

# Thermometry

## 1.1 Concept of Heat and Temperature

It is not possible to speak of work in a body. Similarly, it is not possible to speak of heat in a body. Work is either done on a body or by a body. Similarly, heat can flow from a body or to a body. If a body is at a constant temperature, it has both mechanical and thermal energies due to the molecular agitation and it is not possible to separate them. So, in this case, we cannot talk of heat energy. It means, if the flow of heat stops, the word heat cannot be used. It is only used when there is transfer of energy between two or more systems. Therefore, heat can be defined as *energy in transit*.

*Temperature of a system can be defined as the property that determines whether or not the body is in thermal equilibrium with the neighbouring systems.* If a number of systems are in thermal equilibrium, this common property of the system can be represented by a single numerical value called the temperature. It means that if two systems are not in thermal equilibrium, they are at different temperatures.

**Example.** In a mercury in glass thermometer the pressure above the mercury column is zero and volume of mercury measures the temperature. If a thermometer shows a constant reading in two systems *A* and *B* separately, it will show the same reading even when *A* and *B* are brought in contact.

Measurement of temperature of a body accurately is one of the important branches of heat in physics. It also becomes necessary to measure high temperatures and low temperatures. To make this measurement possible, it is necessary to construct a suitable scale of temperature. The scale chosen must be precise and consistent and the temperatures measured on this scale must be accurate. Assessing the temperature of a body by mere sense of touch or comparing the degree of hotness of a body with respect to another body does not help in measuring the temperature quantitatively and accurately.

## 1.2 Thermometry

The branch of heat relating to the measurement of temperature



of a body is called *thermometry*. Thermometer is an instrument used to measure the temperature of a body.

The essential requisites of a thermometer are :

- (1) Construction,
- (2) Calibration, and
- (3) Sensitiveness.

(1) *Construction*. The physical property of a substance plays an important role in the construction of a thermometer. In a mercury thermometer, the principle of expansion of mercury with rise in temperature is used. The platinum resistance thermometer is based on the principle of the change in resistance with change in temperature. The gas thermometer is based on the principle of change in volume or pressure with change in temperature. Thus, for the construction of a thermometer, the proper choice of a substance, whose physical property varies uniformly with rise in temperature, is essential.

(2) *Calibration*. When a thermometer is constructed, it should be properly calibrated. The standard fixed points are selected for calibrating a thermometer. Melting point of ice, boiling point of water, melting point of silver and melting point of gold are taken as fixed points. The scales are built by dividing the interval between the two fixed points into equal parts. Centigrade scale is built by dividing the interval between the melting point of ice and the boiling point of water (under normal pressure) into 100 equal parts and each part represents  $1^{\circ}\text{C}$ . Similarly, Fahrenheit scale is built by dividing this interval into 180 equal parts.

(3) *Sensitiveness*. The instrument, once constructed and calibrated, should also be sensitive. The thermometer will be sensitive if (i) it can detect even small changes in temperature, (ii) it shows the temperature of a body in a short time and (iii) it does not take large quantity of heat for its own heating from the body whose temperature is being measured.

Fixed points	Degrees Celsius
Boiling point of oxygen	-182.97
Ice point	0.00
Steam point	100.00
Boiling point of sulphur	444.60
Melting point of silver	960.80
Melting point of gold	1063.00
Melting point of cobalt	1492.00
Melting point of platinum	1769.00
Melting point of rhodium	1960.00
Melting point of tungsten	3380.00

### 1.3 Types of Thermometers

There are different kinds of thermometers :

(1) *Liquid thermometers*. These thermometers are based on the principle of change in volume of a liquid with change in temperature. Mercury and alcohol thermometers are based on this principle.

(2) *Gas thermometers*. These are based on the principle of change in pressure or volume with change in temperature, e.g., Callendar's constant pressure thermometer, constant volume hydrogen thermometer etc.

(3) *Resistance thermometers*. These are based on the principle of change in resistance with change in temperature, e.g., platinum resistance thermometer.

(4) *Thermo-electric thermometers*. These are based on the principle of thermo-electricity, i.e., production of thermo-E.M.F. in a thermo-couple when the two junctions are at different temperatures.

The various thermocouples commonly used are :—

- (1) Copper and constantan
- (2) Iron and constantan
- (3) Chromel and constantan
- (4) Chromel and alumel
- (5) Platinum and Rhodium.

(6) *Radiation thermometers*. These are based on the quantity of heat radiations emitted by a body e.g., furnaces. These instruments are known as pyrometer.

(7) *Vapour pressure thermometers*. These are based on the principle of change of vapour pressure with change in temperature. These are used to measure low temperatures, e.g., helium vapour pressure thermometer etc.

(8) *Bimetallic thermometers*. These thermometers are based on the principle of expansion of solids. A bimetallic strip is taken in the form of a spiral. Its one end is fixed and the other end is attached to a long pointer. The pointer moves on a scale, calibrated in degrees. These thermometers are used in meteorology for recording the changes in temperature during the day. They are also used to measure temperatures at high altitudes.

(9) *Magnetic thermometers*. These thermometers are based on the principle of change in the susceptibility of a substance with temperature. These thermometers are useful for measuring low temperatures near the absolute zero temperature.

### 1.4 Centigrade and Fahrenheit Scales

The earliest thermometer was constructed by Galileo in 1593. Newton suggested the necessity of the fixed points. The temperature



of the melting point of ice is taken as the lower fixed point and the temperature of steam at a pressure of 76 cm of Hg (normal pressure) is taken as the upper fixed point.

**Centigrade (or Celsius) scale.** Celsius, in 1742, suggested the centigrade system of temperature. He marked zero at the lower fixed point and 100 at the upper fixed point. The interval between the two fixed points is divided into 100 equal parts. Each part or degree represents  $1^\circ\text{C}$  or  $1^\circ$  Celsius. The scale is also known as Celsius scale.

**Fahrenheit scale.** Fahrenheit, in 1720, suggested this scale by taking zero as the temperature of the freezing mixture. It appears that he took 100 degrees as the temperature of the human body. Later the correct temperature of the human body on this scale was found to be  $98.4^\circ\text{F}$ . The lower fixed point is marked as 32 and the upper fixed point is marked as 212. The interval is divided into 180 equal parts. Each part or degree represents  $1^\circ\text{F}$ .

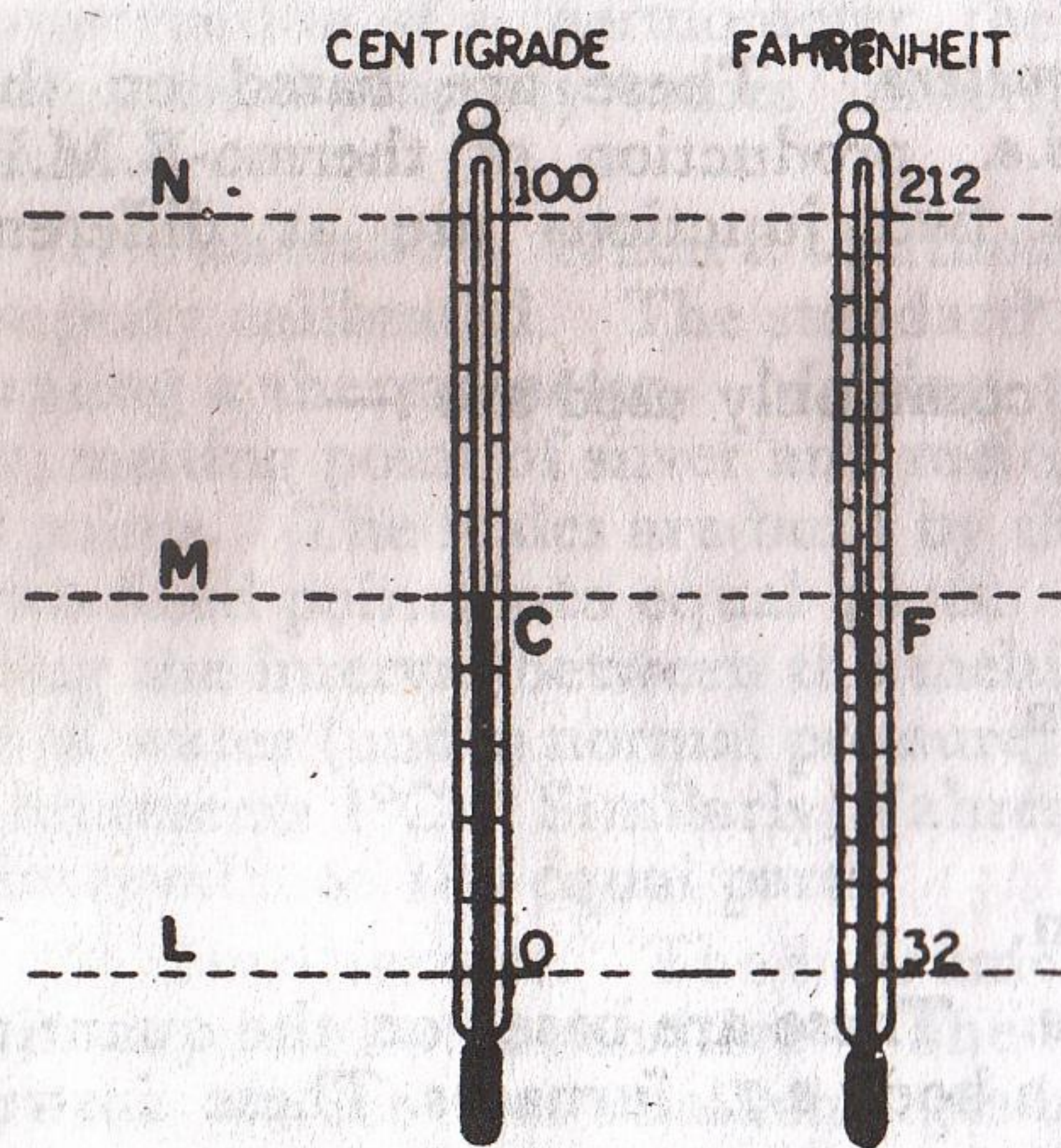


Fig. 1.1

$$\therefore \frac{ML}{NL} = \frac{C-0}{100-0}$$

$$= \frac{F-32}{212-32}$$

$$\text{or } \frac{C}{100} = \frac{F-32}{180}$$

### 1.5. Relation between Celsius, Kelvin, Fahrenheit and Rankine Scales of Temperature

In Fig. 1.2, the temperatures of the absolute zero, the melting point of ice and the boiling point of water as measured on the Celsius (Centigrade), Kelvin (absolute), Fahrenheit and Rankine scales are shown.

Celsius and Fahrenheit scales show the same reading at  $-40^\circ$  i.e.,  $-40^\circ\text{C} = -40^\circ\text{F}$ . The Kelvin scale and the Rankine scale agree at zero degree only.

The temperatures represented in Fig. 1.2, have been rounded off to the nearest degree.

**Relation.**

$$\frac{C-0}{100} = \frac{F-32}{180} = \frac{K-273}{100} = \frac{R-492}{180}$$

$$\text{Also } \frac{K}{100} = \frac{R}{180}$$

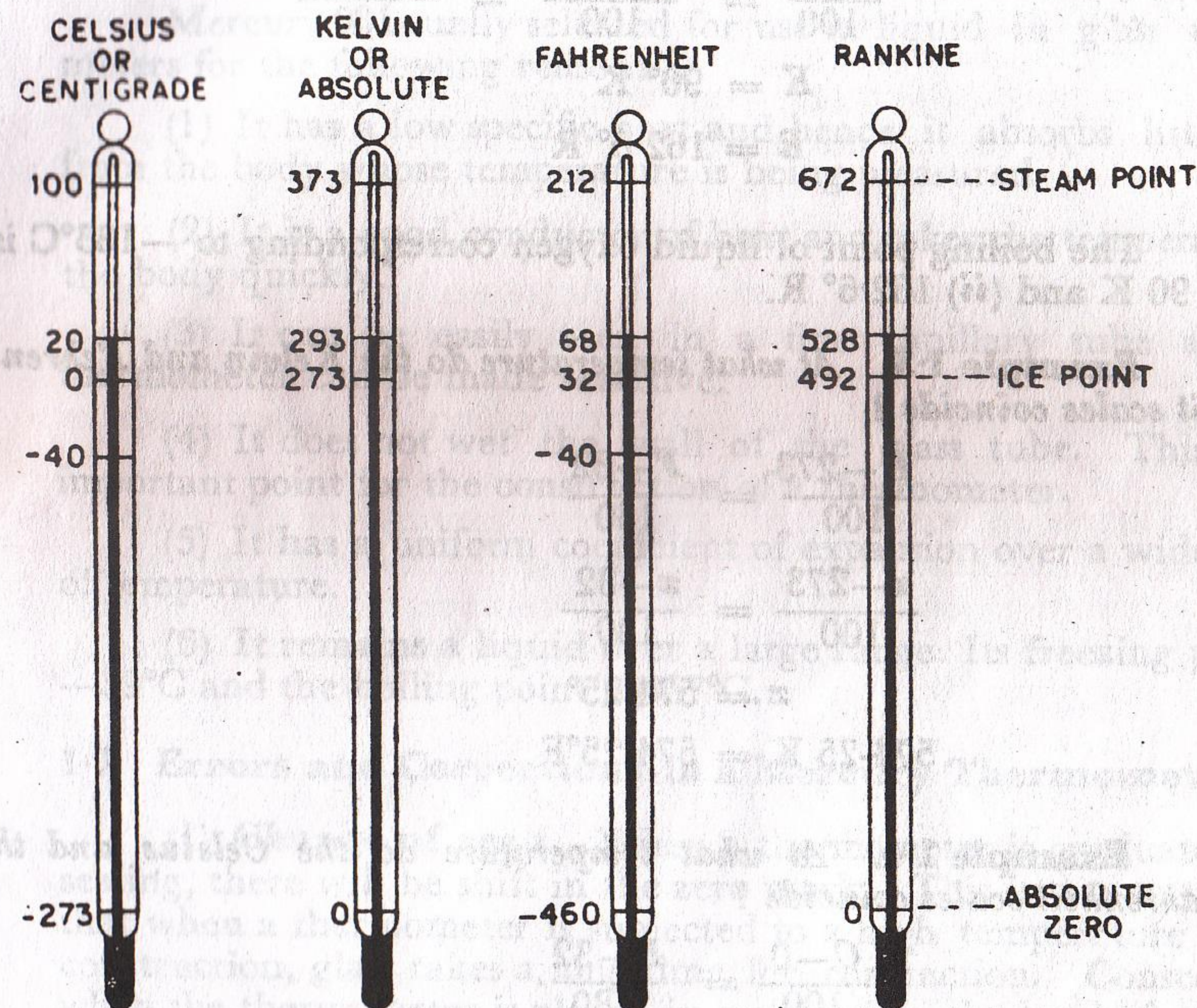


Fig. 1.2

**Example 1.1.** The temperature of the surface of the sun is about  $6500^\circ\text{C}$ . What is this temperature (i) on the Rankine scale and (ii) on the Kelvin scale?

$$\frac{C-0}{100} = \frac{K-273}{100} = \frac{R-492}{180}$$

Here,  $C = 6500^\circ\text{C}$

$$\frac{6500}{100} = \frac{K-273}{100} = \frac{R-492}{180}$$

$$K = 6500 + 273 = 6773 \text{ K}$$

$$R = 12,192^\circ \text{ R}$$

The temperature of the surface of the sun corresponding to  $6500^\circ\text{C}$  is (i) 6773 K and (ii) 12,192° R.



**Example 1.2.** The normal boiling point of liquid oxygen is  $-183^{\circ}\text{C}$ . What is this temperature on (i) Kelvin scale and (ii) Rankine scale?

$$\frac{C-0}{100} = \frac{K-273}{100} = \frac{R-492}{180}$$

Here,  $C = -183^{\circ}\text{C}$

$$\therefore \frac{-183}{100} = \frac{K-273}{100} = \frac{R-492}{180}$$

$$K = 90^{\circ}\text{K}$$

$$R = 162.6^{\circ}\text{R}$$

The boiling point of liquid oxygen corresponding to  $-183^{\circ}\text{C}$  is (i)  $90\text{ K}$  and (ii)  $162.6^{\circ}\text{R}$ .

**Example 1.3.** At what temperature do the Kelvin and Fahrenheit scales coincide?

$$\frac{K-273}{100} = \frac{F-32}{180}$$

$$\frac{x-273}{100} = \frac{x-32}{180}$$

$$x = 574.25^{\circ}$$

$$574.25\text{ K} = 574.25^{\circ}\text{F}$$

**Example 1.4.** At what temperature do the Celsius and the Fahrenheit scales coincide?

$$\frac{C-0}{100} = \frac{F-32}{180}$$

$$\frac{x-0}{100} = \frac{x-32}{180}$$

$$x = -40^{\circ}$$

$$\therefore -40^{\circ}\text{C} = 40^{\circ}\text{F}$$

**Example 1.5.** The boiling point of liquid hydrogen is  $20.2\text{ K}$ . Convert this temperature into degree Rankine.

$$\frac{K-273}{100} = \frac{R-492}{180}$$

Here  $K = 20.2^{\circ}\text{K}$

$$\therefore \frac{20.2-273}{100} = \frac{R-492}{180}$$

$$R = 36.96^{\circ}\text{R}$$

$$\therefore 20.2\text{ K} = 36.96^{\circ}\text{R}$$

## 1.6. Liquid Thermometers

Liquid thermometers are based on the principle of change of volume of the liquid with change in temperature. These thermometers are suitable for a narrow range of measurement. The most commonly used liquids are (i) mercury and (ii) alcohol. The range of mercury thermometer is  $-39^{\circ}\text{C}$  to  $357^{\circ}\text{C}$ . Alcohol thermometers are used only to measure temperature near ice point.

### Mercury Thermometer

Mercury is usually selected for use in liquid in glass thermometers for the following reasons:

- (1) It has a low specific heat and hence it absorbs little heat from the body whose temperature is being measured.
- (2) It is a good conductor of heat and takes the temperature of the body quickly.
- (3) It can be easily seen in a fine capillary tube and the thermometer can be made sensitive.
- (4) It does not wet the wall of the glass tube. This is an important point for the construction of a thermometer.
- (5) It has a uniform coefficient of expansion over a wide range of temperature.
- (6) It remains a liquid over a large range. Its freezing point is  $-39^{\circ}\text{C}$  and the boiling point is  $357^{\circ}\text{C}$ .

## 1.7 Errors and Corrections in a Mercury Thermometer

(1) **Change of zero.** When a thermometer is graduated after sealing, there will be shift in the zero mark. This is due to the fact that when a thermometer is subjected to a high temperature during construction, glass takes a long time for contraction. Consequently when the thermometer is placed in melting ice, the level of mercury will be above the zero mark (say at  $+0.5^{\circ}\text{C}$ ). To correct the error,  $0.5^{\circ}\text{C}$  is subtracted from the observed reading. While manufacturing, the sealed thermometer tubes are allowed sufficient time and are graduated after a number of years.

(2) **Recent heating and cooling.** When a thermometer is used to measure a very high temperature, the thermometer bulb expands. If the same thermometer is immediately used to measure low temperature, the level of mercury will be lower than the correct reading. Suppose the thermometer when placed in melting ice, reads  $-0.3^{\circ}\text{C}$ , the correction to be applied is  $+0.3^{\circ}\text{C}$ .

(3) **Due to exposed stem.** While calibrating a thermometer, the thermometer as a whole is kept in steam. While measuring the temperature, only the bulb is immersed in the bath whose temperature is to be measured. Due to this reason the observed reading is lower than the correct reading. To correct this error, maximum possible length of the stem should be immersed in the bath whose temperature is to be measured.



(4) **Inequality of the bore.** The thermometer is calibrated after marking the lower and the upper fixed points. The distance between the two fixed marks is divided into equal parts assuming that the bore of the thermometer is uniform. If the bore is not uniform, at positions where the bore has lesser diameter, the observed reading is higher than the actual reading. Similarly, when the bore has a higher diameter, the observed reading is less. To correct this error the capillary tube is initially calibrated and a graph is provided which gives the correct readings against the observed readings.

(5) **Position and pressure.** While graduating, if the thermometer was held in the horizontal position and while taking the temperature it is kept in the vertical position, the observed readings will be lower than the correct readings. This is due to the pressure of mercury column. To avoid this error, the thermometer is used in the same position in which it was calibrated.

(6) **Surface tension.** Mercury is depressed in a capillary tube because it does not wet the glass tube. The depression is different at different positions of the stem if the bore is not uniform. To correct this error, the reading of the thermometer is taken with a rising mercury column.

(7) **Thermal capacity.** If the thermal capacity of the bulb is large, a part of the heat from the bath is used in heating the bulb. Therefore, the thermometer reads a lower temperature. To correct this error, small thermometer bulbs with low thermal capacity are used.

(8) **Effect of the bulb.** In a mercury thermometer the glass bulb has an insulating effect. Due to this reason the mercury inside the bulb may not attain the temperature of the bath.

### 1.8 Gas Equation

Consider a perfect gas at a pressure  $P$ , temperature  $T$  and volume  $V$ .

Let the pressure of the gas change from  $P$  to  $P_1$  at constant temperature  $T$  such that the volume changes from  $V$  to  $v$ .

Applying Boyle's law,

$$P_1 v = PV$$

$$\text{or } v = \frac{PV}{P_1} \quad \dots(i)$$

Now, let the temperature of the gas change from  $T$  to  $T_1$  at constant pressure.

Initial Pressure =  $P$ , Volume =  $v$ , Temperature =  $T$

Final Pressure =  $P_1$ , Volume =  $V_1$ , Temperature =  $T_1$

Applying Charles' law,

$$\therefore \frac{v}{T} = \frac{V_1}{T_1} \quad \dots(ii)$$

Substituting the value of  $v$  in equation (ii)

$$\frac{PV}{TP_1} = \frac{V_1}{T_1} \quad \text{or} \quad \frac{PV}{T} = \frac{P_1 V_1}{T_1}$$

$$\therefore \frac{PV}{T} = \text{constant} \quad \dots(iii)$$

**Universal gas constant.** When one gram molecule of a gas at N.T.P. is taken, the constant in equation (iii) is  $R$  and is called 'universal gas constant'. Its value is the same for all gases.

$$\frac{PV}{T} = R \quad \text{or} \quad PV = RT \quad \dots(i)$$

One gram molecule of a gas at N.T.P. has a volume = 22,400 cc  
At normal pressure

$$P = 76 \times 13.6 \times 980 \text{ dynes/sq cm}$$

$$V = 22,400 \text{ cc}$$

$$T = 273 \text{ K}$$

$$R = \frac{76 \times 13.6 \times 980 \times 22,400}{273}$$

$$= 8.31 \times 10^7 \text{ ergs/g-mol-K}$$

$$R = \frac{8.31 \times 10^7}{4.2 \times 10^7} = 1.986 \text{ cal/g-mol-K}$$

For  $n$  gram molecules of a gas, the gas equation is

$$PV = nRT \quad \dots(ii)$$

**Ordinary gas constant.** When one gram of a gas at N.T.P. is taken, the gas constant is  $r$  and is known as ordinary gas constant. Its value is different for different gases.

$$\text{Here } \frac{PV}{T} = r \quad \text{or} \quad PV = rT$$

(i) **For Oxygen.** One gram of oxygen at N.T.P. has a volume

$$= \frac{22,400}{32} \text{ cc}$$

$$r = \frac{PV}{T} = \frac{76 \times 13.6 \times 980 \times 22,400}{273 \times 32}$$

$$= \left( \frac{8.31}{32} \right) \times 10^7 \text{ ergs/g-K}$$

(ii) **For Hydrogen.** One gram of hydrogen at N.T.P. has a volume

$$= \frac{22,400}{2} \text{ cc}$$



$$r = \frac{PV}{T} = \frac{76 \times 13.6 \times 980 \times 22,400}{273 \times 2}$$

$$= \left( \frac{8.31}{2} \right) \times 10^7 \text{ ergs/g-K}$$

Similarly  $r$  for nitrogen

$$= \left( \frac{8.31}{28} \right) \times 10^7 \text{ ergs/g-K}$$

and

$$r \text{ for CO}_2 = \left( \frac{8.31}{44} \right) \times 10^7 \text{ ergs/g-K}$$

In general,

$$r = \frac{R}{\text{Molecular wt. of the gas in grams}}$$

### 1.9 Advantages of a Gas Thermometer

1. The coefficient of expansion of gases is very large as compared to liquids. Therefore, gas thermometers are sensitive.
2. The coefficient and the rate of expansion of all gases are the same under similar conditions.
3. The coefficient of expansion of the material of the bulb of the thermometer is negligible in comparison to the coefficient of expansion of a gas.
4. The gases expand uniformly and regularly over a wide range of temperature.
5. The thermal capacity of a gas is low as compared to liquids. Hence even small changes of temperature can be recorded accurately.
6. Gases can be obtained in a pure form.
7. Gas thermometers can be used over a wide range of temperature. They are suitable to measure high and low temperatures.
8. The temperatures measured with a gas thermometer agree with the temperatures on the thermodynamic scale.

Gas thermometers are not suitable for routine work. They are large and cumbersome. They can be used only in one position. They are mainly used to standardize and calibrate other thermometers.

### 1.10 Callendar's Compensated Constant Pressure Air Thermometer

It is based on the principle that, pressure remaining constant, the volume of a given mass of gas varies directly as its absolute temperature. It consists of a silica bulb  $A$  connected to the reservoir  $R$  containing mercury (Fig. 1.3).  $EF$  is the connecting tube. The compensating bulb  $B$  is exactly similar to the silica bulb  $A$ .  $C_1C_2$  is the compensating tube having a volume equal to the tube  $EF$ .

Initially, the reservoir  $R$  is filled with mercury up to the zero mark and the stop-cock is closed. The bulbs  $A$ ,  $B$  and  $R$  are immers-

ed in melting ice. The tubes are sealed when the pressure on the two sides as shown by the manometer is the same. Therefore, pressure of air in  $A$  and  $B$  is the same.

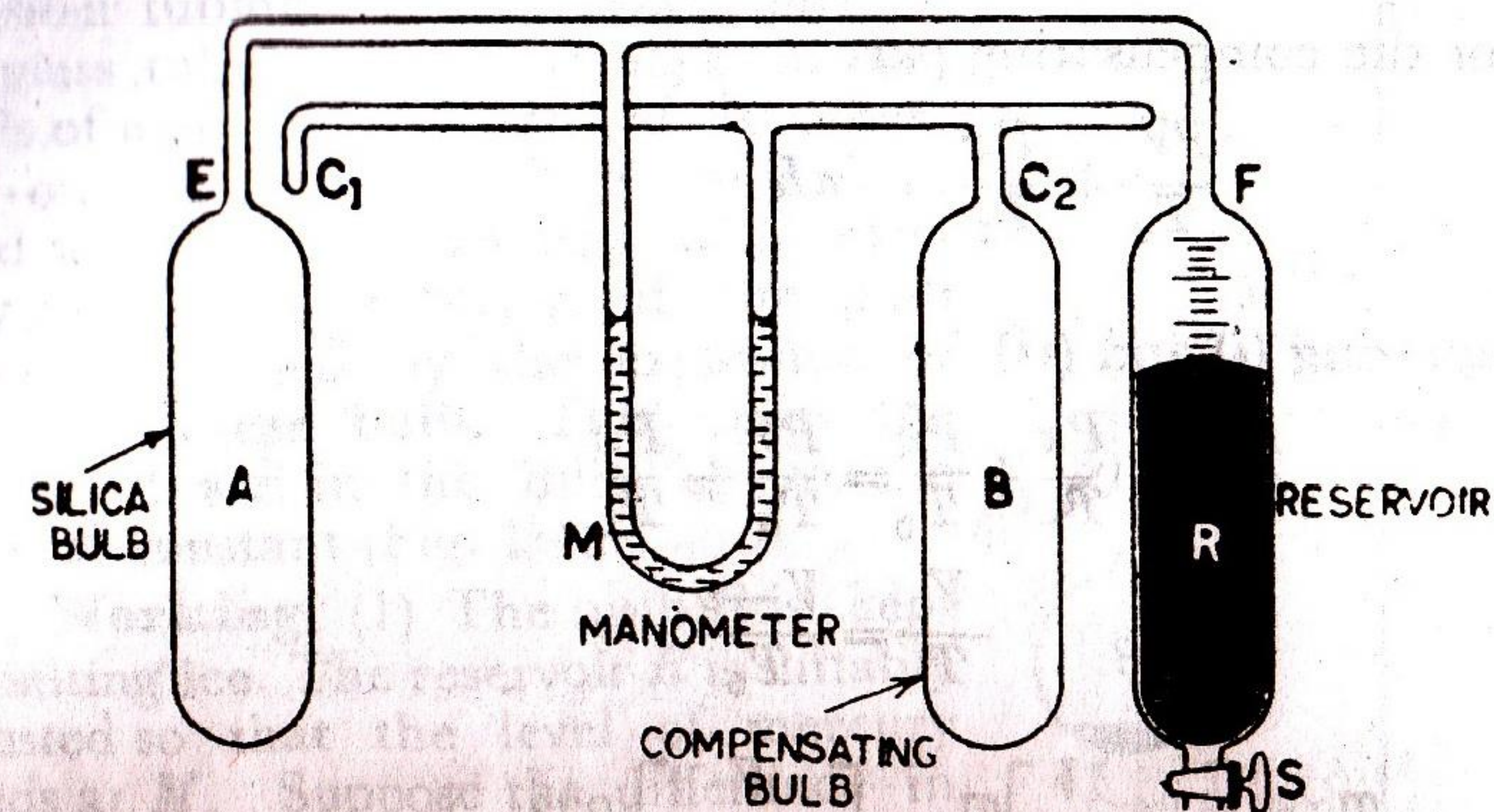


Fig. 1.3

The manometer  $M$  contains sulphuric acid. When the pressure on the two sides of the manometer is the same, the levels of sulphuric acid in the two limbs will be the same. In this way the pressure of the gas or air can be maintained constant at all temperatures.

Suppose, the bulb  $A$  and the connecting tube  $EF$  contain  $n$  gram molecules of air. Also the bulb  $B$  and the compensating tube  $C_1C_2$  contain  $n$  gram molecules of air. The bulb  $A$  is immersed in the bath whose temperature is to be measured. The bulb  $B$  is kept in melting ice. Tubes  $EF$  and  $C_1C_2$  are at the room temperature. The air in the bulb  $A$  attains the temperature of the bath. Pressure of air in  $A$  increases and the manometer shows the difference in level. Mercury is allowed to flow out of the reservoir  $R$  through the stop-cock  $S$  and the stop-cock is closed just at the moment when the pressure on the two sides of the manometer is the same.

$$\text{Volume of the bulb } A = \text{Volume of the bulb } B = V$$

$$\text{Volume of the connecting tube } EF = \text{Volume of the compensating tube } C_1C_2 = x$$

$$\text{Volume of air in the reservoir } R \text{ at the end} = v$$

$$\text{Pressure of air on each side} = P$$

$$\text{Temperature of the bath} = T$$

$$\text{Room temperature} = T_1$$

$$\text{Temperature of melting ice} = T_0$$



For the thermometric part,

$$\frac{PV}{T} + \frac{P_x}{T_1} + \frac{P_v}{T_0} = nR \quad \dots(i)$$

For the compensating part

$$\frac{PV}{T_0} + \frac{P_x}{T_1} = nR \quad \dots(ii)$$

Equating (i) and (ii)

$$\frac{PV}{T} + \frac{P_x}{T_1} + \frac{P_v}{T_0} = \frac{PV}{T_0} + \frac{P_x}{T_1}$$

$$\frac{V}{T} = \frac{V-v}{T_0}$$

$$T = \left[ \frac{V}{V-v} \right] T_0 = \left[ \frac{V}{V-v} \right] 273 \quad \dots(iii)$$

Here  $T$  will be obtained in degrees Kelvin

Let the temperature of the bath be  $t$  °C

Then,  $T = (273 + t)$

$$\therefore 273 + t = \left( \frac{V}{V-v} \right) 273$$

$$t = \left[ \left( \frac{V}{V-v} \right) 273 \right] \text{°C} \quad \dots(iv)$$

This thermometer is useful to measure temperatures upto 600 °C. However, the results obtained with this thermometer are not very accurate.

**Example 1.6.** The bulb of the Callendar's compensated constant pressure air thermometer is 1,000 cm<sup>3</sup>. When the bulb is immersed in a bath, 100 cm<sup>3</sup> of mercury has to be drawn out of the reservoir. Calculate the temperature of the bath on the Celsius scale.

Here  $V = 1,000 \text{ cm}^3$

$v = 100 \text{ cm}^3$

$$t = \left( \frac{V}{V-v} \right) 273$$

$$= \left( \frac{1000}{1000-100} \right) 273$$

$$= 303.3 \text{ °C}$$

### 1.11 Jolly's Constant Volume Air Thermometer

It consists of a glass bulb  $B$  connected to a glass tubing. The end of the glass tube is connected to the reservoir of mercury through a rubber tubing.  $M$  is a fixed mark on the glass tube. The difference in the levels of mercury in  $R$  and  $M$  is observed on the scale  $S$ . The bulb  $B$  is filled with 1/7 of its volume with mercury so that the expansion of the bulb  $B$  is compensated by the expansion of mercury in the bulb. This keeps the volume of air in the bulb up to the mark  $M$  constant (Fig. 1.4).

**Working.** (1) The bulb  $B$  is kept in melting ice. The reservoir  $R$  is suitably adjusted so that the level of mercury stands at  $M$ . Suppose the difference in level between  $R$  and  $M$  is  $h_0$ . If  $P$  is the atmospheric pressure, then

$$P_0 = P + h_0 \quad (i)$$

(2) The bulb  $B$  is kept in boiling water. The reservoir  $R$  is adjusted so that the level of mercury is at  $M$ . Let  $h_{100}$  be the difference in levels.

$$P_{100} = P + h_{100} \quad \dots(ii)$$

(3) The bulb  $B$  is kept in the bath whose temperature is to be measured. The reservoir  $R$  is adjusted so that the level of mercury is at  $M$ . Let  $h_t$  be the difference in levels.

$$\therefore P_t = P + h_t \quad \dots(iii)$$

Volume remaining constant, the pressure increases according to the relation

$$P_t = P_0 (1 + \gamma t)$$

$$\text{Also } P_{100} = P_0 (1 + \gamma 100)$$

$$\text{or } P_t - P_0 = P_0 \gamma t \quad \dots(iv)$$

$$\text{and } P_{100} - P_0 = P_0 \gamma 100 \quad \dots(v)$$

Dividing (iv) by (v)

$$t = \left( \frac{P_t - P_0}{P_{100} - P_0} \right) \times 100 \quad \dots(vi)$$

$$t = \left( \frac{h_t - h_0}{h_{100} - h_0} \right) \times 100 \quad \dots(vii)$$

Hence  $t$  can be calculated.

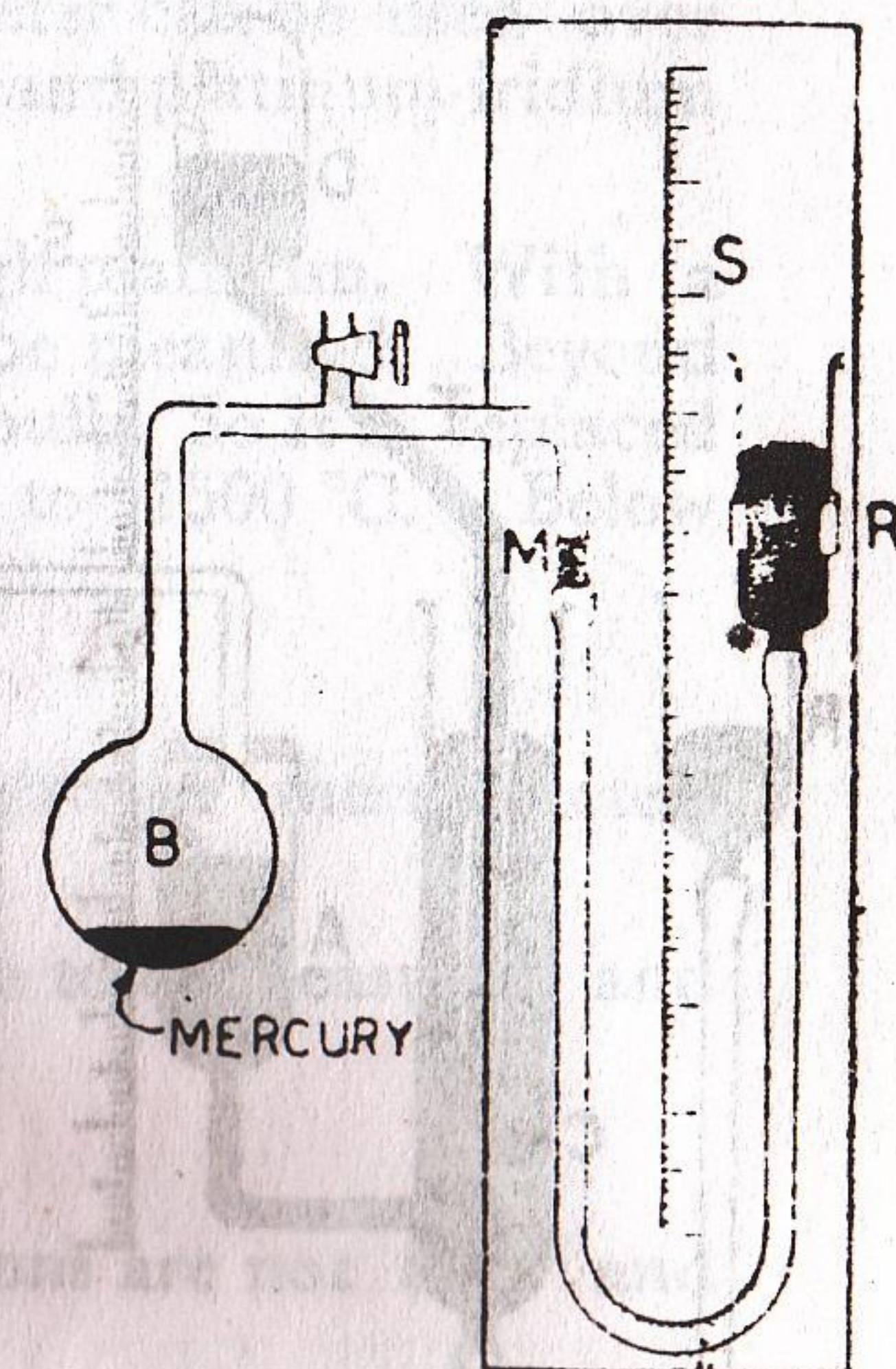


Fig. 1.4



## 1.12 Constant Volume Hydrogen Thermometer

It was first designed by Harker and Chappius and is known as the International standard thermometer. It is based on the principle that when the volume is kept constant, the pressure of a gas increases with rise in temperature according to the relation

$$P_t = P_0 (1 + \gamma t)$$

It consists of a platinum-iridium bulb  $B$  connected to the tube  $A$ . The reservoir  $K$  containing mercury is connected to the tubes  $C$  and  $A$  as shown in Fig. 1.5. A movable barometric tube  $T$  with a bulb  $D$  is clamped to a stand. It can be moved vertically up or down as desired.  $P_1$  and  $P_2$  are the two ivory tips in the same vertical line. Above the level of mercury in  $D$ , there is Toricellian vacuum (Fig. 1.5).

**Working.** Initially the bulb is filled with pure dry hydrogen at a pressure higher than the atmospheric pressure.

(1) The bulb  $B$  is kept in melting ice. The reservoir  $R$  is adjusted so that the level of mercury in  $A$  just touches the tip of the ivory peg  $P_1$ . The tube  $T$  is adjusted so that the level of mercury in  $D$  just touches the tip of the ivory peg  $P_2$ . Suppose the difference in levels of mercury in  $A$  and  $C$  is  $h_0$  and the atmospheric pressure is  $P$ .

Then  $P_0 = P + h_0$ . The difference in levels of mercury in  $C$  and  $D$  is equal to the atmospheric pressure  $P$ . It means the distance between the tips of  $P_1$  and  $P_2$  as measured on the scale (or with the help of a cathetometer) gives  $P_0$  directly.

(2) The bulb  $B$  is kept in steam or boiling water.  $R$  and  $T$  are adjusted so that the level of mercury in  $A$  just touches the tip of  $P_1$  and that in  $D$  just touches the tip of  $P_2$ . The difference in the levels of  $P_1$  and  $P_2$  gives  $P_{100}$ .

(3) Place the bulb  $B$  in the bath whose temperature is to be measured.  $R$  and  $T$  are adjusted in the same way and the difference in levels of  $P_1$  and  $P_2$  gives  $P_t$ .

Here 
$$P_t = P_0 (1 + \gamma t)$$

$$P_{100} = P_0 (1 + \gamma 100)$$

$$\therefore P_t - P_0 = P_0 \gamma t \quad \dots (i)$$

$$P_{100} - P_0 = P_0 \gamma 100 \quad \dots (ii)$$

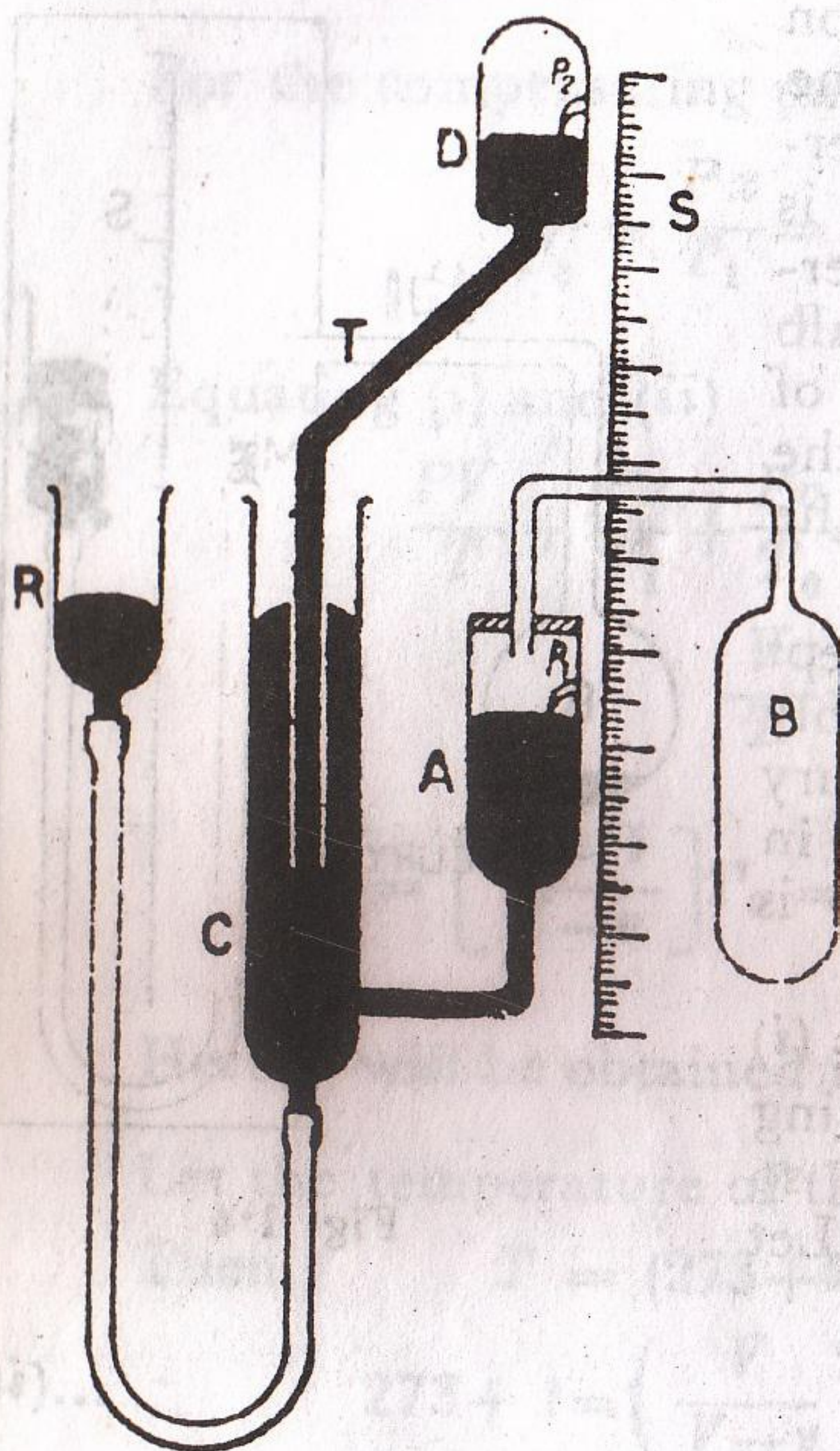


Fig. 1.5

Dividing (i) by (ii),

$$t = \left( \frac{P_t - P_0}{P_{100} - P_0} \right) \times 100 \quad \dots (iii)$$

The constant volume hydrogen thermometer can be used over a wide range of temperature. With hydrogen and platinum-iridium bulb, the range is  $-200^\circ\text{C}$  to  $500^\circ\text{C}$ .

Beyond  $500^\circ\text{C}$ , hydrogen diffuses through platinum. With a porcelain bulb temperatures up to  $110^\circ\text{C}$  can be measured. Beyond  $1100^\circ\text{C}$  hydrogen creeps along the wall of the bulb. So it is replaced by nitrogen for measuring temperatures up to  $1500^\circ\text{C}$ . Below  $-200^\circ\text{C}$  hydrogen is replaced by helium.

## 1.13 Gas Thermometer Corrections

While discussing the constant volume hydrogen thermometer, the following assumptions were made:—

(i) The whole gas attains the temperature to be measured and that the volume of the gas remains constant.

(ii) The gas obeys the perfect gas laws.

In practice, however, these two assumptions are not true and corrections are to be applied.

**First correction.** The main sources of error in a constant volume gas thermometer are:

(i) The gas in the dead space is not at the same temperature as the gas in the bulb. The dead space consists of the space inside the capillary and in the manometer between the mercury level and the index.

(ii) There is increase in the volume of the bulb due to rise in temperature.

(iii) There is a change in the volume of the bulb due to change in pressure.

Let the volume of the bulb at  $0^\circ\text{C}$  be  $V_0$  and  $P_0$  the pressure of the gas at  $0^\circ\text{C}$ . Suppose the volume of the dead space is  $v$  and the temperature of the dead space throughout the experiment is  $\theta^\circ\text{C}$ . The reduced volume of the whole gas at N.T.P.

$$= \left( V_0 + \frac{v}{1 + \gamma \theta} \right) \frac{P_0}{76} \quad \dots (i)$$

Here  $\gamma$  is the coefficient of expansion of the gas.

When the bulb is heated to  $t^\circ\text{C}$ , the pressure of the gas becomes  $(P_0 + h)$ .

The reduced volume of the gas at N.T.P. will be,

$$\left\{ \left( \frac{V_0(1 + \gamma_0 t) + bh}{1 + \gamma t} \right) + \frac{v}{1 + \gamma \theta} \right\} \left( \frac{P_0 + h}{76} \right) \quad \dots (ii)$$

Here,  $\gamma_0$  is the volume coefficient of expansion of the bulb and  $b$  is the volume coefficient of expansion of the bulb due to internal pressure.



Equations (i) and (ii) represent the reduced volume of the gas at NTP at 0 °C and  $t$  °C. As the mass of the enclosed gas is constant, these two volumes must be equal.

Equating (i) and (ii) we get,

$$V_0 P_0 = \frac{\{V_0(1+\gamma_0 t) + bh\}(P_0 + h)}{1+\gamma t} + \frac{vh}{1+\gamma\theta} \quad \dots(iii)$$

$$\text{or } \gamma t = \left(\frac{P_0 + h}{P_0}\right) \left(\frac{V_0(1+\gamma_0 t) + bh}{V_0}\right) + \frac{hv}{P_0 V_0} \left(\frac{1+\gamma t}{1+\gamma\theta}\right) - 1 \quad \dots(iv)$$

Here  $t$  is the temperature of the gas and it occurs on both sides of the equation (iv). In case the above corrections are not taken into account, equation (iv) reduces to,

$$\gamma t = \frac{h}{P_0}$$

$$\text{or } t = \frac{h}{\gamma P_0} \quad \dots(v)$$

Initially the value of  $t$  is calculated from equation (v) and this approximate value is substituted on the right hand side of equation (iv). After substituting, a new value of  $t$  is obtained for the left hand side. This new value is again substituted on the right hand side and the value of  $t$  on the left hand side is once again calculated. This method of successive substitution is performed two or three times depending upon the accuracy desired.

#### 1.14. Second Correction (Perfect or Ideal Gas Scale)

All practical gases *viz.* hydrogen, oxygen, nitrogen etc., show deviations from Boyle's law at high pressures and low temperatures. Hence the temperatures obtained with a constant volume or a constant pressure gas thermometer have to be corrected so as to compensate for the departure from the ideal gas. On the constant volume scale, the temperature  $t_v$ , is written as,

$$t_v = \left[ \frac{P_t - P_0}{P_{100} - P_0} \right] \times 100 \quad \dots(i)$$

Similarly on the constant pressure scale the temperature  $t_p$  is written as

$$t_p = \left( \frac{V_t - V_0}{V_{100} - V_0} \right) \times 100 \quad \dots(ii)$$

If the mass of the gas enclosed is the same in both the thermometers, then at 0 °C, both will have the same pressure  $P_0$  and volume  $V_0$ . In the case of constant volume gas thermometer, when the temperature is increased to  $t$  °C, the pressure becomes  $P_t$ . In the constant pressure gas thermometer, at  $t$  °C, the volume is  $V_t$ . If the gas is ideal, in that case

$$V_0 P_t = (PV)_t = V_t P_0 \quad \dots(iii)$$

Multiplying and dividing the right hand side of equation (i) by  $V_0$ ,

$$t_v = \left[ \frac{(P_t - P_0)V_0}{(P_{100} - P_0)V_0} \right] \times 100$$

$$t_v = \left[ \frac{P_t V_0 - P_0 V_0}{P_{100} V_0 - P_0 V_0} \right] \times 100 \quad \dots(iv)$$

But from equation (iii),

$$P_t V_0 = (PV)_t$$

$$P_0 V_0 = (PV)_0$$

$$P_{100} V_0 = (PV)_{100}$$

and

$$t_v = \left[ \frac{(PV)_t - (PV)_0}{(PV)_{100} - (PV)_0} \right] \times 100 \quad \dots(v)$$

Similarly it can be shown that

$$t_p = \left[ \frac{(PV)_t - (PV)_0}{(PV)_{100} - (PV)_0} \right] \times 100 \quad \dots(vi)$$

Thus for an ideal gas, the temperatures measured on the constant pressure scale and constant volume scale are identical and both can be written as

$$t = \left[ \frac{(PV)_t - (PV)_0}{(PV)_{100} - (PV)_0} \right] \times 100 \quad \dots(v)$$

But in actual practice no gas is ideal.

In the case of real gases, an empirical relation suggested by K-Onnes is

$$PV = A + BP + CP^2 + DP^3 + \dots \quad \dots(vi)$$

Here  $A, B, C, D$  etc., are constants.

These constants are functions of temperature but are independent of pressure. The values of these coefficients are determined by performing compression experiments. These values for oxygen at 0 °C are

$$A = 1.00130$$

$$B = -1.30143 \times 10^{-3}$$

$$C = 3.6898 \times 10^{-6}$$

Here  $P$  is measured in metres of  $Hg$  and when  $P = 1$ ,  $V$  is also equal to one.

It can be seen that the constants  $A, B, C, D$  are of rapidly decreasing magnitude. In actual practice, for the measurement of temperature, constants  $C$  and  $D$  are usually neglected. Therefore,

$$PV = A + BP \quad \dots(vii)$$

Using equation (vii), it is clear that when  $P \rightarrow 0$ ,  $PV = A$  or the gas is ideal and obeys Boyle's law.



In order to obtain the temperatures corresponding to an ideal gas, observations are made with a real gas and the values of the temperature,  $t_{lim}$  when  $P \rightarrow 0$  are obtained. Here  $t_{lim}$  will correspond to the ideal gas temperature.

### Constant Volume Scale

On the constant volume scale

$$t_v = \left( \frac{P_t - P_0}{P_{100} - P_0} \right) \times 100 \quad \dots (viii)$$

From the equation

$$\left. \begin{aligned} PV &= A + BP, \text{ we have} \\ P_0 V_0 &= A_0 + B_0 P_0 \\ P_{100} V_0 &= A_{100} + B_{100} P_{100} \\ P_t V_0 &= A_t + B_t P_t \end{aligned} \right\} \dots (ix)$$

Multiplying and dividing the right hand side of equation (viii) by  $V_0$ ,

$$\begin{aligned} t_v &= \left( \frac{P_t V_0 - P_0 V_0}{P_{100} V_0 - P_0 V_0} \right) \times 100 \\ \therefore t_v &= \left[ \frac{(A_t + B_t P_t) - (A_0 + B_0 P_0)}{(A_{100} + B_{100} P_{100}) - (A_0 + B_0 P_0)} \right] \times 100 \\ t_v &= \left[ \frac{(A_t - A_0) + (B_t P_t - B_0 P_0)}{(A_{100} - A_0) + (B_{100} P_{100} - B_0 P_0)} \right] \times 100 \\ t_v &= \left( \frac{A_t - A_0}{A_{100} - A_0} \right) \times 100 \left[ \frac{\left( 1 + \frac{B_t P_t - B_0 P_0}{A_t - A_0} \right)}{\left( 1 + \frac{B_{100} P_{100} - B_0 P_0}{A_{100} - A_0} \right)} \right] \\ t_v &= \left( \frac{A_t - A_0}{A_{100} - A_0} \right) \times 100 \left( 1 + \frac{B_t P_t - B_0 P_0}{A_t - A_0} \right) \times \\ &\quad \left( 1 - \frac{B_{100} P_{100} - B_0 P_0}{A_{100} - A_0} \right) \\ t_v &= \left( \frac{A_t - A_0}{A_{100} - A_0} \right) \times 100 \left( 1 + \frac{B_t P_t - B_0 P_0}{A_t - A_0} - \frac{B_{100} P_{100} - B_0 P_0}{A_{100} - A_0} \right) \quad \dots (x) \end{aligned}$$

When  $P \rightarrow 0$

$$t_{lim} = \left( \frac{A_t - A_0}{A_{100} - A_0} \right) \times 100$$

$$\therefore t_v = t_{lim} \left( 1 + \frac{B_t P_t - B_0 P_0}{A_t - A_0} - \frac{B_{100} P_{100} - B_0 P_0}{A_{100} - A_0} \right) \quad \dots (xi)$$

$$\text{or } t_{lim} - t_v = t_{lim} \left( \frac{B_{100} P_{100} - B_0 P_0}{A_{100} - A_0} - \frac{B_t P_t - B_0 P_0}{A_t - A_0} \right) \quad \dots (xii)$$

Equation (xii) gives the correction that has to be applied. The value of  $t_v$  is obtained with a thermometer and the value of the constants  $A_0, B_0, A_t, B_t, A_{100}, B_{100}$  are obtained by actual compression experiments.

Initially the value of  $t_v$  is substituted for  $t_{lim}$  on the right hand side of equation (xii) and the correction  $t_{lim} - t_v$  is calculated. This gives a new value of  $t_{lim}$ . This is again substituted on the right hand side and this process of successive substitution is done two or three times. Finally  $t_{lim}$  gives the accurate value of temperature on the **ideal gas scale**.

### Constant Pressure Scale.

On the constant pressure scale

$$t_p = \left( \frac{V_t - V_0}{V_{100} - V_0} \right) \times 100 \quad \dots (xiii)$$

From the equation

$$\left. \begin{aligned} PV &= A + BP \text{ we have} \\ P_0 V_0 &= A_0 + B_0 P_0 \\ P_0 V_{100} &= A_{100} + B_{100} P_0 \\ P_0 V_t &= A_t + B_t P_0 \end{aligned} \right\}$$

Multiplying and dividing the right hand side of equation (xiii) by  $P_0$ ,

$$\begin{aligned} t_p &= \left( \frac{P_0 V_t - P_0 V_0}{P_0 V_{100} - P_0 V_0} \right) \times 100 \\ t_p &= \left[ \frac{(A_t + B_t P_0) - (A_0 + B_0 P_0)}{(A_{100} + B_{100} P_0) - (A_0 + B_0 P_0)} \right] \times 100 \\ t_p &= \left( \frac{A_t - A_0}{A_{100} - A_0} \right) \times 100 \left[ 1 + P_0 \left( \frac{B_t - B_0}{A_t - A_0} \right) - P_0 \left( \frac{B_{100} - B_0}{A_{100} - A_0} \right) \right] \end{aligned}$$

When  $P \rightarrow 0$

$$t_{lim} = \left( \frac{A_t - A_0}{A_{100} - A_0} \right) \times 100$$

$$\therefore t_p = t_{lim} \left[ 1 + P_0 \left( \frac{B_t - B_0}{A_t - A_0} \right) - P_0 \left( \frac{B_{100} - B_0}{A_{100} - A_0} \right) \right]$$

$$\therefore t_{lim} - t_p = t_{lim} P_0 \left[ \left( \frac{B_{100} - B_0}{A_{100} - A_0} \right) - \left( \frac{B_t - B_0}{A_t - A_0} \right) \right] \quad \dots (xiv)$$

Equation (xiv) gives the expression for the correction to be applied. The value of  $t_p$  is obtained with a thermometer. By the method of successive substitution, the value of  $t_{lim}$  is obtained.

From equations (xii) and (xiv) it is clear that in both the cases



the value of

$$t_{lim} = \left( \frac{A_t - A_0}{A_{100} - A_0} \right) \times 100$$

Thus the ideal or perfect gas scale (Celsius scale) can be defined as follows:

$$t = \left[ \frac{(PV)_t - (PV)_0}{(PV)_{100} - (PV)_0} \right]_{\lim P \rightarrow 0} \times 100 \quad \dots (xv)$$

These corrections obtained from equations (xii) and (xiv) are applicable in the range of  $-183^\circ\text{C}$  to  $400^\circ\text{C}$ . Beyond  $400^\circ\text{C}$ , the corrections are negligibly small.

**Example 1.7.** The pressure of air in a constant volume thermometer is 80 cm and 109.3 cm at  $0^\circ\text{C}$  and  $100^\circ\text{C}$  respectively. When the bulb is placed in hot water, the pressure is 100 cm. Find the temperature of the hot water. (Punjab 1966; Delhi 1973)

$$P_t = 100 \text{ cm of Hg}$$

$$P_{100} = 109.3 \text{ cm of Hg}$$

$$P_0 = 80 \text{ cm of Hg}$$

$$t = \left( \frac{P_t - P_0}{P_{100} - P_0} \right) \times 100 = \left( \frac{100 - 80}{109.3 - 80} \right) \times 100$$

$$t = 68.25^\circ\text{C}$$

**Example 1.8.** A constant volume hydrogen thermometer is used to measure the temperature of a furnace. The excess pressure in the bulb over the atmospheric pressure is found to be equal to 152 cm of Hg. At  $0^\circ\text{C}$  the pressure in the bulb is equal to that of the atmosphere. Calculate the temperature of the furnace, assuming that the atmospheric pressure throughout the experiment remains constant.

Here

$$P_0 = 76 \text{ cm of Hg}$$

$$P_t = 152 + 76 = 228 \text{ cm of Hg}$$

$$T_0 = 273 \text{ K}$$

$$T = ?$$

$$\frac{P_t}{T} = \frac{P_0}{T_0}$$

$$T = \frac{P_t}{P_0} \times T_0$$

$$T = \frac{228 \times 273}{76}$$

$$T = 819 \text{ K} = 546^\circ\text{C}$$

### 1.15 Platinum Resistance Thermometer

(It is based on the principle of change of resistance with change in temperature.) It was first designed by Siemen in 1871 and later on improved by Callendar and Griffiths.

A platinum resistance thermometer consists of a pure platinum wire wound in a double spiral to avoid inductive effects. The wire is wound on a mica plate. The two ends of the platinum wire are connected to thick copper leads (for lower temperatures) and connected to the binding terminals  $B_1 B_2$ . For higher temperatures the leads are of platinum.  $C_1$  and  $C_2$  are the compensating leads exactly similar and of the same resistance as the leads used with the platinum wire. (The platinum wire and the compensating leads are enclosed in a glazed porcelain tube. The tube is sealed and binding terminals are provided at the top. The leads pass through mica discs which offer the best insulation and also prevent convection currents (Fig. 1.6).)

(The resistance of a wire at  $t^\circ\text{C} = R_t$  and at  $0^\circ\text{C} = R_0$ . These resistances are connected by the relation

$$R_t = R_0 (1 + \alpha t + \beta t^2) \quad \dots (i)$$

Here  $\alpha$  and  $\beta$  are constants. (The values of  $\alpha$  and  $\beta$  depend on the nature of the material used.) To find the values of  $\alpha$  and  $\beta$ , the resistance of the platinum wire is determined at three fixed points (i) melting point of ice, (ii) boiling point of water, (iii) boiling point of sulphur  $444.6^\circ\text{C}$  for high temperature measurement and (iv) boiling point of oxygen  $-182.5^\circ\text{C}$  for low temperature measurement.

Using these values of resistance in equation (i)

$$R_{100} = R_0 [1 + \alpha 100 + \beta 100^2] \quad \dots (ii)$$

and  $R_{444.6} = R_0 [1 + \alpha 444.6 + \beta (444.6)^2] \quad \dots (iii)$

The values of  $\alpha$  and  $\beta$  can be determined by solving the simultaneous equations (ii) and (iii).

From (i)  $R_t = R_0 [1 + \alpha t + \beta t^2]$

Neglecting  $\beta t^2$  (because  $\beta$  is very small)

$$R_t = R_0 [1 + \alpha t] \quad \dots (iv)$$

and  $R_{100} = R_0 [1 + \alpha \times 100]$

$$\therefore R_t - R_0 = R_0 \alpha t \quad \dots (v)$$

$$R_{100} - R_0 = R_0 \alpha \cdot 100 \quad \dots (vi)$$

Dividing (v) and (vi)

$$\frac{R_t - R_0}{R_{100} - R_0} = \frac{t}{100}$$

$$t = \left( \frac{R_t - R_0}{R_{100} - R_0} \right) \times 100 \quad \dots (vii)$$

Knowing the values of  $R_0$ ,  $R_{100}$  and  $R_t$ ,  $t$  can be calculated.



The resistance of the platinum wire is found accurately using Callendar and Griffiths' bridge.

**Callendar and Griffith's Bridge**

This is a modified form of Wheatstone's bridge, used to measure the change in resistance with temperature of the platinum wire used in a platinum resistance thermometer.

$P$  and  $Q$  are two resistances of equal value and  $S$  is a standard fractional resistance box. The platinum wire whose resistance is to be calculated is connected in one of the arms of the Wheatstone's bridge as shown in Fig. 1.6. The compensating leads are connected in series with the resistance  $S$ .  $XY$  is the bridge wire of length  $2l$  and of resistance per unit length  $\rho$ . Let  $r$  be the resistance of the compensating leads and  $O$  the mid-point of the bridge wire.  $R$  is the resistance of the platinum wire at the temperature of the bath. Let  $D$  be the balance point, with some suitable value of  $S$ ,  $x$  cm to the left of  $O$ .

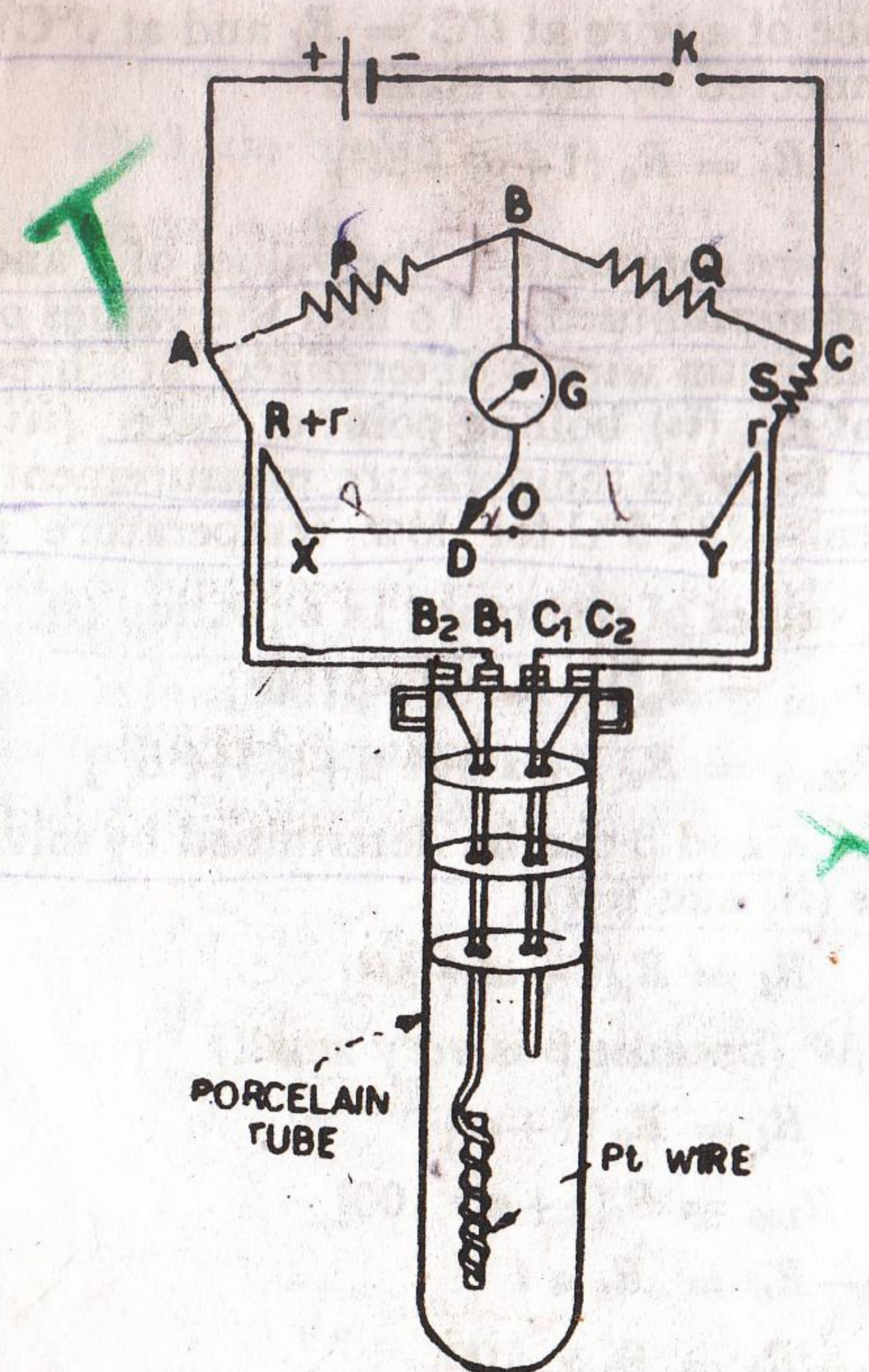


Fig. 1.6

Then length  $XD = l - x$   
 $DY = l + x$

As  $P = Q$ , the resistance of the arm  $AD =$  resistance of the arm  $CD$

$\therefore R + r + (l - x)\rho = S + r + (l + x)\rho$

$$R + r + l\rho - x\rho = S + r + l\rho + x\rho$$

$$R = S + 2x\rho$$

(If the balance point is obtained to the right of  $O$ , it can be shown that  $R = S - 2x\rho$ .)

Thus, using a Callendar and Griffith's bridge the resistance  $R_t$  of the platinum wire can be determined at temperature  $t$ .

A scale can be fixed along the bridge wire  $XY$  to read platinum scale temperatures directly. The range will be different for different values of  $S$ .

**Correction.** The temperature  $t$  measured on the platinum scale according to equation (vii) is not accurate because  $\beta t^2$  term has been neglected. Therefore, the correct thermodynamic temperature  $\theta$  will differ from  $t$ .

Here  $\theta - t = \delta \left[ \left( \frac{\theta}{100} \right)^2 - \left( \frac{t}{100} \right)^2 \right] \dots (viii)$

The value of  $\delta$  can be calculated by knowing the values of  $\alpha$  and  $\beta$ .

Equation (viii) can be derived as follows :

$$\theta - t = \theta - \left( \frac{R_t - R_0}{R_{100} - R_0} \right) \times 100$$

Take

$$R_t = R_0(1 + \alpha\theta + \beta\theta^2)$$

and

$$R_{100} = R_0[1 + \alpha(100) + \beta(100)^2]$$

$$\therefore \frac{R_t - R_0}{R_{100} - R_0} = \frac{\alpha\theta + \beta\theta^2}{\alpha(100) + \beta(100)^2}$$

$$\therefore \theta - t = \theta - \left( \frac{\alpha\theta + \beta\theta^2}{\alpha + \beta(100)} \right)$$

$$\theta - t = \frac{\alpha\theta + 100\beta\theta - \alpha\theta - \beta\theta^2}{\alpha + \beta(100)}$$

$$\theta - t = \frac{100\beta\theta - \beta\theta^2}{(\alpha + \beta(100))}$$

$$\theta - t = \frac{-(100)^2 \beta}{(\alpha + \beta(100))} \left[ \left( \frac{\theta}{100} \right)^2 - \frac{\theta}{100} \right]$$

$$\theta - t = \delta \left[ \left( \frac{\theta}{100} \right)^2 - \left( \frac{t}{100} \right)^2 \right] \dots (ix)$$

Here

$$\delta = \frac{-(100)^2 \beta}{(\alpha + \beta(100))}$$

The value of  $\delta$  for pure platinum is approximately equal to 1.5.

**Advantages.** The platinum resistance thermometer can be used with great accuracy to measure temperatures ranging from  $-200^\circ\text{C}$  to  $1200^\circ\text{C}$ . Its accuracy is  $0.1^\circ\text{C}$ . Once the platinum resistance thermometer has been standardised with a constant volume hydrogen thermometer, it can be used as a standard thermometer.



Its main disadvantage is that the adjustment of the Callendar and Griffiths' bridge takes a long time. Moreover, the wire may not attain the temperature of the bath in a short time.

**Example 1.9.** The resistance of the platinum wire of a platinum resistance thermometer at the ice point is 5 ohms and at the steam point 5.93 ohms. The pressure exerted by the gas in a constant volume gas thermometer is (i) 100 cm of Hg at ice point (ii) 136.6 cm of Hg at the steam point. When both the thermometers are inserted in a hot bath the resistance of the platinum wire is 5.795 ohms and the pressure of the gas is 131.11 cm of Hg. Calculate Celsius temperature of the bath (i) on the platinum scale and (ii) on the gas scale (Berhampur 1972)

(i) On the platinum scale

$$R_0 = 5 \text{ ohms}$$

$$R_{100} = 5.93 \text{ ohms}$$

$$R_t = 5.795 \text{ ohms}$$

$$t_p = \left( \frac{R_t - R_0}{R_{100} - R_0} \right) \times 100$$

$$= \left( \frac{5.795 - 5}{5.93 - 5} \right) \times 100 = 85.48^\circ\text{C}$$

(ii) On the gas scale

$$P_0 = 100 \text{ cm Hg}$$

$$P_{100} = 136.6 \text{ cm Hg}$$

$$P_t = 131.11 \text{ cm Hg}$$

$$t = \left( \frac{P_t - P_0}{P_{100} - P_0} \right) \times 100$$

$$= \left( \frac{131.11 - 100}{136.6 - 100} \right) \times 100 = 85^\circ\text{C}$$

**Example 1.10.** If the platinum temperature, corresponding to  $50^\circ\text{C}$  on the gas scale is  $50.25^\circ$ , what will be the temperature on the platinum scale corresponding to  $150^\circ\text{C}$  on the gas scale?

Let  $\theta$  be the temperature on the gas scale and  $t$  on the platinum scale.

$$\text{Let } \theta - t = \delta \left[ \left( \frac{\theta}{100} \right)^2 - \left( \frac{t}{100} \right)^2 \right]$$

$$\theta = 50^\circ; t = 50.25$$

$$\therefore 50 - 50.25 = \delta \left[ \left( \frac{50}{100} \right)^2 - \left( \frac{50.25}{100} \right)^2 \right] \quad \dots(i)$$

$$(ii) \quad \theta = 150^\circ; t = x$$

$$\therefore 150 - x = \delta \left[ \left( \frac{150}{100} \right)^2 - \left( \frac{x}{100} \right)^2 \right] \quad \dots(ii)$$

Dividing (ii) by (i) and simplifying

$$x = 149.25^\circ\text{C}$$

**Example 1.11.** The resistance of a platinum wire at  $0^\circ\text{C}$ ,  $100^\circ\text{C}$  and  $444.6^\circ\text{C}$  is found to be 5.5, 7.5 and 14.5 ohms respectively. The resistance of a wire at a temperature  $t^\circ\text{C}$  is given by the equation

$$R_t = R_0(1 + \alpha t + \beta t^2)$$

Find the values of  $\alpha$  and  $\beta$ .

Here

$$R_0 = 5.5 \text{ ohms}$$

$$R_{100} = 7.5 \text{ ohms}$$

$$R_{444.6} = 14.5 \text{ ohms}$$

$$R_t = R_0(1 + \alpha t + \beta t^2)$$

$$\therefore R_{100} = R_0[1 + \alpha(100) + \beta(100)^2] \quad \dots(i)$$

$$R_{444.6} = R_0[1 + \alpha(444.6) + \beta(444.6)^2] \quad \dots(ii)$$

$\therefore$  Substituting the values in equations (i) and (ii)

$$7.5 = 5.5[1 + 100\alpha + (100)^2\beta] \quad \dots(iii)$$

$$14.5 = 5.5[1 + 444.6\alpha + (444.6)^2\beta] \quad \dots(iv)$$

Simplifying equations (iii) and (iv)

$$\alpha + 100\beta = \frac{2}{5.5 \times 100} \quad \dots(v)$$

$$\alpha + 444.6\beta = \frac{9}{5.5 \times 444.6} \quad \dots(vi)$$

Subtracting equation (v) from (vi)

$$344.6\beta = \frac{900 - 2 \times 444.6}{5.5 \times 444.6 \times 100}$$

$$\therefore \beta = 1.281 \times 10^{-7}/^\circ\text{C}$$

Substituting the value of  $\beta$  in equation (v)

$$\alpha + 100(1.281 \times 10^{-7}) = \frac{2}{5.50 \times 100}$$

$$\alpha = 363.6 \times 10^{-5} - 1.281 \times 10^{-5}$$

$$\alpha = 362.319 \times 10^{-5}$$

$$\alpha = 3.62 \times 10^{-3}/^\circ\text{C}$$

### 1.16 Seebeck Effect

In 1821, Seebeck found that a current flows in a circuit consisting of two dissimilar metals when one junction is heated while the other junction is kept cold. This was a remarkable experiment be-



cause *no cell* was used in the circuit. He connected a plate of bismuth between copper wires connected to a galvanometer [Fig. 1.7 (i)]. He found that if one of the junctions was heated while the other was kept cold, then a current flowed through the galvanometer. He repeated his experiment by taking a thermo-couple of Fe and Cu [Fig. 1.7 (ii)].

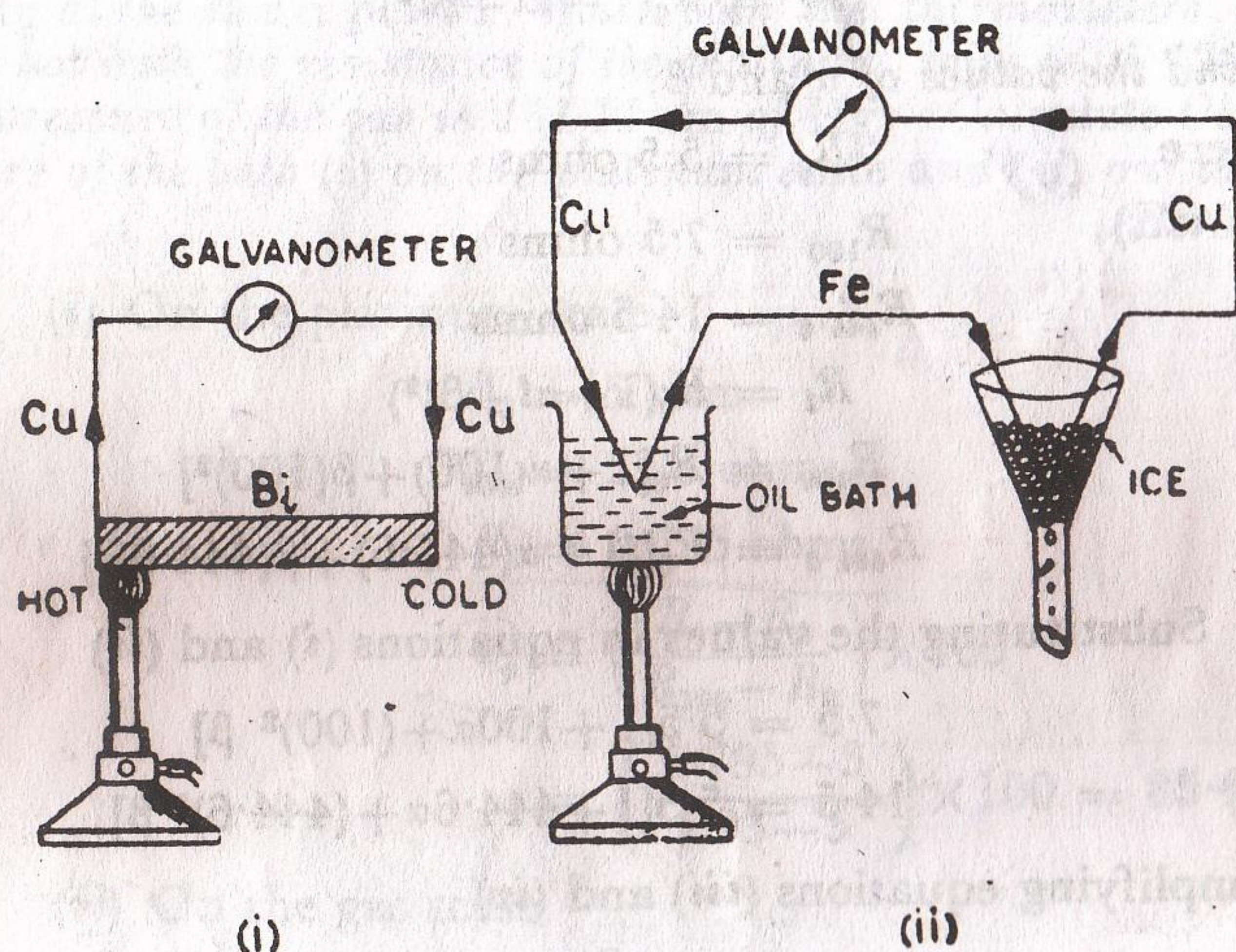


Fig. 1.7

If both the junctions are at  $0^\circ\text{C}$ , there is no deflection in the galvanometer. When one of the junctions is kept at  $0^\circ\text{C}$ , i.e., at the temperature of melting point of ice and the other junction is heated gradually, current flows in the circuit. It was found that current flows from copper to iron at the hot junction and iron to copper at the cold junction. The current increases until the hot junction is at a temperature of  $270^\circ\text{C}$ . If the heating is continued beyond  $270^\circ\text{C}$ , the current decreases and finally at  $540^\circ\text{C}$  the current is zero.

It was discovered by *Cumming* in 1823 that, on increasing the temperature of the hot junction beyond  $540^\circ\text{C}$ , the direction of the current is reversed. It flows from iron to copper through the hot junction and copper to iron through the cold junction.

The current produced in this way without the use of a cell or a battery is known as *thermo-electric current* and this branch of electricity is known as *thermo-electricity*. The effect is known as *Seebeck effect*.

The temperature of the hot junction at which maximum current flows in a circuit is known as *neutral temperature* for that

couple. The neutral temperature for a given thermocouple is fixed and remains constant whatever may be the temperature of the cold junction. The E.M.F. produced in this way is called *thermo-E.M.F.*

If a graph is plotted between the temperature of the hot junction and the thermo-E.M.F., the cold junction being kept at  $0^\circ\text{C}$  the

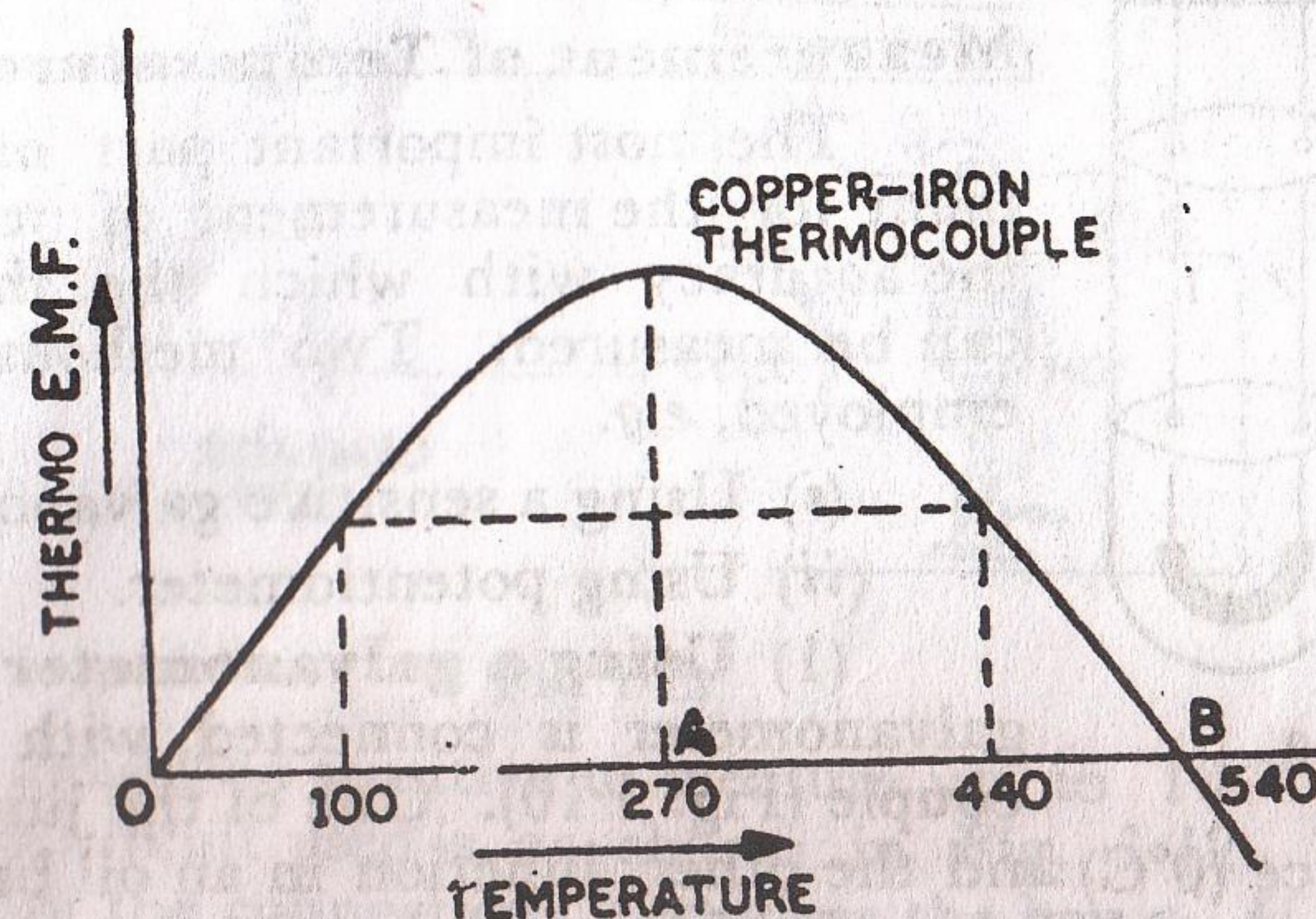


Fig. 1.8

graph is a parabolic curve. The thermo-E.M.F.  $E$  varies with temperature according to  $E = at + bt^2$ , where  $a$  and  $b$  are constants. The point  $A$  represents the neutral temperature. The point  $B$  is the *temperature of inversion*, beyond which the current is reversed (Fig. 1.8).

The temperature of inversion is not fixed. It is as much above the neutral temperature as the cold junction is below the neutral temperature.

Suppose the temperature of the cold junction =  $\theta_1$

Neutral temperature =  $\theta_n$

Temperature of inversion =  $\theta_2$

Then,  $\theta_n - \theta_1 = \theta_2 - \theta_n$ , or  $\theta_n = \left( \frac{\theta_1 + \theta_2}{2} \right)$

In the case of Cu-Fe thermocouple, if the cold junction is at  $100^\circ\text{C}$  temperature of inversion =  $440^\circ\text{C}$ .

### 1.17 Thermo-Electric Thermometer

It is based on the principle of Seebeck effect [Refer article 1.16]. Two dissimilar metals are selected. The thermo-E.M.F. produced, depends upon the difference of temperature between the hot and the cold junctions. For measuring temperatures up to  $300^\circ\text{C}$ , copper-constantan thermocouple is used. Iron-nickel is used between  $300^\circ\text{C}$  and  $600^\circ\text{C}$  and nickel-nichrome is used between  $600^\circ\text{C}$  and  $1000^\circ\text{C}$ . For temperatures between  $1000^\circ\text{C}$  and  $1600^\circ\text{C}$ , a thermocouple of platinum and an alloy of platinum and rhodium is used. Beyond  $1600^\circ\text{C}$ , up to  $2000^\circ\text{C}$  iridium and an alloy of iridium and rhodium is used. For measuring temperatures between  $2000^\circ\text{C}$  and  $3000^\circ\text{C}$ , tungsten and molybdenum thermocouple is used.



The hot junction is made by welding the junction of two metals and the junction is enclosed in a fine capillary tube. The whole arrangement is enclosed in an outer porcelain tube (Fig. 1-9). The wires 1 and 2 inside the tube are kept separate with the help of mica strips.

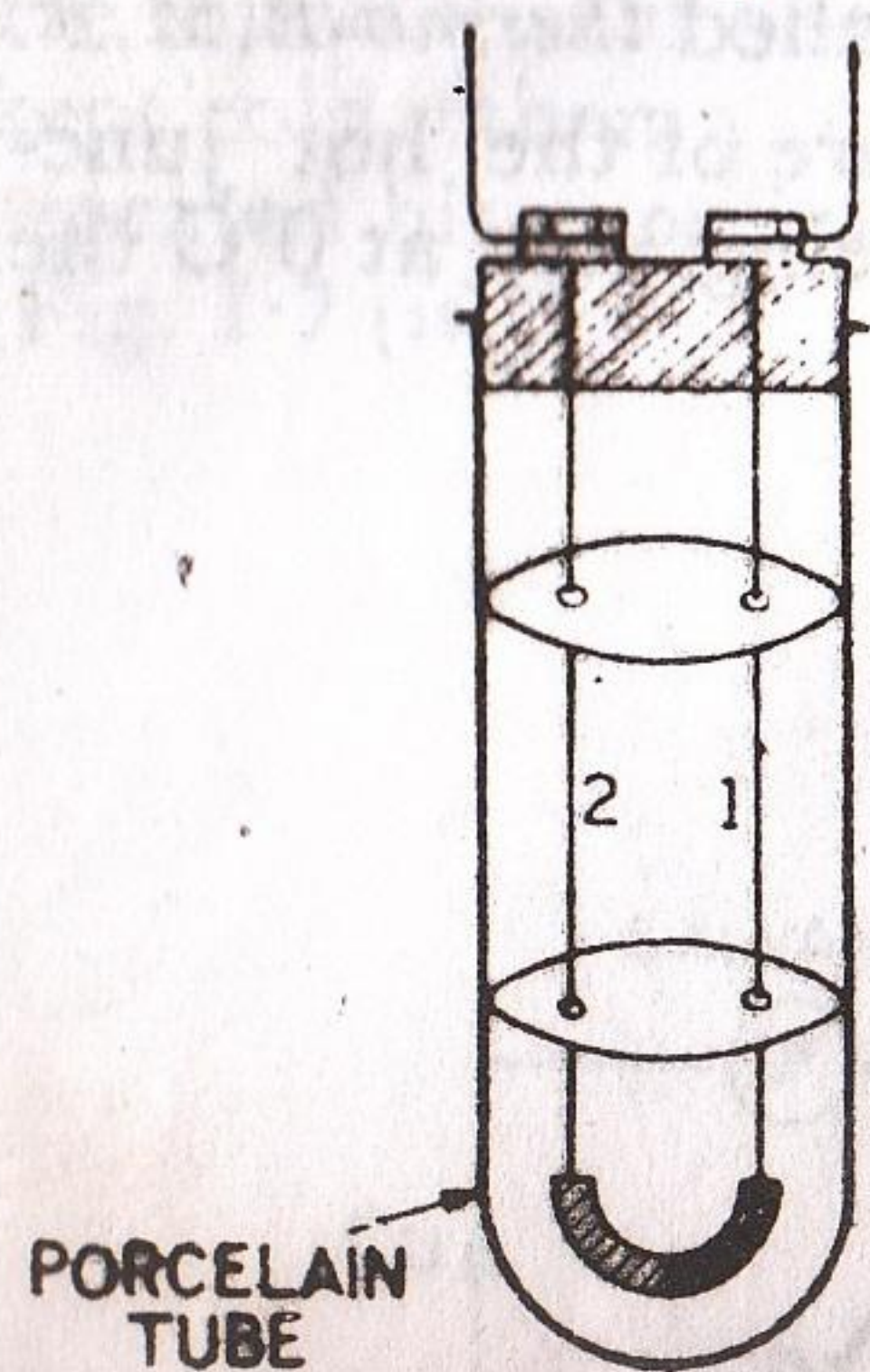


Fig. 1-9

**Measurement of Temperature**

The most important part of the experiment for the measurement of temperature is the accuracy with which the thermo-E.M.F. can be measured. Two methods are usually employed, e.g.

- (i) Using a sensitive galvanometer,
- (ii) Using potentiometer.

(1) **Using a galvanometer.** A sensitive galvanometer is connected with the thermocouple (Fig. 1-10). One of the junctions is kept in melting ice (0°C) and the other junction in an oil bath. The oil bath is heated and thermo-E.M.F. is produced in the circuit. The temperature of the oil bath is determined by using a constant volume hydrogen thermometer or some other sensitive thermometer.

Corresponding to the various temperatures of the hot junction (oil bath) the corresponding deflections in the galvanometer are observed. A graph is plotted between the deflection along the y-axis and the temperature of the hot junction along the x-axis (Fig. 1-11). Thus for the given thermocouple the galvanometer is calibrated.

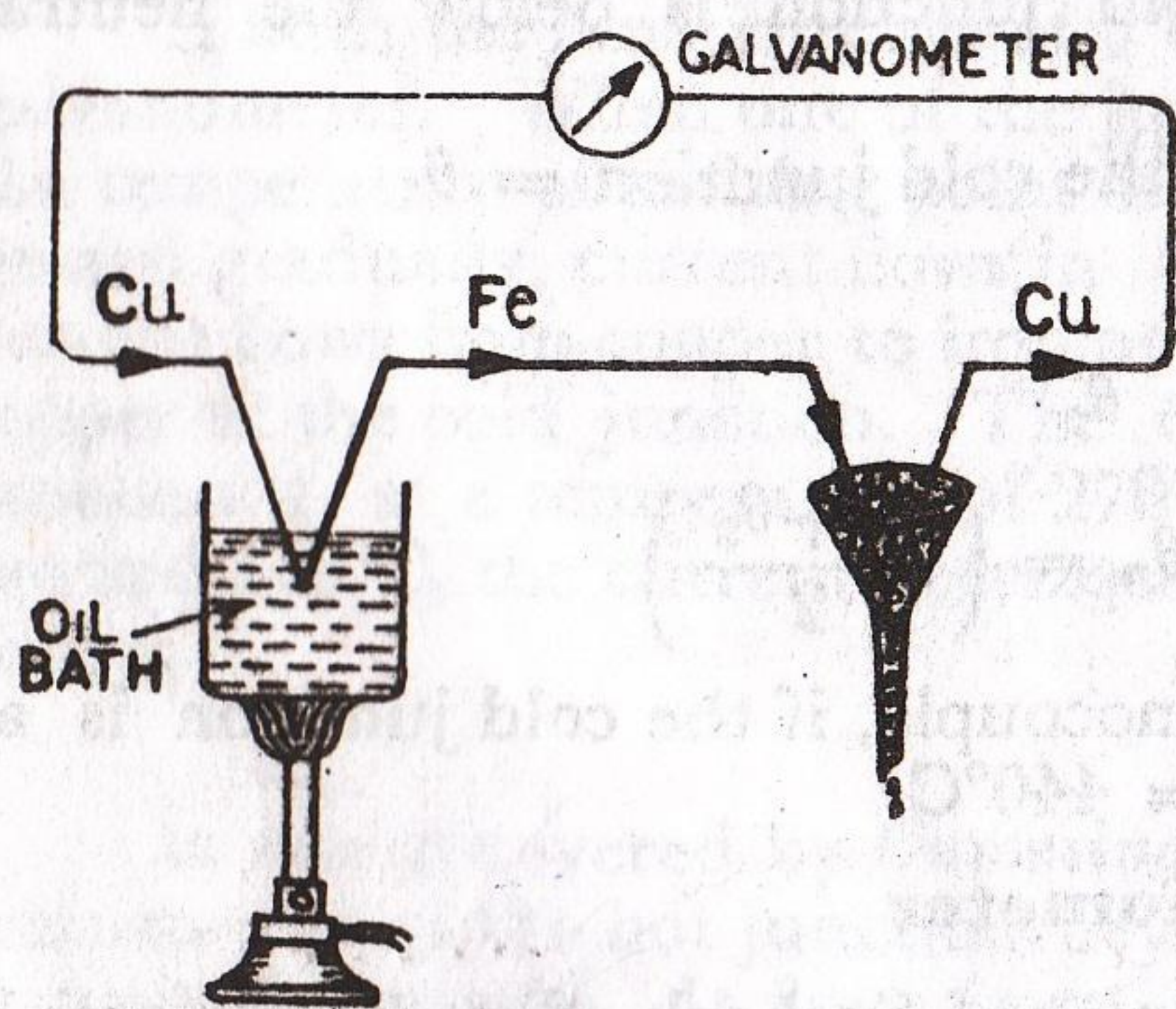


Fig. 1-10

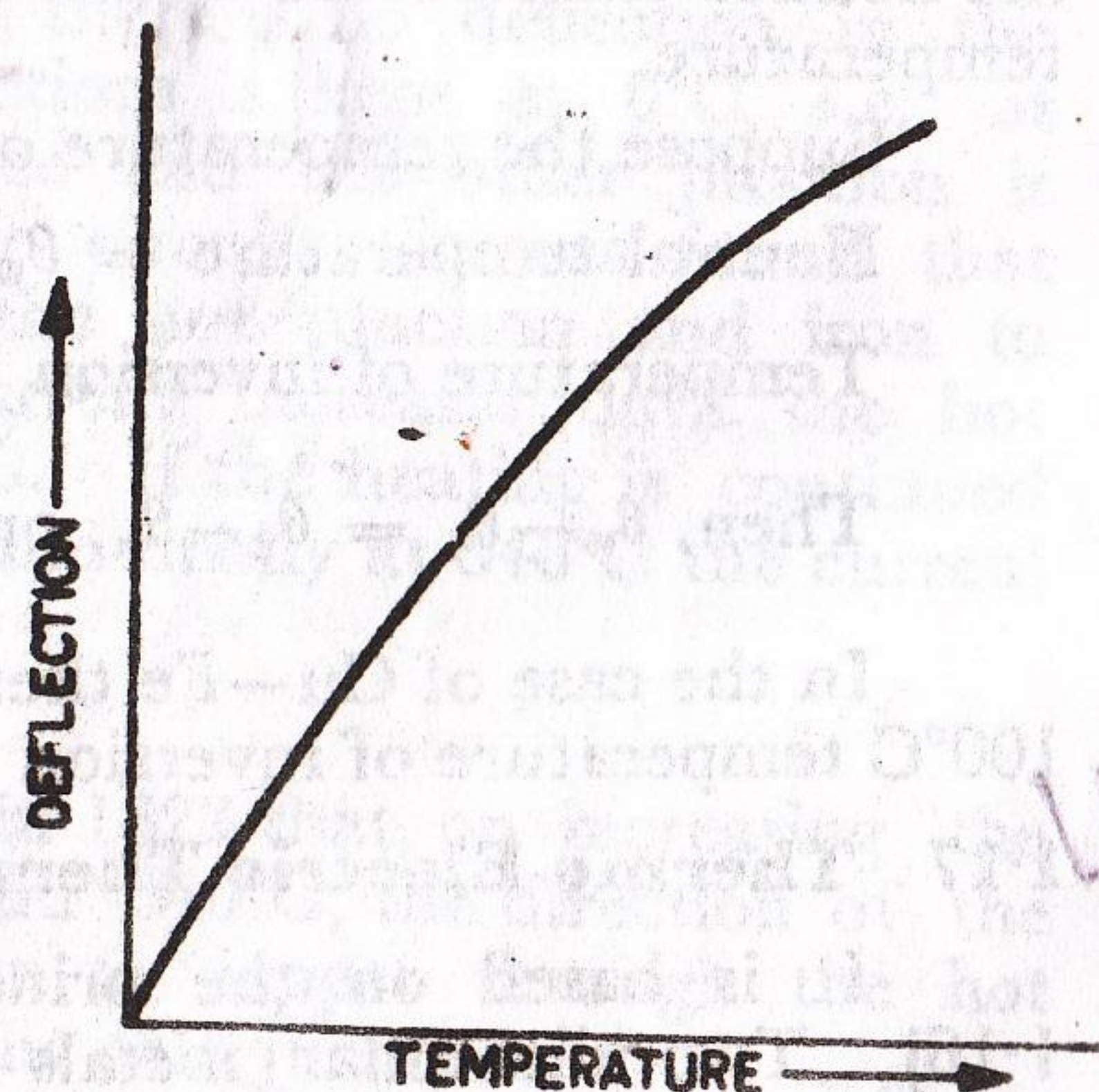


Fig. 1-11

The bath whose temperature is to be measured, is taken. The hot junction of the thermocouple is immersed in the bath, other junction being kept in melting ice. The deflection produced in the galvanometer is observed. From the graph, the corresponding value of temperature is determined. Moreover, the scale of the galvanometer can be directly marked in °C to show the temperatures directly. Such a thermo-electric thermometer is not very sensitive.

(2) **Using a Potentiometer.** The thermo-E.M.F. produced in a thermocouple is of the order of 10<sup>-5</sup> volt. Therefore the

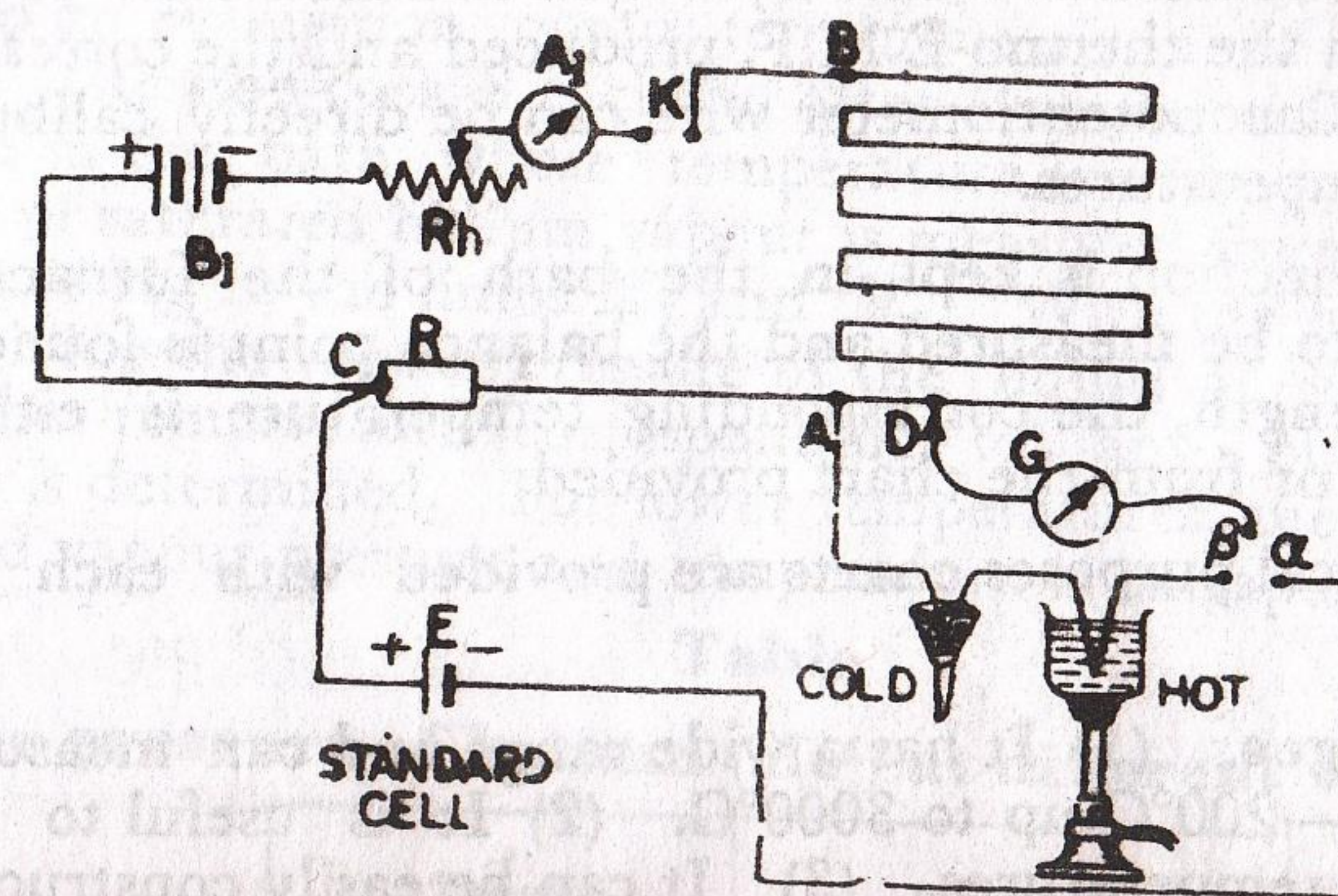


Fig. 1-12

potentiometer should be sensitive to measure up to 10<sup>-6</sup> volt. For this purpose the circuit used is as shown in Fig. 1-12. To check the correctness of the connections, first close the key α. If on closing the key K, the deflection in the galvanometer decreases, then the connections are correct.

The standard cadmium cell E has an E.M.F. = 1.0183 volts. The potentiometer has ten wires of length one metre each. The potentiometer whose wire has a resistance of one ohm per metre is used. The distance AD = 30 cm. The jockey is kept at D.

A resistance of 1018 ohms is introduced in R and the key α is closed. The rheostat Rh is adjusted so that the deflection in the galvanometer is zero. Then, the P.D. across CD = 1.0183 volts.

$$\text{The resistance of } AD = \frac{1 \times 30}{100} = 0.3 \text{ ohm}$$

$$\text{The resistance of } CD = 1018 + 0.3 = 1018.3 \text{ ohm}$$

Fall of potential for one ohm-resistance

$$= \frac{1.0183}{1018.3} = 10^{-3} \text{ volt}$$

The potentiometer wire has a resistance of one ohm per metre. Therefore the fall of potential for one metre length of the potentiometer wire = 10<sup>-3</sup> volt and the fall of potential per mm length of the potentiometer wire

$$= \frac{10^{-3}}{1000} = 10^{-6} \text{ volt per mm.}$$

Thus the potentiometer wire is calibrated.

The key α is taken out and the key β is introduced. For the various temperatures of the hot junction, the corresponding balance points are observed. The lengths are measured from the point A. Thus the corresponding thermo-E.M.F. produced in the circuit can



be calculated. The temperatures of the hot junction are measured with a standard constant volume hydrogen thermometer. A graph is plotted between the thermo-E.M.F. produced and the corresponding temperature. The potentiometer wire can be directly calibrated to measure the temperatures.

The hot junction is kept in the bath of the furnace whose temperature is to be measured and the balance point is found. From the balancing length, the corresponding temperature is either read from the graph or from the chart provided.

For practical purposes charts are provided with each thermo-couple.

**Advantages.** (1) It has a wide range and can measure temperatures from  $-200^{\circ}\text{C}$  up to  $3000^{\circ}\text{C}$ . (2) It is useful to measure rapidly varying temperatures. (3) It can be easily constructed and it is cheap. (4) It is useful to measure the temperature of hot furnaces and does not require any calculations, once it has been calibrated.

**Limits and Drawbacks.** This thermometer does not give accurate temperatures over a wide range. Different thermocouples are to be used for different ranges. The potentiometer is to be calibrated for different thermo-couples separately. Moreover, the neutral temperature of a thermo-couple also limits its range.

### 1.18 Helium Vapour Pressure Thermometer

This thermometer is used to measure temperature up to  $0.7\text{ K}$ . The apparatus consists of a bulb  $A$  containing liquid helium. This bulb is connected to the manometer limbs  $M_1$  and  $M_2$  through a connecting tube  $C$ . The tube  $C$  is surrounded by a copper tube  $B$  to ensure uniform temperature of the vapour,  $R$  is a reservoir containing mercury (Fig. 1.13).

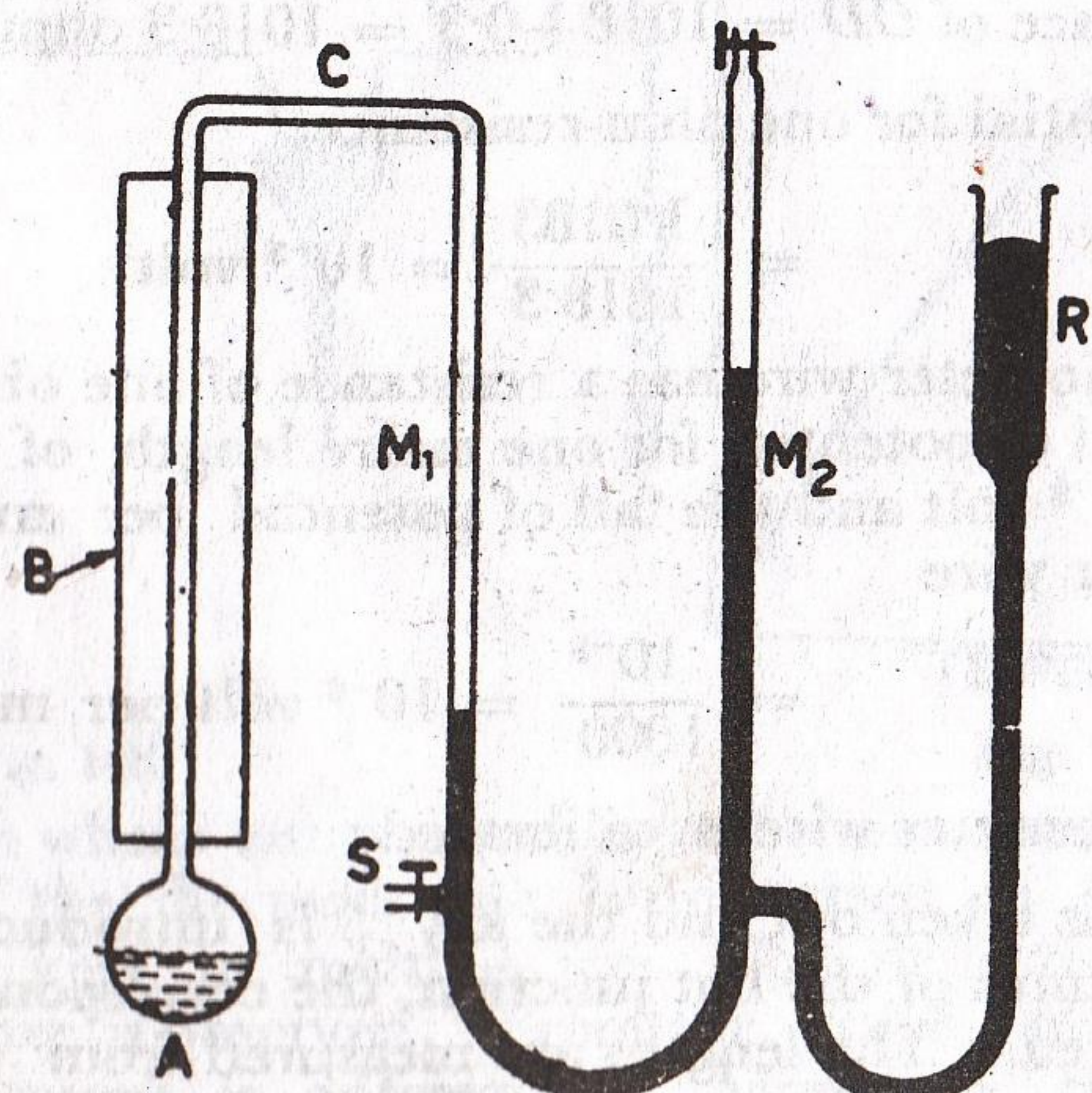


Fig. 1.13

Initially the reservoir  $R$  is lowered so that the mercury in the manometers  $M_1$  and  $M_2$  is below the stop-cock  $S$ . The tube is connected to an evacuation pump to remove air in the tube  $C$  and the bulb  $A$ . The stop-cock  $S$  is closed after evacuation and the bulb  $A$  is placed in the bath whose temperature is to be measured. The pressure of saturated helium vapour is measured from the difference in levels of Hg in the limbs  $M_1$  and  $M_2$ . With the help of constant tables, giving the vapour pressure of the liquid at various temperatures, the temperature corresponding to any observed vapour pressure is determined. For lower temperatures, the graph between saturated vapour pressure and temperature is extrapolated.

Table

Vapour pressure of helium ( $\text{He}^4$ ) at different temperatures

Temp. K	Pressure mm of Hg
5.00	1460
4.50	980
4.00	615
3.50	353
3.00	181
2.50	77
2.00	23.2
1.50	3.6
1.00	0.12
0.40	$1.6 \times 10^{-5}$
0.10	$3.4 \times 10^{-32}$

### 1.19 Standardization and Temperature Scale

The various thermometers depend upon the property of a substance. The temperature scale shown by a thermometer will also depend on the property of the substance selected. Due to this reason, the scale of temperature is arbitrary depending on the property of the substance. The thermometers thus manufactured agree only at the fixed points and not at any other temperature.

Let the magnitude of the property selected be

$$x_t, x_{ice}, x_{steam} \text{ at } t^{\circ}\text{C}, 0^{\circ}\text{C} \text{ and } 100^{\circ}\text{C}.$$

The temperature  $t$  can be calculated from the relation

$$\frac{t}{100} = \frac{x_t - x_{ice}}{x_{steam} - x_{ice}} \quad \dots (i)$$



It means that the temperature scale selected and the property of the substance chosen vary uniformly. But it has been found that the properties of a substance e.g. expansion with rise in temperature in mercury thermometer, change of resistance with temperature in a platinum resistance thermometer etc. do not vary uniformly at all temperatures. Due to this reason, all the thermometers do not show the same temperature readings at one fixed temperature. Suppose the temperature of the liquid is measured by (i) a mercury thermometer (ii) a constant volume hydrogen thermometer and (iii) a platinum resistance thermometer at the same time. All these thermometers will not show the same reading. The readings may be  $40.11^{\circ}\text{C}$ ,  $40^{\circ}\text{C}$ , and  $40.36^{\circ}\text{C}$ . Thus it is found that the temperature shown by the thermometer depends, upon the nature of the thermometer used. However, these thermometers will show the same temperatures at fixed points e.g. melting point of ice and boiling point of water.

#### Comparison of different thermometers

Constant volume hydrogen thermometer	Pt. resistance thermometer	Mercury thermometer
$0^{\circ}\text{C}$	$0^{\circ}\text{C}$	$0^{\circ}\text{C}$
$20^{\circ}\text{C}$	$20.24^{\circ}\text{C}$	$20.09^{\circ}\text{C}$
$60^{\circ}\text{C}$	$60.36^{\circ}\text{C}$	$60.00^{\circ}\text{C}$
$100^{\circ}\text{C}$	$100^{\circ}\text{C}$	$100^{\circ}\text{C}$

In 1887, an international committee of scientists suggested that a scale of temperature shown by constant volume hydrogen thermometer be taken as a standard scale,  $0^{\circ}\text{C}$  being that of melting ice and  $100^{\circ}\text{C}$  being the temperature of steam at normal pressure. But hydrogen also is not a standard or a perfect gas.

Later on it was thought that temperature scale suggested by Kelvin be used as a standard scale because it does not depend upon property of a substance. The Kelvin scale of temperature (K) [work scale] agrees with the absolute gas scale.

As it appears difficult to realize the Kelvin scale in practice it is suggested in 1933 that an international scale of temperature be adopted. The melting point of ice and boiling point of pure water at normal pressure etc. are given the fixed values and the temperatures between these points are calculated by a specific formula. The international scale of temperature is the nearest practical approach to the Kelvin scale of temperature.

#### Fixed Points

(1) **Oxygen point.** Temperature of equilibrium between liquid oxygen and gaseous oxygen at normal pressure =  $-182.97^{\circ}\text{C}$ .

(2) **Ice point.** Temperature of equilibrium between ice and air saturated water at normal pressure =  $0.000^{\circ}\text{C}$ .

(3) **Steam point.** Temperature of equilibrium between liquid water and its vapour at normal pressure =  $100.000^{\circ}\text{C}$ .

(4) **Sulphur point.** Temperature of equilibrium between liquid sulphur and its vapour at normal pressure =  $444.60^{\circ}\text{C}$ .

(5) **Silver point.** Temperature of equilibrium between solid silver and liquid silver at normal pressure =  $960.8^{\circ}\text{C}$ .

(6) **Gold point.** Temperature of equilibrium between solid gold and liquid gold at normal pressure =  $1063^{\circ}\text{C}$ .

The scale is divided into four parts for the purpose of determining the temperature between two fixed points.

(a) **From  $-190^{\circ}\text{C}$  to  $0^{\circ}\text{C}$ .** The platinum resistance thermometer is used and the relation is

$$R_t = R_0 [1 + \alpha t + \beta t^2 - \gamma (t-100) t^3]$$

(b) **From  $0^{\circ}\text{C}$  to  $660^{\circ}\text{C}$ .** The platinum resistance thermometer is used and the relation is

$$R_t = R_0 (1 + \alpha t + \beta t^2)$$

(c) **From  $660^{\circ}\text{C}$  to  $1063^{\circ}\text{C}$ .** The thermocouple of platinum and an alloy of platinum-rhodium is used. One junction is kept in melting ice. The thermo-E.M.F. produced is measured and the relation used is

$$E = \alpha + \beta t + \gamma t^2$$

(d) **Beyond  $1063^{\circ}\text{C}$ .** The ratio of the intensities of monochromatic heat radiation emitted by the black body at  $t^{\circ}\text{C}$  and at  $1063^{\circ}\text{C}$  is determined and the temperature is calculated using Planck's radiation law.

#### 1.20 Absolute Zero and Ice Point

For approximate purposes, the value of absolute zero or zero degree Kelvin is taken as  $-273^{\circ}\text{C}$ . But near about absolute zero its correct determination is a necessity. For accurate work the value of absolute zero is taken as  $-273.16^{\circ}\text{C}$ . Similarly the temperature of melting point of ice was suggested by the International Advisory Committee of Thermometry in 1948, as  $273.15\text{ K}$ . But in 1954, the international committee fixed  $273.16\text{ K}$  (triple point of water) as ice point. Thus for accurate purposes,

$$\text{Ice point} = 0^{\circ}\text{C} = 273.16\text{ K}$$

$$\text{Steam point} = 100^{\circ}\text{C} = 373.16\text{ K}$$

$$\text{Zero degree Kelvin} = -273.16^{\circ}\text{C}$$

#### 1.21 Low Temperature Measurement

The various thermometers used to measure low temperatures are discussed below :

(1) **Liquid Thermometers.** With alcohol thermometer temperatures up to  $-100^{\circ}\text{C}$  can be measured. With mercury in glass, temperatures up to  $-30^{\circ}\text{C}$  can be measured. The accuracy is about  $0.1^{\circ}\text{C}$ . These are convenient and their response is quick.



(2) *Gas Thermometers.* Constant volume hydrogen thermometer can be used to measure up to  $-250^{\circ}\text{C}$  and with constant volume helium thermometer temperatures up to  $-268^{\circ}\text{C}$  can be determined. These thermometers are quite accurate but they are quite bulky.

(3) *Resistance Thermometers.* With platinum resistance thermometers temperatures up to  $-190^{\circ}\text{C}$  can be measured accurately. Its accuracy is  $0.01^{\circ}\text{C}$ . It is quite accurate but its response is slow.

(4) *Thermo-electric Thermometers.* With copper-constantan thermocouple or platinum and silver thermocouple, temperatures up to  $-250^{\circ}\text{C}$  can be measured. Its accuracy is  $0.05^{\circ}\text{C}$ .

(5) *Vapour Pressure Thermometers.* These can be used to measure up to  $-268^{\circ}\text{C}$ . Below  $-268^{\circ}\text{C}$ , helium vapour pressure thermometer is used. It can be used up to  $-272^{\circ}\text{C}$ .

(6) *Magnetic Thermometer.* Near about the absolute zero temperature, magnetic thermometers are used. They are based upon the principle of change in susceptibility with temperature according to Curie's law.

### 1.22 High Temperature Measurement

The various thermometers used to measure high temperatures are given below :

(1) *Liquid Thermometers.* Mercury in glass can measure up to  $300^{\circ}\text{C}$ . If the space above mercury is filled with some inert gas like nitrogen or helium, it can be used up to  $600^{\circ}\text{C}$ .

(2) *Gas Thermometers.* Constant volume hydrogen thermometer having a platinum bulb, can be used to measure up to  $500^{\circ}\text{C}$ . With a porcelain bulb temperatures up to  $1100^{\circ}\text{C}$  can be measured. Beyond  $1100^{\circ}\text{C}$ , using nitrogen in place of hydrogen, temperatures up to  $1500^{\circ}\text{C}$  can be measured. They are quite bulky for use.

(3) *Resistance Thermometers.* The platinum thermometer can be used up to  $1200^{\circ}\text{C}$  when properly calibrated. Its accuracy is  $0.1^{\circ}\text{C}$ . It is a slow measuring instrument.

(4) *Thermo-electric Thermometers.* For measuring up to  $300^{\circ}\text{C}$  copper-constantan thermocouple is used. Iron-nickel thermocouple is used between  $300^{\circ}\text{C}$  and  $600^{\circ}\text{C}$ . Nickel-nichrome thermocouple is used between  $600^{\circ}\text{C}$  and  $1000^{\circ}\text{C}$ . A thermocouple of platinum and rhodium is used between  $1000^{\circ}\text{C}$  and  $1600^{\circ}\text{C}$ . Between  $1600^{\circ}\text{C}$  and  $2000^{\circ}\text{C}$  a thermocouple of iridium and an alloy of iridium and rutherfordium is used. For measuring between  $2000^{\circ}$  and  $3000^{\circ}\text{C}$ , tungsten and molybdenum thermocouple is used.

(5) *Pyrometers.* For measuring the temperature of furnaces and the sun, optical pyrometers are used.

### Exercises 1

1. Write an essay on the measurement of high and low temperatures. (Agra 1960, 1961)
2. State with reasons the type of thermometer which you

consider most suitable for use at temperatures (a)  $-250^{\circ}\text{C}$  (b)  $700^{\circ}\text{C}$  and (c)  $2000^{\circ}\text{C}$ . Indicate briefly the methods of their use.

(Agra 1962)

3. Describe a platinum resistance thermometer. Explain how it works with the help of Callendar and Griffith's bridge. How does the platinum temperature of a body differ from its true temperature ?

(Agra 1963, 1966; Delhi 1960; Lucknow 1963; Berhampur 1972)

4. What do you understand by the absolute scale of temperature ? Is the negative temperature possible on this scale ? (Agra 1964)

5. Describe the Callendar and Griffith's method of determining the temperature coefficient of platinum. In what respects it is superior to Carey Foster's method ? What do you understand by (i) platinum leads (ii) compensating arm (iii) non-inductive winding of the platinum wire ?

(Agra 1965)

6. Give the theory and construction of a constant volume gas thermometer. In what respects is this thermometer superior to mercury in glass thermometer ?

(Punjab 1962)

7. Give an account of the thermoelectric thermometry and discuss the range, sensitivity and usefulness of some important thermocouples.

(Lucknow 1964)

8. Describe carefully the methods for measuring low temperatures in the range  $-100^{\circ}\text{C}$  to  $-273^{\circ}\text{C}$ . Explain the concept of absolute zero of temperature.

(Rajasthan 1964)

9. State the advantages of using a permanent gas as a thermometric substance. Describe the working of a constant volume hydrogen thermometer.

(Madras 1974; Delhi 1968)

10. Describe a resistance thermometer. Explain how it is used to measure temperatures accurately. Discuss its advantages over a thermo-electric thermometer.

(Delhi 1969)

11. Describe and explain how the platinum resistance thermometer can be used to measure temperatures accurately in the region  $200^{\circ}\text{C}$  to  $500^{\circ}\text{C}$ .

(Delhi 1971, 76)

12. Give the construction of a thermoelectric thermometer and compare its performance with a standard gas thermometer. What precautions should be taken in measuring temperatures with a thermoelectric thermometer ?

(Delhi 1972, 76)

13. Describe the construction and working of a standard gas thermometer.

(Delhi 1973, 76)

14. What is meant by a scale of temperature ? On what does the definition of any particular scale depend ?

(Delhi 1975)

15. Give a detailed account of the experiment you would perform to determine the temperature of boiling aniline on the platinum resistance scale of temperature. By how much does the temperature measured by this thermometer differ from that measured by gas thermometer ?

(Delhi 1975)



16. Discuss the advantages of using one of the permanent gases as a thermometric substance for defining a scale of temperature. Describe some convenient and accurate form of gas thermometer. Explain its mode of use and show how the temperature is calculated from the observations made with it. (Delhi 1975)

17. State briefly the principles underlying the working of the following thermometers :

- (i) Constant volume gas thermometer.
- (ii) Resistance thermometer.
- (iii) Thermo-electric thermometer
- (iv) Vapour-pressure thermometer. (Delhi 1974)

18. Describe Callendar and Griffith's bridge for measuring the resistance of a platinum resistance thermometer at various temperatures. Deduce by how much a temperature measured by this thermometer differs from that measured by a gas thermometer. (Delhi 1974)

19. Describe a platinum resistance thermometer. How would you calibrate and use it for measuring the temperature of a body? Mention the advantages of this thermometer. (Delhi 1976)

20. On what principle is the working of a platinum resistance thermometer based? Describe Callendar and Griffiths bridge for the accurate measurement of resistance. How is the true temperature deduced from the measured platinum temperature? (Delhi, 1978)

21. Write short notes on :—

- (i) Callendar's compensated constant pressure air thermometer.
- (ii) Platinum resistance thermometer.
- (iii) Constant volume hydrogen thermometer.
- (iv) Thermo-electric thermometer.
- (v) Measurement of high temperature.
- (vi) Measurement of low temperature.
- (vii) Rankine and Kelvin scales of temperature.
- (viii) Errors and corrections in mercury thermometers.
- (ix) Advantages of gas thermometers.
- (x) Universal gas constant.

22. The temperature of a furnace is  $2000^{\circ}\text{C}$ . What is this temperature (i) on the Rankine scale and (ii) on the Kelvin scale?

[Ans. (i)  $4092^{\circ}\text{R}$  ; (ii)  $2273\text{ K}$ ]

23. The normal boiling point of liquid hydrogen is  $-253^{\circ}\text{C}$ . What is this temperature (i) on the Kelvin scale and (ii) on the Rankine scale?

[Ans. (i)  $20\text{ K}$  ; (ii)  $36.6^{\circ}\text{R}$ ]

24. At what temperature will the Kelvin scale reading be double the Fahrenheit reading?

[Ans.  $176.7^{\circ}\text{F} = 353.4\text{ K}$ ]

25. Find the value of the universal gas constant.

[Ans.  $R = 8.31 \times 10^7$  ergs/per g mol. K]

26. Find the value of the ordinary gas constant for nitrogen.

[Ans.  $r = 2.97 \times 10^6$  ergs per g-K]

27. The resistance of a platinum resistance thermometer at the ice point is 5 ohms and at the steam point is 6.93 ohms. The pressure exerted by the gas in constant volume gas thermometer at the ice point is 100 cm of Hg and at the steam point it is 136.6 cm of Hg. When both the thermometers are placed in a bath, the resistance of a resistance thermometer is 5.795 ohms and the pressure of the gas is 114.9 cm of Hg. Calculate the Celsius temperature of the liquid (i) on the platinum scale and (ii) on the gas scale.

[Ans.  $t_p = 41.19^{\circ}\text{C}$  ;  $\theta = 40.71^{\circ}\text{C}$ ]

28. The bulb of the Callendar's compensated constant pressure air thermometer is  $800\text{ cm}^3$ . When the bulb is immersed in a bath,  $200\text{ cm}^3$  of mercury has to be drawn out of the reservoir. Calculate the temperature of the bath on the Celsius scale.

[Ans.  $33.33^{\circ}\text{C}$ ]

29. If the platinum temperature corresponding to  $60^{\circ}\text{C}$  on the gas scale is  $60.25^{\circ}\text{C}$ , what will be the temperature on the platinum scale corresponding to  $120^{\circ}\text{C}$  on the gas scale?

[Ans.  $119.25^{\circ}\text{C}$ ]

30. If the platinum temperature corresponding to  $60^{\circ}\text{C}$  on the gas scale is  $60.36^{\circ}\text{C}$ , what is the platinum scale temperature corresponding to  $151.7^{\circ}\text{C}$  on the gas scale?

[Ans.  $150^{\circ}\text{C}$ ]



## Expansion

### 2.1 Introduction

All bodies change in size when they are heated. In general all bodies increase in size on heating except water (between 0°C and 4°C) and some aqueous solutions. This increase in the size of a body by heating or thermal expansion is present in solids, liquids and gases. In the case of solids, the increase will be in length, area or volume. In liquids and gases only expansion in volume is possible as they do not possess any fixed shape. In the case of gases, the state of a gas at any instant is dependent on its volume, pressure and temperature. Hence a gas can be heated at constant volume or at constant pressure. The property of thermal expansion of substances is different for different substances and it also depends on the state of the substance *viz.* solid, liquid or gas. Thermal expansion is an important effect of heat and hence it has many practical applications. Construction of thermometers based on the property of expansion of liquids or gases is one of them, and this application has been discussed in the chapter on *Thermometry*. Some other applications of this property are discussed in this chapter.

### 2.2 Expansion of Solids

Solids can be divided into two categories (i) *isotropic* and (ii) *anisotropic*. Metals, glass and some regular systems of crystals come under isotropic solids. These substances have the same properties in all directions and hence have the same coefficient of expansion in all directions. There are certain irregular crystals, which when heated expand in one direction and contract in a perpendicular direction. Hence their coefficient of expansion is different in different directions. They are called anisotropic bodies. The expansion of isotropic solids is discussed in this chapter.

### 2.3 Coefficient of Linear Expansion

The expansion of a solid can be in length, area or volume. Expansion in length is called linear expansion. As different bodies

expand to different extents for the same rise of temperature, the property of linear expansion is denoted by a term called the *coefficient of linear expansion*. The coefficient of linear expansion of a body (designated as  $\alpha$ ) is defined as the increase in length per unit length per unit degree rise of temperature. If  $L_1$  and  $L_2$  are the lengths of a rod at temperatures  $t_1^\circ\text{C}$  and  $t_2^\circ\text{C}$ , then  $\alpha$  the coefficient of linear expansion of the material is given by

$$\alpha = \frac{L_2 - L_1}{L_1(t_2 - t_1)}$$

$$L_2 = L_1[1 + \alpha(t_2 - t_1)]$$

If  $L_0$  and  $L_t$  refer to the lengths of the rod at  $0^\circ\text{C}$  and  $t^\circ\text{C}$ , then

$$\alpha = \frac{L_t - L_0}{L_0 \times t}$$

or

$$L_t = L_0(1 + \alpha t)$$

The equation,  $L_t = L_0(1 + \alpha t)$  is applicable to the temperature range  $0^\circ\text{C}$  to  $100^\circ\text{C}$ . In this range, the expansion of the material can be taken to be uniform. For a wider range of temperature the length  $L_t$  at temperature  $t$  is given by the equation

$$L_t = L_0(1 + \alpha_1 t + \alpha_2 t^2 + \alpha_3 t^3 + \dots)$$

where  $l_0$  is the length of the rod at  $0^\circ\text{C}$  and  $\alpha_1, \alpha_2, \alpha_3$  etc. are coefficients of a rapidly decreasing order.

Since the values of  $\alpha_2, \alpha_3$  etc. are very small, for all practical purposes the relation  $L_t = L_0(1 + \alpha t)$  is used.

### 2.4 Coefficient of Superficial Expansion

It is defined as the increase in area per unit area per unit degree rise of temperature. If  $A_1$  and  $A_2$  are the areas of a solid at temperatures  $t_1$  and  $t_2$  then  $\beta$  the coefficient of superficial expansion is given by the relation

$$\beta = \frac{A_2 - A_1}{A_1(t_2 - t_1)}$$

If  $A_0$  and  $A_t$  are the areas of a solid at  $0^\circ$  and  $t^\circ\text{C}$  respectively then

$$\beta = \frac{A_t - A_0}{A_0 \times t}$$

or

$$A_t = A_0(1 + \beta t)$$

### 2.5 Relation between $\alpha$ and $\beta$

Consider a square plate of side  $L_0$  at  $0^\circ\text{C}$  and  $L_t$  at  $t^\circ\text{C}$ .

Let the area of the plate be  $A_0$  at  $0^\circ\text{C}$  and  $A_t$  at  $t^\circ\text{C}$

$$\text{Then } L_t = L_0(1 + \alpha t) \dots (i)$$

$$A_0 = L_0^2 \dots (ii)$$

$$A_t = L_t^2 \dots (iii)$$

$$\text{and } A_t = A_0(1 + \beta t) \dots (iv)$$

$$L_t^2 = L_0^2(1 + \beta t) \dots (v)$$

Substituting the value of  $L_t$  from equation (i) in (v)

$$L_0^2(1 + \alpha t)^2 = L_0^2(1 + \beta t)$$

$$\text{or } 1 + 2\alpha t + \alpha^2 t^2 = 1 + \beta t$$



Neglecting the term  $\alpha^2 t^2$ ,

$$\begin{aligned} 2\alpha t &= \beta t \\ \beta &= 2\alpha \end{aligned}$$

Thus the coefficient of superficial expansion of a substance is approximately twice its coefficient of linear expansion.

## 2.6 Coefficient of Cubical Expansion

The coefficient of cubical expansion is defined as the increase in volume per unit volume per unit degree rise of temperature. If  $V_1$  and  $V_2$  are the volumes of a substance at temperatures  $t_1$  and  $t_2$ , then  $\gamma$  the coefficient of cubical expansion of the substance is given by

$$\gamma = \frac{V_2 - V_1}{V_1(t_2 - t_1)}$$

If  $V_0$  and  $V_t$  are the values at  $0^\circ\text{C}$  and  $t^\circ\text{C}$  respectively then

$$\gamma = \frac{V_t - V_0}{V_0 \times t}$$

or  $V_t = V_0(1 + \gamma t)$

## 2.7 Relation between $\alpha$ and $\gamma$

Consider a cube of side  $L_0$  at  $0^\circ\text{C}$  and  $L_t$  at  $t^\circ\text{C}$ . Let the volume of the cube be  $V_0$  at  $0^\circ\text{C}$  and  $V_t$  at  $t^\circ\text{C}$

Then  $L_t = L_0(1 + \alpha t)$  ... (i)

$V_0 = L_0^3$  ... (ii)

$V_t = L_t^3$  ... (iii)

and  $V_t = V_0(1 + \gamma t)$  ... (iv)

$L_t^3 = L_0^3(1 + \gamma t)$

Substituting the value of  $L_t$  from equation (i) in (iv)

$$L_0^3(1 + \alpha t)^3 = L_0^3(1 + \gamma t)$$

or  $1 + 3\alpha t + 3\alpha^2 t^2 + \alpha^3 t^3 = 1 + \gamma t$

Neglecting the higher powers of  $\alpha$

$$3\alpha t = \gamma t$$

or  $\gamma = 3\alpha$

Thus the coefficient of cubical expansion is approximately three times its coefficient of linear expansion.

As a close approximation, the ratio between the three coefficients is

$$\alpha : \beta : \gamma = 1 : 2 : 3$$

As it is somewhat difficult to determine  $\beta$  and  $\gamma$  directly, for solids, the value of  $\alpha$  is determined and the values of  $\beta$  and  $\gamma$  are taken approximately equal to  $2\alpha$  and  $3\alpha$  respectively.

## 2.8 Change of Density of a substance with Temperature

When a solid is heated its volume increases and consequently the density of the substance decreases.

Let  $m$  be the mass of a solid. Let  $v_1$  and  $v_2$  be the volumes of the solid at the temperatures  $t_1$  and  $t_2^\circ\text{C}$ . Let  $\rho_1$  and  $\rho_2$  be the cor-

responding densities. Then, since the mass of a solid remains constant at the two temperatures.

$$m = v_1 \rho_1 = v_2 \rho_2$$

$$\frac{v_1}{v_2} = \frac{\rho_2}{\rho_1}$$

But  $v_1 = v_0(1 + \gamma t_1)$

and  $v_2 = v_0(1 + \gamma t_2)$

$$\frac{v_2}{v_1} = \frac{1 + \gamma t_2}{1 + \gamma t_1}$$

or  $\frac{\rho_1}{\rho_2} = \frac{1 + \gamma t_2}{1 + \gamma t_1}$

or  $\rho_1 = \rho_2[(1 + \gamma t_2)(1 + \gamma t_1)^{-1}]$

$$= \rho_2[1 + \gamma t_2 - \gamma t_1] \text{ (neglecting the } \gamma^2 \text{ term)}$$

$$\rho_1 = \rho_2[1 + \gamma(t_2 - t_1)]$$

$$\rho_2 \gamma(t_2 - t_1) = \rho_1 - \rho_2$$

or  $\gamma = \frac{\rho_1 - \rho_2}{\rho_2(t_2 - t_1)}$

## 2.9 Coefficient of Linear Expansion (Spherometer Method)

This is the simplest method of determining the coefficient of linear expansion in the laboratory. The specimen whose coefficient of linear expansion is to be measured is taken in the form of a rod and its initial length ( $L_1$ ) is measured at room temperature. The rod is placed in the linear expansion apparatus shown in Fig. 2.1. The lower end  $B$  of the rod rests against a fixed support and the upper end  $A$  is free to expand. The central screw of the spherometer is just above the end  $A$  of the rod. Initially, the central leg of the spherometer is rotated gently so that, it is just in contact with the upper end. The just position of contact can be judged with the help of an electrical circuit shown in the figure. The point of contact serves as the electrical switch and when the contact is made the bulb in the circuit glows. The initial reading  $a$  of the spherometer is noted. Let the room temperature be  $t_1^\circ\text{C}$ . The spherometer screw is slightly rotated up so that there is a gap for expansion of the end  $A$ .

Now steam is passed through the outer jacket  $C$  continuously till the temperature of the enclosure becomes constant and equal to the temperature of steam ( $t_2^\circ\text{C}$ ). After the steady state is reached, the spherometer screw is

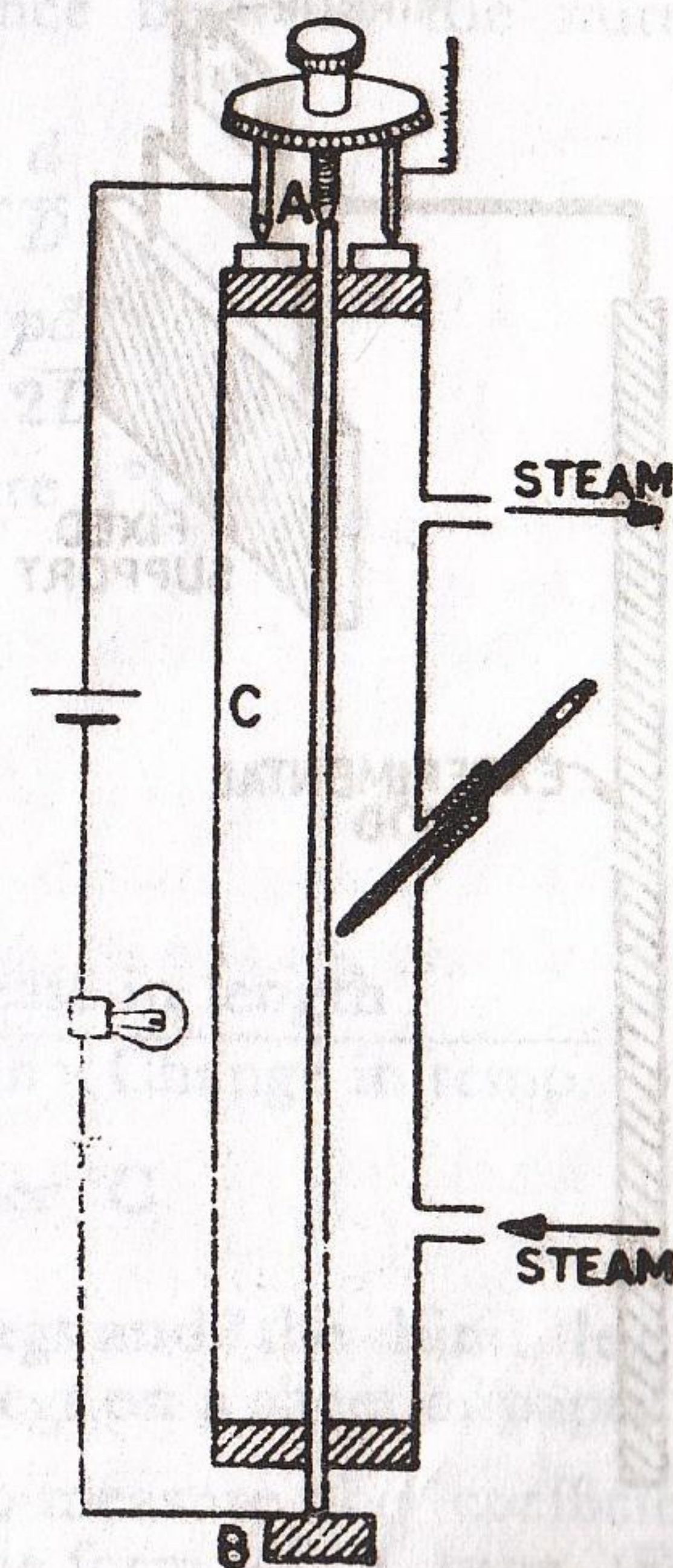


Fig. 2.1



rotated down so that it comes in contact with the end *A* of the rod. Let this reading of the spherometer be *b*.

The coefficient of linear expansion  $\alpha$  can be calculated as follows :

Initial length of the rod at temperature  $t_1^\circ\text{C} = L_1$

Initial temperature =  $t_1^\circ\text{C}$

Initial reading of the spherometer = *a*

Final temperature =  $t_2^\circ\text{C}$

Final reading of the spherometer = *b*

Increase in length =  $(b-a)$

Coefficient of linear expansion

$$\alpha = \frac{\text{Increase in length}}{\text{Original length} \times \text{Change in temp.}}$$

$$= \frac{(b-a)}{L_1(t_2-t_1)} \text{ per } ^\circ\text{C}$$

This method can also be employed to measure the coefficient of linear expansion of a tube.

### 2.10 Optical Lever Method

This is a more sensitive method for determining the coefficient of linear expansion of the material of a rod in the laboratory. If

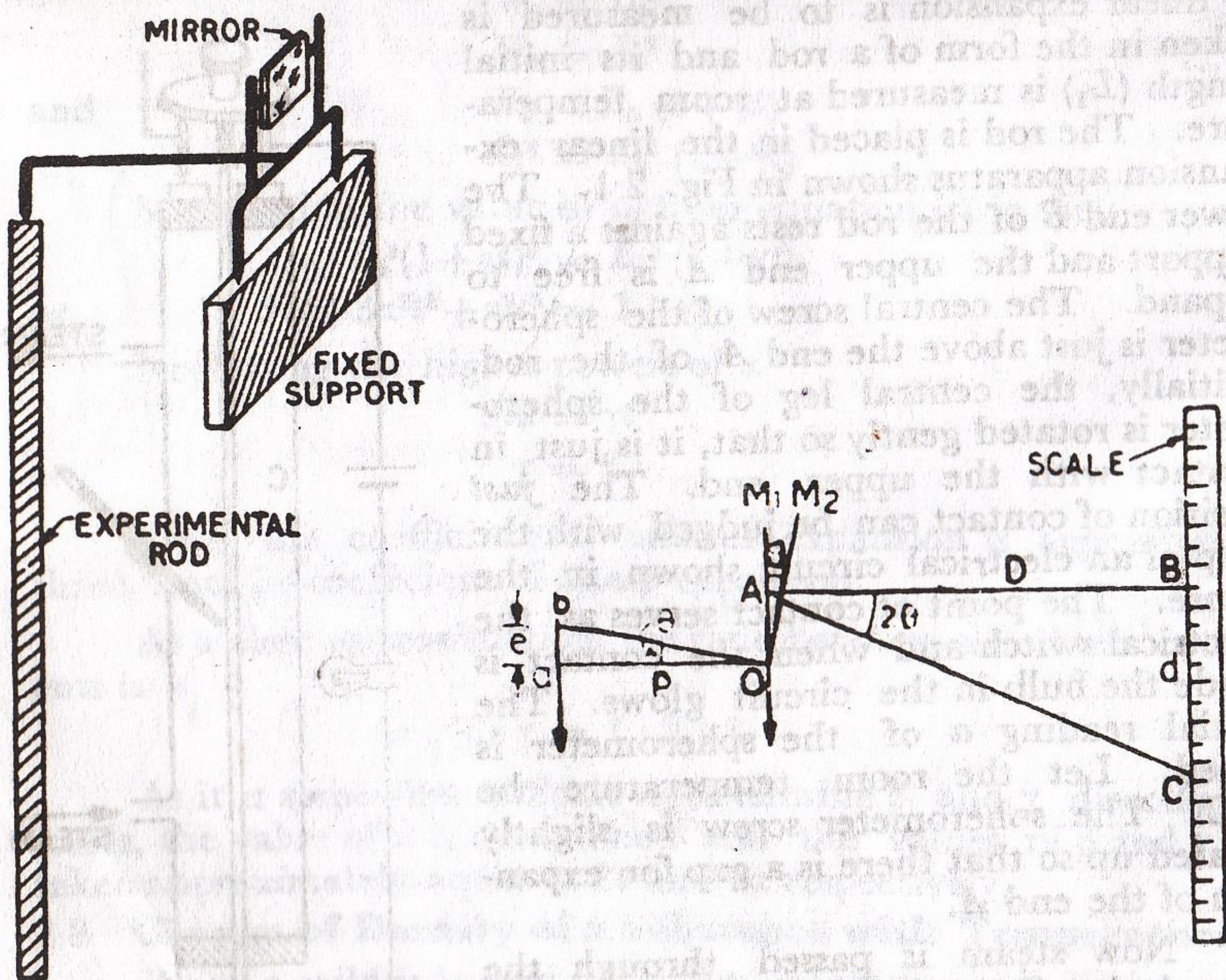


Fig. 2.2

the apparatus is rigid, this method can give very accurate values of  $\alpha$ . This apparatus is the same as in the previous article except that the spherometer is replaced by an optical lever (Fig. 2.2).

The apparatus is shown in Fig. 2.2. The image of the scale in the mirror is viewed with a reading telescope. Let the initial reading of the scale corresponding to the position of the horizontal cross-wires be  $d_1$ .

Now, steam is passed through the outer jacket till the temperature of the whole enclosure becomes constant and equal to the temperature of steam. Due to heating, the rod expands and consequently the leg of the optical lever resting on the rod is tilted up by a distance *e*, equal to the expansion of the rod. Let the mirror of the optical lever be rotated to the position  $OM_2$  by an angle  $\theta$ . Let the reading of the scale, when the temperature of the enclosure is constant, be  $d_2$ . When a mirror is rotated by an angle  $\theta$ , the reflected ray is rotated through  $2\theta$ .

$$\therefore \angle BAC = 2\theta$$

$$\text{In the } \triangle Oab, \quad \theta = \frac{ab}{p} = \tan \theta = \frac{e}{p}$$

where *p* is the perpendicular distance between the front legs and the hind leg.

In the  $\triangle BAC$ ,

$$2\theta = \tan 2\theta = \frac{BC}{D} = \frac{d_2-d_1}{D} = \frac{d}{D}$$

(Here the angle  $\theta$  is small and hence the tangent values can be taken equal to the angles). *D* is the distance between the mirror and the scale and  $d = (d_2-d_1)$

$$\theta = \frac{e}{p} \text{ and } 2\theta = \frac{d}{D}$$

$$\therefore \frac{2e}{p} = \frac{d}{D} \text{ or } e = \frac{pd}{2D}$$

Initial length of the rod at temperature  $t_1^\circ\text{C} = L_1$

Initial temperature =  $t_1^\circ\text{C}$

Final temperature =  $t_2^\circ\text{C}$

Increase in length  $e = \frac{pd}{2D}$

Coefficient of linear expansion

$$\alpha = \frac{\text{Increase in length}}{\text{Original length} \times \text{Change in temp.}}$$

$$= \frac{pd}{2DL_1(t_2-t_1)} \text{ per } ^\circ\text{C}$$

*p* the distance between the front legs and the hind leg is measured by taking the impressions of the legs on a sheet of paper.

The optical lever method is suitable to measure the coefficient of linear expansion of a material given in the form of a tube (Fig. 2.3). One end of the tube is fixed and the other end of the tube is free to expand. The optical lever is fixed over the free end as explained earlier. The tube is covered with cotton or felt so that heat



is not lost to the surroundings. Observations are taken as explained earlier in this article and the value of the coefficient of linear expansion is calculated.

The coefficient of linear expansion of a rod or a tube can also be determined by using a screw gauge instead of a spherometer.

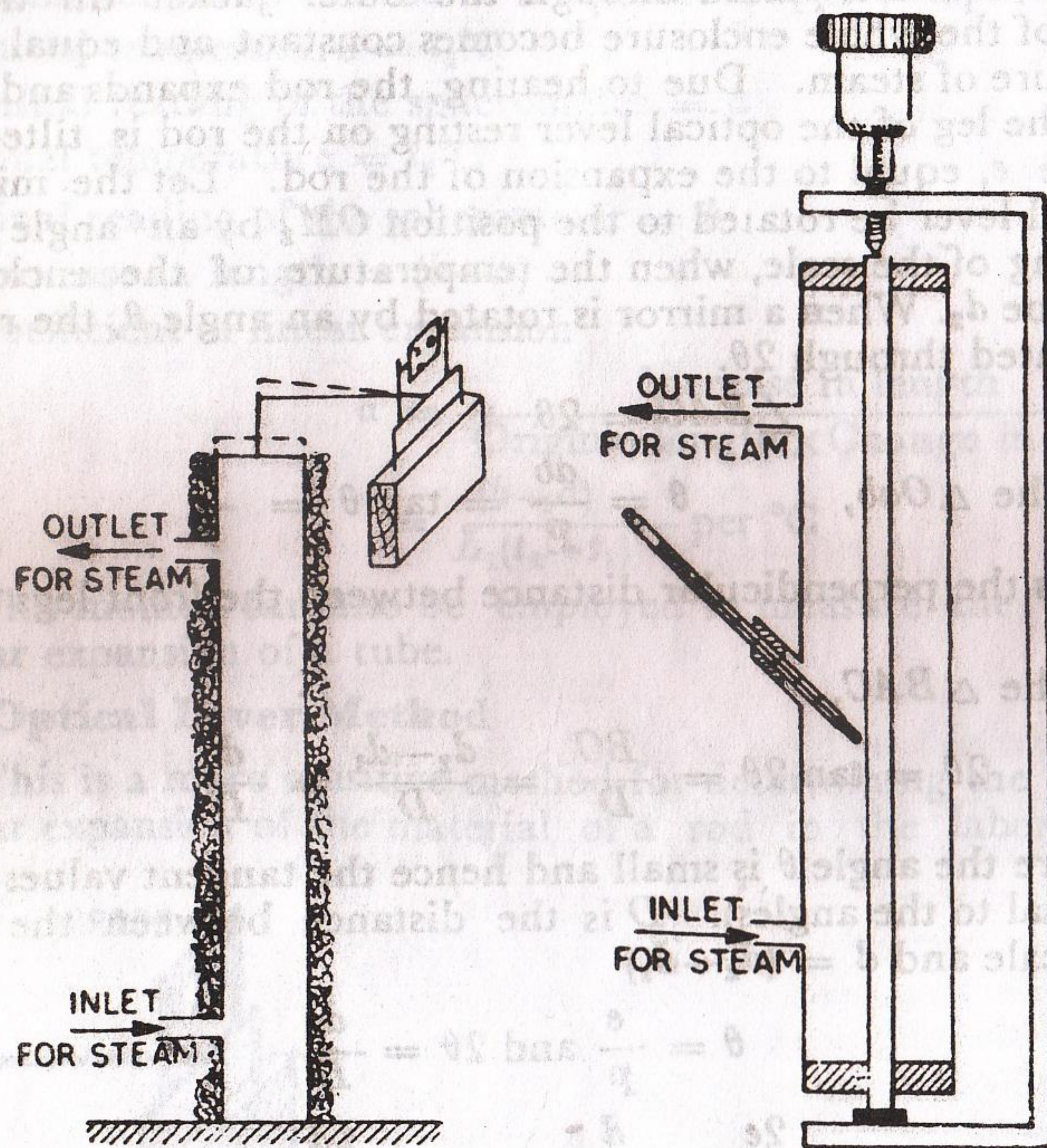


Fig. 2-3.

Fig. 2-4.

In this case, the reading of the screw gauge when it is in contact with the free end at the initial and final temperatures is noted and the difference between the two readings measures the expansion of the rod or tube (Fig. 2-4). Here the tip of the screw serves as the movable jaw and the free end of the rod as the fixed jaw of the screw gauge.

### 2-11 Comparator Method

The comparator method is very accurate for determining the coefficient of linear expansion of rods. The results obtained are of high precision. This method is used by the International Bureau of Weights and Measures for standardising measuring rods.

The method consists of comparing the length of an experimental rod maintained at a constant temperature with a standard rod maintained at the temperature of melting ice. The experimental rod is supported on two rollers and is kept inside a double walled vessel. The rod is surrounded by water and the temperature is maintained constant at any desired value with the help of a thermostat. The standard rod is supported in a similar vessel and the

temperature of the standard rod is maintained constantly at  $0^{\circ}\text{C}$ . The two rods have fine scratches at the two ends approximately at a distance of one metre. The two vessels are supported on wheels capable of movement on two parallel rails fixed on a table.  $T_1$  and  $T_2$  are two microscopes fixed to two sturdy stone pillars  $P_1$  and  $P_2$ . The two scratches can be viewed with the help of the microscopes. The microscopes themselves are capable of movement parallel to the length of the rod with the help of the micrometer screws  $M_1$  and  $M_2$  (Fig. 2-5). The vessels containing the rods can be brought under the microscopes one after the other. The microscopes are fitted with cross-wires.

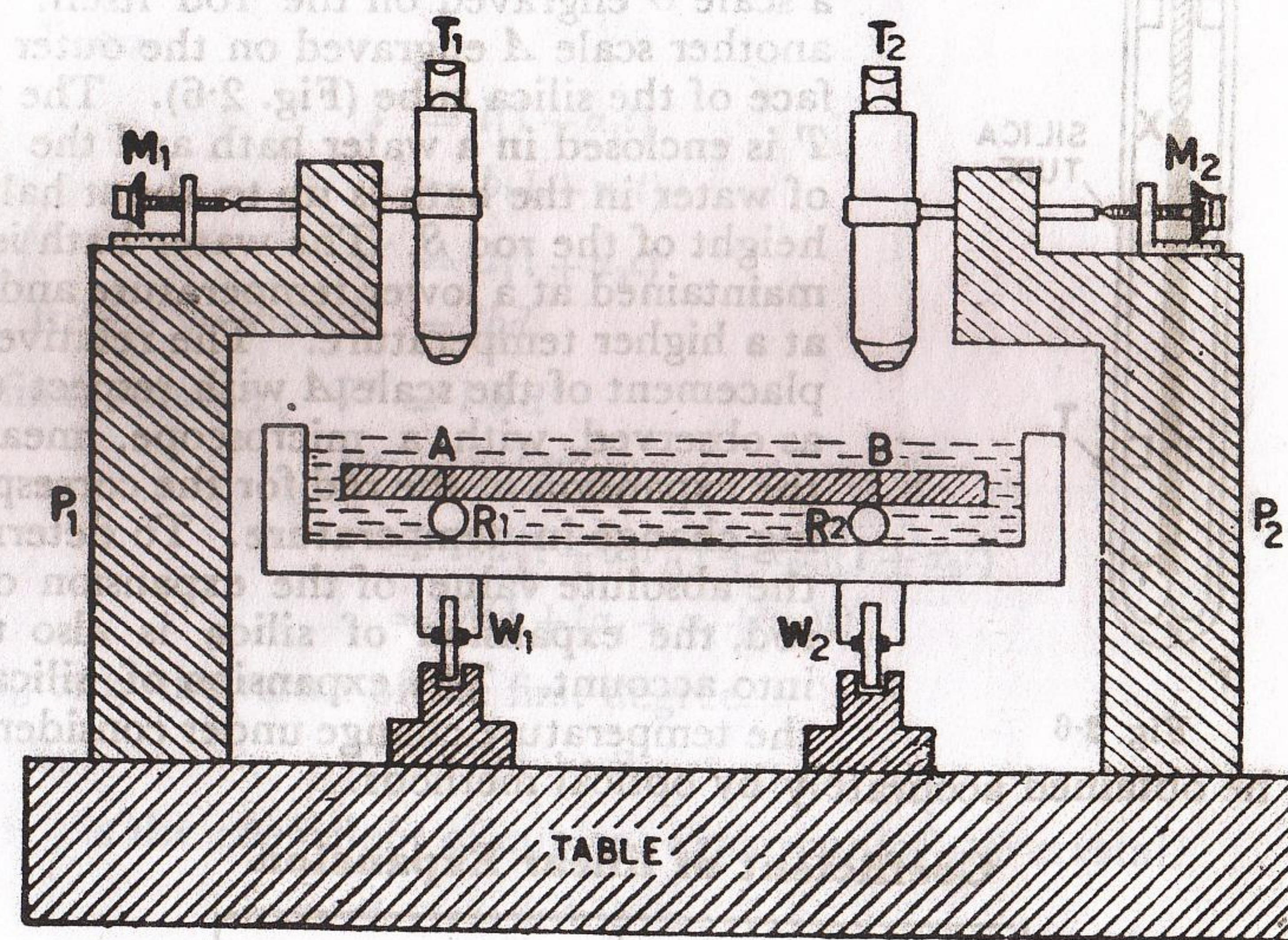


Fig. 2-5

Initially the readings of the micrometer screws when the cross wires are focussed on the scratches are noted with the standard rod first and then with the experimental rod, maintaining the temperatures of both at the temperature of melting ice. Then the experimental rod is maintained at the desired temperature and the readings are again noted at this constant temperature. Thus, the readings corresponding to the two scratches of experimental rod at different successive temperatures are observed. At the end, the standard rod is brought under the microscopes to check any displacement of the support during the experiment.

With the help of the observations mentioned above, it is possible to calculate the increase in length of the rod for different changes in temperature and the coefficient of linear expansion can be calculated in each case. This method also enables us to find the change in the length of the rod at different temperatures in comparison with the length of the standard rod at  $0^{\circ}\text{C}$ .



## 2.12 Henning's Tube Method

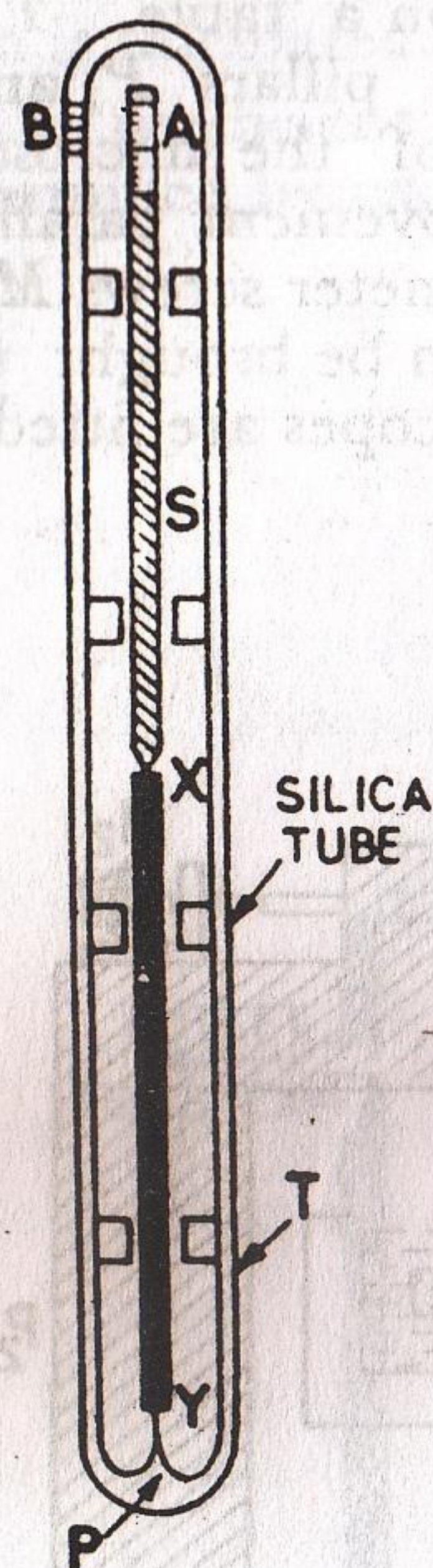


Fig. 2.6

This method is convenient to find the coefficient of linear expansion of the material of a rod accurately. The apparatus consists of a fused silica tube  $T$  having a pointed projection  $P$ .  $XY$  is the experimental rod, one end of which rests on the projection  $P$  and the other end is in contact with a fused silica rod  $S$ . The rod  $XY$  expands upwards and displaces the rod  $S$  upwards. The free end of the rod  $S$  carries a scale  $O$  engraved on the rod itself.  $B$  is another scale  $A$  engraved on the outer surface of the silica tube (Fig. 2.6). The tube  $T$  is enclosed in a water bath and the level of water in the bath is up to about half the height of the rod  $S$ . The water bath is first maintained at a lower temperature and then at a higher temperature. The relative displacement of the scale  $A$  with respect to  $B$  as observed with a microscope, measures the expansion of the rod for the corresponding change in temperature. To determine the absolute value of the expansion of the rod, the expansion of silica is also taken into account. The expansion of silica for the temperature change under consideration

can be obtained accurately by optical methods.

## Coefficient of linear Expansion

Substance	$\alpha \times 10^6$ per $^{\circ}C$
Quartz	0.4
Invar	0.9
Glass	4 to 9
Platinum	8.9
Steel	12
Gold	13.9
Copper	14
Silver	18.8
Brass	20
Aluminium	24
Zinc	26

## 2.13 Expansion of Crystals

In the case of isotropic bodies the coefficient of linear expansion is the same in all directions. The expressions derived earlier in this chapter refer only to isotropic bodies. In the case of anisotropic bodies like the crystals, the coefficient of expansion is *different in different directions*. If a crystal is cut such that its faces are parallel to the three mutually perpendicular axes, then the coefficients of expansion of the crystal are different in the three directions. Let  $l$ ,  $b$  and  $d$  the length, breadth and height of a crystal and  $\alpha_1$ ,  $\alpha_2$ , and  $\alpha_3$  be its coefficients of expansion in the three directions. Let  $l'$ ,  $b'$  and  $d'$  be the dimensions of the crystal when its temperature is increased by  $t^{\circ}C$ .

Then

$$l' = l(1 + \alpha_1 t)$$

$$b' = b(1 + \alpha_2 t)$$

and

$$d' = d(1 + \alpha_3 t)$$

$$\text{Initial volume } V = lbd$$

$$\text{and final volume } V' = l'b'd'$$

$$V' = l'b'd'$$

$$= lbd(1 + \alpha_1 t)(1 + \alpha_2 t)(1 + \alpha_3 t)$$

$$= V\{1 + (\alpha_1 + \alpha_2 + \alpha_3)t\}$$

taking only the terms of the first degree.

But

$$V' = V[1 + \gamma t]$$

where  $\gamma$  is the coefficient of cubical expansion of the crystal.

$$\therefore \gamma = \alpha_1 + \alpha_2 + \alpha_3$$

Uniaxial crystals have an axis of crystalline symmetry and their physical properties are the same in a plane perpendicular to the axis of the crystal. Hence uniaxial crystals have only two coefficients of expansion. Negative coefficients of expansion are obtained with certain negative crystals. However, it is observed that their coefficients of cubical (volume) expansion are positive.

## 2.14 Coefficient of Expansion of a Crystal (Fizeau's method)

This method is suitable to measure the expansion of small specimens *e.g.*, crystals. The experiment is based on the principle of interference of light.

The apparatus consists of a highly polished (plane) metal plate  $MN$ , which is supported on three metal screws passing through the plate.  $PQ$  is a smooth glass plate resting on the tips of the three screws (Fig. 2.7). The specimen (crystal) whose expansion is to be determined is placed on the plate  $MN$  and the screws are rotated such that the thickness of the air film between the upper surface of the crystal  $G$  and the lower surface of the glass plate  $PQ$  is very small.  $S$  is a monochromatic source of light,  $L$  is a condensing lens,  $G$  is a glass plate inclined at  $45^{\circ}$  to the path of the parallel beam of light



from the lens and  $M_1$  is a microscope fitted with cross wires. A parallel beam of light is incident on the glass plate  $PQ$  by this arrangement. Interference takes place due to the light reflected from the lower surface of  $PQ$  and the upper surface of the crystal. If these two surfaces are exactly parallel, the field of view just appears to be coloured without any fringes. But in practice it is not possible to have the two surfaces to be exactly parallel. A wedge shaped air film of very small angle is formed between the two surfaces i.e., the thickness of the air film gradually changes from one point to the other of the crystal surface. This means that the path difference between the two reflected beams gradually increases. According to the

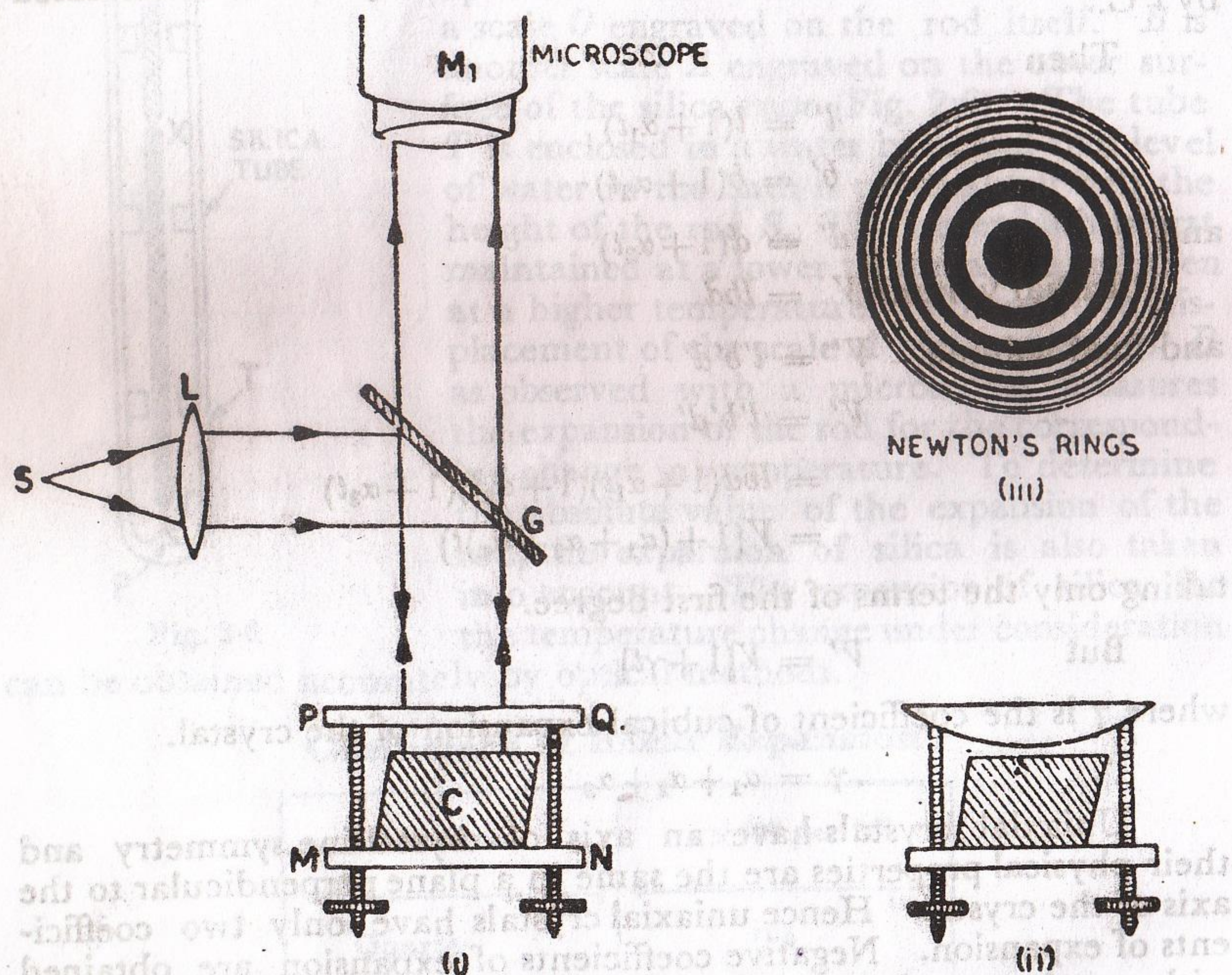


Fig. 2.7

theory of the colours of thin films viewed by reflected light any point in the field of view appears dark or bright depending on whether the path difference  $2\mu T \cos r$  is even or odd multiples of  $\lambda/2$ . Here  $\mu$  is the refractive index of air,  $T$  is the thickness of the air film,  $r$  is the angle of refraction and  $\lambda$  is the wavelength of light used.

$$2\mu T \cos r = 2n \lambda/2 \text{ dark}$$

$$2\mu T \cos r = (2n+1) \lambda/2 \text{ bright}$$

But  $\mu = 1$

and  $r = 0$  for normal incidence

$$\cos r = 1$$

$$2T = 2n \lambda/2 \text{ dark} \quad \dots(i)$$

$$2T = (2n+1) \lambda/2 \text{ bright} \quad \dots(ii)$$

As the thickness of the air film gradually changes, the interference pattern observed in the field of view consists of alternate bright and dark bands parallel to one another. The apparatus is enclosed in an outer bath and its temperature is maintained constant thermostatically at any desired value. The interference pattern is obtained and the cross wire is fixed on one of the dark bands. Now the temperature of the bath is gradually increased and the number of dark bands ( $p$ ) that cross the field of view with reference to the cross wire is observed, till the temperature becomes constant at the desired higher value and till no more fringes cross the field of view. The change in the thickness of the air film can be calculated as follows. If  $t_1$  and  $t_2$  are the thicknesses at the two temperatures, then from equation (i) above

$$T_1 = n \frac{\lambda}{2}$$

and  $T_2 = (n-p) \frac{\lambda}{2}$

or  $T_1 - T_2 = p \frac{\lambda}{2} \quad \dots(iii)$

where  $p$  is the number of fringes that cross the field of view.  $(T_1 - T_2)$  measures the decrease in the thickness of the air film due to the expansion of the crystal and the expansion of the supporting legs between the plates  $MN$  and  $PQ$ . The increase in the thickness of the air film due to the expansion of the supporting legs alone can be determined separately for the same rise of temperature by removing the crystal and bringing the plates  $MN$  and  $PQ$  very near one another. Let the number of fringes that cross the field of view be  $p_1$ .

$$\text{Increase in the thickness of the air film} = \frac{p_1 \lambda}{2}$$

$$\therefore \text{Correct expansion of the crystal} = (p + p_1) \frac{\lambda}{2}$$

This gives the expansion of the crystal. Knowing the initial and final temperatures of the crystal and the thickness of the crystal in the direction of expansion, the coefficient of linear expansion of the crystal can be calculated.

$$\alpha_1 = \frac{(p + p_1) \frac{\lambda}{2}}{l_1(t_2 - t_1)} = \frac{(p + p_1) \lambda}{2(l_1)(t_2 - t_1)}$$

Here  $(t_2 - t_1)$  is the change in temperature.

Similarly, the coefficients of expansion in the other two perpendicular directions also can be determined. If  $\alpha_1$ ,  $\alpha_2$  and  $\alpha_3$  are the coefficients of expansion in the three mutually perpendicular directions, then the coefficient of volume expansion  $\gamma$  is given approximately by  $\gamma = \alpha_1 + \alpha_2 + \alpha_3$ .

If the glass plate  $PQ$  is replaced by a plano-convex lens of large radius of curvature [Fig. 2.7 (ii)] the interference pattern consists



of alternately bright and dark concentric rings called the Newton's rings system. When the temperature of the crystal is increased, the thickness of the air film at any point gradually decreases and the rings appear to expand outwards. With reference to the cross-wire on any particular dark ring, the number of dark rings ( $p$ ) that cross the field of view is noted. Then the decrease in the thickness of the air film ( $T_1 - T_2$ ) is given by

$$T_1 - T_2 = p \frac{\lambda}{2}$$

The displacement of one fringe is equal to a change in the thickness of the air film corresponding to half the wavelength of light used. Even a shift of  $\frac{1}{4}$  of a fringe can be detected. If the wavelength of light used is  $6000 \text{ \AA} = 6 \times 10^{-5} \text{ cm}$ , then a change in the thickness of air film equal to  $6 \times 10^{-6} \text{ cm}$  can be detected. Thus the expansion of the crystal can be measured very accurately.

The experiment is repeated for the same rise of temperature without the crystal. If  $p_1$  is the number of rings that cross the field of view, then the increase in the thickness of the air film due to the expansion of the supporting legs  $= p_1 \lambda / 2$ .

The correct expansion of the crystal  $= (p + p_1) \lambda / 2$

**Tutton's modification.** Tutton modified the apparatus by placing the crystal over a small table having aluminium legs and resting on the plate  $MN$ , the supporting screws are made of platinum [Fig. 2-7 (a)].

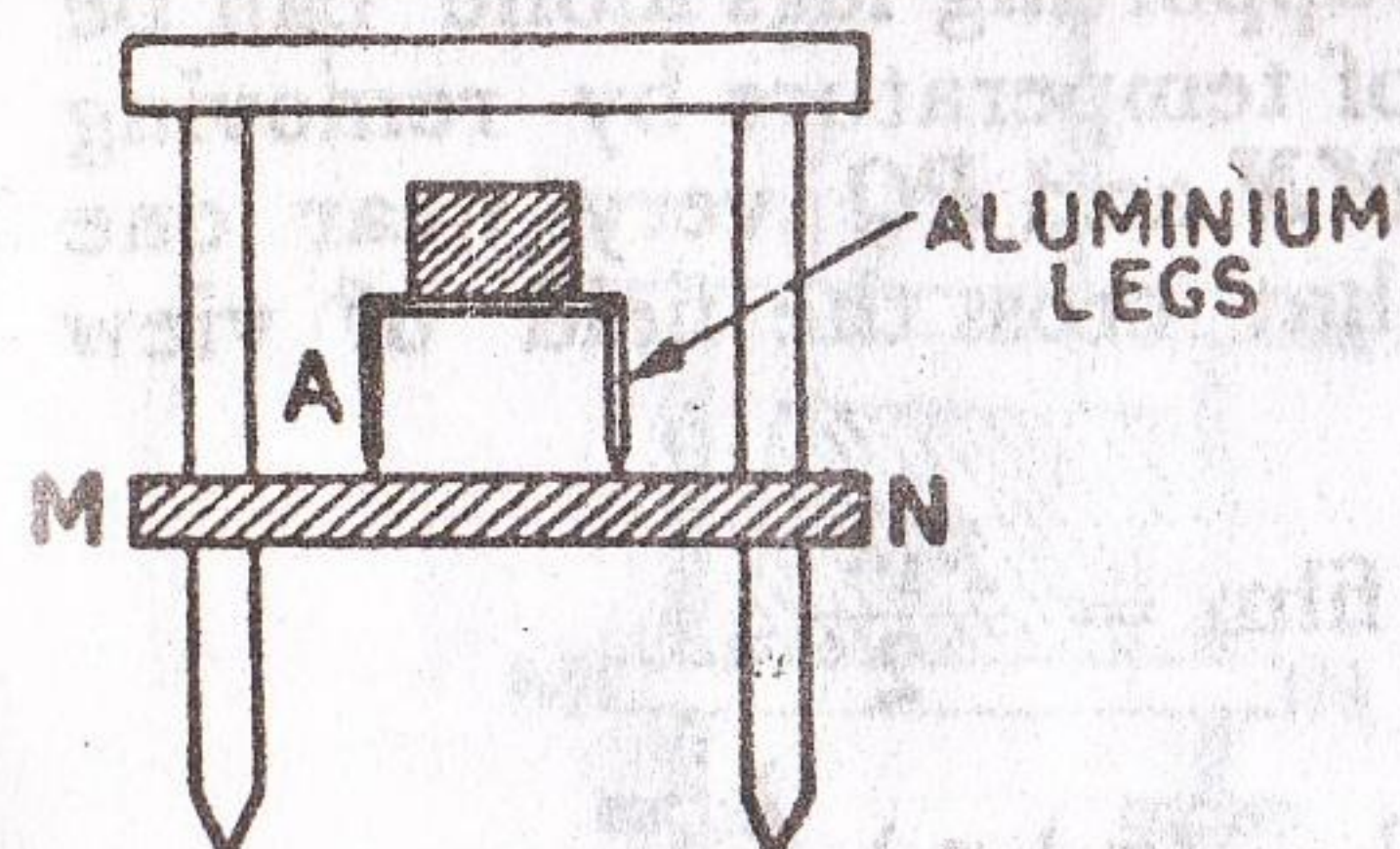


Fig. 2-7(a)

The height of the platinum screws and the height of the legs of aluminium are adjusted such that the increase in length is the same in the two cases. When the experiment is performed, the decrease in the thickness of the air film is only due to the expansion of the crystal.

Let the number of rings that cross the field of view be  $p$

Increase in the length of the crystal  $= p \lambda / 2$

$$\therefore \alpha_1 = \frac{p \lambda}{2l_1(t_2 - t_1)}$$

**Example 2-1.** In Tutton's experiment for determining the coefficient of linear expansion of a crystal the following data was observed. Thickness of the crystal  $= 1 \text{ cm}$ , wavelength of light used  $= 5890 \times 10^{-8} \text{ cm}$ . When the temperature of the crystal was increased from  $20^\circ\text{C}$  to  $50^\circ\text{C}$ , the number of fringes that crossed the field of view was 14. Calculate the coefficient of linear expansion of the crystal.

Here  $\lambda = 5890 \times 10^{-8} \text{ cm}$

$p = 14, l = 1 \text{ cm}$

$t_2 - t_1 = 50 - 20 = 30^\circ\text{C}$

$$\begin{aligned} \alpha &= \frac{p \lambda}{2l(t_2 - t_1)} \\ &= \frac{14 \times 5890 \times 10^{-8}}{2 \times 1 \times 30} \\ &= 13.74 \times 10^{-6} / ^\circ\text{C} \end{aligned}$$

## 2.15. Force due to Expansion or Contraction

The forces of expansion or contraction in metal rods and beams due to changes in temperature are very large. This is the reason why gaps are provided in the railway lines or in the rock pillars for bridges so as to allow for the expansion of the material in summer.

Consider a steel beam of length  $L$  metres. Assuming that this bar is subjected to a temperature change of  $30^\circ\text{C}$ , the increase in length of the bar

$$= L \alpha t$$

Here

$$\alpha = 12 \times 10^{-6} \text{ per } ^\circ\text{C}$$

and

$$t = 30^\circ\text{C}$$

$\therefore$  Increase in length

$$= 12 \times 30 \times 10^{-6} L \text{ metres}$$

Strain produced in the beam

$$\begin{aligned} &= \frac{\text{Increase in length}}{\text{Original length}} \\ &= \frac{12 \times 30 \times 10^{-6} \times L}{L} \\ &= 12 \times 30 \times 10^{-6} \end{aligned}$$

According to Hooke's law (within the elastic limit)

$$Y = \frac{\text{Stress}}{\text{Strain}}$$

or

$$\text{Stress} = Y \times \text{Strain}$$

Young's modulus of elasticity of steel

$$= 2 \times 10^{12} \text{ dynes/sq cm}$$

$\therefore$  Stress  $= 2 \times 10^{12} \times 12 \times 30 \times 10^{-6} \text{ dynes/sq cm}$

$$= 72 \times 10^7 \text{ dynes/sq cm}$$

$$= 745 \text{ kg wt/sq cm}$$

If the bar has a cross-sectional area of  $30 \text{ sq cm}$  then the force due to the expansion of the specimen

$$= 745 \times 30$$

$$= 22,350 \text{ kg wt}$$



The force of expansion is independent of the length of the bar. If the bar is prevented from expansion or contraction, a force of magnitude calculated above will act on the bar. The force of contraction in metals due to increase in temperature has many useful applications. The boiler plates are riveted with red hot rivets and when the rivets cool, they produce steam proof joints. Shrinking of iron tyres on cart wheels is also done the same way.

**Example 2.2.** A steel rod of length 5 metres is fixed rigidly between two supports. The coefficient of linear expansion of steel  $= 12 \times 10^{-6}/^{\circ}\text{C}$ . Calculate the stress in the rod for an increase in temperature of  $40^{\circ}\text{C}$ . The Young's modulus of elasticity of steel  $= 2 \times 10^{12}$  dynes/sq cm.

Here

$$\alpha = 12 \times 10^{-6}/^{\circ}\text{C},$$

$$Y = 2 \times 10^{12} \text{ dynes/sq cm}$$

$$t = 40^{\circ}\text{C}$$

$$L = 500 \text{ cm}$$

Increase in length,

$$l = L \alpha t$$

$$\text{Strain} = \frac{l}{L}$$

$$= \frac{L \alpha t}{L}$$

$$= \alpha t$$

$$\text{Stress} = Y \times \text{Strain}$$

$$= Y \alpha t$$

$$= 2 \times 10^{12} \times 12 \times 10^{-6} \times 40$$

$$= 960 \times 10^6 \text{ dynes/cm}^2$$

**Example 2.3.** A steel wire 8 metres long and 4 mm in diameter is fixed to two rigid supports. Calculate the increase in tension when the temperature falls by  $10^{\circ}\text{C}$ .

$$\alpha = 12 \times 10^{-6}/^{\circ}\text{C}$$

$$Y = 2 \times 10^{12} \text{ dynes/sq cm}$$

Here

$$\alpha = 12 \times 10^{-6}/^{\circ}\text{C}$$

$$Y = 2 \times 10^{12} \text{ dynes/sq cm}$$

$$t = 10^{\circ}\text{C}$$

$$L = 800 \text{ cm}$$

$$r = 0.2 \text{ cm}$$

$$\text{Increase in length, } l = L \alpha t$$

$$\text{Strain} = \frac{l}{L} = \frac{L \alpha t}{L} = \alpha t$$

$$\text{Stress} = Y \times \text{Strain}$$

$$= Y \alpha t$$

$$\text{Area of cross-section} = \pi r^2$$

$$\text{Increase in tension} = \text{stress} \times \text{area of cross-section}$$

$$= Y \alpha t \times \pi r^2 \text{ dynes}$$

$$= \frac{2 \times 10^{12} \times 12 \times 10^{-6} \times 10 \times 22 \times 0.2 \times 0.2}{7}$$

$$= 30.17 \times 10^6 \text{ dynes}$$

**Example 2.4.** A uniform rod of 2 sq mm cross-section is heated from  $0^{\circ}\text{C}$  to  $20^{\circ}\text{C}$ . Find the force which must be exerted to prevent it from expanding. Find also the energy stored per unit volume.  $\alpha = 12 \times 10^{-6}/^{\circ}\text{C}$  and  $Y = 10^{11}$  newtons/sq metre.

Here

$$\alpha = 12 \times 10^{-6}/^{\circ}\text{C},$$

$$Y = 10^{11} \text{ newtons/sq metre}$$

$$t = 20^{\circ}\text{C}, a = 2 \text{ sq mm}$$

$$= 2 \times 10^{-6} \text{ sq metre}$$

$$\text{Increase in length, } l = L \alpha t$$

$$\text{Strain} = \frac{l}{L} = \frac{L \alpha t}{L} = \alpha t$$

$$\text{Stress} = Y \times \text{Strain} = Y \alpha t$$

$$\text{Force} = \text{Stress} \times \text{area of cross-section}$$

$$= Y \alpha t \times a$$

$$= 10^{11} \times 12 \times 10^{-6} \times 20 \times 2 \times 10^{-6}$$

$$= 48 \text{ newtons}$$

$$\text{Energy stored per unit volume} = \frac{1}{2} \times \text{stress} \times \text{strain}$$

$$= \frac{1}{2} \times Y \alpha t \times \alpha t$$

$$= \frac{10^{11} \times (12 \times 10^{-6})^2 \times (20)^2}{2}$$

$$= 2880 \text{ j/m}^3.$$

**Example 2.5.** A steel wire of diameter 1 mm supports a load which is sufficient to keep the wire taut at  $20^{\circ}\text{C}$ . Calculate the additional load that will be required to restore the length of the wire to its initial value when the temperature falls to  $0^{\circ}\text{C}$ .

$$Y = 2 \times 10^{11} \text{ newtons/sq metre}$$



$$\alpha = 10^{-6}/^{\circ}\text{C}, \text{ diameter} = 1 \text{ mm} = 10^{-3} \text{ metre}$$

$$\text{Area of cross-section } a = \frac{\pi d^2}{4} = \frac{22 \times 10^{-6}}{28} \text{ sq metre}$$

$$\text{Decrease in length} = L \alpha t$$

$$\text{Strain} = \frac{L \alpha t}{L} = \alpha t$$

$$\text{Stress} = Y \times \text{Strain} = Y \alpha t$$

Force required to maintain the original length

$$= \text{Stress} \times \text{area of cross-section}$$

$$= y \alpha t \times a$$

$$= \frac{2 \times 10^{11} \times 12 \times 10^{-6} \times 20 \times 22 \times 10^{-6}}{28} \text{ newtons}$$

$$= \frac{44 \times 6}{7 \times 9.8} \text{ kg wt}$$

$$= 3.847 \text{ kg wt}$$

## 2.16 Expansion of Liquids

When a liquid is heated, its volume changes. Liquids have no shape of their own but conform to the shape of the containing vessel. When a liquid is heated, the containing vessel also expands and hence the measured increase in volume of the liquid is the apparent increase in volume. The real increase in volume of the liquid is equal to the sum of the apparent increase in volume of the liquid and the increase in volume of the containing vessel. Thus, a liquid has two coefficients of expansion,  $\gamma_a$  and  $\gamma_r$ , called the coefficients of apparent and real expansion of the liquid respectively.

The coefficient of apparent expansion of a liquid ( $\gamma_a$ ) is defined as the apparent increase in volume per unit volume per unit degree rise in temperature when the liquid is heated in a vessel that expands with heating.

The coefficient of real expansion of a liquid ( $\gamma_r$ ) is defined as the actual increase in volume per unit volume per unit degree rise in temperature.

The expansion of most of the liquids is irregular except mercury whose expansion is uniform. Due to the irregular expansion of liquids, two coefficients of expansion called the *zero coefficient* and the *mean coefficient* are used. Zero coefficient is the increase in volume per unit volume at  $0^{\circ}\text{C}$  per unit degree rise of temperature. If  $V_0$  and  $V_t$  are the volumes of a liquid at  $0^{\circ}\text{C}$  and  $t^{\circ}\text{C}$  respectively, then the zero coefficient is given by

$$\gamma = \frac{V_t - V_0}{V_0 \times t}$$

If  $V_1$  and  $V_2$  are the volumes of a liquid at  $t_1^{\circ}\text{C}$  and  $t_2^{\circ}\text{C}$  respectively then the mean coefficient of expansion is given by

$$\gamma = \frac{V_2 - V_1}{V_1 (t_2 - t_1)}$$

## 2.17 Relation between $\gamma_r$ and $\gamma_a$

Consider a liquid contained in a graduated vessel which reads the volume correctly at  $0^{\circ}\text{C}$ . Let  $V_0$  and  $V_1$  be the volumes of the liquid at  $0^{\circ}\text{C}$  and  $t^{\circ}\text{C}$ . Then the coefficient of apparent expansion of the liquid is given by

$$\gamma_a = \frac{V_1 - V_0}{V_0 t} \text{ or } \frac{V_1}{V_0} = (1 + \gamma_a t)$$

If  $\gamma_v$  is the coefficient of cubical expansion of the containing vessel, then the real volume of the vessel at  $t^{\circ}\text{C}$  corresponding to a reading  $V_1$  will be given by  $V_1[1 + \gamma_v t]$

Hence the coefficient of real expansion of the liquid is given by

$$\gamma_r = \frac{V_1[1 + \gamma_v t] - V_0}{V_0 t}$$

$$= \frac{V_1 - V_0}{V_0 t} + \frac{V_1}{V_0} \cdot \gamma_v$$

$$= \gamma_a + (1 + \gamma_a t) \gamma_v$$

$$= \gamma_a + \gamma_v + \gamma_a \gamma_v t$$

Neglecting the term  $\gamma_a \gamma_v t$ ,

$$\gamma_r = \gamma_a + \gamma_v$$

Thus, the coefficient of real expansion of a liquid is approximately equal to the sum of the coefficient of apparent expansion of the liquid and the coefficient of cubical expansion of the containing vessel. If  $\alpha_v$  is the coefficient of linear expansion of the material of the containing vessel  $\gamma_v = 3\alpha_v$  (approximately).

## 2.18 Determination of $\gamma_a$ with a Dilatometer

A dilatometer consists of a glass bulb attached to a stem. The stem is calibrated to read the volume directly (Fig. 2.8). The dilatometer method is suitable to measure the mean coefficient of apparent expansion of a liquid over different ranges of temperature. The volumes of the liquid  $V_0$  and  $V_t$  at  $0^{\circ}\text{C}$  and  $t^{\circ}\text{C}$  are noted and  $\gamma_a$  is calculated from the formula

$$\gamma_a = \frac{V_t - V_0}{V_0 t}$$

The dilatometer method is most suitable for finding the coefficient of expansion of volatile liquids.

## 2.19 Determination of $\gamma_a$ by weight thermometer Method

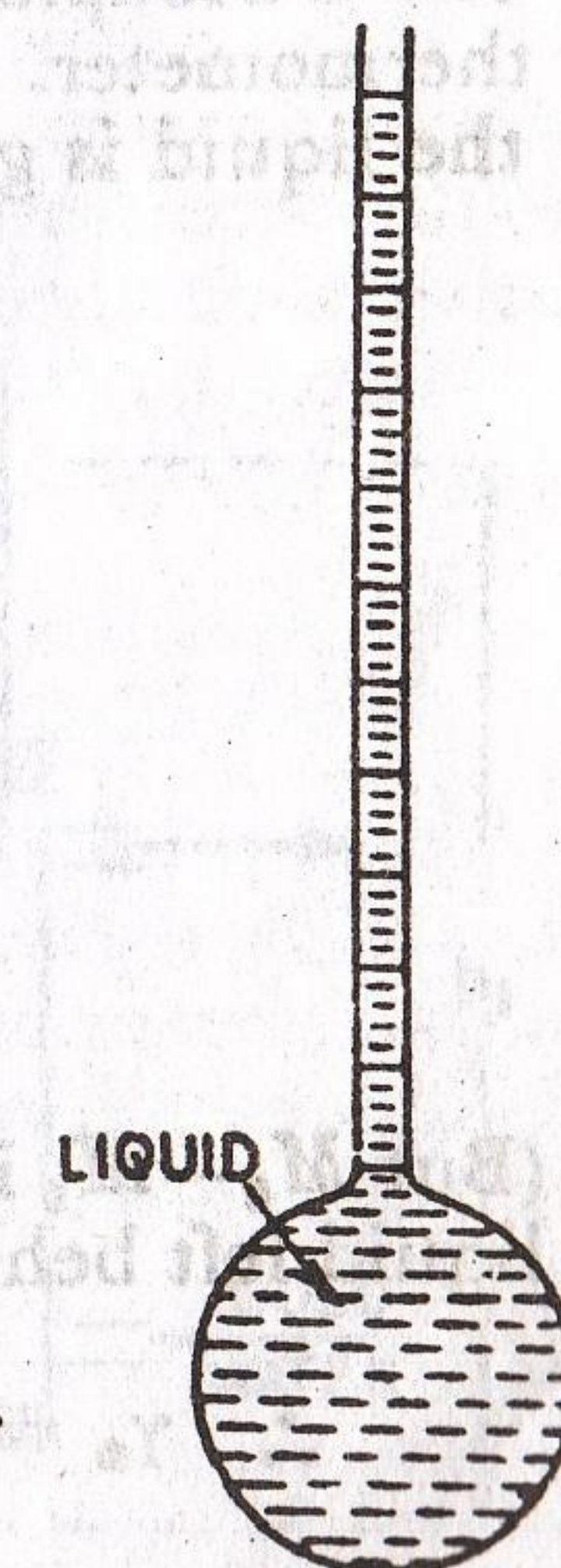
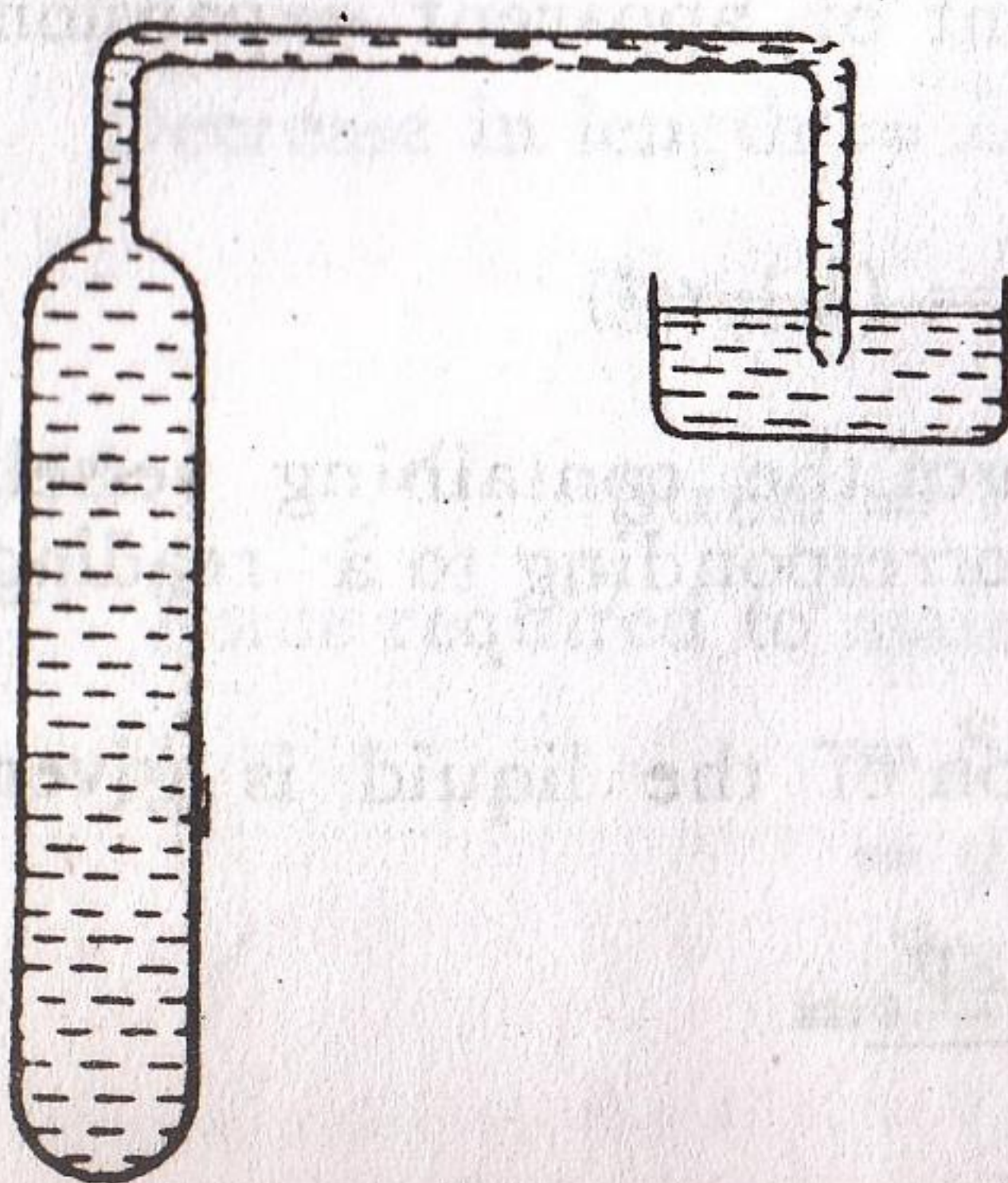


Fig. 2.8



The weight thermometer consists of an elongated glass or fused silica bulb fitted with a narrow glass tube bent twice at right angles (Fig. 2.9). The mass of the empty dry weight thermometer is initially determined ( $W_1$  g). Now the weight thermometer is filled with the given liquid at  $0^\circ\text{C}$  by alternate heating and cooling. The mass of the thermometer when it is filled with the liquid is determined ( $W_2$  g). The mass of the liquid filling the whole bulb at  $0^\circ\text{C}$  is given by  $M_0 = (W_2 - W_1)$ . If  $\rho_0$  is the density of the liquid at  $0^\circ\text{C}$ , then the initial volume of the liquid



$$= V = \frac{M_0}{\rho_0}$$

Now the weight thermometer is heated in a bath to a constant high temperature ( $t^\circ\text{C}$ ) say the boiling point of water. Due to the expansion, the liquid overflows and when the expansion is complete, the bulb is removed from the bath and is cooled. The mass of the weight thermometer and the liquid left behind is noted ( $W_3$  g). Mass of the liquid left behind  $M_1 = W_3 - W_1$ .

Volume of this liquid

$$V_0 = \frac{M_1}{\rho_0}$$

This means that a volume  $V_0$  of the liquid at  $0^\circ\text{C}$  when heated to  $t^\circ\text{C}$  occupies a volume  $V$  viz., the complete volume of weight thermometer. Therefore, the coefficient of apparent expansion of the liquid is given by

$$\gamma_a = \frac{\text{Increase in volume}}{\text{Original volume} \times \text{Change in temperature}}$$

$$\gamma_a = \frac{\frac{M_0}{\rho_0} - \frac{M_1}{\rho_0}}{\frac{M_1}{\rho_0} \times t}$$

$$\gamma_a = \frac{M_0 - M_1}{M_1 \times t}$$

(But  $M_0 - M_1$  is the mass of the liquid expelled and  $M_1$  is the mass of liquid left behind).

$$\therefore \gamma_a = \frac{\text{Mass of the liquid expelled}}{\text{Mass of the liquid left behind} \times \text{rise in temperature}}$$

In the laboratory, a specific gravity bottle may also be used to determine the coefficient of apparent expansion of a liquid.

To find the coefficient of real expansion of the liquid, the coefficient of cubical expansion of the vessel is found from the constant tables.

Then  $\gamma_r = \gamma_a + \gamma_v$ .

### 2.20 Determination of the Coefficient of Real Expansion of a Liquid by Dulong and Petit's Method

Dulong and Petit carried out the first direct determination of the coefficient of real expansion of a liquid in 1817. The apparatus consists of a long glass tube  $ABCDEFGH$  bent in the shape shown in Fig. 2.10. The tube contains the liquid whose coefficient of real expansion has to be determined. The vertical limbs  $CD$  and  $EF$  are surrounded by two outer jackets. Through the jacket of  $CD$ , steam is passed and through the jacket of  $EF$  cold water is circulated. Let the temperatures of the cold and the hot limbs be  $t_1^\circ\text{C}$  and  $t_2^\circ\text{C}$  respectively. The horizontal limb  $DE$  is trapped with a piece of cloth and cold water is poured over it continuously so that there is no conduction of heat from the hot to the cold limb. When the temperatures in the two limbs are constant the liquid in the projecting limbs  $AB$  and  $GH$  stands at constant levels. The difference in levels between the two limbs can be determined. As the points  $D$  and  $E$  are at the same horizontal level, the pressure at  $E =$  the pressure at  $D$ . Let  $h_1$  and  $h_2$  be the heights of the liquid columns and  $\rho_1$  and  $\rho_2$  the densities of the liquid at temperatures  $t_1^\circ\text{C}$  and  $t_2^\circ\text{C}$  respectively.

Pressure at  $E =$  Pressure at  $D$

$$P_A + h_1 \rho_1 g = P_A + h_2 \rho_2 g$$

where  $P_A$  is the atmospheric pressure

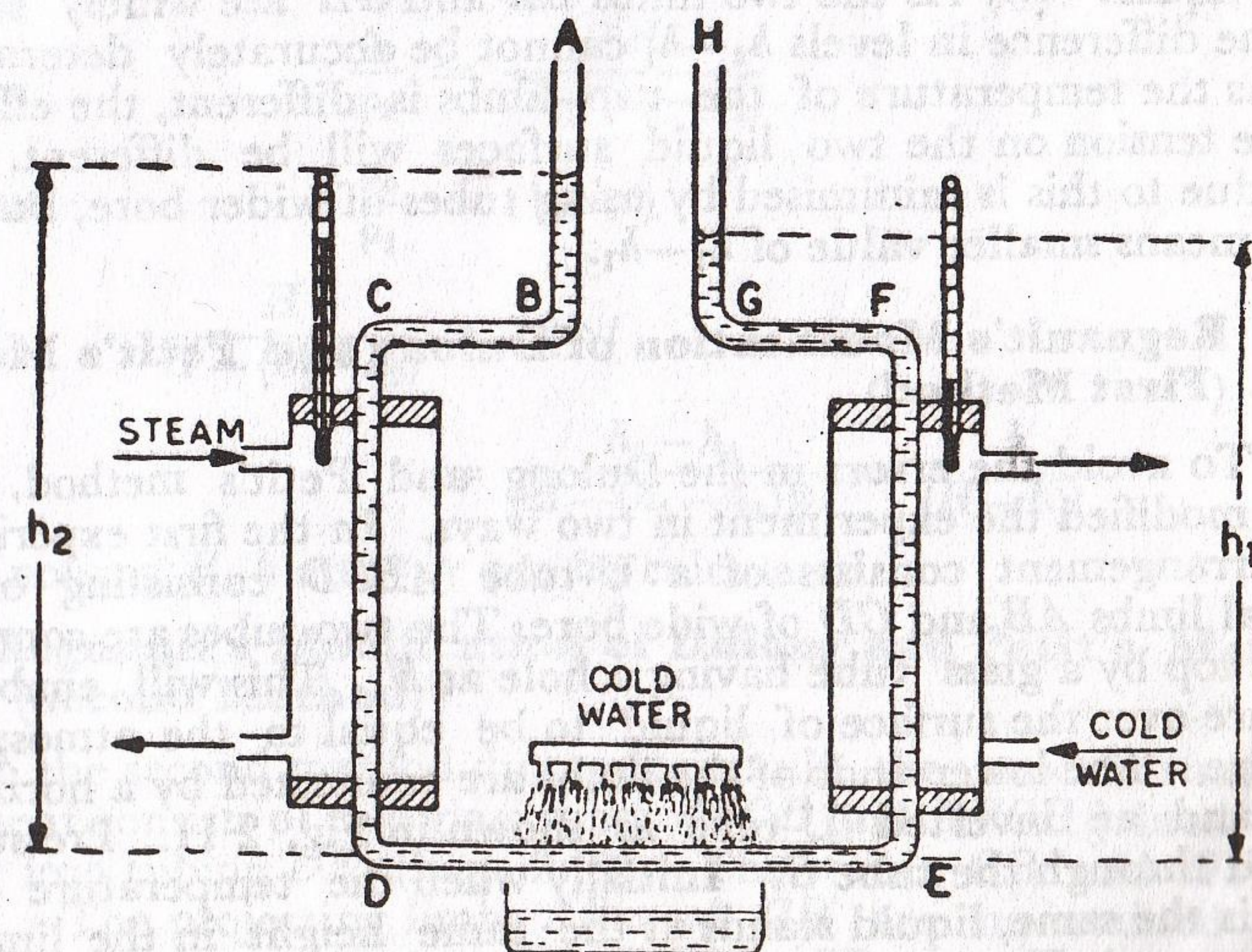


Fig. 2-10



$$\text{or } \frac{h_1}{h_2} = \frac{\rho_2}{\rho_1}$$

$$\text{But } \rho_0 = \rho_1(1 + \gamma_r t_1)$$

$$\text{and } \rho_0 = \rho_2(1 + \gamma_r t_2)$$

$$\therefore \frac{\rho_2}{\rho_1} = \frac{1 + \gamma_r t_1}{1 + \gamma_r t_2}$$

$$\therefore \frac{h_1}{h_2} = \frac{1 + \gamma_r t_1}{1 + \gamma_r t_2}$$

$$h_1 + h_1 \gamma_r t_2 = h_2 + h_2 \gamma_r t_1$$

$$\gamma_r (h_1 t_2 - h_2 t_1) = h_2 - h_1$$

$$\text{OR } \gamma_r = \frac{h_2 - h_1}{h_1 t_2 - h_2 t_1}$$

In the original experiment of Dulong and Petit, the cold limb is surrounded by melting ice and the hot limb is surrounded by an oil bath heated by a furnace. The temperatures are measured with the help of an air thermometer. Heights  $h_1$  and  $h_2$  are measured using a cathetometer.  $\gamma_r$  is calculated on the basis of the pressure of the liquid column which is dependent on the height of the liquid column and the density of the liquid at that temperature. It is not dependent on the volume of the liquid. Hence the value of  $\gamma_r$  obtained from this method measures the coefficient of real expansion of the liquid. It is necessary that the projections  $AB$  and  $GH$  of the glass tube above the level of the jackets are small.

This simple arrangement has the following sources of error :

- (i) Convection currents along the horizontal tube  $DE$  and the necessity of the projecting portions  $AB$  and  $GH$  result in unequal heating of the liquid.
- (ii) As the two limbs  $AB$  and  $GH$  are widely separated, the difference in levels  $h_2 - h_1$  cannot be accurately determined.
- (iii) As the temperature of the two limbs is different, the effect of surface tension on the two liquid surfaces will be different. The error due to this is minimised by using tubes of wider bore, but this again means smaller value of  $h_2 - h_1$ .

### 2-21 Regnault's Modification of Dulong and Petit's Method (First Method)

To avoid the errors in the Dulong and Petit's method, Regnault modified the experiment in two ways. In the first experiment, the arrangement consists of a U-tube  $ABCD$  consisting of two vertical limbs  $AB$  and  $CD$  of wide bore. The two tubes are connected at the top by a glass tube having a hole at  $E$ . This will enable the pressure over the surface of liquid to be equal to the atmospheric pressure. The lower ends of the limbs are connected by a horizontal tube and an inverted U-tube as shown in Fig. 2-11. Pressure is applied through the tube  $G$ . Initially when the temperature of the limbs is the same, liquid stands at the same height in the limbs of the inverted U-tube. The vertical limb  $AB$  is maintained at the

higher temperature and the limb  $CD$  and the inverted U-tube are kept cold. The temperature of the hot limb is measured with an air thermometer.

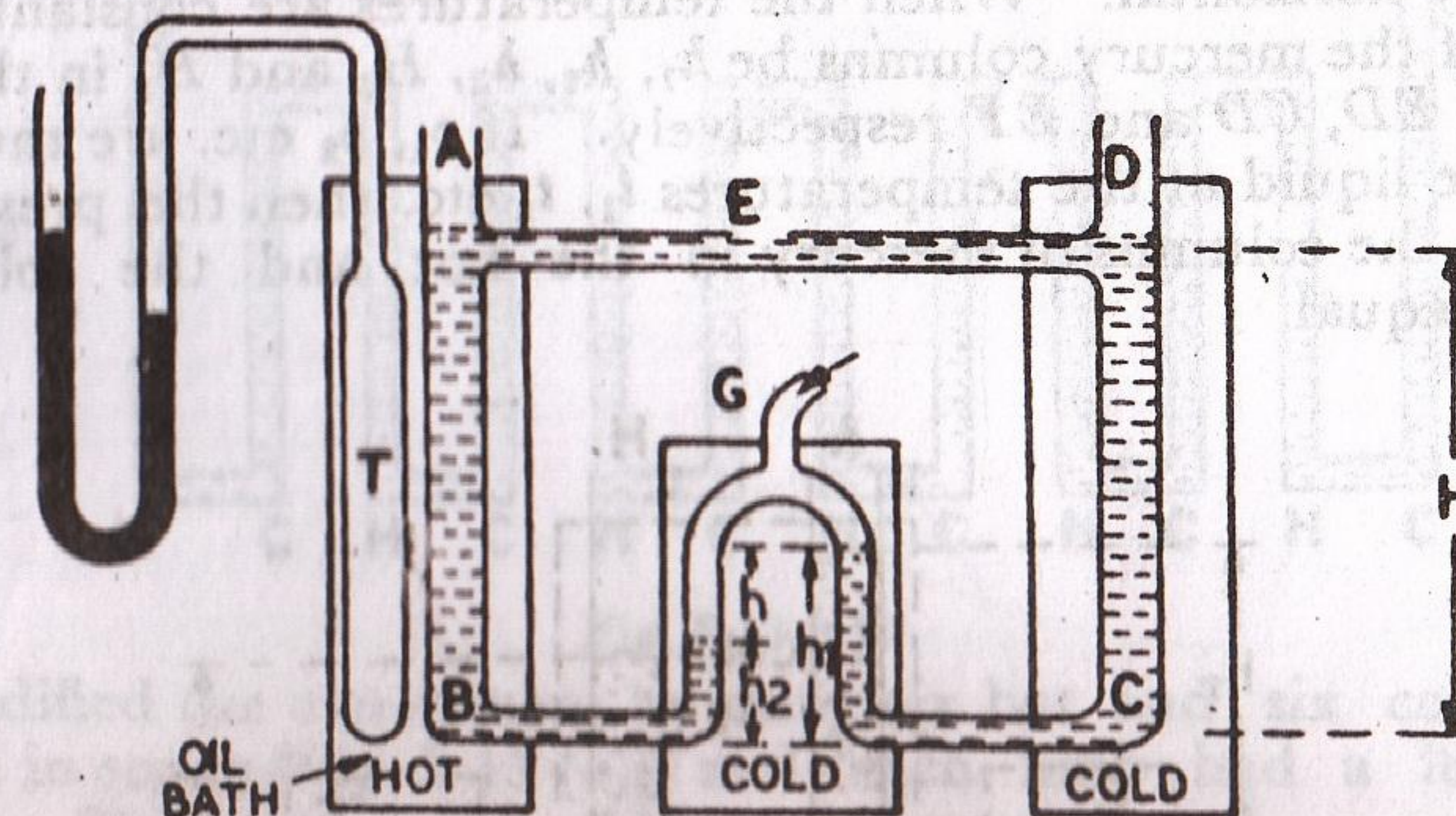


Fig. 2-11

When the temperatures of the limbs are constant, the heights  $h_1$  and  $h_2$  are noted. Let  $\rho_0$  and  $\rho_t$  be the densities of the liquid in the cold and the hot limbs respectively. Let  $P$  be the excess of pressure in the inverted U-tube above the atmospheric pressure.

Then for the hot limb

$$P = H\rho_t - h_2\rho_0$$

and for the cold limb

$$P = H\rho_0 - h_1\rho_0$$

$$\therefore H\rho_t - h_2\rho_0 = H\rho_0 - h_1\rho_0$$

$$H\rho_0 - H\rho_t = (h_1 - h_2)\rho_0$$

$$\rho_0 [H - (h_1 - h_2)] = H\rho_t$$

$$\frac{\rho_0}{\rho_t} = \frac{H}{H - (h_1 - h_2)}$$

$$\text{But } \frac{\rho_0}{\rho_t} = (1 + \gamma t)$$

$$\therefore \frac{H}{H - (h_1 - h_2)} = 1 + \gamma t$$

$$\gamma = \frac{h_1 - h_2}{[H - (h_1 - h_2)]t} = \frac{h}{(H - h)t}$$

Knowing  $H$ ,  $h$  and  $t$ ,  $\gamma$  can be calculated.

### 2-22 Regnault's Modification of Dulong and Petit's Method (Second Method)

In the second method due to Regnault, the experimental arrangement consists of two glass tubes  $ABCD$  and  $EF GH$  joined by a flexible iron tubing  $DE$  (Fig. 2-12). The tubes  $CB$  and  $GF$  are narrow. The projecting limbs  $AB$  and  $GH$  are maintained at a constant temperature  $t_1$ ; the vertical limb  $CD$  is maintained at a



high temperature  $t_1$ ; the vertical limb  $EF$  is maintained at a temperature  $t_3$ ; and the connecting limb  $DE$  is exposed to air and let this temperature be  $t_4$ . The tubes  $BC$  and  $GF$  are adjusted to be accurately horizontal. When the temperatures are constant, let the heights of the mercury columns be  $h_1, h_2, h_3, H_1$  and  $H_2$  in the limbs  $AB, GH, ED, CD$  and  $EF$  respectively. If  $\rho_1, \rho_2$  etc. are the densities of the liquid at the temperatures  $t_1, t_2$  etc. then the pressures at  $D$  due to the columns of mercury in the hot and the cold limbs must be equal.

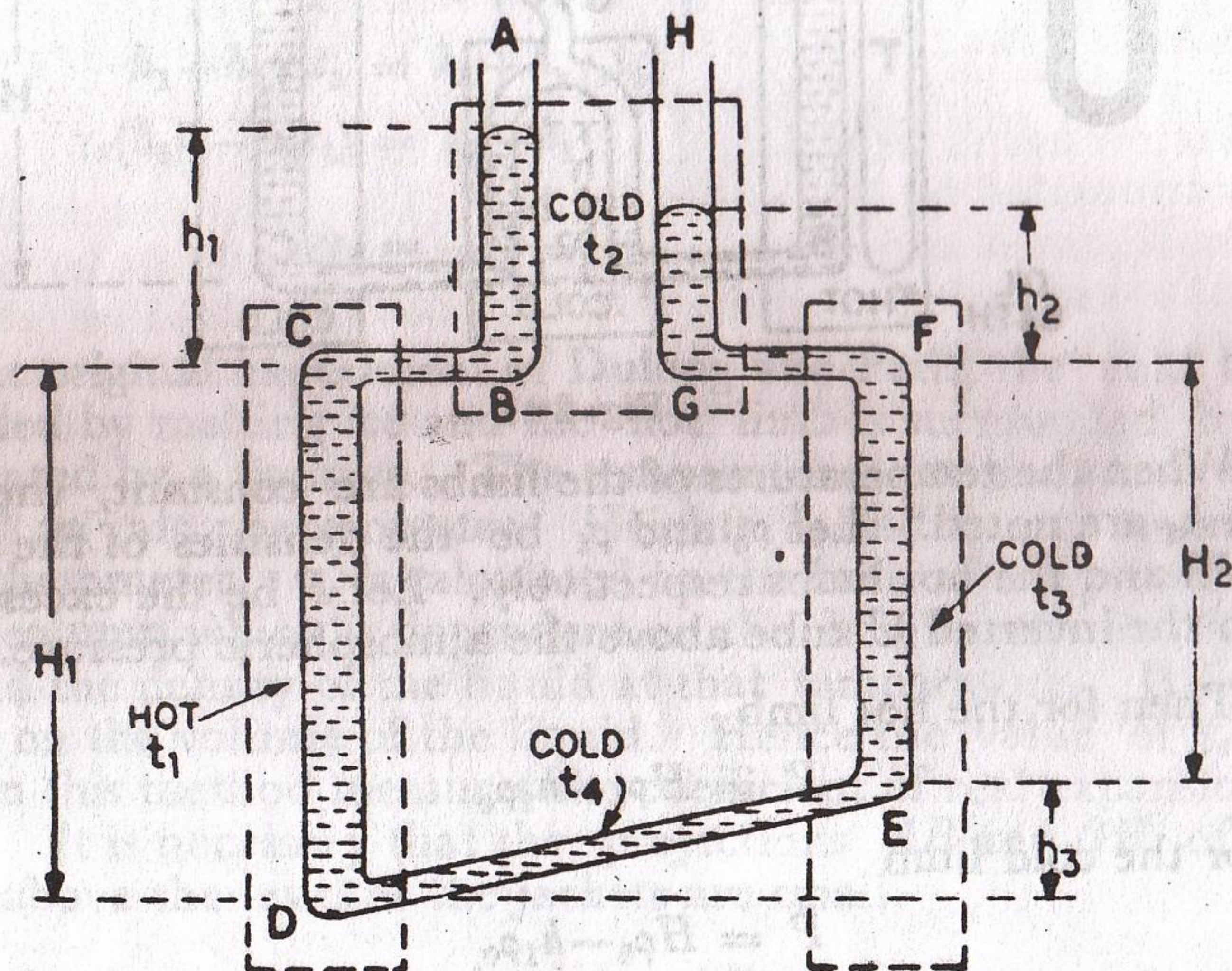


Fig. 2-12

$$\therefore h_1 \rho_2 + H_1 \rho_1 = h_2 \rho_2 + H_2 \rho_3 + h_3 \rho_4$$

But 
$$\rho_1 = \frac{\rho_0}{1 + \gamma t_1}$$

$$\rho_2 = \frac{\rho_0}{1 + \gamma t_2}$$

$$\rho_3 = \frac{\rho_0}{1 + \gamma t_3}$$

$$\rho_4 = \frac{\rho_0}{1 + \gamma t_4}$$

and

$$\therefore \frac{h_1}{1 + \gamma t_2} + \frac{H_1}{1 + \gamma t_1} = \frac{h_2}{1 + \gamma t_2} + \frac{H_2}{1 + \gamma t_3} + \frac{h_3}{1 + \gamma t_4}$$

or

$$\frac{h_1 - h_2}{1 + \gamma t_2} + \frac{H_1}{1 + \gamma t_1} = \frac{H_2}{1 + \gamma t_3} + \frac{h_3}{1 + \gamma t_4}$$

The heights and the temperatures are known in the above equation. Hence the value of  $\gamma$  can be calculated. The temperature  $t_4$  is not accurately known. But this does not involve appreciable error because the height  $h_3$  is very small.

**2-23. Experiment of Callendar and Moss**

In the second method of Regnault described in Art. 2-22, the

total length of the liquid column was 1.5 metres. Callendar and

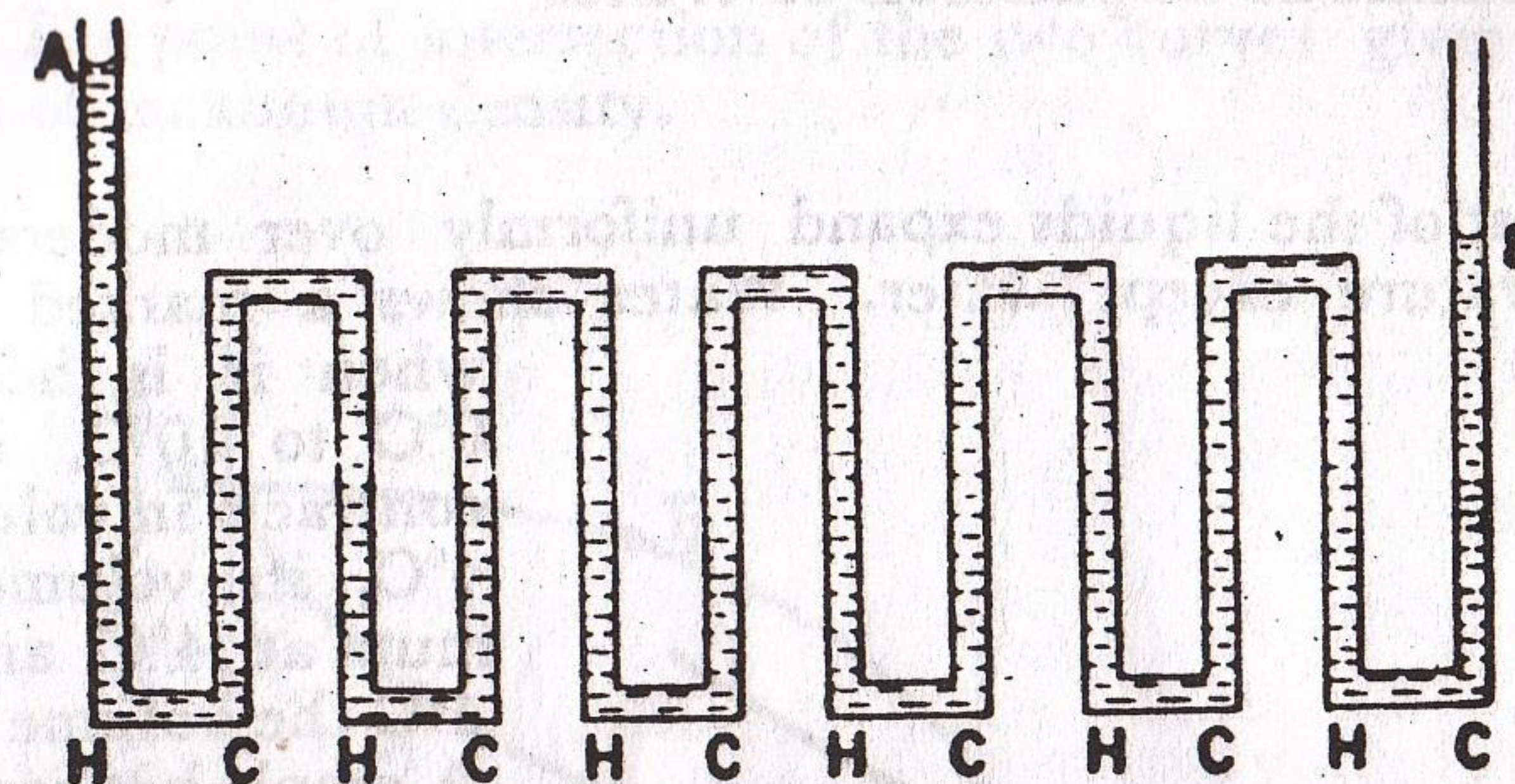
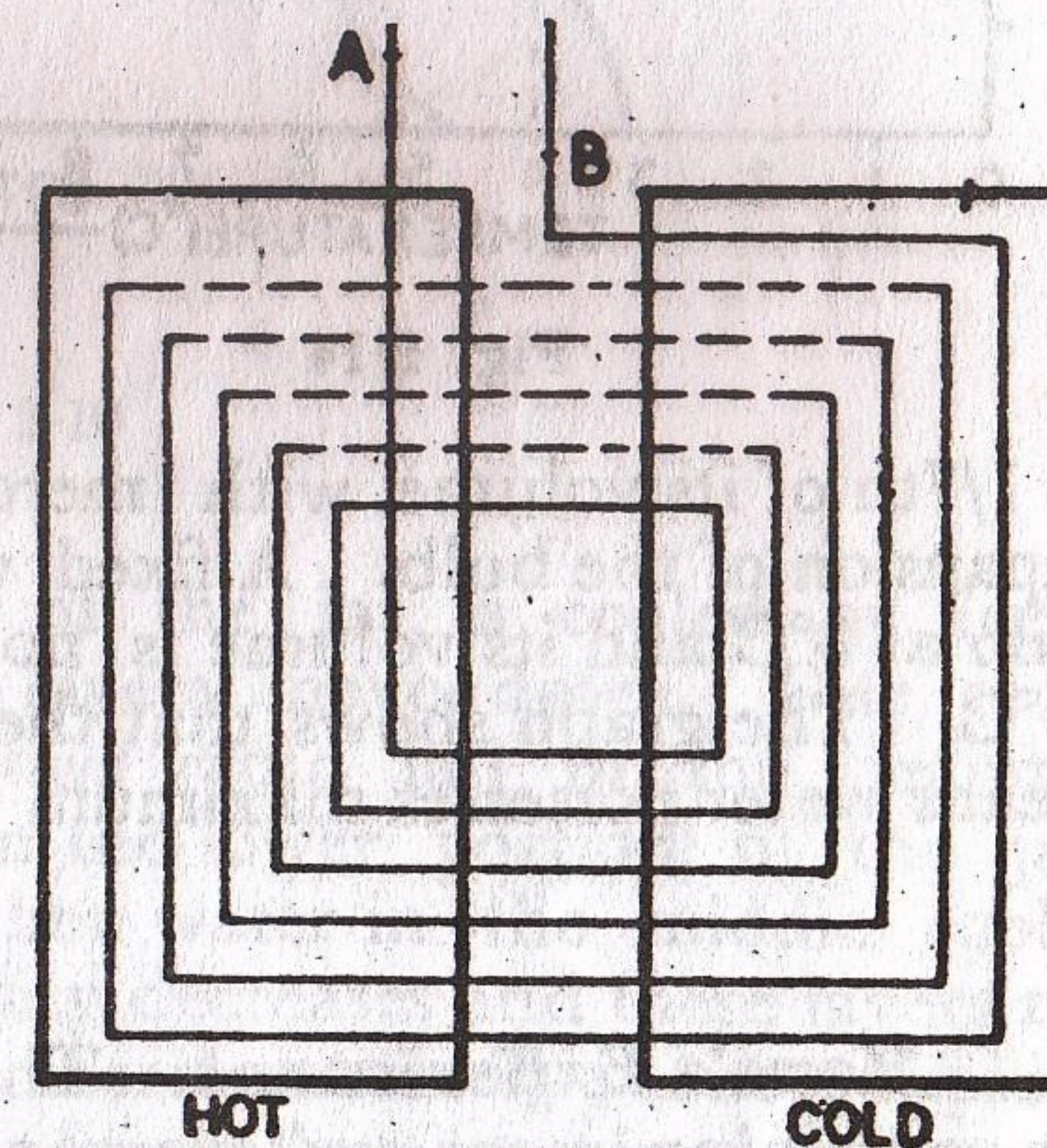


Fig. 2-13 (a)

Moss modified the experiment by using six hot and six cold limbs arranged in series [Fig. 2-13 (a)] and each limb had a length of 2 metres. This will mean that the difference in levels of the liquid in the projecting limbs will be eight times approximately. The hot columns are arranged side by side and enclosed in a constant temperature bath. Similarly all the cold columns are arranged side by side [Fig. 2-13 (b)].



With this experimental arrangement Moss was able to find the absolute coefficient of expansion of mercury from 0°C to 300°C.

Fig. 2-13 (b)

**Coefficient of cubical Expansion**

Substance	$\gamma_r \times 10^3$ per °C
Mercury	0.182
Glycerine	0.485
Ethyl Alcohol	0.745
Petroleum	0.899
Turpentine	0.940
Carbon disulphide	1.140
Water 5°C to 10°C	0.053
Water 10°C to 20°C	0.150
Water 20°C to 40°C	0.302



## 2-24 Anomalous Expansion of Water

Most of the liquids expand uniformly over moderate ranges of temperature except water. Water shows a marked exception

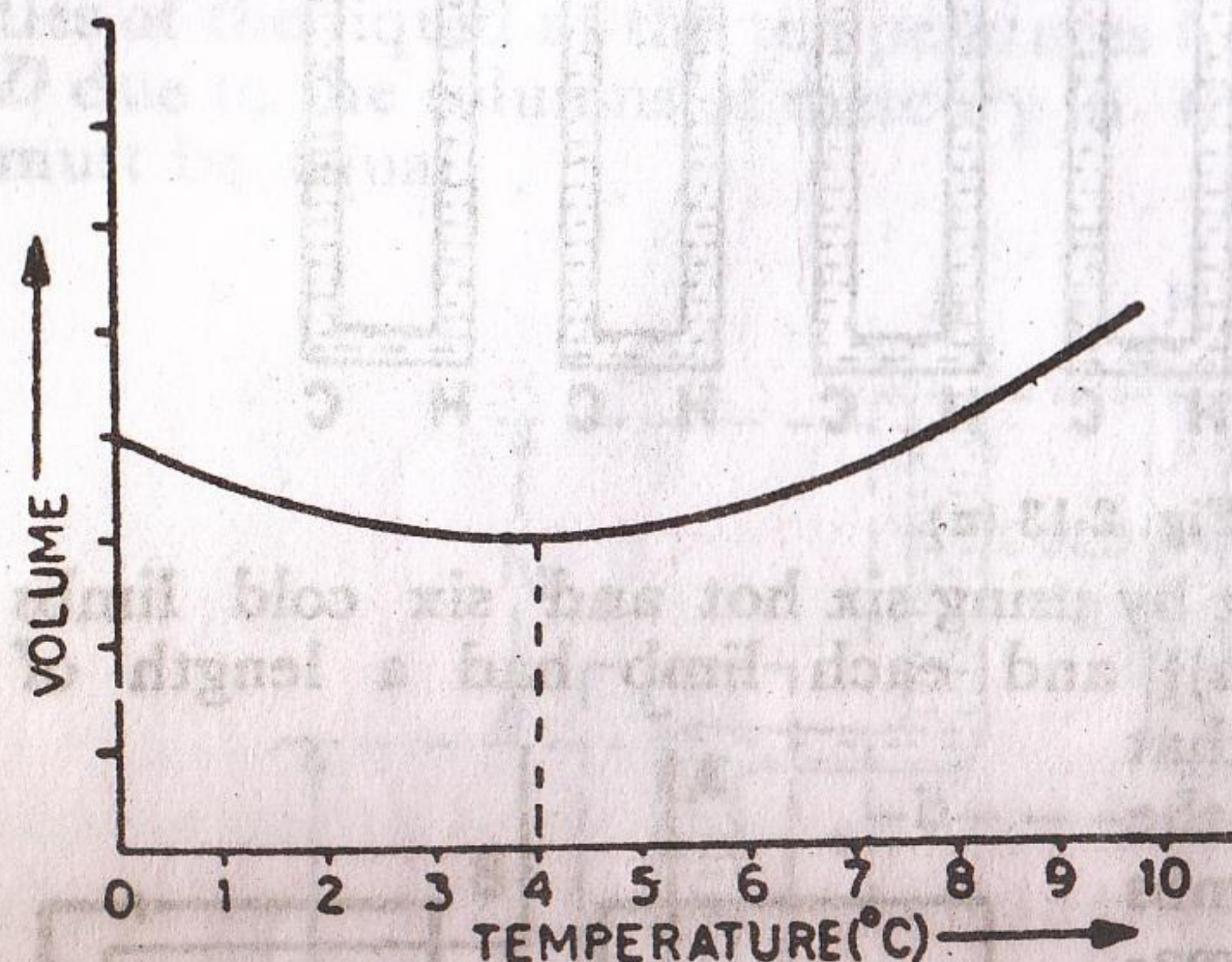


Fig. 2-14

when it is heated from 0°C to 10°C, initially it contracts in volume up to 4°C, its volume is minimum at 4°C and beyond 4°C the volume increases. A graph between volume and temperature is shown in Fig 2-14.

The variation in the volume of water can be studied with a volume dilatometer. It consists of a glass bulb fitted with a narrow graduated glass tube. The bulb is filled

to 1/7th of its volume with mercury so as to compensate for the expansion of the bulb. A fixed volume of water is taken in the bulb at 0°C and its volume is noted at various temperatures up to 10°C. The graph shows that the volume is minimum at 4°C. It means water possesses maximum density at 4°C.

**Hope's Experiment.** The apparatus consists of a cylindrical jar containing water and surrounded by a jacket containing ice in the middle of the jar as shown in Fig. 2-15.  $T_1$  and  $T_2$  are two thermometers to record the temperatures of water below and above the jacket respectively.

Water at 10°C is taken in the jar and the temperature shown by the thermometers  $T_1$  and  $T_2$  are recorded after equal intervals of time (say  $\frac{1}{2}$  minute).

A graph between temperature along the  $y$ -axis and time along the  $x$ -axis is plotted. The graphs for the two thermometers are as shown in Fig. 2-16.

The thermometer  $T_1$  shows a sudden fall of temperature up to 4°C. The thermometer  $T_2$  shows a slow fall in temperature. When the temperature of  $T_1$  remains at 4°C, the temperature  $T_2$  falls rapidly. Then thermometer  $T_2$

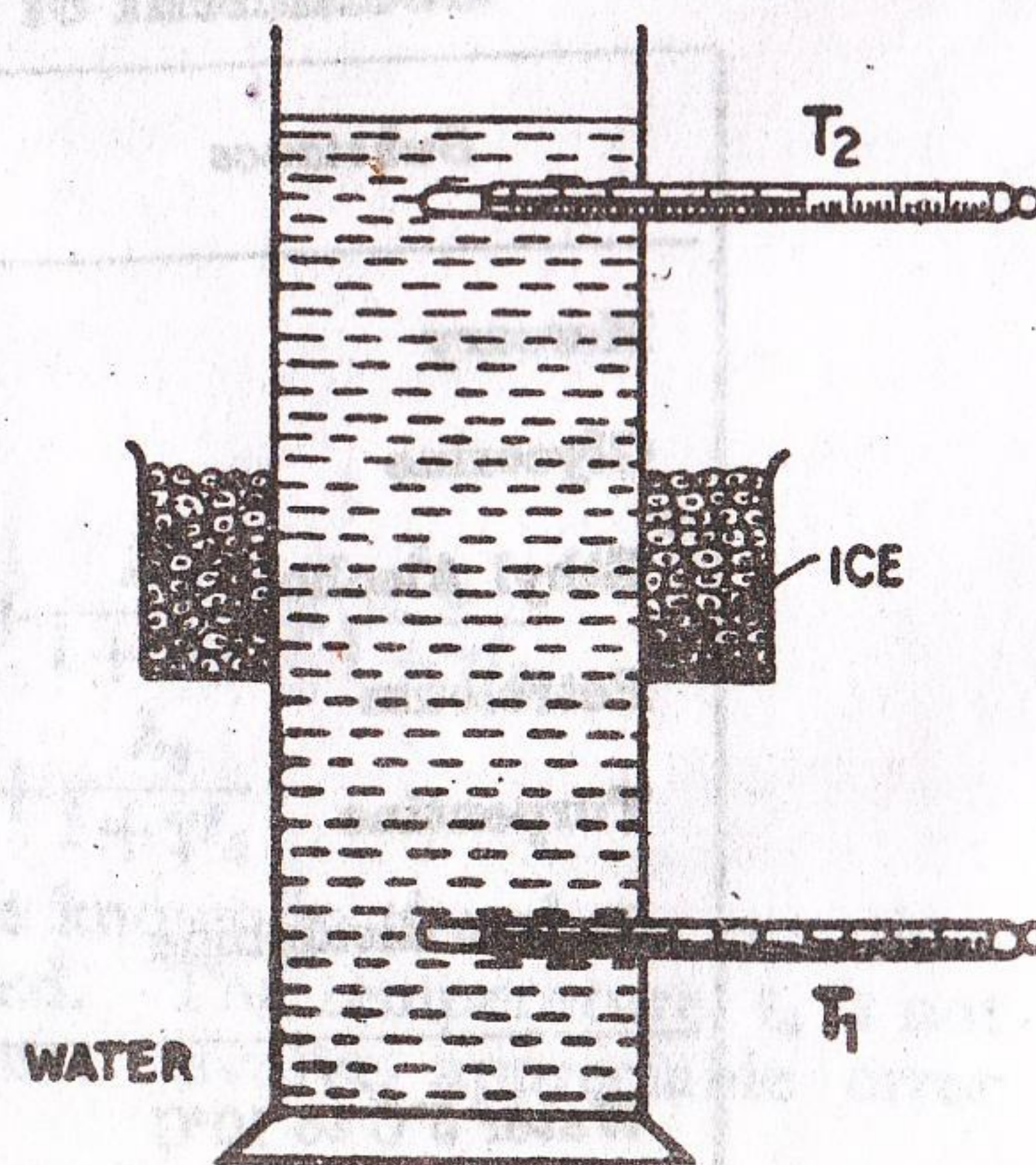


Fig. 2-15

records a temperature of 0°C first and the thermometer  $T_1$  afterwards. The point of intersection of the two curves gives the temperature of maximum density.

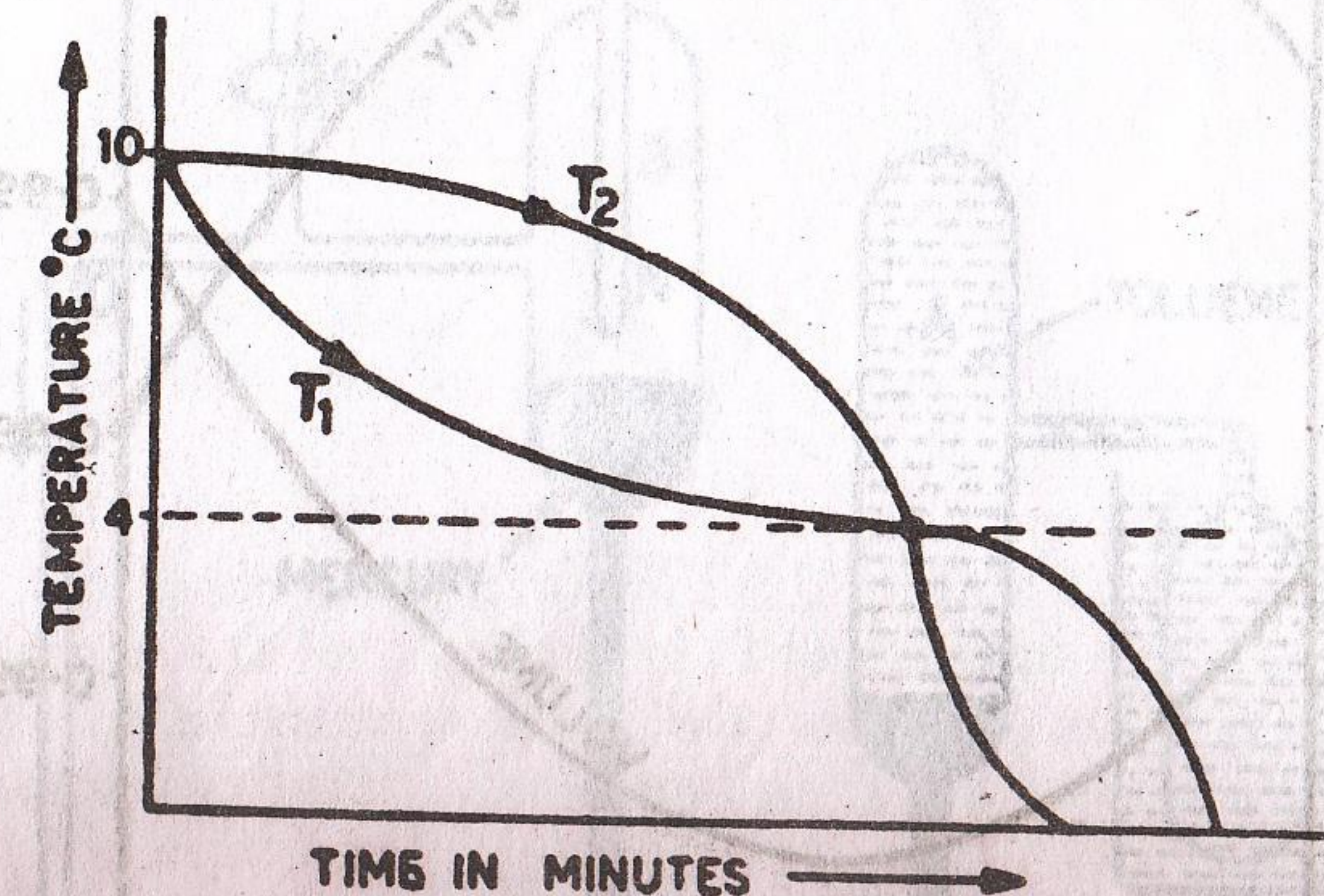


Fig. 2-16

When water in the middle of the jar is cooled by ice, its density increases and the heavier layers move down. Due to this reason the temperature of  $T_1$  shows rapid fall in the beginning. When the temperature of water in the lower portion of the jar is 4°C, its density is maximum. When water in the middle cools to a temperature below 4°C, its density decreases and these layers move upwards. Due to this reason the thermometer  $T_2$  shows a rapid fall of temperature. When the upper layers have attained a temperature of 0°C, the lower layers start cooling below 4°C.

The temperature of maximum density of water has been found by Joule and Playfair. By a series of experiments they found the temperature of maximum density of water to be 3.95°C.

With the increase in pressure over water the temperature of maximum density of water decreases.

$$t_{max} = [3.98 - 0.0225 (P - 1)]^\circ\text{C}$$

where  $P$  is the pressure in atmospheres. According to this relation, 1 kg of water has a volume of 1000.028 cc at 4°C.

Thus 1 cc of water at 4°C has a mass not exactly equal to 1 gram. Therefore a standard kg cannot be taken as the mass of 1 cubic decimetre of water (1000 cc) at 4°C. But it is defined in terms of a prototype kg made of platinum-iridium alloy.



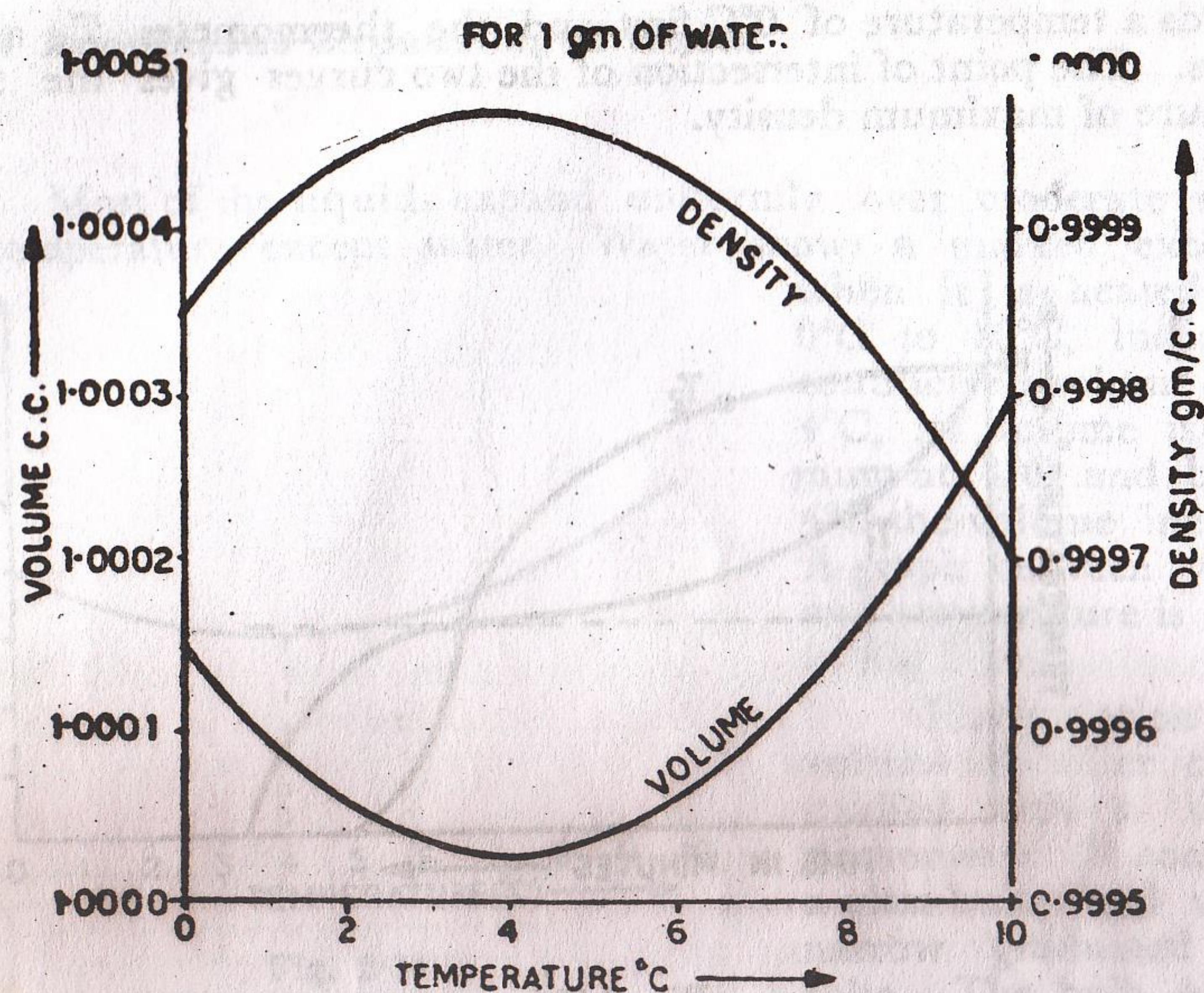


Fig. 2-16 (a)

Density of water at various temperatures

Temperature °C	Density g/cc	Volume of 1 gram of water (cc)
0	0.99987	1.00013
2	0.99997	1.00003
4	1.00000	1.00000
6	0.99997	1.00003
8	0.99988	1.00012
10	0.99973	1.00027
20	0.99823	1.00177
30	0.99567	1.00433
40	0.99220	1.00790
50	0.98810	1.01200

## 2.25. Thermostats

A thermostat is a device used to maintain the temperature of a water bath or an oven at constant temperature.

**Toluene Thermostat.** A liquid thermostat consists of an elongated glass bulb *A* which contains a liquid having high coeffi-

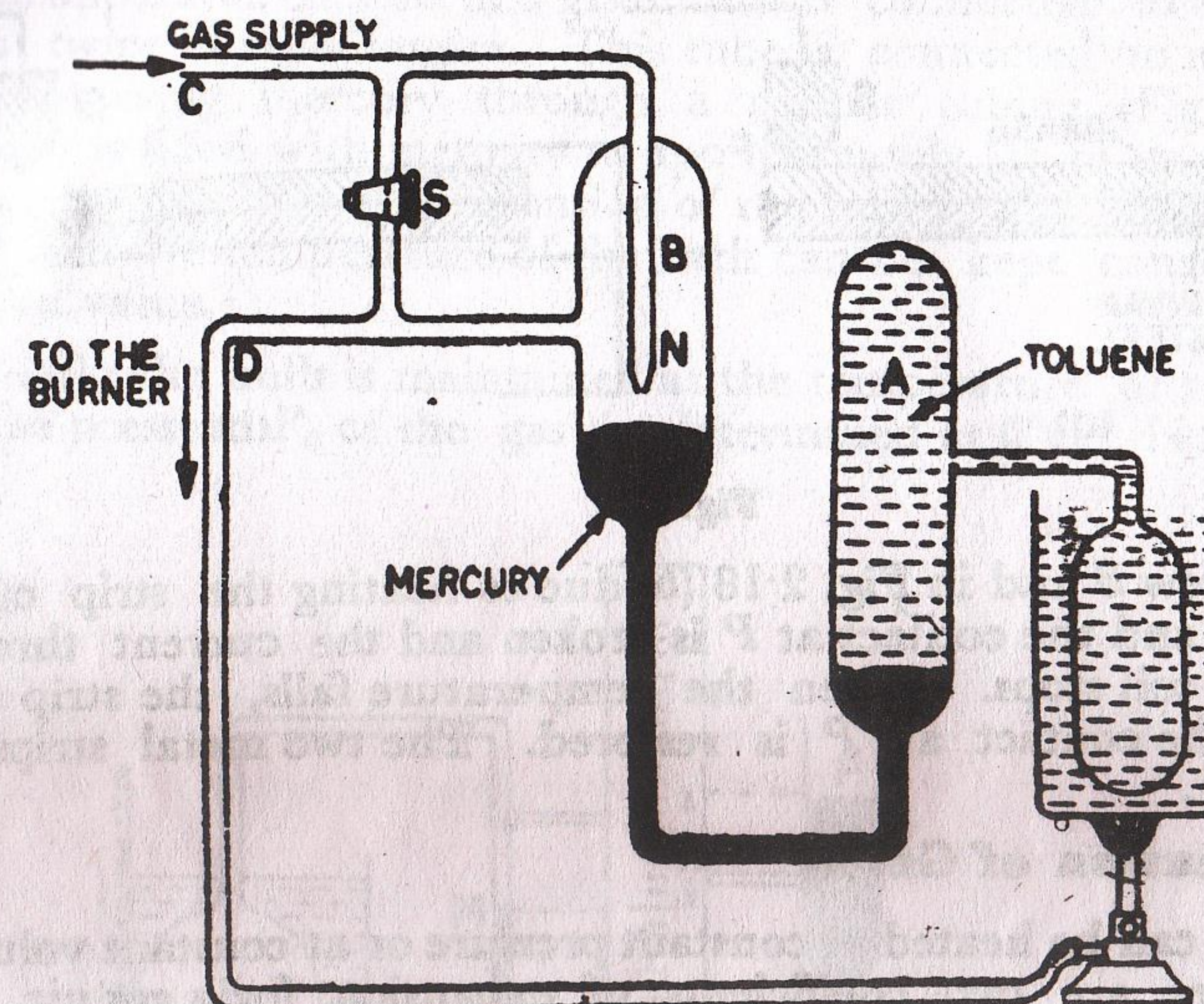


Fig. 2-17

cient of expansion e.g., toluene or alcohol. The bulb *A* is connected to another bulb *B* through a glass tubing as shown and the connecting tube and also partly bulbs *A* and *B* contain mercury. Mercury is in contact with the expansible liquid in *A* and is almost up to the nozzle in *B*. The gas supply is connected at *C* and the burner is connected at *D*. *S* is a stop-cock used for regulating the supply of gas from *C* to *D* (Fig. 2-17). The bulb is immersed in the water bath and when the temperature of the bath increases, toluene expands and pushes mercury down. Consequently the level of mercury in *B* rises and when the desired temperature is reached, the gas supply to the burner through the nozzle is cut off. The stop-cock *S* helps to maintain a feeble supply of the gas so that the burner is not put off. When the gas supply is considerably cut off, the temperature of the bath decreases, toluene contracts and the level of mercury in *B* goes down. Thus the gas supply to the burner through the nozzle is restored and the temperature of the bath increases again. The process repeats and the temperature of the liquid thermostat is limited by the boiling point of the liquid. In the case of toluene, this temperature is 111°C.

**Bimetallic Thermostat.** For higher temperature control bimetallic thermostats are used. The bimetallic strip works as an electric contact breaker in an electrical heating circuit. The circuit is broken when the desired temperature of the bath is reached. Due to the difference in the coefficients of linear expansion of the two metals, when it is heated the metallic strip curves and the circuit is broken. The metal of higher coefficient of expansion



(say brass) bends more than the other of lesser coefficient of expansion say (invar steel). In Fig. 2-18 (a) the metallic strip is in contact

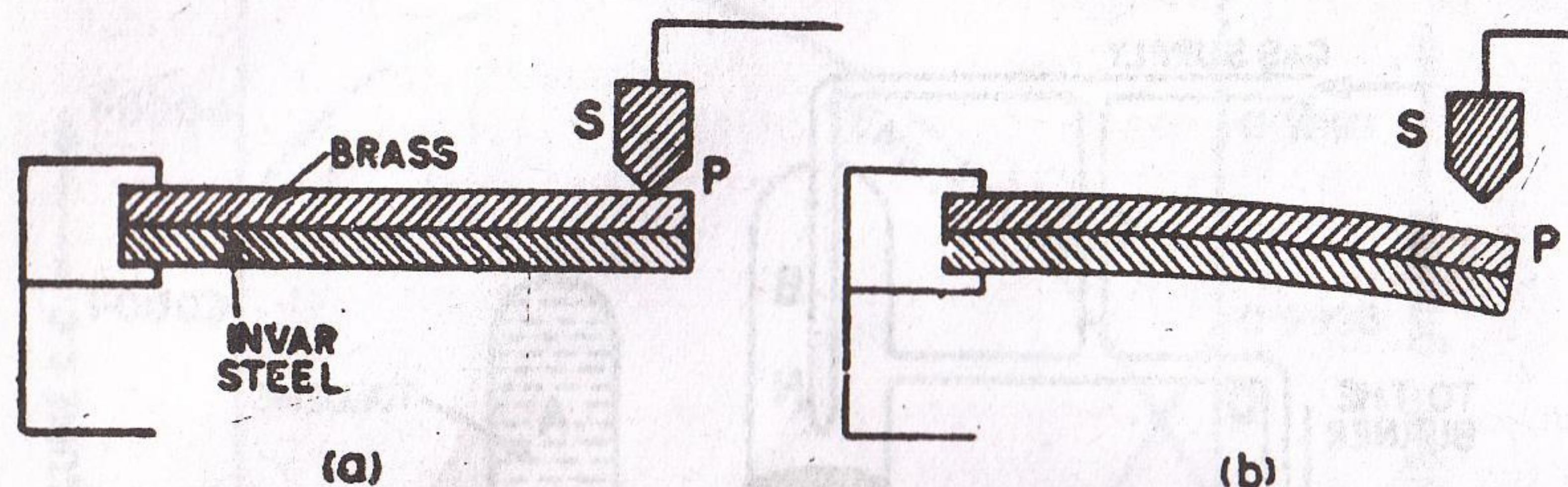


Fig. 2-11

with the screw  $S$  and in Fig. 2-18 (b) due to heating the strip curves downwards and the contact at  $P$  is broken and the current through the heating coil stops. When the temperature falls, the strip contracts and the contact at  $P$  is restored. The two metal strips are well riveted.

### 2-26 Expansion of Gases

A gas can be heated at constant pressure or at constant volume. Hence there are two coefficients of expansion for a gas *viz.*, the pressure coefficient and the volume coefficient.

**Pressure coefficient ( $\gamma_p$ )** is defined as the increase in pressure per unit pressure per unit degree rise of temperature at constant volume. If  $P_0$  and  $P_t$  are the pressures of a given mass of gas at temperatures  $0^\circ\text{C}$  and  $t^\circ\text{C}$  respectively, then

$$\gamma_p = \frac{P_t - P_0}{P_0 \times t} \quad (\text{at constant volume})$$

or

$$P_t = P_0 [1 + \gamma_p t]$$

**Volume coefficient ( $\gamma_v$ )** is defined as the increase in volume per unit volume per unit degree rise of temperature at constant pressure. If  $V_0$  and  $V_t$  are the volumes of a given mass of gas at temperatures  $0^\circ\text{C}$  and  $t^\circ\text{C}$  respectively, then

$$\gamma_v = \frac{V_t - V_0}{V_0 \times t} \quad (\text{at constant pressure})$$

or

$$V_t = V_0 [1 + \gamma_v t]$$

Study of the properties of expansion of gases is as important as the study of the expansion of liquids and solids. The property of expansion of a gas has provided an accurate and standard method of measuring temperature (*vide* chapter 1 on thermometry). Further, the properties of expansion of gases has led to the deduction of the universal gas laws.

The expansion of gases is much more than liquids and the expansion of the container is very small.

### 2-27 Determine of the Pressure Coefficient of a Gas

The apparatus consists of a glass bulb  $B$  connected to a glass tube bent twice at right angles. This tube is connected to a reservoir  $R$  containing mercury through a rubber tubing (Fig. 2-19). The bulb  $B$  is filled with mercury to approximately  $\frac{1}{4}$ th of its volume so as to compensate for the expansion of the bulb. The bulb is kept in a bath and the temperature of the bath can be kept constant at any desired value.

Initially the bulb is maintained at the temperature of melting ice and the pressure  $P_0$  of the gas is determined at  $0^\circ\text{C}$ . Now the

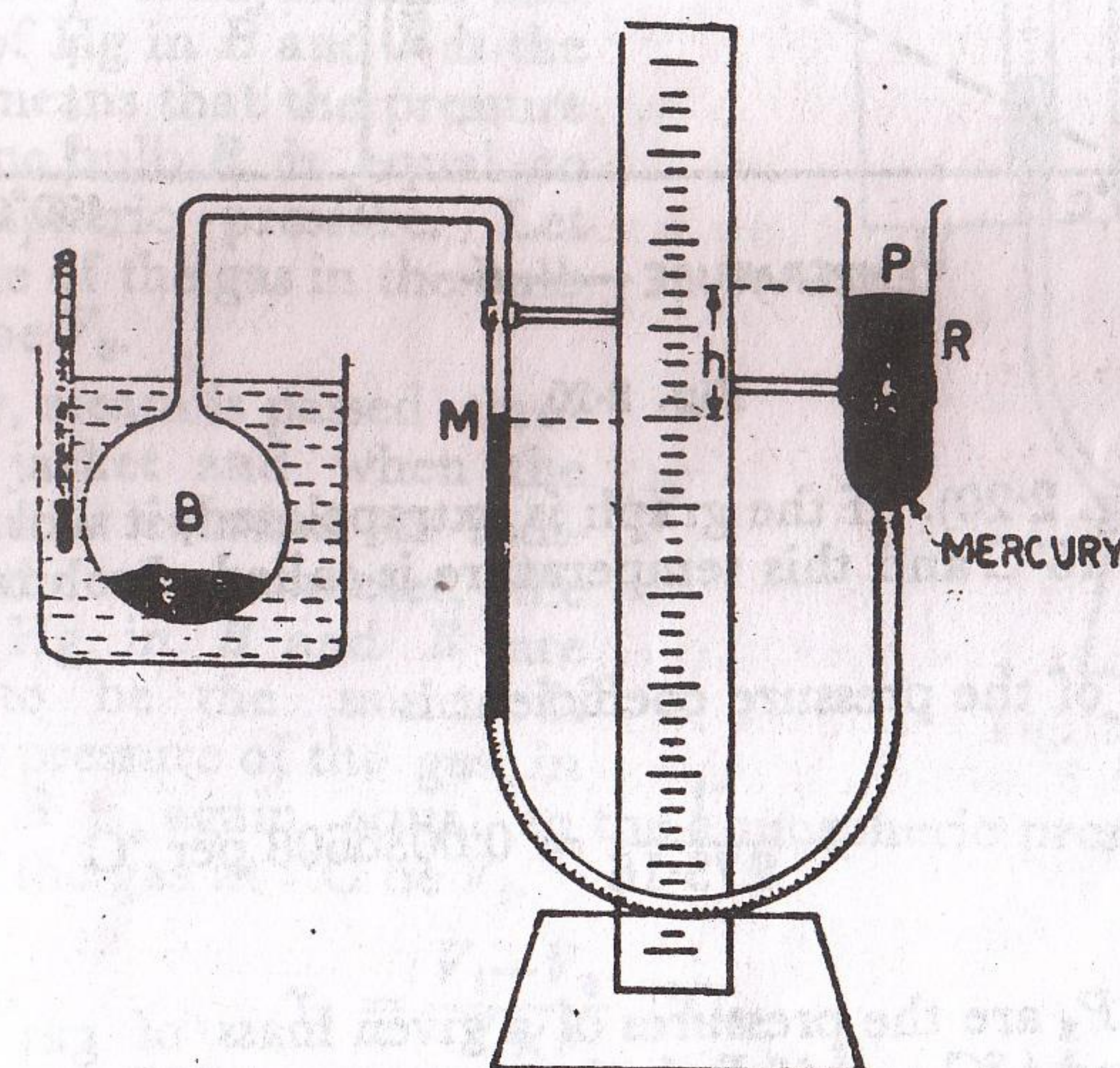


Fig. 2-19

bulb is maintained at different temperatures  $t_1^\circ\text{C}$ ,  $t_2^\circ\text{C}$  etc. and the pressures of the gas  $P_1$ ,  $P_2$  etc. are calculated at each temperature. Before noting the difference in levels at any temperature, the level of mercury in the closed limb is brought to the fixed mark  $M$  so that the volume of the gas is kept constant at each temperature.

If  $h_0$ ,  $h_1$ ,  $h_2$  etc. are the differences in levels at  $0^\circ\text{C}$ ,  $t_1^\circ\text{C}$ ,  $t_2^\circ\text{C}$  etc. and  $P_A$  is the atmospheric pressure, then

$$P_0 = P_A \pm h_0;$$

$$P_1 = P_A \pm h_1;$$

$$P_2 = P_A \pm h_2 \text{ etc.}$$

according as the level of mercury in  $R$  is above or below the fixed mark  $M$ . Then

$$\gamma_p = \frac{P_1 - P_0}{P_0 \times t_1} = \frac{P_2 - P_0}{P_0 \times t_2} \text{ etc.}$$



The value of  $\gamma_p$  is calculated in each case and the mean value is taken.

If a graph is drawn between temperature in  $^{\circ}\text{C}$  along the X-axis and the pressure of the gas along the Y-axis, the graph will be a

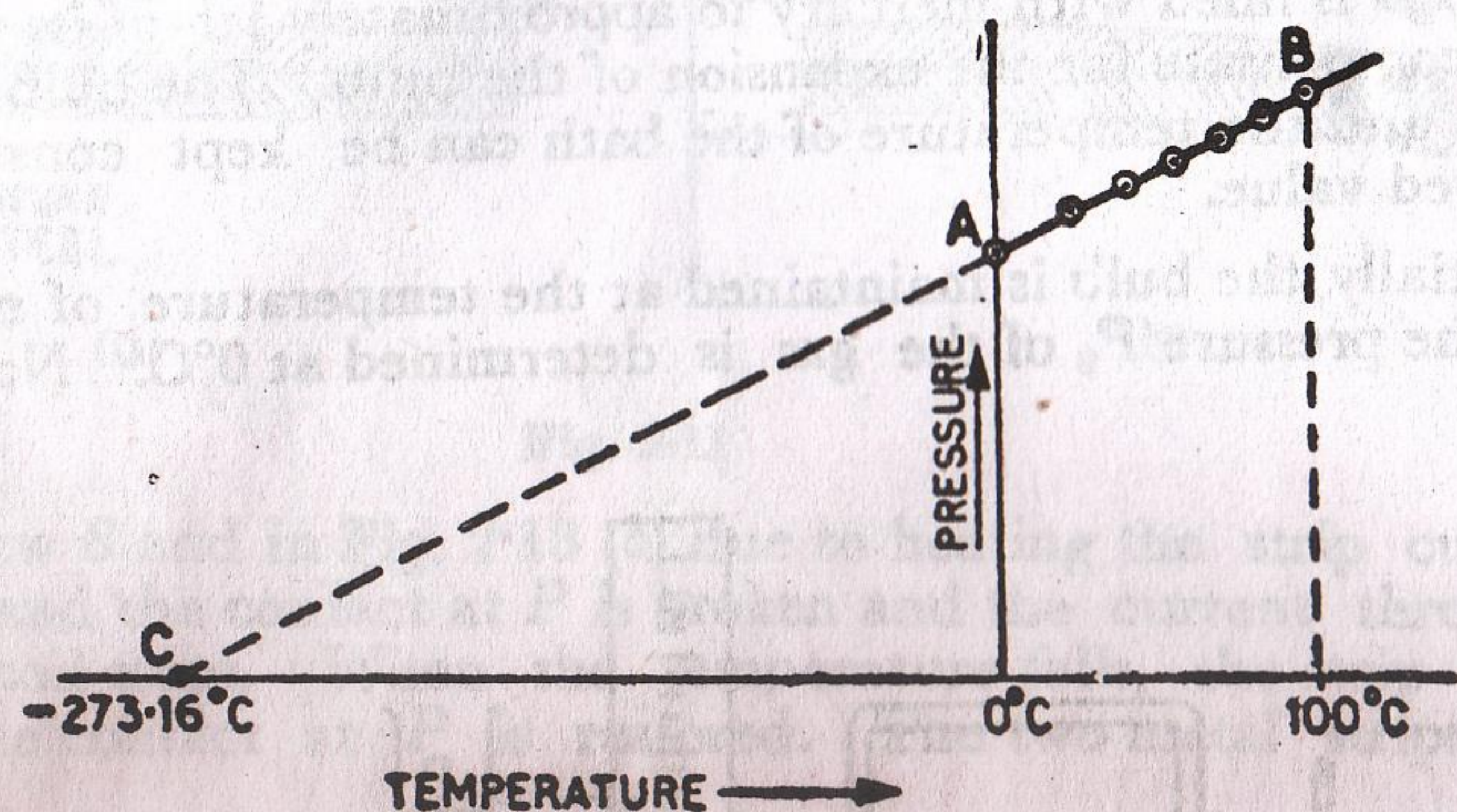


Fig. 2-20

straight line (Fig. 2-20). If the graph is extrapolated, it will meet the X-axis at  $-273.16^{\circ}\text{C}$  and this temperature is called absolute zero.

The value of the pressure coefficient is

$$= \frac{1}{273.16} = 0.0036608 \text{ per } ^{\circ}\text{C}$$

If  $P_1$  and  $P_2$  are the pressures of a given mass of gas at temperatures  $t_1^{\circ}\text{C}$  and  $t_2^{\circ}\text{C}$  and if  $P_0$  is the pressure at  $0^{\circ}\text{C}$

$$P_1 = P_0[1 + \gamma_p t_1]$$

$$P_2 = P_0[1 + \gamma_p t_2]$$

$$\frac{P_2}{P_1} = \frac{1 + \gamma_p t_2}{1 + \gamma_p t_1}$$

$$\text{or } \gamma_p = \frac{P_2 - P_1}{P_1 t_2 - P_2 t_1}$$

Using this relation,  $\gamma_p$  can be calculated for different ranges of temperature.

The value of  $\gamma_p$  can also be calculated by finding the absolute zero from the graph. The reciprocal of this temperature measures  $\gamma_p$ . If the value of absolute zero is  $-273.16^{\circ}\text{C}$ , then

$$\gamma_p = \frac{1}{273.16}$$

## 2-28 Determination of the Volume Coefficient of a Gas

The volume coefficient of a gas can be determined in the laboratory by the following method. The apparatus consists of a glass bulb  $B$  connected to a reservoir  $R$  containing mercury. The bulb  $B$  is surrounded by the outer glass jacket through which steam can be passed. A thermometer is fixed to note the temperature.

Initially water at  $0^{\circ}\text{C}$  is circulated through the jacket and the reservoir  $R$  is adjusted so that the level of Hg in  $B$  and  $R$  is the same. It means that the pressure of air in the bulb  $B$  is equal to the atmospheric pressure. Let the volume of the gas in the bulb  $B$  at  $0^{\circ}\text{C}$  be  $V_0$ .

Now, steam is passed through the jacket and when the temperature is constant as indicated by the thermometer, the levels of Hg in  $B$  and  $R$  are adjusted to be the same. It means the pressure of the gas in the bulb  $B$  is again equal to the atmospheric pressure. Let the volume of the gas at  $t^{\circ}\text{C}$  be  $V_t$ .

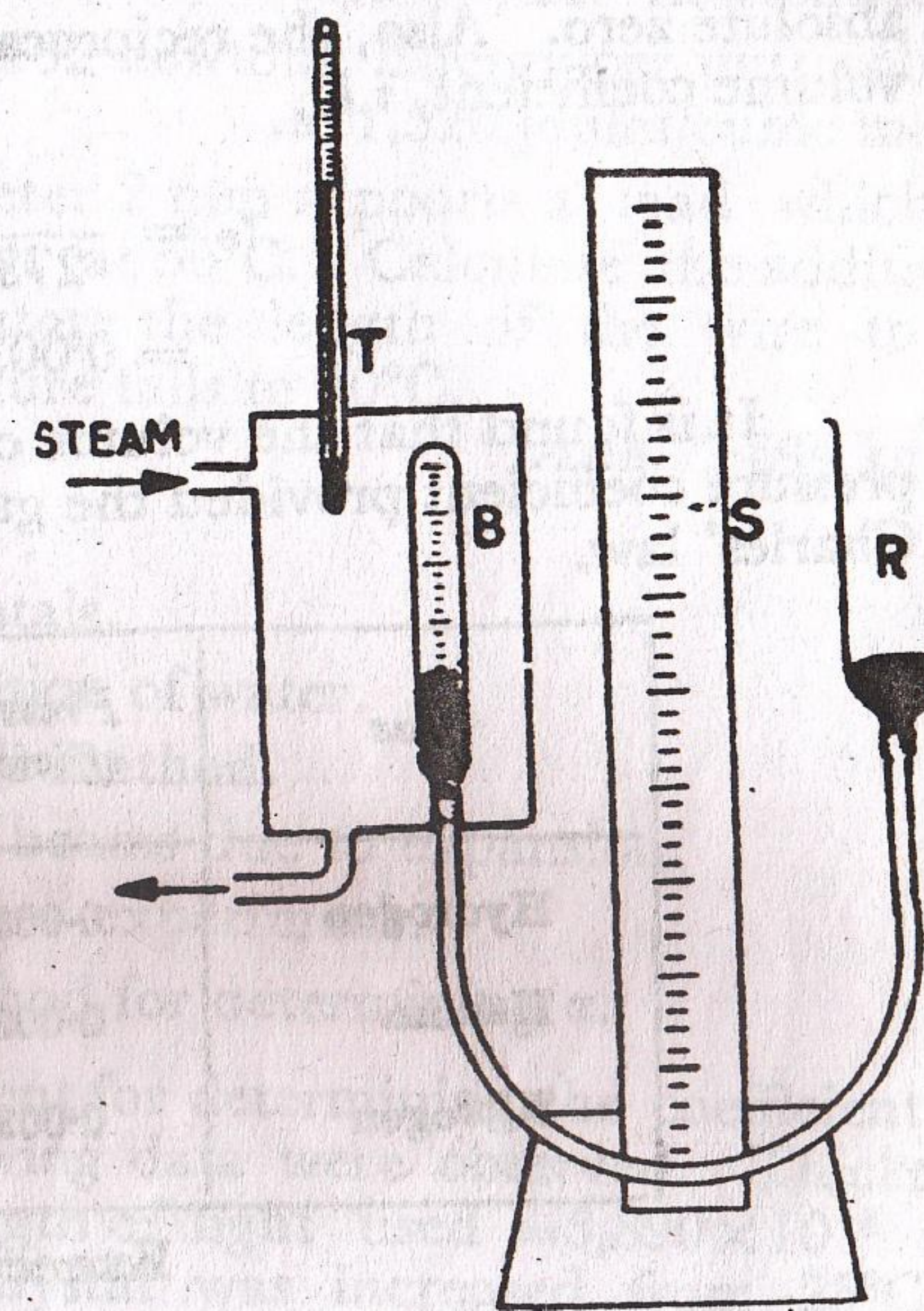


Fig. 2-21

$$\text{Then } \gamma_v = \frac{V_t - V_0}{V_0 t} \text{ } ^{\circ}\text{C}$$

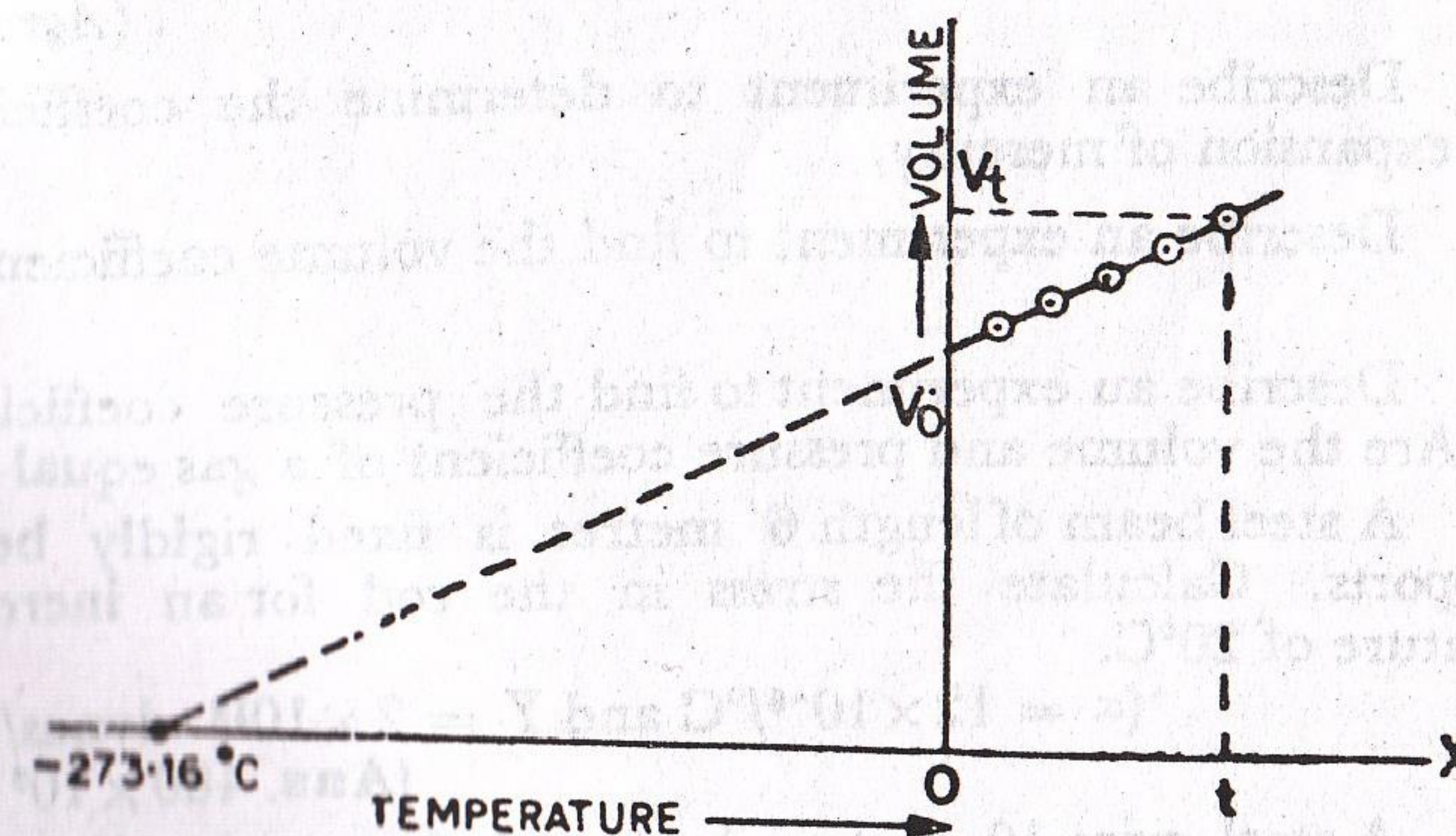


Fig. 2-22

The volume coefficient calculated is about

$$\frac{1}{273.16} = 0.0036608 \text{ } ^{\circ}\text{C}$$



To plot a graph between the volume and temperature, a large number of readings are obtained after the steam is cut off. A graph is drawn between temperature along the X-axis and the volume along the Y-axis. The graph is a straight line. If the graph is extrapolated it meets the X-axis at  $-273.16^\circ\text{C}$ . This corresponds to absolute zero. Also, the reciprocal of this temperature measures the volume coefficient, i.e.,

$$\gamma_v = \frac{1}{273.16} \\ = 0.0036608/^\circ\text{C}$$

It is found that the volume coefficient of a gas is equal to the pressure coefficient provided the gas obeys the Boyle's law and the Charles' law.

Gas	Pressure co-efficient $\gamma_p$	Volume co-efficient $\gamma_v$
Hydrogen	0.0036610	0.0036611
Helium	0.0036609	0.0036607
Nitrogen	0.0036606	0.0036609

### Exercises II

- Define coefficient of linear expansion, cubical expansion and superficial expansion. Derive the relation between them.
- Describe Fizeau's method for determining the coefficient of linear expansion of a crystal. Discuss Tutton's modification.
- Describe how the coefficient of linear expansion of a solid can be determined with the help of an interference method.  
(Agra 1966)
- Describe an experiment to determine the coefficient of cubical expansion of mercury.
- Describe an experiment to find the volume coefficient of a gas.
- Describe an experiment to find the pressure coefficient of a gas. Are the volume and pressure coefficient of a gas equal?
- A steel beam of length 6 metres is fixed rigidly between two supports. Calculate the stress in the rod for an increase in temperature of  $20^\circ\text{C}$ .  
( $\alpha = 12 \times 10^{-6}/^\circ\text{C}$  and  $Y = 2 \times 10^{12}$  dynes/sq cm)  
[Ans.  $480 \times 10^6$  dynes]
- A steel wire 10 metres long and 2 mm in diameter is fixed between two rigid supports. Calculate the increase in tension when the temperature falls through  $30^\circ\text{C}$ .  
( $\alpha = 12 \times 10^{-6}/^\circ\text{C}$  and  $Y = 2 \times 10^{12}$  dynes/sq cm)  
[Ans.  $22.627 \times 10^6$  dynes]

9. A uniform rod 4 sq mm in cross-section is heated from  $0^\circ\text{C}$  to  $40^\circ\text{C}$ . Find the force which must be exerted to prevent it from expanding. Find the energy stored per unit volume.

$$(\alpha = 12 \times 10^{-6}/^\circ\text{C} \text{ and } Y = 10^{11} \text{ newtons/sq m})$$

Ans. Force = 192 newtons, Energy per unit volume = 11,520 joules/cubic metre]

10. A steel wire of diameter 2 mm supports a load which is sufficient to keep the wire taut at  $50^\circ\text{C}$ . Calculate the additional load that will be required to restore the length of the wire to its initial value when the temperature falls to  $40^\circ\text{C}$ .

[Ans. 7.694 kg wt]

11. Write short notes on :

- Expansion of crystals.
- Anomalous expansion of water.
- Dulong and Petit's method.
- Thermal stress in beams due to expansion.
- Callendar and Moss experiment.
- Optical lever method for determining  $\alpha$ .

12. In Tutton's experiment for determining the coefficient of expansion of a crystal, the following data were observed. Thickness of the crystal = 5 mm, wavelength of light used =  $5,890 \times 10^{-8}$  cm. When the temperature of the crystal was increased from  $25^\circ\text{C}$  to  $55^\circ\text{C}$ , the number of fringes that crossed the field of view was found to be 7. Calculate the coefficient of linear expansion of the crystal.

[Ans.  $13.74 \times 10^{-6}/^\circ\text{C}$ ]



## Calorimetry

### 3.1 Definitions

When a body is heated, its temperature rises. If 100 g of copper and 100 g of water are heated by similar burners for the same time, the rise in temperature is not the same in the two cases. The rise in temperature depends on the quantity of heat given to the body and the nature of its material. Let  $H$  be the quantity of heat given to a body of mass  $m$  and let the rise in temperature be  $\theta$ .

$$\text{Then, } H = mC\theta$$

where  $C$  is a constant that depends upon the nature of the substance.  $C$  is called the specific heat of the substance.

**Calorie.** It is the quantity of heat required to raise the temperature of one gram of water from  $14.5^\circ\text{C}$  to  $15.5^\circ\text{C}$ . This is the standard unit recommended by International Committee of Pure Physics.

For ordinary purposes, the specific heat of water is taken as 1 but specific heat of water is not 1 at all temperatures. For practical purposes *Calorie* may be defined as the amount of heat required to raise the temperature of 1 gram of water through  $1^\circ\text{C}$ .

**Kilogram Calorie.** It is the amount of heat required to raise the temperature of 1 kg of water through  $1^\circ\text{C}$ .

$$1 \text{ kg calorie} = 1000 \text{ calories} = 1 \text{ kilocalorie}$$

**British Thermal Unit.** It is the amount of heat required to raise the temperature of 1 pound of water through  $1^\circ\text{F}$ .

$$1 \text{ B.T.U.} = 252 \text{ calories}$$

**Therm.** It is the amount of heat required to raise the temperature of  $10^5$  pounds of water through  $1^\circ\text{F}$ .

$$1 \text{ Therm} = 2.52 \times 10^7 \text{ calories}$$

**Pound Calorie.** It is the amount of heat required to raise the temperature of 1 pound of water through  $1^\circ\text{C}$ .

$$1 \text{ pound calorie} = 453.6 \text{ calories}$$

It is called centigrade heat unit.

### Units of Heat

Unit	Quantity of water	Rise in temp.	Relation
Calorie	1 g	$1^\circ\text{C}$	= 1 Calorie
Kg Calorie	1 kg	$1^\circ\text{C}$	= 1000 Calories
BTU	1 pound	$1^\circ\text{F}$	= 252 ..
Therm	$10^5$ pounds	$1^\circ\text{F}$	= $2.52 \times 10^7$ ..
Pound Calorie	1 pound	$1^\circ\text{C}$	= 453.6 ..

**Specific Heat.** It is defined as the quantity of heat required to raise the temperature of unit mass of a substance through one degree.

Suppose,

$$\text{Mass of the substance} = m$$

$$\text{Sp. heat of the substance} = C$$

$$\text{Rise in temperature} = \theta$$

$$H = mC\theta$$

or

$$C = \frac{H}{m\theta}$$

**Unit of specific heat.** In C.G.S. system, the unit of  $H$  is in calories,  $m$  is in grams and  $\theta$  is in  $^\circ\text{C}$ . Therefore the unit of specific heat will be

$$C = \frac{\text{calorie}}{\text{g } ^\circ\text{C}} \\ = \text{calorie/g } ^\circ\text{C}$$

The specific heat of a substance is not constant and it is different at different temperatures. Ordinarily, the specific heat determined is the mean specific heat. Suppose,  $m$  is the mass of the substance,  $C$  the mean specific heat and if  $H$  units of heat is required to raise its temperature from  $\theta_1$  to  $\theta_2$ , then the mean specific heat

$$C = \frac{H}{m(\theta_2 - \theta_1)}$$

For qualitative work, if  $dQ$  heat is given to raise the temperature of  $m$  g of a substance through  $d\theta$ ,

$$dH = m C d\theta$$

$$C = \frac{1}{m} \cdot \frac{dH}{d\theta}$$

**Thermal Capacity.** It is the quantity of heat required to raise the temperature of the whole of the substance through  $1^\circ\text{C}$ . Let the mass of the substance be  $m$  and its specific heat  $C$ .

$$\text{Thermal capacity} = m \times C \times 1$$

$$= m C \text{ calories/}^\circ\text{C}$$

**Water Equivalent.** It is the amount of water that will absorb the same quantity of heat as the substance for the same rise



in temperature. Let the mass of the substance be  $m$ , specific heat  $C$  and rise in temperature  $\theta$ .

$$H = m C \theta$$

If the water equivalent =  $w$

$$H = w \times 1 \times \theta$$

$$\therefore w \times 1 \times \theta = m C \theta$$

or

$$w = m C \text{ grams}$$

Water equivalent is numerically equal to the thermal capacity but the unit of water equivalent is grams and that of thermal capacity is calories.

### 3.2 Regnault's Method of Mixtures

**Specific heat of solids.** Regnault was the first to devise the apparatus commonly used in the laboratories to find the specific heat of a solid or liquid employing the method of mixtures. In this the given solid is heated to a constant high temperature and then it is quickly transferred into a calorimeter containing water at room temperature. The final temperature of the mixture is noted. From the principle that heat lost by the solid is equal to the heat gained by the calorimeter and contents, the specific heat of the given solid is calculated.

### 3.3 Copper Block Calorimeter

Specific heat of a solid at high temperature cannot be determined by ordinary apparatus used in the laboratory. Steam or vapours produced would cause considerable loss of heat when a hot solid is mixed with water or a liquid. The result obtained will not be accurate. To overcome this difficulty, Nernst and Lindemann designed a calorimeter called the copper block calorimeter.

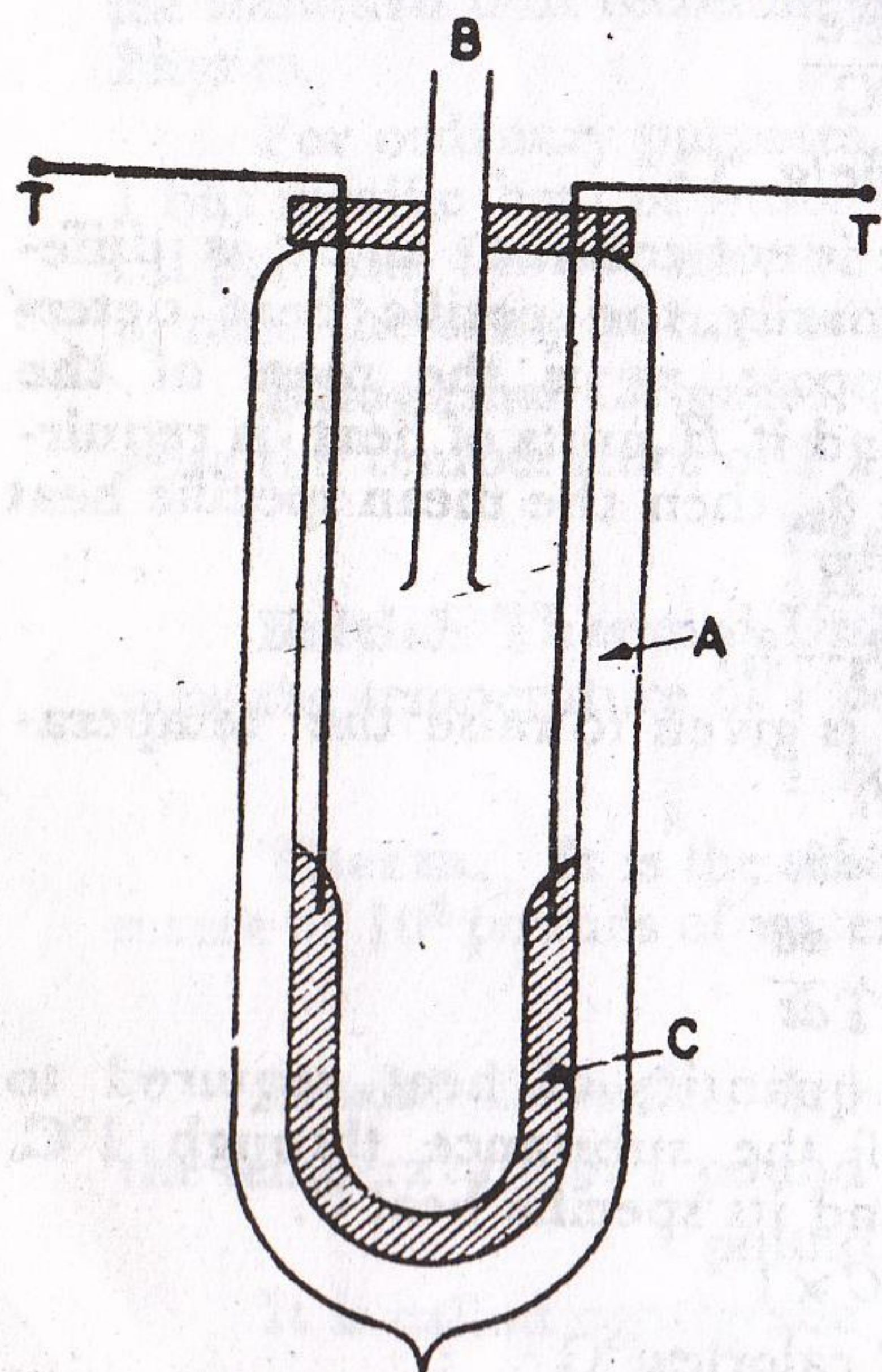


Fig. 3-1

It consists of a Dewar flask  $A$  having a copper block  $C$  fixed inside it.  $T, T$  are the terminals of the leads of the thermocouple to measure the temperature of the block  $C$ . The flask is covered from outside to prevent any loss of heat to the surroundings. Through the tube  $B$ , a hot solid can be dropped (Fig. 3-1).

The substance whose specific heat is to be determined is heated to a known high temperature and then gently dropped through the tube  $B$  into the block  $C$ . The copper block has high thermal conductivity and a uniform final temperature is reached in a short time. The temperature is measured with the help of a calibrated thermocouple, the copper block  $C$  acting

as the hot end. To avoid air currents the tube  $B$  is closed with an automatic electrical arrangement after the substance has been dropped into the block  $C$ . The whole apparatus can be maintained at a desired temperature. From the heat gained by copper block and the heat lost by the substance, the specific heat of the substance can be calculated.

Magnus determined the specific heat of platinum and other metals up to  $900^\circ\text{C}$ . Jaeger and his co-workers determined the specific heat of Pt, tungsten Rh, Ir and other metals up to  $1600^\circ\text{C}$ . In order to find the specific heat at high temperatures, say between  $700^\circ\text{C}$  and  $800^\circ\text{C}$ , the experiment is performed twice, first at  $700^\circ\text{C}$  and then at  $800^\circ\text{C}$ . The mean value of the specific heat is calculated.

### 3.4 Nernst Vacuum Calorimeter

This calorimeter is used to find the specific heat of solids at low temperature. Nernst and Lindemann performed a number of accurate experiments to determine the specific heat of solids at low temperatures.

The Nernst calorimeter consists of an evacuated flask  $D$  in which the metal whose specific heat is to be determined, is suspended as shown in Fig. 3-2.  $P$  and  $Q$  are two non-conducting loops. The cylinder  $B$  and the cylindrical plug  $A$  are made of the same metal. The heating coil  $C$  is made of platinum and is wound around  $A$ . The leads  $LL$  are connected to an electrical heating arrangement. The cylinder  $A$  fits completely inside the cylinder  $B$ . The heating coil  $C$  also serves as a platinum resistance thermometer to measure the temperature of the metal.

Suppose the resistance of the platinum wire is  $R_1$  at temperature  $\theta_1$ .  $R_1$  is found by measuring the current in the wire and the potential difference across the two ends of the wire. Let the potential difference be  $E$  and the current  $I_1$ .

$$\therefore R_1 = \frac{E}{I_1} \text{ (At temperature } \theta_1 \text{)}$$

Similarly resistance  $R_2$  at temperature  $\theta_2$  is given by

$$R_2 = \frac{E}{I_2} \text{ (At temperature } \theta_2 \text{)}$$

The platinum wire is initially calibrated and from these values corresponding to  $R_1$  and  $R_2$ , the values  $\theta_1$  and  $\theta_2$  are found.

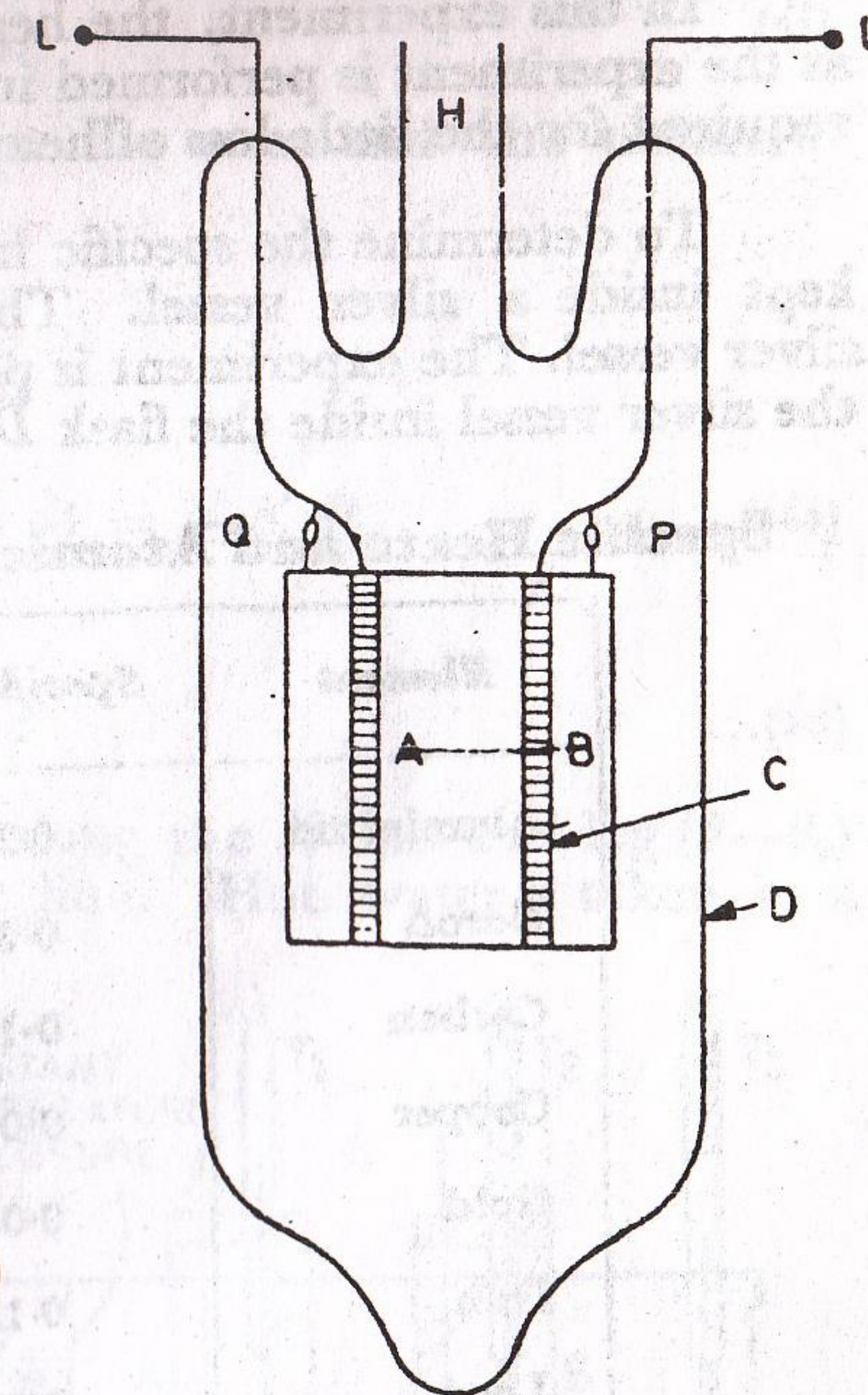


Fig. 3-2



The flask  $D$  is kept in a bath containing ice, liquid air etc., depending upon the low temperature at which the specific heat of the given metal is to be determined. After the flask  $D$  has attained the temperature of the bath, it is evacuated. This initial temperature of the metal is found. Let it be  $\theta_1$ . Electric current is passed through the coil  $C$  for a known time so that the rise in temperature is about  $1^\circ\text{C}$ . Suppose the final temperature of the metal is  $\theta_2$ .

Mass of the metal =  $M$  grams

Specific heat =  $C$

Rise in temperature =  $(\theta_2 - \theta_1)$

Voltmeter reading =  $E$  volts

Ammeter reading =  $I$  amperes

Time for which current is passed =  $t$  seconds

$$\therefore MC(\theta_2 - \theta_1) = \frac{EI t}{4.2}$$

$$C = \frac{E I t}{4.2 M(\theta_2 - \theta_1)}$$

In this experiment, the heat losses are practically eliminated as the experiment is performed in vacuum. However, a correction is required for the little loss of heat due to radiation.

To determine the specific heat of non-metals, the substance is kept inside a silver vessel. The heating coil is wound round the silver vessel. The experiment is performed in vacuume by suspending the silver vessel inside the flask  $D$ .

### Specific Heats and Atomic Heats of Solids at $20^\circ\text{C}$

Element	Specific Heat	Atomic Heat
Aluminium	0.212	5.72
Boron	0.307	3.32
Carbon	0.160	1.92
Copper	0.091	5.79
Gold	0.031	6.11
Iron	0.110	6.12
Lead	0.030	6.21
Silicon	0.182	5.11
Silver	0.056	6.04
Tin	0.054	6.31

### 3.5 Newton's Law of Cooling

Newton's Law of Cooling states that the rate of loss of heat of a body is directly proportional to the difference of temperature of the body and the surroundings. The law holds good only for small difference of temperature. Also, the loss of heat by radiation depends upon the nature of the surface of the body and the area of the exposed surface.

$$\frac{dH}{dt} \propto (\theta - \theta_0) \quad \text{or} \quad -\frac{dH}{dt} = k(\theta - \theta_0)$$

Consider a body of mass  $m$ , specific heat  $C$  and at temperature  $\theta$ . Let  $\theta_0$  be the temperature of the surroundings. Suppose, the temperature falls by a small amount  $d\theta$  in time  $dt$ . Then the amount of heat lost

$$dH = mC d\theta$$

$\therefore$  Rate of loss of heat

$$\frac{dH}{dt} = mC \frac{d\theta}{dt} \quad \dots(i)$$

From Newton's law of cooling

$$-\frac{dH}{dt} = k(\theta - \theta_0) \quad \dots(ii)$$

where  $k$  is a constant depending upon the area and the nature of the surface of the body.

From (i) and (ii)

$$-mC \frac{d\theta}{dt} = k(\theta - \theta_0)$$

$$\text{or} \quad \frac{d\theta}{\theta - \theta_0} = -\frac{k}{mC} dt = -K dt \quad \dots(iii)$$

Integrating,

$$\log(\theta - \theta_0) = -Kt + c \quad \dots(iv)$$

If a graph is plotted between  $t$  along the X-axis and  $\log(\theta - \theta_0)$  along the Y-axis, it is a straight line. Hot water is taken in a

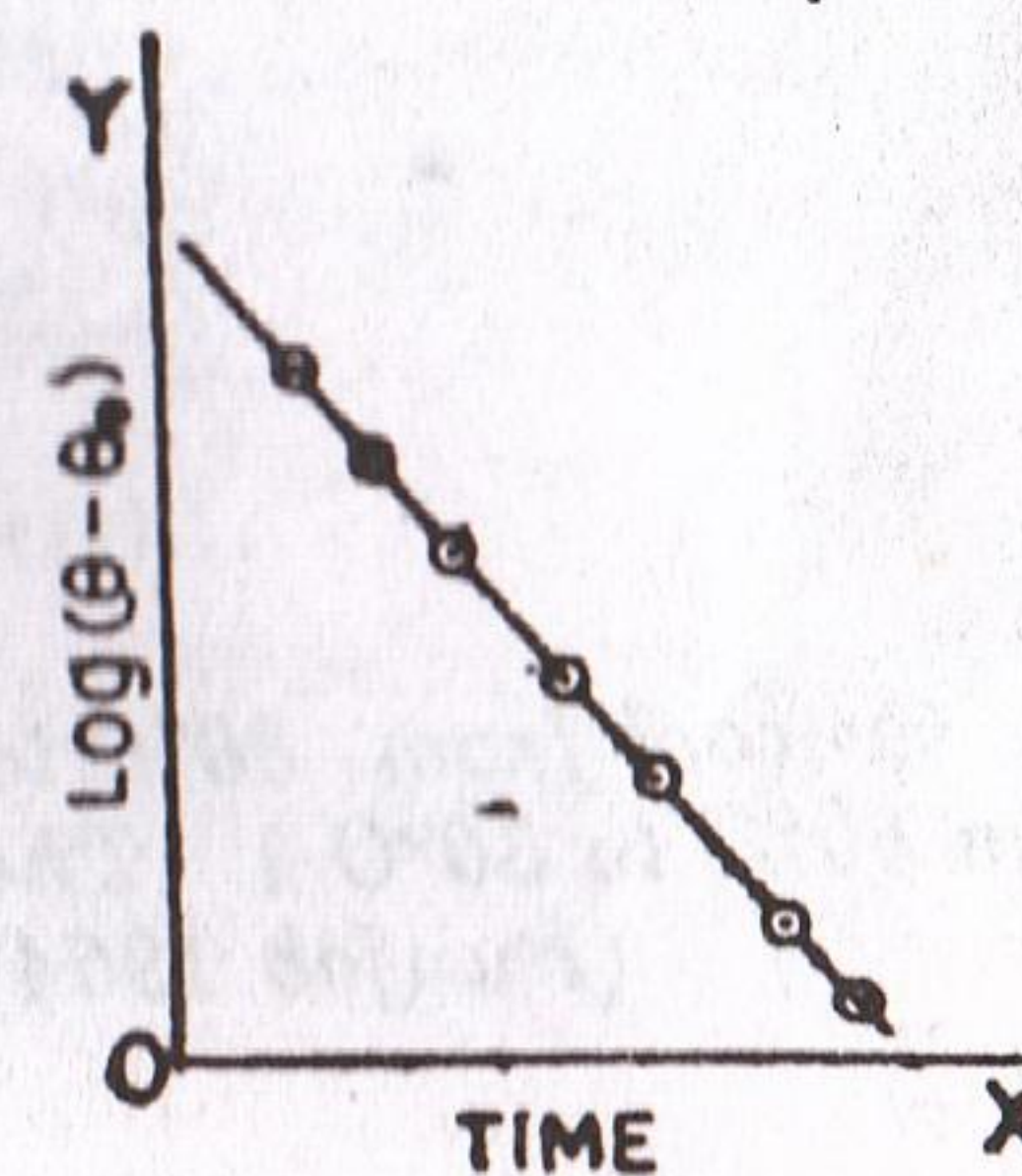


Fig. 3.3 (i)

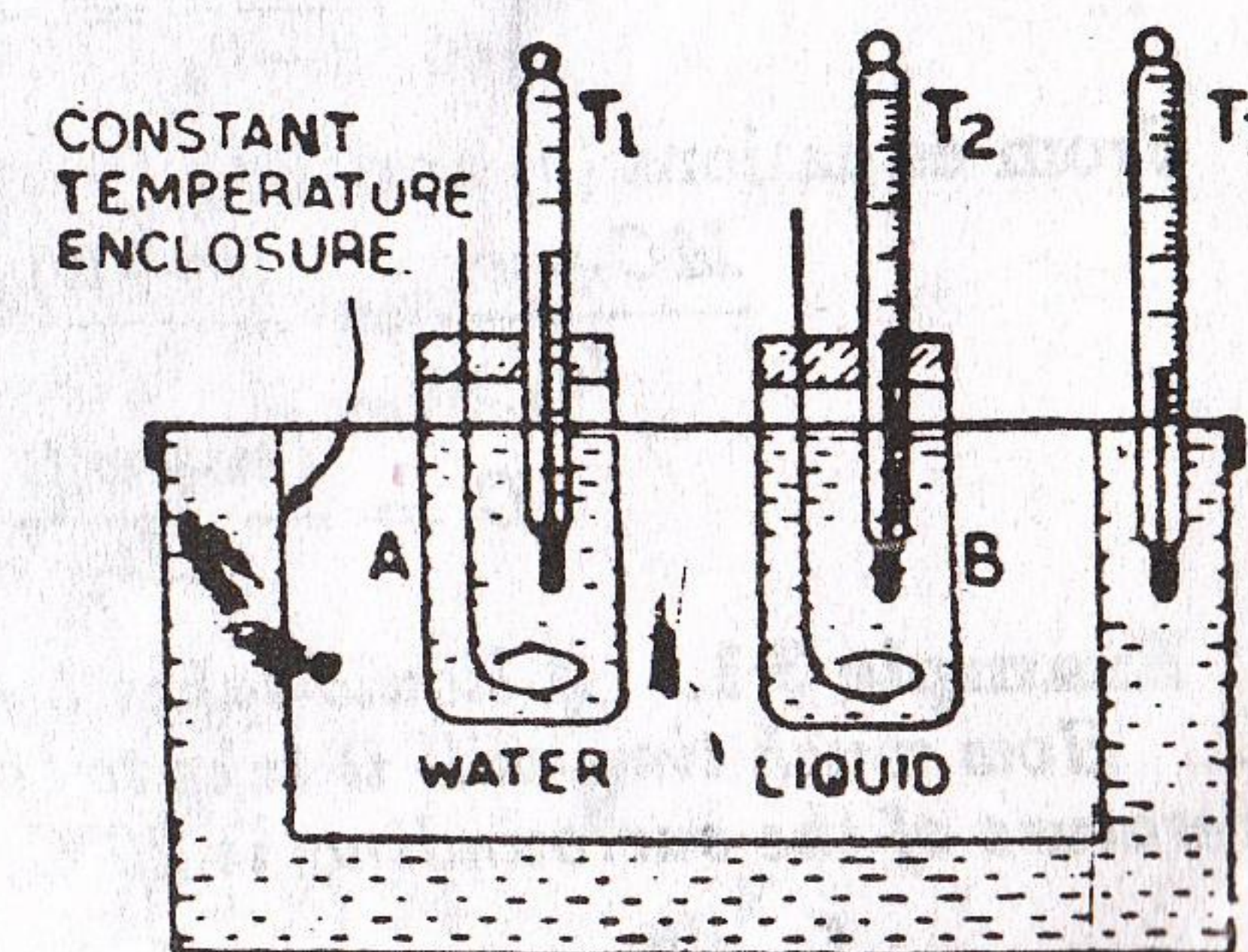


Fig. 3.3 (ii)



calorimeter and is placed in a double walled vessel. Temperature of water after regular intervals of time is noted. A graph between  $\log(\theta - \theta_0)$  and time  $t$  is plotted [Fig. 3.3 (i)]. It is a straight line. This verifies Newton's law of cooling.

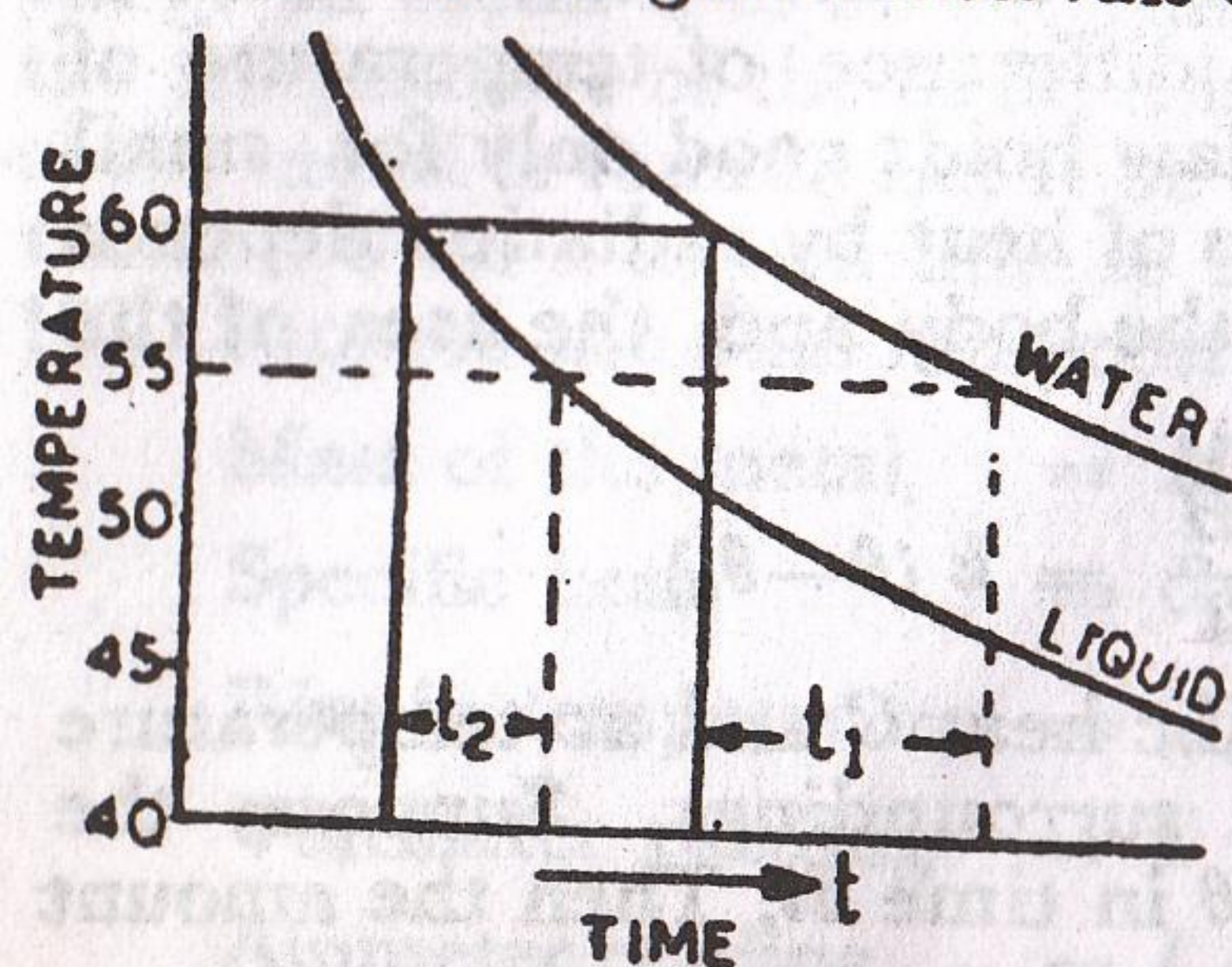


Fig. 3.4

**Specific heat of a liquid.**  $A$  and  $B$  are two identical calorimeters containing equal volumes of hot water and the hot liquid respectively. The two calorimeters are made of the same material and their outer surfaces are equally polished. The calorimeters are kept inside a constant temperature enclosure. The thermometers  $T_1$  and  $T_2$  measure the temperature of water and liquid [Fig. 3.3 (ii)]. The temperature of the two calorimeters are noted after regular intervals of time (say one minute). Graphs are plotted between temperature and time, for water and the liquid (Fig. 3.4).

From equation (iii)

$$\int \frac{d\theta}{\theta - \theta_0} = \frac{k}{mC} \int dt$$

In the case of water, suppose

Mass of water =  $m$

Water equivalent of the calorimeter  $A = w$

Time taken by water to cool from  $60^\circ\text{C}$  to  $55^\circ\text{C} = t_1$

$$\therefore \int_{60}^{55} \frac{d\theta}{\theta - \theta_0} = -\frac{k}{(m+w)} \cdot t_1 \quad \dots(v)$$

Suppose mass of the liquid =  $M$

Water equivalent of the calorimeter  $B = w$

Specific heat of the liquid =  $C$

Time taken by the liquid to cool from  $60^\circ\text{C}$  to  $55^\circ\text{C} = t_2$

$$\therefore \int_{60}^{55} \frac{d\theta}{\theta - \theta_0} = -\frac{k}{(MC+w)} \cdot t_2 \quad \dots(vi)$$

From equations (v) and (vi)

$$\frac{MC+w}{t_2} = \frac{(m+w)}{t_1}$$

$$\text{or} \quad C = \frac{(m+w)t_2}{Mt_1} - \frac{w}{M} \quad \dots(vii)$$

**Example 3.1.** A liquid takes 5 minutes to cool from  $80^\circ\text{C}$  to  $50^\circ\text{C}$ . How much time will it take to cool from  $60^\circ\text{C}$  to  $30^\circ\text{C}$ ? The temperature of the surrounding is  $20^\circ\text{C}$ . (Punjab 1964)

$$\int \frac{d\theta}{\theta - \theta_0} = -K \int dt$$

$\theta_0 = 20^\circ\text{C}$

In the first case,

$$\int_{80}^{50} \frac{d\theta}{\theta - \theta_0} = -K \times 5$$

$$\log \left[ \frac{\theta - \theta_0}{80} \right]_{80}^{50} = 5K$$

$$\log \left( \frac{50 - 20}{80 - 20} \right) = -5K$$

$$\log \frac{30}{60} = -5K$$

$$\log \frac{60}{30} = 5K$$

or

In the second case, suppose the time taken is  $t$  minutes,

$$\int_{60}^{30} \frac{d\theta}{\theta - \theta_0} = -Kt \quad \dots(i)$$

$$\log \left( \frac{30 - 20}{60 - 20} \right) = -Kt$$

$$\log \frac{10}{40} = -Kt$$

or

$$\log \frac{40}{10} = -Kt \quad \dots(ii)$$

Dividing (ii) by (i)

$$\frac{t}{5} = \frac{\log 4}{\log 2} = 2$$

$$t = 10 \text{ minutes}$$

**Example 3.2.** Equal volumes of water (density  $1 \text{ g/cm}^3$ ) and alcohol (density  $0.8 \text{ g/cm}^3$ ) when put in similar calorimeters take 100 seconds and 74 seconds respectively to cool from  $50^\circ\text{C}$  to  $40^\circ\text{C}$ . Calculate the specific heat of alcohol. Thermal capacity of each calorimeter is numerically equal to the volume of either liquid. (Punjab 1966)

Let the volume of either liquid be  $V$

Mass of water =  $m = V \times 1$

Mass of alcohol =  $M = V \times 0.8$

Water equivalent of each calorimeter =  $w = V$

Here

$$t_1 = 100 \text{ s}, t_2 = 74 \text{ s}$$

$$\frac{MC+w}{t_2} = \frac{m+w}{t_1}$$

$$C = \frac{(m+w)t_2}{Mt_1} - \frac{w}{M}$$

$$= \frac{(V+V)74}{V \times 0.8 \times 100} - \frac{V}{V \times 0.8}$$

$$C = 0.6 \text{ calorie/g-K}$$



**Example 3.3.** A body cools in 5 minutes from 60°C to 40°C. What will be its temperature after the next 5 minutes? Temperature of the surroundings = 10°C. Assume that the Newton's law of cooling holds good throughout the process.

$$\int_{\theta_1}^{\theta_2} \frac{d\theta}{\theta - \theta_0} = -K \int dt = -Kt$$

(1) In the first case,  $\theta_0 = 10^\circ\text{C}$ ,

$$\theta_1 = 60^\circ\text{C}, \theta_2 = 40^\circ\text{C}, t = 5 \text{ minutes}$$

$$\int_{60}^{40} \frac{d\theta}{\theta - 10} = -5K$$

$$\left[ \log(\theta - \theta_0) \right]_{60}^{40} = -5K$$

$$\log \left( \frac{40-10}{60-10} \right) = -5K \quad \dots(i)$$

(2) In the second case

$$\theta_0 = 10^\circ\text{C},$$

$$\theta_1 = 40^\circ\text{C}, \theta_2 = x, t = 5 \text{ minutes}$$

$$\int_{40}^x \frac{d\theta}{\theta - 10} = -5K$$

$$\log \left( \frac{x-10}{40-10} \right) = -5K \quad \dots(ii)$$

From equations (i) and (ii)

$$\log \left( \frac{x-10}{40-10} \right) = \log \left( \frac{40-10}{60-10} \right)$$

$$\frac{x-10}{30} = \frac{30}{50}$$

$$x = 28^\circ\text{C}$$

**Example 3.4.** A liquid takes 4 minutes to cool from 70°C to 50°C. How much time will it take to cool from 50°C to 40°C? The temperature of the surroundings is 25°C. Newton's law of cooling is applicable throughout the process.

$$\int_{\theta_1}^{\theta_2} \frac{d\theta}{\theta - \theta_0} = -K \int dt = -Kt$$

(1) In the first case,

$$\theta_0 = 25^\circ\text{C},$$

$$\theta_1 = 70^\circ\text{C}, \theta_2 = 50^\circ\text{C}, t = 4 \text{ minutes}$$

$$\int_{70}^{50} \frac{d\theta}{\theta - 25} = -4K$$

$$\log_e \left( \frac{50-25}{70-25} \right) = -4K \quad \dots(i)$$

(2) In the second case

$$\theta_0 = 25^\circ\text{C},$$

$$\theta_1 = 50^\circ\text{C}, \theta_2 = 40^\circ\text{C}, t = ?$$

$$\int_{50}^{40} \frac{d\theta}{\theta - 25} = -Kt$$

$$\log_e \left( \frac{40-25}{50-25} \right) = -Kt \quad \dots(ii)$$

Dividing (ii) by (i)

$$\frac{t}{4} = \frac{\log_e \left( \frac{15}{25} \right)}{\log_e \left( \frac{25}{45} \right)}$$

$$t = \frac{4 \times 2.3026 \left[ \frac{\log_{10}(0.6)}{\log_{10}(0.55)} \right]}{2.3026}$$

$$= 4 \left[ \frac{\log_{10}(0.6)}{\log_{10}(0.55)} \right]$$

$$= 4 \left[ \frac{1.7782}{1.7404} \right] = 4 \left[ \frac{-0.2218}{-0.2596} \right]$$

$$= \frac{8872}{2596}$$

$$= 3.418 \text{ min.}$$

**Example 3.5.** A liquid cools in 6 minutes from 80°C to 60°C. What will be its temperature after the next 10 minutes? Temperature of the surroundings is 30°C. Assume that the Newton's law of cooling is applicable throughout the process.

$$\int_{\theta_1}^{\theta_2} \frac{d\theta}{\theta - \theta_0} = -K \int dt = -Kt$$

(1) In the first case,

$$\theta_0 = 30^\circ\text{C},$$

$$\theta_1 = 80^\circ\text{C}, \theta_2 = 60^\circ\text{C}, t = 6 \text{ min}$$

$$\int_{80}^{60} \frac{d\theta}{\theta - 30} = -6K$$

$$\log \left( \frac{60-30}{80-30} \right) = -6K$$

$$\log 0.6 = -6K \quad \dots(i)$$

(2) In the second case,

$$\theta_0 = 30^\circ\text{C},$$

$$\theta_1 = 60^\circ\text{C}, \theta_2 = x, t = 10 \text{ min.}$$

$$\int_{60}^x \frac{d\theta}{\theta - 30} = -10K$$



$$\log \left( \frac{x-30}{60-30} \right) = -10K$$

Dividing (ii) by (i)

$$\frac{\log \left( \frac{x-30}{60-30} \right)}{\log (0.6)} = \frac{10}{6}$$

$$\log \left( \frac{x-30}{60-30} \right) = \frac{5}{3} \log (0.6)$$

$$\log \left( \frac{x-30}{30} \right) = \log (0.6)^{5/3}$$

$$\frac{x-30}{30} = (0.6)^{5/3}$$

$$x = 42.80^\circ\text{C}$$

**Example 3.6.** A body initially at  $80^\circ\text{C}$  cools to  $64^\circ\text{C}$  in 5 minutes and to  $52^\circ\text{C}$  in 10 minutes. What will be its temperature after 15 minutes and what is the temperature of the surroundings?

[Delhi (Additional Physics) 1975]

$$\int_{\theta_1}^{\theta_2} \frac{d\theta}{\theta - \theta_0} = -K \int dt = -Kt$$

(1) In the first case

$$\theta_1 = 80^\circ\text{C}, \theta_2 = 64^\circ\text{C}, t = 5 \text{ min}$$

$$\int_{80}^{64} \frac{d\theta}{\theta - \theta_0} = -5K$$

$$\log \left( \frac{64 - \theta_0}{80 - \theta_0} \right) = -5K \quad \dots(i)$$

(2) In the second case,

$$\theta_1 = 64^\circ\text{C}, \theta_2 = 52^\circ\text{C},$$

$$t = 10 - 5 = 5 \text{ min}$$

$$\int_{64}^{52} \frac{d\theta}{\theta - \theta_0} = -5K$$

$$\log \left( \frac{52 - \theta_0}{64 - \theta_0} \right) = -5K \quad \dots(ii)$$

From equations (i) and (ii)

$$\frac{64 - \theta_0}{80 - \theta_0} = \frac{52 - \theta_0}{64 - \theta_0}$$

$$\therefore \theta_0 = 16^\circ\text{C}$$

(iii) Let the temperature after 15 minutes be  $\theta_1$

$$\theta_1 = 52^\circ\text{C}, \theta_0 = 16^\circ\text{C},$$

$$t = 15 - 10 = 5 \text{ minutes}$$

$$\int_{52}^{\theta_2} \frac{d\theta}{\theta - \theta_0} = -5K$$

$$\log \left( \frac{\theta_2 - 16}{52 - 16} \right) = -5K \quad \dots(iii)$$

Equating (ii) and (iii)

$$\frac{\theta_2 - 16}{52 - 16} = \frac{52 - 16}{64 - 16}$$

$$\theta_2 = 43^\circ\text{C}$$

**Example 3.7.** Find the specific heat of a liquid which takes 2 minutes in cooling from  $50^\circ\text{C}$  to  $40^\circ\text{C}$  in a vessel in which the same volume of water takes 5 minutes in cooling through the same range of temperature. Mass of water = 100 g, mass of liquid = 85 g, water equivalent of the vessel = 10 g.

[Delhi, 1976]

$$\frac{MC + w}{t_2} = \frac{m + w}{t_1}$$

Here

$$M = 85 \text{ g}$$

$$C = ?$$

$$w = 10 \text{ g}$$

$$m = 100 \text{ g}$$

$$t_2 = 2 \text{ minutes} = 120 \text{ s}$$

$$t_1 = 5 \text{ minutes} = 300 \text{ s}$$

$$C = \left[ \frac{(m+w)t_2}{Mt_1} \right] - \frac{w}{M}$$

$$C = \left[ \frac{(100+10)120}{85 \times 300} \right] - \frac{10}{85}$$

$$C = 0.4 \text{ calories/g}\cdot\text{K}$$

### 3.6. Specific Heat of a Liquid—Joule's Electrical Method

The apparatus consists of a calorimeter in which a heater coil (wire) of resistance  $R$  is enclosed. The two ends of the wire are connected to the terminals on the lid. The calorimeter is enclosed in a wooden box. The wire  $R$  is connected in series with a battery, a key, a rheostat and an ammeter. A voltmeter is connected parallel to the wire (Fig. 3.5).

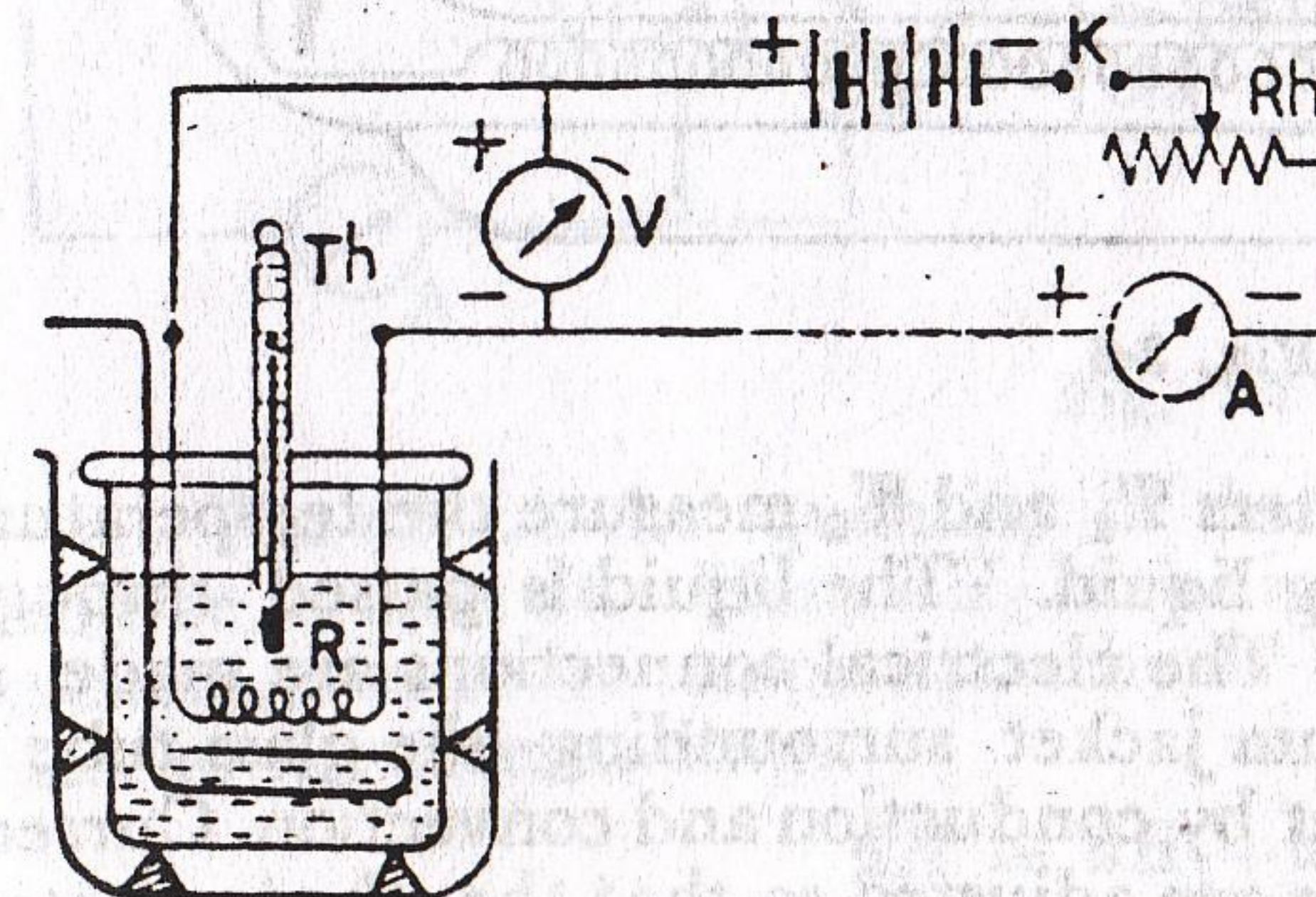


Fig. 3.5.

The liquid whose specific heat is to be determined is taken in the calorimeter. Current is passed



through the wire for a known interval of time. The rise in temperature of the calorimeter and the liquid is noted with the help of a thermometer. The current is passed for such a time that the rise in temperature is about  $10^{\circ}\text{C}$ .

Suppose, the mass of the liquid is  $M$ , specific heat of the liquid is  $C$  and water equivalent of the calorimeter is  $w$ .

Initial temperature of the liquid =  $\theta_1^{\circ}\text{C}$

Final temperature of the liquid =  $\theta_2^{\circ}\text{C}$

P.D. across the wire =  $E$  volts

Current flowing =  $I$  amperes

Time =  $t$  seconds

Heat produced =  $\frac{E.I.t}{4.2}$  calories

Heat gained by the liquid and the calorimeter  
=  $(MC+w)(\theta_2-\theta_1)$

$$\therefore (MC+w)(\theta_2-\theta_1) = \frac{E.I.t}{4.2}$$

$$C = \frac{E.I.t}{4.2(\theta_2-\theta_1)M} - \frac{w}{M}$$

### 3-7 Specific Heat of a Liquid—Callendar and Barnes' Continuous Flow Method

The apparatus consists of a glass tube in which a resistance wire is enclosed. The ends of the wire are connected to the terminals

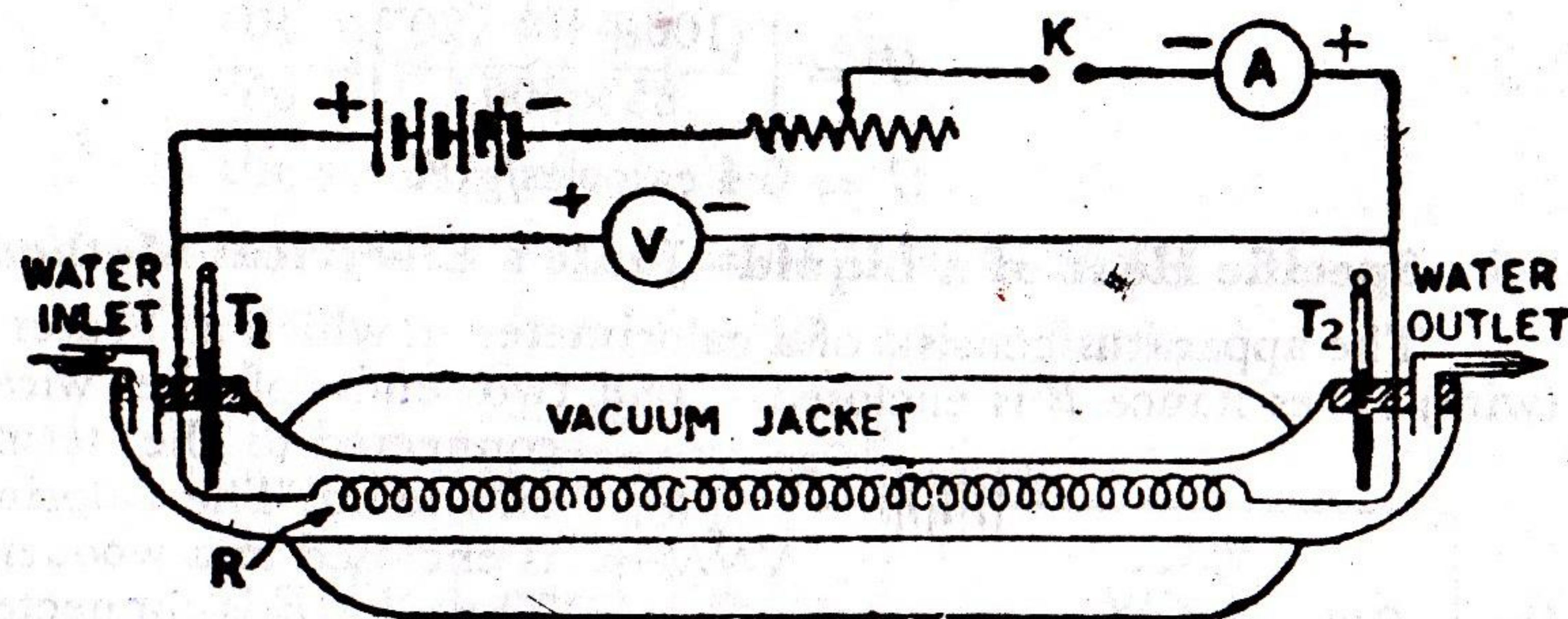


Fig. 3-6

outside the tube. Thermometers  $T_1$  and  $T_2$  measure the temperature of the incoming and outgoing liquid. The liquid is passed through the tube at a uniform rate. The electrical connections are made as shown in Fig. 3-6. A vacuum jacket surrounding the glass tube is provided to avoid loss of heat by conduction and convection. Current and the rate of flow of water are adjusted so that the thermometers show a difference of about  $10^{\circ}\text{C}$ .

Let,  $E$ ,  $I$  and  $t$  be the potential difference, current and time for which the liquid is collected. If the mass of liquid collected is  $m$  and the difference of temperature between the two thermometers is  $(\theta_2-\theta_1)$ , then

$$mC(\theta_2-\theta_1) + R = \frac{E.I.t}{4.2} \quad \dots(i)$$

where  $R$  is the loss of heat by radiation. The current through the wire is changed. The rate of flow is adjusted so that the difference of temperature  $(\theta_2-\theta_1)$  remains the same. Suppose the amount of liquid collected in time  $t$  is  $m'$

$$\therefore m'C(\theta_2-\theta_1) + R = \frac{E'I.t}{4.2} \quad \dots(ii)$$

Subtracting (ii) from (i)

$$(m-m')C(\theta_2-\theta_1) = \frac{(EI-E'I)t}{4.2}$$

$$C = \frac{(EI-E'I)t}{4.2(m-m')(\theta_2-\theta_1)}$$

This is an accurate method because the observations are taken under steady state. The thermal capacity of the apparatus does not occur in the calculations. Moreover, the errors due to loss of heat by conduction, convection and radiation have been practically eliminated.

### 3-8 Experimental Determination of Heat Capacities

In modern methods, measurement of heat capacity of substances involves the supply of heat to the system from an electrical source. From the energy received from the source in a given interval of time, a part of it may be used in raising the temperature of the system and the remainder is lost to the surroundings at a lower temperature (Fig. 3-7).

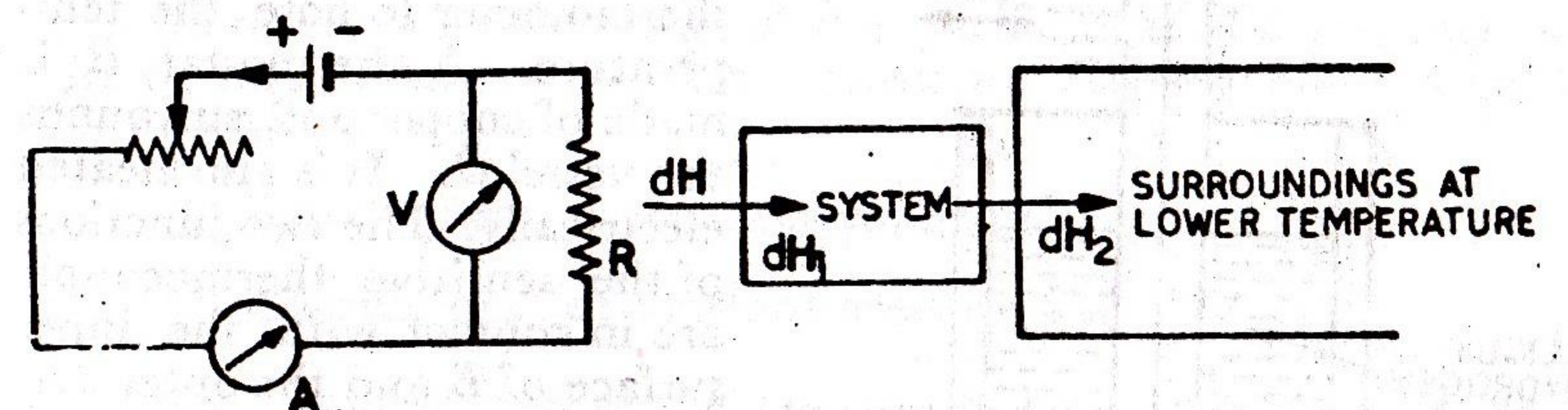


Fig. 3-7

Let  $dH$  be the quantity of heat drawn from the electrical source in time  $dt$ . Here  $dH_1$  is the amount of heat retained by the system to raise its temperature by  $d\theta$  and  $dH_2$  is the amount of heat lost to the surroundings.

$$dH = dH_1 + dH_2$$

$$dH_1 = 1 \times C_p \times dT$$



Here  $C_p$  is the specific heat of a substance at constant pressure. If the potential difference across the heater wire is  $E$  volts and the current flowing is  $I$  amperes, then

$$dH = \frac{EI \cdot dt}{J}$$

$$\therefore dH_1 = dH - dH_2$$

$$= C_p \times dT = \frac{EI \cdot dt}{J} - dH_2 \quad \dots(i)$$

If the system does not lose heat to the surroundings, then

$$dH_2 = 0$$

$$\therefore C_p \times dT = \frac{EI \cdot dt}{J} \quad \dots(ii)$$

$$\frac{EI}{J} = C_p \left[ \frac{dT}{dt} \right] \quad \dots(iii)$$

To find  $\frac{dT}{dt}$ , i.e., the rate of rise of temperature of the system with time, a curve is drawn between temperature ( $T$ ) and time ( $t$ ). From the graph  $\frac{dT}{dt}$  is found for the temperature at which  $C_p$  is to be determined.

### 3.9. Adiabatic Vacuum Calorimeter

This calorimeter was designed by Nernst and later on modified by Simon and Lange. Inside a thin copper vessel  $C$ , the substance is taken. The vessel  $C$  also contains heating coils and a sensitive platinum resistance thermometer to note the temperature. A thermostat  $B$  is made of copper and surrounds the vessel  $C$ . It is also heated electrically. The two junctions of the sensitive thermocouple are in contact with the inner surface of  $B$  and the outer surface of  $C$ .

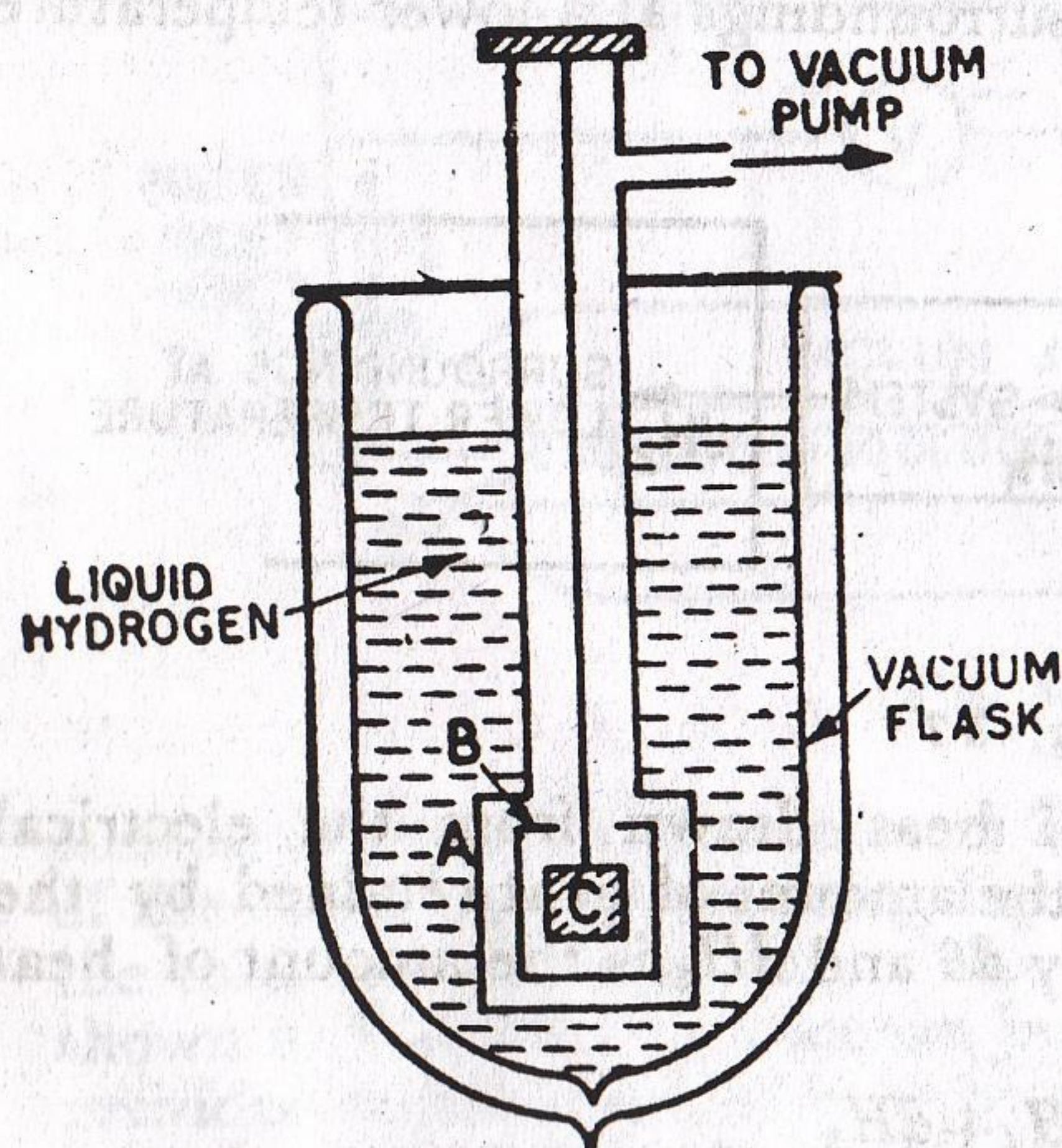


Fig. 3.8

The outer vessel  $A$  is surrounded by liquid hydrogen contained in a vacuum flask. The current through the heater coil of  $B$  is adjusted so that the thermocouple shows no deflection. This ensures a uniform temperature enclosure for  $C$ .

The space inside  $A$  is evacuated with the help of a vacuum pump. If the rise in temperature of 1 gram of the substance is  $dT$  in time  $dt$ ,

$$\text{Then} \quad \frac{EI}{J} = C_p \left[ \frac{dT}{dt} \right] \quad \dots(iv)$$

The method is useful in determining the heat capacity of substances at low temperatures.

To find the value of  $C_p$ , it is difficult to find its value experimentally at low temperature. Its value is calculated from the equation

$$C_p = C_v + \frac{r}{J} \quad \dots(v)$$

### 3.10. Two Specific Heats of a Gas

Consider a gas of mass  $m$  at a pressure  $P$  and volume  $V$ . If the gas is compressed, there is rise in temperature. In this case, no heat has been supplied to the gas to raise its temperature.

$$\therefore \text{Specific heat, } C = \frac{H}{m\theta}$$

$$\text{But} \quad H = 0$$

$$\therefore C = 0$$

On the other hand, if heat is supplied to the gas and the gas is allowed to expand such that there is no rise of temperature, then

$$C = \frac{H}{m\theta}$$

$$\text{Here} \quad \theta = 0$$

$$\therefore C = \frac{H}{m \times 0} = \infty$$

Thus, the specific heat of a gas varies from zero to infinity.

In order to fix the value of the specific heat of a gas, the pressure or volume has to be kept constant. Consequently, a gas has two specific heats.

(1) Specific heat at constant volume  $C_v$ ,

(2) Specific heat at constant pressure  $C_p$ .

$C_v$ . It is defined as the quantity of heat required to raise the temperature of one gram of a gas through  $1^\circ\text{C}$  at constant volume.

$C_p$ . It is defined as the quantity of heat required to raise the temperature of one gram of a gas through  $1^\circ\text{C}$  at constant pressure.

$C_p$  is greater than  $C_v$ . When a gas is heated at constant volume, the heat supplied to the gas is wholly used up to raise its temperature. On the other hand when a gas is heated at constant



pressure, a part of the heat is used to raise its temperature and a part is used to do external work to keep the pressure constant.

$$C_p > C_v$$

**Relation.** Consider one gram of a gas at a pressure  $P$ , volume  $V$  and temperature  $T$ . Heat is supplied to the gas to raise its temperature through  $dT$ . As the pressure has to remain constant,

$$\text{Work done, } W = P \times A \times x = P \times dV$$

where  $dV$  is the change in volume.

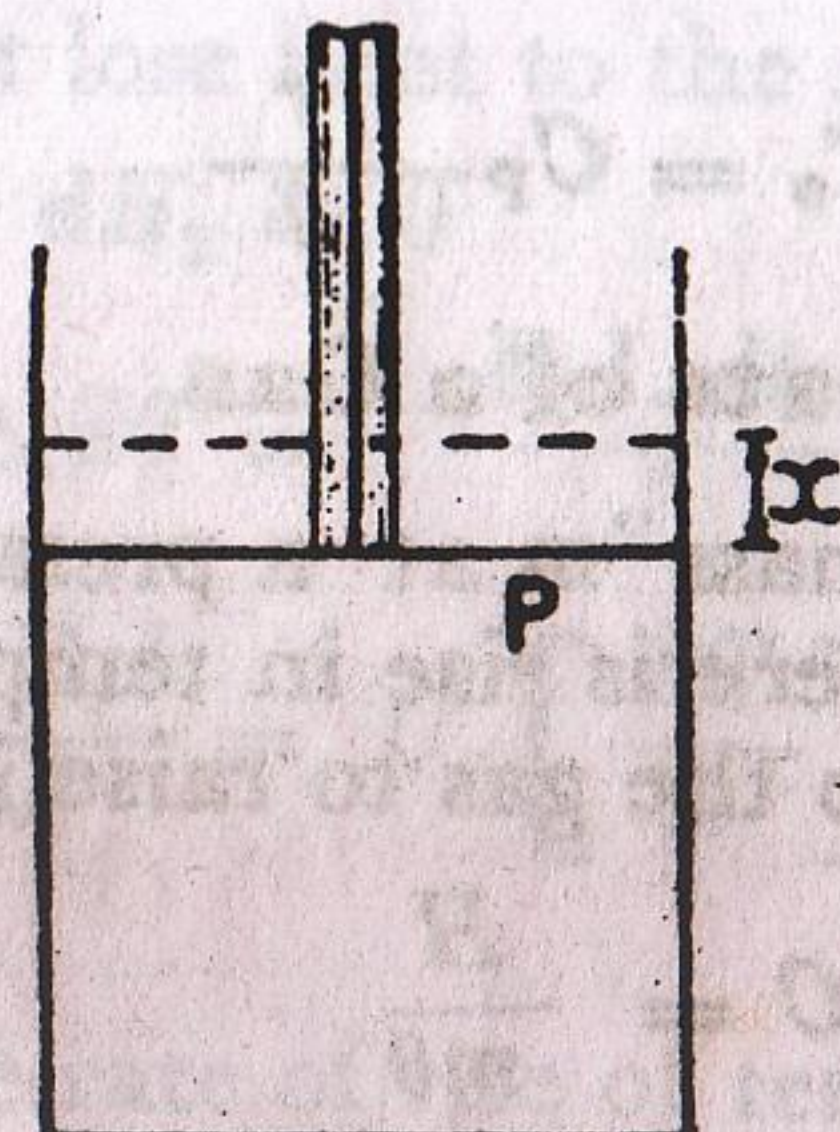


Fig. 3-9

From the gas equation

$$PV = rT,$$

Differentiating,

$$PdV + VdP = r dT$$

But

$$dP = 0$$

$\therefore$

$$PdV = r dT$$

$\therefore$  Work done in heat units

$$= \frac{r \cdot dT}{J} \text{ calories}$$

Heat supplied

$$= 1 \times C_p \times dT$$

$$= 1 \times C_v \times dT + \frac{r \cdot dT}{J}$$

or

$$C_p - C_v = \frac{r}{J}$$

where  $r$  is the gas constant for one gram of a gas. If  $C_p$  and  $C_v$  represent gram molecular specific heats, then

$$C_p - C_v = \frac{R}{J}$$

where  $R$  is the universal gas constant.

**Example 3.8.** Find the value of the universal gas constant  $R$  for one gram molecule of a gas.

One gram molecule of a gas at N.T.P. occupies 22400 cm<sup>3</sup>

$$P = 76 \text{ cm of Hg} = 76 \times 13.6 \times 981 \text{ dynes/cm}^2$$

$$V = 22400 \text{ cm}^3$$

$$T = 273 \text{ K}$$

$$PV = RT$$

or

$$R = \frac{PV}{T} = \frac{76 \times 13.6 \times 981 \times 22400}{273}$$

$$R = 8.31 \times 10^7 \text{ ergs/mole-K}$$

**Note.** The value of  $R$  is the same for all gases, provided the mass of the gas is one gram molecule.

**Example 3.9.** Calculate the specific heat of air at constant volume, given that specific heat at constant pressure is 0.23, density of air at N.T.P. 1.293 gram/litre and  $J = 4.2 \times 10^7$  ergs/cal.

(Punjab 1966)

$$C_p = 0.23, J = 4.2 \times 10^7 \text{ ergs/cal}, C_v = ?$$

Density of air at N.T.P. = 1.293 g/litre

Volume of one gram of air at N.T.P.

$$= \frac{1000}{1.293} \text{ cc}$$

$$PV = rT$$

$$r = \frac{PV}{T}$$

$$= \frac{76 \times 13.6 \times 980 \times 1000}{273 \times 1.293}$$

$$C_p - C_v = \frac{r}{J}$$

$$C_v = C_p - \frac{r}{J}$$

$$= 0.23 - \frac{76 \times 13.6 \times 980 \times 1000}{273 \times 1.293 \times 4.2 \times 10^7}$$

$$C_v = 0.23 - 0.0683$$

$$= 0.1617$$

**Example 3.10.** Calculate the difference in the two specific heats of one gram of helium, given that the molecular weight of helium = 4 and the gram molecular volume of helium at N.T.P. = 22.4 litres.

(Delhi 1975)

$$C_p - C_v = \frac{r}{J}$$

Volume of one gram of helium at N.T.P.

$$V = \frac{22400}{4} = 5600 \text{ cm}^3$$



$$PV = rT$$

$$r = \frac{PV}{T}$$

or

$$r = \frac{76 \times 13.6 \times 980 \times 5600}{273}$$

$$C_p - C_v = \frac{r}{J}$$

$$= \frac{76 \times 13.6 \times 980 \times 5600}{273 \times 4.2 \times 10^7}$$

$$= 0.4946$$

### 3.11. Specific Heat of a Gas at Constant Volume—Joly's Differential Steam Calorimeter

$C$  is a chamber in which steam can be admitted.  $P_1$  and  $P_2$  are scale pans suspended from the scale pans  $S_1$  and  $S_2$  of a balance.

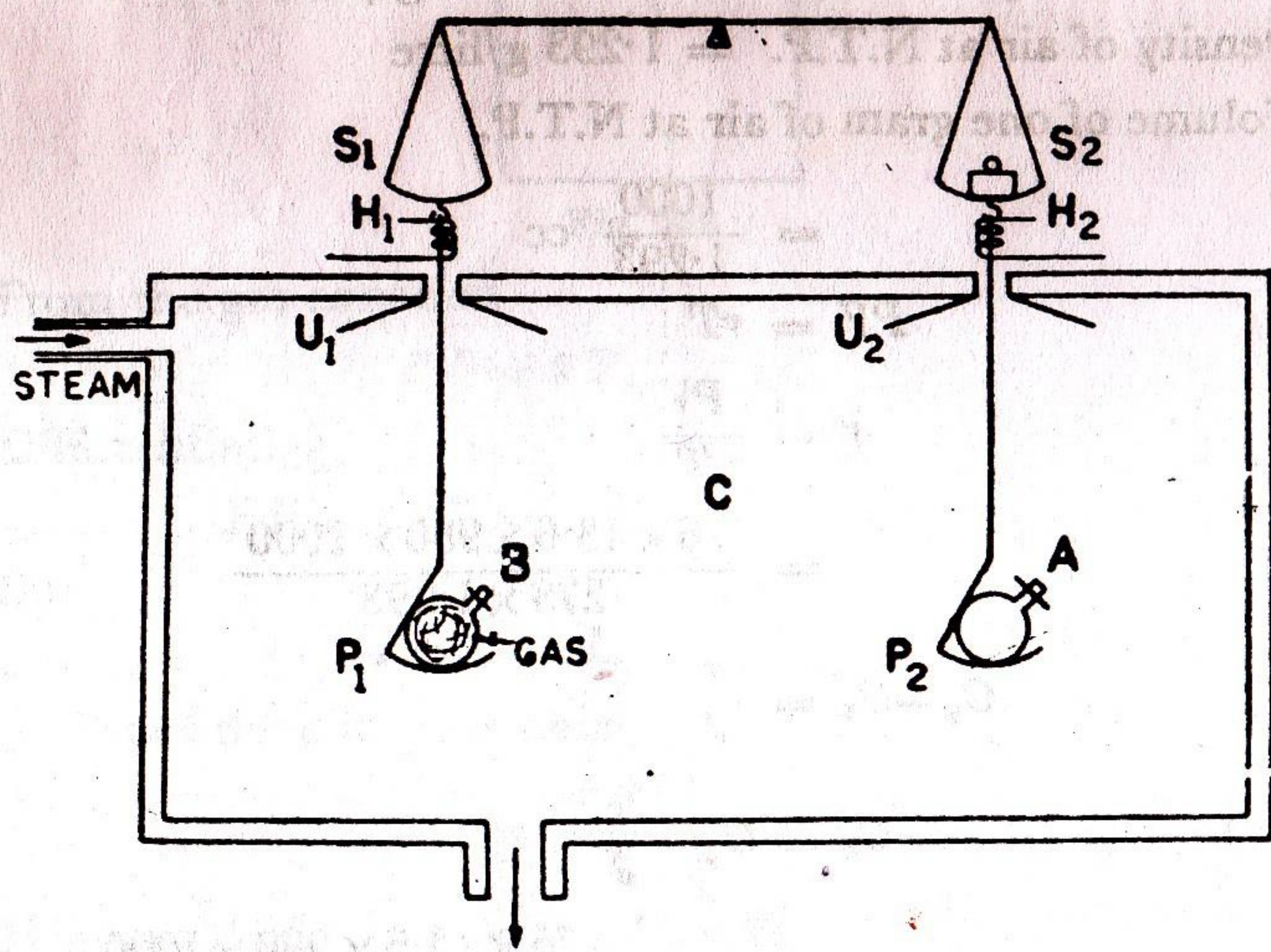


Fig. 3-10

$A$  and  $B$  are two identical hollow metal spheres. The two spheres are initially evacuated and by placing them in the pans  $P_1$  and  $P_2$  the balance is counterpoised. In one of the spheres, the gas whose  $C_v$  is to be determined is admitted at a pressure of about 10 atmospheres and the balance is counterpoised again. Suppose the gas is filled in  $B$ . The balance will tilt to the left and the extra mass that has to be kept in the pan  $S_2$  for counterpoising corresponds to that mass of gas enclosed in  $B$  ( $M$  grams). Now steam is admitted into the chamber and passed continuously till a constant temperature is reached (temperature of steam). Steam condenses and the mass of steam that has condensed on the pan  $P_1$  is more than that condensed on the pan  $P_2$  because in the case of pan  $P_1$  the enclosed gas also has to be heated from the room temperature to the temperature of steam.

The balance tilts and an extra mass has to be kept in the right hand pan to balance it again. This extra mass corresponds to the extra mass of steam condensed on the pan  $P_1$ .

The umbrella-shaped vanes  $U_1$  and  $U_2$  will not allow the steam condensed on the rest of the chamber to fall on the scale pans  $P_1$  and  $P_2$ . The heating coils  $H_1$  and  $H_2$  will heat the suspension wires to a temperature higher than the temperature of steam and therefore, no steam condenses on these wires.

Suppose,

Mass of the gas =  $M$  grams

Sp. heat of gas at constant volume =  $C_v$

Initial temperature of the gas =  $t_1^\circ\text{C}$

Temp. of steam =  $t_2^\circ\text{C}$

Extra mass of steam condensed on the scale pan  $P_1$  =  $m$  grams

Latent heat of vaporisation of water =  $L$  cal/g

Heat lost by steam =  $mL$

Heat gained by the gas =  $MC_v(t_2 - t_1)$

$$MC_v(t_2 - t_1) = mL$$

$$C_v = \frac{mL}{M(t_2 - t_1)} \text{ cal/g}^\circ\text{C}$$

In the second part of the experiment the gas is enclosed in the sphere of pan  $P_2$  and the sphere of pan  $P_1$  is kept empty and  $C_v$  is calculated again. The mean of these two values gives the specific heat of the gas at constant volume.

### 3.12. Specific Heat of a gas at Constant pressure—(Regnault's Method)

The apparatus consists of a reservoir  $R$  containing the gas at high pressure and at a constant temperature. The pressure of the gas in the reservoir is shown by the pressure gauge. The apparatus is as shown in Fig. 3-11.

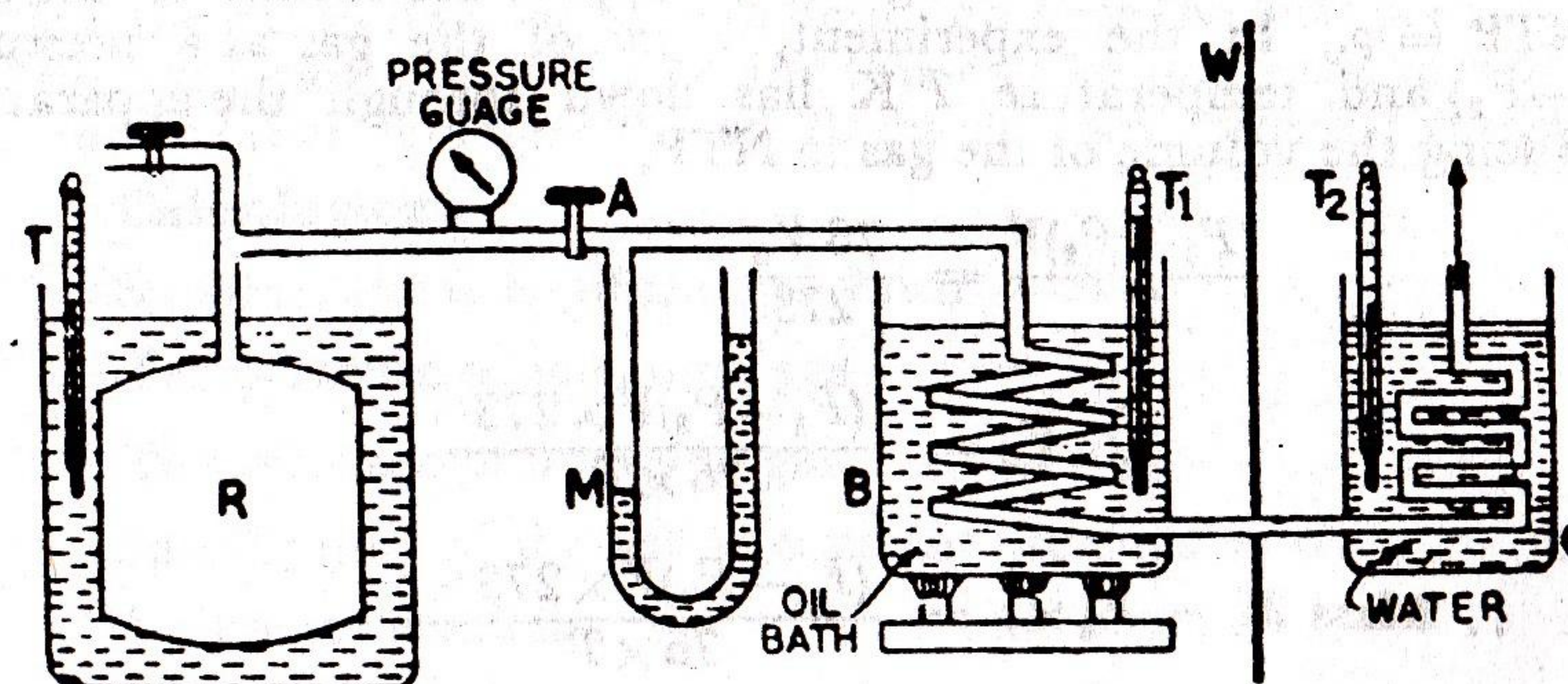


Fig. 3-11



The regulator  $A$  allows the gas to flow at a constant pressure through the spiral tubings immersed in the oil bath  $B$  and the calorimeter. The pressure of the gas flowing through the spiral tubing is shown by the manometer  $M$ . Regulator  $A$  helps in keeping the level of the liquid in the manometer limbs constant.

Suppose the initial pressure of the gas at any instant in the reservoir is  $P_1$ , its temperature is  $T$ , and volume is  $V$ . The temperature shown by the oil bath is  $T_1$  and the calorimeter is at a temperature  $T_2$ . Gas is allowed to flow for about half an hour. The gas after passing through the oil bath gets heated to temperature  $T_1$  and after passing through the calorimeter, gets cooled and gives heat to the calorimeter and its contents. Suppose, the final pressure of the gas in the reservoir is  $P_2$ , volume is  $V$  and the temperature of the calorimeter  $C$  and its contents is  $T_3$ .

### Calculations

Suppose, mass of the gas flown =  $M$

Mass of water in the calorimeter =  $m$

Water equivalent of the calorimeter =  $w$

Rise of temperature of calorimeter and its contents

$$= (T_3 - T_2)$$

Mean fall of temperature of the gas

$$= \left( T_1 - \frac{T_2 + T_3}{2} \right)$$

Heat gained = Heat lost

$$(m + w)(T_3 - T_2) = MC_p \left( T_1 - \frac{T_2 + T_3}{2} \right)$$

$$C_p = \frac{(m + w)(T_3 - T_2)}{M \left( T_1 - \frac{T_2 + T_3}{2} \right)} \quad \dots(i)$$

To find the mass of the gas ( $M$ ), suppose the density of the gas at NTP =  $\rho$ . In the experiment,  $V$  cc of the gas at a pressure ( $P_1 - P_2$ ) and temperature  $T$  K has flown through the apparatus. Reducing the volume of the gas to NTP,

$$\frac{(P_1 - P_2)V}{T} = \frac{76 V_0}{273}$$

or

$$V_0 = \frac{(P_1 - P_2)V \times 273}{76 \times T}$$

$$\therefore \text{Mass } M = V_0 \times \rho = \frac{(P_1 - P_2)V \times 273 \times \rho}{76 \times T}$$

Thus, knowing the value of  $M$ ,  $C_p$  can be calculated from equation (i).

### 3-13 Continuous Flow Electrical Method

The specific heat of a gas at constant pressure by electrical method can be determined by using Callendar and Barnes' continuous flow apparatus. Here  $D$  is a special glass vessel. The heating coil is at the axis of the vessel. The incoming gas takes a long zig zag path as indicated by the arrow heads (Fig. 3-12).

The reservoir  $R$  contains the gas at high pressure and at a constant temperature. The pressure of the gas is read by the pressure gauge. The regulator  $A$  allows the gas to flow at a constant pressure through the vessel  $D$ . The pressure of the gas flowing through  $D$  is indicated by the manometer  $M$ . The regulator helps in keeping the pressure of the gas constant throughout the experiment. The filament is heated by electric current. The incoming gas is heated due to the heat generated by the filament. The inlet and the outlet temperatures of the gas are measured with the help of the platinum resistance thermometers  $T_1$  and  $T_2$ .

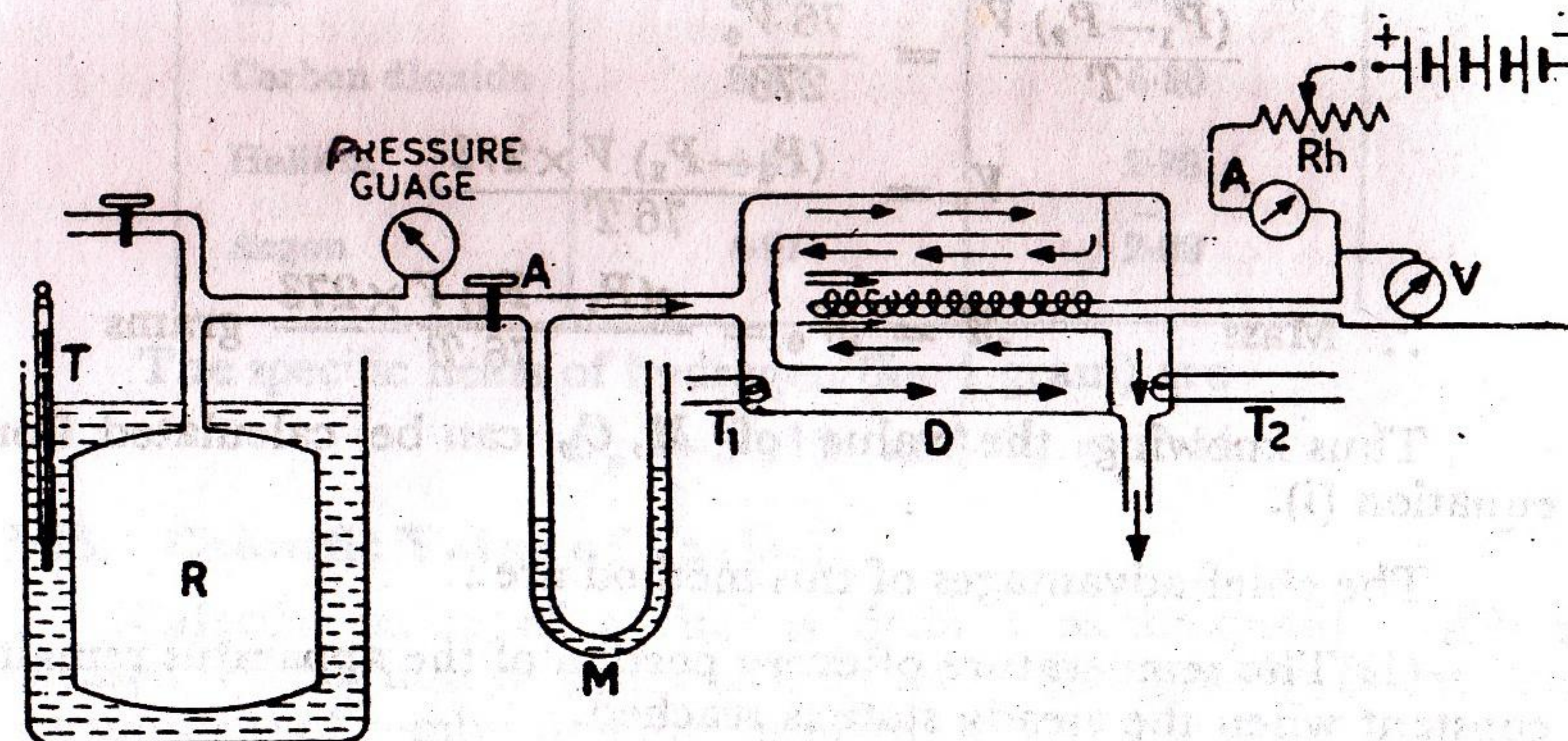


Fig. 3-12

The gas is allowed to flow through the apparatus for some time till the steady state is reached. When the steady state is reached, the thermometers  $T_1$  and  $T_2$  show constant readings. After the steady state, note the pressure ( $P_1$ ) of the gas in the reservoir. Allow the gas to flow for half an hour and note the final pressure ( $P_2$ ) of the gas in the reservoir. During this half an hour, the manometer  $M$  and the thermometers  $T_1$  and  $T_2$  should show constant readings.

### Calculations

Suppose, the mass of the gas flown =  $M$

Temperature of incoming gas =  $T_1$

Temperature of outgoing gas =  $T_2$

Specific heat of the gas at constant pressure =  $C_p$

Voltmeter reading =  $E$  volts

Ammeter reading =  $I$  amperes

Time =  $t$  seconds



$$\text{Heat produced} = \frac{EIt}{4.2} \text{ calories}$$

$$\text{Heat gained by the gas} = MC_p(T_2 - T_1)$$

$$\text{Heat gained} = \text{Heat produced}$$

$$MC_p(T_2 - T_1) = \frac{EIt}{4.2}$$

$$C_p = \frac{EIt}{4.2 M(T_2 - T_1)} \quad \dots(i)$$

To find the mass of the gas ( $M$ ), suppose the density of the gas at N.T.P. =  $\rho$ . In the experiment  $V$  cc of the gas at a pressure ( $P_1 - P_2$ ) and temperature  $T$  has flown through the apparatus. Here  $V$  is the volume of the reservoir  $R$ . Reducing the volume of the gas to N.T.P.

$$\frac{(P_1 - P_2)V}{T} = \frac{76 V_0}{273}$$

$$V_0 = \frac{(P_1 - P_2)V \times 273}{76 T}$$

$$\therefore \text{Mass } M = \rho V_0 = \frac{\rho(P_1 - P_2)V \times 273}{76 T} \text{ grams}$$

Thus knowing the value of  $M$ ,  $C_p$  can be calculated from equation (i).

The chief advantages of this method are :

- (1) The temperature of every portion of the apparatus remains constant when the steady state is reached.
- (2) The thermal capacity of the flow tube and its contents is eliminated in the calculations.
- (3) The loss of heat due to radiation is minimised due to zig-zag path taken by the gas. The gas flows from outer to the inner region of the tube and absorbs any heat radiated from the inner to the outer portion.
- (4) The temperatures can be measured accurately under steady state.
- (5) The heat produced by electrical arrangement can be calculated accurately.

### 3.14. Specific heat of a gas at low temperatures

The continuous flow method can be used to determine the specific heat of a gas at low temperature. Before the gas is passed into the tube  $D$ , the gas is initially cooled to a desired low temperature. The experiment is performed in a similar way as discussed in article 3.13.

This method is also used to find the specific heat of a gas at different high pressures. In this case, the gas at high pressure is

passed through the tube  $D$ . Scheel and Heuse employed this method to find the specific heat of various gases up to  $-180^\circ\text{C}$ . Holborn and Jakob used this method to find the specific heat of gases at high pressures.

### Molecular Heats of Gases at $20^\circ\text{C}$

Gas	$C_p$	$C_v$
Hydrogen	6.87	4.88
Nitrogen	6.95	4.96
Oxygen	7.03	5.04
Chlorine	8.29	6.15
Air	6.95	4.96
Carbon dioxide	8.83	6.80
Helium	4.97	2.98
Argon	4.97	2.98

The specific heats of hydrogen (for 1 gram) are

$$C_p = 3.435 \text{ and } C_v = 2.44$$

### 3.15. Calorific Value of Fuels

Calorific value of a fuel is defined as the quantity of heat released when a unit quantity of the fuel is completely burnt and the products of combustion are brought to the original temperature. It is expressed as calories per gram or B. T. U. per pound. The fuels commonly used are coal, petrol, spirit, diesel oil, wood etc. The calorific value of different fuels can be determined with the help of a bomb calorimeter.

**Bomb Calorimeter.** It consists of a vessel  $D$  made of steel or gun metal. A platinum bowl  $B$  is suspended inside  $D$ . The fuel  $E$  in the form of powder for a solid or a liquid is taken in  $B$ . A heating wire is immersed in the fuel and its ends are connected to the terminals  $T_1$  and  $T_2$ .  $R$  is a regulating valve for the free supply of oxygen inside  $D$ . The vessel  $D$  is closed with an air tight lid. The whole apparatus is immersed in a calorimeter  $C$  containing water. The fuel is ignited by passing current through the heater wire. The heat produced due to the combustion of the fuel is taken by the surrounding water in the calorimeter.

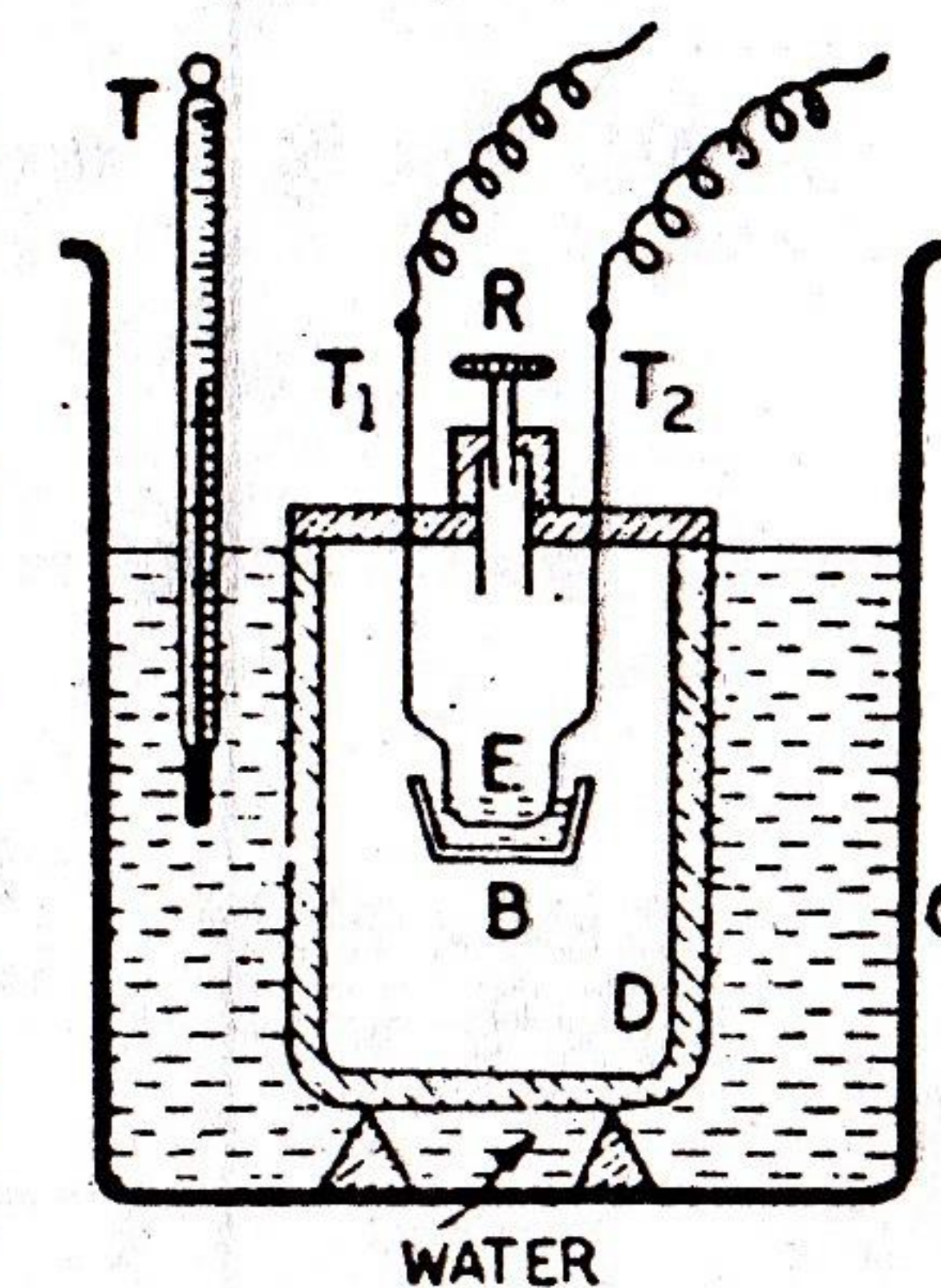


Fig. 3-13

Suppose, the mass of the fuel =  $M$



$$\begin{aligned} \text{Initial temperature of water} &= \theta_1 \\ \text{Final temperature of water} &= \theta_2 \\ \text{Mass of water} &= m \\ \text{Water equivalent of the calorimeter and the vessel } D &= w \\ \text{Heat produced} &= (m+w)(\theta_2-\theta_1) \\ \therefore \text{Calorific value} &= \frac{(m+w)(\theta_2-\theta_1)}{M} \end{aligned}$$

### 3-16 Bell Calorimeter

It is used to find the calorific value of fuels. The apparatus consists of a bell jar *B* arranged inside a calorimeter containing water at room temperature. In the crucible *C*, the substance is taken in the powdered form. The leads *L, L* are connected to a heating filament dipped inside the fuel in the crucible *C* (Fig. 3-14). *D* is a perforated disc through which the burnt gases inside the bell can escape into water. *A* is a pipe through which oxygen is fed into the combustion chamber.

The fuel is ignited by connecting the leads *L, L* to a battery. The fuel burns and the burnt hot gases escape into water in the

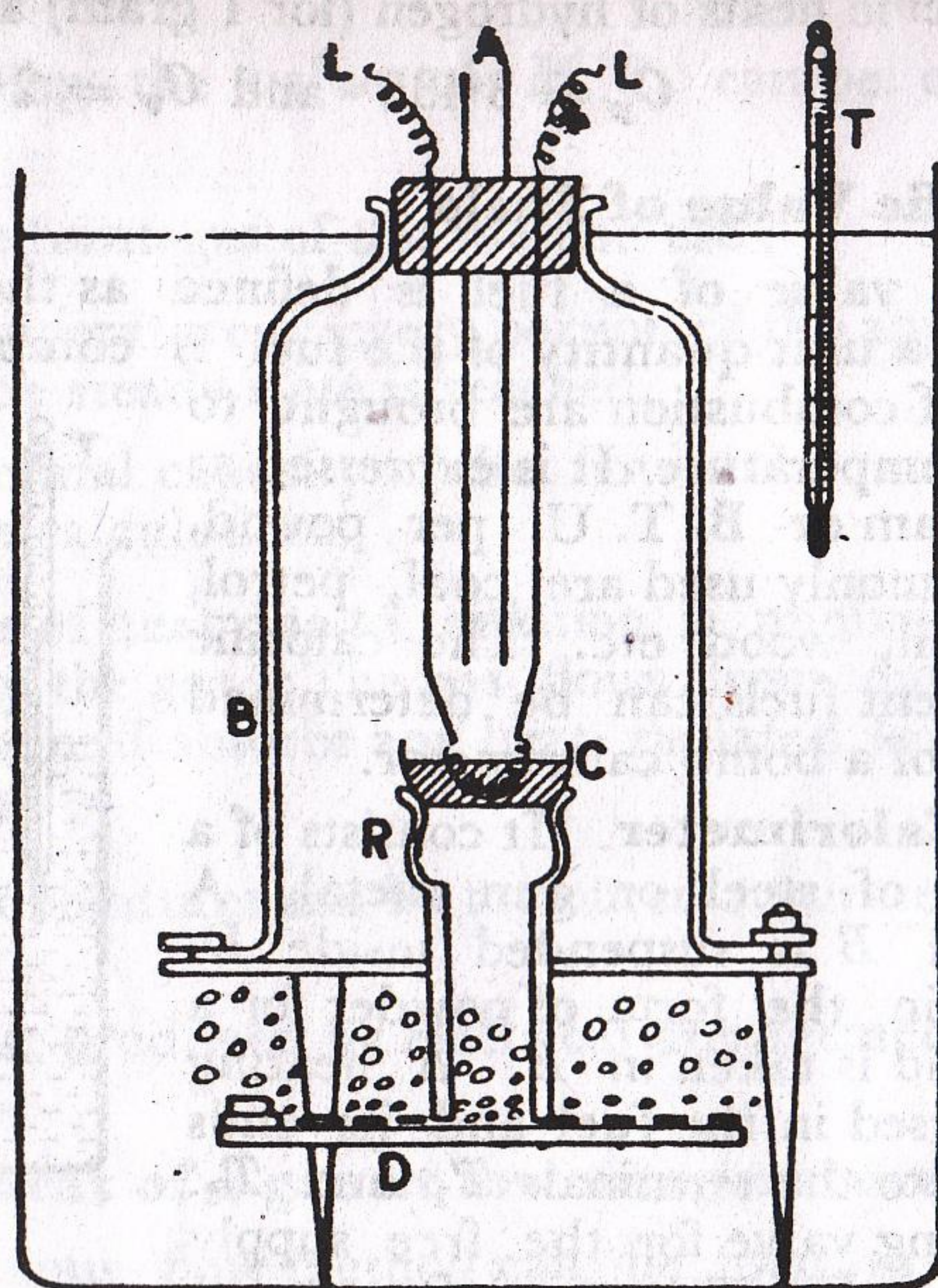


Fig. 3-14

calorimeter, through the disc *D*. When the fuel is completely burnt, the final temperature of water in the calorimeter is noted.

$$\begin{aligned} \text{Suppose, the mass of the fuel} &= M \\ \text{Initial temperature of water} &= \theta_1 \\ \text{Final temperature of water} &= \theta_2 \\ \text{Mass of water} &= m \\ \text{Water equivalent of the calorimeter} &= w \\ \text{Heat produced} &= (m+w)(\theta_2-\theta_1) \\ \text{Calorific value} &= \frac{(m+w)(\theta_2-\theta_1)}{M} \end{aligned}$$

### Calorific Value of Fuels

Fuel	Cal/g
Wood	2500
Gas coke	6000
Methylated spirit	6400
Steam coal	7500
Anthracite coal	8800
Heavy Diesel oil	11350
Petrol	11400
Paraffin oil	11200

**Example 3-11.** An engine consumes 25 litres of gasoline per hour. The calorific value of gasoline is  $6 \times 10^6$  calories per litre. The output of the engine is 35 kilowatts. Calculate the efficiency of the engine.

Total heat produced by gasoline in one hour =  $25 \times 6 \times 10^6$  cal

$$\text{Heat produced per second} = \frac{25 \times 6 \times 10^6}{3600} \text{ cal/s}$$

$$\text{Input} = \frac{25 \times 6 \times 10^6 \times 4.2}{3600} \text{ joules/s, i.e., watts}$$

$$[1 \text{ calorie} = 4.2 \text{ joules}]$$

$$\text{Output} = 35000 \text{ watts}$$

$$\begin{aligned} \text{Efficiency} &= \frac{\text{Useful output}}{\text{Input}} = \frac{35000 \times 3600}{25 \times 6 \times 10^6 \times 4.2} \\ &= \frac{1}{5} \end{aligned}$$

$$\% \text{ efficiency} = 20\%$$

### 3-17 Dulong and Petit's Law

Dulong and Petit, in 1819, studied the specific heat of various elements in a solid state and enunciated a law, called Dulong and Petit's law. According to this law, the product of the specific heat



and the atomic weight i.e. atomic heat of all the elements in the solid state is a constant. The value of this constant was fixed as 6.4 but it is taken as 6 at present. The exact value is 5.96 which also agrees with the value derived from the kinetic theory of matter.

The justification of Dulong and Petit's law was obtained from Boltzmann's consideration of the law of equipartition of energy. According to it, the energy associated with one gram atom of a substance for each degree of freedom at temperature  $T = \frac{1}{2} RT$ . Here  $R$  is the universal gas constant. If the atom is considered to be vibrating about the mean position, its mean kinetic energy will be equal to its mean potential energy.

For each form of energy there are three degrees of freedom. Therefore an atom has got six degrees of freedom. Thus the total energy associated with one gram atom of a substance at a temperature  $T = 3RT$

$$U = 3RT$$

$$A_H = \frac{dU}{dT} = 3R$$

$A_H$  is the atomic heat of the substance

$$A_H = \frac{3 \times 8.31 \times 10^7}{4.18 \times 10^7} \text{ cal/g-atom-K}$$

$$A_H = 5.96 \text{ cal/g-atom-K}$$

#### Atomic Heat of Substances at 20°C

Substance	Atomic Weight	Specific Heat	Atomic Heat
Aluminium	27.0	0.212	5.72
Boron	10.8	0.307	3.32
Carbon	12.0	0.160	1.92
Copper	63.6	0.091	5.79
Gold	197.2	0.031	6.11
Iron	55.8	0.110	6.12
Lead	207.2	0.030	6.21
Silicon	28.1	0.182	5.11
Silver	107.9	0.056	6.04
Zinc	65.4	0.092	6.02

Dulong and Petit's law was modified by Woestyn for metallic compounds. According to him, the *molecular heat of a compound* is equal to the sum of the atomic heat of its constituents. Further, it was found by Newmann that the molecular heat of the compounds of similar nature is also constant. The molecular heat in the case

of NaCl, AgCl and KCl was found to be nearly 13 and that of  $Sb_2O_3$ ,  $Fe_2O_3$ ,  $As_2O_3$  was found to be equal to 26.

#### 3.18 Variation of Specific Heat and Atomic Heat with Temperature.

It is found that Dulong and Petit's law is not true in the case of carbon, boron and silicon. In the case of these elements the

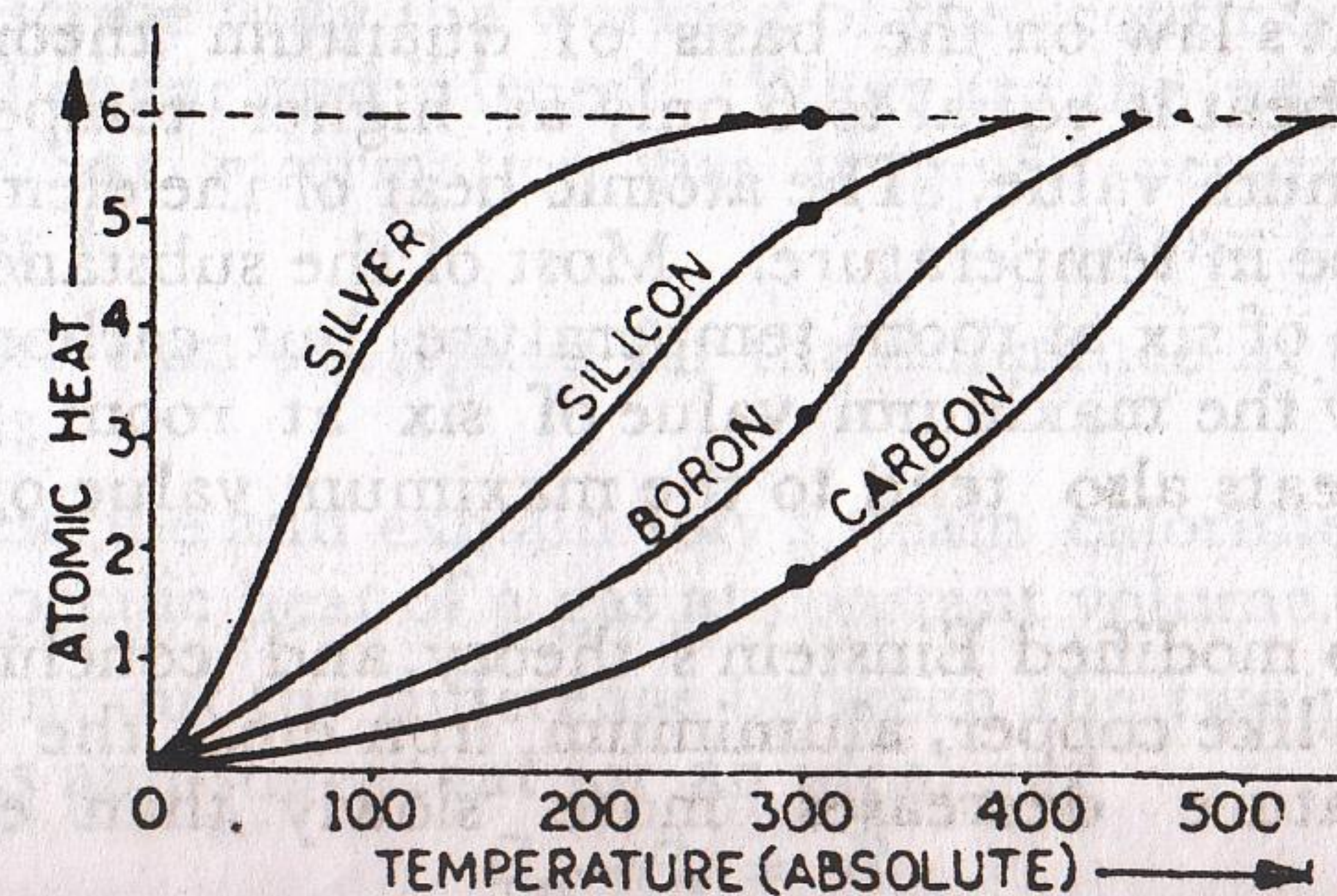


Fig. 3.15

atomic heats at 20°C are 1.92, 3.32 and 5.11. These values differ from the constant value of 6. This variation in atomic heat could not be explained on the basis of kinetic theory of matter. However, it was found by Nernst that the specific heat of a substance decreases with decrease in temperature and at absolute zero the specific heat tends to be zero. Further, he was able to show that the specific heat increases with the rise in temperature and tends to a maximum value. Therefore, the atomic heat of a substance tends to a maximum value of six and decreases with decrease in temperature. In the case of carbon, boron and silicon also, the atomic heat is 6 at high temperatures (Fig. 3.15).

In the case of silver, the atomic heat is 6 at room temperature but it is also less than 6 at lower temperatures.

#### Atomic Heats of Silver at Low Temperature

Temperature (K)	Atomic heat
205	5.61
144	5.37
103	4.80
75	4.04
56	3.19
36	1.89
20	0.40
10	0.05
5	0.005
1.4	0.00025



## 3.19 Quantum Theory

Dulong and Petit's law has been explained on the basis of quantum theory of heat radiation. According to quantum theory, heat is radiated in the form of discrete particles called photons. Each particle has energy equal to  $h\nu$  where  $h$  is the Planck's constant and  $\nu$  is the frequency of heat radiations. Einstein also explained Dulong and Petit's law on the basis of quantum theory and said that the atomic heat is equal to 6 only at higher temperatures, and this is the maximum value. The atomic heat of the elements decreases with decrease in temperature. Most of the substances reach the maximum value of six at room temperature but carbon, boron and silicon are below the maximum value of six at room temperature. Their atomic heats also tend to the maximum value of 6 at higher temperature.

Debye also modified Einstein's theory and concluded that in some substances like copper, aluminium, iron etc., the atomic heat at low temperature decreases more slowly than explained by Einstein.

According to Debye-Einstein theory

- (1) The atomic heat of all the elements tends to the maximum value of six.
- (2) The atomic heat of all the elements decreases with the fall of temperature.
- (3) The atomic heat tends to the value zero, near about absolute zero.
- (4) At low temperatures, the atomic heat varies as the cube of the absolute temperature.

Atomic heat  $\propto T^3$

It is known as Debye's  $T^3$  law.

- (5) The graphs between atomic heats and absolute temperature of all the elements will coincide if the temperature scale is suitably modified according to the expression,

$$A = f\left(\frac{\theta}{T}\right)$$

Here,  $A$  is the atomic heat of the element at absolute temperature  $T$  and  $\theta$  is a parameter called Debye temperature. Debye temperature is a constant for a particular element but is different for different elements.

The value  $f\left(\frac{\theta}{T}\right)$  is the same for all the elements at absolute

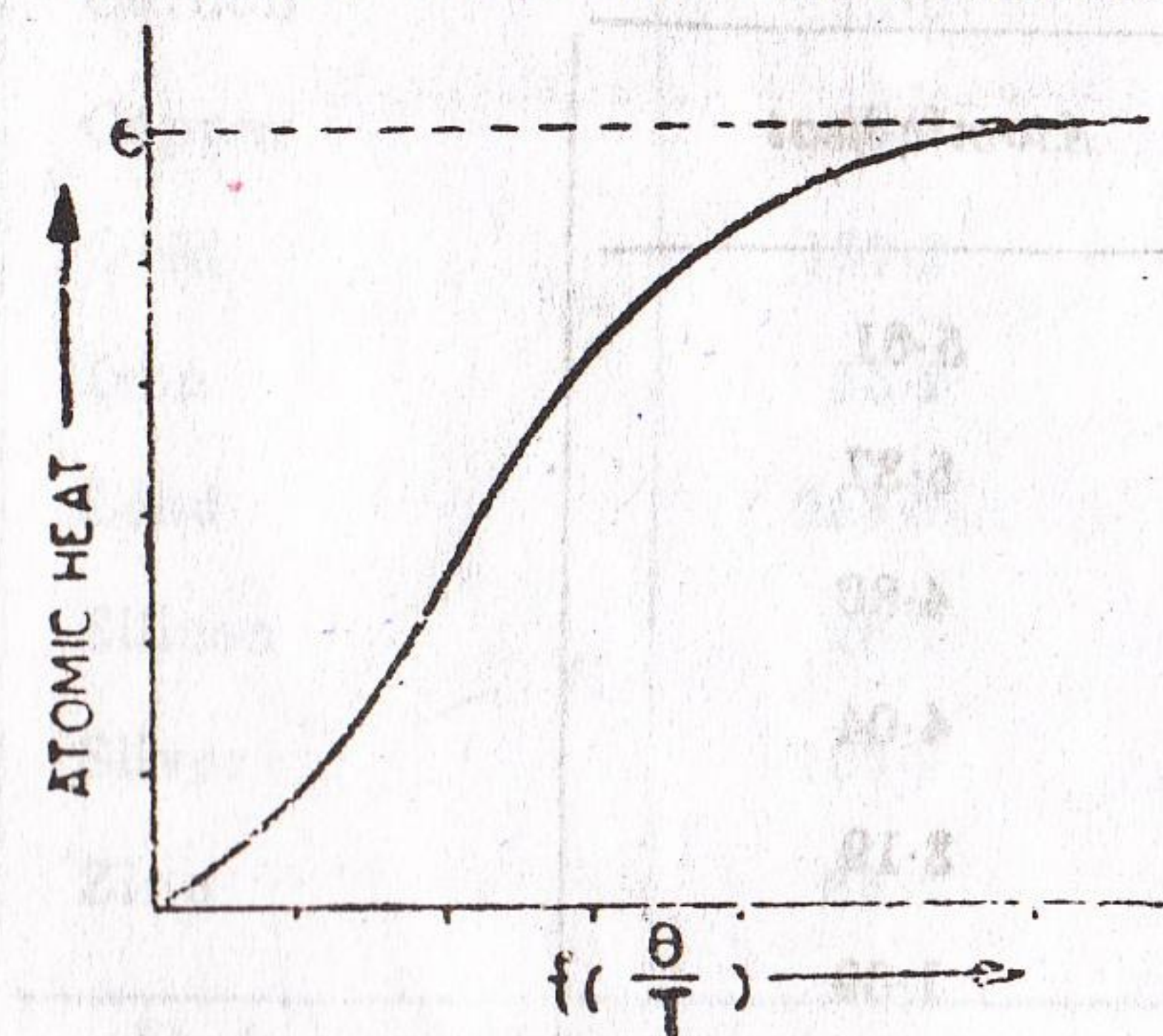


Fig. 3-16

temperature  $T$ . Thus according to this theory a single graph between  $A$  and  $f\left(\frac{\theta}{T}\right)$  for all the elements is obtained (Fig. 3-16).

## Exercises III

1. Describe the continuous flow calorimeter. What are the advantages of this method? How do you determine  $C_p$  of a gas by using this method? (Delhi 1969; Agra 1961)
2. Describe fully the working of the continuous flow calorimeter of 'Callendar and Barnes'. What are the advantages of this method? What information has it given regarding the specific heat of water? (Agra 1963; Delhi 1971)
3. Derive an expression for the difference in the two specific heats of a gas. (Agra 1966)
4. Describe and explain Joly's steam calorimeter method for finding the specific heat of a gas at constant volume. (Agra 1966)
5. Point out the difference between the two principal specific heats of a gas and show that for an ideal gas

$$C_p - C_v = R$$

(Lucknow 1961; Delhi 1972)

6. Describe the bomb calorimeter method to find the calorific value of a fuel. (Lucknow 1972)
7. Indicate the manner in which the specific heat of a solid varies with temperature and outline the theory which explains this variation.
8. Discuss Newton's law of cooling. How will you determine the specific heat of a liquid by Newton's law of cooling? [Delhi (Add Physics) 1975; (Delhi 1973)]
9. Describe the Regnault's method to find the specific heat of a gas at constant pressure.
10. Discuss the working of a bell calorimeter in determining the calorific value of a fuel.
11. Discuss Dulong and Petit's law and explain the variation of the atomic heat of a substance with temperature.
12. How do you explain Dulong and Petit's law according to quantum theory of radiation.
13. Discuss Debye-Einstein theory regarding atomic heats of solids. Discuss Debye's  $T^3$  law. (Madras 1974)
14. Deduce the relation between the specific heat of a gas at constant pressure and at constant volume. (Delhi 1968)
15. Describe how the specific heat of a gas at constant pressure is determined accurately. (Delhi 1968)
16. Describe Nernst vacuum calorimeter and indicate briefly how it may be used to determine the specific heat at low temperatures. (Delhi 1972, 1974)
17. Describe fully the working of a continuous flow electric calorimeter for the measurement of  $C_p$  of a gas. (Delhi 1975)



18. Distinguish between the specific heat of a gas at constant pressure and at constant volume. Describe an accurate method to determine the specific heat of a gas at constant pressure. (Delhi, 1976)
19. Describe Joly's differential steam calorimeter. How will you use this apparatus for determining the specific heat of a gas at constant volume? Mention the sources of error. (Delhi, 1976)
20. Point out the difference between the two principal specific heats of a gas. Describe an accurate method to determine the specific heat of a gas at constant pressure. (Delhi, 1976)
21. State and explain Newton's law of cooling. How will you find the specific heat of a liquid by the method of cooling? (Delhi, 1976)
22. Describe Nernst vacuum calorimeter for determining the specific heat of a good-conducting solid at low temperatures. [Delhi, 1978]
23. Describe the nature of variation of specific heat of solids with temperature. Discuss important regions of this curve. [Delhi, 1978]
24. A liquid takes 10 minutes to cool from  $90^{\circ}\text{C}$  to  $60^{\circ}\text{C}$ . How much time will it take to cool from  $70^{\circ}\text{C}$  to  $40^{\circ}\text{C}$ ? The temperature of the surroundings is  $30^{\circ}\text{C}$ . [Ans. 20 minutes]
25. A body takes 6 minutes to cool from  $80^{\circ}\text{C}$  to  $50^{\circ}\text{C}$ . How much time will it take to cool from  $60^{\circ}\text{C}$  to  $30^{\circ}\text{C}$ ? The temperature of the surroundings is  $20^{\circ}\text{C}$ . [Ans. 12 minutes]
26. A body cools from  $70^{\circ}\text{C}$  to  $50^{\circ}\text{C}$  in 6 minutes. What will be its temperature after the next 6 minutes? The temperature of the surroundings is  $15^{\circ}\text{C}$ . [Ans.  $32.27^{\circ}\text{C}$ ]
27. A liquid cools in 7 minutes from  $60^{\circ}\text{C}$  to  $40^{\circ}\text{C}$ . What will be its temperature after the next 7 minutes? The temperature of the surroundings is  $10^{\circ}\text{C}$ . [Ans.  $28^{\circ}\text{C}$ ]
28. A liquid takes 5 minutes to cool from  $70^{\circ}\text{C}$  to  $50^{\circ}\text{C}$ . How much time will it take to cool from  $50^{\circ}\text{C}$  to  $35^{\circ}\text{C}$ ? The temperature of the surroundings is  $20^{\circ}\text{C}$ . [Ans. 6.785 minutes]
29. A body takes 3 minutes to cool from  $70^{\circ}\text{C}$  to  $60^{\circ}\text{C}$ . What will be its temperature after the next 6 minutes? The temperature of the surroundings is  $25^{\circ}\text{C}$ . [Ans.  $46.17^{\circ}\text{C}$ ]
30. A liquid takes 4 minutes to cool from  $65^{\circ}\text{C}$  to  $50^{\circ}\text{C}$ . What will be its temperature after the next 10 minutes? The temperature of the surroundings is  $35^{\circ}\text{C}$ . [Ans.  $37.65^{\circ}\text{C}$ ]
31. A petrol engine consumes 25 kg of petrol per hour. The calorific value of petrol is  $11.4 \times 10^6$  cal per kg. The power of the

- engine is 99.75 kilowatts. Calculate the efficiency of the engine. [Ans. 30%]
32. A Nernst calorimeter of lead weighing 396.3 grams and surrounded by a bath at temperature 61 K, on being heated electrically for four minutes showed a rise in temperature of 1.219 K under condition of no heat leakage. Calculate the atomic heat of lead at 61 K, if the potential drop and average current during the interval of heating were 1.586 volts and 0.1444 ampere respectively. (Atomic weight of lead = 206.4 and  $J = 4.18$  joules/calorie.) (Delhi 1972) [Ans. 5.619]
33. A copper calorimeter of mass 100 g containing  $150 \text{ cm}^3$  of a liquid of specific heat 0.6 and specific gravity 1.2, is found to cool at the rate of  $2^{\circ}\text{C}$  per minute when its temperature is  $50^{\circ}\text{C}$  above that of the surroundings. If the liquid is emptied out and  $150 \text{ cm}^3$  of a second liquid of specific heat 0.4 and specific gravity 0.9 are substituted, what will be its rate of cooling, when the temperature is  $40^{\circ}\text{C}$  above that of its surroundings? (Sp. heat of copper 0.1.) (Delhi 1973) [Ans.  $2.76^{\circ}\text{C}$  per minute]