Appendix A

Uniqueness of Solutions

A proof establishing the uniqueness of solutions to boundary value problems can often be accomplished by assuming the existence of two solutions and then arriving at a contradiction. We illustrate the procedure by an example involving heat conduction.

Consider a finite closed region $\mathcal{R}$ having surface S. Suppose that the initial temperature inside $\mathcal{R}$ and the surface temperature are specified. Then the boundary value problem for the temperature $u(x,y,z,t)$ at any point (x,y,z) at time t is given by

$$\frac{\partial u}{\partial t} = \kappa \nabla^2 u \quad \text{inside } \mathcal{R} \tag{1}$$

$$u(x,y,z,0) = f(x,y,z) \quad \text{at points } (x,y,z) \text{ of } \mathcal{R} \tag{2}$$

$$u(x,y,z,t) = g(x,y,z,t) \quad \text{at points } (x,y,z) \text{ of } S \tag{3}$$

We shall assume that all functions are at least differentiable at points of $\mathcal{R}$ and S.

Assume the existence of two different solutions, say u_1 and u_2, of the above boundary value problem. Then letting $U = u_1 - u_2$ we find that U satisfies the boundary value problem

$$\frac{\partial U}{\partial t} = \kappa \nabla^2 U \quad \text{inside } \mathcal{R} \tag{4}$$

$$U(x,y,z,0) = 0 \quad \text{at points of } \mathcal{R} \tag{5}$$

$$U(x,y,z,t) = 0 \quad \text{at points of } S \tag{6}$$

Let us now consider

$$W(t) = \frac{1}{2}\iiint_{\mathcal{R}} [U(x,y,z,t)]^2\,dx\,dy\,dz \tag{7}$$

where the integration is performed over the region $\mathcal{R}$. Using (5) we see that

$$W(0) = 0 \tag{8}$$

Also from (7) we have

$$\frac{dW}{dt} = \iiint_{\mathcal{R}} U\frac{\partial U}{\partial t}\,dx\,dy\,dz = \kappa\iiint_{\mathcal{R}} U\nabla^2 U\,dx\,dy\,dz \tag{9}$$

where we have used (4).

We now make use of *Green's theorem* to show that

$$\iiint_{\mathcal{R}} U\nabla^2 U\,dx\,dy\,dz = \iint_{S} U\frac{\partial U}{\partial n}\,dS - \iiint_{\mathcal{R}}\left[\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial U}{\partial y}\right)^2 + \left(\frac{\partial U}{\partial z}\right)^2\right]dx\,dy\,dz \tag{10}$$

where n is a unit outward-drawn normal to S. Since $U = 0$ on S the first integral on the right of (*10*) is zero and we have

$$\iiint_{\mathcal{R}} U \nabla^2 U \, dx \, dy \, dz \;=\; -\iiint_{\mathcal{R}} \left[\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial U}{\partial y}\right)^2 + \left(\frac{\partial U}{\partial z}\right)^2\right] dx \, dy \, dz \tag{11}$$

Thus we have from (*9*),

$$\frac{dW}{dt} \;=\; -\kappa \iiint_{\mathcal{R}} \left[\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial U}{\partial y}\right)^2 + \left(\frac{\partial U}{\partial z}\right)^2\right] dx \, dy \, dz \tag{12}$$

It follows from this that $dW/dt \leqq 0$, i.e. W is a nonincreasing function of t, and, in view of (*8*), that $W(t) \leqq 0$. But from (*7*) we see that $W(t) \geqq 0$. Thus it follows that $W(t) = 0$ identically.

Now if $U(x, y, z, t)$ is not zero at a point of $\mathcal{R}$ it follows by its continuity that there will be a neighborhood of the point in which it is not zero. Then the integral in (*7*) would have to be greater than zero, i.e. $W(t) > 0$. This contradiction with $W(t) = 0$ shows that $U(x, y, z, t)$ must be identically zero, which shows that $u_1 = u_2$ and the solution is unique.

Appendix B

Special Fourier Series

B-1 $f(x) = \begin{cases} 1 & 0 < x < \pi \\ -1 & -\pi < x < 0 \end{cases}$

$$\frac{4}{\pi}\left(\frac{\sin x}{1} + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \cdots\right)$$

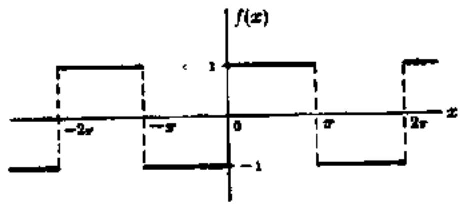

Fig. B-1

B-2 $f(x) = |x| = \begin{cases} x & 0 < x < \pi \\ -x & -\pi < x < 0 \end{cases}$

$$\frac{\pi}{2} - \frac{4}{\pi}\left(\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \cdots\right)$$

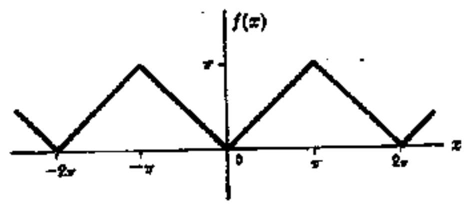

Fig. B-2

B-3 $f(x) = x, \ -\pi < x < \pi$

$$2\left(\frac{\sin x}{1} - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \cdots\right)$$

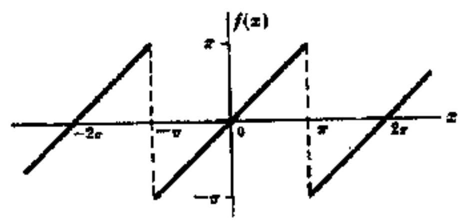

Fig. B-3

B-4 $f(x) = x, \ 0 < x < 2\pi$

$$\pi - 2\left(\frac{\sin x}{1} + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \cdots\right)$$

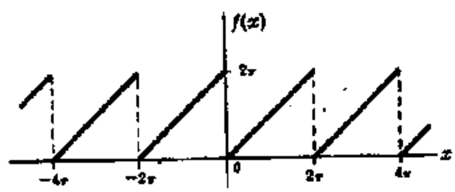

Fig. B-4

B-5 $f(x) = |\sin x|, \ -\pi < x < \pi$

$$\frac{2}{\pi} - \frac{4}{\pi}\left(\frac{\cos 2x}{1 \cdot 3} + \frac{\cos 4x}{3 \cdot 5} + \frac{\cos 6x}{5 \cdot 7} + \cdots\right)$$

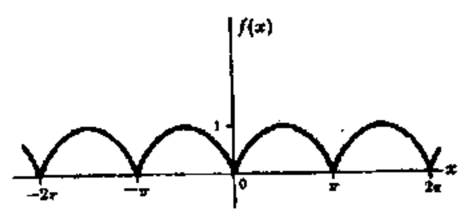

Fig. B-5

B-6 $f(x) = \begin{cases} \sin x & 0 < x < \pi \\ 0 & \pi < x < 2\pi \end{cases}$

$$\frac{1}{\pi} + \frac{1}{2}\sin x - \frac{2}{\pi}\left(\frac{\cos 2x}{1\cdot 3} + \frac{\cos 4x}{3\cdot 5} + \frac{\cos 6x}{5\cdot 7} + \cdots\right)$$

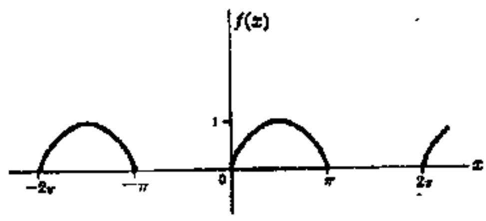

Fig. B-6

B-7 $f(x) = \begin{cases} \cos x & 0 < x < \pi \\ -\cos x & -\pi < x < 0 \end{cases}$

$$\frac{8}{\pi}\left(\frac{\sin 2x}{1\cdot 3} + \frac{2\sin 4x}{3\cdot 5} + \frac{3\sin 6x}{5\cdot 7} + \cdots\right)$$

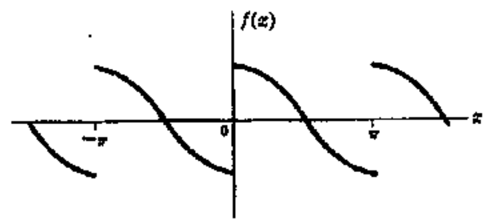

Fig. B-7

B-8 $f(x) = x^2, \quad -\pi < x < \pi$

$$\frac{\pi^2}{3} - 4\left(\frac{\cos x}{1^2} - \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} - \cdots\right)$$

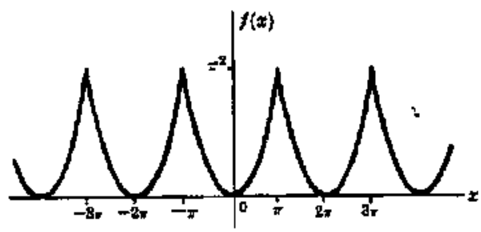

Fig. B-8

B-9 $f(x) = x(\pi - x), \quad 0 < x < \pi$

$$\frac{\pi^2}{6} - \left(\frac{\cos 2x}{1^2} + \frac{\cos 4x}{2^2} + \frac{\cos 6x}{3^2} + \cdots\right)$$

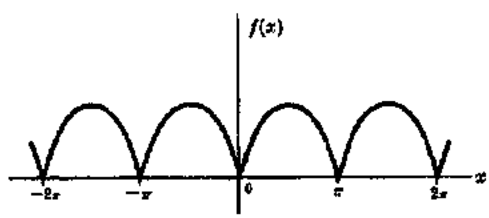

Fig. B-9

B-10 $f(x) = x(\pi - x)(\pi + x), \quad -\pi < x < \pi$

$$12\left(\frac{\sin x}{1^3} - \frac{\sin 2x}{2^3} + \frac{\sin 3x}{3^3} - \cdots\right)$$

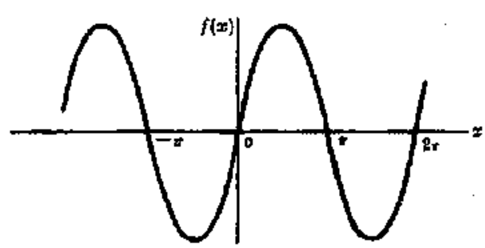

Fig. B-10

B-11 $f(x) = \begin{cases} 0 & 0 < x < \pi - \alpha \\ 1 & \pi - \alpha < x < \pi + \alpha \\ 0 & \pi + \alpha < x < 2\pi \end{cases}$

$$\frac{\alpha}{\pi} - \frac{2}{\pi}\left(\frac{\sin\alpha\cos x}{1} - \frac{\sin 2\alpha\cos 2x}{2} + \frac{\sin 3\alpha\cos 3x}{3} - \cdots\right)$$

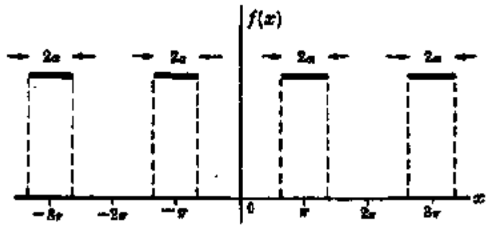

Fig. B-11

B-12 $f(x) = \begin{cases} x(\pi - x) & 0 < x < \pi \\ -x(\pi - x) & -\pi < x < 0 \end{cases}$

$$\frac{8}{\pi}\left(\frac{\sin x}{1^3} + \frac{\sin 3x}{3^3} + \frac{\sin 5x}{5^3} + \cdots\right)$$

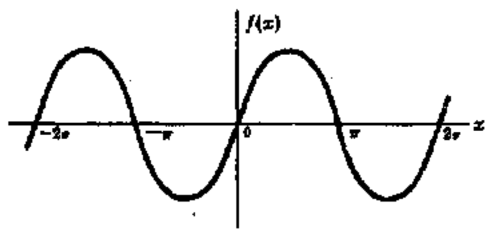

Fig. B-12

B-13 $f(x) = \sin\mu x, \quad -\pi < x < \pi, \quad \mu \neq$ integer

$$\frac{2\sin\mu\pi}{\pi}\left(\frac{\sin x}{1^2 - \mu^2} - \frac{2\sin 2x}{2^2 - \mu^2} + \frac{3\sin 3x}{3^2 - \mu^2} - \cdots\right)$$

B-14 $f(x) = \cos\mu x, \quad -\pi < x < \pi, \quad \mu \neq$ integer

$$\frac{2\mu\sin\mu\pi}{\pi}\left(\frac{1}{2\mu^2} + \frac{\cos x}{1^2 - \mu^2} - \frac{\cos 2x}{2^2 - \mu^2} + \frac{\cos 3x}{3^2 - \mu^2} - \cdots\right)$$

B-15 $f(x) = \tan^{-1}[(a\sin x)/(1 - a\cos x)], \quad -\pi < x < \pi, \quad |a| < 1$

$$a\sin x + \frac{a^2}{2}\sin 2x + \frac{a^3}{3}\sin 3x + \cdots$$

B-16 $f(x) = \ln(1 - 2a\cos x + a^2), \quad -\pi < x < \pi, \quad |a| < 1$

$$-2\left(a\cos x + \frac{a^2}{2}\cos 2x + \frac{a^3}{3}\cos 3x + \cdots\right)$$

B-17 $f(x) = \frac{1}{2}\tan^{-1}[(2a\sin x)/(1 - a^2)], \quad -\pi < x < \pi, \quad |a| < 1$

$$a\sin x + \frac{a^3}{3}\sin 3x + \frac{a^5}{5}\sin 5x + \cdots$$

B-18 $f(x) = \frac{1}{2}\tan^{-1}[(2a\cos x)/(1 - a^2)], \quad -\pi < x < \pi, \quad |a| < 1$

$$a\cos x - \frac{a^3}{3}\cos 3x + \frac{a^5}{5}\cos 5x - \cdots$$

8.19 $f(x) = e^{\mu x}, \quad -\pi < x < \pi$

$$\frac{2\sinh\mu\pi}{\pi}\left(\frac{1}{2\mu} + \sum_{n=1}^{\infty}\frac{(-1)^n(\mu\cos nx - n\sin nx)}{\mu^2 + n^2}\right)$$

8.20 $f(x) = \sinh\mu x, \quad -\pi < x < \pi$

$$\frac{2\sinh\mu\pi}{\pi}\left(\frac{\sin x}{1^2+\mu^2} - \frac{2\sin 2x}{2^2+\mu^2} + \frac{3\sin 3x}{3^2+\mu^2} - \cdots\right)$$

8.21 $f(x) = \cosh\mu x, \quad -\pi < x < \pi$

$$\frac{2\mu\sinh\mu\pi}{\pi}\left(\frac{1}{2\mu^2} - \frac{\cos x}{1^2+\mu^2} + \frac{\cos 2x}{2^2+\mu^2} - \frac{\cos 3x}{3^2+\mu^2} + \cdots\right)$$

8.22 $f(x) = \ln|\sin\tfrac{1}{2}x|, \quad -\pi < x < \pi$

$$-\left(\ln 2 + \frac{\cos x}{1} + \frac{\cos 2x}{2} + \frac{\cos 3x}{3} + \cdots\right)$$

8.23 $f(x) = \ln|\cos\tfrac{1}{2}x|, \quad -\pi < x < \pi$

$$-\left(\ln 2 - \frac{\cos x}{1} + \frac{\cos 2x}{2} - \frac{\cos 3x}{3} + \cdots\right)$$

8.24 $f(x) = \tfrac{1}{6}\pi^2 - \tfrac{1}{2}\pi x + \tfrac{1}{4}x^2, \quad 0 \leqq x \leqq 2\pi$

$$\frac{\cos x}{1^2} + \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} + \cdots$$

8.25 $f(x) = \tfrac{1}{12}x(x-\pi)(x-2\pi), \quad 0 \leqq x \leqq 2\pi$

$$\frac{\sin x}{1^3} + \frac{\sin 2x}{2^3} + \frac{\sin 3x}{3^3} + \cdots$$

8.26 $f(x) = \tfrac{1}{90}\pi^4 - \tfrac{1}{12}\pi^2x^2 + \tfrac{1}{12}\pi x^3 - \tfrac{1}{48}x^4, \quad 0 \leqq x \leqq 2\pi$

$$\frac{\cos x}{1^4} + \frac{\cos 2x}{2^4} + \frac{\cos 3x}{3^4} + \cdots$$

Appendix C

Special Fourier Transforms

SPECIAL FOURIER TRANSFORM PAIRS

	$f(x)$	$F(\alpha)$
C-1	$\begin{cases} 1 & \lvert x\rvert < b \\ 0 & \lvert x\rvert > b \end{cases}$	$\frac{2 \sin b\alpha}{\alpha}$
C-2	$\frac{1}{x^2 + b^2}$	$\frac{\pi e^{-b\alpha}}{b}$
C-3	$\frac{x}{x^2 + b^2}$	$-\frac{\pi i\alpha}{b} e^{-b\alpha}$
C-4	$f^{(n)}(x)$	$i^n \alpha^n F(\alpha)$
C-5	$x^n f(x)$	$i^n \frac{d^n F}{d\alpha^n}$
C-6	$f(bx)e^{itx}$	$\frac{1}{b} F\left(\frac{\alpha - t}{b}\right)$

SPECIAL FOURIER COSINE TRANSFORMS

	$f(x)$	$F_C(\alpha)$
C-7	$\begin{cases} 1 & 0 < x < b \\ 0 & x > b \end{cases}$	$\dfrac{\sin b\alpha}{\alpha}$
C-8	$\dfrac{1}{x^2 + b^2}$	$\dfrac{\pi e^{-b\alpha}}{2b}$
C-9	e^{-bx}	$\dfrac{b}{\alpha^2 + b^2}$
C-10	$x^{n-1} e^{-bx}$	$\dfrac{\Gamma(n) \cos (n \tan^{-1} \alpha/b)}{(\alpha^2 + b^2)^{n/2}}$
C-11	e^{-bx^2}	$\dfrac{1}{2}\sqrt{\dfrac{\pi}{b}}\, e^{-\alpha^2/4b}$
C-12	$x^{-1/2}$	$\sqrt{\dfrac{\pi}{2\alpha}}$
C-13	x^{-n}	$\dfrac{\pi\alpha^{n-1} \sec (n\pi/2)}{2\,\Gamma(n)}, \quad 0 < n < 1$
C-14	$\ln\left(\dfrac{x^2 + b^2}{x^2 + a^2}\right)$	$\dfrac{e^{-a\alpha} - e^{-b\alpha}}{\pi\alpha}$
C-15	$\dfrac{\sin bx}{x}$	$\begin{cases} \pi/2 & \alpha < b \\ \pi/4 & \alpha = b \\ 0 & \alpha > b \end{cases}$
C-16	$\sin bx^2$	$\sqrt{\dfrac{\pi}{8b}}\left(\cos \dfrac{\alpha^2}{4b} - \sin \dfrac{\alpha^2}{4b}\right)$
C-17	$\cos bx^2$	$\sqrt{\dfrac{\pi}{8b}}\left(\cos \dfrac{\alpha^2}{4b} + \sin \dfrac{\alpha^2}{4b}\right)$
C-18	$\operatorname{sech} bx$	$\dfrac{\pi}{2b} \operatorname{sech} \dfrac{\pi\alpha}{2b}$
C-19	$\dfrac{\cosh (\sqrt{\pi}\, x/2)}{\cosh (\sqrt{\pi}\, x)}$	$\sqrt{\dfrac{\pi}{2}}\, \dfrac{\cosh (\sqrt{\pi}\, \alpha/2)}{\cosh (\sqrt{\pi}\, \alpha)}$
C-20	$\dfrac{e^{-b\sqrt{x}}}{\sqrt{x}}$	$\sqrt{\dfrac{\pi}{2\alpha}}\,(\cos (2b\sqrt{\alpha}) - \sin (2b\sqrt{\alpha}))$

SPECIAL FOURIER SINE TRANSFORMS

	$f(x)$	$F_S(\alpha)$
C-21	$\begin{cases} 1 & 0 < x < b \\ 0 & x > b \end{cases}$	$\dfrac{1 - \cos b\alpha}{\alpha}$
C-22	x^{-1}	$\dfrac{\pi}{2}$
C-23	$\dfrac{x}{x^2 + b^2}$	$\dfrac{\pi}{2} e^{-b\alpha}$
C-24	e^{-bx}	$\dfrac{\alpha}{\alpha^2 + b^2}$
C-25	$x^{n-1} e^{-bx}$	$\dfrac{\Gamma(n) \sin (n \tan^{-1} \alpha/b)}{(\alpha^2 + b^2)^{n/2}}$
C-26	xe^{-bx^2}	$\dfrac{\sqrt{\pi}}{4b^{3/2}} \alpha e^{-\alpha^2/4b}$
C-27	$x^{-1/2}$	$\sqrt{\dfrac{\pi}{2\alpha}}$
C-28	x^{-n}	$\dfrac{\pi \alpha^{n-1} \csc (n\pi/2)}{2\,\Gamma(n)} \qquad 0 < n < 2$
C-29	$\dfrac{\sin bx}{x}$	$\dfrac{1}{2} \ln \left(\dfrac{\alpha + b}{\alpha - b}\right)$
C-30	$\dfrac{\sin bx}{x^2}$	$\begin{cases} \pi\alpha/2 & \alpha < b \\ \pi b/2 & \alpha > b \end{cases}$
C-31	$\dfrac{\cos bx}{x}$	$\begin{cases} 0 & \alpha < b \\ \pi/4 & \alpha = b \\ \pi/2 & \alpha > b \end{cases}$
C-32	$\tan^{-1} (x/b)$	$\dfrac{\pi}{2\alpha} e^{-b\alpha}$
C-33	$\csc bx$	$\dfrac{\pi}{2b} \tanh \dfrac{\pi\alpha}{2b}$
C-34	$\dfrac{1}{e^{2x} - 1}$	$\dfrac{\pi}{4} \coth \left(\dfrac{\pi\alpha}{2}\right) - \dfrac{1}{2\alpha}$

Appendix D

Tables of Values for $J_0(x)$ and $J_1(x)$

$J_0(x)$

x	0	1	2	3	4	5	6	7	8	9
0.	1.0000	.9975	.9900	.9776	.9604	.9385	.9120	.8812	.8463	.8075
1.	.7652	.7196	.6711	.6201	.5669	.5118	.4554	.3980	.3400	.2818
2.	.2239	.1666	.1104	.0555	.0025	−.0484	−.0968	−.1424	−.1850	−.2243
3.	−.2601	−.2921	−.3202	−.3443	−.3643	−.3801	−.3918	−.3992	−.4026	−.4018
4.	−.3971	−.3887	−.3766	−.3610	−.3423	−.3205	−.2961	−.2693	−.2404	−.2097
5.	−.1776	−.1443	−.1103	−.0758	−.0412	−.0068	.0270	.0599	.0917	.1220
6.	.1506	.1773	.2017	.2238	.2433	.2601	.2740	.2851	.2931	.2981
7.	.3001	.2991	.2951	.2882	.2786	.2663	.2516	.2346	.2154	.1944
8.	.1717	.1475	.1222	.0960	.0692	.0419	.0146	−.0125	−.0392	−.0653
9.	−.0903	−.1142	−.1367	−.1577	−.1768	−.1939	−.2090	−.2218	−.2323	−.2403

$J_1(x)$

x	0	1	2	3	4	5	6	7	8	9
0.	.0000	.0499	.0995	.1483	.1960	.2423	.2867	.3290	.3688	.4059
1.	.4401	.4709	.4983	.5220	.5419	.5579	.5699	.5778	.5815	.5812
2.	.5767	.5683	.5560	.5399	.5202	.4971	.4708	.4416	.4097	.3754
3.	.3391	.3009	.2613	.2207	.1792	.1374	.0955	.0538	.0128	−.0272
4.	−.0660	−.1033	−.1386	−.1719	−.2028	−.2311	−.2566	−.2791	−.2985	−.3147
5.	−.3276	−.3371	−.3432	−.3460	−.3453	−.3414	−.3343	−.3241	−.3110	−.2951
6.	−.2767	−.2559	−.2329	−.2081	−.1816	−.1538	−.1250	−.0953	−.0652	−.0349
7.	−.0047	.0252	.0543	.0826	.1096	.1352	.1592	.1813	.2014	.2192
8.	.2346	.2476	.2580	.2657	.2708	.2731	.2728	.2697	.2641	.2559
9.	.2453	.2324	.2174	.2004	.1816	.1613	.1395	.1166	.0928	.0684

Appendix E

Zeros of Bessel Functions

The following table lists the first few positive roots of $J_n(x) = 0$ and $J_n'(x) = 0$. Note that for all cases listed successive large roots differ approximately by $\pi = 3.14159\ldots$.

	$n = 0$	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$
$J_n(x) = 0$	2.4048	3.8317	5.1356	6.3802	7.5883	8.7715	9.9361
	5.5201	7.0156	8.4172	9.7610	11.0647	12.3386	13.5893
	8.6537	10.1735	11.6198	13.0152	14.3725	15.7002	17.0038
	11.7915	13.3237	14.7960	16.2235	17.6160	18.9801	20.3208
	14.9309	16.4706	17.9598	19.4094	20.8269	22.2178	23.5861
	18.0711	19.6159	21.1170	22.5827	24.0190	25.4303	26.8202
$J_n'(x) = 0$	0.0000	1.8412	3.0542	4.2012	5.3176	6.4156	7.5013
	3.8317	5.3314	6.7061	8.0152	9.2824	10.5199	11.7349
	7.0156	8.5363	9.9695	11.3459	12.6819	13.9872	15.2682
	10.1735	11.7060	13.1704	14.5859	15.9641	17.3128	18.6374
	13.3237	14.8636	16.3475	17.7888	19.1960	20.5755	21.9317
	16.4706	18.0155	19.5129	20.9725	22.4010	23.8036	25.1839

Answers to Supplementary Problems

CHAPTER 1

1.27. $u(0,t) = T_1, \quad u(L,t) = T_2, \quad u(x,0) = f(x), \quad |u(x,t)| < M$

1.28. (a) $u_x(0,t) = \left.\frac{\partial u}{\partial x}\right|_{x=0} = 0, \quad u_x(L,t) = \left.\frac{\partial u}{\partial x}\right|_{x=L} = 0, \quad u(x,0) = f(x), \quad |u(x,t)| < M$

(b) $u_x(0,t) = \left.\frac{\partial u}{\partial x}\right|_{x=0} = B(u_1 - u_0), \quad u_x(L,t) = \left.\frac{\partial u}{\partial x}\right|_{x=L} = B(u_2 - u_0),$

$u(x,0) = f(x), \quad |u(x,t)| < M$

where $u_1 = u(0,t), \ u_2 = u(L,t)$

1.31. $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}, \quad y(x,0) = \begin{cases} 2hx/L & 0 \leqq x \leqq L/2 \\ 2h(L-x)/L & L/2 \leqq x \leqq L \end{cases}$

$y(0,t) = 0, \quad y(L,t) = 0, \quad y_t(x,0) = 0, \quad |y(x,t)| < M$

1.32. (a) linear, dep. var. u, ind. var. x, y, order 2

(b) linear, dep. var. T, ind. var. x, y, z, order 4

(c) nonlinear, dep. var. ϕ, ind. var. x, y, order 3

(d) linear, dep. var. y, ind. var. x, t, order 2

(e) nonlinear, dep. var. z, ind. var. r, s, order 1

1.33. (a) hyperbolic (b) hyperbolic (c) elliptic (d) parabolic

(e) elliptic if $x^2 + y^2 < 1$, hyperbolic if $x^2 + y^2 > 1$, parabolic if $x^2 + y^2 = 1$

(f) elliptic if $M < 1$, hyperbolic if $M > 1$, parabolic if $M = 1$

1.35. (b) $x(2x + y - 2)^2$

1.36. $3\frac{\partial^2 u}{\partial x^2} - 5\frac{\partial^2 u}{\partial x\,\partial y} - 2\frac{\partial^2 u}{\partial y^2} = 0$

1.37. (a) $2\frac{\partial z}{\partial x} + 3\frac{\partial z}{\partial y} = 2z$ (b) $2\frac{\partial^2 z}{\partial x^2} - 3\frac{\partial^2 z}{\partial x\,\partial y} - 2\frac{\partial^2 z}{\partial y^2} = 0$

1.38. (a) $xz = F(x) + G(y)$ (b) $xz = x^6 + x^2 + 6y^4 - 68$

1.39. (a) $u = F(x+y) + G(x-y)$

(b) $u = e^{2x}F(y - 2x)$

(c) $u = F(x+iy) + G(x-iy)$

(d) $z = F(3x+y) + G(y-x)$

(e) $z = F(x+y) + xG(x+y)$

1.40. (a) $u = F(y - 2x) + \frac{x^2}{2}$

(b) $y = F(x-t) + G(x+t) - t^4$

(c) $u = F(y) + xG(y) + x^2H(y) + I(y - 2x) + \frac{x^4}{6}$

(d) $z = F(x+y) + G(2x+y) - \frac{x}{2}\sin y + \frac{3}{4}\cos y$

1.41. $u = F(x+iy) + G(x-iy) + xH(x+iy) + xJ(x-iy) + (x^2+y^2)^2/4$

1.43. (a) $u = 4e^{(3y-2x)/2}$

(b) $u = 3e^{-5x-3y} + 2e^{-3x-2y}$

(c) $u = 2e^{-36t}\sin 3x - 4e^{-100t}\sin 5x$

(d) $u = 8e^{-9\pi^2 t/16}\cos\frac{3\pi x}{4} - 6e^{-81\pi^2 t/16}\cos\frac{9\pi x}{4}$

(e) $u = 8e^{-2x-6t}$

(f) $u = 10e^{-x-3t} - 6e^{-4x-6t}$

(g) $u = 6e^{-\pi^2 t/4}\sin(\pi x/2) + 3e^{-\pi^2 t}\sin \pi x$

1.44. (a) $y = \frac{5}{2\pi}\sin \pi x \sin 2\pi t$ (b) $y = \frac{3}{4\pi}\sin 2\pi x \sin 4\pi t - \frac{1}{5\pi}\sin 5\pi x \sin 10\pi t$

1.45. $u = e^{-2t}(2e^{-\pi^2 t}\sin \pi x - e^{-16\pi^2 t}\sin 4\pi x)$

CHAPTER 2

2.34. (a) $\frac{16}{\pi}\sum_{n=1}^{\infty}\frac{(1-\cos n\pi)}{n}\sin\frac{n\pi x}{2}$ (c) $20-\frac{40}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{n\pi x}{5}$

(b) $2-\frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{(1-\cos n\pi)}{n^2}\cos\frac{n\pi x}{4}$ (d) $\frac{3}{2}+\sum_{n=1}^{\infty}\left\{\frac{6(\cos n\pi-1)}{n^2\pi^2}\cos\frac{n\pi x}{3}-\frac{6\cos n\pi}{n\pi}\sin\frac{n\pi x}{3}\right\}$

2.35. (a) $x = 0, \pm 2, \pm 4, \ldots; 0$ (c) $x = 0, \pm 10, \pm 20, \ldots; 20$

(b) no discontinuities (d) $x = \pm 3, \pm 9, \pm 15, \ldots; 3$

2.36. $\frac{16}{\pi}\left\{\cos\frac{\pi x}{4}+\frac{1}{3^2}\cos\frac{3\pi x}{4}+\frac{1}{5^2}\cos\frac{5\pi x}{4}+\cdots\right\}$

2.37. (a) $\frac{8}{\pi}\sum_{n=1}^{\infty}\frac{n\sin 2nx}{4n^2-1}$ (b) $f(0) = f(\pi) = 0$

2.38. Same answer as 2.37.

2.39. (a) $\frac{32}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin\frac{n\pi}{2}\sin\frac{n\pi x}{8}$ (b) $2+\frac{16}{\pi^2}\sum_{n=1}^{\infty}\left(\frac{2\cos n\pi/2-\cos n\pi-1}{n^2}\right)\cos\frac{n\pi x}{8}$

2.42. $$u(x,t) = \psi(x)+\frac{2}{L}\sum_{n=1}^{\infty}\left\{\int_0^L[f(u)-\psi(u)]\sin\frac{n\pi u}{L}\,du\right\}e^{-\kappa n^2\pi^2 t/L^2}\sin\frac{n\pi x}{L}$$

where $$\psi(x) = \frac{\beta(1-e^{-\gamma L})}{\kappa L}-\frac{\beta}{\kappa}(1-e^{-\gamma x})$$

2.50. (a) $u(x,t) = -\frac{200}{\pi}\sum_{m=1}^{\infty}\frac{e^{-m^2\pi^2 t/8}\cos m\pi}{m}\sin\frac{m\pi x}{4}$

2.52. $150-5x$

2.53. $u(\rho,\phi) = 120+60\rho^2\cos 2\phi$

2.55. $y(x,t) = \frac{.96}{\pi^3}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}\sin\frac{(2n-1)\pi x}{2}\cos\frac{(2n-1)\pi ct}{2}$

2.57. $$\begin{aligned} u(x,y) = &\sum_{k=1}^{\infty}\left[\frac{2}{a\sinh(k\pi)}\int_0^a f_1(x)\sin\frac{k\pi x}{a}\,dx\right]\sin\frac{k\pi x}{a}\sinh\frac{k\pi y}{a}\\ &+\sum_{l=1}^{\infty}\left[\frac{2}{a\sinh(l\pi)}\int_0^a f_2(x)\sin\frac{l\pi x}{a}\,dx\right]\sin\frac{l\pi x}{a}\sinh\frac{l\pi}{a}(a-y)\\ &+\sum_{m=0}^{\infty}\left[\frac{2}{a\sinh(m\pi)}\int_0^a g_1(y)\sin\frac{m\pi y}{a}\,dy\right]\sin\frac{m\pi y}{a}\sinh\frac{m\pi x}{a}\\ &+\sum_{n=0}^{\infty}\left[\frac{2}{a\sinh(n\pi)}\int_0^a g_2(y)\sin\frac{n\pi y}{a}\,dy\right]\sin\frac{n\pi y}{a}\sinh\frac{n\pi x}{a}\end{aligned}$$

2.59. $$y(x,t) = \sum_{k=1}^{\infty}\left[\frac{2}{k\pi a}\int_0^L g(u)\sin\frac{k\pi u}{L}\sin\frac{k\pi at}{L}\,du+\frac{2}{L}\int_0^L f(u)\sin\frac{k\pi u}{L}\cos\frac{k\pi at}{L}\,du\right]\sin\frac{k\pi x}{L}$$

or $$\frac{1}{2}[f(x+at)+f(x-at)]+\frac{1}{2a}\int_{x-at}^{x+at}g(u)\,du$$

2.61. $$z(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left[A_{mn}\cos\lambda_{mn}at + B_{mn}\sin\lambda_{mn}at\right]\sin\frac{m\pi x}{b}\sin\frac{n\pi y}{c}$$

where $$A_{mn} = \frac{4}{bc}\int_0^b\int_0^c f(x,y)\sin\frac{m\pi x}{b}\sin\frac{n\pi y}{c}\,dx\,dy,$$

$$B_{mn} = \frac{4}{abc\lambda_{mn}}\int_0^b\int_0^c g(x,y)\sin\frac{m\pi x}{b}\sin\frac{n\pi y}{c}\,dx\,dy,$$

and $$\lambda_{mn} = \pi\sqrt{\frac{m^2}{b^2}+\frac{n^2}{c^2}}$$

2.63. $$u(x,t) = \psi(x) - \frac{2}{L}\sum_{n=1}^{\infty} e^{-(\alpha^2+n^2\pi^2/L^2)t}\left[\int_0^L \psi(u)\sin\frac{n\pi u}{L}\,du\right]\sin\frac{n\pi x}{L}$$

where $$\psi(x) = \frac{(u_1e^{\alpha L}-u_2)e^{-\alpha x}-(u_1e^{-\alpha L}-u_2)e^{\alpha x}}{e^{\alpha L}-e^{-\alpha L}}$$

2.64. $$u(x,t) = \psi(x) + \frac{2}{L}\sum_{n=1}^{\infty} e^{-(\alpha^2+n^2\pi^2/L^2)t}\left[\int_0^L (f(u)-\psi(u))\sin\frac{n\pi u}{L}\,du\right]\sin\frac{n\pi x}{L}$$

where $$\psi(x) = \frac{(u_1e^{\alpha L}-u_2)e^{-\alpha x}-(u_1e^{-\alpha L}-u_2)e^{\alpha x}}{e^{\alpha L}-e^{-\alpha L}}$$

2.65. $$y(x,t) = \frac{2}{L}\sum_{n=1}^{\infty}\left[\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx\right]\sin\frac{n\pi x}{L}\cos\frac{n^2\pi^2bt}{L^2}$$

2.66. $$y(x,t) = \frac{2}{L}\sum_{n=1}^{\infty}\left[\left\{\int_0^L f(u)\sin\frac{n\pi u}{L}\,du\right\}\sin\frac{n\pi x}{L}\cos\frac{n^2\pi^2bt}{L^2} + \left\{\frac{L^2}{n^2\pi^2b}\int_0^L g(u)\sin\frac{n\pi u}{L}\,du\right\}\sin\frac{n^2\pi^2bt}{L^2}\sin\frac{n\pi x}{L}\right]$$

2.69. $$u(x,t) = \psi(x) + \frac{2}{L}\sum_{n=1}^{\infty}\left\{\int_0^L [f(u)-\psi(u)]\sin\frac{n\pi u}{L}\,du\right\}e^{-(\kappa n^2\pi^2/L^2)t}\sin\frac{n\pi x}{L}$$

where $$\psi(x) = \frac{B(1-e^{-\gamma L})x}{\gamma^2\kappa L} + \frac{B}{\kappa\gamma^2}(1-e^{-\gamma x})$$

2.70. Same as 2.69 but with $\psi(x) = \dfrac{u_0}{\kappa\alpha^2}\left(\sin\alpha x - \dfrac{x}{L}\sin\alpha L\right)$

2.71. $$y(x,t) = \psi(x) + \frac{2}{L}\sum_{n=1}^{\infty}\left\{\int_0^L (f(u)-\psi(u))\sin\frac{n\pi u}{L}\,du\right\}\sin\frac{n\pi x}{L}\cos\frac{n\pi at}{L} \quad \text{where} \quad \psi(x) = \frac{gx}{2a^2}(x-L)$$

2.72. $$u(x,y,z) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mn}\sin m\pi x\sin n\pi y\sinh\sqrt{m^2+n^2}\,\pi z$$

where $$B_{mn} = \frac{4}{\sinh\sqrt{m^2+n^2}\,\pi}\int_0^1\int_0^1 f(x,y)\sin m\pi x\sin n\pi y\,dx\,dy$$

2.76. $$u(r,\theta) = \frac{2}{\beta}\sum_{n=1}^{\infty}\left(\frac{r}{a}\right)^{n\pi/\beta}\left\{\int_0^\beta f(\phi)\sin\frac{n\pi\phi}{\beta}\,d\phi\right\}\sin\frac{n\pi\theta}{\beta}$$

CHAPTER 3

3.15. (a) $a_0 = 1,\ a_1 = -\sqrt{3},\ a_2 = 2\sqrt{3},\ a_3 = \sqrt{5},\ a_4 = -6\sqrt{5},\ a_5 = 6\sqrt{5}$

(b) $1,\ \sqrt{3}\,(2x-1),\ \sqrt{5}\,(6x^2-6x+1)$

3.17. (b) $1,\ 1-x,\ 1-2x+\frac{1}{2}x^2$

3.19. (b) $\sqrt{\frac{1}{\pi}}\cos(0\cos^{-1}x) = \sqrt{\frac{1}{\pi}},\quad \sqrt{\frac{2}{\pi}}\cos(n\cos^{-1}x) \quad (n = 1, 2, \ldots)$

3.20. $f(x) = \sum_{n=1}^{\infty} c_n\phi_n(x) \quad \text{where} \quad c_n = \int_a^b w(x)\, f(x)\, \phi_n(x)\, dx$

3.21. $f(x) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n\phi_n(x)$

where $c_n = \sqrt{\frac{2}{\pi}}\int_{-1}^{1}(1-x^2)^{-1/2}f(x)\cos(n\cos^{-1}x)\,dx = \sqrt{\frac{2}{\pi}}\int_{-\pi/2}^{\pi/2} f(\cos u)\cos nu\,du$

3.24. (a) $2, -1, \frac{2}{3}$ (b) $\sqrt{(18\pi^2-49)/18\pi}$

3.30. $\sqrt{2},\ \sqrt{\frac{2}{3}}x,\ \sqrt{\frac{2}{5}}\left(\frac{3x^2-1}{2}\right)$, i.e. the normalized Legendre polynomials

3.31. $1-x,\ \frac{1}{2}(2-4x+x^2),\ \frac{1}{6}(6-18x+9x^2-x^3)$

3.33. (b) eigenvalues $(m-\frac{1}{2})^2\pi^2$, eigenfunctions $A_m\cos(m-\frac{1}{2})\pi x$, where $m = 1, 2, \ldots$

(c) $\sqrt{2}\cos(m-\frac{1}{2})\pi x,\ m = 1, 2, \ldots$

3.34. (a) $\frac{(2m-1)^2\pi^2}{4},\ B_m\sin\frac{(2m-1)\pi x}{2},\ \sqrt{2}\sin\frac{(2m-1)\pi x}{2},\ m = 1, 2, \ldots$

(b) $m^2\pi^2,\ A_m\cos m\pi x,\ \sqrt{2}\cos m\pi x,\ m = 1, 2, \ldots$

3.40. $u(x,t) = \frac{2}{L}\sum_{n=1}^{\infty}\left\{\int_0^L f(u)\cos\frac{(2n-1)\pi u}{2L}\,du\right\}\cos\frac{(2n-1)\pi x}{2L}\,e^{-\kappa(2n-1)^2\pi^2 t/4L^2}$

(b) Heat conduction in an infinite strip of width L, with one side at 0°, the other side insulated, and initial temperature distribution given by $f(x)$.

3.41. (a) $y(x,t) = \frac{2}{L}\sum_{n=1}^{\infty}\left[\int_0^L f(u)\sin\frac{(2n-1)}{2L}\pi u\,du\right]\sin\frac{(2n-1)\pi x}{2L}\cos\frac{(2n-1)\pi at}{2L}$

(b) Vibrating string with end $x = 0$ fixed, end $x = L$ free, initial shape $f(x)$, initial speed zero

CHAPTER 4

4.26. (a) 30, (b) 16/105, (c) $\frac{8}{3}\pi^{3/2}$

4.27. (a) 24, (b) $\frac{80}{243}$, (c) $\frac{\sqrt{2\pi}}{16}$

4.28. (a) $\frac{1}{5}\Gamma(\frac{1}{5})$, (b) $\frac{3\sqrt{\pi}}{2}$, (c) $\frac{\Gamma(4/5)}{5\sqrt[3]{16}}$

4.31. (a) 24, (b) −3/128, (c) $\frac{1}{3}\Gamma(\frac{1}{3})$

4.32. (a) $(16\sqrt{\pi})/105$, (b) $-3\,\Gamma(2/3)$

4.37. (a) 1/105, (b) 4/15, (c) $2\pi/\sqrt{3}$

4.38. (a) 1/60, (b) $\pi/2$, (c) 3π

4.39. (a) 12π, (b) π

4.41. (a) $3\pi/256$, (b) $5\pi/8$

4.42. (a) 16/15, (b) 8/105

CHAPTER 5

5.24. (a) $\frac{\sin \alpha\epsilon}{\alpha\epsilon}$ (b) 1

5.25. (a) $\frac{4}{\alpha^3}(\alpha \cos \alpha - \sin \alpha)$ (b) $\frac{3\pi}{16}$

5.26. (a) $\frac{1 - \cos \alpha}{\alpha}$ (b) $\frac{\sin \alpha}{\alpha}$

5.27. (a) $\frac{\alpha}{1 + \alpha^2}$

5.28. $y(x) = (2 + 2\cos x - 4\cos 2x)/\pi x$

5.30. (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{4}$

5.37. $y(u) = \left(\frac{4}{\pi}\right)^{1/4} e^{-2u^2}$

5.44. $u(x, y) = \frac{2}{\pi}\tan^{-1}\frac{x}{y}$

5.45. $u(x, y) = \frac{u_0}{2} + \frac{u_0}{\pi}\tan^{-1}\frac{x}{y}$

5.46. $u(x, y) = \frac{1}{\pi}\left[\tan^{-1}\left(\frac{1 + x}{y}\right) + \tan^{-1}\left(\frac{1 - x}{y}\right)\right]$

CHAPTER 6

6.37. (a) $\sqrt{\frac{2}{\pi x}}\left[\frac{(3 - x^2)\sin x - 3x\cos x}{x^2}\right]$

(b) $\sqrt{\frac{2}{\pi x}}\left[\frac{3x \sin x + (3 - x^2)\cos x}{x^2}\right]$

6.38. $\left(\frac{8 - x^2}{x^2}\right) J_1(x) - \frac{4}{x} J_0(x)$

6.40. (a) $x^3 J_3(x) + c$ (b) $2J_0(1) - 3J_1(1)$ (c) $x^2 J_1(x) + xJ_0(x) - \int J_0(x)\,dx$

6.41. (a) $6\sqrt[3]{x}\, J_1(\sqrt[3]{x}) - 3\sqrt[3]{x^2}\, J_0(\sqrt[3]{x}) + c$ (b) $-\frac{J_2(x)}{3x} - \frac{J_1(x)}{3} + \frac{1}{3}\int J_0(x)\,dx$

6.42. $xJ_0(x)\sin x - xJ_1(x)\cos x + c$

6.57. (a) $-\sqrt{\frac{2}{\pi x}}\cos x$ (b) $\sqrt{\frac{2}{\pi x}}\sin x$ (c) $-\sqrt{\frac{2}{\pi x}}\left(\sin x + \frac{\cos x}{x}\right)$ (d) $\sqrt{\frac{2}{\pi x}}\left(\frac{\sin x}{x} - \cos x\right)$

6.59. (a) $x^3 Y_3(x) + c$ (b) $-Y_2(x) - 2Y_1(x)/x + c$

(c) $-\frac{1}{15}Y_1(x) - \frac{1}{15x}Y_2(x) - \frac{1}{5x^2}Y_3(x) + \frac{1}{15}\int Y_0(x)\,dx$

6.68. (a) $\sqrt{\frac{2}{\pi x}}\, e^{i(x - \pi/4 - n\pi/2)}$ (b) $\sqrt{\frac{2}{\pi x}}\, e^{-i(x - \pi/4 - n\pi/2)}$

6.72. $y = AJ_0(\sqrt{x}) + BY_0(\sqrt{x})$

6.73. (a) $y = \frac{A\sin x + B\cos x}{x}$ (b) $y = \sqrt{x}\,[AJ_{1/4}(\frac{1}{2}x^2) + BJ_{-1/4}(\frac{1}{2}x^2)]$

6.74. $y = AJ_0(e^x) + BY_0(e^x)$

6.75. (b) $y = AJ_0(2\sqrt{x}) + BY_0(2\sqrt{x})$

6.76. (b) $y = A\sqrt{x}\,J_{1/3}(\frac{2}{3}x^{3/2}) + B\sqrt{x}\,J_{-1/3}(\frac{2}{3}x^{3/2})$

6.78. $y = AxJ_1(x) + BxY_1(x)$

6.95. $u(\rho, \phi, t) = \sum_{n=1}^{\infty} A_n J_3(\lambda_n \rho) \cos 3\phi \cos \lambda_n t$ where λ_n are the positive roots of $J_3(\lambda) = 0$ and

$$A_n = \frac{2[(\lambda_n^2 - 8)J_0(\lambda_n) - 6\lambda_n J_1(\lambda_n) + 8]}{\lambda_n^3 J_4^2(\lambda_n)}$$

6.96. $y(x, t) = \sum_{n=1}^{\infty} \frac{J_0(\lambda_n \sqrt{x})}{J_1^2(\lambda_n)} \cos(\frac{1}{2}\lambda_n t) \int_0^1 f(x) J_0(\lambda_n \sqrt{x})\, dx$ where $J_0(\lambda_n) = 0$, $n = 1, 2, \ldots$

6.97. (a) $u(\rho, \phi, z) = \frac{2}{a^2 \pi} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \frac{J_n(\lambda_k \rho) \sinh \lambda_k (l - z)}{J_{n+1}^2(\lambda_k a) \sinh \lambda_k l} (A_{n,k} \sin n\phi + B_{n,k} \cos n\phi)$

where
$$A_{n,k} = \int_0^a \int_0^{2\pi} \rho f(\rho, \phi) J_n(\lambda_k \rho) \sin n\phi \, d\rho \, d\phi$$
$$B_{n,k} = \int_0^a \int_0^{2\pi} \rho f(\rho, \phi) J_n(\lambda_k \rho) \cos n\phi \, d\rho \, d\phi$$

and $J_n(\lambda_k a) = 0$

(b) $u(\rho, \phi, z) = \frac{2}{a^2} \sum_{k=0}^{\infty} \frac{J_1(\lambda_k \rho) \sinh \lambda_k (l - z)}{J_2^2(\lambda_k a) \sinh \lambda_k l} B_k \cos \phi$

where
$$B_k = \frac{1}{\lambda_k^3} \left[(3a - \lambda_k^2 a^3) J_0(\lambda_k a) - 3 \int_0^a J_0(\lambda_k \rho)\, d\rho \right]$$

and $J_1(\lambda_k a) = 0$

6.112. $u(\rho, z, t) = \sum_{k=1}^{\infty} \sum_{m=1}^{\infty} c_{km} e^{-\kappa(r_m^2 + k^2\pi^2)t} J_0(r_m \rho) \sin k\pi z$

where
$$c_{km} = \frac{4}{J_0^2(r_m)} \int_0^1 \int_0^1 \rho J_0(r_m \rho) f(\rho, z) \sin k\pi z \, d\rho \, dz$$

and $J_1(r_m) = 0$

6.117. $z(\rho, \phi, t) = \sum_{m=0}^{\infty} \frac{1}{2} A_{m0} u_0(\lambda_{m0} \rho) \cos \lambda_{m0} c t$

$$+ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (A_{mn} \cos n\phi + B_{mn} \sin n\phi) u_n(\lambda_{mn} \rho) \cos \lambda_{mn} c t$$

where $u_n(\lambda_{mn} \rho) = J_n(\lambda_{mn} \rho) Y_n(\lambda_{mn} a) - J_n(\lambda_{mn} a) Y_n(\lambda_{mn} \rho)$,

$$A_{mn} = \frac{1}{\pi L} \int_0^{2\pi} \int_a^b \rho f(\rho, \phi) u_n(\lambda_{mn} \rho) \cos n\phi \, d\rho \, d\phi,$$
$$B_{mn} = \frac{1}{\pi L} \int_0^{2\pi} \int_a^b \rho f(\rho, \phi) u_n(\lambda_{mn} \rho) \sin n\phi \, d\rho \, d\phi,$$
$$L = \int_a^b \rho [u_n(\lambda_{mn} \rho)]^2 \, d\rho$$

and $c = \sqrt{\tau/\mu}$

CHAPTER 7

7.32. $P_4(x) = \frac{1}{8}(3 - 30x^2 + 35x^4)$

$P_5(x) = \frac{1}{8}(15x - 70x^3 + 63x^5)$

7.33. (a) 0 (b) 2/5 (c) 0

7.43. $Q_3(x) = \frac{x}{4}(5x^2 - 3)\ln\left(\frac{1+x}{1-x}\right) - \frac{5x^2}{2} + \frac{2}{3}$

7.44. $y = Ax + B\left[1 + \frac{x}{2}\ln\left(\frac{1-x}{1+x}\right)\right]$

7.45. $-\frac{4}{5}P_0(x) + P_1(x) - \frac{10}{7}P_2(x) + \frac{8}{35}P_4(x)$

7.46. $P_0(x) + \frac{7}{4}P_1(x) + \frac{5}{8}P_2(x) - \frac{7}{16}P_3(x) + \cdots$

7.48. (a) $v = v_0 + 3v_0 r\cos\theta$

(b) $v = \frac{v_0}{r} + \frac{3v_0}{r^2}\cos\theta$

7.49. (a) $v = \frac{2v_0}{3}[1 - r^2 P_2(\cos\theta)]$

(b) $v = \frac{2v_0}{3}\left[\frac{1}{r} - \frac{P_2(\cos\theta)}{r^3}\right]$

7.50. $2u_0\left(1 - \frac{a}{r}\right)$

7.51. m/r where $r > a$ is the distance from the center of the sphere

7.56. (a) $3(1 - x^2)$

(b) $-\frac{5}{2}(3 - 24x^2 - 21x^4)$

(c) $105x(1 - x^2)^{3/2}$

7.57. (a) $-\sqrt{1-x^2}\left[\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) + \frac{x}{1-x^2}\right]$

(b) $\frac{2}{1-x^2}$

7.63. Inside, $v = v_0 r^4 \sin^3\theta\cos\theta\cos 3\phi$

Outside, $v = \frac{v_0}{r^5}\sin^3\theta\cos\theta\cos 3\phi$

7.71. $u = \frac{bu_2 - au_1}{b-a} + \frac{ab(u_1 - u_2)}{(b-a)r}$

7.73. $u = [A_1 J_{n+1/2}(\lambda r) + B_1 J_{-n-1/2}(\lambda r)] \cdot [A_2 P_n(\cos\theta) + B_2 Q_n(\cos\theta)]e^{-\kappa\lambda^2 t}$

CHAPTER 8

8.17. $1,\ 2x,\ 4x^2 - 2,\ 8x^3 - 12x$

8.20. $4x^2 - 2,\ 8x^3 - 12x$

8.22. $\frac{1}{2}\sqrt{\pi}$ if $n = 0$, $2\sqrt{\pi}$ if $n = 2$,
0 otherwise

8.26. (a) $-\frac{5}{2}H_0(x) + \frac{7}{4}H_1(x) - \frac{3}{4}H_2(x) + \frac{1}{8}H_3(x)$

8.27. (a) $y = c_1 + c_2\int e^{x^2}dx$

(b) $y = c_1 x + c_2 x\int \frac{e^{x^2}}{x^2}dx$

8.28. $L_4(x) = 24 - 96x + 72x^2 - 16x^3 + x^4$

8.30. $L_2(x) = 2 - 4x + x^2$

$L_3(x) = 6 - 18x + 9x^2 - x^3$

8.34. $2L_0(x) - 8L_1(x) + 6L_2(x) - L_3(x)$

8.36. (a) $y = c_1 + c_2\int \frac{e^x}{x}dx$

(b) $y = c_1(1-x) + c_2(1-x)\int \frac{e^x}{x(1-x)^2}dx$

8.38. $L_4^2(x) = 144 - 96x + 12x^2$

$L_3^3(x) = -1296 + 600x - 60x^2$

8.41. 180

8.43. $y = A\sin^{-1}x + B$

8.44. (a) $T_4(x) = 8x^4 - 8x^2 + 1$

(b) $T_5(x) = 16x^5 - 20x^3 + 2x$

8.45. $\frac{1}{4}[T_3(x) + 2T_2(x) - 13T_1(x) + 10T_0(x)]$

8.49. (a) $y = AH_n(x) + BH_n(x) \int \frac{e^{x^2}\,dx}{[H_n(x)]^2}$

8.51. (a) $y = AL_n(x) + BL_n(x) \int \frac{e^x\,dx}{x[L_n(x)]^2}$

8.55. (a) $y = AT_n(x) + BT_n(x) \int \frac{dx}{\sqrt{1-x^2}\,[T_n(x)]^2}$

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