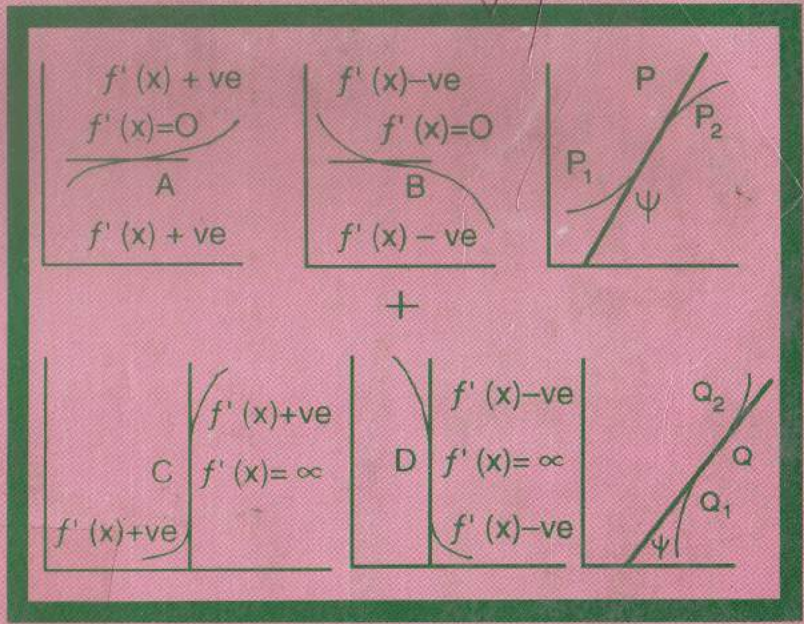
বি. এ ও বি. এস-সি শ্রেণী (পাস ও সম্মান) পর্য্যায়ের অন্যান্য বই-

A TEXT BOOK ON DIFFERENTIAL CALCULUS

(PASS & HONOURS)



MOHAMMAD, BHATTACHARJEE & LATIF

Paper I (প্রথম পত্র)

(Marks 50)

ANALYTIC AND VECTOR GEOMETRY

See Rahman and Bhattacharjee's. A Text Book on Coordinate Geometry and vector Analysis.

A-I **Two-dimensional Geometry :** (দ্বিমাত্রিক স্থানাঙ্ক জ্যামিতি)

Transformation of Coordinates etc : Chapter IV. Pairs of Straightlines (যুগল সরলরেখা অধ্যায়-৫ (chapter-V). (Homogeneous 2nd degree equation Art 36.General 2nd degree equation Art 40. Angle between pairs of St. lines Art 38. Art 41. Bisectors of angles between pairs of St. lines Art 39. Art 41 (d).

General Eq. of 2nd degree Chapter VI (অধ্যায়-৬)

(Reduction to standard forms Art 47. Art 48. At 49. Art 51 Art 51 (a).

A-2 **Three dimensional Geometry chapter I,** অধ্যায়-১ (ত্রিমাত্রিক জ্যামিতি) Coordinates in three dimensions (Art 1. Art 2) Distance (Art 3) Direction Cosines (দিক কোসাইন) (Art 8; Art 9. Art 10. Art 11). Planes (সমতল) ঃ Equation of plane Art 18. Art 19. Angle between two planes (Art 22). Distance of a point from a plane (Art 25 (a)

Straight lines : chapter III (অধ্যায়-৩)

Equation of a line (Art 28. 29. 30(a) 32(a)

Relationship between planes and lines

Art 33. Art 34. Art 35

Shortest distance Art 38

Sphere (গোলক) ঃ Chapter IV অধ্যায়-৪ Art 41. 42. 43

Tangent to the sphere Art 46, Art 47

A-3 **Vector Analysis :**

অধ্যায়-১ অধ্যায়-২, Art 18. অধ্যায়-৩

B CALCULUS- I (Marks 50)

A Text Book on Differential calculus By Mohammed, Bhattacharjee and Latif.

অধ্যায়-১, অধ্যায়-২ অধ্যায়-৩ (বাদ Uniform continuity) বিবিধ প্রশ্নমালা, অধ্যায়-iv অধ্যায়-IV(a) IV(b) অধ্যায়-(৫) Art 5.5, Art 5.6, Art 5.7, Art 5.8 etc.

অধ্যায়-৭ Art 7.2 Art 7.3 Art 7.4 Art 7.5 এবং এদের নিয়ে অংক অধ্যায়-X (A) Art 102, Art 103 অধ্যায়-X (B) Art 108. অধ্যায়-XI Art 11.1 Art 11.2 Art 11.3 Art 113(a), Art 11.5

Integral Calculus: A Text Book on Intgral Calculus By Mohammed, Bhattacharjee and Latif.

অধ্যায়-১ থেকে অধ্যায়-৬ Definite Integrals অধ্যায়-৭ Art 22 অধ্যায়-৭(B) অধ্যায়-VII(c)

Paper II (দ্বিতীয় পত্র)

A **BASIC ALGEBRA :** (Marks 50)

See Shahidullah and Bhattacharjee's A Text book on Higher Algebra, Modern Algebra, Theory of Numbers and Trigonometry.

Elements of Logic (মৌলিক যুক্তিবিদ্যা) (Mathematical statements etc ....Deductic reasoning) see Appendix (উল্লিখিত বই এর প্রথমে) Basic Algebra এর জন্য।

Elements of set Theory : উল্লিখিত বই এর মডার্ণ এলজেবরা (Sets and Subsets, Relation-Orders Equivalence; Functions etc. অধ্যায় পাঁচ পর্যন্ত)

Real Number System : Field and order properties Natural Numbers, Absolute value (See First Chapter of **A Test Book on Differential Calculus by Mohammed, Bhattacharjee and Latif**)

Inequalities : Basic Inequalities :

Weier strass's; Tcheby chef's Cauchy's; A. M. (Arithmetic Mean) and Geometric Means উল্লিখিত এলজেবরা বইএর অসমতা (Inequalities অধ্যায় এক, Art 2(i), 2(ii), 2(iii) Art 3 Art 4. Art 5, Art 8, Art 9. Holder's Inequality Art 14. এর অঙ্কগুলি)

Complex number: System, field of complex numbers De moiver's theorem and its application- See Algebra বই এর **Trigonometry.** (ত্রিকোণমিতি) অধ্যায় এক) অধ্যায় দুই, অধ্যায় তিন।

Elementary Number Theory : Divisibility, Fundamental Theorem Arithmetic, Congruence. See Theory of Number of Higher Algebra-এর সংখ্যা তত্ত্ব (Theory of Numbers) অধ্যায়।

Summation of finite Series : Arithmetico-Geometric Series, Method of Difference, Successive differences. Use of mathematical induction : **See Higher Algebra** (অধ্যায়-৩ III (A), III(B), III(C):

Theory of Equations : Synthetic division, Number of roots of Polynomial, equations. Relation between roots and coefficients. Multiplicitiy of roots, Symmetric functions of roots. Transformation of equations. **See Higher Algebra** অধ্যায়-৪ (ch. IV) Art 50 থেকে প্রশ্নমালা IV পর্যন্ত।

B **Linear Algebra :**

Matrix, Determinant. থেকে Applications of matrices and determinants in solving system of linear equtions—**See Higher Algebra by Shahidulla & Bhattacharjee :**

অধ্যায় নির্ণয়াক (Determinants) অধ্যায়-৬ ঃ মেট্রিক্স (Matrix)

General vector space থেকে Cayley-Hameltion's Theorem. Application-এর জন্য যে কোন একটি Linear algebra বই যথা Schaums Serice এর বই আলোচনা করা যেতে পারে। দেশী বইও দেখা যেতে পারে, তবে কেলী-হেমিষ্টন উপপাদ্য ও ব্যবহার **See Shahidullah and Bhattacharjee's Higher Algebra** ঃ তে Matrix অধ্যায়ের Art 115.11 থেকে Art 115.14 পর্যন্ত পাওয়া যাবে।

প্রকাশকের কথা

জাতীয় বিশ্ববিদ্যালয়ের পাস ও সম্মান শ্রেণীর উচ্চতর গণিতের ১ম পত্র, ২য় পত্র ও ৩য় পত্রের বিষয়াবলী আমাদের প্রকাশিত বইগুলোতে যেভাবে ব্যবহৃত হয়েছে তার একটি নির্দেশিকা দেওয়া হল। বর্তমানে জাতীয় বিশ্ববিদ্যালয়ের পাঠ্যসূচি অনুযায়ী ১ম পত্রের জন্য যেমন, জ্যামিতি ও ভেক্টর এবং ক্যালকুলাসের কিছু অংশ আছে। অবশিষ্ট অংশ ২য় ও ৩য় পত্রে আছে। ছাত্র//ছাত্রীদের বর্তমান পাঠ্যসূচি বুঝার সহজ উপায় নির্দ্ধারণের জন্য এই নির্দেশিকা দেয়া হল।

আশা করি এতে ছাত্র/ছাত্রীদের বিশেষ সুবিধা হলে আমাদের শ্রমি সার্থক হবে।

A TEXT BOOK ON DIFFERENTIAL CALCULUS

(Pass And Honours)

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PREFACE TO 10TH EDITION

In this edition Set Algebra has been used in defining Functions, Relations and related problems. A list of formulae of Differential Calculus and set Algebra is placed at the beginning of the subject matters. Almost all the Chapters are thoroughly revised, altered and adjusted. New sums both worked out and in exercise have been added. Due to readjustment, addition and alteration there may be some errors and maladjustments which we desire to correct in the next edition of the book.

Suggestions for the improvement of the book and intimation of errors will be cordially recieeved.

May, 2001 **AUTHORS**

PUBLISHER'S NOTE

The Present edition of "A Text book on Differential Calculus" is published after thorough revesion of the whole book with a veiw to enricing the contents with modern ideas at home and abroad.

Md. Abdul Latif Dept. of Mathematics, Rajshahi University, Rajshahi, has extended his whole hearted active cooperation to us in this respect.

With a mark of apperciation to Mr. Abdul Latif, he is included as the third author of the book.

1992 **Publisher**

PREFACE TO THE 1ST EDITION

The book is intended to serve as a companion book to our "Integral Calculus" and is designed to meet the requirements of the Pass and Honours Students of our Universities.

The book consists of sixteen chapters covering entire syllabus of Universities. Attempts have been made to elaborate each article with a good numbers of examples. The pass students may drop harder sums and the sums marked with asterisks. Honours students should read the book thoroughly. A good number of examples are given in the book and for their collection foreign books, University question papers of various examinations have been consulted.

We are much thankful to Prof. A. B. Mitra and Prof. H. N. Datta and many others for their reviewing of some chapters of the book.

Suggestions for the improvement of the book, intimation of errors and misprints will be thankfully recieved.

1968 **AUTHORS**

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APPENDIX

Uniform continuity

University Questions

1. **Convergency and Divergency see Higher Algebra**
   BY Shahidullah & Bhattacharjee

   Convergency and Divergency of Imporper Integrals. see Integral Calculus
   BY Mohammad and Bhattacharjee

2. **For Vector Analysis see Co-ordinate Geometry and Vector Analysis.**
   By Rahman & Bhattacharjee

APPENDIX

UNIFORM CONTINUITY

Definition : A function $f(x)$ is said to be uniformly continuous in $[a,b]$ if and only if (iff) for a given arbitrary small positive number ε, there exists a number δ, depending only on ε, such that $x_1, x_2 \,\varepsilon\, [a,b]$ and $|x_1-x_2|<\delta \Rightarrow |f(x_1)-f(x_2)| < \varepsilon$

Difference between continuity and uniform continuity.

Ans: *Uniform continuity is a property associated with an interval not with a single point.*

Note: The uniform continuity is defined with reference to a finite closed interval $[a,b]$. But no such restriction is necessary for uniform continuity. The definition is also true for intervals (a,b), (a,∞) Open intervals.

Art. A function which is continuous in a closed and bounded interval [a,b] is uniformly continuous in [a,b].

Proof: Let the interval $[a,b]$ be divided into sub intervals $[a,x_1]$, $[x_1,x_2]$ .... $[x_{n-1},b]$ ... (1)

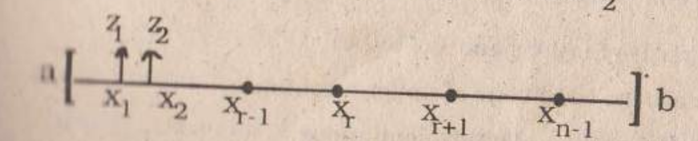
such that if $z, z_2 \,\varepsilon\, [x_1,x_2]$ or belong to any subinterval, we have

$$|f(z_1)-f(z_2)| < \frac{1}{2}\varepsilon \quad ..... (2)$$

Let us consider a small positive number δ such that it is not greater than the least of the numbers

$$x_1-a,\ x_2-x_1,\ x_3-x_2,\ ....\ b-x_{n-1}\ ....\ (3)$$

Let us consider z_1, z_2 two points in the same subinterval say x_2-x_1, then by (2) $|z_2-z_1| < \delta\ (> 0)$ and $|f(z_1)-f(z_2)| < \frac{1}{2}\varepsilon$



If z_1, z_2 do not belong to the same interval $(x_2-x_1) = [x_1, x_2]$, let z_1 lies in $x_2-x_1=[x_1,x_2]$ and z_2 in $(x_3-x_2)=[x_2,x_3]$

i. e; one in each of the two econsecutive intervals

$[x_1, x_2]$ and $[x_2, x_3]$. then we have

$x_1< z_1<x_2<z_2< x_3$ i. e. in general if z_1, z_2 be the points in

$[x_{r-1}, x_r]$ and $[x_r, x_{r+1}]$ respectively, i. e; x_{r-1} $z_1<x_r<z_2<x_{r+1}$;

then $|f(z_1)-f(z_2)| = |f(z_1)-f(x_r)+f(x_r)-f(z_2)|$

$\leq |f(z_1)-f(x_r)| + |f(x_r)-f(z_2)| \quad \leq \frac{1}{2}\varepsilon+\frac{1}{2}\varepsilon=\varepsilon$

But we have noticed that for given $\varepsilon>0$, there exists a small positive number $\delta>0$ such that $|f(z_1)-f(z_2)|<\varepsilon$ for any two points in $[a,b]$ such that $|z_1-z_2|<\delta$

Hence $f(x)$ is uniformly continuous in $[a,b]$

Ex.1. Prove that $f(x) = x^2$ for all $x \varepsilon$ R is uniformly continuous on (0,1].

Ans: Let x_1, x_2 be any two points in (0,1], then

$|f(x_1)-f(x_2)| = |x^2_1-x^2_2|=|x_1-x_2|\ |x_1+x_2|$ ....(1)

$x_1, x_2 \,\varepsilon\, (0, 1] \Rightarrow |x_1+x_2|<2$, whatever be the values of x_1 and x_2 between 0 and 1. Now from (1)

$\therefore \quad |f(x_1)-f(x_2)|<2|x_1-x_2|$ .... (2)

Again if $|x_1-x_2|<\delta$, then from (2)

$|f(x_1)-f(x_2)| <2|x_1-x_2| <2\delta$ ...... from (3)

If $\varepsilon>0$ is given, let $\delta=\frac{1}{2}\varepsilon$, then from (3)

$|f(x_1)-f(x_2)|<2.\frac{1}{2}\varepsilon=\varepsilon$ and $|x_1-x_2|<\frac{1}{2}\varepsilon=\delta$

or, $|f(x_1)-f(x_2)<\varepsilon$ and $|x_1-x_2|<\delta$

which are the conditions of uniform continuity.

Hence $f(x)=x^2$ is uniformly continuous.

Ex. 2. Show that $f(x)=\frac{1}{x}$; $(x>0)$ is not uniformly ontinuous' in (0,1]

Ans: Let us consider two points x_1 and x_2 such that

$x_1, x_2 \,\varepsilon\, (0,1]$, $|x_1-x_2|<\delta \Rightarrow |f(x_1)-f(x_2)|<\varepsilon$

If $\varepsilon=\frac{1}{2}$ and $\delta>0$, any positive number for $n\geq 1/\delta$.

If we take $x_1=1/n$, $x_2=1/(n+1)$; $x_1, x_2 \,\varepsilon\, (0,1]$, then

$|f(x_1)-f(x_2)| = |\frac{1}{x_1}-\frac{1}{x_2}| = |n-(n+1)| = 1>\varepsilon$

when $|x_1-x_2|<\delta$

Hence $f(x)=1/x$ is not uniformly continuous on (0,1]

Note. 1. The function is uniformly continuous on $[a, \infty)$, where $a>0$

Ex. Show that $f(x) = x^3+3x^2-2x+7$ in [-2,3] is uniformly continuous or not.

Ex. Let $x_1 x_2$ be any two points on [-2,3], then

$|f(x_1)-f(x_2)| = |x_1^3+3x_1^2-2x_1+7-x^3_2-3x_2^2+2x_2-7|$

$= |(x_1^3-x_2^3)-3(x_1^2-x_2^2)-2(x_1-x_2)|$

$= |(x_1-x_2)(x_1^2+x_1x_2+x_2^2)-3(x_1+x_2)(x_1-x_2)-2(x_1-x_2)|$

$= |(x_1-x_2)\{(x_1+x_2)^2-x_1x_2\}-3(x_1+x_2)(x_1-x_2)-2(x_1-x_2)|$...(1)

Now $x_1, x_2 \,\varepsilon\, [-2,3] \to |x_1+x_2|<3$, $x_1x_2<3$.

If $|x_1-x_2|<|\delta$, then from (1)

$|f(x_1)-f(x_2)| <|\delta(3-3)-3.3\delta-2\delta| = |\delta.0-9\delta-2\delta|=11\delta$

or, $|f(x_1)-f(x)|<11\delta$

$|f(x_1-f(x_2)| < 11. \varepsilon/11=\varepsilon$ where $\varepsilon=11\delta$

and $|x_1-x_2|<\delta$

If $\varepsilon=\frac{1}{2}$, then $\frac{1}{2}=11\delta$ or, $\delta=1/22$ which lies between -2 and 3.

Hence $f(x)$ is uniformly continuous in [-2,3]

Exercise. Test the uniform continuity of the following functions

(i) $f(x) = x^2+3x$ in $[-2,2]$

(ii) $f(x) = \dfrac{x+2}{2x+3}$ in $[-1,3]$

(iii) Show that $f(x) = x^2$ for all $x \varepsilon R$, $f(x)$ is uniformly continuous on R

(iv) $f(x) = \sqrt{x}$ in $[0,2]$ Ans. yes

(v) $f(x) = \dfrac{x}{1+x^2}$ on R Ans. yes

(vi) $f(x) = \sin \pi x$ for $x \varepsilon (1,2)$ Ans. yes
$= x^2-1$ for $x \varepsilon (1,2)$

(vii) $f(x) = \sin x$ on $[0,\infty)$ Ans. yes

(viii) $f(x) = \tan^{-1}x$ on R Ans. yes

2. Prove that

$$f(x) = \sin\frac{1}{x},\ x\neq 0$$
$$= 0\ ,\ x=0$$

is not uniformly continuous on $[0,\infty)$

Sol: Let $\varepsilon=1/3$ and $\delta>0$ be such that $1/z<\delta$ for all $n\geq m$.

Let $x=\dfrac{4}{4m\pi}$, $y=\dfrac{4}{(4m+1)\pi}$ be any two points of $[0,\infty)$, then.

$$|x-y|=\left|\frac{4}{4m\pi}-\frac{4}{(4m+1)\pi}\right|=\left|\frac{4m+1-4m}{m(4m+1)\ \pi}\right|=\frac{1}{m(4m+1)\pi}=\frac{1}{z}<\delta$$

$$\text{Now}\ |f(x)-f(y)|=\left|\sin m\pi-\sin\frac{(4m+1)\pi}{4}\right|$$

$$=\left|\sin m\pi-\sin\left(m\pi+\frac{\pi}{4}\right)\right|<|\sin\pi/4|=1/\sqrt{2}=\frac{\sqrt{2}}{2}=0.7/2$$

but $\varepsilon=\dfrac{1}{3}=0.33\ \therefore\not< 0.7/2$ i. e.,

$|f(x)-f(y)|>\varepsilon$ which does not satisfy the condition of uniform continuity

Ex. Show that the function

$f(x) = 1-x+[x]-[1-x]$ is not continuous at $x=0$. $[x]$ denotes the greatest integer positive or negative but not numerically greater than x.

Sol: We know

$[x]<x$, $[1-x]<1-x$ or, $-[1-x]>-1+x$

$\therefore\ [0]=0,\ 0\leq x<1$	$[1-0]=1-0=1$ or, $-[1-0]=-1, 0\leq x\leq 1$
$[1]=1,\ 1\leq x<2$	$[1-2]=1-2=-1$, or, $-[1-2]=1, 1\leq x\leq 2$
$[2]=2,\ 2\leq x<3$	$[1-3]=1-3=-2$ or, $-[1-3]=2, 2\leq x<3$
$[3]=3,\ 3\leq x<4$	

$f(x)=1-x+[x]-[1-x]=1-x+x-(1-x)=1-1+x=x$

$f(h)=1-h+h-1+h=h$ $\qquad \therefore \operatorname{Lt}_{h\to 0} h=0$

$f(-h)=1+h+0-h-(1-0+h)=1-1-h=-h$ $\qquad \therefore \operatorname{Lt}_{h\to 0} -h=0$

$f(0)=1-0+[0]-[1-0]=1-0+0-1=0$

$\therefore\ f(0)=f(0+h)=f(0-h)=0$

At $x=0$, $f(x)$ is continuous.

Ex—iv(B)

142. If $y = \sin^{-1}\{x\sqrt{1-x} - \sqrt{x(1-x^2)}\}$ then show that

$$\frac{dy}{dx} = \frac{1}{\sqrt{(1-x^2)}} - \frac{1}{2\sqrt{(x-x^2)}}$$

143. If $\tan y = \dfrac{\sqrt{(1+x^2)} + \sqrt{(1-x^2)}}{\sqrt{(1+x^2)} - \sqrt{(1-x^2)}}$

then $\dfrac{dy}{dx} = -\dfrac{x}{\sqrt{(1-x^4)}}$

$\therefore$ If $y = \tan^{-1} \dfrac{\sqrt{(1-x^2)} + \sqrt{(1-x^2)}}{\sqrt{(1=x^2)} - \sqrt{(1=-x^2)}}$

then $\dfrac{dy}{dx} = -\dfrac{x}{\sqrt{(1-x^4)}}$

144. If $\cot y = \dfrac{\sqrt{(1+\sin x)} + \sqrt{(1-\sin x)}}{\sqrt{(1+\sin x)} + \sqrt{(1-\sin x)}}$, then

$\dfrac{dy}{dx} = \dfrac{1}{2}$

145. If $y = \tan^{-1} \dfrac{\sqrt{(1-x)} - \sqrt{(1-x)}}{\sqrt{(1+x)} + \sqrt{(1-x)}}$, then

$\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{(1-x^4)}}$

146. If $\sqrt{(1-x^4)} + \sqrt{(1-y^4)} = a(x^2 - y^2)$, then

$\dfrac{dy}{dx} = \dfrac{x\sqrt{(1-y^4)}}{y\sqrt{(1-x^4)}}$

147. If $y = \log \dfrac{\sqrt{(1+x)} + \sqrt{(1-x)}}{\sqrt{(1+x)} - \sqrt{(1-x)}}$, then

$\dfrac{dy}{dx} = \dfrac{-1}{x\sqrt{(1-x^2)}}$

148. If $y = \tan\text{-}1\left(\sqrt{\dfrac{a-b}{a+b}} \tan \dfrac{2}{2}\right)$, then

$\dfrac{dy}{dx} = \dfrac{\sqrt{(a^2-b^2)}}{2(a + b\cos x)}$

149. If $\sqrt{(1-x^2)} + \sqrt{(1-y^2)} = a(x-y)$, then

$\dfrac{dy}{dx} = \dfrac{\sqrt{(1-y^2)}}{\sqrt{(1-x^2)}}$

150. If $u\sqrt{(1+v)} + v\sqrt{(1+u)} = 0$,

then show that $\dfrac{du}{dv} = -\dfrac{1}{(1+u)^2}$, $u \neq v$

151. If $y = \sqrt{[\sin x + \sqrt{\{\sin x + \ldots\ldots \text{ to infinity}\}}]}$

Show that $\dfrac{dy}{dx} = \dfrac{\cos x}{2y-1}$

152. If $y = \cos^{-1} \dfrac{3 + 5\cos x}{5 + 3\cos x}$, then

Show that $\dfrac{dy}{dx} = \dfrac{4}{5 + 3\cos x}$

153. If $y = x^{x^{\cdots ad.\ infinity}}$, prove that

$\dfrac{xdy}{dx} = \dfrac{y^2}{1 - y\log x}$

154. If $y = (\sin x)^{x^{\cdots}}$ ....ad. infinity.).

Show that $\dfrac{dy}{dx} = \dfrac{y^2 \cot x}{1 - y \log \sin x}$

155. If $y = e^{x + e^{x + e^{x}}}$ .... *ad.* infinity.

Show that $\dfrac{dy}{dx} = \dfrac{y}{1-y}$

Ex—v এর Example

Ex 5. Separate the intervals in which the polynomials $2x^3 - 15x^2 + 36x + 1$ is increasing or decreasing.

Ans Let $f(x) = 2x^3 - 15x^2 + 36x + 1$

$\therefore\ f'(x) = 6x^2 - 30x + 36 = 6(x^2 - 5x + 6)$
$= 6(x-3)(x-2)$

Now $f'(x) > 0$ for $x > 2$

$f'(x) < 0$ for $2 < x < 3$

$f'(x) > 0$ for $x > 3$

$f'(x) = 0$ for $x = 2$ and 3.

Hence we see that $f'(x)$ is positive in the interval $]-\infty, 2[$ and $]3, \infty[$ and negative in the interval $]2, 3[$ Thus $f(x)$ is monotonically increasing in the intervals $]-\infty, 2[$. $]3, \infty[$ and monotonically decreasing in the intervaL $]2, 3[$

Ex : 6. Let $f(x) = (x^4 + 6x^3 + 17x^2 + 32x + 32)\, e^{-x}$

$f'(x) = (x^4 + 6x^3 + 17x^2 + 32x + 32)(-e^{x} + (4x^3 + 18x^2 + 34x + 32)\, e^{-x}$

$= -e^{-x}(x^4 + 2x^3 - x^2 - 2x) = -xe^{-x}(x^3 + 2x^2 - x - 2)$

$= -x(x+2)(x-1)(x+1)\, e^{-x}$

$= x(1-x)(1+x)(2+x)\, e^{-x}$

The function $f'(x)$ is positive in the ntervals $[-2, -1]$ and $[0,1]$ and negative in $]-\infty,-2]$, $[-1,0]$ and $[1,\infty[$ Hence $f(x)$ is monotonicly increasing in the intervals $[-2,-1]$ and $[0,1]$ and monotonically decreasing in intervals $]-\infty, -2[$, $[-1,0]$ and $[1,\infty]$

Ex. 7. Find the intervals in which the function $f(x) = 8x^3 - 60x^2 + 144x + 15$ is increasing or decreasing.

Sol : $f(x) = 8x^3 - 60x^2 + 144x + 5$

$f'(x) = 24x^2 - 120x + 144$
$= 24(x^2 - 5x + 16)$
$= 24(x-2)(x-3)$

Now $f'(x) > 0$ if $x < 2$ .... (1)
$f'(x) < 0$ if $2 < x < 3$ .... (2)
$f'(x) > 0$ if $x > 3$ .... (3)
and $f'(x) = 0$ for $x = 2, 3$ .... (4)

Hence $f'(x)$ is positive in the intervals from (1) and (3), $(-\infty, 2)$ and $(3, \infty)$ i. e; negative excluding $x = 2$ and $x = 3$ Thus $f(x)$ is increasing monotonically in $(-\infty, 2)$, $(3, \infty)$ open intervals and monotonically decreasing in $(2, 3)$ open interval.

Art. 17.23. A Function is twice differentiable and satisfies the inequalities.

$|f(x)|<A$, $|f''(x)|<B$. in the range $x>a$;

Where A and B are constants. Prove that $|f'(x)|<2\sqrt{(AB)}$.

Ans. For positive number h, and $x>a$:

$$f(x+h)= f(x)+hf'(x)+\frac{h^2}{\lfloor 2}f''(x+\theta h),\ 0<\theta<1$$

$$\therefore\ |hf'(x)| = |f(x+h)- f(x) - \frac{h^2}{\lfloor 2}f''(x+\theta h)|$$

$$\le |f(x+h)| + |f(x)| + |\frac{h^2}{\lfloor 2}f''(x+\theta h)|$$

$$<A+A+Bh^2/2$$

or; $|f'(x)| < \frac{2A}{h} + Bh/2$; h is+ve

$|f'(x)$ is free from h and also less than $(2A/h+Bh/2)$ for all positive value of h. Thus $|f'(x)|$ must be less than the least value of $(2A/h+Bh/2)$

Thus $(2A/h+Bh/2)=\{\sqrt{(2A/h)}-\sqrt{(Bh/2)}\}^2+2\sqrt{(AB)}$

such that $2\sqrt{(AB)} = (2A/h)+Bh/2$ least value

Hence $|f'(x)|<2A/h+Bh/2\ge 2\sqrt{(AB)}$

$\therefore\ |f'(x)|<2\sqrt{(AB)}$

Ex—vii এর Art. 22, 23, 24, 25.

22. A function f(x) is defined in [0;2] as follows
$f(x) = 2$ for $0 < x < 1$
$= 3$ for $1 \le x \le 2$

Show that $f(x)$ satisfies none of the conditions of Rolle's Theorem yet $f'(x) = 1$ for many points in [1,2]

Sol

Here we note that $f(a-h) = f(1-0) = 2$. $f(1+0) = 3$
i.e. $f(1-0) \ne f(1+0)$
Though $f(1+0) = 3 = f(1) \ne f(1-0)$ ....(1)

Hence $f(x)$ is discontinuous at $x = 1$

But we know that continuity is a necessary condition for a finite derivative, so the function $f'(x)$ does not exist for every point in $1 \le x \le 2$ ....(2)

Also $f(1) = 2$ and $f(2) = 3$ given
So $f(1) \ne f(2)$ .... (3)

Hence all the three conditions of Rolle's Theorem are not satisfied by $f(x)$ in [1,2]

Here $f(x)$ is a function free from x *i.e.* a constant in [1,2]. There is possibitlity of value of $f'(x)$ to be one at many points in [1,2]

23. Discuss the applicability of Rolle's Theorem to the function

$f(x) = x^2 + 1$ when $0< x \le 1$
$= 3-x$ when $1\le x \le 2$

Sol : $f(0) = 0^2 + 1 = 1$, $f(2) = 3 - x = 3 - 2 = 1$
$f(0) = 2 = f(2)$ ....(1)

Let us test the continuity of $f(x)$ at $x = 1$,

$$f(1+0) = \underset{x\to 0}{Lt}(3-x) = \underset{h\to 0}{Lt}(3-1-h) = 2$$

$$f(1-0) = \underset{x\to 0}{Lt}x^2 + 1 = \underset{h\to 0}{Lt}(1-h)^2 + 1 = 2$$

$\therefore\ f(1-0) = f(1+0) = f(0) = 2$

It is continuous at $x = 1$, so we infer that $f(x)$ is continuous in the interval [0,2]

Again $f'(x) = 2x,\ 0 \le x \le 1$
$= -1,\ 1 < x \le 2$

Let us suppose that f(x) is differentiable in the interval (0,2) exceft at x = 1

one

$$Rf'(1) = \underset{h\to 0}{Lt} \frac{f(1+h) - f(1)}{h} = \underset{h\to 0}{Lt} \frac{\{3 - (1+h)\} - 2}{h}$$

$$= \underset{h\to 0}{Lt} \frac{2-h-2}{h} = \underset{h\to 0}{Lt} (-1) = -1$$

$$Lf'(1) = \underset{h\to 0}{Lt} \frac{f(1+h) - f(1)}{-h} = \underset{h\to 0}{Lt} \frac{(1-h)^2 + 1 - 2}{-h}$$

$$= \underset{h\to 0}{Lt} \frac{2h-h^2}{h} = 2$$

$\therefore Rf'(1) \neq Lf'(1)$; so $f'(1)$ does not exist. $f(x)$ is not differentiable in the entire region (0,2) and therefore Rolle's Theorem is not applicable to the given function $f(x)$ in (0,2)-

24. A function $f(x)$ is continuous in closed interval [2,3] and differentiable in the open interval (2,3). Prove that

$f'(\theta) = f(3) - f(2)$ where $2 < \theta < 3$.

The conditions of Mean value Theorem are $f(x)$ is continuous in the closed intersol [2,3] and differentiable in the open interval (2,3). if c a value of x such that $2 < c < 3$.

$f(b)-f(a) = (b-a) f'(c)$

or; $f(3) - f(2) = (3-2) f'(c)$

or; $f'(c) = f(3) - f(2)$

Ex. 25. Meaning of the sign of derivative

Let c be a interior point a < c < b of the function $f(x)$. if $f'(c) > 0$

$$\underset{x\to c}{Lt} \frac{f(x) - f(c)}{x - c} = f'(c) > 0$$

If $\varepsilon > 0$ be any number $< f'(c)$. there exists a positive number $\partial > 0$ such that

$$|x-c| < \partial \Rightarrow \left| \frac{f(x) - f(c)}{x-c} - f'(c) \right| < \epsilon$$

Where $c - \partial < x < c + \partial$

$\Rightarrow \frac{f(x) - f(c)}{x-c} \subset c\,(f'(c) - \varepsilon, f'(c) + \varepsilon)$ in the open interval. Since $\varepsilon < f'(c)$, then we conclude that

$$\frac{f(x) - f(c)}{x - c} > 0 \text{ when } x\,\varepsilon\,[c - \partial, c+\delta].\ x \neq c$$

We infer that

$f(x) - f(c) > 0$ when $c < x \le x + \partial$

$f(x) - f(c) < 0$ When $c - \partial \le x < c$

If $f'(c) > 0$, there exists a neighbourhood $[c - \partial, c+\partial]$ of c such that $f(x) > f(c)$ for all $x\,\varepsilon\,(c, c+\partial)$

$f(x) < f(c)$ for all $x\,\varepsilon\,(c, -\partial, c)$

If $f'(c) < 0$, then there exist a neighbourhood $[c-\delta, c+\delta]$ of c such that $f(x) > f(c)$ for all $x\varepsilon[c-\delta, c]$

$f(x) < f(c)$ for all $x\,\varepsilon\,[c, c+\delta]$

For the end points a and b, it can be shown that there exist intervals $(a, a+\partial]$, $(b, b-\partial]$ such that

$f'(a) > 0 \Rightarrow f(x) > f(a)$ for all $x\,\varepsilon\,(a, a+\partial]$

$f'(a) < 0 \Rightarrow f(x) < f(a)$ for all $x\,\varepsilon\,(a, a+\partial]$

$f'(b) > 0 \Rightarrow f(x) < f(b)$ for all $x\,\varepsilon\,[b-\partial, b)$

$f'(b) < 0 \Rightarrow f(x) > f(b)$ for all $x\,\varepsilon\,[b-\partial, b)$

Ex—vii

52. Prove that $\phi'(x) = F'(\{f(x)\} f'(x)$. $\phi(x) = f\{f(x)\}$

assume that the derivatives which are continuous and apply the mean value theorem.

53. Examine whether all the conditions of Rolle's Theorem are satisfied by the function $f(x) = 1 - |x|$ in $[-1,1]$. What is your conclusion?

54. A function $f(x)$ is continuous in the closed interval $0 < x < 1$ and differentiable in the open interval $0 < x < 1$ Prove that $f'(x_1) = f(1) - f(0)$ where $0 < x_1 < 1$

55. Show that Rolle's theorem is not valid for the function $f(x) = x$ in $[-1,1]$ as $f'(x)$ does not exist for a value of x in $[-1,1]$

56. If $f''(x)$ is continuous and not zero at $x = a$. Show that $\underset{h\to 0}{Lt}\ \theta = \frac{1}{2}$ Where $f(x+h) = f(x) + hf'((x+\theta h)$; $0 < \theta < 1$

57. Prove that if $f''(x)$ is continuous

$$\underset{h\to 0}{Lt} \frac{f(x+2h) - 2f(x+h) + f(x)}{h^2} = f''(x)$$

Ex—viii

75. Examine whether the function $f(x)$ is continuous at $x=0$
Where $f(x) = x^{2x}, x \neq 0$, $f(0) = 1$

Sol. প্রদত্ত রাশি, $f(x) = x^{2x}$

$$\therefore \log f(x) = 2x \log x = 2\frac{\log x}{1/x}$$

$$\therefore \underset{x \to 0}{Lt} \log f(x) = 2 \underset{x \to 0}{Lt} \frac{\log x}{1/x} \therefore \text{form } \frac{\infty}{\infty}$$

$$= -2 \underset{x \to 0}{Lt} \frac{1/x}{1/x^2} = 0$$

এখন $\underset{x \to 0}{Lt} \log f(x) = \log \underset{x \to 0}{Lt} f(x)$

অতএব $\log \underset{x \to 0}{Lt} \log f(x) = 0$, ফলে

$$\underset{x \to 0}{Lt} f(x) = e^{\circ} = 1$$

দেওয়া আছে $f(0) = 1$ ফলে $\underset{x \to 0}{Lt} f(x) = f(0)$

অতএব $x=0$ বিন্দুতে $f(x)$ অবিচ্ছিন্ন।

76. মান নির্ণয় কর $\dfrac{a^x \sin bx - b^x \sin ax}{\tan bx - \tan ax}$ R.U. 1988

Ex—ix

97. If $z = x + f(u)$ and $u = xy$, the show that $x\frac{\delta z}{\delta x} - \frac{\delta z}{\delta y} = x$.

[যদি $z = x + f(u)$ এবং $u = xy$ হয়, তাহা হইলে দেখাও যে $x \frac{\delta z}{\delta x} - y\frac{\delta z}{\delta y} = x$]

98. If $\phi(x,y) = 0$, $\psi(y,z) = 0$, show that $\frac{\delta\phi}{dy}\frac{\delta\psi}{\delta z}\frac{dz}{dx} = \frac{\delta\alpha}{\delta x}\frac{\delta\psi}{\delta y}$

Interpret the result geometrically (জ্যামিতি সাহায্যে ব্যাখ্যা কর)

99. If $F(x,y) = x^4y^2 \sin \frac{y}{x} + \tan^{-1}\frac{y}{x}$, then $x\frac{\delta F}{\delta x} + y\frac{\delta F}{\delta y} = 6F$

100. If $z = \frac{x}{y} + x \tan y$, then $\frac{\delta^2 z}{\delta x \delta y} = \frac{\delta^2 z}{\delta y \delta x}$

INTRODUCTION

"The word **Calculus** is the latin name for a stone which was employed by the Romans for reckoning i. e, for "**Calculation**". When used as in the title of the book, it is an abbreviation for "**Infinitesimal Calculus**" which implies a reckoning or Calculation with numbers which are infinitesimally small.

One of the most powerful methods in modern mathematics is that of the Calculus, the ideas of which were conceived by Archimedes in the third century B. C. By dividing a segment of a parabola into thin strips and adding together thier areas, he found an approximation to the area of the segment. He then obtained closer and closer approximations by taking more and thinner strips. By this method of exhaustion he found the area of the segment exactly.

About 1586 Stevinus of Bruges used a method similar to that of Archimedes to find the thrust of a liquid on a surface, and a little later a Jesuit priest, Cavalieri, extended the method to find the volumes of solids. The methods used by these mathematicians to find the whole area or, volume, by dividing it up into small parts is now called "integration' i. e. finding the whole.

In the beginning of the 17th century the French mathematician Fermat considered the ratio of infinitesimally small increments so laid the foundation on which Newton (1642—1727) and Leibnitz (1646—1716) later built the theory of "differentiation" or finding rates of change from the ratios of small differences It was due to the genius of Newton and Leibnitz that a great advance was made. Newton conceived the idea of continuous change and rate of change at an instant or flux, and he described his new subject as "fluxions". He found that his knowledge of rates of change could be applied to calculate areas and volumes that is to perform integration much more easily than by the method of exhaustion described above, Leibnitz discoverd the method of differentiation about the same time and we are specially indebted to him for his notation which is essentially that now in general use. In the last two centuries calculus has been developed to such an extent that it is now used to deal with problems in branch of technical Science.'

Before taking up the subject matters of Calculus let us first discuss something about numbers and related quantities.

উপরে বর্ণিত ক্যালকুলাসের উৎপত্তি ও ব্যাখ্যা হইতে আমরা নিম্নলিখিত মূল অংশে মনোনিবেশ করিতে পারি। ক্যালকুলাসের সাহায্যে আমরা মূলত দুইটি গুরুত্বপূর্ণ বিষয়ের অনুসন্ধান করিতে পারি।

(ক) একটি রেখার (curve) ঢালুতার (solpe) অর্থের অনুসন্ধান করিয়া উহার মান নির্ণয় করিতে পারি।

(খ) কোন রেখার দ্বারা পরিবেষ্টিত স্থানের ক্ষেত্রফলের অর্থ বাহির করিয়া উহার মান নির্ণয় করিতে পারি।

ডিফারেন্সিয়াল ক্যালকুলাস (ক) প্রথমোক্ত বিষয়ে এবং (খ) দ্বিতীয়োক্ত বিষয়ে আমরা ইন্টিগ্র্যাল ক্যালকুলাসে আলোচনা করিয়া থাকি।

পরিভাষা

ইংরেজী-বাংলা

Abscissa —ভুজ
Absolute value —পরম মান
Acceleration —ত্বরণ
Algebraic —বীজগাণিতিক
Alternative —বিকল্প
Analytical —বিশ্লেষক
Angle of incidence —আপতণ কোণ
Angle of reflection —প্রতিফলন কোণ
Angular displacement — কৌণিক সরণ
Applied —ফলিত
Arbitrary —অবাধ
Arch —খিলান
Arc length —চাপ— দৈর্ঘ্য
Assumption —প্রতিজ্ঞা
Astroid —আসট্রোয়েড
asymptote —অসীমতট
axiom —স্বতঃসিদ্ধ
Beam —কড়িকাঠ
Binomial —দ্বিপদী
Bisector —দিখণ্ডক
Chain rule —শৃঙ্খল নিয়ম
Counter clockwise —বামাবর্ত
Critical value —সন্ধিমান
Cross multiplication —বজ্রগুণন
Cross secton —আড়চ্ছেদ
Change of variables —চলক পরিবর্তন
Chord of curvature —বক্রতা জ্যা
Circular function —বৃত্তীয় ফাংশন
Clockwise —দক্ষিণাবর্ত
Co-efficiant —সহগ

Co-efficent of viscosity —আঠালুতা সহগ
Coincidence —সমাপতন
Co-ordinate —স্থানাঙ্ক
Corollary —অনুসিদ্ধান্ত
Concave downward —নিম্নদিকে চন্দ্রাকৃতি/ নিম্ন অবতল
Concave upward —উর্ধ্বদিকে চন্দ্রাকৃতি/উর্ধ্ব অবতল
Cone — কোণক
Conic section — কোণচ্ছেদ
Constant —ধ্রুবক
Continuity — ছেদহীন
Continuity — ছেদহীনতা
Convergence —সীমামুখিতা
Convergent —সীমামুখী
Cubical parabola —ত্রিঘাত পরাবৃত্ত
Curvature —বক্রতা
Curve — রেখা, বক্র
Cycle —চক্র
Cube —ঘনক
Cylinder —সিলিন্ডার/স্তম্ভক
Decreasing —হ্রাসমানী
Deflection —ব্যতায়
Determinant —নির্ণায়ক
Differentiability —ডিফারেন্সিয়েশন যোগ্যতা / অন্তরীকরণীয়ে
Differential —ডিফারেন্সিয়েল /অন্তরক
Differentiate —ডিফারেন্সিয়েশন করা/ অন্তরীকরণ করা
Differentiation —ডিফারেন্সিয়েশন/অন্তরকরণ
Dimension —মাত্রা
Direction cosines —গতিকোসাইন
Discontinuity — ছেদযুক্ততা
discontinuous — ছেদযুক্ত
Discrete — ছেদযুক্ত
Divergence —সীমাবিমুখিত
divergent —সীমাবিমুখ

Domain —এলাকা
Eccentricity —বিকেন্দ্রিকতা
Dlectromotive force —তড়িৎ চালক শক্তি
Ellipsoid —উপবৃত্তক
Element —উপাদান
Ellipse —উপবৃত্ত
Eliminate —অপসারণ করা
Envelope —আচ্ছাদন
Equiangular spiral —সদৃশকোণী স্পাইরেল
Even — জোড়া/যুগ্ম
Expansion —বিস্তারন
Exlicit —ব্যক্ত
Exponential —সূচক
Cycloid —সাইক্লয়েড
Focal chord —ফোকাস জ্যা
Focus — ফোকাস/নাভিবিন্দু
folium —ফলিয়াম
Formula —সূত্র
function —ফাংশন / নির্ণায়ক
General formula —সাধারণ সুত্র
Genralised —ব্যাপকীকৃত
Generalization —ব্যাপকীকরণ
Gradient —ঢালুতা/গ্র্যাডিয়েন্ট
Graph —চিত্র/লেখচিত্র
Homogenous —সমমাত্রিক/সুষম
Horizontal —আনুভূমিক
Hyperbola —অধিবৃত্তি
Hyperboloid —অধিবৃত্তক
Illumination —দীপন
Imaginary number —কাল্পনিক সংখ্যা
Implicit —অব্যক্ত
Improper fraction —অপ্রকৃত ভাগ্গাংশ
Indeterminate form —অনিশ্চিত আকার

Inequality —অসমতা
Infinite —অপরিমেয়/অসীম
Infinitesimal —অণু
Inscribed —অন্তলখিত
Intensity —তীব্রতা
Integer —পূর্ণসংখ্যা
Interval —ব্যবধি
Finite number —পরিমেয় সংখ্যা
Finite series —সসীম ধারা
Intrinsi —স্বকীয়
Inverse cicular function —বিপরীত বৃত্তীয় ফাংশন
Invrsely proportional —বিপরীতক্রমে সমানুপাতিক
Involute —ইনভোলিউট
Irrational —অবাস্তব
Kinetic energy —গতিশক্তি
Left hand limit —বামসীমা
Limit —সীমা
Limiting point —সীমায়িত বিন্দু
Limitsign —সীমা চিহ্ন
Limiting value —সীমায়িত মান
Limiting value of a limit —সীমার সীমায়িত মান
Loop —লুপ/ফাঁস
Major axis —বৃহৎ অক্ষ
Maximum —গুরুমান
Mean value theorem —গড়মান উপপাদ্য
Method of induction —আরোহ পদ্ধতি
Minor axis —ক্ষুদ্র অক্ষ
Moment —ঘুরণ বল
Motion —গতি
Multiple valued —বহুমানী
Negative —বিয়োগ বোধক
Node. নোড, বা গিট
Numerical —সঙ্কেত

Interval of convergence —সীমা মুখিতায় ব্যবধি
Operator —অপারেটর / ঘটক
Order —ক্রম
Ordinate — কোটি
Parabola —পরাবৃত্ত
Paraboloid —পরাবৃত্তক
Parametric —প্যারামিটারযুক্ত
Differentiation —ডিফারেন্সিশন, অন্তরীকরণ
Partial fraction —আংশিক ভগ্নাংশ
Particular solution —নির্দিষ্ট সমাধান
Pendulum — দোলক
Perimeter —পরিসীমা
Periodic —আবর্তনকাল
Plane —সমতল
Point of inflection —ইনফ্লেকশন বিন্দু
Point of intersection — ছেদবিন্দু
Polygon —বহুভুজ
Polynomial —বহুপদী
Positive — যোগবোধক
Powere series —শক্তি ধারা
Probabltity curve —সম্ভাব্যতা রেখা
Process of summation — যোগপ্রক্রিয়া
Proper fraction —প্রকৃত ভগ্নাংশ
Properties —ধর্মাবলী
Proportional— সমানুপাতিক
Projection —প্রক্ষেপ
Range —ব্যাপ্তি
Rational —আনুপাতিক
Rationalization —আনুপাতিকরণ
Real number —বাস্তব সংখ্যা
Numerical —সাংখ্যিক
Odd — বেজোড়/অযুগ্ম
One dimensional —একমাত্রিক

One to one correspondence —এক এক সম্পর্ক
Operation —প্রক্রিয়া/অপরেশন
Resistance —প্রতিবন্ধকতা
Repeated —পুনরাবৃত্তিক
Retardation —মন্দন
Regular pyramid —সুষম পিরামিড
Right hand limit —ডানসীমা
Root —মূল
Sag —ঝুলন
Secant — ছেদক
Semi-axis —অর্ধক্ষ
Series —ধারা
Set — সেট
Sequence —অনুক্রম
Sign —চিহ্ন
Similar —সদৃশ
Single valued —একমানী
singular —বিশিষ্ট, ব্যাত্রিম
Solution —সমাধান
slope —ঢালুতা/ঢাল
Space —স্থান
Speed —দ্রুতি
Sphere — গোলক
Spherical shell — গোলকীয় খোলস
Spiral —স্পাইরেল/ কুণ্ডলী
Standard —প্রামান্য
Strenght —শক্তিমাত্রা
Strophoid — ষ্ট্রোফয়েড
Rectangular hyperbola —আয়াতাকর অধিবৃত্ত
" Parallelopiped " বাক্স
Reference line —নিয়ন্ত্রণ রেখা
Relative —আপেক্ষিক
Remainder —অবশিষ্ট

Sub-interval —উপব্যবধি
Substitution —প্রতিস্থাপন
Subset —উপসেট
Successive differentiation—ক্রমিক ডিফারেন্সিয়েশন ব অন্তরীকরণ
Surface —তল
Suffix —নিম্নসূচক
Symbolic —সাংকেতিক
Symmetrical —প্রতিসম
Symmetry —প্রতিসাম্য
Tangent —স্পর্শক
Temperature —উষ্ণতা
Tetrahedron — টেট্রাহেড্রন
Theorem —উপপাদ্য
Total differential —সার্বিক অন্তরকীয়
Transcendental —তুরীয়া/অবীজগণিতীয়
Uniformly —সমভাবে
Variable —চলক
Velocity — বেগ
Vice-versa —বিপরীতক্রমে
Viscosity —আঠালুতা
Viscous —আঠালু
Vertex —শীর্ষবিন্দু
Vertical —ভূলম্ব রেখা

Important Formulae

1. $|x + y| \le |x| + |y|$
2. $|x - y| \ge |x| - |y|$
3. Continuity at $x = a$ of $f(x)$

$$\lim_{x \to a+} f(x) = \lim_{x \to a-} f(x) = f(a)$$

3. (a) $\lim_{h \to 0} f(a+h) = \lim_{h \to 0} f(a-h) = f(a)$

3. (b) or, $|f(x) - f(a)| < \varepsilon, \ |x - a| \le \delta, \ \varepsilon > 0$

4. Limit

If $\lim_{x \to a} f(x) = l$, $\lim_{x \to a} \phi(x) = m$, then

(a) $\lim_{x \to a} \{ f(x) \pm \phi(x) \} = l \pm m$

(b) $\lim_{x \to a} f(x)\,\phi(x) = lm$

(c) $\lim_{x \to a} \dfrac{f(x)}{\phi(x)} = \dfrac{l}{m}$ if $\lim_{x \to a} \phi(x) \ne o$

Limit of a function of a function

$$\lim_{x \to a} \phi(f(x)) = \phi\left\{ \lim_{x \to a} f(x) \right\}$$

5. (a) $\lim_{x \to 0} \dfrac{\sin x}{x} = 1$ (b) $\lim_{x \to 0} \left(1 + \dfrac{1}{x}\right)^x = e$

(c) $\lim_{x \to 0} \dfrac{a^x - 1}{x} = \log_e a$ (d) $\lim_{x \to 0} \dfrac{1}{x} \log(1 + x) = 1$

(e) $\lim_{x \to 0} \dfrac{e^x - 1}{x} = 1$ (f) $\lim_{x \to 0} \dfrac{x^n - a^n}{x - a} = na^{n-1}$

(g) $\lim_{x \to 0} \dfrac{(1+x)^n - 1}{x} = n$

6. Dfferential Co-efficient of elementary functions

$$\lim_{h \to 0} \frac{(x+h) - f(x)}{h} = \frac{d}{dx}\{f(x)\} = f'(x)$$

7. $\dfrac{d}{dx}(x^n) = nx^{n-1}$ $\dfrac{d}{dx}(x^{-n}) = -\dfrac{n}{x^{n+1}}$

$\dfrac{d}{dx}(\sqrt{x}) = \dfrac{1}{2\sqrt{x}}$ $\dfrac{d}{dx}(e^x) = e^x$

$\dfrac{d}{dx}(a^x) = a^x \log a$ $\dfrac{d}{dx}(\log x) = 1/x$

$\dfrac{d}{dx}(\sin x) = \cos x$ $\dfrac{d}{dx}(\cos x) = -\sin x$

$\dfrac{d}{dx}(\tan x) = \sec^2 x$ $\dfrac{d}{dx}(\cot x) = -\mathrm{cosec}^2 x$

$\dfrac{d}{dx}(\sec x) = \sec x \tan x$ $\dfrac{d}{dx}(\mathrm{cosec}\, x) = -\mathrm{cosec}\, x \cot x$

$\dfrac{d}{dx}(\sin^{-1} x) = \dfrac{1}{\sqrt{(1-x^2)}}$ $\dfrac{d}{dx}(\cos^{-1} x) = -\dfrac{1}{\sqrt{(1-x^2)}}$

$\dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1+x^2}$ $\dfrac{d}{dx}(\cot^{-1}) = \dfrac{-1}{1+x^2}$

$\dfrac{d}{dx}(\sec^{-1} x) = \dfrac{1}{x\sqrt{(x^2-1)}}$ $\dfrac{d}{dx}(\mathrm{cosec}^{-1} x) = \dfrac{-1}{x\sqrt{(x^2-1)}}$

$\dfrac{d}{dx}(\sinh x) = \cosh x$ $\dfrac{d}{dx}(\cosh x) = \sinh x$

$\dfrac{d}{dx}(\tanh x) = \mathrm{sech}^2 x$ $\dfrac{d}{dx}(\coth x) = -\mathrm{cosec}\, h^2 x.$

$\dfrac{d}{dx}(\sec hx) = -\sec hx \tan hx$ $\dfrac{d}{dx}(\mathrm{cosec}\, hx) = -\mathrm{cosec}\, hx \cot hx.$

$\dfrac{d}{dx}(\sin h^{-1} x) = \dfrac{1}{\sqrt{(1+x^2)}}$ $\dfrac{d}{dx}(\cosh^{-1} x) = \dfrac{-1}{\sqrt{(x^2-1)}}, \ x > 1$

$\dfrac{d}{dx}(\tan h^{-1} x) = \dfrac{1}{1-x^2}; \ x < 1$ $\dfrac{d}{dx}(\cot h^{-1} x) = \dfrac{-1}{\sqrt{(x^2-1)}}, \ x > 1$

$\dfrac{d}{dx}(\sec h^{-1} x) = \dfrac{-1}{x\sqrt{(1-x^2)}}, \ x < 1$

$\dfrac{d}{dx}(\mathrm{cosec}\, h^{-1} x) = \dfrac{-1}{x\sqrt{(x^2+1)}}$

8. $\frac{d}{dx}(c)=0,\ \frac{d}{dx}cf(x)=c\frac{d}{dx}f(x),\quad \frac{d}{dx}[f(x)\pm\phi(x)]=f'(x)\pm\phi'(x)$

9. $\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$

10. $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$; $v\neq 0$

11. If $y=f(t),\ t=\psi(x)$

$\frac{dy}{dx}=\frac{dy}{dt}\cdot\frac{dt}{dx}$

Successive Differentiation

12. $D^n(ax+b)^m=\frac{m!}{(m-n)!}a^n(ax+b)^{m-n}$; $(m>n)$

$D^n(ax+b)^m=0,\ (n>m)$

$D^n(ax+b)^n=a^n n!$ if $m=n$.

13. $D^n(e^{ax})=a^n e^{ax}$

$D^n(a^x)=(\log a)^n a^x$

14. $D^n\frac{1}{(ax+b)}=\frac{(-1)^n n!\ a^n}{(ax+b)^{n+1}}$

15. $D^n\left[\frac{1}{(ax+b)^m}\right]=\frac{(-1)^n(m+n-1)!\ a^n}{(m-1)!\ (ax+b)^{m+1}}$

16. $D^n\log(ax+b)=\frac{(-1)^{n-1}(n-1)!\ a^n}{(ax+b)^n}$

17. $D^n\sin(ax+b)=a^n\sin\left(ax+b+\frac{1}{2}n\Pi\right)$

18. $D^n\cos(ax+b)=a^n\cos\left(ax+b+\frac{1}{2}n\Pi\right)$

19. $D^n(e^{ax}\cos bx)=\sqrt{(a^2+b^2)^n}e^{ax}\cos(bx+n\tan^{-1}b/a)$

20. $D^n(e^{ax}\sin bx)=\sqrt{(a^2+b^2)^n}e^{ax}\sin(bx+n\tan^{-1}b/a)$

21. $D^n\left(\frac{1}{x^2+a^2}\right)=\frac{(-1)^n n!}{a^{n+2}}\sin^{n+1}\theta(\sin(n+1)\theta$

Where $\theta=\tan^{-1}a/x$ or, $\cot^{-1}x/a$.

22. $D^n\left(\tan^{-1}\frac{x}{a}\right)=\frac{(-1)^{n-1}(n-1)!}{a^n}\sin^n\theta\ \sin n\theta$

Where $\theta=\tan^{-1}a/x$ or, $\cot^{-1}x/a$

23. Leibnitz's Theorem

$D^n(uv)=u_n v+{}^nc_1u_{n-1}v_1+{}^nc_2u_{n-2}v_2+\ldots+{}^nc_ru_{n-2}v_r+\ldots+uv_n$

Expansions

24. Machlaurin's Series

$f(x)=f(0)+xf'(0)+\frac{x^2}{2!}f''(0)+\ldots+\frac{x^nf^n(0)}{n!}+\ldots$

25. Taylor's Series

$f(x+h)=f(x)+hf'(x)+\frac{h^2}{\lfloor 2}f''(x)+\ldots+\frac{h^nf^n(x)}{n!}+\ldots$

26. $f(x)=f(a)+(x-a)f'(a)+\frac{(x-a)^2}{2!}f(a)+\ldots$

$+\frac{(x-a)^n}{n!}f^n(a)+\ldots$

27. $e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots\frac{x^r}{r!}+\ldots$

$e^{-x}=1-\frac{x}{1!}+\frac{x^2}{2!}-\frac{x^3}{3!}+\ldots+(-1)^r\frac{x^r}{r!}+\ldots$

$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\ldots$

$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\ldots$

$\log(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}-\ldots$

$\log(1-x)=-x-\frac{x^2}{2}-\frac{x^3}{3}-\frac{x^4}{4}+\ldots$

$\tan^{-1}x=x-\frac{x^3}{x}+\frac{x^5}{5}-\frac{x^7}{7}+\ldots$

$$\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \ldots\ldots\ldots\ldots$$

$$\sin^{-1}x = x + \frac{x^3}{2.3} + \frac{1.3x^5}{2.4.5} + \ldots\ldots\ldots\ldots$$

$$(1-x)^{-1} = 1 + x + x^2 + x^3 + x^4 + \ldots\ldots\ldots\ldots$$

$$(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 - \ldots\ldots\ldots\ldots$$

$$(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \ldots\ldots\ldots\ldots$$

$$(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \ldots\ldots\ldots\ldots$$

$$(1-x)^{-1/2} = 1 + \frac{1}{2}x + \frac{1.3}{2.4}x^2 + \frac{1.3.5}{2.4.6}x^3 + \ldots\ldots\ldots$$

$$(1+x)^{-1/2} = 1 - \frac{1}{2} + \frac{1.3}{2.4}x^2 - \frac{1.3.5}{2.4.6}x^3 + \ldots$$

Partial Differentiation

28. If $u = f(x,y)$,

$$\frac{\delta^2 u}{\delta x \delta y} = \frac{\delta^2 u}{\delta y \delta x}$$

29. if $u = f(x,y)$, $x = f(t)$, $y = \psi(t)$, then

$$\frac{du}{dt} = \frac{\delta u}{\delta x}\frac{dx}{dt} + \frac{\delta u}{\delta y}\frac{dy}{dx}$$ (Totall differntial co-efficient for a variable t.)

cor : If $u = f(x,y)$, $y=f(x)$, then

30. $$\frac{\delta u}{\delta x} = \frac{\delta u}{\delta x} + \frac{\delta u}{\delta y} = \frac{\delta v}{\delta x}$$

30. (a) Total Differential of $u = f(x,y)$

$$du = \frac{\delta u}{\delta x}dx + \frac{\delta u}{\delta y}dy.$$

If $x = \phi(r, s, t)$, $y = \psi(r, s, t)$, then

$$dx = \frac{\delta x}{\delta r}dr + \frac{\delta x}{\delta s}ds + \frac{\delta x}{\delta t}dt$$

$$dy = \frac{\delta y}{\delta r}dr + \frac{\delta y}{\delta s}ds + \frac{\delta y}{\delta t}dt$$

31. If $u = f(x, y)$, $x = \phi(r, s, t)$, $y = \psi(r, s, t)$, the Partial differential co-efficient of u are

$$\frac{\delta u}{\delta r} = \frac{\delta u}{\delta x}\frac{\delta x}{\delta r} + \frac{\delta u}{\delta y}\frac{\delta v}{\delta r}$$

$$\frac{\delta u}{\delta s} = \frac{\delta u}{\delta x}\frac{\delta x}{\delta s} + \frac{\delta u}{\delta y}\frac{\delta y}{\delta s}$$

$$\frac{\delta u}{\delta t} = \frac{\delta u}{\delta x}\frac{\delta x}{\delta t} + \frac{\delta u}{\delta y}\frac{\delta y}{\delta t}$$

32. If $f(x,y)=0$ or c, then $\frac{dy}{dx} = -\frac{f_x}{f_y}$, $f_y \neq 0$.

33. Euler Theorem

If $f(x,y)$ be a homogeneous function of degree n, then

$$x\frac{\delta f}{\delta x} + y\frac{\delta f}{\delta y} = nu$$

34. If $u = f(x,y)$, then $du = \left(\frac{\delta f}{\delta x}dx + \frac{\delta f}{\delta x}\ \frac{\delta}{\delta y}dy\right)u.$

and $d^n u = \left(\frac{\delta f}{\delta x}dx + \frac{\delta f}{\delta y}dy\right)^n u.$

35. Taylor's Theorem for n variables

$$\phi(x+h, y+k, z+l) = e^{\left(x\frac{\delta f}{\delta x} + y\frac{\delta f}{\delta y} + z\frac{\delta f}{\delta z} + ..\right)}\ \phi(x,y,z,\ldots\ldots)$$

36. Jacobian

$$J_n(u_1, u_2, \ldots u_n) = \frac{\delta(u_1, x_2, \ldots\ldots u_n)}{\delta(x_1, x_2, \ldots\ldots x_n)} = \begin{vmatrix} \frac{\delta u_1}{\delta x_1} & \frac{\delta u_1}{\delta x_2} & \cdots\cdots\cdots & \frac{\delta u_1}{\delta x_n} \\ \frac{\delta u_2}{\delta x_1} & \frac{\delta u_2}{\delta x_2} & \cdots\cdots\cdots & \frac{\delta u_2}{\delta x_n} \\ \cdots & \cdots & & \cdots \\ \frac{\delta u_n}{\delta x_1} & \frac{\delta u_n}{\delta x_2} & \cdots\cdots\cdots & \frac{\delta u_1}{\delta x_n} \end{vmatrix}$$

37. Equation of tangent at (x,y)

$$Y - y = \frac{dy}{dx}(X - x) \text{ for } y = f(x)$$

$$(Y - y) f_y + (X - x) f_x = 0 \text{ for } f(x,y) = 0$$

(a) Slope for the tangent, $\frac{dy}{dx}$ or, $f'(x)$

(b) tangent is parallel to x—axis : $f'(x)=0$ or, $f_x=0$

(c) tangent is perpendicular to x—axis is

$f'(x)=\infty$ or, $f_y=0$

38. Angle of intersection of two curves

$y=f(x),\ y=\varphi(x)$

$\tan\alpha=\frac{f_x\varphi_y-\varphi_x f_y}{f_x\varphi_x+f_y\varphi_y}$, α is the angle of intersection

(a) Two curves touch

$\frac{f_x}{f_y}=\frac{\varphi_x}{\varphi_y}$ or, $f'(x)=\varphi'(x)$

(b) Two curves intersect orthogonally

$f_x\varphi_x+f_y\varphi_y=0$ or, $f'(x)\,\varphi'(x)=-1$.

39. Length of the tangent$=y\operatorname{cosec}\psi=\frac{y}{y_1}\sqrt{(1+y_1^2)}$; $y_1=dy/dx$

Length of the normal $=y\sec\psi$

40. Subtangent $=y\cot\psi=y/y_1$

Subnormal$=y\tan\psi=yy_1$

41. φ, the angle between tne radius vector and tangent,

$\tan\phi=r\frac{d\theta}{dr}=\frac{f(\theta)}{f_1(\theta)}$, $f(\theta)=r$

42. Pedal Equation

p= r sin φ, p is the perpendicular from pole to the tangent.

43. $\frac{1}{p^2}=\frac{1}{r^2}+\left(\frac{1}{r^2}\frac{dr}{d\theta}\right)^2=u^2+\left(\frac{du}{d\theta}\right)^2,\ u=\frac{1}{r}$

44. Polar subtangent $=r\tan\phi=r^2\,d\theta/dr$

Polar Subbnormal $=r\cot\phi=dr/d\theta$

45. $\alpha=\phi_1\sim\phi_2$, α is the angle of intersection between two curves $r=f(\theta)$, $r=\phi(\theta)$

(a) Two curves tauch if $\phi_1=\phi_2$

(b) Two curves cut orthogonally $\phi_1\sim\phi_2=\frac{1}{2}\pi$

46. Arc-length

$\frac{ds}{dx}=\sqrt{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}}$, $\frac{ds}{dy}=\sqrt{\left\{1+\left(\frac{dx}{dy}\right)^2\right\}}$

$(ds)^2=(dx)^2+(dy)^2$, $(ds)^2=(rd\theta)^2+dr^2$

47. $\tan\psi=\frac{dy}{dx}$, $\sin\psi=\frac{dy}{dx}$, $\cos\psi=\frac{dx}{ds}$

$\tan\varphi=r\frac{d\theta}{dr}$, $\sin\varphi=r\frac{d\theta}{ds}$, $\cos\varphi=\frac{dr}{ds}$

48. Negative Pedals

Put $p=r$ and $r=p^2/r$ in $f(p,r)=0$ i. e,

$f(r,p^2/r)=0$ is the First Negative pedal.

Repeat this proves for 2nd negative pedal, 3rd negative pedal etc.

48. (a) Inverse curve

$f\left(\frac{k^2x}{x^2+y^2}\quad\frac{k^2}{x^2+y^2}\right)=0$ for $f(x,y)=0$

(b) $f\left(\frac{k^2}{r},\ \theta\right)=0$ for $f(r,\theta)=0$

(c) $p=\frac{r^2}{k^2}\,f\left(\frac{k^2}{r}\right)$ for $p=f(r)$.

48. (d) Pedal Equations of well known curve

Circle : $x^2+y^2=a^2$ (centre), $r=p$

Circle : $x^2+y^2=a^2$ (point on the circumference, $r^2=2ap$

Parabola : $y^2=4ax$ (focus), $p^2=ar$

Ellipse : $\frac{x^2}{a^2}+\frac{y^2}{a^2}=1$ (focus), $\frac{b^2}{p^2}=\frac{2a}{r}-1$

Ellipse : $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (centre), $\frac{a^2b^2}{p^2}+r^2=a^2+b^2$

Hyperbola, $\frac{y^2}{a^2}-\frac{y^2}{b^2}=1$ (focus), $\frac{b^2}{p^2}=\frac{2a}{r}+1$

Hyperbola, $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ (centre), $\frac{a^2b^2}{p^2}-r^2=a^2-b^2$

Rect. Hyperbola, $x^2-y^2=a^2$(centre), $pr=a^2$

Parabola, $r=\frac{2a}{1\pm\cos}$ (focous), $p^2=ar$.

Cardioide $r=a(1\pm\cos\theta)$ (pole), $r^3=2ap^2$

Lemniscate $\left.\begin{matrix} r^2=a^2\cos 2\theta \\ r^2=a^2\sin 2\theta \end{matrix}\right\} r^3=a^2p$

$\left.\begin{matrix} r^n=a^n\cos n\theta \\ r^n=a^n\sin n\theta \end{matrix}\right\}$ (pole), $r^{n+1}=a^np$.

49. Indeterminate Forms

(a) If $f(a)=\phi(a)=0$, then

$$\underset{x\to a}{Lt}\frac{f(x)}{\phi(x)}=\underset{x\to a}{Lt}\frac{f'(x)}{\phi'(x)} \text{ form } \frac{0}{0}$$

(b) If $f(a)=\phi(a)=\infty$, then

$$\underset{x\to a}{Lt}\frac{f(x)}{\phi(x)}=\underset{x\to a}{Lt}\frac{f'(x)}{\phi'(x)} \text{ for } \frac{\infty}{\infty}$$

Maxima and Minima

50. If $f'(a)=0$, $f''(a)\neq 0$ for $y=f(x)$, then

(a) $f(a)$ is maximum if $f''(a)$ is negative

(b) $f(a)$ is minimum if $f''(a)$ is positive.

(c) If $f'(a)=f''(a)=\ldots\ldots\ldots\ldots=f^{n-1}(a)=0$ and $f^n(a)\neq 0$,

(i) $f(x)$ is maximum if $f^n(a)$ is negative n is even

(ii) $f(x)$ is minimum if $f^n(a)$ is negaive n is even

(iii) $f(x)$ is neither maximum or minimum if n is odd.

51. $\phi(x, y)$ be a functions of two variables x and y and $r=\frac{\delta^2\phi}{\delta x^2}$, $s=\frac{\delta^2\phi}{\delta x\delta y}$, $t=\frac{\delta^2\phi}{\delta y^2}$, then for (a, b)

If $rt-s^2$ is positive, $\phi(a, b)$ is maximum or minimum according as r and t are both negative or both positive.

If $rt-s^2$ is negative, $\phi(a, b)$ is neither maximum or minimum.

52. For $\phi(x, y, z)$ for a point $\phi(a, b, c)$,

$$A=\frac{\delta^2\phi}{\delta x^2},\ B=\frac{\delta^2\phi}{\delta y^2},\ C=\frac{\delta^2\varphi}{\delta z^2},\ F=\frac{\delta^2\varphi}{\delta y\delta z},\ G=\frac{\delta^2\varphi}{\delta z\delta x},\ H=\frac{\delta^2\varphi}{\delta x\delta y}.$$

$\varphi(a, b, c)$ is minimum

if A, $\begin{vmatrix} A & H \\ H & B \end{vmatrix}$, $\begin{vmatrix} A & H & G \\ H & B & F \\ G & F & C \end{vmatrix}$ are all positive

and $\varphi(a, b, c)$ is maximum if

A, $\begin{vmatrix} A & H \\ H & B \end{vmatrix}$ $\begin{vmatrix} A & H & G \\ H & B & F \\ G & F & C \end{vmatrix}$ are alternately negative and positive.

Asymptotes

53. $y=mx+c$ be the asymptote of the curve $y=f(x)$ if

$m=\underset{x\to\infty}{Lt}\ y/x$ and $c=\underset{x\to\infty}{Lt}\ (y-mx)$

54 $\varphi(x, y)=p_n+P_{n-1}+P_{n-2}+\ldots\ldots\ldots\ldots+P_o$, n indicated the homogeneous function in x and y of degree n.

Asymptotes.

$$y-m_1x+\underset{x\to\infty}{Lt}\frac{F_{n-1}}{\varphi_{n-1}}=0$$

(a) If $y-m_1x$ is repeated, then asymptotes are

$$(y-m_1x)^2+(y-m_1x)\underset{x\to\infty}{Lt}\frac{R_{n-1}}{\varphi_{n-1}}+\underset{x\to\infty}{Lt}\frac{F_{n-2}}{\varphi_{n-2}}=0$$

(b) In $\varphi_n(y/x)$, Put $y=m$, $x=1$, find $\varphi_n(m)$

Differentiate, $\varphi'(m)$.

Then put $\varphi_n(m)=0$, then m_1, m_2 etc are obtained.

For the roots of m, $c\,\varphi'_n(m) + \varphi_{n-1}(m) = 0$, then c, s are obtained.

Thus $y = mx + c$ be the asymptote.

(c) For repeated roots of m, say two equal roots, the

$$\frac{c^2}{2\,!}\varphi''_n(m) + c\,\varphi'_{n-1}(m) + \varphi_{n-2}(m) = 0$$

then put the valus of c's in $y = mx + c$. two asymptotes will be obtained.

In the same way for three equal roots etc.

55. Asymptotes in Polar Coordinates

If α be a root of $f(\theta) = 0$, then

$r \sin(\theta - \alpha) = 1/f'(\alpha)$ is an asymptote of the curve $1/r = f(\theta)$

Curvature

$$\rho = \frac{ds}{d\psi} \quad \therefore s = f(\psi)$$

$$\rho = \frac{(1 + y_1^2)^{3/2}}{y_2} \qquad \therefore y = f(x),\ y_2 \neq 0$$

$$\rho = \frac{(1 + x^2)^{3/2}}{x_2} \qquad \therefore x = f(y),\ x_2 \neq 0$$

$$\rho = \frac{(x_1^2 + y_1^2)^{3/2}}{y_2}\ .\ x = \varphi(t),\ y = \psi(t)$$

$$\rho = \frac{(f_x^2 + f_y^2)^{3/2}}{f_{xx}f_y^2 - 2f_{xy}f_xf_y + f_{yy}f_x^2} \quad \therefore f(x, y) = 0$$

$$\rho = \frac{(r^2 + r_1^2)^{3/2}}{r^2 + 2r_1^2 - rr_2},\ r = f(\theta)$$

$$\rho = \frac{(u^2 + u_1^2)^{3/2}}{u^3(u + u_2)},\ u = f(\theta)$$

$$\rho = \frac{rdr}{dp},\ r = f(r)\ ;\ \rho = p + \frac{d^2p}{d\psi^2} \quad \rho = f(\psi)$$

$\rho = \mathrm{Lt}\,\dfrac{y^2}{2x}$ (at the origin, y-axis ($x = 0$) being tangent)

$\rho = \mathrm{Lt}\,\dfrac{x^2}{2y}$ (at the origin, x axis ($y = 0$) being tangent.

$\rho = \sqrt{(a^2 + b^2)}\ \mathrm{Lt}\,\dfrac{x^2 + y^2}{ax + by}$ (at the origin $ax + by = 0$ being tangent)

Chord of curvature through the pole = $2\rho \sin \psi$

Chord of curvature Parallel to x-axis is $2\rho \sin \psi$

Parallel to y-axis is $2\rho \cos \psi$

Centre of curvature

$$\bar{x} = x - \frac{y_1(1 + y_1^2)}{y_2}\ ;\ \bar{y} = y + \frac{1 + y_1^2}{y_2} \quad \therefore y = f(x)$$

$$\bar{x} = x + \frac{1 + x_1^2}{x_2},\ \bar{y} = y - \frac{x_1(1 + x_1^2)}{x_2} \quad \therefore x = f(y)$$

$\bar{x} = x - \rho \sin y$, $y = \bar{y} + \rho \cos \psi$

Sets

1.1. Finite sets : $A = \{1, 2, 3, 4\}$

2. Infinite sets : $B = \{1, 2, 3, \ldots\ldots\ldots\ldots\}$

3. Empty set : $A = A\ (\phi)$ or, ϕ or $\{0\}$

4. $a \in A$ means a contains in A or belongs to A.

5. $a \notin A$ means a does not belong to A.

6. Unit set : $A = \{a\}$

7. Subset : $A\ \ B$ Proper subset : $A \subset B$ if $A \neq B$

8. Union of sets : $A \cup B = \{x \in A$ or $x \in B\}$ $\therefore$ Read A cup B

9. Intersection of sets : $A \cap B = \{x \in A$ and $x \in B\}$ $\therefore$ Read A cup B

10. Disjoint sets : $A \cap B = \phi = \{x \notin A$ and $x \notin B\}$

11. Difference of two sets : A and B. $A - B = \{x \in A$ and $x \notin B\}$

12. Universal set = U (Union of all sets)

13. complement of a set : $A^c + A' = \{x \in X : x \notin A\}$ i. e, $x \in A^c = A'$ i. e; $x \notin A$.

14. Power set of $S = 2^s$

15. Countable set : A set is countable if it finite or denumerable.

16. $A \cup B = B \cup A$, $A \cap B = B \cap A$,

17. $(A \cup B) \cup C = A \cup (B \cup C)$;

18. $(A \cap B) \cap C = A \cap (B \cap C)$

19. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

20. $A \cup (B \cap C) = A \cup B) \cap (A \cup C)$

21. $(A')' = A$, $(A \cup B)' = A' \cap B'$; $(A \cap B)' = A' \cup B'$

22. $A-B = A-(A \cap B)$

23. $(A-B) \cap A=A$, $(A-B) \cap B = \emptyset$

24. $A -B = A \cap B' = B'-A'$

25. $B-A' = B \cap A$, $A-B= A \cup B$, $A \cup (B-A) = B$

Set Theory

11. Meaning of Sets : An object which belongs to a given set is called a number or an element of the set. We designate sets by the Capital letters *A, B, C* etc and elements of a set by small letters *a, b, c* etc. Generally we say a is an element or member of *A i. e;* $a \in A$. Sets may be finite and infinite. Set which does not contain any element is called an empty set and is denoted by φ.

Description of Sets : *A* set generally dscribed by two methods :

(i) Roster method, (ii) Rule method :

In **Roster method,** We include a set by listing the elements and enclosing them with braces { }. Thus the set consisting of Rahim, Jack, Ram be written as {Rahim, Jack, Ram]

In the **Rule method,** we describe the set by a phrase "the set of all books of Rajshahi University Library." It is written as {*x* is a book in the Rajshahi University Library} or, as {*x*/*x* is a book in the Rajshahi University Library}

The oblique line standing for **"such that"**

Roster method is used for finite sets while Rule method is used for infinite set or sets containg large number of elements.

Subsets : A set A is a subset of B if every element of A is also as element of *B i. e;* $A \subseteq B$ or $B \subseteq A$

Example : Let A = {1, 2, 3, 4} and B={1, 2, 3, 4, 5}

Here every element of *A* is in *B i. e.* $A \subseteq B$

Every set is a subset of itself thus $A \subseteq A$, because *A* certainly contains *A i. e.,* $x \subset A$ implies *x* in *A'*

The null set ø is a subset of every set. Thus $\emptyset \subseteq A$ because *A* certainly 'contains' ø.

2.2 Equality of sets :

Two sets A and B are equal (svmbolically $A = B$) if and only if $A \subseteq B$ and $B \subseteq A$ *i. e., A* is a subset of *B* and B is also a subset of A.

Example : Let A = {2, 3, 5} and {3, 5, 2}

Here every element of A is in B and every evlement of B is in A. Here $A \subseteq B$ and $B \subseteq A$ ∴ $A = B$

2.3 Proper subsets :

A set A is a Proper subset of a set B (symbloically, $A \subseteq B$) if $A \subseteq B$ and $A \neq B$

Thus $A \subset B$ means that A is a subset of B but B is not a subset of *A, i. e.* every element of *A* is in B but B has at least one element which is not in *A.*

Example : Let A = {1, 2, 3, 4} and *B* ={ 1,2, 3, 4, 5}

Here every element of *A* is in *B*, but *B*, has the element 5 which is not in A.

Here $A \subseteq B$ but $A \neq B$. ∴ $A \subseteq B$,

Example : Let *A*={ 2, 3, 5}, *B*={3, 5,2} and *C*= {3, 2}.

Here every element of A is in *B* and every element of *B* is in *A*. $A \subseteq B$ and $B \subseteq A$, and hence $A = B$

But though every element of C is in A or B the element 5 of / or B is not in C.

∴ $C \subseteq A$ and $C \subseteq B$ but $C \neq A$ or $C \neq B$ ∴ $C \subset A$, $C \subset B$.

2.4 disjoint sets :

Two sets A and B are disjoint if A and B have no commo elements.

Example : The sets {0, 2, 3} and {4, 5, 6} are disjoint.

2.5 Number of Subsets or a set; Power set P (S)

The set of all subsets s of a set S is called the power set of S and is denoted by $P(S) = (s/s \subset S)$ if S contains n elements, the power (S) = P set 2^n or the number of subsets s of a set S.

Example : Let $A = \{a, b\}$ and $B = \{a, b, c\}$

(i) the subsets of A are ø, {a}, {b}, {a, b}

there being $^2C_0 = 1$ subset with no element. $^2C_1 = 2$ subsets with one element and $^2C_2 = 1$ subset with two elements.

Thus the set A containing 2 elements

has $4(=2^2= {}^2C_0 + {}^2C_1 + {}^2C_2)$ subsets.

(ii) the subsets of B are

ø, {a}, {b}, {c} : {a, b} : {a, c}; {b, c}; {a, b, c}

there being $^3C_0 = 1$ subset with no elements, and $^3C_1 = 1$ subset with three elements.

Thus the set B containing 2 elements

has $8(= 2^3 = {}^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3)$ elements)

from the above consideration, it is clear that a set containing n elements has

$^nC_0 = 1$ subset with no element.

$^nC_1 = n$ subsets with one element.

$^nC_2 = \frac{n(n-1)}{\lfloor 2}$ subsets with two elements and so on, the total

number of subsets thus being

$^nC_0 + {}^nC_1 + {}^nC_2 \ldots\ldots\ldots\ldots + {}^nC_n = (1 + 1)^n 2^n.$

2.6. about the Symbols ε and ⊂

The two symbols ε and ⊂ should not be confused with each other. The symbols ε denotes the relationship between a set and its elements. where as the symbol ⊂ denotes the relationship between two sets.

Thus if $D = \{5, 2, -3\}$, then it is true that $\{5\} \subset D$ It is not, however, correct to write 5 c D or, | 5 | CD.

2.7 Finite Set : A set is finite if it consists of a specific number of different elements.

Infinite set : A set is infinite if it is equivalent to a proper subswet of itself otherwise, a set is finite.

Example : A = { S.M.T.W.Th. F sat} A is finite

B = { x | x is an integer}, B contains infinite numbers of intergers, B is infinite.

Power Set : The family of all the subsets of any set S is called the power set of S. The power set of S is denoted by 2^s

Let S = {2, 3}, then subsets are { 2, 3}, {2}, {3}, ϕ, there are four subsets = 2^s, Hence Power set $2^s = 2^2 = 4$.

Countable Set : a set is called countable if it is finite or denumerable *e.g.* {(1,1), (4,8), (9,27)} ....$(n^2 n^3)$}.

3.1 Union of two Sets :

The union of two sets A and B is the set of elements which are in at least one of the sets A and B *i.e.*, which belongs to either A or B.

Symbolically we write the union of A and B as $A \cup B$. read **'A union B, or ' A cup B'**

Thus $A \cup B = B\ \{x/x \in A$ or, $x \in B)\}$ It follows from the definition that, $A \cup B = B \cup A$ i ε., the equation of union is commutative.

Example : Let $A=\{a,b,c\}$, $B=\{d,e\}$, $C=(b,e)$, $D=(a,c)$ then $A \cup B= \{a, b, c, d, e\}$, $B \cup C = \{b, d, e\}$ $A \cup D = \{a, b, c\}$,

Note—In the above example.

$D \cup A$ and $A \cup D$ = A. This is true for two such sets. In particular $A \cup A = A$ and $A \cup ø = A$ for any set A.

3.2 Intersection of two sets;

The intersection of two sets A and B is the set of elements which belong to both A and B.

Symbolically we write the intersection of two sets A and B as ***$A \cap B$*; *A* intersection B or *A* cup B.**

Thus $A \cap B = \{x/x \in A$ and $x \in B\}$. If follows form the definition that $A \cap B = B \cap A$, i.e., the operation of its intersection is commutative.

Example : Let *A, B, C, D* be sets as in the previous example in Art. 3.1

Then $A \cap B = \emptyset$. $A \cap C = \{b\}$, $A \cap D = \{a, c\}$

Note- in the sbove example

A and B are disjoint sets and $A \cap B = \emptyset$. This is true for any two such sets. Also $D \subseteq A$ and $A \cap D = D$. This is also true for aby two such sets. In particular $A \cap A = A$ and $A \cap \emptyset = \emptyset$ for any set A.

3. 3. Complement of a set.

If A is a subset of a universal set U then the complement of A is the set of elements which belongs to U are not contained in A.

The complement of a set A is denoted by A' read 'A prime' and is defined relative to a particular universal set U.

Thus $A' = \{x/x \in$ U and x, does not belong to A\}

Example — Let U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}

$A = \{1, 7, 6\}$, and $B = \{2, 5, 8, 9, 0, 3, 1\}$

Thus $A' = \{0, 2, 3, 4, 5, 8, 9\}$ and $B' = \{4, 6, 7\}$

Note- In the above example.

we change U, A' and B' will be changed. This is true in all cases. the following results directly obtained from definitions.

(1) $U' = \emptyset$ *i. e.*, the complement of the universal set is the null set

(2) $\emptyset' = U$, *i. e.*, the complement of the null set is the universal set

(3) $(A')' = A$, *i, e.* the complement of the complement of any set is the set itself.

(4) $A \cup A' = U$, *i. e.*, the union of any set and complement of the universal set.

(5) $A \cap A' = \emptyset$, *i.e.*, the intersection of any set and its complement is of the null set.

4. 1. Venn Diagrams.

It is often convenient to draw diagrams to represent relation between sets or operations on sets. Such diagrams are called Venn diagrams.

In the following illustions, we have represented the univer- sal set U as the set of points in a rectangle and other sets as sets of points a circle within the rectangle.

Laws of Algebra of Sets

Art.4.2. *A. B, C* are sets A', B', C' are the complements of A,B and C respectively U-universal set, $\emptyset$ = null set

Identity Laws

1. (i) $A \cup \emptyset = A$ (ii) $A \cap \emptyset = \emptyset$

(iii) $A \cup U = U$ (iv) $A \cap U = A$

Idempotent Laws

2. (i) $A \cup A = A$ (ii) $A \cap A = A$

Complement Laws

3. (i) $A \cup A' = U$ (ii) $A \cap A' = \emptyset$

(iii) $(A')' = A$ (iv) $\emptyset' = U$ (v) $U' = \emptyset$

Commutative Laws

4. (i) $A \cup B = B \cup A$ (ii) $A \cap B = B \cap A$

Associative Laws

5. (i) $(A \cup B) \cup C = A \cup (B \cup C)$

(ii) $A \cap (B \cap C) = (A \cap B) \cap C$

Distributive Laws

6. (i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cup C)$

The distributive laws take on the general forms

$A \cap (B_1 \cup B_2 \cup \ldots\ldots\ldots \cup B_n) = (A \cup B_1) \cup (A \cup B_2) \cup \ldots \cup (A \cap B_n)$.

$A \cup (B_1 \cap B_2 \cap \ldots\ldots\ldots \cap B_n) = (A \cup B_1) \cap (A \cup B_2) \cap \ldots\ldots \cap (A \cup B_n)$

De Morgan's Laws

7. (i) $(B \cup B)' = B' \cap B'$

(ii) $(B \cap B)' = B' \cup B'$

Art. 4.3 (1) Prove De Morgan's Theorem :

$(A \cup B)' = A' \cap B' = \{x / x \in (A \cup B)'\}$

Proof : $(A \cup B)' = \{x \mid x \notin A \cup B\} = \{x \mid -(x \in A \text{ or } x \in B\}$.

$= \{x \mid x \notin A' \text{ and } x \notin B\} = x \mid \{x \notin A \text{ and } x \notin B\} = A \cap B$

Alternative Method:-Let $x \notin (A \cup B)$, then $x \notin (A \cup B)$. Therefore $x \notin A$ or $x \notin B$ thus $x \notin A'$ and $x \in B'$. So $a \in A' \cap B'$ But our assumption is that $x \in (A \cup B')$ Hence $(A \cup B)' \subseteq A' \cup B$. (1)

Again if $y \in A' \cap B'$ then, $y \in A'$ and $y \in B'$ then $y \notin A$ and $y \notin B$. hence $y \notin A \cup B)$ so $y \in (A \cup B)'$ we have shown that $y \in A' \cap B'$ implies that $y \in (A \cup B)'$ Hence $A' \cap B' \subseteq (A \cup B)'$ .... (2)

From (1) and (2). $(A \cup B)' = A' \cap B'$ proved.

44.(2) Prove that $(A \cap B)' A' \cup B'$

Proof : $(A \cap B)' = \{x \mid (x \in (A \cap B)'\}$

$= \{x \mid x \notin (A \cup B)\} - \{x \mid (x \in = A \text{ and } x \in B)\}$

$= x \mid x \notin A \text{ or } x \notin B\} - (x \mid x \in A' \text{ or } x \notin$ Œ$B') = A' \cup B'$

De Morgan's Laws can be extended and this has been stated without proof here :-

$(A_1 \cup A_2 \cup A_3 \cup ......A_n)' = (A_1)' \cap (A_2)' \cap (A_3)' \cap \; .... \cap (A_n)'$

$(A_1 \cap A_2 \cap A_3 \cap \; ...... \cap A_n), = (A_1)' \cup (A_2)' \cup (A_3)' \cup \; ... \cup \; (A_n)'$

4.5 (1) difference : The difference of two sets A and B (relative to A) is the set of elements which belong to A but do not belong to B. The difference is writen as $A-B$.

which reads' "A difference B" or, simple, 'A minus B"

i. e. $A-B = \{x . \mid x \in A \text{ and } x \notin B\}$

Similarly, the difference of two sets A and B (relative to B) is defined as $B-A = \{x \mid x \in B \text{ and } x \notin A\}$

For example, if $A = \{a, b, c, d\}$ and $B = \{a, e, f, d\}$, then

$A-B = \{b, c\}$

But $B-A = \{e, f\}$

Art. 2. In Venn Diagram, the difference $A-B$ and $B-A$ are shown. The dotted area is the difference $A-B$ and cross cut area is the difference $B-A$, U represents the universe.

Art. 2. (i) Prove that $A-B = A-(A \cap B)$

Let $x \in (A-B)$ (if and only if)$\leftrightarrow x \in A$ and $A \notin B \leftrightarrow x \in A$ and $x \notin (A \cap B) \leftrightarrow x \in A-(A \cap B)$

$\therefore A-B = A-(A \cap B)$

Art. 3. Prove that $(A-B) \subset A$

i. e, set A contains A-B as a subset

Proof : Let x be any element of $A-B$. Then we have $x \in A$ and $x \notin B$ *i.e.* x belong to A but we have shown that $x \in A-B$ implies that $x \in A$. Hence $(A-B) \subset A$

Art. 4. Prove that $(A-B) \cap B = \emptyset$

Let x belong to $(A-B) \cap B$ *i. e.*, $x \in (A-B) \cap B$ by the intersection of two sets. we have $x \in A-B$ and $x \in B$ but by the definition of difference $(A-B)$, we have $x \in A$ and $x \in B$. Hence there is no element satisfies both $x \in B$ and $x \notin B$ then

$(A-B) \cap B = \emptyset$

Art. 5. rove that A-B = A ∩ B′ = B′-A′ where A′ is the complement of A, B′ is the complement of B.

Proof : $A-B = \{x : x \in A \text{ and } x \notin B\}$

$= \{x : x \in A \text{ and } x \in B'\} = A \cap B'$

$= \{x : x \in A' \text{ and } x \in B'\}$

$= \{x : x \in B' \text{ and } x \notin A'\} = B'-A'$

Alse $B-A = A'-B'$

Note : The complement of a set A is the set of elements which are not present in A, *i. e,* the difference of the universal set U and A. we represent the complement of A by A', concisely we define A' By

$A' = \{ x$ is such that $x \in U, x \notin A\}$

$= \{x \mid x \in U, x \notin A\}$

or, simply, $A' = \{x \mid x \notin A\}$.

Art. 6 Prove B–A is a subset of A′

Proof : Suppose $x \in B\text{–}A$. . e. $x \in B$ and $x \notin A$. Hence $x \in B$ and $x \in A'$ i. e, x belongs to B' but $x \in B\text{–}A$ impies that $x \in A'$. $B\text{–}A$ is a subset of A', i. e.. $B\text{–}A \subseteq A'$.

Art. 7. Prove that B–A′ = B ∩ A.

Proof : Let $x \in B\text{–}A'$ which means that $x \in B$ and $x \notin A'$., i. e, $x \in B$ and $x \in A$ which means that $x \in B \cap A$. Hence $B\text{–}A' = \{x \in B$ and $x \notin A\} = \{x \mid x \in B$ and $A\} = B \cap A$

$\therefore$ B–A′ = B ∩ a.

Art. 8 Prove that A–B is subset of AUB.

Proof : Let $x \in$ A–B means $x \in$ A and $x \notin$ B, therefore $x \in$ AUB i. e, A–B ⊆AUB.

Art. 9. Prove that A ⊆ B inplies that subset of AU(B–A) = B

Proof : $A \cup (B\text{–}A) = \{ x \mid x \in a$ or, $x \in (B\text{–}A)\}$

$= \{ x \mid x \in A$ or, $(x \in B$ and $x \notin A)\}$

$= \{ x \mid x \in A \cap x \in B\}$

$= \{ x \mid x \in B\}$ since $A \subset B$

Hence A is the Proper subset of B i. e, $A \subset B$.

Example 10. To show thar for any three sets A, B and C $(A \cap B) \cap C = A \cap (B \cap C)$.

we show that (1) $(A \cap B) \cap C \subseteq A \cap (B \cap C)$

and (ii) $A \cap (B \cap C) \subseteq (A \cap B) \cap C$

(i) Let x be any element of the set $(A \cap B) \cap C$, Then $x \in (A \cap B)$ and $x \in C$. But $x \in (A \cap B)$ implies that $x \in A$ and $x \in B$. Thus $x \in A$ and $x \in B$ and $x \in C$ and as such $x \in (B \cap C)$ as well. Hence $x \in A \cap (B \cap C)$. Therefore

$(A \cap B) \cap C \subseteq A \cap (B \cap C)$.

(ii) Let y be any element of the set $A \cap (B \cap C)$. Then, $y \in (B \cap C)$. But $y \in (B \cap C)$ implies that $y \in B$ and $y \in C$. Thus $y \in A$ and $y \in B$ and $y \in C$, and as such $y \in (A \cap B)$ as well.

Hence $(A \cap B) \cap C$

Therefore $A \cap (B \cap C) \subseteq (A \cap B) \cap C$

Hence the conclusion

Ex. 11. prove that $(A\text{–}B) \cup (B\text{–}A) = (A \cup B) = (A \cup B)\text{–}(A \cap B)$ when A = {1, 2, 3, 4, 5} and B = {2, 3, 5, 6, 7} C.U. 1982

Ans. $A - B = \{ x : x \in A$ and $x \notin B\} = \{1, 4\}$

$B\text{–}A = \{x : x \in B$ and $x \notin A\} = \{6, 7\}$

Now $(A\text{–}B) \cup (B\text{–}A) = \{1, 4\} \cup \{6, 7\} = \{1, 4, 6, 7\}$ ....(1)

Again $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$

$A \cap B = \{2, 3, 5\}$

$(A \cup B) - (A \cap B) = \{x : x \in (A \cup B)$ and $x \in (A \cap B)$

$= \{1, 4, 6, 7\}$...........(2)

From (1) and (2)

$(A - B) \cup (B\text{–}A) = (A \cup B)\text{–}(A \cap B)$.

Ex. 11. (a) If U = {1, 2, 3, ...9}, A = {1, 2, 3, 4}, B = {2, 3, 4, 5}

Find $A \cap B$ and the complement of A–B. C. U. 1982.

2. A few examples :

Example. 1. Find the union and intersection of the sets

A = {n/n is and integer and $n \geq 9$}.

and B = {n/n is an interger and $n \geq 2$}.

Answer. Since A is the set of integers ≥9 and B is the set of integers ≥2 . ∴ $A \subseteq B$.

Every element of A being also in B.

Hence $A \cup B = B$ and $A \cap B = A$.

Example 2. For the set A = {a, b, c}, B = {b, d, e} and C = {d, f, g}, veryfy that

$A \cap (A \cup C) = (A \cap B) \cup (A \cap C)$,

Answer : Here $B \cup C = \{a,b,e\} \cup \{d, f,g\} = \{b, d, e,f,g\}$

$\therefore A \cap (B \cup C) = \{a,b,c\} \cup \{b,d,f,g\} = \{b\}$

Also $A \cap B = \{a, b, c\} \cap \{b, d, e, f, g\} = \{b\}$

And $A \cap C = \{a, b, c\} \cap \{d, f, g\} = \emptyset$

$\therefore (A \cap B) \cup (A \cap C) = (b \cup \emptyset) = \{b\}$.

Hernce the conclusion.

Example 3. Given any set A in the universe $\cup$. show that there exists a unique set X satisfying the conditions

(a) $A \cup X = U$ and (b) $A \cap X = \emptyset$.

Ans : There exists a set $X = A$ satisfying the conditions because $A \cup A' = U$ and $A \cap A' = \emptyset$

[**Art.** 3,3; Properties (4) and (5)]

If Y be any other set satisfying the conditions, then

$A \cup Y = \cup$ and $A \cap Y = \emptyset$

But $Y = Y \cap U$ [since $Y \subset U$]

$= Y \cap (A \cup X) = (Y \cap A) \cup (Y \cap X)$ [Property (5) **Art. 4.2**]

$= \emptyset \cup (Y \cap X)$ (since $Y \cap A = A \cap Y = \emptyset$)

$= Y \cap X$ (since $\emptyset \cup k = k$ for any set K)

$\therefore Y \subseteq X$ Art. 4.1

Also, by similar argument, $X \subseteq Y$

$Y = X$

Hence there the set X satisfying the given condition is unique.

Example 4. Show that for any two sets A and $B : A \subseteq B$ if and only if $B' \subseteq A'$

Let $A \subseteq B$. Then $x \in A$ implies that $x \in B$ and as such x is not in B'. Hence A and B are disjoint sets. Having no common elements so that $B' \cap A = \emptyset$

Now $B' = B \cap U$ (U is the universal set)

$= B' \cap (A \cup A)'$ (since $A \cup A' = U$)

$= (B' \cap A) \cup (B' \cap A')$ (property (5) Art. (4.2)

$= \emptyset \cup (B' \cap A') = (B' \cap A')$ ($\emptyset \cup K = k$ for any set (K)

$\therefore B' \subseteq A'$

Example 5. show by means of Venn Diagram that is n(x) denotes the number of elements of the finite set x, then

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

A and B being two finite sets. How is the result modified if A and B are disjoint sets?

Ans : Visualising the sets A and B as the sets of points within the two circles, let p, q, r denote the number of points in the regions.

P, Q, remarked in the diagram. Then

n(A) = number of points in the regions P and $Q = p + q$

n (B) = number of points in the regions Q and $R = q + r$

$n(A \cap B)$ = number of points in the regions $Q = q$

$n(A \cap B)$ = number of points in the regions P. Q and $R = p + q + r$.

Hence $n(A \cup B) = p + q + r = (p + q) + (q + r) - q = n(A) + n(B) - n(A \cap B)$

If A and B are disjoint sets then $A \cap B = \emptyset$ so that $n(A \cap B) = o$. In that case, $n(A \cup B) = n(A) + n(B)$ simply.

6.2 Ordered Pairs :

A pair element is said to form an ordered pair if it is specified which of the two comes first and which comes second.

An ordered pair in which a is the first component and b is the second component is denoted by the notation (a, b)

It should be noted that $(a,b) \neq (b, a)$

unless $a = b$ although $\{a, b\} = \{b, a\}$. Morover. we speak of orderd pairs of the form (a, b) even though there is no set (a, a)

Two ordered pairs (a, b) and (c, d) are said to be equal *i, e.* $(a, b) = (c, d)$ if and only if a =c and $b = d$.

Example : In plane analytic geometry, the position of a point in the plane is determined by an ordered pair (x,y) of real numbers, the first component x being the abscissa of the point and the second component y being the ordinate of the point. The ordered pair (2, 3) is certainly different from the ordered pair (3, 2) as they represent differents points in the plane.

Example. Given a set A ={a, b} the ordered pairs which formed with elements of A are (a, a). (a, b). (b, a). (b, b)

Example. Given two sets $a = \{a, b\}$ and $X = \{x, y\}$ the ordered pairs which can be formed with an element of A as the first component and an element of X as the second component are (a, x). (a, y). (b, x) . (b, y)

7.1 MAPPINGS

Let X and Y be two sets not necessarily distinct. *A* mapping of X into Y is a correspondence that associates with each element of X with unique element of Y.

A mapping is usually denoted by a single letter such as f, g, a, etc. The fact that f is a mapping of X into Y is often indicated by the symbol $f : X \to Y$.

If $f: X \to Y$, then for each $x \in X$, the corresponding element of Y is called the image of x under the mapping f and is denoted by (x). The set X is called the domain of the mapping f and the set Y is called its image space. The subset of Y consisting of those elements of Y which are image of some $x \in X$ *i e.* $\{ y/y \in Y$ and $y = f(x)$ for some $x \in X\}$ is called the range of the mapping f.

If may be noted that under a mapping f of X into Y, every x ∈ X has one and exactly one image in Y. Where as the same $y \in Y$ may be the image of more than one x ∈ X and there may be some $y \in Y$ which is not the image of any $x \in X$.

A mapping of X into Y is defined if we know the image of of each $x \in X$. The notation f : x→ f(x) is used to indicate that under the mapping of X into Y, x is mapped into f(x), *i. e.* $f(x)$ is the image of x.

A mapping $f : X \to Y$ can be pictorially represented by listing the elements of X and Y inside two closed curves and drawing arrows from each $x \in X$ the coriesponking image $y \in Y$.

Example : Let a mapping $f : Y \to Y$ be defined as follows. 1→3, 2→2, 3→1, 4→1, 3→3, 7→2, 9→5.

Here the domain of f is $x = \{1, 2, 4, 7, 9\}$ and the range of f is $\{1, 2, 3, 5\}$.

Example : Let x = {1, 2, 3, 4, 5, 6, 7, 8} and f : x→X be a mapping defined as follows :

$f(1) = 1$. $f(2) = 5$ $f(3)=4$ $f(4)=8$.
$f(5)=6$ $f(6)=3$, $f(7)=7$, $f(8)=2$.

Hence the domain as range of f is the x.

Example : Let X= {1, 2, 3} and Y = {a, b, c}.

The correspondence defined as.

$1 \to a$, $1 \to b$, $2 \to b$, $3 \to c$, is not a mapping because under the correspondence, two distinct elements Y correspond to the elements 1 of X.

CHAPTER–1

NUMBERS

1.1 The set of Real Numbers

(a) Integers (পূর্ণসংখ্যা) **:** The numbers 1, 2, 3, ... are known as the ***natural*** or ***counting*** numbers. The natural numbers, their negatives and zero form the set of integers Z. Thus

$$Z = \{ \ldots\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\ldots \}$$

(b) Rational numbers (আনুপাতিক সংখ্যা) **:** Any number which can be expressed in the form $\frac{p}{q}$, where p and q are integers with $q \neq 0$, is called a rational number. Clearly any integer is a rational number (correspo nding to $q = 1$).

Examples of rational numbers are 2, $\frac{5}{7}$, $\frac{-3}{8}$, 1.2, etc.

In *decimal representation* of a rational number, the steps will either *terminate* or *a certain part of the steps will repeat.* For example,

$\frac{1}{8} = 0{\cdot}125$; here the steps terminate.

$\frac{1}{3} = 0{\cdot}3333 \ldots\ldots\ldots = 0.\dot{3}$; here the steps do not terminate, but 3 is repeated which is indicated by putting a dot over 3·

$\frac{1}{7} = 0.142857\ 142857\ 142857 \cdots\cdots = 0{\cdot}\dot{1}4285\dot{7}$; the dots over 1 and 7 show that the part '142857' is repeated.

(c) Irrational numbers (অমেয় সংখ্যা) **:** A number which represents a certain length on a straight line but cannot be represented in the form $\frac{p}{q}$ (p, q being integers $q \neq 0$) is called an irrational number.

In decimal representation of an irrational number the steps involved one non-terminating and non-recurring.

$\sqrt{2}$, $\sqrt{3}$, π, e, etc. are irrational numbers.

All the rational and irrational number together are said to form the **continuum of Real numbers** or **the Set of real numbers**, denoted by $\mathbb{R}$.