

Psychrometry

1. Introduction. 2. Psychrometric Terms. 3. Dalton's Law of Partial Pressures. 4. Psychrometric Relations. 5. Enthalpy (Total Heat) of Moist Air. 6. Thermodynamic Wet Bulb Temperature or Adiabatic Saturation Temperature. 7. Psychrometric Chart. 8. Psychrometric Processes. 9. Sensible Cooling. 10. Sensible Heating. 11. Humidification and Dehumidification. 12. Sensible Heat Factor. 13. Cooling and Dehumidification. 12. Heating and Humidification. 15. Adiabatic Mixing of Two Air Streams.

37.1. Introduction

The psychrometry is that branch of engineering science, which deals with the study of moist air *i.e.* dry air mixed with water vapour or humidity. It also includes the study of behaviour of dry air and water vapour mixture under various sets of conditions. Though the earth's atmosphere is a mixture of gases including nitrogen (N_2), oxygen (O_2), argon (Ar) and carbon dioxide (CO_2), yet for the purpose of psychrometry, it is considered to be a mixture of dry air and water vapour only.

37.2. Psychrometric Terms

Though there are many psychrometric terms, yet the following are important from the subject point of view :

F. Dry air. The pure dry air is a mixture of number of gases such as nitrogen, oxygen, carbon-dioxide, hydrogen, argon, neon, helium etc. But the nitrogen and oxygen have the major portion of the combination.

The dry air is considered to have the composition as given in the following table.

Table 37.1. Composition of dry air.

S.No.	Constituent	By volume	By mass
1.	Nitrogen (N_2)	78.03%	75.47%
2.	Oxygen (O_2)	20.99%	23.19%
3.	Argon (Ar)	0.94%	1.29%
4.	Carbon-dioxide (CO_2)	0.03%	0.05%
5.	Hydrogen (H_2)	0.01%	—

The molecular mass of dry air is taken as 28.966 and the gas constant for air (R_a) is equal to 0.287 kJ/kg K.

Notes : (a) The pure dry air does not, ordinarily, exist in nature because it always contain some water vapour.

(b) The term air, wherever used in this text, means dry air containing moisture in the vapour form.

(c) Both dry air and water vapour can be considered as perfect gases because both exist in the atmosphere at low pressure. Thus all the perfect gas terms can be applied to them individually.

(d) The density of dry air is taken as 1.293 kg/m³ at pressure 1.0135 bar or 101.35 kN/m² and at temperature 0° C (273 K).

2. *Moist air.* It is a mixture of dry air and water vapour. The amount of water vapour, present in the air, depends upon the absolute pressure and temperature of the mixture.

3. *Saturated air.* It is a mixture of dry air and water vapour, when the air has diffused the maximum amount of water vapour into it. The water vapours, usually, occur in the form of superheated steam as an invisible gas. However, when the saturated air is cooled, the water vapour in the air starts condensing, and the same may be visible in the form of moist, fog or condensation on cold surfaces.

4. *Degree of saturation.* It is the ratio of actual mass of water vapour in a unit mass of dry air to the mass of water vapour in the same mass and pressure of dry air when it is saturated at the same temperature.

5. *Humidity.* It is the mass of water vapour present in 1 kg of dry air, and is generally expressed in terms of gm per kg of dry air. It is also called *specific humidity* or *humidity ratio*.

6. *Absolute humidity.* It is the mass of water vapour present in 1 m³ of dry air, and is generally expressed in terms of gm per cubic metre of dry air. It is also expressed in terms of grains per cubic metre of dry air. Mathematically, one kg of water vapour is equal to 15 430 grains.

7. *Relative humidity.* It is the ratio of actual mass of water vapour in a given volume of moist air to the mass of water vapour in the same volume of saturated air at the same temperature and pressure.

8. *Dry bulb temperature.* It is the temperature of air recorded by a thermometer, when it is not affected by the moisture present in the air. The dry bulb temperature is generally denoted by t_d or t_{db} .

9. *Wet bulb temperature.* It is the temperature of air recorded by a thermometer, when its bulb is surrounded by a wet cloth exposed to the air. Such a thermometer is called *Wet bulb thermometer. The wet bulb temperature is generally denoted by t_w or t_{wb} .

10. *Wet bulb depression.* It is the difference between dry bulb temperature and wet bulb temperature at any point. The wet bulb depression indicates relative humidity of the air.

11. *Dew point temperature.* It is the temperature of air recorded by a thermometer, when the moisture (water vapour) present in it begins to condense. In other words, the dew point temperature is the saturation temperature (t_{sat}) corresponding to the partial pressure of water vapour (p_v). It is, usually, denoted by t_{dp} .

Note : For saturated air, the dry bulb temperature, wet bulb temperature and dew point temperature is same.

12. *Dew point depression.* It is the difference between the dry bulb temperature and dew point temperature of air.

13. *Psychrometer.* There are many types of psychrometers, but the sling psychrometer, as shown in Fig. 37.1, is widely used. It consists of a dry bulb thermometer and a wet

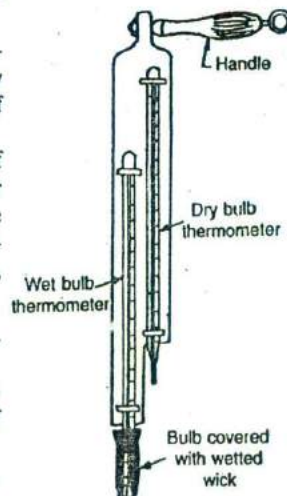


Fig. 37.1. Sling psychrometer.

* A wet bulb thermometer has its bulb covered with a piece of soft cloth (or silk wick) which is exposed to the air. The lower part of this cloth is dipped in a small basin of water. The water from the basin rises up in the cloth by the capillary action, and then gets evaporated. It may be noted that if relative humidity of air is high (i.e. the air contains more water vapour), there will be little evaporation and thus there will be a small cooling effect. On the other hand, if relative humidity of air is low (i.e. the air contains less water vapour), there will be more evaporation and thus there will be more cooling effect.

bulb thermometer mounted side by side in a protective case that is attached to a handle by a swivel connection so that the case can be easily rotated. The dry bulb thermometer is directly exposed to air and measures the actual temperature of the air. The bulb of the wet bulb thermometer is covered by a wick thoroughly wetted by distilled water. The temperature measured by this wick covered bulb of a thermometer is the temperature of liquid water in the wick and is called wet bulb temperature.

The sling psychrometer is rotated in the air for approximately one minute after which the readings from both the thermometers are taken. This process is repeated several times to assure that the lowest possible wet bulb temperature is recorded.

37.3. Dalton's Law of Partial Pressures

It states, "The total pressure exerted by the mixture of air and water vapour is equal to the sum of the pressures, which each constituent would exert, if it occupied the same space by itself". In other words, the total pressure exerted by air and water vapour mixture is equal to the barometric pressure. Mathematically, barometric pressure of the mixture,

$$p_b = p_a + p_v$$

where

$$p_a = \text{Partial pressure of dry air, and}$$

$$p_v = \text{Partial pressure of water vapour.}$$

37.4. Psychrometric Relations

We have already discussed some psychrometric terms in Art. 37.2. These terms have some relations between one another. The following psychrometric relations are important from the subject point of view :

1. *Humidity, specific or absolute humidity, humidity ratio or moisture content.* It is the mass of water vapour present in 1 kg of dry air and is generally expressed in kg/kg of dry air. It may also be defined as the ratio of mass of water vapour to the mass of dry air in a given volume of the air-vapour mixture. Mathematically,

Humidity, specific or absolute humidity, humidity ratio or moisture content,

$$W = 0.622 \times \frac{p_v}{p_a} = \frac{0.622 p_v}{p_b - p_v}$$

where

$$p_v = \text{Partial pressure of water vapour,}$$

$$p_a = \text{Partial pressure of dry air, and}$$

$$p_b = \text{Barometric pressure.}$$

For saturated air (*i.e.* when the air is holding maximum amount of water vapour), the humidity ratio or maximum specific humidity,

$$W_s = W_{max} = 0.622 \times \frac{p_s}{p_b - p_s}$$

where

$$p_s = \text{Partial pressure of air corresponding to saturation temperature (i.e. dry bulb temperature, } t_d).$$

2. *Degree of saturation or percentage humidity.* We have already discussed that the degree of saturation is the ratio of actual mass of water vapour in a unit mass of dry air to the mass of water vapour in the same mass of dry air when it is saturated at the same temperature (dry bulb temperature). In other words, it may be defined as the ratio of actual specific humidity to the specific humidity of saturated air at the same dry bulb temperature. It is, usually, denoted by μ .

Mathematically, degree of saturation,

$$\mu = \frac{W}{W_s} = \frac{0.622 p_v}{\frac{p_b - p_v}{0.622 p_s}} = \frac{p_v}{p_s} \left(\frac{p_b - p_s}{p_b - p_v} \right) = \frac{p_v}{p_s} \left[\frac{1 - \frac{p_s}{p_b}}{1 - \frac{p_v}{p_b}} \right]$$

Notes : (a) The partial pressure of saturated air (p_s) is obtained from the steam tables corresponding to dry bulb temperature t_d .

(b) If the relative humidity, $\phi = p_v/p_s$ is equal to zero, then the humidity ratio, $W = 0$, i.e., for dry air, degree of saturation, $\mu = 0$.

(c) If the relative humidity, $\phi = p_v/p_s$ is equal to 1, then $W = W_s$, and $\mu = 1$. Thus, degree of saturation, μ varies between 0 and 1.

3. *Relative humidity.* We have already discussed that the relative humidity is the ratio of actual mass of water vapour (m_v) in a given volume of moist air to the mass of water vapour (m_s) in the same volume of saturated air at the same temperature and pressure.

The relative humidity may also be defined as the ratio of actual partial pressure of water vapour in moist air (p_v) at a given temperature (dry bulb temperature) to the saturation pressure (p_s) of water vapour (or partial pressure of water vapour in saturated air) at the same temperature. It is usually denoted by ϕ . Mathematically, relative humidity,

$$\phi = \frac{m_v}{m_s} = \frac{p_v}{p_s}$$

The relative humidity may also be obtained as discussed below :

We know that degree of saturation,

$$\mu = \frac{p_v}{p_s} \left[\frac{1 - \frac{p_s}{p_b}}{1 - \frac{p_v}{p_b}} \right] = \phi \left[\frac{1 - \frac{p_s}{p_b}}{1 - \phi \times \frac{p_s}{p_b}} \right] \quad \dots \left(\because \phi = \frac{p_v}{p_s} \right)$$

$$\therefore \phi = \frac{\mu}{1 - (1 - \mu) \frac{p_s}{p_b}}$$

Note : For saturated air, the relative humidity is 100%.

4. *Pressure of water vapour.* According to Carrier's equation, the partial pressure of water vapour,

$$p_v = p_w - \frac{(p_b - p_w)(t_d - t_w)}{1544 - 1.44 t_w}$$

where

p_w = Saturation pressure corresponding to wet bulb temperature (from steam tables),

p_b = Barometric pressure,

t_d = Dry bulb temperature, and

t_w = Wet bulb temperature.

5. *Vapour density or absolute humidity.* We have already discussed that the vapour density or absolute humidity is the mass of water vapour present in 1 m^3 of dry air. Mathematically, vapour density or absolute humidity,

$$\rho_v = \frac{W p_a}{R_a T_d} = \frac{W (p_b - p_v)}{R_a T_d} \quad \dots (\because p_b = p_a + p_v)$$

where

p_a = Pressure of air in kN/m^2 ,

W = Humidity ratio,

R_a = Gas constant for air = 0.287 kJ/kg K , and

T_d = Dry bulb temperature in K.

Example 37.1. *The atmospheric air has a dry bulb temperature of 21°C and wet bulb temperature 18°C . If the barometer reads 750 mm of Hg, determine : 1. partial pressure of water vapour ; 2. Relative humidity ; and 3. Dew point temperature.*

Solution. Given : $t_d = 21^\circ \text{C}$; $t_w = 18^\circ \text{C}$; $p_b = 750 \text{ mm}$ of Hg

1. Partial pressure of water vapour

From steam tables, we find that saturation pressure of vapour corresponding to wet bulb temperature of 18°C , is

$$p_w = 0.02062 \text{ bar} = \frac{0.02062}{0.00133} = 15.5 \text{ mm of Hg.}$$

$\dots (\because 1 \text{ mm of Hg} = 0.00133 \text{ bar})$

We know that partial pressure of water vapour,

$$\begin{aligned} p_v &= p_w - \frac{(p_b - p_w)(t_d - t_w)}{1544 - 1.44 t_w} \\ &= 15.5 - \frac{(750 - 15.5)(21 - 18)}{1544 - 1.44 \times 18} = 14 \text{ mm of Hg Ans.} \end{aligned}$$

2. Relative humidity

From steam tables, we find that saturation pressure of vapour, corresponding to dry bulb temperature of 21°C , is

$$p_s = 0.02485 \text{ bar} = \frac{0.02485}{0.00133} = 18.7 \text{ mm of Hg}$$

We know that relative humidity,

$$\phi = \frac{p_v}{p_s} = \frac{14}{18.7} = 0.748 \text{ or } 74.8 \% \text{ Ans.}$$

3. Dew point temperature

We know that dew point temperature has a saturation pressure equal to the pressure of water vapour in the air. From steam tables, we find that the temperature corresponding to 14 mm of Hg is 16.4°C . Therefore dew point temperature,

$$t_{dp} = 16.4^\circ \text{C Ans.}$$

Example 37.2. A sling psychrometer gives reading of 25° C dry bulb temperature and 15° C wet bulb temperature. The barometer indicates 760 mm of Hg. Assuming partial pressure of the vapour as 10 mm of Hg, determine : 1. Specific humidity, and 2. Saturation ratio.

Solution. Given : $t_d = 25^\circ \text{C}$; $t_w = 15^\circ \text{C}$; $p_b = 760 \text{ mm of Hg}$; $p_v = 10 \text{ mm of Hg}$

1. Specific humidity

We know that specific humidity,

$$W = \frac{0.622 p_v}{p_b - p_v} = \frac{0.622 \times 10}{760 - 10} = 0.0083 \text{ kg/kg of dry air Ans.}$$

2. Saturation ratio

From steam tables, corresponding to a dry bulb temperature of 25° C, we find that partial pressure,

$$p_s = 0.03166 \text{ bar} = \frac{0.03166}{0.00133} = 23.8 \text{ mm of Hg } (\because 1 \text{ mm of Hg} = 0.00133 \text{ bar})$$

We know that saturation ratio,

$$\mu = \frac{p_v (p_b - p_s)}{p_s (p_b - p_v)} = \frac{10 (760 - 23.8)}{23.8 (760 - 10)} = 0.41 \text{ Ans.}$$

37.5. Enthalpy (Total heat) of Moist Air

The enthalpy of moist air is numerically equal to the enthalpy of dry air *plus* the enthalpy of water vapour associated with dry air. Let us consider one kg of dry air. We know that enthalpy of 1 kg of dry air,

$$h_a = c_{pa} \times t_d \quad \dots (i)$$

where

c_{pa} = Specific heat of dry air which is normally taken as 1.005 kJ/kg K, and
 t_d = Dry bulb temperature.

Enthalpy of water vapour associated with 1 kg of dry air,

$$h_v = W h_s \quad \dots (ii)$$

where

W = Specific humidity (*i.e.* mass of water vapour) in kg per kg of dry air, and

h_s = Enthalpy of water vapour in kJ per kg of dry air at dew point temperature (t_{dp}).

If the moist air is superheated, then the enthalpy of water vapour

$$= W c_{ps} (t_d - t_{dp}) \quad \dots (iii)$$

where

c_{ps} = Specific heat of superheated water vapour which is normally taken as 1.9 kJ/kg K, and

$t_d - t_{dp}$ = Degree of superheat of the water vapour.

\therefore Total enthalpy of superheated water vapour,

$$\begin{aligned} h &= c_{pa} t_d + W h_s + W c_{ps} (t_d - t_{dp}) \\ &= c_{pa} t_d + W [h_{fdp} + h_{fgdp} + c_{ps} (t_d - t_{dp})] \quad \dots (\because h_s = h_{fdp} + h_{fgdp}) \\ &= c_{pa} t_d + W [4.2 \times t_{dp} + h_{fgdp} + c_{ps} (t_d - t_{dp})] \quad \dots (\because h_{fdp} = 4.2 t_{dp}) \\ &= (c_{pa} + W c_{ps}) t_d + W [h_{fgdp} + (4.2 - c_{ps}) t_{dp}] \end{aligned}$$

The term $(c_{pa} + W c_{ps})$ is called *humid specific heat* (c_{um}). It is the specific heat or heat capacity of moist air, i.e. $(1 + W)$ kg/kg of dry air. At low temperature of air conditioning range, the value of W is very small. The general value of humid specific heat in air conditioning range is taken as 1.022 kJ/kg K.

$$\begin{aligned} \therefore h &= 1.022 t_d + W [h_{fgdp} + (4.2 - 1.9) t_{dp}] \quad \dots (\because c_{ps} = 1.9 \text{ kJ/kg K}) \\ &= 1.022 t_d + W [h_{fgdp} + 2.3 t_{dp}] \text{ kJ} \end{aligned}$$

where h_{fgdp} = Latent heat of vaporisation of water corresponding to dew point temperature (from steam tables).

Example 37.3. A sleeve psychrometer reads 40°C dry bulb temperature and 28°C wet bulb temperature. Assuming the barometric pressure as 1.013 bar, determine : 1. Humidity ratio ; 2. Relative humidity ; 3. Dew point temperature ; and 4. Enthalpy of the mixture per kg of dry air.

Solution. Given : $t_d = 40^\circ \text{C}$; $t_w = 28^\circ \text{C}$; $p_b = 1.013 \text{ bar}$

1. Humidity ratio

First of all, let us find the partial pressure of vapour (p_v). From steam tables, we find that the saturation pressure corresponding to wet bulb temperature of 28°C is

$$p_w = 0.03778 \text{ bar}$$

$$\begin{aligned} \therefore p_v &= p_w - \frac{(p_b - p_w)(t_d - t_w)}{1544 - 1.44 t_w} \\ &= 0.03778 - \frac{(1.013 - 0.03778)(40 - 28)}{1544 - 1.44 \times 28} \\ &= 0.03778 - 0.00778 = 0.03 \text{ bar Ans.} \end{aligned}$$

We know that humidity ratio,

$$W = \frac{0.622 p_v}{p_b - p_v} = \frac{0.622 \times 0.03}{1.013 - 0.03} = 0.019 \text{ kg/kg of dry air Ans.}$$

2. Relative humidity

From steam tables, we find that the saturation pressure of vapour, corresponding to dry bulb temperature of 40°C is

$$p_s = 0.07375 \text{ bar}$$

\therefore Relative humidity,

$$\phi = \frac{p_v}{p_s} = \frac{0.03}{0.07375} = 0.407 \text{ or } 40.7\% \text{ Ans.}$$

3. Dew point temperature

Since the dew point temperature is the saturation temperature corresponding to the partial pressure of water vapour (p_v), therefore from steam tables, corresponding to 0.03 bar, dew point temperature is

$$t_{dp} = 24^\circ \text{C Ans.}$$

4. Enthalpy of mixture per kg of dry air

From steam tables, we find that the latent heat of vaporisation of water at dew point temperature of 24°C is

$$h_{fgdp} = 2445 \text{ kJ/kg}$$

∴ Enthalpy of mixture per kg of dry air,

$$\begin{aligned} &= 1.022 t_d + W [h_{fgdp} + 2.3 t_{dp}] \\ &= 1.022 \times 40 + 0.019 [2445 + 2.3 \times 24] \text{ kJ/kg of dry air} \\ &= 88.4 \text{ kJ/kg of dry air Ans.} \end{aligned}$$

Example 37.4. In a laboratory test, a sling psychrometer recorded dry bulb and wet bulb temperatures as 30°C and 25°C respectively. Calculate : 1. vapour pressure, 2. relative humidity, 3. specific humidity, 4. degree of saturation, 5. dew point temperature, and 6. enthalpy of the mixture.

Solution. Given : $t_d = 30^\circ \text{C}$; $t_w = 25^\circ \text{C}$

1. Vapour pressure

From steam tables, we find that saturation pressure of vapour corresponding to wet bulb temperature of 25°C is

$$p_w = 0.03166 \text{ bar}$$

We know that vapour pressure,

$$\begin{aligned} p_v &= p_s - \frac{(p_s - p_w)(t_d - t_w)}{1544 - 1.44 t_w} \\ &= 0.03166 - \frac{(1.0133 - 0.03166)(30 - 25)}{1544 - 1.44 \times 25} \\ &= 0.03166 - 0.00326 = 0.0284 \text{ bar Ans.} \end{aligned}$$

2. Relative humidity

From steam tables, we also find that the saturation pressure of vapour at 30°C ,

$$p_s = 0.04242 \text{ bar}$$

We know that relative humidity,

$$\phi = \frac{p_v}{p_s} = \frac{0.0284}{0.04242} = 0.66 \text{ or } 66\% \text{ Ans.}$$

3. Specific humidity

We know that specific humidity,

$$\begin{aligned} W &= \frac{0.622 p_v}{(p_b - p_v)} = \frac{0.622 \times 0.0284}{1.0133 - 0.0284} \text{ kg / kg of dry air} \\ &= 0.018 \text{ kg / kg of dry air Ans.} \end{aligned}$$

4. Degree of saturation

We know that degree of saturation,

$$\begin{aligned} \mu &= \frac{p_v (p_b - p_s)}{p_s (p_b - p_v)} \\ &= \frac{0.0284 (1.0133 - 0.04242)}{0.04242 (1.0133 - 0.0284)} = 0.651 \text{ Ans.} \end{aligned}$$

5. Dew point temperature

We know that dew point temperature has a saturation pressure equal to the pressure of water vapour in the air. From steam tables, we find that the temperature corresponding to 0.0284 bar is 23°C . Therefore dew point temperature,

$$t_{dp} = 23^\circ \text{C Ans.}$$

6. Enthalpy of the mixture

From steam tables, we find that the latent heat of vaporisation of water at dew point temperature of 23°C , is

$$h_{fgdp} = 2447.2 \text{ kJ/kg}$$

\therefore Enthalpy of the mixture

$$\begin{aligned} &= 1.022 t_d + W (h_{fgdp} + 2.3 t_{dp}) \\ &= 1.022 \times 30 + 0.018 (2447.2 + 2.3 \times 23) = 75.66 \text{ kJ Ans.} \end{aligned}$$

37.6. Thermodynamic Wet Bulb Temperature or Adiabatic Saturation Temperature

The thermodynamic wet bulb temperature or adiabatic saturation temperature is the temperature at which the air can be brought to saturation state, adiabatically, by the evaporation of water into the flowing air.

The equipment used for the adiabatic saturation of air, in its simplest form, consists of an insulated chamber containing adequate quantity of water. There is also an arrangement for extra water (known as make-up water) to flow into the chamber from its top, as shown in Fig. 37.2.

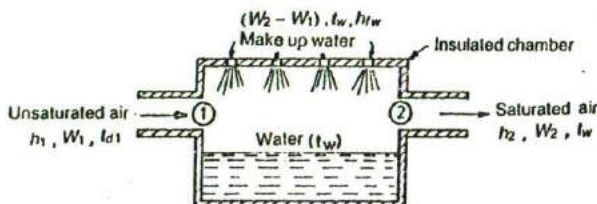


Fig. 37.2. Adiabatic saturation of air.

Let the unsaturated air enter the chamber at section 1. As the air passes through the chamber over a long sheet of water, the water evaporates which is carried with the flowing stream of air, and the specific humidity of the air increases. The make up water is added to the chamber at this temperature to make the water level constant. Both the air and water are cooled as the evaporation takes place. This process continues until the energy transferred from the air to the water is equal to the energy required to vaporise the water. When steady conditions are reached, the air flowing at section 2 is saturated with water vapour. The temperature of the saturated air at section 2 is known as *thermodynamic wet bulb temperature* or *adiabatic saturation temperature*.

The adiabatic saturation process can be represented on T - s diagram as shown by the curve 1-2 in Fig. 37.3. During the adiabatic saturation process, the partial pressure of vapour increases, although the total pressure of the air-vapour mixture remains constant. The unsaturated air initially at dry bulb temperature t_{d1} is cooled adiabatically to dry bulb temperature t_{d2} which is equal to the adiabatic saturation temperature t_w . It may be noted that the adiabatic saturation temperature is taken equal to the wet bulb temperature for all practical purposes.

- Let
- h_1 = Enthalpy of unsaturated air at section 1,
 - W_1 = Specific humidity of air at section 1,
 - h_2, W_2 = Corresponding values of saturated air at section 2, and
 - h_w = Sensible heat of water at adiabatic saturation temperature.

Balancing the enthalpies of air at inlet and outlet (i.e. at sections 1 and 2),

$$h_1 + (W_2 - W_1) h_{fw} = h_2 \quad \dots (i)$$

or
$$h_1 - W_1 h_{fw} = h_2 - W_2 h_{fw} \quad \dots (ii)$$

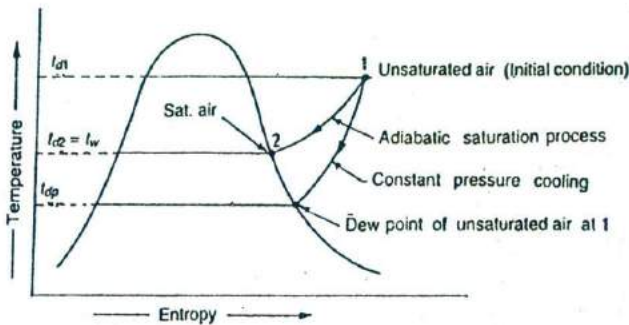


Fig. 37.3. T - s diagram for adiabatic saturation process.

The term $(h_2 - W_2 h_{fw})$ is known as *sigma heat* and remains constant during the adiabatic process.

We know that
$$h_1 = h_{a1} + W_1 h_{s1}$$

and
$$h_2 = h_{a2} + W_2 h_{s2}$$

where

h_{a1} = Enthalpy of 1 kg of dry air at dry bulb temperature t_{d1} ,

h_{s1} = Enthalpy of superheated vapour at t_{d1} per kg of vapour,

h_{a2} = Enthalpy of 1 kg of air at wet bulb temperature t_w , and

h_{s2} = Enthalpy of saturated vapour at wet bulb temperature t_w per kg of vapour.

Now the equation (ii) may be written as :

$$(h_{a1} + W_1 h_{s1}) - W_1 h_{fw} = (h_{a2} + W_2 h_{s2}) - W_2 h_{fw}$$

$$(h_{a1} + W_1 h_{s1}) + W_2 h_{fw} - W_1 h_{fw} = h_{a2} + W_2 h_{s2}$$

$$W_1 (h_{s1} - h_{fw}) = W_2 (h_{s2} - h_{fw}) + h_{a2} - h_{a1}$$

$$\therefore W_1 = \frac{W_2 (h_{s2} - h_{fw}) + h_{a2} - h_{a1}}{h_{s1} - h_{fw}}$$

Example 37.5. Atmospheric air at 0.965 bar enters the adiabatic saturator. The wet bulb temperature is 20°C and dry bulb temperature is 31°C during adiabatic saturation process. Determine 1. humidity ratio of the entering air, 2. vapour pressure and relative humidity at 31°C , and 3. dew point temperature.

Solution. Given : $p_h = 0.965$ bar ; $t_w = 20^\circ\text{C}$; $t_d = 31^\circ\text{C}$

In psychrometry, the enthalpy of superheated vapour at dry bulb temperature t_{d1} is taken equal to the enthalpy of saturated vapour corresponding to dry bulb temperature t_{d1} .

1. Humidity ratio of the entering air

Let W_1 = Humidity ratio of the entering air, and

W_2 = Humidity ratio of the saturated air.

First of all, let us find the value of W_2 . From steam tables, we find that saturation pressure of vapour at 20° C,

$$p_{v2} = 0.023\ 37\ \text{bar}$$

Enthalpy of saturated vapour at 20° C,

$$h_{s2} = h_{g2} = 2538.2\ \text{kJ/kg}$$

Sensible heat of water at 20° C,

$$h_{fw} = 83.9\ \text{kJ/kg}$$

and enthalpy of saturated vapour at 31° C,

$$h_{s1} = h_{g1} = 2558.2\ \text{kJ/kg}$$

We know that enthalpy of unsaturated air corresponding to dry bulb temperature of 31° C,

$$h_{a1} = m\ c_p\ t_d = 1 \times 1.005 \times 31 = 31.155\ \text{kJ/kg}$$

... (Taking c_p for air = 1.005 kJ/kg K)

Enthalpy of 1 kg of saturated air corresponding to wet bulb temperature of 20° C,

$$h_{a2} = m\ c_p\ t_w = 1 \times 1.005 \times 20 = 20.1\ \text{kJ/kg}$$

We know that

$$W_2 = \frac{0.622\ p_{v2}}{p_b - p_{v2}} = \frac{0.622 \times 0.023\ 37}{0.965 - 0.023\ 37} = 0.0154\ \text{kg / kg of dry air}$$

∴

$$\begin{aligned} W_1 &= \frac{W_2 (h_{s2} - h_{fw}) + h_{a2} - h_{a1}}{h_{s1} - h_{fw}} \\ &= \frac{0.0154 (2538.2 - 83.9) + 20.1 - 31.155}{2558.2 - 83.9} \end{aligned}$$

$$= 0.0108\ \text{kg / kg of dry air Ans.}$$

2. Vapour pressure and relative humidity at 31° C

Let p_{v1} = Vapour pressure at 31° C.

We know that $W_1 = \frac{0.622\ p_{v1}}{p_b - p_{v1}}$

$$0.0108 = \frac{0.622\ p_{v1}}{0.965 - p_{v1}}$$

$$0.0104 - 0.0108\ p_{v1} = 0.622\ p_{v1}$$

∴ $p_{v1} = 0.0164\ \text{bar Ans.}$

From steam tables, we find that the saturation pressure corresponding to dry bulb temperature of 31° C is

$$p_s = 0.044\ 91\ \text{bar}$$

∴ Relative humidity, $\phi = \frac{p_{v1}}{p_s} = \frac{0.0164}{0.044\ 91} = 0.365$ or 36.5% Ans.

3. Dew point temperature

Since the dew point temperature (t_{dp}) is the saturation temperature corresponding to the partial pressure of water vapour (p_{v1}), therefore from steam tables, corresponding to 0.0164 bar, we find that

$$t_{dp} = 14.5^\circ \text{C Ans.}$$

37.7. Psychrometric Chart

It is a graphical representation of the various thermodynamic properties of moist air. The psychrometric chart is very useful for finding out the properties of air (which are required in the field of air conditioning) and eliminates a lot of calculations. There is a slight variation in the charts prepared by different air-conditioning manufacturers but basically they are all alike. The psychrometric chart is normally drawn for standard atmospheric pressure of 760 mm of Hg (or 1.01325 bar).

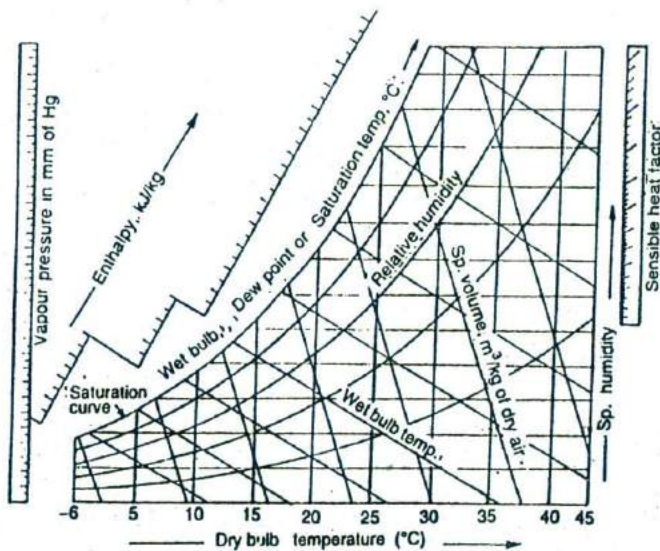


Fig. 37.4. Psychrometric chart.

In a psychrometric chart, dry bulb temperature is taken as abscissa and specific humidity *i.e.* moisture contents as ordinate as shown in Fig. 37.4. Now the saturation curve is drawn by plotting the various saturation points at corresponding dry bulb temperatures. The saturation curve represents 100% relative humidity at various dry bulb temperatures. It also represents the wet bulb and dew point temperatures.

Though the psychrometric chart has a number of details, yet the following lines are important from the subject point of view :

1. **Dry bulb temperature lines.** The dry bulb temperature lines are vertical *i.e.* parallel to the ordinate and uniformly spaced as shown in Fig. 37.5. Generally, the temperature range of these lines on psychrometric chart is from -6°C to 45°C . The dry bulb temperature lines are drawn with difference of every 5°C and up to the saturation curve as shown in the figure. The values of dry bulb temperature are also shown on the saturation curve.

2. **Specific humidity or moisture content lines.** The specific humidity (moisture content) lines are horizontal *i.e.* parallel to the abscissa and are also uniformly spaced as shown in Fig. 37.6. Generally, moisture content range of these lines on psychrometric chart is from 0 to 30 g / kg of dry

air (or from 0 to 0.030 kg / kg of dry air). The moisture content lines are drawn with a difference of every 1 g (or 0.001 kg) and up to the saturation curve as shown in the figure.

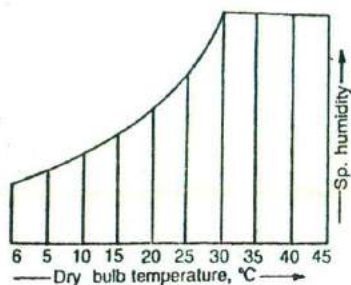


Fig. 37.5. Dry bulb temperature lines.

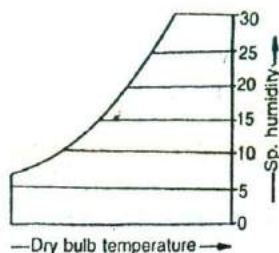


Fig. 37.6. Specific humidity lines.

3. *Dew point temperature lines.* The dew point temperature lines are horizontal *i.e.* parallel to the abscissa and non-uniformly spaced as shown in Fig. 37.7. At any point on the saturation curve, the dry bulb and dew point temperatures are equal.

The values of dew point temperatures are generally given along the saturation curve of the chart as shown in the figure.

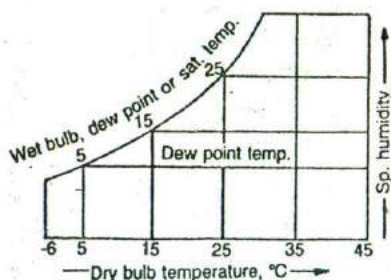


Fig. 37.7. Dew point temperature lines.

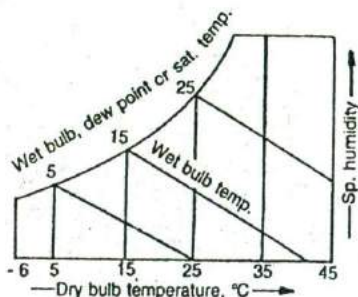


Fig. 37.8. Wet bulb temperature lines.

4. *Wet bulb temperatures lines.* The wet bulb temperature lines are inclined straight lines and non-uniformly spaced as shown in Fig. 37.8. At any point on the saturation curve, the dry bulb and wet bulb temperatures are equal.

The values of wet bulb temperatures are generally given along the saturation curve of the chart as shown in the figure.

5. *Enthalpy (total heat) lines.* The enthalpy (or total heat) lines are inclined straight lines and uniformly spaced as shown in Fig. 37.9. These lines are parallel to the wet bulb temperature lines, and are drawn up to the saturation curve. Some of these lines coincide with the wet bulb temperature lines also.

The values of total enthalpy are given on a scale above the saturation curve as shown in the figure.

6. *Specific volume lines.* The specific volume lines are obliquely inclined straight lines and uniformly spaced as shown in Fig. 37.10. These lines are drawn up to the saturation curve.

The values of volume lines are generally given at the base of the chart.

7. *Vapour pressure lines.* The vapour pressure lines are horizontal and uniformly spaced. Generally, the vapour pressure lines are not drawn in the main chart. But a scale showing vapour pressure in mm of Hg is given on the extreme left side of the chart as shown in Fig. 37.11.

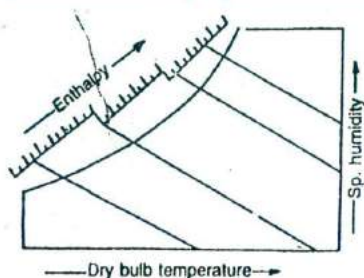


Fig. 37.9. Enthalpy lines.

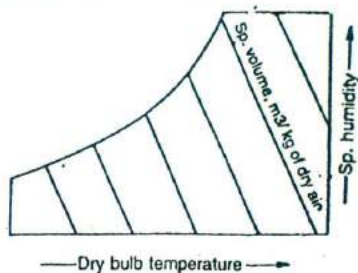


Fig. 37.10. Specific volume lines.

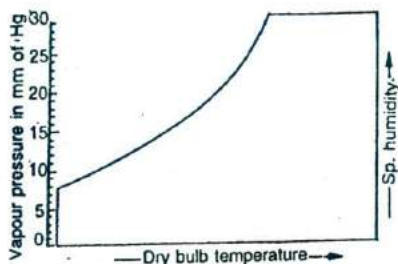


Fig. 37.11. Vapour pressure lines.

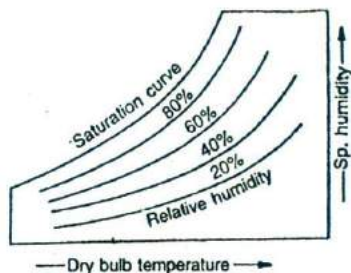


Fig. 37.12. Relative humidity lines.

8. *Relative humidity lines.* The relative humidity lines are curved lines and follows the saturation curve. Generally, these lines are drawn with values 10%, 20%, 30% etc., and upto 100%. The saturation curve represents 100% relative humidity. The values of relative humidity lines are, generally given along the lines themselves as shown in Fig. 37.12.

Example 37.6. Atmospheric air at 760 mm of Hg barometric pressure has 25° C dry bulb temperature and 15° C wet bulb temperature. By using psychrometric chart, determine : 1. Relative humidity, 2. Humidity ratio, and 3. Dew point temperature.

Solution. * $p_b = 760$ mm of Hg ; $t_d = 25^\circ\text{C}$; $t_w = 15^\circ\text{C}$

The initial condition of the air, i.e. 25° C dry bulb temperature and 15° C wet bulb temperature is marked on the psychrometric chart at point P as shown in Fig. 37.13.

1. Relative humidity

The relative humidity, as read along the relative humidity curve, is 35%. **Ans.**

2. Humidity ratio

From point P, draw a horizontal line meeting the humidity ratio line at point Q, as shown in Fig. 37.13. Now, the humidity ratio, as read on the scale at point Q, is 6.7 g/kg of dry air. **Ans.**

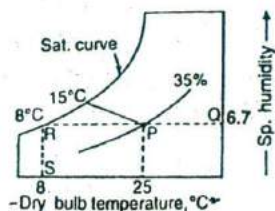


Fig. 37.13

* Superfluous data.

3. Dew point temperature

From point P , draw a horizontal line meeting the saturation curve at point R , as shown in Fig. 37.13. Now, the dew point temperature, as read on the saturation curve at point R is 8°C .

Note : The dew point temperature may also be read on the dry bulb temperature scale. From point R , draw a vertical line meeting the dry bulb temperature scale at point S , as shown in Fig. 37.13. Now the reading at point S is 8°C .

Example 37.7. With the help of psychrometric chart, find : 1. dew point temperature ; 2. Enthalpy ; and 3. Vapour pressure of air having a dry bulb temperature of 30°C and 50% relative humidity.

Solution. Given : $t_d = 30^\circ\text{C}$; $\phi = 50\%$

The initial condition of air, i.e. 30°C dry bulb temperature and 50% relative humidity is marked on the psychrometric chart at point P , as shown in Fig. 37.14.

1. Dew point temperature

From point P , draw a horizontal line meeting the saturation curve at point Q , as shown in Fig. 37.14. Now the dew point temperature, as read on the saturation curve at point Q , is 18.5°C . **Ans.**

2. Enthalpy

From point P , draw a line parallel to enthalpy line (or wet bulb line) meeting the enthalpy scale at point R . Now the enthalpy as read on the scale at point R is 64.7 kJ/kg of dry air. **Ans.**

3. Vapour pressure

Extend the horizontal line PQ to meet the vapour pressure scale at point S . Now the vapour pressure, as read on the scale at point S , is 16 mm of Hg . **Ans.**

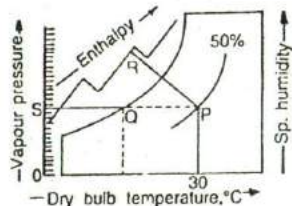


Fig. 37.14

37.8. Psychrometric Processes

Though there are many psychrometric processes, yet the following are important from the subject point of view :

1. Sensible cooling.
2. Sensible heating.
3. Humidification and dehumidification.

We shall now discuss these processes, in detail, in the following pages.

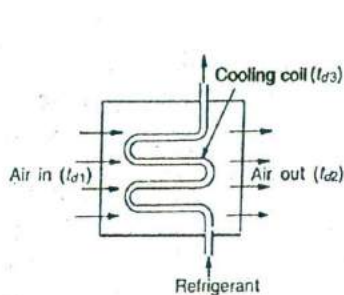
37.9. Sensible Cooling

The cooling of air, without any change in its specific humidity, is known as *sensible cooling*. Let air at temperature t_{d1} passes over a cooling coil of temperature t_{d3} , as shown in Fig. 37.15 (a). A little consideration will show that the temperature of air leaving the cooling coil (t_{d2}) will be more than t_{d3} . The process of sensible cooling, on the psychrometric chart, is shown by a horizontal line 1-2 extending from right to left, as shown in Fig. 37.15 (b). The point 3 represents the surface temperature of the cooling coil.

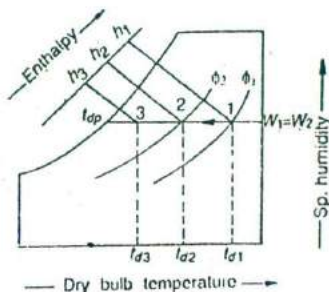
The heat rejected by air during sensible cooling may be obtained from the psychrometric chart by the enthalpy difference ($h_1 - h_2$), as shown in Fig. 37.15 (b).

It may be noted that the specific humidity during the sensible cooling remains constant (i.e. $W_1 = W_2$). The dry bulb temperature reduces from t_{d1} to t_{d2} and relative humidity increases from ϕ_1 to ϕ_2 , as shown in Fig. 37.15 (b). The amount of heat rejected during sensible cooling may also be obtained from the relation :

$$\begin{aligned} \text{Heat rejected, } h &= h_1 - h_2 = c_{pa}(t_{d1} - t_{d2}) + W c_{ps}(t_{d1} - t_{d2}) \\ &= (c_{pa} + W c_{ps})(t_{d1} - t_{d2}) = c_{pm}(t_{d1} - t_{d2}) \end{aligned}$$



(a) Psychrometric process.



(b) Psychrometric chart.

Fig. 37.15. Sensible cooling.

The term $(c_{pa} + W c_{ps})$ is called *humid specific heat* (c_{pm}) and its value is taken as 1.022 kJ/kg K.

$$\therefore \text{Heat rejected, } h = 1.022 (t_{d1} - t_{d2}) \text{ kJ/kg}$$

We know that *by pass factor in sensible cooling,

$$\text{B.P.F.} = \frac{t_{d2} - t_{d3}}{t_{d1} - t_{d3}}$$

$$\text{and coil efficiency, } \eta_c = 1 - \text{B.P.F.} = 1 - \frac{t_{d2} - t_{d3}}{t_{d1} - t_{d3}}$$

Notes : 1. For sensible cooling, the cooling coil may have refrigerant, cooling water or cool gas flowing through it.

2. The sensible cooling can be done only upto the dew point temperature (t_{dp}), as shown in Fig. 37.15 (b). The cooling below this temperature will result in the condensation of moisture.

Example 37.8. Atmospheric air with dry bulb temperature of 28° C and a wet bulb temperature of 17° C is cooled to 15° C without changing its moisture content. Find : 1. Original relative humidity ; 2. Final relative humidity ; and 3. Final wet bulb temperature.

Solution. Given : $t_{d1} = 28^\circ \text{C}$; $t_{w1} = 17^\circ \text{C}$; $t_{d2} = 15^\circ \text{C}$

The initial condition of air, i.e. 20° C dry bulb temperature and 17° C wet bulb temperature is marked on the psychrometric chart at point 1, as shown in Fig. 37.16. Now mark the final condition of air by drawing a horizontal line through point 1 (because there is no change in moisture content of the air) to meet the 15° C dry bulb temperature line at point 2, as shown in Fig. 37.16.

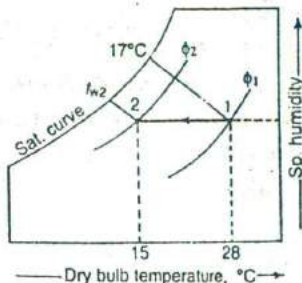


Fig. 37.16

* Under ideal conditions, the dry bulb temperature of the air leaving the apparatus (t_{d2}) should be equal to that of the cooling coil (t_{d3}). But it is not so, because of the inefficiency of the cooling coil. This phenomenon is known as by pass factor.

1. Original relative humidity

From the psychrometric chart, we find that the original relative humidity at point 1,

$$\phi_1 = 34\% \quad \text{Ans.}$$

2. Final relative humidity

From psychrometric chart, we find that the final relative humidity at point 2,

$$\phi_2 = 73\% \quad \text{Ans.}$$

3. Final wet bulb temperature

From the psychrometric chart, we find that the wet bulb temperature at point 2,

$$t_{w2} = 12.2^\circ \text{C} \quad \text{Ans.}$$

37.10. Sensible Heating

The heating of air, without any change in its specific humidity, is known as *sensible heating*. Let air at temperature t_{d1} pass over a heating coil of temperature t_{d3} , as shown in Fig. 37.17 (a). A little consideration will show that the temperature of air leaving the heating coil (*i.e.* t_{d2}) will be less than t_{d3} . The process of sensible heating, on the psychrometric chart, is shown by a horizontal line 1-2 extending from left to right, as shown in Fig. 37.17 (b). The point 3 represents the surface temperature of the heating coil.

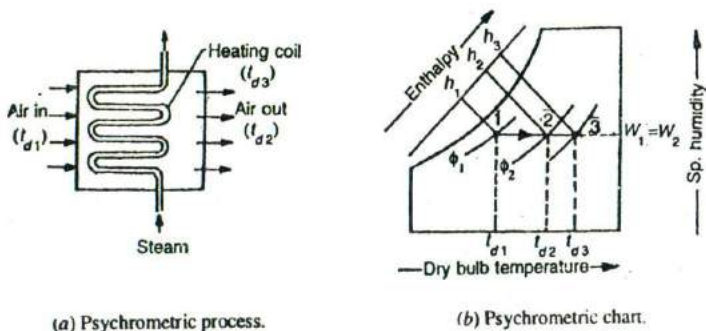


Fig. 37.17. Sensible heating.

The heat absorbed by the air during sensible heating may be obtained from the psychrometric chart by the enthalpy difference ($h_2 - h_1$), as shown in Fig. 37.17 (b). It may be noted that the specific humidity during the sensible heating remains constant (*i.e.* $W_1 = W_2$). The dry bulb temperature increases from t_{d1} to t_{d2} and relative humidity reduces from ϕ_1 to ϕ_2 , as shown in Fig. 37.17 (b). The amount of heat added during sensible heating may also be obtained from the relation :

$$\begin{aligned} \text{Heat added,} \quad h &= h_2 - h_1 \\ &= c_{pa} (t_{d2} - t_{d1}) + W c_{ps} (t_{d2} - t_{d1}) \\ &= (c_{pa} + W c_{ps}) (t_{d2} - t_{d1}) = c_{pm} (t_{d2} - t_{d1}) \end{aligned}$$

The term $(c_{pa} + W c_{ps})$ is called *humid specific heat* (c_{pm}) and its value is taken as 1.022 kJ/kg K.

$$\therefore \text{Heat added,} \quad h = 1.022 (t_{d2} - t_{d1}) \text{ kJ/kg}$$

We know that by pass factor in sensible heating,

$$\text{B.P.F.} = \frac{t_{d3} - t_{d2}}{t_{d3} - t_{d1}}$$

and coil efficiency, $\eta_c = 1 - \text{B.P.F.} = 1 - \frac{t_{d3} - t_{d2}}{t_{d3} - t_{d1}}$

Notes : 1 For sensible heating, steam or hot water is passed through the heating coil. The heating coil may be electric resistance coil.

2. The sensible heating of moist air can be done to any desired temperature.

Example 37.9. The atmospheric air at 760 mm of Hg, dry bulb temperature 15°C and wet bulb temperature 11°C enters a heating coil whose temperature is 41°C . Assuming by-pass factor of heating coil as 0.5, determine dry bulb temperature, wet bulb temperature and relative humidity of the air leaving the coil. Also determine the sensible heat added to the air per kg of dry air.

Solution. Given : $p_b = 760$ mm of Hg ; $t_{d1} = 15^\circ\text{C}$; $t_{w1} = 11^\circ\text{C}$; $t_{d3} = 41^\circ\text{C}$; BPF = 0.5

The initial condition of air entering the coil at dry bulb temperature of 15°C , and wet bulb temperature of 11°C , is marked by point 1 on the psychrometric chart as shown in Fig. 37.18. Now draw a constant specific humidity line from point 1 to intersect the vertical line drawn through 41°C at point 3. The point 2 lies on the line 1-3.

Dry bulb temperature of the air leaving the coil

t_{d2} = Dry bulb temperature of the air leaving the coil.

We know that by pass factor (BPF),

$$\begin{aligned} 0.5 &= \frac{t_{d3} - t_{d2}}{t_{d3} - t_{d1}} = \frac{41 - t_{d2}}{41 - 15} \\ &= \frac{41 - t_{d2}}{26} \end{aligned}$$

$\therefore t_{d2} = 28^\circ\text{C}$ Ans.

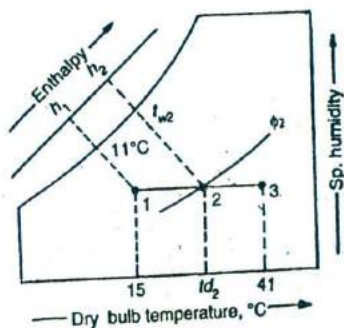


Fig. 37.18

Wet bulb temperature of the air leaving the coil

From the psychrometric chart, we find that the wet bulb temperature of the air leaving the coil at point 2 is

$$t_{w2} = 16.1^\circ\text{C} \text{ Ans.}$$

Relative humidity of the air leaving the coil

From the psychrometric chart, we find that the relative humidity of the air leaving the coil at point 2 is,

$$\phi_2 = 29\% \text{ Ans.}$$

Sensible heat added to the air per kg of dry air

From the psychrometric chart, we find that enthalpy of air at point 2,

$$h_2 = 46 \text{ kJ/kg of dry air}$$

and enthalpy of air at point 1, $h_1 = 31.8$ kJ/kg of dry air

\therefore Sensible heat added to the air per kg of dry air,

$$h = h_2 - h_1 = 46 - 31.8 = 14.2 \text{ kJ/kg of dry air} \text{ Ans.}$$

37.11. Humidification and Dehumidification

The addition of moisture to the air, without change in its dry bulb temperature, is known as *humidification*. Similarly, removal of moisture from the air, without change in its dry bulb temperature is known as *dehumidification*. The heat added during humidification process and heat removed during dehumidification process is shown on the psychrometric chart in Fig. 37.19 and 37.20 respectively.

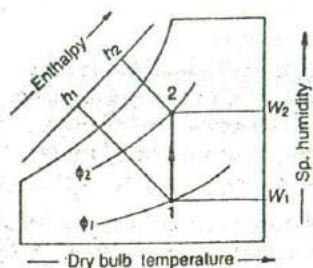


Fig. 37.19. Humidification.

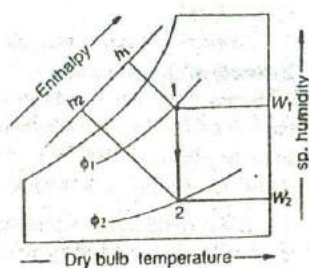


Fig. 37.20. Dehumidification.

It may be noted that in humidification, the relative humidity increases from ϕ_1 to ϕ_2 and specific humidity also increases from W_1 to W_2 , as shown in Fig. 37.19. Similarly, in dehumidification, the relative humidity decreases from ϕ_1 to ϕ_2 and specific humidity also decreases from W_1 to W_2 , as shown in Fig. 37.20.

A little consideration will show, that in humidification, change in enthalpy is shown by the intercept $(h_2 - h_1)$ on the psychrometric chart. Since the dry bulb temperature of air during the humidification remains constant, therefore its sensible heat also remains constant. It is thus obvious that the change in enthalpy per kg of dry air is the latent heat of vaporisation of the increased moisture content equal to $(W_2 - W_1)$ kg per kg of dry air. Mathematically,

$$(h_2 - h_1) = h_{f_{rd}}(W_2 - W_1)$$

where $h_{f_{rd}}$ is the latent heat of vaporisation at dry bulb temperature (t_d).

Notes : 1. For dehumidification, the above equation may be written as :

$$(h_1 - h_2) = h_{f_{rd}}(W_1 - W_2)$$

2. Absolute humidification and dehumidification processes are rarely found in practice. These are always accompanied by heating or cooling processes.

37.12. Sensible Heat Factor

As a matter of fact, the heat added during a psychrometric process may be split up into sensible heat and latent heat. The ratio of the sensible heat to the total heat is known as *sensible heat factor* (briefly written as SHF) or *sensible heat ratio* (briefly written as SHR). Mathematically,

$$\text{SHF} = \frac{\text{Sensible heat}}{\text{Total heat}} = \frac{SH}{SH + LH}$$

where

SH = Sensible heat, and

LH = Latent heat.

The sensible heat factor scale is shown on the right hand side of the psychrometric chart.

37.13. Cooling and Dehumidification

This process is generally used in summer air conditioning to cool and dehumidify the air. The process of cooling and dehumidification is shown by line 1-2 on the psychrometric chart, as shown in Fig. 37.21. The air enters at condition 1 and leaves at condition 2. In this process, the dry bulb temperature of air decreases from t_{d1} to t_{d2} (i.e. cooling) and specific humidity of air decreases from W_1 to W_2 (i.e. dehumidification). The final relative humidity of the air is generally higher than that of the entering air.

Actually, the cooling and dehumidification process follows the path as shown by a dotted curve in Fig. 37.21, but for the calculation of psychrometric properties, only end points are important. Thus the cooling and dehumidification process shown by a line 1-2 may be assumed to have followed a path 1-A (i.e. sensible cooling) and A-2 (i.e. dehumidification) as shown in Fig. 37.21. We see that the total heat removed from the air during the cooling and dehumidification process is

$$h = h_1 - h_2 \\ = (h_1 - h_A) + (h_A - h_2) = SH + LH$$

where

$$SH = h_1 - h_A = \text{Sensible heat removed, and}$$

$$LH = h_A - h_2 = \text{Latent heat removed due to condensation of vapour of the reduced moisture content } (W_1 - W_2).$$

We know that sensible heat factor,

$$SHF = \frac{\text{Sensible heat}}{\text{Total heat}} = \frac{SH}{SH + LH} = \frac{h_1 - h_A}{h_1 - h_2}$$

Example 37.10. Atmospheric air at 30°C dry bulb temperature and 45% R.H. is to be conditioned to 17°C dry bulb temperature and 15°C wet bulb temperature. Find the amount of heat rejected by the air. Also find the sensible heat factor of the process.

Solution. Given : $t_{d1} = 30^\circ\text{C}$; $\phi_1 = 45\%$; $t_{d2} = 17^\circ\text{C}$; $t_{w2} = 15^\circ\text{C}$

Amount of heat rejected by the air

First of all, mark the initial condition of air, i.e. at 30°C dry bulb temperature and 45% relative humidity on the psychrometric chart as point 1. Then mark the final condition of air at 17°C dry bulb temperature and 15°C wet bulb temperature on the psychrometric chart as point 2 shown in Fig 37.22. Now locate the point A by drawing horizontal line through point 1 and vertical line through point 2.

From the psychrometric chart, we find that enthalpy of air at point 1,

$$h_1 = 60 \text{ kJ/kg of dry air}$$

Similarly $h_A = 49 \text{ kJ/kg of dry air}$

and

$$h_2 = 41 \text{ kJ/kg of dry air}$$

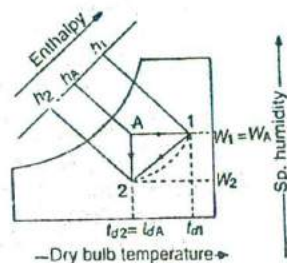


Fig. 37.21. Cooling and dehumidification.

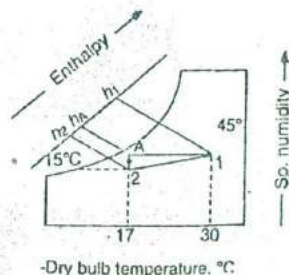


Fig. 37.22

We know that amount of heat rejected by the air

$$= h_1 - h_2 = 60 - 41 = 19 \text{ kJ/kg of dry air Ans.}$$

Sensible heat factor

We know that sensible heat factor,

$$\text{SHF} = \frac{h_1 - h_A}{h_1 - h_2} = \frac{60 - 49}{60 - 41} = 0.58 \text{ Ans.}$$

37.14. Heating and Humidification

This process is generally used in winter air conditioning to warm and humidify the air. The process of heating and humidification is shown by line 1-2 on the psychrometric chart, as shown in Fig. 37.23. The air enters at condition 1 and leaves at condition 2. In this process, the dry temperature of air increases from t_{d1} to t_{d2} (i.e. heating) and specific humidity of air increases from W_1 to W_2 (i.e. humidification). The final relative humidity of the air can be lower or higher than that of the entering air.

Actually, the heating and humidification process follows the path as shown by dotted curve in Fig. 37.23, but for the calculation of psychrometric properties, only the end points are important. Thus, the heating and humidification process shown by a line 1-2 on the psychrometric chart may be assumed to have followed the path 1-A (i.e. sensible heating) and A-2 (i.e. humidification), as shown in Fig. 37.23. We see that the total heat added to the air during heating and humidification is

$$\begin{aligned} h &= h_2 - h_1 \\ &= (h_2 - h_A) + (h_A - h_1) = LH + SH \end{aligned}$$

where

$$LH = h_2 - h_A = \text{Latent of vaporisation of the increased moisture content } (W_2 - W_1), \text{ and}$$

$$SH = h_A - h_1 = \text{Sensible heat added.}$$

We know that sensible heat factor,

$$\text{SHF} = \frac{\text{Sensible heat}}{\text{Total heat}} = \frac{SH}{SH + LH} = \frac{h_A - h_1}{h_2 - h_1}$$

Example 37.11. Atmospheric air at a dry bulb temperature of 16°C and 25% relative humidity passes through a furnace and then through a humidifier, in such a way that the final dry bulb temperature is 30°C and 50% relative humidity. Find the heat and moisture added to the air. Also determine the sensible heat factor of the process.

Solution. Given : $t_{d1} = 16^\circ \text{C}$; $\phi_1 = 25\%$; $t_{d2} = 30^\circ \text{C}$; $\phi_2 = 50\%$

Heat added to the air

First of all, mark the initial condition of air i.e. at 16°C dry bulb temperature and 25% relative humidity on the psychrometric chart at point 1, as shown in Fig. 37.24. Then mark the final condition of air at 30°C dry bulb temperature and 50% relative humidity on the psychrometric chart as point

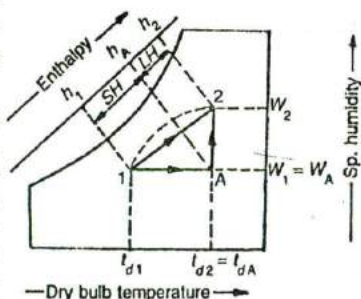


Fig. 37.23. Heating and humidification.

2. Now locate the point A by drawing horizontal line through point 1 and vertical line through point 2. From the psychrometric chart, we find that enthalpy of air at point 1,

$$h_1 = 23 \text{ kJ/kg of dry air}$$

Enthalpy of air at point A,

$$h_A = 37.5 \text{ kJ/kg of dry air}$$

and enthalpy of air at point 2,

$$h_2 = 64.5 \text{ kJ/kg of dry air}$$

∴ Heat added to the air

$$= h_2 - h_1 = 64.5 - 23$$

$$= 41.5 \text{ kJ/kg of dry air Ans.}$$

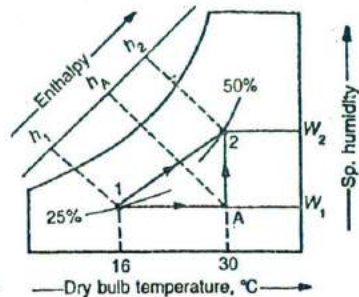


Fig. 37.24

Moisture added to the air

From the psychrometric chart, we find that the specific humidity in the air at point 1,

$$W_1 = 0.0028 \text{ kg / kg of dry air}$$

and specific humidity in the air at point 2,

$$W_2 = 0.0134 \text{ kg / kg of dry air}$$

∴ Moisture added to the air

$$= W_2 - W_1 = (0.0134 - 0.0028) = 0.0106 \text{ kg / kg of dry air Ans.}$$

Sensible heat factor of the process

We know that sensible heat factor of the process,

$$\text{SHF} = \frac{h_A - h_1}{h_2 - h_1} = \frac{37.5 - 23}{64.5 - 23} = 0.35 \text{ Ans.}$$

37.15. Adiabatic Mixing of Two Air Streams

When two quantities of air having different enthalpies and different specific humidities are mixed, the final condition of the air mixture depends upon the masses involved, and on the enthalpy and specific humidity of each of the constituent masses which enter the mixture.

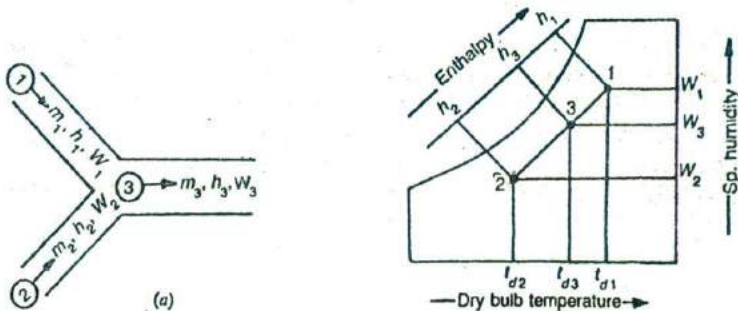


Fig. 37.25. Adiabatic mixing of two air streams (b)

Now consider two air streams 1 and 2 mixing adiabatically, as shown in Fig. 37.25 (a).

- Let m_1 = Mass of air entering at 1,
 h_1 = Enthalpy of air entering at 1,
 W_1 = Specific humidity of air entering at 1,
 m_2, h_2, W_2 = Corresponding values of air entering at 2, and
 m_3, h_3, W_3 = Corresponding values of the mixture leaving at 3.

Assuming no loss of enthalpy and specific humidity during the air mixing process, we have for the mass balance,

$$m_1 + m_2 = m_3 \quad \dots (i)$$

For the energy balance,

$$m_1 h_1 + m_2 h_2 = m_3 h_3 \quad \dots (ii)$$

and for the mass balance of water vapour,

$$m_1 W_1 + m_2 W_2 = m_3 W_3 \quad \dots (iii)$$

From equations (i) and (ii),

$$m_1 h_1 + m_2 h_2 = (m_1 + m_2) h_3 = m_1 h_3 + m_2 h_3$$

or
$$m_1 h_1 - m_1 h_3 = m_2 h_3 - m_2 h_2$$

$$m_1 (h_1 - h_3) = m_2 (h_3 - h_2)$$

$$\therefore \frac{m_1}{m_2} = \frac{h_3 - h_2}{h_1 - h_3} \quad \dots (iv)$$

Similarly, from equations (i) and (iii),

$$\frac{m_1}{m_2} = \frac{W_3 - W_2}{W_1 - W_3} \quad \dots (v)$$

Now from equations (iv) and (v),

$$\frac{m_1}{m_2} = \frac{h_3 - h_2}{h_1 - h_3} = \frac{W_3 - W_2}{W_1 - W_3} \quad \dots (vi)$$

The adiabatic mixing process is represented on the psychrometric chart, as shown in Fig. 37.25 (b). The final condition of the mixture (point 3) lies on the straight line 1-2. The point 3 divides the line 1-2 in the inverse ratio of the mixing masses. By calculating the value of W_3 from equation (vi), the point 3 is plotted on the line 1-2.

Example 37.12. One kg of air at 40°C dry bulb temperature and 50% relative humidity is mixed with 2 kg of air at 20°C dry bulb temperature and 20°C dew point temperature. Calculate the temperature and specific humidity of the mixture.

Solution. Given : $m_1 = 1\text{ kg}$; $t_{d1} = 40^\circ\text{C}$; $\phi_1 = 50\%$; $m_2 = 2\text{ kg}$; $t_{d2} = 20^\circ\text{C}$; $t_{dp} = 20^\circ\text{C}$

Specific humidity of the mixture

Let W_3 = Specific humidity of the mixture.

The condition of first mass of air at 40°C dry bulb temperature and 50% relative humidity is marked on the psychrometric chart at point 1, as shown in Fig. 37.26. Now mark the condition of

second mass of air at 20°C dry bulb temperature and 20°C dew point temperature at point 2, as shown in the figure. This point lies on the saturation curve. Join the points 1 and 2. From the psychrometric chart, we find that specific humidity of the first mass of air,

$$W_1 = 0.0238 \text{ kg / kg of dry air}$$

and specific humidity of the second mass of air,

$$W_2 = 0.0148 \text{ kg / kg of dry air}$$

We know that

$$\frac{m_1}{m_2} = \frac{W_3 - W_2}{W_1 - W_3}$$

$$\frac{1}{2} = \frac{W_3 - 0.0148}{0.0238 - W_3}$$

or $0.0238 - W_3 = 2W_3 - 0.0296$

$$\therefore W_3 = 0.0178 \text{ kg / kg of dry air Ans.}$$

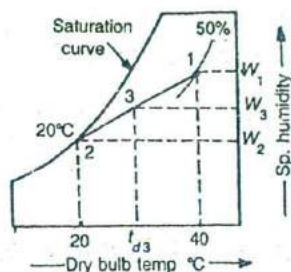


Fig. 37.26

Temperature of the mixture

Now plot point 3 on the line joining the points 1 and 2 corresponding to specific humidity $W_3 = 0.0178 \text{ kg / kg of dry air}$ as shown in Fig. 37.26. We find that at point 3, the dry bulb temperature of the mixture is

$$t_{db} = 26.8^\circ \text{C Ans.}$$

EXERCISES

1. A sling psychrometer reads 44°C dry bulb temperature and 30°C wet bulb temperature. Calculate 1. vapour pressure; 2. specific humidity; 3. relative humidity; 4. dew point temperature; and 5. enthalpy of the mixture per kg of dry air. [Ans. 0.033 bar; 0.021 kg/kg of dry air; 36.7%; 25.9°C; 99.3 kJ]
2. The humidity ratio of atmospheric air at 28°C dry bulb temperature and 760 mm of mercury is 0.016 kg/kg of dry air. Determine: 1. partial pressure of water vapour, 2. relative humidity, 3. dew point temperature, and 4. specific enthalpy. [Ans. 0.0253 bar; 67%; 21.1°C; 68.6 kJ/kg of dry air]
3. The atmospheric air enters the adiabatic saturator at 33°C dry bulb temperature and 23°C wet bulb temperature. The barometric pressure is 740 mm of Hg. Determine the specific humidity and vapour pressure at 33°C. [Ans. 0.012 kg/kg of dry air; 13 mm of Hg]
4. A sample of air has 22°C DBT, relative humidity 30 percent at barometric pressure of 760 mm of Hg. Calculate: 1. Vapour pressure, 2. Humidity ratio, 3. Vapour density, and 4. Enthalpy.

Verify your results by psychrometric chart.

$$[\text{Ans. } 0.008 \text{ bar; } 0.005 \text{ kg/kg of dry air; } 0.00582 \text{ kg/m}^3 \text{ of dry air; } 34.8 \text{ kJ/kg of dry air}]$$

5. A quantity of air having a volume of 300 m³ at 30°C dry bulb temperature and 25°C wet bulb temperature is heated to 40°C dry bulb temperature. Estimate the amount of heat added, final relative humidity and wet bulb temperature. The air pressure is 1.013 25 bar.

$$[\text{Ans. } 3533.4 \text{ kJ/kg; } 39\%; 27.5^\circ \text{C}]$$

6. 2 kg of air at 40°C dry bulb temperature and 50% relative humidity is mixed with 3 kg of air at 20°C dry bulb temperature and 12°C dew point temperature. Calculate specific humidity and the dry bulb temperature of the mixture. [Ans. 14.5 g/kg of dry air; 28°C]

7. 800 m³/min of recirculated air at 22°C DBT and 10°C dew point temperature is to be mixed with 300 m³/min of fresh air at 30°C DBT and 50% RH. Determine the enthalpy, specific volume, humidity ratio and dew point temperature of the mixture. [Ans. 47.6 kJ/kg; 0.856 m³/kg; 9.2 g/kg of dry air; 13°C]

QUESTIONS

1. What do you understand by the term 'psychrometry'.
2. Define the following :
 1. Specific humidity ;
 2. Absolute humidity ;
 3. Relative humidity ; and
 4. Dew point temperature.
3. What is a sling psychrometer ? Make a neat sketch and explain its use.
4. Establish the following expression for air-vapour mixture :

$$\text{Specific humidity, } W = 0.622 \times \frac{P_v}{P_b - P_v}$$

where

P_v = Partial pressure of water vapour,

and

P_b = Barometric pressure.

5. How does the wet bulb temperature differ from thermodynamic wet bulb temperature ?
6. Prove that the partial pressure of water vapour in the atmospheric air remains constant as long as the specific humidity remains constant.
7. Prove that the enthalpy of the humid air remains constant along a wet bulb temperature line on the psychrometric chart.
8. When is dehumidification of air necessary and how it is achieved.
9. Define sensible heat factor.
10. Show the following processes on the skeleton psychrometric chart :
 - (a) Dehumidification of moist air by cooling ; and
 - (b) Adiabatic mixing of two streams.

OBJECTIVE TYPE QUESTIONS

1. A mixture of dry air and water vapour, when the air has diffused the maximum amount of water vapour into it, is called
 - (a) dry air
 - (b) moist air
 - (c) saturated air
 - (d) specific humidity
2. The temperature of air recorded by a thermometer, when it is not effected by the moisture present in it, is called
 - (a) wet bulb temperature
 - (b) dry bulb temperature
 - (c) dew point temperature
 - (d) none of these
3. For unsaturated air, the dew point temperature is wet bulb temperature.
 - (a) equal to
 - (b) less than
 - (c) more than
4. The difference between dry bulb temperature and wet bulb temperature, is called
 - (a) dry bulb depression
 - (b) wet bulb depression
 - (c) dew point depression
 - (d) degree of saturation
5. The vertical and uniformly spaced lines on a psychrometric chart indicates
 - (a) dry bulb temperature
 - (b) wet bulb temperature
 - (c) dew point temperature
 - (d) specific humidity
6. The curved lines on a psychrometric chart indicates
 - (a) dry bulb temperature
 - (b) wet bulb temperature
 - (c) specific humidity
 - (d) relative humidity
7. During sensible cooling of air, the specific humidity
 - (a) increases
 - (b) decreases
 - (c) remains constant

8. During sensible cooling of air, the dry bulb temperature
(a) increases (b) decreases (c) remains constant
9. During sensible cooling of air, the wet bulb temperature
(a) increases (b) decreases (c) remains constant
10. The process generally used in winter air conditioning to warm and humidify the air, is called
(a) humidification (b) dehumidification
(c) heating and humidification (d) cooling and dehumidification

ANSWERS

- | | | | | |
|--------|--------|--------|--------|---------|
| 1. (c) | 2. (b) | 3. (b) | 4. (b) | 5. (a) |
| 6. (d) | 7. (c) | 8. (b) | 9. (b) | 10. (c) |

Air Conditioning Systems

1. Introduction. 2. Factors Affecting Comfort Air Conditioning. 3. Air Conditioning System. 4. Equipments used in an Air Conditioning System. 5. Classifications of Air Conditioning Systems. 6. Comfort Air Conditioning System. 7. Industrial Air Conditioning System. 8. Winter Air Conditioning System. 9. Summer Air Conditioning System. 10. Year-round Air Conditioning System. 11. Unitary Air Conditioning System. 12. Central Air Conditioning System.

38.1. Introduction

The air conditioning is that branch of engineering science which deals with the study of conditioning of air for *human comfort. This subject, in its broad sense, also deals with the conditioning of air for industrial purposes, food processing, storage of food and other materials.

38.2. Factors Affecting Comfort Air Conditioning

The four important factors for comfort air conditioning are discussed as below :

1. *Temperature of air.* In air conditioning, the control of temperature means the maintenance of any desired temperature within an enclosed space even though the temperature of the outside air is above or below the desired room temperature. This is accomplished either by the addition or removal of heat from the enclosed space as and when demanded. It may be noted that a human being feels comfortable when the air is at 21° C with 56% relative humidity.

2. *Humidity of air.* The control of humidity of air means the increasing or decreasing of moisture contents of air during summer or winter respectively in order to produce comfortable and healthy conditions. The control of humidity is not only necessary for human comfort but it also increases the efficiency of the workers. In general, for summer air conditioning, the relative humidity should not be less than 60% whereas for winter air conditioning it should not be more than 40%.

3. *Purity of air.* It is an important factor for the comfort of a human body. It has been noticed that people do not feel comfortable when breathing contaminated air, even if it is within acceptable temperature and humidity ranges. It is thus obvious that proper filtration, cleaning and purification of air is essential to keep it free from dust and other impurities.

4. *Motion of air.* The motion or circulation of air is another important factor which should be controlled, in order to keep constant temperature throughout the conditioned space. It is, therefore, necessary that there should be equi-distribution of air throughout the space to be air conditioned.

* Strictly speaking, the human comfort depends upon physiological and psychological conditions. Thus it is difficult to define the term 'human comfort'. There are many definitions given for this term by different bodies. But the most accepted definition, from the subject point of view, is given by the American Society of Heating, Refrigeration and Air conditioning Engineers (ASHRAE) which states "human comfort is that condition of mind, which expresses satisfaction with the thermal environment."

38.3. Air Conditioning System

We have already discussed in Art. 38.2, the four important factors which effect the human comfort. The system which effectively controls these conditions to produce the desired effects upon the occupants of the space, is known as an *air conditioning system*.

38.4. Equipments Used in an Air Conditioning System

Following are the main equipments or parts used in an air conditioning system :

1. *Circulation fan.* The main function of this fan is to move air to and from the room.
2. *Air conditioning units.* It is a unit, which consists of cooling and dehumidifying processes for summer air conditioning or heating and humidification processes for winter air conditioning.
3. *Supply duct.* It directs the conditioned air from the circulating fan to the space to be air conditioned at proper point.
4. *Supply outlets.* These are grills, which distribute the conditioned air evenly in the room.
5. *Return outlets.* These are the openings in a room surface which allow the room air to enter the return duct.
6. *Filters.* The main function of the filters is to remove dust, dirt and other harmful bacteria from the air.

38.5. Classifications of Air Conditioning Systems

The air conditioning systems may be broadly classified as follows :

1. *According to the purpose*
 - (a) Comfort air conditioning system, and
 - (b) Industrial air conditioning system.
2. *According to season of the year*
 - (a) Winter air conditioning system,
 - (b) Summer air conditioning system, and
 - (c) Year-round air conditioning system.
3. *According to the arrangement of equipment*
 - (a) Unitary air conditioning system, and
 - (b) Central air conditioning system.

In this chapter, we shall discuss all the above mentioned air conditioning systems, one by one.

38.6. Comfort Air Conditioning System

In comfort air conditioning, the air is brought to the required dry bulb temperature and relative humidity. If sufficient data of the required condition is not given, then it is assumed to be 21°C dry bulb temperature and 50% relative humidity. The sensible heat factor is, generally, kept as following :

For residence or private office	= 0.9
For restaurant or busy office	= 0.8
Auditorium or cinema hall	= 0.7
Ball room dance hall etc.	= 0.6

The comfort air conditioning may be adopted for small cabins, office halls or big halls like cinema houses.

Example 38.1. An air conditioning plant is required to supply 60 m^3 of air per minute at a DBT of 21°C and 55% R.H. The outside air is at DBT of 28°C and 60% RH. Determine the mass of water drained and capacity of the cooling coil. Assume the air conditioning plant first to dehumidify and then to cool the air.

Solution. Given : $v_2 = 60 \text{ m}^3/\text{min}$; $t_{d2} = 21^\circ \text{C}$; $\phi_2 = 55\%$; $t_{d1} = 28^\circ \text{C}$; $\phi_1 = 60\%$

Mass of water drained

First of all, mark the initial condition of air at 28°C dry bulb temperature and 60% relative humidity on the psychrometric chart as point 1, as shown in Fig. 38.1. Now mark the final condition of air at 21°C dry bulb temperature and 55% relative humidity as point 2. From the psychrometric chart, we find that

Specific humidity of air at point 1,

$$W_1 = 0.0142 \text{ kg / kg of dry air}$$

Specific humidity of air at point 2,

$$W_2 = 0.0088 \text{ kg / kg of dry air}$$

and specific volume of air at point 2,

$$v_{s2} = 0.845 \text{ m}^3/\text{kg of dry air}$$

We know that mass of air circulated,

$$m_a = \frac{v_2}{v_{s2}} = \frac{60}{0.845} = 71 \text{ kg / min}$$

\therefore Mass of water drained

$$= m_a (W_1 - W_2) = 71 (0.0142 - 0.0088) = 0.3834 \text{ kg / min}$$

$$= 23 \text{ kg / h Ans.}$$

Capacity of the cooling coil

From the psychrometric chart, we find that

Enthalpy of air at point 1,

$$h_1 = 63.8 \text{ kJ / kg of dry air}$$

and enthalpy of air at point 2,

$$h_2 = 44.5 \text{ kJ / kg of dry air}$$

\therefore Capacity of the cooling coil

$$= m_a (h_1 - h_2) = 71 (63.8 - 44.5) = 1370.3 \text{ kJ / min Ans.}$$

38.7. Industrial Air Conditioning System

It is an important system of air conditioning these days in which the inside dry bulb temperature and relative humidity of the air is kept constant for proper working of the machines. Some of the sophisticated electronic and other machines need a particular dry bulb temperature and relative humidity. Sometimes, these machines also require a particular method of psychrometric processes.

Example 38.2. Following data refers to an air conditioning system to be designed for an industrial process for hot and wet climate :

Outside conditions = 30°C DBT and 75% RH

Required inside conditions = 20°C DBT and 60% RH

The required condition is to be achieved first by cooling and dehumidifying and then by heating. If 20 m^3 of air is absorbed by the plant every minute, find : 1. capacity of the cooling coil in tonne of refrigeration ; 2. capacity of the heating coil in kW ; and 3. amount of water removed per hour.

Solution. Given : $t_{d1} = 30^\circ \text{C}$; $\phi_1 = 75\%$; $t_{d2} = 20^\circ \text{C}$; $\phi_2 = 60\%$; $v_1 = 20 \text{ m}^3/\text{min}$

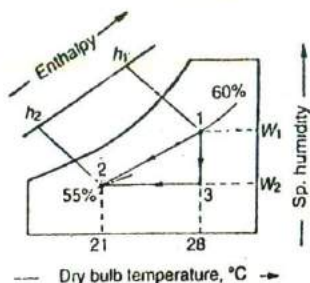


Fig. 38.1

1. Capacity of the cooling coil in tonne of refrigeration

First of all, mark the initial condition of air at 30° C dry bulb temperature and 75% relative humidity on the psychrometric chart as point 1, as shown in Fig. 38.2. Then mark the final condition of air at 20° C dry bulb temperature and 60% relative humidity on the chart as point 3.

Now locate the points 2' and 2 on the saturation curve by drawing horizontal lines through points 1 and 3, as shown in Fig. 38.2. On the chart, the process 1-2'-2 represents the cooling and dehumidifying process and 2-3 represents the heating process. From the psychrometric chart, we find that the specific volume of air at point 1,

$$v_{s1} = 0.886 \text{ m}^3/\text{kg of dry air}$$

Enthalpy of air at point 1,

$$h_1 = 81.8 \text{ kJ / kg of dry air}$$

and enthalpy of air at point 2,

$$h_2 = 34.2 \text{ kJ / kg of dry air}$$

We know that mass of air absorbed by the plant,

$$m_a = \frac{v_1}{v_{s1}} = \frac{20}{0.886} = 22.6 \text{ kg / min}$$

∴ Capacity of the cooling coil

$$\begin{aligned} &= m_a (h_1 - h_2) = 22.6 (81.8 - 34.2) = 1075.76 \text{ kJ / min} \\ &= \frac{1075.76}{210} = 5.1 \text{ TR Ans.} \end{aligned}$$

2. Capacity of the heating coil in kW

From the psychrometric chart, we find that enthalpy of air at point 3,

$$h_3 = 42.6 \text{ kJ / kg of dry air}$$

∴ Capacity of the heating coil

$$\begin{aligned} &= m_a (h_3 - h_2) = 22.6 (42.6 - 34.2) = 189.84 \text{ kJ / min} \\ &= \frac{189.84}{60} = 3.16 \text{ kW Ans.} \end{aligned}$$

3. Amount of water removed per hour

From the psychrometric chart, we find that specific humidity of air at point 1,

$$W_1 = 0.0202 \text{ kg / kg of dry air}$$

and specific humidity of air at point 2,

$$W_2 = 0.0088 \text{ kg / kg of dry air}$$

∴ Amount of water removed per hour

$$\begin{aligned} &= m_a (W_1 - W_2) = 22.6 (0.0202 - 0.0088) = 0.258 \text{ kg / min} \\ &= 15.48 \text{ kg / h Ans.} \end{aligned}$$

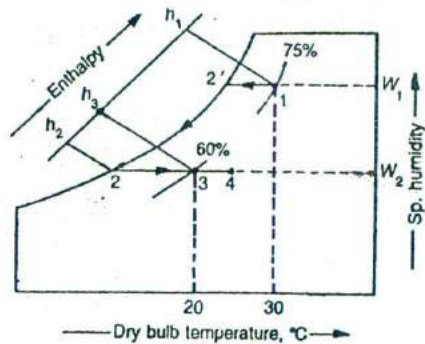


Fig 38.2

38.8. Winter Air Conditioning System

In winter air conditioning, the air is heated, which is generally accompanied by humidification. The schematic arrangement of the system is shown in Fig. 38.3.

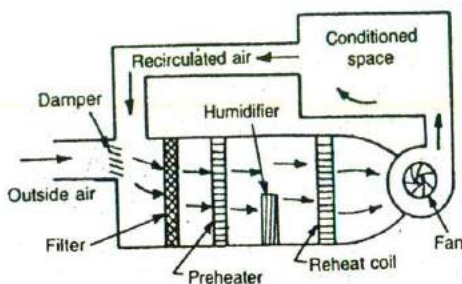


Fig. 38.3. Winter air conditioning system.

The outside air flows through a damper and mixes up with the recirculated air (which is obtained from the conditioned space). The mixed air passes through a filter to remove dirt, dust and other impurities. The air now passes through a preheat coil in order to prevent the possible freezing of water and to control the evaporation of water in the humidifier. After that, the air is made to pass through a reheat coil to bring the air to the designed dry bulb temperature. Now, the conditioned air is supplied to the conditioned space by a fan. From the conditioned space, a part of the used air is exhausted to the atmosphere by the exhaust fans or ventilators. The remaining part of the used air (known as recirculated air) is again conditioned, as shown in Fig. 38.3.

The outside air is sucked and made to mix with the recirculated air, in order to make up for the loss of conditioned (or used) air through exhaust fans or ventilation from the conditioned space.

Example 38.3. In a winter air conditioning system, 100 m^3 of air per minute at 15°C dry bulb temperature and 80% RH is heated until its dry bulb temperature is 22°C . Find heat added to the air per minute.

Solution. Given : $v_1 = 100 \text{ m}^3/\text{min}$; $t_{d1} = 15^\circ \text{C}$; $\phi_1 = 80\%$; $t_{d2} = 22^\circ \text{C}$

First of all, mark the initial condition of air at 15°C dry bulb temperature and 80% relative humidity on the psychrometric chart as point 1. Now mark final condition of air by drawing a horizontal line through point 1 up to 22°C dry bulb temperature, as shown in Fig. 38.4. From the psychrometric chart, we find that enthalpy of air at point 1,

$$h_1 = 36.5 \text{ kJ / kg of dry air}$$

Similarly, $h_2 = 43.5 \text{ kJ / kg of dry air}$

Moreover, specific volume of air at point 1,

$$v_{s1} = 0.827 \text{ m}^3/\text{kg}$$

\therefore Mass of air heated per minute,

$$m_a = \frac{v_1}{v_{s1}} = \frac{100}{0.827} = 120.9 \text{ kg}$$

and heat added to the air per minute

$$= m_a (h_2 - h_1) = 120.9 (43.5 - 36.5) = 846.3 \text{ kJ Aus.}$$

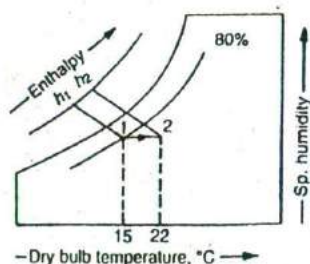


Fig. 38.4

38.9. Summer Air Conditioning System

It is the most important type of air conditioning, in which the air is cooled and generally dehumidified. The schematic arrangement of a typical summer air conditioning system is shown in Fig. 38.5.

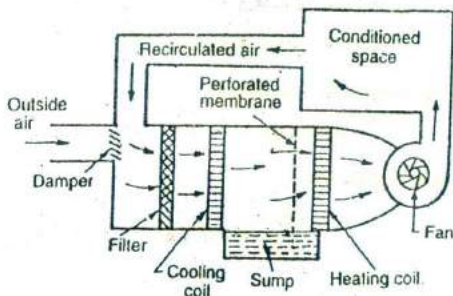


Fig. 38.5. Summer air conditioning system.

The outside air flows through the damper, and mixes up with recirculated air (which is obtained from the conditioned space). The mixed air passes through a filter to remove dirt, dust and other impurities. The air now passes through a cooling coil. The coil has a temperature much below the required dry bulb temperature of the air in the conditioned space. The cooled air passes through a perforated membrane and loses its moisture in the condensed form which is collected in a sump. After that, the air is made to pass through a heating coil which heats up the air slightly. This is done to bring the air to the designed dry bulb temperature and relative humidity.

Now the conditioned air is supplied to the conditioned space by a fan. From the conditioned space a part of the used air is exhausted to the atmosphere by the exhaust fans or ventilators. The remaining part of the used air (known as recirculated air) is again conditioned, as shown in Fig. 38.5. The outside air is sucked and made to mix with the recirculated air in order to make up for the loss of conditioned (or used) air through exhaust fans or ventilation from the conditioned space.

Example 38.4. A theatre of 1200 seating capacity is to be air conditioned for summer conditions with the following data :

Outdoor conditions	30° C DBT and 55% RH
Required conditions	20° C DBT and 60% RH
Amount of air supplied	0.25 m ³ /min/person

Find the sensible heat, latent heat removed from the air per minute and sensible heat factor for the system.

Solution. Given : Seating capacity = 1200 persons ; $t_{d1} = 30^\circ \text{C}$; $\phi_1 = 55\%$; $t_{d2} = 22^\circ \text{C}$; $\phi_2 = 60\%$; $v_1 = 0.25 \text{ m}^3/\text{min}/\text{person} = 0.25 \times 1200 = 300 \text{ m}^3/\text{min}$

Sensible heat removed from the air

First of all, mark the initial condition of air at 30° C dry bulb temperature and 55% relative humidity on the psychrometric chart as point 1, as shown in Fig. 38.6. Now mark the final condition of air at 20° C dry bulb temperature and 60% relative humidity on the chart as point 2. Now locate point 3 on the chart by

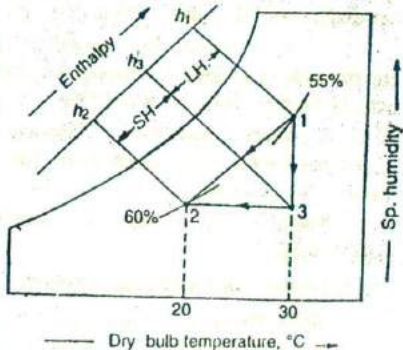


Fig. 38.6

drawing horizontal line through point 2 and vertical line through point 1. From the psychrometric chart, we find that enthalpy of air at point 1,

$$h_1 = 68 \text{ kJ / kg of dry air}$$

Similarly $h_2 = 42 \text{ kJ / kg of dry air}$

and $h_3 = 52.5 \text{ kJ / kg of dry air}$

Moreover, specific volume of air at point 1,

$$v_{s1} = 0.88 \text{ m}^3/\text{kg}$$

\therefore Mass of air cooled per minute

$$m_a = \frac{v_1}{v_{s1}} = \frac{300}{0.88} = 340.9 \text{ kg}$$

and sensible heat removed from the air,

$$SH = m_a (h_3 - h_2) = 340.9 (52.5 - 42) = 3579.5 \text{ kJ / min Ans.}$$

Latent heat removed from the air

We know that the latent heat removed from the air,

$$LH = m_a (h_1 - h_3) = 340.9 (68 - 52.5) = 5284 \text{ kJ / min Ans.}$$

Sensible heat factor for the system

We also know that sensible heat factor for the system,

$$= \frac{SH}{SH + LH} = \frac{3579.5}{3579.5 + 5284} = 0.4 \text{ Ans.}$$

38.10. Year-round Air Conditioning System

An year-round air conditioning system should have equipment for both the winter and summer air conditioning. There can be so many combinations for providing year-round air conditioning. The arrangement of the combination should be such that one part is working in winter and the other in summer. For example, heating and humidification is needed in winter, whereas cooling and dehumidification is needed in summer.

38.11. Unitary Air Conditioning System

In this system, factory assembled air conditioners are installed in or adjacent to the space to be conditioned. The unitary air conditioning systems are of the following two types :

1. *Window units.* These are self-contained units of small capacity of 1 tonne to 3 tonnes, and are mounted in a window or through the wall. They are employed to condition the air of one room only. If the room is bigger in size, then two or more units are installed.

2. *Vertical packed units.* These are also self-contained units of bigger capacity of 5 to 10 tonnes and are installed adjacent to the space to be conditioned. This is very useful for conditioning the air of a restaurant, bank or small office.

The unitary air conditioning system may be adopted for winter, summer or year-round air conditioning.

38.12. Central Air Conditioning System

This is the most important type of air conditioning system, which is adopted when the cooling capacity required is 25 tonnes or more. The central air conditioning system is also adopted when the air flow is more than 300 m³/min or different zones in a building are to be conditioned.

Example 38.5. An air conditioning plant is to be designed for a small office for winter conditions :

Out-door conditions = $10^{\circ}\text{C DBT } 8^{\circ}\text{C WBT}$

Required indoor conditions = $20^{\circ}\text{C DBT and } 60\% \text{ RH}$

Amount of air circulation = $0.3 \text{ m}^3/\text{min}/\text{person}$

Seating capacity of the office = 50 persons

The required conditions is achieved first by heating and then by adiabatic humidifying. Find :
1. heating capacity of the coil in kW ; and 2. capacity of the humidifier.

Solution. Given : $t_{d1} = 10^{\circ}\text{C}$; $t_{w1} = 8^{\circ}\text{C}$; $t_{d2} = 20^{\circ}\text{C}$; $\phi_2 = 60\%$; Seating capacity = 50 persons ; $v_1 = 0.3 \text{ m}^3/\text{min} / \text{person} = 0.3 \times 50 = 15 \text{ m}^3/\text{min}$

First of all, mark the initial condition of air at 10°C dry bulb temperature and 8°C wet bulb temperature on the psychrometric chart as point 1, as shown in Fig. 38.7. Now mark the final condition of air at 20°C dry bulb temperature and 60% relative humidity on the chart as point 2. Now locate point 3 on the chart by drawing horizontal line through point 1 and constant enthalpy line through point 2. From the psychrometric chart, we find that the specific volume at point 1,

$$v_{s1} = 0.81 \text{ m}^3/\text{kg of dry air}$$

\therefore Mass of air supplied per minute,

$$m_a = \frac{v_1}{v_{s1}} = \frac{15}{0.81} = 18.52 \text{ kg/min}$$

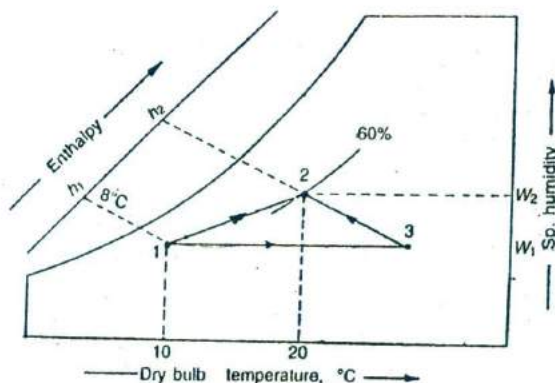


Fig. 38.7

1. Heating capacity of the coil in kW

From the psychrometric chart, we find that enthalpy at point 1,

$$h_1 = 24.8 \text{ kJ/kg of dry air}$$

and enthalpy at point 2, $h_2 = 42.6 \text{ kJ/kg of dry air}$

We know that heating capacity of the coil

$$= m_a (h_2 - h_1) = 18.52 (42.6 - 24.8) = 329.6 \text{ kJ/min}$$

$$= \frac{329.6}{60} = 5.49 \text{ kW Ans.}$$

2. Capacity of the humidifier

From the psychrometric chart, we find that specific humidity at point 1,

$$W_1 = 0.0058 \text{ kg / kg of dry air}$$

and specific humidity at point 2,

$$W_2 = 0.0088 \text{ kg / kg of dry air}$$

We know that capacity of the humidifier,

$$\begin{aligned} &= m_a (W_2 - W_1) = 18.52(0.0088 - 0.0058) = 0.055 \text{ kg / min} \\ &= 3.3 \text{ kg / h Ans.} \end{aligned}$$

Example 38.6. A small office hall of 25 persons capacity is provided with summer air conditioning system with the following data :

Outside conditions

$$= 34^\circ \text{ C DBT and } 28^\circ \text{ C WBT}$$

Inside conditions

$$= 24^\circ \text{ C DBT and } 50\% \text{ RH}$$

Volume of air supplied

$$= 0.4 \text{ m}^3/\text{min/person}$$

Sensible heat load in room

$$= 125\,600 \text{ kJ/h}$$

Latent heat load in the room

$$= 42\,000 \text{ kJ/h}$$

Find the sensible heat factor of the plant.

Solution. Given : Seating capacity = 25 persons ; $t_{d1} = 34^\circ \text{ C}$; $t_{w1} = 28^\circ \text{ C}$; $t_{d2} = 24^\circ \text{ C}$; $\phi_2 = 50\%$; $v_1 = 0.4 \text{ m}^3/\text{min} / \text{person} = 0.4 \times 25 = 10 \text{ m}^3/\text{h}$; SH load = 125 600 kJ/h ; LH load = 42 000 kJ/h

First of all, mark the initial condition of air at 34° C dry bulb temperature and 28° C wet bulb temperature on the psychrometric chart as point 1, as shown in Fig. 38.8. Now mark the final condition of air at 24° C dry bulb temperature and 50% relative humidity on the chart as point 2. Now locate point 3 on the chart by drawing horizontal line through point 2 and vertical line through point 1. From the psychrometric chart, we find that the specific volume at point 1,

$$v_{s1} = 0.9 \text{ m}^3/\text{kg of dry air}$$

Enthalpy of air at point 1,

$$h_1 = 90 \text{ kJ / kg of dry air}$$

Enthalpy of air at point 2,

$$h_2 = 48.2 \text{ kJ / kg of dry air}$$

and enthalpy of air at point 3,

$$h_3 = 59 \text{ kJ / kg of dry air}$$

We know that mass of air supplied per min,

$$m_a = \frac{v_1}{v_{s1}} = \frac{10}{0.9} = 11.1 \text{ kg / min}$$

and sensible heat removed from the air

$$\begin{aligned} &= m_a (h_3 - h_2) = 11.1 (59 - 48.2) = 119.88 \text{ kJ / min} \\ &= 7192.8 \text{ kJ/h} \end{aligned}$$

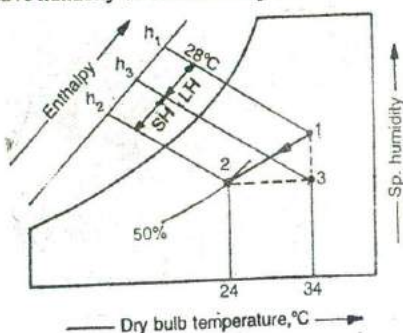


Fig. 38.8

∴ Total sensible heat of the room,

$$SH = 7192.8 + 125\,600 = 132\,792.8 \text{ kJ/h}$$

We know that latent heat removed from the air

$$= m_a (h_1 - h_3) = 11.1 (90 - 59) = 344.1 \text{ kJ/min} = 20\,646 \text{ kJ/h}$$

∴ Total latent heat of the room

$$LH = 20\,646 + 42\,000 = 62\,646 \text{ kJ/h}$$

We know that sensible heat factor,

$$SHF = \frac{SH}{SH + LH} = \frac{132\,792.8}{132\,792.8 + 62\,646} = 0.679 \text{ Ans.}$$

EXERCISES

1. An air conditioning plant is handling 30 m^3 of air per minute at 32°C dry bulb temperature and 22°C wet bulb temperature. If the final conditions of air are 22°C dry bulb temperature and 50% relative humidity, find the heating capacity of the dehumidifier. [Ans. 7.3 kg/h]

2. A cinema hall of seating capacity 1500 persons has been provided with an air conditioned plant with the following data :

Outdoor conditions	= 40°C DBT and 20°C WBT
Required conditions	= 20°C DBT and 60% RH
Amount of air supplied	= $0.3 \text{ m}^3/\text{min}/\text{person}$

If the required condition is achieved first by adiabatic humidifying and then cooling, find the capacity of cooling coil and capacity of the humidifier. [Ans. 37.6 TR ; 54.2 kg/h]

3. An office hall of 70 persons capacity is to be provided with summer air conditioning plant with the following data :

Outside conditions	= 35°C DBT and 26°C WBT
Inside conditions	= 20°C DBT and 45% RH
Volume of air supplied	= $0.3 \text{ m}^3/\text{min}/\text{person}$

Find the sensible heat factor for the plant.

[Ans. 0.35]

4. The amount of air supplied to an air conditioned hall is $300 \text{ m}^3/\text{min}$. The atmospheric conditions are 35°C DBT and 55% RH. The required conditions are 20°C DBT and 60% RH. Find out the sensible heat and latent heat removed from the air per minute. Also find sensible heat factor for the system.

[Ans. 5040 kJ/min ; 9517 kJ/min ; 0.346]

5. An auditorium of 300 seating capacity is to be provided with air conditioning with the following data :

Outdoor conditions	= 12°C DBT and 70% RH
Indoor conditions	= 20°C DBT and 50% RH
Amount of air supplied	= $0.25 \text{ m}^3/\text{min}/\text{person}$

Find the heating capacity of the coil.

[Ans. 13.6 kW]

6. A restaurant with a capacity of 100 persons is to be air-conditioned with the following conditions :

Outside conditions	= 30°C DBT, 70% RH
Desired inside conditions	= 23°C DBT, 55% RH
Quantity of air supplied	= $0.5 \text{ m}^3/\text{min}/\text{person}$

The desired conditions are achieved by cooling, dehumidifying and then heating. Determine : 1. Capacity of cooling coils in tonnes of refrigeration ; 2. Capacity of heating coil ; and 3. Amount of water removed by dehumidifier.

[Ans. 9.3 TR ; 8.7 kW ; 31.5 kg/h]

QUESTIONS

1. Define comfort. What are the factors which affect comfort air conditioning ?
2. How does industrial air conditioning is different from comfort air conditioning ?
3. Draw a line diagram of air conditioning system required in winter season. Explain the working of different components in the circuit.
4. Draw a block diagram of a simple year round air conditioning system and briefly explain the process on a psychrometric chart.
5. Describe unitary and central air conditioning systems.

OBJECTIVE TYPE QUESTIONS

1. For summer air conditioning, the relative humidity should not be less than
(a) 40% (b) 60% (c) 75% (d) 90%
2. For winter air conditioning, the relative humidity should not be more than
(a) 40% (b) 60% (c) 75% (d) 90%
3. The sensible heat factor for auditorium or cinema hall is generally kept as
(a) 0.6 (b) 0.7 (c) 0.8 (d) 0.9
4. The conditioned air supplied to the room must have the capacity to take up
(a) room sensible heat load only
(b) room latent heat load only
(c) both room sensible heat and latent heat loads

ANSWERS

1. (b) 2. (a) 3. (b) 4. (c)

General Thermodynamic Relations

1. Introduction. 2. Helmholtz Function. 3. Gibbs Function. 4. Mathematical Fundamentals. 5. Maxwell's Equations. 6. Co-efficient of Volume Expansion. 7. Isothermal Compressibility. 8. Adiabatic Compressibility. 9. Ratio of Co-efficient of Volume Expansion and Isothermal Compressibility. 10. General Equations for Change in Internal Energy. 11. TdS Equations. 12. Specific Heats. 13. Specific Heat Relations. 14. Joule Thomson Co-efficient. 15. Equations of State. 16. Compressibility Factor.

39.1. Introduction

We have already discussed, in the previous chapters, the six thermodynamic properties, *i.e.* pressure (p), volume (v), temperature (T), internal energy (U), enthalpy (H) and entropy (S). There are other two important properties also which are known as Helmholtz function (A) and Gibbs function (G). In this chapter, we shall discuss some important thermodynamic relations between different properties. It may be noted that only the three thermodynamic properties (*i.e.* pressure, volume and temperature) are directly measurable while the other properties are evaluated in terms of these measurable properties. Thus, it is very important to develop the thermodynamic relations for the properties which cannot be measured directly in terms of the combination of properties which can be measured directly or evaluated experimentally. These all combinations are defined as the rate of change of one property with respect to another while the third property is held constant. The thermodynamic relations have been derived with the help of first and second laws of thermodynamics and the mathematical concepts of differentiation, particularly partial differentiation.

39.2. Helmholtz Function

Helmholtz function (A) is a property of a system and is given by subtracting the product of absolute temperature (T) and entropy (S) from the internal energy (U). Mathematically,

$$\text{Helmholtz function, } A = U - TS \quad \dots (i)$$

Since ($U - TS$) is made up entirely of properties, therefore Helmholtz function (A) is also a property.

For per unit mass, specific Helmholtz function,

$$a = u - Ts$$

where

u = Specific internal energy, and

s = Specific entropy.

The general gas energy equation is

$$\delta Q = dU + \delta W \quad \text{or} \quad \delta W = \delta Q - dU \quad \dots (ii)$$

The equation (ii) implies that the energy converted into work is provided partly by the heat reservoir with which the system is in contact and which gives up a quantity of heat δQ and partly by the system whose internal energy decreases by $-dU$.

Let us now, derive the relationship for the *maximum work* that can be done when a system undergoes a process between two equilibrium states. Assume that the system exchanges heat only with a single heat reservoir at a temperature T_0 .

Let

- dS = Increase in entropy of the system,
- dS_0 = Increase in entropy of the surroundings (reservoir),
- δQ = Heat absorbed by the system from the reservoir, and
- T_0 = Temperature of the reservoir.

From the *principle of increase of entropy, it is known that the sum of the increase in entropy of the system and that of the surroundings is equal to or greater than zero.

$$\therefore dS + dS_0 \geq 0 \quad \dots (iii)$$

We know that

$$dS_0 = \frac{-\delta Q}{T_0} \quad \dots (\because \text{Reservoir has given out heat, therefore } \delta Q \text{ is -ve})$$

Substituting this value of dS_0 in equation (iii), we get

$$dS - \frac{\delta Q}{T_0} \geq 0$$

$$\text{or} \quad T_0 dS \geq \delta Q \quad \dots (iv)$$

From equations (ii) and (iv), we get

$$\delta W \leq T_0 dS - dU \quad \dots (v)$$

On integrating the above equation from state 1 to state 2,

$$\int_1^2 \delta W \leq T_0 \int_1^2 dS - \int_1^2 dU \quad \dots (\because T_0 \text{ is a constant})$$

$$\text{or} \quad W_{1-2} \leq T_0 (S_2 - S_1) - (U_2 - U_1)$$

$$W_{1-2} \leq (U_1 - U_2) - T_0 (S_1 - S_2) \quad \dots (vi)$$

When initial and final temperatures are equal and are the same as that of the heat reservoir, then

$$T = T_0$$

where T = Temperature of the system.

Now, equation (vi) can be written as

$$(W_T)_{1-2} \leq (U_1 - U_2)_T - T (S_1 - S_2)_T \quad \dots (vii)$$

* Refer Chapter 4, Art. 4.7.

General Thermodynamic Relations

We know that, Helmholtz function,

$$A = U - TS$$

For a small change, the Helmholtz function can be written as

$$dA = dU - d(TS) \quad \dots \text{(viii)}$$

On integrating the above equation from state 1 to state 2, keeping temperature (T) constant, we have

$$\int_1^2 dA = \int_1^2 dU - T \int_1^2 dS$$

$$(A_2 - A_1)_T = (U_2 - U_1)_T - T(S_2 - S_1)_T$$

or

$$(A_1 - A_2)_T = (U_1 - U_2)_T - T(S_1 - S_2)_T \quad \dots \text{(ix)}$$

From equations (vii) and (ix), we have

$$(W_T)_{1-2} \leq (A_1 - A_2)_T \quad \dots \text{(x)}$$

Thus, the work done in any process between two equilibrium states keeping the temperature constant is equal to the decrease in Helmholtz function of the system but the energy converted into work is provided partly by the system and the remaining heat is taken from the heat reservoir.

The work done is maximum when the process is reversible. If the process is irreversible, the work done is less than the maximum. Therefore, for maximum work, equation (x) may be written as :

$$(W_T)_{max} = (W_T)_{rev} = (A_1 - A_2)_T$$

Note: Helmholtz function (A) is useful when considering chemical reactions and is of fundamental importance in statistical thermodynamics.

39.3. Gibbs Function

The Gibbs function (also known as thermodynamic potential) is also a property of a system and is denoted by G . Mathematically, Gibbs function,

$$G = U - TS + p v$$

$$= H - TS$$

$$\dots (\because H = U + p v)$$

Since $(H - TS)$ is made up entirely of properties, therefore Gibbs function (G) is also a property.

For per unit mass, specific Gibbs function,

$$g = h - T s$$

where

h = Specific enthalpy, and

s = Specific entropy.

Consider a system that can do other forms of work, in addition to $(p dv)$ like electrical work, magnetic work etc.

Let δW_0 = Some other form of work in addition to $p dv$.

$$\delta W = p dv + \delta W_0$$

On integrating the above equation from state 1 to state 2, keeping the pressure p (equal to

the external pressure, p_0) constant, we have

$$\int_1^2 \delta W = p \int_1^2 dv + \int_1^2 \delta W_0$$

$$\text{or } W_{1-2} = p_0(v_2 - v_1) + W_0^* \quad \dots (\because p = p_0) \quad \dots (i)$$

We have already discussed that when a system exchanges heat with a reservoir whose temperature is T_0 , then work done,

$$W_{1-2} \leq (U_1 - U_2) - T_0(S_1 - S_2) \quad \dots (ii)$$

Substituting the value of W_{1-2} from equation (i) in equation (ii), we get

$$p_0(v_2 - v_1) + W_0 \leq (U_1 - U_2) - T_0(S_1 - S_2)$$

$$\text{or } W_0 \leq (U_1 - U_2) - T_0(S_1 - S_2) - p_0(v_2 - v_1) \quad \dots (iii)$$

When the initial and final equilibrium states are at the same temperature and pressure, then

$$T_0 = T \quad \text{and} \quad p_0 = p$$

Thus, equation (iii) can be written as

$$(W_0)_{p,T} \leq (U_1 - U_2)_{p,T} - T(S_1 - S_2)_{p,T} + p(v_1 - v_2)_{p,T} \quad \dots (iv)$$

We know that, Gibbs function,

$$G = U - TS + p v$$

For a small change, the Gibbs function can be written as

$$dG = dU - d(TS) + d(pv)$$

On integrating from state 1 to state 2, keeping both pressure (p) and temperature (T) constant, we have

$$\int_1^2 dG = \int_1^2 dU - T \int_1^2 dS + p \int_1^2 dv \quad \dots (\because p \text{ and } T \text{ are constant})$$

$$G_2 - G_1 = (U_2 - U_1) - T(S_2 - S_1) + p(v_2 - v_1)$$

$$\text{or } (G_1 - G_2)_{p,T} = (U_1 - U_2)_{p,T} - T(S_1 - S_2)_{p,T} + p(v_1 - v_2)_{p,T} \quad \dots (v)$$

From equations (iv) and (v), we have

$$(W_0)_{p,T} \leq (G_1 - G_2)_{p,T} \quad \dots (vi)$$

Thus, the difference between Gibbs function of a system between two equilibrium states sets the maximum limit to the work that can be performed in addition to $p dv$ work, provided that two equilibrium states are at the same pressure and temperature and the system exchange heat with a single heat reservoir.

The work done is maximum when the process is reversible. If the process is irreversible,

* W_0 may be electrical work as in case of voltaic cell which is equal to $-E dl$ or for a magnetic material, it may be magnetic work which is equal to $-mdH$.

General Thermodynamic Relations

the work done is less than the maximum. Therefore, for maximum work, equation (vi) can be written as :

$$(W_0)_{p,T} = (W_0)_{p,T} = (G_1 - G_2)_{p,T}$$

Note : The Gibbs function (G) is also useful when considering chemical reactions. It is also useful when considering processes involving a change of phase.

39.4. Mathematical Fundamentals

The following mathematical fundamentals are important from the subject point of view :

1. *Exact differential.* Suppose z is a function of two independent properties x and y , such that

$$z = f(x, y) \quad \dots (i)$$

According to the condition of exact differentials,

$$dz = \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy \quad \dots (ii)$$

where

$$\left(\frac{\partial z}{\partial x}\right)_y = \text{Partial derivative of } z \text{ with respect to } x, \text{ when } y \text{ is held constant, and}$$

$$\left(\frac{\partial z}{\partial y}\right)_x = \text{Partial derivative of } z \text{ with respect to } y, \text{ when } x \text{ is held constant.}$$

Now, the above equation (ii) can be written as

$$dz = Mdx + Ndy \quad \dots (iii)$$

where

$$M = \left(\frac{\partial z}{\partial x}\right)_y \quad \text{and} \quad N = \left(\frac{\partial z}{\partial y}\right)_x$$

We know that,

$$\left(\frac{\partial M}{\partial y}\right)_x = \left[\frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}\right)_y \right]_x = \left(\frac{\partial^2 z}{\partial x \partial y}\right) \quad \dots (iv)$$

and

$$\left(\frac{\partial N}{\partial x}\right)_y = \left[\frac{\partial}{\partial x} \left(\frac{\partial z}{\partial y}\right)_x \right]_y = \left(\frac{\partial^2 z}{\partial y \partial x}\right) \quad \dots (v)$$

According to calculus, the magnitude of second mixed partial derivatives of a function are independent of the order of differentiation and hence are equal.

$$\therefore \left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

2. *Cyclic relation.* We know that, when z is a function of two independent variables x and y , then we can write

$$z = f(x, y) \quad \dots (i)$$

It may be noted that from above equation, x can be similarly expressed explicitly in terms of other two variables *i.e.* y and z , such that

$$x = f(y, z) \quad \dots (ii)$$

According to the condition of exact differentials,

$$dx = \left(\frac{\partial x}{\partial y} \right)_z dy + \left(\frac{\partial x}{\partial z} \right)_y dz \quad \dots (iii)$$

Similarly y can be expressed explicitly in terms of x and z , so that

$$y = f(x, z) \quad \dots (iv)$$

According to the condition of exact differentials,

$$dy = \left(\frac{\partial y}{\partial x} \right)_z dx + \left(\frac{\partial y}{\partial z} \right)_x dz \quad \dots (v)$$

Now from equation (iii), we have

$$dx = \left(\frac{\partial x}{\partial z} \right)_y dz + \left(\frac{\partial x}{\partial y} \right)_z dy$$

Substituting the value of dy from equation (v) in the above equation, we have

$$\begin{aligned} dx &= \left(\frac{\partial x}{\partial z} \right)_y dz + \left(\frac{\partial x}{\partial y} \right)_z \left[\left(\frac{\partial y}{\partial x} \right)_z dx + \left(\frac{\partial y}{\partial z} \right)_x dz \right] \\ &= \left(\frac{\partial x}{\partial z} \right)_y dz + \left(\frac{\partial x}{\partial y} \right)_z \left(\frac{\partial y}{\partial x} \right)_z dx + \left(\frac{\partial x}{\partial y} \right)_z \left(\frac{\partial y}{\partial z} \right)_x dz \\ &= \left[\left(\frac{\partial x}{\partial z} \right)_y + \left(\frac{\partial x}{\partial y} \right)_z \left(\frac{\partial y}{\partial z} \right)_x \right] dz + dx \end{aligned}$$

$$\text{or} \quad \left(\frac{\partial x}{\partial y} \right)_z \left(\frac{\partial y}{\partial z} \right)_x + \left(\frac{\partial x}{\partial z} \right)_y = 0$$

$$\text{or} \quad \left(\frac{\partial x}{\partial y} \right)_z \left(\frac{\partial y}{\partial z} \right)_x = - \left(\frac{\partial x}{\partial z} \right)_y$$

$$\text{or} \quad \left(\frac{\partial x}{\partial y} \right)_z \left(\frac{\partial y}{\partial z} \right)_x \left(\frac{\partial z}{\partial x} \right)_y = -1$$

The above relation is known as *cyclic relation*.

Note : For thermodynamic properties p , v and T , the cyclic relation is

$$\left(\frac{\partial p}{\partial v} \right)_T \left(\frac{\partial v}{\partial T} \right)_p \left(\frac{\partial T}{\partial p} \right)_v = -1$$

39.5. Maxwell's Equations

Now let us derive the Maxwell's equation.

We know that for a system undergoing an infinitesimal reversible process from one equilibrium state to another :

$$1. \text{ Internal energy,} \quad dU = \delta Q - \delta W \quad \dots (\text{General gas energy equation})$$

$$= T dS - p dv \quad \dots (\because \delta Q = T dS \text{ and } \delta W = p dv) \quad \dots (i)$$

The above equation (i) is of the form

$$dz = M dx + N dy$$

where $M = T, N = -p, x = S$ and $y = v$

$$\therefore \left(\frac{\partial T}{\partial v} \right)_S = - \left(\frac{\partial p}{\partial S} \right)_v \quad \dots (ii)$$

2. Enthalpy,

$$\begin{aligned} dH &= dU + d(pv) \\ &= dU + p dv + v dp \\ &= (T dS - p dv) + p dv + v dp \quad \dots (\because dU = T dS - p dv) \\ &= T ds + v dp \quad \dots (iii) \end{aligned}$$

The above equation (iii) is of the form

$$dz = M dx + N dy$$

where

$$M = T, N = v, x = S \text{ and } y = p$$

$$\therefore \left(\frac{\partial T}{\partial p} \right)_S = \left(\frac{\partial v}{\partial S} \right)_p \quad \dots (iv)$$

3. Helmholtz function (A),

$$\begin{aligned} dA &= dU - d(TS) \\ &= dU - T dS - S dT \\ &= (T dS - p dv) - T dS - S dT \quad \dots (\because dU = T dS - p dv) \\ &= -p dv - S dT \quad \dots (v) \end{aligned}$$

The above equation (v) is of the form

$$dz = M dx + N dy$$

where

$$M = -p, N = -S, x = v, y = T$$

$$\therefore - \left(\frac{\partial p}{\partial T} \right)_v = - \left(\frac{\partial S}{\partial v} \right)_T$$

or

$$\left(\frac{\partial p}{\partial T} \right)_v = \left(\frac{\partial S}{\partial v} \right)_T \quad \dots (vi)$$

4. Gibbs function (G), $dG = dH - d(TS)$

$$\begin{aligned} &= dH - T dS - S dT \\ &= (T dS + v dp) - T dS - S dT \quad \dots (\because dH = T dS + v dp) \\ &= v dp - S dT \quad \dots (vii) \end{aligned}$$

The above equation (vii) is of the form

$$dz = M dx + N dy$$

where

$$M = v, N = -S, x = p, y = T$$

$$\therefore \left(\frac{\partial v}{\partial T} \right)_p = - \left(\frac{\partial S}{\partial p} \right)_T \quad \dots (viii)$$

The equations (ii), (iv), (vi) and (viii) are known as *Maxwell's equations* in thermodynamics.

39.6. Co-efficient of Volume Expansion

The co-efficient of volume expansion is defined as the change in volume with change in

temperature per unit volume keeping the pressure constant. It is denoted by β . Mathematically, co-efficient of volume expansion,

$$\beta = \frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_p$$

For specific volume, the coefficient of volume expansion is written as,

$$\beta = \frac{1}{v_s} \left(\frac{\partial v_s}{\partial T} \right)_p$$

where $v_s =$ Specific volume in m^3/kg .

The volume-temperature ($v-T$) diagram at constant pressure is shown in Fig. 39.1. The slope at any point A is given by $\left(\frac{\partial v}{\partial T} \right)_p$. If this slope

is divided by the volume at that point, then we get co-efficient of volume expansion (β).

39.7. Isothermal Compressibility

The isothermal compressibility of a substance is defined as the change in volume with the change in pressure per unit volume, keeping the temperature constant. It is denoted by K . Mathematically, isothermal compressibility,

$$K = -\frac{1}{v} \left(\frac{\partial v}{\partial p} \right)_T$$

For specific volume, the isothermal compressibility is written as

$$K = -\frac{1}{v_s} \left(\frac{\partial v_s}{\partial p} \right)_T$$

where $v_s =$ Specific volume in m^3/kg .

The pressure-volume ($p-v$) diagram at constant temperature is shown in Fig. 39.2. The slope at a point is given by $-\left(\frac{\partial v}{\partial p} \right)_T$. If this slope is divided by the volume at that point, then we get isothermal compressibility (K).

39.8. Adiabatic Compressibility

The adiabatic compressibility is defined as the change in volume with change in pressure per unit volume when entropy is kept constant. It is denoted by K_s . Mathematically, adiabatic compressibility,

$$K_s = -\frac{1}{v} \left(\frac{\partial v}{\partial p} \right)_s$$

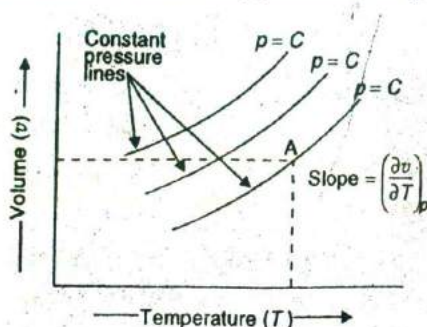


Fig 39.1. Coefficient of volume expansion.

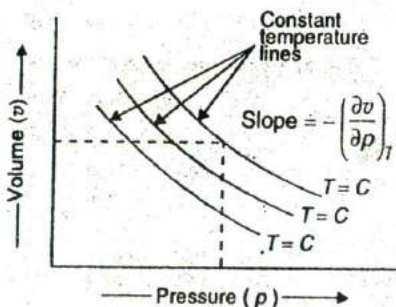


Fig 39.2. Isothermal compressibility.

* The minus sign in the equation indicates that the slope is decreasing.

For specific volume, the adiabatic compressibility is written as

$$K_S = -\frac{1}{v_s} \left(\frac{\partial v_s}{\partial p} \right)_S$$

where

$v_s =$ Specific volume in m^3/kg .

Note : It may be noted that the ratio of specific heats (i.e. c_p/c_v) is equal to the ratio of isothermal compressibility and adiabatic compressibility. Mathematically, adiabatic index,

$$\gamma = \frac{c_p}{c_v} = \frac{K}{K_S} = \frac{-(1/v) (\partial v/\partial p)_T}{-(1/v) (\partial v/\partial p)_S} = \frac{(\partial p/\partial v)_S}{(\partial p/\partial v)_T}$$

For specific volume, the adiabatic index is written as

$$\gamma = \frac{(\partial p/\partial v_s)_S}{(\partial p/\partial v_s)_T}$$

Since γ is always greater than unity, therefore K is always greater than K_S .

39.9. Ratio of Co-efficient of Volume Expansion and Isothermal Compressibility

We know that the co-efficient of volume expansion,

$$\beta = \frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_p \quad \text{or} \quad \left(\frac{\partial v}{\partial T} \right)_p = \beta v \quad \dots (i)$$

and isothermal compressibility,

$$K = -\frac{1}{v} \left(\frac{\partial v}{\partial p} \right)_T \quad \text{or} \quad \left(\frac{\partial v}{\partial p} \right)_T = -Kv \quad \dots (ii)$$

We also know by cyclic property of differentials,

$$\left(\frac{\partial p}{\partial T} \right)_v \left(\frac{\partial T}{\partial v} \right)_p \left(\frac{\partial v}{\partial p} \right)_T = -1$$

Substituting the value of $\left(\frac{\partial v}{\partial T} \right)_p$ and $\left(\frac{\partial v}{\partial p} \right)_T$ from equations (i) and (ii), we get

$$\left(\frac{\partial p}{\partial T} \right)_v \left(\frac{1}{\beta v} \right) (-Kv) = -1$$

$$\text{or} \quad \left(\frac{\partial p}{\partial T} \right)_v = \frac{\beta v}{Kv} = \frac{\beta}{K} \quad \dots (iii)$$

The ratio of co-efficient of volume expansion and isothermal compressibility is defined as the change in pressure with temperature when volume is kept constant. Thus

$$\frac{\beta}{K} = \left(\frac{\partial p}{\partial T} \right)_v$$

39.10. General Equations for Change in Internal Energy

We have already discussed that internal energy (U) is a property of a system which cannot be measured directly or experimentally. Hence thermodynamic general equations are derived in terms of pressure (p), volume (v) and temperature (T) and specific heats to measure change in internal energy (dU).

Let us now derive the following equations for the change in internal energy.

1. When pressure (p) and volume (v) are independent. When p and v are taken as independent variables, then change in internal energy,

$$dU = \left(\frac{\partial U}{\partial p} \right)_v dp + \left(\frac{\partial U}{\partial v} \right)_p dv \quad \dots (i)$$

It may be noted that

$$\left(\frac{\partial U}{\partial p} \right)_v = \left(\frac{\partial U}{\partial T} \right)_v \left(\frac{\partial T}{\partial p} \right)_v = m c_v \times \frac{K}{\beta} \quad \dots (ii)$$

$$\dots \left[\because \left(\frac{\partial U}{\partial T} \right)_v = m c_v, \text{ and } \left(\frac{\partial T}{\partial p} \right)_v = \frac{K}{\beta} \right]$$

We know that $H = U + p v \quad \dots (iii)$

Taking the partial derivative of equation (iii) with respect to volume (v) keeping the pressure (p) constant, we get

$$\left(\frac{\partial H}{\partial v} \right)_p = \left(\frac{\partial U}{\partial v} \right)_p + \frac{\partial}{\partial v} (p v) = \left(\frac{\partial U}{\partial v} \right)_p + p$$

or $\left(\frac{\partial U}{\partial v} \right)_p = \left(\frac{\partial H}{\partial v} \right)_p - p \quad \dots (iv)$

It may be noted that

$$\left(\frac{\partial H}{\partial v} \right)_p = \left(\frac{\partial H}{\partial T} \right)_p \left(\frac{\partial T}{\partial v} \right)_p = \frac{m c_p}{\beta v} \quad \dots (v)$$

$$\dots \left[\left(\frac{\partial H}{\partial T} \right)_p = m c_p \text{ and } \left(\frac{\partial T}{\partial v} \right)_p = \frac{1}{\beta v} \right]$$

Substituting this value of $\left(\frac{\partial H}{\partial v} \right)_p$ in equation (iv), we get

$$\left(\frac{\partial U}{\partial v} \right)_p = \frac{m c_p}{\beta v} - p \quad \dots (vi)$$

Now substituting the values of $\left(\frac{\partial U}{\partial p} \right)_v$ from equation (ii) and $\left(\frac{\partial U}{\partial v} \right)_p$ from equation (vi), in equation (i), we get

$$dU = \left(m c_v \times \frac{K}{\beta} \right) dp + \left(\frac{m c_p}{\beta v} - p \right) dv$$

Note: The change in specific internal energy (du) is given as

$$du = \left(c_v \times \frac{K}{\beta} \right) dp + \left(\frac{c_p}{v_s \beta} - p \right) dv_s$$

where

$v_s =$ Specific volume of the gas in m^3/kg .

2. When volume (v) and temperature (T) are independent. When v and T are taken as independent variables, then change in internal energy,

$$dU = \left(\frac{\partial U}{\partial v} \right)_T dv + \left(\frac{\partial U}{\partial T} \right)_v dT \quad \dots (i)$$

By definition, we know that,

$$\left(\frac{\partial U}{\partial T}\right)_v = m c_v$$

$$\therefore dU = \left(\frac{\partial U}{\partial v}\right)_T dv + m c_v dT \quad \dots (ii)$$

From the first law of thermodynamics,

$$\delta Q = dU + \delta W = dU + p dv \quad \dots (iii)$$

and from the second law of thermodynamics,

$$dS = \frac{\delta Q}{T} \quad \text{or} \quad \delta Q = T dS \quad \dots (iv)$$

From equations (iii) and (iv),

$$T dS = dU + p dv \quad \dots (v)$$

We know that enthalpy,

$$H = U + p v$$

Differentiating the above equation, we get

$$dH = dU + d(p v) = dU + p dv + v dp = T dS + v dp \quad \dots (\because T dS = dU + p dv)$$

$$\text{or} \quad T dS = dH - v dp \quad \dots (vi)$$

When T and p are taken as independent variables, then change in enthalpy,

$$dH = \left(\frac{\partial H}{\partial T}\right)_p dT + \left(\frac{\partial H}{\partial p}\right)_T dp = m c_p dT + \left(\frac{\partial H}{\partial p}\right)_T dp \quad \dots (vii)$$

$$\dots \left[\because \left(\frac{\partial H}{\partial p}\right)_T = m c_p \right]$$

From equations (v) and (vi),

$$dU + p dv = dH - v dp$$

Substituting the values of dU and dH in above equation, we get

$$\left(\frac{\partial U}{\partial v}\right)_T dv + m c_v dT + p dv = m c_p dT + \left(\frac{\partial H}{\partial p}\right)_T dp - v dp$$

$$\text{or} \quad m c_v dT + \left[p + \left(\frac{\partial U}{\partial v}\right)_T \right] dv = m c_p dT - \left[v - \left(\frac{\partial H}{\partial p}\right)_T \right] dp$$

When pressure is assumed to be constant, then $dp = 0$.

$$\therefore m (c_p - c_v) (dT)_p = \left[p + \left(\frac{\partial U}{\partial v}\right)_T \right] (dv)_p$$

$$\text{or} \quad m (c_p - c_v) = \left[p + \left(\frac{\partial U}{\partial v}\right)_T \right] \left(\frac{dv}{dT}\right)_p$$

$$\text{or} \quad m(c_p - c_v) = \left[p + \left(\frac{\partial U}{\partial v} \right)_T \right] \beta v \quad \dots \left[\because \left(\frac{\partial v}{\partial T} \right)_p = \beta v \right]$$

$$\frac{m(c_p - c_v)}{\beta v} = p + \left(\frac{\partial U}{\partial v} \right)_T$$

$$\therefore \left(\frac{\partial U}{\partial v} \right)_T = \frac{m(c_p - c_v)}{\beta v} - p \quad \dots \text{(viii)}$$

Substituting this value in equation (ii), we get

$$dU = \left[\frac{m(c_p - c_v)}{\beta v} - p \right] dv + m c_v dT$$

Note : The change in specific internal energy (du) is given as

$$du = \left[\frac{(c_p - c_v)}{\beta v_s} - p \right] dv_s + c_v dT$$

where

v_s = Specific volume of the gas in m^3/kg .

3. When temperature (T) and pressure (p) are independent. When T and p are taken as independent variables, then change in internal energy,

$$dU = \left(\frac{\partial U}{\partial T} \right)_p dT + \left(\frac{\partial U}{\partial p} \right)_T dp \quad \dots \text{(i)}$$

By definition, we know that,

$$m c_p = \left(\frac{\partial H}{\partial T} \right)_p = \left[\frac{\partial (U + pv)}{\partial T} \right]_p = \left(\frac{\partial U}{\partial T} \right)_p + v \left(\frac{\partial p}{\partial T} \right)_p + p \left(\frac{\partial v}{\partial T} \right)_p \quad \dots (\because H = U + pv)$$

$$= \left(\frac{\partial U}{\partial T} \right)_p + p \left(\frac{\partial v}{\partial T} \right)_p \quad \dots (\because \text{Pressure is constant})$$

$$\therefore \left(\frac{\partial U}{\partial T} \right)_p = m c_p - p \left(\frac{\partial v}{\partial T} \right)_p = m c_p - p \beta v \quad \dots \left[\because \left(\frac{\partial v}{\partial T} \right)_p = \beta v \right] \quad \dots \text{(ii)}$$

Also, from the first law of thermodynamics,

$$\delta Q = dU + \delta W = dU + p dv$$

If T and p are taken as independent variables, then we can say

$$\begin{aligned} \delta Q &= \left(\frac{\partial U}{\partial T} \right)_p dT + \left(\frac{\partial U}{\partial p} \right)_T dp + p \left[\left(\frac{\partial v}{\partial T} \right)_p dT + \left(\frac{\partial v}{\partial p} \right)_T dp \right] \\ &= \left[\left(\frac{\partial U}{\partial T} \right)_p + p \left(\frac{\partial v}{\partial T} \right)_p \right] dT + \left[\left(\frac{\partial U}{\partial p} \right)_T + p \left(\frac{\partial v}{\partial p} \right)_T \right] dp \end{aligned}$$

We know that

$$m c_p = \left(\frac{\partial U}{\partial T} \right)_p + p \left(\frac{\partial v}{\partial T} \right)_p$$

$$\therefore \delta Q = m c_p dT + \left[\left(\frac{\partial U}{\partial p} \right)_T + p \left(\frac{\partial v}{\partial p} \right)_T \right] dp$$

If volume is assumed to be constant, then

$$(\delta Q)_v = m c_v (dT)_v = m c_p (dT)_v + \left[\left(\frac{\partial U}{\partial p} \right)_T + p \left(\frac{\partial v}{\partial p} \right)_T \right] (dp)_v$$

$$\therefore m c_v = m c_p + \left[\left(\frac{\partial U}{\partial p} \right)_T + p \left(\frac{\partial v}{\partial p} \right)_T \right] \left(\frac{dp}{dT} \right)_v$$

But $\left(\frac{\partial v}{\partial p} \right)_T = -K v$ and $\left(\frac{dp}{dT} \right)_v = \frac{\beta}{K}$

$$\therefore m c_v = m c_p + \left[\left(\frac{\partial U}{\partial p} \right)_T - p K v \right] \frac{\beta}{K}$$

Thus $\left(\frac{\partial U}{\partial p} \right)_T = p K v - \frac{m (c_p - c_v) K}{\beta}$... (iii)

From equations (ii) and (iii),

$$dU = (m c_p - p \beta v) dT + \left[p K v - \frac{m (c_p - c_v) K}{\beta} \right] dp$$

Note : For per unit mass, the change in specific internal energy (du) is given as

$$du = (c_p - p \beta v_s) dT + \left[p K v_s - \frac{(c_p - c_v) K}{\beta} \right] dp$$

where

$$v_s = \text{Specific volume of the gas in m}^3/\text{kg}.$$

39.11. $T dS$ Equations

The First Law of Thermodynamics as applied to a closed system undergoing a reversible process is given by

$$\delta Q = dU + \delta W = dU + p dv \quad \dots (\because \delta W = p dv) \quad \dots (i)$$

According to Second Law of Thermodynamics, we know that

$$dS = \frac{\delta Q}{T} \quad \text{or} \quad \delta Q = T dS \quad \dots (ii)$$

From equations (i) and (ii),

$$T dS = dU + p dv \quad \dots (iii)$$

The above equation (iii) is sometimes called as *general TdS equation*. Let us derive the following useful relationships for the TdS equation.

1. When temperature (T) and volume (v) are independent. When T and v are taken as independent variables, then change in internal energy,

$$dU = \left(\frac{\partial U}{\partial T} \right)_v dT + \left(\frac{\partial U}{\partial v} \right)_T dv \quad \dots (i)$$

We know that

$$T dS = dU + p dv$$

Substituting the value of dU in the above equation, we get

$$TdS = \left[\left(\frac{\partial U}{\partial T} \right)_v dT + \left(\frac{\partial U}{\partial v} \right)_T dv \right] + p dv$$

or

$$\begin{aligned} dS &= \frac{1}{T} \left[\left(\frac{\partial U}{\partial T} \right)_v dT + \left(\frac{\partial U}{\partial v} \right)_T dv \right] + \frac{p}{T} dv \\ &= \frac{1}{T} \left(\frac{\partial U}{\partial T} \right)_v dT + \frac{1}{T} \left[p + \left(\frac{\partial U}{\partial v} \right)_T \right] dv \end{aligned} \quad \dots (ii)$$

But, we know that for dS , taking T and v as independent variables, we can write

$$dS = \left(\frac{\partial S}{\partial T} \right)_v dT + \left(\frac{\partial S}{\partial v} \right)_T dv \quad \dots (iii)$$

Comparing equations (ii) and (iii), we get

$$\left(\frac{\partial S}{\partial T} \right)_v = \frac{1}{T} \left(\frac{\partial U}{\partial T} \right)_v \quad \dots (iv)$$

and

$$\left(\frac{\partial S}{\partial v} \right)_T = \frac{1}{T} \left[p + \left(\frac{\partial U}{\partial v} \right)_T \right] \quad \dots (v)$$

Taking the partial derivative of equation (iv) with respect to volume (v), we get

$$\frac{\partial^2 S}{\partial v \partial T} = \frac{1}{T} \left[\frac{\partial^2 U}{\partial v \partial T} \right] \quad \dots (vi)$$

Taking the partial derivative of equation (v) with respect to temperature (T), we get

$$\frac{\partial^2 S}{\partial T \partial v} = -\frac{1}{T^2} \left[p + \left(\frac{\partial U}{\partial v} \right)_T \right] + \frac{1}{T} \left[\left(\frac{\partial p}{\partial T} \right)_v + \frac{\partial^2 U}{\partial T \partial v} \right] \quad \dots (vii)$$

We have already derived that the magnitude of second mixed partial derivative of a function are independent of the order of differentiation and hence are equal.

$$\therefore \frac{\partial^2 S}{\partial v \partial T} = \frac{\partial^2 S}{\partial T \partial v}$$

$$\text{or} \quad \frac{1}{T} \left[\frac{\partial^2 U}{\partial v \partial T} \right] = -\frac{1}{T^2} \left[p + \left(\frac{\partial U}{\partial v} \right)_T \right] + \frac{1}{T} \left[\left(\frac{\partial p}{\partial T} \right)_v + \frac{\partial^2 U}{\partial T \partial v} \right]$$

$$\text{or} \quad \frac{1}{T^2} \left[p + \left(\frac{\partial U}{\partial v} \right)_T \right] = \frac{1}{T} \left(\frac{\partial p}{\partial T} \right)_v$$

$$\text{or} \quad \frac{1}{T} \left[p + \left(\frac{\partial U}{\partial v} \right)_T \right] = \left(\frac{\partial p}{\partial T} \right)_v = \frac{\beta}{K} \quad \dots \left[\because \frac{\beta}{K} = \left(\frac{\partial p}{\partial T} \right)_v \right] \quad \dots (viii)$$

Also, we know that

$$m c_v = T \left(\frac{\partial S}{\partial T} \right)_v \quad \text{or} \quad \left(\frac{\partial S}{\partial T} \right)_v = \frac{m c_v}{T}$$

where $m =$ Mass of the gas in kg.

From equation (iv), we know that

$$\left(\frac{\partial S}{\partial T}\right)_v = \frac{1}{T} \left(\frac{\partial U}{\partial T}\right)_v$$

or
$$\frac{1}{T} \left(\frac{\partial U}{\partial T}\right)_v = \frac{m c_v}{T} \quad \dots \left[\because \left(\frac{\partial S}{\partial T}\right)_v = \frac{m c_v}{T} \right] \quad \dots (ix)$$

Substituting the values from equation (viii) and (ix) in equation (ii), we get

$$dS = \frac{m c_v}{T} dT + \frac{\beta}{K} dv$$

or
$$T dS = m c_v dT + \frac{T \beta}{K} dv \quad \dots (x)$$

The above equation (x) is called the *first TdS equation*.

2. When temperature (T) and pressure (p) are independent. When T and p are taken as independent variables, then change in internal energy,

$$dU = \left(\frac{\partial U}{\partial T}\right)_p dT + \left(\frac{\partial U}{\partial p}\right)_T dp \quad \dots (i)$$

and change in volume,

$$dv = \left(\frac{\partial v}{\partial T}\right)_p dT + \left(\frac{\partial v}{\partial p}\right)_T dp \quad \dots (ii)$$

But, we have discussed when T and v are taken as independent variables, then change in internal energy (dU) can be expressed as

$$dU = (m c_p - p \beta v) dT + \left[p K v - \frac{m(c_p - c_v)K}{\beta} \right] dp \quad \dots (iii)$$

We also know that

$$m(c_p - c_v) = \left[p + \left(\frac{\partial U}{\partial v}\right)_T \right] \left(\frac{\partial v}{\partial T}\right)_p = \frac{T \beta}{K} \left(\frac{\partial v}{\partial T}\right)_p = \frac{v T \beta^2}{K} \quad \dots (iv)$$

$$\dots \left[\because p + \left(\frac{\partial U}{\partial v}\right)_T = \frac{T \beta}{K} \text{ and } \left(\frac{\partial v}{\partial T}\right)_p = \beta v \right]$$

Substituting this value of $m(c_p - c_v)$ in equation (iii) we get

$$dU = (m c_p - p \beta v) dT + (p K v - v T \beta) dp \quad \dots (v)$$

We also know that

$$\left(\frac{\partial v}{\partial T}\right)_p = \beta v \quad \text{and} \quad \left(\frac{\partial v}{\partial p}\right)_T = -K v$$

Substituting these values in equation (ii), we get

$$dv = \beta v dT - K v dp \quad \dots (vi)$$

We know that

$$T dS = dU + p dv$$

Substituting the values of dU and dv in the above equation, we get

$$\begin{aligned} T dS &= (m c_p - p \beta v) dT + (p K v - v T \beta) dp + p (\beta v dT - K v dp) \\ &= m c_p dT - v T \beta dp \end{aligned} \quad \dots (vii)$$

The above equation (vii) is called the *second TdS equation*.

3. When pressure (p) and volume (v) are independent. When p and v are taken as independent variables, then change in internal energy,

$$dU = \left(\frac{\partial U}{\partial p} \right)_v dp + \left(\frac{\partial U}{\partial v} \right)_p dv \quad \dots (i)$$

But we have already discussed that when p and v are taken as independent variables, then change in internal energy,

$$dU = \frac{K m c_v}{\beta} dp + \left(\frac{m c_p}{\beta v} - p \right) dv \quad \dots (ii)$$

We know that

$$\begin{aligned} TdS &= dU + pdv \\ &= \frac{K m c_v}{\beta} dp + \left(\frac{m c_p}{\beta v} - p \right) dv + pdv \quad \dots \text{ [From equation (ii)]} \end{aligned}$$

$$= \frac{K m c_v}{\beta} dp + \frac{m c_p}{\beta v} dv \quad \dots (iii)$$

The above equation (iii) is called the *third TdS equation*.

39.12. Specific Heat

We know that, there are two specific heats of a gas, i.e. specific heat at a constant volume (c_v) and specific heat at a constant pressure (c_p). These two specific heats can be measured experimentally. *Joly's differential steam calorimeter* is used to determine specific heat of a gas at constant volume and *Regnault's method* is used to determine specific heat of a gas at constant pressure experimentally*. Now let us discuss the two specific heats as follows:

1. *Specific heat at constant volume (c_v)*. The specific heat at constant volume is given by

$$c_v = \left(\frac{\partial u}{\partial T} \right)_v \quad \dots (i)$$

$$= \left(\frac{\partial u}{\partial s} \right)_v \left(\frac{\partial s}{\partial T} \right)_v = T \left(\frac{\partial s}{\partial T} \right)_v \quad \dots \left[\because T = \left(\frac{\partial u}{\partial s} \right)_v \right] \quad \dots (ii)$$

2. *Specific heat at constant pressure (c_p)*. The specific heat at constant pressure is given by

$$c_p = \left(\frac{\partial h}{\partial T} \right)_p \quad \dots (i)$$

$$= \left(\frac{\partial h}{\partial s} \right)_p \left(\frac{\partial s}{\partial T} \right)_p = T \left(\frac{\partial s}{\partial T} \right)_p \quad \dots \left[\because T = \left(\frac{\partial h}{\partial s} \right)_p \right] \quad \dots (ii)$$

39.13. Specific Heat Relations

We have already derived the results of first and second TdS equations in Art.39.11.

The first TdS equation is given by

$$\begin{aligned} TdS &= m c_v dT + \frac{T\beta}{K} dv \\ &= m c_v dT + T \left(\frac{\partial p}{\partial T} \right)_v dv \quad \dots \left[\because \frac{\beta}{K} = \left(\frac{\partial p}{\partial T} \right)_v \right] \quad \dots (i) \end{aligned}$$

and the second TdS equation is given by

$$\begin{aligned} TdS &= m c_p dT - vT\beta dp \\ &= m c_p dT - T \left(\frac{\partial v}{\partial T} \right)_p dp \quad \dots \left[\because \beta v = \left(\frac{\partial v}{\partial T} \right)_p \right] \quad \dots (ii) \end{aligned}$$

Equating the equations (i) and (ii), we get

$$m c_v dT + T \left(\frac{\partial p}{\partial T} \right)_v dv = m c_p dT - T \left(\frac{\partial v}{\partial T} \right)_p dp$$

or
$$m (c_p - c_v) dT = T \left(\frac{\partial p}{\partial T} \right)_v dv + T \left(\frac{\partial v}{\partial T} \right)_p dp$$

$$\therefore dT = \frac{T \left(\frac{\partial p}{\partial T} \right)_v}{m (c_p - c_v)} dv + \frac{T \left(\frac{\partial v}{\partial T} \right)_p}{m (c_p - c_v)} dp \quad \dots (iii)$$

Taking p and v as independent variables, the change in temperature (dT) can be expressed as

$$dT = \left(\frac{\partial T}{\partial v} \right)_p dv + \left(\frac{\partial T}{\partial p} \right)_v dp \quad \dots (iv)$$

Comparing equations (iv) and (iii), we get

$$\left(\frac{\partial T}{\partial v} \right)_p = \frac{T \left(\frac{\partial p}{\partial T} \right)_v}{m (c_p - c_v)} \quad \text{or} \quad m (c_p - c_v) = T \left(\frac{\partial p}{\partial T} \right)_v \left(\frac{\partial v}{\partial T} \right)_p \quad \dots (v)$$

and
$$\left(\frac{\partial T}{\partial p} \right)_v = \frac{T \left(\frac{\partial v}{\partial T} \right)_p}{m (c_p - c_v)} \quad \text{or} \quad m (c_p - c_v) = T \left(\frac{\partial v}{\partial T} \right)_p \left(\frac{\partial p}{\partial T} \right)_v \quad \dots (vi)$$

Thus from both equations (v) and (vi), it may be concluded that

$$m (c_p - c_v) = T \left(\frac{\partial p}{\partial T} \right)_v \left(\frac{\partial v}{\partial T} \right)_p \quad \dots (vii)$$

We know that by cyclic property of differentials,

$$\left(\frac{\partial p}{\partial T} \right)_v \left(\frac{\partial T}{\partial v} \right)_p \left(\frac{\partial v}{\partial p} \right)_T = -1$$

or

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{-1}{\left(\frac{\partial T}{\partial v}\right)_p \left(\frac{\partial p}{\partial T}\right)_T} = -\left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial p}{\partial v}\right)_T$$

Substituting this value of $\left(\frac{\partial p}{\partial T}\right)_v$ in equation (vii), we get

$$m(c_p - c_v) = T \left[-\left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial p}{\partial v}\right)_T \right] \left(\frac{\partial v}{\partial T}\right)_p = -T \left(\frac{\partial v}{\partial T}\right)_p^2 \left(\frac{\partial p}{\partial v}\right)_T$$

or

$$c_p - c_v = -\frac{T}{m} \left(\beta^2 v^2 \right) \times \frac{-1}{Kv} = \frac{T \beta^2 v}{mK} = \frac{T \beta^2 v_s}{K} \dots \left[\because \left(\frac{\partial v}{\partial T}\right)_p = \beta v \text{ and } \left(\frac{\partial p}{\partial v}\right)_T = Kv \right]$$

where

$$v_s = \frac{v}{m} = \text{Specific volume of gas in m}^3/\text{kg.}$$

39.14. Joule Thomson Co-efficient

The change in temperature with drop in pressure at constant enthalpy is termed as *Joule-Thomson coefficient* (μ). Mathematically,

$$\mu = \left(\frac{dT}{dp}\right)_h \dots (i)$$

It varies with both the temperature and pressure of the gas.

The magnitude of the Joule-Thomson coefficient is a measure of the imperfection of a gas or its deviation from perfect gas behaviour. For real gases, μ may be either positive or negative depending upon the thermodynamic state of the gas. When μ is zero, the temperature of the gas remains constant with throttling. The temperature at which $\mu=0$, is called the *inversion temperature* for a given pressure. If μ is greater than zero, the temperature of the gas decreases with throttling and when μ is less than zero, the temperature of the gas increases with throttling. Thus, in cooling of a gas by throttling, we require that the gas shows a large positive value of μ .

The Joule-Thomson coefficient is not a constant but is a function of both pressure and temperature. We shall now derive the functional relationship for the coefficient.

We know that enthalpy,

$$H = U + p v \dots (ii)$$

Differentiating equation (ii), we get

$$dH = dU + d(pv) = dU + p dv + v dp \dots (iii)$$

We also know that

$$T dS = dU + p dv \text{ or } dU = T dS - p dv \dots (iv)$$

Substituting the value of dU in equation (iii), we get

$$dH = T dS - p dv + p dv + v dp = T dS + v dp$$

When T and p are taken as independent variables, then we can write

* For details, refer Chapter 3, Art. 3.21.

$$\begin{aligned}
 dH &= T \left[\left(\frac{\partial S}{\partial T} \right)_p dT + \left(\frac{\partial S}{\partial p} \right)_T dp \right] + v dp \\
 &= T \left(\frac{\partial S}{\partial T} \right)_p dT + \left[T \left(\frac{\partial S}{\partial p} \right)_T + v \right] dp \quad \dots (v)
 \end{aligned}$$

We know that

$$T \left(\frac{\partial S}{\partial T} \right)_p = m c_p \quad \dots (vi)$$

where m = Mass of the gas in kg.

Also, from Maxwell relations,

$$\left(\frac{\partial S}{\partial p} \right)_T = - \left(\frac{\partial v}{\partial T} \right)_p \quad \dots (vii)$$

Substituting the value from equations (vi) and (vii) in equation (v), we get

$$dH = m c_p dT + \left[v - T \left(\frac{\partial v}{\partial T} \right)_p \right] dp \quad \dots (viii)$$

Solving the above equation (viii) for dT , we have

$$\begin{aligned}
 dT &= \frac{dH}{m c_p} + \frac{T \left(\frac{\partial v}{\partial T} \right)_p - v}{m c_p} dp \\
 &= \frac{dh}{c_p} + \frac{T \left(\frac{\partial v_s}{\partial T} \right)_p - v_s}{c_p} dp \quad \dots (ix)
 \end{aligned}$$

where $dh = dH/m$ = Change in specific enthalpy in kJ/kg,

$\frac{\partial v_s}{\partial T} = \frac{\partial}{\partial T} \left(\frac{v}{m} \right)$ = Change in specific volume with temperature, and

$v_s = v/m$ = Specific volume of gas in m^3/kg .

When specific enthalpy (h) and pressure (p) are taken as independent variables, then for dT we can write

$$dT = \left(\frac{\partial T}{\partial h} \right)_p dh + \left(\frac{\partial T}{\partial p} \right)_h dp \quad \dots (x)$$

Comparing equations (ix) and (x), we get

$$\begin{aligned}
 \left(\frac{\partial T}{\partial h} \right)_p &= \frac{1}{c_p} \\
 \left(\frac{\partial T}{\partial p} \right)_h &= \frac{T \left(\frac{\partial v_s}{\partial T} \right)_p - v_s}{c_p}
 \end{aligned}$$

and

We know that

$$\left(\frac{\partial T}{\partial p}\right)_h = \text{Joule-Thomson coefficient, } \mu$$

$$\therefore \mu = \left(\frac{\partial T}{\partial p}\right)_h = \frac{T^2}{c_p} \left[\frac{T \left(\frac{\partial v_s}{\partial T}\right) - v_s}{T^2} \right] = \frac{T^2}{c_p} \left[\frac{\partial}{\partial T} \left(\frac{v_s}{T}\right) \right]_p$$

... (Multiplying the numerator and denominator by T^2)

Note : We have derived above that

$$\mu = \frac{T^2}{c_p} \left[\frac{\partial}{\partial T} \left(\frac{v_s}{T}\right) \right]_p$$

For an ideal gas,

$$p v_s = R T \quad \text{or} \quad \frac{v_s}{T} = \frac{R}{p}$$

Substituting this value of (v_s/T) in the above equation,

$$\mu = \frac{T^2}{c_p} \left[\frac{\partial}{\partial T} \left(\frac{R}{p}\right) \right]_p = 0 \quad \dots (\because R \text{ and } p \text{ are constants})$$

Thus, the Joule Thomson coefficient for an ideal gas is zero and the ideal gas passing through the porous gas does not show any change in temperature.

39.15. Equations of State

Practically there is no real gas which obeys the gas laws perfectly. The deviation is small at ordinary temperatures and pressures, but it is large at high pressure and low temperatures. In deriving the perfect gas laws on the basis of kinetic theory of gases, the following assumptions were made which do not hold true for real gases :

1. The molecules of gas are mere mass-points occupying no space; and
2. There is no attraction or repulsion between the molecules.

In actual practice, the molecules of all actual or real gases do occupy some space and do attract each other. Hence no real gas equation conforms to the perfect gas equation $p v_s = R T$.

Some of the equations used to correct the perfect gas equation are defined below.

(a) *Van der Waal's equation**. The Dutch physicist J.D. Van der Waal was the first scientist to correct the perfect gas equation $p v_s = R T$. Van der Waal proposed an equation of state, to account for non ideal gas behaviour. Thus the equation given by Van der Waal has the form

$$\left(p + \frac{a}{v_s^2}\right) (v_s - b) = R T$$

where a is a constant of proportionality which takes into account the molecular force-field interactions and b is a constant of proportionality which takes account of the volume occupied by all molecules per kg of gas.

(b) *Clausius equation*. The Clausius equation is a modification of Van der Waal's equation. It neglects molecular force-field interactions, a . Thus, the equation has the form

$$p (v_s - b) = R T$$

* For details, refer Chapter 5, Art. 5.11.

where $b =$ Volume occupied by all molecules.

(c) *Beattie Bridgeman equation*. Beattie Bridgeman in 1928 proposed an equation of state which gives accurate results. Thus, the equation has the form

$$p = \frac{RT(1-\epsilon)}{v_s} (v_s + B) - \frac{A}{v_s}$$

where

$$A = A_0 \left(1 - \frac{a}{v_s} \right),$$

$$B = B_0 \left(1 - \frac{b}{v_s} \right),$$

$$\epsilon = \left(\frac{C}{v_s T^3} \right),$$

and A_0, a, B_0, b and c are constants whose values are different for different gases.

Beattie Bridgeman equation of state gives properties accurately (within 2 percent) in regions where the density is less than 0.8 times the critical density.

(d) *Bertholet equation*. Bertholet equation is similar to the Van der Waals equation except for the inclusion of the temperature in the denominator of second term. Thus, the equation has the form

$$\left(p - \frac{a}{T v_s^2} \right) (v_s - b) = RT$$

where a and b are constants, as in Van-der Waals equation.

Bertholet equation gives accurate properties within 1 percent.

(e) *Dieterici equation*. Dieterici equation of state gives properties very near to exact values of the properties in the neighbourhood of the critical point and on the critical isotherm (constant temperature). This equation does not give satisfactory results in other region and may produce large errors if applied away from critical region. Thus, the equation has the form

$$p = \frac{RT}{v_s - b} e^{-a/RT v_s}$$

where a and b are constants.

(f) *Redlich-Kwong equation*. The equation given by Redlich-Kwong has the form

$$\left(p - \frac{a}{\sqrt{T} v_s (v_s + b)} \right) (v_s - b) = RT$$

where a and b are constants.

(g) *Benedict-Webb-Rubin equation*. Benedict-Webb-Rubin equation of state is used for high density regions upto approximately two times the critical density. Thus, the equation involves eight experimentally determined constants and has the form

$$p = \frac{RT}{v_s} + \frac{B_0 RT - A_0 - C_0/T^2}{v_s^2} + \frac{bRT - a}{v_s^3} + \frac{a\alpha}{v_s^6} + \frac{C}{v_s^3 T^2} \left(1 + \frac{\gamma}{v_s^2} \right) e^{-\gamma/v_s^2}$$

where $A_0, B_0, C_0, a, b, c, \alpha$ and γ are constants.

(h) *Viral equation.* The viral equation of state was first introduced by the Dutch physicist Kammerlingh Onnes, and is only applicable to gases of low and medium densities. Basically, the viral equation of state represents an expansion of the $p v_s$ product in infinite series form. It is given as

$$p v_s = RT \left(1 + \frac{B}{v_s} + \frac{C}{v_s^2} + \frac{D}{v_s^3} + \dots \right) \quad \dots (i)$$

It is sometimes convenient to express the viral equation of state with pressure as the independent variable, i.e.

$$p v_s = RT \left(1 + B' p + C' p^2 + D' p^3 + \dots \right) \quad \dots (ii)$$

In the above equations, B, B', C, C', D, D' etc. are called viral coefficients. B and B' are called second viral coefficients, C and C' are called third viral coefficients, D and D' are called fourth viral coefficients and so on. Also, there exists a relation between the viral coefficients in equation (i) to the corresponding viral coefficients in equation (ii), such that

$$B' = \frac{B}{RT}, \quad C' = \frac{C - B^2}{(RT)^2}, \quad D' = \frac{D - 3BC + 2B^3}{(RT)^3}, \quad \text{and so on.}$$

39.16. Compressibility Factor

We have discussed that the viral equation of state is given by

$$p v_s = RT \left(1 + \frac{B}{v_s} + \frac{C}{v_s^2} + \frac{D}{v_s^3} + \dots \right)$$

or

$$\frac{p v_s}{RT} = \left(1 + \frac{B}{v_s} + \frac{C}{v_s^2} + \frac{D}{v_s^3} + \dots \right)$$

The ratio of $p v_s / RT$ is called the *compressibility factor*. It is denoted by Z . For the ideal gas, $Z = 1$ and for real gases, Z is determined from experimental data. The magnitude of Z for a certain gas at particular pressure and temperature is a measure of deviation of the real gas from the ideal gas behaviour.

QUESTIONS

1. Define the following :

- (a) Co-efficient of volume expansion ; (b) Isothermal compressibility; and
(c) Adiabatic compressibility.

2. Derive the Maxwell's thermodynamic relations and explain their importance in thermodynamics.

3. Show from the consideration of Maxwell's thermodynamic relations that

(a) For a perfect gas, $c_p - c_v = R$

(b) For a gas obeying Van der Waal's equation,

$$c_p - c_v = \frac{R}{1 - 2a(v_s - b)^2 / RT v_s^3}$$

4. Show that for specific helmholtz function, $a = u - T_s$

$$(a) \left(\frac{\partial a}{\partial v_s} \right)_T = -p \qquad (b) \left(\frac{\partial a}{\partial T} \right)_{v_s} = -s$$

$$(c) u = a - T \left(\frac{\partial a}{\partial T} \right)_{v_s} \qquad (d) c_v = -T \left(\frac{\partial^2 a}{\partial T^2} \right)_{v_s}$$

5. Show that for specific Gibbs function, $g = h - T_s$

$$(a) \left(\frac{\partial g}{\partial T} \right)_p = -s \qquad (b) \left(\frac{\partial g}{\partial p} \right)_T = -v_s$$

$$(c) u = g - T \left(\frac{\partial g}{\partial T} \right)_p - p \left(\frac{\partial g}{\partial p} \right)_T \qquad (d) c_v = -T \left(\frac{\partial^2 g}{\partial T^2} \right)_{v_s}$$

6. Show that the equation of state of a substance may be written in the form

$$\frac{dv}{v} = -K dp + \beta dT$$

7. A substance has the volume expansivity (β) and isothermal compressibility (K) as

$$\beta = \frac{1}{T} \quad \text{and} \quad K = \frac{1}{p}$$

Show that the equation of state is $p v_s / T = \text{constant}$.

8. Using the cyclic relation, prove that

$$\left(\frac{\partial p}{\partial T} \right)_v = \frac{\beta}{KT}$$

9. Derive the following thermodynamic relations

$$(a) \left(\frac{\partial h}{\partial p} \right)_T = -T \left(\frac{\partial v_s}{\partial T} \right)_{v_s} + v_s = -c_p \left(\frac{\partial T}{\partial p} \right)_h \qquad (b) \left(\frac{\partial h}{\partial v_s} \right)_T = T \left(\frac{\partial p}{\partial T} \right)_{v_s} + v_s \left(\frac{\partial p}{\partial v_s} \right)_T$$

$$(c) \left(\frac{\partial h}{\partial s} \right)_{v_s} = T - v_s \left(\frac{\partial T}{\partial v_s} \right)_s \qquad (d) \left(\frac{\partial u}{\partial v_s} \right) = T \left(\frac{\partial p}{\partial T} \right)_{v_s} - p$$

10. Show that for a Van der Waal's gas

$$\left(\frac{\partial c_v}{\partial v} \right)_T = 0$$

OBJECTIVE TYPE QUESTIONS

1. Helmholtz function (A) is given as

$$(a) A = U - TS$$

$$(b) A = U + TS$$

$$(c) A = TS - U$$

$$(d) A = U - T/S$$

2. Gibbs function (G) is given as

(a) $G = H + TS$

(b) $G = H - TS$

(c) $G = TS - H$

(d) $G = H + T/S'$

3. Difference in specific heat at constant pressure (c_p) and specific heat at constant volume (c_v) i.e. ($c_p - c_v$) is

(a) $\frac{T\beta^2}{Kv_s}$

(b) $\frac{T\beta^2K}{v_s}$

(c) $\frac{Tv_s}{\beta^2K}$

(d) $\frac{T\beta^2v_s}{k}$

4. TdS equation taking temperature (T) and volume (v) as independent variables is

(a) $m c_v dT + T \frac{K}{\beta} dv$

(b) $m c_v dT + T \beta K dv$

(c) $m c_v dT + T \frac{\beta}{K} dv$

(d) $m c_v dT + T \beta dv$

5. Van der Waal's equation of state is

(a) $p v_s = RT$

(b) $p v_s = 1 + \frac{B}{v_s} + \frac{C}{v_s^2}$

(c) $p = \frac{RT}{v_s - b} - \frac{a}{v_s^2}$

(d) $v_s = \frac{RT}{p} - \frac{K}{T^3}$

ANSWERS

1. (a)

2. (b)

3. (d)

4. (c)

5. (c)

Variable Specific Heat

1. Introduction. 2. Molar Specific Heats of a Gas. 3. Variation of Specific Heat with Temperature. 4. Atmospheric Air. 5. Heat Transfer during a Process with Variable Specific Heat. 6. Heat Transfer during Polytropic Expansion or Compression Process. 7. Change in Thermodynamic Properties with Variable Specific Heat. 8. Isentropic Expansion with Variable Specific Heats. 9. Effect of Variable Specific Heat in I.C. Engines. 10. Effect of Variable Specific Heat on Otto Cycle. 11. Effect of Variable Specific Heat on Diesel Cycle.

40.1. Introduction

We have already discussed in Chapter 1 (Art. 1.40), that specific heat of a substance may be broadly defined as the amount of heat required to raise the temperature of a unit mass of any substance through one degree. It is generally denoted by c . In S.I. system of units, the unit of specific heat (c) is taken as kJ/kg K.

Since the solids and liquids does not have appreciable change in their volume on heating therefore they have only one specific heat. But the gases have the following two *specific heats depending upon the process adopted for heating the gas.

1. Specific heat at constant pressure (c_p), and
2. Specific heat at constant volume (c_v).

In general, specific heat is not constant. The specific heat of any gas increases significantly with rise in temperature, but it does not increase significantly with pressure, except at high pressure. Thus, for simple calculations, the change in specific heat with pressure is neglected.

40.2. Molar Specific Heats of a Gas

The molar or volumetric specific heat of a gas may be defined as the amount of heat required to raise the temperature of unit mole of gas through one degree. It is generally denoted by c_m . In S.I. system of units, the unit of molar specific heat (c_m) is kJ/kg-mol K. Mathematically, molar specific heat (c_m) is given as

$$c_m = M \times c$$

where

M = Molar mass of the gas, and

c = Specific heat of the gas.

* For further details, refer Chapter 2 (Art. 2.11).

In the similar way, the molar specific heat at constant volume

$$c_{mv} = M c_v$$

and molar specific heat at constant pressure,

$$c_{mp} = M c_p$$

40.3. Variation of Specific Heat with Temperature

We have already discussed that the specific heat of any gas increases with increase in temperature. The specific heat increases because of increase in energy of vibration of molecules at high temperature. The vibrations are caused because of collisions among molecules which are significant at higher temperature. Therefore, the energy of vibration of a polyatomic gas will undergo a considerable change at ordinary temperature, thus increasing the value of specific heat.

It is assumed for gases that in a temperature range of 300 K to 1500 K, the specific heat is a linear function of temperature and may be expressed as

$$c_p = a + K T, \text{ and}$$

$$c_v = b + K T$$

where a , b and K are constants.

We know that characteristic gas constant,

$$\begin{aligned} R &= c_p - c_v \\ &= (a + K T) - (b + K T) = a - b \end{aligned}$$

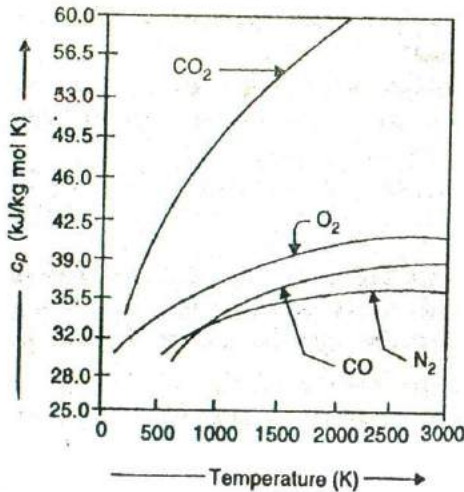


Fig. 40.1. Variation of molar specific heat at constant pressure with temperature.

Above 1500 K, the specific heats increase very rapidly and is given by the following expressions :

$$c_p = a + K_1 T + K_2 T^2, \text{ and}$$

$$c_v = b + K_1 T + K_2 T^2$$

* For details, refer Chapter 2 (Art. 2.17).

In the above equations, if the term involving T^2 is neglected, then the expression for c_p and c_v becomes same as for the temperature range of 300 K to 1500 K. Fig. 40.1 shows the variation of molar specific heat (c_{mp}) for CO_2 , O_2 , CO and N_2 with temperature.

The following table shows the expressions for variable molar specific heat at low pressure.

Table 40.1. Variable molar specific heat at low pressure.

S.No.	Name of gas	Molecular mass (M)	Temp. range (K)	c_{mp} (kJ/kg-mol K)
1.	Air	29	280-1500	$26.63 + 7473.44 \times 10^{-4} T - 115.97 \times 10^{-8} T^2$
2.	Carbon dioxide (CO_2)	44	280-3500	$67.83 - 15156.22 T^{-1} + 181.71 \times 10^4 T^{-2}$
3.	Oxygen (O_2)	32	280-2800	$48.21 - 535.917 T^{-0.5} + 3516.91 T^{-1}$
4.	Nitrogen (N_2)	28	280-5000	$39.65 - 8080.52 T^{-1} + 1494.69 T^{-2}$
5.	Ammonia (NH_3)	17	280-1000	$25.92 + 330.76 \times 10^{-4} T - 305.64 \times 10^{-8} T^2$
6.	Carbon monoxide (CO)	28	280-2800	$39.61 - 7640.91 T^{-1} + 138.16 \times 10^4 T^{-2}$
7.	Hydrogen (H_2)	2	300-2200	$24.12 + 43.54 \times 10^{-4} T + 62.38 T^{-0.5}$
8.	Sulphur dioxide (SO_2)	64	300-1900	$49.78 + 46.05 \times 10^{-4} T + 110.11 \times 10^{-4} T^{-2}$
9.	Methane (CH_4)	16	280-1500	$14.15 + 753.62 \times 10^{-4} T - 1808.70 \times 10^{-8} T^2$
10.	Water vapour	18	280-3000	$83.15 - 1863.13 T^{-0.5} - 173.75 T^{-1}$

40.4. Atmospheric Air

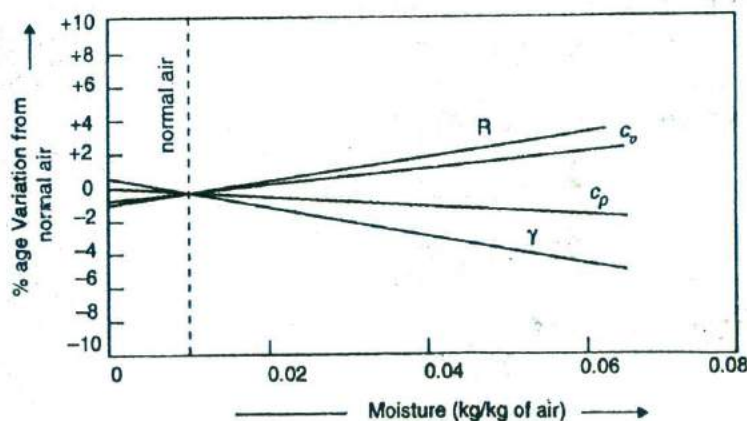


Fig. 40.2. Effect of moisture on properties of air.

The atmospheric air is a mixture of a number of gases such as nitrogen, oxygen, carbon-dioxide, hydrogen, argon, neon, helium etc. But the nitrogen and oxygen have the major portion of the combination. The atmospheric air consists of 23.19% oxygen and 75.47% nitrogen by mass.

However, the atmospheric air also contains water vapours known as *moisture*. The amount of water vapour, present in the air, depends upon the absolute pressure and temperature of the mixture. Fig. 40.2 shows the effect of moisture on the properties of air, like characteristic gas constant (R), specific heat at constant volume (c_v), specific heat at constant pressure (c_p), and ratio of specific heats (γ).

40.5. Heat Transfer during a Process with Variable Specific Heat

We know that for a small change in the state of a working substance, the general gas energy equation is

$$\delta Q = dU + \delta W$$

or

$$\delta Q = m c_v dT + p dv \quad \dots (i)$$

where

$$dT = \text{Change in temperature, and}$$

$$dv = \text{Change in volume.}$$

Dividing both sides of equation (i) by dv ,

$$\frac{\delta Q}{dv} = m c_v \times \frac{dT}{dv} + p \quad \dots (ii)$$

We know that according to general gas equation,

$$p v = m R T$$

or

$$T = \frac{p v}{m R} \quad \dots (iii)$$

Differentiating equation (iii),

$$dT = \frac{1}{m R} [p dv + v dp]$$

Dividing both sides of the above equation by dv ,

$$\frac{dT}{dv} = \frac{1}{m R} \left(p + \frac{v dp}{dv} \right) \quad \dots (iv)$$

Substituting the value of $\frac{dT}{dv}$ in equation (ii),

$$\frac{\delta Q}{dv} = m c_v \left[\frac{1}{m R} \left(p + \frac{v dp}{dv} \right) \right] + p$$

or

$$\frac{\delta Q}{dv} = \frac{c_v}{R} \left(p + \frac{v dp}{dv} \right) + p \quad \dots (v)$$

We know that

$$c_p = a + K T,$$

$$c_v = b + K T, \text{ and}$$

$$R = c_p - c_v = (a + K T) - (b + K T) = a - b$$

Substituting this value of c_v and R in equation (v),

$$\frac{\delta Q}{dv} = \frac{b + K T}{a - b} \left(p + \frac{v dp}{dv} \right) + p$$

$$\begin{aligned}
 &= \frac{1}{a-b} \left(b p + \frac{b v dp}{dv} + K T p + \frac{K T v dp}{dv} + a p - b p \right) \\
 &= \frac{b}{a-b} \left(\frac{v dp}{dv} + \frac{K T p}{b} + \frac{K T v dp}{b dv} + \frac{a p}{b} \right) \\
 &= \frac{1}{\frac{a}{b} - 1} \left(\frac{v dp}{dv} + \frac{K T p}{b} + \frac{K T v dp}{b dv} + \frac{a p}{b} \right) \\
 &= \frac{1}{\frac{a}{b} - 1} \left(\frac{a p}{b} + \frac{v dp}{dv} \right) + \frac{K T}{a-b} \left(p + \frac{v dp}{dv} \right)
 \end{aligned}$$

Let $\frac{a^*}{b} = \frac{c_p - K T}{c_v - K T} = \gamma'$

$$\therefore \frac{\delta Q}{dv} = \frac{1}{\gamma' - 1} \left(\gamma' p + \frac{v dp}{dv} \right) + \frac{K T}{a-b} \left(p + \frac{v dp}{dv} \right) \quad \dots (vi)$$

Note: For constant specific heats, $K = 0$. Thus the above equation (vi) becomes

$$\frac{\delta Q}{dv} = \frac{1}{\gamma' - 1} \left(\gamma' p + \frac{v dp}{dv} \right)$$

40.6. Heat Transfer during Polytropic Expansion or Compression Process

We know that for a polytropic process,

$$p v^n = \text{Constant}$$

Differentiating the above equation with respect to v ,

$$n p v^{n-1} + v^n \times \frac{dp}{dv} = 0 \quad \dots \left[\because \frac{d}{dx} (u \times v) = u \times \frac{dv}{dx} + v \times \frac{du}{dx} \right]$$

$$v^n \times \frac{dp}{dv} = -n p v^{n-1}$$

$$v \times \frac{dp}{dv} = -n p$$

We have already derived that

$$\frac{\delta Q}{dv} = \frac{1}{\gamma' - 1} \left(\gamma' p + \frac{v dp}{dv} \right) + \frac{K T}{a-b} \left(p + \frac{v dp}{dv} \right)$$

Substituting the value of $v \times \frac{dp}{dv}$ in the above equation, we get

$$\begin{aligned}
 \frac{\delta Q}{dv} &= \frac{1}{\gamma' - 1} \left(\gamma' p - n p \right) + \frac{K T}{a-b} \left(p - n p \right) \\
 &= \frac{\gamma' - n}{\gamma' - 1} p + \frac{K T}{a-b} \left(1 - n \right) p
 \end{aligned}$$

* We know that

$$c_p = a + K T \quad \text{or} \quad a = c_p - K T$$

$$c_v = b + K T \quad \text{or} \quad b = c_v - K T$$

$$\therefore \frac{a}{b} = \frac{c_p - K T}{c_v - K T}$$

$$= \left(\frac{\gamma' - n}{\gamma' - 1} - K T \frac{n-1}{a-b} \right) p \quad \dots (vii)$$

Note : Since for constant specific heats, $K = 0$, therefore the above equation (vii) becomes

$$\frac{\delta Q}{dv} = \frac{\gamma' - n}{\gamma' - 1} \times p$$

40.7. Change in Thermodynamic Properties with Variable Specific Heat

We have already discussed that when a gas is heated, its specific heat increases. Since the thermodynamic properties like internal energy, enthalpy and entropy depends upon specific heat also, thus there is a considerable change in these properties with change in temperature. Consider m kg of gas being heated from the initial state 1 to the final state 2.

Let $m =$ Mass of the gas,

p_1, v_1 and $T_1 =$ Pressure, volume and temperature respectively at the initial state 1,

p_2, v_2 and $T_2 =$ Pressure, volume and temperature respectively at the final state 2,

$c_p =$ Specific heat at constant pressure $= a + K T$, and

$c_v =$ Specific heat at constant volume $= b + K T$

Now, let us derive the following relations for the change in thermodynamic properties with variable specific heat.

1. Change in internal energy

We know that change in internal energy of gas,

$$dU = mc_v dT \quad \dots \text{ (Joule's law)}$$

$$= m (b + K T) dT \quad \dots \text{ (} \because c_v = b + K T \text{)}$$

$$= m b dT + m K T dT$$

On integrating from state 1 to state 2,

$$\int_1^2 dU = m b \int_1^2 dT + m K \int_1^2 T dT$$

$$\begin{aligned} \text{or } U_2 - U_1 &= m b (T_2 - T_1) + m K \times \frac{(T_2^2 - T_1^2)}{2} \\ &= m (T_2 - T_1) \left[b + K \times \frac{(T_2 + T_1)}{2} \right] = m (T_2 - T_1) (b + K T_m) \\ &= m c_{vm} (T_2 - T_1) \end{aligned}$$

where

$$T_m = \text{Mean temperature} = \frac{T_2 + T_1}{2}, \text{ and}$$

$$c_{vm} = \text{Mean specific heat at constant volume} = b + K T_m$$

Note : For per unit mass of gas, the expression for change in internal energy may be written as

$$u_2 - u_1 = c_{vm} (T_2 - T_1)$$

where

$$u_2 - u_1 = \text{Change in specific internal energy in kJ/kg.}$$

Variable Specific Heat

2. Change in enthalpy

We know that change in enthalpy of gas,

$$\begin{aligned} dH &= mc_p dT = m(a + K T) dT & \dots (\because c_p = a + K T) \\ &= m a dT + m K T dT \end{aligned}$$

On integrating from state 1 to state 2,

$$\int_1^2 dH = m a \int_1^2 dT + m K \int_1^2 T dT$$

or

$$\begin{aligned} H_2 - H_1 &= m a (T_2 - T_1) + m K \times \frac{(T_2^2 - T_1^2)}{2} \\ &= m (T_2 - T_1) \left[a + K \times \frac{(T_2 + T_1)}{2} \right] \\ &= m (T_2 - T_1) (a + K T_m) \\ &= m c_{pm} (T_2 - T_1) \end{aligned}$$

where

$$T_m = \text{Mean temperature} = \frac{T_2 + T_1}{2}, \text{ and}$$

$$c_{pm} = \text{Mean specific heat at constant pressure} = a + K T_m.$$

Note : For per unit mass of gas, the expression for change in enthalpy can be written as

$$h_2 - h_1 = c_{pm} (T_2 - T_1)$$

where

$$h_2 - h_1 = \text{Change in specific enthalpy in kJ/kg.}$$

3. Change in entropy

We know that change in entropy of the gas,

$$dS = \frac{\delta Q}{T} \quad \dots (i)$$

where

$$\delta Q = \text{Heat transferred in kJ, and}$$

$$T = \text{Absolute temperature in K.}$$

Now the relation for the change of entropy for variable specific heat can be expressed in the following three ways :

(a) In terms of volume and absolute temperature

We know that for a small change in the state of a working substance, the general gas energy equation is

$$\delta Q = dU + \delta W \quad \dots (\text{First Law of Thermodynamics})$$

$$= m c_v dT + p dv \quad \dots (ii)$$

where

$$dT = \text{Change in temperature, and}$$

$$dv = \text{Change in volume.}$$

Dividing throughout equation (ii) by T ,

$$\frac{\delta Q}{T} = \frac{m c_v dT}{T} + \frac{p dv}{T}$$

Since $p v = m R T$ or $\frac{p}{T} = \frac{m R}{v}$ and $\frac{\delta Q}{T} = dS$, therefore

$$dS = \frac{m c_v dT}{T} + \frac{m R dv}{v} \quad \dots (iii)$$

$$= m (b + K T) \frac{dT}{T} + m (a - b) \frac{dv}{v}$$

$\dots (\because c_v = b + K T \text{ and } R^* = a - b)$

$$= m b \times \frac{dT}{T} + m K T \times \frac{dT}{T} + m (a - b) \frac{dv}{v}$$

$$= m b \times \frac{dT}{T} + m K \times dT + m (a - b) \frac{dv}{v} \quad \dots (iv)$$

On integrating equation (iv) from state 1 to state 2,

$$\int_1^2 dS = m b \int_1^2 \frac{dT}{T} + m K \int_1^2 dT + m (a - b) \int_1^2 \frac{dv}{v}$$

$$\begin{aligned} \text{or } S_2 - S_1 &= m b \log_e \left(\frac{T_2}{T_1} \right) + m K (T_2 - T_1) + m (a - b) \log_e \left(\frac{v_2}{v_1} \right) \quad \dots (v) \\ &= 2.3 m b \log \left(\frac{T_2}{T_1} \right) + m K (T_2 - T_1) + 2.3 m (a - b) \log \left(\frac{v_2}{v_1} \right) \end{aligned}$$

Note : For per unit mass, the above expression may be written as

$$s_2 - s_1 = 2.3 b \log \left(\frac{T_2}{T_1} \right) + K (T_2 - T_1) + 2.3 (a - b) \log \left(\frac{v_{s2}}{v_{s1}} \right)$$

where

$$s_2 - s_1 = \text{Change in specific entropy in kJ/kg K, and}$$

v_{s1} and v_{s2} = Specific volume at states 1 and 2 in m^3/kg of gas.

(b) In terms of pressure and absolute temperature

We know from the general gas equation,

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} \quad \text{or} \quad \frac{v_2}{v_1} = \frac{p_1}{p_2} \times \frac{T_2}{T_1}$$

$$\text{or } \log_e \left(\frac{v_2}{v_1} \right) = \log_e \left(\frac{p_1}{p_2} \right) + \log_e \left(\frac{T_2}{T_1} \right)$$

Substituting the value of $\log_e \left(\frac{v_2}{v_1} \right)$ in equation (v),

$$S_2 - S_1 = m b \log_e \left(\frac{T_2}{T_1} \right) + m K (T_2 - T_1) + m (a - b) \left[\log_e \left(\frac{p_1}{p_2} \right) + \log_e \left(\frac{T_2}{T_1} \right) \right]$$

$$\begin{aligned}
 &= m \log_e \left(\frac{T_2}{T_1} \right) (b + a - b) + m K (T_2 - T_1) + m (a - b) \log_e \left(\frac{p_1}{p_2} \right) \\
 &= m a \log_e \left(\frac{T_2}{T_1} \right) + m K (T_2 - T_1) + m (a - b) \log_e \left(\frac{p_1}{p_2} \right) \\
 &= 2.3 m a \log \left(\frac{T_2}{T_1} \right) + m K (T_2 - T_1) + 2.3 m (a - b) \log \left(\frac{p_1}{p_2} \right)
 \end{aligned}$$

Note : For per unit mass, the above expression may be written as :

$$s_2 - s_1 = 2.3 a \log \left(\frac{T_2}{T_1} \right) + K (T_2 - T_1) + 2.3 (a - b) \log \left(\frac{p_1}{p_2} \right)$$

where

$$s_2 - s_1 = \text{Change in specific entropy in kJ/kg K.}$$

(c) In terms of pressure and volume

We know from the general gas equation,

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} \quad \text{or} \quad \frac{T_2}{T_1} = \frac{p_2}{p_1} \times \frac{v_2}{v_1}$$

$$\text{or} \quad \log_e \left(\frac{T_2}{T_1} \right) = \log_e \left(\frac{p_2}{p_1} \right) + \log_e \left(\frac{v_2}{v_1} \right)$$

Substituting the value of $\log_e \left(\frac{T_2}{T_1} \right)$ in equation (v),

$$\begin{aligned}
 S_2 - S_1 &= m b \left[\log_e \left(\frac{p_2}{p_1} \right) + \log_e \left(\frac{v_2}{v_1} \right) \right] + m K (T_2 - T_1) + m (a - b) \log_e \left(\frac{v_2}{v_1} \right) \\
 &= m b \log_e \left(\frac{p_2}{p_1} \right) + m \log_e \left(\frac{v_2}{v_1} \right) (b + a - b) + m K (T_2 - T_1) \\
 &= m b \log_e \left(\frac{p_2}{p_1} \right) + m a \log_e \left(\frac{v_2}{v_1} \right) + m K (T_2 - T_1) \\
 &= 2.3 m b \log \left(\frac{p_2}{p_1} \right) + 2.3 m a \log \left(\frac{v_2}{v_1} \right) + m K (T_2 - T_1)
 \end{aligned}$$

Note : For per unit mass, the above expression may be written as

$$s_2 - s_1 = 2.3 b \log \left(\frac{p_2}{p_1} \right) + 2.3 a \log \left(\frac{v_2}{v_1} \right) + K (T_2 - T_1)$$

where

$$s_2 - s_1 = \text{Change in specific entropy in kJ/kg K, and}$$

$$v_{s1} \text{ and } v_{s2} = \text{Change in specific volume at states 1 and 2 in m}^3/\text{kg of gas.}$$

Example 40.1. 2 kg of a certain gas is heated from 400 K to 1000 K. Calculate the change in internal energy and enthalpy.

Take $c_p = 0.946 + 0.184 \times 10^{-3} T$ kJ/kg K, and $c_v = 0.653 + 0.184 \times 10^{-3} T$ kJ/kg K.

Solution. Given : $m = 2$ kg ; $T_1 = 400$ K ; $T_2 = 1000$ K ; $a = 0.946$ kJ/kg K ; $b = 0.653$ kJ/kg K ; $K = 0.184 \times 10^{-3}$ kJ/kg K²

We know that mean temperature,

$$T_m = \frac{T_1 + T_2}{2} = \frac{400 + 1000}{2} = 700 \text{ K}$$

∴ Mean specific heat at constant volume,

$$c_{vm} = b + K T_m = 0.653 + 0.184 \times 10^{-3} \times 700 = 0.782 \text{ kJ/kg K}$$

and mean specific heat at constant pressure,

$$c_{pm} = a + K T_m = 0.946 + 0.184 \times 10^{-3} \times 700 = 1.075 \text{ kJ/kg K}$$

Change in internal energy

We know that change in internal energy,

$$\begin{aligned} dU &= U_2 - U_1 = m c_{vm} (T_2 - T_1) \\ &= 2 \times 0.782 (1000 - 400) = 938.4 \text{ kJ Ans.} \end{aligned}$$

Change in enthalpy

We know that change in enthalpy,

$$\begin{aligned} dH &= H_2 - H_1 = m c_{pm} (T_2 - T_1) \\ &= 2 \times 1.075 (1000 - 400) = 1290 \text{ kJ Ans.} \end{aligned}$$

Example 40.2. A certain quantity of gas having a volume of 5 m^3 at 1 bar and 27°C is heated at constant pressure to 55°C . Determine : 1. mass of the gas, 2. heat transfer during the process, and 3. change of entropy.

Take $c_p = 0.946 + 0.184 \times 10^{-3} T$ kJ/kg K, and $c_v = 0.653 + 0.184 \times 10^{-3} T$ kJ/kg K

Solution. Given : $v_1 = 5 \text{ m}^3$; $p_1 = 1 \text{ bar} = 100 \text{ kN/m}^2$; $T_1 = 27^\circ\text{C} = 27 + 273 = 300 \text{ K}$;
 $T_2 = 55^\circ\text{C} = 55 + 273 = 328 \text{ K}$; $a = 0.946 \text{ kJ/kg K}$; $b = 0.653 \text{ kJ/kg K}$; $K = 0.184 \times 10^{-3} \text{ kJ/kg K}^2$

In Fig. 40.3, 1-2 represents the constant pressure process.

1. *Mass of the gas*

Let $m =$ Mass of the gas.

First of all, let us find the characteristic gas constant (R). We know that

$$\begin{aligned} R &= c_p - c_v = a - b = 0.946 - 0.653 \\ &= 0.293 \text{ kJ/kg K} \end{aligned}$$

We know that

$$p_1 v_1 = m R T_1$$

$$\therefore m = \frac{p_1 v_1}{R T_1} = \frac{100 \times 5}{0.293 \times 300} = 5.69 \text{ kg Ans.}$$

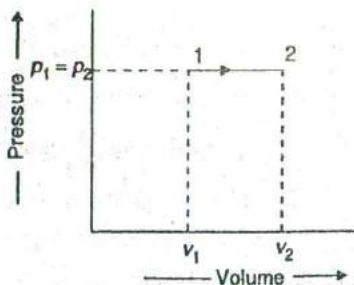


Fig. 40.3

2. Heat transfer during the process

First of all, let us find the value of mean temperature (T_m). We know that

$$T_m = \frac{T_1 + T_2}{2} = \frac{300 + 328}{2} = 314 \text{ K}$$

\therefore Mean specific heat at constant pressure,

$$c_{pm} = a + K T_m = 0.946 + 0.184 \times 10^{-3} \times 314 = 1 \text{ kJ/kg K}$$

We know that heat transfer during the process,

$$\begin{aligned} Q_{1-2} &= m c_{pm} (T_2 - T_1) \\ &= 5.69 \times 1 (328 - 300) = 159.32 \text{ kJ Ans.} \end{aligned}$$

3. Change in entropy

We know that change in entropy,

$$\begin{aligned} S_2 - S_1 &= 2.3 m a \log \left(\frac{T_2}{T_1} \right) + m K (T_2 - T_1) + 2.3 m (a - b) \log \left(\frac{p_1}{p_2} \right) \\ &= 2.3 \times 5.69 \times 0.946 \log \left(\frac{328}{300} \right) + 5.69 \times 0.184 \times 10^{-3} (328 - 300) \\ &= 0.48 + 0.029 = 0.509 \text{ kJ/K Ans.} \end{aligned}$$

40.8. Isentropic Expansion with Variable Specific Heat

Isentropic expansion is a reversible adiabatic process, in which no heat enters or leaves the gas. Thus

$$\delta Q = 0$$

We know that change in entropy,

$$dS = \frac{\delta Q}{T}$$

Since $\delta Q = 0$, therefore $dS = 0$.

The expression for isentropic expansion with variable specific heat may be expressed in the following ways :

(a) In terms of volume and absolute temperature

We know that for a small change in the state of a working substance, the general gas energy equation is,

$$\begin{aligned} \delta Q &= dU + \delta W && \dots \text{ (First Law of Thermodynamics) } \\ &= m c_v dT + p dv && \dots \text{ (i) } \end{aligned}$$

where

dT = Small change in temperature, and

dv = Small change in volume.

Since $p_1 = p_2$, therefore $\log \left(\frac{p_1}{p_2} \right) = 0$

Dividing throughout equation (i) by T

$$\frac{\delta Q}{T} = m c_v \times \frac{dT}{T} + \frac{p dv}{T}$$

Since $p v = m R T$ or $\frac{p}{T} = \frac{mR}{v}$ and $\frac{\delta Q}{T} = dS = 0$, therefore

$$m c_v \times \frac{dT}{T} + mR \times \frac{dv}{v} = 0$$

$$\text{or } m(b + KT) \frac{dT}{T} + m(a - b) \frac{dv}{v} = 0 \quad \dots (\because c_v = b + KT \text{ and } R = a - b)$$

$$m b \times \frac{dT}{T} + m K dT + m(a - b) \frac{dv}{v} = 0 \quad \dots (ii)$$

On integrating equation (ii),

$$m b \int \frac{dT}{T} + m K \int dT + m(a - b) \int \frac{dv}{v} = \int 0$$

$$m b \log_e T + m K T + m(a - b) \log_e v = \text{Constant}$$

Dividing throughout by mb , we get

$$\log_e T + \frac{KT}{b} + \left(\frac{a}{b} - 1\right) \log_e v = \text{Constant}$$

$$\text{or } \log_e T + \log_e e^{\frac{KT}{b}} + \log_e v^{\left(\frac{a}{b} - 1\right)} = \text{Constant}$$

$$\text{or } T e^{\frac{KT}{b}} v^{\left(\frac{a}{b} - 1\right)} = \text{Constant} \quad \dots (iii)$$

$$\therefore \frac{T}{v} e^{\frac{KT}{b}} v^{\frac{a}{b}} = \text{Constant} \quad \dots (iv)$$

The more useful form of above equation is

$$\frac{T_1}{v_1} e^{\frac{KT_1}{b}} v_1^{\frac{a}{b}} = \frac{T_2}{v_2} e^{\frac{KT_2}{b}} v_2^{\frac{a}{b}} = \frac{T_3}{v_3} e^{\frac{KT_3}{b}} v_3^{\frac{a}{b}} = \dots = \text{Constant}$$

where suffixes 1, 2 and 3 ... refer to different sets of condition.

Note : For per unit mass, the above expression may be written as

$$\frac{T_1}{v_{s1}} e^{\frac{KT_1}{b}} v_{s1}^{\frac{a}{b}} = \frac{T_2}{v_{s2}} e^{\frac{KT_2}{b}} v_{s2}^{\frac{a}{b}} = \frac{T_3}{v_{s3}} e^{\frac{KT_3}{b}} v_{s3}^{\frac{a}{b}} = \dots = \text{Constant}$$

where v_{s1} , v_{s2} and v_{s3} = Specific volumes at states 1, 2 and 3 respectively in m^3/kg of gas.

(b) In terms of pressure and volume

We know that the general gas equation is

$$p v = m R T \quad \text{or} \quad \frac{T}{v} = \frac{p}{mR} = \frac{p}{m(a - b)}$$

Substituting this value of T/v in equation (iv), we get

$$\frac{p}{m(a-b)} e^{\frac{KT}{b}} v^{\frac{a}{b}} = \text{Constant}$$

or $p e^{\frac{KT}{b}} v^{\frac{a}{b}} = \text{Constant}$... [$m(a-b)$ is constant for a particular gas]

The more useful form of above equation is

$$p_1 e^{\frac{KT_1}{b}} v_1^{\frac{a}{b}} = p_2 e^{\frac{KT_2}{b}} v_2^{\frac{a}{b}} = p_3 e^{\frac{KT_3}{b}} v_3^{\frac{a}{b}} = \dots = \text{Constant}$$

where suffixes 1, 2 and 3 refer to different sets of condition.

Note : For per unit mass, the above expression may be written as

$$p_1 e^{\frac{KT_1}{b}} v_{s1}^{\frac{a}{b}} = p_2 e^{\frac{KT_2}{b}} v_{s2}^{\frac{a}{b}} = p_3 e^{\frac{KT_3}{b}} v_{s3}^{\frac{a}{b}} = \dots = \text{Constant}$$

where v_{s1}, v_{s2} and v_{s3} = Specific volumes at states 1, 2 and 3 respectively in m^3/kg of gas.

Example 40.3. A mass of 1.5 kg and volume of 0.14 m^3 of certain gas at 40 bar is expanded isentropically such that temperature falls to 500 K. Determine : 1. Initial temperature of the gas, 2. Work done during the process, and 3. Pressure at the end of expansion.

Take $c_p = 0.946 + 0.000184 T$ kJ/kg K, and $c_v = 0.653 + 0.000184 T$ kJ/kg K.

Solution. Given : $m = 1.5 \text{ kg}$; $v_1 = 0.14 \text{ m}^3$; $p_1 = 40 \text{ bar} = 4 \times 10^6 \text{ N/m}^2 = 4 \times 10^3 \text{ kN/m}^2$, $T_2 = 500 \text{ K}$; $a = 0.946 \text{ kJ/kg K}$; $b = 0.653 \text{ kJ/kg K}$; $K = 0.000184 \text{ kJ/kg K}^2$

In Fig. 40.4, the process 1-2 represents the isentropic expansion of the gas.

1. Initial temperature of the gas

Let T_1 = Initial temperature of the gas.

First of all, let us find the characteristic gas constant (R). We know that

$$\begin{aligned} R &= c_p - c_v \\ &= a - b = 0.946 - 0.653 \\ &= 0.293 \text{ kJ/kg K} \end{aligned}$$

We know that

$$p_1 v_1 = m R T_1$$

$$\therefore T_1 = \frac{p_1 v_1}{m R} = \frac{4 \times 10^3 \times 0.14}{1.5 \times 0.293} = 1274.2 \text{ K Ans.}$$

2. Work done by the gas

Let W_{1-2} = Work done by the gas.

We know that

$$\delta Q = dU + \delta W$$

... (First Law of Thermodynamics)

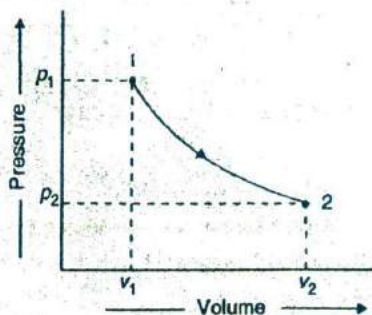


Fig. 40.4

In case of isentropic expansion, no heat enters or leaves the gas.

$$\therefore \delta Q = 0 \text{ or } \delta W = -dU$$

On integrating from state 1 to state 2,

$$W_{1-2} = -(U_2 - U_1)$$

Now, let us find the change in internal energy ($U_2 - U_1$).

We know that mean temperature

$$T_m = \frac{T_1 + T_2}{2} = \frac{1274.2 + 500}{2} = 887.1 \text{ K}$$

and mean specific heat at constant volume,

$$c_{vm} = b + K T_m = 0.653 + 0.000184 \times 887.1 = 0.816 \text{ kJ kg}^{-1} \text{ K}^{-1}$$

We know that change in internal energy,

$$\begin{aligned} dU &= U_2 - U_1 = m c_{vm} (T_2 - T_1) \\ &= 1.5 \times 0.816 \times (500 - 1274.2) = -947.6 \text{ kJ} \end{aligned}$$

\therefore Work done by the gas,

$$W_{1-2} = -(U_2 - U_1) = 947.6 \text{ kJ Ans.}$$

3. Pressure at the end of expansion

Let p_2 = Pressure at the end of expansion.

First of all, let us find out the volume at the end of expansion.

We know that for isentropic process,

$$\begin{aligned} T_1 e^{\frac{KT_1}{b}} v_1^{\left(\frac{a}{b}-1\right)} &= T_2 e^{\frac{KT_2}{b}} v_2^{\left(\frac{a}{b}-1\right)} \\ 1274.2 \times e^{\frac{0.000184 \times 1274.2}{0.653}} \times 0.14^{0.653-1} &= 500 \times e^{\frac{0.000184 \times 500}{0.653}} \times v_2^{\frac{0.946}{0.653}-1} \\ 1274.2 \times 1.43 \times 0.41 &= 500 \times 1.15 \times v_2^{0.45} \end{aligned}$$

or

$$v_2 = \left(\frac{1274.2 \times 1.43 \times 0.41}{500 \times 1.15} \right)^{\frac{1}{0.45}} = 0.99 \text{ m}^3$$

We know that $p_2 v_2 = m R T_2$

$$\therefore p_2 = \frac{m R T_2}{v_2} = \frac{1.5 \times 0.293 \times 500}{0.99} = 221.97 \text{ kN/m}^2 = 2.2197 \text{ bar Ans.}$$

40.9. Effect of Variable Specific Heat in I.C. Engines

We have already discussed that with the increase in temperature, the specific heat of any gas also increases. In an I.C. engine, the pressure and temperature developed during combustion are much less than that calculated by assuming specific heat to be constant. This results in lower efficiency of the cycle. This effect of variable specific heat on air standard efficiencies of Otto and Diesel cycles is discussed in the following pages.

40.10. Effect of Variable Specific Heat on Otto Cycle

The effect of variable specific heat on Otto cycle is shown on $p-v$ and $T-s$ diagram in Fig. 40.5. The ideal Otto cycle is represented by 1-2-3-4. When the variation in specific heat is taken into account, the cycle is represented by 1'-2'-3-4' (i.e. by dotted lines).

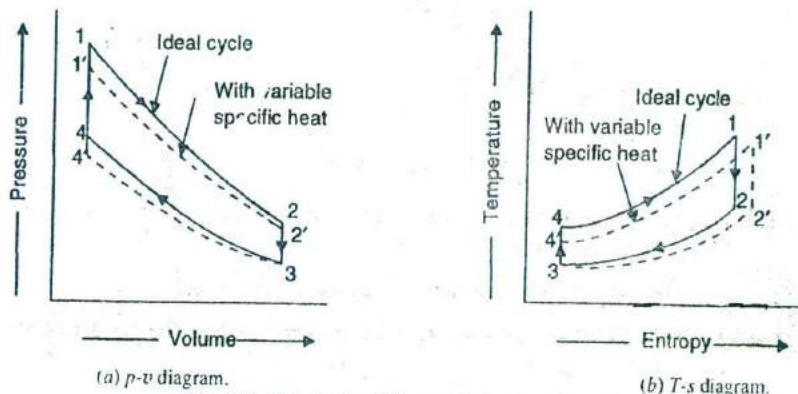


Fig. 40.5. Effect of variable specific heat on Otto cycle.

Let us derive the relation for the variation in air standard efficiency of Otto cycle with percentage variation in c_v . We have already discussed in Chapter 6 (Art. 6.16), that air standard efficiency of Otto cycle is

$$\eta = 1 - \frac{1}{(r)^{\gamma-1}} = 1 - (r)^{-(\gamma-1)} = 1 - (r)^{-R^*/c_v}$$

where

r = Compression ratio, and

γ = Ratio of specific heats = c_p/c_v

$$\therefore 1 - \eta = (r)^{\frac{-R}{c_v}}$$

Taking \log_e on both sides, we get

$$\log_e (1 - \eta) = \frac{-R}{c_v} \log_e r$$

Differentiating the above equation, we get

$$-\frac{1}{1 - \eta} d\eta = -R \log_e r \left(\frac{-1}{c_v^2} dc_v \right) = R \log_e r \times \frac{dc_v}{c_v}$$

* We know that

$$c_p - c_v = R$$

Dividing throughout by c_v ,

$$\frac{c_p}{c_v} - 1 = \frac{R}{c_v} \text{ or } \gamma - 1 = \frac{R}{c_v}$$

$$\therefore \left(\because \gamma = c_p/c_v \right)$$

$$\text{or} \quad -d\eta = (1-\eta) \frac{R}{c_v} \log_e r \times \frac{dc_v}{c_v}$$

Dividing both sides by η , we get

$$-\frac{d\eta}{\eta} = \left(\frac{1-\eta}{\eta} \right) \frac{R}{c_v} \log_e r \times \frac{dc_v}{c_v}$$

$$\text{or} \quad \frac{d\eta}{\eta} = - \left(\frac{1-\eta}{\eta} \right) \frac{R}{c_v} \log_e r \times \frac{dc_v}{c_v}$$

$$= -2.3 \left(\frac{1-\eta}{\eta} \right) \frac{R}{c_v} \log r \times \frac{dc_v}{c_v}$$

Hence the above equation shows the percentage variation in air standard efficiency of Otto cycle with the percentage variation in c_v .

The minus sign indicates that the efficiency decreases with increase in c_v .

Example 40.4. In an air standard Otto cycle, the compression ratio is 7. Calculate the change in efficiency if the specific heat at constant volume increases by 2%. Take $\gamma = 1.4$.

Solution. Given : $r = 7$; $dc_v/c_v = 2\% = 0.02$; $\gamma = 1.4$

We know that air standard efficiency for Otto cycle,

$$\eta = 1 - \frac{1}{(r)^{\gamma-1}} = 1 - \frac{1}{(7)^{1.4-1}} = 1 - \frac{1}{2.18}$$

$$= 1 - 0.46 = 0.54 \quad \text{or} \quad 54\%$$

\therefore Change in efficiency,

$$\frac{d\eta}{\eta} = -2.3 \left(\frac{1-\eta}{\eta} \right) \frac{R}{c_v} \log r \times \frac{dc_v}{c_v}$$

$$= -2.3 \left(\frac{1-0.54}{0.54} \right) (1.4-1) \log 7 \times 0.02 \quad \left(\because \frac{R}{c_v} = \frac{c_p - c_v}{c_v} = \gamma - 1 \right)$$

$$= -0.0133 = -1.33\% \quad \text{Ans.}$$

The minus sign indicates that there is a decrease in the efficiency of Otto cycle.

Example 40.5. In an Otto cycle, the compression ratio is 7 and the compression begins at 1 bar and 325 K. The air fuel ratio is 15 : 1 and the calorific value of fuel is 46 000 kJ/kg. Determine the maximum pressure in the cycle, if the mean index of compression is 1.37 and specific heat at constant volume is given by

$$c_v = 0.7116 + 0.21 \times 10^{-3} \text{ kJ/kg K}$$

If value of c_v remains constant as 0.7116 kJ/kg, and the value of compression index is unaltered, what will be the maximum pressure ?

Solution. Given : $r = v_3/v_4 = v_2/v_1 = 7$; $p_3 = 1 \text{ bar} = 100 \text{ kN/m}^2$; $T_3 = 325 \text{ K}$; Air-fuel ratio = 15 : 1 ; C.V. = 46 000 kJ/kg ; $n = 1.37$; $b = 0.7116 \text{ kJ/kg K}$; $K = 0.21 \times 10^{-3} \text{ kJ/kg K}^2$.

The p - v diagram for the Otto cycle with variable specific heat (c_p) is shown in Fig. 40.6.

First of all, let us find the pressure and temperature at the end of compression.

Let p_4 = Pressure at the end of compression, and
 T_4 = Temperature at the end of compression.

We know that, during compression process 3-4,

$$p_3 (v_3)^n = p_4 (v_4)^n$$

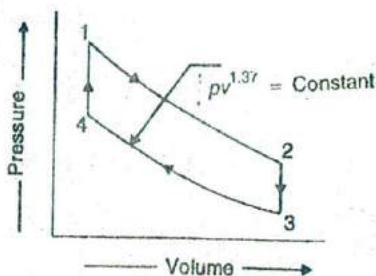


Fig. 40.6

or
$$p_4 = p_3 \left(\frac{v_3}{v_4} \right)^n = p_3 \times r^n = 100 (7)^{1.37} = 1438 \text{ kN/m}^2 \quad \dots (\because v_3/v_4 = r)$$

Also,
$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3} \right)^{\frac{n-1}{n}}$$

$$\therefore T_4 = T_3 \left(\frac{p_4}{p_3} \right)^{\frac{n-1}{n}} = 325 \left(\frac{1438}{100} \right)^{\frac{1.37-1}{1.37}} = 667.54 \text{ K}$$

Now, let us find the temperature at the end of constant volume heat addition process 4-1.

Let T_1 = Temperature at the end of constant volume heat addition process.

We know that mean temperature,

$$T_m = \frac{T_4 + T_1}{2} = \frac{667.54 + T_1}{2}$$

\therefore Mean specific heat at constant volume,

$$c_{vm} = b + K T_m = 0.7716 + 0.21 \times 10^{-3} \left(\frac{667.54 + T_1}{2} \right) \text{ kJ/kg K}$$

Now, let us find out the amount of heat added during the process 4-1.

Let Q_{4-1} = Amount of heat added in kJ.

We know that, Air fuel ratio = 15 : 1

... (Given)

i.e. One kg of fuel mixes with 15 kg of air and thus total mass of mixture (m) = 15 + 1 = 16 kg.

Since the calorific value of fuel is 46 000 kJ/kg of fuel therefore,

$$Q_{4-1} = \text{Mass of fuel} \times C.V. = 1 \times 46\,000 = 46\,000 \text{ kJ}$$

We know that in constant volume process, the amount of heat added is equal to change in internal energy, i.e.

$$Q_{4-1} = U_1 - U_4$$

$$\begin{aligned} \therefore Q_{4-1} &= m c_{vm} (T_1 - T_4) && \dots [\because U_1 - U_4 = m c_{vm} (T_1 - T_4)] \\ 46\,000 &= 16 \times \left[0.7116 + 0.21 \times 10^{-3} \left(\frac{667.54 + T_1}{2} \right) \right] (T_1 - 667.54) \\ \frac{46\,000}{16} &= \left(0.7116 + 0.105 \times 10^{-3} \times 667.54 + 0.105 \times 10^{-3} \times T_1 \right) (T_1 - 667.54) \\ 2875 &= \left(0.7116 + 0.0701 + 0.105 \times 10^{-3} \times T_1 \right) (T_1 - 667.54) \\ &= 0.7116 T_1 + 0.0701 T_1 + 0.105 \times 10^{-3} (T_1)^2 - 475.02 - 46.79 - 0.0701 T_1 \\ &= 0.105 \times 10^{-3} (T_1)^2 + 0.7116 T_1 - 521.81 \end{aligned}$$

$$\text{or } 0.105 \times 10^{-3} (T_1)^2 + 0.7116 T_1 - 3396.81 = 0$$

$$(T_1)^2 + 6.78 \times 10^3 T_1 - 32.34 \times 10^6 = 0$$

$$\begin{aligned} \therefore T_1 &= \frac{-6.78 \times 10^3 \pm \sqrt{(6.78 \times 10^3)^2 - 4 \times (-32.34 \times 10^6)}}{2} \\ &= \frac{-6.78 \times 10^3 \pm \sqrt{(45.97 \times 10^6) + (129.36 \times 10^6)}}{2} \\ &= \frac{-6.78 \times 10^3 \pm 13.24 \times 10^3}{2} \end{aligned}$$

$$\therefore T_1 = 3230 \text{ K or } -10\,010 \text{ K}$$

Taking the positive value of T_1 , we get

$$T_1 = 3230 \text{ K}$$

Maximum pressure in the cycle

Let $p_3 =$ Maximum pressure in the cycle.

We know that for the process 4-1,

$$\frac{p_1}{T_1} = \frac{p_4}{T_4} \quad \dots (\because \text{Volume is constant})$$

$$\therefore p_1 = p_4 \left(\frac{T_1}{T_4} \right) = 1438 \left(\frac{3230}{667.54} \right) = 6958 \text{ kN/m}^2 = 69.58 \text{ bar Ans.}$$

Maximum pressure in the cycle if c_v remains constant

Let $p_1' =$ Maximum pressure in the cycle if c_v remains constant.

First of all, let us find temperature at the end of compression again with c_v as constant.

Let $T_1' =$ Temperature at the end of compression.

We know that

$$Q_{4-1} = m c_v (T_1' - T_4)$$

$$46\,000 = 16 \times 0.7116 (T_1' - 667.54)$$

$$T_1' - 667.54 = \frac{46\,000}{16 \times 0.7116} = 4040.2$$

or

$$T_1' = 4040.2 + 667.54 = 4707.74 \text{ K}$$

∴ Maximum pressure air the cycle,

$$p_1' = p_4 \left(\frac{T_1'}{T_4} \right) = 1438 \left(\frac{4707.74}{667.54} \right) = 10.141 \times 10^3 \text{ kN/m}^2$$

$$= 101.41 \text{ bar Ans.}$$

Example 40.6. An engine working on Otto cycle (constant volume cycle) has a compression ratio of 10. The temperature rise during compression is from 320 K to 820 K and at the beginning and end of expansion, the temperatures are 1720 K and 688 K, both the compression and expansion being adiabatic and occupying the whole stroke.

Assuming that the specific heat at constant volume has the form $b + KT$ where b and K are constants and T is the absolute temperature, and the value of $R = 287 \text{ J/kg K}$, find: 1. The values of b and K , and 2. the theoretical efficiency.

Solution. Given: $r = v_3/v_4 = v_2/v_1 = 10$; $T_3 = 320 \text{ K}$; $T_4 = 820 \text{ K}$; $T_1 = 1720 \text{ K}$; $T_2 = 688 \text{ K}$; $R = 287 \text{ J/kg K}$

1. The values of b and K

The p - v diagram for Otto cycle with variable specific heat (c_v) is shown in Fig. 40.7.

We know that during the expansion process 1-2,

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}$$

or

$$\frac{p_1}{p_2} = \left(\frac{v_2}{v_1} \right) \times \left(\frac{T_1}{T_2} \right) = 10 \times \frac{1720}{688} = 25$$

We also know that for compression process 3-4,

$$\frac{p_3 v_3}{T_3} = \frac{p_4 v_4}{T_4} \quad \text{or} \quad \frac{p_4}{p_3} = \left(\frac{v_3}{v_4} \right) \times \left(\frac{T_4}{T_3} \right) = 10 \times \frac{820}{320} = 25.625$$

We know that the equation for reversible adiabatic process with variable specific heat is

$$p e^{\frac{KT}{b}} v^{\frac{a}{b}} = \text{Constant}$$

Hence for the expansion process 1-2,

$$p_1 e^{\frac{KT_1}{b}} v_1^{\frac{a}{b}} = p_2 e^{\frac{KT_2}{b}} v_2^{\frac{a}{b}}$$

or

$$\log_e p_1 + \frac{KT_1}{b} + \frac{a}{b} \log_e v_1 = \log_e p_2 + \frac{KT_2}{b} + \frac{a}{b} \log_e v_2$$

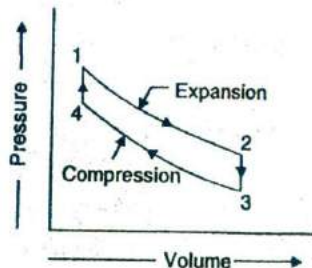


Fig. 40.7

$$\begin{aligned}
 -2.3 \log \left(\frac{p_1}{p_2} \right) + 2.3 \times \frac{a}{b} \log \left(\frac{v_2}{v_1} \right) &= \frac{K}{b} (T_1 - T_2) \\
 \therefore -2.3 \log (25) + 2.3 \times \frac{a}{b} \log (10) &= \frac{K}{b} (1720 - 688) \\
 -3.22b + 2.3 \times a &= 1032K \quad \dots (i)
 \end{aligned}$$

Similarly for the compression process 3-4,

$$\begin{aligned}
 p_3 e^{\frac{KT_3}{b}} \cdot \frac{a}{v_3^{\frac{a}{b}}} &= p_4 e^{\frac{KT_4}{b}} \frac{a}{v_4^{\frac{a}{b}}} \\
 \text{or } \log_e p_3 + \frac{KT_3}{b} + \frac{a}{b} \times \log_e v_3 &= \log_e p_4 + \frac{KT_4}{b} + \frac{a}{b} \log_e v_4 \\
 \text{or } -2.3 \log \left(\frac{p_4}{p_3} \right) + 2.3 \times \frac{a}{b} \log \left(\frac{v_3}{v_4} \right) &= \frac{K}{b} (T_4 - T_1) \\
 -2.3 \log (25.625) + 2.3 \times \frac{a}{b} \log (10) &= \frac{K}{b} (820 - 320) \\
 -3.24 + 2.3 \times \frac{a}{b} &= 500 \times \frac{K}{b} \\
 \text{or } -3.24b + 2.3a &= 500K \quad \dots (ii)
 \end{aligned}$$

From equations (i) and (ii), we get

$$0.02b = 532K \quad \text{or} \quad b = 26\,600K \quad \dots (iii)$$

We know that

$$c_v = b + KT; \quad \text{and} \quad c_p = a + KT$$

$$\therefore R = c_p - c_v = a - b$$

$$\text{or} \quad a = R + b = 287 + 26\,600K \quad \dots (iv)$$

Substituting the value of b and a from equations (iii) and (iv) respectively in equation (ii), we get

$$-3.24 \left(26\,600K \right) + 2.3 \left(287 + 26\,600K \right) = 500K$$

$$-86\,184K + 660.1 + 61\,180K = 500K$$

$$\text{or} \quad 25\,504K = 660.1 \quad \text{or} \quad K = 0.0259 \text{ J/kg K}^2 \text{ Ans.}$$

$$\therefore b = 26\,600K = 26\,600 \times 0.0259 = 688.9 \text{ J/kg Ans.} \quad \dots [\text{From equation (iii)}]$$

2. Theoretical efficiency

We know that theoretical efficiency,

$$\eta = 1 - \frac{1}{(r)^{\gamma-1}} = 1 - \frac{1}{(10)^{1.4-1}} = 1 - 0.40 = 0.60 \quad \text{or} \quad 60\% \text{ Ans.}$$

40.11. Effect of Variable Specific Heat on Diesel Cycle

The effect of variable specific heat on Diesel cycle is shown on $p-v$ and $T-s$ diagram in Fig. 40.8. The ideal Diesel cycle is represented by 1-2-3-4. When the variation in specific heat is taken into account, the cycle is represented by 1'-2'-3'-4.

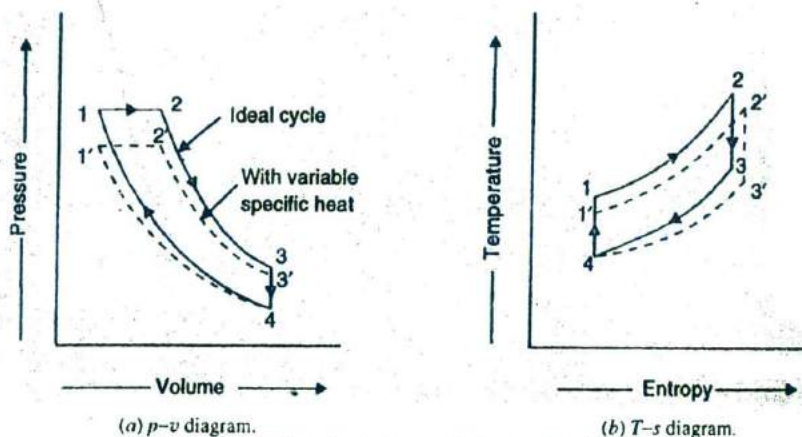


Fig. 40.8. Effect of variable specific heat on Diesel cycle.

Let us derive the relation for the percentage variation in air standard efficiency of Diesel cycle with the percentage variation of c_v .

We have already discussed in Chapter 6 (Art. 6.17) that the efficiency of Diesel cycle is

$$\eta = 1 - \frac{1}{r^{\gamma-1}} \left[\frac{\rho^\gamma - 1}{\gamma(\rho - 1)} \right] \quad \dots (i)$$

where

r = Compression ratio,

ρ = Cut-off ratio, and

γ = Ratio of specific heats = c_p/c_v .

$$\therefore 1 - \eta = \frac{1}{r^{\gamma-1}} \left[\frac{\rho^\gamma - 1}{\gamma(\rho - 1)} \right]$$

or

$$1 - \eta = r^{-(\gamma-1)} \left[\frac{\rho^\gamma - 1}{\gamma(\rho - 1)} \right]$$

Taking \log_e on both sides, we get

$$\log_e (1 - \eta) = -(\gamma - 1) \log_e r + \log_e (\rho^\gamma - 1) - \log_e \gamma - \log_e (\rho - 1)$$

Now, differentiate the above equation with respect to γ ,

$$-\frac{1}{1 - \eta} \times \frac{d\eta}{d\gamma} = -\log_e r + \frac{\rho^\gamma \log_e \rho}{\rho^\gamma - 1} - \frac{1}{\gamma}$$

$$\text{or } d\eta = (1 - \eta) \left[\log_e r - \frac{\rho^\gamma \log_e \rho}{\rho^\gamma - 1} + \frac{1}{\gamma} \right] d\gamma$$

Dividing both sides by η , we get

$$\frac{d\eta}{\eta} = \left[\frac{1 - \eta}{\eta} \right] \left[\log_e r - \frac{\rho^\gamma \log_e \rho}{\rho^\gamma - 1} + \frac{1}{\gamma} \right] d\gamma \quad \dots (ii)$$

We know that

$$c_p - c_v = R \quad \dots (iii)$$

Dividing equation (iii) by c_v , we get

$$\frac{c_p}{c_v} - 1 = \frac{R}{c_v}$$

$$\text{or } \gamma - 1 = \frac{R}{c_v} \quad \dots \left(\because \gamma = \frac{c_p}{c_v} \right)$$

Differentiating the above equation, we get

$$d\gamma = -\frac{R}{c_v^2} dc_v = -\frac{R}{c_v} \times \frac{dc_v}{c_v}$$

$$\text{or } d\gamma = -(\gamma - 1) \frac{dc_v}{c_v} \quad \dots \left(\because \gamma - 1 = \frac{R}{c_v} \right)$$

Substituting this value of $d\gamma$ in equation (ii), we get

$$\frac{d\eta}{\eta} = -\left[\frac{1 - \eta}{\eta} \right] \left[\log_e r - \frac{\rho^\gamma \log_e \rho}{\rho^\gamma - 1} + \frac{1}{\gamma} \right] (\gamma - 1) \frac{dc_v}{c_v}$$

$$\begin{aligned} \text{or } \frac{d\eta}{\eta} &= -\left[\frac{1 - \eta}{\eta} \right] (\gamma - 1) \left[\log_e r - \frac{\rho^\gamma \log_e \rho}{\rho^\gamma - 1} + \frac{1}{\gamma} \right] \frac{dc_v}{c_v} \\ &= -\left[\frac{1 - \eta}{\eta} \right] (\gamma - 1) \left[2.3 \log_e r - \frac{2.3 \rho^\gamma \log \rho}{\rho^\gamma - 1} + \frac{1}{\gamma} \right] \frac{dc_v}{c_v} \end{aligned}$$

Hence the above equation shows the percentage variation in air standard efficiency of Diesel cycle with the percentage variation in c_v .

The minus sign indicates that there is a decrease in efficiency with increase in c_v .

Example 40.7. In an air standard Diesel cycle, the compression ratio is 13 and the fuel is cut-off at 8% of the stroke. Calculate the change in efficiency if the specific heat at constant volume increases by 2%. Take $\gamma = 1.4$.

Solution. Given : $r = 13$; Cut-off = 8% of stroke volume ; $dc_v/c_v = 2\% = 0.02$; $\gamma = 1.4$

First of all, let us find out the cut-off ratio. We know that cut-off ratio,

$$\begin{aligned} \rho &= 1 + \frac{\text{Cut-off \%}}{100} (r - 1) \\ &= 1 + \frac{8}{100} (13 - 1) = 1 + 0.08 \times 12 = 1.96 \end{aligned}$$

∴ Air standard efficiency for Diesel cycle,

$$\begin{aligned} \eta &= 1 - \frac{1}{(r)^{\gamma-1}} \left[\frac{\rho^{\gamma} - 1}{\gamma(\rho - 1)} \right] = 1 - \frac{1}{(13)^{1.4-1}} \left[\frac{(1.96)^{1.4} - 1}{1.4(1.96 - 1)} \right] \\ &= 1 - 0.417 = 0.583 \text{ or } 58.3\% \end{aligned}$$

We know that change in efficiency for Diesel cycle,

$$\begin{aligned} \frac{d\eta}{\eta} &= - \left[\frac{1 - \eta}{\eta} \right] (\gamma - 1) \left[2.3 \log r - \frac{2.3 \rho^{\gamma} \log \rho}{\rho^{\gamma} - 1} + \frac{1}{\gamma} \right] \frac{dc_v}{c_v} \\ &= - \left[\frac{1 - 0.583}{0.583} \right] (1.4 - 1) \left[2.3 \log 13 - \frac{2.3 \times (1.96)^{1.4} \log (1.96)}{(1.96)^{1.4} - 1} + \frac{1}{1.4} \right] 0.02 \\ &= -0.0124 = -1.24\% \text{ Ans.} \end{aligned}$$

The minus sign indicates that there is a decrease in the efficiency of Diesel cycle.

Example 40.8. In an Diesel cycle, the compression ratio is 15 and the compression begins at 1 bar and 420 K. The air-fuel ratio is 25 : 1 and the calorific value of fuel is 42 000 kJ/kg. The law of compression follows the relation $p v^{1.35} = \text{constant}$. Determine the percentage at which the constant pressure combustion stops. Neglect molecular changes during combustion.

Take $c_v = 0.7116 + 0.21 \times 10^{-3} T$ kJ/kg K, and $R = 0.287$ kJ/kg K.

Solution. Given : $r = v_4/v_1 = 15$; $p_4 = 1 \text{ bar} = 100 \text{ kN/m}^2$; $T_4 = 420 \text{ K}$; Air-fuel ratio = 25 : 1 ; C.V. = 42 000 kJ/kg ; $n = 1.35$; $b = 0.7116 \text{ kJ/kg K}$; $K = 0.21 \times 10^{-3} \text{ kJ/kg K}^2$; $R = 0.287 \text{ kJ/kg K}$

The p - v diagram for Diesel cycle with variable specific heat is shown in Fig. 40.9.

First of all, let us find out the temperature at the end of compression process 4-1.

Let T_1 = Temperature at the end of compression process 4-1.

We know that volume at cut-off,

$$v_2 = v_1 + \frac{\text{Cut-off \%}}{100} \times \text{Stroke volume}$$

Dividing throughout by v_1 ,

$$\frac{v_2}{v_1} = 1 + \frac{\text{Cut-off \%}}{100} \left(\frac{v_4}{v_1} - 1 \right) \quad \dots (\because \text{Stroke volume} = v_4 - v_1)$$

$$\text{or} \quad \rho = 1 + \frac{\text{Cut-off \%}}{100} (r - 1) \quad \dots \left(\because \frac{v_2}{v_1} = \rho \text{ and } \frac{v_4}{v_1} = r \right)$$

We know that during compression process 4-1,

$$\frac{T_1}{T_4} = \left(\frac{v_4}{v_1}\right)^{\gamma-1}$$

$$\therefore T_1 = T_4 \left(\frac{v_4}{v_1}\right)^{\gamma-1}$$

$$= 420 (15)^{1.35-1} = 1083.63 \text{ K}$$

Now let us find out the amount of heat added during the constant pressure heat addition process 1-2.

Let Q_{1-2} = Amount of heat added in kJ.

We know that air fuel ratio = 25 : 1, i.e. one kg of fuel mixes with 25 kg of air and thus the total mass of the mixture (m) = 25 + 1 = 26 kg.

Since the calorific value of fuel is 42 000 kJ/kg of fuel, therefore

$$Q_{1-2} = \text{Mass of fuel} \times C.V. = 1 \times 42\,000 = 42\,000 \text{ kJ}$$

Now, let us find out the change in internal energy during the constant pressure heat addition process.

Let $U_2 - U_1$ = Change in internal energy.

We know that mean temperature (T_m) during process 1-2,

$$T_m = \frac{T_1 + T_2}{2} = \frac{1083.63 + T_2}{2}$$

\therefore Mean specific heat at constant volume,

$$c_{vm} = b + K T_m = 0.7116 + 0.21 \times 10^{-3} \left(\frac{1083.63 + T_2}{2} \right) \text{ kJ/kg K}$$

$$= 0.7116 + 0.105 \times 10^{-3} (1083.63 + T_2)$$

We also know that change in internal energy,

$$U_2 - U_1 = m c_{vm} (T_2 - T_1)$$

$$= 26 \left[0.7116 + 0.105 \times 10^{-3} (1083.63 + T_2) \right] (T_2 - 1083.63)$$

$$= (21.46 + 2.73 \times 10^{-3} T_2) (T_2 - 1083.63)$$

$$= 2.73 \times 10^{-3} (T_2)^2 + 18.5 T_2 - 23254.7$$

Now let us find out the work done during the process 1-2. We know that for constant pressure process 1-2, work done

$$W_{1-2} = p(v_2 - v_1) = m R (T_2 - T_1) = 16 \times 0.287 (T_2 - 1083.63)$$

$$= 4.6 T_2 - 4984.7 \text{ kJ}$$

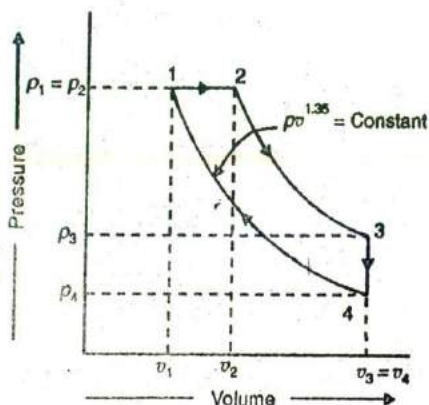


Fig. 40.9

Variable Specific Heat

Now, let us find out the temperature at the end of constant pressure process 1-2.

Let T_2 = Temperature at the end of process 1-2.

We know that for a process, the general gas energy equation is

$$Q_{1-2} = U_2 - U_1 + W_{1-2}$$

Substituting the values, in the above equation, we get

$$42\,000 = (2.73 \times 10^{-3} (T_2)^2 + 18.5 T_2 - 23\,254.7) + (4.6 T_2 - 4984.7)$$

$$2.73 \times 10^{-3} (T_2)^2 + 23.1 T_2 - 70239.4 = 0$$

$$(T_2)^2 + 8.46 \times 10^3 T_2 - 25.73 \times 10^6 = 0$$

$$\begin{aligned} \therefore T_2 &= \frac{-8.46 \times 10^3 \pm \sqrt{(8.46 \times 10^3)^2 - 4 \times 1 \times (-25.73 \times 10^6)}}{2} \\ &= \frac{-8.46 \times 10^3 \pm \sqrt{71.57 \times 10^6 + 102.92 \times 10^6}}{2} \\ &= \frac{-8.46 \times 10^3 \pm 13.21 \times 10^3}{2} \end{aligned}$$

$$\therefore T_2 = 2375 \text{ K or } T_2 = -10\,830 \text{ K}$$

Taking positive value of T_2 , we get

$$T_2 = 2375 \text{ K}$$

We know that for constant pressure process 1-2,

$$\frac{v_2}{v_1} = \frac{T_2}{T_1} = \frac{2375}{1083.63} = 2.19$$

Percentage at which the constant pressure combustion stops

We know that the percentage at which the constant pressure combustion stops

$$\begin{aligned} &= \frac{v_2 - v_1}{v_4 - v_1} \times 100 = \frac{\frac{v_2}{v_1} - 1}{\frac{v_4}{v_1} - 1} \times 100 \\ &= \frac{2.19 - 1}{15 - 1} \times 100 = 8.5\% \text{ Ans.} \end{aligned}$$

EXERCISES

1. One kg of a certain gas is heated from 500 K to 1300 K. Calculate the change in internal energy and enthalpy.

¹ Take $c_p = 0.946 + 0.000184 T$ kJ/kg K and $c_v = 0.653 + 0.000184 T$ kJ/kg K.

[Ans. 655.2 kJ; 889.6 kJ]

2. A certain quantity of gas having a volume of 4 m^3 at 1.2 bar and 30°C is heated at constant pressure to 50°C . Determine : 1. Mass of the gas ; 2. Heat transfer during the process and 3. Change of entropy.

$$\text{Take } c_p = 0.946 + 0.184 \times 10^{-3} T \text{ kJ/kg K and } c_v = 0.946 + 0.184 \times 10^{-3} T \text{ kJ/kg K}$$

[Ans. 5.41 kg ; 108.2 kJ]

3. A mass of 1.8 kg and volume of 0.15 m^3 of certain gas at 45 bar is expanded isentropically such that the temperature falls to 600 K. Determine : 1. Initial temperature of the gas, 2. Work done during the process, and 3. Pressure at the end of expansion.

$$\text{Take } c_p = 0.946 + 0.000184 T \text{ kJ/kg K and } c_v = 0.653 + 0.000184 T \text{ kJ/kg K}$$

[Ans. 1279.86 K ; 1010.82 kJ ; 2.532 bar]

4. In an air standard Otto cycle, the compression ratio is 7.5. Calculate the change in efficiency if the specific heat at constant volume increases by 3%. Take $\gamma = 1.4$

[Ans. -1.95%]

5. In an air standard Diesel cycle, the compression ratio is 18 and the fuel is cut-off at 5% of the stroke. Calculate the change in efficiency if the specific heat at constant volume increases by 2.5%. Take $\gamma = 1.4$.

[Ans. -1.44%]

QUESTIONS

1. What do you understand by molar specific heat?
2. Explain the effect of variation of specific heat with temperature.
3. Describe how moisture content of air affects the specific heats of air.
4. Derive the relations for the change in internal energy and enthalpy during a process with variable specific heats.
5. The specific heats of a gas are of the form $c_p = a + K T$ and $c_v = b + K T$, where a, b and K are constants and T is in K. Derive the formula $T^b v_s^{a-b} e^{KT} = \text{constant}$, for the adiabatic expansion of the gas.
6. Explain the effect of variable specific heat on air standard efficiencies of Otto and Diesel cycle.
7. Derive the following relation for Otto cycle, by taking variation of specific heat in account :

$$\frac{d\eta}{\eta} = - \left[\frac{1-\eta}{\eta} \right] \frac{R}{c_v} \log_e r \times \frac{dc_v}{c_v}$$

OBJECTIVE TYPE QUESTIONS

1. Gases have

(a) one specific heat	(b) two specific heats
(c) three specific heats	(d) four specific heats
2. The value of specific heat at constant pressure (c_p) with increase in temperature.

(a) increases	(b) decreases	(c) remains same
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3. Molar specific heat at constant pressure (
- c_{mp}
-) is

(a) $M + c_p$

(b) $M - c_p$

(c) $M c_p$

(d) M/c_p

where

 $M =$ Molecular mass of gas, and $c_p =$ Specific heat at constant pressure.

4. Molar specific heat at constant volume (
- c_{mv}
-) is

(a) $M + c_v$

(b) $M - c_v$

(c) $M c_v$

(d) M/c_v

where

 $M =$ Molecular mass of gas, and $c_v =$ Specific heat at constant volume.

5. The ratio of specific heats
- $\left(\gamma = \frac{c_p}{c_v}\right)$
- for air

(a) increases with increase in moisture content in air

(b) decreases with increase in moisture content in air

(c) remains constant irrespective of the increase in moisture content of air

(d) increases first and then decreases later with moisture content in air

6. The ratio of specific heats
- $\left(\gamma = \frac{c_p}{c_v}\right)$
- for real gas

(a) increases with increase in temperature

(b) decreases with increase in temperature

(c) remains same irrespective of increase in temperature

(d) increases first with increase in temperature and then decreases with further increase in temperature

7. The gas constant (
- R
-) is equal to

(a) ratio of two specific heats

(b) sum of two specific heats

(c) difference of two specific heats

(d) product of two specific heats

8. The change in internal energy during a process with variable specific heats is equal to

(a) $m c_v (T_2 - T_1)$

(b) $m c_{vm} (T_2 - T_1)$

(c) $m c_v / (T_2 - T_1)$

(d) $m c_{vm} (T_2 - T_1)$

9. The efficiency of Otto cycle

(a) increases with increase in specific heat

(b) decreases with increase in specific heat

(c) remains same with increase in specific heat

(d) first increases then decreases with increase in specific heat

10. The efficiency of Diesel cycle

- (a) increases with increase in specific heat
- (b) decreases with increase in specific heat
- (c) remains same with increase in specific heat
- (d) first increases and then decreases with increase in specific heat

ANSWERS

- | | | | | |
|--------|--------|--------|--------|---------|
| 1. (b) | 2. (a) | 3. (c) | 4. (c) | 5. (b) |
| 6. (b) | 7. (c) | 8. (d) | 9. (b) | 10. (b) |