SERIES-PARALLEL AC NETWORKS

Objectives
- Develop confidence in the analysis of series-parallel ac networks.
- Become proficient in the use of calculators and computer methods to support the analysis of ac series-parallel networks.
- Understand the importance of proper grounding in the operation of any electrical system.

16.1 INTRODUCTION

In this chapter, we shall use the fundamental concepts of the previous chapter to develop a technique for solving **series-parallel ac networks**. A brief review of Chapter 7 may be helpful before considering these networks since the approach here is quite similar to that undertaken earlier. The circuits to be discussed have only one source of energy, either potential or current. Networks with two or more sources are considered in Chapters 17 and 18, using methods previously described for dc circuits.

In general, when working with series-parallel ac networks, consider the following approach:

1. *Redraw the network, using block impedances to combine obvious series and parallel elements, which will reduce the network to one that clearly reveals the fundamental structure of the system.*
2. *Study the problem and make a brief mental sketch of the overall approach you plan to use. Doing this may result in time- and energy-saving shortcuts. In some cases, a lengthy, drawn-out analysis may not be necessary. A single application of a fundamental law of circuit analysis may result in the desired solution.*
3. *After the overall approach has been determined, it is usually best to consider each branch involved in your method independently before tying them together in series-parallel combinations. In most cases, work back from the obvious series and parallel combinations to the source to determine the total impedance of the network. The source current can then be determined, and the path back to specific unknowns can be defined. As you progress back to the source, continually define those unknowns that have not been lost in the reduction process. It will save time when you have to work back through the network to find specific quantities.*
4. *When you have arrived at a solution, check to see that it is reasonable by considering the magnitudes of the energy source and the elements in the circuit. If not, either solve the network using another approach or check over your work very carefully. At this point, a computer solution can be an invaluable asset in the validation process.*

16.2 ILLUSTRATIVE EXAMPLES

EXAMPLE 16.1 For the network in Fig. 16.1:

a. Calculate Z_T
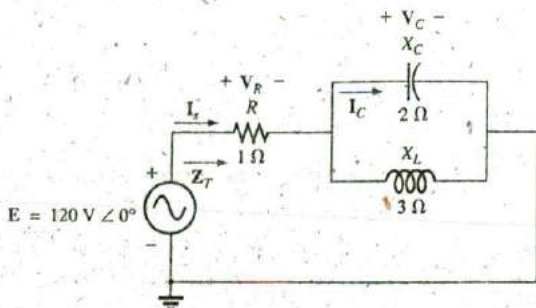b. Determine I_s
c. Calculate V_R and V_C.
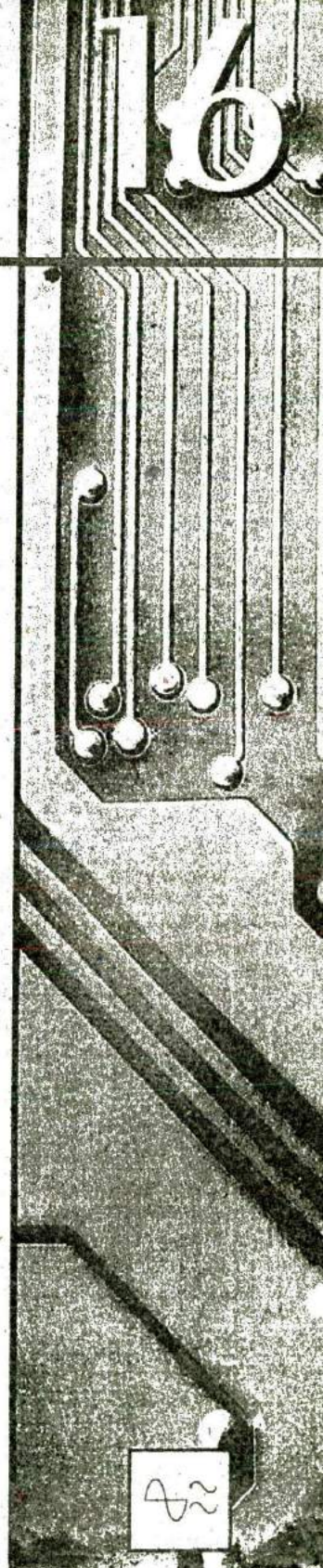
FIG. 16.1
Example 16.1.

FIG. 16.2
Network in Fig. 16.1 after assigning the block impedances.

d. Find I_C.
e. Compute the power delivered.
f. Find F_p of the network.

Solutions:

a. As suggested in the introduction, the network has been redrawn with block impedances, as shown in Fig. 16.2. The impedance Z_1 is simply the resistor R of 1 Ω, and Z_2 is the parallel combination of X_C and X_L. The network now clearly reveals that it is fundamentally a series circuit, suggesting a direct path toward the total impedance and the source current. For many such problems, you must work back to the source to find first the total impedance and then the source current. When the unknown quantities are found in terms of these subscripted impedances, the numerical values can then be substituted to find the magnitude and phase angle of the unknown. In other words, try to find the desired solution solely in terms of the subscripted impedances before substituting numbers. This approach will usually enhance the clarity of the chosen path toward a solution while saving time and preventing careless calculation errors. Note also in Fig. 16.2 that all the unknown quantities except I_C have been preserved, meaning that we can use Fig. 16.2 to determine these quantities rather than having to return to the more complex network in Fig. 16.1.

The total impedance is defined by

$$Z_T = Z_1 + Z_2$$

with

$Z_1 = R \angle 0° = 1\,\Omega \angle 0°$

$$Z_2 = Z_C \| Z_L = \frac{(X_C \angle -90°)(X_L \angle 90°)}{-jX_C + jX_L} = \frac{(2\,\Omega \angle -90°)(3\,\Omega \angle 90°)}{-j\,2\,\Omega + j\,3\,\Omega}$$

$$= \frac{6\,\Omega \angle 0°}{j\,1} = \frac{6\,\Omega \angle 0°}{1 \angle 90°} = 6\,\Omega \angle -90°$$

and

$$Z_T = Z_1 + Z_2 = 1\,\Omega - j\,6\,\Omega = 6.08\,\Omega \angle -80.54°$$

b. $I_s = \dfrac{E}{Z_T} = \dfrac{120\,V \angle 0°}{6.08\,\Omega \angle -80.54°} = 19.74\,A \angle 80.54°$

c. Referring to Fig. 16.2, we find that V_R and V_C can be found by a direct application of Ohm's law:

$$V_R = I_s Z_1 = (19.74 \text{ A } \angle 80.54°)(1 \text{ } \Omega \angle 0°) = 19.74 \text{ V} \angle 80.54°$$
$$V_C = I_s Z_2 = (19.74 \text{ A } \angle 80.54°)(6 \text{ } \Omega \angle -90°)$$
$$= 118.44 \text{ V} \angle -9.46°$$

d. Now that V_C is known, the current I_C can also be found using Ohm's law:

$$I_C = \frac{V_C}{Z_C} = \frac{118.44 \text{ V} \angle -9.46°}{2 \text{ } \Omega \angle -90°} = 59.22 \text{ A } \angle 80.54°$$

e. $P_{del} = I_s^2 R = (19.74 \text{ A})^2(1 \text{ } \Omega) = 389.67 \text{ W}$

f. $F_p = \cos \theta = \cos 80.54° = 0.164 \text{ leading}$

The fact that the total impedance has a negative phase angle (revealing that I_s leads E) is a clear indication that the network is capacitive in nature and therefore has a leading power factor. The fact that the network is capacitive can be determined from the original network by first realizing that, for the parallel L-C elements, the smaller impedance predominates and results in an R-C network.

EXAMPLE 16.2 For the network in Fig. 16.3:

a. If I is 50 A $\angle 30°$ calculate I_1 using the current divider rule.
b. Repeat part (a) for I_2.
c. Verify Kirchhoff's current law at one node.

Solutions:

a. Redrawing the circuit as in Fig. 16.4, we have

$$Z_1 = R + jX_L = 3 \text{ } \Omega + j4 \text{ } \Omega = 5 \text{ } \Omega \angle 53.13°$$
$$Z_2 = -jX_C = -j8 \text{ } \Omega = 8 \text{ } \Omega \angle -90°$$

Using the current divider rule yields

$$I_1 = \frac{Z_2 I}{Z_2 + Z_1} = \frac{(8 \text{ } \Omega \angle -90°)(50 \text{ A } \angle 30°)}{(-j8 \text{ } \Omega) + (3 \text{ } \Omega + j4 \text{ } \Omega)} = \frac{400 \angle -60°}{3 - j4}$$
$$= \frac{400 \angle -60°}{5 \angle -53.13°} = 80 \text{ A } \angle -6.87°$$

b. $I_2 = \frac{Z_1 I}{Z_2 + Z_1} = \frac{(5 \text{ } \Omega \angle 53.13°)(50 \text{ A } \angle 30°)}{5 \text{ } \Omega \angle -53.13°} = \frac{250 \angle 83.13°}{5 \angle -53.13°}$
$$= 50 \text{ A } \angle 136.26°$$

c. $\quad I = I_1 + I_2$
$50 \text{ A } \angle 30° = 80 \text{ A } \angle -6.87° + 50 \text{ A } \angle 136.26°$
$$= (79.43 - j9.57) + (-36.12 + j34.57)$$
$$= 43.31 + j25.0$$
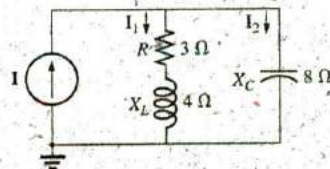$50 \text{ A } \angle 30° = 50 \text{ A } \angle 30° \quad \text{(checks)}$
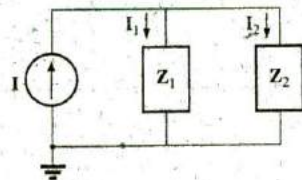

FIG. 16.3
Example 16.2.

FIG. 16.4
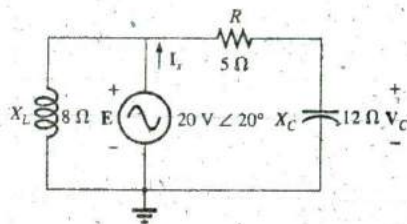Network in Fig. 16.3 after assigning the block impedances.
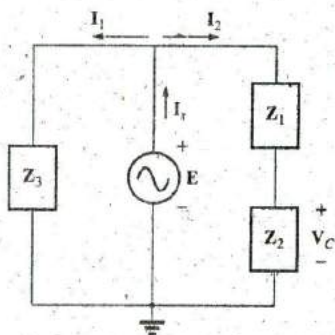
FIG. 16.5
Example 16.3.

FIG. 16.6
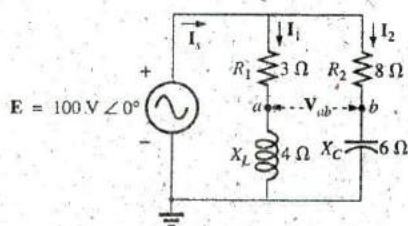Network in Fig. 16.5 after assigning the block
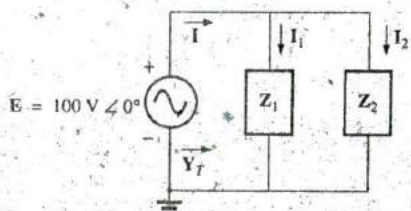impedances.

FIG. 16.7
Example 16.4.

FIG. 16.8
Network in Fig. 16.7 after assigning the block
impedances.

EXAMPLE 16.3 For the network in Fig. 16.5:

a. Calculate the voltage V_C using the voltage divider rule.
b. Calculate the current I_s.

Solutions:

a. The network is redrawn as shown in Fig. 16.6, with

$$Z_1 = 5\,\Omega = 5\,\Omega\,\angle 0°$$
$$Z_2 = -j\,12\,\Omega = 12\,\Omega\,\angle -90°$$
$$Z_3 = +j\,8\,\Omega = 8\,\Omega\,\angle 90°$$

Since V_C is desired, we will not combine R and X_C into a single block impedance. Note also how Fig. 16.6 clearly reveals that E is the total voltage across the series combination of Z_1 and Z_2, permitting the use of the voltage divider rule to calculate V_C. In addition, note that all the currents necessary to determine I_s have been preserved in Fig. 16.6, revealing that there is no need to ever return to the network of Fig. 16.5—everything is defined by Fig. 16.6.

$$V_C = \frac{Z_2 E}{Z_1 + Z_2} = \frac{(12\,\Omega\,\angle -90°)(20\,V\,\angle 20°)}{5\,\Omega - j\,12\,\Omega} = \frac{240\,V\,\angle -70°}{13\,\angle -67.38°}$$
$$= 18.46\,V\,\angle -2.62°$$

b. $I_1 = \dfrac{E}{Z_3} = \dfrac{20\,V\,\angle 20°}{8\,\Omega\,\angle 90°} = 2.5\,A\,\angle -70°$

$$I_2 = \frac{E}{Z_1 + Z_2} = \frac{20\,V\,\angle 20°}{13\,\Omega\,\angle -67.38°} = 1.54\,A\,\angle 87.38°$$

and

$$I_s = I_1 + I_2$$
$$= 2.5\,A\,\angle -70° + 1.54\,A\,\angle 87.38°$$
$$= (0.86 - j\,2.35) + (0.07 + j\,1.54)$$
$$I_s = 0.93 - j\,0.81 = 1.23\,A\,\angle -41.05°$$

EXAMPLE 16.4 For Fig. 16.7:

a. Calculate the current I_s.
b. Find the voltage V_{ab}.

Solutions:

a. Redrawing the circuit as in Fig. 16.8, we obtain

$$Z_1 = R_1 + jX_L = 3\,\Omega + j\,4\,\Omega = 5\,\Omega\,\angle 53.13°$$
$$Z_2 = R_2 - jX_C = 8\,\Omega - j\,6\,\Omega = 10\,\Omega\,\angle -36.87°$$

In this case the voltage V_{ab} is lost in the redrawn network, but the currents I_1 and I_2 remain defined for future calculations necessary to determine V_{ab}. Fig. 16.8 clearly reveals that the total impedance can be found using the equation for two parallel impedances:

$$Z_T = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(5\,\Omega\,\angle 53.13°)(10\,\Omega\,\angle -36.87°)}{(3\,\Omega + j\,4\,\Omega) + (8\,\Omega - j\,6\,\Omega)}$$
$$= \frac{50\,\Omega\,\angle 16.26°}{11 - j\,2} = \frac{50\,\Omega\,\angle 16.26°}{11.18\,\angle -10.30°}$$
$$= 4.472\,\Omega\,\angle 26.56°$$

and

$$I_s = \frac{E}{Z_T} = \frac{100 \text{ V} \angle 0°}{4.472 \text{ }\Omega \angle 26.56°} = 22.36 \text{ A} \angle -26.56°$$

b. By Ohm's law,

$$I_1 = \frac{E}{Z_1} = \frac{100 \text{ V} \angle 0°}{5 \text{ }\Omega \angle 53.13°} = 20 \text{ A} \angle -53.13°$$

$$I_2 = \frac{E}{Z_2} = \frac{100 \text{ V} \angle 0°}{10 \text{ }\Omega \angle -36.87°} = 10 \text{ A} \angle 36.87°$$

Returning to Fig. 16.7, we have

$$V_{R_1} = I_1 Z_{R_1} = (20 \text{ A} \angle -53.13°)(3 \text{ }\Omega \angle 0°) = 60 \text{ V} \angle -53.13°$$
$$V_{R_2} = I_1 Z_{R_2} = (10 \text{ A} \angle +36.87°)(8 \text{ }\Omega \angle 0°) = 80 \text{ V} \angle +36.87°$$

Instead of using the two steps just shown, we could have determined V_{R_1} or V_{R_2} in one step using the voltage divider rule:

$$V_{R_1} = \frac{(3 \text{ }\Omega \angle 0°)(100 \text{ V} \angle 0°)}{3 \text{ }\Omega \angle 0° + 4 \text{ }\Omega \angle 90°} = \frac{300 \text{ V} \angle 0°}{5 \angle 53.13°} = 60 \text{ V} \angle -53.13°$$

To find V_{ab}, Kirchhoff's voltage law must be applied around the loop (Fig. 16.9) consisting of the 3 Ω and 8 Ω resistors. By Kirchhoff's voltage law,

$$V_{ab} + V_{R_1} - V_{R_2} = 0$$

or
$$\begin{aligned} V_{ab} &= V_{R_2} - V_{R_1} \\ &= 80 \text{ V} \angle 36.87° - 60 \text{ V} \angle -53.13° \\ &= (64 + j\,48) - (36 - j\,48) \\ &= 28 + j\,96 \\ V_{ab} &= 100 \text{ V} \angle 73.74° \end{aligned}$$
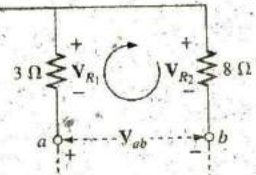

FIG. 16.9
Determining the voltage V_{ab} for the network in Fig. 16.7.

EXAMPLE 16.5 The network in Fig. 16.10 is frequently encountered in the analysis of transistor networks. The transistor equivalent circuit includes a current source I and an output impedance R_o. The resistor R_C is a biasing resistor to establish specific dc conditions, and the resistor R_i represents the loading of the next stage. The coupling capacitor is designed to be an open circuit for dc and to have as low an impedance as possible for the frequencies of interest to ensure that V_L is a maximum
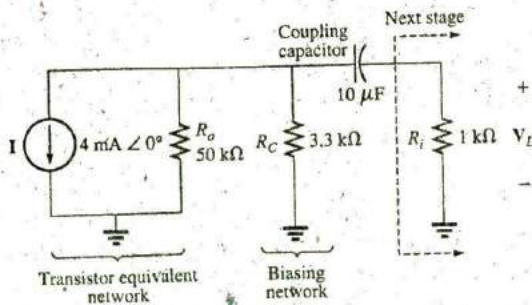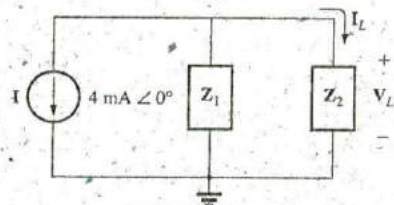

FIG. 16.10
Basic transistor amplifier.

value. The frequency range of the example includes the entire audio (hearing) spectrum from 100 Hz to 20 kHz. The purpose of the example is to demonstrate that, for the full audio range, the effect of the capacitor can be ignored. It performs its function as a dc blocking agent but permits the ac to pass through with little disturbance.

a. Determine V_L for the network in Fig. 16.10 at a frequency of 100 Hz.
b. Repeat part (a) at a frequency of 20 kHz.
c. Compare the results of parts (a) and (b).

Solutions:

a. The network is redrawn with subscripted impedances in Fig. 16.11:

$$Z_1 = 50 \text{ k}\Omega \angle 0° \parallel 3.3 \text{ k}\Omega \angle 0° = 3.096 \text{ k}\Omega \angle 0°$$
$$Z_2 = R_i - jX_C$$

At $f = 100$ Hz, $X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(100 \text{ Hz})(10 \text{ }\mu\text{F})} = 159.16 \text{ }\Omega$

and
$$Z_2 = 1 \text{ k}\Omega - j\,159.16 \text{ }\Omega$$

Using the current divider rule gives

$$
\begin{aligned}
I_L &= \frac{-Z_1 I}{Z_1 + Z_2} = \frac{-(3.096 \text{ k}\Omega \angle 0°)(4 \text{ mA} \angle 0°)}{3.096 \text{ k}\Omega + 1 \text{ k}\Omega - j\,159.16 \text{ }\Omega} \\
&= \frac{-12.384 \text{ A} \angle 0°}{4096 - j\,159.16} = \frac{-12.384 \text{ A} \angle 0°}{4099 \angle -2.225°} \\
&= -3.02 \text{ mA} \angle 2.23° = 3.02 \text{ mA} \angle 2.23° + 180° = 3.02 \text{ mA} \angle 182.23°
\end{aligned}
$$

and
$$
\begin{aligned}
V_L &= I_L Z_R \\
&= (3.02 \text{ mA} \angle 182.23°)(1 \text{ k}\Omega \angle 0°) \\
&= \mathbf{3.02 \text{ V} \angle 182.23°}
\end{aligned}
$$

b. At $f = 20$ kHz, $X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(20 \text{ kHz})(10 \text{ }\mu\text{F})} = 0.796 \text{ }\Omega$

Note the dramatic change in X_C with frequency. Obviously, the higher the frequency, the better is the short-circuit approximation for X_C for ac conditions. We have

$$Z_2 = 1 \text{ k}\Omega - j\,0.796 \text{ }\Omega$$

Using the current divider rule gives

$$
\begin{aligned}
I_L &= \frac{-Z_1 I}{Z_1 + Z_2} = \frac{-(3.096 \text{ k}\Omega \angle 0°)(4 \text{ mA} \angle 0°)}{3.096 \text{ k}\Omega + 1 \text{ k}\Omega - j\,0.796 \text{ }\Omega} \\
&= \frac{-12.384 \text{ A} \angle 0°}{4096 - j\,0.796 \text{ }\Omega} = \frac{-12.384 \text{ A} \angle 0°}{4096 \angle -0.011°} \\
&= -3.02 \text{ mA} \angle 0.01° = 3.02 \text{ mA} \angle 0.01° + 180° = 3.02 \text{ mA} \angle 180.01°
\end{aligned}
$$

and
$$
\begin{aligned}
V_L &= I_L Z_R \\
&= (3.02 \text{ mA} \angle 180.01°)(1 \text{ k}\Omega \angle 0°) \\
&= \mathbf{3.02 \text{ V} \angle 180.01°}
\end{aligned}
$$

c. The results clearly indicate that the capacitor had little effect on the frequencies of interest. In addition, note that most of the supply current reached the load for the typical parameters employed.

FIG. 16.11
Network in Fig. 16.10 following the assignment of the block impedances.

EXAMPLE 16.6 For the network in Fig. 16.12:

a. Determine the current **I**.
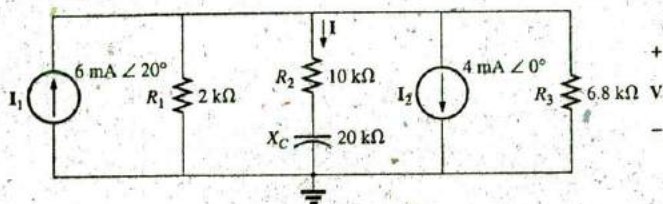b. Find the voltage **V**.

FIG. 16.12
Example 16.6.

Solutions:

a. The rules for parallel current sources are the same for dc and ac networks. That is, the equivalent current source is their sum or difference (as phasors). Therefore,

$$I_T = 6 \text{ mA} \angle 20° - 4 \text{ mA} \angle 0°$$
$$= 5.638 \text{ mA} + j\, 2.052 \text{ mA} - 4 \text{ mA}$$
$$= 1.638 \text{ mA} + j\, 2.052 \text{ mA}$$
$$= 2.626 \text{ mA} \angle 51.402°$$

Redrawing the network using block impedances results in the network in Fig. 16.13, where

$$Z_1 = 2 \text{ k}\Omega \angle 0° \, 6.8 \text{ k}\Omega \angle 0° = 1.545 \text{ k}\Omega \angle 0°$$

and $$Z_2 = 10 \text{ k}\Omega - j\, 20 \text{ k}\Omega = 22.361 \text{ k}\Omega \angle -63.435°$$

Note that **I** and **V** are still defined in Fig. 16.13.

Using the current divider rule gives

$$I = \frac{Z_1 I_T}{Z_1 + Z_2} = \frac{(1.545 \text{ k}\Omega \angle 0°)(2.626 \text{ mA} \angle 51.402°)}{1.545 \text{ k}\Omega + 10 \text{ k}\Omega - j\, 20 \text{ k}\Omega}$$
$$= \frac{4.057 \text{ A} \angle 51.402°}{11.545 \times 10^3 - j\, 20 \times 10^3} = \frac{4.057 \text{ A} \angle 51.402°}{23.093 \times 10^3 \angle -60.004°}$$
$$= 0.18 \text{ mA} \angle 111.41°$$

b. $$V = IZ_2$$
$$= (0.176 \text{ mA} \angle 111.406°)(22.36 \text{ k}\Omega \angle -63.435°)$$
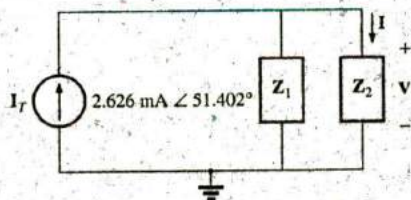$$= 3.94 \text{ V} \angle 47.97°$$

FIG. 16.13
Network in Fig. 16.12 following the assignment of
the subscripted impedances.

EXAMPLE 16.7 For the network in Fig. 16.14:

a. Compute **I**.
b. Find I_1, I_2, and I_3.
c. Verify Kirchhoff's current law by showing that

$$I = I_1 + I_2 + I_3$$
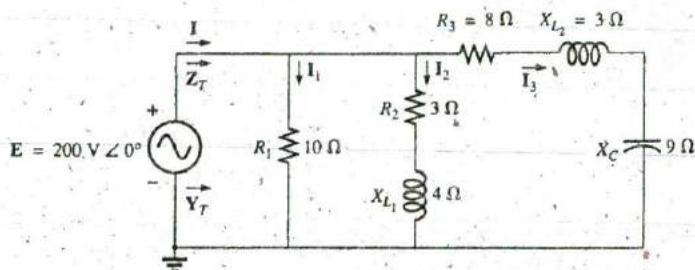
d. Find the total impedance of the circuit.

FIG. 16.14
Example 16.7.

Solutions:

a. Redrawing the circuit as in Fig. 16.15 reveals a strictly parallel network where

$$Z_1 = R_1 = 10\ \Omega\ \angle 0°$$
$$Z_2 = R_2 + jX_{L_1} = 3\ \Omega + j4\ \Omega$$
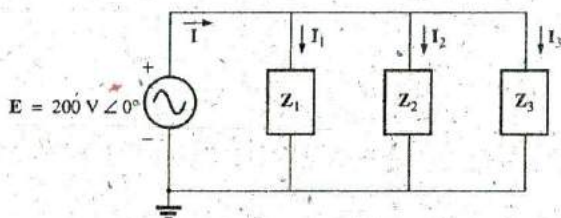$$Z_3 = R_3 + jX_{L_2} - jX_C = 8\ \Omega + j\ 3\ \Omega - j\ 9\ \Omega = 8\ \Omega - j\ 6\ \Omega$$

FIG. 16.15
Network in Fig. 16.14 following the assignment of the sub-
scripted impedances.

The total admittance is

$$Y_T = Y_1 + Y_2 + Y_3$$
$$= \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} = \frac{1}{10\ \Omega} + \frac{1}{3\ \Omega + j4\ \Omega} + \frac{1}{8\ \Omega - j6\ \Omega}$$
$$= 0.1\ S + \frac{1}{5\ \Omega\ \angle 53.13°} + \frac{1}{10\ \Omega\ \angle -36.87°}$$
$$= 0.1\ S + 0.2\ S\ \angle -53.13° + 0.1\ S\ \angle 36.87°$$
$$= 0.1\ S + 0.12\ S - j\ 0.16\ S + 0.08\ S + j\ 0.06\ S$$
$$= 0.3\ S - j\ 0.1\ S = 0.316\ S\ \angle -18.435°$$

Calculator The above mathematical exercise presents an excellent
opportunity to demonstrate the power of some of today's calculators. For
the TI-89, the above operation is as shown in Fig. 16.16.

300.0E-3-100.0E-3i

FIG. 16.16
Finding the total admittance for the network in Fig. 16.14 using the TI-89 calculator.

Be sure to use the negative sign for the complex number from the subtraction option and not the sign selection (–). The sign selection is used for negative angles in the polar form.

Converting to polar form requires the sequence shown in Fig. 16.17.

((• 3 – • 1 i)) ▶ Polar ENTER ENTER 316.2E–3 ∠ –18.43E0

FIG. 16.17
Converting the rectangular form in Fig. 16.16 to polar form.

Convert to polar form:
The current **I** is given by

$$\mathbf{I} = \mathbf{EY}_T = (200 \text{ V} \angle 0°)(0.326 \text{ S} \angle -18.435°)$$
$$= 63.2 \text{ A} \angle -18.44°$$

b. Since the voltage is the same across parallel branches,

$$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1} = \frac{200 \text{ V} \angle 0°}{10 \text{ }\Omega \angle 0°} = 20 \text{ A} \angle 0°$$

$$\mathbf{I}_2 = \frac{\mathbf{E}}{\mathbf{Z}_2} = \frac{200 \text{ V} \angle 0°}{5 \text{ }\Omega \angle 53.13°} = 40 \text{ A} \angle -53.13°$$

$$\mathbf{I}_3 = \frac{\mathbf{E}}{\mathbf{Z}_3} = \frac{200 \text{ V} \angle 0°}{10 \text{ }\Omega \angle -36.87°} = 20 \text{ A} \angle +36.87°$$

c.
$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3$$
$$60 - j\,20 = 20 \angle 0° + 40 \angle -53.13° + 20 \angle +36.87°$$
$$= (20 + j\,0) + (24 - j\,32) + (16 + j\,12)$$
$$60 - j\,20 = 60 - j\,20 \qquad \text{(checks)}$$

d.
$$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.316 \text{ S} \angle -18.435°}$$
$$= 3.17 \text{ }\Omega \angle 18.44°$$

EXAMPLE 16.8 For the network in Fig. 16.18:
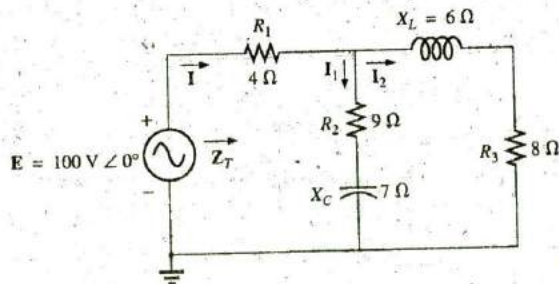
FIG. 16.18
Example 16.8.

a. Calculate the total impedance $\mathbf{Z}_T$.
b. Compute **I**.
c. Find the total power factor.
d. Calculate $\mathbf{I}_1$ and $\mathbf{I}_2$.
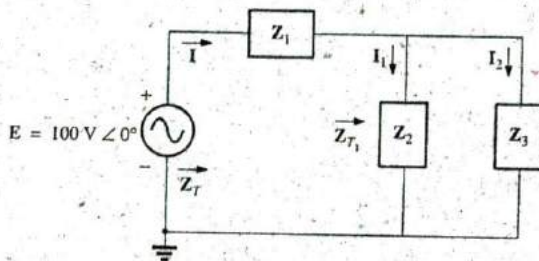e. Find the average power delivered to the circuit.

Solutions:

a. Redrawing the circuit as in Fig. 16.19, we have

$$Z_1 = R_1 = 4\,\Omega\,\angle 0°$$
$$Z_2 = R_2 - jX_C = 9\,\Omega - j\,7\,\Omega = 11.40\,\Omega\,\angle -37.87°$$
$$Z_3 = R_3 + jX_L = 8\,\Omega + j\,6\,\Omega = 10\,\Omega\,\angle +36.87°$$

FIG. 16.19
Network in Fig. 16.18 following the assignment of the subscripted impedances.

Notice that all the desired quantities were conserved in the redrawn network. The total impedance is

$$Z_T = Z_1 + Z_{T_1}$$
$$= Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3}$$
$$= 4\,\Omega + \frac{(11.4\,\Omega\,\angle -37.87°)(10\,\Omega\,\angle 36.87°)}{(9\,\Omega - j\,7\,\Omega) + (8\,\Omega + j\,6\,\Omega)}$$
$$= 4\,\Omega + \frac{114\,\Omega\,\angle -1.00°}{17.03\,\angle -3.37°} = 4\,\Omega + 6.69\,\Omega\,\angle 2.37°$$
$$= 4\,\Omega + 6.68\,\Omega + j\,0.28\,\Omega = 10.68\,\Omega + j\,0.28\,\Omega$$
$$Z_T = 10.68\,\Omega\,\angle 1.5°$$

Calculator Another opportunity to demonstrate the versatility of the calculator! For the above operation, however, you must be aware of the priority of the mathematical operations, as demonstrated in the calculator display in Fig. 16.20. In most cases, the operations are performed in the same order they would be if you wrote them longhand.
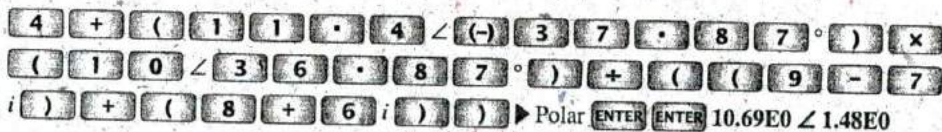
FIG. 16.20
Finding the total impedance for the network in Fig. 16.18 using the TI-89 calculator.

b. $I = \dfrac{E}{Z_T} = \dfrac{100\,V\,\angle 0°}{10.684\,\Omega\,\angle 1.5°} = 9.36\,A\,\angle -1.5°$

c. $F_p = \cos\theta_T = \dfrac{R}{Z_T} = \dfrac{10.68\,\Omega}{10.684\,\Omega} \cong 1$

(essentially resistive, which is interesting, considering the complexity of the network)

d. Using the current divider rule gives

$$I_2 = \frac{Z_2 I}{Z_2 + Z_3} = \frac{(11.40 \ \Omega \ \angle -37.87°)(9.36 \ A \ \angle -1.5°)}{(9 \ \Omega - j \ 7 \ \Omega) + (8 \ \Omega + j \ 6 \ \Omega)}$$

$$= \frac{106.7 \ A \ \angle -39.37°}{17 - j \ 1} = \frac{106.7 \ A \ \angle -39.37°}{17.03 \ \angle -3.37°}$$

$$I_2 = 6.27 \ A \ \angle -36°$$

Applying Kirchhoff's current law (rather than another. application of the current divider rule) yields

$$I_1 = I - I_2$$

or

$$I = I_1 - I_2$$
$$= (9.36 \ A \ \angle -1.5°) - (6.27 \ A \ \angle -36°)$$
$$= (9.36 \ A - j \ 0.25 \ A) - (5.07 \ A - j \ 3.69 \ A)$$
$$I_1 = 4.29 \ A + j \ 3.44 \ A = 5.5 \ A \ \angle 38.72°$$

e. $P_T = EI \cos \theta_T$
$$= (100 \ V)(9.36 \ A) \cos 1.5°$$
$$= (936)(0.99966)$$
$$P_T = 935.68 \ W$$

16.3 LADDER NETWORKS

Ladder networks were discussed in some detail in Chapter 7. This section will simply apply the first method described in Section 7.6 to the general sinusoidal ac ladder network in Fig. 16.21. The current I_6 is desired.
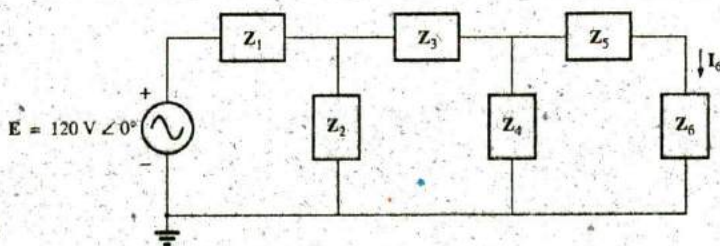
FIG. 16.21
Ladder network.

Impedances Z_T, Z'_T, and Z''_T and currents I_1 and I_3 are defined in Fig. 16.22. We have

$$Z''_T = Z_5 + Z_6$$

and

$$Z'_T = Z_3 + Z_4 \| Z''_T$$

with

$$Z_T = Z_1 + Z_2 \| Z'_T$$

Then

$$I = \frac{E}{Z_T}$$

and

$$I_3 = \frac{Z_2 I}{Z_2 + Z'_T}$$

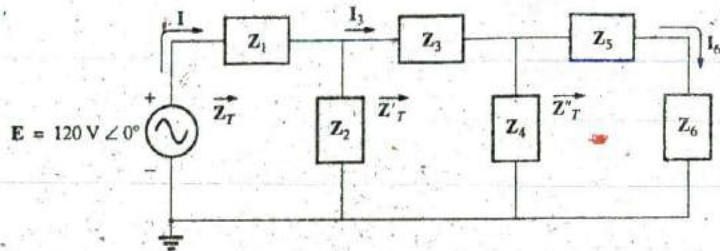with

$$I_6 = \frac{Z_4 I_3}{Z_4 + Z''_T}$$

FIG. 16.22

Defining an approach to the analysis of ladder networks.

16.4 GROUNDING

Although usually treated too lightly in most introductory electrical or electronics texts, the impact of the ground connection and how it can provide a measure of safety to a design are very important topics. *Ground potential is zero volts at every point in a network that has a ground symbol.* Since all points are at the same potential, they can all be connected together, but for purposes of clarity, most are left isolated on a large schematic. On a schematic, the voltage levels provided are always with respect to ground. A system can therefore be checked quite rapidly by simply connecting the black lead of the voltmeter to the ground connection and placing the red lead at the various points where the typical operating voltage is provided. A close match normally implies that that portion of the system is operating properly.

There are various types of grounds, whose use depends on the application. An *earth ground* is one that is connected directly to the earth by a low-impedance connection. Under typical environmental conditions, *local* ground potentials are fairly uniform and can be defined as equal to zero volts. This local uniformity is due to sufficient conductive agents in the soil such as water and electrolytes to ensure that any difference in voltage on the surface is equalized by a flow of charge between the two points. However, between long distances on the earth's surface there can be significant changes in potential level. Every home has an earth ground, usually established by a long conductive rod driven into the ground and connected to the power panel. The electrical code requires a direct connection from earth ground to the cold-water pipes of a home for safety reasons. A "hot" wire touching a cold-water pipe draws sufficient current because of the low-impedance ground connection to throw the breaker. Otherwise, people in the bathroom could pick up the voltage when they touched the cold-water faucet, thereby risking bodily harm. Because water is a conductive agent, any area of the home with water, such as a bathroom or the kitchen, is of particular concern. Most electrical systems are connected to earth ground primarily for safety reasons. All the power lines in a laboratory, at industrial locations, or in the home are connected to earth ground.

A second type is referred to as a *chassis ground*, which may be *floating* or connected directly to an earth ground. A chassis ground simply stipulates that the chassis has a reference potential for all points of the network. If the chassis is not connected to earth potential (0 V), it is said to be *floating* and can have any other reference voltage for the other voltages to be compared to. For instance, if the chassis is sitting at 120 V, all measured voltages of the network will be referenced to this level. A reading of 32 V between a point in the network and the chassis ground will

therefore actually be at 152 V with respect to earth potential. Most high-voltage systems are not left floating, however, because of loss of the safety factor. For instance, if someone should touch the chassis and be standing on a suitable ground, the full 120 V would fall across that individual.

Grounding can be particularly important when working with numerous pieces of measuring equipment in the laboratory. For instance, the supply and oscilloscope in Fig. 16.23(a) are each connected directly to an earth ground through the negative terminal of each. If the oscilloscope is connected as shown in Fig. 16.23(a) to measure the voltage V_{R_1}, a dangerous situation will develop. The grounds of each piece of equipment are connected together through the earth ground, and they effectively short out the resistor. Since the resistor is the primary current-controlling element in the network, the current will rise to a very high level and possibly damage the instruments or cause dangerous side effects. In this case, the supply or scope should be used in the floating mode or the resistors interchanged as shown in Fig. 16.23(b). In Fig. 16.23(b), the grounds have a common point and do not affect the structure of the network.
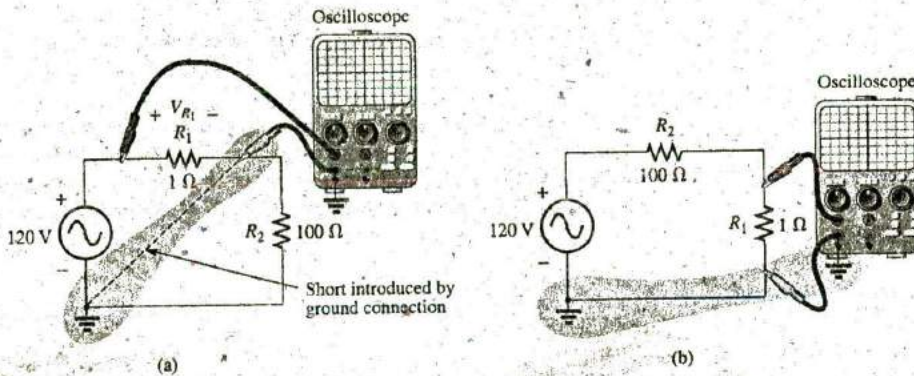
FIG. 16.23

Demonstrating the effect of the oscilloscope ground on the measurement of the voltage across resistor R_1.

The National Electrical Code requires that the "hot" (or *feeder*) line that carries current to a load be *black* and the line (called the *neutral*) that carries the current back to the supply be *white*. Three-wire conductors have a ground wire that must be *green* or *bare*, which ensures a common ground but which is not designed to carry current. The components of a three-prong extension cord and wall outlet are shown in Fig. 16.24. Note
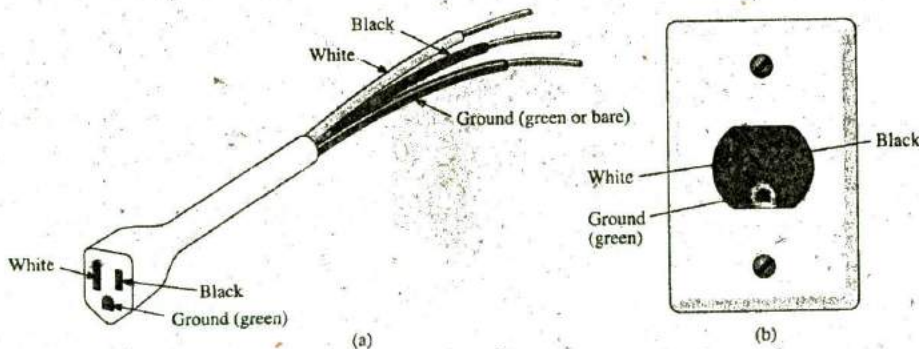

FIG. 16.24

Three-wire conductors: (a) extension cord; (b) home outlet.

that on both fixtures, the connection to the hot lead is smaller than the return leg and that the ground connection is partially circular.

The complete wiring diagram for a household outlet is shown in Fig. 16.25. Note that the current through the ground wire is zero and that both the return wire and the ground wire are connected to an earth ground. The full current to the loads flows through the feeder and return lines.
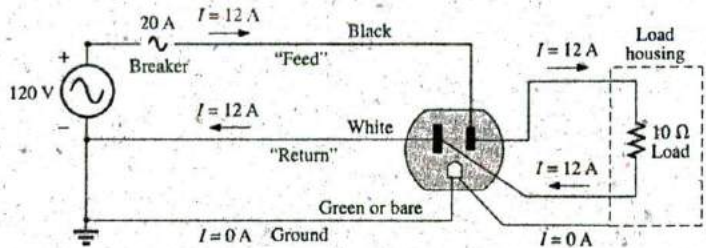
FIG. 16.25

Complete wiring diagram for a household outlet with a 10 Ω load.

The importance of the ground wire in a three-wire system can be demonstrated by the toaster in Fig. 16.26, rated 1200 W at 120 V. From the power equation $P = EI$, the current drawn under normal operating conditions is $I = P/E = 1200 \text{ W}/120 \text{ V} = 10$ A. If a two-wire line were

(a)

(b)

(c)
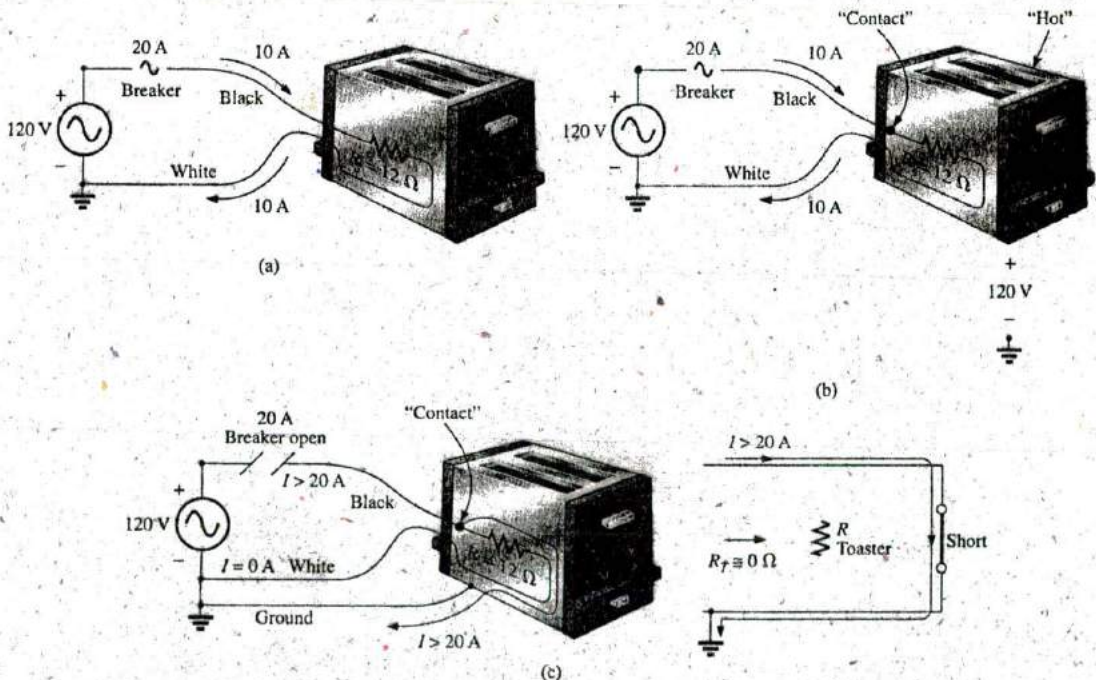
FIG. 16.26

Demonstrating the importance of a properly grounded appliance: (a) ungrounded; (b) ungrounded and undesirable contact; (c) grounded appliance with undesirable contact.

used as shown in Fig. 16.26(a), the 20 A breaker would be quite comfortable with the 10 A current, and the system would perform normally. However, if abuse to the feeder caused it to become frayed and to touch the metal housing of the toaster, the situation depicted in Fig. 16.26(b) ..uld result. The housing would become "hot," yet the breaker would not "pop" because the current would still be the rated 10 A. A dangerous condition would exist because anyone touching the toaster would feel the full 120 V to ground. If the ground wire were attached to the chassis as shown in Fig. 16.26(c), a low-resistance path would be created between the short-circuit point and ground, and the current would jump to very high levels. The breaker would "pop," and the user would be warned that a problem exists.

Although the above discussion does not cover all possible areas of concern with proper grounding or introduce all the nuances associated with the effect of grounds on a system's performance, you should understand the importance of the impact of grounds.

16.5 APPLICATIONS

The vast majority of the applications appearing throughout the text have been of the series-parallel variety. The following are series-parallel combinations of elements and systems used to perform important everyday tasks. The ground fault circuit interrupter outlet employs series protective switches and sensing coils and a parallel control system, while the ideal equivalent circuit for the coax cable employs a series-parallel combination of inductors and capacitors.

GFCI (Ground Fault Circuit Interrupter)

The National Electric Code, the "bible" for all electrical contractors, now requires that ground fault circuit interrupter (GFCI) outlets be used in any area where water and dampness could result in serious injury, such as in bathrooms, pools, marinas, and so on. The outlet looks like any other except that it has a reset button and a test button in the center of the unit as shown in Fig. 16.27(a). The primary difference between it and an ordinary outlet is that it will shut the power off much more quickly than the breaker all the way down in the basement could. You may still feel a shock with a GFCI outlet, but the current cuts off so quickly (in a few milliseconds) that a person in normal health should not receive a serious electrical injury. Whenever in doubt about its use, remember that its cost (relatively inexpensive) is well worth the increased measure of safety.

The basic operation is best described by the network in Fig. 16.27(b). The protection circuit separates the power source from the outlet itself. Note in Fig. 16.27(b) the importance of grounding the protection circuit to the central ground of the establishment (a water pipe, ground bar, and so on; connected to the main panel). In general, the outlet will be grounded to the same connection. Basically, the network shown in Fig. 16.27(b) senses both the current entering (I_i) and the current leaving (I_o) and provides a direct connection to the outlet when they are equal. If a fault should develop such as that caused by someone touching the hot leg while standing on a wet floor, the return current will be less than the feed current (just a few milliamperes is enough). The protection circuitry senses this difference, establishes an open circuit in the line, and cuts off the power to the outlet.
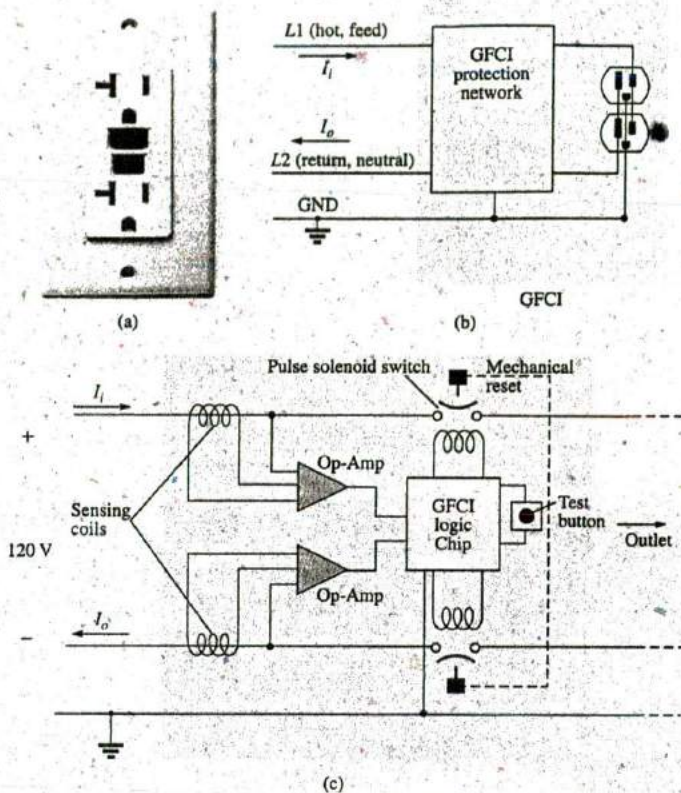
(a)

(b)

GFCI

(c)

FIG. 16.27

GFCI outlet: (a) wall-mounted appearance; (b) basic operation; (c) schematic.

Fig. 16.28(a) shows the feed and return lines passing through the sensing coils. The two sensing coils are separately connected to the printed circuit board. There are two pulse control switches in the line and a return to establish an open circuit under errant conditions. The two contacts in Fig. 16.28(a) are the contacts that provide conduction to the outlet. When a fault develops, another set of similar contacts in the housing slides away, providing the desired open-circuit condition. The separation is created by the solenoid appearing in Fig. 16.28(b). When the solenoid is energized due to a fault condition, it pulls the plunger toward the solenoid, compressing the spring. At the same time, the slots in the lower plastic piece (connected directly to the plunger) shift down, causing a disconnect by moving the structure inserted in the slots. The test button is connected to the brass bar across the unit in Fig. 16.28(c) below the reset button. When pressed, it places a large resistor between the line and ground to "unbalance" the line and cause a fault condition. When the button is released, the resistor is separated from the line, and the unbalance condition is removed. The resistor is actually connected directly to one end of the bar and moves down with pressure on the bar as shown in Fig. 16.28(d). Note in Fig. 16.28(c) how the metal ground connection passes right through the entire unit and how it is connected to the ground terminal of an applied plug. Also note how it is separated from the rest of the network with the plastic housing. Although this unit appears simple on the outside and is relatively small in size, it is beautifully designed and contains a great deal of technology and innovation.
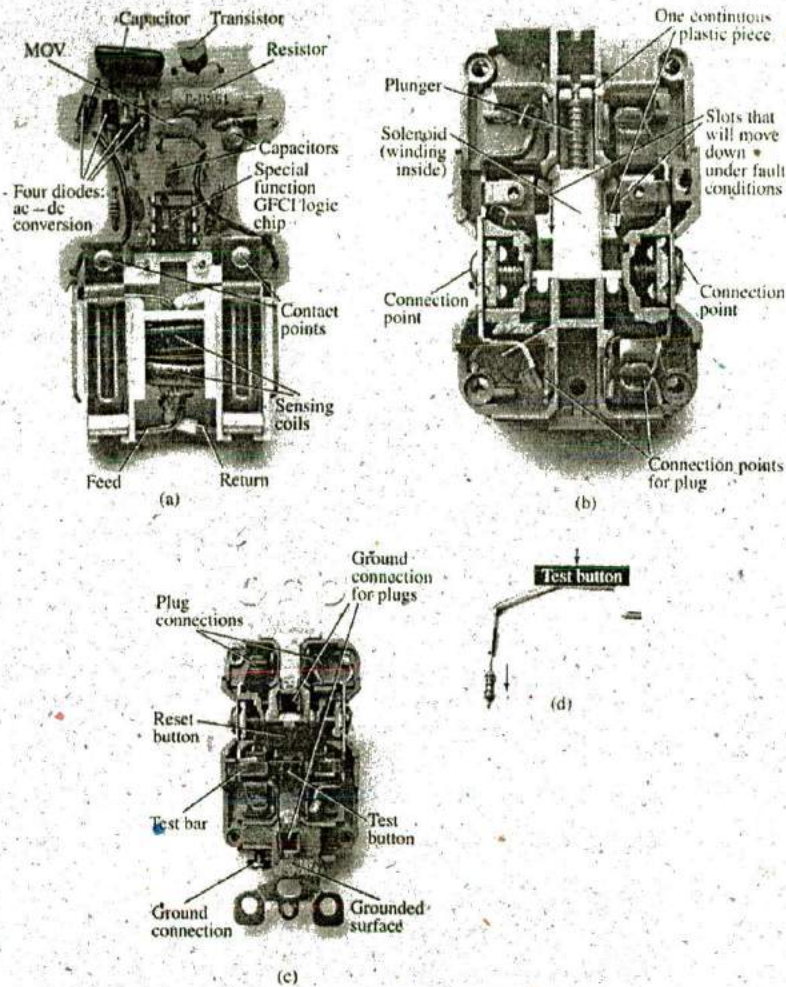
FIG. 16.28

GFCI construction: (a) sensing coils; (b) solenoid control (bottom view); (c) grounding (top view); (d) test bar.

Before leaving the subject, note the logic chip in the center of Fig. 16.28(a) and the various sizes of capacitors and resistors. Note also the four diodes in the upper left region of the circuit board used as a bridge rectifier for the ac-to-dc conversion process. The transistor is the black element with the half-circle appearance. It is part of the driver circuit for the controlling solenoid. Because of the size of the unit, there wasn't a lot of room to provide the power to quickly open the circuit. The result is the use of a pulse circuit to control the motion of the controlling solenoid. In other words, the solenoid is pulsed for a short period of time to cause the required release. If the design used a system that would hold the circuit open on a continuing basis, the power requirement would be greater and the size of the coil larger. A small coil can handle the required power pulse for a short period of time without any long-term damage.

As mentioned earlier, if unsure, install a GFCI. It provides a measure of safety—at a very reasonable cost—that should not be ignored.

16.6 COMPUTER ANALYSIS

PSpice

ac Bridge Network We will use Example 16.4 to demonstrate the power of the **VPRINT** option in the **SPECIAL** library. It permits a direct determination of the magnitude and angle of any voltage in an ac network. Similarly, the **IPRINT** option does the same for ac currents. In Example 16.4, the ac voltages across R_1 and R_2 were first determined, and then Kirchhoff's voltage law was applied to determine the voltage between the two known points. Since PSpice is designed primarily to determine the voltage at a point with respect to ground, the network in Fig. 16.7 is entered as shown in Fig. 16.29 to permit a direct calculation of the voltages across R_1 and R_2.

The source and network elements are entered using a procedure that has been demonstrated several times in previous chapters, although for the **AC Sweep** analysis to be performed in this example, the source must carry an **AC** level also. Fortunately, it is the same as **VAMPL** as shown in Fig. 16.29. It is introduced into the source description by double-clicking on the source symbol to obtain the **Property Editor** dialog box. Select the **AC** column and enter 100 V in the box below. Then select **Display** and choose **Name and Value.** Click **OK** followed by **Apply**, and you can exit the dialog box. The result is **AC** = 100 V added to the source description on the diagram and in the system. For the reactance values in Fig. 16.7, the values for L and C were determined using a frequency of 1 kHz. The voltage across R_1 and R_2 can be determined using the **Trace** command in the same manner as described in the previous chapter or by using the **VPRINT** option. Both approaches are discussed in this section because they have application to any ac network.
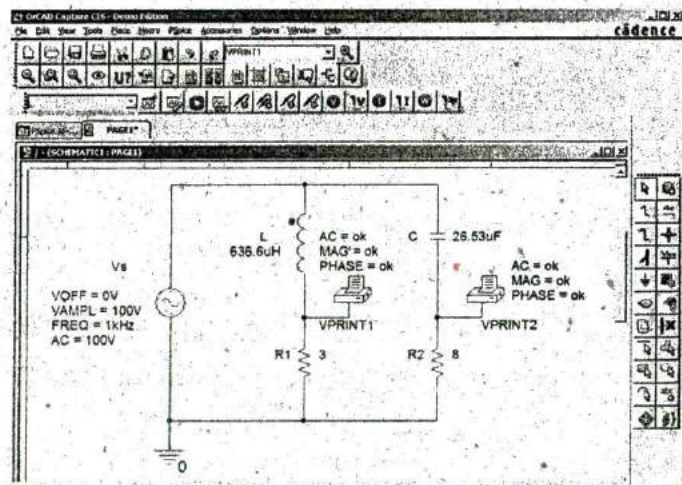
FIG. 16.29

*Determining the voltage across R_1 and R_2 using the **VPRINT** option of a PSpice analysis.*

The **VPRINT** option is under the **SPECIAL** library in the **Place Part** dialog box. Once selected, the printer symbol appears on the screen next to the cursor, and it can be placed near the point of interest. Once the printer symbol is in place, double-click on it to display the **Property Editor** dialog box. Scrolling from left to right, type the word **ok** under **AC,**

MAG, and PHASE. When each is active, select the Display key and choose the option Name and Value followed by OK. When all the entries have been made, choose Apply and exit the dialog box. The result appears in Fig. 16.29 for the two applications of the VPRINT option. If you prefer, VPRINT1 and VPRINT2 can be added to distinguish between the two when you review the output data. To do this, return to the Property Editor dialog box for each by double-clicking on the printer symbol of each and selecting Value and then Display followed by Value Only. Do not forget to select Apply after each change in the Property Editor dialog box. You are now ready for the simulation.

The simulation is initiated by selecting the New Simulation Profile icon and entering PSpice 16-1 as the Name. Then select Create to bring up the Simulation Settings dialog box. This time, you want to analyze the network at 1 kHz but are not interested in plots against time. Thus, select the AC Sweep/Noise option under Analysis type in the Analysis section. An AC Sweep Type region then appears asking for the Start Frequency. Since you are interested in the response at only one frequency, the Start and End Frequency will be the same: 1 kHz. Since you need only one point of analysis, the Points/Decade will be 1. Click OK, and select the Run PSpice icon. The SCHEMATIC1 screen appears, and the voltage across R_1 can be determined by selecting Trace followed by Add Trace and then V(R1:1). The result is the bottom display in Fig. 16.30 with only one plot point at 1 kHz. Since you fixed the
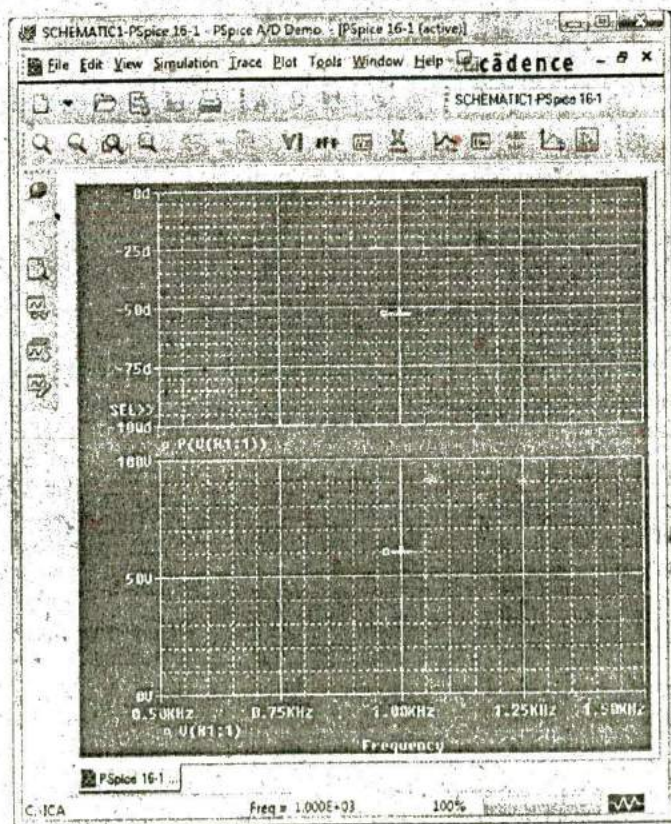
FIG. 16.30

The resulting magnitude and phase angle for the voltage V_{R_1} in Fig. 16.29.

frequency of interest at 1 kHz, this is the only frequency with a response. The magnitude of the voltage across R_1 is 60 V to match the longhand solution of Example 16.4. The phase angle associated with the voltage can be determined by the sequence **Plot-Add Plot to Window-Trace-Add Trace-P( )** from the **Functions or Macros** list and then **V(R1:1)** to obtain **P(V(R1:1))** in the **Trace Expression** box. Click **OK**, and the resulting plot shows that the phase angle is near just less than $-50°$ which is certainly a close match with the $-53.13°$ obtained in Example 16.4.

The **VPRINT** option just introduced offers another method for analyzing voltage in a network. When the **SCHEMATIC1** window appears after the simulation, exit the window using the **X**, and select **PSpice** on the top menu bar of the resulting screen. Select **View Output File** from the list that appears. You will see a long list of data about the construction of the network and the results obtained from the simulation. In Fig. 16.31, the portion of the output file listing the resulting magnitude and phase angle for the voltages defined by **VPRINT1** and **VPRINT2** is provided. Note that the voltage across R_1 defined by **VPRINT1** is 60 V at an angle of $-53.13°$. The voltage across R_2 as defined by **VPRINT2** is 80 V at an angle of $36.87°$. Both are exact matches of the solutions in Example 16.4. In the future, therefore, if the **VPRINT** option is used, the results will appear in the output file.

```
** Profile: "SCHEMATIC1-PSpice 16-1"
[ C:\ICA12\PSpice\pspice 16-1-pspicefiles\schematic1\pspice 16-1.sim ]

****    AC ANALYSIS                    TEMPERATURE =  27.000 DEG C
*****************************************************************************
FREQ      VM(N00879) VP(N00879)
1.000E+03  6.000E+01 -5.313E+01

****    AC ANALYSIS                    TEMPERATURE =  27.000 DEG C
*****************************************************************************
FREQ      VM(N00875) VP(N00875)
1.000E+03  8.000E+01  3.687E+01
```

FIG. 16.31

The VPRINT1 (V_{R_1}) and VPRINT2 (V_{R_2}) response for the network in Fig. 16.29.

Now you can determine the voltage across the two branches from point a to point b. Return to **SCHEMATIC1**, and select **Trace** followed by **Add Trace** to obtain the list of **Simulation Output Variables.** Then, by applying Kirchhoff's voltage law around the closed loop, you find that the desired voltage is **V(R1:1)-V(R2:1)** which when followed by **OK**, results in the plot point in the screen in the bottom of Fig. 16.32. Note that it is exactly 100 V as obtained in the longhand solution. Determine the phase angle through **Plot-Add Plot to Window-Trace-Add Trace**, creating the expression **P(V(R1:1)-V(R2:1))**. Remember that the expression can be generated using the lists of **Output variables** and **Functions**, but it can also be typed in from the keyboard. However, always remember that parentheses must be in sets—a left and a right. Click **OK**, and a solution near $-105°$ appears. A better reading can be obtained by using **Plot-Axis Settings-Y Axis-User Defined** and changing the scale to $-100°$ to $-110°$. The result is the top screen in Fig. 16.32 with an angle closer to $-106.5°$ or $73.5°$, which is very close to the theoretical solution of $73.74°$.

Finally, the last way to find the desired bridge voltage is to remove the **VPRINT2** option and place the ground at that point as shown in Fig. 16.33. Be absolutely sure to remove the original ground from the network. Now the voltage generated from a point above R_1 to ground
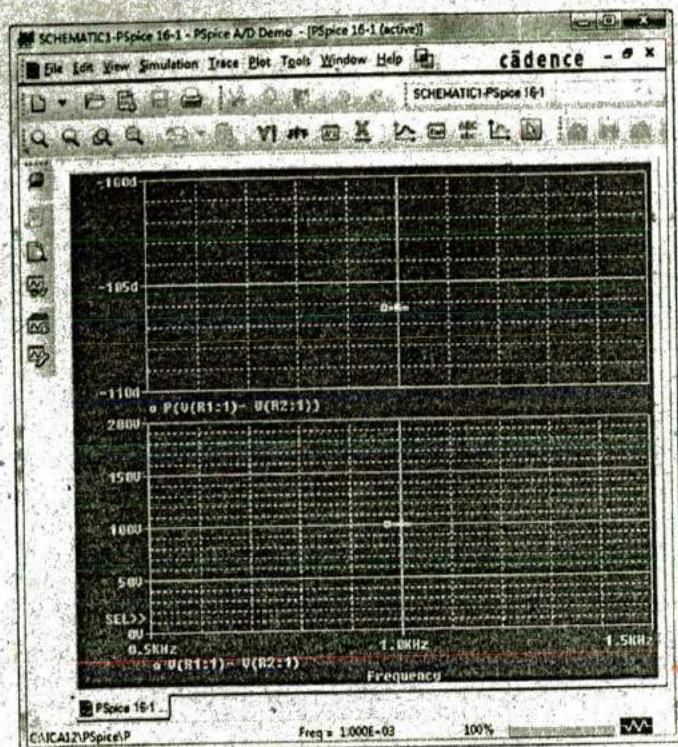
FIG. 16.32

The PSpice response for the voltage between the two points above resistors
R_1 and R_2.
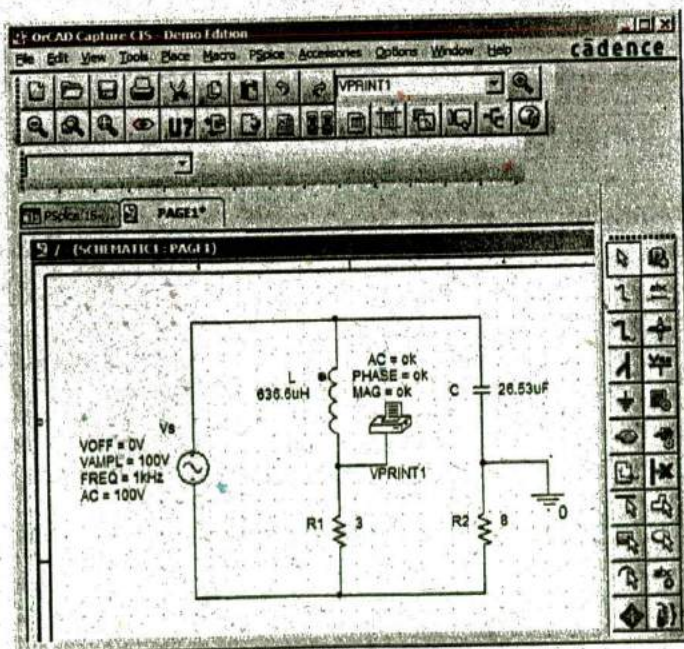
FIG. 16.33

Determining the voltage between the two points above resistors R_1 and R_2 by
moving the ground connection in Fig. 16.29 to the position of VPRINT2.

will be the desired voltage. Repeating a full simulation results in the plot in Fig. 16.34 with the same results as Fig. 16.32. Note, however, that even though the two figures look the same, the quantities listed in the bottom left of each plot are different.
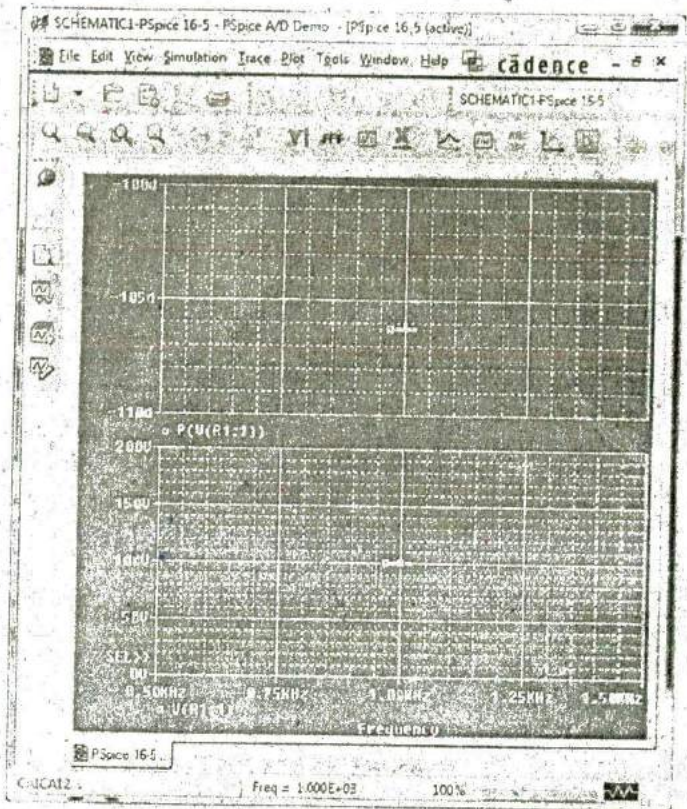
FIG. 16.34

PSpice response to the simulation of the network in Fig. 16.33.

Multisim

Multisim is now used to determine the voltage across the last element of the ladder network in Fig. 16.35. The mathematical content of this chapter suggests that this analysis would be a lengthy exercise in complex algebra, with one mistake (a single sign or an incorrect angle) enough to invalidate the results. However, it takes only a few minutes to "draw" the network on the screen and only a few seconds to generate the results—results you can usually assume are correct if all the parameters were entered correctly. The results are certainly an excellent check against a longhand solution.

Our first approach is to use an oscilloscope to measure the amplitude and phase angle of the output voltage as shown in Fig. 16.35. The oscilloscope settings include a **Time base** of 20 μs/div. since the period of the 10 kHz signal is 100 μs. Channel A is set on 10 V/div. so that the full 20 V of the applied signal will have a peak value encompassing two divisions. Note that Channel A in Fig. 16.35 is connected directly to the source Vs and to the **Trigger** input for synchronization. Expecting the output voltage to have a smaller amplitude resulted in a vertical sensitivity
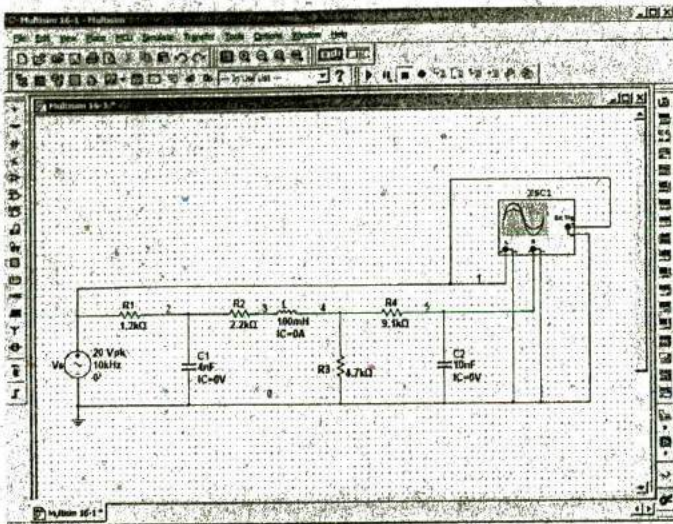
FIG. 16.35
Using the Multisim oscilloscope to determine the voltage across the capacitor C_2.

of 1 V/div. for **Channel B.** The analysis was initiated by placing the **Simulation** switch in the **1** position. It is important to realize that

when simulation is initiated, it will take time for networks with reactive elements to settle down and for the response to reach its steady-state condition. It is therefore wise to let a system run for a while after simulation before selecting Sing. (Single) on the oscilloscope to obtain a steady waveform for analysis.

The resulting plots in Fig. 16.36 clearly show that the applied voltage has an amplitude of 20 V and a period of 100 μs (5 div. at 20 μs/div).
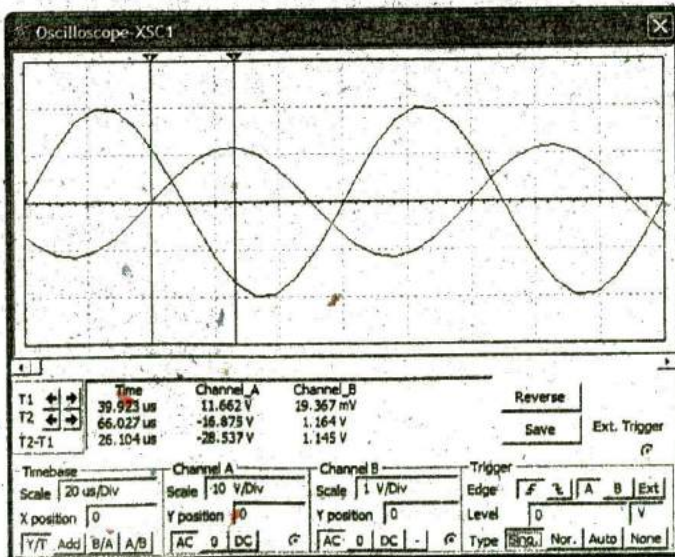
FIG. 16.36
Using Multisim to display the applied voltage and voltage across the capacitor C_2 for the network in Fig. 16.35.

The cursors sit ready for use at the left and right edges of the screen. Clicking on the small red arrow (with number 1) at the top of the oscilloscope screen allows you to drag it to any location on the horizontal axis. As you move the cursor, the magnitude of each waveform appears in the **T1** box below. By comparing positive slopes through the origin, you should see that the applied voltage is leading the output voltage by an angle that is more than 90°. Setting the cursor at the point where the output voltage on channel B passes through the origin with a positive slope, you find that you cannot achieve exactly 0 V, but 0.01 V is certainly very close at 39.8 μs (**T1**).

Knowing that the applied voltage passed through the origin at 0 μs permits the following calculation for the phase angle:

$$\frac{39.8 \ \mu s}{100 \ \mu s} = \frac{\theta}{360°}$$
$$\theta = 143.28°$$

with the result that the output voltage has an angle of $-143.28°$ associated with it. The second cursor at the right edge of the screen is blue. Selecting it and moving it to the peak value of the output voltage results in 1.16 V at 66.33 μs (**T2**). The result of all the above is

$$V_{C_2} = 1.16 \text{ V} \angle -143.28°$$

The second approach is to use the **AC Analysis** option under the **Simulate** heading. First, realize that when you use the oscilloscope as you just did, you did not need to pass through the sequence of dialog boxes to choose the desired analysis. All that was necessary was to simulate using either the switch or the **Simulate Run** sequence—the oscilloscope was there to measure the output voltage. The **AC Analysis** approach requires that you first return to the **AC_VOLTAGE** dialog box and set the **AC Analysis** magnitude to 20 V. Then use the sequence **Simulate-Analyses-AC Analysis** to obtain the **AC Analysis** dialog box and set the **Start** and **Stop frequencies** at 10 kHz and the **Selected variable for analysis** as V(5). Selecting **Simulate** results in a magnitude-phase plot with small Δ indicators at 10 kHz.

At this point, it is difficult to get a good reading of the magnitude of the voltage. This can be corrected by first selecting each graph and adding the grid with **Show Grid** and adding the legend for each with **Show Legend**. Always remember that most dialog boxes, such as those associated with the **Legend** and **Cursor**, can be moved by simply selecting the blue heading bar and moving them to the desired position. If we now select the **Magnitude** plot and then perform a right-click of the mouse, a listing of options will appear. Selecting **Properties** and then **Left Axis** will allow us to change the range and scale of the left axis. If we choose a **Min** of 1 and a **Max** of 2 followed by **OK**, the result will appear between the two closer to the 1. Returning to the **Graph Properties** dialog box and selecting **Left Axis** again, we can choose **Linear** with a range of **Min** = 1.05 and **Max** = 1.35, and the **Total Ticks** can be 6, **Minor Ticks** 1, and **Precision** 2 (hundredths place on vertical scale). The result clearly shows that we are close to 1.6 V in magnitude. Additional accuracy can be obtained by **Show Cursors** to obtain the **AC Analysis** dialog box for V(5). By moving one cursor to **x1** = 10 kHz, we find **y1** = 1.1946 V for a very high degree of accuracy, as shown in Fig. 16.37. If we now select the **Phase(deg)** plot and use the cursor control, we find with **x1** = 10 kHz that **y1** = $-142.147°$, which is very close to the result obtained above.
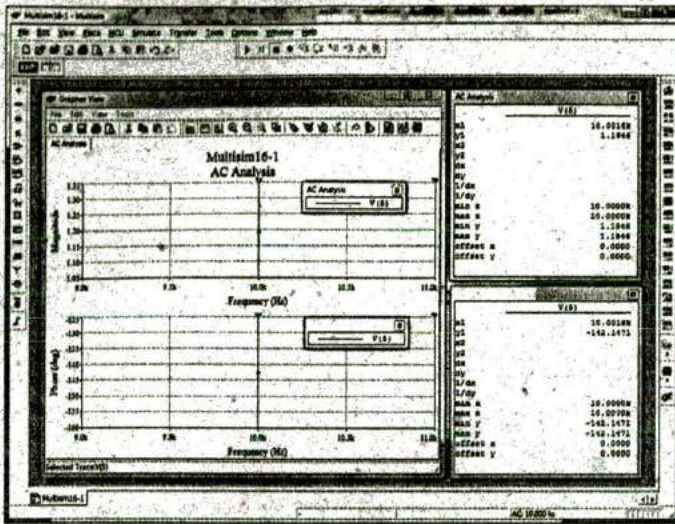
FIG. 16.37

Using the AC Analysis option in Multisim to determine the magnitude and phase angle for the voltage V_{C_2} for the network in Fig. 16.35.

In total, therefore, you have two methods to obtain an ac voltage in a network—one by instrumentation and the other through the computer methods. Both are valid, although, as expected, the computer approach has a higher level of accuracy.

PROBLEMS

SECTION 16.2 Illustrative Examples

1. For the series-parallel network in Fig. 16.38:
   a. Calculate Z_T.
   b. Determine I_s.
   c. Determine I_1.
   d. Find I_2.
   e. Find V_L.
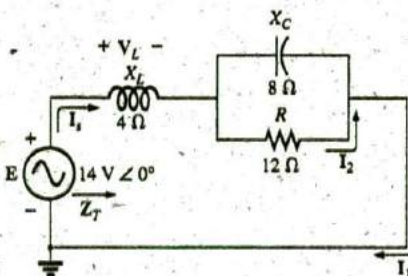
2. For the network in Fig. 16.39:
   a. Find the total impedance Z_T.
   b. Determine the current I_s.
   c. Calculate I_C using the current divider rule.
   d. Calculate V_L using the voltage divider rule.
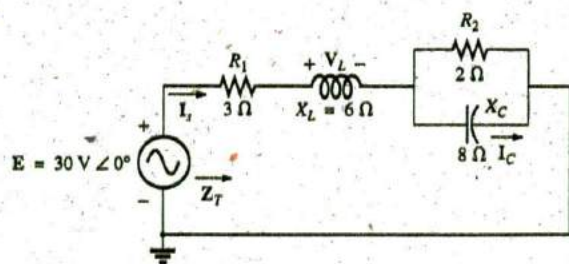
FIG. 16.38

Problem 1.

FIG. 16.39

Problems 2 and 15.

3. For the network in Fig. 16.40:
   a. Find the total impedance Z_T.
   b. Find the current I_s.
   c. Calculate I_2 using the current divider rule.
   d. Calculate V_C using the voltage divider rule.
   e. Calculate the average power delivered to the network.
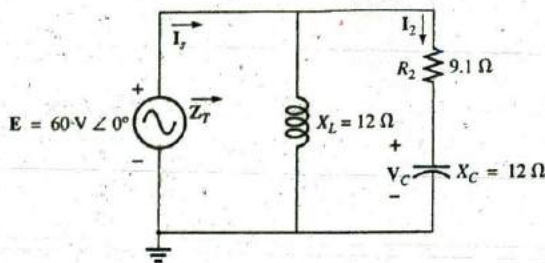
FIG. 16.40
Problem 3.

4. For the network in Fig. 16.41:
   a. Find the total impedance Z_T
   b. Calculate the voltage V_2 and the current I_L
   c. Find the power factor of the network.
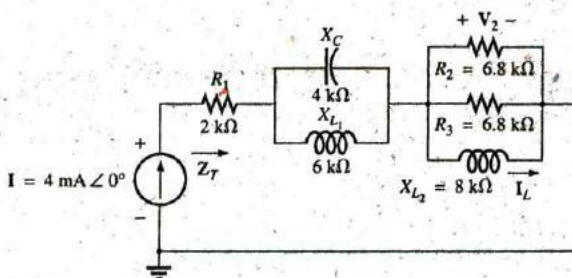
FIG. 16.41
Problem 4.

5. For the network in Fig. 16.42:
   a. Find the current I.
   b. Find the voltage V_C.
   c. Find the average power delivered to the network.
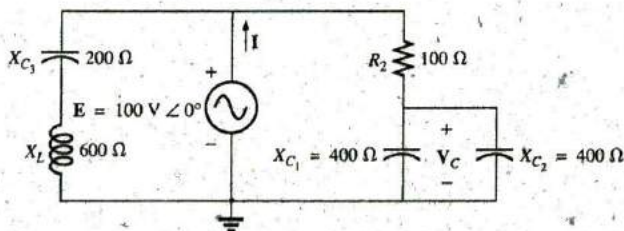
FIG. 16.42
Problem 5.

*6. For the network in Fig. 16.43:
   a. Find the current I_1.
   b. Calculate the voltage V_C using the voltage divider rule.
   c. Find the voltage V_{ab}
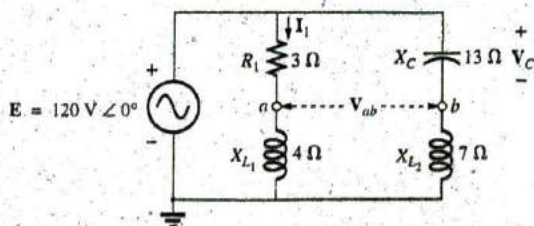
FIG. 16.43
Problem 6.

*7. For the network in Fig. 16.44:
  a. Find the current I_1.
  b. Find the voltage V_1.
  c. Calculate the average power delivered to the network.
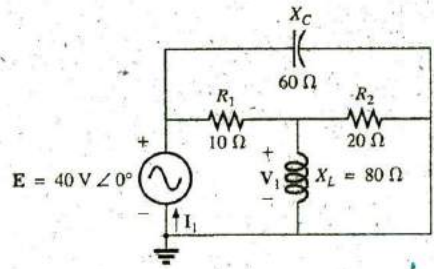
FIG. 16.44
Problems 7 and 16.

8. For the network in Fig. 16.45:
  a. Find the total impedance Z_T and the admittance Y_T.
  b. Find the currents I_1, I_2, and I_3.
  c. Verify Kirchhoff's current law by showing that $I_s = I_1 + I_2 + I_3$.
  d. Find the power factor of the network, and indicate whether it is leading or lagging.
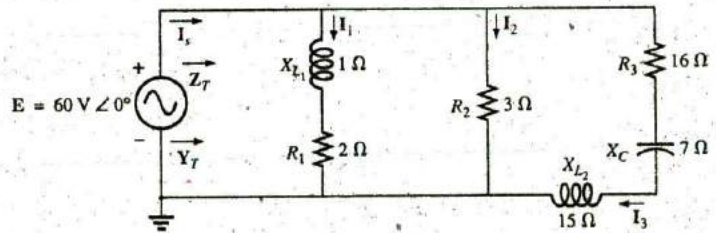
FIG. 16.45
Problem 8.

*9. For the network in Fig. 16.46:
  a. Find the total impedance Z_T.
  b. Find the source current I_s in phasor form.
  c. Find the currents I_1 and I_2 in phasor form.
  d. Find the voltages V_1 and V_{ab} in phasor form.
  e. Find the average power delivered to the network.
  f. Find the power factor of the network, and indicate whether it is leading or lagging.
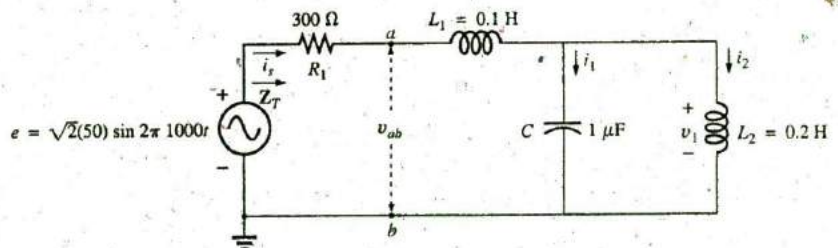
FIG. 16.46
Problem 9.

*10. For the network of Fig. 16.47:
  a. Find the total impedance Z_T.
  b. Find the voltage V_1 in phasor form.
  c. Find the current I_1 in phasor form.
  d. Find the voltage V_2 in phasor form.
  e. Find the source voltage V_s in phasor form.

  $I = 20$ mA $\angle 0°$
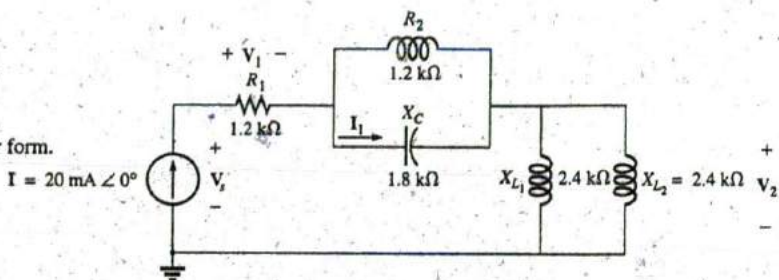
FIG. 16.47
Problem 10.

11. For the network of Fig. 16.48:
  a. Find the total impedance Z_T.
  b. Find the voltage V_1 across the 2 Ω resistor using the voltage divider rule.
  c. Find the current I_1 using Ohm's law.
  d. Find the current I_s.

  $E = 60$ V $\angle 0°$
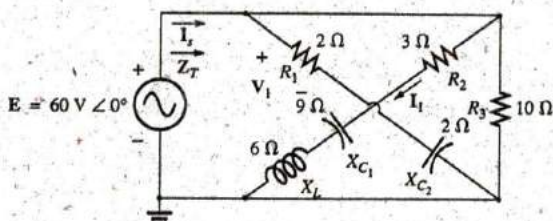
FIG. 16.48
Problems 11 and 17.

SECTION 16.3 Ladder Networks

12. Find the current I_5 for the network in Fig. 16.49. Note the effect of one reactive element on the resulting calculations.

  $E = 100$ V $\angle 0°$
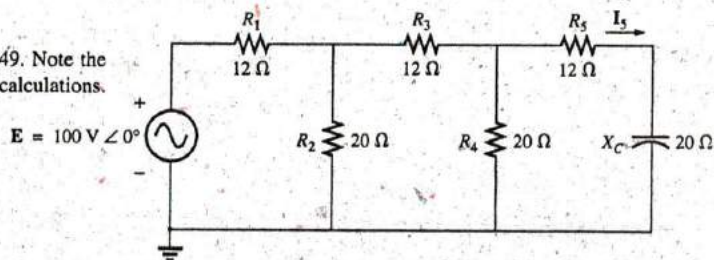
FIG. 16.49
Problem 12.

13. Find the average power delivered to R_4 in Fig. 16.50.

  $I = 20$ mA $\angle 0°$
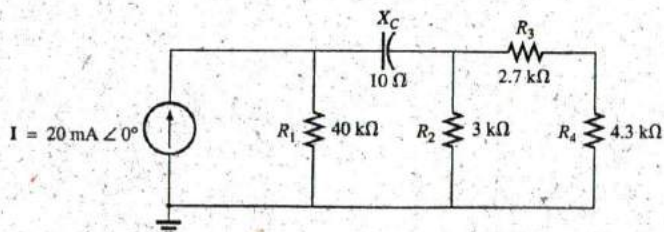
FIG. 16.50
Problem 13.

14. Find the current I_1 for the network in Fig. 16.51.
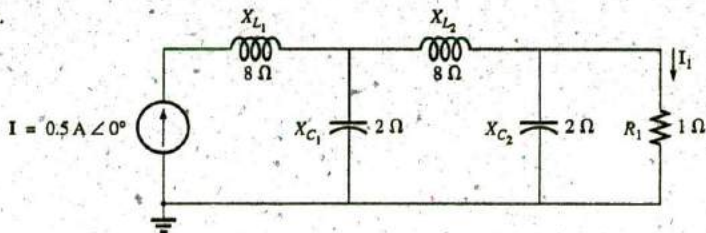
FIG. 16.51
Problems 14 and 18.

SECTION 16.6 Computer Analysis

PSpice or Multisim

For Problems 15 through 18, use a frequency of 1 kHz to determine the inductive and capacitive levels required for the input files. In each case, write the required input file.

*15. Repeat Problem 2 using PSpice or Multisim.

*16. Repeat Problem 7, parts (a) and (b), using PSpice or Multisim.

*17. Repeat Problem 11 using PSpice or Multisim.

*18. Repeat Problem 14 using PSpice or Multisim.

GLOSSARY

Ladder network A repetitive combination of series and parallel branches that has the appearance of a ladder.

Series-parallel ac network A combination of series and parallel branches in the same network configuration. Each branch may contain any number of elements, whose impedance is dependent on the applied frequency.

Methods of Analysis and Selected Topics (AC)

Objectives

- Understand the differences between independent and dependent sources and how the magnitude and angle of a controlled source is determined by the dependent variable.

- Be able to convert between voltage and current sources and vice versa in the ac domain.

- Become proficient in the application of mesh and nodal analysis to ac networks with independent and controlled sources.

- Be able to define the relationship between the elements of an ac bridge network that will establish a balance condition.

17.1 INTRODUCTION

For networks with two or more sources that are not in series or parallel, the methods described in the last two chapters cannot be applied. Rather, methods such as mesh analysis or nodal analysis must be used. Since these methods were discussed in detail for dc circuits in Chapter 8, this chapter considers the variations required to apply these methods to ac circuits. Dependent sources are also introduced for both mesh and nodal analysis.

The branch-current method is not discussed again because it falls within the framework of mesh analysis. In addition to the methods mentioned above, the bridge network and Δ-Y, Y-Δ conversions are also discussed for ac circuits.

Before we examine these topics, however, we must consider the subject of independent and controlled sources.

17.2 INDEPENDENT VERSUS DEPENDENT (CONTROLLED) SOURCES

In the previous chapters, each source appearing in the analysis of dc or ac networks was an **independent source,** such as E and I (or E and I) in Fig. 17.1.

The term independent specifies that the magnitude of the source is independent of the network to which it is applied and that the source displays its terminal characteristics even if completely isolated.

A dependent or controlled source is one whose magnitude is determined (or controlled) by a current or voltage of the system in which it appears.
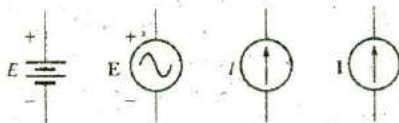
FIG. 17.1
Independent sources
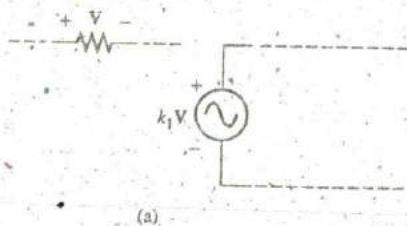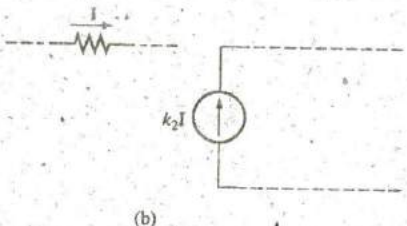
FIG. 17.2

Controlled or dependent sources.
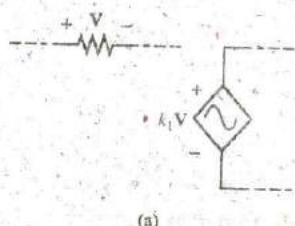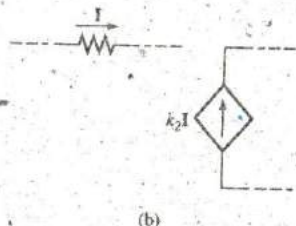
FIG. 17.3

Special notation for controlled or dependent sources.

Voltage source   Current source

FIG. 17.5

Source Conversion.

Currently two symbols are used for controlled sources. One simply uses the independent symbol with an indication of the controlling element, as shown in Fig. 17.2. In Fig. 17.2(a), the magnitude and phase of the voltage are controlled by a voltage $\hat{V}$ elsewhere in the system, with the magnitude further controlled by the constant k_1. In Fig. 17.2(b), the magnitude and phase of the current source are controlled by a current I elsewhere in the system, with the magnitude further controlled by the constant k_2. To distinguish between the dependent and independent sources, the notation in Fig. 17.3 was introduced. In recent years, many respected publications on circuit analysis have accepted the notation in Fig. 17.3, although a number of excellent publications in the area of electronics continue to use the symbol in Fig. 17.2, especially in the circuit modeling for a variety of electronic devices such as the transistor and FET. This text uses the symbols in Fig. 17.3.

Possible combinations for controlled sources are indicated in Fig. 17.4. Note that the magnitude of current sources or voltage sources can be controlled by a voltage and a current, respectively. Unlike with the independent source, isolation such that V or I = 0 in Fig. 17.4(a) results in the short-circuit or open-circuit equivalent as indicated in Fig. 17.4(b). Note that the type of representation under these conditions is controlled by whether it is a current source or a voltage source, not by the controlling agent (V or I).

FIG. 17.4

Conditions of V = 0 V and I = 0 A for a controlled source.

17.3 SOURCE CONVERSIONS

When applying the methods to be discussed, it may be necessary to convert a current source to a voltage source or a voltage source to a current source. This **source conversion** can be accomplished in much the same manner as for dc circuits, except that now we shall be dealing with phasors and impedances instead of just real numbers and resistors.

Independent Sources

In general, the format for converting one type of independent source to another is as shown in Fig. 17.5.

EXAMPLE 17.1 Convert the voltage source in Fig. 17.6(a) to a current source.

Source conversion
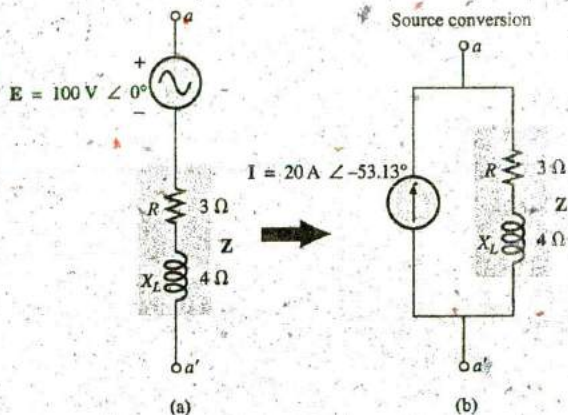
FIG. 17.6
Example 17.1.

Solution:

$$I = \frac{E}{Z} = \frac{100 \text{ V} \angle 0°}{5 \text{ Ω} \angle 53.13°}$$
$$= 20 \text{ A} \angle -53.13° \qquad [\text{Fig. 17.6(b)}]$$

EXAMPLE 17.2 Convert the current source in Fig. 17.7(a) to a voltage source.
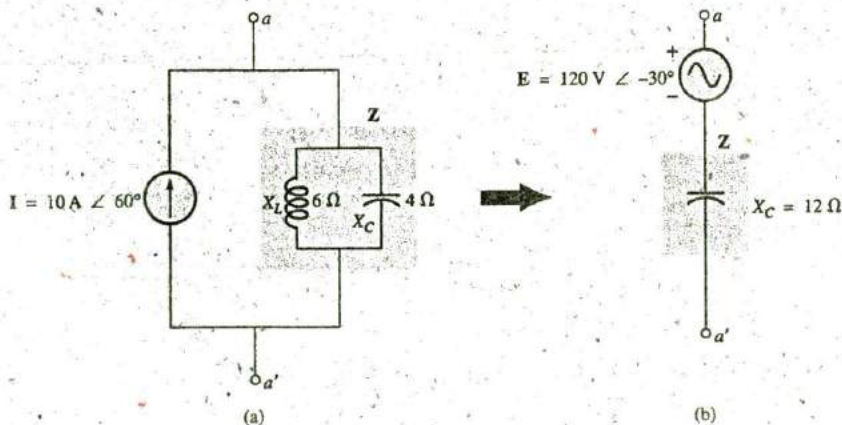
FIG. 17.7
*Example 17.2.*

Solution:

$$Z = \frac{Z_C Z_L}{Z_C + Z_L} = \frac{(X_C \angle -90°)(X_L \angle 90°)}{-jX_C + jX_L}$$

$$= \frac{(4\ \Omega \angle -90°)(6\ \Omega \angle 90°)}{-j4\ \Omega + j6\ \Omega} = \frac{24\ \Omega \angle 0°}{2 \angle 90°}$$

$$= 12\ \Omega \angle -90°$$

$$E = IZ = (10\ A \angle 60°)(12\ \Omega \angle -90°)$$

$$= 120\ V \angle -30° \qquad [\text{Fig. 17.7(b)}]$$

Dependent Sources

For dependent sources, the direct conversion in Fig. 17.5 can be applied if the controlling variable (**V** or **I** in Fig. 17.4) is not determined by a portion of the network to which the conversion is to be applied. For example, in Figs. 17.8 and 17.9, **V** and **I**, respectively, are controlled by an external portion of the network. Conversions of the other kind, where **V** and **I** are controlled by a portion of the network to be converted, are considered in Sections 18.3 and 18.4.

EXAMPLE 17.3 Convert the voltage source in Fig. 17.8(a) to a current source.
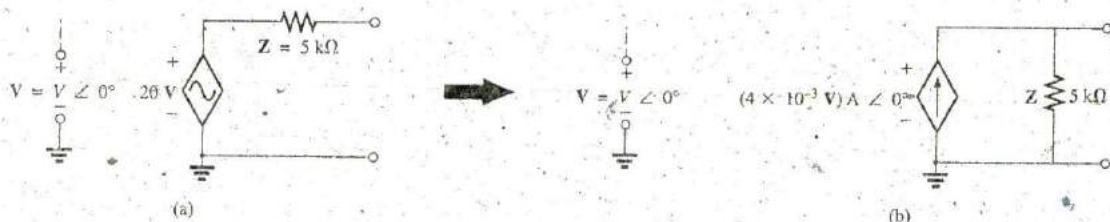
FIG. 17.8
Source conversion with a voltage-controlled voltage source.

Solution:

$$I = \frac{E}{Z} = \frac{(20\ V)\ V \angle 0°}{5\ k\Omega \angle 0°}$$

$$= (4 \times 10^{-3}\ V)A \angle 0° \qquad [\text{Fig. 17.8(b)}]$$

EXAMPLE 17.4 Convert the current source in Fig. 17.9(a) to a voltage source.
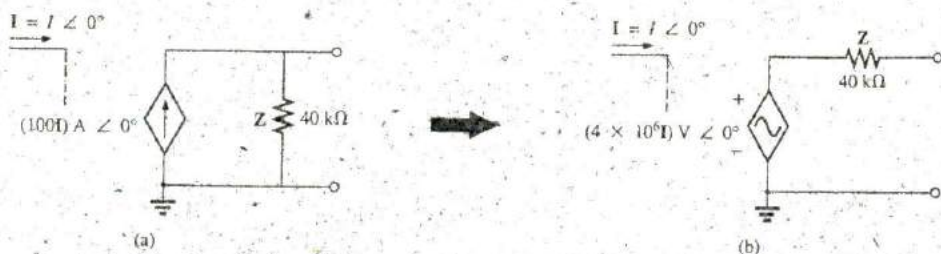
FIG. 17.9
Source conversion with a current-controlled current source.

Solution:

$$E = IZ = [(100\ I)\ A\ \angle 0°][40\ k\Omega\ \angle 0°]$$
$$= (4 \times 10^6\ I)\ V\ \angle 0°$$

17.4 MESH ANALYSIS
General Approach

Independent Voltage Sources   Before examining the application of the method to ac networks, the student should first review the appropriate sections on **mesh analysis** in Chapter 8 since the content of this section will be limited to the general conclusions of Chapter 8.

The general approach to mesh analysis for independent sources includes the same sequence of steps appearing in Chapter 8. In fact, throughout this section the only change from the dc coverage is to substitute impedance for resistance and admittance for conductance in the general procedure.

1. *Assign a distinct current in the clockwise direction to each independent closed loop of the network. It is not absolutely necessary to choose the clockwise direction for each loop current. However, it eliminates the need to have to choose a direction for each application. Any direction can be chosen for each loop current with no loss in accuracy as long as the remaining steps are followed properly.*
2. *Indicate the polarities within each loop for each impedance as determined by the assumed direction of loop current for that loop.*
3. *Apply Kirchhoff's voltage law around each closed loop in the clockwise direction. Again, the clockwise direction was chosen to establish uniformity and to prepare us for the format approach to follow.*

   a. *If an impedance has two or more assumed currents through it, the total current through the impedance is the assumed current of the loop in which Kirchhoff's voltage law is being applied, plus the assumed currents of the other loops passing through in the same direction, minus the assumed currents passing through in the opposite direction.*
   b. *The polarity of a voltage source is unaffected by the direction of the assigned loop currents.*
4. *Solve the resulting simultaneous linear equations for the assumed loop currents.*

The technique is applied as above for all networks with independent sources or for networks with *dependent sources where the controlling variable is not a part of the network under investigation.* If the controlling variable is part of the network being examined, a method to be described shortly must be applied.

EXAMPLE 17.5   Using the general approach to mesh analysis, find the current I_1 in Fig. 17.10.

Solution: When applying these methods to ac circuits, it is good practice to represent the resistors and reactances (or combinations thereof) by subscripted impedances. When the total solution is found in terms of these subscripted impedances, the numerical values can be substituted to find the unknown quantities.
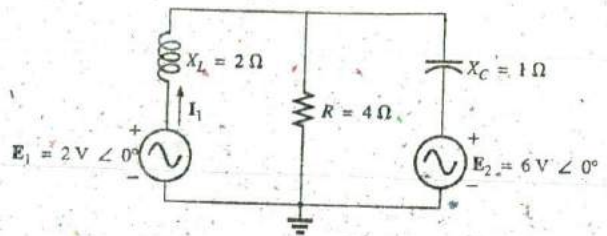
FIG. 17.10
Example 17.5.
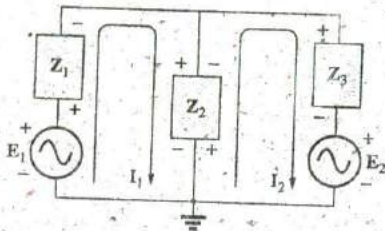
FIG. 17.11
Assigning the mesh currents and subscripted imped-
ances for the network in Fig. 17.10.

The network is redrawn in Fig. 17.11 with subscripted impedances:

$$Z_1 = +jX_L = +j2\,\Omega \qquad E_1 = 2\,V \angle 0°$$
$$Z_2 = R = 4\,\Omega \qquad\qquad E_2 = 6\,V \angle 0°$$
$$Z_3 = -jX_C = -j1\,\Omega$$

Steps 1 and 2 are as indicated in Fig. 17.11.

Step 3:

$$+E_1 - I_1Z_1 - Z_2(I_1 - I_2) = 0$$
$$-Z_2(I_2 - I_1) - I_2Z_3 - E_2 = 0$$

or

$$E_1 - I_1Z_1 - I_1Z_2 + I_2Z_2 = 0$$
$$-I_2Z_2 + I_1Z_2 - I_2Z_3 - E_2 = 0$$

so that

$$I_1(Z_1 + Z_2) - I_2Z_2 = E_1$$
$$I_2(Z_2 + Z_3) - I_1Z_2 = -E_2$$

which are rewritten as

$$I_1(Z_1 + Z_2) - I_2Z_2 \qquad\quad = E_1$$
$$-I_1Z_2 \qquad\quad + I_2(Z_2 + Z_3) = -E_2$$

Step 4: Using determinants, we obtain

$$I_1 = \frac{\begin{vmatrix} E_1 & -Z_2 \\ -E_2 & Z_2 + Z_3 \end{vmatrix}}{\begin{vmatrix} Z_1 + Z_2 & -Z_2 \\ -Z_2 & Z_2 + Z_3 \end{vmatrix}}$$

$$= \frac{E_1(Z_2 + Z_3) - E_2(Z_2)}{(Z_1 + Z_2)(Z_2 + Z_3) - (Z_2)^2}$$

$$= \frac{(E_1 - E_2)Z_2 + E_1Z_3}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}$$

Substituting numerical values yields

$$I_1 = \frac{(2\,V - 6\,V)(4\,\Omega) + (2\,V)(-j1\,\Omega)}{(+j2\,\Omega)(4\,\Omega) + (+j2\,\Omega)(-j2\,\Omega) + (4\,\Omega)(-j2\,\Omega)}$$

$$= \frac{-16 - j2}{j8 - j^2 2 - j4} = \frac{-16 - j2}{2 + j4} = \frac{16.12\,A \angle -172.87°}{4.47 \angle 63.43°}$$

$$= 3.61\,A \angle -236.30° \quad \text{or} \quad 3.61\,A \angle 123.70°$$

Dependent Voltage Sources For dependent voltage sources, the procedure is modified as follows:

1. Steps 1 and 2 are the same as those applied for independent voltage sources.
2. Step 3 is modified as follows: Treat each dependent source like an independent source when Kirchhoff's voltage law is applied to each independent loop. However, once the equation is written, substitute the equation for the controlling quantity to ensure that the unknowns are limited solely to the chosen mesh currents.
3. Step 4 is as before.

EXAMPLE 17.6 Write the mesh currents for the network in Fig. 17.12 having a dependent voltage source.

Solution:

Steps 1 and 2 are defined in Fig. 17.12.

Step 3:
$$E_1 - I_1 R_1 - R_2(I_1 - I_2) = 0$$
$$R_2(I_2 - I_1) + \mu V_x - I_2 R_3 = 0$$

Then substitute $V_x = (I_1 - I_2)R_2$.
The result is two equations and two unknowns:

$$E_1 - I_1 R_1 - R_2(I - I_2) = 0$$
$$R_2(I_2 - I_1) + \mu R_2(I_1 - I_2) - I_2 R_3 = 0$$
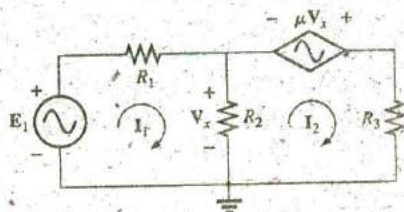
FIG. 17.12

Applying mesh analysis to a network with a voltage-controlled voltage source.

Independent Current Sources For independent current sources, the procedure is modified as follows:

1. Steps 1 and 2 are the same as those applied for independent sources.
2. Step 3 is modified as follows: Treat each current source as an open circuit (recall the *supermesh* designation in Chapter 8), and write the mesh equations for each remaining independent path. Then relate the chosen mesh currents to the dependent sources to ensure that the unknowns of the final equations are limited to the mesh currents.
3. Step 4 is as before.

EXAMPLE 17.7 Write the mesh currents for the network in Fig. 17.13 having an independent current source.

Solution:

Steps 1 and 2 are defined in Fig. 17.13.

Step 3: $E_1 - I_1 Z_1 + E_2 - I_2 Z_2 = 0$ (only remaining independent path)

with $I_1 + I = I_2$

The result is two equations and two unknowns.
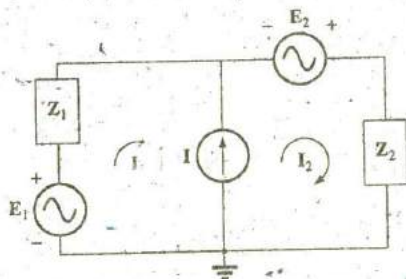
FIG. 17.13

Applying mesh analysis to a network with an independent current source.

FIG. 17.14

Applying mesh analysis to a network with a current-controlled current source.

Dependent Current Sources For dependent current sources, the procedure is modified as follows:

1. Steps 1 and 2 are the same as those applied for independent sources.
2. Step 3 is modified as follows: The procedure is essentially the same as that applied for independent current sources, except now the dependent sources have to be defined in terms of the chosen mesh currents to ensure that the final equations have only mesh currents as the unknown quantities.
3. Step 4 is as before.

EXAMPLE 17.8 Write the mesh currents for the network in Fig. 17.14 having a dependent current source.

Solution:

Steps 1 and 2 are defined in Fig. 17.14.

Step 3:
$$E_1 - I_1 Z_1 - I_2 Z_2 + E_2 = 0$$

and
$$kI = I_1 - I_2$$

Now $I = I_1$ so that $\quad kI_1 = I_1 - I_2 \quad$ or $\quad I_2 = I_1(1 - k)$

The result is two equations and two unknowns.

Format Approach

The format approach was introduced in Section 8.8. The steps for applying this method are repeated here with changes for its use in ac circuits:

1. *Assign a loop current to each independent closed loop (as in the previous section) in a clockwise direction.*
2. *The number of required equations is equal to the number of chosen independent closed loops. Column 1 of each equation is formed by summing the impedance values of those impedances through which the loop current of interest passes and multiplying the result by that loop current.*
3. *We must now consider the mutual terms that are always subtracted from the terms in the first column. It is possible to have more than one mutual term if the loop current of interest has an element in common with more than one other loop current. Each mutual term is the product of the mutual impedance and the other loop current passing through the same element.*
4. *The column to the right of the equality sign is the algebraic sum of the voltage sources through which the loop current of interest passes. Positive signs are assigned to those sources of voltage having a polarity such that the loop current passes from the negative to the positive terminal. Negative signs are assigned to those potentials for which the reverse is true.*
5. *Solve the resulting simultaneous equations for the desired loop currents.*

The technique is applied as above for all networks with independent sources or for networks with dependent sources where the controlling variable is not a part of the network under investigation. If the controlling variable is part of the network being examined, additional care must be taken when applying the above steps.

EXAMPLE 17.9 Using the format approach to mesh analysis, find the current I_2 in Fig. 17.15.
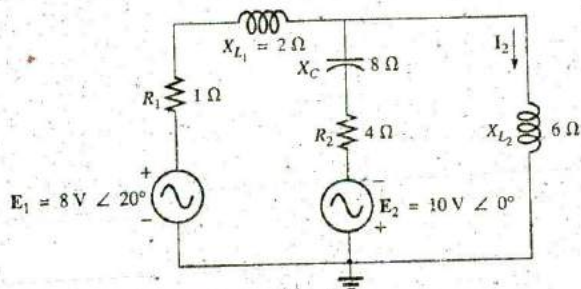
FIG. 17.15
Example 17.9.

Solution: The network is redrawn in Fig. 17.16:

$$Z_1 = R_1 + jX_{L_1} = 1\,\Omega + j\,2\,\Omega \qquad E_1 = 8\text{ V }\angle 20°$$
$$Z_2 = R_2 - jX_C = 4\,\Omega - j\,8\,\Omega \qquad E_2 = 10\text{ V }\angle 0°$$
$$Z_3 = +jX_{L_2} = +j\,6\,\Omega$$

Note the reduction in complexity of the problem with the substitution of the subscripted impedances.

Step 1 is as indicated in Fig. 17.16.

Steps 2 to 4:

$$I_1(Z_1 + Z_2) - I_2 Z_2 = E_1 + E_2$$
$$I_2(Z_2 + Z_3) - I_1 Z_2 = -E_2$$

which are rewritten as

$$I_1(Z_1 + Z_2) - I_2 Z_2 \qquad\qquad = E_1 + E_2$$
$$-I_1 Z_2 \qquad +I_2(Z_2 + Z_3) = -E_2$$

FIG. 17.16
Assigning the mesh currents and subscripted impedances for the network in Fig. 17.15.

Step 5: Using determinants, we have

$$I_2 = \frac{\begin{vmatrix} Z_1 + Z_2 & E_1 + E_2 \\ -Z_2 & -E_2 \end{vmatrix}}{\begin{vmatrix} Z_1 + Z_2 & -Z_2 \\ -Z_2 & Z_2 + Z_3 \end{vmatrix}}$$

$$= \frac{-(Z_1 + Z_2)E_2 + Z_2(E_1 + E_2)}{(Z_1 + Z_2)(Z_2 + Z_3) - Z_2^2}$$

$$= \frac{Z_2 E_1 - Z_1 E_2}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}$$

Substituting numerical values yields

$$I_2 = \frac{(4\,\Omega - j\,8\,\Omega)(8\text{ V }\angle 20°) - (1\,\Omega + j\,2\,\Omega)(10\text{ V }\angle 0°)}{(1\,\Omega + j\,2\,\Omega)(4\,\Omega - j\,8\,\Omega) + (1\,\Omega + j\,2\,\Omega)(+j\,6\,\Omega) + (4\,\Omega - j\,8\,\Omega)(+j\,6\,\Omega)}$$

$$= \frac{(4 - j\,8)(7.52 + j\,2.74) - (10 + j\,20)}{20 + (j\,6 - 12) + (j\,24 + 48)}$$

$$= \frac{(52.0 - j\,49.20) - (10 + j\,20)}{56 + j\,30} = \frac{42.0 - j\,69.20}{56 + j\,30} = \frac{80.95\ A\ \angle -58.74°}{63.53\ \angle 28.18°}$$

$$= 1.27\ A\ \angle 286.92°$$

Calculator Solution: The calculator can be an effective tool in performing the long, laborious calculations involved with the final equation appearing above. However, you must be very careful to use brackets to define the order of the arithmetic operations (remember that each open bracket must be followed by a close bracket). With the TI-89 calculator, the sequence in Fig. 17.17(a) provides the solution for the numerator.

For the denominator, the sequence appears in Fig. 17.17(b).

The solution is then determined in Fig. 17.17(c).

(a)

(b)

(c)

FIG. 17.17

Determining I_2 for the network of Fig. 17.15.

EXAMPLE 17.10 Write the mesh equations for the network in Fig. 17.18. Do not solve.
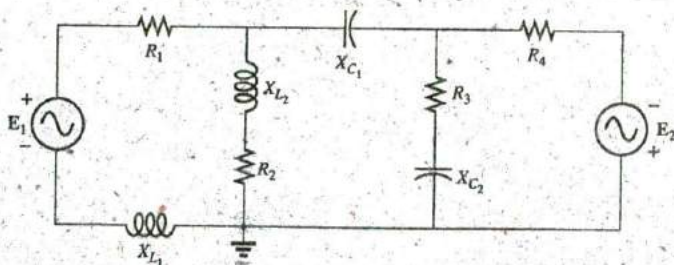
FIG. 17.18

Example 17.10.

Solution: The network is redrawn in Fig. 17.19. Again note the reduced complexity and increased clarity provided by the use of subscripted impedances:

$$Z_1 = R_1 + jX_{L_1} \quad Z_4 = R_3 - jX_{C_2}$$
$$Z_2 = R_2 + jX_{L_2} \quad Z_5 = R_4$$
$$Z_3 = jX_{C_1}$$

and

$$I_1(Z_1 + Z_2) - I_2 Z_2 = E_1$$
$$I_2(Z_2 + Z_3 + Z_4) - I_1 Z_2 - I_3 Z_4 = 0$$
$$I_3(Z_4 + Z_5) - I_2 Z_4 = E_2$$

or

$$\begin{array}{lll} I_1(Z_1 + Z_2) - I_2(Z_2) & +0 & = E_1 \\ -I_1 Z_2 & -I_2(Z_2 + Z_3 + Z_4) = I_3(Z_4) & = 0 \\ 0 & -I_2(Z_4) & -I_3(Z_4 + Z_5) = E_2 \end{array}$$
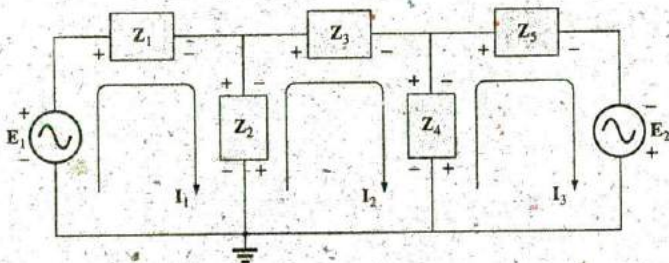
FIG. 17.19
Assigning the mesh currents and subscripted impedances for the network in Fig. 17.18.

EXAMPLE 17.11 Using the format approach, write the mesh equations for the network in Fig. 17.20.

Solution: The network is redrawn as shown in Fig. 17.21, where

$$Z_1 = R_1 + jX_{L_1} \quad Z_3 = jX_{L_2}$$
$$Z_2 = R_2 \quad Z_4 = jX_{L_3}$$

and

$$I_1(Z_2 + Z_4) - I_2 Z_2 - I_3 Z_4 = E_1$$
$$I_2(Z_1 + Z_2 + Z_3) - I_1 Z_2 - I_3 Z_3 = 0$$
$$I_3(Z_3 + Z_4) - I_2 Z_3 - I_1 Z_4 = E_2$$

or

$$\begin{array}{lll} I_1(Z_2 + Z_4) - I_2 Z_2 & -I_3 Z_4 & = E_1 \\ -I_1 Z_2 & +I_2(Z_1 + Z_2 + Z_3) - I_3 Z_3 & = 0 \\ -I_1 Z_4 & -I_2 Z_3 & +I_3(Z_3 + Z_4) = E_2 \end{array}$$

Note the symmetry *about* the diagonal axis; that is, note the location of $-Z_2, -Z_4,$ and $-Z_3$ off the diagonal.

17.5 NODAL ANALYSIS

General Approach

Independent Sources Before examining the application of the method to ac networks, a review of the appropriate sections on **nodal**
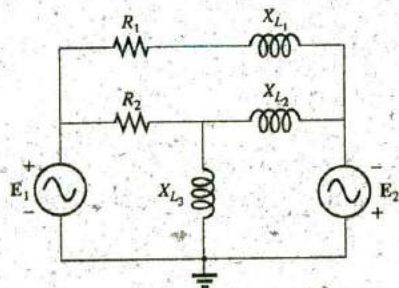
FIG. 17.20
Example 17.11.
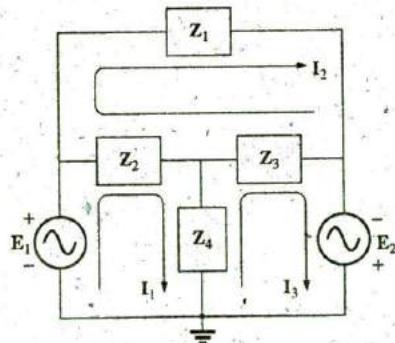
FIG. 17.21
Assigning the mesh currents and subscripted impedances for the network in Fig. 17.20.

analysis in Chapter 8 is suggested since the content of this section is limited to the general conclusions of Chapter 8.

The fundamental steps are the following:

1. *Determine the number of nodes within the network.*
2. *Pick a reference node and label each remaining node with a sub-scripted value of voltage: V_1, V_2, and so on.*
3. *Apply Kirchhoff's current law at each node except the reference. Assume that all unknown currents leave the node for each application of Kirchhoff's current law.*
4. *Solve the resulting equations for the nodal voltages.*

A few examples will refresh your memory about the content of Chapter 8 and the general approach to a nodal-analysis solution.

EXAMPLE 17.12 Determine the voltage across the inductor for the network in Fig. 17.22.
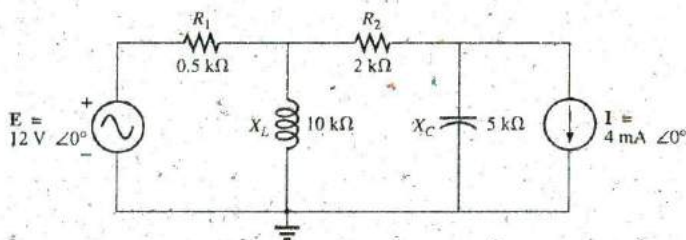
FIG. 17.22
Example 17.12.

Solution:

Steps 1 and 2 are as indicated in Fig. 17.23.
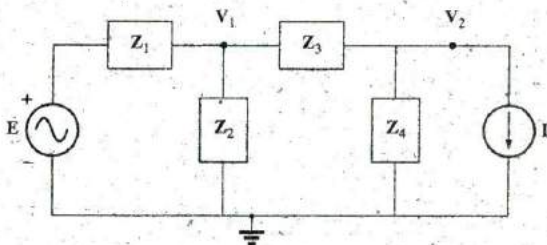
FIG. 17.23
Assigning the nodal voltages and subscripted impedances to the network in Fig. 17.22.
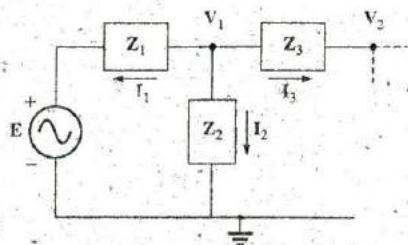
FIG. 17.24.
Applying Kirchhoff's current law to the node V_1 in Fig. 17.23.

Step 3: Note Fig. 17.24 for the application of Kirchhoff's current law to node V_1:

$$\Sigma I_i = \Sigma I_o$$
$$0 = I_1 + I_2 + I_3$$
$$\frac{V_1 - E}{Z_1} + \frac{V_1}{Z_2} + \frac{V_1 - V_2}{Z_3} = 0$$

Rearranging terms gives

$$V_1\left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right] - V_2\left[\frac{1}{Z_3}\right] = \frac{E_1}{Z_1} \qquad (17.1)$$

Note Fig. 17.25 for the application of Kirchoff's current law to node V_2.

$$0 = I_3 + I_4 + I$$

$$\frac{V_2 - V_1}{Z_3} + \frac{V_2}{Z_4} + I = 0$$

Rearranging terms gives

$$V_2\left[\frac{1}{Z_3} + \frac{1}{Z_4}\right] - V_1\left[\frac{1}{Z_3}\right] = -I \qquad (17.2)$$

Grouping equations gives

$$V_1\left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right] - V_2\left[\frac{1}{Z_3}\right] = \frac{E}{Z_1}$$

$$V_1\left[\frac{1}{Z_3}\right] \qquad - V_2\left[\frac{1}{Z_3} + \frac{1}{Z_4}\right] = I$$

$$\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} = \frac{1}{0.5\ k\Omega} + \frac{1}{j\ 10\ k\Omega} + \frac{1}{2\ k\Omega} = 2.5\ mS\ \angle -2.29°$$

$$\frac{1}{Z_3} + \frac{1}{Z_4} = \frac{1}{2\ k\Omega} + \frac{1}{-j\ 5\ k\Omega} = 0.539\ mS\ \angle 21.80°$$

and

$$V_1[2.5\ mS\ \angle -2.29°] - V_2[0.5\ mS\ \angle 0°] = 24\ mA\ \angle 0°$$
$$V_1[0.5\ mS\ \angle 0°] - V_2[0.539\ mS\ \angle 21.80°] = 4\ mA\ \angle 0°$$

with

$$V_1 = \frac{\begin{vmatrix} 24\ mA\ \angle 0° & -0.5\ mS\ \angle 0° \\ 4\ mA\ \angle 0° & -0.539\ mS\ \angle 21.80° \end{vmatrix}}{\begin{vmatrix} 2.5\ mS\ \angle -2.29° & -0.5\ mS\ \angle 0° \\ 0.5\ mS\ \angle 0° & -0.539\ mS\ \angle 21.80° \end{vmatrix}}$$

$$= \frac{(24\ mA\ \angle 0°)(-0.539\ mS\ \angle 21.80°) + (0.5\ mS\ \angle 0°)(4\ mA\ \angle 0°)}{(2.5\ mS\ \angle -2.29°)(-0.539\ mS\ \angle 21.80°) + (0.5\ mS\ \angle 0°)(0.5\ mS\ \angle 0°)}$$

$$= \frac{-12.94 \times 10^{-6}\ V\ \angle 21.80° + 2 \times 10^{-6}\ V\ \angle 0°}{-1.348 \times 10^{-6}\ \angle 19.51° + 0.25 \times 10^{-6}\ \angle 0°}$$

$$= \frac{-(12.01 + j\ 4.81) \times 10^{-6}\ V + 2 \times 10^{-6}\ V}{-(1.271 + j\ 0.45) \times 10^{-6} + 0.25 \times 10^{-6}}$$

$$= \frac{-10.01\ V - j\ 4.81\ V}{-1.021 - j\ 0.45} = \frac{11.106\ V\ \angle -154.33°}{1.116\ \angle -156.21°}$$

$$V_1 = 9.95\ V\ \angle 1.88°$$

Dependent Current Sources For dependent current sources, the procedure is modified as follows:

1. Steps 1 and 2 are the same as those applied for independent sources.
2. Step 3 is modified as follows: Treat each dependent current source like an independent source when Kirchhoff's current law
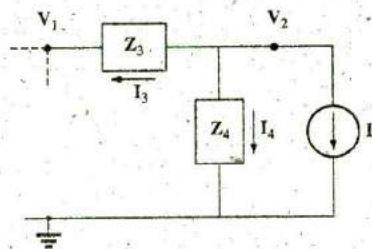
FIG. 17.25

Applying Kirchhoff's current law to the node V_2 in Fig. 17.23.

is applied to each defined node. However, once the equations are established, substitute the equation for the controlling quantity to ensure that the unknowns are limited solely to the chosen nodal voltages.

3. Step 4 is as before.

**EXAMPLE 17.13** Write the nodal equations for the network in Fig. 17.26 having a dependent current source.
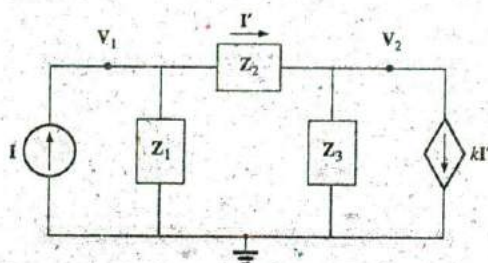
FIG. 17.26

Applying nodal analysis to a network with a current-controlled current source.

Solution:

Steps 1 and 2 are as defined in Fig. 17.26.

Step 3: At node V_1,

$$I = I_1 + I_2$$

$$\frac{V_1}{Z_1} + \frac{V_1 - V_2}{Z_2} - I = 0$$

and

$$V_1\left[\frac{1}{Z_1} + \frac{1}{Z_2}\right] - V_2\left[\frac{1}{Z_2}\right] = I$$

At node V_2,

$$I_2 + I_3 + kI = 0$$

$$\frac{V_2 - V_1}{Z_2} + \frac{V_2}{Z_3} + k\left[\frac{V_1 - V_2}{Z_2}\right] = 0$$

and

$$V_1\left[\frac{1 - k}{Z_2}\right] - V_2\left[\frac{1 - k}{Z_2} + \frac{1}{Z_3}\right] = 0$$

resulting in two equations and two unknowns.

Independent Voltage Sources between Assigned Nodes For independent voltage sources between assigned nodes, the procedure is modified as follows:

1. Steps 1 and 2 are the same as those applied for independent sources.

2. Step 3 is modified as follows: Treat each source between defined nodes as a short circuit (recall the *supernode* classification in Chapter 8), and write the nodal equations for each remaining independent node. Then relate the chosen nodal voltages to the

independent voltage source to ensure that the unknowns of the final equations are limited solely to the nodal voltages.
3. Step 4 is as before.

EXAMPLE 17.14 Write the nodal equations for the network in Fig. 17.27 having an independent source between two assigned nodes:
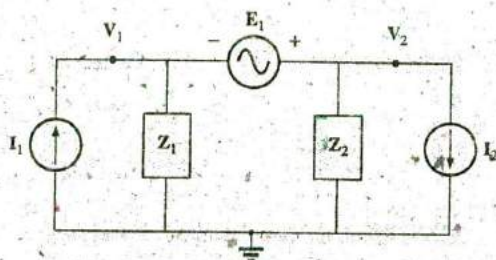
FIG. 17.27
Applying nodal analysis to a network with an independent voltage source between defined nodes.

Solution:

Steps 1 and 2 are defined in Fig. 17.27.

Steps 3: Replacing the independent source E_1 with a short-circuit equivalent results in a supernode that generates the following equation when Kirchhoff's current law is applied to node V_1:

$$I_1 = \frac{V_1}{Z_1} + \frac{V_2}{Z_2} + I_2$$

with

$$V_2 - V_1 = E_1$$

and we have two equations and two unknowns.

Dependent Voltage Sources between Defined Nodes For dependent voltage sources between defined nodes, the procedure is modified as follows:

1. Steps 1 and 2 are the same as those applied for independent voltage sources.
2. Step 3 is modified as follows: The procedure is essentially the same as that applied for independent voltage sources, except that now the dependent sources have to be defined in terms of the chosen nodal voltages to ensure that the final equations have only nodal voltages as their unknown quantities.
3. Step 4 is as before.

EXAMPLE 17.15 Write the nodal equations for the network in Fig. 17.28 having a dependent voltage source between two defined nodes.

Solution:

Steps 1 and 2 are defined in Fig. 17.28.
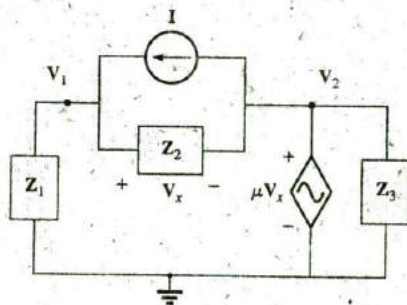
FIG. 17.28
Applying nodal analysis to a network with a voltage-controlled voltage source.

Step 3: Replacing the dependent source μV_x with a short-circuit equivalent results in the following equation when Kirchhoff's current law is applied at node V_1:

$$I = I_1 + I_2,$$

$$\frac{V_1}{Z_1} + \frac{(V_1 - V_2)}{Z_2} - I = 0$$

and

$$V_2 = \mu V_x = \mu[V_1 - V_2]$$

or

$$V_2 = \frac{\mu}{1 + \mu} V_1$$

resulting in two equations and two unknowns. Note that because the impedance Z_3 is in parallel with a voltage source, it does not appear in the analysis. It will, however, affect the current through the dependent voltage source.

Format Approach

A close examination of Eqs. (17.1) and (17.2) in Example 17.12 reveals that they are the same equations that would have been obtained using the format approach introduced in Chapter 8. Recall that the approach required that the voltage source first be converted to a current source, but the writing of the equations was quite direct and minimized any chances of an error due to a lost sign or missing term.

The sequence of steps required to apply the format approach is the following:

1. *Choose a reference node and assign a subscripted voltage label to the $(N - 1)$ remaining independent nodes of the network.*
2. *The number of equations required for a complete solution is equal to the number of subscripted voltages $(N - 1)$. Column 1 of each equation is formed by summing the admittances tied to the node of interest and multiplying the result by that subscripted nodal voltage.*
3. *The mutual terms are always subtracted from the terms of the first column. It is possible to have more than one mutual term if the nodal voltage of interest has an element in common with more than one other nodal voltage. Each mutual term is the product of the mutual admittance and the other nodal voltage tied to that admittance.*
4. *The column to the right of the equality sign is the algebraic sum of the current sources tied to the node of interest. A current source is assigned a positive sign if it supplies current to a node and a negative sign if it draws current from the node.*
5. *Solve the resulting simultaneous equations for the desired nodal voltages. The comments offered for mesh analysis regarding independent and dependent sources apply here also.*

EXAMPLE 17.16 Using the format approach tq nodal analysis, find the voltage across the $4\,\Omega$ resistor in Fig. 17.29.

Solution: Choosing nodes (Fig. 17.30) and writing the nodal equations, we have

$$Z_1 = R = 4\,\Omega \qquad Z_2 = jX_L = j5\,\Omega \qquad Z_3 = -jX_C = -j2\,\Omega$$
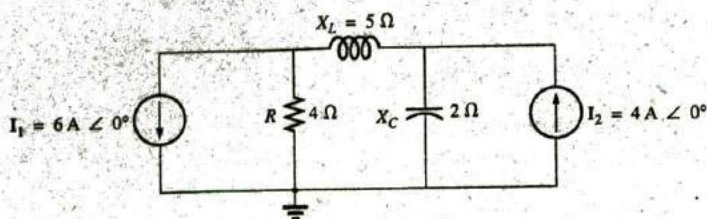
FIG. 17.29
Example 17.16.
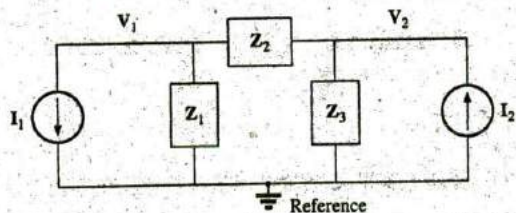
FIG. 17.30
*Assigning the nodal voltages and subscripted impedances
for the network in Fig. 17.29.*

$$V_1(Y_1 + Y_2) - V_2(Y_2) = -I_1$$
$$V_2(Y_3 + Y_2) - V_1(Y_2) = +I_2$$

or

$$V_1(Y_1 + Y_2) - V_2(Y_2) \quad = -I_1$$
$$-V_1(Y_2) \quad + V_2(Y_3 + Y_2) = +I_2$$

$$Y_1 = \frac{1}{Z_1} \qquad Y_2 = \frac{1}{Z_2} \qquad Y_3 = \frac{1}{Z_3}$$

Using determinants yields

$$V_1 = \frac{\begin{vmatrix} -I_1 & -Y_2 \\ +I_2 & Y_3 + Y_2 \end{vmatrix}}{\begin{vmatrix} Y_1 + Y_2 & -Y_2 \\ -Y_2 & Y_3 + Y_2 \end{vmatrix}} = \frac{\begin{vmatrix} -6\,A + j\,0.2\,S \\ +4\,A + j\,0.3\,S \end{vmatrix}}{\begin{vmatrix} 0.25\,S - j\,0.2\,S & +j\,0.2\,S \\ +j\,0.2\,S & +j\,0.3\,S \end{vmatrix}}$$

$$= \frac{-(Y_3 + Y_2)I_1 + I_2 Y_2}{(Y_1 + Y_2)(Y_3 + Y_2) - Y_2^2}$$

$$= \frac{-(Y_3 + Y_2)I_1 + I_2 Y_2}{Y_1 Y_3 + Y_2 Y_3 + Y_1 Y_2}$$

Substituting numerical values, we have

$$V_1 = \frac{-[(1/-j\,2\,\Omega) + (1/j\,5\,\Omega)]6\,A\,\angle 0° + 4\,A\,\angle 0°(1/j\,5\,\Omega)}{(1/4\,\Omega)(1/-j\,2\,\Omega) + (1/j\,5\,\Omega)(1/-j\,2\,\Omega) + (1/4\,\Omega)(1/j\,5\,\Omega)}$$

$$= \frac{-(+j\,0.5 - j\,0.2)6\,\angle 0° + 4\,\angle 0°(-j\,0.2)}{(1/-j\,8) + (1/10) + (1/j\,20)}$$

$$= \frac{(-0.3\,\angle 90°)(6\,\angle 0°) + (4\,\angle 0°)(0.2\,\angle -90°)}{j\,0.125 + 0.1 - j\,0.05}$$

$$= \frac{-1.8\,\angle 90° + 0.8\,\angle -90°}{0.1 + j\,0.075}$$

Introductory, C.-49A

$$= \frac{2.6 \text{ V.} \angle -90°}{0.125 \angle 36.87°}$$

$$V_1 = 20.80 \text{ V} \angle 2126.87°$$

Calculator Solution: Using the TI-89 calculator, enter the parameters for the determinant form for V_1 as shown by the sequence in Fig. 17.31. Note the different negative signs used to enter the data.

det ([(-) 6 . . 2 i ; 4 . . 3 i]) +

det ([. 2 5 - . 2 i . . 2 i ; . 2 i .

. 3 i]) ▶ Polar ENTER 20.80E0 ∠ –126.87E0

FIG. 17.31
Determining V_1 using the TI-89 calculator.

EXAMPLE 17.17 Using the format approach, write the nodal equations for the network in Fig. 17.32.
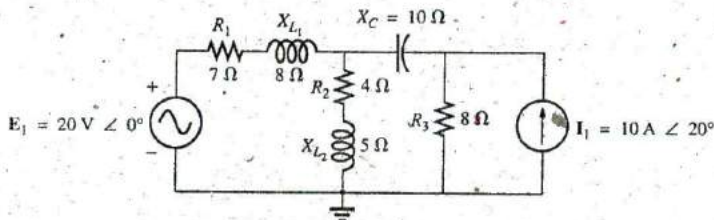
FIG. 17.32
Example 17.17.

Solution: The circuit is redrawn in Fig. 17.33, where
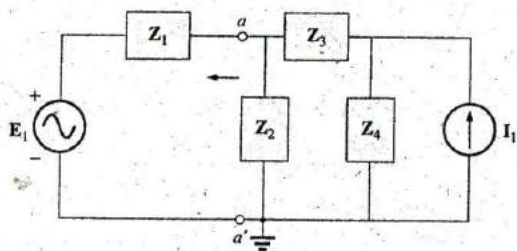
FIG. 17.33
Assigning the subscripted impedances for the network in Fig. 17.32.

$$\mathbf{Z}_1 = R_1 + jX_{L_1} = 7 \,\Omega + j \, 8 \,\Omega \qquad \mathbf{E}_1 = 20 \text{ V} \angle 0°$$
$$\mathbf{Z}_2 = R_2 + jX_{L_2} = 4 \,\Omega + j \, 5 \,\Omega \qquad \mathbf{I}_1 = 10 \text{ A} \angle 20°$$
$$\mathbf{Z}_3 = -jX_2 = -j \, 10 \,\Omega$$
$$\mathbf{Z}_4 = R_3 = 8 \,\Omega$$

Converting the voltage source to a current source and choosing nodes, we obtain Fig. 17.34. Note the "neat" appearance of the network using
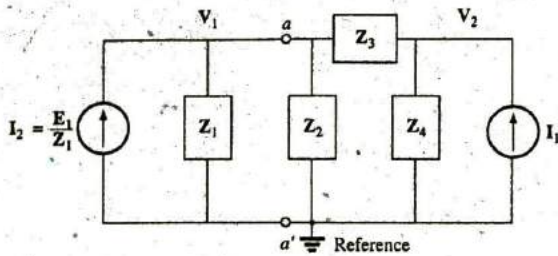
FIG. 17.34

Converting the voltage source in Fig. 17.33 to a current source and defining the nodal voltages.

the subscripted impedances. Working directly with Fig. 17.32 would be more difficult and could produce errors.

Write the nodal equations:

$$V_1(Y_1 + Y_2 + Y_3) - V_2(Y_3) = +I_2$$
$$V_2(Y_3 + Y_4) - V_1(Y_3) = +I_1$$

$$Y_1 = \frac{1}{Z_1} \qquad Y_2 = \frac{1}{Z_2} \qquad Y_3 = \frac{1}{Z_3} \qquad Y_4 = \frac{1}{Z_4}$$

which are rewritten as

$$V_1(Y_1 + Y_2 + Y_3) - V_2(Y_3) \qquad = +I_2$$
$$-V_1(Y_3) \qquad +V_2(Y_3 + Y_4) = +I_1$$

EXAMPLE 17.18 Write the nodal equations for the network in Fig. 17.35. Do not solve.
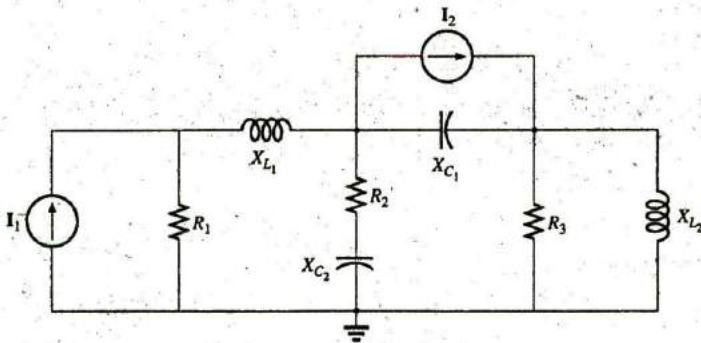
FIG. 17.35

Example 17.18.

Solution: Choose nodes (Fig. 17.36):

$$Z_1 = R_1 \qquad Z_2 = jX_{L_1} \qquad Z_3 = R_2 - jX_{C_2}$$
$$Z_4 = -jX_{C_1} \qquad Z_5 = R_3 \qquad Z_6 = jX_{L_2}$$

and write the nodal equations:

$$V_1(Y_1 + Y_2) - V_2(Y_2) = +I_1$$
$$V_2(Y_2 + Y_3 + Y_4) - V_1(Y_2) - V_3(Y_4) = -I_2$$
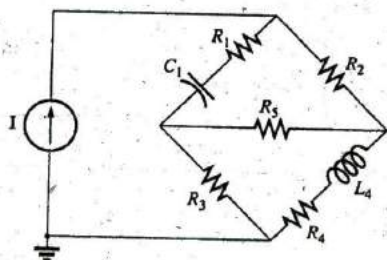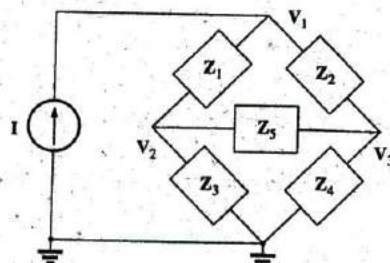$$V_3(Y_4 + Y_5 + Y_6) - V_2(Y_4) = +I_2$$
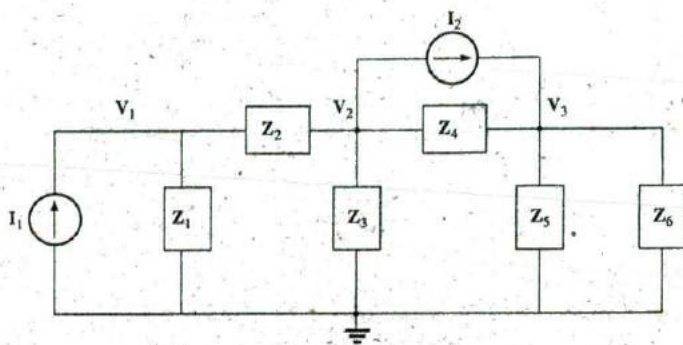
FIG. 17.36
Assigning the nodal voltages and subscripted impedances for the network in Fig. 17.35.

which are rewritten as

$$
\begin{aligned}
\mathbf{V}_1(\mathbf{Y}_1 + \mathbf{Y}_2) &- \mathbf{V}_2(\mathbf{Y}_2) && + 0 && = +\mathbf{I}_1 \\
-\mathbf{V}_1(\mathbf{Y}_2) &+ \mathbf{V}_2(\mathbf{Y}_2 + \mathbf{Y}_3 + \mathbf{Y}_4) - \mathbf{V}_3(\mathbf{Y}_4) && = -\mathbf{I}_2 \\
0 &- \mathbf{V}_2(\mathbf{Y}_4) && + \mathbf{V}_3(\mathbf{Y}_4 + \mathbf{Y}_5 + \mathbf{Y}_6) && = +\mathbf{I}_2
\end{aligned}
$$

$$
\mathbf{Y}_1 = \frac{1}{R_1} \qquad \mathbf{Y}_2 = \frac{1}{jX_{L_1}} \qquad \mathbf{Y}_3 = \frac{1}{R_2 - jX_{C_2}}
$$

$$
\mathbf{Y}_4 = \frac{1}{-jX_{C_1}} \qquad \mathbf{Y}_5 = \frac{1}{R_3} \qquad \mathbf{Y}_6 = \frac{1}{jX_{L_2}}
$$

Note the symmetry about the diagonal for this example and those preceding it in this section.

EXAMPLE 17.19 Apply nodal analysis to the network in Fig. 17.37. Determine the voltage $\mathbf{V}_L$.
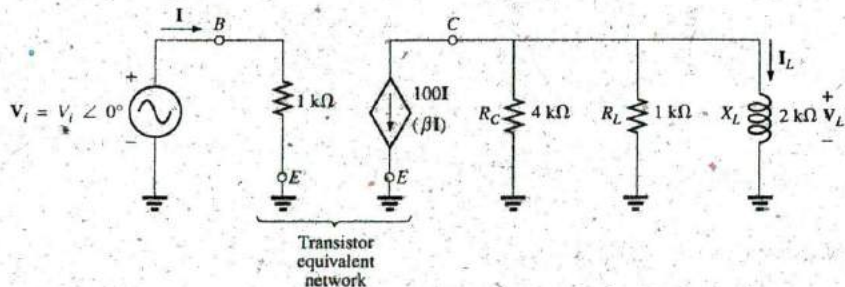
FIG. 17.37
Example 17.19

Solution: In this case, there is no need for a source conversion. The network is redrawn in Fig. 17.38 with the chosen nodal voltage and subscripted impedances.

Apply the format approach:

$$Y_1 = \frac{1}{Z_1} = \frac{1}{4 \text{ k}\Omega} = 0.25 \text{ mS } \angle 0° = G_1 \angle 0°$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{1 \text{ k}\Omega} = 1 \text{ mS } \angle 0° = G_2 \angle 0°$$

$$Y_3 = \frac{1}{Z_3} = \frac{1}{2 \text{ k}\Omega \angle 90°} = 0.5 \text{ mS } \angle -90°$$
$$= -j\, 0.5 \text{ mS} = -jB_L$$

$$V_1\text{:}(Y_1 + Y_2 + Y_3)V_1 = -100\, I$$

and

$$V_1 = \frac{-100\, I}{Y_1 + Y_2 + Y_3}$$

$$= \frac{-100\, I}{0.25 \text{ mS} + 1 \text{ mS} - j\, 0.5 \text{ mS}}$$

$$= \frac{-100 \times 10^3\, I}{1.25 - j\, 0.5} = \frac{-100 \times 10^3\, I}{1.3463 \angle -21.80°}$$

$$= -74.28 \times 10^3\, I \angle 21.80°$$

$$= -74.28 \times 10^3 \left(\frac{V_i}{1 \text{ k}\Omega}\right) \angle 21.80°$$

$$V_1 = V_L = -(74.28 V_i) V \angle 21.80°$$
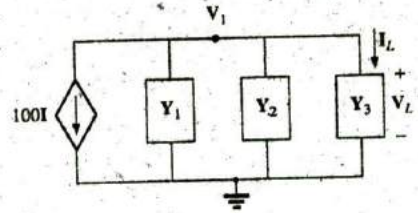
FIG. 17.38
Assigning the nodal voltage and subscripted impedances for the network in Fig. 17.37.

17.6 BRIDGE NETWORKS (ac)

The basic bridge configuration was discussed in some detail in Section 8.11 for dc networks. We now continue to examine **bridge networks** by considering those that have reactive components and a sinusoidal ac voltage or current applied.

We first analyze various familiar forms of the bridge network using mesh analysis and nodal analysis (the format approach). The balance conditions are investigated throughout the section.

Apply **mesh analysis** to the network in Fig. 17.39. The network is redrawn in Fig. 17.40, where

$$Z_1 = \frac{1}{Y_1} = \frac{1}{G_1 + jB_C} = \frac{G_1}{G_1^2 + B_C^2} - j\frac{B_C}{G_1^2 + B_C^2}$$
$$Z_2 = R_2 \quad Z_3 = R_3 \quad Z_4 = R_4 + jX_L \quad Z_5 = R_5$$

Applying the format approach:

$$(Z_1 + Z_3)I_1 - (Z_1)I_2 - (Z_3)I_3 = E$$
$$(Z_1 + Z_2 + Z_5)I_2 - (Z_1)I_1 - (Z_5)I_3 = 0$$
$$(Z_3 + Z_4 + Z_5)I_3 - (Z_3)I_1 - (Z_5)I_2 = 0$$

which are rewritten as

$$
\begin{array}{llll}
I_1(Z_1 + Z_3) & - I_2 Z_1 & - I_3 Z_3 & = E \\
-I_1 Z_1 & + I_2(Z_1 + Z_2 + Z_5) & - I_3 Z_5 & = 0 \\
-I_1 Z_3 & - I_2 Z_5 & + I_3(Z_3 + Z_4 + Z_5) & = 0
\end{array}
$$

Note the symmetry about the diagonal of the above equations. For balance, $I_{Z_5} = 0$ A, and

$$I_{Z_5} = I_2 - I_3 = 0$$
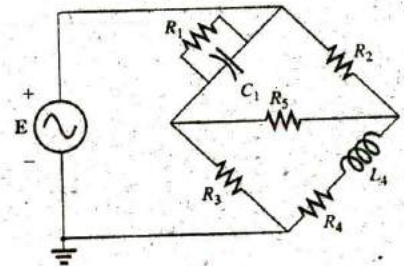
FIG. 17.39
Maxwell bridge.
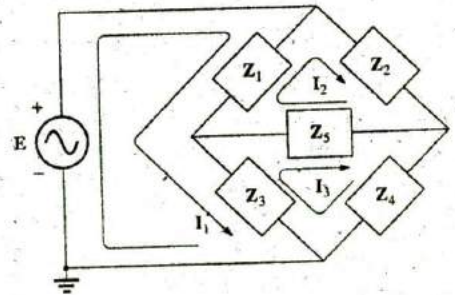
FIG. 17.40
Assigning the mesh currents and subscripted impedances for the network in Fig. 17.39.

From the above equations,

$$I_2 = \frac{\begin{vmatrix} Z_1 + Z_3 & E & -Z_3 \\ -Z_1 & 0 & -Z_5 \\ -Z_3 & 0 & (Z_3 + Z_4 + Z_5) \end{vmatrix}}{\begin{vmatrix} Z_1 + Z_3 & -Z_1 & -Z_3 \\ -Z_1 & (Z_1 + Z_2 + Z_5) & -Z_5 \\ -Z_3 & -Z_5 & (Z_3 + Z_4 + Z_5) \end{vmatrix}}$$

$$= \frac{E(Z_1Z_3 + Z_1Z_4 + Z_1Z_5 + Z_3Z_5)}{\Delta}$$

where Δ signifies the determinant of the denominator (or coefficients). Similarly,

$$I_3 = \frac{E(Z_1Z_3 + Z_3Z_2 + Z_1Z_5 + Z_3Z_5)}{\Delta}$$

and

$$I_{Z_5} = I_2 - I_3 = \frac{E(Z_1Z_4 - Z_3Z_2)}{\Delta}$$

For $I_{Z_5} = 0$, the following must be satisfied (for a finite Δ not equal to zero):

$$\boxed{Z_1Z_4 = Z_3Z_2} \qquad I_{Z_5} = 0 \qquad (17.3)$$

This condition is analyzed in greater depth later in this section.

Applying **nodal analysis** to the network in Fig. 17.41 results in the configuration in Fig. 17.42, where

$$Y_1 = \frac{1}{Z_1} = \frac{1}{R_1 - jX_C} \qquad Y_2 = \frac{1}{Z_2} = \frac{1}{R_2}$$

$$Y_3 = \frac{1}{Z_3} = \frac{1}{R_3} \qquad Y_4 = \frac{1}{Z_4} = \frac{1}{R_4 + jX_L} \qquad Y_5 = \frac{1}{R_5}$$

and

$$(Y_1 + Y_2)V_1 - (Y_1)V_2 - (Y_2)V_3 = I$$
$$(Y_1 + Y_3 + Y_5)V_2 - (Y_1)V_1 - (Y_5)V_3 = 0$$
$$(Y_2 + Y_4 + Y_5)V_3 - (Y_2)V_1 - (Y_5)V_2 = 0$$

FIG. 17.41
Hay bridge.

FIG. 17.42
Assigning the nodal voltages and subscripted
impedances for the network in Fig. 17.41.

which are rewritten as

$$
\begin{array}{llll}
V_1(Y_1 + Y_2) & - V_2(Y_1) & - V_3Y_2 & = I \\
-V_1(Y_2) & + V_2(Y_1 + Y_3 + Y_5) & - V_3Y_5 & = 0 \\
-V_1Y_2 & - V_2Y_5 & + V_3(Y_2 + Y_4 + Y_5) & = 0
\end{array}
$$

Again, note the symmetry about the diagonal axis. For balance, $V_{Z_5} = 0$ V, and

$$V_{Z_5} = V_2 - V_3 = 0$$

From the above equations,

$$V_2 = \frac{\begin{vmatrix} Y_1 + Y_2 & I & -Y_2 \\ -Y_1 & 0 & -Y_5 \\ -Y_2 & 0 & (Y_2 + Y_4 + Y_5) \end{vmatrix}}{\begin{vmatrix} Y_1 + Y_2 & -Y_1 & -Y_2 \\ -Y_1 & (Y_1 + Y_3 + Y_5) & -Y_5 \\ -Y_2 & -Y_5 & (Y_2 + Y_4 + Y_5) \end{vmatrix}}$$

$$= \frac{I(Y_1Y_3 + Y_1Y_4 + Y_1Y_5 + Y_3Y_5)}{\Delta}$$

Similarly,

$$V_3 = \frac{I(Y_1Y_3 + Y_3Y_2 + Y_1Y_5 + Y_3Y_5)}{\Delta}$$

Note the similarities between the above equations and those obtained for mesh analysis. Then

$$V_{Z_5} = V_2 - V_3 = \frac{I(Y_1Y_4 - Y_3Y_2)}{\Delta}$$

For $V_{Z_5} = 0$, the following must be satisfied for a finite Δ not equal to zero:

$$\boxed{Y_1Y_4 = Y_3Y_2} \qquad V_{Z_5} = 0 \qquad (17.4)$$

However, substituting $Y_1 = 1/Z_1$, $Y_2 = 1/Z_2$, $Y_3 = 1/Z_3$, and $Y_4 = 1/Z_4$, we have

$$\frac{1}{Z_1Z_4} = \frac{1}{Z_3Z_2}$$

or

$$\boxed{Z_1Z_4 = Z_3Z_2} \qquad V_{Z_5} = 0$$

corresponding with Eq. (17.3) obtained earlier.

Let us now investigate the balance criteria in more detail by considering the network in Fig. 17.43, where it is specified that I and V = 0.

Since $I = 0$,

$$\boxed{I_1 = I_3} \qquad (17.5)$$

and

$$\boxed{I_2 = I_4} \qquad (17.6)$$

In addition, for $V = 0$,

$$\boxed{I_1Z_1 = I_2Z_2} \qquad (17.7)$$

and

$$\boxed{I_3Z_3 = I_4Z_4} \qquad (17.8)$$

Substituting the preceding current relations into Eq. (17.8), we have

$$I_1Z_3 = I_2Z_4$$
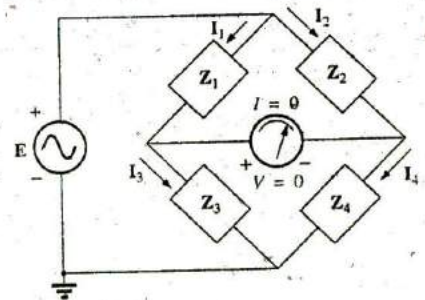
FIG. 17.43

Investigating the balance criteria for an ac bridge configuration.

and

$$I_2 = \frac{Z_3}{Z_4} I_1$$

Substituting this relationship for I_2 into Eq. (17.7) yields

$$I_1 Z_1 = \left(\frac{Z_3}{Z_4} I_1\right) Z_2$$

and

$$Z_1 Z_4 = Z_2 Z_3$$

as obtained earlier. Rearranging, we have

$$\boxed{\frac{Z_1}{Z_3} = \frac{Z_2}{Z_4}}$$

(17.9)

corresponding to Eq. (8.2) for dc resistive networks.

For the network in Fig. 17.41, which is referred to as a **Hay bridge** when Z_5 is replaced by a sensitive galvanometer,

$$Z_1 = R_1 - jX_C$$
$$Z_2 = R_2$$
$$Z_3 = R_3$$
$$Z_4 = R_4 + jX_L$$

This particular network is used for measuring the resistance and inductance of coils in which the resistance is a small fraction of the reactance X_L.

Substitute into Eq. (17.9) in the following form:

$$Z_2 Z_3 = Z_4 Z_1$$
$$R_2 R_3 = (R_4 + jX_L)(R_1 - jX_C)$$

or

$$R_2 R_3 = R_1 R_4 + j(R_1 X_L - R_4 X_C) + X_C X_L$$

so that

$$R_2 R_3 + j0 = (R_1 R_4 + X_C X_L) + j(R_1 X_L - R_4 X_C)$$

For the equations to be equal, *the real and imaginary parts must be equal.* Therefore, for a balanced Hay bridge,

$$\boxed{R_2 R_3 = R_1 R_4 + X_C X_L}$$

(17.10)

and

$$\boxed{0 = R_1 X_L - R_4 X_C}$$

(17.11)

or substituting $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$

we have

$$X_C X_L = \left(\frac{1}{\omega C}\right)(\omega L) = \frac{L}{C}$$

and

$$R_2 R_3 = R_1 R_4 + \frac{L}{C}$$

with

$$R_1 \omega L = \frac{R_4}{\omega C}$$

Solving for R_4 in the last equation yields

$$R_4 = \omega^2 L C R_1$$

and substituting into the previous equation, we have

$$R_2 R_3 = R_1(\omega^2 LCR_1) + \frac{L}{C}$$

Multiply through by C and factor:

$$CR_2 R_3 = L(\omega^2 C^2 R_1^2 + 1)$$

and

$$L = \frac{CR_2 R_3}{1 + \omega^2 C^2 R_1^2} \qquad (17.12)$$

With additional algebra this yields:

$$R_4 = \frac{\omega^2 C^2 R_1 R_2 R_3}{1 + \omega^2 C^2 R_1^2} \qquad (17.13)$$

Eqs. (17.12) and (17.13) are the balance conditions for the Hay bridge. Note that each is frequency dependent. For different frequencies, the resistive and capacitive elements must vary for a particular coil to achieve balance. For a coil placed in the Hay bridge as shown in Fig. 17.41, the resistance and inductance of the coil can be determined by Eqs. (17.12) and (17.13) when balance is achieved.

The bridge in Fig. 17.39 is referred to as a **Maxwell bridge** when Z_5 is replaced by a sensitive galvanometer. This setup is used for inductance measurements when the resistance of the coil is large enough not to require a Hay bridge.

Applying Eq. (17.9) in the form

$$Z_2 Z_3 = Z_4 Z_1$$

and substituting

$$Z_1 = R_1 \angle 0° \parallel X_{C_1} \angle -90° = \frac{(R_1 \angle 0°)(X_{C_1} \angle -90°)}{R_1 - jX_{C_1}}$$

$$= \frac{R_1 X_{C_1} \angle -90°}{R_1 - jX_{C_1}} = \frac{-jR_1 X_{C_1}}{R_1 - jX_{C_1}}$$

$$Z_2 = R_2$$

$$Z_3 = R_3$$

and $Z_4 = R_4 + jX_{L_4}$

we have

$$(R_2)(R_3) = (R_4 + jX_{L_4})\left(\frac{-jR_1 X_{C_1}}{R_1 - jX_{C_1}}\right)$$

$$R_2 R_3 = \frac{-jR_1 R_4 X_{C_1} + R_1 X_{C_1} X_{L_4}}{R_1 - jX_{C_1}}$$

or

$$(R_2 R_3)(R_1 - jX_{C_1}) = R_1 X_{C_1} X_{L_4} - jR_1 R_4 X_{C_1}$$

and

$$R_1 R_2 R_3 - jR_2 R_3 X_{C_1} = R_1 X_{C_1} X_{L_4} - jR_1 R_4 X_{C_1}$$

so that for balance

$$R_1 R_2 R_3 = R_1 X_{C_1} X_{L_4}$$

$$R_2 R_3 = \left(\frac{1}{2\pi f C_1}\right)(2\pi f L_4)$$

and

$$L_4 = C_1 R_2 R_3 \qquad (17.14)$$

and

$$R_2R_3X_{C_1} = R_1R_4X_{C_1}$$

so that

$$R_4 = \frac{R_2R_3}{R_1}$$ (17.15)

Note the absence of frequency in Eqs. (17.14) and (17.15).

One remaining popular bridge is the **capacitance comparison bridge** of Fig. 17.44. An unknown capacitance and its associated resistance can be determined using this bridge. Application of Eq. (17.9) yields the following results:

$$C_4 = C_3\frac{R_1}{R_2}$$ (17.16)
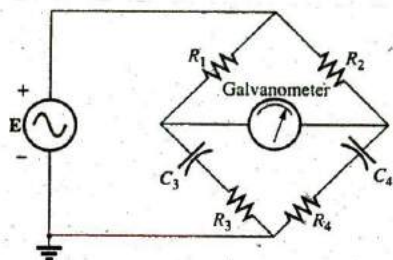
$$R_4 = \frac{R_2R_3}{R_1}$$ (17.17)

FIG. 17.44
Capacitance comparison bridge.

The derivation of these equations appears as a problem at the end of the chapter.

17.7 Δ-Y, Y-Δ CONVERSIONS

The Δ-Y, Y-Δ (or π-T, T-π as defined in Section 8.12) conversions for ac circuits are not derived here since the development corresponds exactly with that for dc circuits. Taking the **Δ-Y configuration** shown in Fig. 17.45, we find the general equations for the impedances of the Y in terms of those for the Δ:
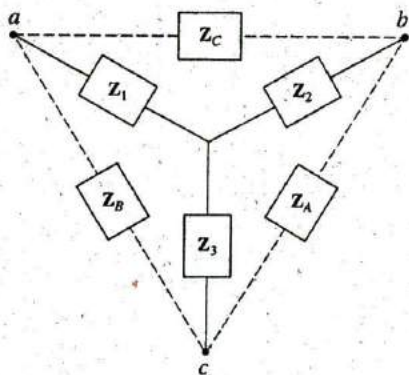
$$Z_1 = \frac{Z_B Z_C}{Z_A + Z_B + Z_C}$$ (17.18)

$$Z_2 = \frac{Z_A Z_C}{Z_A + Z_B + Z_C}$$ (17.19)

$$Z_3 = \frac{Z_A Z_B}{Z_A + Z_B + Z_C}$$ (17.20)

FIG. 17.45
Δ-Y configuration.

For the impedances of the Δ in terms of those for the Y, the equations are

$$Z_B = \frac{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}{Z_2}$$ (17.21)

$$Z_A = \frac{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}{Z_1}$$ (17.22)

$$Z_C = \frac{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}{Z_3}$$ (17.23)

Note that each impedance of the Y is equal to the product of the impedances in the two closest branches of the Δ, divided by the sum of the impedances in the Δ.

Further, the value of each impedance of the Δ is equal to the sum of the possible product combinations of the impedances of the Y, divided by the impedances of the Y farthest from the impedance to be determined.

Drawn in different forms (Fig. 17.46), they are also referred to as the T and π configurations.
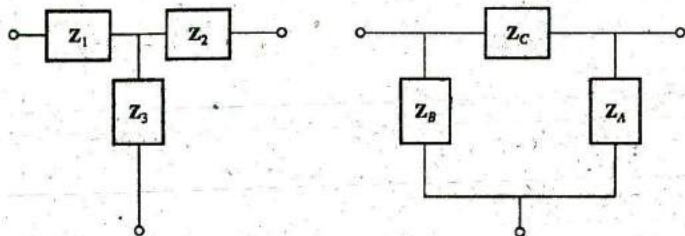
FIG. 17.46
The T and π configurations.

In the study of dc networks, we found that if all of the resistors of the Δ or Y were the same, the conversion from one to the other could be accomplished using the equation

$$R_\Delta = 3R_Y \quad \text{or} \quad R_Y = \frac{R_\Delta}{3}$$

For ac networks,

$$Z_\Delta = 3Z_Y \quad \text{or} \quad Z_Y = \frac{Z_\Delta}{3} \qquad (17.24)$$

Be careful when using this simplified form. It is not sufficient for all the impedances of the Δ or Y to be of the same magnitude: *The angle associated with each must also be the same.*

EXAMPLE 17.20 Find the total impedance Z_T of the network in Fig. 17.47.
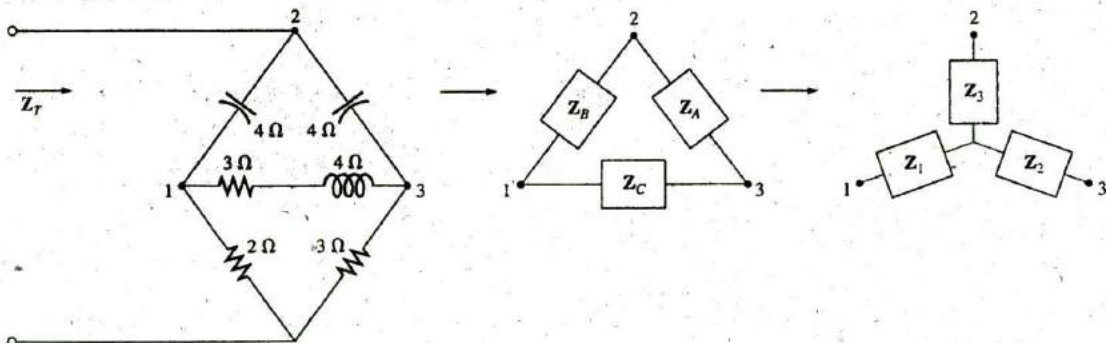
FIG. 17.47
Converting the upper Δ of a bridge configuration to a Y.

Solution:

$$\mathbf{Z}_B = -j4 \quad \mathbf{Z}_A = -j4 \quad \mathbf{Z}_C = 3 + j4$$

$$\mathbf{Z}_1 = \frac{\mathbf{Z}_B \mathbf{Z}_C}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C} = \frac{(-j4\,\Omega)(3\,\Omega + j4\,\Omega)}{(-j4\,\Omega) + (-j4\,\Omega) + (3\,\Omega + j4\,\Omega)}$$

$$= \frac{(4\angle-90°)(5\angle53.13°)}{3 - j4} = \frac{20\angle-36.87°}{5\angle-53.13°}$$

$$= 4\,\Omega\angle16.13° = 3.84\,\Omega + j1.11\,\Omega$$

$$\mathbf{Z}_2 = \frac{\mathbf{Z}_A \mathbf{Z}_C}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C} = \frac{(-j4\,\Omega)(3\,\Omega + j4\,\Omega)}{5\,\Omega\angle-53.13°}$$

$$= 4\,\Omega\angle16.13° = 3.84\,\Omega + j1.11\,\Omega$$

Recall from the study of dc circuits that if two branches of the Y or Δ are the same, the corresponding Δ or Y, respectively, will also have two similar branches. In this example, $\mathbf{Z}_A = \mathbf{Z}_B$. Therefore, $\mathbf{Z}_1 = \mathbf{Z}_2$, and

$$\mathbf{Z}_3 = \frac{\mathbf{Z}_A \mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C} = \frac{(-j4\,\Omega)(-j4\,\Omega)}{5\,\Omega\angle-53.13°}$$

$$= \frac{16\,\Omega\angle-180°}{5\angle-53.13°} = 3.2\,\Omega\angle-126.87° = -1.92\,\Omega - j2.56\,\Omega$$

Replace the Δ by the Y (Fig. 17.48):

$$\mathbf{Z}_1 = 3.84\,\Omega + j1.11\,\Omega \qquad \mathbf{Z}_2 = 3.84\,\Omega + j1.11\,\Omega$$
$$\mathbf{Z}_3 = -1.92\,\Omega - j2.56\,\Omega \qquad \mathbf{Z}_4 = 2\,\Omega$$
$$\mathbf{Z}_5 = 3\,\Omega$$

Impedances $\mathbf{Z}_1$ and $\mathbf{Z}_4$ are in series:

$$\mathbf{Z}_{T_1} = \mathbf{Z}_1 + \mathbf{Z}_4 = 3.84\,\Omega + j1.11\,\Omega + 2\,\Omega = 5.84\,\Omega + j1.11\,\Omega$$
$$= 5.94\,\Omega\angle10.76°$$

Impedances $\mathbf{Z}_2$ and $\mathbf{Z}_5$ are in series:

$$\mathbf{Z}_{T_2} = \mathbf{Z}_2 + \mathbf{Z}_5 = 3.84\,\Omega + j1.11\,\Omega + 3\,\Omega = 6.84\,\Omega + j1.11\,\Omega$$
$$= 6.93\,\Omega\angle9.22°$$

Impedances $\mathbf{Z}_{T_1}$ and $\mathbf{Z}_{T_2}$ are in parallel:

$$\mathbf{Z}_{T_3} = \frac{\mathbf{Z}_{T_1}\mathbf{Z}_{T_2}}{\mathbf{Z}_{T_1} + \mathbf{Z}_{T_2}} = \frac{(5.94\,\Omega\angle10.76°)(6.93\,\Omega\angle9.22°)}{5.84\,\Omega + j1.11\,\Omega + 6.84\,\Omega + j1.11\,\Omega}$$

$$= \frac{41.16\,\Omega\angle19.98°}{12.68 + j2.22} = \frac{41.16\,\Omega\angle19.98°}{12.87\angle9.93°} = 3.198\,\Omega\angle10.05°$$

$$= 3.15\,\Omega + j0.56\,\Omega$$

Impedances $\mathbf{Z}_3$ and $\mathbf{Z}_{T_3}$ are in series. Therefore,

$$\mathbf{Z}_T = \mathbf{Z}_3 + \mathbf{Z}_{T_3} = -1.92\,\Omega - j2.56\,\Omega + 3.15\,\Omega + j0.56\,\Omega$$
$$= 1.23\,\Omega - j2.0\,\Omega = 2.35\,\Omega\angle-58.41°$$
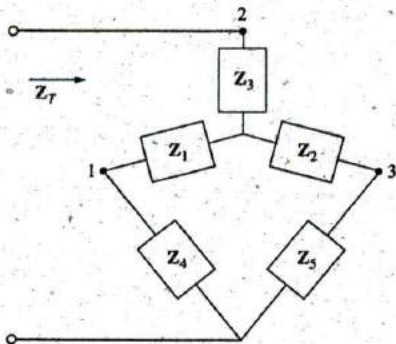
FIG. 17.48
The network in Fig. 17.47 following the substitution of the Y configuration.

EXAMPLE 17.21 Using both the Δ-Y and Y-Δ transformations, find the total impedance $\mathbf{Z}_T$ for the network in Fig. 17.49.

Solution: *Using the Δ-Y transformation, we obtain Fig. 17.50. In this case, since both systems are balanced (same impedance in each branch),*
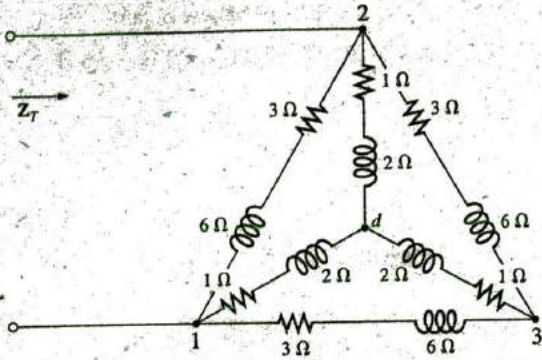
FIG. 17.49
Example 17.21.
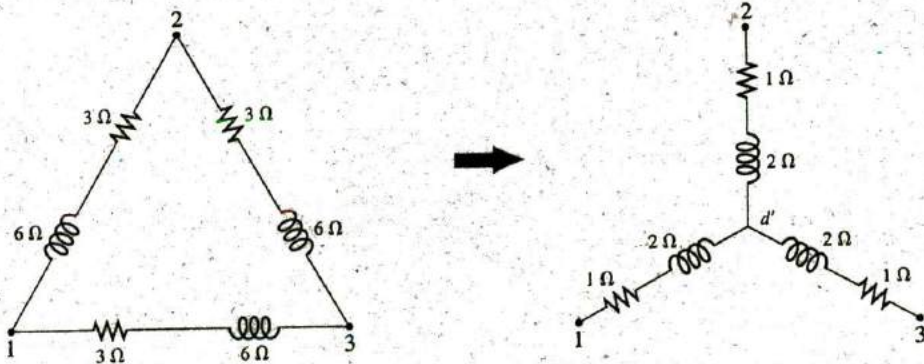
FIG. 17.50
Converting a Δ configuration to a Y configuration.

the center point d' of the transformed Δ will be the same as point d of the original Y:

$$\mathbf{Z_Y} = \frac{\mathbf{Z_\Delta}}{3} = \frac{3\,\Omega + j\,6\,\Omega}{3} = 1\,\Omega + j\,2\,\Omega$$

and (Fig. 17.51)

$$\mathbf{Z_T} = 2\left(\frac{1\,\Omega + j\,2\,\Omega}{2}\right) = 1\,\Omega + j\,2\,\Omega$$
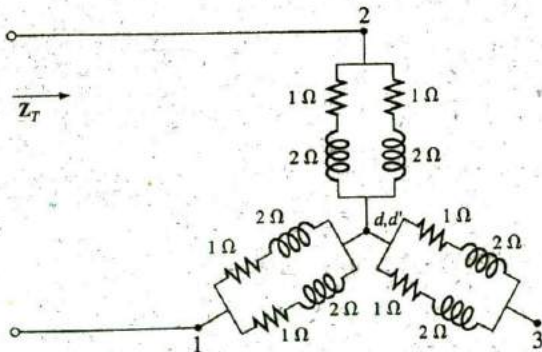
FIG. 17.51
Substituting the Y configuration in Fig. 17.50 into the network in Fig. 17.49.

Using the Y-Δ transformation (Fig. 17.52), we obtain

$$Z_\Delta = 3Z_Y = 3(1\,\Omega + j\,2\,\Omega) = 3\,\Omega + j\,6\,\Omega$$
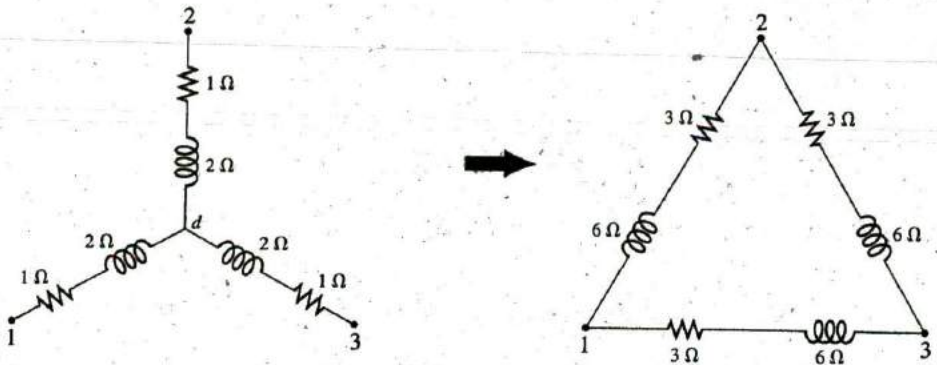
FIG. 17.52
Converting the Y configuration in Fig. 17.49 to a Δ.

Each resulting parallel combination in Fig. 17.53 will have the following impedance:

$$Z' = \frac{3\,\Omega + j\,6\,\Omega}{2} = 1.5\,\Omega + j\,3\,\Omega$$

and

$$Z_T = \frac{Z'(2Z')}{Z' + 2Z'} = \frac{2(Z')^2}{3Z'} = \frac{2Z'}{3}$$

$$= \frac{2(1.5\,\Omega + j\,3\,\Omega)}{3} = 1\,\Omega + j\,2\,\Omega$$

which compares with the above result.
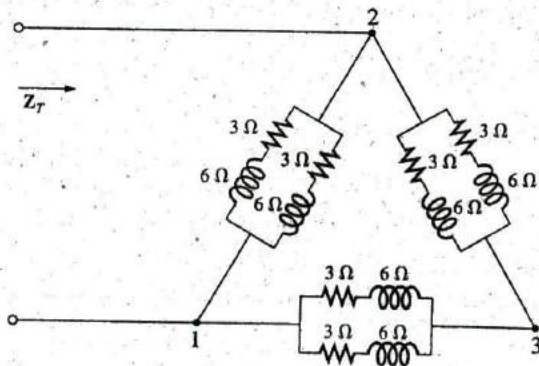
FIG. 17.53
Substituting the Δ configuration in Fig. 17.54 into the network in Fig. 17.49.

17.8 COMPUTER ANALYSIS

PSpice

Nodal Analysis The first application of PSpice is to determine the nodal voltages for the network in Example 17.16 and compare solutions. The network appears as shown in Fig. 17.54 using elements that were
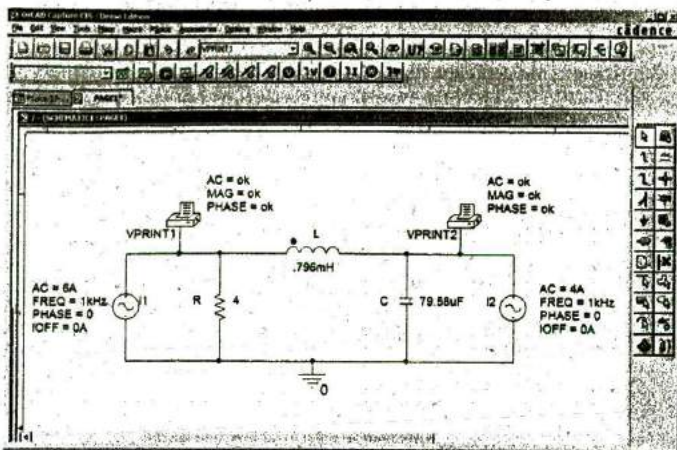
FIG. 17.54
Using PSpice to verify the results in Example 17.16.

determined from the reactance level at a frequency of 1 kHz. There is no
need to continually use 1 kHz. Any frequency will do, but remember to
use the chosen frequency to find the network components and when set-
ting up the simulation.

For the current sources, choose **ISIN** so that the phase angle can be
specified (even though it is 0°), although the symbol does not have the
arrow used in the text material. The direction must be recognized as
pointing from the + to − sign of the source. That requires that the
sources I_1 and I_2 be set as shown in Fig. 17.54. Reverse the source I_2
by using the **Mirror Vertically** option obtained by right-clicking the
source symbol on the screen. Setting up the **ISIN** source is the same
as that used with the **VSIN** source. It can be found under the
SOURCE library, and its attributes are the same as for the **VSIN**
source. For each source, set **IOFF** to 0 A; the amplitude is the peak
value of the source current. The frequency will be the same for each
source. Then select **VPRINT1** from the **SPECIAL** library and place
it to generate the desired nodal voltages. Finally add the remaining el-
ements to the network as shown in Fig. 17.54. For each source, dou-
ble-click the symbol to generate the **Property Editor** dialog box. Set
AC at the 6 A level for the I_1 source and at 4 A for the I_2 source, fol-
lowed by **Display** and **Name and Value** for each. It appears as shown
in Fig. 17.54. Double-clicking on each **VPRINT1** option also pro-
vides the **Property Editor**, so OK can be added under AC, MAG, and
PHASE. For each quantity, select **Display** followed by **Name and
Value** and OK. Then select Value and **VPRINT1** is displayed as
Value only. Selecting **Apply** and leaving the dialog box results in the
listing next to each source in Fig. 17.54. For **VPRINT2**, first change
the listing on **Value** from VPRINT1 to VPRINT2 before selecting
Display and **Apply**.

Now select the **New Simulation Profile** icon, and enter **PSpice 17-1**
as the **Name** followed by **Create**. In the **Simulation Settings** dialog
box, select **AC Sweep**, and set the **Start Frequency** and **End Fre-
quency** at 1 kHz with 1 for the **Points/Decade**. Click **OK**, and select
the **Run PSpice** icon; a SCHEMATIC1 screen results. Exiting (**X**)

brings you back to the **Orcad Capture** window. Selecting **PSpice** followed by **View Output File** results in the display in Fig. 17.55, providing exactly the same results as obtained in Example 17.16 with $V_1 = 20.8$ V $\angle -126.9°$. The other nodal voltage is 8.617 V $\angle -15.09°$.

```
** Profile: "SCHEMATIC1-PSpice 17-1"
[ C:\ICA11\PSpice\pspice 17-1-pspicefiles\schematic1\pspice 17-1.sim ]

****   AC ANALYSIS            TEMPERATURE =  27.000 DEG C
************************************************************
***
FREQ        VM(N01900)   VP(N01900)
1.000E+03   2.080E+01    -1.269E+02


FREQ        VM(N01908)   VP(N01908)
1.000E+03   8.617E+00    -1.509E+01
```

FIG. 17.55

Output file for the nodal voltages for the network in Fig. 17.54.

Current-Controlled Current Source (CCCS)   Our interest now turns to controlled sources in the PSpice environment. Controlled sources are not particularly difficult to apply once a few important elements of their use are understood. The network in Fig. 17.14 has a current-controlled current source in the center leg of the configuration. The magnitude of the current source is k times the current through resistor R_1, where k can be greater or less than 1. The resulting schematic, appearing in Fig. 17.56, seems quite complex in the area of the controlled source, but once you understand the role of each component, the schematic is not that difficult to understand. First, since it is the only new element in the schematic, let us concentrate on the controlled source. Current-controlled current sources (CCCS) are called up under the **ANALOG** library as **F** and appear as shown in the center in Fig. 17.56. Pay attention to the direction of the current in
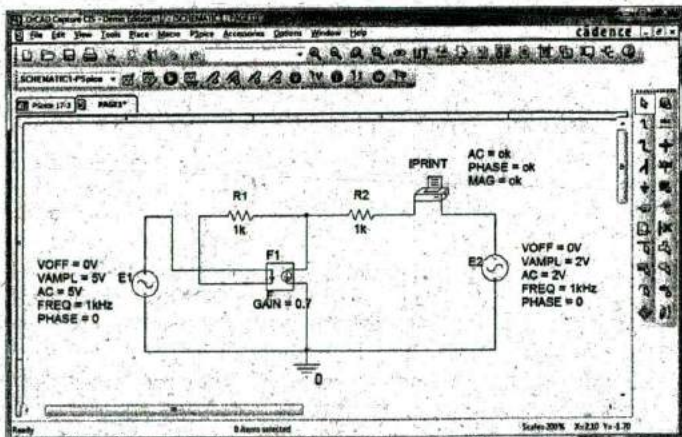
FIG. 17.56

Using PSpice to verify the results in Example 17.8.

each part of the symbol. In particular, note that the sensing current of F has the same direction as the defining controlling current in Fig. 17.14. In addition, note that the controlled current source also has the same direction as the source in Fig. 17.14. If you double-click on the CCCS symbol, the **Property Editor** dialog box appears with the **GAIN** (k as described above) set at 1. In this example, the gain must be set at **0.7**, so click on the region below the **GAIN** label and enter **0.7**. Then select **Display** followed by **Name and Value-OK**. Exit the **Property Editor**, and **GAIN** $= 0.7$ appears with the CCCS as shown in Fig. 17.56.

The other new component in this schematic is **IPRINT**; it can be found in the **SPECIAL** library. It is used to tell the program to list the current in the branch of interest in the output file. If you fail to tell the program which output data you would like, it will simply run through the simulation and list specific features of the network but will not provide any voltages or currents. In this case, the current I_2 through the resistor R_2 is desired. Double-clicking on the **IPRINT** component results in the **Property Editor** dialog box with a number of elements that need to be defined—much like that for **VPRINT**. First enter **OK** beneath **AC** and follow with **Display-Name and Value-OK**. Repeat for **MAG** and **PHASE**, and then select **Apply** before leaving the dialog box. The **OK** tells the software program that these are the quantities that it is "ok" to generate and provide. The purpose of the **Apply** at the end of each visit to the **Property Editor** dialog box is to "apply" the changes made to the network under investigation. When you exit the **Property Editor**, the three chosen parameters appear on the schematic with the **OK** directive. You may find that the labels will appear all over the **IPRINT** symbol. No problem—just click on each, and move to a more convenient location.

The remaining components of the network should be fairly familiar, but don't forget to **Mirror Vertically** the voltage source E2. In addition, do not forget to call up the **Property Editor** for each source and set the level of AC, FREQ, VAMPL, and VOFF and be sure that the PHASE is set on the default value of $0°$. The value appears with each parameter in Fig. 17.58 for each source. Always be sure to select **Apply** before leaving the **Property Editor**. After placing all the components on the screen, you must connect them with a **Place wire** selection. Normally, this is pretty straightforward. However, with controlled sources, it is often necessary to cross over wires *without* making a connection. In general, when you're placing a wire over another wire and you don't want a connection to be made, click a spot on one side of the wire to be crossed to create the temporary red square. Then cross the wire, and click again to establish another red square. If the connection is done properly, the crossed wire should not show a connection point (a small red dot). In this example, the top of the controlling current was connected first from the E1 source. Then a wire was connected from the lower end of the sensing current to the point where a $90°$ turn up the page was to be made. The wire was clicked in place at this point before crossing the original wire and clicked again before making the right turn to resistor R_1. You will not find a small red dot where the wires cross.

Now for the simulation. In the **Simulation Settings** dialog box, select AC Sweep/Noise with a **Start** and **End Frequency** of 1 kHz. There will be **1 Point/Decade**. Click OK, and select the **Run Spice** key; a **SCHEMATIC1** results that should be exited to obtain the Orcad Capture screen. Select PSpice followed by View Output File,

and scroll down until you read **AC ANALYSIS** (see Fig. 17.57). The magnitude of the desired current is 1.615 mA with a phase angle of $0°$, a perfect match with the theoretical analysis to follow. One would expect a phase angle of $0°$ since the network is composed solely of resistive elements.

```
** Profile: "SCHEMATIC1-PSpice 17-3"
[ C:\ICA11\PSpice\PSpice 17-3-PSpiceFiles\SCHEMATIC1\PSpice 17-3.sim ]

****    AC ANALYSIS                    TEMPERATURE = 27.000 DEG C
*****************************************************************************
***
FREQ        IM(V_PRINT1)        IP(V_PRINT1)
1.000E+03   1.615E-03           0.000E+00
```

FIG. 17.57
The output file for the mesh current I_2 in Fig. 17.14.

The equations obtained earlier using the supermesh approach were

$$E - I_1 Z_1 - I_2 Z_2 + E_2 = 0 \quad \text{or} \quad I_1 Z_1 + I_2 Z_2 = E_1 + E_2$$

and

$$kI = kI_1 = I_1 - I_2$$

resulting in $\quad I_1 = \dfrac{I_2}{1-k} = \dfrac{I_2}{1-0.7} = \dfrac{I_2}{0.3} = 3.333 I_2$

so that $\quad I_1(1\ k\Omega) + I_2(1\ k\Omega) = 7\ V \quad$ (from above)

becomes $\quad (3.333 I_2)1\ k\Omega + I_2(1\ k\Omega) = 7\ V$

or $\quad (4.333\ k\Omega)I_2 = 7\ V$

and $\quad I_2 = \dfrac{7\ V}{4.333\ k\Omega} = 1.615\ mA\ \angle 0°$

confirming the computer solution.

PROBLEMS

SECTION 17.2 Independent versus Dependent (Controlled) Sources

1. Discuss, in your own words, the difference between a controlled and an independent source.

SECTION 17.3 Source Conversions

2. Convert the voltage source in Fig. 17.58 to a current source.

$E = 60\ V\ \angle 30°$
$2\ \Omega$
$5\ \Omega$
$(5\ \Omega$
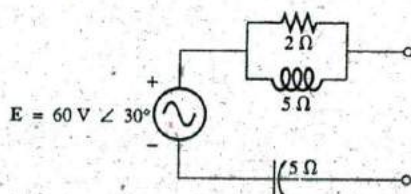
FIG. 17.58
Problem 2

3. Convert the current source in Fig. 17.59 to a voltage source.

$10\ \Omega \quad 6\ \Omega \quad I \quad 2\ A\ \angle 120°$
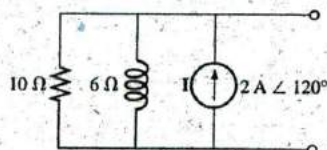
FIG. 17.59
Problem 3.

4. Convert the voltage source in Fig. 17.60(a) to a current source and the current source in Fig. 17.60(b) to a voltage source.
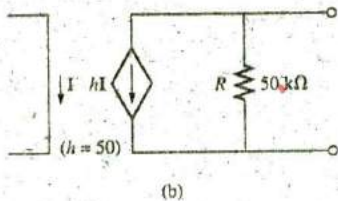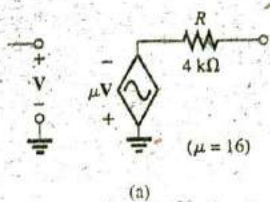
FIG. 17.60
Problem 4.

SECTION 17.4 Mesh Analysis

5. Write the mesh equations for the network of Fig. 17.61. De--
termine the current through the resistor R.
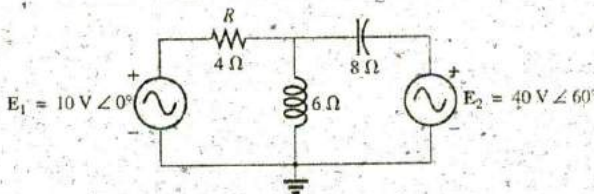
FIG. 17.61
Problems 5 and 40.

6. Write the mesh equations for the network of Fig. 17.62.
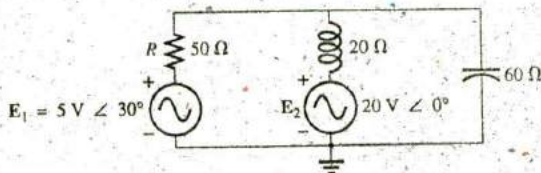
FIG. 17.62
Problem 6.

7. Write the mesh equations for the network of Fig. 17.63.
Determine the current through the resistor R_1.
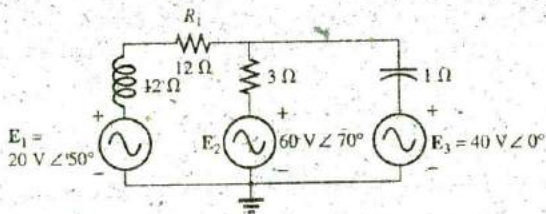
FIG. 17.63
Problems 7 and 21.

*8. Write the mesh equations for the network of Fig. 17.64.
Determine the current through the resistor R_1.
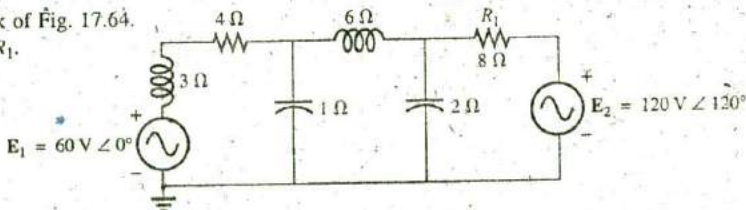
FIG. 17.64
Problem 8.

*9. Write the mesh equations for the network of Fig. 17.65. Determine the current through the resistor R_1..
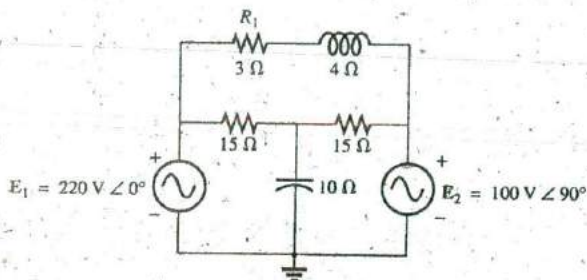
FIG. 17.65
Problems 9 and 41.

*10. Write the mesh equations for the network of Fig. 17.66. Determine the current through the resistor R_1.
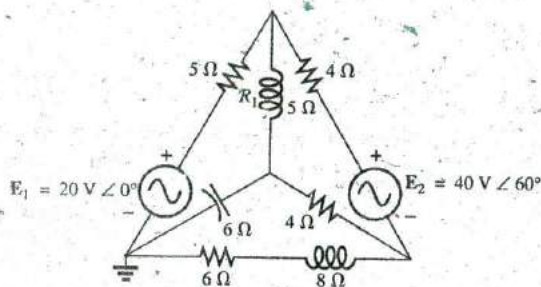
FIG. 17.66
Problem 10.

11. Write the mesh equations for the network of Fig. 17.67. Determine the current through the resistor R_1.
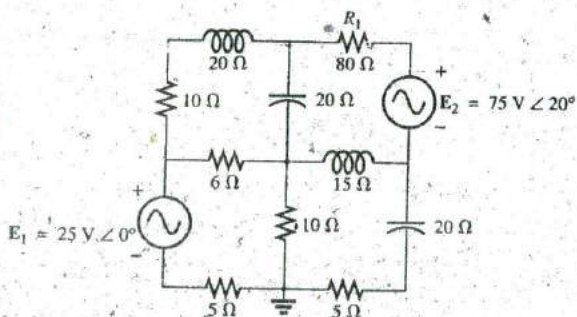
FIG. 17.67
Problems 11 and 22.

12. Using mesh analysis, determine the current I_L (in terms of V) for the network of Fig. 17.68.
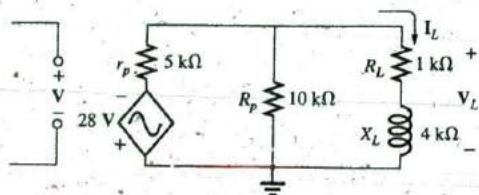
FIG. 17.68
Problem 12.

13. Using mesh analysis, determine the current I_L (in terms of I) for the network of Fig. 17.69.
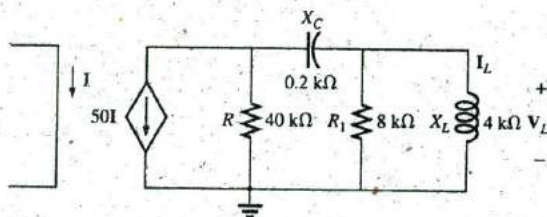
FIG. 17.69
Problem 13.

*14. Write the mesh equations for the network of Fig. 17.70, and determine the current through the 1 kΩ and 2 kΩ resistors.
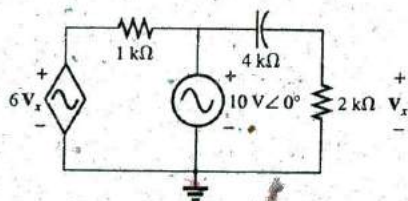
FIG. 17.70
Problems 14 and 42.

*15. Write the mesh equations for the network of Fig. 17.71, and determine the current through the 10 kΩ resistor.
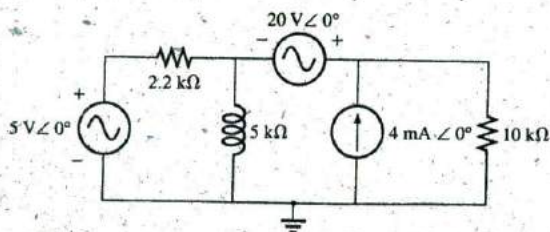
FIG. 17.71
Problems 15 and 43.

*16. Write the mesh equations for the network of Fig. 17.72, and determine the current through the inductive element.
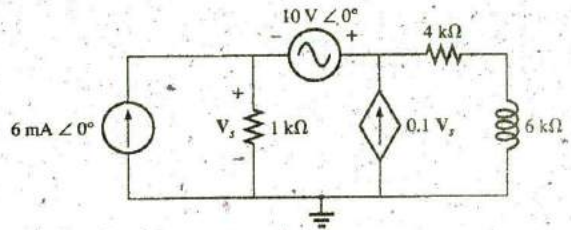
FIG. 17.72
Problems 16 and 44.

SECTION 17.5 Nodal Analysis

17. Determine the nodal voltages for the network of Fig. 17.73.
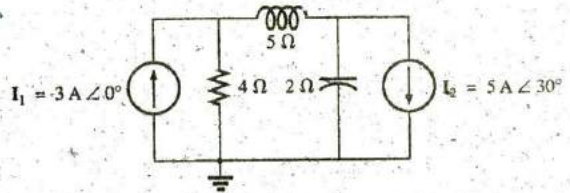
FIG. 17.73
Problem 17.

18. Determine the nodal voltages for the network of Fig. 17.74.
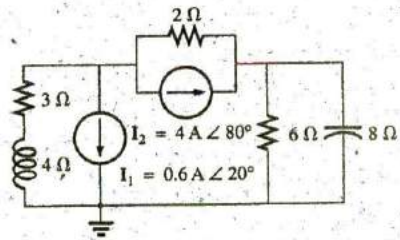
FIG. 17.74
Problems 18 and 45.

19. Determine the nodal voltages for the network of Fig. 17.75.
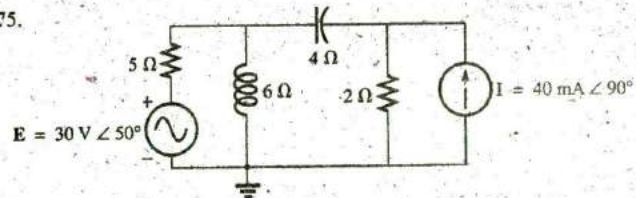
FIG. 17.75
Problem 19.

20. Determine the nodal voltages for the network of Fig. 17.76.
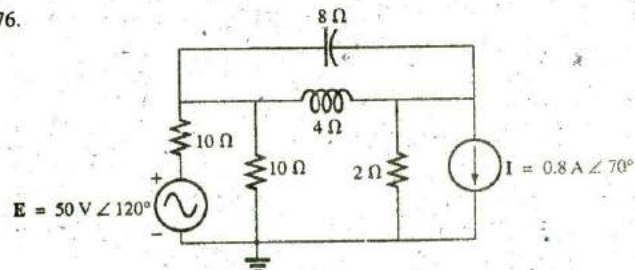
FIG. 17.76
Problem 20.

21. Determine the nodal voltages for the network of Fig. 17.65.

*22. Determine the nodal voltages for the network of Fig. 17.67.

*23. Determine the nodal voltages for the network of Fig. 17.77.
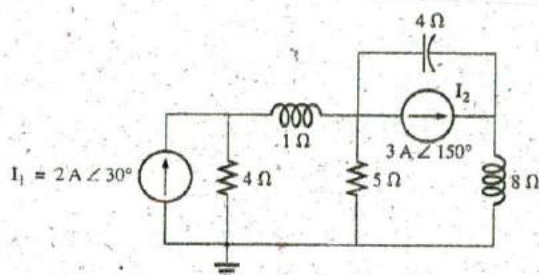
FIG. 17.77
Problem 23.

*24. Determine the nodal voltages for the network of Fig. 17.78.
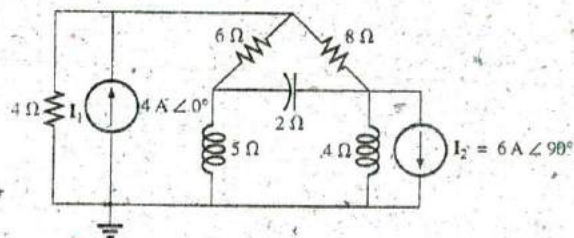
FIG. 17.78
Problem 24.

*25. Write the nodal equations for the network in Fig. 17.79, and find the voltage across the 1 kΩ resistor.
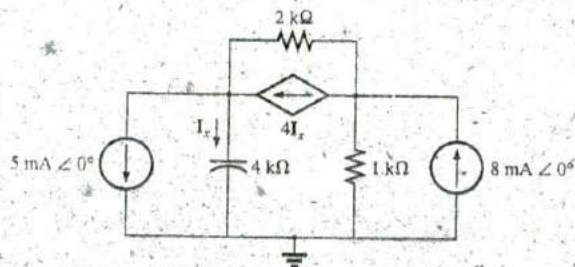
FIG. 17.79
Problems 25 and 46.

*26. Write the nodal equations for the network of Fig. 17.80, and find the voltage across the capacitive element.
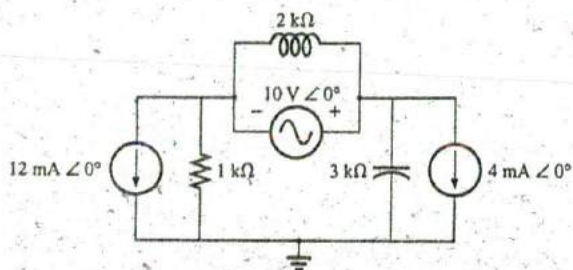
FIG. 17.80
Problems 26 and 47.

*27. Write the nodal equations for the network of Fig. 17.81, and find the voltage across the 2 kΩ resistor.
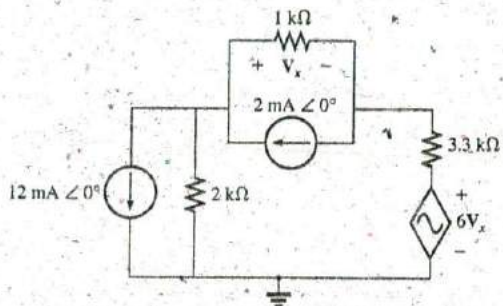
FIG. 17.81
Problems 27 and 48.

*28. Write the nodal equations for the network of Fig. 17.82, and find the voltage across the 2 kΩ resistor.
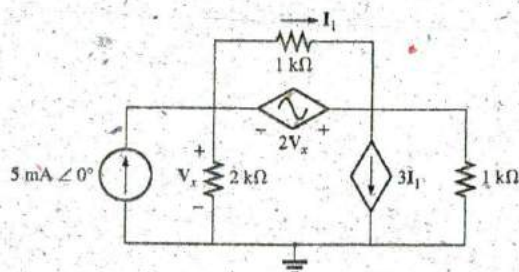
FIG. 17.82
Problems 28 and 49.

*29. For the network of Fig. 17.83, determine the voltage V_L in terms of the voltage E_i.
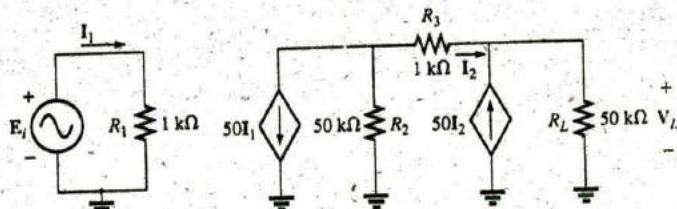
FIG. 17.83
Problem 29.

SECTION 17.6 Bridge Networks (ac)

30. For the bridge network in Fig. 17.84:
    a. Is the bridge balanced?
    b. Using mesh analysis, determine the current through the capacitive reactance.
    c. Using nodal analysis, determine the voltage across the capacitive reactance.
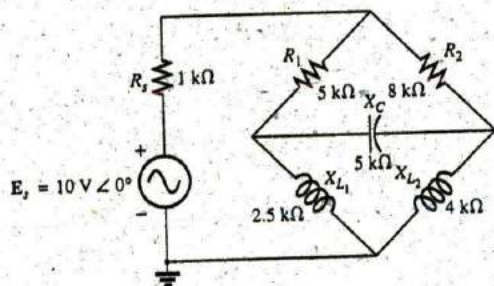
FIG. 17.84
Problem 30.

31. For the bridge network in Fig. 17.85:
    a. Is the bridge balanced?
    b. Using mesh analysis, determine the current through the capacitive reactance.
    c. Using nodal analysis, determine the voltage across the capacitive reactance.

FIG. 17.85
Problem 31.

32. The Hay bridge in Fig. 17.86 is balanced. Using Eq. (17.3), determine the unknown inductance L_x and resistance R_x

FIG. 17.86
Problem 32.
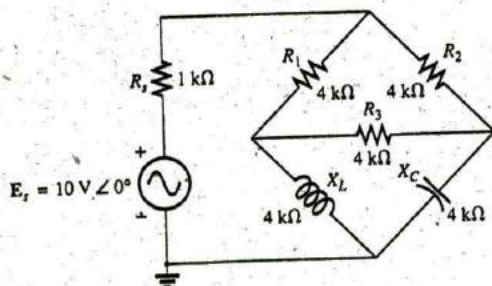
33. Determine whether the Maxwell bridge in Fig. 17.87 is balanced ($\omega = 1000$ rad/s).
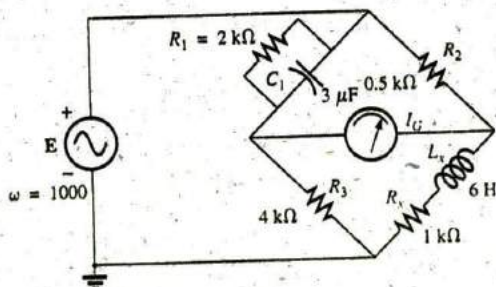
FIG. 17.87
Problem 33.

34. Derive the balance equations (17.16) and (17.17) for the capacitance comparison bridge.

35. Determine the balance equations for the inductance bridge in Fig. 17.88.
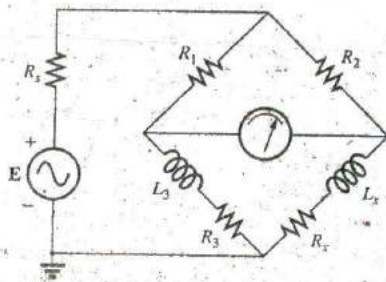
FIG. 17.88
Problem 35.
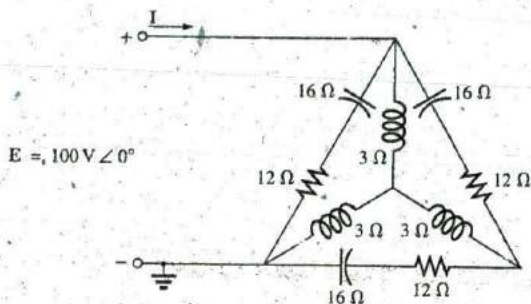
SECTION 17.7 Δ-Y, Y-Δ Conversions

36. Using the Δ-Y or Y-Δ conversion, determine the current I for the network of Fig. 17.89.
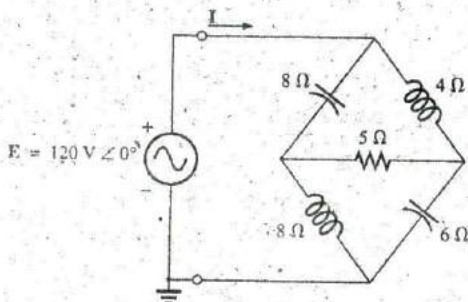
FIG. 17.89
Problem 36.
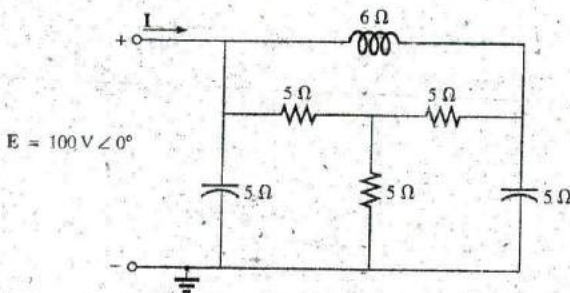
37. Using the Δ-Y or Y-Δ conversion, determine the current I for the network of Fig. 17.90.

FIG. 17.90
Problem 37.

38. Using the Λ-Y or Y-Δ conversion, determine the current I for the network of Fig. 17.91.

FIG. 17.91
Problem 38.

39. Using the Δ-Y or Y-Δ conversion, determine the current I for the network of Fig. 17.92.

FIG. 17.92
Problem 39.
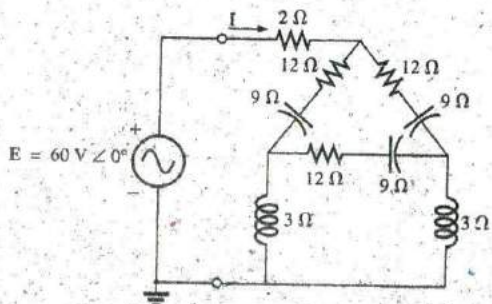
SECTION 17.8 Computer Analysis PSpice or Multisim

40. Determine the mesh currents for the network of Fig. 17.61.
41. Determine the mesh currents for the network of Fig. 17.65.
*42. Determine the mesh currents for the network of Fig. 17.70.
*43. Determine the mesh currents for the network of Fig. 17.71.
*44. Determine the mesh currents for the network of Fig. 17.72.
45. Determine the nodal voltages for the network of Fig. 17.74.
*46. Determine the nodal voltages for the network of Fig. 17.79.
*47. Determine the nodal voltages for the network of Fig. 17.80.
*48. Determine the nodal voltages for the network of Fig. 17.81.
*49. Determine the nodal voltages for the network of Fig. 17.82.

GLOSSARY

Bridge network A network configuration having the appearance of a diamond in which no two branches are in series or parallel.

Capacitance comparison bridge A bridge configuration having a galvanometer in the bridge arm that is used to determine an unknown capacitance and associated resistance.

Delta (Δ) configuration A network configuration having the appearance of the capital Greek letter *delta*.

Dependent (controlled) source A source whose magnitude and/or phase angle is determined (controlled) by a current or voltage of the system in which it appears.

Hay bridge A bridge configuration used for measuring the resistance and inductance of coils in those cases where the resistance is a small fraction of the reactance of the coil.

Independent source A source whose magnitude is independent of the network to which it is applied. It displays its terminal characteristics even if completely isolated.

Maxwell bridge A bridge configuration used for inductance measurements when the resistance of the coil is large enough not to require a Hay bridge.

Mesh analysis A method through which the loop (or mesh) currents of a network can be determined. The branch currents of the network can then be determined directly from the loop currents.

Nodal analysis A method through which the nodal voltages of a network can be determined. The voltage across each element can then be determined through application of Kirchhoff's voltage law.

Source conversion The changing of a voltage source to a current source, or vice versa, which will result in the same terminal behavior of the source. In other words, the external network is unaware of the change in sources.

Wye (Y) configuration A network configuration having the appearance of the capital letter Y.

Network Theorems (AC)

Objectives

- *Be able to apply the superposition theorem to ac networks with independent and dependent sources.*

- *Become proficient in applying Thévenin's theorem to ac networks with independent and dependent sources.*

- *Be able to apply Norton's theorem to ac networks with independent and dependent sources.*

- *Clearly understand the conditions that must be met for maximum power transfer to a load in an ac network with independent or dependent sources.*

18.1 INTRODUCTION

This chapter parallels Chapter 9, which dealt with network theorems as applied to dc networks. Reviewing each theorem in Chapter 9 before beginning this chapter is recommended because many of the comments offered there are not repeated here.

Due to the need for developing confidence in the application of the various theorems to networks with controlled (dependent) sources, some sections have been divided into two parts: independent sources and dependent sources.

Theorems to be considered in detail include the superposition theorem, Thévenin's and Norton's theorems, and the maximum power transfer theorem. The substitution and reciprocity theorems and Millman's theorem are not discussed in detail here because a review of Chapter 9 will enable you to apply them to sinusoidal ac networks with little difficulty.

18.2 SUPERPOSITION THEOREM

You will recall from Chapter 9 that the **superposition theorem** eliminated the need for solving simultaneous linear equations by considering the effects of each source independently. To consider the effects of each source, we had to remove the remaining sources. This was accomplished by setting voltage sources to zero (short-circuit representation) and current sources to zero (open-circuit representation). The current through, or voltage across, a portion of the network produced by each source was then added algebraically to find the total solution for the current or voltage.

The only variation in applying this method to ac networks with independent sources is that we are now working with impedances and phasors instead of just resistors and real numbers.

The superposition theorem is not applicable to power effects in ac networks since we are still dealing with a nonlinear relationship. It can be applied to networks with sources of different frequencies only if the total response for *each* frequency is found independently and the results are expanded in a nonsinusoidal expression, as appearing in Chapter 25.

One of the most frequent applications of the superposition theorem is to electronic systems in which the dc and ac analyses are treated separately and the total solution is the sum of the two. It is an important application of the theorem because the impact of the reactive elements
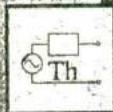
changes dramatically in response to the two types of independent sources. In addition, the dc analysis of an electronic system can often define important parameters for the ac analysis. Example 18.4 demonstrates the impact of the applied source on the general configuration of the network.

We first consider networks with only independent sources to provide a close association with the analysis of Chapter 9.

Independent Sources

EXAMPLE 18.1 Using the superposition theorem, find the current I through the 4 Ω reactance (X_{L_2}) in Fig. 18.1.

FIG. 18.1
Example 18.1.

Solution: For the redrawn circuit (Fig. 18.2),

$$Z_1 = +jX_{L_1} = j\,4\Omega$$
$$Z_2 = +jX_{L_2} = j\,4\Omega$$
$$Z_3 = -jX_C = -j\,3\Omega$$

Considering the effects of the voltage source E_1 (Fig. 18.3), we have

$$Z_{2\|3} = \frac{Z_2 Z_3}{Z_2 + Z_3} = \frac{(j\,4\,\Omega)(-j\,3\,\Omega)}{j\,4\,\Omega - j\,3\,\Omega} = \frac{12\,\Omega}{j} = -j\,12\,\Omega$$
$$= 12\,\Omega\,\angle{-90°}$$

$$I_{s_1} = \frac{E_1}{Z_{2\|3} + Z_1} = \frac{10\,V\,\angle0°}{-j\,12\,\Omega + j\,4\,\Omega} = \frac{10\,V\,\angle0°}{8\,\Omega\,\angle{-90°}}$$
$$= 1.25\,A\,\angle90°$$
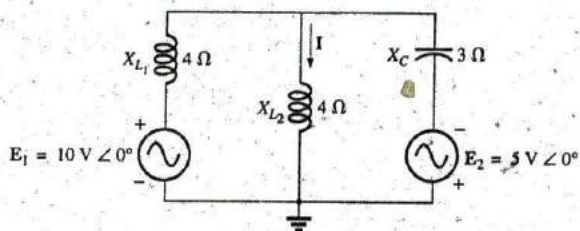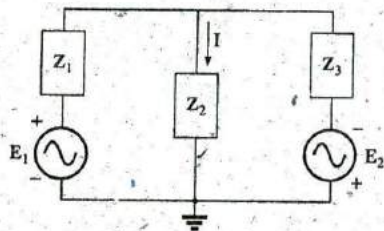
FIG. 18.2
Assigning the subscripted impedances to the network in Fig. 18.1.
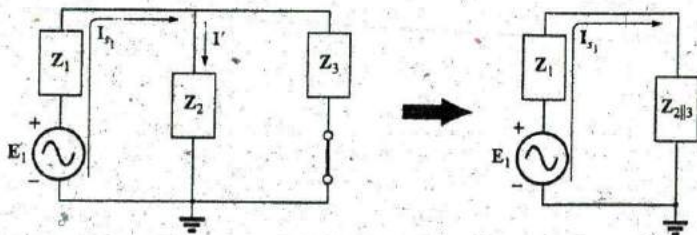
FIG. 18.3
Determining the effect of the voltage source E_1 on the current I of the network in Fig. 18.1.

and

$$I' = \frac{Z_3 I_{s_1}}{Z_2 + Z_3} \quad \text{(current divider rule)}$$

$$= \frac{(-j3\,\Omega)(j1.25\text{ A})}{j4\,\Omega - j3\,\Omega} = \frac{3.75\text{ A}}{j1} = 3.75\text{ A} \angle -90°$$

Considering the effects of the voltage source E_2 (Fig. 18.4), we have
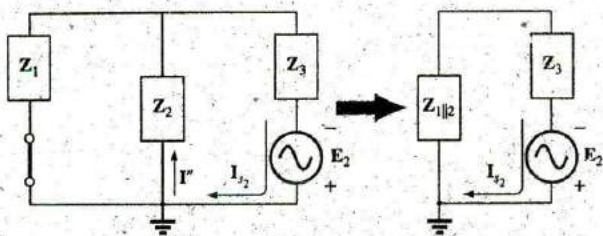
FIG. 18.4
*Determining the effect of the voltage source E_2 on the current I
of the network in Fig. 18.1.*

$$Z_{1\|2} = \frac{Z_1}{N} = \frac{j4\,\Omega}{2} = j2\,\Omega$$

$$I_{s_2} = \frac{E_2}{Z_{1\|2} + Z_3} = \frac{5\text{ V} \angle 0°}{j2\,\Omega - j3\,\Omega} = \frac{5\text{ V} \angle 0°}{1\,\Omega \angle -90°} = 5\text{ A} \angle 90°$$

and

$$I'' = \frac{I_{s_2}}{2} = 2.5\text{ A} \angle 90°$$

The resultant current through the 4 Ω reactance X_{L_2} (Fig. 18.5) is

$$I = I' - I''$$
$$= 3.75\text{ A} \angle -90° - 2.50\text{ A} \angle 90° = -j3.75\text{ A} - j2.50\text{ A}$$
$$= -j6.25\text{ A}$$
$$I = 6.25\text{ A} \angle -90°$$
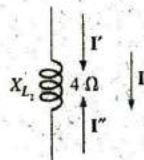
FIG. 18.5
*Determining the resultant current for the network in
Fig. 18.1.*

EXAMPLE 18.2 Using superposition, find the current I through the 6 Ω resistor in Fig. 18.6.
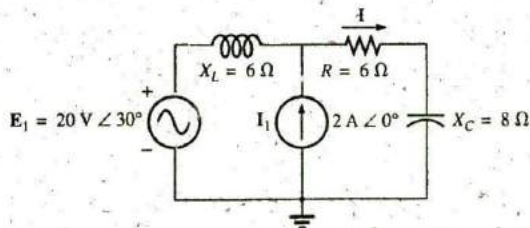
FIG. 18.6
Example 18.2.
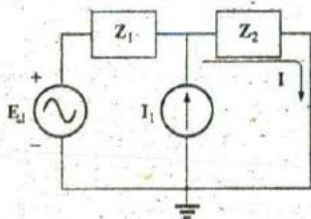
FIG. 18.7

Assigning the subscripted impedances to the network in Fig. 18.6.
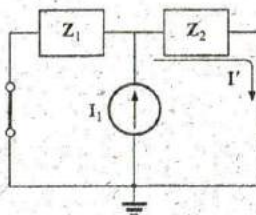
FIG. 18.8

Determining the effect of the current source I_1 on the current I of the network in Fig. 18.6.
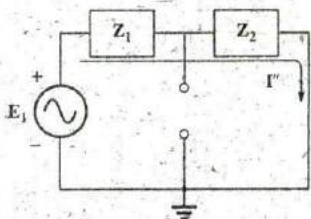
FIG. 18.9

Determining the effect of the voltage source E_1 on the current I of the network in Fig. 18.6.
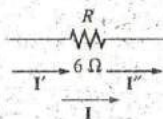
FIG. 18.10

Determining the resultant current I for the network in Fig. 18.6.

Solution: For the redrawn circuit (Fig. 18.7),

$$Z_1 = j6\,\Omega \qquad Z_2 = 6\,\Omega - j8\,\Omega$$

Consider the effects of the current source (Fig. 18.8). Applying the current divider rule, we have

$$I' = \frac{Z_1 I_1}{Z_1 + Z_2} = \frac{(j6\,\Omega)(2\,A)}{j6\,\Omega + 6\,\Omega - j8\,\Omega} = \frac{j12\,A}{6 - j2}$$

$$= \frac{12\,A\,\angle 90°}{6.32\,\angle -18.43°}$$

$$I' = 1.9\,A\,\angle 108.43°$$

Consider the effects of the voltage source (Fig. 18.9). Applying Ohm's law gives us

$$I'' = \frac{E_1}{Z_T} = \frac{E_1}{Z_1 + Z_2} = \frac{20\,V\,\angle 30°}{6.32\,\Omega\,\angle -18.43°}$$

$$= 3.16\,A\,\angle 48.43°$$

The total current through the $6\,\Omega$ resistor (Fig. 18.10) is

$$I = I' + I''$$

$$= 1.9\,A\,\angle 108.43° + 3.16\,A\,\angle 48.43°$$

$$= (-0.60\,A + j1.80\,A) + (2.10\,A + j2.36\,A)$$

$$= 1.50\,A + j4.16A$$

$$I = 4.42\,A\,\angle 70.2°$$

EXAMPLE 18.3 Using superposition, find the voltage across the $6\,\Omega$ resistor in Fig. 18.6. Check the results against $V_{6\Omega} = I(6\,\Omega)$, where I is the current found through the $6\,\Omega$ resistor in Example 18.2.

Solution: For the current source,

$$V'_{6\Omega} = I'(6\,\Omega) = (1.9\,A\,\angle 108.43°)(6\,\Omega) = 11.4\,V\,\angle 108.43°$$

For the voltage source,

$$V''_{6\Omega} = I''(6) = (3.16\,A\,\angle 48.43°)(6\,\Omega) = 18.96\,V\,\angle 48.43°$$

The total voltage across the $6\,\Omega$ resistor (Fig. 18.11) is

$$V_{6\Omega} = V'_{6\Omega} + V''_{6\Omega}$$

$$= 11.4\,V\,\angle 108.43° + 18.96\,V\,\angle 48.43°$$

$$= (-3.60\,V + j10.82\,V) + (12.58\,V + j14.18\,V)$$

$$= 8.98\,V + j25.0\,V$$

$$V_{6\Omega} = 26.5\,V\,\angle 70.2°$$

FIG. 18.11

Determining the resultant voltage $V_{6\Omega}$ for the network in Fig. 18.6.

Checking the result, we have

$$V_{6\Omega} = I(6\,\Omega) = (4.42\,A\,\angle70.2°)(6\,\Omega)$$
$$= 26.5\,V\,\angle70.2°\quad\text{(checks)}$$

EXAMPLE 18.4 For the network in Fig. 18.12, determine the sinusoidal expression for the voltage v_3 using superposition.
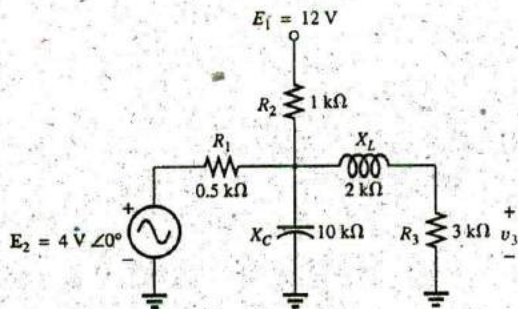
FIG. 18.12
Example 18.4.

Solution: For the dc analysis, the capacitor can be replaced by an open-circuit equivalent and the inductor by a short-circuit equivalent. The result is the network in Fig. 18.13.

The resistors R_1 and R_3 are then in parallel, and the voltage V_3 can be determined using the voltage divider rule:

$$R' = R_1 \parallel R_3 = 0.5\,k\Omega \parallel 3\,k\Omega = 0.429\,k\Omega$$

and

$$V_3 = \frac{R'E_1}{R' + R_2}$$
$$= \frac{(0.429\,k\Omega)(12\,V)}{0.429\,k\Omega + 1\,k\Omega} = \frac{5.148\,V}{1.429}$$
$$V_3 \cong 3.6\,V$$

For the ac analysis, the dc source is set to zero and the network is redrawn, as shown in Fig. 18.14.
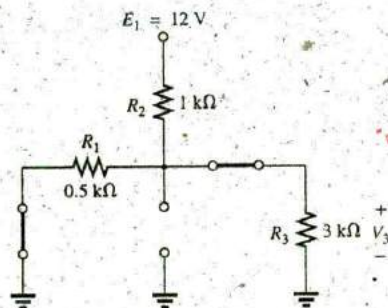
FIG. 18.13
Determining the effect of the dc voltage source E_1 on the voltage v_3 of the network in Fig. 18.12.
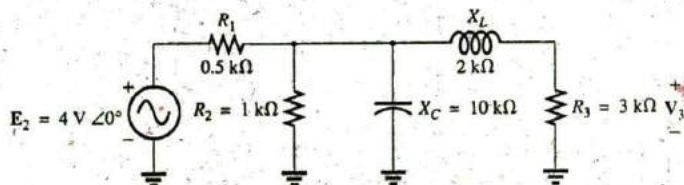
FIG. 18.14
Redrawing the network in Fig. 18.12 to determine the effect of the ac voltage source E_2.
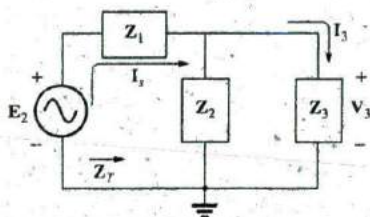
FIG. 18.15
Assigning the subscripted impedances to the network in Fig. 18.14.

The block impedances are then defined as in Fig. 18.15, and series-parallel techniques are applied as follows:

$$Z_1 = 0.5 \text{ k}\Omega \angle 0°$$
$$Z_2 = (R_2 \angle 0° \| (X_C \angle -90°)$$
$$= \frac{(1 \text{ k}\Omega \angle 0°)(10 \text{ k}\Omega \angle -90°)}{1 \text{ k}\Omega - j\,10 \text{ k}\Omega} = \frac{10 \text{ k}\Omega \angle -90°}{10.05 \angle -84.29°}$$
$$= 0.995 \text{ k}\Omega \angle -5.71°$$
$$Z_3 = R_3 + jX_L = 3 \text{ k}\Omega + j\,2 \text{ k}\Omega = 3.61 \text{ k}\Omega \angle 33.69°$$

and

$$Z_T = Z_1 + Z_2 \| Z_3$$
$$= 0.5 \text{ k}\Omega + (0.995 \text{ k}\Omega \angle -5.71°) \| (3.61 \text{ k}\Omega \angle 33.69°)$$
$$= 1.312 \text{ k}\Omega \angle 1.57°$$

Calculator Solution: Performing the above on the TI-89 calculator requires the sequence of steps in Fig. 18.16.
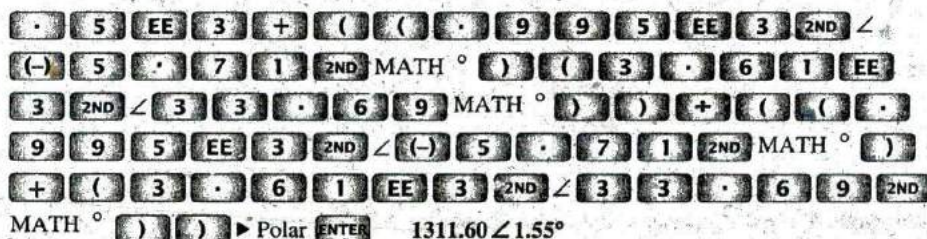
FIG. 18.16
Determining the total impedance for the network of Fig. 18.12.

$$I_s = \frac{E_2}{Z_T} = \frac{4 \text{ V} \angle 0°}{1.312 \text{ k}\Omega \angle 1.57°} = 3.05 \text{ mA} \angle -1.57°$$

The current divider rule gives

$$I_3 = \frac{Z_2 I_s}{Z_2 + Z_3} = \frac{(0.995 \text{ k}\Omega \angle -5.71°)(3.05 \text{ mA} \angle -1.57°)}{0.995 \text{ k}\Omega \angle -5.71° + 3.61 \text{ k}\Omega \angle 33.69°}$$
$$= 0.686 \text{ mA} \angle -32.74°$$

with

$$V_3 = (I_3 \angle \theta)(R_3 \angle 0°)$$
$$= (0.686 \text{ mA} \angle -32.74°)(3 \text{ k}\Omega \angle 0°)$$
$$= 2.06 \text{ V} \angle -32.74°$$

The total solution is

$$v_3 = v_3(\text{dc}) + v_3(\text{ac})$$
$$= 3.6 \text{ V} + 2.06 \text{ V} \angle -32.74°$$
$$v_3 = 3.6 + 2.91 \sin(\omega t - 32.74°)$$

The result is a sinusoidal voltage having a peak value of 2.91 V riding on an average value of 3.6 V, as shown in Fig. 18.17.
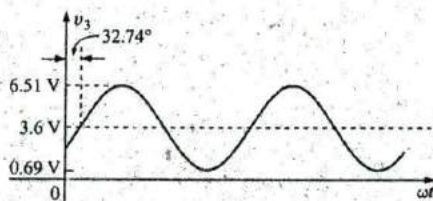
FIG. 18.17
The resultant voltage v_3 for the network in Fig. 18.12.

Dependent Sources

For dependent sources in which *the controlling variable is not determined by the network to which the superposition theorem is to be applied,* the application of the theorem is basically the same as for independent sources. The solution obtained will simply be in terms of the controlling variables.

EXAMPLE 18.5 Using the superposition theorem, determine the current I_2 for the network in Fig. 18.18. The quantities μ and h are constants.
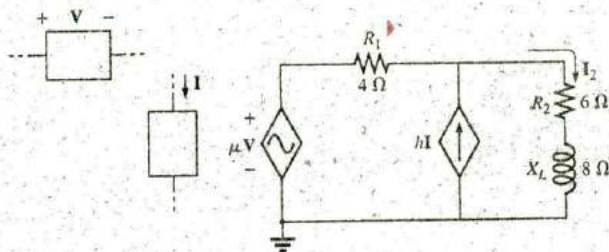
FIG. 18.18
Example 18.5.

Solution: With a portion of the system redrawn (Fig. 18.19),

$$Z_1 = R_1 = 4\,\Omega \qquad Z_2 = R_2 + jX_L = 6\,\Omega + j\,8\,\Omega$$

For the voltage source (Fig. 18.20),

$$I' = \frac{\mu V}{Z_1 + Z_2} = \frac{\mu V}{4\,\Omega + 6\,\Omega + j\,8\,\Omega} = \frac{\mu V}{10\,\Omega + j\,8\,\Omega}$$

$$= \frac{\mu V}{12.8\,\Omega\,\angle 38.66^\circ} = 0.078\,\mu V/\Omega\,\angle -38.66^\circ$$

For the current source (Fig. 18.21),

$$I'' = \frac{Z_1(hI)}{Z_1 + Z_2} = \frac{(4\,\Omega)(hI)}{12.8\,\Omega\,\angle 38.66^\circ} = 4(0.078)hI\,\angle -38.66^\circ$$
$$= 0.312hI\,\angle -38.66^\circ$$

The current I_2 is

$$I_2 = I' + I''$$
$$= 0.078\,\mu\,V/\Omega\,\angle -38.66^\circ + 0.312hI\,\angle -38.66^\circ$$

For $V = 10\,V\,\angle 0^\circ$, $I = 20\,mA\,\angle 0^\circ$, $\mu = 20$, and $h = 100$,

$$I_2 = 0.078(20)(10\,V\,\angle 0^\circ)/\Omega\,\angle -38.66^\circ$$
$$+0.312(100)(20\,mA\,\angle 0^\circ)\angle -38.66^\circ$$
$$= 15.60\,A\,\angle -38.66^\circ + 0.62\,A\,\angle -38.66^\circ$$
$$I_2 = 16.22\,A\,\angle -38.66^\circ$$

For dependent sources in which *the controlling variable is determined by the network to which the theorem is to be applied,* the dependent source cannot be set to zero unless the controlling variable is also zero. For networks containing dependent sources (as in Example 18.5)
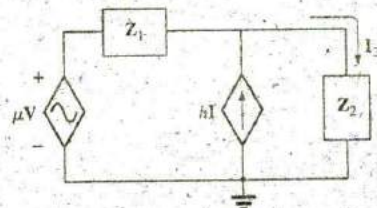
FIG. 18.19
Assigning the subscripted impedances to the network in Fig. 18.18.
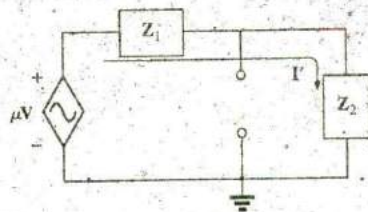
FIG. 18.20
Determining the effect of the voltage-controlled voltage source on the current I_2 for the network in Fig. 18.18.
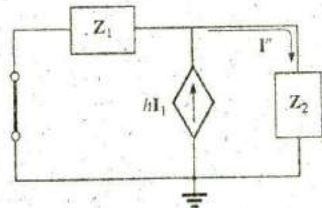
FIG. 18.21
Determining the effect of the current-controlled current source on the current I_2 for the network in Fig. 18.18.
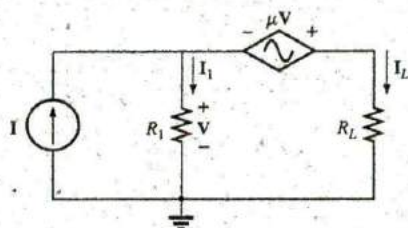
and dependent sources of the type just introduced above, the superposition theorem is applied for each independent source and each dependent source not having a controlling variable in the portions of the network under investigation. It must be reemphasized that dependent sources are not sources of energy in the sense that, if all independent sources are removed from a system, all currents and voltages must be zero.

EXAMPLE 18.6 Determine the current I_L through the resistor R_L in Fig. 18.22.

Solution: Note that the controlling variable V is determined by the network to be analyzed. From the above discussions, it is understood that the dependent source cannot be set to zero unless V is zero. If we set I to zero, the network lacks a source of voltage, and $V = 0$ with $\mu V = 0$. The resulting I_L under this condition is zero. Obviously, therefore, the network must be analyzed as it appears in Fig. 18.22, with the result that neither source can be eliminated, as is normally done using the superposition theorem.

Applying Kirchhoff's voltage law, we have

$$V_L = V + \mu V = (1 + \mu)V$$

and

$$I_L = \frac{V_L}{R_L} = \frac{(1 + \mu)V}{R_L}$$

The result, however, must be found in terms of I since V and μV are only dependent variables.

Applying Kirchhoff's current law gives us

$$I = I_1 + I_L = \frac{V}{R_1} + \frac{(1 + \mu)V}{R_L}$$

and

$$I = V\left(\frac{1}{R_1} + \frac{1 + \mu}{R_L}\right)$$

or

$$V = \frac{I}{(1/R_1) + [(1 + \mu)/R_L]}$$

Substituting into the above yields

$$I_L = \frac{(1 + \mu)V}{R_L} = \frac{(1 + \mu)}{R_L}\left(\frac{I}{(1/R_1) + [(1 + \mu)/R_L]}\right)$$

Therefore,

$$I_L = \frac{(1 + \mu)R_1 I}{R_L + (1 + \mu)R_1}$$

FIG. 18.22
Example 18.6.

18.3 THÉVENIN'S THEOREM

Thévenin's theorem, as stated for sinusoidal ac circuits, is changed only to include the term *impedance* instead of *resistance;* that is,

any two-terminal linear ac network can be replaced with an equivalent circuit consisting of a voltage source and an impedance in series, as shown in Fig. 18.23.

Since the reactances of a circuit are frequency dependent, the Thévenin circuit found for a particular network is applicable only at *one* frequency.

FIG. 18.23
Thévenin equivalent circuit for ac networks.

Introductory C 510

The steps required to apply this method to dc circuits are repeated here with changes for sinusoidal ac circuits. As before, the only change is the replacement of the term *resistance* with *impedance*. Again, dependent and independent sources are treated separately.

Example 18.9, the last example of the independent source section, includes a network with dc and ac sources to establish the groundwork for possible use in the electronics area.

Independent Sources

1. *Remove that portion of the network across which the Thévenin equivalent circuit is to be found.*
2. *Mark (o, •, and so on) the terminals of the remaining two-terminal network.*
3. *Calculate Z_{Th} by first setting all voltage and current sources to zero (short circuit and open circuit, respectively) and then finding the resulting impedance between the two marked terminals.*
4. *Calculate E_{Th} by first replacing the voltage and current sources and then finding the open-circuit voltage between the marked terminals.*
5. *Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the Thévenin equivalent circuit.*

EXAMPLE 18.7 Find the Thévenin equivalent circuit for the network external to resistor R in Fig. 18.24.
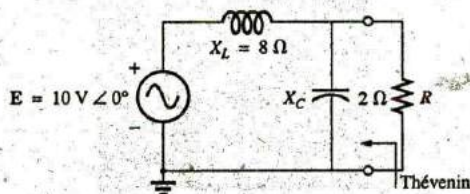
FIG. 18.24
Example 18.7.

Solution:

Steps 1 and 2 (Fig. 18.25):

$$Z_1 = jX_L = j8\,\Omega \qquad Z_2 = -jX_C = -j2\,\Omega$$
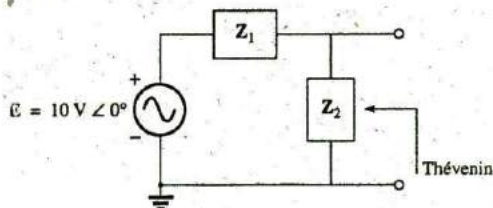
FIG. 18.25
Assigning the subscripted impedances to the network in Fig. 18.24.
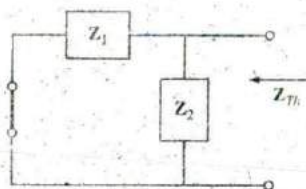
FIG. 18.26
Determining the Thévenin impedance
for the network in Fig. 18.24.
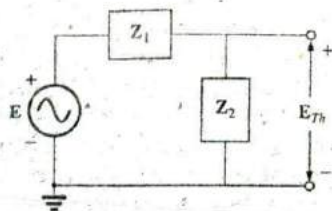
FIG. 18.27
Determining the open-circuit Thévenin
voltage for the network in Fig. 18.24.

Step 3 (Fig. 18.26):

$$\mathbf{Z}_{Th} = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(j\,8\,\Omega)(-j\,2\,\Omega)}{j\,8\,\Omega - j\,2\,\Omega} = \frac{-j^2\,16\,\Omega}{j6} = \frac{16\,\Omega}{6\,\angle 90°}$$
$$= 2.67\,\Omega\,\angle -90°$$

Step 4 (Fig. 18.27):

$$\mathbf{E}_{Th} = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_2} \qquad \text{(voltage divider rule)}$$
$$= \frac{(-j\,2\,\Omega)(10\,\text{V})}{j\,8\,\Omega - j\,2\,\Omega} = \frac{-j\,20\,\text{V}}{j6} = 3.33\,\text{V}\,\angle -180°$$

Step 5: The Thévenin equivalent circuit is shown in Fig. 18.28.
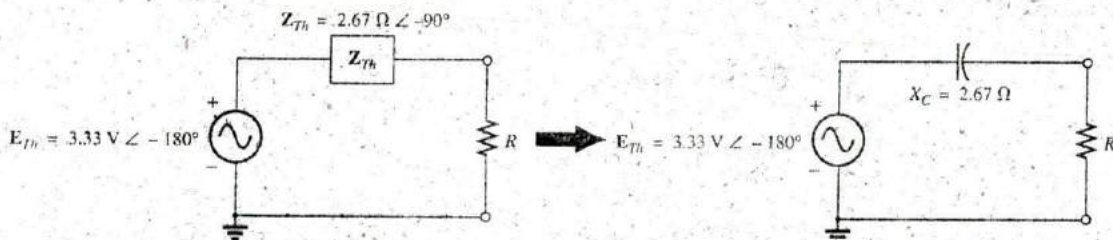
FIG. 18.28
The Thévenin equivalent circuit for the network in Fig. 18.24.

EXAMPLE 18.8 Find the Thévenin equivalent circuit for the network external to branch a-a' in Fig. 18.29.
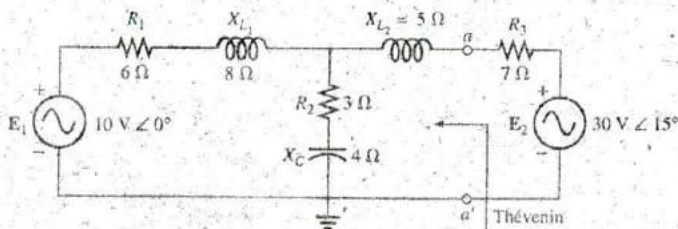
FIG. 18.29
Example 18.8.

Solution:

Steps 1 and 2 (Fig. 18.30): Note the reduced complexity with subscripted impedances:

$$Z_1 = R_1 + jX_{L_1} = 6\,\Omega + j\,8\,\Omega$$

$$Z_2 = R_2 - jX_C = 3\,\Omega - j\,4\,\Omega$$

$$Z_3 = +jX_{L_2} = j\,5\,\Omega$$
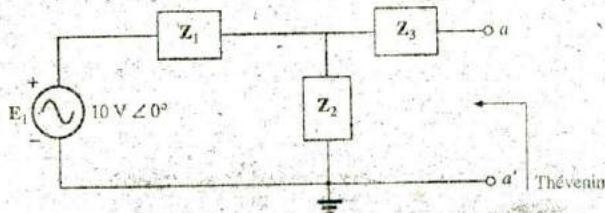
FIG. 18.30

Assigning the subscripted impedances for the network in Fig. 18.29.

Step 3 (Fig. 18.31):

$$Z_{Th} = Z_3 + \frac{Z_1 Z_2}{Z_1 + Z_2} = j\,5\,\Omega + \frac{(10\,\Omega\,\angle 53.13°)(5\,\Omega\,\angle -53.13°)}{(6\,\Omega + j\,8\,\Omega) + (3\,\Omega - j\,4\,\Omega)}$$

$$= j\,5 + \frac{50\,\angle 0°}{9 + j\,4} = j\,5 + \frac{50\,\angle 0°}{9.85\,\angle 23.96°}$$

$$= j\,5 + 5.08\,\angle -23.96° = j\,5 + 4.64 - j\,2.06$$

$$Z_{Th} = 4.64\,\Omega + j\,2.94\,\Omega = 5.49\,\Omega\,\angle 32.36°$$
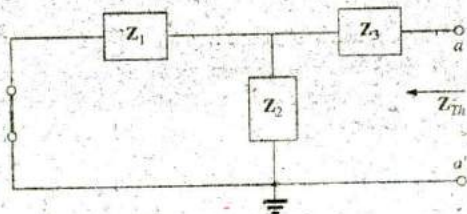
FIG. 18.31

Determining the Thévenin impedance for the network in Fig. 18.29.

Step 4 (Fig. 18.32): Since a-a' is an open circuit, $I_{Z_3} = 0$. Then E_{Th} is the voltage drop across Z_2:

$$E_{Th} = \frac{Z_2 E}{Z_2 + Z_1} \qquad \text{(voltage divider rule)}$$

$$= \frac{(5\,\Omega\,\angle -53.13°)(10\,V\,\angle 0°)}{9.85\,\Omega\,\angle 23.96°}$$

$$E_{Th} = \frac{50\,V\,\angle -53.13°}{9.85\,\angle 23.96°} = 5.08\,V\,\angle -77.09°$$
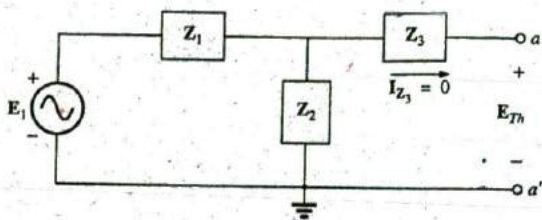
FIG. 18.32

Determining the open-circuit Thévenin voltage for the network in Fig. 18.29.

Step 5: The Thévenin equivalent circuit is shown in Fig. 18.33.
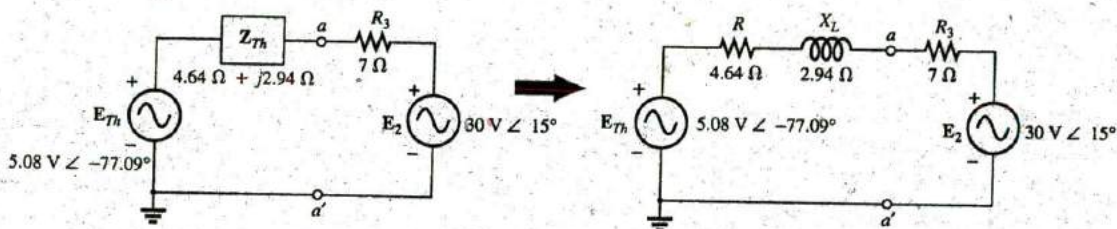
FIG. 18.33

The Thévenin equivalent circuit for the network in Fig. 18.29.

The next example demonstrates how superposition is applied to electronic circuits to permit *a separation of the dc and ac analyses.* The fact that the controlling variable in this analysis is not in the portion of the network connected directly to the terminals of interest permits an analysis of the network in the same manner as applied above for independent sources.

EXAMPLE 18.9 Determine the Thévenin equivalent circuit for the transistor network external to the resistor R_L in the network in Fig. 18.34. Then determine V_L.
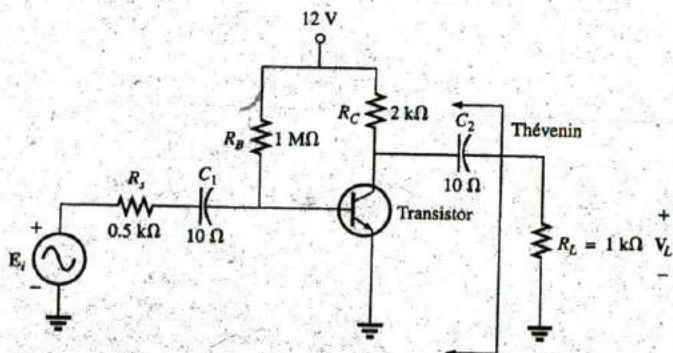
FIG. 18.34

Example 18.9.

Solution: Applying superposition.

dc Conditions Substituting the open-circuit equivalent for the coupling capacitor C_2 will isolate the dc source and the resulting currents from the load resistor. The result is that for dc conditions, $V_L = 0$ V. Although the output dc voltage is zero, the application of the dc voltage is important to the basic operation of the transistor in a number of important ways, one of which is to determine the parameters of the "equivalent circuit" to appear in the ac analysis to follow.

ac Conditions For the ac analysis, an equivalent circuit is substituted for the transistor, as established by the dc conditions above, that will behave like the actual transistor. A great deal more will be said about equivalent circuits and the operations performed to obtain the network in Fig. 18.35, but for now we limit our attention to the manner in which the Thévenin equivalent circuit is obtained. Note in Fig. 18.35 that the equivalent circuit includes a resistor of 2.3 kΩ and a controlled current source whose magnitude is determined by the product of a factor of 100 and the current I_1 in another part of the network.
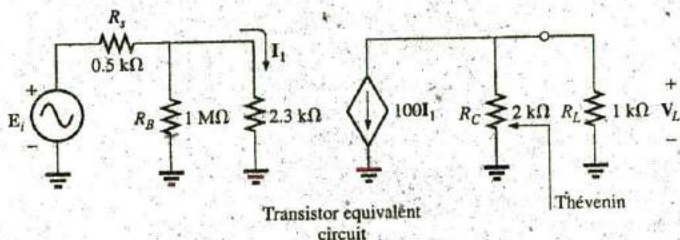
FIG. 18.35
The ac equivalent network for the transistor amplifier in Fig. 18.34.

Note in Fig. 18.35 the absence of the coupling capacitors for the ac analysis. In general, coupling capacitors are designed to be open circuits for dc analysis and short circuits for ac analysis. The short-circuit equivalent is valid because the other impedances in series with the coupling capacitors are so much larger in magnitude that the effect of the coupling capacitors can be ignored. Both R_B and R_C are now tied to ground because the dc source was set to zero volts (superposition) and replaced by a short-circuit equivalent to ground.

For the analysis to follow, the effect of the resistor R_B will be ignored since it is so much larger than the parallel 2.3 kΩ resistor.

Z_{Th} When E_i is set to zero volts, the current I_1 will be zero amperes, and the controlled source $100 I_1$ will be zero amperes also. The result is an open-circuit equivalent for the source, as appearing in Fig. 18.36.

It is fairly obvious from Fig. 18.36 that

$$Z_{Th} = 2 \text{ k}\Omega$$

E_{Th} For E_{Th}, the current I_1 in Fig. 18.35 will be

$$I_1 = \frac{E_i}{R_s + 2.3 \text{ k}\Omega} = \frac{E_i}{0.5 \text{ k}\Omega + 2.3 \text{ k}\Omega} = \frac{E_i}{2.8 \text{ k}\Omega}$$

and

$$100 I_1 = (100)\left(\frac{E_i}{2.8 \text{ k}\Omega}\right) = \frac{E_i}{28 \text{ }\Omega}$$
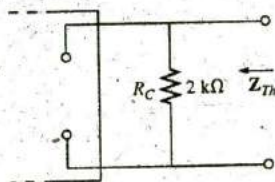
FIG. 18.36
Determining the Thévenin impedance for the network in Fig. 18.35.
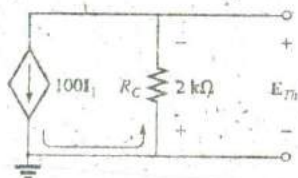
FIG. 18.37

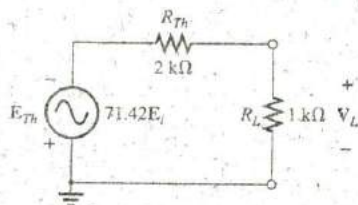*Determining the Thévenin voltage for the network in Fig. 18.35.*

FIG. 18.38

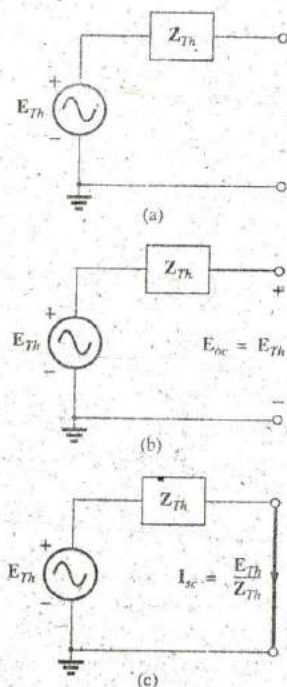*The Thévenin equivalent circuit for the network in Fig. 18.35.*

(a)

(b)

(c)

FIG. 18.39

Defining an alternative approach for determining the Thévenin impedance.

Referring to Fig. 18.37, we find that

$$E_{Th} = -(100I_1)R_C$$
$$= -\left(\frac{E_i}{28\ \Omega}\right)(2 \times 10^3\ \Omega)$$
$$E_{Th} = -71.42E_i$$

The Thévenin equivalent circuit appears in Fig. 18.38 with the original load R_L.

Output Voltage V_L

$$V_L = \frac{-R_L E_{Th}}{R_L + R_{Th}} = \frac{-(1\ k\Omega)(71.42E_i)}{1\ k\Omega + 2\ k\Omega}$$

and

$$V_L = -23.81\ E_i$$

revealing that the output voltage is 23.81 times the applied voltage with a phase shift of 180° due to the minus sign.

Dependent Sources

For dependent sources with a *controlling variable not in the network under investigation*, the procedure indicated above can be applied. However, for dependent sources of the other type, where the *controlling variable is part of the network to which the theorem is to be applied*, another approach must be used. The necessity for a different approach is demonstrated in an example to follow. The method is *not limited to dependent sources* of the latter type. It can also be applied to any dc or sinusoidal ac network. However, for networks of independent sources, the method of application used in Chapter 9 and presented in the first portion of this section is generally more direct, with the usual savings in time and errors.

The new approach to Thévenin's theorem can best be introduced at this stage in the development by considering the Thévenin equivalent circuit in Fig. 18.39(a). As indicated in Fig. 18.39(b), the open-circuit terminal voltage (E_{oc}) of the Thévenin equivalent circuit is the Thévenin equivalent voltage; that is,

$$\boxed{E_{oc} = E_{Th}} \tag{18.1}$$

If the external terminals are short circuited as in Fig. 18.39(c), the resulting short-circuit current is determined by

$$\boxed{I_{sc} = \frac{E_{Th}}{Z_{Th}}} \tag{18.2}$$

or, rearranged,

$$Z_{Th} = \frac{E_{Th}}{I_{sc}}$$

and

$$\boxed{Z_{Th} = \frac{E_{oc}}{I_{sc}}} \tag{18.3}$$

Eqs. (18.1) and (18.3) indicate that for any linear bilateral dc or ac network with or without dependent sources of any type, if the open-circuit terminal voltage of a portion of a network can be determined along with the short-circuit current between the same two terminals, the Thévenin

equivalent circuit is effectively known. A few examples will make the method quite clear. The advantage of the method, which was stressed earlier in this section for independent sources, should now be more obvious. The current I_{sc}, which is necessary to find Z_{Th}, is in general more difficult to obtain since all of the sources are present.

There is a third approach to the Thévenin equivalent circuit that is also useful from a practical viewpoint. The Thévenin voltage is found as in the two previous methods. However, the Thévenin impedance is obtained by applying a source of voltage to the terminals of interest and determining the source current as indicated in Fig. 18.40. For this method, the source voltage of the original network is set to zero. The Thévenin impedance is then determined by the following equation:

$$Z_{Th} = \frac{E_g}{I_g} \qquad (18.4)$$

Note that for each technique, $E_{Th} = E_{oc}$, but the Thévenin impedance is found in different ways.

FIG. 18.40

Determining Z_{Th} using the approach $Z_{Th} = E_g/I_g$.

EXAMPLE 18.10 Using each of the three techniques described in this section, determine the Thévenin equivalent circuit for the network in Fig. 18.41.

Solution: Since for each approach the Thévenin voltage is found in exactly the same manner, it is determined first. From Fig. 18.41, where $I_{X_C} = 0$,

Due to the polarity for V and
defined terminal polarities

$$V_{R_1} = E_{Th} = E_{oc} = \frac{R_2(\mu V)}{R_1 + R_2} = -\frac{\mu R_2 V}{R_1 + R_2}$$

The following three methods for determining the Thévenin impedance appear in the order in which they were introduced in this section.

Method 1: See Fig. 18.42.

$$Z_{Th} = R_1 \| R_2 - jX_C$$

Method 2: See Fig. 18.43. Converting the voltage source to a current source (Fig. 18.44), we have (current divider rule)

$$I_{sc} = \frac{-(R_1 \| R_2)\dfrac{\mu V}{R_1}}{(R_1 \| R_2) - jX_C} = \frac{\dfrac{R_1 R_2}{R_1 + R_2}\left(\dfrac{\mu V}{R_1}\right)}{(R_1 \| R_2) - jX_C}$$

$$= \frac{\dfrac{-\mu R_2 V}{R_1 + R_2}}{(R_1 \| R_2) - jX_C}$$

and

$$Z_{Th} = \frac{E_{oc}}{I_{sc}} = \frac{\dfrac{-\mu R_2 V}{R_1 + R_2}}{\dfrac{-\mu R_2 V}{R_1 + R_2}} = \frac{1}{\dfrac{1}{(R_1 \| R_2) - jX_C}}$$

$$= R_1 \| R_2 - jX_C$$

FIG. 18.41

Example 18.10.

FIG. 18.42

Determining the Thévenin impedance for the network in Fig. 18.41.
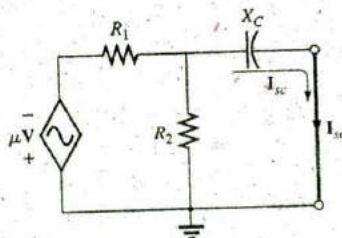
FIG. 18.43

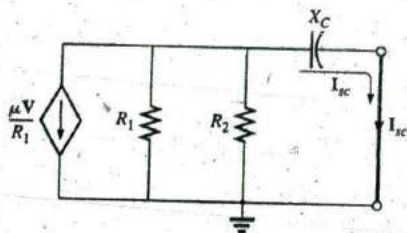*Determining the short-circuit current for the network in Fig. 18.41.*

FIG. 18.44

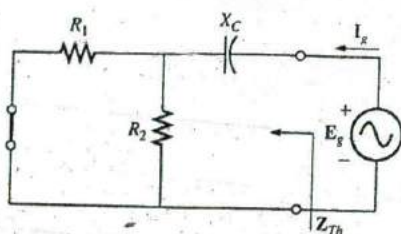*Converting the voltage source in Fig. 18.43 to a current source.*

FIG. 18.45

Determining the Thévenin impedance for the network in Fig. 18.41 using the approach $Z_{Th} = E_g/I_g$.

Method 3: See Fig. 18.45.

$$I_g = \frac{E_g}{(R_1 \| R_2) - jX_C}$$

and

$$Z_{Th} = \frac{E_g}{I_g} = R_1 \| R_2 - jX_C$$

In each case, the Thévenin impedance is the same. The resulting Thévenin equivalent circuit is shown in Fig. 18.46.
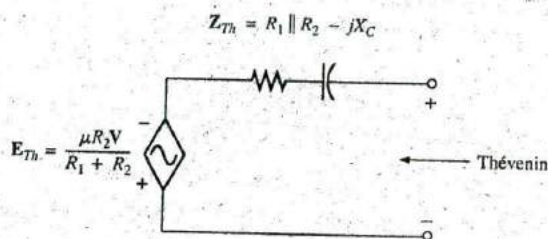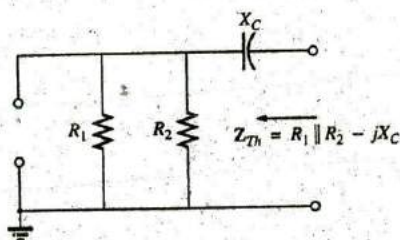

FIG. 18.46

The Thévenin equivalent circuit for the network in Fig. 18.41.

EXAMPLE 18.11 Repeat Example 18.10 for the network in Fig. 18.47.
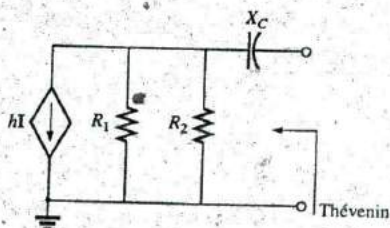
FIG. 18.47

Example 18.11.

Solution: From Fig. 18.47, E_{Th} is

$$E_{Th} = E_{oc} = -hI(R_1 \| R_2) = -\frac{hR_1R_2I}{R_1 + R_2}$$

Method 1: See Fig. 18.48.

$$Z_{Th} = R_1 \| R_2 - jX_C$$

Note the similarity between this solution and that obtained for the previous example.

FIG. 18.48
Determining the Thévenin impedance for the
network in Fig. 18.47.
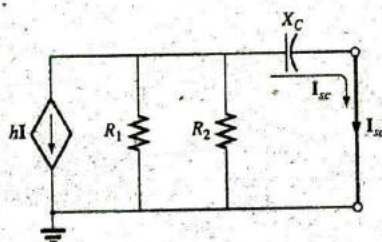
FIG. 18.49
Determining the short-circuit current for the network
in Fig. 18.47.

Method 2: See Fig. 18.49.

$$I_{sc} = \frac{-(R_1 \| R_2)hI}{(R_1 \| R_2) - jX_C}$$

and

$$Z_{Th} = \frac{E_{oc}}{I_{sc}} = \frac{-hI(R_1 \| R_2)}{\dfrac{-(R_1 \| R_2)hI}{(R_1 \| R_2) - jX_C}} = R_1 \| R_2 - jX_C$$

Method 3: See Fig. 18.50.

$$I_g = \frac{E_g}{(R_1 \| R_2) - jX_C}$$

and

$$Z_{Th} = \frac{E_g}{I_g} = R_1 \| R_2 - jX_C$$
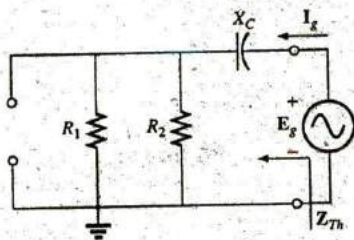
FIG. 18.50
Determining the Thévenin impedance using the
approach $Z_{Th} = E_g/I_g$.

The following example has a dependent source that will not permit the use of the method described at the beginning of this section for independent sources. All three methods will be applied, however, so that the results can be compared.

EXAMPLE 18.12 For the network in Fig. 18.51 (introduced in Example 18.6), determine the Thévenin equivalent circuit between the indicated terminals using each method described in this section. Compare your results.

Solution: First, using Kirchhoff's voltage law, we write E_{Th} (which is the same for each method)

$$E_{th} = V + \mu V = (1 + \mu)V$$
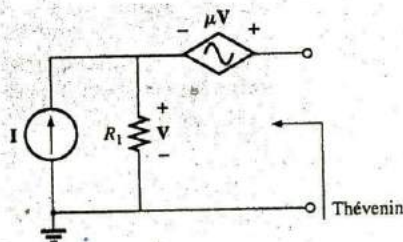
FIG. 18.51
Example 18.12.
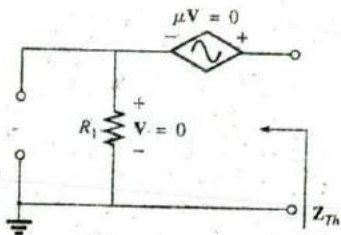
FIG. 18.52
Determining Z_{Th} incorrectly.
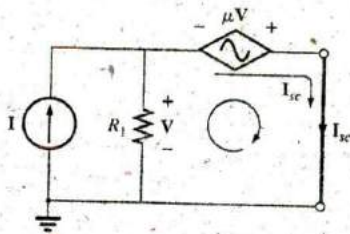
FIG. 18.53
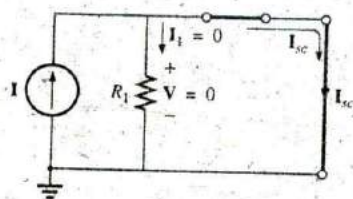*Determining I_{sc} for the network in Fig. 18.51.*

FIG. 18.54
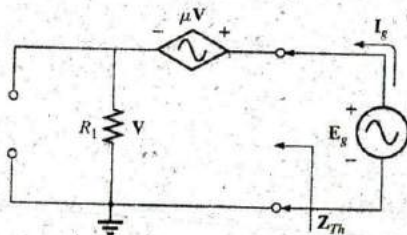*Substituting $V = 0$ into the network in Fig. 18.53.*

However,
$$V = IR_1$$
so
$$E_{Th} = (1 + \mu)IR_1$$

Z_{Th}

Method 1: See Fig. 18.52. Since $I = 0$, V and $\mu V = 0$, and

$$Z_{Th} = R_1 \quad \text{(incorrect)}$$

Method 2: See Fig. 18.53. Kirchhoff's voltage law around the indicated loop gives us

$$V = \mu V = 0$$
and
$$V(1 + \mu) = 0$$

Since μ is a positive constant, the above equation can be satisfied only when $V = 0$. Substitution of this result into Fig. 18.53 yields the configuration in Fig. 18.54, and

$$I_{sc} = I$$

with

$$Z_{Th} = \frac{E_{oc}}{I_{sc}} = \frac{(1 + \mu)IR_1}{I} = (1 + \mu)R_1 \quad \text{(correct)}$$

Method 3: See Fig. 18.55.

$$E_g = V + \mu V = (1 + \mu)V$$

or

$$V = \frac{E_g}{1 + \mu}$$

and

$$I_g = \frac{V}{R_1} = \frac{E_g}{(1 + \mu)R_1}$$

and

$$Z_{Th} = \frac{E_g}{I_g} = (1 + \mu)R_1 \quad \text{(correct)}$$

FIG. 18.55
Determining Z_{Th} using the approach $Z_{Th} = E_g/I_g$.
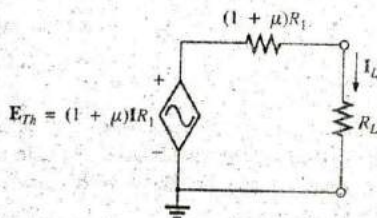
FIG. 18.56
The Thévenin equivalent circuit for the network in Fig. 18.51.

The Thévenin equivalent circuit appears in Fig. 18.56, and

$$I_L = \frac{(1 + \mu)R_1 I}{R_L + (1 + \mu)R_1}$$

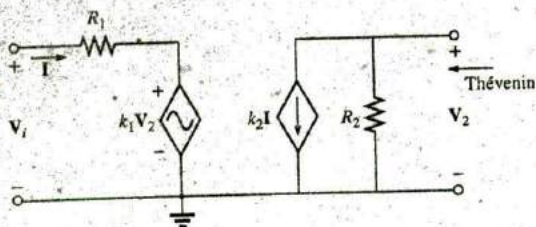which compares with the result in Example 18.6.

FIG. 18.57
Example 18.13: Transistor equivalent network.

The network in Fig. 18.57 is the basic configuration of the transistor equivalent circuit applied most frequently today (although most texts in electronics use the circle rather than the diamond outline for the source). Obviously, it is necessary to know its characteristics and to be adept in its use. Note that there are both a controlled voltage and a controlled current source, each controlled by variables in the configuration.

EXAMPLE 18.13 Determine the Thévenin equivalent circuit for the indicated terminals of the network in Fig. 18.57.

Solution: Apply the second method introduced in this section.

E$_{Th}$

$$E_{oc} = V_2$$

$$I = \frac{V_i - k_1 V_2}{R_1} = \frac{V_i - k_1 E_{oc}}{R_1}$$

and

$$E_{oc} = -k_2 I R_2 = -k_2 R_2 \left(\frac{V_i - k_1 E_{oc}}{R_1} \right)$$

$$= \frac{-k_2 R_2 V_i}{R_1} + \frac{k_1 k_2 R_2 E_{oc}}{R_1}$$

or

$$E_{oc} \left(1 - \frac{k_1 k_2 R_2}{R_1} \right) = \frac{-k_2 R_2 V_i}{R_1}$$

and

$$E_{oc} \left(\frac{R_1 - k_1 k_2 R_2}{R_1} \right) = \frac{-k_2 R_2 V_i}{R_1}$$

so

$$\boxed{E_{oc} = \frac{-k_2 R_2 V_i}{R_1 - k_1 k_2 R_2} = E_{Th}}$$ (18.5)

I$_{sc}$ For the network in Fig. 18.58, where

$$V_2 = 0 \qquad k_1 V_2 = 0 \qquad I = \frac{V_i}{R_1}$$

and

$$I_{sc} = -k_2 I = \frac{-k_2 V_i}{R_1}$$

so

$$Z_{Th} = \frac{E_{oc}}{I_{sc}} = \frac{\dfrac{-k_2 R_2 V_i}{R_1 - k_1 k_2 R_2}}{\dfrac{-k_2 V_i}{R_1}}$$
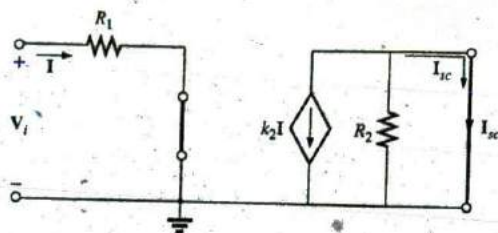
FIG. 18.58

Determining I_{sc} for the network in Fig. 18.57.

and

$$Z_{Th} = \frac{R_1 R_2}{R_1 - k_1 k_2 R_2} \qquad (18.6)$$

Frequently, the approximation $k_1 \cong 0$ is applied. Then the Thévenin voltage and impedance are, respectively,

$$E_{Th} = \frac{-k_2 R_2 V_1}{R_1} \qquad k_1 = 0 \qquad (18.7)$$

$$Z_{Th} = R_2 \qquad k_1 = 0 \qquad (18.8)$$

Apply $Z_{Th} = E_g/I_g$ to the network in Fig. 18.59, where
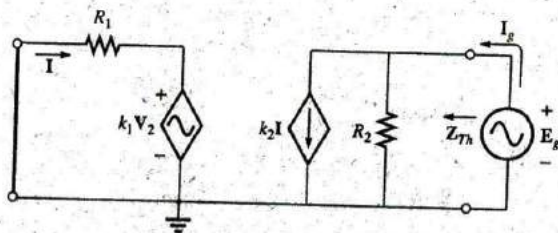
$$I = \frac{-k_1 V_2}{R_1}$$

FIG. 18.59

Determining Z_{Th} using the procedure $Z_{Th} = E_g/I_g$.

But

$$V_2 = E_g$$

so

$$I = \frac{-k_1 E_g}{R_1}$$

Applying Kirchhoff's current law, we have

$$I_g = k_2 I + \frac{E_g}{R_2} = k_2 \left(-\frac{k_1 E_g}{R_1} \right) + \frac{E_g}{R_2}$$

$$= E_g \left(\frac{1}{R_2} - \frac{k_1 k_2}{R_1} \right)$$

and

$$\frac{I_g}{E_g} = \frac{R_1 - k_1 k_2 R_2}{R_1 R_2}$$

or

$$Z_{Th} = \frac{E_g}{I_g} = \frac{R_1 R_2}{R_1 - k_1 k_2 R_2}$$

as obtained above.

The last two methods presented in this section were applied only to networks in which the magnitudes of the controlled sources were dependent on a variable within the network for which the Thévenin equivalent circuit was to be obtained. Understand that both of these methods can also be applied to any dc or sinusoidal ac network containing only independent sources or dependent sources of the other kind.

18.4 NORTON'S THEOREM

The three methods described for Thévenin's theorem will each be altered to permit their use with **Norton's theorem.** Since the Thévenin and Norton impedances are the same for a particular network, certain portions of the discussion are quite similar to those encountered in the previous section. We first consider independent sources and the approach developed in Chapter 9, followed by dependent sources and the new techniques developed for Thévenin's theorem.

You will recall from Chapter 9 that Norton's theorem allows us to replace any two-terminal linear bilateral ac network with an equivalent circuit consisting of a current source and an impedance, as in Fig. 18.60.

The Norton equivalent circuit, like the Thévenin equivalent circuit, is applicable at only one frequency since the reactances are frequency dependent.
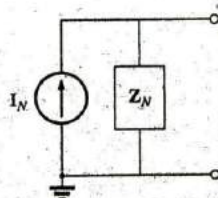
FIG. 18.60

The Norton equivalent circuit for ac networks.

Independent Sources

The procedure outlined below to find the Norton equivalent of a sinusoidal ac network is changed (from that in Chapter 9) in only one respect: the replacement of the term *resistance* with the term *impedance:*

1. *Remove that portion of the network across which the Norton equivalent circuit is to be found.*
2. *Mark (o, •, and so on) the terminals of the remaining two-terminal network.*
3. *Calculate Z_N by first setting all voltage and current sources to zero (short circuit and open circuit, respectively) and then finding the resulting impedance between the two marked terminals.*
4. *Calculate I_N by first replacing the voltage and current sources and then finding the short-circuit current between the marked terminals.*
5. *Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the Norton equivalent circuit.*

The Norton and Thévenin equivalent circuits can be found from each other by using the source transformation shown in Fig. 18.61. The source transformation is applicable for any Thévenin or Norton equivalent circuit determined from a network with any combination of independent or dependent sources.
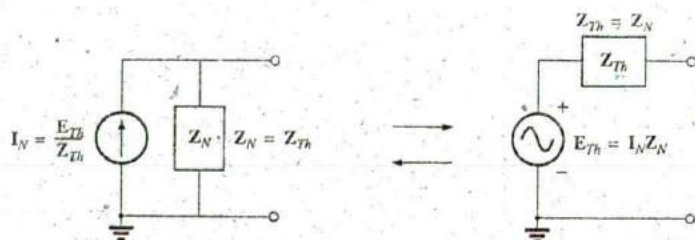
FIG. 18.61

Conversion between the Thévenin and Norton equivalent circuits.

EXAMPLE 18.14 Determine the Norton equivalent circuit for the network external to the 6 Ω resistor in Fig. 18.62.
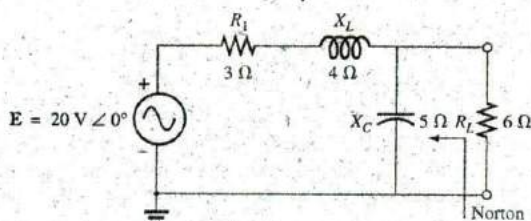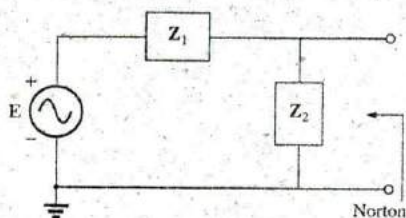

FIG. 18.62

Example 18.14.

Solution:

Steps 1 and 2 (Fig. 18.63):

$$Z_1 = R_1 + jX_L = 3\,\Omega + j\,4\,\Omega = 5\,\Omega\ \angle53.13°$$
$$Z_2 = -jX_C = -j\,5\,\Omega$$

Step 3 (Fig. 18.64):

$$Z_N = \frac{Z_1Z_2}{Z_1 + Z_2} = \frac{(5\,\Omega\ \angle53.13°)(5\,\Omega\ \angle-90°)}{3\,\Omega + j\,4\,\Omega - j\,5\,\Omega} = \frac{25\,\Omega\ \angle-36.87°}{3 - j\,1}$$

$$= \frac{25\,\Omega\ \angle-36.87°}{3.16\ \angle-18.43°} = 7.91\,\Omega\ \angle-18.44° = 7.50\,\Omega - j\,2.50\,\Omega$$

Step 4 (Fig. 18.65):

$$I_N = I_1 = \frac{E}{Z_1} = \frac{20\,\text{V}\ \angle0°}{5\,\Omega\ \angle53.13°} = 4\,\text{A}\ \angle-53.13°$$

FIG. 18.63

Assigning the subscripted impedances to the network in Fig. 18.62.
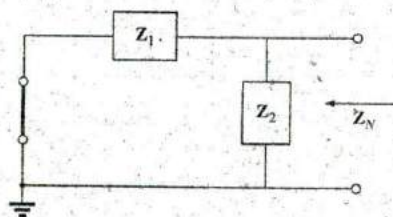
FIG. 18.64

Determining the Norton impedance for the network in Fig. 18.62.
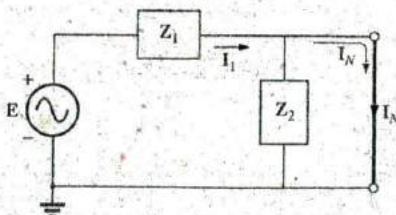
FIG. 18.65

Determining I_N for the network in Fig. 18.62.

Step 5: The Norton equivalent circuit is shown in Fig. 18.66.
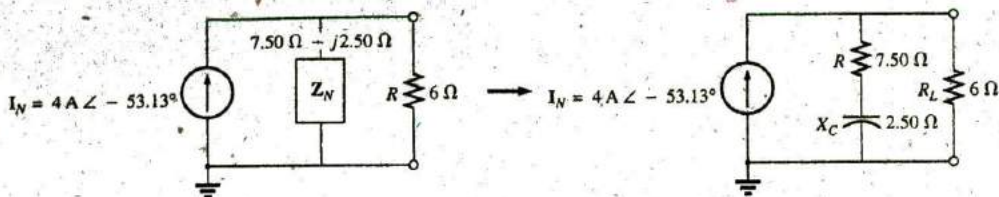
FIG. 18.66
The Norton equivalent circuit for the network in Fig. 18.62.

EXAMPLE 18.15 Find the Norton equivalent circuit for the network
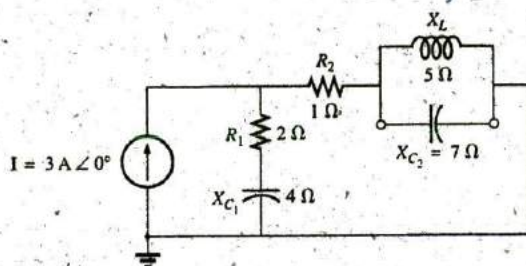external to the 7 Ω capacitive reactance in Fig. 18.67.

FIG. 18.67
Example 18.15.

Solution:

Steps 1 and 2 (Fig. 18.68):

$$Z_1 = R_1 - jX_{C_1} = 2\,\Omega - j4\,\Omega$$
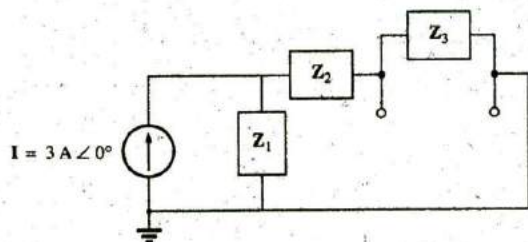$$Z_2 = R_2 = 1\,\Omega$$
$$Z_3 = +jX_L = j5\,\Omega$$

FIG. 18.68
Assigning the subscripted impedances to the network in Fig. 18.67.

Step 3 (Fig. 18.69):

$$Z_N = \frac{Z_3(Z_1 + Z_2)}{Z_3 + (Z_1 + Z_2)}$$

$$Z_1 + Z_2 = 2\,\Omega - j4\,\Omega + 1\,\Omega = 3\,\Omega - j4\,\Omega = 5\,\Omega\angle{-53.13°}$$

Introductory, C.- 52A
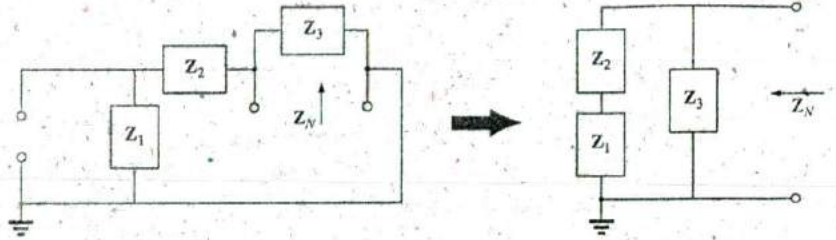
FIG. 18.69
Finding the Norton impedance for the network in Fig. 18.67.

$$Z_N = \frac{(5\ \Omega\ \angle 90°)(5\ \Omega\ \angle -53.13°)}{j\,5\ \Omega + 3\ \Omega - j\,4\ \Omega} = \frac{25\ \Omega\ \angle 36.87°}{3 + j\,1}$$

$$= \frac{25\ \Omega\ \angle 36.87°}{3.16\ \angle +18.43°}$$

$$Z_N = 7.91\ \Omega\ \angle 18.44° = 7.50\ \Omega + j\,2.50\ \Omega$$

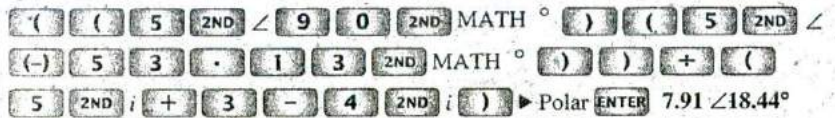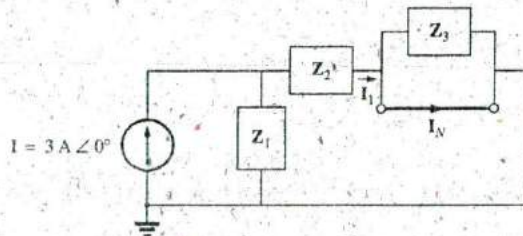**Calculator Solution:** Performing the above on the TI-89 calculator results in the sequence in Fig. 18.70:

FIG. 18.70
Determining Z_N for the network of Fig. 18.67.

Step 4 (Fig. 18.71):

$$I_N = I_1 = \frac{Z_1 I}{Z_1 + Z_2} \quad \text{(current divider rule)}$$

$$= \frac{(2\ \Omega - j\,4\ \Omega)(3\ \text{A})}{3\ \Omega - j\,4\ \Omega} = \frac{6\ \text{A} - j\,12\ \text{A}}{5\ \angle -53.13°} = \frac{13.4\ \text{A}\ \angle -63.43°}{5\ \angle -53.13°}$$

$$I_N = 2.68\ \text{A}\ \angle -10.3°$$

FIG. 18.71
Determining I_N for the network in Fig. 18.67.

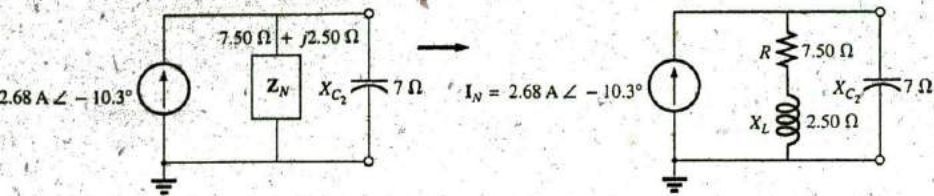*Step 5:* The Norton equivalent circuit is shown in Fig. 18.72.

FIG. 18.72

The Norton equivalent circuit for the network in Fig. 18.67.

\MPLE 18.16 Find the Thévenin equivalent circuit for the network
'nal to the 7 Ω capacitive reactance in Fig. 18.67.

tion: Using the conversion between sources (Fig. 18.73), we obtain

$$\mathbf{Z}_{Th} = \mathbf{Z}_N = 7.50 \ \Omega + j \, 2.50 \ \Omega$$
$$\mathbf{E}_{Th} = \mathbf{I}_N \mathbf{Z}_N = (2.68 \ \text{A} \ \angle -10.3°)(7.91 \ \Omega \ \angle 18.44°)$$
$$= 21.2 \ \mathbf{V} \angle 8.14°$$

Thévenin equivalent circuit is shown in Fig. 18.74.
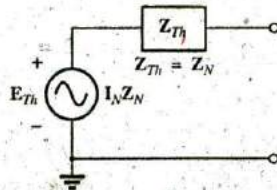
FIG. 18.73

Determining the Thévenin equivalent circuit for the Norton equivalent in Fig. 18.72.
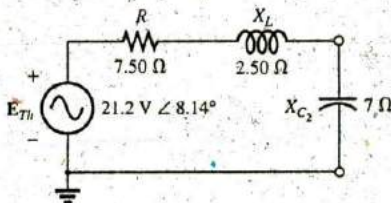
pendent Sources

tated for Thévenin's theorem, *dependent sources in which the con-
ing variable is not determined by the network* for which the Norton
valent circuit is to be found do not alter the procedure outlined
e.

or dependent sources of the other kind, one of the following proce-
s must be applied. Both of these procedures can also be applied to
orks with any combination of independent sources and dependent
ces not controlled by the network under investigation.
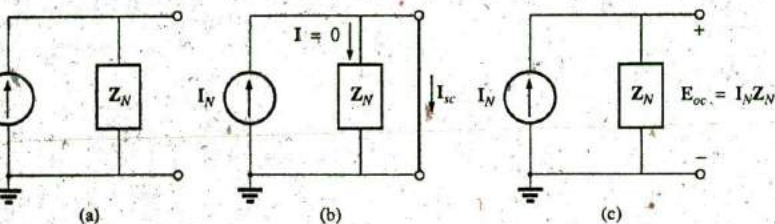he Norton equivalent circuit appears in Fig. 18.75(a). In Fig. 18.75(b),
nd that

FIG. 18.74

The Thévenin equivalent circuit for the network in Fig. 18.67.

(a)   (b)   (c)

FIG. 18.75

Defining an alternative approach for determining $\mathbf{Z}_N$.

$$\boxed{\mathbf{I}_{sc} = \mathbf{I}_N} \qquad\qquad (18.9)$$

n Fig. 18.75(c) that

$$\mathbf{E}_{oc} = \mathbf{I}_N \mathbf{Z}_N$$

ranging, we have

$$\mathbf{Z}_N = \frac{\mathbf{E}_{oc}}{\mathbf{I}_N}$$
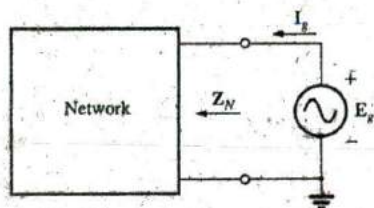
FIG. 18.76
Determining the Norton impedance using the approach $Z_N = E_g/I_g$.

and
$$Z_N = \frac{E_{oc}}{I_{sc}} \qquad (18.$$

The Norton impedance can also be determined by applying a sou of voltage E_g to the terminals of interest and finding the resulting I_g shown in Fig. 18.76. All independent sources and dependent sources controlled by a variable in the network of interest are set to zero, and

$$Z_N = \frac{E_g}{I_g} \qquad (18.$$

For this latter approach, the Norton current is still determined by short-circuit current.

EXAMPLE 18.17 Using each method described for dependent sourc find the Norton equivalent circuit for the network in Fig. 18.77.

Solution:

I_N For each method, I_N is determined in the same manner. Fr Fig. 18.78 using Kirchhoff's current law, we have

$$0 = I + hI + I_{sc}$$

or
$$I_{sc} = -(1 + h)I$$

FIG. 18.77
Example 18.17.

Applying Kirchhoff's voltage law gives us

$$E + IR_1 - I_{sc}R_2 = 0$$

and
$$IR_1 = I_{sc}R_2 - E$$

or
$$I = \frac{I_{sc}R_2 - E}{R_1}$$

so
$$I_{sc} = -(1 + h)I = -(1 + h)\left(\frac{I_{sc}R_2 - E}{R_1}\right)$$

or
$$R_1 I_{sc} = -(1 + h)I_{sc}R_2 + (1 + h)E$$
$$I_{sc}[R_1 + (1 + h)R_2] = (1 + h)E$$
$$I_{sc} = \frac{(1 + h)E}{R_1 + (1 + h)R_2} = I_N$$

FIG. 18.78
Determining I_{sc} for the network in Fig. 18.77.

Z_N

Method 1: E_{oc} is determined from the network in Fig. 18.79. By Kir hoff's current law,

$$0 = I + hI \qquad \text{or} \qquad 1(h + 1) = 0$$

For h, a positive constant I must equal zero to satisfy the abo Therefore,

$$I = 0 \qquad \text{and} \qquad hI = 0$$

and
$$E_{oc} = E$$
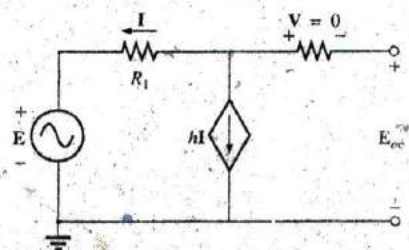
FIG. 18.79
Determining E_{oc} for the network in Fig. 18.77.

with
$$Z_N = \frac{E_{oc}}{I_{sc}} = \frac{E}{\dfrac{(1 + h)E}{R_1 + (1 + h)R_2}} = \frac{R_1 + (1 + h)R_2}{(1 + h)}$$

Method 2: Note Fig. 18.80. By Kirchhoff's current law,

$$I_g = I + hI = (1 + h)I$$

By Kirchhoff's voltage law,

$$E_g - I_g R_2 - IR_1 = 0$$

$$I = \frac{E_g - I_g R_2}{R_1}$$

Substituting, we have

$$I_g = (1 + h)I = (1 + h)\left(\frac{E_g - I_g R_2}{R_1}\right)$$

and

$$I_g R_1 = (1 + h)E_g - (1 + h)I_g R_2$$

$$E_g(1 + h) = I_g[R_1 + (1 + h)R_2]$$

$$Z_N = \frac{E_g}{I_g} = \frac{R_1 + (1 + h)R_2}{1 + h}$$

which agrees with the above.

FIG. 18.80
Determining the Norton impedance using the approach $Z_N = E_g/E_g$.

EXAMPLE 18.18 Find the Norton equivalent circuit for the network configuration in Fig. 18.57.

Solution: By source conversion,

$$I_N = \frac{E_{Th}}{Z_{Th}} = \frac{\dfrac{-k_2 R_2 V_i}{R_1 - k_1 k_2 R_2}}{\dfrac{R_1 R_2}{R_1 - k_1 k_2 R_2}}$$

and

$$\boxed{I_N = \frac{-k_2 V_i}{R_1}} \tag{18.12}$$

which is I_{sc} as determined in Example 18.13, and

$$\boxed{Z_N = Z_{Th} = \frac{R_2}{1 - \dfrac{k_1 k_2 R_2}{R_1}}} \tag{18.13}$$

For $k_1 \cong 0$, we have

$$\boxed{I_N = \frac{-k_2 V_i}{R_1}} \qquad k_1 = 0 \tag{18.14}$$

$$\boxed{Z_N = R_2} \qquad k_1 = 0 \tag{18.15}$$

18.5 MAXIMUM POWER TRANSFER THEOREM

When applied to ac circuits, the **maximum power transfer theorem** states that

maximum power will be delivered to a load when the load impedance is the conjugate of the Thévenin impedance across its terminals.
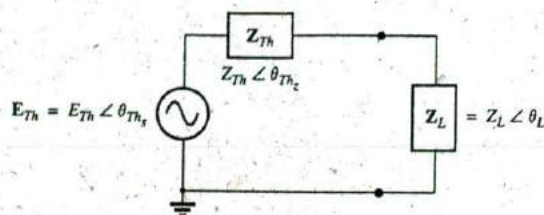
FIG. 18.81

Defining the conditions for maximum power transfer to a load.

That is, for Fig. 18.81, for maximum power transfer to the load,

$$\boxed{Z_L = Z_{Th} \quad \text{and} \quad \theta_L = -\theta_{Th_Z}} \tag{18.}$$

or, in rectangular form,

$$\boxed{R_L = R_{Th} \quad \text{and} \quad \pm jX_{\text{load}} = \mp jX_{Th}} \tag{18.}$$

The conditions just mentioned will make the total impedance of the circuit appear purely resistive, as indicated in Fig. 18.82:-

$$Z_T = (R \pm jX) + (R \mp jX)$$

and

$$\boxed{Z_T = 2R} \tag{18.1}$$
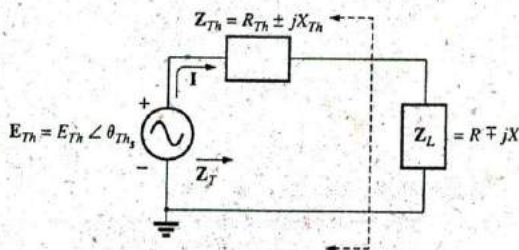
FIG. 18.82

Conditions for maximum power transfer to Z_L.

Since the circuit is purely resistive, the power factor of the circuit under maximum power conditions is 1; that is,

$$\boxed{F_p = 1} \quad \text{(maximum power transfer)} \tag{18.1}$$

The magnitude of the current I in Fig. 18.82 is

$$I = \frac{E_{Th}}{Z_T} = \frac{E_{Th}}{2R}$$

The maximum power to the load is

$$P_{\text{max}} = I^2 R = \left(\frac{E_{Th}}{2R}\right)^2 R$$

and

$$P_{max} = \frac{E_{Th}^2}{4R}$$

(18.20)

EXAMPLE 18.19 Find the load impedance in Fig. 18.83 for maximum power to the load, and find the maximum power.

Solution: Determine Z_{Th} [Fig. 18.84(a)]:

$Z_1 = R - jX_C = 6\,\Omega - j8\,\Omega = 10\,\Omega\,\angle-53.13°$

$Z_2 = +jX_L = j8\,\Omega$

$Z_{Th} = \dfrac{Z_1 Z_2}{Z_1 + Z_2} = \dfrac{(10\,\Omega\,\angle-53.13°)(8\,\Omega\,\angle90°)}{6\,\Omega - j8\,\Omega + j8\,\Omega} = \dfrac{80\,\Omega\,\angle36.87°}{6\,\angle0°}$

$= 13.33\,\Omega\,\angle36.87° = 10.66\,\Omega + j8\,\Omega$
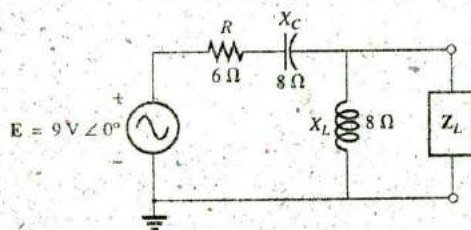
FIG. 18.83
Example 18.19.
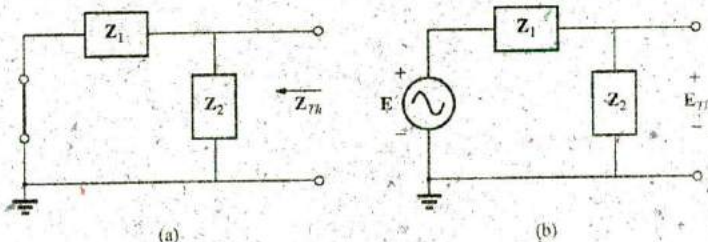

(a)   (b)

FIG. 18.84
Determining (a) Z_{Th} and (b) E_{Th} for the network external
to the load in Fig. 18.83.

and

$Z_{L} = 13.3\,\Omega\,\angle-36.87° = 10.66\,\Omega - j8\,\Omega$

To find the maximum power, we must first find E_{Th} [Fig. 18.84(b)], as follows:

$E_{Th} = \dfrac{Z_2 E}{Z_2 + Z_1}$ (voltage divider rule)

$= \dfrac{(8\,\Omega\,\angle90°)(9\,V\,\angle0°)}{j8\,\Omega + 6\,\Omega - j8\,\Omega} = \dfrac{72\,V\,\angle90°}{6\,\angle0°} = 12\,V\,\angle90°$

Then

$P_{max} = \dfrac{E_{Th}^2}{4R} = \dfrac{(12\,V)^2}{4(10.66\,\Omega)} = \dfrac{144}{42.64} = 3.38\,W$

FIG. 18.85
Example 18.20.

EXAMPLE 18.20 Find the load impedance in Fig. 18.85 for maximum power to the load, and find the maximum power.

Solution: First we must find Z_{Th} (Fig. 18.86).

$Z_1 = +jX_L = j9\,\Omega$ $Z_2 = R = 8\,\Omega$

Converting from a Δ to a Y (Fig. 18.87), we have

$Z'_1 = \dfrac{Z_1}{3} = j3\,\Omega$ $Z_2 = 8\,\Omega$
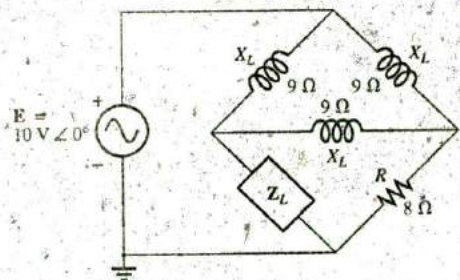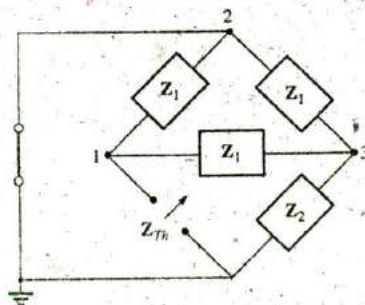

FIG. 18.86
Defining the subscripted impedances for the network
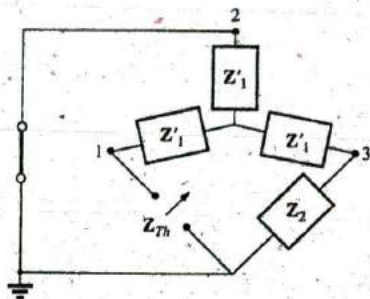in Fig. 18.85.

FIG. 18.87

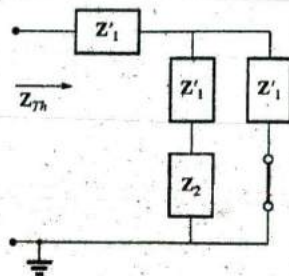*Substituting the Y equivalent for the upper Δ configuration in Fig. 18.86.*

FIG. 18.88

Determining Z_{Th} for the network in Fig. 18.85.

The redrawn circuit (Fig. 18.88) shows

$$Z_{Th} = Z'_1 + \frac{Z'_1(Z'_1 + Z_2)}{Z'_1 + (Z'_1 + Z_2)}$$

$$= j\,3\,\Omega + \frac{3\,\Omega\,\angle 90°\,(j\,3\,\Omega + 8\,\Omega)}{j\,6\,\Omega + 8\,\Omega}$$

$$= j\,3 + \frac{(3\,\angle 90°)(8.54\,\angle 20.56°)}{10\,\angle 36.87°}$$

$$= j\,3 + \frac{25.62\,\angle 110.56°}{10\,\angle 36.87°} = j\,3 + 2.56\,\angle 73.69°$$

$$= j\,3 + 0.72 + j\,2.46$$

$$Z_{Th} = 0.72\,\Omega + j\,5.46\,\Omega$$

and

$$Z_L = 0.72\,\Omega - j\,5.46\,\Omega$$

For E_{Th}, use the modified circuit in Fig. 18.89 with the voltage source replaced in its original position. Since $I_1 = 0$, E_{Th} is the voltage across the series impedance of Z'_2 and Z_2. Using the voltage divider rule gives us

FIG. 18.89

Finding the Thévenin voltage for the network in Fig. 18.85.

$$E_{Th} = \frac{(Z'_1 + Z_2)E}{Z'_1 + Z_2 + Z'_1} = \frac{(j\,3\,\Omega + 8\,\Omega)(10\text{ V }\angle 0°)}{8\,\Omega + j\,6\,\Omega}$$

$$= \frac{(8.54\,\angle 20.56°)(10\text{ V }\angle 0°)}{10\,\angle 36.87°}$$

$$E_{Th} = 8.54\text{ V }\angle -16.31°$$

and

$$P_{max} = \frac{E_{Th}^2}{4R} = \frac{(8.54\text{ V})^2}{4(0.72\,\Omega)} = \frac{72.93}{2.88}\text{ W}$$
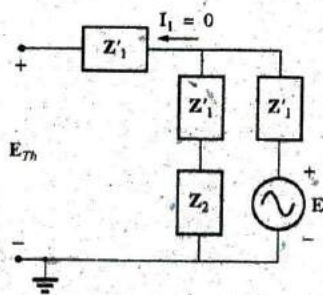
$$= 25.32\text{ W}$$

If the load resistance is adjustable but the magnitude of the load reactance is not, then the maximum power *that can be delivered* to the load will occur when the load resistance is set to the following value:

$$\boxed{R_L = \sqrt{R_{Th}^2 + (X_{Th} + X_{load})^2}} \qquad (18.21)$$

as derived in Appendix F.

In Eq. (18.21), each reactance carries a positive sign if inductive and a negative sign if capacitive.

The power delivered is then determined by

$$P = E_{Th}^2/4R_{av} \qquad\qquad (18.22)$$

where

$$R_{av} = \frac{R_{Th} + R_L}{2} \qquad\qquad (18.23)$$

The derivation of the above equations is given in Appendix F. The following example demonstrates the use of the above.

EXAMPLE 18.21 For the network in Fig. 18.90:

a. Determine the value of R_L for maximum power to the load if the load reactance is fixed at 4 Ω.
b. Find the power delivered to the load under the conditions of part (a).
c. Find the maximum power to the load if the load reactance is made adjustable to any value, and compare the result to part (b) above.
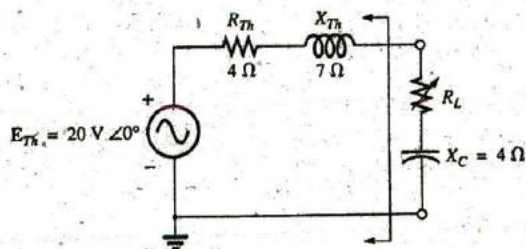

FIG. 18.90
Example 18.21.

Solutions:

a. Eq. (18.21):
$$R_L = \sqrt{R_{Th}^2 + (X_{Th} + X_{load})^2}$$
$$= \sqrt{(4\,\Omega)^2 + (7\,\Omega - 4\,\Omega)^2}$$
$$= \sqrt{16 + 9} = \sqrt{25}$$
$$R_L = 5\,\Omega$$

b. Eq. (18.23):
$$R_{av} = \frac{R_{Th} + R_L}{2} = \frac{4\,\Omega + 5\,\Omega}{2}$$
$$= 4.5\,\Omega$$

Eq. (18.22):
$$P = \frac{E_{Th}^2}{4R_{av}}$$
$$= \frac{(20\text{ V})^2}{4(4.5\,\Omega)} = \frac{400}{18}\text{ W}$$
$$\cong 22.22\text{ W}$$

c. For $Z_L = 4\,\Omega - j7\,\Omega$,

$$P_{max} = \frac{E_{Th}^2}{4R_{Th}} = \frac{(20\text{ V})^2}{4(4\,\Omega)}$$
$$= 25\text{ W}$$

exceeding the result of part (b) by 2.78 W.

18.6 SUBSTITUTION, RECIPROCITY, AND MILLMAN'S THEOREMS

As indicated in the introduction to this chapter, the **substitution** and **reciprocity theorems** and **Millman's theorem** will not be considered here in detail. A careful review of Chapter 9 will enable you to apply these theorems to sinusoidal ac networks with little difficulty. A number of problems in the use of these theorems appear in the Problems section at the end of the chapter.

18.7 APPLICATION

Electronic Systems

One of the blessings in the analysis of electronic systems is that the superposition theorem can be applied so that the dc analysis and ac analysis can be performed separately. The analysis of the dc system will affect the ac response, but the analysis of each is a distinct, separate process. Even though electronic systems have not been investigated in this text, a number of important points can be made in the description to follow that support some of the theory presented in this and recent chapters, so inclusion of this description is totally valid at this point. Consider the network in Fig. 18.91 with a transistor power amplifier and a source with an internal resistance of 800 Ω. Note that each component of the design was isolated by a color box to emphasize the fact that each component must be carefully weighed in any good design.
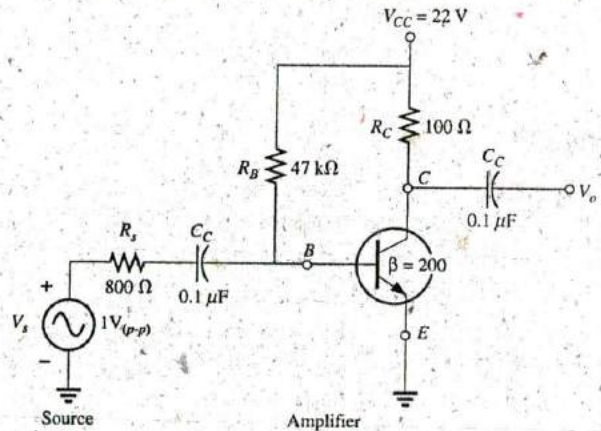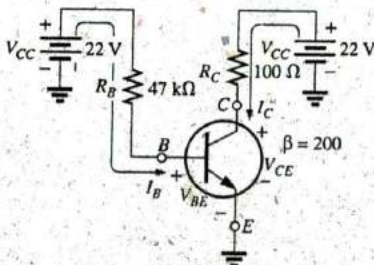

FIG. 18.91
Transistor amplifier.

As mentioned above, the analysis can be separated into a dc and an ac component. For the dc analysis, the two capacitors can be replaced by an open-circuit equivalent (Chapter 10), resulting in an isolation of the amplifier network as shown in Fig. 18.92. Given the fact that V_{BE} will be about 0.7 V dc for any operating transistor, the base current I_B can be found as follows using Kirchhoff's voltage law:

$$I_B = \frac{V_{R_B}}{R_B} = \frac{V_{CC} - V_{BE}}{R_B} = \frac{22\ \text{V} - 0.7\ \text{V}}{47\ \text{k}\Omega} = 453.2\ \mu\text{A}$$

FIG. 18.92
dc equivalent of the transistor network in Fig. 18.91.

For transistors, the collector current I_C is related to the base current by $I_C = \beta I_B$, and

$$I_C = \beta I_B = (200)(453.2 \ \mu A) = 90.64 \text{ mA}$$

Finally, through Kirchhoff's voltage law, the collector voltage (also the collector-to-emitter voltage since the emitter is grounded) can be determined as follows:

$$V_C = V_{CE} = V_{CC} - I_C R_C = 22 \text{ V} - (90.64 \text{ mA})(100 \ \Omega) = 12.94 \text{ V}$$

For the dc analysis, therefore,

$$I_B = 453.2 \ \mu A \qquad I_C = 90.64 \text{ mA} \qquad V_{CE} = 12.94 \text{ V}$$

which will define a point of dc operation for the transistor. This is an important aspect of electronic design since the dc operating point will have an effect on the ac gain of the network.

Now, using superposition, we can analyze the network from an ac viewpoint by setting all dc sources to zero (replaced by ground connections) and replacing both capacitors by short circuits as shown in Fig. 18.93. Substituting the short-circuit equivalent for the capacitors is valid because at 10 kHz (the midrange for human hearing response), the reactance of the capacitor is determined by $X_C = 1/2\pi fC = 15.92 \ \Omega$, which can be ignored when compared to the series resistors at the source and load. In other words, the capacitor has played the important role of isolating the amplifier for the dc response and completing the network for the ac response.

Redrawing the network as shown in Fig. 18.94 permits an ac investigation of its response. The transistor has now been replaced by an equivalent network that represents the behavior of the device. This process will be covered in detail in your basic electronics courses. This transistor configuration has an input impedance of 200 Ω and a current source whose magnitude is sensitive to the base current in the input circuit and to the amplifying factor for this transistor of 200. The 47 kΩ resistor in parallel with the 200 Ω input impedance of the transistor can be ignored, so the input current I_i and base current I_b are determined by

$$I_i \cong I_b = \frac{V_s}{R_s + R_i} = \frac{1 \text{ V}(p\text{-}p)}{800 \ \Omega + 200 \ \Omega} = \frac{1 \text{ V}(p\text{-}p)}{1 \text{ k}\Omega} = 1 \text{ mA}(p\text{-}p)$$

The collector current I_C is then

$$I_c = \beta I_b = (200)(1 \text{ mA } (p\text{-}p)) = 200 \text{ mA } (p\text{-}p)$$

and the output voltage V_o across the resistor R_C is

$$V_o = -I_c R_C = -(200 \text{ mA } (p\text{-}p))(100 \ \Omega)$$
$$= -20 \text{ V } (p\text{-}p)$$
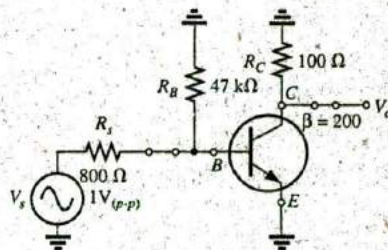
FIG. 18.93

ac equivalent of the transistor network in Fig. 18.9

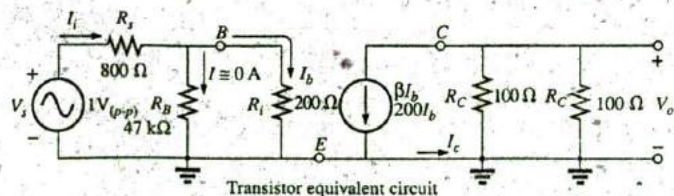Transistor equivalent circuit

FIG. 18.94

Network in Fig. 18.93 following the substitution of the transistor equivalent network.

The gain of the system is

$$A_v = \frac{V_o}{V_s} = \frac{-20 \text{ V } (p\text{-}p)}{1 \text{ V } (p\text{-}p)}$$
$$= -20$$

where the minus sign indicates that there is a 180° phase shift between input and output.

18.8 COMPUTER ANALYSIS

PSpice

Thévenín's Theorem This application parallels the methods used to determine the Thévenin equivalent circuit for dc circuits. The network in Fig. 18.29 appears as shown in Fig. 18.95 when the open-circuit Thévenin voltage is to be determined. The open circuit is simulated by using a resistor of 1 T (1 million MΩ). The resistor is necessary to es-tablish a connection between the right side of inductor L_2 and ground—nodes cannot be left floating for OrCAD simulations. Since the magnitude and the angle of the voltage are required, **VPRINT1** is introduced as shown in Fig 18.95. The simulation was an **AC Sweep** simulation at 1 kHz, and when the **Orcad Capture** window was ob-tained, the results appearing in Fig. 18.96 were taken from the listing
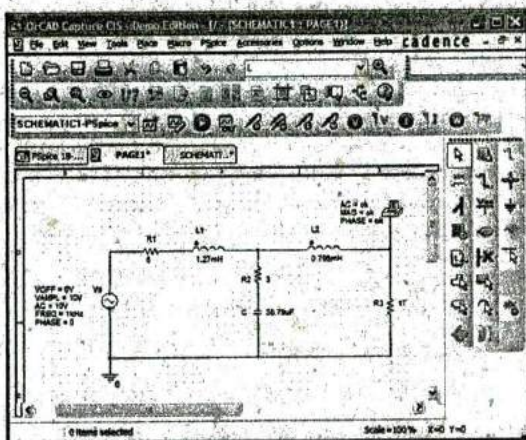
FIG. 18.95
Using PSpice to determine the open-circuit Thévenin voltage.

** Profile: "SCHEMATIC1-Pspice 18-1" [C:\ICA12\PSpice\PSpice 18-1-
PSpiceFiles\SCHEMATIC1\PSpice 18-1.sim]

**** AC ANALYSIS   TEMPERATURE = 27.000 DEG C

***

FREQ	VM(N00235)	VP(N00235)
1.000E+03	5.187E+00	-7.713E+01

FIG. 18.96
The output file for the open-circuit Thévenin voltage for the network in Fig. 18.95.

resulting from the **PSpice-View Output File.** The magnitude of the Thévenin voltage is 5.187 V to compare with the 5.08 V of Example 18.8, while the phase angle is $-77.13°$ to compare with the $-77.09°$ of the same example—excellent results.

Next, the short-circuit current is determined using **IPRINT** as shown in Fig. 18.97, to permit a determination of the Thévenin impedance. The resistance R_{coil} of 1 $\mu\Omega$ had to be introduced because inductors cannot be treated as ideal elements when using PSpice; they must all show some series internal resistance. Note that the short-circuit current will pass directly through the printer symbol for **IPRINT.** Incidentally, there is no need to exit the **SCHEMATIC1** developed above to determine the Thévenin voltage. Simply delete **VPRINT** and **R3,** and insert **IPRINT.** Then run a new simulation to obtain the results in Fig. 18.98. The magnitude of the short-circuit current is 936.1 mA at an angle of $-108.6°$. The Thévenin impedance is then defined by

$$Z_{Th} = \frac{E_{Th}}{I_{sc}} = \frac{5.187 \text{ V} \angle -77.13°}{936.1 \text{ mA} \angle -108.6°} = 5.54 \ \Omega \angle 31.47°$$

which is an excellent match with 5.49 $\Omega \angle 32.36°$ obtained in Example 18.8.
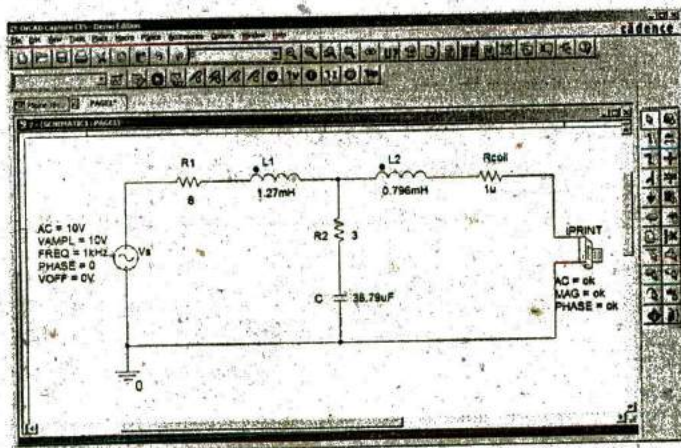
FIG. 18.97

Using PSpice to determine the short-circuit current.

```
****  AC ANALYSIS                 TEMPERATURE =  27.000 DEG C
*************************************************************************

***
FREQ         IM(V_PRINT1)     IP(V_PRINT1)
1.000E+03    9.361E-01        -1.086E+02
```

FIG. 18.98

The output file for the short-circuit current for the network in Fig. 18.97.

VCVS The next application will verify the results in Example 18.12 and provide some practice using controlled (dependent) sources. The network in Fig. 18.51, with its **voltage-controlled voltage source** (VCVS), will have the schematic appearance in Fig. 18.99. The VCVS appears as **E** in the **ANALOG** library, with the voltage E1 as the controlling voltage and **E** as the controlled voltage. In the **Property Editor** dialog box, change the **GAIN** to 20, but leave the rest of the columns as is. After **Display-Name and Value**, select **Apply** and exit the dialog box. This results in **GAIN = 20** near the controlled source. Take particular note of the second ground inserted near **E** to avoid a long wire to ground that may overlap other elements. For this exercise, the current source **ISRC** is used because it has an arrow in its symbol, and frequency is not important for this analysis since there are only resistive elements present. In the **Property Editor** dialog box, set the **AC** level to 5 mA and the **DC** level to 0 A; both are displayed using **Display-Name and value**. VPRINT1 is set up as in past exercises. The resistor **Roc** (open circuit) was given a very large value so that it appears as an open circuit to the rest of the network. VPRINT1 provides the open circuit Thévenin voltage between the points of interest. Running the simulation in the **AC Sweep** mode at 1 kHz results in the output file appearing in Fig. 18.100, revealing that the Thévenin voltage is 210 V∠ 0°. Substituting the numerical values of this example into the equation obtained in Example 18.12 confirms the result:
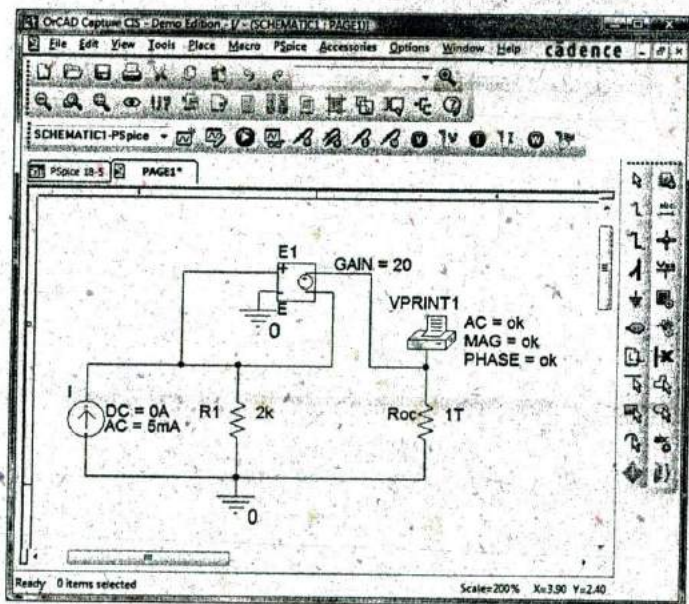

FIG. 18.99

Using PSpice to determine the open-circuit Thévenin voltage for the network in Fig. 18.51.

$$E_{Th} = (1 + \mu)IR_1 = (1 + 20)(5 \text{ mA} \angle 0°)(2 \text{ k}\Omega)$$
$$= 210 \text{ V} \angle 0°$$

Next, determine the short-circuit current using the **IPRINT** option. Note in Fig. 18.101 that the only difference between this network and

```
** Profile: "SCHEMATIC1-PSpice 18-5"
[ C:\ICA11\PSpice\pspice 18-5-pspic  files\schematic1\pspice 18-5.sim ]

****   AC ANALYSIS                  TEMPERATURE = 27.000 DEG C
*************************************************************************
***
FREQ        VM(N01477)  VP(N01477)
1.000E+03   2.100E+02   0.000E+00
```

FIG. 18.100

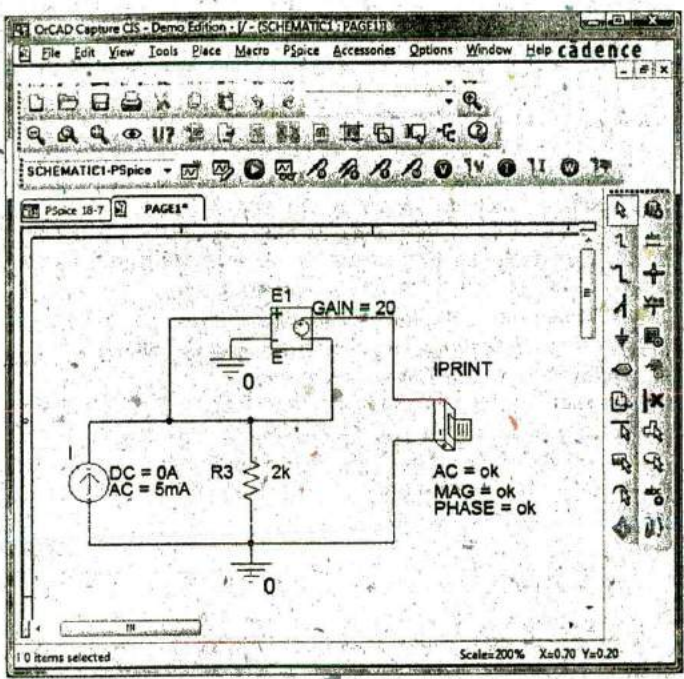*The output file for the open-circuit Thévenin voltage for the network in Fig. 18.99.*

.FIG. 18.101

Using PSpice to determine the short-circuit current for the network in Fig. 18.51.

that in Fig. 18.102 is the replacement of **Roc** with **IPRINT** and the removal of **VPRINT1**. Therefore, you do not need to completely "redraw" the network. Just make the changes and run a new simulation. The result of the new simulation as shown in Fig. 18.102 is a current of 5 mA at an angle of 0°.

```
** Profile: "SCHEMATIC1-PSpice 18-7"  [ C:\ICA11\PSpice\pspice 18-7-
pspicefiles\schematic1\PSpice 18-7.sim ]

****   AC ANALYSIS                  TEMPERATURE = 27.000 DEG C
*************************************************************************
***
FREQ        IM(V_PRINT2)    IP(V_PRINT2)
1.000E+03   5.000E-03       0.000E+00
```

FIG. 18.102

The output file for the short-circuit current for the network in Fig. 18.101.

The ratio of the two measured quantities results in the Thévenin impedance:

$$Z_{Th} = \frac{E_{oc}}{I_{sc}} = \frac{E_{Th}}{I_{sc}} = \frac{210 \text{ V} \angle 0°}{5 \text{ mA} \angle 0°} = 42 \text{ k}\Omega$$

which also matches the longhand solution in Example 18.12:

$$Z_{Th} = (1 + \mu)R_1 = (1 + 20)2 \text{ k}\Omega = (21)2 \text{ k}\Omega = 42 \text{ k}\Omega$$

Multisim

Superposition This analysis begins with the network in Fig. 18.12 from Example 18.4 because it has both an ac and a dc source. You will find in the analysis to follow that it is not necessary to set up a separate network for each source. Once the network is set up, the dc levels will appear during simulation, and the ac response can be found from a **View** option.

The resulting schematic appears in Fig. 18.103. The construction is quite straightforward with the parameters of the ac source set as follows: In the **Select a Component** dialog box, choose **POWER SOURCES,** followed by **AC_POWER.** Then set **Voltage(RMS)** to 4 V and **AC Analysis Magnitude** to 4V, followed by **Phase:** 0 Degrees; **AC Analysis Phase:** 0 Degrees; **Frequency (F):** 1 kHz; **Voltage Offset:** 0 V; and **Time Delay:** 0 Seconds. The dc voltage across R3 can be displayed using the **Indicator** option obtained by selecting the **Place Indicator** key pad that looks like a red number 8 on an IC package. The **VOLTMETER_V** option was chosen so that the plus sign of the potential difference was directly above the negative sign.
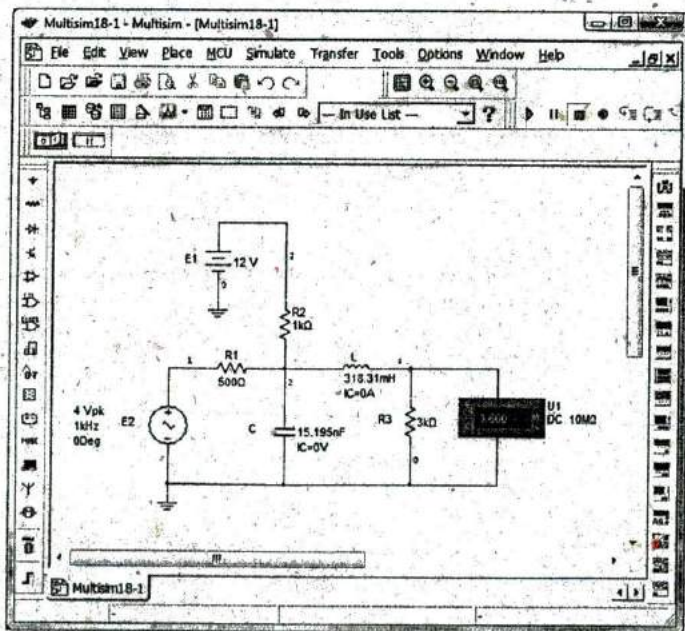
FIG. 18.103

Using Multisim to apply superposition to the network in Fig. 18.12.

To perform the analysis, use the following sequence to obtain the AC Analysis dialog box: **Simulate-Analyses-AC Analysis**. In the dialog box, make the following settings under the **Frequency Parameters** heading: **Start frequency:** 1 kHz; **Stop frequency:** 1 kHz; **Sweep Type:** Decade; **Number of points:** 1000; **Vertical scale:** Linear. Then shift to the **Output** option and select **V(4)** under **Variables in circuit**, followed by **Add** to place it in the **Selected variables for analysis** column. Move any other variables in the selected list back to the variable list using the **Remove** option. Then select **Simulate**, and the **Grapher View** response of Fig. 18.104 results. During the simulation process, the dc solution of 3.6 V appears on the voltmeter display (an exact match with the longhand solution). There are two plots in Fig. 18.104: o.. f magnitude versus frequency and the other of phase versus frequency. Left-click to select the upper graph, and a red arrow shows up along the left edge of the plot. The arrow reveals which plot is current... tive. To change the label for the vertical axis from **Magnitude** to Vo. ge (V) as shown in Fig. 18.104, select the **Properties** key from the top toolbar and choose **Left Axis**. Then change the label to **Voltage (V)** followed by **OK**, and the label appears as shown in Fig. 18.106. Next, to read the levels indicated on each graph with a high degree of accuracy, select the **Show/Hide Cursor** keypad on the toolbar. The keypad has a small red sine wave with two vertical markers. The result is a set of markers at the left edge of each figure. By selecting a marker from the left edge of the voltage plot and moving it to 1 kHz, you can find the value of the voltage in the accompanying table. Note that at a frequency of 1 kHz or essentially 1 kHz, the voltage is 2.06 V which is an exact match with the longhand solution in Example 18.4. If you then drop down to the phase plot, you find at the same frequency that the phase angle is -32.72, which is very close to the -32.74 in the longhand solution.
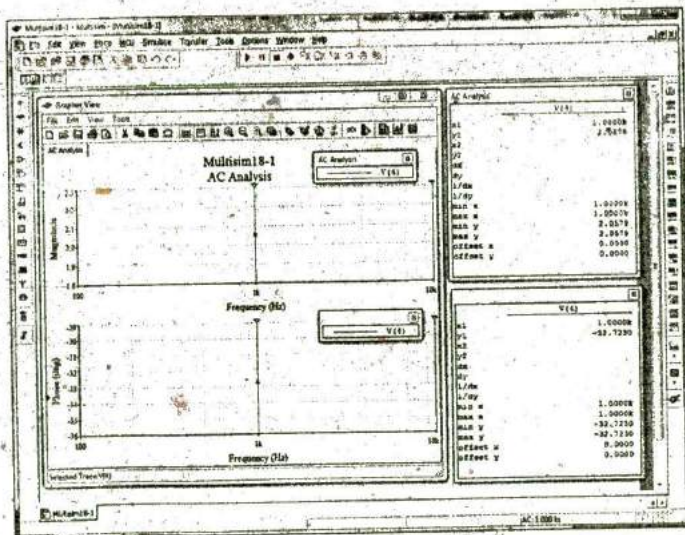
FIG. 18.104

The output results from the simulation of the network in Fig. 18.103.

In general, therefore, the results are an excellent match with the solutions in Example 18.4 using techniques that can be applied to a wide va-... of networks that have both dc and ac sources.

PROBLEMS

SECTION 18.2 Superposition Theorem

1. Using superposition, determine the current through the inductance X_L for the network of Fig. 18.105.
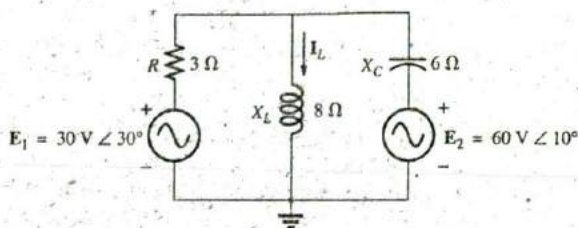

FIG. 18.105
Problem 1.

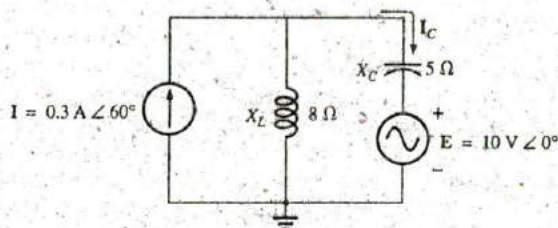2. Using superposition, determine the current through the capacitance X_C in Fig. 18.106.

FIG. 18.106
Problem 2.

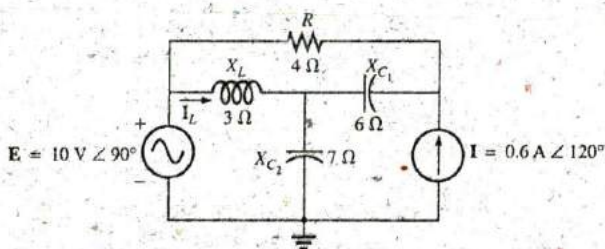*3. Using superposition, determine the current I_L for the network of Fig. 18.107.

FIG. 18.107
Problem 3.

*4. Using superposition, find the sinusoidal expression for the current i for the network of Fig. 18.108.

$e = 20 \sin(1000t + 60°)$
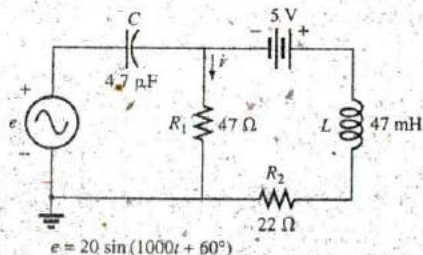
FIG. 18.108
Problems 4, 19, and 35.

5. Using superposition, find the sinusoidal expression for the voltage v_C for the network of Fig. 18.109.
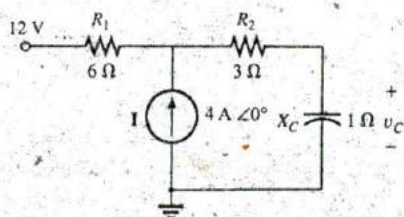
FIG. 18.109
Problems 5, 20, 36, and 50.

*6. Using superposition, find the current I for the network of Fig. 18.110.
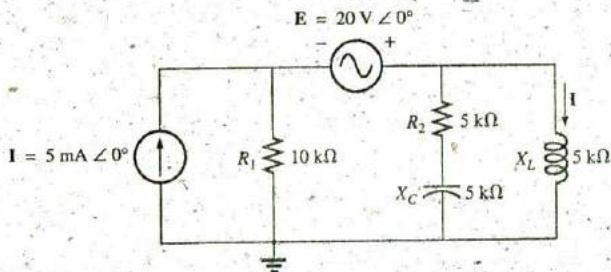
FIG. 18.110
Problems 6, 21, 37, and 51.

7. Using superposition, determine the current I_L ($h = 100$) for the network of Fig. 18.111.
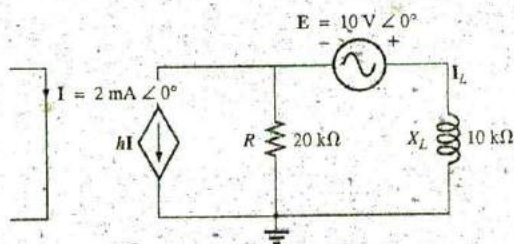
FIG. 18.111
Problems 7 and 24.

8. Using superposition, for the network of Fig. 18.112, determine the voltage V_L ($\mu = 20$).
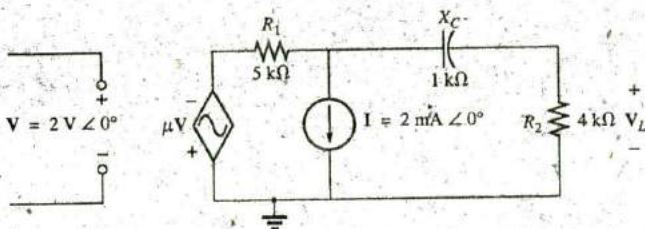
FIG. 18.112
Problems 8, 25, and 40.

*9. Using superposition, determine the current I_L for the network of Fig. 18.113 ($\mu = 20$; $h = 100$).
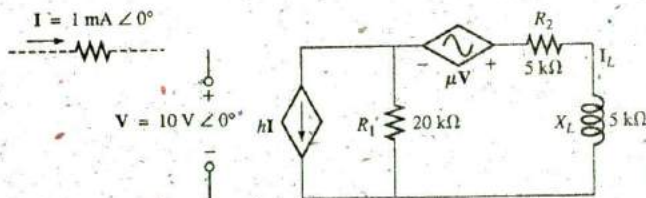
FIG. 18.113
Problems 9, 26, and 41.

*10. Determine V_L for the network of Fig. 18.114 ($h = 50$).
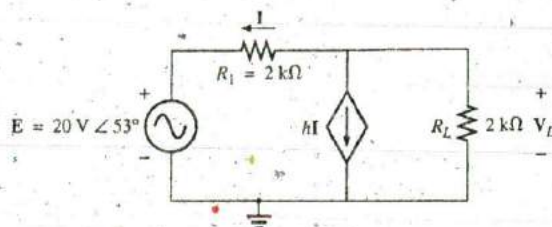
FIG. 18.114
Problems 10 and 27.

*11. Calculate the current I for the network of Fig. 18.115.
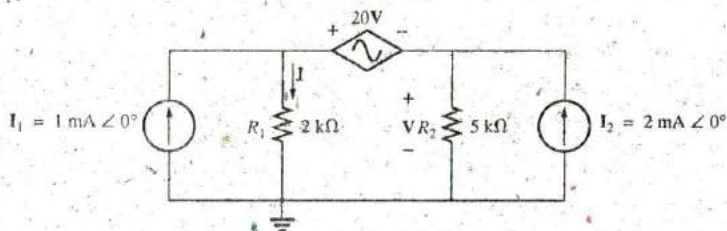
FIG. 18.115
Problems 11, 28, and 42.

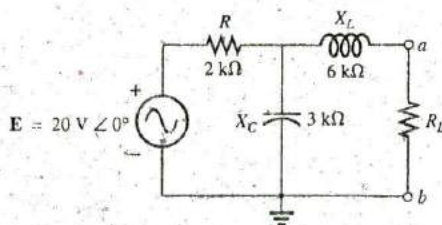12. Find the voltage V_s for the network in Fig. 18.116.
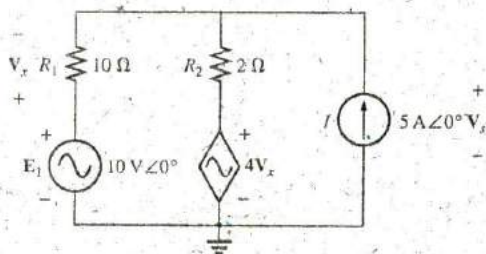
FIG. 18.116
Problem 12.

SECTION 18.3 Thévenin's Theorem

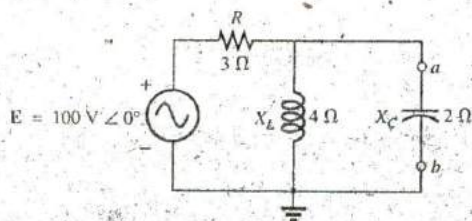13. Find the Thévenin equivalent circuit for the portion of the network of Fig. 18.117 external to the elements between points a and b.

FIG. 18.117
Problems 13 and 30.

14. Find the Thévenin equivalent circuit for the portion of the network of Fig. 18.118 external to the elements between a and b.

FIG. 18.118
Problems 14 and 49.

*15. Find the Thévenin equivalent circuit for the portion of the networks of Fig. 18.119 external to the elements between points a and b.
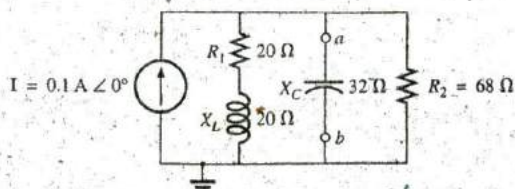

FIG. 18.119
Problems 15 and 31.

*16. Find the Thévenin equivalent circuit for the portion of the network of Fig. 18.120 external to the elements between points a and b.
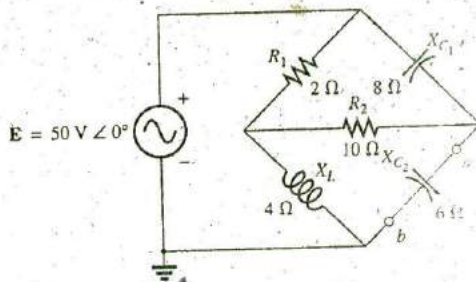
$E = 50 \text{ V} \angle 0°$

FIG. 18.120
Problem 16.

*17. Find the Thévenin equivalent circuit for the portion of the networks of Fig. 18.121 external to the elements between points a and b.
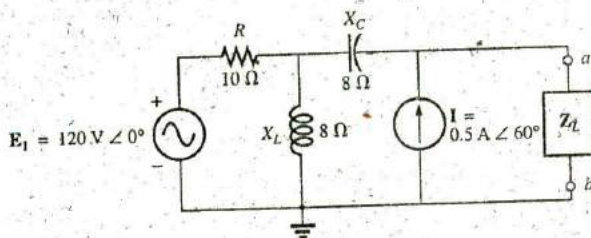
$E_1 = 120 \text{ V} \angle 0°$

$R = 10 \Omega$ $X_C = 8 \Omega$ $X_L = 8 \Omega$ $I = 0.5 \text{ A} \angle 60°$ Z_L

FIG. 18.121
Problems 17 and 32.

*18. Find the Thévenin equivalent circuit for the portion of the network of Fig. 18.122 external to the elements between points a and b.

$I = 0.6 \text{ A} \angle 90°$ $R_1 = 9 \Omega$ $I_2 = 0.8 \angle 60°$ $X_C = 10 \Omega$ $R_2 = 10 \Omega$ $E = 20 \text{ V} \angle 40°$
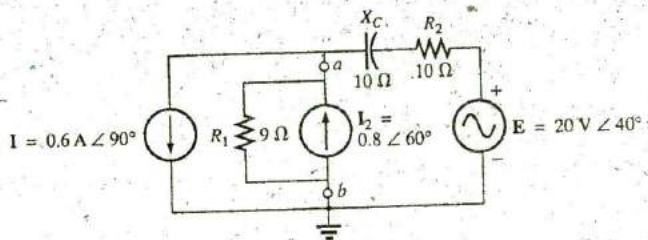
FIG. 18.122
Problem 18.

*19. a. Find the Thévenin equivalent circuit for the network external to the resistor R_1 in Fig. 18.108.
    b. Using the results of part (a), determine the current i of the same figure.

20. a. Find the Thévenin equivalent circuit for the network external to the capacitor in Fig. 18.109.
    b. Using the results of part (a), determine the voltage v_C for the same figure.

*21. a. Find the Thévenin equivalent circuit of the network external to the inductor in Fig. 18.110.
    b. Using the results of part (a), determine the current I of the same figure.

22. Determine the Thévenin equivalent circuit for the network external to the 5 kΩ inductive reactance in Fig. 18.123 (in terms of V).
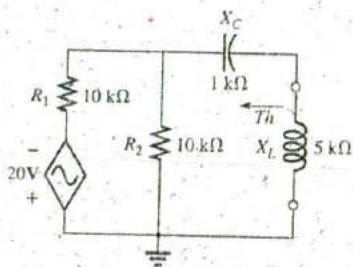
FIG. 18.123
Problems 22 and 38.

23. Determine the Thévenin equivalent circuit for the network external to the 4 kΩ inductive reactance in Fig. 18.124 (in terms of I).
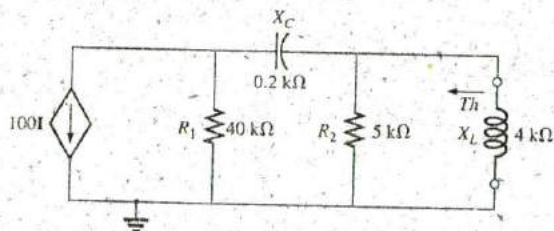

FIG. 18.124
Problems 23 and 39.

SECTION 18.4 Norton's Theorem

30. Find the Norton equivalent circuit for the network external to the elements between *a* and *b* for the network of Fig. 18.117.

31. Find the Norton equivalent circuit for the network external to the elements between *a* and *b* for the network of Fig. 18.119.

32. Find the Norton equivalent circuit for the network external to the elements between *a* and *b* for the network of Fig. 18.121.

*33. Find the Norton equivalent circuit for the portion of the network of Fig. 18.126 external to the elements between points *a* and *b*.

*34. Find the Norton equivalent circuit for the portion of the network of Fig. 18.127 external to the elements between points *a* and *b*.

24. Find the Thévenin equivalent circuit for the network external to the 10 kΩ inductive reactance in Fig. 18.111.

25. Determine the Thévenin equivalent circuit for the network external to the 4 kΩ resistor in Fig. 18.112.

*26. Find the Thévenin equivalent circuit for the network external to the 5 kΩ inductive reactance in Fig. 18.113.

*27. Determine the Thévenin equivalent circuit for the network external to the 2 kΩ resistor in Fig. 18.114.

*28. Find the Thévenin equivalent circuit for the network external to the resistor R_1 in Fig. 18.115.

*29. Find the Thévenin equivalent circuit for the network to the left of terminals *a-a'* in Fig. 18.125.
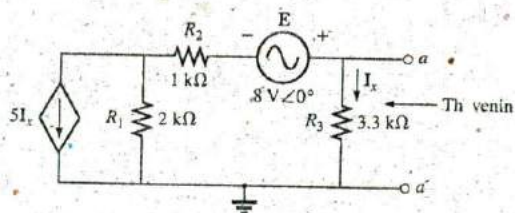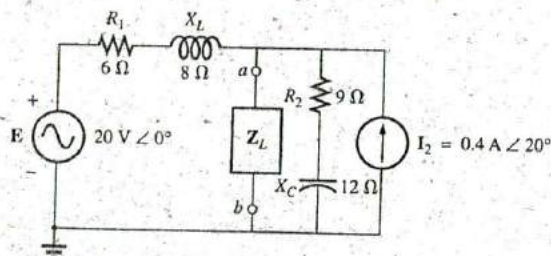

FIG. 18.125
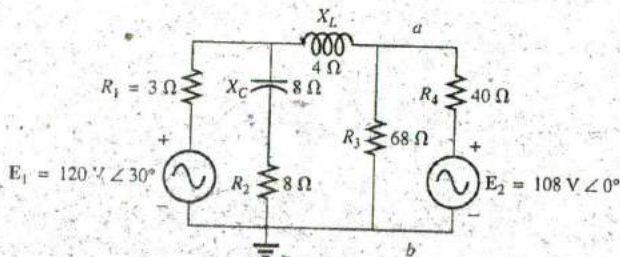*Problem 29.*

FIG. 18.126
Problem 33.

FIG. 18.127
Problem 34.

*35. a. Find the Norton equivalent circuit for the network external to the resistor R_1 in Fig. 18.108.
   b. Using the results of part (a), determine the current i of the same figure.

*36. a. Find the Norton equivalent circuit for the network external to the capacitor in Fig. 18.109.
   b. Using the results of part (a), determine the voltage V_C for the same figure.

*37. a. Find the Norton equivalent circuit for the network external to the inductor in Fig. 18.110.
   b. Using the results of part (a), determine the current I of the same figure.

38. Determine the Norton equivalent circuit for the network external to the 5 kΩ inductive reactance in Fig. 18.123.

39. Determine the Norton equivalent circuit for the network external to the 4 kΩ inductive reactance in Fig. 18.124.

40. Find the Norton equivalent circuit for the network external to the 4 kΩ resistor in Fig. 18.112.

*41. Find the Norton equivalent circuit for the network external to the 5 kΩ inductive reactance in Fig. 18.113.

*42. Find the Norton equivalent circuit for the network external to the I_1 current source in Fig. 18.115.

*43. For the network in Fig. 18.128, find the Norton equivalent circuit for the network external to the 2 kΩ resistor.
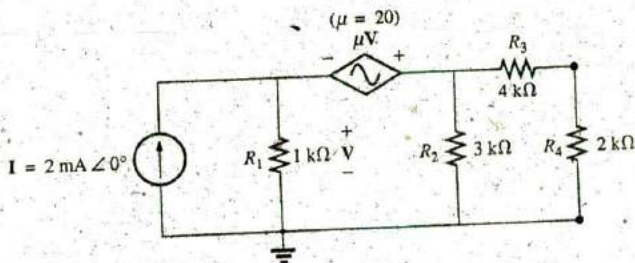

FIG. 18.128
Problem 43.

SECTION 18.5 Maximum Power Transfer Theorem

44. Find the load impedance Z_L for the network of Fig. 18.129 for maximum power to the load, and find the maximum power to the load.
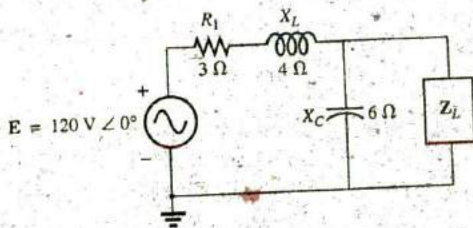

FIG. 18.129
Problem 44.

45. Find the load impedance Z_L for the network of Fig. 18.130 for maximum power to the load, and find the maximum power to the load.
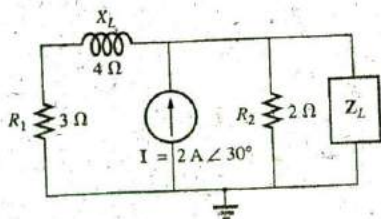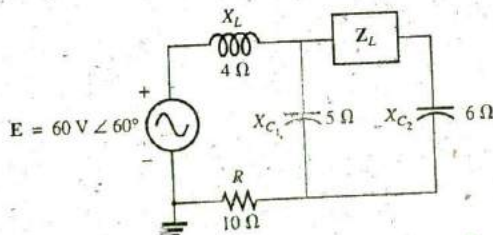
*46. Find the load impedance Z_L for the network of Fig. 18.131 for maximum power to the load, and find the maximum power to the load.

FIG. 18.130
Problem 45.

FIG. 18.131
Problem 46.

Th

*47. Find the load impedance Z_L for the network of Fig. 18.132 for maximum power to the load, and find the maximum power to the load.
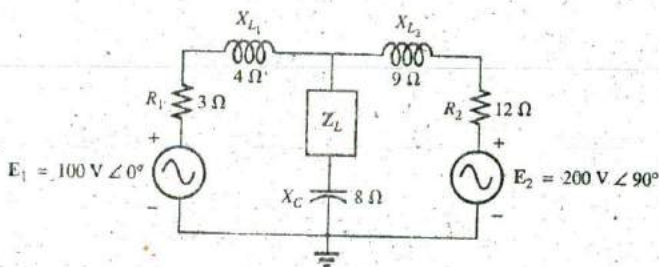

FIG. 18.132
Problem 47.

48. Find the load impedance R_L for the network in Fig. 18.133 for maximum power to the load, and find the maximum power to the load.
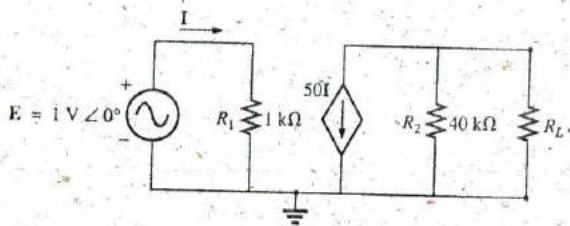

FIG. 18.133
Problem 48.

*49. a. Determine the load impedance to replace the resistor R_L in Fig. 18.118 to ensure maximum power to the load.
b. Using the results of part (a), determine the maximum power to the load.

*50. a. Determine the load impedance to replace the capacitor X_C in Fig. 18.109 to ensure maximum power to the load.
b. Using the results of part (a), determine the maximum power to the load.

*51. a. Determine the load impedance to replace the inductor X_L in Fig. 18.110 to ensure maximum power to the load.

b. Using the results of part (a), determine the maximum power to the load.

52. a. For the network in Fig. 18.134, determine the value of R_L that will result in maximum power to the load.
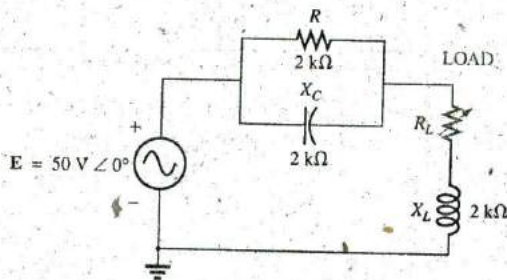b. Using the results of part (a), determine the maximum power delivered.

FIG. 18.134
Problem 52.

*53. a. For the network in Fig. 18.135, determine the level of capacitance that will ensure maximum power to the load if the range of capacitance is limited to 1 nF to 10 nF.
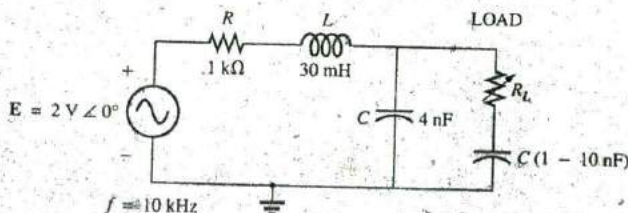

FIG. 18.135
Problem 53.

b. Using the results of part (a), determine the value of R_L that will ensure maximum power to the load.

c. Using the results of parts (a) and (b), determine the maximum power to the load.

SECTION 18.6 Substitution, Reciprocity, and Millman's Theorems

54. For the network in Fig. 18.136, determine two equivalent branches through the substitution theorem for the branch a-b.
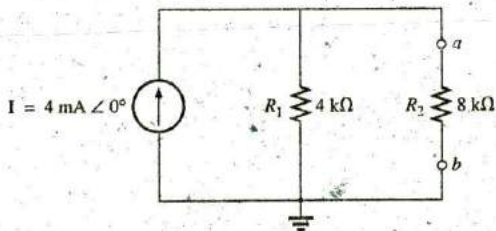

FIG. 18.136
Problem 54.

55. a. For the network of Fig. 18.137(a), find the current **I.**
b. Repeat part (a) for the network of Fig. 18.137(b).
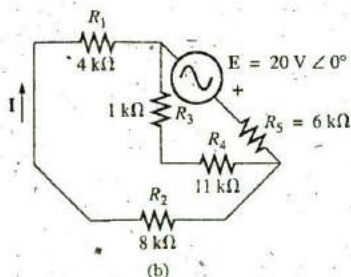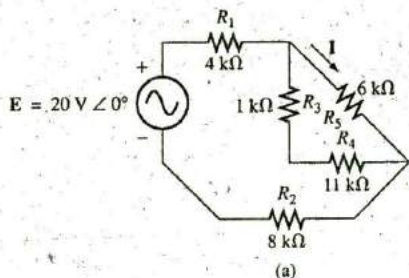**c.** How do the results of parts (a) and (b) compare?

FIG. 18.137
Problem 55.

56. .Using Millman's theorem, determine the current through, the 4 kΩ capacitive reactance of Fig. 18.138.
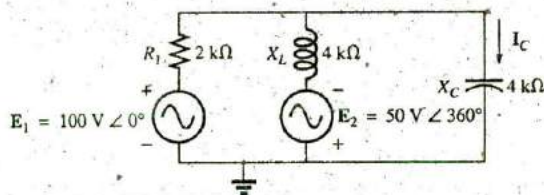

FIG. 18.138
Problem 56.

SECTION 18.8 Computer Analysis

PSpice or Multisim

57. Apply superposition to the network in Fig. 18.6. That is, determine the current **I** due to each source, and then find the resultant current.

__58.__ Determine the current I_L for the network in Fig. 18.22 using schematics.

__59.__ Using schematics, determine V_2 for the network in Fig. 18.57 if $V_i = 1$ V $\angle 0°$, $R_1 = 0.5$ kΩ, $k_1 = 3 \times 10^{-4}$, $k_2 =$ 50, and $R_2 = 20$ kΩ.

__60.__ Find the Norton equivalent circuit for the network in Fig. 18.77 using schematics.

__61.__ Using schematics, plot the power to the R-C load in Fig. 18.90 for values of R_L from 1 Ω to 10 Ω.

GLOSSARY

Maximum power transfer theorem A theorem used to determine the load impedance necessary to ensure maximum power to the load.

Millman's theorem A method using voltage-to-current source conversions that will permit the determination of unknown variables in a multiloop network.

Norton's theorem A theorem that permits the reduction of any two-terminal linear ac network to one having a single current source and parallel impedance. The resulting configuration can then be used to determine a particular current or voltage in the original network or to examine the effects of a specific portion of the network on a particular variable.

Reciprocity theorem A theorem stating that for single-source networks, the magnitude of the current in any branch of a network, due to a single voltage source anywhere else in the network, will equal the magnitude of the current through the branch in which the source was originally located if the source is placed in the branch in which the current was originally measured.

Substitution theorem A theorem stating that if the voltage across and current through any branch of an ac bilateral network are known, the branch can be replaced by any

combination of elements that will maintain the same voltage across and current through the chosen branch.

Superposition theorem A method of network analysis that permits considering the effects of each source independently. The resulting current and/or voltage is the phasor sum of the currents and/or voltages developed by each source independently.

Thévenin's theorem A theorem that permits the reduction of any two-terminal linear ac network to one having a single

voltage source and series impedance. The resulting configuration can then be employed to determine a particular current or voltage in the original network or to examine the effects of a specific portion of the network on a particular variable.

Voltage-controlled voltage source (VCVS) A voltage source whose parameters are controlled by a voltage elsewhere in the system.