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Rotary Air Compressors

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29.1. Introduction

In the previous chapter, we have discussed the reciprocating compressors, in which the pressure of the air is increased in its cylinder with the help of a moving piston. But in a rotary air compressor, the air is entrapped between two sets of engaging surface and the pressure of air is increased by squeezing action or back flow of the air.

29.2. Comparison of Reciprocating and Rotary Air Compressors

Following are the main points of comparison of reciprocating and rotary air compressors :

S.No.	*Reciprocating air compressors*	*Rotary air compressors*
1.	The maximum delivery pressure may be as high as 1000 bar.	The maximum delivery pressure is 10 bar only.
2.	The maximum free air discharge is about 300 m^3/min.	The maximum free air discharge is as high as 3000 m^3/min.
3.	They are suitable for low discharge of air at very high pressure.	They are suitable for large discharge of air at low pressure.
4.	The speed of air compressor is low.	The speed of air compressor is high.
5.	The air supply is intermittent.	The air supply is continuous.
6.	The size of air compressor is large for the given discharge.	The size of air compressor is small for the same discharge.
7	The balancing is a major problem.	There is no balancing problem.
8.	The lubricating system is complicated.	The lubricating system is simple.
9.	The air delivered is less clean, as it comes in contact with the lubricating oil.	The air delivered is more clean, as it does not come in contact with the lubricating oil.
10.	Isothermal efficiency is used for all sorts of calculations.	Isentropic efficiency is used for all sorts of calculations.

29.3. Types of Rotary Air Compressors

Though there are many types of rotary air compressors, yet the following are important from the subject point of view :

1. Roots blower compressor ; 2. Vane blower compressor ; 3. Centrifugal blower compressor and 4. Axial flow compressor.

The first two compressors are popularly known as positive displacement compressors, whereas the last two as non-positive displacement. We shall discuss all the above mentioned rotary compressors one by one.

Note : The positive displacement compressors (*i.e.* roots blower and vane blower) are not very popular from the practical point of view. However, they have some academic importance. The only important rotary compressor is the centrifugal blower compressor.

29.4. Roots Blower Compressor

A roots blower compressor, in its simplest form, consists of two rotors with lobes rotating in an air tight casing which has inlet and outlet ports. Its action resembles with that of a gear pump. There are many designs of wheels, but they generally have two or three lobes (and sometimes even more). In all cases, their action remains the same as shown in Fig. 29.1 (*a*) and (*b*). The lobes are so designed that they provide an air tight joint at the point of their contact.

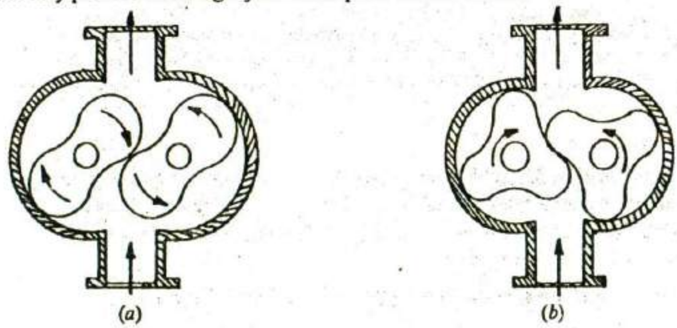


Fig. 29.1. Roots blower compressor.

The mechanical energy is provided to one of the rotors from some external source, while the other is gear driven from the first. As the rotors rotate, the air, at atmospheric pressure, is trapped in the pockets formed between the lobes and casing. The rotary motion of the lobes delivers the entrapped air into the receiver. Thus more and more flow of air into the receiver increases its pressure. Finally, the air at a higher pressure is delivered from the receiver.

It will be interesting to know that when the rotating lobe uncovers the exit port, some air (under high pressure) flows back into the pocket from the receiver. It is known as backflow process. The air, which flows from the receiver to the pocket, gets mixed up with the entrapped air. The backflow of air continues, till the pressure in the pocket and receiver is equalised. Thus the pressure of air entrapped in the pocket is increased at constant volume entirely by the backflow of air. The backflow process is shown in Fig. 29.2. Now the air is delivered to the receiver by the rotation of the lobes. Finally, the air at a higher pressure is delivered from the receiver.

Let p_1 = Intake pressure of air,

p_2 = Discharge pressure of air,

γ = Isentropic index for air, and

v_1 = Volume of air compressed.

We know that theoretical work done in compressing the air,

$$W = \frac{\gamma}{\gamma - 1} \times p_1 v_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right] \qquad \ldots (i)$$

and actual work $= v_1 (p_2 - p_1)$ ... (ii)

∴ Efficiency of roots blower (also known as roots efficiency),

$$\eta = \frac{\frac{\gamma}{\gamma-1} \times p_1 v_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]}{v_1 (p_2 - p_1)}$$

$$= \frac{\gamma}{\gamma - 1} \times \frac{\left[r^{\frac{\gamma-1}{\gamma}} - 1 \right]}{(r-1)}$$

where r is the pressure ratio (*i.e.* p_2 / p_1). Now the power required to drive the compressor may be found out from the work done as usual. Thus we see that the efficiency of roots blower decreases with the increase in pressure ratio.

Notes : 1. Sometimes, air at high pressure it obtained by placing two or more roots blower in series, and having intercoolers between each stage.

2. The air is delivered four times in one revolution in case of two-lobbed rotor. Similarly, the air is delivered six times in one revolution in case of three-lobbed rotor.

Example 29.1. *A Roots blower compressor compresses 0.05 m³ of air from 1 bar to 1.5 bar per revolution. Find the compressor efficiency.*

Solution. Given : $v_1 = 0.05$ m³ ; $p_1 = 1$ bar $= 1 \times 10^5$ N/m² ; $p_2 = 1.5$ bar $= 1.5 \times 10^5$ N/m²

We know that actual work done per revolution,

$$W_1 = v_1 (p_2 - p_1) = 0.05 (1.5 \times 10^5 - 1 \times 10^5) = 2500 \text{ N-m}$$

and ideal work done per revolution.

$$W_2 = \frac{\gamma}{\gamma - 1} \times p_1 v_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

$$= \frac{1.4}{1.4 - 1} \times 1 \times 10^5 \times 0.05 \left[\left(\frac{1.5}{1} \right)^{\frac{1.4-1}{1.4}} - 1 \right] = 2150 \text{ N-m}$$

∴ Compressor efficiency,

$$\eta = \frac{W_2}{W_1} = \frac{2150}{2500} = 0.86 \text{ or } 86\% \text{ **Ans.**}$$

29.5. Vane Blower Compressor

A vane blower, in its simplest form, consists of a disc rotating eccentrically in an air tight casing with inlet and outlet ports. The disc has a number of slots (generally 4 to 8) containing vanes. When the rotor rotates the disc, the vanes are pressed against the casing, due to centrifugal force, and form air tight pockets.

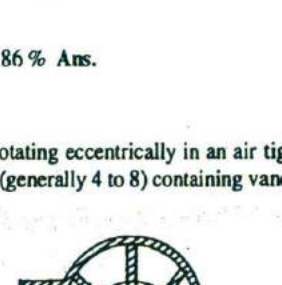


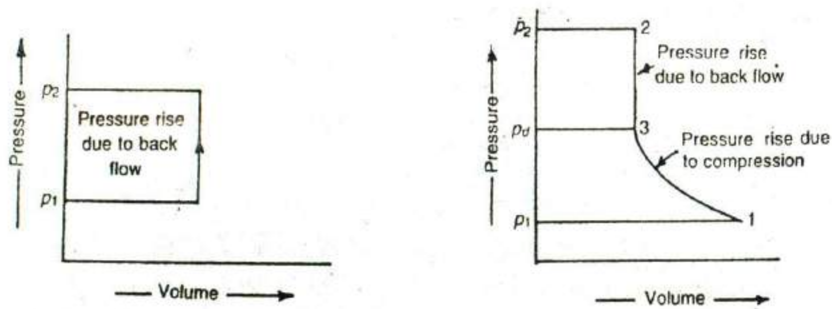
Fig. 29.2. Vane blower compressor.

The mechanical energy is provided to the disc from some external source. As the disc rotates, the air is trapped in the pockets formed between the vanes and casing. First of all, the rotary motion of the vanes compresses the air. When the rotating vane uncovers the exit port, some air (under high pressure) flows back into the pocket in the same way as discussed in the case of roots blower compressor. Thus the pressure of air, entrapped in the pocket, is increased first by decreasing the volume and then by the backflow of air as shown in Fig. 29.2. Now the air is delivered to the receiver by the rotation of the vanes. Finally, the air at a high pressure is delivered from the receiver.

29.6. Backflow in Positive Displacement Air Compressors

We have discussed two important types of positive displacement air compressors *viz.*, Roots blower compressor and vane blower compressor in the last articles. In both the cases, the air is delivered to the receiver by the rotating lobes or vanes. It will be interesting to know, that when the rotating lobe (in case of Roots blower) or vane (in case of vane blower) uncovers the exit port, some air (under high pressure) from the receiver flows back into the pockets formed between lobes and casing or vanes and casing. This backflow of air mixes up with the entrapped air, and continues until the pressure in the pockets and receiver are equalised. Thus the pressure of air delivered from the pocket to the receiver is taken to be equal to the receiver pressure. The process of backflow of air is an irreversible process, and called irreversible compression.

It may be noted that the increase of pressure in a Roots blower is entirely due to backflow, and this process is explained on *p-v* diagram as shown in Fig. 29.3 (*a*).



(*a*) Roots blower compressor. (*b*) Vane blower compressor.

Fig. 29.3. *p-v* diagram of air compressor.

The increase of pressure in a vane blower takes place first due to compression and then due to backflow as shown in Fig. 29.3 (*b*). Strictly speaking, the Roots blower compressor is of academic interest only. but vane blower compressor has been used, but with little success. Now consider a vane blower compressor compressing air as shown in Fig. 29.3 (*b*).

Let p_1 = Intake pressure of air,

p_2 = Discharge pressure of air,

p_d = Pressure at point 3.

γ = Isentropic index for air, and

v_1 = Volume of air compressed.

We know that work done due to compression (1-3),

$$W_1 = \frac{\gamma}{\gamma-1} \times p_1 v_1 \left[\left(\frac{p_d}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \qquad \ldots (i)$$

and work done due to backflow (3-2),

$$W_2 = v_2 (p_2 - p_d) \qquad \ldots (ii)$$

$\therefore$ Total work done, $W = W_1 + W_2$

∴ Efficiency of the vane blower (also known as vane blower efficiency)

$$\eta = \frac{W_2}{W_1 + W_2} \qquad \ldots (iii)$$

Now the power required to drive to compressor may be found out from the work done as usual.

Note : The value of v_2 or p_d in equation (*ii*) may be found out from the relation,

$$v_2 = v_1 \left(\frac{p_1}{p_d} \right)^{1/\gamma}$$

Example 29.2. *A rotary vane compressor compresses 4.5 m^3 of air per minute from 1 bar to 2 bar when running at 450 r.p.m. Find the power required to drive the compressor when 1. the ports are so placed that there is no internal compression ; and 2. the ports are so placed that there is 50% increase in pressure due to compression before the backflow occurs.*

Solution. Given : $v_1 = 4.5$ m³/min ; $p_1 = 1$ bar $= 1 \times 10^5$ N/m² ; $p_2 = 2$ bar $= 2 \times 10^5$ N/m² ; $N = 450$ r.p.m.

1. *Power required to drive the compressor when there is no internal compression*

We know that work done without internal compression,

$$W = v_1 (p_2 - p_1) = 4.5 (2 \times 10^5 - 1 \times 10^5) = 450\,000 \text{ N-m/min}$$

$$= 450 \text{ kN-m/min}$$

∴ Power required to drive the compressor,

$$P = 450/60 = 7.5 \text{ kW} \textbf{ Ans.}$$

2. *Power required to drive the compressor when there is 50% increase in pressure due to compression*

Since there is 50% increase in the pressure due to compression, therefore delivery pressure before backflow,

$$p_d = 1 + 0.5 (2 - 1) = 1.5 \text{ bar} = 1.5 \times 10^5 \text{ N/m}^2$$

$$\therefore \qquad v_2 = v_1 \times \left(\frac{p_1}{p_d} \right)^{1/\gamma} = 4.5 \left(\frac{1}{1.5} \right)^{1/1.4} = 3.37 \text{ m}^3\text{/min}$$

We know that theoretical work done in compressing the air from 1 bar to 1.5 bar,

$$W_1 = \frac{\gamma}{\gamma - 1} \times p_1 v_1 \left[\left(\frac{p_d}{p_1} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right]$$

$$= \frac{1.4}{1.4 - 1} \times 1 \times 10^5 \times 4.5 \left[\left(\frac{1.5}{1} \right)^{\frac{1.4 - 1}{1.4}} - 1 \right]$$

$$= 193\,500 \text{ N-m/min} = 193.5 \text{ kN-m/min}$$

and work done in backflow $\quad W_2 = v_2 (p_2 - p_d) = 3.37 (2 \times 10^5 - 1.5 \times 10^5)$

$$= 168\,500 \text{ N-m/min} = 168.5 \text{ kN-m/min}$$

∴ Total work done, $\quad W = W_1 + W_2 = 193.5 + 168.5 = 362$ kN-m/min

and power required to drive the compressor,

$$P = 362/60 = 6.03 \text{ kW} \textbf{ Ans.}$$

29.7. Centrifugal Compressor

A centrifugal blower compressor, in its simplest form, consists of a rotor (or impeller) to which a number of curved vanes are fitted symmetrically. The rotor rotates in an air tight volute casing with inlet and outlet points. The casing for the compressor is so designed that the kinetic energy of the air is converted into pressure energy before it leaves the casing as shown in Fig. 29.4.

The mechanical energy is provided to the rotor from some external source. As the rotor rotates, it sucks air through its eye, increases its pressure due to centrifugal force and forces the air to flow over the diffuser. The pressure of air is further increased during its flow over the diffuser.

Finally, the air at a high pressure is delivered to the receiver. It will be interesting to know that the air enters the impeller radially and leaves the vanes axially.

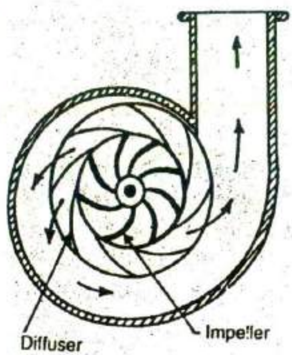


Fig. 29.4. Centrifugal compressor.

Notes : 1. The curved vanes as well as the diffuser are so designed that the air enters and leaves their tips tangentially *i.e.* without shock. Their surface is made very smooth in order to minimise the frictional losses.

2. The workdone by a centrifugal compressor (or power required to drive it) may be found out either by the velocity triangles or otherwise.

29.8. Workdone by a Centrifugal Air Compressor

We have already discussed in Art. 28.6 the work done by a single acting reciprocating air compressor. The equations for work done or power required to drive the reciprocating compressor are applicable for the work done or power required by a rotary compressor also. Thus workdone by a rotary compressor,

$$W = 2.3\, p_1 v_1 \log\left(\frac{v_1}{v_2}\right) \quad \text{... (For isothermal compression)}$$

$$= 2.3\, m\, R\, T_1 \log r \quad \text{... (where } r = v_1/v_2 \text{ or } p_2/p_1)$$

$$= \frac{n}{n-1} \times p_1 v_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] \quad \text{... (For polytropic compression)}$$

$$= \frac{n}{n-1} \times m\, R\, T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] \quad \text{... } (\because pv = mRT)$$

$$= \frac{\gamma}{\gamma-1} \times p_1 v_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \quad \text{... (For isentropic compression)}$$

$$= m\, c_p (T_2 - T_1)$$

where p_1 = Initial pressure of air,

v_1 = Initial volume of air,

T_1 = Initial temperature of air,

p_2, v_2, T_2 = Corresponding values for the final condition,

m = Mass of air compressed per minute,

n = Polytropic index,

γ = Isentropic index, and

c_p = Specific heat at constant pressure.

Example 29.3. *A centrifugal compressor delivers 50 kg of air per minute at a pressure of 2 bar and 97° C. The intake pressure and temperature of the air is 1 bar and 15° C. If no heat is lost to the surrounding, find : 1. index of compression ; and 2. power required, if the compression is isothermal. Take R = 287 J/kg K.*

Solution. Given : $m = 50$ kg/min ; $p_2 = 2$ bar ; $T_2 = 97°\text{C} = 97 + 273 = 370$ K ; $p_1 = 1$ bar ; $T_1 = 15°\text{C} = 15 + 273 = 288$ K ; $R = 287$ J/kg K

1. *Index of compression*

Let n = Index of compression.

We know that $$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

$$\frac{370}{288} = \left(\frac{2}{1}\right)^{\frac{n-1}{n}} = (2)^{\frac{n-1}{n}}$$

$$1.285 = (2)^{\frac{n-1}{n}}$$

$$\log 1.285 = \frac{n-1}{n} \times \log 2 \qquad \text{. . . (Taking log of both sides)}$$

$$0.1088 = \frac{n-1}{n} \times 0.301$$

$$0.1088\, n = 0.301\, n - 0.301$$

$$0.1922\, n = 0.301 \quad \text{or} \quad n = 1.57 \textbf{ Ans.}$$

2. *Power required if the compression is isothermal*

We know that work done by the compressor if the compression is isothermal,

$$W = 2.3\, mRT_1 \log r$$

$$= 2.3 \times 50 \times 287 \times 288 \log 2 \text{ J/min} \qquad \ldots (\because r = p_2 / p_1)$$

$$= 9\,505\,440 \times 0.301 = 2\,861\,140 \text{ J/min} = 2861.14 \text{ kJ/min}$$

$\therefore$ Power required, $P = 2861.14 / 60 = 47.7$ kW **Ans.**

Example 29.4. *A centrifugal air compressor having a pressure compression ratio of 5 compresses air at the rate of 10 kg/s. If the initial pressure and temperature of the air is 1 bar and 20° C, find : 1. the final temperature of the gas, and 2. power required to drive the compressor. Take $\gamma = 1.4$ and $c_p = 1$ kJ/kg K.*

Solution. Given : $p_2 / p_1 = 5$; $m = 10$ kg/s ; $p_1 = 1$ bar ; $T_1 = 20°\text{C} = 20 + 273 = 293$ K ; $\gamma = 1.4$; $c_p = 1$ kJ/kg K

1. *Final temperature of the gas*

Let T_2 = Final temperature of the gas.

We know that $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (5)^{\frac{1.4-1}{1.4}} = 1.584$

$\therefore$ $T_2 = T_1 \times 1.584 = 293 \times 1.584 = 464\text{ K} = 191°\text{C}$ **Ans.**

2. *Power required by the compressor*

We know that the workdone by the compressor,

$$W = m\, c_p (T_2 - T_1) = 10 \times 1\,(464 - 293) = 1710\text{ kJ/s}$$

$\therefore$ Power required to drive the compressor,

$$P = 1710\text{ kW} \quad \textbf{Ans.} \qquad \ldots (\because 1\text{ kJ/s} = 1\text{ kW})$$

Example 29.5. *A rotary air compressor receives air at a pressure of 1 bar and 17° C, and delivers it at a pressure of 6 bar. Determine, per kg of air delivered, work done by the compressor and heat exchanged with the jacket water when the compression is isothermal, isentropic and by the relation $pv^{1.6}$ = Constant.*

Solution. Given : $p_1 = 1$ bar ; $T_1 = 17°\text{C} = 17 + 273 = 290\text{ K}$; $p_2 = 6$ bar ; $m = 1$ kg

Isothermal compression

We know that work done by the compressor,

$$W = 2.3\, mRT_1 \log\left(\frac{p_2}{p_1}\right) = 2.3 \times 1 \times 287 \times 290 \log\left(\frac{6}{1}\right)\text{J}$$

$$= 148\,970\text{ J} = 148.97\text{ kJ} \quad \textbf{Ans.}$$

We also know that in isothermal compression, the temperature of air during the process remains constant. Thus the entire work done is carried away by the jacket water in the form of heat. Therefore, heat exchanged with the jacket water

$$= 148.97\text{ kJ} \quad \textbf{Ans.}$$

Isentropic compression

Let T_2 = Final temperature of the air.

We know that $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{6}{1}\right)^{\frac{1.4-1}{1.4}} = 1.668$

$\therefore$ $T_2 = T_1 \times 1.668 = 290 \times 1.668 = 484\text{ K}$

We know that work done by the compressor

$$= m\, c_p (T_2 - T_1) = 1 \times 1\,(484 - 290) = 194\text{ kJ} \quad \textbf{Ans.}$$

We also know that in isentropic compression, heat exchanged with the jacket water is zero. **Ans.**

Compression by the relation $pv^{1.6}$ = Constant

We know that $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{6}{1}\right)^{\frac{1.6-1}{1.6}} = 1.958$

$\therefore$ $T_2 = T_1 \times 1.958 = 290 \times 1.958 = 568\text{ K}$

We know that work done by the compressor

$$= m\,c_p\,(T_2 - T_1) = 1 \times 1\,(568 - 290) = 278 \text{ kJ} \;\; \textbf{Ans.}$$

We also know that in polytropic process, heat exchanged

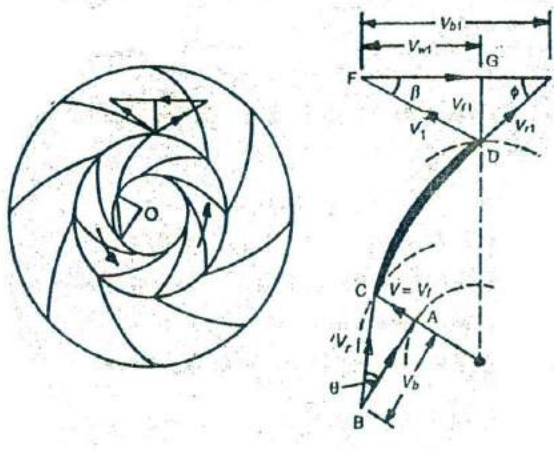
$$= \frac{\gamma - n}{\gamma - 1} \times \text{Work done} = \frac{1.4 - 1.6}{1.4 - 1} \times 278 = -139 \text{ kJ} \;\; \textbf{Ans.}$$

The minus sign means that heat is taken by the air from the jacket water

29.9. Velocity Triangles for Moving Blades of a Centrifugal Compressor

We have already discussed that in a centrifugal compressor, the air enters radially and leaves axially. Moreover, the blades ana diffuser are so designed that the air enters and leaves them tangentially for the shockless entry and exit.

Consider a stream of air, entering the curved blade at C, and leaving it at D as shown in Fig. 29.5 (*a*) and (*b*).



(*a*) (*b*)

Fig. 29.5. Velocity triangles for a centrifugal compressor.

Now let us draw the velocity triangles at the inlet and outlet tips of the blades as shown in Fig. 29.5 (*a*) and (*b*).

Let V_b = Linear velocity of the moving blade at inlet (BA),

V = Absolute velocity of the air entering the blade (AC),

V_r = Relative velocity of air to the moving blade at inlet (BC). It is vectorial difference between V_b and V,

V_f = Velocity of flow at inlet,

θ = Angle which the relative velocity (V_r) makes with the direction of motion of the blade, and

$V_{b1}, V_1, V_{r1}, V_{f1}, \phi$ = Corresponding values at outlet.

It may be seen from the above, that the original notations (*i.e.* V_b, V, V_r, V_f) stand for the inlet triangle. The notations with suffix 1 (*i.e.* V_{b1}, V_1, V_{r1}, V_{f1}) stand for the outlet triangle. A little consideration will show, that as the air enters and leaves the blades without any shock (or in other words tangentially), therefore shape of the blades will be such that V_r and V_{r1} will be along the tangents to the blades at inlet and outlet respectively.

The air enters the blades along AC with a velocity (V). Since the air enters the blades at right angle (*i.e.* radially) to the direction of motion of the blade, therefore velocity of flow (V_f) will be equal to the air velocity (V). Moreover, velocity of whirl at inlet (V_w) will be zero. The linear velocity or mean velocity of blades (V_b) is represented by BA in magnitude and direction. The length BC represents the relative velocity (V_r) of the air with respect to the blade. The air now glides over and leaves the blade with a relative velocity (V_{r1}) which is represented by DE.

The absolute velocity of air (V_1) as it leaves the blade is represented by DF inclined at an angle β with the direction of the blade motion. The tangential component of V_1 (represented by FG) is known as velocity of whirl at exit (V_{w1}). The axial component of V_1 (represented by DG) is known as velocity of flow at exit (V_{f1}).

Let m = Mass of air compressed by the compressor in kg/s.

We know that according to Newton's second law of motion, force in the direction of motion of blades (in newtons),

$$F = \text{Mass of air flowing in kg/s} \times \text{Change in the velocity of whirl in m/s}$$

$$= m(V_w + V_{w1}) = m\,V_{w1} \qquad \ldots (\because V_w = 0)$$

and work done in the direction of motion of the blades,

$$W = \text{Force} \times \text{Distance} = m\,V_{w1}\,V_{b1} \text{ N-m/s or J/s}$$

Now power required to drive the compressor may be found out, as usual, by the relation,

$$P = \text{Work done in J/s} = m\,V_{w1}\,V_{b1} \text{ watts} \qquad \ldots (1 \text{ J/s} = 1 \text{ watt})$$

Notes : 1. The blade velocity at inlet or outlet (V_b or V_{b1}) may be found out by the relation,

$$V_b = \frac{\pi DN}{60} \text{ and } V_{b1} = \frac{\pi D_1 N}{60}$$

where D and D_1 are the internal and external diameters of the impeller.

2. Under ideal conditions (or in other words for maximum work) $V_{w1} = V_{b1}$.

Therefore ideal work done

$$= m(V_{w1})^2 = m(V_{b1})^2 \text{ J/s}$$

Example 29.6. *A centrifugal compressor running at 2000 r.p.m. receives air at 17° C. If the outer diameter of the blade tip is 750 mm ; find the temperature of the air leaving the compressor. Take c_p = 1 kJ/kg K.*

Solution. Given : N = 2000 r.p.m. ; T_1 = 17° C = 17 + 273 = 290 K ; D_1 = 750 mm = 0.75 m ; c_p = 1 kJ/kg K

Temperature of the air leaving the compressor

Let T_2 = Temperature of the air leaving the compressor

We know that tangential velocity of the outer blade tip,

$$V_{b1} = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.75 \times 2000}{60} = 78.6 \text{ m/s}$$

∴ Work done per kg of air,

$$W = m\,(V_{b1})^2 = 1\,(78.6)^2 = 6178 \text{ J/s} = 6.178 \text{ kJ/s}$$

We also know that workdone (*W*),

$$6.178 = m\,c_p\,(T_2 - T_1) = 1 \times 1\,(T_2 - 290)$$

$$\therefore \quad T_2 = 6.178 + 290 = 296.178 \text{ K} = 23.178° \text{C} \textbf{ Ans.}$$

Example 29.7. *A rotary air compressor working between 1 bar and 2.5 bar has internal and external diameters of impeller as 300 mm and 600 mm respectively. The vane angle at inlet and outlet are 30° and 45° respectively. If the air enters the impeller at 15 m/s, find : 1. speed of the impeller in r.p.m. ; and 2. workdone by the compressor per kg of air.*

Solution. Given : $p_1 = 1$ bar ; * $p_2 = 2.5$ bar ; $D = 300$ mm $= 0.3$ m ; $D_1 = 600$ mm $= 0.6$ m ; $\theta = 30°$; $\phi = 45°$; $V = 15$ m/s

1. *Speed of the impeller*

Let N = Speed of the impeller in r.p.m.

From the inlet velocity triangle, as shown in Fig. 29.6, we find that the blade velocity,

$$V_b = \frac{V}{\tan 30°} = \frac{15}{0.5774} = 25.98 \text{ m/s}$$

We know that speed of the impeller (V_b),

$$25.98 = \frac{\pi DN}{60} = \frac{\pi \times 0.3\,N}{60} = 0.0157\,N$$

$$\therefore \quad N = 1655 \text{ r.p.m.} \textbf{ Ans.}$$

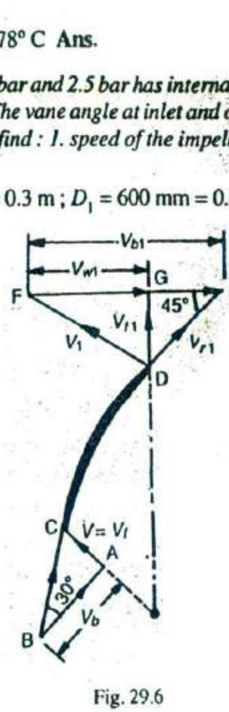


Fig. 29.6

2. *Work done by the compressor per kg of air*

From the outlet triangle, as shown in Fig. 29.6, we find that the blade velocity at outlet,

$$V_{b1} = V_b \times \frac{D_1}{D} = V_b \times \frac{0.6}{0.3} = 25.98 \times \frac{0.6}{0.3} = 51.96 \text{ m/s}$$

and velocity of whirl at outlet,

$$V_{w1} = V_{b1} - \frac{V_{f1}}{\tan 45°} = 51.96 - \frac{15}{1} = 36.96 \text{ m/s} \quad \ldots (\because V_{f1} = V_f = V)$$

Since the velocity of blade at outlet (51.96 m/s) is more than velocity of whirl at outlet (36.96 m/s), therefore shape of the outlet triangle will be as shown in Fig. 29.6.

We know that workdone by the compressor per kg of air,

$$W = m\,V_{w1}\,V_{b1} = 1 \times 36.96 \times 51.96 = 1920.44 \text{ J/s} \textbf{ Ans.}$$

29.10. Width of Impeller Blades

The width of impeller blades at inlet or outlet of a rotary air compressor is found out from the fact that mass of air flowing through the blades at inlet and outlet is constant. Now consider a rotary air compressor compressing the air, whose blade widths at inlet or outlet is required to be found out

* Superfluous data

Let b = Width of the impeller blades at inlet.

D = Diameter of impeller at inlet,

V_f = Velocity of flow at inlet,

v_s = Specific volume of air at inlet,

b_1, D_1, V_{f1}, v_{s1} = Corresponding values at outlet, and

m = Mass of the air flowing through the impeller.

We know that the mass of air flowing through the impeller at inlet,

$$m = \frac{\pi D b V_f}{v_s} \quad \ldots (i)$$

Similarly, mass of air flowing through the impeller at outlet,

$$m = \frac{\pi D_1 b_1 V_{f1}}{v_{s1}} \quad \ldots (ii)$$

Since the mass of air flowing through the impeller is constant, therefore,

$$\frac{\pi D b V_{f1}}{v_s} = \frac{\pi D_1 b_1 V_{f1}}{v_{s1}} \quad \ldots (iii)$$

Note : Sometimes, number and thickness of the blades is also taken into consideration. In such a case, mass of air flowing through the impeller at inlet,

$$m = \frac{(\pi D - nb) V_f}{v_s}$$

where n stands for the number of blades.

Example 29.8. *A centrifugal air compressor having internal and external diameters of 250 mm and 500 mm respectively compresses 30 kg of air per minute while running at 4000 r.p.m. The vane angles at inlet and outlet are 30° and 40° respectively. Find the necessary thickness of the blade, if the impeller contains 40 blades. Take specific volume of air as 0.8 m³/kg.*

Solution. Given : $D = 250$ mm $= 0.25$ m ; $D_1 = 500$ mm $= 0.5$ m ; $m = 30$ kg/min $= 0.5$ kg/s ; $N = 4000$ r.p.m. ; $\theta = 30°$; $\phi = 40°$; $n = 40$; $v_s = 0.8$ m³/kg

We know that impeller velocity at inlet,

$$V_b = \frac{\pi D N}{60} = \frac{\pi \times 0.25 \times 4000}{60} = 52.4 \text{ m/s}$$

and velocity of flow at inlet, $V_f = V_b \tan\theta = 52.4 \tan 30° = 30.2$ m/s

Let b = Thickness of the blades.

We know that mass of air flowing through the impeller (m),

$$0.5 = \frac{(\pi D - nb) V_f}{v_s} = \frac{(\pi \times 0.25 - 40\,b)\,30.2}{0.8}$$

or $$0.0132 = 0.7855 - 40b$$

$\therefore$ $$b = 0.0193 \text{ m} = 19.3 \text{ mm} \quad \textbf{Ans.}$$

29.11. Prewhirl

It has been observed that tangential velocity of the inlet impeller end is very high due to its exceedingly high revolutions per minute (sometimes, as high as 20 000 r.p.m.). At this point, there is always a tendency for the air stream to break away from the trailing face of the curved part of the impeller vane. This phenomenon, under certain set of conditions*causes the shock waves to form. The shock waves increase the loss of energy.

In order to eliminate (or reduce) the shock waves, the air is made to rotate before it enters the impeller blades. This process, which causes the air to enter the impeller blades at a reduced velocity (without effecting the mass of air to flow and velocity of flow), is known as pre-rotation or *prewhirl*.

29.12. Axial Flow Compressors

An axial flow compressor, in its simplest form, consists of a number of rotating blade rows fixed to a rotating drum. The drum rotates inside an air tight casing to which are fixed stator blade rows, as shown in Fig. 29.7. The blades are made of aerofoil section to reduce the loss caused by turbulence and boundary separation.

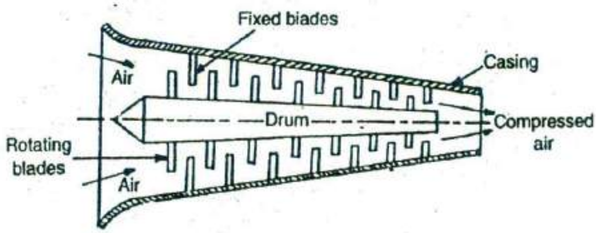


Fig. 29.7. Axial flow compressor.

The mechanical energy is provided to the rotating shaft, which rotates the drum. The air enters from the left side of the compressor. As the drum rotates, the air flows through the alternately arranged stator and rotor. As the air flows from one set of stator and rotor to another, it gets compressed. Thus successive compression of the air, in all the sets of stator and rotor, the air is delivered at a high pressure at the outlet point.

29.13. Comparison of Centrifugal and Axial Flow Air Compressors

Following are the main points of comparison of the centrifugal and axial flow air compressors :

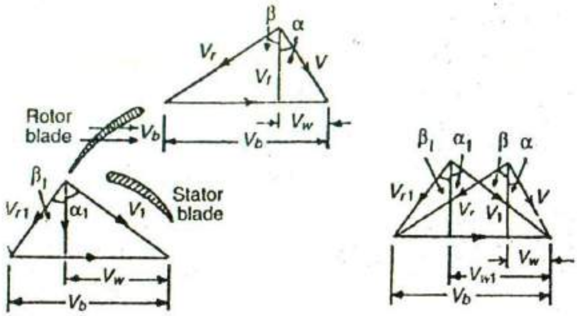
S.No.	*Centrifugal compressor*	*Axial flow compressor*
1.	The flow of air is perpendicular to the axis of compressor.	The flow of air is parallel to the axis of compressor.
2.	It has low manufacturing and running cost.	It has high manufacturing and running cost.
3.	It requires low starting torque.	It requires high starting torque.
4.	It is not suitable for multi-staging.	It is suitable for multi-staging.
5.	It requires large frontal area for a given rate of flow.	It requires less frontal area for a given rate of flow. It makes the compressor suitable for air crafts.

* This condition is popularly defined in terms of Mach number. If the Mach number is less than unity, the flow is known as subsonic. But if it is equal to unity, the flow is known as sonic. Similarly, if the Mach number is more than unity, the flow is known as supersonic. The shock waves are formed when the Mach number exceeds 0.90. The value of Mach number is mathematically given by the relation.

$$M_N = \frac{V_b}{\sqrt{VgRT}}$$

29.14. Velocity Diagrams for Axial Flow Air Compressors

We have already discussed in Art. 29.12 that in an axial flow compressor, the drum with rotor blades, rotates inside a casing with a fixed or stator blades. The inlet and outlet velocity triangles for the rotor blades are shown in Fig. 29.8 (*a*) and (*b*). The general relations between the inlet and outlet velocity triangles are as below :



(*a*) Separate velocity diagrams. (*b*) Combined velocity diagram.

Fig. 29.8. Velocity diagrams for axial flow compressor.

1. Blade velocity (V_b) for both the triangles is equal.
2. Velocity of flow (V_f) for both the triangles is also equal.
3. Relative velocity in outlet triangle (V_{r1}) is less than that in inlet triangle (V_r) due to friction.

Notes : 1. Work done by the compressor per kg of air,

$$w = V_b (V_{w1} - V_w) \text{ in N-m or J}$$

2. Sometimes, work factor or work input factor is also given. In such a case, work done by the compressor per kg of air,

$$w = V_b (V_{w1} - V_w) \times \text{Work factor}$$

29.15. Degree of Reaction

It is an important term in the field of axial flow compressor which may be defined as the ratio of pressure rise in the rotor blades to the pressure rise in the compressor in one stage.

As a matter of fact, the degree of reaction is usually kept as 50% or 0.5 for all types of axial flow compressors.

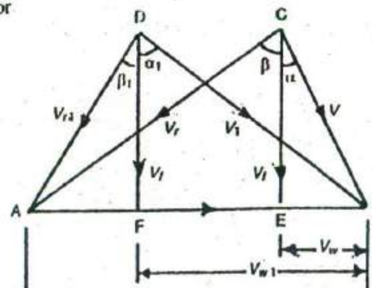


Fig. 29.9. Combined velocity diagram with 50% degree of reaction

Mathematically, degree of reaction,

$$= \frac{\text{Pressure rise in rotor blades}}{\text{Pressure rise in compressor}}$$

$$= \frac{\dfrac{(V_r)^2 - (V_{r1})^2}{2}}{V_b (V_{w1} - V_w)} = \frac{(V_r)^2 - (V_{r1})^2}{2V_b (V_{w1} - V_w)} \quad \ldots (i)$$

First of all, let us draw a combined velocity diagram for an axial flow compressor (with degree of reaction as 0.5), as shown in Fig. 29.9.

From the geometry of the figure, we find that

$$V_w = AB - AE = V_b - V_f \tan\beta\ ;\ \text{and}\ V_{w1} = AB - AF = V_b - V_f \tan\beta_1$$

$$\therefore \quad (V_{w1} - V_w) = (V_b - V_f \tan\beta_1) - (V_b - V_f \tan\beta)$$

$$= V_f(\tan\beta - \tan\beta_1) \quad \ldots (ii)$$

Moreover, from the geometry of the figure, we also find that

$$(V_r)^2 = (V_f)^2 + (V_f \tan\beta)^2$$

and

$$(V_{r1})^2 = (V_f)^2 + (V_f \tan\beta_1)^2$$

$$\therefore \quad (V_r)^2 - (V_{r1})^2 = [(V_f)^2 + (V_f \tan\beta)^2] - [(V_f)^2 + (V_f \tan\beta_1)^2]$$

$$= (V_f \tan\beta)^2 - (V_f \tan\beta_1)^2 = (V_f)^2 (\tan^2\beta - \tan^2\beta_1) \quad \ldots (iii)$$

Now substituting the values of $(V_{w1} - V_w)$ and $[(V_r)^2 - (V_{r1})^2]$ from equations (*ii*) and (*iii*) in equation (*i*), we have degree of reaction,

$$R = \frac{(V_f)^2(\tan^2\beta - \tan^2\beta_1)}{2\,V_b\,V_f(\tan\beta - \tan\beta_1)} = \frac{V_f(\tan\beta + \tan\beta_1)}{2\,V_b}$$

Now substituting the value of degree of reaction as 0.5, we have

$$0.5 = \frac{V_f(\tan\beta + \tan\beta_1)}{2\,V_b}$$

$$\therefore \quad \frac{V_b}{V_f} = \tan\beta + \tan\beta_1$$

From the geometry of the figure, we find that

$$\frac{V_b}{V_f} = \tan\alpha + \tan\alpha_1 = \tan\alpha + \tan\beta = \tan\alpha_1 + \tan\beta_1$$

$$\therefore \quad \angle\beta = \angle\alpha_1 \text{ and } \angle\beta_1 = \angle\alpha$$

It is thus obvious, that for 50% reaction, the compressor will have symmetrical blades.

Example 29.9. *An axial flow compressor, with compression ratio as 5, draws air at 20° C delivers it at 50° C. Assuming 50% degree of reaction, find the velocity of flow if the blade velocity is 100 m/s. Also find the number of stages. Take work factor = 0.85 ; $\alpha = 10°$; $\beta = 40°$ and $c_p = 1$ kJ/kg K.*

Solution. Given : *$p_2/p_1 = 5$; $T_1 = 20°\text{C} = 20 + 273 = 293$ K ; $T_2 = 50°\text{C} = 323$ K ; $R = 50\% = 0.5$; $V_b = 100$ m/s ; Work factor = 0.85 ; $\alpha = 10°$; $\beta = 40°$; $c_p = 1$ kJ/kg K

Velocity of flow

Let $\quad V_f$ = Velocity of flow.

From the geometry of the velocity triangle (Fig. 29.9), we know that

$$\frac{V_b}{V_f} = \tan\alpha + \tan\beta = \tan 10° + \tan 40°$$

* Superfluous data

$$\frac{100}{V_f} = 0.1763 + 0.8391 = 1.0154$$

$\therefore$ $V_f = 98.5$ m/s **Ans.**

Number of stages

We know that total work required per kg of air

$$= c_p (T_2 - T_1) = 1\,(323 - 293) = 30 \text{ kJ/kg}$$

From the geometry of the velocity triangles, we also know that

$$V_w = V_f \tan\alpha = 98.5 \times \tan 10^\circ = 98.5 \times 0.1763 = 17.4 \text{ m/s}$$

and $$V_{w1} = V_f \tan\alpha_1 = 98.5 \tan 40^\circ = 98.5 \times 0.8391 = 82.7 \text{ m/s}$$

... (with 50% reaction, $\angle\alpha_1 = \angle\beta$)

We know that workdone per kg of air per stage,

$$= V_b (V_{w1} - V_w) \times \text{Work factor}$$

$$= 100\,(82.7 - 17.4) \times 0.85 = 5550 \text{ J} = 5.55 \text{ kJ/kg}$$

$\therefore$ Number of stages $= \dfrac{\text{Total work required}}{\text{Work done per stage}} = \dfrac{30}{5.55} = 5.4$ say 6 **Ans.**

EXERCISES

1. Air at 1 bar and 30° C is to be compressed to 1.2 bar at the rate of 50 m³/min. Find the power required by a Root's blower. **[Ans. 15.45 kW]**

2. Compare the work inputs required for a Roots blower and a vane type compressor having the same induced volume of 0.03 m³ per revolution, the inlet pressure being 1.013 bar and the pressure ratio 1.5 to 1. For the vane type, assume that internal compression takes place through half the pressure range.

[Ans. 1.52 kJ, 1.352 kJ]

3. A centrifugal compressor having compression ratio of 2 delivers air at the rate of 1.5 kg/s. Find the power required to drive the compressor with isothermal compression, if the intake temperature is 300 K.

[Ans. 89.5 kW]

4. A centrifugal air compressor receives air at 1 bar and deliver it at 3.5 bar. Find the final temperature of air, if the initial temperature of air is 310 K. The compressor compresses 2 kg of air per second. Take γ as 1.4.

[Ans. 443 K]

5 A rotary air compressor compresses 100 kg of air per minute from 1.2 bar and 20° C to 4.8 bar. Find the power required by the compressor, if the compression is isentropic and by the relation $pv^{1.5} = C$. Take $c_p = 1.008$ kJ/kg K. **[Ans. 247 kW ; 300 kW]**

6. An axial flow compressor, with compression ratio as 4, draws air at 20° C and delivers it at 197° C. The mean blade speed and flow velocity are constant throughout the compressor. Assuming 50 percent reaction blading and taking blade velocity as 180 m/s; find the flow velocity and the number of stages. Take work factor = 0.82 ; $\alpha = 12^\circ$; $\beta = 42^\circ$ and $c_p = 1.005$ kJ/kg K. **[Ans. 162 m/s ; 10]**

QUESTIONS

1. What is the difference between rotary and reciprocating compressor ?

2. Derive an expression for efficiency of a Roots blower in terms of pressure ratio and ratio of specific heats.

3. Describe with a neat sketch, the working of a vane blower compressor and show its *p-v* diagram. For what applications, it is used.

4. Explain, with a neat sketch, the working of a centrifugal compressor and obtain an expression for the workdone.

5. Discuss the method of finding the width of impeller blades in a rotary air compressor.

6. Define 'prewhirl'. Explain its effect on the impeller of a centrifugal pump.

7. Explain, with a neat sketch, the working of an axial flow compressor.

8. Differentiate between centrifugal compressor and axial flow compressor.

OBJECTIVE TYPE QUESTIONS

1. The positive displacement compressor is
   (*a*) roots blower compressor (*b*) vane blower compressor
   (*c*) centrifugal compressor (*d*) axial flow compressor
   (*e*) both (*a*) and (*b*) (*f*) both (*c*) and (*d*)

2. The rotary compressors are used for delivering
   (*a*) small quantities of air at high pressures (*b*) large quantities of air at high pressures
   (*c*) small quantities of air at low pressures (*d*) large quantities of air at low pressures

3. The maximum delivery pressure in a rotary air compressor is
   (*a*) 10 bar (*b*) 20 bar (*c*) 30 bar (*d*) 40 bar

4. The speed of a rotary compressor is ..... as compared to reciprocating air compressor.
   (*a*) high (*b*) low

5. The type of rotary compressor used in gas turbines is of
   (*a*) centrifugal type (*b*) axial flow type (*c*) radial flow type (*d*) none of these

6. If the flow of air through the compressor is perpendicular to its axis, then it is a
   (*a*) reciprocating compressor (*b*) centrifugal compressor
   (*c*) axial flow compressor (*d*) turbo compressor

7. In a centrifugal compressor, an increase in speed at a given pressure ratio causes
   (*a*) increase in flow (*b*) decrease in flow
   (*c*) increase in efficiency (*d*) decrease in efficiency
   (*e*) increase in flow and decrease in efficiency

8. In an axial flow compressor, the ratio of pressure in the rotor blades to the pressure rise in the compressor in one stage is known as
   (*a*) work factor (*b*) slip factor (*c*) degree of reaction (*d*) pressure coefficient

9. A compressor mostly used for supercharging of I.C. engines is
   (*a*) radial flow compressor (*b*) axial flow compressor
   (*c*) roots blower (*d*) reciprocating compressor

10. Which of the following statement is correct as regard to centrifugal compressors ?
    (*a*) The flow of air is parallel to the axis of the compressor.
    (*b*) The static pressure of air in the impeller increases in order to provide centripetal force on the air.
    (*c*) The impeller rotates at high speeds.
    (*d*) The maximum efficiency is higher than multi-stage axial flow compressors.

ANSWERS

1. (*e*)	2. (*d*)	3. (*a*)	4. (*a*)	5. (*b*)
6. (*b*)	7. (*e*)	8. (*c*)	9. (*a*)	10. (*b*)

30

Performance of Air Compressors

1. Introduction. 2. Efficiencies of Reciprocating and Centrifugal Air Compressors. 3. Efficiencies of Reciprocating Air Compressor. 4. Volumetric Efficiency of a Reciprocating Air Compressor with Clearance Volume. 5. Thermodynamic Cycle for a Rotary Air Compressor. 6. Efficiencies of a Centrifugal Air Compressor. 7. Static and Total Head Quantities. 8. Slip Factor. 9. Comparison of Turbine and Centrifugal Compressor Blades.

30.1. Introduction

In the last two chapters, we nave discussed reciprocating and rotary air compressors. Now in this chapter, we shall discuss their efficiencies and other important performances.

30.2. Efficiencies of Reciprocating and Centrifugal Air Compressors

The efficiency of any machine is the general term used for the ratio of work done to the energy supplied. The criterion for the thermodynamic efficiency of the reciprocating air compressor is isothermal; whereas that for the centrifugal air compressor is isentropic. The reason for the same is that in case of reciprocating air compressors, due to slow speed of the piston and cooling of the cylinder, the compression of air is approximately isothermal. But in case of centrifugal air compressor, due to high speed of the rotor and without any cooling arrangement, the compression of air is approximately isentropic. Now we shall discuss the efficiencies of both the compressors, in the following pages.

30.3. Efficiencies of Reciprocating Air Compressor

We have already discussed in the last article that the criterion for thermodynamic efficiency of a reciprocating air compressor is isothermal. But in general, the following efficiencies of reciprocating air compressor are important from the subject point of view :

1. *Isothermal efficiency (or compressor efficiency).* It is the ratio of work (or power) required to compress the air isothermally to the actual work required to compress the air for the same pressure ratio. Mathematically, isothermal efficiency or compressor efficiency,

$$\eta_c = \frac{\text{Isothermal power}}{\text{Indicated power}} = \frac{\text{Isothermal work done}}{\text{Indicated work done}}$$

We nave already discussed for a single stage reciprocating compressor that the isothermal work done

$$= 2.3\, p_1 v_1 \log\left(\frac{p_2}{p_1}\right)$$

and indicated work done by the compressor

$$= \text{Work done during polytropic compression}$$

$$= \frac{n}{n-1} \times p_1 v_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

∴ Isothermal efficiency for a single stage reciprocating compressor

$$= \frac{2.3 \log \left(\frac{p_2}{p_1} \right)}{\frac{n}{n-1} \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]}$$

2. *Overall isothermal efficiency.* It is the ratio of the isothermal power to the shaft power or brake power of the motor or engine required to drive the compressor. Mathematically, overall isothermal efficiency,

$$\eta_0 = \frac{\text{Isothermal power}}{\text{Shaft power or B.P. of motor}}$$

3. *Mechanical efficiency.* It is the ratio of the indicated power to the shaft power or brake power of the motor or engine required to drive the compressor. Mathematically, mechanical efficiency,

$$\eta_m = \frac{\text{Indicated power}}{\text{Shaft power or B.P. of motor}}$$

Note : The shaft power or brake power of the motor or engine

$$= \frac{\text{Indicated power}}{\text{Mechanical efficiency}}$$

4. *Isentropic efficiency.* It is the ratio of the isentropic power to the brake power required to drive the compressor. Mathematically, isentropic efficiency,

$$\eta_i = \frac{\text{Isentropic power}}{\text{B.P. required to drive the compressor}}$$

5. *Volumetric efficiency.* It is the ratio of volume of free air delivery per stroke to the swept volume of the piston. The volumetric efficiency of a reciprocating air compressor is different when it is with or without clearance volume.

Note : Since it is difficult to visualise the N.T.P. conditions of the swept air, therefore the general trend is to define the volumetric efficiency as the ratio of actual volume of air sucked by the compressor to the swept volume of the piston.

Example 30.1. *A reciprocating air compressor draws in 6 kg of air per minute at 25° C. It compresses the air polytropically and delivers it at 105° C. Find the air power. If the shaft power is 14 kW ; find the mechanical efficiency. Assume R = 0.287 kJ/kg K and n = 1.3.*

Solution. Given : $m = 6$ kg/min ; $T_1 = 25°\text{ C} = 25 + 273 = 298$ K ; $T_2 = 105°\text{ C} = 105 + 273 = 378$ K ; Shaft power = 14 kW ; $R = 0.287$ kJ/kg K ; $n = 1.3$

Air Power

We know that workdone by the compressor,

$$W = \frac{n}{n-1} \times mR\,(T_2 - T_1) = \frac{1.3}{1.3-1} \times 6 \times 0.287\,(378 - 298) \text{ kJ/min}$$

$$= 597 \text{ kJ/min}$$

∴ Air power $= 597/60 = 9.95$ kW **Ans.**

Mechanical efficiency

We know that mechanical efficiency,

$$\eta_m = \frac{\text{Air power}}{\text{Shaft power}} = \frac{9.95}{14} = 0.7107 \text{ or } 71.07\% \textbf{ Ans.}$$

Example 30.2. *A compressor draws 42.5 m^3 of air per minute into the cylinder at a pressure of 1.05 bar. It is compressed polytropically ($pv^{1.3} = C$) to a pressure of 4.2 bar before being delivered to a receiver. Assuming a mechanical efficiency of 80%, find ;*

1. Indicated power ; 2. Shaft power ; and 3. Overall isothermal efficiency.

Solution. Given : $v_1 = 42.5$ m³/min ; $p_1 = 1.05$ bar $= 1.05 \times 10^5$ N/m² ; $n = 1.3$; $p_2 = 4.2$ bar $= 4.2 \times 10^5$ N/m² ; $\eta_m = 80\% = 0.8$

1. *Indicated power*

We know that indicated workdone by the compressor,

$$W = \frac{n}{n-1} \times p_1 v_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{1.3}{1.3-1} \times 1.05 \times 10^5 \times 42.5 \left[\left(\frac{4.2}{1.05} \right)^{\frac{1.3-1}{1.3}} - 1 \right] \text{ J/min}$$

$$= 7287 \times 10^3 \text{ J/min} = 7287 \text{ kJ/min}$$

$\therefore$ Indicated power $= 7287/60 = 121.5$ kW **Ans.**

2. *Shaft power*

We know that shaft power,

$$= \frac{\text{Indicated power}}{\text{Mechanical efficiency}} = \frac{121.5}{0.8} = 151.87 \text{ kW } \textbf{Ans.}$$

3. *Overall isothermal efficiency*

We know that isothermal work done/min,

$$W = 2.3\, p_1 v_1 \log \left(\frac{p_2}{p_1} \right) = 2.3 \times 1.05 \times 10^5 \times 42.5 \log \left(\frac{4.2}{1.05} \right) \text{ J/min}$$

$$= 6180 \times 10^3 \text{ J/min} = 6180 \text{ kJ/min}$$

$\therefore$ Isothermal power $= 6180/60 = 103$ kW

We also know that overall isothermal efficiency,

$$\eta_0 = \frac{\text{Isothermal power}}{\text{Shaft power}} = \frac{103}{151.87} = 0.678 \text{ or } 67.8\% \textbf{ Ans.}$$

30.4. Volumetric Efficiency of a Reciprocating Air Compressor with Clearance Volume

We have already discussed in Art. 30.3, that the volumetric efficiency of a reciprocating air compressor is given by

$$\eta_v = \frac{\text{Volume of free air delivery per stroke}}{\text{Swept volume of the piston}}$$

$$= \frac{\text{Actual volume of air sucked referred to ambient conditions}}{\text{Swept volume of the piston}}$$

Now let us derive an expression for it when the air compressor has clearance volume. Consider a *p-v* diagram of a single acting reciprocating air compressor with clearance volume as shown in Fig. 30.1.

Let p_1 = Initial pressure of air (before compression),

v_1 = Initial volume of air (before compression),

T_1 = Initial temperature of air (before compression),

p_2, v_2, T_2 = Corresponding values for the final conditions (*i.e.* at the delivery point),

p_a, v_a, T_a = Corresponding values for the ambient (*i.e.* N.T.P.) conditions

v_c = Clearance volume,

v_s = Swept volume of the piston, and

n = Polytropic index.

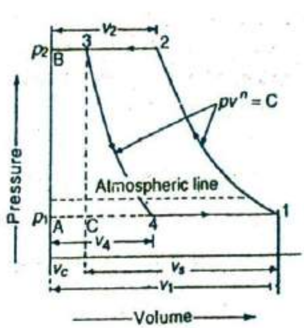


Fig. 30.1. *p-v* diagram with clearance volume.

In actual practice, the temperature at the end of suction *i.e.* at point 1 is not atmospheric because the fresh air passes over hot valves and mixes with the residual air. Also, the pressure at point 1 is not atmospheric as there are obstructions in suction of fresh air. Applying general gas equation to the atmospheric condition of air and the condition of air before compression, we have

$$\frac{p_a v_a}{T_a} = \frac{p_1 (v_1 - v_4)}{T_1}$$

∴ Volume of air sucked referred to ambient conditions,

$$v_a = \frac{p_1 T_a}{p_a T_1}(v_1 - v_4)$$

We know that volumetric efficiency,

$$\eta_v = \frac{v_a}{v_s} = \frac{p_1 T_a}{p_a T_1}\left(\frac{v_1 - v_4}{v_s}\right)$$

$$= \frac{p_1 T_a}{p_a T_1}\left(\frac{v_s + v_c - v_4}{v_s}\right) \qquad \ldots (\because v_1 = v_s + v_c)$$

$$= \frac{p_1 T_a}{p_a T_1}\left(1 + \frac{v_c}{v_s} - \frac{v_4}{v_c} \times \frac{v_c}{v_s}\right)$$

$$= \frac{p_1 T_a}{p_a T_1}\left(1 + K - K \times \frac{v_4}{v_c}\right) \qquad \ldots (i)$$

where K = Clearance ratio = v_c / v_s

Now for the polytropic expansion process 3-4,

$$p_2 v_c^n = p_1 v_4^n$$

$$\therefore \qquad \frac{v_4}{v_c} = \left(\frac{p_2}{p_1}\right)^{1/n}$$

Substituting the value of v_4 / v_c in equation (*i*), we have volumetric efficiency referred to ambient conditions,

$$\eta_v = \frac{p_1 T_a}{p_a T_1}\left[1 + K - K\left(\frac{p_2}{p_1}\right)^{1/n}\right] \quad \ldots (ii)$$

When the ambient and suction conditions are same, then $p_a = p_1$ and $T_a = T_1$. In such a case,

$$\eta_v = 1 + K - K\left(\frac{p_2}{p_1}\right)^{1/n} \quad \ldots (iii)$$

Example 30.3. *A single stage air compressor receives air at 1 bar and 27° C and delivers at 6.5 bar. The atmospheric pressure and temperature are 1.013 bar and 15° C. The compression follows the law $p v^{1.25}$ = constant and the clearance volume is 5 percent of the stroke volume. Calculate the volumetric efficiency referred to the atmospheric condition.*

Solution. Given : $p_1 = 1$ bar ; $T_1 = 27° \text{C} = 27 + 273 = 300$ K ; $p_2 = 6.5$ bar ; $p_a = 1.013$ bar ; $T_a = 15° \text{C} = 15 + 273 = 288$ K ; $n = 1.25$; $v_c = 5\%\ v_s = 0.05\ v_s$

We know that clearance ratio,

$$K = v_c / v_s = 0.05\, v_s / v_s = 0.05$$

∴ Volumetric efficiency referred to the atmospheric condition,

$$\eta_v = \frac{p_1 T_a}{p_a T_1}\left[1 + K - K\left(\frac{p_2}{p_1}\right)^{1/n}\right]$$

$$= \frac{1 \times 288}{1.013 \times 300}\left[1 + 0.05 - 0.05\left(\frac{6.5}{1}\right)^{1/1.25}\right]$$

$$= 0.783 \text{ or } 78.3\% \text{ Ans.}$$

Example 30.4. *A single stage reciprocating air compressor takes in 7.5 m³/min of air at 1 bar and 30° C and delivers it at 5 bar. The clearance is 5 percent of the stroke. The expansion and compression are polytropic, n = 1.3. Calculate : 1. the temperature of delivered air ; 2. volumetric efficiency, and 3. power of the compressor.*

Solution. Given : $v_1 - v_4 = 7.5$ m³/min ; $p_1 = 1$ bar $= 1 \times 10^5$ N/m² ; $T_1 = 30° \text{C} = 30 + 273 = 303$ K ; $p_2 = 5$ bar $= 5 \times 10^5$ N/m² ; $v_c = 5\%\ v_s = 0.05\ v_s$; $n = 1.3$

1. *Temperature of delivered air*

Let T_2 = Temperature of the delivered air.

We know that $$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{5}{1}\right)^{\frac{1.3-1}{1.3}} = 1.45$$

∴ $$T_2 = T_1 \times 1.45 = 303 \times 1.45 = 439.3 \text{ K} = 166.3° \text{C Ans.}$$

2. *Volumetric efficiency*

We know that clearance ratio,

$$K = \frac{v_c}{v_s} = \frac{0.05\, v_s}{v_s} = 0.05$$

∴ Volumetric efficiency,

$$\eta_v = 1 + K - K\left(\frac{p_2}{p_1}\right)^{1/n} = 1 + 0.05 - 0.05\left(\frac{5}{1}\right)^{1/1.3}$$

$$= 1.05 - 0.172 = 0.878 \text{ or } 87.8\% \textbf{ Ans.}$$

3. *Power of the compressor*

We know that workdone by the compressor,

$$W = \frac{n}{n-1} \times p_1 (v_1 - v_4)\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{1.3}{1.3-1} \times 1 \times 10^5 \times 7.5\left[\left(\frac{5}{1}\right)^{\frac{1.3-1}{1.3}} - 1\right] \text{ J/min}$$

$$= 1462.5 \times 10^3 \text{ J/min} = 1462.5 \text{ kJ/min}$$

∴ Power of the compressor

$$= 1462.5/60 = 24.4 \text{ kW} \textbf{ Ans.}$$

Example 30.5. *A single stage single acting reciprocating air compressor is required to handle 30 m^3 of free air per hour measured at 1 bar. The delivery pressure is 6.5 bar and the speed is 450 r.p.m.*

Allowing a volumetric efficiency of 75% ; an isothermal efficiency of 76% and a mechanical efficiency of 80% ; Calculate the indicated mean effective pressure and the power required to drive the compressor.

Solution. Given : $v_1 = 30$ m³/h ; $p_1 = 1$ bar $= 1 \times 10^5$ N/m² ; $p_2 = 6.5$ bar $= 6.5 \times 10^5$ N/m² ; $N = 450$ r.p.m. ; $\eta_v = 75\% = 0.75$; $\eta_i = 76\% = 0.76$; $\eta_m = 80\% = 0.8$

Indicated mean effective pressure

We know that isothermal work done

$$= 2.3\, p_1 v_1 \log\left(\frac{p_2}{p_1}\right) = 2.3 \times 1 \times 10^5 \times 30 \log\left(\frac{6.5}{1}\right) \text{ J/h}$$

$$= 5609 \times 10^3 \text{ J/h} = 5609 \text{ kJ/h}$$

and indicated workdone $= \dfrac{\text{Isothermal work done}}{\text{Isothermal efficiency}} = \dfrac{5609}{0.76} = 7380$ kJ/h

We know that swept volume of the piston,

$$v_s = \frac{\text{Volume of free air}}{\text{Volumetric efficiency}} = \frac{30}{0.75} = 40 \text{ m}^3\text{/h}$$

∴ Indicated mean effective pressure,

$$p_m = \frac{\text{Indicated work done}}{\text{Swept volume}} = \frac{7380}{40} = 184.5 \text{ kJ/m}^3$$

$$= 184.5 \text{ kN/m}^2 \qquad \ldots\left(\because \frac{1 \text{ kJ}}{\text{m}^3} = \frac{1 \text{ kN-m}}{\text{m}^3} = 1 \text{ kN/m}^2\right)$$

$$= 1.845 \text{ bar} \textbf{ Ans.}$$

Power required to drive the compressor

We know that work done by the compressor

$$= \frac{\text{Indicated work done}}{\text{Mechanical efficiency}} = \frac{7380}{0.8} = 9225 \text{ kJ/h}$$

∴ Power required to drive the compressor

$$= \frac{9225}{3600} = 2.56 \text{ kW} \textbf{ Ans.}$$

Example 30.6. *A single stage double acting air compressor delivers 3 m³ of free air per minute at 1.013 bar and 20° C to 8 bar with the following data ;*

R.P.M. = 300 ; Mechanical efficiency = 0.9 ; Pressure loss in passing through intake valves = 0.04 bar ; Temperature rise of air during suction stroke = 12° C ; Clearance volume = 5% of stroke volume ; Index of compression and expansion, n = 1.35 ; Length of the stroke = 1.2 times the cylinder diameter.

Calculate : 1. power input to the shaft ; 2. the volumetric efficiency ; and 3. the cylinder diameter.

Solution. Given : $v_a = 3$ m³/min ; $p_a = 1.013$ bar ; $T_a = 20°$ C $= 20 + 273 = 293$ K ; $p_2 = 8$ bar ; $N = 300$ r.p.m. ; $\eta_m = 0.9$; Pressure loss = 0.04 bar ; Temperature rise = 12° C ; $v_c = 5\%\ v_s = 0.05\ v_s$; $n = 1.35$; $L = 1.2\ D$

Let v_1 = Volume of free air at the suction conditions.

Since there is a pressure loss of 0.04 bar in passing through intake valves, therefore suction pressure,

$$p_1 = p_a - 0.04 = 1.013 - 0.04 = 0.973 \text{ bar} = 0.973 \times 10^5 \text{ N/m}^2$$

Also there is a temperature rise of air of 12° C during suction stroke, therefore temperature of air at the beginning of compression,

$$T_1 = T_a + 12 = 20 + 12 = 32° \text{C} = 32 + 273 = 305 \text{ K}$$

We know that $\frac{p_a v_a}{T_a} = \frac{p_1 v_1}{T_1}$

$$\therefore \quad v_1 = \frac{p_a v_a T_1}{p_1 T_a} = \frac{1.013 \times 3 \times 305}{0.973 \times 293} = 3.25 \text{ m}^3/\text{min}$$

1. *Power input to the shaft*

We know that indicated workdone

$$= \frac{n}{n-1} \times p_1 v_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{1.35}{1.35 - 1} \times 0.973 \times 10^5 \times 3.25 \left[\left(\frac{8}{0.973} \right)^{\frac{1.35-1}{1.35}} - 1 \right] \text{J/min}$$

$$= 885 \times 10^3 \text{ J/min} = 885 \text{ kJ/min}$$

and indicated power $= 885/60 = 14.75$ kW

We know that power input to the shaft

$$= \frac{\text{Indicated power}}{\eta_m} = \frac{14.75}{0.9} = 16.4 \text{ kW } \textbf{Ans.}$$

2. *Volumetric efficiency*

We know that clearance ratio,

$$K = v_c / v_s = 0.05\, v_s / v_s = 0.05$$

$\therefore$ Volumetric efficiency,

$$\eta_v = 1 + K - K\left(\frac{p_2}{p_1}\right)^{1/n} = 1 + 0.05 - 0.05\left(\frac{8}{0.973}\right)^{1/1.35}$$

$$= 0.812 \text{ or } 81.2\% \textbf{ Ans.}$$

3. *Cylinder diameter*

Let D = Cylinder diameter, and

L = Stroke length = $1.2\, D$ ...(Given)

We know that swept volume per stroke

$$= \frac{\pi}{4} \times D^2 \times L = \frac{\pi}{4} \times D^2 \times 1.2\, D = 0.9426\, D^3$$

Since the compressor is double acting, therefore number of working strokes per minute

$$= 2\,N = 2 \times 300 = 600$$

and swept volume per minute

$$v_s = 0.9426\, D^3 \times 600 = 565.56\, D^3$$

We know that volumetric efficiency (η_v),

$$0.812 = \frac{v_1}{v_s} = \frac{3.25}{565.56\, D^3}$$

$\therefore$ $D^3 = 0.00707$ or $D = 0.192 \text{ m} = 192 \text{ mm}$ **Ans.**

and $L = 1.2\, D = 1.2 \times 192 = 230.4 \text{ mm}$ **Ans.**

Example 30.7. *A single acting two-stage air compressor deals with air measured at atmospheric conditions of 1.013 bar and 15° C. At suction, the pressure is 1 bar and the temperature is 30° C. The final delivery pressure is 17 bar, the interstage pressure is 4 bar and perfect intercooling is to be assumed. If the L.P. cylinder bore is 230 mm, the common stroke is 150 mm and the speed of the compressor is 350 r.p.m. ; calculate 1. the volumetric efficiency of the compressor ; 2. the volume of atmospheric air dealt with per minute ; and 3. the power of the driving motor required. Assume the clearance volume of L.P. cylinder to be 5% and the indices of compression and expansion in the L.P. and H.P. cylinder to be 1.25 ; the mechanical efficiency being 85%.*

Solution. Given : $p_a = 1.013$ bar ; $T_a = 15°\text{C} = 15 + 273 = 288$ K ; $p_1 = 1 \text{ bar} = 1 \times 10^5 \text{ N/m}^2$; $T_1 = 30°\text{ C} = 30 + 273 = 303$ K ; $p_3 = 17$ bar ; $p_2 = 4$ bar ; $D_1 = 230 \text{ mm} = 0.23$ m ; $L = 150$ mm $= 0.15$ m ; $N = 350$ r.p.m. ; $K = v_{c1} / v_{s1} = 5\% = 0.05$; $n = 1.25$; $\eta_m = 85\% = 0.85$

1. *Volumetric efficiency of the compressor*

We know that volumetric efficiency of the compressor,

$$\eta_v = \frac{p_1 T_a}{p_a T_1}\left[1 + K - K\left(\frac{p_2}{p_1}\right)^{1/n}\right]$$

$$= \frac{1 \times 288}{1.013 \times 303}\left[1 + 0.05 - 0.05\left(\frac{4}{1}\right)^{1/1.25}\right]$$

$$= 0.843 \text{ or } 84.3\% \textbf{ Ans.}$$

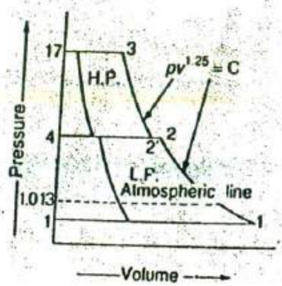


Fig. 30.2

2. *Volume of atmospheric air dealt with per minute*

We know that swept volume of the L.P. cylinder per minute,

$$v_s = \text{Swept volume per stroke} \times \text{No. of working strokes/min}$$

$$= \frac{\pi}{4}(D_1)^2 L \times N_w$$

$$= \frac{\pi}{4}(0.23)^2\, 0.15 \times 350 = 2.18 \text{ m}^3\text{/min} \qquad \ldots [\because \text{For single acting, } N_w = N]$$

∴ Volume of atmospheric air dealt with per minute,

$$v_a = v_s \times \eta_v = 2.18 \times 0.843 = 1.838 \text{ m}^3\text{/min } \textbf{Ans.} \qquad \ldots (\because \eta_v = v_a / v_s)$$

3. *Power of driving motor required*

Let T_2 = Temperature of air leaving the L.P. cylinder or entering the intercooler,

T_2' = Temperature of air leaving the intercooler or entering the H.P. cylinder, and

T_3 = Temperature of air delivered by the H.P. cylinder

= T_1, for perfect intercooling

We know that for polytropic compression 1-2 in the L.P. cylinder,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{4}{1}\right)^{\frac{1.25-1}{1.25}} = 1.3195$$

$$\therefore \quad T_2 = T_1 \times 1.3195 = 303 \times 1.3195 = 400 \text{ K}$$

Similarly, for polytropic compression 2′-3 in the H.P. cylinder,

$$\frac{T_3}{T_2'} = \left(\frac{p_3}{p_2'}\right)^{\frac{n-1}{n}} = \left(\frac{17}{4}\right)^{\frac{1.25-1}{1.25}} = 1.3356 \qquad \ldots (\because p_2' = p_2)$$

$$\therefore \quad T_3 = T_2' \times 1.3356 = 303 \times 1.3356 = 405 \text{ K} \qquad \ldots (\because T_2' = T_1)$$

We know that mass of air dealt with per minute,

$$m = \frac{p_a v_a}{R T_a} = \frac{1.013 \times 10^5 \times 1.838}{287 \times 288} = 2.25 \text{ kg/min}$$

∴ Indicated workdone by L.P. compressor,

$$W_L = \frac{n}{n-1} \times mR\,(T_2 - T_1) = \frac{1.25}{1.25-1} \times 2.25 \times 287\,(400 - 303) \text{ J/min}$$

$$= 313\,190 \text{ J/min} = 313.19 \text{ kJ/min}$$

and indicated workdone by H.P. compressor,

$$W_H = \frac{n}{n-1} \times mR\,(T_3 - T_2') = \frac{1.25}{1.25-1} \times 2.25 \times 287\,(405 - 303) \text{ J/min}$$

$$= 329\,330 \text{ J/min} = 329.33 \text{ kJ/min} \qquad \ldots (T_2' = T_1)$$

We know that total indicated work done,

$$W = W_L + W_H = 313.19 + 329.33 = 642.52 \text{ kJ/min}$$

∴ Power of the driving motor required,

$$P = 642.52 / 60 = 10.7 \text{ kW} \quad \textbf{Ans.}$$

Note : The total workdone (W) by a two-stage air compressor with perfect intercooling may also be determined by using the relation,

$$W = \frac{n}{n-1} \times mRT_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2} \right)^{\frac{n-1}{n}} - 2 \right]$$

Example 30.8. *A two stage, single acting air compressor compresses air to 20 bar. The air enters the L.P. cylinder at 1 bar and 27° C and leaves it at 4.7 bar. The air enters the H.P. cylinder at 4.5 bar and 27° C. The size of L.P. cylinder is 400 mm diameter and 500 mm stroke. The clearance volume in both cylinders is 4% of the respective stroke volume. The compressor runs at 200 r.p.m. Taking index of compression and expansion in the two cylinders as 1.3, estimate 1. the indicated power required to run the compressor ; and 2. the heat rejected in the intercooler per minute.*

Solution. Given : $p_4 = 20$ bar ; $p_1 = 1$ bar $= 1 \times 10^5$ N/m^2 ; $T_1 = 27°$ C $= 27 + 273 = 300$ K ; $p_2 = 4.7$ bar ; $p_3 = 4.5$ bar $= 4.5 \times 10^5$ N/m^2 ; $T_3 = 27°$ C $= 27 + 273 = 300$ K ; $D_1 = 400$ mm $= 0.4$ m ; $L_1 = 500$ mm $= 0.5$ m ; $K = v_{c1} / v_{s1} = v_{c3} / v_{s3} = 4\% = 0.04$; $N = 200$ r.p.m. ; $n = 1.3$

1. *Indicated power required to run the compressor*

We know that swept volume of L.P. cylinder,

$$v_{s1} = \frac{\pi}{4} (D_1)^2 L_1 = \frac{\pi}{4} (0.4)^2\, 0.5 \text{ m}^3$$

$$= 0.062\,84 \text{ m}^3$$

and volumetric efficiency,

$$\eta_v = 1 + K - K \left(\frac{p_2}{p_1} \right)^{1/n}$$

$$= 1 + 0.04 - 0.04 \left(\frac{4.7}{1} \right)^{1/1.3}$$

$$= 0.9085 \text{ or } 90.85\,\%$$

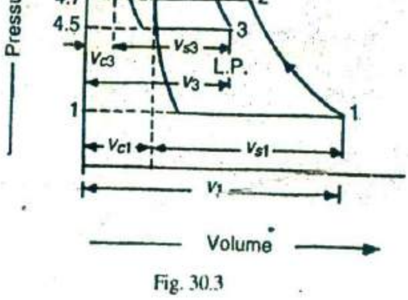


Fig. 30.3

∴ Volume of air sucked by L.P. compressor,

$$v_1 = v_{s1} \times \eta_v = 0.062\,84 \times 0.9085 = 0.0571 \text{ m}^3\text{/stroke}$$

$$= 0.0571 \times N_w = 0.0571 \times 200 = 11.42 \text{ m}^3\text{/min}$$

. . . ($\because$ For single acting, no. of working strokes per min, $N_w = N = 200$)

and volume of air sucked by H.P compressor,

$$v_3 = \frac{p_1 v_1}{p_3} = \frac{1 \times 11.42}{4.5} = 2.54 \text{ m}^3\text{/min} \qquad \ldots (\because p_1 v_1 = p_3 v_3)$$

We know that indicated workdone by L.P. compressor,

$$W_L = \frac{n}{n-1} \times p_1 v_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{1.3}{1.3-1} \times 1 \times 10^5 \times 11.42 \left[\left(\frac{4.7}{1} \right)^{\frac{1.3-1}{1.3}} - 1 \right] \text{ J/min}$$

$$= 2123.3 \times 10^3 \text{ J/min} = 2123.3 \text{ kJ/min}$$

and indicated workdone by H.P. compressor,

$$W_H = \frac{n}{n-1} \times p_3 v_3 \left[\left(\frac{p_4}{p_3} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{1.3}{1.3-1} \times 4.5 \times 10^5 \times 2.54 \left[\left(\frac{20}{4.5} \right)^{\frac{1.3-1}{1.3}} - 1 \right] \text{ J/min}$$

$$= 2034.5 \times 10^3 \text{ J/min} = 2034.5 \text{ kJ/min}$$

Total indicated workdone by the compressor,

$$W = W_L + W_H = 2123.3 + 2034.5 = 4157.8 \text{ kJ/min}$$

∴ Indicated power required to run the compressor

$$= 4157.8 / 60 = 69.3 \text{ kW} \quad \textbf{Ans.}$$

2. ***Heat rejected in the intercooler per minute***

Let T_2 = Temperature of air after compression in the L.P. cylinder.

We know that mass of air dealt for compression in the L.P. cylinder,

$$m = \frac{p_1 v_1}{R\,T_1} = \frac{1 \times 10^5 \times 11.42}{287 \times 300} = 13.26 \text{ kg/min}$$

. . . ($\because$ R for air = 287 J/kg K)

and

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} = \left(\frac{4.7}{1} \right)^{\frac{1.3-1}{1.3}} = 1.429$$

∴

$$T_2 = T_1 \times 1.429 = 300 \times 1.429 = 428.7 \text{ K}$$

We know that heat rejected in the intercooler

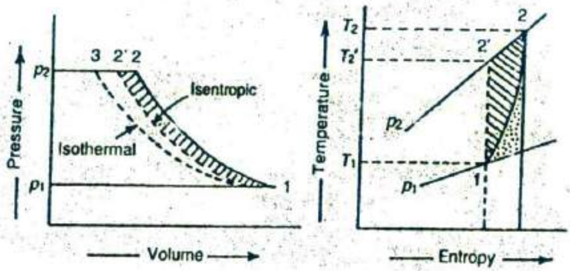
$$= m\,c_p\,(T_2 - T_1)$$

$$= 13.26 \times 1\,(428.7 - 300) = 1706.6 \text{ kJ/min} \quad \textbf{Ans.}$$

$$\ldots (\because\ c_p \text{ for air} = 1 \text{ kJ/kg K})$$

30.5. Thermodynamic Cycle for a Rotary Air Compressor

As a matter of fact, the ideal compression in a rotary compressor is isentropic (*i.e.* frictionless adiabatic) which is shown by the graph 1-2′ in Fig. 30.4 (*a*) and (*b*).



(*a*) *p-v* diagram. (*b*) *T-s* diagram.

Fig. 30.4. Thermodynamic cycle for a rotary air compressor.

But in actual practice, there is always some friction among the air molecules as well as between the air and the compressor casing. Moreover, there is always some shock at the entry and exit of air. It results in the formation of eddies at the entry and exit of the air. The above factors cause an increase in the temperature of the air at the exit without increasing its pressure. As a result of this, the temperature of air coming out of the compressor is more than that if it would have been compressed isentropically. A little consideration will show, that increase in the air temperature causes increase in its volume. Thus the amount of work done is also increased.

In Fig. 30.4 (*a*) and (*b*), the graph 1-2′ shows the ideal isentropic compression from pressure p_1 to p_2 (with an increase in temperature from T_1 to T_2'). The graph 1-2 shows the actual polytropic process (*i.e.* pv^n = constant).

Note : In actual polytropic process, the value of index n is greater than γ (about 1.7).

30.6. Efficiencies of a Centrifugal Air Compressor

We have already discussed in Art. 30.2 that the criterion for thermodynamic efficiency of a centrifugal air compressor is isentropic. But in general, the following efficiencies of a centrifugal air compressor are important from the subject point of view :

1. *Isentropic efficiency (or compressor efficiency).* It is the ratio of work (or power) required to compress the air isentropically to the actual work required to compress the air for the same pressure ratio. Mathematically, isentropic efficiency,

$$\eta_i = \frac{h_2' - h_1}{h_2 - h_1} = \frac{T_2' - T_1}{T_2 - T_1}$$

where h_2' = Enthalpy of air at exit for isentropic compression,

h_2 = Enthalpy of air at exit for actual compression,

h_1 = Enthalpy of air at inlet, and

T_2', T_2, T_1 = Temperatures at corresponding points.

2. *Polytropic efficiency.* It is the ratio of work (or power) required to compress the air polytropically to the actual work required to compress the air for the same pressure ratio. Mathematically, polytropic efficiency,

$$\eta_p = \left(\frac{\gamma - 1}{\gamma}\right)\left(\frac{n}{n-1}\right)$$

where γ = Ratio of specific heats, and

n = Polytropic index.

Note : The value of an n is always greater than γ.

Example 30.9. *A centrifugal compressor delivers 0.5 kg of air per second at a pressure of 1.8 bar and 100° C. The intake conditions are 20° C and 1 bar. Find the isentropic efficiency of the compressor and the power required to drive it. Take n = 1.65 and c_p = 1 kJ/kg K.*

Solution. Given : $m = 0.5$ kg/s ; $p_2 = 1.8$ bar ; $T_2 = 100°$ C = 100 + 273 = 373 K ; $T_1 = 20°$ C = 20 + 273 = 293 K ; $p_1 = 1$ bar ; $n = 1.65$; $c_p = 1$ kJ/kg K

Isentropic efficiency

Let T_2' = Temperature of air at exit for isentropic compression.

We know that $$\frac{T_2'}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{1.8}{1}\right)^{\frac{1.65-1}{1.65}} = (1.8)^{0.391} = 1.261$$

$\therefore$ $$T_2' = T_1 \times 1.261 = 293 \times 1.261 = 369 \text{ K}$$

We know that isentropic efficiency,

$$\eta_i = \frac{T_2' - T_1}{T_2 - T_1} = \frac{369 - 293}{373 - 293} = 0.95 \quad \text{or} \quad 95\% \text{ Ans.}$$

Power required to drive the compressor

We know that work done in compressing the air isentropically,

$$W = m\, c_p (T_2 - T_1) = 0.5 \times 1\,(373 - 293) = 40 \text{ kJ/s}$$

$\therefore$ Power required to drive the compressor,

$$= 40 \text{ kW Ans.} \qquad \ldots (\because 1 \text{ kJ/s} = 1 \text{ kW})$$

Example 30.10. *A centrifugal compressor with 70% isentropic efficiency delivers 20 kg of air per minute at a pressure of 3 bar. If the compressor receives air at 20° C and at a pressure of 1 bar, find the actual temperature of the air at exit. Also find the power required to run the compressor, if its mechanical efficiency is 95%. Take γ and c_p for air as 1.4 and 1 kJ/kg K respectively.*

Solution. Given : $\eta_i = 70\% = 0.7$; $m = 20$ kg/min ; $p_2 = 3$ bar ; $T_1 = 20°$ C = 20 + 273 = 293 K ; $p_1 = 1$ bar ; $\eta_m = 95\% = 0.95$; $\gamma = 1.4$; $c_p = 1$ kJ/kg K

Actual temperature of the air at exit

Let T_2 = Actual temperature of the air at exit, and

T_2' = Temperature of the air at exit for isentropic compression.

We know that $\frac{T_2'}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{3}{1}\right)^{\frac{1.4-1}{1.4}} = (3)^{0.286} = 1.369$

$\therefore \quad T_2' = T_1 \times 1.369 = 293 \times 1.369 = 401.1 \text{ K}$

We also know that isentropic efficiency (η_i),

$$0.7 = \frac{T_2' - T_1}{T_2 - T_1} = \frac{401.1 - 293}{T_2 - 293}$$

$$0.7\, T_2 - 205.1 = 401.1 - 293 = 108.1$$

$\therefore \quad T_2 = 447.4 \text{ K} = 174.4°\text{C}$ **Ans.**

Power required to run the compressor

We know that work done in compressing the air isentropically,

$$W = m\, c_p (T_2 - T_1) = 20 \times 1\,(447.4 - 293) = 3088 \text{ kJ/min}$$

$$= 51.47 \text{ kJ/s}$$

$\therefore$ Power required to run the compressor

$$= \frac{51.47}{\eta_m} = \frac{51.47}{0.95} = 54.25 \text{ kW} \quad \textbf{Ans.}$$

Example 30.11. *A centrifugal compressor having compression ratio of 2.4 compresses the air polytropically according to law $pv^{1.6}$ = constant. Find the polytropic efficiency of the compressor, if c_p = 0.995 kJ/kg K and c_v = 0.71 kJ/kg K.*

Solution. Given : *$p_2 / p_1 = 2.4$; $n = 1.6$; $c_p = 0.995$ kJ/kg K ; $c_v = 0.71$ kJ/kg K

We know that ratio of specific heats,

$$\gamma = c_p / c_v = 0.995 / 0.71 = 1.4$$

$\therefore$ Polytropic efficiency of the compressor,

$$\eta_p = \left(\frac{\gamma - 1}{\gamma}\right)\left(\frac{n}{n-1}\right) = \frac{1.4 - 1}{1.4} \times \frac{1.6}{1.6 - 1} = 0.762 \text{ or } 76.2\% \quad \textbf{Ans.}$$

30.7. Static and Total Head Quantities

As a matter of fact, the velocities encountered in a centrifugal compressor are very large as compared with those of a reciprocating air compressor. It is, therefore, very essential to take into consideration the velocities for the analysis purpose.

Now consider a horizontal passage of varying area through which 1 kg of air is flowing as shown in Fig. 30.5.

Let T_1 = Temperature at section 1 in K,

h_1 = Enthalpy at section 1 in kJ/kg,

p_1 = Pressure of air at section 1 in bar,

V_1 = Velocity of air at section 1 in m/s, and

T_2, h_2, p_2, V_2 = Corresponding values at section 2.

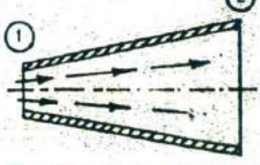


Fig. 30.5. Static and total head quantities.

* Superfluous data

Now let us assume a steady flow of air from section 1 to 2, so that no heat is transferred as well as no work is done, as the air flows through the passage. Applying the steady flow energy equation to the system,

$$h_1 + \frac{10^5 p_1 v_1}{1000} + \frac{V_1^2}{2000} = h_2 + \frac{10^5 p_2 v_2}{1000} + \frac{V_2^2}{2000}$$

$$c_p T_1 + \frac{V_1^2}{2000} = c_p T_2 + \frac{V_2^2}{2000} \quad \ldots (\text{Assuming } p_1 v_1 = p_2 v_2)$$

or in other words,

$$c_p T + \frac{V^2}{2000} = \text{Constant}$$

In the above general expression, the temperature (T) stands for the actual temperature of the air recorded by a thermometer which is also moving in the air with the same velocity as that of air. A little consideration will show, that if the moving air is brought to rest under reversible adiabatic conditions, the total kinetic energy will be converted into heat energy, which will increase its temperature and pressure. The new temperature and pressure of the air are called total heat or stagnation temperature (T_0) and pressure (p_0) respectively.

$$\therefore \quad c_p T + \frac{V^2}{2000} = c_p T_0 \quad \text{or} \quad T_0 - T = \frac{V^2}{2000\, c_p}$$

and

$$h_0 - h = \frac{V^2}{2000} \quad \ldots (\because h = c_p T)$$

where h_0 is the stagnation enthalpy.

The total head pressure may be obtained from the equation,

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}} \quad \text{or} \quad \frac{T_0}{T} = \left(\frac{p_0}{p}\right)^{\frac{\gamma-1}{\gamma}}$$

where γ is the usual ratio of specific heats.

Notes : 1. The term p_0 / p is called static pressure ratio.

2. This relation may be used for any two sections in a centrifugal compressor also.

Example 30.12. *A centrifugal air compressor having isentropic efficiency of 70% receives air at 17° C. If the outer diameter of the blade tip is 1 m and the compressor runs at 5000 r.p.m., find : 1. the temperature rise of the air ; and 2. the static pressure ratio.*

Solution. Given : $\eta_i = 70\% = 0.7$; $T_1 = 17°\text{C} = 17 + 273 = 290$ K ; $D = 1$ m ; $N = 5000$ r.p.m.

We know that blade velocity,

$$V_b = \frac{\pi DN}{60} = \frac{\pi \times 1 \times 5000}{60} = 261.8 \text{ m/s}$$

Temperature rise of the air

Let $(T_2 - T_1)$ = Temperature rise of the air.

We know that work done by the compressor per kg of air,

$$w = \frac{V_b^2}{1000} = \frac{(261.8)^2}{1000} = 68.5 \text{ kJ}$$

We also know that work done by the compressor per kg of air (w),

$$68.5 = c_p (T_2 - T_1) = 1 (T_2 - T_1) \quad \ldots (\text{For air}, c_p = 1 \text{ kJ/kg K})$$

$$\therefore \quad (T_2 - T_1) = 68.5° \text{C or K} \textbf{ Ans.}$$

Static pressure ratio

Let $\frac{p_2}{p_1}$ = Static pressure ratio, and

T_2' = Temperature of air at exit for isentropic compression.

We know that isentropic efficiency (η_i),

$$0.7 = \frac{T_2' - T_1}{T_2 - T_1} = \frac{T_2' - 290}{68.5}$$

$$\therefore \quad T_2' = (0.7 \times 68.5) + 290 = 337.95 \text{ K}$$

and

$$\frac{p_2}{p_1} = \left(\frac{T_2'}{T_1}\right)^{\frac{\gamma}{\gamma - 1}} = \left(\frac{337.95}{290}\right)^{\frac{1.4}{1.4-1}} = (1.165)^{3.5} = 1.71 \textbf{ Ans.}$$

30.8. Slip Factor

We have already discussed in Art. 29.9 that the ideal or maximum work done by a centrifugal rotary compressor

$$= m (V_{w1})^2 = m (V_b)^2$$

The above relation has been derived under the assumption that $V_{w1} = V_b$. But in actual practice, V_{w1} is a always less than V_b. The difference between V_b and V_{w1} (*i.e.* $V_b - V_{w1}$) is known as slip and the ratio of V_{w1} to V_b (*i.e.* V_{w1}/V_b) is known as *slip factor.*

30.9. Comparison of Turbine and Centrifugal Compressor Blades

Following are the main points of comparison of the turbine and centrifugal compressor blades.

S. No.	*Turbine blades*	*Centrifugal compressor blades*
1.	Passage between the blades is converging.	Passage between the blades is diverging.
2.	Due to converging passage, the flow gets accelerated. But the pressure decreases.	Due to diverging passage, the flow gets diffused or decelerated. But the pressure increases.
3.	The flow is more stable.	The flow is less stable.
4.	The flow always takes place in one direction only.	Sometimes, the flow breaks away and reverses its direction.
5.	The blades are simple in design and construction, as their profile consists of circular arc and straight line.	The blades are complicated in design and construction, as their profile consists of aerofile section based on aerodynamic theory.

EXERCISES

1. A single stage reciprocating air compressor takes in air at 1 bar and 15° C. The conditions at the end of suction are 0.97 bar and 30° C. The discharge pressure is 6 bar. The clearance is 5% of the stroke. The compression and expansion follows $pv^{1.3}$ = Constant. Find the volumetric efficiency of the compressor.

[Ans. 78.1%]

2. A single acting single stage reciprocating air compressor with 5% clearance volume compresses air from 1 bar to 5 bar. Find the change in volumetric efficiency of the compressor, if the exponents of expansion process change from 1.25 to 1.4. **[Ans. 2.5%]**

3. Air is compressed by a reciprocating compressor from 1.05 bar and 27° C to 7.9 bar. During the suction and discharge, there are inlet and outlet pressure losses at the valves of 0.05 bar and 0.1 bar respectively and the atmospheric air is heated up after induction to 37° C. Determine the volumetric efficiency of the compressor. Assume law of compression and expansion to be the same, $pv^{1.3}$ = Constant and percentage of clearance volume 4%. **[Ans. 84.2%]**

4. A compressor has 150 mm bore and 200 mm stroke and the linear clearance is 10 mm. Calculate the theoretical volume of air taken in m^3 per stroke when working between 1 bar and 7 bar. Take $n = 1.25$.

[Ans. 2.87×10^{-3} m^3**]**

5. A compressor is used to compress air from a pressure of 1.013 bar to 7.21 bar. The polytropic exponent for both compression and expansion is $n = 1.35$. The clearance volume of the compressor is 200×10^{-6} m^3. If the volumetric efficiency of the compressor is 80 percent and the stroke is 250 mm, determine the cylinder diameter of the compressor. **[Ans.** 129.2 mm**]**

6. A single stage reciprocating air compressor takes in air at 1 bar and 15° C. The conditions at the end of suction are 0.97 bar and 30° C. The discharge pressure is 6 bar. The clearance is 5 percent of stroke. The compression and expansion follow $p\,v^{1.3}$ = Constant and the mass of air handled is 1 kg/min. Estimate the stroke volume and power needed in kW. Assume compressor speed as 1000 r.p.m. **[Ans.** 1.147×10^{-3} m^3 ; 3.28 kW**]**

7. A single stage, double acting air compressor delivers 15 m^3 of air per minute measured at 1.013 bar and temperature 300 K and delivers at 7 bar. The conditions at the end of suction stroke are pressure 0.98 bar and temperature 313 K. The clearance volume is 4 percent of the swept volume and the stroke/bore ratio is 1.3/1. The compressor runs at 300 r.p.m. Calculate 1. volumetric efficiency ; 2. cylinder dimensions ; 3. indicated power ; and 4. isothermal efficiency of this compressor. Take the index of compression and expansion as 1.3.

[Ans. 79.6% ; 313 mm, 407 mm ; 65.8 kW ; 78.8%**]**

8. A single acting two-stage air compressor deals with 4 m^3/min of air under atmospheric conditions of 1.013 bar and 15° C with a speed of 250 r.p.m. The delivery pressure is 80 bar. Assuming complete intercooling, find the minimum power required by the compressor and the bore and stroke of the compressor. Assume a piston speed of 3 m/s, mechanical efficiency of 75 percent, and volumetric efficiency of 80 percent per stage. Assume the polytropic index of compression in both the stages to be $n = 1.25$ and neglect clearance. **[Ans.** 50 kW**]**

9. A single stage double acting air compressor delivers air at 7 bar. The pressure and temperature at the end of suction stroke are 1 bar and 27° C. It delivers 2 m^3 of free air per minute when the compressor is running at 300 r.p.m. The clearance volume is 5 percent of the stroke volume. The pressure and temperature of ambient air are 1.013 bar and 20° C. The index of compression is 1.3 and index of expansion is 1.35. Find : 1. Volumetric efficiency of the compressor ; and 2. Diameter and stroke of the cylinder if both are equal.

[Ans. 83,8% ; 175 mm**]**

QUESTIONS

1. Explain the following terms :

   (*a*) Isothermal efficiency ; (*b*) Isentropic efficiency ; and (*c*) Volumetric efficiency.

2. Discuss the effect of clearance volume on the volumetric efficiency of a reciprocating air compressor.

3. Prove that the volumetric efficiency of a compressor is given by

$$\frac{p_1\,T_a}{p_a\,T_1}\left[1 + K - K\left(\frac{p_2}{p_1}\right)^{1/n}\right]$$

where suffix a and 1 represent ambient and before compression conditions respectively and K is the ratio of clearance volume to the swept volume.

4. Describe 'thermodynamic cycle for a rotary air compressor'.
5. Define static and total head quantities.
6. What do you understand by the term 'slip factor'?

OBJECTIVE TYPE QUESTIONS

1. The ratio of the indicated power to the shaft power or brake power of the motor or engine required to drive the compressor, is called

(*a*) compressor efficiency (*b*) volumetric efficiency

(*c*) isentropic efficiency (*d*) mechanical efficiency

2. The ratio of the volume of free air delivery per stroke to the swept volume of the piston, is known as

(*a*) compressor efficiency (*b*) volumetric efficiency

(*c*) isentropic efficiency (*d*) mechanical efficiency

3. If the clearance ratio for a reciprocating air compressor is K, then its volumetric efficiency is given by

(*a*) $1 - K + K\left(\frac{p_1}{p_2}\right)^{1/n}$ (*b*) $1 + K - K\left(\frac{p_1}{p_2}\right)^{1/n}$

(*c*) $1 + K - K\left(\frac{p_2}{p_1}\right)^{1/n}$ (*d*) $1 - K + K\left(\frac{p_2}{p_1}\right)^{1/n}$

4. The volumetric efficiency of a compressor

(*a*) increases with decrease in compression ratio

(*b*) decreases with decrease in compression ratio

(*c*) increases with increase in compression ratio

(*d*) decreases with increase in compression ratio

5. The volumetric efficiency for reciprocating air compressors is about

(*a*) 10 to 40% (*b*) 40 to 60% (*c*) 60 to 70% (*d*) 70 to 90%

ANSWERS

1. (*d*) 2. (*b*) 3. (*c*) 4. (*d*) 5. (*d*)

31

Air Motors

1. Introduction. 2. Workdone by Air in an Air Motor. 3. Combined Air Motor and Air Compressor (Compressed Air System). 4. Efficiency of Compressed Air System. 5. Preheating of Compressed Air.

31.1. Introduction

In the last three chapters, we have discussed air compressors and their performance. Though the air compressors are used for innumerable purposes these days in various fields, yet one of their uses is in air motors attached to portable tools. An air motor is used as an alternative to an electric motor, especially when sparks from the electric motor (or cables) might prove dangerous *e.g.* in explosive factories and mines.

The operation of a reciprocating air motor is similar to that of a reciprocating steam engine ; but reverse to that of reciprocating air compressor.

31.2. Workdone by Air in an Air Motor

As a matter of fact, the compressed air (from an air compressor) is made to enter the cylinder of an air motor which pushes its piston forward in the same way as of a reciprocating steam engine. Now the actual work is done by the movement of the piston. Now consider an air motor working with the help of compressed air.

Let p_1 = Pressure of the compressed air, and

v_1 = Volume of the compressed air.

The theoretical indicator diagram of a reciprocating air motor without clearance, compression and pressure drop at release is shown in Fig. 31.1.

The compressed air from the compressor is admitted into an air motor at A with pressure p_1. It drives the piston forward. But after a part stroke is performed, the air supply is cut-off at B and the expansion occurs from B to C. After the stroke is completed, the air which has done some work is exhausted into the atmosphere at a constant pressure p_2.

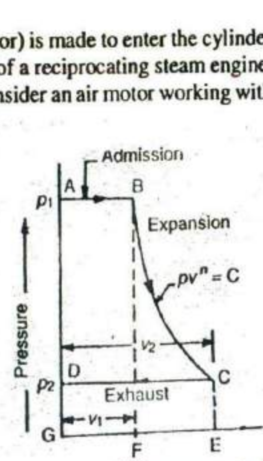


Fig. 31.1. Workdone by air motor.

We know that work done by the air per cycle,

$$W = \text{Area } ABCD$$

$$= \text{Area } ABFG + \text{Area } BCEF - \text{Area } CEGD$$

$$= p_1 v_1 + \frac{p_1 v_1 - p_2 v_2}{n-1} - p_2 v_2$$

$$= \frac{n}{n-1}(p_1v_1 - p_2v_2)$$

$$= \frac{n}{n-1} \times p_1v_1\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right]$$

$$= \frac{n}{n-1} \times mRT_1\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right] \qquad \ldots(\because pv = mRT)$$

Example 31.1. *An air motor receives air at 3.5 bar and 425 K, and exhaust it at 1 bar. Find the amount of work done per kg of air if the air expands according to the law $pv^{1.35}$ = constant.*

Solution. Given : p_1 = 3.5 bar ; T_1 = 425 K ; p_2 = 1 bar ; m = 1 kg ; n = 1.35

We know that work done per kg of air,

$$W = \frac{n}{n-1} \times mRT_1\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right]$$

$$= \frac{1.35}{1.35-1} \times 1 \times 287 \times 425\left[1-\left(\frac{1}{3.5}\right)^{\frac{1.35-1}{1.35}}\right] \text{J}$$

$$= 130\,320 \text{ J} = 130.32 \text{ kJ} \textbf{ Ans.}$$

Example 31.2. *An air motor is supplied with compressed air at 6.5 bar and 157° C. It is expanded to 1.04 bar and then exhausted at constant pressure. Determine the amount of work done by 1 kg of air and the temperature of air at the end of expansion. Assume the expansion according to $pv^{1.3}$ = constant and neglect clearance.*

Solution. Given : p_1 = 6.5 bar ; T_1 = 157° C = 157 + 273 = 430 K ; p_2 = 1.04 bar ; m = 1 kg ; n = 1.3

Work done by 1 kg of air

We know that work done by 1 kg of air,

$$W = \frac{n}{n-1} \times mRT_1\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right]$$

$$= \frac{1.3}{1.3-1} \times 1 \times 287 \times 430\left[1-\left(\frac{1.04}{6.5}\right)^{\frac{1.3-1}{1.3}}\right] \text{J}$$

$$= 184\,500 \text{ J} = 184.5 \text{ kJ} \textbf{ Ans.}$$

Temperature at the end of the expansion

Let T_1 = Temperature at the end of expansion.

We know that
$$\frac{T_2}{T_1} = \left(\frac{p_1}{p_2}\right)^{\frac{n-1}{n}} = \left(\frac{1.04}{6.5}\right)^{\frac{1.3-1}{1.3}} = (0.16)^{\frac{0.3}{1.3}} = 0.655$$

$$T_2 = 430 \times 0.655 = 281.6 \text{ K} = 8.6° \text{ C} \textbf{ Ans.}$$

31.3. Combined Air Compressor and Air Motor (Compressed Air System)

In the last article, we have discussed that the compressed air is carried, from the air compressor, to the air motor. Sometimes, air compressor and air motor are installed as two separate units. But sometimes they are installed as one unit. A little consideration will show, that if the air compressor and air motor are installed at some distance apart, the hot* compressed air, flowing through the duct, will get cooled to some extent. But if they are installed as one unit, there is no time for the air to get cooled. In such a case, some cooling arrangement is provided between the two units. *i.e.* after the air compressor or in other words before the motor. Such a system is known as compressed air system.

In a compressed air system, the air is first compressed in an air compressor from pressure p_1 to p_2 with a corresponding rise in its temperature. The hot air, leaving the compressor, is now cooled to the initial compressor temperature. The air is then made to expand in the air motor cylinder from pressure p_2 to p_1 with a corresponding fall in its temperature. Thus the temperature of air discharged from the air motor is less than the initial compressor intake temperature.

31.4. Efficiency of Compressed Air System

The theoretical indicator diagram of a compressed air system is shown in Fig. 31.2. The compression of air, in a compressor cylinder from pressure p_1 to p_2 is represented by the curve 1-2. The hot air leaving the compressor is cooled down in an air cooler to original compressor intake temperature.

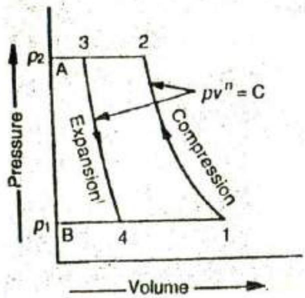


Fig. 31.2. Compressed air system.

The air now enters the air motor cylinder, and expands from pressure p_2 to p_1 as shown by the curve 3-4 in Fig. 31.2. Now let us assume the compression and expansion according to pv^n = constant and neglect clearance.

∴ Work done on the air compressor

$$= \text{Area } B\,12\,A = \frac{n}{n-1}(p_2v_2 - p_1v_1)$$

$$= \frac{n}{n-1} \times mR\,(T_2 - T_1)$$

and work done by the air in the motor

$$= \text{Area } A\,34\,B = \frac{n}{n-1}(p_3v_3 - p_4v_4)$$

$$= \frac{n}{n-1} \times mR\,(T_3 - T_4)$$

Now, let η_m = Efficiency of the air motor, and

η_c = Efficiency of the compressor.

∴ Shaft output of the air motor

$$= \text{Work done by the air} \times \eta_m$$

$$= \frac{n}{n-1} \times mR\,(T_3 - T_4)\,\eta_m \qquad \ldots (i)$$

and shaft input to the compressor

$$= \frac{n}{n-1} \times mR\,(T_2 - T_1)/\eta_c \qquad \ldots (ii)$$

* When the air is compressed, its temperature is increased.

The overall efficiency of the compressed air system is the ratio of the shaft output of the air motor to the shaft input to the compressor. Mathematically, overall efficiency of the compressed air system,

$$= \frac{\text{Shaft output of the air motor}}{\text{Shaft input to the compressor}}$$

$$= \frac{\frac{n}{n-1} \times mR\,(T_3 - T_4)\,\eta_m}{\frac{n}{n-1} \times mR\,(T_2 - T_1)/\eta_c} = \frac{(T_3 - T_4)\,\eta_m}{(T_2 - T_1)/\eta_c}$$

$$= \frac{T_3\left[1 - \frac{T_4}{T_3}\right]\eta_m}{T_1\left[\frac{T_2}{T_1} - 1\right]\eta_c} = \frac{1 - \frac{T_4}{T_3}}{\frac{T_2}{T_1} - 1} \times \eta_m \times \eta_c \qquad \ldots(\because T_3 = T_1)\ldots(iii)$$

We know that $\frac{T_4}{T_3} = \left(\frac{p_1}{p_2}\right)^{\frac{n-1}{n}}$ and $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$

$\therefore$ Equation (*iii*) may also be written as,

$$\eta_0 = \frac{1 - \left(\frac{p_1}{p_2}\right)^{\frac{n-1}{n}}}{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1} \times \eta_m \times \eta_c$$

Example 31.3. *A compressed air system consists of a single stage compressor of efficiency 75 % and air motor of efficiency 65%. If the compression and expansion follows the law $pv^{1.25}$ = constant, find the overall efficiency of the system. Take pressure ratio for both the machines as 3.5.*

Solution. Given : $\eta_c = 75\,\% = 0.75$; $\eta_m = 65\% = 0.65$; $n = 1.25$; $p_2/p_1 = 3.5$ or $p_1/p_2 = 1/3.5$

We know that overall efficiency of the system,

$$\eta_0 = \frac{1 - \left(\frac{p_1}{p_2}\right)^{\frac{n-1}{n}}}{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1} \times \eta_m \times \eta_c = \frac{1 - \left(\frac{1}{3.5}\right)^{\frac{1.25-1}{1.25}}}{(3.5)^{\frac{1.25-1}{1.25}} - 1} \times 0.65 \times 0.75$$

$$= \frac{1 - 0.778}{1.285 - 1} \times 0.65 \times 0.75 = 0.38 \quad \text{or} \quad 38\% \ \textbf{Ans.}$$

Example 31.4. *A system using compressed air for power transmission consists of a single stage compressor and air motor both having mechanical efficiency of 80%. The compression and expansion takes place according to $pv^{1.2}$ = constant. The higher and lower pressure for both compressor and air motor are 5 bar and 1 bar respectively. The air is cooled during its passage from the compressor to the motor to the initial temperature of 15° C. Calculate :*

1. Work done in compressor and motor cylinders ; and 2. Overall efficiency of the system.

Solution. Given : $\eta_m = \eta_c = 80\% = 0.8$; $n = 1.2$; $p_2 = p_3 = 5$ bar $= 5 \times 10^5$ N/m² ; $p_1 = p_4 = 1$ bar $= 1 \times 10^5$ N/m² ; $T_3 = T_1 = 15°$C $= 15 + 273 = 288$ K

Let T_2 = Temperature of air at the end of compression ; and

T_4 = Temperature of air at the end of expansion.

We know that
$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{5}{1}\right)^{\frac{1.2-1}{1.2}} = 1.308$$

$\therefore$
$$T_2 = T_1 \times 1.308 = 288 \times 1.308 = 376.7 \text{ K}$$

Similarly
$$\frac{T_4}{T_3} = \left(\frac{p_1}{p_2}\right)^{\frac{n-1}{n}} = \left(\frac{1}{5}\right)^{\frac{1.2-1}{1.2}} = 0.765$$

$\therefore$
$$T_4 = T_3 \times 0.765 = 288 \times 0.765 = 220.3 \text{ K}$$

1. *Work done in compressor and motor cylinder*

We know that mass of air,

$$m = \frac{p_1 v_1}{R T_1} = \frac{1 \times 10^5 \times 1}{287 \times 288} = 1.21 \text{ kg} \qquad \ldots (\text{Taking } v_1 = 1 \text{ m}^3)$$

$\therefore$ Work done in compressor cylinder,

$$W_{1-2} = \frac{n}{n-1} \times m R (T_2 - T_1)$$
$$= \frac{1.2}{1.2-1} \times 1.21 \times 287 (376.7 - 288) = 184\,820 \text{ J}$$
$$= 184.82 \text{ kJ} \quad \textbf{Ans.}$$

and work done in motor cylinder,

$$W_{3-4} = \frac{n}{n-1} \times m R (T_3 - T_4)$$
$$= \frac{1.2}{1.2-1} \times 1.21 \times 287 (288 - 220.3) = 141\,060 \text{ J}$$
$$= 141.06 \text{ kJ} \quad \textbf{Ans.}$$

2. *Overall efficiency of the system*

We know that overall efficiency of the system,

$$\eta_0 = \frac{(T_3 - T_4)\eta_m}{(T_2 - T_1)/\eta_c} = \frac{(288 - 220.3)\,0.8}{(376.7 - 288)/0.8} = 0.488 \quad \text{or} \quad 48.8\% \quad \textbf{Ans.}$$

Note : The overall efficiency may also be found out from the relation,

$$\eta_0 = \frac{\text{Work done in air motor cylinder} \times \eta_m}{\text{Work done in air compressor cylinder}/\eta_c}$$
$$= \frac{141.06 \times 0.8}{184.82/0.8} = 0.488 \quad \text{or} \quad 48.8\%$$

31.5. Preheating of Compressed Air

We have already discussed in the previous articles that compressed air (from the air compressor) is supplied to an air motor. A little consideration will show, that when the air expands in the motor cylinder, its temperature decreases. It will be interesting to know that if proper care in the design of compressed air system is not taken, then the temperature of exhaust air (from the air motor) may be below the freezing point. As a result of this, if there is any moisture present in the air, the same will be deposited in the form of ice, which will block exhaust valves of the motor. In order to prevent ice formation in the motor cylinder, the air is warmed by the steam in a preheater, at a constant pressure, before admission to the motor. Since the volume of air is proportional to the absolute temperature, a part of this heat energy is converted into additional work in the motor cylinder.

Let W = Work done in the motor cylinder for the given mass of air without preheating, and

W' = Work done for the same mass of air after preheating.

Then assuming that the work done is proportional to the volume of air used, therefore

$$\frac{W'}{W} = \frac{T_1'}{T_1}$$

Now the temperature, at which the air must be heated (T_1') in order to avoid freezing may be found out from the relation :

$$\frac{T_2}{T_1'} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

where T_2 = Temperature of the exhaust air from the motor.

Example 31.5. *The initial pressure of the air in air motor is 5 bar and final pressure is 1 bar. Find the temperature at which the air must be preheated in order that the temperature after expansion may be 2° C. Assume the expansion to be according to $pv^{1.3}$ = Constant.*

Solution. Given : $p_1 = 5$ bar ; $p_2 = 1$ bar ; $T_2 = 2°\text{C} = 2 + 273 = 275$ K

Let T_1' = Temperature at which the air must be heated.

We know that
$$\frac{T_2}{T_1'} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{1}{5}\right)^{\frac{1.3-1}{1.3}} = (0.2)^{\frac{0.3}{1.3}} = 0.69$$

$\therefore$ $T_1' = T_2 / 0.69 = 275 / 0.69 = 398.6\text{ K} = 125.6°\text{C}$ **Ans.**

Example 31.6. *An air motor receives air at a pressure of 4 bar and 50° C and exhausts at 1 bar. Determine the temperature to which the air should be preheated, so that exhaust temperature may be 5° C. Also find the ratio of work done due to preheating and otherwise. Take polytropic index as 1.35.*

Solution. Given : $p_1 = 4$ bar ; $T_1 = 50°\text{C} = 50 + 273 = 323$ K ; $p_2 = 1$ bar ; $T_2 = 5°\text{C} = 5 + 273 = 278$ K ; $n = 1.35$

Temperature to which the air should be preheated

Let T_1' = Temperature to which the air should be preheated.

We know that
$$\frac{T_2}{T_1'} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{1}{4}\right)^{\frac{1.35-1}{1.35}} = (0.25)^{\frac{0.35}{1.35}} = 0.698$$

$\therefore$ $T_1' = T_2 / 0.698 = 278 / 0.698 = 398.3\text{ K} = 125.3°\text{C}$ **Ans.**

Ratio of work done due to preheating and otherwise

Let W = Work done in the motor cylinder without preheating, and

W' = Work done in the motor cylinder after preheating.

We know that the ratio of work done due to preheating and otherwise,

$$\frac{W'}{W} = \frac{T_1'}{T_1} = \frac{398.3}{323} = 1.233 \text{ Ans.}$$

EXERCISES

1. An air motor receives air at a pressure of 5.5 bar and delivers it at 1 bar. Find the amount of work done per kg of air, if the expansion follows the law $pv^{1.3}$ = Constant. Take temperature of inlet air as 100° C. [Ans. 155 kJ]

2. In a compressed air system, the compression and expansion of air takes place according to the law $pv^{1.5}$ = Constant. Find its overall efficiency if both the machines work within pressure limits of 4.5 bar and 1 bar. Take efficiencies of compressor and motor alike. [Ans. 74%]

3. The initial pressure of the air in an air motor is 5.25 bar and final pressure is 1.05 bar. Find the temperature at which the air must be preheated in order that the temperature after expansion may be 3° C. Assuming the expansion to be according to $pv^{1.3}$ = Constant. [Ans. 127.2° C]

4. A single acting motor works on compressed air at 10.5 bar and 37° C, supplied at the rate of 1 kg/min. The cut-off takes place at 20% of the stroke and the expansion follows adiabatic and frictionless down to 1.03 bar. Determine the cylinder volume, mean effective pressure and indicated power, if the machine runs at 300 r.p.m. Neglect clearance.

QUESTIONS

1. What is air motor ? On what principle does it work ?
2. Obtain expression for the workdone by air in an air motor.
3. Explain the working of compressed air system.
4. Derive an expression for the overall efficiency of a compressed air system.
5. What is preheating of air ? Explain its uses.

OBJECTIVE TYPE QUESTIONS

1. The operation of a reciprocating air motor is similar to that of

   (*a*) reciprocating steam engine (*b*) reciprocating air compressor

   (*c*) both (*a*) and (*b*) (*d*) none of these

2. Air motors work on the cycle which is the ..... of the reciprocating air compressor cycle.

   (*a*) same as that (*b*) reverse

3. In a compressed air system, the temperature of air discharged from the air motor is ..... than the initial compressor intake temperature.

   (*a*) more (*b*) less

4. The overall efficiency of the compressed air system is

   (*a*) the ratio of shaft output of the air motor to the shaft input to the compressor

   (*b*) the ratio of shaft input to the compressor to the shaft output of the air motor

   (*c*) the product of the shaft output of the air motor and the shaft input to the compressor

   (*d*) none of the above

ANSWERS

1. (*a*) 2. (*b*) 3. (*b*) 4. (*a*)

32

Gas Turbines

1. Introduction. 2. Comparison of Gas Turbines and Steam Turbines. 3. Comparison of Gas Turbines and I.C. Engines. 4. Classification of Gas Turbines. 5. Closed Cycle Gas Turbines. 6. Gas Turbines with Intercooling. 7. Gas Turbines with Reheating. 8. Open Cycle Gas Turbines. 9. Comparison of Closed Cycle and Open Cycle Gas Turbines. 10. Semi-closed Cycle Gas Turbines. 11. Constant Pressure Gas Turbines. 12. Constant Volume Gas Turbines.

32.1. Introduction

The idea of gas turbine is the oldest one, and its working principle is an improved version of the wind mill, which was used several centuries back. In order to achieve an efficient working of the turbine, the movement of gas (or air) is properly controlled and then directed on the blades fixed to the turbine runner. The air, under pressure, is supplied to the turbine by an air compressor, which is run by the turbine itself.

In a gas turbine, first of all, the air is obtained from the atmosphere and compressed in an air compressor. The compressed air is then passed into the combustion chamber, where it is heated considerably. The hot air is then made to flow over the moving blades of the gas turbine, which imparts rotational motion to the runner. During this process, the air gets expanded and finally it is exhausted into the atmosphere. A major part of the power developed by the turbine is consumed for driving the compressor (which supplies compressed air to the combustion chamber). The remaining power is utilised for doing some external work.

32.2. Comparison of Gas Turbines and Steam Turbines

Following are the points of comparison between gas turbines and steam turbines :

S.No.	*Gas turbines*	*Steam turbines*
1.	The important components are compressor and combustion chamber.	The important components are steam boiler and accessories.
2.	The mass of gas turbine per kW developed is less.	The mass of steam turbine per kW developed is more.
3.	It requires less space for installation.	It requires more space for installation.
4.	The installation and running cost is less.	The installation and running cost is more.
5.	The starting of gas turbine is very easy and quick.	The starting of steam turbine is difficult and takes long time.
6.	Its control, with the changing load conditions, is easy.	Its control, with the changing load conditions, is difficult.
7.	A gas turbine does not depend on water supply.	A steam turbine depends on water supply.
8.	Its efficiency is less.	Its efficiency is higher.

32.3. Comparison of Gas Turbines and I.C. Engines

Following are the points of comparison between gas turbines and I.C. engines :

S.No.	Gas turbines	I.C. engines
1.	The mass of gas turbine per kW developed is less.	The mass of an I.C. engine per kW deveoped is more.
2.	The installation and running cost is less.	The installation and running cost is more.
3.	Its efficiency is higher.	Its efficiency is less.
4.	The balancing of a gas turbine is perfect.	The balancing of an I.C. engine is not perfect.
5.	The torque produced is uniform. Thus no flywheel is required.	The torque produced is not uniform. Thus flywheel is necessary.
6.	The lubrication and ignition systems are simple.	The lubrication and ignition systems are difficult.
7.	It can be driven at a very high speed.	It can not be driven at a very high speed.
8.	The pressures used are very low (about 5 bar).	The pressures used are high (above 60 bar).
9.	The exhaust of a gas turbine is free from smoke and less polluting.	The exhaust of an I.C. engine is more polluting.
10.	They are very suitable for air crafts.	They are less suitable for air crafts.
11.	The starting of a gas turbine is not simple.	The starting of an I.C. engine is simple.

32.4. Classification of Gas Turbines

Though the gas turbines may be classified in many ways,yet the following are important from the subject point of view :

1. *According to path of the working substance*

   (*a*) Closed cycle gas turbines, (*b*) Open cycle gas turbines, and

   (c) Semi-closed gas turbines.

2. *According to process of heat absorption*

   (*a*) Constant pressure gas turbines, and (*b*) Constant volume gas turbines.

In the following pages, we shall discuss all the above mentioned gas turbines one by one.

32.5. Closed Cycle Gas Turbines

A closed cycle gas turbine, in its simplest form, consists of a compressor, heating chamber, gas turbine which drives the generator and compressor, and a cooling chamber.

The schematic arrangement of a closed cycle gas turbine is shown in Fig. 32.1. In this turbine, the air is compressed isentropically (generally in rotary compressor) and then passed into the heating chamber. The compressed air is heated with the help of some external source, and made to flow over the turbine blades (generally reaction type). The gas, while flowing over the blades, gets expanded. From the turbine, the gas is passed to the cooling chamber where it is cooled at constant pressure with the help of circulating water to its original temperature. Now the air is made to flow into the compressor again. It

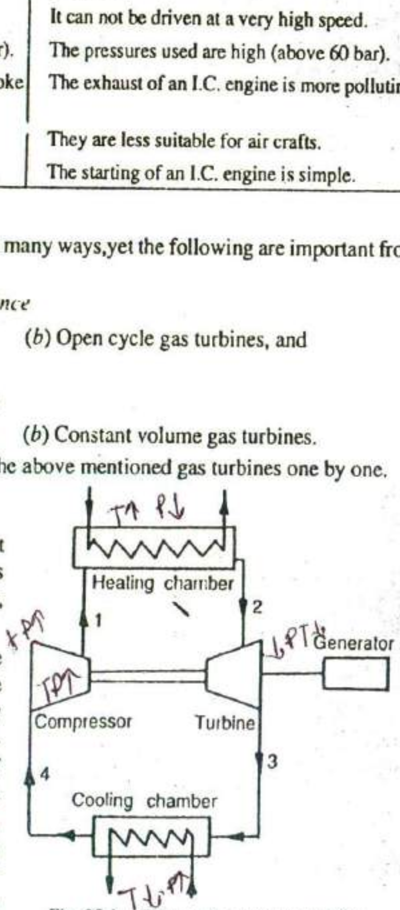
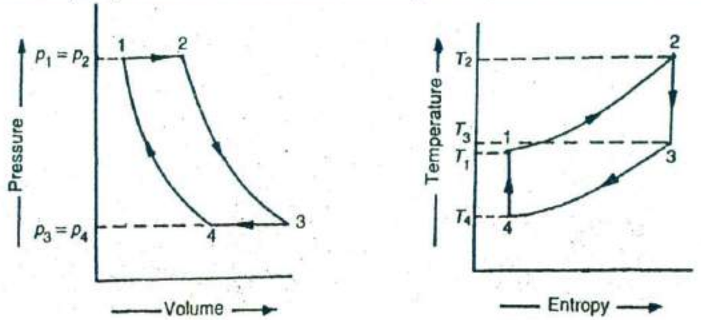


Fig. 32.1. Schematic arrangement of a closed cycle gas turbine.

is thus obvious, that in a closed cycle gas turbine, the air is continuously circulated within the turbine.

A closed cycle gas turbine works on Joule's or Brayton's cycle as shown in Fig. 32.2.



(*a*) *p-v* diagram. (*b*) *T-s* diagram.

Fig. 32.2 Constant pressure closed cycle gas turbine.

The process 1-2 shows heating of the air in heating chamber at constant pressure. The process 2-3 shows isentropic expansion of air in the turbine. Similarly, the process 3-4 shows cooling of the air at constant pressure in cooling chamber. Finally, the process 4-1 shows isentropic compression of the air in the compressor.

∴ Work done by the turbine per kg of air,

$$W_T = c_p (T_2 - T_3) \qquad \ldots (i)$$

and work required by the compressor per kg of air,

$$W_C = c_p (T_1 - T_4) \qquad \ldots (ii)$$

Now the net work available,

$$W = W_T - W_C$$

Notes : 1. In the above expressions, c_p is taken in kJ/kg K.

2. The power available (or net power of the installation) may be found out from the work available as usual.

Example 32.1. *A simple closed cycle gas turbine plant receives air at 1 bar and 15° C, and compresses it to 5 bar and then heats it to 800° C in the heating chamber. The hot air expands in a turbine back to 1 bar. Calculate the power developed per kg of air supplied per second. Take c_p for the air as 1 kJ/kg K.*

Solution. Given : $p_3 = p_4 = 1$ bar ; $T_4 = 15°\text{C} = 15 + 273 = 288$ K ; $p_1 = p_2 = 5$ bar ; $T_2 = 800°\text{C} = 800 + 273 = 1073$ K ; $c_p = 1$ kJ/kg K

The *p-v* and *T-s* diagram for the closed cycle gas turbine is shown in Fig. 32.3.

Let T_1 and T_3 = Temperature of air after isentropic compression and expansion (*i.e.* at points 1 and 3 respectively).

We know that for isentropic expansion 2-3,

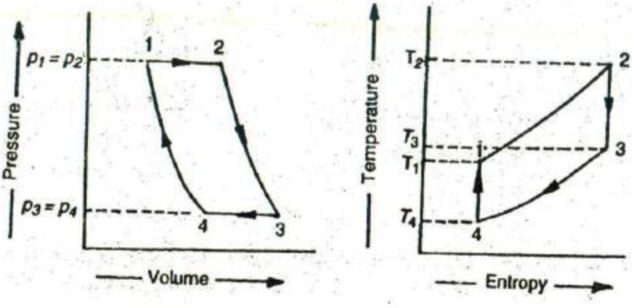
$$\frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{5}\right)^{\frac{1.4-1}{1.4}} = (0.2)^{0.286} = 0.631$$

$$\therefore \quad T_3 = T_2 \times 0.631 = 1073 \times 0.631 = 677 \text{ K}$$

Similarly, for isentropic compression 4-1,

$$\frac{T_4}{T_1} = \left(\frac{p_4}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{5}\right)^{\frac{1.4-1}{1.4}} = 0.631$$

$$\therefore \quad T_1 = T_4 / 0.631 = 288 / 0.631 = 456 \text{ K}$$



(*a*) *p-v* diagram. (*b*) *T-s* diagram.

Fig. 32.3

We know that work developed by the turbine,

$$W_T = c_p (T_2 - T_3) = 1 (1073 - 677) = 396 \text{ kJ/s}$$

and work required by the compressor,

$$W_C = c_p (T_1 - T_4) = 1 (456 - 288) = 168 \text{ kJ/s}$$

∴ Net work done by the turbine,

$$W = W_T \pm W_C = 396 - 168 = 228 \text{ kJ/s}$$

and power developed, $P = 228$ kW **Ans.** ... ($\because$ 1 kJ/s = 1 kW)

Example 32.2. *In an oil-gas turbine installation, it is taken at pressure of 1 bar and 27° C and compressed to a pressure of 4 bar. The oil with a calorific value of 42 000 kJ/kg is burnt in the combustion chamber to raise the temperature of air to 550° C. If the air flows at the rate of 1.2 kg/s ; find the net power of the installation. Also find air fuel ratio. Take $c_p = 1.05$ kJ/kg K.*

Solution. Given : $p_3 = p_4 = 1$ bar ; $T_4 = 27°$ C = 27 + 273 = 300 K ; $p_1 = p_4 = 4$ bar ; $C = 42\,000$ kJ/kg ; $T_2 = 550°$ C = 550 + 273 = 823 K ; $m = 1.2$ kg/s ; $c_p = 1.05$ kJ/kg K

Net power of the Installation

Let T_1 and T_3 = Temperature of air after isentropic compression and expansion respectively.

We know that for isentropic expansion 2-3 (Refer Fig. 32.3),

$$\frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{4}\right)^{\frac{1.4-1}{1.4}} = (0.25)^{0.286} = 0.673$$

$$\therefore \quad T_3 = T_2 \times 0.673 = 823 \times 0.673 = 553.9 \text{ K}$$

Similarly, for isentropic compression 4-1,

$$\frac{T_4}{T_1} = \left(\frac{p_4}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{4}\right)^{\frac{1.4-1}{1.4}} = 0.673$$

$$\therefore \quad T_1 = T_4 / 0.673 = 300 / 0.673 = 445.8 \text{ K}$$

We know that work done by the turbine,

$$W_T = m\,c_p\,(T_2 - T_3) = 1.2 \times 1.05\,(823 - 553.9) = 339.1 \text{ kJ/s}$$

and work done by the compressor,

$$W_C = m\,c_p\,(T_1 - T_4)$$
$$= 1.2 \times 1.05\,(445.8 - 300) = 183.7 \text{ kJ/s}$$

$\therefore$ Net power of the installation,

$$= 339.1 - 183.7 = 154.4 \text{ kJ/s} = 154.4 \text{ kW } \textbf{Ans.}$$

Air-fuel ratio

We know that heat supplied by the oil

$$= m\,c_p\,(T_2 - T_1)$$
$$= 1.2 \times 1.05(823 - 445.8) = 475.3 \text{ kJ/s}$$

$\therefore$ Mass of fuel burnt per second

$$= \frac{\text{Heat supplied}}{\text{Calorific value}} = \frac{475.3}{42\,000} = 0.011 \text{ kg}$$

and air-fuel ratio $= \dfrac{\text{Mass of air}}{\text{Mass of fuel}} = \dfrac{1.2}{0.011} = 109.1$ **Ans.**

32.6. Gas Turbine with Intercooling

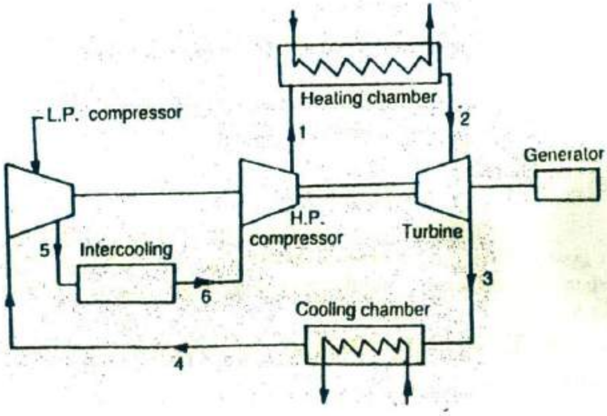


Fig. 32.4. Schematic arrangement of a closed cycle gas turbine with intercooler.

We have already discussed that a major portion of the power developed by the gas turbine is utilised by the compressor. It can be reduced by compressing the air in two stages with an intercooler between the two. This improves the efficiency of the gas turbine. The schematic arrangement of a closed cycle gas turbine with an intercooler is shown in Fig. 32.4.

In this arrangement*, first of all, the air is compressed in the first compressor, known as low pressure (L.P.) compressor. We know that as a result of this compression, the pressure and temperature of the air is increased. Now the air is passed to an intercooler which reduces the temperature of the compressed air to its original temperature, but keeping the pressure constant. After that, the compressed air is once again compressed in the second compressor known as high pressure (H.P.) compressor. Now the compressed air is passed through the heating chamber and then through the turbine. Finally, the air is cooled in the cooling chamber and again passed into the low pressure compressor as shown in Fig. 32.4.

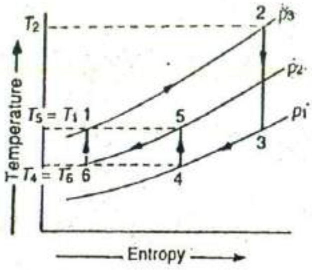


Fig. 32.5. *T-s* diagram for intercooling.

The process of intercooling the air in two stages of compression is shown on *T-s* diagram in Fig. 32.5. The process 1-2 shows heating of the air in heating chamber at constant pressure. The process 2-3 shows isentropic expansion of air in the turbine. The process 3-4 shows cooling of the air in the cooling chamber at constant pressure. The process 4-5 shows compression of air in the L.P. compressor. The process 5-6 shows cooling of the air in the intercooler at constant pressure. Finally, the process 6-1 shows compression of air in the H.P. compressor.

∴ Work done by the compressor per kg of air,

$$W_T = c_p (T_2 - T_3) \qquad \ldots (i)$$

and work done by the compressor per kg of air,

$$W_C = c_p [(T_1 - T_6) + (T_5 - T_4)] \qquad \ldots (ii)$$

Now the net work available,

$$W = W_T - W_C \qquad \ldots (iii)$$

Notes: 1. The power available (or net power of the installation) may be found out from the work available as usual.

2. For perfect intercooling, the intermediate pressure may be found out from the relation,

$$p_6 = p_5 = \sqrt{p_1 \times p_4} = \sqrt{p_2 \times p_3}$$

3. For perfect intercooling,

$$T_4 = T_6 \,; \text{ and } T_5 = T_1$$

Example 32.3. *A gas turbine plant consists of two stage compressor with perfect intercooler and a single stage turbine. If the plant works between the temperature limits of 300 K and 1000 K and 1 bar and 16 bar ; find the net power of the plant per kg of air. Take specific heat at constant pressure as 1 kJ/kg K.*

Solution. Given : $T_4 = 300$ K ; $T_2 = 1000$ K ; $p_3 = p_4 = 1$ bar ; $p_1 = p_2 = 16$ bar ; $c_p = 1$ kJ/kg K

The *T-s* diagram is shown in Fig. 32.6.

Let T_1, T_3, T_5 and T_6 = Temperature of air at corresponding points.

We know that for perfect intercooling, the intermediate pressure,

$$p_5 = p_6 = \sqrt{p_1 \times p_4} = \sqrt{16 \times 1} = 4 \text{ bar}$$

* Please refer Chapter 28 (Art. 28.14) also.

Now for the isentropic process 2-3,

$$\frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= \left(\frac{1}{16}\right)^{\frac{1.4-1}{1.4}} = 0.453$$

$$\therefore \quad T_3 = T_2 \times 0.453$$

$$= 1000 \times 0.453 = 453 \text{ K}$$

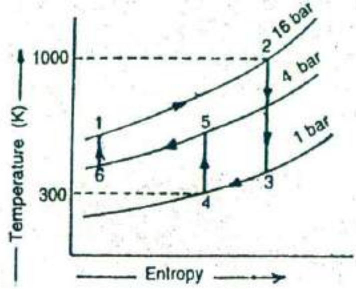


Fig. 32.6

Similarly for the isentropic process 4-5,

$$\frac{T_4}{T_5} = \left(\frac{p_4}{p_5}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= \left(\frac{1}{4}\right)^{\frac{1.4-1}{1.4}} = 0.673$$

$$\therefore \quad T_5 = T_4 / 0.673 = 300 / 0.673 = 446 \text{ K}$$

We know that for perfect inter cooling,

$$T_1 = T_5 = 446 \text{ K}$$

and for isentropic process 6-1, $\frac{T_6}{T_1} = \left(\frac{p_6}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{4}{16}\right)^{\frac{1.4-1}{1.4}} = 0.673$

$$\therefore \quad T_6 = T_1 \times 0.673 = 446 \times 0.673 = 300 \text{ K}$$

Now work done by the turbine per kg of air,

$$W_T = c_p (T_2 - T_3) = 1 (1000 - 453) = 547 \text{ kJ/s}$$

and work absorbed by the compressor per kg of air,

$$W_C = c_p [(T_1 - T_6) + (T_5 - T_4)]$$

$$= 1 [(446 - 300) + (446 - 300)] = 292 \text{ kJ/s}$$

We know that work done by the plant per kg of air,

$$W = W_T - W_C = 547 - 292 = 255 \text{ kJ/s}$$

$\therefore$ Net power of the plant, $P = 255$ kW **Ans.**

32.7. Gas Turbine with Reheating

The output of a gas turbine can be considerably improved by expanding the hot air in two stages with a reheater between the two. The schematic arrangement of a closed cycle gas turbine with reheating is shown in Fig. 32.7.

In this arrangement, the air is first compressed in the compressor, passed into the heating chamber, and then to the first turbine. The air is once again passed on to another heating chamber and then to the second turbine. Finally, the air is cooled in the cooling chamber and again passed into the compressor as shown in Fig. 32.7.

The process of reheating in the two turbines is shown on T-s diagram in Fig. 32.8. The process 1-2 shows heating of the air in the first heating chamber at constant pressure. The process2-3 shows isentropic expansion of air in the first turbine. The process 3-4 shows heating of the air in the second heating chamber at constant pressure. The process 4-5 shows isentropic expansion of air in the second turbine. The process 5-6 shows cooling of the air in the intercooler at constant pressure. Finally, the process 6-1 shows compression of air in the compressor.

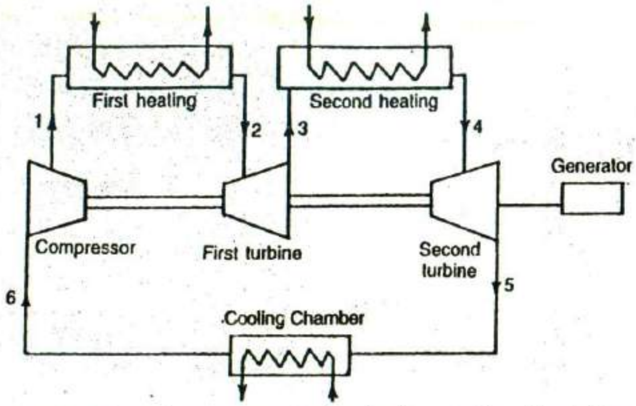


Fig. 32.7. Schematic arrangement of a closed cycle gas turbine with reheating.

∴ Work done by the turbine per kg of air,

$$W_T = c_p [(T_2 - T_3) + (T_4 - T_5)] \quad \ldots (i)$$

and work done by the compressor per kg of air,

$$W_C = c_p [(T_1 - T_6)] \quad \ldots (ii)$$

Now net work available,

$$W = W_T - W_C \quad \ldots (iii)$$

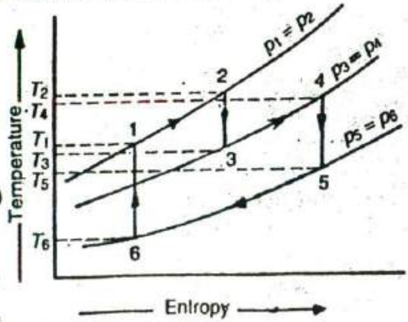


Fig. 32.8. T-s diagram for reheating.

Notes : 1. The power available (or net power of the installation may be found out from the work available as usual.

2. For maximum work, the reheating should be done to an intermediate pressure,

$$p_3 = p_4 = \sqrt{p_2 \times p_5} = \sqrt{p_1 \times p_6} \quad \ldots (\because p_1 = p_2 \text{ and } p_5 = p_6)$$

3. Sometimes, cooling and reheating is provided simultaneously in gas turbines. In such a case, the corresponding values should be used.

Example 32.4. *In a gas turbine plant, the air is compressed in a single stage compressor from 1 bar to 9 bar and from an initial temperature of 300 K. The same air is then heated to a temperature of 800 K and then expanded in the turbine. The air is then reheated to a temperature of 800 K and then expanded in the second turbine. Find the maximum power that can be obtained from the installation, if the mass of air circulated per second is 2 kg. Take c_p = 1 kJ/kg K.*

Solution. Given : $p_6 = p_5 = 1$ bar ; $p_1 = p_2 = 9$ bar ; $T_6 = 300$ K ; $T_2 = T_4 = 800$ K ; $m = 2$ kg/s ; $c_p = 1$ kJ/kg K

The T-s diagram of the reheat cycle is shown in Fig. 32.9.

Let T_1, T_3, T_5 = Temperature of air at the corresponding points.

We know that for maximum power (or work), the intermediate pressure,

$$p_3 = p_4 = \sqrt{p_1 \times p_6}$$
$$= \sqrt{9 \times 1} = 3 \text{ bar}$$

We also know that for isentropic compression of air in the compressor (process 6-1),

$$\frac{T_1}{T_6} = \left(\frac{p_1}{p_6}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{9}{1}\right)^{\frac{1.4-1}{1.4}}$$
$$= (9)^{0.286} = 1.873$$

$\therefore$ $$T_1 = T_6 \times 1.873$$
$$= 300 \times 1.873 = 562 \text{ K}$$

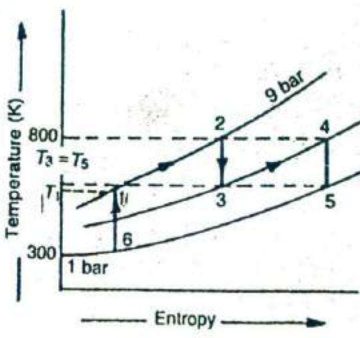


Fig. 32.9

For isentropic expansion of air in the first turbine (process 2-3),

$$\frac{T_2}{T_3} = \left(\frac{p_2}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{9}{3}\right)^{\frac{1.4-1}{1.4}} = (3)^{0.286} = 1.369$$

$\therefore$ $$T_3 = T_2 / 1.369 = 800 / 1.369 = 584 \text{ K}$$

Similarly, for isentropic expansion of air in the second turbine (process 4-5),

$$\frac{T_4}{T_5} = \left(\frac{p_4}{p_5}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{3}{1}\right)^{\frac{1.4-1}{1.4}} = (3)^{0.286} = 1.369$$

$\therefore$ $$T_5 = T_4 / 1.369 = 800 / 1.369 = 584 \text{ K}$$

We know that work done by the turbine,

$$W_T = m\,c_p\,[(T_2 - T_3) + (T_4 - T_5)]$$
$$= 2 \times 1\,[(800 - 584) + (800 - 584)] = 864 \text{ kJ/s}$$

and work absorbed by the compressor,

$$W_C = m\,c_p\,(T_1 - T_6) = 2 \times 1\,(562 - 300) = 524 \text{ kJ/s}$$

We also know that net work available,

$$W = W_T - W_C = 864 - 524 = 340 \text{ kJ/s}$$

$\therefore$ Power that can be obtained from the installation,

$$P = 340 \text{ kJ/s} = 340 \text{ kW} \quad \textbf{Ans.}$$

Example 32.5 *A closed cycle gas turbine consists of a two stage compressor with perfect intercooler and a two stage turbine, with a reheater. All the components are mounted on the same shaft. The pressure and temperature at the inlet of the low pressure compressor are 2 bar and 300 K. The maximum pressure and temperature are limited to 8 bar and 1000 K. The gases are heated in the reheater to 1000 K. Calculate mass of fluid circulated in the turbine, if the net power developed by the turbine is 370 kW. Also find the amount of heat supplied per second from the external source.*

Solution. Given : $p_6 = p_5 = 2$ bar ; $T_6 = 300$ K ; $p_1 = p_2 = 8$ bar ; $T_2 = 1000$ K ; $T_4 = 1000$ K ; $P = 370$ kW

The *T-s* diagram of the reheat cycle is shown in Fig. 32.10.

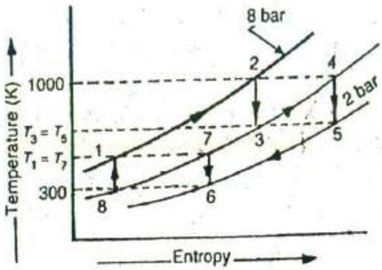


Fig. 32.10

Mass of fluid circulated in the turbine

Let m = Mass of air circulated in the turbine,

T_1, T_3, T_5, T_7, T_8 = Temperature of air at the corresponding points.

We know that for perfect cooling, the intermediate pressure,

$$p_8 = p_7 = p_3 = p_4 = \sqrt{p_1 \times p_6} = \sqrt{8 \times 2} = 4 \text{ bar}$$

Now for the isentropic process 6-7,

$$\frac{T_6}{T_7} = \left(\frac{p_6}{p_7}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{2}{4}\right)^{\frac{1.4-1}{1.4}} = (0.5)^{0.286} = 0.82$$

$$\therefore \quad T_7 = T_6 / 0.82 = 300 / 0.82 = 366 \text{ K}$$

We know that for perfect cooling,

$$T_1 = T_7 = 366 \text{ K}$$

Again, for the isentropic process 8-1,

$$\frac{T_8}{T_1} = \left(\frac{p_8}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{4}{8}\right)^{\frac{1.4-1}{1.4}} = (0.5)^{0.286} = 0.82$$

$$\therefore \quad T_8 = T_1 \times 0.82 = 366 \times 0.82 = 300 \text{ K}$$

and for the isentropic process 2-3,

$$\frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{4}{8}\right)^{\frac{1.4-1}{1.4}} = 0.82$$

$$\therefore \quad T_3 = T_2 \times 0.82 = 1000 \times 0.82 = 820 \text{ K}$$

Similarly, for the isentropic process 4-5,

$$\frac{T_5}{T_4} = \left(\frac{p_5}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{2}{4}\right)^{\frac{1.4-1}{1.4}} = 0.82$$

$$\therefore \quad T_5 = T_4 \times 0.82 = 1000 \times 0.82 = 820 \text{ K}$$

We know that work done by the turbine,

$$W_T = m\,c_p\,[(T_2 - T_3) + (T_4 - T_5)]$$

$$= m \times 1\,[(1000 - 820) + (1000 - 820)] = 360\,m \text{ kJ/s}$$

and work absorbed by the compressor,

$$W_C = m\,c_p\,[(T_1 - T_8) + (T_7 - T_6)]$$
$$= m \times 1\,[(366 - 300) + (366 - 300)] = 132\,m \text{ kJ/s}$$

$\therefore$ Net work done by the turbine,

$$W = W_T - W_C = 360\,m - 132\,m = 228\,m \text{ kJ/s}$$

We also know that power developed by the turbine (P),

$$370 = 228\,m \quad \text{or} \quad m = 1.62 \text{ kg/s} \textbf{ Ans.}$$

Heat supplied from the external source

We know that heat supplied from the external source

$$= m\,c_p\,[(T_2 - T_1) + (T_4 - T_3)]$$
$$= 1.62 \times 1\,[(1000 - 366) + (1000 - 820)] \text{ kJ/s}$$
$$= 1318.7 \text{ kJ/s} \textbf{ Ans.}$$

32.8. Open Cycle Gas Turbines

An open cycle gas turbine, in its simplest form, consists of a compressor, combustion chamber and a gas turbine which drives the generator and compressor.

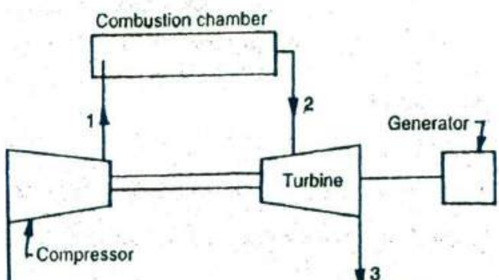


Fig. 32.11. Schematic arrangement of an open cycle gas turbine.

The schematic arrangement of an open cycle gas turbine is shown in Fig. 32.11. In this turbine, the air is first sucked from the atmosphere and then compressed isentropically (generally in a rotary compressor) and then passed into the combustion chamber. The compressed air is heated by the combustion of fuel and the products of combustion (*i.e.* hot gases formed by the combustion of fuel) also get mixed up with the compressed air, thus increasing the mass of compressed air. The hot gas is then made to flow over the turbine blades (generally of reaction type). The gas, while flowing over the blades, gets expanded and finally exhausted into the atmosphere.

An open cycle gas turbine is also called a continuous combustion gas turbine as the combustion of fuel takes place continuously. This turbine also works on Joule's cycle. The relations for work done by the compressor and turbine are same as those of closed cycle gas turbine.

Note : In an open cycle gas turbine, the process 3-4 has no practical importance, as the air is exhausted into the atmosphere at point 3 and fresh air is sucked in the compressor at point 4.

32.9. Comparison of Closed Cycle and Open Cycle Gas Turbines

Following are the points of comparison between closed and open cycle gas turbines.

S.No.	Closed cycle gas turbine	Open cycle gas turbine
1.	The compressed air is heated in a heating chamber. Since the gas is heated by an external source, so the amount of gas remains the same.	The compressed air is heated in a combustion chamber. The products of combustion get mixed up in the heated air.
2.	The gas from the turbine is passed into the cooling chamber.	The gas from the turbine is exhausted into the atmosphere.
3.	The working fluid is circulated continuously.	The working fluid is replaced continuously.
4.	Any fluid with better thermodynamic properties can be used.	Only air can be used as the working fluid.
5.	The turbine blades do not wear away earlier, as the enclosed gas does not get contaminated while flowing through the heating chamber.	The turbine blades wear away earlier, as the air from the atmosphere gets contaminated while flowing through the combustion chamber.
6.	Since the air, from the turbine, is cooled by circulating water, it is best suited for stationary installation or marine uses.	Since the air, from the turbine, is discharged into the atmosphere, it is best suited for moving vehicle.
7.	Its maintenance cost is high.	Its maintenance cost is low.
8.	The mass of installation per kW is more.	The mass of installation per kW is less.

Example 32.6. *A constant pressure open cycle gas turbine plant works between temperature range of 15° C and 700° C and pressure ratio of 6. Find the mass of air circulating in the installation, if it develops 1100 kW. Also find the heat supplied by the heating chamber.*

Solution. Given : $T_4 = 15° \text{C} = 15 + 273 = 288 \text{ K}$; $T_2 = 700° \text{C} = 700 + 273 = 973 \text{ K}$; $p_2 / p_3 = p_1 / p_4 = 6$; $P = 1100 \text{ kW}$

Mass of air circulating in the installation

Let m = Mass of air circulating in the installation.

T_1 and T_3 = Temperatures of air after isentropic compression and expansion respectively.

We know that for isentropic expansion process 2-3 (Refer Fig. 32.2),

$$\frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{6}\right)^{\frac{1.4-1}{1.4}} = \left(\frac{1}{6}\right)^{0.286} = 0.599$$

$$\therefore \quad T_3 = T_2 \times 0.599 = 973 \times 0.599 = 583 \text{ K}$$

Similarly, for isentropic compression process 4-1,

$$\frac{T_4}{T_1} = \left(\frac{p_4}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{6}\right)^{\frac{1.4-1}{1.4}} = \left(\frac{1}{6}\right)^{0.286} = 0.599$$

$$\therefore \quad T_1 = T_4 / 0.599 = 288 / 0.599 = 481 \text{ K}$$

Now work done by the turbine per kg of air,

$$W_T = m\, c_p (T_2 - T_3) = m \times 1\,(973 - 583) = 390\, m \text{ kJ/s}$$

and work absorbed by the compressor per kg of air,

$$W_C = m\, c_p (T_1 - T_4) = m \times 1\,(481 - 288) = 193\, m \text{ kJ/s}$$

$\therefore$ Net work done by the turbine per kg of air,

$$W = W_T - W_C = 390\,m - 193\,m = 197\,m \text{ kJ/s}$$

and power developed by the installation (P),

$$1100 = 197\,m \quad \text{or} \quad m = 1100/197 = 5.58 \text{ kg/s} \textbf{ Ans.}$$

Heat supplied by the heating chamber

We know that heat supplied by the heating chamber

$$= m\,c_p\,(T_2 - T_1)$$

$$= 5.58 \times 1\,(973 - 481) = 2745.4 \text{ kJ/s} \textbf{ Ans.}$$

32.10. Semi-closed Cycle Gas Turbines

A semi-closed cycle gas turbine, as the name indicates, is a turbine which is a combination of two turbines, one working on open cycle and the other on closed cycle. The open cycle turbine is used to drive the main generator and works within the pressure limits of atmospheric and about 16 bar. The closed cycle turbine is used to drive the air compressor and works within the pressure limits of about 2 bar and 16 bar.

Strictly speaking, the semi-closed cycle gas turbines are not used on commercial basis, though they are important from academic point of view only.

32.11. Constant Pressure Gas Turbines

A turbine in which the air is heated in the combustion (or heating) chamber at constant pressure, is known as *constant pressure gas turbine*. Almost all the turbines, manufactured today, are constant pressure gas turbines.

32.12. Constant Volume Gas Turbines

A turbine in which the air is heated in combustion (or heating) chamber at constant volume is known as *constant volume gas turbine*. These turbines are not used on commercial basis, though they have academic importance only.

EXERCISES

1. A simple closed cycle gas turbine installation works between the temperature limits of 300 K and 1000 K and pressure limits of 1 bar and 5 bar. If 1.25 kg of air is circulated per second, determine the power developed by the turbine. For air, take $c_p = 1.008$ kJ/kg K and $\gamma = 1.4$. **[Ans. 680.5 kW]**

2. In a gas turbine plant, operating on Brayton cycle, air enters the compressor at 1 bar and 27° C. The pressure ratio in the cycle is 6. Calculate the maximum temperature in the cycle and the power developed by the turbine. Assume the turbine work as 2.5 times the compressor work. Take $\gamma = 1.4$ **[Ans. 703.5 kW]**

3. A gas turbine plant consists of two stage compressor (with perfect intercooler) and a single stage turbine. The plant receives air at 1 bar and 290 K. If the maximum pressure and temperature of air in the plant is 12.25 bar and 950 K, find the power developed by the plant per kg of air. Take specific heat at constant pressure as 1 kJ/kg K. **[Ans. 235 kW]**

4. In a gas turbine installation, the air is compressed in a single stage compressor from 1 bar to 6.25 bar and from an initial temperature of 20° C. The air after compression is heated in a chamber to a temperature of 750° C. The hot air is expanded in the turbine and then reheated to a temperature of 750° C. The hot air is once again expanded in the second turbine. Find the power that can be developed per kg of air. **[Ans. 269 kW]**

QUESTIONS

1. What is a gas turbine ? How does it differ from a steam turbine ?
2. How does a gas turbine compare with the internal combustion engine power plant ?
3. List the methods of improving the efficiency and specific output of a simple gas turbine.

4. Draw the layout of a gas turbine plant which has two stage compression with complete intercooling. The high pressure turbine develops power enough only to drive the high pressure compressor. The L.P. turbine drives both the L.P. compressor and the load. Indicate the ideal process of this plant on a *T-s* diagram.

5. What are the essential components of a simple open cycle gas turbine plant ?

6. Differentiate clearly between a closed cycle gas turbine and an open cycle gas turbine.

7. Write a short note on semi-closed cycle gas turbine.

OBJECTIVE TYPE QUESTIONS

1. A closed cycle gas turbine works on
   (*a*) Carnot cycle (*b*) Rankine cycle
   (*c*) Ericsson cycle (*d*) Joule cycle
2. In a closed cycle gas turbine, the air is compressed
   (*a*) isothermally (*b*) isentropically
   (*c*) polytropically (*d*) none of these
3. The gas in cooling chamber of a closed cycle gas turbine is cooled at
   (*a*) constant volume (*b*) constant temperature
   (*c*) constant pressure (*d*) none of these
4. A closed cycle gas turbine gives....efficiency as compared to an open cycle gas turbine.
   (*a*) same (*b*) lower (*c*) higher
5. Reheating in a gas turbine
   (*a*) increases the thermal efficiency (*b*) increases the compressor work
   (*c*) increases the turbine work (*d*) decreases the thermal efficiency

ANSWERS

1. (*d*) 2. (*b*) 3. (*c*) 4. (*c*) 5. (*a*)