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Introduction

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1.1. Definition

The field of science, which deals with the energies possessed by gases and vapours, is known as *Thermodynamics*. It also includes the conversion of these energies in terms of heat and mechanical work and their relationship with properties of the system. A machine, which converts heat into mechanical work or vice versa, is known as *Heat Engine*. The field of engineering science, which deals with the applications of thermodynamics and its laws to work producing and work absorbing devices, in order to understand their functions and improve their performance, is known as *Thermal Engineering*.

The heat is, usually, generated by the combustion of fuel which may be solid, liquid or gas. It is supplied to the working substance (a source of conveying heat to the heat engine for doing work in the engine cylinder) at a higher temperature. A part of the heat energy is converted into mechanical work by expanding the working substance, within the engine cylinder. The remaining heat energy is rejected at a lower temperature.

The working substances, widely used in the heat engines, are fluids in the gaseous or liquid state. A mixture of air and fuel is used as a working substance in internal combustion engines, and water vapour (steam) in the steam engines or steam turbines.

1.2. Fundamental Units

The measurement of physical quantities is one of the most important operations in engineering. Every quantity is measured in terms of some arbitrary, but internationally accepted units, called *fundamental units*.

1.3. Derived Units

Some units are expressed in terms of other units, which are derived from fundamental units are known as *derived units e.g.* the unit of area, velocity, acceleration, pressure etc.

1.4. Systems of Units

There are only four systems of units, which are commonly used and universally recognised. These are known as :

1. C.G.S. units, 2. F.P.S. units, 3. M.K.S. units, and 4. S.I. units.

1.5. C.G.S. Units

In this system, the fundamental units of length, mass and time are *centimetre*, *gram* and *second* respectively. The C.G.S. units are known as absolute units or physicist's units.

1.6. F.P.S. Units

In this system, the fundamental units of length, mass and time are *foot*, *pound* and *second* respectively.

1.7. M.K.S. Units

In this system, the fundamental units of length, mass and time are *metre*, *kilogram* and *second* respectively. The M.K.S. units are known as gravitational units or engineer's units.

1.8. S.I. Units (International System of Units)

The 11th General Conference* of Weights and Measures have recommended a unified and systematically constituted system of fundamental and derived units for international use. This system is now being used in many countries. In India, the standards of Weights and Measures Act, 1956 (vide which we switched over to M.K.S. units) has been revised to recognise all the S.I. units in industry and commerce.

In this system of units, there are seven fundamental units and two supplementary units, which cover the entire field of science and engineering. These units are shown in the following table.

Table 1.1. Fundamental and supplementary units.

S.No.	*Physical quantity*	*Unit*
	Fundamental units	
1.	Length (l)	Metre (m)
2.	Mass (m)	Kilogram (kg)
3.	Time (t)	Second (s)
4.	Temperature (T)	Kelvin (K)
5.	Electric current (I)	Ampere (A)
6.	Luminous intensity (I_v)	Candela (cd)
7.	Amount of substance (n)	Mole (mol)
	Supplementary units	
1.	Plane angle ($\alpha, \beta, \theta, \phi$)	Radian (rad)
2.	Solid angle (Ω)	Steradian (sr)

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* It is known as General Conference of Weights and Measures (C.G.P.M.). It is an international organisation, of which most of the advanced and developing countries (including India) are members. The conference has been entrusted with the task of prescribing definitions for various units of weights and measures, which are the very basic of science and technology today.

The derived units, which will be commonly used in this book, are given in the following table :

Table 1.2. Derived units.

S.No.	*Quantity*	*Symbol*	*Unit*
1.	Linear velocity	V	m/s
2.	Linear acceleration	a	m/s^2
3.	Angular velocity	ω	rad/s
4.	Angular acceleration	α	rad/s^2
5.	Mass density	ρ	kg/m^3
6.	Force, Weight	F, W	N ; 1N = 1kg-m/s^2
7.	Pressure	p	N/m^2
8.	Work, Energy, Enthalpy	W, E, H	J ; 1J = 1N-m
9.	Power	P	W ; 1W = 1J/s
10.	Absolute or dynamic viscosity	μ	N-s/m^2
11.	Kinematic viscosity	ν	m^2/s
12.	Frequency	f	Hz ; 1Hz = 1 cycle/s
13.	Gas constant	R	J/kg K
14.	Thermal conductance	h	W/m^2 K
15.	Thermal conductivity	k	W/m K
16.	Specific heat	c	J/kg K
17.	Molar mass or Molecular mass	M	kg/mol

1.9. Metre

The metre is defined as the length equal to 1 650 763.73 wavelengths in vacuum of the radiation corresponding to the transition between the levels $2\,p_{10}$ and $5\,d_5$ of the Krypton - 86 atom.

1.10. Kilogram

The kilogram is defined as the mass of the international prototype (standard block of platinum - iridium alloy) of the kilogram, kept at the international Bureau of Weights and Measures at Sevres, near Paris.

1.11. Second

The second is defined as the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium - 133 atom.

1.12. Kelvin

The kelvin is defined as the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.

Note. The triple point of water is taken as a fundamental fixed point having a temperature 273.16 K.

1.13. Presentation of Units and their Values

The frequent changes in the present day life are facilitated by an international body known as International Standard Organisation (ISO) which makes recommendations regarding international standard procedures. The implementation of ISO recommendation, in a country, is assisted by its organisation appointed for the purpose. In India, Bureau of Indian Standards (BIS) previously known as Indian Standards Institution (ISI) has been created for this purpose. We have already discussed that the fundamental units in M.K.S. and S.I. units for length, mass and time is metre, kilogram and second respectively. But in actual practice, it is not necessary to express all lengths in metres, all masses in kilograms and all times in seconds. We shall, sometimes, use the convenient units, which are multiples or divisions of our basic units in tens. As a typical example, although the metre is the unit of length, yet a small length of one-thousandth of a metre proves to be more convenient unit, especially in the dimensioning of drawings. Such convenient units are formed by using a prefix in front of the basic units to indicate the multiplier. The full list of these prefixes is given in the following table.

Table 1.3. Prefixes used in basic units.

Factor by which the units is multiplied	*Standard form*	*Prefix*	*Abbreviation*
1 000 000 000 000	10^{12}	tera	T
1 000 000 000	10^{9}	giga	G
1 000 000	10^{6}	mega	M
1 000	10^{3}	kilo	k
100	10^{2}	hecto*	h
10	10^{1}	deca*	da
0.1	10^{-1}	deci*	d
0.01	10^{-2}	centi*	c
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n
0.000 000 000 001	10^{-12}	pico	p

1.14. Rules for S.I. Units

The eleventh General Conference of Weights and Measures recommended only the fundamental and derived units for S.I. system. But it did not elaborate the rules for the usage of the units. Later on many scientists and engineers held a number of meetings for the style and usage of S.I. units. Some of the decisions of the meetings are as follows :

1. For numbers having five or more digits, the digits should be placed in groups of three separated by spaces** (instead of commas) counting both to the left and right to the decimal point.

2. In a four digit number,*** the space is not required unless the four digit number is used in a column of numbers with five or more digits.

3. A dash is to be used to separate units that are multiplied together. For example, newton × metre is written as N-m. It should not be confused with mN, which stands for millinewton.

4. Plurals are never used with symbols. For example, metre or metres are written as m.

5. All symbols are written in small letters except the symbols derived from the proper names. For example, N for newton and W for watt.

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* These prefixes are generally becoming obsolete probably due to possible confusion. Moreover it i becoming a conventional practice to use only those powers of ten which conform to 10^{3x}, where x is positive or negative whole number.

** In certain countries, comma is still used as the decimal mark.

*** In certain countries, a space is used even in a four digit number.

6. The units with names of scientists should not start with capital letter when written in full. For example, 90 newton and not 90 Newton.

At the time of writing this book, the authors sought the advice of various international authorities, regarding the use of units and their values. Keeping in view the international reputation of the authors, as well as international popularity of their books, it was decided to present units* and their values as per recommendations of ISO and BIS. It was decided to use :

4500	not	4 500	or	4,500
7 589 000	not	7589000	or	7,58,90,00
0.012 55	not	0.01255	or	.01255
30×10^6	not	3,00,00,000	or	3×10^7

The above mentioned figures are meant for numerical values only. Now let us discuss about the units. We know that the fundamental units in S.I. system of units for length, mass and time are metre, kilogram and second respectively. While expressing these quantities, we find it time consuming to write the units such as metres, kilograms and seconds, in full, every time we use them. As a result of this, we find it quite convenient to use some standard abbreviations.

We shall use :

m	for metre or metres
km	for kilometre or kilometres
kg	for kilogram or kilograms
t	for tonne or tonnes
s	for second or seconds
min	for minute or minutes
N-m	for newton × metres (*e.g.* work done)
kN-m	for kilonewton × metres
rev	for revolution or revolutions
rad	for radian or radians

1.15. Newton's Laws of Motion

Newton has formulated three laws of motion, which are the basic postulates or assumptions on which the whole system of dynamics is based. Like other scientific laws, these are also justified as the results, so obtained, agree with the actual observations. These three laws of motion are as follows :

1. *Newton's First Law of Motion.* It states, "*Every body continues in its state of rest or of uniform motion in a straight line, unless it is acted upon by some external force.*" *This is also known as Law of inertia.*

The inertia is that property of a matter, by virtue of which a body cannot move of itself, nor change the motion imparted to it.

2. *Newton's Second Law of Motion.* It states, "*The rate of change of momentum is directly proportional to the impressed force and takes place in the same direction in which the force acts.*"

3. *Newton's Third Law of Motion.* It states "*To every action, there is always an equal and opposite reaction.*"

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* In some of the question papers of the universities and other examining bodies, standard values are not used. The authors have tried to avoid such questions in the text of the book. However, at certain places the questions with sub-standard values have to be included, keeping in view the merits of the question from the reader's angle.

1.16. Mass and Weight

Sometimes much confusion and misunderstanding is created, while using the various systems of units in the measurement of force and mass. This happens, because of the lack of clear under-standing of the difference between mass and weight. The following definitions of mass and weight should be clearly understood.

1. *Mass.* It is the amount of matter contained in a given body, and does not vary with the change in its position on the earth's surface. The mass of a body is measured by direct comparison with a standard mass by using a lever balance.

2. *Weight.* It is the amount of pull, which the earth exerts upon a given body. Since the pull varies with the distance of the body from the centre of the earth, therefore weight of the body will also vary with its position on the earth's surface (say latitude and elevation). It is thus obvious, that the weight is a *force*.

The earth's pull in metric units, at sea level and 45° latitude, has been adopted as one force unit and named one kilogram of force. Thus it is a definite amount of force. But, unfortunately, it has the same name as the unit of mass. The weight of a body is measured by the use of a spring balance, which indicates the varying tension in the spring as the body is moved from place to place.

Note. The confusion in the units of mass and weight is eliminated, to a great extent, in S.I. units. In this system, mass is taken in kg and weight in newtons. The relation between the mass (m) and the weight (W) of a body is

$$W = m\,g \quad \text{or} \quad m = W/g$$

where W is in newtons, m is in kg and g is the acceleration due to gravity in m/s^2.

1.17. Force

It is an important factor in the field of Engineering science, which may be defined as an agent which produces or tends to produce, destroy or tends to destroy the motion. According to Newton's Second Law of Motion, the applied force or impressed force is directly proportional to the rate of change of momentum. We know that

$$\text{Momentum} = \text{Mass} \times \text{Velocity}$$

Let m = Mass of the body,

u = Initial velocity of the body,

v = Final velocity of the body,

a = Constant acceleration, and

t = Time required to change the velocity from u to v.

$$\therefore \text{ Change of momentum } = m\,v - m\,u$$

$$\text{and rate of change of momentum } = \frac{m\,v - m\,u}{t} = \frac{m\,(v-u)}{t} = m\,a \qquad \ldots\left(\because \frac{v-u}{t} = a\right)$$

or

$$\text{Force, } F \propto m\,a \text{ or } F = k\,m\,a$$

where k is a constant of proportionality.

For the sake of convenience, the unit of force adopted is such that it produces a unit acceleration to a body of unit mass.

$$\therefore \qquad F = m\,a = \text{Mass} \times \text{Acceleration}$$

In S.I. system of units, the unit of force is called newton (briefly written as N). A newton may be defined as the force while acting upon a mass of one kg produces an acceleration of 1 m/s^2 in the direction of which it acts. Thus

$$1\text{ N} = 1\text{ kg} \times 1\text{ m/s}^2 = 1\text{ kg-m/s}^2$$

1.18. Absolute and Gravitational Units of Force

We have already discussed that when a body of mass 1 kg is moving with an acceleration of 1 m/s^2, the force acting on the body is 1 newton (briefly written as 1 N). Therefore when the same body is moving with an acceleration of 9.81 m/s^2, the force acting on the body is 9.81 N. But we denote 1 kg mass attracted towards the earth with an acceleration of 9.81 m/s^2 as 1 kilogram-force (briefly written as kgf) or 1 kilogram-weight (briefly written as kg-wt). It is thus obvious, that

$$1 \text{ kgf} = 1 \text{ kg} \times 9.81 \text{ m/s}^2 = 9.81 \text{ kg-m/s}^2 = 9.81 \text{ N} \qquad \ldots(\because 1\text{N} = 1 \text{ kg-m/s}^2)$$

The above unit of force *i.e.* kilogram force (kgf) is called *gravitational or engineer's units of force*, whereas newton is the *absolute* or *scientific* or *S.I. units of force*. It is thus obvious, that the gravitational or engineer's units of force are 'g' times greater than the unit of force in the absolute or S.I. units.

It will be interesting to know that the *mass of the body in absolute units is numerically equal to the weight of the same body in gravitational units*. For example, consider a body whose mass,

$$m = 100 \text{ kg}$$

Therefore the force, with which the body will be attracted towards the centre of the earth,

$$F = m\,a = m\,g = 100 \times 9.81 = 981 \text{ N}$$

Now, as per definition, we know that the weight of a body is the force, by which it is attracted towards the centre of the earth. Therefore weight of the body,

$$W = 981 \text{ N} = 981 / 9.81 = 100 \text{ kgf} \qquad \ldots(\because 1 \text{ kgf} = 9.81 \text{ N})$$

In brief, the weight of a body of mass m kg at a place where gravitational acceleration is 'g' m/s^2 is $m.g$ newtons.

1.19. Thermodynamic Systems

The *thermodynamic system* (or simply known as *system*) may be broadly defined as a *definite area* or *a space* where some thermodynamic process* is taking place. It is a region where our attention is focussed for studying a thermodynamic process. A little observation will show that a thermodynamic system has its boundaries and anything outside the boundaries is called its *surroundings* as shown in Fig. 1.1. These boundaries may be *fixed* like that of a tank enclosing a certain mass of compressed gas, or *movable* like boundary of a certain volume of liquid in a pipe line.

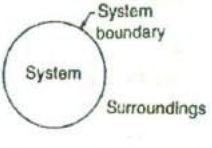

Fig. 1.1. Thermodynamic system.

1.20. Classification of Thermodynamic Systems

The thermodynamic systems may be classified into the following three groups :

1. Closed system ; 2. Open system ; and 3. Isolated system.

These systems are discussed, in detail, as follows :

1. *Closed system.* This is a system of fixed mass and identity whose boundaries are determined by the space of the matter (working substance) occupied in it.

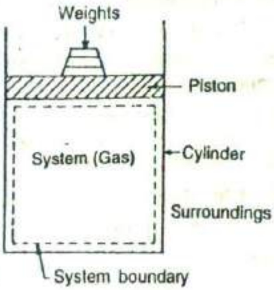

Fig. 1.2. Closed thermodynamic system.

A closed system is shown in Fig. 1.2. The gas in the cylinder is considered as a system. If heat is supplied to the cylinder from some external source, the temperature of the gas will increase and the piston will rise.

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* Refer Art. 1.25

As the piston rises, the boundary of the system moves. In other words, the heat and work energy crosses the boundary of the system during this process, but there is no addition or loss of the original mass of the working substance. It is thus obvious, that the mass of the working substance, which comprises the system, is fixed.

Thus, a closed system does not permit any mass transfer across its boundary, but it permits transfer of energy (heat and work).

2. *Open system*. In this system, the mass of the working substance crosses the boundary of the system. Heat and work may also cross the boundary. Fig. 1.3 shows the diagram of an air compressor which illustrates an open system.

The working substance crosses the boundary of the system as the low pressure (L.P.) air enters the compressor and leaves the high pressure (H.P.) air. The work crosses the boundary of the system through the driving shaft and the heat is transferred across the boundary from the cylinder walls.

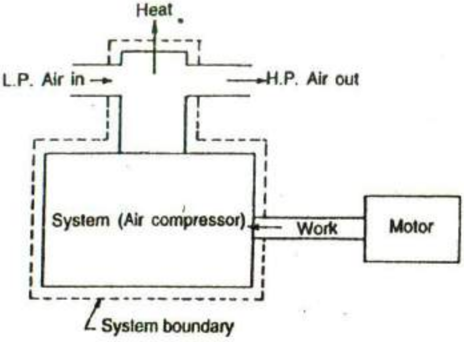

Fig. 1.3. Open thermodynamic system.

Thus, an open system permits both mass and energy (heat and work) transfer across the boundaries and the mass within the system may not be constant.

Note. An open system may be referred to as *control volume*. An open system is equivalent in every respect to a control volume, but the term open system is used throughout this text as it specifically implies that the system can have mass and energy crossing the system boundary.

3. *Isolated system*. A system which is completely uninfluenced by the surrounding is called an isolated system. It is a system of fixed mass and no heat or work energy cross its boundary. In other words, an isolated system does not have transfer of either mass or energy (heat or work) with the surroundings. An open system with its surroundings (known as an universe) is an example of an isolated system.

Note. The practical examples of isolated systems are rare. The concept of this system is particularly useful in formulating the principles derived from the Second Law of Thermodynamics.

1.21. Properties of a System

The *state of a system may be identified or described by certain observable quantities such as volume, temperature, pressure and density etc. All the quantities, which identify the state of a system, are called *properties*.

Note. Thermodynamics deals with those quantities also which are not properties of any system. For example, when there is a flow of energy between a system and its surroundings, the energy transferred is not a property of the system or its surroundings.

1.22. Classification of Properties of a System

The thermodynamic properties of a system may be divided into the following two general classes :

1. Extensive properties, and 2. Intensive properties.

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* Refer Art. 1.23.

1. Extensive properties. A quantity of matter in a given system is divided, notionally into a number of parts. The properties of the system, whose value for the entire system is equal to the sum of their values for the individual parts of the system are called *extensive properties, e.g.* total volume, total mass and total energy of a system are its extensive properties.

2. Intensive properties. It may be noticed that the temperature of the system is not equal to the sum of the temperatures of its individual parts. It is also true for pressure and density of the system. Thus properties like temperature, pressure and density are called *intensive properties*.

Note. The ratio of any extensive property of a system to the mass of the system is called an average specific value of that property (also known as intensive property) *e.g.* specific volume of a system (v_s) is the ratio of the total volume (v) of the system to its total mass (m). Mathematically,

$$v_s = v/m$$

The specific volume is an intensive property.

1.23. State of a System

The state of a system (when the system is in thermodynamic equilibrium) is the condition of the system at any particular moment which can be identified by the statement of its properties, such as pressure, volume, temperature etc. The number of properties which are required to describe a system depends upon the nature of the system.

Consider a system (gas) enclosed in a cylinder and piston arrangement as shown in Fig. 1.4. Let the system is initially in equilibrium when the piston is at position 1, represented by its properties p_1, v_1 and T_1. When the system expands, the piston moves towards right and occupies the final position at 2. At this, the system is finally in the equilibrium state represented by the properties p_2, v_2 and T_2. The initial and final states, on the pressure-volume diagram, are shown in Fig. 1.4.

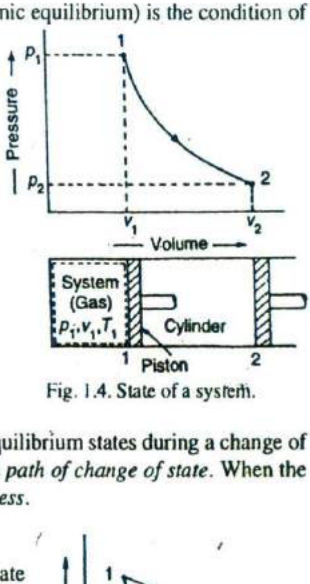

Fig. 1.4. State of a system.

1.24. Path of Change of State

When a system passes through the continuous series of equilibrium states during a change of state (from the initial state to the final state), then it is known as *path of change of state*. When the path is completely specified, it is then known as *path of the process*.

1.25. Thermodynamic Process

When a system changes its state from one equilibrium state to another equilibrium state, then the path of successive states through which the system has passed is known as *thermodynamic process*. In Fig. 1.4, 1-2 represents a thermodynamic process.

1.26. Thermodynamic Cycle or Cyclic Process

When a process or processes are performed on a system in such a way that the final state is identical with the initial state, it is then known as a *thermodynamic cycle* or *cyclic process*. In Fig. 1.5, 1-*A*-2 and 2-*B*-1 are processes whereas 1-*A*-2-*B*-1 is a thermodynamic cycle or cyclic process.

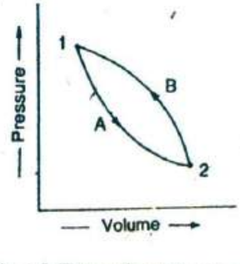

Fig. 1.5. Thermodynamic process or cyclic process.

1.27. Quasi-static or Quasi-equilibrium Process

When the process is carried out in such a way that at every instant, the system deviation from the thermodynamic equilibrium is infinitesimal, then the process is known as *quasi-static* or *quasi-equilibrium process* and each state in the process may be considered as an equilibrium state.

Consider a system (gas) enclosed in a cylinder and piston arrangement as shown in Fig. 1.6 (*a*). Let the system is initially in equilibrium state when the piston is at *A*, where the pressure is p_A, volume v_A and temperature T_A as shown in Fig. 1.6 (*b*). The weight (*W*) on the piston is composed of number of small weights which balances the upward force exerted by the system. If the whole weight is removed from the piston, then there will be unbalanced force between the system and the surroundings and the piston will move upwards till it hits the stops at *B*. At this point *B*, the system again comes to an equilibrium state where the pressure is p_B, volume v_B and temperature T_B. But the intermediate states through which the system has passed, are non-equilibrium states whose properties (pressure, volume and temperature) are not uniform throughout the system and thus the state of the system cannot be well defined. Such a process is called *irreversible* or *non-equilibrium process*, as shown by a broken line in Fig. 1.6 (*b*).

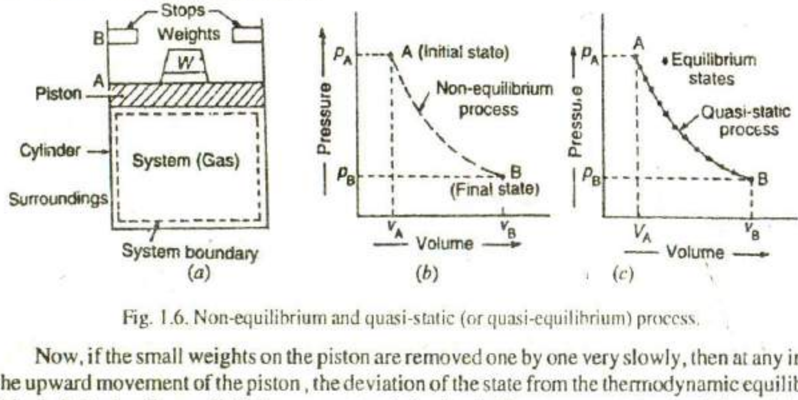

Fig. 1.6. Non-equilibrium and quasi-static (or quasi-equilibrium) process.

Now, if the small weights on the piston are removed one by one very slowly, then at any instant of the upward movement of the piston, the deviation of the state from the thermodynamic equilibrium will be infinitesimally small, if the gas system is isolated. Thus, every state passed through by the system will be in equilibrium state. Such a process, which is the locus of all these equilibrium points passed through the system, is known as *quasi-static* or *quasi-equilibrium process*.

Note : The quasi-static or quasi-equilibrium process is also known as *reversible process*. A process which can be reversed in direction and the system retraces the same equilibrium states is known as reversible process.

1.28. Temperature

It is an intensive thermodynamic property, which determines the degree of hotness or the level of heat intensity of a body. A body is said to be at a *high temperature* or *hot*, if it shows high level of heat intensity in it. Similarly, a body is said to be at a *low temperature* or *cold*, if it shows a low level of heat intensity.

The temperature of a body is measured with the help of an instrument known as *thermometer* which is in the form of a glass tube containing mercury in its stem. Following are the two commonly used scales for measuring the temperature of a body :

1. Celsius or centigrade scale, and 2. Fahrenheit scale.

Each of these scales is based on two fixed points known as freezing point of water under atmospheric pressure or *ice point* and the boiling point of water or *steam point*.

1. *Celsius or centigrade scale*. This scale was first used by Celsius in 1742. This scale is mostly used by engineers and scientists. The freezing point of water on this scale is marked as zero, and the boiling point of water as 100. The space between these two points has 100 equal divisions, and each division represents one degree Celsius (written as °C).

2. *Fahrenheit scale*. This scale was first used in 1665. In this scale, the freezing point of water is marked as 32 and the boiling point of water as 212. The space between these two points has 180 equal divisions and each division represents one degree Fahrenheit (written as °F).

Note. The relation between Celsius scale and Fahrenheit scale is given by :

$$\frac{C}{100} = \frac{F-32}{180} \quad \text{or} \quad \frac{C}{5} = \frac{F-32}{9}$$

Example 1.1. *Find the temperature which has the same value on both the Celsius and Fahrenheit scales.*

Solution.

Let x = Temperature which has the same value on both the Celsius and Fahrenheit scales.

We know that $\frac{C}{5} = \frac{F-32}{9}$ or $9C = 5(F-32)$

$$\therefore \quad 9x = 5(x-32) = 5x-160$$

$$9x-5x = -160 \quad \text{or} \quad 4x = -160$$

$$\therefore \quad x = -160/4 = -40$$

Hence – 40° on the Celsius scale is equal to – 40° on the Fahrenheit scale. **Ans.**

1.29. Absolute Temperature

As a matter of fact, the zero readings of Celsius and Fahrenheit scales are chosen arbitrarily for the purpose of simplicity. It helps us in our calculations, when changes of temperature in a process are known. But, whenever the value of temperature is used in equations relating to fundamental laws, then the value of temperature, whose reference point is true zero or absolute zero, is used. The temperature, below which the temperature of any substance can not fall, is known as absolute zero temperature.

The absolute zero temperature, for all sorts of calculations, is taken as – 273°C in case of Celsius scale and – 460°F in case of Fahrenheit scale. The temperatures measured from this zero are called *absolute temperatures*. The absolute temperature in Celsius scale is called degree Kelvin (briefly written as K)* ; such that K = °C + 273. Similarly, absolute temperature in Fahrenheit scale is called degrees Rankine (briefly written as °R) ; such that °R = °F + 460.

1.30. Thermodynamic Equilibrium

A system is said to be in thermodynamic equilibrium, if it satisfies the following three requirements of equilibrium.

1. *Mechanical equilibrium.* A system is said to be in mechanical equilibrium, when there is no unbalanced forces acting on any part of the system or the system as a whole.

2. *Thermal equilibrium.* A system is said to be in thermal equilibrium, when there is no temperature difference between the parts of the system or between the system and the surroundings.

3. *Chemical equilibrium.* A system is said to be in chemical equilibrium, when there is no chemical reaction within the system and also there is no movement of any chemical constituent from one part of the system to the other.

1.31. Equality of Temperature

Consider two bodies of the same or different materials, one hot and the other cold. When these bodies are brought in contact, the hot body becomes colder, and the cold body becomes warmer. If these bodies remain in contact for some time, a state reaches when there is no further observable change in the properties of the two bodies. This is a state of thermal equilibrium, and at this stage the two bodies have the equal temperatures. It thus follows that when two bodies are in thermal equilibrium with each other, their temperatures are equal.

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* In S.I. units, degrees Kelvin is not written as °K but only K.

1.32. Pressure

The term 'pressure' may be defined as the normal force per unit area. The unit of pressure depends upon the units of force and area.

In S.I. system of units, the practical unit of pressure is N/mm^2, N/m^2, kN/m^2, MN/m^2 etc. But sometimes a bigger unit of pressure (known as bar) is used, such that,

$$1 \text{ bar} = 1 \times 10^5 \text{ N/m}^2 = 0.1 \times 10^6 \text{ N/m}^2 = 0.1 \text{ MN/m}^2$$

Sometimes the pressure is expressed in another unit, called Pa (named after Pascal) and kPa, such that

$$1 \text{ Pa} = 1 \text{ N/m}^2 \quad \text{and} \quad 1 \text{ kPa} = 1 \text{ kN/m}^2$$

1.33. Gauge Pressure and Absolute Pressure

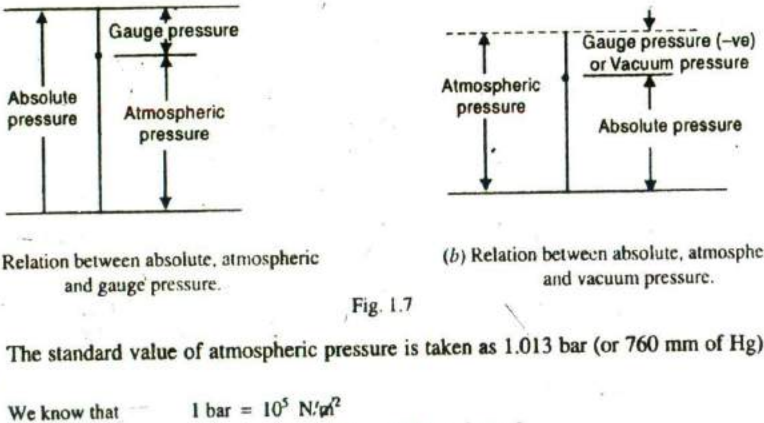
All the pressure gauges read the difference between the actual pressure in any system and the atmospheric pressure. The reading of the pressure gauge is known as *gauge pressure*, while the actual pressure is called *absolute pressure*. Mathematically,

Absolute pressure = Atmospheric pressure + Gauge pressure

This relation is used for pressures above atmospheric, as shown in Fig. 1.7 (*a*). For pressures below atmospheric, the gauge pressure will be negative. This negative gauge pressure is known as *vacuum pressure*. Therefore

Absolute pressure = Atmospheric pressure – Vacuum pressure

This relation is shown in Fig. 1.7 (*b*).



(*a*) Relation between absolute, atmospheric and gauge pressure.

(*b*) Relation between absolute, atmospheric and vacuum pressure.

Fig. 1.7

The standard value of atmospheric pressure is taken as 1.013 bar (or 760 mm of Hg) at sea level.

Note. We know that $1 \text{ bar} = 10^5 \text{ N/m}^2$

$\therefore$ Atmospheric pressure $= 1.013 \times 10^5 = 1013 \times 10^2 \text{ N/m}^2$

We also know that atmospheric pressure

$= 760$ mm of Hg

$\therefore$ $1 \text{ mm of Hg} = 1013 \times 10^2 / 760 = 133.3 \text{ N/m}^2$

or $1 \text{ N/m}^2 = 760 / 1013 \times 10^2 = 7.5 \times 10^{-3} \text{ mm of Hg}$

1.34. Normal Temperature and Pressure (N.T.P.)

The conditions of temperature and pressure at 0°C (273 K) temperature and 760 mm of Hg pressure are termed as *normal temperature* and *pressure* (briefly written as N.T.P.)

1.35. Standard Temperature and Pressure (S.T.P.)

The temperature and pressure of any gas, under standard atmospheric conditions, is taken as 15°C (288 K) and 760 mm of Hg respectively.

Example 1.2. *The pressure of steam inside a boiler, as measured by pressure gauge, is 1 N/mm². The barometric pressure of the atmosphere is 765 mm of mercury. Find the absolute pressure of steam in N/m², kPa, bar and N/mm².*

Solution. Given : Gauge pressure = $1\ \text{N/mm}^2 = 1 \times 10^6\ \text{N/m}^2$; Atmospheric pressure = 765 mm of Hg

We know that atmospheric pressure

$$= 765 \text{ mm of Hg}$$

$$= 765 \times 133.3 = 0.102 \times 10^6\ \text{N/m}^2 \qquad \ldots(\because 1 \text{ mm of Hg} = 133.3\ \text{N/m}^2)$$

∴ Absolute pressure of steam

$$= \text{Atmospheric pressure} + \text{Gauge pressure}$$

$$= 0.102 \times 10^6 + 1 \times 10^6 = 1.102 \times 10^6\ \text{N/m}^2 \ \textbf{Ans.}$$

$$= 1102 \text{ kPa } \textbf{Ans.} \qquad \ldots(\because 1 \text{ kPa} = 10^3\ \text{N/m}^2)$$

$$= 11.02 \text{ bar } \textbf{Ans.} \qquad \ldots(\because 1 \text{ bar} = 10^5\ \text{N/m}^2)$$

$$= 1.102\ \text{N/mm}^2 \ \textbf{Ans.} \qquad \ldots(\because 1\ \text{N/mm}^2 = 10^6\ \text{N/m}^2)$$

Example 1.3. *In a condenser of a steam power plant, the vacuum is recorded as 700 mm of mercury. If the barometer reading is 760 mm of mercury, find the absolute pressure in the condenser in N/m², kPa, bar and N/mm².*

Solution. Given : Vacuum pressure = 700 mm of Hg ; Barometer reading = 760 mm of Hg

We know that absolute pressure in the condenser

$$= \text{Atmospheric pressure} - \text{Vacuum pressure}$$

$$= \text{Barometric pressure} - \text{Vacuum pressure}$$

$$= 760 - 700 = 60 \text{ mm of Hg}$$

$$= 60 \times 133.3 = 7998\ \text{N/m}^2 \ \textbf{Ans.} \qquad \ldots(\because 1 \text{ mm of Hg} = 133.3\ \text{N/m}^2)$$

$$= 7.998 \text{ kPa } \textbf{Ans.} \qquad \ldots(\because 1 \text{ kPa} = 10^3\ \text{N/m}^2)$$

$$= 0.079\,98 \text{ bar } \textbf{Ans.} \qquad \ldots(\because 1 \text{ bar} = 10^5\ \text{N/m}^2)$$

$$= 0.007\,998\ \text{N/mm}^2 \ \textbf{Ans.} \qquad \ldots(\because 1\ \text{N/mm}^2 = 10^6\ \text{N/m}^2)$$

1.36. Energy

The energy is defined as the capacity to do work. In other words, a system is said to possess energy when it is capable of doing work. The energy possessed by a system is of the following two types :

1. Stored energy, and 2. Transit energy (or energy in transition)

The *stored energy* is the energy possessed by a system within its boundaries. The potential energy, kinetic energy and internal energy are the examples of stored energy.

The *transit energy* (or energy in transition) is the energy possessed by a system which is capable of crossing its boundaries. The heat, work and electrical energy are the examples of transit energy.

It may be noted that only the stored energy is a thermodynamic property whereas the transit energy is not a thermodynamic property as it depends upon the path.

1.37. Types of Stored Energy

We have discussed above that the potential energy, kinetic energy and internal energy are the different types of stored energy. These energies are discussed, in detail, as follows :

1. *Potential energy*. It is the energy possessed by a body or a system for doing work, by virtue of its position above the ground level. For example, a body raised to some height above the ground level possesses potential energy because it can do some work by falling on earth's surface.

Let W = Weight of the body,

m = Mass of the body,

z = Distance through which the body falls, and

g = Acceleration due to gravity = $9.81\ \text{m/s}^2$.

$\therefore$ Potential energy,

$$PE = Wz = mgz$$

It may be noted that

(*a*) When W is in newtons and z in metres, then potential energy will be in N-m.

(*b*) When m is in kg and z in metres, then the potential energy will also be in N-m, as discussed below :

We know that potential energy,

$$PE = mgz = \text{kg} \times \frac{\text{m}}{\text{s}^2} \times \text{m} = \text{N-m} \qquad \ldots \left(\because 1\ \text{N} = \frac{1\ \text{kg-m}}{\text{s}^2} \right)$$

2. *Kinetic energy*. It is the energy possessed by a body or a system, for doing work, by virtue of its mass and velocity of motion.

Let m = Mass of the body, and

V = Velocity of the body.

When m is in kg and V is in m/s, then kinetic energy will be in N-m, as discussed below :

We know that kinetic energy,

$$KE = \frac{1}{2}\, m V^2 = kg \times \frac{m^2}{s^2} = \frac{\text{kg-m}}{\text{s}^2} \times \text{m} = \text{N-m} \ \ldots \left(\because 1\text{N} = \frac{1\ \text{kg-m}}{\text{s}^2} \right)$$

3. *Internal energy*. It is the energy possessed by a body or a system due to its molecular arrangement and motion of the molecules. It is usually represented by U.

In the study of thermodynamics, we are mainly concerned with the change in internal energy (dU) which depends upon the change in temperature of the system.

Notes. 1. The total energy of the system (E) is equal to the sum of the above three types of energies. Mathematically

$$E = PE + KE + U = mgz + \tfrac{1}{2} \times m V^2 + U$$

Any other form of the energy such as chemical energy, electrical energy etc. is neglected.

For unit mass, the above expression is written as

$$e = pe + ke + u = gz + \frac{V^2}{2} + u$$

2. When the system is stationary and the effect of gravity is neglected, then $PE = 0$, and $KE = 0$. In such a case

$$E = U \quad \text{or} \quad e = u$$

1.38. Law of Conservation of Energy

It states. "*The energy can neither be created nor destroyed, though it can be transformed from one form to any other form, in which the energy can exist.*"

1.39. Heat

The heat is defined as the energy transferred, without transfer of mass, across the boundary of a system because of a temperature difference between the system and the surroundings. It is usually represented by Q and is expressed in joule (J) or kilo-joule (kJ).

The heat can be transferred in three distinct ways, *i.e.* conduction, convection and radiation. The transfer of heat through solids takes place by *conduction*, while the transfer of heat through fluids is by *convection*. The *radiation* is an electromagnetic wave phenomenon in which energy can be transported through transparent substances and even through a vacuum. These three modes of heat transfer are quite different, but they have one factor in common. All these modes occur across the surface area of a system because of a temperature difference between the system and the surroundings.

The following points are worth noting about heat :

1. The heat is transferred across a boundary from a system at a higher temperature to a system at a lower temperature by virtue of the temperature difference.
2. The heat is a form of transit energy which can be identified only when it crosses the boundary of a system. It exists only during transfer of energy into or out of a system.
3. The heat flowing *into* a system is considered as *positive* and heat flowing *out* of a system is considered *negative*.

1.40. Specific Heat

The specific heat of a substance may be broadly defined as the amount of heat required to raise the temperature of a unit mass of any substance through one degree. It is generally denoted by c. In S.I. system of units, the unit of specific heat (c) is taken as kJ/kg K. If m kg of a substance of specific heat c is required to raise the temperature from an initial temperature of T_1 to a final temperature of T_2, then

$$\text{Heat required} = m\,c\,(T_2 - T_1)\text{ kJ}$$

where T_1 and T_2 may be either in °C or in K.

Since the solids and liquids do not change the volume on heating, therefore they have only one specific heat. But the gases have the following two *specific heats depending upon the process adopted for heating the gas.

1. Specific heat at constant pressure (c_p), and
2. Specific heat at constant volume (c_v).

It may be noted that c_p is always greater than c_v. The average values of specific heats for some commonly used substances are given in the following table.

Table 1.4. Values of Specific heat for some commonly used substances.

Solids	*Specific heat (kJ/kg K)*	*Fluids*	*Specific heat (kJ/kg K)*	*Gases*	***Specific heat at constant pressure (kJ/kg K)***
Steel	0.490	Water	4.187	Air	**1.000**
Copper	0.406	Ice	2.110	Carbon dioxide	**0.846**
Zinc	0.389	Steam	2.094	Nitrogen	**1.043**
Mercury	0.138	Petrol	1.817	Oxygen	**0.913**
Coal	1.010	Alcohol	2.512	Carbon monoxide	**1.047**
Coke	0.837	Paraffin oil	2.140	Hydrogen	**14.257**

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* For further details, please refer Art. 2.11.

1.41. Thermal or Heat Capacity

The thermal or heat capacity of a substance may be defined as the heat required to raise the temperature of whole mass of a substance through one degree. Mathematically,

Thermal or heat capacity = $m\,c$ kJ

where m = Mass of the substance in kg, and

c = Specific heat of the substance in kJ/ kg K.

1.42. Water Equivalent

The water equivalent of a substance may be defined as the quantity of water, which requires the same quantity of heat as the substance to raise its temperature through one degree. Mathematically,

Water equivalent of a substance

$= m\,c$ kg

where m = Mass of the substance in kg, and

c = Specific heat of the substance in kJ/kg K.

Note. The numerical value of the thermal capacity and the water equivalent of the substance are the same but they are expressed in different units.

Example 1.4. *Calculate the quantity of heat required to raise the temperature of a steel forging of mass 180 kg from 300 K to 1265 K. The specific heat of steel = 0.49 kJ/kg K.*

Solution. Given : $m = 180$ kg ; $T_1 = 300$ K ; $T_2 = 1265$ K ; $c = 0.49$ kJ/kg K

We know that the quantity of heat required

$= \text{Mass} \times \text{Sp. heat} \times \text{Rise in temp.} = m\,c\,(T_2 - T_1)$

$= 180 \times 0.49\,(1265 - 300) = 85\,113$ kJ **Ans.**

1.43. Mechanical Equivalent of Heat

It was established by Joule that heat and mechanical energies are mutually convertible. He established, experimentally, that there is a numerical relation between the unit of heat and unit of work. This relation is denoted by J (named after Joule) and is known as Joule's equivalent or mechanical equivalent of heat.

Note. In S.I. system of units, the unit of work done is joule or kilo joule (such that 1 J = 1 N-m or 1 kJ = 1 kN-m). The unit of heat is also joule or kilo joule. So we can straightway convert heat units into mechanical units and vice versa.

1.44. Work

In mechanics, work is defined as the product of the force (F) and the distance moved (x) in the direction of the force. Mathematically, work done,

$$W = F \times x$$

The unit of work depends upon the unit of force and the distance moved. In S.I. system of units, the practical unit of work is newton-metre (briefly written as N-m). The work of 1 N-m is known as joule (briefly written as J) such that 1 N-m = 1 J.

In thermodynamics, work may be defined as follows :

1. According to Obert, work is defined as *the energy transferred (without the transfer of mass) across the boundary of a system because of an intensive property difference other than temperature that exists between the system and surroundings.*

In engineering practice, the intensive property difference is the pressure difference. The pressure difference (between the system and the surrounding) at the surface of the system gives rise to a force and the action of this force over a distance is called mechanical work.

In some cases, the intensive property difference may be the electrical potential difference between the system and the surrounding. In this case, the resulting energy transfer across the system and boundary is known as electrical work.

2. According to Keenan, *work is said to be done by a system during a given operation if the sole effect of the system on things external to the system (surroundings) can be reduced to the raising of a weight.*

The weight may not be actually raised but the net effect external to the system should be the raising of a weight.

For example, consider a system consisting of a storage battery, as shown in Fig. 1.8. The terminals connected to a resistance through a switch constitute external to the system (*i.e.* surroundings). When the switch is closed for a certain period of time, then the current will flow through the battery and the resistance, as a result the resistance becomes warmer. This clearly shows that the system (battery) has interaction with the surroundings. In other words, the energy transfer (electrical energy) has taken place between the system and the surroundings because of potential difference (not the temperature).

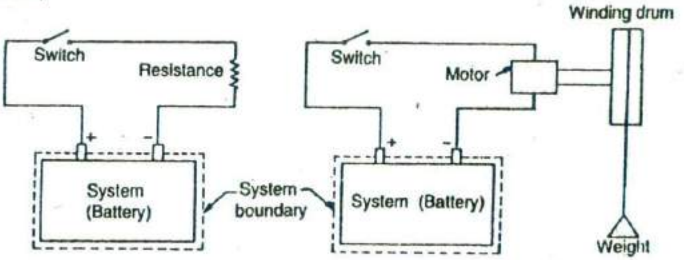


Fig. 1.8. Thermodynamic work.

Now according to the mechanics definition of work, there is no force which moves through a distance. Thus no work is done by the system. However, according to the thermodynamic definition, the work is done by the system because the resistance can be replaced by an ideal motor (100% efficient) driving a winding drum, thereby raising a weight. Thus, the sole effect external to the system (surroundings) has been reduced to the raising of a weight. Hence, thermodynamic work is done by the system.

Note. The work done *by* the system is considered as *positive* work, while the work done *on* the system is considered as *negative* work.

1.45. Heat and Work—A Path Function

Consider that a system from an initial equilibrium state 1 reaches to a final equilibrium state 2 by two different paths 1-A-2 and 1-B-2, as shown in Fig. 1.9. The processes are quasi-static.

When the system changes from its initial state 1 to final state 2, the quantity of heat transfer will depend upon the intermediate stages through which the system passes, *i.e.* its path. In other words, heat is a path function. Thus, heat is an inexact differential and is written as δQ. On integrating, for the path 1-A-2,

$$\int_1^2 \delta Q = [Q]_1^2 = Q_{1-2} \quad \text{or} \quad {}_1Q_2$$

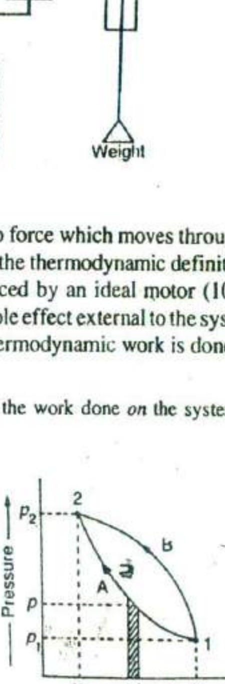


Fig. 1.9. Heat and work-a path function.

It may be noted that $\int_1^2 \delta Q \neq Q_2 - Q_1$, because heat is not a point function. Thus, it is meaningless to say 'heat in a system or heat of a system'. The heat can not be interpreted similar to temperature and pressure.*

The work, like heat, is not a thermodynamic property, therefore it is a path function as its value depends upon the particular path followed during the process. Since the areas under the curves 1-*A*-2 and 1-*B*-2 are different, therefore work done by these two processes will also be different**. Hence, work is an inexact differential and is written as δW. On integration, for the path 1-*A*-2,

$$\int_1^2 \delta W = [W]_1^2 = W_{1-2} \quad \text{or} \quad {}_1W_2$$

As discussed above, $\int_1^2 \delta W \neq W_2 - W_1$, because work is not a point function. Thus, it is meaningless to say 'work in a system or work of a system'. Since the work can not be interpreted similar to temperature and pressure of the system, therefore it is a path function and it depends upon the process. It is not a point function as the temperature and pressure. The work done in taking the system from state 1 to state 2 will be different for different paths.

1.46. Comparison of Heat and Work

There are many similarities between heat and work. These are

1. The heat and work are both transient phenomena. The systems do not possess heat or work. When a system undergoes a change, heat transfer or work done may occur.

2. The heat and work are boundary phenomena. They are observed at the boundary of the system.

3. The heat and work represent the energy crossing the boundary of the system.

4. The heat and work are path functions and hence they are inexact differentials. They are written as δQ and δW.

1.47. Power

It may be defined as the rate of doing work or work done per unit time. Mathematically,

$$\text{Power} = \frac{\text{Work done}}{\text{Time taken}}$$

In S.I. system of units, the unit of power is watt (briefly written as W) which is equal to 1 J/s or 1 N-m/s. Generally, a bigger unit of power called kilowatt (briefly written as kW) is used which is equal to 1000 W.

Notes. 1. If T is the torque transmitted in N-m or J and ω is the angular speed in rad/s, then

$$\text{Power,} \quad P = T\omega = T \times 2\pi N/60 \text{ watts} \qquad \ldots (\because \omega = 2\pi N/60)$$

where N is the speed in r.p.m.

2. The ratio of power output to power input is known as efficiency. It is denoted by a Greek letter eta (η). It is always less than unity and is represented as percentage. Mathematically,

$$\text{Efficiency}, \eta = \frac{\text{Power output}}{\text{Power input}}$$

---

* Heat is not a thermodynamic property whereas the temperature and pressure are thermodynamic properties.

** The area under the pressure - volume (p-v) diagram represents the work done during the process and is given by $p\,dv$.

1.48. Laws of Thermodynamics

The following three laws of thermodynamics are important from the subject point of view :

1. Zeroth law of thermodynamics, 2. First law of thermodynamics, and 3. Second law of thermodynamics.

These laws are discussed, in detail, as follows :

1.49. Zeroth Law of Thermodynamics

This law states, "*When two systems are each in thermal equilibrium with a third system, then the two systems are also in thermal equilibrium with one another.*"

This law provides the basis of temperature measurement.

1.50. First Law of Thermodynamics

This law may be stated as follows :

(*a*) "*The heat and mechanical work are mutually convertible*". According to this law, when a closed system undergoes a thermodynamic cycle, the net heat transfer is equal to the net work transfer. In other words, the cyclic integral of heat transfers is equal to the cyclic integral of work transfers. Mathematically,

$$\oint \delta Q = \oint \delta W$$

where symbol $\oint$ stands for cyclic integral (integral around a complete cycle), and δQ and δW represent infinitesimal elements of heat and work transfers respectively. It may be noted that δQ and δW are expressed in same units.

(*b*) *The energy can neither be created nor destroyed though it can be transformed from one form to another.* According to this law, when a system undergoes a change of state (or a thermodynamic process), then both heat transfer and work transfer takes place. The net energy transfer is stored within the system and is known as stored energy or total energy of the system. Mathematically

$$\delta Q - \delta W = dE$$

The symbol δ is used for a quantity which is inexact differential and symbol d is used for a quantity which is an exact differential. The quantity E is an extensive property and represents the total energy of the system at a particular state.

On integrating the above expression for a change of state from 1 to 2, we have

$$Q_{1-2} - W_{1-2} = E_2 - E_1 \qquad \ldots (Q,\ W \text{ and } E \text{ are in same units})$$

For a unit mass, this expression is written as

$$q_{1-2} - w_{1-2} = e_2 - e_1$$

where Q_{1-2} = Heat transferred to the system during the process from state 1 to state 2,

W_{1-2} = Workdone by the system on the surroundings during the process, and

E_1 = Total energy of the system at state 1

$$= PE_1 + KE_1 + U_1 = {}^{*}m\,g\,z_1 + \frac{m\,V_1^2}{2} + U_1$$

E_2 = Total energy of the system at state 2

$$= PE_2 + KE_2 + U_2 = m\,g\,z_2 + \frac{m\,V_2^2}{2} + U_2$$

---

* Refer Art. 1.37

Thus the above expression may be written as

$$Q_{1-2} - W_{1-2} = E_2 - E_1 \quad \ldots (i)$$

$$= (PE_2 + KE_2 + U_2) - (PE_1 + KE_1 + U_1)$$

$$= (PE_2 - PE_1) + (KE_2 - KE_1) + (U_2 - U_1) \quad \ldots (ii)$$

$$= m\,(g\,z_2 - g\,z_1) + m\left(\frac{V_2^2}{2} - \frac{V_1^2}{2}\right) + (U_2 - U_1)$$

For unit mass, this expression is written as

$$q_{1-2} - w_{1-2} = (g\,z_2 - g\,z_1) + \left(\frac{V_2^2}{2} - \frac{V_1^2}{2}\right) + (u_2 - u_1)$$

Notes. 1. When there is no change in potential energy of the system (*i.e.* when the height of the system from the datum level is same), then $PE_1 = PE_2$. Thus, the above equation (*ii*) is written as

$$Q_{1-2} - W_{1-2} = (KE_2 - KE_1) + (U_2 - U_1) \quad \ldots (iii)$$

2. When there is no change of *PE* and also there is no flow of the mass into or out of the system, then $PE_1 = PE_2$ and $KE_1 = KE_2$. Thus, the above equation (*ii*) is written as

$$Q_{1-2} - W_{1-2} = U_2 - U_1 = dU \quad \ldots (iv)$$

In other words, in a closed or non-flow thermodynamic system,

$$PE = 0 \quad \text{and} \quad KE = 0$$

Thus the equation (*iv*) is known as *Non-flow energy equation.*

3. For an isolated system for which $Q_{1-2} = W_{1-2} = 0$, the above equation (*i*) becomes

$$E_2 = E_1$$

This shows that the first law of thermodynamics is the law of conservation of energy.

1.51. Limitations of First Law of Thermodynamics

We have already discussed that according to first law of thermodynamics that

1. *When a closed system undergoes a thermodynamic cycle, the net heat transfer is equal to the net work transfer.* This statement does not specify the direction of flow of heat and work (*i.e.* whether the heat flows from a hot body to a cold body or from a cold body to a hot body). It also does not give any condition under which these transfers take place.

2. *The heat energy and mechanical work are mutually convertible.* Though the mechanical work can be fully converted into heat energy, but only a part of heat energy can be converted into mechanical work. This means that the heat energy and mechanical work are not fully mutually convertible. In other words, there is a limitation on the conversion of one form of energy into another form.

A machine which violates the first law of thermodynamics (*i.e.* energy can neither be created nor destroyed, but can be transformed from one form to another) is known as *perpetual motion machine of the first kind* (briefly written as PMM-I). It is defined as a machine which produces work energy without consuming an equivalent of energy from other source. Such a machine, as shown in Fig. 1.10, is impossible to obtain in actual practice, because no machine can produce energy of its own without consuming any other form of energy.

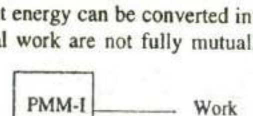


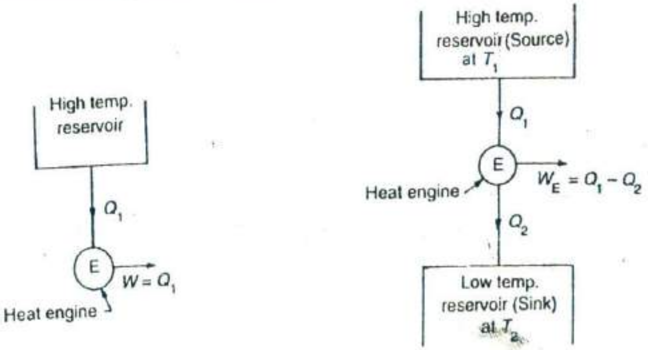
Fig. 1.10. Perpetual motion machine of the first kind.

1.52. Second Law of Thermodynamics

The second law of thermodynamics may be defined in many ways, but the two common statements according to Kelvin - Planck and Clausius are as follows :

1. *Kelvin - Planck Statement.* According to Kelvin-Planck '*It is impossible to construct an engine working on a cyclic process, whose sole purpose is to convert heat energy from a single thermal reservoir* into an equivalent amount of work*'. In other words, no actual heat engine, working on a cyclic process, can convert whole of the heat supplied to it, into mechanical work. It means that there is a degradation of energy in the process of producing mechanical work from the heat supplied. Thus the Kelvin - Planck statement of the second law of thermodynamics, is sometimes known as *law of degradation of energy.*

A heat engine which violates this statement of the second law of thermodynamics (*i.e.* a heat engine which converts whole of the heat energy into mechanical work) is known as ***perpetual motion machine of the second kind* (briefly written as PMM-II) or 100 percent efficient machine which is impossible to obtain in actual practice, because no machine can convert whole of the heat energy supplied to it, into its equivalent amount of work.



(*a*) Perpetual motion machine of the second kind (impossible).

(*b*) Heat engine.

Fig. 1.11

Thus for the satisfactory operation of a heat engine which is a device used for converting heat energy into mechanical work, there should be at-least two reservoirs of heat, one at a ***higher temperature and the other at a lower temperature, as shown in Fig. 1.11 (*b*). In this case, consider that heat energy (Q_1) from the high temperature reservoir (or source) at temperature T_1 is supplied to the engine. A part of this heat energy is rejected to the low temperature reservoir (or sink) at temperature T_2. If Q_2 is the heat rejected to the sink, then the remaining heat (*i.e.* $Q_1 - Q_2$) is converted into mechanical work. The ratio of the maximum mechanical work obtained to the total heat supplied to the engine is known as maximum thermal efficiency (η_{max}) of the engine. Mathematically,

$$\eta_{max} = \frac{\text{Maximum work obtained}}{\text{Total heat supplied}} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

Note. For a reversible engine, $Q_1 / T_1 = Q_2 / T_2$.

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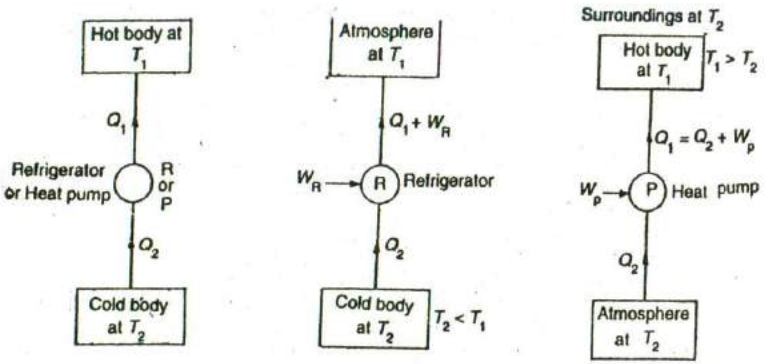
* A thermal reservoir is a body of infinite heat capacity which is capable of absorbing or rejecting an unlimited quantity of heat without affecting its temperature.

** A perpetual motion machine of the second kind (PMM-II) does not violate the first law of thermodynamics as such a machine would not create or destroy energy.

*** In a heat engine, the reservoir (or body) at a higher temperature is known as a *source* and the reservoir at a lower temperature is called a *sink*.

2. *Clausius Statement.* According to Clausius statement "*It is impossible for a self acting machine, working in a cyclic process, to transfer heat from a body at a lower temperature to a body at a higher temperature without the aid of an external agency.*" In other words, heat cannot flow itself from a cold body to a hot body without the help of an external agency (*i.e.* without the expenditure of mechanical work).

The device (such as a refrigerator or a heat pump). as shown in Fig. 1.12 (*a*), violates the Clausius statement because no input work is supplied to the device to transfer heat from a cold body to a hot body. Such a device is called perpetual motion machine of the second kind.



(*a*) Perpetual motion machine of the second kind. (*b*) Refrigerator. (*c*) Heat pump.

Fig. 1.12

In order to achieve the object of transferring heat from a cold body to a hot body, the refrigerator and a heat pump, while operating in a cyclic process, require an input work, as shown in Fig. 1.12 (*b*) and (*c*) respectively. Though there is no difference between the cycle of operations of the refrigerator and a heat pump and achieve the same overall objective, but the basic purpose of each is quite different. A refrigerator is a device which operating in a cyclic process, maintains the temperature of a cold body (refrigerated space) at a temperature lower than the temperature of the surroundings. On the other hand, a heat pump is a device which operating in a cyclic process, maintains the temperature of a hot body (heated space) at a temperature higher than the temperature of surroundings. In other words, a refrigerator works between the cold body temperature and the *atmospheric temperature whereas a heat pump operates between the hot body temperature and the atmospheric temperature.

The **performance of refrigerator and heat pump is measured in terms of coefficient of performance which is defined as the ratio of the maximum heat transferred (*i.e.* heat taken from the cold body) to the amount of work required to produce the desired effect. Mathematically, maximum coefficient of performance for a refrigerator,

$$(\text{C.O.P})_R = \frac{Q_2}{W_R} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}$$

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* In case of a refrigerator, the atmosphere acts as a hot body while in case of a heat pump, the atmosphere acts as a cold body.

* The performance of a heat engine is measured in terms of thermal efficiency.

and maximum coefficient of performance for a heat pump,

$$(\text{C.O.P})_P = \frac{Q_1}{W_P} = \frac{Q_1}{Q_1 - Q_2} = \frac{T_1}{T_1 - T_2} = \frac{T_2}{T_1 - T_2} + 1$$

$$= (\text{C.O.P})_R + 1$$

We see that C.O.P of a heat pump is greater than C.O.P of a refrigerator by unity.

1.53. Equivalence of Kelvin-Planck and Clausius Statements

Though Kelvin-Planck and Clausius statements of the second law of thermodynamics appear to be different, from each other, but these two statements are virtually equivalent in all respects. The equivalence of the Kelvin-Planck and Clausius statements can be proved if it can be shown that the violation of Kelvin-Planck statement implies the violation of Clausius statement and vice versa. This is discussed as follows :

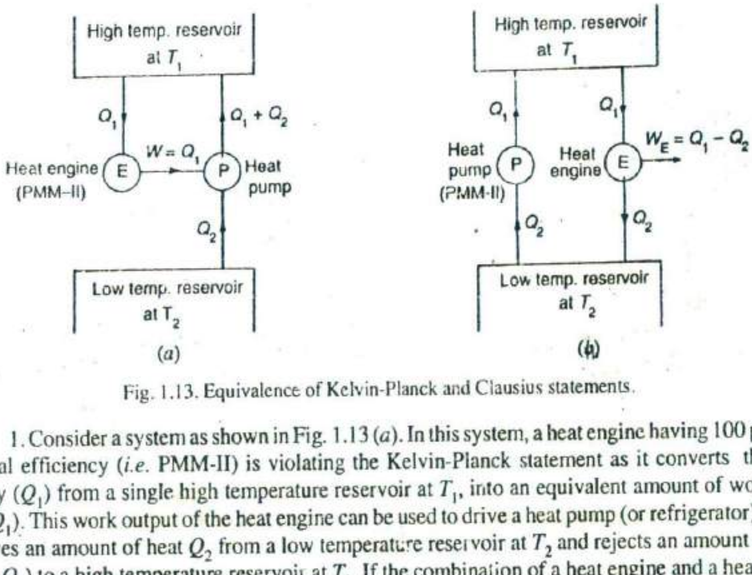

Fig. 1.13. Equivalence of Kelvin-Planck and Clausius statements.

1. Consider a system as shown in Fig. 1.13 (*a*). In this system, a heat engine having 100 percent thermal efficiency (*i.e.* PMM-II) is violating the Kelvin-Planck statement as it converts the heat energy (Q_1) from a single high temperature reservoir at T_1, into an equivalent amount of work (*i.e.* $W = Q_1$). This work output of the heat engine can be used to drive a heat pump (or refrigerator) which receives an amount of heat Q_2 from a low temperature reservoir at T_2 and rejects an amount of heat ($Q_1 + Q_2$) to a high temperature reservoir at T_1. If the combination of a heat engine and a heat pump (or refrigerator) is considered as a single system, as shown in Fig. 1.13 (*a*), then the result is a device that operates in a cycle and has no effect on the surroundings other than the transfer of heat Q_2 from a low temperature reservoir to a high temperature reservoir, thus violating the Clausius statement. Hence, a violation of Kelvin-Planck statement leads to a violation of Clausius statement.

2. Consider a system as shown in Fig. 1.13 (*b*). In this system, a heat pump or refrigerator (*i.e.* PMM-II) is violating the Clausius statement as it transfers heat from a low temperature reservoir at T_2 to a high temperature reservoir at T_1 without any expenditure of work. Now let a heat engine, operating between the same heat reservoirs, receives an amount of heat Q_1 (as discharged by the heat pump) from the high temperature reservoir at T_1, does work ($W_E = Q_1 - Q_2$) and rejects an amount of heat Q_2 to the low temperature reservoir at T_2. If the combination of the heat pump (or refrigerator) and the heat engine is considered as a single system, as shown in Fig. 1.13 (*b*), then the result is a device that operates in a cycle whose sole effect is to remove heat at the rate of ($Q_1 - Q_2$) and convert it completely into an equivalent amount of work, thus violating the Kelvin-Planck statement. Hence, a violation of Clausius statement leads to a violation of Kelvin-Planck statement.

From above, we see that the Kelvin-Planck and Clausius statements of the second law of thermodynamics are complimentary to each other. The truth of the first statement implies the truth of the second statement and vice versa.

Example 1.5. *An engine works between the temperature limits of 1775 K and 375 K. What can be the maximum thermal efficiency of this engine ?*

Solution. Given : $T_1 = 1775\text{ K}$; $T_2 = 375\text{ K}$

We know that maximum thermal efficiency of the engine,

$$\eta_{max} = \frac{T_1 - T_2}{T_1} = \frac{1775 - 375}{1775} = 0.7887 \text{ or } 78.87\% \textbf{ Ans.}$$

Example 1.6. *A reversible engine is supplied with heat from two constant temperature sources at 900 K and 600 K and rejects heat to a constant temperature sink at 300 K. The engine develops work equivalent to 90 kJ/s and rejects heat at the rate of 56 kJ/s. Estimate : 1. Heat supplied by each source, and 2. Thermal efficiency of the engine.*

Solution. Given : $T_1 = 900\text{ K}$; $T_3 = 600\text{ K}$; $T_2 = T_4 = 300\text{ K}$; $W_E = 90\text{ kJ/s}$; $Q_2 + Q_4 = 56\text{ kJ/s}$

1. *Heat supplied by each source*

Let Q_1 = Heat supplied by the first source, and

Q_3 = Heat supplied by the second source.

We know that efficiency of the engine when the heat is supplied from the first source,

$$\eta_1 = \frac{\text{Work obtained}}{\text{Heat supplied}} = \frac{W_1}{Q_1}$$

$$= \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1}$$

$$\ldots\left(\because \text{ For a reversible engine, } \frac{Q_1}{T_1} = \frac{Q_2}{T_2}\right)$$

$$= \frac{900 - 300}{900} = 0.67$$

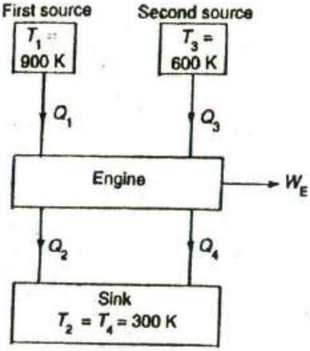

Fig. 1.14

$\therefore$ Work obtained by the engine from the first source,

$$W_1 = Q_1 - Q_2 = 0.67\, Q_1$$

and heat rejected to the sink,

$$Q_2 = Q_1 - W_1 = Q_1 - 0.67\, Q_1 = 0.33\, Q_1$$

Similarly, efficiency of the engine when the heat is supplied from the second source

$$\eta_2 = \frac{W_2}{Q_3} = \frac{Q_3 - Q_4}{Q_3} = \frac{T_3 - T_4}{T_3} = \frac{600 - 300}{600} = 0.5$$

$\therefore$ Work obtained by the engine from the second source,

$$W_2 = Q_3 - Q_4 = 0.5\, Q_3$$

and heat rejected to the sink,

$$Q_4 = Q_3 - W_2 = Q_3 - 0.5\,Q_3 = 0.5\,Q_3$$

We know that total work obtained from the engine (W_E),

$$90 = W_1 + W_2 = 0.67\,Q_1 + 0.5\,Q_3 \qquad \ldots (i)$$

and total heat rejected to the sink,

$$56 = Q_2 + Q_4 = 0.33\,Q_1 + 0.5\,Q_3 \qquad \ldots (ii)$$

From equations (*i*) and (*ii*),

$$Q_1 = 100 \text{ kJ/s} \text{ and } Q_3 = 46 \text{ kJ/s} \quad \textbf{Ans.}$$

2. *Thermal efficiency of the engine*

We know that maximum thermal efficiency of the engine,

$$\eta_{\max} = \frac{\text{Work obtained}}{\text{Heat supplied}} = \frac{W_E}{Q_1 + Q_3}$$

$$= \frac{90}{100 + 46} = 0.616 \text{ or } 61.6\ \% \quad \textbf{Ans.}$$

Example 1.7. *A cold storage is to be maintained at – 5°C while the surroundings are at 35°C. The heat leakage from the surroundings into the cold storage is estimated to be 29 kW. The actual C.O.P of the refrigeration plant is one -third of an ideal plant working between the same temperatures. Find the power required to drive the plant.*

Solution. Given : $T_2 = -5°\text{C} = -5 + 273 = 268$ K ; $T_1 = 35°\text{C} = 35 + 273 = 308$ K

$Q_2 = 29$ kW ; $(\text{C.O.P})_{actual} = \frac{1}{3}\,(\text{C.O.P})_{ideal}$

The refrigerating plant operating between the temperatures T_1 and T_2 is shown in Fig. 1.15.

Let W_R = Work or power required to drive the plant.

We know that the coefficient of performance of an ideal refrigeration plant,

$$(\text{C.O.P})_{ideal} = \frac{T_2}{T_1 - T_2}$$

$$= \frac{268}{308 - 268} = 6.7$$

Fig. 1.15

∴ Actual coefficient of performance,

$$(\text{C.O.P})_{actual} = \frac{1}{3} \times (\text{C.O.P})_{ideal} = \frac{1}{3} \times 6.7 = 2.233$$

We also know that C.O.P.= $\dfrac{Q_2}{W_R}$

$$\therefore \qquad W_R = \frac{Q_2}{(\text{C.O.P})_{actual}} = \frac{29}{2.233} = 12.987 \text{ kW} \quad \textbf{Ans.}$$

Example 1.8. *A reversible heat engine operates between two reservoirs at temperatures of 600°C and 40°C. The engine drives a reversible refrigerator which operates between reservoirs at temperatures of 40°C and – 20°C. The heat transfer to the engine is 2 MJ and the net work output of the combined engine and refrigerator plant is 360 kJ. Find the heat transfer to the refrigerant and*

the net heat transfer to the reservoir at 40°C. Also find these values if the efficiency of the heat engine and C.O.P of the refrigerator are each 40% of their maximum possible values.

Solution. Given : $T_1 = 600°\text{C} = 600 + 273 = 873$ K ; $T_2 = T_4 = 40°\text{C} = 40 + 273 = 313$ K ; $T_3 = -20°\text{C} = -20 + 273 = 253$ K ; $Q_1 = 2$ MJ $= 2000$ kJ ; $W = 360$ kJ

The combined heat engine and refrigerator system is shown in Fig. 1.16.

Heat transfer to the refrigerant

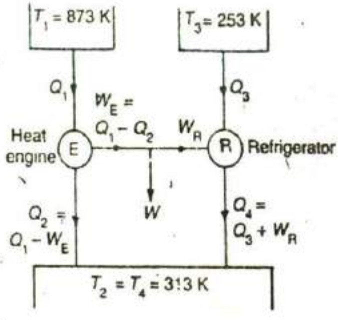

Fig. 1.16

Let Q_3 = Heat transfer to the refrigerant.

We know that maximum efficiency of the heat engine,

$$\eta_{max} = 1 - \frac{T_2}{T_1} = 1 - \frac{313}{873} = 0.6415$$

We also know that

$$\eta_{max} = \frac{\text{Heat supplied} - \text{Heat rejected}}{\text{Heat supplied}}$$

$$= \frac{\text{Work done}}{\text{Heat supplied}} = \frac{Q_1 - Q_2}{Q_1}$$

∴ Work done by the heat engine,

$$W_E = Q_1 - Q_2 = \eta_{max} \times Q_1 = 0.6415 \times 2000 = 1283 \text{ kJ}$$

Since the net work output of the combined heat engine and refrigerator plant i $W = W_E - W_R = 360$ kJ, therefore work required for the refrigerator,

$$W_R = W_E - W = 1283 - 360 = 923 \text{ kJ}$$

We know that maximum C.O.P of the refrigerator,

$$(\text{C.O.P})_{max} = \frac{T_3}{T_4 - T_3} = \frac{253}{313 - 253} = 4.217$$

We also know that maximum C.O.P. of the refrigerator,

$$(\text{C.O.P})_{max} = \frac{Q_3}{Q_4 - Q_3} = \frac{Q_3}{W_R}$$

$$\therefore \quad Q_3 = (\text{C.O.P.})_{max} \times W_R = 4.217 \times 923 = 3892.3 \text{ kJ} \quad \textbf{Ans.}$$

Net heat transfer to the reservoir at 40°C

We know that $Q_4 = Q_3 + W_R = 3892.3 + 923 = 4815.3$ kJ

and $Q_2 = Q_1 - W_E = 2000 - 1283 = 717$ kJ

∴ Net heat transfer (*i.e.*, heat rejected) to the reservoir at 40°C

$$= Q_2 + Q_4 = 717 + 4815.3 = 5532.3 \text{ kJ} \quad \textbf{Ans.}$$

When efficiency of the heat engine and C.O.P. of the refrigerator are each 40% of their maximum possible values

We know that the efficiency of the actual heat engine cycle,

$$\eta_{actual} = 40\% \, \eta_{max} = 0.4 \, \eta_{max} = 0.4 \times 0.6415 = 0.2566$$

$$\therefore \quad W_E = \eta_{actual} \times Q_1 = 0.2566 \times 2000 = 513.2 \text{ kJ}$$

$$W_R = W_E - W = 513.2 - 360 = 153.2 \text{ kJ}$$

We know that C.O.P. of the actual refrigerator cycle,

$$(\text{C.O.P})_{actual} = 40\% \, (\text{C.O.P})_{max} = 0.4 \, (\text{C.O.P})_{max}$$

$$= 0.4 \times 4.217 = 1.6868$$

$\therefore$ Heat transfer to the refrigerant,

$$Q_3 = (\text{C.O.P})_{actual} \times W_R$$

$$= 1.6868 \times 153.2 = 258.4 \text{ kJ} \quad \textbf{Ans.}$$

We know that $$Q_4 = Q_3 + W_R = 258.4 + 153.2 = 411.6 \text{ kJ}$$

and $$Q_2 = Q_1 - W_E = 2000 - 513.2 = 1486.8 \text{ kJ}$$

$\therefore$ Net heat transfer (*i.e.*, heat rejected) to the reservoir at 40°C

$$= Q_2 + Q_4 = 1486.8 + 411.6 = 1898.4 \text{ kJ} \quad \textbf{Ans.}$$

EXERCISES

1. The pressure of steam inside a boiler is recorded by a pressure gauge which shows 1.2 N/mm^2. If the barometer reads the atmospheric pressure as 770 mm of mercury, find the absolute pressure of steam inside the boiler in N/m^2, kPa and bar. [**Ans.** 1.3026×10^6 N/m^2 ; 1302.6 kPa ; 13.026 bar]

2. In a condenser, the vacuum is found to be 145 mm of mercury and the barometer reads 735 mm of mercury. Find the absolute pressure in a condenser in N/m^2 ; kPa and N/mm^2. [**Ans.** 78 647 N/m^2 ; 78.647 kPa ; 0.078 647 N/mm^2]

3. A copper vessel of mass 1.35 kg contains 6.75 kg of water at a temperature of 25°C. Find the heat required to warm the vessel and water to 90°C. Take specific heat of copper = 0.406 kJ/kg K and specific heat of water = 4.187 kJ/kg K. [**Ans.** 1872.6 kJ]

4. The net work output of a cyclic process is 45 kN-m. If the heat input is 125 kJ, determine the efficiency of the cycle. [**Ans.** 36%]

5. One kg of air at a temperature of 20°C is heated to a temperature of 60°C. Find the heat supplied to air when heated at constant pressure. The specific heat for air at constant pressure = 1 kJ/kg K. [**Ans.** 40 kJ]

6. A system receives 10×10^6 J in the form of heat energy in a specified process and it produces work of 4×10^6 J. The system velocity changes from 10 m/s to 25 m/s. For 50 kg mass of the system, determine the change in internal energy of the system. [**Ans.** 119.7375 kJ/kg]

[**Hint:** $$q_{1-2} = \frac{10 \times 10^6}{50} \text{ J/kg}; \quad w_{1-2} = \frac{4 \times 10^6}{50} \text{ J/kg}$$

$$ke_1 = \frac{1}{2} \times m \, (V_1)^2 = \frac{1}{2} \times 1 \, (10)^2 = 50 \text{ J/kg}$$

$$ke_2 = \frac{1}{2} \times m \, (V_2)^2 = \frac{1}{2} \times 1 \, (25)^2 = 312.5 \text{ J/kg}$$

We know that

$$q_{1-2} - w_{1-2} = (pe_2 - pe_1) + (ke_2 - ke_1) + (u_2 - u_1)$$

$$\frac{10 \times 10^6}{50} - \frac{4 \times 10^6}{50} = 0 + (312.5 - 50) + (u_2 - u_1)$$

. . . (Taking same datum level, $pe_2 = pe_1$)

$$\therefore \quad u_2 - u_1 = 119\,737.5 \text{ J/kg} = 119.7375 \text{ kJ/kg}$$

7. A reversible engine receives heat from a reservoir at 700°C and rejects heat at temperature T_2. A second reversible engine receives the heat rejected by the first engine to a sink at a temperature 37°C. Calculate

the temperature T_2 for 1. equal efficiency of both the engines, and 2. equal output of both the engines. [Ans. 276.2°C ; 368.5°C]

8. A domestic food freezer is to be maintained at temperature of − 15°C. The ambient air temperature is 30°C. If the heat leaks into the freezer at the continuous rate of 1.75 kJ/s, find the power required to pump this heat out continuously. [Ans. 0.305 kJ/s]

9. A heat pump is used for heating the interior of a house in a cold climate. The ambient temperature is − 5°C and the desired interior temperature is 25°C. The compressor of the heat pump is to be driven by a heat engine working between 1000°C and 25°C. Treating both the cycles as reversible, calculate the ratio in which the heat pump and the heat engine share the heating load. [Ans. 7.606]

10. A heat engine is used to drive a heat pump. The heat transfer from the heat engine and from the heat pump are used to heat the water circulating through the radiators of a building. The efficiency of the heat engine is 27% and C.O.P of the heat pump is 4. Show that the ratio of the heat transfer to the circulating water to the heat transfer to the engine is 1.81.

QUESTIONS

1. Define a thermodynamic system. Explain its different types.

2. What do you understand by property of a system ? Distinguish between extensive and intensive properties of a system.

3. Define the following properties :

(*a*) Specific weight (*b*) Pressure (*c*) Volume

(*d*) Temperature (*e*) Specific volume (*f*) Density

4. What is a thermodynamic process and a cyclic process ?

5. Explain the non-equilibrium and quasi-static process. Is the quasi-static process a reversible process ?

6. Define temperature. Name the different temperature scales in common use. Establish relation between Celsius and Fahrenheit scales ?

7. What is absolute temperature ? How it is obtained for Celsius and Fahrenheit scales ?

8. Distinguish between gauge pressure and absolute pressure. How the gauge pressure is converted into absolute pressure ?

9. What do you understand by N.T.P. and S.T.P. ? What are their values ?

10. Define energy. What is stored energy and transit energy ? Discuss the types of stored energy.

11. How heat and work is defined ? Are these quantities a path function or point function ?

12. Explain the three laws of thermodynamics.

OBJECTIVE TYPE QUESTIONS

1. A definite area or a space where some thermodynamic process takes place, is known as

(*a*) thermodynamic cycle (*b*) thermodynamic process

(*c*) thermodynamic system (*d*) thermodynamic law

2. When neither mass nor energy is allowed to cross the boundary of a system, it is then called

(*a*) closed system (*b*) open system

(*c*) isolated system (*d*) none of these

3. Which of the following is the extensive property of a thermodynamic system ?

(*a*) pressure (*b*) volume

(*c*) temperature (*d*) density

4. Which of the following is an intensive property of a thermodynamic system ?

(*a*) volume
(*b*) temperature
(*c*) mass
(*d*) energy

5. Which of the following is not a thermodynamic property ?

(*a*) pressure
(*b*) temperature
(*c*) heat
(*d*) specific volume

6. When a process or processes are performed on a system in such a way that the final state is identical with the initial state, it is then known as

(*a*) thermodynamic cycle
(*b*) thermodynamic property
(*c*) thermodynamic process
(*d*) zeroth law of thermodynamics

7. Atmospheric pressure is equal to

(*a*) 1.013 bar
(*b*) 101.3 kN/m^2
(*c*) 760 mm of Hg
(*d*) all of these

8. First law of thermodynamics deals with

(*a*) conservation of heat
(*b*) conservation of momentum
(*c*) conservation of mass
(*d*) conservation of energy

9. Second law of thermodynamics defines

(*a*) heat
(*b*) work
(*c*) entropy
(*d*) internal energy

10. Kelvin-Planck's law deals with

(*a*) conservation of work
(*b*) conservation of heat
(*c*) conservation of mass
(*d*) conversion of heat into work

ANSWERS

1. (*c*) 2. (*c*) 3. (*b*) 4. (*b*) 5. (*c*)
6. (*a*) 7. (*d*) 8. (*d*) 9. (*c*) 10. (*d*)

2

Properties of Perfect Gases

1. Introduction. 2. Laws of Perfect Gases. 3. Boyle's Law. 4. Charles' Law. 5. Gay-Lussac Law. 6. General Gas Equation. 7. Joule's Law. 8. Characteristic Equation of Gas. 9. Avogadro's Law. 10. Universal Gas Constant or Molar Constant. 11. Specific Heat of a Gas. 12. Specific Heat at Constant Volume. 13. Specific Heat at Constant Pressure. 14. Enthalpy of a Gas. 15. Molar Specific Heats of a Gas. 16. Regnaults' Law. 17. Relation between Specific Heats. 18. Ratio of Specific Heats.

2.1. Introduction

A perfect gas (or an ideal gas) may be defined as a state of a substance, whose evaporation from its liquid state is complete*, and strictly obeys all the gas laws under all conditions of temperature and pressure. In actual practice, there is no real or actual gas which strictly obeys the gas laws over the entire range of temperature and pressure. But, the real gases which are ordinarily difficult to liquify, such as oxygen, nitrogen, hydrogen and air, within certain temperature and pressure limits, may be regarded as perfect gases.

2.2. Laws of Perfect Gases

The physical properties of a gas are controlled by the following three variables :

1. Pressure exerted by the gas, 2. Volume occupied by the gas, and 3. Temperature of the gas.

The behaviour of a perfect gas, undergoing any change in the above mentioned variables, is governed by the following laws which have been established from experimental results.

1. Boyle's law, 2. Charles' law, and 3. Gay-Lussac law.

These laws are discussed, in detail, in the following pages.

2.3. Boyle's Law

This law was formulated by Robert Boyle in 1662. It states, *"The absolute pressure of a given mass of a perfect gas varies inversely as its volume, when the temperature remains constant."* Mathematically,

$$p \propto \frac{1}{v} \quad \text{or} \quad p\,v = \text{Constant}$$

The more useful form of the above equation is :

$$p_1 v_1 = p_2 v_2 = p_3 v_3 = \ldots.. = \text{Constant}$$

where suffixes $_{1}$, $_{2}$ and $_{3}$ .... refer to different sets of conditions.

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* If its evaporation is partial, the substance is called vapour. A vapour, therefore, contains some particles of liquid in suspension. It is thus obvious, that steam, carbon dioxide, sulphur dioxide and ammonia are regarded as vapours. It may be noted that a vapour becomes dry, when it is completely evaporated. If the dry vapour is further heated, the process is called *super heating* and the vapour is called *superheated vapour*. The behaviour of superheated vapour is similar to that of a perfect gas.

2.4. Charles' Law

This law was formulated by a Frenchman Jacques A.C. Charles in about 1787. It may be stated in the following two different forms :

(*i*) *"The volume of a given mass of a perfect gas varies directly as its absolute temperature, when the absolute pressure remains constant."* Mathematically,

$$v \propto T \quad \text{or} \quad \frac{v}{T} = \text{Constant}$$

or $$\frac{v_1}{T_1} = \frac{v_2}{T_2} = \frac{v_3}{T_3} = \ldots. = \text{Constant}$$

where suffixes $_{1}$, $_{2}$ and $_{3}$ ..... refer to different sets of conditions.

(*ii*) *"All perfect gases change in volume by 1/273th of its original volume at 0° C for every 1° C change in temperature, when the pressure remains constant."*

Let v_0 = Volume of a given mass of gas at 0° C, and

v_t = Volume of the same mass of gas at $t°$ C.

Then, according to the above statement,

$$v_t = v_0 + \frac{1}{273} v_0 t = v_0 \left(\frac{273 + t}{273} \right) = v_0 \times \frac{T}{T_0}$$

or $$\frac{v_t}{T} = \frac{v_0}{T_0}$$

where T = Absolute temperature corresponding to $t°$ C.

T_0 = Absolute temperature corresponding to 0° C.

A little consideration will show, that the volume of a gas goes on decreasing by 1/273th of its original volume for every 1° C decrease in temperature. It is thus obvious, that at a temperature of − 273° C, the volume of the gas would become *zero. The temperature at which the volume of a gas becomes zero is called *absolute zero temperature*.

Note. In all calculations of a perfect gas, the pressure and temperature values are expressed in absolute units.

2.5. Gay-Lussac Law

This law states, *"The absolute pressure of a given mass of a perfect gas varies directly as its absolute temperature, when the volume remains constant."* Mathematically

$$p \propto T \text{ or } \frac{p}{T} = \text{Constant}$$

or $$\frac{p_1}{T_1} = \frac{p_2}{T_2} = \frac{p_3}{T_3} = \ldots. = \text{Constant}$$

where suffixes $_{1}$, $_{2}$ and $_{3}$ ... refer to different sets of conditions.

2.6. General Gas Equation

In the previous section we have discussed the gas laws which give us the relation between the two variables when the third variable is constant. But in actual practice, all the three variables *i.e.*, pressure, volume and temperature, change simultaneously. In order to deal with all practical cases, the Boyle's law and Charles' law are combined together, which give us a *general gas equation*.

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* It is only theoretical. Its exact value is − 273.16 °C. But for all practical purposes, this value is taken as − 273 °C..

According to Boyle's law

$$p \propto \frac{1}{v} \quad \text{or} \quad v \propto \frac{1}{p} \qquad \text{. . . (Keeping } T \text{ constant)}$$

and according to Charles' law

$$v \propto T \qquad \text{. . . (Keeping } p \text{ consta}$$

It is thus obvious that

$$v \propto \frac{1}{p} \text{ and } T \text{ both} \quad \text{or } v \propto \frac{T}{p}$$

$$\therefore \qquad p\,v \propto T \quad \text{or} \quad p\,v = C\,T$$

where C is a constant, whose value depends upon the mass and properties of the gas concerned.

The more useful form of the general gas equation is :

$$\frac{p_1\,v_1}{T_1} = \frac{p_2\,v_2}{T_2} = \frac{p_3\,v_3}{T_3} = \ldots = \text{Constant}$$

where suffixes $_{1}$, $_{2}$ and $_{3}$ refer to different sets of conditions.

Example 2.1. *A gas occupies a volume of 0.1 m^3 at a temperature of 20° C and a pressure of 1.5 bar. Find the final temperature of the gas, if it is compressed to a pressure of 7.5 bar and occupies a volume of 0.04 m^3.*

Solution. Given : $v_1 = 0.1\ \text{m}^3$; $T_1 = 20°\text{C} = 20 + 273 = 293\ \text{K}$; $p_1 = 1.5\ \text{bar} = {}^*0.15 \times 10^6\ \text{N/m}^2$; $p_2 = 7.5\ \text{bar} = 0.75 \times 10^6\ \text{N/m}^2$; $v_2 = 0.04\ \text{m}^3$

Let T_2 = Final temperature of the gas.

We know that

$$\frac{p_1\,v_1}{T_1} = \frac{p_2\,v_2}{T_2}$$

$$\therefore \qquad T_2 = \frac{p_2\,v_2\,T_1}{p_1\,v_1} = \frac{0.75 \times 10^6 \times 0.04 \times 293}{0.15 \times 10^6 \times 0.1} = 586\ \text{K}$$

$$= 586 - 273 = 313°\text{C} \ \textbf{Ans.}$$

2.7. Joule's Law

It states, "*The change of internal energy of a perfect gas is directly proportional to the change of temperature.*" Mathematically

$$dE \propto dT \quad \text{or} \quad dE = m\,c\,dT = m\,c\,(T_2 - T_1)$$

where m = Mass of the gas, and

c = A constant of proportionality, known as specific heat.

An important consequence of this law is that if the temperature of a given mass m of a gas changes from T_1 to T_2, then the internal energy will change from E_1 to E_2 and the change in internal energy $(E_2 - E_1)$ will be same irrespective of the manner how the pressure (p) and volume (v) of the gas have changed.

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[*] $1\ \text{bar} = 0.1 \times 10^6\ \text{N/m}^2$.

2.8. Characteristic Equation of a Gas

It is a modified form of general gas equation. If the volume (v) in the general gas equation is taken as that of 1 kg of gas (known as its specific volume, and denoted by v_s), then the constant C (in the general gas equation) is represented by another constant R (in the characteristic equation of gas). Thus the general gas equation may be rewritten as :

$$pv_s = RT$$

where R is known as *characteristic gas constant* or simply *gas constant.*

For any mass m kg of a gas, the characteristic gas equation becomes :

$$m\,p\,v_s = m\,R\,T$$

or

$$p\,v = m\,R\,T \qquad \ldots (\because m\,v_s = v)$$

Notes : 1. The units of gas constant (R) may be obtained as discussed below:

$$R = \frac{p\,v}{m\,T} = \frac{\text{N/m}^2 \times \text{m}^3}{\text{kg} \times \text{K}} = \frac{\text{N-m}}{\text{kg} \times \text{K}} = \text{N-m/kg K} = \text{J/kg K}$$

$$\ldots (\because 1 \text{ N-m} = 1 \text{ J})$$

2. The value of gas constant (R) is different for different gases. In S.I. units, its value for atmospheric air is taken 287 J/kg K or 0.287 kJ/kg K.

3. The equation $pv = m\,R\,T$ may also be expressed in another form *i.e.*,

$$p = \frac{m}{v} R\,T = \rho\,R\,T \qquad \ldots \left(\because \frac{m}{v} = \rho\right)$$

where ρ (rho) is the density of the given gas.

Example 2.2. *A vessel of capacity 3 m³ contains air at a pressure of 1.5 bar and a temperature of 25° C. Additional air is now pumped into the system until the pressure rises to 30 bar and temperature rises to 60° C. Determine the mass of air pumped in and express the quantity as a volume at a pressure of 1.02 bar and a temperature of 20° C.*

If the vessel is allowed to cool until the temperature is again 25° C, calculate the pressure in the vessel.

Solution. Given : $v_1 = 3 \text{ m}^3$; $p_1 = 1.5 \text{ bar} = 0.15 \times 10^6 \text{ N/m}^2$; $T_1 = 25° \text{C} = 25 + 273 = 298 \text{ K}$; $p_2 = 30 \text{ bar} = 3 \times 10^6 \text{ N/m}^2$; $T_2 = 60° \text{C} = 60 + 273 = 333 \text{ K}$; $p_3 = 1.02 \text{ bar} = 0.102 \times 10^6 \text{ N/m}^2$; $T_3 = 20° \text{C} = 20 + 273 = 293 \text{ K}$

Mass of air pumped in

Let m_1 = Mass of air initially filled in the vessel, and

m_2 = Mass of air in the vessel after pumping.

We know that $p_1\,v_1 = m_1\,R\,T_1$

$$\therefore \qquad m_1 = \frac{p_1\,v_1}{R\,T_1} = \frac{0.15 \times 10^6 \times 3}{287 \times 298} = 5.26 \text{ kg}$$

... (Taking R for air = 287 J/kg K)

Similarly, $p_2\,v_2 = m_2\,R\,T_2$

$$\therefore \qquad m_2 = \frac{p_2\,v_2}{R\,T_2} = \frac{3 \times 10^6 \times 3}{287 \times 333} = 94.17 \text{ kg} \qquad \ldots (\because v_2 = v_1)$$

$\therefore$ Mass of air pumped in,

$$m = m_2 - m_1 = 94.17 - 5.26 = 88.91 \text{ kg} \quad \textbf{Ans.}$$

Volume of air pumped in at a pressure of 1.02 bar and temperature of 20° C

Let v_3 = Volume of air pumped in.

We know that

$$p_3 v_3 = m R T_3$$

$$\therefore \quad v_3 = \frac{m R T_3}{p_3} = \frac{88.91 \times 287 \times 293}{0.102 \times 10^6} = 73.3 \text{ m}^3 \quad \textbf{Ans.}$$

Pressure in the vessel after cooling

Let p_4 = Pressure in the vessel after cooling.

We know that the temperature after cooling,

$$T_4 = T_1 = 25^\circ \text{C} = 298 \text{ K}$$

Since the cooling is at constant volume, therefore

$$\frac{p_4}{p_2} = \frac{T_4}{T_2}$$

$$\therefore \quad p_4 = \frac{T_4 p_2}{T_2} = \frac{298 \times 3 \times 10^6}{333} = 2.68 \times 10^6 \text{ N/m}^2 = 26.8 \text{ bar} \quad \textbf{Ans.}$$

Example 2.3. *A spherical vessel of 1.5 m diameter, containing air at 40° C is evacuated till the vacuum inside the vessel becomes 735 mm of Hg. Calculate the mass of air pumped out. If the tank is then cooled to 3° C, what will be the final pressure in the tank ? Take atmospheric pressure as 760 mm of Hg.*

Solution. Given : $d = 1.5$ m ; $T_1 = 40^\circ \text{C} = 40 + 273 = 313$ K ; $p_v = 735$ mm of Hg ; $T_3 = 3^\circ \text{C} = 3 + 273 = 276$ K; $p_1 = 760$ mm of Hg

Mass of air pumped out

Let m = Mass of air pumped out.

First of all, let us find out the initial mass of air (m_1) in a vessel. We know that volume of a spherical vessel,

$$v_1 = \frac{\pi d^3}{6} = \frac{\pi (1.5)^3}{6} = 1.767 \text{ m}^3$$

We know that $p_1 v_1 = m_1 R T_1$

$$\therefore \quad m_1 = \frac{p_1 v_1}{R T_1} = \frac{(760 \times 133.3)\, 1.767}{287 \times 313} = 1.993 \text{ kg}$$

. . . ($\because$ 1 mm of Hg = 133.3 N/m^2 ; and R for air = 287 J/kg K)

Let m_2 = Mass of air left in the vessel after evacuation.

We know that pressure after evacuation,

$$p_2 = \text{Atmospheric pressure} - \text{Vacuum pressure}$$

$$= 760 - 735 = 25 \text{ mm of Hg} = 25 \times 133.3 = 3332.5 \text{ N/m}^2$$

$$\therefore \qquad m_2 = \frac{p_2\,v_2}{R\,T_2} = \frac{3332.5 \times 1.767}{287 \times 313} = 0.066 \text{ kg}$$

$$\ldots(\because v_2 = v_1 \text{ and } T_2 = T_1)$$

$\therefore$ Mass of air pumped out,

$$m = m_1 - m_2 = 1.993 - 0.066 = 1.927 \text{ kg} \quad \textbf{Ans.}$$

Final pressure in the tank

Let p_3 = Final pressure in the tank.

Since the cooling is at constant volume, therefore

$$\frac{p_3}{T_3} = \frac{p_2}{T_2}$$

or

$$p_3 = \frac{p_2\,T_3}{T_2} = \frac{3332.5 \times 276}{313} = 2938 \text{ N/m}^2 = 22 \text{ mm of Hg} \quad \textbf{Ans.}$$

2.9. Avogadro's Law

It states, *"Equal volumes of all gases, at the same temperature and pressure, contain equal number of molecules."*

Thus, according to Avogadro's law, 1 m^3 of oxygen (O_2) will contain the same number of molecules as 1 m^3 of hydrogen (H_2) when the temperature and pressure is the same. Since the molecular mass of hydrogen is 2 and that of oxygen is 16, therefore a molecule of oxygen has a mass which is 32/2 = 16 times the mass of hydrogen molecules. Moreover, as 1 m^3 of these two gases contain the same number of molecules, and a molecule of oxygen has a mass 16 times than that of hydrogen molecule, therefore it is evident that density of oxygen is 16 times the density of hydrogen. Hence, the Avogadro's law indicates that the density of any two gases is directly proportional to their molecular masses, if the gases are at the same temperature and pressure.

The density of oxygen at Normal Temperature and Pressure (briefly written as N.T.P.) *i.e.* at 0^0 C and 1.013 bar is 1.429 kg/m^3.

$\therefore$ Specific volume (of 1 kg) of oxygen at N.T.P.,

$$v_s = \frac{1}{1.429}\text{ m}^3\text{/kg} \qquad \ldots\left[\because \text{ Specific volume} = \frac{1}{\text{Density}}\right]$$

and volume of 32 kg (or 1 kg molecule briefly written as 1 kg-mol)

$$= \frac{1}{1.429} \times 32 = 22.4 \text{ m}^3$$

Similarly, it can be proved that the volume of 1 kg mol of any gas at N.T.P. is 22.4 m^3.

Note : 1 g-mole (molecular mass expressed in gram) of all gases occupies a volume of 22.4 litres at N.T.P.

The values of molecular mass for some common gases are given in the following table :

Table 2.1. Molecular mass for some common gases.

S.No.	*Gas*	*Molecular mass*	*S.No.*	*Gas*	*Molecular mass*
1.	Hydrogen (H_2)	2	5.	Carbon dioxide (CO_2)	44
2.	Oxygen (O_2)	32	6.	Methane (CH_4)	16
3.	Nitrogen (N_2)	28	7.	Acetylene (C_2H_2)	26
4.	Carbon monoxide (CO)	28	8.	Sulphur dioxide (SO_2)	64

2.10. Universal Gas Constant or Molar Constant

The *universal gas constant* or *molar constant* (generally denoted by R_u) of a gas is the product of the gas constant and the molecular mass of the gas. Mathematically,

$$R_u = MR$$

where M = Molecular mass of the gas expressed in kg-mole, and

R = Gas constant.

In general, if M_1, M_2, M_3, etc. are the molecular masses of different gases and R_1, R_2, R_3, etc. are their gas constants respectively, then

$$M_1 R_1 = M_2 R_2 = M_3 R_3 = ... = R_u$$

Notes : 1. The value of R_u is same for all gases.

2. In S.I. units, the value of R_u is taken as 8314 J/kg-mol K or 8.314 kJ/kg-mol K.

3. The characteristic gas equation (*i.e.* $p\,v = R\,T$) may be written in terms of molecular mass as :

$$p\,v = M\,R\,T$$

Example 2.4. *A mass of 2.25 kg of nitrogen occupying 1.5 m^3 is heated from 25° C to 200° C at a constant volume. Calculate the initial and final pressures of the gas. Take universal gas constant as 8314 J/kg mol K. The molecular mass of nitrogen is 28.*

Solution. Given : $m = 2.25$ kg ; $v_1 = 1.5\ \text{m}^3$; $T_1 = 25°\text{C} = 25 + 273 = 298$ K ; $T_2 = 200°\text{C} = 200 + 273 = 473$ K ; $R_u = 8314$ J/kg mol K ; $M = 28$

We know that gas constant,

$$R = \frac{R_u}{M} = \frac{8314}{28} = 297\ \text{J/kg K}$$

Initial pressure of the gas

Let p_1 = Initial pressure of the gas.

We know that

$$p_1 v_1 = m\,R\,T_1$$

$$\therefore \quad p_1 = \frac{m\,R\,T_1}{v_1} = \frac{2.25 \times 297 \times 298}{1.5} = 0.133 \times 10^6\ \text{N/m}^2 = 1.33\ \text{bar}\ \textbf{Ans.}$$

Final pressure of the gas

Let p_2 = Final pressure of the gas.

Since the volume is constant, therefore

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \quad \text{or} \quad p_2 = \frac{p_1\,T_2}{T_1} = \frac{1.33 \times 473}{298} = 2.11\ \text{bar}\ \textbf{Ans.}$$

Example 2.5. *Nitrogen is to be stored at pressure 140 bar, temperature 27° C in a steel flask of 0.05 m^3 volume. The flask is to be protected against excessive pressure by a fusible plug which will melt and allow the gas to escape if the temperature rises too high. Find :*

1. How many kg of nitrogen will the flask hold at the designed conditions ? Take molecular mass of nitrogen as 28 ; and

2. At what temperature must the fusible plug melt in order to limit the pressure of the full flask to a maximum of 168 bar ?

Solution. Given : $p_1 = 140 \text{ bar} = 14 \times 10^6 \text{ N/m}^2$; $T_1 = 27° \text{C} = 27 + 273 = 300 \text{ K}$; $v_1 = 0.05 \text{ m}^3$; $M = 28$

1. Mass of nitrogen

Let m = Mass of nitrogen in kg which the flask will hold.

We know that gas constant,

$$R = \frac{\text{Universal gas constant}}{\text{Molecular mass}} = \frac{R_u}{M} = \frac{8314}{28} = 297 \text{ J/kg K}$$

. . . (R_u for all gases = 8314 J/kg K)

We also know that

$$p_1 v_1 = m R T_1$$

$$\therefore \quad m = \frac{p_1 v_1}{R T_1} = \frac{14 \times 10^6 \times 0.05}{297 \times 300} = 7.86 \text{ kg} \quad \textbf{Ans.}$$

2. Melting temperature of fusible plug

Let T_2 = Melting temperature of fusible plug, and

p_2 = Maximum pressure = 168 bar = $16.8 \times 10^6 \text{ N/m}^2$ . . . (Given)

Since the gas is heated at constant volume, therefore

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

$$\therefore \quad T_2 = \frac{p_2 T_1}{p_1} = \frac{16.8 \times 10^6 \times 300}{14 \times 10^6} = 360 \text{ K} = 87° \text{C} \quad \textbf{Ans.}$$

2.11. Specific Heats of a Gas

The specific heat of a substance may be broadly defined as the amount of heat required to raise the temperature of its unit mass through one degree. All the liquids and solids have one specific heat only. But a gas can have any number of specific heats (lying between zero and infinity) depending upon the conditions, under which it is heated. The following two types of specific heats of a gas are important from the subject point of view :

1. Specific heat at constant volume, and 2. Specific heat at constant pressure.

These specific heats are discussed, in detail, as follows :

2.12. Specific Heat at Constant Volume

It is the amount of heat required to raise the temperature of a unit mass of gas through one degree when it is heated at a constant volume. It is generally denoted by c_v.

Consider a gas contained in a container with a fixed lid as shown in Fig. 2.1. Now, if this gas is heated, it will increase the temperature and pressure of the gas in the container. Since the lid of the container is fixed, therefore the volume of gas remains unchanged.

Let m = Mass of the gas,

T_1 = Initial temperature of the gas, and

T_2 = Final temperature of the gas.

Fig. 2.1. Heat being supplied at constant volume.

∴ Total heat supplied to the gas at constant volume,

$$Q_{1-2} = \text{Mass} \times \text{Sp. heat at constant volume} \times \text{Rise in temperature}$$

$$= mc_v (T_2 - T_1)$$

It may be noted that whenever a gas is heated at constant volume, no work is done by the gas.* The whole heat energy is utilised in increasing the temperature and pressure of the gas. In other words, all the amount of heat supplied remains within the body of the gas, and represents the *increase in internal energy of the gas.*

2.13. Specific Heat at Constant Pressure

It is the amount of heat required to raise the temperature of a unit mass of a gas through one degree, when it is heated at constant pressure. It is generally denoted by c_p.

Consider a gas contained in a container with a movable lid as shown in Fig. 2.2. Now if this gas is heated, it will increase the temperature and pressure of the gas in the container. Since the lid of the container is movable, therefore it will move upwards, in order to counterbalance the tendency for pressure to rise.

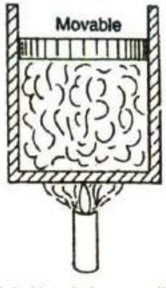

Fig. 2.2. Heat being supplied at constant pressure.

Let m = Mass of the gas,

T_1 = Initial temperature of the gas,

v_1 = Initial volume of the gas, and

T_2, v_2 = Corresponding values for the final condition. of the gas.

∴ Total heat supplied to the gas, at constant pressure,

$$Q_{1-2} = \text{Mass} \times \text{Sp. heat at constant pressure} \times \text{Rise in temperature}$$

$$= mc_p (T_2 - T_1)$$

Whenever a gas is heated at a constant pressure, the heat supplied to the gas is utilised for the following two purposes :

1. To raise the temperature of the gas. This heat remains within the body of the gas, and represents the increase in internal energy Mathematically, increase in internal energy,

$$dU = mc_v \ (T_2 - T_1)$$

2. To do some external work during expansion. Mathematically, workdone by the gas,

$$W_{1-2} = p\,(v_2 - v_1) = mR\,(T_2 - T_1)$$

It is thus obvious, that the specific heat at constant pressure is higher than the specific heat at constant volume.

From above, we may write as

$$Q_{1-2} = dU + W_{1-2} \quad \text{or} \quad {}^{**}Q_{1-2} - W_{1-2} = dU \quad \text{... (First Law of Thermodynamics)}$$

---

* We know that workdone by the gas,

$$W = pdv = p\,(v_2 - v_1)$$

where p = Pressure of the gas, and

dv = Change in volume = $v_2 - v_1$

When there is no change in volume, then $dv = 0$. Therefore $W = 0$.

** Refer Art. 1.49. note 2 equation (*iv*).

2.14. Enthalpy of a Gas

In thermodynamics, one of the basic quantities most frequently recurring is the *sum* of the internal energy (U) and the product of pressure and volume ($p\,v$). This sum ($U + p\,v$) is termed as *enthalpy* and is written as H. Mathematically,

Enthalpy, $H = U + p\,v$

Since ($U + p\,v$) is made up entirely of properties, therefore enthalpy (H) is also a property.

For a unit mass, specific enthalpy,

$$h = u + p\,v_s$$

where u = Specific internal energy, and

v_s = Specific volume.

We know that $Q_{1-2} = dU + W_{1-2} = dU + p\,dv$

When gas is heated at constant pressure from an initial condition 1 to a final condition 2, then change in internal energy,

$$dU = U_2 - U_1$$

and workdone by the gas,

$$W_{1-2} = p\,dv = p\,(v_2 - v_1)$$

$$\therefore \quad Q_{1-2} = (U_2 - U_1) + p\,(v_2 - v_1)$$

$$= (U_2 + p\,v_2) - (U_1 + p\,v_1) = H_2 - H_1$$

and for a unit mass, $q_{1-2} = h_2 - h_1$

Thus, for a constant pressure process, the heat supplied to the gas is equal to the change of enthalpy.

2.15. Molar Specific Heats of a Gas

The molar or volumetric specific heat of a gas may be defined as the amount of heat required to raise the temperature of unit mole of gas through one degree. Mathematically, molar specific heat,

$$c_m = Mc$$

where M = Molecular mass of the gas.

In the similar way as discussed in Art. 2.11, the molar specific heat at constant volume,

$$c_{vm} = M\,c_v$$

and molar specific heat at constant pressure,

$$c_{pm} = M\,c_p$$

Example 2.6. *A closed vessel contains 2 kg of carbon dioxide at temperature 20° C and pressure 0.7 bar. Heat is supplied to the vessel till the gas acquires a pressure of 1.4 bar. Calculate : 1. Final temperature ; 2. Work done on or by the gas ; 3. Heat added ; and 4. Change in internal energy. Take specific heat of the gas at constant volume as 0.657 kJ/kg K.*

Solution. Given : $m = 2$ kg ; $T_1 = 20° \text{C} = 20 + 273 = 293$ K ; $p_1 = 0.7$ bar ; $p_2 = 1.4$ bar

1. Final temperature

Let T_2 = Final temperature.

Since the gas is heated in a closed vessel, therefore the volume of gas will remain constant.

We know that $\dfrac{p_1}{T_1} = \dfrac{p_2}{T_2}$

$$\therefore \qquad T_2 = \frac{p_2 T_1}{p_1} = \frac{1.4 \times 293}{0.7} = 586 \text{ K} = 586 - 273 = 313^\circ \text{C} \quad \textbf{Ans.}$$

2. Workdone on or by the gas

Since there is no change in volume, therefore workdone on or by the gas (W_{1-2}) is *zero*. **Ans.**

3. Heat added

We know that heat added at constant volume,

$$Q_{1-2} = m\,c_v\,(T_2 - T_1) = 2 \times 0.657\,(586 - 293) = 385 \text{ kJ} \quad \textbf{Ans.}$$

4. Change in internal energy

Let dU = Change in internal energy.

We know that $Q_{1-2} = W_{1-2} + dU$

$$\therefore \qquad dU = Q_{1-2} = 385 \text{ kJ} \quad \textbf{Ans.} \qquad \ldots (\because W_{1-2} = 0)$$

Example 2.7. *A mass of 0.25 kg of air in a closed system expands from 2 bar, 60° C to 1 bar and 40° C while receiving 1.005 kJ of heat from a reservoir at 100° C. The surrounding atmosphere is at 0.95 bar and 27° C. Determine the maximum work. How much of this work would be done on the atmosphere ?*

Solution. Given : $m = 0.25$ kg ; $p_1 = 2$ bar $= 0.2 \times 10^6$ N/m^2 ; $T_1 = 60^\circ$ C $= 60 + 273 = 333$ K ; $p_2 = 1$ bar $= 0.1 \times 10^6$ N/m^2 ; $T_2 = 40^\circ$ C $= 40 + 273 = 313$ K ; $Q = 1.005$ kJ ; *$T_R = 100^\circ$ C ; $p = 0.95$ bar $= 0.095 \times 10^6$ N/m^2 ; *$T = 27^\circ$ C

Maximum Workdone

First of all, let us find the values of initial volume (v_1) and final volume (v_2) of air. We know that

$$v_1 = \frac{m R T_1}{p_1} \qquad \ldots (\because pv = mRT)$$

$$= \frac{0.25 \times 287 \times 333}{0.2 \times 10^6} = 0.119 \text{ m}^3 \qquad \ldots (\text{Taking } R = 287 \text{ J/kg K})$$

Similarly

$$v_2 = \frac{m R T_2}{p_2} = \frac{0.25 \times 287 \times 313}{0.1 \times 10^6} = 0.224 \text{ m}^3$$

$\therefore$ Workdone on the atmosphere,

$$W_1 = p\,(v_2 - v_1) = 0.095 \times 10^6\,(0.224 - 0.119) = 9975 \text{ J} = 9.975 \text{ kJ}$$

We know that change of internal energy,

$$dU = m\,c_v\,(T_2 - T_1) = 0.25 \times 0.712\,(313 - 333) = -3.56 \text{ kJ}$$

$$\ldots (\text{Taking } c_v = 0.712 \text{ kJ/kg K})$$

The –ve sign shows that there is a *decrease* of internal energy.

Net workdone, $W_2 = Q - dU = 1.005 - (-3.56) = 4.565$ kJ

$\therefore$ Maximum workdone,

$$W = W_1 + W_2 = 9.975 + 4.565 = 14.54 \text{ kJ} \quad \textbf{Ans.}$$

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* Superfluous data

Workdone on the atmosphere

We have calculated above that the workdone on the atmosphere,

$$W_1 = 9.975 \text{ kJ} \quad \textbf{Ans.}$$

Example 2.8. *3 kg of an ideal gas is expanded from a pressure 7 bar and volume 1.5 m³ to a pressure 1.4 bar and volume 4.5 m³. The change in internal energy is 525 kJ. The specific heat at constant volume for the gas is 1.047 kJ/kg K. Calculate : 1. Gas constant ; 2. Change in enthalpy ; and 3. Initial and final temperatures.*

Solution. Given : $m = 3$ kg ; $p_1 = 7$ bar $= 0.7 \times 10^6$ N/m² ; $v_1 = 1.5$ m³ ; $p_2 = 1.4$ bar $= 0.14 \times 10^6$ N/m² ; $v_2 = 4.5$ m³ ; $dU = 525$ kJ ; $c_v = 1.047$ kJ/kg K

1. *Gas constant*

Let R = Gas constant,

T_1 and T_2 = Initial and final temperatures.

We know that $p_1 v_1 = m R T_1$

$$\therefore \quad R T_1 = \frac{p_1 v_1}{m} = \frac{0.7 \times 10^6 \times 1.5}{3} = 0.35 \times 10^6 \quad \ldots (i)$$

Similarly
$$R T_2 = \frac{p_2 v_2}{m} = \frac{0.14 \times 10^6 \times 4.5}{3} = 0.21 \times 10^6 \quad \ldots (ii)$$

Subtracting equation (*ii*) from equation (*i*),

$$R(T_1 - T_2) = (0.35 - 0.21)\, 10^6 = 0.14 \times 10^6 \quad \ldots (iii)$$

We also know that change in internal energy

$$dU = m\, c_v (T_2 - T_1)$$

Since during expansion, there is a decrease in internal energy, therefore the change in internal energy is *negative.

$$\therefore \quad -525 = 3 \times 1.047\, (T_2 - T_1) = -3.141\, (T_1 - T_2)$$

$$T_1 - T_2 = 525 / 3.141 = 167.14 \quad \ldots (iv)$$

Dividing equation (*iii*) by equation (*iv*), we get

$$R = 0.14 \times 10^6 / 167.14 = 838 \text{ J/kg K} = 0.838 \text{ kJ/kg K} \quad \textbf{Ans.}$$

2. *Change in enthalpy*

First of all, let us find the value of specific heat at constant pressure (c_p). We know that

$$\therefore \quad c_p - c_v = R \quad \text{or} \quad c_p = R + c_v = 0.838 + 1.047 = 1.885 \text{ kJ/kg K}$$

$\therefore$ Change in enthalpy,

$$dH = m\, c_p (T_2 - T_1) = 3 \times 1.885\, (-167.14) = -945 \text{ kJ} \quad \textbf{Ans.}$$

The –ve sign indicates that there is a decease in enthalpy.

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* We may also say as follows :
From equation (*iii*), we see that $R(T_1 - T_2)$ is a *positive* equation. This shows that T_1 is greater than T_2, because R is always positive. Thus there is a decrease is internal energy or the change in internal energy is *negative* (*i.e.* $dU = -525$ kJ).

3. Initial and final temperatures

From equation (*i*), we find that initial temperature,

$$T_1 = \frac{0.35 \times 10^6}{R} = \frac{0.35 \times 10^6}{838} = 417 \text{ K} \quad \textbf{Ans.}$$

and from equation (*iv*), final temperature,

$$T_2 = T_1 - 167.14 = 417 - 167.14 = 249.86 \text{ K} \quad \textbf{Ans.}$$

2.16. Regnault's Law

This law states, "*The two specific heats of a gas (i.e. the specific heat at constant pressure,* c_p *and specific heat at constant volume,* c_v*) do not change with the change in pressure and temperature of the gas.*"

2.17. Relation between Specific Heats

Consider a gas enclosed in a container and being heated, at a constant pressure, from the initial state 1 to the final state 2.

Let m = Mass of the gas,
T_1 = Initial temperature of the gas,
T_2 = Final temperature of the gas,
v_1 = Initial volume of the gas,
v_2 = Final volume of the gas,
c_p = Specific heat at constant pressure,
c_v = Specific heat at constant volume, and
p = Constant pressure.

We know that the heat supplied to the gas at constant pressure,

$$Q_{1-2} = m\,c_p\,(T_2 - T_1)$$

As already discussed, a part of this heat is utilised in doing the external work, and the rest remains within the gas, and is used in increasing the internal energy of the gas.

∴ Heat utilised for external work,

$$W_{1-2} = p\,(v_2 - v_1) \quad \ldots (i)$$

and increase in internal energy,

$$dU = m\,c_v\,(T_2 - T_1) \quad \ldots (ii)$$

We know that

$$Q_{1-2} = W_{1-2} + dU \quad \ldots (iii)$$

$$\therefore \quad m\,c_p\,(T_2 - T_1) = p\,(v_2 - v_1) + m\,c_v\,(T_2 - T_1) \quad \ldots (iv)$$

Using characteristic gas equation (*i.e.* $p\,v = m\,R\,T$), we have

$$p\,v_1 = m\,R\,T_1 \quad \ldots \text{(for initial conditions)}$$

and

$$p\,v_2 = m\,R\,T_2 \quad \ldots \text{(for final conditions)}$$

$$\therefore \quad p\,(v_2 - v_1) = m\,R\,(T_2 - T_1)$$

Now substituting the value of $p\,(v_2 - v_1)$ in equation (*iv*),

$$m\,c_p\,(T_2 - T_1) = m\,R\,(T_2 - T_1) + m\,c_v\,(T_2 - T_1)$$

$$c_p = R + c_v \quad \text{or} \quad c_p - c_v = R \quad \ldots (v)$$

The above equation may be rewritten as :

$$c_p - c_v = R \quad \text{or} \quad c_v(\gamma - 1) = R \qquad \ldots \left(\text{where } \gamma = \frac{c_p}{c_v} \right)$$

$$\therefore \qquad c_v = \frac{R}{(\gamma - 1)} \qquad \ldots (vi)$$

Notes. 1. The equation (*v*) gives an important result, as it proves that characteristic constant of a gas (R) is equal to the difference of its two specific heats (*i.e.* $c_p - c_v$).

2. The value of R is taken as 287 J/kg K or 0.287 kJ/kg K.

3. In terms of molar specific heats, the equation (*v*) may be written as

$$c_{pm} - c_{vm} = R_u$$

where R_u is the universal gas constant and its value is taken 8314 J/kg K or 8.314 kJ/kg K.

2.18. Ratio of Specific Heats

The ratio of two specific heats (*i.e.* c_p/c_v) of a gas is an important constant in the field of Thermodynamics and is represented by a Greek letter gamma (γ). It is also known as *adiabatic index*. Since c_p is always greater than c_v, the value of γ is always greater than unity.

We have seen in Art. 2.17 that

$$c_p - c_v = R \quad \text{or} \quad c_p = c_v + R$$

Dividing both sides by c_v,

$$\frac{c_p}{c_v} = 1 + \frac{R}{c_v} \quad \text{or} \quad \gamma = 1 + \frac{R}{c_v}$$

The values of c_p, c_v and γ for some common gases are given below :

Table 2.2. Values of c_p and c_v for some common gases.

S.No.	*Name of gas*	c_p *(kJ/kg K)*	c_v *(kJ/kg K)*	$\gamma = \frac{c_p}{c_v}$
1.	Air	1.000	0.720	1.40
2.	Carbon dioxide (CO_2)	0.846	0.657	1.29
3.	Oxygen (O_2)	0.913	0.653	1.39
4.	Nitrogen (N_2)	1.043	0.745	1.40
5.	Ammonia (NH_3)	2.177	1.692	1.29
6.	Carbon monoxide (CO)	1.047	0.749	1.40
7.	Hydrogen (H_2)	14.257	10.133	1.40
8.	Argon (A)	0.523	0.314	1.67
9.	Helium (He)	5.234	3.153	1.66
10.	Methane (CH_4)	2.169	1.650	1.31

Example 2.9. *One kg of a perfect gas occupies a volume of 0.85 m³ at 15° C and at a constant pressure of 1 bar. The gas is first heated at a constant volume, and then at a constant pressure. Find the specific heat at constant volume and constant pressure of the gas. Take* $\gamma = 1.4$.

Solution. Given : $m = 1$ kg ; $v = 0.85$ m^3 ; $T = 15°$ C $= 15 + 273 = 288$ K ; $p = 1$ bar $= 0.1 \times 10^6$ N/m^2 ; $\gamma = c_p/c_v = 1.4$

Specific heat of gas at constant volume

Let c_v = Specific heat of gas at constant volume, and

R = Characteristic gas constant.

We know that

$$p\,v = m\,R\,T$$

$$\therefore \quad R = \frac{p\,v}{m\,T} = \frac{0.1 \times 10^6 \times 0.85}{1 \times 288} = 295 \text{ J/kg K} = 0.295 \text{ kJ/kg K}$$

We also know that

$$c_v = \frac{R}{\gamma - 1} = \frac{0.295}{1.4 - 1} = 0.7375 \text{ kJ/kg K} \text{ **Ans.**}$$

Specific heat of gas at constant pressure

We know that specific heat of gas at constant pressure,

$$c_p = 1.4\,c_v = 1.4 \times 0.7375 = 1.0325 \text{ kJ/kg K} \text{ **Ans.**} \quad \ldots (\because c_p/c_v = 1.4)$$

Example 2.10. *A gas mixture obeying perfect gas law has a molecular mass of 26.7. Assuming a mean molar specific heat at constant volume of 21.1 kJ/kg K, determine the values of characteristic gas constant, molar specific heat at constant pressure and the ratio of specific heats.*

Solution. Given : $M = 26.7$; $c_{vm} = 21.1$ kJ/kg K

Characteristic gas constant

We know that characteristic gas constant,

$$R = \frac{\text{Universal gas constant}}{\text{Molecular mass}} = \frac{R_u}{M} = \frac{8.314}{26.7} = 0.3114 \text{ kJ/kg K} \text{ **Ans.**}$$

$$\ldots (\because R_u \text{ for all gases} = 8.314 \text{ kJ / kg K})$$

Molar specific heat at constant pressure

Let c_{pm} = Molar specific heat at constant pressure.

We know that

$$c_{pm} - c_{vm} = R_u \text{ or } c_{pm} = R_u + c_{vm} = 8.314 + 21.1 = 29.414 \text{ kJ/kg K} \text{ **Ans.**}$$

Ratio of specific heats

We know that ratio of specific heats,

$$\gamma = \frac{c_{pm}}{c_{vm}} = \frac{29.414}{21.1} = 1.394 \text{ **Ans.**}$$

Example 2.11. *One kg of ideal gas is heated from 18.3° C to 93.4° C. Assuming $R = 0.264$ kJ/kg K and $\gamma = 1.18$ for the gas, find : 1. Specific heats ; 2. Change in internal energy ; and 3. Change in enthalpy.*

Solution. Given : $m = 1$ kg ; $T_1 = 18.3° \text{C} = 18.3 + 273 = 291.3$ K ; $T_2 = 93.4° \text{C} = 93.4 + 273 = 366.4$ K ; $R = 0.264$ kJ/kg K ; $\gamma = c_p/c_v = 1.18$

Specific heats

Let c_p = Specific heat at constant pressure, and

c_v = Specific heat at constant volume.

We know that $$c_v = \frac{R}{\gamma - 1} = \frac{0.264}{1.18 - 1} = 1.47 \text{ kJ/kg K} \quad \textbf{Ans.}$$

$$\therefore \quad c_p = \gamma c_v = 1.18 \times 1.47 = 1.73 \text{ kJ/kg K} \quad \textbf{Ans.}$$

Change in internal energy

We know that change in internal energy,

$$dU = m\, c_v (T_2 - T_1) = 1 \times 1.47\,(366.4 - 291.3) = 110.4 \text{ kJ} \quad \textbf{Ans.}$$

Change in enthalpy

We know that change in enthalpy,

$$dH = m\, c_p (T_2 - T_1) = 1 \times 1.73\,(366.4 - 291.3) = 130 \text{ kJ} \quad \textbf{Ans.}$$

Example 2.12. *A gas, having initial pressure, volume and temperature as 275 kN/m^2, 0.09 m^3 and 185° C respectively, is compressed at constant pressure until its temperature is 15° C. Calculate the amount of heat transferred and work done during the process. Take R = 290 J/kg K and c_p = 1.005 kJ/kg K.*

Solution. Given : p_1 = 275 kN/m^2 = 275 × 10^3 N/m^2 ; v_1 = 0.09 m^3 ; T_1 = 185° C = 185 + 273 = 458 K ; T_2 = 15° C = 15 + 273 = 288 K ; R = 290 J/kg K ; c_p = 1.005 kJ/kg K

Amount of heat transferred

First of all, let us find the mass of the gas (m). We know that

$$p_1 v_1 = m R T_1$$

$$\therefore \quad m = \frac{p_1 v_1}{R T_1} = \frac{275 \times 10^3 \times 0.09}{290 \times 458} = 0.186 \text{ kg}$$

We know that the amount of heat transferred,

$$Q_{1-2} = m\, c_p (T_2 - T_1) = 0.186 \times 1.005\,(288 - 458) \text{ kJ}$$

$$= -31.78 \text{ kJ} \quad \textbf{Ans.}$$

The –ve sign indicates that the heat has been extracted from the gas during the process. In other words, the gas is compressed.

Workdone during the process

First of all, let us find the final volume of the gas (v_2). Since the process takes place at constant pressure, therefore

$$\frac{v_1}{T_1} = \frac{v_2}{T_2} \quad \text{or} \quad v_2 = \frac{v_1 T_2}{T_1} = \frac{0.09 \times 288}{458} = 0.056 \text{ m}^3$$

We know that the workdone during the process,

$$W_{1-2} = p\,(v_2 - v_1) = 275 \times 10^3\,(0.056 - 0.09) = -9350 \text{ J}$$

$$= -9.35 \text{ kJ} \quad \textbf{Ans.}$$

The –ve sign indicates the work is done on the gas. In other words, the gas is compressed.

Example 2.13. *A certain gas has c_p = 1.96 kJ/kg K and c_v = 1.5 kJ/kg K. Find its molecular mass and gas constant. A constant volume chamber of 0.3 m^3 capacity contains 2 kg of this gas at 5° C. The heat is transferred to the gas until the temperature is 100° C. Find the workdone, heat transferred and change in internal energy.*

Solution. Given : $c_p = 1.96$ kJ/kg K ; $c_v = 1.5$ kJ/kg K ; $v = 0.3\ \text{m}^3$; $m = 2$ kg ; $T_1 = 5°$ C $= 5 + 273 = 278$ K ; $T_2 = 100°$ C $= 100 + 273 = 373$ K

Molecular mass and gas constant

Let M = Molecular mass, and

R = Gas constant.

We know that gas constant,

$$R = c_p - c_v = 1.96 - 1.5 = 0.46 \text{ kJ/kg K}$$

and molecular mass,

$$M = \frac{\text{Universal gas constant}}{\text{Gas constant}} = \frac{R_u}{R} = \frac{8.314}{0.46} = 18 \text{ kg} \quad \textbf{Ans.}$$

. . . ($\because R_u$ for all gases = 8.314 kJ/kg K)

Workdone

Since the volume is constant, therefore workdone (W_{1-2}) is zero. **Ans.**

Heat transferred

We know that heat transferred,

$$Q_{1-2} = m\,c_v\,(T_2 - T_1) = 2 \times 1.5\,(373 - 278) = 285 \text{ kJ} \quad \textbf{Ans.}$$

Change in internal energy

Let dU = Change in internal energy.

We know that $Q_{1-2} = W_{1-2} + dU = 0 + dU = dU$

$$\therefore \quad dU = Q_{1-2} = 285 \text{ kJ} \quad \textbf{Ans.}$$

Example 2.14. *A vessel of 2.5 m^3 capacity contains one kg-mole of nitrogen at 100° C. Evaluate the specific volume and pressure. If the gas is cooled to 30° C, calculate final pressure, change in specific internal energy and specific enthalpy.*

The ratio of specific heats is 1.4 and one kg-mole nitrogen is 28 kg.

Solution. Given : $v_1 = 2.5\ \text{m}^3$; $M = 1$ kg-mole $= 28$ kg ; $T_1 = 100°$ C $= 100 + 273 = 373$ K ; $T_2 = 30°$ C $= 30 + 273 = 303$ K ; $\gamma = c_p/c_v = 1.4$

Specific volume and pressure

Let v_s = Specific volume of the gas, and

p_1 = Pressure of the gas.

We know that specific volume of the gas,

$$v_s = \frac{v_1}{M} = \frac{2.5}{28} = 0.089 \text{ m}^3\text{/kg} \quad \textbf{Ans.}$$

Gas constant,

$$R = \frac{R_u}{M} = \frac{8314}{28} = 297 \text{ J/kg K}$$

. . . ($\because R_u = 8314$ J/kg mol K)

We know that $p_1 v_1 = M R T_1$

$$\therefore \quad p_1 = \frac{M R T_1}{v_1} = \frac{28 \times 297 \times 373}{2.5} = 1.24 \times 10^6 \text{ N/m}^2$$

$$= 12.4 \text{ bar} \quad \textbf{Ans.}$$

. . . ($\because 1 \text{ bar} = 0.1 \times 10^6 \text{ N/m}^2$)

Final pressure, change in specific internal energy and specific enthalpy

Let p_2 = Final pressure of the gas.

We know that $\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}$ or $\frac{p_1}{T_1} = \frac{p_2}{T_2}$ $\quad \ldots (\because v_1 = v_2)$

$$\therefore \quad p_2 = \frac{p_1 T_2}{T_1} = \frac{12.4 \times 303}{373} = 10.07 \text{ bar } \textbf{Ans.}$$

Now, let us find out the values of specific heat at constant pressure (c_p) and specific heat at constant volume (c_v).

We know that $c_p - c_v = R$

$$1.4\, c_v - c_v = 297 \quad \ldots (\because c_p / c_v = 1.4)$$

$$\therefore \quad c_v = 297/0.4 = 742.5 \text{ J/kg} = 0.7425 \text{ kJ/kg K}$$

and

$$c_p = 1.4\, c_v = 1.4 \times 0.7425 = 1.04 \text{ kJ/kg K}$$

We know that change in specific internal energy,

$$du = c_v (T_2 - T_1) = 0.7425\,(303 - 373) = -52 \text{ kJ/kg } \textbf{Ans.}$$

The –ve sign indicates that the specific internal energy is reduced after the gas is cooled.

We also know that change in specific enthalpy,

$$dh = c_p (T_2 - T_1) = 1.04\,(303 - 373) = -72.8 \text{ kJ/kg } \textbf{Ans.}$$

The –ve sign indicates that the specific enthalpy is reduced after the gas is cooled.

EXERCISES

1. Determine the final pressure of a gas when 2 m^3 of gas at 6 bar is heated by keeping the temperature constant. The final volume is 6 m^3. [**Ans.** 2 bar]

2. A certain quantity of air is cooled at a constant pressure from 300 K to 280 K. If the initial volume of the air is 0.15 m^3, find by how much the volume will diminish ? [**Ans.** 0.01 m^3]

3. A gas at a temperature of 333° C and 20 bar has a volume of 0.06 m^3. It is expanded to a volume of 0.54 m^3. Determine the final pressure of the gas if the temperature of the gas after expansion is 30° C. [**Ans.** 1.33 bar]

4. A gas at a temperature of 20° C and pressure of 1.5 bar occupies a volume of 0.105 m^3. If the gas is compressed to a pressure of 7.5 bar and volume of 0.04 m^3, what will be the final temperature of the gas ? [**Ans.** 285° C]

5. A cylinder contains 3 kg of air at a pressure of 300 bar and a temperature of 27° C. Find the volume of air occupied by the gas. Assume R for air as 287 J/kg K. [**Ans.** 0.0086 m^3]

6. A vessel of capacity 5 m^3 contains 20 kg of an ideal gas having a molecular mass of 25. If the temperature of the gas is 15° C, find its pressure. [**Ans.** 3.83 bar]

$$\left[\text{Hint. } R = \frac{R_u}{M} = \frac{8314}{25} = 332.56 \text{ J/kg K} \quad (\because R_u \text{ for an ideal gas} = 8314 \text{ J/kg K})\right]$$

7. A certain gas occupies 0.15 m^3 at a temperature of 20° C and a pressure of 1.2 bar. If the gas has mass of 200 g, calculate (*i*) value of gas constant, and (*ii*) molecular mass of the gas. [**Ans.** 307.2 J/kg K ; 27.06]

8. A certain gas has $c_p = 1.96$ kJ/kg K and $c_v = 1.5$ kJ/kg K. Find its molecular mass and the gas constant. The universal gas constant is 8.315 kJ/kg K. [**Ans.** 18 ; 0.46 kJ/kg K]

9. The volume of air at a pressure of 5 bar and 47° C is 0.5 m^3. Calculate the mass of the air, if the specific heats at constant pressure and volume are 1 kJ/kg K and 0.72 kJ/kg K respectively. [**Ans.** 2.8 kg]

10. The heated nitrogen gas expands from 0.2 m^3 to 0.85 m^3 in a quasi-static process at a constant pressure of 1000 kPa. For 1 kg mass of gas, determine the amount of workdone by the gas and the final temperature. $R = 296.8$ J/kg K, for nitrogen. [**Ans.** 650 kJ ; 2863.88 K]

11. The gas constant for atmospheric air is 0.287 kJ/kg K and the specific heat at constant volume is 0.713 kJ/kg K. Find the specific heat at constant pressure and the ratio of specific heats.

[**Ans.** 1 kJ/kg K ; 1.403]

12. A certain quantity of gas occupies 0.14 m^3 at 12.6 bar and 100° C. Calculate the change in internal energy if the gas is heated to a temperature of 300° C. Take $c_p = 1$ kJ/kg K and $c_v = 0.72$ kJ/kg K.

[**Ans.** 245 kJ]

13. The temperature of 3.5 kg of gas is raised from 95° C to 225° C at a constant pressure. Find the amount of heat supplied to the gas and the amount of the external workdone. The specific heats at constant pressure and volume are 1 kJ/kg K and 0.72 kJ/kg K respectively. [**Ans.** 455 kJ ; 127.4 kJ]

14. An ideal gas 0.9 kg having gas constant 287 J/kg K is heated at constant pressure of 8 bar from 30° C to 200° C. If the specific heat at constant volume is 0.72 kJ/kg K, find 1. specific heat at constant pressure, 2. total heat supplied to the gas, 3. increase in internal energy, and 4. workdone in expansion.

[**Ans.** 1.007 kJ/kg K ; 154.1 kJ ; 110.16 kJ ; 43.94 kJ]

15. One kg mole of nitrogen (molecular mass = 28) is contained in a vessel of volume 2.5 m^3 at 100° C. 1. Evaluate the mass, the pressure and the specific volume of the gas ; 2. If the ratio of specific heats is 1.4, evaluate c_p and c_v. 3. If the gas cools to the atmospheric temperature of 30° C, evaluate the final pressure of the gas. 4. Find the increase in specific internal energy and the increase in specific enthalpy.

[**Ans.** 28 kg, 12.4 bar, 0.089 m^3/kg ; 1.04 kJ/kg K, 0.7425 kJ/kg K ; 10.07 bar ; 52 kJ/kg, 72.8 kJ/kg]

QUESTIONS

1. What is a perfect gas ? Under what conditions does a real gas behave as a perfect gas?

2. Name the variables which control the physical properties of a perfect gas.

3. State Boyle's law and Charles' law and prove that the characteristic gas equation is $p \cdot v = m R T$.

4. What is the difference between universal gas constant and characteristic gas constant ?

5. Define the specific heat at constant volume and at constant pressure.

6. What do you understand by enthalpy ? Show that for a constant pressure process, the heat supplied to the gas is equal to the change of enthalpy.

7. Prove that the difference between two specific heats (c_p and c_v) is equal to characteristic gas constant (R).

8. What is an adiabatic index ? Why its value is always greater than unity ?

OBJECTIVE TYPE QUESTIONS

1. If the temperature remains constant, the volume of a given mass of a gas is inversely proportional to the pressure. This is known as

(*a*) Charles' law (*b*) Boyle's law (*c*) Joule's law (*d*) Gay-Lussac's law

2. The state of a substance whose evaporation from its liquid state is complete, is known as

(*a*) steam (*b*) vapour (*c*) air (*d*) perfect gas

3. The characteristic equation of a gas is

(*a*) $pv = \text{constant}$ (*b*) $pv = mR$ (*c*) $pv = mRT$ (*d*) $pv = RT^m$

where p, v, T and m = Pressure, volume, temperature and mass of the gas respectively, and R = Gas constant.

4. The value of gas constant (R) is

(*a*) 287 J/kg K (*b*) 28.7 J/kg K (*c*) 2.87 J/kg K (*d*) 0.287 J/kg K

5. The value of universal gas constant (R_u) is

   (*a*) 8.314 J/kg K (*b*) 83.14 J/kg K (*c*) 831.4 J/kg K (*d*) 8314 J/kg K

6. The gas constant (R) is equal to the................of two specific heats.

   (*a*) sum (*b*) difference (*c*) product (*d*) ratio

7. The specific heat at constant pressure is ....that of specific heat at constant volume.

   (*a*) equal to (*b*) less than (*c*) more than

8. The ratio of specific heat at constant pressure (c_p) and specific heat at constant volume (c_v) is

   (*a*) equal to one (*b*) less than one (*c*) more than one (*d*) none of these

9. The value of c_p/c_v for air is

   (*a*) 1 (*b*) 1.4 (*c*) 1.8 (*d*) 2.3

10. When the gas is heated at constant pressure, then the heat supplied

    (*a*) raises the temperature of the gas

    (*b*) increases the internal energy of the gas

    (*c*) does some external work during expansion

    (*d*) both (*a*) and (*b*)

    (*e*) both (*b*) and (*c*)

ANSWERS

1. (*b*) 2. (*d*) 3. (*c*) 4. (*a*) 5. (*d*)
6. (*b*) 7. (*c*) 8. (*c*) 9. (*b*) 10. (*e*)

3

Thermodynamic Processes of Perfect Gases

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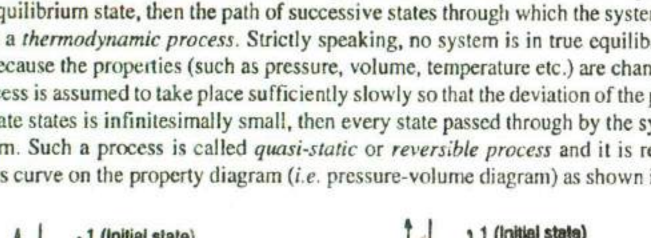
1. Introduction. 2. Classification of Thermodynamic Processes. 3. Workdone During a Non-flow Process. 4. Application of First Law of Thermodynamics to a Non-flow Process. 5. Heating and Expansion of Gases in Non-flow Processes. 6. Constant Volume Process (or Isochoric Process). 7. Constant Pressure Process (or Isobaric Process). 8. Hyperbolic Process. 9. Constant Temperature Process (or Isothermal Process). 10. Adiabatic Process (or Isentropic Process). 11. Polytropic Process. 12. Rate of Heat Transfer (Absorption or Rejection) per Unit Volume During a Polytropic Process. 13. Determination of Polytropic Index. 14. Free Expansion (or Un-resisted Expansion) Process. 15. General Laws for Expansion and Compression. 16. Summary of Formulae for Heating and Expansion of Perfect Gases in Reversible Non-flow Processes. 17. Flow Processes. 18. Application of First Law of Thermodynamics to a Steady Flow Process. 19. Workdone in a Steady Flow Process. 20. Workdone for Various Steady Flow Processes. 21. Throttling Process. 22. Application of Steady Flow Energy Equation to Engineering Systems.

3.1. Introduction

We have already discussed that when a system changes its state from one equilibrium state to another equilibrium state, then the path of successive states through which the system has passed, is known as a *thermodynamic process*. Strictly speaking, no system is in true equilibrium during the process because the properties (such as pressure, volume, temperature etc.) are changing. However, if the process is assumed to take place sufficiently slowly so that the deviation of the properties at the intermediate states is infinitesimally small, then every state passed through by the system will be in equilibrium. Such a process is called *quasi-static* or *reversible process* and it is represented by a continuous curve on the property diagram (*i.e.* pressure-volume diagram) as shown in Fig. 3.1 (*a*).



(*a*) Reversible process. (*b*) Irreversible process.

Fig. 3.1. Reversible and irreversible process.

If the process takes place in such a manner that the properties at the intermediate states are not in equilibrium state (except the initial and final state), then the process is said to be ***non-equilibrium*** or *irreversible process*. This process is represented by the broken lines on the property diagram as shown in Fig. 3.1 (*b*).

3.2. Classification of Thermodynamic Processes

All the thermodynamic processes are classified into the following two groups :

1. Non-flow processes, and 2. Flow processes.

The processes occurring in closed systems which do not permit the transfer of mass across their boundaries, are known as *non-flow processes*. It may be noted that in a non-flow process, the energy crosses the system boundary in the form of heat and work, but there is no mass flow into or out of the system.

The processes occurring in open systems which permit the transfer of mass to and from the system, are known as *flow processes*. It may be noted that in a flow process, the mass enters the system and leaves after enhancing energy. The flow processes may be steady flow and non-steady flow processes. The common examples of steady flow processes are flow through nozzles, turbines compressors etc. and the common examples of non-steady flow processes are filling or evacuation of vessels.

3.3. Workdone During a Non-flow Process

Consider a system contained in a frictionless piston and cylinder arrangement as shown in Fig. 3.2. As the system expands from its original state 1, it overcomes the external resistance (such as rotation of the flywheel) which opposes the motion of the piston by exerting a force through a distance. The variation of the volume and pressure of the system as it expands to final state 2, is drawn on the pressure-volume diagram (briefly called p-v diagram) as shown in Fig. 3.2.

Let at any small section (shown shaded), the pressure (p) of the system is constant. If A is the cross-sectional area of the piston, then force on the piston ($F = p\,A$) causes the piston to move through a distance dx. Thus, workdone by the system,

$$\delta W = F\,dx = p\,A\,dx = p\,dv \qquad \ldots (\because dv = A\,dx)$$

$\therefore$ Workdone for non-flow process from state 1 to state 2,

$$W_{1-2} = \int_1^2 \delta W = \int_1^2 p\,dv$$

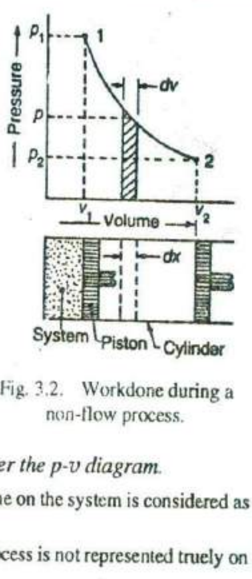


Fig. 3.2. Workdone during a non-flow process.

From above, we see that the *workdone is given by the area under the p-v diagram.*

Notes : 1. The workdone by the system is taken as *positive* while the workdone on the system is considered as *negative.*

2. For an irreversible process, $\delta W \neq p\,dv$, because the path of the process is not represented truely on the p-v diagram due to its non-equilibrium states in the process.

3.4. Application of First Law of Thermodynamics to a Non-flow Process

We have already discussed in Chapter 1 (Art. 1.49) that when a system undergoes a change of state or a thermodynamic process, then both the heat transfer and work transfer takes place. The net energy transfer is stored within the system and is known as *stored* or *total energy* of the system. Mathematically

$$Q_{1-2} - W_{1-2} = dE = E_2 - E_1 \qquad \ldots (i)$$

where Q_{1-2} = Heat transferred or heat supplied to the system during the process *i.e.* from state 1 to state 2,

W_{1-2} = Workdone by the system on the surrounding during the process *i.e.* from state 1 to state 2,

E_2 = *Total energy or stored energy of the system at the end of the process *i.e.* at state 2, and

E_1 = Total energy or stored energy of the system at the start of the process *i.e.* at state 1

For a non-flow process, the stored energy is the internal energy only. Thus equation (*i*) of the first law of thermodynamics, when applied to a non-flow process or a static system, may be written as

$$Q_{1-2} - W_{1-2} = dU = U_2 - U_1$$

where dU = Change in internal energy $= U_2 - U_1$

It may be noted that heat and work are not a property of the system, but their difference $(Q_{1-2} - W_{1-2})$ during a process is the numerical equivalent of stored energy. Since the stored energy is a property, therefore $(Q_{1-2} - W_{1-2})$ is also a property.

Example 3.1. *When a system is taken from the state A to the state B, in Fig. 3.3, along the path ACB, 80 kJ of heat flows into the system and the system does 30 kJ of work.*

1. How much heat flows into the system along the path ADB, if the workdone is 10 kJ.

2. When the system is returned from the state B to the state A along the curved path, the workdone on the system is 20 kJ. Does the system absorb or liberate heat and how much heat is absorbed or liberated.

3. If the internal energy at A $(U_A) = 0$ *and at D* $(U_D) = 40$ *kJ, find the heat absorbed in the processes AD and DB.*

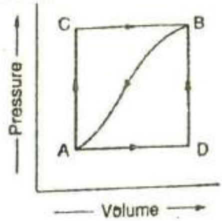


Fig. 3.3

Solution. Given : $Q_{ACB} = 80$ kJ ; $W_{ACB} = 30$ kJ

1. *Heat flowing into the system along the path ADB, if workdone* $(W_{ADB}) = 10$ *kJ*

Let Q_{ADB} = Heat flowing into the system along the path *ADB*,

U_A = Internal energy at *A*, and

U_B = Internal energy at *B*.

We know that for path *ACB*,

$$Q_{ACB} = {}^{**}(U_B - U_A) + W_{ACB}$$

$$\therefore \quad U_B - U_A = Q_{ACB} - W_{ACB} = 80 - 30 = 50 \text{ kJ}$$

We also know that for path *ADB*,

$$Q_{ADB} = (U_B - U_A) + W_{ADB} = 50 + 10 = 60 \text{ kJ} \quad \textbf{Ans.}$$

$$\ldots (\because W_{ADB} = 10 \text{ kJ})$$

2. *Heat absorbed or liberated*

Let Q_{B-A} = Heat absorbed or liberated along the curved path *BA*.

Since the work is done on the system, therefore

$$W_{B-A} = -20 \text{ kJ} \qquad \ldots \text{(Given)}$$

---

* The total energy of a system is the sum of potential energy (*PE*), kinetic energy (*KE*) and internal energy (*U*) plus any other form of the energy such as chemical energy, electrical energy etc.

** Since internal energy is a point function, and depends on the initial and final states, therefore dU is taken as $(U_B - U_A)$ where as heat and work are path functions.

We know that for the curved path *B-A*,

$$Q_{B-A} = (U_A - U_B) + W_{B-A} = -(U_B - U_A) + W_{B-A}$$

$$= -50 - 20 = -70 \text{ kJ} \ \textbf{Ans.}$$

The –ve sign indicates that heat is liberated by the system.

3. Heat absorbed in the processes AD and DB

Let Q_{AD} = Heat absorbed in the process *AD*,

Q_{DB} = Heat absorbed in the process *DB*,

U_A = Internal energy at $A = 0$ . . . (Given)

U_D = Internal energy at $D = 40$ kJ . . . (Given)

We know that work done along the path *ADB*,

$$W_{ADB} = W_{A-D} + W_{D-B}$$

Since the process *D-B* is a constant volume process, therefore $W_{D-B} = 0$. Thus, workdone during the process *A-D*,

$$W_{A-D} = W_{ADB} = 10 \text{ kJ}$$

We know that $Q_{A-D} = (U_D - U_A) + W_{A-D}$

$$= (40 - 0) + 10 = 50 \text{ kJ} \ \textbf{Ans.}$$

Similarly $Q_{D-B} = (U_B - U_D) + W_{D-B} = U_B - U_D$ $\quad \ldots (\because W_{D-B} = 0)$

$$= 50 - 40 = 10 \text{ kJ} \ \textbf{Ans.}$$

$\ldots (\because U_B - U_A = 50 \text{ kJ}$ and $U_A = 0$, therefore $U_B = 50 \text{ kJ})$

Example 3.2. *A fluid is confined in a cylinder by a spring loaded frictionless piston so that the pressure in the fluid is a linear function of the volume ($p = a + b\,v$). The internal energy of the fluid is given by the following equation :*

$$U = 33.5 + 3\,p\,v$$

where U is in kJ, p in kN/m² and v in m³. If the fluid changes from an initial state of 1.7 bar, 0.03 m³ to a final state of 4 bar, 0.06 m³, with no work other than that done on the piston, find the direction and magnitude of the work and heat transfer.

Solution. Given : $p = a + b\,v$; $U = 33.5 + 3\,p\,v$; $p_1 = 1.7 \text{ bar} = 170 \text{ kN/m}^2$; $v_1 = 0.03 \text{ m}^3$; $p_2 = 4 \text{ bar} = 400 \text{ kN/m}^2$; $v_2 = 0.06 \text{ m}^3$

We know that change in internal energy of the fluid,

$$dU = U_2 - U_1 = (33.5 + 3\,p_2 v_2) - (33.5 + 3\,p_1 v_1)$$

$$= 3\,(p_2 v_2 - p_1 v_1) = 3\,(400 \times 0.06 - 170 \times 0.03) \text{ kJ}$$

$$= 3\,(24 - 5.1) = 56.7 \text{ kJ}$$

We also know that $p = a + b\,v$

∴ For the initial state of fluid,

$$170 = a + b \times 0.03 \quad \ldots (i)$$

and for the final state of fluid,

$$400 = a + b \times 0.06 \quad \ldots (ii)$$

Subtracting equation (*i*) from equation (*ii*),

$$400 - 170 = 0.06\,b - 0.03\,b \quad \text{or} \quad 230 = 0.03\,b$$

$$\therefore \qquad b = 7667$$

Substituting the value of b in equation (*i*),

$$170 = a + 7667 \times 0.03 = a + 230$$

$$\therefore \qquad a = -60$$

We know that work transfer,

$$W_{1-2} = \int_{v_1}^{v_2} p\,dv = \int_{v_1}^{v_2} (a + b\,v)\,dv$$

$$= \left[a\,v + \frac{b\,v^2}{2} \right]_{v_1}^{v_2} = a\,(v_2 - v_1) + b\left(\frac{v_2^2 - v_1^2}{2} \right)$$

$$= -60\,(0.06 - 0.03) + 7667\left[\frac{(0.06)^2 - (0.03)^2}{2} \right]$$

$$= -1.8 + 10.35 = 8.55 \text{ kJ} \quad \textbf{Ans.}$$

$\therefore$ Heat transfer, $Q_{1-2} = W_{1-2} + dU = 8.55 + 56.7 = 65.25$ kJ **Ans.**

Since both work transfer and heat transfer are *positive*, therefore work is done by the fluid and heat is supplied to the fluid.

3.5. Heating and Expansion of Gases in Non-flow Processes

The heating and expanding of a gas may be performed in many ways. But the following are the different non-flow processes (reversible and irreversible) as applied to perfect gas :

1. *Reversible non-flow processes.* These processes are as follows :

   (*a*) Constant volume process (or Isochoric process),

   (*b*) Constant pressure process (or Isobaric process),

   (*c*) Hyperbolic process,

   (*d*) Constant temperature process (or Isothermal process)

   (*e*) Adiabatic process (or Isentropic process), and

   (*f*) Polytropic process

2. *Irreversible non-flow processes.* The free expansion process is an irreversible non-flow process.

These above mentioned processes are discussed, in detail, in the following pages.

Note: The above mentioned processes are also applicable to the cooling and compression of gases. Cooling is regarded as negative heating, and compression as negative expansion.

3.6. Constant Volume Process (or Isochoric Process)

We have already discussed that when a gas is heated at a constant volume, its temperature and pressure will increase. Since there is no change in its volume, therefore no work is done by the gas. All the heat supplied to the gas is stored within the gas in the form of internal energy. It may be noted that this process is governed by Gay-Lussac law. Now consider m kg of a certain gas being heated at constant volume from initial state 1 to a final state 2.

Let p_1, v_1 and T_1 = Pressure, volume and temperature at the initial state 1, and

p_2, v_2 and T_2 = Pressure, volume and temperature at the final state 2.

The process is shown on the pressure-volume (p-v) and pressure-temperature (p-T) diagrams in Fig. 3.4 (*a*) and (*b*) respectively.

(*a*) p-v diagram. (*b*) p-T diagram.

Fig. 3.4. Constant volume process.

Now let us derive the following relations for the reversible constant volume process.

1. *Pressure-volume-temperature (p-v-T) relationship*

We know that the general gas equation is

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} \qquad \ldots (i)$$

Since the gas is heated at constant volume, therefore $v_1 = v_2$.

$$\therefore \qquad \frac{p_1}{T_1} = \frac{p_2}{T_2} \text{ or } \frac{p}{T} = \text{Constant} \qquad \ldots \text{[From equation } (i)]$$

Thus the constant volume process is governed by Gay-Lussac law.

2. *Workdone by the gas*

We know that $\delta W = p\, dv$

On integrating from state 1 to state 2,

$$\int_1^2 \delta W = \int_1^2 p\, dv = p \int_1^2 dv$$

or

$$W_{1-2} = p\,(v_2 - v_1) = 0 \qquad \ldots (\because v_1 = v_2)$$

3. *Change in internal energy*

We know that change in internal energy,

$$dU = m\, c_v\, dT \qquad \ldots \text{(Joule's law)}$$

On integrating from state 1 to state 2,

$$\int_1^2 dU = m\, c_v \int_1^2 dT$$

or

$$U_2 - U_1 = m\, c_v\, (T_2 - T_1)$$

4. *Heat supplied or heat transfer*

We know that $\delta Q = dU + \delta W$

On integrating from state 1 to state 2,

$$\int_1^2 \delta Q = \int_1^2 dU + \int_1^2 \delta W$$

or

$$Q_{1-2} = (U_2 - U_1) + W_{1-2}$$

Since $W_{1-2} = 0$, therefore heat supplied or heat transfer,

$$Q_{1-2} = U_2 - U_1 = m\,c_v\,(T_2 - T_1)$$

This shows that all the heat supplied to the gas is utilised in increasing the internal energy of the gas.

5. Change in enthalpy

We know that the change in enthalpy,

$$dH = dU + d\,(pv)$$

On Integrating from state 1 to state 2,

$$\int_1^2 dH = \int_1^2 dU + \int_1^2 d\,(p\,v)$$

or

$$H_2 - H_1 = (U_2 - U_1) + (p_2 v_2 - p_1 v_1)$$

$$= m\,c_v\,(T_2 - T_1) + m\,R\,(T_2 - T_1)$$

$$\ldots (\because p_1 v_1 = m\,R\,T_1 \,;\text{and}\, p_2 v_2 = m\,R\,T_2)$$

$$= m\,(T_2 - T_1)\,(c_v + R) = m\,c_p\,(T_2 - T_1) \qquad \ldots (\because c_p - c_v = R)$$

Notes : 1. The change in internal energy (dU) and the change in enthalpy (dH) have the same expression for each process.

2. During expansion or heating process, work is done by the gas (*i.e.* W_{1-2} is + ve) ; internal energy of the gas decreases (*i.e.* dU is −ve) and heat is supplied to the gas (*i.e.* Q_{1-2} is +ve).

3. During compression or cooling process, work is done on the gas (*i.e.* W_{1-2} is −ve) ; internal energy of the gas increases (*i.e.* dU is +ve) and heat is rejected by the gas (*i.e.* Q_{1-2} is −ve).

Example 3.3. *A certain gas occupies a space of 0.3 m³ at a pressure of 2 bar and a temperature of 77° C. It is heated at a constant volume, until the pressure is 7 bar. Determine : 1. temperature at the end of the process ; 2. mass of the gas ; 3. change in internal energy ; and 4. change in enthalpy during the process.*

Assume $c_p = 1.005$ *kJ/kg K ;* $c_v = 0.712$ *kJ/kg K ; and* $R = 287$ *J/kg K.*

Solution. Given : $v_1 = 0.3\ \text{m}^3$; $p_1 = 2\ \text{bar} = 0.2 \times 10^6\ \text{N/m}^2$; $T_1 = 77°\text{C} = 77 + 273 = 350\ \text{K}$; $p_2 = 7\ \text{bar} = 0.7 \times 10^6\ \text{N/m}^2$; $c_p = 1.005$ kJ/kg K ; $c_v = 0.712$ kJ/kg K ; $R = 287$ J/kg K

1. Temperature at the end of the process

Let T_2 = Temperature at the end of the process.

We know that $\dfrac{p_1}{T_1} = \dfrac{p_2}{T_2}$

$$\therefore \quad T_2 = \frac{p_2 T_1}{p_1} = \frac{0.7 \times 10^6 \times 350}{0.2 \times 10^6} = 1225\ \text{K} = 1225 - 273 = 952°\ \text{C} \quad \textbf{Ans.}$$

2. *Mass of the gas*

Let m = Mass of the gas.

We know that $p_1 v_1 = m R T_1$

$$\therefore \quad m = \frac{p_1 v_1}{R T_1} = \frac{0.2 \times 10^6 \times 0.3}{287 \times 350} = 0.597 \text{ kg } \textbf{Ans.}$$

3. *Change in internal energy*

We know that change in internal energy,

$$dU = U_2 - U_1 = m c_v (T_2 - T_1)$$
$$= 0.597 \times 0.712 (1225 - 350) = 372 \text{ kJ } \textbf{Ans.}$$

4. *Change in enthalpy*

We know that change in enthalpy,

$$dH = H_2 - H_1 = m c_p (T_2 - T_1)$$
$$= 0.597 \times 1.005 (1225 - 350) = 525 \text{ kJ } \textbf{Ans.}$$

3.7. Constant Pressure Process (or Isobaric Process)

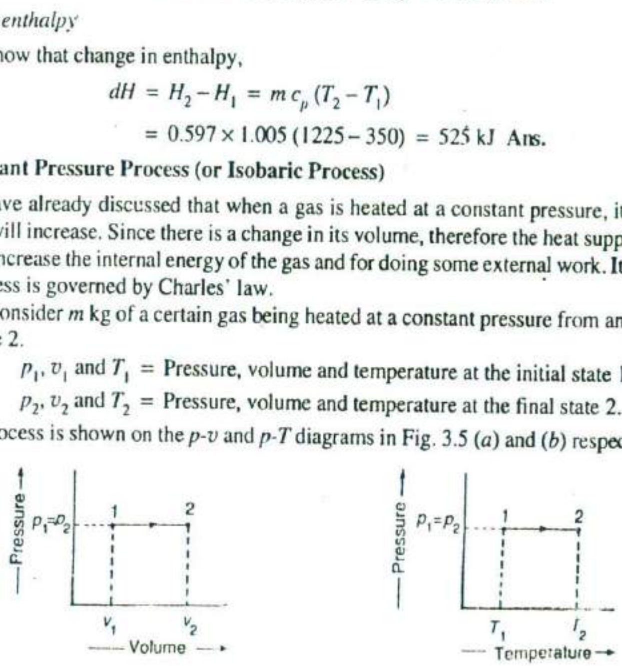
We have already discussed that when a gas is heated at a constant pressure, its temperature and volume will increase. Since there is a change in its volume, therefore the heat supplied to the gas is utilised to increase the internal energy of the gas and for doing some external work. It may be noted that this process is governed by Charles' law.

Now consider m kg of a certain gas being heated at a constant pressure from an initial state 1 to a final state 2.

Let p_1, v_1 and T_1 = Pressure, volume and temperature at the initial state 1, and

p_2, v_2 and T_2 = Pressure, volume and temperature at the final state 2.

The process is shown on the p-v and p-T diagrams in Fig. 3.5 (*a*) and (*b*) respectively.



(*a*) p-v diagram. (*b*) p-T diagram

Fig. 3.5. Constant pressure process.

Now let us derive the following relations for the reversible constant pressure process.

1. *Pressure -volume-temperature (p-v-T relationship)*

We know that the general gas equation is

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} \quad \ldots (i)$$

Since the gas is heated at constant pressure, therefore $p_1 = p_2$.

$$\therefore \quad \frac{v_1}{T_1} = \frac{v_2}{T_2} \text{ or } \frac{v}{T} = \text{Constant} \quad \ldots [\text{From equation } (i)]$$

Thus, the constant pressure process is governed by Charles' law.

2. *Workdone by the gas*

We know that $\delta W = p\,dv$

On integrating from state 1 to state 2,

$$\int_1^2 \delta W = \int_1^2 p\,dv = p\int_1^2 dv$$

or $$W_{1-2} = p\,(v_2 - v_1) = m\,R\,(T_2 - T_1)$$

$$\ldots (\because\ p\,v_1 = m\,R\,T_1 \text{ and } p\,v_2 = m\,R\,T_2)$$

3. *Change in internal energy*

We have discussed in the previous article that the change in internal energy (dU) is same for all the processes. Therefore change in internal energy,

$$dU = U_2 - U_1 = m\,c_v\,(T_2 - T_1)$$

4. *Heat supplied or heat transferred*

We know that $\delta Q = dU + \delta W$

On integrating from state 1 to state 2,

$$\int_1^2 \delta Q = \int_1^2 dU + \int_1^2 \delta W$$

$$\therefore \quad Q_{1-2} = (U_2 - U_1) + W_{1-2} \qquad \ldots (ii)$$

$$= m\,c_v\,(T_2 - T_1) + m\,R\,(T_2 - T_1)$$

$$= m\,(T_2 - T_1)\,(c_v + R)$$

$$= m\,c_p\,(T_2 - T_1) \qquad \ldots (\because\ c_p - c_v = R)$$

The equation (*ii*) shows that the heat supplied to the gas is utilised in increasing the internal energy of the gas and for doing some external work.

5. *Change in enthalpy*

We have discussed in the previous article that the change in enthalpy (dH) is same for all the processes. Therefore, change in enthalpy,

$$dH = H_2 - H_1 = m\,c_p\,(T_2 - T_1)$$

We see that change in enthalpy is equal to the heat supplied or heat transferred.

Note : If the gas is cooled at a constant pressure, then there will be a compression. It is thus obvious that, during cooling, the temperature and volume will decrease and work is said to be done on the gas. In this case,

Workdone on the gas, $W_{1-2} = p\,(v_1 - v_2) = m\,R\,(T_1 - T_2)$

Decrease in internal energy, $dU = U_1 - U_2 = m\,c_v\,(T_1 - T_2)$

and heat rejected by the gas, $Q_{1-2} = m\,c_p\,(T_1 - T_2)$

Example 3.4. *The values of specific heats at constant pressure and at constant volume for an ideal gas are 0.984 kJ/kg K and 0.728 kJ/kg K. Find the values of characteristic gas constant (R) and ratio of specific heats (γ) for the gas. If one kg of this gas is heated at constant pressure from 25° C to 200° C, estimate the heat added, ideal workdone and change in internal energy. Also calculate the pressure and final volume, if the initial volume was 2 m³.*

Solution. Given : $c_p = 0.984$ kJ/kg K ; $c_v = 0.728$ kJ/kg K ; $m = 1$ kg ; $T_1 = 25°\text{ C} = 25 + 273 = 298$ K ; $T_2 = 200°\text{ C} = 200 + 273 = 473$ K ; $v_1 = 2\text{ m}^3$

The heating of gas at constant pressure is shown in Fig. 3.6.

Characteristic gas constant

We know that characteristic gas constant,

$$R = c_p - c_v = 0.984 - 0.728$$

$$= 0.256 \text{ kJ/kg K} \textbf{ Ans.}$$

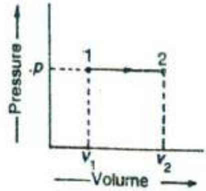


Fig. 3.6

Ratio of specific heats

We know that ratio of specific heats,

$$\gamma = c_p / c_v = 0.984 / 0.728 = 1.35 \textbf{ Ans.}$$

Heat added

We know that heat added during constant pressure operation,

$$Q_{1-2} = m\,c_p\,(T_2 - T_1) = 1 \times 0.984\,(473 - 298) = 172.2 \text{ kJ} \textbf{ Ans.}$$

Workdone

We know that workdone during constant pressure operation,

$$W_{1-2} = p\,(v_2 - v_1) = m\,R\,(T_2 - T_1) \qquad \ldots (\because pv = mRT)$$

$$= 1 \times 0.256\,(473 - 298) = 44.8 \text{ kJ} \textbf{ Ans.}$$

Change in internal energy

*We know that change in internal energy,

$$dU = U_2 - U_1 = m\,c_v\,(T_2 - T_1) = 1 \times 0.728\,(473 - 298) \text{ kJ}$$

$$= 127.4 \text{ kJ} \textbf{ Ans.}$$

Pressure and final volume of the gas if the initial volume, $v_1 = 2$ m^3

Let $p_1 = p_2$ = Pressure of the gas, and

v_2 = Final volume of the gas.

We know that $$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} \quad \text{or} \quad \frac{v_1}{T_1} = \frac{v_2}{T_2} \qquad \ldots (\because \text{Pressure is constant})$$

$$\therefore \quad v_2 = \frac{v_1 T_2}{T_1} = \frac{2 \times 473}{298} = 3.17 \text{ m}^3 \textbf{ Ans.}$$

We also know that

$$p_1 v_1 = m\,R\,T_1$$

$$\therefore \quad p_1 = \frac{m\,R\,T_1}{v_1} = \frac{1 \times 256 \times 298}{2} = 38\,140 \text{ N/m}^2 \qquad \ldots (R \text{ is taken in J/kg K})$$

$$= 0.3814 \text{ bar} \textbf{ Ans.} \qquad \ldots (\because 1 \text{ bar} = 10^5 \text{ N/m}^2)$$

Example 3.5. *A quantity of gas has a volume of 0.14 m^3, pressure 1.5 bar and a temperature 100° C. If the gas is compressed at a constant pressure, until its volume becomes 0.112 m^3, determine :*

---

* The change in internal energy may also be obtained from the relation

$$Q_{1-2} = dU + W_{1-2} \quad \text{or} \quad dU = Q_{1-2} - W_{1-2} = 172.2 - 44.8 = 127.4 \text{ kJ}$$

1. the temperature at the end of compression ; 2. work done in compressing the gas ; 3. decrease in internal energy ; and 4. heat given out by the gas.

Assume $c_p = 1.005$ kJ/kg K ; $c_v = 0.712$ kJ/kg K ; and $R = 285$ J/kg K

Solution. Given : $v_1 = 0.14\ \text{m}^3$; $p = 1.5$ bar $= 0.15 \times 10^6\ \text{N/m}^2$; $T_1 = 100°\text{C} = 100 + 273 = 373$ K ; $v_2 = 0.112\ \text{m}^3$; $c_p = 1.005$ kJ/kg K ; $c_v = 0.712$ kJ/kg K ; $R = 285$ J/kg K

The compression of gas at constant pressure is shown in Fig. 3.7.

1. Temperature at the end of compression

Let T_2 = Temperature at the end of compression.

We know that $\dfrac{v_1}{T_1} = \dfrac{v_2}{T_2}$

$$\therefore \quad T_2 = \frac{v_2 T_1}{v_2} = \frac{0.112 \times 373}{0.14} = 298.4 \text{ K}$$

$$= 298.4 - 273 = 25.4°\text{C} \ \textbf{Ans.}$$

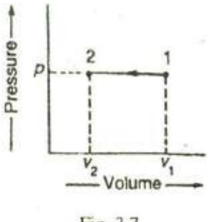


Fig. 3.7

2. Workdone is compressing the gas

We know that workdone in compressing the gas,

$$W_{1-2} = p\,(v_1 - v_2) = 0.15 \times 10^6\,(0.14 - 0.112) = 4200 \text{ J} = 4.2 \text{ kJ} \ \textbf{Ans.}$$

3. Decrease in internal energy

First of all, let us find the mass of gas (m) admitted for compression. We know that

$$p_1 v_1 = m R T_1$$

$$\therefore \quad m = \frac{p_1 v_1}{R T_1} = \frac{0.15 \times 10^6 \times 0.14}{285 \times 373} = 0.197 \text{ kg} \qquad \ldots (\because p_1 = p)$$

We know that decrease in internal energy,

$$dU = U_1 - U_2 = m\,c_v\,(T_1 - T_2)$$

$$= 0.197 \times 0.712\,(373 - 298.4) = 10.46 \text{ kJ} \ \textbf{Ans.}$$

4. Heat given out by the gas

We know that heat given out by the gas,

$$Q_{1-2} = m\,c_p\,(T_1 - T_2)$$

$$= 0.197 \times 1.005\,(373 - 298.4) = 14.77 \text{ kJ} \ \textbf{Ans.}$$

3.8. Hyperbolic Process

A process, in which the gas is heated or expanded in such a way that the product of its pressure and volume (*i.e.* $p \times v$) remains constant, is called a *hyperbolic process.*

It may be noted that the hyperbolic process is governed by Boyle's law *i.e.* $p\,v$ = constant. If we plot a graph for pressure and volume, during the process as shown in Fig. 3.8, we shall get a rectangular hyperbola. Hence, this process is termed as hyperbolic process. It is merely a theoretical case, and has a little importance from the subject point of view. Its practical application is isothermal process, which is discussed below.

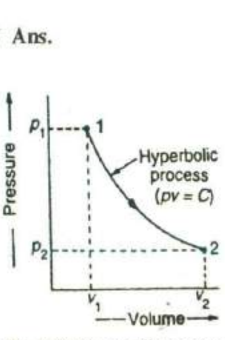


Fig. 3.8. Hyperbolic process.

3.9. Constant Temperature Process (or Isothermal Process)

A process, in which the temperature of the working substance remains constant during its expansion or compression, is called *constant temperature process* or *isothermal process*. This will happen when the working substance remains in a perfect thermal contact with the surroundings, so that the heat 'sucked in' or 'squeezed out' is compensated exactly for the work done by the gas or on the gas respectively. It is thus obvious that in an isothermal process :

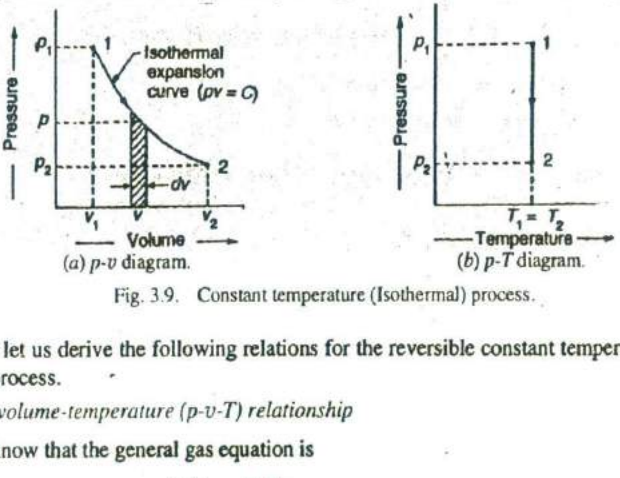
1. there is no change in temperature,
2. there is no change in internal energy, and
3. there is no change in enthalpy.

Now consider *m* kg of a certain gas being heated at constant temperature from an initial state 1 to final state 2.

Let p_1, v_1 and T_1 = Pressure, volume and temperature at the initial state 1, and

p_2, v_2 and T_2 = Pressure, volume and temperature at the final state 2.

The process is shown on the *p-v* and *p-T* diagrams in Fig. 3.9 (*a*) and (*b*) respectively.



(*a*) *p-v* diagram. (*b*) *p-T* diagram.

Fig. 3.9. Constant temperature (Isothermal) process.

Now let us derive the following relations for the reversible constant temperature process or isothermal process.

1. *Pressure-volume-temperature (p-v-T) relationship*

We know that the general gas equation is

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} \qquad \ldots (i)$$

Since the gas is heated at constant temperature, therefore $T_1 = T_2$.

$$\therefore \quad p_1 v_1 = p_2 v_2 \quad \text{or} \quad p\,v = \text{Constant} \qquad \ldots \text{[From equation (i)]}$$

Thus, the constant temperature process or isothermal process is governed by Boyle's law.

2. *Workdone by the gas*

We know that $\delta W = p\,dv$

On integrating from state 1 to state 2,

$$\int_1^2 \delta W = \int_1^2 p\,dv$$

or

$$W_{1-2} = \int_1^2 p\,dv \qquad \ldots (ii)$$

Since the expansion of the gas is isothermal, *i.e.* $p\,v = C$, therefore

$$p\,v = p_1\,v_1 \quad \text{or} \quad p = \frac{p_1\,v_1}{v}$$

Substituting this value of p in equation (*ii*), we have

$$W_{1-2} = \int_{v_1}^{v_2} \frac{p_1\,v_1}{v}\,dv = p_1\,v_1 \int_{v_1}^{v_2} \frac{dv}{v}$$

$$= p_1\,v_1 \Big[\log_e v\Big]_{v_1}^{v_2} = p_1\,v_1\,\log_e\left(\frac{v_2}{v_1}\right) \quad \ldots(iii)$$

The above equation may be expressed in terms of corresponding logarithm to the base 10, *i.e.*

$$W_{1-2} = 2.3\,p_1\,v_1 \log\left(\frac{v_2}{v_1}\right) = 2.3\,p_1\,v_1 \log r \quad \ldots(iv)$$

where $r = \dfrac{v_2}{v_1}$, and is known as *expansion ratio.*

The equation (*iv*) may also be written as follows :

We know that $p_1\,v_1 = p_2\,v_2 = m\,R\,T$

$$\therefore \text{ Work done, } \quad W_{1-2} = 2.3\,m\,R\,T \log\left(\frac{v_2}{v_1}\right) = 2.3\,m\,R\,T \log r$$

Since $p_1\,v_2 = p_2\,v_2$, therefore $\dfrac{v_2}{v_1} = \dfrac{p_1}{p_2}$

$$\therefore \text{ Work done, } \quad W_{1-2} = 2.3\,p_1\,v_1 \log\left(\frac{p_1}{p_2}\right)$$

Notes : (*a*) Expansion ratio, $r = \dfrac{\text{Volume at the end of expansion}}{\text{Volume at the beginning of expansion}}$

(*b*) Compression ratio, $r = \dfrac{\text{Volume at the beginning of compression}}{\text{Volume at the end of compression}}$

3. *Change in internal energy*

We know that change in internal energy,

$$dU = U_2 - U_1 = m\,c_v\,(T_2 - T_1)$$

Since it is a constant temperature process, *i.e.* $T_1 = T_2$, therefore

$$dU = U_2 - U_1 = 0 \quad \text{or} \quad U_1 = U_2$$

4. *Heat supplied or heat transferred*

We know that heat supplied or heat transferred from state 1 to state 2,

$$Q_{1-2} = dU + W_{1-2} = W_{1-2} \quad \ldots(\because dU = 0)$$

This shows that all the heat supplied to the gas is equal to the workdone by the gas.

5. *Change in enthalpy*

We know that change in enthalpy,

$$dH = H_2 - H_1 = m\,c_p\,(T_2 - T_1)$$

Since it is a constant temperature process, *i.e.* $T_1 = T_2$, therefore

$$dH = H_2 - H_1 = 0 \text{ or } H_1 = H_2$$

Example 3.6. *A quantity of air has a volume of 0.4 m^3 at a pressure of 5 bar and a temperature of 80° C. It is expanded in a cylinder at a constant temperature to a pressure of 1 bar. Determine the amount of work done by the air during expansion.*

Solution. Given : $v_1 = 0.4\ \text{m}^3$; $p_1 = 5\ \text{bar} = 0.5 \times 10^6\ \text{N/m}^2$; *$T = 80°\text{C}$; $p_2 = 1\ \text{bar} = 0.1 \times 10^6\ \text{N/m}^2$

First of all, let us find the volume of air at the end of expansion (*i.e.* v_2). We know that

$$p_1 v_1 = p_2 v_2 \text{ or } v_2 = \frac{p_1 v_1}{p_2} = \frac{0.5 \times 10^6 \times 0.4}{0.1 \times 10^6} = 2\ \text{m}^3$$

and expansion ratio, $r = v_2 / v_1 = 2 / 0.4 = 5$

We know that workdone by the air during expansion,

$$W_{1-2} = 2.3\, p_1 v_1 \log r = 2.3 \times 0.5 \times 10^6 \times 0.4 \log 5\ \text{J}$$

$$= 0.46 \times 10^6 \times 0.699 = 321\,540\ \text{J} = 321.54\ \text{kJ} \textbf{ Ans.}$$

Example 3.7. *0.1 m^3 of air at a pressure of 1.5 bar is expanded isothermally to 0.5 m^3. Calculate the final pressure of the gas and heat supplied during the process.*

Solution. Given : $v_1 = 0.1\ \text{m}^3$; $p_1 = 1.5\ \text{bar} = 0.15 \times 10^6\ \text{N/m}^2$; $v_2 = 0.5\ \text{m}^3$

Final pressure of the gas

Let p_2 = Final pressure of the gas.

We know that $p_1 v_1 = p_2 v_2$

$$\therefore \quad p_2 = \frac{p_1 v_1}{v_2} = \frac{0.15 \times 10^6 \times 0.1}{0.5} = 0.03 \times 10^6\ \text{N/m}^2 = 0.3\ \text{bar} \textbf{ Ans.}$$

Heat supplied during the process

We know that expansion ratio,

$$r = v_2 / v_1 = 0.5 / 0.1 = 5$$

∴ Workdone during the process,

$$W_{1-2} = 2.3\, p_1 v_1 \log r = 2.3 \times 0.15 \times 10^6 \times 0.1 \log 5\ \text{J}$$

$$= 0.0345 \times 10^6 \times 0.699 = 24\,115\ \text{J} = 24.115\ \text{kJ}$$

We know that in an isothermal process, heat supplied (Q_{1-2}) is equal to the workdone during the process.

$$\therefore \quad Q_{1-2} = W_{1-2} = 24.115\ \text{kJ} \textbf{ Ans.}$$

3.10. Adiabatic Process (or Isentropic Process)

A process, in which the working substance neither receives nor gives out heat to its surroundings, during its expansion or compression, is called an ***adiabatic process*. This will happen when

---

* Superfluous data

** It may be noted that the adiabatic process may be reversible or irreversible. The reversible adiabatic process or frictionless adiabatic process is known as *isentropic process* (or constant entropy process). But when friction is involved in the process, then the adiabatic process is said to be irreversible, in which case the entropy does not remain constant *i.e.* the entropy increases (See Chapter 4, Art. 4.7).

the working substance remains thermally insulated, so that no heat enters or leaves it during the process. It is thus obvious, that in an adiabatic or isentropic process :

1. No heat leaves or enters the gas,
2. The temperature of the gas changes, as the work is done at the cost of internal energy, and
3. The change in internal energy is equal to the work done.

Now consider m kg of a certain gas being heated adiabatically from an initial state 1 to a final state 2.

Let p_1, v_1 and T_1 = Pressure, volume and temperature at the initial state 1, and

p_2, v_2 and T_2 = Pressure, volume and temperature at the final state 2.

The process is shown on the p-v diagram in Fig. 3.10.

Now let us derive the following relations for a reversible adiabatic process.

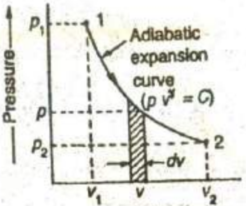


Fig. 3.10. Adiabatic process.

1. *Pressure-volume-temperature (p-v-T) relationship*

We know that $\delta Q = \delta W + dU$ (First law of thermodynamics) . . . (*i*)

Since in an adiabatic process, no heat transfer takes places, therefore

$$\delta Q = 0$$

$$\therefore \quad \delta W + dU = 0$$

or

$$p\,dv + m\,c_v\,dT = 0$$

$$\therefore \quad dT = \frac{-p\,dv}{m\,c_v} \qquad \ldots (ii)$$

We know that $p\,v = m\,R\,T$

Differentiating this expression, we get

$$p\,dv + v\,dp = m\,R\,dT$$

$$\therefore \quad dT = \frac{p\,dv + v\,dp}{m\,R} = \frac{p\,dv + v\,dp}{m\,(c_p - c_v)} \qquad \ldots (iii)$$

$$\ldots (\because R = c_p - c_v)$$

Equating equations (*ii*) and (*iii*),

$$\frac{-p\,dv}{m\,c_v} = \frac{p\,dv + v\,dp}{m\,(c_p - c_v)}$$

$$\frac{c_p - c_v}{c_v} = \frac{p\,dv + v\,dp}{-p\,dv} = -1 - \frac{v\,dp}{p\,dv}$$

$$\frac{c_p}{c_v} - 1 = -1 - \left(\frac{v}{dv} \times \frac{dp}{p}\right)$$

$$\gamma = -\left(\frac{v}{dv} \times \frac{dp}{p}\right) \qquad \ldots \left(\because \frac{c_p}{c_v} = \gamma\right)$$

$$\therefore \qquad \gamma \times \frac{dv}{v} = -\frac{dp}{p}$$

$$\gamma \times \frac{dv}{v} + \frac{dp}{p} = 0$$

Integrating both sides,

$$\gamma \log_e v + \log_e p = \text{Constant} \quad \text{or} \quad \log_e pv^\gamma = \log_e C$$

$$\therefore \qquad pv^\gamma = C \quad \text{or} \quad p_1 v_1^\gamma = p_2 v_2^\gamma = \ldots = C \qquad \ldots (iv)$$

The equation (*iv*) may also be expressed in the following forms ;

$$\frac{p_1}{p_2} = \left(\frac{v_2}{v_1}\right)^\gamma \qquad \ldots (v)$$

From the general gas equation, we know that

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} \quad \text{or} \quad \frac{p_1}{p_2} = \frac{T_1}{T_2} \times \qquad \ldots (vi)$$

Equating equations (*v*) and (*vi*),

$$\left(\frac{v_2}{v_1}\right)^\gamma = \frac{T_1}{T_2} \times \frac{v_2}{v_1} \quad \text{or} \quad \frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^\gamma \times \frac{v_1}{v_2} = \left(\frac{v_2}{v_1}\right)^\gamma \left(\frac{v_2}{v_1}\right)^{-1}$$

$$\therefore \qquad \frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{\gamma - 1} \qquad \ldots (vii)$$

From equation (*iv*), we also know that

$$\frac{v_1}{v_2} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} \qquad \ldots (viii)$$

From the general gas equation, we know that

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} \quad \text{or} \quad \frac{v_1}{v_2} = \frac{T_1}{T_2} \times \frac{p_2}{p_1} \qquad \ldots (ix)$$

Equating equations (*viii*) and (*ix*),

$$\left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} = \frac{T_1}{T_2} \times \frac{p_2}{p_1} \quad \text{or} \quad \frac{T_1}{T_2} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} \times \frac{p_1}{p_2} = \left(\frac{p_1}{p_2}\right)^{-\frac{1}{\gamma} + 1}$$

$$\therefore \qquad \frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma - 1}{\gamma}} \qquad \ldots (x)$$

2. *Workdone during adiabatic expansion*

We have already discussed that workdone,

$$\delta W = p\, dv$$

On integrating from state 1 to state 2,

$$\int_1^2 \delta W = \int_1^2 p\, dv \quad \text{or} \quad W_{1-2} = \int_1^2 p\, dv \qquad \ldots (xi)$$

Since adiabatic expansion of the gas follows the law,

$$pv^{\gamma} = p_1 v_1^{\gamma} \quad \text{or} \quad p = \frac{p_1 v_1^{\gamma}}{v^{\gamma}}$$

Substituting this value of p in equation (*xi*),

$$W_{1-2} = \int_1^2 \frac{p_1 v_1^{\gamma}}{v^{\gamma}}\, dv = p_1 v_1^{\gamma} \int_1^2 v^{-\gamma}\, dv = p_1 v_1^{\gamma} \left[\frac{v^{-\gamma+1}}{-\gamma+1} \right]_1^2$$

$$= \frac{p_1 v_1^{\gamma}}{1-\gamma} \left[v_2^{1-\gamma} - v_1^{1-\gamma} \right]$$

$$= \frac{p_1 v_1^{\gamma} v_2^{1-\gamma} - p_1 v_1^{\gamma} v_1^{1-\gamma}}{1-\gamma}$$

$$= \frac{p_2 (v_2^{\gamma} v_2^{1-\gamma}) - p_1 (v_1^{\gamma} v_1^{1-\gamma})}{1-\gamma} \qquad \ldots (\because p_1 v_1^{\gamma} = p_2 v_2^{\gamma})$$

$$= \frac{p_2 v_2 - p_1 v_1}{1-\gamma}$$

$$= \frac{p_1 v_1 - p_2 v_2}{\gamma - 1} \qquad \ldots \text{(For expansion)}$$

$$= \frac{p_2 v_2 - p_1 v_1}{\gamma - 1} \qquad \ldots \text{(For compression)}$$

Note : The above equation for work done may also be expressed as :

(*a*) We know that $p_1 v_1 = m\,R\,T_1$ and $p_2 v_2 = m\,R\,T_2$. Substituting these values in the equation for expansion,

$$W_{1-2} = \frac{m\,R\,T_1 - m\,R\,T_2}{\gamma - 1} = \frac{m\,R\,(T_1 - T_2)}{\gamma - 1} \qquad \ldots \text{(For expansion)}$$

$$= \frac{m\,R\,(T_2 - T_1)}{\gamma - 1} \qquad \ldots \text{(For compression)}$$

(*b*) We also know that work done during expansion,

$$W_{1-2} = \frac{p_1 v_1 - p_2 v_2}{\gamma - 1} = \frac{p_1 v_1}{\gamma - 1} \left[1 - \frac{p_2 v_2}{p_1 v_1} \right]$$

$$= \frac{m\,R\,T_1}{\gamma - 1} \left[1 - \frac{p_2 v_2}{p_1 v_1} \right] \qquad \ldots (\because p\,v = m\,R\,T)$$

3. *Change in internal energy*

We know that change in internal energy,

$$dU = U_2 - U_1 = m\,c_v\,(T_2 - T_1)$$

4. *Heat supplied or heat transferred*

We know that head supplied or heat transferred in case of adiabatic process is zero, therefore

$$Q_{1-2} = 0$$

5. *Change in enthalpy*

We know that change in enthalpy,

$$dH = H_2 - H_1 = m\,c_p\,(T_2 - T_1)$$

Example 3.8. *One litre of hydrogen at 0° C is suddenly compressed to one-half of its volume. Find the change in temperature of the gas, if the ratio of two specific heats for hydrogen is 1.4.*

Solution. Given : $v_1 = 1$ litre ; $T_1 = 0^\circ \text{ C} = 0 + 273 = 273$ K ; $v_2 = v_1/2 = 1/2 = 0.5$ litre ; $\gamma = 1.4$

Let T_2 = Final temperature of the gas.

We know that $$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{\gamma - 1} = \left(\frac{0.5}{1}\right)^{1.4-1} = (0.5)^{0.4} = 0.758$$

$$\therefore \quad T_2 = T_1 / 0.758 = 273 / 0.758 = 360.16 \text{ K}$$

$$= 360.16 - 273 = 87.16^\circ \text{ C} \quad \textbf{Ans.}$$

Example 3.9. *The initial volume of 0.18 kg of a certain gas was 0.15 m^3 at a temperature of 1 ° C and a pressure of 1 bar. After adiabatic compression to 0.056 m^3, the pressure was found to be bar. Find ;*

1. Gas constant ; 2. Molecular mass of the gas ; 3. Ratio of specific heats ; 4. Two specific heats, one at a constant pressure and the other at a constant volume ; and 5. Change of internal energy.

Solution. Given : $m = 0.18$ kg ; $v_1 = 0.15 \text{ m}^3$; $T_1 = 15^\circ \text{ C} = 15 + 273 = 288$ K ; $p_1 = 1$ bar $= .1 \times 10^6 \text{ N/m}^2$; $v_2 = 0.056 \text{ m}^3$; $p_2 = 4 \text{ bar} = 0.4 \times 10^6 \text{ N/m}^2$

The *p-v* diagram is shown in Fig. 3.11.

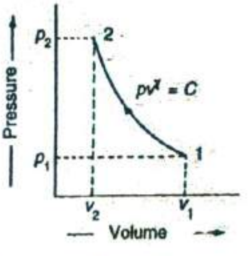


Fig. 3.11

1. Gas constant

Let R = Gas constant.

We know that

$$p_1 v_1 = m R T_1$$

$$\therefore \quad R = \frac{p_1 v_1}{m T_1} = \frac{0.1 \times 10^6 \times 0.15}{0.18 \times 288}$$

$$= 289.4 \text{ J/kg K} = 0.2894 \text{ kJ/kg K} \quad \textbf{Ans.}$$

2. Molecular mass of the gas

We know that molecular mass of the gas,

$$M = \frac{\text{Universal gas constant } (R_u)}{\text{Characteristic gas constant } (R)} = \frac{8314}{289.4} = 28.73 \text{ kg} \quad \textbf{Ans.}$$

$$\ldots (\because R_u = 8314 \text{ J/kg K, for all gases})$$

3. Ratio of specific heats

We know that ratio of specific heats,

$$\gamma = \frac{\log\left(\frac{p_2}{p_1}\right)}{\log\left(\frac{v_1}{v_2}\right)} = \frac{\log\left(\frac{0.4 \times 10^6}{0.1 \times 10^6}\right)}{\log\left(\frac{0.15}{0.056}\right)} = \frac{\log 4}{\log 2.678} = \frac{0.6020}{0.4278}$$

$$= 1.407 \quad \textbf{Ans.}$$

4. *Specific heat at a constant volume and constant pressure*

Let c_v = Specific heat at a constant volume, and

c_p = Specific heat at a constant pressure.

We know that $c_p - c_v = R$ or $1.407\,c_v - c_v = 0.2894$ $\quad (\because \gamma = c_p/c_v = 1.407)$

$$\therefore \quad c_v = 0.2894 / 0.407 = 0.711 \text{ kJ/kg K} \textbf{ Ans.}$$

and $$c_p = 1.407\, c_v = 1.407 \times 0.711 = 1 \text{ kJ/kg K} \textbf{ Ans.}$$

5. *Change in internal energy*

First of all, let us find the final temperature (T_2). We know that

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{4}\right)^{\frac{1.407-1}{1.407}} = (0.25)^{0.289} = 0.67$$

$$\therefore \quad T_2 = T_1 / 0.67 = 288 / 0.67 = 430 \text{ K}$$

We know that change in internal energy,

$$dU = U_2 - U_1 = m\,c_v\,(T_2 - T_1) = 0.18 \times 0.711\,(430 - 288) \text{ kJ}$$

$$= 18.17 \text{ kJ} \textbf{ Ans.}$$

Example 3.10. *A system contains 0.15 m^3 of a gas at a pressure of 3.8 bar and 150° C. It is expanded adiabatically till the pressure falls to 1 bar. The gas is then heated at a constant pressure till its enthalpy increases by 70 kJ. Determine the total work done. Take c_p = 1 kJ/kg K and c_v = 0.714 kJ/kg K.*

Solution. Given : $v_1 = 0.15 \text{ m}^3$; $p_1 = 3.8 \text{ bar} = 0.38 \times 10^6 \text{ N/m}^2$; $T_1 = 150° \text{ C} = 150 + 273 = 423 \text{ K}$; $p_2 = 1 \text{ bar} = 0.1 \times 10^6 \text{ N/m}^2$; $dH = 70 \text{ kJ}$; $c_p = 1 \text{ kJ/kg K}$; $c_v = 0.714 \text{ kJ/kg K}$

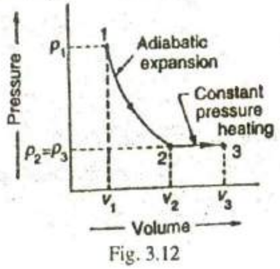


Fig. 3.12

In Fig. 3.12, process 1-2 represents adiabatic expansion of the gas and the process 2-3 represents heating at constant pressure.

First of all, let us find the temperature (T_2) and volume (v_2) after the adiabatic expansion.

We know that adiabatic index,

$$\gamma = c_p / c_v = 1 / 0.714 = 1.4$$

$$\therefore \quad \frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{3.8}{1}\right)^{\frac{1.4-1}{1.4}} = (3.8)^{0.286} = 1.465$$

or $$T_2 = T_1 / 1.465 = 423 / 1.465 = 288.7 \text{ K}$$

and $$\frac{v_1}{v_2} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} = \left(\frac{1}{3.8}\right)^{\frac{1}{1.4}} = (0.263)^{0.714} = 0.385$$

$$\therefore \quad v_2 = v_1 / 0.385 = 0.15 / 0.385 = 0.39 \text{ m}^3$$

Now let us find the temperature (T_3) and volume (v_3) after constant pressure heating.

Let m = Mass of gas contained in the system.

We know that gas constant,

$$R = c_p - c_v = 1 - 0.714 = 0.286 \text{ kJ/kg K} = 286 \text{ J/kg K}$$

and

$$p_1 v_1 = m R T_1$$

$$\therefore \quad m = \frac{p_1 v_1}{R T_1} = \frac{0.38 \times 10^6 \times 0.15}{286 \times 423} = 0.47 \text{ kg}$$

We also know that increase in enthalpy (dH),

$$70 = m\, c_p (T_3 - T_2) = 0.47 \times 1 (T_3 - 288.7) \text{ kJ}$$

$$\therefore \quad T_3 = \frac{70}{0.47} + 288.7 = 437.6 \text{ K}$$

Since the heating is at constant pressure, therefore

$$\frac{v_2}{T_2} = \frac{v_3}{T_3} \text{ or } v_3 = \frac{v_2 T_3}{T_2} = \frac{0.39 \times 437.6}{288.7} = 0.59 \text{ m}^3$$

We know that work done during adiabatic expansion,

$$W_{1-2} = \frac{p_1 v_1 - p_2 v_2}{\gamma - 1} = \frac{0.38 \times 10^6 \times 0.15 - 0.1 \times 10^6 \times 0.39}{1.4 - 1} \text{ J}$$

$$= \frac{57 \times 10^3 - 39 \times 10^3}{0.4} = 45\,000 \text{ J} = 45 \text{ kJ}$$

and workdone during constant pressure heating,

$$W_{2-3} = p_2 (v_3 - v_2) = 0.1 \times 10^6 (0.59 - 0.39) = 20\,000 \text{ J} = 20 \text{ kJ}$$

$\therefore$ Total work done, $W = W_{1-2} + W_{2-3} = 45 + 20 = 65$ kJ **Ans.**

Example 3.11. *0.336 m^3 of gas at 10 bar and 150° C expands adiabatically, until its pressure is 4 bar. It is then compressed, isothermally, to its original volume. Find the final temperature and pressure of the gas. Also determine the change in internal energy. Take $c_p = 0.996$ kJ/kg K ; and $c_v = 0.703$ kJ/kg K.*

Solution. Given : $v_1 = 0.336 \text{ m}^3$; $p_1 = 10 \text{ bar} = 1 \times 10^6 \text{ N/m}^2$; $T_1 = 150° \text{C} = 150 + 273 = 423 \text{ K}$; $p_2 = 4 \text{ bar} = 0.4 \times 10^6 \text{ N/m}^2$; $v_3 = v_1 = 0.336 \text{ m}^3$; $c_p = 0.996$ kJ/kg K ; $c_v = 0.703$ kJ/kg K

In Fig. 3.13, process 1-2 represents the adiabatic expansion of the gas and the process 2-3 represents the isothermal compression to its original volume.

We know that adiabatic index,

$$\gamma = c_p / c_v = 0.996/0.703 = 1.417$$

Final temperature of the gas

Let T_3 = Final temperature of the gas, and

T_2 = Temperature of the gas after adiabatic expansion.

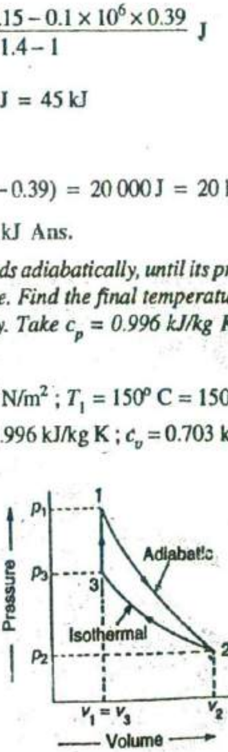


Fig. 3.13

We know that

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{10}{4}\right)^{\frac{1.417-1}{1.417}} = (2.5)^{0.294} = 1.31$$

$$T_2 = T_1/1.31 = 423/1.31 = 323 \text{ K}$$

Since the compression is isothermal from 2 to 3, therefore

$$T_3 = T_2 = 323 \text{ K} = 50^\circ \text{C} \ \textbf{Ans.}$$

Final pressure of the gas

Let p_3 = Final pressure of the gas.

We know that for a constant volume process 3-1,

$$\frac{p_3}{T_3} = \frac{p_1}{T_1}$$

$$\therefore \quad p_3 = \frac{T_3 p_1}{T_1} = \frac{323 \times 1 \times 10^6}{423} = 0.76 \times 10^6 \text{ N/m}^2 = 7.6 \text{ bar} \ \textbf{Ans.}$$

Change in internal energy

First of all, let us find the mass of the gas (m).

We know that gas constant,

$$R = c_p - c_v = 0.996 - 0.703 = 0.293 \text{ kJ/kg K} = 293 \text{ J/kg K}$$

We also know that

$$p_1 v_1 = m R T_1$$

or

$$m = \frac{p_1 v_1}{R T_1} = \frac{1 \times 10^6 \times 0.336}{293 \times 423} = 2.7 \text{ kg}$$

∴ Change in internal energy,

$$dU = U_3 - U_1 = m c_v (T_3 - T_1) = 2.7 \times 0.703 (323 - 423) \text{ kJ}$$

$$= -189.8 \text{ kJ} \ \textbf{Ans.}$$

The *negative* sign indicates that there is a decrease in internal energy.

Example 3.12. *0.75 m^3 of hydrogen gas is initially at a pressure of 1 bar and temperature 290 K. It is compressed isentropically to 15 bar. Next it is expanded at constant temperature to original volume. Finally heat rejection takes place at constant volume and the gas pressure is restored to the original condition of pressure. Find : 1. Pressure, volume and temperature at the end of each operation ; 2. Heat added during isothermal expansion ; and 3. Change of internal energy during each process.*

For hydrogen, $R = 4126$ J/kg K ; $c_p = 14.26$ kJ/kg K.

Solution. Given : $v_1 = 0.75 \text{ m}^3$; $p_1 = 1 \text{ bar} = 0.1 \times 10^6 \text{ N/m}^2$; $T_1 = 290$ K ; $p_2 = 15 \text{ bar} = 1.5 \times 10^6 \text{ N/m}^2$; $v_3 = v_1 = 0.75 \text{ m}^3$; $R = 4126$ J/kg K = 4.126 kJ/kg K ; $c_p = 14.26$ kJ/kg K

In p-v diagram, as shown in Fig. 3.14, process 1-2 represents isentropic compression, process 2-3 represents expansion at constant temperature (isothermal expansion) and process 3-1 represents rejection of heat at constant volume.

1. *Pressure, volume and temperature at the end of each operation*

Let p_2, v_2 and T_2 = Pressure, volume and temperature respectively at the end of isentropic operation 1-2,

p_3, v_3 and T_3 = Pressure, volume and temperature respectively at the end of isothermal operation 2-3.

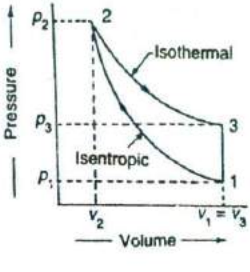


Fig. 3.14

First of all, let us find the value of isentropic index (γ). We know that

$$c_p - c_v = R$$

$$c_v = c_p - R = 14.26 - 4.126 = 10.134 \text{ kJ/kg K}$$

$$\gamma = c_p / c_v = 14.26 / 10.134 = 1.407$$

We know that for isentropic process 1-2,

$$p_1 (v_1)^\gamma = p_2 (v_2)^\gamma \quad \text{or} \quad \left(\frac{v_1}{v_2}\right)^\gamma = \frac{p_2}{p_1}$$

$$\therefore \qquad v_2 = v_1 \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}} = 0.75\left(\frac{1}{15}\right)^{\frac{1}{1.407}} = 0.75 \times 0.146 = 0.11 \text{ m}^3 \textbf{ Ans.}$$

and
$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}$$

or
$$T_2 = T_1 \times \frac{p_2 v_2}{p_1 v_1} = 290 \times \frac{1.5 \times 10^6 \times 0.11}{0.1 \times 10^6 \times 0.75} = 638 \text{ K} \textbf{ Ans.}$$

Now for isothermal process 2-3,

$$p_2 v_2 = p_3 v_3$$

or
$$p_3 = p_2 \times \frac{v_2}{v_3} = 1.5 \times 10^6 \times \frac{0.11}{0.75} = 0.22 \times 10^6 \text{ N/m}^2 \simeq 2.2 \text{ bar} \textbf{ Ans.}$$

$$\ldots (\because v_3 = v_1)$$

2. *Heat added during isothermal expansion*

We know that heat added during isothermal expansion 2-3,

$$Q_{2-3} = 2.3\, p_2 v_2 \log\left(\frac{v_3}{v_2}\right)$$

$$= 2.3 \times 1.5 \times 10^6 \times 0.11 \log\left(\frac{0.75}{0.11}\right) = 0.38 \times 10^6 \times 0.834 \text{ J}$$

$$= 0.317 \times 10^6 \text{ J} = 317 \text{ kJ} \textbf{ Ans.}$$

3. *Change of internal energy during each process*

First of all, let us find the mass of hydrogen (m). We know that

$$p_1 v_1 = m R T_1 \quad \text{or} \quad m = \frac{p_1 v_1}{R T_1} = \frac{0.1 \times 10^6 \times 0.75}{4126 \times 290} = 0.063 \text{ kg}$$

We know that change of internal energy during isentropic process,

$$dU = U_2 - U_1 = m c_v (T_2 - T_1) = 0.063 \times 10.134 (638 - 290) \text{ kJ}$$

$$= 222.18 \text{ kJ} \quad \textbf{Ans.}$$

Since in an isothermal process 2-3, the temperature is constant ($T_2 = T_3$), therefore change in internal energy during isothermal process is zero. **Ans.**

.11. Polytropic Process

The polytropic process is also known as the general law for the expansion and compression of gases, and is given by the relation :

$$pv^n = \text{Constant}$$

where n is a polytropic index, which may have any value from zero to infinity, depending upon the manner, in which the expansion or compression has taken place.

The various equations for polytropic process may be expressed by changing the index n for γ in the adiabatic process.

Now consider m kg of a certain gas being heated polytropically from an initial state 1 to a final state 2.

Let p_1, v_1 and T_1 = Pressure, volume and temperature at the initial state 1, and

p_2, v_2 and T_2 = Pressure, volume and temperature at the final state 2.

The process is shown on the p-v diagram in Fig. 3.15. Now let us derive the following relations for the polytropic process.

1. *Pressure-volume-temperature (p-v-T) relationship*

The following relations for the polytropic process are derived in the similar way as discussed for adiabatic process.

(*a*) $p_1 v_1^n = p_2 v_2^n = \ldots = C$

(*b*) $$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{n-1}$$

(*c*) $$\frac{v_1}{v_2} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

(*d*) $$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{n-1}{n}}$$

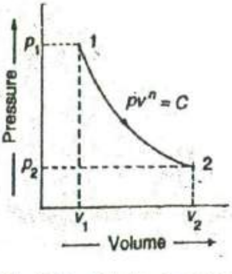


Fig. 3.15. Polytropic process.

2. *Workdone during polytropic expansion*

The equations for the work done during a polytropic process may also be expressed by changing the index n for γ in the adiabatic process.

∴ Workdone during a polytropic process from state 1 to state 2,

$$W_{1-2} = \frac{p_1 v_1 - p_2 v_2}{n-1} = \frac{m\,R\,(T_1 - T_2)}{n-1} \qquad \text{. . . (For expansion)}$$

$$= \frac{p_2 v_2 - p_1 v_1}{n-1} = \frac{m\,R\,(T_2 - T_1)}{n-1} \qquad \text{. . . (For compression)}$$

3. *Change in internal energy*

We know that change in internal energy,

$$dU = U_2 - U_1 = m\,c_v\,(T_2 - T_1)$$

4. *Heat supplied or heat transfer*

We know that the heat supplied or heat transferred,

$$Q_{1-2} = W_{1-2} + dU$$

$$= \frac{p_1 v_1 - p_2 v_2}{n-1} + m\,c_v\,(T_2 - T_1)$$

$$= \frac{m\,R\,(T_1 - T_2)}{n-1} + m \times \frac{R}{\gamma - 1}\,(T_2 - T_1) \qquad \ldots \left(\because\ {}^{*}c_v = \frac{R}{\gamma - 1} \right)$$

$$= m\,R\,(T_1 - T_2) \left[\frac{1}{n-1} - \frac{1}{\gamma - 1} \right]$$

$$= m\,R\,(T_1 - T_2) \left[\frac{(\gamma - 1) - (n-1)}{(n-1)(\gamma - 1)} \right]$$

$$= m\,R\,(T_1 - T_2) \left[\frac{\gamma - n}{(n-1)(\gamma - 1)} \right]$$

$$= \frac{\gamma - n}{\gamma - 1} \times \frac{m\,R\,(T_1 - T_2)}{n-1}$$

5. *Change in enthalpy*

We know that change in enthalpy,

$$dH = H_2 - H_1 = m\,c_p\,(T_2 - T_1)$$

Notes : 1. The equations for heat transfer may also be expressed as :

(*a*)

$$Q_{1-2} = \frac{\gamma - n}{\gamma - 1} \times \text{Workdone} = \frac{\gamma - n}{\gamma - 1} \times \frac{p_1 v_1 - p_2 v_2}{(n-1)}$$

(*b*) We know that

$$Q_{1-2} = \frac{\gamma - n}{\gamma - 1} \times \frac{m\,R\,(T_1 - T_2)}{n-1} = \frac{\gamma - n}{n-1} \times m\,c_v\,(T_1 - T_2) \qquad \ldots \left(\because\ \frac{R}{\gamma - 1} = c_v \right)$$

$$= \frac{\gamma - n}{n-1} \times \text{Change in internal energy}$$

---

* We know that $c_p - c_v = R$

Dividing throughout by c_v, we have

$$\frac{c_p}{c_v} - 1 = \frac{R}{c_v} \text{ or } \gamma - 1 = \frac{R}{c_v}, \qquad \therefore c_v = \frac{R}{\gamma - 1}$$

2. The above equations give the amount of heat, which has passed into the gas through the cylinder walls as the gas expands. It may be noted that this will happen only when n is less than γ. If however, n is greater than γ, then heat is rejected by the gas.

3. Similarly, during compression, work done will be negative, *i.e.* work will be done on the gas. Moreover, heat will be rejected by the gas. This will happen only when n is less than γ.

4. The relation for work done may also be expressed as :

$$W_{1\text{-}2} = \frac{p_1 v_1 - p_2 v_2}{(n-1)} = \frac{p_1 v_1 \left(1 - \frac{p_2 v_2}{p_1 v_1}\right)}{(n-1)}$$

$$= \frac{p_1 v_1 (1 - r^{n-1})}{(n-1)} \qquad \left[\because \frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^n = r^n \text{ and } \frac{p_2 v_2}{p_1 v_1} = r^n \times \frac{1}{r} = r^{n-1}\right]$$

Example 3.13. *A certain quantity of air has a volume of 0.028 m³ at a pressure of 1.25 bar and 25° C. It is compressed to a volume of 0.0042 m³ according to the law $pv^{1.3}$ = Constant. Find the final temperature and work done during compression. Also determine the reduction in pressure at a constant volume required to bring the air back to its original temperature.*

Solution. Given : $v_1 = 0.028$ m³ ; $p_1 = 1.25$ bar $= 0.125 \times 10^6$ N/m² ; $T_1 = 25°$ C $= 25 + 273 = 298$ K ; $v_2 = 0.0042$ m³ ; $n = 1.3$

The p-v diagram is shown in Fig. 3.16.

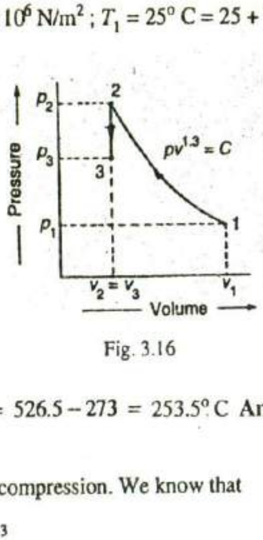


Fig. 3.16

Final temperature

Let T_2 = Final temperature.

We know that

$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{n-1} = \left(\frac{0.0042}{0.028}\right)^{1.3-1}$$

$$= (0.15)^{0.3} = 0.566$$

$$\therefore \quad T_2 = T_1 / 0.566 = 298 / 0.566 = 526.5 \text{ K} = 526.5 - 273 = 253.5° \text{C} \textbf{ Ans.}$$

Workdone during compression

First of all, let us find the final pressure (p_2) at the end of compression. We know that

$$p_1 v_1^n = p_2 v_2^n \text{ or } \frac{p_1}{p_2} = \left(\frac{v_2}{v_1}\right)^n = \left(\frac{0.0042}{0.028}\right)^{1.3} = 0.085$$

$$\therefore \quad p_2 = p_1 / 0.085 = 1.25 / 0.085 = 14.7 \text{ bar} = 1.47 \times 10^6 \text{ N/m}^2$$

We know that workdone during compression,

$$W_{1\text{-}2} = \frac{p_2 v_2 - p_1 v_1}{n-1} = \frac{1.47 \times 10^6 \times 0.0042 - 0.125 \times 10^6 \times 0.028}{1.3 - 1}$$

$$= \frac{6174 - 3500}{0.3} = 8913 \text{ J} = 8.913 \text{ kJ} \textbf{ Ans.}$$

Pressure at a constant volume

Let p_3 = Pressure at a constant volume required to bring the air back to its initial temperature, $T_1 = 298$ K.

We know that for a constant volume process 2-3,

$$\frac{p_2}{T_2} = \frac{p_3}{T_3} \quad \text{or} \quad p_3 = \frac{p_2 T_3}{T_2} = \frac{14.7 \times 298}{526.5} = 8.32 \text{ bar Ans.}$$

$$\ldots (\because v_2 = v_3 \text{ and } T_3 = T_1)$$

Example 3.14. *A gas mixture obeying perfect gas law has a molecular mass of 26.7. The gas mixture is compressed through a compression ratio of 12 according to the law $pv^{1.25}$ = Constant, from initial conditions of 0.9 bar and 333 K. Assuming a mean molar specific heat at constant volume of 21.1 kJ/kg K, find, per kg of mass, the workdone and heat flow across the cylinder walls.*

For the above gas, determine the value of characteristic gas constant, molar specific heat at a constant pressure and ratio of specific heats.

Solution. Given : $M = 26.7$; $r = v_1/v_2 = 12$; $n = 1.25$; $p_1 = 0.9 \text{ bar} = 0.09 \times 10^6 \text{ N/m}^2$; $T_1 = 333 \text{ K}$; $c_{vm} = 21.1 \text{ kJ/kg K}$; $m = 1 \text{ kg}$

The *p-v* diagram is shown in Fig. 3.17.

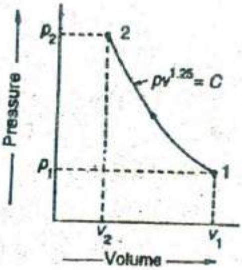


Fig. 3.17

Workdone per kg of gas

First of all, let us find the initial volume (v_1), final volume (v_2) and final pressure (p_2).

We know that $p_1 v_1^n = p_2 v_2^n$

$$\therefore \quad p_2 = p_1 \left(\frac{v_1}{v_2}\right)^n = 0.9\,(12)^{1.25}$$

$$= 20.1 \text{ bar} = 2.01 \times 10^6 \text{ N/m}^2$$

We also know that gas constant,

$$R = \frac{R_u}{M} = \frac{8314}{26.7} = 311.4 \text{ J/kg K}$$

and

$$p_1 v_1 = m R T_1 \quad \text{or} \quad v_1 = \frac{m R T_1}{p_1} = \frac{1 \times 311.4 \times 333}{0.09 \times 10^6} = 1.15 \text{ m}^3$$

$$\therefore \quad v_2 = v_1/12 = 1.15/12 = 0.096 \text{ m}^3 \qquad \ldots (\because v_1/v_2 = 12)$$

We know that workdone

$$W_{1-2} = \frac{p_1 v_1 - p_2 v_2}{n-1} = \frac{0.09 \times 10^6 \times 1.15 - 2.01 \times 10^6 \times 0.096}{1.25 - 1}$$

$$= \frac{103\,500 - 192\,960}{0.25} = -357\,840 \text{ J} = -357.84 \text{ kJ Ans.}$$

The *negative* sign indicates that the work is done on the gas.

Heat flow across the cylinder walls

Let T_2 = Final temperature.

We know that
$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{n-1} = \left(\frac{1}{12}\right)^{1.25-1} = 0.537$$

$$\therefore \quad T_2 = T_1 / 0.537 = 333/0.537 = 620 \text{ K}$$

and specific heat at constant volume,

$$c_v = c_{vm} / M = 21.1/26.7 = 0.79 \text{ kJ/kg K}$$

$\therefore$ Change in internal energy,

$$dU = U_2 - U_1 = m\,c_v (T_2 - T_1) = 1 \times 0.79\,(620 - 333) = 226.7 \text{ kJ}$$

We know that heat flow across the cylinder walls,

$$Q_{1-2} = W_{1-2} + dU = -357.84 + 226.7 = -131.1 \text{ kJ} \quad \textbf{Ans.}$$

The *negative* sign indicates that the heat is rejected through the cylinder walls.

Characteristic gas constant

We know that characteristic gas constant,

$$R = \frac{\text{Universal gas constant}}{\text{Molecular mass}} = \frac{R_u}{M} = \frac{8314}{26.7}$$

$$\ldots (\because R_u = 8314 \text{ J/kg K, for all gases})$$

$$= 311.4 \text{ J/kg K} = 0.3114 \text{ kJ/kg K} \quad \textbf{Ans.}$$

Molar specific heat at a constant pressure

Let c_{pm} = Molar specific heat at a constant pressure.

We know that $c_p - c_v = R$ or $c_p - 0.79 = 0.3114$

$\therefore$ $c_p = 0.3114 + 0.79 = 1.1014$ kJ/kg K

and $c_{pm} = M\,c_p = 26.7 \times 1.1014 = 29.4$ kJ/kg K **Ans.**

Ratio of specific heats

We know that ratio of specific heats,

$$\gamma = c_p / c_v = 1.1014/0.79 = 1.394 \quad \textbf{Ans.}$$

Example 3.15. *An internal combustion engine has the following dimensions :*

Diameter of cylinder = 550 mm ; Stroke = 750 mm ; Compression ratio = 13.5. At the end of the suction stroke, the pressure is 1 bar and temperature is 316 K. The compression follows the law $pv^{1.37} = C$. *Determine :*

1. the pressure and temperature at the end of compression ; 2. the mass of the charge ; 3. the work done during compression ; and 4. the heat rejected during compression.

Take $c_p = 0.996$ *kJ/kg K and* $c_v = 0.707$ *kJ/kg K.*

Solution. Given : $D = 550$ mm $= 0.55$ m ; $L = 750$ mm $= 0.75$ m ; $v_1/v_2 = 13.5$; $p_1 = 1$ bar $= 0.1 \times 10^6$ N/m^2 ; $T_1 = 316$ K ; $n = 1.37$; $c_p = 0.996$ kJ/kg K ; $c_v = 0.707$ kJ/kg K

In the p-v diagram, as shown in Fig. 3.18, point 1 refers to the end of suction stroke and point 2 refers to the end of compression stroke.

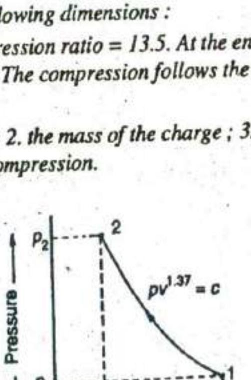


Fig. 3.18

1. *Pressure and temperature at the end of compression*

Let p_2 and T_2 = Pressure and temperature at the end of compression respectively.

We know that

$$p_1 v_1^n = p_2 v_2^n$$

$$\therefore \qquad p_2 = p_1 \left(\frac{v_1}{v_2} \right)^n = 0.1 \times 10^6 \, (13.5)^{1.37} \text{ N/m}^2$$

$$= 3.536 \times 10^6 \text{ N/m}^2 = 3.536 \text{ bar} \quad \textbf{Ans.}$$

We also know that

$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1} \right)^{n-1} = \left(\frac{1}{13.5} \right)^{1.37-1} = (0.074)^{0.37} = 0.3816$$

$$\therefore \qquad T_2 = T_1 / 0.3816 = 316 / 0.3816 = 828 \text{ K} \quad \textbf{Ans.}$$

2. Mass of the charge

Let m = Mass of the charge.

We know that swept volume,

$$v_1 - v_2 = \frac{\pi}{4} \times D^2 \times L = \frac{\pi}{4} (0.55)^2 \, 0.75 = 0.178 \text{ m}^3$$

or

$$v_1 - \frac{v_1}{13.5} = 0.178 \qquad \ldots \left(\because \frac{v_1}{v_2} = 13.5 \right)$$

$$\therefore \qquad v_1 = 0.178 \times 13.5 / 12.5 = 0.192 \text{ m}^3$$

and gas constant, $R = c_p - c_v = 0.996 - 0.707 = 0.289 \text{ kJ/kg K} = 289 \text{ J/kg K}$

We know that $p_1 v_1 = m R T_1$

$$\therefore \qquad m = \frac{p_1 v_1}{R T_1} = \frac{0.1 \times 10^6 \times 0.192}{289 \times 316} = 0.21 \text{ kg} \quad \textbf{Ans.}$$

3. Workdone during compression

We know that workdone during compression,

$$W_{1-2} = \frac{m R (T_2 - T_1)}{n-1} = \frac{0.21 \times 0.289 \, (828 - 316)}{1.37 - 1} = 84 \text{ kJ} \quad \textbf{Ans.}$$

4. Heat rejected during compression

We know that adiabatic index,

$$\gamma = c_p / c_v = 0.996 / 0.707 = 1.41$$

∴ Heat rejected during compression,

$$Q_{1-2} = \frac{\gamma - n}{\gamma - 1} \times \text{Workdone during compression}$$

$$= \frac{1.41 - 1.37}{1.41 - 1} \times 84 = 8.195 \text{ kJ} \quad \textbf{Ans.}$$

Example 3.16. *0.2 m³ of mixture of fuel and air at 1.2 bar and 60° C is compressed until its pressure becomes 12 bar and temperature becomes 270° C. Then, it is ignited suddenly at constant volume and its pressure becomes twice the pressure at the end of compression. Calculate the maximum temperature reached and change in internal energy. Also compute the heat transfer during the compression process. Consider mixture as a prefect gas and take* $c_p = 1.072$ *kJ/kg K ; and* $R = 294$ *J/kg K.*

Solution. Given : $v_1 = 0.2 \text{ m}^3$; $p_1 = 1.2 \text{ bar} = 0.12 \times 10^6 \text{ N/m}^2$; $T_1 = 60° \text{C} = 60 + 273 = 333 \text{ K}$; $p_2 = 12 \text{ bar} = 1.2 \times 10^6 \text{ N/m}^2$; $T_2 = 270° \text{C} = 270 + 273 = 543 \text{ K}$; $p_3 = 2\,p_2$; $c_p = 1.072 \text{ kJ/kg K}$; $R = 294 \text{ J/kg K} = 0.294 \text{ kJ/kg K}$.

In the *p-v* diagram, as shown in Fig. 3.19, process 1-2 represents polytropic compression (*i.e.* according to the general law $p\,v^n = C$) and the process 2-3 represents the constant volume process.

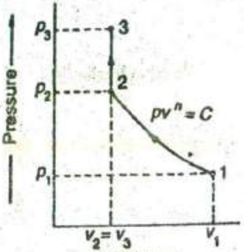


Fig. 3.19

Maximum temperature

Let $\qquad T_3$ = Maximum temperature.

Since the process 2-3 is a constant volume process, therefore

$$\frac{p_2}{T_2} = \frac{p_3}{T_3}$$

or
$$T_3 = T_2 \times \frac{p_3}{p_2} = 543 \times \frac{2\,p_2}{p_2} = 1086 \text{ K} = 813° \text{C} \ \ \textbf{Ans.}$$

Change in internal energy

First of all, let us find the mass of the mixture compressed (*i.e.* m) and c_v.

We know that $\quad p_1\,v_1 = m\,R\,T_1 \quad$ or $\quad m = \dfrac{p_1\,v_1}{R\,T_1} = \dfrac{0.12 \times 10^6 \times 0.2}{294 \times 333} = 0.245 \text{ kg}$

and
$$c_v = c_p - R = 1.072 - 0.294 = 0.778 \text{ kJ/kg K} \qquad \ldots (\because c_p - c_v = R)$$

We know that change in internal energy,

$$dU = U_3 - U_1 = m\,c_v\,(T_3 - T_1) = 0.245 \times 0.778\,(1086 - 333) \text{ kJ}$$
$$= 143.5 \text{ kJ} \ \ \textbf{Ans.}$$

Heat transfer during compression process

First of all, let us find the value of polytropic index (n) for the compression process 1-2.

We know that $\quad \dfrac{T_1}{T_2} = \left(\dfrac{p_1}{p_2}\right)^{\frac{n-1}{n}} \quad$ or $\quad \dfrac{333}{543} = \left(\dfrac{1.2}{12}\right)^{\frac{n-1}{n}}$

Taking log on both sides,

$$\therefore \quad \log\left(\frac{333}{543}\right) = \frac{n-1}{n} \log\left(\frac{1.2}{12}\right) \quad \text{or} \quad \log 0.613 = \frac{n-1}{n} \log 0.1$$

$$\therefore \qquad -0.2125 = \frac{n-1}{n}(-1) \quad \text{or} \quad n = 1.27$$

We know that workdone,

$$W_{1\text{-}2} = \frac{m\,R\,(T_1 - T_2)}{n-1} = \frac{0.245 \times 0.294\,(333 - 543)}{1.27 - 1} = -56 \text{ kJ}$$

The *negative* sign shows that the gas is compressed and the work is done on the gas.

We know that change in internal energy,

$$dU = (U_2 - U_1) = m\,c_v\,(T_2 - T_1) = 0.245 \times 0.778\,(543 - 333) = 40 \text{ kJ}$$

∴ Heat transfer during compression process,

$$Q_{1-2} = W_{1-2} + dU = -56 + 40 = -16 \text{ kJ} \text{ Ans.}$$

The *negative* sign indicates that the heat is rejected by the gas.

Example 3.17. *An oil engine has a volume of 60 litres and a compression ratio of 14.2 to 1. At the beginning of the compression stroke, the pressure and temperature are 1 bar and 80° C respectively. At the end of compression, the pressure is 30 bar. The charge is now heated at constant pressure until the volume is doubled. Find :*

1. The index of compression ; 2. The temperature at the end of compression, 3. The heat transfer ; and 4. The heat received in constant pressure operation.

Assume $c_v = 0.712$ *kJ/kg K and* $R = 0.293$ *kJ/kg K.*

Solution. Given : $v_1 = 60$ litres $= 60 \times 10^{-3}$ m^3 ; $v_1/v_2 = 14.2$; $p_1 = 1$ bar $= 0.1 \times 10^6$ N/m^2 ; $T_1 = 80°$ C $= 80 + 273 = 353$ K ; $p_2 = 30$ bar $= 3 \times 10^6$ N/m^2 ; $v_3 = 2\,v_2$; $c_v = 0.712$ kJ/kg K ; $R = 0.293$ kJ/kg K $= 293$ J/kg K

In Fig. 3.20, 1-2 represents the polytropic compression process and 2-3 represents the constant pressure heating process.

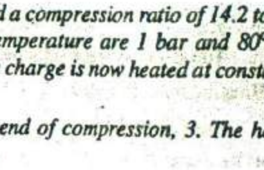


Fig. 3.20

1. Index of compression

Let n = Index of compression.

We know that $p_1 v_1^n = p_2 v_2^n$

$$\left(\frac{v_1}{v_2}\right)^n = \frac{p_2}{p_1}$$

$$(14.2)^n = \frac{3 \times 10^6}{0.1 \times 10^6} = 30$$

Taking log on both sides,

$$n \log 14.2 = \log 30$$

$$n \times 1.152 = 1.477$$

$$\therefore \quad n = 1.282 \text{ Ans.}$$

2. Temperature at the end of compression

Let T_2 = Temperature at the end of compression.

We know that $$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{n-1} = \left(\frac{1}{14.2}\right)^{1.282-1} = (0.0704)^{0.282} = 0.473$$

$$\therefore \quad T_2 = T_1 / 0.473 = 353 / 0.473 = 746.3 \text{ K} = 473.3° \text{C} \text{ Ans.}$$

3. Heat transfer

First of all, let us find the mass of the charge (m) and the ratio of specific heats (γ). We know that

$$p_1 v_1 = m R T_1 \quad \text{or} \quad m = \frac{p_1 v_1}{R T_1} = \frac{0.1 \times 10^6 \times 60 \times 10^{-3}}{293 \times 353} = 0.058 \text{ kg}$$

We also know that

$$c_p - c_v = R \quad \text{or} \quad c_p = c_v + R = 0.712 + 0.293 = 1.005 \text{ kJ/kg K}$$

$$\therefore \quad \gamma = c_p / c_v = 1.005 / 0.712 = 1.41$$

We know that heat transfer,

$$Q_{1-2} = \frac{\gamma - n}{\gamma - 1} \times \frac{m\,R\,(T_1 - T_2)}{n-1} = \frac{1.41 - 1.282}{1.41 - 1} \times \frac{0.058 \times 0.293\,(353 - 746.3)}{1.282 - 1}$$

$$= 0.3122 \times -23.7 = -7.4 \text{ kJ} \ \textbf{Ans.}$$

The *negative* sign indicates that the heat is rejected.

4. Heat received in constant pressure operation

First of all, let us find the temperature (T_3) at the end of constant pressure operation. We know that, for constant pressure process 2-3,

$$\frac{v_2}{T_2} = \frac{v_3}{T_3} \quad \text{or} \quad T_3 = \frac{v_3\,T_2}{v_2} = \frac{2\,v_2 \times 746.3}{v_2} = 1492.6 \text{ K} \qquad \ldots(\because v_3 = 2\,v_2)$$

We know that heat received,

$$Q_{2-3} = m\,c_p\,(T_3 - T_2) = 0.058 \times 1.005\,(1492.6 - 746.3) = 43.5 \text{ kJ} \ \textbf{Ans.}$$

Example 3.18. *A system contains 0.15 m^3 of air at 4 bar and 423 K. A reversible adiabatic expansion takes place till the pressure falls to 1 bar. The air is then heated at constant pressure till enthalpy increases by 67 kJ. Determine the total workdone.*

If these processes are replaced by a single reversible polytropic process giving the same work between the same initial and final states, determine the index of expansion. Take $c_p = 1.009$ kJ/kg K.

Solution. Given : $v_1 = 0.15 \text{ m}^3$; $p_1 = 4 \text{ bar} = 0.4 \times 10^6 \text{ N/m}^2$; $T_1 = 423$ K ; $p_2 = 1 \text{ bar} = 0.1 \times 10^6 \text{ N/m}^2$; $dH = 67$ kJ ; $c_p = 1.009$ kJ/kg K

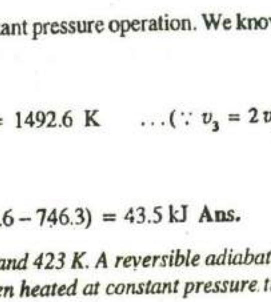


Fig. 3.21

In the *p-v* diagram, as shown in Fig. 3.21, 1-2 represents reversible adiabatic expansion, 2-3 represents heating at constant pressure.

Total workdone

Let m = Mass of air,

v_2 and T_2 = Volume and temperature of air after adiabatic expansion,

v_3 and T_3 = Volume and temperature of air after heating at constant pressure.

We know that

$$p_1\,v_1 = m\,R\,T_1 \quad \text{or} \quad m = \frac{p_1\,v_1}{R\,T_1} = \frac{0.4 \times 10^6 \times 0.15}{287 \times 423} = 0.494 \text{ kg}$$

...(Taking $R = 287$ J/kg K)

and

$$p_1\,(v_1)^\gamma = p_2\,(v_2)^\gamma \quad \text{or} \quad \left(\frac{v_2}{v_1}\right)^\gamma = \frac{p_1}{p_2}$$

$$\therefore \quad v_2 = v_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}} = 0.15\left(\frac{4}{1}\right)^{\frac{1}{1.4}} = 0.4036 \text{ m}^3 \qquad \ldots(\because \gamma \text{ for air} = 1.4)$$

We know that the workdone during adiabatic expansion 1-2,

$$W_{1-2} = \frac{p_1 v_1 - p_2 v_2}{\gamma - 1} = \frac{0.4 \times 10^6 \times 0.15 - 0.1 \times 10^6 \times 0.4036}{1.4 - 1} \text{ J}$$

$$= \frac{60\,000 - 40\,360}{0.4} = 49\,100 \text{ J} = 49.1 \text{ kJ}$$

Now let us find the workdone during heating at constant pressure.

We know that

$$p_2 v_2 = m R T_2 \text{ or } T_2 = \frac{p_2 v_2}{m R} = \frac{0.1 \times 10^6 \times 0.4036}{0.494 \times 287} = 284.6 \text{ K}$$

and the increase in enthalpy (*dH*),

$$67 = m c_p (T_3 - T_2) = 0.494 \times 1.009 (T_3 - 284.6) = 0.5 (T_3 - 284.6)$$

$$\therefore \qquad T_3 = \frac{67}{0.5} + 284.6 = 418.6 \text{ K}$$

Since the heating is at constant pressure from 2 to 3, therefore

$$\frac{v_2}{T_2} = \frac{v_3}{T_3} \text{ or } v_3 = \frac{v_2 T_3}{T_2} = \frac{0.4036 \times 418.6}{284.6} = 0.594 \text{ m}^3$$

We know that workdone during constant pressure process 2-3,

$$W_{2-3} = p_2 (v_3 - v_2) = 0.1 \times 10^6 (0.594 - 0.4036) = 19\,040 \text{ J} = 19.04 \text{ kJ}$$

∴ Total workdone,

$$W_{1-2-3} = W_{1-2} + W_{2-3} = 49.1 + 19.04 = 68.14 \text{ kJ} \textbf{ Ans.}$$

Index of expansion

Let n = Index of expansion.

The process 1-2 and 2-3 are replaced by a single polytropic process 1-3 giving the same work *i.e.* 68.14 kJ or 68.14×10^3 J.

We know that workdone during a polytropic process 1-3 (W_{1-3}),

$$68.14 \times 10^3 = \frac{p_1 v_1 - p_3 v_3}{n - 1} = \frac{0.4 \times 10^6 \times 0.15 - 0.1 \times 10^6 \times 0.594}{n - 1} = \frac{600}{n - 1}$$

$$\therefore \qquad n - 1 = 600 / 68.14 \times 10^3 = 0.0088 \text{ or } n = 1.0088 \textbf{ Ans.}$$

3.12. Rate of Heat Transfer (Absorption or Rejection) per Unit Volume During a Polytropic Process

In the previous article, we have seen that heat transfer (absorbed or rejected) during a polytropic process,

$$Q_{1-2} = \frac{\gamma - n}{\gamma - 1} \times W_{1-2}$$

where W_{1-2} is the work done during polytropic process.

If dQ is the small quantity of heat transfer during small change of pressure and volume, then

$$dQ = \frac{\gamma - n}{\gamma - 1} \times p \, dv$$

∴ Rate of heat transfer per unit volume,

$$\frac{dQ}{dv} = \frac{\gamma - n}{\gamma - 1} \times p$$

and rate of heat transfer per second,

$$\frac{dQ}{dt} = \frac{dQ}{dv} \times \frac{dv}{dt} = \frac{\gamma - n}{\gamma - 1} \times p \times \frac{dv}{dt}$$

where $\frac{dv}{dt}$ is the swept volume of the piston per second.

Example 3.19. *The law of the expansion curve of a gas engine indicator is found to be $pv^{1.3}$ = Constant, and the ratio of specific heats of the mixture is 1.37. If the piston sweeps out 2 m^3/min, when the pressure on this expansion curve is 14 bar, what is the rate of heat transfer per second at this instant ?*

Solution. Given : $n = 1.3$; $\gamma = 1.37$; $dv/dt = 2 \text{ m}^3/\text{min} = 0.0333 \text{ m}^3/\text{s}$; $p = 14$ bar $= 1.4 \times 10^6 \text{ N/m}^2$

We know that the rate of heat transfer per second,

$$\frac{dQ}{dt} = \frac{\gamma - n}{\gamma - 1} \times p \times \frac{dv}{dt} = \frac{1.37 - 1.3}{1.37 - 1} \times 1.4 \times 10^6 \times 0.0333 = 8820 \text{ J}$$

$$= 8.82 \text{ kJ} \textbf{ Ans.}$$

3.13. Determination of polytropic Index

We know that for a polytropic process,

$$p_1 v_1^n = p_2 v_2^n$$

Taking logarithms on both sides,

$$\log p_1 + n \log v_1 = \log p_2 + n \log v_2$$

or

$$n \log v_1 - n \log v_2 = \log p_2 - \log p_1$$

$$n (\log v_1 - \log v_2) = \log p_2 - \log p_1$$

$$n \log\left(\frac{v_1}{v_2}\right) = \log\left(\frac{p_2}{p_1}\right)$$

∴

$$n = \frac{\log\left(\frac{p_2}{p_1}\right)}{\log\left(\frac{v_1}{v_2}\right)}$$

Note : Proceeding in the similar way, we can find out that adiabatic index,

$$\gamma = \frac{\log\left(\frac{p_2}{p_1}\right)}{\log\left(\frac{v_1}{v_2}\right)}$$

Example 3.20. *A gas initially at 603 K expands until its volume is 5.2 times the initial volume, according to pv^n = Constant. If the initial and final pressures are observed to be 8.5 bar and 1 bar, determine : 1. the index of expansion, 2. work done per kg of gas, and 3. heat exchange per kg of gas.*

Assume $c_v = 0.712$ kJ/kg K and $\gamma = 1.4$.

Solution. Given : $T_1 = 603$ K ; $v_2 = 5.2\, v_1$; $p_1 = 8.5$ bar $= 0.85 \times 10^6$ N/m^2 ; $p_2 = 1$ bar $= 0.1 \times 10^6$ N/m^2 ; $c_v = 0.712$ kJ/kg K ; $\gamma = c_p / c_v = 1.4$

The *p-v* diagram is shown in Fig. 3.22.

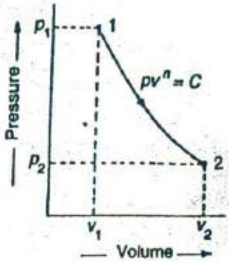


Fig. 3.22

1. *Index of expansion*

We know that index of expansion,

$$n = \frac{\log\left(\dfrac{p_2}{p_1}\right)}{\log\left(\dfrac{v_1}{v_2}\right)} = \frac{\log\left(\dfrac{0.1 \times 10^6}{0.85 \times 10^6}\right)}{\log\left(\dfrac{v_1}{5.2\, v_1}\right)}$$

$$= \frac{\log (0.1176)}{\log (0.1923)} = \frac{-0.9296}{-0.7160} = 1.3 \text{ Ans.}$$

Workdone per kg of gas

First of all, let us find the initial volume (v_1) and final volume (v_2).

We know that gas constant,

$$R = c_p - c_v = 1.4\, c_v - c_v = 0.4\, c_v = 0.4 \times 0.712 = 0.2848 \text{ kJ/kg K}$$

$$= 284.8 \text{ J/kg K} \qquad \ldots (\because c_p / c_v = \gamma = 1.4)$$

and $$p_1 v_1 = m R T_1 \text{ or } v_1 = \frac{m R T_1}{p_1} = \frac{1 \times 284.8 \times 603}{0.85 \times 10^6} = 0.202 \text{ m}^3$$

$$\ldots (\because m = 1 \text{ kg})$$

$$\therefore \qquad v_2 = 5.2\, v_1 = 5.2 \times 0.202 = 1.05 \text{ m}^3$$

We know that workdone per kg of gas,

$$W_{1\text{-}2} = \frac{p_1 v_1 - p_2 v_2}{n - 1} = \frac{0.85 \times 10^6 \times 0.202 - 0.1 \times 10^6 \times 1.05}{1.3 - 1} \text{ J}$$

$$= 0.222 \times 10^6 \text{ J} = 222 \text{ kJ Ans.}$$

3. *Heat exchange per kg of gas*

We know that heat exchange per kg of gas,

$$Q_{1\text{-}2} = \frac{\gamma - n}{\gamma - 1} \times \text{Workdone per kg of gas} = \frac{1.4 - 1.3}{1.4 - 1} \times 222 = 55.5 \text{ kJ Ans.}$$

3.14. Free Expansion (or Unresisted Expansion) Process

The free expansion (or unresisted expansion) process is an irreversible non-flow process. A free expansion occurs when a fluid is allowed to expand suddenly into a vacuum chamber through an orifice of large dimensions.

Consider two chambers *A* and *B* separated by a partition as shown in Fig. 3.23 (*a*). Let the chamber *A* contains a perfect gas having volume v_1, pressure p_1, and temperature T_1 and the chamber *B* is completely evacuated. These chambers are perfectly insulated so that no heat transfer takes place from or to its surroundings. Now, if the partition is removed, the gas will expand freely and occupy the whole space as shown in Fig. 3.23 (*b*). By this, the volume of the gas increases to v_2, pressure decreases to p_2 and the temperature may also decrease to T_2.

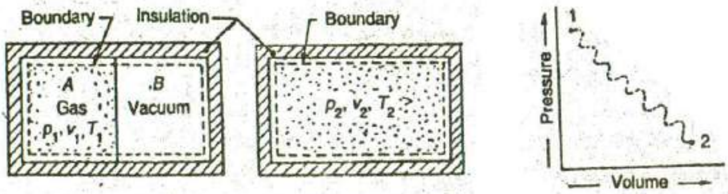
Since there is no expansion of the boundary of the system, because it is rigid, therefore no work is done. Thus, for a free expansion process,

$$Q_{1-2} = 0\,;\; W_{1-2} = 0 \text{ and } dU = 0$$

The following points may be noted regarding the free expansion of a gas :

1. Since the system is perfectly insulated so that no heat transfer takes place (*i.e.* $Q_{1-2} = 0$), therefore the expansion of gas may be called as an adiabatic expansion.

2. Since the free expansion of the gas from the equilibrium state 1 to the equilibrium state 2 takes place suddenly, therefore the intermediate states will not be in equilibrium states, as shown on the *p-v* diagram in Fig. 3.23 (*c*). Thus the process is irreversible and the expansion is, therefore, known as irreversible adiabatic expansion.



(*a*) Before expansion (*b*) After expansion. (*c*) *p-v* diagram.

Fig. 3.23. Free expansion.

3. Since there is no resistance to overcome during free expansion process, therefore no work is done by the system (*i.e.* $W_{1-2} = 0$). Thus, the free expansion process is also known as * *unresisted expansion process.*

4. According to the first law of thermodynamics,

$$Q_{1-2} = W_{1-2} + dU$$

Since for the free expansion, $Q_{1-2} = 0$ and $W_{1-2} = 0$, therefore the change in internal energy,

$$dU = U_2 - U_1 = 0 \quad \text{or} \quad U_2 = U_1$$

In other words, the internal energy of the system, in a free expansion process remains constant. Thus, the free expansion process is also known as *constant internal energy process.*

5. We know that change in internal energy,

$$dU = m\,c_v\,dT = m\,c_v\,(T_2 - T_1)$$

Since $dU = 0$, therefore $dT = 0$ or $T_2 = T_1$, *i.e.* there is no change in temperature of the system. In other words, the temperature of the system, in a free expansion process remains constant.

Note : It can not be called an isothermal process because in an actual isothermal process, work is done by the gas during expansion.

6. We know that change in enthalpy,

$$dH = H_2 - H_1 = m\,c_p\,dT$$

Since $dT = 0$, therefore $dH = 0$ or $H_2 = H_1$. In other words, the enthalpy of the system in a free expansion process remains constant. Thus the free expansion process may also be called *constant enthalpy process.*

---

* In the previous non-flow processes discussed, the expansion was against the resistance offered by the piston. So all the non-flow processes discussed earlier are resisted expansion processes.

3.15. General Laws for Expansion and Compression

The general law of expansion or compression of a perfect gas is pv^n = Constant (Art. 3.11). It gives the relationship between pressure and volume of a given quantity of gas. The value of n depends upon the nature of gas, and condition under which the changes (*i.e.* expansion or compression) take place. The value of n may be between zero and infinity. But the following values of n are important from the subject point of view :

1. When $n = 0$. This means pv^0 = Constant, *i.e.* p = Constant. In other words, for the expansion or compression of a perfect gas at *constant pressure*, $n = 0$.
2. When $n = 1$; then pv = Constant, *i.e.* the expansion or compression is *isothermal* or *hyperbolic*.
3. When n lies between 1 and n, the expansion or compression is *polytropic*, *i.e.* pv^n = Constant.
4. When $n = \gamma$, the expansion or compression is *adiabatic*, *i.e.* pv^γ = Constant.
5. When $n = \infty$, the expansion or compression is at constant volume, *i.e.* v = Constant.

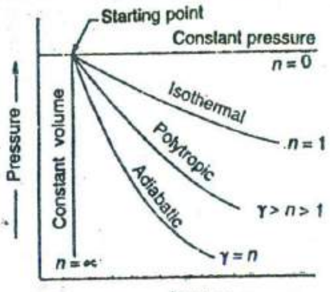


Fig. 3.24. Curves for various values of n.

Fig. 3.24 shows the curves of expansion of a perfect gas for different values of n. It is obvious that greater the value of n, steeper the curve of expansion.

3.16. Summary of Formulae for Heating and Expansion of Perfect Gases in Reversible Non-flow processes

The following table shows the summary of formulae for heating and expansion of perfect gases in reversible non-flow processes :

Table 3.1. Summary of formulae for heating and expansion of perfect gases.

S. No.	*Type of reversible non-flow process*	*p-v-T relation*	*Workdone* (W_{1-2})	*Change of internal energy* ($dU = U_2 - U_1$)	*Heat supplied* (Q_{1-2}) $= W_{1-2} + dU$	*Change in enthalpy* ($dH = H_2 - H_1$)
1.	Constant volume or Isochoric (v = Constant)	$\frac{p_1}{T_1} = \frac{p_2}{T_2}$ (Gay-Lussac Law)	0	$m\,c_v(T_2 - T_1)$	$m\,c_v(T_2 - T_1)$	$m\,c_p(T_2 - T_1)$
2.	Constant pressure or Isobaric (p = Constant)	$\frac{v_1}{T_1} = \frac{v_2}{T_2}$ (Charles' Law)	$p(v_2 - v_1)$ or $mR(T_2 - T_1)$	$m\,c_v(T_2 - T_1)$	$m\,c_p(T_2 - T_1)$	$m\,c_p(T_2 - T_1)$
3.	Hyperbolic or Constant temperature or Isothermal (T = Constant)	$p_1v_1 = p_2v_2$ (Boyle's Law)	$p_1v_1 \log_e\left(\frac{v_2}{v_1}\right)$ or $mRT\log_e\left(\frac{v_2}{v_1}\right)$	0	$p_1v_1 \log_e\left(\frac{v_2}{v_1}\right)$ or $mR\,T\log_e\left(\frac{v_2}{v_1}\right)$	0
4.	Adiabatic or Isentropic (pv^γ = Constant)	$p_1v_1^\gamma = p_2v_2^\gamma$ $\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{\gamma-1}$ $\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}}$	$\frac{p_1v_1 - p_2v_2}{\gamma - 1}$ or $\frac{mR(T_1 - T_2)}{\gamma - 1}$	$mc_v(T_2 - T_1)$	0	$m\,c_p(T_2 - T_1)$

5.	Polytropic or General expansion (pv^n = Constant)	$p_1v_1^n = p_2v_2^n$ $\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{n-1}$ $\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{n-1}{n}}$	$\frac{p_1v_1 - p_2v_2}{n-1}$ or $\frac{mR(T_1 - T_2)}{n-1}$	$mc_v(T_2 - T_1)$	$\frac{p_1v_1 - p_2v_2}{n-1}$ $+ mc_v(T_2 - T_1)$ or $\frac{\gamma - n}{\gamma - 1} \times$ $\frac{mR(T_1 - T_2)}{n-1}$	$mc_p(T_2 - T_1)$

3.17. Flow Processes

We have already discussed in Art. 3.2, that the processes occuring in *open system which permit the transfer of mass to and from the system, are known as *flow processes*. In a flow process, the mass (working substance) enters the system and leaves after doing the work. The flow process may be classified as

1. Steady flow process, and 2. Unsteady flow process.

In a steady flow process, the following conditions must be satisfied ;

(*a*) The rate of mass flow at inlet and outlet is same, *i.e.* the mass flow rate through the system remains constant.

(*b*) The rate of heat transfer is constant.

(*c*) The rate of work transfer is constant.

(*d*) The state of working substance at any point within the system is same at all times.

(*e*) There is no change in the chemical composition of the system. Thus no chemical energy is involved.

If any one of these conditions are not satisfied, then the process is said to be non-steady flow process. In engineering, we are mainly concerned with steady flow processes, therefore only these processes are discussed in the following pages.

3.18. Application of First Law of Thermodynamics to a Steady Flow Process

Consider an open system through which the working substance flows at a steady rate, as shown in Fig. 3.25. The working substance enters the system at section 1 and leaves the system at section 2.

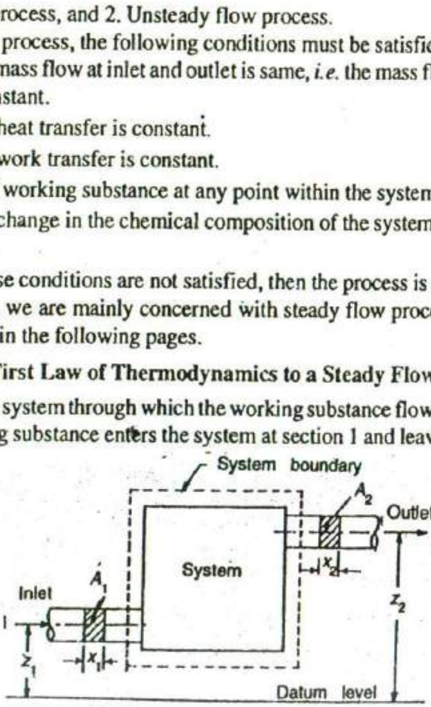


Fig. 3.25. Steady flow process.

Let p_1 = Pressure of the working substance entering the system in N/m^2,

v_{s1} = Specific volume of the working substance entering the system in m^3/kg

V_1 = Velocity of the working substance entering the system in m/s,

u_1 = Specific internal energy of the working substance entering the system in J/kg,

z_1 = Height above datum level for inlet in metres,

p_2, v_{s2}, V_2, u_2 and z_2 = Corresponding values for the working substance leaving the system.

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* Some authors use the term control volume instead of open system.

q_{1-2} = Heat supplied to the system in J/kg, and

w_{1-2} = Work delivered by the system in J/kg.

Consider 1kg of mass of the working substance.

We know that total energy entering the system per kg of the working substance,

e_1 = Internal energy + *Flow or displacement energy + Kinetic energy + Potential energy + Heat supplied

$$= u_1 + p_1 v_{s1} + \frac{V_1^2}{2} + g z_1 + q_{1-2} \text{ (in J/kg)}$$

Similarly, total energy leaving the system per kg of the working substance,

$$e_2 = u_2 + p_2 v_{s2} + \frac{V_2^2}{2} + g z_2 + w_{1-2} \text{ (in J/kg)}$$

Assuming no loss of energy during flow, then according to First Law of Thermodynamics (*i.e.* Law of Conservation of Energy), $e_1 = e_2$.

$$\therefore \quad u_1 + p_1 v_{s1} + \frac{V_1^2}{2} + g z_1 + q_{1-2} = u_2 + p_2 v_{s2} + \frac{V_2^2}{2} + g z_2 + w_{1-2}$$

We know that

$u_1 + p_1 v_{s1} = h_1$ = Enthalpy of the working substance entering the system in J/kg, and

$u_2 + p_2 v_{s2} = h_2$ = Enthalpy of the working substance leaving the system in J/kg.

Thus, the above expression may be written as

$$h_1 + \frac{V_1^2}{2} + g z_1 + q_{1-2} = h_2 + \frac{V_2^2}{2} + g z_2 + w_{1-2} \quad \ldots (i)$$

or

$$h_1 + ke_1 + pe_1 + q_{1-2} = h_2 + ke_2 + pe_2 + w_{1-2}$$

It may be noted that all the terms in equaiton (*i*) represent the energy flow per unit mass of the working substance (*i.e.* in J/kg). When the equation (*i*) is multiplied through by the mass of the working substance (m) in kg/s, then all the terms will represent the energy flow per unit time (*i.e.* in J/s).

Thus the equation (*i*) may also be written as

$$m\left(h_1 + \frac{V_1^2}{2} + g z_1 + q_{1-2}\right) = m\left(h_2 + \frac{V_2^2}{2} + g z_2 + w_{1-2}\right) \quad \ldots (ii)$$

---

* The *flow or displacement energy* is the energy required to flow or move the working substance against its pressure. It is also known as *flow work.*

For example, let the working substance with pressure p_1 (in N/m^2) flows through area A_1 (in m^2) and moves through a distance x_1 (in metres).

∴ Energy or work required to flow the working substance,

$$FE = \text{Force} \times \text{Distance} = (p_1 A_1)\, x_1 = p_1 v_1 \text{ (in joules)} \quad \ldots (\because v_1 = A_1 x_1)$$

where v_1 = Volume of the working substance in m^3.

For 1kg mass of the working substance,

$v_1 = v_{s1}$ = Specific volume of the working substance in m^3/kg.

$FE = p_1 v_{s1}$ (in J/kg)

Both the equations (*i*) and (*ii*) are known as ***steady flow energy equations.***

Notes : 1. In a steady flow, the mass flow rate (m) of the working substance entering and leaving the system is given by

$$m = \frac{A_1 V_1}{v_{s1}} = \frac{A_2 V_2}{v_{s2}} \text{ (in kg/s)}$$

This equation is known as ***equation of continuity.***

2. The steady flow energy equation (*i*), for unit mass flow may be written as

$$q_{1-2} - w_{1-2} = (h_2 - h_1) + \left(\frac{V_2^2}{2} - \frac{V_1^2}{2}\right) + (g\,z_2 - g\,z_1) \qquad \ldots (iii)$$

$$= (h_2 - h_1) + (ke_2 - ke_1) + (pe_2 - pe_1)$$

In differential form, this expression is written as

$$\delta q - \delta w = dh + d\,(ke) + d\,(pe)$$

3. In thermodynamics, the effect of gravity is generally neglected, therefore equation (*iii*) may be written as

$$q_{1-2} - w_{1-2} = (h_2 - h_1) + \left(\frac{V_2^2}{2} - \frac{V_1^2}{2}\right) \qquad \ldots (iv)$$

If $V_1 = V_2$, then equation (*iv*) reduces to

$$q_{1-2} - w_{1-2} = h_2 - h_1 \qquad \ldots (v)$$

4. In a non-flow process, the flow or displacement energy at inlet and outlet is zero, *i.e.*, $p_1 v_{s1} = 0$ and $p_2 v_{s2} = 0$. Therefore $h_2 = u_2$ and $h_1 = u_1$.

Thus the equation (*v*) may be written as $q_{1-2} - w_{1-2} = u_2 - u_1$; which is same as for non-flow process.

Example 3.21. *A steady flow thermodynamic system receives fluid at the rate of 6 kg/min with an initial pressure of 2 bar, velocity 150 m/s, internal energy 800 kJ/kg and density 27 kg/m³. The fluid leaves the system with a final pressure of 8 bar, velocity 200 m/s, internal energy 800 kJ/kg and density 5 kg/m³. If fluid receives 80 kJ/kg of heat during passing through the system and rises through 60 metres, determine the workdone during the process.*

Solution. Given : $m = 6$ kg/min $= 0.1$ kg/s ; $p_1 = 2$ bar $= 0.2 \times 10^6$ N/m² ; $V_1 = 150$ m/s ; $u_1 = 800$ kJ/kg ; $\rho_1 = 27$ kg/m³ ; $p_2 = 8$ bar $= 0.8 \times 10^6$ N/m² ; $V_2 = 200$ m/s ; $u_2 = 800$ kJ/kg ; $\rho_2 = 5$ kg/m³ ; $q_{1-2} = 80$ kJ/kg ; $z_2 = 60$ m

Consider 1 kg of mass flow.

We know that initial flow energy

$$= p_1 v_{s1} = p_1/\rho_1 = 0.2 \times 10^6/27 = 7407 \text{ J/kg} = 7.407 \text{ kJ/kg}$$

$$\ldots (\because v_{s1} = 1/\rho_1)$$

and final flow energy $= p_2 v_{s2} = p_2/\rho_2 = 0.8 \times 10^6/5 = 160 \times 10^3 \text{ J/kg} = 160 \text{ kJ/kg}$

∴ Initial enthalpy,

$$h_1 = u_1 + p_1 v_{s1} = 800 + 7.407 = 807.407 \text{ kJ/kg}$$

and final enthalpy, $h_2 = u_2 + p_2 v_{s2} = 800 + 160 = 960$ kJ/kg

Initial kinetic energy,

$$ke_1 = (V_1)^2/2 = (150)^2/2 = 11\,250 \text{ J/kg} = 11.25 \text{ kJ/kg}$$

Final kinetic energy, $ke_2 = (V_2)^2/2 = (200)^2/2 = 20\,000 \text{ J/kg} = 20 \text{ kJ/kg}$

Initial potential energy, $pe_1 = g\,z_1 = 0$ $\quad \ldots (\because z_1 = 0)$

Final potential energy, $pe_2 = g\,z_2 = 9.81 \times 60 = 588.6 \text{ J/kg} = 0.5886 \text{ kJ/kg}$

We know that the steady flow energy equation for unit mass flow is

$$h_1 + ke_1 + pe_1 + q_{1-2} = h_2 + ke_2 + pe_2 + w_{1-2}$$

or

$$q_{1-2} - w_{1-2} = (h_2 - h_1) + (ke_2 - ke_1) + (pe_2 - pe_1)$$

$$= (960 - 807.407) + (20 - 11.25) + (0.5886 - 0)$$

$$= 161.9316 \text{ kJ/kg}$$

$\therefore$

$$-w_{1-2} = 161.9316 - q_{1-2} = 161.9316 - 80 = 81.9316 \text{ kJ/kg}$$

or

$$w_{1-2} = -81.9316 \times 0.1 = -8.19316 \text{ kJ/s} \quad \textbf{Ans.} \quad (\because m = 0.1 \text{ kg/s})$$

The – *ve* sign indicates that the work is done on the system.

3.19. Workdone in a Steady Flow Process

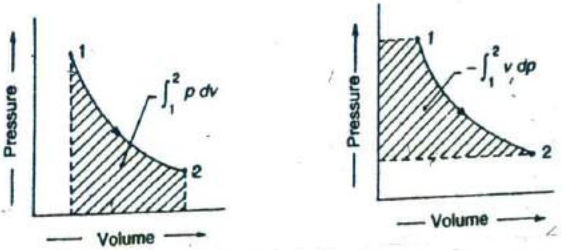
We know that the steady flow equation for unit mass flow, in the differential form, is

$$\delta q - \delta w = dh + d\,(ke) + d\,(pe) \quad \ldots (i)$$

We also know that $h = u + p\,v_s = u + p\,v$ $\quad \ldots (\because \text{For unit mass, } v_s = v)$

Differentiating this expression,

$$dh = du + d\,(p\,v) = du + p\,dv + v\,dp \quad \ldots (ii)$$



(*a*) Non-flow process. (*b*) Steady flow process.

Fig. 3.26. Workdone in a steady flow process.

According to First Law of Thermodynamics for a closed system, we know that

$$\delta q = du + p\,dv$$

Now the equation (*ii*) may be written as

$$dh = \delta q + v\,dp$$

Substituting this value of *dh* in equation (*i*), we have

$$\delta q - \delta w = (\delta q + v\,dp) + d\,(ke) + d\,(pe)$$

or

$$-\delta w = v\,dp + d\,(ke) + d\,(pe) \quad \ldots (iii)$$

If the changes in kinetic and potential energies are negligible [*i.e.* $d\,(ke) = 0$ and $d\,(pe) = 0$], then equation (*iii*) may be written as

$$-\delta w = v\,dp \text{ or } \delta w = -v\,dp$$

On integrating, $\int_1^2 \delta w = -\int_1^2 v\,dp$ or $w_{1-2} = -\int_1^2 v\,dp$ $\quad \dots (iv)$

Thus in a steady flow process, the workdone (w_{1-2}) is $-\int_1^{2'} v\,dp$ instead of $\int_1^2 p\,dv$ in a non-flow process. Fig. 3.26 shows the difference between workdone in a non-flow and steady flow processes.

Note : The –ve sign in equation (*iv*) makes the integral positive during expansion process. In other words, $-\int_1^2 v\,dp$ is a positive quantity and represents the workdone by the system.

3.20. Workdone for Various Steady Flow Processes

The various steady flow processes, like non-flow processes, are as follows :

1. Constant volume process, 2. Constant pressure process, 3. Constant temperature process, 4. Adiabatic process, and 5. Polytropic process.

We shall now derive the expressions for workdone during these processes, as discussed below. The suffixes $_{1,2}$ represents the initial and final conditions respectively.

1. *Constant volume process*

We know that workdone,

$$w_{1-2} = -\int_1^2 v\,dp = -v\int_1^2 dp = -v(p_2 - p_1) = v(p_1 - p_2)$$

2. *Constant pressure process*

We know that workdone,

$$w_{1-2} = -\int_1^2 v\,dp = -v\int_1^2 dp = v(p_1 - p_2) = 0 \quad \dots (\because p_1 = p_2 = \text{Constant})$$

3. *Constant temperature process*

Since the temperature is constant, therefore for a perfect gas,

$$p\,v = p_1\,v_1 = p_2\,v_2 = \text{Constant}$$

or

$$v = \frac{p_1\,v_1}{p}$$

We know that workdone

$$w_{1-2} = -\int_1^2 v\,dp = -\int_1^2 \frac{p_1\,v_1}{p}\,dp = -p_1 v_1 \int_1^2 \frac{dp}{p}$$

$$= -p_1\,v_1\,[\log_e p_2 - \log_e p_1] = p_1\,v_1\,[\log_e p_1 - \log_e p_2]$$

$$= p_1\,v_1 \log_e\left(\frac{p_1}{p_2}\right) = 2.3\,p_1\,v_1 \log\left(\frac{p_1}{p_2}\right)$$

$$= p_1 v_1 \log_e\left(\frac{v_2}{v_1}\right) = 2.3\,p_1\,v_1 \log\left(\frac{v_2}{v_1}\right)$$

$$\dots\left(\because p_1\,v_1 = p_2\,v_2 \text{ or } \frac{p_1}{p_2} = \frac{v_2}{v_1}\right)$$

4. *Adiabatic process*

We know that for an adiabatic process,

$$p v^{\gamma} = p_1 v_1^{\gamma} = p_2 v_2^{\gamma} = \text{Constant}$$

or
$$v = v_1 \left(\frac{p_1}{p} \right)^{1/\gamma}$$

We know that workdone,

$$w_{1-2} = -\int_1^2 v \, dp = -\int_1^2 v_1 \left(\frac{p_1}{p} \right)^{1/\gamma} dp$$

$$= -v_1 p_1^{1/\gamma} \int_1^2 p^{-1/\gamma} dp = -v_1 p_1^{1/\gamma} \left[\frac{p^{-1/\gamma+1}}{-\frac{1}{\gamma}+1} \right]_1^2$$

$$= -v_1 p_1^{1/\gamma} \left[\frac{p^{\frac{\gamma-1}{\gamma}}}{\frac{\gamma-1}{\gamma}} \right]_1^2 = \frac{\gamma}{\gamma-1} \times -v_1 p_1^{1/\gamma} \left[p_2^{\frac{\gamma-1}{\gamma}} - p_1^{\frac{\gamma-1}{\gamma}} \right]$$

$$= \frac{\gamma}{\gamma-1} (p_1 v_1 - p_2 v_2)$$

5. *Polytropic process*

We know that for a polytropic process,

$$p v^{n} = p_1 v_1^{n} = p_2 v_2^{n} = \text{Constant}$$

$\therefore$ Workdone, $w_{1-2} = \frac{n}{n-1} (p_1 v_1 - p_2 v_2)$ ... (Substituting $\gamma = n$, for polytropic process)

3.21. Throttling Process

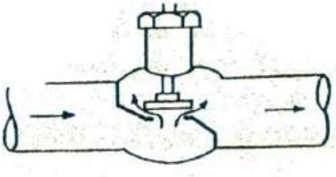


Fig. 3.27. Throttling process.

The throttling process is an irreversible steady flow expansion process in which a perfect gas is expanded through an orifice of minute dimensions such as a narrow throat or a slightly opened valve as shown in Fig. 3.27. Due to the fall in pressure during expansion, the gas should come out with a large velocity, but due to high frictional resistance between the gas and the walls of the aperture, there is no considerable change in velocity. The kinetic energy of the gas is converted into heat which is utilised in warming the gas to its initial temperature. Since no heat is supplied or rejected during the throttling process, and also no work is done, therefore

$$q_{1-2} = 0 \quad \text{and} \quad w_{1-2} = 0$$

We know that steady flow energy equation for unit mass flow is

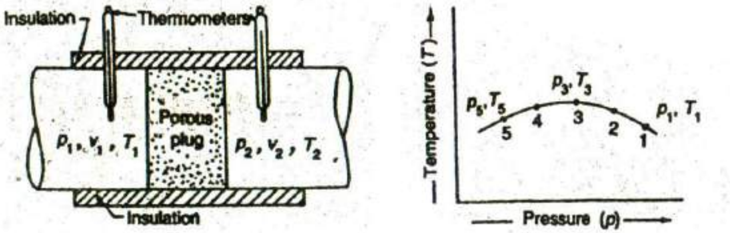
$$h_1 + \frac{V_1^2}{2} + gz_1 + q_{1-2} = h_2 + \frac{V_2^2}{2} + gz_2 + w_{1-2}$$

Since there is no considerable change in velocity and the inlet and outlet are at the same level, therefore $V_1 = V_2$ and $z_1 = z_2$. Now the steady flow energy equation is written as

$$h_1 = h_2 \qquad \ldots (\because\ q_{1-2} = 0 \text{ and } w_{1-2} = 0)$$

Thus, the throttling process is a *constant enthalpy process*. We have discussed in Art. 3.14, that in a free expansion process, the enthalpy also remains constant. But the difference between the throttling process and the free expansion process is that in the former case, the gas leaves with negligible velocity where as in the latter case, the gas leaves with a large velocity.

The throttling process was investigated by Joule and Thompson during an experiment known as Joule Thompson porous plug experiment, as shown in Fig. 3.28 (*a*).



(*a*) Joule Thompson porous plug experiment. (*b*) Constant enthalpy curve.

Fig. 3.28

In this experiment, a stream of gas at pressure p_1 and temperature T_1 is forced continuously through one side of the porous plug, as shown in Fig. 3.28 (*a*). The gas comes out from the other side of the porous plug at a pressure p_2 and temperature T_2. The whole apparatus is completely insulated so that no heat transfer takes place. In the similar way as discussed above, the enthalpy of gas before and after the process, remains constant, *i.e.* $h_1 = h_2$.

If the pressure p_1 and temperature T_1 is kept constant and the pressure on the downstream of the porous plug is varied to p_2, p_3, p_4, p_5 etc., then the graph between the pressures and the corresponding temperatures T_2, T_3, T_4, T_5 etc. will be a curve of constant enthalpy, as shown in Fig. 3.28 (*b*) because $h_1 = h_2 = h_3 = h_4 = h_5$ etc.

The slope of a constant enthalpy line is called *Joule Thompson coefficient* and is denoted by μ whose value is given by

$$\mu = \left(\frac{\partial T}{\partial p}\right)_h$$

The value of Joule Thompson coefficient (μ) at a particular state may be positive, zero or negative. For a perfect gas, the value of μ is zero.

3.22. Application of Steady Flow Energy Equation to Engineering Systems

The application of steady flow energy equation to some of the engineering systems such as boilers, condensers, nozzles, diffusers, compressors and turbines are discussed below :

1. *Boiler*. A boiler, as shown in Fig. 3.29, is a device which supplies heat to water and generates steam. In this system, there is no change in kinetic and potential energies.

Also there is no workdone by the system. In other words,

$$(ke_2 - ke_1) = 0,\ (pe_2 - pe_1) = 0 \text{ and } w_{1-2} = 0$$

We know that the steady flow energy equation for a unit mass flow is

$$q_{1-2} - w_{1-2} = (h_2 - h_1) + (ke_2 - ke_1) + (pe_2 - pe_1)$$

Thus, for a boiler, the steady flow energy equation is written as

$$q_{1-2} = h_2 - h_1$$

This shows that the heat supplied to the system in a boiler increases the enthalpy of the system.

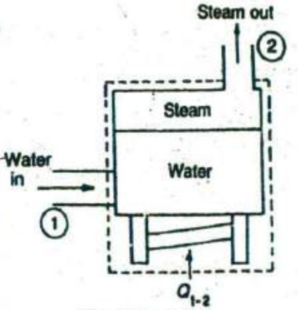


Fig. 3.29. Boiler.

2. *Condenser.* A condenser, as shown in Fig. 3.30. is a device used to condense steam in case of steam power plants using water as the cooling medium, whereas in refrigeration systems, it is used to condense refrigerant vapour using air as the cooling medium. For such a system, there is no change in kinetic and potential energies. Also there is no workdone by the system. In other words,

$$(ke_2 - ke_1) = 0\ ;\ (pe_2 - pe_1) = 0$$

and $$w_{1-2} = 0$$

We know that the steady flow energy equation for a unit mass flow is,

$$q_{1-2} - w_{1-2} = (h_2 - h_1) + (ke_2 - ke_1) + (pe_2 - pe_1)$$

Thus, for a condenser, the steady flow energy equation becomes

$$-q_{1-2} = h_2 - h_1 \quad \text{or} \quad q_{1-2} = h_1 - h_2$$

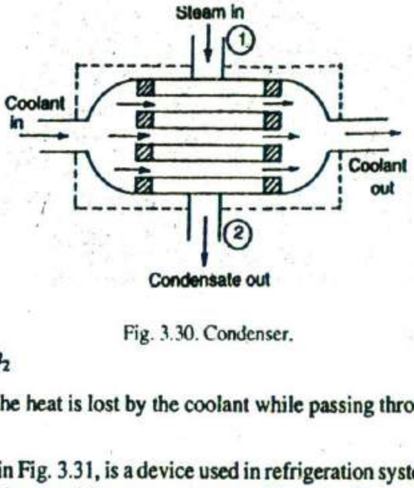


Fig. 3.30. Condenser.

The –ve sign with q_{1-2} is taken because the heat is lost by the coolant while passing through the condenser.

3. *Evaporator.* The evaporator, as shown in Fig. 3.31, is a device used in refrigeration systems in which the liquid refrigerant passes, receives heat and leaves as vapour refrigerant. For such a system, the change in kinetic and potential energies is negligible. Also there is no workdone by the system. In other words,

$$(ke_2 - ke_1) = 0,\ (pe_2 - pe_1) = 0$$

and $$w_{1-2} = 0$$

We know that the steady flow energy equation for a unit mass flow is

$$q_{1-2} - w_{1-2} = (h_2 - h_1) + (ke_2 - ke_1) + (pe_2 - pe_1)$$

Fig. 3.31. Evaporator.

Thus, for an evaporator, the steady flow energy equation becomes,

$$q_{1-2} = h_2 - h_1$$

Note : The process occuring in an evaporator is the reverse of that of a condenser.

4. *Nozzle.* A nozzle, as shown in Fig. 3.32, is a device which increases the velocity or kinetic energy of the working substance at the expense of its pressure drop. The nozzle is insulated so that no heat enters or leaves the system (*i.e.* $q_{1-2}=0$). In other words, the flow through nozzles is considered adiabatic. Further, the system does not deliver any work *i.e.* $w_{1-2}=0$ and there is no change in potential energy, *i.e.* $(pe_2-pe_1)=0$.

We know that the steady flow energy equation for a unit mass flow is

$$q_{1-2}-w_{1-2} = (h_2-h_1)+(ke_2-ke_1)+(pe_2-pe_1) = 0$$

Thus, for a nozzle, the steady flow energy equation becomes

$$0 = (h_2-h_1)+(ke_2-ke_1)$$

or
$$\frac{V_2^2}{2}-\frac{V_1^2}{2} = h_1-h_2$$

This shows that the increase in kinetic energy will result in decrease in enthalpy. If the process is reversed, it is obvious that the decrease in kinetic energy will result in increase of enthalpy. Such a system is known as ***diffuser***.

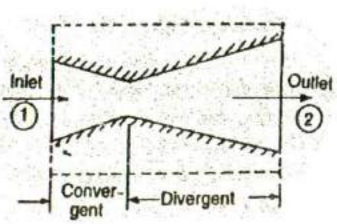


Fig. 3.32. Convergent-divergent nozzle.

From the above expression, we have

$$V_2^2-V_1^2 = 2(h_1-h_2) \quad \text{or} \quad V_2^2 = V_1^2+2(h_1-h_2)$$

$$\therefore \quad V_2 = \sqrt{V_1^2+2(h_1-h_2)}$$

If the initial velocity V_1 (also known as velocity of approach) is very small as compared to outlet velocity V_2, then V_1 may be neglected.

$$\therefore \quad V_2 = \sqrt{2(h_1-h_2)}$$

Note : If A_1 = Cross-sectional area of the nozzle at inlet,

V_1 = Velocity of the fluid entering the nozzle,

v_{s1} = Specific volume of the fluid at inlet,

A_2, V_2 and v_{s2} = Corresponding values at outlet.

∴ For continuous steady flow, mass flow rate,

$$m = \frac{A_1 V_1}{v_{s1}} = \frac{A_2 V_2}{v_{s2}}$$

5. *Turbine.* A turbine, as shown in Fig. 3.33, is a device which converts energy of the working substance (gas or steam) into work in the turbine. The turbine is insulated so that there is no transfer of heat (*i.e.* $q_{1-2}=0$). In other words, the flow through a turbine is considered adiabatic.

We know that the steady flow energy equation for a unit mass flow is

$$q_{1-2}-w_{1-2} = (h_2-h_1)+(ke_2-ke_1)+(pe_2-pe_1)$$

Thus, for a turbine, the steady flow energy equation becomes

$$-w_{1-2} = (h_2-h_1)+(ke_2-ke_1)+(pe_2-pe_1)$$

In case the changes in kinetic and potential energies are negligible [*i.e.* $(ke_2-ke_1)=0$ and $(pe_2-pe_1)=0$], then the above expression is written as

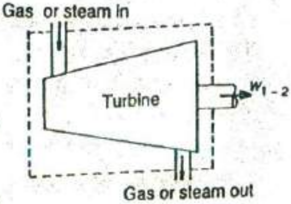


Fig. 3.33. Turbine.

$$-w_{1-2} = h_2 - h_1 \quad \text{or} \quad w_{1-2} = h_1 - h_2$$

This shows that the work is done by the system due to decrease in enthalpy of the working substance.

6. *Rotary compressor.* A rotary compressor, as shown in Fig. 3.34, is a device which compresses air and supplies the same at moderate pressure and in large quantities. The rotary compressor is insulated so that no heat transfer takes place (*i.e.* $q_{1-2} = 0$). In other words, the flow through a rotary compressor is adiabatic.

We know that the steady flow energy equation for a unit mass flow is

$$q_{1-2} - w_{1-2} = (h_2 - h_1) + (ke_2 - ke_1) + (pe_2 - pe_1)$$

Thus, for a rotary compressor, the steady flow energy equation becomes,

$$-(-w_{1-2}) = (h_2 - h_1) + (ke_2 - ke_1) + (pe_2 - pe_1)$$

The – *ve* sign is used because the work is done on the system. In case the changes in kinetic and potential energies are negligible [*i.e.* $ke_2 - ke_1 = 0$ and $(pe_2 - pe_1) = 0$], then the above expression is written as

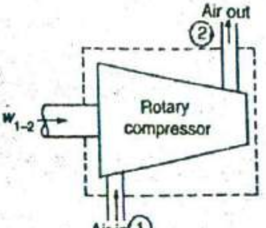


Fig. 3.34. Rotary compressor.

$$w_{1-2} = h_2 - h_1$$

This shows that the work is done due to increase in enthalpy.

7. *Reciprocating compressor.* A reciprocating compressor, as shown in Fig. 3.35, is a device which compresses air and supplies the same at a considerable higher pressure and in small quantities. The reciprocating compressor is considered as a steady flow system provided it includes the receiver which reduces the fluctuation of flow considerably.

We know that the steady flow energy equation for a unit mass flow is

$$q_{1-2} - w_{1-2} = (h_2 - h_1) + (ke_2 - ke_1) + (pe_2 - pe_1)$$

Since in a reciprocating compressor, the changes in kinetic and potential energies are negligible, therefore $(ke_2 - ke_1) = 0$ and $(pe_2 - pe_1) = 0$

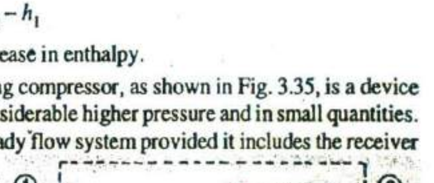


Fig. 3.35. Reciprocating compressor.

Thus, for a reciprocating compressor, the steady flow energy equation becomes

$$-q_{1-2} - (-w_{1-2}) = h_2 - h_1 \quad \text{or} \quad w_{1-2} = q_{1-2} + (h_2 - h_1)$$

The – *ve* sign to q_{1-2} and w_{1-2} is used because the heat is rejected and the work is done on the system.

Example 3.22. *A gas expands through an ideally, insulated nozzle following a reversible polytropic law $p\,v^{1.2} = C$. There is no change in potential energy but the pressure drops from 20 bar to 2 bar and specific volume increases from 0.05 m^3 to 0.3 m^3. If the entrance velocity is 80 m/s, determine the exit velocity.*

Solution. Given : *$n = 1.2$; $p_1 = 20$ bar $= 2 \times 10^6$ N/m^2 ; $p_2 = 2$ bar $= 0.2 \times 10^6$ N/m^2 ; $v_{x1} = 0.05$ m^3 ; $v_{x2} = 0.3$ m^3 ; $V_1 = 80$ m/s

---

* Superfluous data

Let V_2 = Exit velocity in m/s.

We know that the steady flow energy equation for a unit mass flow is

$$h_1 + ke_1 + pe_1 + q_{1-2} = h_2 + ke_2 + pe_2 + w_{1-2}$$

Since the nozzle is insulated so that no heat transfer takes place, therefore $q_{1-2} = 0$. Also there is no workdone during expansion of gas through the nozzle (*i.e.* $w_{1-2} = 0$) and there is no change in potential energy (*i.e.* $pe_1 = pe_2$). Thus the steady flow energy equation is written as

$$h_1 + ke_1 = h_2 + ke_2$$

$$u_1 + p_1 v_{s1} + \frac{V_1^2}{2} = u_2 + p_2 v_{s2} + \frac{V_2^2}{2}$$

or $$p_1 v_{s1} + \frac{V_1^2}{2} = p_2 v_{s2} + \frac{V_2^2}{2}$$ . . . (∵ For an insulated nozzle, $u_1 = u_2$)

$$2 \times 10^6 \times 0.05 + \frac{(80)^2}{2} = 0.2 \times 10^6 \times 0.3 + \frac{V_2^2}{2}$$

$$103\,200 = 60\,000 + \frac{V_2^2}{2}$$

∴ $$V_2^2 = 2\,(103\,200 - 60\,000) = 86\,400$$

or $$V_2 = 294 \text{ m/s}$$ **Ans.**

Example 3.23. *The velocity and enthalpy of fluid at the inlet of a certain nozzle are 50 m/s and 2800 kJ/kg respectively. The enthalpy at the exit of nozzle is 2600 kJ/kg. The nozzle is horizontal and insulated so that no heat transfer takes place from it. Find : 1. velocity of the fluid at exit of the nozzle ; 2. mass flow rate, if the area at inlet of nozzle is 0.09 m² and the specific volume is 0.185 m³/kg ; and 3. exit area of the nozzle, if the specific volume at the exit of nozzle is 0.495 m³/kg.*

Solution. Given : $V_1 = 50$ m/s ; $h_1 = 2800$ kJ/kg $= 2800 \times 10^3$ J/kg ; $h_2 = 2600$ kJ/kg $= 2600 \times 10^3$ J/kg ; $q_{1-2} = 0$; $A_1 = 0.09$ m² ; $v_{s1} = 0.185$ m³/kg ; $v_{s2} = 0.495$ m³/kg

1. *Velocity of fluid at exit of nozzle*

Let V_2 = Velocity of fluid at exit of nozzle.

We know that the steady flow energy equation for a unit mass flow is

$$h_1 + ke_1 + pe_1 + q_{1-2} = h_2 + ke_2 + pe_2 + w_{1-2}$$

Since the nozzle is insulated so that no heat transfer takes place, therefore $q_{1-2} = 0$. Also there is no workdone, *i.e.* $w_{1-2} = 0$. Neglecting the potential energy at inlet and outlet of the nozzle because of at same level from datum, the steady flow energy equation is written as

$$h_1 + ke_1 = h_2 + ke_2 \quad \text{or} \quad ke_2 - ke_1 = h_1 - h_2$$

∴ $$\frac{V_2^2}{2} - \frac{V_1^2}{2} = h_1 - h_2 \quad \text{or} \quad V_2^2 - V_1^2 = 2\,(h_1 - h_2)$$

and $$V_2 = \sqrt{V_1^2 + 2\,(h_1 - h_2)} = \sqrt{(50)^2 + 2\,(2800 \times 10^3 - 2600 \times 10^3)}$$

$$= \sqrt{2500 + 400 \times 10^3} = 634.4 \text{ m/s}$$ **Ans.**

2. *Mass flow rate*

Let m = Mass flow rate in kg/s.

We know that $$m = \frac{A_1 V_1}{v_{s1}} = \frac{0.09 \times 50}{0.185} = 24.3 \text{ kg/s} \quad \textbf{Ans.}$$

3. *Exit area of nozzle*

Let A_2 = Exit area of nozzle in m².

We know that $$m = \frac{A_2 V_2}{v_{s2}} \quad \text{or} \quad A_2 = \frac{m \times v_{s2}}{V_2} = \frac{24.3 \times 0.495}{634.4} = 0.019 \text{ m}^2 \quad \textbf{Ans.}$$

Example 3.24. *In an air compressor, air flows steadily at the rate of 15 kg per minute. The air enters the compressor at 5 m/s with a pressure of 1 bar and a specific volume of 0.5 m³/kg. It leaves the compressor at 7.5 m/s with a pressure of 7 bar and a specific volume of 0.15 m³/kg. The internal energy of the air leaving the compressor is 165 kJ/kg greater than that of the air entering. The cooling water in the compressor jackets absorbs heat from the air at the rate of 125 kJ/s. Find : 1. power required to drive the compressor ; and 2. ratio of the inlet pipe diameter to outlet pipe diameter.*

Solution. Given : $m = 15$ kg/min $= 0.25$ kg/s ; $V_1 = 5$ m/s ; $p_1 = 1$ bar $= 0.1 \times 10^6$ N/m² ; $v_{s1} = 0.5$ m³/kg ; $V_2 = 7.5$ m/s ; $p_2 = 7$ bar $= 0.7 \times 10^6$ N/m² ; $v_{s2} = 0.15$ m³/kg ; $u_2 - u_1 = 165$ kJ/kg $= 165 \times 10^3$ J/kg ; $q_{1-2} = 125$ kJ/s $= 125/m = 125/0.25 = 500$ kJ/kg $= 500 \times 10^3$ J/kg

1. *Power required to drive the compressor*

Let w_{1-2} = Work required to drive the compressor in J/kg.

We know that flow energy at inlet

$$= p_1 v_{s1} = 0.1 \times 10^6 \times 0.5 = 50 \times 10^3 \text{ J/kg}$$

Flow energy at outlet $= p_2 v_{s2} = 0.7 \times 10^6 \times 0.15 = 105 \times 10^3$ J/kg

Kinetic energy at inlet,

$$ke_1 = (V_1)^2/2 = 5^2/2 = 12.5 \text{ J/kg}$$

and kinetic energy at outlet,

$$ke_2 = (V_2)^2/2 = (7.5)^2/2 = 28.1 \text{ J/kg}$$

We know that the steady flow energy equation for a unit mass flow is

$$u_1 + p_1 v_{s1} + ke_1 + pe_1 - q_{1-2} = u_2 + p_2 v_{s2} + ke_2 + pe_2 - w_{1-2}$$

In this expression, q_{1-2} and w_{1-2} are taken – *ve*, because heat is *rejected* by the air and work is done *on* the air. Neglecting the potential energy, the steady flow energy equation may be written as

$$w_{1-2} - q_{1-2} = (u_2 - u_1) + (p_2 v_{s2} - p_1 v_{s1}) + (ke_2 - ke_1)$$

$$w_{1-2} - 500 \times 10^3 = 165 \times 10^3 + (105 \times 10^3 - 50 \times 10^3) + (28.1 - 12.5)$$

$$= 220 \times 10^3 \text{ J/kg}$$

. . . [Neglecting $(ke_2 - ke_1)$, as it is very small as compared to other terms]

or $$w_{1-2} = 720 \times 10^3 \text{ J/kg}$$

∴ Power required to drive the compressor

$$= m \times w_{1-2} = 0.25 \times 720 \times 10^3 \text{ J/s} = 80 \times 10^3 \text{ J/s}$$

$$= 180 \text{ kJ/s} = 180 \text{ kW} \textbf{ Ans.} \qquad \ldots (\because 1 \text{ kJ/s} = 1 \text{kW})$$

2. *Ratio of the inlet pipe diameter to outlet pipe diameter*

Let D_1 = Inlet pipe diameter, and

D_2 = Outlet pipe diameter.

We know that $\dfrac{A_1 V_1}{v_{s1}} = \dfrac{A_2 V_2}{v_{s2}}$ or $\dfrac{A_1}{A_2} = \dfrac{V_2 \times v_{s1}}{V_1 \times v_{s2}}$

$$\therefore \quad \frac{\frac{\pi}{4}(D_1)^2}{\frac{\pi}{4}(D_2)^2} = \frac{V_2 \times v_{s1}}{V_1 \times v_{s2}} \text{ or } \frac{D_1}{D_2} = \sqrt{\frac{V_2 \times v_{s1}}{V_1 \times v_{s2}}} = \sqrt{\frac{7.5 \times 0.5}{5 \times 0.15}} = 2.236 \textbf{ Ans.}$$

Example 3.25. *In a gas turbine, the gases flow at the rate of 5 kg/s. The gases enter the turbine at a pressure 7 bar with a velocity 120 m/s and leaves at a pressure 2 bar with velocity 250 m/s. The turbine is insulated. If the enthalpy of the gas at inlet is 900 kJ/kg and at outlet 600 kJ/kg, determine the capacity of the turbine.*

Solution. Given : $m = 5$ kg/s ; $p_1 = 7 \text{ bar} = 0.7 \times 10^6 \text{ N/m}^2$; $V_1 = 120$ m/s ; $p_2 = 2$ bar $= 0.2 \times 10^6 \text{ N/m}^2$; $V_2 = 250$ m/s ; $h_1 = 900 \text{ kJ/kg} = 900 \times 10^3$ J/kg ; $h_2 = 600 \text{ kJ/kg} = 600 \times 10^3$ J/kg

We know that the steady flow energy equation for a unit mass flow is

$$h_1 + ke_1 + pe_1 + q_{1-2} = h_2 + ke_2 + pe_2 + w_{1-2}$$

Since the turbine is insulated so that no heat transfer takes place, therefore $q_{1-2} = 0$ Neglecting the potential energy at inlet and outlet, the steady flow energy equation becomes

$$h_1 + ke_1 = h_2 + ke_2 + w_{1-2}$$

or

$$w_{1-2} = (h_1 - h_2) + (ke_1 - ke_2) = (h_1 - h_2) + \left(\frac{V_1^2}{2} - \frac{V_2^2}{2}\right)$$

$$= (900 \times 10^3 - 600 \times 10^3) + \left[\frac{(120)^2}{2} - \frac{(250)^2}{2}\right]$$

$$= 300 \times 10^3 - 24.05 \times 10^3 = 275.95 \times 10^3 \text{ J/kg}$$

∴ Capacity of the turbine

$$= m \times w_{1-2} = 5 \times 275.95 \times 10^3 = 1379.75 \times 10^3 \text{ J/s}$$

$$= 1379.75 \text{ kJ/s} = 1379.75 \text{ kW} \textbf{ Ans.}$$

EXERCISES

1. One kg of air is heated in a closed vessel, *i.e.*, at a constant volume from a pressure of 2 bar to 5 bar. If the initial temperature of the air is 300 K, determine the change in internal energy. $c_v = 0.712$ kJ/kg K. **[Ans. 320.4 kJ]**

2. A volume of 0.5 m^3 of gas at a pressure of 10 bar and 200° C is expanded in a cylinder to 1.2 m^3 at a constant pressure. Calculate the amount of work done by the gas and the increase in internal energy. Assume $c_p = 1.005$ kJ/kg K and $c_v = 0.712$ kJ/kg K. **[Ans. 700 kJ ; 1697.35 kJ]**

3. A piston cylinder containing air expands at a constant pressure of 150 kPa from a temperature of 285 K to a temperature of 550 K. The mass of air is 0.05 kg. Find the heat transfer, work transfer and the change

in internal energy during the process. Take $c_p = 1$ kJ/kg K and $R = 0.287$ kJ/kg K.

[Ans. 13.25 kJ ; 3.8 kJ ; 9.45 kJ]

4. One kg of air at a temperature of 40° C is compressed isothermally from a pressure of 1.5 bar to 6 bar. Determine the heat rejected by the air during the process of compression. For air, $c_p = 1.005$ kJ/kg K and $c_v = 0.712$ kJ/kg K. [Ans. 124.2 kJ]

5. An ideal gas at 30° C and 1 bar is compressed adiabatically from 5 m^3 to 1 m^3. Find the temperature, pressure and the work done. Take $\gamma = 1.4$. [Ans. 304° C ; 9.5 bar ; 1.125 kJ]

6. A 0.568 m^3 capacity insulated vessel of oxygen at the pressure of 13.6 bar is stirred by internal paddle until the pressure becomes 21 bar. Find out the heat transferred and the work input.

Take $c_v = 0.658$ kJ/kg K, and $R = 260$ J/kg K. [Ans. zero ; 1063.8 kJ/kg]

7. One kg of a gas expands reversibly and adiabatically. Its temperature during the process falls from 515 K to 390 K, while the volume is doubled. The gas does 92 kJ of work in this process. Find : 1. The value of c_p and c_v ; and 2. The molecular mass of the gas. [Ans. 1.0326 kJ/kg K, 0.736 kJ/kg K ; 28.03]

8. A gas initially at 14.4 bar and 360° C is expanded isothermally to a pressure of 2.24 bar. It is then cooled at constant volume till the pressure falls to 1.02 bar. Finally an adiabatic compression brings the gas back to the initial stage. The mass of the gas is 0.23 kg and $c_p = 1$ kJ/kg K. Draw the p-v diagram and determine : 1. the value of the adiabatic index of compression ; and 2. the change of internal energy of the gas during the adiabatic process. [Ans. 1.423 ; 55.8 kJ (decrease)]

9. A cylinder contains 0.084 m^3 of hydrogen at 1.05 bar and 18° C. It is compressed adiabatically to 14 bar and then expanded isothermally to the original volume of 0.084 m^3. The characteristic constant for hydrogen is 4200 J/kg K and its specific heat at constant pressure is 14.28 kJ/kg K.

Determine the final pressure of the gas and the amount of heat which must be added to the gas during isothermal expansion. Also calculate the heat which must be abstracted from the gas after expansion in order to reduce it to its initial state of pressure. [Ans. 2.25 bar ; 34.5 kJ ; 24.14 kJ]

10. A quantity of gas is compressed according to $pv^{1.25}$ = Constant. The initial temperature and pressure of the gas is 15° C and 1 bar respectively. Find the work done in compressing 1 kg of air at 3 bar and the heat rejected through the walls of the cylinder. $\gamma = 1.4$ for air. [Ans. 82.65 kJ ; 103.3 kJ]

11. A quantity of air has a volume of 56.5 litres and a pressure of 7.03 bar. It is expanded in a cylinder to a pressure of 1.05 bar. Compute the workdone if the expansion is 1. hyperbolic, 2. adiabatic, and 3. $p\,v^{1.2} = C$. Take $\gamma = 1.4$. [Ans. 75.4 kJ ; 42.5 kJ ; 55 kJ]

12. 1.4 m^3 of a gas at a pressure of 1.26 bar is compressed to a volume of 0.28 m^3. The final pressure is 7 bar. Assuming the compression to be polytropic, calculate the heat transfer and change in internal energy. Assume $\gamma = 1.4$. [Ans. 252.54 kJ (rejected) ; 49 kJ (decrease)]

13. An ideal gas of molecular mass 30 and specific heat ratio 1.38 is compressed according to the law $p\,v^{1.25}$ = constant, from a pressure of 1 bar and 15° C to a pressure of 16 bar. Calculate the temperature at the end of compression, the heat received or rejected and workdone by the gas during the process. Assume 1 kg mass of the gas. Use only calculated values of c_p and c_v. [Ans. 228.7° C ; 81 kJ (rejected) ; 236.78 kJ]

14. A cylinder contains 0.113 m^3 of air at 1 bar and 90° C. The air is compressed to a volume of 0.028 m^3, the final pressure being 5.8 bar. Determine : 1. mass of the air in the cylinder, 2. value of index (n) for the compression process, 3. increase in internal energy, and 4. heat received or rejected by air during compression.

If, after the above process, the air is cooled at a constant pressure to its original temperature of 90° C, find the further work of compression required. $\gamma = 1.4$ and $R = 287$ J/kg K.

[Ans. 0.108 kg ; 1.26 ; 12.3 kJ ; 6.62 kJ (rejected) ; 4.912 kJ]

15. An internal combustion engine cylinder has a diameter 240 mm and length of the stroke 400 mm. The clearance volume is one-fourth of the swept volume. The pressure at the beginning of expansion stroke is 16 bar and the expansion follows the law $pv^{1.3}$ = Constant. Determine : 1. the pressure at the end of expansion stroke, and 2. the work done during the expansion. [Ans. 1.974 bar ; 9.2 kJ]

16. The internal energy and equation of state of a closed gas system, are given by

$$U = (188.4 + 1.256\,t)\ \text{kJ/kg} \ ; \text{and}\ p\,v = 600\,T$$

where t is the temperature in °C and T in Kelvin. p is the pressure in bar and v is the specific volume in m^3.

If the temperature of 2 kg of gas is raised from 100° C to 200° C at 1. constant pressure, and 2. according to the law $p\,v^{1.2}$ = constant ; find out the heat flow, work flow and change in internal energy of the system. [Ans. 251.2 kJ ; 371.2 kJ ; 600 kJ, 348.8 kJ]

17. Gas at 1.5 bar and 295 K in a closed vessel is compressed to 10 bar. Its temperature then becomes 455 K. If the compression follows the law $pv^n = C$, find the value of n. [Ans. 1.3]

18. One kg of fluid enters the steady flow apparatus at a pressure of 6 bar, velocity 16 m/s and specific volume 0.4 m^3/kg. The inlet is 30 m above the ground level. The fluid leaves the apparatus at a pressure of 1 bar, velocity 275 m/s ; and specific volume 0.6 m^3/kg. The outlet is at the ground level. The total heat loss between the inlet and outlet is 10 kJ/kg of fluid. If 140 kJ/kg of work is done by the system, find the change in specific internal energy and indicate whether this is a increase or decrease. [Ans. 7.4 kJ/kg (decrease)]

19. Air at the rate of 12 kg/min flows steadily through a nozzle. The pressure and temperature of air at the inlet to the nozzle are 20 bar and 390 K respectively. The pressure of air at the exit of nozzle is 5 bar. Assuming adiabatic flow with initial velocity of 100 m/s, determine the exit velocity and the inlet and exit areas.

Take c_p = 1.005 kJ/kg K and $\gamma = 1.4$, for air. [Ans. 516.4 m/s ; 112 mm^2, 58 mm^2]

20. An air compressor draws air at 1 bar and 20° C and discharges into a line having an inside diameter of 10 mm. The average air velocity in the line at a point close to the discharge is 7.5 m/s and the discharge pressure is 3 bar. Assuming that compression takes place adiabatically, determine the power required to drive the compressor. The velocity of air entering the compressor has negligible velocity. Take c_p = 1.005 kJ/kg K ; R = 287 J/kg K and $\gamma = 1.4$. [Ans. 0.1665 kW]

21. Air is expanded reversibly and adiabatically in a turbine from 3.5 bar and 260° C to 1 bar. The turbine is insulated and the inlet velocity is negligible. The exit velocity is 150 m/s. Find the work output of the turbine per unit mass of air flow. Take for air, c_p = 1.005 kJ/kg K, and $\gamma = 1.4$. [Ans. 15 5 kJ/kg]

QUESTIONS

1. What do you understand by a 'thermodynamic process' ? Distinguish between reversible and irreversible process.

2. Explain the difference between non-flow process and a flow process. Derive the equation for workdone during a non-flow process.

3. What is an isothermal process ? Derive an expression for the workdone during an isothermal process.

4. Explain the adiabatic process. Derive an expression for the workdone during the adiabatic expansion of an ideal gas.

5. What is polytropic process ? How does it differ from an adiabatic process.

6. Prove that the heat absorbed or rejected during a polytropic process is $\frac{\gamma - n}{\gamma - 1} \times$ work done, where γ is the ratio of specific heat and n is the polytropic index.

7. Explain what is meant by "polytropic" operation. Starting from the fundamental, show that during a polytropic compression, according to the law pv^n = Constant, the rate of heat rejection per unit change in volume is given by $\left(\frac{\gamma - n}{\gamma - 1}\right) \times p$, where γ is the ratio of specific heat and p is the pressure at a particular point (a mean pressure during the process) at which heat rejection is considered.

8. Explain free expansion process. What is the difference between throttling process and a free expansion process ?

9. What are the assumptions for a steady flow process ? Write the general energy equation for a steady flow system.

10. Write down the simplified steady flow energy equation for a unit mass flow for (*a*) condenser ; (*b*) compressor ; and (*c*) turbine.

11. Apply steady flow energy equation to a nozzle and derive an equation for velocity at exit.

OBJECTIVE TYPE QUESTIONS

1. When a gas is heated at constant volume,
   (*a*) its temperature will increase
   (*b*) its pressure will increase
   (*c*) both temperature and pressure will increase
   (*d*) neither temperature nor pressure will increase

2. The heating of a gas at constant pressure is governed by
   (*a*) Boyle's law (*b*) Charles' law (*c*) Gay-Lussac law (*d*) Joule's law

3. A process, in which the gas is heated or expanded in such a way that the product of it pressure and volume remains constant, is called
   (*a*) isothermal process (*b*) isobaric process
   (*c*) adiabatic process (*d*) polytropic process

4. The hyperbolic process is governed by
   (*a*) Boyle's law (*b*) Charles' law (*c*) Gay-Lussac law (*d*) Joule's law

5. The heating of gas at constant ....is governed by Boyle's law.
   (*a*) volume (*b*) pressure (*c*) temperature

✓ 6. In an isothermal process,
   (*a*) internal energy increases
   (*b*) internal energy decreases
   (*c*) there is no change in internal energy
   (*d*) internal energy first decreases and then increases

7. The expansion ratio (r) is the ratio of
   (*a*) $\frac{v_1}{v_2}$ (*b*) $\frac{v_2}{v_1}$ (*c*) $\frac{v_1+v_2}{v_1}$ (*d*) $\frac{v_1+v_2}{v_2}$

where v_1 = Volume at the beginning of expansion, and
v_2 = Volume at the end of expansion.

8. When the expansion or compression of the gas takes places according to the law $pv^n = C$, then the process is known as
   (*a*) isothermal process (*b*) isobaric process
   (*c*) adiabatic process (*d*) polytropic process

9. An adiabatic process is one in which
   (*a*) no heat enters or leaves the gas
   (*b*) the temperature of the gas changes
   (*c*) the change in internal energy is equal to the workdone
   (*d*) all of the above

10. The general law of expansion or compression is $pv^n = C$. The process is said to be ıyperbolic, if n is equal to
   (*a*) 0 (*b*) 1 (*c*) γ (*d*) ∞

11. If the value of $n = 0$ in the general law $pv^n = C$, then the process is called
   (*a*) isochoric process (*b*) isobaric process
   (*c*) isothermal process (*d*) isentropic process

12. The workdone in a free expansion process is
    (*a*) zero (*b*) minimum (*c*) maximum (*d*) positive

13. In a steady flow process,
    (*a*) the mass flow rate is constant (*b*) the heat transfer rate is constant
    (*c*) the work transfer rate is constant (*d*) all of the above

14. The workdone in steady flow process is given by
    (*a*) $\int_1^2 p\,dv$ (*b*) $-\int_1^2 p\,dv$ (*c*) $\int_1^2 v\,dp$ (*d*) $-\int_1^2 v\,dp$

15. The throttling process is a
    (*a*) non-flow process (*b*) steady flow process
    (*c*) non-steady flow process

ANSWERS

1. (*c*)	2. (*b*)	3. (*a*)	4. (*a*)	5. (*c*)
6. (*c*)	7. (*b*)	8. (*d*)	9. (*d*)	10. (*b*)
11. (*b*)	12. (*a*)	13. (*d*)	14. (*d*)	15. (*b*)

4

Entropy of Perfect Gases

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1. Introduction. 2. Relation between Heat and Entropy. 3. Importance of Entropy. 4. Available and Unavailable Heat Energy. 5. Units of Entropy. 6. Clausius Inequality. 7. Principle of Increase of Entropy. 8. General Expression for Change of Entropy of a Perfect Gas. 9. Change of Entropy of a Perfect Gas during Various Thermodynamic Processes. 10. Change of Entropy during Constant Volume Process (or Isochoric Process). 11. Change of Entropy during Constant Pressure Process (or Isobaric Process). 12. Change of Entropy during Constant Temperature Process (or Isothermal Process). 13. Change of Entropy during Reversible Adiabatic Process (or Isentropic Process). 14. Change of Entropy during Polytropic Process (pv^n = Constant). 15. Approximate Method for Heat Absorbed.

4.1. Introduction

The term 'entropy' which literally means transformation, was first introduced by Clausius. It is an important thermodynamic property of a working substance, which increases with the addition of heat, and decreases with its removal. As a matter of fact, it is tedious to define the term entropy. But it is comparatively easy to define change of entropy of a working substance. In a reversible process, over a small range of temperature, the increase or decrease of entropy, when multiplied by the absolute temperature, gives the heat absorbed or rejected by the working substance. Mathematically, heat absorbed by the working substance,

$$\delta Q = TdS$$

where T = Absolute temperature, and

dS = Increase in entropy.

Note : The above relation also holds good for heat rejected by the working substance. In that case, dS will be decrease in entropy.

The engineers and scientists use it for providing quick solution, to problems dealing with reversible adiabatic expansion. The entropy is usually represented by S.

4.2. Relation between Heat and Entropy

Consider the heating of a working substance by a reversible process as shown by a curve from 1 to 2, on a graph, whose base represents the entropy and the vertical ordinate represents the absolute temperature as shown in Fig. 4.1. This diagram is known as temperature-entropy (T-S) diagram.

Now consider any point A on the curve 1-2. At this point, let a small quantity of heat (δQ) be supplied to the working substance, which will increase the entropy by dS. Let the absolute temperature at this instant be T. Then according to the definition of entropy,

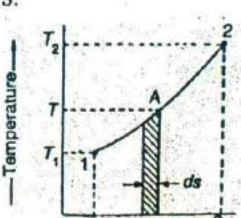

Fig. 4.1. Temperature-entropy diagram.

$$\delta Q = TdS \quad \ldots (i)$$

From Fig. 4.1, we see that the term TdS represents the area under the curve during this change of entropy. Now the total area under the curve 1-2 may be found out by integrating the equation (*i*), *i.e.*

$$\int \delta Q = \int TdS = \text{Total heat supplied or absorbed}$$

From equation (*i*), we get

$$dS = \frac{\delta Q}{T}$$

The total change in entropy may be obtained by integrating the above expression from state 1 to state 2,

$$\therefore \qquad \int_1^2 dS = \int_1^2 \frac{\delta Q}{T} \qquad \ldots (ii)$$

Notes : 1. The area under the T-S diagram of any thermodynamic process represents the heat absorbed or rejected during that process.

2. Since $\int \frac{\delta Q}{T}$ is same for all reversible paths between states 1 and 2, so we conclude that this quantity is independent of a path and is a function of end states only. In other words, the entropy is a point function and thus it is a property of the system. The entropy may be expressed as a function of other thermodynamic properties of the system, such as the pressure and temperature or pressure and volume.

3. We know that according of First Law of Thermodynamics,

$$\delta Q = dU + \delta W = dU + p\,dv \qquad \ldots (\because \delta W = p\,dv) \quad \ldots (iii)$$

and

$$\delta Q = TdS \qquad \ldots (iv)$$

From equations (*iii*) and (*iv*),

$$TdS = dU + pdv \qquad \ldots (v)$$

It is very interesting to note that in equations (*iii*) and (*iv*), δQ and δW are path functions, therefore these equations are true only for reversible processes. But in equation (*v*), dS, dU and dv are point functions as they depend upon the initial and final equilibrium states, therefore equation (*v*) is true for reversible as well as irreversible processes.

4. The entropy remains constant in a reversible cyclic process and increases in an irreversible cyclic process (see Art. 4.7)

4.3. Importance of Entropy

The maximum possible efficiency obtainable by any engine working on a reversible * Carnot cycle is given by

$$\eta = \frac{T_1 - T_2}{T_1} \qquad \ldots (i)$$

where T_1 = Highest absolute temperature, and

T_2 = Lowest absolute temperature.

In general, efficiency is given by

$$\eta = \frac{\text{Maximum work obtained}}{\text{Heat supplied or absorbed}} = \frac{\delta W}{\delta Q}$$

or

$$\delta W = \delta Q \times \eta = \delta Q \left(\frac{T_1 - T_2}{T_1} \right) \qquad \ldots \text{(From equation } (i))$$

---

* For details, please refer to Chapter 6 on Thermodynamic Air Cycles.

For one degree temperature drop, the above expression may be written as

$$\delta W = \frac{\delta Q}{T} = dS = \text{Change in entropy}$$

From this expression, it can be easily understood that

1. The change in entropy represents the maximum amount of work obtainable per degree drop in temperature.
2. The change in entropy may be regarded as a measure of the rate of the availability or *unavailability of heat for transformation into work.
3. The increase in entropy is obtained from a given quantity of heat at a low temperature.

4.4. Available and Unavailable Heat Energy

The heat energy of a system (or heat supplied to the working substance) is considered to have the following two parts :

1. Available heat energy ; and 2. Unavailable heat energy.

The *available heat energy* is that part of the heat energy (or heat supplied) which can be converted into mechanical work by ideal processes which reduces the system in a state of equilibrium.

The *unavailable heat energy* is that part of heat energy (or heat supplied) which can not be converted into mechanical work even by ideal process which reduces the system in a state of equilibrium. The common term used for unavailable heat energy, according to Second Law of Thermodynamics, is the heat rejected by the system to the surroundings.

From above, we have total heat energy or heat supplied to the system,

$$\delta Q = \text{Available heat energy} + \text{Unavailable heat energy}$$

$$= \text{A.H.E} + \text{U.H.E} = \text{Workdone} + \text{Heat rejected}$$

We know that the maximum possible efficiency obtainable by any engine working on a Carnot cycle is given by

$$\eta = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1} \qquad \ldots (i)$$

where T_1 = Maximum absolute temperature, and

T_2 = Lowest absolute temperature.

We also know that efficiency,

$$\eta = \frac{\text{Maximum work obtained}}{\text{Heat supplied or absorbed}} = \frac{\delta W}{\delta Q} \qquad \ldots (ii)$$

From equations (*i*) and (*ii*),

$$\frac{\delta W}{\delta Q} = 1 - \frac{T_2}{T_1} \quad \text{or} \quad \delta W = \delta Q\left(1 - \frac{T_2}{T_1}\right)$$

Since the lowest practical temperature of heat rejection is the temperature of surroundings (T_0), therefore the above expression may be written as

$$\delta W = \delta Q\left(1 - \frac{T_0}{T_1}\right) = \delta Q - \delta Q \times \frac{T_0}{T_1} \qquad \ldots (iii)$$

---

* Refer Art 4.4 and 4.13

We also know that workdone,

$$\delta W = \text{Heat supplied} - \text{Unavailable heat energy or heat rejected.}$$

$$= \delta Q - \text{U.H.E.} \qquad \ldots(iv)$$

From equations (*iii*) and (*iv*),

$$\delta Q - \delta Q \times \frac{T_0}{T_1} = \delta Q - \text{U.H.E}$$

or

$$\text{U.H.E} = T_0\left(\frac{\delta Q}{T_1}\right) = T_0 \times dS$$

Thus the unavailable heat energy (U.H.E) or the heat rejected is the product of the lowest temperature of heat rejection and the change of entropy of the system during the process of supplying heat. In other words, the *change in entropy may be regarded as a measure of unavailable form of heat energy* or *irreversibility of the process.*

Note : In the above discussion, the heat rejection takes place from the system to the surrounding (*i.e.* from a higher temperature to a lower temperature). At the end of the process, by virtue of Second Law of Thermodynamics, it is not possible to transfer heat from the system at a lower temperature to the surroundings at a higher temperature. Thus, the above process of heat transfer is *irreversible process.*

4.5. Units of Entropy

The unit of entropy depends upon the unit of heat employed and the absolute temperature. We know that

$$\text{Change in entropy } (dS) = \frac{\text{Heat supplied or rejected } (\delta Q)}{\text{Absolute temperature } (T)}$$

Therefore, if the heat supplied or rejected is in kJ and the temperature is in K, then the unit of entropy is kJ/K. The entropy may be expressed in so many units entropy without assigning any dimensional units. Since the entropy is expressed per unit mass of the working substance, it would be more correct to speak **specific entropy*. The absolute values of entropy cannot be determined, but only the change in entropy may be obtained by using equation (*ii*) in Art. 4.2.

Theoretically, the entropy of a substance is *zero* at *absolute zero temperature*. Hence, in entropy calculations, some convenient datum should be selected from which measurement may be made.

It may be noted that water at 0° C is assumed to have zero entropy, and changes in its entropy are reckoned from this temperature.

4.6. Clausius Inequality

The Clausius inequality states that 'whenever a closed system undergoes a cyclic process, the cyclic integral of $\delta Q/T$ is less than zero (*i.e.* negative) for an irreversible cyclic process and equal to zero for a reversible cyclic process, Mathematically,

$$\oint \frac{\delta Q}{T} < 0, \text{ for an irreversible cyclic process} \qquad \ldots(i)$$

and

$$\oint \frac{\delta Q}{T} = 0, \text{ for a reversible cyclic process} \qquad \ldots(ii)$$

Combining the equations (*i*) and (*ii*), the equation for the Clausius inequality is written as

$$\oint \frac{\delta Q}{T} \leq 0 \qquad \ldots(iii)$$

---

* The entropy is an extensive property of the system. The ratio of the extensive property of the system to the mass of the system is the specific value of that property as explained in Chapter 1.

The Clausius inequality not only gives mathematical expression to the second law of thermodynamics, but it also gives the quantitative measure of irreversibility of the system. For example, the equation (i) for an irreversible cyclic process may be written as

$$\oint \frac{\delta Q}{T} + I = 0$$

where I represents the amount by which the given cyclic process is irreversible. When I is equal to zero, then the given cyclic process will be reversible. Moreover, a cyclic process in which $\oint dQ/T$ is more than zero, is impossible because it voilates the second law of thermodynamics.

4.7. Principle of Increase of Entropy

We have discussed in Art. 4.6 that the equation for Clausius inequality is

$$\oint \frac{\delta Q}{T} \leq 0 \qquad \ldots (i)$$

We know that the change in entropy,

$$dS = \frac{\delta Q}{T}$$

Since the entropy is a thermodynamic property and the cyclic integral of a thermodynamic property is zero, therefore equation (i) may be written as

$$\oint \frac{\delta Q}{T} \leq \oint dS$$

or

$$\frac{\delta Q}{T} \leq dS$$

and

$$dS \geq \frac{\delta Q}{T} \qquad \ldots (ii)$$

When the process is reversible, then

$$dS = \frac{\delta Q}{T}$$

and when the process is irreversible, then

$$dS > \frac{\delta Q}{T}$$

If we apply the equation (ii) to an isolated system like universe, for which $\delta Q = 0$, then the equation (ii) may be written as

$$dS \geq 0$$

For a reversible cyclic process,

$$dS = 0 \text{ or } S = \text{Constant}$$

In other words, the entropy for a reversible cyclic process remains constant. Now for an irreversible cyclic process,

$$dS > 0$$

Since, in practice, all processes are irreversible, therefore the entropy of such a system like universe goes on increasing.

This is known as the *principle of increase of entropy.*

Note : The principle of change of entropy may also be discussed as follows :

Consider a given quantity of heat energy Q rejected by a hot body at temperature T_1 and absorbed by a cold body at temperature T_2.

$\therefore$ Loss of entropy by the hot body $= Q/T_1$

and gain of entropy by the cold body $= Q/T_2$

Since T_1 is greater than T_2, therefore the gain of entropy by the cold body is greater than the loss of entropy by the hot body. In other words, we can say that when the temperature *falls in a system (*i.e.* irreversible process) the entropy increases.

This conclusion can be extended to any isolated system (say universe) in which the heat exchange between the system (at a lower temperature) and the surroundings (at a higher temperature) takes place in an irreversible manner. Thus the entropy of an isolated system (universe) increases.

4.8. General Expression for Change of Entropy of a Perfect Gas

Consider a certain quantity of a perfect gas being heated by any thermodynamic process.

Let m = Mass of the gas,

p_1 = Initial pressure of the gas,

v_1 = Initial volume of the gas,

T_1 = Initial temperature of the gas, and

p_2, v_2, T_2 = Corresponding values for the final conditions.

Now the relation for the change of entropy during the process may be expressed in the following three ways :

(a) In terms of volume and absolute temperature

We know that for a small change in the state of a working substance, the general gas energy equation is,

$$\delta Q = dU + \delta W = mc_v\, dT + p\, dv \qquad \dots (i)$$

where dT = Small change in temperature, and

dv = Small change in volume.

Dividing throughout equation (*i*) by T,

$$\frac{\delta Q}{T} = mc_v \frac{dT}{T} + \frac{p\, dv}{T}$$

Since $pv = mRT$ or $\frac{p}{T} = \frac{mR}{v}$ and $\frac{\delta Q}{T} = dS$, therefore

$$dS = mc_v \frac{dT}{T} + \frac{mR}{v} dv \qquad \dots (ii)$$

Integrating equation (*ii*) within appropriate limits,

$$\int_{S_1}^{S_2} dS = mc_v \int_{T_1}^{T_2} \frac{dT}{T} + mR \int_{v_1}^{v_2} \frac{dv}{v}$$

$$[\,S\,]_{S_1}^{S_2} = mc_v \Big[\log_e T\Big]_{T_1}^{T_2} + mR \Big[\log_e v\Big]_{v_1}^{v_2}$$

---

* The cold body which is at a lower temperature receives heat from a hot body which is at a higher temperature. The temperature of the hot body falls. It is similar to a case when we supply heat at constant volume or constant pressure. After the process, by virtue of second law of thermodynamics, it not possible to transfer heat from a cold body to a hot body. Such a process is irreversible process.

$$\therefore \quad S_2 - S_1 = mc_v (\log_e T_2 - \log_e T_1) + mR(\log_e v_2 - \log_e v_1)$$

$$= mc_v \log_e \left(\frac{T_2}{T_1}\right) + mR \log_e \left(\frac{v_2}{v_1}\right) \qquad \ldots (iii)$$

$$= 2.3\, m \left[c_v \log \left(\frac{T_2}{T_1}\right) + R \log \left(\frac{v_2}{v_1}\right)\right]$$

$$= 2.3\, m \left[c_v \log \left(\frac{T_2}{T_1}\right) + (c_p - c_v) \log \left(\frac{v_2}{v_1}\right)\right]$$

(*b*) *In terms of pressure and absolute temperature*

We know from the general gas equation,

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} \quad \text{or} \quad \frac{v_2}{v_1} = \frac{p_1}{p_2} \times \frac{T_2}{T_1}$$

Substituting the value of $\left(\frac{v_2}{v_1}\right)$ in equation (*iii*),

$$S_2 - S_1 = mc_v \log_e \left(\frac{T_2}{T_1}\right) + mR \log_e \left(\frac{p_1}{p_2} \times \frac{T_2}{T_1}\right)$$

$$= mc_v \log_e \left(\frac{T_2}{T_1}\right) + mR \log_e \left(\frac{p_1}{p_2}\right) + mR \log_e \left(\frac{T_2}{T_1}\right)$$

$$= m \log_e \left(\frac{T_2}{T_1}\right)\left(c_v + R\right) + mR \log_e \left(\frac{p_1}{p_2}\right) \qquad \ldots (iv)$$

Now substituting $R = c_p - c_v$ in the above equation,

$$S_2 - S_1 = mc_p \log_e \left(\frac{T_2}{T_1}\right) + m(c_p - c_v) \log_e \left(\frac{p_1}{p_2}\right) \qquad \ldots (v)$$

$$= 2.3\, m \left[c_p \log \left(\frac{T_2}{T_1}\right) + (c_p - c_v) \log \left(\frac{p_1}{p_2}\right)\right]$$

(*c*) *In terms of pressure and volume*

We know from the general gas equation,

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} \quad \text{or} \quad \frac{T_2}{T_1} = \frac{p_2}{p_1} \times \frac{v_2}{v_1}$$

Substituting the value of $\left(\frac{T_2}{T_1}\right)$ in equation (*iii*),

$$S_2 - S_1 = mc_v \log_e \left(\frac{p_2}{p_1} \times \frac{v_2}{v_1}\right) + mR \log_e \left(\frac{v_2}{v_1}\right)$$

Now substituting $R = c_p - c_v$ in the above equation,

$$S_2 - S_1 = mc_v \log_e\left(\frac{p_2}{p_1}\right) + mc_v \log_e\left(\frac{v_2}{v_1}\right) + m\,(c_p - c_v) \log_e\left(\frac{v_2}{v_1}\right)$$

$$= mc_v \log_e\left(\frac{p_2}{p_1}\right) + mc_v \log_e\left(\frac{v_2}{v_1}\right) + mc_p \log_e\left(\frac{v_2}{v_1}\right) - mc_v \log_e\left(\frac{v_2}{v_1}\right)$$

$$= mc_v \log_e\left(\frac{p_2}{p_1}\right) + mc_p \log_e\left(\frac{v_2}{v_1}\right) \qquad \ldots (vi)$$

$$= 2.3\,m\left[c_v \log\left(\frac{p_2}{p_1}\right) + c_p \log\left(\frac{v_2}{v_1}\right)\right]$$

Notes : 1. The expression (*vi*) is valid for both reversible as well as irreversible processes.

2. The change of entropy is *positive* when heat is *absorbed* by the gas and there is *increase of entropy*.

3. The change of entropy is *negative* when heat is *removed* from the gas and there is *decrease of entropy*.

Example 4.1. *0.05 m^3 of air at a pressure of 8 bar and temperature 280° C expands to eight times its original volume and the final temperature after expansion is 25° C. Calculate change of entropy of air during the process. Assume $c_p = 1.005$ kJ/kg K and $c_v = 0.712$ kJ/kg K.*

Solution. Given : $v_1 = 0.05\ \text{m}^3$; $p_1 = 8\ \text{bar} = 0.8 \times 10^6\ \text{N/m}^2$; $T_1 = 280°\text{C} = 280 + 273 = 553\ \text{K}$; $v_2 = 8\,v_1 = 8 \times 0.05 = 0.4\ \text{m}^3$; $T_2 = 25°\text{C} = 25 + 273 = 298\ \text{K}$; $c_p = 1.005$ kJ/kg K ; $c_v = 0.712$ kJ/kg K

Let m = Mass of air in kg.

We know that gas constant,

$$R = c_p - c_v = 1.005 - 0.712 = 0.293\ \text{kJ/kg K} = 293\ \text{J/kg K}$$

and

$$p_1 v_1 = m R T \text{ or } m = \frac{p_1 v_1}{R\,T_1} = \frac{0.8 \times 10^6 \times 0.05}{293 \times 553} = 0.247\ \text{kg}$$

∴ Change of entropy,

$$S_2 - S_1 = 2.3\,m\left[c_v \log\left(\frac{T_2}{T_1}\right) + R \log\left(\frac{v_2}{v_1}\right)\right]$$

$$= 2.3 \times 0.247\left[0.712 \log\left(\frac{298}{553}\right) + 0.293 \log\left(\frac{0.4}{0.05}\right)\right]$$

$$= 0.568\,(-0.19 + 0.26) = 0.04\ \text{kJ/K}\ \textbf{Ans.}$$

4.9. Change of Entropy of a Perfect Gas during Various Thermodynamic Processes

We have already discussed in Chapter 3, the various thermodynamic processes of a perfect gas and have derived the equations for work done, change of internal energy and heat supplied. Now we shall derive expressions for the change of entropy during the following thermodynamic processes :

1. Constant volume process (or isochoric process) ; 2. Constant pressure process (or Isobaric process) ; 3. Constant temperature process (or Isothermal process) ; 4. Adiabatic process (or Isentropic process) ; and 5. Polytropic process.

4.10. Change of Entropy during Constant Volume Process (or Isochoric Process)

Consider a certain quantity of a perfect gas being heated at a constant volume.

Let m = Mass of the gas,

p_1 = Initial pressure of the gas,

T_1 = Initial temperature of the gas,

p_2, T_2 = Corresponding values for the final conditions.

Let this process be represented by the curve 1-2 on *T-S* diagram as shown in Fig. 4.2.

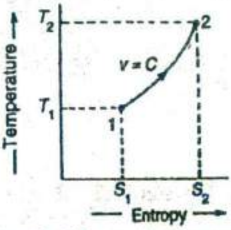

Fig. 4.2. *T-S* curve during constant volume process.

We know that for a small change of temperature (dT), the heat supplied,

$$\delta Q = mc_v\,dT$$

Dividing both sides of the above equation by T,

$$\frac{\delta Q}{T} = mc_v\frac{dT}{T}$$

or $$^{*}dS = mc_v\frac{dT}{T} \qquad \ldots\left(\because \frac{\delta Q}{T} = dS\right)$$

Integrating this expression for the total change of entropy,

$$\int_{S_1}^{S_2} dS = mc_v\int_{T_1}^{T_2}\frac{dT}{T} \quad \text{or} \quad [\,S\,]_{S_1}^{S_2} = mc_v\Big[\log_e T\Big]_{T_1}^{T_2}$$

$$\therefore \quad S_2 - S_1 = mc_v\log_e\left(\frac{T_2}{T_1}\right) = 2.3\,mc_v\log\left(\frac{T_2}{T_1}\right) \qquad \ldots(i)$$

The above relation may also be expressed in terms of pressure. We know from the general gas equation,

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} \quad \text{or} \quad \frac{T_2}{T_1} = \frac{p_2}{p_1} \qquad \ldots(\because v_1 = v_2)$$

Substituting the value of $\left(\frac{T_2}{T_1}\right)$ in equation (*i*),

$$S_2 - S_1 = 2.3\,mc_v\log\left(\frac{p_2}{p_1}\right) \qquad \ldots(ii)$$

The equations (*i*) and (*ii*) are valid for both reversible as well as irreversible process.

Alternate proofs for change of entropy

We have seen in Art. 4.8 that the general expression for change of entropy in terms of volume and absolute temperature is,

$$S_2 - S_1 = 2.3\,m\left[c_v\log\left(\frac{T_2}{T_1}\right) + R\log\left(\frac{v_2}{v_1}\right)\right]$$

---

* We know that $dS = m\,c_v\frac{dT}{T}$ or $\frac{dT}{dS} = \frac{T}{m\,c_v}$

For 1 kg of a perfect gas, $\frac{dT}{ds} = \frac{T}{c_v}$

The term $\frac{dT}{ds}$ is known as slope of the curve 1-2 on the *T-s* diagram as shown in Fig. 4.2.

Since $v_1 = v_2$, therefore $v_2/v_1 = 1$. Moreover, $\log 1 = 0$

$$\therefore \qquad S_2 - S_1 = 2.3\, mc_v \log\left(\frac{T_2}{T_1}\right)$$

Similarly, the general expression for change of entropy in terms of pressure and volume is,

$$S_2 - S_1 = 2.3\, m\left[c_v \log\left(\frac{p_2}{p_1}\right) + c_p \log\left(\frac{v_2}{v_1}\right)\right]$$

Since $v_1 = v_2$, therefore $v_2/v_1 = 1$. Moreover, $\log 1 = 0$.

$$\therefore \qquad S_2 - S_1 = 2.3\, mc_v \log\left(\frac{p_2}{p_1}\right)$$

Example 4.2. *A vessel of 2.5 m^3 capacity contains 1 kg-mole of N_2 at 100° C. If the gas is cooled to 30° C, calculate the change in specific entropy.*

The ratio of specific heats is 1.4 and one kg-mole nitrogen is 28 kg.

Solution. Given : $^*v = 2.5\ \text{m}^3$; $M = 1$ kg-mole $= 28$ kg ; $T_1 = 100°\text{ C} = 100 + 273 = 373$ K ; $T_2 = 30°\text{ C} = 30 + 273 = 303$ K ; $\gamma = c_p/c_v = 1.4$

Since the universal gas constant (R_u) for all gases is 8.314 kJ/kg K, therefore characteristic gas constant,

$$R = R_u / M = 8.314/28 = 0.297 \text{ kJ/kg K}$$

and

$$c_p - c_v = R \quad \text{or} \quad 1.4\, c_v - c_v = 0.297 \qquad \ldots(\because c_p/c_v = \gamma = 1.4)$$

$$\therefore \qquad c_v = 0.297/0.4 = 0.74 \text{ kJ/kg K}$$

We know that change in specific entropy (*i.e.* per kg of gas),

$$s_2 - s_1 = 2.3\, m\, c_v \log\left(\frac{T_2}{T_1}\right) = 2.3 \times 1 \times 0.74 \log\left(\frac{303}{373}\right) \text{ kJ/kg K}$$

$$= -0.1536 \text{ kJ/kg K} \quad \textbf{Ans.}$$

The – *ve* sign indicates that there is a decrease in entropy.

Example 4.3. *A vessel of capacity 3 m^3 contains air at a pressure of 1.5 bar and a temperature of 25° C. Additional air is now pumped into the system until the pressure rises to 30 bar and the temperature rises to 60° C. Determine the mass of air pumped in, and express the quantity as a volume at a pressure of 1.02 bar and a temperature of 20° C.*

If the vessel is allowed to cool until the temperature is again 25° C, calculate the pressure in the vessel. Determine the quantity of heat transferred and change of entropy of the gas during the cooling process only. Neglect the effect of heat capacity of the vessel. Assume air as an ideal gas.

Solution. Given : $v_1 = 3\ \text{m}^3$; $p_1 = 1.5\ \text{bar} = 0.15 \times 10^6\ \text{N/m}^2$; $T_1 = 25°\text{ C} = 25 + 273 = 298$ K ; $p_2 = 30\ \text{bar} = 3 \times 10^6\ \text{N/m}^2$; $T_2 = 60°\text{ C} = 60 + 273 = 333$ K

Mass of air pumped in

Let m_1 = Mass of air initially filled in the vessel, and

m_2 = Mass of air in the vessel after pumping.

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* Superfluous data

We know that $p_1 v_1 = m_1 R T_1$

$$\therefore \quad m_1 = \frac{p_1 v_1}{R T_1} = \frac{0.15 \times 10^6 \times 3}{287 \times 298} = 5.26 \text{ kg} \quad \ldots \text{(Taking } R \text{ for air} = 287 \text{ J/kg K)}$$

Similarly $p_2 v_2 = m_2 R T_2$

$$\therefore \quad m_2 = \frac{p_2 v_2}{R T_2} = \frac{3 \times 10^6 \times 3}{287 \times 333} = 94.17 \text{ kg} \quad \ldots (\because v_2 = v_1)$$

$\therefore$ Mass of air pumped in,

$$m = m_2 - m_1 = 94.17 - 5.26 = 88.91 \text{ kg} \quad \textbf{Ans.}$$

Volume of air pumped in at a pressure of 1.02 bar and temperature of 20° C

Given : $p = 1.02 \text{ bar} = 0.102 \times 10^6 \text{ N/m}^2$; $T = 20° \text{C} = 20 + 273 = 293 \text{ K}$

Let v = Volume of air pumped in.

We know that $p\,v = m\,R\,T$

$$\therefore \quad v = \frac{m R T}{p} = \frac{88.91 \times 287 \times 293}{0.102 \times 10^6} = 73.3 \text{ m}^3 \quad \textbf{Ans.}$$

Pressure in the vessel after cooling

Let p_3 = Pressure in the vessel after cooling.

We know that the temperature after cooling,

$$T_3 = T_1 = 25° \text{C} = 298 \text{ K}$$

Since the cooling is at constant volume, therefore

$$\frac{p_3}{p_2} = \frac{T_3}{T_2}$$

or

$$p_3 = \frac{T_3 p_2}{T_2} = \frac{298 \times 3 \times 10^6}{333} = 2.68 \times 10^6 \text{ N/m}^2 = 26.8 \text{ bar} \quad \textbf{Ans.}$$

Heat transferred during cooling

Since the vessel is cooled from $T_2 = 333$ K to the initial temperature $T_3 = T_1 = 298$ K, therefore change in internal energy during cooling,

$$dU = m_2 c_v (T_3 - T_2) = 94.17 \times 0.172\,(298 - 333) = -567 \text{ kJ} \quad \textbf{Ans.}$$

The – *ve* sign indicates that the internal energy decreases during cooling.

We know that heat transferred is equal to change in internal energy, when the process takes place at constant volume (because work done is zero). Therefore heat transferred,

$$\delta Q = -567 \text{ kJ} \quad \textbf{Ans.}$$

The – *ve* sign indicates that heat is rejected by the gas.

Change of entropy during cooling process

We know that change of entropy during cooling process,

$$S_3 - S_2 = 2.3\, m_2\, c_v \log\left(\frac{T_3}{T_2}\right) = 2.3 \times 94.17 \times 0.712 \log\left(\frac{298}{333}\right) \text{kJ/K}$$

$$= -7.44 \text{ kJ/K} \quad \textbf{Ans.} \qquad \ldots \text{(Taking } c_v = 0.712 \text{ kJ/kg K)}$$

The – *ve* sign indicates that there is a decrease of entropy.

Example 4.4. *An insulated vessel of capacity 0.056 m^3 is divided into two compartments A and B by a conducting diaphragm. Each compartment has a capacity of 0.028 m^3. The compartment A contains air at a pressure of 1.5 bar and 25° C and the compartment B contains air at a pressure 4.2 bar and 175° C. Find : 1. final equilibrium temperature, 2. final pressure on each side of the diaphragm, and 3. change of entropy of the system.*

Solution. Given : $v_A = v_B = 0.028\ \text{m}^3$; $p_A = 1.5\ \text{bar} = 0.15 \times 10^6\ \text{N/m}^2$; $T_A = 25°\text{C} = 25 + 273 = 298\ \text{K}$; $p_B = 4.2\ \text{bar} = 0.42 \times 10^6\ \text{N/m}^2$; $T_B = 175°\text{C} = 175 + 273 = 448\ \text{K}$

1. *Final equilibrium temperature*

Let T_F = Final equilibrium temperature.

First all, let us find the mass of air in compartment A (m_A) and the mass of air in compartment B (m_B). We know that

$$p_A v_A = m_A R T_A \text{ or } m_A = \frac{p_A v_A}{R T_A} = \frac{0.15 \times 10^6 \times 0.028}{287 \times 298} = 0.049 \text{ kg}$$

... ($\because R$ for air = 287 J/kg K)

$$p_B v_B = m_B R T_B \text{ or } m_B = \frac{p_B v_B}{R T_B} = \frac{0.42 \times 10^6 \times 0.028}{287 \times 448} = 0.091 \text{ kg}$$

Since the diaphragm is conducting, therefore

Heat gained by air in compartment A

= Heat rejected by air in compartment B

or

$$m_A c_v (T_F - T_A) = m_B c_v (T_B - T_F)$$

$$0.049 \times c_v (T_F - 298) = 0.091 \times c_v (448 - T_F)$$

$$0.049\, T_F - 14.6 = 40.77 - 0.091\, T_F \quad \text{or} \quad 0.14\, T_F = 55.37$$

$$\therefore \quad T_F = 395.5 \text{ K} \quad \textbf{Ans.}$$

2. *Final pressure on each side of the diaphragm*

Let p_{AF} = Final pressure in compartment A, and

p_{BF} = Final pressure in compartment B.

Since the volume of each compartment is same, therefore

$$\frac{p_A}{T_A} = \frac{p_{AF}}{T_F} \quad \text{or} \quad p_{AF} = \frac{p_A T_F}{T_A} = \frac{1.5 \times 395.5}{298} = 1.99 \text{ bar} \quad \textbf{Ans.}$$

and

$$\frac{p_B}{T_B} = \frac{p_{BF}}{T_F} \quad \text{or} \quad p_{BF} = \frac{p_B T_F}{T_B} = \frac{4.2 \times 395.5}{448} = 3.708 \text{ bar} \quad \textbf{Ans.}$$

3. *Change of entropy of the system*

We know that change of entropy for compartment A, whose temperature has been increased from T_A to T_F,

$$(dS)_A = 2.3\, m_A c_v \log\left(\frac{T_F}{T_A}\right) = 2.3 \times 0.049 \times 0.712 \log\left(\frac{395.5}{298}\right)$$

... (Taking $c_v = 0.712$ kJ/kg K)

$$= 0.08 \log (1.327) = 0.0098 \text{ kJ/K}$$

and change of entropy for compartment B, whose temperature has been decreased from T_B to T_F,

$$(dS)_B = 2.3\, m_B\, c_v \log\left(\frac{T_F}{T_B}\right) = 2.3 \times 0.091 \times 0.712 \log\left(\frac{395.5}{448}\right) \text{kJ/K}$$

$$= 0.149 \log (0.883) = -0.0081 \text{ kJ/K}$$

$\therefore$ Change of entropy of the system,

$$dS = (dS)_A + (dS)_B = 0.0098 - 0.0081 = 0.0017 \text{ kJ/K } \textbf{Ans.}$$

4.11. Change of Entropy during Constant Pressure Process (or Isobaric Process)

Consider a certain quantity of a perfect gas being heated at constant pressure.

Let m = Mass of the gas,

v_1 = Initial volume of the gas,

T_1 = Initial temperature of the gas,

v_2, T_2 = Corresponding values for the final conditions.

Let this process be represented by the curve 1-2 on T-S diagram as shown in Fig. 4.3.

We know that for a small change of temperature (dT), the heat supplied,

$$\delta Q = mc_p\, dT$$

Dividing both sides of the above equation by T,

$$\frac{\delta Q}{T} = mc_p\left(\frac{dT}{T}\right)$$

or

$$^{*}dS = mc_p\left(\frac{dT}{T}\right)$$

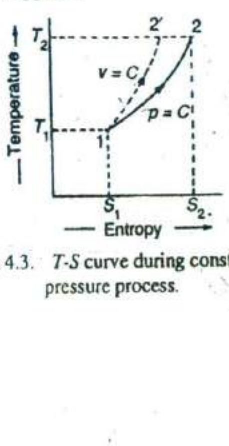

Fig. 4.3. *T-S* curve during constant pressure process.

Integrating this expression for the total change of entropy,

$$\int_{S_1}^{S_2} dS = mc_p \int_{T_1}^{T_2} \frac{dT}{T}$$

---

* We know that $dS = mc_p \dfrac{dT}{T}$ or $\dfrac{dT}{dS} = \dfrac{T}{mc_p}$

For 1 kg of a perfect gas, $\dfrac{dT}{ds} = \dfrac{T}{c_p}$

The term $\dfrac{dT}{ds}$ is known as slope of the curve 1-2 on the T-S diagram as shown in Fig. 4.3.

We have already discussed that slope of the curve for constant volume process is

$$\frac{dT}{ds} = \frac{T}{c_v}$$

Since for a perfect gas, $c_v < c_p$, therefore $\dfrac{1}{c_v} > \dfrac{1}{c_p}$ or $\dfrac{T}{c_v} > \dfrac{T}{c_p}$

Thus, the slope of the curve on the T-S diagram for constant volume process 1-2′ is higher than that of constant pressure process 1-2, as shown in Fig. 4.3.

or $$S_2 - S_1 = mc_p \log_e\left(\frac{T_2}{T_1}\right) = 2.3\ mc_p\ \log\left(\frac{T_2}{T_1}\right) \qquad \ldots (i)$$

The above relation may also be expressed in terms of volume. We know from the general gas equation,

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} \quad \text{or} \quad \frac{T_2}{T_1} = \frac{v_2}{v_1} \qquad \ldots (\because p_1 = p_2)$$

Substituting this value of T_2 / T_1 in equation (*i*),

$$S_2 - S_1 = 2.3\, mc_p \log\left(\frac{v_2}{v_1}\right) \qquad \ldots (ii)$$

The equations (*i*) and (*ii*) are valid for both reversible as well as irreversible process.

Alternate proof for change of entropy

We have seen in Art. 4.8 that the general expression for change of entropy in terms of pressure and absolute temperature,

$$S_2 - S_1 = 2.3\, m\left[c_p \log\left(\frac{T_2}{T_1}\right) + (c_p - c_v)\log\left(\frac{p_1}{p_2}\right)\right]$$

Since $p_1 = p_2$, therefore $p_1 / p_2 = 1$. Moreover $\log 1 = 0$.

$$\therefore \qquad S_2 - S_1 = 2.3\, mc_p \log\left(\frac{T_2}{T_1}\right)$$

Similarly, general expression for change of entropy in terms of pressure and volume,

$$S_2 - S_1 = 2.3\, m\left[c_v \log\left(\frac{p_2}{p_1}\right) + c_p \log\left(\frac{v_2}{v_1}\right)\right]$$

Since $p_1 = p_2$, therefore $p_1 / p_2 = 1$. Moreover $\log 1 = 0$.

$$\therefore \qquad S_2 - S_1 = 2.3\, mc_p \log\left(\frac{v_2}{v_1}\right)$$

Example 4.5. *0.5 kg of a perfect gas is heated from 100° C to 300° C at a constant pressure of 2.8 bar. It is then cooled to 100° C at constant volume. Find the overall change in entropy. Take* $c_p = 1$ *kJ/kg K and* $c_v = 0.72$ *kJ/kg K.*

Solution. Given : $m = 0.5$ kg ; $T_1 = 100°\text{C} = 100 + 273 = 373$ K ; $T_2 = 300°\text{C} = 300 + 273 = 573$ K ; $p = 2.8$ bar $= 0.28 \times 10^6$ N/m^2 ; $T_3 = 100°\text{C} = 100 + 273 = 373$ K ; $c_p = 1$ kJ/kg K ; $c_v = 0.72$ kJ/kg K

We know that change of entropy during constant pressure heating,

$$S_2 - S_1 = 2.3\, mc_p\ \log\left(\frac{T_2}{T_1}\right) = 2.3 \times 0.5 \times 1 \times \log\left(\frac{573}{373}\right) \text{ kJ/K}$$
$$= 0.214 \text{ kJ/K}$$

and change of entropy during constant volume cooling,

$$S_3 - S_2 = 2.3\, mc_v \log\left(\frac{T_3}{T_2}\right) = 2.3 \times 0.5 \times 0.72 \log\left(\frac{373}{573}\right) \text{ kJ/K}$$
$$= -0.154 \text{ kJ/K}$$

The – *ve* sign indicates that there is a decrease of entropy.

∴ Overall change in entropy,

$$S_3 - S_1 = (S_2 - S_1) + (S_3 - S_2) = 0.214 - 0.154 = 0.06 \text{ kJ/K} \quad \textbf{Ans.}$$

Example 4.6. *Cold air from atmosphere is circulated through an air heating system, where the temperature of air is increased from 7.2° C to 21.2° C without any pressure loss. If the hourly consumption of warm air at 21.2° C is 850 m³, calculate how much heat per hour must be imparted to the air ?*

What will be the change in entropy of air circulated per hour ? Assume proper values of the specific heat of air.

Solution. Given : $T_1 = 7.2° \text{C} = 7.2 + 273 = 280.2 \text{ K}$; $T_2 = 21.2° \text{C} = 21.2 + 273 = 294.2 \text{ K}$; $v = 850 \text{ m}^3/\text{h}$

Heat imparted to the air per hour

Let $p_1 = p_2$ = Atmospheric pressure (constant).

$= 1.013 \text{ bar} = 0.1013 \times 10^6 \text{ N/m}^2$ ...(Given)

R = Gas constant = 287 J/kg K ...(Assume)

c_p = Specific heat at constant pressure = 1.005 kJ/kg K ...(Assume)

First of all, let us find the mass of air (m). We know that

$$p_2 v_2 = m R T_2 \quad \text{or} \quad m = \frac{p_2 v_2}{R T_2} = \frac{0.1013 \times 10^6 \times 850}{287 \times 294.2} = 1020 \text{ kg/h}$$

We know that heat imparted to the air

$$Q = mc_p (T_2 - T_1) = 1020 \times 1.005 (294.2 - 280.2) \text{ kJ/h}$$

$$= 14\,351.4 \text{ kJ/h} \quad \textbf{Ans.}$$

Change in entropy of air circulated per hour

We know that change in entropy of air,

$$S_2 - S_1 = 2.3\, mc_p \log\left(\frac{T_2}{T_1}\right) = 2.3 \times 1020 \times 1.005 \log\left(\frac{294.2}{280.2}\right) \text{ kJ/K/h}$$

$$= 50 \text{ kJ/K/h} \quad \textbf{Ans.}$$

Example 4.7. *A mass of m_1 kg of a certain gas at a temperature T_1 is mixed at constant pressure with m_2 kg of mass of the same gas at a temperature T_2 ($T_1 > T_2$). The system is thermally insulated. Find the change in entropy of the universe and deduce the same for equal masses of the gas. Show that the change is necessarily positive.*

Solution. First of all, let us find the common temperature (T_c) of the mixture of the gas. This temperature T_c is less than T_1 and greater than T_2. We know that

Heat lost by the gas at temperature T_1

= Heat gained by the gas at temperature T_2

i.e. $$m_1 c_p (T_1 - T_c) = m_2 c_p (T_c - T_2)$$

...(where c_p is the specific heat of the gas at constant pressure)

$$T_c = \frac{m_1 T_1 + m_2 T_2}{m_1 + m_2}$$

We know that the change in entropy for the gas whose temperature has been decreased from T_1 to T_c,

$$(dS)_1 = m_1 c_p \log_e \left(\frac{T_c}{T_1}\right) \qquad \text{... (This will be negative as } T_1 > T_c)$$

Similarly, the change in entropy for the gas whose temperature has been increased from T_2 to T_c,

$$(dS)_2 = m_2 c_p \log_e \left(\frac{T_c}{T_2}\right) \qquad \text{... (This will be positive as } T_c > T_2)$$

∴ Change in entropy of the universe (*i.e.* isolated system),

$$(dS)_u = (dS)_1 + (dS)_2$$

$$= m_1 c_p \log_e \left(\frac{T_c}{T_1}\right) + m_2 c_p \log_e \left(\frac{T_c}{T_2}\right)$$

$$= m_1 c_p \log_e \left[\frac{m_1 T_1 + m_2 T_2}{T_1 (m_1 + m_2)}\right] + m_2 c_p \log_e \left[\frac{m_1 T_1 + m_2 T_2}{T_2 (m_1 + m_2)}\right] \quad \textbf{Ans.}$$

If $m_1 = m_2 = m$, then the change in entropy of the universe,

$$(dS)_u = mc_p \log_e \left(\frac{T_1 + T_2}{2\,T_1}\right) + mc_p \log_e \left(\frac{T_1 + T_2}{2\,T_2}\right)$$

$$= mc_p \log_e \left[\left(\frac{T_1 + T_2}{2\,T_1}\right)\left(\frac{T_1 + T_2}{2\,T_2}\right)\right]$$

$$\ldots \left[\because \log_e x + \log_e y = \log_e (x \times y)\right]$$

$$= mc_p \log_e \left(\frac{T_1 + T_2}{2\sqrt{T_1 T_2}}\right)^2 = 2\,mc_p \log_e \left(\frac{T_1 + T_2}{2\sqrt{T_1 T_2}}\right)$$

The term within the bracket is the ratio of arithmetic mean and the geometric mean of the temperatures T_1 and T_2. Since the arithmetic mean is always greater than the geometric mean, therefore $\frac{T_1 + T_2}{2} > \sqrt{T_1 T_2}$ or $\frac{T_1 + T_2}{2\sqrt{T_1 T_2}}$ is positive. Thus the change in entropy is always positive.

Example 4.8. *One kg of air at 310 K is heated at constant pressure by bringing it in contact with a hot reservoir at 1150 K. Find the entropy change of air, hot reservoir and of the universe.*

If the air is heated from 310 K to 1150 K by first bringing it in contact with a reservoir at 730 K and then with a reservoir at 1150 K, what will be the change of entropy of the universe ?

Solution. Given : $m = 1$ kg ; $T_A = 310$ K ; $T_R = 1150$ K

Entropy change of air, hot reservoir and of the universe

We know that entropy change of air,

$$(dS)_A = 2.3\,mc_p \log \left(\frac{T_R}{T_A}\right) = 2.3 \times 1 \times 1 \times \log \left(\frac{1150}{310}\right) = 1.309 \text{ kJ/K} \quad \textbf{Ans.}$$

... (Taking c_p for air = 1 kJ/kg K)

We also know that the heat absorbed by air or heat rejected by hot reservoir,

$$Q_A = Q_R = -mc_p(T_R - T_A) = -1 \times 1\,(1150 - 310) = -840 \text{ kJ}$$

. . . (–ve sign because of heat rejection)

$\therefore$ Change of entropy of the hot reservoir,

$$(dS)_R = \frac{\text{Heat rejected } (Q_R)}{\text{Absolute temperature } (T_R)} = \frac{-840}{1150} = -0.73 \text{ kJ/K} \quad \textbf{Ans.}$$

We know that change of entropy of the universe,

$$(dS)_u = (dS)_A + (dS)_R = 1.309 - 0.73 = 0.579 \text{ kJ/K} \quad \text{Ans.}$$

Change of entropy of universe when air is heated in two stages

The air is heated in two stages, first from 310 K to 730 K by bringing it in contact with a first reservoir at 730 K and then from 730 K to 1150 K by bringing it in contact with a second reservoir at 1150 K.

$\therefore$ Heat absorbed by air when heated from 310 K to 730 K or heat rejected by first reservoir,

$$Q_{A1} = Q_{R1} = -mc_p(T_{R1} - T_{A1}) = -1 \times 1\,(730 - 310) = -420 \text{ kJ}$$

. . . (Here $T_{R1} = 730$ K and $T_{A1} = T_A = 310$ K)

and heat absorbed by air when heated from 730 K to 1150 K or heat rejected by the second reservoir,

$$Q_{A2} = Q_{R2} = -mc_p(T_{R2} - T_{A2}) = -1 \times 1\,(1150 - 730) = -420 \text{ kJ}$$

. . . (Here $T_{R2} = 1150$ K and $T_{A2} = 730$ K)

We know that change of entropy of the air,

$$(dS)_A = 2.3\, mc_p \log\left(\frac{T_{R1}}{T_{A1}}\right) + 2.3\, mc_p \log\left(\frac{T_{R2}}{T_{A2}}\right)$$

$$= 2.3 \times 1 \times 1\left[\log\left(\frac{730}{310}\right) + \log\left(\frac{1150}{730}\right)\right]$$

$$= 2.3\,(0.372 + 0.197) = 1.309 \text{ kJ/K}$$

Change of entropy for the first reservoir,

$$(dS)_{R1} = \frac{Q_{R1}}{T_{R1}} = \frac{-420}{730} = -0.575 \text{ kJ/K}$$

Change of entropy for the second reservoir,

$$(dS)_{R2} = \frac{Q_{R2}}{T_{R2}} = \frac{-420}{1150} = -0.365 \text{ kJ/K}$$

$\therefore$ Change of entropy of the universe,

$$(dS)_u = (dS)_A + (dS)_{R1} + (dS)_{R2}$$

$$= 1.309 - 0.581 - 0.575 = 0.369 \text{ kJ/K} \quad \textbf{Ans.}$$

4.12. Change of Entropy during Constant Temperature Process (or Isothermal Process)

Consider a certain quantity of a perfect gas being heated at constant temperature.

Let m = Mass of the gas,
p_1 = Initial pressure of gas,
v_1 = Initial volume of gas,
p_2, v_2 = Corresponding values for the final conditions.

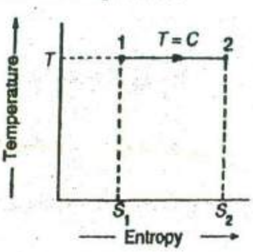

Fig. 4.4. T-S curve during constant temperature process

Let this process be represented by the line 1-2 on T-S diagram as shown in Fig. 4.4. We know that during constant temperature process (*i.e.* isothermal process), there is no change in internal energy, and the heat supplied is equal to the work done by the gas. We also know that work done during an isothermal process.

$$W_{1-2} = 2.3\, mRT \log\left(\frac{v_2}{v_1}\right)$$

$$\therefore \text{ Heat supplied, } \quad Q_{1-2} = W_{1-2} = 2.3\, mRT \log\left(\frac{v_2}{v_1}\right)$$

We know that change of entropy

$$= \frac{\text{Heat supplied}}{\text{Absolute temperature}}$$

or

$$S_2 - S_1 = 2.3\,\frac{mRT}{T} \log\left(\frac{v_2}{v_1}\right)$$

$$= 2.3\, mR \log\left(\frac{v_2}{v_1}\right) = 2.3\, m\,(c_p - c_v) \log\left(\frac{v_2}{v_1}\right) \qquad \dots (i)$$

The above relation may also be expressed in terms of pressure. We know from the general gas equation,

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} \quad \text{or} \quad \frac{v_2}{v_1} = \frac{p_1}{p_2} \qquad \dots (\because T_1 = T_2)$$

Substituting the value of $\left(\frac{v_2}{v_1}\right)$ in equation (*i*),

$$S_2 - S_1 = 2.3\, mR \log\left(\frac{p_1}{p_2}\right) = 2.3\, m\,(c_p - c_v) \log\left(\frac{p_1}{p_2}\right) \qquad \dots (ii)$$

The equations (*i*) and (*ii*) are valid for both reversible and irreversible processes.

Alternate proofs for change of entropy

We have seen in Art. 4.8 that the general expression for change of entropy in terms of volume and absolute temperature is,

$$S_2 - S_1 = 2.3\, m\left[c_v \log\left(\frac{T_2}{T_1}\right) + (c_p - c_v) \log\left(\frac{v_2}{v_1}\right)\right]$$

Since $T_1 = T_2$, therefore $T_2 / T_1 = 1$. Moreover $\log 1 = 0$.

$$\therefore \qquad S_2 - S_1 = 2.3\, m\, (c_p - c_v) \log\left(\frac{v_2}{v_1}\right)$$

Similarly, general expression for change of entropy in terms of pressure and absolute temperature is,

$$S_2 - S_1 = 2.3\, m\left[c_p \log\left(\frac{T_2}{T_1}\right) + (c_p - c_v)\log\left(\frac{p_1}{p_2}\right)\right]$$

Since $T_1 = T_2$, therefore $T_2 / T_1 = 1$. Moreover $\log 1 = 0$.

$$\therefore \qquad S_2 - S_1 = 2.3\, m\, (c_p - c_v) \log\left(\frac{p_1}{p_2}\right)$$

Example 4.9. *A certain quantity of a perfect gas is heated in a reversible isothermal process from 1 bar and 40° C to 10 bar. Find the work done per kg of gas and the change of entropy per kg of gas. Take R = 287 J/kg K.*

Solution. Given : $p_1 = 1$ bar $= 0.1 \times 10^6$ N/m^2 ; $T_1 = 40°$ C $= 40 + 273 = 313$ K ; $p_2 = 10$ bar $= 1 \times 10^6$ N/m^2 ; $R = 287$ J/kg K

Workdone per kg of gas

We know that workdone per kg of gas,

$$w_{1-2} = 2.3\, mRT_1 \log\left(\frac{v_2}{v_1}\right) = 2.3\, mRT_1 \log\left(\frac{p_1}{p_2}\right) \quad \ldots (\because p_1 v_1 = p_2 v_2)$$

$$= 2.3 \times 1 \times 287 \times 313 \log\left(\frac{0.1 \times 10^6}{1 \times 10^6}\right) = -206\,610 \text{ J/kg}$$

$$= -206.61 \text{ kJ/kg} \quad \textbf{Ans.}$$

The – *ve* sign indicates that the work is done on the gas.

Change in entropy per kg of gas

We know that the change in entropy (*i.e.* change in specific entropy),

$$s_2 - s_1 = 2.3\, mR \log\left(\frac{p_1}{p_2}\right) = 2.3 \times 1 \times 287 \log\left(\frac{0.1 \times 10^6}{1 \times 10^6}\right) \text{ J/kg K}$$

$$= -660.1 \text{ J/kg K} = -0.6601 \text{ kJ/kg K} \quad \textbf{Ans.}$$

The – *ve* sign indicates that there is a decrease in entropy.

Note : The change in entropy may also be obtained as follows :

We know that the heat supplied (q_{1-2}) in an isothermal process is equal to the workdone (w_{1-2}).

$\therefore$ Heat supplied $= q_{1-2} = w_{1-2} = -206.61$ kJ/kg

$\therefore$ Change in specific entropy,

$$s_2 - s_1 = \frac{\text{Heat supplied}}{\text{Absolute temperature}} = \frac{-206.61}{313} = -0.6601 \text{ kJ/kg K} \quad \textbf{Ans.}$$

Example 4.10. *One kg of air occupies 0.084 m^3 at 12.5 bar and 537° C. It is expanded at a constant temperature to a final volume of 0.336 m^3. Calculate :*

1. the pressure at the end of expansion, 2. work done during expansion, 3. heat absorbed by the air, and 4. change of entropy.

Solution. Given : $m = 1$ kg ; $v_1 = 0.084$ m^3 ; $p_1 = 12.5$ bar $= 1.25 \times 10^6$ N/m^2 ; $T_1 = 537°$ C $= 537 + 273 = 810$ K ; $v_2 = 0.336$ m^3

1. *Pressure at the end of expansion*

Let p_2 = Pressure at the end of expansion.

We know that

$$p_1 v_1 = p_2 v_2$$

$$\therefore \quad p_2 = \frac{p_1 v_1}{v_2} = \frac{1.25 \times 10^6 \times 0.084}{0.336} = 0.3125 \times 10^6 \text{ N/m}^2$$

$$= 3.125 \text{ bar } \textbf{Ans.}$$

2. *Workdone during expansion*

We know that workdone during expansion,

$$W_{1-2} = 2.3\, mRT_1 \log\left(\frac{v_2}{v_1}\right) = 2.3\, p_1 v_1 \log\left(\frac{v_2}{v_1}\right) \quad \ldots (\because p_1 v_1 = m R T_1)$$

$$= 2.3 \times 1.25 \times 10^6 \times 0.084 \log\left(\frac{0.336}{0.084}\right) = 145\,400 \text{ J}$$

$$= 145.4 \text{ kJ } \textbf{Ans.}$$

3. *Heat absorbed by the air*

We know that during constant temperature process, there is no change in internal energy and the heat absorbed is equal to the amount of work done by the air.

∴ Heat absorbed by the air,

$$Q_{1-2} = \text{Work done by the air} = 145.4 \text{ kJ } \textbf{Ans.}$$

4. *Change of entropy*

We know that change of entropy

$$S_2 - S_1 = \frac{\text{Heat absorbed}}{\text{Absolute temperature}} = \frac{145.4}{810} = 0.18 \text{ kJ/K } \textbf{Ans.}$$

Example 4.11. *One kg of hydrogen (molecular mass 2) is expanded from 1 m^3 to 5 m^3 during a free expansion process. Calculate the change in entropy of the gas and the surroundings. If the expansion between the same two states is carried out by a reversible isothermal process, find the change in entropy of the gas and the surroundings. What will be the net change of entropy of the universe?*

Solution. Given : $m = 1$ kg ; $M = 2$; $v_1 = 1$ m^3 ; $v_2 = 5$ m^3

We know that the characteristic gas constant,

$$R = \frac{\text{Universal gas constant } (R_u)}{\text{Molecular mass } (M)} = \frac{8.314}{2} = 4.157 \text{ kJ/kg K}$$

$$\ldots (\because R_u \text{ for all gases} = 8.314 \text{ kJ/kg K})$$

Change in entropy of the gas and the surroundings

Since in a *free expansion process, the temperature of the gas remains constant, therefore the process is assumed as reversible isothermal process.

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* Refer Art. 3.14

We know that change in entropy of the gas,

$$(dS)_G = 2.3\, mR \log\left(\frac{v_2}{v_1}\right) = 2.3 \times 1 \times 4.157 \log\left(\frac{5}{1}\right) = 6.67 \text{ kJ/K} \quad \textbf{Ans.}$$

Also, in a free expansion process, no heat transfer takes place with the surroundings, therefore entropy of surroundings remains constant.

∴ Change of entropy of the surroundings,

$$(dS)_S = 0 \quad \textbf{Ans.}$$

Change in entropy of the gas and the surroundings for reversible isothermal process

We know that change in entropy of the gas during reversible isothermal expansion,

$$(dS)_G = 2.3\, mR \log\left(\frac{v_2}{v_1}\right) = 2.3 \times 1 \times 4.157 \log\left(\frac{5}{1}\right) = 6.67 \text{ kJ/K} \quad \textbf{Ans.}$$

In an isothermal expansion, the heat transfer takes place between the gas and the surroundings. The gas absorbs heat and an equal amount of heat is rejected by the surroundings.

∴ Change in entropy of the surroundings,

$$(dS)_S = -6.67 \text{ kJ/K} \quad \textbf{Ans.}$$

and net change in entropy of the universe,

$$(dS)_U = (dS)_G + (dS)_S = 6.67 - 6.67 = 0 \quad \textbf{Ans.}$$

Example 4.12. *2 kg of oxygen at 60° C is mixed with 6 kg of nitrogen at the same temperature. The initial pressure of oxygen and nitrogen is 1.03 bar and remains same after mixing. Find the increase in entropy.*

Solution. Given : $m_O = 2$ kg ; $T_O = T_N = 60°$ C $= 60 + 273 = 333$ K ; $m_N = 6$ kg ; $p_{O1} = p_{N1} = p_M = 1.03 \text{ bar} = 0.103 \times 10^6 \text{ N/m}^2$

We know that the molecular mass of oxygen (M_O) is 32 and the molecular mass of nitrogen (M_N) is 28. Since the density (*i.e.* mass per unit volume) is directly proportional to the molecular mass, therefore

Initial volume of oxygen,

$$v_O = \frac{m_O}{M_O} = \frac{2}{32} = 0.0625 \text{ m}^3 \qquad \ldots\left(\because v = \frac{m}{\rho} = \frac{m}{M}\right)$$

and initial volume of nitrogen,

$$v_N = \frac{m_N}{M_N} = \frac{6}{28} = 0.2143 \text{ m}^3$$

∴ Volume of the mixture,

$$v_M = v_O + v_N = 0.0625 + 0.2143 = 0.2768 \text{ m}^3$$

We know that the characteristic gas constant for oxygen,

$$R_O = \frac{\text{Universal gas constant } (R_u)}{\text{Molecular mass of oxygen } (M_O)} = \frac{8.314}{32} = 0.26 \text{ kJ/kg K}$$

$$\ldots(\because R_u \text{ for all gases} = 8.314 \text{ kJ/kg K})$$

and characteristic gas constant for nitrogen,

$$R_N = \frac{R_u}{M_N} = \frac{8.314}{28} = 0.297 \text{ kJ/kg K}$$

We know that change in entropy for oxygen

$$(dS)_O = 2.3\, m_O\, R_O \log\left(\frac{v_M}{v_O}\right) = 2.3 \times 2 \times 0.26 \log\left(\frac{0.2768}{0.0625}\right) \text{kJ/K}$$

$$= 1.196 \log (4.429) = 0.773 \text{ kJ/K (increase)}$$

and change in entropy for nitrogen,

$$(dS)_N = 2.3\, m_N\, R_N \log\left(\frac{v_M}{v_N}\right) = 2.3 \times 6 \times 0.297 \log\left(\frac{0.2768}{0.2143}\right) \text{kJ/K}$$

$$= 4.1 \log (1.2916) = 0.456 \text{ kJ/K (increase)}$$

$\therefore$ Increase in entropy,

$$dS = (dS)_O + (dS)_N = 0.773 + 0.456 = 1.229 \text{ kJ/K} \quad \textbf{Ans.}$$

4.13. Change of Entropy during Reversible Adiabatic Process (or Isentropic Process)

We have already discussed in Art. 3.10, that in a reversible adiabatic process, no heat enters or leaves the gas. Mathematically,

$$\delta Q = 0$$

$$\therefore \qquad dS = 0 \qquad \ldots\left(\because dS = \frac{\delta Q}{T}\right)$$

In other words, change of entropy during a reversible adiabatic process is zero. The reversible adiabatic process on *T-S* graph is shown by a vertical straight line 1-2, as shown in Fig. 4.5.

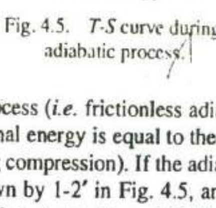

Fig. 4.5. *T-S* curve during adiabatic process.

Since the entropy of the gas remains constant during reversible adiabatic expansion or compression of the gas, this process is said to be isentropic (*i.e.* frictionless adiabatic process). This fact makes the *T-S* diagram quite useful in solving problems on adiabatic expansion.

We have also discussed in Art. 3.10, that in an isentropic process (*i.e.* frictionless adiabatic process), the temperature of the gas changes and the change in internal energy is equal to the work done by the gas during expansion (or work is done on the gas during compression). If the adiabatic process is irreversible (*i.e.* adiabatic process with *friction) as shown by 1-2′ in Fig. 4.5, and the expansion takes places within the same temperature limits T_1 and T_2, then due to internal friction, the internal energy of the gas at the end of the expansion (*i.e.* at point 2′) will be more than that of at point 2 of reversible process. If $\delta Q'$ is the amount of heat absorbed by the gas due to the internal friction, then the ratio $\delta Q'/T$ will be more. Thus the entropy is more at point 2′ (of irreversible process) than at point 2 (of reversible process). Thus, an *irreversible process always results in increase in entropy i.e.* $dS > 0$ (Refer Art. 4.7).

Since the internal energy of the gas at the end of irreversible adiabatic expansion is more than that of isentropic expansion, therefore the amount of work done by the gas will be less than that for

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* It may be noted that friction makes the process irreversible as it increases the heat contents of the gas.

reversible expansion. In other words, higher entropy at the end of irreversible process, makes less availability of heat energy for transformation into work. Thus entropy may be regarded as a measure of rate of the availability or non-availability of heat energy for transformation into work.

Example 4.13. *A 0.568 m³ capacity insulated vessel of oxygen at a pressure of 2 bar is stirred by an internal paddle until the pressure becomes 2.4 bar. Find out 1. Heat transferred, 2. Work output, and 3. Change in entropy per kg.*

Take $c_v = 0.657$ *kJ/kg K, and* $R = 260$ *J/kg K.*

Solution. Given : $v_1 = 0.568 \text{ m}^3$; $p_1 = 2 \text{ bar} = 0.2 \times 10^6 \text{ N/m}^2$; $p_2 = 2.4 \text{ bar} = 0.24 \times 10^6 \text{ N/m}^2$; $c_v = 0.657$ kJ/kg K ; $R = 260$ J/kg K

1. Heat transferred

Since the vessel is insulated, therefore it is an adiabatic process. We know that in an adiabatic process no heat is transferred. **Ans.**

2. Work output per kg

First of all, let us find the initial and final temperature of the gas *i.e.* T_1 and T_2 respectively. Consider 1 kg of mass of the gas.

We know that $$p_1 v_1 = m R T_1 \text{ or } T_1 = \frac{p_1 v_1}{m R} = \frac{0.2 \times 10^6 \times 0.568}{1 \times 260} = 437 \text{ K}$$

Similarly $$p_2 v_2 = m R T_2 \text{ or } T_2 = \frac{p_2 v_2}{m R} = \frac{0.24 \times 10^6 \times 0.568}{1 \times 260} = 524.3 \text{ K}$$

$$\ldots (\because v_2 = v_1)$$

∴ Change in internal energy,

$$dU = mc_v (T_2 - T_1) = 1 \times 0.657\,(524.3 - 437) = 57.3 \text{ kJ}$$

We know that in adiabatic process, the work output is at the cost of change in internal energy.

∴ Work output $= dU = 57.3$ kJ **Ans.**

3. Change in entropy per kg

We know that in an adiabatic process, there is no change in heat, therefore, there is no change in entropy also. **Ans.**

Example 4.14. *An ideal gas of mass 0.25 kg has a pressure of 3 bar, a temperature of 80° C and a volume of 0.07 m³. The gas undergoes an irreversible adiabatic process to a final pressure of 3 bar and a final volume of 0.10 m³, during which the workdone on the gas is 25 kJ. Evaluate* c_p *and* c_v *of the gas and increase in entropy of the gas.*

Solution. Given : $m = 0.25$ kg ; $p_1 = 3 \text{ bar} = 0.3 \times 10^6 \text{ N/m}^2$; $T_1 = 80° \text{C} = 80 + 273 = 353 \text{ K}$; $v_1 = 0.07 \text{ m}^3$; $p_2 = 3 \text{ bar} = 0.3 \times 10^6 \text{ N/m}^2$; $v_2 = 0.1 \text{ m}^3$; $W_{1-2} = -25$ kJ

Value of c_p *and* c_v *of the gas*

First of all, let us find the value of gas constant (R) and the final temperature of the gas (T_2).

We know that $$p_1 v_1 = m R T_1 \text{ or } R = \frac{p_1 v_1}{m T_1} = \frac{0.3 \times 10^6 \times 0.07}{0.25 \times 353} = 238 \text{ J/kg K}$$

and $$p_2 v_2 = m R T_2 \text{ or } T_2 = \frac{p_2 v_2}{m R} = \frac{0.3 \times 10^6 \times 0.1}{0.25 \times 238} = 504 \text{ K}$$

We know that change in internal energy,

$$dU = mc_v (T_2 - T_1) = 0.25 \times c_v (504 - 353) = 37.75\, c_v$$

We also know that heat transfer,

$$^*Q_{1-2} = dU + W_{1-2}$$

$$0 = 37.75\, c_v - 25 \quad \text{or} \quad c_v = 25 / 37.75 = 0.662 \text{ kJ/kg K} \textbf{ Ans.}$$

Now $$c_p - c_v = R = 238 \text{ J/kg K} = 0.238 \text{ kJ/kg K}$$

$\therefore$ $$c_p = R + c_v = 0.238 + 0.662 = 0.9 \text{ kJ/kg K} \textbf{ Ans.}$$

Increase in entropy of the gas

We know that the change in entropy,

$$S_2 - S_1 = 2.3\, m \left[c_v \log \left(\frac{p_2}{p_1} \right) + c_p \log \left(\frac{v_2}{v_1} \right) \right] \qquad \text{... (General equation)}$$

$$= 2.3 \times 0.25 \left[0.662 \log \left(\frac{3}{3} \right) + 0.9 \log \left(\frac{0.1}{0.07} \right) \right]$$

$$= 0.575\, [\, 0.662 \log 1 + 0.9 \log 1.428]$$

$$= 0.575 \times 0.9 \times 0.155 = 0.08 \text{ kJ/K} \textbf{ Ans.}$$

The *+ve* sign indicates that there is an increase in entropy.

4.14. Change of Entropy during Polytropic Process (pv^n = Constant)

Consider a certain quantity of a perfect gas being heated by polytropic process.

Let m = Mass of the gas,

p_1 = Initial pressure of the gas,

v_1 = Initial volume of the gas,

T_1 = Initial temperature of the gas, and

p_2, v_2, T_2 = Corresponding values for the final conditions.

We have already discussed in Art. 3.11 that whenever a gas is heated according to the general law pv^n = constant, the small amount of heat absorbed by the gas during its expansion is given by :

$$\delta Q = \frac{\gamma - n}{\gamma - 1} \times \delta W = \frac{\gamma - n}{\gamma - 1} \times p\, dv \qquad \ldots \left(\because \text{ Workdone, } \delta W = p dv \right)$$

Dividing this equation throughout by T,

$$\frac{\delta Q}{T} = \frac{\gamma - n}{\gamma - 1} \times \frac{p\, dv}{T}$$

Substituting $\frac{\delta Q}{T} = dS$, and $\frac{p}{T} = \frac{mR}{v}$, we have $\qquad \ldots (\because pv = mRT)$

$$dS = \frac{\gamma - n}{\gamma - 1} \times mR \times \frac{dv}{v}$$

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* For an adiabatic process, $Q_{1-2} = 0$ and W_{1-2} is $-ve$ as work is done on the gas.

Integrating this expression for the total change of entropy, we have

$$\int_{S_1}^{S_2} dS = \frac{\gamma-n}{\gamma-1} \times mR \int_{v_1}^{v_2} \frac{dv}{v}$$

or

$$S_2-S_1 = \frac{\gamma-n}{\gamma-1} \times mR \log_e\left(\frac{v_2}{v_1}\right) = 2.3\, m \times \frac{\gamma-n}{\gamma-1} \times R \log\left(\frac{v_2}{v_1}\right) \quad \ldots (i)$$

$$= 2.3\, m \times \frac{\frac{c_p}{c_v}-n}{\gamma-1} \times c_v(\gamma-1)\log\left(\frac{v_2}{v_1}\right) \quad \ldots [\because R = c_v(\gamma-1)]$$

$$= 2.3\, m\,(c_p - n\, c_v)\log\left(\frac{v_2}{v_1}\right) \quad \ldots (ii)$$

The equations (*i*) and (*ii*) are valid for both reversible and irreversible processes.

The above relation may also be expressed in terms of absolute temperature and pressure. We know that in a polytropic process,

$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{n-1} \quad \text{or} \quad \frac{v_2}{v_1} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{n-1}}$$

Substituting this value of v_2 / v_1 in equation (*i*),

$$S_2-S_1 = 2.3 \times \frac{\gamma-n}{\gamma-1} \times mR \log\left(\frac{T_1}{T_2}\right)^{\frac{1}{n-1}}$$

$$= 2.3\, m \times \frac{\gamma-n}{\gamma-1} \times R \times \frac{1}{n-1} \log\left(\frac{T_1}{T_2}\right)$$

$$= 2.3\, m \times \frac{\gamma-n}{\gamma-1} \times c_v(\gamma-1) \times \frac{1}{n-1} \log\left(\frac{T_1}{T_2}\right)$$

$$= 2.3\, m\, c_v \times \frac{\gamma-n}{n-1} \log\left(\frac{T_1}{T_2}\right)$$

We also know that in a polytropic process,

$$\frac{v_1}{v_2} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} \quad \text{or} \quad \frac{v_2}{v_1} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}}$$

Substituting this value of v_2 / v_1 in equation (*i*),

$$S_2-S_1 = 2.3\, m \times \frac{\gamma-n}{\gamma-1} \times R \log\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}}$$

$$= 2.3\, m \times \frac{\gamma-n}{\gamma-1} \times R \times \frac{1}{n} \log\left(\frac{p_1}{p_2}\right)$$

$$= 2.3\,m \times \frac{\gamma - n}{\gamma - 1} \times c_v\,(\gamma - 1) \times \frac{1}{n}\log\left(\frac{p_1}{p_2}\right)$$

$$= 2.3\,m\,c_v \times \frac{\gamma - n}{n}\log\left(\frac{p_1}{p_2}\right)$$

Example 4.15. *A perfect gas is compressed according to the law $pv^{1.25}$ = constant from an initial pressure of 1 bar and volume of 0.9 m^3 to a final volume of 0.6 m^3. Determine the final pressure and change of entropy per kg of gas during the process. Take $\gamma = 1.4$ and $R = 287$ J/kg K.*

Solution. Given : $n = 1.25$; $p_1 = 1$ bar ; $v_1 = 0.9\ \text{m}^3$; $v_2 = 0.6\ \text{m}^3$; $\gamma = c_p / c_v = 1.4$; $R = 287$ J/kg K = 0.287 kJ/kg K

Final pressure of the gas

Let p_2 = Final pressure of the gas.

We know that

$$p_1 v_1^{1.25} = p_2 v_2^{1.25}$$

$$\therefore \quad p_2 = p_1\left(\frac{v_1}{v_2}\right)^{1.25} = 1\left(\frac{0.9}{0.6}\right)^{1.25} = 1.66\ \text{bar} \quad \textbf{Ans.}$$

Change of entropy per kg of gas

We know that change of entropy per kg of gas,

$$S_2 - S_1 = 2.3\,m \times \frac{\gamma - n}{\gamma - 1} \times R\log\left(\frac{v_2}{v_1}\right)$$

$$= 2.3 \times 1 \times \frac{1.4 - 1.25}{1.4 - 1} \times 0.287\log\left(\frac{0.6}{0.9}\right)\text{kJ/K}$$

$$= 0.247\log(0.6667) = -0.0435\ \text{kJ/K} \quad \textbf{Ans.}$$

The –ve sign indicates that there is a decrease of entropy.

Example 4.16. *A certain volume of gas at 320 K and 6.5 bar is expanded to four times its original volume, according to $pv^{1.25}$ = constant. Determine the final temperature of the gas and change of entropy per kg of gas, assuming $c_p = 0.996$ kJ/kg K and $c_v = 0.707$ kJ/kg K.*

Solution. Given : $T_1 = 320$ K ; $^*p_1 = 6.5$ bar ; $v_2 = 4\,v_1$; $n = 1.25$; $c_p = 0.996$ kJ/kg K ; $c_v = 0.707$ kJ/kg K

Final temperature of the gas

Let T_2 = Final temperature of the gas.

We know that $$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{n-1} = \left(\frac{4v_1}{v_1}\right)^{1.25-1} = (4)^{0.25} = 1.414$$

$$\therefore \quad T_2 = T_1 / 1.414 = 320 / 1.414 = 226.3\ \text{K} \quad \textbf{Ans.}$$

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* Superfluous data

Change of entropy per kg of gas

We know that change of entropy per kg of gas,

$$S_2 - S_1 = 2.3\, m\,(c_p - n\,c_v)\log\left(\frac{v_2}{v_1}\right)$$

$$= 2.3 \times 1\,(0.996 - 1.25 \times 0.707)\log\left(\frac{4\,v_1}{v_1}\right)\ \text{kJ/K}$$

$$= 0.2576 \log 4 = 0.155\ \text{kJ/K} \quad \textbf{Ans.}$$

Example 4.17. *0.2 kg of air with $p_1 = 1.5$ bar and $T_1 = 300$ K is compressed to a pressure of 15 bar, according to the law $pv^{1.25}$ = constant. Determine :*

1. Initial and final parameters of the air, 2. Workdone on or by the air ; 3. Heat flow to or from the air ; and 4. Change of entropy stating whether it is an increase or decrease.

Solution. Given : $m = 0.2$ kg ; $p_1 = 1.5$ bar $= 0.15 \times 10^6$ N/m^2 ; $T_1 = 300$ K ; $p_2 = 15$ bar $= 1.5 \times 10^6$ N/m^2 ; $n = 1.25$

1. Initial and final parameters of the air

Let T_2 = Final temperature,

v_1 = Initial volume, and

v_2 = Final volume.

We know that

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{n-1}{n}} = \left(\frac{1.5}{15}\right)^{\frac{1.25-1}{1.25}} = (0.1)^{0.2} = 0.631$$

$$\therefore \quad T_2 = T_1 / 0.631 = 300 / 0.631 = 475.4\ \text{K} \quad \textbf{Ans.}$$

We also know that $p_1 v_1 = m R T_1$

$$\therefore \quad v_1 = \frac{m R T_1}{p_1} = \frac{0.2 \times 287 \times 300}{0.15 \times 10^6} = 0.115\ \text{m}^3 \quad \textbf{Ans.}$$

... (Taking R for air = 287 J/kg K)

and

$$p_1 v_1^n = p_2 v_2^n \quad \text{or} \quad v_2 = v_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} = 0.115\left(\frac{1.5}{15}\right)^{\frac{1}{1.25}}\ \text{m}^3$$

$$= 0.018\ \text{m}^3 \quad \textbf{Ans.}$$

2. Workdone on or by the air

We know that workdone,

$$W_{1-2} = \frac{p_1 v_1 - p_2 v_2}{n-1} = \frac{0.15 \times 10^6 \times 0.115 - 1.5 \times 10^6 \times 0.018}{1.25 - 1}$$

$$= -39\,000\ \text{J} = -39\ \text{kJ} \quad \textbf{Ans.}$$

The –ve sign indicates that work is done on the air.

3. *Heat flow to or from the air*

We know that change in internal energy,

$$dU = mc_v(T_2 - T_1) = 0.2 \times 0.712\,(475.4 - 300) = 25\text{ kJ}$$

... (Taking $c_v = 0.712$ kJ/kg K)

$\therefore$ Heat flow, $\quad Q_{1-2} = dU + W_{1-2} = 25 - 39 = -14$ kJ **Ans**

The –ve sign indicates that heat flows from the air.

4. *Change of entropy*

We know that change of entropy

$$S_2 - S_1 = 2.3\,mc_v \times \frac{\gamma - n}{n-1}\log\left(\frac{T_1}{T_2}\right)$$

$$= 2.3 \times 0.2 \times 0.712 \times \times \frac{1.4 - 1.25}{1.25 - 1}\log\left(\frac{300}{475.4}\right)\text{ kJ/K}$$

$$= 0.1965\log(0.631) = -0.04\text{ kJ/K } \textbf{Ans.}$$

The –ve sign indicates that there is a *decrease* in entropy.

Example 4.18. *A mass of 9 kg of air at 1.75 bar and 13° C is compressed to 24.5 bar according to the law* $pv^{1.32}$ = *constant, and then cooled at constant volume to 15° C. Determine : 1. Volume and temperature at the end of compression, and 2. Change of entropy during compression and during constant volume cooling.*

For air, take $c_p = 0.996$ *kJ/kg K and* $c_v = 0.712$ *kJ/kg K.*

Solution. Given : $m = 9$ kg ; $p_1 = 1.75$ bar $= 0.175 \times 10^6$ N/m² ; $T_1 = 13°\text{C} = 13 + 273 = 286$ K ; $p_2 = 24.5$ bar $= 2.45 \times 10^6$ N/m² ; $n = 1.32$; $T_3 = 15°\text{C} = 15 + 273 = 288$ K ; $c_p = 0.996$ kJ/kg K ; $c_v = 0.712$ kJ/kg K

In the *p-v* diagram, as shown in Fig.4.6, the process 1-2 represents the compression of air according to $pv^{1.32} = C$ and the process 2-3 represents cooling of air at constant volume.

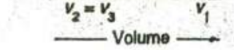


Fig. 4.6

1. *Volume and temperature at the end of compression*

Let $\quad v_2$ = Volume at the end of compression, and

T_2 = Temperature at the end of compression.

We know that gas constant,

$$R = c_p - c_v = 0.996 - 0.712 = 0.284\text{ kJ/kg K} = 284\text{ J/kg K}$$

and

$$p_1 v_1 = m\,R\,T_1 \quad\text{or}\quad v_1 = \frac{m\,R\,T_1}{p_1} = \frac{9 \times 284 \times 286}{0.175 \times 10^6} = 4.18\text{ m}^3$$

We also know that

$$p_1 v_2^n = p_2 v_2^n$$

$$\therefore \qquad v_2 = v_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} = 4.18\left(\frac{0.175\times10^6}{2.45\times10^6}\right)^{\frac{1}{1.32}}$$

$$= 4.18\,(0.0714)^{0.758} = 0.565\,\text{m}^3 \textbf{ Ans.}$$

We know that
$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{n-1}{n}} = \left(\frac{0.175\times10^6}{2.45\times10^6}\right)^{\frac{1.32-1}{1.32}} = (0.0714)^{0.242} = 0.528$$

$$\therefore \qquad T_2 = T_1/0.528 = 286/0.528 = 542\text{ K} = 269^\circ\text{C} \textbf{ Ans.}$$

2. *Change of entropy*

We know that change of entropy during compression (process 1-2),

$$S_2 - S_1 = 2.3\,m\,(c_p - n\,c_v)\log\left(\frac{v_2}{v_1}\right)$$

$$= 2.3\times9\,(0.996 - 1.32\times0.712)\log\left(\frac{0.565}{4.18}\right)\text{kJ/K}$$

$$= 1.16\log(0.135) = -1.008\text{ kJ/K} \textbf{ Ans.}$$

The –ve sign indicates that there is a decrease of entropy.

We also know that change of entropy during constant volume cooling (process 2-3),

$$S_3 - S_2 = 2.3\,mc_v\log\left(\frac{T_3}{T_2}\right) = 2.3\times9\times0.712\log\left(\frac{288}{542}\right)\text{kJ/K}$$

$$= 14.74\log(0.5314) = -4.04\text{ kJ/K} \textbf{ Ans.}$$

The –ve sign indicates that there is a decrease of entropy.

Example 4.19. *A volume of 0.14 m³ of air at 1 bar and 90° C is compressed to 0.014 m³ according to $pv^{1.3}$ = Constant. Heat is then added at a constant volume until the pressure is 66 bar. Determine : 1. Heat exchange with cylinder walls during compression, and 2. Change of entropy during each portion of the process.*

Assume $\gamma = 1.4$ and $R = 286$ J/kg K.

Solution. Given : $v_1 = 0.14\text{ m}^3$; $p_1 = 1\text{ bar} = 0.1\times10^6\text{ N/m}^2$; $T_1 = 90^\circ\text{C} = 90 + 273 = 363\text{ K}$; $v_2 = 0.014\text{ m}^3$; $n = 1.3$; $p_3 = 66\text{ bar} = 6.6\times10^6\text{ N/m}^2$; $\gamma = c_p/c_v = 1.4$; $R = 286\text{ J/kg K} = 0.286\text{ kJ/kg K}$

In the *p-v* diagram, as shown in Fig. 4.7, the process 1-2 represents compression according to $pv^{1.3} = C$ and the process 2-3 represents heating at constant volume.

1. *Heat exchange with cylinder walls during compression*

First of all, let us find the pressure at the end of compression (*i.e.* p_2)

We know that
$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^n = \left(\frac{0.14}{0.014}\right)^{1.3}$$

$$= (10)^{1.3} = 19.95$$

$$\therefore \qquad p_2 = p_1\times19.95 = 1\times19.95$$

$$= 19.95\text{ bar} = 1.995\times10^6\text{ N/m}^2$$

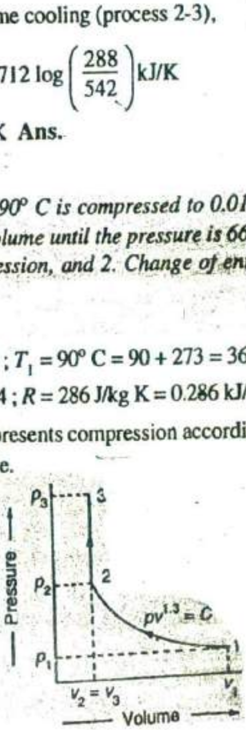


Fig. 4.7

We know that workdone during compression,

$$W_{1-2} = \frac{p_2 v_2 - p_1 v_1}{n-1} = \frac{1.995 \times 10^6 \times 0.014 - 0.1 \times 10^6 \times 0.14}{1.3 - 1} \text{ J}$$

$$= 44\,635 \text{ J} = 44.635 \text{ kJ}$$

∴ Heat exchange with cylinder walls during compression,

$$Q_{1-2} = \frac{\gamma - n}{\gamma - 1} \times W_{1-2} = \frac{1.4 - 1.3}{1.4 - 1} \times 44.635 = 11.16 \text{ kJ} \quad \textbf{Ans.}$$

2. *Change of entropy during each portion of the process*

First of all let us find out the mass of air (m) and the specific heat at constant volume (c_v).

We know that $p_1 v_1 = m R T_1$

$$\therefore \quad m = \frac{p_1 v_1}{R T_1} = \frac{0.1 \times 10^6 \times 0.14}{286 \times 363} = 0.135 \text{ kg}$$

and $c_p - c_v = R$ or $1.4\, c_v - c_v = 0.286$

$$\therefore \quad c_v = 0.286 / 0.4 = 0.715 \text{ kJ/kg K}$$

We know that change of entropy during compression process 1-2,

$$S_2 - S_1 = 2.3\, m \times \frac{\gamma - n}{\gamma - 1} \times R \log \left(\frac{v_2}{v_1} \right)$$

$$= 2.3 \times 0.135 \times \frac{1.4 - 1.3}{1.4 - 1} \times 0.286 \log \left(\frac{0.014}{0.14} \right) \text{kJ/K}$$

$$= 0.0222 \log (0.1) = -0.0222 \text{ kJ/K} \quad \textbf{Ans.}$$

The – *ve* sign indicates that there is a decrease of entropy.

We also know that change of entropy during constant volume process 2-3,

$$S_3 - S_2 = 2.3\, m c_v \log \left(\frac{p_3}{p_2} \right) = 2.3 \times 0.135 \times 0.715 \log \left(\frac{6.6 \times 10^6}{1.995 \times 10^6} \right)$$

$$= 0.222 \log (3.31) = 0.115 \text{ kJ/K} \quad \textbf{Ans.}$$

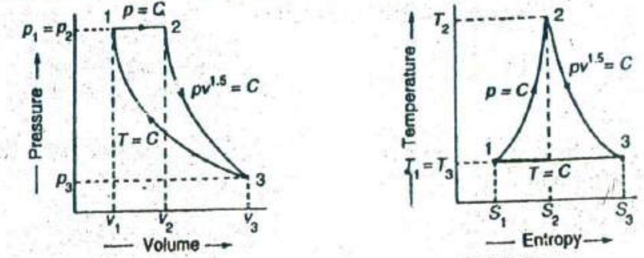
Example 4.20. *A mass of air is initially at 206° C at pressure of 7 bar and having volume of 0.03 m³. The air is expanded at constant pressure to 0.09 m³, a polytropic process with n = 1.5 is then carried out, followed by a constant temperature process which completes the cycle. All processes are reversible. Sketch the cycle on pressure-volume and temperature-entropy planes and find : 1. change in entropy during each process ; 2. Heat received and heat rejected in the cycle ; and 3. efficiency of the cycle.*

Take $R = 287$ J/kg K and $c_v = 0.713$ kJ/kg K.

Solution. Given : $T_1 = 206°\text{ C} = 206 + 273 = 479 \text{ K}$; $p_1 = p_2 = 7 \text{ bar} = 0.7 \times 10^6 \text{ N/m}^2$; $v_1 = 0.03 \text{ m}^3$; $v_2 = 0.09 \text{ m}^3$; $n = 1.5$; $R = 287 \text{ J/kg K} = 0.287 \text{ kJ/kg K}$; $c_v = 0.713 \text{ kJ/kg K}$

The pressure-volume (p-v) and temperature-entropy (T-S) diagrams are shown in Fig. 4.8 (a) and (b) respectively. The process 1-2 represents the expansion of air at constant pressure ; process

2-3 represents the polytropic expansion *i.e.* according to $pv^{1.5} = C$ and the process 3-1 represents the compression of air at constant temperature (*i.e.* isothermal compression).



(*a*) *p-v* diagram. (*b*) *T-S* diagram.

Fig. 4.8

1. *Change of entropy during each process*

First of all, let us find the mass of air (m). We know that

$$p_1 v_1 = m R T_1 \quad \text{or} \quad m = \frac{p_1 v_1}{R T_1} = \frac{0.7 \times 10^6 \times 0.03}{287 \times 479} = 0.153 \text{ kg}$$

Let T_2 = Temperature at the end of constant pressure process 1-2,

v_3 = Volume at the end of polytropic process 2-3 or at the beginning of constant temperature process 3-1, and

p_3 = Pressure at the end of polytropic process 2-3 or at the beginning of constant temperature process 3-1.

We know that for a constant pressure process 1-2,

$$\frac{v_1}{T_1} = \frac{v_2}{T_2} \quad \text{or} \quad T_2 = \frac{v_2 T_1}{v_1} = \frac{0.09 \times 479}{0.03} = 1437 \text{ K}$$

For a polytropic process 2-3,

$$\frac{T_2}{T_3} = \left(\frac{p_2}{p_3}\right)^{\frac{n-1}{n}} \quad \text{or} \quad \frac{p_2}{p_3} = \left(\frac{T_2}{T_1}\right)^{\frac{n}{n-1}} = \left(\frac{1437}{479}\right)^{\frac{1.5}{1.5-1}} = 3^3 = 27$$

$$\ldots(\because T_3 = T_1)$$

$$\therefore \quad p_3 = p_2 / 27 = 7/27 = 0.26 \text{ bar} = 0.026 \times 10^6 \text{ N/m}^2$$

Now for a constant temperature process 3-1,

$$p_1 v_1 = p_3 v_3 \quad \text{or} \quad v_3 = \frac{p_1 v_1}{p_3} = \frac{7 \times 0.03}{0.26} = 0.807 \text{ m}^3$$

We know that change in entropy during constant pressure process 1-2,

$$S_2 - S_1 = 2.3\, m c_p \log\left(\frac{T_2}{T_1}\right) = 2.3 \times 0.153 \times 1 \log\left(\frac{1437}{479}\right) \text{kJ/K}$$

$$\ldots(\because c_p = R + c_v = 0.287 + 0.713 = 1 \text{ kJ/kg K})$$

$$= 0.352 \log 3 = 0.168 \text{ kJ/K (increase)} \quad \textbf{Ans.}$$

Change in entropy during polytropic process 2-3,

$$S_3 - S_2 = 2.3\ m\,(c_p - n\,c_v)\log\left(\frac{v_3}{v_2}\right)$$

$$= 2.3 \times 0.153\,(1 - 1.5 \times 0.713)\log\left(\frac{0.807}{0.09}\right)\ \text{kJ/K}$$

$$= -0.0246\log 8.97 = -0.024\ \text{kJ/K (decrease)}\quad \textbf{Ans.}$$

and change in entropy during constant temperature process 3-1,

$$S_1 - S_3 = 2.3\,mR\log\left(\frac{v_1}{v_3}\right) = 2.3 \times 0.153 \times 0.287\log\left(\frac{0.03}{0.807}\right)\ \text{kJ/K}$$

$$= 0.101\log(0.0372) = -0.144\ \text{kJ/K (decrease)}\quad \textbf{Ans.}$$

Note : From above, we find that net change of entropy during a cycle,

$$dS = (S_2 - S_1) + (S_3 - S_2) + (S_1 - S_3)$$

$$= 0.168 - 0.024 - 0.144 = 0 \qquad \text{. . . (See Art. 4.7)}$$

2. Heat received and heat rejected in a cycle

We know that heat transferred during a constant pressure process 1-2,

$$Q_{1-2} = mc_p\,(T_2 - T_1) = 0.153 \times 1\,(1437 - 479) = 146.6\ \text{kJ}\quad \textbf{Ans.}$$

The +ve sign indicates that the heat is received by the air.

Workdone during a polytropic process 2-3,

$$W_{2\text{-}3} = \frac{p_2 v_2 - p_3 v_3}{n-1} = \frac{0.7 \times 10^6 \times 0.09 - 0.026 \times 10^6 \times 0.807}{1.5 - 1}\ \text{J}$$

$$= \frac{63\,000 - 20\,982}{0.5} = 84\,036\ \text{J} = 84.036\ \text{kJ}$$

Change in internal energy during a polytropic process 2-3,

$$dU = mc_v\,(T_3 - T_2) = 0.153 \times 0.713\,(479 - 1437) = -104.5\ \text{kJ}$$

We know that heat transferred during a polytropic process 2-3,

$$Q_{2\text{-}3} = W_{2-3} + dU = 84.036 - 104.5 = -20.464\ \text{kJ}$$

The –ve sign indicates that the heat is rejected by the air.

We also know that heat transferred during a constant temperature process 3-1,

$$Q_{3\text{-}1} = \text{Workdone} = 2.3\,mRT_3\log\left(\frac{v_2}{v_3}\right),$$

$$= 2.3 \times 0.153 \times 0.287 \times 479\log\left(\frac{0.03}{0.807}\right)\text{kJ}$$

$$= 48.4\log(0.0372) = -69.2\ \text{kJ}$$

The –ve sign indicates that the heat is rejected by the air.

∴ Total heat rejected in a cycle

$$= 20.464 + 69.2 = 89.664\ \text{kJ}\quad \textbf{Ans.}$$

3. *Efficiency of the cycle*

We know that efficiency of the cycle

$$\eta = \frac{\text{Workdone}}{\text{Heat supplied}} = \frac{\text{Heat supplied} - \text{Heat rejected}}{\text{Heat supplied}}$$

$$= \frac{146.6 - 89.664}{146.6} = 0.3884 \text{ or } 38.84\% \textbf{ Ans.}$$

4.15. Approximate Method for Heat Absorbed

Consider 1 kg of a perfect gas being heated by any process. Let this process be represented by the curve 1-2 on *T-S* diagram as shown in Fig. 4.9.

Let T_1 = Initial temperature of the gas,

T_2 = Final temperature of the gas,

S_1 = Initial entropy of the gas, and

S_2 = Final entropy of the gas.

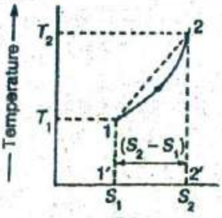


Fig. 4.9. Heat absorbed due to change in entropy.

We know that the heat absorbed during the process is equal to the area of the *T-S* diagram under the curve 1-2 *i.e.* 1-2-2′-1′. Assuming 1-2 to be a straight line (as shown by the dotted line), we find that

Heat absorbed = Area 1-2-2′-1′ = Base × Mean height

or
$$Q_{1-2} = (S_2 - S_1)\left(\frac{T_1 + T_2}{2}\right)$$

Thus, the heat absorbed is approximately equal to the change of entropy multiplied by the mean absolute temperature.

Note : This method is called approximate method, as we have taken the curve 1-2 to be a straight line.

Example 4.21. *One kg of air is compressed in a cylinder according to the law $pv^{1.3}$ = Constant. If the initial temperature is 100° C and compression ratio is 15, find the change of entropy of the air.*

Also find the percentage error, if the change in entropy is calculated by the approximate method. Take c_p = 1 kJ/kg K and c_v = 0.714 kJ/kg K.

Solution. Given : $m = 1$ kg ; $n = 1.3$; $T_1 = 100°\text{C} = 100 + 273 = 373$ K ; $v_1/v_2 = 15$; $c_p = 1$ kJ/kg K ; $c_v = 0.714$ kJ/kg K

Change of entropy

First of all, let us find the final temperature of the air (*i.e.* T_2).

We know that
$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{n-1} = \left(\frac{1}{15}\right)^{1.3-1} = 0.4438 \qquad \ldots(\because v_1/v_2 = 15)$$

$\therefore$
$$T_2 = T_1 / 0.4438 = 373 / 0.4438 = 840.5 \text{ K}$$

We know that change of entropy,

$$S_2 - S_1 = 2.3\, mc_v \times \frac{\gamma - n}{n-1} \log\left(\frac{T_1}{T_2}\right)$$

$$= 2.3 \times 1 \times 0.714 \times \frac{1.4 - 1.3}{1.3 - 1} \log\left(\frac{373}{840.5}\right) \text{ kJ/K}$$

$$= 0.547 \log (0.4438) = -0.193 \text{ kJ/K} \quad \textbf{Ans.}$$

The −ve sign indicates that there is a decrease of entropy.

Percentage error

We know that heat transferred during the process,

$$Q_{1-2} = (S_2 - S_1)\left(\frac{T_1 + T_2}{2}\right)$$

or

$$S_2 - S_1 = \frac{2 \times Q_{1-1}}{T_1 + T_2} = \frac{2\,(-111.26)}{373 + 840.5} = -0.1834 \text{ kJ/K}$$

The −ve sign indicates the heat is rejected by the air.

We also know that heat transferred by approximate method,

$$Q_{1-2} = (S_2 - S_1)\left(\frac{T_1 + T_2}{2}\right)$$

or

$$S_2 - S_1 = \frac{2 \times Q_{1-2}}{T_1 + T_2} = \frac{2\,(-111.26)}{373 + 840.5} = -0.1834 \text{ kJ/K}$$

The −ve sign indicates that there is a decrease of entropy.

$$\therefore \text{ Percentage error } = \frac{0.193 - 0.1834}{0.193} = 0.05 \quad \text{or} \quad 5\% \quad \textbf{Ans.}$$

EXERCISES

1. Calculate the change in entropy per kg when 3 kg of air at a pressure of 2.5 bar and temperature 20° C are expanded to a pressure of 1 bar and temperature of 110° C. Take $R = 0.287$ kJ/kg K ; and $c_v = 0.707$ kJ/kg K. **[Ans.** 0.528 kJ/K]

2. One kg of air is heated at a constant volume from 1 bar and 27° C to a pressure of 5 bar. Calculate the change of entropy. Assume $R = 286$ J/kg K and $c_v = 0.712$ kJ/kg K. **[Ans.** 1.145 kJ/K]

3. A constant volume chamber of 0.3 m^3 capacity contains 2 kg of the gas at 5° C. The heat is transferred to the gas until the temperature is 100° C. Find the heat transferred and the change in internal energy, enthalpy and entropy. Take $c_p = 1.985$ kJ/kg K and $c_v = 1.507$ kJ/kg K.

[Ans. 286.3 kJ ; 286.3 kJ ; 377.15 kJ ; 0.885 kJ/K]

4. 0.28 m^3 of gas at a pressure of 10.5 bar and temperature of 538° C is expanded at a constant pressure to a volume of 0.34 m^3. Determine the change of entropy, assuming $c_v = 0.69$ kJ/kg K and $R = 287$ J/kg K.

[Ans. 0.24 kJ/K]

5. A mixture of ideal gases consists of 3 kg of nitrogen (N_2) and 5 kg of carbon dioxide (CO_2) at a pressure of 3 bar and a temperature of 20° C. If the ratio of specific heats (γ) for the nitrogen is 1.4 kJ/kg K and for carbon dioxide is 1.286 kJ/kg K, find : 1. c_p and c_v for nitrogen, carbon dioxide and of the mixture, and 2. The changes in internal energy, enthalpy and entropy of the mixture, if the mixture is heated to 40° C (*a*) at constant volume, and (*b*) at constant pressure.

The molecular masses for nitrogen and carbon dioxide is 28 and 44 respectively.

[Ans. For nitrogen, $c_p = 1.039$ kJ/kg K, $c_v = 0.742$ kJ/kg K, For carbon dioxide, $c_p = 0.85$ kJ/kg K, $c_v = 0.661$ kJ/kg K, For mixture, $c_p = 0.92$ kJ/kg K, $c_v = 0.69$ kJ/kg K ; 110.4 kJ, 147.2 kJ, 0.368 kJ/kg ; 110.4 kJ, 147.2 kJ, 0.49 kJ/ K]

6. A certain quantity of gas occupies 0.56 m^3 at 400° C and 28 bar. Determine the gain in entropy if the gas expands isothermally to a final volume of 2.8 m^3. $R = 287$ J/kg K. **[Ans.** 3.746 kJ/K]

7. Calculate the changes of entropy per kg of air in the following cases :

(*a*) Air is heated at constant volume till its final pressure is three times the initial pressure, (*b*) Air is compressed at constant pressure till its final volume is one-half the initial volume ; and

(*c*) Air expands isothermally from 6 bar to 3 bar.

Find the change in entropy if the air undergoes the above three process in sequence.

Take $c_p = 1.01$ kJ/kg K and $c_v = 0.72$ kJ/kg K.

[**Ans.** 0.79 kJ/kg K ; – 0.699 kJ/kg K ; 0.2008 kJ/kg K ; 0.2918 (increase)]

8. A perfect gas is contained in a cylinder and undergoes an isothermal expansion according to the law $p = A + B\,v$, where p is the pressure in bar, v is the volume in m^3 and A and B are constants. The initial and final pressures are 8.4 bar and 2.8 bar and the corresponding volumes are 0.056 m^3 and 0.168 m^3. Find 1. workdone by the gas, 2. heat transferred during the process, and 3. change in entropy per kg of gas during expansion. Take $R = 275$ J/kg K. [**Ans.** 62.72 kJ ; 62.72 kJ ; 0.3018 kJ/kg K]

9. Calculate the change of entropy when 0.14 kg of gas initially at 170° C expands with a volume ratio of 5.4 according to $pv^{1.24}$ = Constant. Take $\gamma = 1.4$ and $R = 287$ kJ/kg K. [**Ans.** 0.027 kJ/K]

10. One kg of air at a pressure of 7 bar and a temperature of 363 K undergoes a reversible polytropic process which may be represented by $pv^{1.1}$ = Constant. The final pressure is 1.4 bar. Evaluate : 1. The final specific volume, temperature and increase in entropy ; and 2. The workdone and heat transfer during the process. Assume $R = 287$ J/kg K and $\gamma = 1.4$. [**Ans.** 0.643 m^3, 313.3 K, 0.316 kJ/K ; 142.64 kJ, 107.04 kJ]

11. One kg of air at 1 bar and 15° C is compressed according to $pv^{1.25}$ = Constant to a pressure of 16 bar. Calculate the temperature at the end of compression, the heat received or rejected by the air during the process and the change of entropy. Sketch the operation on temperature-entropy diagram.

Take $c_p = 1.005$ kJ/kg K, $c_v = 0.716$ kJ/kg K.

[**Ans.** 228.7° C ; 92.6 kJ (rejected) ; 0.238 kJ/K (decrease)]

12. An ideal gas of molecular mass 30 and specific heat ratio 1.38 is compressed according to the law $pv^{1.25}$ = Constant, from a pressure of 1 bar and 15° C to a pressure of 16 bar. Calculate the temperature at the end of compression, the heat received or rejected and workdone by the gas during the process and the change in entropy. Assume 1 kg mass of the gas. Use only calculated values of c_p and c_v.

[**Ans.** 228.7° C ; 81 kJ (rejected) ; 0.21 kJ/K (decrease)]

13. A gas engine mixture at 95° C and 1 bar is compressed with index of compression 1.3, the volume compression ratio being 6 : 1. The maximum pressure is 25 bar. Assuming the ratio of specific heats as 1.38 and the specific heat at constant volume as 0.754 kJ/kg K, find the change in entropy during compression stroke and during combustion which takes place at constant volume. Represent the process on p-v and T-s planes.

[**Ans.** 0.108 kJ/K (decrease) ; 0.67 kJ/K]

14. An ideal gas at temperature T_1 is heated at constant pressure to T_2 and then expanded reversibly according to the law pv^n = Constant, until the temperature is again T_1. Find the value of n, if the changes in entropy during the separate processes are equal.

$$\left[\textbf{Ans. } \frac{2\gamma}{\gamma + 1}\right]$$

15. The workdone by 0.07 kg of air when it expands according to pv^n = Constant is 7.6 kJ. The temperature of air falls from an initial value of 105° C to a final value of 13° C during the process. Determine : 1. the heat supplied or rejected by the air during the expansion ; 2. the value of index n ; and 3. the change of entropy, stating whether this is an increase or decrease. $\gamma = 1.4$ and $c_v = 0.712$ kJ/kg K.

[**Ans.** 3 kJ ; 1.24 ; 0.009 26 kJ/K (increase)]

16. One kg of air at 1 bar and 27° C is compressed isothermally to one-fifth the original volume. It is then heated at constant volume to a condition such that isentropic expansion from that state will return the system to the original state. Determine the pressure and temperature at the end of constant volume heating. Represent the processes on pressure-volume and temperature-entropy diagrams, and find : 1. the change in entropy during each process ; and 2. net workdone during the cycle.

[**Ans.** 9.518 bar, 571.08 K ; – 0.46 kJ/K, 0.46 kJ/K, zero ; 56.08 kJ]

17. 0.056 m^3 of carbon monoxide is contained in a cylinder at 37° C and 1.4 bar. The gas is compressed to 0.0224 m^3 during the inward stroke of the piston. If the compression process is (*a*) isothermal, and (*b*) adiabatic, find : 1. final temperature and pressure, 2. workdone, and 3. change of entropy. Take $c_p = 1.047$ kJ/kg K and $c_v = 0.749$ kJ/kg K. [**Ans.** 310 K, 3.5 bar, – 7.176 kJ, – 0.0232 kJ/K ; 447.2 K, 5.05 bar, – 8.68 kJ, zero]

18. Calculate the change of entropy when 1 kg of air changes from a temperature of 330 K and a volume of 0.14 m^3 to a temperature of 550 K and a volume of 0.56 m^3. If the gas expands according to the law pv^n = Constant, determine the value of index n and the heat absorbed or rejected by the air during the expansion. Show that it is approximately equal to the change of entropy multiplied by the mean absolute temperature. $R = 286$ J/kg K ; and $c_v = 0.712$ kJ/kg K. **[Ans. 1.365 ; 15.74 kJ]**

QUESTIONS

1. Explain clearly what is meant by 'entropy' of a gas.

2. Show that the specific entropy change for a perfect gas in a process is given by

$$s_2 - s_1 = c_p \log_e \left(\frac{v_2}{v_1} \right) + c_v \log_e \left(\frac{p_2}{p_1} \right)$$

where the subscripts $_1$ and $_2$ relate to the initial and final states and other symbols have their usual meanings.

3. Deduce the expressions in terms of initial and final temperatures and pressures, for the increase in entropy of a perfect gas when heated at a constant volume and at a constant pressure.

4. Show that for an ideal gas, the slope of the constant volume line on a temperature-entropy diagram is higher than that of constant pressure line.

5. Derive an expression for the change of entropy for the isothermal process in terms of volumes.

6. Starting from the fundamental, show that the change of entropy of a gas undergoing a polytropic process according to the law $pv^n = C$ is given by

$$S_2 - S_1 = m\,R \times \frac{\gamma - n}{(\gamma - 1)(n - 1)} \log_e \left(\frac{T_1}{T_2} \right)$$

where $S_2 - S_1$ = Entropy,
γ = Ratio of specific heats,
n = Index of polytropic operation,
T = Absolute temperature of a gas, and
R = Characteristic gas constant.

7. Establish the equation for the change in entropy of m kg of a perfect gas during a change according to the law pv^n = Constant,

$$S_2 - S_1 = m\,c_v \times \frac{\gamma - n}{n - 1} \log_e \left(\frac{T_1}{T_2} \right)$$

where γ is the ratio of c_p / c_v.

8. Show that when one kg of a perfect gas expands according to pv^n = constant, the change in entropy is given by

$$s_2 - s_1 = (c_p - n\,c_v) \log_e \left(\frac{v_2}{v_1} \right)$$

9. A mass of m kg of a fluid at a temperature T_1 is mixed with an equal mass of the same fluid at a temperature T_2. The system is thermally insulated. Show that the entropy change of the universe is given by

$$dS = 2\,mc_p \log_e \left(\frac{T_1 + T_2}{2\sqrt{T_1 T_2}} \right)$$

and show that it is necessarily positive.

10. Derive an expression for the approximate heat absorbed by a gas when heated from an absolute temperature T_1 to T_2 and an initial entropy of S_1 to S_2.

OBJECTIVE TYPE QUESTIONS

1. The heat absorbed or rejected by the working substance is given by

(*a*) $\delta Q = TdS$ (*b*) $\delta Q = T/dS$ (*c*) $\delta Q = dS/T$

where δQ = Heat absorbed or rejected,

dS = Increase or decrease of entropy, and

T = Absolute temperature.

2. The property of a working substance which increases or decreases as the heat is supplied or removed in a reversible manner, is known as

(*a*) enthalpy (*b*) internal energy (*c*) entropy (*d*) external energy

3. The entropy may be expressed as a function of

(*a*) pressure and temperature (*b*) temperature and volume

(*c*) heat and work (*d*) all of these

4. The entropy of water at 0° C is assumed to be

(*a*) 1 (*b*) 0 (*c*) -1 (*d*) 10

5. The change of entropy when heat is absorbed by the gas is

(*a*) positive (*b*) negative (*c*) positive or negative

ANSWERS

1. (*a*) 2. (*c*) 3. (*a*) 4. (*b*) 5. (*a*)