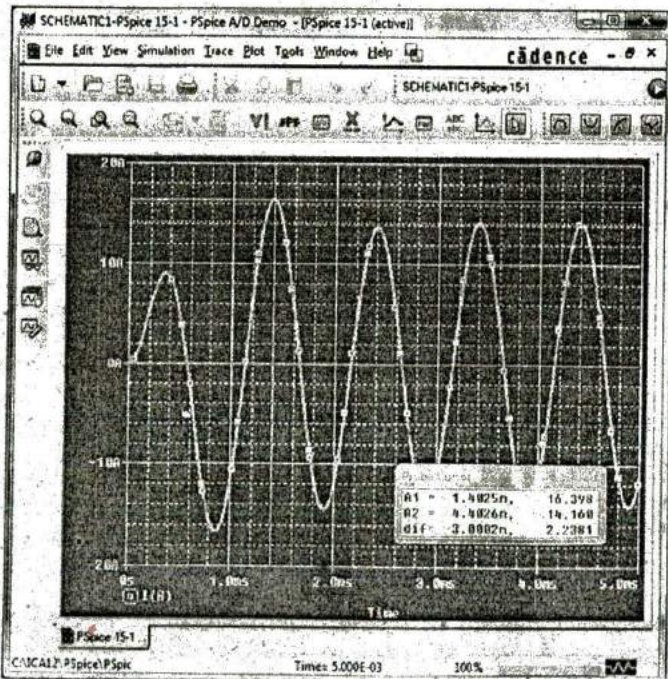
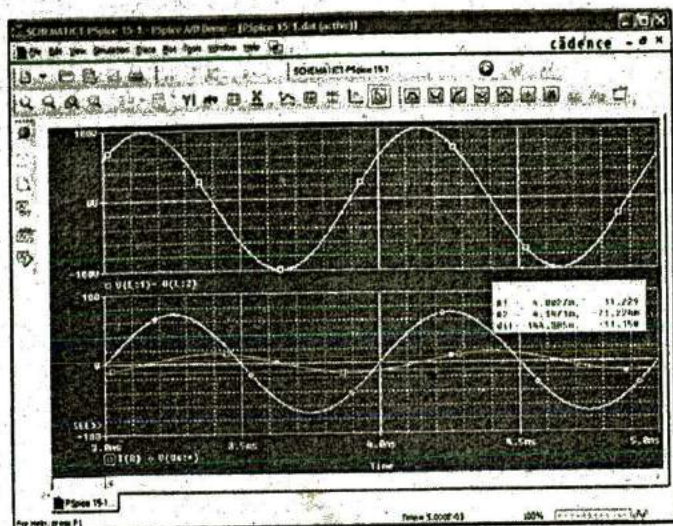


FIG. 15.117

A plot of the current for the circuit in Fig. 15.116 showing the transition from the transient state to the steady-state response.


FIG. 15.118

A plot of the steady-state response ($t > 3$ ms) for v_L , v_p , and i for the circuit in Fig. 15.116.

would appear that after 3 ms, steady-state conditions have been essentially established. Select the Toggle cursor key, and left-click; a cursor appears that can be moved along the axis near the maximum value around 1.4 ms. In fact, the cursor reveals a maximum value of 16.4 A, which exceeds the steady-state solution by over 2 A. Right-click to establish a second cursor on the screen that can be placed near the steady-state peak around 4.4 ms. The resulting peak value is about 14.15 A, which is a match with the longhand solution for Fig. 15.35. We will therefore assume that steady-state conditions have been established for the circuit after 4 ms.

Now add the source voltage through **Trace-Add Trace-V(Vs:+) - OK** to obtain the multiple plot at the bottom of Fig. 15.118. For the voltage across the coil, the sequence **Plot-Add Plot to Window-Trace-Add Trace-V(L:1)-V(L:2)** results in the plot appearing at the top of Fig. 15.118. Take special note that the Trace Expression is $V(L:1) - V(L:2)$ rather than just $V(L:1)$ because $V(L:1)$ would be the voltage from that point to ground, which would include the voltage across the capacitor. In addition, the $-$ sign between the two comes from the **Functions or Macros** list at the right of the **Add Traces** dialog box. Finally, since we know that the waveforms are fairly steady after 3 ms, cut away the waveforms before 3 ms with **Plot-Axis Settings-X axis-User Defined-3ms to 5ms-OK** to obtain the two cycles of Fig. 15.118. Now you can clearly see that the peak value of the voltage across the coil is 100 V to match the analysis of Fig. 15.35. It is also clear that the applied voltage leads the input current by an angle that can be determined using the cursors. First activate the cursor option by selecting the cursor key (a red plot through the origin) in the second toolbar down from the menu bar. Then select **V(Vs: +)** at the bottom left of the screen with a left click, and set it at that point where the applied voltage passes through the horizontal axis with a positive slope. The result is $A1 = 4$ ms at $-4.243 \mu\text{V} \cong$

0 V. Then select **I(R)** at the bottom left of the screen with a right click, and place it at the point where the current waveform passes through the horizontal axis with a positive slope. The result is $A2 = 4.15 \text{ ms}$ at $-55.15 \text{ mA} = 0.55 \text{ A} \cong 0 \text{ A}$ (compared to a peak value of 14.14 A). At the bottom of the **Probe Cursor** dialog box, the time difference is $147.24 \mu\text{s}$.

Now set up the ratio

$$\frac{147.24 \mu\text{s}}{1000 \mu\text{s}} = \frac{\theta}{360^\circ}$$

$$\theta = 52.99^\circ$$

The phase angle by which the applied voltage leads the source is 52.99° , which is very close to the theoretical solution of 53.13° obtained in Fig. 15.39. Increasing the number of data points for the plot would have increased the accuracy level and brought the results closer to 53.13° .

Multisim

We now examine the response of a network versus frequency rather than time using the network in Fig. 15.86, which now appears on the schematic in Fig. 15.119. The ac current source appears as **AC_CURRENT_SOURCE** under the **SIGNAL_CURRENT_SOURCES** Family listing. Note that the current source was given an amplitude of 1 A to establish a magnitude match between the response of the voltage across the network and the impedance of the network. That is,

$$|Z_T| = \left| \frac{V_s}{I_s} \right| = \left| \frac{V_s}{1 \text{ A}} \right| = |V_s|$$

Before applying computer methods, we should develop a rough idea of what to expect so that we have something to which to compare the

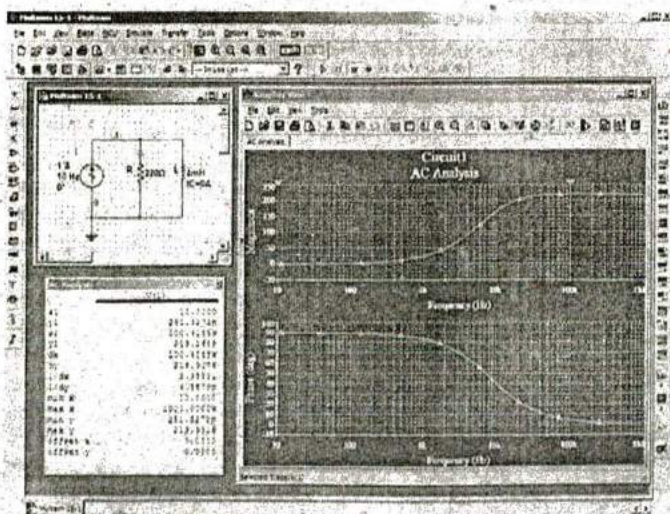


FIG. 15.119

Obtaining an impedance plot for a parallel R-L network using Multisim.

computer solution. At very high frequencies such as 1 MHz, the impedance of the inductive element will be about 25 k Ω , which when placed in parallel with the 220 Ω will look like an open circuit. The result is that as the frequency gets very high, we should expect the impedance of the network to approach the 220 Ω level of the resistor. In addition, since the network will take on resistive characteristics at very high frequencies, the angle associated with the input impedance should also approach 0°. At very low frequencies the reactance of the inductive element will be much less than the 220 Ω of the resistor, and the network will take on inductive characteristics. In fact, at, say, 10 Hz, the reactance of the inductor is only about 0.25 Ω , which is very close to a short-circuit equivalent compared to the parallel 220 Ω resistor. The result is that the impedance of the network is very close to 0 Ω at very low frequencies. Again, since the inductive effects are so strong at low frequencies, the phase angle associated with the input impedance should be very close to 90°.

Now for the computer analysis. The current source, the resistor element, and the inductor are all placed and connected using procedures described in detail in earlier chapters. However, there is one big difference this time: Since the output will be plotted versus frequency, the **AC Analysis Magnitude** in the **AC_CURRENT** dialog box for the source must be set to 1 A. In this case, the default level of 1A matches that of the applied source, so you were set even if you failed to check the setting. In the future, however, a voltage or current source may be used that does not have an amplitude of 1, and proper entries must be made to this listing.

For the simulation, first apply the sequence **Simulate-Analyses-AC Analysis** to obtain the **AC Analysis** dialog box. Set the **Start frequency** at 10 Hz so that you have entries at very low frequencies, and set the **Stop frequency** at 1MHz so that you have data points at the other end of the spectrum. The **Sweep type** can remain **Decade**, but the number of points per decade will be 1000 so that you obtain a detailed plot. Set the **Vertical scale** on **Linear**. Within **Output variables**, V(1). Shifting it over to the **Selected variables for analysis** column using the **Add** keypad and then hitting the **Simulate** key results in the two plots in Fig. 15.119. Select the **Show/Hide Grid** key to place the grid on the graph, and select the **Show/Hide Cursors** key to place the **AC Analysis** dialog box appearing in Fig. 15.119. Since two graphs are present, define the one you are working on by clicking on the **Voltage** or **Phase** heading on the left side of each plot. A small red arrow appears when selected so you know which is the active plot. When setting up the cursors, be sure that you have activated the correct plot. When the red cursor is moved to 10 Hz (x1), you find that the voltage across the network is only 0.251 V (y1), resulting in an input impedance of only 0.25 Ω —quite small and matching your theoretical prediction. In addition, note that the phase angle is essentially at 90° in the other plot, confirming your other assumption above—a totally inductive network. If you set the blue cursor near 100 kHz (x2 = 102.3 kHz), you will find that the impedance at 219.2 Ω (y2) is closing in on the resistance of the parallel resistor of 220 Ω , again confirming the preliminary analysis above. As noted in the bottom of the **AC Analysis** box, the maximum value of the voltage is 219.99 Ω or essentially 220 Ω at 1 MHz. Before leaving the plot, note the advantages of using a log axis when you want a response over a wide frequency range.

PROBLEMS

SECTION 15.2 Impedance and the Phasor Diagram

1. Express the impedances in Fig. 15.120 in both polar and rectangular forms.

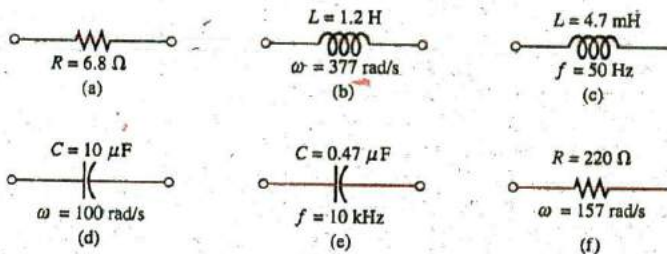


FIG. 15.120

Problem 1.

2. Find the current i for the elements in Fig. 15.121 using complex algebra. Sketch the waveforms for v and i on the same set of axes.

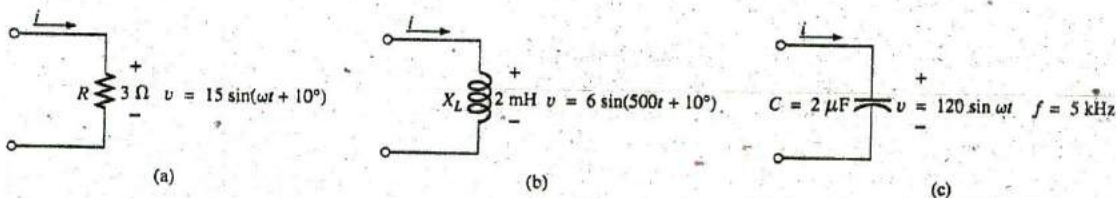


FIG. 15.121

Problem 2.

3. Find the voltage v for the elements in Fig. 15.122 using complex algebra. Sketch the waveforms of v and i on the same set of axes.

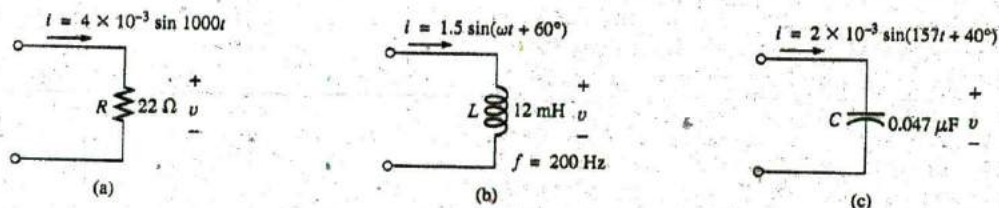
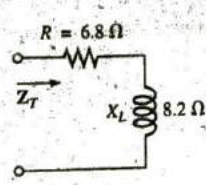


FIG. 15.122

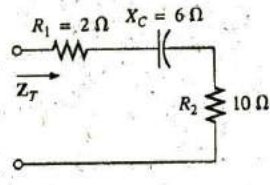
Problem 3.

SECTION 15.3 Series Configuration

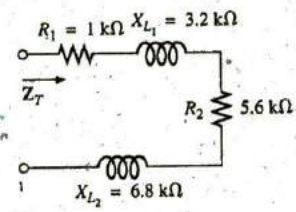
4. Calculate the total impedance of the circuits in Fig. 15.123. Express your answer in rectangular and polar forms, and draw the impedance diagram.



(a)



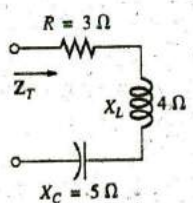
(b)



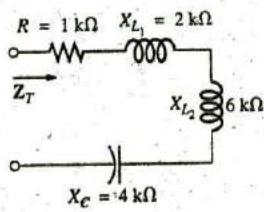
(c)

FIG. 15.123
Problem 4.

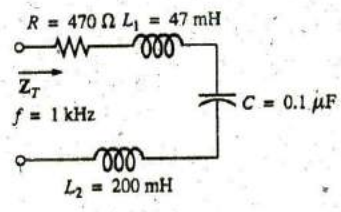
5. Calculate the total impedance of the circuits in Fig. 15.124. Express your answer in rectangular and polar forms, and draw the impedance diagram.



(a)



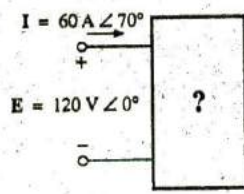
(b)



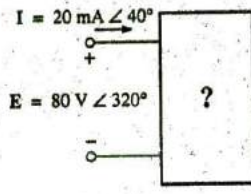
(c)

FIG. 15.124
Problem 5.

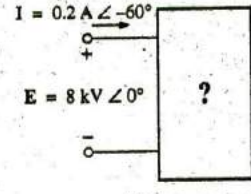
6. Find the type and impedance in ohms of the series circuit elements that must be in the closed container in Fig. 15.125 for the indicated voltages and currents to exist at the input terminals. (Find the simplest series circuit that will satisfy the indicated conditions.)



(a)



(b)



(c)

FIG. 15.125
Problems 6 and 26.

7. For the circuit in Fig. 15.126:
a. Find the total impedance Z_T in polar form.
b. Draw the impedance diagram.

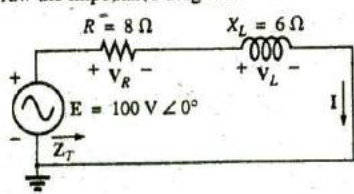


FIG. 15.126
Problems 7 and 48.

- Find the current I and the voltages V_R and V_L in phasor form.
- Draw the phasor diagram of the voltages E , V_R , and V_L , and the current I .
- Verify Kirchhoff's voltage law around the closed loop.
- Find the average power delivered to the circuit.
- Find the power factor of the circuit, and indicate whether it is leading or lagging.
- Find the sinusoidal expressions for the voltages and current if the frequency is 60 Hz.
- Plot the waveforms for the voltages and current on the same set of axes.

8. Repeat Problem 7 for the circuit in Fig. 15.127, replacing V_L with V_C in parts (c) and (d).

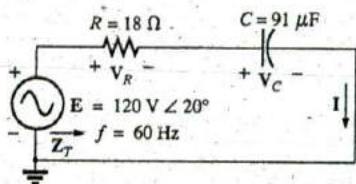


FIG. 15.127
Problem 8.

11. Using the oscilloscope reading in Fig. 15.130, determine the resistance R (closest standard value).

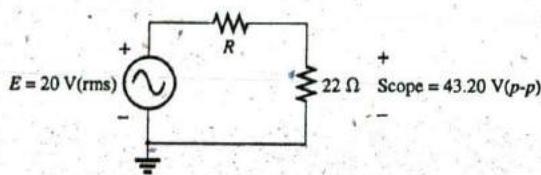


FIG. 15.130
Problem 11.

9. For the circuit in Fig. 15.128:
- Find the total impedance Z_T in polar form.
 - Draw the impedance diagram.
 - Find the value of C in microfarads and L in henries.
 - Find the current I and the voltages V_R , V_L , and V_C in phasor form.
 - Draw the phasor diagram of the voltages E , V_R , V_L , and V_C and the current I .
 - Verify Kirchhoff's voltage law around the closed loop.
 - Find the average power delivered to the circuit.
 - Find the power factor of the circuit, and indicate whether it is leading or lagging.
 - Find the sinusoidal expressions for the voltages and current.
 - Plot the waveforms for the voltages and current on the same set of axes.

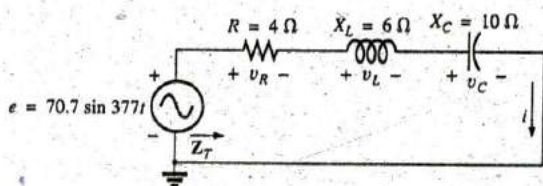


FIG. 15.128
Problem 9.

10. Repeat Problem 9 for the circuit in Fig. 15.129 except for part (c).

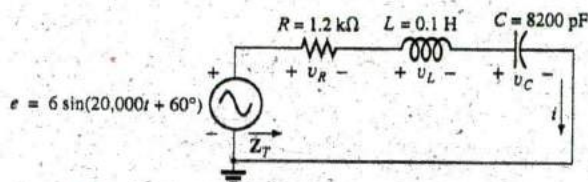


FIG. 15.129
Problem 10.

- *12. Using the DMM current reading and the oscilloscope measurement in Fig. 15.131:
- Determine the inductance L .
 - Find the resistance R .
 - Find the closest standard value for the inductance found in part (a).

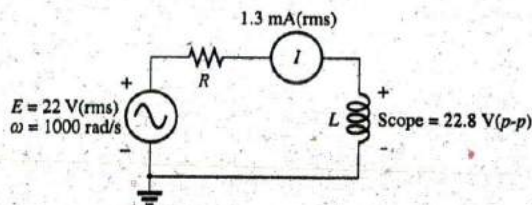


FIG. 15.131
Problem 12.

- *13. Using the oscilloscope reading in Fig. 15.132:
- Find the rms value of the current in the series circuit.
 - Determine the capacitance C .

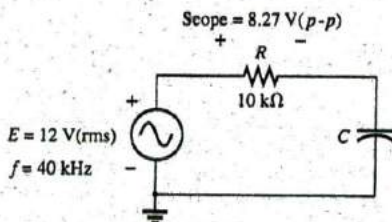


FIG. 15.132
Problem 13.

SECTION 15.4 Voltage Divider Rule

14. Calculate the voltages V_1 and V_2 for the circuits in Fig. 15.133 in phasor form using the voltage divider rule.

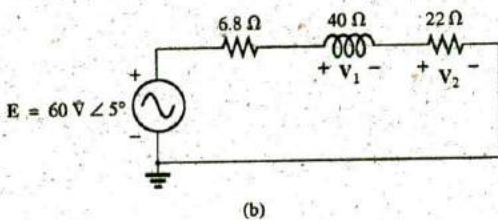
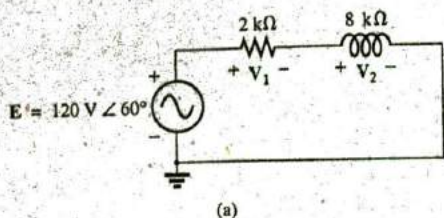


FIG. 15.133
Problem 14.

15. Calculate the voltages V_1 and V_2 for the circuits in Fig. 15.134 in phasor form using the voltage divider rule.

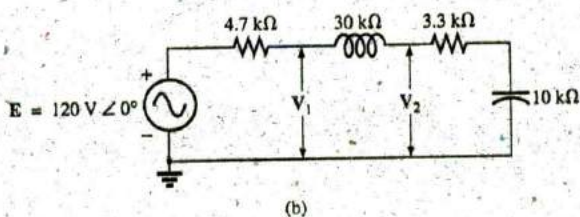
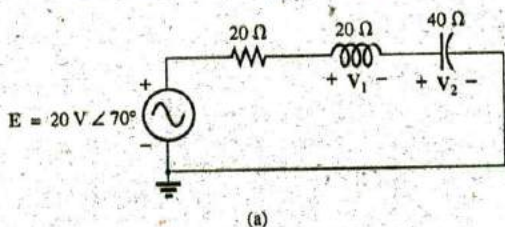


FIG. 15.134
Problem 15.

*16. For the circuit in Fig. 15.135:

- Determine I , V_R , and V_C in phasor form.
- Calculate the total power factor, and indicate whether it is leading or lagging.
- Calculate the average power delivered to the circuit.
- Draw the impedance diagram.
- Draw the phasor diagram of the voltages E , V_R , and V_C , and the current I .
- Find the voltages V_R and V_C using the voltage divider rule, and compare them with the results of part (a).
- Draw the equivalent series circuit of the above as far as the total impedance and the current i are concerned.

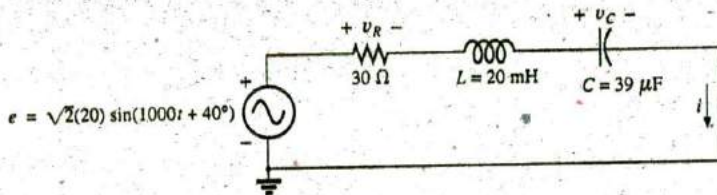


FIG. 15.135
Problems 16 and 47.

17. An electrical load has a power factor of 0.8 lagging. It dissipates 8 kW at a voltage of 200 V. Calculate the impedance of this load in rectangular coordinates.

*18. Find the series element or elements that must be in the enclosed container in Fig. 15.136 to satisfy the following conditions:

- Average power to circuit = 300 W.
- Circuit has a lagging power factor.

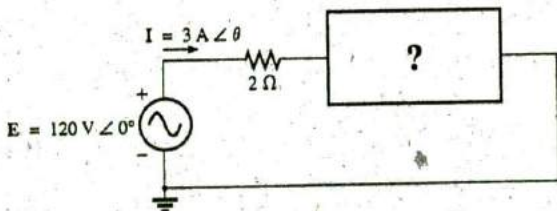


FIG. 15.136
Problem 18.

SECTION 15.5 Frequency Response for Series ac Circuits

*19. For the circuit in Fig. 15.137:

- Plot Z_T and θ_T versus frequency for a frequency range of zero to 20 kHz.
- Plot V_L versus frequency for the frequency range of part (a).
- Plot θ_L versus frequency for the frequency range of part (a).
- Plot V_R versus frequency for the frequency range of part (a).

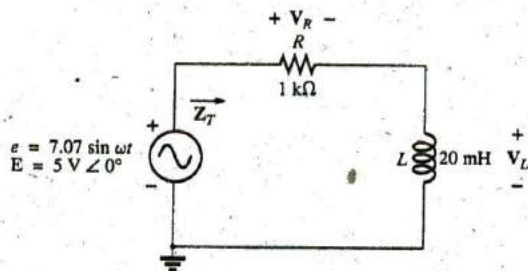


FIG. 15.137
Problem 19.

*20. For the circuit in Fig. 15.138:

- Plot Z_T and θ_T versus frequency for a frequency range of zero to 10 kHz.
- Plot V_C versus frequency for the frequency range of part (a).
- Plot θ_C versus frequency for the frequency range of part (a).
- Plot V_R versus frequency for the frequency range of part (a).

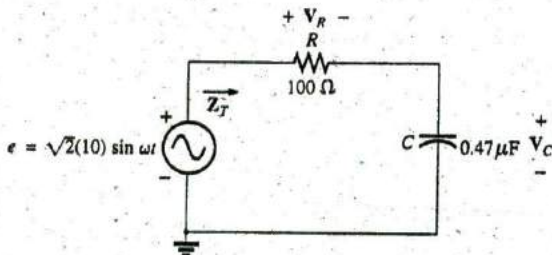


FIG. 15.138
Problem 20.

*21. For the series R-L-C circuit in Fig. 15.139:

- Plot Z_T and θ_T versus frequency for a frequency range of zero to 20 kHz in increments of 1 kHz.
- Plot V_C (magnitude only) versus frequency for the same frequency range of part (a).
- Plot I (magnitude only) versus frequency for the same frequency range of part (a).

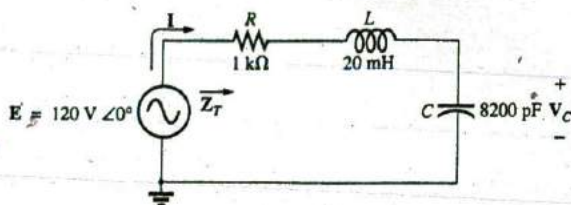


FIG. 15.139
Problem 21.

22. For the series R-C circuit in Fig. 15.140:

- Determine the frequency at which $X_C = R$.
- Develop a mental image of the change in total impedance with frequency without resorting to a single calculation.
- Find the total impedance at 100 Hz and 10 kHz, and compare your answer with the assumptions of part (b).
- Plot the curve of V_C versus frequency.
- Find the phase angle of the total impedance at $f = 40$ kHz. Is the network resistive or capacitive at this frequency?

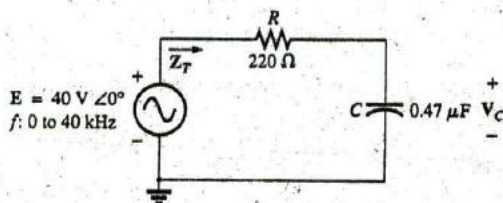


FIG. 15.140
Problem 22.

SECTION 15.7 Admittance and Susceptance

23. Find the total admittance of the branches in Fig. 15.141. Identify the values of conductance and susceptance, and draw the admittance diagram.

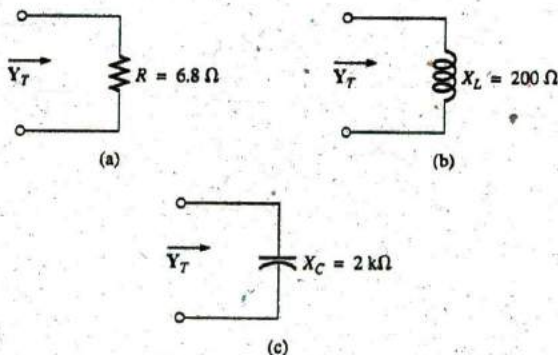


FIG. 15.141
Problem 23.

24. Find the total admittance for the networks of Fig. 15.142. Identify the values of total conductance and admittance, and draw the admittance diagram.

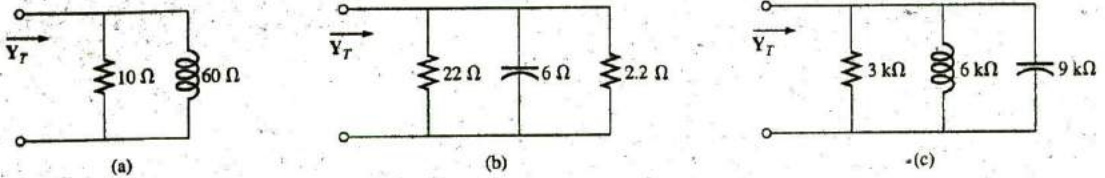


FIG. 15.142
Problem 24.

25. Find the total admittance for the networks of Fig. 15.143. Identify the values of total conductance and admittance, and draw the admittance diagram.

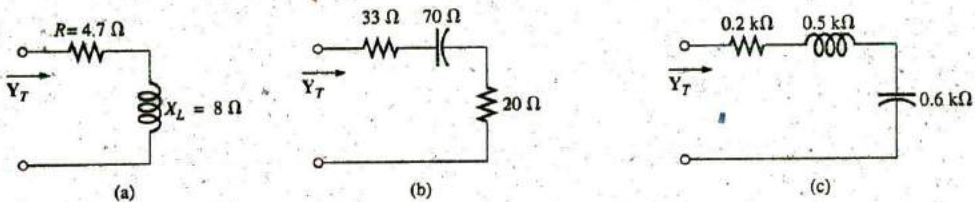


FIG. 15.143
Problem 25.

26. Repeat Problem 6 for the parallel circuit elements that must be in the closed container for the same voltage and current to exist at the input terminals. (Find the simplest parallel circuit that will satisfy the conditions indicated.)

SECTION 15.8 Parallel ac Networks

27. For the circuit in Fig. 15.144:
- Find the total admittance Y_T in polar form.
 - Draw the admittance diagram.
 - Find the voltage E and the currents I_R and I_L in phasor form.
 - Draw the phasor diagram of the currents I_T , I_R , and I_L , and the voltage E .
 - Verify Kirchoff's current law at one node.
 - Find the average power delivered to the circuit.

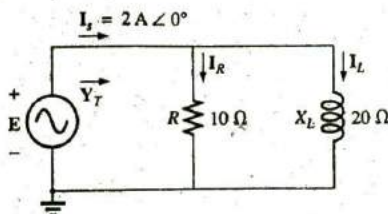


FIG. 15.144
Problem 27.

- Find the power factor of the circuit, and indicate whether it is leading or lagging.
 - Find the sinusoidal expressions for the currents and voltage if the frequency is 60 Hz.
 - Plot the waveforms for the currents and voltage on the same set of axes.
28. Repeat Problem 27 for the circuit in Fig. 15.145, replacing I_L with I_C in parts (c) and (d).

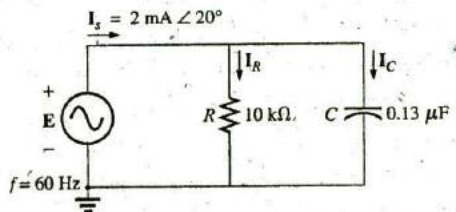


FIG. 15.145
Problem 28.

29. For the circuit in Fig. 15.146:
- Find the total admittance and impedance in polar form.
 - Draw the admittance and impedance diagrams.
 - Find the value of C in microfarads and L in henries.
 - Find the voltage E and currents I_R , I_L , and I_C in phasor form.
 - Draw the phasor diagram of the currents I_T , I_R , I_L , and I_C , and the voltage E .

- f. Verify Kirchoff's current law at one node.
 - g. Find the average power delivered to the circuit.
 - h. Find the power factor of the circuit, and indicate whether it is leading or lagging.
 - i. Find the sinusoidal expressions for the currents and voltage.
 - j. Plot the waveforms for the currents and voltage on the same set of axes.
30. Repeat Problem 29 for the circuit in Fig. 15.147 except for part (c).

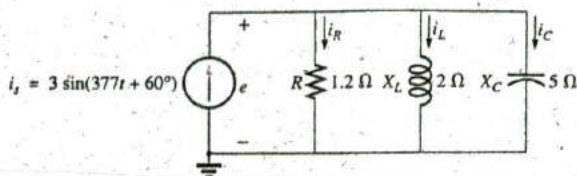


FIG. 15.146
Problem 29.

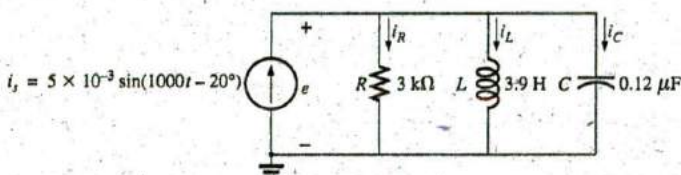


FIG. 15.147
Problem 30.

SECTION 15.9 Current Divider Rule

31. Calculate the currents I_1 and I_2 in Fig. 15.148 in phasor form using the current divider rule.

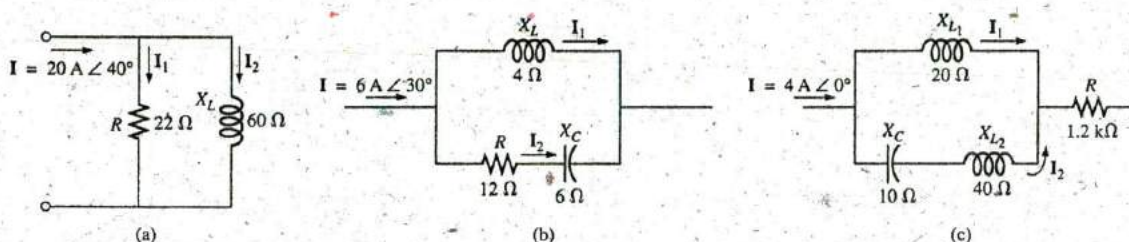


FIG. 15.148
Problem 31.

SECTION 15.10 Frequency Response of Parallel Elements

- *32. For the parallel R - C network in Fig. 15.149:
- a. Plot Z_T and θ_T versus frequency for a frequency range of zero to 20 kHz.
 - b. Plot V_C versus frequency for the frequency range of part (a).

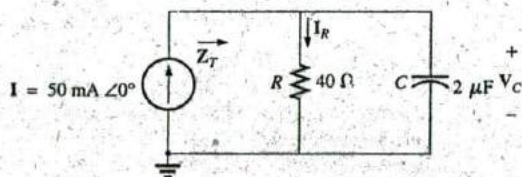


FIG. 15.149
Problems 32 and 34.

- c. Plot I_R versus frequency for the frequency range of part (a).

- *33. For the parallel R - L network in Fig. 15.150:
- a. Plot Z_T and θ_T versus frequency for a frequency range of zero to 10 kHz.
 - b. Plot I_L versus frequency for the frequency range of part (a).
 - c. Plot I_R versus frequency for the frequency range of part (a).

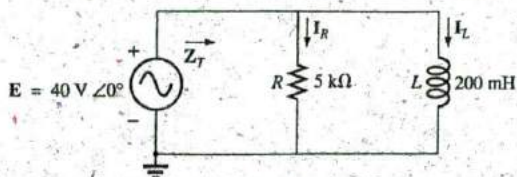


FIG. 15.150
Problems 33 and 35.

34. Plot Y_T and θ_T (of $Y_T = Y_T \angle \theta_T$) for a frequency range of zero to 20 kHz for the network in Fig. 15.149.
35. Plot Y_T and θ_T (of $Y_T = Y_T \angle \theta_T$) for a frequency range of zero to 10 kHz for the network in Fig. 15.150.
36. For the parallel R - L - C network in Fig. 15.151:

- a. Plot Y_T and θ_T (of $Y_T = Y_T \angle \theta_T$) for a frequency range of zero to 20 kHz.
- b. Repeat part (a) for Z_T and θ_T (of $Z_T = Z_T \angle \theta_T$).
- c. Plot V_C versus frequency for the frequency range of part (a).
- d. Plot I_L versus frequency for the frequency range of part (a).

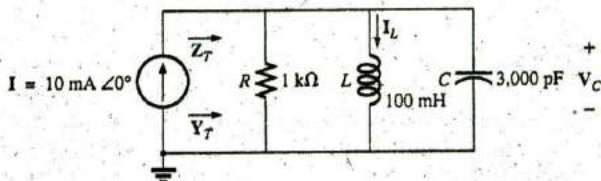


FIG. 15.151
Problem 36.

SECTION 15.12 Equivalent Circuits

37. For the series circuits in Fig. 15.152, find a parallel circuit that will have the same total impedance (Z_T).

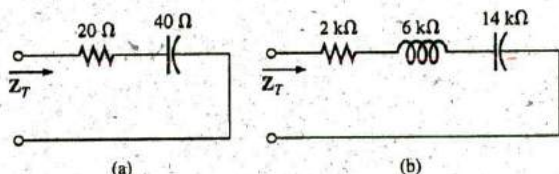


FIG. 15.152
Problem 37.

38. For the parallel circuits in Fig. 15.153, find a series circuit that will have the same total impedance.

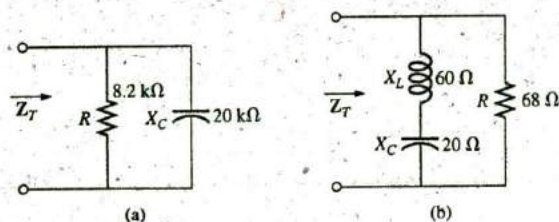


FIG. 15.153
Problem 38.

- *39. For the network in Fig. 15.154:
 - a. Calculate E , I_R , and I_L in phasor form.
 - b. Calculate the total power factor, and indicate whether it is leading or lagging.
 - c. Calculate the average power delivered to the circuit.
 - d. Draw the admittance diagram.
 - e. Draw the phasor diagram of the currents I_T , I_R , and I_L , and the voltage E .
 - f. Find the current I_C for each capacitor using only Kirchhoff's current law.
 - g. Find the series circuit of one resistive and reactive element that will have the same impedance as the original circuit.

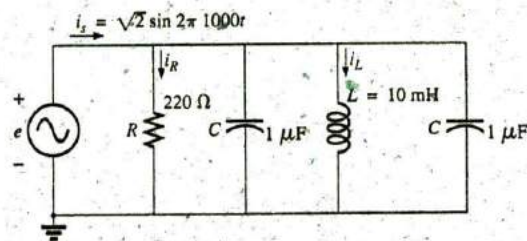


FIG. 15.154
Problem 39.

40. Find the element or elements that must be in the closed container in Fig. 15.155 to satisfy the following conditions. (Find the simplest parallel circuit that will satisfy the indicated conditions.)
 - a. Average power to the circuit = 3000 W.
 - b. Circuit has a lagging power factor.

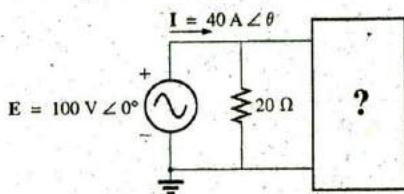


FIG. 15.155
Problem 40.

SECTION 15.13 Phase Measurements

41. For the circuit in Fig. 15.156, determine the phase relationship between the following using a dual-trace oscilloscope. The circuit can be reconstructed differently for each part, but do not use sensing resistors. Show all connections on a redrawn diagram.
 - a. e and v_C
 - b. e and i_T
 - c. e and v_L

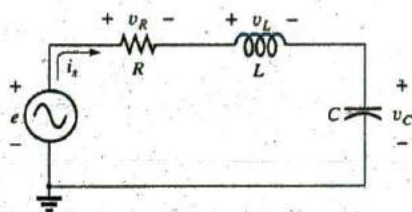
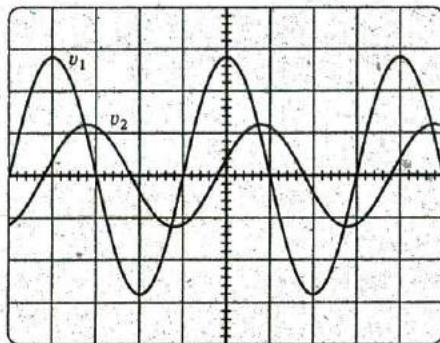


FIG. 15.156

Problem 41.

42. For the network in Fig. 15.157, determine the phase relationship between the following using a dual-trace oscilloscope. The network must remain as constructed in Fig. 15.157, but sensing resistors can be introduced. Show all connections on a redrawn diagram.
- e and v_{R_2}
 - e and i_s
 - i_L and i_C



Vertical sensitivity = 0.5 V/div.
Horizontal sensitivity = 0.2 ms/div.

(a)

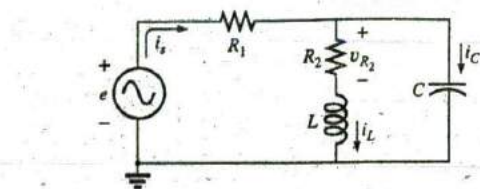
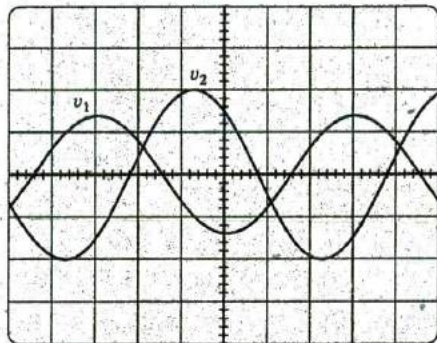


FIG. 15.157

Problem 42.

43. For the oscilloscope traces in Fig. 15.158:
- Determine the phase relationship between the waveforms, and indicate which one leads or lags.
 - Determine the peak-to-peak and rms values of each waveform.
 - Find the frequency of each waveform.



Vertical sensitivity = 2 V/div.
Horizontal sensitivity = 10 μ s/div.

(b)

FIG. 15.158

Problem 43.

SECTION 15.15 Computer Analysis

PSpice or Multisim

44. For the network in Fig. 15.124 (use $f = 1$ kHz):
- Determine the rms values of the voltages V_R and V_L and the current I .
 - Plot v_R , v_L , and i versus time on separate plots.
 - Place e , v_R , v_L , and i on the same plot, and label accordingly.
45. For the network in Fig. 15.144:
- Determine the rms values of the currents I_s , I_R , and I_L .
 - Plot i_s , i_R , and i_L versus time on separate plots.
 - Place e , i_s , i_R , and i_L on the same plot, and label accordingly.
46. For the network in Fig. 15.127:
- Plot the impedance of the network versus frequency from 0 to 10 kHz.
 - Plot the current i versus frequency for the frequency range zero to 10 kHz.

- *47. For the network in Fig. 15.135:

- Find the rms values of the voltages v_R and v_C at a frequency of 1 kHz.
- Plot v_C versus frequency for the frequency range zero to 10 kHz.
- Plot the phase angle between e and i for the frequency range zero to 10 kHz.

GLOSSARY

Admittance A measure of how easily a network will "admit" the passage of current through that system. It is measured in siemens, abbreviated S, and is represented by the capital letter Y .

Admittance diagram A vector display that clearly depicts the magnitude of the admittance of the conductance, capacitive susceptance, and inductive susceptance and the magnitude and angle of the total admittance of the system.

Current divider rule A method by which the current through either of two parallel branches can be determined in an ac network without first finding the voltage across the parallel branches.

Equivalent circuits For every series ac network, there is a parallel ac network (and vice versa) that will be "equivalent" in the sense that the input current and impedance are the same.

Impedance diagram A vector display that clearly depicts the magnitude of the impedance of the resistive, reactive, and capacitive components of a network and the magnitude and angle of the total impedance of the system.

Parallel ac circuits A connection of elements in an ac network in which all the elements have two points in common. The voltage is the same across each element.

Phasor diagram A vector display that provides at a glance the magnitude and phase relationships among the various voltages and currents of a network.

Series ac configuration A connection of elements in an ac network in which no two impedances have more than one terminal in common and the current is the same through each element.

Susceptance A measure of how "susceptible" an element is to the passage of current through it. It is measured in siemens, abbreviated S, and is represented by the capital letter *B*.

Voltage divider rule A method through which the voltage across one element of a series of elements in an ac network can be determined without first having to find the current through the elements.