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MURRAY R. SPIEGEL

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SCHAUM'S OUTLINE OF

THEORY AND PROBLEMS

of

FOURIER ANALYSIS

with Applications to
Boundary Value Problems

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by

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Preface

In the early years of the 19th century the French mathematician J. B. J. Fourier in his researches on heat conduction was led to the remarkable discovery of certain trigonometric series which now bear his name. Since that time Fourier series, and generalizations to Fourier integrals and orthogonal series, have become an essential part of the background of scientists, engineers and mathematicians from both an applied and theoretical point of view.

The purpose of this book is to present the fundamental concepts and applications of Fourier series, Fourier integrals and orthogonal functions (Bessel, Legendre, Hermite, and Laguerre functions, as well as others).

The book is designed to be used either as a textbook for a formal course in Fourier Analysis or as a comprehensive supplement to all current standard texts. It should be of considerable value to those taking courses in engineering, science or mathematics in which these important methods are frequently used. It should also prove useful as a book of reference to research workers employing Fourier methods or to those interested in the field for self-study.

Each chapter begins with a clear statement of pertinent definitions, principles and theorems, together with illustrative and other descriptive material. The solved problems serve to illustrate and amplify the theory and to provide the repetition of basic principles so vital to effective learning. Numerous proofs of theorems and derivations of formulas are included among the solved problems. The large number of supplementary problems with answers serve as a complete review of the material of each chapter.

Considerably more material has been included here than can be covered in most first courses. This has been done to make the book more flexible, to provide a more useful book of reference, and to stimulate further interest in the topics.

I wish to take this opportunity to thank Henry Hayden and David Beckwith for their splendid cooperation.

M. R. SPIEGEL

CONTENTS

	Page
Chapter 1 BOUNDARY VALUE PROBLEMS	1
Mathematical Formulation and Solution of Physical Problems. Definitions Pertaining to Partial Differential Equations. Linear Partial Differential Equations. Some Important Partial Differential Equations. The Laplacian in Different Coordinate Systems. Methods of Solving Boundary Value Problems.	
<hr/>	
Chapter 2 FOURIER SERIES AND APPLICATIONS	20
The Need for Fourier Series. Periodic Functions. Piecewise Continuous Functions. Definition of Fourier Series. Dirichlet Conditions. Odd and Even Functions. Half-Range Fourier Sine or Cosine Series. Parseval's Identity. Uniform Convergence. Integration and Differentiation of Fourier Series. Complex Notation for Fourier Series. Double Fourier Series. Applications of Fourier Series.	
<hr/>	
Chapter 3 ORTHOGONAL FUNCTIONS	52
Definitions Involving Orthogonal Functions. Orthonormal Sets. Orthogonality with Respect to a Weight Function. Expansion of Functions in Orthonormal Series. Approximations in the Least-Squares Sense. Parseval's Identity for Orthonormal Series. Completeness. Sturm-Liouville Systems. Eigenvalues and Eigenfunctions. The Gram-Schmidt Orthonormalization Process. Applications to Boundary Value Problems.	
<hr/>	
Chapter 4 GAMMA, BETA AND OTHER SPECIAL FUNCTIONS	67
Special Functions. The Gamma Function. Table of Values and Graph of the Gamma Function. Asymptotic Formula for $\Gamma(x)$. Miscellaneous Results Involving the Gamma Function. The Beta Function. Other Special Functions. Asymptotic Series or Expansions.	
<hr/>	
Chapter 5 FOURIER INTEGRALS AND APPLICATIONS	80
The Need for Fourier Integrals. The Fourier Integral. Equivalent Forms of Fourier's Integral Theorem. Fourier Transforms. Fourier Sine and Cosine Transforms. Parseval's Identities for Fourier Integrals. The Convolution Theorem for Fourier Transforms. Applications of Fourier Integrals and Transforms.	
<hr/>	
Chapter 6 BESSEL FUNCTIONS AND APPLICATIONS	97
Bessel's Differential Equation. The Method of Frobenius. Bessel Functions of the First Kind. Bessel Functions of the Second Kind. Generating Function for $J_n(x)$. Recurrence Formulas. Functions Related to Bessel Functions. Equations Transformable into Bessel's Equation. Asymptotic Formulas for Bessel Functions. Zeros of Bessel Functions. Orthogonality of Bessel Functions of the First Kind. Series of Bessel Functions of the First Kind. Orthogonality and Series of Bessel Functions of the Second Kind. Solutions to Boundary Value Problems Using Bessel Functions.	

CONTENTS

	Page
Chapter 7 LEGENDRE FUNCTIONS AND APPLICATIONS	130
Legendre's Differential Equation. Legendre Polynomials. Generating Function for Legendre Polynomials. Recurrence Formulas. Legendre Functions of the Second Kind. Orthogonality of Legendre Polynomials. Series of Legendre Polynomials. Associated Legendre Functions. Orthogonality of Associated Legendre Functions. Solutions to Boundary Value Problems Using Legendre Functions.	
Chapter 8 HERMITE, LAGUERRE AND OTHER ORTHOGONAL POLYNOMIALS	154
Hermite's Differential Equation. Hermite Polynomials. Generating Function for Hermite Polynomials. Recurrence Formulas for Hermite Polynomials. Orthogonality of Hermite Polynomials. Series of Hermite Polynomials. Laguerre's Differential Equation. Laguerre Polynomials. Some Important Properties of Laguerre Polynomials. Miscellaneous Orthogonal Polynomials and Their Properties.	
Appendix A Uniqueness of Solutions	167
Appendix B Special Fourier Series	169
Appendix C Special Fourier Transforms	173
Appendix D Tables of Values for $J_0(x)$ and $J_1(x)$	176
Appendix E Zeros of Bessel Functions	177
ANSWERS TO SUPPLEMENTARY PROBLEMS	
179	
INDEX	
187	