Transformers

Objectives

- Become familiar with the flux linkages that exist between the coils of a transformer and how the voltages across the primary and secondary are established.

- Understand the operation of an iron-core and air-core transformer and how to calculate the currents and voltages of the primary and secondary circuits.

- Be aware of how the transformer is used for impedance matching purposes to ensure a high level of power transfer.

- Become aware of all the components that make up the equivalent circuit of a transformer and how they affect its performance and frequency response.

- Understand how to use and interpret the dot convention of mutually coupled coils in a network.

22.1 INTRODUCTION

Chapter 11 discussed the *self-inductance* of a coil. We shall now examine the **mutual inductance** that exists between coils of the same or different dimensions. Mutual inductance is a phenomenon basic to the operation of the *transformer,* an electrical device used today in almost every field of electrical engineering. This device plays an integral part in power distribution systems and can be found in many electronic circuits and measuring instruments. In this chapter, we discuss three of the basic applications of a transformer: to build up or step down the voltage or current, to act as an impedance matching device, and to isolate (no physical connection) one portion of a circuit from another. In addition, we will introduce the **dot convention** and will consider the transformer equivalent circuit. The chapter concludes with a word about writing mesh equations for a network with mutual inductance.

22.2 MUTUAL INDUCTANCE

A transformer is constructed of two coils placed so that the changing flux developed by one links the other, as shown in Fig. 22.1. This results in an induced voltage across each coil. To distinguish between the coils, we will apply the transformer convention that

the coil to which the source is applied is called the primary, and the coil to which the load is applied is called the secondary.

For the primary of the transformer in Fig. 22.1, an application of Faraday's law [Eq. (11.9)] results in

$$e_p = N_p \frac{d\phi_p}{dt} \quad \text{(volts, V)} \qquad (22.1)$$
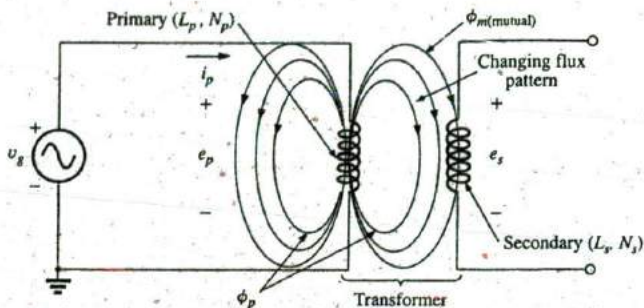
FIG. 22.1

Defining the components of a transformer.

revealing that the voltage induced across the primary is directly related to the number of turns in the primary and the rate of change of magnetic flux linking the primary coil. Or, from Eq. (11.11),

$$e_p = L_p \frac{di_p}{dt} \qquad \text{(volts, V)} \qquad \textbf{(22.2)}$$

revealing that the induced voltage across the primary is also directly related to the self-inductance of the primary and the rate of change of current through the primary winding.

The magnitude of e_s, the voltage induced across the secondary, is determined by

$$e_s = N_s \frac{d\phi_m}{dt} \qquad \text{(volts, V)} \qquad \textbf{(22.3)}$$

where N_s is the number of turns in the secondary winding and ϕ_m is the portion of the primary flux ϕ_p that links the secondary winding.

If all of the flux linking the primary links the secondary, then

$$\phi_m = \phi_p$$

and

$$e_s = N_s \frac{d\phi_p}{dt} \qquad \text{(volts, V)} \qquad \textbf{(22.4)}$$

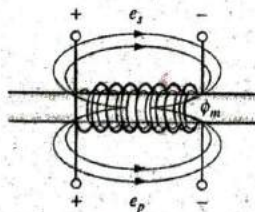The **coefficient of coupling** (k) between two coils is determined by

$$k \text{ (coefficient of coupling)} = \frac{\phi_m}{\phi_p} \qquad \textbf{(22.5)}$$

Since the maximum level of ϕ_m is ϕ_p, the coefficient of coupling between two coils can never be greater than 1.

The coefficient of coupling between various coils is indicated in Fig. 22.2. In Fig. 22.2(a), the ferromagnetic steel core ensures that most of the flux linking the primary also links the secondary, establishing a coupling coefficient very close to 1. In Fig. 22.2(b), the fact that both coils are overlapping results in the flux of one coil linking the other coil, with the result that the coefficient of coupling is again very close to 1. In Fig. 22.2(c), the absence of a ferromagnetic core results

Steel core
$k \cong 1$
(a)

$\phi_m \cong \phi_p$

Any core
$k \cong 1$
(b)

Air core
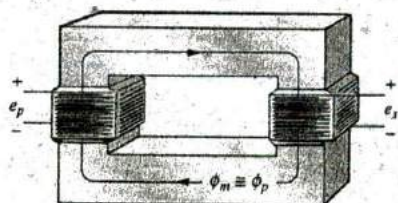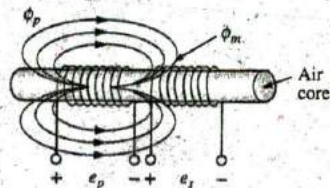
$k = \frac{\phi_m}{\phi_p} \ll 1 \, (0.01 \rightarrow 0.3)$

(c)

FIG. 22.2

Windings having different coefficients of coupling.

in low levels of flux linkage between the coils. The closer the two coils are, the greater is the flux linkage, and the higher is the value of k, although it will never approach a level of 1. Those coils with low coefficients of coupling are said to be **loosely coupled**.

For the secondary, we have

$$e_s = N_s \frac{d\phi_m}{dt} = N_s \frac{dk\phi_p}{dt}$$

and

$$\boxed{e_s = kN_s \frac{d\phi_p}{dt}} \quad \text{(volts, V)} \qquad \text{(22.6)}$$

The mutual inductance between the two coils in Fig. 22.1 is determined by

$$\boxed{M = N_s \frac{d\phi_m}{di_p}} \quad \text{(henries, H)} \qquad \text{(22.7)}$$

or

$$\boxed{M = N_p \frac{d\phi_p}{di_s}} \quad \text{(henries, H)} \qquad \text{(22.8)}$$

Note in the above equations that the symbol for mutual inductance is the capital letter M and that its unit of measurement, like that of self-inductance, is the *henry*. In words, Eqs. (22.7) and (22.8) state that the

mutual inductance between two coils is proportional to the instantaneous change in flux linking one coil due to an instantaneous change in current through the other coil.

In terms of the inductance of each coil and the coefficient of coupling, the mutual inductance is determined by

$$\boxed{M = k\sqrt{L_p L_s}} \quad \text{(henries, H)} \qquad \text{(22.9)}$$

The greater the coefficient of coupling (greater flux linkages), or the greater the inductance of either coil, the higher is the mutual inductance between the coils. Relate this fact to the configurations in Fig. 22.2.

The secondary voltage e_s can also be found in terms of the mutual inductance if we rewrite Eq. (22.3) as

$$e_s = N_s \left(\frac{d\phi_m}{di_p} \right) \left(\frac{di_p}{dt} \right)$$

and, since $M = N_s (d\phi_m/di_p)$, it can also be written

$$\boxed{e_s = M \frac{di_p}{dt}} \quad \text{(volts, V)} \qquad \text{(22.10)}$$

Similarly,

$$\boxed{e_p = M \frac{di_s}{dt}} \quad \text{(volts, V)} \qquad \text{(22.11)}$$

$L_p = 200$ mH
$N_p = 50$ turns

$L_s = 800$ mH
$N_s = 100$ turns

i_p

$+$
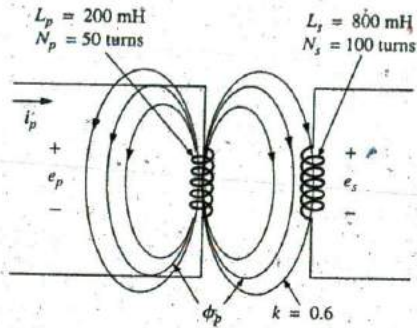
e_p

$-$

$+$

e_s

$-$

ϕ_p $k = 0.6$

FIG. 22.3
Example 22.1

EXAMPLE 22.1 For the transformer in Fig. 22.3:

a. Find the mutual inductance M.
b. Find the induced voltage e_p if the flux ϕ_p changes at the rate of 450 mWb/s.
c. Find the induced voltage e_s for the same rate of change indicated in part (b).
d. Find the induced voltages e_p and e_s if the current i_p changes at the rate of 0.2 A/ms.

Solutions:

a. $M = k\sqrt{L_p L_s} = 0.6\sqrt{(200 \text{ mH})(800 \text{ mH})}$
   $= 0.6\sqrt{16 \times 10^{-2}} = (0.6)(400 \times 10^{-3}) = 240 \text{ mH}$

b. $e_p = N_p \dfrac{d\phi_p}{dt} = (50)(450 \text{ mWb/s}) = 22.5 \text{ V}$

c. $e_s = kN_s \dfrac{d\phi_p}{dt} = (0.6)(100)(450 \text{ mWb/s}) = 27 \text{ V}$

d. $e_p = L_p \dfrac{di_p}{dt} = (200 \text{ mH})(0.2 \text{ A/ms})$
   $= (200 \text{ mH})(200 \text{ A/s}) = 40 \text{ V}$

   $e_s = M \dfrac{di_p}{dt} = (240 \text{ mH})(200 \text{ A/s}) = 48 \text{ V}$

22.3 THE IRON-CORE TRANSFORMER

An iron-core transformer under loaded conditions is shown in Fig. 22.4. The iron core will serve to increase the coefficient of coupling between the coils by increasing the mutual flux ϕ_m. Recall from Chapter 11 that magnetic flux lines always take the path of least reluctance, which in this case is the iron core.

In the analyses in this chapter, we assume that all of the flux linking coil 1 will link coil 2. In other words, the coefficient of coupling is its
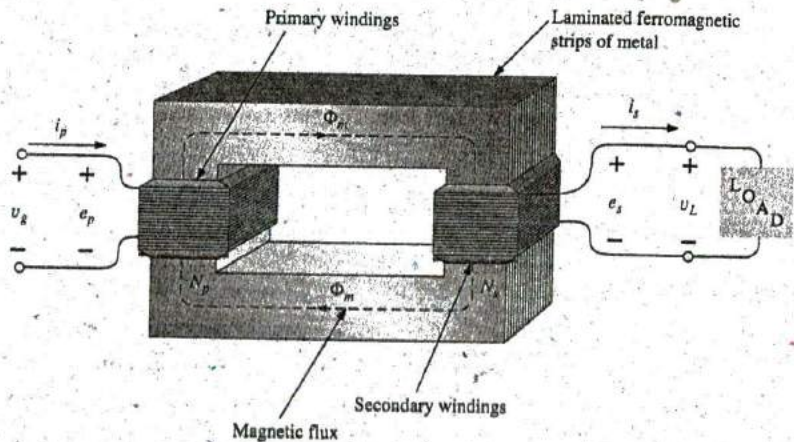

Primary windings

Laminated ferromagnetic strips of metal

I_s

i_p

$+$ $+$

v_g e_p

$-$ $-$

Φ_m

N_p

Φ_m

$+$ $+$

e_s v_L

$-$ $-$

LOAD

N_s

Secondary windings

Magnetic flux

FIG. 22.4
Iron-core transformer.

maximum value, 1, and $\phi_m = \phi_p = \phi_s$. In addition, we first analyze the transformer from an ideal viewpoint; that is, we neglect losses such as the geometric or dc resistance of the coils, the leakage reactance due to the flux linking either coil that forms no part of ϕ_m, and the hysteresis and eddy current losses. This is not to convey the impression, however, that we are far from the actual operation of a transformer. Most transformers manufactured today can be considered almost ideal. The equations we develop under ideal conditions are, in general, a first approximation to the actual response, which is never off by more than a few percentage points. The losses are considered in greater detail in Section 22.6.

When the current i_p through the primary circuit of the iron-core trans-former is a maximum, the flux ϕ_m linking both coils is also a maximum. In fact, the magnitude of the flux is directly proportional to the current through the primary windings. Therefore, the two are in phase, and for sinusoidal inputs, the magnitude of the flux varies as a sinusoid also. That is, if

$$i_p = \sqrt{2}I_p \sin \omega t$$

then

$$\phi_m = \Phi_m \sin \omega t$$

The induced voltage across the primary due to a sinusoidal input can be determined by Faraday's law:

$$e_p = N_p \frac{d\phi_p}{dt} = N_p \frac{d\phi_m}{dt}$$

Substituting for ϕ_m gives us

$$e_p = N_p \frac{d}{dt}(\Phi_m \sin \omega t)$$

and differentiating, we obtain

$$e_p = \omega N_p \Phi_m \cos \omega t$$

or

$$e_p = \omega N_p \Phi_m \sin (\omega t + 90°)$$

indicating that the induced voltage e_p leads the current through the pri-mary coil by 90°.

The effective value of e_p is

$$E_p = \frac{\omega N_p \Phi_m}{\sqrt{2}} = \frac{2\pi f N_p \Phi_m}{\sqrt{2}}$$

and

$$\boxed{E_p = 4.44 f N_p \Phi_m} \qquad (22.12)$$

which is an equation for the rms value of the voltage across the primary coil in terms of the frequency of the input current or voltage, the number of turns of the primary, and the maximum value of the magnetic flux linking the primary.

For the case under discussion, where the flux linking the secondary equals that of the primary, if we repeat the procedure just described for the induced voltage across the secondary, we get

$$\boxed{E_s = 4.44 f N_s \Phi_m} \qquad (22.13)$$

Dividing Eq. (22.12) by Eq. (22.13) as

$$\frac{E_p}{E_s} = \frac{4.44 f N_p \Phi_m}{4.44 f N_s \Phi_m}$$

we obtain

$$\frac{E_p}{E_s} = \frac{N_p}{N_s}$$

(22.14)

revealing an important relationship for transformers:

The ratio of the magnitudes of the induced voltages is the same as the ratio of the corresponding turns.

If we consider that

$$e_p = N_p \frac{d\phi_m}{dt} \quad \text{and} \quad e_s = N_s \frac{d\phi_m}{dt}$$

and divide one by the other, that is,

$$\frac{e_p}{e_s} = \frac{N_p(d\phi_m/dt)}{N_s(d\phi_m/dt)}$$

then

$$\frac{e_p}{e_s} = \frac{N_p}{N_s}$$

The *instantaneous* values of e_1 and e_2 are therefore related by a constant determined by the turns ratio. Since their instantaneous magnitudes are related by a constant, the induced voltages are in phase, and Eq. (22.14) can be changed to include phasor notation; that is,

$$\frac{\mathbf{E}_p}{\mathbf{E}_s} = \frac{N_p}{N_s}$$

(22.15)

or, since $\mathbf{V}_g = \mathbf{E}_1$ and $\mathbf{V}_L = \mathbf{E}_2$ for the ideal situation,

$$\frac{\mathbf{V}_g}{\mathbf{V}_L} = \frac{N_p}{N_s}$$

(22.16)

The ratio N_p/N_s, usually represented by the lowercase letter a, is referred to as the **transformation ratio:**

$$a = \frac{N_p}{N_s}$$

(22.17)

If $a < 1$, the transformer is called a **step-up transformer** since the voltage $E_s > E_p$; that is,

$$\frac{E_p}{E_s} = \frac{N_p}{N_s} = a \quad \text{or} \quad E_s = \frac{E_p}{a}$$

and, if $a < 1$, $\qquad\qquad\qquad E_s > E_p$

If $a > 1$, the transformer is called a **step-down transformer** since $E_s < E_p$; that is,
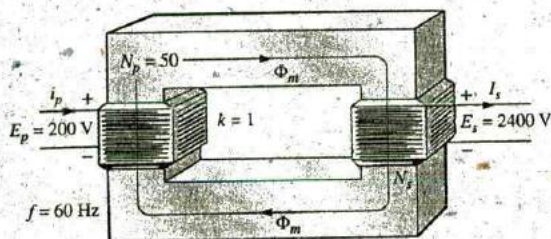
$$E_p = aE_s$$

and, if $a > 1$, then $\qquad\qquad\qquad E_p > E_s$

EXAMPLE 22.2 For the iron-core transformer in Fig. 22.5:

a. Find the maximum flux Φ_m.
b. Find the secondary turns N_s.

FIG. 22.5
Example 22.2.

Solutions:

a. $E_p = 4.44 N_p f \Phi_m$

Therefore, $\Phi_m = \dfrac{E_p}{4.44 N_p f} = \dfrac{200 \text{ V}}{(4.44)(50 \text{ t})(60 \text{ Hz})}$

and $\Phi_m = 15.02 \text{ mWb}$

b. $\dfrac{E_p}{E_s} = \dfrac{N_p}{N_s}$

Therefore, $N_s = \dfrac{N_p E_s}{E_p} = \dfrac{(50 \text{ t})(2400 \text{ V})}{200 \text{ V}}$

$= 600 \text{ turns}$

The induced voltage across the secondary of the transformer in Fig. 22.4 establishes a current i_s through the load Z_L and the secondary windings. This current and the turns N_s develop an mmf $N_s i_s$ that is not present under no-load conditions since $i_s = 0$ and $N_s i_s = 0$. Under loaded or unloaded conditions, however, the net ampere-turns on the core produced by both the primary and the secondary must remain unchanged for the same flux ϕ_m to be established in the core. The flux ϕ_m must remain the same to have the same induced voltage across the primary and to balance the voltage impressed across the primary. To counteract the mmf of the secondary, which is tending to change ϕ_m, an additional current must flow in the primary. This current is called the *load component of the primary current* and is represented by the notation i'_p.

For the balanced or equilibrium condition,

$$N_p i'_p = N_s i_s$$

The total current in the primary under loaded conditions is

$$i_p = i'_p + i_{\phi_m}$$

where i_{ϕ_m} is the current in the primary necessary to establish the flux ϕ_m. For most practical applications, $i'_p > i_{\phi_m}$. For our analysis, we assume $i_p \cong i'_p$, so

$$N_p i_p = N_s i_s$$

Since the instantaneous values of i_p and i_s are related by the turns ratio, the phasor quantities I_p and I_s are also related by the same ratio:

$$N_p I_p = N_s I_s$$

or

$$\boxed{\frac{I_p}{I_s} = \frac{N_s}{N_p}}$$

(22.18)

◄ *The primary and secondary currents of a transformer are therefore related by the inverse ratios of the turns.*

Keep in mind that Eq. (22.18) holds true only if we neglect the effects of i_{ϕ_m}. Otherwise, the magnitudes of I_p and I_s are not related by the turns ratio, and I_p and I_s are not in phase.

For the step-up transformer, $a < 1$, and the current in the secondary, $I_s = aI_p$, is less in magnitude than that in the primary. For a step-down transformer, the reverse is true.

22.4 REFLECTED IMPEDANCE AND POWER

In the previous section we found that

$$\frac{V_g}{V_L} = \frac{N_p}{N_s} = a \qquad \text{and} \qquad \frac{I_p}{I_s} = \frac{N_s}{N_p} = \frac{1}{a}$$

Dividing the first by the second, we have

$$\frac{V_g / V_L}{I_p / I_s} = \frac{a}{1/a}$$

or

$$\frac{V_g / I_p}{V_L / I_s} = a^2 \qquad \text{and} \qquad \frac{V_g}{I_p} = a^2 \frac{V_L}{I_s}$$

However, since

$$Z_p = \frac{V_g}{I_p} \qquad \text{and} \qquad Z_L = \frac{V_L}{I_s}$$

then

$$\boxed{Z_p = a^2 Z_L}$$

(22.19)

That is, the impedance of the primary circuit of an ideal transformer is the transformation ratio squared times the impedance of the load. If a transformer is used, therefore, an impedance can be made to appear larger or smaller at the primary by placing it in the secondary of a step-down ($a > 1$) or step-up ($a < 1$) transformer, respectively. Note that if the load is capacitive or inductive, the **reflected impedance** is also capacitive or inductive.

For the ideal iron-core transformer,

$$\frac{E_p}{E_s} = a = \frac{I_s}{I_p}$$

or

$$\boxed{E_p I_p = E_s I_s}$$

(22.20)

and

$$\boxed{P_{in} = P_{out}}$$ (ideal conditions)

(22.21)

EXAMPLE 22.3 For the iron-core transformer in Fig. 22.6:

a. Find the magnitude of the current in the primary and the impressed voltage across the primary.
b. Find the input resistance of the transformer.

Solutions:

a. $\dfrac{I_p}{I_s} = \dfrac{N_s}{N_p}$

$I_p = \dfrac{N_s}{N_p}I_s = \left(\dfrac{5\,t}{40\,t}\right)(0.1\,\text{A}) = \textbf{12.5 mA}$

$V_L = I_s Z_L = (0.1\,\text{A})(2\,\text{k}\Omega) = 200\,\text{V}$

Also, $\dfrac{V_g}{V_L} = \dfrac{N_p}{N_s}$

$V_g = \dfrac{N_p}{N_s}V_L = \left(\dfrac{40\,t}{5\,t}\right)(200\,\text{V}) = \textbf{1600 V}$

b. $Z_{p} = a^2 Z_L$

$a = \dfrac{N_p}{N_s} = 8$

$Z_p = (8)^2(2\,\text{k}\Omega) = R_p = \textbf{128 k}\Omega$

Denotes iron core

$I_s = 100$ mA

$N_p = 40$ t $N_s = 5$ t
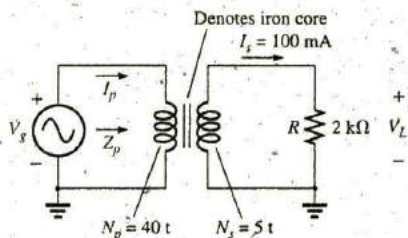
FIG. 22.6
Example 22.3.

EXAMPLE 22.4 For the residential supply appearing in Fig. 22.7, determine (assuming a totally resistive load) the following:

a. the value of R to ensure a balanced load
b. the magnitude of I_1 and I_2
c. the line voltage V_L
d. the total power delivered for a balanced three-phase load
e. the turns ratio $a = N_p/N_s$

Ten 60 W bulbs

TV
200 W

Air conditioner
2000 W

Main service ← | Residential service:
120/240 V, 3-wire,
single-phase
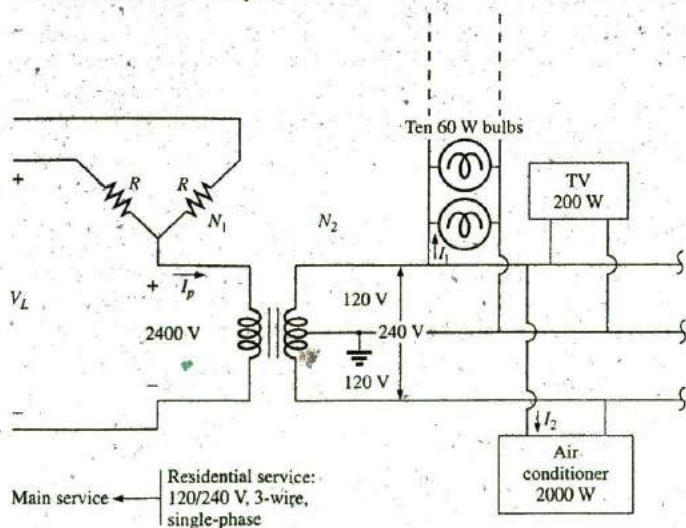
FIG. 22.7
Single-phase residential supply.

Solutions:

a. $P_T = (10)(60 \text{ W}) + 200 \text{ W} + 2000 \text{ W}$
   $= 600 \text{ W} + 200 \text{ W} + 2000 \text{ W} = 2800 \text{ W}$

$P_{in} = P_{out}$

$V_p I_p = V_s I_s = 2800 \text{ W}$ (purely resistive load)
$(2400 \text{ V})I_p = 2800 \text{ W}$ and $I_p = 1.17 \text{ A}$

$$R = \frac{V_\phi}{I_p} = \frac{2400 \text{ V}}{1.17 \text{ A}} = 2051.28 \ \Omega$$

b. $P_1 = 600 \text{ W} = VI_1 = (120 \text{ V})I_1$

and $\qquad\qquad I_1 = 5 \text{ A}$

$\qquad\qquad\qquad P_2 = 2000 \text{ W} = VI_2 = (240 \text{ V})I_2$

and $\qquad\qquad I_2 = 8.33 \text{ A}$

c. $V_L = \sqrt{3}V_\phi = 1.73(2400 \text{ V}) = 4152 \text{ V}$

d. $P_T = 3P_\phi = 3(2800 \text{ W}) = 8.4 \text{ kW}$

e. $a = \dfrac{N_p}{N_s} = \dfrac{V_p}{V_s} = \dfrac{2400 \text{ V}}{240 \text{ V}} = 10$

22.5 IMPEDANCE MATCHING, ISOLATION, AND DISPLACEMENT

Transformers can be particularly useful when you are trying to ensure that a load receives maximum power from a source. Recall that maximum power is transferred to a load when its impedance is a match with the internal resistance of the supply. Even if a perfect match is unattainable, the closer the load matches the internal resistance, the greater is the power to the load and the more efficient is the system. Unfortunately, unless it is planned as part of the design, most loads are not a close match with the internal impedance of the supply. However, transformers have a unique relationship between their primary and secondary impedances that can be put to good use in the impedance matching process. Example 22.5 demonstrates the significant difference in the power delivered to the load with and without an impedance matching transformer.

EXAMPLE 22.5

a. The source impedance for the supply in Fig. 22.8(a) is 500 Ω, which is a poor match with the 8 Ω input impedance of the speaker. You can expect only that the power delivered to the speaker will be significantly less than the maximum possible level. Determine the power to the speaker under the conditions in Fig. 22.8(a).
b. In Fig. 22.8(b), a commercially available 500 Ω to 8 Ω audio impedance matching transformer was introduced between the speaker and the source. Determine the input impedance of the transformer and the power delivered to the speaker.
c. Compare the power delivered to the speaker under the conditions of parts (a) and (b).
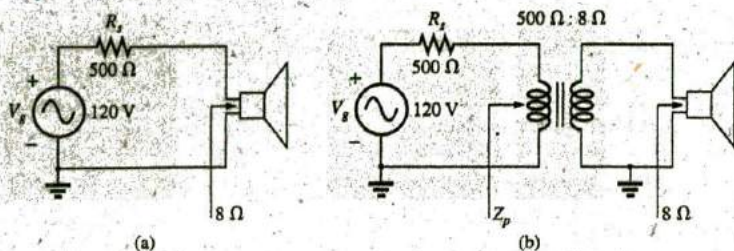d. Find the approximate turns ratio for the transformer.

FIG. 22.8
Example 22.5.

Solutions:

a. The source current:

$$I_s = \frac{E}{R_T} = \frac{120 \text{ V}}{500 \,\Omega + 8 \,\Omega} = \frac{120 \text{ V}}{508 \,\Omega} = 236.2 \text{ mA}$$

The power to the speaker:

$$P = I^2R = (236.2 \text{ mA})^2 \cdot 8 \,\Omega \cong 446.3 \text{ mW} \cong 0.45 \text{ W}$$

or less than ½ W.

b. Since the input impedance of the transformer matches that of the source, maximum power transfer conditions have been established, and the source current is now determined by

$$I_s = \frac{E}{R_T} = \frac{120 \text{ V}}{500 \,\Omega + 500 \,\Omega} = \frac{120 \text{ V}}{1000 \,\Omega} = 120 \text{ mA}$$

The power to the primary (which equals that to the secondary for the ideal transformer) is

$$P = I^2R = (120 \text{ mA})^2 \cdot 500 \,\Omega = 7.2 \text{ W}$$

The result is not in milliwatts, as obtained above, and exceeds 7 W, which is a significant improvement.

c. Comparing levels, we see that 7.2 W/446.3 mW = 16:1, or more than 16 times the power is delivered to the speaker using the impedance matching transformer.

d. $Z_p = a^2Z_L$

$$a = \sqrt{\frac{Z_p}{Z_L}} = \sqrt{\frac{500 \,\Omega}{8 \,\Omega}} = 7.91 \cong 8:1$$

Another important application of the impedance matching capabilities of a transformer is the matching of the 300 Ω twin line transmission line from a television antenna to the 75 Ω input impedance of a television (ready-made for the 75 Ω coaxial cable), as shown in Fig. 22.9. A match must be made to ensure the strongest signal to the television receiver.

Using the equation $Z_p = a^2Z_L$, we find

$$300 \,\Omega = a^2 75 \,\Omega$$

and

$$a = \sqrt{\frac{300 \,\Omega}{75 \,\Omega}} = \sqrt{4} = 2$$

with $N_p : N_s = 2:1$ (a step-down transformer)
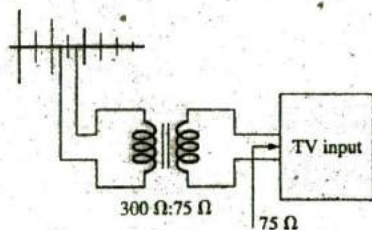
FIG. 22.9
Television impedance matching transformer.

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EXAMPLE 22.6 Impedance matching transformers are also quite evident in public address systems, such as the one appearing in the 70.7 V system in Fig. 22.10. Although the system has only one set of output terminals, up to four speakers can be connected to this system (the number is a function of the chosen system). Each 8 Ω speaker is connected to the 70.7 V line through a 10 W audio-matching transformer (defining the frequency range of linear operation).

a. If each speaker in Fig. 22.10 can receive 10 W of power, what is the maximum power drain on the source?
b. For each speaker, determine the impedance seen at the input side of the transformer if each is operating under its full 10 W of power.
c. Determine the turns ratio of the transformers.
d. At 10 W, what are the speaker voltage and current?
e. What is the load seen by the source with one, two, three, or four speakers connected?
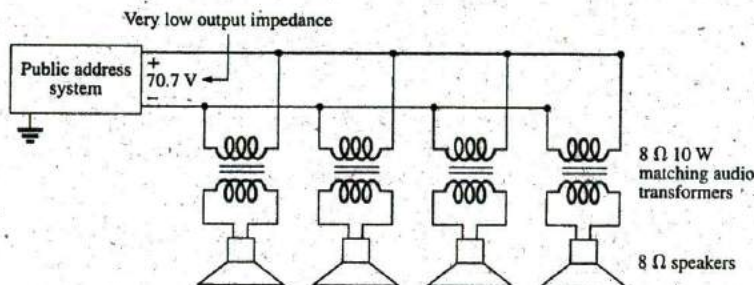
FIG. 22.10
Public address system.

Solutions:

a. Ideally, the primary power equals the power delivered to the load, resulting in a maximum of **40 W** from the supply.
b. The power at the primary is

$$P_p = V_p I_p = (70.7 \text{ V}) I_p = 10 \text{ W}$$

and

$$I_p = \frac{10 \text{ W}}{70.7 \text{ V}} = 141.4 \text{ mA}$$

so that

$$Z_p = \frac{V_p}{I_p} = \frac{70.7 \text{ V}}{141.4 \text{ mA}} = 500 \ \Omega$$

c. $Z_p = a^2 Z_L \Rightarrow a = \sqrt{\dfrac{Z_p}{Z_L}} = \sqrt{\dfrac{500 \ \Omega}{8 \ \Omega}} = \sqrt{62.5} = 7.91 \cong 8{:}1$

d. $V_s = V_L = \dfrac{V_p}{a} = \dfrac{70.7 \text{ V}}{7.91} = 8.94 \text{ V} \cong 9 \text{ V}$

e. All the speakers are in parallel. Therefore,

One speaker: $R_T = 500 \ \Omega$

Two speakers: $R_T = \dfrac{500 \ \Omega}{2} = 250 \ \Omega$

Introductory, C.- 64B

Three speakers: $\quad R_T = \dfrac{500\ \Omega}{3} = 167\ \Omega$

Four speakers: $\quad R_T = \dfrac{500\ \Omega}{4} = 125\ \Omega$

Even though the load seen by the source varies with the number of speakers connected, the source impedance is so low (compared to the lowest load of 125 Ω) that the terminal voltage of 70.7 V is essentially constant. This is not the case where the desired result is to match the load to the input impedance; rather, it was to ensure 70.7 V at each primary, no matter how many speakers were connected, and to limit the current drawn from the supply.

The transformer is frequently used to isolate one portion of an electrical system from another. *Isolation* implies the absence of any direct physical connection. As a first example of its use as an isolation device, consider the measurement of line voltages on the order of 40,000 V (Fig. 22.11).
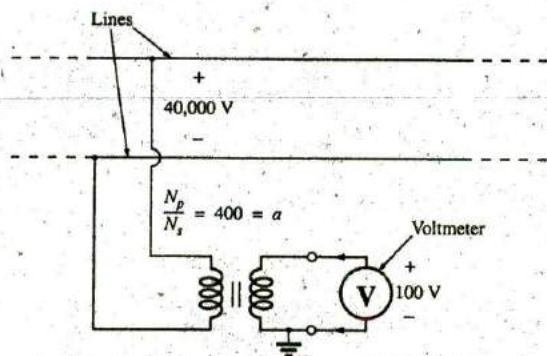
FIG. 22.11

Isolating a high-voltage line from the point of measurement.

To apply a voltmeter across 40,000 V would obviously be a dangerous task due to the possibility of physical contact with the lines when making the necessary connections. Including a transformer in the transmission system as original equipment can bring the potential down to a safe level for measurement purposes and can determine the line voltage using the turns ratio. Therefore, the transformer serves both to isolate and to step down the voltage.

As a second example, consider the application of the voltage v_x to the vertical input of the oscilloscope (a measuring instrument) in Fig. 22.12. If the connections are made as shown, and if the generator and oscilloscope have a common ground, the impedance Z_2 has been effectively shorted out of the circuit by the ground connection of the oscilloscope. The input voltage to the oscilloscope is therefore meaningless as far as the voltage v_x is concerned. In addition, if Z_2 is the current-limiting impedance in the circuit, the current in the circuit may rise to a level that causes severe damage to the circuit. If a transformer is used as shown in Fig. 22.13, this problem is eliminated, and the input voltage to the oscilloscope will be v_x.
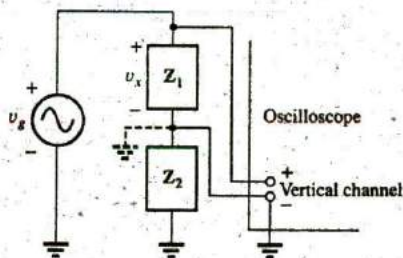

FIG. 22.12

Demonstrating the shorting effect introduced by the grounded side of the vertical channel of an oscilloscope.
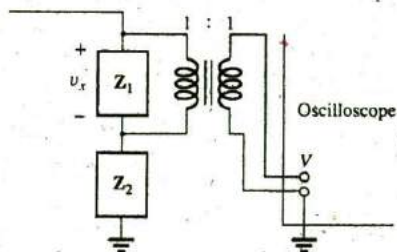

FIG. 22.13

Correcting the situation of Fig. 22.12 using an isolation transformer.

The linear variable differential transformer (LVDT) is a sensor that can reveal displacement using transformer effects. In its simplest form, the LVDT has a central winding and two secondary windings, as shown in Fig. 22.14(a). A ferromagnetic core inside the windings is free to move as dictated by some external force. A constant, low-level ac voltage is applied to the primary, and the output voltage is the difference between the voltages induced in the secondaries. If the core is in the position shown in Fig. 22.14(b), a relatively large voltage is induced across the secondary winding labeled coil 1, and a relatively small voltage is induced across the secondary winding labeled coil 2 (essentially an air-core transformer for this position). The result is a relatively large secondary output voltage. If the core is in the position shown in Fig. 22.14(c), the flux linking each coil is the same, and the output voltage (being the difference) will be quite small. In total, therefore, the position of the core can be related to the secondary voltage, and a position-versus-voltage graph can be developed as shown in Fig. 22.14(d). Due to the nonlinearity of the B-H curve, the curve becomes somewhat nonlinear if the core is moved too far out of the unit.
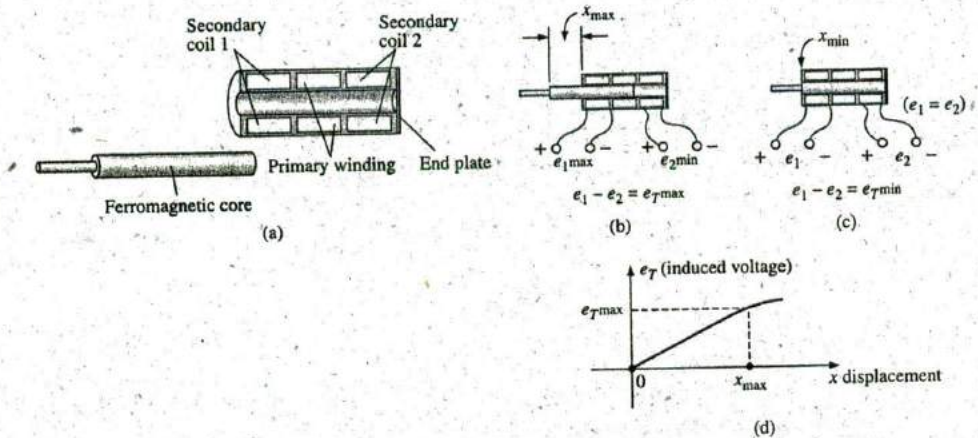
FIG. 22.14

LVDT transformer: (a) construction; (b) maximum displacement; (c) minimum displacement; (d) graph of induced voltage versus displacement.

22.6 EQUIVALENT CIRCUIT (IRON-CORE TRANSFORMER)

For the nonideal or practical iron-core transformer, the equivalent circuit appears as in Fig. 22.15. As indicated, part of this equivalent circuit includes an ideal transformer. The remaining elements of Fig. 22.15 are those elements that contribute to the nonideal characteristics of the device. The resistances R_p and R_s are simply the dc or geometric resistance of the primary and secondary windings, respectively. For the primary and secondary coils of a transformer, there is a small amount of flux that links each coil but does not pass through the core, as shown in Fig. 22.16 for the primary winding. This **leakage flux**, representing a definite loss in the system, is represented by an inductance L_p in the primary circuit and an inductance L_s in the secondary.
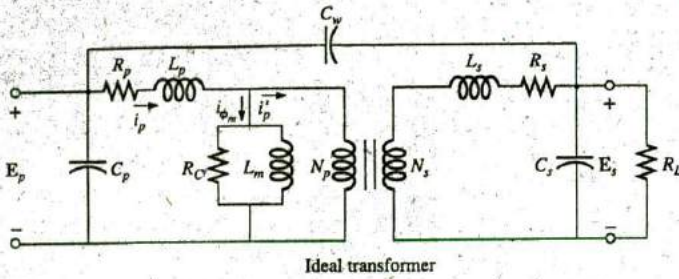
FIG. 22.15

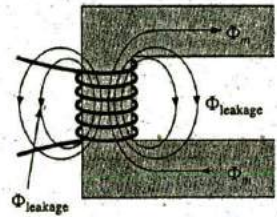*Equivalent circuit for the practical iron-core transformer.*

FIG. 22.16

Identifying the leakage flux of the primary.

The resistance R_c represents the hysteresis and eddy current losses (core losses) within the core due to an ac flux through the core. The inductance L_m (magnetizing inductance) is the inductance associated with the magnetization of the core, that is, the establishing of the flux ϕ_m in the core. The capacitances C_p and C_s are the lumped capacitances of the primary and secondary circuits, respectively, and C_w represents the equivalent lumped capacitances between the windings of the transformer.

Since i'_p is normally considerably larger than i_{ϕ_m} (the magnetizing current), we will ignore i_{ϕ_m} for the moment (set it equal to zero), resulting in the absence of R_c and L_m in the reduced equivalent circuit in Fig. 22.17. The capacitances C_p, C_w, and C_s do not appear in the equivalent circuit in Fig. 22.17 since their reactance at typical operating frequencies do not appreciably affect the transfer characteristics of the transformer.
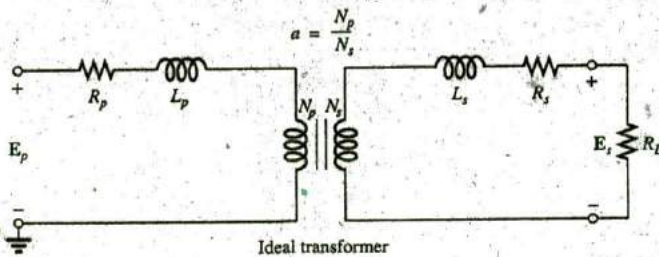
FIG. 22.17

Reduced equivalent circuit for the nonideal iron-core transformer.

If we now reflect the secondary circuit through the ideal transformer using Eq. (22.19), as shown in Fig. 22.18(a), we will have the load and generator voltage in the same continuous circuit. The total resistance and inductive reactance of the primary circuit are determined by

$$R_{equivalent} = R_e = R_p + a^2 R_s \qquad (22.22)$$

and

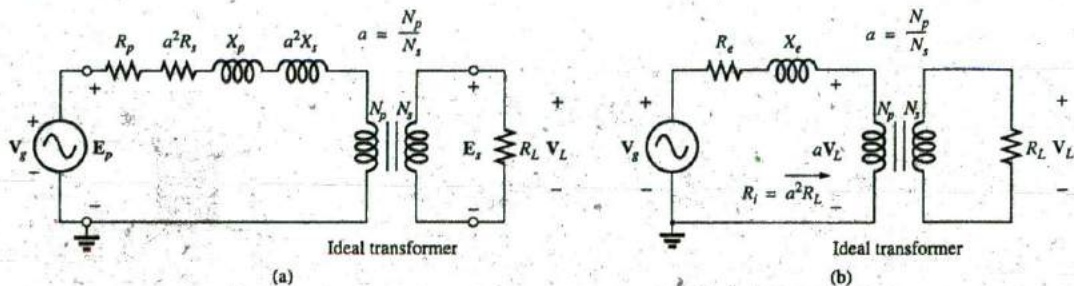$$X_{equivalent} = X_e = X_p + a^2 X_s \qquad (22.23)$$

FIG. 22.18

Reflecting the secondary circuit into the primary side of the iron-core transformer.

which result in the useful equivalent circuit of Fig. 22.18(b). The load voltage can be obtained directly from the circuit in Fig. 22.18(b) through the voltage divider rule:

$$a V_L = \frac{R_i V_g}{(R_e + R_i) + jX_e}$$

and

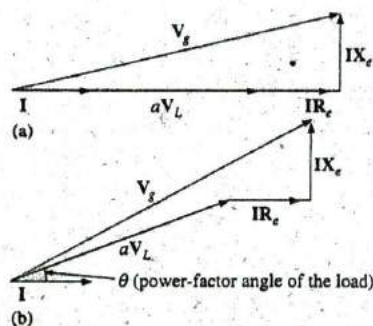$$\boxed{V_L = \frac{a^2 R_L V_g}{(R_e + a^2 R_L) + jX_e}}$$

(22.24)

The network in Fig. 22.18(b) also allows us to calculate the generator voltage necessary to establish a particular load voltage. The voltages across the elements in Fig. 22.18(b) have the phasor relationship indicated in Fig. 22.19(a). Note that the current is the reference phasor for drawing the phasor diagram. That is, the voltages across the resistive elements are *in phase* with the current phasor, while the voltage across the equivalent inductance leads the current by 90°. The primary voltage, by Kirchhoff's voltage law, is then the phasor sum of these voltages, as indicated in Fig. 22.19(a). For an inductive load, the phasor diagram appears in Fig. 22.19(b). Note that aV_L leads I by the power-factor angle of the load. The remainder of the diagram is then similar to that for a resistive load. (The phasor diagram for a capacitive load is left to the reader as an exercise.)

The effect of R_e and X_e on the magnitude of V_g for a particular V_L is obvious from Eq. (22.24) or Fig. 22.19. For increased values of R_e or X_e, an increase in V_g is required for the same load voltage. For R_e and X_e = 0, V_L and V_g are simply related by the turns ratio.

FIG. 22.19

Phasor diagram for the iron-core transformer with (a) unity power-factor load (resistive) and (b) lagging power-factor load (inductive).

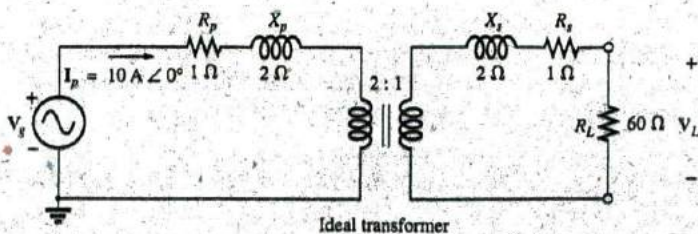**EXAMPLE 22.7** For a transformer having the equivalent circuit in Fig. 22.20:

FIG. 22.20

Example 22.7

a. Determine R_e and X_e.
b. Determine the magnitude of the voltages V_L and V_g.
c. Determine the magnitude of the voltage V_g to establish the same load voltage in part (b) if R_e and $X_e = 0\,\Omega$. Compare with the result of part (b).

Solutions:

a. $R_e = R_p + a^2R_s = 1\,\Omega + (2)^2(1\,\Omega) = 5\,\Omega$

$X_e = X_p + a^2X_s = 2\,\Omega + (2)^2(2\,\Omega) = 10\,\Omega$

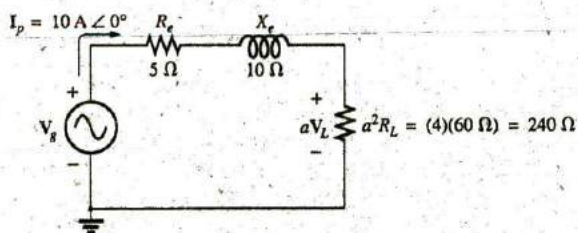b. The transformed equivalent circuit appears in Fig. 22.21.

$$aV_L = (I_p)(a^2R_L) = 2400\ \text{V}$$

Thus,

$$V_L = \frac{2400\ \text{V}}{a} = \frac{2400\ \text{V}}{2} = 1200\ \text{V}$$

and

$V_g = I_p(R_e + a^2R_L + jX_e)$
$\quad = 10\ \text{A}(5\,\Omega + 240\,\Omega + j\,10\,\Omega) = 10\ \text{A}(245\,\Omega + j\,10\,\Omega)$
$V_g = 2450\ \text{V} + j\,100\ \text{V} = 2452.04\ \text{V}\ \angle 2.34°$
$\quad = 2452.04\ \text{V}\ \angle 2.34°$

FIG. 22.21

Transformed equivalent circuit of Fig. 22.20.

c. For R_e and $X_e = 0$, $V_g = aV_L = (2)(1200\ \text{V}) = 2400\ \text{V}$.

Therefore, it is necessary to increase the generator voltage by 52.04 V (due to R_e and X_e) to obtain the same load voltage.

22.7 FREQUENCY CONSIDERATIONS

For certain frequency ranges, the effect of some parameters in the equivalent circuit of the iron-core transformer in Fig. 22.15 should not be ignored. Since it is convenient to consider a low-, mid-, and high-frequency region, the equivalent circuits for each are now introduced and briefly examined.

For the low-frequency region, the series reactance $(2\pi fL)$ of the primary and secondary leakage reactances can be ignored since they are small in magnitude. The magnetizing inductance must be included, however, since it appears in parallel with the secondary reflected circuit, and small impedances in a parallel network can have a dramatic impact on the terminal characteristics. The resulting equivalent network for the low-frequency region is provided in Fig. 22.22(a). As the frequency decreases, the reactance of the magnetizing inductance reduces in magnitude, causing a reduction in the voltage across the secondary circuit. For $f = 0$ Hz, L_m is ideally a short circuit, and $V_L = 0$. As the frequency
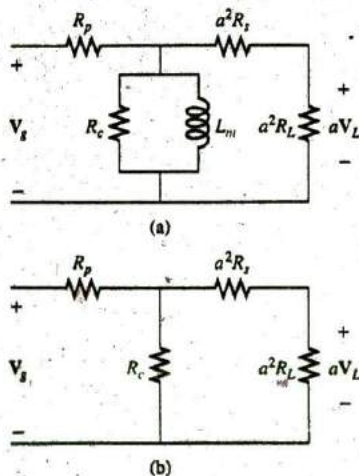
FIG. 22.22

(a) Low-frequency reflected equivalent circuit;
(b) mid-frequency reflected circuit.

increases, the reactance of L_m is eventually sufficiently large compared with the reflected secondary impedance to be neglected. The mid-frequency reflected equivalent circuit then appears as shown in Fig. 22.22(b). Note the absence of reactive elements, resulting in an *in-phase* relationship between load and generator voltages.

For higher frequencies, the capacitive elements and primary and secondary leakage reactances must be considered, as shown in Fig. 22.23. For discussion purposes, the effects of C_w and C_s appear as a lumped capacitor C in the reflected network in Fig. 22.23; C_p does not appear since the effect of C predominates. As the frequency of interest increases, the capacitive reactance $(X_C = 1/2\pi fC)$ decreases to the point that it will have a shorting effect across the secondary circuit of the transformer, causing V_L to decrease in magnitude.
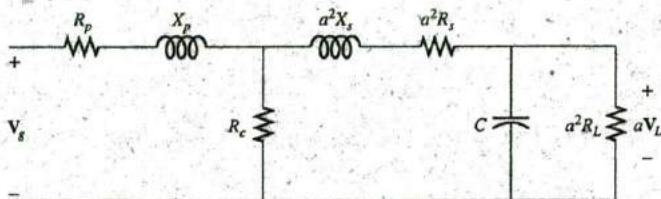
FIG. 22.23
High-frequency reflected equivalent circuit.

A typical iron-core transformer-frequency response curve appears in Fig. 22.24. For the low- and high-frequency regions, the primary element responsible for the drop-off is indicated. The peaking that occurs in the high-frequency region is due to the series resonant circuit established by the inductive and capacitive elements of the equivalent circuit. In the peaking region, the series resonant circuit is in, or near, its resonant or tuned state.
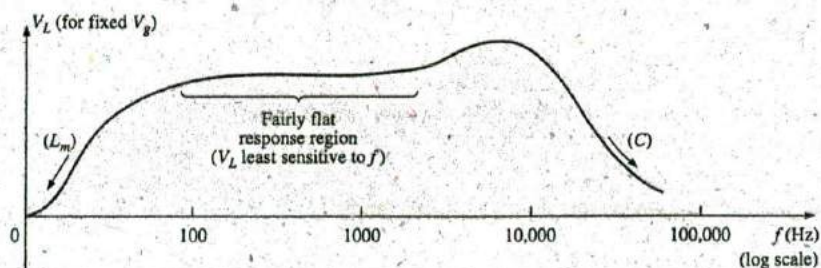
FIG. 22.24
Transformer-frequency response curve.

22.8 SERIES CONNECTION OF MUTUALLY COUPLED COILS

In Chapter 11, we found that the total inductance of series isolated coils was determined simply by the sum of the inductances. For two coils that are connected in series but also share the same flux linkages, such as those in Fig. 22.25(a), a mutual term is introduced that alters the total inductance of the series combination. The physical picture of how the coils are connected is indicated in Fig. 22.25(b). An iron core is included,

although the equations to be developed are for any two mutually coupled coils with any value of coefficient of coupling k. When referring to the voltage induced across the inductance L_1 (or L_2) due to the change in flux linkages of the inductance L_2 (or L_1, respectively), the mutual inductance is represented by M_{12}. This type of subscript notation is particularly important when there are two or more mutual terms.

Due to the presence of the mutual term, the induced voltage e_1 is composed of that due to the self-inductance L_1 and that due to the mutual inductance M_{12}. That is,

$$e_1 = L_1 \frac{di_1}{dt} + M_{12} \frac{di_2}{dt}$$

However, since $i_1 = i_2 = i$,

$$e_1 = L_1 \frac{di}{dt} + M_{12} \frac{di}{dt}$$

or

$$\boxed{e_1 = (L_1 + M_{12}) \frac{di}{dt}} \qquad \text{(volts, V)} \qquad \textbf{(22.25)}$$

and, similarly,

$$\boxed{e_2 = (L_2 + M_{12}) \frac{di}{dt}} \qquad \text{(volts, V)} \qquad \textbf{(22.26)}$$

For the series connection, the total induced voltage across the series coils, represented by e_T, is

$$e_T = e_1 + e_2 = (L_1 + M_{12}) \frac{di}{dt} + (L_2 + M_{12}) \frac{di}{dt}$$

or

$$e_T = (L_1 + L_2 + M_{12} + M_{12}) \frac{di}{dt}$$

and the total effective inductance is

$$\boxed{L_{T(+)} = L_1 + L_2 + 2M_{12}} \qquad \text{(henries, H)} \qquad \textbf{(22.27)}$$

The subscript $(+)$ was included to indicate that the mutual terms have a positive sign and are added to the self-inductance values to determine the total inductance. If the coils are wound such as shown in Fig. 22.26, where ϕ_1 and ϕ_2 are in opposition, the induced voltages due to the mutual terms oppose that due to the self-inductance, and the total inductance is determined by

$$\boxed{L_{T(-)} = L_1 + L_2 - 2M_{12}} \qquad \text{(henries, H)} \qquad \textbf{(22.28)}$$

Through Eqs. (22.27) and (22.28), the mutual inductance can be determined by

$$\boxed{M_{12} = \frac{1}{4}(L_{T(+)} - L_{T(-)})} \qquad \textbf{(22.29)}$$

Eq. (22.29) is very effective in determining the mutual inductance between two coils. It states that the mutual inductance is equal to one-quarter the difference between the total inductance with a positive and negative mutual effect.
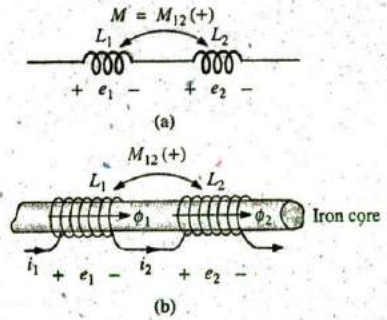
FIG. 22.25
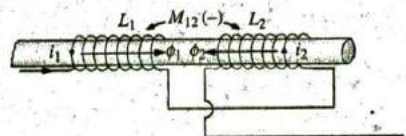
Mutually coupled coils connected in series.

FIG. 22.26

Mutually coupled coils connected in series with negative mutual inductance.
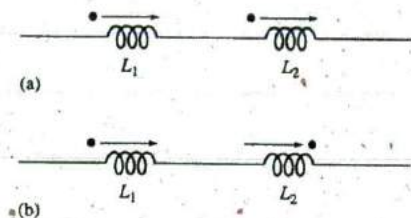
(a)

(b)

FIG. 22.27

Dot convention for the series coils in (a) Fig. 22.25 and (b) Fig. 22.26.

From the preceding, it should be clear that the mutual inductance directly affects the magnitude of the voltage induced across a coil since it determines the net inductance of the coil. Additional examination reveals that the sign of the mutual term for each coil of a coupled pair is the same. For $L_{T(+)}$ they are both positive, and for $L_{T(-)}$ they are both negative. On a network schematic where it is inconvenient to indicate the windings and the flux path, a system of dots is used that determines whether the mutual terms are to be positive or negative. The dot convention is shown in Fig. 22.27 for the series coils in Figs. 22.25 and 22.26.

If the current through *each* of the mutually coupled coils is going away from (or toward) the dot as it *passes through the coil*, the mutual term will be positive, as shown for the case in Fig 22.27(a). If the arrow indicating current direction through the coil is leaving the dot for one coil and entering the dot for the other, the mutual term is negative.

A few possibilities for mutually coupled transformer coils are indicated in Fig. 22.28(a). The sign of M is indicated for each. When determining the sign, be sure to examine the current direction within the coil itself. In Fig. 22.28(b), one direction is indicated outside for one coil and through for the other. It initially may appear that the sign should be positive since both currents enter the dot, but the current *through* coil 1 is leaving the dot; hence a negative sign is in order.

$M(+)$ $M(+)$ $M(-)$ $M(-)$

(a)   (b)
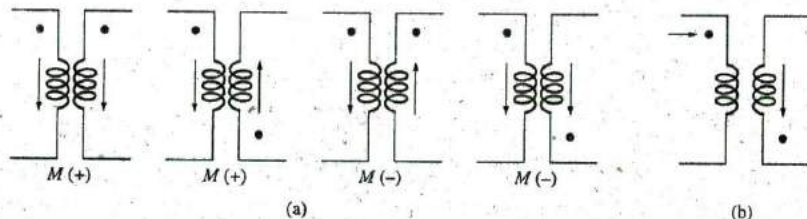
FIG. 22.28

Defining the sign of M for mutually coupled transformer coils.

The dot convention also reveals the polarity of the *induced* voltage across the mutually coupled coil. If the reference direction for the current *in* a coil leaves the dot, the polarity at the dot for the induced voltage of the mutually coupled coil is positive. In the first two figures in Fig. 22.28(a), the polarity at the dots of the induced voltages is positive. In the third figure in Fig. 22.28(a), the polarity at the dot of the right coil is negative, while the polarity at the dot of the left coil is positive, since the current enters the dot (within the coil) of the right coil. The comments for the third figure in Fig. 22.28(a) can also be applied to the last figure in Fig. 22.28(a).

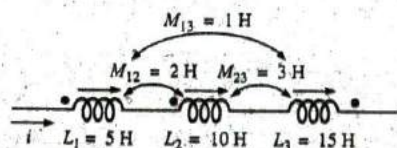**EXAMPLE 22.8** Find the total inductance of the series coils in Fig. 22.29.

Solution:

FIG. 22.29

Example 22.8.

Current vectors leave dot.

Coil 1: $L_1 + M_{12} - M_{13}$

One current vector enters dot, while one leaves:

Coil 2: $L_2 + M_{12} - M_{23}$

Coil 3: $L_3 - M_{23} - M_{13}$

and

$$L_T = (L_1 + M_{12} - M_{13}) + (L_2 + M_{12} - M_{23}) + (L_3 - M_{23} - M_{13})$$
$$= L_1 + L_2 + L_3 + 2M_{12} - 2M_{23} - 2M_{13}$$

Substituting values, we find

$$L_T = 5\,H + 10\,H + 15\,H + 2(2\,H) - 2(3\,H) - 2(1\,H)$$
$$= 34\,H - 8\,H = 26\,H$$

EXAMPLE 22.9 Write the mesh equations for the transformer network in Fig. 22.30.

Solution: For each coil, the mutual term is positive, and the sign of M in $X_m = \omega M \angle 90°$ is positive, as determined by the direction of I_1 and I_2. Thus,

$$E_1 - I_1 R_1 - I_1 X_{L_1} \angle 90° - I_2 X_m \angle 90° = 0$$

or
$$E_1 - I_1(R_1 + jX_{L_1}) - I_2 X_m \angle 90° = 0$$

For the other loop,

$$-I_2 X_{L_2} \angle 90° - I_1 X_m \angle 90° - I_2 R_L = 0$$

or
$$I_2(R_L + jX_{L_2}) - I_1 X_m \angle 90° = 0$$
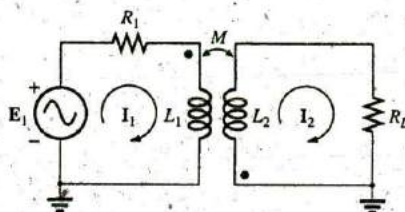
FIG. 22.30
Example 22.9.

22.9 AIR-CORE TRANSFORMER

As the name implies, the air-core transformer does not have a ferromagnetic core to link the primary and secondary coils. Rather, the coils are placed sufficiently close to have a mutual inductance that establishes the desired transformer action. In Fig. 22.31, current direction and polarities have been defined for the air-core transformer. Note the presence of a mutual inductance term M, which is positive in this case, as determined by the dot convention.
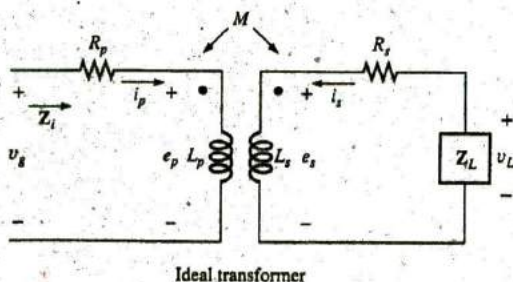
Ideal transformer

FIG. 22.31
Air-core transformer equivalent circuit.

From past analysis in this chapter, we now know that

$$e_p = L_p \frac{di_p}{dt} + M \frac{di_s}{dt} \qquad (22.30)$$

for the primary circuit.

We found in Chapter 11 that for the pure inductor, with no mutual inductance present, the mathematical relationship

$$v_1 = L \frac{di_1}{dt}$$

resulted in the following useful form of the voltage across an inductor:

$$\mathbf{V}_1 = \mathbf{I}_1 X_L \angle 90° \quad \text{where} \quad X_{L_1} = \omega L$$

Similarly, it can be shown, for a mutual inductance, that

$$v_1 = M \frac{di_2}{dt}$$

results in

$$\boxed{\mathbf{V}_1 = \mathbf{I}_2 X_m \angle 90° \quad \text{where } X_m = \omega M} \tag{22.31}$$

Eq. (22.30) can then be written (using phasor notation) as

$$\boxed{\mathbf{E}_p = \mathbf{I}_p X_{L_p} \angle 90° + \mathbf{I}_s X_m \angle 90°} \tag{22.32}$$

and

$$\mathbf{V}_g = \mathbf{I}_p R_p \angle 0° + \mathbf{I}_p X_{L_p} \angle 90° + \mathbf{I}_s X_m \angle 90°$$

or

$$\boxed{\mathbf{V}_g = \mathbf{I}_p(R_p + jX_{L_p}) + \mathbf{I}_s X_m \angle 90°} \tag{22.33}$$

For the secondary circuit,

$$\boxed{\mathbf{E}_s = \mathbf{I}_s X_{L_s} \angle 90° + \mathbf{I}_p X_m \angle 90°} \tag{22.34}$$

and

$$\mathbf{V}_L = \mathbf{I}_s R_s \angle 0° + \mathbf{I}_s X_{L_s} \angle 90° + \mathbf{I}_p X_m \angle 90°$$

or

$$\boxed{\mathbf{V}_L = \mathbf{I}(R_s + jX_{L_s}) + \mathbf{I}_p X_m \angle 90°} \tag{22.35}$$

Substituting

$$\mathbf{V}_L = -\mathbf{I}_s \mathbf{Z}_L$$

into Eq. (22.35) results in

$$0 = \mathbf{I}_s(R_s + jX_{L_s} + \mathbf{Z}_L) + \mathbf{I}_p X_m \angle 90°$$

Solving for $\mathbf{I}_s$, we have

$$\mathbf{I}_s = \frac{-\mathbf{I}_p X_m \angle 90°}{R_s + jX_{L_s} + \mathbf{Z}_L}$$

and substituting into Eq. (22.33), we obtain

$$\mathbf{V}_g = \mathbf{I}_p(R_p + jX_{L_p}) + \left(\frac{-\mathbf{I}_p X_m \angle 90°}{R_s + jX_{L_s} + \mathbf{Z}_L} \right) X_m \angle 90°$$

Thus, the input impedance is

$$\mathbf{Z}_i = \frac{\mathbf{V}_g}{\mathbf{I}_p} = R_p + jX_{L_p} - \frac{(X_m \angle 90°)^2}{R_s + jX_{L_s} + \mathbf{Z}_L}$$

or, defining

$$\mathbf{Z}_p = R_p + jX_{L_p} \qquad \mathbf{Z}_s = R_s + jX_{L_s} \qquad \text{and} \qquad X_m \angle 90° = +j\omega M$$

we have

$$Z_i = Z_p - \frac{(+j\omega M)^2}{Z_s + Z_L}$$

and

$$\boxed{Z_i = Z_p - \frac{(\omega M)^2}{Z_s + Z_L}}$$ (22.36)

The term $(\omega M)^2/(Z_s + Z_L)$ is called the *coupled impedance*, and it is independent of the sign of M since it is squared in the equation. Consider also that since $(\omega M)^2$ is a constant with $0°$ phase angle, if the load Z_L is resistive, the resulting coupled impedance term appears capacitive due to division of $(Z_s + R_L)$ into $(\omega M)^2$. This resulting capacitive reactance opposes the series primary inductance L_p, causing a reduction in Z_i. Including the effect of the mutual term, the input impedance to the network appears as shown in Fig. 22.32.
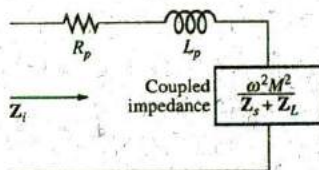
FIG. 22.32

Input characteristics for the air-core transformer.

EXAMPLE 22.10 Determine the input impedance to the air-core transformer in Fig. 22.33.
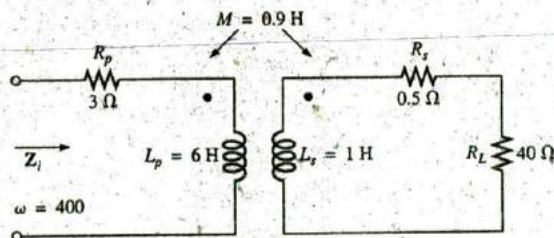
FIG. 22.33

Example 22.10.

Solution:

$$Z_i = Z_p + \frac{(\omega M)^2}{Z_s + Z_L}$$

$$= R_p + jX_{L_p} + \frac{(\omega M)^2}{R_s + jX_{L_s} + R_L}$$

$$= 3\,\Omega + j\,2.4\ \text{k}\Omega + \frac{((400\ \text{rad/s})(0.9\ \text{H}))^2}{0.5\,\Omega + j\,400\,\Omega + 40\,\Omega}$$

$$\cong j\,2.4\ \text{k}\Omega + \frac{129.6 \times 10^3\,\Omega}{40.5 + j\,400}$$

$$= j\,2.4\ \text{k}\Omega + \underset{\text{capacitive}}{322.4\,\Omega\ \angle{-84.22°}}$$

$$= j\,2.4\ \text{k}\Omega + (0.0325\ \text{k}\Omega - j\,0.3208\ \text{k}\Omega)$$

$$= 0.0325\ \text{k}\Omega + j\,(2.40 - 0.3208)\ \text{k}\Omega$$

and $Z_i = R_i + jX_{L_i} = 32.5\,\Omega + j\,2079\,\Omega = 2079.25\,\Omega\ \angle 89.10°$

22.10 NAMEPLATE DATA

A typical iron-core power transformer rating included in the **nameplate data** for the transformer might be the following:

<center>5 kVA   2000/100 V   60 Hz</center>

The 2000 V or the 100 V can be either the primary or the secondary voltage; that is, if 2000 V is the primary voltage, then 100 V is the secondary voltage, and vice versa. The 5 kVA is the apparent power $(S = VI)$ rating of the transformer. If the secondary voltage is 100 V, then the maximum load current is

$$I_L = \frac{S}{V_L} = \frac{5000 \text{ VA}}{100 \text{ V}} = 50 \text{ A}$$

and if the secondary voltage is 2000 V, then the maximum load current is

$$I_L = \frac{S}{V_L} = \frac{5000 \text{ VA}}{2000 \text{ V}} = 2.5 \text{ A}$$

The transformer is rated in terms of the apparent power rather than the average, or real, power for the reason demonstrated by the circuit in Fig. 22.34. Since the current through the load is greater than that determined by the apparent power rating, the transformer may be permanently damaged. Note, however, that since the load is purely capacitive, the average power to the load is zero. The wattage rating is therefore meaningless regarding the ability of this load to damage the transformer.

The transformation ratio of the transformer under discussion can be either of two values. If the secondary voltage is 2000 V, the transformation ratio is $a = N_p/N_s = V_g/V_L = 100 \text{ V}/2000 \text{ V} = 1/20$, and the transformer is a step-up transformer. If the secondary voltage is 100 V, the transformation ratio is $a = N_p/N_s = V_g/V_L = 2000 \text{ V}/100 \text{ V} = 20$, and the transformer is a step-down transformer.

The rated primary current can be determined by applying Eq. (22.18):

$$I_p = \frac{I_s}{a}$$

which is equal to $[2.5 \text{ A}/(1/20)] = 50 \text{ A}$ if the secondary voltage is 2000 V, and $(50 \text{ A}/20) = 2.5 \text{ A}$ if the secondary voltage is 100 V.

To explain the necessity for including the frequency in the nameplate data, consider Eq. (22.12):

$$E_p = 4.44 f_p N_p \Phi_m$$

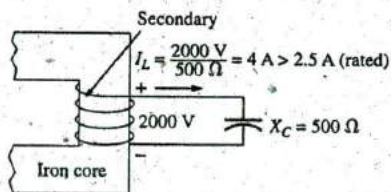and the B-H curve for the iron core of the transformer (Fig. 22.35).

Secondary

$I_L = \dfrac{2000 \text{ V}}{500 \ \Omega} = 4 \text{ A} > 2.5 \text{ A (rated)}$

2000 V $X_C = 500 \ \Omega$

Iron core

FIG. 22.34
Demonstrating why transformers are rated in kVA rather than kW.

$B = \dfrac{\Phi_m}{A_{core}}$

$\Delta B = \dfrac{\Delta \Phi_m}{A_{core}}$

Knee of curve

$\Delta H = \dfrac{N_1 \Delta I_1}{l_{core}}$ $H = \dfrac{N_1 I_1}{l_{core}}$
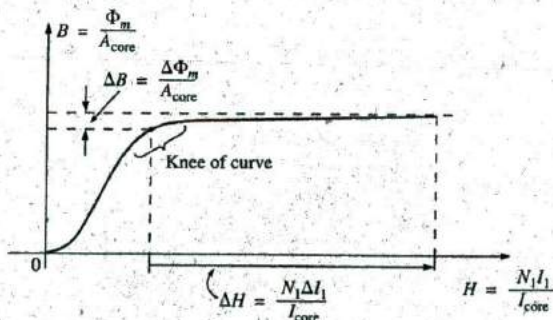
FIG. 22.35
*Demonstrating why the frequency of application is important for transformers.*

The point of operation on the *B-H* curve for most transformers is at the knee of the curve. If the frequency of the applied signal drops and N_p and E_p remain the same, then Φ_m must increase in magnitude, as determined by Eq. (22.12):

$$\Phi_m \uparrow = \frac{E_p}{4.44 f_p \downarrow N_p}$$

The result is that *B* increases, as shown in Fig. 22.35, causing *H* to increase also. The resulting ΔI could cause a very high current in the primary, resulting in possible damage to the transformer.

22.11 TYPES OF TRANSFORMERS

Transformers are available in many different shapes and sizes. Some of the more common types include the power transformer, audio transformer, IF (intermediate frequency) transformer, and RF (radio frequency) transformer. Each is designed to fulfill a particular requirement in a specific area of application. The symbols for some of the basic types of transformers are shown in Fig. 22.36.

Air-core   Iron-core   Variable-core
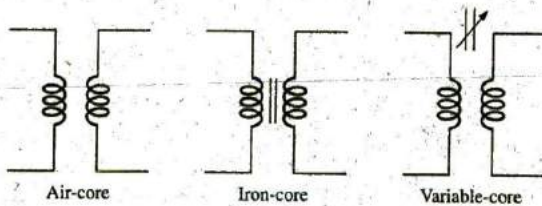
FIG. 22.36
Transformer symbols.

The method of construction varies from one transformer to another. Two of the many different ways in which the primary and secondary coils can be wound around an iron core are shown in Fig. 22.37. In either case, the core is made of laminated sheets of ferromagnetic material

(a) Core type   (b) Shell type
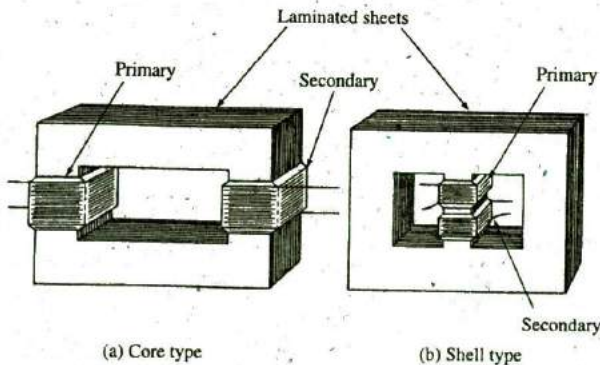
FIG. 22.37
Types of ferromagnetic core construction.
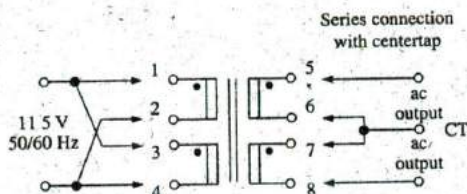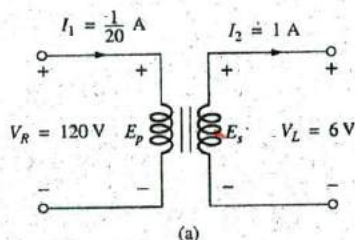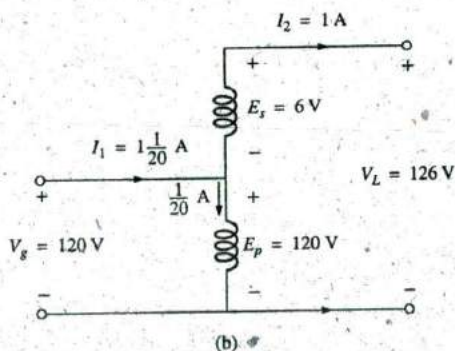
separated by an insulator to reduce the eddy current losses. The sheets themselves also contain a small percentage of silicon to increase the electrical resistivity of the material and further reduce the eddy current losses.

A variation of the core-type transformer appears in Fig. 22.38. This transformer is designed for low-profile (the 1.1 VA size has a maximum height of only 1 in.) applications in power, control, and instrumentation applications. There are actually two transformers on the same core, with the primary and secondary of each wound side by side. The schematic is provided in Fig. 22.38 for a single 115 V, 50/60 Hz input using a series connection with centertap for the output. For this unit, the output voltage is 10 V line to centertap with a current rating of 0.11 A, satisfying the condition that (10 V) (0.11 A) = 1.1 VA as indicated above. Note the dot convention and the commercial representation of the transformer coils.

The **autotransformer** [Fig. 22.39(b)] is a type of power transformer that, instead of employing the two-circuit principle (complete isolation between coils), has one winding common to both the input and the output circuits. The induced voltages are related to the turns ratio in the same manner as that described for the two-circuit transformer. If the proper connection is used, a two-circuit power transformer can be used as an autotransformer. The advantage of using it as an autotransformer is that a larger apparent power can be transformed. This can be demonstrated by the two-circuit transformer of Fig. 22.39(a), shown in Fig. 22.39(b) as an autotransformer.

For the two-circuit transformer, note that $S = (\frac{1}{20} \text{A})(120 \text{ V}) = 6 \text{ VA}$, whereas for the autotransformer, $S = (1\frac{1}{20} \text{A})(120 \text{ V}) = 126 \text{ VA}$, which is many times that of the two-circuit transformer. Note also that the current and voltage of each coil are the same as those for the two-circuit configuration. The disadvantage of the autotransformer is obvious: loss of the isolation between the primary and secondary circuits.

A pulse transformer designed for printed-circuit applications where high-amplitude, long-duration pulses must be transferred without saturation appears in Fig. 22.40. Turns ratios are available from 1 : 1 to 5 : 1 at maximum line voltages of 240 V rms at 60 Hz. The upper unit is for printed-circuit applications with isolated dual primaries; whereas the lower unit is the bobbin variety with a single primary winding.
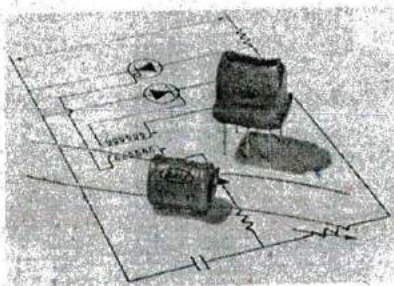

FIG. 22.38

Laminated power transformer.
(Courtesy of the Tamura Corp. of America.)

(a)

(b)

FIG. 22.39
(a) Two-circuit transformer; (b) autotransformer.

FIG. 22.40
Pulse transformers.
(Courtesy of DALE Electronics, Inc.)

The ultra-wideband audio transformer in Fig. 22.41 is designed to work in the frequency range of 300 Hz to 100 kHz. The unit is quite small, with dimensions of 0.31 in. by 0.41 in. for the area and 0.465 in. for the height. The ac impedance at the primary is 200 kΩ CT, and it is 1 kΩ CT at secondary. The turns ratio is 14.1 CT:1 CT, with a maximum power rating of 10 mW. The dc resistance of the primary is 5.3 kΩ, with 120 Ω at the secondary.
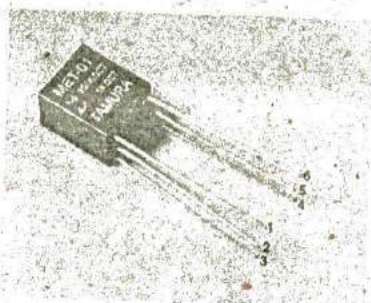
22.12 TAPPED AND MULTIPLE-LOAD TRANSFORMERS

For the **center-tapped** (primary) transformer in Fig. 22.42, where the voltage from the center tap to either outside lead is defined as $E_p/2$, the relationship between E_p and E_s is

$$\boxed{\frac{E_p}{E_s} = \frac{N_p}{N_s}} \tag{22.37}$$

FIG. 22.41
Ultra-wideband transformer.
(Courtesy of the Tamura Corp. of America)
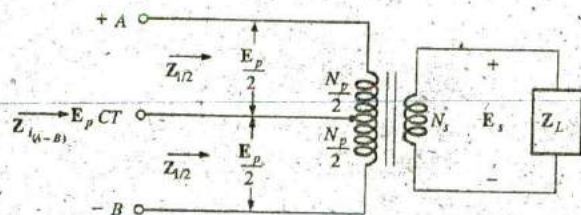
FIG. 22.42
Ideal transformer with a center-tapped primary.

For each half-section of the primary,

$$Z_{1/2} = \left(\frac{N_p/2}{N_s}\right)^2 Z_L = \frac{1}{4}\left(\frac{N_p}{N_s}\right)^2 Z_L$$

with

$$Z_{i_{(A-B)}} = \left(\frac{N_p}{N_s}\right)^2 Z_L$$

Therefore,

$$\boxed{Z_{1/2} = \frac{1}{4} Z_i} \tag{22.38}$$

For the **multiple-load transformer** in Fig. 22.43, the following equations apply:

$$\boxed{\frac{E_i}{E_2} = \frac{N_1}{N_2} \qquad \frac{E_i}{E_3} = \frac{N_1}{N_3} \qquad \frac{E_2}{E_3} = \frac{N_2}{N_3}} \tag{22.39}$$

The total input impedance can be determined by first noting that, for the ideal transformer, the power delivered to the primary is equal to the power dissipated by the load; that is,

$$P_1 = P_{L_2} + P_{L_3}$$
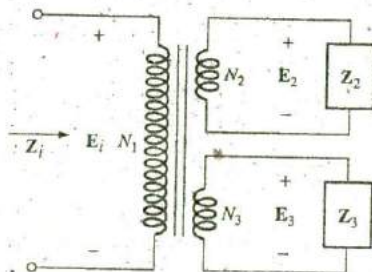
FIG. 22.43
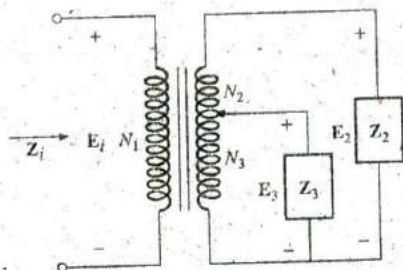*Ideal transformer with multiple loads.*

Introductory, C.- 65A

FIG. 22.44

Ideal transformer with a tapped secondary and multiple loads.

and for resistive loads ($Z_i = R_i$, $Z_2 = R_2$, and $Z_3 = R_3$),

$$\frac{E_i^2}{R_i} = \frac{E_2^2}{R_2} = \frac{E_3^2}{R_3}$$

or, since

$$E_2 = \frac{N_2}{N_1}E_i \quad \text{and} \quad E_3 = \frac{N_3}{N_1}E_1$$

then

$$\frac{E_i^2}{R_i} = \frac{[(N_2/N_1)E_i]^2}{R_2} + \frac{[(N_3/N_1)E_i]^2}{R_3}$$

and

$$\frac{E_i^2}{R_i} = \frac{E_i^2}{(N_1/N_2)^2 R_2} + \frac{E_i^2}{(N_1/N_3)^2 R_3}$$

Thus,

$$\frac{1}{R_i} = \frac{1}{(N_1/N_2)^2 R_2} + \frac{1}{(N_1/N_3)^2 R_3} \qquad (22.40)$$

indicating that the load resistances are reflected in parallel.

For the configuration in Fig. 22.44, with E_2 and E_3 defined as shown, Eqs. (22.39) and (22.40) are applicable.

22.13 NETWORKS WITH MAGNETICALLY COUPLED COILS

For multiloop networks with magnetically coupled coils, the mesh-analysis approach is most frequently applied. A firm understanding of the dot convention discussed earlier should make the writing of the equations quite direct and free of errors. Before writing the equations for any particular loop, first determine whether the mutual term is positive or negative, keeping in mind that it will have the same sign as that for the other magnetically coupled coil. For the two-loop network in Fig. 22.45, for example, the mutual term has a positive sign since the current through each coil leaves the dot. For the primary loop,

$$E_1 - I_1 Z_1 - I_1 Z_{L_1} - I_2 Z_m - Z_2(I_1 - I_2) = 0$$

where M of $Z_m = \omega M \angle 90°$ is positive, and

$$I_1(Z_1 + Z_{L_1} + Z_2) - I_2(Z_2 - Z_m) = E_1$$

FIG. 22.45

Applying mesh analysis to magnetically coupled coils.

Note in the above that the mutual impedance was treated as if it were an additional inductance in series with the inductance L_1 having a sign determined by the dot convention and the voltage across which is determined by the current in the magnetically coupled loop.

For the secondary loop,

$$-Z_2(I_2 - I_1) - I_2 Z_{L_2} - I_1 Z_m - I_2 Z_3 = 0$$

or

$$I_2(Z_2 + Z_{L_2} + Z_3) - I_1(Z_2 - Z_m) = 0$$

For the network in Fig. 22.46, we find a mutual term between L_1 and L_2 and L_1 and L_3, labeled M_{12} and M_{13}, respectively.

For the coils with the dots (L_1 and L_3), since each current through the coils leaves the dot, M_{13} is positive for the chosen direction of I_1 and I_3. However, since the current I_1 leaves the dot through L_1, and I_2 enters the dot through coil L_2, M_{12} is negative. Consequently, for the input circuit,

$$E_1 - I_1 Z_1 - I_1 Z_{L_1} - I_2(-Z_{m_{12}}) - I_3 Z_{m_{13}} = 0$$

or

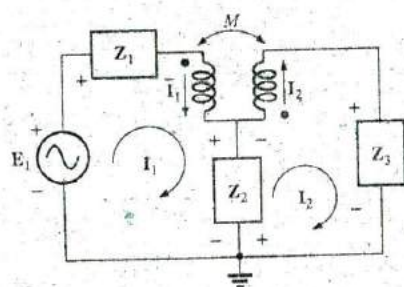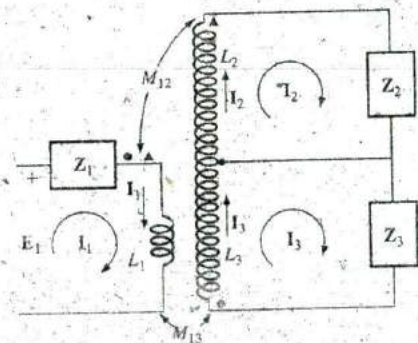$$E_1 - I_1(Z_1 + Z_{L_1}) + I_2 Z_{m_{12}} - I_3 Z_{m_{13}} = 0$$

FIG. 22.46

Applying mesh analysis to a network with two magnetically coupled coils.

For loop 2,

$$-I_2Z_2 - I_2Z_{L_2} - I_1(-Z_{m_{12}}) = 0$$
$$-I_1Z_{m_{12}} + I_2(Z_2 + Z_{L_2}) = 0$$

and for loop 3,

$$-I_3Z_3 - I_3Z_{L_3} - I_1Z_{m_{13}} = 0$$

or

$$I_1Z_{m_{13}} + I_3(Z_3 + Z_{L_3}) = 0$$

In determinant form,

$$
\begin{array}{llll}
I_1(Z_1 + Z_{L_1}) & - I_2Z_{m_{12}} & + I_3Z_{m_{13}} & = E_1 \\
-I_1Z_{m_{12}} & + I_2(Z_2 + Z_{L_2}) & + 0 & = 0 \\
I_1Z_{m_{13}} & + 0 & + I_3(Z_3 + Z_{13}) & = 0
\end{array}
$$

22.14 APPLICATIONS

The transformer has appeared throughout the text in a number of described applications, from the basic dc supply to the flyback transformer of a simple flash camera. Transformers were used to increase or decrease the voltage or current level, to act as an impedance matching device, or in some-cases to play a dual role of transformer action and reactive element. They are so common in such a wide variety of systems that it is important to become very familiar with their general characteristics. For most applications, transformer design can be considered 100% efficient. That is, the power applied is the power delivered to the load. In general, however, transformers are frequently the largest element of a design and, because of the nonlinearity of the B-H curve, can cause some distortion of the transformed waveform. Therefore, they are useful only in situations where the applied voltage is changing with time. The application of a dc voltage to the primary results in 0 V at the secondary, but the application of a voltage that changes with time, no matter what its general appearance, results in a voltage on the secondary. Remember that even though it can provide isolation between the primary and secondary circuits, a transformer can transform the load impedance to the primary circuit at a level that can significantly affect the behavior of the network. Even the smallest impedance in the secondary can be made to appear very large in the primary when a step-down transformer is used.

Transformers, like every other component you may use, have power ratings. The larger the power rating, the larger is the resulting transformer, primarily because of the larger conductors in the windings to handle the current. The size of a transformer is also a function of the frequency involved. The lower the frequency, the larger is the required transformer, as is easily recognized by the size of large power transformers (also affected by the current levels as mentioned above). For the same power level, the higher the frequency of transformation, the smaller the transformer can be. Because of eddy current and hysteresis losses in a transformer, the design of the core is quite important. A solid core would introduce high levels of such losses, whereas a core constructed of sheets of high-permeability steel with the proper insulation between the sheets would reduce the losses significantly.

Although very fundamental in their basic structure, transformers are among the basic building blocks of electric and electronic systems. There is not a publication on new components that does not include a new design for the variety of applications being developed every day.

Soldering Gun

Soldering and welding are two operations that are best performed by the application of heat that is unaffected by the thermal characteristics of the materials involved. In other words, the heat applied should not be sensitive to the changing parameters of the welding materials, the metals involved, or the welding conditions. The arc (a heavy current) established in the welding process should remain fixed in magnitude to ensure an even weld. This is best accomplished by ensuring a fixed current through the system even though the load characteristics may change—that is, by ensuring a constant current supply of sufficient amperage to establish the required arc for the welding equipment or even heating of the soldering iron tip. A further requirement for the soldering process is that the heat developed be sufficient to raise the solder to its melting point of about 800°F.

The soldering gun of Fig. 22.47(a) employs a unique approach to establishing a fixed current through the soldering tip. The soldering tip is actually part of a secondary winding of transformer having only one turn as its secondary, as shown in Fig. 22.47(b). Because of the heavy currents
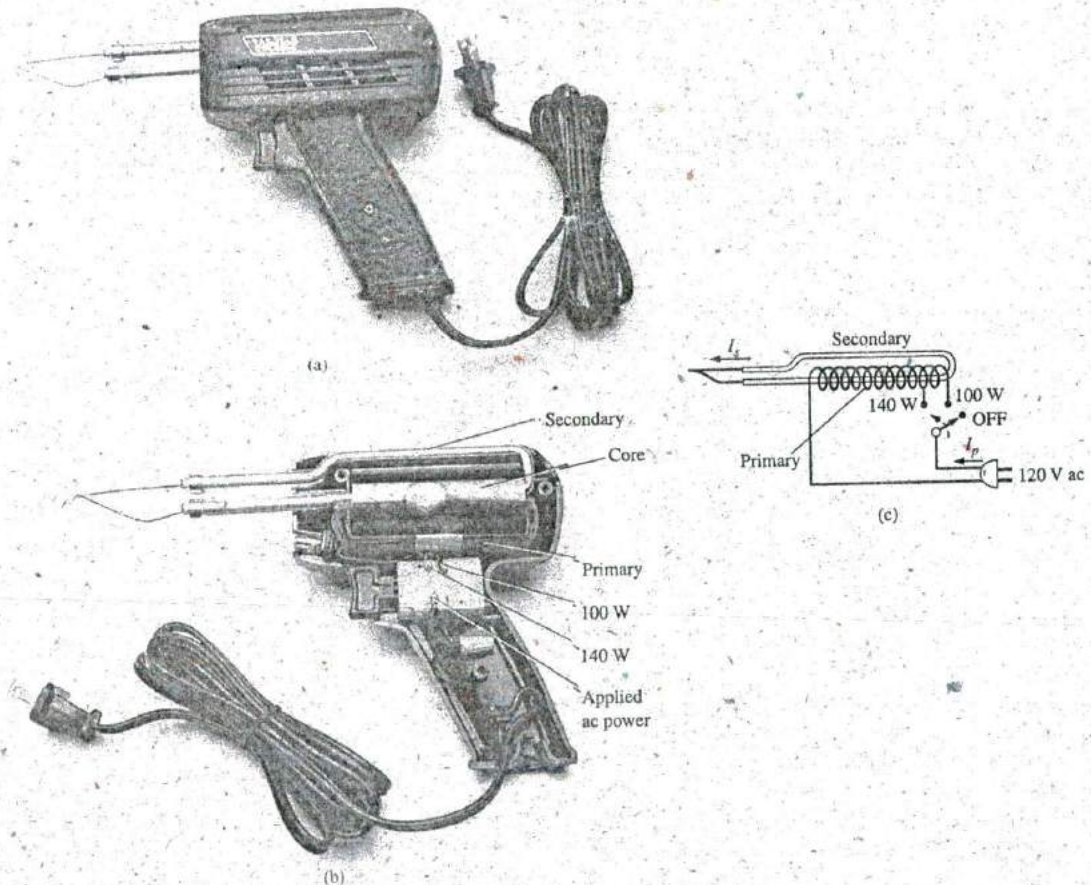
FIG. 22.47

Soldering gun: (a) appearance; (b) internal construction; (c) turns ratio control.

that will be established in this single-turn secondary, it is quite large in size to ensure that it can handle the current and to minimize its resistance level. The primary of the transformer has many turns of thinner wire to establish the turns ratio necessary to establish the required current in the secondary. The Universal® unit of Fig. 22.47 is rated 140 W/100 W, indicating that it has two levels of power controlled by the trigger. As you pull the trigger, the first-setting will be at 100 W, and a fully depressed trigger will provide 140 W of power. The inductance of the primary is 285 mH at the 140 W setting and 380 mH at the 100 W setting, indicating that the switch controls how many windings of the primary will be part of the transformer action for each wattage rating, as shown in Fig. 22.47(c). Since inductance is a direct function of the number of turns, the 140 W setting has fewer turns than the 100 W setting. The dc resistance of the primary was found to be about 11.2 Ω for the 140 W setting and 12.8 Ω for the 100 W setting, which makes sense also since more turns will require a longer wire, and the resistance should increase accordingly.

Under rated operating conditions, the primary current for each setting can be determined using Ohm's law in the following manner:
For 140 W,

$$I_p = \frac{P}{V_p} = \frac{140\ W}{120\ V} = 1.17\ A$$

For 100 W,

$$I_p = \frac{P}{V_p} = \frac{100\ W}{120\ V} = 0.83\ A$$

As expected, the current demand is more for the 140 W setting that for the 100 W setting. Using the measured values of input inductance and resistance for the 140 W setting gives the equivalent circuit of Fig. 22.48(a). Using the applied 60 Hz to determine the reactance of the coil and then determining the total impedance seen by the primary results in the following for the source current:

$$X_L = 2\pi fL = 2\pi(60\ Hz)(285\ mH) = 107.44\ \Omega$$

and $\quad Z_T = R + jX_L = 11.2\ \Omega + j\,107.44\ \Omega = 108.02\ \Omega\ \angle 84.05°$

so that $\quad |I_p| = \left|\dfrac{E}{Z_T}\right| = \dfrac{120\ V}{108.02\ \Omega} = 1.11\ A$

which is a close match with the rated level.

For the 100 W level of Fig. 22.48(b), the following analysis would result:

$$X_L = 2\pi fL = 2\pi(60\ Hz)(380\ mH) = 143.26\ \Omega$$

and $\quad Z_T = R + jX_L = 12.8\ \Omega + j\,143.26\ \Omega = 143.83\ \Omega\ \angle 84.89°$

so that $\quad |I_p| = \left|\dfrac{E}{Z_T}\right| = \dfrac{120\ V}{143.83\ \Omega} = 0.83\ A$

which is a match to hundredths place with the value calculated from rated conditions.

Removing the tip and measuring the primary and secondary voltages resulted in 120 V/0.38 V for the 140 W setting and 120 V/0.31 V for the 100 W setting, respectively. Since the voltages of a transformer are directly related to the turns ratio, the ratio of the number of turns in
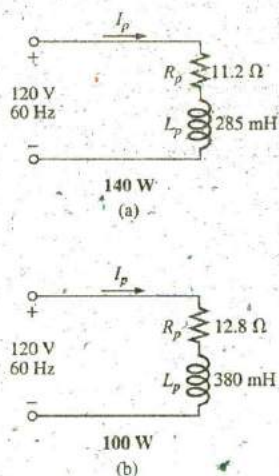
140 W
(a)

100 W
(b)

FIG. 22.48

Equivalent circuits for the soldering iron of Fig. 22.47(a): at 140-W setting; (b) at 100-W setting.

the primary (N_p) to that of the secondary (N_s) can be estimated by the following for each setting:

For 140 W,

$$\frac{N_p}{N_s} = \frac{120 \text{ V}}{0.38 \text{ V}} \cong 316$$

For 100 W,

$$\frac{N_p}{N_s} = \frac{120 \text{ V}}{0.31 \text{ V}} \cong 387$$

Looking at the photograph of Fig. 22.47(b), one would certainly consider that there are 300 or more turns in the primary winding.

The currents of a transformer are related by the turns ratio in the following manner, permitting a calculation of the secondary current for each setting:

For 140 W,

$$I_{sh} = \frac{N_p}{N_s} I_p = 316(1.17 \text{ A}) \cong 370 \text{ A}$$

For 100 W,

$$I_s = \frac{N_p}{N_s} I_p = 387(0.83 \text{ A}) \cong 321 \text{ A}$$

Quite clearly, the secondary current is much higher for the 140 W setting. Using an Amp-Clamp® showed that the current in secondary exceeded 300 A when the power was first applied and the soldering tip was cold. However, as the tip heated up because of the high current levels, the current through the primary dropped to about 215 A for the 140 W setting and to 180 A for the 100 W setting. These high currents are part of the reason that the lifetime of most soldering tips on soldering guns is about 20 hours. Eventually, the tip will simply begin to melt. Using these levels of current and the given power rating, we can approximate the resistance of the secondary as follows:

For 140 W,

$$R = \frac{P}{I^2} = \frac{140 \text{ W}}{(215 \text{ A})^2} \cong 3 \text{ m}\Omega$$

For 100 W,

$$R = \frac{P}{I^2} = \frac{100 \text{ W}}{(180 \text{ A})^2} \cong 3 \text{ m}\Omega$$

which is low, as expected when you consider the cross-sectional area of the secondary and the fact that the tip is a short section of low-resistance, tin-plated copper.

One of the obvious advantages of the soldering gun versus the iron is that the iron is off when you release the trigger, thus reducing energy costs and extending the life of the tip. Applying dc current rather than ac to develop a constant current would be impractical because the high current demand would require a series of large batteries in parallel.

The above investigation was particularly interesting because of the manner in which the constant current characteristic was established, the levels of current established, and the excellent manner in which some of the theory introduced in the text was verified.

Low-Voltage Compensation

At times during the year, peak demands from the power company can result in a reduced voltage down the line. In midsummer, for example, the line voltage may drop from 120 V to 100 V because of the heavy load often due primarily to air conditioners. However, air conditioners do not run as well under low-voltage conditions, so the following option using an autotransformer may be the solution.

In Fig. 22.49(a), an air conditioner drawing 10 A at 120 V is connected through an autotransformer to the available supply, which has dropped to 100 V. Assuming 100% efficiency, the current drawn from the line would have to be 12 A to ensure that $P_i = P_o = 1200$ W. Using the analysis introduced in Section 22.11, we find that the current in the primary winding is 2 A with 10 A in the secondary. The 12 A exist only in the line connecting the source to the primary. If the voltage level is increased using the traditional step-up transformer shown in Fig. 22.49(b), the same currents result at the source and load. However, note that the current through the primary is now 12 A, which is 6 times that in the autotransformer. The result is that the winding in the autotransformer can be much thinner due to the significantly lower current level.
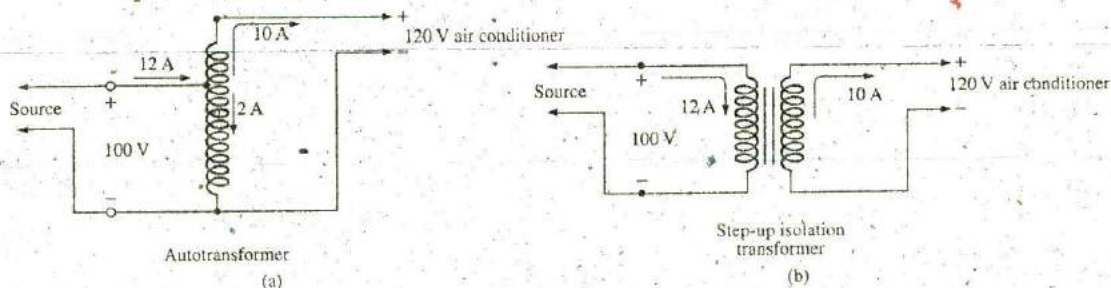
FIG. 22.49

Maintaining a 120 V supply for an air conditioner: (a) using an autotransformer;
(b) using a traditional step-up transformer.

Let us now examine the turns ratio required and the number of turns involved for each setup (associating one turn with each volt of the primary and secondary).

For the autotransformer,

$$\frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{10\ V}{100\ V} \Rightarrow \frac{10\ t}{100\ t}$$

For the traditional transformer,

$$\frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{120\ V}{100\ V} \Rightarrow \frac{120\ t}{100\ t}$$

In total, therefore, the autotransformer has only 10 turns in the secondary, whereas the traditional has 120. For the autotransformer, we need only 10 turns of heavy wire to handle the current of 10 A, not the full 120 required for the traditional transformer. In addition, the total number of turns for the autotransformer is 110, compared to 220 for the traditional transformer.

The net result of all the above is that even though the protection offered by the isolation feature is lost, the autotransformer can be much smaller in size and weight and, therefore, less costly.

Ballast Transformer

Until just recently, all fluorescent lights like those in Fig. 22.50(a) had a ballast transformer as shown in Fig. 22.50(b). In many cases, its weight alone is almost equal to that of the fixture itself. In recent years, a solid-state equivalent transformer has been developed that in time will replace most of the ballast transformers.

(a)

(b)
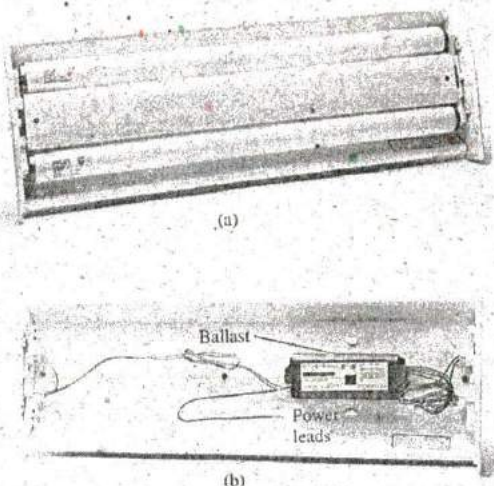
FIG. 22.50

Fluorescent lamp: (a) general appearance;
(b) internal view with ballast.

The basic connections for a single-bulb fluorescent light are provided in Fig. 22.51(a). Note that the transformer is connected as an autotransformer with the full applied 120 V across the primary. When the switch is closed, the applied voltage and the voltage across the secondary will add and establish a current through the filaments of the fluorescent bulb. The starter is initially a short circuit to establish the continuous path through the two filaments. In older fluorescent bulbs, the starter was a cylinder with two contacts, as shown in Fig. 22.51(b), which had to be replaced on occasion. It sat right under the fluorescent bulb near one of the bulb connections. Now, as shown by the sketch of the inside of a ballast transformer in Fig. 22.51(c), the starter is now commonly built into the ballast and can no longer be replaced. The voltage established by the auto-transformer action is sufficient to heat the filaments but not light the fluorescent bulb. The fluorescent lamp is a long tube with a coating of fluorescent paint on the inside. It is filled with an inert gas and a small amount of liquid mercury. The distance between the electrodes at the ends of the lamp is too much for the applied autotransformer voltage to establish conduction. To overcome this problem, the filaments are first heated as described above to convert the mercury (a good conductor)
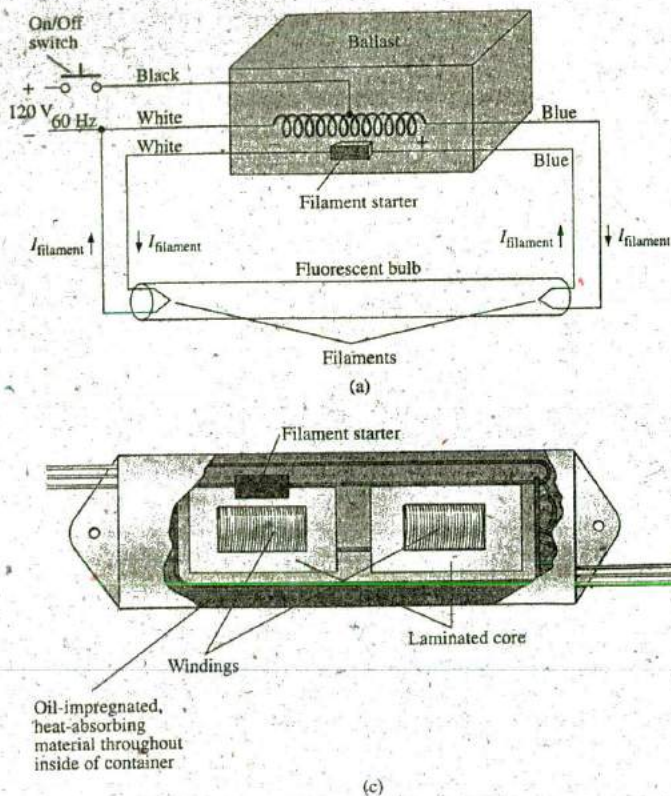
FIG. 22.51

(a) Schematic of single-bulb fluorescent lamp; (b) starter;
(c) internal view of ballast transformer.

from a liquid to a gas. Conduction can then be established by the application of a large potential across the electrodes. This potential is established when the starter (a thermal switch that opens when it reaches a particular temperature) opens and causes the inductor current to drop from its established level to zero amperes. This quick drop in current establishes a very high spike in voltage across the coils of the autotransformer as determined by $v_L = L(di_L/dt)$. This significant spike in voltage also appears across the bulb and establishes current between the electrodes. Light is then given off as the electrons hit the fluorescent surface on the inside of the tube. It is the persistence of the coating that helps hide the oscillation in conduction level due to the low-frequency (60 Hz) power that can result in a flickering light. The starter remains open until the next time the bulb is turned on. The flow of charge between electrodes is then maintained solely by the voltage across the autotransformer. This current is relatively low in magnitude because of the reactance of the secondary winding in the resulting series circuit. In other words, the autotransformer has shifted to one that is now providing a reactance to the secondary circuit to limit the current through the bulb. Without this limiting factor, the current through the bulb would be too high, and the bulb would quickly burn out. This action of the coils of the transformer generating the required voltage and then acting as a coil to limit the current has resulted in the general terminology of *swinging choke.*

The fact that the light is not generated by an *IR* drop across a filament of a bulb is the reason fluorescent lights are so energy efficient. In fact, in an incandescent bulb, about 75% of the applied energy is lost in heat, with only 25% going to light emission. In a fluorescent bulb, more than 70% goes to light emission and 30% to heat losses. As a rule of thumb, the lighting from a 40 W fluorescent lamp [such as the unit in Fig. 22.50(a) with its two 20 W bulbs] is equivalent to that of a 100 W incandescent bulb.

One other interesting difference between incandescent and fluorescent bulbs is the method of determining whether they are good or bad. For the incandescent light, it is immediately obvious when it fails to give light at all. For the fluorescent bulb, however, assuming that the ballast is in good working order, the bulb begins to dim as its life wears on. The electrodes become coated and less efficient, and the coating on the inner surface begins to deteriorate.

Rapid-start fluorescent lamps are different in operation only in that the voltage generated by the transformer is sufficiently large to atomize the gas upon application and initiate conduction, thereby removing the need for a starter and eliminating the warm-up time of the filaments. In time, the solid-state ballast will probably be the unit of choice because of its quick response, higher efficiency, and lighter weight, but the transition will take some time. The basic operation will remain the same, however.

Because of the fluorine gas (hence the name *fluorescent* bulb) and the mercury in fluorescent lamps, they must be discarded with care. Ask your local disposal facility where to take bulbs. Breaking them for insertion in a plastic bag could be very dangerous. If you happen to break a bulb and get cut in the process, go immediately to a medical facility since you could sustain fluorine or mercury poisoning.

Recent Developments

As pointed out in Chapter 4, the compact flourescent bulb (CFL) has had a tremendous impact on the "green" movement, with entire countries determined to be fully flourescent within the next 3 to 4 years. However, the design of the CFL is electronic in nature and does not use the ballast and starter mechanism described above, although its actual mode of operation is the same. Note in Fig. 4.25 the package of electronics crowded into each CFL, resulting in the higher cost compared to incandescent lighting. In fact, take particular note of the small transformer in the center of the package that is used to help establish the required firing voltage. Further impact on the lighting industry is bound to come from the growing interest in light-emitting diodes (LEDs), which have even longer lifetimes than CFLs and do not require a firing mechanism of any kind. As with any new source of light, however, there are both positive and negative aspects to each approach. Some suggest that the future holds a mixture of use, depending on the application, cost involved, and effect on the environment. On the very positive side, however, it is nice to have options, and those being develops seem to be full of promise.

22.15 COMPUTER ANALYSIS

PSpice

Transformer (Controlled Sources) The simple transformer configuration in Fig. 22.52 is now investigated using controlled sources to mimic the behavior of the transformer as defined by its basic voltage and current relationships.
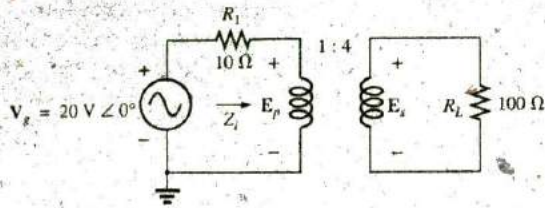
FIG. 22.52

Applying PSpice to a step-up transformer.

For comparison purposes, a theoretical solution of the network yields the following:

$$Z_i = a^2 Z_L$$
$$= \left(\frac{1}{4}\right)^2 \cdot 100 \ \Omega$$
$$= 6.25 \ \Omega$$

and

$$E_p = \frac{(6.25 \ \Omega)(20 \ V)}{6.25 \ \Omega + 10 \ \Omega} = 7.692 \ V$$

with

$$E_s = \frac{1}{a}E_p = \frac{1}{(1/4)}(7.692 \ V) = 4(7.692 \ V) = 30.77 \ V$$

and

$$V_L = E_s = 30.77 \ V$$

For the ideal transformer, the secondary voltage is defined by $E_s = N_s/N_p(E_p)$ which is $E_s = 4E_p$ for the network in Fig. 22.52. The fact that the magnitude of one voltage is controlled by another requires that we use the **Voltage-Controlled Voltage Source (VCVS)** source in the **ANALOG** library. It appears as **E** in the **Parts List** and has the format appearing in Fig. 22.53. The sensing voltage is **E1**, and the controlled
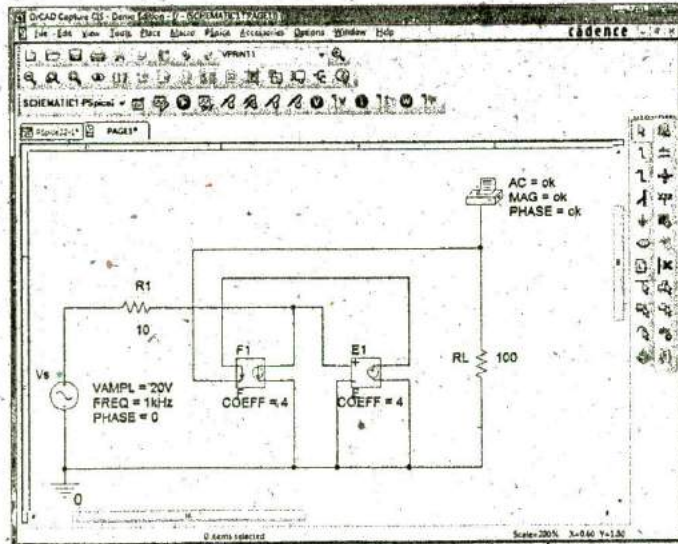
FIG. 22.53

Using PSpice to determine the magnitude and phase angle for the load voltage of the network in Fig. 22.52.

voltage appears across the two terminals of the circular symbol for a voltage source. Double-click on the source symbol to set the **GAIN** to 4 for this example. Note in Fig. 22.53 that the sensing voltage is the primary voltage of the circuit in Fig. 22.52, and the output voltage is connected directly to the load resistor **RL.** There is no real problem making the necessary connections because of the format of the **E** source.

The next step is to set up the current relationship for the transformer. Since the magnitude of one current is controlled by the magnitude of another current in the same configuration, a **Current-Controlled Current Source (CCCS)** must be employed. It also appears in the **ANALOG** library under the **Parts List** as **F** and has the format appearing in Fig. 22.53. Note that both currents have a direction associated with them. For the ideal transformer, $I_p = N_s/N_p (I_s)$ which is $I_p = 4I_s$ for the network in Fig. 22.52. The gain for the part can be set using the same procedure defined for the **E** source. Since the secondary current is the controlling current, its level must be fed into the **F** source in the same direction as indicated in the controlled source. When making this connection, be sure to click the wire in location before crossing the wire of the primary circuit and then clicking it again after crossing the wire. If you do this properly, a connection point indicated by a small red dot will not appear. The controlled current I_{R_1} can be connected as shown because the connection **E1** is only sensing a voltage level, essentially has infinite impedance, and can be looked upon as an open circuit. In other words, the current through **R1** will be the same as through the controlled source of **F.**

A simulation was set up with **AC Sweep** and 1 kHz for the **Start** and **End Frequencies.** One data point per decade was selected, and the simulation was initiated. After the **SCHEMATIC1** screen appeared, the window was exited, and **PSpice-View Output File** was selected to result in the **AC ANALYSIS** solution of Fig. 22.54. Note that the voltage is 30.77 V, which is an exact match with the theoretical solution.

```
**  Profile: "SCHEMATIC1-PSpice 22-1"
 [ C:\ICA12\PSpice\PSpice 22-1-PSpiceFiles\SCHEMATIC1\PSpice 22-1.sim ]


****    AC ANALYSIS              TEMPERATURE =  27.000 DEG C

*******************************************************************************

 FREQ        VM(N00908)              VP (N00908)
 1.000E+03   3.077E+01               0.000E+00
```

FIG. 22.54
The output file for the analysis indicated in Fig. 22.53.

Multisim

Transformer(Library)   Multisim is now used to analyze the same transformer configuration just investigated using PSpice. In Fig. 22.55, obtain the source by first selecting **Place Source** to open the **Select a Component** dialog box. Select **SIGNAL_VOLTAGE** followed by **AC_VOLTAGE** and click **OK.** For the source, peak values are set, hence the difference in the set value in Fig. 22.55 and the rms multimeter reading. Obtain the transformer by selecting **Place Basic-Family-BASIC_VIRTUAL-TS_VIRTUAL.** Then select the turns ratio of 2 to open the **TS_VIRTUAL** dialog box. Change the **Primary-to-Secondary Turns Ratio:** to 0.25 for this example.
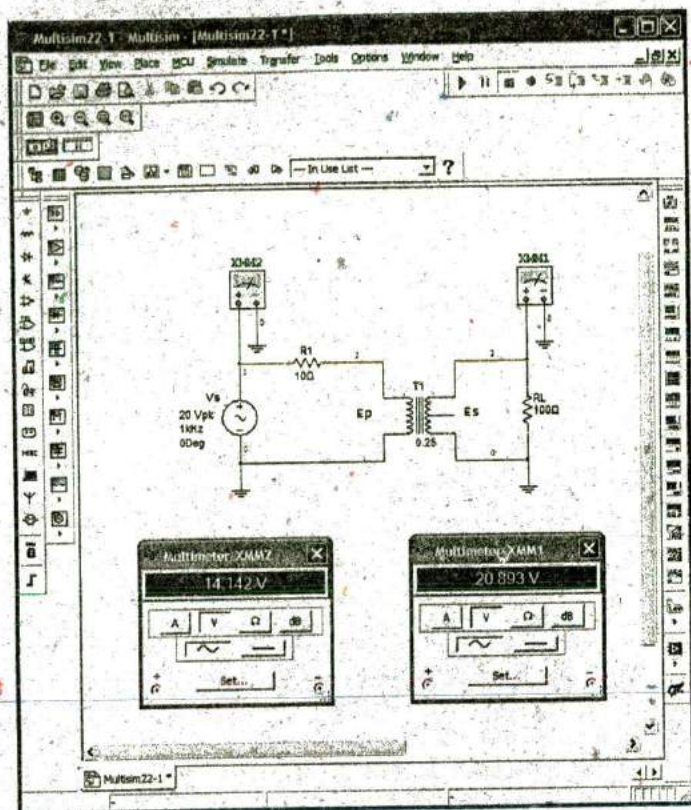
FIG. 22.55

The rest of the configuration is constructed using techniques described earlier. A simulation results in the meter displays in Fig. 22.55. Changing the rms reading of 20.893 V to a peak value results in 29.54 V which is a close match to that obtained using PSpice.

PROBLEMS

SECTION 22.2 Mutual Inductance

1. For the air-core transformer in Fig. 22.56:
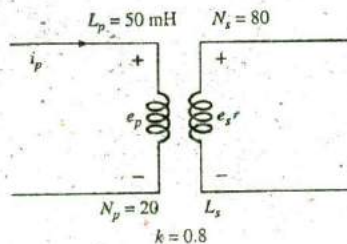   a. Find the value of L_s if the mutual inductance M is equal to 40 mH.

$L_p = 50$ mH $\qquad N_s = 80$

$N_p = 20 \qquad L_s$

$k = 0.8$

FIG. 22.56

Problems 1, 2 and 3.

b. Find the induced voltages e_p and e_s if the flux linking the primary coil changes at the rate of 0.08 Wb/s,
   c. Find the induced voltages e_p and e_s if the current i_p changes at the rate of 0.3 A/ms.

2. a. Repeat Problem 1 if k is changed to 1.
   b. Repeat Problem 1 if k is changed to 0.2.
   c. Compare the results of parts (a) and (b).

3. Repeat Problem 1 for $k = 0.3$, $N_p = 300$ turns, and $N_s = 25$ turns.

SECTION 22.3 The Iron-Core Transformer

4. For the iron-core transformer ($k = 1$) in Fig. 22.57:
   a. Find the magnitude of the induced voltage E_s.
   b. Find the maximum flux Φ_m.

5. Repeat Problem 4 for $N_p = 240$ and $N_s = 30$.

6. Find the applied voltage of an iron-core transformer if the secondary voltage is 240 V, and $N_p = 60$ with $N_s = 720$.

7. If the maximum flux passing through the core of Problem 4 is 3.75 mWb, find the frequency of the input voltage.
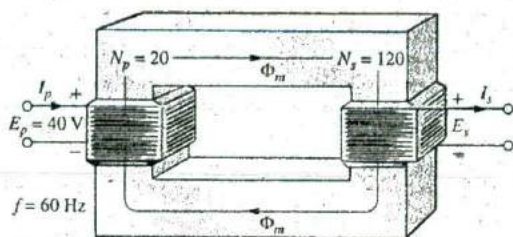
FIG. 22.57

Problems 4, 5 and 7.

SECTION 22.4 Reflected Impedance and Power

8. For the iron-core transformer in Fig. 22.58:
   a. Find the magnitude of the current I_L and the voltage V_L if $a = 1/5$, $I_p = 2$ A, and $Z_L = 2\ \Omega$ resistor.
   b. Find the input resistance for the data specified in part (a).
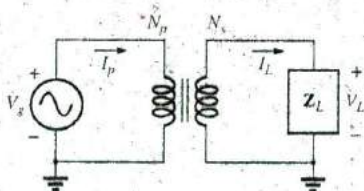
FIG. 22.58

Problems 8 through 12.

9. Find the input impedance for the iron-core transformer of Fig. 22.58 if $a = 2$, $I_p = 4$ A, and $V_g = 120$ V.

10. Find the voltage V_g and the current I_p if the input impedance of the iron-core transformer in Fig. 22.58 is $4\ \Omega$, and $V_L = 600$ V and $a = 1/4$.

11. If $V_L = 240$ V, $Z_L = 20\ \Omega$ resistor, $I_p = 0.05$ A, and $N_s = 50$, find the number of turns in the primary circuit of the iron-core transformer in Fig. 22.58.

12. a. If $N_p = 400$, $N_s = 1200$, and $V_g = 100$ V, find the magnitude of I_p for the iron-core transformer in Fig. 22.58 if $Z_L = 12\ \Omega + j\ 12\ \Omega$.
    b. Find the magnitude of the voltage V_L and the current I_L for the conditions of part (a).

SECTION 22.5 Impedance Matching, Isolation, and Displacement

13. a. For the circuit in Fig. 22.59, find the transformation ratio required to deliver maximum power to the speaker.
    b. Find the maximum power delivered to the speaker.
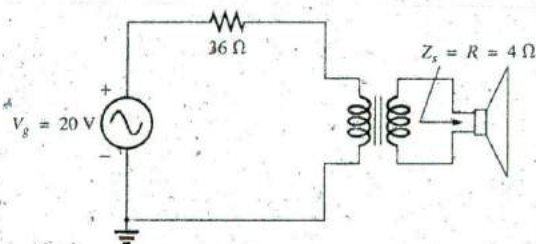
FIG. 22.59

Problem 13.

SECTION 22.6 Equivalent Circuit (Iron-Core Transformer)

14. For the transformer in Fig. 22.60, determine
    a. the equivalent resistance R_e.
    b. the equivalent reactance X_e.
    c. the equivalent circuit reflected to the primary.
    d. the primary current for $V_g = 50$ V $\angle 0°$.
    e. the load voltage V_L.
    f. the phasor diagram of the reflected primary circuit.
    g. the new load voltage if we assume the transformer to be ideal with a 4 : 1 turns ratio. Compare the result with that of part (e).

15. For the transformer in Fig. 22.60, if the resistive load is replaced by an inductive reactance of $20\ \Omega$:
    a. Determine the total reflected primary impedance.
    b. Calculate the primary current, I_p.
    c. Determine the voltage across R_e and X_e, and find the reflected load.
    d. Draw the phasor diagram.

Ideal transformer

FIG. 22.60

Problem 14 through 16, 30, and 31.

16. Repeat Problem 15 for a capacitive load having a reactance of 20 Ω.

SECTION 22.7 Frequency Considerations

17. Discuss in your own words the frequency characteristics of the transformer. Use the applicable equivalent circuit and frequency characteristics appearing in this chapter.

SECTION 22.8 Series Connection of Mutually Coupled Coils

18. Determine the total inductance of the series coils in Fig. 22.61.
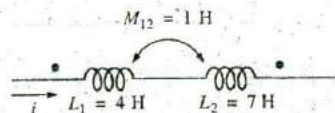
FIG. 22.61
Problem 18.

19. Determine the total inductance of the series coils in Fig. 22.62.
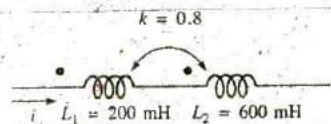
FIG. 22.62
Problem 19.

20. Determine the total inductance of the series coils in Fig. 22.63.
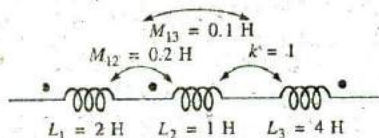
FIG. 22.63
Problem 20.

21. Write the mesh equations for the network in Fig. 22.64.
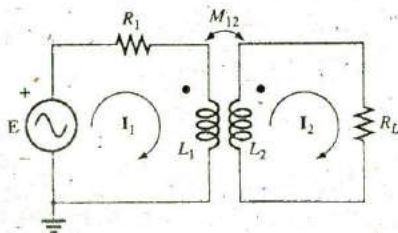
FIG. 22.64
Problem 21.

SECTION 22.9 Air-Core Transformer

22. Determine the input impedance to the air-core transformer in Fig. 22.65. Sketch the reflected primary network.
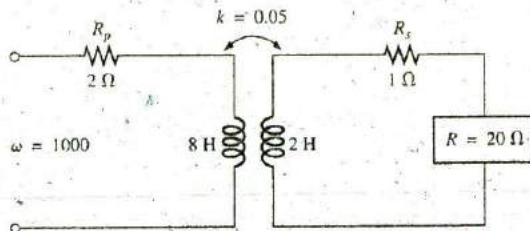
FIG. 22.65
Problems 22 and 32.

SECTION 22.10 Nameplate Data

23. An ideal transformer is rated 10 kVA, 2400/120 V, 60 Hz.
    a. Find the transformation ratio if the 120 V is the secondary voltage.
    b. Find the current rating of the secondary if the 120 V is the secondary voltage.
    c. Find the current rating of the primary if the 120 V is the secondary voltage.
    d. Repeat parts (a) through (c) if the 2400 V is the secondary voltage.

SECTION 22.11 Types of Transformers

24. Determine the primary and secondary voltages and currents for the autotransformer in Fig. 22.66.
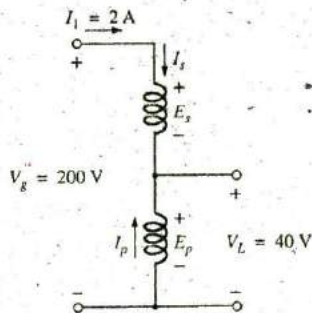
FIG. 22.66
Problem 24.

SECTION 22.12 Tapped and Multiple-Load Transformers

25. For the center-tapped transformer in Fig. 22.42, where $N_p = 100$, $N_s = 25$, $Z_L = R \angle 0° = 5 \Omega \angle 0°$, and $E_p = 100 \text{ V} \angle 0°$:
    a. Determine the load voltage and current.
    b. Find the impedance Z_i.
    c. Calculate the impedance $Z_{1/2}$.

26. For the multiple-load transformer in Fig. 22.43, where $N_1 = 90$, $N_2 = 15$, $N_3 = 45$, $Z_2 = R_2 \angle 0° = 8 \, \Omega \angle 0°$, $Z_3 = R_L \angle 0° = 5 \, \Omega \angle 0°$, and $E_i = 60$ V $\angle 0°$:
   a. Determine the load voltages and currents.
   b. Calculate Z_1.

27. For the multiple-load transformer in Fig. 22.44, where $N_1 = 120$, $N_2 = 40$, $N_3 = 30$, $Z_2 = R_2 \angle 0° = 12 \, \Omega \angle 0°$, $Z_3 = R_3 \angle 0° = 10 \, \Omega \angle 0°$, and $E_1 = 120$ V $\angle 60°$:
   a. Determine the load voltages and currents.
   b. Calculate Z_1.

SECTION 22.13 Networks with Magnetically Coupled Coils

28. Write the mesh equations for the network of Fig. 22.67.
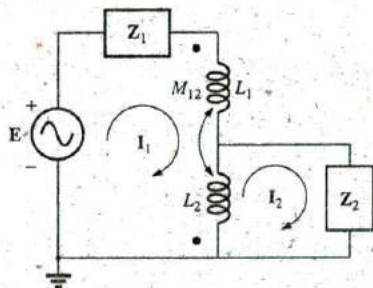
FiG. 22.67
Problem 28.

29. Write the mesh equations for the network of Fig. 22.68.
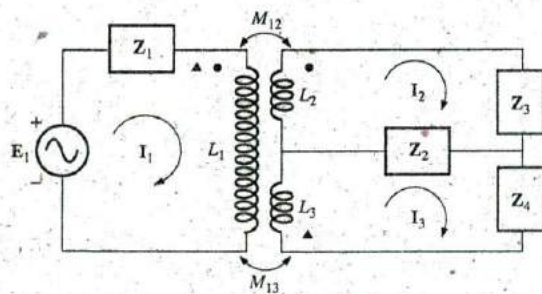
FIG. 22.68
Problem 28.

SECTION 22.15 Computer Analysis

PSpice or Multisim

*30. Generate the schematic for the network in Fig. 22.60, and find the voltage V_L.

*31. Develop a technique using PSpice or Multisim to find the input impedance at the source for the network in Fig. 22.60.

*32. Using a transformer from the library, find the load voltage for the network in Fig. 22.65 for an applied voltage of 40 V $\angle 0°$.

GLOSSARY

Autotransformer A transformer with one winding common to both the primary and the secondary circuits. A loss in isolation is balanced by the increase in its kilovolt-ampere rating.

Coefficient of coupling (k) A measure of the magnetic coupling of two coils that ranges from a minimum of 0 to a maximum of 1.

Dot convention A technique for labeling the effect of the mutual inductance on a net inductance of a network or system.

Leakage flux The flux linking the coil that does not pass through the ferromagnetic path of the magnetic circuit.

Loosely coupled A term applied to two coils that have a low coefficient of coupling.

Multiple-load transformers Transformers having more than a single load connected to the secondary winding or windings.

Mutual inductance The inductance that exists between magnetically coupled coils of the same or different dimensions.

Nameplate data Information such as the kilovolt-ampere rating, voltage transformation ratio, and frequency of application that is of primary importance in choosing the proper transformer for a particular application.

Primary The coil or winding to which the source of electrical energy is normally applied.

Reflected impedance The impedance appearing at the primary of a transformer due to a load connected to the secondary. Its magnitude is controlled directly by the transformation ratio.

Secondary The coil or winding to which the load is normally applied.

Step-down transformer A transformer whose secondary voltage is less than its primary voltage. The transformation ratio a is greater than 1.

Step-up transformer A transformer whose secondary voltage is greater than its primary voltage. The magnitude of the transformation ratio a is less than 1.

Tapped transformer A transformer having an additional connection between the terminals of the primary or secondary windings.

Transformation ratio (a) The ratio of primary to secondary turns of a transformer.

Polyphase Systems

Objectives

- *Become familiar with the operation of a three-phase generator and the magnitude and phase relationship connecting the three phase voltages.*

- *Be able to calculate the voltages and currents for a three-phase Y-connected generator and Y-connected load.*

- *Understand the significance of the phase sequence for the generated voltages of a three-phase Y-connected or Δ-connected generator.*

- *Be able to calculate the voltages and currents for a three-phase Δ-connected generator and Δ-connected load.*

- *Understand how to calculate the real, reactive, and apparent power to all the elements of a Y- or Δ-connected load and be able to measure the power to the load.*

23.1 INTRODUCTION

An ac generator designed to develop a single sinusoidal voltage for each rotation of the shaft (rotor) is referred to as a **single-phase ac generator.** If the number of coils on the rotor is increased in a specified manner, the result is a **polyphase ac generator,** which develops more than one ac phase voltage per rotation of the rotor. In this chapter, the three-phase system is discussed in detail since it is the system most frequently used for power transmission.

In general, three-phase systems are preferred over single-phase systems for the transmission of power for many reasons, including the following:

1. Thinner conductors can be used to transmit the same kVA at the same voltage, which reduces the amount of copper required (typically about 25% less) and in turn reduces construction and maintenance costs.
2. The lighter lines are easier to install, and the supporting structures can be less massive and farther apart.
3. Three-phase equipment and motors have preferred running and starting characteristics compared to single-phase systems because of a more even flow of power to the transducer than can be delivered with a single-phase supply.
4. In general, most larger motors are three phase because they are essentially self-starting and do not require a special design or additional starting circuitry.

The frequency generated is determined by the number of poles on the *rotor* (the rotating part of the generator) and the speed with which the shaft is turned. In the United States, the line frequency is 60 Hz, whereas in Europe the chosen standard is 50 Hz. Both frequencies were chosen primarily because they can be generated by a relatively efficient and stable mechanical design that is sensitive to the size of the generating systems and the demand that must be met during peak periods. On aircraft and ships, the demand levels permit the use of a 400 Hz line frequency.

Introductory, C, SS1
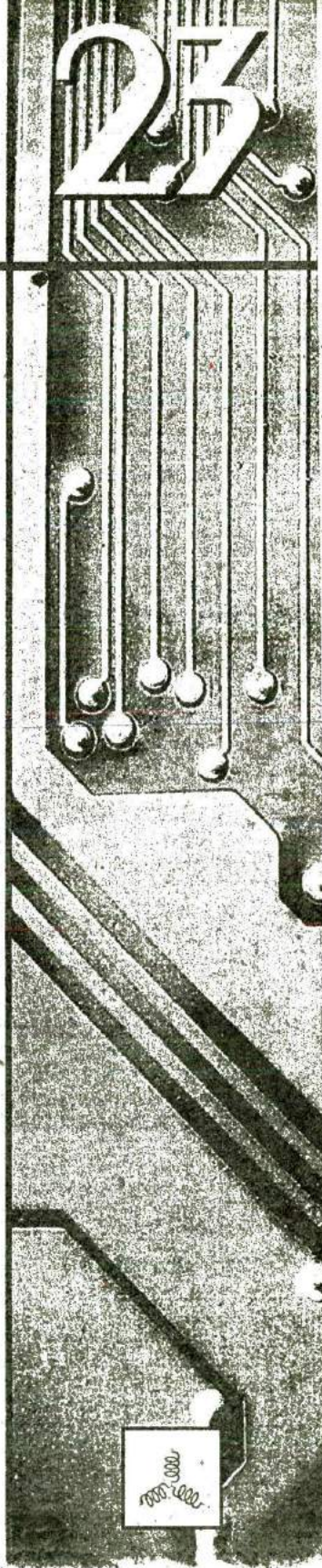
The three-phase system is used by almost all commercial electric generators. This does not mean that single-phase and two-phase generating systems are obsolete. Most small emergency generators, such as the gasoline type, are one-phase generating systems. The two-phase system is commonly used in servomechanisms, which are self-correcting control systems capable of detecting and adjusting their own operation. Servomechanisms are used in ships and aircraft to keep them on course automatically, or, in simpler devices such as a thermostatic circuit, to regulate heat output. In many cases, however, where single-phase and two-phase inputs are required, they are supplied by one and two phases of a three-phase generating system rather than generated independently.

The number of **phase voltages** that can be produced by a polyphase generator is not limited to three. Any number of phases can be obtained by spacing the windings for each phase at the proper angular position around the stator. Some electrical systems operate more efficiently if more than three phases are used. One such system involves the process of rectification, which is used to convert an alternating output to one having an average, or dc, value. The greater the number of phases, the smoother is the dc output of the system.

23.2 THREE-PHASE GENERATOR

The three-phase generator in Fig. 23.1(a) has three induction coils placed 120° apart on the stator, as shown symbolically by Fig. 23.1(b). Since the three coils have an equal number of turns, and each coil rotates with the same angular velocity, the voltage induced across each coil has the same peak value, shape, and frequency. As the shaft of the generator is turned by some external means, the induced voltages e_{AN}, e_{BN}, and e_{CN} are generated simultaneously, as shown in Fig. 23.2. Note the 120° phase shift between waveforms and the similarities in appearance of the three sinusoidal functions.

(a)   (b)
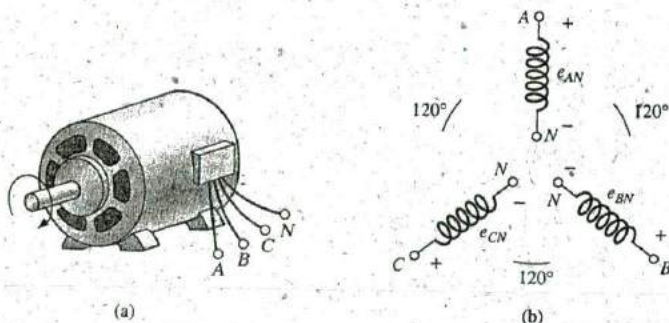
FIG. 23.1

(a) Three-phase generator; (b) induced voltages of a three-phase generator.

In particular, note that

at any instant of time, the algebraic sum of the three phase voltages of a three-phase generator is zero.

This is shown at $\omega t = 0$ in Fig. 23.2, where it is also evident that *when one induced voltage is zero, the other two are 86.6% of their positive or negative maximums. In addition, when any two are equal in magnitude*
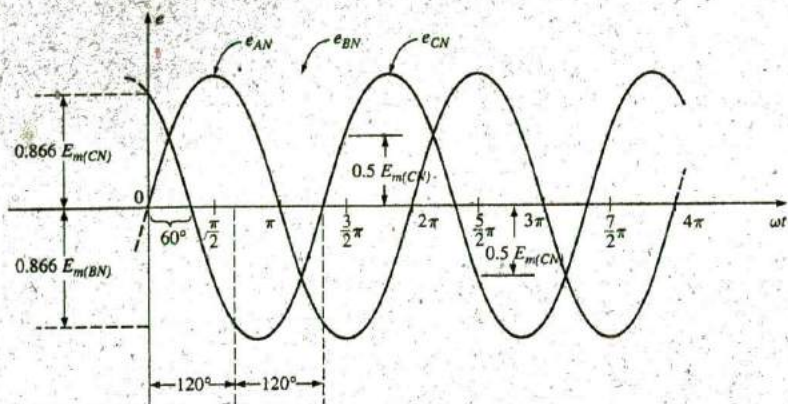
FIG. 23.2

Phase voltages of a three-phase generator.

and sign (at $0.5E_m$), the remaining induced voltage has the opposite po-
larity and a peak value.

The respective sinusoidal expressions for the induced voltages in Fig.
23.2 are

$$e_{AN} = E_{m(AN)} \sin \omega t$$
$$e_{BN} = E_{m(BN)} \sin(\omega t - 120°) \qquad (23.1)$$
$$e_{CN} = E_{m(CN)} \sin(\omega t - 240°) = E_{m(CN)} \sin(\omega t + 120°)$$

The phasor diagram of the induced voltages is shown in Fig. 23.3,
where the effective value of each is determined by

$$E_{AN} = 0.707 E_{m(AN)}$$
$$E_{BN} = 0.707 E_{m(BN)}$$
$$E_{CN} = 0.707 E_{m(CN)}$$

and

$$E_{AN} = E_{AN} \angle 0°$$
$$E_{BN} = E_{BN} \angle -120°$$
$$E_{CN} = E_{CN} \angle +120°$$

By rearranging the phasors as shown in Fig. 23.4 and applying a law
of vectors that states that *the vector sum of any number of vectors drawn
such that the "head" of one is connected to the "tail" of the next and the
head of the last vector is connected to the tail of the first is zero*, we can
conclude that the phasor sum of the phase voltages in a three-phase sys-
tem is zero. That is,

$$E_{AN} + E_{BN} + E_{CN} = 0 \qquad (23.2)$$
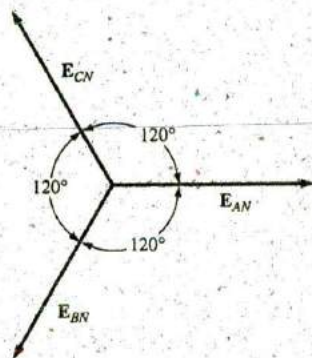
FIG. 23.3

*Phasor diagram for the phase voltages of a
three-phase generator.*
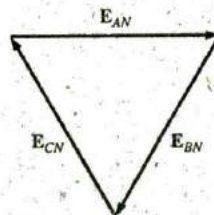
FIG. 23.4

*Demonstrating that the vector sum of the phase
voltages of a three-phase generator is zero.*

23.3 Y-CONNECTED GENERATOR

If the three terminals denoted N in Fig. 23.1(b) are connected together,
the generator is referred to as a **Y-connected three-phase generator**
(Fig. 23.5). As indicated in Fig. 23.5, the Y is inverted for ease of notation
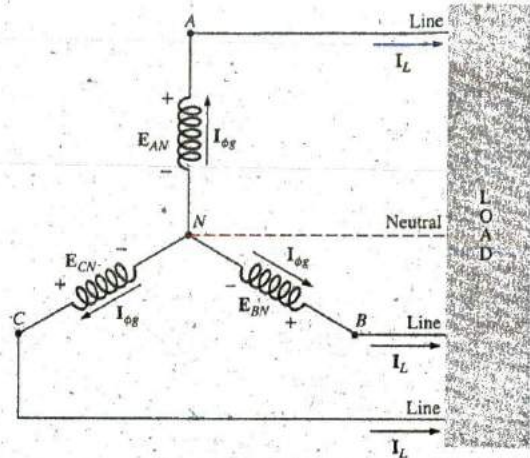
FIG. 23.5
Y-connected generator.
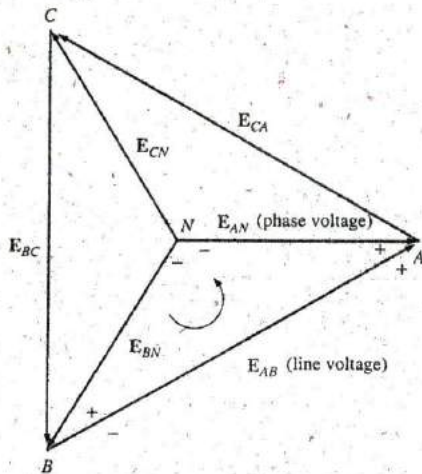
FIG. 23.6
Line and phase voltages of the Y-connected three-phase generator.
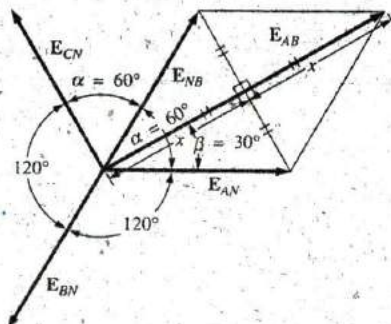
FIG. 23.7
Determining a line voltage for a three-phase generator.

and for clarity. The point at which all the terminals are connected is called the *neutral point*. If a conductor is not attached from this point to the load, the system is called a *Y-connected, three-phase, three-wire generator*. If the neutral is connected, the system is a *Y-connected, three-phase, four-wire generator*. The function of the neutral will be discussed in detail when we consider the load circuit.

The three conductors connected from A, B, and C to the load are called *lines*. For the Y-connected system, it should be obvious from Fig. 23.5 that the **line current** equals the **phase current** for each phase; that is,

$$\boxed{I_L = I_{\phi g}} \tag{23.3}$$

where ϕ is used to denote a phase quantity, and g is a generator parameter.

The voltage from one line to another is called a **line voltage.** On the phasor diagram (Fig. 23.6), it is the phasor drawn from the end of one phase to another in the counterclockwise direction.

Applying Kirchhoff's voltage law around the indicated loop in Fig. 23.6, we obtain

$$E_{AB} - E_{AN} + E_{BN} = 0$$

or

$$E_{AB} = E_{AN} - E_{BN} = E_{AN} + E_{NB}$$

The phasor diagram is redrawn to find E_{AB} as shown in Fig. 23.7. Since each phase voltage, when reversed (E_{NB}), bisects the other two, $\alpha = 60°$. The angle β is 30° since a line drawn from opposite ends of a rhombus divides in half both the angle of origin and the opposite angle. Lines drawn between opposite corners of a rhombus also bisect each other at right angles.

The length x is

$$x = E_{AN} \cos 30° = \frac{\sqrt{3}}{2} E_{AN}$$

and

$$E_{AB} = 2x = (2)\frac{\sqrt{3}}{2} E_{AN} = \sqrt{3}E_{AN}$$

Noting from the phasor diagram that θ of $E_{AB} = \beta = 30°$, we obtain the result

$$E_{AB} = E_{AB} \angle 30° = \sqrt{3}E_{AN} \angle 30°$$

and
$$E_{CA} = \sqrt{3}E_{CN} \angle 150°$$
$$E_{BC} = \sqrt{3}E_{BN} \angle 270°$$

In words, the magnitude of the line voltage of a Y-connected generator is $\sqrt{3}$ times the phase voltage:

$$\boxed{E_L = \sqrt{3}E_\phi} \qquad (23.4)$$

with the phase angle between any line voltage and the nearest phase voltage at 30°.

In sinusoidal notation,

$$e_{AB} = \sqrt{2}E_{AB} \sin(\omega t + 30°)$$
$$e_{CA} = \sqrt{2}E_{CA} \sin(\omega t + 150°)$$
and
$$e_{BC} = \sqrt{2}E_{BC} \sin(\omega t + 270°)$$

The phasor diagram of the line and phase voltages is shown in Fig. 23.8. If the phasors representing the line voltages in Fig. 23.8(a) are re-arranged slightly, they will form a closed loop [Fig. 23.8(b)]. Therefore, we can conclude that the sum of the line voltages is also zero; that is,

$$\boxed{E_{AB} + E_{CA} + E_{BC} + 0} \qquad (23.5)$$
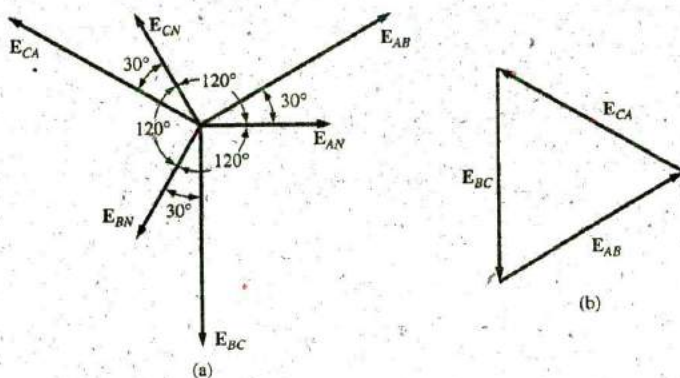
FIG. 23.8

(a) Phasor diagram of the line and phase voltages of a three-phase generator;
(b) demonstrating that the vector sum of the line voltages of a
three-phase system is zero.

23.4 PHASE SEQUENCE
(Y-CONNECTED GENERATOR)

The **phase sequence** can be determined by the order in which the phasors representing the phase voltages pass through a fixed point on the phasor diagram if the phasors are rotated in a counterclockwise direction. For example, in Fig. 23.9 the phase sequence is ABC. However, since the fixed point can be chosen anywhere on the phasor
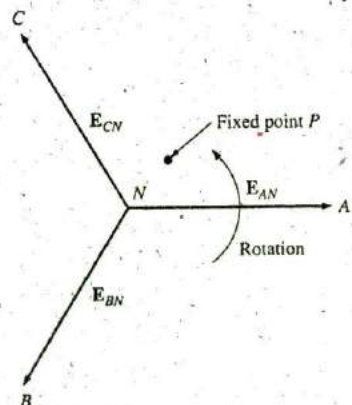
FIG. 23.9

Determining the phase sequence from the phase
voltages of a three-phase generator.
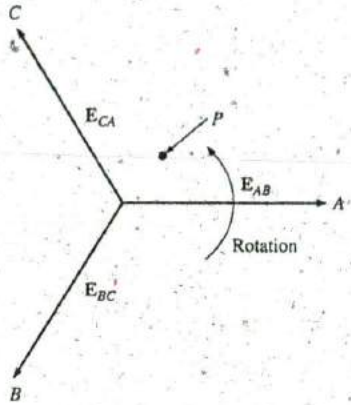
FIG. 23.10

Determining the phase sequence from the line voltages of a three-phase generator.

diagram, the sequence can also be written as *BCA* or *CAB*. The phase sequence is quite important in the three-phase distribution of power. In a three-phase motor, for example, if two phase voltages are interchanged, the sequence will change, and the direction of rotation of the motor will be reversed. Other effects will be described when we consider the loaded three-phase system.

The phase sequence can also be described in terms of the line voltages. Drawing the line voltages on a phasor diagram in Fig. 23.10, we are able to determine the phase sequence by again rotating the phasors in the counterclockwise direction. In this case, however, the sequence can be determined by noting the order of the passing first or second subscripts. In the system in Fig. 23.10, for example, the phase sequence of the first subscripts passing point *P* is *ABC*, and the phase sequence of the second subscripts is *BCA*. But we know that *BCA* is equivalent to *ABC*, so the sequence is the same for each. Note that the phase sequence is the same as that of the phase voltages described in Fig. 23.9.

If the sequence is given, the phasor diagram can be drawn by simply picking a reference voltage, placing it on the reference axis, and then drawing the other voltages at the proper angular position. For a sequence of *ACB*, for example, we might choose E_{AB} to be the reference [Fig. 23.11(a)] if we wanted the phasor diagram of the line voltages, or E_{AN} for the phase voltages [Fig. 23.11(b)]. For the sequence indicated, the phasor diagrams would be as in Fig. 23.11. In phasor notation,

$$\text{Line voltages} \begin{cases} \mathbf{E}_{AB} = E_{AB} \angle 0° & \text{(reference)} \\ \mathbf{E}_{CA} = E_{CA} \angle -120° \\ \mathbf{E}_{BC} = E_{BC} \angle +120° \end{cases}$$

$$\text{Phase voltages} \begin{cases} \mathbf{E}_{AN} = E_{AN} \angle 0° & \text{(reference)} \\ \mathbf{E}_{CN} = E_{CN} \angle -120° \\ \mathbf{E}_{BN} = E_{BN} \angle +120° \end{cases}$$

(a)

(b)
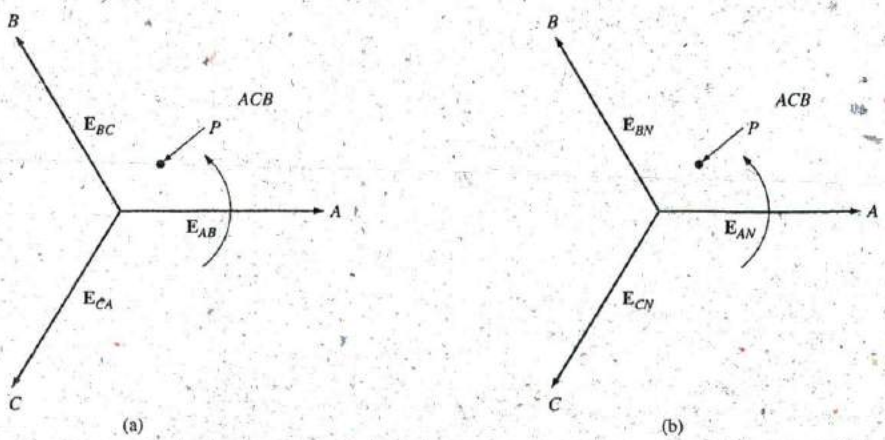
FIG. 23.11

Drawing the phasor diagram from the phase sequence.

23.5 Y-CONNECTED GENERATOR WITH A Y-CONNECTED LOAD

Loads connected to three-phase supplies are of two types: the Y and the Δ. If a Y-connected load is connected to a Y-connected generator, the system is symbolically represented by Y-Y. The physical setup of such a system is shown in Fig. 23.12.
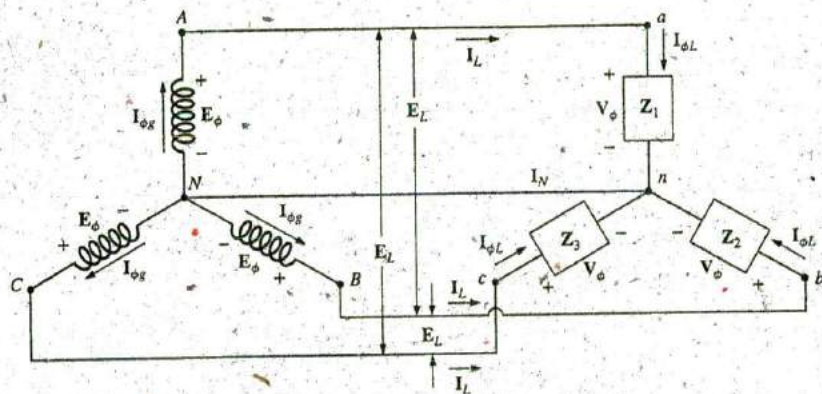
FIG. 23.12

Y-connected generator with a Y-connected load.

If the load is balanced, the **neutral connection** can be removed without affecting the circuit in any manner; that is, if

$$Z_1 = Z_2 = Z_3$$

then I_N will be zero. (This will be demonstrated in Example 23.1.) Note that in order to have a balanced load, the phase angle must also be the same for each impedance—a condition that was unnecessary in dc circuits when we considered balanced systems.

In practice, if a factory, for example, had only balanced, three-phase loads, the absence of the neutral would have no effect since, ideally, the system would always be balanced. The cost would therefore be less since the number of required conductors would be reduced. However, lighting and most other electrical equipment use only one of the phase voltages, and even if the loading is designed to be balanced (as it should be), there is never perfect continuous balancing since lights and other electrical equipment are turned on and off, upsetting the balanced condition. The neutral is therefore necessary to carry the resulting current away from the load and back to the Y-connected generator. This is demonstrated when we consider unbalanced Y-Y-connected systems.

We shall now examine the *four-wire Y-Y-connected system.* The current passing through each phase of the generator is the same as its corresponding line current, which in turn for a Y-connected load is equal to the current in the phase of the load to which it is attached:

$$\boxed{I_{\phi g} = I_L = I_{\phi L}} \qquad (23.6)$$

For a balanced or an unbalanced load, since the generator and load have a common neutral point, then

$$\boxed{V_\phi = E_\phi} \qquad (23.7)$$

In addition, since $I_{\phi L} = V_\phi/Z_\phi$, the magnitude of the current in each phase is equal for a balanced load and unequal for an unbalanced load. Recall that for the Y-connected generator, the magnitude of the line voltage is equal to $\sqrt{3}$ times the phase voltage. This same relationship can be applied to a balanced or an unbalanced four-wire Y-connected load:

$$\boxed{E_L = \sqrt{3}V_\phi} \tag{23.8}$$

For a voltage drop across a load element, the first subscript refers to that terminal through which the current enters the load element, and the second subscript refers to the terminal from which the current leaves. In other words, the first subscript is, by definition, positive with respect to the second for a voltage drop. Note Fig. 23.13, in which the standard double subscripts for a source of voltage and a voltage drop are indicated.

EXAMPLE 23.1 The phase sequence of the Y-connected generator in Fig. 23.13 is ABC.

a. Find the phase angles θ_2 and θ_3.
b. Find the magnitude of the line voltages.
c. Find the line currents.
d. Verify that, since the load is balanced, $I_N = 0$.
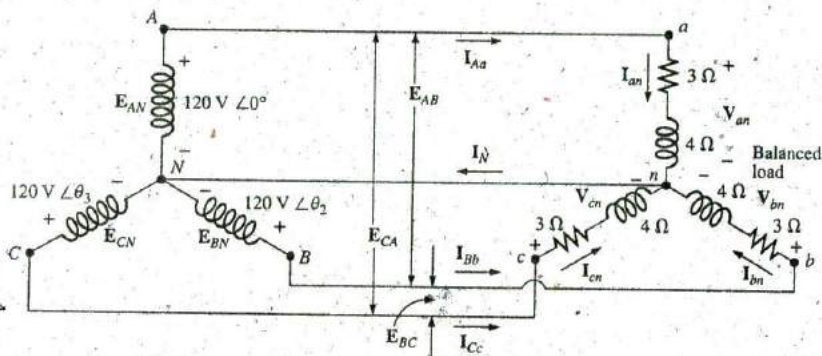
FIG. 23.13
Example 23.1.

Solutions:

a. For an ABC phase sequence,

$$\theta_2 = -120° \quad \text{and} \quad \theta_3 = +120°$$

b. $E_L = \sqrt{3}E_\phi = (1.73)(120 \text{ V}) = 208$ V. Therefore,

$$E_{AB} = E_{BC} = E_{CA} = 208 \text{ V}$$

c. $V_\phi = E_\phi$. Therefore,

$$V_{an} = E_{AN} \qquad V_{bn} = E_{BN} \qquad V_{cn} = E_{CN}$$

$$I_{\phi L} = I_{an} = \frac{V_{an}}{Z_{an}} = \frac{120 \text{ V} \angle 0°}{3\,\Omega + j4\,\Omega} = \frac{120 \text{ V} \angle 0°}{5\,\Omega \angle 53.13°}$$

$$= 24 \text{ A} \angle -53.13°$$

$$I_{bn} = \frac{V_{bn}}{Z_{bn}} = \frac{120\,V\,\angle-120°}{5\,\Omega\,\angle53.13°} = 24\,A\,\angle-173.13°$$

$$I_{cn} = \frac{V_{cn}}{Z_{cn}} = \frac{120\,V\,\angle+120°}{5\,\Omega\,\angle53.13°} = 24\,A\,\angle66.87°$$

and, since $I_L = I_{\phi L}$,

$$I_{Aa} = I_{an} = 24\,A\,\angle-53.13°$$
$$I_{Bb} = I_{bn} = 24\,A\,\angle-173.13°$$
$$I_{Cc} = I_{cn} = 24\,A\,\angle66.87°$$

d. Applying Kirchhoff's current law, we have

$$I_N = I_{Aa} + I_{Bb} + I_{Cc}$$

In rectangular form,

$$
\begin{array}{rcl}
I_{Aa} = 24\,A\,\angle-53.13° & = & 14.40\,A \quad -j\,19.20\,A \\
I_{Bb} = 24\,A\,\angle-173.13° & = & -22.83\,A - j\,2.87\,A \\
I_{Cc} = 24\,A\,\angle66.87° & = & 9.43\,A \quad +j\,22.07\,A \\
\Sigma(I_{Aa} + I_{Bb} + I_{Cc}) & = & 0 + j\,0
\end{array}
$$

and I_N is in fact equals to **zero**, as required for a balanced load.

23.6 Y-Δ SYSTEM

There is no neutral connection for the Y-Δ system in Fig. 23.14. Any variation in the impedance of a phase that produces an unbalanced system simply varies the line and phase currents of the system.

For a balanced load,

$$\boxed{Z_1 = Z_2 = Z_3} \tag{23.9}$$

The voltage across each phase of the load is equal to the line voltage of the generator for a balanced or an unbalanced load:

$$\boxed{V_\phi = E_L} \tag{23.10}$$
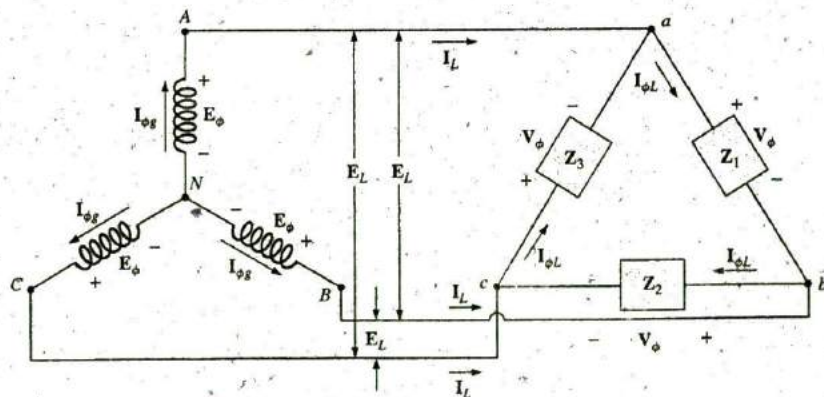
FIG. 23.14
Y-connected generator with a Δ-connected load.

The relationship between the line currents and phase currents of a balanced Δ load can be found using an approach very similar to that used in Section 23.3 to find the relationship between the line voltages and phase voltages of a Y-connected generator. For this case, however, Kirchhoff's current law is used instead of Kirchhoff's voltage law.

The result is

$$I_L = \sqrt{3}I_\phi \qquad\qquad\qquad (23.11)$$

and the phase angle between a line current and the nearest phase current is 30°. A more detailed discussion of this relationship between the line and phase currents of a Δ-connected system can be found in Section 23.7.

For a balanced load, the line currents will be equal in magnitude, as will the phase currents.

EXAMPLE 23.2 For the three-phase system in Fig. 23.15:

a. Find the phase angles θ_2 and θ_3.
b. Find the current in each phase of the load.
c. Find the magnitude of the line currents.
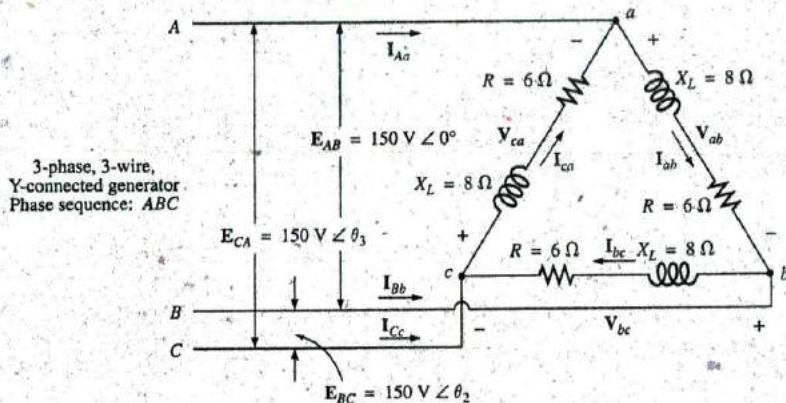
FIG. 23.15
Example 23.2.

Solutions:

a. For an *ABC* sequence,

$$\theta_2 = -120° \quad \text{and} \quad \theta_3 = +120°$$

b. $V_\phi = E_L$. Therefore,

$$V_{ab} = E_{AB} \qquad V_{ca} = E_{CA} \qquad V_{bc} = E_{BC}$$

The phase currents are

$$I_{ab} = \frac{V_{ab}}{Z_{ab}} = \frac{150\ V\ \angle 0°}{6\ \Omega + j8\ \Omega} = \frac{150\ V\ \angle 0°}{10\ \Omega\ \angle 53.13°} = 15\ A\ \angle -53.13°$$

$$I_{bc} = \frac{V_{bc}}{Z_{bc}} = \frac{150\ V\ \angle -120°}{10\ \Omega\ \angle 53.13°} = 15\ A\ \angle -173.13°$$

$$I_{ca} = \frac{V_{ca}}{Z_{ca}} = \frac{150\ V\ \angle +120°}{10\ \Omega\ \angle 53.13°} = 15\ A\ \angle 66.87°$$

c. $I_L = \sqrt{3}I_\phi = (1.73)(15 \text{ A}) = 25.95 \text{ A}$. Therefore,

$$I_{Aa} = I_{Bb} = I_{Cc} = 25.95 \text{ A}$$

23.7 Δ-CONNECTED GENERATOR

If we rearrange the coils of the generator in Fig. 23.16(a) as shown in Fig. 23.16(b), the system is referred to as a *three-phase, three-wire, Δ-connected ac generator.* In this system, the phase and line voltages are equivalent and equal to the voltage induced across each coil of the generator; that is,

$$\left. \begin{array}{l} E_{AB} = E_{AN} \text{ and } e_{AN} = \sqrt{2}E_{AN} \sin \omega t \\ E_{BC} = E_{BN} \text{ and } e_{BN} = \sqrt{2}E_{BN} \sin(\omega t - 120°) \\ E_{CA} = E_{CN} \text{ and } e_{CN} = \sqrt{2}E_{CN} \sin(\omega t + 120°) \end{array} \right\} \begin{array}{l} \text{Phase} \\ \text{sequence} \\ ABC \end{array}$$

or $$\boxed{E_L = E_{\phi g}}$$ (23.12)
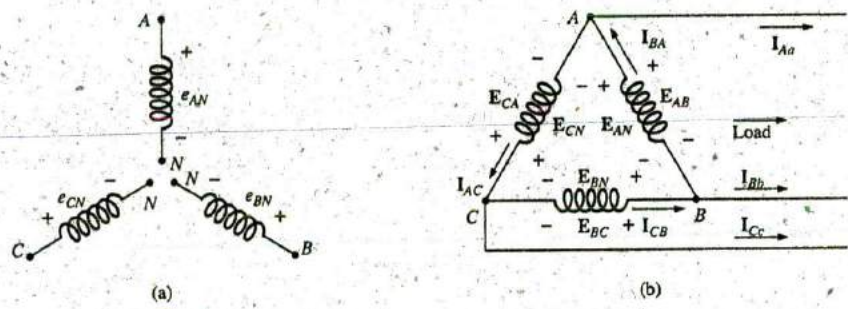
FIG. 23.16
Δ-connected generator.

Note that only one voltage (magnitude) is available instead of the two available in the Y-connected system.

Unlike the line current for the Y-connected generator, the line current for the Δ-connected system is not equal to the phase current. The relationship between the two can be found by applying Kirchhoff's current law at one of the nodes and solving for the line current in terms of the phase currents; that is, at node A,

$$I_{BA} = I_{Aa} + I_{AC}$$

or $$I_{Aa} = I_{BA} - I_{AC} = I_{BA} + I_{CA}$$

The phasor diagram is shown in Fig. 23.17 for a balanced load.

Using the same procedure to find the line current as was used to find the line voltage of a Y-connected generator produces the following:

$$I_{Aa} = \sqrt{3}I_{BA} \angle -30°$$
$$I_{Bb} = \sqrt{3}I_{CB} \angle -150°$$
$$I_{Cc} = \sqrt{3}I_{AC} \angle 90°$$

In general,

$$\boxed{I_L = \sqrt{3}I_{\phi g}}$$ (23.13)
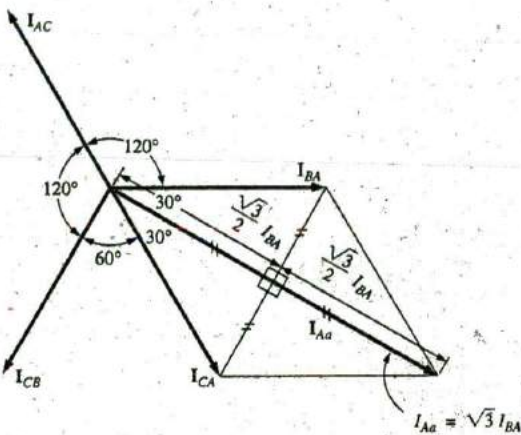
FIG. 23.17

Determining a line current from the phase currents of a Δ-connected, three-phase generator.
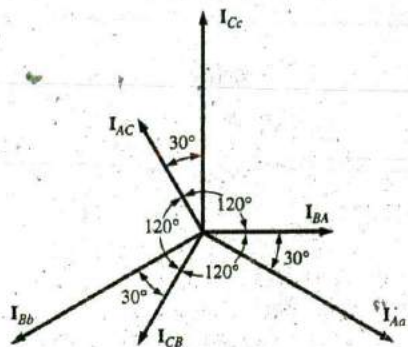
FIG. 23.18

The phasor diagram of the currents of a three-phase, Δ-connected generator.

with the phase angle between a line current and the nearest phase current at 30°. The phasor diagram of the currents is shown in Fig. 23.18.

Just as for the voltages of a Y-connected generator, the phasor sum of the line currents or phase currents for Δ-connected systems with balanced loads is zero.
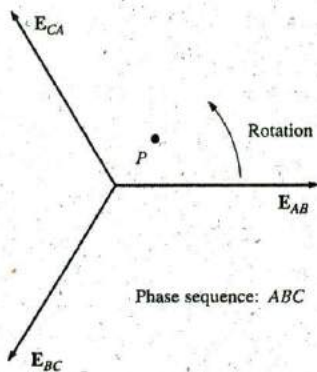
FIG. 23.19

Determining the phase sequence for a Δ-connected, three-phase generator.

23.8 PHASE SEQUENCE (Δ-CONNECTED GENERATOR)

Even though the line and phase voltages of a Δ-connected system are the same, it is standard practice to describe the phase sequence in terms of the line voltages. The method used is the same as that described for the line voltages of the Y-connected generator. For example, the phasor diagram of the line voltages for a phase sequence ABC is shown in Fig. 23.19. In drawing such a diagram, one must take care to have the sequence of the first and second subscripts the same. In phasor notation,

$$E_{AB} = E_{AB} \angle 0°$$
$$E_{BC} = E_{BC} \angle -120°$$
$$E_{CA} = E_{CA} \angle 120°$$

23.9 Δ-Δ, Δ-Y THREE-PHASE SYSTEMS

The basic equations necessary to analyze either of the two systems (Δ-Δ, Δ-Y) have been presented in this chapter. Following are two descriptive examples, one with a Δ-connected load, and one with a Y-connected load.

EXAMPLE 23.3 For the Δ-Δ system shown in Fig. 23.20:

a. Find the phase angles θ_2 and θ_3 for the specified phase sequence.
b. Find the current in each phase of the load.
c. Find the magnitude of the line currents.
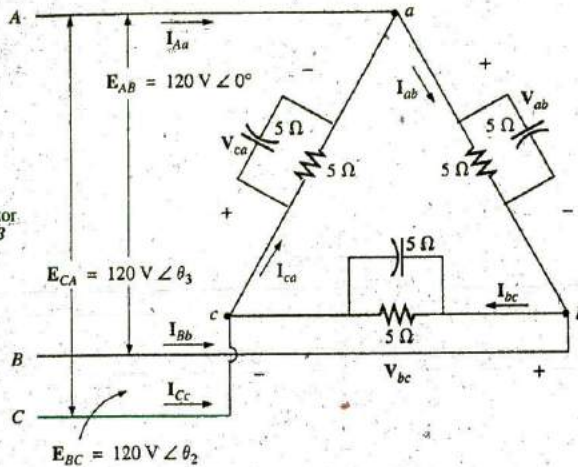
FIG. 23.20
Example 23.3: Δ-Δ system.

Solutions:

a. For an *ACB* phase sequence,

$$\theta_2 = 120° \quad \text{and} \quad \theta_3 = -120°$$

b. $\mathbf{V}_\phi = \mathbf{E}_L$. Therefore,

$$\mathbf{V}_{ab} = \mathbf{E}_{AB} \quad \mathbf{V}_{ca} = \mathbf{E}_{CA} \quad \mathbf{V}_{bc} = \mathbf{E}_{BC}$$

The phase currents are

$$\mathbf{I}_{ab} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_{ab}} = \frac{120 \text{ V} \angle 0°}{\dfrac{(5 \text{ Ω} \angle 0°)(5 \text{ Ω} \angle -90°)}{5 \text{ Ω} - j5 \text{ Ω}}} = \frac{120 \text{ V} \angle 0°}{\dfrac{25 \text{ Ω} \angle -90°}{7.071 \angle -45°}}$$

$$= \frac{120 \text{ V} \angle 0°}{3.54 \text{ Ω} \angle -45°} = 33.9 \text{ A} \angle 45°$$

$$\mathbf{I}_{bc} = \frac{\mathbf{V}_{bc}}{\mathbf{Z}_{bc}} = \frac{120 \text{ V} \angle 120°}{3.54 \text{ Ω} \angle -45°} = 33.9 \text{ A} \angle 165°$$

$$\mathbf{I}_{ca} = \frac{\mathbf{V}_{ca}}{\mathbf{Z}_{ca}} = \frac{120 \text{ V} \angle -120°}{3.54 \text{ Ω} \angle -45°} = 33.9 \text{ A} \angle -75°$$

c. $I_L = \sqrt{3} I_\phi = (1.73)(34 \text{ A}) = 58.82 \text{ A}$. Therefore,

$$I_{Aa} = I_{Bb} = I_{Cc} = 58.82 \text{ A}$$

EXAMPLE 23.4 For the Δ-Y system shown in Fig. 23.21:

a. Find the voltage across each phase of the load.
b. Find the magnitude of the line voltages.

Solutions:

a. $\mathbf{I}_{\phi L} = \mathbf{I}_L$. Therefore,

$$\mathbf{I}_{an} = \mathbf{I}_{Aa} = 2 \text{ A} \angle 0°$$
$$\mathbf{I}_{bn} = \mathbf{I}_{Bb} = 2 \text{ A} \angle -120°$$
$$\mathbf{I}_{cn} = \mathbf{I}_{Cc} = 2 \text{ A} \angle 120°$$

FIG. 23.21

Example 23.4: Δ-Y system.
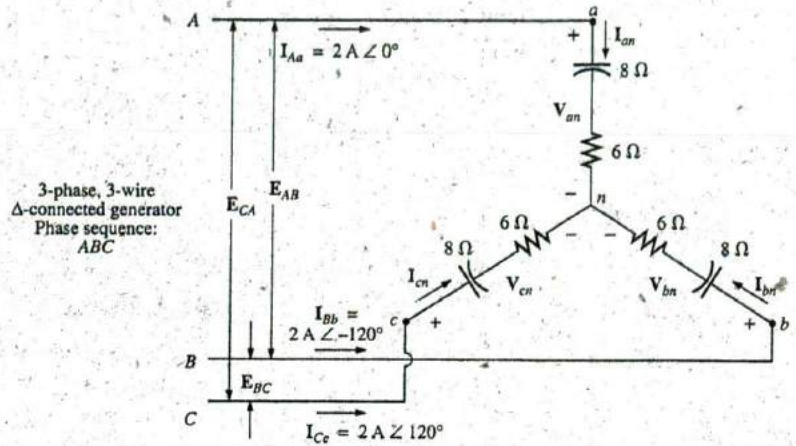
The phase voltages are

$$V_{an} = I_{an}Z_{an} = (2 \text{ A } \angle 0°)(10 \text{ } \Omega \angle -53.13°) = 20 \text{ V } \angle -53.13°$$
$$V_{bn} = I_{bn}Z_{bn} = (2 \text{ A } \angle -120°)(10 \text{ } \Omega \angle -53.13°) = 20 \text{ V } \angle -173.13°$$
$$V_{cn} = I_{cn}Z_{cn} = (2 \text{ A } \angle 120°)(10 \text{ } \Omega \angle -53.13°) = 20 \text{ V } \angle 66.87°$$

b. $E_L = \sqrt{3}V_\phi = (1.73)(20 \text{ V}) = 34.6 \text{ V}$. Therefore,

$$E_{BA} = E_{CB} = E_{AC} = 34.6 \text{ V}$$

23.10 POWER

Y-Connected Balanced Load

Please refer to Fig. 23.22 for the following discussion.
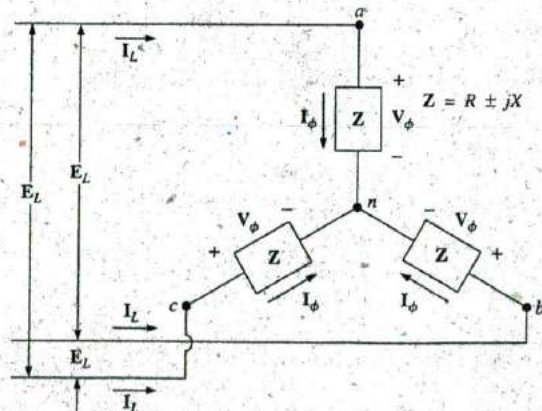
FIG. 23.22

Y-connected balanced load.

Average Power The average power delivered to each phase can be determined by

$$P_\phi = V_\phi I_\phi \cos \theta_{I_\phi}^{V_\phi} = I_\phi^2 R_\phi = \frac{V_R^2}{R_\phi} \qquad \text{(watts, W)} \qquad \textbf{(23.14)}$$

where $\theta_{I_\phi}^{V_\phi}$ indicates that θ is the phase angle between V_ϕ and I_ϕ.

The total power delivered can be determined by Eq. (23.15) or Eq. (23.16):

$$P_T = 3P_\phi \qquad \text{(W)} \qquad \textbf{(23.15)}$$

or, since $\qquad V_\phi = \dfrac{E_L}{\sqrt{3}} \qquad$ and $\qquad I_\phi = I_L$

then $\qquad P_T = 3\dfrac{E_L}{\sqrt{3}} I_L \cos \theta_{I_\phi}^{V_\phi}$

But $\qquad \left(\dfrac{3}{\sqrt{3}}\right)(1) = \left(\dfrac{3}{\sqrt{3}}\right)\left(\dfrac{\sqrt{3}}{\sqrt{3}}\right) = \dfrac{3\sqrt{3}}{3} = \sqrt{3}$

Therefore,

$$P_T = \sqrt{3}E_L I_L \cos \theta_{I_\phi}^{V_\phi} = 3I_L^2 R_\phi \qquad \text{(W)} \qquad \textbf{(23.16)}$$

Reactive Power The reactive power of each phase (in volt-amperes reactive) is

$$Q_\phi = V_\phi I_\phi \sin \theta_{I_\phi}^{V_\phi} = I_\phi^2 X_\phi = \frac{V_\phi^2}{X_\phi} \qquad \text{(VAR)} \qquad \textbf{(23.17)}$$

The total reactive power of the load is

$$Q_T = 3Q_\phi \qquad \text{(VAR)} \qquad \textbf{(23.18)}$$

or, proceeding in the same manner as above, we have

$$Q_T = \sqrt{3}E_L I_L \sin \theta_{I_\phi}^{V_\phi} = 3I_L^2 X_\phi \qquad \text{(VAR)} \qquad \textbf{(23.19)}$$

Apparent Power The apparent power of each phase is

$$S_\phi = V_\phi I_\phi \qquad \text{(VA)} \qquad \textbf{(23.20)}$$

The total apparent power of the load is

$$S_T = 3S_\phi \qquad \text{(VA)} \qquad \textbf{(23.21)}$$

or, as before,

$$S_T = \sqrt{3}E_L I_L \qquad \text{(VA)} \qquad \textbf{(23.22)}$$

Power Factor The power factor of the system is given by

$$F_p = \frac{P_T}{S_T} = \cos \theta_{I_\phi}^{V_\phi} \quad \text{(leading or lagging)} \quad (23.23)$$

EXAMPLE 23.5 For the Y-connected load in Fig. 23.23:
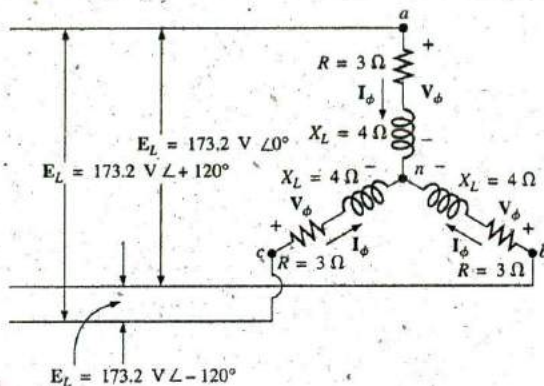
FIG. 23.23
Example 23.5.

a. Find the average power to each phase and the total load.
b. Determine the reactive power to each phase and the total reactive power.
c. Find the apparent power to each phase and the total apparent power.
d. Find the power factor of the load.

Solutions:

a. The *average power* is

$$P_\phi = V_\phi I_\phi \cos \theta_{I_\phi}^{V_\phi} = (100 \text{ V})(20 \text{ A}) \cos 53.13° = (2000)(0.6)$$
$$= \mathbf{1200 \ W}$$

$$P_\phi = I_\phi^2 R_\phi = (20 \text{ A})^2(3 \ \Omega) = (400)(3) = \mathbf{1200 \ W}$$

$$P_\phi = \frac{V_R^2}{R_\phi} = \frac{(60 \text{ V})^2}{3 \ \Omega} = \frac{3600}{3} = \mathbf{1200 \ W}$$

$$P_T = 3P_\phi = (3)(1200 \text{ W}) = \mathbf{3600 \ W}$$

or

$$P_T = \sqrt{3} E_L I_L \cos \theta_{I_\phi}^{V_\phi} = (1.732)(173.2 \text{ V})(20 \text{ A})(0.6) = \mathbf{3600 \ W}$$

b. The *reactive power* is

$$Q_\phi = V_\phi I_\phi \sin \theta_{I_\phi}^{V_\phi} = (100 \text{ V})(20 \text{ A}) \sin 53.13° = (2000)(0.8)$$
$$= \mathbf{1600 \ VAR}$$

or $\quad Q_\phi = I_\phi^2 X_\phi = (20 \text{ A})^2(4 \ \Omega) = (400)(4) = \mathbf{1600 \ VAR}$
$$Q_T = 3Q_\phi = (3)(1600 \text{ VAR}) = \mathbf{4800 \ VAR}$$

or

$$Q_T = \sqrt{3} E_L I_L \sin \theta_{I_\phi}^{V_\phi} = (1.732)(173.2 \text{ V})(20 \text{ A})(0.8) = \mathbf{4800 \ VAR}$$

c. The *apparent power* is

$$S_\phi = V_\phi I_\phi = (100 \text{ V})(20 \text{ A}) = 2000 \text{ VA}$$
$$S_T = 3S_\phi = (3)(2000 \text{ VA}) = 6000 \text{ VA}$$

or $S_T = \sqrt{3}E_L I_L = (1.732)(173.2 \text{ V})(20 \text{ A}) = 6000 \text{ VA}$

d. The *power factor* is

$$F_p = \frac{P_T}{S_T} = \frac{3600 \text{ W}}{6000 \text{ VA}} = 0.6 \text{ lagging}$$

Δ-Connected Balanced Load

Please refer to Fig. 23.24 for the following discussion.
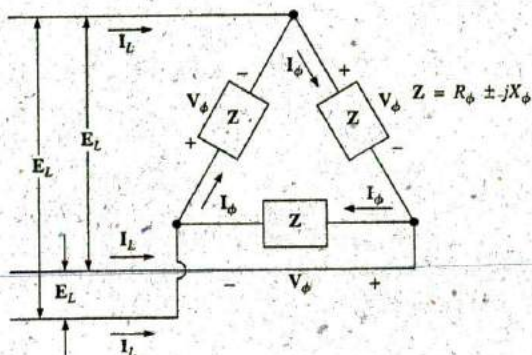
FIG. 23.24
Δ-connected balanced load.

Average Power

$$P_\phi = V_\phi I_\phi \cos \theta_{I_\phi}^{V_\phi} = I_\phi^2 R_\phi = \frac{V_R^2}{R_\phi} \quad \text{(W)} \qquad \textbf{(23.24)}$$

$$P_T = 3P_\phi \quad \text{(W)} \qquad \textbf{(23.25)}$$

Reactive Power

$$Q_\phi = V_\phi I_\phi \sin \theta_{I_\phi}^{V_\phi} = I_\phi^2 X_\phi = \frac{V_\phi^2}{X_\phi} \quad \text{(VAR)} \qquad \textbf{(23.26)}$$

$$Q_T = 3Q_\phi \quad \text{(VAR)} \qquad \textbf{(23.27)}$$

Apparent Power

$$S_\phi = V_\phi I_\phi \quad \text{(VA)} \qquad \textbf{(23.28)}$$

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$$S_T = 3S_\phi = \sqrt{3}E_L I_L \qquad \text{(VA)} \qquad (23.29)$$

Power Factor

$$F_p = \frac{P_T}{S_T} \qquad (23.30)$$

EXAMPLE 23.6 For the Δ-Y connected load in Fig. 23.25, find the total average, reactive, and apparent power. In addition, find the power factor of the load.
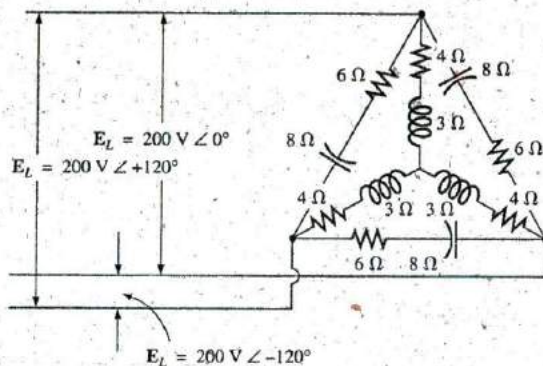
FIG. 23.25
Example 23.6.

Solution: Consider the Δ and Y separately.

For the Δ:

$$Z_\Delta = 6\,\Omega - j\,8\,\Omega = 10\,\Omega\,\angle{-53.13°}$$

$$I_\phi = \frac{E_L}{Z_\Delta} = \frac{200\text{ V}}{10\,\Omega} = 20\text{ A}$$

$$P_{T_\Delta} = 3I_\phi^2 R_\phi = (3)(20\text{ A})^2(6\,\Omega) = 7200\text{ W}$$

$$Q_{T_\Delta} = 3I_\phi^2 X_\phi = (3)(20\text{ A})^2(8\,\Omega) = 9600\text{ VAR }(C).$$

$$S_{T_\Delta} = 3V_\phi I_\phi = (3)(200\text{ V})(20\text{ A}) = 12{,}000\text{ VA}$$

For the Y:

$$Z_Y = 4\,\Omega + j\,3\,\Omega = 5\,\Omega\,\angle{36.87°}$$

$$I_\phi = \frac{E_L/\sqrt{3}}{Z_Y} = \frac{200\text{ V}/\sqrt{3}}{5\,\Omega} = \frac{116\text{ V}}{5\,\Omega} = 23.12\text{ A}$$

$$P_{T_Y} = 3I_\phi^2 R_\phi = (3)(23.12\text{ A})^2(4\,\Omega) = 6414.41\text{ W}$$

$$Q_{T_Y} = 3I_\phi^2 X_\phi = (3)(23.12\text{ A})^2(3\,\Omega) = 4810.81\text{ VAR }(L)$$

$$S_{T_Y} = 3V_\phi I_\phi = (3)(116\text{ V})(23.12\text{ A}) = 8045.76\text{ VA}$$

For the total load:

$$P_T = P_{T_\Delta} + P_{T_Y} = 7200 \text{ W} + 6414.41 \text{ W} = 13{,}614.41 \text{ W}$$

$$Q_T = Q_{T_\Delta} - Q_{T_Y} = 9600 \text{ VAR } (C) - 4810.81 \text{ VAR } (I)$$
$$= 4789.19 \text{ VAR } (C)$$

$$S_T = \sqrt{P_T^2 + Q_T^2} = \sqrt{(13{,}614.41 \text{ W})^2 + (4789.19 \text{ VAR})^2}$$
$$= 14{,}432.2 \text{ VA}$$

$$F_p = \frac{P_T}{S_T} = \frac{13{,}614.41 \text{ W}}{14{,}432.20 \text{ VA}} = 0.943 \text{ leading}$$

EXAMPLE 23.7 Each transmission line of the three-wire, three-phase system in Fig. 23.26 has an impedance of $15 \ \Omega + j20 \ \Omega$. The system delivers a total power of 160 kW at 12,000 V to a balanced three-phase load with a lagging power factor of 0.86.
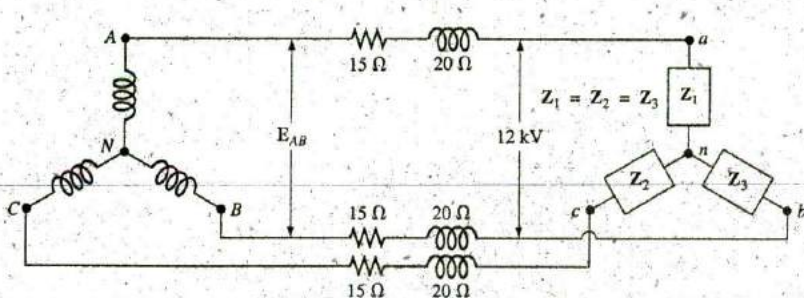
FIG. 23.26
Example 23.7.

a. Determine the magnitude of the line voltage E_{AB} of the generator.
b. Find the power factor of the total load applied to the generator.
c. What is the efficiency of the system?

Solutions:

a. $V_\phi \text{ (load)} = \dfrac{V_L}{\sqrt{3}} = \dfrac{12{,}000 \text{ V}}{1.73} = 6936.42 \text{ V}$

$P_T \text{ (load)} = 3V_\phi I_\phi \cos \theta$

and

$$I_\phi = \frac{P_T}{3V_\phi \cos \theta} = \frac{160{,}000 \text{ W}}{3(6936.42 \text{ V})(0.86)}$$
$$= 8.94 \text{ A}$$

Since $\theta = \cos^{-1} 0.86 = 30.68°$, assigning V_ϕ an angle of $0°$ or $V_\phi = V_\phi \angle 0°$, a lagging power factor results in

$$I_\phi = 8.94 \text{ A} \angle -30.68°$$

For each phase, the system will appear as shown in Fig. 23.27, where

$$E_{AN} - I_\phi Z_{\text{line}} - V_\phi = 0$$
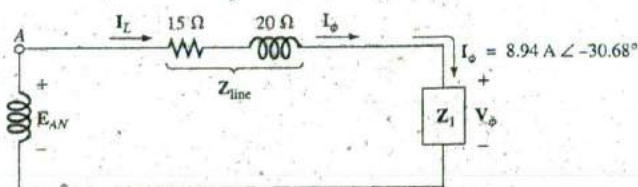
FIG. 23.27

The loading on each phase of the system in Fig. 23.26.

or

$$E_{AN} = I_\phi Z_{line} + V_\phi$$
$$= (8.94 \text{ A} \angle -30.68°)(25 \text{ } \Omega \angle 53.13°) + 6936.42 \text{ V} \angle 0°$$
$$= 223.5 \text{ V} \angle 22.45° + 6936.42 \text{ V} \angle 0°$$
$$= 206.56 \text{ V} + j \, 85.35 \text{ V} + 6936.42 \text{ V}$$
$$= 7142.98 \text{ V} + j \, 85.35 \text{ V}$$
$$= 7143.5 \text{ V} \angle 0.68°$$

Then
$$E_{AB} = \sqrt{3}E_{\phi g} = (1.73)(7143.5 \text{ V})$$
$$= \mathbf{12,358.26 \text{ V}}$$

b. $P_T = P_{load} + P_{lines}$
$$= 160 \text{ kW} + 3(I_L)^2 R_{line}$$
$$= 160 \text{ kW} + 3(8.94 \text{ A})^2 15 \text{ } \Omega$$
$$= 160,000 \text{ W} + 3596.55 \text{ W}$$
$$= 163,596.55 \text{ W}$$

and
$$P_T = \sqrt{3}V_L I_L \cos \theta_T$$

or
$$\cos \theta_T = \frac{P_T}{\sqrt{3}V_L I_L} = \frac{163,596.55 \text{ W}}{(1.73)(12,358.26 \text{ V})(8.94 \text{ A})}$$

and
$$F_p = \mathbf{0.856} < 0.86 \text{ of load}$$

c. $\eta = \dfrac{P_o}{P_i} = \dfrac{P_o}{P_o + P_{losses}} = \dfrac{160 \text{ kW}}{160 \text{ kW} + 3596.55 \text{ W}} = 0.978$
$$= \mathbf{97.8\%}$$

23.11 THREE-WATTMETER METHOD

The power delivered to a balanced or an unbalanced four-wire, Y-connected load can be found by the **three-wattmeter method**, that is, by using three wattmeters in the manner shown in Fig. 23.28. Each wattmeter measures the power delivered to each phase. The potential coil of each wattmeter is connected parallel with the load, while the current coil is in series with the load. The total average power of the system can be found by summing the three wattmeter readings; that is,

$$\boxed{P_{T_Y} = P_1 + P_2 + P_3} \tag{23.31}$$

For the load (balanced or unbalanced), the wattmeters are connected as shown in Fig. 23.29. The total power is again the sum of the three wattmeter readings:

$$\boxed{P_{T_\Delta} = P_1 + P_2 + P_3} \tag{23.32}$$
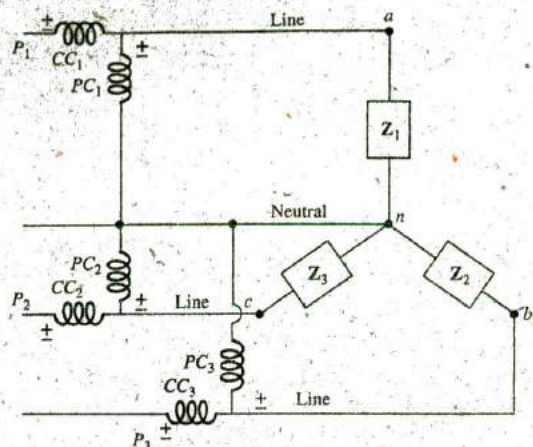
FIG. 23.28
Three-wattmeter method for a Y-connected load.
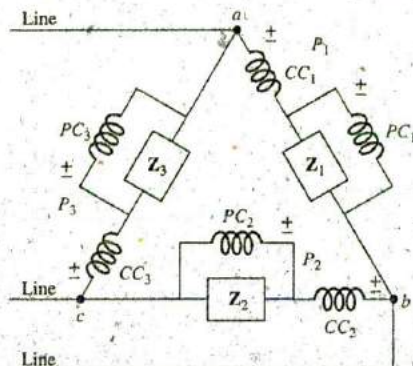
FIG. 23.29
Three-wattmeter method for a Δ-connected load.

If in either of the cases just described the load is balanced, the power delivered to each phase will be the same. The total power is then just three times any one wattmeter reading.

23.12 TWO-WATTMETER METHOD

The power delivered to a three-phase, three-wire, Δ- or Y-connected, balanced or unbalanced local can be found using only two wattmeters if the proper connection is employed and if the wattmeter readings are interpreted properly. The basic connections of this **two-wattmeter method** are shown in Fig. 23.30. One end of each potential coil is connected to the same line. The current coils are then placed in the remaining lines.

The connection shown in Fig. 23.31 also satisfies the requirements. A third hookup is also possible, but this is left to the reader as an exercise.

The total power delivered to the load is the algebraic sum of the two wattmeter readings. For a *balanced* load, we now consider two methods of determining whether the total power is the sum or the difference of the two wattmeter readings. The first method to be described requires
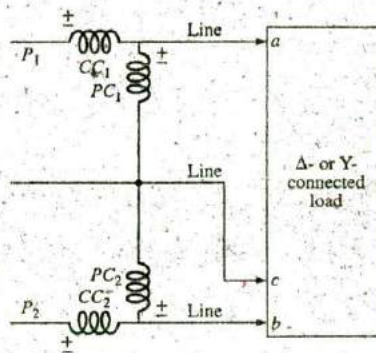
FIG. 23.30
Two-wattmeter method for a Δ- or a Y-connected load.
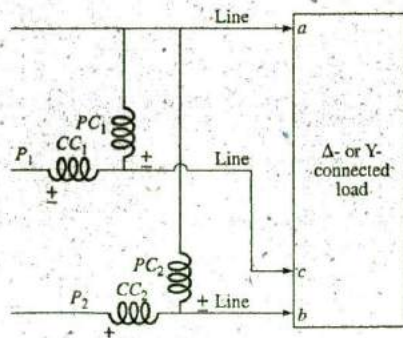
FIG. 23.31
Alternative hookup for the two-wattmeter method.
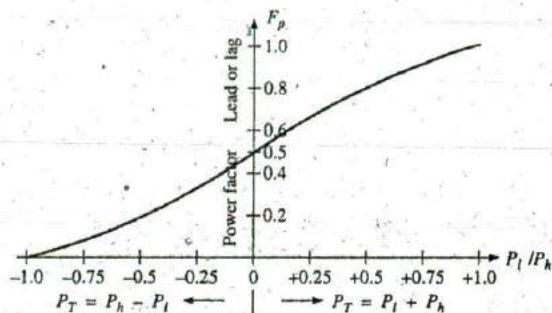
FIG. 23.32

Determining whether the readings obtained using the two-wattmeter method should be added or subtracted.

that we know or are able to find the power factor (leading or lagging) of any one phase of the load. When this information has been obtained, it can be applied directly to the curve in Fig. 23.32.

The curve in Fig. 23.32 is a plot of the power factor of the load (phase) versus the ratio P_l/P_h, where P_l and P_h are the magnitudes of the lower- and higher-reading wattmeters, respectively. Note that for a power factor (leading or lagging) greater than 0.5, the ratio has a positive value. This indicates that both wattmeters are reading positive, and the total power is the sum of the two wattmeter readings; that is, $P_T = P_l + P_h$. For a power factor less than 0.5 (leading or lagging), the ratio has a negative value. This indicates that the smaller-reading wattmeter is reading negative, and the total power is the difference of the two wattmeter readings; that is, $P_T = P_h - P_l$.

A closer examination reveals that, when the power factor is 1 ($\cos 0° = 1$), corresponding to a purely resistive load, $P_l/P_h = 1$ or $P_l = P_h$, and both wattmeters have the same wattage indication. At a power factor equal to 0 ($\cos 90° = 0$), corresponding to a purely reactive load, $P_l/P_h = -1$ or $P_l = -P_h$, and both wattmeters again have the same wattage indication but with opposite signs. The transition from a negative to a positive ratio occurs when the power factor of the load is 0.5 or $\theta = \cos^{-1} 0.5 = 60°$. At this power factor, $P_l/P_h = 0$, so that $P_l = 0$, while P_h reads the total power delivered to the load.

The second method for determining whether the total power is the sum or difference of the two wattmeter readings involves a simple laboratory test. For the test to be applied, both wattmeters must first have an up-scale deflection. If one of the wattmeters has a below-zero indication, an up-scale deflection can be obtained by simply reversing the leads of the current coil of the wattmeter. To perform the test:

1. Take notice of which line does not have a current coil sensing the line current.
2. For the lower-reading wattmeter, disconnect the lead of the potential coil connected to the line without the current coil.
3. Take the disconnected lead of the lower-reading wattmeter's potential coil, and touch a connection point on the line that has the current coil of the higher-reading wattmeter.
4. If the pointer deflects downward (below zero watts), the wattage reading of the lower-reading wattmeter should be subtracted from that of the higher-reading wattmeter. Otherwise, the readings should be added.

For a *balanced system,* since.

$$P_T = P_h \pm P_l = \sqrt{3} E_L I_L \cos \theta_{I_\phi}^{V_\phi}$$

the power factor of the load (phase) can be found from the wattmeter readings and the magnitude of the line voltage and current:

$$F_p = \cos \theta_{I_\phi}^{V_\phi} = \frac{P_h \pm P_l}{\sqrt{3} E_L I_L} \qquad (23.33)$$

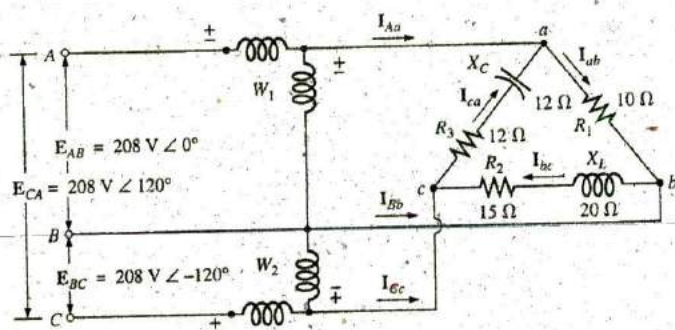**EXAMPLE 23.8** For the unbalanced Δ-connected load in Fig. 23.33 with two properly connected wattmeters:

FIG. 23.33
Example 23.8.

a. Determine the magnitude and angle of the phase currents.
b. Calculate the magnitude and angle of the line currents.
c. Determine the power reading of each wattmeter.
d. Calculate the total power absorbed by the load.
e. Compare the result of part (d) with the total power calculated using the phase currents and the resistive elements.

• Solutions:

a. $\mathbf{I}_{ab} = \dfrac{\mathbf{V}_{ab}}{\mathbf{Z}_{ab}} = \dfrac{\mathbf{E}_{AB}}{\mathbf{Z}_{ab}} = \dfrac{208 \text{ V} \angle 0°}{10 \,\Omega \angle 0°} = 20.8 \text{ A} \angle 0°$

$\mathbf{I}_{bc} = \dfrac{\mathbf{V}_{bc}}{\mathbf{Z}_{bc}} = \dfrac{\mathbf{E}_{BC}}{\mathbf{Z}_{bc}} = \dfrac{208 \text{ V} \angle -120°}{15 \,\Omega + j\,20 \,\Omega} = \dfrac{208 \text{ V} \angle -120°}{25 \,\Omega \angle 53.13°}$

$\quad = 8.32 \text{ A} \angle -173.13°$

$\mathbf{I}_{ca} = \dfrac{\mathbf{V}_{ca}}{\mathbf{Z}_{ca}} = \dfrac{\mathbf{E}_{CA}}{\mathbf{Z}_{ca}} = \dfrac{208 \text{ V} \angle +120°}{12 \,\Omega + j\,12 \,\Omega} = \dfrac{208 \text{ V} \angle +120°}{16.97 \,\Omega \angle -45°}$

$\quad = 12.26 \text{ A} \angle 165°$

b. $\mathbf{I}_{Aa} = \mathbf{I}_{ab} - \mathbf{I}_{ca}$

$\quad = 20.8 \text{ A} \angle 0° - 12.26 \text{ A} \angle 165°$

$\quad = 20.8 \text{ A} - (-11.84 \text{ A} + j\,3.17 \text{ A})$

$\quad = 20.8 \text{ A} + 11.84 \text{ A} - j\,3.17 \text{ A} = 32.64 \text{ A} - j\,3.17 \text{ A}$

$\quad = 32.79 \text{ A} \angle -5.55°$

$$I_{Bb} = I_{bc} - I_{ab}$$
$$= 8.32 \text{ A} \angle -173.13° - 20.8 \text{ A} \angle 0°$$
$$= (-8.26 \text{ A} - j1 \text{ A}) - 20.8 \text{ A}$$
$$= -8.26 \text{ A} - 20.8 \text{ A} - j1 \text{ A} = -29.06 \text{ A} - j1 \text{ A}$$
$$= \mathbf{29.08 \text{ A} \angle -178.03°}$$

$$I_{Cc} = I_{ca} - I_{bc}$$
$$= 12.26 \text{ A} \angle 165° - 8.32 \text{ A} \angle -173.13°$$
$$= (-11.84 \text{ A} + j3.17 \text{ A}) - (-8.26 \text{ A} - j1 \text{ A})$$
$$= -11.84 \text{ A} + 8.26 \text{ A} + j(3.17 \text{ A} + 1 \text{ A}) = -3.58 \text{ A} + j4.17 \text{ A}$$
$$= \mathbf{5.5 \text{ A} \angle 130.65°}$$

c. $P_1 = V_{ab}I_{Aa} \cos \theta_{I_{Aa}}^{V_{ab}}$ $V_{ab} = 208 \text{ V} \angle 0°$
$$I_{Aa} = 32.79 \text{ A} \angle -5.55°$$
$$= (208 \text{ V})(32.79 \text{ A}) \cos 5.55°$$
$$= \mathbf{6788.35 \text{ W}}$$

$$V_{bc} = E_{BC} = 208 \text{ V} \angle -120°$$

but $V_{cb} = E_{CB} = 208 \text{ V} \angle -120° + 180°$
$$= 208 \text{ V} \angle 60°$$

with $I_{Cc} = 5.5 \text{ A} \angle 130.65°$
$$P_2 = V_{cb}I_{Cc} \cos \theta_{I_{Cc}}^{V_{cb}}$$
$$= (208 \text{ V})(5.5 \text{ A}) \cos 70.65°$$
$$= \mathbf{379.1 \text{ W}}$$

d. $P_T = P_1 + P_2 = 6788.35 \text{ W} + 379.1 \text{ W}$
$$= \mathbf{7167.45 \text{ W}}$$

e. $P_T = (I_{ab})^2 R_1 + (I_{bc})^2 R_2 + (I_{ca})^2 R_3$
$$= (20.8 \text{ A})^2 10 \, \Omega + (8.32 \text{ A})^2 15 \, \Omega + (12.26 \text{ A})^2 12 \, \Omega$$
$$= 4326.4 \text{ W} + 1038.34 \text{ W} + 1803.69 \text{ W}$$
$$= \mathbf{7168.43 \text{ W}}$$

(The slight difference is due to the level of accuracy carried through the calculations.)

23.13 UNBALANCED, THREE-PHASE, FOUR-WIRE, Y-CONNECTED LOAD

For the three-phase, four-wire, Y-connected load in Fig. 23.34, conditions are such that *none* of the load impedances are equal—hence we have an **unbalanced polyphase load**. Since the neutral is a common point between the load and source, no matter what the impedance of each phase of the load and source, the voltage across each phase is the phase voltage of the generator:

$$\boxed{V_\phi = E_\phi} \tag{23.34}$$

The phase currents can therefore be determined by Ohm's law:

$$\boxed{i_{\phi_1} = \frac{V_{\phi_1}}{Z_1} = \frac{E_{\phi_1}}{Z_1} \quad \text{and so on}} \tag{23.35}$$
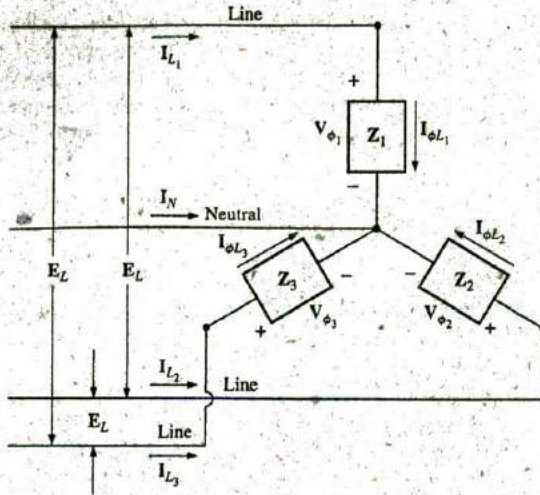
FIG. 23.34

Unbalanced Y-connected load.

The current in the neutral for any unbalanced system can then be found by applying Kirchhoff's current law at the common point n:

$$I_N = I_{\phi_1} + I_{\phi_2} + I_{\phi_3} = I_{L_1} + I_{L_2} + I_{L_3} \qquad (23.36)$$

Because of the variety of equipment found in an industrial environment, both three-phase power and single-phase power are usually provided with the single-phase obtained off the three-phase system. In addition, since the load on each phase is continually changing, a four-wire system (with a neutral) is normally used to ensure steady voltage levels and to provide a path for the current resulting from an unbalanced load. The system in Fig. 23.35 has a three-phase transformer dropping the line voltage from 13,800 V to 208 V. All the lower-power-demand loads, such as lighting, wall outlets, security, and so on, use the single-phase, 120 V line to neutral voltage. Higher power loads, such as air conditioners, electric ovens or dryers, and so on, use the single-phase, 208 V available from line to line. For larger motors and special high-demand equipment, the full three-phase power can be taken directly off the system, as shown in
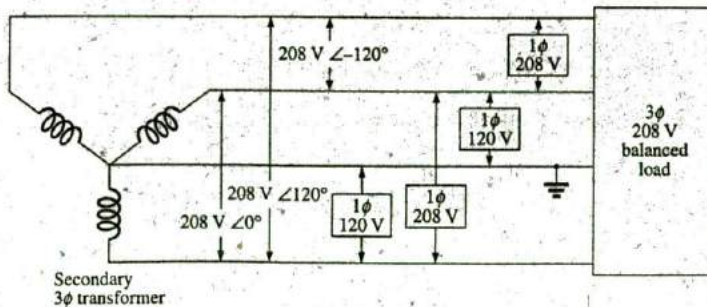
FIG. 23.35

$3\phi/1\phi$, 208 V/120 V *industrial supply.*

Fig. 23.35. In the design and construction of a commercial establishment, the National Electric Code requires that every effort be made to ensure that the expected loads, whether they be single- or multiphase, result in a total load that is as balanced as possible between the phases, thus ensuring the highest level of transmission efficiency.

23.14 UNBALANCED, THREE-PHASE, THREE-WIRE, Y-CONNECTED LOAD

For the system shown in Fig. 23.36, the required equations can be derived by first applying Kirchhoff's voltage law around each closed loop to produce

$$E_{AB} - V_{an} + V_{bn} = 0$$
$$E_{BC} - V_{bn} + V_{cn} = 0$$
$$E_{CA} - V_{cn} + V_{an} = 0$$
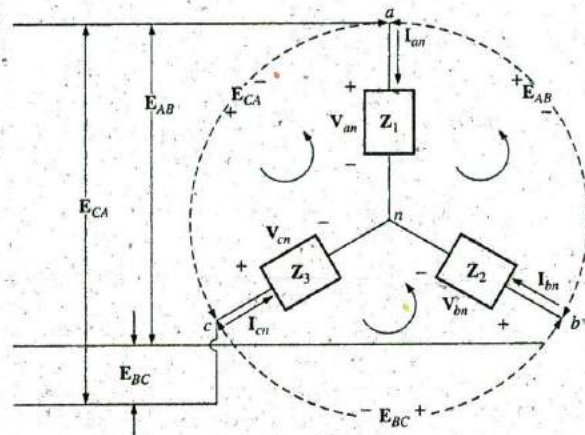
FIG. 23.36
Unbalanced, three-phase, three-wire, Y-connected load.

Substituting, we have

$$V_{an} = I_{an}Z_1 \qquad V_{bn} = I_{bn}Z_2 \qquad V_{cn} = I_{cn}Z_3$$

$$\boxed{\begin{aligned} E_{AB} &= I_{an}Z_1 - I_{bn}Z_2 \\ E_{BC} &= I_{bn}Z_2 - I_{cn}Z_3 \\ E_{CA} &= I_{cn}Z_3 - I_{an}Z_1 \end{aligned}}$$

(23.37a)
(23.37b)
(23.37c)

Applying Kirchhoff's current law at node n results in

$$I_{an} + I_{bn} + I_{cn} = 0 \qquad \text{and} \qquad I_{bn} = -I_{an} - I_{cn}$$

Substituting for I_{bn} in Eqs. (23.37a) and (23.37b) yields

$$E_{AB} = I_{an}Z_1 - [-(I_{an} + I_{cn})]Z_2$$
$$E_{BC} = -(I_{an} + I_{cn})Z_2 - I_{cn}Z_3$$

which are rewritten as

$$E_{AB} = I_{an}(Z_1 + Z_2) + I_{cn}Z_2$$
$$E_{BC} = I_{an}(-Z_2) + I_{cn}[-(Z_2 + Z_3)]$$

Using determinants, we have

$$I_{an} = \frac{\begin{vmatrix} E_{AB} & Z_2 \\ E_{BC} & -(Z_2 + Z_3) \end{vmatrix}}{\begin{vmatrix} Z_1 + Z_2 - & Z_2 \\ -Z_2 & -(Z_2 + Z_3) \end{vmatrix}}$$

$$= \frac{-(Z_2 + Z_3)E_{AB} - E_{BC}Z_2}{-Z_1Z_2 - Z_1Z_3 - Z_2Z_3 - Z_2^2 + Z_2^2}$$

$$I_{an} = \frac{-Z_2(E_{AB} + E_{BC}) - Z_3E_{AB}}{-Z_1Z_2 - Z_1Z_3 - Z_2Z_3}$$

Applying Kirchhoff's voltage law to the line voltages gives

$$E_{AB} + E_{CA} + E_{BC} = 0 \quad \text{or} \quad E_{AB} + E_{BC} = -E_{CA}$$

Substituting for $(E_{AB} + E_{CB})$ in the above equation for I_{an} gives

$$I_{an} = \frac{-Z_2(-E_{CA}) - Z_3E_{AB}}{-Z_1Z_2 - Z_1Z_3 - Z_2Z_3}$$

and

$$\boxed{I_{an} = \frac{E_{AB}Z_3 - E_{CA}Z_2}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}} \tag{23.38}$$

In the same manner, it can be shown that

$$\boxed{I_{cn} = \frac{E_{CA}Z_2 - E_{BC}Z_1}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}} \tag{23.39}$$

Substituting Eq. (23.39) for I_{cn} in the right-hand side of Eq. (23.37b), we obtain

$$\boxed{I_{bn} = \frac{E_{BC}Z_1 - E_{AB}Z_3}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}} \tag{23.40}$$

EXAMPLE 23.9 A *phase-sequence indicator* is an instrument such as shown in Fig. 23.37(a) that can display the phase sequence of a polyphase circuit. A network that performs this function appears in Fig. 23.37(b). The applied phase sequence is *ABC*. The bulb corresponding to this phase sequence burns more brightly than the bulb indicating the *ACB* sequence because a greater current is passing through the *ABC* bulb. Calculating the phase currents demonstrates that this situation does in fact exist:

$$Z_1 = X_C = \frac{1}{\omega C} = \frac{1}{(377 \text{ rad/s})(16 \times 10^{-6} \text{ F})} = 166 \ \Omega$$

By Eq. (23.39),

$$I_{cn} = \frac{E_{CA}Z_2 - E_{BC}Z_1}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}$$

$$= \frac{(200 \text{ V } \angle 120°)(200 \ \Omega \ \angle 0°) - (200 \text{ V } \angle -120°)(166 \ \Omega \ \angle -90°)}{(166 \ \Omega \ \angle -90°)(200 \ \Omega \ \angle 0°) + (166 \ \Omega \ \angle -90°)(200 \ \Omega \ \angle 0°) + (200 \ \Omega \ \angle 0°)(200 \ \Omega \ \angle 0°)}$$
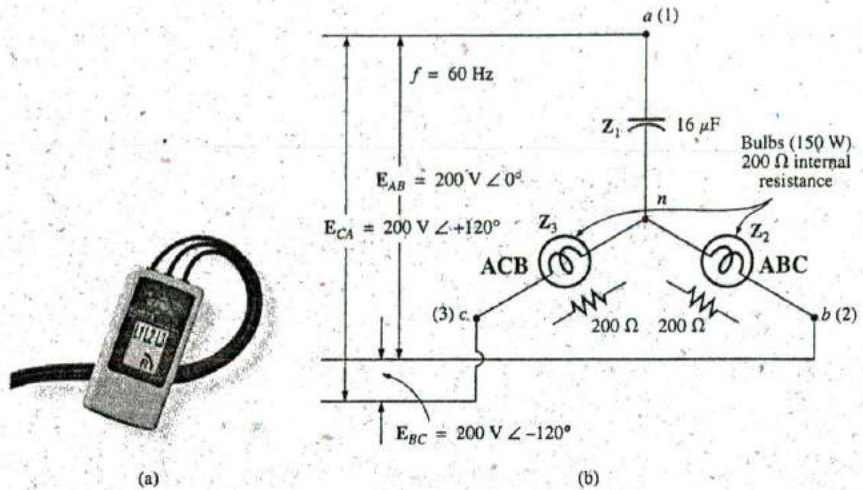
FIG. 23.37

(a) Phase sequence indicator. (b) Phase sequence detector network.
[Part (a) courtesy of Fluke Corporation.]

$$I_{cn} = \frac{40,000 \text{ V} \angle 120° + 33,200 \text{ V} \angle -30°}{33,200 \text{ }\Omega \angle -90° + 33,200 \text{ }\Omega \angle -90° + 40,000 \text{ }\Omega \angle 0°}$$

Dividing the numerator and denominator by 1000 and converting both to the rectangular domain yields

$$I_{cn} = \frac{(-20 + j\,34.64) + (28.75 - j\,16.60)}{40 - j\,66.4}$$

$$= \frac{8.75 + j\,18.04}{77.52 \angle -58.93°} = \frac{20.05 \angle 64.13°}{77.52 \angle -58.93°}$$

$$I_{cn} = 0.259 \text{ A} \angle 123.06°$$

By Eq. (23.40),

$$I_{bn} = \frac{E_{BC}Z_1 - E_{AB}Z_3}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}$$

$$= \frac{(200 \text{ V} \angle -120°)(166 \angle -90°) - (200 \text{ V} \angle 0°)(200 \angle 0°)}{77.52 \times 10^3 \text{ }\Omega \angle -58.93°}$$

$$I_{bn} = \frac{33,200 \text{ V} \angle -210° - 40,000 \text{ V} \angle 0°}{77.52 \times 10^3 \text{ }\Omega \angle -58.93°}$$

Dividing by 1000 and converting to the rectangular domain yields

$$I_{bn} = \frac{-28.75 + j\,16.60 - 40.0}{77.52 \angle -58.93°} = \frac{-68.75 + j\,16.60}{77.52 \angle -58.93°}$$

$$= \frac{70.73 \angle 166.43°}{77.52 \angle -58.93°} = 0.91 \text{ A} \angle 225.36°$$

and $I_{bn} > I_{cn}$ by a factor of more than 3 : 1. Therefore, the bulb indicating an *ABC* sequence will burn more brightly due to the greater current. If the phase sequence were *ACB*, the reverse would be true.

PROBLEMS

SECTION 23.5 Y-Connected Generator with a Y-Connected Load
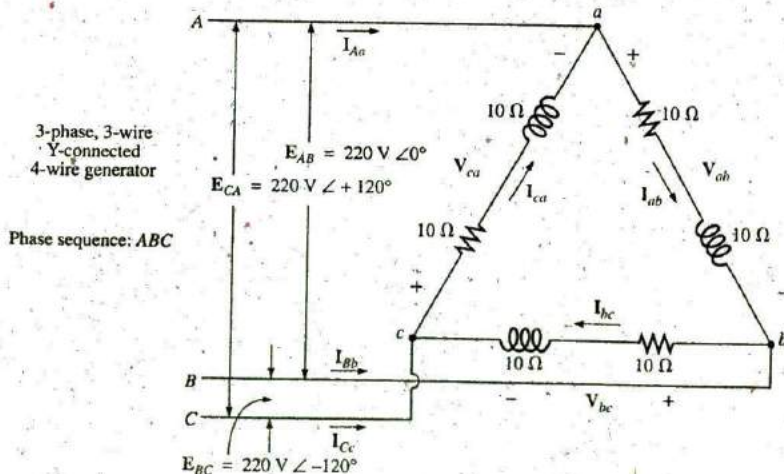
1. A balanced Y load having a 10 Ω resistance in each leg is connected to a three-phase, four-wire, Y-connected generator having a line voltage of 208 V. Calculate the magnitude of
   a. the phase voltage of the generator.
   b. the phase voltage of the load.
   c. the phase current of the load.
   d. the line current.

2. Repeat Problem 1 if each phase impedance is changed to a 12 Ω resistor in series with a 16 Ω capacitive reactance.

3. Repeat Problem 1 if each phase impedance is changed to a 10 Ω resistor in parallel with a 10 Ω capacitive reactance.

4. The phase sequence for the Y-Y system in Fig. 23.38 is ABC.
   a. Find the angles θ_2 and θ_3 for the specified phase sequence.
   b. Find the voltage across each phase impedance in phasor form.
   c. Find the current through each phase impedance in phasor form.
   d. Draw the phasor diagram of the currents found in part (c), and show that their phasor sum is zero.
   e. Find the magnitude of the line currents.
   f. Find the magnitude of the line voltages.

5. Repeat Problem 4 if the phase impedances are changed to a 9 Ω resistor in series with a 12 Ω inductive reactance.

6. Repeat Problem 4 if the phase impedances are changed to a 6 Ω resistance in parallel with an 8 Ω capacitive reactance.

7. For the system in Fig. 23.39, find the magnitude of the unknown voltages and currents.
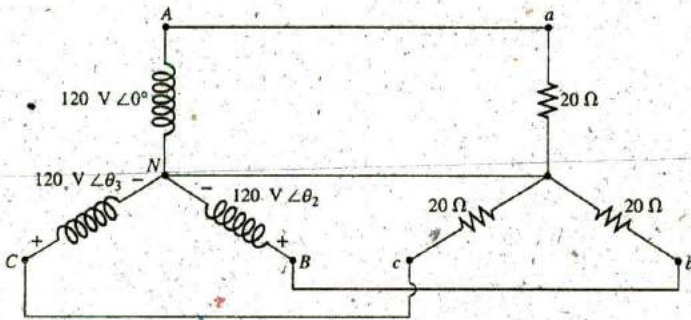
FIG. 23.38

Problems 4, 5, 6, and 31.
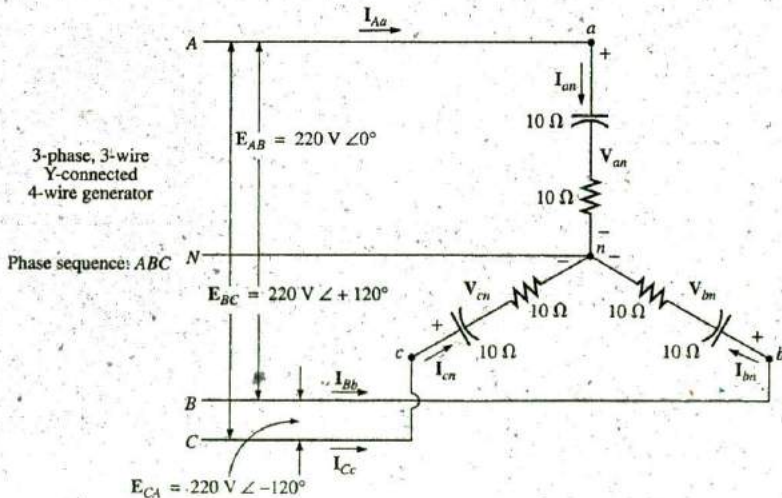
FIG. 23.39

Problems 7, 32, and 44.

*8. Compute the magnitude of the voltage E_{AB} for the balanced three-phase system in Fig. 23.40.
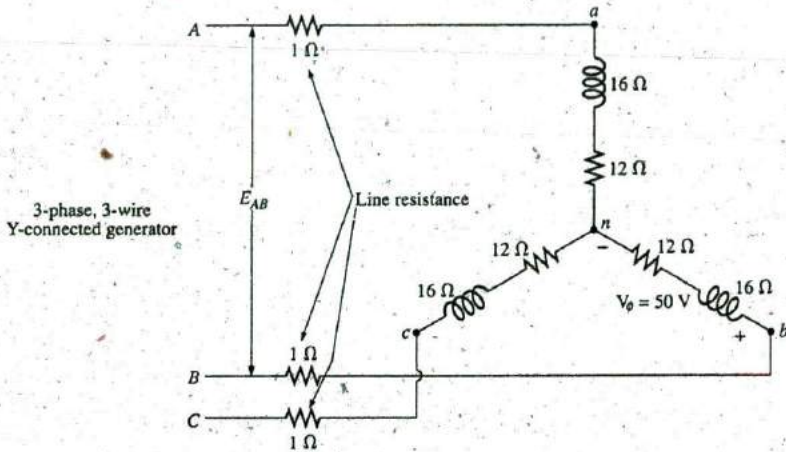
FIG. 23.40
Problem 8.

*9. For the Y-Y system in Fig. 23.41:
  a. Find the magnitude and angle associated with the voltages E_{AN}, E_{BN}, and E_{CN}.
  b. Determine the magnitude and angle associated with each phase current of the load: I_{an}, I_{bn}, and I_{cn}.
  c. Find the magnitude and phase angle of each line current: I_{Aa}, I_{Bb}, and I_{Cc}.
  d. Determine the magnitude and phase angle of the voltage across each phase of the load: V_{an}, V_{bn}, and V_{cn}.
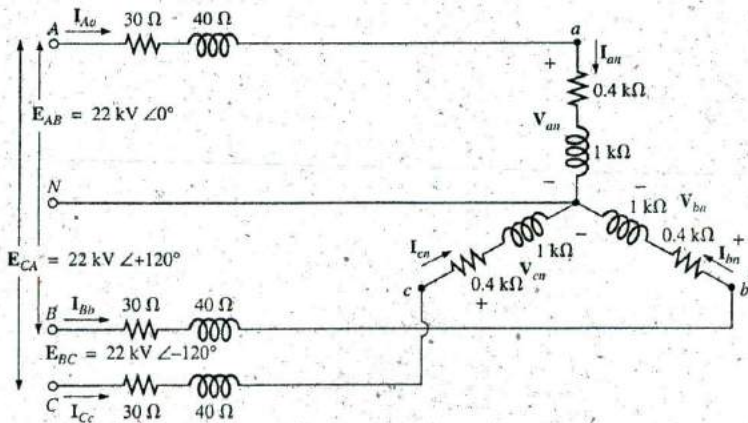
FIG. 23.41
Problem 9.

SECTION 23.6 Y-Δ System

10. A balanced Δ load having a 20 Ω resistance in each leg is connected to a three-phase, three-wire, Y-connected generator having a line voltage of 208 V. Calculate the magnitude of
    a. the phase voltage of the generator.
    b. the phase voltage of the load.
    c. the phase current of the load.
    d. the line current.

11. Repeat Problem 10 if each phase impedance is changed to a 6.8 Ω resistor in series with a 14 Ω inductive reactance.

12. Repeat Problem 10 if each phase impedance is changed to an 18 Ω resistance in parallel with an 18 Ω capacitive reactance.

13. The phase sequence for the Y-Δ system in Fig. 23.42 is ABC.
    a. Find the angles θ_2 and θ_3 for the specified phase sequence.
    b. Find the voltage across each phase impedance in phasor form.
    c. Draw the phasor diagram of the voltages found in part (b), and show that their sum is zero around the closed loop of the Δ load.
    d. Find the current through each phase impedance in phasor form.
    e. Find the magnitude of the line currents.
    f. Find the magnitude of the generator phase voltages.

14. Repeat Problem 13 if the phase impedances are changed to a 100 Ω resistor in series with a capacitive reactance of 100 Ω.

15. Repeat Problem 13 if the phase impedances are changed to a 3 Ω resistor in parallel with an inductive reactance of 4 Ω.

16. For the system in Fig. 23.43, find the magnitude of the unknown voltages and currents.
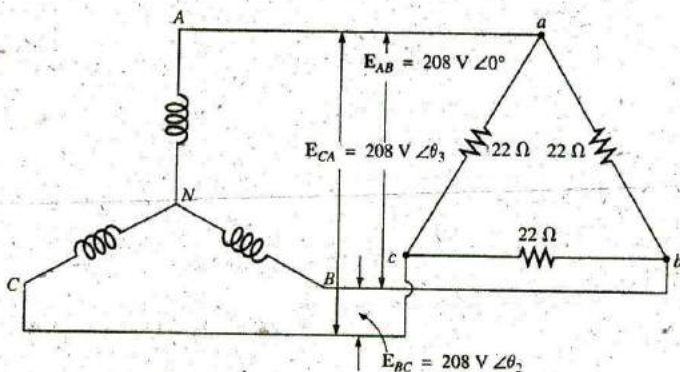
FIG. 23.42
Problems 13, 14, 15, 34, and 45.

$E_{AB} = 208 \text{ V } \angle 0°$
$E_{CA} = 208 \text{ V } \angle \theta_3$
$E_{BC} = 208 \text{ V } \angle \theta_2$
22 Ω 22 Ω 22 Ω

3-phase, 3-wire
Y-connected
4-wire generator

Phase sequence: ABC

$E_{AB} = 220 \text{ V } \angle 0°$
$E_{CA} = 220 \text{ V } \angle +120°$
$E_{BC} = 220 \text{ V } \angle -120°$

10 Ω

FIG. 23.43
Problems 16, 35, and 47.

*17. For the Δ-connected load in Fig. 23.44:
   a. Find the magnitude and angle of each phase current I_{ab}, I_{bc}, and I_{ca}.
   b. Calculate the magnitude and angle of each line current I_{Aa}, I_{Bb}, and I_{Cc}.
   c. Determine the magnitude and angle of the voltages E_{AB}, E_{BC}, and E_{CA}.

$V_{ab} = 16$ kV $\angle 0°$
$V_{bc} = 16$ kV $\angle -120°$
$V_{ca} = 16$ kV $\angle +120°$
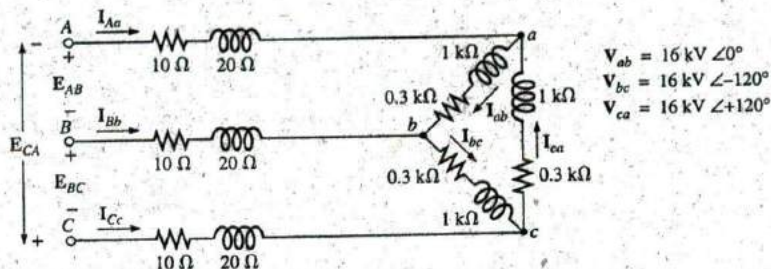
FIG. 23.44
Problem 17.

SECTION 23.9 Δ-Δ, Δ-Y Three-Phase Systems

18. A balanced Y load having a 30 Ω resistance in each leg is connected to a three-phase, Δ-connected generator having a line voltage of 208 V. Calculate the magnitude of
   a. the phase voltage of the generator.
   b. the phase voltage of the load.
   c. the phase current of the load.
   d. the line current.

19. Repeat Problem 18 if each phase impedance is changed to a 12 Ω resistor in series with a 12 Ω inductive reactance.

20. Repeat Problem 18 if each phase impedance is changed to a 15 Ω resistor in parallel with a 20 Ω capacitive reactance.

*21. For the system in Fig. 23.45, find the magnitude of the unknown voltages and currents.

22. Repeat Problem 21 if each phase impedance is changed to a 10 Ω resistor in series with a 20 Ω inductive reactance.

23. Repeat Problem 21 if each phase impedance is changed to a 20 Ω resistor in parallel with a 15 Ω capacitive reactance.
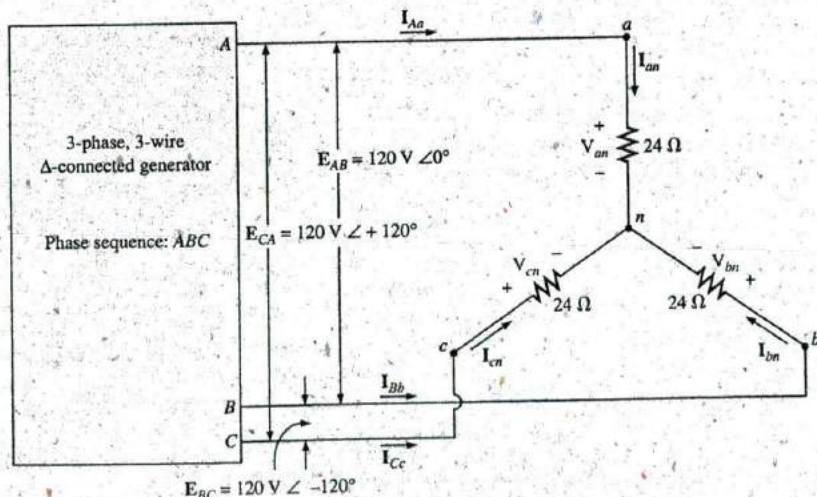
FIG. 23.45
Problems 21, 22, 23, and 37.

24. A balanced Δ load having a 220 Ω resistance in each leg is connected to a three-phase, Δ-connected generator having a line voltage of 440 V. Calculate the magnitude of
 a. the phase voltage of the generator.
 b. the phase voltage of the load.
 c. the phase current of the load.
 d. the line current.

25. Repeat Problem 24 if each phase impedance is changed to a 12 Ω resistor in series with a 9 Ω capacitive reactance.

26. Repeat Problem 24 if each phase impedance is changed to a 22 Ω resistor in parallel with a 22 Ω inductive reactance.

27. The phase sequence for the Δ-Δ system in Fig. 23.46 is *ABC*.
 a. Find the angles θ_2 and θ_3 for the specified phase sequence.
 b. Find the voltage across each phase impedance in phasor form.
 c. Draw the phasor diagram of the voltages found in part (b), and show that their phasor sum is zero around the closed loop of the Δ load.
 d. Find the current through each phase impedance in phasor form.
 e. Find the magnitude of the line currents.

28. Repeat Problem 25 if each phase impedance is changed to a 12 Ω resistor in series with a 16 Ω inductive reactance.

29. Repeat Problem 25 if each phase impedance is changed to a 20 Ω resistor in parallel with a 20 Ω capacitive reactance.
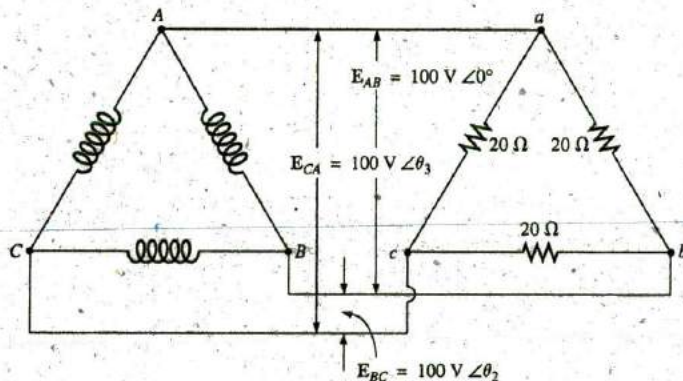
FIG. 23.46
Problem 27.

SECTION 23.10 Power

30. Find the total watts, volt-amperes reactive, volt-amperes, and F_p of the three-phase system in Problem 2.

31. Find the total watts, volt-amperes reactive, volt-amperes, and F_p of the three-phase system in Problem 4.

32. Find the total watts, volt-amperes reactive, volt-amperes, and F_p of the three-phase system in Problem 7.

33. Find the total watts, volt-amperes reactive, volt-amperes, and F_p of the three-phase system in Problem 12.

34. Find the total watts, volt-amperes reactive, volt-amperes, and F_p of the three-phase system in Problem 14.

35. Find the total watts, volt-amperes reactive, volt-amperes, and F_p of the three-phase system in Problem 16.

36. Find the total watts, volt-amperes reactive, volt-amperes, and F_p of the three-phase system in Problem 20.

37. Find the total watts, volt-amperes reactive, volt-amperes, and F_p of the three-phase system in Problem 22.

38. Find the total watts, volt-amperes reactive, volt-amperes, and F_p of the three-phase system in Problem 26.

39. Find the total watts, volt-amperes reactive, volt-amperes, and F_p of the three-phase system in Problem 28.

40. A balanced, three-phase, Δ-connected load has a line voltage of 200 and a total power consumption of 4800 W at a lagging power factor of 0.8. Find the impedance of each phase in rectangular coordinates.

41. A balanced, three-phase, Y-connected load has a line voltage of 208 and a total power consumption of 1200 W at a leading power factor of 0.6. Find the impedance of each phase in rectangular coordinates.

*42. Find the total watts, volt-amperes reactive, volt-amperes, and F_p of the system in Fig. 23.47.

*43. The Y-Y system in Fig. 23.48 has a balanced load and a line impedance $Z_{line} = 4 \, \Omega + j \, 20 \, \Omega$. If the line voltage at the generator is 16,000 V and the total power delivered to the load is 1200 kW at 80 A, determine each of the following:
 a. The magnitude of each phase voltage of the generator.
 b. The magnitude of the line currents.
 c. The total power delivered by the source.
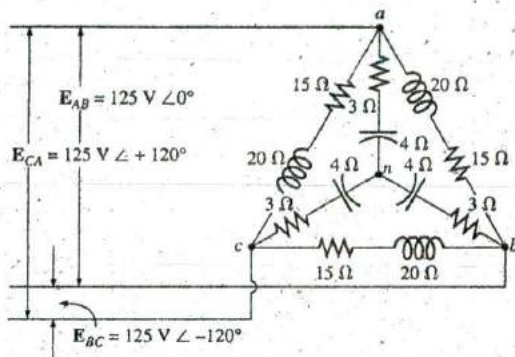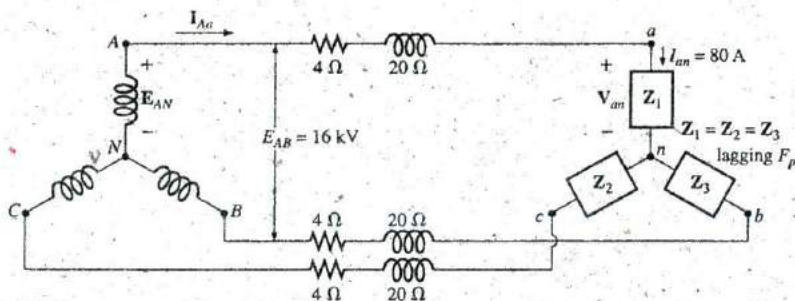 d. The power factor angle of the entire load "seen" by the source.

FIG. 23.47
Problem 42.

FIG. 23.48
Problem 43.

e. The magnitude and angle of the current I_{Aa} if $E_{AN} = E_{AN} \angle 0°$.

f. The magnitude and angle of the phase voltage V_{an}.

g. The impedance of the load of each phase in rectangular coordinates.

h. The difference between the power factor of the load and the power factor of the entire system (including Z_{line}).

i. The efficiency of the system.

SECTION 23.11 Three-Wattmeter Method

44. **a.** Sketch the connections required to measure the total watts delivered to the load in Fig. 23.39 using three wattmeters.

    b. Determine the total wattage dissipation and the reading of each wattmeter.

45. Repeat Problem 44 for the network in Fig. 23.42.

SECTION 23.12 Two-Wattmeter Method

46. **a.** For the three-wire system in Fig. 23.49, properly connect a second wattmeter so that the two measure the total power delivered to the load.

    b. If one wattmeter has a reading of 200 W and the other a reading of 85 W, what is the total dissipation in watts if the total power factor is 0.8 leading?
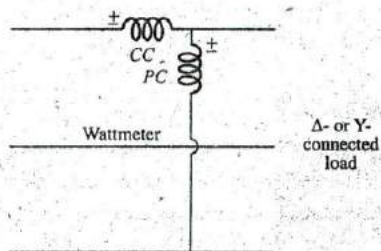
FIG. 23.49
Problem 46.

c. Repeat part (b) if the total power factor is 0.2 lagging and $P_t = 100$ W.

47. Sketch three different ways that two wattmeters can be connected to measure the total power delivered to the load in Problem 16.

*48. For the Y-Δ system in Fig. 23.50:

    a. Determine the magnitude and angle of the phase currents.

    b. Find the magnitude and angle of the line currents.

    c. Determine the reading of each wattmeter.

    d. Find the total power delivered to the load.
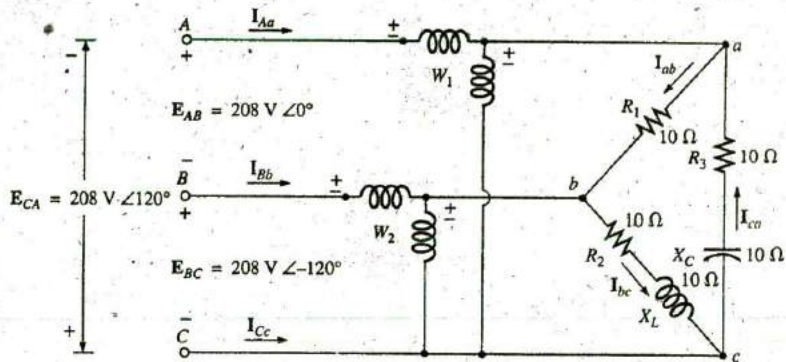
FIG. 23.50
Problem 48.

SECTION 23.13 Unbalanced, Three-Phase, Four-Wire, Y-Connected Load

*49. For the system in Fig. 23.51:
  a. Calculate the magnitude of the voltage across each phase of the load.
  b. Find the magnitude of the current through each phase of the load.
  c. Find the total watts, volt-amperes reactive, volt-amperes, and F_p of the system.
  d. Find the phase currents in phasor form.
  e. Using the results of part (c), determine the current I_N.
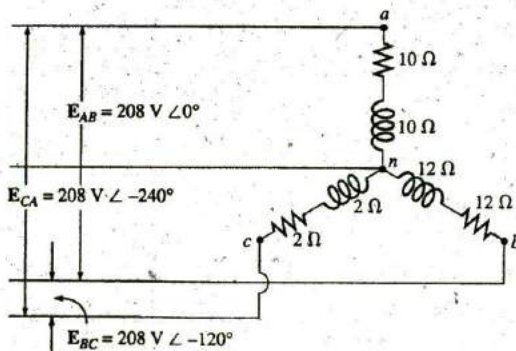
FIG. 23.52
Problem 50.

FIG. 23.51
Problem 49.

SECTION 23.14 Unbalanced, Three-Phase, Three-Wire, Y-Connected Load

*50. For the three-phase, three-wire system in Fig. 23.52, find the magnitude of the current through each phase of the load, and find the total watts, volt-amperes reactive, volt-amperes, and F_p of the load.
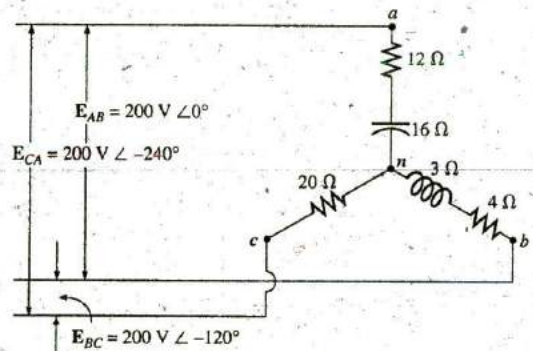
GLOSSARY

Δ-connected ac generator A three-phase generator having the three phases connected in the shape of the capital Greek letter *delta* (Δ).

Line current The current that flows from the generator to the load of a single-phase or polyphase system.

Line voltage The potential difference that exists between the lines of a single-phase or polyphase system.

Neutral connection The connection between the generator and the load that, under balanced conditions, will have zero current associated with it.

Phase current The current that flows through each phase of a single-phase (or polyphase) generator or load.

Phase sequence The order in which the generated sinusoidal voltages of a polyphase generator will affect the load to which they are applied.

Phase voltage The voltage that appears between the line and neutral of a Y-connected generator and from line to line in a Δ-connected generator.

Polyphase ac generator An electromechanical source of ac power that generates more than one sinusoidal voltage per rotation of the rotor. The frequency generated is determined by the speed of rotation and the number of poles of the rotor.

Single-phase ac generator An electromechanical source of ac power that generates a single sinusoidal voltage having a frequency determined by the speed of rotation and the number of poles of the rotor.

Three-wattmeter method A method for determining the total power delivered to a three-phase load using three wattmeters.

Two-wattmeter method A method for determining the total power delivered to a Δ- or Y-connected three-phase load using only two wattmeters and considering the power factor of the load.

Unbalanced polyphase load A load not having the same impedance in each phase.

Y-connected three-phase generator A three-phase source of ac power in which the three phases are connected in the shape of the letter Y.

Pulse Waveforms and the R-C Response

Objectives

- Become familiar with the specific terms that define a pulse waveform and how to calculate various parameters such as the pulse width, rise and fall times, and tilt.

- Be able to calculate the pulse repetition rate and the duty cycle of any pulse waveform.

- Become aware of the parameters that define the response of an R-C network to a square-wave input.

- Understand how a compensator probe of an oscilloscope is used to improve the appearance of an output pulse waveform.

24.1 INTRODUCTION

Our analysis thus far has been limited to alternating waveforms that vary in a sinusoidal manner. This chapter introduces the basic terminology associated with the pulse waveform and examines the response of an R-C circuit to a square-wave input. The importance of the pulse waveform to the electrical/electronics industry cannot be overstated. A vast array of instrumentation, communication systems, computers, radar systems, and so on, all use pulse signals to control operation, transmit data, and display information in a variety of formats.

The response to a pulse signal of the networks described thus far is quite different from that obtained for sinusoidal signals. In fact, we must refer to the dc chapter on capacitors (Chapter 10) for a few fundamental concepts and equations that will help us in the analysis to follow. This chapter is just an introduction, designed to provide the fundamentals that will be helpful when the pulse waveform is encountered in specific areas of application.

24.2 IDEAL VERSUS ACTUAL

The **ideal pulse** in Fig. 24.1 has vertical sides, sharp corners, and a flat peak characteristic; it starts instantaneously at t_1 and ends just as abruptly at t_2.
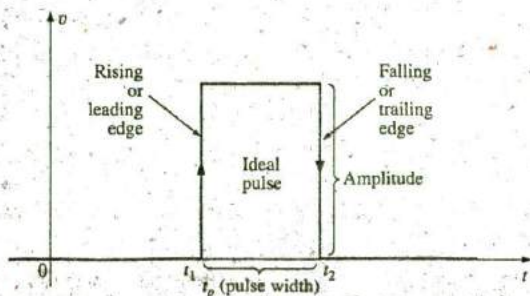
FIG. 24.1
Ideal pulse waveform.

The waveform in Fig. 24.1 is applied in the analysis in this chapter and probably in the initial investigation of areas of application beyond the scope of this text. Once the fundamental operation of a device, package, or system is clearly understood using ideal characteristics, the effect of an **actual** (or **true** or **practical**) **pulse** must be considered. If an attempt were made to introduce all the differences between an ideal and actual pulse in a single figure, the result would probably be complex and confusing. A number of waveforms are therefore used to define the critical parameters.

The reactive elements of a network, in their effort to prevent instantaneous changes in voltage (capacitor) and current (inductor), establish a slope to both edges of the pulse waveform, as shown in Fig. 24.2. The *rising edge* of the waveform in Fig. 24.2 is defined as the edge that increases from a lower to a higher level.
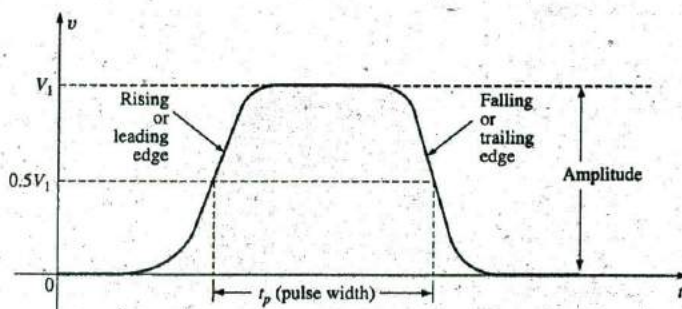
FIG. 24.2
Actual pulse waveform.

The falling edge is defined by the region or edge where the waveform decreases from a higher to a lower level. Since the rising edge is the first to be encountered (closest to t = 0 s), it is also called the leading edge. The falling edge always follows the leading edge and is therefore often called the trailing edge.

Both regions are defined in Figs. 24.1 and 24.2.

Amplitude

For most applications, the **amplitude of a pulse waveform** is defined as the peak-to-peak value. Of course, if the waveforms all start and return to the zero-volt level, then the peak and peak-to-peak values are synonymous.

For the purposes of this text, the amplitude of a pulse waveform is the peak-to-peak value, as illustrated in Figs. 24.1 and 24.2.

Pulse Width

The pulse width (t_p), or pulse duration, is defined by a pulse level equal to 50% of the peak value.

For the ideal pulse in Fig. 24.1, the pulse width is the same at any level, whereas t_p for the waveform in Fig. 24.2 is a very specific value.

Base-Line Voltage

The base-line voltage (V_b) is the voltage level from which the pulse is initiated.

The waveforms in Figs. 24.1 and 24.2 both have a 0 V base-line voltage. In Fig. 24.3(a) the base-line voltage is 1 V, whereas in Fig. 24.3(b) the base-line voltage is −4 V.
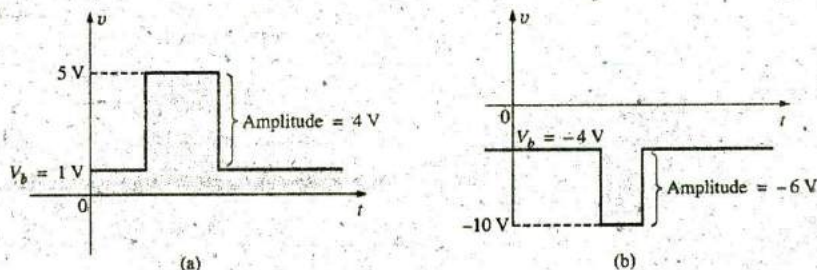
FIG. 24.3
Defining the base-line voltage.

Positive-Going and Negative-Going Pulses

A positive-going pulse increases positively from the base-line voltage, whereas a negative-going pulse increases in the negative direction from the base-line voltage.

The waveform in Fig. 24.3(a) is a positive-going pulse, whereas the waveform in Fig. 24.3(b) is a negative-going pulse.

Even though the base-line voltage in Fig. 24.4 is negative, the waveform is positive-going (with an amplitude of 10 V) since the voltage increased in the positive direction from the base-line voltage.
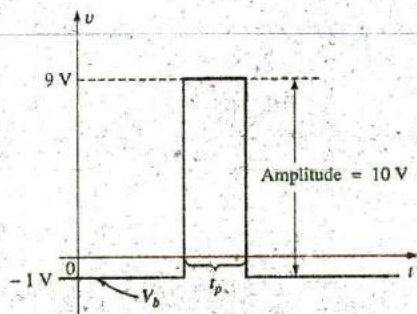
FIG. 24.4
Positive-going pulse.

Rise Time (t_r) and Fall Time (t_f)

The time required for the pulse to shift from one level to another is of particular importance. The *rounding* (defined in Fig. 24.5) that occurs at the beginning and end of each transition makes it difficult to define the exact point at which the rise time should be initiated and terminated. For this reason,
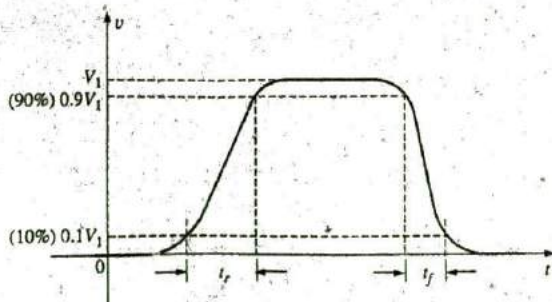
FIG. 24.5
Defining t_r and t_f.
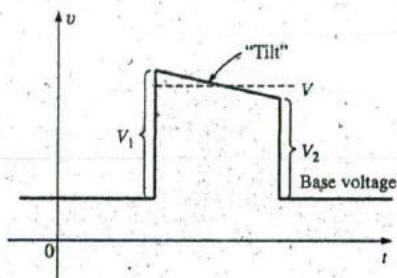
FIG. 24.6
Defining tilt.
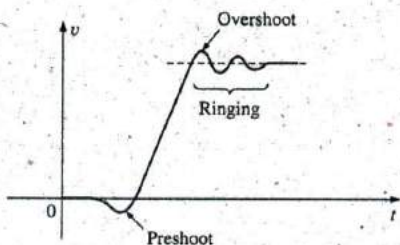
FIG. 24.7
Defining preshoot, overshoot, and ringing.

the rise time and the fall time are defined by the 10% and 90% levels, as indicated in Fig. 24.5.

Note that there is no requirement that t_r equal t_f.

Tilt

Fig. 24.6 illustrates an undesirable but common distortion normally occurring due to a poor low-frequency response characteristic of the system through which a pulse has passed. The drop in peak value is called **tilt, droop,** or **sag.** The percentage tilt is defined by

$$\% \text{ tilt} = \frac{V_1 - V_2}{V} \times 100\% \qquad (24.1)$$

where V is the average value of the peak amplitude as determined by

$$V = \frac{V_1 + V_2}{2} \qquad (24.2)$$

Naturally, the less the percentage tilt or sag, the more ideal is the pulse. Due to rounding, it may be difficult to define the values of V_1 and V_2. It is then necessary to approximate the sloping region by a straight-line approximation and use the resulting values of V_1 and V_2.

Other distortions include the *preshoot* and *overshoot* appearing in Fig. 24.7, normally due to pronounced high-frequency effects of a system, and *ringing*, due to the interaction between the capacitive and inductive elements of a network at their natural or resonant frequency.

EXAMPLE 24.1 Determine the following for the pulse waveform in Fig. 24.8:

    a. whether it is positive- or negative-going
    b. base-line voltage
    c. pulse width
    d. maximum amplitude
    e. tilt
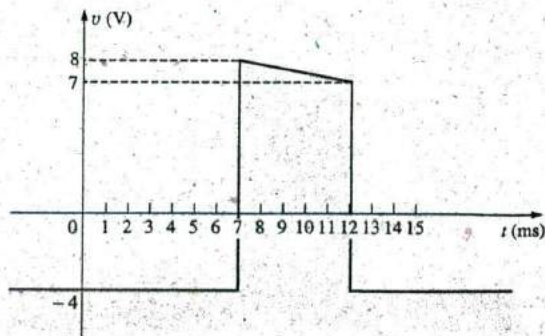
FIG. 24.8
Example 24.1.

Solutions:

a. **positive-going**
b. $V_b = -4\,V$
c. $t_p = (12 - 7)$ ms $= 5$ ms
d. $V_{max} = 8\,V + 4\,V = 12\,V$
e. $V = \dfrac{V_1 + V_2}{2} = \dfrac{12\,V + 11\,V}{2} = \dfrac{23\,V}{2} = 11.5\,V$

$\%\ tilt = \dfrac{V_1 - V_2}{V} \times 100\% = \dfrac{12\,V - 11\,V}{11.5\,V} \times 100\% = 8.7\%$

(Remember, V is defined by the average value of the peak amplitude.)

EXAMPLE 24.2 Determine the following for the pulse waveform in Fig. 24.9:

a. whether it is positive- or negative-going
b. base-line voltage
c. tilt
d. amplitude
e. t_p
f. t_r and t_f

Solutions:

a. **positive-going**
b. $V_b = 0\,V$
c. **% tilt = 0%**
d. amplitude = (4 div.)(10 mV/div.) = **40 mV**
e. $t_p = $ (3.2 div.)(5 μs/div.) = **16 μs**
f. $t_r = $ (0.4 div.)(5 μs/div.) = **2 μs**
$t_f = $ (0.8 div.)(5 μs/div.) = **4 μs**

Vertical sensitivity = 10 mV/div.
Horizontal sensitivity = 5 μs/div.
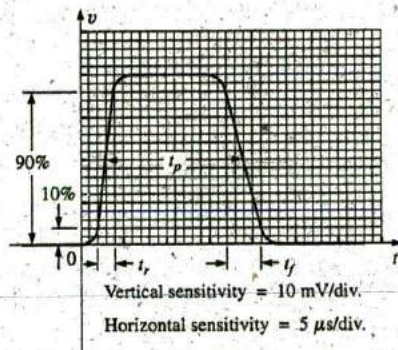
FIG. 24.9
Example 24.2.

24.3 PULSE REPETITION RATE AND DUTY CYCLE

A series of pulses such as those appearing in Fig. 24.10 is called a **pulse train.** The varying widths and heights may contain information that can be decoded at the receiving end.

If the pattern repeats itself in a periodic manner as shown in Fig. 24.11(a) and (b), the result is called a **periodic pulse train.**

The *period (T)* of the pulse train is defined as the time differential between any two similar points on the pulse train, as shown in Figs. 24.11(a) and (b).
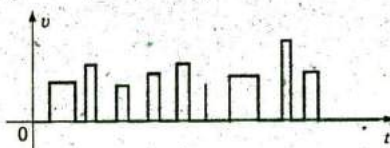
FIG. 24.10
Pulse train.

(a)

(b)
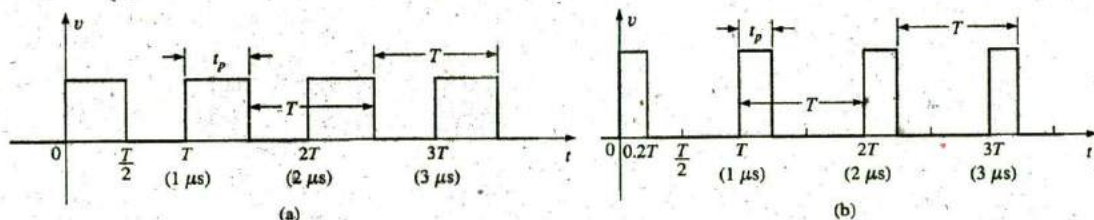
FIG. 24.11
Periodic pulse trains.

The **pulse repetition frequency** (prf), or **pulse repetition rate** (prr), is defined by

$$\boxed{\text{prf (or prr)} = \frac{1}{T}} \quad \text{Hz or pulses/s} \qquad (24.3)$$

Applying Eq. (24.3) to each waveform in Fig. 24.11 results in the same pulse repetition frequency since the periods are the same. The result clearly reveals that

the shape of the periodic pulse does not affect the determination of the pulse repetition frequency.

The pulse repetition frequency is determined solely by the period of the repeating pulse. The factor that reveals how much of the period is encompassed by the pulse is called the **duty cycle**, defined as follows:

$$\text{Duty cycle} = \frac{\text{pulse width}}{\text{period}} \times 100\%$$

or

$$\boxed{\text{Duty cycle} = \frac{t_p}{T} \times 100\%} \qquad (24.4)$$

For Fig. 24.11(a) (a square-wave pattern),

$$\text{Duty cycle} = \frac{0.5T}{T} \times 100\% = 50\%$$

and for Fig. 24.11(b),

$$\text{Duty cycle} = \frac{0.2T}{T} \times 100\% = 20\%$$

The above results clearly reveal that

the duty cycle provides a percentage indication of the portion of the total period encompassed by the pulse waveform.

EXAMPLE 24.3 Determine the pulse repetition frequency and the duty cycle for the periodic pulse waveform in Fig. 24.12.

Solution:

$$T = (15 - 6)\ \mu s = 9\ \mu s$$

$$\text{prf} = \frac{1}{T} = \frac{1}{9\ \mu s} \cong 111.11 \text{ kHz}$$
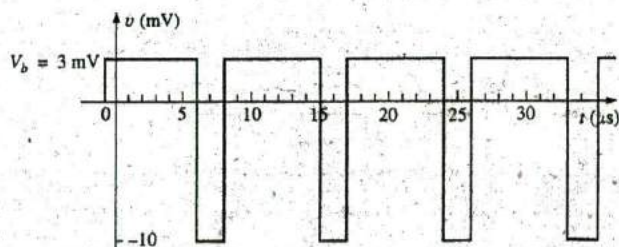
FIG. 24.12
Example 24.3.

$$\text{Duty cycle} = \frac{t_p}{T} \times 100\% = \frac{(8-6)\,\mu s}{9\,\mu s} \times 100\%$$

$$= \frac{2}{9} \times 100\% \cong 22.22\%$$

EXAMPLE 24.4 Determine the pulse repetition frequency and the duty cycle for the oscilloscope pattern in Fig. 24.13 having the indicated sensitivities.

Vertical sensitivity = 0.2 V/div.
Horizontal sensitivity = 1 ms/div.

div.
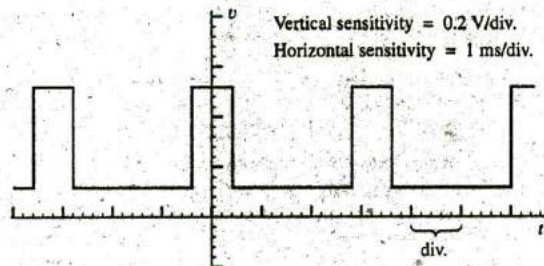
FIG. 24.13
Example 24.4.

Solution:

$$T = (3.2\text{ div.})(1\text{ ms/div.}) = 3.2\text{ ms}$$
$$t_p = (0.8\text{ div.})(1\text{ ms/div.}) = 0.8\text{ ms}$$

$$\text{prf} = \frac{1}{T} = \frac{1}{3.2\text{ ms}} = 312.5\text{ Hz}$$

$$\text{Duty cycle} = \frac{t_p}{T} \times 100\% = \frac{0.8\text{ ms}}{3.2\text{ ms}} \times 100\% = 25\%$$

EXAMPLE 24.5 Determine the pulse repetition rate and duty cycle for the trigger waveform in Fig. 24.14.

Solution:

$$T = (2.6\text{ div.})(10\,\mu s/\text{div.}) = 26\,\mu s$$

$$\text{prf} = \frac{1}{T} = \frac{1}{26\,\mu s} = 38,462\text{ kHz}$$

0.5 V

div.

Horizontal sensitivity = 10 μs/div.
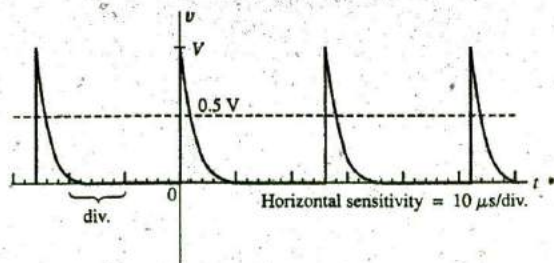
FIG. 24.14
Example 24.5.

$$t_p \cong (0.2 \text{ div.})(10 \ \mu\text{s/div.}) = 2 \ \mu\text{s}$$

$$\text{Duty cycle} = \frac{t_p}{T} \times 100\% = \frac{2 \ \mu\text{s}}{26 \ \mu\text{s}} \times 100\% = 7.69\%$$

24.4 AVERAGE VALUE

The average value of a pulse waveform can be determined using one of two methods. The first is the procedure outlined in Section 13.7, which can be applied to any alternating waveform. The second can be applied only to pulse waveforms since it utilizes terms specifically related to pulse waveforms; that is,

$$V_{av} = (\text{duty cycle})(\text{peak value}) + (1 - \text{duty cycle})(V_b) \quad \textbf{(24.5)}$$

In Eq. (24.5), the peak value is the maximum deviation from the reference or zero-volt level, and the duty cycle is in decimal form. Eq. (24.5) does not include the effect of any tilt pulse waveforms with sloping sides.

EXAMPLE 24.6 Determine the average value for the periodic pulse waveform in Fig. 24.15.
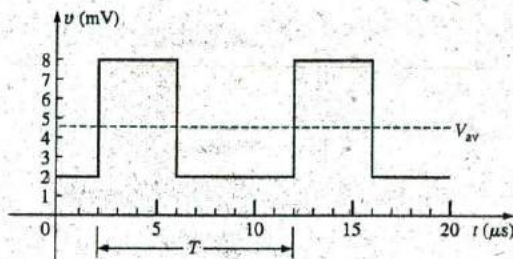
FIG. 24.15
Example 24.6.

Solution: By the method in Section 13.7,

$$G = \frac{\text{area under curve}}{T}$$

$$T = (12 - 2) \ \mu\text{s} = 10 \ \mu\text{s}$$

$$G = \frac{(8 \text{ mV})(4 \ \mu\text{s}) + (2 \text{ mV})(6 \ \mu\text{s})}{10 \ \mu\text{s}} = \frac{32 \times 10^{-9} + 22 \times 10^{-9}}{10 \times 10^{-6}}$$

$$= \frac{44 \times 10^{-9}}{10 \times 10^{-6}} = \textbf{4.4 mV}$$

By Eq. (24.5),

$$V_b = +2 \text{ mV}$$

$$\text{Duty cycle} = \frac{t_p}{T} = \frac{(6 - 2) \ \mu\text{s}}{10 \ \mu\text{s}} = \frac{4}{10} = 0.4 \quad \text{(decimal form)}$$

$$\text{Peak value (from 0 V reference)} = 8 \text{ mV}$$

V_{av} = (duty cycle)(peak value) + (1 − duty cycle)(V_b)
   = (0.4)(8 mV) + (1 − 0.4)(2 mV)
   = 3.2 mV + 1.2 mV = 4.4 mV

as obtained above.

FIG. 24.16
Solution to part (b) of Example 24.7.

EXAMPLE 24.7 Given a periodic pulse waveform with a duty cycle of 28%, a peak value of 7 V, and a base-line voltage of −3 V:

a. Determine the average value.
b. Sketch the waveform.
c. Verify the result of part (a) using the method of Section 13.7.

Solutions:

a. By Eq. (24.5),

V_{av} = (duty cycle)(peak value) + (1 − duty cycle)(V_b)
   = (0.28)(7 V) + (1 − 0.28)(−3 V) = 1.96 V + (−2.16 V)
   = **−0.2 V**

b. See Fig. 24.16.

c. $G = \dfrac{(7\text{ V})(0.28T) - (3\text{ V})(0.72T)}{T} = 1.96\text{ V} - 2.16\text{ V}$

   = **−0.2 V**

as obtained above.

(a)

(b)

FIG. 24.17
Determining the average value of a pulse waveform
using an oscilloscope.

Instrumentation

The average value (dc value) of any waveform can be easily determined using the oscilloscope. If the mode switch of the scope is set in the ac position, the average or dc component of the applied waveform is blocked by an internal capacitor from reaching the screen. The pattern can be adjusted to establish the display in Fig. 24.17(a). If the mode switch is then placed in the dc position, the vertical shift (positive or negative) reveals the average or dc level of the input signal, as shown in Fig. 24.17(b).

24.5 TRANSIENT R-C NETWORKS

In Chapter 10, the general solution for the transient behavior of an R-C network with or without initial values was developed. The resulting equation for the voltage across a capacitor is repeated here for convenience:

$$v_C = V_f + (V_i - V_f)e^{-t/RC} \qquad (24.6)$$

Recall that V_i is the initial voltage across the capacitor when the transient phase is initiated as shown in Fig. 24.18. The voltage V_f is the steady-state (resting) value of the voltage across the capacitor when the transient phase has ended. The transient period is approximated as 5τ, where τ is the time constant of the network and is equal to the product RC.

FIG. 24.18
Defining the parameters of Eq. (24.6).

For the situation where the initial voltage is zero volts, the equation reduces to the following familiar form, where V_f is often the applied voltage:

$$v_C = V_f(1 - e^{-t/RC})\Big|_{V_i = 0V} \qquad (24.7)$$

For the case in Fig. 24.19, $V_i = -2$ V, $V_f = +5$ V, and

$$v_C = V_i + (V_f - V_i)(1 - e^{-t/RC})$$
$$= -2 \text{ V} + [5 \text{ V} - (-2 \text{ V})](1 - e^{-t/RC})$$
$$v_C = -2 \text{ V} + 7 \text{ V}(1 - e^{-t/RC})$$

For the case where $t = \tau = RC$,

$$v_C = -2 \text{ V} + 7 \text{ V}(1 - e^{-t/\tau}) = -2 \text{ V} + 7 \text{ V}(1 - e^{-1})$$
$$= -2 \text{ V} + 7 \text{ V}(1 - 0.368) = -2 \text{ V} + 7 \text{ V}(0.632)$$
$$v_C = 2.424 \text{ V}$$

as verified by Fig. 24.19.
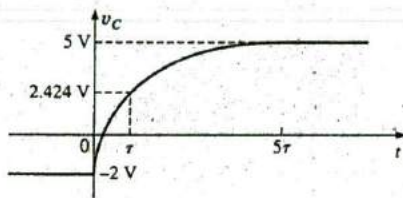
FIG. 24.19
Example of the use of Eq. (24.6).
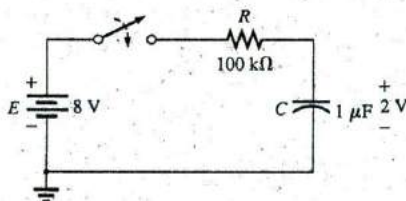
EXAMPLE 24.8 The capacitor in Fig. 24.20 is initially charged to 2 V before the switch is closed. The switch is then closed.

a. Determine the mathematical expression for v_C.
b. Determine the mathematical expression for i_C.
c. Sketch the waveforms of v_C and i_C.

Solutions:

a. $V_i = 2$ V
V_f (after 5τ) $= E = 8$ V
$\tau = RC = (100 \text{ k}\Omega)(1 \text{ }\mu\text{F}) = 100$ ms

By Eq. (24.6),

$$v_C = V_f + (V_i - V_f)e^{-t/RC}$$
$$= 8 \text{ V} + (2 \text{ V} - 8 \text{ V})e^{-t/\tau}$$

and $\qquad v_C = 8 \text{ V} - 6 \text{ V}e^{-t/\tau}$

FIG. 24.20
Example 24.8.

b. When the switch is first closed, the voltage across the capacitor cannot change instantaneously, and $V_R = E - V_i = 8 \text{ V} - 2 \text{ V} = 6$ V. The current therefore jumps to a level determined by Ohm's law:

$$I_{R_{max}} = \frac{V_R}{R} = \frac{6 \text{ V}}{100 \text{ k}\Omega} = 0.06 \text{ mA}$$

The current then decays to zero amperes with the same time constant calculated in part (a), and

$$i_C = 0.06 \text{ mA}e^{-t/\tau}$$

c. See Fig. 24.21.
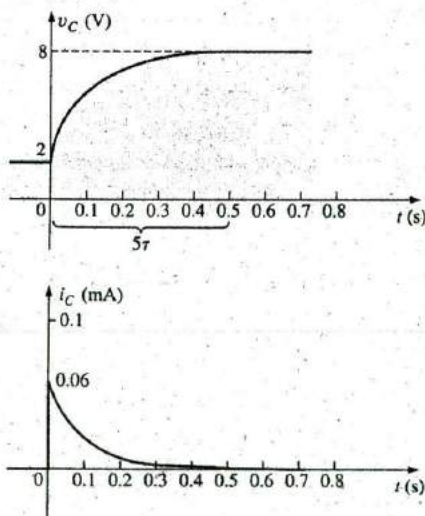
FIG. 24.21
v_C and i_C for the network in Fig. 24.20.

EXAMPLE 24.9 Sketch v_C for the step input shown in Fig. 24.22. Assume that the -4 mV has been present for a period of time in excess of five time constants of the network. Then determine when $v_C = 0$ V if the step changes levels at $t = 0$ s.
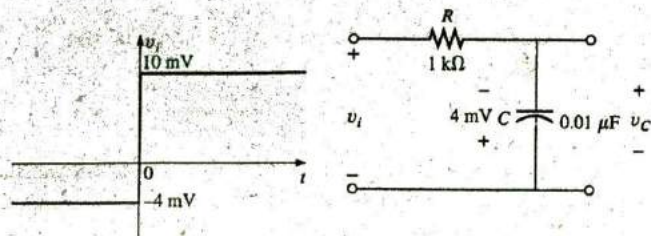
FIG. 24.22
Example 24.9.

Solution:

$$V_i = -4 \text{ mV} \qquad V_f = 10 \text{ mV}$$
$$\tau = RC = (1 \text{ k}\Omega)(0.01 \text{ }\mu\text{F}) = 10 \text{ }\mu\text{s}$$

By Eq. (24.6),

$$v_C = V_f + (V_i - V_f)e^{-t/RC}$$
$$= 10 \text{ mV} + (-4 \text{ mV} - 10 \text{ mV})e^{-t/10\mu s}$$

and $\qquad v_C = 10 \text{ mV} - 14 \text{ mV } e^{-t/10\mu s}$

The waveform appears in Fig. 24.23.
Substituting $v_C = 0$ V into the above equation yields

$$v_C = 0 \text{ V} = 10 \text{ mV} - 14 \text{ mV } e^{-t/10\mu s}$$

and $\qquad \dfrac{10 \text{ mV}}{14 \text{ mV}} = e^{-t/10\mu s}$

or $\qquad 0.714 = e^{-t/10\mu s}$

but $\qquad \log_e 0.714 = \log_e(e^{-t/10\mu s}) = \dfrac{-t}{10 \text{ }\mu\text{s}}$

and $\quad t = -(10 \text{ }\mu\text{s})\log_e 0.714 = -(10 \text{ }\mu\text{s})(-0.377) = 3.37 \text{ }\mu\text{s}$
as indicated in Fig. 24.23.
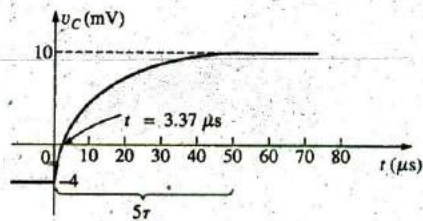
FIG. 24.23
v_C *for the network in Fig. 24.22.*

24.6 R-C RESPONSE TO SQUARE-WAVE INPUTS

The **square wave** in Fig. 24.24 is a particular form of pulse waveform. It has a duty cycle of 50% and an average value of zero volts, as calculated as follows:
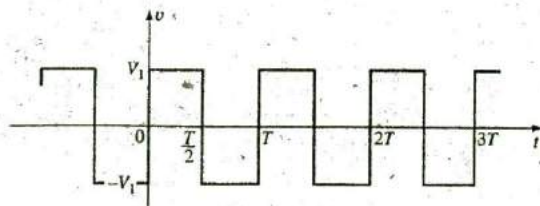
FIG. 24.24
Periodic square wave.

$$\text{Duty cycle} = \frac{t_p}{T} \times 100\% = \frac{T/2}{T} \times 100\% = 50\%$$

$$V_{av} = \frac{(V_1)(T/2) + (-V_1)(T/2)}{T} = \frac{0}{T} = 0 \text{ V}$$

The application of a dc voltage V_1 in series with the square wave in Fig. 24.24 can raise the base-line voltage from $-V_1$ to zero volts and the average value to V_1 volts.

If a square wave such as developed in Fig. 24.25 is applied to an R-C circuit as shown in Fig. 24.26, the period of the square wave can have a pronounced effect on the resulting waveform for v_C.
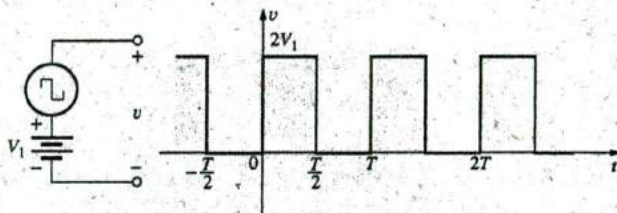
FIG. 24.25

Raising the base-line voltage of a square wave to zero volts.
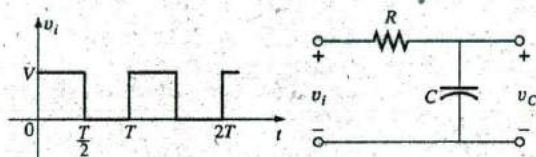
FIG. 24.26

Applying a periodic square-wave pulse train to an R-C network.

For the analysis to follow, we will assume that steady-state conditions will be established after a period of five time constants has passed. The types of waveforms developed across the capacitor can then be separated into three fundamental types: $T/2 > 5\tau$, $T/2 = 5\tau$, and $T/2 < 5\tau$.

$T/2 > 5\tau$

The condition $T/2 > 5\tau$, or $T > 10\tau$, establishes a situation where the capacitor can charge to its steady-state value in advance of $t = T/2$. The resulting waveforms for v_C and i_C appear as shown in Fig. 24.27. Note how

(a)   (b)
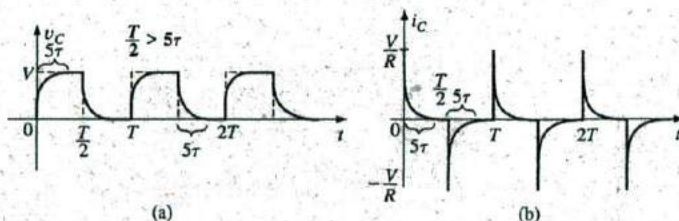
FIG. 24.27

v_C and i_C for $T/2 > 5\tau$.

closely the voltage v_C shadows the applied waveform and how i_C is nothing more than a series of very sharp spikes. Note also that the change of V_i from V to zero volts during the trailing edge results in a rapid discharge of v_C to zero volts. In essence, when $V_i = 0$, the capacitor and resistor are in parallel and the capacitor discharges through R with a time constant equal to that encountered during the charging phase but with a direction of charge flow (current) opposite to that established during the charging phase.

$T/2 = 5\tau$

If the frequency of the square wave is chosen such that $T/2 = 5\tau$ or $T = 10\tau$, the voltage v_C reaches its final value just before beginning its discharge phase, as shown in Fig. 24.28. The voltage v_C no longer resembles the square-wave input and, in fact, has some of the characteristics of a triangular waveform. The increased time constant has resulted in a more rounded v_C, and i_C has increased substantially in width to reveal the longer charging period.

(a)   (b)
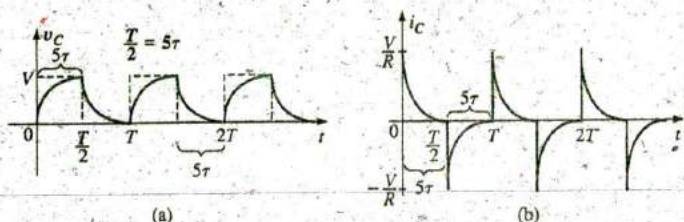
FIG. 24.28
v_C and i_C for $T/2 = 5\tau$.

$T/2 < 5\tau$

If $T/2 < 5\tau$ or $T < 10\tau$, the voltage v_C will not reach its final value during the first pulse (Fig. 24.29), and the discharge cycle will not return to zero volts. In fact, the initial value for each succeeding pulse changes until steady-state conditions are reached. In most instances, it is a good approximation to assume that steady-state conditions have been established in five cycles of the applied waveform.
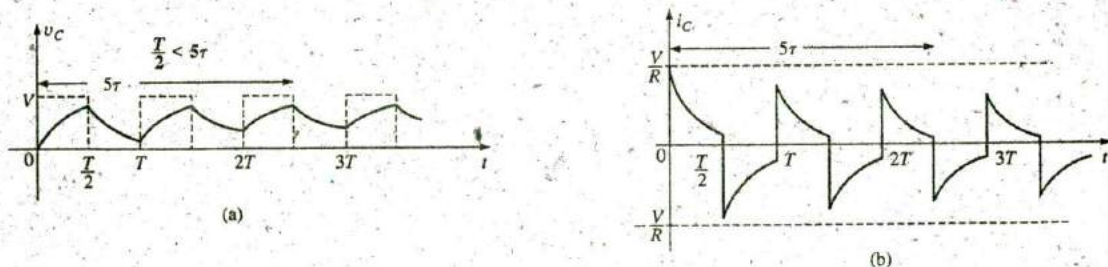
(a)   (b)

FIG. 24.29
v_C and i_C for $T/2 < 5\tau$.

As the frequency increases and the period decreases, there will be a flattening of the response for v_C until a pattern like that in Fig. 24.30 results. Fig. 24.30 begins to reveal an important conclusion regarding the response curve for v_C:
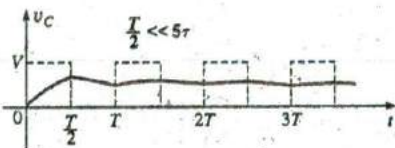
Introductory, C.- 69A

FIG. 24.30

v_C for $T/2 \ll 5\tau$ or $T \ll 10\tau$.

Under steady-state conditions, the average value of v_C will equal the average value of the applied square wave.

Note in Figs. 24.29 and 24.30 that the waveform for v_C approaches an average value of V/2.

EXAMPLE 24.10 The 1000 Hz square wave in Fig. 24.31 is applied to the R-C circuit of the same figure.

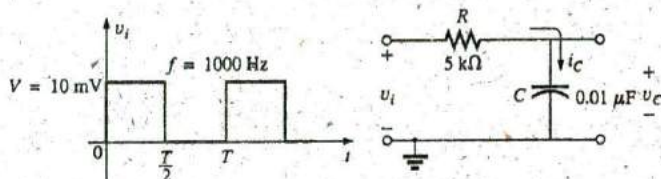a. Compare the pulse width of the square wave to the time constant of the circuit.
b. Sketch v_C.
c. Sketch i_C.

FIG. 24.31
Example 24.10.

Solutions:

a. $T = \dfrac{1}{f} = \dfrac{1}{1000} = 1$ ms

$t_p = \dfrac{T}{2} = 0.5$ ms

$\tau = RC = (5 \times 10^3\,\Omega)(0.01 \times 10^{-6}\,F) = 0.05$ ms

$\dfrac{t_p}{\tau} = \dfrac{0.5\ \text{ms}}{0.05\ \text{ms}} = 10$ and

$t_p = 10\tau = \dfrac{T}{2}$

The result reveals that v_C charges to its final value in half the pulse width.

b. For the charging phase, $V_i = 0$ V and $V_f = 10$ mV, and

$$v_C = V_f + (V_i - V_f)e^{-t/RC}$$
$$= 10\ \text{mV} + (0 - 10\ \text{mV})e^{-t/\tau}$$

and $\qquad v_C = 10\ \text{mV}(1 - e^{-t/\tau})$

For the discharge phase, $V_i = 10$ mV and $V_f = 0$ V, and

$$v_C = V_f + (V_i - V_f)e^{-t/\tau}$$
$$= 0\ \text{V} + (10\ \text{mV} - 0\ \text{V})e^{-t/\tau}$$

and $\qquad v_C = 10\ \text{mV}e^{-t/\tau}$

The waveform for v_C appears in Fig. 24.32.

c. For the charging phase at $t = 0$ s, $V_R = V$ and $I_{R_{max}} = V/R = 10\ \text{mV}/5\ \text{k}\Omega = 2\ \mu A$ and

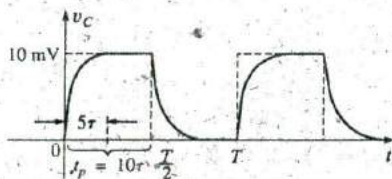$$i_C = I_{max}e^{-t/\tau} = 2\ \mu A e^{-t/\tau}$$

FIG. 24.32

v_C for the R-C network in Fig. 24.31.

For the discharge phase, the current will have the same mathematical formulation but the opposite direction, as shown in Fig. 24.33.

EXAMPLE 24.11 Repeat Example 24.10 for $f = 10$ kHz.

Solution:

$$T = \frac{1}{f} = \frac{1}{10 \text{ kHz}} = 0.1 \text{ ms}$$

and

$$\frac{T}{2} = 0.05 \text{ ms}$$

with

$$\tau = t_p = \frac{T}{2} = 0.05 \text{ ms}$$

FIG. 24.33
i_C for the R-C network in Fig. 24.31.

In other words, the pulse width is exactly equal to the time constant of the network. The voltage v_C will not reach the final value before the first pulse of the square-wave input returns to zero volts.
For t in the range $t = 0$ to $T/2$, $V_i = 0$ V and $V_f = 10$ mV, and

$$v_C = 10 \text{ mV}(1 - e^{-t/\tau})$$

Recall from Chapter 10 that at $t = \tau$, $v_C = 63.2\%$ of the final value. Substituting $t = \tau$ into the equation above yields

$$v_C = (10 \text{ mV})(1 - e^{-1}) = (10 \text{ mV})(1 - 0.368)$$
$$= (10 \text{ mV})(0.632) = 6.32 \text{ mV}$$

as shown in Fig. 24.34.

FIG. 24.34
v_C response for $t_p = \tau = T/2$.

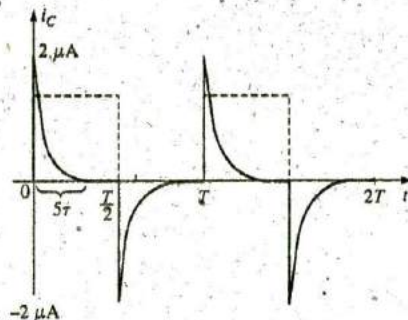For the discharge phase between $t = T/2$ and T, $V_i = 6.32$ mV and $V_f = 0$ V, and

$$v_C = V_f + (V_i - V_f)e^{-t/\tau}$$
$$= 0 \text{ V} + (6.32 \text{ mV} - 0 \text{ V})e^{-t/\tau}$$
$$v_C = 6.32 \text{ mV}e^{-t/\tau}$$

with t now being measured from $t = T/2$ in Fig. 24.34. In other words, for each interval in Fig. 24.34, the beginning of the transient waveform is defined as $t = 0$ s. The value of v_C at $t = T$ is therefore determined by substituting $t = \tau$ into the above equation, and not 2τ as defined by Fig. 24.34.
Substituting $t = \tau$ gives

$$v_C = (6.32 \text{ mV})(e^{-1}) = (6.32 \text{ mV})(0.368)$$
$$= 2.33 \text{ mV}$$

as shown in Fig. 24.34.

For the next interval, $V_i = 2.33$ mV and $V_f = 10$ mV, and

$$v_C = V_f + (V_i - V_f)e^{-t/\tau}$$
$$= 10 \text{ mV} + (2.33 \text{ mV} - 10 \text{ mV})e^{-t/\tau}$$
$$v_C = 10 \text{ mV} - 7.67 \text{ mV}e^{-t/\tau}$$

At $t = \tau$ (since $t = T = 2\tau$ is now $t = 0$ s for this interval),

$$v_C = 10 \text{ mV} - 7.67 \text{ mV}e^{-1}$$
$$= 10 \text{ mV} - 2.82 \text{ mV}$$
$$v_C = 7.18 \text{ mV}$$

as shown in Fig. 24.34.

For the discharge interval, $V_i = 7.18$ mV and $V_f = 0$ V, and

$$v_C = V_f + (V_i - V_f)e^{-t/\tau}$$
$$= 0 \text{ V} + (7.18 \text{ mV} - 0)e^{-t/\tau}$$
$$v_C = 7.18 \text{ mV}e^{-t/\tau}$$

At $t = \tau$ (measured from 3τ in Fig. 24.34),

$$v_C = (7.18 \text{ mV})(e^{-1}) = (7.18 \text{ mV})(0.368)$$
$$= 2.64 \text{ mV}$$

as shown in Fig. 24.34.

Continuing in the same manner, we generate the remaining waveform for v_C as depicted in Fig. 24.34. Note that repetition occurs after $t = 8\tau$, and the waveform has essentially reached steady-state conditions in a period of time less than 10τ, or five cycles of the applied square wave.

A closer look reveals that both the peak and the lower levels continued to increase until steady-state conditions were established. Since the exponential waveforms between $t = 4T$ and $t = 5T$ have the same time constant, the average value of v_C can be determined from the steady-state 7.31 mV and 2.69 mV levels as follows:

$$V_{av} = \frac{7.31 \text{ mV} + 2.69 \text{ mV}}{2} = \frac{10 \text{ mV}}{2} = 5 \text{ mV}$$

which equals the average value of the applied signal, as stated earlier in this section.

We can use the results in Fig. 24.34 to plot i_C. At any instant of time,

$$v_i = v_R + v_C \qquad \text{or} \qquad v_R = v_i - v_C$$

and

$$i_R = i_C = \frac{v_i - v_C}{R}$$

At $t = 0^+$, $v_C = 0$ V, and

$$i_R = \frac{v_i - v_C}{R} = \frac{10 \text{ mV} - 0}{5 \text{ k}\Omega} = 2 \text{ }\mu\text{A}$$

as shown in Fig. 24.35.

As the charging process proceeds, the current i_C decays at a rate determined by

$$i_C = 2 \text{ }\mu\text{A}e^{-t/\tau}$$

At $t = \tau$,

$$i_C = (2 \text{ }\mu\text{A})(e^{-\tau/\tau}) = (2 \text{ }\mu\text{A})(e^{-1}) = (2 \text{ }\mu\text{A})(0.368)$$
$$= 0.736 \text{ }\mu\text{A}$$
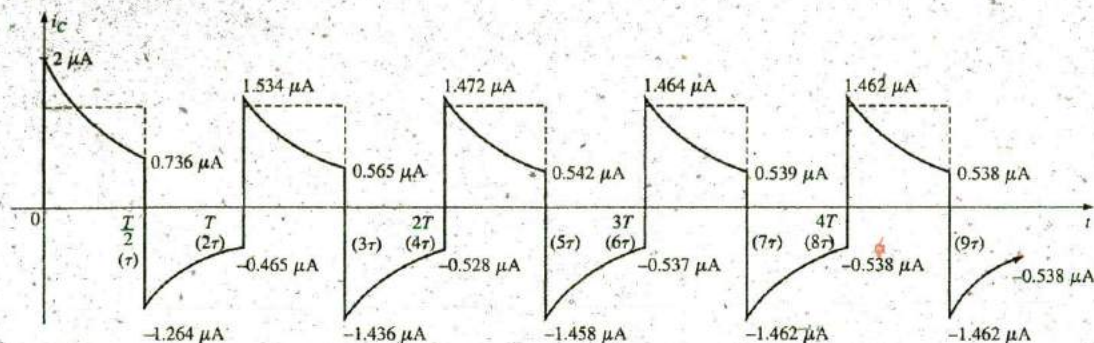
as shown in Fig. 24.35.

FIG. 24.35

i_C response for $t_p = \tau = T/2$.

For the trailing edge of the first pulse, the voltage across the capacitor cannot change instantaneously, resulting in the following when v_i drops to zero volts:

$$i_C = i_R = \frac{v_i - v_C}{R} = \frac{0 - 6.32 \text{ mV}}{5 \text{ k}\Omega} = -1.264 \text{ } \mu A$$

as illustrated in Fig. 24.35. The current then decays as determined by

$$i_C = -1.264 \text{ } \mu A e^{-t/\tau}$$

and at $t = \tau$ (actually $t = 2\tau$ in Fig. 24.35),

$$i_C = (-1.264 \text{ } \mu A)(e^{-\tau/\tau}) = (-1.264 \text{ } \mu A)(e^{-1})$$
$$= (-1.264 \text{ } \mu A)(0.368) = -0.465 \text{ } \mu A$$

as shown in Fig. 24.35.

At $t = T$ ($t = 2\tau$), $v_C = 2.33$ mV, and v_i returns to 10 mV, resulting in

$$i_C = i_R = \frac{v_i - v_C}{R} = \frac{10 \text{ mV} - 2.33 \text{ mV}}{5 \text{ k}\Omega} = 1.534 \text{ } \mu A$$

The equation for the decaying current is now

$$i_C = 1.534 \text{ } \mu A e^{-t/\tau}$$

and at $t = \tau$ (actually $t = 3\tau$ in Fig. 24.35),

$$i_C = (1.534 \text{ } \mu A)(0.368) = 0.565 \text{ } \mu A$$

The process continues until steady-state conditions are reached at the same time they were attained for v_C. Note in Fig. 24.35 that the positive peak current decreased toward steady-state conditions while the negative peak became more negative. Note that the current waveform becomes symmetrical about the axis when steady-state conditions are established. The result is that the net average current over one cycle is zero, as it should be in a series R-C circuit. Recall from Chapter 10 that the capacitor under dc steady-state conditions can be replaced by an open-circuit equivalent, resulting in $I_C = 0$ A.

Although both examples just provided started with an uncharged capacitor, the same approach can be used effectively for initial conditions. Simply substitute the initial voltage on the capacitor as V_i in Eq. (24.6) and proceed as before.

24.7 OSCILLOSCOPE ATTENUATOR AND COMPENSATING PROBE

The ×10 **attenuator probe** used with oscilloscopes is designed to reduce the magnitude of the input voltage by a factor of 10. If the input impedance to a scope is 1 MΩ, the ×10 attenuator probe will have an internal resistance of 9 MΩ, as shown in Fig. 24.36.
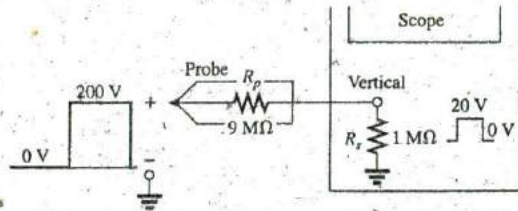

FIG. 24.36
×10 attenuator probe.

Applying the voltage divider rule gives

$$V_{scope} = \frac{(1 \text{ M}\Omega)(V_i)}{1 \text{ M}\Omega + 9 \text{ M}\Omega} = \frac{1}{10}V_i$$

In addition to the input resistance, oscilloscopes have some internal input capacitance, and the probe adds an additional capacitance in parallel with the oscilloscope capacitance, as shown in Fig. 24.37. The probe capacitance is typically about 10 pF for a 1 m (3.3 ft) cable, reaching about 15 pF for a 3 m (9.9 ft) cable. The total input capacitance is therefore the sum of the two capacitive elements, resulting in the equivalent network in Fig. 24.38.
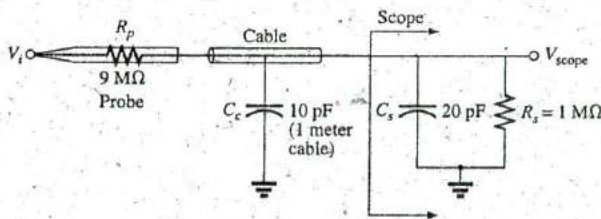

FIG. 24.37
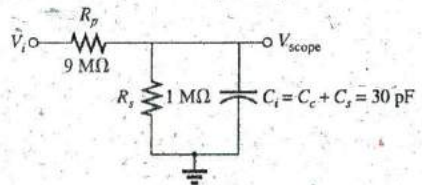*Capacitive elements present in an attenuator probe arrangement.*

FIG. 24.38
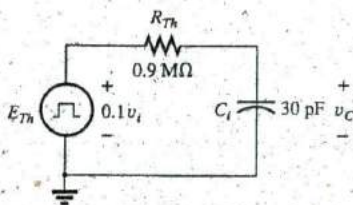*Equivalent network in Fig. 24.37.*

FIG. 24.39
Thévenin equivalent for C_i in Fig. 24.38.

For the analysis to follow, let us determine the Thévenin equivalent circuit for the capacitor C_i:

$$E_{Th} = \frac{(1 \text{ M}\Omega)(V_i)}{1 \text{ M}\Omega + 9 \text{ M}\Omega} = \frac{1}{10}V_i$$

and

$$R_{Th} = 9 \text{ M}\Omega \parallel 1 \text{ M}\Omega = 0.9 \text{ M}\Omega$$

The Thévenin network is shown in Fig. 24.39.
For $v_i = 200$ V (peak),

$$E_{Th} = 0.1v_i = 20 \text{ V (peak)}$$

and for v_C, $V_f = 20$ V and $V_i = 0$ V, with

$$\tau = RC = (0.9 \times 10^6 \, \Omega)(30 \times 10^{-12} \, F) = 27 \, \mu s$$

For an applied frequency of 5 kHz,

$$T = \frac{1}{f} = 0.2 \text{ ms} \quad \text{and} \quad \frac{T}{2} = 0.1 \text{ ms} = 100 \, \mu s$$

with $5\tau = 135 \, \mu s > 100 \, \mu s$, as shown in Fig. 24.40, clearly producing a severe rounding distortion of the square wave and a poor representation of the applied signal.

To improve matters, a variable capacitor is often added in parallel with the resistance of the attenuator, resulting in a **compensated attenuator probe** such as the one shown in Fig. 24.41. In Chapter 22, it was demonstrated that a square wave can be generated by a summation of sinusoidal signals of particular frequency and amplitude. If we therefore design a network such as the one shown in Fig. 24.42 that ensures that V_{scope} is $0.1v_i$ for any frequency, then the rounding distortion is removed, and V_{scope} has the same appearance as v_i.

Applying the voltage divider rule to the network in Fig. 24.42 gives

$$V_{scope} = \frac{Z_s V_i}{Z_s + Z_p} \qquad (24.8)$$

If the parameters are chosen or adjusted such that

$$R_p C_p = R_s C_s \qquad (24.9)$$

the phase angle of Z_s and Z_p will be the same, and Eq. (24.8) will reduce to

$$V_{scope} = \frac{R_s V_i}{R_s + R_p} \qquad (24.10)$$

which is insensitive to frequency since the capacitive elements have dropped out of the relationship.

In the laboratory, simply adjust the probe capacitance using a standard or known square-wave signal until the desired sharp corners of the square wave are obtained. If you avoid the calibration step, you may make a rounded signal look square since you assumed a square wave at the point of measurement.

Too much capacitance results in an overshoot effect, whereas too little continues to show the rounding effect.

24.8 APPLICATION

TV Remote

The TV remote works in many ways like a garage door opener or car alarm transmitter. There is no visible connection between the transmitter and the receiver, and each transmitter is linked to its receiver with a special code. The only major difference is that the TV remote uses an infrared frequency, while the other two use a much lower radio frequency.

The TV remote in Fig. 24.43(a) has been opened to reveal the internal construction of its keypad and face in Fig. 24.43(b). The three components in Fig. 24.43(b) are lined up to show how the holes in the cover
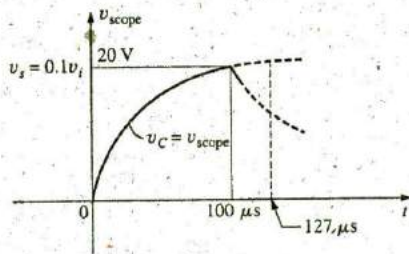

FIG. 24.40

The scope pattern for the conditions in Fig. 24.38 with $v_i = 200$ V peak.

FIG. 24.41

Commercial compensated 10 : 1 attenuator probe. (Courtesy of Tektronix, Inc.)
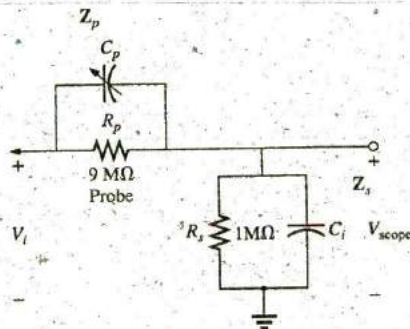

FIG. 24.42

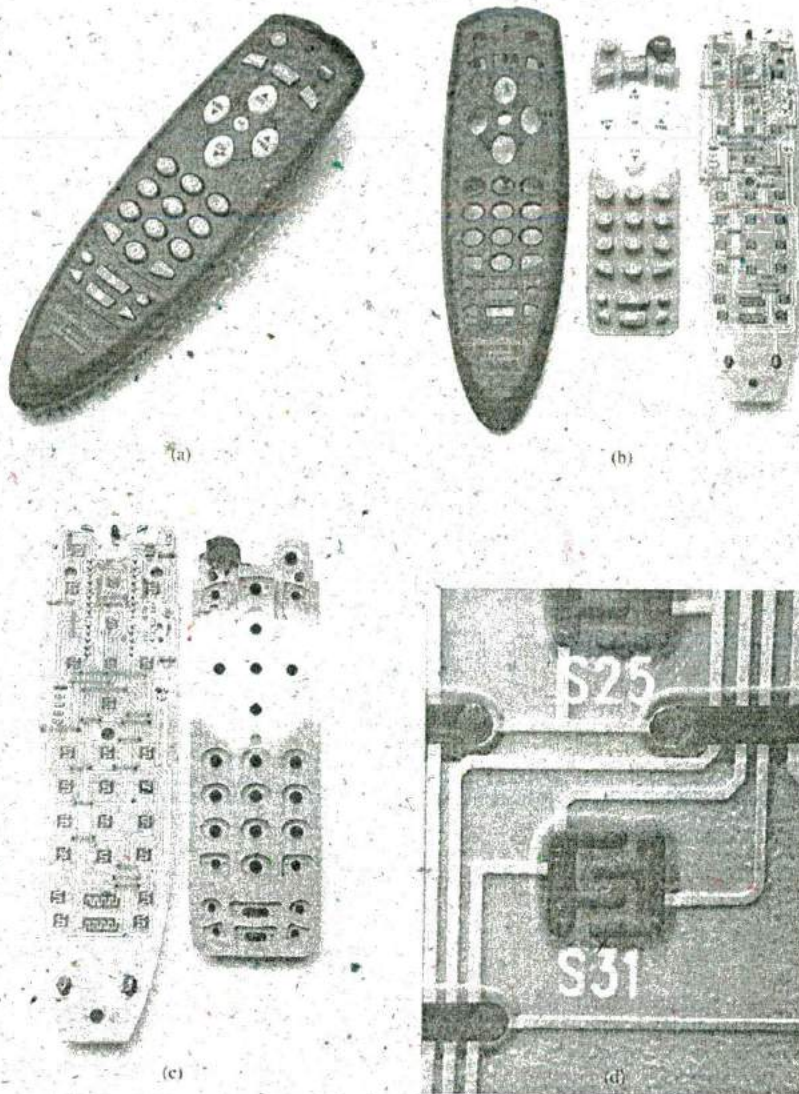Compensated attenuator and input impedance to a scope, including the cable capacitance.

FIG. 24.43
TV remote: (a) external appearance; (b) internal construction; (c) carbon keypads;
(d) enlarged view of S31 keypad.

match the actual keys in the switch membrane and where each button on the keypad hits on the face of the printed circuit board. Note on the printed circuit board that there is a black pad to match each key on the membrane. The back side of the switch membrane in Fig. 24.43(c) shows the soft carbon contacts that make contact with the carbon contacts on the printed board when the buttons are depressed. An enlarged view of one of the contacts (S31) in Fig. 24.43(c) is shown in Fig. 24.43(d) to illustrate the separation between circuits and the pattern used to ensure continuity when the solid round carbon pad at the bottom of the key is put in place.

All the connections established when a key is pressed are passed on to a relatively large switch-matrix-encoder IC chip appearing on the back side

of the printed circuit board as shown in Fig. 24.44. For the pad (S31) in Fig. 24.43(d), three wires of the matrix appearing in Fig. 24.43(b) are connected when the corresponding key (number 5) is pressed. The encoder then reacts to this combination and sends out the appropriate signal as an infrared (IR) signal from the IR LED appearing at the end of the remote control, as shown in Fig. 24.43(b) and Fig. 24.44. The second, smaller LED (red on actual unit) appearing at the top of Fig. 24.43(b) blinks during transmission. Once the batteries are inserted, the CMOS electronic circuitry that controls the operation of the remote is *always on*. This is possible only because of the very low power drain of CMOS circuitry. The power (PWR) button is used only to turn the TV on and activate the receiver.

The signal sent out by the majority of remotes is one of the two types appearing in Fig. 24.45. In each case there is a *key pulse* to initiate the signal sequence and to inform the receiver that the coded signal is about to arrive. In Fig. 24.45(a), a 4-bit binary-coded signal is transmitted using pulses in specific locations to represent the "ones" and using the absence of a pulse to represent the "zeros." That coded signal can then be interpreted by the receiver unit and the proper operation performed. In Fig. 24.45(b), the signal is frequency controlled. Each key has a different frequency associated with it. The result is that each key has a specific transmission frequency. Since each TV receiver responds to a different pulse train, a remote must be coded for the TV under control. There are *fixed program* remotes that can be used with only one TV. Then there are *smart* remotes that are preprogrammed internally with a number of remote control codes. You have to set up remotes of this type according to the TV you have, using a three-digit coding system accessed through the TV setup screen. *Learning* remotes are those that can use the old remote to learn the code and then store it for future use. In this case, one remote is set directly in front of the other, and the information is transferred from one to the other when both are energized. Remotes are also available that are a combination of the last two.

The remote in Fig. 24.43 uses four AAA batteries in series for a total of 6 V. It has its own local crystal oscillator separate from the IC, as shown by the discrete elements to the top right and midleft of the printed circuit board in Fig. 24.43(c). The crystal itself, which is relatively large
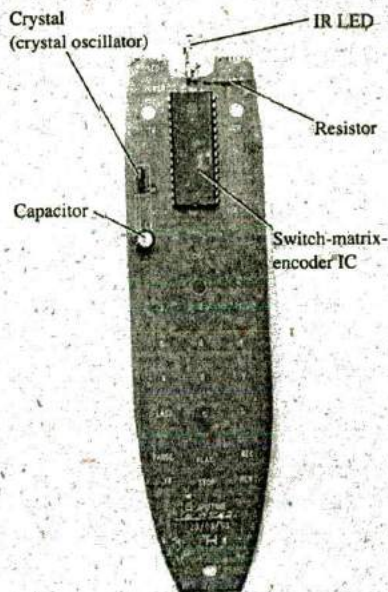

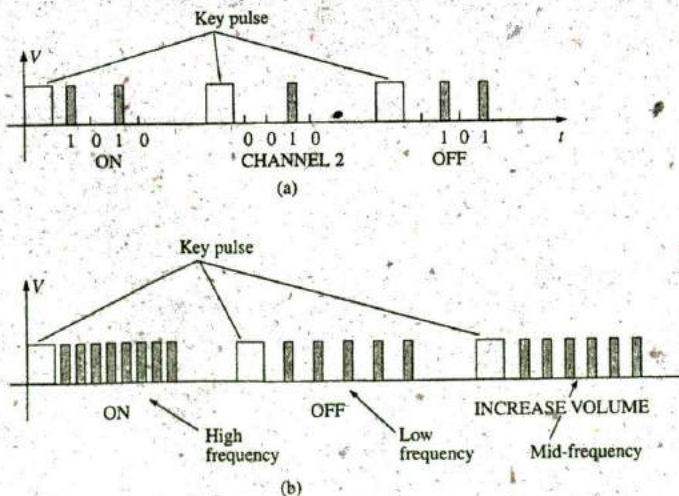**FIG. 24.44**

Back side of TV remote in Fig. 24.43.

FIG. 24.45

Signal transmission: (a) pulse train; (b) variation.

FIG. 24.46
Range and coverage arc for TV remote in Fig. 24.43.

compared to the other elements, appears on the other side of the board just above the electrolytic capacitor in Fig. 24.44. The oscillator generates the pulse signal required for proper IC operation. Note how flush most of the discrete elements are in Fig. 24.43(b) and the rather large electrolytic capacitor on the back of the printed circuit board in Fig. 24.44. The specifications on the unit give it a range control of 25 ft with a 30° coverage arc as shown in Fig. 24.46. The arc coverage of your unit can easily be tested by pointing if directly at the TV and then moving it in any direction until it no longer controls the TV.

24.9 COMPUTER ANALYSIS

PSpice

R-C Response Our analysis begins with a verification of the results of Example 24.10, which examined the response of the series *R-C* circuit appearing on the schematic in Fig. 24.47. The source is one used in Chapters
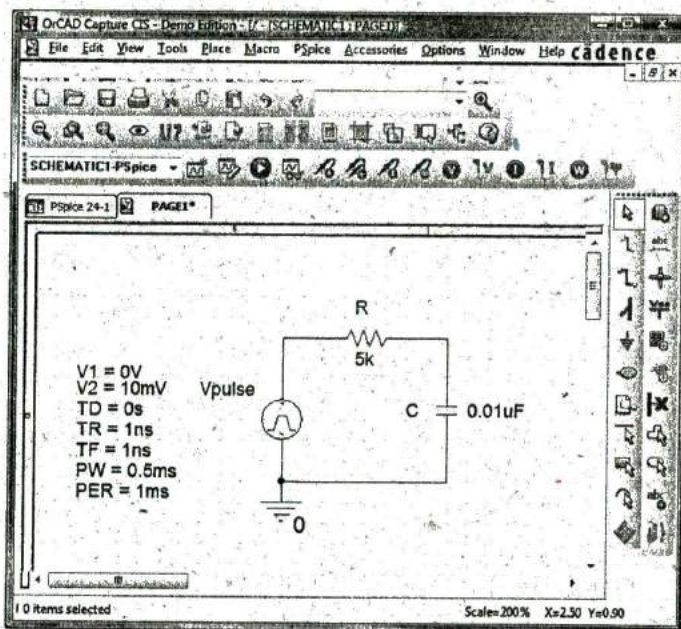

FIG. 24.47
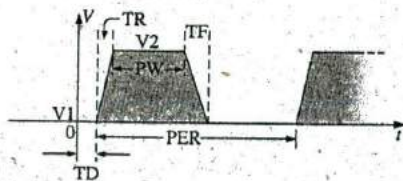*Using PSpice to verify the results of Example 24.10.*

FIG. 24.48
Defining the PSpice Vpulse parameters.

10 and 11 to replicate the action of a switch in series with a dc source. The defining attributes for the pulse waveform are repeated for convenience in Fig. 24.48. Recall that the PW was made long enough so that the full transient period could be examined. In this analysis, the pulse width is adjusted to permit viewing the transient behavior of an *R-C* network between changing levels of the applied pulse. Initially the **PW** is set at 10 times the time constant of the network so that the full transient response can occur between changes in voltage level. The time constant of the network is $\tau = RC = (5\ k\Omega)(0.01\ \mu F) = 0.05$ ms, resulting in a **PW** of 0.5 ms in Fig. 24.47. To establish a square-wave appearance, the period was chosen as twice the pulse width or 1 ms as shown in the **VPulse** listing.

⌐⌐

In the **Simulation Settings** dialog box, select **Time Domain(Transient)** to get a response versus time. Select the **Run to time** at 2 ms so that two full cycles result. Leave the **Start saving data after** on the default value of 0 s, and set the **Maximum step size** at 2 ms/1000 = 2 μs. After simulation, apply **Trace-Add Trace-I(C)-OK**, and the bottom plot in Fig. 24.49 is the result. Note that the maximum current is 2 μA as determined by $I_{C_{max}}$ = 10 mV/5 kΩ, and the full transient response appears within each pulse. Note also that the current dropped below the axis to reveal a change in direction when the applied voltage dropped from the 10 mV level to 0 V. Through **Plot-Add Plot to Window-Trace-Add Trace-V(Vpulse:+)-OK-Trace-Add Trace-V(C:1)-OK**, the plots of the applied voltage and the voltage across the capacitor can be displayed in the upper graph in Fig. 24.49. First, select the upper graph in Fig. 24.49 so that you can move the **SEL>>**, and then select the **Toggle cursor** key. Now left-click on **V(C:1)** at the bottom right of the graph and left-click again to set a cursor on the graph. Setting the cursor at five time constants reveals that the transient voltage has reached 10 mV. Setting the right-click cursor at ten time constants reveals that V_C has also reached the 10 mV level.
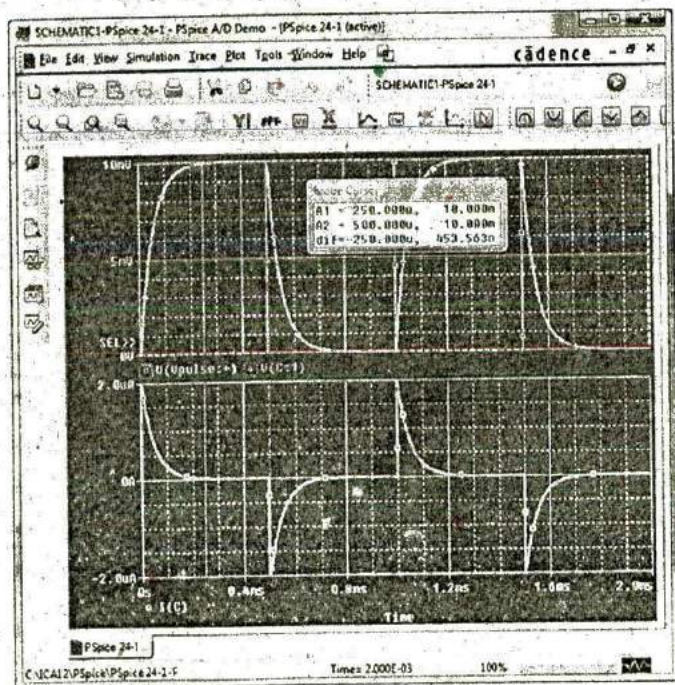

FIG. 24.49

Plot of v_{pulse}, v_C, and i_C for the circuit in Fig. 24.47.

Setting $t_p = \tau = T/2$ The parameters of the source will now be modified by changing the frequency of the pulse waveform to 10 kHz with a period of 0.1 ms and a pulse width of 0.05 ms. For **Vpulse**, the changes are **PW** = 0.05 ms and **PER** = 0.1 ms. The time constant of the network remains the same at 0.05 ms, so the pulse width equals the time constant of the circuit. The result is that it will take a number of pulses before the voltage across the capacitor reaches its final value of 10 mV. Under the **Simulation Settings**, change the **Run to time** to 0.5 ms = 500 μs, or
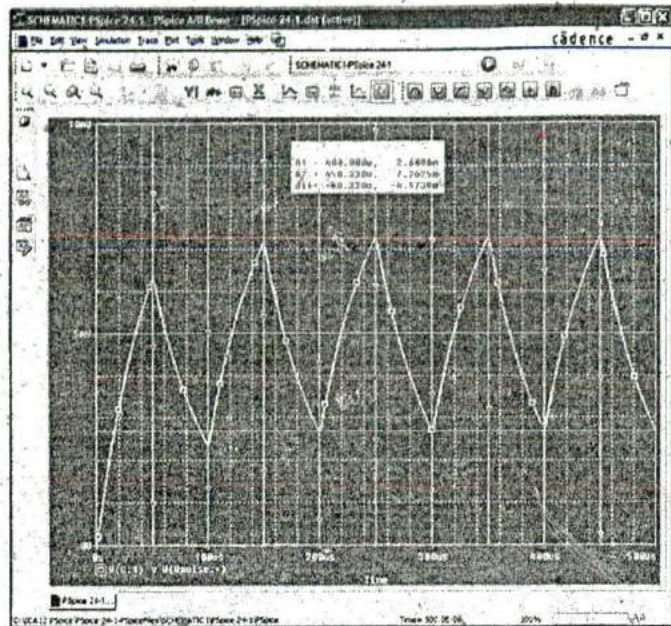
FIG. 24.50

Plot of v_C for the circuit in Fig. 24.47 with $t_p = \tau = T/2$.

five cycles of the applied voltage. Change the **Maximum step size** to 500 μs/1000 = 500 ns = 0.5 μs. Under the **SCHEMATIC1** window, select **Trace-Add Trace-V(C:1)-OK** to obtain the transient voltage across the capacitor. Select **Trace-Add Trace-V(Vpulse:+)-OK** to place the applied voltage on the same screen, Note in the resulting plots in Fig. 24.50 that the voltage builds up from 0 V until it appears to reach a fairly steady state after 400 μs. At 400 μs, use a left cursor (A1) to find the minimum point with 2.69 mV resulting—an exact match with the longhand calculation of Example 24.11 at 2.69 mV. At 450 μs, the right-click cursor (A2) provides a level of 7.26 mV, which is again a close match with the calculated level of 7.31 mV.

PROBLEMS

SECTION 24.2 Ideal versus Actual

1. Determine the following for the pulse waveform in Fig. 24.51:
   a. whether it is positive- or negative-going
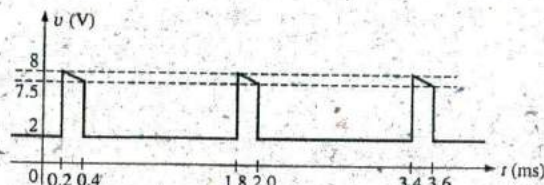   b. base-line voltage

c. pulse width
d. amplitude
e. % tilt
f. prf
g. duty cycle

2. Repeat Problem 1 for the pulse waveform in Fig. 24.52.

FIG: 24.51
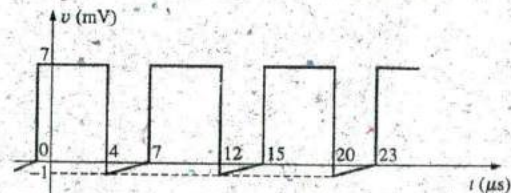
Problems 1, 8, and 12,

FIG. 24.52

Problems 2 and 9.

3. Repeat Problem 1 for the pulse waveform in Fig. 24.53.

Vertical sensitivity = 10 mV/div.
Horizontal sensitivity = 2 ms/div.
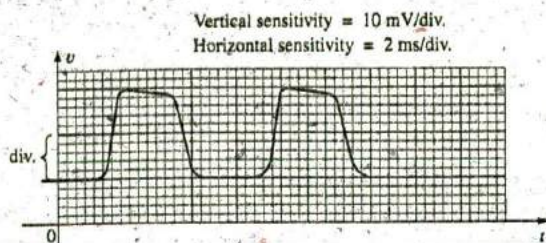
FIG. 24.53
Problems 3, 4, 10, and 13.

4. Determine the rise and fall times for the waveform in Fig. 24.53.

5. Sketch a pulse waveform that has a base-line voltage of −5 mV, a pulse width of 2 μs, an amplitude of 15 mV, a 10% tilt, a period of 10 μs, and vertical sides and is positive-going.

6. For the waveform in Fig. 24.54, established by straight-line approximations of the original waveform:
   a. Determine the rise time.
   b. Find the fall time.
   c. Find the pulse width.
   d. Calculate the frequency.
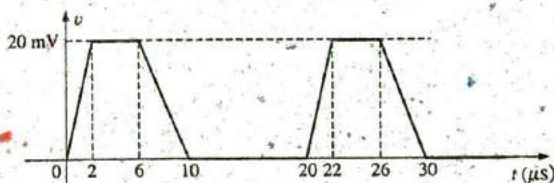
FIG. 24.54
Problems 6 and 14.

7. For the waveform in Fig. 24.55:
   a. Determine the period.
   b. Find the frequency.
   c. Find the maximum and minimum amplitudes.

Vertical sensitivity = 0.2 V/div.
Horizontal sensitivity = 50 μs/div.
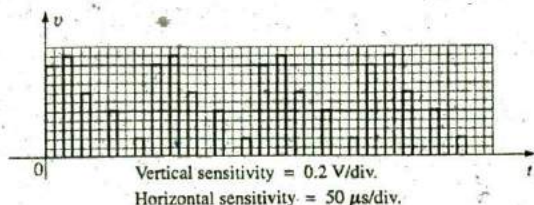
FIG. 24.55
Problems 7 and 15.

SECTION 24.3 Pulse Repetition Rate and Duty Cycle

8. Determine the pulse repetition frequency and duty cycle for the waveform in Fig. 24.51.

9. Determine the pulse repetition frequency and duty cycle for the waveform in Fig. 24.52.

10. Determine the pulse repetition frequency and duty cycle for the waveform in Fig. 24.53.

SECTION 24.4 Average Value

11. For the waveform in Fig. 24.56, determine the
    a. period.
    b. pulse width.
    c. pulse repetition frequency.
   d. average value.
    e. effective value.
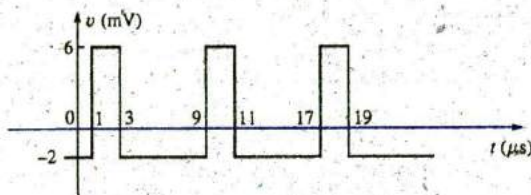
FIG. 24.56
Problem 11.

12. Determine the average value of the periodic pulse waveform in Fig. 24.51.

13. To the best accuracy possible, determine the average value of the waveform in Fig. 24.53.

14. Determine the average value of the waveform in Fig. 24.54.

15. Determine the average value of the periodic pulse train in Fig. 24.55.

SECTION 24.5 Transient R-C Networks

16. The capacitor in Fig. 24.57 is initially charged to 5 V, with the polarity indicated in the figure. The switch is then closed at $t = 0$ s.
    a. What is the mathematical expression for the voltage v_C?
    b. Sketch v_C versus t.
    c. What is the mathematical expression for the current i_C?
    d. Sketch i_C versus t.
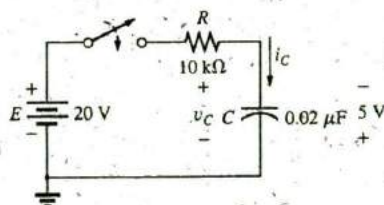
FIG. 24.57
Problem 16.

17. For the input voltage v_i appearing in Fig. 24.58, sketch the waveform for v_o. Assume that steady-state conditions were established with $v_i = 8$ V.
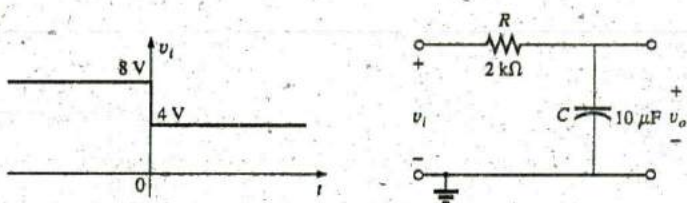

FIG. 24.58
Problem 17.

18. The switch in Fig. 24.59 is in position 1 until steady-state conditions are established. Then the switch is moved (at $t = 0$ s) to position 2. Sketch the waveform for the voltage v_C.
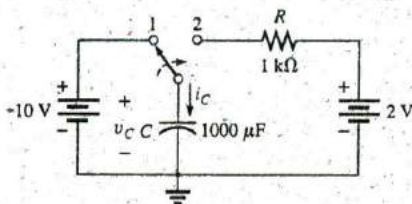

FIG. 24.59
Problems 18 and 19.

19. Sketch the waveform for i_C for Problem 18.

SECTION 24.6   R-C Response to Square-Wave Inputs

20. Sketch the voltage v_C for the network in Fig. 24.60 due to the square-wave input of the same figure with a frequency of
   a. 500 Hz.
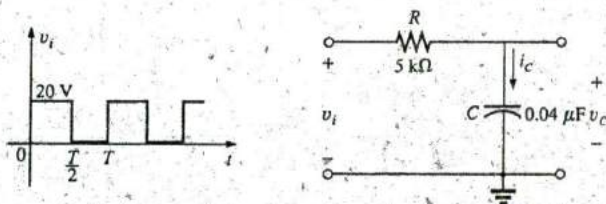   **b.** 100 Hz.
   c. 5000 Hz.

FIG. 24.60.
Problem 20, 21, 23, 24, 27, and 28.

21. Sketch the current i_C for each frequency in Problem 20.

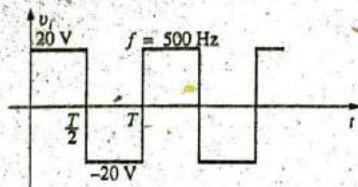22. Sketch the response v_C of the network in Fig. 24.60 to the square-wave input in Fig. 24.61.

FIG. 24.61
Problem 22.

23. If the capacitor in Fig. 24.60 is initially charged to 20 V, sketch the response v_C to the same input signal (in Fig. 24.60) at a frequency of 500 Hz.

24. Repeat Problem 23 if the capacitor is initially charged to −10 V.

SECTION 24.7 Oscilloscope Attenuator and Compensating Probe

25. Given the network in Fig. 24.42 with $R_p = 9$ MΩ and $R_s = 1$ MΩ, find V_{scope} in polar form if $C_p = 3$ pF, $C_s = 18$ pF, $C_c = 9$ pF, and $v_i = \sqrt{2}\,(100)\sin 2\pi\,10{,}000t$. That is, determine Z_s and Z_p, substitute into Eq. (24.8), and compare the results obtained with Eq. (24.10). Is it verified that the phase angle of Z_s and Z_p is the same under the condition $R_pC_p = R_sC_s$?

26. Repeat Problem 25 at $\omega = 10^5$ rad/s.

SECTION 24.9 Computer Analysis

PSpice

27. Using schematics, obtain the waveforms for v_C and i_C for the network in Fig. 24.60 for a frequency of 1 kHz.

*28. Using schematics, place the waveforms of v_i, v_C, and i_C on the same printout for the network in Fig. 24.60 at a frequency of 2 kHz.

*29. Using schematics, obtain the waveform appearing on the scope in Fig. 24.37 with a 20 V pulse input at a frequency of 5 kHz.

*30. Place a capacitor in parallel with R_p in Fig. 24.37 that will establish an in-phase relationship between v_{scope} and v_i. Using schematics, obtain the waveform appearing on the scope in Fig. 24.37 with a 20 V pulse input at a frequency of 5 kHz.

GLOSSARY

Actual (true, practical) pulse A pulse waveform having a leading edge and a trailing edge that are not vertical, along with other distortion effects such as tilt, ringing, or overshoot.

Amplitude of a pulse waveform The peak-to-peak value of a pulse waveform.

Attenuator probe A scope probe that will reduce the strength of the signal applied to the vertical channel of a scope.

Base-line voltage The voltage level from which a pulse is initiated.

Compensated attenuator probe A scope probe that can reduce the applied signal and balance the effects of the input capacitance of a scope on the signal to be displayed.

Duty cycle Factor that reveals how much of a period is encompassed by the pulse waveform.

Fall time (t_f) The time required for the trailing edge of a pulse waveform to drop from the 90% to the 10% level.

Ideal pulse A pulse waveform characterized as having vertical sides, sharp corners, and a flat peak response.

Negative-going pulse A pulse that increases in the negative direction from the base-line voltage.

Periodic pulse train A sequence of pulses that repeats itself after a specific period of time.

Positive-going pulse A pulse that increases in the positive direction from the base-line voltage.

Pulse repetition frequency (pulse repetition rate) The frequency of a periodic pulse train.

Pulse train A series of pulses that may have varying heights and widths.

Pulse width (t_p) The pulse width defined by the 50% voltage level.

Rise time (t_r) The time required for the leading edge of a pulse waveform to travel from the 10% to the 90% level.

Square wave A periodic pulse waveform with a 50% duty cycle.

Tilt (droop, sag) The drop in peak value across the pulse width of a pulse waveform.