33

Performance of Gas Turbines

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33.1. Introduction

In the last chapter, we have discussed the working of gas turbines under ideal conditions *i.e.* with isentropic expansion and compression. But in actual practice, the ideal conditions do not prevail. Now, in this chapter, we shall discuss the performance and uses of gas turbines under various sets of conditions.

33.2. Assumptions for Overall (Thermal) Efficiency of an Ideal Gas Turbine Plant

Following simplifying assumptions are made to obtain an expression for the overall (thermal) efficiency of an ideal gas turbine installation plant :

1. The compression and expansion processes are considered as isentropic.
2. The specific heat of the working fluid remains constant throughout the cycle.
3. The pressure losses in the cycle are neglected.
4. The heat losses due to various factors are neglected.
5. The kinetic energy of the working fluid, while entering the compressor and leaving the turbine, is equal.

33.3. Overall (Thermal) Efficiency of an Ideal Gas Turbine Plant

It is the ratio of net work done by a turbine (*i.e.* work developed by turbine – work required by compressor) to the heat supplied. Mathematically, thermal efficiency,

$$\eta_{th} = \frac{\text{Net work done}}{\text{Heat supplied}} \qquad \ldots (i)$$

Now consider a gas turbine comprising of air compressor and turbine working on Joule's or Brayton's cycle as shown in Fig. 33.1 (*a*) and (*b*).

We know that net work done by the turbine

$$= c_p\,[(T_2 - T_3) - (T_1 - T_4)] \qquad \ldots (ii)$$

and heat supplied

$$= c_p\,(T_2 - T_1) \qquad \ldots (ii)$$

$$\therefore \quad \eta_{th} = \frac{c_p\,[(T_2 - T_3) - (T_1 - T_4)]}{c_p\,(T_2 - T_1)} \qquad \ldots (iv)$$

$$= \frac{(T_2 - T_3) - (T_1 - T_4)}{(T_2 - T_1)} \qquad \ldots (v)$$

We know that pressure ratio,

$$r = \frac{p_1}{p_4} = \frac{p_2}{p_3}$$

and for isentropic compression (4-1),

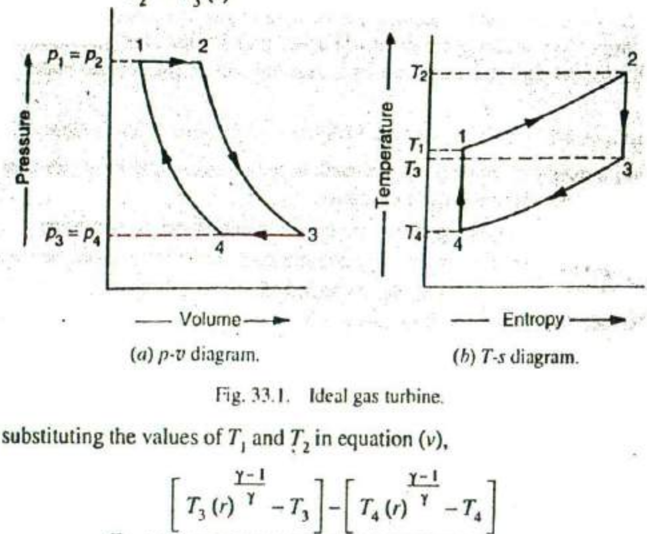
$$\frac{T_1}{T_4} = \left(\frac{p_1}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = (r)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore \quad T_1 = T_4\,(r)^{\frac{\gamma-1}{\gamma}}$$

Similarly, for isentropic expansion (2-3),

$$\frac{T_2}{T_3} = \left(\frac{p_2}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = (r)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore \quad T_2 = T_3\,(r)^{\frac{\gamma-1}{\gamma}}$$



(*a*) *p-v* diagram. (*b*) *T-s* diagram.

Fig. 33.1. Ideal gas turbine.

Now substituting the values of T_1 and T_2 in equation (*v*),

$$\eta_{th} = \frac{\left[T_3\,(r)^{\frac{\gamma-1}{\gamma}} - T_3\right] - \left[T_4\,(r)^{\frac{\gamma-1}{\gamma}} - T_4\right]}{\left[T_3\,(r)^{\frac{\gamma-1}{\gamma}} - T_4\,(r)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$= \frac{T_3\left[(r)^{\frac{\gamma-1}{\gamma}} - 1\right] - T_4\left[(r)^{\frac{\gamma-1}{\gamma}} - 1\right]}{(T_3 - T_4)\,(r)^{\frac{\gamma-1}{\gamma}}}$$

$$= \frac{[T_3 - T_4]\left[(r)^{\frac{\gamma-1}{\gamma}} - 1\right]}{(T_3 - T_4)\,(r)^{\frac{\gamma-1}{\gamma}}} = \frac{\left[r^{\frac{\gamma-1}{\gamma}} - 1\right]}{(r)^{\frac{\gamma-1}{\gamma}}} = 1 - \left(\frac{1}{r}\right)^{\frac{\gamma-1}{\gamma}}$$

Notes : 1. The thermal efficiency may also be obtained by substituting the values in equations (*i*) or (*iv*).

2. If the heat supplied is obtained from the mass of fuel × calorific value, then the corresponding efficiency will be overall efficiency.

Example 33.1. *A gas turbine plant receives air at 1 bar and 290 K and compresses it to 5 bar. If the temperature of air after compression is 1000 K ; find the thermal efficiency of the turbine. Take γ for the air as 1.4.*

Solution. Given : $p_3 = p_4 = 1$ bar ; $^*T_4 = 290$ K ; $p_1 = p_2 = 5$ bar ; $^*T_2 = 1000$ K ; $\gamma = 1.4$

We know that pressure ratio,

$$r = \frac{p_1}{p_4} = \frac{p_2}{p_3} = \frac{5}{1} = 5$$

∴ Thermal efficiency of the turbine,

$$\eta_{th} = 1 - \left(\frac{1}{r}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \left(\frac{1}{5}\right)^{\frac{1.4-1}{1.4}} = 1 - (0.2)^{0.286} = 1 - 0.631$$

$$= 0.369 \text{ or } 36.9\ \% \textbf{ Ans.}$$

Example 33.2. *Air enters the compressor of a gas turbine plant operating on Brayton cycle at 1 bar and 27° C. The pressure ratio in the cycle is 6. Calculate the maximum temperature in the cycle and the cycle efficiency. Assume the turbine work as 2.5 times the compressor work. Take γ = 1.4.*

Solution. Given : $p_3 = p_4 = 1$ bar ; $T_4 = 27°\text{ C} = 27 + 273 = 300$ K ; $r = p_1 / p_4 = p_2 / p_3 = 6$; $W_T = 2.5\ W_C$; $\gamma = 1.4$

The *T-s* diagram of the Brayton cycle is shown in Fig. 33.2.

1. *Maximum temperature in the cycle*

Let T_2 = Maximum temperature in the cycle.

We know that for isentropic compression 4-1,

$$\frac{T_1}{T_4} = \left(\frac{p_1}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = (6)^{\frac{1.4-1}{1.4}} = 1.67$$

$$\therefore \quad T_1 = T_4 \times 1.67 = 300 \times 1.67 = 501 \text{ K}$$

and for isentropic expansion 2-3,

$$\frac{T_2}{T_3} = \left(\frac{p_2}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = (6)^{\frac{1.4-1}{1.4}} = 1.67$$

$$\therefore \quad T_3 = T_2 / 1.67 \qquad \ldots (i)$$

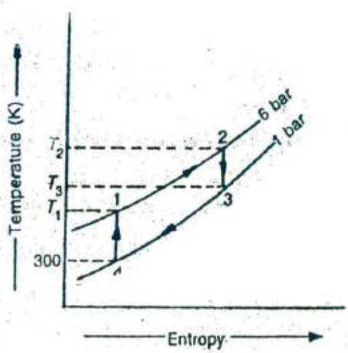


Fig. 33.2

We also know that turbine work per kg of air,

$$W_T = c_p (T_2 - T_3)$$

and compressor work per kg of air, $W_C = c_p (T_1 - T_4)$

Since $W_T = 2.5\ W_C$, therefore

$$c_p (T_2 - T_3) = 2.5\ c_p (T_1 - T_4)$$

* Superfluous data

or
$$T_2 - T_3 = 2.5\,(T_1 - T_4)$$
$$= 2.5\,(501 - 300) = 502.5 \text{ K} \quad \ldots (ii)$$

From equations (*i*) and (*ii*),
$$T_2 = 1252.5 \text{ K} = 979.5^\circ \text{C} \textbf{ Ans.}$$
and
$$T_3 = 750 \text{ K}$$

Cycle efficiency

We know that cycle efficiency,
$$\eta = \frac{(T_2 - T_3) - (T_1 - T_4)}{T_2 - T_1}$$
$$= \frac{(1252.5 - 750) - (501 - 300)}{1252.5 - 501} = \frac{301.5}{751.5}$$
$$= 0.40 \text{ or } 40\% \textbf{ Ans.}$$

Note : The cycle efficiency may also be obtained by using the relation,
$$\eta = 1 - \left(\frac{1}{r}\right)^{\frac{\gamma - 1}{\gamma}} = 1 - \left(\frac{1}{6}\right)^{\frac{1.4 - 1}{1.4}} = 1 - \frac{1}{1.67} = 1 - 0.6$$
$$= 0.40 \text{ or } 40\% \textbf{ Ans.}$$

Example 33.3. *In an oil gas turbine installation, air is taken at 1 bar and 30° C. The air is compressed to 4 bar and then heated by burning the oil to a temperature of 500° C. If the air flows at the rate of 90 kg/minute, find the power developed by the plant. Take γ for air as 1.4 and c_p as 1 kJ/kg K.*

If 2.4 kg of oil having calorific value of 40 000 kJ/kg is burnt in the combustion chamber per minute, find the overall efficiency of the plant.

Solution. Given : $p_4 = p_3 = 1$ bar ; $T_4 = 30°\text{ C} = 30 + 273 = 303$ K ; $p_1 = p_2 = 4$ bar ; $T_2 = 500°\text{ C} = 500 + 273 = 773$ K ; $m_a = 90$ kg/min $= 1.5$ kg/s ; $\gamma = 1.4$; $c_p = 1$ kJ/kg K ; $m_f = 2.4$ kg/min $= 0.04$ kg/s ; $C = 40\,000$ kJ/kg

The *T-s* diagram of the cycle is shown in Fig. 33.3.

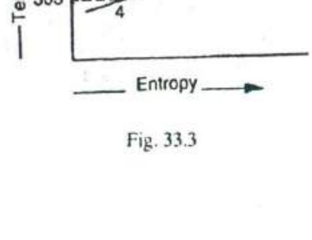


Fig. 33.3

Power developed by the plant

Let T_1, T_3 = Temperature of air at points 1 and 3.

We know that for isentropic expansion 2-3,
$$\frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma - 1}{\gamma}} = \left(\frac{1}{4}\right)^{\frac{1.4 - 1}{1.4}} = 0.673$$
$$\therefore \quad T_3 = T_2 \times 0.673 = 773 \times 0.673 = 520 \text{ K}$$

Similarly, for isentropic compression 4-1,
$$\frac{T_4}{T_1} = \left(\frac{p_4}{p_1}\right)^{\frac{\gamma - 1}{\gamma}} = \left(\frac{1}{4}\right)^{\frac{1.4 - 1}{1.4}} = 0.673$$
$$\therefore \quad T_1 = T_4 / 0.673 = 303 / 0.673 = 450 \text{ K}$$

We know that work developed by the turbine,

$$W_T = m\,c_p\,(T_2 - T_3) = 1.5 \times 1\,(773 - 520) = 379.5 \text{ kJ/s}$$

and work required by compressor,

$$W_C = m\,c_p\,(T_1 - T_4) = 1.5 \times 1\,(450 - 303) = 220.5 \text{ kJ/s}$$

$\therefore$ Net work or power of the turbine,

$$P = W_T - W_C = 379.5 - 220.5 = 159 \text{ kJ/s} = 159 \text{ kW} \quad \textbf{Ans.}$$

Overall efficiency of the plant

We know that heat supplied per second

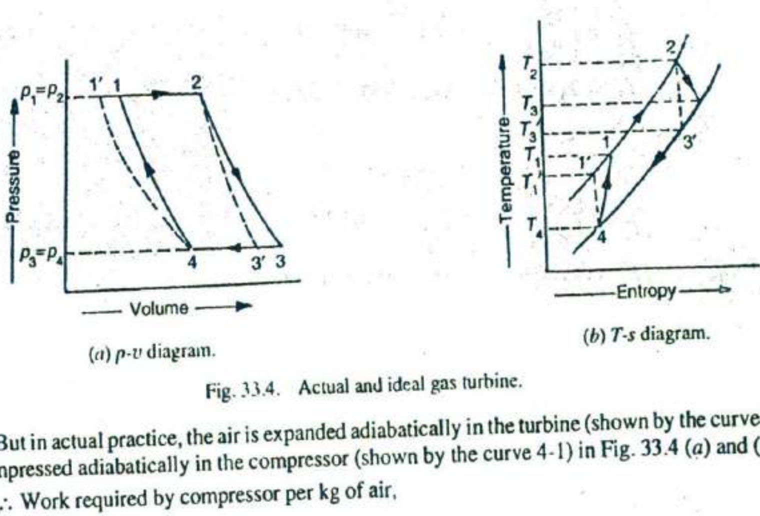
$$= m_f \times C = 0.04 \times 40\,000 = 1600 \text{ kJ/s}$$

$\therefore$ Overall efficiency of the plant,

$$\eta_0 = \frac{159}{1600} = 0.099 \text{ or } 9.9\% \quad \textbf{Ans.}$$

33.4. Actual Gas Turbine

We have already discussed in Art. 32.5, an ideal closed cycle gas turbine. In this article, we have shown isentropic expansion of air in the turbine by the curve 2-3′ and isentropic compression of air in the compressor by the curve 4-1′ as shown in Fig. 33.4 (*a*) and (*b*).



(*a*) *p-v* diagram.

(*b*) *T-s* diagram.

Fig. 33.4. Actual and ideal gas turbine.

But in actual practice, the air is expanded adiabatically in the turbine (shown by the curve 2-3) and compressed adiabatically in the compressor (shown by the curve 4-1) in Fig. 33.4 (*a*) and (*b*).

$\therefore$ Work required by compressor per kg of air,

$$W_C = c_p\,(T_1 - T_4) \qquad \ldots (i)$$

and work done by turbine per kg of air,

$$W_T = c_p\,(T_2 - T_3) \qquad \ldots (ii)$$

Now net work available,

$$W = W_T - W_C \qquad \ldots (iii)$$

Notes : 1. The power developed by the turbine may be found out from the work done as usual.

2. In this case, isentropic efficiency of the compressor,

$$\eta_C = \frac{T_1' - T_4}{T_1 - T_4}.$$

and isentropic efficiency of the turbine (alone),

$$\eta_T = \frac{T_2 - T_3}{T_2 - T_3'}$$

Example 33.4. *A gas turbine plant with a pressure ratio of 1 : 5 takes in air at 15° C. The maximum temperature is 600° C and develops 2200 kW. The turbine and compressor efficiencies are equal to 0.85. Taking $c_p = 1$ kJ/kg K and $c_v = 0.714$ kJ/kg K; determine 1. Actual overall efficiency of the turbine ; and 2. Mass of air circulated by the turbine.*

Solution. Given : $r = p_1/p_4 = p_2/p_3 = 5$; $T_4 = 15°\text{ C} = 288\text{ K}$; $T_2 = 600°\text{ C} = 873\text{ K}$; $P = 2200\text{ kW}$; $\eta_T = \eta_C = 0.85$; $c_p = 1$ kJ/kg K ; $c_v = 0.714$ kJ/kg K

We know that ratio of specific heats,

$$\gamma = c_p / c_v = 1/0.714 = 1.4$$

Overall efficiency of the turbine

Let T_1', T_1, T_3, T_3' = Temperature of air at corresponding points.

We know that

$$\frac{T_3'}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{5}\right)^{\frac{1.4-1}{1.4}} = 0.631$$

$$\therefore \quad T_3' = T_2 \times 0.631 = 873 \times 0.631 = 551\text{ K}$$

We also know that

$$\frac{T_4}{T_1'} = \left(\frac{p_4}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{5}\right)^{\frac{1.4-1}{1.4}} = 0.631$$

$$\therefore \quad T_1' = T_4/0.631 = 288/0.631 = 456\text{ K}$$

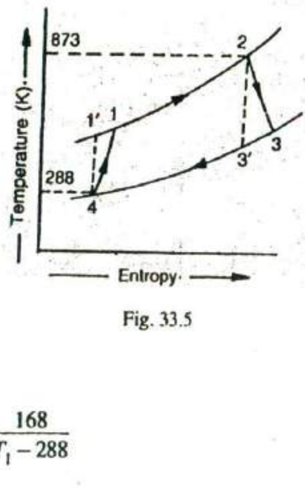


Fig. 33.5

We know that compressor efficiency (η_C),

$$0.85 = \frac{T_1' - T_4}{T_1 - T_4} = \frac{456 - 288}{T_1 - 288} = \frac{168}{T_1 - 288}$$

$$\therefore \quad T_1 = 486\text{ K}$$

and turbine efficiency (η_T),

$$0.85 = \frac{T_2 - T_3}{T_2 - T_3'} = \frac{873 - T_3}{873 - 551} = \frac{873 - T_3}{322}$$

$$\therefore \quad T_3 = 599\text{ K}$$

We know that work done by the turbine per kg of air,

$$W_T = c_p (T_2 - T_3) = 1\,(873 - 599) = 274\text{ kJ}$$

and work required by the compressor per kg of air,

$$W_C = c_p(T_1 - T_4) = 1\,(486 - 288) = 198 \text{ kJ}$$

$\therefore$ Net work done by the turbine per kg of air,

$$W = W_T - W_C = 274 - 198 = 76 \text{ kJ}$$

and heat supplied per kg of air $= c_p(T_2 - T_1) = 1\,(873 - 486) = 387$ kJ

$\therefore$ Overall efficiency of the turbine,

$$\eta_0 = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{76}{387} = 0.1964 \text{ or } 19.64\ \% \quad \textbf{Ans.}$$

Mass of air circulated by the turbine

Let m_a = Mass of air circulated by the turbine in kg/s.

We know that net workdone by the turbine,

$$W = 76 \text{ kJ/kg of air} = 76 \times m_a \text{ kJ/s} = 76\, m_a \text{ kW}$$

We also know that power developed (P),

$$2200 = 76\, m_a \quad \text{or} \quad m_a = 2200/76 = 28.95 \text{ kg/s} \quad \textbf{Ans.}$$

Example 33.5. *A gas turbine unit receives air at 100 kPa and 300 K and compresses it adiabatically to 620 kPa with efficiency of the compressor 88%. The fuel has a heating value of 44 180 kJ/kg and the fuel/air ratio is 0.017 kg fuel/kg air. The turbine internal efficiency is 90%. Calculate the compressor work ; turbine work and thermal efficiency.*

Solution. Given : $p_3 = p_4 = 100$ kPa $= 1$ bar ; $T_4 = 300$ K ; $p_1 = p_2 = 620$ kPa $= 6.2$ bar ; $\eta_C = 88\% = 0.88$; $C = 44\ 180$ kJ/kg ; $\eta_T = 90\ \% = 0.9$

The T-s diagram is shown in Fig. 33.6.

Let T_1, T_1', T_2, T_3 and T_3' = Temperature at corresponding points.

We know that

$$\frac{T_4}{T_1'} = \left(\frac{p_4}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{6.2}\right)^{\frac{1.4-1}{1.4}} = 0.593$$

$$\therefore \quad T_1' = T_4 / 0.593 = 300 / 0.593 = 506 \text{ K}$$

and efficiency of the compressor (η_C),

$$0.88 = \frac{T_1' - T_4}{T_1 - T_4} = \frac{506 - 300}{T_1 - 300} = \frac{206}{T_1 - 300}$$

$$\therefore \quad T_1 = 534 \text{ K}$$

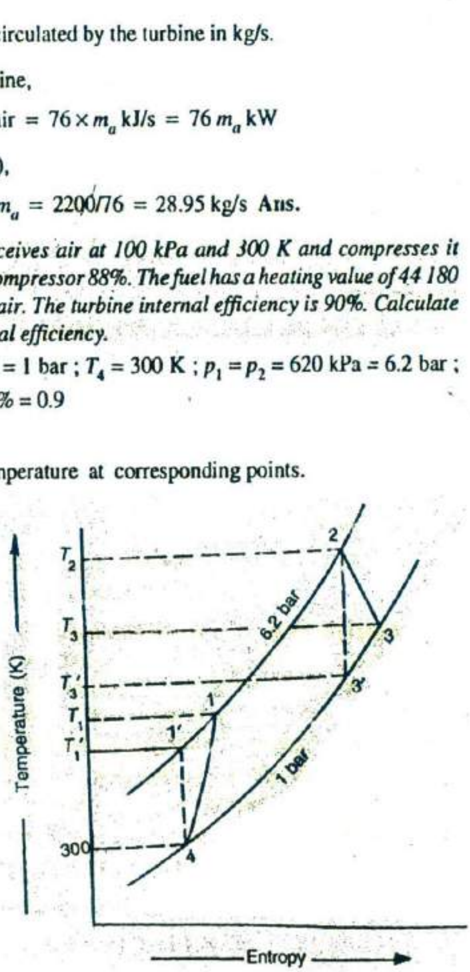


Fig. 33.6

We know that heat supplied by the fuel

$$= m_f \times C = 0.017 \times 44\ 180 = 751 \text{ kJ}$$

We also know that heat supplied per kg of air,

$$751 = m\,c_p\,(T_2 - T_1) = (1 + 0.017)\,1.005\,(T_2 - 534)$$

. . . (Taking $c_p = 1.005$ kJ/kg K)

$$\therefore \qquad T_2 = 735 + 534 = 1269 \text{ K}$$

Now for isentropic process 2-3′,

$$\frac{T_3'}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{6.2}\right)^{\frac{1.4-1}{1.4}} = 0.593$$

$$\therefore \qquad T_3' = 0.593\,T_2 = 0.593 \times 1269 = 752 \text{ K}$$

and turbine efficiency (η_T),

$$0.9 = \frac{T_2 - T_3}{T_2 - T_3'} = \frac{1269 - T_3}{1269 - 752}$$

$$\therefore \qquad T_3 = 804 \text{ K}$$

Compressor work

We know that compressor work per kg of air

$$W_C = m\,c_p\,(T_1 - T_4) = (1 + 0.017)\,1.005\,(534 - 300) = 239 \text{ kJ} \quad \textbf{Ans.}$$

Turbine work

We know that turbine work per kg of air,

$$W_T = m\,c_p\,(T_2 - T_3) = (1 + 0.017)\,1.005\,(1269 - 804) = 475 \text{ kJ} \quad \textbf{Ans.}$$

Thermal efficiency

We know that net workdone by the turbine per kg of air,

$$W = W_T - W_C = 475 - 239 = 236 \text{ kJ}$$

$\therefore$ Thermal efficiency,

$$\eta_{th} = \frac{\text{Net work done}}{\text{Heat supplied}} = \frac{236}{751} = 0.314 \text{ or } 31.4\% \quad \textbf{Ans.}$$

Example 33.6. *In a gas turbine plant, the intake temperature and pressure are 1 bar and 18° C respectively. The air is then compressed to a pressure of 4.1 bar by a compressor, whose isentropic efficiency is 80%. The temperature of the gas, whose properties may be assumed to resemble with those of air, is raised to 645° C in the combustion chamber where there is a pressure drop of 0.1 bar. Expansion to atmospheric pressure then occurs.*

If the thermal efficiency of the plant is to be 19%, what must be the isentropic efficiency of the turbine ? The mass of fuel may be neglected. Take $\gamma = 1.4$.

Solution. Given : $p_4 = p_3 = 1$ bar ; $T_4 = 18° \text{ C} = 291$ K ; $p_1 = 4.1$ bar ; $\eta_C = 80\% = 0.8$; $T_2 = 645° \text{ C} = 918$ K ; Pressure drop $= 0.1$ bar ; $\eta_{th} = 19\% = 0.19$; $\gamma = 1.4$

Since the pressure drop in the combustion chamber is 0.1 bar, therefore

$$p_2 = 4.1 - 0.1 = 4 \text{ bar}$$

The *T-s* diagram is shown in Fig. 33.7.

Let T_1, T_1', T_3, T_3' = Temperature at corresponding points.

We know that for isentropic process 2-3',

$$\frac{T_3'}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{4}\right)^{\frac{1.4-1}{1.4}} = 0.673$$

$$\therefore \quad T_3' = T_2 \times 0.673 = 918 \times 0.673 = 618 \text{ K}$$

and for isentropic process 4-1',

$$\frac{T_4}{T_1'} = \left(\frac{p_4}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{4.1}\right)^{\frac{1.4-1}{1.4}} = 0.668$$

$$\therefore \quad T_1' = T_4 / 0.668 = 291 / 0.668 = 436 \text{ K}$$

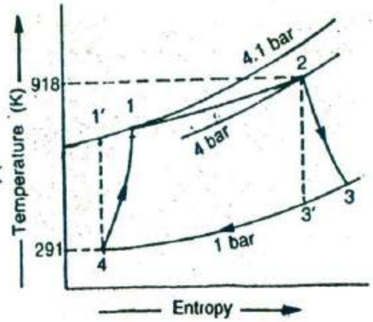


Fig. 33.7

We know that isentropic efficiency of compressor (η_C),

$$0.8 = \frac{T_1' - T_4}{T_1 - T_4} = \frac{436 - 291}{T_1 - 291} = \frac{145}{T_1 - 291}$$

$$\therefore \quad T_1 = 472 \text{ K}$$

We also know that work done by turbine per kg of air,

$$W_T = c_p (T_2 - T_3) = c_p (918 - T_3)$$

and work required by compressor per kg of air,

$$W_C = c_p (T_1 - T_4) = c_p (472 - 291) = 181\, c_p$$

$\therefore$ Net work done by turbine per kg of air

$$= c_p (918 - T_3) - 181\, c_p = c_p (737 - T_3)$$

and heat supplied per kg of air $= c_p (T_2 - T_1) = c_p (918 - 472) = 446\, c_p$

We know that thermal efficiency of the turbine (η_{th}),

$$0.19 = \frac{c_p (737 - T_3)}{446\, c_p} = \frac{737 - T_3}{446}$$

$$\therefore \quad T_3 = 652 \text{ K}$$

and isentropic efficiency of the turbine,

$$\eta_T = \frac{T_2 - T_3}{T_2 - T_3'} = \frac{918 - 652}{918 - 618} = 0.887 \text{ or } 88.7\% \text{ Ans.}$$

33.5. Heat Exchanger

As a matter of fact, the exhaust gases, leaving the turbine at the end of expansion, are still at high temperature. If these gases are allowed to pass into the atmosphere, then it amounts to the loss of available heat energy.

In order to achieve fuel economy, some of the available heat energy is recovered by passing the gases from the turbine through a heat exchanger. A heat exchanger, in its simplest form, consists of a chamber having two passages. In one of the passages, air from the compressor flows to the combustion chamber. In the second passage, hot gases from the turbine are made to flow before exhaust to the atmosphere as shown in Fig. 33.8.

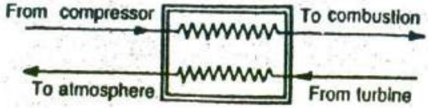


Fig. 33.8. Heat exchanger.

In a heat exchanger, hot gases from the turbine loose some energy and gets cooler, whereas air from compressor gets heated up before entering the combustion chamber.

33.6. Efficiency of Heat Exchanger

We have already discussed in the last article that hot gases from the turbine loose some heat energy in the heat exchanger. This heat energy is taken up by the air flowing from the compressor to the combustion chamber. Both these processes are shown on T-s diagram in Fig. 33.9.

Let T_X be the temperature of the compressed air and T_Y be the temperature of hot gases after passing through the heat exchanger.

In an ideal heat exchanger, the compressed air would be heated from T_1 to T_X (equal to T_3). Similarly, the hot gases will be cooled from T_3 to T_Y (equal to T_1). But in actual practice, it is not possible, as a finite temperature difference is required at all points in the heat exchanger for the heat to flow. In this case, this finite temperature difference is equal to $(T_3 - T_X)$ or $(T_Y - T_1)$.

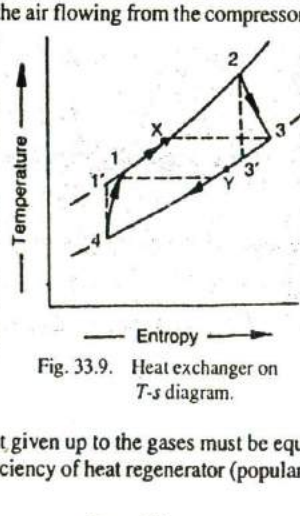


Fig. 33.9. Heat exchanger on T-s diagram.

If there is no loss of heat in the exchanger, then the heat given up to the gases must be equal to the heat taken up by the air. But it is rarely possible. The efficiency of heat regenerator (popularly known as effectiveness) is given by the relation,

$$\eta_e = \frac{\text{Actual heat transfer}}{\text{Maximum possible heat transfer}} = \frac{c_p (T_X - T_1)}{c_p (T_3 - T_1)}$$

Example 33.7. *A gas turbine plant receives air at a pressure of 1 bar and 290 K. The air is then compressed in a rotary compressor to a pressure of 4 bar and then heated to a temperature of 840 K. The efficiencies of compressor and turbine are 82% and 86% respectively. Neglecting the pressure drop, find overall efficiency of the plant (i) without heat exchanger ; and (ii) with heat exchanger of 70% effectiveness. Take γ and c_p for air and hot gases as 1.4 and 1 kJ/kg K respectively.*

Solution. Given : $p_4 = p_3 = 1$ bar ; $T_4 = 290$ K ; $p_1 = p_2 = 4$ bar ; $T_2 = 840$ K ; $\eta_C = 82\% = 0.82$; $\eta_T = 86\% = 0.86$; $\eta_e = 70\% = 0.7$; $\gamma = 1.4$; $c_p = 1$ kJ/kg K

The T-s diagram without heat exchanger and with heat exchanger is shown in Fig. 33.10 (*a*) and (*b*) respectively.

Efficiency of the plant without heat exchanger

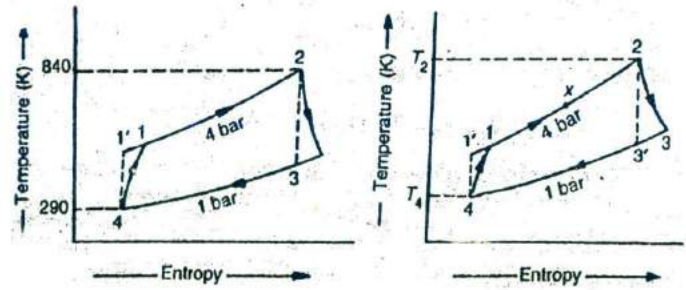
Let T_1', T_1, T_3, T_3' = Temperature of air at correspondingpoints.

We know that $\frac{T_3'}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{4}\right)^{\frac{1.4-1}{1.4}} = 0.673$

$\therefore \quad T_3' = T_2 \times 0.673 = 840 \times 0.673 = 565.3 \text{ K}$

and $\frac{T_4}{T_1'} = \left(\frac{p_4}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{4}\right)^{\frac{1.4-1}{1.4}} = 0.673$

$\therefore \quad T_1' = T_4 / 0.673 = 290 / 0.673 = 430.9 \text{ K}$



(*a*) Without heat exchanger. (*b*) With heat exchanger.

Fig. 33.10

We know that compressor efficiency (η_C),

$$0.82 = \frac{T_1' - T_4}{T_1 - T_4} = \frac{430.9 - 290}{T_1 - 290} = \frac{140.9}{T_1 - 290}$$

$\therefore \quad T_1 = 461.8 \text{ K}$

and turbine efficiency (η_T),

$$0.86 = \frac{T_2 - T_3}{T_2 - T_3'} = \frac{840 - T_3}{840 - 565.3} = \frac{840 - T_3}{274.7}$$

$\therefore \quad T_3 = 603.8 \text{ K}$

We also know that work done by the turbine per kg of air,

$$W_T = c_p (T_2 - T_3) = 1\,(840 - 603.8) = 236.2 \text{ kJ/s}$$

and work required by the compressor,

$$W_C = c_p (T_1 - T_4) = 1\,(461.8 - 290) = 171.8 \text{ kJ/s}$$

$\therefore$ Net work done by the turbine,

$$W = W_T - W_C = 236.2 - 171.8 = 64.4 \text{ kJ/s}$$

and heat supplied per kg of air

$$= c_p (T_2 - T_1) = 1\,(840 - 461.8) = 378.2 \text{ kJ/s}$$

$\therefore$ Thermal efficiency,

$$\eta_{th} = \frac{\text{Net work done}}{\text{Heat supplied}} = \frac{64.4}{378.2} = 0.17 \text{ or } 17\% \textbf{ Ans.}$$

Thermal efficiency of the plant with heat exchanger

Given. Effectiveness of heat exchanger,

$$\eta_e = 70\% = 0.7$$

We know that heat available in exhaust gases

$$= c_p (T_3 - T_1) = 1\,(603.8 - 461.8) = 142 \text{ kJ/s}$$

$\therefore$ Actual heat recovered from exhaust gases

$$= 0.7 \times 142 = 99.4 \text{ kJ/s}$$

and heat supplied by the combustion chamber per kg of air

$$= 378.2 - 99.4 = 278.8 \text{ kJ/s}$$

$\therefore$ Thermal efficiency with heat exchanger,

$$\eta_{th} = \frac{64.4}{278.8} = 0.231 \text{ or } 23.1\ \% \textbf{ Ans.}$$

Example 33.8. *Determine the efficiency of a gas turbine plant fitted with a heat exchanger of 75% effectiveness. The pressure ratio is 4 : 1 and the compression is carried out in two stages of equal pressure ratio with intercooling back to initial temperature of 290 K. The maximum temperature is 925 K. The turbine isentropic efficiency is 88% and each compressor isentropic efficiency is 85%. For air, $\gamma = 1.4$ and $c_p = 1.005$ kJ/kg K.*

Solution. Given : $\eta_e = 75\% = 0.75$; $r = p_1 / p_4 = p_2 / p_3 = 4$; $T_4 = 290$ K ; $T_2 = 925$ K ; $\eta_T = 88\% = 0.88$; $\eta_C = 85\% = 0.85$; $\gamma = 1.4$; $c_p = 1.005$ kJ/kg K

The *T-s* diagram is shown in Fig. 33.11.

Let $T_1, T_3, T_3', T_5, T_5', T_6$ = Temperature of air at corresponding points.

We know that for equal pressure ratio, the intermediate pressure,

$$p_6 = p_5 = p_5' = \sqrt{p_1 \times p_4} = \sqrt{4 \times 1} = 2 \text{ bar}$$

We also know that

$$\frac{T_3'}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{4}\right)^{\frac{1.4-1}{1.4}} = 0.673$$

$$\therefore \quad T_3' = T_2 \times 0.673 = 925 \times 0.673 = 622.6 \text{ K}$$

and

$$\frac{T_4}{T_5'} = \left(\frac{p_4}{p_5}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{2}\right)^{\frac{1.4-1}{1.4}} = 0.82$$

$$\therefore \quad T_5' = T_4 / 0.82 = 290 / 0.82 = 353.7 \text{ K}$$

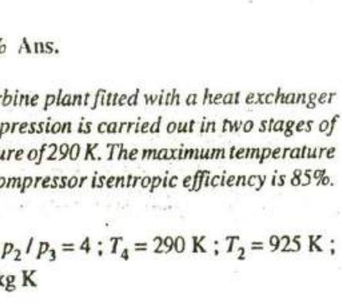


Fig. 33.11

We know that for equal pressure ratio,

$$T_5' = T_1' = 353.7 \text{ K}$$

and

$$T_6 = T_4 = 290 \text{ K}$$

We also know that compressor efficiency (η_C),

$$0.85 = \frac{T_1' - T_6}{T_1 - T_6} = \frac{353.7 - 290}{T_1 - 290} = \frac{63.7}{T_1 - 290}$$

$$\therefore \qquad T_1 = 364.9 \text{ K}$$

and turbine efficiency (η_T), $$0.88 = \frac{T_2 - T_3}{T_2 - T_3'} = \frac{925 - T_3}{925 - 622.6} = \frac{925 - T_3}{302.4}$$

$$\therefore \qquad T_3 = 658.9 \text{ K}$$

We know that for equal pressure ratio,

$$T_5 = T_1 = 364.9 \text{ K}$$

$\therefore$ Work done by the turbine per kg of air,

$$W_T = c_p (T_2 - T_3) = 1.005 (925 - 658.9) = 267.4 \text{ kJ}$$

and work required by the compressor,

$$W_C = c_p [(T_1 - T_6) + (T_5 - T_4)]$$

$$= 1.005 [(364.9 - 290) + (364.9 - 290)] = 150.5 \text{ kJ}$$

$\therefore$ Net work done by the turbine,

$$W = W_T - W_C = 267.4 - 150.5 = 116.9 \text{ kJ}$$

and heat supplied per kg of air $= c_p (T_2 - T_1) = 1.005 (925 - 364.9) = 562.9 \text{ kJ}$

We also know that heat available in exhaust gases

$$= c_p (T_3 - T_1) = 1.005 (658.9 - 364.6) = 295.8 \text{ kJ}$$

$\therefore$ Actual heat recovered from exhaust gases

$$= 0.75 \times 295.8 = 221.8 \text{ kJ}$$

and heat supplied by the combustion chamber per kg of air

$$= 562.9 - 221.8 = 341.1 \text{ kJ}$$

$\therefore$ Efficiency of the turbine plant,

$$\eta_{th} = \frac{116.9}{341.1} = 0.343 \text{ or } 34.3\% \textbf{ Ans.}$$

33.7. Uses of Gas Turbines

Though there are innumerable uses of gas turbines these days, yet the following are important from the subject point of view :

1. *Generation of electric power.* The gas turbines are extensively used in the generation of electric power The largest gas turbine power plant is installed in Switzerland. It consists of two turbines of total capacity 40 MW.

2. *Turbojet and turbo propeller engines.* The gas turbines are used to drive air compressors in turbojets. They are also used to drive air compressor and turbines in turbo propeller engines. The gas turbines in turbojet and turbo propeller engines are operated in the temperature range of 800° C to 1000° C.

3. *Supercharger.* The gas turbines are also used to drive superchargers fitted in the aviation gasoline engines as well as for heavy duty diesel engines.

4. *Marine engines.* The gas turbines are used in marine engines. Unlike steam turbine or steam engines, the gas turbines do not require water storage tanks or distillation plants.

5. *Railway engines.* The gas turbines have also entered the field of railway engines. The first gas turbine locomotive was put into service in 1941 in Switzerland. The turbine developed 5800 kW out of which 4200 kW is consumed to drive the air compressor.

33.8. Recent Trends in Gas Turbines

The earlier gas turbines were designed to work on closed cycle with air as the working fluid. In recent years, lot of research is being done all over the world to improve the working of gas turbines.

The main focus of the research is aimed to generate more power from the plant as well as to effect economy in the installation and maintenance expenses. It has lead to the usage of various working fluids such as carbon dioxide, nitrogen, argon etc. Helium has proved to be the most suitable working fluid because of its lower density and higher (about five times) specific heat at constant pressure. Moreover, it requires about 1/3 surface area for heating in combustion chamber and heat exchanger than the turbine using air as the working fluid for the same temperature range and pressure ratio. The second important aspect of the research is aimed to heat the air in the combustion chambers. The scientists have tried many fuels under variable sets of conditions, but the recent trend is to use the heat generated by nuclear fusion in a reactor.

EXERCISES

1. A gas turbine plant receives air at 1 bar and 20° C. Find the thermal efficiency of the plant, if the compression ratio is 4 and specific heat at constant pressure for the working fluid is 1 kJ/kg K. **[Ans. 32.7%]**

2. Find the actual overall efficiency of a simple combustion turbine, the following data of which are available :

Compression ratio = 5 ; Initial temperature of air = 15° C ; γ for air = 1.4 ; Compressor and turbine efficiency = 85%. Maximum temperature of combustion = 600° C and c_p = 1.005 kJ/kg K. **[Ans. 20.5%]**

3. A gas turbine plant works in temperature limits of 300 K and 900 K and the pressure limits are 1 bar and 4 bar. The internal efficiency of the compressor is 0.8 and that of the turbine is 0.85. Estimate the thermal efficiency of the plant and the power available in kilowatts, if the air consumption is 1 kg/s. The heating value of the fuel is 42 000 kJ/kg. **[Ans. 16.2% ; 71 kW]**

4. In a gas turbine plant comprising a single stage compressor, combustion chamber and turbine, the compressor takes in air at 15°C and compresses it to 4 times the initial pressure with an isentropic efficiency of 85 percent. The fuel-air ratio is 0.0125 and the calorific value of the fuel is 42 000 kJ/kg. If the isentropic efficiency of the turbine is 82 percent, find the overall thermal efficiency and the air intake for a power output of 260 kW.

Take the mass of the fuel into account. The turbine inlet temperature is 1000 K.

[Ans. 21.5% ; 2.28 kg/s]

5. In a gas turbine plant, air at 10° C and atmospheric pressure is compressed through a pressure ratio of 4 : 1. In a heat exchanger and combustion chamber, the air is heated to 700° C. After expansion through the turbine, the air passes through a heat exchanger which cools the air through 75% of the maximum range possible, while the pressure drops 0.14 bar and the air is finally exhausted to atmosphere. The isentropic efficiency of the compressor is 0.80 and that of turbine 0.85. Find the efficiency of the plant. **[Ans. 19%]**

6. A gas turbine takes in air at 27° C and 1 bar. The pressure ratio is 4 and the maximum temperature in the cycle is 560° C. The compressor and turbine efficiencies are 0.83 and 0.85 respectively. Determine the overall efficiency if the regenerator effectiveness is 0.75. **[Ans. 21.2%]**

7. In a gas turbine plant, air is compressed from 1 bar and 15° C through a pressure ratio of 4 : 1. It is then heated to 650° C in a combustion chamber and expanded back to a pressure of 1 bar in a turbine. Calculate the cycle efficiency and work ratio, if a perfect heat exchanger is used. Assume isentropic efficiency of the turbine and compressor as 85% and 80% respectively. **[Ans. 31.9% ; 31.9%]**

8. A gas turbine plant consists of two stage compressor with perfect intercooler and a set of H.P. and L.P. turbines. The exhaust from L.P. turbine passes through a heat exchanger which heats up the air leaving the H.P. compressor. The overall pressure ratio is 10 and temperature range is 20° C to 600° C. Assuming isentropic efficiencies for compressors, turbines and heat exchanger as 80%, 85% and 70% respectively, find the power developed and thermal efficiency of the plant. Take mass of air flow as 1.15 kg/s and 2% work of each turbine is lost in overcoming friction. **[Ans. 144.6 kW ; 25.7%]**

QUESTIONS

1. State the assumptions made for thermal efficiency of a gas turbine plant.

2. Derive an expression for the thermal efficiency of a gas turbine plant, and show that it is independent of the mass of air circulated in it.

3. Describe the difference between an ideal gas turbine plant and an actual gas turbine plant. Give relations for the isentropic efficiencies of compressor and turbine.

4. What is heat exchanger ? Describe its utility.

5. Obtain an expression for the effectiveness of a heat exchanger.

6. Write briefly the uses of gas turbines.

OBJECTIVE TYPE QUESTIONS

1. In an ideal gas turbine plant, it is assumed that the compression and expansion processes are

   (*a*) isothermal (*b*) isentropic (*c*) polytropic

2. The thermal efficiency of an ideal gas turbine plant is given by

   (*a*) $r^{\gamma-1}$ (*b*) $1-r^{\gamma-1}$ (*c*) $1-\left(\frac{1}{r}\right)^{\frac{\gamma}{\gamma-1}}$ (*d*) $1-\left(\frac{1}{r}\right)^{\frac{\gamma-1}{\gamma}}$

3. The gas turbine cycle with regenerator improves

   (*a*) thermal efficiency (*b*) work ratio

   (*c*) avoids pollution (*d*) none of these

4. High air-fuel ratio is gas turbines

   (*a*) increases power output (*b*) improves thermal efficiency

   (*c*) reduces exhaust temperature (*d*) do not damage turbine blades

ANSWERS

1. (*b*) 2. (*d*) 3. (*a*) 4. (*c*)

34

Introduction to Heat Transfer

1. Introduction. 2. Methods of Heat Transfer. 3. Newton's Law of Cooling. 4. Fourier's Law of Heat Conduction. 5. Heat Transfer by Conduction through a Slab. 6. Thermal Conductivity. 7. Temperature Gradient. 8. Heat Transfer by Conduction through a Composite Wall. 9. Radial Heat Transfer by Conduction through a Thick Cylinder. 10. Heat Transfer by Conduction through a Thick Sphere. 11. Overall Coefficient of Heat Transfer.

34.1. Introduction

As a matter of fact, the subject of heat transfer is getting more and more importance in the sphere of science and technology these days. The mechanical engineers deal with the problems of heat transfer in the design of their I.C. engines, refrigeration and air-conditioning system as well as various types of steam generation plants. The electrical engineers deal with the problems of heat transfer in the cooling systems of power generation machines, transformers and various types of huge electrical installations. Similarly, the civil engineers require the knowledge of heat transfer in the design of dams, tunnels, cold storages, cinema halls and other important structures.

The subject of heat transfer is very vast, and its detailed study is beyond the scope of this book. However, its basic principles will be discussed in this book.

34.2. Methods of Heat Transfer

The heat transfer may be broadly defined as the transmission of heat energy from one region to another due to the temperature difference between these two regions.

The following methods, of heat transfer, from one body to another, are important from the subject point of view :

1. *Conduction.* It is a process of heat transfer from one particle of the body to another in the direction of fall of temperature. The particles themselves remain in fixed position relative to each other.

2. *Convection.* It is a process of heat transfer from one particle of the body to another by convection current. In this case, the particles of the body move relative to each other.

3. *Radiation.* It is a process of heat transfer from a hot body to a cold body, in a straight line, without affecting the intervening medium.

In this chapter, we shall discuss the heat transfer by conduction only.

34.3. Newton's Law of Cooling

It is an important law in the field of heat transfer which states, "*Heat transfer from a hot body to a cold body is directly proportional to the surface area and difference of temperatures between the two bodies*".

It is a general law, for the heat transfer which can not be applied to all sets of conditions. But it paved the way for other laws dealing in the heat loss.

34.4. Fourier's* Law of Heat Conduction

It is also an important law in heat conduction, which is represented by the equation,

$$Q \propto A \times \frac{dT}{dx} = k\,A \times \frac{dT}{dx}$$

where Q = Amount of heat flow through the body in a unit time.

A = Surface area of heat flow. It is taken at right angles to the direction of flow.

dT = Temperature difference on the two faces of the body.

dx = Thickness of the body through which the heat flows. It is taken along the direction of heat flow.

k = Constant of proportionality known as *thermal conductivity* of the body

34.5. Heat Transfer by Conduction through a Slab

Consider a solid slab having one of its face (say left) at a higher temperature and the other (say right) at a lower temperature as shown in Fig. 34.1.

Let T_1 = Temperature of the left face (*i.e.* higher temperature) in K,

T_2 = Temperature of the right face (*i.e.* lower temperature) in K,

x = Thickness of the slab,

A = Area of the slab.

k = Thermal conductivity of the body,

t = Time through which the heat flow has taken place.

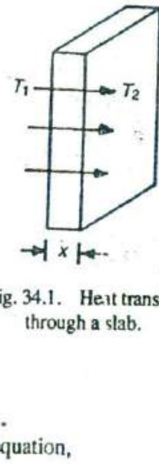


Fig. 34.1. Heat transfer through a slab.

As per the Fourier's law of heat conduction, the heat flow (assuming no loss of heat from the sides) through the slab,

$$Q = k\,A \times \frac{dT}{dx} = \frac{k\,A\,(T_1 - T_2)}{dx}$$

Now the total amount of heat flow in time t may be found out by the equation,

$$Q = \frac{k\,A\,(T_1 - T_2)\,t}{x}$$

Notes : 1. Since the temperature of the slab decreases as x increases, therefore sometimes negative sign is put on the right hand side of the above equation.

2. The rate of heat flow per second is given by the relation,

$$Q = \frac{k\,A\,(T_1 - T_2)}{x}$$

34.6. Thermal Conductivity

We have discussed in the previous article that the amount of heat flow through a body.

$$Q = \frac{k\,A\,(T_1 - T_2)\,t}{x}$$

* The fundamental law of heat conduction was proposed by Biot in 1804. But is could not get proper recognition until 1822, when Fourier confirmed it by his outstanding contribution in the field. After confirmation, the law is popularly known as Fourier's Law (or Fourier's equation) of heat conduction.

In the above equation, if we substitute $A = 1\ m^2$; $(T_1 - T_2) = 1$ K ; $t = 1$ s and $x = 1$ m, then $Q = k$.

It is thus obvious, that the thermal conductivity of a material is numerically equal to the quantity of heat (in joules) which flows in one second through a slab of the material of area 1 m^2 and thickness 1 m when its faces differ in temperature by 1 K. It may also be defined as the quantity of heat in joules that flows in one second through one metre cube of a material when opposite faces are maintained at a temperature difference of 1 K.

Notes : 1. The unit of thermal conductivity depends upon the units of the quantities on the right side of the above equation. In S.I. units, the unit of thermal conductivity is J/m/K/s = W/m K ($\because$ 1 J/s = 1W).

2. The above expression for rate of heat flow may also be written as :

$$Q = \frac{T_1 - T_2}{x / k A}$$

The term $x / k A$ is known as *thermal resistance* and corresponds to resistance 'R' in Ohm's law.

34.7. Temperature Gradient

We know that the rate of heat flow through a body,

$$Q = \frac{k A (T_1 - T_2)}{x}$$

The term $(T_1 - T_2)/x$ is known as *temperature gradient*. If the temperature falls by dT over a small distance dx, then temperature gradient may be written as dT/dx. Hence the quantity of heat flowing in a differential form may be written as :

$$Q = k A \times \frac{dT}{dx}$$

Example 34.1. *The glass windows of a room have a total area of 10 m^2 and the glass is 4 mm thick. Calculate the quantity of heat that escapes from the room by conduction per second when the inside surfaces of windows are at 25° C and the outside surfaces at 10° C. The value of k is 0.84 W/m K.*

Solution. Given : $A = 10\ m^2$; $x = 4$ mm = 0.004 m ; $T_1 = 25°$ C = 298 K ; $T_2 = 10°$ C = 283 K ; $k = 0.84$ W/m K

We know that the quantity of heat that escapes from the room per second,

$$Q = \frac{k A (T_1 - T_2)}{x} = \frac{0.84 \times 10\,(298 - 283)}{0.004} = 31\,500 \text{ J}$$

$$= 31.5 \text{ kJ } \textbf{Ans.}$$

Example 34.2. *A boiler is made of iron plates 12 mm thick. If the temperature of the outside surface be 120° C and that of the inner 100° C, calculate the mass of water evaporated per hour. Assume that the area of heating surface is 5 m^2, and k for iron as 84 W/m K.*

Solution. Given : $x = 12$ mm = 0.012 m ; $T_1 = 120°$ C = 393 K ; $T_2 = 100°$ C = 373 K ; $t = 1$h = 3600 s ; $A = 5\ m^2$; $k = 84$ W/m K

We know that amount of heat transferred,

$$Q = \frac{k A (T_1 - T_2)\, t}{x} = \frac{84 \times 5\,(393 - 373)\,3600}{0.012} = 2520 \times 10^6 \text{ J/h}$$

$$= 2520 \times 10^3 \text{ kJ/h}$$

$\because$ We know that the heat required to evaporate 1 kg of water at 100° C is equal to its latent heat, *i.e.* 2260 kJ.

$\therefore$ Mass of water evaporated per hour

$$= \frac{\text{Total amount of heat transferred}}{\text{Heat reqd. to evaporate 1 kg of water}}$$

$$= \frac{2520 \times 10^3}{2260} = 1115 \text{ kg } \textbf{Ans.}$$

34.8. Heat Transfer by Conduction through a Composite Wall

Consider a composite wall consisting of two different materials through which the heat is being transferred by conduction, as shown in Fig. 34.2.

Let x_1 = Thickness of first material,

k_1 = Thermal conductivity of first material,

x_2, k_2 = Corresponding values for the second material,

T_1, T_3 = Temperatures of the two outer surfaces,

T_2 = Temperature at junction point, and

A = Surface area of the wall.

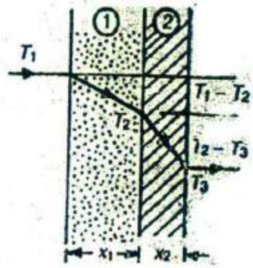


Fig. 34.2. Heat transfer through a composite wall.

Now assuming T_1 to be higher than T_2, the heat will flow from left to right as shown in the figure. Under steady conditions, the rate of heat flow through section 1 is equal to that through section 2. We know that heat flowing through section 1,

$$Q = \frac{k_1 A (T_1 - T_2)}{x_1}$$

or
$$(T_1 - T_2) = \frac{Q}{A} \times \frac{x_1}{k_1} \qquad \ldots (i)$$

Similarly for section 2,

$$(T_2 - T_3) = \frac{Q}{A} \times \frac{x_2}{k_2} \qquad \ldots (ii)$$

Adding equations (*i*) and (*ii*),

$$(T_1 - T_3) = \frac{Q}{A}\left(\frac{x_1}{k_1} + \frac{x_2}{k_2}\right)$$

or
$$Q = \frac{A(T_1 - T_3)}{\frac{x_1}{k_1} + \frac{x_2}{k_2}} = \frac{(T_1 - T_3)}{\frac{x_1}{k_1 A} + \frac{x_2}{k_2 A}} = \frac{(T_1 - T_3)}{\sum \frac{x}{kA}}$$

Notes : 1. We have taken the composite wall consisting of two different materials for simplicity. But this relation may be extended for any number of materials.

2. Now the total heat flow in any time (t) may be found out by the equation :

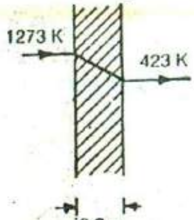
$$Q = \frac{(T_1 - T_3)\, t}{\sum \frac{x}{kA}}$$

3. The term $\Sigma \frac{x}{kA}$ is known as thermal resistance of the wall.

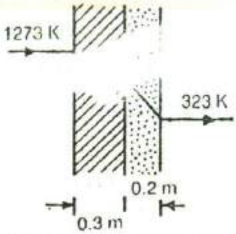
Example 34.3. *A furnace wall is made up of refractory bricks of 300 mm thick. The inner and outer surfaces of the wall have temperature of 1000° C and 150° C. Find the heat loss per square metre per hour.*

If the outside temperature becomes 50° C, the furnace wall is covered with insulating bricks of 200 mm thickness. Find the reduction in heat loss. Take thermal conductivities of refractory and insulating bricks as 4.5 and 0.5 W/m K.

Solution. Given : $x_1 = 300$ mm $= 0.3$ m ; $T_1 = 1000°$ C $= 1273$ K ; $T_2 = 150°$ C $= 423$ K ; $t = 1$ h $= 3600$ s ; $T_3 = 50°$ C $= 323$ K ; $x_2 = 200$ mm $= 0.2$ m ; $k_1 = 4.5$ W/m K ; $k_2 = 0.5$ W/mK



(*a*) Only refractory bricks.



(*b*) Lined with insulating bricks.

Fig. 34.3

Heat loss when the furnace wall is made of refractory bricks

We know that heat loss per square metre per hour,

$$Q_1 = \frac{k_1 A\,(T_1 - T_2)\,t}{x_1} = \frac{4.5 \times 1\,(1273 - 423)\,3600}{0.3}\ \text{J/m}^2\,\text{h}$$

$$= 45.9 \times 10^6\ \text{J/m}^2\,\text{h} = 45.9\ \text{MJ/m}^2\,\text{h}\ \textbf{Ans.}$$

Reduction in heat loss when the furnace wall is covered with insulating bricks

We know that heat loss per square metre per hour,

$$Q_2 = \frac{A\,(T_1 - T_3)\,t}{\dfrac{x_1}{k_1} + \dfrac{x_2}{k_2}} = \frac{1\,(1273 - 323)\,3600}{\dfrac{0.3}{4.5} + \dfrac{0.2}{0.5}}\ \text{J/m}^2\,\text{h}$$

$$= 7.3 \times 10^6\ \text{J/m}^2\,\text{h} = 7.3\ \text{MJ/m}^2\text{h}$$

$\therefore$ Reduction in heat loss $= Q_1 - Q_2 = 45.9 - 7.3 = 38.6$ MJ/m^{2}h **Ans.**

Example 34.4. *Heat is conducted through a compound plate composed of two parallel plates of different materials A and B of conductivities 134 W/mK and 60 W/mK and each of thickness 36 and 42 mm respectively. If the temperature of the outer face of the slab A and that of B are found to be steady at 96° C and 8° C respectively, find the temperature of the interface A/B.*

Solution. Given : $k_1 = 134$ W/mK ; $k_2 = 60$ W/mK ; $x_1 = 36$ mm $= 0.036$ m ; $x_2 = 42$ mm $= 0.042$ m ; $T_1 = 96°$ C $= 369$ K ; $T_3 = 8°$ C $= 281$ K

Let T_2 = Temperature of the interface *A*/*B*, and

A = Area of plates *A* and *B*.

We know that the rate of heat flow through the compound plate,

$$Q = \frac{k\,A\,(T_1 - T_2)}{x}$$

$$\therefore \qquad Q_A = \frac{134\,A\,(369 - T_2)}{0.036} \qquad \ldots (i)$$

Similarly,
$$Q_B = \frac{60\,A\,(T_2 - 281)}{0.042} \qquad \ldots (ii)$$

We know that under steady conditions, the rate of heat flow through A and B is same, therefore equating (i) and (ii),

$$\frac{134\,A\,(369 - T_2)}{0.036} = \frac{60\,A\,(T_2 - 281)}{0.042}$$

$$3722\,(369 - T_2) = 1429\,(T_2 - 281)$$

$$2.6\,(369 - T_2) = T_2 - 281$$

$$\therefore \qquad T_2 = 344.5\text{ K} = 71.5^\circ\text{ C} \quad \textbf{Ans.}$$

Example 34.5. *The walls of a room consist of parallel layers in contact of cement, brick and wood of thickness 20 mm, 300 mm and 10 mm respectively. Find the quantity of heat that passes through each m^2 of wall per minute, if the temperature of air in contact with the wall is 5° C and 30° C inside. The values of k for cement, brick and wood are 0.294, 0.252 and 0.168 W/mK respectively.*

Solution. Given : $x_1 = 20$ mm $= 0.02$ m ; $x_2 = 300$ mm $= 0.3$ m ; $x_3 = 10$ mm $= 0.01$ m ; $T_1 = 30^\circ$ C $= 303$ K ; $T_2 = 5^\circ$ C $= 278$ K ; $k_1 = 0.294$ W/mK ; $k_2 = 0.252$ W/m K ; $k_3 = 0.168$ W/m K ; $A = 1\text{ m}^2$; $t = 1$ min $= 60$ s

We know that heat passing through the wall,

$$Q = \frac{A\,(T_1 - T_2)\,t}{\dfrac{x_1}{k_1} + \dfrac{x_2}{k_2} + \dfrac{x_3}{k_3}} = \frac{1\,(303 - 278)\,60}{\dfrac{0.02}{0.294} + \dfrac{0.3}{0.252} + \dfrac{0.01}{0.168}}$$

$$= \frac{1500}{0.068 + 1.19 + 0.059} = 1139\text{ J} \quad \textbf{Ans.}$$

34.9. Radial Heat Transfer by Conduction through a Thick Cylinder

The heat transfer through boiler tubes or refrigerator pipings are the examples of conduction of heat transferred radially through the walls of hollow thick cylindrical pipes.

Consider a thick pipe of length l carrying steam or a hot liquid at a higher temperature as shown in Fig 34.4.

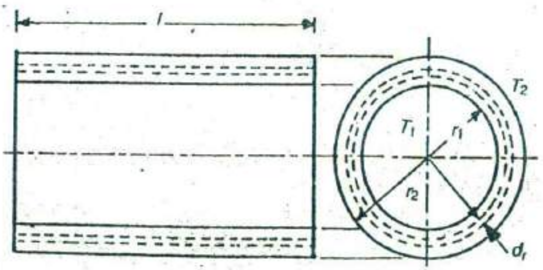


Fig. 34.4. Heat transfer through a thick cylinder.

Let $\qquad T_1$ = Inside (higher) temperatureof liquid,

T_2 = Outside (lower) temperatureof the surroundings,

r_1 = Inside diameter of the pipe, and

r_2 = Outside diameter of the pipe.

$\therefore \quad (r_2 - r_1)$ = Thickness of the pipe.

This thick pipe may be imagined to consist of a large number of thin concentric cylinders of increasing radii. Now consider any thin imaginary cylinder of infinitesimal thickness (dr) at a distance (r) from the axis of the pipe as shown in Fig. 34.4. Let the temperature drop across the thickness be dT.

We know that the surface area of this imaginary cylinder,

$$A = 2\pi rl$$

and heat conduction through this elementary cylinder,

$$Q = kA\left(\frac{-dT^*}{dr}\right) = -k \times 2\pi rl\left(\frac{dT}{dr}\right)$$

or

$$\frac{dr}{r} = \left(\frac{-2\pi lk}{Q}\right)dT$$

Integrating tne above expression,

$$\int_{r_1}^{r_2}\frac{dr}{r} = \frac{-2\pi lk}{Q}\int_{T_1}^{T_2}dT$$

$$\left[\log_e r\right]_{r_1}^{r_2} = \frac{-2\pi lk}{Q}\left[T\right]_{T_1}^{T_2}$$

$$\therefore \quad \log_e\left(\frac{r_2}{r_1}\right) = \frac{-2\pi lk}{Q}(T_2 - T_1) = \frac{2\pi lk}{Q}(T_1 - T_2)$$

or

$$Q = \frac{2\pi lk\,(T_1 - T_2)}{\log_e\left(\frac{r_2}{r_1}\right)} = \frac{2\pi lk\,(T_1 - T_2)}{2.3\log\left(\frac{r_2}{r_1}\right)}$$

Notes : 1. The above relation also holds good when the outside temperature is higher than the inside temperature and the heat flows from outside to inside of the pipe.

2. In case of a composite cylinder, the heat transfer,

$$Q = \frac{2\pi l\,(T_1 - T_2)}{\sum \frac{2.3}{k}\log\left(\frac{r_2}{r_1}\right)}$$

Example 34.6. *A metal pipe having an external diameter of 150 mm carries steam at 200° C. The pipe is covered by a layer 25 mm thick of an insulating material whose conductivity is 0.21 W/mK. If the outer surface is at 100° C, find the amount of heat lost per metre length per minute.*

Solution. Given : $d_1 = 150$ mm or $r_1 = 75$ mm = 0.075 m ; $T_1 = 200°$ C = 473 K ; Thickness of insulating material = 25 mm = 0.025 m ; $k = 0.21$ W/mK ; $T_2 = 100°$ C = 373 K ; $l = 1$ m ; $t = 1$ min = 60 s

* The negative sign indicates that as the distance (r) increases from the centre outwards, the temperature decreases.

We know that amount of heat lost per minute,

$$Q = \frac{2\pi lk\,(T_1 - T_2)\,t}{2.3 \log\left(\dfrac{r_2}{r_1}\right)} = \frac{2\pi \times 1 \times 0.21\,(473 - 373)\,60}{2.3 \log\left(\dfrac{0.075 + 0.025}{0.075}\right)} \text{ J}$$

$$= 27\,600 \text{ J} = 27.6 \text{ kJ} \textbf{ Ans.} \qquad \ldots (\because r_2 = r_1 + 0.025)$$

Example 34.7. *Water is pumped through an iron pipe (k = 67.2 W/mK), 2 metres long at the rate of 1000 kg/min. The inner and outer diameters of the tube are 50 mm and 60 mm respectively. Calculate the rise in temperature of water when the outside of the tube is heated to a temperature of 600° C. The initial temperature of the water is 30° C.*

Solution. Given : $k = 67.2$ W/mK ; $l = 2$ m ; $m = 1000$ kg/min ; $d_1 = 50$ mm or $r_1 = 25$ mm $= 0.025$ m ; $d_2 = 60$ mm or $r_2 = 30$ mm $= 0.03$ m ; $T_1 = 600°$ C $= 873$ K ; $T_{w1} = 30°$ C $= 303$ K

Let T_{w2} = Final temperature of water in K.

We know that heat transferred through the tube per second,

$$Q = \text{Mass} \times \text{Sp. heat} \times \text{Rise in temp.}$$

$$= \frac{1000 \times 4.2\,(T_{w2} - 303)}{60} = 70\,(T_{w2} - 303) \text{ kJ} \qquad \ldots (i)$$

We also know that

$$Q = \frac{2\pi lk\,(T_1 - T_2)\,t}{2.3 \log\left(\dfrac{r_2}{r_1}\right)} = \frac{2\pi \times 2 \times 67.2\left[873 - \dfrac{(303 + T_{w2})}{2}\right]}{2.3 \log\left(\dfrac{0.03}{0.025}\right)} \text{ J}$$

$$\ldots (\because t = 1 \text{ s})$$

$$= 4640\left[873 - \frac{(303 + T_{w2})}{2}\right] = 2320\,(1443 - T_{w2}) \text{ J}$$

$$= 2.32\,(1443 - T_{w2}) \text{ kJ} \qquad \ldots (ii)$$

Equating equations (*i*) and (*ii*),

$$70\,(T_{w2} - 303) = 2.32\,(1443 - T_{w2})$$

or

$$T_{w2} = 339.6 \text{ K} = 66.6° \text{C}$$

$\therefore$ Rise in temperature $= T_{w2} - T_{w1} = 66.6 - 30 = 36.6°$ C **Ans.**

Example 34.8. *A steam pipe 20 m long, 100 mm internal diameter and 40 mm thick is covered by a layer of lagging of 25 mm thick. The coefficient of thermal conductivities for the pipe material and lagging are 0.07 W/m K and 0.1 W/mK respectively. If the steam is conveyed at a pressure of 17 bar with 30° C superheat and the outside temperature of the lagging is 24° C, determine : 1. the heat lost per hour ; and 2. the interface temperature. Neglect the pressure drop across the steam pipe.*

Solution. Given : $l = 20$ m ; $d_1 = 100$ mm or $r_1 = 50$ mm $= 0.05$ m ; $r_2 = r_1$ + Pipe thickness $= 50 + 40 = 90$ mm $= 0.09$ m ; $r_3 = r_2$ + Lagging thickness $= 90 + 25 = 115$ mm $= 0.115$ m ; $k_1 = 0.07$ W/mK ; $k_2 = 0.1$ W/mK ; $p = 17$ bar ; Degree of superheat $= 30°$ C ; $T_3 = 24°$ C $= 297$ K

1. *Total heat lost per hour*

From steam tables, we find that the temperature of steam corresponding to pressure of 17 bar is 204.3° C. Therefore, temperature of steam,

$$T_1 = 204.3° + 30° = 234.3° \text{C} = 507.3 \text{ K}$$

We know that total heat lost per second,

$$Q = \frac{2\pi l\,(T_1 - T_3)}{\frac{1}{k_1} \times 2.3 \log\left(\frac{r_2}{r_1}\right) + \frac{1}{k_2} \times 2.3 \log\left(\frac{r_3}{r_2}\right)}$$

$$= \frac{2\pi \times 20\,(507.3 - 297)}{\left[\frac{1}{0.07} \times 2.3 \log\left(\frac{0.09}{0.05}\right)\right] + \left[\frac{1}{0.1} \times 2.3 \log\left(\frac{0.115}{0.09}\right)\right]}$$

$$= \frac{2\pi \times 20 \times 210.3}{8.39 + 2.45} = 2438 \text{ J/s}$$

$\therefore$ Heat lost per hour $= 2438 \times 3600 = 8\,776\,800 \text{ J/h} = 8776.8 \text{ kJ/h}$ **Ans.**

2. *Interface temperature*

Let T_2 = Interface temperature in K.

Now we shall consider, the flow of heat (*i.e.* 2438 J/s) through the steam pipe*. We know that

$$Q = \frac{2\pi\, lk\,(T_1 - T_2)}{2.3 \log\left(\frac{r_2}{r_1}\right)}$$

$$2438 = \frac{2\pi \times 20 \times 0.07\,(T_1 - T_2)}{2.3 \log\left(\frac{0.09}{0.05}\right)} = \frac{8.8\,(507.3 - T_2)}{0.587}$$

$$1431 = 4464 - 8.8\,T_2 \quad \text{or} \quad T_2 = 344.6 \text{ K} = 71.6^\circ \text{ C} \textbf{ Ans.}$$

34.10. Heat Transfer by Conduction through a Thick Sphere

Consider a hollow thick spherical shell containing liquid at a higher temperature as shown in Fig. 34.5.

Let T_1 = Inside (higher) temperature of the liquid,

T_2 = Outside (lower) temperature of the surroundings,

r_1 = Inside diameter of the sphere, and

r_2 = Outside diameter of the sphere.

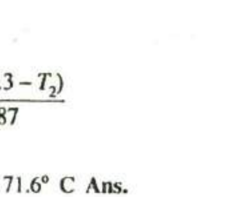


Fig. 34.5. Heat transfer through a thick sphere.

This thick sphere may be imagined to consist of a large number of thin concentric spheres of increasing radii. Now consider any thin imaginary sphere of infinitesimal thickness (dr) at a distance (r) from the centre of the sphere. Let the temperature across the thickness be dT.

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* It may also be found by considering the flow of heat through the pipe lagging as discussed below :

$$Q = \frac{2\pi lk\,(T_2 - T_3)}{2.3 \log\left(\frac{r_3}{r_2}\right)}$$

$$2438 = \frac{2\pi \times 20 \times 0.1\,(T_2 - 297)}{2.3 \log\left(\frac{0.115}{0.09}\right)} = \frac{12.57\,(T_2 - 297)}{0.245}$$

$$597.3 = 12.57\,T_2 - 3733 \quad \text{or} \quad T_2 = 344.6 \text{ K} = 71.6^\circ \text{C} \textbf{ Ans.}$$

We know that surface area of the imaginary sphere,

$$A = 4\pi r^2$$

and heat conduction through the imaginary sphere,

$$Q = kA\left(\frac{-dT}{dr}\right)^{*} = -k \times 4\pi r^2\left(\frac{dT}{dr}\right)$$

$$\frac{dr}{r^2} = \left(\frac{-4\pi k}{Q}\right)dT$$

Integrating the above expression,

$$\int_{r_1}^{r_2}\frac{dr}{r^2} = \frac{-4\pi k}{Q}\int_{T_1}^{T_2} dT$$

$$\left[\frac{r^{-1}}{-1}\right]_{r_1}^{r_2} = \frac{-4\pi k}{Q}[T]_{T_1}^{T_2}$$

$$\frac{1}{r_1} - \frac{1}{r_2} = \frac{-4\pi k}{Q}(T_2 - T_1)$$

$$\frac{r_2 - r_1}{r_1 r_2} = \frac{4\pi k}{Q}(T_1 - T_2)$$

$$\therefore \qquad Q = \frac{4\pi k\, r_1 r_2 (T_1 - T_2)}{(r_2 - r_1)}$$

Notes : 1. The above relation also holds good when the outside temperature is higher than the inside temperature, and the heat flows from outside to inside of the sphere.

2. In case of a composite sphere, the heat transfer,

$$Q = \frac{4\pi (T_1 - T_2)}{\sum \frac{r_2 - r_1}{k_1 r_1 r_2}}$$

Example. 34.9. *A spherical shaped vessel of 1 m outside diameter is 100 mm thick. Find the rate of heat leakage, if the temperature difference between the inner and outer surface is 150 K. Take k for the vessel material as 0.2 kJ/mh K.*

Solution. Given : $d_2 = 1$ m or $r_2 = 0.5$ m ; $r_1 = r_2 -$ Thickness $= 0.5 - 0.1 = 0.4$ m ; $T_1 - T_2 = 150$ K ; $k = 0.2$ kJ/mh K

We know that rate of heat leakage,

$$Q = \frac{4\pi k r_1 r_2 (T_1 - T_2)}{(r_2 - r_1)} = \frac{4\pi \times 0.2 \times 0.4 \times 0.5 \times 150}{(0.5 - 0.4)} = 754 \text{ kJ/h} \quad \textbf{Ans.}$$

Example 34.10. *An air-conditioned hall has one of its walls 0.7 m thick. The inside space is required to be maintained at 20° C. The thermal conductivity of the wall is variable, and is given by the relation :*

$$k = 0.93 + 1.163 \times 10^{-4}\, T^2$$

* The negative sign indicates that as the distance (r) increases from the centre outwards, the temperature decreases.

where T is in degrees centigrade and k in W/m °C. Find the heat lost per hour through one of its walls 10 m × 5 m when the outside temperature is 40° C.

Solution. Given : $x = 0.7$ m ; $T_2 = 20°$ C ; $k = 0.93 + 1.163 \times 10^{-4} T^2$; $A = 10 \times 5 = 50$ m^2 ; $T_1 = 40°$ C

We know that heat lost,

$$Q = -kA \times \frac{dT}{dx}$$

$$\therefore \qquad Q\,dx = -kA\,dT = -(0.93 + 1.163 \times 10^{-4} T^2)\,50 \times dT$$

Integrating this expression for the wall thickness,

$$Q\int_0^{0.7} dx = -50\int_{T_1}^{T_2} (0.93 + 1.163 \times 10^{-4} T^2)\,dT$$

$$Q \times 0.7 = -50\left[0.93\,T + \frac{1.163 \times 10^{-4} \times T^3}{3}\right]_{T_1}^{T_2}$$

$$= -50\left[0.93\,(T_2 - T_1) + \frac{1.163 \times 10^{-4}}{3}(T_2^3 - T_1^3)\right]$$

$$= -50\left[0.93\,(20 - 40) + \frac{1.163 \times 10^{-4}}{3}(20^3 - 40^3)\right]$$

$$= -50\,[-18.6 - 2.17] = 1038.5$$

$$\therefore \qquad Q = 1038.5 / 0.7 = 1483.6 \text{ J/s } \textbf{Ans.}$$

34.11. Overall Coefficient of Heat Transfer

In the previous articles, we have discussed only the theoretical cases of heat transfer through conduction. But in actual practice, the heat from a hot body is transferred to the cold body by the combined effect of conduction and convection.

Consider a wall through which heat is transferred from a hot surface to a cold surface as shown in Fig. 34.6.

Let $(T_1 - T_2)$ = Difference of temperatures,

A = Surface area of the wall,

x = Thickness of the wall, and

k = Thermal conductivity of the wall material.

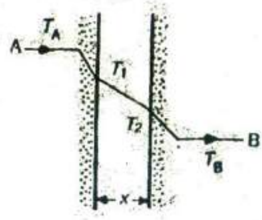

Fig. 34.6. Overall coefficient of heat transfer.

As a matter of fact, there will be a thin film of air on both the hot as well as cold faces of the wall, which will act as transition layers adjacent to the wall surface, and through which the heat also has to flow in addition to the wall as shown in Fig. 34.6. Let A and B be the effective film of air for the heat flow.

Let T_A and T_B = Temperatures at ends of two thin films of air A and B respectively.

h_A and h_B = Coefficients of heat transfer for A and B respectively.

U = Overall coefficient of heat transfer.

We know that the rate of heat flow through air film A,

$$Q = h_A A (T_A - T_1) \text{ or } (T_A - T_1) = \frac{Q}{h_A A} \qquad \ldots (i)$$

Similarly, rate of heat flow through the wall,

$$Q = \frac{kA (T_1 - T_2)}{x} \text{ or } (T_1 - T_2) = \frac{Q x}{k A} \qquad \ldots (ii)$$

and rate of heat flow through the film B,

$$Q = h_B A (T_2 - T_B) \text{ or } (T_2 - T_B) = \frac{Q}{h_B A} \qquad \ldots (iii)$$

Adding equations (*i*), (*ii*) and (*iii*), we get

$$(T_A - T_B) = \frac{Q}{A}\left[\frac{1}{h_A} + \frac{x}{k} + \frac{1}{h_B}\right]$$

$$Q = \frac{A (T_A - T_B)}{\left(\frac{1}{h_A} + \frac{x}{k} + \frac{1}{h_B}\right)} \qquad \ldots (iv)$$

We know that the rate of heat flow,

$$Q = UA (T_A - T_B) \qquad \ldots (v)$$

Now equating the equations (*iv*) and (*v*),

$$U A (T_A - T_B) = \frac{A (T_A - T_B)}{\left(\frac{1}{h_A} + \frac{x}{k} + \frac{1}{h_B}\right)}$$

$$\therefore \qquad U = \frac{1}{\left(\frac{1}{h_A} + \frac{x}{k} + \frac{1}{h_B}\right)}$$

Notes : 1. For the sake of simplicity, we have considered a simple wall of one material only. But this relation may be extended to all the cases discussed earlier, *e.g.* for a composite wall,

$$U = \frac{1}{\left(\frac{1}{h_A} + \frac{x_1}{k_1} + \frac{x_2}{k_2} + \ldots + \frac{1}{h_B}\right)}$$

Similarly, for a simple cylinder,

$$U = \frac{1}{\left\{\frac{1}{r_1 h_A} + 2.3 \log\left(\frac{r_2}{r_1}\right) + \frac{1}{r_2 h_B}\right\}}$$

2. If ambient temperature (*i.e.* atmospheric temperature) is existing on one side of the wall or cylinder, then thickness of air film is zero on that side.

Example 34.11. *A furnace wall 200 mm thick is made of a material having thermal conductivity of 1.45 W/m² K. The inner and outer surfaces are exposed to average temperatures of 350° C and 40° C respectively. If the gas and air film coefficients are 58 and 11.63 W/m² K respectively, find the rate of heat transfer through a wall of 2.5 square metre. Also, find the temperatures on the two sides of the wall.*

Solution. Given : $x = 200$ mm $= 0.2$ m ; $k = 1.45$ W/m^2 K ; $T_A = 350°$ C $= 623$ K ; $T_B =$ 4° C $= 313$ K ; $h_A = 58$ W/m^2 K ; $h_B = 11.63$ W/m^2 K ; $A = 2.5$ m^2

Rate of heat transfer

We know that rate of heat transfer,

$$Q = \frac{A\,(T_A - T_B)}{\frac{1}{h_A} + \frac{x}{k} + \frac{1}{h_B}} = \frac{2.5\,(623 - 313)}{\frac{1}{58} + \frac{0.2}{1.45} + \frac{1}{11.63}} = \frac{775}{0.017 + 0.138 + 0.086} \text{ J/s}$$

$$= 3216 \text{ J/s} \quad \textbf{Ans.}$$

Temperature on two sides of the wall

Let T_1 = Temperature on the inner side of the wall, and

T_2 = Temperature on the outside of the wall.

We know that the rate of heat transfer through the gas film, wall and air film is the same. So we shall first consider the flow of heat through the gas film and then through the air film.

We know that $Q = h_A\,A\,(T_A - T_1)$

$\therefore$ $3216 = 58 \times 2.5\,(623 - T_1) = 145\,(623 - T_1)$

$623 - T_1 = 3216/145 = 22.2$ or $T_1 = 600.8 \text{ K} = 327.8° \text{C}$ **Ans.**

Similarly $Q = h_B\,A\,(T_2 - T_B)$

$3216 = 11.63 \times 2.5\,(T_2 - 313) = 29\,(T_2 - 313)$

$\therefore$ $T_2 = \frac{3216}{29} + 313 = 423.9 \text{ K} = 150.9° \text{C}$ **Ans.**

Example 34.12. *A composite wall is made up of brickwork, fiberglass and insulating board with thickness and thermal conductivities as given below :*

Brick work	*110 mm thick*	*1.15 W/mK*
Fiberglass	*75 mm thick*	*0.04 W/mK*
Board	*25 mm thick*	*0.06 W/mK*

Find the overall coefficient of heat transfer, if the coefficient of heat transfer for the outside and inside walls are 3.1 W/m^2 K and 2.5 W/m^2 K respectively. Also find the heat lost per hour through such a wall of 20 square metre, when the temperature difference is 27 K.

Solution. Given : For brick work : $x_1 = 110$ mm $= 0.11$ m ; $k_1 = 1.15$ W/mK ; For fibre glass : $x_2 = 75$ mm $= 0.075$ m ; $k_2 = 0.04$ W/m K ; For board : $x_3 = 25$ mm $= 0.025$ m ; $k_3 = 0.06$ W/mK ; $h_A = 3.1$ W/m^2 K ; $h_B = 2.5$ W/m^2 K ; $A = 20$ m^2 ; $T_A - T_B = 27$ K

Overall coefficient of heat transfer

We know that overall coefficient of heat transfer,

$$U = \frac{1}{\left(\frac{1}{h_A} + \frac{x_1}{k_1} + \frac{x_2}{k_2} + \frac{x_3}{k_3} + \frac{1}{h_B}\right)}$$

$$= \frac{1}{\left(\frac{1}{3.1} + \frac{0.11}{1.15} + \frac{0.075}{0.04} + \frac{0.025}{0.06} + \frac{1}{2.5}\right)} = 0.322 \text{ W/m}^2\text{ K} \quad \textbf{Ans.}$$

Heat loss per hour through the wall

We know that heat loss per hour through the wall,

$$Q = UA\,(T_A - T_B)$$

$$= 0.322 \times 20 \times 27 = 173.9 \text{ J/s} \quad \textbf{Ans.}$$

Example 34.13. *A 200 mm diameter steel pipe, conveying saturated steam at a pressure of 9.8 bar, is covered by a layer of lagging material of thickness 60 mm and thermal conductivity 0.116 W/mK. If the heat transfer coefficient of steam is 9.3 W/m² K and ambient temperature is 25° C, find the loss of heat per metre length of the pipe.*

Solution. Given : $d_1 = 200$ mm or $r_1 = 100$ mm = 0.1 m ; $p = 9.8$ bar ; $r_2 = r_1$ + Lagging thickness = 100 + 60 = 160 mm = 0.16 m ; $k = 0.116$ W/m K ; $h = 9.3$ W/m² K ; $T_B = 25°$ C = 298 K

From steam tables, we find that the temperature of steam at 9.8 bar,

$$t_A = 179° \text{ C} = 452 \text{ K}$$

We know that loss of heat per metre length of the pipe,

$$Q = \frac{2\pi\, l\,(T_A - T_B)}{\left[\dfrac{2.3}{k}\log\left(\dfrac{r_2}{r_1}\right) + \dfrac{1}{r_2 h}\right]}$$

$$= \frac{2\pi \times 1\,(452 - 298)}{\left[\dfrac{2.3}{0.116}\log\left(\dfrac{0.16}{0.10}\right) + \dfrac{1}{0.16 \times 9.3}\right]} \text{ J/s}$$

$$= \frac{967.74}{4.05 + 0.672} = 205 \text{ J/s} \quad \textbf{Ans.}$$

Note : In this example, thickness and thermal conductivity of pipe as well as heat transfer coefficient of steam is not given. So the example has to be solved ignoring these values.

EXERCISES

1. The temperatures inside and outside of a room are 5° C and 18° C respectively. Calculate the rate at which heat is being transferred through a glass window of area 1 m² and thickness 5 mm. Take k for glass as 0.84 W/mK. **[Ans. 2.18 kJ/s]**

2. The glass windows of a room have total area of 10 square metres, while thickness is 4 mm. Find the rate at which the heat escapes from the room per second by conduction when the temperature difference is 20° C. Take k for glass as 3 W/mK. **[Ans. 41.67 kJ/s]**

3. An iron boiler 12.5 mm thick contains water at atmospheric pressure. The heated surface is 2 m² in area and the temperature of the inner side is 120° C. If the thermal conductivity of iron is 84 W/mK and the latent heat of evaporation of water is 22.5 J, find the mass of water evaporated. **[Ans. 370 kg]**

4. An aluminium plate ($k = 193.2$) and a gold plate ($k = 294$) are joined with faces in contact. The outer face of the aluminium plate is kept at a temperature of 100° C and the gold face is at 60° C. Compare the thickness required, if the temperature drop through aluminium plate is three times that of gold plate. The surface areas of the two plates is same. **[Ans. 197 : 1]**

5. A tube has internal radius 20 mm and external radius 25 mm. The inside of the tube is maintained at 100° C and the outside at 20° C. Calculate the quantity of heat conducted through unit length of the tube per second. Take the value of k as 380 W/mK **[Ans. 8.6 kJ]**

6. A spherical storage vessel of 3 m internal diameter and 150 mm thick has a liquid at a temperature of 120° C. If the thermal conductivity of the vessel material is 10.5 W/m² K, find the heat transfer rate.

[Ans. 2395 J/h]

7. A furnace wall is built up with 200 mm thick refractory bricks and 150 mm insulating bricks. The temperature of the surrounding is 40° C, whereas that inside the furnace is 1000° C. The thermal conductivities of the refractory bricks and insulating bricks are 5 W/m K and 0.5 W/m K respectively. If the coefficients of heat transfer for the furnace gas and air is 80 and 40 W/m^2 K, determine the rate of heat flow per square metre.

[Ans. 2543 kJ/h]

QUESTIONS

1. What are the three methods of heat transfer ?
2. How do you define the thermal conductivity of a material ?
3. Deduce an expression for the quantity of heat flow through solid bodies when joined together.
4. Derive an expression for the quantity of heat flow through a thick cylindrical pipe.
5. Define the overall coefficient of heat transfer.

OBJECTIVE TYPE QUESTIONS

1. The process of heat transfer from one particle of the body to another is called conduction, when the particles of the body

   (*a*) move actually (*b*) do not move actually

   (*c*) affect the intervening medium (*d*) does not affect the intervening medium

2. The heat transfer takes place according to

   (*a*) zeroth law of thermodynamics (*b*) first law of thermodynamics

   (*c*) second law of thermodynamics (*d*) Stefan's law

3. The rate of heat flow through a body is $Q = \dfrac{k\,A\,(T_1 - T_2)}{x}$. The term $x/k\,A$ is known as

   (*a*) thermal coefficient (*b*) thermal resistance

   (*c*) thermal conductivity (*d*) none of these

4. The thermal conductivity of sold metals ..... with rise in temperature.

   (*a*) remains same (*b*) decreases (*c*) increases

5. The overall coefficient of heat transfer is used in problems of

   (*a*) conduction (*b*) convection

   (*c*) radiation (*d*) conduction and convection

   (*e*) conduction and radiation.

ANSWERS

1. (*b*) 2. (*c*) 3. (*b*) 4. (*b*) 5. (*d*)

35

Air Refrigeration Cycles

1. Introduction. 2. Air Refrigeration Cycle. 3. Units of Refrigeration. 4. Coefficient of Performance of a Refrigerator. 5. Difference between a Heat Engine, Refrigerator and Heat Pump. 6. Open Air Refrigeration Cycle. 7. Closed or Dense Air Refrigeration Cycle. 8. Air Refrigerator Working on Reversed Carnot Cycle. 9. Temperature Limitations for Reversed Carnot Cycle. 10. Air Refrigerator Working on a Bell-Coleman Cycle (or Reversed Joule or Brayton Cycle).

35.1. Introduction

The term 'refrigeration' in a broad sense is used for the process of removing heat (*i.e.* cooling) from a substance. It also includes the process of reducing and maintaining the temperature of a body below the general temperature of its surroundings. In other words, the refrigeration means a continued extraction of heat from a body, whose temperature is already below the temperature of its surroundings.

For example, if some space (say in cold storage) is to be kept at $-2°$ C, we must continuously extract heat which flows into it due to leakage through the walls and also the heat, which is brought into it with the articles stored after the temperature is once reduced to $-2°$ C. Thus in a refrigerator, heat is virtually being pumped from a lower temperature to a higher temperature. According to second* law of thermodynamics, this process can only be performed with the aid of some external work. It is thus obvious, that supply of power (say electric motor) is regularly required to drive a refrigerator. Theoretically, the refrigerator is a reversed heat engine. or a heat pump which pumps heat from a cold body and delivers it to a hot body. The substance which works in a heat pump to extract heat from a cold body and to deliver it to a hot body is called *refrigerant.*

The refrigeration system is known to the man, since the middle of nineteenth century. The scientists, of the time, developed a few stray machines to achieve some pleasure. But it paved the way by inviting the attention of scientists for proper studies and research. They were able to build a reasonably reliable machine by the end of nineteenth century for the refrigeration jobs. But with the advent of efficient rotary compressors and gas turbines, the science of refrigeration reached its present height. Today, it is used for the manufacture of ice and other similar products. It is also widely used for the cooling of storage chambers in which perishable food, drinks and medicines are stored. The refrigeration also has wide applications in sub-marine ships, rockets and aircrafts.

35.2. Air Refrigeration Cycle

In an air refrigeration cycle, the air is used as a refrigerant. In olden days, air was widely used in commercial applications because of its availability at free of cost. Since air does not change its phase *i.e.* remains gaseous throughout the cycle, therefore its heat carrying capacity per kg of air is

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* Second law of thermodynamics states, "*It is impossible for a self-acting machine, working in a cyclic process, to transfer heat from a body at a lower temperature to a body at a higher temperature, without the aid of an external agency.*" For more details, please refer Chapter 1.

very small as compared to vapour absorbing systems. The air-cycle refrigeration systems, as originally designed and installed, are now practically obsolete because of their low coefficient of performance and high power requirements. However, this system continues to be favoured for aircraft refrigeration because of the low weight and volume of the equipment. The basic elements of an air cycle refrigeration system are the compressor, the cooler or heat exchanger, the expander and the refrigerator.

Before discussing the air refrigeration cycles, we should first know about the unit of refrigeration, coefficient of performance of a refrigerator and the difference between the heat engine, a refrigerator and a heat pump.

35.3. Units of Refrigeration

The practical unit of refrigeration is expressed in terms of 'tonne of refrigeration' (briefly written as TR). A tonne of refrigeration is defined as the amount of refrigeration effect produced by the uniform melting of one tonne (1000 kg) of ice from and at 0° C in 24 hours. Since the latent heat of ice is 335 kJ/kg, therefore one tonne of refrigeration,

$$1 \text{ TR} = 1000 \times 335 \text{ kJ in 24 hours}$$

$$= \frac{1000 \times 335}{24 \times 60} = 232.6 \text{ kJ/min}$$

In actual practice, one tonne of refrigeration is taken as equivalent to 210 kJ/min or 3.5 kW (*i.e.* 3.5 kJ/s).

35.4. Coefficient of Performance of a Refrigerator

The coefficient of performance (briefly written as C.O.P.) is the ratio of heat extracted in the refrigerator to the work done on the refrigerant. It is also known as theoretical coefficient of performance. Mathematically,

$$\text{Theoretical C.O.P.} = \frac{Q}{W}$$

where Q = Amount of heat extracted in the refrigerator (or the amount of refrigeration effect produced, or the capacity of a refrigerator), and

W = Amount of work done.

Notes : 1. The coefficient of performance is the reciprocal of the efficiency (*i.e.* $1/\eta$) of a heat engine. It is thus obvious, that the value of C.O.P. is always greater than unity.

2. The ratio of the actual C.O.P. to the theoretical C.O.P. is known as relative coefficient of performance. Mathematically,

$$\text{Relative C.O.P.} = \frac{\text{Actual C.O.P.}}{\text{Theoretical C.O.P.}}$$

Example 35.1. *An ice plant produces 10 tonnes of ice per day at 0° C using water at room temperature of 20° C. Estimate the power rating of the compressor-motor, if the C.O.P. of the plant is 2.5 and overall electro-mechanical efficiency is 90%.*

Solution. Given : $m = 10 \text{ t/day} = 10 \times 1000 / 24 \times 60 = 6.94$ kg/min ; $T_1 = 0° \text{C} = 273$ K ; $T_2 = 20° \text{C} = 293$ K ; C.O.P. = 2.5 ; $\eta_0 = 90\ \% = 0.9$

Let W = Work required to drive the compressor / min

We know that heat extracted from 1 kg of water at 20° C to produce 1 kg of ice at 0° C

$$= 1 \times 4.187 (20 - 0) + 335 = 418.74 \text{ kJ/kg}$$

... ($\because$ Latent heat of ice = 335 kJ/kg)

$\therefore$ Total heat extracted,

$$Q = 418.74 \times 6.94 = 2906 \text{ kJ/min}$$

We know that C.O.P. of the plant,

$$2.5 = \frac{Q}{W} = \frac{2906}{W}$$

$$\therefore \quad W = 2906 / 2.5 = 1162.4 \text{ kJ/min}$$

and power,
$$P = \frac{1162.4}{60 \times \eta} = \frac{1162.4}{60 \times 0.9} = 21.5 \text{ kW} \quad \textbf{Ans.}$$

Example 35.2. *Five hundred kg of fruits are supplied to a cold storage at 20° C. The cold storage is maintained at –5° C and the fruits get cooled to the storage temperature in 10 hours. The latent heat of freezing is 105 kJ/kg and specific heat of fruit is 1.26. Find the refrigeration capacity of the plant.*

Solution. Given : $m = 500$ kg ; $T_2 = 20°\text{ C} = 293$ K ; $T_1 = -5°\text{ C} = 268$ K ; $h_{fg} = 105$ kJ/kg ; $c_F = 1.26$

We know that heat removed from the fruit in 10 hrs,

$$Q_1 = m\, c_F (T_2 - T_1) = 500 \times 1.26\,(293 - 268) = 15\,750 \text{ kJ}$$

and total latent heat of freezing,

$$Q_2 = m\, h_{fg} = 500 \times 105 = 52\,500 \text{ kJ}$$

∴ Total heat removed in 10 hrs,

$$Q = Q_1 + Q_2 = 15\,750 + 52\,500 = 68\,250 \text{ kJ}$$

and total heat removed in one minute,

$$= \frac{68\,250}{10 \times 60} = 113.75 \text{ kJ/min}$$

∴ Refrigeration capacity of the plant

$$= \frac{113.75}{210} = 0.542 \text{ TR} \quad \textbf{Ans.} \qquad \ldots (\because 1 \text{ TR} = 210 \text{ kJ/min})$$

35.5. Difference between a Heat Engine, Refrigerator and Heat Pump

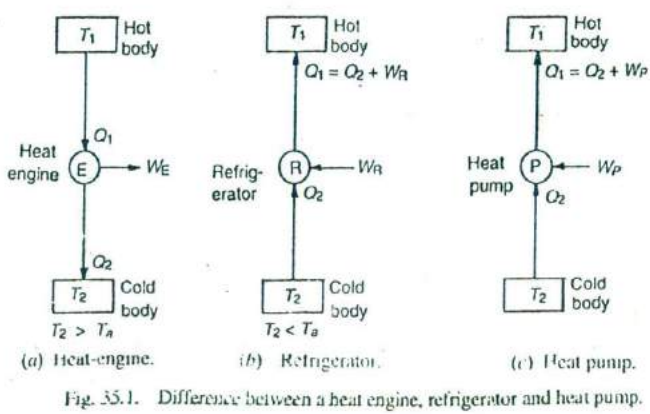


Fig. 35.1. Difference between a heat engine, refrigerator and heat pump.

In a heat engine, as shown in Fig. 35.1 (*a*), the heat supplied to the engine is converted into useful work. If Q_1 is the heat supplied to the engine and Q_2 is the heat rejected from the engine, then the net work done by the engine is given by

$$W_E = Q_1 - Q_2$$

The performance of a heat engine is expressed by its efficiency. We know that the efficiency or coefficient of performance of a heat engine,

$$\eta_E \text{ or } (\text{C.O.P.})_E = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{W_E}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$

A refrigerator, as shown in Fig. 35.1 (*b*), is a reversed heat engine which either cool or maintain the temperature of a body (T_2) lower than the atmospheric temperature (T_a). This is done by extracting the heat (Q_2) from a cold body and delivering it to a hot body (Q_1). In doing so, work W_R is required to be done on the system. According to first law of thermodynamics,

$$W_R = Q_1 - Q_2$$

The performance of a refrigerator is expressed by the ratio of amount of heat taken from the cold body (Q_2) to the amount of work required to be done on the system (W_R). This ratio is called coefficient of performance. Mathematically, coefficient of performance of a refrigerator,

$$(\text{C.O.P.})_R = \frac{Q_2}{W_R} = \frac{Q_2}{Q_1 - Q_2}$$

Any refrigerating system is a heat pump, as shown in Fig. 35.1 (*c*), which extracts heat (Q_2) from a cold body and delivers it to a hot body. Thus there is no difference between the cycle of operations of a heat pump and a refrigerator. The main difference between the two is in their operating temperatures. A refrigerator works between the cold body temperature (T_2) and the atmospheric temperature (T_a) whereas the heat pump operates between the hot body temperature (T_1) and the atmospheric temperature (T_a). A refrigerator used for cooling in summer can be used as a heat pump for heating in winter.

In the similar way, as discussed for refrigerator, we have

$$W_P = Q_1 - Q_2$$

The performance of a heat pump is expressed by the ratio of the amount of heat delivered to the hot body (Q_1) to the amount of work required to be done on the system (W_P). This ratio is called coefficient of performance or energy performance ratio (E.P.R.) of a heat pump. Mathematically, coefficient of performance or energy performance ratio of a heat pump,

$$(\text{C.O.P.})_P \text{ or E.P.R.} = \frac{Q_1}{W_P} = \frac{Q_1}{Q_1 - Q_2}$$

$$= \frac{Q_2}{Q_1 - Q_2} + 1 = (\text{C.O.P.})_R + 1$$

From above, we see that the C.O.P. may be less than one or greater than one depending on the type of refrigeration system used. But the C.O.P. of a heat pump is always greater than one.

35.6. Open Air Refrigeration Cycle

In an open air refrigeration cycle, the air is directly led to the space to be cooled (*i.e.* a refrigerator), allowed to circulate through the cooler and then returned to the compressor to start another cycle. Since the air is supplied to the refrigerator at atmospheric pressure, therefore, volume of air handled by the compressor and expander is large. Thus the size of compressor and expander should be large. Another disadvantage of the open cycle system is that the moisture is regularly carried away by the air circulated through the cooled space. This leads to the formation of frost at the end of expansion process and clog the line. Thus in an open cycle system, a drier should be used.

35.7. Closed or Dense Air Refrigeration Cycle

In a closed or dense air refrigeration cycle, the air is passed through the pipes and component parts of the system at all times. The air, in this system, is used for absorbing heat from the other fluid (say brine) and this cooled brine is circulated into the space to be cooled. The air in the closed system does not come in contact directly with the space to be cooled.

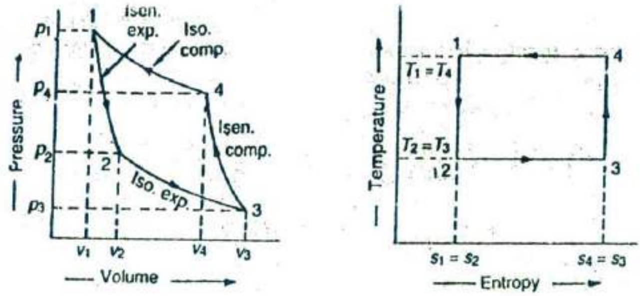
The closed air refrigeration cycle has the following thermodynamic advantages :

1. Since it can work at a suction pressure higher than that of atmospheric pressure, therefore the volume of air handled by the compressor and expander are smaller as compared to an open air refrigeration cycle system.
2. The operating pressure ratio can be reduced, which results in higher coefficient of performance.

35.8. Air Refrigerator Working on Reversed Carnot Cycle

In refrigerating systems, the Carnot cycle considered is the reversed Carnot cycle. We know that a heat engine working on Carnot cycle has the highest possible efficiency. Similarly, a refrigerating system working on the reversed Carnot cycle, will have the maximum possible coefficient of performance. We also know that it is not possible to make an engine working on the Carnot cycle. Similarly, it is also not possible to make a refrigerating machine working on the reversed Carnot cycle. However, it is used as the ultimate standard of comparison.

A reversed Carnot cycle, using air as working medium (or refrigerant) is shown on p-v and T-s diagram in Fig. 35.2 (a) and (b) respectively. At point 1, let p_1, v_1 and T_1 be the pressure, volume and temperature of air respectively.



(a) p-v diagram. (b) T-s diagram.

Fig. 35.2. Reversed Carnot cycle.

The four stages of the cycle are as follows :

1. *First stage (Isentropic expansion).* The air is expanded isentropically as shown by the curve 1-2 on p-v and T-s diagrams. The pressure of air decreases from p_1 to p_2, specific volume increases from v_1 to v_2 and the temperature decreases from T_1 to T_2. We know that during isentropic expansion, no heat is absorbed or rejected by the air.

2. *Second stage (Isothermal expansion).* The air is now expanded isothermally (*i.e.* at constant temperature, $T_2 = T_3$) as shown by the curve 2-3 on p-v and T-s diagrams. The pressure of air decreases from p_2 to p_3 and the specific volume increases from v_2 to v_3. We know that the heat absorbed by the air (or heat extracted from the cold body) during isothermal expansion per kg of air,

$$q_A = q_{2\text{-}3} = T_2(s_3 - s_2) = T_3(s_3 - s_2)$$

3. *Third stage (Isentropic compression).* The air is compressed isentropically as shown by the curve 3-4 on p-v and T-s diagrams. During this process, the pressure of air increases from p_3 to p_4, specific volume decreases from v_3 to v_4 and temperature increases from T_3 to T_4. We know that during isentropic compression, no heat is absorbed or rejected by the air.

4. *Fourth stage (Isothermal compression).* The air is now compressed isothermally (*i.e.* at constant temperature, $T_4 = T_1$) as shown by the curve 4-1 on p-v and T-s diagrams. During this process, the pressure of air increases from p_4 to p_1 and specific volume decreases from v_4 to v_1. We know that the heat rejected by the air during isothermal compression per kg of air,

$$q_R = q_{4-1} = T_4 (s_4 - s_1) = T_1 (s_3 - s_2)$$

We know that work done during the cycle per kg of air,

$$w_R = {}^*\text{Heat rejected} - \text{Heat absorbed} = q_R - q_A$$

$$= T_1 (s_3 - s_2) - T_3 (s_3 - s_2) = (T_1 - T_3)(s_3 - s_2)$$

∴ Coefficient of performance of the refrigeration system working on reversed Carnot cycle,

$$(\text{C.O.P.})_R = \frac{\text{Heat absorbed}}{\text{Work done}} = \frac{q_A}{q_R - q_A}$$

$$= \frac{T_3 (s_3 - s_2)}{(T_1 - T_3)(s_3 - s_2)} = \frac{T_3}{T_1 - T_3} = \frac{T_2}{T_1 - T_2} \qquad \ldots (\because T_3 = T_2)$$

Though the reversed Carnot cycle is the most efficient between the fixed temperature limits, yet no refrigerator has been made using this cycle. This is due to the reason that the isentropic processes of the cycle require high speed while the isothermal processes require an extremely low speed. This variation in speed of air is not practicable.

Note : We have already discussed that C.O.P. of a heat pump,

$$(\text{C.O.P.})_P = \frac{\text{Heat rejected}}{\text{Work done}} = \frac{q_R}{q_R - q_A} = \frac{T_1 (s_3 - s_2)}{T_1 (s_3 - s_2) - T_2 (s_3 - s_2)}$$

$$= \frac{T_1}{T_1 - T_2} = \frac{T_2}{T_1 - T_2} + 1 = (\text{C.O.P.})_R + 1$$

and C.O.P. or efficiency of a heat engine,

$$(\text{C.O.P.})_E = \frac{\text{Work done}}{\text{Heat rejected}} = \frac{q_R - q_A}{q_R}$$

$$= \frac{T_1 (s_3 - s_2) - T_2 (s_3 - s_2)}{T_1 (s_3 - s_2)} = \frac{T_1 - T_2}{T_1}$$

35.9. Temperature Limitations for Reversed Carnot Cycle

We have seen in the previous article that the C.O.P. of a reversed Carnot cycle is given by

$$\text{C.O.P} = \frac{T_2}{T_1 - T_2}$$

where T_1 = Higher temperature, and

T_2 = Lower temperature.

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* In a refrigerating machine, heat rejected is more than heat absorbed.

The C.O.P. of the reversed Carnot cycle may be improved by

1. decreasing the higher temperature (*i.e.* temperature of hot body, T_1), or
2. increasing the lower temperature (*i.e.* temperature of cold body, T_2).

This applies to all refrigerating machines, both theoretical and practical. It may be noted that temperatures T_1 and T_2 cannot be varied at will, due to certain functional limitations. It should be kept in mind that the higher temperature T_1 is the temperature of cooling water or air available for rejection of heat and the lower temperature T_2 is the temperature to be maintained in the refrigerator. The heat transfer will take place in the right direction only when the higher temperature is more than the temperature of cooling water or air to which heat is to be rejected, while the lower temperature must be less than the temperature of substance to be cooled.

Thus if the temperature of cooling water or air (*i.e.* T_1) available for heat rejection is low, the C.O.P. of the Carnot refrigerator will be high. Since T_1 in winter is less than T_1 in summer, therefore, C.O.P. in winter will be higher than C.O.P. in summer. In other words, the Carnot refrigerators work more efficiently in winter than in summer. Similarly, if the lower temperature fixed by the refrigeration application is high, the C.O.P. of the Carnot refrigerator will be high. Thus a Carnot refrigerator used for making ice at 0° C (273 K) will have less C.O.P. than a Carnot refrigerator used for air-conditioned plant in summer at 20° C when the atmospheric temperature is 40° C. In other words, we can say that the Carnot C.O.P. of a domestic refrigerator is less than the Carnot C.O.P. of a domestic air-conditioner.

Example 35.3. *A machine working on a Carnot cycle operates between 305 K and 260 K. Determine the C.O.P. when it is operated as 1. a refrigerating machine, 2. a heat pump, and 3. a heat engine.*

Solution. Given : $T_1 = 305$ K ; $T_2 = 260$ K

1. *C.O.P. of a refrigerating machine*

We know that C.O.P. of a refrigerating machine,

$$(\text{C.O.P.})_R = \frac{T_2}{T_1 - T_2} = \frac{260}{305 - 260} = 5.78 \textbf{ Ans.}$$

2. *C.O.P. of a heat pump*

We know that C.O.P. of a *heat pump

$$(\text{C.O.P.})_P = \frac{T_1}{T_1 - T_2} = \frac{305}{305 - 260} = 6.78 \textbf{ Ans.}$$

3. *C.O.P. of a heat engine*

We know that C.O.P. of a heat engine,

$$(\text{C.O.P.})_E = \frac{T_1 - T_2}{T_1} = \frac{305 - 260}{305} = 0.147 \textbf{ Ans.}$$

Example 35.4. *A Carnot refrigeration cycle absorbs heat at − 3° C and rejects it at 27° C.*

1. Calculate the coefficient of performance of this refrigeration cycle.

2. If the cycle is absorbing 1130 kJ/min at − 3° C, how many kJ of work is required per second ?

3. If the Carnot heat pump operates between the same temperatures as the above refrigeration cycle, what is the coefficient of performance ?

4. How many kJ/min will the heat pump deliver at 27° C if it absorbs 1130 kJ/min at − 3° C ?

---

* We know that C.O.P. of a heat pump,

$$(\text{C.O.P.})_P = (\text{C.O.P})_R + 1 = 5.78 + 1 = 6.78 \textbf{ Ans.}$$

Solution. Given : $T_2 = -3°\text{C} = 270\text{ K}$; $T_1 = 27°\text{C} = 300\text{ K}$

1. *Coefficient of performance of Carnot refrigeration cycle*

We know that coefficient of performance of Carnot refrigeration cycle,

$$(\text{C.O.P.})_R = \frac{T_2}{T_1 - T_2} = \frac{270}{300-270} = 9 \textbf{ Ans.}$$

2. *Work required per second*

Let W_R = Work required per second.

Heat absorbed at $-3°\text{C}$ (*i.e.* T_2),

$$Q_2 = 1130 \text{ kJ/min} = 18.83 \text{ kJ/s} \qquad \text{. . . (Given)}$$

We know that $(\text{C.O.P.})_R = \dfrac{Q_2}{W_R}$

$$\therefore \quad 9 = \frac{18.83}{W_R} \quad \text{or} \quad W_R = 2.1 \text{ kJ/s } \textbf{Ans.}$$

3. *Coefficient of performance of Carnot heat pump*

We know that coefficient of performance of a Carnot heat pump,

$$(\text{C.O.P.})_P = \frac{T_1}{T_1 - T_2} = \frac{300}{300-270} = 10 \textbf{ Ans.}$$

4. *Heat delivered by heat pump at 27° C*

Let Q_1 = Heat delivered by heat pump at 27° C.

Heat absorbed at $-3°\text{C}$ (*i.e.* T_2),

$$Q_2 = 1130 \text{ kJ/min} \qquad \text{. . . (Given)}$$

We know that $(\text{C.O.P.})_P = \dfrac{Q_1}{Q_1 - Q_2}$

$$\therefore \quad 10 = \frac{Q_1}{Q_1 - 1130}$$

$$10\,Q_1 - 11\,300 = Q_1$$

$$\therefore \quad Q_1 = 1256 \text{ kJ/min } \textbf{Ans.}$$

Example 35.5. *1.5 kW per tonne of refrigeration is required to maintain the temperature of −40° C in the refrigerator. If the refrigeration cycle works on Carnot cycle, determine the following :*

1. C.O.P. of the cycle, 2. Temperature of the sink, 3. Heat rejected to the sink per tonne of refrigeration, and 4. Heat supplied and E.P.R., if the cycle is used as a heat pump.

Solution. Given : $T_2 = -40°\text{C} = 233\text{ K}$

1. *C.O.P. of the cycle*

Since 1.5 kW per tonne of refrigeration is required to maintain the temperature in the refrigerator, therefore amount of work required to be done,

$$W_R = 1.5 \text{ kW} = 1.5 \text{ kJ/s} = 1.5 \times 60 = 90 \text{ kJ/min}$$

and heat extracted from the cold body,

$$Q_2 = 1 \text{ TR} = 210 \text{ kJ/min}$$

We know that $\qquad (\text{C.O.P.})_R = \dfrac{Q_2}{W_R} = \dfrac{210}{90} = 2.33$ **Ans.**

2. *Temperature of the sink*

Let $\qquad T_1$ = Temperature of the sink.

We know that $\qquad (\text{C.O.P.})_R = \dfrac{T_2}{T_1 - T_2}$ or $2.33 = \dfrac{233}{T_1 - 233}$

$\therefore \qquad T_1 = \dfrac{233}{2.33} + 233 = 333\text{ K} = 60°\text{ C}$ **Ans.**

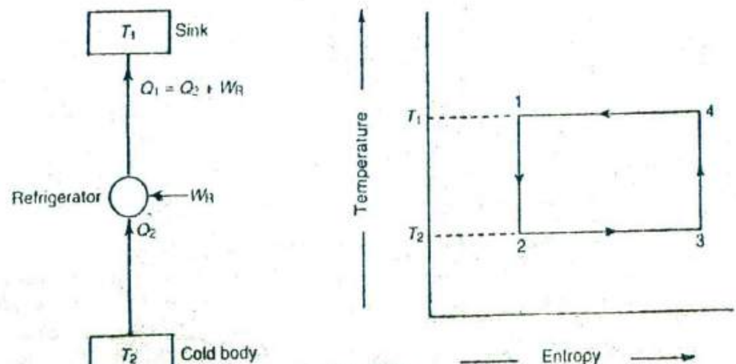


Fig. 35.3

3. *Heat rejected to the sink per tonne of refrigeration*

We know that heat rejected to the sink,

$$Q_1 = Q_2 + W_R = 210 + 90 = 300\text{ kJ/min } \textbf{Ans.}$$

4. *Heat supplied and E.P.R., if the cycle is used as a heat pump*

We know that heat supplied when the cycle is used as a heat pump is $Q_1 = 300$ kJ/min **Ans.**

and $\qquad \text{E.P.R.} = (\text{C.O.P.})_R + 1 = 2.33 + 1 = 3.33$ **Ans.**

Example 35.6 *The capacity of a refrigerator is 200 TR when working between – 6° C and 25° C. Determine the mass of ice produced per day from water at 25° C. Also find the power required to drive the unit. Assume that the cycle operates on reversed Carnot cycle and latent heat of ice is 335 kJ/kg.*

Solution. Given : $Q = 200$ TR ; $T_2 = -6°\text{ C} = 267\text{ K}$; $T_1 = 25°\text{ C} = 298\text{ K}$, $T_w = 25°\text{ C}$; $h_{fg(ice)} = 335$ kJ/kg

Mass of ice produced per day

We know that heat extraction capacity of the refrigerator

$$= 200 \times 210 = 42\,000\text{ kJ/min} \qquad \ldots (\because 1\text{ TR} = 210\text{ kJ/min})$$

and heat removed from 1 kg of water at 25° C (298 K) to form ice at 0° C (273 K)

$$= \text{Mass} \times \text{Sp. heat} \times \text{Rise in temperature} + h_{fg(ice)}$$

$$= 1 \times 4.187\,(298 - 273) + 335 = 439.7\text{ kJ/kg}$$

∴ Mass of ice produced per min

$$= \frac{42\,000}{439.7} = 95.52 \text{ kg/min}$$

and mass of ice produced per day

$$= 95.52 \times 60 \times 24 = 137\,550 \text{ kg} = 137.55 \text{ tonnes} \quad \textbf{Ans.}$$

Power required to drive the unit

We know that C.O.P. of the reversed Carnot cycle,

$$= \frac{T_2}{T_1 - T_2} = \frac{267}{298 - 267} = 8.6$$

Also $\quad \text{C.O.P.} = \dfrac{\text{Heat extraction capacity}}{\text{Work done per min}}$

∴ $\quad 8.6 = \dfrac{42\,000}{\text{Work done per min}}$

or $\quad$ Work done per min $= 42\,000 / 8.6 = 4884$ kJ/min

∴ Power required to drive the unit

$$= 4884 / 60 = 81.4 \text{ kW} \quad \textbf{Ans.}$$

Example 35.7. *A cold storage plant is required to store 20 tonnes of fish. The fish is supplied at a temperature of 30° C. The specific heat of fish above freezing point is 2.93 kJ/kg K. The specific heat of fish below freezing point is 1.26 kJ/kg K. The fish is stored in cold storage which is maintained at – 8° C. The freezing point of fish is – 4° C. The latent heat of fish is 235 kJ/kg. If the plant requires 75 kW to drive it, find*

1. The capacity of the plant, and 2. Time taken to achieve cooling.

Assume actual C.O.P. of the plant as 0.3 of the Carnot C.O.P.

Solution. Given : $m = 20 \text{ t} = 20\,000$ kg ; $T_1 = 30°\text{ C} = 303$ K ; $c_{AF} = 2.93$ kJ/kg K ; $c_{BF} = 1.26$ kJ/kg K ; $T_2 = -8°\text{ C} = 265$ K ; $T_3 = -4°\text{ C} = 269$ K ; $h_{fg\,(fish)} = 235$ kJ/kg ; $P = 75$ kW $= 75$ kJ/s

1. *Capacity of the plant*

We know that Carnot C.O.P.

$$= \frac{T_2}{T_1 - T_2} = \frac{265}{303 - 265} = 6.97$$

∴ $\quad$ Actual C.O.P. $= 0.3 \times 6.97 = 2.091$

and heat removed by the plant

$$= \text{Actual C.O.P.} \times \text{Work required}$$

$$= 2.091 \times 75 = 156.8 \text{ kJ/s} = 9408 \text{ kJ/min}$$

∴ Capacity of the plant

$$= 9408 / 210 = 44.8 \text{ TR} \quad \textbf{Ans.} \qquad \ldots (\because 1 \text{ TR} = 210 \text{ kJ/min})$$

2. *Time taken to achieve cooling*

We know that heat removed from the fish above freezing point,

$$Q_1 = m \times c_{AF} (T_1 - T_3)$$

$$= 20\,000 \times 2.93\,(303 - 269) = 1.992 \times 10^6 \text{ kJ}$$

Similarly, heat removed from the fish below freezing point,

$$Q_2 = m \times c_{BF}(T_3 - T_2)$$
$$= 20\,000 \times 1.26\,(269 - 265) = 0.101 \times 10^6 \text{ kJ}$$

and total latent heat of fish,

$$Q_3 = m \times h_{fg\,(fish)} = 20\,000 \times 235 = 4.7 \times 10^6 \text{ kJ}$$

∴ Total heat removed by the plant

$$= Q_1 + Q_2 + Q_3$$
$$= 1.992 \times 10^6 + 0.101 \times 10^6 + 4.7 \times 10^6 = 6.793 \times 10^6 \text{ kJ}$$

and time taken to achieve cooling

$$= \frac{\text{Total heat removed by the plant}}{\text{Heat removed by the plant per min}}$$
$$= \frac{6.793 \times 10^6}{9408} = 722 \text{ min} = 12.03 \text{ h} \quad \textbf{Ans.}$$

35.10. Air Refrigerator Working on a Bell-Coleman Cycle (or Reversed Joule or Brayton Cycle)

A Bell-Coleman air refrigerating machine was developed by Bell-Coleman and Light Foot by reversing the Joule's air cycle. It was one of the earliest types of refrigerators used in ships carrying frozen meat. Fig. 35.4 shows a schematic diagram of such a machine which consists of a compressor, a cooler, an expander and a refrigerator.

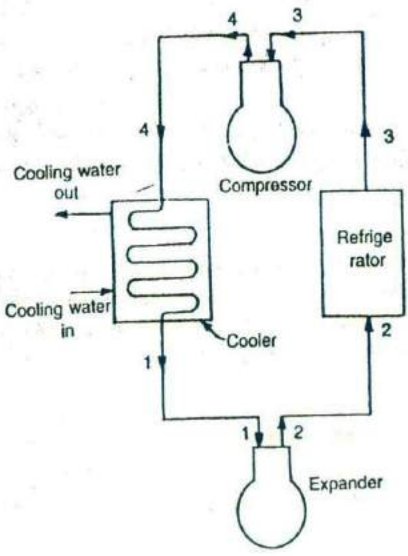


Fig. 35.4. Open cycle air Bell-Coleman refrigerator.

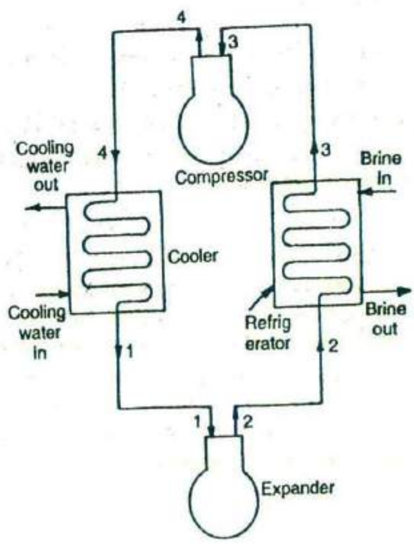
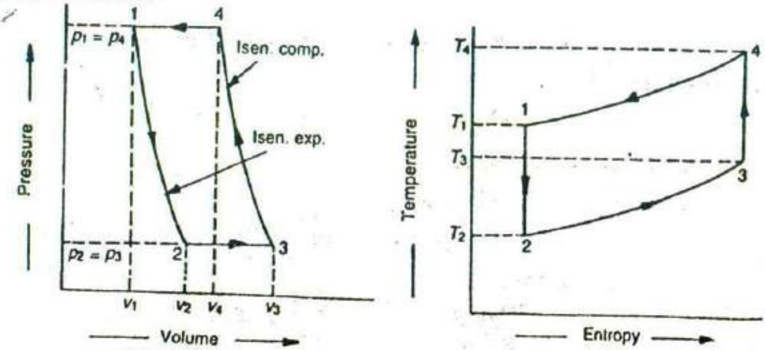


Fig. 35.5. Closed cycle or dense air Bell-Coleman refrigerator.

The Bell-Coleman cycle (also known as *reversed Joule* or *Brayton cycle*) is a modification of reversed Carnot cycle. The cycle is shown on *p-v* and *T-s* diagrams in Fig.35.6 (*a*) and (*b*). At point

1, let p_1, v_1, and T_1 be the pressure, volume and temperature of air respectively. The four stages of the cycle are as follows :



(a) p-v diagram. (b) T-s diagram.

Fig. 35.6. Bell-Coleman cycle.

1. *First stage (Isentropic expansion).* The air from the cooler is drawn into the expander cylinder where it is expanded isentropically from pressure p_1 to the refrigerator pressure p_2 which is equal to the atmospheric pressure.The temperature of air during expansion falls from T_1 to T_2 (*i.e.* the temperature much below the temperature of cooling water, T_1). The expansion process is shown by the curve 1-2 on the p-v and T-s diagrams. The specific volume of air at entry to the refrigerator increases from v_1 to v_2. We know that during isentropic expansion of air, no heat is absorbed or rejected by the air.

2. *Second stage (Constant pressure expansion).* The cold air from the expander is now passed to the refrigerator where it is expanded at constant pressure p_3 (equal to p_2). The temperature of air increases from T_2 to T_3. This process is shown by the curve 2-3 on the p-v and T-s diagrams. Due to heat from the refrigerator, the specific volume of the air changes from v_2 to v_3. We know that the heat absorbed by the air (or heat extracted from the refrigerator) during constant pressure expansion per kg of air is

$$q_A = q_{2-3} = c_p (T_3 - T_2)$$

3. *Third stage (Isentropic compression).* The cold air from the refrigerator is drawn into the compressor cylinder, where it is compressed isentropically as shown by the curve 3-4 on p-v and T-s diagrams. During the compression stroke, both the pressure and temperature increases and the specific volume of air at delivery from compressor reduces from v_3 to v_4. We know the during isentropic compression process, no heat is absorbed or rejected by the air.

4. *Fourth stage (Constant pressure cooling).* The warm air from the compressor is now passed into the cooler where it is cooled at constant pressure p_4 (equal to p_1), reducing the temperature from T_4 to T_1 (the temperature of cooling water) as shown by the curve 4-1 on p-v and T-s diagrams. The specific volume also reduces from v_4 to v_1. We know that heat rejected by the air during constant pressure cooling per kg of air,

$$q_R = q_{4-1} = c_p (T_4 - T_1)$$

We know that work done during the cycle per kg of air

$$w = \text{Heat rejected} - \text{Heat absorbed} = q_R - q_A$$

$$= c_p (T_4 - T_1) - c_p (T_3 - T_2)$$

$\therefore$ Coefficient of performance,

$$\text{C.O.P.} = \frac{\text{Heat absorbed}}{\text{Work done}} = \frac{q_A}{q_R - q_A}$$

$$= \frac{c_p(T_3 - T_2)}{c_p(T_4 - T_1) - c_p(T_3 - T_2)} = \frac{T_3 - T_2}{(T_4 - T_1) - (T_3 - T_2)}$$

$$= \frac{T_2\left(\frac{T_3}{T_2} - 1\right)}{T_1\left(\frac{T_4}{T_1} - 1\right) - T_2\left(\frac{T_3}{T_2} - 1\right)} \qquad \ldots (i)$$

We know that for isentropic expansion process 1-2,

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} \qquad \ldots (ii)$$

Similarly, for isentropic compression process 3-4,

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} \qquad \ldots (iii)$$

Since $p_1 = p_4$ and $p_2 = p_3$, therefore, from equations (*ii*) and (*iii*),

$$\frac{T_1}{T_2} = \frac{T_4}{T_3} \text{ or } \frac{T_3}{T_2} = \frac{T_4}{T_1} \qquad \ldots (iv)$$

Now substituting these values in equation (*i*), we get

$$\text{C.O.P.} = \frac{T_2}{T_1 - T_2} = \frac{1}{\frac{T_1}{T_2} - 1}$$

$$= \frac{1}{\left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} - 1} = \frac{1}{\left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} - 1}$$

$$= \frac{1}{(r_P)^{\frac{\gamma-1}{\gamma}} - 1} \qquad \ldots (v)$$

where r_P = Expansion or compression ratio $= \frac{p_1}{p_2} = \frac{p_4}{p_3}$

Sometimes, the compression and expansion processes take place according to the law pv^n = constant. In such a case, the C.O.P. is obtained from the fundamentals as discussed below :

We know that work done by the expander during the process 1-2 per kg of air,

$$w_E = w_{1-2} = \frac{n}{n-1}(p_1 v_1 - p_2 v_2) = \frac{n}{n-1}(RT_1 - RT_2) \qquad \ldots (\because pv = RT)$$

and work done by the compressor during the process 3-4 per kg of air,

$$w_C = w_{3-4} = \frac{n}{n-1}(p_4 v_4 - p_3 v_3) = \frac{n}{n-1}(RT_4 - RT_3)$$

$\therefore$ Net work done during the cycle per kg of air,

$$w = w_C - w_E = \frac{n}{n-1} \times R[(T_4 - T_3) - (T_1 - T_2)]$$

We also know that heat absorbed during constant pressure process 2-3

$$= c_p (T_3 - T_2)$$

$$\therefore \qquad \text{C.O.P.} = \frac{\text{Heat absorbed}}{\text{Work done}} = \frac{q_A}{w}$$

$$= \frac{c_p (T_3 - T_2)}{\frac{n}{n-1} \times R[(T_4 - T_3) - (T_1 - T_2)]} \qquad \ldots (vi)$$

We know that $\quad R = c_p - c_v = c_v(\gamma - 1)$

Substituting the value of R in equation (*vi*),

$$\text{C.O.P.} = \frac{c_p (T_3 - T_2)}{\frac{n}{n-1} \times c_v (\gamma - 1)[(T_4 - T_3) - (T_1 - T_2)]}$$

$$= \frac{T_3 - T_2}{\frac{n}{n-1} \times \frac{\gamma - 1}{\gamma}[(T_4 - T_3) - (T_1 - T_2)]}$$

$$= \frac{T_3 - T_2}{\frac{n}{n-1} \times \frac{\gamma - 1}{\gamma}[(T_4 - T_1) - (T_3 - T_2)]} \qquad \ldots (vii)$$

Notes : 1. In this case, the values of T_2 and T_4 are to be obtained from the relations

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{n-1}{n}}, \quad \text{and} \quad \frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{n-1}{n}}$$

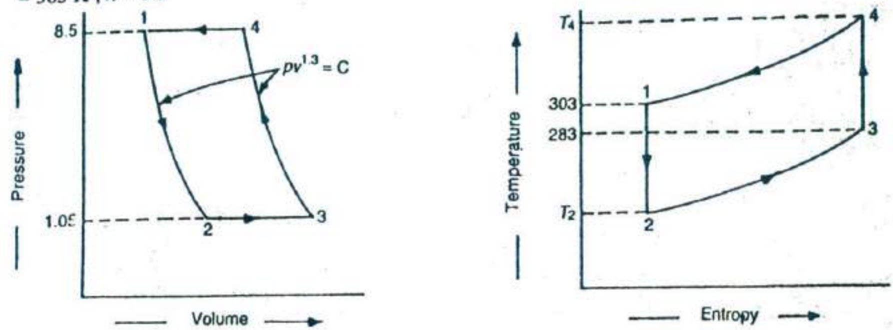
2. For isentropic expansion or compression $n = \gamma$. Therefore, the equation (*vii*) may be written as

$$\text{C.O.P.} = \frac{T_3 - T_2}{(T_4 - T_1) - (T_3 - T_2)} \qquad \ldots \text{(same as before)}$$

3. We have already discussed that the main drawback of the open cycle air refrigerator is freezing of the moisture in the air during expansion stroke which is liable to choke up the valves. Due to this reason, a closed cycle or dense air Bell-Coleman refrigerator, as shown in Fig. 35.5, is preferred. In this case, the cold air does not come in direct contact of the refrigerator. The cold air is passed through the pipes and it is used for absorbing heat from the brine and this cooled brine is circulated into the refrigerated space. The term dense air system, is derived from the fact that the suction to the compressor is at higher pressure than the open cycle system (which is atmospheric).

Example 35.8. *A refrigerator working on Bell-Coleman cycle operates between pressure limits of 1.05 bar and 8.5 bar. Air is drawn from the cold chamber at 10° C, compressed and then it is cooled to 30° C before entering the expansion cylinder. The expansion and compression follows the law $pv^{1.3}$ = constant. Determine the theoretical C.O.P. of the system.*

Solution. Given : $p_3 = p_2 = 1.05$ bar ; $p_4 = p_1 = 8.5$ bar ; $T_3 = 10°$ C = 283 K ; $T_1 = 30°$C = 303 K ; $n = 1.3$



(*a*) *p-v* diagram. (*b*) *T-s* diagram.

Fig. 35.7

The *p-v* and *T-s* diagram for a refrigerator working on the Bell-Coleman cycle is shown in Fig. 35.7 (*a*) and (*b*) respectively.

Let T_2 and T_4 = Temperature of air at the end of expansion and compression respectively.

Since the expansion and compression follows the law $pv^{1.3} = C$, therefore

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{n-1}{n}} = \left(\frac{8.5}{1.05}\right)^{\frac{1.3-1}{1.3}} = (8.1)^{0.231} = 1.62$$

$\therefore$ $$T_2 = T_1 / 1.62 = 303 / 1.62 = 187 \text{ K}$$

Similarly, $$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{n-1}{n}} = \left(\frac{8.5}{1.05}\right)^{\frac{1.3-1}{1.3}} = 1.62$$

$\therefore$ $$T_4 = T_3 \times 1.62 = 283 \times 1.62 = 458.5 \text{ K}$$

We know that theoretical coefficient of performance,

$$\text{C.O.P.} = \frac{T_3 - T_2}{\frac{n}{n-1} \times \frac{\gamma - 1}{\gamma}\left[(T_4 - T_1) - (T_3 - T_2)\right]}$$

$$= \frac{283 - 187}{\frac{1.3}{1.3-1} \times \frac{1.4-1}{1.4}\left[(458.5 - 303) - (283 - 187)\right]} \quad \text{. . . (Taking } \gamma = 1.4)$$

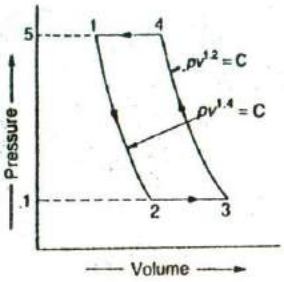
$$= \frac{96}{1.24 \times 59.5} = 1.3 \text{ Ans.}$$

Example 35.9. *The atmospheric air at pressure 1 bar and temperature – 5°C is drawn in the cylinder of the compressor of a Bell-Coleman refrigeration machine. It is compressed isentropically to a pressure of 5 bar. In the cooler, the compressed air is cooled to 15° C, pressure remaining the same. It is then expanded to a pressure of 1 bar in an expansion cylinder, from where it is passed to the cold chamber. Find 1. the work done per kg of air ; and 2. the C.O.P. of the plant.*

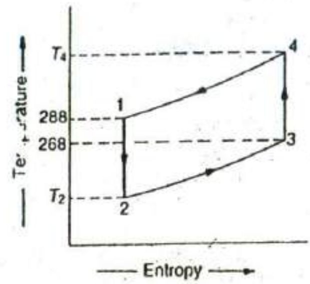
For air, assume law for expansion $pv^{1.2}$ = constant, law for compression $pv^{1.4}$ = constant and specific heat of air at constant pressure = 1 kJ/kg K.

Solution. Given : $p_3 = p_2 = 1$ bar ; $T_3 = -5°$ C = 268 K ; $p_4 = p_1 = 5$ bar ; $T_1 = 15°$ C = 288 K ; $n = 1.2$; $\gamma = 1.4$; $c_p = 1$ kJ/kg K

The *p-v* and *T-s* diagram for a refrigerating machine working on Bell-Coleman cycle is shown in Fig. 35.8 (*a*) and (*b*) respectively.



(*a*) *p-v* diagram.



(*b*) *T-s* diagram.

Fig. 35.8

1. *Work done per kg of air*

Let T_2 and T_4 = Temperatures at the end of expansion and compression respectively.

The expansion process 1-2 follows the law $pv^{1.2}$ = constant.

$$\therefore \quad \frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{n-1}{n}} = \left(\frac{5}{1}\right)^{\frac{1.2-1}{1.2}} = (5)^{0.167} = 1.31$$

or
$$T_2 = T_1 / 1.31 = 288 / 1.31 = 220 \text{ K}$$

The compression process 3-4 is isentropic and follows the law $pv^{1.4}$ = constant.

$$\therefore \quad \frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{5}{1}\right)^{\frac{1.4-1}{1.4}} = (5)^{0.286} = 1.585$$

or
$$T_4 = T_3 \times 1.585 = 268 \times 1.585 = 424.8 \text{ K}$$

We know that work done by the expander during the process 1-2 per kg of air,

$$w_E = w_{1-2} = \frac{n}{n-1} \times R\,(T_1 - T_2)$$

$$= \frac{1.2}{1.2-1} \times 0.29\,(288 - 220) = 118.3 \text{ kJ/kg}$$

... (Taking $R = 0.29$ kJ/kg K)

and work done by the compressor during the isentropic process 3-4 per kg of air,

$$w_C = w_{3-4} = \frac{\gamma}{\gamma-1} \times R\,(T_4 - T_3)$$

$$= \frac{1.4}{1.4-1} \times 0.29\,(424.8 - 268) = 159 \text{ kJ/kg}$$

$\therefore$ Net work done per kg of air,

$$w = w_C - w_E = 159 - 118.3 = 40.7 \text{ kJ/kg} \quad \textbf{Ans.}$$

2. C.O.P. of the plant

We know that heat absorbed during constant pressure process 2-3 per kg of air,

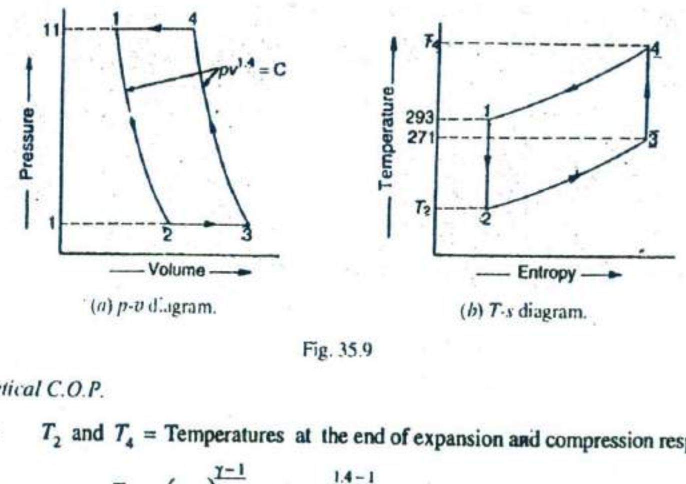
$$q_A = c_p (T_3 - T_2) = 1\,(268 - 220) = 48 \text{ kJ/kg}$$

$$\therefore \text{C.O.P. of the plant} = \frac{\text{Heat absorbed}}{\text{Work done}} = \frac{q_A}{w} = \frac{48}{40.7} = 1.18 \quad \textbf{Ans.}$$

Example 35.10. *In an open cycle air refrigeration machine, air is drawn from a cold chamber at $-2°$ C and 1 bar and compressed to 11 bar. It is then cooled, at this pressure, to the cooler temperature of $20°$ C and then expanded in expansion cylinder and returned to the cold room. The compression and expansion are isentropic, and follows the law $pv^{1.4}$ = constant. Sketch the p-v and T-s diagrams of the cycle and for a refrigeration of 15 tonnes, find : 1. theoretical C.O.P. ; 2. rate of circulation of the air in kg/min ; 3. piston displacement per minute in the compressor and expander; and 4. theoretical power per tonne of refrigeration.*

Solution. Given : $T_3 = -2°\text{ C} = 271 \text{ K}$; $p_3 = p_2 = 1 \text{ bar} = 1 \times 10^5 \text{ N/m}^2$; $p_4 = p_1 = 11$ bar ; $T_1 = 20° \text{ C} = 293 \text{ K}$; $\gamma = 1.4$; $Q = 15$ TR

The p-v and T-s diagrams of the cycle are shown in Fig. 35.9 (*a*) and (*b*) respectively.



(*a*) p-v diagram. (*b*) T-s diagram.

Fig. 35.9

1. Theoretical C.O.P.

Let T_2 and T_4 = Temperatures at the end of expansion and compression respectively.

We know that
$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma - 1}{\gamma}} = \left(\frac{11}{1}\right)^{\frac{1.4-1}{1.4}} = 1.985$$

$$\therefore \quad T_2 = T_1 / 1.985 = 293 / 1.985 = 147.6 \text{ K}$$

Similarly,
$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma - 1}{\gamma}} = \left(\frac{11}{1}\right)^{\frac{1.4-1}{1.4}} = (11)^{0.286} = 1.985$$

$$\therefore \quad T_4 = T_3 \times 1.985 = 271 \times 1.985 = 538 \text{ K}$$

We also know that theoretical C.O.P.

$$= \frac{T_3 - T_2}{(T_4 - T_1) - (T_3 - T_2)}$$

$$= \frac{271 - 147.6}{(538 - 293) - (271 - 147.6)} = \frac{123.4}{121.6} = 1.015 \text{ Ans.}$$

2. *Rate of circulation of the air in kg/min*

Refrigeration capacity $= 15$ TR ... (Given)

$\therefore$ Heat extracted/min $= 15 \times 210 = 3150$ kJ/min ... ($\because$ 1 TR = 210 kJ/min)

We know that heat extracted from cold chamber per kg

$$= c_p (T_3 - T_2) = 1 (271 - 147.6) = 123.4 \text{ kJ/kg}$$

... ($\because$ c_p for air = 1 kJ/kg K)

$\therefore$ Rate of circulation of air,

$$m_a = \frac{\text{Heat extracted /min}}{\text{Heat extracted /kg}} = \frac{3150}{123.4} = 25.5 \text{ kg/min Ans.}$$

3. *Piston displacement per minute in the compressor and expander*

Let v_3 and v_2 = Piston displacement per minute in the compressor and expander respectively.

We know that $p_3 v_3 = m_a R_a T_3$

$$\therefore \quad v_3 = \frac{m_a R_a T_3}{p_3} = \frac{25.5 \times 287 \times 271}{1 \times 10^5} = 19.8 \text{ m}^3\text{/min Ans.}$$

... (Taking $R_a = 287$ kJ/kg K)

For constant pressure process 2-3,

$$\frac{v_2}{T_2} = \frac{v_3}{T_3}$$

$$\therefore \quad v_2 = v_3 \times \frac{T_2}{T_3} = 19.8 \times \frac{147.6}{271} = 10.8 \text{ m}^3 \text{ Ans.}$$

4. *Theoretical power per tonne of refrigeration*

We know that net work done on the refrigeration machine per minute

$$= m_a \text{ (Heat rejected – Heat extracted)}$$

$$= m_a c_p [(T_4 - T_1) - (T_3 - T_2)]$$

$$= 25.5 \times 1 [(538 - 293) - (271 - 147.6)] = 3100 \text{ kJ/min}$$

$\therefore$ Theoretical power of the refrigerating machine

$$= 3100/60 = 51.57 \text{ kW}$$

and theoretical power per tonne of refrigeration

$$\doteq 51.67/15 = 3.44 \text{ kW / TR Ans.}$$

EXERCISES

1. A refrigerating plant is required to produce 2.5 tonnes of ice per day at – 4° C from water at 20° C. If the temperature range in the compressor is between 25° C and – 6° C, calculate power required to drive the compressor. Latent heat of ice = 335 kJ/kg and specific heat of ice = 2.1 kJ/kg K. [Ans. 1.437 kW]

2. A refrigeration system has working temperature of – 30° C and 40° C. What is the maximum C.O.P. possible? If the actual C.O.P. is 75% of the maximum, calculate the actual refrigerating effect produced per kW hour. [Ans. 0.743 TR]

3. A Carnot cycle machine operates between the temperature limits of 47° C and – 30°C. Determine the C.O.P. when it operates as 1. a refrigerating machine ; 2. a heat pump ; and 3. a heat engine.

[Ans. 3.16 ; 4.16 ; 0.24]

4. A refrigerator using Carnot cycle requires 1.25 kW per tonne of refrigeration to maintain a temperature of – 30° C. Find : 1. C.O.P. of Carnot refrigerator ; 2. Temperature at which heat is rejected ; and 3. heat rejected per tonne of refrigeration. [Ans. 2.8 ; 55.4°C ; 284 kJ/min]

5. A refrigerator storage is supplied with 30 tonnes of fish at a temperature of 27° C. The fish has to be cooled to – 9° C for preserving it for long period without deterioration. The cooling takes place in 10 hours. The specific heat of fish is 2.93 kJ/kg K above freezing point of fish and 1.26 kJ/kg K below freezing point of fish which is – 3° C. The latent heat of freezing is 232 kJ/kg. What is the capacity of the plant in tonne of refrigeration for cooling the fish ? What would be the ideal C.O.P. between this temperature range ? If the actual C.O.P. is 40% of the ideal, find the power required to run the cooling plant. [Ans. 78 TR ; 7.33 ; 93.3 kW]

6. A refrigerating system working on Bell-Coleman cycle receives air from cold chamber at – 5° C and compresses it from 1 bar to 4.5 bar. The compressed air is then cooled to a temperature of 37° C before it is expanded in the expander. Calculate the C.O.P. of the system when compression and expansion are (*i*) isentropic, and (*ii*) follow the law $pv^{1.25}$ = constant. [Ans. 1.86 ; 1.98]

7. A Bell-Coleman refrigerator works between 4 bar and 1 bar pressure limits. After compression, the cooling water reduces the air temperature to 17° C. What is the lowest temperature produced by the ideal machine ?

Compare the coefficient of performance of this machine with that of the ideal Carnot cycle machine working between the same pressure limits, the temperature at the beginning of compression being – 13° C.

[Ans. – 78°C ; 2.07, 1.02]

8. A dense air refrigerating system operating between pressures of 17.5 bar and 3.5 bar is to produce 10 tonne of refrigeration. Air leaves the refrigerating coils at – 7° C and it leaves the air cooler at 15.5° C. Neglecting losses and clearance, calculate the net work done per minute and the coefficient of performance. For air, c_p = 1.005 kJ/kg K and γ = 1.4. [Ans. 1237 kJ/min ; 1.7]

9. In a refrigeration plant using Bell-Coleman cycle, air at 0.8 bar, 6° C enters the compressor. The conditions at entry to the air turbine are 3.2 bar and 30° C. Assuming the isentropic efficiencies of compressor and turbine to be 83% and 85%, estimate the C.O.P. and air flow rate per tonne of refrigeration.

[Ans. 0.76 ; 3.46 kg/min]

10. An air refrigeration used for food storage provides 25 TR. The temperature of air entering the compressor is 7° C and the temperature at exit of cooler is 27° C. Find (*a*) C.O.P. of the cycle, and (*b*) power per tonne of refrigeration required by the compressor. The quantity of air circulated in the system is 3000 kg/h. The compression and expansion both follows the law $pv^{1.3}$ = constant and take γ = 1.4 and c_p = 1 for air.

[Ans. 1.13 ; 3.1 kW/TR]

11. A dense air machine operates on reversed Brayton cycle and is required for a capacity of 10 TR. The cooler pressure is 4.2 bar and the refrigerator pressure is 1.4 bar. The air is cooled in the cooler to a temperature of 50° C and the temperature of air at inlet to compressor is – 20° C. Determine for the ideal cycle (*a*) C.O.P. (*b*) mass of air circulated per minute, (*c*) theoretical piston displacement of compressor, (*d*) theoretical piston displacement of expander, and (*e*) net power per tonne of refrigeration. Show the cycle on *p-v* and *T-s* planes.

[Ans. 2.83 ; 123.5 kg/min ; 64 m³ ; 60 m³ ; 1.235 kW/TR]

QUESTIONS

1. Define the following terms :

(*a*) Coefficient of performance ; (*b*) tonne of refrigeration.

2. Discuss the advantages of the dense air refrigerating system over an open air rerrigeration system.

3. What is the difference between a refrigerator and a heat pump ? Derive an expression for the performance factor for both if they are running on reversed Carnot cycle.

4. Describe the Bell-Coleman cycle and obtain an expression for the C.O.P. of the cycle.

5. Show that C.O.P. of a Bell-Coleman cycle is given by the expression :

$$\text{C.O.P.} = \frac{1}{\left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}} - 1}$$

where r_p is the compression ratio and γ is the usual ratio of specific heats.

OBJECTIVE TYPE QUESTIONS

1. One tonne of refrigeration is equal to

   (*a*) 21 kJ/min (*b*) 210 kJ/min (*c*) 420 kJ/min (*d*) 620 kJ/min

2. One tonne refrigerating machine means that

   (*a*) one tonne is the total mass of the machine

   (*b*) one tonne of refrigerant is used

   (*c*) one tonne of water can be converted into ice

   (*d*) one tonne of ice when melts from and at 0° C in 24 hours, the refrigeration effect produced is equivalent to 210 kJ/min

3. The coefficient of performance is always .... one.

   (*a*) equal to (*b*) less than (*c*) greater than

4. The relative coefficient of performance is equal to

   (*a*) $\frac{\text{Theoretical C.O.P.}}{\text{Actual C.O.P.}}$ (*b*) $\frac{\text{Actual C.O.P.}}{\text{Theoretical C.O.P.}}$

   (*c*) Theoretical C.O.P. × Actual C.O.P.

5. In a closed or dense air refrigeration cycle, the operating pressure ratio can be reduced, which results in .... coefficient of performance.

   (*a*) lower (*b*) higher

6. Air refrigeration cycle is used in

   (*a*) commercial refrigerators (*b*) domestic refrigerators

   (*c*) air-conditioning (*d*) gas liquefaction

7. In a refrigerating machine, heat rejected is .... heat absorbed.

   (*a*) equal to (*b*) less than (*c*) greater than

8. Air refrigerator works on

   (*a*) Carnot cycle (*b*) Rankine cycle

   (*c*) reversed Carnot cycle (*d*) Bell-Coleman cycle

   (*e*) both (*a*) and (*b*) (*f*) both (*c*) and (*d*)

9. In air-conditioning of aeroplanes, using air as a refrigerant, the cycle used is

   (*a*) reversed Carnot cycle (*b*) reversed Joule cycle

   (*c*) reversed Brayton cycle (*d*) reversed Otto cycle

ANSWERS

1. (*b*)	2. (*d*)	3. (*c*)	4. (*b*)	5. (*b*)
6. (*d*)	7. (*c*)	8. (*f*)	9. (*c*)	

36

Vapour Compression Refrigeration Systems

1. Introduction. 2. Advantages and Disadvantages of Vapour Compression Refrigeration System over Air Refrigeration System. 3. Mechanism of a Simple Vapour Compression Refrigeration System. 4. Pressure-Enthalpy (p-h) Chart. 5. Types of Vapour Compression Cycles. 6. Theoretical Vapour Compression Cycle with Dry Saturated Vapour after Compression. 7. Theoretical Vapour Compression Cycle with Wet Vapour after Compression. 8. Theoretical Vapour Compression Cycle with Superheated Vapour after Compression. 9. Theoretical Vapour Compression Cycle with Superheated Vapour before Compression. 10. Theoretical Vapour Compression Cycle with Under-cooling or Sub-cooling of Refrigerant. 11. Actual Vapour Compression Cycle. 12. Vapour Absorption Refrigeration System. 13. Advantages of Vapour Absorption Refrigeration System over Vapour Compression Refrigeration System. 14. Ammonia-Hydrogen (Electrolux) Refrigerator. 15. Properties of a Refrigerant. 16. Refrigerants Commonly Used in Practice.

36.1. Introduction

A vapour compression refrigeration system* is an improved type of air refrigeration system in which a suitable working substance, termed as refrigerant, is used. It condenses and evaporates at temperature and pressures close to the atmospheric conditions. The refrigerants, usually, used for this purpose are ammonia, carbon dioxide and sulphur dioxide. The refrigerant used, does not leave the system, but is circulated throughout the system alternately condensing and evaporating. In evaporating, the refrigerant absorbs its latent heat from the brine** (salt water) which is used for circulating it around the cold chamber. While condensing, it gives out its latent heat to the circulating water of the cooler. The vapour compression refrigeration system is, therefore, a latent heat pump, as it pumps its latent heat from the brine and delivers it to the cooler.

The vapour compression refrigeration system is now-a-days used for all purpose refrigeration. It is generally used for all industrial purposes from a small domestic refrigerator to a big air conditioning plant.

36.2. Advantages and Disadvantages of Vapour Compression Refrigeration System over Air Refrigeration System

Following are the advantages and disadvantages of the vapour compression refrigeration system over air refrigeration system :

Advantages

1. It has smaller size for the given capacity of refrigeration.
2. It has less running cost.

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* Since low pressure vapour refrigerant from the evaporator is changed into high pressure vapour refrigerant in the compressor, therefore it is named as vapour compression refrigeration system.

** Brine is used as it has a very low freezing temperature.

3. It can be employed over a large range of temperatures.
4. The coefficient of performance is quite high.

Disadvantages

1. The initial cost is high.
2. The prevention of leakage of the refrigerant is the major problem in vapour compression system.

36.3. Mechanism of a Simple Vapour Compression Refrigeration System

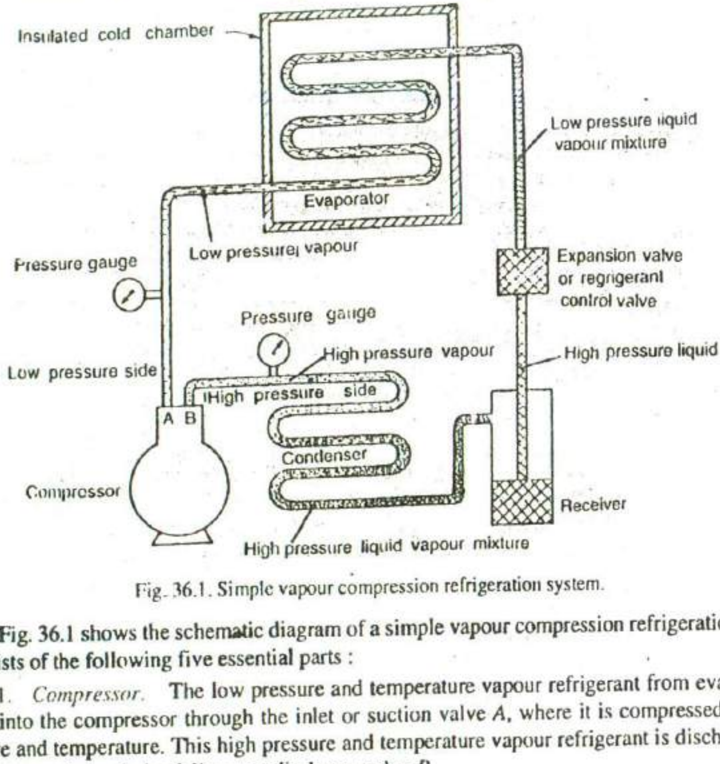


Fig. 36.1. Simple vapour compression refrigeration system.

Fig. 36.1 shows the schematic diagram of a simple vapour compression refrigeration system. It consists of the following five essential parts :

1. *Compressor.* The low pressure and temperature vapour refrigerant from evaporator is drawn into the compressor through the inlet or suction valve *A*, where it is compressed to a high pressure and temperature. This high pressure and temperature vapour refrigerant is discharged into the condenser through the delivery or discharge valve *B*.

2. *Condenser.* The condenser or cooler consists of coils of pipe in which the high pressure and temperature vapour refrigerant is cooled and condensed. The refrigerant, while passing through the condenser, gives up its latent heat to the surrounding condensing medium which is normally air or water.

3. *Receiver.* The condensed liquid refrigerant from the condenser is stored in a vessel known as receiver from where it is supplied to the evaporator through the expansion valve or refrigerant control valve.

4. *Expansion valve.* It is also called throttle valve or refrigerant control valve. The function of the expansion valve is to allow the liquid refrigerant under high pressure and temperature to pass at a controlled rate after reducing its pressure and temperature. Some of the liquid refrigerant evaporates as it passes through the expansion valve, but the greater portion is vaporised in the evaporator at the low pressure and temperature.

5. *Evaporator*. An evaporator consists of coils of pipe in which the liquid-vapour refrigerant at low pressure and temperature is evaporated and changed into vapour refrigerant at low pressure and temperature. In evaporating, the liquid vapour refrigerant absorbs its latent heat of vaporisation from the medium (air, water or brine) which is to be cooled.

Note · In any compression refrigeration system, there are two different pressure conditions. One is called the ***high pressure side*** and the other is known as *low pressure side*. The high pressure side includes the discharge line (*i.e.* piping from delivery valve *B* to the condenser), receiver and expansion valve. The low pressure side includes the evaporatior, piping from the expansion valve to the evaporator and the suction line (*i.e.* piping from the evaporator to the suction valve *A*).

36.4. Pressure - Enthalpy (p-h) Chart

The most convenient chart for studying the behavior of a refrigerant is the p-h chart in which the vertical ordinates represent pressure and horizontal ordinates represent enthalpy (*i.e.* total heat). A typical chart is shown in Fig. 36.2, in which a few important lines of the complete chart are drawn. The saturated liquid line and the saturated vapour line merge into one another at the critical point. A

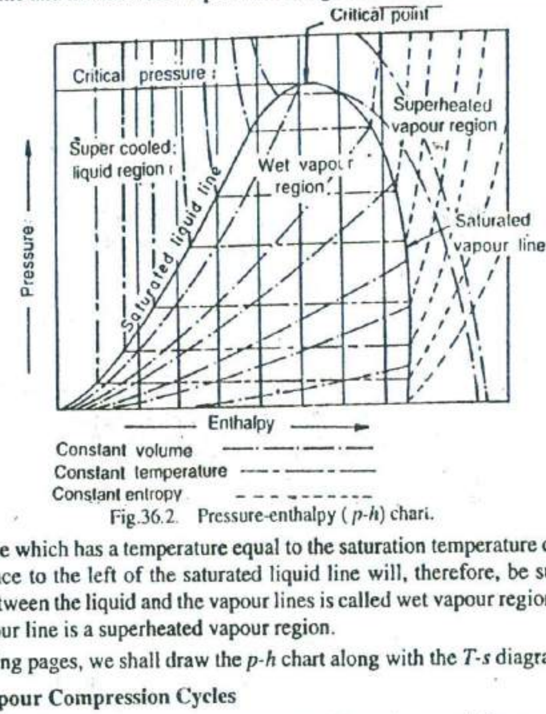


Fig.36.2. Pressure-enthalpy (p-h) chart.

saturated liquid is one which has a temperature equal to the saturation temperature corresponding to its pressure. The space to the left of the saturated liquid line will, therefore, be sub-cooled liquid region. The space between the liquid and the vapour lines is called wet vapour region and to the right of the saturated vapour line is a superheated vapour region.

In the following pages, we shall draw the p-h chart along with the T-s diagram of the cycles.

36.5. Types of Vapour Compression Cycles

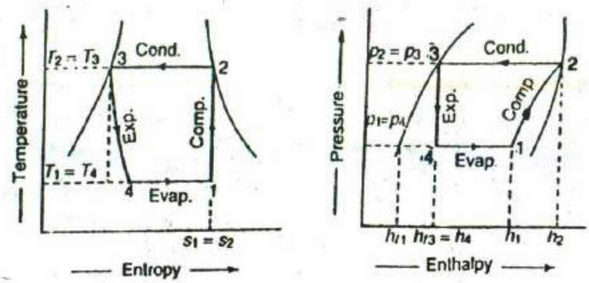
We have already discussed that a vapour compression cycle essentially consists of compression, condensation, throttling and evaporation. Many scientists have focussed their attention to increase the coefficient of performance of the cycle. Though there are many cycles, yet the following are important from the subject point of view :

1. Cycle with dry saturated vapour after compression,
2. Cycle with wet vapour after compression,
3. Cycle with superheated vapour after compression,
4. Cycle with superheated vapour before compression, and
5. Cycle with undercooling or subcooling of refrigerant.

Now we shall discuss all the above mentioned cycles, one by one, in the following pages.

36.6. Theoretical Vapour Compression Cycle with Dry Saturated Vapour after Compression

A vapour compression cycle with dry saturated vapour after compression is shown on T-s and p-h diagrams in Fig. 36.3 (*a*) and (*b*) respectively. At point 1, let T_1, p_1 and s_1 be the temperature, pressure and entropy of the vapour refrigerant respectively. The four processes of the cycle are as follows :



(*a*) T-s diagram. (*b*) p-h diagram.

Fig. 36.3. Theoretical vapour compression cycle with dry saturated vapour after compression.

1. *Compression process.* The vapour refrigerant at low pressure p_1 and temperature T_1 is compressed isentropically to dry saturated vapour as shown by the vertical line 1-2 on T-s diagram and by the curve 1-2 on p-h diagram. The pressure and temperature rises from p_1 to p_2 and T_1 to T_2 respectively.

The work done during isentropic compression per kg of refrigerant is given by

$$w_{1-2} = h_2 - h_1$$

where h_1 = Enthalpy of vapour refrigerant (in kJ/kg) at temperature T_1, *i.e.* at suction of the compressor, and

h_2 = Enthalpy of the vapour refrigerant (in kJ/kg) at temperature T_2, *i.e.* at discharge of the compressor.

2. *Condensing process.* The high pressure and temperature vapour refrigerant from the compressor is passed through the condenser where it is completely condensed at constant pressure p_2 and temperature T_2, as shown by the horizontal line 2-3 on T-s and p-h diagrams. The vapour refrigerant is changed into liquid refrigerant. The refrigerant, while passing through the condenser, gives its latent heat to the surrounding condensing medium.

3. *Expansion process.* The liquid refrigerant at pressure $p_3 = p_2$ and temperature $T_3 = T_2$ is expanded by throttling process* through the expansion valve to a low pressure $p_4 = p_1$ and temperature $T_4 = T_1$, as shown by the curve 3-4 on T-s diagram and by the vertical line 3-4 on p-h diagram. We have already discussed that some of the liquid refrigerant evaporates as it passes through the expansion valve, but the greater portion is vaporised in the evaporator. We know that during the throttling process, no heat is absorbed or rejected by the liquid refrigerant.

Notes : (*a*) In case an expansion cylinder is used in place of throttle or expansion valve to expand the liquid refrigerant, then the refrigerant will expand isentropically as shown by dotted vertical line on T-s diagram in Fig. 36.3 (*a*). The isentropic expansion reduces the external work being expanded in running the compressor and increases the refrigerating effect. Thus, the net result of using the expansion cylinder is to increase the coefficient of performance.

* The throttling process is an irreversible process.

Since the expansion cylinder system of expanding the liquid refrigerant is quite complicated and involves greater initial cost, therefore its use is not justified for small gain in cooling capacity. Moreover, the flow rate of the refrigerant can be controlled with throttle valve which is not possible in case of expansion cylinder which has a fixed cylinder volume.

(*b*) In modern domestic refrigerators, a capillary (small bore tube) is used in place of an expansion valve.

4. *Vaporising process.* The liquid-vapour mixture of the refrigerant at pressure $p_4 = p_1$ and temperature $T_4 = T_1$ is evaporated and changed into vapour refrigerant at constant pressure and temperature, as shown by the horizontal line 4-1 on *T-s* and *p-h* diagrams. During evaporation, the liquid-vapour refrigerant absorbs its latent heat of vaporisation from the medium (air, water or brine) which is to be cooled. This heat which is absorbed by the refrigerant is called *refrigerating effect*. The process of vaporisation continues upto point 1 which is the starting point and thus the cycle is completed.

We know that the refrigerating effect or the heat absorbed or extracted by the liquid-vapour refrigerant during evaporation per kg of refrigerant is given by

$$R_E = h_1 - h_4 = h_1 - h_{f3} \qquad \ldots (\because h_{f3} = h_4)$$

where h_{f3} = Sensible heat at temperature T_3.

It may be noticed from the cycle that the liquid-vapour refrigerant has extracted heat during evaporation and the work will be done by the compressor for isentropic compression of the high pressure and temperature vapour refrigerant.

∴ Coefficient of performance,

$$\text{C.O.P.} = \frac{\text{Refrigerating effect}}{\text{Work done}} = \frac{h_1 - h_{f3}}{h_2 - h_1}$$

Example 36.1. *The temperature limits of an ammonia refrigerating system are 25° C and − 10° C. If the gas is dry at the end of compression, calculate the coefficient of performance of the cycle assuming no undercooling of the liquid ammonia. Use the following table for properties of ammonia :*

Temperature (°C)	*Liquid heat (kJ/kg)*	*Latent heat (kJ/kg)*	*Liquid entropy (kJ/kg K)*
25	298.9	1166.94	1.1242
− 10	135.37	1297.68	0.5443

Solution. Given : $T_2 = T_3 = 25°\text{C} = 298$ K ; $T_1 = T_4 = -10°\text{C} = 263$ K ; $h_{f3} = h_{f2} = 298.9$ kJ/kg ; $h_{fg2} = 1166.94$ kJ/kg ; $s_{f2} = 1.1242$ kJ/kg K ; $h_{f1} = 135.37$ kJ/kg ; $h_{fg1} = 1297.68$ kJ/kg ; $s_{f1} = 0.5443$ kJ/kg K

The *T-s* and *p-h* diagrams are shown in Fig. 36.4 (*a*) and (*b*) respectively.

Let x_1 = Dryness fraction at point 1.

We know that entropy at point 1,

$$s_1 = s_{f1} + \frac{x_1 h_{fg1}}{T_1} = 0.5443 + \frac{x_1 \times 1297.68}{263}$$

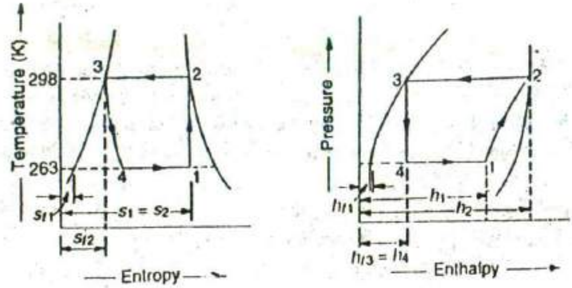
$$= 0.5443 + 4.934\, x_1 \qquad \ldots (i)$$

Similarly, entropy at point 2,

$$s_2 = s_{f2} + \frac{h_{fg2}}{T_2} = 1.1242 + \frac{1166.94}{298} = 5.04 \qquad \ldots (ii)$$

Since the entropy at point 1 is equal to entropy at point 2, therefore equating equations (*i*) and (*ii*),

$$0.5443 + 4.934\, x_1 = 5.04 \quad \text{or} \quad x_1 = 0.91$$



(*a*) *T-s* diagram. (*b*) *p-h* diagram.

Fig. 36.4

We know that enthalpy at point 1,

$$h_1 = h_{f1} + x_1\, h_{fg1} = 135.37 + 0.91 \times 1297.68 = 1316.26 \text{ kJ/kg}$$

and enthalpy at point 2, $\quad h_2 = h_{f2} + h_{fg2} = 298.9 + 1166.94 = 1465.84$ kJ/kg

$\therefore$ Coefficient of performance of the cycle

$$= \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{1316.26 - 298.9}{1465.84 - 1316.26} = 6.8 \text{ Ans.}$$

Example 36.2. *A vapour compression refrigerator works between the pressure limits of 60 bar and 25 bar. The working fluid is just dry at the end of compression and there is no under cooling of the liquid before the expansion valve. Determine : 1. C.O.P. of the cycle; and 2. Capacity of the refrigerator if the fluid flow is at the rate of 5 kg/min. Data :*

Pressure, bar	*Saturation temperature, K*	*Enthalpy, kJ/kg*		*Entropy, kJ/kg K*	
		Liquid	*Vapour*	*Liquid*	*Vapour*
60	295	61.9	208.1	0.197	0.703
25	261	−18.4	234.5	−0.075	0.896

Solution. Given : $p_2 = p_3 = 60$ bar ; $p_1 = p_4 = 25$ bar ; $T_2 = T_3 = 295$ K ; $T_1 = T_4 = 261$ K ; $h_{f3} = h_4 = 61.9$ kJ/kg ; $h_{f1} = -18.4$ kJ/kg ; $h_{g2} = h_2 = 208.1$ kJ/kg ; $h_{g1} = 234.5$ kJ/kg ; *$s_{f2} = 0.197$ kJ/kg K ; $s_{f1} = -0.075$ kJ/kg K ; $s_{g1} = 0.896$ kJ/kg K ; $s_{g2} = s_2 = 0.703$ kJ/kg K

1. *C.O.P. of the cycle*

The *T-s* and *p-h* diagrams are shown in Fig. 36.5 (*a*) and (*b*) respectively.

Let $\quad x_1$ = Dryness fraction of the vapour refrigerant entering the compressor at point 1.

* Superfluous data

We know that entropy at point 1 (s_1)

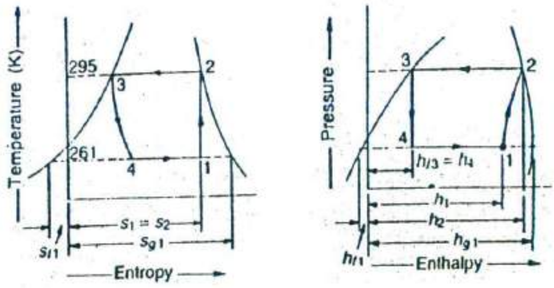
$$= \text{Entropy at point } 2\ (s_2)$$

$$s_{f1} + x_1 s_{fg1} = s_2$$

$$s_{f1} + x_1 (s_{g1} - s_{f1}) = s_{g2} \qquad \ldots(\because s_{g1} = s_{f1} + s_{fg1} \text{ and } s_{g2} = s_2)$$

$$-0.075 + x_1 [0.896 - (-0.075)] = 0.703$$

$$\therefore \quad 0.971\, x_1 = 0.778 \text{ or } x_1 = 0.8$$



(*a*) *T-s* diagram. (*b*) *p-h* diagram.

Fig. 36.5

We know that enthalpy at point 1,

$$h_1 = h_{f1} + x_1 h_{fg1} = h_{f1} + x_1 (h_{g1} - h_{f1}) \quad \ldots(h_{g1} = h_{f1} + h_{fg1})$$

$$= -18.4 + 0.8 [234.5 - (-18.4)] = 183.9 \text{ kJ/kg}$$

$$\therefore \text{C.O.P. of the cycle} = \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{183.9 - 61.9}{208.1 - 183.9} = 5.04 \textbf{ Ans.}$$

2. *Capacity of the refrigerator*

We know that the heat extracted or refrigerating effect produced per kg of refrigerant

$$= h_1 - h_{f3} = 183.9 - 61.9 = 122 \text{ kJ/kg}$$

Since the fluid flow is at the rate of 5 kg/min, therefore total heat extracted

$$= 5 \times 122 = 610 \text{ kJ/min}$$

$$\therefore \text{Capacity of the refrigerator} = \frac{610}{210} = 2.9 \text{ TR } \textbf{Ans.} \qquad \ldots(\because 1 \text{ TR} = 210 \text{ kJ/min})$$

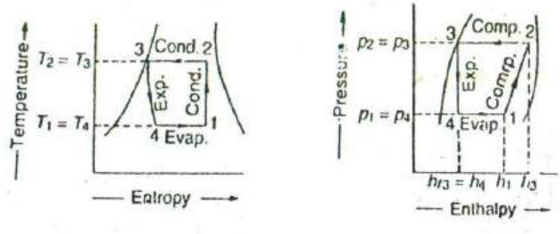
36.7. Theoretical Vapour Compression Cycle with Wet Vapour after Compression

A vapour compression cycle with wet vapour after compression is shown on *T-s* and *p-h* diagrams in Fig 36.6 (*a*) and (*b*) respectively. In this cycle, the enthalpy or total heat at point 2 is found out with the help of dryness fraction at this point. The dryness fraction at point 1 and 2 may be obtained by equating entropies at points 1 and 2.

Now the coefficient of performance may be found out as usual from the relation,

$$\text{C.O.P.} = \frac{\text{Refrigerating effect}}{\text{Work done}} = \frac{h_1 - h_{f3}}{h_2 - h_1}$$

Note : The remaining cycle is the same as discussed in the last article.



(*a*) *T-s* diagram. (*b*) *p-h* diagram.

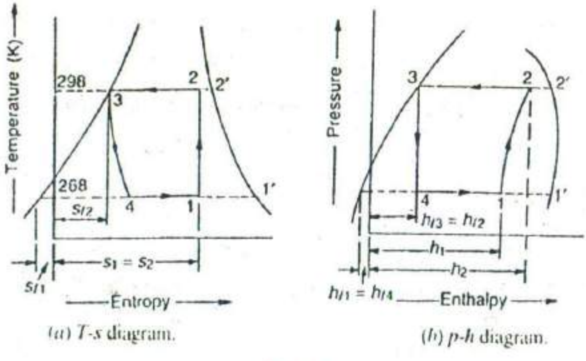
Fig. 36.6. Theoretical vapour compression cycle with wet vapour after compression.

Example 36.3. *Find the theoretical C.O.P. for a CO_2 machine working between the temperature range of 25° C and –5° C. The dryness fraction of CO_2 gas during the suction stroke is 0.6. Following properties of CO_2 are given :*

Temperature °C	*Liquid*		*Vapour*		*Latent heat kJ/kg*
	Enthalpy kJkg	*Entropy kJ/kg K*	*Enthalpy kJ/kg*	*Entropy kJ/kg K*	
25	81.3	0.251	202.6	0.63	121.4
– 5	– 7.54	– 0.042	237	0.84	245.3

Solution. Given : $T_2 = T_3 = 25°\text{ C} = 298\text{ K}$; $T_1 = T_4 = -5°\text{C} = 268\text{ K}$; $x_1 = 0.6$; $h_{f3} = h_{f2} = 81.3\text{ kJ/kg}$; $h_{f1} = h_{f4} = -7.54\text{ kJ/kg}$; $s_{f2} = 0.251\text{ kJ/kg K}$; $s_{f1} = -0.042\text{ kJ/kg K}$; $h_2' = 202.6$ kJ/kg ; $^*h_1' = 237$ kJ/kg ; $^*s_2' = 0.63$ kJ/kg K ; $^*s_1' = 0.84$ kJ/kg K ; $h_{fg2} = 121.4$ kJ/kg ; $h_{fg1} = 245.3$ kJ/kg

The *T-s* and *p-h* diagrams are shown in Fig. 36.7 (*a*) and (*b*) respectively.



(*a*) *T-s* diagram. (*b*) *p-h* diagram.

Fig. 36.7

First of all, let us find the dryness fraction at point 2, *i.e.* x_2. We know that the entropy at point 1,

* Superfluous data

$$s_1 = s_{f1} + \frac{x_1 h_{fg1}}{T_1} = -0.042 + \frac{0.6 \times 245.3}{268} = 0.507 \qquad \ldots (i)$$

Similarly, entropy at point 2,

$$s_2 = s_{f2} + \frac{x_2 h_{fg2}}{T_2} = 0.251 + \frac{x_2 \times 121.4}{298} = 0.251 + 0.407\,x_2 \quad \ldots (ii)$$

Since the entropy at point 1 (s_1) is equal to entropy at point 2 (s_2), therefore equating equations (i) and (ii),

$$0.507 = 0.251 + 0.407\,x_2 \quad \text{or} \quad x_2 = 0.629$$

$\therefore$ Enthalpy at point 1, $h_1 = h_{f1} + x_1 h_{fg1} = -7.54 + 0.6 \times 245.3 = 139.64$ kJ/kg

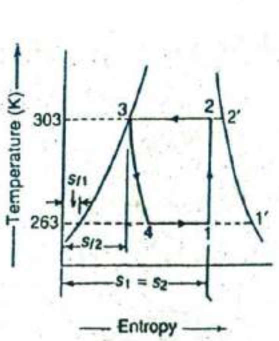
and enthalpy at point 2, $h_2 = h_{f2} + x_2 h_{fg2} = 81.3 + 0.629 \times 121.4 = 157.66$ kJ/kg

$\therefore$ Theoretical C.O.P. $= \dfrac{h_1 - h_{f3}}{h_2 - h_1} = \dfrac{139.64 - 81.3}{157.66 - 139.64} = 3.24$ **Ans.**

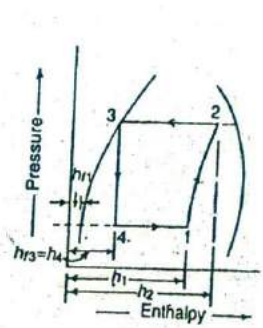
Example 36.4. *An ammonia refrigerating machine fitted with an expansion valve works between the temperature limits of $-10°$ C and $30°$ C. The vapour is 95% dry at the end of isentropic compression and the fluid leaving the condenser is at $30°$ C. Assuming actual C.O.P. as 60% of the theoretical, calculate the kilograms of ice produced per kW hour at $0°$ C from water at $10°$ C. Latent heat of ice is 335 kJ/kg. Ammonia has the following properties :*

Temperature °C	Liquid heat kJ/kg	Latent heat kJ/kg	Liquid entropy	Total entropy of dry saturated vapour
30	323.08	1145.80	1.2037	4.9842
–10	135.37	1297.68	0.5443	5.4770

Solution. Given : $T_1 = T_4 = -10°$ C = 263 K ; $T_2 = T_3 = 30°$ C = 303° K ; $x_2 = 0.95$
$h_{f3} = h_{f2} = 323.08$ kJ/kg ; $h_{f1} = h_{f4} = 135.37$ kJ/kg ; $h_{fg2} = 1145.8$ kJ/kg ; $h_{fg1} = 1297.68$ kJ/kg ; $s_{f2} = 1.2037$; $s_{f1} = 0.5443$; $*s_2' = 4.9842$; $*s_1' = 5.4770$



(a) T-s diagram.



(b) p-h diagram.

Fig.36.8

The T-s and p-h diagrams are shown in Fig. 36.8 (a) and (b) respectively.

* Superfluous data

Let x_1 = Dryness fraction at point 1.

We know that entropy at point 1,

$$s_1 = s_{f1} + \frac{x_1 h_{fg1}}{T_1} = 0.5443 + \frac{x_1 \times 1297.68}{263} = 0.5443 + 4.934\, x_1 \quad \ldots (i)$$

Similarly, entropy at point 2,

$$s_2 = s_{f2} + \frac{x_2 h_{fg2}}{T_2} = 1.2037 + \frac{0.95 \times 1145.8}{303} = 4.796 \quad \ldots (ii)$$

Since the entropy at point 1 (s_1) is equal to entropy at point 2 (s_2), therefore equating equations (i) and (ii),

$$0.5443 + 4.934\, x_1 = 4.796 \quad \text{or} \quad x_1 = 0.86$$

$\therefore$ Enthalpy at point 1, $h_1 = h_{f1} + x_1 h_{fg1} = 135.37 + 0.86 \times 1297.68 = 1251.4$ **kJ/kg**

and enthalpy at point 2, $h_2 = h_{f2} + x_2 h_{fg2} = 323.08 + 0.95 \times 1145.8 = 1411.6$ **kJ/kg**

We know that theoretical C.O.P.

$$= \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{1251.4 - 323.08}{1411.6 - 1251.4} = 5.8$$

$\therefore$ Actual C.O.P. $= 0.6 \times 5.8 = 3.48$

Work to be spent corresponding to 1 kW hour,

$$W = 3600 \text{ kJ}$$

$\therefore$ Actual heat extracted or refrigeration effect produced per kW hour

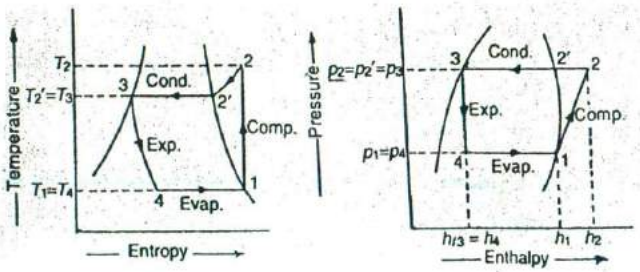
$$= W \times \text{Actual C.O.P.} = 3600 \times 3.48 = 12\,528 \text{ kJ}$$

We know that heat extracted from 1 kg of water at 10°C for the formation of 1 kg of ice at 0°C

$$= 1 \times 4.187 \times 10 + 335 = 376.87 \text{ kJ}$$

$\therefore$ Amount of ice produced $= \dfrac{12\,528}{376.87} = 33.2$ kg/kW hour **Ans.**

36.8. Theoretical Vapour Compression Cycle with Superheated Vapour after Compression



(a) T-s diagram. (b) p-h diagram.

Fig. 36.9. Theoretical vapour compression cycle with superheated vapour after compression.

A vapour compression cycle with superheated vapour after compression is shown on T-s and p-h diagrams in Fig. 36.9 (a) and (b) respectively. In this cycle, the enthalpy or total heat at point 2 is found out with the help of degree of superheat. The degree of superheat may be found out by equating the entropies at points 1 and 2.

Now the coefficient of performance may be found out as usual from the relation,

$$\text{C.O.P.} = \frac{\text{Refrigerating effect}}{\text{Work done}} = \frac{h_1 - h_{f3}}{h_2 - h_1}$$

A little consideration will show that the superheating increases the refrigerating effect and the amount of work done in the compressor. Since the increase in refrigerating effect is less as compared to the increase in work done, therefore, the net effect of superheating is to have low coefficient of performance.

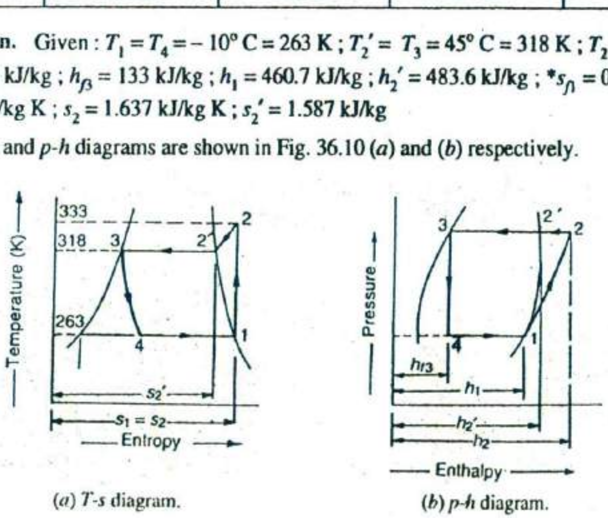
Note : In this cycle, the cooling of superheated vapour will take place in two stages. Firstly, it will be condensed to dry saturated state at constant pressure (shown by graph 2-2′) and secondly it will be condensed at constant temperature (shown by graph 2′-3). The remaining cycle is same as discussed in the last article.

Example 36.5. *A vapour compression refrigerator uses methyl chloride (R-40) and operates between temperature limits of – 10° C and 45° C. At entry to the compressor, the refrigerant is dry saturated and after compression it acquires a temperature of 60° C. Find the C.O.P. of the refrigerator. The relevant properties of methyl chloride are as follows :*

Saturation Temperature in °C	*Enthalpy in kJ/kg*		*Entropy in kJ/kg K*	
	Liquid	*Vapour*	*Liquid*	*Vapour*
–10	*45.4*	*460.7*	*0.183*	*1.637*
45	*133.0*	*483.6*	*0.485*	*1.587*

Solution. Given : $T_1 = T_4 = -10°\text{C} = 263\text{ K}$; $T_2' = T_3 = 45°\text{C} = 318\text{ K}$; $T_2 = 60°\text{C} = 333\text{ K}$; $^*h_{f1} = 45.4$ kJ/kg ; $h_{f3} = 133$ kJ/kg ; $h_1 = 460.7$ kJ/kg ; $h_2' = 483.6$ kJ/kg ; $^*s_{f1} = 0.183$ kJ/kg K ; $^*s_{f3} = 0.485$ kJ/kg K ; $s_2 = 1.637$ kJ/kg K ; $s_2' = 1.587$ kJ/kg

The *T-s* and *p-h* diagrams are shown in Fig. 36.10 (*a*) and (*b*) respectively.



(*a*) *T-s* diagram. (*b*) *p-h* diagram.

Fig. 36.10

Let c_p = Specific heat at constant pressure for superheated vapour.

We know that entropy at point 2,

$$s_2 = s_2' + 2.3\, c_p \log\left(\frac{T_2}{T_2'}\right)$$

* Superfluous data

$$1.637 = 1.587 + 2.3\, c_p \log\left(\frac{333}{318}\right)$$

$$= 1.587 + 2.3\, c_p \times 0.02 = 1.587 + 0.046\, c_p$$

$\therefore$ $$c_p = 1.09$$

and enthalpy at point 2, $$h_2 = h_2' + c_p \times \text{Degree of superheat} = h_2' + c_p (T_2 - T_2')$$

$$= 483.6 + 1.09\,(333 - 318) = 500 \text{ kJ/kg}$$

$$\therefore \text{ C.O.P. of the refrigera tor} = \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{460.7 - 133}{500 - 460.7} = 8.34 \textbf{ Ans.}$$

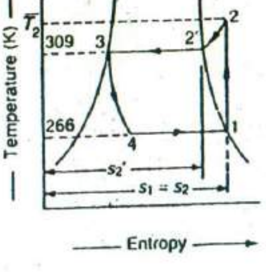
Example 36.6. *A refrigeration machine using R-12 as refrigerant operates between the pressures 2.5 bar and 9 bar. The compression is isentropic and there is no undercooling in the condenser.*

The vapour is in dry saturated condition at the beginning of the compression. Estimate the theoretical coefficient of performance. If the actual coefficient of performance is 0.65 of theoretical value, calculate the net cooling produced per hour. The refrigerant flow is 5 kg per minute. Properties of refrigerant are :

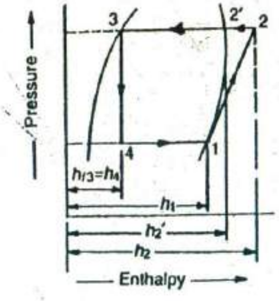
Pressure, bar	*Saturation temperature, °C*	*Enthalpy, kJ/kg*		*Entropy of saturated vapour, kJ/kg K*
		Liquid	*Vapour*	
9.0	*36*	*456.4*	*585.3*	*4.74*
2.5	*–7*	*412.4*	*570.3*	*4.76*

Take c_p for superheated vapour at 9 bar as 0.67 kJ/kg K.

Solution. Given : $T_2' = T_3 = 36°\text{ C} = 309\text{ K}$; $T_1 = T_4 = -7°\text{ C} = 266\text{ K}$; $(\text{C.O.P.})_{actual} = 0.65\,(\text{C.O.P.})_{th}$; $m = 5$ kg/min = 300 kg/h ; $h_{f3} = h_4 = 456.4$ kJ/kg ; $^*h_{f1} = h_{f4} = 412.4$ kJ/kg ; $h_2' = 585.3$ kJ/kg ; $h_1 = 570.3$ kJ/kg ; $s_2' = 4.74$ kJ/kg K ; $s_1 = s_2 = 4.76$ kJ/kg K ; $c_p = 0.67$ kJ/kg K



(*a*) *T-s* diagram.



(*b*) *p-h* diagram.

Fig. 36.11

The *T-s* and *p-h* diagrams are shown in Fig. 36.11 (*a*) and (*b*) respectively.

* Superfluous data

Theoretical coefficient of performance

First of all, let us find the temperature at point 2 (T_2).

We know that entropy at point 2,

$$s_2 = s_2' + 2.3\, c_p \log\left(\frac{T_2}{T_2'}\right)$$

$$4.76 = 4.74 + 2.3 \times 0.67 \log\left(\frac{T_2}{309}\right)$$

$$\therefore \quad \log\left(\frac{T_2}{309}\right) = \frac{4.76 - 4.74}{2.3 \times 0.67} = 0.013$$

$$\frac{T_2}{309} = 1.03 \qquad \text{...(Taking antilog of 0.013)}$$

$$\therefore \quad T_2 = 309 \times 1.03 = 318.3 \text{ K}$$

We know that enthalpy of superheated vapour at point 2,

$$h_2 = h_2' + c_p(T_2 - T_2')$$

$$= 585.3 + 0.67\,(318.3 - 309) = 591.5 \text{ kJ/kg}$$

$\therefore$ Theoretical coefficient of performance,

$$(\text{C.O.P.})_{th} = \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{570.3 - 456.4}{591.5 - 570.3} = 5.37 \textbf{ Ans.}$$

Net cooling produced per hour

We also know that actual C.O.P. of the machine,

$$(\text{C.O.P.})_{actual} = 0.65 \times (\text{C.O.P.})_{th} = 0.65 \times 5.37 = 3.49$$

and actual work done,

$$W_{actual} = h_2 - h_1 = 591.5 - 570.3 = 21.2 \text{ kJ/kg}$$

We know that net cooling (or refrigerating effect) produced per kg of refrigerant

$$= W_{actual} \times (\text{C.O.P.})_{actual} = 21.2 \times 3.49 = 74 \text{ kJ/kg}$$

$\therefore$ Net cooling produced per hour

$$= m \times 74 = 300 \times 74 = 22\,200 \text{ kJ/h}$$

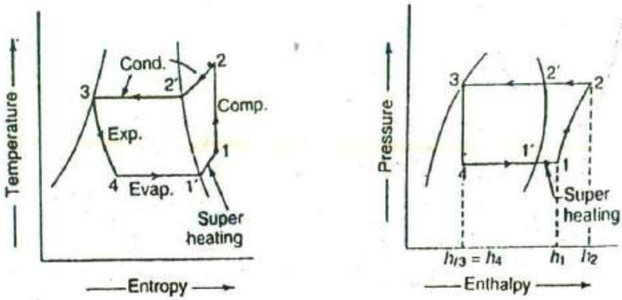
$$= \frac{22\,200}{210 \times 60} = 1.76 \text{ TR} \textbf{ Ans.} \qquad \text{...}(\because 1 \text{ TR} = 210 \text{ kJ/min})$$

36.9. Theoretical Vapour Compression Cycle with Superheated Vapour before Compression

A vapour compression cycle with superheated vapour before compression is shown on *T-s* and *p-h* diagrams in Fig. 36.12 (*a*) and (*b*) respectively. In this cycle, the evaporation starts at point 4 and continues upto point 1′, when it is dry saturated. The vapour is now superheated before entering the compressor upto the point 1.

The coefficient of performance may be found out as usual from the relation,

$$\text{C.O.P.} = \frac{\text{Refrigerating effect}}{\text{Work done}} = \frac{h_1 - h_{f3}}{h_2 - h_1}$$



(a) T-s diagram. (b) p-h diagram.

Fig. 36.12. Theoretical vapour compression cycle with superheated vapour before compression.

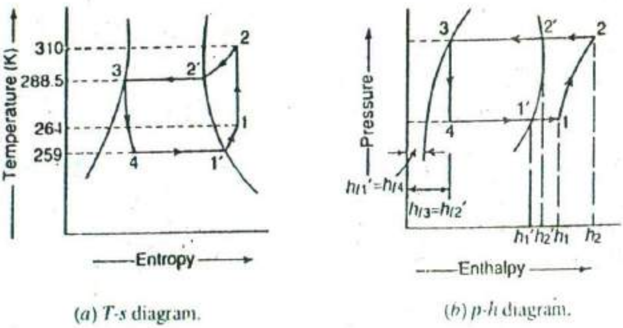
Note : In this cycle, the heat is absorbed (or extracted) in two stages. Firstly from point 4 to point 1′ and from point 1′ to point 1. The remaining cycle is same as discussed in the previous article.

Example 36.7. *A vapour compression refrigeration plant works between pressure limits of 5.3 bar and 2.1 bar. The vapour is superheated at the end of compression, its temperature being 37° C. The vapour is superheated by 5° C before entering the compressor.*

If the specific heat of superheated vapour is 0.63 kJ/kg K, find the coefficient of performance of the plant. Use the data given below :

Pressure, bar	*Saturation temperature, °C*	*Liquid heat, kJ/kg*	*Latent heat, kJ/kg*
5.3	*15.5*	*56.15*	*144.9*
2.1	*– 14.0*	*25.12*	*158.7*

Solution. Given : $p_2 = 5.3$ bar ; $p_1 = 2.1$ bar ; $T_2 = 37°\text{ C} = 310$ K ; $T_1 - T_1' = 5°$ C ; $c_{pv} = 0.63$ kJ/kg K ; $T_2' = 15.5°\text{ C} = 288.5$ K ; $T_1' = -14°\text{ C} = 259$ K ; $h_{f3} = h_{f2}' = 56.15$ kJ/kg ; $h_{f1}' = 25.12$ kJ/kg ; $h_{fg2}' = 144.9$ kJ/kg ; $h_{fg1}' = 158.7$ kJ/kg



(a) T-s diagram. (b) p-h diagram.

Fig. 36.13

The T-s and p-h diagrams are shown in Fig. 36.13 (a) and (b) respectively.

We know that enthalpy of vapour at point 1,

$$h_1 = h_1' + c_{pv}(T_1 - T_1') = (h_{f1}' + h_{fg1}') + c_{pv}(T_1 - T_1')$$

$$= (25.12 + 158.7) + 0.63 \times 5 = 186.97 \text{ kJ/kg}$$

Similarly, enthalpy of vapour at point 2,

$$h_2 = h_2' + c_{pv}(T_2 - T_2') = (h_{f2}' + h_{fg2}') + c_{pv}(T_2 - T_2')$$

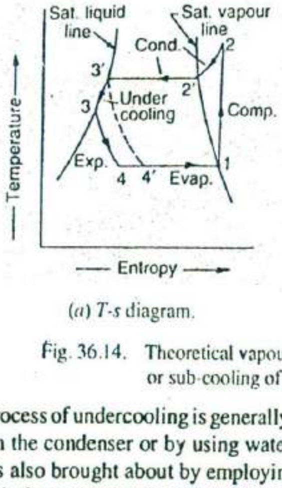
$$= (56.15 + 144.9) + 0.63\,(310 - 288.5) = 214.6 \text{ kJ/kg}$$

∴ Coefficient of performance of the plant,

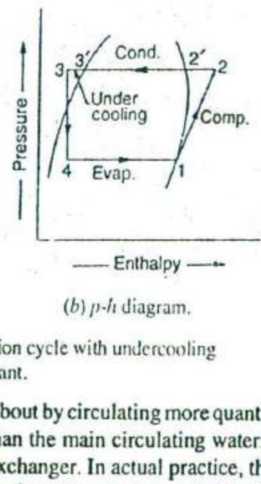
$$\text{C.O.P.} = \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{186.97 - 56.15}{214.6 - 186.97} = \frac{130.82}{27.63} = 4.735 \text{ Ans.}$$

36.10. Theoretical Vapour Compression Cycle with Undercooling or Sub-cooling of Refrigerant

Sometimes, the refrigerant, after condensation process 2′-3′, is cooled below the saturation temperature (T_3') before expansion by throttling. Such a process is called *undercooling* or *subcooling* of the refrigerant and is generally done along the liquid line as shown in Fig. 36.14 (*a*) and (*b*). The ultimate effect of the undercooling is to increase the value of coefficient of performance under the same set of conditions.



(*a*) *T-s* diagram.



(*b*) *p-h* diagram.

Fig. 36.14. Theoretical vapour compression cycle with undercooling or sub-cooling of the refrigerant.

The process of undercooling is generally brought about by circulating more quantity of cooling water through the condenser or by using water colder than the main circulating water. Sometimes, this process is also brought about by employing a heat exchanger. In actual practice, the refrigerant is superheated after compression and undercooled before throttling, as shown in Fig. 36.14 (*a*) and (*b*). A little consideration will show, that the refrigerating effect is increased by adopting both the superheating and undercooling process as compared to a cycle without them, which is shown by dotted lines in Fig. 36.14 (*a*).

In this case, the refrigerating effect or heat absorbed or extracted,

$$R_E = h_1 - h_4 = h_1 - h_{f3} \qquad \ldots (\because h_4 = h_{f3})$$

and work done, $W = h_2 - h_1$

$$\therefore \quad \text{C.O.P.} = \frac{\text{Refrigerating effect}}{\text{Work done}} = \frac{h_1 - h_{f3}}{h_2 - h_1}$$

Note : The value of h_{f3} may be found out from the relation,

$$h_{f3} = h_{f3}' - c_p \times \text{Degree of undercooling}$$

Example 36.8. *A vapour compression refrigerator uses R-12 as refrigerant and the liquid evaporates in the evaporator at – 15°C. The temperature of this refrigerant at the delivery from the compressor is 15° C when the vapour is condensed at 10° C. Find the coefficient of performance if 1. there is no undercooling ; and 2. the liquid is cooled by 5° C before expansion by throttling.*

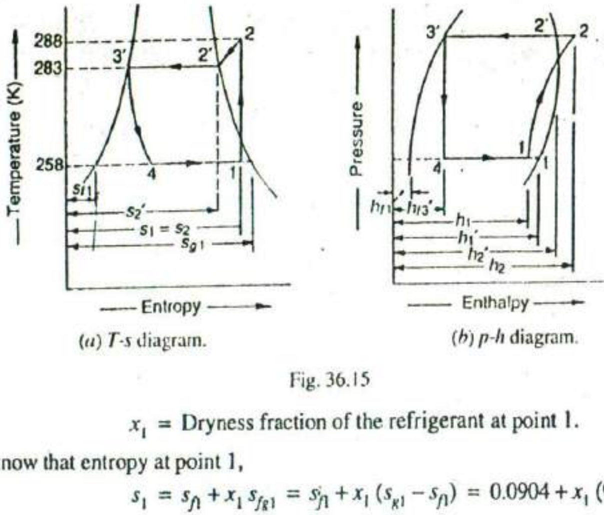
Take specific heat at constant pressure for the superheated vapour as 0.64 kJ/kg K and that for liquid as 0.94 kJ/kg K. The other properties of refrigerant are as follows :

Temperature in °C	*Enthalpy in kJ/kg*		*Entropy in kJ/kg K*	
	Liquid	*Vapour*	*Liquid*	*Vapour*
–15	*22.3*	*180.88*	*0.0904*	*0.7051*
+10	*45.4*	*191.76*	*0.1750*	*0.6921*

Solution. Given : $T_1 = T_4 = -15°\text{ C} = 258\text{ K}$; $T_2 = 15°\text{ C} = 288\text{ K}$; $T_2' = 10°\text{ C} = 283\text{ K}$; $c_{pv} = 0.64$ kJ/kg K ; $c_{pl} = 0.94$ kJ/kg K ; $h_{f1} = 22.3$ kJ/kg ; $h_{f3}' = 45.4$ kJ/kg ; $h_1' = 180.88$ kJ/kg ; $h_2' = 191.76$ kJ/kg ; $s_{f1} = 0.0904$ kJ/kg K ; $^*s_{f2} = 0.1750$ kJ/kg K ; $s_{g1} = 0.7051$ kJ/kg K ; $s_2' = 0.6921$ kJ/kg K

1. *Coefficient of performance if there is no undercooling*

The *T-s* and *p-h* diagrams, when there is no undercooling, are shown in Fig. 36.15 (*a*) and (*b*) respectively.



(*a*) *T-s* diagram. (*b*) *p-h* diagram.

Fig. 36.15

Let x_1 = Dryness fraction of the refrigerant at point 1.

We know that entropy at point 1,

$$s_1 = s_{f1} + x_1 s_{fg1} = s_{f1} + x_1 (s_{g1} - s_{f1}) = 0.0904 + x_1 (0.7051 - 0.0904)$$

$$= 0.0904 + 0.6147\, x_1 \qquad \ldots (i)$$

and entropy at point 2,

$$s_2 = s_2' + 2.3\, c_{pv} \log \left(\frac{T_2}{T_2'} \right) = 0.6921 + 2.3 \times 0.64 \log \left(\frac{288}{283} \right)$$

$$= 0.6921 + 2.3 \times 0.64 \times 0.0077 = 0.7034 \qquad \ldots (ii)$$

* Superfluous data

Since the entropy at point 1 is equal to entropy at point 2, therefore equating equations (*i*) and (*ii*),

$$0.0904 + 0.6147\, x_1 = 0.7034 \quad \text{or} \quad x_1 = 0.997$$

We know that the enthalpy at point 1,

$$h_1 = h_{f1} + x_1\, h_{fg1} = h_{f1} + x_1\,(h_1' - h_{f1}) \qquad \ldots (\because h_{fg1} = h_1' - h_{f1})$$

$$= 22.3 + 0.997\,(180.88 - 22.3) = 180.4 \text{ kJ/kg}$$

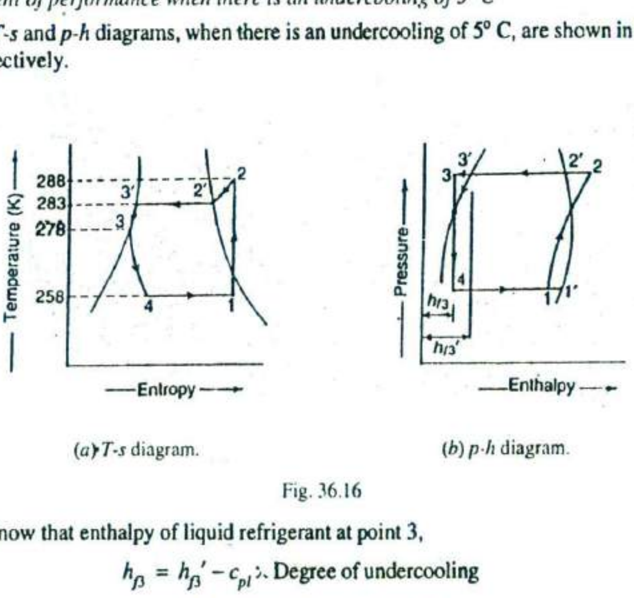
and enthalpy at point 2, $h_2 = h_2' + c_{pv}\,(T_2 - T_2')$

$$= 191.76 + 0.64\,(288 - 283) = 194.96 \text{ kJ/kg}$$

$$\therefore \qquad \text{C.O.P.} = \frac{h_1 - h_{f3}'}{h_2 - h_1} = \frac{180.4 - 45.4}{194.96 - 180.4} = 9.27 \ \textbf{Ans.}$$

2. *Coefficient of performance when there is an undercooling of 5° C*

The *T-s* and *p-h* diagrams, when there is an undercooling of 5° C, are shown in Fig. 36.16 (*a*) and (*b*) respectively.



(*a*) *T-s* diagram. (*b*) *p-h* diagram.

Fig. 36.16

We know that enthalpy of liquid refrigerant at point 3,

$$h_{f3} = h_{f3}' - c_{pl} \times \text{Degree of undercooling}$$

$$= 45.4 - 0.94 \times 5 = 40.7 \text{ kJ/kg}$$

$$\therefore \qquad \text{C.O.P.} = \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{180.4 - 40.7}{194.96 - 180.4} = 9.59 \ \textbf{Ans.}$$

Example 36.9. *A food storage locker requires a refrigeration capacity of 12 TR and works between the evaporating temperature of – 8° C and condensing temperature of 30° C. The refrigerant R-12 is subcooled by 5° C before entry to expansion valve and the vapour is superheated to – 2° C before leaving the evaporator coils. Determine : 1. coefficient of performance ; and 2. theoretical power per tonne of refrigeration.*

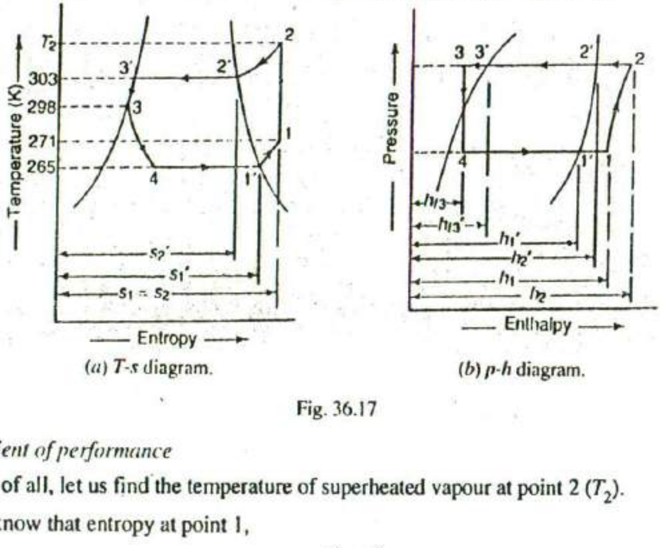
Use the following data for R-12

Saturation temperature, °C	Pressure, bar	Enthalpy, kJ/kg		Entropy, kJ/kg K	
		Liquid	Vapour	Liquid	Vapour
–8	2.354	28.72	184.07	0.1149	0.7007
30	7.451	64.59	199.62	0.2400	0.6853

The specific heat of liquid R-12 is 1.235 kJ/kg K, and of vapour R-12 is 0.733 kJ/kg K.

Solution. Given : Q = 12 TR ; $T_1' = -8°C$ = 265 K; $T_2' = 30°C$ = 303 K; $T_3' - T_3 = 5°C$; $T_1 = -2°C$ = 271 K ; h_{f1} = 28.72 kJ/kg ; h_{f3}' = 64.59 kJ/kg ; h_1' = 184.07 kJ/kg ; h_2' = 199.62 kJ/kg ; *s_{f1} = 0.1149 kJ/kg K ; *s_{f3} = 0.2400 kJ/kg K ; s_1' = 0.7007 kJ/kg K ; s_2' = 0.6853 kJ/kg K ; c_{pl} = 1.235 kJ/kg K ; c_{pv} = 0.733 kJ/kg K

The T-s and p-h diagrams are shown in Fig. 36.17 (*a*) and (*b*) respectively.



(*a*) T-s diagram. (*b*) p-h diagram.

Fig. 36.17

1. *Coefficient of performance*

First of all, let us find the temperature of superheated vapour at point 2 (T_2).

We know that entropy at point 1,

$$s_1 = s_1' + 2.3\, c_{pv} \log \left(\frac{T_1}{T_1'} \right)$$

$$= 0.7007 + 2.3 \times 0.733 \log \left(\frac{271}{265} \right) = 0.7171 \qquad \ldots (i)$$

and entropy at point 2,

$$s_2 = s_2' + 2.3\, c_{pv} \log \left(\frac{T_2}{T_2'} \right)$$

$$= 0.6853 + 2.3 \times 0.733 \log \left(\frac{T_2}{303} \right)$$

$$= 0.6853 + 1.686 \log \left(\frac{T_2}{303} \right) \qquad \ldots (ii)$$

* Superfluous data

Since the entropy at point 1 is equal to entropy at point 2, therefore equating equations (*i*) and (*ii*),

$$0.7171 = 0.6853 + 1.686 \log\left(\frac{T_2}{303}\right)$$

or
$$\log\left(\frac{T_2}{303}\right) = \frac{0.7171 - 0.6853}{1.686} = 0.0188$$

$$\frac{T_2}{303} = 1.0444 \qquad \text{...(Taking antilog of 0.0188)}$$

$$\therefore \qquad T_2 = 316.4\text{ K or } 43.4\,°\text{C}$$

We know that enthalpy at point 1,

$$h_1 = h_1' + c_{pv}(T_1 - T_1')$$
$$= 184.07 + 0.733\,(271 - 265) = 188.47\text{ kJ/kg}$$

Enthalpy at point 2,
$$h_2 = h_2' + c_{pv}(T_2 - T_2')$$
$$= 199.62 + 0.733\,(316.4 - 303) = 209.44\text{ kJ/kg}$$

and enthalpy of liquid refrigerant at point 3,

$$h_{f3} = h_{f3}' - c_{pl}(T_3' - T_3)$$
$$= 64.59 - 1.235 \times 5 = 58.42\text{ kJ/kg}$$

$$\therefore \qquad \text{C.O.P.} = \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{188.47 - 58.42}{209.44 - 188.47} = \frac{130.05}{20.97} = 6.2 \textbf{ Ans.}$$

2. *Theoretical power per tonne of refrigeration*

We know that the heat extracted or refrigerating effect per kg of the refrigerant,

$$R_E = h_1 - h_{f3} = 188.47 - 58.42 = 130.05\text{ kJ/kg}$$

and the refrigerating capacity of the system,

$$Q = 12\text{ TR} = 12 \times 210 = 2520\text{ kJ/min} \qquad \text{...(Given)}$$

$\therefore$ Mass flow of the refrigerant,

$$m_R = \frac{Q}{R_E} = \frac{2520}{130.05} = 19.4\text{ kg/min}$$

Work done during compression of the refrigerant

$$= m_R(h_2 - h_1) = 19.4\,(209.44 - 188.47) = 406.82\text{ kJ/min}$$

$\therefore$ Theoretical power per tonne of refrigeration

$$= \frac{406.82}{60 \times 12} = 0.565\text{ kW/TR} \textbf{ Ans.}$$

36.11. Actual Vapour Compression Cycle

The actual vapour compression cycle differs from the theoretical vapour compression cycle in many ways, some of which are unavoidable and cause losses. The main deviations between the theoretical cycle and actual cycle are as follows :

1. The vapour refrigerant leaving the evaporator is in superheated state.

2. The compression of refrigerant is neither isentropic nor polytropic.
3. The liquid refrigerant before entering the expansion valve is sub-cooled in the condenser.
4. The pressure drops in the evaporator and condenser.

The actual vapour compression cycle on *T-s* diagram is shown in Fig. 36.18. The various processes are discussed below :

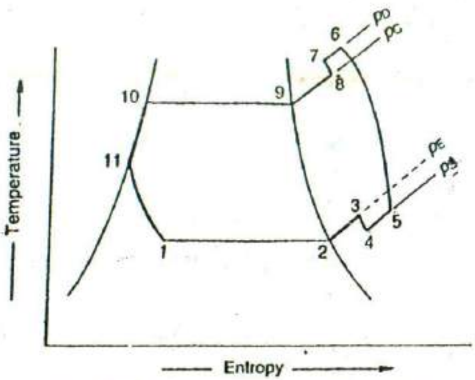


Fig. 36.18. Actual vapour compression cycle.

(*a*) *Process 1-2-3.* This process shows the flow of refrigerant in the evaporator. The point 1 represents the entry of refrigerant into the evaporator and the point 3 represents the exit of refrigerant from the evaporator in a superheated state. The point 3 also represents the entry of refrigerant into the compressor in a superheated condition. The superheating of vapour refrigerant from point 2 to point 3 may be due to :

(*i*) automatic control of expansion valve so that the refrigerant leaves the evaporator as the superheated vapour.

(*ii*) picking up of larger amount of heat from the evaporator through pipes located within the cooled space.

(*iii*) picking up of heat from the suction pipe, *i.e.* the pipe connecting the evaporator delivery and the compressor suction valve.

In the first and second case of superheating the vapour refrigerant, the refrigerating effect as well as the compressor work is increased. The coefficient of performance, as compared to saturation cycle at the same suction pressure may be greater, less or unchanged.

The superheating also causes increase in the required displacement of compressor and load on the compressor and condenser. This is indicated by 2-3 on *T-s* diagram as shown in Fig. 36.18.

(*b*) *Process 3-4-5-6-7-8.* This process represents the flow of refrigerant through the compressor. When the refrigerant enters the compressor through the suction valve at point 3, the pressure falls to point 4 due to frictional resistance to flow. Thus the actual suction pressure (p_S) is lower than the evaporator pressure (p_E). During suction and prior to compression, the temperature of the cold refrigerant vapour rises to point 5 when it comes in contact with the compressor cylinder walls. The actual compression of the refrigerant is shown by 5-6 in Fig. 36.18, which is neither isentropic nor polytropic. This is due to the heat transfer between the cylinder walls and the vapour refrigerant. The temperature of the cylinder walls is some-what in between the temperature of cold suction vapour refrigerant and hot discharge vapour refrigerant. It may be assumed that the heat absorbed by the vapour refrigerant from the cylinder walls during the first part of the compression stroke is equal to heat rejected by the vapour refrigerant to the cylinder walls. Like the heating effect at suction given by 4-5 in Fig. 36.18, there is a cooling effect at discharge as given by 6-7. These heating and cooling effects take place at constant pressure. Due to the frictional resistance of flow, there is a pressure drop *i.e.* the actual discharge pressure (p_D) is more than the condenser pressure (p_C).

(*c*) *Process 8-9-10-11.* This process represents the flow of refrigerant through the condenser. The process 8-9 represents the cooling of superheated vapour refrigerant to the dry saturated state. The process 9-10 shows the removal of latent heat which changes the dry saturated refrigerant into liquid refrigerant. The process 10-11 represents the sub-cooling of liquid refrigerant in the condenser before passing through the expansion valve. This is desirable as it increases the refrigerating effect per kg of the refrigerant flow. It also reduces the volume of the refrigerant partially evaporated from the liquid refrigerant while passing through the expansion valve. The increase in refrigerating effect can be obtained by large quantities of circulating cooling water which should be at a temperature much lower than the condensing temperatures.

(*d*) *Process 11-1* This process represents the expansion of sub-cooled liquid refrigerant by throttling from the condenser pressure to the evaporator pressure.

36.12. Vapour Absorption Refrigeration System

The idea of a vapour absorption refrigeration system is to avoid compression of the refrigerant. In this type of refrigeration system, the vapour produced by the evaporation of the refrigerant, in the cold chamber, passes into a vessel containing a homogeneous mixture of ammonia and water (known as aqua-ammonia). In this chamber, the vapour is absorbed, which maintains constant low pressure, thus facilitating its further evaporation.

The refrigerant is liberated in the vapour state subsequently by the direct application of heat, and at such a pressure that condensation can be effected at the temperature of the air or by cold water. Fig. 36.19 shows a schematic arrangement of the essential elements of such a system.

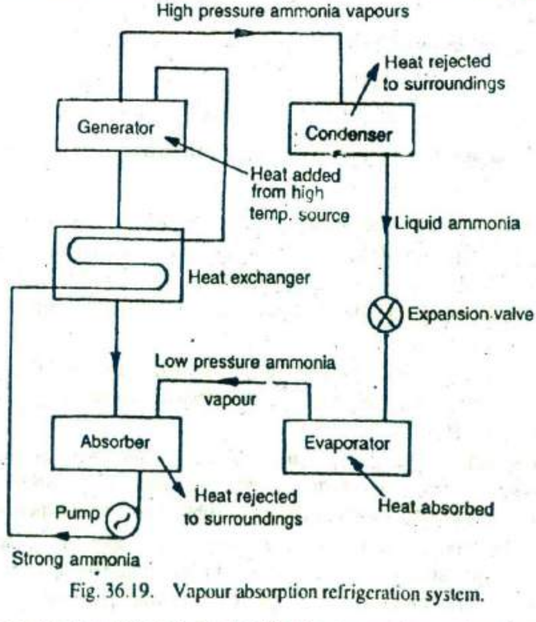


Fig. 36.19. Vapour absorption refrigeration system.

The low pressure ammonia vapour, leaving the evaporator, enters the absorber where it is absorbed in the weak ammonia solution. This process takes place at a temperature slightly above than that of the surroundings. In this process, some heat is transferred to the surroundings. The strong ammonia solution is then pumped through a heat exchanger to the generator, where a high pressure and temperature is maintained. Under these conditions, the ammonia vapour is driven from the solution. This happens because of the heat transfer from a high temperature source. The ammonia vapour enters into the condenser, where it gets condensed, in the same way as in the vapour compression system. The weak ammonia solution returns back to the absorber through a heat exchanger.

The equipment used in a vapour absorption system is somewhat complicated than in a vapour compression system. It can be economically justified only in those cases where a suitable source of heat is available which would otherwise be wasted.

The coefficient of performance of this refrigerator is given by :

$$\text{C.O.P.} = \frac{\text{Heat absorbed during evaporation}}{\text{Work done by pump} + \text{Heat supplied in heat exchanger}}$$

Mathematically,
$$\text{C.O.P.} = \frac{T_3\,(T_1 - T_2)}{T_1\,(T_2 - T_3)}$$

where T_1 = Temperature at which the working substance receives heat,

T_2 = Temperature of the cooling water, and

T_3 = Evaporator temperature.

Example 36.10. *In an absorption type refrigerator, heating, cooling and refrigeration takes place at the temperature of 100° C ; 20° C and – 5° C respectively. Find the theoretical C.O.P. of the system.*

Solution. Given : T_1 = 100° C = 373 K ; T_2 = 20° C = 293 K ; T_3 = – 5° C = 268 K

We know that C.O.P. of the system

$$= \frac{T_3\,(T_1 - T_2)}{T_1\,(T_2 - T_3)} = \frac{268\,(373 - 293)}{373\,(293 - 268)} = 2.3 \text{ Ans.}$$

36.13. Advantages of Vapour Absorption Refrigeration System over Vapour Compression Refrigeration System

Following are the advantages of vapour absorption system over vapour compression system :

1. In the vapour absorption system, the only moving part of the entire system is a pump which has a small motor. Thus, the operation of this system is essentially quiet and is subjected to little wear.

The vapour compression system of the same capacity has more wear, tear and noise due to moving parts of the compressor.

2. The vapour absorption system uses heat energy to change the condition of the refrigerant from the evaporator. The vapour compression system uses mechanical energy to change the condition of the refrigerant from the evaporator.

3. The vapour absorption systems are usually designed to use steam, either at high pressure or low pressure. The exhaust steam from furnaces and solar energy may also be used. Thus, this system can be used where the electric power is difficult to obtain or is very expensive.

4. The vapour absorption systems can operate at reduced evaporator pressure and temperature by increasing the steam pressure to the generator, with little decrease in capacity. But the capacity of vapour compression system drops rapidly with lowered evaporator pressure.

5. The load variations does not effect the performance of a vapour absorption system. The load variations are met by controlling the quantity of aqua circulated and the quantity of steam supplied to the generator.

The performance of a vapour compression system at partial loads is poor.

6. In the vapour absorption system, the liquid refrigerant leaving the evaporator has no bad effect on the system except that of reducing the refrigerating effect. In the vapour compression system, it is essential to superheat the vapour refrigerant leaving the evaporator so that no liquid may enter the compressor.

7. The vapour absorption systems can be built in capacities well above 1000 tonne of refrigeration each which is the largest size for single compressor units.

8. The space requirements and automatic control requirements favour the absorption system more and more as the desired evaporator temperature drops.

36.14. Ammonia-Hydrogen (Electrolux) Refrigerator

In small domestic installations, working on the ammonia absorption process, the pump may be omitted by the introduction of hydrogen into the low pressure side. The ammonia acts, normally, under its partial pressure. The total pressure is arranged to be practically uniform throughout the system. Thus the weak solution, passing from the boiler to the absorber, moves under gravity. The flow of strong solution, in the opposite direction, is assisted by a vertical pipe between the boiler and absorber, which is heated at its lower end by a small heating coil or gas jet. The electrolux refrigerator, as shown in Fig. 36.20, makes use of this principle.

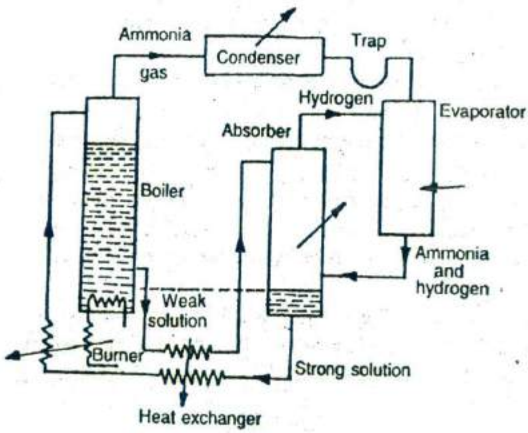


Fig. 36.20. Ammonia-Hydrogen refrigerator.

The chief advantage of this type of refrigerator is that no compressor, pump or fan is required in it. Therefore, there is no noise due to moving parts. Moreover, there is no machinery to give mechanical trouble. The coefficient of performance of this refrigerator is given by :

$$\text{C.O.P.} = \frac{\text{Heat absorbed by evaporator}}{\text{Heat supplied by burner}}$$

36.15. Properties of a Refrigerant

A substance which absorbs heat through expansion or vaporisation is termed as a refrigerant. An ideal refrigerant should possess chemical, physical and thermodynamic properties which permit its efficient application in the refrigerating system. An ideal refrigerant should have the following properties :

1. Low boiling point.
2. High critical temperature.

3. High latent heat of vaporisation.
4. Low specific heat of liquid.
5. Low specific volume of vapour.
6. Non-corrosive to metal.
7. Non-flammable and non-explosive.
8. Non-toxic.
9. Easy to liquify at moderate pressure and temperature.
10. Easy of locating leaks by odour or suitable indicator.
11. Low cost.
12. Mixes well with oil.

36.16. Refrigerants Commonly Used in Practice

Through there are many refrigerants which are commonly used, yet the following are important from the subject point of view :

1. *Ammonia* (NH_3). It is one of the oldest and the most commonly used of all the refrigerants. It is highly toxic and flammable. It has a boiling point of – 333° C and a liquid specific gravity of 0.684 at atmospheric pressure. It is widely used in larger industrial and commercial reciprocating compression systems, where high toxicating is of secondary importance. It is also widely used as a refrigerant in absorption systems.

2. *Carbon dioxide* (CO_2). It is a colourless and odourless gas, and is heavier than atmospheric air. It has a boiling point of – 77.6° C and a liquid specific gravity of 1.56 at atmospheric pressure. It is nontoxic and non-flammable, but has extremely high operating pressure (70 bar). It is not widely used, because of its high power requirements per tonne of refrigeration and high operating pressure.

3. *Sulphur dioxide* (SO_2). It is a colourless gas or liquid. It is a extremely toxic and has a pungent irritating odour. It is non-explosive and non-flammable. It has a boiling point of – 10.5° C and a liquid specific gravity of 1.36. It is used in small-tonnage commercial machines (hermetically sealed units).

4. *Freon-12**. The entire Freon group is white in colour and odourless. They are all non-flammable and non-toxic. Freon-12 is mostly used out of all the Freon group. It has a boiling point of – 30° C and operating pressure of 8 bar. It is widely used for domestic refrigerators.

EXERCISES

1. A refrigerator works between – 7° C and – 27° C. The vapour is dry at the end of isentropic compression. There is no undercooling, and the evaporation is by throttle valve. Find : 1. C.O.P. ; and 2. power of the compressor to remove 175 kJ/min.

The properties of the refrigerant are as under ;

Temperature °C	*Sensible heat kJ/kg*	*Latent heat kJ/kg*	*Entropy Liquid kJ/kg K*	*Entropy of dry saturated vapour, kJ/kg K*
– 7	– 29.4	1298	– 0.1088	4.748
27	124.8	1172.4	0.427	4.334

[Ans. 7.51 ; 0.55]

2. An ammonia vapour compression refrigerator works between the temperature limits of – 6.7° C and 26.7° C. The vapour is dry at the end of compression, and there is no undercooling of the liquid, which is

* Other types of refrigerants are Freon-11, Freon-13, Freon-21, and Freon-22, etc. (abbreviated as F-11, F-13 and so on). These are also abbreviated as Refrigerant-11, Refrigerant-13, etc. or simply R-11, R-13, etc.

throttled to the lower temperature. Estimate the C.O.P. of the machine. Properties of ammonia given below should be used :

Temperature, °C	*Enthalpy, kJ/kg*			*Entropy, kJ/kg K*	
	Sensible (h_f)	*Latent (h_{fg})*	*Vapour (h)*	*Liquid (s_f)*	*Vapour (s_g)*
− 6.7	−29.3	1293.8	1264.5	− 0.113	4.752
26.7	125.6	1172.4	1297.9	0.427	4.334

[Ans. 7.1]

3. Determine the theoretical coefficient of performance for CO_2 refrigerating machine working between the limits of pressures of 65.1 bar and 30.8 bar. The CO_2 during the suction stroke has a dryness fraction of 0.6. Properties of refrigerant are :

Pressure, bar	*Temperature, °C*	*Liquid heat, kJ/kg*	*Latent heat, kJ/kg*	*Entropy of liquid, kJ/kg K*
65.1	25	81.23	121.42	0.2512
30.8	− 5	− 7.54	245.36	− 0.042

How many tonnes of ice would a machine, working between the same limits and having a relative coefficient of performance of 40%, make in 24 hours ? The water for the ice is supplied at 10° C and the compressor takes 6.8 kg of CO_2 per minute. Latent heat of ice is 335 kJ/kg. [Ans. 3.3 ; 0.606 tonnes]

4. An ammonia refrigerator produces 30 tonnes of ice from and at 0° C in 24 hours. The temperature range of the compressor is from 25° C to − 15° C. The vapour is dry saturated at the end of compression and an expansion valve is used. Assume a coefficient of performance to be 60% of the theoretical value. Calculate the power required to drive the compressor. Latent heat of ice = 335 kJ/kg.

Properties of ammonia are :

Temperature, °C	*Enthalpy, kJ/kg.*		*Entropy, kJ/kg K*	
	Liquid	*Vapour*	*Liquid*	*Vapour*
25	298.9	1465.84	1.1242	5.0391
− 15	112.34	1426.54	0.4572	5.5490

[Ans. 33.24 kW]

5. An ammonia refrigerating machine fitted with an expansion valve works between the temperature limits of − 10° C and 30° C. The vapour is 95% dry at the end of isentropic compression and the fluid leaving the condenser is at 30° C. If the actual coefficient of performance is 60% of the theoretical, find the ice produced per kW hour at 0° C from water at 10° C. The latent heat of ice is 335 kJ/kg. The ammonia has the following properties :

Temperature, °C	*Liquid heat kJ/kg*	*Latent heat kJ/kg*	*Entropy, kJ/kg K*	
			Liquid	*Vapour*
30	323.08	1145.79	1.2037	4.9842
− 10	135.37	1297.68	0.5443	5.4770

[Ans. 33.24 kg/kWh]

6. A R-12 refrigerating machine works on vapour compression cycle. The temperature of refrigerant in the evaporator is − 20° C. The vapour is dry saturated when it enters the compressor and leaves it in a superheated condition. The condenser temperature is 30° C. Assuming specific heat at constant pressure for R-12 in the superheated condition as 1.884 kJ/kg K, determine :

1. condition of vapour at the entrance to condenser,

2. condition of vapour at the entrance to the evaporator, and
3. theoretical C.O.P. of the machine.

The properties of R-12 are :

Temperature, °C	*Enthalpy, kJ/kg*		*Entropy, kJ/kg K*	
	Liquid	*Vapour*	*Liquid*	*Vapour*
−20	17.82	178.73	0.0731	0.7087
30	64.59	199.62	0.2400	0.6853

[**Ans.** 33.8° C, 29% dry ; 4.07]

7. A CO_2 refrigerating plant fitted with an expansion valve, works between the pressure limits of 56.25 bar and 21.2 bar. The vapour is compressed isentropically and leaves the compressor cylinder at 32° C and with a total heat of 246.2 kJ/kg. The condensation takes place at 18.5° C in the condenser and there is no undercooling of the liquid. Determine the theoretical coefficient of performance of the plant. The properties of CO_2 are :-

Pressure bar	*Saturation temperature °C*	*Enthalpy, kJ/kg*		*Entropy, kJ/kg K*	
		Liquid	*Vapour*	*Liquid*	*Vapour*
56.25	18.5	52.75	214.37	0.1675	0.7244
21.2	−18.0	−37.68	234.90	−0.1507	0.9170

[**Ans.** 4.84]

8. A vapour compression refrigerator works between the temperature limits of − 20° C and 25° C. The refrigerant leaves the compressor in dry saturated condition. If the liquid refrigerant is undercooled to 20° C before entering the throttle valve, determine :

1. work required to drive the compressor,
2. refrigerating effect produced per kg of the refrigerant, and
3. theoretical C.O.P.

Assume specific heat of the refrigerant as 1.15. The properties of the refrigerant are :

Temperature, °C	*Enthalpy, kJ/kg*		*Entropy, kJ/kg K*	
	Liquid	*Vapour*	*Liquid*	*Vapour*
−20	327.4	1655.9	3.8416	9.09
25	536.3	1703.3	4.5956	8.50

[**Ans.** 189.7 kJ/kg ; 990.2 kJ/kg ; 5.01]

9. A food storage chamber requires a refrigeration system of 12 TR capacity with an evaporator temperature of − 8° C and condenser temperature of 30° C. The refrigerant R-12 is subcooled by 5° C before entering the throttle valve, and the vapour is superheated by 6° C before entering the compressor. If the liquid and vapour specific heats are 1.235 and 0.733 kJ/kg K respectively, find : 1. refrigerating effect per kg ; 2. mass of refrigerant circulated per minute ; and 3. coefficient of performance.

The relevant properties of the refrigerant R-12 are given below :

Saturation temperature, °C	*Enthalpy, kJ/kg*		*Entropy, kJ/ K*	
	Liquid	*Vapour*	*Liquid*	*Vapour*
−8	28.72	184.07	0.1149	0.7007
30	64.59	199.62	0.2400	0.6853

[**Ans.** 130.05 kJ/kg ; 19.4 kg ; 6.2]

QUESTIONS

1. Mention the advantages of vapour compression refrigeration system over air refrigeration system.

2. Draw the layouts of a vapour compression refrigerating system. State the function of each of the component and show the thermodynamic processes on a pressure-enthalpy diagram.

3. Sketch the *T-s* and *p-h* diagrams for the vapour compression cycles when the vapour after compression is (*i*) dry saturated, and (*ii*) wet.

4. Why in practice a throttle valve is used in vapour compression refrigerator rather than an expansion cylinder to reduce pressure between the condenser and the evaporator ?

5. What is sub-cooling and superheating ? Explain with the help of diagram. Why is superheating considered to be good in certain cases ?

6. Establish how an actual cycle differs from a theoretical vapour compression cycle.

7. Describe briefly with the help of a diagram, the vapour absorption system of refrigeration. In what way this system is advantageous over the vapour compression system ?

8. State the properties of a good refrigerant. What are the normal refrigerants used.

OBJECTIVE TYPE QUESTIONS

1. During a refrigeration cycle, heat is rejected by the refrigerant in a
   (*a*) compressor (*b*) condenser (*c*) evaporator (*d*) expansion valve
2. In a vapour compression system, the condition of refrigerant before entering the compres sor is
   (*a*) saturated liquid (*b*) wet vapour
   (*c*) dry saturated liquid (*d*) superheated vapour
3. The highest temperature during the cycle, in a vapour compression system, occurs afte
   (*a*) compression (*b*) condensation (*c*) expansion (*d*) evaporation
4. In a vapour compression system, the lowest temperature during the cycle occurs after
   (*a*) compression (*b*) condensation (*c*) expansion (*d*) evaporation
5. The sub-cooling in a refrigeration cycle
   (*a*) does not alter C.O.P. (*b*) increases C.O.P. (*c*) decreases C.O.P.
6. The refrigerant, commonly used in vapour absorption refrigeration systems, is
   (*a*) sulphur dioxide (*b*) ammonia (*c*) freon (*d*) aqua-ammonia
7. In ammonia-hydrogen refrigerator,
   (*a*) ammonia is absorbed in hydrogen (*b*) ammonia is absorbed in water
   (*c*) ammonia evaporates in hydrogen (*d*) hydrogen evaporates in ammonia
8. The boiling point of ammonia is
   (*a*) $-10.5°$ C (*b*) $-30°$ C (*c*) $-33.3°$ C (*d*) $-77.6°$ C
9. Which of the following refrigerant has the lowest boiling point ?
   (*a*) Ammonia (*b*) Carbon dioxide (*c*) Sulphur dioxide (d) Freon-12
10. Which of the following refrigerant is highly toxic and flammable ?
   (*a*) Ammonia (*b*) Carbon dioxide (*c*) Sulphur dioxide (*d*) Freon-12

ANSWERS

1. (*b*)	2. (*d*)	3. (*a*)	4. (*d*)	5. (*b*)
6. (*d*)	7. (*c*)	8. (*c*)	9. (*b*)	10. (*a*)